On the Motion of Comet Halley

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This work will be published in *Astronomy and Astrophysics* or in *Astronomische Nachrichten* with exception of the appendix, some particular results have been published already on the quoted references. It also represents the report to ESTEC contract EP/14.7/6184 which can be used for preliminary reference. It have been distributed to several libraries.

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Summary.

We report several results of orbit determinations of comet Halley. Some problems which appear are considered, especially in regard to the nongravitational forces and the differences between the light center and the nucleus of the comet. Improved orbital elements have been computed for different assumptions about these and other bases. Further results are: a radial light offset can most simply to be eliminated by using only the position angles to sun of the observations (instead α and δ); due to the decay of the comet the nongravitational forces increase by about 1% per revolution, the nongravitational forces decrease much slower at large heliocentric distances than according to the nongravitational models, and observations back to at least 1759 should be used for orbit determinations until the comet is on larger distances. The perihelion time in A.D. 837 has been determined very precisely and later can be used as a check of improved models of the nongravitational forces. A backward integration of the comet back to 2300 B.C. has been carried out and compared with the observed perihelion times.

Zusammenfassung.

Es wird über Ergebnisse der Bahnberechnungen des Halley'schen Kometen berichtet. Die auftretenden Schwierigkeiten werden erörtert, insbesondere in Hinblick auf die nichtgravitativen Kräfte und die Differenz zwischen dem Lichtschwerpunkt und dem Kern des Kometen. Zu verschiedenen Annahmen über diese und andere Grundlagen wurden verbesserte Bahnelemente abgeleitet. Weitere Befunde sind: einen radialen light offset kann man am einfachsten eliminieren, indem man nur die Positionswinkel zur Sonne (statt α und δ) von den Beobachtungen verwendet, durch Alterung des Kometen vergrößern sich die nichtgravitativen Parameter um etwa 1% pro Umlauf, die nichtgravitativen Kräfte nehmen mit großer Entfernung erheblich langsamer ab als gemäß den vorhandenen Modellen, und bei Bahnrechnungen sollten deshalb Beobachtungen zurück bis mindestens 1759 verwendet werden, solange der Komet in größeren Distanzen ist. Die Perihelzeit im Jahre 837 wurde sehr genau bestimmt und kann zur Überprüfung verbesserter Modelle für die nichtgravitativen Kräfte verwendet werden. Eine Rückrechnung des Kometen bis 2300 v.Chr. wurde durchgeführt und mit den beobachteten Periheldurchgängen verglichen.

Резюмэ

Сообщены результаты вычислений орбиты кометы Галлея. Обуждаются возникающие трудности, особенно в звязи с негравитационными силами и с разностью между центром максимальнай яркости и ядром кометы. Приведены исправленные элементы орбиты под разными предположениями. Дальнейшие результаты: Можно элиминировать сдвиг обусловлен разносью центры яркости и ядра, если пользуется из наблюдении только позиционным углом в направлении к солнцу. Через одно появление кометы Галлея негравитационные параметры увеличиваются в среднем на 1%. Обязательно учитывать наблюдения до 1759 года. С увеличивающим расстоянием от солнца негравитационные силы уменьшаются слабее чем в существующих моделях. Время прохода через перигелий в 837 году определено очень точно. Им можно пользоваться за проверку разных моделей. Орбита кометы была вычислена до 2300 г. до н. э. и сравнена с наблюдениями

0. Introduction

After the rediscovery of comet Halley in October 1982, the author started computations with the aim to derive improved orbital elements for the comet *(W.Landgraf, 1983a)*. The present report reviews the proper results of these computations, together with comments on the bases of them and their uncertainties and difficulties, especially in regard to the nongravitational forces. In addition, some other aspects are pointed out which should be taken into account in similar computations.

1. The observations

Reliable Reports refering to comet Halley extend back to 466 B.C. The results presented here refer only to the 1607 to 1984 appearences. However, they exhibit significant differences to the earliest reports, and thus the latter are principially of value for more extended investigations of the long-term motion of the comet.

From 1909 to 1911 about five thousand observations have been reported and about half of their most precise ones have been reduced to the system of GC and collected to 33 normal places *(P.Zadunaisky, 1966)*. These normals has been choosen for our computations and reduced from GC to the FK_4 system. Although the time of brighter magnitude of the comet is covered by a larger number of observations, the greater observational uncertainty there and consideration of the residuals of the normals favours weighting all normal places with one unit weight, which *a posteriori* corresponds to the mean error of $1''.2$. In addition, four observations recently measured *(E.Bowell 1982)* were used with unit weight in α and a mean weight of 0.6 in δ.

About 400 observations are published on the apparition of 1835. These are, however, of very different quality. The observations by *J.Maclear (1837)* at Cape, for example, immediately show a scattering of some 1^s in subsequent right ascensions, while in those by *J.F.W.Herrschel (1837)* the observation times are given only to one minute. Other observations by different observers are either from rather poor observation series, or have shown both before and after re-reduction large residuals. Some of the most favourable observation series are these by *J.Encke (1838)* at Berlin and *J.Lamont (1843)* at Bogenhausen. From the first observer, after removing each three right asecnsions and declinations with much large residuals of the 25 observations which were re-reduceable using modern comparison star positions, there remains 44 measurements between 1835 September 18 and 1836 March 19 with a mean error of 4".7 . From the latter observer, 23 observations 1836 January 14 to May 17 are available, but for the largest part the comparison stars are not contained in modern star catalogues. However, after a preliminary re-reduction using star positions partly from earlier sources, the mean error of about the half of the observations was 5".4 so that principially this observational serie can be used. For the present investigations, however, it appeared senseful to use exclusively the rather precise observations by *F.W.Bessel (1844)* at Königsberg and *F.G.W.Struve (1839)* at Dorpat (two very experienced observers) instead of a lot of much more uncertain or only limited re-reduceable observations. An attempt was made to re-reduce their observations by means of modern comparison star positions. But first, only the half of the used comparison stars are contained in modern star catalogues, and secondly, after the reduction and a subsequent orbit fit the residuals of these observations have been increased, so that obviously the star positions computed back over one century are worse than the positions obtained with special effort in that time by the two observers. Thus, finally 27 normal places performed by *H.Westphalen (1847)* were used and reduced from the system of the 36 clock stars to FK_4. They received weights of 0.6 each.

From 1759, 150 observations are published which scatters by about $0\overset{'}{.}6$. With completely sufficient accuracy, differences to an ephemeris obtained during an earlier investigation *(O.A.Rosenberger, 1830)* have been plotted and the ephemeris corrections for three normals taken from this figure. The calculated mean errors are 4" to 8" and thus weights of 0.1 in mean are adjoined.

For the appearence 1682 only measurements of distances of the comet to surrounding stars are published instead of equatoreal position values, so that here three normals were generated by computing them from orbital elements which have been derivated earlier from these measurements by *O.A.Rosenberger (1830)*. Because the accuracy of the measured distances is of order 1', this orbit and the normals represent them without loss of information. The weights have been choosen to be 0.1 . Similar like to 1759, the times of the three normal places were choosen under consideration of the distribution of the observations.

For 1607, one normal place was generated by means of ten observations reduced by *F.W.Bessel (1804)*. Two observations each in α and δ and the last observation completely have been ignored, and the time of the first observation was corrected *(T.Kiang, 1972)*. The corresponding weight was calculated to 0.0004, but the perihelion time is determined to 1607 October 27.5196 $\pm 0\overset{d}{.}0043$ TDB which strongly suggests taking this appearence into account in the computations, too.

For the present apparition, all observations published at the time of the individual computations were processed, up to those given on

IAU-Circular No. 3914 used for the most recent results. Because of their uncertainty, the observations of the recovery night made at Palomar Mountain, and on 1983 December 31 at Hawaii were not used. Although they have lower residual noise, all these observations at larger heliocentric distances received one weight unit only, so as to avoid forging too many the results due to the errors of the used nongravitational force models (cf. chapter 2 and 3 below).

Besides these standard data, there exist a collection of 663 individual observations from 1759 on (cf. table 10), which is in use by the 'International Halley Watch' (IHW) as base for its computations. These data correspond, with a few exceptions, to the values originally published by the observers, and have been reduced to the equinox 1950 and into astrometric positions, but without any correction to a common reference system. Moreover, accuracy and weight of the different observational series are not valued sufficiently. Because of these and some other considerations, these data were not prefered about the normal places explained above, and have been used only for comparisonal purposes.

2. Some aspects refered to orbit determinations of comets

With this chapter it is intend to examine some problems in connection to orbit computations of comets, with especial regard to the present work.

2.1. Nongravitational forces

Besides the attraction by sun and the planets, the motion of the comets is also influenced essentially by the repulsion forces due to gas sublimation on the surface of the cometary nucleus. Quantitative theories for the forces are given by *F.W.Bessel (1836)*, *B.G.Marsden and Z.Sekanina (1968-1972)* and especially for comet Halley by *H.Rickman and C.Froeschle (1982)*. The components of the nongravitational acceleration can be written in the form

$$b_i = g_i(r) \cdot A_i(t,r) \qquad 1)$$

(i=1,2,3 for the radial, toroidal and normal direction to the orbital motion). The $g_i(r)$ should be choice so that they contain the dependence of the acceleration components on the heliocentric distance completely, and should be normalized so that $g_i(r = 1\ \mathrm{A.U.}) = 1$. Then the A_i correspond to the nongravitational acceleration components at r = 1 A.U. and only include an explicite (secular) time dependence of the nongravitational forces.

The accurate shape of the $g_i(r)$ depends on the direction of the sublimation center at the nucleus and thus on the orientation of the rotational axis, on the rotational period, and on the thermal inertia of the cometary nucleus, as the sublimation center is shifted from the subsolar point by an lag angle $\alpha(r)$ in direction of the rotation. This, in general, causes an asymmetrical lapse of the $g_i(r)$ before and after the perihelion transit, if not then the orbital normal vector and the rotational axis lie in one plane at perihelion transit. For the individual comets, these circumstances in particular are widely unknown, and thus only a very rough overall treatment on computation of the nongravitational forces is presently possible.

Bessel assumed constant values of the g_i and A_i during short time spans. *Marsden and Sekanina* equalized $g_i(r)$ with the sublimation rate of water ice according to *A.H.Delsemme (1982)*. This implies a rotation axis perpendicular to the orbital plane and thus a constant lag angle $\alpha = \arctan(A_2/A_1)$ and $A_3 = 0$. However, in general, $A_3 \neq 0$ and a secular variation $A_i \sim e^{-B_i t}$ was considered. *Rickman and Froeschle* carried out model computations for several assumptions about rotational period and axis orientation, chemical and physical composition and other parameters. From the local surface temperature and the resulting sublimation rate and velocity, the magnitude and direction of the nongravitational accelerations $b_i(r)$ is obtained by integration over the whole surface. For seperation of the A_i, the $g_i(r)$ *ad hoc* were equilized to *Delsemme's* formula, too, so that here the A_i depend considerably on r. Except for uncertainties in the other assumptions, the results depend mainly on the thermal inertia I_{th} of the cometary nucleus. This especially is the case for the ratio $A_2(r)/A_1(r)$ whose average value can be determined from positional observations. Furthermore, these model computations gave negligible effects into the the orbital motion due to A_3.

The nongravitational forces produce difficulties on the orbit computations due to the following reasons.

I. The models quoted above satisfy only marginally the entangling circumstances. On the theory by *Marsden and Sekanina*, the usually more or less unknown rotational parameters remains *a priori* out of account , and a point of large uncertainty on the more explicite models. The chemical composition and the accurate surface temperature distribution, and correspondingly magnitude and direction of the sublimation are known only very approximately. On the models by *Rickman and Froeschle*, in the present state also further essential effects have not been taken into account, e.g. a dust layer on the surface, multiple scattering within the coma, and inertial hot sources in the nucleus. Observations from 1984, 1910 and 1835, especially the careful observations by *F.G.W.Struve (1839)*, show evidence that the widely accepted rotation period of 10^h, which has been adopted for a part of the model computations, probably is much too small. One of the above authors *(H.Rickman)* kindly communicated to the author the results of model computations using 50^h rotation period, and remarked that further model computations under consideration of the explained and further new aspects are in work.

Besides the global dependence of the nongravitational forces on the heliocentric distance, also the essence and magnitude of short- and long-term fluctuations of the forces are unknown, but principially we have to expect such in connection with the observed optical activities like magnitude bursts, jets etc. Considerable activities on comet Halley were observed in 1910 and 1835, and these, together with reports from earlier apparitions about a tail division, suggest considerable activity and essential changes on this comet. On the other hand, the nongravitational parameters are comparatively small - the mean acceleration is of order 100 m/d^2 - so that fluctuations even of five times the averaged forces would need about one month to produce an observable position shift of order 1", and this would, for the most part, be canceled by continued observations and orbital fits. Because of such fluctuations, if they do not become a part of the global models of the forces, we have to consider much shorter durations as they produce no positional errors of importance (cf. chapter 5.2.1).

II. Even if the $b_i(r,t)$ would be accurately known, difficulties would appear in the estimation of the parameters we have to compute, e.g. A_1 and A_2 according to *Marsden and Sekanina*, or in the necessary correction coefficients of the $A_1(r)$, $A_2(r)$ by *Rickman and Froeschle* or similar models, which hereafter also are designated with A_1 and A_2.

To better explain these circumstances, one might reflect upon table 1. It refers particulary to nongravitational forces according to *Delsemme*'s formula, however, the essential conclusions are qualitatively valid for any symmetric force model. The nongravitational perturbations of the single orbital elements during one revolution can ensue before and after the perihelion in the same or in opposite sense. In the latter case, the difference in the value on time of perihelion transit at one and the same heliocentric distance is the same before and after it, so that the nongravitational forces only produce an inequal motion during one revolution. In the first case, however, contrarily is produced a remaining secular change at each revolution.

Table 2 gives the corresponding values for comet Halley. As we have to expect for any central force $b = r^n$ with $n \neq 1,-2$, the only secular perturbation by A_1 is a perihelion motion. The small change of the perihelion time we have to interprete as the duration which the comet need to pass this 1.4 perihelion shift. A_2, however, produces an increase of q and e on each revolution, corresponding to a delay of 4.2 days each revolution. Temporary changes during one revolution caused by both A_1 and A_2 arise in all elements. The changes due to A_1 in q and e from perihel to aphel, which disappear until the next perihel again, amounts approximately to three times of the corresponding changes due to A_2, which again corresponds to the half of the delay of four days until the next perihel. Thus, A_1 produces a considerable deviation of a few days in the motion of the comet near its aphel compared with the unperturbed motion; in case of a negative value of A_1 the comet is too late. The intristic cause is, that in large distances the radial nongravitational force component acts nearly parallel to the motion of the comet, and thus any positive A_1 accelerates the comet towards its aphel until that is reaches, but after this it counteracts its free fall to sun. In smaller heliocentric distances ($r < 3$ A.U.), the temporarly changes in the motion by both A_1 and A_2

are only poor perceptible and vastly representable by a slightly changed value of the excentricity. In practice, this causes a strong correlation of values determined for e, A_1 and A_2 if we have only observations in close heliocentric distances.

During these considerations it was assumed, that the nongravitational forces lapse symmetricaly in the ascending and descending part of the orbit. If this is not the case, also the perturbations in the elements progress asymmetrically, and, especially, a secular change in the revolution time produced by A_1 must be expected. On comet Halley, if A_1 before the perihel is by 0.01 larger than after it, this effect already would amount to +0.58 days.

