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ADVANCED
ARITHMETIC.



COMPILED UNDER THE DIRECTION
OF THE
STATE BOARD OF EDUCATION.

SACRAMENTO, CALIFORNIA.
PRINTED AT THE STATE PRINTING OFFICE.

Entered according to Act of Congress, in the year 1887, by the
STATE OF CALIFORNIA,
In the Office of the Librarian of Congress, at Washington.

36291

PREFACE.

The State Board of Education expect to make no revolution in teaching the old subject of Arithmetic, by the issuance of a new book. They feel, however, that arithmetics have been too much given to talking and not enough to doing—that a student seldom or never masters the thought in a long and minute explanation. He cannot understand it before working the examples, and does not need it afterward. Hence, the explanations in the present volume have been made brief, and may be enlarged by the teacher as the occasion demands.

Let no one despise the book on account of its small size, but work a class carefully through it, making it familiar by frequent reviews, and observe the effect. We respectfully invite the candid criticism of those who have done this, that the defects of the present volume may be remedied in the near future.

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CALIFORNIA SERIES.

ADVANCED

ARITHMETIC.

NOTATION AND NUMERATION.

A single, whole thing is called a **unit**; as, *one, one apple, one pencil.*

Several things taken together as a whole may be a unit; as, *one dozen pencils, one pile of books, one class of boys.*

A **number** consists of one or more units; as, *one, one cent, seven, seven books, ten pens.*

Writing numbers is called **NOTATION**.

The notation in common use is the **decimal notation**, which employs ten different characters, or *figures*, to form all numbers.

All numbers are properly followed by a point ($.$), called the **decimal point**; as in the table below. In writing numbers in a series or in a sentence, the decimal point is omitted to avoid confusion with the *period*.

The following table gives the ten characters of the decimal notation in the upper horizontal row and their names beneath. Then follow their combinations, forming numbers of two figures. Put this diagram on the slate and fill out completely, writing (1) the figure (2) the combination (3) the name.

Figure Name	0 Zero	1. One	2. Two	3. Three
Figure Combination Name	10. 1 ten Ten	11. { 1 ten } { 1 unit } Eleven	12. { 1 ten } { 2 units } Twelve	13. { 1 ten } { 3 units } Thirteen
Figure Combination Name	20. 2 tens Twenty	21. { 2 tens } { 1 unit } Twenty-one	22. { 2 tens } { 2 units } Twenty-two	
Figure Combination Name	30. 3 tens Thirty			
Figure Combination Name	40. 4 tens Forty			
Figure Combination Name	50. 5 tens Fifty			
Figure Combination Name	60. 6 tens Sixty			
Figure Combination Name	70. 7 tens Seventy			
Figure Combination Name	80. 8 tens Eighty			
Figure Combination Name	90. 9 tens Ninety			

Observe { *Units form the first figure at the left of the decimal*
In column 1, the absence of units is marked by 0,
any decimal place is always marked by 0.
How many units make 1 ten?

EXERCISE 1.

Draw a diagram like the preceding and write the names of the following numbers and the combinations which make them:

94.	18.	82.	40.	90.	95.	98.
72.	27.	97.	77.	49.	59.	36.
13.	55.	47.	30.	37.	46.	41.
29.	14.	79.	62.	20.	67.	75.
85.	32.	28.	71.	88.	92.	58.
44.	66.	38.	54.	50.	12.	39.
51.	74.	70.	87.	19.	21.	96.

EXERCISE 2.

Write the following combinations in figures and their names in words:

- 2 tens 4 units, 3 tens 1 unit, 6 tens, 5 tens 3 units.
- 8 tens 4 units, 4 tens 2 units, 8 tens, 3 tens 5 units, 6 tens 8 units, 5 tens 6 units, 6 tens 3 units.
- 8 tens 6 units, 5 tens 7 units, 7 tens, 4 tens 5 units, 1 ten 6 units, 7 tens 3 units, 6 tens 1 unit.
- 1 ten 7 units, 9 tens, 6 tens 9 units, 4 tens, 1 ten 8 units, 7 tens 8 units, 8 tens 1 unit.
- 3 tens, 2 tens 6 units, 4 tens 8 units, 9 tens 9 units, 6 tens 4 units, 2 tens 5 units, 5 tens 8 units.
- 4 tens 3 units, 9 tens 1 unit, 8 tens 9 units, 6 tens 5 units, 2 tens 3 units, 3 tens 4 units, 5 tens 2 units.
- 4 tens 1 unit, 7 tens 5 units, 1 ten 7 units, 9 tens 6 units, 3 tens 3 units, 5 tens 8 units, 1 ten 5 units.

The third figure at the left of the decimal point is called **hundreds**; thus, *236 is 2 hundreds 3 tens 6 units, or two hundred thirty-six.*

* How many tens make 1 hundred?

These three places of figures—units, tens, and hundreds—form the first group of numbers, called **units**.

The fourth, fifth, and sixth places at the left of the decimal point form the second group, called **thousands**; units, tens, and hundreds of thousands, respectively.

The seventh, eighth, and ninth places at the left of the decimal point, form the third group, called **millions**; units, tens, and hundreds of millions, respectively.

The following table shows the scheme for *reading numbers*, or NUMERATION. In *reading*, begin at the left, read each group, and add the group name; thus, *one hundred twenty-four sextillion, seven hundred thirty quintillion, etc.*, omitting the name of the unit group:

TABLE.

of sextillions.			of quintillions.			of quadrillions.			of trillions.			of billions.			of millions.			of thousands.			of units.		
hundreds	tens	units	hundreds	tens	units	hundreds	tens	units	hundreds	tens	units	hundreds	tens	units	hundreds	tens	units	hundreds	tens	units	hundreds	tens	units
1	2	4	7	3	0	2	1	8	6	9	3	0	1	3	9	7	8	2	1	0	4	5	3

NOTE.—The omission of “and” between hundreds and tens is the better usage, although many writers and speakers still use it.

SUGGESTION.—Require oral exercise by the class upon the preceding table until it is familiar to all.

EXERCISE 3.

Read, or write on slates or blackboard, in words:

- | | | | |
|----------|----------|----------|------------|
| 1. 208. | 4. 727. | 7. 7051. | 10. 3108. |
| 2. 523. | 5. 4009. | 8. 555. | 11. 4018. |
| 3. 1001. | 6. 300. | 9. 476. | 12. 23760. |

13.	1414.	22.	1211.	31.	525.	40.	5729.
14.	2007.	23.	41407.	32.	800.	41.	100010.
15.	105.	24.	270.	33.	805.	42.	74179.
16.	8248.	25.	643077.	34.	3104.	43.	85128.
17.	5678.	26.	21190.	35.	7228.	44.	7300.
18.	179.	27.	758.	36.	720.	45.	211.
19.	24198.	28.	7112.	37.	5000.	46.	2419.
20.	179226.	29.	987.	38.	2726.	47.	43200.
21.	473.	30.	3721.	39.	54100.	48.	7290.

EXERCISE 4.

Write the following in figures, to be read in the class:

1. Five hundred seventy-two, one thousand seventeen, five thousand ninety, four hundred sixty-four, twenty-four thousand eight, three hundred forty-six, nine thousand ninety-nine.

How many *groups* are employed in writing the first number? How many in writing the second? The third? What places are vacant in each group? They should be occupied by zero.

2. Eleven thousand seven hundred eighty-five, seventeen thousand twenty-nine, eight hundred eight, three thousand fifteen, eighteen thousand thirty, twenty-five thousand four hundred, seven hundred six.

3. Forty thousand nine hundred three, sixty-one thousand three hundred thirty-three, one hundred four thousand twenty, seven thousand forty-six, eight hundred eighty-eight thousand eight, nine hundred sixty-nine, two thousand four hundred thirteen.

4. Fourteen thousand seven hundred forty-five, two hundred fifty-one thousand one hundred sixteen, thirty-four thousand one hundred eleven, five thousand sixty-six, thirty-one thousand nine hundred fifty-two, eighty-two thousand three hundred twelve.

5. Nineteen thousand five hundred, seven thousand four hundred twenty-three, six hundred nine, six hundred nine

thousand, six thousand nine, fifty-nine thousand five, five thousand nine hundred five.

6. Three thousand thirteen, three hundred thirteen, three hundred thousand thirteen, thirty thousand thirteen, eight hundred eighty-one, eight thousand eighty-one, eighty thousand eighty-one.

EXERCISE 5.

Write, on your slates, through the group of millions, a table like that on page 9, and place under it in vertical column 20 numbers of your own selection, containing from 3 to 9 places each, for reading and dictation in the class.

EXERCISE 6.

Read the numbers under Exercises 22 and 24.

EXERCISE 7.

Write the following in figures:

1. 5 thousand 2 hundred 10, 24 thousand 6 hundred 3, 11 thousand 29, 7 hundred 63, 16 thousand 8 hundred, 4 hundred 44.

2. 123 thousand 123, 14 hundred 14, 73 thousand 5 hundred 8, 17 hundred, 141 thousand, 3 million 3 thousand 3 hundred 3.

3. 7 thousand 7, 7 hundred 7, 7 million 7 thousand 7, 7 million 7, 13 hundred 30, 13 thousand 30.

4. 115 thousand 7 hundred 74, 10 hundred 10, 10 thousand 10, 1 thousand 10, 5 hundred 91, 8 thousand 4 hundred 20.

5. 404 thousand 44, 23 thousand 213, 180 thousand 180, 47 thousand 474, 3 thousand 206, eighty-one.

6. 826 thousand 013, 15 thousand 411, 111 thousand 111, 400 thousand 400, 328 thousand 910, 50 thousand 50.

7. 501 thousand 107, 55 thousand 76, 28 thousand 1.

8. 101 thousand 10, 101 million 1 thousand 6.

9. 110 thousand 11, 20 million 11 thousand 11.

EXERCISE 8.

Dictation exercise by the class, each giving his own numbers without reference to book, slate, or paper. Repeat this exercise until the class dictate and write rapidly.

EXERCISE 9.

Place the following in tabular form, as in Exercise 5, for reading in the class:

224368192, 1724261, 2004101, 7264180, 2010194, 3762108, 23101, 47266, 4004, 20801, 76001, 2108, 17007, 100100.

Another notation, called the **Roman notation**, is sometimes used for writing dates, headings of chapters, and the like; but it is too cumbrous for ordinary computations. The Roman notation employs *seven capital letters*, with their combinations, to represent numbers, viz.:

I	V	X	L	C	D	M
One,	five,	ten,	fifty,	one hundred,	five hundred,	one thousand.
1	5	10	50	100	500	1000

The following table shows the method of combining:

I . . . one.	VII . . . seven.	LX . . . sixty.
II . . . two.	VIII . . . eight.	XC . . . ninety.
III . . . three.	IX . . . nine.	XL . . . forty.
IV . . . four.	X . . . ten.	L . . . fifty.
V . . . five.	XI . . . eleven.	C . one hundred.
VI . . . six.	XX . . . twenty.	D . five hundred.
M one thousand.	\bar{M} one million.	

Observe { *Repeating a letter repeats its value.*
If a letter of smaller value precedes one of larger, the difference of their values is indicated; if the reverse, the sum.
A dash (—) above a letter indicates so many thousand; thus, \bar{L} = fifty thousand.

EXERCISE 10.

Write in Roman notation:

8, 14, 27, 144, 1875, 599, 1620, 35, 178, 83, 124000, 753,
16, 222, 1888, 7, 12, 79.

EXERCISE 11.

Read the following numbers:

XIX, XXIX, XXXVI, CCCI, CX, DCLII, CDXIV,
MDLXXXIV, MDCCCLXXXVI, CXLVII, $\overline{\text{MC}}$, XCIX,
CCCXXV, LXXII, DIV, MCCXVIII, CXI, DCCXLVII,
MDCCLXXXIX, MCDXCII, CL, CCXV.

EXERCISE 12.

Prepare 3 columns on your slate as follows:

First column, 10 numbers written in words;

Second column, the same numbers in decimal notation;

Third column, the same in Roman notation.

Model :

No.	Words.	Decimal Notation.	Roman Notation.
1.	Twenty-five.	25.	XXV.
2.			
3.			
4.			
5.			
6.			
7.			
8.			
9.			
10.			

ADDITION.

If you have 8 apples and a schoolmate gives you 5 more, how many will you have?

The process of putting together two or more numbers of the same kind into one is called **ADDITION**.

The result obtained is called the **sum** or **amount**.

The sign (+), called **plus** or **and**, is used to indicate addition. Thus,

$8+5=13$ is read *8 plus 5 equals 13*, or *8 and 5 are 13*.

SUGGESTION.—With beginners “and” is preferable to plus.

EXERCISE 13. (ORAL.)

TO THE TEACHER.—Give pupils pebbles, beans, peas; or, better, pasteboard cut into strips $\frac{3}{8}$ in. wide and 3 in. long, to find out the results by going through with the combinations. Drill on the following until the pupil recognizes at sight the sum of each pair:

1	2	3	1	3	2	2	1	4	3	5	3	5	2	6	4	2	8
2	1	1	3	2	3	2	4	1	3	2	6	5	5	3	2	6	3
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6	5	7	8	6	9	4	5	3	1	7	9	8	5	2	6	6	4
5	9	6	8	4	3	7	4	5	6	2	9	7	3	9	8	6	9
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
2	4	7	7	2	7	3	6	5	2	9	8	1	3				
8	4	9	7	4	5	4	2	1	7	6	5	9	7				
—	—	—	—	—	—	—	—	—	—	—	—	—	—				

EXERCISE 14. (WRITTEN.)

Write each of the pairs and their sum, in Exercise 13, horizontally, using the signs (+) and (=); thus, $1+2=3$.

Bring to the class to read.

EXERCISE 15. (ORAL.)

Add these columns, taking the figures in pairs, and call-

ing only the sums of the pairs; thus, in the first, *10, 15*; also add across the page from left to right as indicated by the sign (+):

$$1+3+4+3+8+7+8+7+5+8=?$$

$$4+2+3+9+4+6+5+8+5+1=?$$

$$7+5+7+4+3+9+8+5+6+4=?$$

$$\underline{3+6+9+6+1+5+9+4+3+9=?}$$

$$7+9+7+8+2+2+9+6+2+2=?$$

$$1+1+7+8+8+9+9+9+1+6=?$$

$$9+6+3+5+4+2+2+3+8+4=?$$

$$\underline{3+6+8+7+4+7+5+3+6+7=?}$$

EXERCISE 16. (ORAL.)

Extend Exercise 15 by beginning with the second figure on the left in each horizontal row and adding through the row. Then begin with the third, and so on.

EXERCISE 17. (WRITTEN.)

Copy the following, fill out as indicated, and write results:

1.	2.	3.	5.	8.
$2+5=$	$3+7=$	$7+8=$	$3+9=$	$9+9=$
$12+5=$	$13+7=$	$17+8=$	$13+9=$	$19+9=$
$22+5=$	$23+7=$	$27+8=$	and so on.	and so on.
$32+5=$	$33+7=$	$37+8=$	6.	9.
$42+5=$	$43+7=$	and so on.	$6+6=$	$7+7=$
$52+5=$	$53+7=$	4.	$16+6=$	$17+7=$
$62+5=$	$63+7=$	$7+6=$	and so on.	and so on.
$72+5=$	$73+7=$	$17+6=$	7.	10.
$82+5=$	$83+7=$	$27+6=$	$4+7=$	$1+9=$
$92+5=$	$93+7=$	and so on.	and so on.	and so on.

EXERCISE 18. (ORAL.)

1. Begin with 0 and add by 2's to 50; thus, *0, 2, 4, 6*, etc. Do the same, beginning with 1, to 51; thus, *1, 3, 5*, etc.

2. Add by 3's from 0 to 51; from 50 to 98; from 2 to 50; from 51 to 99.

3. Add by 4's from 0 to 52; from 51 to 99; from 2 to 50; from 52 to 96.

4. Add by 5's from 0 to 100; from 1 to 101; from 2 to 102; from 3 to 103; from 4 to 104.

EXERCISE 19. (WRITTEN.)

Write as in Exercise 17:

1.	2.	3.	4.	5.
$4+9=$	$3+8=$	$9+7=$	$8+8=$	$8+5=$
$14+9=$	$13+8=$	$19+7=$	$18+8=$	$18+5=$
$24+9=$	$23+8=$	$29+7=$	$28+8=$	$28+5=$
and so on.	and so on.	and so on.	and so on.	and so on.

6.	7.	8.	9.	10.
$5+8=$	$9+2=$	$2+9=$	$6+5=$	$5+6=$
$15+8=$	$19+2=$	$12+9=$	$16+5=$	$15+6=$
$25+8=$	$29+2=$	$22+9=$	$26+5=$	$25+6=$
and so on.	and so on.	and so on.	and so on.	and so on.

SUGGESTION.—Let the teacher give further oral work of the same kind.

EXERCISE 20. (ORAL.)

1. Add by 6's from 0 to 60; from 52 to 100; from 1 to 61; from 43 to 103; from 5 to 65.

2. Add by 7's from 0 to 70; from 3 to 73; from 32 to 102; from 16 to 86; from 31 to 101.

3. Add by 8's from 0 to 80; from 21 to 101; from 5 to 85; from 17 to 97; from 6 to 86; from 43 to 123.

4. Add by 9's from 0 to 90; from 2 to 101; from 5 to 104; from 13 to 103; from 8 to 98; from 7 to 106.

EXERCISE 21. (ORAL.)

(1) Begin with the bottom of each column and add upward in pairs (2) begin with the top and add downward in pairs (3) add across from left to right:

$$\begin{array}{r}
7+5+3+6+7+2+9+3+8+4=? \\
2+2+3+8+4+9+2+9+1+6=? \\
9+9+9+1+6+2+9+6+2+2=? \\
2+9+6+2+2+4+3+9+3+6=? \\
4+3+9+3+6+5+6+4+9+6=? \\
5+6+4+9+6+8+5+1+1+1=? \\
8+5+1+1+1+7+5+8+7+9=? \\
\underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad}
\end{array}$$

$$\begin{array}{r}
5+3+9+3+6+8+7+4+7+5=? \\
8+6+4+9+6+3+5+4+2+2=? \\
7+5+1+1+1+7+8+8+9+9=? \\
3+5+8+7+9+7+8+2+2+9=? \\
7+6+9+6+1+5+9+5+3+6=? \\
4+5+7+4+3+9+8+8+7+5=? \\
1+2+3+9+4+6+7+1+4+2=? \\
4+3+4+3+8+7+8+1+1+3=? \\
\underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad}
\end{array}$$

$$\begin{array}{r}
1+4+7+3+3+2+5+6+5+5=? \\
3+2+5+6+4+3+7+9+8+1=? \\
4+3+7+9+3+9+4+6+7+1=? \\
3+9+4+6+8+4+3+1+9+1=? \\
8+4+3+1+7+6+9+5+7+7=? \\
7+6+9+5+8+7+8+9+8+8=? \\
8+7+8+9+1+1+8+5+2+8=? \\
1+1+8+5+7+8+5+4+2+9=? \\
7+8+5+4+5+5+6+3+9+9=? \\
\underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad}
\end{array}$$

This exercise may be extended by beginning at any intermediate point and adding onward.

To write and add numbers of two or more figures.

SUGGESTION.—Before allowing the pupils to study this or any similar *explanation*, the teacher should take the work orally with them and let them make as many of the suggestions as they can.

Find the sum of 327, 48, and 452.

FULL WORK.	EXPLANATION.—We cannot add 7 pencils and 6 pens,
327	because they are unlike. Likewise, we cannot add
48	tens to units. Therefore, write units under units, tens
452	under tens, etc. The sum of the units is 17; of the
<hr/>	tens, 11; of the hundreds, 7. Adding these results
17	gives 827.
110	Why is <i>zero</i> (0) placed after 11?
700	Why are two <i>zeros</i> (00) placed after 7?
<hr/>	
827	

But 17 units are 1 ten 7 units. As the 1 ten belongs
 CONTRACTED. in the tens' column, we may write only the 7 units,
 327 as in the *contracted* operation, and add the 1 ten to the
 48 tens' column; thus, 1, 6, 10, 12. Again, 12 tens are 1
 452 hundred, 2 tens. As before, write, in the result, only
 the 2 tens and add the 1 hundred to the column of
 827 hundreds.

Test, or prove, the correctness of the work by adding downward.

EXERCISE 22. (WRITTEN.)

Write in columns properly, add, and test the work:

- | | |
|------------------------------------|-----------------------------|
| 1. 424, 236, 38, 120. | 11. 1234, 4321, 1324, 4231. |
| 2. 34, 108, 246, 5. | 12. 3579, 9753, 3795, 9573. |
| 3. 402, 1728, 526, 100. | 13. 908, 7098, 9708, 987. |
| 4. 3756, 11, 153, 4005. | 14. 7890, 798, 8790, 809. |
| 5. 271, 109, 9019, 49. | 15. 4796, 7694, 976, 479. |
| 6. 7310, 101, 476, 1203, 45. | 16. 3251, 1523, 5237, 8. |
| 7. 423, 13, 9, 237, 2314, 103. | 17. 487, 9217, 1499, 7. |
| 8. 19, 500, 275, 2406, 2728, 2010. | 18. 534, 434, 898, 10. |
| 9. 9019, 428, 1300, 23, 99, 3003. | 19. 921, 651, 1397, 14. |
| 10. 1314, 810, 278, 4130, 44, 176. | 20. 1455, 1085, 95, 117. |

EXERCISE 23.

Add the columns of figures in Exercise 1, and test. Add the same across the page.

Write each example of Exercise 2 in column, add, and test.

EXERCISE 24.

Add and prove, in columns and in rows:

- | | 1. | 2. | 3. | 4. | 5. |
|-----|--------------|-----------------|-----------------|-----------------|-------------------|
| 15. | 4298 | + 1029 | + 428 | + 7296 | + 49 = ? |
| 16. | 376 | + 76 | + 5001 | + 98 | + 1311 = ? |
| 17. | 107 | + 237 | + 19 | + 402 | + 205 = ? |
| 18. | 25 | + 4196 | + 1279 | + 13 | + 15 = ? |
| 19. | <u>3178</u> | + <u>703</u> | + <u>499</u> | + <u>720</u> | + <u>3146</u> = ? |
| | | | | | |
| | 6. | 7. | 8. | 9. | 10. |
| 20. | 9 | + 207 | + 575 | + 3209 | + 1712 = ? |
| 21. | 79 | + 3426 | + 82 | + 729 | + 5726 = ? |
| 22. | 4327 | + 127 | + 426 | + 48 | + 209 = ? |
| 23. | 214 | + 5728 | + 1350 | + 7216 | + 8702 = ? |
| 24. | <u>903</u> | + <u>4019</u> | + <u>407</u> | + <u>590</u> | + <u>435</u> = ? |
| | | | | | |
| | 11. | 12. | 13. | 14. | |
| 25. | 375409 | + 72496 | + 718409 | + 419009 | = ? |
| 26. | 23216 | + 570203 | + 20171 | + 21060 | = ? |
| 27. | 25100 | + 30206 | + 376219 | + 1199 | = ? |
| 28. | 5196 | + 175410 | + 4211010 | + 519257 | = ? |
| 29. | 576206 | + 76228 | + 5176159 | + 4219219 | = ? |
| 30. | <u>61070</u> | + <u>481112</u> | + <u>172105</u> | + <u>728400</u> | = ? |

EXERCISE 25.

Add the columns of figures in Exercise 3, and test. Add the same across the page. Write each example, Exercises 4 and 7, in columns, add, and test.

EXERCISE 26.

Write 10 examples of your own, of 10 numbers each, perform, and prove, and bring into the class to dictate to the others for board-work.

EXERCISE 27.

Dictate numbers of your own, without reference to book,

slate, or paper, for the other members of the class to perform. Repeat the exercise until each dictates rapidly.

Accountants and business men, by constant practice, add two or even three columns at once with great rapidity.

To add two columns, it is customary to add the tens first and then the units in each successive number.

24	Thus, 3 tens 4 units + 4 tens (= 7 tens) 6 units = 8 tens 0 units, + 5 tens (= 13 tens) 7 units = 13 tens 7 units, + 2 tens (= 15 tens) 4 units = 16 tens 1 unit. In reading omit parts in parenthesis and the words, tens and units; thus, 3, 4;
57	
46	
34	
161	
	8, 0; 13, 7; 16, 1.

24	83	19	5721	3333
17	40	12	4804	3214
50	26	4141	6789	1818
75	23	2414	7117	7642
29	81	7236	2104	4261
31	44	4004	5016	1781
<hr style="width: 100%;"/>	<hr style="width: 100%;"/>	<hr style="width: 100%;"/>	<hr style="width: 100%;"/>	<hr style="width: 100%;"/>

(WRITTEN OR ORAL.)

Copy and fill out, or read, as in the first example:

$30 + 20 = 50 = 5 \text{ tens.}$

$40 + 30 =$	$70 + 50 =$	$100 + 20 =$	$120 + 60 =$
$40 + 20 =$	$90 + 40 =$	$100 + 30 =$	$140 + 70 =$
$70 + 20 =$	$80 + 20 =$	$110 + 40 =$	$130 + 80 =$
$80 + 10 =$	$50 + 40 =$	$110 + 70 =$	$110 + 60 =$
$50 + 30 =$	$30 + 20 =$	$130 + 40 =$	$150 + 90 =$
$60 + 40 =$	$60 + 30 =$	$150 + 70 =$	$100 + 40 =$
$90 + 70 =$	$80 + 40 =$	$110 + 50 =$	$140 + 50 =$
$70 + 40 =$	$40 + 10 =$	$120 + 70 =$	$160 + 80 =$
$80 + 50 =$	$90 + 60 =$	$100 + 50 =$	$170 + 90 =$
$30 + 10 =$	$60 + 20 =$	$140 + 60 =$	$150 + 80 =$

Observe { *Adding 10's to 10's gives 10's, as adding units to units gives units.*

SUBTRACTION.

Copy the following on your slates, and put in place of each blank the number that you must add to the one above the line to make the one below:

2 pencils	4 pens	5 apples	9 books	8 marbles	3 pins
— pencils	— pens	— apples	— books	— marbles	— pins
7 pencils	11 pens	9 apples	17 books	16 marbles	9 pins

Copy the following, also. Place below each line the number that will be left, if you take away the lower from the upper number:

7 pencils	11 pens	9 apples	17 books	16 marbles	9 pins
5 pencils	7 pens	4 apples	8 books	8 marbles	6 pins
— pencils	— pens	— apples	— books	— marbles	— pins

Compare these two exercises. What did you do in the first? In the second? Since 5 put with 2 makes 7, 5 taken from 7 will leave 2.

The process of taking one number from another of the same kind is called **SUBTRACTION**.

The number from which we take is called the **minuend**.

The number taken away is called the **subtrahend**.

The number left is called the **difference** or **remainder**.

Pick out each in the second part you copied above.

The sign ($-$), called **minus** or **less**, is used to indicate subtraction; thus,

$7-5=2$ is read *7 minus 5 equals 2*, or *7 less 5 are 2*.

SUGGESTION.—With beginners, *less* is to be preferred to *minus*.

EXERCISE 28. (WRITTEN.)

Place in each blank the number that must be added to the number above the line to make the one below:

4	7	2	1	5	6	7	5	8	4	8	6	2	7
$\overline{12}$	$\overline{16}$	$\overline{17}$	$\overline{13}$	$\overline{11}$	$\overline{18}$	$\overline{14}$	$\overline{13}$	$\overline{17}$	$\overline{15}$	$\overline{18}$	$\overline{14}$	$\overline{12}$	$\overline{19}$
5	9	1	2	7	3	10	6	13	9	2	5	8	
$\overline{18}$	$\overline{13}$	$\overline{16}$	$\overline{14}$	$\overline{15}$	$\overline{17}$	$\overline{15}$	$\overline{12}$	$\overline{17}$	$\overline{18}$	$\overline{15}$	$\overline{12}$	$\overline{11}$	
3	13	5	12	14	5	3	9	6	7	10	4	12	
$\overline{18}$	$\overline{16}$	$\overline{14}$	$\overline{17}$	$\overline{18}$	$\overline{16}$	$\overline{14}$	$\overline{11}$	$\overline{13}$	$\overline{18}$	$\overline{19}$	$\overline{14}$	$\overline{16}$	

EXERCISE 29. (ORAL.)

Perform Exercise 18 backward. That is, in (1) begin with 50 and take away 2 each time; thus, 50, 48, 46, etc. Then begin with 51; 51, 49, 47, etc.

Then in (2) begin with 51 and subtract by 3's; and so on. Repeat until all subtract readily.

EXERCISE 30. (WRITTEN.)

Copy the following, fill out as indicated, and write the remainders:

1.	2.	3.	4.	5.
7—5=	10—7=	15—8=	13—6=	12—9=
17—5=	20—7=	25—8=	23—6=	22—9=
27—5=	30—7=	35—8=	33—6=	32—9=
37—5=	40—7=	45—8=	43—6=	42—9=
and so on	and so on	and so on	and so on	and so on
to 97—5.	to 100—7.	to 95—8.	to 93—6.	to 92—9.
6.	7.	8.	9.	10.
12—6=	11—7=	18—9=	14—7=	10—9=
22—6=	21—7=	28—9=	24—7=	20—9=
32—6=	31—7=	38—9=	34—7=	30—9=
42—6=	41—7=	48—9=	44—7=	40—9=
and so on.	and so on.	and so on.	and so on.	and so on.

Compare this work with that of Exercise 17.

EXERCISE 31. (ORAL.)

Subtract the lower number from the upper:

20	32	31	25	21	23	33	22	16	43
<u>9</u>	<u>5</u>	<u>3</u>	<u>5</u>	<u>11</u>	<u>8</u>	<u>9</u>	<u>8</u>	<u>9</u>	<u>3</u>
39	18	26	59	47	64	71	74	41	55
<u>4</u>	<u>1</u>	<u>0</u>	<u>8</u>	<u>7</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>5</u>	<u>6</u>
72	85	91	67	54	35	46	69	56	44
<u>8</u>	<u>5</u>	<u>10</u>	<u>8</u>	<u>7</u>	<u>8</u>	<u>5</u>	<u>7</u>	<u>3</u>	<u>8</u>
77	86	98	83	90	49	57	43	38	25
<u>9</u>	<u>4</u>	<u>7</u>	<u>3</u>	<u>1</u>	<u>8</u>	<u>5</u>	<u>9</u>	<u>9</u>	<u>9</u>
40	52	71	50	41	53	61	84	91	50
<u>2</u>	<u>3</u>	<u>5</u>	<u>5</u>	<u>2</u>	<u>4</u>	<u>7</u>	<u>5</u>	<u>3</u>	<u>4</u>
70	23	87	85	91	41	53	28	93	96
<u>6</u>	<u>4</u>	<u>5</u>	<u>7</u>	<u>8</u>	<u>6</u>	<u>4</u>	<u>7</u>	<u>5</u>	<u>9</u>

EXERCISE 32. (WRITTEN.)

Fill out as directed in Exercise 30:

1.	2.	3.	4.	5.
13-4=	11-3=	11-9=	16-9=	16-8=
23-4=	21-3=	21-9=	26-9=	26-8=
and so on	and so on	and so on	and so on	and so on
to 93-4.	to 91-3.	to 91-9.	to 96-9.	to 96-8.
6.	7.	8.	9.	10.
13-8=	14-8=	10-6=	11-6=	17-8=
23-8=	24-8=	20-6=	21-6=	27-8=
and so on	and so on	and so on	and so on	and so on
to 93-8.	to 94-8.	to 90-6.	to 91-6.	to 97-8.

EXERCISE 33. (ORAL.)

Perform the work of Exercise 20 backwards, as you were directed in Exercise 29.

EXERCISE 34. (WRITTEN OR ORAL.)

Copy and fill out, or read, as in the first example:

$30-20=10=1$ ten.

$40-30=$	$70-50=$	$100-20=$	$120-60=$
$40-20=$	$90-40=$	$100-30=$	$140-70=$
$70-20=$	$80-20=$	$110-40=$	$130-80=$
$80-10=$	$50-40=$	$110-70=$	$110-60=$
$50-30=$	$30-20=$	$130-40=$	$150-90=$
$60-40=$	$60-30=$	$150-70=$	$100-40=$
$90-70=$	$80-40=$	$110-50=$	$140-50=$
$70-40=$	$40-10=$	$120-70=$	$160-80=$
$80-50=$	$90-60=$	$100-50=$	$170-90=$
$30-10=$	$60-50=$	$140-60=$	$150-80=$

Observe { *Subtracting 10's from 10's leaves 10's, as subtracting units from units leaves units.*

EXERCISE 35. (ORAL.)

- $15-7+4-10+9+6-10+1-4+9+3+10-5=?$
- $5+9-1+7-3-5-2+11+4-9+3-10+1=?$
- $1+9-10+5+3-7+10-11+3-2+8+9-8=?$
- $43-5+2-10-9+4-10+3-7+3-9+5-10=?$
- $17+5-11-1+8-16+4-6+11+9-3-2+5=?$
- $44+6-10+1-9+8-10+3-8+2-9+2-11=?$
- $90-20+5-10-5+1-10+2-9+3-8-9-10=?$
- $1+17+2-9-10+7+8-11+5-10+0+1+4=?$
- $3+9-11+10+4-3+7+1-7+2-8+2+1=?$
- $2+6-7+10+4+3-9-1+3+2-6-7=?$

To subtract numbers of two or more figures.

Take 416 from 829.

OPERATION.

$$\begin{array}{r} 829 \\ 416 \\ \hline 413 \end{array}$$

EXPLANATION.—Write the subtrahend under the minuend, units under units, etc., as in Addition, and for the same reason. (What reason?) Begin with units.

EXERCISE 36. (WRITTEN.)

- | | | |
|-------------|--------------|----------------|
| 1. 178—134= | 6. 447—336= | 11. 4391—1290= |
| 2. 495—274= | 7. 678—567= | 12. 7448—5346= |
| 3. 982—471= | 8. 595—494= | 13. 8254—3223= |
| 4. 778—545= | 9. 309—207= | 14. 9725—2501= |
| 5. 904—503= | 10. 828—721= | 15. 3486—1376= |

Take 479 from 627.

FULL OPERATION.

$$\begin{array}{r} 500 + 110 + 17 = 627 \\ 400 + 70 + 9 = 479 \\ \hline 100 + 40 + 8 = 148 \end{array}$$

units, which we write below in the units column. We can not take 7 tens from 1 ten (left in the minuend); hence take 1 hundred from the 6 hundred in the minuend and put it with the 1 ten, making 11 tens. 7 tens from 11 tens leave 4 tens. 4 hundreds from 5 hundreds leave 1 hundred.

EXPLANATION.—We can not take 9 units from 7 units. Take away 1 ten from the 2 tens of the minuend and put it with the 7 units, making 17 units. 9 units from 17 units leave 8

Test by adding the remainder and subtrahend; the result should be the minuend.

EXERCISE 37. (WRITTEN.)

Write properly, find the differences, and prove :

- | | | |
|-------------------|-----------------|-------------------|
| 1. 738 and 542. | 4. 500 and 430. | 7. 1247 and 8146. |
| 2. 239 and 410. | 5. 378 and 909. | 8. 598 and 399. |
| 3. 5786 and 4310. | 6. 246 and 725. | 9. 979 and 451. |

Sometimes, when our minuend figure is too small, it happens that the next minuend figure is 0, or nothing to take from. In such a case go to the first minuend figure, not 0, to the left, and reduce down. Thus,

Subtract 2378 from 5005.

OPERATION.

$$\begin{array}{r} 49915 \\ 5005 \\ 2378 \\ \hline 2627 \end{array}$$

EXPLANATION.—We can not take 8 from 5, and the next two minuend figures are 0's. We, therefore, take 1 thousand from the 5 thousands, leaving 4 thousands, as shown by the small figure above. 1 thousand is 10 hundreds. Again, take 1 hundred from the 10 hundreds, leaving 9 hundreds, as shown above. 1 hun-

dred is 10 tens. Take 1 ten from 10 tens, leaving 9 tens. 1 ten 5 units are 15 units. Now subtract the subtrahend figures from the small figures above the minuend.

EXERCISE 38. (ORAL AND WRITTEN.)

Take the lower from the upper numbers; also subtract as indicated by the sign ($-$); prove your work.

- | | | | | | |
|---|----|---|-----|--|-----|
| 1. | 2. | 3. | 4. | 5. | 6. |
| 13. $800 - 143 = ?$ | | 15. $1467 - 300 = ?$ | | 17. $671 - 420 = ?$ | |
| 14. $\begin{array}{r} 75 \\ \underline{\quad} \end{array} - \begin{array}{r} 29 \\ \underline{\quad} \end{array} = ?$ | | 16. $\begin{array}{r} 229 \\ \underline{\quad} \end{array} - \begin{array}{r} 85 \\ \underline{\quad} \end{array} = ?$ | | 18. $\begin{array}{r} 176 \\ \underline{\quad} \end{array} - \begin{array}{r} 89 \\ \underline{\quad} \end{array} = ?$ | |
| 7. | 8. | 9. | 10. | 11. | 12. |
| 19. $1100 - 240 = ?$ | | 21. $1728 - 1128 = ?$ | | 23. $990 - 871 = ?$ | |
| 20. $\begin{array}{r} 73 \\ \underline{\quad} \end{array} - \begin{array}{r} 19 \\ \underline{\quad} \end{array} = ?$ | | 22. $\begin{array}{r} 411 \\ \underline{\quad} \end{array} - \begin{array}{r} 301 \\ \underline{\quad} \end{array} = ?$ | | 24. $\begin{array}{r} 747 \\ \underline{\quad} \end{array} - \begin{array}{r} 75 \\ \underline{\quad} \end{array} = ?$ | |

EXERCISE 39. (WRITTEN.)

Write and find the difference between the first two numbers of each example in Exercise 22; prove your work. Thus,

$$1. \begin{array}{r} 424 \\ \underline{236} \end{array} \quad 2. \begin{array}{r} 108 \\ \underline{34} \end{array}$$

Do the same with the last two numbers of each example.

EXERCISE 40. (ORAL OR WRITTEN.)

Find the difference between each number, except the last, and the next one below it in examples 1 to 14, Exercise 24. Finish with the numbers of Example 1, then take those of Example 2, and so on. Number your examples as you write them.

EXERCISE 41.

Find the difference between each number, of the first two, and the second number below it in examples 1 to 14, Exercise 24; between the first number and the third number below it. Work in the same order as in Exercise 40.

EXERCISE 42.

Find the difference between the first number of Example

1, Exercise 22, and the first number of each of the other examples. Thus,

1. $424-34$; 2. $424-402$; 3. $3756-424$; and so on.

Difference between the second number of Example 1 and the first number of each of the other examples; the third number of Example 1 and the first number of each of the other examples; the fourth number of Example 1 and the first number of each of the other examples.

EXERCISE 43.

Find the difference between each number, except the last, and the next number to the right in examples 15 to 30, Exercise 24. Finish with each line before proceeding to the next.

EXERCISE 44.

Difference between each number, of the first two, and the second number to the right in examples 15 to 30, Exercise 24; between the first number and the third number to the right. Work in the same order as in Exercise 43.

EXERCISE 45.

Write, perform, and prove 20 examples of your own in Subtraction. Bring to the class to dictate to the others.

EXERCISE 46.

Dictate, without writing them and without help, numbers of your own, to the others of your class.

PRACTICAL WORK IN ADDITION AND SUBTRACTION.

All examples in Addition and Subtraction may be reduced to one of the following *general forms* :

General { **A.—Find the sum of 327, 48, and 452.**
Forms. { **B.—Find the difference between 479 and 627.**

Illustration 1.—A man has 256 trees in one orchard and 375 in another ; how many has he in both?

We are to put together, or add, the trees in both orchards ; hence, the *general form* for the example is:

A. Find the sum of 256 and 375.

Illustration 2.—A man having 324 oranges sold 108 of them ; how many had he left?

We are to take away the number of oranges sold from the whole number he had ; hence, the *general form* for this example is:

B. Find the difference between 324 and 108.

EXERCISE 47.

Think of each example carefully, find out what is asked, and then write the *general form* for each of the first 20 examples below:

1. A man had a ranch of 4750 acres, from which he sold 1287 acres ; how many acres had he left?

2. A man sets out an orchard of 156 pear trees, 273 apricot trees, 195 peach trees, 390 apple trees, and 312 almond trees ; how many trees were in the orchard?

3. A boy saves \$83 the first year after leaving school, and \$147 the second ; how much does he save in both?

4. Two men walk a three days' race. One travels 263 miles ; the other, 197. How many more miles does one walk than the other?

5. There are 31 days in January, 28 in February, 31 in

March, 30 in April, 31 in May, 30 in June, 31 in July, 31 in August, 30 in September, 31 in October, 30 in November, and 31 in December. How many days are there in the whole year?

6. I paid \$2500 for a house, \$350 for a horse and buggy, \$65 for a cow, \$119 for furniture, and \$47 for groceries; what did I pay for all?

7. The number of people in Sacramento in 1870 was 16283; in 1880, 21420. How many more people were in Sacramento in 1880 than in 1870?

8. Gen. Grant was born in 1822 and died in 1885; how old was he when he died?

9. The Mississippi River is 2816 miles long; the Missouri, 3047. Which is the longer and how much?

10. In 1882, Alameda County cast 4617 votes for George Stoneman for governor; Los Angeles, 3943; Sacramento, 3248; San Francisco, 24257; Santa Clara, 3308. How many votes did these 5 counties cast for Mr. Stoneman?

11. How many more votes were cast by San Francisco County than by the other 4 counties put together?

12. A man having \$2375 in the bank drew out \$187 at one time and \$298 at another; what did he draw out in all, and what was still remaining in the bank?

13. In 1880 there were 16120 Indians and 75025 Chinese in California; there were how many of both, and how many more of one than of the other?

14. In a certain orchard containing 425 trees, 187 are orange trees, 153 are lemon trees, and the rest are nut trees; how many nut trees are in the orchard?

15. Bought a horse for \$185 and sold it for \$212; how much did I gain?

16. George Washington was born in 1732 and died in 1799. Abraham Lincoln was born in 1809 and died in 1865. Which lived the longer, and how many years longer?

17. A farmer raises 1276 centals of wheat; his neighbor on the right raises 125 centals more than he; his left-hand neighbor raises 375 centals more than both the others. Find the number of centals raised by each, and by all together.

18. Daniel Webster died in 1852 at the age of 70; in what year was he born?

19. A stock-raiser has 1483 sheep in one corral, 578 in a second, 230 in a third, and 1020 in a fourth; how many sheep has he?

20. Sold a carriage for \$145, which was \$65 less than it cost me; what did it cost me?

21. California became a State in 1850; how many years has it been a State?

22. From the sum of 309 and 576 subtract their difference.

23. A speculator bought a lot of cattle for \$2375, paid \$450 to get them to market, and sold them for \$3100; how much did he gain?

24. The distance by rail from San Francisco to Ogden is 602 miles; from Ogden to Omaha, 1312; from Omaha to Chicago, 490; from Chicago to New York, 963. Find the distance by rail from San Francisco to Chicago; from San Francisco to New York.

25. Which is the longer distance by rail, from San Francisco to Omaha, or from Omaha to New York, and how much longer?

26. How many years is it since Columbus discovered America?

27. The votes cast in California at the presidential election of 1884 were as follows: For Cleveland, 89225; for Blaine, 102406; for St. John, 2960; for Butler, 2010; scattering, 356. What was the total vote of California?

28. Blaine received how many more votes than Cleveland?

29. Blaine received how many more than all the rest put together?

30. I bought a carpet for \$17, a chamber suit for \$26, a spring mattress for \$8, a lounge for \$18, an extension table for \$11, and a parlor stove for \$7; gave in payment \$100. What change should I receive?

31. The smaller of two numbers is 173, and their difference is 49; what is the larger?

32. The sum of two numbers is 1208, and the larger is 749; what is the smaller?

33. The larger of two numbers is 970, and their difference is 127; what is the smaller?

34. A man bought 4 house lots for \$4000. He paid \$800 for the first, \$125 more for the second than for the first, and \$250 more for the third than for the second; what did he pay for the fourth?

35. A boy said if he had 23 more marbles he would have 100. How many had he?

36. What number taken from 1728 leaves 209?

37. Should a man die to-day at the age of 69, in what year was he born?

38. If you live till the year 1922, how old will you be?

39. Three men go into business together. The first puts in \$2500; the second, \$1550; the third, \$1325. They gain \$725 during the year. How much money have they in all at the close of the year?

40. A certain school has 7 grades. In the first are 57 pupils; in the second, 73; in the third, 61; in the fourth, 93; in the fifth, 84; in the sixth, 101; in the seventh, 112. How many pupils are in the school?

41. If 273 pupils in the above school are boys, how many are girls?

42. Benj. Franklin was born in 1706 and lived 84 years; in what year did he die?

43. The population of the United States in 1870 was

38567617; in 1880, 50267519. How much had it gained in 10 years?

44. There were 6608 miles of railroad built in 1883 in the United States, and 11591 miles in 1882. How many miles were built in both years? How many more in 1882 than in 1883?

45. How many days from Jan. 1 to July 1?

46. Two boys have each 145 cents; one gives the other 25 cents. How many cents has each now, and how many more has one than the other?

47. A man has \$2783 on hand and owes \$1296; how much is he really worth?

48. A man receives \$125 a month for 3 months; he spends during that time \$171. How much does he save?

49. A man lays up \$370 a year for 4 years; how much has he at the end of the time?

50. A merchant bought 3 lots of wheat containing 1250, 498, and 726 centals respectively. He sold 550 centals at one time and 1500 at another; how many centals remained?

51. The city of Rome was founded 753 years before Christ (B. C.). How old is it?

52. The date given for the creation is 4004 B. C. The Flood occurred 1652 years later. In what year was the Flood?

53. Mt. Everest is 29062 feet high; Mt. Whitney, 14900. What is the difference in their heights?

54. A lady went on a journey, traveling 175 miles by steamer, 213 by rail, and 94 by stage. What was the length of the journey?

55. A man gained \$45 by selling a horse for \$190. What did the horse cost him?

56. How much will a man have left from \$1000, if he spends \$125 at one time, \$256 at another, and \$114 at another?

57. A man sells 130 sheep for \$325, 115 sheep for \$345,

and 58 sheep for \$203. How many sheep did he sell, and what did he get for all?

58. A sells a house to B for \$2375; B sells it to C at a gain of \$250; C sells it to D at a loss of \$175. What does D pay for the house?

59. A man dying leaves \$3400 to his wife, \$1700 to each of his two sons, and \$1500 apiece to his three daughters. How much money does he leave?

60. During the year ending July 1, 1885, 2114 arrests were made in Oakland, of which all but 906 were caused by drunkenness. Find the number thus caused.

61. How many days from August 1 to the end of the year?

62. How many years was it from the birth of Moses 1571 B. C. to the founding of Rome?

63. A man exchanged a lot of wheat and \$725 for cattle valued at \$2700. What was the value of the wheat?

64. How many more days are there from June 1 to October 1 than from Jan. 1 to May 1?

65. The first Spanish mission founded in California was at San Diego in 1769. 79 years later, gold was discovered in the State. In what year was gold discovered?

66. Mt. Everest is 29062 feet above the sea level; the Dead Sea is 1317 feet below the sea level. How many feet does Mt. Everest rise above the Dead Sea?

