Drawing

Perspective & Geometrical

McGUIRL

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PERSPECTIVE

AND

GEOMETRICAL

DRAWING

ADAPTED TO THE USE OF CANDIDATES FOR

SECOND AND THIRD-CLASS TEACHERS' CERTIFICATES.

BY

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PREFACE.

DRAWING having been at length recognized by the Education Department as an essential feature in High School education, it is necessary that a work, at once simple and concise, should be prepared on this subject. The incompleteness and want of definiteness in the existing works on perspective, have induced me to place this book before High School pupils. It consists mainly of problems, etc., that have been given from time to time in my own classes. To obviate the necessity of copying problems from the blackboard, I have added a number in Geometrical Drawing, which will be found useful.

Believing that this work supplies a want long felt in our schools, I have consented to place it before the public.

T. H. M.

THE INSTITUTE, *March*, 1887.

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Geometrical Drawing—*Continued.*

DRAWING.

DRAWING may be defined as the representation of an object or collection of objects on a plain or level surface.

There are two kinds of Drawing: Perspective, or the representation of an object as it *appears* to the eye; and Geometrical Drawing, as it *actually* is.

Our knowledge of the size of an object can be known only by experiment. We must either see the object near the eye, and observe its size, or know its distance from the eye.

From long practice we are enabled to tell the height of a hill, breadth of a river, or capacity of a ship, though at a considerable distance. If we hold a rule of definite length close to the eye, and then withdraw it six or eight feet away, we notice that it is *apparently* smaller. Experience teaches us that it is not really smaller, the *apparent diminution* being only the effect of distance.

Perspective aims, then, at measuring and representing objects as they appear at a distance.

The horizon always bounds our vision.

If we look out upon a large lake we find that the sky and water appear to meet; this line of apparent union is called the horizon. No matter how large an object is, if it recedes far enough from us on a lake, it would at length appear on the horizon as a point.

If a person stands on a level plain he can see over a range of 60 degrees in every direction without moving his head. The point on the horizon directly in front of him is called the Centre of Vision.

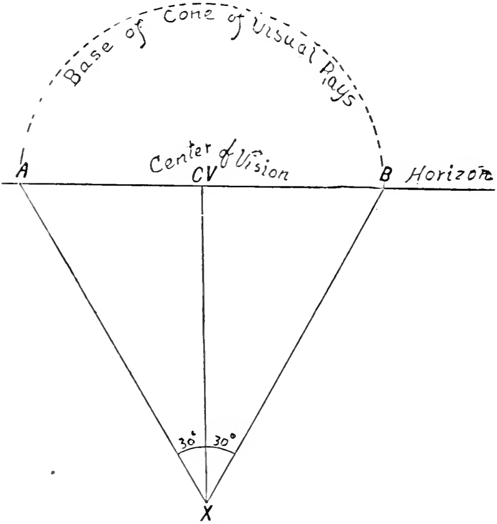

Fig. 1.

Now, if we take this point as a centre, and join it with the eye of the observer, and draw lines at an angle of 30° with it from the observer's eye, we shall form a hollow cone, which represents his range of vision: thus, if the observer's eye be at the point X, and AB represent the horizon, and C the point on it directly in front of X, if CX be joined, and AX, BX be drawn at an angle of 30 degrees with CX, AB will be the horizontal range of the spectator's vision. If we describe a circle from centre C, and distance CA or CB, such circle will be called the base of the cone of visual rays; for it will be observed that from point X the spectator can see just as far as the edge of this circle. The ground as a plane is generally supposed to extend to the horizon, and is marked for sake of abbreviation, G. P. The horizontal line is unlimited in length, and is written H. L. (Fig. 1.)

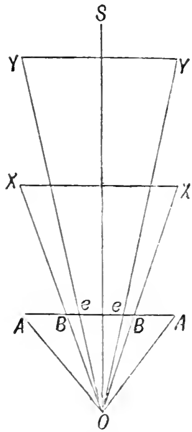

Fig. 2.

All representations of objects are supposed to be made on a plane (unlimited in extent), perpendicular to, and resting on, the ground plane, and directly in front of the spectator. This plane is called the Picture Plane, and is written P. P.

It must be distinctly borne in mind that the bottom of the picture plane touches the ground plane, and this line is always at right angles to that joining the spectator's eye with the centre of vision.

The line joining the spectator's eye with the centre of vision is usually denoted L. D. (line of direction, and sometimes length of distance).

The position of the spectator's eye is called the point of station, or P. S. (Fig. 2.)

If a spectator stand at O and observe a stick at AA, then carry the stick back and place it at XX, parallel to AA, and draw lines OX, OX; it is plain the stick cannot appear as large as AA, but will be represented in length by BB. If again withdrawn to YY, it will appear as *ee*, and so on. As we remove it the apparent length diminishes, or, in other words, the angle made by the line with O constantly decreases. (Fig. 2.)

If the stick were removed to an infinite distance, its apparent length would vanish to a point S, and the lines drawn to O would coincide, forming an angle of O degrees.

Objects appear to diminish in size as they recede from the eye, and *vice versa*.

At the centre of vision all objects have no apparent magnitude, and must be represented by a point, and steadily increased in size till they approach the eye. This is why rails on a straight piece of railway appear to meet in the distance, though everywhere the distance between them is the same.

Now, if we stood on a straight piece of track, and if other tracks were laid on each side and parallel with it, every track would appear to vanish at the same point directly in front of us, hence the rule:

Lines parallel to the line of direction will vanish at the centre of vision. (Fig. 3.)

Thus, let C be centre of vision, and AA, BB the ends of parallel lines: they will all meet at C, and all the lines drawn from C to AB will *really* be right angles, however different they may *appear* to be. Now draw DD parallel to AA, and since the distance from A to A is always the same as from D to D, we have the rule:

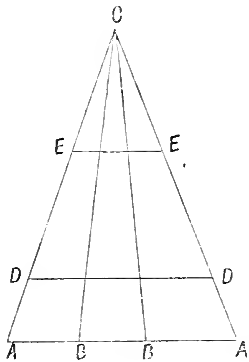

Fig. 3.

Parallel lines drawn between vanishing lines are of equal length, and, conversely, the lines joining the extremities of equal parallel lines will vanish to a point.

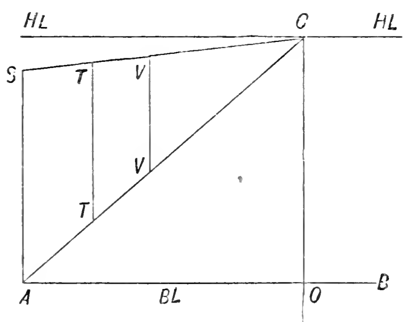

Fig. 4.

Again, if the spectator stood on the ground at O, where the picture plane rests, and looked towards C on the horizon, it is clear that he could see objects lying on the ground anywhere between AB and HL. Hence, if we take AB as the ground line, or base of picture plane, and OC as the height of spectator's eye above the ground, the horizontal line will pass through C and be parallel to AB. (Fig. 4.)

The ground line is denoted G. L.

If a stake AS be placed perpendicularly in the ground plane at A, it will necessarily touch P.P. throughout its length. If placed as TT, still erect, it is said to be *within* the picture plane, and parallel to it. Now, if the stake be same height throughout, TT will be equal to AS ; so also will VV be equal to AS ; and being placed on the ground, their extremities will lie in the straight lines SV, AV (being parallel). But lines joining equal parallel lines will vanish, hence SV and AV, being produced, will meet in C.

As nothing is ever supposed to be drawn *nearer* to the eye than the picture plane, we use the picture plane as a basis of measurement,—for in the figure we can estimate the length of TT or VV only by referring them to AS, which is drawn on the picture plane.

Objects to the right of OC are said to be to the *right ;* objects left of OC, to the *left*. Objects which are at an equal distance on each side are said to be *directly in front.*

Perspective is of four kinds :—

1. Parallel, in which some side or face of the object is perpendicular or parallel to the P.P., and also to the G.P.

2. Angular, in which a side or face makes an angle less than a right angle with the P.P., but is parallel to the G.P.

3. Oblique, in which the sides or faces make angles less than right angles with both P.P. and G.P.

4. Aerial, or the perspective of distinctness in a view. It is related to shading and painting.

DRAWING TO A SCALE.

Unless objects were very small, and our drawing surface large, we could not represent the size of the object as it is. It is usual to draw the figure in miniature, or a certain number of times smaller. Every line in the drawing must bear this fixed proportion to the corresponding one in the figure or object. This is called drawing to a scale.

Usually, one quarter-inch for each foot is the proportion in perspective, but any other ratio may be used.

Thus, if a line eight feet in length were to be drawn, it would be represented by a line (on the picture plane) two *inches* in length.

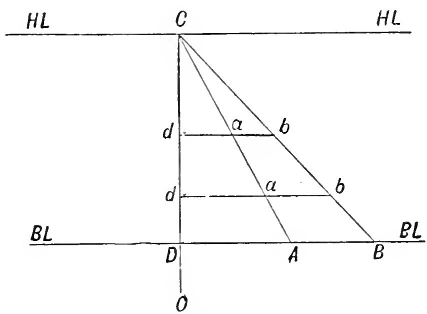

Fig. 5.

Draw any horizontal line H.L., take any point O, draw OC perpendicular to H.L. Let CD = height of spectator; through D draw B.L. parallel to H.L. If O be spectator, C will be centre of vision, B.L. base line, and H.L. horizontal line. OC will be line of direction and OD length of distance. (Fig. 5.)

In B.L. take any points, A, B, and join them with centre of vision (C.V.). If *dab* be drawn parallel to DAB, and touching CD and CB, it will be equal to DB, and $da = DA$ and $ab = AB$; also, if the distance of B to the right of D be known, the distance of *b* from *d* is known, for $db = DB$; or, in other words, every part of BC is the same distance from CD that B is.

If we wish to find the position of a point within the plane, we first find its distance on B.L. from D, and then join the point marking this distance with C; the latter line would pass through it.

Exercise I.

1. Find a point on the ground plane, at base of picture plane, 4 feet to right.
2. Find a point 3 feet above G.P., 4 feet to right, and touching P.P.
3. Find a point directly in front touching P.P., and 6 feet above it.
4. Find position of a point 3 feet under G.P., 3 feet to right in P.P. produced.

(In foregoing examples assume spectator's height to be six feet.)

All lines at *right angles* with the base line (B.L.)., or, *parallel* with the direction we are looking (L.D) will vanish at the centre of vision (C.V.); but lines parallel with the base line (B.L.) or horizon (H.L.) will never vanish, but always appear parallel. This is an important rule.

To find the distance of a point within the plane.

In fig. 6, let AB DC represent the face of a cube resting on the ground, and let the given face touch the picture plane. The near lower edge will then coincide with base line AB. Now, if we suppose the cube placed to the left of the spectator, he will be able to see the side BD FE, or the top, if he be as high as the cube.

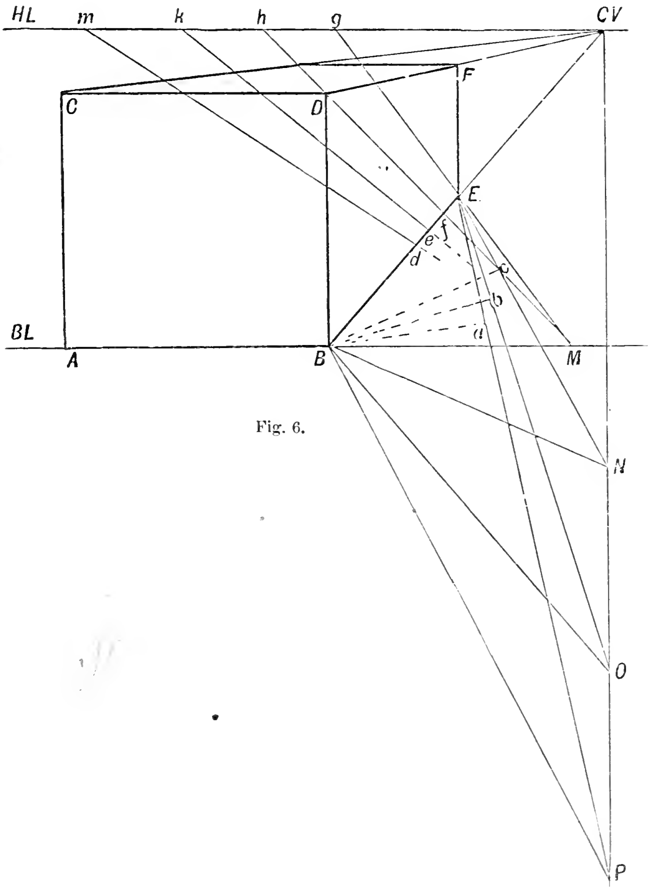

Fig. 6.

We have said that all lines at right angles to AB will vanish at point C.V.; and since all angles of a cube are right angles, the line BE, if produced, will reach C.V., and so will line DF, for the same reason. But we want to find the position of E, the extremity of the line BE.

Draw a line from C.V. perpendicular to AB, and produce it indefinitely. A spectator stationed at N would see the edge BE represented in size by the dotted line B*c* perpendicular from B on EN. If he were placed at O, BE would appear as B*b*, perpendicular to EO from B; also, if placed at P, BE would appear as B*a*; and it will be noticed that, as the spectator recedes, the apparent size of BE decreases, *i.e.*, B*a* is less than B*b*, and B*b* less than B*c*, and so on. (Fig. 6.)

Now from B measure lengths of B*c*, B*b*, B*a* on BE; thus, B*f* = B*c*, B*e* = B*b*, B*d* = B*a*, etc.

Take a point M on base line at a distance from B equal to required length of BE; this point may be on either side of B.

In given case, since BE = AB or BD, make BM = AB, and suppose M to be drawn to right of B. Join M*f*, M*e*, M*d*, etc., and produce them backward to horizontal line touching it in *h*, *k*, *m*, etc.

Now, if M be a fixed point for a distance from B, it will be seen that as the spectator *recedes* from the object the points *h*, *k*, *m*, etc., recede from C.V.; hence, if the points *h*, *k*, *m*, etc., be given, and knowing position of M, we can find apparent distance of BM within the plane, as shown by B*f*, B*e*, etc., and this is done by the following method:

Take C.V. as centre, and distances N, O, P, etc., of spectators from object, as distance; and describe a semicircle cutting horizontal line in *m*, *k*, or *h* on one side, and corresponding points on the other. The points where the semicircle cuts the horizontal line, are called the measuring points, and are denoted by RMP, LMP, according as they are to the right or left of C V.

To find the measuring points, having given the height of the spectator and his distance from the picture plane or base line.

In fig. 7, let H.L. be horizontal line, which is unlimited in

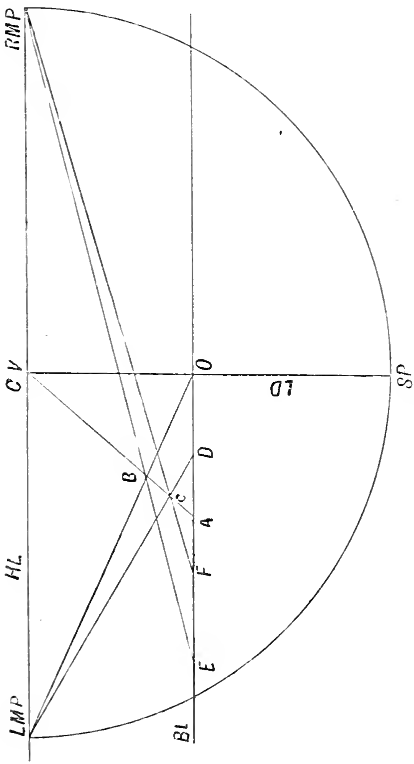

Fig. 7.

length; take any point C.V. in it, and draw line L.D. from it at right angles. Make C.V. O = 6 feet, or whatever may be the spectator's height, and through O draw B.L. parallel to H.L.; produce C.V. O to S.P., so that O S.P. equals distance of spectator from base line. Then with centre C.V. and distance C.V. S.P. describe a semicircle, cutting H.L. in LMP and RMP, the right and left measuring points respectively.

In fig 7, let any point A be taken, say 4 feet to the left, on the base line; join A C.V.

Suppose we wish to find a point the same distance to the left that A is, but 4 feet within the plane: we know that the point lies somewhere on A C.V., because every part of this line is the same distance from C.V. O that A is. Now, we proceed by measuring the required distance to right or left of A, and joining the point thus found with the measuring point opposite—that is, if point be taken to the right of A, as O, we join O LMP; if to the left, as E, we join E RMP. O LMP and E RMP will always cut A C.V. at the same point B if EA = AO.

So also, if we take a lesser distance, as AD, and make AF = AD; join F RMP and D LMP; they will intersect in *c*. Now AO or AE = AB, hence B is four feet within the plane and four feet to the left.

In practice it is not necessary to draw to both measuring points, one (the nearest) will answer every purpose.

Example 1.—Find position of a point on the ground 6 feet directly in front, as seen by a spectator 6 feet in height, and 4 feet from picture plane.

In fig. 8, we draw H.L. and B.L. 6 feet apart, and C.V. S.P. perpendicular to B.L. from C.V., and make O S.P. the spectator's distance from base line. Draw a semicircle to cut H.L. in LMP and RMP. Now, since point required is on line between C.V. and O, we measure 6 feet either way on B.L., as A or B, and join to measuring point as B LMP or A RMP; they will intersect in X, the point required.

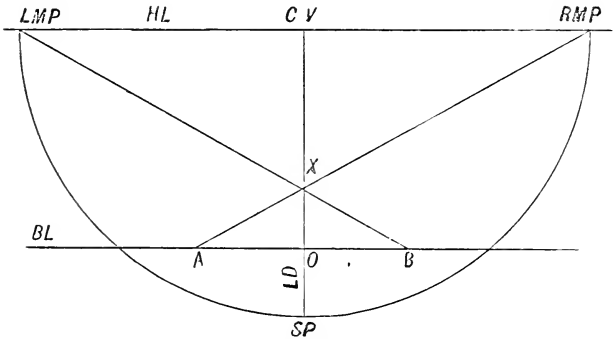

Fig. 8.

Example 2.—Find a point 3 feet (3′) to right, 4 feet (4′) within the P.P., and 5 feet (5′) above it. Height of spectator 5 feet 6 inches (5′ 6″), and his distance from P.P. 4 feet (4′).

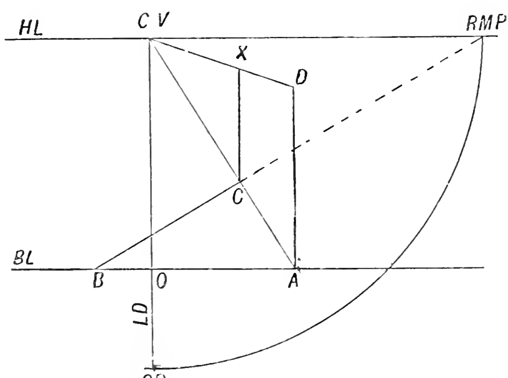

Fig. 9.

Draw H.L. and B.L. 5′ 6″ apart; draw C.V. S.P. cutting B.L. in O; make O S.P. = 4′. Find RMP (C.V. RMP must always be equal to C.V. S.P.); make OA = 3′, from A measure off AB equal to the required distance of point within the plane (4′). Join A C.V. and B RMP, intersecting in C; draw AD perpendicular to B.L., and make AD equal to required height (5′). Join D C.V. Draw CX parallel to AD and meeting D C.V. in X. X is position required. For, since AC = AB, and any point C in A C.V. is same distance to the right that A is, then C is 3′ to the right and 4′ within P.P.

Now, D C.V. and A C.V. are vanishing lines, and CX and AD are parallel lines drawn between them, then CX = AD; but point D is 5′ high, then X is the same height, and is vertically above C also; therefore X is position of required point. (Fig. 9.)

NOTE.—All measurements must be made on the P.P.

EXERCISE II.

(In the following examples take height of spectator 6 feet, and his distance from P.P. 4 feet, and make scale $\frac{1}{4}$ inch to one foot.)

1. Find position of a point directly in front, and 8 feet within P.P. on G.P.
2. Find position of a point 4′ to left (L), 4′ within P.P. on G.P.
3. Find position of a point 6′ to right (R), 6′ within P.P., and 4′ above G.P.
4. Find position of a bird flying 10′ to R., 12′ within P.P., and 8′ above G.P.
5. Find position of a fish resting in water 4′ beneath G.P., 6′ to R., and 8′ within P.P.
6. In Ex. 1, show that however far the point be away, it must always be nearer than the point C.V.

THE PERSPECTIVE LINE.

A straight line is the shortest distance between any two points. If we know the position of any two points we can locate the line between them.

Example 1.—Draw a staff 8′ high placed erect on ground plane, 6′ to R., and 4′ within P.P.; distance 4′, height of spectator 6′ (H = 6′), scale $\frac{1}{4}'' = 1'$.

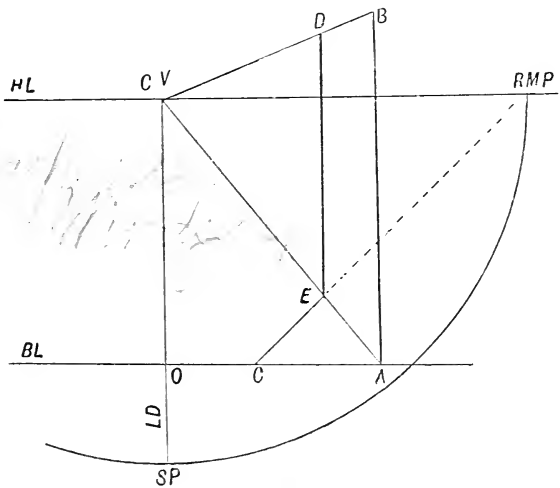

Fig. 10.

Draw H.L. and B.L. 6′ apart; draw C.V. S.P., making O S.P. (LD) 4′, and find RMP as before. Take A, 6′ to right of O, and erect perpendicular AB 8′ in height; join B C.V.

and A C.V.; take C 4′ to left of A, and join C RMP, cutting A C.V. in E. Through E draw ED parallel to AB; ED is line required. Now, since AE = AC, E is 6′ to right and 4′ within P.P.; but ED = AB, then ED is 4′ within P.P., 8′ high, and 6′ to right. (Fig. 10.)

Example 2.—Draw a line 3′ long lying on G.P., parallel to P.P. (or base line), and 4′ within it, near extremity of line being 2′ to left. H = 6′, L.D. = 4′, scale $\frac{1}{4}''$ = 1′.

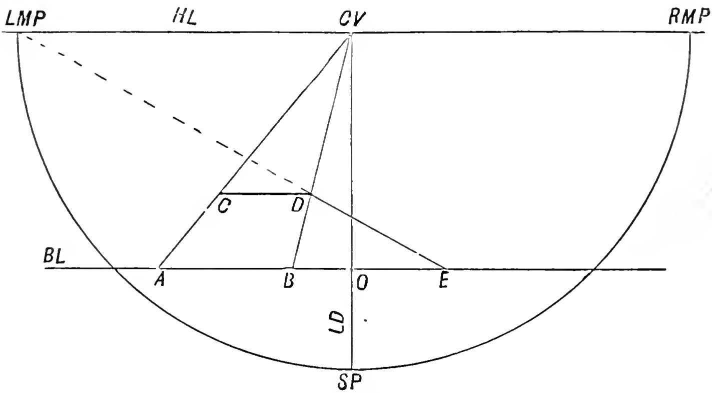

Fig. 11.

Draw H.L., B.L. and L.D. as before, and find also LMP. From O mark off OB = 2′ (distance of near extremity from O), and make BA = 3′ (length of line); join B C.V. and A C.V. From B measure off BE = 4′ (distance within P.P.); join E LMP, cutting B C.V. in D; through D draw DC to line A C.V. and parallel to AB. CD is line required, for it is equal to AB, and its near extremity D is same distance to the left that B is, and BE = BD. (Fig. 11.)

Example 3.—Draw a line 3′ in length lying on G.P., perpendicular to P.P. (base line), 4′ to R., and nearest extremity 2′ within P.P. H = 6′, L.D. = 4′, scale $\frac{1}{4}'' = 1'$.

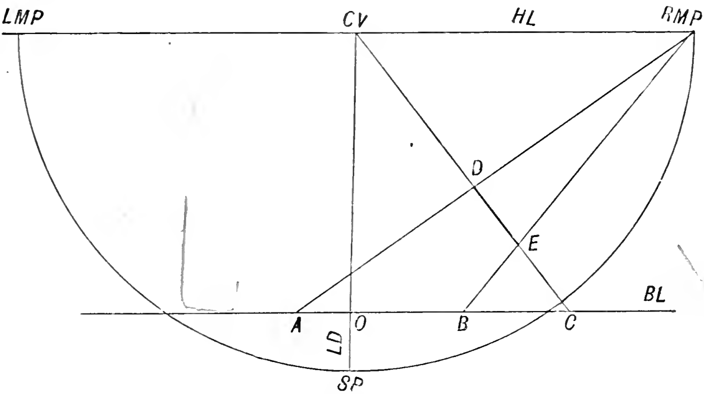

Fig. 12.