Moreover, we might shortly consider the perpendicular force parameter A_3. If the excentricity is not small, the perturbations mainly happens close the perihelion and on each revolution in the same direction, so that we have to expect secular changes of the orbital plane orientation. The temporarly changes due to A_3 are much smaller than these by A_1 and A_2, the difference of q and e on the aphel compared with the values on perihel only amounts to $+1.9\cdot10^{-9}$ and $+6.4\cdot10^{-9}$ per $A_3/0.10$, respectively. The perturbations in Ω and i before and after the perihel lapse differently, because the nodes are not located symmetrically to the apside line (ω=112°). The secular perturbations per revolution amounts to $\Delta\omega = +1''.97$, $\Delta\Omega = +2''.07$ and $\Delta i = -0''.25$ per $A_3/0.10$. The change in the arguement of perihel is essentially changed by that of the node; in a resting reference frame the perihelion moves essentially perpendicularly to the orbital plane, corresponding to Δi.

For the determination of the nongravitational parameters in case of certainly advanced $g_i(r)$, the above considerations allow the following conclusions to be drawn. A_2 is well determined with a high degree of accuracy by three or more observed appearences of the comet and the perihelion times implied thereby, because a secular change of the revolution period is explainable neither by the classical orbital elements nor by A_1 (assuming symmetric g_i), and thus no correlation of A_2 with other unknowns occurs.

For comet Halley, by the perihelion times in 1759, 1835 and 1910 the increase of the revolution period of about four days is determined accurately to a few minutes, which corresponds to a relative accuracy of 0.1% in A_2. This effect is the most essential nongravitational effect in the motion of comet Halley *(P.H.Cowell and A.C.D.Crommelin 1910, T.Kiang 1972)*.

Principially, A_1 can be computed most accurate from the perihelion shift between at least two apparitions. On comet Halley, however, this is not possible with sufficient accuracy before the perihelion transit in 1986, because the longitude of the perihel in 1910 is determined with a mean error of $\pm 0''.5$, that in 1835 by $\pm 1''.4$, and thus presently the uncertainty still is of order of the perihelion shift we have to expect for an amount of $A_1 = 0.10$. Although in practice, in case of accurate calculation of the equations of condition this effect certainly is taken into account, too, at the present state the determination and the results of A_1 mainly depend on the few recent observations in large heliocentric distances. Whilst a seperation of A_1 and e is always uncertain if observations are available only from low heliocentric distances, thanks to these far distanted observations this becomes possible because the considerable deviation from mean motion in larger distances, as explained above. In case of symmetric g_i, the analogous effect caused by A_2 do not make trouble because its certain knowledge.

Because a motion of the orbital plane is not obtainable by another one of the unknowns, A_3 is determinable without principial difficulties if it is not very small and produces only unobservable effects. However, until there remain small residuals due to our insufficient force models, especially at large distances and not lapsing along the line of variation precisely, on orbit determinations it can easily happen that these will be partly compensated by a small change of the orbital orientation for the different appearences and thus by a falsified value for A_3. Apart from this, in case of comet Halley the limits of accuracy for A_3 mainly are set by that of the available observations.

III. The present situation in practice is, however, that neither the true lapse of the $g_i(r)$ is known, nor have we enough observational information to seperate well all unknowns even in case of an ascertained lapse. There are many examples for periodic comets in which, after orbit improvements, systematic residuals remain. This strongly permit doubts on the adopted force law, even if these observations provide only marginal information for improvements only.

The fact that, besides the rather poor determination of A_1 especially on using observations from 1835 to 1984 only, we have to consider also errors of the adopted model for the $b_i(r)$ and its assumed symmetry, changes the aspects of parameter estimation discussed above essentially. First, the observed increase of 4.2 days in the revolution period must not necessarily be caused by A_2, but can also originate partly by A_1. Thus, A_2 is no longer very precisely determined and, similary, its influence on the observations at large distances what we need for the separation of A_1. Whilst in case of knowledge of the true force lapse, the far observations 1982 - 4 would be very important for seperate A_1 and e, in case of considerable erroneously models they become vast valueless, because A_1 or its values acting at, and also these computed from, large and low distances have nothing to do with each other. Because our lack of ability to recognize the accurate development of the forces until the perihelion transit in 1986, these far observations are presently inapplicable for accurate predictions. This is the result of fitting a wrong model to these observations, as we then would have to expect a corresponding error in the result.

These essential differences between the previously assumed ideal but not present conditions must be admitted. On comet Halley we presently have the situation that sometimes from observations 1835, 1909 - 11 and 1982 - 4 , elements inclusive the perihelion time for 1986 have been computed and the results published. These observations are quite sufficient to seperate the unknowns within a certain force model, so that it is not surprising that there remains no systematical residuals. However, first this does not permit the conclusion that thus the model used *ad hoc* (*Delsemme*'s formula) certainly reflects the true force lapse well. Secondly, the obtained low mean errors calculated for the unknowns are only correct within a certain model and yield of a too high accuracy of the results. The formal mean error of the perihelion time in 1986 appears to ±0.008 days, but the full error of the model enters in the calculated perihelion time as explained above, so that its true error can be ten times larger. If we use the observations of three or more previous apparitions, the perihelion time follows much more precisely and rather independently on the adopted force laws and their errors, and differences between the far observations permit conclusions about the favourable ones of the models instead about the perihelion time in 1986. In fact, presently the latter is not better known than before the recovery of the comet in 1982.

Any quasistatic force model, which assumes the same force lapse on each revolution without secular or essential short-term changes, produces independently on its explicite form on every revolution the same delay in the subsequent perihelion transit time. On comet Halley, however, the amount of 4.2 days does not remain constant, but increased by about one hour each revolution, as followed significantly by the observations back to at least 1531. This cannot be explained due to our inability to recognize the force lapse, but suggests a secular increase of the nongravitational forces by 1.1% per revolution. Thus, a further parameter B has been introduced, in order to describe the time dependence by

$$A_i(t) = A_i(\text{epoch})\cdot(1-B\cdot t) \qquad 2)$$

for which sign and time unit (10000 days from the epoch) have been choosen in accordance to a similar parameter used by *Marsden and Sekanina*. In order to avoid a secular change in the lag angle, as well as in the delay of one hour which on the asymmetrical force models can be caused by both A_1 and A_2, B has been refered to both parameters commonly.

During application of different models to the observations, the results for B changed only minutely and were always determined well. There are some physical reasons which let us expect such a change of the nongravitational parameters. Due to sublimation, on each appearence the cometary nucleus decreases in size. If its constitution, and thus the sublimation rate per surface unit element, is assumed to being constant, the nongravitational acceleration increases with the ratio of surface to mass or volumina, or indirectly proportional to the radius of the nucleus.

In the case of a radius of 3 km, the sublimation of a 30 m thick layer per revolution, which is necessary to explain the force increasement of 1%, appears somewhat too large. However, at least 20% or so of the observed effect, or about ten minutes in the perihelion time, must be expected. Another effect is, that with progressing decay of the comet, the lag angle increases. If we assume - for a very rough valuation - that $A_2 \ll A_1$ and that the increase of the ratio A_2/A_1 by a factor ten, as observed on many old comets, happens within thousand revolutions, there would appear an increase of A_2 and of the corresponding revolution time increment of 1% per revolution. Certainly, under bad circumstances this conclusion is invalid (e.g., if the force law is very assymetrical and thus the revolution time delay mainly is caused by A_1 instead of A_2, we would have to expect an decrease instead), but in any case it is obvious that, in general, secular changes of the forces of this amount are possible and have to be considered. A further cause can be a change of the inclination between the orbital plane and the cometary equator, caused either by planetary perturbations of the orbit or by precession of the nucleus. The different reasons which allow us to expect changes of one hour in the nongravitational revolution time delay are a further strong approach to prefer solutions from observations 1607 to 1984 before such from 1835 to 1984, because rather independent on the particularly physical reason, the half of this effect (corresponding to a half hour in the perihelion time) still will be originated until February 1986 and thus is not contained already in the 1982 - 4 observations.

2.2. Displacement of the light center on observations to the nucleus of the comet

If we consider the observational residuals after orbit improvements of comets, systematic values during some time intervals become visible in some cases and are not explainable in the usual way, e.g. by errors in the comparison star positions. Usually this is interpreted as the presence of an offset between either the image center or its brightest point measured, to the cometary nucleus, which we have to expect because the gas emission and points preferably in direction towards to sun. However, neither quantitative investigations nor a theory of this effect has been presented. The residuals, however, permit the following conclusions: a) The light shift approximately points along the radius vector to sun. b) Its magnitude is not in simple relationship to the heliocentric distance, but obviously depends strongly on the observers and on observational circumstances. On very short exposed plates, the effect in whole is smaller than on longer exposed ones, and in many cases none systematical residuals appear in observation series with very short exposure times and stellar images of the comet. But also it might be important, if the position was obtained by a densitiometric measurement which gives always the brightest point of the coma nearly independent on the exposure time, or by visual measurement of the geometric center of the image. In the latter case, in addition to the dependence on the exposure time, physiological errors along symmetry direction of the comet (or correspondingly approximately along the radius vector to sun) might possibly occur. Probably there is also a dependence on the spectral range of the exposure because the different positions of brightness center.

Although in medium-term sight we do not expect that these circumstances will be cleared in detail, the question arises if this effect cannot be taken into account anyhow during orbit computations. In the literature, an offset along the heliocentric radius vector is usually assumed, and for its magnitude one adopts a radius dependence $S(r) = S_\circ \cdot s(r)$, where $S_\circ$ is an unknown parameter which we have to determine together with the orbital elements using different assumptions for $s(r)$. Because of the above considerations, however, such an onset probably is rather valueless as the dependence on r is obviously much less than that on other facts.

A possibility to eliminate the effect rather independently in assumptions on its magnitude, is a transformation of the equations of condition for the orbit improvement from right ascension and declination to position angle and appearent angular distance to sun, and the exclusive use of the equations for the position angles only. The latter ones are not influenced by any radial offsets, whilst the final residuals in the angular distances to sun reflect the magnitude of the offset in the single observations and perhaps permit conclusions on its dependence from observational circumstances. The transformation of the equations of conditions to position angle θ and angular distance ψ can most simply be performed by

$$\begin{pmatrix} d\Theta/\psi \\ d\psi \end{pmatrix} = \frac{1}{\sqrt{a^2+b^2}} \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \begin{pmatrix} d\alpha \cos\delta \\ d\delta \end{pmatrix} \qquad 3)$$

with

$$a = \frac{\partial\alpha \cos\delta}{\partial S} = \frac{y\cos\alpha - x\sin\alpha}{\Delta r} \qquad 4)$$

$$b = \frac{\partial\delta}{\partial S} = \frac{(-x\cos\alpha + y\sin\alpha)\sin\delta + z\cos\delta}{\Delta r} \qquad 5)$$

(x,y,z,r heliocentric coordinates and distance of the comet), or immediately by numerical elimination of S_o in the both equations for $\Delta\alpha \cos \delta$ and $\Delta\delta$ of each observation considered, but in regard to the normalisation of the weights.

By application of this method, the mean errors of the elements increased a few times. This is rather unchanged even if both positional values are used for some observations in larger heliocentric distances. Using observations from two or more appearences, the mean errors increased only minutely, most that of the perihelion time, and only in case of more than three apparitions independently if nongravitational parameters are estimated or not. The cause is, that by more than one apparition, the semimajor axis, and by the position angles during the single appearences the angular motion in the orbital plane and thus the perihelion distance and excentricity are determined well, better determinations than these being possible only by observations of the true or appearent distance to sun. Hence, the elimination of the offset is possible, as the concerned distances enter only minutely into the computations in case of more than one apparition. However, this is only correct for sufficient orbital inclinations, because on the limit $\sin i \to 0$ the position angles to sun become meaningless and thus on low inclinations the uncertainty in e,q and especially T would be increased considerably. Because the uncertainty of predicted ephemeris places mainly depends on that of the perihalion time, then it also can increase several times. In practice, we have to check individually if, in the considered case, the increase in position uncertainty on ignoring the elongations becomes larger than the light offset we have to expect.

A remaining question is, how accurate the assumption of a light offset along the radial direction is at all. Presently, at least based on the available astrometric data of comets, this cannot be ascertained better than over the indication quoted above, and thus this assumption must be used in order to having a working hypothesis. Perhaps in the future the application of different assumptions to observations of comet Halley on smaller heliocentric distances will give improved verdicts.

3. Further details on the performance of the computations

The used observations had been discussed in section 1, and here it is intended to explain some further bases and details of the computations.

The computations were carried out by means of a computer programme which the author has performed six years ago and which has established itself well since without essential modifications. First, initial values of the planets are read in and are integrated to the osculation epoch choosen for the comet. To integrate the planets, too, has of course the disadvantage of much increased computation time, but on the other hand it is not necessary to use tapes containing planetary coordinates in case of long-term integrations and, moreover, the procedure remains flexible in regard to the integration step width and to the choosen planetary theory. Afterwards, preliminary values for orbital elements of the comet and further unknown parameters, e.g. the nongravitational parameters, are read in, and also the observations. These are reduced to the FK_5 system so far as presently possible by correction of the main term of elliptic aberration, of the equinox and precession, and for the proper local errors of the FK_4 system according to modern meridian observations of stars. For the nongravitational force lapse in the programme different theories are incooperated which are coiceable by an input digit (presently those according to *Marsden and Sekanina / Delsemme* and *Rickman and Froeschle*) and which can easily be exchanged for other ones on demand. Values of the parameters $A_1, B_1, A_2, B_2, A_3, B_3$, the orbital elements, and further ones, or of any linear combination of these, can be either considered or estimated. The computation of the coefficients of the equations of condition have to be performed rather precisely because of the strong correlation between some of the unknowns, as explained in chapter 2. A method of great flexibility and accuracy, well established also in difficult cases and on a large number of unknowns, which calculates the partials by integration simultaneously together with that of the celestial bodies, was used for this *(Landgraf, 1983b)*. In order to avoid large residuals and to support the convergence during the searching of sufficient initial values of the unknowns for each force model, in most cases the osculation epoch has been placed on 1910. First was integrated forewards, then backwards, to compute both the cometary position and the equations of condition for all observation times decreased for the light time. An integration step size of 0.625 days was found to being most favourable for integrations back to 837, and used throughout. For solving the conditional equations, the Method of Least Squares was adopted. The complete procedure was repeated until a certain accuracy or number of iterations was attained. The used criterion was the comparison of the remaining error square sum guessed during the solution of the normal equations with the value later obtained by the new residuals of the observations. Because of the accurate computation of the conditional equations, the convergence was very good. Also in cases of rather bad starting values, only rarely more than two iterations and one computation of the equations was necessary.

For the planets Mercury to Neptune, initial values obtained by the Institute for Theoretical Astronomy at Leningrad (ITA) have been used *(G.A.Krasinsky, E.V.Pitjeva, M.L.Sveshnikov and E.S.Sveshnikova 1982)* and refered to the FK_5 system and reciproce mass values for Jupiter, Saturn and Uranus of 1047.348, 3498.0 and 23030, respectively. For comparison purposes, initial values of the theory DE119 by the Jet Propulsion Laboratory at Pasadena (JPL) have been applied. In the first case, standard coordinates of the Schwarzschild metric and in the second case isotropic coordinates are used, so that the motion of the comet was integrated in, and the presented results refered to, the corresponding ones. The position of the moon have been computed geocentrically, and the perturbations by earth and moon seperately taken into account to all other bodies and the earth-moon barycenter. For the transformation of the observation times from UT to TDB, values obtained recently by comparison of the lunar theory LE200 with observations have been kindly communicated by *P.K.Seidelmann*, US Naval Observatory, Washington.