67. A boy has 175 cents but gives away 30 to a boy who had none. After the gift, how many more has the first boy than the second?

68. A fruit grower has 4 rows of trees in a certain orchard, containing 32 trees each. 72 are orange trees and the remainder are lemon; how many lemon trees are there?

EXERCISE 48.

Make up 10 examples of your own like the preceding, work out, and bring into the class for dictation to the others.

MULTIPLICATION.

I bought 2 apples for which I paid 2 cents each; what did I pay for both apples?

How do you find it?

At 2 cents each, what must I pay for 3 apples? For 4? For 5? For 6? For 7? For 8?

Compare your work with the first direction in Example 1, Exercise 18. In this work you are adding by what number? How many times do you take 2 to get the price of 2 apples? To get the price of 3? Of 4? Of 6? Of 7?

To find how much any number of apples, oranges, pencils, etc., costs at 2c. each, we add by 2's as many times as there are apples, oranges, pencils, etc.

At 3 cents each what will 2 pencils cost? 3 pencils? 4? 5? 6? 7? 8? 9?

Compare with the first direction in Example 2, Exercise 18. You are now adding by what number? How many times, for 2 pencils? For 3? 5? 7? 9?

Instead of adding from 0 up, every time, when we wish to perform examples like the preceding, it is better to commit to memory these results for all numbers up to 10.

The process of taking any number of times a given number is called **MULTIPLICATION**.

The number to be taken a number of times is called the **multiplicand**.

The number showing how many times the multiplicand is to be taken is called the **multiplier**.

The result of multiplying is called the **product**.

Pick out each in the above illustrations.

The sign (\times), called **times**, is used to indicate multiplication. Thus,

$3 \times 3 = 9$ is read *3 times 3 are 9*, or *3 3's are nine*.

The multiplicand and multiplier are sometimes called **factors** of the product. Thus, in the phrase,

3 2's are 6, 3 and 2 are factors of 6.

In general, any whole numbers, which, multiplied together, will produce a given number, are called *factors* of that number.

EXERCISE 49. (ORAL AND WRITTEN.)

Add by 2's from 0 to 20, write out the work in column as indicated below, and commit to memory, reading as directed above. Thus,

$$1 \times 2 = 2$$

$$2 \times 2 = 4$$

$$3 \times 2 = 6$$

$$4 \times 2 = 8$$

$$5 \times 2 = 10$$

and so on

$$\text{to } 10 \times 2.$$

Do the same with 3's from 0 to 30; 4's from 0 to 40; 5's from 0 to 50; 6's from 0 to 60; 7's from 0 to 70; 8's from 0 to 80; 9's from 0 to 90.

EXERCISE 50. (ORAL AND WRITTEN.)

After the thorough memorizing of the tables, give them backward, writing them backward, also.

EXERCISE 51. (ORAL.)

$$3 \times 2 = \quad 4 \times 3 = \quad 5 \times 6 = \quad 4 \times 9 = \quad 2 \times 9 =$$

$$2 \times 3 = \quad 3 \times 4 = \quad 6 \times 5 = \quad 9 \times 4 = \quad 3 \times 6 =$$

$$4 \times 2 = \quad 5 \times 3 = \quad 8 \times 5 = \quad 7 \times 7 = \quad 7 \times 9 =$$

$$2 \times 4 = \quad 3 \times 5 = \quad 5 \times 8 = \quad 8 \times 7 = \quad 9 \times 7 =$$

$$7 \times 2 = \quad 3 \times 7 = \quad 4 \times 7 = \quad 7 \times 8 = \quad 8 \times 9 =$$

$$2 \times 7 = \quad 7 \times 3 = \quad 7 \times 4 = \quad 8 \times 8 = \quad 9 \times 8 =$$

$$9 \times 2 = \quad 9 \times 3 = \quad 6 \times 7 = \quad 6 \times 6 = \quad 9 \times 9 =$$

$$2 \times 9 = \quad 3 \times 9 = \quad 7 \times 6 = \quad 4 \times 4 = \quad 5 \times 9 =$$

$$6 \times 2 = \quad 10 \times 3 = \quad 8 \times 6 = \quad 2 \times 8 = \quad 9 \times 5 =$$

$$2 \times 6 = \quad 3 \times 10 = \quad 6 \times 8 = \quad 8 \times 2 = \quad 5 \times 5 =$$

EXERCISE 52. (WRITTEN.)

Write all the factors of the following numbers, and bring in to the class for reading. Thus,

21 has 3 and 7 for its factors; hence, $7 \times 3 = 21$.

21, 42, 32, 36, 16, 45, 27, 80, 48, 18, 15, 10, 56, 30, 9, 40, 25, 64, 14, 28, 56, 20, 60, 63, 81, 70, 24, 72, 90, 8, 12, 54, 11, 17, 23, 29.

EXERCISE 53. (ORAL.)

Use 2 as a multiplier with each of the following numbers; then use 3, 4, 5, 6, 7, 8, 9 in turn:

7 3 8 5 2 9 6 4 1 0

A number applied to a particular object or thing is called a **concrete number**; as, *7 books, 3 yards, 71 days.*

A number used with no reference to any object or thing is called an **abstract number**; as, *7, 3, 71.*

EXERCISE 54. (WRITTEN.)

Write, in one column, the concrete numbers, and, in another, the abstract numbers in the following:

51, 29 inches, 7 pencils, 147, 512, 14 cows, 28 horses, 28, 158, 12 months, 10 cents, 12 knives, 159, 6 dozen pens, 1200, 496.

Write 10 abstract and 10 concrete numbers of your own.

When both factors are abstract, either may be the multiplicand.

When one factor is concrete, it is the multiplicand, and the product is like it.

Thus,

M-r. M-d.

P-t.

7×3 units = 21 units; 7×3 tens = 21 tens; $7 \times \$3 = \21 .

At 5 cents apiece what will 7 pencils cost?

MODEL FOR ANALYSIS.—If 1 pencil costs 5 cents, 7 pencils will cost 7×5 cents, or 35 cents.

Pick out (1) the multiplicand (2) the multiplier (3) an abstract number (4) a concrete number. What is the product like in name?

EXERCISE 55. (WRITTEN.)

Write the analysis of the following like the preceding model:

1. At 10 cents a dozen what will 9 dozen oranges cost?
2. If a watch ticks 3 times in 1 second, how many times will it tick in 6 seconds?
3. If 1 yard contains 3 feet, how many feet do 8 yards contain?
4. If 1 ton of coal costs \$8 what will 7 tons cost?
5. What will 3 pairs of shoes cost at \$2 a pair?
6. If a man can walk 4 miles an hour, how far can he walk in 9 hours?
7. What cost 7 cords of wood at \$7 a cord?
8. How many trees are there in an orchard containing 9 rows of 8 trees each?
9. There are 7 days in 1 week; how many days are in 9 weeks?
10. 8 boys have 6 marbles each; how many have all?

Repeat the analysis orally in the class.

EXERCISE 56. (ORAL ANALYSIS.)

1. Find the cost of a dozen pencils at 3 cents each.
2. There are 4 quarts in a gallon. How many quarts are there in 7 gallons? In 4 gallons? In 9 gallons? In 5 gallons?
3. At 9 cents a yard what must be paid for 4 yards of calico? For 7 yards? For 2 yards? For 8 yards?
4. If a man works 8 hours a day, how many hours does he work in 5 days? In 6 days? In 8 days? In 3 days?
5. I pay \$5 a week for board. What do I pay for board for 2 weeks? For 4 weeks? For 6 weeks? For 7 weeks?

6. There are 10 tens in 1 hundred. There are how many tens in 5 hundreds? In 7 hundreds? In 9 hundreds?

7. A horse travels 6 hours at the rate of 7 miles an hour. How far does he travel?

8. What cost 7 2-cent postage stamps? 5? 8? 4?

9. If 6 men can dig a ditch in 6 days, how long will it take 1 man?

10. At 7 cents a yard what will 10 yards of ribbon come to? 8 yards? 3 yards? 2 yards? 6 yards?

As the multiplicand may be concrete, representing hours, cents, etc., any example in multiplication may be made a practical example like the above. Hence, instead of

$$6 \times 3 = 18,$$

write

At 3 cents each what will 6 apples cost?

EXERCISE 57.

Form 10 examples of your own, like the above, from the following, and analyze:

1. $9 \times 2 = ?$ 3. $5 \times 9 = ?$ 5. $7 \times 7 = ?$ 7. $8 \times 5 = ?$ 9. $3 \times 8 = ?$
 2. $6 \times 7 = ?$ 4. $9 \times 3 = ?$ 6. $4 \times 4 = ?$ 8. $10 \times 7 = ?$ 10. $8 \times 7 = ?$

Dictate similar examples in the class, on the spur of the moment.

To multiply a number, of several figures, by units.

Multiply 423 by 2.

OPERATION.

423.

2.

846.

EXPLANATION.—Write the numbers as in Addition.
 Write the several products in their proper column.

EXERCISE 58.

1. $724 \times 2 =$ 4. $123 \times 3 =$ 7. $821 \times 3 =$
 2. $522 \times 3 =$ 5. $443 \times 2 =$ 8. $610 \times 4 =$
 3. $321 \times 4 =$ 6. $711 \times 5 =$ 9. $7122 \times 4 =$

To multiply by any number of 10's, 100's, 1000's, etc.

Write in figures and read by common names the following:

7 10's=	155 100's (hundreds)=
15 10's=	176 100's " =
25 10's=	3141 100's " =
230 10's=	7 1000's (thousands)=
175 10's=	12 1000's " =
3126 10's=	36 1000's " =
6 100's (hundreds)=	172 1000's " =
17 100's " =	230 1000's " =
20 100's " =	1756 1000's " =

But 7 10's or 7×10 , is the same as 10 7's or 10×7 ; 15 10's, the same as 10×15 ; 175 10's, the same as 10×175 ; 6 100's= 100×6 ; 3141 100's= 100×3141 ; 12 1000's= 1000×12 ; 172 1000's= 1000×172 ; and so on.

Hence, to multiply a number by 10, what will you do? By 100? By 1000? By 10000? By 100000?

7×2 tens=? 7×2 hundreds=? 7×2 thousands=?

7×2 10's is the same as 2 10's $\times 7$, or 20×7 .

7×2 100's is 200×7 .

$27 \times 200 = 200 \times 27$.

$77 \times 2000 = 2000 \times 77$.

Hence, to multiply by 2 10's, 3 10's, 4 10's, etc., what can you do? By 2 100's, 3 100's, 4 100's, etc.? By 2 1000's, 3 1000's, 4 1000's, etc.?

EXERCISE 62. (WRITTEN.)

Perform the multiplications indicated below. Also, multiply by two other numbers of 10's, 100's, 1000's, besides those given. The multiplication by numbers with 0's is usually written as below.

1.	2.	3.	4.	5.	6.
43	225	75	239	3141	729
30	70	400	600	500	3000
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>

7.	8.	9.	10.	11.
4280	2146	404	75	675
50	7000	900	10000	800
<hr style="width: 80%; margin: 0 auto;"/>	<hr style="width: 80%; margin: 0 auto;"/>	<hr style="width: 80%; margin: 0 auto;"/>	<hr style="width: 80%; margin: 0 auto;"/>	<hr style="width: 80%; margin: 0 auto;"/>

To multiply by numbers of two or more figures.

Multiply 5847 by 3075.

FULL WORK.	CONTRACTED.
5847	5847
3075	3075
<hr style="width: 80%; margin: 0 auto;"/>	<hr style="width: 80%; margin: 0 auto;"/>
29235 = 5 × 5847	29235
409290 = 70 × 5847	40929
17541000 = 3000 × 5847	17541
<hr style="width: 80%; margin: 0 auto;"/>	<hr style="width: 80%; margin: 0 auto;"/>
17979525 = 3075 × 5847	17979525

Since 0's count nothing in adding, it shortens the work to omit writing them, as in the contracted operation.

Test by using the multiplicand for the multiplier; that is, multiply 3075 by 5847.

EXERCISE 63.

Multiply each multiplicand, of examples 31 to 39, inclusive, Exercise 59, by each multiplicand of examples 1 to 10, inclusive, and prove.

The same may be done with examples 11 to 30 of the same exercise.

EXERCISE 64.

Write, perform, and prove 10 similar examples of your own, and bring into the class for dictation.

EXERCISE 65. (WRITTEN.)

See "Model for Analysis," p. 36, and analyze the following:

1. Bought 25 cows at \$37 a head; what was paid for all?
2. There are 24 hours in one day; how many hours are in 32 days?
3. A lady paid \$16 a month for board for 11 months; what was her board bill?

4. There are 5280 feet in a mile; how many feet are there in 18 miles?
5. At an average rate of 23 miles an hour, how far will a railroad train go in a day?
6. A man sets out 93 acres of fruit trees, setting 104 trees to the acre; find the number of trees in his orchard.
7. What will be the cost of building 35 miles of railroad at an average expense of \$33275 a mile?
8. Find the cost of a ranch of 960 acres at \$75 an acre.
9. How many pounds of tea are in 346 chests, each chest containing 65 pounds?
10. A clerk's salary is \$75 a month; what does he receive in a year?
11. At \$23 each, what does a furniture dealer receive for 24 lounges?
12. A grocer's sales average \$19 a day for the month of March; leaving out 5 days for Sundays, what were his receipts during the month?
13. Each workman in an iron foundry is paid \$525 a year; what do 12 men receive at that rate?

DIVISION.

Copy the following exercise on your slates, and in place of each blank put how many times the number above the line must be taken to make the one below:

$$\begin{array}{cccccccccc} 7 & 8 & 6 & 3 & 8 & 9 & 10 & 3 & 7 & 5 \\ \hline 35 & 72 & 42 & 18 & 48 & 54 & 60 & 27 & 63 & 45 \end{array}$$

Copy the following, and place below the line the number of times the number on the left of the curve is contained in the number on the right:

$$\begin{array}{cccccccc} 5 \curvearrowright 40 & 7 \curvearrowright 56 & 9 \curvearrowright 81 & 8 \curvearrowright 32 & 5 \curvearrowright 50 & 4 \curvearrowright 20 & 10 \curvearrowright 30 & 6 \curvearrowright 54 & 5 \curvearrowright 27 & 8 \curvearrowright 59 \end{array}$$

How do the last two differ from the others? Find the largest number below 27 that is an exact number of times 5. Find the remainder after taking away 5 5's from 27. Do the same with the 8's in 59 and find what remains.

The process of finding how many times one number contains another, is called **DIVISION**.

The containing number is called the **dividend**.

The number contained is called the **divisor**.

The number of times the dividend contains the divisor is called the **quotient**.

The part of the dividend left over, when the divisor is not contained an exact number of times, is called the **remainder**. It is always like the dividend.

Pick out each in the above exercise.

The sign (\div) is used to indicate division. Thus,

$$35 \div 7 = 5,$$

Is read,

35 divided by 7 is 5.

It may also be written $\frac{35}{7} = 5$.

What precedes the sign (\div) in the first expression? Where is the same found in the second? What follows the sign (\div) and stands below the line?

EXERCISE 66. (WRITTEN.)

Write, in vertical column, the numbers from 2 to 20, by 2's. Write the sign (\div) after each, and, using 2 as a divisor of each, find the quotient. After writing, practice upon the exercise. Thus,

$$\text{First Form. } \begin{cases} 2 \div 2 = 1 \\ 4 \div 2 = 2, \text{ and so on.} \end{cases}$$

The same by 3's from 3 to 30 with 3 as a divisor, expressing the division in the second form. Thus,

$$\text{Second Form. } \begin{cases} \frac{3}{3} = 1 & \frac{9}{3} = 3 \\ \frac{6}{3} = 2 & \frac{12}{3} = 4; \text{ and so on.} \end{cases}$$

The same by 4's from 4 to 40, with divisor 4, using the sign \div ;

By 5's from 5 to 50, using divisor 5 and the line;

By 6's from 6 to 60, using divisor 6 and the line;

By 7's from 7 to 70, using divisor 7 and sign (\div);

By 8's from 8 to 80, using divisor 8 and sign (\div);

By 9's from 9 to 90, using divisor 9 and line.

Bring into the class to read.

To divide a number is to separate it into equal parts.

Thus, when we divide an apple into 2 equal parts, each part is one-half; when into 3 equal parts, each part is one-third; when into 4 equal parts, each part is one-fourth; and so on.

So when we divide a number by 2, or into 2 equal parts, we get one-half the number; into 3 equal parts, one-third the number; into 4 equal parts, one-fourth the number. Thus, dividing 12 by 2 is taking one-half of it; or,

$$12 \div 2 = \text{one-half of } 12 = 6, \quad 12 \div 4 = \text{one-fourth of } 12 = 3,$$

$$12 \div 3 = \text{one-third of } 12 = 4, \quad 12 \div 6 = \text{one-sixth of } 12 = 2,$$

and so on.

What is each part when we divide a number into 5 equal parts? Into 6? 7? 8? 9? 10? 11? 12? 15? 17? 19? 25? 36? 50? 100? 136? 175?

Read the following exercise thus, *One-fifth of 30 is 6, etc.:*

5)30 4)28 7)35 6)42 9)54 3)21 2)14 8)72 10)90

EXERCISE 67. (ORAL.)

Use the numbers in the left hand column for divisors and the other numbers in the same row for dividends.

Name quotients and remainders. Read,

3 in 15, 5; 3 in 29, 9 and 2 over; and so on.

Also read,

$\frac{1}{3}$ (one-third) of 15 is 5; $\frac{1}{3}$ of 29 is 9 and 2 over, or $9\frac{2}{3}$ (two-thirds); $\frac{1}{7}$ (one-seventh) of 44 is 6 and 2 over, or $6\frac{2}{7}$ (two-sevenths).

3	15	29	31	23	18	9	0	24	17	27	14	11
7	44	37	28	19	7	14	24	32	46	56	38	43
9	22	35	41	80	72	56	19	7	26	81	93	45
6	36	43	14	8	49	53	62	25	33	18	42	51
8	21	17	16	80	66	44	55	0	38	14	35	76
5	35	43	52	24	37	15	8	28	49	36	21	18
4	15	22	28	35	18	7	11	24	31	17	10	0
8	48	79	46	64	15	23	59	75	41	12	9	28
2	5	2	11	13	19	7	8	12	10	3	6	17
7	42	21	9	17	33	55	66	35	63	71	13	11
9	85	44	27	10	36	51	75	25	13	63	97	17
10	23	30	17	46	10	87	54	95	70	36	0	14
6	48	27	10	19	30	66	54	47	13	24	11	59
7	40	62	22	72	45	67	27	15	54	59	73	18
8	22	43	16	25	71	85	53	29	78	19	49	87

EXERCISE 68. (WRITTEN ANALYSIS.)

If 7 pencils cost 35 cents, what costs 1 pencil?

MODEL.—If 7 pencils cost 35 cents, 1 pencil costs $\frac{1}{7}$ of 35 cents, or 5 cents.

Observe $\left\{ \begin{array}{l} \text{In this form of analysis the dividend and quotient} \\ \text{are concrete numbers of the same kind.} \\ \text{The divisor is abstract and corresponds to the mul-} \\ \text{tiplier.} \end{array} \right.$

Write analyses of the following :

1. If 9 dozen oranges cost 90 cents, what will 1 dozen cost?

2. If a watch ticks 18 times in 6 seconds, how many times does it tick in 1 second?

3. How many feet in 1 yard, if 8 yards are 24 feet?

4. What is the price per ton, when 7 tons of coal cost \$56?

5. If three pairs of shoes sell for \$6, what do they bring a pair?

6. How far does a man walk in 1 hour, if he goes 36 miles in 9 hours?

7. Paid \$49 for 7 cords of wood; how much was the wood a cord?

8. 9 rows of trees in an orchard contain 72 trees; how many trees in a row?

9. If there are 63 days in 9 weeks, how many days in a week?

10. I divided 48 marbles equally among 8 boys; how many did they receive apiece?

Compare the above work with that of Exercises 69 and 55.

EXERCISE 69. (WRITTEN ANALYSIS.)

At 5 cents apiece, how many pencils can I get for 35 cents?

MODEL.—If 1 pencil costs 5 cents, I can get as many pencils for 35 cents as 5 cents is contained times in 35 cents, or 7.

Observe { *In this form of analysis the dividend and divisor are concrete numbers of the same kind.*
The quotient is an abstract number and corresponds to the multiplier in Multiplication.

Write analyses of the following :

1. How many dozen oranges at 10 cents a dozen can be bought for 90 cents?
2. How many nickel watches at \$3 each can you buy for \$18?
3. There are 3 feet in a yard; how many yards in 24 feet?
4. At \$8 a ton, how many tons of coal can be bought for \$56?
5. At \$2 a pair, how many pairs of shoes can be bought for \$6?
6. At the rate of 4 miles an hour, how long will it take a man to walk 36 miles?
7. How many cords of wood at \$7 a cord can you buy for \$49?
8. An orchard of 72 trees has 8 trees in a row; how many rows are there?
9. In 63 days how many weeks?
10. I divided 48 marbles among some boys, giving 6 marbles to each boy; how many boys were there?

Compare work with that of Exercise 55.

EXERCISE 70. (ORAL ANALYSIS.)

1. At 3 cents each, how many pencils can you buy for 36 cents? For 24 cents? For 18 cents? For 30 cents?
2. There are 4 quarts in 1 gallon; how many gallons will 28 quarts make? 16 quarts? 32 quarts? 24 quarts?
3. At 9 cents a yard, how many yards of calico can be bought for 36 cents? For 63 cents? For 18 cents? For 72 cents?
4. If a man works 8 hours a day, how many days' work will 40 hours make? 48 hours? 64 hours? 24 hours?

5. I pay \$5 a week for board; how many weeks' board can I get for \$10? For \$20? For \$30? For \$35?

6. There are 10 tens in 1 hundred; how many hundreds will 50 tens make? 70 tens? 90 tens?

7. A horse travels 7 miles an hour; in how many hours will he travel 42 miles? 63 miles?

8. How many 2-cent postage stamps can you get for 14 cents? 10 cents? 16 cents? 8 cents?

9. If 1 man can dig a certain ditch in 36 days, how many men will it take to do it in 4 days? In 9 days? In 6 days? In 12 days?

10. At 7 cents a yard, how many yards of ribbon can be bought for 70 cents? For 21 cents? For 56 cents? For 14 cents? For 42 cents?

As the divisor and dividend may represent cents, dollars, hours, etc., any example in Division may become a practical example. Thus, instead of

$$18 \div 3 = 6,$$

Write,

At 3 cents each, how many apples can be bought for 18 cents?

EXERCISE 71. (WRITTEN.)

Form, from the following, 8 examples like the above, and analyze:

1. $18 \div 2 = ?$ 3. $45 \div 5 = ?$ 5. $49 \div 7 = ?$ 7. $40 \div 8 = ?$
 2. $42 \div 6 = ?$ 4. $27 \div 9 = ?$ 6. $16 \div 4 = ?$ 8. $70 \div 10 = ?$

Give additional examples in the class.

EXERCISE 72. (ORAL ANALYSIS.)

1. If 9 pencils cost $\left\{ \begin{array}{l} 36 \\ 63 \\ 90 \\ 81 \end{array} \right\}$ cents, what does 1 pencil cost?

NOTE.—Use each of the numbers in the braces for one example.

2. If 28 quarts make 7 gallons, how many quarts are in 1 gallon?

3. If 4 yards of calico cost $\left\{ \begin{array}{l} 36 \\ 44 \\ 48 \\ 40 \end{array} \right\}$ cents, what costs 1 yard?

4. If $\left\{ \begin{array}{l} 40 \\ 48 \\ 56 \\ 64 \end{array} \right\}$ hours make 8 days' work, how many hours' work to a day?

5. If I pay \$ $\left\{ \begin{array}{l} 10 \\ 8 \\ 12 \\ 14 \end{array} \right\}$ for 2 weeks' board, what is the rate per week?

6. If there are 50 tens in 5 hundred, how many tens in 1 hundred?

7. If a horse travels $\left\{ \begin{array}{l} 36 \\ 42 \\ 54 \\ 48 \\ 60 \end{array} \right\}$ miles in 6 hours, what is his rate per hour?

8. I bought 7 postage stamps for 14 cents; what did each stamp cost?

9. 1 man can dig a ditch in 36 days; how long will it take 6 men? 4 men? 9 men? 3 men? 12 men? 2 men? 18 men? 36 men?

10. If 10 yards of calico cost $\left\{ \begin{array}{l} 80 \\ 90 \\ 100 \end{array} \right\}$ cents, what is the price of 1 yard?

11. If 7 pen holders cost $\left\{ \begin{array}{l} 35 \\ 14 \\ 49 \\ 63 \\ 56 \end{array} \right\}$ cents, what does 1 pen holder cost?

12. If 9 Plymouth Rock chickens cost \$27, what is the price of 1?

EXERCISE 73. (WRITTEN.)

From the examples given in Exercise 70, form 8 examples of your own, similar to those in Exercise 72.

SHORT DIVISION.

To divide numbers, of several figures each, by units.

Divide 8484 by 4.

OPERATION.

$$\begin{array}{r} 4 \overline{)8484} \\ \underline{2121} \end{array}$$

DIRECTION.—Write the divisor at the left of the dividend for convenience and begin with the left figures of the dividend. Write the quotients in their respective columns.

This method of dividing has been named SHORT DIVISION.

EXERCISE 74.

- | | | |
|--------------------|--------------------|---------------------|
| 1. $3699 \div 3 =$ | 5. $455 \div 5 =$ | 9. $2505 \div 5 =$ |
| 2. $3699 \div 9 =$ | 6. $217 \div 7 =$ | 10. $7007 \div 7 =$ |
| 3. $8484 \div 2 =$ | 7. $1282 \div 2 =$ | 11. $1402 \div 2 =$ |
| 4. $728 \div 8 =$ | 8. $1596 \div 3 =$ | 12. $903 \div 3 =$ |

Divide 43457 by 7.

FULL OPERATION.

$$\begin{array}{r} 7 \overline{)43457} \\ 42000 \div 7 = \underline{6000} \\ 1400 \div 7 = \underline{200} \\ 57 \div 7 = \underline{8} \text{ 1 over} \\ \hline 43457 \div 7 = \underline{6208} \text{ 1 over} \end{array}$$

EXPLANATION.— $\frac{1}{7}$ of 43 thousand is 6 thousand, and 1 thousand over. But 1 thousand is 10 hundred, which, with the 4 hundred in the dividend, make 14 hundred. $\frac{1}{7}$ of 14 hundred is 2 hundred. $\frac{1}{7}$ of 57 is 8, and 1 over. The

ciphers counting nothing in adding, they are omitted in the work, as in the contracted operation. In explaining the work, read thus: 7 in 43, 6, and 1 over; 7 in 14, 2; 7 in 5, 0, and 5 over; 7 in 57, 8, and 1 over, or $\frac{1}{7}$.

Test by multiplying divisor and quotient, and adding the remainder. The result should be the dividend.

EXERCISE 75. (WRITTEN.)

Give oral explanation in the class, reading as in the explanation above, and prove the work.

- | | | | |
|------------------|------------------|------------------|--------------------|
| 1. $59 \div 2 =$ | 4. $88 \div 7 =$ | 7. $84 \div 4 =$ | 10. $67 \div 3 =$ |
| 2. $78 \div 3 =$ | 5. $99 \div 6 =$ | 8. $93 \div 5 =$ | 11. $796 \div 9 =$ |
| 3. $97 \div 5 =$ | 6. $51 \div 3 =$ | 9. $79 \div 2 =$ | 12. $576 \div 8 =$ |

- | | | |
|---------------------|----------------------|-----------------------|
| 13. $479 \div 6 =$ | 22. $8000 \div 9 =$ | 31. $436208 \div 3 =$ |
| 14. $510 \div 5 =$ | 23. $3796 \div 4 =$ | 32. $58436 \div 9 =$ |
| 15. $708 \div 7 =$ | 24. $1001 \div 9 =$ | 33. $90000 \div 8 =$ |
| 16. $429 \div 2 =$ | 25. $4296 \div 3 =$ | 34. $723506 \div 5 =$ |
| 17. $233 \div 4 =$ | 26. $6511 \div 8 =$ | 35. $117452 \div 6 =$ |
| 18. $420 \div 7 =$ | 27. $2458 \div 2 =$ | 36. $89001 \div 9 =$ |
| 19. $129 \div 2 =$ | 28. $9400 \div 8 =$ | 37. $76400 \div 3 =$ |
| 20. $5309 \div 7 =$ | 29. $8057 \div 7 =$ | 38. $14700 \div 2 =$ |
| 21. $7002 \div 5 =$ | 30. $23809 \div 7 =$ | 39. $518206 \div 9 =$ |

EXERCISE 76. (WRITTEN.)

Divide each of the dividends in examples 31 to 39, Exercise 75, by each number from 2 to 9, inclusive, except the one used as divisor in the example. Finish with the dividend of Example 31 by all the divisors, before going to 32.

This exercise may be extended to as many of examples 1 to 30 as the teacher may desire.

EXERCISE 77. (ORAL REVIEW.)

- $7 + 3, \div 5, \times 4, - 1, \times 4, - 3, \div 5, + 7, \div 4, \times 9, + 3, \div 3 = ?$
- $15 \div 3, - 1, \times 4, \div 8, + 7, \div 3, \times 4, \div 2, \div 2, \times 5, - 1, \div 7 = ?$
- $9 \times 4, \div 6, - 1, \times 8, + 2, \div 7, \times 4, \div 8, + 7, \times 6, - 4, \div 8 = ?$
- $17 + 4, \div 3, - 2, \times 7, - 3, \div 4, + 4, \div 3, \times 9, + 6, + 3, \div 5, - 9, + 0 = ?$
- $48 \div 8, \times 9, - 4, \div 10, - 3, \times 7, + 1, \div 5, - 3, \times 5, \div 7, + 1, \times 4, + 1 = ?$
- $9 \times 9, - 1, \div 8, - 3, \times 9, + 1, \div 8, - 2, \times 9, + 6, \div 10, - 5, \times 7, - 2, + 5 = ?$
- $3 + 6, \times 8, - 2, \div 7, + 3, - 5, \times 10, + 10, \div 10, - 3, \times 5, - 2, \div 4, + 6, - 10 = ?$
- $41 + 7, \div 6, \times 2, - 1, \div 3, \times 4, \div 2, \div 2, + 1, + 4, \times 3, + 5, \div 7, - 5 = ?$
- $7 \times 0, + 9, \times 5, - 3, \div 6, - 4, \times 8, - 18, \times 2, - 12, + 3, \times 9, + 9, \times 6 = ?$
- $11 - 10, + 17, \div 9, + 14, \div 8, + 19, \div 3, + 20, \div 3, + 40, \div 7, + 15, \div 2 = ?$

To divide by any number of 10's, 100's, 1000's, etc.

How many 10's in 85, and what remainder? In 97? In 117? In 376? In 475?

If now you move the decimal point from the right of the number one place toward the left, you have, at the left of the point, the quotient arising from dividing by 10, and at the right of the point, the remainder. Thus,

$$8.5, 9.7, 37.6,$$

Should be read,

8 and 5 over; and so on.

How many 100's in 395, and what over? In 510? In 708? In 1576? In 9301?

How many places to the left shall we move the decimal point to show a division by 100? By 1000? By 10000?

To divide by 20, since 20 is 2 10's, we divide first by 10 by moving the point one place to the left, and then this result by 2. Thus,

$$395 \div 20 = 39.5 \div 2 = 19 \text{ } 15 \text{ over.}$$

So with 30, 40, 50, and on to 90.

To divide by 200, since 200 is 2 100's, divide first by 100 by moving the point two places to the left, and this result by 2. Thus,

$$3784 \div 200 = 37.84 \div 2 = 18 \text{ } 184 \text{ over.}$$

EXERCISE 78. (WRITTEN.)

Divide each dividend in examples 21 to 25, inclusive, Exercise 75, by 20; by 30; by 40; and so on to 90.

Divide each dividend in examples 26 to 30, inclusive, Exercise 75, by 200; 300; and so on to 900.

Divide each dividend in examples 31 to 35, inclusive, Exercise 75, by 2000; 3000; and so on to 9000.

EXERCISE 79.

Construct 10 examples of your own like the examples of Exercise 77, and bring into the class for dictation.

LONG DIVISION.

To divide by numbers of two or more figures.

All operations in division are performed like Short Division; but, in divisions by one figure, we easily recognize the quotients and remainders. With large numbers, however, the operations must be written, as we cannot tell at sight, but must find by trial, the quotients and remainders. This process has received the name of LONG DIVISION.

Divide 1459774 by 239.

OPERATION.

$$\begin{array}{r}
 239 \overline{) 1459774} \quad (6107 \text{ Quo.} \\
 \underline{1434} \\
 257 \\
 \underline{239} \\
 187 \\
 \underline{000} \\
 1874 \\
 \underline{1673} \\
 201 \text{ Rem.}
 \end{array}$$

EXPLANATION.—For convenience write the quotient to the right of the dividend, as we need the space below. We find by trial multiplication that we can take 6 239's, or 1434, out of 1459 with 25 remainder. Write down the next dividend figure, 7. There is 1 239 in 257 with a remainder 18; 0 239's in 187 with a remainder 187; 7 239's, or 1673, in 1874, with a remainder 201.

The following hints will be found useful :

1. To find quotient figures, use the first figure of the divisor and the first one or two figures of the dividend. Thus, in the above example,

$$2 \text{ in } 14; \quad 2 \text{ in } 2; \quad 2 \text{ in } 1; \quad 2 \text{ in } 18.$$

2. One or two dividend figures can be used only in case as many more are left in the dividend as are left in the divisor. Thus, in dividend 187, 18 cannot be used, as it leaves one figure only in the dividend, while there are two others in the divisor. In such a case, the figure in the quotient is 0.

3. If the second or third divisor figure is large, take one

or two less than the number of times the first divisor figure is contained in the first dividend figure. Thus,

2 in 14, 7; but 7 239's are 1673, which is too large.

4. Remember: There must always be a quotient figure for every figure brought down from the dividend.

EXERCISE 80. (WRITTEN.)

Divide each dividend in examples 26 to 35, Exercise 75, by each dividend in examples 1 to 10. Prove. Finish with the dividend of example 26 by all the divisors before going to example 27.

Divide each dividend in examples 36 to 39, Exercise 75, by each dividend in examples 11 to 20.

Divide each dividend in examples 32, 35, 36, and 39, by each dividend in examples 26 to 30.

EXERCISE 81. (ANALYSIS.)

Analyze as in Exercise 69:

1. How many cows at \$37 a head can be bought for \$925?
2. How many days are there in 744 hours?
3. I have \$176; how many months' board will it pay at \$16 a month?
4. At the rate of 23 miles an hour, how many hours will it take a railroad train to go 552 miles?
5. How many acres will 9672 fruit trees require, allowing 104 trees to the acre?
6. At \$33275 a mile, how many miles of railroad can be built for \$1164625?
7. Find the number of acres, at \$75, that can be bought for \$72000.
8. How many chests will 22490 pounds of tea fill, allowing 65 pounds to a chest?
9. A clerk received \$900 salary, at \$75 a month; how many months did he work?
10. Find how many ponies worth \$55 each are in a band sold for \$605.

GENERAL PRINCIPLES OF DIVISION.

$$\mathbf{A.} \left\{ \begin{array}{l} \text{Dividend } 48 \text{ Quotient} \\ \text{Divisor } 4 = 12 \end{array} \right.$$

Multiply the dividend of **A** by 2, and divide that product by the divisor 4. How does this quotient compare with the quotient of **A**? How many times greater?

Multiply dividend **A** by 3, and divide the result by divisor **A**. Compare your quotient with quotient **A**. Try 4 for a multiplier in the same way. Try 5.

Fill out the blanks properly in the following :

Multiplying the dividend by any number — the quotient by — number.

Divide dividend **A** by 2, and divide the result by divisor **A**. How does your quotient compare with the quotient **A**? Try 3 for a divisor and describe the result. Try 4.

Fill out :

Dividing the dividend by any number — the quotient by — number.

Multiply divisor **A** by 2, and divide dividend **A** by the product. Compare with quotient **A** and describe the result. Try 3 and 4.

Fill out :

Multiplying the divisor by any number — the quotient by — number.

Divide divisor **A** by 2, and divide dividend **A** by the result. Compare with quotient **A** and describe. Try 4.

Fill out :

Dividing the divisor by any number — the quotient by — number.

Multiply both dividend **A** and divisor **A** by 2, and divide the results. Multiply both by 3; by 4; by 5. Divide both by 2 and divide the results; by 4.

Fill out:

Multiplying or dividing both dividend and divisor by the same number ——— the quotient.

PRINCIPLES.

From these illustrations we see that—

(1) *Multiplying the dividend or dividing the divisor by any number, multiplies the quotient by the same number.*

(2) *Dividing the dividend or multiplying the divisor by any number, divides the quotient by the same number.*

(3) *Multiplying or dividing both dividend and divisor by the same number does not change the quotient.*

With the following, write on your slate, operations similar to those performed on **A** :

$$1. \frac{60}{10} = 6$$

$$3. \frac{144}{18} = 8$$

$$5. \frac{100}{10} = 10$$

$$2. \frac{72}{12} = 6$$

$$4. \frac{96}{24} = 4$$

$$6. \frac{90}{9} = 10$$

PRACTICAL WORK IN MULTIPLICATION AND DIVISION.

Examples in Multiplication and Division may be reduced to one of the following *general forms* :

General Forms. $\left\{ \begin{array}{l} \text{C.—What are 12 times 198?} \\ \text{D.—How many times is 12 contained in 144? or,} \\ \quad \text{What is } \frac{1}{12} \text{ of 144?} \end{array} \right.$

EXERCISE 82.

Determine what is to be found in the following examples, and write the first 20 in *general form* before performing. Then analyze.

1. A merchant sold 50 pieces of cloth, each containing 45 yards; how many yards did he sell?

2. Sold hay to the amount of \$1728 at \$8 a ton; how many tons did I sell?
3. What will 190 acres of land cost at \$112 an acre?
4. A mile is 320 rods; how many rods are 15 miles?
5. Bought 11 chests of tea at \$31 a chest; what did the tea cost me?
6. At \$14 a ton, how many tons of coal can be bought for \$1500?
7. Bought 15 bales of hay, averaging 235 pounds to the bale; how many pounds were there?
8. How many bales of hay will 19600 pounds make, allowing 240 pounds to a bale?
9. A field of 600 acres produced 8700 bags of wheat; how many bags to the acre was that?
10. Sent to San Francisco 5 boxes of eggs, each containing 360 eggs; how many eggs did I send?
11. How many days are in 9785 hours?
12. How many hours are in 365 days?
13. Sold 850 head of cattle at \$28 a head; how much money did I receive?
14. How many calves at \$14 can I purchase for \$1974?
15. I walked 3 days, 8 hours per day, and found I had gone 72 miles; what was my average rate per hour?
16. A man saves \$175 a year for 11 years; how much does he save in the time?
17. In one year in the United States 132890 tons of lead were produced, worth \$95 a ton; find the whole value.
18. 137 mills in California in 1882 produced 1246453 sacks of flour; what was the average per mill?
19. The State raised 3672 centals of buckwheat from 297 acres; find the average per acre.
20. At 25 cents a day what will a man's cigars cost him in 1 year?
21. Paid \$512 for 64 tons of hay; what will 25 tons cost at the same rate?

22. If a family spend 15 cents a day for beer, how much is spent in 4 weeks? How many loaves of bread at 10 cents each would the money buy?

23. Bought 15 cows at \$25 a head, 11 horses at \$95, and 50 sheep at \$3; what did the whole cost me?

24. Bought 15 acres of land of one man for \$1575, and 25 acres of another for \$2750; which cost me the more per acre, and how much?

25. I have \$2000. I buy a lot of land for \$295; build a house on it for \$1275, a shed for \$96, and set out trees which cost me \$12; buy a horse for \$115, and 5 tons of hay at \$12 a ton. With the remainder of my money I buy 3 acres of pasturage; what do I pay per acre?

26. Exchanged 8 rolls of butter for 2 pounds of tea at 65 cents, 1 pound of coffee at 35 cents, 10 pounds of sugar at 8 cents, 25 pounds of flour at 3 cents, and 2 pounds of honey at 20 cents; what did I receive a roll for my butter?

27. Bought 15 sacks of potatoes for 1275 cents, and sold them for 10 cents a sack more than I paid for them; what did I get for them, and how much more than I paid?

28. Bought 95 centals of wheat at \$1 a cental; if I give 5 20-dollar pieces in payment, what change do I receive?

29. What must I pay for 10 pounds of oatmeal at 5 cents, 4 rolls of butter at 75 cents, and 2 dozen eggs at 25 cents?

30. After taking 37 oranges from a box, there were 13 more than twice as many left in the box; how many oranges were in the box before any were taken out?

31. At \$21 a barrel how many barrels of sugar can be bought for \$3675?

32. Suppose each barrel in the 31st example contained 265 pounds; what was the total weight?

33. A ship sails 4032 miles in 14 days; how many miles a day does she sail? How many miles an hour?

34. Sold 35 bales of cloth, each bale containing 41 yards; how many yards did I sell?

35. A cattle dealer bought 175 head of cattle at \$24 a head, paying \$3500 cash down; how much remained to be paid?

36. \$3 a day amounts to how much in 4 weeks?

37. Subtract 3 thousand 8 hundred 79 from 4 thousand, multiply the remainder by 1 hundred 21, add 17 hundred 81 to the product, and divide the sum by 23; what is the quotient?

38. Two men start from the same place and travel in the same direction, one at the rate of 23 miles a day, the other, 28 miles; how far apart are they at the end of 13 days? Draw a diagram on your slate to show this.

39. Two men start from places 600 miles apart and walk towards each other; one travels 20 miles a day, the other 29 miles; how far apart are they at the end of 11 days? Draw diagram.

40. The first census of the United States was taken in 1790; since then a census has been taken every 10 years; how many had been taken, up to 1887?

41. The first president of the United States was inaugurated in 1789; since then a president has been inaugurated every 4 years; how many inaugurations had there been, up to 1887?

42. In California, in the year ending June 30, 1880, 5 factories produced \$159175 worth of silk goods; what was the average per factory?

43. If a man spends 20 cents a day for whisky, and 25 cents for cigars, what will he spend in 4 years? At 50 cents a day, how many days' board would the money furnish to a disabled soldier?

44. How many bales of cotton in 259186 pounds, allowing 312 pounds to a bale?

45. Which goes farther, a railroad train in 6 days at the rate of 22 miles an hour, or a steamship in 7 days at the rate of 16 miles an hour? How much?

46. Find the cost of 137 cords of wood at \$13 a cord.

47. I have two fields of 160 acres each. From one I cut 2 tons of hay to the acre and sell it for \$11 a ton; from the other I get 16 centals of wheat to the acre and sell it at \$1 a cental; find what I received for both, and how much more for one than for the other.

48. I feed my cow 32 pounds of hay a day; how long will 8 bales, of 240 pounds each, last?

49. How many sacks of flour will 2750 pounds make, at 50 pounds to a sack?

50. Bought 10 horses at \$125 each and paid for them in wood at \$10 a cord; how many cords did it take?

51. A fruit-grower sets out 11984 trees on 107 acres of land; how many trees to the acre?

52. If 12 men can dig a ditch in 12 days, how long will it take 1 man?

53. How many boxes of oranges at \$1 a box can I buy for \$375?

54. At an average of 750 oranges to a tree, how many oranges are in an orchard of 84 trees? How many dozen?

55. What are they worth at 12 cents a dozen?

56. If 12 men can build a house in 16 days, how long will it take 6 men?

57. Bought 150 barrels of flour for \$750; sold 125 barrels at \$6 a barrel, and the remainder at \$4; how much did I gain?

58. How many 65-dollar gold watches can you buy for \$1000, and how many 5-dollar gold rings for the remainder?

59. A man receives \$120 a month; his expenses are \$60 a month; how long will it take him to pay for a house that cost \$1920?

60. A man's salary is \$1500 a year; he pays \$22 a month for board, and \$42 a month for additional expenses; what will he save in 4 years?

61. A man bought a horse for \$175; he kept him 24

weeks at an expense of \$2 a week and then sold him for \$225; what did he gain?

62. If a man deposits \$15 in the bank every month from the time he is 21 years old until he is 70, how much will he then have deposited?

63. The President of the United States receives \$50000 a year; what does he get a month? In 1 term of office?

64. I pay \$1974 for 141 head of calves; how much do I pay per head?

65. A man bought 150 calves at \$14 a head and sold them so as to gain \$450; what did he get a head?

66. What is the average value of 5 horses, worth respectively \$85, \$95, \$105, \$115, and \$120?

67. A man receives a salary of \$1750; his expenses are \$3 a day; how much does he save in a year?

68. If 23 acres of land cost \$1955, what will 33 acres cost at the same rate?

69. Bought a certain number of watches for \$432; sold them for \$20 apiece, gaining \$2 on the cost; how many watches were there?

70. A farmer had 784 sheep; he sold 200 at one time and 375 at another; what are the remainder worth at \$2 a head?

71. I have 3 fields containing 320 acres in all and worth \$30000; the first contains 160 acres worth \$125 an acre, the second 80 acres worth \$75 an acre; what is the value per acre of the third?

72. Bought 31 hogs at \$3 each, and gave in payment eight 10-dollar bills and three 5-dollar gold pieces; what change should I receive?

73. A liquor dealer bought 15 casks of brandy, each containing 38 gallons, at \$4 a gallon; find the cost.

74. Find the average value of 4 lots of land worth respectively \$195, \$210, \$255, and \$300.

75. In 1 gallon there are 231 cubic inches; how many cubic inches in 63 gallons?

76. A miller has 11 tons of flour valued at \$45 per ton; he adds to it 4 tons at the same value, and then sells 8 tons; what is the whole value of the part left?

77. I paid \$2160 for 16 horses; 4 of them being stolen, for what must I sell the rest apiece that nothing may be lost?

78. If 35 yards of cloth cost \$105, what cost 25 yards?

79. A merchant sold two pieces of cloth for \$296; one piece contained 32 yards, the other, 42 yards; what average price per yard did he receive?

80. James has 94 marbles, which are 2 less than 4 times as many as John has; how many has John?

81. Mary washes dishes for her mother 15 minutes every morning; if she receives 10 cents for an hour's work, how much money will she earn in 4 weeks?

82. John wished to know his father's and mother's ages; his father told him the product of their ages was 1755, and his mother's age was 39; how old was his father?

83. How many pounds of cheese at 15 cents a pound are worth 135 gallons of milk at 25 cents a gallon?

84. A certain school has 4 rooms, with an average of 65 scholars to a room; if 105 scholars are boys, how many girls are in the school?

85. A field has two of its sides 105 rods each, and the other two 108 rods each; how long is the fence surrounding the field? Draw a diagram of the field.

86. A tower is 148 feet high; how many steps, each 6 inches high, will it take to reach the top?

87. A certain quantity of barley lasts 11 horses 15 days; how long would it last 5 horses?

88. Bought 9 horses for \$1530, and sold them for \$1665; how much did I gain on each horse?

Make up 10 examples of your own like the above, work, and bring to the class for dictation.

FACTORS.

What are Factors? (See p. 35.)