Draw H.L., B.L., L.D., as before, and mark point RMP. Make OC = 4′, join C C.V. From C measure off CB = 2′, and from B mark off BA = 3′ (length of line); join B RMP and A RMP, to cut C C.V. in E, D. ED is the line required, for DE is parallel to LD, and therefore perpendicular to BL. EC = BC and DC = AC, then DE = AB, and point E is the same distance from L.D. that C is. (Fig. 12.)

Example 4.—Draw a line 3′ in length parallel to P.P. and 4′ within it, and parallel to G.P., and 4′ above it, line to be drawn with near extremity 2′ to left. H = 6′, LD = 4′, scale $\frac{1}{4}'' = 1'$.

In fig. 13 draw H.L., B.L. and L.D., and find LMP, take B 2′ to left and A 4′ to left of B, also S 4′ to right of B. Erect AC and BD perpendiculars to AB, and each 4′ in height; join CD, C C.V., D C.V., B C.V., and S LMP. Let S LMP cut B C.V. in G; draw GF parallel to BD, and FE parallel to CD: then EF is line required. For since BG = BS, and BD = FG, then DF = BG, so also CE = DF. But EF = CD, then EF is parallel to P.P., is same height above G.P. as BD, and the same distance to the left as BD.

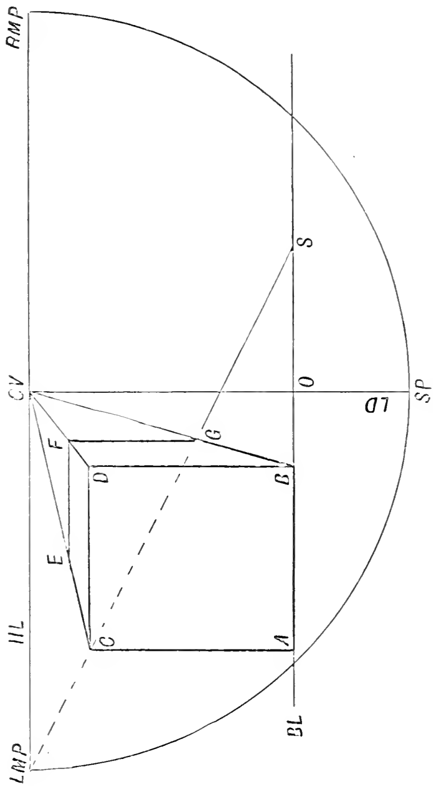

Fig. 13.

Exercise III.

(H = 6′, L.D. = 4′, scale $\frac{1}{4}''$ = 1′.)

1. Draw a line 4′ in length on G.P., parallel to P.P. and 4′ within it, directly in front.

2. Draw a line 4′ in length parallel to G.P. and 4′ above it, touching P.P. 4′ to left, and perpendicular to it.

3. Draw a line 3′ in length perpendicular to G.P., and 3′ above it; line to be 4′ to right.

4. Draw a line 4′ in length parallel to G.P. and 5′ above it, and in contact with P.P.; line to have one extremity 3′ to right.

5. A line 5′ in length is drawn parallel to P.P. and 6′ within it; it is parallel to the G.P., one extremity being 3′ to right, the other 2′ to left. It is 4′ above the G.P.

SURFACES IN PERSPECTIVE.

RECTANGULAR SURFACES.

The Square.

A square is a parallelogram having two adjacent sides equal, and the included angle a right angle.

We said previously that an object appears to decrease in size as it recedes from the eye. If, therefore, a square is placed on the ground plane with one side touching the picture plane, it is clear that the side most removed will appear smaller than that touching the picture plane, and so the square may not appear to have even one right angle.

Let AB, BC, CD, DE be all taken of equal length, and let C be on L.D., it is plain if lines parallel to AB and of equal length be drawn on G.P. within the P.P., they will appear shorter than AB. Find points M, N and join BM, CM, etc. Then AF = AB, GB = BC = AB, etc. Hence FG = AB, GH = BC, etc., also AF = BG, BG = CH, etc., and the squares AG, BH, CK, and DL will all be equal, and the only real right angles will be BCH and DCH ; all the other angles, FAB, ABG, etc., though really right angles will not appear so. (Fig. 14.)

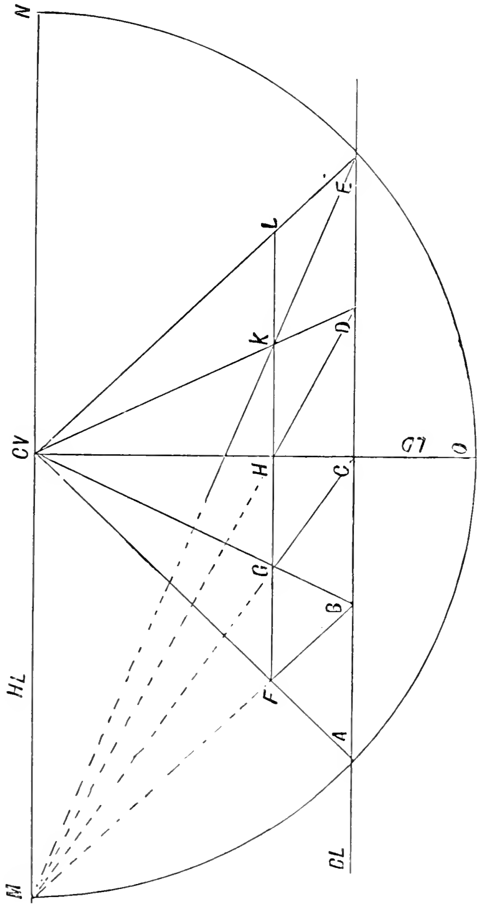

Fig. 14.

To represent a square perpendicular to G.P. and P.P.

Let H.L. and B.L. be drawn, also L.D., and find X. Take

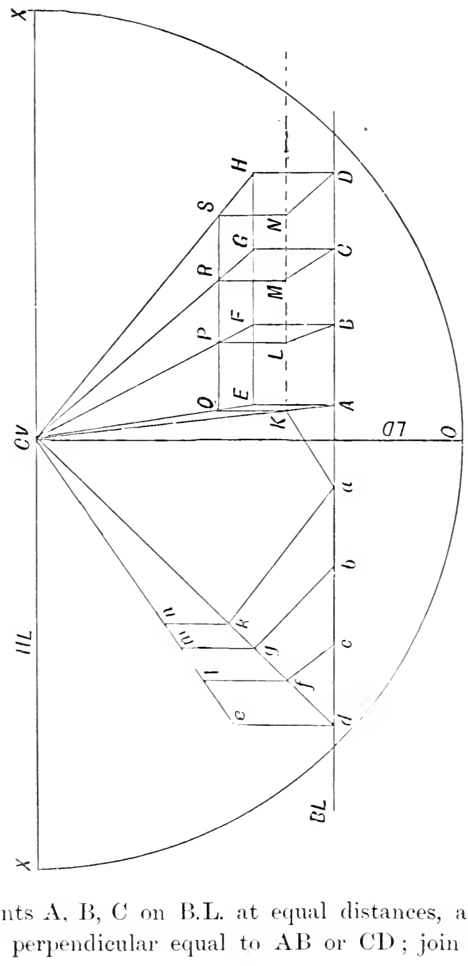

Fig. 15.

any points A, B, C on B.L. at equal distances, and at each erect a perpendicular equal to AB or CD; join EH, which will be parallel to AD; join A C.V., B C.V., C C.V. and D C.V. Take a point *a* at a distance from A equal to AB (on left), and join AX to cut A C.V. in K; draw KN parallel to AD, intersecting the vanishing lines in L, M, N; at K, L, M, N, erect perpendiculars to meet the vanishing lines in Q, P, R, S; join QS. Now, since AB = AE, AF is a square,

(*a*) AF is said to be drawn to right, perpendicular to G.P., and touching or coincident with P.P.

(*b*) Since A*a* = AK = QE, then KE is a square drawn to right, perpendicular to G.P., perpendicular to and touching P.P. and parallel to L D.

(*c*) Since AK = AB = KL = LB, AL is a square drawn to right, resting on G.P., perpendicular to and touching P.P.

(*d*) Since AB = AK = QE = PF, QF is a square drawn to right, parallel to G.P. and raised above it, and perpendicular to and touching P.P. and parallel to L.D.

The cube AP or BR will show the square in every position in parallel perspective, provided it touch the picture plane.

(*e*) Similarly, square QL is drawn to right, perpendicular to G.P., parallel to P.P., and *within it*. (Fig. 15)

NOTE.—If a figure be drawn *parallel* to the P.P. it will always be drawn in its true shape, but smaller, hence QL will be a true square. In above figure, take points *a*, *b*, *c*, *d*, etc., at equal distances to the left; erect perpendicular *de* at *d*, and equal to *dc*. Join *e* C.V. and *d* C.V., also *c*X, *b*X, *a*X, to meet *d* C.V. in *f*, *g*, *h*. Through *f*, *g*, *h* draw parallels to *de*, meeting *e* C.V. in *l*, *m*, *n*. Then, since *de*, *fl*, etc., are parallels between vanishing lines, they are all equal to each other and to *dc*, *cb*, etc., for *de* = *dc*. But *df* = *dc*, and *fg* = *cb*, etc., hence *ef*, *lg*, *mn* are equal squares, and are said to be drawn perpendicular to G.P. and also to picture plane. One of them, *ef*, touches the P.P., the others are within it. It will be seen that as the squares recede they appear smaller.

Example 1.—Draw a square, side 4′, 4′ to R., 4′ within, lying on G.P., $\perp$ to P.P. H = 6′, L.D. = 4′, scale $\frac{1}{4}'' = 1'$.

Construct figure as before. Take AC one inch (4′) and CD one inch, take AB to left one inch, join C C.V., D C.V., B MP, A MP, cutting C C.V. in F, E. Draw EK and FG, parallel to AC, and meeting D C.V. in K, G: then FK is square required, for FG = EK = CD, EC = CA, and EF = AB. (Fig. 16.)

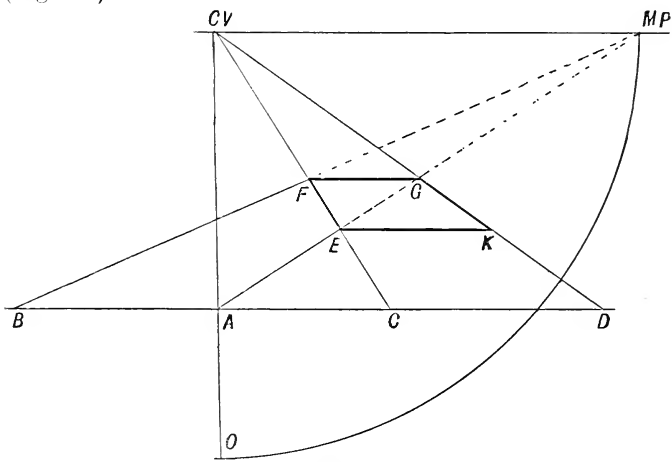

Fig. 16.

Example 2.—Draw a square, side 4′ and 6′ to left, $\perp$ to P.P. and 3′ within it, and $\perp$ to G.P. and 1′ above it. Other specifications as in last example.

Construct figure as before. Take C $1\frac{1}{2}''$ (6′) to left of A, take B 1″ (4′) to right of A, erect CE $\perp$ to AC at C and $1\frac{1}{4}''$ (5′) in height, and mark off D $\frac{1}{4}''$ (1′) above C; join E C.V. D C.V., C C.V., A MP and B MP. Through F, G draw FL and GS parallel to CE, and meeting vanishing lines in L, S and H, K respectively: LK is the figure required. For

LS = HK = FG = AB = 4′, SK = LH = ED = 4′ and HF = DC = 1′, also FC = AC = 6′. (Fig. 17.)

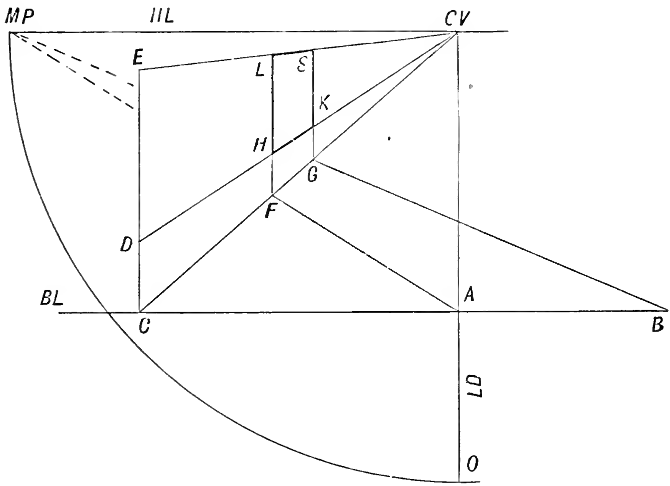

Fig. 17.

Example 3.—Draw a square 4′ side directly in front, parallel to G.P. and 3′ above it, and ⊥ to P.P. and 3′ within it. Same specifications as before.

Construct figure and find M as before. From A measure off $\frac{1}{2}''$ both ways to B and C (BC = 4′); erect perpendiculars BF and CG, each $\frac{3}{4}''$ (3′); join FG, B C.V., F C.V., and G C.V. From B measure off BD $\frac{3}{4}''$ (3′), and from D measure off DE 1″ (4′); join DM and EM, cutting B C.V. in H, K. Draw HL, KN parallel to BF, and meeting F C.V. in L, N; through L, N draw LJ and NP parallel to FG: then LP is the square required. For FL = BH = BD = 3′, and NP = LJ = FG = BC = 4′, LN = HK = DE = 4′, and LH = FB = 3′; also SL = SJ = XF = XG = AB = AC = 2′. (Fig. 18.)

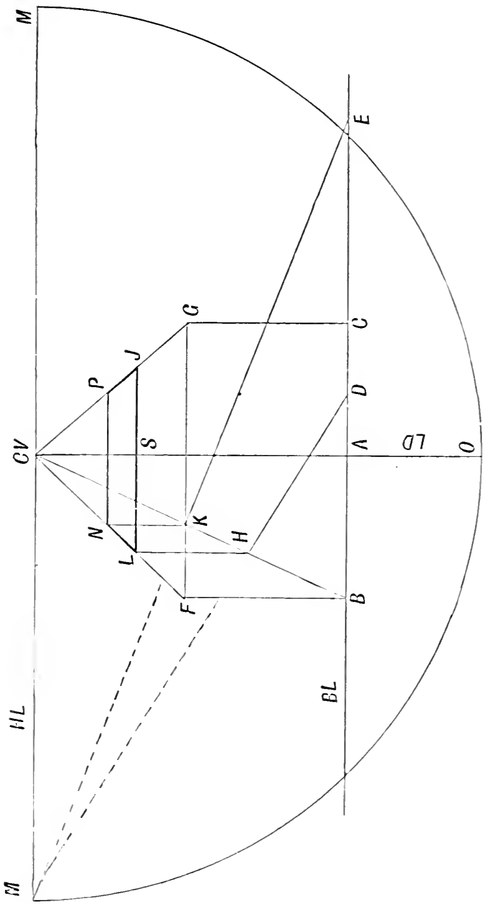

Fig. 18.

Example 4.—Draw a square 4′ side, 4′ to right, lying on G.P., ⊥ to P.P. and 4′ within it, and within this square place centrally a square whose sides are 2′.

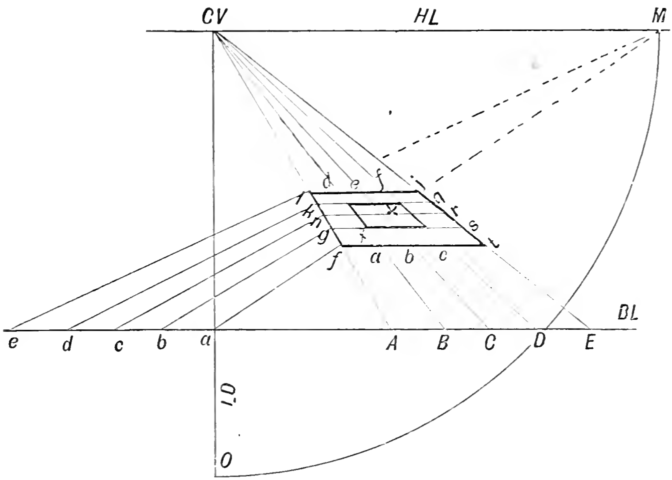

Fig. 19.

Note.—Figures are said to be placed *centrally* when their centres coincide and their like sides are parallel to each other. Concentric circles are always placed centrally with respect to each other.

Draw H.L., B.L., L.D., as before, and find M. Measure off *a*A, *ae*, and AE, each one inch; bisect AE and *ae* in C and *c*, and bisect CE, CA, *ce* and *ca* in D, B, *d*, *b* respectively. Join A C.V., B C.V., C C.V., D C.V., *a*M, *b*M, *c*M, *d*M, and *e*M, cutting A C.V. in *f*, *g*, *n*, *k*, *l*; and through these latter points draw parallels to AE, cutting E C.V. in *t*, *s*, *r*, *q* and *j*. *lt* will form the outer square, and XX the inner square. For *ac* = BD = 2′, and *kg* = *db* = 2′, and *f*A = A*a* and *ft* = AE, etc. (Fig. 19.)

NOTE.—If aM be joined, it will always pass through j if the figure be a square. It will also pass through X and X.

EXERCISE IV.

(H = 6′, L.D. = 4′, scale $\frac{1}{4}''$ = 1′.)

1. Draw a square 4′ side, 3′ to left, resting on G.P., $\perp$ to and touching P.P.

2. Draw a square 5′ side, directly in front, lying on G.P., $\perp$ to and touching P.P.

3. Draw a square 6′ side, $\perp$ to G.P., $\perp$ to P.P., touching both, and 6′ to left.

4. Draw a square 3′ side, parallel to G.P. and 2′ above it, 4′ to R., perpendicular to P.P. and 2′ within it.

5. Draw a square 4′ side, parallel to P.P. and 4′ within it, resting on ground plane, left corner touching L.D.

6. Draw a square 4′ side, parallel to G.P. and 2′ above it, right corner 1′ to right; square to have side $\perp$ to P.P. and 1′ within it.

7. Draw a square 4′ side, $\perp$ to G.P. and 1′ above it, 3′ to right, $\perp$ to P.P. and 2′ within it.

8. Draw a square 6′ side, directly in front, $\perp$ to P.P. and 2′ within it, parallel to G.P. and 4′ above it; and place a square of one-fourth its area centrally within it.

9. Draw a square 3′ side, 4′ to right, $\perp$ to P.P. and 3′ within it, $\perp$ to G.P. and just its own height *below* it.

10. Draw a cube (edge 4′) touching P.P. 4′ to left, resting on G.P.

11. Draw a cube (4′ edge) directly in front, and to rest on G.P., one edge parallel to P.P. and 2′ within it.

The Oblong.

An oblong is a figure whose opposite sides are equal and parallel, and whose angles are right angles.

The drawing of the oblong differs little from that of the square, care being required only to distinguish the sides.

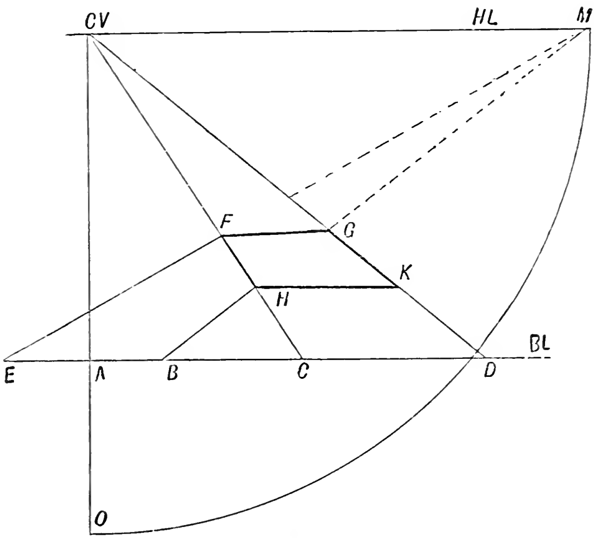

Fig. 20.

Example 1.—Draw an oblong 3′ × 2′, lying on G.P., ⊥ to P.P., side 3′, parallel to, and 2′ within it; oblong to be 4′ to right. Specifications as before.

Construct figure as before. Find M, and measure off AC 4′ =(distance to right), CD = 3′ (length of side), and measure off BE 2′ (breadth); measure off BC = 2′ (distance within P.P.). Join EM, BM, C C.V., D C.V.; through H, F draw HK, FG parallel to CD, meeting D C.V. in G, K: then FK will be oblong required. For FG = HK = CD = 3′, GK = FH = EB = 2′, and HC = CB = 2′. (Fig. 20.)

Exercise V.

(H = 6′, L.D. = 4′, scale $\frac{1}{4}''$ = 1′.)

1. Draw an oblong 4′ × 2′ $\perp$ to G.P., and 4′ to left, $\perp$ to P.P. and 3′ within it; oblong to rest on end.

2. Draw an oblong 7′ × 5′, standing on end on G.P., parallel with P.P. and 2′ within it, directly in front.

3. Draw an oblong 3′ 6″ × 4′ 6″, parallel to G.P. and 2′ 6″ above it, right corner 3′ 6″ to left; end of oblong to be parallel to P.P. and 6″ within it.

4. An oblong 6′ × 4′ is buried in the ground to a depth of 2′; it is parallel with LD and 4′ to right, and 2′ within P.P.

5. An oblong 6′ × 4′, with side resting on G.P. 4′ to left, and $\perp$ to it, intersects a square of 4′ side, resting on G.P. and parallel to P.P.; the oblong divides the square into two equal portions, and the square divides the oblong into parts of 4′ and 2′ respectively, the greater portion being nearest. The square is 4′ within the P.P.

6. Draw an oblong 4′ × 2′, end touching P.P. 4′ to R.; oblong lying on G.P.

7. Draw an oblong 5′ × 3′, lying on G.P. directly in front; end parallel with P.P. and 2′ within it.

8. A wall, whose height is 8′, begins at a point 10′ to left, and stretches inwards indefinitely; at distances of 10′ and 20′ doors 5′ × 3′ are made in it. Draw it.

9. Draw an oblong 6′ × 4′, lying on ground, sides parallel to P.P. and 2′ from it, oblong 3′ to right; and within it place an oblong 4′ × 2′ centrally.

10. Draw an oblong 3′ × 2′, lying on G.P. 4′ to left, end parallel to P.P. and 2′ from it; and *about* it draw an oblong 5′ × 4′ centrally.

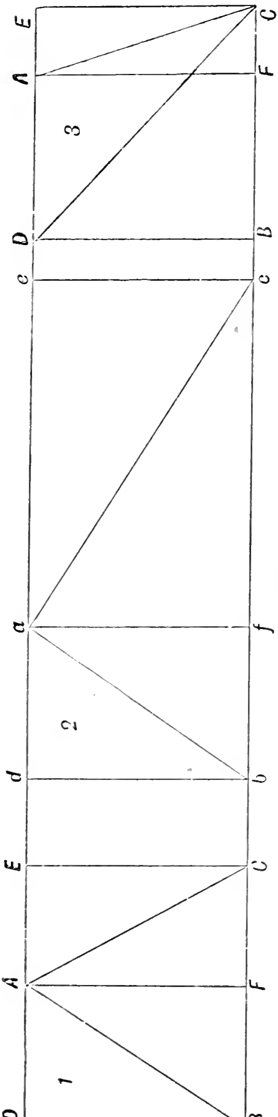

Fig. 21.

The Triangle.

A triangle is a figure enclosed by three straight lines; these may be of uniform length, but the length of any two taken together must be greater than the third.

The triangle cannot be conveniently drawn alone, but is drawn with reference to an oblong.

Triangles are equilateral, with three equal sides;

Isosceles, with two equal sides;

Scalene, with unequal sides.

Right angled, or containing a right angle;

Obtuse angled, or containing an obtuse angle;

Acute angled, or containing three acute angles.

To draw a triangle we first draw a *plan*, showing the triangle and its bounding oblong.

Thus, 1 would be a plan for an equilateral triangle ABC, in which DA = AE = BF = FC.