4. Results of the computations

Since the commencement of the work on comet Halley, a large number of trials and computations were carried out during which the influence of the unknowns on each other and these on the bases have been lighted out, and several perceptions for further work have been collected.

However, because of the expense, it neither was possible nor necessary to always repeat previous computations after some additional observations were published. Thus, a few of the results represented below are refered not to the most recent state of the available observations, but this does not essentially defeat the mainly conclusions of these computations.

4.1. Elements refered to nongravitational forces according to the theory by Marsden and Sekanina (Formula by Delsemme)

The last results obtained during application of the formula by *Delsemme* for the force lapse are given in table 3. The elements no. 1 and 3 have already been published in MPC 8665.

The first two orbits have been computed using observations from 1607 to 1984. In contrary to the solutions based on 1835 - 1984, the observations in 1984 exhibit a systematic residual of $-1\overset{''}{.}8$ in right ascension and $+0\overset{''}{.}7$ in declination in mean. However, under consideration of the connections explained in chapter 2, we must assume that the perihelion time in 1986 is determined better by the previous perihelion transits than by far distanced observations connected with an *ad hoc* assumption about the force lapse until the perihel, and that any residuals have to traced back to insufficiency of the latter. This also is confirmed by the fact, that, on adopting other models (cf. chapter 4.2) , these residuals decrease by $+0\overset{''}{.}4$ and $-0\overset{''}{.}3$ respectively in both coordinates, while the corresponding perihelion transit predictions based on the same observations differs only within a few $0\overset{d}{.}001$ without any obviously dependence on the remaining residuals (see table 6). These residuals let us conclude that the nongravitational at larger distances decrease much more heavily than according the formula by *Delsemme* (and also than according to the models of *Rickman and Froeschle*), in agreement with the observed activity of comets at large heliocentric distances, and even perihelion distances of about 6 A.U. Because the residuals try to decrease the result for the perihelion time, the true value might be later than computed, and perhaps falls into the interval 1986 February 9.51 to 9.55 TDB. We cannot expect that this will be cleared accurately before the comet reachs lower distances ($r < 3$ A.U.). Then, due to the lack of improved force models, most precise computations of the observations in large distances should be either excluded completely, or, the two equations of each far observation should be transformed to one equation for the unknowns except of the perihelion time, by elimination of the latter. Moreover, solutions from observations 1835 to 1985 ... may then become preferable because of better elimination of systematic errors due to long-term variation of the nongravitational parameters.

On consideration of the previous and following results, the first orbit in table 1 must be considered as the most favourable result on the present state of the observations.

The second solution corresponds to the first one, with the exception that A_3 have been added as a further parameter. For that, however, a solution from observations 1835 to 1984 might give a more trustworthly result (orbit no. 6).

The third orbit is based on observations and normal places 1835 to 1984 only. In this case, the residuals in 1984 are $+0\overset{''}{.}2$

in α and -0".2 in δ. Alltogether with the observations 1982-3, no systematic residual remains, but, as explained above, this is no reason to prefer this solution and the following ones to solutions 1607 - 1984. This is confirmed again by the backward integration results given in table 5 and their comparison with the observations. The differences of -0.5, +0.7 and +8 hours in 1759, 1682 and 1607, respective, due to an average of B by other unknowns in solutions 1835 - 1984 does not correspond to the values of 1 h/rev.^{2} we would expect, but neverthless they clearly suggest the need of taking the time dependence of the parameters for precise predictions into account. After introduction of B as an additional parameter, not only the motion 1607 - 1984 can be described without significant differences to the observations, but also the well determined perihelion time in 837 (see part 4.4 below and table 9) is represented accurate to -0.9 days, so that this parameter is vindicated and determined well and in good aggreement from several earlier apparitions (cf. table 4).

The fourth orbit, like all further ones, bases on the IHW data for 1835 to 1984, and on adoption of the weights like advanced in table 10. On the fifth orbit, the planetary initial values have been taken from DE119. It is remarkable that the improved orbits computed in connection to the ITA initial values, now in 1835 exhibit deviation from the observations by up to 0°.5 . Like the differences between the backward integration of orbits no. 4 and 5 according to table 5, this is mainly due to the different mass values for Uranus (1:23030 on orbit 4, 1:22960 on orbit 5, cf. also part 4.3 below). This has to be taken into account in case of employment of the results presented here in connection with other planetary initial values.

The sixth orbit corresponds to the fourth one but with addition of A_3. The latter is similarly well determined like in orbit no. 2, but differs in the result. In cases of essential differences between the adopted and the true lapse of nongravitational forces, the results for the parameters depend very much on the distribution of the used observations and on their residuals. Because in the solutions 1607 - 1984 the residuals due to insufficient modellation of the forces are partly fitted by changes of A_3, too, but on the other hand only in 1835 and 1910 the orbital plane orientation is ascertained well enough to permit a determination of A_3 and thus for this parameter (in contrary to the computation of A_2 and B) the earlier apparitions do not achieve an improvement, the result for A_3 from 1835 - 1984 has to prefer to that from solution 2. However, the only thing presently ascertained is that the absolute value of A_3 is very probably less than 0.1 , in agreement with the expections considering the low values of A_1 and A_2.

The seventh and eight solutions have been carried out under consideration of a possible light shift in the observations. In the first case, in analogy to the literature, a dependence s(r) only on the heliocentric distance of the comet was assumed. For this, the formula of *Delsemme* was choosen again (with $s(r = 1 \text{ A.U.}) = 1$). The result was $S_\circ = -326 \pm 75$ km , so that the brightest point of the coma is displaced towards to the sun, in agreement with our expections. The nongravitational parameters and their mean errors changed only minutely, in agreement with the fact that we do not expect any strong correlation with $S_\circ$. These results are contradictionarly to those obtained at the European Space Operations Centre at Darmstadt (ESOC). An offset of nearly the same amount, but in the opposite direction, and an increase of the mean error of A_1 by 2.5 times *(T.A.Morley, 1984)* was computed under somewhat different suppositions. In case of orbit 8, at heliocentric distances below 2.0 A.U. only the position angles of the observed positions to sun were used. In agreement with the expections on the use of this decreased observation matter, the mean error of an unit weight decreased slightly (2%), and that of the different elements increased slightly, by up to 1.5 times (for A_1 and T). The differences in the residuals of the elongations between this solution and solution no. 3 amounts to only a few 0"1 . This is much below the strong scattering of the residuals of the IHW observations, so that they obviously are mainly caused by the different sources of the comparison star positions used, or by other errors. Because of this, but also because of the fact that single observational circumstances like the exposure time are in most cases unknown, it is not possible to draw any essential conclusions about the light shift from these data.

In general, during the progress of the investigations it was noted that more and more decreasing values for A_1 (down to negative values) and for the secular increment of the forces, -B, resulted due to an increase of the observations used (either by adding earlier apparitions, or additional recent observations). A few examples for this are given in table 4.

The only published orbit which is also based on observations 1835 to 1984 was computed at ESOC *(T.A.Morley, 1984)*. In contrary to our solutions no. 4ff., the four recently measured observations of 1911 mentioned in chapter 1 have not been used, and the weights differed slightly (see table 10). The planetary coordinates have been taken from the theory DE118, which corresponds to DE119 but is refered to the FK_4 system at epoch 1972.5 . These earth coordinates, inertially at rest, have been combined with cometary observations refered to the rotating FK_4 system. Because of the somewhat different bases, these results are only very limited comparable with ours. The difference of $0\overset{d}{.}005$ in the result for the perihelion time to our solutions 4 to 8 corresponds to a position shift of $0\overset{''}{.}1$ in the 1982 - 4 observations, and thus is within the limits to be expected because of the corrections to the FK_5 system on the latter solutions. The other elements agree with ours within their noise, and also the mean error and the residuals of the observations are in good agreement.

4.2. Elements refered to nongravitational forces according to the models by Rickman and Froeschle

The results obtained on application of the models by *Rickman and Froeschle (1982 and private communication)* are given in table 6; A_1 and A_2 are the necessary correction factors of the model values. The results for A_2 correspond to the ratio of the value yielded by *Delsemme*'s formula to that according to the applied models at r = 0.6 ... 0.8 A.U., so that this interval points out the averaged value of the forces.

All the solutions 1835 - 1984 fit equally well the observations at large heliocentric distances. The representation of the earlier appearences differs largely. Although the representation is clearly better than by the *Delsemme* formula (no. 3 in table 5), it is not possible to prefer some of the models from these results. Considering the results for A_1 and A_2, the models of higher thermal inertia, which also gave the lowest mean residual of the observations, appears to be most favourable.

By introduction of B and using observations from 1607 to 1984, again it was possible to represent all observations well. The good agreement of the well determined result for B confirm it's justification again. The mean residuals of the observations are only a little better than on application of *Delsemme*'s formula, but the representation of the far distanced observations is significantly better, especially on the models of higher thermal inertia. Because the results for A_1 have been very low during the application of the rather different models, we must conclude that the radial force component is truely negligible. Furthermore, on use of these models A_1 and B again decrease with increasing number of apparitions and observations used. For example, using the first one of these models, from observations 1759 - 1983 resulted $A_1 = +0.43$, from 1607 - 1983 $A_1 = +0.12$.

Altogether, we can conclude that the secular increase of the nongravitational forces of comet Halley is well determined, that A_1 is nearly zero and thus the position errors at far distances are mainly caused by an essentially asymmetrical lapse of A_2 instead of any A_1 , that the nongravitational forces reach to larger distances than represented by all available models, and that the *Rickman - Froeschle* models are clearly favourable to the *Delsemme* formula, but that also these models still are far away from a representation of the true force lapse. For more detailed conclusions, however, further observations and models are urgently necessary.

4.3. The influence of the masses of Uranus and Neptune

During the computations it have been noted that, besides other influences, the results rather depended on changes of the adopted mass values for Uranus and Neptune within the limits of their uncertainties. This is caused by the fact that these planets have approximately the same and double revolution time like the comet, respectively, and that on each or every second one of the last stayings of the comet on the far parts of its orbit, these planets have been in similar heliocentric direction.

Table 7 gives the changes in earlier perihelion times corresponding to solutions 1759 - 1983, due to variation of the reciproce mass values by +50 units and due to variation of B_2, respectively. At the time of performation of these computations the latter have been used, and contrarily to the later used B it is only refered to A_2; however, because of the poor determination of A_1 both values coincide.

Here would be the wrong place to carry through a discussion of the most probable mass values for Uranus and Neptune and their accuracy. Under consideration of the results of the different determinations (a review, for example, is given by *L.Ballani, 1981*), however, we can say that the uncertainty that is to be expected in the mass values for Uranus and Neptune, and correspondingly those in the computed perihelion time, are approximately one and two times of the range of table 7, respectively. A variation of the mass of Uranus within acceptable limits can make amends only for a small part of the results of B_2 or B, but the vast correlation between both parameters has the practical advantage that errors in the adopted mass for Uranus are compensated by inclusion of B as a further unknown and use four or more apparitions.

From the observations 1607 - 1984 in connection with the perihelion time in 837 (see part 4.4), a good seperation of all parameters considered in table 7 is possible, and the reciproce mass of Uranus is presently determined accurately to ±40 units. For reliable results, however, it is better to wait until improved force models and observations close the present perihelion are available.

4.4. The perihelion transit in 837 and the long-term motion of the comet

To both check the different force models and investigate the long-term motion of the comet, a well determined perihelion time of a much earlier apparition would be of very large value. It was noted that for this the apparition in 837 can be used.

The comet passed the earth on 837 April 10.63 TDB at only 0.0325 A.U. distance and, because of the differential perturbations, a variation of the accurate time of the encounter by only 0.1 day would produce differences of some days in the previous perihelion times. In table 8 the results of backward computations until 141 are compared with the observed perihelion times *(T.Kiang 1972, I.Hasegawa 1979)*, starting with the assumptions T = 837 February 28.40 and 28.44, respectively. All other elements are taken from a backward computation of an earlier 1607 - 1983 solution (see table 9). Corresponding to the observed perihelion times in 607, 530, 374 and 141, the perihelion time in 837 has been between 837 February 28.43 to 28.48, and the good agreement suggests that the influence of possible inequalities in the motion of the comet to the result can be only very small. By exclusive use of the observed perihelion time in 141, which formaly gives the most accurate result and, moreover, must be prefered as base of a continued backward integration, we get 837 February 28.427 TDB, or 28.424 TDB at epoch 837 March 10, with an accuracy which is very probably better than ±0.010 days. Because, in particular, not the perihelion time, but the circumstances of the close encounter to the earth are determined by the previous motion, the above result has to be corrected by $\Delta T_{837} = -0\overset{d}{.}322 \cdot (\pi - 304\overset{\circ}{.}140)$, where $\pi = \Omega - \omega$ refers to the epoch 837 March 10 and to the FK_5 equinox at B1950 *(W.Landgraf, 1983c)*. This result for the perihelion transit in 837 is slightly later than that obtained by the observations which have been made in that time *(I.Hasegawa, 1979)*. Backward integrations of several solutions 1607 to 1983 - 4 gave 837 February 27.1 - 27.6 TDB and thus are approximate one day too early.

