Notes on the Tunings

We microtonalists inhabit a very different musical universe from that of most "normal" musicians. To a normal musician, the line between dissonance and consonance is clear-cut, and all of the chords in his/her musical vocabulary unambiguously fit into one category or another, as dictated by common practice music theory. But when we abandon the grid of 12-tone equal temperament, our ideas about consonance and dissonance become a lot more complex—we encounter chords that don't sound anything like those of the common practice, and thus don't obviously fit into either category. We are also forced to find new ways of organizing pitches, which lead to new forms of harmonic "functionality" that are quite unexplored. Since these new harmonic structures are so unlike those of the common practice, we might feel as though we are lost in a strange world, with neither map nor compass—save for our trusty ears, that is.

To date, there exists nothing that I would call "microtonal music theory"—no one has yet written anything meant to guide composition in microtonal tuning systems. However, there is a *great* abundance, a plethora even, of what I'll call "tuning theory"—theory describing how to select tunings for the making of music. It is all based, to a greater or lesser degree, on the practice of **Just intonation**, which involves tuning intervals to exact simple-integer frequency ratios, for the purpose of producing harmony that is smooth, fused, and beatless. This sort of tuning theory has in many ways remained unchanged since the ancient Greeks, who discovered the "special" properties of simple-integer rational intervals by playing with string lengths on their monochords (and other instruments). Just intonation presented a number of compositional difficulties to Western composers (which I won't address here), and eventually **temperament** was discovered. Though temperament does not rely on simple integer ratios, it does *approximate* them, and there have long been debates among tuning theorists over whether it is better to have greater accuracy of approximation at the expense of a more complicated temperament, or lesser complexity at the expense of a less accurate temperament. In any case, ratios have always been the ideal that temperament is meant to approach, and this remains unchanged among tuning theorists to this day.

One area in which modern theories have advanced beyond the Greeks is in the explanation of *why* intervals defined by simple ratios sound good. For starters, it has been discovered that all sounds get their timbre (or "tonal color") from their overtone structure, which is to say the multiple and varied frequencies that make up the sound. Particularly, most musical sounds used for harmonic music—voices, vibrating strings, resonating columns of air, etc.—possess overtone structures wherein all the frequencies are related to each other by simple integer ratios. Then the venerable Plomp and Levelt in 1965 discovered something called the "critical band" when examining interactions between two sine tones: when the frequency of one sine tone is tuned to fall within the critical band of the other sine tone, strong "roughness" is produced and the two tones sound dissonant together. William Sethares later expanded Plomp and Levelt's model to work with complex tones, i.e. those which possess overtones, by modeling critical band interactions between the interacting partials of any number of complex tones in harmony. Sethares' model shows that for any timbre, the most consonant musical harmonies will be those that mirror the frequency relationships within the timbre's overtone structure—thus explaining why the ancient Greeks found intervals based on simple ratios to be the most consonant.

Along with Sethares' model, Paul Erlich produced a model of discordance known as "harmonic entropy". This theory models consonance not based on critical band effects or overtone structures, but based on information theory. This model is based on the idea that the brain, when confronted with an auditory stimulus representing multiple pitches, it will try to intelligize these pitches *as* harmonics of some root or "fundamental" pitch (whether or not the fundamental itself is present in the stimulus). If the pitches are harmonically related by simple integer ratios that follow the natural overtone series (an infinite ratio of the form 1:2:3:4:5:6:7:8...), the brain can easily produce a "virtual fundamental", to the point where the VF may even be audible despite its absence in the stimulus. On the other hand, if the pitches are *not* harmonically related in a way suggestive of a single overtone series, the brain will have difficulty producing a single virtual fundamental—and this difficulty leads to perceiving the harmony as "discordant". In other words, the VF is considered to be "information" that is to be conveyed by harmony, so that the lowest-entropy harmony is a unison (the fundamental can be perceived directly without any conflicting or confusing additional information). All other harmonies, because they contain additional information, are by definition more uncertain and thus higher in entropy, and according to Erlich's model, the entropy goes up the further the harmony gets from suggesting an overtone series rooted on a single fundamental.

Both Erlich's and Sethares' theories are not, however, theories of *music*. They are *psychoacoustic* theories, which is to say they are scientific, not artistic. Both predict that the most "consonant" interval (for a normal musical timbre) is the perfect unison, followed by the octave, then the perfect fifth, and so on up the overtone series; this quantification of musical sound seems to obscure the fact that music (as an art-form) requires contrast, and that music consisting solely of "maximally consonant" harmony would be utterly boring and unpleasant. Assuming we need a scale of at least 7 distinct notes to make music, and that we are making music with normal harmonic timbres, both theories suggest that the diatonic scale as tuned in 1/4-comma meantone is the absolute pinnacle of tuning. Likewise, both agree that the maximally-consonant 12-tone scale is...12-tone equal temperament. Thus, if we are attempting to find a new scale to use with normal musical instruments, the inescapable conclusion produced by both of these theories is that *whatever we find will be inferior to what is already established*. It will be either more complex or less accurate at approximating the simplest ratios, and therefore *worse*. So while both theorists are themselves microtonalists, seeking for new scales that will function as compelling alternatives to the common practice, the theories they have produced suggest that this is impossible—for what is compelling about "second-best" scales?