Review Exercise 52. What is peculiar in the last 4 numbers of that exercise?

An integral exact divisor of a number is a **factor** of it.

A number that contains factors is a **composite number**.

A number that contains no factors is a **prime number**.

Factors which are themselves prime numbers are **prime factors**.

Pick out illustrations of each in Exercise 52.

To find the prime factors of a number.

Find the prime factors of 60.

OPERATION.

2	60	EXPLANATION.—Divide the given number by its smallest prime factor; the quotient by its smallest prime factor; and so on until the quotient is prime. The quotient and the several divisors are the prime factors.
2	30	
3	15	
	5	

Test by taking the product of the prime factors, which should be the number itself.

When the same number occurs in another several times as a factor, the number of times it occurs is shown by a small figure placed to the right and above it, and called the **power**. Thus, 2, as a factor, occurs twice in 60. Write it— 2^2 ; read it—*2 second power*.

The following hints will be found useful in factoring.

A number is divisible:

By 2 or 5, if its units figure is divisible by 2 or 5, respectively;

By 3, if the sum of its figures is divisible by 3.

All higher prime factors than these are usually found by trial division. Try the prime numbers as divisors in their

order upward, commencing with 2, until you reach one whose quotient is no larger than itself. If none of these are contained, the number to be factored is prime.

Composite numbers need not be used as divisors, since every composite number is made up of some smaller prime numbers than itself, which prime numbers you have already tried; and no number contains a composite number as a factor, unless it contains all the prime factors of that composite number. Thus, a number divisible by 6 is also divisible by 2 and 3, the factors of 6.

EXERCISE 83.

Make a list of prime numbers to 100, as follows:

1. Write all the numbers in order, from 2 to 100;
2. Since every second number after 2 is divisible by 2, cross it out;
3. Cross out every third number after 3, because divisible by 3;
4. Every fifth number after 5, because divisible by 5;
5. Lastly, every seventh number after 7, because divisible by 7.

Those not crossed are the prime numbers sought.

EXERCISE 84. (ORAL.)

Determine by inspection which of the following numbers are divisible by 2, 3, or 5:

- | | | | |
|--|--|---|--|
| 1. $\left\{ \begin{array}{l} 135 \\ 270 \end{array} \right\} a$ | 5. $\left\{ \begin{array}{l} 725 \\ 4350 \end{array} \right\} c$ | 9. $\left\{ \begin{array}{l} 975 \\ 555 \end{array} \right\} e$ | 13. $\left\{ \begin{array}{l} 246 \\ 438 \end{array} \right\} g$ |
| 2. $\left\{ \begin{array}{l} 207 \\ 1017 \end{array} \right\}$ | 6. $\left\{ \begin{array}{l} 87 \\ 203 \end{array} \right\}$ | 10. $\left\{ \begin{array}{l} 510 \\ 714 \end{array} \right\}$ | 14. $\left\{ \begin{array}{l} 720 \\ 256 \end{array} \right\}$ |
| 3. $\left\{ \begin{array}{l} 702 \\ 4706 \end{array} \right\} b$ | 7. $\left\{ \begin{array}{l} 750 \\ 129 \end{array} \right\} d$ | 11. $\left\{ \begin{array}{l} 378 \\ 1818 \end{array} \right\} f$ | 15. $\left\{ \begin{array}{l} 981 \\ 711 \end{array} \right\} h$ |
| 4. $\left\{ \begin{array}{l} 3003 \\ 11011 \end{array} \right\}$ | 8. $\left\{ \begin{array}{l} 597 \\ 237 \end{array} \right\}$ | 12. $\left\{ \begin{array}{l} 1011 \\ 1101 \end{array} \right\}$ | 16. $\left\{ \begin{array}{l} 846 \\ 329 \end{array} \right\}$ |
| 17. 279, 496. | | 18. 213, 284. | |

EXERCISE 85. (WRITTEN.)

Find the prime factors of all the numbers of Exercise 86; same in Exercise 84.

GREATEST COMMON FACTOR.

Pick out a factor that is found in all the numbers in each of the following sets:

- | | | |
|---------------|------------------|-------------------|
| 1. 16 and 20. | 3. 12 and 16. | 5. 6, 12, and 24. |
| 2. 25 and 20. | 4. 4, 8, and 12. | 6. 9, 12, and 18. |

Pick out the largest factor found in all the numbers of each of the preceding sets.

A factor contained in each of several numbers is called a **common factor** of those numbers.

The greatest factor common to several numbers is called their **greatest common factor (g. c. f.)**.

Name each in the above sets.

Numbers that have no common factors are *prime to each other*.

To find the g. c. f. of several numbers.

What is the greatest factor common to 30, 45, and 75?

OPERATION.

$$30 = 2 \times 3 \times 5$$

$$45$$

$$75 \quad 3 \times 5 = 15 = \text{g. c. f.}$$

$$\begin{array}{r} 3 \overline{) 30 - 45 - 75} \\ \underline{30} \\ 0 \end{array}$$

$$\begin{array}{r} 5 \overline{) 10 15 25} \\ \underline{10} \\ 0 \end{array}$$

$$\begin{array}{ccc} 2 & 3 & 5 \end{array}$$

$$3 \times 5 = 15 \text{ g. c. f.}$$

$$5 - A$$

EXPLANATION.—Separate the simplest number into its prime factors. Of these, reject such as are not contained in all the other numbers. The product of the remaining factors is the **g. c. f.** Or,

Divide by as many common prime factors as can be found; their product is the **g. c. f.**

EXERCISE 86. (WRITTEN.)

Find the *g. c. f.* of:

- | | | |
|---------------------|-----------------------|--------------------|
| 1. 24, 36, 42. | 13. 105, 140, 175. | 25. 55, 110. |
| 2. 33, 44, 77, 187. | 14. 99, 180, 252. | 26. 81, 120, 141. |
| 3. 120, 144, 216. | 15. 132, 154, 165. | 27. 78, 169, 130. |
| 4. 135, 180, 90. | 16. 60, 80, 100, 120. | 28. 150, 210, 330. |
| 5. 108, 45, 81. | 17. 864, 420, 600. | 29. 99, 132. |
| 6. 85, 95. | 18. 75, 105, 120. | 30. 120, 165. |
| 7. 72, 168. | 19. 108, 252. | 31. 120, 252. |
| 8. 119, 132. | 20. 39, 52, 65. | 32. 85, 102. |
| 9. 24, 33, 120. | 21. 84, 132. | 33. 42, 77, 91. |
| 10. 36, 44, 144. | 22. 168, 539. | 34. 34, 44. |
| 11. 105, 120, 135. | 23. 112, 147, 168. | 35. 28, 98. |
| 12. 144, 180. | 24. 287, 369. | 36. 110, 210. |

EXERCISE 87. (ORAL.)

Name the *g. c. f.* at sight:

- | | | |
|-----------------|-------------|-----------------|
| 1. 4, 6, 8. | 11. 15, 21. | 21. 56, 77. |
| 2. 9, 12, 15. | 12. 21, 28. | 22. 27, 45. |
| 3. 10, 15, 20. | 13. 16, 20. | 23. 35, 45, 48. |
| 4. 4, 8, 12. | 14. 40, 45. | 24. 32, 40, 56. |
| 5. 7, 14, 21. | 15. 36, 45. | 25. 72, 32. |
| 6. 9, 18, 27. | 16. 36, 54. | 26. 18, 32, 40. |
| 7. 10, 20, 30. | 17. 16, 32. | 27. 25, 50. |
| 8. 16, 24, 32. | 18. 63, 81. | 28. 30, 60. |
| 9. 3, 4, 5. | 19. 56, 64. | 29. 12, 32, 44. |
| 10. 18, 30, 36. | 20. 33, 44. | 30. 24, 27, 33. |

EXERCISE 88. (WRITTEN.)

Find the *g. c. f.* of the sets of numbers included under the same figure, Exercise 84.

Also of each set of numbers included under the same letter, Exercise 84.

Sometimes the prime factors are not easily recognized, nor found except by long trial. In such case the work may

be shortened by dividing the larger number by the smaller, and factoring the remainder; ascertaining whether any of its factors are common to the smaller number.

Thus;

Find the *g. c. f.* of 629 and 731.

$$\begin{array}{r} \text{OPERATION.} \\ 629)731(1 \\ \underline{629} \\ 102=2 \times 3 \times 17 \\ 17)629 \\ \underline{37} \end{array}$$

$\therefore 17 = \text{g. c. f.}$

$$\begin{array}{r} 37 \times 17 = 629 \\ 6 \times 17 = 102 \\ \underline{43 \times 17 = 731} \end{array}$$

A common factor of two numbers is also a factor of their difference.

EXPLANATION.—Multiplication is the successive additions of the same number a certain number of times. If two numbers have a common factor, each may be obtained by successive additions of that factor. Here 629 is obtained by adding 37 17's. Any number above 629 containing the factor 17 is obtained by adding a sufficient number of 17's to 629. But the sum of the 17's added to 629 to produce 731 is the same as the difference between 629 and 731. That is,

A common factor of two numbers is also a

EXERCISE 89. (WRITTEN.)

Find by the previous method the *g. c. f.* of:

- | | | |
|--------------|--------------|--------------|
| 1. 168, 539. | 3. 287, 369. | 5. 371, 636. |
| 2. 147, 168. | 4. 78, 169. | 6. 279, 961. |

Also, sets 2, 4, 6, 11, 15, and 16, Exercise 84.

MULTIPLES.

Name 2 numbers that have both 4 and 3 as factors; 7 and 2; 9, 6, and 3; 8, 4, and 6; 5 and 10.

Name the smallest number in each case that contains the given factors above.

A number which contains another as a factor is called a **multiple** of that factor.

A number which contains several others as factors is a **common multiple** of those factors; if the smallest number, the **least common multiple (l. c. m.)**.

Name an example of each in the above sets.

To find the l. c. m. of several numbers.

Find the l. c. m. of 9, 12, 15, and 20.

OPERATION.

$$\left. \begin{array}{l} 20 \\ 15 \\ 9 \\ 12 \end{array} \right\} 20 \times 3 \times 3 = 180$$

EXPLANATION.—Multiply the largest number by all the prime factors found in the other numbers, but not contained in this.

$$\begin{array}{r} 2 \overline{) 9-12-15-20} \\ 2 \overline{) 9-6-15-10} \\ 3 \overline{) 9-3-15-5} \\ 5 \overline{) 3-5-5} \\ 3 \end{array}$$

Divide by any prime factor common to two or more, and the quotients in the same way until prime to each other. The product of the divisors and final quotients is the l. c. m.

EXERCISE 90. (WRITTEN.)

Find the l. c. m. of:

- | | | |
|-------------------------|---------------------|-----------------|
| 1. 30, 45, 90. | 14. 12, 16, 20, 24. | 27. 18, 27, 36. |
| 2. 24, 36, 42. | 15. 28, 42, 35. | 28. 26, 39, 65. |
| 3. 4, 8, 10, 5. | 16. 50, 75, 125. | 29. 33, 44, 55. |
| 4. 5, 12, 15, 30. | 17. 9, 10, 12. | 30. 84, 96. |
| 5. 7, 12, 18, 24. | 18. 24, 30, 36, 40. | 31. 12, 13. |
| 6. 75, 100. | 19. 108, 132, 144. | 32. 13, 16. |
| 7. 20, 30, 40. | 20. 7, 11, 14, 21. | 33. 23, 25, 30. |
| 8. 33, 44, 21. | 21. 72, 84, 132. | 34. 9, 11. |
| 9. 105, 120. | 22. 75, 105, 120. | 35. 24, 26. |
| 10. 18, 27, 12. | 23. 30, 42, 126. | 36. 24, 25. |
| 11. 14, 21, 15. | 24. 120, 140, 210. | 37. 64, 84. |
| 12. 3, 4, 5, 6, 10. | 25. 15, 21, 35. | 38. 34, 36. |
| 13. 12, 18, 24, 36, 72. | 26. 38, 57, 95. | 39. 17, 18, 30. |

EXERCISE 91. (ORAL.)

Name at sight the l. c. m. of:

- | | | |
|----------------|-----------|---------------------------|
| 1. 2, 3, 4, 6. | 2. 2, 11. | 3. 2, 3, 4, 6, 8, 12, 16. |
|----------------|-----------|---------------------------|

- | | | |
|-----------------|-------------------|--------------------------------|
| 4. 6, 9, 18. | 13. 6, 8, 12, 24. | 22. 5, 10, 25, 50. |
| 5. 4, 5, 10. | 14. 5, 6, 10, 15. | 23. 2, 3, 4, 6, 9, 12, 18. |
| 6. 3, 5. | 15. 5, 7. | 24. 2, 3, 4, 5, 6, 10, 12, 15. |
| 7. 5, 8, 10. | 16. 4, 9, 6. | 25. 2, 3, 6, 7, 14. |
| 8. 7, 9. | 17. 5, 10, 20. | 26. 3, 5, 9, 15. |
| 9. 6, 8, 9, 12. | 18. 4, 5, 6. | 27. 2, 3, 6, 9, 18. |
| 10. 5, 10, 15. | 19. 5, 10, 12. | 28. 3, 6, 9, 12, 36. |
| 11. 3, 5, 15. | 20. 5, 11. | 29. 2, 3, 4, 6, 8, 9, 12. |
| 12. 4, 8, 16. | 21. 3, 7. | 30. 3, 7, 9. |

When, in finding the **l. c. m.** of two numbers, the numbers are not easily factored, it is well to find the **g. c. f.** by the division method, divide one of the numbers by it, and multiply the quotient by the other. Thus,

Find the **l. c. m.** of 119 and 187.

OPERATION.

$$119)187(1$$

$$\frac{119}{68=2 \times 2 \times 17}$$

$$17)119(7$$

$$7 \times 187 = 1309 \text{ l. c. m.}$$

EXERCISE 92. (WRITTEN.)

Find, by the preceding method, the **l. c. m.** of:

- | | | | |
|--------------|--------------|--------------|--------------|
| 1. 105, 189. | 3. 78, 169. | 5. 168, 539. | 7. 147, 168. |
| 2. 91, 169. | 4. 119, 132. | 6. 287, 369. | 8. 279, 124. |

Also, examples 11 to 20, Exercise 87, and numbers marked 2, 4, 6, 7, 11, 15, and 16, Exercise 84.

EXERCISE 93.

Write and perform 10 examples of your own in **g. c. f.** and 10 in **l. c. m.**, and bring to the class for dictation.

PRACTICAL WORK IN FACTORING.

- General Forms. { **E.—Find the largest factor common (g. c. f.) to 48 and 60.**
- { **F.—Find the smallest number that will contain (l. c. m. of) 12 and 16.**

EXERCISE 94.

Write out the following examples in proper *general form* before performing:

1. I have two rooms respectively 15 and 18 feet wide; what is the widest carpeting that will exactly fit the rooms?

2. A man wishes to fence a field having three sides respectively 120, 128, and 144 feet long; what is the length of the longest rail he can use, and not cut the rails?

3. Three men can walk 3, 4, and 5 miles an hour respectively; what is the length of the shortest journey they can walk, and each walk an exact number of hours?

4. A man has two lots of land 360 and 480 feet wide respectively, which he wishes to divide into house-lots of equal widths; what is the greatest width of house-lot he can make, and how many will there be?

5. Two boys travel around a race-track 80 rods in circumference, starting together, one making the circuit in 15 minutes, the other in 20; in what time will they be together again at the point of starting? How many rods will each have traveled?

6. What is the smallest tank that can be filled by using a 4-quart, 6-quart, 8-quart, or 10-quart measure?

7. A man sends to market 525 pounds of barley and 945 pounds of wheat in the largest sized bags he can use and have each contain the same number of pounds: how many pounds were there to a bag? What was each kind of grain worth at \$2 a bag?

8. I wish to spend the smallest like sum for pencils at 5, 6, 7, 8, and 10 cents each; find it.

9. A man spends equal sums in buying 2-cent, 3-cent, 5-cent, and 10-cent postage stamps, using the smallest sum possible; what did all the stamps come to?

10. Place 112 oranges and 140 lemons in piles, without mixing, so that each pile shall have the same number, and that the largest possible.

11. Draw from a basket of nuts 3 lots of equal numbers, the smallest possible, so as to arrange the 3 lots in piles of 7, 9, and 12 nuts, respectively.

12. A boy has the same number of marbles in each of 4 boxes; the first he arranges in piles of 3 each, the second in piles of 4 each, the third in piles of 5 each, and the fourth in piles of 6 each; on calculation he found he had the fewest marbles to a box that could be so arranged; how many marbles had he?

13. A dressmaker wishes to buy a piece of silk which she can cut into patterns of either 8, 10, or 12 yards; how large *must* the piece be?

14. Find the smallest number that can be divided by 12, 14, or 16, and leave 4 remainder.

15. Find the largest number that is contained in 47 and 77, with a remainder of 2.

16. What are the least equal sums a man can spend in buying sheep at \$3, calves at \$12, cows at \$30, and horses at \$75? How much for all?

17. A teacher distributes 56 cards to one class, 63 to another, and 77 to a third, giving the same number to each pupil; how many pupils, and how many cards to each?

18. Find the least number of cards that can be equally distributed among 7, 11, 14, or 22 girls.

19. A kind lady has 18 pears and 33 apples which she wishes to give to some poor children in equal numbers; how many can she give to each, and to how many can she give?

FRACTIONS.

How would you divide 5 apples equally among 2 boys?

If you divide 7 apples among 2 boys, how many would each receive? Among 3 boys? 8 apples among 3 boys?

$5 \div 2 = ?$ $5 \div 3 = ?$ $7 \div 2 = ?$ $7 \div 3 = ?$ $8 \div 3 = ?$

What does the form of expression $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, indicate?

A FRACTION is an indicated division. Thus, the indicated division of the remainder in Division is a *fraction*.

If you divide 1 apple equally among 3 boys, what part does each receive? If you divide 2 apples equally among 3 boys, what part of each apple does 1 boy receive? From both apples how many pieces does he receive?

Which would he prefer, one piece from each of the apples or two pieces from one apple?

Then $\frac{1}{3}$ of 2 apples is the same as $\frac{2}{3}$ of 1 apple.

A fraction, therefore, may be regarded as an equal part or as equal parts of a unit or one. When so regarded:

(1) The **denominator** (*namer*) names the parts and shows their size;

(2) The **numerator** (*numberer*) shows the number of parts.

Thus, $\frac{3}{8} = \frac{\text{three (number)}}{\text{eighths (name)}}$.

In the written expression is the denominator above or below the line? The numerator? To what term in division does each correspond? (See pages 43 and 44.)

The numerator and denominator are the **terms** of the fraction.

The value of a fraction is the quotient obtained by performing the indicated division.

A whole number, also called an **integer**, may be expressed in fractional form by using 1 for a denominator. Thus,

For $\frac{7}{1}$, $\frac{27}{1}$, read 7 1's, 27 1's.

A combination of a whole number and a fraction forms a **mixed number**; as $5\frac{2}{3}$, $117\frac{5}{19}$.

A quotient in division, with the remainder at the right over the divisor, is a *mixed number*.

When the dividend is equal to, or larger than, the divisor, the indicated division is an **improper fraction**; for the division may be performed and the result expressed as a whole or a mixed number.

Thus, $\frac{3}{2}$, $\frac{7}{3}$, $\frac{12}{4}$, $\frac{13}{13}$.

What is the quotient in each of these *improper* fractions? How do you prove the division? (See test, p. 50.)

EXERCISE 95. (WRITTEN.)

In the following examples write on your slate, in separate columns, the fractions (proper), the improper fractions, and the mixed numbers; change the improper fractions to mixed or whole numbers, and prove.

- | | | |
|--|---|---|
| 1. $\frac{7}{8}$, $4\frac{3}{5}$, $\frac{15}{4}$. | 8. $\frac{54}{33}$, $25\frac{1}{11}$, $\frac{153}{3}$. | 15. $29\frac{7}{8}$, $4\frac{9}{14}$, $\frac{27}{42}$. |
| 2. $\frac{119}{7}$, $\frac{7}{119}$, $5\frac{13}{17}$. | 9. $\frac{41}{164}$, $\frac{117}{10}$, $49\frac{3}{25}$. | 16. $\frac{11}{14}$, $\frac{17}{51}$, $\frac{29}{58}$. |
| 3. $72\frac{1}{4}$, $18\frac{5}{9}$, $\frac{17}{8}$. | 10. $19\frac{11}{50}$, $\frac{33}{77}$, $\frac{48}{75}$. | 17. $\frac{260}{11}$, $\frac{501}{29}$, $\frac{7}{11}$. |
| 4. $\frac{27}{36}$, $\frac{45}{22}$, $\frac{3}{8}$. | 11. $\frac{25}{80}$, $15\frac{3}{9}$, $12\frac{5}{16}$. | 18. $11\frac{5}{7}$, $\frac{28}{49}$, $21\frac{7}{16}$. |
| 5. $\frac{4}{8}$, $\frac{27}{8}$, $\frac{127}{5}$. | 12. $23\frac{5}{12}$, $\frac{187}{7}$, $\frac{209}{11}$. | 19. $\frac{130}{7}$, $\frac{58}{16}$, $7\frac{1}{56}$. |
| 6. $\frac{125}{120}$, $13\frac{13}{15}$, $\frac{180}{225}$. | 13. $\frac{401}{130}$, $\frac{375}{390}$, $\frac{209}{220}$. | 20. $\frac{133}{19}$, $\frac{18}{32}$, $13\frac{9}{12}$. |
| 7. $144\frac{2}{3}$, $\frac{125}{3}$, $\frac{112}{144}$. | 14. $23\frac{7}{9}$, $\frac{111}{3}$, $17\frac{5}{8}$. | 21. $41\frac{1}{2}$, $47\frac{3}{4}$, $\frac{171}{10}$. |

Select the first 10 proper fractions to analyze orally in the class. Thus,

$\frac{7}{8}$ is a fraction because _____; 8 is the denominator because _____; 7 is the numerator because _____.

EXERCISE 96. (ORAL.)

Form improper fractions on your slate by placing the dividends in the first 3 columns of dividends, Exercise 67, as numerators, and their corresponding divisors in the divisor column as denominators, and bring into the class for oral work in changing to mixed numbers.

EXERCISE 97. (ORAL.)

Change to improper fractions:

- | | | | |
|----------------------------------|------------------------------------|------------------------------------|--------------------------------------|
| 1. $4\frac{1}{5}, 7\frac{1}{2}.$ | 5. $3\frac{5}{8}, 1\frac{3}{4}.$ | 9. $8\frac{2}{5}, 7\frac{7}{10}.$ | 13. $9\frac{5}{12}, 8\frac{7}{8}.$ |
| 2. $3\frac{2}{3}, 5\frac{4}{5}.$ | 6. $5\frac{5}{9}, 11\frac{2}{3}.$ | 10. $12\frac{2}{3}, 7\frac{5}{6}.$ | 14. $7\frac{0}{11}, 7\frac{6}{7}.$ |
| 3. $9\frac{1}{8}, 9\frac{3}{8}.$ | 7. $9\frac{2}{5}, 7\frac{3}{8}.$ | 11. $4\frac{1}{2}, 5\frac{3}{4}.$ | 15. $12\frac{3}{6}, 10\frac{1}{10}.$ |
| 4. $7\frac{5}{6}, 8\frac{4}{7}.$ | 8. $5\frac{3}{10}, 9\frac{1}{11}.$ | 12. $6\frac{6}{7}, 7\frac{2}{3}.$ | 16. $15\frac{2}{3}, 5\frac{8}{9}.$ |

EXERCISE 98. (WRITTEN.)

Change to improper fractions:

- | | | |
|---------------------------------------|--------------------------------------|---------------------------------------|
| 1. $45\frac{11}{12}, 72\frac{9}{10}.$ | 5. $13\frac{9}{13}, 17\frac{5}{8}.$ | 9. $15\frac{3}{6}, 18\frac{5}{4}.$ |
| 2. $109\frac{7}{9}, 25\frac{7}{12}.$ | 6. $85\frac{8}{15}, 63\frac{7}{10}.$ | 10. $104\frac{1}{5}, 106\frac{5}{6}.$ |
| 3. $58\frac{2}{9}, 19\frac{1}{8}.$ | 7. $49\frac{4}{7}, 20\frac{13}{14}.$ | 11. $78\frac{7}{8}, 49\frac{1}{2}.$ |
| 4. $140\frac{1}{8}, 14\frac{5}{4}.$ | 8. $240\frac{1}{3}, 10\frac{7}{15}.$ | 12. $10\frac{9}{50}, 19\frac{7}{20}.$ |

Also all mixed numbers in Exercise 95, and prove.

EXERCISE 99.

Write ten mixed numbers of your own and reduce them to improper fractions. Also ten improper fractions and reduce them to mixed numbers.

To reduce to lower or higher terms.

If you cut each half of an apple into two equal parts, what kind of pieces do you get? How many 4ths in each half? Cut each fourth into 2 equal parts, and you get what kind of pieces? How many in each fourth? How many in $\frac{2}{4}$? Cut each half into 3 equal pieces, and you get how many pieces in all? Name of each piece? How many in $\frac{1}{3}$? Cut each third into 2 equal parts, and you get how many pieces in all? How many in each third? How many in $\frac{2}{3}$? Write these results in a row on your slates; thus,

$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}, \quad \frac{1}{2} = \frac{3}{6}, \quad \frac{1}{3} = \frac{2}{6}, \quad \frac{2}{3} = \frac{4}{6}.$$

Also, in a second row under these, write each expression backward; thus,

$$\frac{4}{8} = \frac{2}{4} = \frac{1}{2}, \quad \frac{3}{6} = \frac{1}{2}, \quad \frac{2}{6} = \frac{1}{3}, \quad \frac{4}{6} = \frac{2}{3}.$$

In the upper row, what do you do, in each case, with the numerators and denominators of the fractions on the left of the sign ($=$) to get those on the right? What in the lower row? Which is the larger, $\frac{1}{2}$ or $\frac{2}{4}$? $\frac{1}{2}$ or $\frac{3}{6}$? $\frac{1}{3}$ or $\frac{2}{6}$? $\frac{2}{3}$ or $\frac{4}{6}$?

Multiplying both numerator and denominator of a fraction by the same number is reducing the fraction to **higher terms**, as in the upper row above.

Dividing both numerator and denominator of a fraction by the same number, is reducing the fraction to **lower terms**.

If both numerator and denominator are divided by all their common factors, or their **g. c. f.**, the fraction is reduced to its **lowest terms**.

If there is no common factor, the fraction is already in its *lowest terms*.

Either of the above processes does or does not change the value of the fraction, and why? (See Prin. 3, p. 56.)

EXERCISE 100. (ORAL.)

Reduce to lowest terms:

- | | | | |
|--|--|---|---|
| 1. $\frac{8}{12}, \frac{7}{14}, \frac{6}{10}$. | 5. $\frac{6}{9}, \frac{4}{18}, \frac{12}{48}$. | 9. $\frac{12}{36}, \frac{16}{32}, \frac{28}{32}$. | 13. $\frac{36}{42}, \frac{36}{45}, \frac{33}{55}$. |
| 2. $\frac{9}{15}, \frac{14}{21}, \frac{18}{24}$. | 6. $\frac{18}{30}, \frac{15}{30}, \frac{20}{30}$. | 10. $\frac{16}{24}, \frac{16}{20}, \frac{18}{20}$. | 14. $\frac{32}{44}, \frac{36}{54}, \frac{5}{10}$. |
| 3. $\frac{18}{27}, \frac{20}{32}, \frac{18}{36}$. | 7. $\frac{30}{36}, \frac{24}{30}, \frac{24}{36}$. | 11. $\frac{8}{18}, \frac{20}{60}, \frac{5}{60}$. | 15. $\frac{16}{48}, \frac{30}{60}, \frac{45}{54}$. |
| 4. $\frac{5}{15}, \frac{8}{48}, \frac{6}{48}$. | 8. $\frac{15}{35}, \frac{42}{49}, \frac{21}{28}$. | 12. $\frac{10}{35}, \frac{7}{35}, \frac{14}{35}$. | 16. $\frac{15}{45}, \frac{12}{60}, \frac{10}{40}$. |

EXERCISE 101. (WRITTEN.)

Form examples by writing the smaller number in each numbered couplet of Exercise 84 for a numerator, and the larger for a denominator, and reduce to lowest terms.

In Exercise 86, select the smallest and largest numbers of each example for numerator and denominator, respectively, and reduce to lowest terms.

Also, reduce to lowest terms the proper fractions in Exercise 95.

EXERCISE 102.

Write and perform 10 examples of your own like those of the preceding exercise, and bring to the class for dictation.

To reduce to a common denominator.

By what number must you multiply both numerator and denominator of $\frac{1}{2}$ to change it to 6ths? To 8ths? To 10ths? To 12ths?

$\frac{2}{3}$ to change it to 6ths? To 9ths? To 12ths? To 15ths?

$\frac{3}{4}$ to change it to 8ths? To 12ths? To 16ths? To 20ths?

$\frac{2}{5}$ to change it to 10ths? To 15ths? To 20ths?

Give results in each case.

Of the above fractions, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{5}$, which have you changed to 6ths? Write on your slate thus:

$$\frac{1}{2} = \frac{3}{6}$$

$$\frac{2}{3} = \frac{4}{6}$$

Of the denominators 2 and 3, what is 6?

Which have you changed to 8ths? To 10ths? To 12ths? Of the denominators 2, 3, and 4, what is 12?

Changing fractions of different denominators to equivalent fractions having the same denominator is **reducing to a common denominator**.

When the new denominator is the smallest number (l. c. m.) that can be used, it is the **least common denominator**.

EXERCISE 103.

Write the fractions of each example, Exercise 100, in lowest terms, numbering the examples as in the exercise.

Do the same with the fractional parts of the examples in Exercises 97 and 98. Bring into the class for oral work in reducing to least common denominator.

Name, at sight, several common denominators for each example, and decide which is the least common denominator.

EXERCISE 104. (WRITTEN.)

Change the fractions of each example, Exercise 95, to equivalent fractions having their least common denominator.

EXERCISE 105.

Write 10 examples of your own, each containing 3 fractions in their lowest terms. Bring to the class for dictation.

ADDITION AND SUBTRACTION.

What kinds of objects can be put together, or added? What subtracted?

Fractions of the same name (*denominator*) may be added and subtracted. Thus,

5 apples and 3 apples are how many apples?

5 sevenths and 3 sevenths are how many sevenths?

Express the latter in the written form; thus, $\frac{5}{7} + \frac{3}{7} = \frac{8}{7} = 1\frac{1}{7}$.

EXERCISE 106. (ORAL.)

The fractions in the answers must be proper, and in lowest terms:

1. $\frac{5}{8} + \frac{7}{8} - \frac{3}{8} - \frac{2}{8} = ?$
2. $\frac{6}{13} + \frac{11}{13} + \frac{8}{13} + \frac{1}{13} = ?$
3. $\frac{7}{30} - \frac{1}{30} + \frac{11}{30} - \frac{7}{30} = ?$
4. $\frac{19}{125} + \frac{6}{125} - \frac{24}{125} + \frac{4}{125} = ?$
5. $\frac{21}{67} + \frac{19}{67} + \frac{30}{67} - \frac{2}{67} = ?$
6. $\frac{8}{9} + \frac{5}{9} - \frac{4}{9} - \frac{3}{9} = ?$
7. $\frac{16}{17} - \frac{5}{17} + \frac{4}{17} - \frac{8}{17} = ?$
8. $\frac{3}{41} + \frac{7}{41} - \frac{9}{41} + \frac{40}{41} = ?$
9. $\frac{21}{23} - \frac{10}{23} - \frac{5}{23} - \frac{3}{23} = ?$
10. $\frac{7}{160} + \frac{7}{160} - \frac{4}{160} + \frac{10}{160} = ?$
11. $\frac{43}{88} - \frac{40}{88} + \frac{5}{88} + \frac{3}{88} = ?$
12. $\frac{14}{15} + \frac{5}{15} + \frac{7}{15} + \frac{9}{15} = ?$
13. $\frac{8}{19} + \frac{11}{19} - \frac{3}{19} + \frac{10}{19} = ?$
14. $\frac{20}{21} + \frac{20}{21} - \frac{2}{21} - \frac{2}{21} = ?$
15. $\frac{17}{18} + \frac{17}{18} + \frac{17}{18} - \frac{6}{18} = ?$

We have seen that fractions to be added or subtracted must have the same name, or denominator. What did you

find could be done to fractions of different names on page 76? Then to add or subtract such fractions what must first be done?

EXERCISE 107.

In Exercise 100, write the fractions of each example in lowest terms and add. Perform mentally as far as possible, combining first those most easily reduced. Thus, in

$$\text{Example 1. } \frac{2}{3} + \frac{1}{2} = \frac{7}{6} = 1\frac{1}{6}. \quad 1\frac{1}{6} + \frac{2}{5} = 1\frac{23}{30}.$$

Also find the difference between the third fraction and the sum of the first and second in each example.

If any or all of the numbers are mixed numbers, add or subtract the fractional parts first. In adding, reduce sums, if improper fractions, to whole or mixed numbers and add the whole part to the sum of the whole numbers.

In subtraction, if the fraction in the subtrahend is larger than the fraction in the minuend, take one from the whole number of the minuend and add it to the fraction of the minuend, making an improper fraction; then subtract.

EXERCISE 108. (ORAL.)

Give the sum and the difference of the mixed numbers in each example of Exercises 97 and 98.

EXERCISE 109. (WRITTEN.)

Add the expressions in each example, Exercise 95.

EXERCISE 110. (ORAL.)

Subtract $\frac{5}{11}$ from each of the following; also $3\frac{2}{3}$:

10 23 41 36 84 93 102 170

Subtract $4\frac{3}{4}$ from each of the following; also $10\frac{1}{2}$:

$20\frac{1}{4}$ $32\frac{1}{2}$ $44\frac{3}{8}$ $55\frac{1}{2}$ $61\frac{1}{3}$ $78\frac{1}{8}$ $93\frac{2}{3}$

EXERCISE 111. (WRITTEN.)

Subtract $\frac{1}{3}$ from each mixed number of Exercise 98.

PRACTICAL WORK IN ADDITION AND SUBTRACTION.

EXERCISE 112. (ORAL.)

1. Lucy gives away $\frac{1}{2}$ of her apples to Sarah and $\frac{1}{4}$ to Jane; what part has she left?
2. I cut 3 pieces of cloth from 3 yards; the first contained $\frac{2}{3}$ of a yard, the second $\frac{1}{2}$, and the third $\frac{3}{4}$; what part of the 3 yards was still left?
3. John gave away $\frac{1}{3}$ and $\frac{1}{5}$ of his marbles, respectively, to two playmates; what did he give to both?
4. From $\frac{2}{3}$ of a yard of ribbon I cut $\frac{5}{12}$ of a yard; what length of piece remained?
5. Mr. A. buys $2\frac{5}{16}$ acres of land, and fences $1\frac{1}{4}$ acres; what remains unfenced?
6. Frank received on Christmas $\$2\frac{1}{5}$ from his father, $\$1\frac{3}{4}$ from his mother, and $\$\frac{1}{2}$ from his sister; what did he receive in all?
7. How much more from his father than from his mother?
8. Than from his sister?
9. Than from both mother and sister?
10. He spent $\$\frac{3}{10}$ for marbles and $\$\frac{1}{20}$ for a top; what had he left?
11. George lives $2\frac{7}{8}$ miles from the school-house; how many miles does he have to go a day, going and coming once?
12. William jumped $8\frac{2}{3}$ feet, and John $7\frac{1}{8}$ feet; how much farther did William jump than John?
13. Charles earns $\$1\frac{1}{4}$ Monday, $\$1\frac{7}{10}$ Tuesday, $\$1\frac{2}{5}$ Wednesday, $\$1\frac{3}{10}$ Thursday, $\$1\frac{1}{2}$ Friday, and $\$1\frac{2}{5}$ Saturday; he spends $\$3\frac{3}{4}$ during the week; how much does he save?
14. I have 3 pieces of carpeting containing $9\frac{2}{3}$, $7\frac{1}{4}$, and $8\frac{5}{9}$ yards, respectively; what is the length of the three pieces sewed together, allowing $\frac{1}{6}$ yard for laps?
15. I have a journey of $13\frac{5}{16}$ miles to go; after walking $3\frac{5}{8}$ miles, how much farther have I to go?

16. A farmer has $4\frac{1}{4}$ acres in orange trees and $2\frac{3}{8}$ acres in lemon trees; how much has he in both?

17. How much more in one than in the other?

18. I read $3\frac{4}{5}$ hours on Monday, $2\frac{3}{10}$ hours on Tuesday, and $3\frac{1}{6}$ hours on Wednesday; how many hours did I read in three days?

19. How much longer on Monday than on Tuesday or Wednesday?

EXERCISE 113.

Form 5 examples, like the preceding, in addition, and 5 in subtraction, using the mixed numbers in the first 10 examples, Exercise 97, and bring to the class for dictation.

EXERCISE 114. (WRITTEN.)

Rewrite the first 10 examples in *general form* (A or B, page 28), before performing.

1. I paid $\$41\frac{1}{4}$ for a watch, $\$3\frac{1}{2}$ for a chain, and $\$\frac{3}{4}$ for a ring; what was my bill?

2. I raised on my land last year $155\frac{7}{8}$ centals of wheat, $76\frac{3}{4}$ centals of oats, and $111\frac{4}{5}$ centals of barley; how many centals of grain did I raise?

3. From a piece of land containing $723\frac{7}{10}$ acres, I sold $149\frac{9}{20}$ acres; what had I left?

4. Sold a horse for $\$125$, which was $\$13\frac{3}{4}$ more than he cost me; what did he cost me?

5. I start on a journey, going $\frac{2}{11}$ of the distance the first day, and $\frac{2}{9}$ the second day; what part of the journey have I yet to travel?

6. In a certain school $\frac{43}{110}$ of the pupils are boys; what part are girls?

7. A three-sided field has its sides $31\frac{3}{11}$, $46\frac{2}{3}$, and $59\frac{9}{8}$ rods long, respectively; how far is it around the field? Draw a diagram to illustrate.

8. A merchant buys a barrel of sugar containing $237\frac{11}{16}$ pounds; he sells $17\frac{1}{2}$ pounds at one time, $23\frac{5}{8}$ at another,

and $41\frac{3}{4}$ at a third; how many pounds of sugar are still in the barrel?

9. A man traveled $8\frac{7}{5}$ hours on Monday, $9\frac{1}{3}$ on Tuesday, $11\frac{1}{2}$ on Wednesday, $8\frac{7}{6}$ on Thursday, $13\frac{1}{6}$ on Friday, and $13\frac{5}{6}$ on Saturday; how many hours did he travel in all?

10. He traveled $27\frac{4}{5}$ miles on Monday, $34\frac{3}{4}$ on Tuesday, $31\frac{5}{2}$ miles on Wednesday, and 179 $\frac{3}{4}$ miles in all; how far did he travel the last three days?

11. From a piece of cloth containing $108\frac{1}{4}$ yards, 3 pieces of $17\frac{2}{3}$, $18\frac{1}{9}$, and $14\frac{7}{8}$ yards, respectively, were cut; what remained?

12. A church steeple reaches $101\frac{7}{2}$ feet from the ground; the roof of the church is $53\frac{1}{3}$ feet high; how high does the steeple rise above the roof?

13. Two men travel around a pond; the first goes $\frac{1}{7}$ of the distance, and the second $\frac{2}{13}$ of the distance, in one hour; how much of the distance has one gained upon the other?

14. 4 piles of wood contain $37\frac{3}{6}$, $41\frac{7}{8}$, $29\frac{1}{2}\frac{5}{8}$, and $54\frac{1}{3}\frac{9}{2}$ cords, respectively; how many cords are in the 4 piles?

15. One of two stations is $171\frac{7}{5}$ miles east of a certain point, and the other is $235\frac{5}{11}$ miles west of the same point; how far apart are they?

16. How far apart if both stations are east of the point?

17. I bought of one man $119\frac{4}{5}$ acres of land, of another $91\frac{5}{2}$ acres, and of a third $75\frac{1}{6}$ acres; what amount did I buy in all?

18. A man had $\frac{1}{2}$ of his sheep in one pasture, $\frac{1}{5}$ in another, and $\frac{3}{7}$ in a third, and the remainder in a fourth; what part are in a fourth?

19. A room is $17\frac{5}{2}$ feet long, and $14\frac{1}{4}$ feet wide; how far is it round the room? Draw diagram.

20. Sold a horse for $\$117\frac{7}{10}$ at a loss of $\$7\frac{1}{5}$; what did I pay for him?

21. A ship was $53\frac{1}{5}$ hours in sailing to a certain port, and $41\frac{3}{5}$ in returning; how long was she gone?

To multiply a fraction by a whole number, or the reverse.

How many apples are 5×2 apples? 2 apples multiplied by 5? What are 5×2 thirds? 2 thirds times 5? Write this latter operation in proper form. Does it make a difference in the product in multiplication as to which term stands first?

EXERCISE 115. (ORAL.)

Leave no improper fractions for answers.

- | | | | |
|--------------------------------|---------------------------------|--------------------------------|--------------------------------|
| 1. $9 \times \frac{3}{7}$. | 9. $\frac{11}{12} \times 11$. | 17. $\frac{2}{5} \times 21$. | 25. $31 \times \frac{2}{5}$. |
| 2. $7 \times \frac{3}{4}$. | 10. $\frac{12}{13} \times 12$. | 18. $\frac{1}{3} \times 25$. | 26. $25 \times \frac{4}{11}$. |
| 3. $\frac{2}{3} \times 5$. | 11. $9 \times \frac{9}{10}$. | 19. $\frac{2}{3} \times 29$. | 27. $\frac{8}{9} \times 8$. |
| 4. $10 \times \frac{7}{11}$. | 12. $7 \times \frac{13}{15}$. | 20. $\frac{3}{5} \times 18$. | 28. $\frac{3}{10} \times 21$. |
| 5. $13 \times \frac{3}{8}$. | 13. $\frac{2}{11} \times 18$. | 21. $\frac{5}{7} \times 20$. | 29. $\frac{2}{7} \times 40$. |
| 6. $15 \times \frac{4}{11}$. | 14. $13 \times \frac{1}{2}$. | 22. $19 \times \frac{2}{11}$. | 30. $30 \times \frac{4}{11}$. |
| 7. $\frac{5}{13} \times 100$. | 15. $17 \times \frac{3}{4}$. | 23. $17 \times \frac{3}{11}$. | 31. $50 \times \frac{2}{7}$. |
| 8. $\frac{7}{10} \times 9$. | 16. $11 \times \frac{3}{7}$. | 24. $\frac{4}{9} \times 11$. | 32. $\frac{1}{9} \times 91$. |

EXERCISE 116. (WRITTEN.)

Multiply each mixed number, of examples 1 to 4, Exercise 98, by 11. Multiply the fractional part first, and add the product to the product of the whole part.

Multiply examples 5 to 8, by 13; 9 to 12, by 17.

Mixed numbers may be changed to improper fractions before multiplying, when preferable.

EXERCISE 117. (ORAL.)

Multiply the mixed numbers of examples 1 to 6, Exercise 97, by 3; 7 to 12, by 5; 13 to 16, by 7.

Multiplying the numerator by a whole number multiplies the fraction by that number. Why? (See Prin. 1, p. 56, Division.) What other operation does that Prin. say will multiply the fraction?

Which term of the fraction is the divisor?

In what two ways, then, can you multiply a fraction by a whole number?

EXERCISE 118. (ORAL.)

In the following, divide the denominator by the whole number:

1. $9 \times \frac{4}{27}$.

6. $9 \times \frac{79}{81}$.

11. $\frac{27}{8} \times 7$.

2. $7 \times \frac{11}{14}$.

7. $\frac{7}{15} \times 5$.

12. $20 \times \frac{19}{6}$.

3. $15 \times \frac{29}{36}$.

8. $\frac{15}{16} \times 8$.

13. $\frac{41}{2} \times 6$.

4. $\frac{19}{100} \times 25$.

9. $11 \times \frac{21}{2}$.

14. $\frac{197}{110} \times 11$.

5. $7 \times \frac{39}{42}$.

10. $5 \times \frac{49}{50}$.

15. $\frac{72}{5} \times 25$.

EXERCISE 119. (WRITTEN ANALYSIS.)

See analysis, page 36, Multiplication.

- At $\$7\frac{3}{4}$ a cord what will 25 cords of wood cost?
- Sold 19 yards of cloth at $\$2\frac{3}{5}$ a yard; find the whole selling price.
- What cost 160 acres of land at $\$65\frac{1}{2}$?
- A barrel contains $31\frac{1}{2}$ gallons; how many gallons in 12 barrels?
- What will 175 centals of wheat cost at $\$1\frac{1}{10}$ a cental?
- How far can I walk in 11 hours, at the rate of $3\frac{1}{3}\frac{1}{2}$ miles an hour?
- If a certain number of shoes can be sewed in $18\frac{7}{15}$ hours on 12 machines, how long will it take, using 1 machine?
- If I can copy $12\frac{1}{3}\frac{2}{3}$ pages in one day, how many pages can I copy in 6 days?
- 12 men buy a mill together, each paying $\$728\frac{5}{6}$; what is the cost of the mill?
- One rod contains $5\frac{1}{2}$ yards; how many yards are in 80 rods?

EXERCISE 120.

Write 10 examples of your own, like the preceding exercise, using 5 examples each from Exercises 115 and 118.

EXERCISE 121. (ORAL ANALYSIS.)

1. At $\$ \frac{2}{5}$ a roll what will 10 rolls of butter cost?
2. What cost 9 yards of cloth at $\$ \frac{3}{4}$ per yard?
3. What cost 5 rings at $\$ 2 \frac{1}{4}$ apiece?
4. How many centals of grain will 12 bags hold, they averaging $1 \frac{1}{3}$ centals to a bag?
5. What does a man earn in a week, at $\$ 1 \frac{3}{4}$ a day?
6. If $1 \frac{1}{2}$ pounds of butter pay for a yard of cloth, how many pounds of butter will it take to pay for 16 yards of cloth?
7. What cost a dozen oranges at $2 \frac{1}{3}$ cents each?
8. I gave 7 boys $\$ \frac{3}{5}$ each; how much to all?
9. A wheel turns $2 \frac{1}{6}$ times in going a rod; how many times will it turn in going 18 rods?
10. Find the price of 13 sacks of wheat at $\$ 1 \frac{3}{4}$ each.

Perform the work of the 3 upper rows in Exercise 67, reading $\frac{1}{3}$ of 15, $\frac{1}{3}$ of 29, etc.

$\frac{1}{3}$ of 29 = ? $9 \frac{2}{3}$ are how many thirds? $\frac{1}{3}$ of 29 = $2 \frac{2}{3}$.

$\frac{1}{7}$ of 44 = ? $6 \frac{2}{7}$ are how many sevenths? $\frac{1}{7}$ of 44 = $4 \frac{4}{7}$.

$\frac{2}{3}$ are how many times $\frac{1}{3}$? If $\frac{1}{3}$ of 15 is 5, $\frac{2}{3}$ of 15 is what? If $\frac{1}{3}$ of 29 is $2 \frac{2}{3}$, $\frac{2}{3}$ of 29 = what? $\frac{3}{7}$ of 44 = ?