Starting from this improved perihelion time for 837, subsequently the motion of the comet was computed back to 2300 B.C. The results are given in table 9. As is to be expected, the perihelion times back to 141 are satisfied completely, and also the well observed appearence in 12 B.C. with an difference of four days. The earliest ascertained apparition is that of 466 B.C. In Greece, the comet was observed in the second year of the 78^{th} olympiad in western direction and has been described among others by *Pline* and *Aristoteles* *(S.Lubienietzky, 1668, A.G.Pingre 1783, A.A.Barret 1978)*. In China it was observed on the second reign year of emporer *Ting Wang (P.Y.Ho, 1962)*. This corresponds to the time spans July 467 to June 466 B.C. and February 467 to January 466 B.C., respective, so that the comet must have been observed between July -466 and January -465. This is in agreement with our computed perihelion time, because, according to this, the comet would have been observable during the winter in western direction before caming into conjunction with the sun. The comet was also possibly observed in 618 B.C., but the corresponding report and its date are rather poor *(A.G.Pingre, 1783)*. For the earlier calculated perihelion times, no corresponding reports of an observed comet was found. Reliable evidence about the accuracy of the back computations are first possible after a repetition using an improved theory for the nongravitational forces. Because the comet often very closely (to a few 0.01 A.U.) encountered the earth, the back computations already in 466 B.C. are possibly so uncertain that a close encounter could have occured on this or a previously apparition so that the earlier motion happened rather differently than according to our integration.

q	dT/ A_1 (+)	dq/ A_1 (−)	de/ A_1 (−)	dω/ A_1 (+)
3.0	$-0^{d}.000044$	−0.00000009	+0.00000006	$-0^{\circ}.000013$
2.0	−0.000920	−0.00000419	+0.00000419	−0.000537
1.5	−0.001150	−0.00001053	+0.00001404	−0.001267
1.0	−0.000372	−0.00001511	+0.00003023	−0.001825
0.75	+0.000161	−0.00001506	+0.00004016	−0.001938
0.50	+0.000585	−0.00001290	+0.00005161	−0.001925
0.40	+0.000692	−0.00001138	+0.00005691	−0.001881
0.30	+0.000752	−0.00000945	+0.00006299	−0.001813
0.20	+0.000754	−0.00000706	+0.00007055	−0.001715
0.025	+0.000581	−0.00000130	+0.00010392	−0.001505

q	dT/ A_2 (−)	dq/ A_2 (+)	de/ A_2 (+)	dω/ A_2 (−)
3.0	$+0^{d}.000027$	+0.00000004	+0.00000045	$+0^{\circ}.000006$
2.0	+0.000969	+0.00000289	+0.00002006	+0.000375
1.5	+0.002086	+0.00001020	+0.00005013	+0.001294
1.0	+0.002347	+0.00002202	+0.00008093	+0.002943
0.75	+0.001906	+0.00002800	+0.00009437	+0.004088
0.50	+0.001128	+0.00003235	+0.00010764	+0.005607
0.40	+0.000754	+0.00003312	+0.00011354	+0.006410
0.30	+0.000369	+0.00003289	+0.00012043	+0.007426
0.20	+0.000004	+0.00003104	+0.00012952	+0.008853
0.025	−0.000287	+0.00001563	+0.00017904	+0.016923

Table 1 -- Perturbations in the orbital elements by nongravitational forces

The perturbations Δ(per-orig) of the elements in nearly parabolic orbits from great heliocentric distances until the perihel by nongravitational parameters A_1 and A_2 of the model by Delsemme are presented. If a (+) or (−) is indicated, the perturbations Δ(fut-per) from the perihelion until the following aphel are of the same or different sign, respectively. In the first case, a secular effect twice the given amount originates on each perihel, in the second case only a temporary perturbation. The proper part of the perturbations happens in the part with heliocentric distance below 4 AU.

a)

dT	dq	de	dω	dv	
$+0\overset{d}{.}00007$	-	-	$-1\overset{''}{.}39$	-	per $A_1/0.10$
-	+0.0000010	+0.0000033	-	$+4\overset{d}{.}15$	per $A_2/0.0160$

b)

dT	dq	de	dω	dv	
$+0\overset{d}{.}00003$	+0.0000014	-0.0000047	$-0\overset{''}{.}70$	$-5\overset{d}{.}76$	per $A_1/0.10$
-0.00003	+0.0000005	+0.0000016	-0.29	+2.07	per $A_2/0.0160$

Table 2 -- Perturbations in the elements of comet Halley by nongravitational forces according to Delsemme's formula

Part a): Secular changes of the elements $\Delta(per^{+}-per)$ between two revolutions. Part b): Changes in the elements from perihel until subsequent aphel, $\Delta(fut-per)$. The perturbations of the elements are performed mainly below 3 AU heliocentric distance, whilst the deviation Δv of the true anomaly from the unperturbed motion, caused by Δe and Δq accumulate until the aphel.

Nr.	T(TDB)	q	e	ω	Ω	i
1	1986 Feb 9.50762	0.5871048	0.96727910	111.84718	58.14364	162.23917
2	1986 Feb 9.50236	0.5871049	0.96727841	111.84768	58.14427	162.23928
3	1986 Feb 9.44912	0.5871014	0.96727328	111.84651	58.14386	162.23940
4	1986 Feb 9.45034	0.5871033	0.96727517	111.84690	58.14417	162.23933
5	1986 Feb 9.45032	0.5871030	0.96727592	111.84703	58.14424	162.23932
6	1986 Feb 9.45103	0.5871033	0.96727535	111.84654	58.14378	162.23928
7	1986 Feb 9.44843	0.5871041	0.96727510	111.84685	58.14418	162.23933
8	1986 Feb 9.45894	0.5871061	0.96727645	111.84720	58.14431	162.23929
ESOC	1986 Feb 9.44367	0.5871022	0.9672750	111.84690	58.14414	162.23938

Nr.	A_1	A_2	B	
1	-0.0138 ±0.0092	0.015964 ±0.000016	-0.00495 ±0.00018	
2	+0.0192 ±0.0100	0.015935 ±0.000016	-0.00472 ±0.00019	A_3=+0.0556 ±0.0159
3	+0.1232 ±0.0205	0.015516 ±0.000024		
4	+0.0768 ±0.0205	0.015519 ±0.000017		
5	+0.0581 ±0.0204	0.015504 ±0.000017		
6	+0.0728 ±0.0205	0.015521 ±0.000017		A_3=-0.0396 ±0.0143
7	+0.0768 ±0.0204	0.015512 ±0.000017		S_0=-326 ±75 km
8	+0.0439 ±0.0301	0.015549 ±0.000021		
ESOC	+0.080	0.0155		

Nr.	Observations arc	no.	σ	μ	K	S	P
1	1607 - 1984	91	1".43	1".25	S	FK5	ITA
2	1607 - 1984	91	1.43	1.25	S	FK5	ITA
3	1835 - 1984	84	0.94	0.92	S	FK5	ITA
4	1835 - 1984	662	2.15	1.02	S	FK5	ITA
5	1835 - 1984	662	2.15	1.02	I	FK5	DE119
6	1835 - 1984	662	2.15	1.02	S	FK5	ITA
7	1835 - 1984	662	2.15	1.02	S	FK5	ITA
8	1835 - 1984	(662)	2.11	1.00	S	FK5	ITA
ESOC	1835 - 1984	658	2.20	1.07	N	(FK4)	DE118

Epoch 1986 Feb 19.0 TDB, Equinox B1950.

Table 3 -- Orbital elements of comet Halley with nongravitational forces according to Delsemme's formula

K *reference coordinates for the elements:* N *newtonean,* S *standard- and* I *isotropic coordinates of the Schwarzschild metric*

S *reference system*

P *adopted initial values for the planets:* ITA *from ITA Leningrad (Mercury to Neptune),* DE118, DE119 *from JPL Pasadena (Mercury to Pluto)*

σ,μ *root mean square and unit weight residual of the used observations (cf. table 10)*

See also table 16.

Observations	T(TDB)	A_1	A_2	B
1835 - 1984	1986 Feb 9.449	+0.12	+0.01552	0
1607 - 1984	9.508	-0.01	+0.01596	-0.0050
1607 - 1983	9.529	+0.01	+0.01609	-0.0059
1759 - 1983	9.535	+0.08	+0.01610	(-0.0059 assumed)
1759 - 1982	9.474	+0.13	+0.01561	0
1682 - 1982	9.549	+0.10	+0.01619	-0.0067

Epoch 1986 Feb 19.0 TDB

Table 4 -- The dependence of the results for A_1, A_2, B and T_{1986} on the employed observation matter

Epoch (TDB)	T_{obs}	1	3	4	5
1986 Feb 19	1986 Feb 9	9.50762	9.44912	9.45034	9.45082
1910 May 9	1910 Apr 20	20.17871	20.17859	20.17849	20.17852
1835 Nov 18	1835 Nov 16	16.43953	16.43961	16.43953	16.43956
1759 Mar 21	1759 Mar 13.0628 $\pm$0.0012	13.05932	13.03703	13.04513	13.04255
1682 Aug 31	1682 Aug 15.2806 $\pm$0.0022	15.28158	15.30807	15.32121	15.30713
1607 Nov 13	1607 Oct 27.5196 $\pm$0.0043	27.51776	27.89802	27.91202	27.75982

Table 5 -- Comparison of the perihel times of the single apparitions

The given no. corresponds to those in table 3. In contradiction to the solution 1 from observations 1607 - 1984, the other orbits from observations 1835 - 1984 exhibit significant differences to the previously perihel times. The large differences for 1607 on orbit 5 compared with orbit 3 and 4 are mainly caused by the differently adopted mass values for Uranus (cf. table 7).

Table 6 -- Orbital elements of comet Halley with nongravitational forces according to the models by Rickman-Froeschle

All represented orbits were computed from the same observation matter 1607 - 1984 and the same further bases like solution 1 in table 3. $\Delta\alpha$ and $\Delta\delta$ are the difference in the representation of the observations from 1984 to that solution. Orbit 1 - 3 refers to models assuming $P_{rot}=10^h$ and $I_{th}=130, 500$ and 1000, orbit 4 and 5 to $P_{rot}=50^h$ and $I_{th}=130$ and 1000.

Table 6

a) 1607 – 1984

No.	T(TDB)	q	e	ω	Ω	i
1	1986 Feb 9.50517	0.5871077	0.96727826	111°.84716	58°.14373	162°.23921
2	1986 Feb 9.50390	0.5871061	0.96727842	111.84716	58.14370	162.23920
3	1986 Feb 9.50458	0.5871054	0.96727858	111.84716	58.14367	162.23919
4	1986 Feb 9.50556	0.5871085	0.96727780	111.84716	58.14374	162.23922
5	1986 Feb 9.50304	0.5871062	0.96727839	111.84716	58.14371	162.23921

No.	A_1	A_2	B	Δα	Δδ	μ	obs.
1	-0.020 ±0.061	+3.415 ±0.002	-0.00395 ±0.00019	+0".37	-0".28	1".26	91
2	-0.032 ±0.060	+1.585 ±0.002	-0.00400 ±0.00019	+0.30	-0.20	1.25	91
3	-0.049 ±0.061	+1.088 ±0.001	-0.00426 ±0.00018	+0.19	-0.12	1.27	91
4	+0.036 ±0.063	+6.445 ±0.006	-0.00390 ±0.00019	+0.44	-0.33	1.27	91
5	-0.026 ±0.062	+1.802 ±0.002	-0.00391 ±0.00019	+0.35	-0.24	1.24	91

Epoch	T	1	2	3	4	5	observed
1986 02 19	1986 Feb	9.50517	9.50390	9.50458	9.50556	9.50304	
1910 05 09	1910 Apr	20.17869	20.17869	20.17870	20.17868	20.17869	
1835 11 18	1835 Nov	16.43952	16.43953	16.43953	16.43952	16.43953	
1759 03 21	1759 Mar	13.05989	13.05957	13.05931	13.06015	13.05965	13.0628 ±0.0012
1682 08 31	1682 Sep	15.27975	15.28001	15.27998	15.28009	15.28063	15.2806 ±0.0022
1607 11 13	1607 Nov	27.51840	27.52749	27.51359	27.52247	27.52035	27.5196 ±0.0043

b) 1835 – 1984

No.	T(TDB)	q	e	ω	Ω	i
1	1986 Feb 9.44889	0.5871029	0.96727146	111°.84632	58°.14387	162°.23942
2	1986 Feb 9.45022	0.5871021	0.96727322	111.84644	58.14387	162.23942
3	1986 Feb 9.45092	0.5871017	0.96727419	111.84651	58.14387	162.23942
4	1986 Feb 9.44817	0.5871034	0.96727012	111.84624	58.14387	162.23942
5	1986 Feb 9.45024	0.5871021	0.96727318	111.84644	58.14387	162.23942

No.	A_1	A_2	μ	obs.
1	+1.095 ±0.136	+3.331 ±0.005	0".91	84
2	+0.934 ±0.132	+1.545 ±0.002	0.90	84
3	+0.854 ±0.131	+1.060 ±0.002	0.89	84
4	+1.251 ±0.140	+6.300 ±0.009	0.91	84
5	+0.968 ±0.136	+1.777 ±0.002	0.89	84

Epoch	T	1	2	3	4	5	observed
1986 02 19	1986 Feb	9.44889	9.45022	9.45092	9.44817	9.45024	
1910 05 09	1910 Apr	20.17857	20.17859	20.17861	20.17855	20.17859	
1835 11 18	1835 Nov	16.43962	16.43960	16.43959	16.43963	16.43960	
1759 03 21	1759 Mrz	13.02589	13.03025	13.03342	13.02377	13.03026	13.0628 ±0.0012
1682 08 31	1682 Sep	15.27062	15.28074	15.29092	15.26572	15.28004	15.2806 ±0.0022
1607 11 13	1607 Nov	27.71973	27.75112	27.79830	27.70633	27.74671	27.5196 ±0.0043

Epoch	m'_U=+50	m'_N=+50	B_2=+0.0005
1986	$-0^d.00715$	$-0^d.00029$	$-0^d.00530$
1682	+0.00545	+0.00455	+0.00588
1607	+0.04465	+0.03566	+0.04428
837	+0.174	-0.022	+0.316

Table 7 -- Dependence of the perihel times in 1986, 1682, 1607 and 837 predicted by solutions 1759 - 1983, under variation of the mass values for Uranus and Neptune and of B_2

Given are the changes in perihel time in days after variation of the reciproce mass values for Uranus and Neptune by +50 units, and of B_2 by +0.0005 .

I	II	T_{obs}
837 Feb 28.40	837 Feb 28.44	
760 May 20.93	760 May 20.47	
684 Oct 2.36	684 Sep 30.98	
607 Mar 14.13	607 Mar 13.29	607 Mar 12.5 ±1.5
530 Sep 25.15	530 Sep 25.90	530 Sep 25.5 ±1.5
451 Jun 26.04	451 Jun 27.88	
374 Feb 14.08	374 Feb 15.93	374 Feb 16 ±1.5
295 Apr 21.98	295 Apr 19.76	
218 May 23.20	218 May 14.16	
141 Mar 31.03	141 Mar 16.27	141 Mar 21.1 ±1.5

Table 8 -- The representation of the apparitions back to 141 on variation of the perihelion time in 837

a)

Epoch(TDB)	T(TDB)	q	e	ω	Ω	i
2284 06 16	2284 06 06.84231	0.59211254	0.96681684	115°.03325	61°.98002	161°.75497
2209 02 10	2209 02 05.49280	0.59018100	0.96702148	114.70857	61.50899	161.77262
2134 03 15	2134 03 28.66037	0.59322261	0.96664479	113.98893	60.59178	161.74586
2061 08 04	2061 07 28.86064	0.59278730	0.96657957	112.03456	58.67675	161.96180
1986 02 19	1986 02 09.50762	0.58710485	0.96727910	111.84718	58.14364	162.23917
1910 05 09	1910 04 20.17871	0.58720991	0.96730558	111.71809	57.84577	162.21550
1835 11 18	1835 11 16.43953	0.58656496	0.96739858	110.68479	56.80149	162.25562
1759 03 21	1759 03 13.05932	0.58446927	0.96769098	110.68921	56.52871	162.36927
1682 08 31	1682 09 15.28158	0.58262489	0.96793295	109.20287	54.85078	162.26146
1607 11 13	1607 10 27.51776	0.58364892	0.96750342	107.52900	53.05286	162.89776
1531 08 14	1531 08 26.25852	0.58122493	0.96776025	106.95515	52.34172	162.90943
1456 06 28	1456 06 09.49917	0.57974055	0.96800529	105.81548	51.15294	162.88167
1378 11 05	1378 11 10.62287	0.57627478	0.96838164	105.28010	50.31277	163.10367
1301 11 09	1301 10 25.18611	0.57277443	0.96893884	104.48707	49.44723	163.06644
1222 10 15	1222 09 28.55316	0.57429114	0.96885062	103.83558	48.60076	163.18236
1145 04 02	1145 04 18.11662	0.57489917	0.96878998	103.69886	48.36140	163.21368
1066 03 08	1066 03 20.06530	0.57458508	0.96887294	102.46544	46.92993	163.10153
989 08 19	989 09 04.09034	0.58201659	0.96789357	101.47334	45.86508	163.38898
912 07 14	912 07 17.00485	0.58027612	0.96807602	100.77435	44.95956	163.29993
837 03 10	837 02 27.53654	0.58245958	0.96779633	100.09855	44.24273	163.43779