Frankly, though I greatly admire the scientific and mathematical prowess of these two theorists, as well as the music they have produced, I find their theoretical work to be somewhat of an insult to microtonality (for lack of a better word). I do not believe that there is such thing as an "objectively bad" tuning or scale. Neither is there such thing as an "objectively dissonant" tuning or scale. One can compose accessible, compelling, even *beautiful* music in just about any tuning or scale imaginable. I could argue this till I'm blue in the face, but when it comes to music, words are worth less than nothing. Instead, I sought to compose an album of emotionally-varied, intelligible, and compelling music, using scales and harmonies that would be described as *maximally dissonant* by both models of harmonic consonance. The scales and harmonies used throughout this album are among the theoretical "worst" I could find, not only through my own research but with the aid of several microtonalists who either subscribe to the above models or have produced similar models of their own. In almost every case, critical band roughness and harmonic entropy are off-the-charts, or in any case well above what is considered "acceptable". There are a few scales that turned out to be less dissonant than they were supposed to be, because of "lurking consonances" hiding out in different modes of the scale or because of the theorist's mistaken ideas about how to achieve maximum dissonance in a scale, but for the most part it would be difficult to produce a collection of scales less consonant than the ones on this album. I think the results speak for themselves, but for those interested in a deeper analysis, I have included the following track-by-track breakdown of each tuning used (and how it violates the various theoretical principles of consonance).

Track 01: "Uncanny Valley": 24 Equal Divisions of the Octave

(limited to a scale of 0—250—300—550—600—850—900—1150—1200 cents)

This scale represents the absolute worst harmonic scale I could come up with myself. It has only one other modal rotation, and that is:

0—50—300—350—600—650—900—950—1200 cents

I devised this scale to ensure as many intervals as possible would be near the exact mid-point between two Just intervals. 50 cents is almost exactly the global maximum of both dyadic harmonic entropy and root-tone critical band roughness; 250 cents is almost exactly the midpoint between 7/6 and 8/7, 300 cents is close to the midpoint between 7/6 and 6/5, 350 cents is nearly the exact midpoint between 6/5 and 5/4, 550 cents is nearly the exact midpoint between 4/3 and 7/5, 600 cents is the exact midpoint between 7/5 and 10/7, 900 cents is near the midpoint between 5/3 and 12/7, 950 cents is nearly the exact midpoint between 12/7 and 7/4, and 1150 cents is near the next-largest global maximum of dyadic harmonic entropy after 50 cents. There is nothing near a 3/2 or a 5/4, the nearest thing to 7/4 is 20 cents off, and the simplest near-"Just" triadic harmony possible is 8:11:13, which is not very simple or smooth at all even when pure.

The piece that came out of this scale, "Uncanny Valley", is the first time I've ever used actual quartertones as a microtonalist, and I have to say I'm kind of disappointed that this scale didn't sound worse. Contrary to everything I expected, most of the triads I tried (excepting those that involved intervals of 50 cents or 1150 cents) sounded pretty smooth. The triads I used in most of the first half of the piece are 0-300-550 cents, 0-250-550 cents, and 0-250-600-cents. Despite the differences of a full quartertone, these chords all felt more or less like regular 12-TET diminished chords, perhaps a bit mellower if anything. The sine-wave sections are full of intervals apart by 50 cents or 1150 cents, and while some of the chords sound pretty cringe-worthy, the slow beating of the lower and sparser parts is kind of relaxing to my ears, reminiscent of those "binaural beating" recordings that are supposed to induce meditative brain-wave patterns. The two solo sections that follow the first sine section feel very natural (if somewhat exotic) to me, and my general outlook on 24-EDO has actually been improved by writing this piece. So apparently I kind of suck at creating impossibly-dissonant scales!

Track 02: "Broken Dream Jar": 13 Equal Divisions of the Octave:
0—92.31—184.62—276.92—369.23—461.54—553.85—646.15—738.46-830.77—923.08—1015.38—1107.69—1200 cents
(limited to a scale of 0—184.62—369.23—461.54—646.15—830.77—923.08—1107.69—1200 ¢)

13-EDO has a reputation of being almost comically horrid, owing to its total lack of a 3/2, yet I find it to be a very pleasant and even beautiful tuning. It approximates ratios of 10/9, 7/6, 11/8, 12/7, and 9/5 quite well, is not half-bad at 13/8, and is not much further off than 12-TET in approximating 5/4 and 8/5. The "flat 4th" of 461.54 cents approximates 17/13, and the major 7th is very near a ratio of 17/9, and while neither of these ratios are properly consonances, there is some vague "regularity" about them that makes them not entirely unpleasant to my ears. A stack of 13-EDO "whole-tones" approximates 8:9:10, as it does in 12-TET, and adding an extra whole-tone gets us to a decent 8:9:10:11. A chord of 0-369-831 cents approximates 8:10:13 fairly well also, and 0-554-831 is spitting-distance from 8:11:13, so 13-EDO gives lots of strong implications of overtones 11 and 13. These overtones are not smooth and powerful like 3 and 5 are, but they can have an intriguingly-complex beauty when treated with sensitivity.

Of course, all this talk of ratios is just window-dressing, as in 13-EDO I find the most consonant harmonies to be 0-185-462 and 0-276-462, two triads which can be chained together into pentads like 0-185-462-646-923 or 0-277-462-738-923. These chords are not "beatless", and I am utterly confounded as to the best way to translate them into ratios from the overtone series, but nevertheless I believe them to be some of the smoothest and most "restful" chords 13-EDO has to offer. They are beautiful, evocative, and totally unfamiliar. I used a few of these in the minimal piano interlude of this song, and have explored them more fully in other compositions.