Write these on your slate, in a row; thus,

$$\frac{2}{3} \text{ of } 15 = 10, \quad \frac{2}{3} \text{ of } 29 = 5 \frac{8}{3}, \quad \frac{3}{7} \text{ of } 44 = 1 \frac{32}{7}.$$

Below these write in a row, with results:

$$\frac{2}{3} \times 15 = \quad , \quad \frac{2}{3} \times 29 = \quad , \quad \frac{3}{7} \times 44 = \quad .$$

Compare answers. "Of," between fractions, is therefore equivalent to what sign?

EXERCISE 122. (WRITTEN.)

Write and find $\frac{2}{3}$ of each dividend in the upper row, Exercise 67. In cases similar to the first, write thus,

$$\begin{array}{c} 5 \\ \frac{2}{3} \text{ of } 15 = 10. \end{array}$$

$\frac{3}{7}$ of numbers in the second row; $\frac{5}{9}$ in the third; $\frac{5}{6}$ in the fourth.

EXERCISE 123. (ORAL ANALYSIS.)

If 1 dozen eggs cost 30 cents, what will $\frac{2}{3}$ of a dozen cost?

MODEL.— $\frac{1}{3}$ dozen will cost $\frac{1}{3}$ of 30 cents = 10 cents. $\frac{2}{3}$ dozen will cost $2 \times 10 = 20$ cents.

1. What cost $\frac{3}{4}$ yard of cloth at 20 cents a yard?
2. Bought $\frac{7}{8}$ of a cord of wood at \$8 a cord; what was my bill?
3. Sold $\frac{7}{16}$ of an acre of land at \$48 an acre; what did it cost?
4. Sold $\frac{7}{10}$ of a ton of hay at \$8 a ton; what did I receive?
5. From a piece of cloth containing 55 yards, $\frac{5}{11}$ of it was cut; how many yards were cut?
6. Of 63 children in a certain district, $\frac{4}{9}$ attend school; how many attend?
7. Bought $\frac{5}{9}$ of a yard of ribbon at 20 cents a yard; what did the ribbon cost me?
8. A row of trees contains 28 trees; how many trees are in $\frac{1}{4}$ the row?
9. In a certain school containing 56 scholars, $\frac{5}{8}$ are in the first grade and $\frac{3}{8}$ in the second; how many in each?
10. Received \$2 for a day's work of 10 hours; what did I receive per hour?
11. If $\frac{2}{5}$ of 10 questions are missed, how many are missed?

To multiply a fraction by a fraction.

The upper lines divide the whole line into how many parts? The lower lines? Into how many parts do the lower lines divide each fifth? Then $\frac{1}{3}$ of $\frac{1}{5} = \frac{1}{15}$; or (of = \times) $\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$.

$\frac{1}{3}$ of $\frac{2}{5} = ?$ $\frac{2}{3}$ of $\frac{1}{5} = ?$ $\frac{2}{3}$ of $\frac{2}{5} = ?$ $\frac{1}{3}$ of $\frac{4}{5} = ?$ $\frac{2}{3}$ of $\frac{4}{5} = ?$

Use the sign (\times), writing both ways. How can you obtain your new numerator in each case from those of the multiplicand and multiplier? The new denominator?

EXERCISE 124. (ORAL.)

- | | | | |
|--|--|---|---|
| 1. $\frac{3}{7} \times \frac{2}{5}$. | 9. $\frac{3}{20}$ of $\frac{9}{10}$. | 17. $\frac{3}{5} \times \frac{3}{10}$. | 25. $\frac{2}{9}$ of $\frac{2}{5}$. |
| 2. $\frac{5}{6} \times \frac{5}{6}$. | 10. $\frac{7}{30}$ of $\frac{4}{5}$. | 18. $\frac{2}{5}$ of $\frac{2}{9}$. | 26. $\frac{2}{5}$ of $\frac{2}{9}$. |
| 3. $\frac{3}{4}$ of $\frac{5}{11}$. | 11. $\frac{3}{4}$ of $\frac{6}{7}$. | 19. $\frac{5}{6} \times \frac{5}{12}$. | 27. $\frac{3}{20} \times \frac{3}{5}$. |
| 4. $\frac{7}{10}$ of $\frac{7}{8}$. | 12. $\frac{10}{11}$ of $\frac{10}{11}$. | 20. $\frac{4}{7} \times \frac{5}{14}$. | 28. $\frac{3}{5} \times \frac{3}{20}$. |
| 5. $\frac{5}{8} \times \frac{7}{16}$. | 13. $\frac{5}{7} \times \frac{5}{9}$. | 21. $\frac{7}{9} \times \frac{7}{3}$. | 29. $\frac{5}{7}$ of $\frac{5}{14}$. |
| 6. $\frac{3}{11}$ of $\frac{6}{7}$. | 14. $\frac{8}{11} \times \frac{8}{11}$. | 22. $\frac{8}{9}$ of $\frac{8}{11}$. | 30. $\frac{5}{14}$ of $\frac{5}{7}$. |
| 7. $\frac{5}{13}$ of $\frac{10}{11}$. | 15. $\frac{15}{16} \times \frac{1}{2}$. | 23. $\frac{4}{3} \times \frac{2}{3}$. | 31. $\frac{3}{8} \times \frac{3}{8}$. |
| 8. $\frac{2}{3}$ of $\frac{8}{11}$. | 16. $\frac{2}{3}$ of $\frac{7}{9}$. | 24. $\frac{8}{11}$ of $\frac{8}{9}$. | 32. $\frac{9}{13} \times \frac{1}{3}$. |

What is $\frac{1}{3}$ of 9 apples? $\frac{2}{3}$ of 9 apples?

What is $\frac{1}{3}$ of 9 tenths? $\frac{2}{3}$ of 9 tenths? Write the last two expressions in full.

$$\frac{1}{3} \text{ of } \frac{9}{10} = \frac{3}{10}. \quad (\text{See Exercise 122.})$$

$$\frac{2}{3} \text{ of } \frac{9}{10} \text{ are } 2 \times \frac{3}{10} = \frac{3}{5}. \quad (\text{See Exercise 115.})$$

Hence, in writing the work, shorten; thus, $\frac{2}{3}$ of $\frac{9}{10} = \frac{3}{5}$.

This method of work is called **cancellation**.

A common factor of any numerator and denominator, in multiplying, may be canceled by Prin. 3, p. 56. Thus,

$$\frac{2}{3} \times \frac{5}{7} = \frac{10}{21}; \quad \text{dividing 4 and 14 by 2, and 9 and 15 by 3, before multiplying.}$$

What is Prin. 3, referred to?

EXERCISE 125. (WRITTEN.)

- | | | |
|---|---|--|
| 1. $\frac{55}{56} \times \frac{7}{11}$. | 7. $\frac{85}{108}$ of $\frac{81}{95}$. | 13. $\frac{32}{75} \times \frac{25}{48}$. |
| 2. $\frac{13}{25} \times \frac{5}{39}$. | 8. $\frac{84}{119}$ of $\frac{17}{24}$. | 14. $\frac{325}{7} \times \frac{1}{25}$. |
| 3. $\frac{14}{15}$ of $\frac{25}{28}$. | 9. $\frac{55}{81} \times \frac{141}{143}$. | 15. $\frac{3}{160} \times \frac{32}{9}$. |
| 4. $\frac{11}{16} \times \frac{7}{8}$. | 10. $\frac{39}{60}$ of $\frac{40}{65}$. | 16. $\frac{7}{64} \times \frac{24}{35}$. |
| 5. $\frac{57}{108} \times \frac{12}{19}$. | 11. $\frac{21}{26} \times \frac{13}{14}$. | 17. $\frac{150}{151} \times \frac{3}{5}$. |
| 6. $\frac{75}{105} \times \frac{84}{120}$. | 12. $\frac{19}{20}$ of $\frac{13}{20}$. | 18. $\frac{19}{38} \times \frac{7}{14}$. |

EXERCISE 126. (WRITTEN.)

Multiply together the mixed numbers of each example in Exercises 97 and 98.

EXERCISE 127. (ORAL.)

Write on your slates, in their lowest terms, with "of" or sign (\times) between, the fractions of each example in Exercise 100. Bring to the class to multiply.

EXERCISE 128. (WRITTEN ANALYSIS.)

1. What cost $18\frac{3}{4}$ yards of carpet at $\$2\frac{7}{10}$ a yard?
2. How far will a railroad train go in $21\frac{2}{3}$ hours at the rate of $19\frac{2}{3}$ miles an hour?
3. What is the value of a pile of wood containing $45\frac{5}{16}$ cords, at $\$13\frac{3}{4}$ a cord?
4. At $\$1\frac{1}{4}$ a pound what will $9\frac{3}{4}$ pounds of coffee cost?
5. Find the price of $21\frac{1}{2}$ yards of ribbon at $12\frac{1}{2}$ cents.
6. What is the weight of $11\frac{2}{3}$ barrels of flour averaging $197\frac{2}{3}$ pounds each?
7. At $\$4\frac{3}{4}$ a barrel what will $8\frac{1}{2}$ barrels of flour cost?
8. What is the cost of building a fence $29\frac{7}{8}$ rods long at $\$2\frac{1}{4}$ a rod?
9. A certain river flows $7\frac{7}{16}$ miles an hour; how far will a boat float on it in $20\frac{1}{7}$ hours?
10. A wind blowing $27\frac{4}{5}$ miles an hour blows how far in $9\frac{1}{2}$ hours?

EXERCISE 129.

Write 10 examples similar to the preceding, using the last 10 examples of Exercise 97.

To divide a fraction or a mixed number by a whole number.

Write the following with answers on your slate:

- | | |
|---------------------------|---|
| 9 apples $\div 3 =$ | 18 dollars $\div 6 =$ |
| 9 tenths $\div 3 =$ | 18 twenty-fifths $\div 6 =$ |
| 12 thirteenths $\div 3 =$ | $(2\frac{8}{11})$ 30 elevenths $\div 6 =$ |

$$25 \text{ cents} \div 5 =$$

$$(3\frac{1}{8}) 25 \text{ eighths} \div 5 =$$

$$(2\frac{6}{7}) 20 \text{ sevenths} \div 5 =$$

Dividing anything by 2 is taking what part of it? By 4? By 7? Taking $\frac{1}{2}$ of anything is the same as multiplying by what? Then dividing by 2 is the same as multiplying by what? By 4?

Write on your slate the following:

$$\frac{3}{7} \quad \frac{5}{9} \quad \frac{6}{13} \quad \frac{8}{19} \quad \frac{11}{5} \quad \frac{14}{15} \quad \frac{16}{17} \quad \frac{9}{2}$$

Divide each by 2 by taking $\frac{1}{2}$ of it; by 4 by taking $\frac{1}{4}$ of it; by 7 by taking $\frac{1}{7}$ of it.

EXERCISE 130. (ORAL.)

Divide the following by each number in turn from 2 to 9, choosing the better of the two methods above:

$$\frac{7}{11} \quad \frac{9}{10} \quad 2\frac{1}{7} \quad \frac{6}{7} \quad \frac{1}{2} \quad \frac{4}{9} \quad \frac{8}{11} \quad 5\frac{1}{7}$$

When the dividend is a mixed number whose whole number is larger than the divisor, divide as in whole numbers, reduce the remainder to an improper fraction and divide it. Thus, 4 in $5\frac{1}{7} = 1$, and $1\frac{1}{7}$, or $\frac{8}{7}$, over. 4 in $\frac{8}{7} = \frac{2}{7}$. Ans. $4\frac{2}{7}$.

EXERCISE 131. (WRITTEN.)

Select 10 expressions, either improper fractions, proper fractions, or mixed numbers, reducing mixed numbers to improper fractions, and divide by numbers that are factors of the numerators; also 10 others, with divisors that are not factors of the numerators.

EXERCISE 132. (ORAL ANALYSIS.)

Models on p. 46, Division.

1. If 7 dozen eggs cost \$1 $\frac{2}{3}$, what are eggs a dozen?
2. If 5 pounds of butter pay for 11 $\frac{1}{4}$ yards of cloth, how many yards does 1 pound pay for?

3. When I pay $\$42\frac{1}{2}$ for 5 cords of wood, what is the price per cord?

4. At $\$5$ a yard how many yards of silk can I get for $\$18\frac{3}{4}$?

5. Allowing 9 hours a day, how many days' work will $47\frac{1}{4}$ hours make?

6. If 1 man can do a piece of work in $9\frac{1}{3}$ days, in what time can 7 men do it? 9 men?

7. When 5 gold rings cost $\$11\frac{1}{4}$, what are they apiece?

8. At $\$9$ a cord how many cords of wood can be bought for $\$20\frac{1}{4}$?

9. At $\$2$ a day how long will it take a man to earn $\$12\frac{2}{5}$?

10. For $\$22\frac{1}{2}$ I bought cloth at $\$3$ a yard; how many yards did I buy?

To divide a whole number or a fraction by a fraction.

How many times are 3 dollars contained in 15 dollars?
In 17 dollars? In 2 dollars?

How many times are 3 fifths contained in 15 fifths? In 17 fifths? In 2 fifths?

But $\frac{15}{3}=5$ and $\frac{17}{3}=5\frac{2}{3}$. Hence 3, or $\frac{15}{3}$, $\div \frac{3}{3}=5$; $3\frac{2}{3}$, or $\frac{17}{3}$, $\div \frac{3}{3}=5\frac{2}{3}$. $\frac{2}{3} \div \frac{3}{3}=\frac{2}{3}$.

$\frac{3}{3}$ are how many 15ths? $\frac{2}{3}$ are how many 15ths? Then $\frac{2}{3} \div \frac{3}{3}=\frac{10}{15} \div \frac{9}{15}=\frac{10}{9}$, or $1\frac{1}{9}$.

EXERCISE 133. (WRITTEN.)

Divide each of the following by $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{5}{6}$ in turn :

$\frac{1}{2}$ $\frac{7}{8}$ $2\frac{1}{4}$ $\frac{11}{12}$ $1\frac{1}{9}$ $\frac{2}{3}$ $\frac{5}{11}$ $\frac{1}{6}$ 7 3

INVERTING THE DIVISOR.

By Prin. 1 and 2, p. 56, Division, and their explanations, we find that the smaller the divisor the larger the quotient. Dividing by $\frac{2}{3}$, then, will give a quotient 5 times larger than

dividing by $\frac{2}{3}$, because $\frac{2}{3}$ is 5 times smaller than (or $\frac{1}{5}$ of) 3. But dividing any number by 3 is taking $\frac{1}{3}$ of it; therefore dividing by $\frac{2}{3}$ is $5 \times \frac{1}{3}$, or $\frac{5}{3}$ of it; or multiplying by $\frac{3}{2}$ ($\frac{2}{3}$ inverted). Thus, $\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times \frac{3}{2} = 1\frac{1}{2}$. With fractions of different denominators this is the shorter process, except in cases where the numerator and denominator of the dividend are respectively divisible by the numerator and denominator of the divisor; as

$$\frac{\frac{28}{2} \div \frac{2}{3}}{5} = \frac{28}{5} = 5\frac{3}{5} = 11\frac{1}{5}.$$

EXERCISE 134.

Repeat Exercise 133 orally by this method.

Also divide the first mixed number by the second in each example, Exercise 97.

EXERCISE 135. (WRITTEN ANALYSIS.)

1. At $\$2\frac{2}{3}$ each how many chairs will $\$28\frac{6}{10}$ buy?
2. At $\$3\frac{1}{2}$ a day how many days must a man work to earn $\$245\frac{1}{2}$?
3. How long will it take a tree to grow 30 feet high at an average of $3\frac{5}{12}$ feet a year?
4. Allowing $9\frac{3}{4}$ yards to a dress, how many dresses will $126\frac{3}{4}$ yards of cloth make?
5. $28\frac{1}{6}$ dozen buttons will be sufficient for how many dresses, allowing $2\frac{1}{6}$ dozen to a dress?
6. A man divided $458\frac{1}{4}$ A. of land among his sons, giving each $114\frac{9}{16}$ A.; how many sons had he?
7. I divide a pole 3 yards long into divisions $\frac{1}{9}$ of a yard long; how many divisions?
8. If you take $1\frac{5}{6}$ feet to a step, how many steps will you take in going $16\frac{1}{2}$ feet?
9. A man digging a ditch $38\frac{2}{3}$ feet long, digs $7\frac{1}{4}$ feet a day; how many days will it take him?
10. A cubic foot of air weighs $1\frac{1}{4}$ ounces; how many cubic feet of air will weigh 16 ounces?

EXERCISE 136.

In the following **complex** fractions, perform indicated operations:

$$1. \frac{\frac{1}{3} \text{ of } \frac{7}{9}}{\frac{5}{9}} \quad 2. \frac{1\frac{1}{2}}{\frac{2}{3} \text{ of } \frac{3}{4}} \quad 3. \frac{7 \times 3 \times \frac{5}{9}}{2\frac{1}{4} \times \frac{8}{9}} \quad 4. \frac{11 \times \frac{2}{3}}{3\frac{1}{7}}$$

In the following, multiply each expression of the *complex* fraction by the **l. c. m.** of the denominators of all the fractions above and below the main line, combining as you go. Thus, in **5**, 6 is the **l. c. m.** of 2, 3, and 6. $6 \times \frac{1}{2} = 3$, $6 \times \frac{1}{3} = 2$, $2 + 3 = 5$; $6 \times \frac{1}{6} = 1$; $\frac{5}{1} = 5$. Work mentally.

$$\begin{array}{llll} 5. \frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{6}} & 6. \frac{2\frac{1}{3}}{9\frac{2}{7}} & 7. \frac{8\frac{1}{5} - 3\frac{2}{3}}{1\frac{2}{3} + 1\frac{2}{5}} & 8. \frac{4\frac{1}{2} - \frac{3}{4}}{2\frac{1}{7}} \\ 9. \frac{7\frac{1}{2}}{\frac{1}{15}} & 10. \frac{\frac{2}{3} + \frac{3}{4}}{\frac{5}{6} - \frac{3}{4}} & 11. \frac{8\frac{1}{8}}{3\frac{1}{4}} & 12. \frac{\frac{1}{2} + \frac{2}{3} - \frac{3}{4}}{\frac{1}{12}} \\ 13. \frac{12\frac{1}{2}}{100} & 14. \frac{37\frac{1}{2}}{100} & 15. \frac{33\frac{1}{3}}{100} & 16. \frac{62\frac{1}{2}}{100} \\ 17. \frac{66\frac{2}{3}}{100} & 18. \frac{16\frac{2}{3}}{100} & 19. \frac{87\frac{1}{2}}{100} & 20. \frac{83\frac{1}{3}}{100} \end{array}$$

To find what part, or fraction, one number is of another.

What is $\frac{1}{7}$ of 7? $\frac{2}{7}$ of 7? $\frac{3}{4}$ of 7? $\frac{1}{15}$ of 15? $\frac{4}{15}$ of 15?
 $\frac{8}{15}$ of 15?

1 is what part of 7? 2 what part of 7? What part of 7 is 3?

1 is what part of 15? What part of 15 is 4? 8 what part of 15?

The numbers 7 and 15, of which you are finding parts, are found where in the resulting fractions?

Where are the numbers which are parts of 7 and 15 found in the results?

Hence we see that, to find what part or fraction one number is of another, we form a fraction by placing the number that is a part as the numerator and the number of which it is a part as the denominator. The fraction thus formed should be reduced to lowest terms, when not so.

EXERCISE 137. (ORAL.)

What fraction—

- | | | |
|-----------------|------------------|------------------|
| 1. of 20 is 8? | 9. of 24 is 21? | 17. is 8 of 20? |
| 2. of 21 is 9? | 10. of 27 is 20? | 18. of 27 is 20? |
| 3. is 18 of 32? | 11. of 20 is 19? | 19. is 8 of 32? |
| 4. of 45 is 15? | 12. is 20 of 32? | 20. is 12 of 16? |
| 5. is 12 of 18? | 13. of 21 is 14? | 21. of 29 is 27? |
| 6. of 25 is 20? | 14. is 15 of 25? | 22. is 33 of 44? |
| 7. is 18 of 24? | 15. is 24 of 25? | 23. of 30 is 25? |
| 8. is 13 of 20? | 16. of 30 is 25? | 24. of 24 is 18? |

To find the whole when a part is given.

How does $\frac{1}{3}$ of an apple compare in size with $\frac{2}{3}$?

$\frac{1}{5}$ with $\frac{2}{5}$? $\frac{1}{5}$ with $\frac{3}{5}$? $\frac{1}{5}$ with $\frac{4}{5}$?

What is $\frac{1}{3}$ of 9? $\frac{2}{3}$ of 9? $\frac{1}{3}$ of 9, or 3, is what part of $\frac{2}{3}$ of 9, or 6?

If 6 is $\frac{2}{3}$ of some number, $\frac{1}{3}$ of that number is what part of 6? If $\frac{1}{3}$ is 3, $\frac{2}{3}$ are what?

EXERCISE 138. (WRITTEN.)

18 is $\frac{2}{3}$ of what number?

MODEL.—If 18 is $\frac{2}{3}$, $\frac{1}{3}$ is $\frac{1}{2}$ of 18, or 9; $\frac{3}{3}$ are 3×9 , or 27.

- | | |
|---|--------------------------------------|
| 1. 125 is $\frac{5}{6}$ of what number? | 6. 642 is $\frac{107}{100}$ of what? |
| 2. 144 is $\frac{9}{11}$ " " ? | 7. 840 is $\frac{7}{12}$ " " ? |
| 3. 321 is $\frac{3}{4}$ " " ? | 8. 59 is $\frac{1}{7}$ " " ? |
| 4. 45 is $\frac{1}{9}$ " " ? | 9. 189 is $\frac{9}{13}$ " " ? |
| 5. 540 is $\frac{5}{12}$ " " ? | 10. 910 is $\frac{10}{11}$ " " ? |

EXERCISE 139. (ORAL.)

- | | |
|----------------------------------|---------------------------------|
| 1. 16 is $\frac{8}{11}$ of what? | 7. 20 is $\frac{1}{5}$ of what? |
| 2. 49 is $\frac{7}{11}$ " " ? | 8. 19 is $\frac{19}{100}$ " " ? |
| 3. 32 is $\frac{2}{3}$ " " ? | 9. 64 is $\frac{8}{9}$ " " ? |
| 4. 44 is $\frac{11}{100}$ " " ? | 10. 50 is $\frac{1}{3}$ " " ? |
| 5. 28 is $\frac{7}{4}$ " " ? | 11. 12 is $\frac{3}{100}$ " " ? |
| 6. 72 is $\frac{8}{100}$ " " ? | 12. 25 is $\frac{5}{7}$ " " ? |

- | | | | | | |
|-----|------------------------|----------|-----|-----------------------|----------|
| 13. | 36 is $\frac{9}{5}$ | of what? | 17. | 33 is $\frac{11}{12}$ | of what? |
| 14. | 14 is $\frac{14}{100}$ | “ “ ? | 18. | 144 is $\frac{12}{7}$ | “ “ ? |
| 15. | 25 is $\frac{1}{4}$ | “ “ ? | 19. | 108 is $\frac{9}{10}$ | “ “ ? |
| 16. | 28 is $\frac{7}{3}$ | “ “ ? | 20. | 7 is $\frac{1}{10}$ | “ “ ? |
-

PRACTICAL WORK IN FRACTIONAL ANALYSIS.

- General Forms. $\left\{ \begin{array}{l} \text{G.—What is } \frac{3}{4} \text{ of } 16 ? \\ \text{H.—} 12 \text{ is } \frac{3}{4} \text{ of what number ?} \\ \text{I.—} 12 \text{ is what part (fraction) of } 16 ? \end{array} \right.$

EXERCISE 140. (WRITTEN.)

Write the following 20 examples in proper *general form* before performing, and analyze:

1. A man sold a watch for \$36, which was $\frac{4}{5}$ of what it cost him; what did it cost?
2. A broker having \$875 lost \$175 in speculating; what part of his money did he lose?
3. A stock-raiser sold 250 sheep, which were $\frac{2}{3}$ of all he had; how many had he?
4. I bought a horse for \$1575 and sold it for $\frac{6}{5}$ of the cost; what did I get for it?
5. $\frac{2}{3}$ of a ranch is worth \$12300; what is the whole worth?
6. What is $\frac{7}{15}$ of the above ranch worth?
7. What part of it is worth \$10250?
8. At \$7 $\frac{1}{2}$ a ton what is $\frac{3}{5}$ of a ton of hay worth?
9. What is $\frac{2}{3}$ of an acre of land worth if $\frac{3}{4}$ of it is worth \$75?
10. A man owning $\frac{5}{8}$ of a mill sells $\frac{2}{3}$ of his part for \$5760; what is the value of the mill at that rate?
11. What is the value of the part he has left?
12. Bought a watch for \$65 and sold it at a gain of \$5; what fraction of the cost did I gain?
13. A man having a journey of 248 miles to perform goes

31 miles the first day; what part of the journey is that? What part has he left to walk?

14. A farmer sold 108 acres of land, which was $\frac{9}{20}$ of his whole ranch; how large was his ranch?

15. After selling $\frac{2}{3}$ of my sheep I have 1200 left; how many had I at first?

16. At $\$5\frac{2}{3}$ a yard what will $\frac{5}{9}$ of a yard of cloth cost?

17. I buy a place for \$2325 and pay \$1800 down; what part of the money do I still owe?

18. A man earns \$1575 a year and spends $\frac{2}{3}$ of it; what does he save?

19. A ship having 320 tons of coal on board sprung a leak, and 128 tons were thrown overboard; what part of the coal was lost?

20. $\frac{5}{13}$ of an army was lost in battle and 8800 men were left; how many men were in the army?

ORAL REVIEW IN FRACTIONS.

EXERCISE 141.

1. A man gave $\frac{1}{5}$ of a dollar to John, twice as much to Eddie, and half as much to Elmer as to the other two; what did he give to all?

2. If he gave the remainder of the dollar to Peter, what did Peter get?

3. Daniel's kite string is $31\frac{1}{2}$ feet long, and he ties a piece $\frac{1}{7}$ as long to it; how long is it now?

4. Joseph's kite-string is $\frac{8}{9}$ as long as Daniel's was at first; find the length of Joseph's kite-string.

5. Elmer can walk $2\frac{5}{8}$ miles an hour, and Charlie $3\frac{1}{2}$ miles; how far can both walk in 5 hours?

6. Katie and Nellie have 10 examples to work; it takes them $8\frac{3}{4}$ minutes each to perform an example; how long will it take both to perform the 10 examples?

7. Mamie, by devoting $\frac{2}{3}$ of an hour to each lesson, studied $2\frac{2}{3}$ hours; how many lessons had she to get?

8. Agnes devotes $2\frac{1}{3}$ hours a day to study, $1\frac{1}{2}$ hours to music, and 1 hour to sewing; what does she spend on all in 5 days?

9. Four girls, Bell, Mell, Nellie, and Susie, agree to lay by $\frac{1}{20}$ of a dollar each week for the Sunday school; what do they all lay by in 20 weeks?

10. Angie finds that, by working $1\frac{4}{5}$ hours on her dress each day, she can make it in 5 days; how many hours does it take?

11. Edith and Hazel have each $\$1\frac{1}{2}$; they agree to buy in equal shares a book costing $\$1\frac{1}{2}$ as a Christmas present for their mother; what has each left?

12. Florence and Leona together lack $\frac{1}{3}$ of a dollar to buy a picture worth $\$1\frac{1}{2}$; if each has the same sum, what has each?

13. Hattie and Mary give $\frac{1}{3}$ of an apple to each of two playmates and divide the rest equally between themselves; what part has each?

14. Susie receives $\frac{1}{10}$ of a dollar a day from her mother for work; she wishes to buy a dress worth $\$6\frac{1}{2}$ and a hat worth $\$3\frac{1}{4}$ at the close of the school term; can she do it, counting the time 20 weeks of 5 days each?

15. Antone drives the cows to pasture in the morning and John gets them at night; if the distance from their house to the pasture is $\frac{2}{3}$ of a mile, how far do both travel in 1 week?

16. Marvin lives $2\frac{1}{3}$ miles from the school-house; if it takes him $5\frac{1}{2}$ minutes to go $\frac{1}{3}$ of a mile how long is he in going to school?

17. It takes Willie $1\frac{1}{2}$ minutes to distribute the copy-books to 40 pupils; suppose each scholar were allowed to get his own copy-book, taking $\frac{1}{2}$ minute, how much time would be lost?

18. Joseph finds he has 20 pages of his grammar to learn

to meet the requirements of his class; if he learns $1\frac{1}{2}$ pages a day, how long will it take him?

19. Alfred and Charles together have 60 cents; Charles has $\frac{1}{2}$ as much as Alfred; what has each?

20. $\frac{1}{2}$ of A's money is $\frac{1}{3}$ of B's, and together they have \$75; what has each?

21. At $\$ \frac{4}{5}$ a yard how many yards of cloth can I get for \$12?

22. How many sacks of potatoes at $\$1\frac{1}{4}$ a sack will pay for a barrel of sugar at $\$18\frac{3}{4}$?

23. At $\frac{2}{3}$ of a dollar a pound, how many pounds of coffee are worth $\$15\frac{2}{3}$?

24. A can do a piece of work in 3 days and B the same in 6 days; what part can each do in 1 day? How long will it take both together to do the work?

25. I deposited $\$32\frac{7}{10}$ in the bank, which was $\frac{2}{11}$ of what I had there already; how much had I there?

26. 1 man builds a barn in $8\frac{1}{2}$ days; how long will it take 4 men?

27. John can do a piece of work in 8 days, and Bertie in 12 days; how long will it take both to do it?

28. A pole is $\frac{1}{3}$ in the mud, $\frac{1}{4}$ in the water, and 10 feet above the water; how long is it?

29. A boy has 52 eggs in his basket; what are they worth at 21 cents a dozen?

30. A can dig a ditch in 6 days, B in 8 days, and C in 12 days; in what time can all do it?

31. At $1\frac{1}{2}$ cents apiece, how many oranges will pay for 11 yards of print at 9 cents a yard?

32. Bought 90 centals of wheat at $\$1\frac{1}{10}$ a cental; if I give 5 20-dollar pieces in payment, what change do I receive?

33. At $\$1\frac{2}{3}$ a day, what does a man earn in 5 weeks?

34. How many oranges can I buy for 105 cents at $1\frac{3}{4}$ cents apiece?

35. John had 17 marbles, which were 5 less than $\frac{1}{12}$ of James's; how many had James?

36. A can do a piece of work in 10 days, C in 12 days, and B in 15 days; in what time can all do it?

37. In what time can A and B do it? A and C? B and C?

WRITTEN REVIEW IN FRACTIONS.

EXERCISE 142.

1. At $\$1\frac{5}{8}$ a yard, how many yards of cloth can I get for $\$9\frac{3}{4}$?

2. Bought of one man $11\frac{1}{2}$ acres of land at $\$37\frac{1}{2}$, and of another $17\frac{3}{4}$ acres at $\$42$; how many acres had I and how much did I pay for the whole?

3. If $43\frac{1}{2}$ yards of silk cost $\$108\frac{3}{4}$, what must I pay for $12\frac{1}{2}$ yards at the same price?

4. Multiply $\frac{2}{3}$ of $9\frac{3}{4}$ by $\frac{4}{5}$ of $2\frac{1}{2}$.

5. A man having $175\frac{1}{2}$ acres of land sold $\frac{1}{3}$ of it at one time, and $\frac{1}{6}$ at another; what is the remainder worth at $\$45$ an acre?

6. Sold 20 dozen eggs at $\$\frac{1}{5}$ a dozen, and received in payment butter at $\$\frac{1}{4}$ a pound; how many pounds did I receive?

7. Sold my farm for $\$2250$, which was $\frac{5}{7}$ of its value; what was its value?

8. How many coats can be made from $175\frac{3}{8}$ yards of cloth, allowing $2\frac{7}{8}$ yards to a coat?

9. The length of a room is $17\frac{2}{3}$ feet, and the width is $12\frac{5}{8}$ feet; what will be the cost of a moulding around it at $3\frac{1}{2}$ cents a foot?

10. How many pounds of butter at $22\frac{1}{2}$ cents a pound will pay for $18\frac{3}{4}$ pounds of sugar at 12 cents a pound?

11. A sold $\frac{2}{5}$ of his farm of 475 acres to B, and B $\frac{2}{5}$ of his part to C; how many acres did B sell?

12. A man contracts to do a job in 60 days; how much of the work should be done in $22\frac{1}{2}$ days?

13. A lady has $\$63\frac{3}{4}$ in her purse; she spends $\$17\frac{1}{2}$ for a shawl, $\$3\frac{4}{5}$ for cloth, $\$7\frac{1}{4}$ for a bonnet, and $\$5\frac{1}{4}$ for lace; how much has she left?

14. There are $5\frac{1}{2}$ yards in a rod; how many rods are in $104\frac{1}{2}$ yards?

15. How many yards in 320 rods?

16. Sold wheat for $\$517\frac{9}{10}$, gaining $\$27\frac{3}{5}$ on the cost; what did I pay for it?

17. A man owning $7\frac{1}{5}$ acres of land, divided it into house lots containing $\frac{9}{10}$ acres each; how many lots did he make?

18. If $\$3\frac{2}{5}$ buys a yard of cloth, how many yards will $\$7\frac{1}{2}$ buy?

19. If $3\frac{2}{3}$ pounds of coffee costs 99 cents, what will $\frac{3}{4}$ of a pound cost?

20. How many tons of hay are in 19 loads, each containing $\frac{1}{2}\frac{9}{10}$ of a ton?

21. What is the price per yard, when $7\frac{7}{8}$ yards of cloth cost $\$14$?

22. A man owning $\frac{5}{9}$ of a mill sold $\frac{3}{10}$ of his share to one man and $\frac{4}{5}$ to another; what had he left?

23. A has 324 head of cattle, and $\frac{1}{18}$ of his herd is $\frac{2}{19}$ of B's; how many has B?

24. A lot of goods was sold for $\$4774$, of which A owns $\frac{3}{11}$, B $\frac{9}{22}$, and C the remainder; find the money each should receive.

25. A tailor wishes to put $2\frac{7}{8}$ yards of cloth into a coat, $2\frac{1}{8}$ into a pair of pants, and $\frac{7}{8}$ into a vest; how many suits can be made from a piece of cloth containing $60\frac{1}{2}$ yards, and how many vests from the remainder?

26. A can do a piece of work in 13 days, and B in 14 days; in what time can both do it?

27. When $5\frac{1}{2}$ centals of wheat cost $\$6\frac{1}{10}$, how many centals can be bought for $\$12\frac{1}{10}$?

28. A man having sold $\frac{2}{3}$ of his hogs, and lost $\frac{1}{8}$ by disease, had 150 left; how many had he at first?

29. Find the whole value of $127\frac{1}{2}$ centals of wheat at $\$1\frac{1}{3}$ a cental, 18 centals of oats at $\$1\frac{1}{4}$ a cental, and 75 centals of barley at $\$1\frac{9}{10}$ a cental.

30. If 4 acres of land cost $\$321$, what are $11\frac{5}{16}$ acres worth?

31. Divide 1 by $47\frac{2}{3}$.

32. Bought 35 yards of carpeting at $\$1\frac{9}{10}$ a yard, 3 curtains at $\$1\frac{1}{5}$ each, 5 chairs at $\$1\frac{3}{4}$ each; what was my bill?

33. A can walk a mile in $\frac{1}{4}$ of an hour, and B in $\frac{3}{11}$ of an hour; in a race of 22 miles, which will win, and by how much?

34. A and B can do a piece of work in 10 days, A and C in 12 days, and B and C in 15 days; in what time can the three working together do it? In what time can each do it working alone?

35. A man has $49\frac{3}{4}$ acres of land; he sold all but $9\frac{1}{5}$ acres of land for $\$3190$; how much did he get an acre?

36. When $33\frac{1}{3}$ yards of cloth cost $\$20$, what is the price per yard?

37. At $\$2\frac{1}{4}$ a yard, how much cloth will $\$1\frac{1}{2}$ buy?

38. How many sheep must I sell at $\$3\frac{1}{4}$ to get $\$169$?

39. A lady divided $\$3\frac{2}{3}$ among some poor children, giving them $\$1\frac{9}{10}$ each; what number of children were there?

40. If $\frac{5}{16}$ of an acre of land is worth $\$23\frac{3}{4}$, what is 1 acre worth?

41. Bought 50 sacks of potatoes for $\$62\frac{1}{2}$; what will 12 sacks cost at the same rate?

42. Paid $\frac{1}{7}$ of my money for a lounge, and $\frac{1}{10}$ of it for a stove, when I had $\$106$ left; what had I at first?

43. A's money is $\frac{1}{3}$ of B's, and together they have $\$1728$; what has each?

44. What do I receive by selling $17\frac{1}{2}$ bales of cotton, each containing $5\frac{1}{3}$ hundred weight, at $\$18\frac{3}{4}$ per hundred weight?

45. How many dipperfuls, each $\frac{2}{3}$ of a quart, will empty a tub containing $81\frac{1}{3}$ quarts?

46. A stockman buys a certain number of cattle at $\$24\frac{1}{2}$ each for $\$588$, and sells them at $\$27\frac{3}{4}$; what does he gain on each and on the whole?

47. Divide the sum of $31\frac{3}{8}$ and $5\frac{7}{8}$ by $\frac{4}{7}$.

48. A has $\frac{1}{2}$ as much money as B, and $\frac{2}{3}$ as much as C; the three have $\$2835$; what has each?

49. From a chest of tea containing $63\frac{1}{3}$ pounds, $\frac{1}{9}$ was sold for $\$34$; what price per pound was obtained?

50. B's money is $1\frac{2}{3}$ times A's, and C's is $1\frac{2}{3}$ times B's; all together have $\$15300$; what has each?

51. If $\frac{2}{3}$ of a cord of wood costs $\$5\frac{1}{2}$, what will $17\frac{1}{2}$ cords cost?

52. I buy $35\frac{9}{16}$ acres of land at one time, $47\frac{3}{4}$ acres at another, and $17\frac{3}{8}$ acres at a third; I sell it all at an average rate of $\$40$ an acre; what do I receive for the whole?

53. At $\$1\frac{1}{4}$ a cental, how many centals of wheat can be bought for $\$1000$?

54. Find the sum, difference, and product of $\frac{1}{2}\frac{3}{6}$ and $\frac{3}{8}\frac{8}{9}$.

55. I exchanged $5\frac{1}{2}$ rolls of butter, worth 40 cents a roll, and $10\frac{1}{3}$ dozen eggs, worth 18 cents a dozen, for sugar worth $7\frac{1}{4}$ cents a pound; how many pounds of sugar did I receive?

56. Gained $\$3\frac{1}{3}$ by selling $12\frac{1}{2}$ yards of cloth for $\$41\frac{3}{10}$; what was the cost per yard?

57. $8\frac{1}{2}$ tons of Wellington coal at $\$15$ per ton, and $9\frac{2}{3}$ cords of wood at $\$7\frac{1}{2}$ a cord, amount to what?

58. Bought 40 acres of land at $\$63$ an acre. Sold $\frac{5}{16}$ at $\$72$ an acre, $\frac{3}{10}$ at $\$59\frac{1}{2}$ an acre, and the remainder for $\$2\frac{1}{2}$ more per acre than I paid for it; what did I gain on the whole?

59. $\frac{3}{7}$ of 189 is what fraction of 567?

60. I lend A a certain sum and B twice as much. A pays me back $\frac{1}{2}$ of his and B $\frac{1}{2}$ of his, making $\$150$ received from both; what did I lend each?

61. A merchant sold $35\frac{1}{2}$ pieces of cloth, each piece containing $47\frac{2}{3}$ yards; how many yards did he sell?

62. Bought 40 bales of hay, averaging $2\frac{7}{10}$ hundred weight a bale; how many hundred weight were there?

63. How many tons in the preceding example, at 20 hundred weight to a ton?

64. There is an average of $365\frac{1}{4}$ days in a year; how many hours?

65. Bought 14 cows at $\$23\frac{1}{2}$ a head, 11 horses at $\$85\frac{3}{4}$ a head, and 50 sheep at $\$2\frac{3}{4}$ a head; what had I left from $\$1500$?

66. Bought 15 sacks of potatoes for $\$12\frac{3}{4}$, and sold them for $\frac{1}{10}$ dollar a sack more than I paid; what did I receive for them, and how much more than I paid?

67. The distance by rail from San Francisco to Los Angeles is $9\frac{3}{10}$ times the distance from San Francisco to San José. The sum of the distances is 533 miles; what is each distance?

68. I pay 3 men $\$12.30$ for doing a piece of work. The second works 3 times as long as the first, and the third $\frac{1}{2}$ as long as the first and second together; if they are paid the same rate per day what should each receive?

69. Two men starting at the same point travel in opposite directions for $13\frac{2}{3}$ hours. One travels $3\frac{1}{2}$ miles and the other $3\frac{2}{3}$ miles per hour; how far apart are they at the end of the time? Draw a diagram to show it.

70. Suppose the men in the preceding example traveled in the same direction, how far apart would they be? Draw diagram.

71. Allowing $225\frac{3}{4}$ pounds to a barrel, how many barrels of sugar will $2483\frac{1}{4}$ pounds make?

72. How many collars at $\$\frac{2}{3}$ each can I get for $\$2\frac{1}{4}$?

73. What is the average value of 4 horses worth $\$81\frac{1}{2}$, $\$98\frac{4}{5}$, $\$105\frac{3}{4}$, and $\$112\frac{3}{4}$, respectively?

74. A man bought 35 watches for $\$15\frac{1}{2}$ apiece and sold

them so as to gain \$17½ on the whole; what did he get apiece for them?

75. A boy has 375 oranges. He sells $\frac{4}{5}$ of them at one time and $\frac{3}{5}$ at another; what are the remainder worth at $1\frac{3}{4}$ cents each?

76. A merchant sold two pieces of cloth for \$241½. One piece contained $30\frac{1}{4}$ yards, the other $42\frac{1}{5}$ yards; what average price per yard did he get?

77. Richard can walk $\frac{4}{5}$ as fast as Walter. In a certain time both together walked $5\frac{2}{3}$ miles; what part of the distance did each walk?

78. At \$ $\frac{3}{4}$ a day what will a man earn in 1 year, leaving out 60 days for Sundays and holidays?

79. If $11\frac{1}{2}$ boxes of oranges cost \$28 $\frac{3}{4}$, how many boxes can I get for \$22½?

80. Fred worked $2\frac{3}{4}$ times as long as Frank at $\frac{3}{4}$ as much per day. They received \$24½; what part should each have?

DECIMAL FRACTIONS.

If you divide a unit, or 1, into 10 equal parts, what is each part called? How many tenths make a unit? If you divide each of the 10ths into 10 equal parts, how many pieces will there be and what is each called? How many hundredths in 1 tenth? If you divide each hundredth into 10 equal parts, how many pieces will there be and what is each called? How many thousandths in 1 hundredth?

Review Obs. on p. 6.

What is the first figure on the left of the decimal point called? The second? The third? The fourth? How many units make 1 ten? How many tens make 1 hundred? What part of 100 is 10? Of 10 is 1?

Since numbers decrease by 10 fold from left to right we may go on in our decimal notation beyond the decimal

point on the right, and make the name of each succeeding place $\frac{1}{10}$ the value of the preceding. Thus, starting with units, the first figure on the right of the point will be $\frac{1}{10}$ of units or 10ths. 5.7 is 5 and $\frac{7}{10}$. What will the second figure on the right of the point be called? The third? The fourth? How do we mark the absence of number in any decimal place? (See Obs., p. 6.)

Fractions, then, whose denominators are 10, 100, 1000, etc., may be written *decimally*; the denominator being indicated by the number of places on the right of the decimal point, and not expressed, as in the *common form*.

Put a diagram similar to the following on your slate, extending it further if necessary; under it place the mixed numbers and fractions of Exercise 143 in their proper places, and practice reading. In reading, use "and" at the decimal point only.

Read the fractional part as a whole number first, and add the decimal name of the last figure; thus,

32575 and 4763 hundred-thousandths.

3	2	5	7	5	.0	4	7	6	3
ten thousands.	thousands.	hundreds.	tens.	units.	tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.

Observe. { *The number of figures on the right of the point corresponds to the number of 0's in the denominator of the fraction.*

EXERCISE 143.

- | | | | |
|----------|------------|-----------|-----------|
| 1. 75.14 | 3. 131.131 | 5. 7.007 | 7. 1389.9 |
| 2. .125 | 4. .0785 | 6. .13147 | 8. .0091 |

9.	857.14	15.	2.0404	21.	480.7	27.	3150.071
10.	85.0714	16.	7814.002	22.	526.114	28.	4090.07
11.	.07408	17.	7.0707	23.	.070107	29.	293.0293
12.	.00291	18.	20.0003	24.	.1410	30.	47.141
13.	405.01	19.	171.4112	25.	82.1073	31.	29.641
14.	78.78	20.	27141.75	26.	1.01010	32.	10.1

EXERCISE 144.

Write each example of Exercise 143 with denominators, thus,

$$75\frac{14}{100}.$$

Write each in words, also; thus,

Seventy-five and fourteen hundredths.

EXERCISE 145.

Write the following in decimal and in common forms, and reduce the fractional parts in the common form to lowest terms:

1. Twenty-five and twenty-five hundredths, 9 and 114 thousandths, 7 and 5 tenths, 11 and 8 thousandths.

2. 74 and 99 ten thousandths, 11 and 45 hundred thousandths, 4 thousandths, 4 hundredths.

3. 75 hundredths, 75 ten thousandths, 40 and 40 hundredths, 4 thousand and 4 thousandths.

4. 91 hundredths, 91 tenths, 400 thousandths, 121 hundredths.

5. 90 tenths, 57 thousandths, 5 and 11 thousandths, 72 and 6 tenths.

6. 87 and 54 hundredths, 90 and 8 tenths, 117 and 41 thousandths, 25 and 9 thousandths.

7. 238 and 12 thousandths, 171 and 125 thousandths, 328 and 10 thousandths, 190 and 8 thousandths.

8. 2 tenths, 24 tenths, 120 tenths, 175 tenths.

9. 830 hundredths, 375 hundredths, 57 hundredths, 9 hundredths.

10. 2496 thousandths, 7125 thousandths, 125 ten thousandths, 25 thousandths.

EXERCISE 146.

(1) Write 10 mixed numbers or fractions of your own in the common form, using 10, 100, 1000, etc., for denominators; (2) the same in decimal form; (3) the same in words; (4) write the first examples with the fractions in their lowest terms. Bring to the class for dictation.

EXERCISE 147.

Write the following in common form and reduce to lowest terms:

- | | |
|---|-----------------------|
| 1. .25, .75, .125. | 6. .144, .0256, .075. |
| 2. .375, .088, .048. | 7. .84, .164, .175. |
| 3. .0175, .35, .015. | 8. .8, .50, .0625. |
| 4. .16, .016, 1.75. | 9. .1875, .625, .18. |
| 5. $.33\frac{1}{3}$, $.66\frac{2}{3}$, .78. | 10. .95, .3125, .105. |

DOLLARS AND CENTS WRITTEN DECIMALLY.

The decimal notation is employed in writing dollars and cents in United States money. There are 100 cents in 1 dollar. Hence any number of cents are so many hundredths of a dollar; thus,

5 cents are $\frac{5}{100}$, or .05 of a dollar; 20 cents $\frac{20}{100}$, or .20.