A_1 -0.0138 A_2 +0.015964 B_2 -0.00495 (Epoch 1986 02 19)

Ecliptic and Equinox B1950.

b)

Epoch(TDB)	T(TDB)	q	e	ω	Ω	i
837 03 10	837 02 28.4241	0.5824596	0.9677998	100°.10398	44°.24514	163°.43737
760 02 03	760 05 20.615	0.581957	0.967864	100.0022	44.0035	163.4330
684 09 29	684 10 01.430	0.579734	0.968153	99.1603	43.1252	163.4084
607 03 18	607 03 13.571	0.581007	0.968041	98.8116	42.5888	163.4666
530 07 20	530 09 25.625	0.575779	0.968706	97.5970	41.3071	163.3851
451 04 06	451 06 27.230	0.573927	0.968916	97.0499	40.5498	163.4692
374 03 01	374 02 15.292	0.577417	0.968581	96.5341	39.9211	163.5323
295 04 25	295 04 20.632	0.576120	0.968752	95.2667	38.4563	163.3577
218 03 20	218 05 17.709	0.581677	0.967964	94.1737	37.2550	163.5640
141 02 12	141 03 21.076	0.583379	0.967841	93.7197	36.5671	163.4265
66 03 18	66 01 21.896	0.585360	0.967556	92.6798	35.4781	163.5670
-11 12 27	-11 10 05.995	0.587443	0.967381	92.5830	35.2523	163.5790
-86 08 23	-86 08 03.536	0.585875	0.967670	90.8193	33.3840	163.3324
-163 12 25	-163 10 30.106	0.584770	0.967668	89.1685	31.4448	163.6947
-239 04 28	-239 04 16.516	0.585340	0.967653	88.0191	30.0449	163.4238
-314 06 01	-314 05 15.216	0.587653	0.967311	86.7582	28.7325	163.5810
-390 03 12	-390 04 28.980	0.588377	0.967269	86.6838	28.5122	163.5798
-465 04 15	-465 04 11.147	0.588596	0.967407	85.0953	26.8145	163.2738
-541 12 10	-541 12 17.112	0.590201	0.967168	83.5615	25.1016	163.5488
-617 09 20	-617 09 19.971	0.590725	0.967172	82.9646	24.3105	163.4449
-692 10 23	-691 01 08.037	0.590124	0.967354	81.5788	22.8803	163.3756
-768 02 25	-768 02 02.102	0.587300	0.967652	81.0201	22.0952	163.4349
-845 06 29	-845 05 20.163	0.583601	0.968270	79.9831	20.9705	163.1551
-923 01 29	-923 02 21.150	0.584879	0.968047	78.7296	19.4916	163.3239
-1001 12 25	-1001 10 12.603	0.580168	0.968686	77.9033	18.5697	163.1327
-1081 10 26	-1081 12 22.048	0.580099	0.968692	77.0967	17.5479	163.0869
-1158 04 13	-1158 06 15.891	0.586248	0.967932	76.6012	16.9607	163.1110
-1236 04 22	-1236 04 08.118	0.582026	0.968710	74.6496	14.8214	162.5231
-1315 06 16	-1315 04 26.872	0.585688	0.968162	72.9303	12.9420	162.6629
-1393 12 03	-1393 10 11.979	0.585227	0.968372	72.5614	12.4545	162.5847
-1472 03 12	-1472 03 08.117	0.586410	0.968301	71.7378	11.4107	162.3520
-1550 08 29	-1550 08 23.932	0.592668	0.967509	70.8977	10.4604	162.3886
-1627 02 14	-1627 01 07.750	0.593452	0.967747	69.9578	9.2762	161.8652
-1705 08 03	-1705 08 23.832	0.598066	0.967097	68.8350	8.0064	161.9380
-1782 12 05	-1782 11 03.000	0.602695	0.966898	67.6358	6.5454	161.3854
-1858 09 15	-1858 10 21.969	0.603160	0.966699	66.5011	5.2945	161.4205
-1935 08 10	-1935 10 17.220	0.604625	0.966532	66.3097	4.8877	161.3508
-2009 02 20	-2009 01 06.670	0.605355	0.966513	65.5906	4.1540	161.2762
-2086 01 15	-2086 03 27.721	0.605553	0.966485	65.2602	3.6731	161.2362
-2162 09 11	-2162 10 31.405	0.607084	0.966682	64.3009	2.6268	160.7396
-2238 01 13	-2239 12 30.106	0.607594	0.966544	62.8200	0.9533	160.6292
-2316 12 08	-2316 12 10.342	0.609346	0.966774	61.8347	359.7110	159.9643

A_1 +0.0557 A_2 +0.01200 B_2 -0.00653   (Epoch 837 03 10)

Ecliptic and Equinox B1950.

Table 9 -- The motion of the comet during 2300 BC to 2300 AD

The results for the time span 837 to 2300 AD (part a) base on orbit 1 on table 3. The results for 2300 BC until 837 AD base on a back integration of a solution 1607 - 1983 and correction of the perihel time in 837 (part b). Because the insufficiancy of the force model, the uncertainty in the predicted perihel times for 2061 and 2209 are $0^d.1$ and 1^d, respective.

	0	0.04 (5".5	0.2 2".0	0.4 1".5	1.0 1".0)	σ	μ	$\langle\varepsilon p\rangle_\alpha$	$\langle\varepsilon p\rangle_\delta$
1835	2	173	-	-	-	4".44	0".39	+1".17	-0".61
1910	1	-	409	31	12	2.20	1.07	+0.29	+0.17
1986	6	-	-	-	37	0.57	0.57	-0.09	-0.09
1835 - 1984	9	173	409	31	49	2.15	1.02	+0.23	+0.07

Table 10 -- Short error analysis of the IHW data for 1835 to 1984

For the apparitions 1835,1910,1986 and all observations together, is given the distribution over different weight classes (in parentheses enclosed the mean error σ according to IHW and used by ESOC on the elements given in table 3), the root mean square and a posteriori unit weight mean error, $\sigma^2=\Sigma\varepsilon^2 p/\Sigma p$ and $\mu^2=\Sigma\varepsilon^2/(n-u)$, *and the average residuals in α and δ.*

5. The ephemeris uncertainty for comet Halley in March 1986

With regard to the space missions to comet Halley in March 1986, it is not without interest to know the position of the comet and its uncertainty. Below the uncertainty we have to expect in the position determinations for 13 March 1986 is considered because at this time the *Giotto* spacecraft of *ESA* will be targetted with high precision to flyby the nucleus of comet Halley at a distance of 500 km.

5.1. Mean error (variance) of the estimated position values

The mean errors of unknown parameters (e.g., $T, q, e, \omega, \Omega, i, A_1, A_2$) to be determined from observations (e.g. α, δ), as well as the mean errors of functions of the unknown parameters (e.g. the ephemeris place at the flyby or the miss vector), depend on the accuracy of the observations and the functional dependence of the parameters from the observations. In particular, the mean errors depend on the assumed model, the equations of conditions in it, and on the number, distribution and accuracy of the observations, but not on their accurate values or residuals. Therefore, the uncertainty in the position predictions in March 1986 can be predetermined, but the results depend essentially on the assumptions about the observations until March 1986.

In table 11 are given some different assumptions for the observations until 1986, which are the bases for the subsequent error estimations. Six different cases assuming different sums of weights are considered, from a large number of observations to very bad expections. An unit weight corresponds to one observation with a mean error of ±1"0, and given is the assumed sum of weights collected for time spans up to one month and placed to the date of greatest elongation between the comet and the moon.

Case no. 1 represents most favourable expections. It was assumed that the lapse of the nongravitational forces is known with sufficient accuracy, so that the systematic error in representation of the observations in large heliocentric distances is much below the mean error of its whole (approximately 0"2), and thus these observations can be used for the orbit determinations. Furthermore it was assumed, that no light shift exists between the nucleus of the comet and the neasured positions, so that also the observations in small heliocentric distances can be used. The total weight was choosen to approximately 1200 weight units, based on the number of observations obtained during the last apparition of the comet.

In case 2, the same assumptions on the nongravitational modelling and the light offset were made, but fewer observations were assumed. In addition to the observations already existing until February 1984, a weight sum of 260 till end 1985 and of additional 6 for the beginning of March 1986 was assumed.

In case 3 it was assumed that the modelling of the forces is too uncertain for the observations at large distances can be used. The limit was set to $r = 2.8$ A.U. Similar to case 2, a low number of observations was assumed, with the exception of about twenty weight units for the beginning of March 1986.

The three following cases should investigate the situation if one have to consider a radial light offset at low heliocentric distances. Similar to a stiff assumption of a certain lapse for the nongravitational forces, by assuming a certain dependence on heliocentric distance, e.g. $\Delta r(r) = S_o \cdot s(r)$ and solving for S_o as an additional parameter, the resulting increase in position uncertainty is reflected only insufficiently. The results presented subsequently refer to the assumption, that nothing is known about the magnitude of the light shift, and thus only position angles of the corresponding observations will be used (cf. chapter 2.2). The position error is probably even more underestimated also by this method, because of the possible additional presence of a tangential light shift. However, nothing better is presently obtainable, because, depending on whether the error of this assumption is erratic or systematic, either it must be taken into account by increasing the mean observational error or it cannot be considered within the postulates of error computations. Subsequently, for $r < 2.0$ A.U. only the weights for the position angles to the sun have been used. Case 4 corresponds to the same other assumptions as in case 2. Case 5 corresponds to case 3 but with the exception of 30 weight units in total for March 1986. Case 6 finally assumes only 6 weight units in March 1986, and shall represent the worst case.

In addition to these assumed observations in the present apparition, the observations of the previous ones 1835 - 1910 or 1607 - 1910 were added, respectively. On the solutions 1835 - 1986, the unknowns which had to be estimated are the six orbital elements as well as A_1 and A_2; on solutions 1607 - 1986 also B (for details, see chapter 3 and 4). In order not to repeat in each case the computation of their equations of condition, all the observations of the earlier apparitions in all cases were used (also on $r > 2.8$ A.U., and for $r < 2.0$ A.U. all elongations from sun). The thereby slightly decreased mean error in the earlier perihelion times and other elements have only minute influence on that of the present apparition.

The results of the error estimations are collected in table 2. Under very good conditions, one can expect mean errors of below 50 km in the impact plane of *Giotto* and of 120 km along its flight direction. In general one has to expect greater uncertainties, but it was noted that any omitting of observations in $r > 2.8$ A.U. can be easily compensated by increasing observational effort in March 1986. Also, in the most favourable cases, the calculated uncertainty for right ascensions and declinations of about 0".1 is above the correlating errors one might expect in observations of shorter time spans (assuming the use of the new Halley comparison star positions), so that the assumed large number of observations still has statistical significance.

Obviously both the uncertainty in modelling the nongravitational forces as well as the bad determination and separation of the parameters, which presently still are the main source of uncertainty in the prediction of the perihel time for 1986, will not cause much more essential positional uncertainty over shorter time intervals in 1985-6 (see also chapter 2.1). If light biases definitely do not exist then one could expect a position accuracy of 100 ... 150 km.

The main source of position uncertainty, however, will be the possibility of a light shift of unknown magnitude, as suggested by the results of cases 4 to 6. If it cannot be taken into account explicitely, one must expect uncertainties up to 700 km. As is visible from comparison of cases 5 and 6, the accuracy can improve essentially by increased observational effort from the end of 1985 to the first part of March 1986, down to below 200 km uncertainty.

Even if the exclusive use of the position angles appears to be the only possibility to realistically guess the amount of uncertainty in the case of a light shift of unknown lapse for error estimations, it remains to question whether this method should indeed be used on orbit computations in 1986. This question cannot be answered presently because it strongly depends on our knowledge of the essence of the light offset and the observations obtained until March 1986. If the amount of the offset is ascertained to be smaller, or is modelled better than the calculated position uncertainty using all observational information (including the elongations), then this is good so. But if this is not the case the suggested method should be used, because the primary intention is not to decrease the formal position uncertainty but is rather to exclude systematic errors larger or even of order than it. Also, if the light offset does not coincide accurately with the radial direction then probably at least its main part and the corresponding part which causes the most uncertainty in the target plane will be eliminated, whilst tangential parts mainly influence the arrival time. Most desirable, however, is to obtain models for the light shift from the theory of cometary comae and thus a decrease of uncertainty similar to the cases 1 - 3 discussed above.

The presented assignation of the light shift as the main source of position uncertainties is in certain contradiction to the results of *D.K.Yeomans et al. (1982)*. These authors adopted *ad hoc* a certain magnitude S for the light shift depending only on the heliocentric distance (200 and 1000 km at $r = 1$ A.U., varying to r^{-2} and r^{-3}, respective), subtracted it from the simulated observations, fitted through an orbit without solving for S_o, and represented the result for the corresponding position uncertainty for March 1986. Subsequently it was concluded, that the light shift has little influence on the position accuracy in March 1986, because only the projection of the true uncertainty in the line of sight (which then coincide approximately with the radius vector) enters into the observed position. However, such investigations only adress the problem in a very limited way because the assumption of, and the fitting of the observations with, a certain lapse of the light shift causes essential systematic errors not canceled on the fit and not indicated by the somewhat increased mean errors (in a simulation under similar conditions appearing to approximate the half magnitude of S_o). Thus, the effect of the light shift to the geocentric position in March 1986 has only little to do with that to the heliocentric position.

The error ellipsoides given by these authors for the cases without light shift agree in whole with ours in orientation and the ratios of the semi axis, but are a few times larger. This is probably caused by more pessimistic expections for the observations.

Using observations 1835 to February 1984, *T.A.Morley (1984)* gave for the corresponding determination of the position in March 1986, A = 5700 km, B = 54 km and C = 15100 km, which is not in good agreement but of the same magnitude as our results to case 0b in table 12.

Additionally, it was interesting to check how far the position accuracy can increase by special observational effort from end of 1985 until March 1986. One possibility for such, according to a suggestion by *E.Bowell* at Lowell Observatory, could be the observation of appearent close encounters of comet Halley to background stars. If these stars previously have been observed by transit circles, and if star-like images of the comet have been obtained by very short exposures, a position accuracy of 0.″1 is possible. For the error estimations, from the middle of October until the end of 1985, 80 appearent encounters were assumed, for March 1986 further six encounters, of which each follows a position to ±0".10 (the number of stars appropriate by its position and magnitude difference to the comet is much larger). The other assumptions correspond to those of case 1 and 6 above; in particular, in case 6 for only the first twenty encounters the elongations to sun were also used. The results (no. 7 in table 12) correspond approximately to the expections regarding the increment of observation weights. We do not intend to enter into the technical particulars of such an observation project and some limitations in obtainable accuracy, although consideration of these would not change the result of a considerable improvement of the position accuracy by such a project.

A further question was, whether it is imperative to abandon completely the observations of earlier apparitions and use only those of the present one.

The results in table 13 are given only for the best and worst assumed cases. In the latter one, two assumptions about the observations in March 1986 were made to see their influence. Now, some of the parameters should not be taken as unknowns but should be considered with certain mean errors instead.