What I like the most about 13-EDO, though, is the melodies it makes possible. The "major scale" used for this track is just like an ordinary major scale (TTsTTTs) but with an extra semitone added in the middle to become TTsTTsTs. This scale is one of my absolute favorites of all time, because it allows for melodies that start out sounding normal but end up somewhere totally unexpected, and there's always a point in the melody where that "extra" semitone causes a distortion that sounds simultaneously natural and totally wrong. It sounds totally insane when harmonized in parallel thirds, too, as you can hear in the "verse" section prior to the glitch-out breakdown. I can't get enough of it, really. I use this scale all the time in my "non-expositional" music, so of all the tracks on this album, this one was by far the least challenging.

Track 03: "Trilobe": 14 Equal Divisions of the Octave:
0—85.7—171.4—257.1—342.9—428.6—514.3—600—685.7—771.4—857.1—942.9—1028.6—1114.3—1200 cents
(limited to a scale of 0—171.4—257.1—428.6—514.3—685.7—771.4—942.9—1028.6—1200 cents)

This one was requested by Michael Sheiman, author of three of the non-equal tunings to be introduced later and my biggest "inspiration" for making this album. In terms of JI, 14-EDO is spectacularly "meh"...it's not nearly as nasty as some of the other EDOs can be, but it's far enough off that there just aren't any very strong intervals at all. Yeah, there is a recognizable 3/2 and 4/3, but they both beat pretty strongly and are thus not as stable as we're used to. Almost all the intervals are near maxima of harmonic entropy, meaning they are between two Just intervals but not close enough to either to get "sucked in", making them discordant because the brain gets confused by them: 171 cents is between 11/10 and 10/9, 257 cents is between 8/7 and 7/6, 343 is between 5/4 and 6/5, 429 is between 5/4 and 9/7, 771 is between 14/9 and 11/7, 857 is between 8/5 and 5/3, 943 is between 12/7 and 7/4, and 1029 is between 9/5 and 11/6. They all beat pretty noticeably, too, although I don't find the beating too unpleasant (unless I use something with very loud high overtones like a sawtooth wave). Unlike 8, 11, and 13-EDOs, nothing in 14 is close to a standard 5/4 major third OR a 6/5 minor 3rd. It's also a challenge melodically, as the whole-tone is very narrow, almost-but-not-quite a neutral 2nd, and unlike the whole-tones in 13-EDO and 11-EDO, a stack of 14-EDO whole-tones does not sound very good.

And yet, despite all these limitations, I feel like the harmonic progressions in "Trilobe" are very functional and natural, in a very traditional sense. I use the 257-cent interval as a "subminor" 3rd and the 429-cent interval as a "supermajor" 3rd, and though the resulting triads beat pretty noticeably, their harmonic function remains intact. The scale I used is a 9-note "major/minor" scale, TsTsTsTsT, kind of like an octatonic scale with an extra whole-tone squeezed in. This scale is produced by repeatedly stacking the subminor 3rd, and since two subminor 3rds in 14-EDO make up a "perfect" 4th (i.e. a near-4/3), this scale is very well-supplied with subminor and supermajor triads. In fact, seven of the nine roots are tonics of at least one type of these "consonant" triads, and in fact many triads have both a subminor *and* a supermajor 3rd. In point of fact, since this is a 9-note scale, the "supermajor 3rd" is technically a "major 4th"; to better understand this, take a look at this table of modal rotations of this scale:

	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	Oct.
Mode I	0	171.43	257.14	428.57	514.29	685.71	771.43	942.86	1028.57	1200
Mode II	0	85.71	257.14	342.86	514.29	600	771.43	857.14	1028.57	1200
Mode III	0	171.43	257.14	428.57	514.29	685.71	771.43	942.86	1114.29	1200
Mode IV	0	85.71	257.14	342.86	514.29	600	771.43	942.86	1028.57	1200
Mode V	0	171.43	257.14	428.57	514.29	685.71	857.14	942.86	1114.29	1200
Mode VI	0	85.71	257.14	342.86	514.29	685.71	771.43	942.86	1028.57	1200
Mode VII	0	171.43	257.14	428.57	600	685.71	857.14	942.86	1114.29	1200
Mode VIII	0	85.71	257.14	428.57	514.29	685.71	771.43	942.86	1028.57	1200
Mode IX	0	171.43	342.86	428.57	600	685.71	857.14	942.86	1114.29	1200

So you can see, a 1-3-6 pattern usually makes a subminor triad, and a 1-4-6 pattern makes either a supermajor or a neutral triad. The odd thing about the neutral triad is that it tends to sound major in contrast to the subminor triad, but minor in contrast to the supermajor triad, and if you listen closely in this piece (especially during the middle section) you can hear this "flip-flopping" happening in a few places. This is really a fascinating scale to work with compositionally, and I think the chord progressions it makes possible are a great compensation for the...colorful...sound of the harmonies.