2 would be a plan for a right-angled triangle, in which the angle *bac* is theright angle, with *bc* as hypothenuse; the segments *bf*, *fc* of the hypothenuse would determine the position of *a*.

In 3, the triangle DAC being given, CE would be drawn perpendicular to DA, and CB parallel to DA; also DB and AF would be drawn parallel to CE: then position of the angles at A and C would be easily determined.

Example 1.—Draw an equilateral triangle, 3′ to a side, lying on ground, one side touching P.P. near left angle, 4′ to right. Height 6′, distance 4′, scale $\frac{1}{4}'' = 1'$.

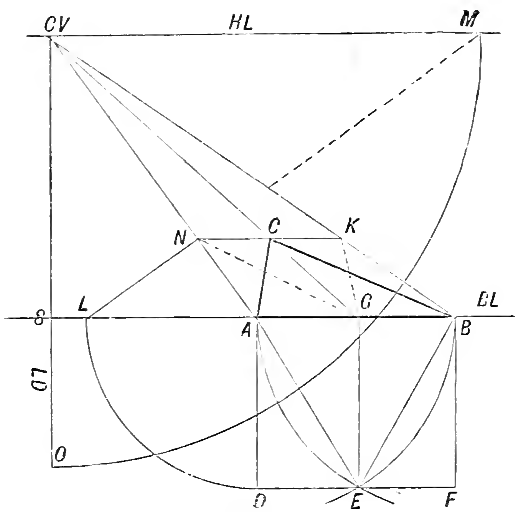

Fig. 22.

Draw H.L., B.L. and L.D. as before. Take A 4′ to right of S, and B 3′ to right of A; join A C.V. and B C.V.; on AB describe equilateral triangle ABE (below AB). Draw AD, BF at right angles to AB, and through E, draw DF parallel to AB, and meeting AD in D and BF in F; draw EG parallel to AD. Join G C.V., and with centre A and distance AD, describe arc DL, cutting B.L. in L. Join LM, cutting

A C.V. in N; through N draw NK parallel to AB, cutting G C.V. in C and B C.V. in K. Join CA and CB: then ABC is the triangle required. For AB = AE and AD = AL = AN = CG = height of triangle; also NC = CK = AG = GB = DE = EF. Then AC = AE = EB = BC, for they are diagonals of equal oblongs. (Fig. 22.)

In this figure the vertex C is directed *away from* the eye.

If we joined NG and KG we would have a similar triangle, but with vertex G directed *towards* the observer.

Example 2.—Draw an isosceles triangle whose sides are 2′, 3′ and 3′ respectively, lying on ground plane, vertex directed towards the spectator, base parallel with P.P. and 5′ away from it; near angle 2′ to left.

Draw B.L. and H.L. as before, find M. Take K 2′ to left of X, and G 2′ to left of K; bisect GK in H: draw GR, HS and KT at right angles to B.L., and 5′ in length; join RST. Take TP 3′, and with centre T and distance TP describe arc PN, cutting HS in N: join NT and NR: NRT will be the plan of the triangle.

Through N draw UNQ parallel to RT, cutting GR in U and KT in Q; with centre K and distance KQ describe arc QL, cutting B.L. in L; and with same centre and distance KT describe arc TV, cutting B.L. in V. Join G C.V., H C.V. and K C.V.; and join also LM and VM to cut K C.V. in E and C. Through E draw EAF, and through C draw CDB, each parallel to B.L., and cutting H C.V. in A and D respectively; join BA and AC: then ABC will be the triangle required. For CK = KV = KT = 5′, and EC = LV = QT = NS = height of triangle, and EK = KL = KQ. Hence AD = NS and AH = HN. But since triangle is isosceles, GH is made equal to HK; hence BD = DC = GH = HK, then CA = NT = NR = BA. (Fig. 23.)

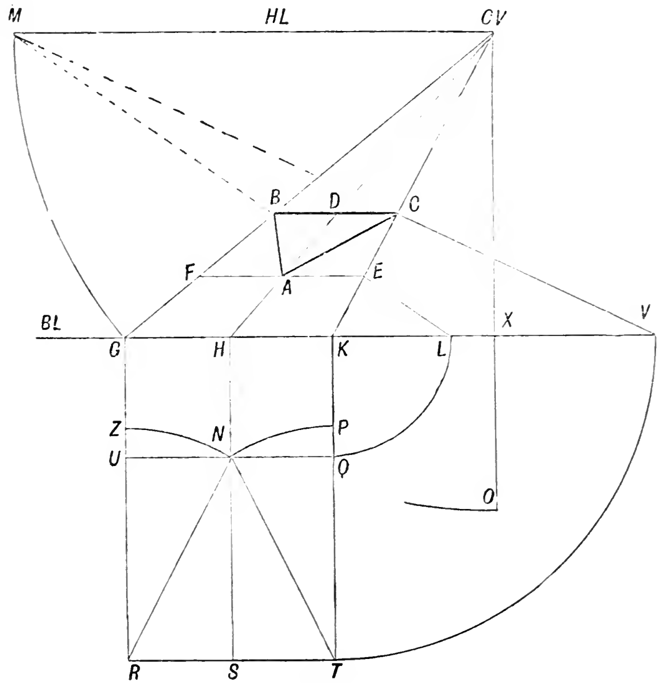

Fig. 23.

Example 3.—Draw an equilateral triangle, each side 4′, one side on ground plane parallel to L.D.; triangle placed ⊥ to ground plane and touching P.P. 3′ to right.

Draw H.L., B.L. and L.D., and find M. Take point D 3′ to R., and B 3′ to right of D; on DB describe the equilateral triangle DGB. Bisect DB in E, join EG; through G draw FH parallel to B.L., and through D, B draw DF, BH parallel to EG, cutting FH in F, H. Join B C.V., draw EM, DM, cutting B C.V. in N and A. With centre B and distance BH describe arc HK, cutting BK, a perpendicular on B.L., at K;

join K C.V. Through A and N draw parallels to BK, meeting K C.V. in L and C; join AC, CB. Then ABC is the triangle required. For LA = CN = KB = BH = EG = altitude of the triangle; and AN = NB = DE = EB, and AB = DB =

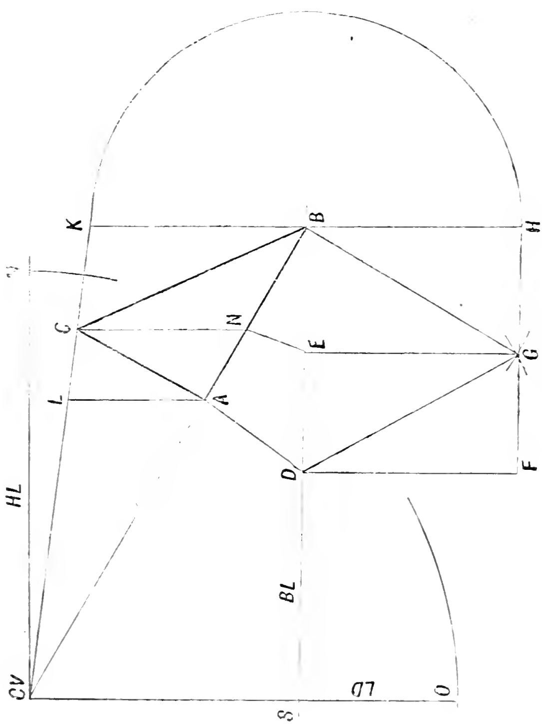

Fig. 24.

length of side; then C corresponds to G, and AC, CB to DG and GB respectively. (Fig. 24.)

Example 4.—Draw a right angled-triangle whose hypothenuse is 4′ and one of the other sides 3′; the right angle is directed away from the observer, and is 5′ to the left, and the hypothenuse is 3′ to the left. The triangle lies on the ground with the hypothenuse in contact with the B.L.

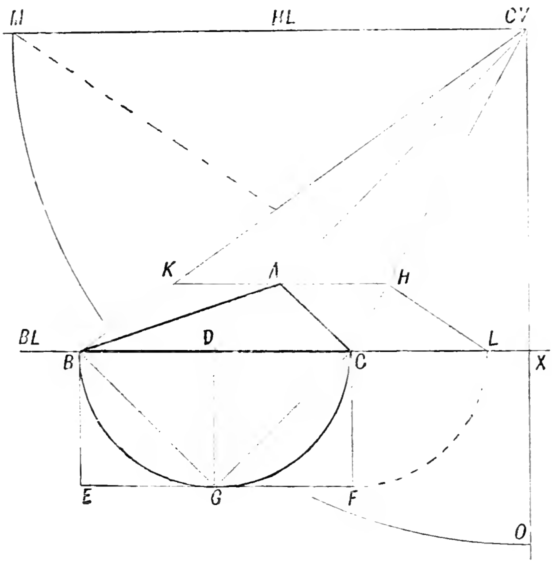

Fig. 25.

Draw H.L., B.L. and L.D. as before. Find M, and take a point D, 5′ to left, C, 3′ to left, and B, 4′ to left of C. On BC describe the semicircle BGC. (The angle BGC in a semicircle is always a right angle.) Join DG, B C.V., D C.V. and C C.V.; through G draw EF parallel to BC, and through B, C draw BE, CF perpendiculars on EF, at E and F. With centre C and distance CF, describe arc FL, cutting B.L. in L; join LM, cutting C C.V. in H; through H draw KAH parallel to BC, cutting D C.V. in A; join AB and AC: then

ABC is the triangle required. For AB = BG and AC = CG, and AD = DG = CF = CL, and KA = BD and AH = DC; then KH = BC, and A corresponds to G; then angle BAC corresponds to angle BCG. (Fig. 25.)

Exercise VI.

(H = 6′, LD = 4′, scale $\frac{1}{4}'' = 1'$.)

1. Draw an equilateral triangle, 3′ side, lying on G.P., one side parallel to L.D., vertex directed to the left and distant from the L.D. 4′; the triangle touches P.P.

2. Draw an equilateral triangle parallel to P.P. and 3′ from it, $\perp$ to G.P., near angle 2′ to left, triangle 3′ to side, one side on ground.

3. Draw a triangle whose sides are 4′, 5′, 6′, respectively, 6′ side on ground, $\perp$ to P.P. and 2′ within it; triangle 3′ to right.

4. Draw an isosceles triangle whose base is 3′ and each equal side 4′, lying on ground plane directly in front, vertex directed towards P.P. and 1′ from it.

5. Draw a right-angled triangle whose hypothenuse is 5′, and the perpendicular on it from the right angle divides it into segments of 3′ and 2′. The triangle is parallel with G.P. and 4′ above it, and the vertex is directed away from P.P. at a distance of 4′. The right angle is directly in front, and the larger segment is to the right.

6. Draw a triangle, each side being 3′, $\perp$ to G.P. and 1′ above it, with a side parallel to it. The triangle is 4′ to left, $\perp$ to P.P. and 2′ within it.

7. Draw an isosceles right-angled triangle, the equal sides being 3′; one equal side is parallel to G.P. and 2′ above it. The triangle is 2′ to right, parallel to P.P., and 2′ within it.

8. Draw an isosceles triangle, base 4′, equal sides 3′ each, directly in front, parallel to P.P. and 4′ from it; vertex touches ground, and base is parallel to it.

9. An equilateral triangle, each of whose sides is 4′, lies on the ground, vertex directed away; one side parallel to P.P.

and 2′ from it. Triangle 2′ to right. Within this place centrally a similar triangle whose sides are 2′.

10. An equilateral triangle, each of whose sides is 4′, is $\perp$ to ground and also to P.P., which it touches at a point 5′ to left; the vertex of the triangle is directed downwards, and one side is horizontal. The triangle is buried one-fourth in the ground. Draw it.

The Hexagon.

A hexagon is a figure bounded by six sides. When the sides and angles are all equal the hexagon is regular. Only regular hexagons will be considered here.

In drawing a hexagon, a plan is made somewhat similar to that for the triangle. The following is the general form of the plan.

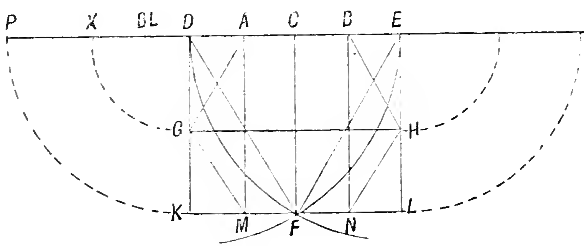

Fig. 26.

Let B.L. = base line, and AB given side of hexagon, and in given position. Bisect AB in C, make AD and BE each equal BC; on DE describe equilateral triangle DFE, and construct oblong DL. Bisect DK in G; draw GH parallel to KL; join AG, GM, BH and HN. With centre D and distances DG and DK describe arcs to cut B.L. in X and P respectively. Like triangles, hexagons may lie flat on the ground, with a side parallel to the P.P. or to L.D., or they may be drawn perpendicular to the ground plane, with a side or an angle touching it; and a side or an angle may touch the P.P. or be within it. (Fig. 26.)

If a side touch the P.P., the extremity of *it* determines the distance of the hexagon to the right or left When an angle touches the P.P., that point determines the distance; so also when the figure is within the P.P.

Example 1.—Draw a hexagon, each side 2′, lying on G.P., one side coincident with P.P. and 4′ to R. H = 6′, L.D. = 4′, scale $\frac{1}{4}''$ = 1′.

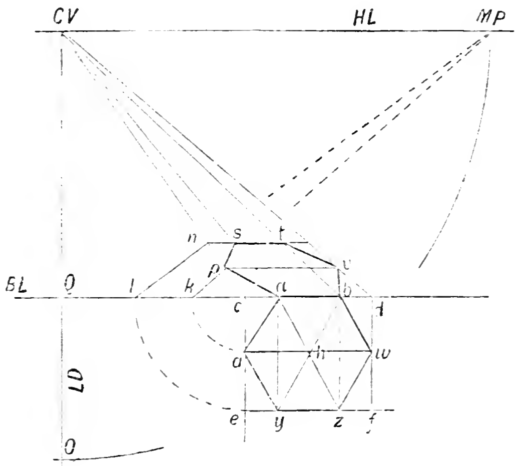

Fig. 27.

Let H.L., B.L. and L.D. be drawn. Take C.V., find O and MP; take AB 4′ to R. of O, make AB = 2′. On AB construct equilateral triangle ABH, and produce to Y, Z, making HZ and HY each equal BH: join AY and YZ, and produce YZ both ways. Through H draw GHW parallel to YZ; through C draw CE parallel to AY; draw also DWF parallel to AY. Join C C.V., A C.V., B C.V. and D C.V.; also L MP and K MP, cutting C C.V. in N and P. Through N, P draw NT and PV: parallel to AB; join SP, PA, TV and VB, then STVBAP is the hexagon required. For PC =

CK = CG, and NP = LK = GE. Then SP = GY, also PA = AG = GY; similarly, TV = VB = BW = WZ = AB, and QA = 4′ and AB = 2′. (Fig. 27.)

Example 2.—Draw a hexagon same as in Example 1, but ⊥ to G.P., and with an angle touching P.P. 4′ to L.

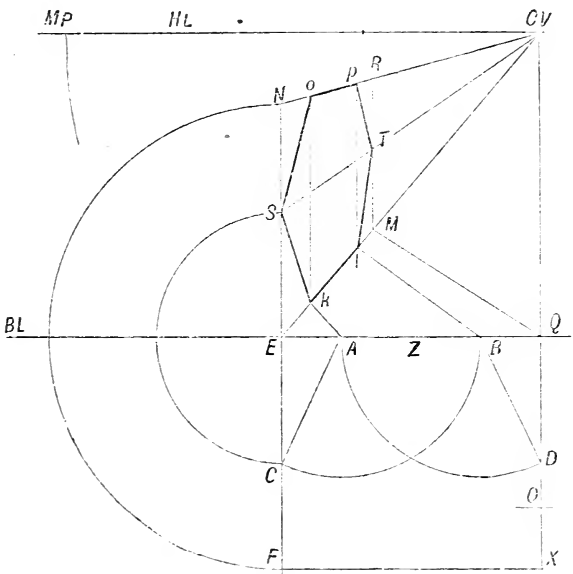

Fig. 28.

Here take E 4′ to left, and for plan of the hexagon we may proceed as follows:—Bisect EQ (4′) in Z, also EZ, QZ in A, B respectively; with centre A and distance AB describe arc BC; similarly, draw arc AD. Draw EC and QD, perpendiculars to AB at E and Q respectively, meeting arcs in C and D; produce EC to F, making CF = EC. Erect perpendicular EN and make it equal EF, and make

ES = EC; join N C.V., S C.V. and E C.V., also A MP, B MP and Q MP, cutting E C.V. in K, L, M. Through K, L, M draw parallels to EN, cutting N C.V. in O, P, R: join OS, SK, LT and TP: then OPTLKS will be hexagon required. For ES = EC and EN = EF, also KL = AB and EM = EQ. Then OS = SK = AB; and PT = TL = BD. (Fig. 28.)

Example 3.—Draw a hexagon, 2′ side, resting on G.P. ⊥ to P.P., having one side coincident with it and 4′ to right. H = 6′, L.D. = 4′, scale $\frac{1}{4}''$ = 1′.

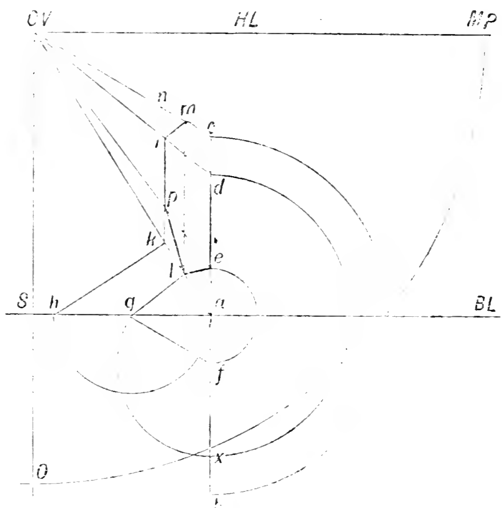

Fig. 29.

We proceed as follows:—After drawing H.L. and B.L. and finding positions of O, MP and C.V., take A 4′ to right of S; at A erect perpendicular AC = 4′, and measure off AE = 1′ and DE = 2′. Produce CA to B, making AB = AC and FX = DE; make FG = FX and GH = GA; join HKMP, GLMP, A C.V., D C.V., E C.V. and C C.V. At K, L erect perpendiculars to meet vanishing lines in N and M; join MR, MD, LE and LP: then RMDELPR will be hexagon required. For AK = AH and AL = AG = FX = 2′, and DE = RP and ML = NK = CA. Then RM = MD = DE = 2′, etc. (Fig. 29.)

Example 4.—Draw a hexagon, 2′ side, lying on ground, near side parallel with P.P. and 2′ within it; hexagon to be directly in front. H = 6′, L.D. = 4′, scale $\frac{1}{4}'' = 1'$.

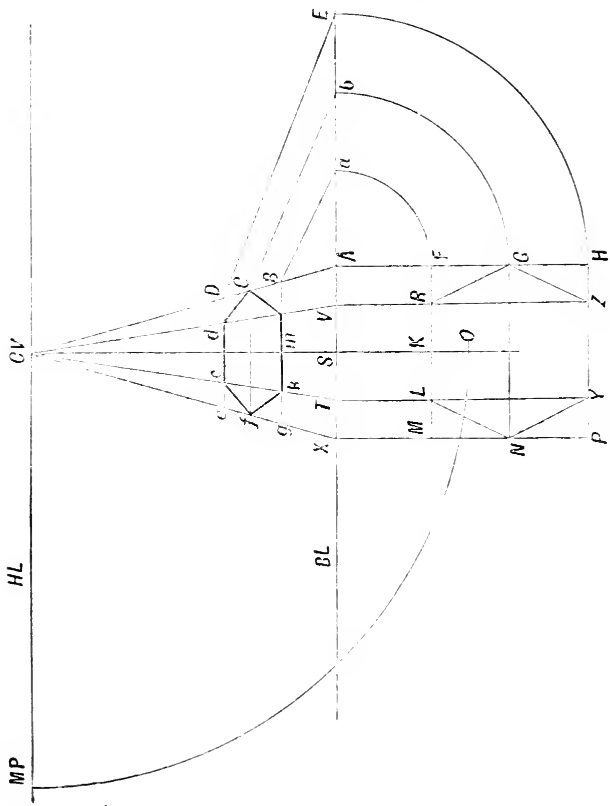

Fig. 30.

Draw H.L., B,L., and find O C.V. and MP as before. Make SV and ST each = 1′, and make TX and VA each = 1′; make AF = 2′, and draw it $\perp$ to AX; draw XP, TY, VZ parallel to AF. Make RG = RL or TV, and GZ also = RL, and complete the oblong MH. Make A*a* = AF, A*b* = AG and AE = AH; join EMP, *b*MP, *a*MP, and A C.V., V C.V., T C.V. and X C.V. Through intersecting points D, C, B, draw parallels to AX, meeting X C.V. in *e*, *f*, *g*, respectively; join *ef*, *fk*, *d*C and C*m*, completing the hexagon: then AB = A*a* = 2′, and BC = CD = *ab* = *b*E = GH, and hence C*m* = GZ and *d*C = YG = 2′, etc. (Fig. 30.)

Exercise VII.

In these examples take H = 6′, L.D. = 4′, and scale $\frac{1}{4}'' = 1'$; but a scale of $\frac{1}{2}'' = 1'$ may be used if thought more convenient.

1. Draw a hexagon, side 2′, lying on ground plane, one side perpendicular to P.P., and an angle touching it at a point 4′ to right.

2. Draw a hexagon, side 3′, standing on edge, $\perp$ to ground plane and P.P., and having an angle touch the P.P. 3′ to left.

3. Draw a hexagon, side 2′, standing on edge, parallel to P.P. $\perp$ to ground plane, directly in front, and 3′ away.

4. Draw a hexagon, 2′ side, lying on G.P., one side parallel to P.P. and 3′ away; hexagon 4′ to left.

5. Draw a hexagon, 3′ side, resting on an angle $\perp$ to G.P. and P.P., one side parallel to P.P. 4′ to right and 4′ within it.

6. Draw a hexagon whose edge is coincident with that of a square, and lying in same plane. The square is 2′ to the side, and is placed $\perp$ to P.P. and G.P. 2′ to right and 2′ within.

7. Draw a hexagon, 3′ side, parallel to G.P. and 4′ above it, 4′ to right, 3′ within P.P., and one side parallel to P.P.

8. Draw a hexagon 3′ to side, 4′ to right, one angle touching P.P.; hexagon to be $\perp$ to P.P. and parallel to G.P.

9. Draw a hexagon 3′ to a side, directly in front, lying on ground plane, one angle touching P.P., and sides $\perp$ to it.

10. Draw a hexagon about an equilateral triangle lying on G.P., vertex directed away from observer; the triangle is 3′ to a side, and one side is parallel to P.P. and 3′ within it. The vertex of the triangle is 4′ to left.

11. Draw a hexagon, 2′ side, placed $\perp$ to P.P. and G.P., 4′ to right, lower side parallel to ground plane and 4′ above it.

12. Draw a hexagon, 4′ side, lying on G.P., near side parallel to P.P. and 2′ within it; hexagon to be 4′ to right. Within this draw (centrally) another hexagon, whose sides shall be 2′ in length.

13. Draw a hexagon, side 2′, parallel to P.P. and 2′ within it, lying on G.P. directly in front.

14. Represent a hexagon, side 3′, half buried vertically in the ground, one side parallel to G.P.; hexagon $\perp$ to P.P. and 3′ within it, 4′ to left.

The Octagon.

The drawing of an octagon differs but little from that of the hexagon, we shall, therefore, merely show the *plan.* The following are methods of drawing the plan:—

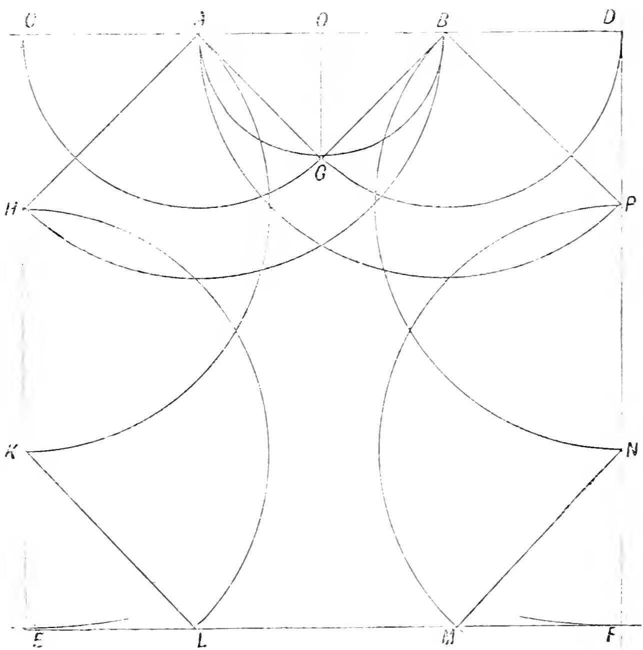

Fig. 31.

