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ROCK OR SAND ?

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ROCK OR SAND ?

I. CONTROLLING PURPOSE

What is the foundation upon which mathematics rests? Is it as solid as rock or as meta-stable as sand? Is it a material foundation derived from the observation of our material universe or is it an abstract basis invented by the mind? What is the fundamental difference between mathematics and the sciences? To answer these questions will be the purpose of this paper. The answer to all these questions is simply that mathematics is built upon conceded assumptions invented by the mind.[1] To fully understand this statement is to understand the foundation of mathematics. The nature in which mathematics is built upon this foundation will not be considered to any extent. Simply speaking, the mathematician does nothing more than to say that if such and such be true, then this is true.

1. The stock definitions of mathematics, altho concise, are usually confussing and will be avoided. For example, Mario Pieri, the Italian mathematician said,"Mathematics is the hypothetico-deductive science". This definition by Pieri was taken from Cassius J. Keyser's book, "The Pastures of Wonder: the Realm of Mathematics and the Realm of Science".

The purpose of this paper is to disclose the nature of this "such and such".

II. BUILT ON UNPROVEN AXIOMS

Mathematics is not a pure, divine study which proves everything, but merely is a game of: if you will concede such and such, I will prove this. An allegory to a mathematical system is the following sentence. If all men are good, then I am good because I am a man. The mathematician then says that he has formulated a proposition - namely "that I am good". The proof of the proposition or theorem is "because I am a man". And lastly, the foundation of the proposition is the axiom "all men are good". The mathematician assumes responsibility only for his deductive powers to reach conclusions, not for the truth of the axion, which he first gets you to concede. To toy with the proving of the axioms is the purpose of the sciences, not of the mathematics. No theorems can be invented nor proven unless something is conceded- namely, the "such and such". Cassius J. Keyser once said "In every field it is true that from nothing assumed, nothing can be derived".[2]

2. Keyser, Cassius J., "Mathematics", _Encyclopedia Americana_, Eighth Edition, XV111, p. 433

III. AXIOMS AND INDEFINABLES

Not only is mathematics built upon unproven assumptions, but these assumptions also contain many indefinables. As Keyser said, "Any discourse, what ever, whether mathematical or non-mathematical, is, and of necessity must be, discourse about terms or symbols that, however much they are or may be described, remain ultimately undefined".[3] For example, in the Eculidian Geometry all terms are defined by the use of indefinables - namely, the point and the straight line. No one has as yet given a satisfactory definition of a point or a straight line. Yet, the propositions or conclusions of the entire study of geometry are derived from a small number of axioms about these indefinables. An example of such an axiom is that through any two points pass one and only one straight line. The true "mathematician regards geometry as simply tracing the consquences of certain axioms dealing with undefined terms".[4]

IV. TRUTH OF AXIOMS

One may now ask if these axioms are a solid foundation.

3. Keyser, Cassuis J., The Pastures of Wonder; the Realm of Mathematics and the Realm of Science, p.13

4. Ramsey, Frank P,, "Foundations of Mathematics", The Encyclopedia Britannica, Fourteenth Edition, XV, p. 83

Precedence stands first in our discussion. How is it that the mathematician's palace has existed twenty thousand years - longer than any other study? Is it possible that it is built upon the sands? This question I leave up to you. If precedence is not convincing evidence of the truth of the axioms, let us turn to the consciencious physicist who will actually make two points on a piece of paper and see just how many straight lines he can pass through them. In additional to experience, there is the slight intuitional force that tells one that the axioms are true. Bertrand Russell said, "The axioms are recommended only by a certain appeal to the imagination".[5] Man's imagination, no doubt, comprises the mightiest force working for their general acceptance. Precedence, experience, and intuition convince one of the truth of the axioms.

V. SOURCE OF AXIOMS

Now that we have an idea of the vital part that the axioms and indefinables play in mathematics, we will try to discover their source. Indeed the predominant factor which distinguishes mathematics from the sciences is not in their conclusions, which usually coincide, but in the source of their axioms.

5. Russell, Bertrand, Principles of Mathematics.

Mathematical axioms are invented by the mind; scientific axioms are discovered by the senses. Here lies the difference. Keyser once said, "arithmetic of counting-house and the geometry of carpenter-are not mathematical but are strictly scientific in even the critic's sense of scientific, for they are discovered and established by observation and experiment long before mathematicians succeeded (only recently) in deducing them from postulates".[6] In general, the difference lies in that mathematics is a product of the mind, not the senses; a product of invention, not of discovery. The laws of science are discovered. Newton did not invent the laws of universal gravitation - they always existed; he merely discovered their presence and wrote them down for posterity. On the other hand, the "fundamental things of mathematics seem to have been created by the mind. The positive integers, for instance, were not found in nature, but were created by the human spirit".[7]

6. Keyser, The Pastures of Wonder; the Realm of Mathematics and the Realm of Science, p. 17
Notice that the word "postulates" is here used synonomously with "axiom",- "Postulate" has been abandoned in favor of "axiom",so as not to confuss it with propositions, which are deduced from postulates.

7. Carmichael, Robert Daniel, The Logic of Discovery, p. 257

VI. SCOPE OF MATHEMATICS

The difference in choice of the axioms of mathematics and the sciences brings about the subject of their scope. The sciences are limited to the sensuous universe with its mere thousands and thousands of galaxies each containing billions of stars and billions upon billions of tiny spheres like our earth. But greater yet is the realm of mathematics-a study which includes not only all of the sensuous universe, but also the infinite meta-physical universe. "Immense indeed and marvelous is our own world of sense but compared with mathematics it is a mere point of light in a shining sky".[8]

8. Keyser, "Mathematics", The Encyclopedia Americana XVIII p. 433