The comparison between the results of the four first cases in table 3, in which always the six orbital elements had to be estimated, again suggests strongly that in 1986 the nongravitational forces will have only very minute influence on the position uncertainty. Also an uncertainty of any perpendicular force parameter A_3 as large as four times its present uncertainty will cause only an increase of the uncertainty in z-direction by 15 km in the favourable observational case, but is completely succombed by other uncertainties in the less favourable cases.

By comparison of these results with those in table 12 it is evident that, by not taking into account the earlier apparitions, the uncertainty is increased two to five times. If values for both the nongravitational parameters and the revolution time are considered instead of estimated, the increase is only 1.5 times. However, even if one solves only for one unknown, the perihelion time, under consideration of all other parameters as known without any uncertainty, the position uncertainty is not significantly below the limits attainable by a general solution including the earlier apparitions and increased observational efforts. Because, furthermore, by considering values and mean errors of several unknowns one would fall back upon the earlier observations implicitely, and because without considering a part of the unknowns, the computed uncertainty is much larger than the systematical errors produced by fitting observations from different apparitions using the available force models, it appears not to be imperative to use observations of the present apparition exclusively, but it could become appropriate later not to use the apparitions before 1835 or 1759 (cf. chapter 2.1).

5.2. Systematic errors in the predicted positions

Although systematic errors are not the object of an error computation, we shall at this point make some remarks as, in few cases, one can guess their amount.

5.2.1. Different models for the nongravitational forces

To get an idea about the influence on the position of the comet due the errors of assumed force models, one could compare the results after application of different models to one and the same observations. In particular, these differences depend completely on model and observation distribution, much more than the formal position uncertainty considered above. Neverthless, some results will be given as examples to show the order of magnitude of these differences.

The differences in position predictions which resulted from a fit using two theories according to *H.Rickman and C.Froeschle (1982)* are given in table 14 and are compared with the results using the sublimation formula by *Delsemme* which was also used for the simulation of the expected observations. The assumed observational distribution approximately corresponds to no.1 on table 11, but in the second case of table 14, after October 1985 only position angles were used. These results suggest that the lack of knowledge of the accurate force lapse in 1986 will produce only minute position uncertainties. Of special interest is the strong dependence of the calculated position uncertainty on the adopted model. This is mainly due to the different lapse of the parameters in the three models, wherein they are determined by very different portion of the observations (e.g., for *Rickman-Froeschle* with thermal inertia of the nucleus I_{th}=130, much more by the far distant observations then on the other models), and also by very different correlation with the other unknowns.

Also, using a model which is sufficiently accurate over the heliocentric distances covered by observations and applying this to the apparitions 1607 - 1986 and 1835 - 1986, respectively, will give some differences in the two results for perihelion time in 1986 and the other parameters. These differences depend mainly on the error of assumptions about the secular behaviour of the parameters, e.g. in the above cases we assumed linear dependence (1607 - 1986) or parameters constant with time (1835 - 1986), respectively. It is not presently possible to conclude anything about the amount of the corresponding position difference in 1986, which is essentially $\Delta \underline{r} = \dot{\underline{r}} \cdot \Delta T + \Delta \underline{\pi}$ ($\underline{r}$ position of the comet in March 1986, $\Delta T, \Delta \underline{\pi}$ differences of both results for the perihelion time and location of the perihel). The corresponding differences for 1835 and 1910 (using observations until 1984) let us conclude that this can amount to a few hundreds of km, but this we must see in 1986. Whilst presently solutions 1607 - 1984 for the predictions appear preferable, it could become possible to prefer solutions 1682 or 1835 - 1986 then because of the better elimination of time dependence of the force parameters and the better fit of the observations although these would only yield a minute increase in the calculated uncertainty.

Because of short-term fluctuations of the nongravitational forces we do not expect position shifts to be important. The magnitude of the nongravitational forces is approximately $|A| = 0.1$ km/d^2. Even if a perturbation of the same magnitude as the main force is acting always in the same direction, after one month it would have produced a position error of only

∿100 km. Such long-term effects, however, would have to be considered as a part of the (global) force theory, and furthermore, by the continuous fit to the observations they would produce only much smaller errors in the position predictions, as discussed above (cf. table 14).

D.K.Yeomans et al. (1982) have investigated extensively the positional effects of fluctuations composed by a decaying earlier fluctuation and a new random one. An amplitude of 20% of the main force and a time scale of one day was choosen. However, considering the above magnitude of the nongravitational forces on comet Halley, one can already compute by head that such perturbations are only of subkilometric amounts, so that similar computations were not made.

5.2.2. Different reference systems for observations and coordinates of earth

A source of essential systematic errors in the use of different reference systems for observations and earth coordinates. This is of special importance, because the navigation of space probes is bound to the earth rotation, so that the position of the comet must be known with reference to the dynamic equator and equinox and errors do not cancel but enter on the targeting accuracy. To the present state, the FK_5 system coincides with the dynamic reference system better than to 0".01 and thus is sufficient for our purpose.

If for both the observations and the earth coordinates the FK_4 system, for example, is used, the error in the position values corresponds to the transformation FK_4-FK_5, although small dynamical inconsistency exists because of the inertial rotation of the FK_4 system. In practice, however, bases refered to different systems, e.g. observations in the rotating FK_4 system and the approximate inertial resting earth coordinates according to new radar theories (DE102, DE118 etc) are used. Then the corresponding errors in the cometary position are no longer independent from the observations and can be estimated only by simulation computations.

Subsequently the case was considered where the positions of earth corresponds to the theory DE118, whilst the observations of the comet refers to the FK_4 system but are corrected for the elliptic aberration. It was noted that such bases are often in use in practice. The calculated positions for the comet were compared with corresponding results, assuming that the earth coordinates are refered to the FK_5 system, and using the same observations of the comet reduced to the FK_5 system, too (by application of the correction of equinox, of precession, of elliptic aberration, and of approximate local corrections).

The results for four different assumptions of the observation distribution are presented in table 15. For essential simplification, the differences explained above were not taken into account on the observations of earlier apparitions again. Certainly, they do not enter into the result for the perihelion time for 1986 but do enter into the other elements, especially into the orbital plane orientation, which is of relevance in this case. Insofar, these results may have an error of 20% or so. However, they are sufficient to show that here one has to expect considerable systematic errors whose accurate values cannot be taken into account other than by direct consideration. Thus, it is urgently necessary to eliminate these discrepancies in the bases.

This point is also of relevance for the later investigation of the nongravitational force lapse by means of positional observations. A correction by some 0".1 in far distant observations corresponds to an considerable correction of the mean anomaly.

Date			1	2	3	4	5	6
1982	10	18	4	4		4		
	11	16	2	2		2		
	12	11	2	2		2		
1983	1	14	1	1		1		
	2	13	2	2		2		
1984	1	29	20	20		20		
	2	28	1	1		1		
	10	29	10					
	11	25	20					
	12	22	60	10		10		
1985	1	18	40	20		20		
	2	14	40	20		20		
	3	13	20	10		10		
	4	10	4					
	8	4	10					
	8	25	60	20	20	20	20	20
	9	22	200	40	40	40	40	40
	10	19	200	40	40	40	40	40
	11	14	200	40	40	(40)	(40)	(40)
	12	7	200	40	40	(40)	(40)	(40)
1986	1	1	80	20	20	(20)	(20)	(20)
	1	10	20					
	3	5	10	2	20	(2)	(10)	(2)
	3	8	10	2	2	(2)	(10)	(2)
	3	11	10	2	2	(2)	(10)	(2)

Table 11 -- Assumed observation distribution until March 1986

Error estimations of the position on 1986 March 13 are provided for six different assumptions about observation distribution until then. The table gives the assumed distributions of observation weights. One weight unit corresponds to one observation with a mean error of 1."0 .

Table 12 -- Mean error (1σ-variance) of the position predictions for comet Halley in March 1986

obs.: *assumed observation distribution until March 1986 (see table 11)*
- a: *included observations 1607 - 1911 and solved for 9 unknown parameters (elements, A_1, A_2, B)*
- b: *included observations 1835 - 1911; 8 unknowns only (without B)*
- c: *included observations 1607 - 1911 and 80 assumed observations for mid October until end December 1985 and 6 further for first part of March 1986 with a mean error of 0."1 each*

Case no. 0 refers to the observations which have been presented until February 1984. For comparison, on top are given the results from the observations 1607 to 1911 only.

$\sigma x, \sigma y, \sigma z, \sigma r$: *mean errors of the heliocentric equatoreal coordinates and distance of the comet at 1986 March 13.60 UT*

A,B,Θ: *semimajor and semiminor axis of the error ellipse in the target plane of 'Giotto', and direction of the semimajor axis (ecliptic 0°, orbital plane of comet Halley 4°)*

C: *mean positional error of the comet in flight direction of 'Giotto'*

$\sigma T, \sigma q, \sigma e, \sigma \omega, \sigma \Omega, \sigma i$: *mean errors of the orbital elements of comet Halley at osculation epoch 1986 February 19*

$\sigma \alpha, \sigma \delta, \sigma \Delta$: *mean errors of the equatoreal geocentric coordinates of comet Halley at 1986 March 13.60 UT*

See also table 16.

Table 13 -- Mean error (1σ-variance) of the position predictions for comet Halley without use of the observations at earlier apparitions

Corresponding to table 2. In the last column are given the parameters which have been considered with certain mean errors, instead of estimated.

No.	obs.	σx	σy	σz	σr	Θ	A	B	C	σT	σq	σe	$\sigma\omega$	$\sigma\Omega$	σi	$\sigma\alpha$	$\sigma\delta$	$\sigma\Delta$
										10^{-6}	10^{-9}	10^{-8}		10^{-7}				10^{-7}
		km	*km*	*km*	*km*	*degr.*	*km*	*km*	*km*	*d*	*AU*			*degree*				*AU*
	a	21200	2090	5310	12770	20	16200	59	14700	6860	508	107	1728	1302	761	$1^{s}.61$	13".6	650
0	0a	9700	920	2450	5880	20	7500	54	6700	2660	388	29	1029	824	366	0.75	6.2	293
	0b	20260	1940	5120	12260	20	15600	57	14000	5560	756	83	1509	855	431	1.56	13.0	614
1	1a	105	77	19	29	45	52	21	119	15	259	14	253	170	179	0.01	0.1	7
	1b	109	80	19	29	45	54	21	123	16	281	22	254	171	206	0.01	0.1	7
2	2a	152	95	36	63	31	89	37	156	31	326	17	422	346	241	0.01	0.1	9
	2b	166	102	37	64	31	95	37	170	33	378	25	423	348	316	0.01	0.1	10
3	3a	135	100	34	60	31	76	40	148	28	328	18	462	365	282	0.01	0.1	9
	3b	144	106	34	60	31	79	41	158	29	369	25	471	386	460	0.01	0.1	9
4	4a	653	106	138	394	17	491	48	462	180	352	19	673	546	252	0.05	0.4	21
	4b	729	124	152	428	17	539	50	526	198	445	32	707	557	337	0.06	0.4	24
5	5a	507	105	107	316	16	388	44	357	144	356	20	702	551	288	0.04	0.3	16
	5b	518	113	109	319	16	393	45	370	145	420	30	714	576	462	0.04	0.3	17
6	6a	719	116	151	436	17	543	48	508	199	368	22	796	626	288	0.06	0.4	23
	6b	769	131	160	456	17	572	53	551	211	453	33	819	647	464	0.06	0.5	25
7	1c	42	36	6	9	59	18	7	52	5	120	13	122	61	97	0.00	0.0	1
	6c	119	73	26	95	5	103	26	93	40	261	14	313	153	134	0.01	0.1	6

Table 12

No.	obs.	σx	σy	σz	σr	Θ	A	B	C	σT	σq	σe	σω	σΩ	σi	σα	σδ	σΔ	considered parameters
		km	km	km	km	degr.	km	km	km	10^{-6} d	10^{-9} AU	10^{-8}	10^{-7} degree					10^{-7} AU	
1	1	218	317	36	135	-49	122	67	360	31	838	94	930	175	222	$0^{s}.01$	0".2	24	A_1, A_2, A_3 ±0
	5	1126	2169	762	2432	-3	2320	180	1060	810	4800	1100	8580	1150	1590	0.19	0.4	110	
	6	1510	2860	950	3040	-4	2900	180	1690	1010	6480	1360	11000	1190	1940	0.23	0.7	150	
2	1	218	320	38	141	-46	125	72	361	31	838	94	930	175	222	0.01	0.2	24	A_1 ±0.05,
	5	same like on no. 1																	A_2, A_3 ±0
	6	"																	
3	1	same like on no. 1																	A_2 $\pm 5\cdot 10^{-5}$,
	5	"																	A_1, A_3 ±0
	6	"																	
4	1	219	317	53	135	-53	127	72	360	31	838	94	930	175	222	0.01	0.2	24	A_3 ±0.05,
	5	1126	2169	763	2432	-3	2320	180	1060	810	4800	1100	8580	1150	1590	0.19	0.4	110	A_1, A_2 ±0
	6	same like on no. 1																	
5	1	147	119	46	49	55	77	47	173	17	375	2	293	174	222	0.01	0.1	10	A_1, A_3 ±0.05,
	5	675	557	178	707	-1	725	150	470	280	2090	12	2410	1150	1010	0.07	0.3	31	A_2 $\pm 5\cdot 10^{-5}$,
	6	1100	720	220	760	9	840	290	995	320	2570	14	2540	1180	1030	0.08	0.7	57	a $\pm 4\cdot 10^{-7}$
6	1	36	4	9	21	18	27	0	25	10	-	-	-	-	-	0.00	0.0	1	q, e, ω, Ω, i,
	5	310	31	78	187	18	238	0	215	85	-	-	-	-	-	0.02	0.2	10	A_1, A_2, A_3 ±0
	6	364	36	92	220	18	280	0	253	99	-	-	-	-	-	0.03	0.2	11	

Table 13

No.	Δx	Δy	Δz	σx	σy	σz
1	0	0	0	276	591	96
2	-3	+24	+6	274	549	85
3	+2	+6	+2	283	587	93
1'	0	0	0	1737	2852	1168
2'	-11	-17	-7	840	1140	462
3'	+20	+20	+11	1423	2174	905

Table 14 -- Systematic differences in position due to application of different models for the nongravitational forces

For the models by Delsemme and Rickman-Froeschlè (I_{th}=130 and 1000) are given (No. = 1,2,3, *resp.):*

Δx,Δy,Δz: *positional difference to the results using the model by Delsemme*
σx,σy,σz: *mean error of the position*

The first three line refers to computations without, the second ones with considering a possibly present light shift between nucleus and light center of the comet, by using for observations at r<2.0 AU only the position angles to sun in the latter case. Moreover the observation assumptions corresponds approximately to case no.1 in table 1.

No.	Δx km	Δy km	Δz km	$\Delta\alpha$	$\Delta\delta$	$\lvert\Delta s\rvert$ km	$\lvert\Delta t\rvert$ km
1	+370	−240	+30	−0".1	−0".1	410	170
3	+280	−150	+23	−0.0	−0.0	290	140
5	+560	−160	+90	−0.1	+0.4	470	350
6	+1040	−204	+190	−0.3	+1.0	810	710
Earth	−48	−380	0				

Table 15 -- Systematic errors in the position predictions due to using observations refered to the FK_4 system and earth coordinates according to DE118

Presented are the differences in sense a)-b) of position predictions based on: a) coordinates of earth refered to the FK_5 system, cometary observations corrected for the correction of equinox (FK_5-FK_4), correction of precession (IAU 1979 - Newcomb), and for the approximate local corrections FK_5-FK_4 , b) earth coordinates corresponding to DE118, observations not corrected.