Let AB = given side (on B.L.); bisect AB in O, and draw OG ⊥ to OA and equal it, and describe semicircle AGB;

join AG and GB; with centres A, B, and distances equal to AG, describe arcs to cut AB produced in C and D: on CD describe square CDFE. With centre A and distance AB describe arc to cut CE in H; find P similarly; with centres H and P and distances equal to HA, describe arcs to cut CE and DF in K and N respectively, and with same distances describe arcs to cut EF in L and M; join AH, KL, MN and BP, which will complete the hexagon. (Fig. 31.)

Another way:

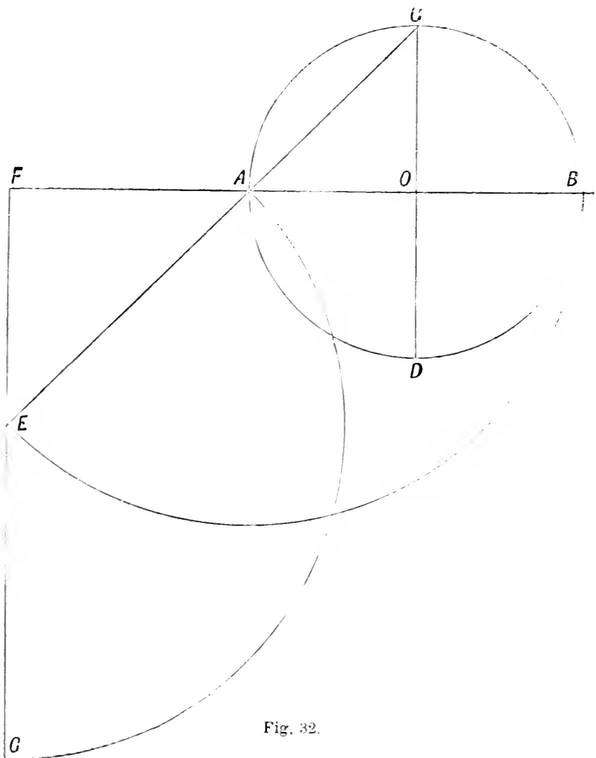

Fig. 32.

Let AB = given side; take centre O, and with distance AO describe circle ACD; draw diameter CD $\perp$ to AB. Join CA and produce it. With centre A and distance AB describe arc to cut CA produced in E; then draw EF $\perp$ to BA produced, and produce FE, making EG = EA, etc. (Fig. 32.)

NOTE.—In parallel perspective a hexagon or an octagon must be supposed to have a side parallel or perpendicular to, the picture plane.

The Circle.

Hitherto we have been dealing exclusively with straight lines, in so far as the appearance of figures is concerned; we now proceed to represent curved lines in perspective. It is evident that a curve cannot be correctly represented, without the aid of straight lines.

There is only one position in which a circle will appear *true* to the eye, and that is, when the eye is in a line exactly perpendicular to its plane, at its centre. In all other positions it will appear an ellipse, varying from a circle to a line. If, for instance, we place a hoop on the ground, and look at it directly, it will appear *true*, but if turned on an imaginary axis it will assume the form of an ellipse. The height or diameter of the hoop corresponding to the imaginary axis will remain the same, while the diameter at right angles to it, or the *revolving axis*, will diminish, till at length it is a mere point. Hence, to know the appearance of a circle *not viewed directly*, we must know the angle the eye makes with its plane, or its appearance in relation to some figure easy of representation, contained by straight lines.

Now, a square answers admirably for this purpose, for if we draw the diameters of a square, and then draw a circle so as to touch its sides at the extremities of the diameters, we can without much difficulty represent the circle, for we will have four *points* as guides. If, however, the diagonals also, of the square be drawn, the four points where they cut the circumference of the circle will furnish additional points, so that we will have altogether, eight points for guidance in drawing the circle. Thus,—

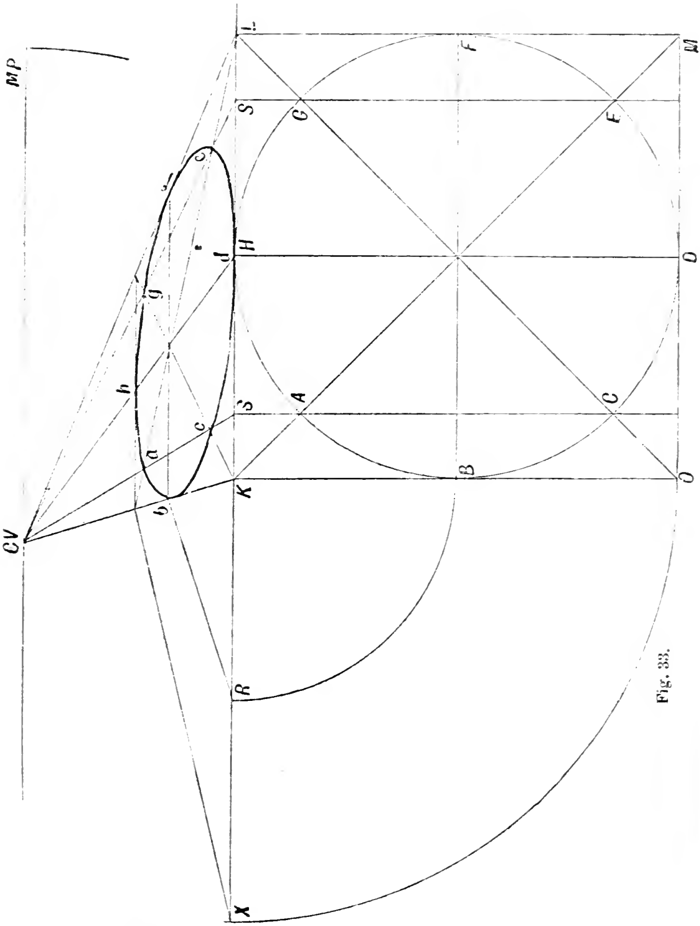

Fig. 33.

Let KLMO = given square, draw diameters and diagonals, and inscribe circle cutting the diagonals in A, C, G, E; join AC and GE, and produce them to meet KL in S, S; join

K C.V., S C.V., etc.; also make KR = KB and RX = BO or KB. Join X MP and R MP, and where they meet K C.V. draw parallels to KL: then *bf* will represent the diameter BF, and it will be easily seen that *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h* will correspond with A, B, C, D, E, F, G, H respectively, and the curve traced between them will represent the circle (in this case) lying on the ground plane. (Fig. 33.)

Example 1.—Draw a circle, diameter 4′, lying on G.P., centre 4′ to right and 2′ within P.P. H = 6′, L.D. = 4′, scale $\frac{1}{4}'' = 1'$.

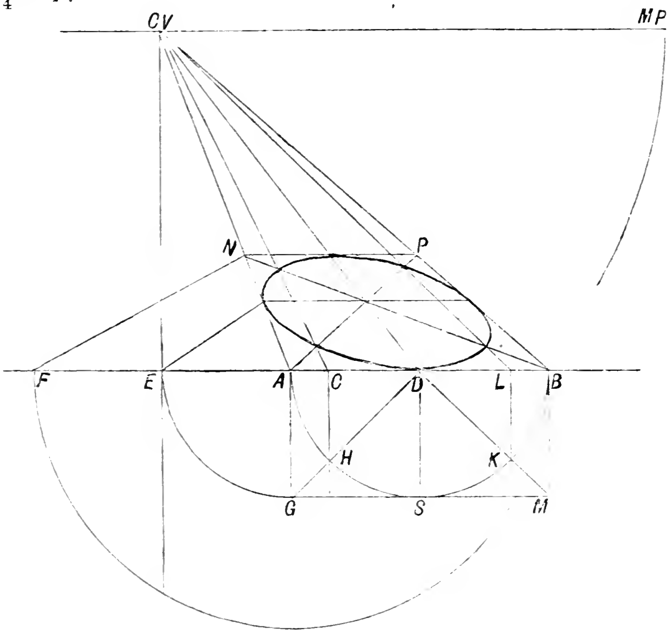

Fig. 34.

Here ED = 4′, EA = 2′, DB = 2′. Describe semicircle ASD, draw AG and BM ⊥ to AB, and draw GSM through S, parallel to AB; make AE = AG and AF = AB; join F MP and E MP, also A C.V., D C.V., B C.V. Join DG and DM, cutting curve in H and K. Draw HC and KL parallel to AG; join C C.V. and L C.V.; complete square NABP, draw diagonals; then on the eight points thus shown draw the curve required. (Fig. 34.)

Example 2.—Draw a circle touching P.P. 4′ to left, standing on G.P. and ⊥ to it and P.P.; circle to be 4′ in diameter. H = 6′, L.D. = 4′, scale $\frac{1}{4}'' = 1'$.

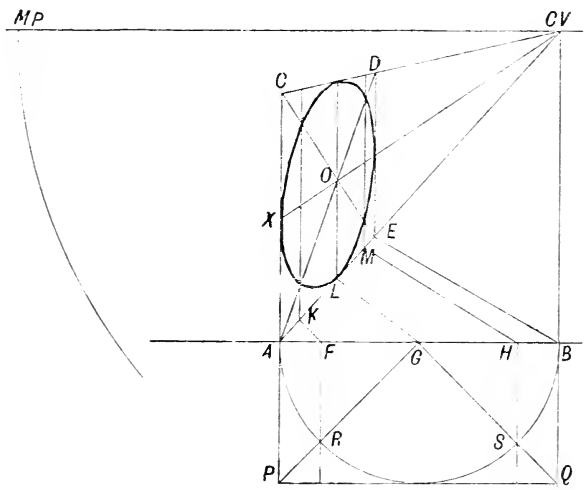

Fig. 35.

Here take A 4′ to left, bisect it in G; describe semicircle, and complete oblong APQB; join GP, GQ, cutting curve in R, S; draw RF, SH ⊥ to AB; join A C.V. Erect at A the perpendicular AC = AB, and bisect it in X; join C C.V., X C.V., also F MP, G MP, H MP, B MP. At points of section K, L, M, E draw parallels to CA; draw diagonals

CE, AD, and trace curve between the eight marked points. (Fig. 35.)

Example 3.—Draw a circle, diameter 4′, directly in front, lying on G.P., centre 4′ within P.P. H = 6′, L.D. = 4′, scale $\frac{1}{4}'' = 1'$.

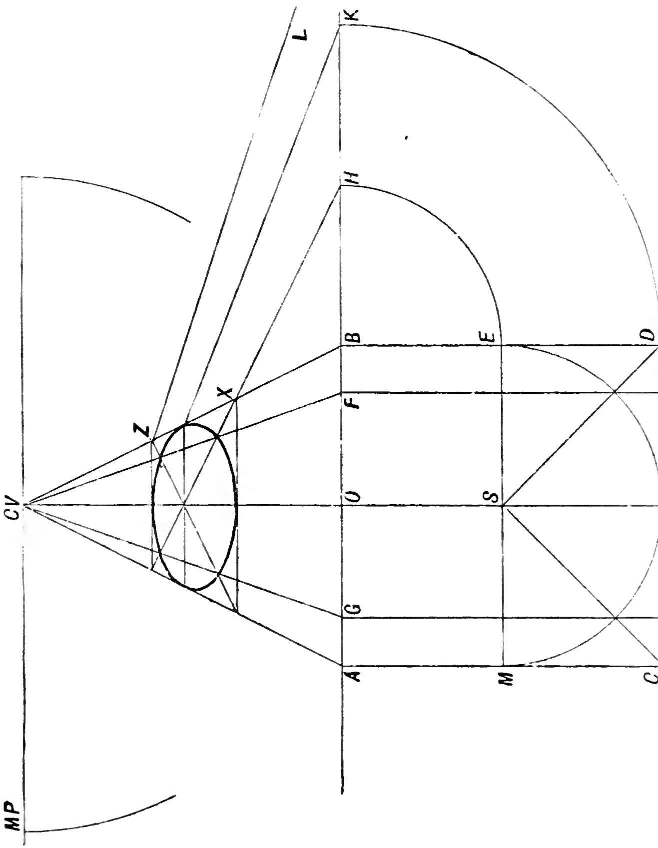

Fig. 36.

Here take OA, OB, each = 2′; upon AB describe square ACDB; bisect BD in E, and draw EM parallel to CD; describe semicircle and find points F, G, as already shown. Make BK = BD, BH = BE and KL = DE: join L MP, K MP, etc.; also A C.V., G C.V., etc.; and on eight points thus formed describe circle required. For BX = BH = BE = 2′, and XZ = HL = BD = 4′, etc.

Example 4.—Draw a circle, diameter = 4′, ⊥ to G.P., parallel with P.P. and 2′ within it; centre of circle 5′ to right. H = 6′, L.D. = 4′, scale $\frac{1}{4}'' = 1'$.

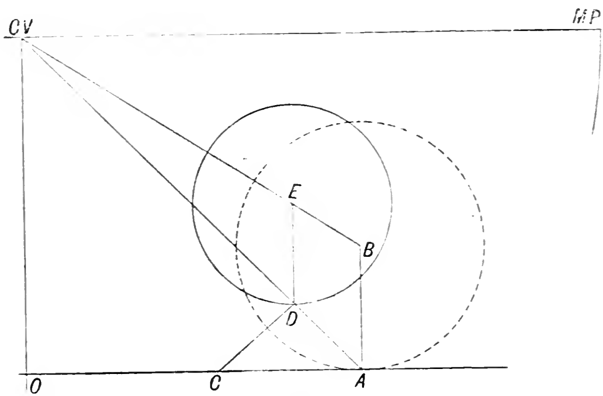

Fig. 37.

Here take A 5′ to right of O; erect the perpendicular AB. 2′ = radius of given circle; join B C.V. and A C.V.; take C 2′ to left of A; join C MP, cutting A C.V. in D. Draw DE parallel to AB; then with centre E and distance ED describe circle required. For AD = AC = BE; then E is 2′ within P.P. and 5′ to right, also DE = AB = 2′, etc. (Fig. 37.)

Exercise VIII.

(H = 6′, L.D. = 4′; scale $\frac{1}{4}''$ = 1′.)

1. Draw a circle, diameter 4′, resting on ground plane and touching P.P. at a point 4′ to right.

2. Draw a circle, diameter 4′, resting on G.P., centre 4′ to left and 4′ within P.P.

3. Draw a circle, diameter 4′ its plane perpendicular to G.P. and touching it at a point 4′ to left; the circle is perpendicular to P.P. and touches it.

4. Draw a circle, diameter 4′, lying on ground plane directly in front, centre 5′ within P.P.

5. Draw a circle, diameter 4′, parallel to G.P. and 2′ above it, placed with centre 4′ to right and 4′ within P.P.

6. Draw a circle, diameter 4′, coincident with P.P. and touching G.P., centre 4′ to right.

7. Draw a circle, diameter 4′, plane perpendicular to G.P. and P.P.; the centre of the circle is 5′ to left and 3′ within P.P.

8. Draw a circle, diameter 4′, parallel to G.P. and 9′ above it, directly in front, centre 4′ within P.P.

9. Draw a circle, diameter 4′, parallel to P.P. and 6′ within it; centre of circle 1′ to right and 3′ below G.P.

10. Draw a circle, diameter 6′, lying on G.P., centre 4′ to right and 4′ within P.P., and within it draw a concentric circle of 3′ diameter.

11. Draw a quadrant, radius 2′, lying on G.P., vertex directed away, and placed 4′ to left and 4′ within P.P.; the radii make an angle of 45° with P.P.

12. Draw a circle, diameter 4′, buried vertically in the ground to a depth of 1′; the circle is perpendicular to P.P., and its centre is placed at a point 5′ to left and 3′ within P.P.

SOLIDS.

It is expected that the pupil will have drawn all the figures mentioned in the exercises. Unless the problems have been thoroughly understood, comparatively little progress can be made in the perspective of solids.

Solids may be classified thus:

I. Those contained by plane surfaces.

II. Those partially or wholly contained by convex surfaces.

They are sometimes classified as solids with Developable, or with Undevelopable surfaces.

Those belonging to Class I. are Cubes, Plinths, Parallelopipeds, Prisms, Pyramids, Wedges, and Frusta.

Of those contained by convex surfaces in part, are Cones, Cylinders, Hemispheres, and frusta of Cones.

Those contained wholly by convex surfaces are Spheres, Spheroids, Ellipsoids, Cylindroids, Spindles, etc.

All the latter have *undevelopable* surfaces, *i.e.*, they cannot be straightened out to a plane surface.

Solids contained by plane surfaces may be subdivided into:

1. Those rectangular throughout.
2. " partly rectangular.
3. " wholly oblique.

The latter class of solids cannot be readily drawn in perspective, and will not be treated of, here.

Of (1) are Cubes and Plinths or Parallelopipeds.

A cube is a solid contained by six equal squares, and all its angles are right angles.

A plinth is a solid contained by three pairs of equal and similar oblongs. Each pair of surfaces may be equal or unequal to one or both of the other pairs, but the angles are right angles.

(2) A prism is a solid contained by two regular polygons whose planes are parallel to each other, and whose like sides are joined by rectangular planes.

A pyramid is a solid formed by joining the angles of a triangle, square, etc., with some external point. If the external point be vertically above the centre of the pyramid, the pyramid is said to be *right;* if in any other position, *oblique.*

A frustum of a pyramid is the part remaining after a smaller pyramid is cut off by a plane parallel to the base.

Of solids with convex surfaces:—

A sphere is formed by the revolution of a semicircle around the diameter, which remains fixed.

A cone is formed by the revolution of a right-angled triangle around one of the *containing* sides, which remains fixed.

A cylinder is formed by the revolution of an oblong around one side, which remains fixed.

A spheroid is formed by the revolution of a semi-ellipse around one of the axes, which remains fixed.

If the fixed axis be *major*, the spheroid is prolate; if *minor*, oblate.

The Cube.

The drawing of the cube is so simple that a single example will suffice for its explanation.

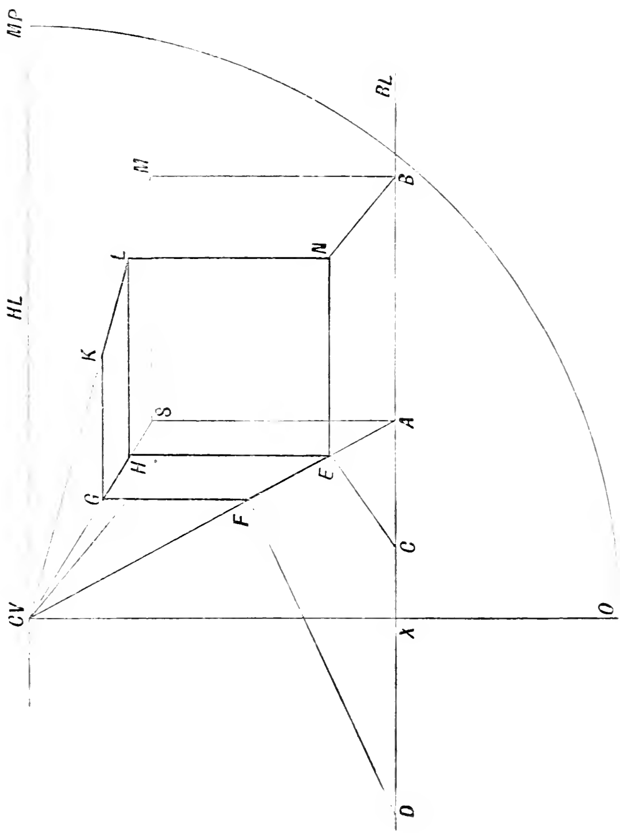

Fig. 38.

Example 1.—Draw a cube, edge 4′, placed on G.P., one side parallel to P.P. and 2′ within it, near angle 3′ to right.

H = 6′, L.D. = 4′, scale $\frac{1}{4}''$ = 1′.

Draw B.L., H.L., and O, C.V. and find MP as before. Take A 3′ to right, and B 4′ to right of A; take C 2′ to left of A, and D 4′ to left of C; join A C.V., D MP and C MP, cutting A C.V. in F and E. On AB describe square ASMB; join S C.V., M C.V. and B C.V.; through E and F draw parallels to AS, meeting S C.V. in H and G; through G, H draw parallels to SM, meeting M C.V. in K and L. Draw LN from L $\perp$ to MB, and EN from E parallel to AB. This will complete the required cube. For XA = 3′, and AE = AC = 2′, AF = AD and EF = CD = AB = 4′; also FG = EH = AS = 4′, and GK = HL = EN = AB = 4′: then EF = GH = KL = 4′, etc. (Fig. 38.)

The Plinth.

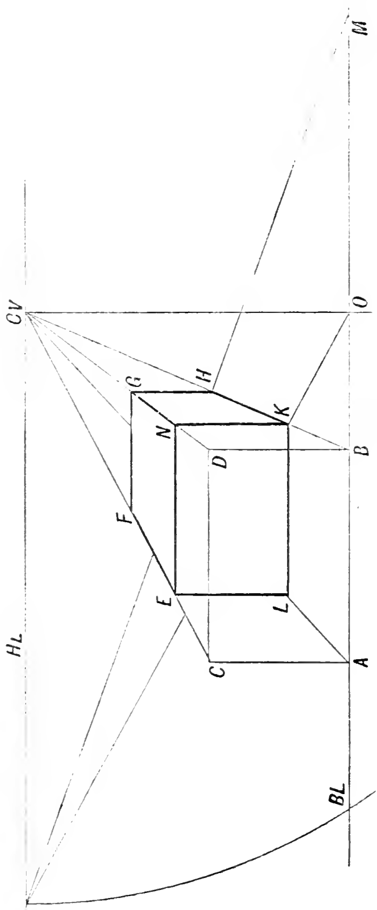

Fig. 39.

The plinth differs from a cube only in the relation of its dimensions; the principle employed in drawing them is the same, but a particular side of the plinth is mentioned in reference to the P.P. or G.P. In the cube this is quite unnecessary, as all the sides are equal.

Example 1.—Draw a plinth whose dimensions are $4' \times 3' \times 2'$ (4′ long, 3′ wide and 2′ thick), the side $4' \times 3'$ rests on the ground plane, and side $3' \times 2'$ is parallel to P.P. and 2′ from it; the plinth is 2′ to the left.

H = 6′, L.D. = 4′, scale $\frac{1}{4}'' = 1'$.

Draw H.L., B.L. and find C.V., O and MP as before. Take B, 2′ to left, and A, 3′ to left of B; also M, 4′ to right of O. On AB construct the oblong ABDC, 3′ × 2′; join C C.V., D C.V., A C.V., B C.V.; also O MP and M MP. From points K, H, where B C.V. intersects O MP and M MP, erect KN and HG parallel to BD; and through N, G draw NE and GF parallel to CD. Draw EL ⊥ to EN and KL ⊥ to EL, which will complete the plinth. For BK = BO and KH = OM; then EF = NG = KH = OM = 4′, and FG = EN = LK = AB = 3′. GH = NK = DB = 2′, etc.

Example 2.—Draw a flight of four steps, each step 4′ × 1′ × 1′. The ends of the steps are coincident with the P.P., and 4′ to right. H = 6′, L.D. = 4′, scale $\frac{1}{4}''$ = 1′.

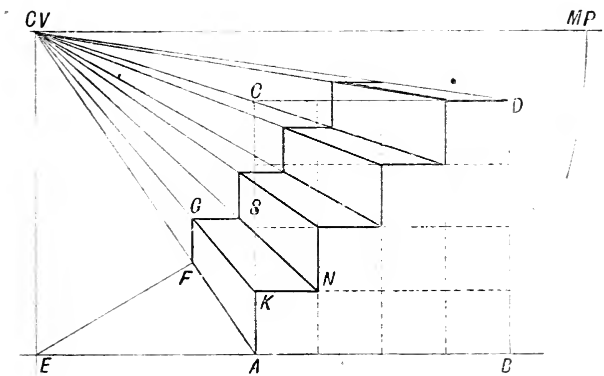

Fig. 40.

Here take A, 4′ to right of E, and B 4′ to right of A: on AB describe square ABDC, and divide it into sixteen equal squares; join each angle, as shown in figure, with the C.V.: join also E MP, cutting A C.V. in F; draw FG parallel to AC, GS ⊥ to GF, etc. Then FG = AK = 1′, and GS = KN = 1′, etc.; and AF = AE = AB = 4′. (Fig. 40.)

Example 3.—Draw same, with ends perpendicular to P.P., one step being coincident with it and 4′ to left.