Track 04: "Anxiously Orbiting in Andromeda": John O'Sullivan's "Worst Scale"

A 12-note scale, mapped 1-to-1 to interval classes of 12-TET as found on a piano keyboard

Mode	1st	m2	M2	m3	M3	4	#4/b5	5	m6	M6	m7	M7	Oct.
I	0	102	*217.6*	*291.3*	351	466.6	524.7	659.8	742.3	849.1	*993.2*	1135.9	1200
II	0	115.6	**189.3**	*249*	364.6	422.7	557.8	640.3	747.1	**891.2**	1033.9	*1098*	1200
III	0	73.7	133.4	*249*	**307.1**	**442.2**	524.7	631.5	**775.6**	*918.3*	**982.4**	**1084.4**	1200
IV	0	59.7	175.3	**233.4**	368.5	451	557.8	**701.9**	844.6	*908.7*	**1010.7**	1126.3	1200
V	0	115.6	173.7	**308.8**	**391.3**	**498.1**	642.2	**784.9**	849	*951*	*1066.6*	1140.3	1200
VI	0	58.1	**193.2**	**275.7**	**382.5**	526.6	669.3	733.4	835.4	*951*	**1024.7**	**1084.4**	1200
VII	0	135.1	*217.6*	**324.4**	468.5	**611.2**	675.3	**777.3**	**892.9**	**966.6**	*1026.3*	1141.9	1200
VIII	0	82.5	**189.3**	333.4	476.1	540.2	642.2	757.8	*831.5*	**891.2**	*1006.8*	*1064.9*	1200
IX	0	106.8	*250.9*	**393.6**	457.7	559.7	675.3	749	**808.7**	*924.3*	**982.4**	1117.5	1200
X	0	144.1	*286.8*	350.9	452.9	568.5	642.2	**701.9**	**817.5**	**875.6**	**1010.7**	*1093.2*	1200
XI	0	142.7	**206.8**	**308.8**	424.4	**498.1**	557.8	673.4	731.5	866.6	*949.1*	**1055.9**	1200
XII	0	64.1	166.1	*281.7*	355.4	*415.1*	530.7	**588.8**	723.9	**806.4**	*913.2*	**1057.3**	1200

John O'Sullivan is the author of *The Mathematics of Music*, a short book documenting his quest for a "better" 12-tone scale that fits his rather stringent criteria for consonance. He spent years experimenting with different formulae to quantify the consonance of Just intervals, found the 12 most consonant intervals that are near to the notes of 12-TET, and then wrote a program to optimally temper each note so that there would be as many intervals as possible within around 7 cents of Just (to be exact, a ratio of 256/255, or 6.77587576937045 cents). He is also one of the main inspirations for this album, so of course I wanted to know what his "worst" scale would look like. He described for me the process he used to arrive at the above 12-note scale:

> "I listed all 20 notes that (I consider) go with 1/1 in harmony. These are: 9/8, 8/7, 7/6, 6/5, 5/4, 9/7, 4/3, 11/8, 7/5, 10/7, 3/2, 11/7, 8/5, 5/3, 12/7, 7/4, 9/5, 11/6, 13/7 and 2/1. Next I identified all notes that are exactly midway between each successive pair of good notes so that the 'bad' notes have the maximum deviation from the nearest good note below them and the nearest good note above them. Then I chose 12 notes out of 20 that have the maximum deviation from a good note."

This approach looks like it worked pretty well if you just look at mode I, but in other modes there were plenty of good intervals that "crept in"; so many, in fact, that rather than list all of them mode-by-mode, I've just highlighted them in dark blue in the above chart. Near-Just intervals are bolded, non-Just-but-still-consonant intervals are italicized. Without fail, every single interval in John's "good" list makes at least one appearance in this scale. I count a total of 64 "accidental consonances" out of 132 intervals (octaves and unisons excluded), so that's just about 48% consonance. The modes with the most consonances are VII and III (with 7 consonant intervals each). Only the interval classes of m2 and #4/b5 fail to include a single consonant interval in all of the modes. For comparison, in 12-TET we'd call the m2 and #4/b5 dissonant in all modes, and all other intervals consonant in all modes, making for a total of 108 consonances, or about 82% consonance. So John *did* succeed in drastically reducing the number of consonances, but he did not succeed in eliminating them all together.

The music that I wrote with this scale is less an example of making dissonance sound good and more an example of finding the accidental consonances and running with them (note that I did not analyze the different modes of the scale until after the tune was finished). I found two good 4-chord progressions, made one the "verse" and the other the "chorus", and then improvised over them. This is the largest unequal "scale" on the album, so I tried to play as chromatically as possible with the melodies to make the texture of the whole scale apparent. There's one part, just prior to the 2nd chorus, where I played a chromatically-ascending vibraphone passage that really gives me goosebumps! All in all, I really liked this scale, as it's full of strong contrasts between consonance and dissonance—a feature I think is really important to have in a scale. John was not a huge fan of the piece but there were parts he liked. I could probably do better if I gave the scale a second go, now that I've analyzed it, but I think that might be cheating. One of my main premises on this album was to avoid analyzing any unfamiliar tunings (I've already analyzed the equal tunings half to death) and to make the tunes as off-the-cuff and uncontrived as possible, because I believe that the real test of a tuning is how easy it is to use in improvisation.