12 dollars 6 cents is written \$12.06; 17 dollars 37 cents, \$17.37; 18 dollars $12\frac{1}{2}$ cents, \$18.12 $\frac{1}{2}$. All rules for operations in decimals are equally true of United States money.

Observe.—*The decimal point is placed at the right of dollars.*

EXERCISE 148.

Read the following as dollars and cents:

- | | | | |
|---------------------------|-------------|----------------------------|---------------------------|
| 1. \$7.02 | 5. \$175.75 | 9. \$708.09 | 13. \$927.06 |
| 2. \$25.50 | 6. \$38.25 | 10. \$150.12 $\frac{1}{2}$ | 14. \$41.62 $\frac{1}{2}$ |
| 3. \$137.37 $\frac{1}{2}$ | 7. \$450.80 | 11. \$45.33 $\frac{1}{3}$ | 15. \$108.03 |
| 4. \$98.01 | 8. \$92.90 | 12. \$128.07 | 16. \$29.66 $\frac{2}{3}$ |

EXERCISE 149.

Write 20 numbers of your own, representing dollars and cents decimally, and bring to the class for reading and dictation.

To change any fraction from the common to the decimal form.

How is a fraction reduced to higher terms? (See pp. 75 and 76.) How do you change $\frac{1}{2}$ to a fraction having a denominator 10? $\frac{1}{2}$ is how many 10ths? $\frac{2}{5}$ are how many 10ths? $\frac{3}{5}$? $\frac{4}{5}$? $\frac{1}{4}$ is how many 100ths? $\frac{3}{4}$? $\frac{1}{8}$ is how many 1000ths? $\frac{3}{8}$? $\frac{5}{8}$? $\frac{7}{8}$?

Write each result in decimal form.

We see from this, that, to change any fraction from the common to the decimal form, we reduce it to a fraction having 10, 100, 1000, etc., for a denominator, and write decimally.

EXERCISE 150. (ORAL.)

Change to decimal denominator and express decimally:

- | | | |
|---|--|---|
| 1. $\frac{3}{10}, \frac{4}{5}, \frac{7}{20}$. | 4. $\frac{2}{5}, \frac{13}{20}, \frac{19}{25}$. | 7. $\frac{2}{4}, \frac{19}{50}, \frac{24}{25}$. |
| 2. $\frac{4}{25}, \frac{11}{50}, \frac{11}{20}$. | 5. $\frac{17}{20}, \frac{3}{4}, \frac{18}{25}$. | 8. $\frac{45}{50}, \frac{1}{20}, \frac{1}{5}$. |
| 3. $\frac{11}{25}, \frac{9}{50}, \frac{27}{25}$. | 6. $\frac{49}{50}, \frac{21}{25}, \frac{17}{50}$. | 9. $\frac{7}{25}, \frac{14}{20}, \frac{37}{50}$. |

Again: $5 = \frac{5}{1} = \frac{50}{10} = \frac{500}{100} = \frac{5000}{1000} = 5.000$

$\frac{5}{8} = \frac{1}{8}$ of 5, or $\frac{1}{8}$ of $\frac{5000}{1000} = \frac{625}{1000} = 8 \overline{)5.000}$
.625

Therefore $\frac{5}{8} = .625$.

Hence, to change common to decimal fractions, annex ciphers to the numerator and divide by the denominator.

If the denominator contains no other prime factors than 2 or 5, the division will be exact: and the number of places will be equal to the largest number of times 2 or 5 is contained as a factor.

Thus, in $\frac{4}{125}$, the division is exact, because 5 is the only prime factor in 125, and there will be 3 places, because 125

contains 5^3 ; in $\frac{13}{375}$ the division is not exact, because 375 has the factor 3. When the division is not exact, carry it to 3 or 4 places and express the remainder in the form of a common fraction.

EXERCISE 151. (ORAL.)

Tell by inspection whether the following are exact decimals; and, if exact, how many places in the decimal:

$$\begin{array}{cccccccc} \frac{23}{40} & \frac{19}{20} & \frac{4}{125} & \frac{13}{50} & \frac{19}{75} & \frac{40}{60} & \frac{17}{150} & \frac{13}{105} \\ \frac{131}{125} & \frac{36}{175} & \frac{19}{200} & \frac{43}{100} & \frac{78}{275} & \frac{37}{300} & \frac{17}{250} & \frac{11}{375} \end{array}$$

In performing work where denominators are as small as in the preceding exercise, it is better to work mentally, reducing as you go; thus,

$$\frac{23}{40} = .5\frac{3}{4} = .57\frac{1}{2} = .575; \quad \frac{1}{375} = .0\frac{2}{75} = .00\frac{4}{15} = .002\frac{2}{3}.$$

Work the preceding exercise in this way.

EXERCISE 152.

Change the fractions in Exercise 150 to decimals by this method.

Also the fractional part in Exercises 97 and 98, and rewrite the mixed numbers in decimal form.

EXERCISE 153.

Write 10 common fractions of your own, and change to decimals. Bring to the class for dictation.

CIRCULATING DECIMALS.

In cases where the division in the preceding work is not exact, the figures of the quotient will begin to repeat at some point of the division, producing what is called a **circulating decimal**. The circulate, or repeating part, is marked by a dot over the first and last of the repeating figures.

The number of places before the circulate begins will equal the greatest number of 2's or 5's in the denominator.

When special accuracy is required, however, it is better to divide until the 2's and 5's are all canceled, and express the remainder as a common fraction. Thus, $\frac{97}{220} = .44\dot{0}\dot{9}$, expressed as a circulate; or $.44\frac{1}{11}$, expressed fractionally.

ADDITION AND SUBTRACTION OF DECIMALS.

Decimals containing fractions are written and worked, for adding and subtracting, like whole numbers. How must they be written, and why? (See explanations, pp. 18 and 24, Addition and Subtraction.)

EXERCISE 154.

Write properly and add the numbers in examples 1 to 8, Exercise 143; same with 9 to 16, 17 to 24, 25 to 32. Also find the sum of the numbers in each row. Add the numbers in each example of Exercise 145.

EXERCISE 155.

Find the difference between Example 1, Exercise 143, and each remaining example in the same column; Example 9 and each remaining example in the same column; Example 17 and each remaining example in the same column; Example 25 and each remaining example in the same column.

EXERCISE 156.

Add the numbers in each example, Exercise 97, as they are; change to decimals and add; compare results. Subtract in the same way.

EXERCISE 157.

Change the common fractions to decimals, and perform examples 1, 2, 3, 8, 10, 17, and 20, Exercise 114.

MULTIPLICATION OF DECIMALS.

Multiply 3.728 by .18.

OPERATION.	EXPLANATION.—Any number of units times a certain
3.728	denomination gives that denomination as a product.
.18	Hence, $18 \times 3.728 = 67.104$; but the multiplier is $\frac{18}{100}$ or
<hr/> 29824	$\frac{1}{100}$ of 18. Therefore the product will be $\frac{1}{100}$ of 67.104
3728	or .67104. Whence the law for multiplying decimals:
<hr/> .67104	<i>Point off from the right as many places in the product</i>
	<i>as there are in both multiplicand and multiplier.</i>

EXERCISE 158.

Use numbers in examples 1 to 8, Exercise 143, as multiplicands, and 75.14 for a multiplier of each: also, .125 as a multiplier; 9 to 16 as multiplicands, and examples 3 and 4 as multipliers; 17 to 24 as multiplicands, and 5 and 6 as multipliers; 25 to 32 as multiplicands, and 7 and 8 as multipliers.

Finish with one multiplier before using another.

EXERCISE 159.

Multiply the numbers in each example, Exercise 97, as they are; change to decimals and multiply. Compare results. Perform the work of Exercise 119, changing common fractions to decimals.

EXERCISE 160. (Analysis, General Form G, p. 93.)

Find:

- | | | |
|-------------------------------|--------------------------------|-------------------|
| 1. .06 of 725. | 8. $.33\frac{1}{3}$ of 515.1. | 15. .8 of 3.55. |
| 2. .8 of 42.5. | 9. .05 of 480. | 16. .025 of 96. |
| 3. .125 of 7.84. | 10. .175 of .764. | 17. .28 of 250. |
| 4. .03 of 17.28. | 11. .04 of 57.75. | 18. 1.05 of 1400. |
| 5. $.12\frac{1}{2}$ of 4.096. | 12. .9 of 1.044. | 19. 1.2 of 380. |
| 6. $.16\frac{2}{3}$ of 256. | 13. $.06\frac{1}{4}$ of 72400. | 20. .45 of 920. |
| 7. .25 of 2.444. | 14. .15 of 245.4. | |

Also perform the above by changing the multiplier in each example to a common fraction. Compare work.

EXERCISE 161.

Perform Exercises 123 and 128, changing the fractions to decimal form.

DIVISION OF DECIMALS.

Divide .67104 by 3.728.

OPERATION.

$$\begin{array}{r} 3.728 \overline{) .67104} \quad (.18 \\ \underline{3728} \\ 29824 \\ \underline{29824} \end{array}$$

EXPLANATION.—Any denomination divided by the same denomination gives units for a quotient; $.671 \div 3.728$ gives 0 units for a quotient; the remaining places in the dividend will be the number of places to point off in the quotient giving .18.

Whence the law for division of decimals:

Point off as many places from the right in the quotient as those in the dividend exceed those in the divisor.

When the division is not exact, carry the answer to 3 or 4 places beyond the point; in money operations, to 2 places, adding 1 to the second figure if the third figure should be 5 or more.

Where special accuracy is required, express the remainder as a common fraction.

In actual practice it is a shorter and surer way to draw a vertical line in the dividend after the figure whose denomination is that of the divisor, first annexing ciphers to the dividend if necessary; when the division has reached this line put a point in the quotient.

Divide 44.232 by .12.

OPERATION.

$$\begin{array}{r} .12 \overline{) 44.23|2} \\ \underline{368.6} \end{array}$$

Draw a vertical line after hundredths, that being the denomination of the divisor. The part of the dividend on the left of the line contains the divisor 368 times. Then comes the point.

EXERCISE 162.

Divide examples 1 to 8, Exercise 143, by .02 (short division); examples 9 to 16 by .095; 17 to 24 by $\frac{3}{27}$; 25 to 32 by 1.01.

EXERCISE 163.

Divide each number in this row by 25:

1. 3. 4. 7. 8. 1.1 7.8 .001

Divisor .1 for this row:

10. .001 75. 7.5 .75 .1 100.

Divisor 150 for this row:

45. .0450 .75 7.5 3. 10. 15.

EXERCISE 164.

Divide, in Exercise 97, the greater number by the less, using the numbers as they are; change to decimals and divide. Compare results.

EXERCISE 165.

Perform the work of Exercises 132 and 135 by decimals.

EXERCISE 166. (Analysis, General Form II, p. 93.)

- | | |
|-----------------------|------------------------|
| 1. 75 is .03 of what? | 9. 120 is .15 of what? |
| 2. 125 is .05 " " ? | 10. 240 is 1.20 " " ? |
| 3. 128 is .2 " " ? | 11. 196 is 1.4 " " ? |
| 4. 296 is .04 " " ? | 12. 28 is .07 " " ? |
| 5. 144 is .12 " " ? | 13. 13 is .08 " " ? |
| 6. 50 is .025 " " ? | 14. 14 is .09 " " ? |
| 7. 150 is .1 " " ? | 15. 15 is .10 " " ? |
| 8. 99 is .011 " " ? | 16. 16 is .11 " " ? |

CONTRACTED MULTIPLICATION OF DECIMALS.

In multiplying decimals having several places on the right of the point, where the product is desired only to 2 or 3 places, the work may be greatly shortened by multiplying only those denominations that produce the required places. Thus,

Multiply 428.9543 by 17.454; 2 decimal places required.

OPERATION.	EXPLANATION.—
$ \begin{array}{r} 428.9543 \\ \underline{17.454} \\ 3002.68 \\ 4289.54 \\ 171.58 \\ 21.45 \\ \underline{1.72} \\ 7486.97 \end{array} $	Place denominations of the same name under each other. For convenience, use the unit figure of the multiplier first, if there is a unit figure, since units times any denomination gives that denomination. Begin with hundredths in the multiplicand, the required denomination in the answer. 7×5 hundredths = 35 hundredths + 3 hundredths (7×4 thousandths = 28 thousandths, which being $2\frac{1}{2}$ or more we call 3) = 38 hundredths; and so on. In multiplying by 1 ten we go one place farther to the right in the multiplicand; by 4 tenths, one place farther to the left; and so on.

This operation saves so much labor that it should be used, when available, throughout the work.

EXERCISE 167. (WRITTEN.)

Perform by contracted multiplication; the first 5, to 2 places, the remainder, to 3 places.

- | | |
|------------------------------|----------------------------------|
| 1. $\$417.87 \times .0783$. | 6. 85.0714×131.131 . |
| 2. $191.45 \times .173$. | 7. $7814.002 \times .0785$. |
| 3. $.956 \times 1.413$. | 8. 20.0003×7.007 . |
| 4. $.7854 \times 3.1416$. | 9. $.13147 \times 480.7$. |
| 5. 91.4726×7.141 . | 10. 171.4112×293.0293 . |

CONTRACTED DIVISION OF DECIMALS.

Divide 4129.7854 by 47.62143; 2 decimal places required.

OPERATION.	EXPLANATION.—
$ \begin{array}{r} 47\overline{)62} 143) 4129.7 854(86.72 \\ \underline{38097} \\ 3201 \\ \underline{2857} \\ 344 \\ \underline{333} \\ 11 \\ \underline{10} \\ 1 \end{array} $	The contraction consists in omitting a figure from the right of the divisor, instead of bringing down one at the right of the dividend, in each successive division. The last divisor is the left-hand figure of the divisor. Now take the dividend to include the same denomination as the highest

PROOF.

$$47.62143 \times 8672 \text{ reserving 1 dec. place.}$$

$$\begin{array}{r} 47.62143 \\ 86.72 \\ \hline 285.7 \\ 38097 \\ 333 \\ 10 \\ \hline 4129.7 \end{array}$$

place in the divisor, and the quotient will be units (see p. 110); therefore go to the right of this denomination as many places as you wish to reserve decimal places in the quotient.

Cut down the divisor from the right until it is contained in this dividend.

1. $3.4268731 \div .284638413$, reserving 2 decimal places.
2. $.04278593 \div .02872539$, reserving 3 decimal places.

PRACTICAL WORK IN DECIMALS.

EXERCISE 168. (WRITTEN.)

1. In 1880 California raised 240.25 acres of cotton, averaging .63 of a bale to the acre; how many bales were produced?
2. If there are 475 pounds of cotton in a bale, what number of pounds to the acre was produced?
3. At \$40 an acre, what are 3 fields worth, containing, respectively, 17.6 acres, 23.25 acres, and 42.625 acres?
4. A man owning .3125 of a ship, sold .2 of his share; what part had he left?
5. What are 36 dozen eggs worth at \$.12 $\frac{1}{2}$ per dozen?
6. A man divided his ranch of 648.96 acres into 8 equal fields; how many acres did each field contain?
7. At \$2.25 each, how many books can you buy for \$27?
8. How far will a horse travel in 11 hours at the rate of 6.75 miles an hour?
9. A bushel contains 2150.42 cu. in.; 5.16 $\frac{2}{3}$ bushels contain how many cu. in.?
10. A man earns \$1.37 $\frac{1}{2}$ a day; if he works 296 days during the year, what will he earn?

11. There are 16.5 feet in 1 rod; how many rods are in 272.25 feet?

12. A man walks 32.75 miles on Monday, 29.8 on Tuesday, 27.41 on Wednesday, 40.5 on Thursday, $31.66\frac{2}{3}$ on Friday, and $25.33\frac{1}{3}$ on Saturday; how far did he walk during the week?

13. What was his average distance per day?

14. A real estate agent having 3218 acres of land to sell, sold, on different occasions, 278.15 acres, 392.14 acres, 171.9 acres, 429.51 acres, and 530.875 acres; what had he left?

15. Allowing 2.625 yards to a pair, how many pairs of pants can be made from a piece of cloth containing 42 yards?

16. There are 231 cubic inches in a gallon, and 31.5 gallons in a barrel; how many cubic inches are in a barrel?

17. How many rods of fence will surround a field 32.0625 rods long and 28.4375 rods wide?

18. How many turns will the driving-wheel of a locomotive make in going 1 mile, the wheel being 21.96 feet in circumference?

19. I bought 12 horses at \$81.875 apiece, and gave a 1000-dollar note in payment; what change did I receive?

20. At \$7.75 per cord, how many cords of wood can be bought for \$162.75?

21. I have 4 fields; the first contains 7.231 acres, the second 9.124 acres, the third 6.715 acres, and the fourth $\frac{1}{3}$ as much as the other 3 together; what do they all contain?

22. What are all worth at \$50 an acre?

23. I spend .08 of my money one day, .16 a second day, $\frac{1}{2}$ of it a third; if I have \$26 left, how much had I at first?

24. A man on a journey goes $\frac{1}{3}$ of it on Monday, and .450 on Tuesday; how much has he left to perform?

25. I have $15\frac{1}{2}$ cords of wood in one pile, $17.66\frac{2}{3}$ in a second, $14\frac{1}{3}$ in a third, and 15.125 in a fourth; how many cords in all?

26. How much is it worth at $\$7\frac{1}{2}$ a cord?
27. The distance around a pond is .59375 of a mile; how many times around it can I travel in 8 hours, traveling $4\frac{3}{4}$ miles per hour?
28. How many fence rails 7.5 feet long will go 3 times around a field $15\frac{5}{16}$ rods long and 10.1875 rods wide? Draw diagram.
29. The rainfall at Sacramento for the year ending with August, 1880, was 26.744 in.; 1881, 26.134 in.; 1882, 16.283 in.; 1883, 18.3 in.; 1884, 24.78 in. What was the average yearly rainfall for that time?
30. We inhale about 2.125 gallons of air every minute; how much do we inhale in an hour?
31. Sold 17.125 tons of hay at $\$9\frac{3}{8}$ per ton; what was received for the hay?
32. Two men start from the same place at the same time and travel in opposite directions; one goes 4.64 miles an hour, the other 5.16; how far apart are they in 13 hours?
33. When they are 107.8 miles apart, how many hours have they traveled?
34. The distance around a circle is 3.1416 times the distance across it. If I can walk across a circular field in 48 seconds, how long will it take me to walk around it?
35. If the distance through the earth is 8000 miles what is the distance around it?

Perform decimally examples 1, 2, 3, 5, 8, 10, 14, 17, 21, 25, 27, 30, 35, 37, 38, 39, 40, 52, 53, 57, 65, 71, 73, 76, Exercise 142.

SHORT METHODS IN MULTIPLICATION.

An **aliquot part** of a number is such a number, whole or fractional, as is contained in it an exact number of times. Thus, 20, 25, and $33\frac{1}{3}$ are *aliquot parts* of 100, being contained respectively 5, 4, and 3 times in 100.

1. To multiply by an aliquot part of 100, 1000, etc.

Multiply 447 by $33\frac{1}{3}$.

OPERATION. EXPLANATION.—Multiplying by 100 gives a product

$$\begin{array}{r} 3) 44700 \\ \hline 14900 \end{array}$$
 3 times too large, since $100=3\times 33\frac{1}{3}$; dividing by 3 gives the true product.

2. To multiply by 9, 99, 999, 9999, etc.

Multiply 5728 by 99.

OPERATION.

$$\begin{array}{r} 572800.=100\times 5728 \\ 5728.=1\times 5728 \\ \hline 567072.=99\times 5728 \end{array}$$

Multiply 387 by 999.

OPERATION.

$$\begin{array}{r} 387000. \\ 387. \\ \hline 386613. \end{array}$$

3. To multiply when a part of the multiplier is a multiple of another part.

Multiply 3216 by 357.

OPERATION.

$$\begin{array}{r} 3216 \\ 357 \\ \hline 22512 \\ 112560 \\ \hline 1148112 \end{array}$$

EXPLANATION.—Multiply by 7 for the first product; then this product by 5, since 5×7 times a number is 35 times that number.

4. To multiply numbers whose tens are alike and the sum of whose units is 10.

$$43\times 47=(50\times 40)+(3\times 7)=2021.$$

$$\begin{array}{r} 43 \\ 47 \\ \hline 21=7\times 3 \\ 280=7\times 40 \\ 120=3\times 40 \\ 1600=40\times 40 \\ \hline 2021 \end{array}$$

EXPLANATION.—The product of the tens by 1 more than itself gives hundreds; and the product of the units, units.

The preceding method may be applied to mixed num-

bers whose integral parts are alike and the sum of whose fractional parts is 1.

$$5\frac{2}{7} \times 5\frac{5}{7} = (6 \times 5) + (\frac{2}{7} \times \frac{5}{7}) = 30\frac{10}{49}.$$

5. To find the product of two numbers whose mean number is easily squared.

$$57 \times 63 = 3600 - 9 = 3591.$$

Here the mean or middle number is 60, it being 3 greater than 57, and 3 less than 63.

$$\text{The result is } 60^2 - 3^2 = 3591.$$

The same process may be applied to mixed numbers.

$$4\frac{3}{4} \times 5\frac{1}{4} = 5^2 - \frac{1}{4}^2 = 24\frac{5}{16}.$$

6. To multiply a number by itself or square it.

$$64^2 = 60^2 + 2 \times 4 \times 60 + 4^2 = 4096.$$

64

64

$$\begin{array}{r} 16 = 4^2 \\ 240 = 4 \times 60 \\ 240 = 4 \times 60 \\ \hline 3600 = 60^2 \\ \hline 4096 \end{array}$$

EXPLANATION.—Square the tens; add the product of the tens by twice the units, and the square of the units.

The same result may be reached by reversing the process in No. 5.

$$64^2 = 60 \times 68 + 4^2 = 4096.$$

EXERCISE 169. (WRITTEN.)

- | | | |
|--|----------------------------------|---|
| 1. $4721 \times 999.$ | 9. $82 \times 78.$ | 17. $41 \times 39.$ |
| 2. $117 \times 113.$ | 10. $104 \times 106.$ | 18. $11\frac{1}{2} \times 12\frac{1}{2}.$ |
| 3. $576 \times 33\frac{1}{3}.$ | 11. $4184 \times 125.$ | 19. $99 \times 4201.$ |
| 4. $875 \times 328.$ | 12. $396 \times 16\frac{2}{3}.$ | 20. $68 \times 62.$ |
| 5. $11\frac{3}{8} \times 11\frac{5}{8}.$ | 13. $3248 \times 12\frac{1}{2}.$ | 21. $75 \times 2320.$ |
| 6. $7\frac{1}{9} \times 8\frac{2}{9}.$ | 14. $81 \times 89.$ | 22. $166\frac{2}{3} \times 891.$ |
| 7. $2160 \times 99.$ | 15. $1064 \times 25.$ | 23. $8\frac{1}{9} \times 8\frac{8}{9}.$ |
| 8. $41 \times 41.$ | 16. $94 \times 94.$ | 24. $72 \times 72.$ |

SHORT METHODS IN DIVISION.

1. To divide by an aliquot part of 100, 1000, etc.

Divide 4256 by $33\frac{1}{3}$.

OPERATION.

$$3 \times 42 + 1 = 127 \text{ Quotient.}$$

$$56 - 33\frac{1}{3} = 22\frac{2}{3} \text{ Remainder.}$$

EXPLANATION.—100 is contained 42 times with 56 Rem. $33\frac{1}{3}$ ($\frac{1}{3}$ of 100) is contained $3 \times 42 + (56 \div 33\frac{1}{3})$ or 127 times with $22\frac{2}{3}$ Rem.

Divide 4256 by $14\frac{2}{7}$.

$$7 \times 42 + 3 = 297 \text{ Quo.}$$

$$56 - (3 \times 14\frac{2}{7}) = 13\frac{1}{7} \text{ Rem.}$$

2. To divide by a number a little less than 100, 1000, etc.

Divide 31241 by 99.

OPERATION.

$$\begin{array}{r} 99 \overline{) 31241} \\ \underline{312} \\ 312 \\ \underline{312} \\ 3 \\ \underline{3} \\ \text{Quo.} = 315 \mid 56 = \text{Rem.} \end{array}$$

EXPLANATION.—100 is contained 312 times with a remainder; 99 is contained 312 times with 312 units additional remainder. Dividing this remainder, 100 is contained 3 times with 12 remainder, so 99 is contained 3 times with 3 more remainder. The sum of the quotients is 315, and the sum of the remainders, 56. When the sum of the remainders equals or exceeds the divisor, it must be again divided in the same way.

If the divisor be 98 or 97 the additional remainder is the quotient times 2 or 3.

Divide 31241 by 998.

$$\begin{array}{r} 998 \overline{) 31241} \\ \underline{312} \\ 312 \\ \underline{312} \\ 303 \\ \underline{303} \\ \text{Quo.} = 31 \mid 303 = \text{Rem.} \end{array}$$

EXERCISE 170.

- | | | |
|--------------------------------|---------------------------------|----------------------------------|
| 1. $5280 \div 16\frac{2}{3}$. | 5. $9825 \div 125$. | 9. $21047 \div 98$. |
| 2. $28171 \div 99$. | 6. $7200 \div 11\frac{1}{9}$. | 10. $7519 \div 166\frac{2}{3}$. |
| 3. $7428 \div 97$. | 7. $41256 \div 997$. | 11. $2763 \div 114\frac{2}{7}$. |
| 4. $7800 \div 25$. | 8. $5386 \div 333\frac{1}{3}$. | 12. $3672 \div 212\frac{1}{2}$. |

BILLS.

When one person sells goods to another, or works for another, he writes to the buyer or employer an account of the things sold or work performed, with dates, prices, and amount. Such a writing is called a BILL.

When the bill is paid, the person receiving the money signs his name, with the words, "Received Payment," or, "Paid," at the end. This is called **receipting** the bill.

The abbreviation Dr. for debtor (ower) is sometimes used, showing that the person first named in the bill owes the money.

LOS ANGELES, Mar. 26, 1886.

MR. F. E. ADAMS,

Bought of ELLIS, WELLS, & Co.

1886			@		\$	ct.	\$	ct.
Feb.	10	10 lb. Gran. Sugar,	@	9 $\frac{1}{2}$ [¢]		95		
"	"	3 " Cheese,	"	15 [¢]		45		
"	15	2 bags Flour,	"	\$1.25	2	50		
Mar.	1	2 lb. Coffee,	"	37 $\frac{1}{2}$ [¢]		75		
"	"	2 " Tea,	"	55 [¢]	1	10		
"	10	3 rolls Butter,	"	60 [¢]	1	80	7	55

Rec'd Payment,

ELLIS, WELLS, & Co.

SAN JOSE, CAL., Mar. 1, 1886.

MR. A. S. AMES,

To J. E. SYMONDS, *Dr.*

To 5 days' Labor, @ \$1.25 \$6.25

Rec'd Payment,

J. E. SYMONDS.

EXERCISE 171.

Copy, on paper or slate, carry out the items, and receipt the bills found on page 120.

1.

OAKLAND, June 1, 1886.

MR. GEO. H. JONES,

To BARNES & COLE, Dr.

1886					
May	30	To	1 bbl. Gran. Sugar, 245 lb. @ 8¢		
			" 1 10-pound sack Oatmeal, . . .		25
			" 3 lb. Honey, . . . @ 12½¢		
			" 4 sacks Flour, . . . " \$1.35		
			" 3 lb. Raisins, . . . " 15¢		
			" 7 doz. Eggs, . . . " 16¢		
			" 10 lb. Crackers, . . . " 8½¢		
			" 1 caddy Japan Tea, 22 lb., " 65¢		
			" 10-lb. sack Salt, . . . " 3½¢		

2.

SAN FRANCISCO, Mar. 1, 1885.

MERCANTILE LIBRARY,

To JAMES LAND, Dr.

		To	binding 27 vol. Atlantic Monthly, . . @ 90¢		
		"	" 2 " Pop. Sci. Mo., . . " 90¢		
		"	" 3 " St. Nicholas, . . . " 75¢		
		"	" 1 " Overland Mo.,		90
		"	" 3 " Harper's Mo., . . . " 90¢		
		"	" 4 " Century, " 75¢		

3.

HOLLISTER, CAL., Sept. 3, 1884.

MR. H. B. CROCKETT,

Bought of SMITH & TYLER.

1884					
June	3	5	Gent's Collars, @ 30¢		
"	"	1	doz. Hdki., " 25¢		
July	5	2	pr. Kid Gloves, " \$1.75		
"	"	3	doz. Buttons, " 40¢		
"	17	3	Fine Linen Shirts, " \$2.25		
Aug.	10	4	pr. Gent's Hose, " 35¢		
"	13	3	pr. Linen Cuffs, " 40¢		
"	28	1	Derby Hat,	3	50

EXERCISE 172.

Make out bills of the following, and receipt them, using your own and classmates' names:

1. 5 yd. Ribbon @ $12\frac{1}{2}$ cents. 11 yd. Black Cashmere @ \$1.60. 4 doz. Buttons @ 30 cents. 2 yd. Silicia @ 20 cents. 10 yd. Sheeting @ 18 cents. 1 pr. Gaiters \$3.50.

2. 5 gal. Kerosene Oil @ 25 cents. 3 pr. Blankets @ \$6.50. 25 lb. Brown Sugar @ 7 cents. 3 doz. Eggs @ 20 cents. 1 Turkey, 12 lb., @ 22 cents. 50 lb. Irish Potatoes @ $1\frac{1}{2}$ cents.

3. Mar. 3, 1880, 2 lb. Steak @ $12\frac{1}{2}$ cents. Mar. 4, $4\frac{1}{2}$ lb. Roast Beef @ 12 cents. March 5, $1\frac{3}{4}$ lb. Sirloin @ 15 cents. March 6, $5\frac{1}{2}$ lb. Mutton @ 10 cents. Mar. 8, 1 15-lb. A. & C. Ham @ 19 cents. Mar. 10, 3 lb. Veal Roast @ 14 cents.

4. John Smith performed 12 days' work for M. S. Johnson at \$1.50 per day.

5. $\frac{1}{2}$ doz. Wooden Chairs @ \$1. 1 Lounge \$12.50. 1 Bed Room Set \$22.75. 3 Fancy Chairs @ \$2.25. 1 Extension Table \$7.50. 1 Center Table \$4.

6. S. Wilson sold Geo. Sims 10 tons of hay @ \$10 a ton.

7. 8 doz. Oranges @ 15 cents. 10 lb. Nuts @ 10 cents. 8 Lemons @ $2\frac{1}{2}$ cents. 5 lb. Mixed Candies @ 20 cents. 1 box Apples \$1. 7 boxes Strawberries @ 45 cents.

8. 1 doz. Lead Pencils @ 5 cents each. $\frac{1}{4}$ ream Note Paper 40 cents. 4 Note Books @ 10 cents. 1 Rubber Eraser 5 cents. 1 package Envelopes 10 cents. 2 Fifth Readers @ 85 cents. 2 School Geographies @ \$1.40.

9. 14 yd. Print @ 12 cents. 3 lb. Butter @ 28 cents. 4 bars Soap @ 10 cents. 1 pr. Child's Shoes \$1.75. 25 lb. Flour @ $2\frac{1}{2}$ cents. 1 can Lard 65 cents. 2 lb. Cheese @ 17 cents.

EXERCISE 173.

Ask your parents for 3 bills they may have, and bring to the class for dictation. They may caution you not to lose them even though paid. Why?

WEIGHTS AND MEASURES.

A *concrete number* when written in terms of one denomination is **simple**; thus,

125 yards; 15.25 dollars; 3.435 hours are simple numbers.

But when expressed in two or more different units it is said to be **compound**; thus,

2 yards 2 feet 3 inches is a compound number.

These compound numbers were used before the decimal notation was known to English-speaking people. Calculations are made much simpler and easier by the use of decimals.

The compound number has properly no unit; although one of any denomination may be taken as the unit. In the example given above of 2 yd. 2 ft. 3 in., the unit may be 1 yd., 1 ft., or 1 in.

15.6 pounds: 7 lb. 6 oz.; 16 ounces; 17 gal. 3 qt. 1 pt.; 13.75 inches; 156 A. 25 rd.; 13 quarts; 7 bu. 3 pk. 7 qt. 1 pt.

From the above select the simple and the compound numbers, and analyze in each case, thus:

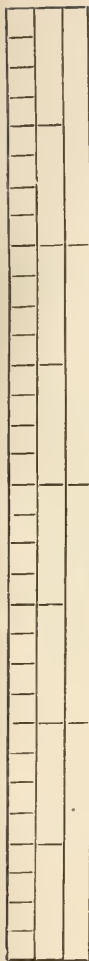
1. — is a *simple number* because it is a — — written in — of one —.

2. — is a *compound number* because it is — — written in — or — —.

LONG OR LINEAR MEASURE.

The denominations of *length* or *line measurement* are given in the following table:

12 inches (in.)	= 1 foot (ft.).
3 ft.	= 1 yard (yd.).
5½ yd.	= 1 rod (rd.).
320 rd.	= 1 mile (mi.).



In the 4-inch scale here given what divisions of the inch do you find?

Draw on paper a line $2\frac{7}{8}$ in. long.

Draw a line that you think is $3\frac{1}{2}$ in. long.

Measure it.

Draw on the blackboard a line 1 ft. long; $1\frac{1}{2}$ ft. long; 1 yd. long.

Divide, without measuring, the last line into parts each 9 in. long.

Now measure them.

Divide your yard line into 3 equal parts. How long should each part be? Measure.

Draw the same line vertical or oblique and divide it into 6 in. parts.

Estimate the width and length of your desk; measure it. The width and height of the window; the door; dimensions of the blackboard; dimensions of the teacher's desk; measure each.

Estimate the length and width of your school-room, and measure, using the yardstick or tape measure.

Pace off the length and width of your school yard, and then measure with the tape-line. Take long steps. Find from this how long your paces are.

A bright, intelligent class well fitted in the work thus far, will need the pencil or crayon to record results only in most of the following work.

EXERCISE 174.

1. How many in. in $1\frac{1}{2}$ yd.? In 2 yd.?
2. How many ft. in 7 yd.? In 4 yd.?
3. How many yd. in 3 rd.? In 4 rd.?
4. In 108 in. how many ft.? In 720 in.?
5. What part of a mi. are 16 rd.? 160 rd.?
6. What part of a ft. are 9 in.? 6 in.? 4 in.?

7. What part of a yard are 2 ft. ? 9 in. ? 18 in. ?
8. In $\frac{1}{2}$ mi. how many rd. ?
9. In $\frac{1}{3}$ yd. how many in. ?
10. In $\frac{1}{8}$ mi. how many yd. ?
11. 2 ft. 6 in. are what part of a yd. ?
12. How many rd. in 1 mi. ? 1 mi. 25 rd. ?
13. How many yd. in 5 rd. ? In 5 rd. 3 yd. ?
14. How many yd. in 1 mi. ? How many ft. in 1 mi. ?
How many in. ?
15. In 1 mi. 20 rd. 4 yd. how many yd. ?
16. How many inches in 1 rd. 2 yd. 2 ft. 7 in. ?
17. How many ft. are there in 2 mi. 35 rd. 2 yd. 2 ft. ?
18. How many ft. and in. are there in 40 in. ?
19. How many yd. ft. and in. are there in 79 in. ?
20. How many rd. yd. ft. and in. are there in 607 in. ?
21. How many yd. ft. and in. are there in 874 in. ?
22. How many yd. ft. and in. in 211 in. ? In 100 in. ?
23. How many rd. yd. and ft. in 373 ft. ?
24. How many rd. are there in 35 yd. ?
25. How many mi. and rd. are there in 650 rd. ?
26. How many rd. and yd. are there in 98 yd. ?
27. How many rd. yd. and ft. in 1000 ft. ? In 179 ft. ?

To reduce to a decimal of any denomination.

28. Reduce 3 mi. 235 rd. 2 yd. 2 ft. 3 in. to rd.

12	3 in.	
3	2.25 ft.	adding the 2 ft.
5 $\frac{1}{2}$	2.75 yd.	adding the 2 yds.
	235.5 rd.	adding the 235 rds.
	960.0 = 3 mi.	
	1195.5 rd.	

29. Express 2 ft. 6 in. in inches. Express decimally, using the ft. as the unit.

30. Express 3 mi. 2 rd. 4 yd. 2 ft. in ft. Express decimally, using the yd. as the unit.

31. Express 3 yd. 2 in. as in. Express decimally, with the yd. as the unit.

32. Express 1 mi. 2 rd. 2 ft. in ft. Express decimally, with the mi. as the unit.

33. Express 3 rd. 4 yd. 2 ft. 6 in. in inches. Express decimally, using the ft. as the unit.

34. Express 2 rd. 1 yd. 2 ft. 6 in. as in. Express decimally, using the ft. as the unit.

35. Express 1 mi. 2 rd. 1 yd. 1 ft. 6 in. as in. Express decimally, using the yd. as the unit.

36. Express 3 mi. 80 rd. in ft. Express decimally, with the mi. as the unit.

37. Express 2 mi. 2 rd. 3 ft. in ft. Express decimally, with the mi. as the unit.

38. Express 3 rd. 2 yd. 2 ft. 3 in. in in. Express decimally, with the rd. as the unit.

39. Express 4 mi. 240 rd. as yd. Express decimally, with the mi. as the unit.

40. Express 3 mi. 8 rd. 3 yd. 2 ft. 3 in. in ft. Express decimally, with the yd. as the unit.

41. Express 7 rd. 2 yd. 2 ft. 3 in. as in.

To reduce to a fraction of higher denomination.

42. 3 yd. 2 ft. 6 in. is what fraction of a rd.?

3 yd. 2 ft. 6 in.

6 in. = $\frac{1}{2}$ ft.

$2\frac{1}{2}$ ft. = $\frac{2\frac{1}{2}}{3} = \frac{5}{6}$ yd.

$3\frac{5}{6}$ yd. = $\frac{3\frac{5}{6}}{5\frac{1}{2}} = \frac{2\frac{2}{3}}{3}$ rd.

43. Change $4\frac{2}{7}$ rd. to a fraction of a mi.

44. Reduce $\frac{5}{8}$ mi. to a decimal of a mi.

45. Reduce .375 mi. to rd.
46. 2 ft. $6\frac{3}{4}$ in. are what decimal of a rd.?
47. Change 65 rd. 2 yd. 2 ft. 6 in. to the decimal of a mi.
48. Express 25 rd. $4\frac{1}{2}$ yd. as a decimal of 35 rd. 3 yd. $2\frac{3}{4}$ ft.
49. Express 42 rd. 2 yd. 4.3 in. as a decimal of a mi.
50. Express 6 ft. 8.5 in. as a decimal of a rd.
51. Express $3\frac{1}{2}$ yd. as a fraction of 7 yd. 4 in.
52. Express 165 rd. 2 yd. 2 ft. 9 in. as a fraction of a mi.
53. Express 2 yd. 2 ft. 2 in. as a fraction of 3 yd.
54. Express 98 rd. 7 yd. 2 ft. 4 in. as a fraction of a mi.

EXERCISE 175.

Bring in 10 examples of your own like those of the preceding exercise, for dictation.

SURVEYOR'S LONG MEASURE.

The land surveyor uses the following table. It has some advantages over the table given above, being partly decimal.

25 links (l.)	= 1 rd.
4 rd.	= 1 chain (ch.).
80 ch.	= 1 mile (mi.).

EXERCISE 176.

1. How many ch. in 2 mi.? In $3\frac{1}{2}$ mi.?
2. How many links in 4 rd.? In 5 rd.?
3. How many rd. in 7 ch.? In $5\frac{1}{2}$ ch.?
4. How many yd. in 2 ch.?
5. How many ch. in $\frac{1}{4}$ of a mi.?
6. In 160 ch. how many mi.? In 320 ch.? In 96 ch.?
In 100 ch.?
7. What part of a ch. are 75 l.? 20 l.?
8. What part of a mi. are 5 rd.? 16 rd.?

9. What part of a rd. are 5 l.? 10 l.?
10. Reduce 1 mi. 2 ch. 1 rd. to l.
11. Reduce 2 mi. 2 rd. 6 l. to l.
12. Reduce 5 mi. 79 ch. 3 rd. to rd.
13. Change 29763 l. to higher denominations.
14. Change 8543 l. to higher denominations.
15. How many mi. ch. rd. and l. in 796328 l.?
16. How many mi. ch. rd. and l. in 76543 l.?
17. Reduce 5 ch. 15 l. to rd.
18. What part of a rd. is 15 l.?
19. How many rd. in $3\frac{1}{4}$ ch.? How many l.?
20. Reduce 1 mi. 1 ch. 1 rd. 1 l. to ch. To rd. To mi. To l.
21. What is the difference between 15 ch. 44 l. and 15.44 ch.?

LONG MEASURE—METRIC SYSTEM.

The French express measures and weights decimally, and their methods and measuring units have been adopted by most of the nations of Europe. In the United States this system has been legalized, and is coming slowly into use.

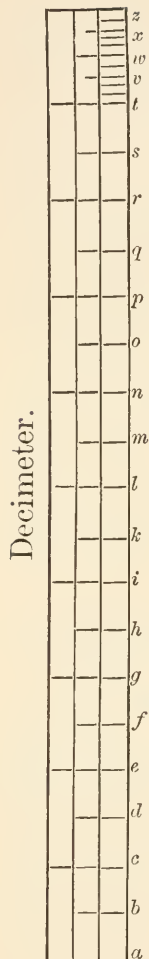
The *unit of length* is one ten-millionth the meridian distance from the equator to the pole; is nearly 39.37 inches, and is called a *meter*, whence this system is called the **metric system**.

The decimal places have received names; thus,

myriameter.	kilometer.	hektometer.	dekameter.	meter.	decimeter.	centimeter.	millimeter.
0	0	0	0	0	0	0	0

Observe. { That *myria, kilo, hekto, deka, deci, centi, milli*, are
 { prefixes used in weights and measures.

EXERCISE 177.



1. How many centimeters are there in 47.265 meters? How many decimeters? How many dekameters?

2. Read 3825.386 meters, by placing in turn each of its names in place of meters and without changing its value.

3. In the above metric scale what is the distance from "c" to "s"?

4. Write the distance from "d" to each of the following points: a, n, q, r, h, t.

5. Write the distance from "x" to each of the following points: v, t, o, l, h, m, b, a, q, g.

6. Draw on paper a line .187 meters long.

7. Draw on the blackboard a line 1 meter long.

8. Divide this line into halves; into tenths. What are the last divisions called?

9. How many feet and inches in 5.24 meters?

10. How many yd. ft. and in. in 35.428 meters?

11. In 5785 meters how many mi.?

12. In 7856918 in. how many meters?

SURFACE MEASURE.

A flat surface of four straight edges and square corners is a **rectangle**.

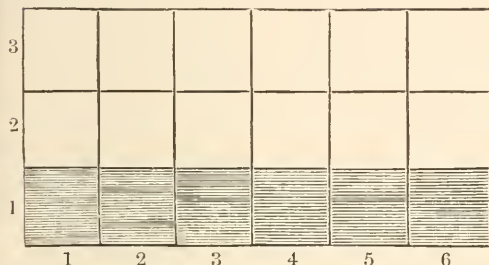
A **square** is a rectangle with equal edges.

The **measuring unit** is a square having a linear unit for its edge: as 1 *square foot*, 1 *square meter*, 1 *square inch*, 1 *square mile*.

TABLE.

144	square (sq.) in.	= 1 sq. ft.
9	sq. ft.	= 1 sq. yd.
$30\frac{1}{4}$	sq. yd.	= 1 sq. rd.
160	sq. rd.	= 1 acre. (A.)
640	A.	= 1 sq. mi. or section of land.

The side or edge of the acre is not a unit of length.



A rectangle 6 units long and 1 unit wide contains 6 square units; 3 units wide it contains 3 times 6 sq. units, or 18 sq. units; hence,

To find the area of a rectangle.

Observe. $\left\{ \begin{array}{l} \text{1st.—The length and the breadth are factors of} \\ \text{the area.} \\ \text{2d.—The multiplier is an abstract number.} \end{array} \right.$

EXERCISE 178.

1. A rectangle 8 in. long and 5 in. wide contains how many sq. in.?
2. How many sq. ft. in a rectangle 17 ft. long and 13 ft. wide?
3. How many sq. yd. in a square whose edge is 36 ft.?
4. How many sq. in. in 2 sq. ft.?
5. In 6 sq. yd. how many sq. ft.?
6. In 3 A. how many sq. rd.?
7. In $1\frac{1}{2}$ sq. mi. how many sq. rd.?
8. In 36 sq. ft. how many sq. yd.?
9. How many sq. ft. in 288 sq. in.?
10. In $2\frac{1}{2}$ acres how many sq. rd.?
11. How many sq. ft. in a table $5\frac{1}{2}$ ft. long and 3 ft. wide?

12. If your reader is $7\frac{1}{2}$ in. long and 5 in. wide, how many sq. in. in the surface of its sides?
13. How many sq. in. in $\frac{3}{4}$ of a sq. ft.?
14. How many sq. ft. in $\frac{2}{3}$ of a sq. yd.?
15. What cost a quarter section of land at \$1.25 an acre?
16. How many sq. yd. in 2 sq. rd.?
17. In a piece of land 9 ft. wide and 12 ft. long, how many sq. yd.?
18. Reduce 1 sq. rd. to sq. in. One A. to sq. ft.
19. Change 2 A. 40 sq. rd. 17 sq. ft. to sq. ft.
20. How many sq. ft. in 3 A.?
21. Find the number of sq. yd. in 3 sq. mi., 17 sq. rd., and 4 sq. yd.
22. Find the number of sq. ft. in 3476 sq. in.
23. How many sq. ft. and in. in 98756 sq. in.?
24. Change 7856 sq. ft. to higher denominations.
25. Reduce 48413 sq. yd. to higher denominations.
26. At $\$75\frac{3}{4}$ an A., what is the value of a farm 189.5 rd. long and 150 rd. wide?
27. If 37 A. 128 sq. rd. are uncultivated in a farm of 170 A. 16 sq. rd., what part of the farm is cultivated?

SURVEYOR'S SURFACE MEASURE.

625 sq. l.	= 1 sq. rd.
16 " rd.	= 1 " ch.
10 " ch.	= 1 A.
640 A.	= 1 sq. mi. or section.

EXERCISE 179.

1. How many sq. rd. in 3 sq. ch.? In $2\frac{1}{2}$ sq. ch. how many sq. rd.?
2. How many sq. ch. in 5 A.? In $6\frac{1}{2}$ A.?
3. In 1 sq. mi. how many sq. ch.?
4. How many sq. ch. in $\frac{1}{5}$ of an A.?