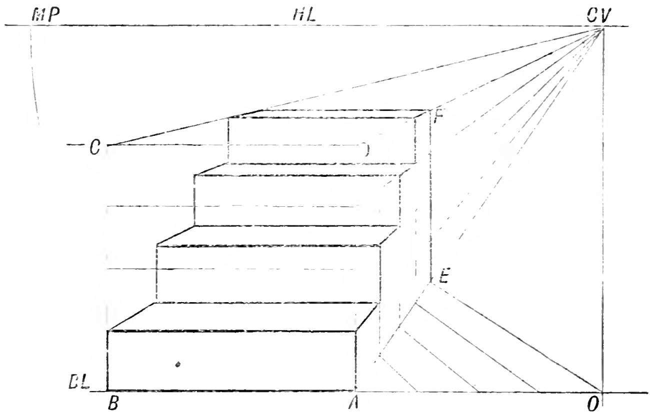

Fig. 41.

Draw H.L., B.L., and find C.V. and MP as before. Take A, 4′ to left, and B, 4′ to left of A: on AB construct square ABCD; join O MP; and on AE. complete square ADFE; then draw steps similar to preceding example.

Exercise IX.

(H = 6′, L.D. = 4′, scale $\frac{1}{4}'' = 1'$.)

1. Draw a cube, edge 4′, 2′ within P.P. parallel to it, and 4′ to right.
2. Draw a cube, edge 3′, directly in front, at a distance of 3′ from P.P., resting on G.P.
3. Draw a cube, edge 5′, parallel to G.P. and 2′ above it; cube 3′ to right, parallel to P.P. and touching it.
4. Place two cubes, each 4′ edge, on a line parallel to L D., cubes to be 4′ apart, and nearest 4′ from P.P. and 4′ to left.
5. Draw a cube, edge 4′, touching P.P. and 4′ to right; and place a cube, 2′ edge, centrally upon it.

6. Draw a plinth 6′ × 4′ × 2′, side 4′ × 2′ on ground, side 6′ × 2′ parallel with P.P. and 4′ to left; figure to be 2′ within P.P.

7. Draw a slab 4′ × 2′ × 2′, lying on ground directly in front, side 4′ × 2′ parallel to P.P. and 4′ from it.

8. Draw a slab 5′ × 5′ × 1′ lying flat on G.P., side 5′ × 1′ parallel to P.P. and 3′ from it; slab to be 4′ to left. Place centrally on this slab a cube whose edge is 3′.

9. Draw a cube, 2′ edge, on each side of L.D., 2′ from it, and touching P.P.; on these cubes place a slab 6′ × 2′ × 1′ coincident with the cubes.

10. A wall 8′ high and 2′ thick starts from a point on the P.P. 4′ to the left, and runs straight forward to the horizon: at distances of 6′ and 12′ doors 5′ × 3′ are placed.

11. Draw a cross whose beams are 7′ × 1′ × 1′ and 5′ × 1′ × 1′ respectively; the cross-beam is placed at a height of 3′. The cross stands erect, its cross-beam parallel to P.P. and 4′ within it; the foot of the cross is 4′ to right.

12. Draw same, with end of cross-beam coincident with P.P., 4′ to left.

13. Draw same, lying on G.P., cross-beam $\perp$ to P.P. and its end coincident with it, 3′ to right.

14. Draw same, lying on ground directly in front, cross-beam directed away, end of main beam coincident with P.P.

15. A circular table 4′ in circumference is supported by four legs 2′ high, which proceed from the edge of the table; the legs form a square whose side is parallel to P.P. and 3′ within it. The centre of the table is 4′ to right. Thickness of neither table nor legs, taken into account.

16. Draw a set of four steps, each 4′ × 1′ × 1′, ends parallel to P.P. and 2′ within it; to be 5′ to left, facing toward right.

17. Draw same, ends perpendicular to P.P. and 2′ within it, and 3′ to right.

18. Draw same, with steps descending as they recede; back coincident with P.P. and 2′ to left.

19. Draw same, directly in front, steps ascending as they recede, and 2′ within P.P.

The Prism.

Prisms are square, triangular, hexagonal, etc., according to their ends or bases.

The square prism may be considered as a mere modification of the plinth.

To draw a prism, we have only to draw the two surfaces forming its ends, and join similar angles.

Example 1.—Draw a triangular (equilateral) prism, length 6′, side of base 2′, lying on G.P., one end perpendicular to P.P. and 3′ to left; prism to touch P.P. H = 6′, L.D. = 4′, scale $\frac{1}{4}''=1'$.

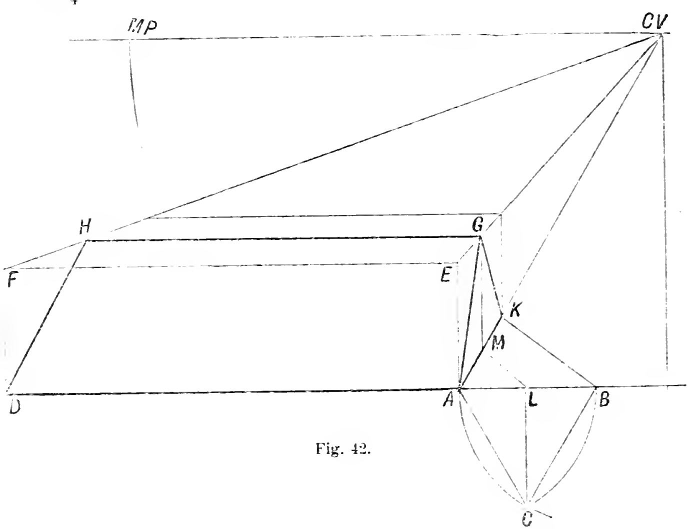

Fig. 42.

Take B, 1′ to left, A, 3′ to left, and D, 6′ to left of A. On AB describe equilateral triangle ABC; draw CL ⊥ to AB and bisecting it. Draw AE ⊥ to AB and equal to CL; similarly draw DF; join F C.V., E C.V. and A C.V., also B MP and L MP; through M draw MG parallel to AE, meeting E C.V. in G; draw GH parallel to EF; join HD, AG and GK, completing the prism. Then AK = AB = AC = BC = 2′. Hence AG = KG = AK, and GM = EA = LC = required height, and HG = FE = DA = 6′, etc. (Fig. 42.)

Example 2.—Draw a hexagonal prism (edge of base 2′) whose length shall be 4′, one side touching P.P. 4′ to right; prism to stand on end. H = 6′; L.D.′ = 4′; scale, $\frac{1}{4}'' = 1$.

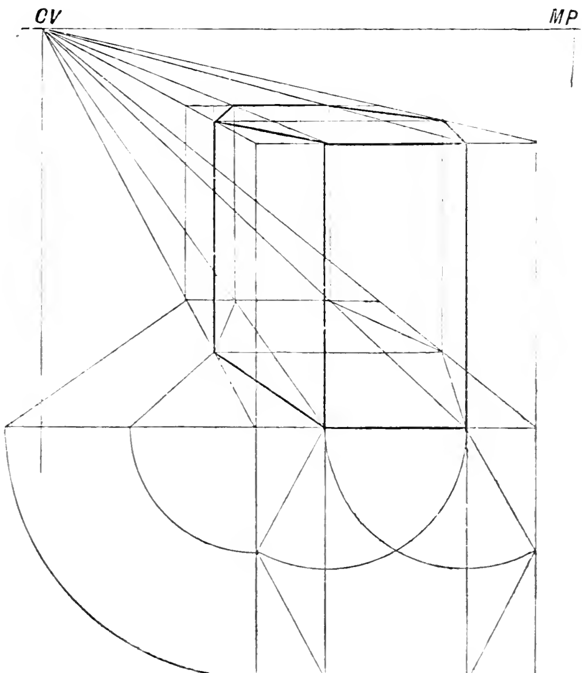

Fig. 43.

Draw the plan in proper situation, as already explained; then draw the hexagons, one on G.P., the other 4′ above it; then join similar angles in each, forming the required hexagon. (Fig. 43.)

The Cylinder.

The drawing of the cylinder differs from that of the prism, only in the plan. Draw the circles, forming the ends, in the proper positions, and then draw tangents to them, forming the cylinder.

Example 1.—Draw a cylinder, length 4′ and diameter 4′, lying on ground plane parallel to P.P. and 2′ within it; the end of cylinder 4′ to left.

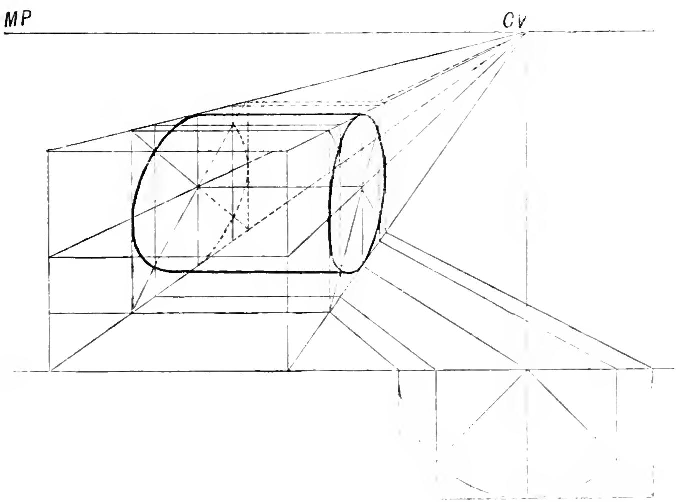

Fig. 44.

Example 2.—Draw a cylinder lying on G.P., 4′ to right, having end perpendicular to P.P., and touching it; cylinder 8′ long and 3′ in diameter.

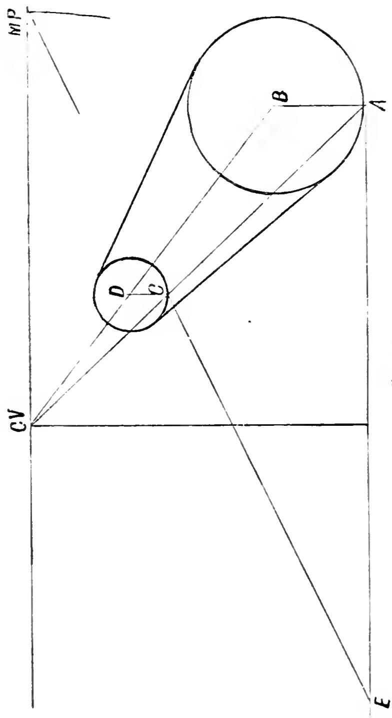

Fig. 45.

Take A, 4′ to right and E, 4′ to left; make AB = 1½′, and describe circle; join B C.V. and A C.V., also E MP, and from C, draw CD parallel to AB, and describe smaller circle; then draw common tangents, completing the cylinder (Fig. 45.)

Example 3.—The figure shows how to draw a common pail, showing staves and hoops.

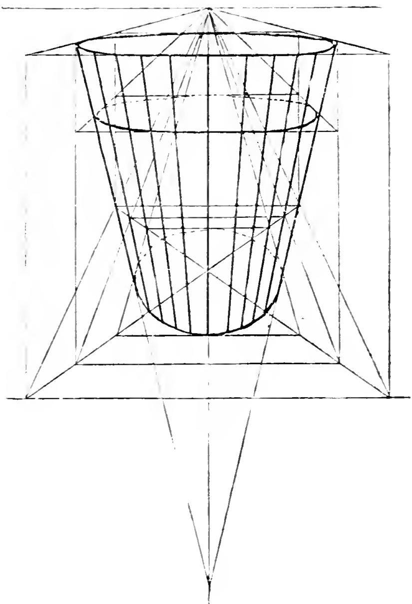

Fig.

The Pyramid.

We now come to consider solids, which are not wholly rectangular; they are cones and pyramids and their frusta.

In speaking of the height of a pyramid or cone, we mean the distance from the vertex perpendicularly to the base. This is important, especially in frusta, where the *slant* height might be mistaken for the real height of the solid.

Example 1.—Draw a pyramid, 8′ high with square base, each side of which is 4′, and touches P.P. 4′ to right. H = 6′, L.D. = 4′, scale $\frac{1}{4}''$ = ′.

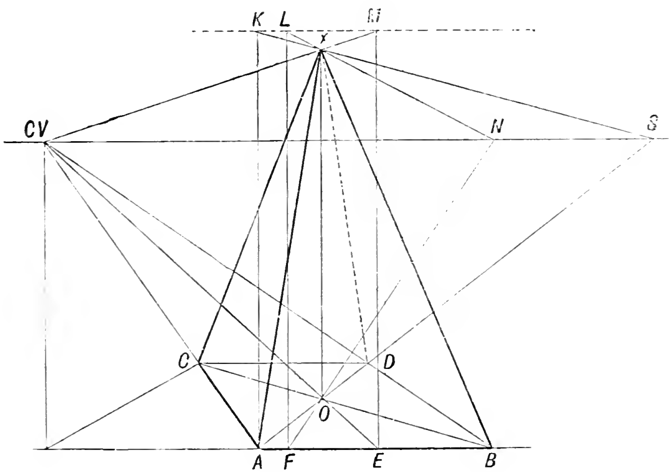

Fig. 47.

Take A, 4′ to right and B, 4′ to right of A; complete the square ABDC; draw diagonals intersecting in O; join O with

C.V. and produce it backward to meet base line in E; at E erect perpendicular, 8′ in height to M; join M C.V.; through O, draw OX, parallel to EM; join XA, XB, XC, completing the pyramid. Now, OX = EM = 8′, and this represents the vertical height. (Fig. 47.)

It is not absolutely necessary to join O with C.V. We may draw it to any point on the H.L., as N or S., and produce it backward to F or A, and erect a perpendicular from either of these points; but it must be carefully remembered, that the so-found point K or L, must be joined to S or N respectively. Such lines, KS, LN, M C.V., etc., will all pass through same point X, which may be considered as a locus for all such lines. For convenience, however, the line O C.V. should be used, unless the solid be directly in front.

The Cone.

The drawing of the cone does not differ materially from that of the pyramid. The circle forming the base being drawn, and the position of the vertex found, it is only neces-

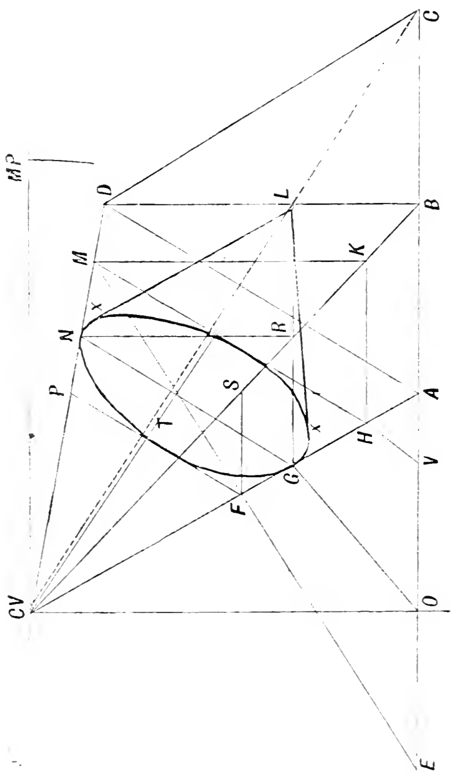

Fig. 48.

sary to draw the tangents from it to the circle. We give a particular example:—

Draw a cone whose base = 6′ in diameter and slant height 6′. The cone lies on its side; plane of base, $\perp$ to P.P., and the line joining the centre of base with the vertex is parallel to the P.P. and 4′ from it. The cone is 4′ to the right. H = 6′, L.D. = 4′, scale $\frac{1}{4}''$ = 1′.

Take A, 4′ to right and C, 6′ to right of A; on AC describe equilateral triangle ADC; draw DB perpendicular to AC; join A C.V., B C.V., C C.V.; take V and E, 3′ from O; join E MP, O MP and V MP; through F, G, H draw parallels to AB; through S, R, K draw parallels to DB; join PF, NG, MH; then in square PH describe circle; produce GR to L, and from L draw tangents LX, LX to circle, completing the cone. (Fig. 48.)

Exercise X.—*On the Prism and the Cylinder*

(H = 6′, L.D. = 4′, scale $\frac{1}{4}''$ = 1′.)

1. Draw a prism 6′ in length, triangular base, each side of which is 2′. The prism stands on end 4′ to right, with one side coincident with P.P.
2. Draw same, 3′ to left, one side perpendicular to P.P.
3. Draw same, directly in front, one side parallel to P.P. and 2′ from it, vertex away.
4. Draw same, lying on ground plane, perpendicular to P.P., 3′ to right and 3′ within P.P.
5. Draw a hexagonal prism 6′ high, each side of base 2′, standing on end directly in front, one side touching P.P.
6. Draw same, lying on ground parallel to P.P. and 2′ within it; one end projecting 2′ to right, and opposite end 4′ to left.
7. Draw same, lying on ground perpendicular to P.P., 3′ to right and 3′ within P.P.
8. Draw a cylinder, diameter of base 4′, height 6′, lying on ground, parallel to P.P. and 4′ within it; left end just in line with L.D.

9. Draw a cylindrical vessel 4′ feet in height and 4′ in diameter, standing on end, touching P.P. 4′ to left; show 4 hoops at distances of 1′ from each other.

10. Draw a hollow cylinder, 4′ in length, outer diameter 4′, inner diameter 3′, lying on ground perpendicular to P.P., 2′ to right and touching P.P.

Exercise XI.—*On the Pyramid and Cone.*

(H = 6′. L.D. = 4′, scale $\frac{1}{4}'' = 1'$.)

1. Draw a square pyramid 5′ high, each side of base 3′, standing on ground, directly in front, touching P.P.

2. Draw same, 4′ to right, parallel to P.P. and 4′ within it.

3. Draw same, standing on a 3′ cube, parallel to P.P. and 2′ within it, 4′ to left.

4. Draw same, 3′ above ground plane and parallel with it, 4′ to right and touching P.P.

5. Draw same, with vertex downwards, base parallel with ground and P.P., vertex, 4′ to left and 3′ within P.P.

6. Draw a cone, height 5′, diameter of base 4′, standing on ground 4′ to right and 4′ within P.P.

7. Draw same, touching P.P., 3′ to left.

8. Draw same, directly in front, 3′ from P.P.

9. Draw same, standing on a cylinder 4′ in diameter and 4′ high, 3′ to right and touching P.P.

10. Draw same, placed centrally on a cylinder of 5′ in diameter, directly in front and touching P.P.

11. Draw a cone, base 4′, slant height 4′, lying on side; base perpendicular to P.P. and touching it, 4′ to left; vertex directed toward left.

Frusta.

The dimensions of a frustum may be given by stating dimensions of each end, and vertical height.

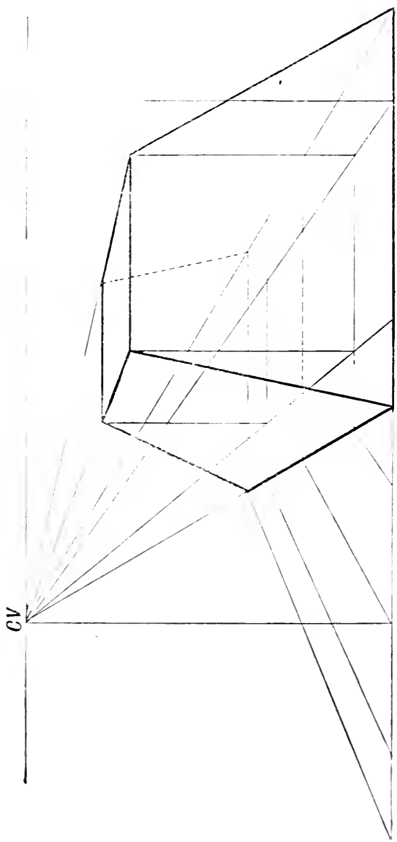

Fig. 49.

Example 1.—Draw the frustum of a square pyramid, whose bases are 6′ and 4′ square, respectively; the frustum touches P.P. 4′ to right, height 4′. (Fig. 49.)

From the above the method of drawing may be easily understood.

Example 2.—A pyramid with square base, each side of which is 4′, stands on the ground plane 4′ to the left, touching P.P. The pyramid is 8′ in height; 3′ from the vertex the pyramid passes through a square plinth 4′ × 4′ × 1′ placed parallel to the ground plane. The pyramid cuts the plinth centrally. H = 6′, L.D. = 4′, scale $\frac{1}{4}''$ = 1′. (Fig. 50.)

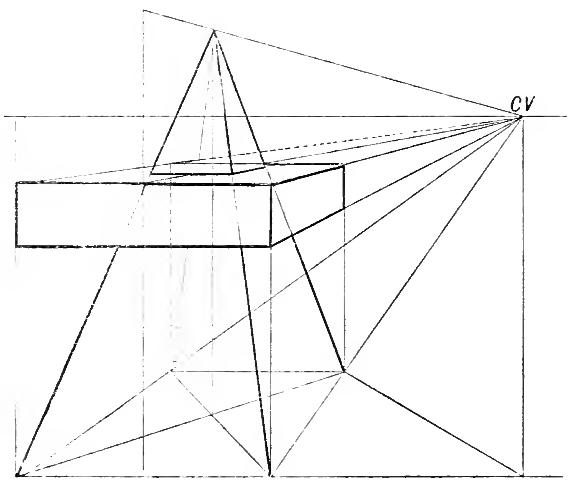

Fig. 50.

By a careful observation of the lines drawn above, the method may be easily seen.

Exercise XII.

(H. = 6′, L.D. = 4′, scale $\frac{1}{4}''$ = 1′.)

1. A pyramidal frustum with square base, and vertical height 4′, touches P.P. 4′ to right, resting on ground plane: the sides of the square are 3′ and 5′ respectively.

2. Draw a pyramidal frustum same as No. 1, 3′ to left, and 3′ within P.P.

3. Draw a triangular frustum (equilateral), sides of base 5′ and 3′ respectively, height 4′; on G.P. 4′ to right and 4′ within P.P., vertex away, one edge parallel to P.P.

4. Draw a square pyramidal frustum, height 4′, sides of square 2′ and 4′ respectively, standing on G.P. reversed, directly in front, 1′ within P.P.

5. Draw a conical frustum, height 5′, diameters 5′ and 3′ respectively, on G.P., and touching P.P. 4′ to right.

6. Draw No. 5, 6′ to left and 3′ within P.P.

7. Draw same, directly in front, 3′ within P.P. and 3′ above G.P.

8. Draw a hexagonal frustum, height 5′, sides of bases 3′ and 2′ respectively, touching P.P., resting on G.P. 4′ to right.

9. Draw same, 4′ to left and 4′ within P.P.

10. Draw an octagonal frustum, height 5′, edges of bases 2′ and 1′ respectively, resting on G.P., and touching P.P. 4′ to left.

Fig. 51 shows a method of laying out a plan for a frustum of a cone.

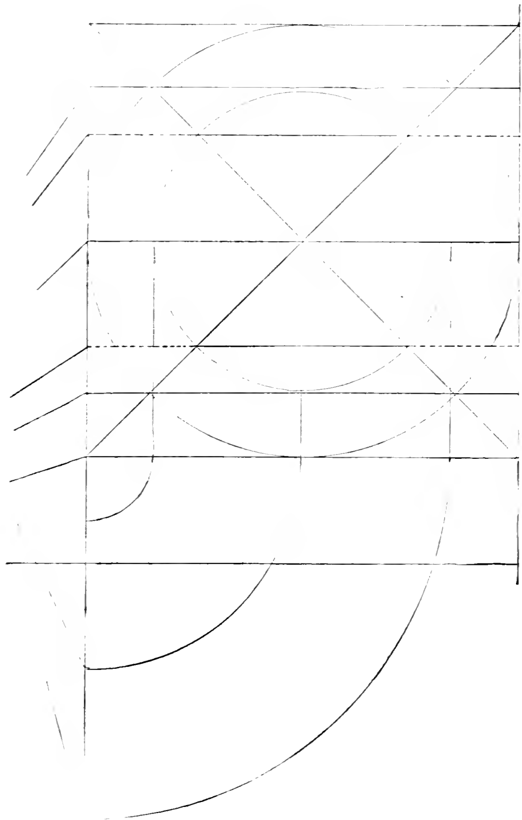

Fig. 51.

Exercise XIII.

(H = 6′, L.D. = 4′, scale $\frac{1}{4}''$ = 1′.)

1. 4′ to right and 2′ within P.P. draw a frustum of a square pyramid, edges of squares 4′ and 2′ respectively, height 4′.

2. Draw same, touching P.P. 2′ to left.

3. Draw same, directly in front, parallel to G.P. and 3′ above it.

4. Draw a conical frustum, height 5′, diameters 3′ and 2′ respectively; frustum rests on ground, with centre of base 3′ to right and 3′ within P.P.