$\Delta x, \Delta y, \Delta z$: *differences in the heliocentric equatoreal coordinates of comet Halley at 1986 March 13.60 UT, refered to mean equinox 1950*

$\Delta\alpha, \Delta\delta$: *differences in the residuals of observations at 1986 March 13*

$\Delta s, \Delta t$: *difference in flight direction and target plane of 'Giotto'*

For comparison, the position difference FK_5-DE118 of earth is given at the bottom.

a) orbital elements (epoch 1986 Feb 19.0)

No.	T(TDB)	q	e	ω	Ω	i
1	1986 Feb 9.50954	0.5871050	0.96727924	111°.84726	58°.14370	162°.23921
2	1986 Feb 9.46133	0.5871017	0.96727381	111.84662	58.14388	162.23943

No.	A_1	A_2	B
1	-0.0172 ±0.0084	0.015973 ±0.000014	-0.00502 ±0.00016
2	+0.1110 ±0.0192	0.015564 ±0.000021	

No.	arc	observations no.	σ	μ	K	S	P
1	1607 - 1984	111	1".49	1".25	S	FK5	ITA
2	1835 - 1984	104	1.01	0.88	S	FK5	ITA

b) mean errors (referred to μ=1".0)

σT	$6148\cdot10^{-6}$ d	σω	$1594\cdot10^{-7}$ °	σx	22390 km	A	17200 km
σq	$771\cdot10^{-9}$	σΩ	$897\cdot10^{-7}$ °	σy	2180 km	B	60 km
σe	$91\cdot10^{-8}$	σi	$481\cdot10^{-7}$ °	σz	5650 km	C	15540 km
σA_1	0.0218			σr	13520 km	Θ	20°
σA_2	$2.349\cdot10^{-5}$						

Table 16 -- Updated orbital elements and error estimation

The orbital elements correspond to those of no. 1 and 3 in table 3, the mean errors to the 1835 - 1984 solution and no. Ob in table 12. In comparison to the earlier solutions, few observations obtained recently have been added, but the weights of some observations made in early 1984 have been decreased, and the declinations of the remeasured 1911 observations (cf. chapter 1) were omitted. Due to this, the weight sum of the observations decreased by 7%, and the actual mean errors (tabulated values times μ) have not become smaller.

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References

Ballani,L.(1981): Die Sterne 57,172.

Barret,A.A.(1978): Journ.RAS Canada 72,86.

Bessel,F.W.(1804): Abhandlungen 1,5.

Bessel,F.W.(1836): AN 13,345.

Bessel,F.W.(1844): Astronomische Beobachtungen ... Königsberg 21,81.

Beutler,G.(1982): Astronomisch-geodät. Arbeiten i.d.Schweiz 34.

Bowell,E.(1982): MPC 6841

Brady,J.L.,Carpenter,E.(1971): AJ 76,728.

Cowell,P.H.,Crommelin,A.C.D.(1910): Publ.d.Astron.Gesellschaft 23,60.

Delsemme,A.H.(1982): Comets.Tucson. *94.*

Encke,J.(1838): Math.Abhandl.d.Königl.Akad.d.Wiss. 1836,103.

Hasegawa,I.(1979): Publ.Astr.Soc.Japan 31,263.

Herrschel,J.F.W.(1837): Mem.RAS 10.325.

Ho,P.Y.(1962): Vistas in Astronomy 4,142.

Kiang,T.(1972): Mem.RAS 76,45.

Krasinsky,G.A.,Pitjeva,E.V.,Sveshnikov,M.L.,Sveshnikova,E.S.(1982): Bull.ITA 15,145.

Lamont,J.(1843): Observ.Astron.in Specula Regia Monachiensi 11,1.

Landgraf,W.(1983a): Sterne und Weltraum 22,59.

Landgraf,W.(1983b): Die Sterne 59,153.

Landgraf,W.(1983c-5): Mitt.Astr.Verein.Südwestd. 22,47,68,92, 23,19

Lubienietzky,S.(1668): Theatrum Cometicum.Amsterdam. *4.*

Maclear,J.(1837): Mem.RAS 10,91.

Marsden,B.G.,Sekanina,Z.(1968-72): AJ 73,367

Morley,T.A.(1984): Giotto Flight Dyn. Report 1.

Pingrè,A.G.(1783): Cometographie.Paris. 1,253,255,574.

Rickman,H.,Froeschle,C.(1982): Cometary Exploration.Budapest. 3,109.

Rosenberger,O.A.(1830-1): AN 8,221,9,53.

Struve,F.G.W.(1839): Beobachtungen des Halley'schen Cometen.St.Petersburg.

Westphalen,H.(1847-8): AN 24,333,365,25,165,181.

Yeomans,D.K.,Jacobson,R.A.,Williams,B.G.,Chodas,P.W.(1982): Cometary Exploration.Budapest. 3,95.

Zadunaisky,P.(1966): AJ 71,70.

Appendix. A program for the computation of the mean error (1σ - variance) of position estimations for comet Halley at the Giotto encounter

I. Introductionary remarks

The calculation of the mean error of parameters determined by application of the Method of Least Squares, and of functions of these unknowns, can, for example, be obtained in the following basical way (e.g. for technical performation, see *G.Beutler, 1982*). From the system of normal equations for the unknown parameters $\underline{u}$,

$$\underline{l} = \underline{\underline{C}} \cdot \underline{u} \qquad \text{a)}$$

the variance-covariance matrix $\underline{\underline{V}}$, correlation matrix $\underline{\underline{K}}$, and the mean errors $\underline{\sigma}$ of the m parameters follows by

$$V_{ij} = \sigma_i K_{ij} \sigma_j = \mu^2 (\underline{\underline{C}}^{-1})_{ij} \qquad \text{b)}$$

with $K_{ii}=1$. The mean error of an unit weight, μ, can be guessed by

$$\mu^2 = \frac{s}{n-m} \approx \frac{s° - \underline{l} \cdot \underline{u}}{n-m} \qquad \text{c)}$$

(n number of equations of condition, s°,s residual squares sum before and after the orbit improvement, respectively). By substitution of one of the unknowns by a function $f = f(\underline{u})$ of them, the transformation of $\underline{\underline{C}}$, $\underline{\underline{V}}$ and $\underline{\sigma}$ results in

$$\sigma_f^2 = \sum_i^m \sum_j^m \frac{\partial f}{\partial u_i} \sigma_i K_{ij} \sigma_j \frac{\partial f}{\partial u_j} \qquad d)$$

which permits the computation of the mean error of a function of the unknowns.

For the calculation of the mean error of ephemeris places obtained after an orbit improvement, the partials of the position values to the unknowns and the normal equations of the orbit improvement are need. Thus, the only suitable place to apply a corresponding programme is to include it to the orbit determination program and call it subsequent to the orbit improvement(s). For this reason this is assumed to be the case on the programme explained below. Moreover, it was assumed that for usual orbit determination problems (e.g., the computation of the mean errors of the elements if using rectangular initial values as parameters) the user already has a subprogramme for computing the mean error of a function of the parameters, or otherwhise can write this quickly by using formula d) above. This subprogram subsequently is refered as MF2. Then the enclosed programme, called ERAN, contains only the calculations necessary in specific regard to the position uncertainty of comet Halley on 1986 March 13.6 UT, the assumed time of the Giotto encounter.

II. Explanation of program ERAN

The mean errors of the heliocentric equatoreal coordinates $\underline{x}$ of the comet, immediately follow by application of MF2 to $\partial \underline{x}/\partial \underline{u}$, the partials between position and parameters $\underline{u}$. The latter have to be computed by the orbit programme in the same way as the equations of condition for an observation at this time, and transfer to ERAN; however, they are already predetermined very precisely and could instead be taken from the example given below (assuming elliptic elements as the parameters).

For computation of the mean error of the heliocentric distance r, the $\partial r/\partial \underline{u}$ follows immediately by

$$\frac{\partial r}{\partial \underline{u}} = \frac{\partial \underline{x}}{\partial \underline{u}} \cdot \frac{\partial r}{\partial \underline{x}} \qquad \text{e)}$$

where $\partial r/\partial \underline{x} = (x,y,z)/r$ for the cometary position is also pre-determined.

Let be $\underline{x}' := (r,s,t)$ the position of the comet in a coordinate system in which the flight direction of Giotto is perpendicular to the r/s-plane (target plane) and parallel to the t-axis. Then the mean errors of $\underline{x}'$ can be computed by application of MF2 to

$$\frac{\partial \underline{x}'}{\partial \underline{u}} = \frac{\partial \underline{x}}{\partial \underline{u}} \cdot \frac{\partial \underline{x}'}{\partial \underline{x}} \qquad \text{f)}$$

The orientation of r,s (miss vector) in the target plane is arbitrarly. In prcatice, the position uncertainty in the target plane has to be computed for different directions of the miss vector.

If the flight direction of Giotto is called $\underline{v} = (v \cos\alpha \cos\delta,\ v \sin\alpha \cos\delta,\ v \sin\delta)$, then the transformation $\underline{x} \to \underline{x}'$ is obtainable by rotating first the z-axis by $+\alpha$, and then the new y-axis by $90°-\delta$. After this, the new z-axis coincides with the flight direction, and all orientations of the miss vector in the target plane can be obtained due to additional rotation of it by an arbitrary angle Θ running from 0° to 360°. Thus, we have

$$\frac{\partial \underline{x}'}{\partial \underline{x}} = \begin{pmatrix} \cos\Theta & \sin\Theta & 0 \\ -\sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sin\delta & 0 & -\cos\delta \\ 0 & 1 & 0 \\ \cos\delta & 0 & \sin\delta \end{pmatrix} \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{g)}$$

The uncertainty in t-direction is independent on Θ, and if Θ runs from 0° to 180°, only the mean error in the r-direction must be computed.

The angle $\alpha(\Theta)$ between the corresponding r-direction $\underline{e}_r$ and a certain equatoreal direction $\underline{n}$ ($|\underline{n}| = 1$) can be computed by

$$\cos\alpha(\Theta) = \underline{e}_r'\cdot\underline{n}' = \begin{pmatrix}1\\0\\0\end{pmatrix}\cdot\frac{\partial\underline{x}'}{\partial\underline{x}}\underline{n} = \sum_{i=1}^{3}\frac{\partial x_1'}{\partial x_i}n_i \qquad h)$$

If $\underline{n}$ is the normal vector to the ecliptical plane, then at $\cos\alpha(\Theta) = 0$ the r-direction lies in this plane, and the corresponding r-uncertainty is that of the miss-vector along the direction of the node between ecliptic and the target plane.

These are essentially the formulas after which ERAN works. A listing of the program is enclosed, as well as a table which give the used notation and references of the variables used in the program.

III. Example

As an example which can be used to check out ERAN before application (and to check a procedure for formulas a),b),d), if necessary), are enclosed the values for the normal equations, correlation coefficients, mean errors of the parameters, and results printed by ERAN, corresponding to case no. 1b on table 12. Note that all results are normed by setting μ=1".0=1/206264.8, instead application of formula c) . The parameters are the elliptic orbital elements and two nongravitational parameters, expressed in the common units. For example, the correlation coefficient between e and A_1 is -0.9977339, and the partials of the cometary position to the perihelion time corresponds to the negative velocity of the comet. Although not necessary, ERAN prints the uncertainty in miss vector until Θ=360° . The node between target plane and ecliptic (used as reference point for the orientation of the error ellipses on table 12 and 13) is at Θ=107°, the semimajor axis at Θ=162°. The semimajor and semiminor axes amount to 54 km and 21 km, respectively, and the uncertainty in flight direction 123 km.

VARIABLES IN SUBPROGRAM ERAN: EXPLANATION

notation in text	*variable name*	*explanation*
	AE	reduction factor from AU to km times the unit weight mean error μ (μ=1" assumed)
$\underline{v}$	X(I=1-3)	equatoreal velocity components of Giotto flight direction
	SD,CD	sin, cos of declination "
	SR,CR	sin, cos of right ascension "
$\partial \underline{x}/\partial \underline{u}$	DXDU(I=1-M,J=1-3)	partial of J-th equatoreal coordinate of comet Halley to the I-th parameter on the orbit improvement
$\partial r/\partial \underline{x}$	DDDX(J=1-3)	partial of heliocentric distance to J-th equatoreal coordinate of the comet
$\partial r/\partial \underline{u}$	DDDU(I=1-M)	" I-th parameter of orbital motion
σr	FD	mean error of heliocentric distance
$\sigma \underline{x}$	FX(J=1-3)	" J-th equatoreal coordinate of the comet
$\underline{x}'$	R(K=1-3)	position of the comet in Giotto target plane and flight direction
$\partial \underline{x}'/\partial \underline{x}$	DRDX(J=1-3,K=1-3)	partial of the K-th coordinate in target system to J-th equatoreal coordinate of the comet
Θ	W	arbitrary angle for the direction in the target plane of the r-axis
$\cos(\alpha)$	SOBLQ1	sin of angle between ecliptical plane and r(Θ)-direction
	SOBLQ2	" Halley orbital plane "
	SOBLQ3	" Giotto orbital plane "
$\partial \underline{x}'/\partial \underline{u}$	DRDU(I=1-M,K=1-3)	partial of the K-th coordinate in the target system to the I-th orbital parameter of the comet
$\sigma \underline{x}'$	FR(K=1-3)	mean error of the K-th coordinate in the target system