5. Twenty-five sq. l. are what part of a sq. rd.?
6. Eight sq. rd. are what part of a sq. ch.?
7. What part of an A. are 5 sq. ch.? 2 sq. ch.?
8. 80 A. are what part of a sq. mi.?
9. In a quarter section land, how many A.?
10. In a section of land, how many sq. l.?
11. Reduce 160 A. to sq. l.
12. How many sq. ch. in 2 sq. mi. 6 A. 9 sq. ch.?
13. Reduce an A. to sq. l.
14. In 1 sq. mi. 1 A. 1 sq. ch. 1 sq. rd., how many sq. l.?
15. Change 842590 sq. l. to higher denominations.
16. Reduce 25373896 sq. l. to higher denominations.
17. In 98754 sq. rd., how many A., sq. ch., and rd.?
18. In 9857 sq. ch., how many sq. mi., A., and ch.?
19. Reduce 75328 sq. rd. to higher denominations.
20. A man owned a piece of land 46 ch. long by 37 ch. wide; he sold a piece containing 42 A. $5\frac{5}{10}$ sq. ch.; what part of the whole was left?
21. A man owns a piece of land containing 12 A.; has an irrigating ditch cut through one part of it, 25 l. wide and 5 ch. long; what part of an A. does he lose by the ditch, and what part of the whole can he cultivate?
22. A surveyor, starting at a certain point, laid his chain 10.6 times to the east, then south 5 times, then west 5.3 times, then south 3 times, then west 5.3 times, then measured north to the place of beginning; how long was his last line, and how many acres in the field?

SURFACE MEASURE—METRIC SYSTEM.

The unit of land measure is a square whose edge is 10 meters; hence the unit contains 100 square meters. The name, **are**, meaning area, has been given to it.

All perfect squares, in the decimal system, end in the

alternate places commencing with units; hence only those places have received names.

hektare.	are.	centiare=sq. meter.	sq. decimeter.	sq. centimeter.
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

Small areas are given in *square meters* or *centiares*, while large ones are usually given in *hektares*.

The hektare=2.47 acres, nearly.

EXERCISE 180.

1. 236.47925 ares: read this number as hektares; as centiares; as square meters; as square decimeters; as square centimeters.

2. In 34652 sq. centimeters, are how many ares?

3. How much land in a field 234.56 meters long and 184.25 meters wide?

4. What is the difference between 6 square meters and 6 meters square?

5. How many ares in 1 quarter section of land?

CARPETING.

The lighter carpets, as ingrains, are 1 yard wide; the heavier, as Brussels, etc., are $\frac{3}{4}$ yd. wide. Carpets waste in matching, according to the pattern, the breadths requiring to be cut longer than the room.

EXERCISE 181. (WRITTEN.)

1. How many yd. of carpeting $\frac{3}{4}$ yd. wide will it take for a room 17 ft. long, 15 ft. 9 in. wide, if the breadths run lengthwise?

2. How much yard-wide carpeting will be required for a room 17 ft. 6 in. wide, 23 ft. 4 in. long, if the breadths run crosswise? If they run lengthwise?

~~3.~~ How many yd. of carpeting $\frac{3}{4}$ yd. wide will it take for a room 11 ft. wide, 15 ft. long, if the breadths run across the room? If they run lengthwise?

~~4.~~ What will it cost to carpet a room 19 ft. wide and 24 ft. long, with carpet a yd. wide costing \$1.25 per yd., the breadths to run crosswise and $\frac{1}{8}$ of a yd. on each breadth allowed for matching?

5. Find the smallest cost at which a room can be carpeted, which is 13 ft. 6 in. wide, 18 ft. long, the carpet $\frac{3}{4}$ yd. wide at \$2.75 per yd., $\frac{1}{4}$ yd. in each breadth allowed for waste in matching.

PLASTERING.

Usually estimated by the *square yard*. Multiply the distance around the room by the height from floor to ceiling; deduct $\frac{1}{2}$ the surface of the openings and add the area of the ceiling.

6. At \$.27 per sq. yd. what will it cost to plaster a room 18 ft. wide, 20 ft. long, 10 ft. high, having 3 windows, each 2 ft. wide, 6 ft. high, and 1 door 3 ft. wide, 7 ft. high?

~~7.~~ How many sq. yd. of plastering in 17 rooms of a hotel, each 11 ft. wide, 12 ft. long, 12 ft. high, there being 1 window $2\frac{1}{2}$ ft. wide and 6 ft. high, and 1 door $2\frac{2}{3}$ ft. wide and 7 ft. high in each room?

8. How many sq. yd. of plastering in a hall 90 ft. long, 65 ft. wide, and 24 ft. high, there being 13 windows, each 3 ft. wide and 10 ft. high, and 4 doors, each 4 ft. wide and 9 ft. high?

9. How many yd. of plastering in a room 16 ft. wide, 24 ft. long, and 9 ft. high, allowing 12 sq. yd. for doors and windows?

10. What will it cost to plaster a room 24 ft. 6 in. long, 15 ft. 3 in. wide, and 10 ft. high, at \$.30 per sq. yd., allowing 14 sq. yd. for doors and windows?

11. Find the cost of papering a room 15 ft. wide, 18 ft.

long, and 10 ft. high, with paper 24 in. wide at \$.95 a roll, 8 yd. in a roll, 20 sq. yd. allowed for doors and windows.

12. Find the cost of plastering a room 17 ft. 6 in. wide, 24 ft. 8 in. long, 10 ft. high, at \$.33 per sq. yd., allowing 50 sq. yd. for doors and windows.

13. What will be the cost of the paper for the same room, 18 in. wide, 8 yd. in a roll, at \$.75 a roll?

14. How many thousand shingles will it take to cover a roof whose rafters are 25 ft. long, and ridge pole 30 ft. long, if 4 in. in width and 5 in. in length of each shingle is exposed to the weather?

15. How many bricks 8 in. long, 4 in. wide, will be required for a sidewalk 100 ft. 4 in. long and 4 ft. wide?

16. If it takes 840 sheets of tin 16 in. wide and 24 in. long, to roof a house, what is the area covered?

17. What will it cost to plaster a room 18 ft. long, 16 ft. wide, and 12 ft. high, at $$.37\frac{1}{2}$ per sq. yd., allowing for 3 windows $2\frac{2}{3}$ by 8 ft. and 2 doors 3 by 8 ft.

18. In a border of tiling around my fireplace, 8 in. wide, 3 ft. 8 in. high, and 4 ft. across the top, how many tiles 4 in. sq.? Draw diagram before working.

EXERCISE 182.

Measure the length, breadth, and height of all the rooms in your school building; also take the dimensions of the windows, doors, and baseboards. Estimate the cost of carpeting and plastering each room.

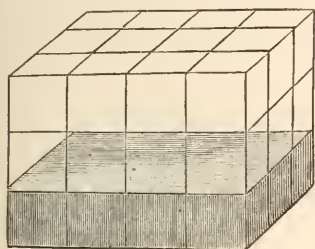
Make the same measurements of three rooms at your homes and bring to the class for similar work.

SOLID OR CUBIC MEASURE.

A solid, all whose faces are rectangles or squares, is a *rectangular solid*; the **cube** has *equal* square faces.

The unit of solid or cubic measure is a cube having a

linear unit for its edge; as *1 cubic inch, 1 cubic yard, 1 cubic meter.*



WORK.—This rectangular solid is 4 units long, 3 units wide, and 3 units high. How many cubic units does it contain?

4 long \times 3 wide gives 12 sq. units in base; for 1 unit high there are 12 cubic units, and for 3 units high 3×12 cubic units = 36 cubic units.

Observe. $\left\{ \begin{array}{l} 1.—\text{The length, width, and height are factors of} \\ \text{the cubic contents.} \\ 2.—\text{The multipliers are abstract numbers.} \end{array} \right.$

Do the same work with the foot as the unit; with the yard; with the meter; with the inch.

TABLE.

1728 cu. in.	= 1 cu. ft.
27 cu. ft.	= 1 cu. yd.
128 cu. ft.	= 1 cd.

EXERCISE 183.

1. How many cu. ft. in a room $17\frac{1}{2}$ ft. long, 14 ft. wide, and 12 ft. high? How many cu. yd.?
2. How many cd. of wood are in a pile 18 ft. long, 12 ft. wide, and $9\frac{1}{4}$ ft. high?
3. How many cu. ft. in $\frac{1}{2}$ cu. yd.?
4. How many cu. in. in a brick a ft. long, 5 in. wide, and $1\frac{1}{2}$ in. thick?
5. How many cu. ft. in 2 cu. yd.?
6. How many cu. ft. in $1\frac{1}{2}$ cd. of wood?
7. How many cu. ft. in 3 cu. yd.?
8. How many cu. yd. in 108 cu. ft.?
9. What part of a cu. yd. are 9 cu. ft.?
10. What part of a cd. are 64 cu. ft.?
11. Four cu. ft. are what part of a cd.?

12. How many cu. ft. in a stick of timber 9 in. wide, 4 in. thick, and 24 ft. long?

13. A trench for a water main is 3 ft. deep, 2 ft. wide, and 21 ft. long; how many cu. ft. have been excavated to form it?

14. How many cd. in a pile of wood 4 ft. wide, 4 ft. high, and 8 ft. long?

15. How many cu. ft. in a tank 8 ft. long, 6 ft. wide, and 3 ft. deep?

16. Change 13 cu. yd. 11 cu. ft. to cu. in.

17. In 9 cu. yd. 4 cu. ft. 13 cu. in. how many cu. in.?

18. Change 159728 cu. in. to higher denominations.

19. If there are 9 cu. ft. 828 cu. in. in one block of stone, and 7 cu. ft. 932 cu. in. in another, how many in both?

20. How many cu. yd. in a room 12 ft. long, 11 ft. wide, and 9 ft. high?

21. A parcel of wrapping paper is 30 in. long, 24 in. wide, and 2 in. thick; how many cu. in. does it contain?

22. If a cu. ft. of stone weighs 175 lb., what will be the weight of a cu. yd.?

23. A water tank is 7 ft. deep, 9 ft. long, and 7 ft. wide. Find its contents in cu. in.

24. If a load of wood is $3\frac{1}{2}$ ft. wide and 5 ft. high, how long must it be to contain $1\frac{1}{2}$ cd.?

25. A block of stone is 7 ft. in each dimension; how many cu. yd. does it contain?

26. How many cd. of wood in a pile 56 ft. long, $4\frac{1}{2}$ ft. high, and 6 ft. wide?

27. A dealer gets \$6.50 a cd. for a pile of wood 16 ft. long, $4\frac{1}{3}$ ft. wide, and 7 ft. 6 in. high; how much does he receive?

28. In 247 cu. ft. how many cu. yd.?

29. The excavation for a block of stores was 63 ft. wide, 157 ft. long, and 8 ft. deep; how many cu. yd. of earth were excavated?

30. Find the value of the wood piled on one half of a vacant lot 60 ft. by 150 ft., the wood being piled 9 ft. high, and worth \$9.50 per cord.

31. Reduce 20 cu. ft. 432 cu. in. to the decimal of a cu. yd.

32. 216 cu. in. are what decimal of a cu. ft.?

33. 648 cu. in. are what fraction of a cu. yd.?

34. Reduce .75 of a cu. yd. to cu. in.

35. What is .975 of a cu. yd. expressed in lower denominations?

36. Express .375 of a cd. in cu. ft.

37. How many cu. ft. of air does your school room contain?

SOLID MEASURE—METRIC SYSTEM.

In the decimal notation of solid measure the **stere**, or cubic meter, is the unit.

EXERCISE 184.

dekastere.	stere = cubic meter.	decistere.	cubic decimeter.	cubic centimeter.	cubic millimeter.
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

The stere = 1.308 cubic yards, or .276 cords.

1. Read 34.6 steres as decisteres; as dekasteres.

2. Read 225.463829731 cubic meters as cubic decimeters; as dekasteres; as cubic millimeters; as cubic centimeters.

3. In 28.5 steres of wood are how many cd.?

4. How much wood in a pile 7.2 meters long, 1.7 meters wide, and 2 meters high?

STONE AND BRICK WORK.

A wall is measured on the outside, no allowance being made for corners. All measurements are made in feet.

Multiply the length thus obtained by the height, deduct the surface of the openings, and multiply by the thickness; divide by $16\frac{1}{2}$ for perches, or multiply by 21 for the number of bricks.

EXERCISE 185.

1. At \$5.25 a perch what will it cost to build a wall around a piece of land 22 ft. by 45 ft., the wall $1\frac{1}{2}$ ft. thick and 8 ft. high?

2. How many bricks would it take to build the walls of a house 30 ft. wide, 45 ft. long, and 20 ft. high, the wall to be 1 ft. thick? There are 10 windows, each $2\frac{1}{2}$ ft. wide and 7 ft. high, and 4 doors, each 3 ft. wide and 8 ft. high.

3. At \$4 per thousand for bricks, what will it cost for the wall of a building 58 ft. long, 25 ft. wide, 44 ft. high, the wall to be $1\frac{1}{2}$ ft. thick; there being 20 windows, each 3 ft. wide, 8 ft. high, and 9 doors, each 3 ft. wide and 8 ft. high?

4. How many perches of masonry in a wall 5 ft. high, $1\frac{1}{2}$ ft. thick, inclosing a garden 9 rd. long, 7 rd. wide?

5. How many perches of stone in a wall 2 ft. thick and 4 ft. high, inclosing a piece of land 40 rd. sq.?

6. How many bricks will it take to build a house 46 ft. long, 34 ft. wide, 20 ft. high, the wall 18 in. thick; allowing for 12 windows, each 8 ft. high and 3 ft. wide, and 6 doors, each 7 ft. 8 in. high and 3 ft. 3 in. wide?

LUMBER MEASURE.

Lumber is commonly estimated by board measure; 1 foot being 1 square foot of surface and 1 inch in thickness.

Less than 1 inch in thickness counts the same as 1 inch. Above 1 inch the thickness is counted by fourths; as, $1\frac{1}{4}$, $1\frac{1}{2}$, $1\frac{3}{4}$, etc. One cubic foot counts as 12 ft. of lumber.

The width of a tapering board is half the sum of the end widths.

7. Find contents and cost of a board 14 ft. long, 1 ft. 4 in. wide, at $7\frac{1}{2}$ cents a ft.

8. Find the contents of a tapering board 15 ft. long, 16 in. wide at one end and 11 in. wide at the other.
9. How many ft. in a stick of timber 30 ft. 6 in. long and 8 in. square?
10. Find the cost of 40 boards 14 ft. long, 11 in. wide, at \$32.50 per thousand.
11. Find cost of 9 planks 12 ft. long, 14 in. wide, and 3 in. thick, at \$40 per thousand.
12. How many ft. in 45 2-by-4 scantlings 18 ft. long?
13. What will 328 inch boards, 12 ft. long, 8 in. wide, cost at \$24 per thousand?
14. How many ft. in a board 12 ft. long, 8 in. wide, $\frac{3}{4}$ in. thick?
15. In a stick of 10-by-12 in. timber 24 ft. long, how many ft.?
16. Find the number of ft. in 8 3-in. planks 14 ft. long and 10 in. wide.
17. In a stick of timber 50 ft. long 12 in. square, how many ft.?
18. In 10 4-by-6 in. joists 18 ft. long, how many ft.?
19. In 8 2-in. planks 16 in. wide 18 ft. long, how many ft.?
20. In a tapering board 11 ft. long 18 in. wide at one end, 11 in. wide at the other, and $\frac{7}{8}$ in. thick, how many ft.?
21. In 2 sticks of timber each 15 in. square and 19 ft. long, how many ft.?

LIQUID MEASURE.

TABLE.

2 pints (pt.)	= 1 quart (qt.).
4 qt.	= 1 gallon (gal.).
31½ gal.	= 1 barrel (bbl.).

The gallon = 231 cubic inches. The barrel is a measure in estimating the capacity of tanks and cisterns. Casks of

all sizes are used for wine, beer, oil, etc., the number of gallons in each being marked on the outside.

EXERCISE 186.

1. How many pt. in 3 qt.? In 3 qt. 1 pt.?
2. How many qt. in 5 gal.? How many pt. in $2\frac{1}{2}$ gal.?
3. How many gal. in 48 qt.? In 96 qt.?
4. What part of a gallon is 1 pt.?
5. What part of 3 qt. are 3 pt.?
6. If milk is worth 8 cents a qt., what will 5 gal. cost?
7. How many pt. of lemonade can be made from the water in a 6-gal. olla which lacks 3 qt. of being full?
8. How many pt. bottles can a druggist fill from 7 gal. of alcohol?
9. How much linseed oil can you buy for \$5, at $12\frac{1}{2}$ ct. per qt.?
10. If a gallon of molasses cost \$.60, what will 3 pt. cost?
11. What will 4 gal. of milk cost at 2 ct. per $\frac{1}{2}$ pt.?
12. A physician uses 1 pt. of distilled water in 1 day; how long will $2\frac{1}{2}$ gal. last him?
13. How many pt. in 1 bbl. 2 qt. 1 pt.?
14. How many qt. in 5 bbl. 6 qt.?
15. How many pt. in a bbl.?
16. Reduce 5 bbl. 2 qt. 1 pt. to pt.
17. Reduce 2 gal. 1 qt. 1 pt. to pt.
18. Change $\frac{1}{2}$ bbl. 3 qt. to qt.
19. Change 1 bbl. $\frac{1}{4}$ gal. 1 pt. to pt.
20. How many bbl. in 7856 qt.?
21. Change 9563 pt. to higher denominations.
22. Change 9543 qt. to higher denominations.
23. Change 86543 pt. to higher denominations.
24. Reduce 6754 gal. to higher denominations.
25. A maker of patent medicine has 16 gal. prepared; how many pt. bottles does he need?
26. Find the cost of 2 bbl. 4 gal. 2 qt. 1 pt. of vinegar at 5 ct. a pt.

27. What will be the cost of the following lots of molasses at \$.75 per gal.: 41 gal. 3 qt. 1½ pt., 25 gal. 7 qt. 1 pt., and 9 gal. 3 qt. 1½ pt.?

28. A man sells in one week 73 gal. 3 qt. of oil, in another 60 gal. 2 qt., in another 40 gal. 1 qt., and in a fourth 65 gal. 2 qt.; what is it worth at \$.17 a gal.?

29. A Ventura oil well yields 150000 gal. of refined oil; how many 5-gal. cans will it fill, and what is the value at \$1.75 a can?

30. 4 gal. 1 pt. is what fraction of a bbl.?

31. What is the value of .75 bbl.?

32. 13 gal. 1 pt. is what fraction of $\frac{2}{3}$ of a bbl.?

33. Reduce $\frac{1}{2}$ of 16 gal. 2 qt. to the fraction of 1½ bbl.

34. Reduce $\frac{3}{4}$ gal. to the decimal of a bbl.

35. How many quarts in .375 of a bbl.?

DRY AND LIQUID MEASURE—METRIC SYSTEM.

The unit, or **liter**, is the cubic decimeter=1.057 quarts.

EXERCISE 187.

myrialiter.	kiloliter.	hektoliter.	dekaliter.	liter.	deciliter.	centiliter.	milliliter.	1. In a cubic meter of water are how many liters?
0	0	0	0	0	0	0	0	2. Read 9852.436 liters with any one of the names in the table as the unit.

3. How many liters are there in a cubic dekameter of water?

4. A rectangular cistern is 2.75 meters long, 1.82 meters wide, and 1.12 meters high, inside measurement. How many liters of water will it hold? How many kilograms?

5. Find the answer to the last example in gallons, and in pounds avoirdupois.

WEIGHT.

This is the measure of the force that draws all bodies downward.

The table in common use is that of

AVOIRDUPOIS WEIGHT.

16 ounces (oz.)	= 1 pound (lb.).
100 lb.	= 1 cental.
20 centals	= 1 ton (T.).

The *cental* is also called the *quintal* and *hundredweight*. 100 lb. of grain or flour is called a *cental*; 100 lb. dried fish, a *quintal*; 100 lb. of other coarse substances, a *hundredweight* (cwt.).

EXERCISE 188.

1. How many oz. in 3 lb.? In 4 lb.?
2. In 5 cwt. 25 lb. how many lb.?
3. How many lb. in $4\frac{1}{2}$ centals?
4. What is a quintal of fish worth at 6 ct. per lb.?
5. How many lb. in 2 T.? In $3\frac{1}{2}$ T.?
6. What part of a ton are 400 lb.? 500 lb.?
7. What part of a lb. are 4 oz.? 12 oz.?
8. What part of a cwt. are 75 lb.? $66\frac{2}{3}$ lb.?
9. 20 lb. are what part of a cental?
10. 60 lb. are what part of a cental?
11. At 5 ct. per lb. what are 7 cwt. of beef worth?
12. At \$. $12\frac{1}{2}$ per lb. what are 12 lb. of cheese worth?
13. How much are $2\frac{1}{2}$ centals of wheat worth at \$.94?
14. A druggist buys powdered bloodroot at \$1.00 per lb. and sells it for $12\frac{1}{2}$ ct. per oz.; what does he gain?
15. A man buys 240 lb. of sugar at the rate of 12 lb. for a dollar, and pays for it in peaches at 2 ct. a lb.; how many lb. of peaches will it take?
16. A woman takes 300 lb. of honey from her hives each month. What is it worth for one year at \$5.00 per cwt.?

TROY WEIGHT.

The following table is used for weighing gold, silver, and precious stones :

24 grains (gr.)	=1 pennyweight (pwt.).
20 pwt.	=1 ounce (oz.).
12 oz.	=1 pound (lb.).

The Avoirdupois pound=7000 Troy grains.

The purity of gold and silver was formerly expressed in 24ths, or carats; thus if $\frac{1}{2}\frac{8}{4}$ of a piece of metal was gold, it was 18 carats fine.

This reckoning of fineness of silver and gold by 24ths, or carats, is now used only for jewelry. In buying and selling, as well as in coining, fineness is now reckoned in thousandths; thus,

925 fine means that $\frac{925}{1000}$ of the entire weight is pure metal.

EXERCISE 189. (WRITTEN.)

1. How many gr. heavier is the Avoirdupois lb. than the Troy lb.?
2. Which is the heavier and by how many gr., the Avoirdupois oz. or the Troy oz.?
3. In $1\frac{1}{2}$ pwt. how many gr.? In 3 pwt.?
4. How many gr. and pwt. in 80 gr.?
5. What part of a pwt. are 12 gr.? 3 gr.? 8 gr.?
6. How many pwt. in 3 oz.? In 5 oz.? In $7\frac{1}{2}$ oz.?
7. In 70 pwt. how many oz.? In 90 pwt.?
8. In 72 oz. how many lb.? In 84 oz.?
9. How many oz. in 9 lb.? In 6 lb.? In $7\frac{1}{2}$ lb.?
10. What part of a lb. are 4 oz.? 6 oz.?
11. What part of a lb. are 40 pwt.? 60 pwt.?
12. If a pwt. of gold is worth \$.95, what are 12 pwt. worth? What are 12 gr. worth?
13. If a salt spoon weighs 5 pwt., how many spoons can be made from 2 lb. of silver?
14. Reduce 35624 avoirdupois oz. to cwt.

15. Change 16256 avoirdupois oz. to higher denominations.
16. How many cwt. in 40607 avoirdupois oz.?
17. In 267235 lb. how many T.?
18. Change 8420724 avoirdupois oz. to T.
19. Reduce 24 lb. 9 oz. 6 gr. Troy, to gr.
20. Change 855 gr. to higher denominations.
21. Change 25 lb. 7 oz. 18 pwt. 9 gr. to gr.
22. Reduce 6 lb. 8 oz. 6 pwt. to gr.
23. Reduce 3756 lb. to gr.
24. Reduce 217 T. 35 lb. to lb.
25. Change 7 T. 9 cwt. 18 lb. to lb.
26. If a man has 987567 lb. of wheat how many centals has he?
27. How many quintals of fish in 9875 lb.?
28. What are 78569 lb. of wheat worth at \$.95 a cental?
29. Find the value of 8564 oz. of tea at \$1.25 per lb.
30. Change 97546 gr. Troy to higher denominations.
31. How many Troy lb., oz., and gr. in 85643 gr.?
32. What are 755 centals of barley worth at \$.84 $\frac{1}{4}$ a cental?
33. Find the cost of 896 lb. of dried apricots at \$25 per cwt.
34. What are 745 lb. of dried ginseng worth at \$.12 $\frac{1}{2}$ per oz.?
35. A lady buys table silver weighing 2 lb. 3 oz. 10 pwt. at the rate of \$1.60 per oz. Find the cost.
36. The Monitor Co. ships by the Wells-Fargo Express Co. 9 bars of silver bullion, each bar weighing 1000 oz., the whole valued at \$9612; what is the value of an oz.?
37. A car-load of ore taken to Denver yields 800 oz. of silver to the ton; if there are 10 tons in the car, what is it worth according to the valuation in the preceding example?
38. A gold mine was sold for \$200000; if the ore yields \$248 a load, how many loads will it take to pay for it, and

how many bricks will it make of 500 oz. each, if an oz. of gold is worth \$20?

39. Reduce 5 oz. 5 pwt. to the fraction of a lb.
40. Express as a decimal $\frac{9}{20}$ of $\frac{45}{100}$ lb.
41. Express .08 lb. as units of lower denominations.
42. What decimal of a lb. is .24 oz.?
43. What decimal of a lb. is 4 oz. 10 pwt.?
44. Change 2 centals 15 oz. to the fraction of a T.
45. Change 3 centals 8 oz. to the decimal of a T.
46. Express in lower denominations $\frac{5}{12}$ of a T.
47. Express in lower denominations .075 of a T.
48. $\frac{7}{15}$ of a T. is what fraction of $\frac{1}{2}$ of $2\frac{1}{2}$ T.?
49. What fraction of 3 T. is .065 of a T.?
50. Reduce 17 centals 50 lb. to the decimal of a T.

WEIGHT—METRIC SYSTEM.

The **gram**, or unit, is the weight of a cubic centimeter of pure water.

ton.	quintal.	myriagram.	kilogram.	hektogram.	dekagram.	gram.	decigram.	centigram.	milligram.	The gram=15.4 grains Troy weight. It is the unit for very small weights. For common purposes the kilogram is the unit.
0	0	0	0	0	0	0	0	0	0	

The kilogram=2.2 pounds avoirdupois.

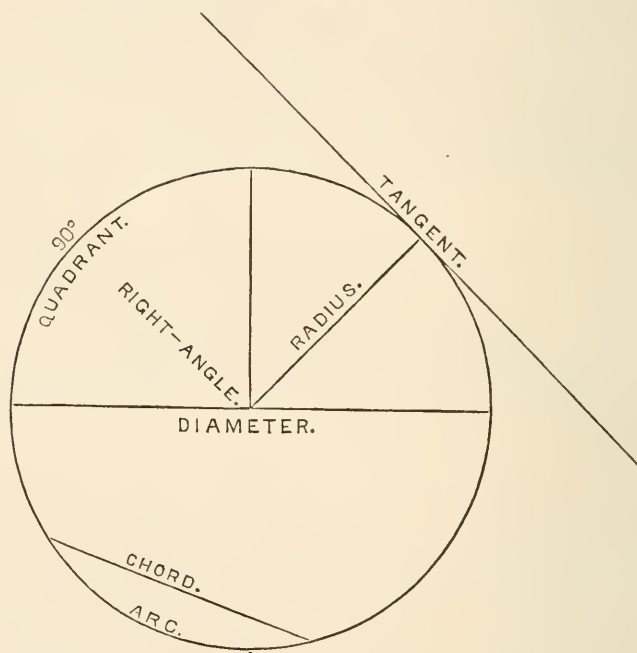
The ton =2204.6 " "

EXERCISE 190.

1. 2784.683 grams: read this value as kilograms; as hektograms; as decigrams.
2. What is the weight of a liter of water?
3. 434.28 grams: give this value in Troy wt.; in Avoirdupois wt.

4. A cistern holds 74625837 grams of water, how many liters of water does it contain?
5. How many liters in a barrel of water?
6. A garden is 115 meters long, 87.5 meters wide; when rain falls on it to the depth of $1\frac{1}{2}$ centimeters, how many liters of water has it received?

CIRCULAR MEASURE.



A circle is a flat surface whose edge is a uniformly curved line.

The edge, or *circumference*, is divided into 360 equal parts called degrees. Learn the names connected with the circle from the figure.

TABLE.

60 seconds (")	= 1 minute (').
60'	= 1 degree (°).
90°	= 1 quadrant.
4 quadrants	= 1 circumference.

EXERCISE 191. (WRITTEN.)

1. What part of a circumference is 90° ? What part is 180° ?
2. How many seconds in $2^\circ 1' 5''$?
3. How many degrees in a quadrant?
4. How many degrees in $\frac{1}{2}$ a circumference?
5. How many degrees in $3600''$?
6. In $2\frac{1}{2}$ minutes, how many seconds?
7. How many degrees in $\frac{1}{2}$ of a quadrant? In $\frac{1}{3}$ of a quadrant? In $\frac{2}{3}$ of a quadrant?
8. $2\frac{1}{2}$ quadrants are what part of a circumference?
9. How many seconds in $29^\circ 35' 26''$?
10. Change $943765''$ to higher denominations.
11. What is $\frac{2}{3}$ of 45° ?
12. What part of a quadrant is $25''$?
13. Reduce .05 of a circumference to the fraction of 3 circumferences.
14. Change .32 of a quadrant to the decimal of a circumference.
15. What part of $5^\circ 2' 3''$ is $1^\circ 40' 41''$?
16. Reduce $9^\circ 25' 48''$ to the decimal of $25^\circ 3' 28''$.
17. What fraction of $9^\circ 8'$ is $\frac{1}{3}$ of $22^\circ 50'$?
18. Change .125 of a degree to minutes and sec.

 TIME.

The earth is a huge pendulum marking time by its motion; turning once around on its axis marks one day.

TABLE.

60 seconds (sec.)	= 1 minute (min.).
60 min.	= 1 hour (hr.).
24 hr.	= 1 day (da.).
7 da.	= 1 week (wk.).
365 da.	= 1 common year.
366 da.	= 1 leap year.

All years divisible by 4 are leap years.

Exception: those divisible by 100 and not by 400 are common years.

The year has 12 months of 28, 29, 30, or 31 days. The following rhymes serve to keep the lengths of the months in the memory:

Thirty days hath September,
April, June, and November,
February twenty-eight,
Thirty-one the others rate.

The extra day for leap year is added to February, making 29.

For school purposes and in working examples, 4 weeks are counted as 1 month.

In reckoning interest 30 days are counted as 1 month.

EXERCISE 192. (WRITTEN.)

1. How many min. in $\frac{1}{4}$ of an hour? In $\frac{1}{2}$ of an hour? In $\frac{3}{4}$ of an hour?
2. How many sec. in $\frac{5}{6}$ of a min.? $\frac{4}{5}$ of a min.? $\frac{2}{3}$ of a min.?
3. How many min. in 5 hr.? How many hr. in 25 da.?
4. Change 3 min. 25 sec. to sec. Change 2 wk. 5 da. to da.; 4 da. to hr.; 7 wk. 3 da. to da.
5. How many wk. in 497 da.? In 427 da.?
6. How many da. in 72 hr.? In 96 hr.?
7. How long is it from 25 min. past 5 A. M. to noon?
8. How many days from Mar. 16 to June 11 of the same year?
9. How many mo. from July 4 to Dec. 4 of the same year?
10. What is $\frac{1}{2}$ of 1 da. 2 hr. 40 min. 20 sec.?
11. If a horse trots 1 mi. in 2 min. 35 sec., how long will it take him to go 3 mi. at the same rate?
12. If a man earns \$3 per day and pays \$5 a week for board and other expenses, what can he save in 6 months?

13. Reduce 5 hr. 15 min. 25 sec. to sec.
14. Reduce 2 yr. 11 da. 12 min. to min.
15. Reduce 3 yr. 37 da. 16 hr. 24 min. 13 sec. to sec.
16. Reduce 58967379 sec. to higher denominations.
17. Change 47675 min. to higher denominations.
18. Change 427329 sec. to higher denominations.
19. Change 157540 min. to higher denominations.
20. Reduce to higher denominations 8567983 sec.
21. How many min. are there from 25 min. past 9 P. M. to 15 min. past 6 the next morning?
22. How much time is there from 9 min. 25 sec. past 3 P. M. to 8 min. 16 sec. of 5 A. M. of the next day?
23. A young lady reads German 25 minutes each day for 5 years; how much time does she spend?
24. If it takes you 25 min. 15 sec. to walk to school, how much time will you spend in 6 mo. if you make two trips a day?
25. Reduce $\frac{5}{6}$ of a common yr. to da.
26. What part of a day are 2 hr. 30 min. 45 sec.?
27. What part of 6 da. 15 hr. 40 min. 36 sec. are 3 da. 7 hr. 50 min. 18 sec.?
28. Reduce .075 of a da.; .625 of a wk.; .378 of a common yr. to lower denominations.
29. What fraction of a wk. is 2 da. 18 hr.?
30. Reduce .58 of a common yr. to lower denominations.
31. Express 4 da. 7 hr. 45 min. 48 sec. as a decimal of 34 da. 14 hr. 6 min. 24 sec.
32. Find the value of .975 of a yr.
33. Express .125 of a yr. in lower denominations.
34. 22 da. 12 hr. is what part of a mo.?
35. 2 hr. 40 min. 36 sec. is what fraction of a mo.?
36. Express 11 hr. 33 min. as a fraction of a wk.
37. Reduce 31 min. 30 sec. to the fraction of a da.
38. Express in integers 4.655 yr.
39. Express in decimals of a week, 3 da. 3 hr.

40. 2 mo. 3 da. 4 hr. 28 min. 28 sec. is what decimal of 6 mo. 1 wk. 2 da. 13 hr. 25 min. ?

41. 3 hr. 37 min. 1 sec. is what decimal of 12 da. 1 hr. ?

LONGITUDE AND TIME.

The earth turns on its axis 360° , or once around, in 24 hours; therefore in one hour it turns $\frac{1}{24}$ of $360^\circ = 15^\circ$.

If then we divide the degrees of longitude between two places by 15, the quotient is the difference of time in hours. Or if we multiply the difference of time, in hours, by 15, the product is the difference in degrees of longitude. Now the hour and degree have the same division into minutes and seconds; hence,

Hours $\times 15 =$ degrees of longitude.

Minutes of time $\times 15 =$ minutes of longitude.

Seconds of time $\times 15 =$ seconds of longitude.

And difference of longitude divided by 15 gives corresponding divisions of difference of time.

EXERCISE 193. (WRITTEN.)

1. What is the difference of longitude of two places whose difference of time is 3 hr. 25 min. 30 sec. ?

2. Two towns have a difference of longitude of $17^\circ 48' 36''$; what is the difference of time ?

In changing difference of time to difference of longitude, we multiply by 15 and divide the products of the minutes and seconds by 60 to reduce. Multiplying by 15 and dividing by 60 is equivalent to dividing by 4, hence we may shorten the work in this way:

1.	3 ^{hr.}	25 ^{min.}	30 ^{sec.}	
	15			
	45°	15'	30''	
	25 ÷ 4 = $\frac{6}{51^\circ}$	30 ÷ 4 = $\frac{7}{22'}$		
			Ans. $51^\circ 22' 30''$.	

Counting from the meridian that passes through the observatory of Greenwich, near London, the longitude of

New York is $74^{\circ} 3' W.$	Paris, $2^{\circ} 20' 22'' E.$
New Orleans, $90^{\circ} 5' W.$	Boston, $71^{\circ} 3' 30'' W.$
Berlin, $13^{\circ} 23' 53'' E.$	Pekin, $116^{\circ} 28' 54'' E.$
Chicago, $87^{\circ} 37' 30'' W.$	Montreal, $73^{\circ} 34' W.$
Cincinnati, $84^{\circ} 26' W.$	St. Petersburg, $30^{\circ} 18' E.$
St. Louis, $90^{\circ} 15' 16'' W.$	St. Paul, $93^{\circ} 5' W.$
Bombay, $72^{\circ} 53' E.$	San Francisco, $122^{\circ} 24' 15'' W.$
Mexico, $99^{\circ} 5' W.$	Omaha, $95^{\circ} 56' W.$
Washington, $77^{\circ} 2' 48'' W.$	Los Angeles, $118^{\circ} 18' W.$
Albany, $73^{\circ} 32' W.$	

15. What is the difference in time between San Francisco and St. Paul?

16. Find the difference in time between New York and New Orleans.

17. What is the difference in time between Berlin and Bombay?

18. Find the difference in time between San Francisco and New York.

19. What is the difference in time between Chicago and St. Petersburg?

20. Find the difference in time between the City of Mexico and St. Louis.

21. What is the difference in time between Cincinnati and Washington, D. C.?

22. What is the difference in time between Pekin and Montreal?

23. When it is noon at San Francisco, what time is it at Paris?

24. When it is six o'clock p. m. in Boston, what time is it at Pekin?

25. When it is midnight in Paris, what time is it in St. Petersburg?

As the earth turns eastward in its daily motion, the sun

appears to move westward 15° per hour. When it is noon at any point, it will be past noon at all places east of that meridian, and before noon at all places west; therefore we reckon to the west for earlier time, and to the east for later.

26. When it is $10\frac{1}{2}$ o'clock A. M. at Sacramento, longitude $121^\circ 26' W.$, what is the longitude of the places having the following times: 7 hr. 20 min. A. M.; 2 hr. 25 min. P. M.; 1 hr. 10 min. P. M.; 5 hr. 15 min. A. M.?

27. What is the longitude of a place where it is 3 hr. 30 min. P. M. when it is 7 hr. 30 min. A. M. in Albany?

28. At Paris it is 4 P. M., and at the same time it is 2 A. M. of the next day in another place; give the longitude of the last place.

29. In Boston it is 1:25 P. M. when it is 1:25 A. M. of the next day at another place; what is the longitude of that place?

30. What is the longitude of the place where it is 9 hr. 25 min. A. M. when it is 6 hr. 30 min. P. M. of the same day at Paris?

31. What is the longitude of a place where it is 15 min. past 8 o'clock A. M., when it is 7 hr. 30 min. A. M. in Omaha?

32. When it is 45 min. past 11 P. M. in Montreal, it is 15 min. past 2 o'clock A. M. next day at another place; what is the longitude of the second place?

33. What place in the above list has 5 hr. 5 min. $21\frac{2}{3}$ sec. earlier time than Paris?

34. A gentleman travels from Boston to Springfield where he finds that, by his watch, the sun rises 6 min. $6\frac{1}{2}$ sec. later than in Boston; what is the longitude of Springfield?

35. At how late an hour may news be telegraphed from New York and reach San Francisco at 3 A. M.?

36. The absolute difference in time between the Bermudas (which are east of New York) and New York is 37 min.; find the longitude of the Bermudas.

37. Find the longitude of the place where it is 20 min.

past 5 P. M. when it is 25 min. past 11 P. M. in St. Petersburg.

The change of time from place to place is very inconvenient for travelers and railways. To do away with this the United States has been divided into four districts by lines running nearly north and south. The eastern district takes the time of the 75th degree of longitude west from Greenwich; the next district takes the 90th degree; the next the 105th, and the western, which includes all the Pacific states and territories, takes the 120th degree.

In traveling across the continent the minute hand of an accurate watch keeps always right; the hour hand alone needs changing.

In changing from local time to the time now kept, what change was made by San Francisco, lon. $122^{\circ} 26' 4''$ W.? What by Los Angeles, $118^{\circ} 18'$? By Sacramento, $121^{\circ} 26'$? By San Diego, $117^{\circ} 10' 40''$?

NOTE.—The examples in this book are wrought for true time and not for the standard time given above.

REFERENCE TABLES.

LENGTH.

7.92 in.	= 1 l.
6 ft.	= 1 fathom.
120 fathoms	= 1 cable-length.
1 nautical mile	= 1.153 common mile.
1 deg. at equator	= 60 nautical miles = 69.16 common miles.
40 rd.	= 1 furlong.
4 in.	= 1 hand.
9 in.	= 1 span.
18 in.	= 1 cubit.
12 lines	= 1 in.
33.39 in.	= 1 vara.

The table of cloth measure seems to be entirely obsolete.

SQUARE OR SURFACE MEASURE.

40 sq. rd. = 1 rood.
100 sq. ft. = 1 square.

CUBIC OR SOLID MEASURE.

16 cu. ft. = 1 cord foot (cd. ft.).
8 cd. ft. = 1 cord (cd.).
 $16\frac{1}{2}$ cu. ft. of masonry = 1 perch.

50 cubic feet of hewn timber, or round timber enough to make 40 ft. of hewn timber = 1 ton.

Masonry is usually estimated by the cubic foot; but the perch is not unfrequently used, and may be that given above, or the old measure of $24\frac{3}{4}$ cubic feet.

Custom in California has reduced the cord of wood to an uncertain quantity, usually called 96 cubic feet or 3 tiers of 12-inch stovewood, 8 ft. long and 4 ft. high.

LIQUID MEASURE.

4 gills (gi.) = 1 pt.
42 gal. = 1 tierce.
2 bbl. = 1 hogshead (hhd.).
2 hhd. = 1 pipe or butt.
2 pipes = 1 tun.

BEER MEASURE.

2 pt. = 1 qt.
4 qt. = 1 gal.
36 gal. = 1 bbl.
 $1\frac{1}{2}$ bbl. = 1 hhd.

The beer gallon = 282 cubic inches.

DRY MEASURE.

2 pt. = 1 qt.
8 qt. = 1 peck (pk.).
4 pk. = 1 bushel (bu.)

The bushel = 2150.42 cubic inches.

The table of dry measure is seldom or never used in California, grain and vegetables being bought and sold by weight. It is still in common use in the eastern states and the Mississippi valley, but the bushel differs somewhat in the different states.

The English standard bushel=2218.19 cu. in.

APOTHECARIES' WEIGHT.

20 gr. = 1 scruple (scr.)

3 scr. = 1 dram.

8 drams = 1 oz.

12 oz. = 1 lb.

Used in mixing medicines but not in buying and selling. The pound, ounce, and grain are the same as in Troy weight.

DIAMOND WEIGHT.

16 parts = 1 carat gr.

4 carat gr. = 1 carat.

1 carat = 3.17 gr. Troy.

Circular measure has been expressed decimally by dividing the quadrant into 100 equal parts called grades, the tenths being called minutes, and the hundredths, seconds. Thus, 45° 30' 15" would be 50.56 grades, but the French people, who invented this, have not generally adopted it.

PAPER AND BOOKS.

24 sheets = 1 quire (qr.).

20 qr. = 1 ream (rm.).

2 rm. = 1 bundle (bun.).

5 bun. = 1 bale.

A sheet folded into 2 leaves forms a folio.

A " " " 4 " " a quarto or 4to.

A " " " 8 " " an octavo or 8vo.

A " " " 12 " " a duodecimo or 12mo.

A " " " 18 " " an 18mo.

A " " " 36 " " a 36mo,

ENGLISH MONEY.

4 farthings (far.)	= 1 penny (d.).
12 d.	= 1 shilling (s.).
20 s.	= 1 pound (£).

The pound = \$4.86.

FRENCH MONEY.

The franc is the unit, tenths being called decimes, and hundredths, centimes. 1 franc = \$.186.

CALIFORNIA MEASURES.

The following approximate measures may be found of practical value:

A cord of stovewood is a pile 8 ft. long, 4 ft. high, and three tiers wide.

Hay is not dried grasses, as in most of the states, but oats, barley, or wheat cut before the grain is fully formed; a ton of hay in the stack, well settled, is a cube each of whose edges is 8 ft., or $8 \times 8 \times 8 = 512$ cu. ft. If loose on the wagon it is $10 \times 10 \times 10 = 1000$ cu. ft.

4 cu. ft. of unshelled corn = 1 cental of shelled corn. If loosely thrown in and not settled, $4\frac{1}{2}$ cu. ft. = 1 cental.

A square whose edge is 70 paces = 1 acre.

Flowing water is measured by the inch in California. The inch, however, is not, at present (1887), a fixed unit, but varies by custom of the different companies supplying water for irrigation or domestic purposes. By statute, the inch is the water flowing through a square inch of vertical surface, the center of the opening being $4\frac{1}{2}$ inches below the surface of the reservoir from which the water is flowing. The amount is given as 1.394 cubic feet per minute.

COUNTING TABLE.

12 units	make	1 dozen (doz.).
12 doz.	“	1 gross (gro.).
12 gro.	“	1 great gro.
20 units	“	1 score (sc.).

LONG TON TABLE.

28 lb.	make	1 quarter.
112 “	“	1 cwt.
20 cwt.	“	1 T.

ADDITION.

EXERCISE 194. (WRITTEN.)

1. Add 69 rd. $2\frac{1}{2}$ yd., 1 mi. 14 rd. 2 yd. 2 ft. 3 in., 16 rd. 9 in., and 25 rd. 11 ft.
2. Add 7 yd. 2 ft., 5 yd. $1\frac{1}{4}$ ft., 2 ft. $9\frac{1}{2}$ in., 3 yd. 1 ft. $6\frac{1}{2}$ in., $2\frac{3}{4}$ ft., and $4\frac{1}{2}$ yd.
3. Add 25 yd. 1 ft. 9 in., 32 yd. 1 ft. 8 in., 35 yd. 6 ft. 4 in., 7 yd. 2 ft. 11 in., and 9 ft.
4. Add 23 mi. 118 rd. 14 ft., 19 mi. 137 rd. 11 ft., 8 mi. 62 ft. 8 in., 23 mi. 147 rd. 6 in., and 9 rd. 7 in.
5. Add 22 rd. 2 yd. 2 ft., 18 rd. 4 yd. 2 ft., 22 rd. 6 yd. 1 ft., and 16 rd. 4 ft. 3 in.
6. Add 7 mi. 59 rd. 6 ft. 7 in., 8 mi. 96 rd. 7 ft. 8 in., 5 mi. 9 rd. 8 in., 26 mi. 87 rd. 8 ft. 3 in.
7. Add 71 mi. 23 rd. $4\frac{1}{2}$ yd., 9 mi. 17 rd. 2 yd. $2\frac{1}{2}$ ft., 23 mi. 3 yd. 9 in.
8. Add $1\frac{1}{2}$ yd. 3 in., 2 ft. 4 in., and $3\frac{1}{4}$ ft.
9. Add $\frac{1}{2}$ yd., $\frac{1}{4}$ ft., and $\frac{1}{2}$ rd.
10. Add $\frac{3}{4}$ mi., $\frac{1}{2}$ rd., $\frac{1}{2}$ yd., and $\frac{3}{4}$ ft.
11. Add 79 ch. 3 rd. 16 l., 65 ch. 2 rd. 11 l., 33 ch. 2 rd. 6 l., 46 ch. 1 rd. 13 l., 75 ch 2 l.
12. Add 75 A. 4 sq. rd. 9 sq. yd. 72 sq. in., 27 A. 48 sq. rd. 18 sq. yd. 92 sq. in., 7 A. 100 sq. rd. 29 sq. yd. 8 sq. ft. 139 sq. in., and 7 sq. yd. 129 sq. in.
13. Add $\frac{1}{2}$ A. and $\frac{5}{9}$ sq. yd.
14. Add $\frac{2}{3}$ A., $\frac{4}{9}$ sq. rd., and $\frac{2}{3}$ of a sq. yd.
15. Add 5 cd. 7 cd. ft., 2 cd. 2 cd. ft. 12 cu. ft., 6 cd. ft. 15 cu. ft., $7\frac{2}{3}$ cd., 3 cd. 2 cu. ft.
16. How many cu. yd. and ft. in three bins, the first containing 95 cu. yd. 26 cu. ft. 985 cu. in., the second, 87 cu. yd. 19 cu. ft. 876 cu. in., the third, 98 cu. yd. 3 cu. ft. 875 cu. in.?
17. A man buys 3 lots of vinegar; the first is 29 gal. 2

qt. 1 pt., the second, 16 gal. 3 qt., the third 11 qt. 1 pt.; how much did he buy, and what will it sell for at 10 cents a qt.?