5. Draw a frustum of a triangular pyramid, edges of ends 3′ and 2′ respectively, height 4′; it rests on G.P. with one edge coincident with P.P., and 3′ to left.

6. Draw a frustum of a hexagonal prism, edges of bases 3′ and 2′ respectively, height 4′; one edge of frustum is perpendicular to P.P., and an angle touches it at a point, 3′ to the right.

7. A cone, whose height is 8′ and diameter of base 4′, touches the P.P. 4′ to left; it is encircled by a rectangular collar whose dimensions are 4′ × 4′ × 1′, placed centrally over it, 4′ above the ground. The cone rests on the ground.

A frustum of a square pyramid, whose bases are 5′ and 3′ respectively, and whose height is 4′, supports a cone placed centrally upon it; the diameter of the cone is 3′ and its height 3′. The edge of the base touches the P.P. 2′ to right.

The Sphere.

The perspective of the sphere must necessarily be represented by a true circle, and little difficulty will be experienced in drawing a complete sphere. However, when a hemisphere

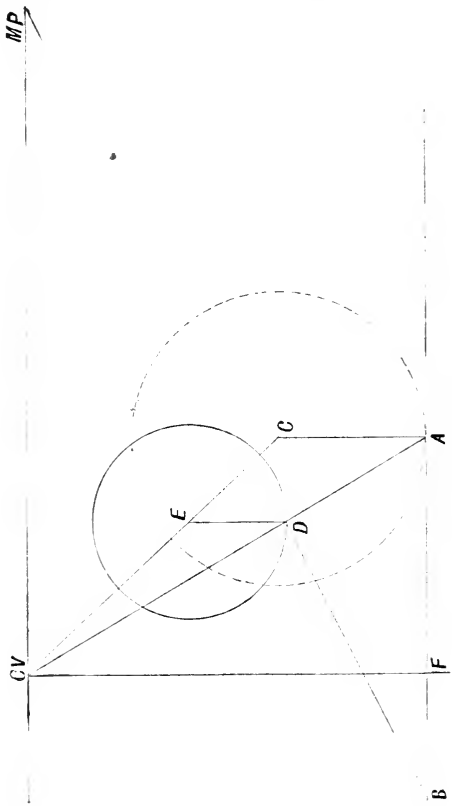

Fig. 52.

is to be represented, an apparent fallacy appears, owing to the representation of the circle that shows the section of the sphere. A sphere must always be supposed to be drawn with the radius of the circle as distance. However, as the perspective of the circle, viewed in any oblique position, shows diameters of varying length, care must be used in drawing the curve of the hemisphere at the *greatest* apparent diameter, and *this* diameter cannot be *definitely* determined in perspective, if drawn in any but a direct view. The sphere rests on the ground at a point directly beneath the centre, and it touches P.P. at a point perpendicular to the vertical, from the centre.

Example 1.—Draw a sphere, radius 2′, resting on ground at a point 3′ to right and 2′ within P.P. H = 6′, L.D. = 4′, scale $\frac{1}{4}''$ = 1′.

Here FA = 3′, BA = AD = 4′ and AC = ED = 2′. (Fig. 52.)

Example 2.—On centre of the top of a cube of 4′ edge, placed 4′ to left and touching P.P., place a sphere of radius $1\frac{1}{2}'$.

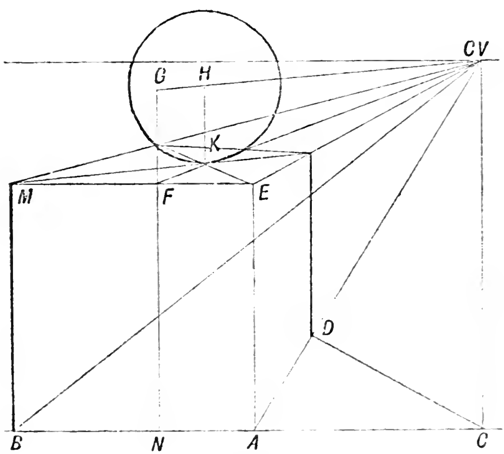

Fig. 53.

Here $CA = AB = AD = AE = 4'$, $EF = FM = 2'$, $GF = 1\frac{1}{2}'$ hence K is centre at intersection of diagonals and $HK = GF$. (Fig. 53.)

NOTE.—The sphere will not touch the P.P. unless $HK = KF$.

Example 3.—Draw a cube, edge 4′, touching P.P. 4′ to right, and in this place a sphere whose radius = 2′. Here the sphere will touch the centre of each side.

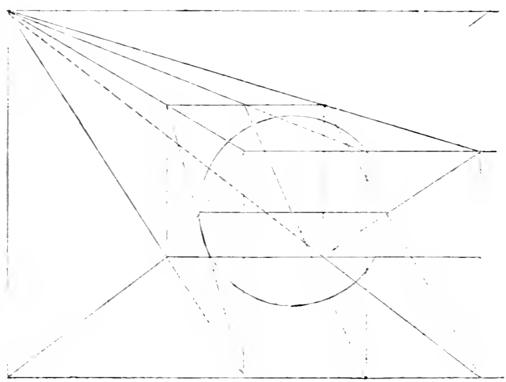

Fig. 54.

Draw cube, and diagonals of those sides, perpendicular to P.P., join intersections, and bisect this horizontal line as shown. The circle drawn on this line as diameter will represent the sphere, and touch the centre of each side. (Fig. 54.)

EXERCISE XIV.

(H = 6′, L.D. = 4′, scale $\frac{1}{4}'' = 1'$.)

1. Draw a sphere, radius 2′, placed centrally on a cylinder (on end), touching P.P. 4′ to left, cylinder 3′ high and diameter 4′.

2. Draw a sphere, diameter 3′, 8′ high, 6′ to right and 6′ within P.P.

3. Draw a sphere, diameter 4′, directly in front, touching P.P.

4. Draw a sphere, radius 2′, buried completely beneath the ground, centre of sphere 6′ to right and 6′ within P.P.

5. Place a sphere in a cubical box of 4′ edge, diameter of sphere 4′; cube to be 4′ to left, 4′ within P.P., parallel to, and 2′ above G.P.

6. A cylinder whose height is 4′ and diameter 3′ stands on end, touching P.P. 3′ to right; this cylinder passes centrally through a sphere whose diameter is 4′ and whose centre coincides with that of the cylinder.

Foreshortening.

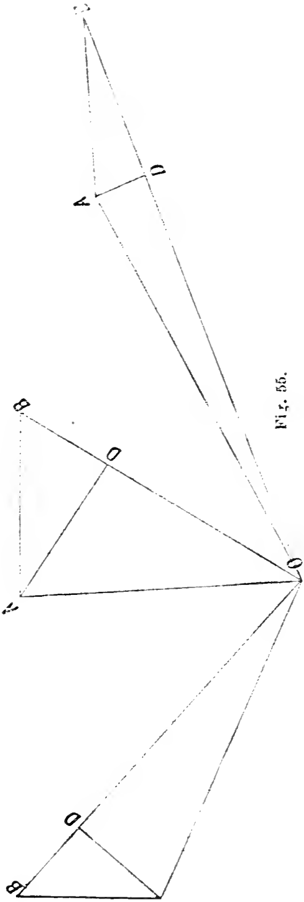

Fig. 55.

Foreshortening consists in representing the apparent length of a visible object. It depends on the distance of the object, and its position with regard to the eye. Thus, a lead-pencil may be so turned as to show only the end, or it may be placed so as to show its whole, or greatest length. Again, if AB represent a line of definite length, and O, the observer, the apparent length of AB as seen from O will be AD. (Fig. 55.)

This representation of a line AB by AD, which is always *less* than AB, is called "foreshortening."

NOTE.—AD is always *perpendicular* to the longest side OB.

Taking an object "out of" perspective means, that when an object is drawn, and the position of the observer's eye given, the size and position of the object may be determined.

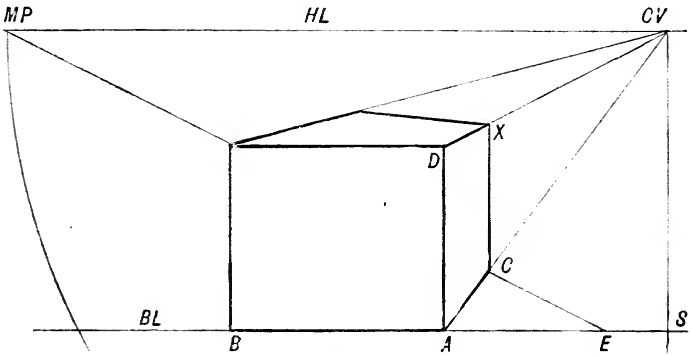

Fig. 56.

Here, the figure BX only, would be given, and it would be assumed to touch the picture plane. We first produce the vanishing lines DX and AC to meet in C.V., then draw C.V. S perpendicular to B.L., and make it equal to height of spectator; then draw H.L. parallel to B L. through C.V.; next take a point MP at a distance to the left equal to the height of spectator and his distance away, combined; then join MP with C, and produce it to base at E. Then, scale being given, find BA, AD and AC, the *dimensions* of the solid, and AS will show its distance to the left. (Fig. 56.)

Perspective Effect.

This consists in showing merely the appearance of an object when placed in a certain position. The dimensions and distance of the object are not taken into account.

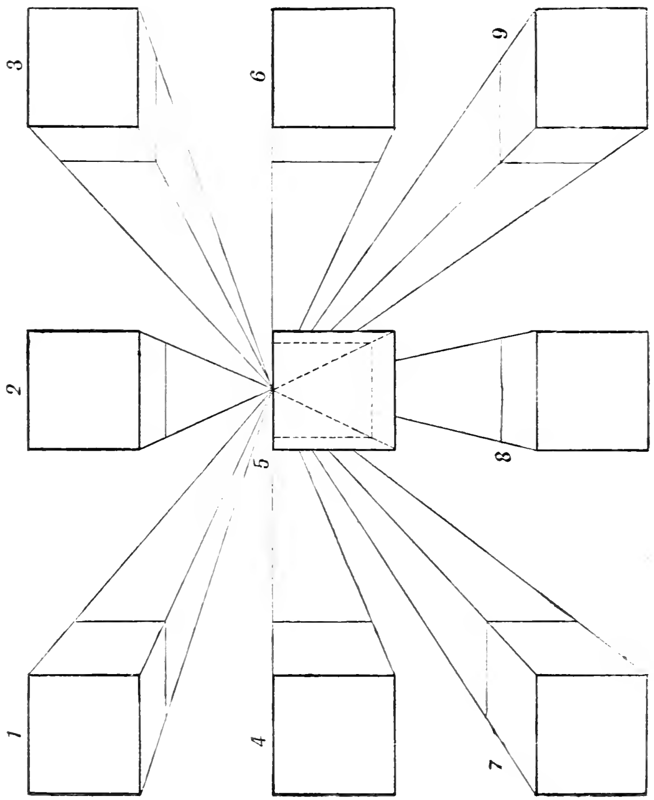

Fig. 57.

It will be remembered that if an object of less height than the observer, be placed on the ground, the observer will be able to see the upper side of it, and if placed above him he will see the under side; if placed on his right, he will see the front and left sides; if placed directly in front, he will see front side and upper or lower sides, according to the height of the object.

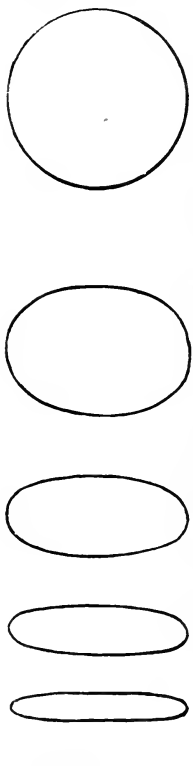

Fig. 58.—Perspective effect of a circle revolving on a vertical axis.

Take a cube, for instance, placed on the ground parallel with the P.P.

Now, if the cube be lower than the observer's eye, the upper face will be visible; if raised above, the lower face, and so on.

Figure 57 will illustrate perspective effect.

(1) Shows object above and left of eye.
(2) Above and directly in front of eye.
(3) Above and to right of eye.
(4) Level with and to left of eye.
(5) Level with and directly in front.
(6) Level with and to right.
(7) Below and to left.
(8) Below and directly in front.
(9) Below and to right.

If an object, as for instance a cube, is to be drawn, say to right and above the eye, draw first a square, then take a point to *left* and below, draw the vanishing lines to this point, and mark off lines for thickness, etc.

ANGULAR PERSPECTIVE.

We now come to consider the rules pertaining to angular perspective, or the perspective of *two* vanishing points. If a rectangular object, as a cube, rests on the ground parallel to P.P., it is evident that its sides, if produced, will appear to vanish directly in front, at the point called the centre of vision. If we move the cube by even a small amount from the parallel position, its sides will no longer vanish at the centre of vision, but at a point to the right or left of it, and at a distance from it, depending on the *angle* which the sides make with the P.P. Now, in parallel perspective we deal with only *one* vanishing point—the centre of vision; but there are really *two:* for all lines *parallel* to the P.P., if produced to an infinitely great distance, will appear to meet at a point to right or left. Hence in parallel perspective only one vanishing point is of practical utility. However, when the cube is moved out of its parallel position, this apparently-hidden vanishing point appears, and strikes the horizon at a distance from the centre of vision, depending on its angle, as already explained. Thus:—

In Fig. 59, AB on left side shows the base of a cube in parallel perspective, while in Fig. 60 AB has been moved around to position of AD, and AC will not now vanish to C.V., but to a point Y to right of it; so also AD will not vanish at a point parallel to AB, but at X, a point in the

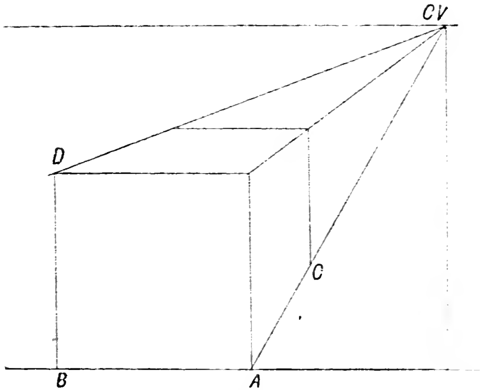

Fig. 59.

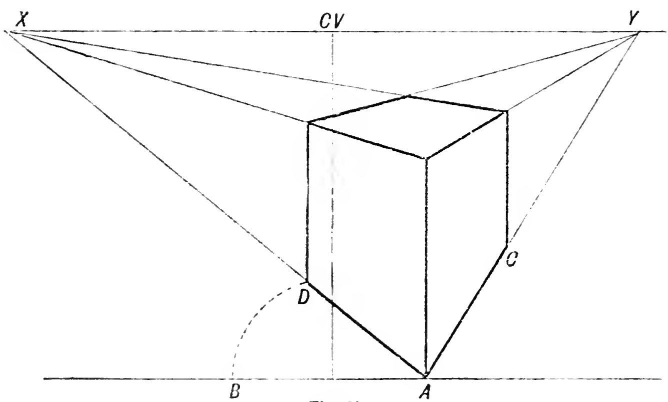

Fig. 60.

horizontal line to left of C.V. We will now proceed to ascertain the positions of these points.

Example 1.—Draw a square (side 4′) lying on ground; sides make an angle of 45° with picture plane, and the angle touches the P.P. at a point 4′ to the right; scale $\frac{1}{4}'' = 1'$.

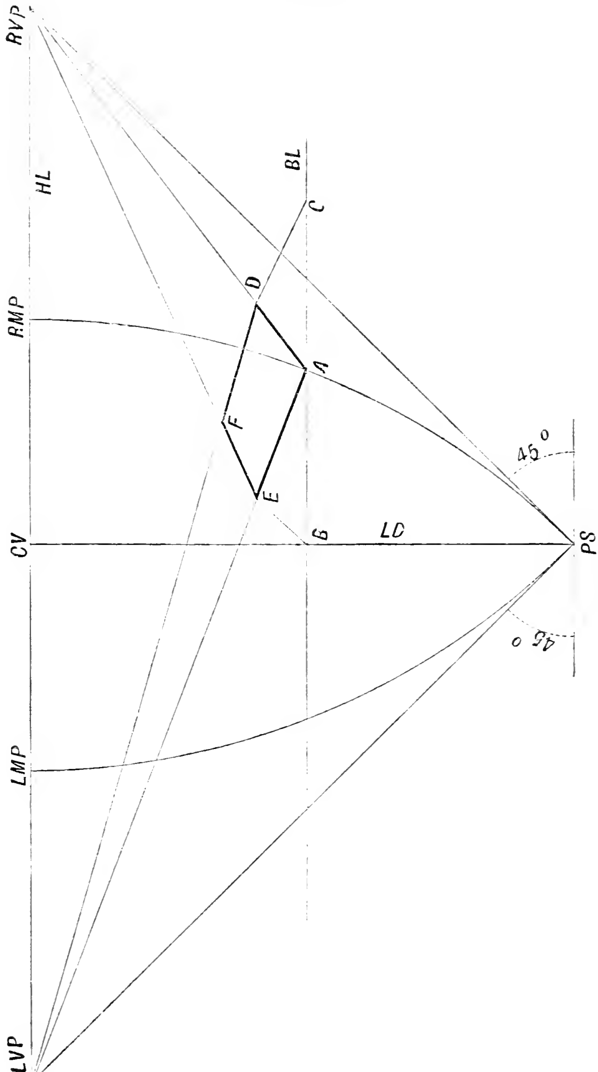

Fig. 61.

Here draw H.L. and B.L. as before, and take P.S. at given distance; then draw a straight line through P.S. parallel to B.L. and on each side of L.D.; lay off the sides at required angle (in this case 45°); produce these lines till they meet the horizon in R V.P. and L V.P. (the vanishing points); with L V.P. as a centre, and P.S. as distance, describe an arc to cut H.L. in R MP.; similarly find L MP. These are called measuring points. Now take A, 4′ to right and draw A L V.P. and A R V.P.; take 4′ on each side of A, namely, B and C, and draw B R MP and C L MP to cut vanishing lines in E and D. Draw D L V.P. and E R V.P. to cut in F. Then ADFE will be the square required. For AD and EF are parallel to P.S. R V.P. placed at given angle, and AE and DF are parallel to P.S. L V.P. also placed at given angle; and DF = AE = AB = 4′, and EF = AD = AC = 4′. (Fig. 61.)

Example 2.—Draw a cube, edges 4′, right face at an angle of 60° and left face at an angle of 30° with P.P. H = 6′, L.D. = 4′, scale $\frac{1}{4}'' = 1'$. Cube to have an angle 4′ to left and 2′ within P.P.

Draw H.L., B.L. and OO as before; find also C.V., R V.P., L V.P., R MP, L MP and MP (parallel perspective), as already shown. Take A, 4′ to left; join A C.V.; erect AK = 4′; join K C.V.; join also B M.P.; through E draw EL parallel to AK; join E RMP and E LMP, and produce them backwards to meet B.L. in Z and X. Mark off XC = 4′, also ZD = 4′; join D RMP and C LMP, also L R V.P. and L L V.P.; through F draw FM, and through G draw GH, parallel to EL; join M L V.P. and H R V.P. to meet at N, completing the required cube. For FM = EL = HG = AK = 4′, and EG = EF = BC = AD = 4′; so also MN = HL = GE, etc. (Fig. 62.)

Note.—The point E must always be determined by parallel perspective; hence necessity for finding MP.

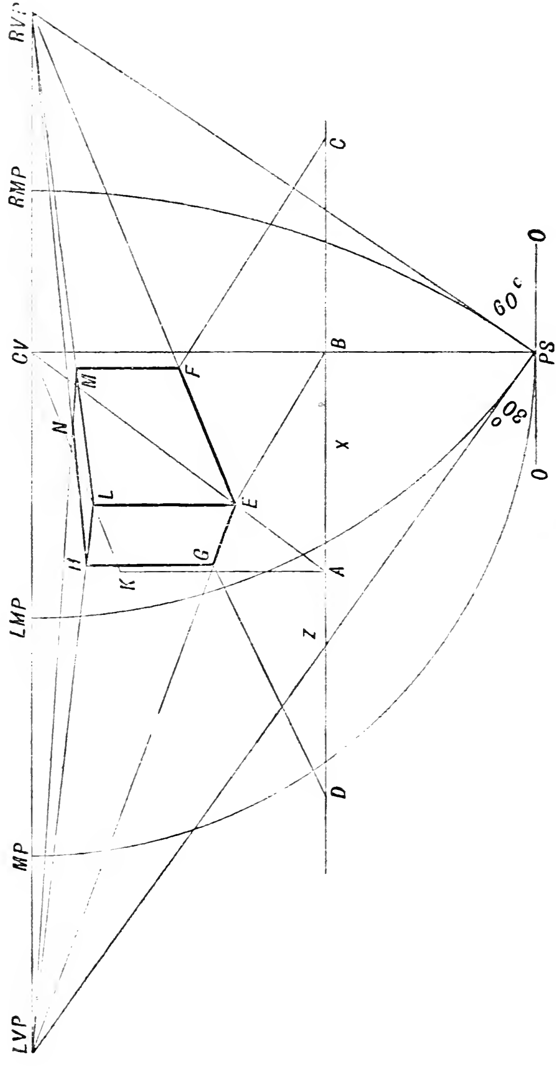

Fig. 62.

Exercise XV.—*Figures in Angular Perspective.*

(In the following consider H = 6′, L.D. = 4′, scale $\frac{1}{4}'' = 1'$.)

1. A square whose sides are 4′, lies on ground; an angle touches P.P. 2′ to right; angle 45°.
2. Draw same, 3′ to left and 2′ within P.P.; right side makes angle of 60°.
3. Draw same, touching P.P. directly in front; angle 45°.
4. Draw a cube, of 4′ edge, touching P.P. at a point 2′ to right; angle 45°.
5. Draw same, 4′ to left and 1′ within P.P.; angle 45°.
6. Draw a square pyramid, edge of base 4′, height 8′, angle 45°; touches P.P. 4′ to left.
7. Draw same, 4′ to right and 2′ within P.P.; angle 45°.
8. Draw same, 2′ to left and 2′ within P.P.; left and right angles, 30° and 60° respectively.
9. Draw a triangular prism, each edge of base 3′ and 5′ long; standing on end, angle touching P.P. 4′ to left; angles 60° and 60°.
10. Draw a square pyramid, edge of base 3′, placed centrally on a cube of 4′ edge; angle 45°; touches P.P. 3′ to left. Height of pyramid 4′.
11. Show perspective effect (angular) of a pyramid to left and above the eye.
12. Show angular perspective effect of a square pyramid placed centrally over a cube of smaller base, to right and below eye.

Miscellaneous Exercises.

(Unless otherwise stated, consider H = 6′, L.D. = 4′, and scale $\frac{1}{4}'' = 1'$.)

1. Two circles, whose diameters are 4′, and intersect at right angles, having their common diameter perpendicular to ground and touching it at a point 4′ to left and 4′ within P.P.
2. Draw an equilateral triangle, lying on ground plane, side 3′, vertex directed away, one side parallel to P.P.; vertex 4′ to right and 4′ within P.P.
3. Draw a hexagon, each side 2′, standing on ground, $\perp$ to P.P., one side touching it 4′ to left.

4. Draw a circle, diameter 4′, touching G.P. and P.P., and perpendicular to both, 4′ to right.

5. A rod is placed obliquely in the ground, and its outside length is 5′; it makes an angle of 30° with the ground and 60° with the P.P. The rod descends toward the left, and lower point is 6′ within P.P. and 6′ to right. Draw it.

6. Draw an octagon, side 2′, lying on ground, touching P.P. 4′ to left.

7. Within a circle, diameter 4′, lying on ground plane, 4′ to right and 4′ within, describe a square whose side shall be parallel to P.P.

8. Draw a triangular prism, length 6′, edges 2′, parallel to P.P. and 3′ within it, one end 4′ to right, other 2′ to left.

9. Draw a cone, diameter 4′, height 4′, standing on ground plane, touching P.P. directly in front.

10. Draw a pyramidal frustum (square), edges 2′ and 4′, height 4′, touching P.P. 4′ to left.

11. Draw same, in angular perspective, angle 45°, 4′ to left, touching P.P.

12. Draw a sphere, diameter 4′, half buried in ground, 6′ to right and 6′ within P.P.

13. Draw a hemisphere, plane directed towards right and perpendicular to P.P. and G.P., touching each; hemisphere to be 4′ diameter and 4′ to left.

14. Draw a cylinder on end, diameter 4′, height 4′, touching P.P. 4′ to right, and on this place a hemisphere centrally, 4′ diameter, convex surface upward.