VARIABLES IN SUBPROGRAM ERAN (CONT.): REFERENCE LISTING

```
                                                    Subroutine eran

                NAMES USED IN THIS PROGRAM UNIT

NAME        TYPE OF NAME          LOC       STORAGE    ATTRIBUTES AND REFERENCES

ae                                000000    automatic double precision initialized ref 1985 1994 1994 2014 2014 2014 2014 2059 2059
cd                                000000    automatic double precision ref 1997 1998 2023 2024 2032
cr                                000000    automatic double precision ref 2000 2001 2001 2023 2030 2031
cw                                000000    automatic double precision ref 2029 2030 2031 2032
dcos        builtin                                   double precision ref 2029
dddu                              000000    automatic double precision array(1:11) ref 1984 2010 2011
dddx                              000000    automatic double precision array(1:3) initialized ref 1984 1990 2010 2010 2010
drdu                              000000    automatic double precision array(1:11,1:3) ref 1983 2052 2054 2054 2054
drdx                              000000    automatic double precision array(1:3,1:3) ref 1983 2023 2024 2025 2030 2031 2032 2044 2044
                                                     2044 2045 2045 2045 2046 2046 2046 2054
dsin        builtin                                   double precision ref 2028
dsqrt       builtin                                   double precision ref 1997 2000
dxdu                              000000    /erran/   double precision array(1:11,1:3) ref 1982 2006 2010 2010 2010 2013 2054
eran        entry point                     constant  on line 1966
erran       common block name                         70 words ref 1982
fd                                000000    automatic double precision ref 2011 2014
fr                                000000    automatic double precision array(1:3) ref 1983 2056 2059 2057
fx                                000000    automatic double precision array(1:3) ref 1983 2013 2014 2014 2014
i                                 000000    automatic integer ref 2006 2006 2012 2013 2013 2051 2052 2054 2054 2054
iw                                000000    automatic integer ref 2026 2027 2057 2059
j                                 000000    automatic integer ref 2009 2010 2010 2010 2010 2049 2050 2050 2052 2054 2054 2054 2056
                                                     2056 2057
k                                 000000    automatic integer ref 2053 2054 2054
l                                 000000    automatic integer ref 2005 2006
m                                 000000    /erran/   integer ref 1982 1995 2002 2006 2009 2011 2013 2051 2056
mf2         external subroutine             constant  ref 2011 2013 2056
sd                                000000    automatic double precision ref 1996 1997 1997 2025 2030 2031
soblq1                            000000    automatic double precision ref 2044 2059
soblq2                            000000    automatic double precision ref 2045 2059
sohlq3                            000000    automatic double precision ref 2045 2059
sr                                000000    automatic double precision ref 1998 2000 2000 2024 2030 2031
sw                                000000    automatic double precision ref 2028 2030 2031
w                                 000000    automatic double precision ref 2027 2023 2029
x                                 000000    automatic double precision array(1:3) initialized ref 1983 1986 1996 1998 2001
x1                                000000    automatic double precision array(1:3) initialized ref 1983 1987 2044 2044 2044
x2                                000000    automatic double precision array(1:3) initialized ref 1983 1988 2045 2045 2045
x3                                000000    automatic double precision array(1:3) initialized ref 1984 1989 2046 2046 2046
zeit                              000000    /erran/   double precision ref 1982 2002 2014
zweipi      named constant                            double precision initialized ref 1985 2027

NAMES DECLARED BUT NOT USED

lu          named constant                            integer declared 1981 1982 1983 1984
r                                 000000    automatic double precision array(1:3) declared 1983

LOC           LABEL  TYPE          LINE         REFERENCES

                1    executable    2061         ref 2026
                2    executable    2013         ref 2012
                5    executable    2054         ref 2051 2053
                7    executable    2050         used in transfer ref 2057
               10    executable    2010         ref 2009
               12    format        2003         ref 2002
               13    executable    2006         ref 2005
               14    format        2007         ref 2006
               20    format        2015         ref 2014
               21    format        2019         ref 2018
               22    format        2060         ref 2059
               23    format        2063         ref 2062
```

LISTING OF SUBPROGRAM ERAN

Subroutine eran

```
      SUBROUTINE ERAN                                                   HALLEY
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C   Berechnet die Unsicherheiten der Positionswerte des Himmelskoerpers,
C   projeziert in eine vorgegebene Richtung (Komponenten X im Aequator-
C   system ausgedrueckt), so dass diese Richtung zur t-Achse eines neuen Koordi-
C   natensystemes (R=r,s,t) und die Ausrichtung dessen r-Achse variiert wird
C   (Drehwinkel W, entspr. Neigung der r-Achse zu einer vorgegebenen Ebene
C   wie etwa Ekliptik oder Kometenbahn, OBLQ).
C   Beispiel: Beim Vorbeiflug einer Sonde an einem Kometen soll die Unsi-
C   cherheit seiner Position in Flugrichtung (t-Richtung) und im
C   miss vector (bei Zaehlung von Ekliptik oder Kometenbahnebene) ange-
C   geben werden.
C   SD,CD,SR,CR Sinus und Kosinus der Rekt. und Dekl. in Richtung X,
C   X1, X2 Normalenvektor  senkrecht auf Ekliptik bzw.  Kometenbahnebene
C   im Aequatorsystem (weitere Ebenen entsprechend einbaubar)
      PARAMETER (LU=11)
      COMMON/ERRAN/M,ZEIT,DXDU(LU,3)
      DIMENSION X(3),R(3),X1(3),X2(3),FR(3),FX(3),DRDX(3,3),DRDU(LU,3)
      DIMENSION X3(3),DDDU(LU),DDDX(3)
      DATA ZWEIPI,AE  /6.283185307179586D0,149597870.65295D0/
      DATA X /+.69821807197800,-.66212926998100,-.27216971506400/
      DATA X1 /.0D0,-.39788115834900,+.91743696450100/
      DATA X2 /+.25909490920500,+.23121270537600,-.93776890164700/
      DATA X3 /-.02876083256500,-.40579948573300,+.91350949195300/
      DATA DDDX /-.5883434D0,-.7322187D0,-.3430858D0/
C   (Entsprechen der Vorbeiflugrichtung von Giotto am Halley'schen Kometen
C    und der Normalen senkrecht zur Ekliptik, Halley- und Giottobahnebene).
C    Bezogen auf mittl. Fehler von 1":
      AE=AE/206264.8D0
      IF(M.EQ.0)RETURN
      SD=X(3)
      CD=DSQRT(1.D0-SD*SD)
      SR=X(2)/CD
C     CR=X(1)/CD
      CR=DSQRT(1.D0-SR*SR)
      IF(X(1).LT..0D0)CR=-CR
      WRITE(6,12)ZEIT,M
   12 FORMAT(/' Ableitungsmatrix des Ortes zur Zeit ',F13.4,
     . ' nach den ',I2,' Unbekannten:')
      DO13L=1,3
   13 WRITE(6,14)(DXDU(I,L),I=1,M)
   14 FORMAT(7(1X,G17.10))
C    Mittl. Fehler FX in aequatorealen Koordinaten X,Y,Z,R  des Kometen
      DO10J=1,M
   10 DDDU(J)=DXDU(J,1)*DDDX(1)+DXDU(J,2)*DDDX(2)+DXDU(J,3)*DDDX(3)
      CALL MF2(M,DDDU,FD)
      DO2I=1,3
    2 CALL MF2(M,DXDU(1,I),FX(I))
      WRITE(6,20)ZEIT,FX(1)*AE,FX(2)*AE,FX(3)*AE,FD*AE
   20 FORMAT(///' Unsicherheit der aequatorealen Koordinaten und ',
     .  'helioz.Distanz: ',//' Zeit:',F15.3,' dx ',F8.1,' dy ',
     .  F8.1,' dz ',F8.1,' dr ',F8.1,' (in km)'/)
      WRITE(6,21)
   21 FORMAT(//20X,'Fehlerellipsoid'//' Winkel',5X,'dr(km)',5X,
     .  'dt(km)',6X,'C1',7X,'C2',7X,'C3'/)
C    Berechnung von DRDX(I,J)=dR(J)/dX(I) mit Variation von W
C    (ergibt die Fehlerellipse in der s,t-Ebene.)
      DRDX(1,3)=CD*CR
      DRDX(2,3)=CD*SR
      DRDX(3,3)=SD
      DO1IW=1,36
      W=ZWEIPI/36.D0*IW
      SW=DSIN(W)
      CW=DCOS(W)
```

LISTING OF SUBPROGRAM ERAN (CONT.)

```
        DRDX(1,1)=CW*SD*CR-SW*SR
        DRDX(2,1)=CW*SD*SR+SW*CR
        DRDX(3,1)=-CW*CD
C       DRDX(1,2)=-SW*SD*CR-CW*SR
C       DRDX(2,2)=-SW*SD*SR+CW*CR
C       DRDX(3,2)=SW*CD
C     Kontrolle: bei Transformation der Richtung X, wird t=1,s=r=0
C       DO9I=1,3
C       R(I)=.0D0
C       DO9J=1,3
C     9 R(I)=R(I)+X(J)*DRDX(J,I)
C     Berechnung des Winkels OBLQ der r-Achse zu den vorgeg. Ebenen (Ekliptik,
C     Kometenbahn usw). Falls cos(OBLQ)=r-Komponente d.Normalen=0 liegt
C     die r-Achse in der betr. Ebene
        SOBLQ1=X1(1)*DRDX(1,1)+X1(2)*DRDX(2,1)+X1(3)*DRDX(3,1)
        SOBLQ2=X2(1)*DRDX(1,1)+X2(2)*DRDX(2,1)+X2(3)*DRDX(3,1)
        SOBLQ3=X3(1)*DRDX(1,1)+X3(2)*DRDX(2,1)+X3(3)*DRDX(3,1)
C     Transformationsmatrix DRDU(I,J)=dR(J)/dU(I), fuer r immer, fuer s nicht,
C     fuer t nur einmal (bei IW=1) zu berechnen
        J=-1
      7 J=J+2
        DO6I=1,M
        DRDU(I,J)=.0D0
        DO6K=1,3
      6 DRDU(I,J)=DRDU(I,J)+DRDX(K,J)*DXDU(I,K)
C     Fehler in r-Richtung
        CALL MF2(M,DRDU(1,J),FR(J))
        IF(IW.EQ.1.AND.J.EQ.1)GOTO7
C     Ausdruck der Ergebnisse
        WRITE(6,22)10.*IW,FR(1)*AE,FR(3)*AE,SOBLQ1,SOBLQ2,SOBLQ3
     22 FORMAT(2X,F5.1,2(3X,F8.1),3(2X,F7.3))
      1 CONTINUE
        WRITE(6,23)
     23 FORMAT(/,' dr,dt mittl. Fehler in r- bzw. t-Richtung,',
       . /' C1-C3 Sinus der Neigung der r-Richtung zu Ekliptik',
       . ' bzw. Kometen- und Giottobahn'//)
        RETURN
C       DEBUG SUBCHK
        END
```

NORMAL EQUATIONS, CORRELATION MATRIX AND MEAN ERRORS OF THE ORBITAL ELEMENTS OF COMET HALLEY FROM OBSERVATIONS 1835 - 1986 (EXAMPLE)

Parameters to be estimated:

$T, q, e, \omega, \Omega, i, A_1, A_2$ (units day, AU, °, 10^{-8} AU/d^2)

at osculation epoch 1986 February 19.0 TDB

Normalgleichungen der Bahnverbesserung:

.35120924492+000	-.16883274232+004	-.30029076886+005	.92815557892+000	-.62380340891+001	-.24027559129+002	-.33051223284+001	-.45103649373+002	.84813536801-006
-.16883274232+004	.93992047326+008	.17780212329+010	.15428176716+006	-.10320196983+006	.14321655191+006	.19594917516+006	-.23604089914+006	-.76089498384-002
-.30029076886+005	.17780212329+010	.31937121259+011	.26788050778+007	-.17586863788+007	.26133822535+007	.35196561503+007	-.41485637604+007	-.13902093538+000
.92815557892+000	.15428176716+006	.26788050778+007	.81163182406+004	-.88048584718+004	-.56730451776+004	.29701432958+003	-.13710606534+005	.26220823871-003
-.62380340891+001	-.10320196983+006	-.17586363788+007	-.88048594718+004	.99538268619+004	.67492807723+004	-.19585716027+003	.15885583354+005	-.31020169395-003
-.24027559129+002	.14321655191+006	.26133822535+007	-.56730451776+004	.67492807723+004	.67898416503+004	.28633466563+003	.14198535940+005	-.31263658472-003
-.33051223284+001	.19594917516+006	.35196561503+007	.29701432958+003	-.19585716027+003	.26633466563+003	.39788700942+003	-.46099662700+003	-.15245278729-004
-.45103649373+002	-.23604089914+006	-.41485637604+007	-.13710406534+005	.15885533354+005	.14198535940+005	-.46099662700+003	.33558656549+005	-.63816661297-003
.84913536801-006	-.76089498384-002	-.13902093538+000	.26220823871-003	-.31020169395-003	-.31263658472-003	-.15245278729-004	-.63816661297-003	.26651317197-008

Korrelationskoeffizient zwischen den Unbekannten: / Mittl. Fehler:

								Mittl. Fehler
1.0000000	.5623554	.4182722	.0127923	.0009879	.0362536	-.4483040	-.2532422	.000016
.5623554	1.0000000	.2897378	-.4917236	-.0152506	.0489722	-.3534782	-.5092978	.000000281
.4182722	.2897378	1.0000000	.3623999	.0250039	-.0913294	-.9977339	-.5772425	.00000023
.0127923	-.4917236	.3623999	1.0000000	.6529793	-.0467699	-.3196597	-.0034441	.0000254
.0009879	-.0152506	.0250039	.6529793	1.0000000	.0165408	-.0234182	-.1408201	.0000171
.0362536	.0489722	-.0913294	-.0467699	.0165408	1.0000000	.0758257	-.6334928	.0000206
-.4483040	-.3534782	-.9977339	-.3196597	-.0234182	.0758257	1.0000000	.6001429	.517154-002
-.2532422	-.5092978	-.5772425	-.0034441	-.1408201	-.6334928	.6001429	1.0000000	.536744-006

OUTPUT FROM SUBPROGRAM ERAN (EXAMPLE)

```
Ableitungsmatrix des Ortes zur Zeit 19860313.6007 nach den  8 Unbekannten:
  .2450560820-001   .1110683575+001  -.1331218339+000  -.6858420709+000   .7237114619+000  -.1781394714-001  -.2512985808-005
 -.1355555905-004
 -.2411877496-002  -.1314316593+001  -.1771911350+000   .5736012191+000  -.4832986097+000  -.1589712942-001  -.1449945412-004
  .2569509844-005
  .5175946604-002  -.1718431911-001  -.9428191812-001  -.4806523480-001  -.2096006257+000   .6447622164-001  -.4269238138-005
 -.3111695139-005
```

Unsicherheit der aequatorealen Koordinaten und helioz.Distanz:

Zeit: 19860313.601 dx   109.0 dy   79.5 dz   18.8 dr   29.3 (in km)

Fehlerellipsoid

winkel	dr(km)	dt(km)	C1	C2	C3
10.0	44.4	123.2	-.993	.941	-.989
20.0	39.4	123.2	-.998	.959	-1.000
30.0	33.8	123.2	-.973	.948	-.979
40.0	28.3	123.2	-.919	.908	-.929
50.0	23.7	123.2	-.837	.840	-.851
60.0	21.5	123.2	-.729	.747	-.747
70.0	22.6	123.2	-.599	.631	-.620
80.0	26.4	123.2	-.451	.496	-.474
90.0	31.7	123.2	-.289	.346	-.314
100.0	37.4	123.2	-.118	.185	-.145
110.0	42.7	123.2	.056	.019	.029
120.0	47.2	123.2	.229	-.149	.203
130.0	50.7	123.2	.394	-.310	.370
140.0	53.0	123.2	.548	-.463	.525
150.0	53.9	123.2	.685	-.602	.665
160.0	53.5	123.2	.801	-.722	.785
170.0	51.7	123.2	.893	-.821	.880
180.0	48.6	123.2	.957	-.895	.949
190.0	44.4	123.2	.993	-.941	.989
200.0	39.4	123.2	.998	-.959	1.000
210.0	33.8	123.2	.973	-.948	.979
220.0	28.3	123.2	.919	-.908	.929
230.0	23.7	123.2	.837	-.840	.851
240.0	21.5	123.2	.729	-.747	.747
250.0	22.6	123.2	.599	-.631	.620
260.0	26.4	123.2	.451	-.496	.474
270.0	31.7	123.2	.289	-.346	.314
280.0	37.4	123.2	.118	-.185	.145
290.0	42.7	123.2	-.056	-.019	-.029
300.0	47.2	123.2	-.229	.148	-.203
310.0	50.7	123.2	-.394	.310	-.370
320.0	53.0	123.2	-.548	.463	-.525
330.0	53.9	123.2	-.685	.602	-.665
340.0	53.5	123.2	-.801	.722	-.785
350.0	51.7	123.2	-.893	.821	-.880
360.0	48.6	123.2	-.957	.895	-.949

dr,dt mittl. Fehler in r- bzw. t-Richtung.
C1-C3 Sinus der Neigung der r-Richtung zu Ekliptik bzw. Kometen- und Giottobahn

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