18. A man sold three lots of beans; the first, 5825 pt., the second, 4285 pt., the third, 3426 pt.; how many bushels did he sell, and what did they amount to if retailed at $12\frac{1}{2}$ ct. a qt.?

19. A woman picked in one day 1 bu. 4 qt. 1 pt. of strawberries, the next day, $\frac{1}{2}$ bu. 3 qt. 1 pt., the third day, 27 qt. 1 pt.; how many qt. boxes can she fill, and what will she receive at $12\frac{1}{2}$ ct. a box?

20. A carpenter worked for \$.35 an hour; on Monday he worked 9 hr. 15 min., Tuesday 8 hr. 20 min., Wednesday 11 hr., Thursday 10 hr. 35 min., Friday 9 hr. 45 min., and Saturday 6 hr. 50 min.; what did he receive for his week's work?

21. Add 1 doz. 3, 2 gro., 3 doz., 1 sc., 3 gro. 5 doz. 4.

22. How many sheets in 2 bun. 1 rm. 3 qr., 3 bun., 17 sheets, 1 bun. 1 qr.?

23. A traveler in England spends £6 17s. 5d. in one week, the next, £7 11s. 4d., the third, £9 7s. 3d.; how much did he spend?

24. A school girl paid $\frac{1}{2}$ dollar for paper, 10 cents for pencils, \$1.37 $\frac{1}{2}$ for a reader, \$.95 for an arithmetic, and \$.25 for a slate; what was the entire cost?

25. A man sold 4 lots of baled hay; the first weighed 14 T. 13 cwt. 75 lb., the second, 25 T. 12 cwt. 26 lb., the third, 2 T. 5 cwt. 14 lb., and the fourth, 17 T. 16 cwt. 29 lb.; how much did it all weigh?

26. Add 84 T. 12 cwt. 74 lb. 6 oz., 23 T. 12 cwt. 26 lb. 8 oz., 51 T. 16 cwt. 45 lb. 15 oz., 81 T. 5 cwt. 4 lb. 7 oz.

27. Three miners have the following amounts of gold dust: the first, 5 lb. 9 oz. 14 pwt., the second, 3 lb. 7 oz. 13 pwt., the third, 2 lb. 4 oz. 11 pwt.; how much have all?

SUBTRACTION.

EXERCISE 195. (WRITTEN.)

1. A man owning a farm of 160 A. sold at one time 25 A. 74 sq. rd., at another $74\frac{1}{4}$ A., and at another $\frac{1}{2}$ as much as at the first sale; how much had he left?

2. Take 3 mi. 110 rd. 4 yd. 2 ft. from 7 mi. 25 rd. 3 yd. 4 ft.

3. Find the difference between two fields: one is 14 ch. 43 l. by 17 ch. 25 l.; the other, 8 ch. 11 l. by 15 ch.

4. From 48 cu. yd. 12 cu. ft. 1236 cu. in. take 28 cu. yd. 24 cu. ft. 1500 cu. in.

5. From 4 gal. 2 qt. of syrup 1 gal. 3 qt. 1 pt. was drawn; what amount was left?

6. A merchant has two barrels of kerosene, one holding $31\frac{1}{2}$ gal., the other 30 gal. 1 qt. He sold at different times 6 gal. 2 qt., 5 gal. 3 qt., $5\frac{1}{2}$ gal., $7\frac{3}{4}$ gal., and 28 gal.; what did his sales amount to at 27 cents per gallon and what amount has he left?

7. A grocer buys at one time 7 cwt. 11 oz. of tea, at another 6 cwt. 38 lb. 7 oz. He sells 11 cwt. 79 lb. 8 oz.; what has he left?

8. From .625 Troy lb. take 4.25 Troy oz.

9. From 1 cwt. take $\frac{1}{3}$ of $\frac{1}{4}$ of 72 lb. 12 oz.

10. A silver butter dish weighs 1 lb. 2 oz. 5 pwt., and 1 doz. teaspoons weigh 11 oz. 17 pwt. 18 gr.; find the difference in weight.

11. From 2 lb. Apothecaries' weight take 9 oz. 1 dr. 2 scr. 7 gr.

12. Take 3 yr. 4 da. 3 hr. from 5 yr. 2 mo. 2 wk. 1 da. 7 hr.

13. From 3 mo. take 2 wk. 4 da. 8 hr. 19 min. 29 sec.

14. Find the difference in days between the first half of the year 1885 and the time from Christmas to the fourth of July, 1884-5.

NOTE.—Count the day *to* which, and omit the one *from* which you reckon.

15. Find the difference between .659 wk. and 2 wk. $3\frac{5}{6}$ da.
16. A lady has \$729 for house furnishing. She buys 23 yd. of carpeting at \$1.75 per yd., 19 yd. at \$1.12 $\frac{1}{2}$ per yd., 47 yd. at \$1.50 per yd., 12 yd. at \$.97; 6 chairs at \$1.25 each, 3 at \$2.75 each, one for \$16, and two for \$19 each. She spends $\frac{1}{6}$ of the remainder for linen and silver, $\frac{1}{4}$ of what still remains for kitchen articles; what has she left?
17. From the sum of $\frac{2}{7}$ of $3\frac{1}{4}$ mi. and $17\frac{1}{7}$ rd. take $120\frac{1}{3}$ rd.
18. How many more seconds from New Years Day to the Fourth of July, 1885, than in the remainder of the year?
19. Find the difference between 3 yr. 17 da. 9 hr. 12 min. 7 sec. multiplied by 4, and 96 yr. 11 mo. 1 wk. 2 da. 4 hr. 12 min. 16 sec. divided by 3.
20. From $\frac{3}{4}$ of 8 T. 16 cwt. $24\frac{3}{4}$ lb. take .25 of a T.
21. From $\frac{2}{11}$ of a sq. rd. take $\frac{3}{4}$ of a sq. yd.
22. From £48 17s. 6d. 2 far. take £39 14s. 9d. 3 far.
23. The latitude of the Cape of Good Hope is $34^{\circ} 22'$ and that of Cape Horn $55^{\circ} 58' 40''$ S.; find their difference.
24. What is the difference between $\frac{3}{10}$ of a lb. and 5 lb. 4 oz. 8 pwt.?
25. Find the difference between £ $\frac{5}{9}$ and $\frac{2}{3}$ of $\frac{3}{4}$ s.
26. Find the difference in the area of two roofs: one is 46 ft. square, the other contains 46 sq. ft.

MULTIPLICATION.

EXERCISE 196. (WRITTEN.)

1. Multiply 5 mi. 28 rd. 3 yd. 2 ft. 11 in. by 9.
2. Multiply 79 ch. 3 rd. 23 l. by 7.
3. Multiply 158 sq. rd. 27 sq. yd. 7 sq. ft. 138 sq. in. by 11.
4. Multiply 98 cd. 13 cu. ft. 758 cu. in. by 13.
5. Multiply 5 bbl. 29 gal. 3 qt. by 23.
6. Multiply 7 oz. 17 pwt. 23 gr. by 96.
7. Multiply 75 centals 15 oz. by 274.

8. Multiply 9 yr. 7 mo. 3 wk. 5 da. 19 hr. 35 min. 28 sec. by 63.

9. What is the length of a fence inclosing a square field one side of which is 17 rd. 3 yd. $2\frac{1}{3}$ ft. long?

10. If a hogshead of sugar weighs 7 cwt. 29 lb. 4 oz., what will 9 hhd. be worth at $9\frac{1}{2}$ ct. per pound?

11. A letter carrier travels 5 mi. 19 rd. 4 yd. each trip; how far does he go in the month of January, mail being delivered twice each day, four Sundays excepted?

12. A workman drinks a pint bottle of wine each day in the year, which costs him 25 cents per bottle; how much has he drunk in 13 years, three of them being leap years, and what has it cost him?

13. His wife buys a pint of milk per day at \$1.25 per month for the same time; which costs the most and how much?

14. \$125 buys 5 A. 24 rd. 19 sq. yd. 7 sq. ft. of land; what will \$1375 buy?

15. A lady has 17 silver spoons, each one weighs 5 pwt. 6 gr.; how much do they all weigh?

DIVISION.

EXERCISE 197. (WRITTEN.)

1. Divide 9 mi. 78 rd. 4 yd. 2 ft. 8 in. by 9.
2. Divide 68 ch. 2 rd. 24 l. by 6.
3. Divide 296 sq. rd. 29 sq. yd. 8 sq. ft. 98 sq. in. by 16.
4. Divide 97 cu. ft. 11 cu. in. by 28.
5. Divide 23 bbl. 28 gal. 3 qt. by 19.
6. Divide 56 lb. 11 oz. 19 pwt. 21 gr. by 15.
7. Divide 87 cwt. 13 oz. by 95.
8. Divide 24 yr. 11 mo. 2 wk. 3 da. 11 hr. 47 min. by 17.
9. If 294 sacks of walnuts weigh 20600 lb. what is the average weight?

10. If two coops of fowls weigh 340 lb. 11 oz. and there are 27 fowls in a coop, what is the average weight?

11. If a township 6 mi. sq. be divided into 62 equal farms, how much land does each contain?

12. The area of a piece of land is 39 sq. rd. 2 sq. yd. 6 sq. ft. 128 sq. in. Its length is 11 rd. 2 ft. 8 in.; what is its width?

13. If 4 men work 5 days to remove 120 cu. yd. 5 cu. ft. of earth, how much does one man remove in a day?

14. How many cups holding one half pt. each can a restaurant keeper fill from a coffee urn holding 2 gal. 3 qt. 1 pt.?

15. How many steel rails 30 ft. long are needed to build one mile of railroad?

REVIEW.

EXERCISE 198. (WRITTEN.)

1. A car wheel is 4 ft. 5 in. in circumference and revolves 59 times a minute; how far does it go in 2 hr. 55 min.?

2. How many cu. yd. of earth have been removed to make an irrigating ditch 1 mi. 8 rd. long, 3 ft. wide, and 2 ft. deep.

2 **Metric.** How many cu. yd. of earth have been removed to make an irrigating ditch 1649.58 meters long, .914 meters wide, and .609 meters deep?

3. A man sells wheat at \$1.50 per cental and receives \$855.95; how much wheat has he sold?

4. If a horse averages a mile in 11 min. 45 sec. how far does he go in a day of 11 hr.?

4 **Metric.** If a horse averages 1609.34 meters in 11 min. 45 sec., how far does he go in 11 hr.?

5. How many cu. ft. in the drawers of a school desk, one of them 3 ft. 2 in. long, 2 ft. 10 in. wide, and 5 in. deep, the other 1 ft. 4 in. wide, 3 ft. long, 5 in. deep?

5 Metric. Find the cubic contents of the drawers of a school desk: one is .965 meters long, .863 meters wide, and .127 meters deep, the other .91 meters long, .406 meters wide, and .127 meters deep.

6. How many square feet in the surface of two blocks of stone, one 4 ft. in each dimension and the other 3 ft. long, 2 ft. 4 in. wide, and 1 ft. thick?

7. In a section of land how many sq. in.?

7 Metric. In a section of land how many hectares?

8. In a pile of wood 16 ft. long, $3\frac{1}{2}$ ft. wide, and 5 ft. high, how many cd.?

8 Metric. In a pile of wood 4.87 meters long, 1.06 meters wide, and 1.52 meters high, how many steres?

9. If a field 260 rd. long contains $9\frac{3}{4}$ A. what is its width?

10. How many spoons weighing 16 pwt. 11 gr. can be made from 5 lb. 1 pwt. 11 gr. of silver?

10 Metric. How many spoons weighing 25.649 grams can be made from 1872.4 grams of silver?

11. From 7 yr. take 1 mo. 2 wk. 3 da. 11 hr. 35 min. 42 sec.

12. At $12\frac{1}{2}$ cents apiece what cost posts to fence a ranch 480 rd. long and 330 rd. wide, posts set $24\frac{3}{4}$ ft. apart?

13. What will it cost to put three wires around the same ranch, if the wire is worth $5\frac{1}{2}$ cents per lb. and weighs $1\frac{1}{2}$ lb. to the rod?

14. A bbl. of kerosene holding 32 gal. loses $.06\frac{1}{4}$ by evaporation. One half of the remainder is sold at \$.29 per gal., $\frac{1}{3}$ of that remainder at \$.27 per gal., and 8 gal. 3 qt. at \$.26 a gal., and the balance at \$.28 per gal. It cost \$.17 per gal. Find the gain.

15. Reduce 660 ft. to the decimal of a mile.

16. A ranchman buys 4 sets of harness at \$31.75 apiece, a wagon for \$175, a bbl. of sugar for \$17.50, and grain sacks to the amount of \$18.42; how much wheat at \$1.50 per cental will it take to pay the bill?

17. From a pile of wood containing 8964 cu. ft., $9\frac{1}{4}$ cd. were sold at one time and $7\frac{1}{2}$ at another; find the worth of the remainder at \$7.25 per cd.

17 **Metric.** A pile of wood contained 253.736413 steres; 33.514492 steres were sold at one time and 27.173913 at another; what is the remainder worth at \$2.001 per stere?

18. If a herder averages 7 mi. 148 rd. travel in a day, how much does he travel in a year?

18 **Metric.** If a herder averages 1609.372 meters a day, how far does he go in a year?

19. From $\frac{4}{5}$ lb. + $4\frac{5}{8}$ oz. + $31\frac{1}{3}$ pwt., take ($\frac{3}{5}$ oz. — $\frac{7}{8}$ pwt.).

20. How many dollars of 25.8 gr. can be made from 2 lb. 6 oz. 17 pwt. 12 gr. of gold?

20 **Metric.** How many dollars of 1.6753 grams can be made from 961.5584 grams of gold?

21. If 6 cu. yd. $2\frac{1}{2}$ cu. ft. of earth are used in grading one rod of street, how much will be used in grading $16\frac{1}{2}$ blocks, allowing 12 blocks to a mile?

22. A milkman starts out with 9 six-gallon cans of milk. He delivers a pt. each to 35 customers, 1 qt. each to 48, 2 qt. each to 69. He sells $\frac{1}{2}$ of what is left, lacking one pt., to a boarding-house keeper; how much remains unsold?

23. How many cu. ft. in a wall one rod long, $5\frac{1}{2}$ ft. high, and 1 ft. thick?

24. How many lots 45 by 150 feet can be made from 10 A., allowing one fourth for streets?

25. At \$1.60 an ounce what is the value of 2 doz. spoons, each weighing 11 pwt. 23 gr.?

25 **Metric.** At \$.50 for 31.168 grams what is the value of 2 doz. spoons, each spoon weighing 18.636 grams?

26. A cistern holds 98 bbl. If 4 gal. run in by one pipe in a minute, and 6 gal. run out in the same time by another, how long will it be in emptying?

27. A man owning a quarter section of land, gave a piece 17 rd. square as a church site; how much has he left?

28. What must be paid for a pile of wood 15 ft. long, 4 ft. high, 4 ft. wide, at \$9.75 per cd.?

29. The small wheel of a bicycle is 3 ft. in circumference, and the large wheel 8 ft. and 3 in.; how many more times does the small wheel turn than the large one in going a mile?

30. From $\frac{1}{2}$ A. take 79 sq. rd. 7 sq. yd. 6 sq. ft. 98 sq. in.

31. How much carpeting $\frac{3}{4}$ of a yd. wide will it take to carpet a room 14 ft. by 27 ft., the breadths to run crosswise?

32. At \$.32 a square yard, what will it cost to plaster a room 11 ft. 3 in. by 15 ft., and 9 ft. high, deducting one half the surface of two doors each 3 ft. wide and 6 ft. 8 in. high, and 3 windows each $2\frac{1}{6}$ ft. wide and 6 ft. high?

33. How many cu. ft. in a tank 9 ft. 3 in. long, 6 ft. 4 in. wide, 4 ft. 9 in. deep, inside measurement?

34. From $\frac{1}{2}$ of 2 A. 159 sq. rd. 13 sq. yd. 4 sq. ft. 138 sq. in. take 100 sq. rd. 24 sq. yd. 7 sq. ft. 96 sq. in.

35. If a celebration on the Fourth of July begins at 10 o'clock A. M. in Chicago, at what hour must it begin at Los Angeles to be at the same time?

36. Buenos Ayres is longitude $58^{\circ} 22'$ W., and the Cape of Good Hope $18^{\circ} 28'$ E.; when it is 6 hr. 30 min. A. M. in Buenos Ayres, what is the time at the Cape of Good Hope?

37. If you sleep 9 hr. each night, what decimal part of your time are you asleep?

38. Add $\frac{5}{8}$ mi., $\frac{1}{3}$ rd., $\frac{5}{6}$ ft.

38 **Metric.** Add 1005.84 meters, 1.67 meters and .25 meters.

39. If a tank, containing 105 cu. ft., is 7 ft. long and 4 ft. deep, what is its width?

40. At 6 cents a square foot, what will it cost for the wainscoting of a room 16 ft. wide and 22 ft. long, if the wainscot is 2 ft. 10 in. high, deducting for 3 doors, which, with their casings, are each 4 ft. 11 in. wide?

41. How much will it take to carpet a room 18 ft. wide

22 ft. long, if the carpeting is $\frac{3}{4}$ yd. wide, and the breadths run across the room?

41 Metric. How much will it take to carpet a room 5.48 meters wide, 6.70 meters long, with carpet .68 meters wide?

42. How much carpet a yard wide will carpet a room 11 ft. 11 in. wide, and 17 ft. 10 in. long, if the breadths run lengthwise?

43. How much will it cost to carpet a church with yard wide carpeting at \$1.50 per yd., the auditorium being 60 ft. wide and 80 ft. long, breadths running lengthwise, and 10 yd. extra allowed for the pulpit platform; one parlor being 20 ft. wide and 24 ft. long, the other 20 ft. wide and 36 ft. long, breadths running crosswise in both parlors?

44. A stage is robbed of two bars of bullion weighing 170 lb. each, worth \$3700.00; how much is it worth an ounce?

45. What will it cost to carpet a room 17 ft. wide and 26 ft. long, with carpet $\frac{3}{4}$ yd. wide, at \$2.75 per yd., allowing $\frac{1}{16}$ of a yd. for matching, if the breadths run across the room, and a border of 1 ft. wide is used, costing \$1.95 per yard? How much border will it take?

46. What part of a yard is $\frac{1}{21} \frac{1}{12}$ of a mile?

47. Reduce to lower denominations $\frac{4}{5}$ of .225 of a mile.

48. Reduce $\frac{1}{4} \frac{2}{9}$ of a cd. ft. to the fraction of a cd.

49. What will it cost to paper a room 16 ft. wide, 22 ft. long, 9 ft. high, with paper at \$.87 $\frac{1}{2}$ per roll, 8 yd. in a roll and 1 $\frac{1}{2}$ ft. wide, allowing 20 sq. yd. for doors, windows, and baseboards?

50. A man having 2 T. 7 cwt. 28 lb. of hay, sold 5 cwt. 91 lb.; what fraction of the whole did he sell?

51. A man has a piece of land 360 ft. long containing 396 sq. rd. 21 sq. yd.; he is offered \$605 an acre; but he runs a 10-foot alley lengthwise through the piece and divides it into 16 equal lots, which he sells at \$175 each; what is the size of his lots, and what does he gain?

UNITED STATES MONEY.

United States money is written decimally, the dollar being the unit. Five decimal places have been named.

eagle.	dollar.	dime.	cent.	mill.	The mill is not represented by any coin. Results should be carried to hundredths only.
0	0	0	0	0	

Gouverneur Morris first recommended making our money in decimal divisions; afterward Jefferson and Hamilton improved upon his plan.

The Spanish silver dollar was chosen as the unit, and coinage commenced in 1792.

Gold and silver are soft, hence the coins are now alloyed with $\frac{1}{10}$ of some other metal to harden them. In gold coins the alloy is a mixture of silver and copper; the dollar weighs 25.8 grains, and all other coins are multiples of this weight. They are 2½-, 3-, 5-, 10-, and 20-dollar pieces.

Silver coins are alloyed with copper. The dollar weighs 412½ grains; the smaller coins are lighter; thus, 2 half dollars, 4 quarters, 10 dimes, or 20 half dimes weigh but 385.8 grains. The half dime is now made of nickel and copper, and the 2- and 1-cent pieces of copper.

$\frac{1}{2} = .50$	$\frac{1}{5} = .20$	$\frac{5}{6} = .83\frac{1}{3}$	$\frac{7}{8} = .87\frac{1}{2}$	$\frac{1}{15} = .06\frac{2}{3}$
$\frac{1}{3} = .33\frac{1}{3}$	$\frac{2}{5} = .40$	$\frac{1}{7} = .14\frac{2}{7}$	$\frac{1}{12} = .08\frac{1}{3}$	$\frac{1}{16} = .06\frac{1}{4}$
$\frac{2}{3} = .66\frac{2}{3}$	$\frac{3}{5} = .60$	$\frac{1}{8} = .12\frac{1}{2}$	$\frac{5}{12} = .41\frac{2}{3}$	$\frac{3}{16} = .18\frac{3}{4}$
$\frac{1}{4} = .25$	$\frac{4}{5} = .80$	$\frac{3}{8} = .37\frac{1}{2}$	$\frac{7}{12} = .58\frac{1}{3}$	$\frac{5}{16} = .31\frac{1}{4}$
$\frac{3}{4} = .75$	$\frac{1}{6} = .16\frac{2}{3}$	$\frac{5}{8} = .62\frac{1}{2}$	$\frac{11}{12} = .91\frac{2}{3}$	$\frac{7}{16} = .43\frac{3}{4}$

EXERCISE 199.

1. At $\$.16\frac{2}{3}$ per pound, what will 96 lb. of coffee cost?

$$$.16\frac{2}{3} = \frac{1}{6}. \quad 96 \times \frac{1}{6} = \$16.$$

2. How many doz. eggs at $\$.12\frac{1}{2}$ per doz. can be bought for \$2.75?

$$$.12\frac{1}{2} = \frac{1}{8}. \quad \$2.75 \div \frac{1}{8} = 22 \text{ doz.}$$

3. What will 128 cents of wheat cost at \$1.25 per cental?

$$$.25 = \frac{1}{4}. \quad \frac{1}{4} \times 128 = 32. \quad 128 \div 32 = \$160.$$

4. Find the cost of 8576 bricks at \$9 per thousand.

$$8576 \div 1000 = 8.576$$

$$8.576 \times 9 = \$77.184.$$

5. What will 7864 lb. of hay cost at \$12.00 per T.?

$$7894 \div 1000 = 7.894$$

$$7.894 \div 2 = 3.947 \text{ T.}$$

$$3.947 \times 12 = \$46.364.$$

6. What part of 100 is $33\frac{1}{3}$? What part is $8\frac{1}{3}$? $37\frac{1}{2}$?

7. What part of 100 is $87\frac{1}{2}$? $6\frac{1}{4}$? $16\frac{2}{3}$?

8. What is $\frac{1}{8}$ of 100? $\frac{1}{5}$ of 100? $\frac{2}{3}$ of 100?

9. What is $\frac{2}{3}$ of 100? $\frac{7}{12}$ of 100? $\frac{9}{16}$ of 100?

10. What part of 100 is $41\frac{2}{3}$?

11. What will 10 yd. of cotton cloth cost at $12\frac{1}{2}$ cents per yd.?

12. Find the cost of 6 centals of wheat at $\$1.16\frac{2}{3}$ per cental.

13. A lady paid \$12.00 for cloth at $\$.16\frac{2}{3}$ per yard; how many yards did she buy?

14. Find the cost of 720 lb. of soap at $\$.08\frac{1}{3}$.

15. How many yards of cloth at $\$1.33\frac{1}{3}$ per yard can be bought for \$128.00?

16. What will 248 dozen eggs cost at $\$.37\frac{1}{2}$ per dozen?

17. At $\$1.62\frac{1}{2}$ apiece how many histories can be bought for \$2613.00?

18. What will 7 cords of wood cost at \$9.75 per cord?

19. At \$11 per cwt. how much sugar can be bought for \$44?

20. What will 500 bricks cost at \$8 per thousand?

21. What will 5500 ft. of boards cost at \$22 per thousand?

22. At $\$.06\frac{1}{4}$ per pound how much honey can be bought for \$16?

23. How many books at \$1.25 apiece will \$48.75 buy?

24. What will 874 grain sacks cost at $6\frac{1}{4}$ cents apiece?
25. When beeswax is \$.25 per lb. how many pounds will \$16 buy?
26. What cost 84 lb. of veal at $12\frac{1}{2}$ cents per pound?
27. At \$.12 $\frac{1}{2}$ per yd. what will 648 yd. of gingham cost?
28. How much alfalfa seed at \$.12 $\frac{1}{2}$ per pound can be bought for \$19?
29. What will 976 boxes of limes cost at \$.75 per box?
30. What will be the cost of 879 centals of wheat at \$1.40 per cental?
31. At \$.87 $\frac{1}{2}$ per yard how much flannel can be bought for \$40?
32. A hotel keeper spends \$9.33 $\frac{1}{3}$ for spring chickens at \$.66 $\frac{2}{3}$ apiece; how many does he buy?
33. What will 376 dozen eggs cost at \$.37 $\frac{1}{2}$ per dozen?
34. At $6\frac{1}{4}$ cents apiece how many paper bags can a confectioner buy for \$32?
35. When hay is \$12.75 per ton how many tons will \$357 buy?
36. Find the cost of 7212 lb. of evaporated apples at 8 $\frac{1}{3}$ cents per lb.
37. At \$1.25 per yard what will 18 yd. of silk cost?
38. What will 189 lb. of coffee cost at \$.33 $\frac{1}{3}$ per pound?
39. How many pounds of tea at \$.62 $\frac{1}{2}$ per lb. can a grocer buy for \$26.87 $\frac{1}{2}$?
40. How many sheep at \$4.75 apiece can be bought for \$95?
41. At \$1.87 $\frac{1}{2}$ per sack what will 408 sacks of flour cost?
42. At \$1.625 per yd. what will 248 yd. of silk cost?
43. A dealer buys butter for \$.87 $\frac{1}{2}$ a roll to the amount of \$100; how many rolls does he buy?
44. What will 249 yd. of velvet cost at \$2.66 $\frac{2}{3}$ per yard?
45. At \$.83 $\frac{1}{3}$ per yard what will 726 yards of cashmere cost?
46. What will 97856 feet of boards cost at \$19.00 per thousand?

47. Find the cost of 785469 bricks at \$8.00 per thousand?
48. At \$6.00 per cwt. what will 9856 lb. of beef cost?
49. What will 764398 lb. of flour cost at \$2.40 per cwt.?
50. Find the cost of 43986 lb. of coal at \$.70 per cwt.
51. A man paid \$95.00 for freight on wool at \$1.25 per cwt.; how much wool had he?
52. What is the freight on 7543 lb. of merchandise at \$2.50 per cwt.?
53. What will 98756 lb. of hay cost at \$12.00 per ton?
54. How many pounds in a load of coal which costs \$8.00, when coal is \$12.00 per T.?
55. What is the value of 3 loads of hay weighing respectively 1975, 1125, and 1240 pounds, at \$12.75 per T.?
56. How many dollars can be made from one lb. of pure gold?
57. How many dollars can be made from one lb. of pure silver?
58. How many dollars can be made from 7 lb. 11 oz. 18 pwt. 3 gr. of pure silver?
59. How many dollars can be made from 1 lb. 1 pwt. 21 gr. of pure gold?
60. How many eagles can be made from 2 lb. 3 pwt. 18 gr. of pure gold?
61. How many half dollars can be made from 6 lb. 6 pwt. 18 gr. of pure silver?
62. How many quarters can be made from 1 lb. 6 oz. 1 pwt. $16\frac{1}{2}$ gr. of pure silver?
63. How many dimes can be made from 3 lb. 3 pwt. 9 gr. of pure silver?
64. A bar of silver bullion is .975 pure; how many half dollars can be made from it? Weight 9 lb. 2 oz. 8 pwt.
65. How many $\$2\frac{1}{2}$ pieces can be made from 3 lb. 8 oz. 11 pwt. of gold bullion?

GENERAL ANALYSIS.

When several successive operations in multiplication and division are to be performed in the same example, each operation may be determined by ordinary analysis and indicated by placing the number concerned above or below the *division line*. Thus,

If 15 acres of land cost \$620, what are 12 acres worth at the same rate?

OPERATION. ANALYSIS.—If 15 acres cost \$620, 1 acre costs

$$\frac{124 \quad 4}{15} \times 620 = \$496$$

 $\frac{1}{15}$ of \$620, or $\frac{\$620}{15}$. 12 acres cost 12 times 1
 acre, or $\frac{\$620}{15} \times 12 = \frac{\$620 \times 12}{15}$. The work is
 shortened by cancellation.

If 18 tons of coal cost \$189, how many tons can be bought for \$105?

OPERATION. ANALYSIS.—For 1 dollar you can buy $\frac{1}{189}$

$$\frac{2 \quad 5}{189} \times 105 = 10 \text{ tons.}$$

 of 18 tons, or $\frac{18}{189}$ tons; and for 105 dollars
 105 times $\frac{18}{189}$ tons, or $\frac{18 \times 105}{189}$ tons.

If 200 bushels of oats will last 30 horses 50 days, how long will 150 bushels last 45 horses?

OPERATION. ANALYSIS.—200 bushels last 50

$$\frac{25 \quad 15}{200 \times 45} \times 150 \times 30 = 25 \text{ days.}$$

 days; 1 bushel will last $\frac{1}{200}$ of 50
 days, or $\frac{50}{200}$ days, and 150 bushels,

$$\frac{2 \quad 3}{200} \times 150 \times 30 = 25 \text{ days.}$$

 150 times as many days, or $\frac{50 \times 150}{200}$
 days; this is the number of days for 30 horses, for 1 horse 30 times
 as many days, or $\frac{50 \times 150 \times 30}{200}$ days, and for 45 horses $\frac{1}{45}$ as many
 days as one horse, or $\frac{50 \times 150 \times 30}{200 \times 45}$ days.

Observe that in these examples, one number is of the same kind as the answer sought, and the others are in like pairs. In the last example, 50 is like the answer, days. Then there are two numbers of bushels, and two of horses. The first two examples have one pair each, beside the number that is like the answer.

The following is a short statement of the method: Begin with the number like the answer; then in each pair reason from the given number of the pair to 1, and from 1 to the number required.

The results all the way through are like the required answer.

EXERCISE 200. (WRITTEN.)

1. If 18 sheep are worth \$45, what are 30 sheep worth at the same rate?
2. If 7 men dig a ditch in 15 days, how long will it take 15 men?
3. If 48 rods of fence cost \$108, what will 84 rods cost?
4. If a locomotive goes 564 miles in 24 hours, how far will it go in 22 hours?
5. If 160 A. of land produce 96 tons of wheat, how many tons will 175 A. produce?
6. If 75 A. of land produce 50 tons of wheat, how many A. will produce 18 tons?
7. If 50 chairs cost \$112.50, how many chairs can be bought for \$90?
8. If $8\frac{3}{4}$ yd. of cloth cost \$17.50, what will $12\frac{1}{2}$ yd. cost?
9. If 12 men earn \$78 in 4 days, how many men will earn \$58 $\frac{1}{2}$ in the same time?
10. If 18 men can do a piece of work in 32 days, how many men will do it in 24 days?
11. If the freight for transporting 18 cwt. of household goods from San José to Los Angeles is \$61.20, what will it cost to transport 42 cwt.?
12. If 30 gal. of oil cost \$3.75, what cost 100 gal.?

13. If a man can perform a journey in 14 days of 10 hours each, how many days of 12 hours each will he need to do the same?

14. If 12 cows can be bought for \$486, for how much can 22 cows be bought?

15. When 12 cows cost \$486, how many cows can be bought for \$891?

16. If $9\frac{1}{2}$ yd. of broadcloth cost $\$44\frac{1}{3}$, how many yd. will cost $\$33\frac{1}{4}$?

17. If it costs \$720 to transport 12 tons of freight 480 miles, what will it cost to transport 15 tons 300 miles?

18. If 16 men earn \$640 in 4 wk., what will 18 men earn in 2 wk.?

19. If 130 lb. of tea cost \$117, what will 80 lb. cost?

20. If a pasture will feed 120 horses 81 da., how many horses will it feed 108 da.?

21. If 12 men in 12 da. of 9 hr. each can perform a certain piece of work, how many days of 8 hr. each will it take 9 men?

22. How many lb. of sugar can you buy for \$380 if 20 lb. cost \$1.90?

23. If it takes 27 yd. of carpeting $\frac{3}{4}$ of a yd. wide to carpet a certain room, how many yards of 1 yd. wide carpeting will it take?

24. If $9\frac{1}{2}$ yd. of cloth $\frac{3}{4}$ of a yd. wide cost \$11.40, what will 10 yd. $1\frac{1}{4}$ yd. wide cost?

25. If 6 bbl. of flour last 80 men 12 da., how long will 9 bbl. last 60 men?

26. If a doz. brooms cost \$4.50, how many brooms can you get for $\$3.37\frac{1}{2}$?

27. How many men will build 35 rd. of wall in the same time that 6 men build 42 rd.?

28. If 7 men dig a ditch 28 feet long in 2 da. of 8 hr. each, how many da. of 10 hr. each will it take 10 men to dig a ditch 25 ft. long?

29. If it cost \$42 to plaster the ceiling of a room 14 ft. long and 12 ft. wide, what will it cost for a room 16 ft. long and 14 ft. wide?

30. If $11\frac{1}{2}$ lb. of coffee cost \$3.45, what will $10\frac{1}{2}$ lb. cost?

31. When the shadow of a post 10 ft. 6 in. high is 12 ft. 3 in. long, what is the length of shadow of a post 8 ft. 9 in. high?

32. The shadow of a post 16 ft. 3 in. high is 5 ft. 5 in. long; what height of post will give a shadow 3 ft. 4 in. long?

33. If 4 men build $12\frac{1}{4}$ rd. of fence in $3\frac{1}{4}$ da., how long will it take 18 men to build $237\frac{6}{13}$ rd.?

34. If a tank 36 in. long, 22 in. wide, and 7 in. deep holds 24 gal., how much will a tank 3 ft. 8 in. long, 1 ft. 2 in. wide, and 1 ft. deep hold?

35. If $\frac{11}{17}$ of an acre of land is worth \$198, what are $\frac{7}{9}$ of an acre worth?

36. At the rate of 14 lb. for \$1, what will 8 bbl. of sugar averaging 259 lb. to a barrel cost?

37. How many oranges at 15 cents a dozen will pay for 7 5-gallon cans of kerosene at $\$ \frac{3}{8}$ a gallon?

38. I buy a certain quantity of rice at \$4.50 per 100 lb. and pay for it with 717 ft. of pine lumber at \$15 per M; what weight of rice did I buy?

39. A farmer bought grain bags worth $7\frac{1}{2}$ cents each for 150 sacks of oats averaging 125 lb. each at \$1.20 a cental; how many grain bags did he receive?

40. Sold a newspaper proprietor 3 bun. of paper, 60 lb. each, at 7 cents per pound, for which he agreed to furnish me his daily paper delivered at 15 cents per week; how long did I receive his paper?

41. Bought 12 doz. glass jars at \$1.75 and paid for them in potatoes at $1\frac{1}{4}$ cents a lb.; how many 80-pound sacks did I give?

PROPORTION.

Examples in General Analysis will be seen to contain one number of the same kind as the thing required in the answer, while the other numbers are arranged in pairs.

A *formula* or *statement* called a PROPORTION is sometimes used in such examples, to precede the performing of the work and take the place of the logical and proper analysis of the example.

3 days is what fraction of 12 days? \$4 is what fraction of \$24? 9 horses of 16 horses?

A **ratio** is a fraction whose terms are of the same kind. Thus, $\frac{3}{12}$ or $\frac{1}{4}$ expresses the ratio of 3 to 12, or of 3 days to 12 days.

Review Exercise 137, examples 1 to 16, reading, "What is the ratio of 8 to 20?" Substitute concrete terms; thus, 8 men to 20 men.

A ratio is often written by using two dots between the terms. Thus, the ratio of 8 to 20 is written 8 : 20.

What is the ratio of 6 to 9 in its lowest terms? Of 12 to 18? What can you say of these two ratios? We will write them equal. $\frac{6}{9} = \frac{12}{18}$, or $(6 : 9) = (12 : 18)$.

Two equal ratios form a **proportion**.

In the written expression four dots (: :) are often used for the sign =.

The first and last terms of a proportion are the **extremes**; the second and third, the **means**.

In any proportion the product of the means equals the product of the extremes; thus, $9 \times 12 = 6 \times 18$. Hence, the product of the means divided by one extreme will give the other extreme; or, the product of the extremes divided by one mean will give the other mean. Thus,

$$\frac{9 \times 12}{6} = 18.$$

$$\frac{9 \times 12}{18} = 6.$$

$$\frac{6 \times 18}{9} = 12.$$

$$\frac{6 \times 18}{12} = 9.$$

To determine and write a proportion.

If 15 A. of land are worth \$620, what are 12 A. worth?

<p>OPERATION.</p> $15 : 12 :: 620 : ?$ $\frac{12 \times 620}{15} = 496.$	<p>EXPLANATION.—\$620 is like the required answer. 15 and 12 are A. The ratio of 15 A. to 12 A. must be the same as the cost of 15 A., \$620, to the cost of 12 A.; or, as written above. Solve as in Analysis.</p>
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To arrange the terms,

1. Place the number which is like the required answer for the third term;

2. If, in the nature of the problem, the answer ought to be larger than the third term, arrange the pair so that the second term shall be larger than the first, but if the answer should be smaller than the third term, let the smaller of the two numbers be the second term. Then divide the product of the means by the given extreme.

Sometimes the result depends upon the relations of several pairs, producing a **compound proportion**. In such case consider the result with reference to each pair separately, as in simple proportion.

Thus, in the third analysis, General Analysis, consider first the horses alone; then the bushels alone.

$$\left. \begin{array}{l} 45 : 30 \\ 200 : 150 \end{array} \right\} = 50 : ?$$

$$\frac{30 \times 150 \times 50}{45 \times 200} = 25.$$

For work, perform the examples in General Analysis.

PARTNERSHIP.

Two men paid \$6 for a pasture 1 month. If each puts in 2 cows what should each pay? If one puts in 2 cows and the other 1 cow what should each pay? If the first had his 2 cows in pasture 4 weeks and the second his 1 cow only 2 weeks how much should each pay?

Thus, we see that each man's share of the expense depends upon the product of the number of cows pastured and the time, if the times are different.

An association of two or more persons together in business is called **PARTNERSHIP**.

The persons associated are called **partners**.

The money subscribed is called the **capital** or **stock**.

Each partner receives the same part or fraction of the *losses* or *gains* that his capital is of the whole capital invested, if all invest for the same time; if for different times, each partner's capital must be multiplied by the time it is in use and the product taken as his share of the capital; the sum of these products being taken as the entire capital, provided no special division has been agreed upon.

Property of all kinds owned by a firm are its **assets**.

Its debts are **liabilities**.

EXERCISE 201. (WRITTEN.)

Find out, if you can, how to prove these examples and prove each.

1. Two men enter into partnership in the grocery business. A furnishes \$2500 capital; B, \$1500. Their gain the first year was \$1840. Find the share of each.

2. A and B trade together. A furnishes $\frac{5}{13}$ of the capital; B, the remainder. Divide their loss of \$637 fairly.

3. Two men hire a pasture for \$96. One pastures 40

sheep for 11 weeks; the other, 65 sheep 8 weeks. What should each pay?

4. Two men engaged in the clothing business with a joint capital of \$6000. The first year's gain was \$2892, of which one received \$964. What amount of capital did each furnish?

5. Three men engage in business. A puts in \$2000 the first of January; B \$3000 the first of March; and C \$4000 the first of April. The profits at the close of the year of \$6045 will be shared how?

6. Divide \$195 among 3 boys, giving them 3, 4, and 6 parts respectively.

7. A bankrupt owes A \$1000, B \$1500, C \$1800, D \$2000, and E \$2700. His assets are \$6000. What sum can he pay each?

8. A man has \$5175 and owes \$6210; what can he pay on every \$1 he owes? What will a man to whom he owes \$1320 receive?

9. A, B, and C sent a ship loaded with Wellington coal to San Francisco. A put on board 180 tons, B 250 tons, and C 400 tons. On account of storm 249 tons were thrown overboard; find the loss of each.

10. In a certain firm B has 3 times as much capital as A, and C has $\frac{1}{2}$ as much as the other two. What is each one's share in a loss of \$786?

11. In a gain of \$600 A received $\frac{1}{2}$, B $\frac{1}{6}$, and C the remainder. If the whole capital was 12 times A's gain what was the capital of each?

12. Two men receive \$1000 for grading. One furnishes 3 teams 20 days and the other 5 teams 30 days. If the first receives \$100 for overseeing the work what does each receive?

13. Two men contract to move 5316 cu. yds. of gravel at 25 cents a cu. yd., and agree to share the profits in the proportion of 2 to 3. They employ 5 teams 45 days at \$4 each per day. What did each make?

14. Divide 1728 in the proportion of 3, 4, and 5.

15. Three men have wheat of different grades in a warehouse. A has 1200 centals worth \$1.10; B has 800 centals worth \$1.25; C has 1600 centals worth \$1.12½. The wheat being damaged, the whole was sold for \$3090. Find each one's share.

EXERCISE 202. (ORAL.)

1. Divide 75 cents among 3 boys, giving to the first 3 cents as often as to the second 5 cents and the third 7 cents.

2. Albert and James buy a book together costing \$1.50, of which Albert paid 50 cents and James the rest. They afterwards sell it for 75 cents. What should each receive?

3. A lady gave \$2 to her children aged 8 and 12 in proportion to their ages. What did each receive?

4. Three girls bought \$1's worth of oranges; the first receiving $\frac{1}{2}$, the second $\frac{1}{3}$, and the third the rest. How much money did each contribute?

5. I hire a pasture, in company with a friend, for \$65. I pasture 8 cows 4 months; my friend, 11 cows 3 months. What is his share of the expense?

6. Divide 50 cents in the proportion of $\frac{1}{2}$ and $\frac{1}{3}$.

7. I owe \$2000 and have \$1500. How much can I pay for every dollar owed?

8. What will a man whom I owe \$100 receive?

9. A man leaves \$5000 to his two sons in the inverse ratio of their ages, 15 and 10. Find what each had.

10. Two men in partnership lose \$800, of which the first bears \$500. Their capital was \$2400; what capital did each furnish?

11. Divide 45 marbles with two companions so that one shall receive 2 to your 1, and the other 3 to your 2.

EXERCISE 203. (WRITTEN.)

Form 10 Partnership examples of your own, perform, and bring to the class for dictation.

PERCENTAGE.

Review Exercises 139, 137, 123, and examples 13 to 20, Exercise 136.

What is $\frac{1}{100}$ of 600? $\frac{3}{100}$? $\frac{5}{100}$? $\frac{8}{100}$? $\frac{10}{100}$? $\frac{20}{100}$?

18 is $\frac{1}{100}$ of what? $\frac{2}{100}$? $\frac{3}{100}$? $\frac{6}{100}$? $\frac{9}{100}$? $\frac{18}{100}$?

Per cent means hundredths (from *per centum*, by the hundred). Thus, $\frac{3}{100}$, or .03, is 3 per cent. $\frac{5}{100}$, or .05, is 5 per cent.

The word **rate** is sometimes used for per cent.

That number of which another is a fraction or per cent is called the **base**.

Name the base in the above illustrations.

Rewrite the above illustrations, using the term *per cent* instead of the denominator 100, with the answers following.

Thus, *What is 1 per cent of 600?* *Ans. 6.*

The sign % means per cent.

EXERCISE 204. (WRITTEN.)

Rewrite the following in decimal and common forms, and reduce the common form to lowest terms.

Thus, $6\% = .06 = \frac{6}{100} = \frac{3}{50}$.

6 per cent.	$14\frac{2}{7}\%$	$5\frac{5}{8}\%$	$6\frac{1}{4}\%$	$\frac{4}{5}\%$
9 " "	48%	$7\frac{1}{7}\%$	$8\frac{1}{3}\%$	$\frac{1}{3}\%$
12 " "	$11\frac{1}{9}\%$	$5\frac{5}{6}\%$	$12\frac{1}{2}\%$	$2\frac{1}{2}\%$
15 " "	$1\frac{2}{3}\%$	$3\frac{3}{4}\%$	$16\frac{2}{3}\%$	$\frac{5}{6}\%$
18 " "	$\frac{2}{3}\%$	$5\frac{5}{9}\%$	30%	$\frac{3}{4}\%$
22 " "	$1\frac{1}{4}\%$	$22\frac{1}{2}\%$	$33\frac{1}{3}\%$	$\frac{1}{2}\%$
27 " "	72%	$17\frac{1}{2}\%$	$37\frac{1}{2}\%$	$3\frac{1}{3}\%$
32 " "	85%	$13\frac{1}{3}\%$	$62\frac{1}{2}\%$	$\frac{7}{8}\%$
36 " "	84%	$23\frac{1}{3}\%$	$66\frac{2}{3}\%$	$87\frac{1}{2}\%$
$9\frac{1}{11}$ " "	96%	$21\frac{1}{9}\%$	$83\frac{1}{3}\%$	$\frac{1}{4}\%$

Practice on the last two columns until familiar.

EXERCISE 205. (ORAL.)

Name the corresponding fractions in lowest terms.

5%	25%	45%	65%	85%	4%
7%	30%	50%	70%	90%	100%
10%	35%	55%	75%	95%	$\frac{1}{6}\%$
20%	40%	60%	80%	2%	$\frac{7}{10}\%$

EXERCISE 206. (WRITTEN.)

How many 100ths, or %, are:

$\frac{1}{3}$	$\frac{7}{8}$	$\frac{5}{6}$	$\frac{3}{800}$	$\frac{2}{7}$	$\frac{7}{11}$	$\frac{1}{160}$	$\frac{3}{41}$
$\frac{2}{3}$	$\frac{1}{12}$	$\frac{1}{16}$	$\frac{3}{400}$	$\frac{5}{19}$	$\frac{4}{9}$	$\frac{13}{200}$	$\frac{7}{45}$
$\frac{3}{8}$	$\frac{1}{200}$	$\frac{5}{8}$	$\frac{7}{400}$	$\frac{4}{13}$	$\frac{5}{16}$	$\frac{11}{900}$	$\frac{12}{61}$
$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{300}$	$\frac{1}{150}$	$\frac{3}{22}$	$\frac{11}{12}$	$\frac{4}{21}$	$\frac{9}{80}$

Drill on the first four columns until familiar.

EXERCISE 207. (ORAL.)