15. Draw a sphere touching sides of a cubical box of 4′ edge, box on ground parallel to P.P., 4′ to left and 4′ within P.P.

16. Draw a pyramid, base 4′ square, 4′ high, 4′ to right and 3′ within P.P.

17. Draw an equilateral triangle, sides 3′, in angular perspective; angle 60°; 4′ to left, 3′ within P.P., on G.P.

18. A square, sides 4′, stands on ground plane perpendicular to it, making an angle of 45° with P.P. and 2′ from it at nearest lower point; it is 3′ to left.

19. Draw a triangular pyramid, height 8′, each side of base 4′, presenting an angle of 60° to the P.P. 4′ to left.

20. A square prism, length 4′, edge 2′, stands on end, an angle touches P.P. directly in front; sides at 45°. This prism

supports a pyramid placed evenly upon it, of equal base and 4′ in height.

21. Draw a plinth 6′ × 4′ × 2′, side 6′ × 4′ on ground, placed at angles of 60° and 30°, 4′ to left, touching P.P.

22. Draw an ordinary Roman cross, beams 6′ and 4′ in length, and 1′ square at ends, at an angle of 45°, 4′ to right and 4′ within P.P.

23. Place a cube of 4′ edge on top of a cylinder (on end), of 4′ diameter, centres coincident; cylinder touches P.P. directly in front.

24. Draw middle zone of a sphere whose radius is 4′, height of zone 2′, plane parallel to ground plane, centre of zone 4′ to right, 4′ within P.P. and 4′ above it. H = 6′, L.D. = 6′, scale $\frac{1}{2}''$ = 1′.

25. Draw four pyramids, each in contact at bases, 4′ square and 4′ in height, standing on ground plane at an angle of 45° with P.P., 4′ to left and 2′ within P.P. H = 6′, L.D. = 6′, scale $\frac{1}{4}''$ = 1′.

26. Draw a frustum of a cone, height 5′, diameters 3′ and 2′ respectively, touching P.P. 4′ to right.

27. A cone whose slant height is 6′ and diameter of base 6′, rests on ground plane, slant touching ground, parallel to P.P. and 6′ within it, vertex directed towards left; cone to be 4′ to left.

28. A cube of 4′ edge contains a cylinder of equal diameter and height; the cube makes an angle of 45° with P.P., and touches P.P. 4′ to right. The cylinder is vertical.

29. A pyramid, whose base is 4′ square and whose height is 6′, presents an angle to the P.P. 4′ to left, the inclination of the sides being 45°. This pyramid passes centrally through a plinth 4′ × 4′ × 1′, placed horizontally upon it at a height of 3′.

30. A cube, whose edges are 4′, is suspended from an angle so as to just touch the ground directly in front, while another angle touches the P.P. directly in front.

31. Draw a stove-pipe elbow, diameter of ends 6″, length of each half (outside) 1′. The elbow rests on the ground, one end touching P.P. 2′ to left, the other bending towards the right and parallel to P.P. H = 6′, L.D. = 4′, scale 1″ = 1′.

32. Show perspective effect of a pipe lying on ground parallel to P.P. and to left.

33. Show same, standing on end to right and below eye.

34. Show angular perspective effect of a square pyramid to right, and below eye.

35. Show perspective effect of a cylinder on end, to right and below.

36. Show perspective effect of a pail with three hoops, below and directly in front.

37. Show perspective effect of a water pitcher, to left and below; lip to right.

38. Show perspective effect of a chair, straight back, directed away, angle to right and below the eye.

39. Show hollow pipe lying on ground, perpendicular to P.P., to right.

40. Show perspective effect of an ink bottle (conical).

41. Show perspective effect of an ink bottle in form of pyramidal frustum, angle to left, below the eye.

42. Show perspective effect of a plinth to left and below, angular.

43. Show perspective effect of a teacup below the eye.

44. Show perspective effect of a sphere placed centrally on a cube, directly in front, below the eye.

45. Show angular perspective effect of a table below and to right.

46. Show perspective effect of a triangular prism on end, one side perpendicular to P.P., below and to right.

47. Show perspective effect of a reversed cone, below the eye.

48. Show perspective effect of a hexagon on one side, perpendicular to ground, to right.

49. Show perspective effect of a hollow conic frustum lying on ground parallel with P.P., larger end to right, smaller end to left.

50. Show perspective effect of a door in three different positions, revolving round an axis through the hinges:

(*a*) When shut, parallel to P.P.
(*b*) When opened at an angle of 45° with P.P.
(*c*) When opened perpendicularly to P.P.

GEOMETRICAL DRAWING.

To construct the following figures, pupils should provide themselves with a pair of good compasses with pen attachment, and a ruler, with marks for inches and fractions of an inch. No proof is necessary, but it will be well to investigate the methods as far as possible, many of which are but modifications of the Euclidian.

No. 1.—To draw a perpendicular to a given line (*a*) from a point on the line.

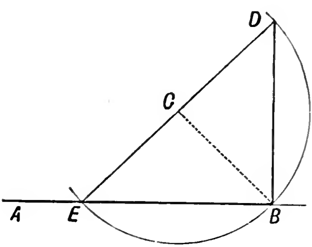

Fig. 63.

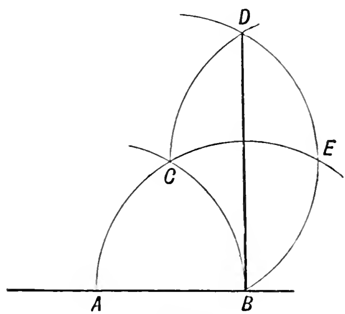

Fig. 64.

Let AB be the given line, and let B be a point at which the perpendicular to AB is to be drawn. In first, take any point C above, and with distance CB describe circle, cutting AB in E; join EC and produce to meet circumference in D; join DB, which will be the perpendicular required. In second, take any point C above, and with B as centre and BC as distance describe arc, cutting AB in A; then with C as centre and CB as distance describe arc, cutting former arc in E; with E as centre and same distance describe arc, cutting in D;

join DB, which will be perpendicular to AB, at B. (Figs. 63 and 64.)

(*b*) From a point above or below AB.

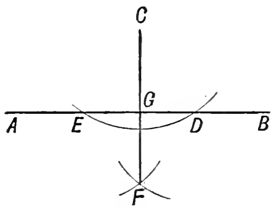

Fig. 65.

Let AB be the given line and C given point above; with centre C describe an arc to cut AB in D and E; with centre D and distance greater than half of DE describe an arc; with centre E and same distance describe an arc to cut former arc in F; join CF, which will cut AB at right angles at G. (Fig. 65.)

No. 2.—To describe a square (*a*) on a given line.

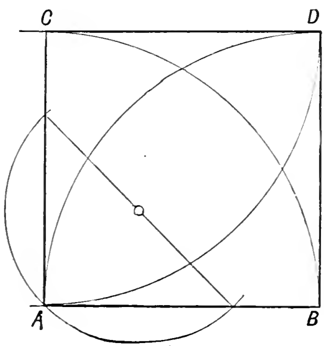

Fig. 66.

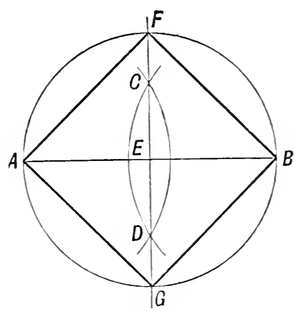

Fig. 67.

In first, let AB be the given line; erect at A a perpendicular and make it equal to AB; with centre C and distance CA describe arc AD; with centre B and distance BA describe arc to meet former arc in D; join CD and BD, completing the square. (Fig. 66.)

(*b*) On a given diagonal AB.

With centre A and distance greater than half of AB, describe arc CD, and with centre B and same distance describe an arc to cut former arc in C and D; join CD and produce both ways; with centre E and distance EA or EB describe a circle cutting diameter in F and G; join FA, FB, GA and GB, completing the square. (Fig. 67.)

No. 3.—To construct an oblong of given dimensions.

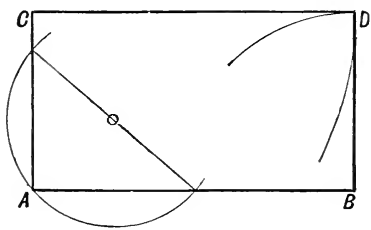

Fig. 68.

Let AB represent the greater side; erect AC perpendicular to AB at A, and with centre C and distance equal to AB describe an arc; with centre B and distance equal to AC describe an arc cutting former arc in D; join DC and DB, completing the required oblong. (Fig. 68.)

No. 4.—To divide a given line into (*a*) two equal parts.

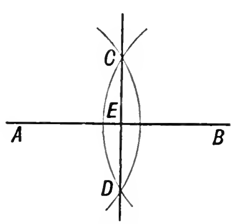

Fig. 69.

Let AB be the given line; with centre A and distance equal to more than half of AB describe an arc, and with centre B and same distance describe an arc cutting former in C and D; join CD, cutting AB in E into two equal parts. (Fig. 69.)

(*b*) Into any number of equal parts.

NOTE.—Before this can be done it is necessary to show how to draw a line parallel to another from an external point.

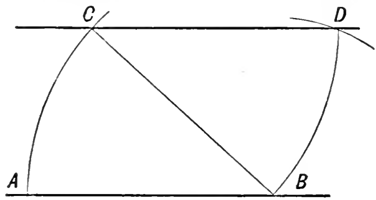

Fig. 70.

Let AB be the given line and C the external point; at any point B and distance BC describe an arc to cut AB; with C as centre and CB as distance describe an arc, and with B as centre and distance equal to AC describe an arc cutting former arc in D; join CD, which will be parallel to AB. (Fig. 70.)

(*b*) To divide a line into any number of equal parts.

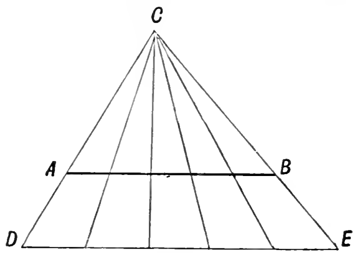

Fig. 71.

Let AB be the given line; draw DE parallel to AB on either side, and on this line set off the required number of equal distances (in this case five); then join each point of section with A, which will divide AB into the same number of equal parts. (Fig. 71.)

(*c*) To divide a given line proportionally to another line.

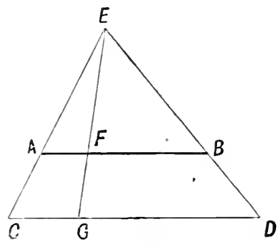

Fig. 72.

Let it be required to so cut AB, that the smaller part shall be to the greater, as the greater is to the whole line. Let any line CD not equal to AB, be cut in G, so that CG : GD :: GD : CD ; place CD parallel to AB, and join CA and DB; produce them to meet in E; join EG to cut AB in F; then will AF : FB :: FB : AB. (Fig. 72.)

No. 5.—To construct a triangle of given dimensions.

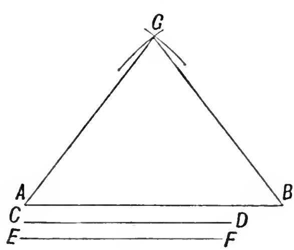

Fig. 73.

Let AB, CD and EF be the given sides, no two of which, taken together, are equal to, or less than, the third. Take one of them AB, and with centre B, and distance equal to CD describe an arc; with centre A and distance equal to EF describe an arc cutting former arc in G; join GA and GB, completing the triangle. (Fig. 73.)

No. 6.—To bisect a given angle.

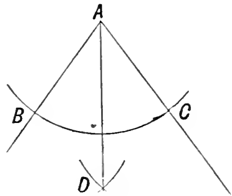

Fig. 74.

Let BAC be a given angle; with centre A and any distance AB describe an arc BC; with centre B and any distance less than half of BC describe an arc; with centre C, and same distance, describe an arc to cut former arc in D; join AD, which will bisect the angle. (Fig. 74.)

No. 7.—To trisect a right angle.

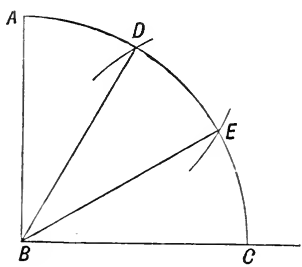

Fig. 75.

Let ABC be the given right angle; with B as centre and at any distance BA describe the quadrant AC; with centre A and distance equal to AB describe an arc to cut AC in E; and with centre C, and distance equal to CB or BA describe an arc to cut arc in D. Then D, E will be points of trisection, and lines from D and E to B will trisect the angle. (Fig. 75.)

No. 8.—To inscribe a circle in a given triangle.

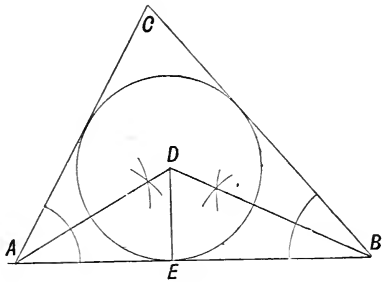

Fig. 76.

Let ABC be the given triangle; bisect the angle at A by AD, and the angle at B by BD, cutting AD in D; draw DE perpendicular to AB at point E; then with centre D and distance DE describe the circle. (Fig. 76.)

No. 9.—To draw a circle through three given points, which, however, cannot be in the same straight line.

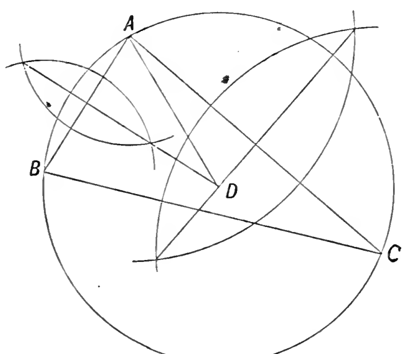

Fig. 77.

Let A, B, C be the given points; join them to form a triangle; with centre A, and distance greater than half of AC, describe an arc; with centre C and same distance, describe an arc to cut former; join points of section of arcs; this line will pass through the centre of the circle; draw similar arcs on AB, and join points of section to meet in D, the centre; then a circle drawn with centre D, and distance DA, will pass through A, B and C respectively. (Fig. 77.)

No. 10.—To find the centre of a whole or part of a circle.

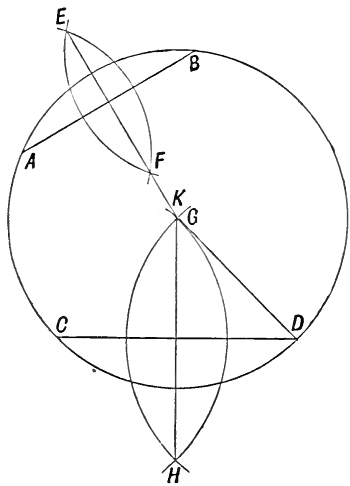

Fig. 78.

Let BACD be a circle or arc; draw any two chords AB, CD, and draw arcs EF and GH, bisecting the chords, respectively; join EF and HG and produce them to meet in K; then K will be the centre, and if K and D be joined, KD will be a radius, and a circle may be thus described with it with the centre thus found. (Fig. 78.)

No. 11.—To draw a tangent to a given circle (*a*) from a point in the circumference.

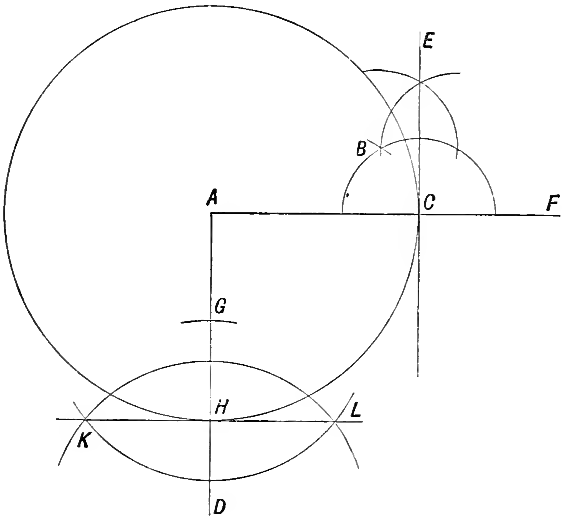

Fig. 79.

Let C be a given point in the circumference; let A be the centre; join CA; at C erect the perpendicular CE, which will be tangent required. Also if H be the given point, join AH and produce it, making HG equal to part produced; bisect this line by KL, which will also be a tangent at point H. If AC be produced, the line FC, perpendicular to the tangent at the point of contact C, is called a "normal." (Fig. 79.)

(*b*) To draw a tangent to a circle from an external point. Join point with centre, and at point where it cuts the circumference draw a perpendicular upon it; thus, if D be given point, HL will be a tangent.

No. 12.—To construct an isosceles triangle of a given altitude.

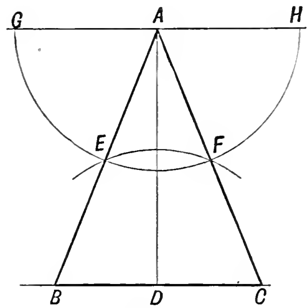

Fig. 80.

Let AD be the given altitude; at A and D draw perpendiculars, and with A as centre and any distance describe an arc EF; with D as centre and any distance less than DA describe an arc to cut arc in E and F; join AE and AF, and produce them to base, forming isosceles triangle ABC. (Fig. 80.)

No. 13.—To construct an equilateral triangle of a given altitude.

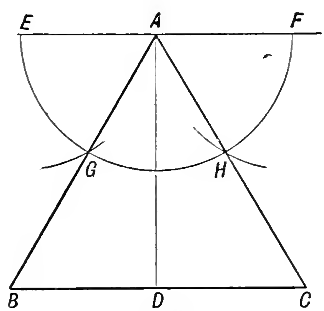

Fig. 81.

Let AD be the given altitude; through A, draw EAF perpendicular to AD; with centre A describe any semicircle EGHF, and with centre F and distance FA, describe arc to cut arc GH in H; also with centre E and same distance describe arc to cut GH in G. Join AG and AH, and produce them to meet BC in B and C: then ABC will be the equilateral triangle required. (Fig. 81.)

No. 14.—(*a*) To draw, from a given point in a straight line, an angle equal to a given angle.

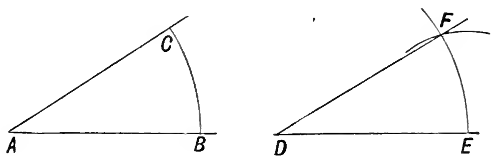

Fig. 82.

Let BAC be given angle and let the given point be at D; with centre A and any distance less than a side describe an arc CB; with centre D and distance equal to AB describe arc EF; with centre E and distance equal to BC describe an arc to cut EF in F; join DF: then the angle EDF will be equal to the angle BAC. (Fig. 82.)

(*b*) Within a given circle to construct a triangle similar to another triangle. (Triangles are similar when the angles in one are equal to the angles in the other, each to each. They are similarly situated when the sides of one are parallel to the sides of the other, each to each. These are called *homologous* sides.)

Let ABC be the given circle and LHK, the given triangle. At any point C, draw a tangent DE, and describe a semicircle DE; with centre H, and any distance HM, describe an arc, and with centre K and same distance describe an arc. Make EG = NO and DF = MO; join CF and CG, and produce them to the circumference in A and B; join AB, completing the triangle required. (Fig. 83.)

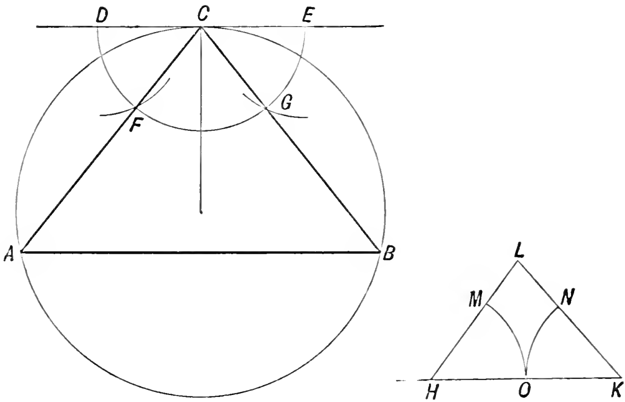

Fig. 83.

No. 15.—To construct an equilateral triangle about a given circle.

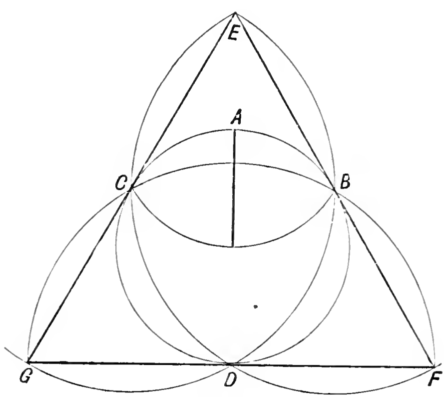

Fig. 84.

Let BCD be given circle; at any point A in the circumference, with distance equal to radius, describe an arc, cutting circle in B and C; with centre C, and same distance describe an arc, cutting circle in D; describe similar arcs with centres B and D; these arcs intersect in points E, F and G; join these, completing the triangle required. (Fig. 84.)

No. 16.—About a given circle to construct a triangle similar to a given triangle.

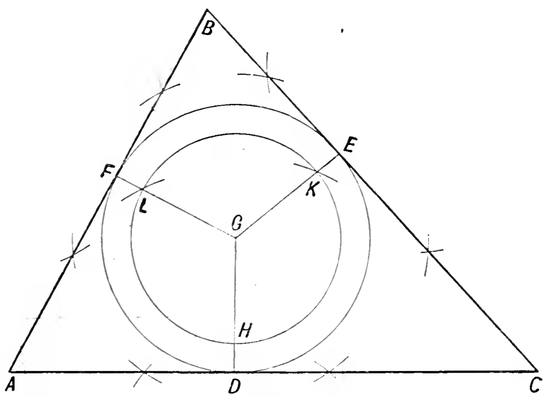

Fig. 85.

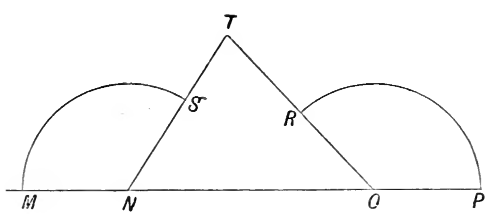

Fig. 86.

Let DEF be the given circle and TON the given triangle; find the centre G, and draw any radius GD; at D, draw a tangent to the circle. With centre N and any distance MN, describe an arc MS, and with centre O, and same distance describe an arc RP; with centre G, and distance equal to MN or OP, describe a circle cutting GD in H. Make arc HK = RP and HL = MS; join GK and GL, and produce them to the circumference in E and F respectively. Draw tangents at E and F to meet the other tangent in A and C; then triangle ABC will be similar to TON. (Figs. 85 and 86.)

No. 17.—Within a circle to draw any number of equal smaller circles, each touching two others and the outer circle.

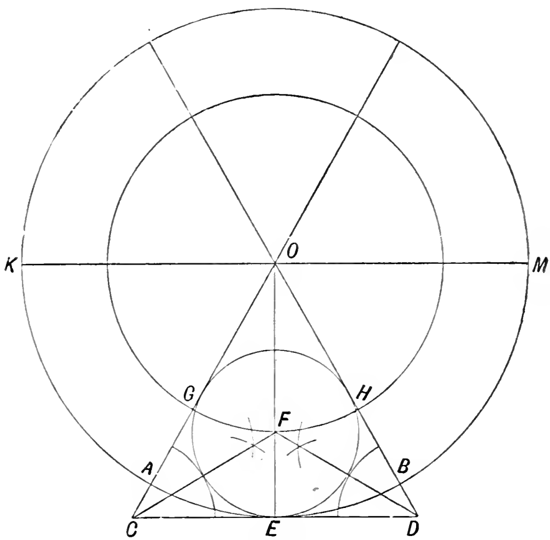

Fig. 87.

Let KME be the given circle, and divide it (in this case) into six equal parts. Take centre O, and join any two, as OA, OB; bisect the angle AOB by OE, and at E draw a tangent CD; produce OA and OB to meet the tangent in C and D.

Bisect the angles at C and D by CF and DF, meeting at F; then with centre O and distance OF describe a circle; also with centre F and distance FE describe a circle: this will be one, and the remaining five may be similarly drawn. (Fig. 87.)

No. 18.—To construct a regular polygon (*a*) on a given line.

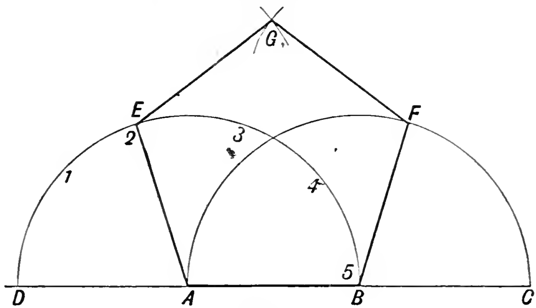

Fig. 88.