Track 05: "Self-Destructing Mechanical Forest": Michael Sheiman's "Bad Fifth" Scale

	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	Oct.
Mode I	0	137.14	274.29	411.43	548.57	651.43	788.57	925.71	1062.86	1200
Mode II	0	137.14	274.29	411.43	514.29	651.43	788.57	925.71	1062.86	1200
Mode III	0	137.14	274.29	377.14	514.29	651.43	788.57	925.71	1062.86	1200
Mode IV	0	137.14	240	377.14	514.29	651.43	788.57	925.71	1062.86	1200
Mode V	0	102.86	240	377.14	514.29	651.43	788.57	925.71	1062.86	1200
Mode VI	0	137.14	274.29	411.43	548.57	685.71	822.86	960	1097.14	1200
Mode VII	0	137.14	274.29	411.43	548.57	685.71	822.86	960	1062.86	1200
Mode VIII	0	137.14	274.29	411.43	548.57	685.71	822.86	925.71	1062.86	1200
Mode IX	0	137.14	274.29	411.43	548.57	685.71	788.57	925.71	1062.86	1200

Michael devised this scale by trying to get as many of his two least-favorite "fifths" (around 650 cents and 685 cents) as possible into a scale, and he definitely succeeded (although technically they're "6ths" in this 9-note scale, as you can see above). The above scale is not the exact same scale he gave me, as his was very very *very* slightly unequal; this one differs from it only by fractions of a cent, and since the resolution of my software synths is only .1 cents, I rounded a little bit for simplicity's sake. Turns out my rounded version of the scale is actually a subset of 35-EDO, based on a circle of consecutive 137.14-cent intervals.

It also turns out that this scale is not actually very dissonant at all, though it is quite "spicy". The 3rds are all very decent 7/6's or 8/7's, the 4ths are all very decent 5/4's or 14/11's, the 7ths are all 11/7's or 8/5's, and the 8ths are all either 12/7's or 7/4's. Four of the 5ths and four of the 6ths are only about 16 cents off from Just, and as a result the 1-3-6 triads on degrees VI, VII, VIII, and IX are all very decent approximations of 6:7:9. On IV and V, 1-3-7 approximates 7:8:11, on II and III 1-3-5 approximates 6:7:8, and on I 1-4-6 approximates 11:14:16 almost perfectly. Truthfully, the 14/11's on the 4th degree of VI, VII, VIII, and IX pair pretty well with the ~686-cent 6ths to make some decent-sounding major triads as well, so we have four roots in this scale that give us four subminor and four major triads that sound pretty decently in tune. In fact, as far as regular 7-limit temperaments that subdivide an approximate 3/2 into 5 equal parts (which is what this scale does), this tuning is very close to optimal! A generator of 136.439 cents would lessen the average error a little bit more (according to Graham Breed's "temperament finder" at http://x31eq.com/temper/).

Needless to say, this piece wasn't challenging at all, and is definitely the "happiest" and brightest-sounding on the album. I think this a great scale, and there are versions of it with similar properties in 17-EDO and 26-EDO (among others) that sound very nice as well. I will admit though that it's kind of weird melodically, and it's strange having all four of the consonant chords clustered up at one end of the scale. I think I ended up tonicizing the VI mode on this track, as it felt most natural to me.

Track 06: "Atlantean Geometry": Michael Sheiman's "Worst Scale #1"

	1st	2nd	3rd	4th	5th	6th	7th	Oct.
Mode I	0	149	334	477	613	764	947	1200
Mode II	0	185	328	464	615	798	1051	1200
Mode III	0	143	279	430	613	866	1015	1200
Mode IV	0	136	287	470	723	872	1057	1200
Mode V	0	151	334	587	736	921	1064	1200
Mode VI	0	183	436	585	770	913	1049	1200
Mode VII	0	253	402	587	730	866	1017	1200

The scale that started it all, Michael produced this in response to my claim that consonance was no indicator of a scale's musical usefulness. I asked him to give me the absolute worst he could come up with in terms of utterly minimizing consonance, and this was his first shot. It's quite irregular and mostly manages to miss approximating 3/2 or 4/3 by a very wide margin (the exception being the 477- and 723-cent intervals, which are close enough to approximate those ratios recognizably). It also misses most of the 5-limit consonances most of the time, though it has one 5/4 and one 6/5 that are not much worse than in 12-TET. In the 7-limit, it has three almost perfect 7/5's (and thus 10/7's) and one excellent 9/7, but 7/6 is at best 13 cents sharp, and 7/4 is at best about 23 cents flat. In the 9-limit, it has two good 9/5's, and in the 11-limit it has three near-perfect 11/6's. From a JI perspective, it's not very good, but not as bad as most of the equal scales.

What this JI analysis does not tell us is that this scale actually sounds really cool. It's got an exotic tropical "latin jazz" kind of flavor to it that I immediately enjoyed. What beating there is in this scale tends to occur between higher (and thus weaker) partials, so it doesn't sound that bad to my ears. When I showed it to some non-microtonalist musician friends, they couldn't believe it was mathematically derived to be as dissonant as possible, and one even thought it sounded "beautiful". I think even Michael was shocked at how good it sounded, especially considering that this was a straight improvisation, with a few extra layers that were all first takes. So not only did I get the scale to sound good, I got it to sound good with next to no compositional effort!

Track 07: "Desperately Seeking Eris": 180-cent Equal Tuning

0—180—360—540—720—900—1080—1260—1440—1620—1800—1980—2160—2340—2520—2700—2880—3060—3240—3420—3600¢

This tuning was independently derived by both Brian Wong and George Secor, at my request for the worst they could come up with. This tuning does not even approximate the *octave* (though it does reach the triple octave of 8/1), and that alone makes it quite a challenge. However, I don't really think it's that bad of a tuning, since it has a recognizable 3/2 at 720 cents that's only 18 cents sharp, and a decent 5/3 at 900 cents that's only 14 cents sharp. Okay, I'll admit that just about everything beats like crazy, and trying to play a melody that goes above the first octave gets really nasty really fast. The fact that it's basically a "squished" version of the whole-tone scale means melodic subtlety is hard to come by, too.