Let AB be the given line, produce it both ways; then with centre A and distance AB, describe the semicircle DEB, describe also a similar semicircle AFC. Divide the circumference DEB into as many equal parts as the polygon is to have sides (in this case five), and join A with the *second* point of division; make the arc FC = DE; join BF and with centres E and F and distance EA and FB describe arcs to intersect at G; join GE and GF, completing the polygon.

Note.—This method will be clear if it be remembered that, if from a point within a polygon straight lines be drawn to the angles, the figure will be divided into as many triangles as it has sides, and each triangle will contain two right angles, but the angles around the common point within, together make four right angles. Then if N represent the number of sides, the number of degrees in the angle of a regular polygon will be $\frac{90(2n-4)}{n}$, that is, $\frac{180\ (n-2)}{n}$. Now, in the above figure the line DB may be called 180°, or two right angles. Then the angle EAB will be represented by $\frac{5-2}{5}$, or $\frac{3}{5}$ of 180°; hence it is *always* necessary to draw through the *second* point of division.

(*b*) In a given circle.

Let ABE be the given circle; draw any diameter FC, and divide it into as many equal parts as the figure is to have sides (in this case five); with centre C and distance CF describe arc FG, and describe similar arc CG, intersecting at G. Draw GA from A, through *second* point of division; join FA, and continue this around the circumference, completing the polygon. (Fig. 89.)

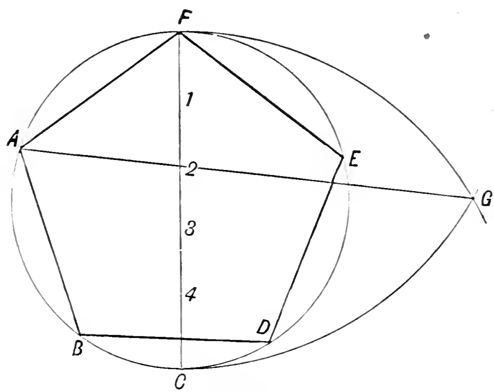

Fig. 89.

No. 19.—To construct a regular pentagon on a given line by a special method.

Let AB be the given line; describe arcs CAD and CBD, with radius AB; join CD; with centre D and distance same as AB describe arc EABF, cutting former arc in E and F.

Join FG and EG, and produce them to meet arcs in H, K; join AH and BK; with centre H and distance HA describe arc; and with centre K and distance KB, describe arc cutting former arc in L. Join LH and LK, completing the pentagon. (Fig. 90.)

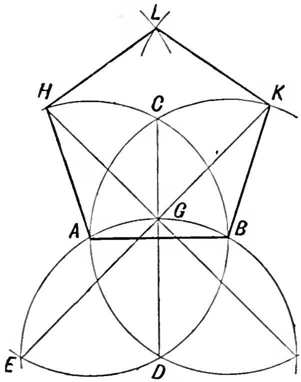

Fig. 90.

No. 20.—To construct a regular hexagon on a given straight line.

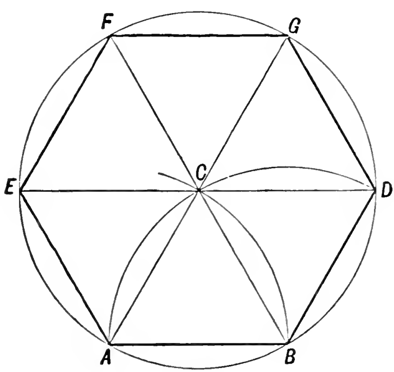

Fig. 91.

Let AB be the given line; with centre A and distance AB describe arc; with centre B and distance BA, describe arc cutting at C; join CA, CB. With centre C and distance CA, describe circle cutting in D, B and A; join DC and produce to E; produce AC to G and BC to F; join AE, EF, FG, GD and DB, completing the hexagon.

No. 21.—To construct a regular octagon (*a*) on a given straight line.

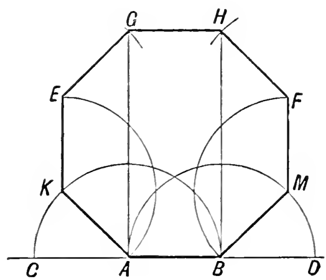

Fig. 92.

Let AB be the given line; produce it both ways, and describe the semicircles CKB and AMD. Erect a perpendicular at A and at B; bisect the right angles CAG and DBH by AK and BM, respectively, and erect perpendiculars KE and MF, at K and M, respectively. With centre K and distance KA describe arc EA cutting KE in E; draw similar arc BF cutting MF in F; with centre E and distance EK describe arc to cut AG in G; similarly find H; join EG, GH and HF, completing the octagon. (Fig. 92.)

(*b*) In a given square.

Let BCDE be the given square; draw diagonals intersecting at A. With centre B, and distance BA, describe arc cutting sides of square in H and P; similarly find points K, N, F, M and G, L; join FG, PN, ML and HK, completing the octagon. (Fig. 93.)

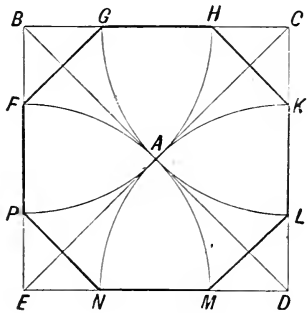

Fig. 93.

No. 22.—To draw a perfect ellipse by means of the foci and intersecting arcs, axes being given.

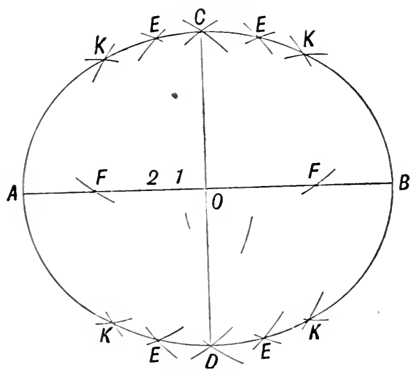

Fig. 94.

Let the axes, AB and CD, be placed centrally, at right angles to each other; then measure from C the distance from A to D, and describe arc cutting AB in F and F: these are the foci. Between O and F, take any number of points, 1, 2, etc.—the more the better; then with centre F (left) and distance equal to distance from 1 to B, describe arcs E, E; and with centre F (right) and same distance describe arcs E, E. Then with centre F (left) and distance equal to that from 1 to A, describe arcs cutting the former, so also describe arcs from centre F (right). Thus for each point between O and F, we get four points. Having thus found a number of points, join them, or rather draw a curve through them: this curve will be an ellipse. (Fig. 94.)

No. 23.—To draw an ellipse (*a*) by means of concentric circles and intersecting perpendiculars.

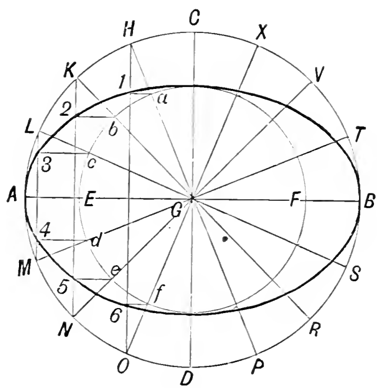

Fig. 95.

Let the concentric circles EF and AB be drawn; draw diameters AB and CD at right angles to each other, divide each quadrant into the same number of equal parts, and join opposite points; draw perpendiculars from the outer points and horizontals from the inner points to meet them; thus draw perpendicular from H and horizontal from *a* to meet in 1; similarly find 2, 3, 4, etc., all around the circle. Draw a curve through the points of intersection thus found, which will form an ellipse. (Fig. 95.)

(*b*) When the major axis (transverse diameter) only is given.

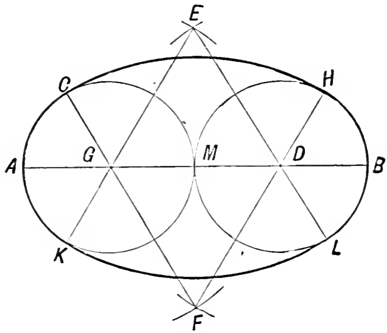

Fig. 96.

Let AB be the given diameter; divide it into four equal parts in G, M, D; with centre D and distance DG describe arc EF, and with centre G and same distance describe arcs to intersect in E and F. Join FG, FD, EG and ED, and produce them to the circumference in C, H, K and L respectively. Then with centre F and distance FC describe arc CH, and with centre E and distance EK describe arc KL, completing the elliptical curve. (Fig. 96.)

NOTE.—No part of a true ellipse is an arc of a circle.

No. 24.—An ellipse being given, to find axis and foci.

Draw any two parallel chords AB and CD; bisect each and join points of section FE, and produce each way to meet the circumference in G and H; bisect GH in K, and with centre K describe a circle to cut the ellipse in four points N, O, R and P; join these to form a rectangular parallelogram; bisect each side and join the opposite points of section, and produce both ways to meet circumference in L, M, T and V; then LM and TV will be the axes; and if the distance TK be taken with centre L or M, the arc will cut TV in S, S, which will be required foci. (Fig. 97.)

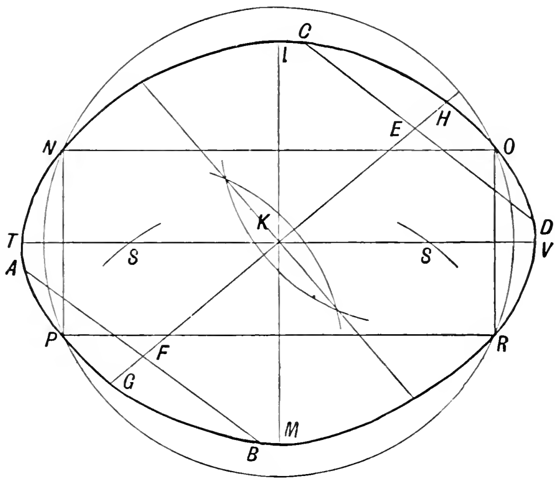

Fig. 97.

No. 25.—To draw a tangent to an ellipse (*a*) from a point in the circumference.

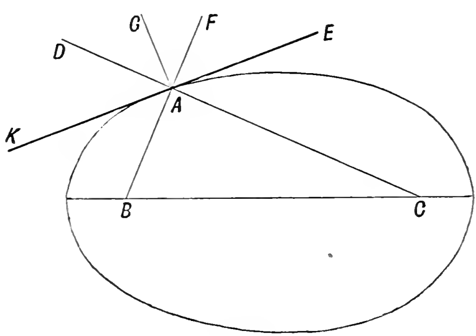

Fig. 98.

Let B, C be the foci and A, given point on the curve; join BA and CA and produce them to D and F; bisect the angle BAD by AK, and the angle DAF by AG. Then will KA be a tangent and GA a perpendicular or normal to it, at the point of contact, A. (Fig. 98.)

(*b*) From an external point.

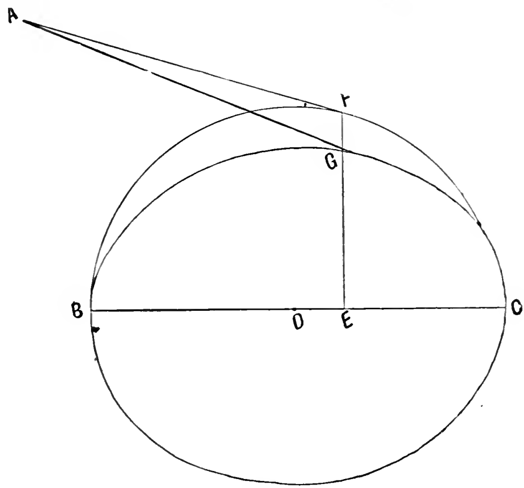

Fig. 99.

Let A be an external point; draw major axis BC, and on it describe the semi-circle BFC; draw a tangent AF to the circle at F; draw FE perpendicular to BC, cutting curve in G; join AG, which will be a tangent, to the curve. (Fig. 99.)

No. 26.—To draw an oval of a given width.

Let AB be the given width; bisect it in C, and on it describe the circle ADB; draw CD at right angles to AB; with centre B and distance BA describe curve AE; similarly describe curve BF; join BD and AD, and produce them to meet curve in E and F; with centre D and distance DE describe curve EF, completing the oval. (Fig. 100.)

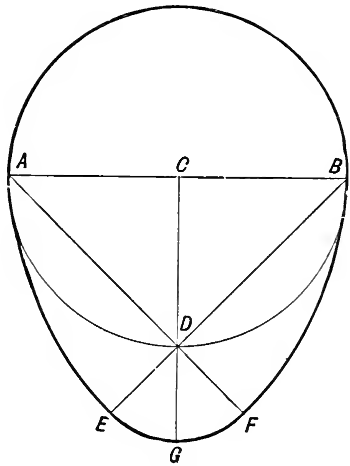

Fig. 100.

No. 27.—To construct the involute of the circle.

Divide the circle into any number of equal parts and draw the radii, numbering them 1, 2, etc. Draw the tangents, making the first the length from 1 to 2, the second twice this length, the third three times, and so on. When all the tangents have been drawn thus, begin again at 1 by producing it, and so get a second series of points. Then draw a curve through the points, commencing with 8, or last, and joining it with 1, then 2, etc. (Fig. 101.)

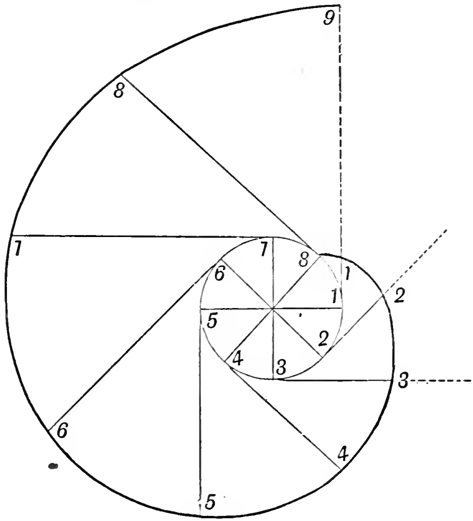

Fig. 101.

No. 28.—To find (*a*) a mean proportional between two given lines.

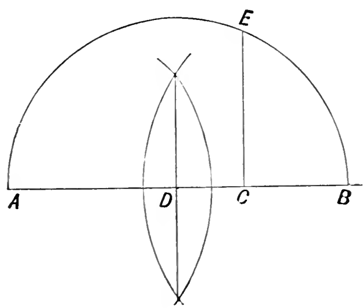

Fig. 102.

Let AC and BC be the given lines; place them in a straight line AB; bisect AB in D, and on AB describe semicircle AEB; through C draw CE at right angles to AB; then will AC : CE : : CE : CB. (Fig. 102.)

(*b*) To draw a third proportional (greater).

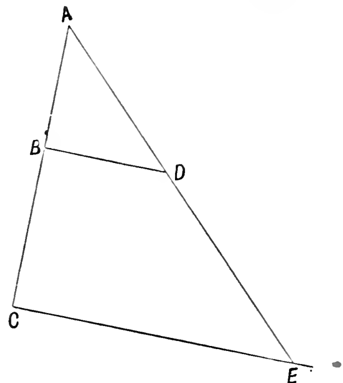

Fig 103

Let AB and BC be the given lines; draw AE greater than AC, making any angle with AC; make AD = BC; join BD, and through C draw CE parallel to BD; then will AB : BC : : BC : DE. (Fig. 103.)

(*c*) (Less). Make AE greater than BC and less than AC.

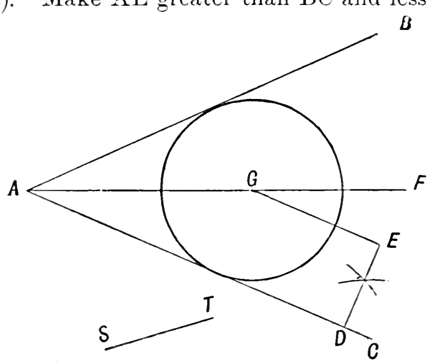

Fig. 104.

No. 29.—To draw a circle of given radius which shall touch both lines of a given angle.

Let BAC be the given angle and ST the given radius; bisect angle by line AF; erect on either line a perpendicular DE equal to ST; through E draw EG parallel to AD, and cutting AF in G; then a circle drawn with centre G and radius equal to DE or ST, will touch the sides AB and AD. (Fig. 104.)

No. 30.—To draw a circle of given radius which shall touch another given circle and a given straight line.

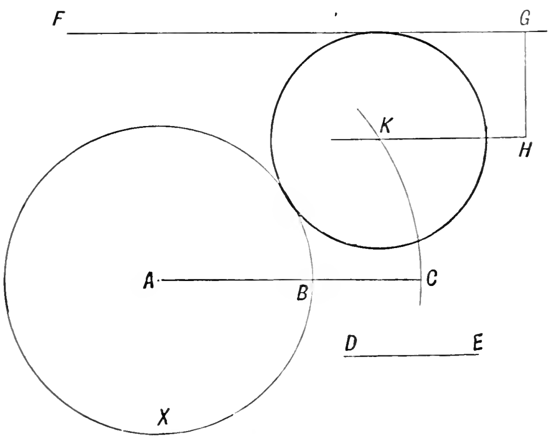

Fig. 105.

Let FG be a given line, BX a given circle, and DE a given radius; draw a line GH perpendicular to FG and equal to DE; through H draw HK parallel to FG; draw any radius AB and produce it, making the part produced equal to DE; then with centre A and distance AC, describe an arc cutting HK in K: the circle drawn with centre K, and at a distance equal to GH or DE will touch the circle BX and the line FG. (Fig. 105.)

Graded Exercises.

1. Construct a square whose side is 2″.

2. Construct a square whose diagonal is 4″.

3. Construct an oblong whose sides are $1\frac{3}{4}''$ and $2\frac{3}{4}''$ respectively.

4. A rectangular field is 900 yds. long and 400 yds. wide; divide it into four equal fields each 400 yds. long. Scale 100 yds. = 1″.

5. Divide a line $3\frac{3}{4}''$ long into two parts in the ratio 3 : 4.

6. Draw a line parallel to and between two other parallel lines $2\frac{1}{4}''$ apart, the line to be twice as near one as the other.

7. Construct an equilateral triangle whose side is 2″.

8. Centrally within the triangle in No. 7 construct a triangle whose side is $1\frac{1}{2}''$.

9. Construct a triangle whose sides are $2\frac{1}{2}''$ $3\frac{1}{4}''$ and $3\frac{3}{4}''$ respectively.

10. Inscribe a circle in a triangle whose sides are same as No. 9.

11. The diagonal of a parallelogram is 4′ and one side is 2′. Draw it.

12. Find the extent of an angle of $22\frac{1}{2}°$,of $37\frac{1}{2}°$, of $41\frac{1}{4}°$.

13. Construct an isosceles triangle whose base is 2″ and vertical angle $37\frac{1}{2}°$.

14. Construct an isosceles triangle whose base is 2″ and altitude $3\frac{1}{2}''$.

15. Construct an equilateral triangle about a circle whose diameter is 4″.

16. Construct a triangle whose sides are in the ratio 3:4:5 about a circle whose diameter is 4″.

17. Describe a circle about a square whose side is 3″.

18. Within a triangle whose sides are 3″, 4″ and 5″ respectively, inscribe a circle.

19. Construct a regular pentagon whose side is $1\frac{1}{4}''$.

20. Construct a regular heptagon in a circle 4′ diameter.

21. Draw a hexagon whose side is 2″.

22. Draw a hexagon within an equilateral triangle of 3″.

23. Construct a regular octagon whose side is $1\frac{1}{2}$″.

24. Construct a regular octagon in a square, side 3″.

25. Construct a regular octagon in a circle of 3″ diameter.

26. Inscribe seven equal circles in a circle of 3″ diameter.

27. The diameters of an ellipse being $3\frac{1}{2}$″ and2 $\frac{1}{2}$″ respectively, draw it.

28. Draw an elliptical curve on a transverse axis of 3″.

29. Draw an elliptical figure from two squares, diagonal of each 3″.

30. A circle and an ellipse touch the angles of an oblong 3″ × 5″, find axes and foci of the ellipse.

31. Draw an oval whose shorter axis is $2\frac{1}{2}$″.

32. Construct an involute to a circle whose diameter is $\frac{1}{2}$″.

33. Find a mean proportional between two lines $2\frac{1}{2}$″ and $3\frac{3}{4}$″ respectively.

34. Draw a third proportional (greater) to two lines $2\frac{1}{2}$″ and $3\frac{1}{2}$″ respectively.

35. Draw same as No. 34, less.

36. Lay out a circular garden whose radius is 30 yds., which shall just touch a fence on one side, and another garden whose radius is 50 yds. on the other. Scale 20 yds. = 1″.

37. Lay out a circular garden, radius 30 yds., which shall touch two fences not parallel. Scale 20 yds. = 1″.

38. Lay out a circular garden, radius 30 yds., which shall touch two fences not parallel, and whose edge shall just touch a tree in a given position. Scale 20 yds. = 1″.

39. The axes of an elliptical flower garden are 40 and 60 yds. respectively, and a point is taken 60 yds. to left of the shorter (produced), and 40 yds. above the longer (produced). Draw a path from this point to touch the elliptical garden. Scale 20 yds. = 1″. Longer axis horizontal.

40. Construct a triangle whose angles are 75°, 45° and 60° respectively, on a line 2″ in length.

41. Two circles, whose diameters are each $3\frac{3}{4}$″, intersect, the intercepted arc of each being one-fifth of the whole circumference.

42. The base of a right-angle triangle being 2″, and the

perpendicular on the hypothenuse from the right angle being $1\frac{1}{2}''$, construct the triangle.

43. Two lines meet at a point. Find a point between them that will be 2″ from one and 3″ from the other.

44. Show that the number of degrees in the angle of any regular polygon is represented by $\frac{180\,(n-2)}{n}$, where n = number of sides.

45. Show that a circle is but a particular form of an ellipse.

46. If an ellipse be drawn with a string around two fixed pins as foci, show that the sum of the distances from any point in the curve to the foci is the same.

47. From the point found in No. 43 draw two equal straight lines to the given line.

48. Draw a parallelogram $3\frac{1}{2}'' \times 5\frac{1}{2}''$, and within this draw one of half the size similar and similarly situated.

49. If one hexagon be inscribed in, and another inscribed about, a circle, show that their areas are in ratio 3 : 4.

50. Three circular gardens, diameters 20 yds., 30 yds. and 40 yds. respectively, are to be placed with walks of 5 yds. between them. Scale 20 yds. = 1″.

51. Divide a triangle whose sides are 3″, 4″ and 5″ respectively, into four equal and similar triangles.

52. From the vertex of a scalene triangle draw a straight line to the base which shall exceed the less side as much as it is exceeded by the greater.

53. One of the acute angles of a right-angled triangle is three times as great as the other; trisect the smaller.

54. One side of a right-angled triangle is 4″, and the difference between the hypothenuse and the sum of the other two sides is 2″; construct the triangle.

55. The altitude of an equilateral triangle is 2″; construct it.

56. Place a straight line, 3″ in length, between two straight lines, each 2″ in length, which meet, so that it shall be equally inclined to each of them.

57. Describe an isosceles triangle upon a given base, having each of the sides double of the base.

58. Draw a square equal in area to two unequal oblongs.

59. Given the base, the vertical angle and the perpendicular of a plane triangle, construct it.

60. Cut off two-thirds of an isosceles triangle by a line parallel to the base.

61. Describe two circles with given radii which shall cut each other and have the line between the points of section equal to a given, limited line.

62. Describe a circle with a given centre cutting a given circle in the extremities of the diameter.

63. Describe a circle which shall pass through a given point and which shall touch a given straight line in a given point.

64. Describe a circle which shall touch a given straight line at a given point and bisect the circumference of a given circle.

65. Two circles are described about the same centre; draw a chord to the outer circle, which shall be divided into three equal parts by the inner one. What are the limits of the diameters?

66. The perimeter of an oblong is 20″, and the sides are in ratio 3 : 2 ; construct it.

67. Construct an isosceles triangle of given vertical angle and given altitude.

68. Draw a square equal in area to an oblong $3'' \times 2''$.

69. Describe a circle of given radius to touch two points. What limits the position of the points?

70. Show how to draw a similar triangle within another.

71. Two equal ellipses, axes 5″ and 3″, cut each other at right angles, and their centres are coincident; draw them.

72. Trisect a given line.

73. Show how an angle may be trisected (mechanically).

74. Divide a square into three equal parts by lines drawn from an angle.

75. Divide an oblong into three equal parts by lines drawn from an angle.

76. Find the number of degrees in the angle of a regular duodecagon.