I almost didn't use this tuning, despite the fact that two theorists suggested it as the "worst" they could do, because at the time I lacked any software that could play non-octave scales (or scales with more than 12 notes). However, it turns out that three degrees of 20-EDO spans exactly 180 cents, and I own a 20-EDO guitar, so I decided to write a strictly-guitar piece (the only synth sounds are ambiguously-pitched, meant to add "texture" but not harmony). Turns out that on guitar, this scale is basically "instant Sonic Youth", so that's the direction I went. I did "cheat" a little bit and use harmonics on the guitar, which I'm sure led to some out-of-scale notes, but they certainly don't add any consonance. All in all I think this scale served the late-'80s/early-'90s post-punk oeuvre I was aiming at, so I count this as a success.

Track 08: "Seven-Legged Bicycle": 18 Equal Divisions of the Octave

0—66.7—133.3—200—266.7—333.3—400—466.7—533.3—600—666.7—733.3—800—866.7—933.3—1000—1066.7—1133.3—1200¢

(limited to a scale of 0—200—333.3—533.3—733.3—866.7—1066.7—1200 cents)

This is one of my own concoctions. I do find 18-EDO to be a beautiful tuning for the most part, and it has a few very near-Just harmonies: 9/8, 7/6, 12/7, and 16/9, as well as decent 5/4 and 8/5 (borrowed from 3-EDO), and two very colorful "fourths" that more or less are described by the ratios 21/16 (the lower 466.7-cent fourth) and 15/11 (the higher 533.3-cent fourth). However, the "fifths" are just plain nasty, and the 333.3-cent and 866.7-cent intervals are quite unpleasant as well. The scale I picked out systematically avoids most of the nice-sounding intervals, as it is generated by repeatedly stacking the 333.3-cent interval and wrapping to fit within an octave. The sweet 7/6 is nowhere to be found, and the awkwardly-large semitone of 133.3 cents is in full effect. Take a look at the modal rotations of this scale and you'll see how successfully this scale avoids producing any triad that is remotely close to being consonant from a JI perspective:

	1st	2nd	3rd	4th	5th	6th	7th	Oct.
Mode I	0	200	333.33	533.33	733.33	866.67	1066.67	1200
Mode II	0	133.33	333.33	533.33	666.67	866.67	1000	1200
Mode III	0	200	400	533.33	733.33	866.67	1066.67	1200
Mode IV	0	200	333.33	533.33	666.67	866.67	1000	1200
Mode V	0	133.33	333.33	466.67	666.67	800	1000	1200
Mode VI	0	200	333.33	533.33	666.67	866.67	1066.67	1200
Mode VII	0	133.33	333.33	466.67	666.67	866.67	1000	1200

The only triad in the whole scale that might remotely be called "consonant" is the 0-200-400-cent triad formed on the III degree, which is roughly an 8:9:10. Indeed, the first half of "7-Legged Bicycle" gives a nice exposition of how dissonant any arbitrarily-chosen set of notes in this scale is likely to be: I wrote it by playing several different monophonic lines in isolation, and then arbitrarily overlapping them with each other.

The second half of the piece, however, demonstrates something interesting. I wanted to see what would happen if I just "pretended" this scale was a regular major scale, so I tried writing a homophonous "root-third-fifth" chord progression with an overlaid melodic line harmonized in parallel thirds. The progression I came up with turned out to be V-I-IV-VII-VI, which loops nicely and resolves with surprising strength to the I chord; in fact, it seemed to be a perfectly functional progression, especially with the melody providing some subtle voice-leading. Most fascinating to me was that I seem to hear the near-ubiquitous 333.3-cent third as "flipping" between "major" and "minor" qualities, so that the chord qualities change across the progression to fit what I'm accustomed to hearing in diatonic music. Your mileage may vary, but to my ears it's a compelling auditory illusion.

Track 09: "Lurid Occlusion": Michael Sheiman's "Worst Scale #2"

	1st	2nd	3rd	4th	5th	6th	7th	Oct.
Mode I	0	112.3	271.8	672.9	862.8	953.9	1034.8	1200
Mode II	0	159.5	560.6	750.5	841.6	922.5	1087.7	1200
Mode III	0	401.1	591	682.1	763	928.2	1040.5	1200
Mode IV	0	189.9	281	361.9	527.1	639.4	798.9	1200
Mode V	0	91.1	172	337.2	449.5	609	1010.1	1200
Mode VI	0	80.9	246.1	358.4	517.9	919	1108.9	1200
Mode VII	0	165.2	277.5	437	838.1	1028	1119.1	1200

This scale was Michael's second attempt to run his dyadic consonance optimization program in reverse, and it produced a whopper. Not only are the intervals in this scale mostly quite dissonant, the scale is very very "lumpy" and very "improper," meaning its interval classes don't just overlap, they pass through each other— for example, the largest 2nd is larger than several 4ths, and the smallest 2nd is less than 1/4 the size of the largest 2nd. According to theorist David Rothenberg, who generalized properties of the diatonic scale and sought to explain what makes the diatonic scale so darn nice, improper scales are a challenge to normal modes of musical cognition because they make it hard to mentally categorize intervals due to the overlapping classes. Another result of this is that there are no consistent chord shapes and no symmetric step-patterns, and that alone makes it challenging enough to work with.

However, there are two approximate 3/2's that are between 20 and 30 cents flat, two or three decent 7/6's, one very good 9/7, an excellent 13/8, an excellent 9/5, a 5/4 that's not really any worse than the one in 12-TET, and a decent 7/5. Okay, that's not much, but it's not nothing, either, and shows just how hard it is to ruin one harmony without improving another one (an issue I often ran into myself when experimenting with irregular scales, and the reason why I stuck to equal scales or distributionally-even scales).

The piece that grew out of this scale was also entirely improvised, in a few different layers, and like his first scale, I believe this is a testament to its "failure to suck". I basically threw "Lurid Occlusion" together in the span of an hour or two, just hacking away at my keyboard, and I think it came out as a surprisingly catchy and dynamic little number. There are some parts that really lay the dissonance on thick, but plenty of parts that feel really natural and almost "triumphant", and I swear the final resolution sounds as natural to me as anything I've ever heard.

Track 10: "Malebolge": 8 Equal Divisons of the Octave:
0—150—300—450—600—750—900—1050—1200 cents

Among the EDOs, none avoids the basic Just consonances of 3/2, 5/4, and 7/4 by as wide a margin as 8-EDO. However, it is not without consonances: 6/5, 5/3, and 11/6 are all decently-approximated, and 13/10 (which may or may not be a true consonance) is also well-approximated. Since the smallest step is 150 cents, the maximum root-tone critical band dissonance is also quite low, compared to larger EDOs like 12 and 19. That being said, the most consonant chord is 0-300-600-900 cents, exactly a 12-TET diminished 7th, and the intervals near the middle of the octave (450, 600, and 750 cents) are near local maxima of dyadic harmonic entropy. Theoretically, it's nasty.

...and in practice, it's pretty nasty too, I can't deny it. Played softly, as in the first half of the piece, it's got a dark, creepy, kind of "alien" feel to it that is somewhat reminiscent of the Bohlen-Pierce scale (which is also an equal scale with a step-size of about 146.3 cents—melodically very similar to 8-EDO, but with very near-Just 3:5:7:9 harmonies and a pure 3/1 "tritave"). I found that it felt "bug-like", tentacular, hazy, clammy and somewhat foul. I wanted to conjure the feeling of being explorer on a mostly-deserted alien planet where the few resident life-forms were incomprehensible yet hostile, and I think that really comes through with the first half of the piece.

Increasing the volume and switching to more overtone-rich timbres, one gets the feeling of breaking through the haze into a panorama of infernal suffering. Out of all the scales I've ever played, 8-EDO is the one most deserving of the title of "the Devil's own scale"—*diabolus in musica* indeed! The second half of the piece, when the pipe organ and fuzz guitar kick in, is what I imagine music would sound like in hell (if such a place existed). The name "Malebolge" comes from Dante's *Inferno*, and is the name (fittingly enough) of the eighth circle of hell. It is pure, unadulterated suffering like nothing I've ever heard! I'm sorry, but you just *can't* evoke pain and misery like this with any other tuning. That, to me, is what is most useful about 8-EDO, and makes it quite important as a tuning.

Track 11: "Concrete Monolith": 20 Equal Divisions of the Octave

0—60—120—180—240—300—360—420—480—540—600—660—720—780—840—900—960—1020—1080—1140—1200 cents

(limited to a scale of 0—120—420—540—660—960—1080—1200 cents)

I'll spare you the commentary on 20-EDO in general (it's got a bad but not horrible reputation) and skip right to the scale selected from it (yes, this is another one of my own devising). This scale is created by stacking a series of 540-cent intervals and wrapping to fit within an octave, and it produces some truly devilish chords. It is very improper, as you can see in the following table:

	1st	2nd	3rd	4th	5th	6th	7th	Oct.
Mode I	0	120	420	540	660	960	1080	1200
Mode II	0	300	420	540	840	960	1080	1200
Mode III	0	120	240	540	660	780	900	1200
Mode IV	0	120	420	540	660	780	1080	1200
Mode V	0	300	420	540	660	960	1080	1200
Mode VI	0	120	240	360	660	780	900	1200
Mode VII	0	120	240	540	660	780	1080	1200

Impropriety is just one strike this scale has against it. From a Just intonation perspective, this scale does a "decent" job of approximating the ratios 8/7, 6/5, 11/7, 5/3, 7/4, and 15/8, but the 4ths and 5ths are quite nasty, as are the 420-cent major 3rds. Root-third-fifth triads sound very out-of-tune, and the only remotely-Just triad approximated is 7:8:11—a rather tense triad in JI, which is not even very well approximated here. From the perspective of dyadic harmonic entropy, the majority of the intervals are near local maxima: 240 cents lies between 8/7 and 7/6, 300 cents between 7/6 and 6/5, 360 cents between 6/5 and 5/4, 420 cents between 5/4 and 9/7, 540 cents between 4/3 and 7/5, 660 cents between 10/7 and 3/2, and 840 cents between 8/5 and 5/3. Theoretically, this should add to the ambiguity caused by the scale's impropriety, making for a very "lumpy" and "ambiguous" scale.

Somehow, this scale fails to sound as bad as it theoretically should. Rather than a lumpy ambiguous mess, I hear it as "bluesy" and somewhat "floaty" or "ethereal". Perhaps it's the quasi-augmented triad on the II chord and the three approximate-7/4's, as well as the multiple clusters of consecutive semitones, but regardless I found this scale to be very evocative. It's not a "strong"-sounding scale at all, and the fact that the nearest thing to a 4/3 and the nearest thing to a 3/2 are only 120 cents apart makes a I-IV-V progression sound pretty strange, but I quite liked the mysterious dream-like feeling this scale conjured.

Track 12: "I Am Stretched On Your Grave": 11 Equal Divisions of the Octave:

0—109.09—218.18—327.27—436.36—545.45—654.55—763.64—872.73—981.82—1090.91—1200 cents

11-EDO was long thought to be an inescapably-dissonant and atonal tuning, but analyzing it according to JI, it doesn't actually look that bad. For starters, a chord of 0-218.18-436.36-545.45 cents is a pretty solid approximation of harmonics 7:8:9:11, done about as accurately as a 12-TET 8:10:12:15 major 7th chord. On top of that, 5/3 and 6/5 are both decently approximated. That said, 11-EDO solidly fails to approximate 3/2 and 5/4, so it is impossible to produce conventional-sounding harmony; and while 7:8:9:11 is technically a "Just" sonority, even in perfect tune it sounds very tense and unsettled. Dropping the 11th harmonic to make a simple 7:8:9 triad sounds more "restful", and that is indeed the most restful-sounding triad in 11-EDO. Because of this, using large vertical sonorities and a generally-homophonous approach (as is often done in 12-TET) in 11-EDO pretty much sucks, to put it bluntly. Such an approach limits the "mood" severely, and is probably one reason why 11-EDO gets such a bad rap among Western composers.

The key to unlocking decent-sounding music and versatile "moods" in 11-EDO is to make use of heterophony, having multiple voices spread out across different registers playing interweaving lines. This works because 11-EDO has very nice melodic properties, and because its harmonies (though tense) are not especially discordant—voices still blend pleasantly enough, and if the focus is drawn to the melodies rather than to the harmonies, 11 has the potential to sound only slightly more dissonant than 12-TET.

I really went to town with this approach here, and I'm quite pleased with how it turned out. The vocal track is the essential element, I think. I used an a cappella vocal version of the Irish traditional song "I Am Stretched On Your Grave", courtesy of Queenie, with a sort of jury-rigged auto-tuning in Logic. While Logic's soft-synths natively support microtuning, the auto-tuner does not, so I had to "cheat" a little and reduce the melody to three notes, which I then manually tweaked to be as close to 11-EDO as possible. It's a shame that I had to limit the melody so much, as the original melody was much more expressive (and translated wonderfully into 11-EDO); but as I'm not wealthy enough to afford Antares or Melodyne, it's the best I could do. I did attempt a vocoder version, which allowed the melody to be retained, but it destroyed the gorgeous tone of Queenie's voice and completely altered the emotional tone of the song*. Thus I decided to stick with the melodically-limited auto-tuned version, since it at least preserved the tone and emotion of the original a cappella track.

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* A16-EDO remix of this track using the vocoder approach was featured on the compilation Mad Love in Crazy Times, available through the lyfstyl music blog at: http://lyfstyl.bandcamp.com/album/mad-love-in-crazy-times

Closing Thoughts

Microtonal music theory has a long way to go before it can actually be called a *music* theory. The psychoacoustic understanding of consonance is, I believe, a red herring in the search for a tuning-nonspecific theory of music. That is not to say that psychoacoustic consonance is irrelevant to music, just that its true compositional importance is still poorly understood (as I hope the music on this album has demonstrated). I personally feel that I was deceived early on into believing consonance to be all-important; I came to microtonality seeking sounds that were radically different, and in ideally-consonant tunings of 31 and 22-EDO (the first two non-12-TET tunings I explored) I did not find what I sought. I caught glimpses, but only when I abandoned the familiar consonances. This led me to believe that, because 12-TET does a very respectable job of representing the first 5 (and maybe even the first 7) harmonics of the overtone series, anything else that does so will remind me of 12-TET. In other words, anything that is comparably consonant with 12-TET, sounds like 12-TET. So it was not simply 12-TET I needed to escape, but the overall ubiquity of consonance as well.

This is not to say that I am against consonance, but it is laid on so thickly in 12-TET (and more thickly still in the meantone temperaments that preceded it) that I believe it needs to be balanced by a hearty serving of some dissonant flavors. More importantly, reductionistic quantifications obscure the fact that the *varieties* of consonances and dissonances are of vital importance, as are their relationships to each other. A tuning's overall or average consonance tells you nothing about how to use it or what it's good for, you have to open it up and look at *which* intervals are consonant and dissonant, and how they work together to create the overall mood of the tuning. This is where the music happens, in the interaction of consonance and dissonance, and to date there is no psycoacoustic theory that can usefully address this.

In the meantime, we still have our ears, and we can still make microtonal music by taking shots in the dark and seeing what targets we end up hitting. Frankly I suspect this is the *only* approach that will ever lead to the development of a true microtonal *music* theory, because without music to analyze, we can only speculate about the effects of microtuning on music. In other words, I believe the only way to advance the theory is by advancing the practice. So I heartily encourage my readers and listeners to make microtonal music of their own, not necessarily in disregard to psychoacoustic principles, but with an attitude of skepticism toward their relevance to music-making.

-Igliashon Jones, June 25, 2011