How many 100ths, or %, are:

$\frac{1}{2}$	$\frac{1}{25}$	$\frac{1}{80}$	$\frac{7}{50}$	$\frac{4}{5}$	$\frac{3}{40}$	$\frac{6}{5}$	$\frac{7}{20}$
$\frac{1}{4}$	$\frac{1}{30}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{9}{10}$	$\frac{17}{20}$	$\frac{5}{4}$	$\frac{24}{5}$
$\frac{1}{5}$	$\frac{1}{40}$	$\frac{2}{5}$	$\frac{7}{10}$	$\frac{7}{25}$	$\frac{4}{25}$	$\frac{6}{25}$	$\frac{11}{50}$
$\frac{1}{10}$	$\frac{1}{50}$	$\frac{3}{10}$	$\frac{11}{20}$	$\frac{13}{20}$	$\frac{21}{50}$	$\frac{31}{50}$	$\frac{7}{30}$
$\frac{1}{20}$	$\frac{1}{75}$	$\frac{3}{20}$	$\frac{8}{25}$	$\frac{9}{50}$	$\frac{19}{20}$	$\frac{21}{25}$	$\frac{5}{40}$

EXERCISE 208. (WRITTEN.)

Change the fractions in Exercise 123 to 100ths, and rewrite the examples, using "per cent" instead of "100ths." Thus, in Example 1, $\frac{3}{4} = \frac{75}{100}$: hence, rewrite thus,

What cost 75 per cent of a yard of cloth at 20 cents a yard?

Same with Exercise 160, orally.

EXERCISE 209. (WRITTEN.)

Rewrite in fractional form, and analyze, using the decimal and common forms:

1. \$75 is 3 per cent of what sum?
2. What is 25% of \$1728?
3. \$750 is 20% of what?
4. \$640 is what per cent of \$3200?

5. What is $\frac{2}{3}\%$ of \$9900?
6. \$75 is $1\frac{2}{3}\%$ of what sum?
7. \$25.92 is $1\frac{1}{2}\%$ of what sum?
8. \$102.50 is what % of \$20500?
9. What is $62\frac{1}{2}$ per cent of \$7288?
10. What is $16\frac{2}{3}\%$ of \$36?
11. \$490 is what % of \$5000?
12. \$6.50 is $12\frac{1}{2}\%$ of what sum?
13. What is 40 per cent of \$1683.25?
14. \$150 is $33\frac{1}{3}\%$ of what?
15. \$729.80 is $66\frac{2}{3}$ per cent of how much?
16. \$2.50 is what per cent of \$20?
17. What is $2\frac{1}{2}\%$ of \$400?
18. What is $1\frac{1}{3}\%$ of \$1500?
19. \$13.50 is what per cent of \$81?
20. \$37.50 is 6% of what amount?

EXERCISE 210. (ORAL.)

Perform Exercise 137, changing each answer to 100ths, or %.

EXERCISE 211. (WRITTEN.)

Change the fractions in Exercise 138 to 100ths, and rewrite the examples, using per cent instead of 100ths.

PRACTICAL WORK IN PERCENTAGE.

General forms for Percentage: $\left\{ \begin{array}{l} \text{J.—What is } \frac{3}{4} (.75) \text{ of } 16? \\ \text{K.—} 12 \text{ is } \frac{3}{4} (.75) \text{ of what number?} \\ \text{L.—} 12 \text{ is what fraction (\%) of } 16? \end{array} \right.$

(Compare with "General Forms," p. 93.)

EXERCISE 212. (WRITTEN.)

Change the per cents to fractions in their lowest terms, rewrite in *general form*, and analyze:

1. A man having \$3300 lost 3 per cent of it; how much did he lose?

2. A man had an annual income of \$2500. He spent 10% of it for board; 5% for clothing, and 18% for incidentals; how much did he spend for each?

3. A man lost \$120, which was 40% of all he had; how much had he?

4. A man having \$5800 worth of hay lost \$870 worth by fire; what fraction and what per cent of the whole was the part lost?

5. If you buy eggs at 20 cents a dozen and sell them at a gain of $2\frac{1}{2}$ cents a dozen, what fraction and what per cent. of the cost do you gain?

6. A merchant sells a barrel of flour for \$6.25, which was 125% of what it cost him; what did it cost him?

7. A jeweler sold a watch for \$36, which was 90 per cent of its cost; find the cost.

8. A ship carrying 8750 tons of coal sprung a leak, on account of which it was found necessary to throw overboard 1250 tons; what per cent of the coal was thus lost?

9. A man spent in one year \$2150, which was $5\frac{2}{3}$ % of what he had; how much had he?

10. My salary is \$2400; if I spend $37\frac{1}{2}$ % of it, how much money do I spend?

EXERCISE 213. (ORAL ANALYSIS.)

1. A man having 800 boxes of oranges lost 3% by decay; how many boxes did he lose?

2. \$25 is 25% of what sum?

3. In a school of 150 pupils 3 were absent; what per cent was absent?

4. A man having spent $33\frac{1}{3}$ % of his money has \$600 left; what had he at first?

5. A boy increasing his money by 25% of itself has \$1; what had he at first?

6. A man owning 75% of a ship sold $33\frac{1}{3}$ % of his share for \$6000; find the value of the ship.

7. 20 is 40% of what number?

8. Sold a horse for \$100 at 20% above cost; find the cost.
9. \$18 is what per cent of \$72?
10. Bought a cow for \$35 and sold her for 20% above cost; what did I receive for her?

PROFIT AND LOSS.

Gains, losses, and selling-price, are always a per cent or fraction of the *cost*.

The *cost*, then, in Profit and Loss, is always the *base*.

EXERCISE 214. (WRITTEN.)

Label everything given in the first 10 examples, with the word *gain, loss, selling-price, or cost*, rewrite in *general form*, and then perform.

1. A man sold a harness for \$35, gaining 40% on the cost; find the cost.

Model: \$35 = S. P. 100% = Cost. 40% = Gain.
\$35 is 140% of what number?

2. I wish to make $37\frac{1}{2}\%$ on a ton of hay which cost me \$7.20; for what must I sell it?

3. By selling a house for \$3500 I lose \$500 on the cost; what fraction, and per cent, of the cost did I lose?

4. A merchant sells cloth for \$3.75, losing $16\frac{2}{3}\%$; what was the cost?

5. A broker bought cotton to the amount of \$3840. The price falling, he was obliged to sell at $2\frac{1}{2}\%$ loss; find his loss and selling price.

6. A man bought 144 pounds of sugar at the rate of 12 pounds for a dollar and sold it at 10 cents a pound. What per cent did he gain?

7. Bought tea at $37\frac{1}{2}$ cents and sold it at 50 cents. What was gained per cent?

8. Sold wheat at \$1.05, losing $12\frac{1}{2}\%$; what did it cost?

9. A merchant marked cloth at 25% advance on the cost. The goods being damaged, he was obliged to take off 20% of the marked price, selling it at \$1.50 per yard; what was the cost?

10. I sold $\frac{3}{4}$ of an acre of land for what the whole acre cost me; what was my gain %?

11. What per cent is gained in buying goods by long ton weight and selling them at the same price per ton by short ton weight?

12. If 20% is lost by selling wheat at \$1, for what must it be sold to gain 10%?

13. By selling a cow for \$7 less than she cost I lose $14\frac{2}{7}\%$; what was her cost and selling price?

14. How shall a merchant mark cloth that cost $16\frac{2}{3}$ cents per yard so as to gain 20%?

15. I buy a box of oranges containing 300 oranges for \$1.50; for how much must I sell them per dozen to gain $41\frac{2}{3}\%$?

16. Sold goods for \$3.50 less than cost and lost 14%; what should I have gained per cent by selling for \$2.75 above cost?

17. A man sold a sack of potatoes at a loss of $12\frac{1}{2}\%$, thereby losing 10 cents; find the cost.

18. A man sold a buggy for $11\frac{1}{9}\%$ above cost, and with the money bought another which he sold for \$160, losing $11\frac{1}{9}\%$. Did he gain or lose on the whole and how much per cent?

19. Bought 12 acres of land for \$840. Sold $\frac{1}{3}$ of it at \$85 per acre, $\frac{1}{4}$ of it at \$75 per acre, and the remainder at a loss of $14\frac{2}{7}\%$ on an acre; what per cent was gained or lost on the whole?

20. Two sets of furniture were sold at \$35 each. On one there was a gain of $16\frac{2}{3}\%$; on the other a loss of $16\frac{2}{3}\%$; was there a gain or loss on both, and how much %?

21. A merchant bought carpetings at 75 cents, 95 cents, and \$1.10; for what must he sell each to make 20%?

22. A furniture dealer sold 10 dozen chairs for \$96; if he paid 55 cents apiece for them, and 5 cents each for transportation, what % was his profit?

23. An oil company paid 8 cents a gallon for a cask of crude oil containing $31\frac{1}{2}$ gallons; if $11\frac{1}{3}\%$ of it leaked out, at what price must it be sold per gallon to gain $11\frac{1}{3}\%$ on the cost?

24. A grocer sells $\frac{2}{3}$ of a barrel of sugar for \$7.82, losing 8%; for how much must he sell the remainder to gain 8% on the whole?

25. By selling a suit of clothes for 5% less than cost, a tailor gets \$5.55 less than if he had sold them for 10% above cost; find the cost.

26. The labor in making a machine will cost \$37.50, and the whole cost is \$65; the laborers strike and get an advance of 10% on their wages; for what must the machine be sold to gain 20%?

27. A merchant bought wheat at 96 cents a cental, and marked it for sale at \$1.12 $\frac{1}{2}$. He afterwards marked up the price $6\frac{2}{3}\%$, and sold 240 centals. The buyer failed, however, and settled by paying 75 cents for every dollar he owed. Did the merchant gain or lose, how much, and how much per cent?

28. A hardware merchant bought three dozen agate basins at the rate of 3 for \$5, and sold them at a gain of \$10 on the whole; what was the average selling price of each, and what was the gain per cent?

29. A merchant sold 25 yards of cloth for \$31.25, at a loss of $16\frac{2}{3}\%$; find the cost per yard.

30. A boy bought oranges at 40 cents a hundred, lost 5% by decay, and sold them at the rate of 3 for 2 cents; what was his gain %?

EXERCISE 215. (ORAL.)

1. A man bought a horse for \$75 and sold him at a gain of 20%; find the selling price.

2. Find the gain per cent on sugar bought at 8 cents and sold at 9 cents.

3. Sold calico at 16 cents, gaining 4 cents; what was the gain %?

4. I wish to make $37\frac{1}{2}\%$ on a suit of clothes that cost \$16; for what must I sell them?

5. A grocer sold tea for 30 cents a pound; if he lost $16\frac{2}{3}\%$, what did the tea cost?

6. Sold a carriage for \$40 less than cost, losing 40%; find the cost.

7. If a dozen lemons are bought for 25 cents and sold for 35 cents, what is the per cent of gain?

8. Gained 10 cents by selling a penknife at 25% profit; what did it cost?

9. Sold goods for $\frac{1}{4}$ more than I paid for them; what was the gain %?

10. A grocer makes 10% by selling coffee at $2\frac{1}{2}$ cents above cost; what is the cost and the selling price?

11. A boy sells newspapers at 5 cents, which is $66\frac{2}{3}\%$ above cost; find the cost.

12. A boy buys pencils for 25 cents a dozen, and sells them for 5 cents apiece; what is his gain %?

13. Bought oranges for 1 cent apiece, and sold them at the rate of 2 for 3 cents; what was the rate of gain?

14. Bought 4 books for \$2.40, lost one, and sold the remainder at \$1 each; find my gain %.

15. A man bought a hat for \$4, and traded it for \$3 and a box of 6 collars, worth 25 cents each; what was his rate of gain?

16. A furniture dealer bought a second-hand set of chairs at 32 cents each, spent 8 cents each in repairs, and then sold them at a gain of 25%; what did he receive for them?

17. Bought a suit for \$25, which was $16\frac{2}{3}\%$ less than the asking price, and the asking price was 50% above the cost; find the cost.

18. 20% of my sales is profit; what is my gain %?

19. Sold $\frac{1}{3}$ of my stock for what the whole cost; what did I gain per cent?

EXERCISE 216.

Find everything not given; gain or loss, rate of gain or loss, cost, and selling price.

- | | |
|--|---|
| 1. Cost \$1500; loss $7\frac{1}{2}\%$. | 13. Loss \$125; S. P. $83\frac{1}{3}\%$. |
| 2. Gain \$500 at $2\frac{1}{2}\%$. | 14. S. P. \$480 or $73\frac{1}{3}\%$. |
| 3. S. P. \$1320; loss 1%. | 15. Cost \$920; loss \$15%. |
| 4. Loss \$75; cost \$2000. | 16. Loss $13\frac{1}{3}\% = \$840$. |
| 5. Gain $7\frac{2}{7}\%$; cost \$1085. | 17. Gain \$5.50; S. P. \$95.50. |
| 6. Cost \$2375; S. P. \$3050. | 18. Cost \$175; S. P. \$200. |
| 7. Gain 7% or \$147. | 19. Gain 14%; cost \$7000. |
| 8. Loss $16\frac{2}{3}\%$; S. P. \$2085. | 20. Profit 50%; gain \$25.50. |
| 9. Cost \$12.50; S. P. \$10. | 21. S. P. \$175; cost \$150. |
| 10. S. P. \$18.50; loss $6\frac{1}{4}\%$. | 22. Cost \$15; loss 20%. |
| 11. Profit \$45 or $3\frac{1}{3}\%$. | 23. S. P. \$15; loss $16\frac{2}{3}\%$. |
| 12. Cost \$1300; S. P. 130%. | 24. Gain 3%; S. P. \$1030. |

EXERCISE 217.

Select 10 examples from Exercise 209 and form practical examples in P. and L. Perform and bring to the class for dictation.

Model: *Example 1.* I gained \$75 by selling goods at 3% profit; what did they cost?

 COMMISSION.

Some men are employed in transacting business for others, such as buying and selling goods or lands, renting houses, collecting money.

These men are found in our business centers under various names, including commission merchants, brokers, auctioneers, real estate agents, collectors, and the like.

The money received for their services is called **COMMISSION**.

It is usually a percentage (1) on the money paid for property bought, (2) received for property sold, (3) on the amount of money collected.

The sum left after taking out the commission is called the **proceeds**.

A broker's commission is called **brokerage**.

$$\text{Base} = \begin{cases} \text{Cost.} \\ \text{Selling price.} \\ \text{Money collected.} \end{cases}$$

EXERCISE 218. (WRITTEN.)

Label numbers given in the first 10 examples with the words *cost*, *S. P.*, *money collected*, *proceeds*, *commission*, or *rate of commission*; rewrite in *general form*, and perform.

1. I send 50 tons of baled hay to a commission merchant in San Francisco, who sells it for me at \$14 a ton and charges 3% commission; what is the amount of his commission, and what do I receive?

Model: $50 \times \$14 = \700 S. P. 3% = Rate of commission.

What is 3% of \$700?

What is 97% of \$700?

2. If I pay \$10.50 per ton for the hay and \$1.14 per ton for freight, what % do I make on the whole cost?

3. A merchant buys 100 barrels of flour for me, paying \$5.50 per barrel. If he charges 3% commission what sum of money must I send him to pay for the flour and his services?

4. I send \$3120 to a commission merchant to buy flour at 4% commission; for what does the \$3120 pay? What is the cost of the flour? Commission?

5. For what must I sell the above flour per barrel to gain 20% on the whole cost, supposing I received 750 barrels?

6. A farmer sends 72 dozen eggs to an agent who sells

them @ 32 cents at a commission of $6\frac{1}{4}\%$; what does the farmer receive for his eggs per dozen?

7. An auctioneer sells at auction a farm, buildings, stock, and tools. He receives \$14000 for the farm and buildings, \$2700 for the stock, and \$1300 for the tools; what is his commission at $1\frac{3}{4}\%$?

8. A man sends \$31500 to a broker to buy cotton at 5% commission; how many bales at \$100 each does he buy?

9. A grocer sends \$2490 to a commission merchant to buy sugar at $3\frac{3}{4}\%$ commission. If he pays 8 cents a pound for the sugar, for what must the grocer sell the whole to gain $16\frac{2}{3}\%$ on the whole cost, and at how much per pound?

10. An agent sells Blaine's "Twenty Years in Congress," at \$5 a volume, receiving 35% commission; how many volumes must he sell to make \$1400?

11. A collector collected rents at 3% commission and received \$87.60 for his services; what sum of money did he collect?

12. A farmer sends 3000 centals of wheat to a commission merchant in San Francisco, who sells it at \$1.16 $\frac{2}{3}$ per cental at a commission of $2\frac{2}{7}\%$; what is his commission and what does the farmer receive for his wheat?

13. My commission for selling flour for \$5150 is \$128.75; what %?

14. I sent \$5115 to an agent who buys goods for a commission of \$165; what %?

15. My agent received \$123 for collecting rents at 3%; how much money did he collect?

16. I pay \$275 for a house lot and build on it a house costing \$1720, which my agent rents for \$25 a month, charging 5% commission; what per cent do I make a year on the money laid out?

17. A lawyer collects 75% of a bill of \$5600 and charges $6\frac{1}{4}\%$ for collecting; what is his commission and what does the creditor receive?

18. A town owes a debt of \$1890 which is to be collected from the people of the town. If the collector charges 10% for collecting, what sum must be collected to pay the debt?

19. I wish to gain 25% on cloth for which I paid \$1.20 per yard, 5% commission to my agent, and $1\frac{1}{2}$ cents per yard for freight; what must be the selling price?

20. I send \$2689.75 to my agent to buy pork at $1\frac{1}{2}$ % commission; how many pounds can he buy at $3\frac{1}{3}$ cents a pound?

21. How many pounds of sugar at $8\frac{1}{2}$ cents does an agent purchase for me, if his commission at $3\frac{1}{3}$ % amounts to \$25? What does the sugar cost me per pound?

22. How many barrels of flour at \$5 can a commission merchant purchase with \$5150 on a commission of 3%?

23. Find the commission on the sale of 100 bales of cotton, averaging 480 lb. to a bale, at \$18 per cwt., the commission being 5%.

24. An agent sells 450 tons of hay at \$13 a ton, commission 5%, and with the proceeds bought wool at $22\frac{1}{2}$ cents per pound, commission 4%; what was his whole commission and how many pounds of wool did he buy?

25. Bought 500 boxes of oranges at \$2.50 a box, and paid \$12 freight. My whole bill was \$1287; what % commission did I pay for buying?

EXERCISE 219. (ORAL.)

1. My agent sells \$250 worth of goods for me at 4% commission; what do I receive?

2. I send an agent sufficient money to buy \$75 worth of shoes at 4%; what do I send him?

3. A commission merchant sold a bill of goods at 3% commission, receiving \$30 for his services; what was the value of the goods sold?

4. A commission merchant sells goods for me for \$200, receiving \$4 commission; what %?

5. An agent receives \$2 for buying eggs on a commission of 2%; what does he pay for the eggs?

6. An auctioneer sells a sewing machine for \$20, receiving 5% for his services; what is the sum received by the owner?

7. If a commission merchant sells flour for \$5 a barrel on a commission of 5%, how many barrels must he sell to realize \$100?

8. \$10 commission; 10% rate; find the cost.

9. Bought a lot of clocks through an agent, paying \$50 for the clocks and \$2 commission; what was the rate of commission?

10. For how much a yard must cloth be sold to gain $33\frac{1}{3}\%$, if the cloth was bought @ 20 cents on a commission of 5%?

11. Sent \$30 to an agent to buy lead pencils at 50% commission; how many at 2 cents apiece can he get?

12. Sales \$2000; commission \$10; find the rate.

13. What amount of money must I send my agent that he may buy 100 pr. of shoes at \$1 and pay himself a commission of 3%?

14. Remittance \$2020; commission 1%; find cost.

15. Cost \$300; remittance \$309; find commission %.

EXERCISE 220.

Find everything not given of the following; rate of commission, commission, cost, S. P., proceeds.

1. Com. \$165; (S. P.) \$6600.
2. Com. for buying \$140 at $1\frac{3}{4}\%$.
3. Remittance to agent \$5600; com. $2\frac{1}{2}\%$.
4. Com. for selling at $1\frac{1}{2}\%$ is \$13.50.
5. Auction sale \$8732; com. 2%.
6. Com. \$14.21; cost \$568.38.
7. Sum collected \$14000; com. \$420.
8. Com. \$141; proceeds \$2209.

9. Remittance to agent \$4000; com. \$250.
10. Remittance to agent \$2182.80; com. 2%.
11. Cost \$4800; remittance \$4872.
12. Com. for selling \$48.29 at $2\frac{3}{4}\%$.
13. Remittance to agent \$1500; com. \$41.12.
14. Proceeds \$4975; com. \$25.
15. Com. for buying \$74.25 at $6\frac{1}{4}\%$.
16. Rate of com. $\frac{1}{4}\%$; S. P. \$2000.

EXERCISE 221.

Select 10 examples from Exercise 209 and construct practical examples in commission. Perform and bring to the class for dictation.

INSURANCE.

I build a house for \$3000. A company gives me a written promise to pay me \$2000 if the house burns, and charges me $\frac{4}{5}\%$ per year of the promised sum for the promise. What do I pay for the promise? What does the company lose if the house burns within a year?

A company is made up of two or more persons joining for the transaction of business.

INSURANCE is a guaranty of a sum of money to be paid in case of loss of property or life.

The company making the written contract to pay losses is an **Insurance Company**.

The written contract is called the **policy**.

The sum paid by the owners of property for insurance is called the **premium**.

The *premium* is a certain fraction, or per cent, of the sum insured, and is *paid in advance*.

Fire Insurance Companies rarely insure property for more than $\frac{2}{3}$ of its value, and in no case pay for more than the

value of the property destroyed, whatever may be the face of the policy.

In Life Insurance the premium is a sum of money varying with the amount of insurance and the age of the individual.

A fee of \$1 or more is sometimes charged for making out the policy.

Do insurance companies lose money by paying these losses? Why?

EXERCISE 222.

1. A merchant has his store and contents insured for \$5500 at $\frac{4}{5}\%$ premium; what is the cost to him? If the store and contents are destroyed, what sum does the insurance company lose?

2. A trader paid \$110 premium to have a shipment of horses insured at $2\frac{3}{4}\%$ of their value; what was their value?

3. A sea captain insures his vessel for \$48000, paying \$360; what is the rate of insurance?

4. A farmer has his standing grain, worth \$4000, insured for $\frac{1}{2}$ its value at $\frac{1}{2}\%$ per month. At the end of a month and a half the grain burns; what does the insurance company lose?

5. I pay \$62.50 to insure my house for $\frac{2}{3}$ its value for 3 years at $2\frac{1}{2}\%$. What is the value of my house?

6. An insurance company loses \$3528 by the wreck of a carload of flour which it had insured for \$3600. What was the rate of insurance?

7. A man has a policy of \$7600 placed on his house, which sum includes the insurance value and the premium at $1\frac{1}{3}\%$; what is the premium, and the value of the house?

8. I have a house worth \$6000, a barn worth \$1800, and personal property worth \$1200; on all which I am insured for $\frac{2}{3}$ value, paying \$106 including \$1 fee for the policy. What is my rate of insurance?

9. I buy a house for \$6500, expend \$500 in repairs, and

insure it at $\frac{3}{4}\%$ on $\frac{5}{7}$ of the whole cost including repairs. I then sell it at a loss of 4% on my whole expense. What is my selling price?

10. Sent \$2846.25 to my agent, who buys flour at \$5 $\frac{1}{2}$ a barrel, charging 3 $\frac{1}{2}\%$ commission. I insure it at 1 $\frac{1}{4}\%$ on the cost; for how much must I sell it per barrel to gain 10% on the whole cost?

11. A house, insured for \$2400, at $\frac{3}{4}\%$, burns. The owner buys another with the insurance money and gets it insured for \$30; find the rate on the latter house.

12. Which is cheaper, to get my building insured in two different companies for \$1500 each, at $\frac{3}{4}\%$, or in one company for \$3200 at $\frac{7}{8}\%$?

13. Paid 3% every 3 years to get an insurance of \$2400 on my house. If it burns at the end of 8 years, what is the loss of the insurance company?

14. A merchant imports a cargo from Liverpool, England, worth £1500 and insures it at $\frac{4}{5}\%$; find the premium in \$'s.

15. Paid \$42 to get an insurance of \$2562 on my stock, the insurance covering the premium; what was the rate?

16. I insure my life for \$8000, paying \$19.80 per \$1000 per year; what do I pay the company if I live 20 years after insurance?

17. Paid 1 $\frac{1}{3}\%$ to get my library insured; premium 6 $\frac{2}{3}\%$; what was the value of the library?

18. A grocer insures 200 barrels of flour for 66 $\frac{2}{3}\%$ of their cost at 1 $\frac{3}{4}\%$, paying a premium of \$10.50; what price per barrel must the flour bring to gain 16 $\frac{2}{3}\%$ on the cost exclusive of insurance?

19. For what sum must a policy be made out to cover the insurance on a property of \$2100, at $\frac{4}{5}\%$?

20. I buy a house for \$6000 and pay 1 $\frac{1}{4}\%$ to get it insured. I also pay \$900 for repairs. I then sell it at a gain of 33 $\frac{1}{3}\%$ on all it has cost me. The money thus received I send to

a commission merchant, who buys flour for me at $3\frac{1}{3}\%$ commission, paying \$4.50 per barrel. He finally sells the flour for \$4 a barrel at $3\frac{1}{8}\%$ commission and sends the balance to me. What % have I gained on all I paid out for the house?

EXERCISE 223. (ORAL.)

1. Paid \$7 on an insurance of \$1400; find the rate.
2. Paid \$20 to get stock insured for $\frac{2}{3}\%$; find the value insured.
3. If I pay $1\frac{2}{3}\%$ a year to get my house insured for \$1500, how many years will it take to pay its value in premiums?
4. Paid \$3 on an insurance of \$400; find the rate.
5. Paid \$12 to insure merchandise at $1\frac{1}{5}\%$; find its value.
6. Insured $\frac{1}{5}$ share in a ship worth \$100000 for $\frac{1}{2}$ value at $\frac{1}{2}\%$; find the premium.
7. Gained 25% by selling flour for \$505, the cost including an insurance of 1%; find the first cost.
8. I insure my house for $\frac{2}{3}$ of its value of \$3000, and my stock for $\frac{5}{6}$ of its value of \$600, for $\frac{4}{5}\%$ premium; find the premium.
9. Paid \$2.50 on an insurance of \$500; find the rate.
10. Paid $\frac{2}{3}\%$ on an insurance of \$900; find the premium.
11. Paid \$5 to get an insurance at $2\frac{1}{2}\%$; find the insurance.
12. Insured standing grain worth \$2000 for $\frac{1}{3}$ its value at $\frac{1}{2}\%$ a month; what do I pay for 2 months' insurance?
13. Paid $\frac{4}{5}\%$ a year for 5 years on property insured for \$5000; if it burns then, what is the company's loss?
14. Paid \$13, including \$1 for policy, on an insurance of \$1200; what was the rate?
15. \$1010 policy including 1% premium; find the value.

EXERCISE 224. (WRITTEN.)

Construct 10 examples of your own from Exercise 209, perform, and bring to the class for dictation.

TAXES.

Towns, counties, and states are at expense to maintain schools, courts, roads, public buildings, officers, and the like. To meet these expenses the people are required to pay a per cent of the value of their property.

The money paid by an individual for public expense is called a **TAX**.

Taxable property is of two kinds: (1) **Personal**, or movable property; as, *money, tools, carriages, stock*; (2) **Real estate**, or immovable property; as *lands, buildings*.

In California, men between the ages of 21 and 60 are also taxed so much a head without regard to property. This is called **poll tax**. The amount raised by poll tax makes the property tax so much less.

EXERCISE 225. (WRITTEN.)

1. Suppose the property of this county to be valued at \$4000000, and its expenses for this year to be \$21800. If 1200 men pay a \$1.50 poll tax, what will be the fraction, and %, of tax on the property?

2. If your parents own real estate valued at \$4000 and personal property valued at \$1800, and pay 1 poll tax, what is their whole tax?

3. If your next door neighbor pays a tax of \$16, including 1 poll, what is his property valued at?

4. Suppose Mr. B pays 3 polls and has real estate valued at \$5500 and personal property valued at \$1700; what is his tax?

5. A county builds a bridge for \$4500. The property is valued at \$1000000; what is the tax per \$100?

6. A tax of \$8500 was raised on a town at \$16 on a \$1000 worth of property. If there were 500 polls at \$1 each, what was the value of the town property?

7. A school district is taxed \$3000 to build a school

house, which sum is a tax of $\frac{3}{10}\%$ of the property value; what is the property value and what is the tax on a dollar?

8. The road tax on a road district was 2 mills on a dollar; what was the rate per cent of tax, and the amount on \$3500 worth of property?

9. A town whose property value is \$450000 has an expense of \$4750. If a collector charges 5% for collecting, what rate of taxation must be made?

10. At the rate of 8 mills on \$1, and \$2 poll tax, find a man's tax on \$7500 real estate, \$2750 personal property, and 2 polls.

11. A poll tax of \$2 for road improvements is assessed on a town of 485 polls; 10% is paid for collecting. What sum of money will be left for improving roads?

12. I buy a house lot for \$400 and build a house on it for \$2000. I pay an insurance on the house of $\frac{4}{5}\%$ on $\frac{3}{4}$ its value, and a tax on the whole of \$14 on \$1000, the property valuation being $\frac{2}{3}$ the cost. For how much must I rent the house per month to realize 20% a year on my money?

13. A tax of \$2850 is to be raised on a town and sufficient besides to pay for collecting at 5%. If the rate is $\frac{1}{2}$ cent on a dollar, what is the property worth?

14. I buy a house for \$6500 and spend \$500 for repairs. I rent it for \$77.50 a month, out of which I pay a yearly insurance of $\frac{3}{4}\%$ on $\frac{5}{7}$ of its whole cost, including repairs, and a yearly tax of 1% on $\frac{3}{4}$ of the same. What per cent of income a year do I realize on the whole cost?

EXERCISE 226. (WRITTEN.)

Ask your parents or guardian for their last tax bill. Bring to the class for dictation.

DUTIES.

The expenses of the U. S. Government are mostly paid by taxes on imported goods.

Such taxes are called **customs** or **duties**.

Another object of duties is "protection to home industry." Find out what you can about "protection."

Duties are of two kinds: **specific** and **ad valorem**.

A **specific duty** is a charge on goods by weight, number, or measure without regard to value.

An **ad valorem duty** is a per cent of the cost of the goods at the port from which shipped. Both classes of duties are laid upon some goods.

Gross weight is the weight of goods including the boxes or other packing material.

Net weight is the weight after deducting the weight of the packing material.

Duties are estimated on the net weight, and all custom-house weights are *long ton* weights.

EXERCISE 227. (WRITTEN.)

State in connection with each example whether the duty is specific or ad valorem.

1. Find the duty on 100 boxes of oranges at 25 ct. per box and 60 boxes of lemons at 30 ct. per box.

2. What is the duty on 100 French watches valued at \$15 each, duty 25%?

3. Imported 11 tons of iron T rails, duty $\frac{9}{16}$ ct. ¢ lb. What was the whole duty?

4. A merchant imported 12 cases woolen shawls, each case averaging 255 lb. valued at 80 ct. ¢ lb., duty 35 ct. ¢ lb. and 35% ad valorem. Charges \$72.50. Find the whole cost.

5. A liquor dealer imported 80 doz. quart bottles of champagne, duty \$7 per doz. bottles; 3 casks of French brandy, 30 gal. each, duty \$2 per gal.; 3 casks wine, 31½ gal. each, duty 50 ct. per gal.; and 50 doz. pint bottles of ale, duty 35 ct. per gal. Find the whole duty.

6. Paid a duty of \$2283.60 on an invoice of silks at 60% ad valorem. What was the value of the goods?

7. Find the duty on 1280 sq. yd. Brussels carpet valued at \$725, duty 30 ct. per sq. yd. and 30% ad valorem; 1440 sq. yd. tapestry valued at \$650, duty 20 ct. per sq. yd. and 30% ad valorem.

8. Duty on 840 lb. flaxseed at 20 ct. per bu. of 56 lb.

9. A coal dealer imported from Sydney, Australia, 1000 tons of coal, paying 75 ct. per ton duty. What was the cost of importing?

10. A ship brought into port 200 tons of rock salt. What was the duty at 8 ct. per cwt.?

11. Imported 50 boxes tin plate, 108 lb. to the box net, on which I paid a duty of 1 ct. per lb. What did the duty amount to?

12. A merchant imported 25 tons of coke invoiced at \$94, duty 20%. Find the duty.

13. Find the duty at 20% on an importation of Bath brick of 200 boxes valued at 45 ct. per box.

STOCKS.

When a number of men wish to form a railroad company, insurance company, bank, or the like, they obtain permission by law, and subscribe a sum of money for the undertaking.

Companies authorized by law to carry on business are called **corporations**.

The money invested is called the **Stock**.

The stock is divided into equal parts, commonly of \$100 each, called **shares**. It may be bought and sold in the market like other property, its value depending mainly upon the prosperity of the company.

The nominal value of the stock is called the **par value**.

The price which the stock brings in the market is called the **market value**.

If the market value is above par, the stock is *at a premium*; if below, *at a discount*. Thus, stock selling for \$118 per share is at 18% premium; for \$82, at 18% discount.

Usually, the broker's commission for buying or selling is a per cent of the *par* value of the stock dealt in; but in mining stocks it is reckoned upon the *market* value of the stock bought or sold.

The earnings of the company, after deducting the expenses, are divided among the stockholders, and are called **dividends**.

Dividends, premium, and discount are reckoned on the par value.

The following table is taken from the stock quotations (market value) found in the daily papers, par value \$100:

N. Y. Central R. R.,	\$102	Western Union Tel.,	\$66½
Mich. Central R. R.,	70	Home Mut. Ins.,	145
Lake Shore R. R.,	81½	Bank California,	169¼
St. Paul R. R.,	87¾	First National,	125
So. Pacific R. R.,	109¼	Spring Valley Water,	91½
Geary St. R.,	107	Oregon Navigation,	99
Giant Powder,	60	Bodie Mining,	1¼
Wells, Fargo Ex.,	118	Mono Mining,	2½

EXERCISE 228. (WRITTEN.)

1. Make separate lists, from the above, of stocks at a premium and those at a discount.

2. What must I pay for 10 shares of N. Y. Central, brokerage $\frac{1}{4}\%$?

3. After buying the above stock, a dividend of 4% was declared; what rate per cent of income did I realize on the money invested?

4. A friend received \$320 in dividends at the same time; how many shares did he own, and what were they worth at the market value?

5. The Giant Powder Co. declares an annual dividend of 9%; how many shares at the above quotation must I buy to get an annual income of \$720, and what will they cost me?

6. What per cent on investment is realized by a dividend of 7% in the Home Mutual if $\frac{1}{4}$ % is paid for brokerage?

7. Which is the better investment: Bank Cal., paying 12% dividends, or First National, paying 9%?

8. I bought a certain number of shares of Lake Shore at $81\frac{7}{8}$, and sold them at par, brokerage $\frac{1}{8}$ % on each transaction, thereby gaining \$2860 on the whole; how many shares were there, and what did they cost me?

9. Find my gain % in the preceding example.

10. Bought a house for \$5000, and rented it for a year for \$275; out of the rent paid taxes at the rate of $\frac{5}{8}$ % on $\frac{4}{5}$ cost. At the end of the year I sold the house for 12% advance on cost, and invested the sum in Michigan Central, paying $3\frac{1}{2}$ % dividends. Which was the better investment?

11. I send a broker \$1468 to buy Sp. V. W., at $\frac{1}{4}$ % commission: how many shares can he buy me?

12. What cost 150 shares Mono at $\frac{1}{2}$ % brokerage?

13. What dividend would have to be declared to realize $11\frac{1}{9}$ % on money invested in Oregon Nav.?

14. What is the cost of 50 shares of Mono and 100 shares of Bodie, brokerage $\frac{1}{2}$ %?

15. Lost \$340 by buying S. P. R. R. at the above quotation and selling at $101\frac{1}{4}$, brokerage $\frac{1}{4}$ % each way; how many shares?

16. How many shares of Western Union must I own to realize \$570 on a 6% dividend, and for what will they sell at the above quotation?

17. Which is better, St. Paul R. R., paying 5%, or Oregon Nav., paying 6%?

18. How many shares in Geary St. R. could you buy for the money you receive by selling 24 shares of Electric Light at $53\frac{1}{2}$, no brokerage?

19. How many shares of Wells, Fargo can I buy for \$1182.50 at $\frac{1}{4}$ % commission?

INTEREST.

Suppose a friend of yours wishes to buy a house for \$3000. Not having that sum in ready money, he borrows \$1000 of you and agrees to pay you 8% of the sum per year so long as he keeps it. What does he pay you a year for the use of the money? What for 2 yr.? If he keeps it for 1 year, what is the whole sum of money he pays you? If he keeps it 2 yr.? 3 yr.? $2\frac{1}{2}$ yr.? 3 yr. 6 mo.? 2 yr. 3 mo.?

Money paid for the use of money is called **INTEREST**.

The sum lent is called the **principal**.

The per cent of interest for a given time is called the **rate**. It is understood as being for a year unless otherwise specified.

The principal and interest added make the **amount**.

Banks usually loan money by the month; and sometimes pay on deposits from 3 to 5% a year. They reckon 12 months of 30 days each, or 360 days, to a year.

The United States laws count 365 days to a year, but this reckoning is not in common use among business men.

EXERCISE 229. (ORAL.)

Find the interest and amount of:

1. \$100, at 6%, for 2 yr.; 3 yr.; $3\frac{1}{2}$ yr.
2. \$200, at 5%, for 4 yr.; 4 yr. 6 mo.
3. \$300, at 8%, for 6 mo.; 1 mo.; 3 mo.
4. \$250, at 6%, for 2 yr.; $2\frac{1}{2}$ yr.; 2 yr. 4 mo.
5. \$150, at 4%, for 2 mo.; 6 mo.
6. \$1, at 6%, for 1 yr.; 1 yr. 4 mo.
7. \$2, at 5%, for $3\frac{1}{2}$ yr.; $2\frac{1}{2}$ yr.
8. \$5, at 8%, for 3 mo.; 3 yr.
9. \$10, at 3%, for 5 mo.; 7 mo.
10. \$20, at 6%, for 9 mo.; 8 mo.
11. \$2.40, at 5%, for 2 yr.; $2\frac{1}{2}$ yr.

12. \$7, at 8%, for 6 mo.; for 2 yr.
13. \$25, at 4%, for 2 yr.; at 8%.
14. \$50, at 5%, for 6 mo.; at 6%.
15. \$800, at 3%, for 1 mo.; at 6%.
16. \$800, at 5%, for 9 mo.; at 10%.
17. \$750, for $2\frac{1}{2}$ yr., at 4%; at 8%.
18. \$1000, for 3 mo., at 4%; at 5%; at 6%.
19. \$1000, for 2 yr. 9 mo., at 4%; at 6%.
20. \$1500, for 4 yr., at 5%; at 6%; at 8%.
21. \$2000, for $2\frac{1}{10}$ yr., at 5%; at 4%; at 6%.
22. \$2500, for 2 mo., at 1% a month.
23. \$450, for 3 mo., at 1% a month.
24. \$40, for 5 mo., at $\frac{1}{2}$ % a month.

EXERCISE 230. (ORAL.)

To find the years, months, and days between two dates.

1. Find the time from January 16th, 1884, to May 27th, 1886.

Method: Jan. 16th, 1884, to Jan. 16th, 1886, . . . 2 yr.
 Jan. 16th, 1886, to May 16th, 1886, . . . 4 mo.
 May 16th, 1886, to May 27th, 1886, . . . 11 da.

Ans.—2 yr. 4 mo. 11 da.

2. Find the time from October 25th, 1885, to May 10th, 1887.

Oct. 25th, 1885, to Oct. 25th, 1886, 1 yr.
 Oct. 25th, 1886, to Apr. 25th, 1887, 6 mo.
 In April after 25th, 5 days, and 10 days in May = 15 da.

Ans.—1 yr. 6 mo. 15 da.

3. Find the time from each date except the last to all the following dates in this list:

January 7th, 1880; May 3d, 1881; August 25th, 1882;
 Sept. 4th, 1883; Dec. 1st, 1884; Dec. 27th, 1885.

Find the time between January 3d, 1885, and each of the above dates. Also from August 7th, 1881, to each of the above dates.

SUGGESTION.—The teacher will give additional examples as needed until the class is quick in the work of finding the time, using the pencil or crayon to record results only.

SIX PER CENT METHOD.

EXERCISE 231. (WRITTEN.)

6 hundredths of the principal per year means half as many hundredths as months; therefore add $\frac{1}{2}$ the number of months to 6 times the number of years for the hundredths of the multiplier. An odd month gives $\frac{1}{2}$ hundredth, or 5 thousandths; and since a month, or 30 days, gives 5 thousandths, $\frac{1}{3}$ of 30 days, or 6 days, gives 1 thousandth. Therefore,

To form a multiplier.

Take 6 times the years and $\frac{1}{2}$ the months as hundredths, and $\frac{1}{6}$ the* days as thousandths.

Example: Find the interest of \$275.75 for 2 yr. 5 mo. 18 da. at 6% yearly.

WORK.

$$\begin{array}{r} 2 \times 6 + \frac{4}{2} = .14 \\ 5 + \frac{18}{6} = .008 \\ \hline .148 \end{array}$$

$$\begin{array}{r} 275.75 \text{ Principal.} \\ \quad .148 \text{ Multiplier.} \\ \hline 2758 \\ 1103 \\ \quad 220 \\ \hline \$40.81 \text{ Interest.} \end{array}$$

A little practice will enable the student to form a 6% multiplier very quickly. A good arithmetician will find the time between two dates and form a 6% multiplier from it in 30 seconds or less. The contraction in multiplication, p. 112, should always be used.

Form 6% multipliers from each of the differences between dates found in Example 3, Exercise 230.

Fill out the following table, rate 6%:

No.	Date.	Date.	Principal.	Interest.	Amount.
1	Aug. 4, 1881.	Sept. 12, 1882.	\$179.50	?	?
2	March 19, 1879.	February 25, 1882.	325.00	?	?
3	July 8, 1883.	Sept. 24, 1886.	758.75	?	?
4	Jan. 16, 1884.	May 8, 1886.	1024.25	?	?
5	Oct. 28, 1885.	Jan. 12, 1886.	584.50	?	?
6	Dec. 1, 1885.	March 12, 1887.	725.84	?	?
7	June 10, 1883.	Jan. 4, 1887.	387.95	?	?
8	April 14, 1887.	May 8, 1887.	42.20	?	?

EXERCISE 232. (WRITTEN.)

To find the interest at other rates than 6 per cent.

First find the interest at 6%; for 5%, subtract $\frac{1}{6}$ of this interest from itself; for 4% subtract $\frac{1}{3}$.

For 7% add $\frac{1}{6}$ of the 6% interest to itself; for 8% add $\frac{1}{3}$; for 9% add $\frac{1}{2}$; for 10% divide by 6, removing the decimal point one place to the right.

If higher rates are needed, form a 12% multiplier with the months as hundredths, and $\frac{1}{3}$ the days as thousandths.

Find the interest on:

- \$450, from Mar. 7, 1885, to July 7, 1885, at 4%; 5%; 6%.
- \$387, from May 3, 1884, to Aug. 3, 1886, at 5%; 7%; 8%.
- \$718.25, from Jan. 1, 1885, to Jan. 1, 1887, at 4%.
- \$410, for 3 yr. 3 mo. 10 da., at 7%.
(3 mo. 10 da. = 100 da. = $\frac{100}{360}$ = $\frac{5}{18}$ year.)
- \$718, from May 11, 1882, to May 31, 1885, at 5%.
- \$380, from February 10, 1883, to May 5, 1885, at 7%.
- \$425, for 2 yr. 5 mo. 17 da., at 8%.
(Divide interest of 1 yr. by 12 to get int. for 1 mo. 2 yr. 5 mo. 17 da. = $29\frac{17}{12}$ mo.)
- \$910.50, from Jan. 1, 1885, to Mar. 15, 1885, at 6%.
- \$748, from April 3, 1886, to Aug. 24, 1886, at 5%.

10. \$875, from July 7, 1886, to Jan. 1, 1887, at 4%.
11. \$2512, from May 1, 1884, to May 10, 1885, at 7%.
12. \$3850, from Mar. 9, 1885, to Sept. 9, 1885, at 8%.

EXERCISE 233. (WRITTEN.)

Find the interest on:

1. \$431, for 3 yr. 2 mo. 12 da., at 6%; at 7%.
2. \$1515, for 1 yr. 1 mo. 1 da., at 6%; at 4%.
3. \$495, for 5 mo. 24 da., at $7\frac{1}{2}\%$.
4. \$218.50, for 1 yr. 3 mo. 15 da., at $4\frac{1}{2}\%$.
5. \$729, for 2 mo. at 8%; at 3%.
6. \$435, for 4 mo., at 7%; at 5%.
7. \$760, for 1 yr. 9 mo. 27 da., at 6%; at 3%.
8. \$129.40, for 7 mo. 16 da., at 4%; at 5%.
9. \$240.50, for 19 mo. 18 da., at $7\frac{1}{2}\%$.
10. \$528, from Jan. 1, 1884, to May 16, 1886, at $4\frac{1}{2}\%$.
11. \$1150, from Mar. 19, 1884, to July 25, 1884, at 7%.
12. \$1425, from May 3, 1885, to Sept. 30, 1886, at 6%.
13. \$45, from Aug. 7, 1885, to Jan. 13, 1886, at 5%.
14. \$75, from Apr. 28, 1884, to Apr. 10, 1885, at 6%.
15. \$110, from May 23, 1880, to Sept. 13, 1884, at 4%.
16. \$434.20, from Dec. 1, 1881, to Nov. 1, 1884, at $4\frac{1}{2}\%$.
17. \$290, for 1 yr. 11 mo., at $8\frac{1}{2}\%$.
18. \$4050, for 5 mo. 10 da., at 5%.
19. \$1235, from May 19, 1886, to Sept. 1, 1886, at 6%.
20. \$1425, from Jan. 25, 1884, to May 10, 1885, at 5%.
21. \$475, for 8 mo. 8 da., at 8%.
22. \$2150, for 21 da., at 6%.
23. \$1240, for 17 da., at 4%.
24. \$1345, from May 1, to May 25, at 6%.

Work the examples of Exercise 229 by this method.

EXERCISE 234. (WRITTEN.)

Find the amount of:

1. \$980, for 7 mo. 10 da., at 6%.
2. \$418.25, for 3 mo., at $\frac{3}{4}\%$ per mo.

3. \$7280, from Mar. 1 to May 13, at 1% per mo.
4. \$1212.50, for 1 yr. 1 mo. 14 da., at $4\frac{1}{2}\%$.
5. \$976.10, from May 27 to Nov. 19, at 5%.
6. \$3200, for 9 mo. 9 da., at 8%.
7. \$225, from June 29 to Dec. 1, at $7\frac{1}{2}\%$.
8. \$850, from Feb. 1 to Sept. 1, at $1\frac{1}{4}\%$ per mo.
9. \$230, for 1 mo. 10 da., at 10%.
10. \$1925, for 4 mo. 4 da., at 5%.
11. \$458, from Jan. 1, 1887, to Mar. 11, 1888, at $6\frac{1}{2}\%$.
12. \$319.50, for 3 mo., at 8%.
13. \$112.75, for 2 yr. 5 mo. 25 da., at 6%.
14. \$550, from Apr. 3 to Nov. 9, at 5%.
15. \$336, from Sept. 20, 1885, to Mar. 1, 1886, at 6%.
16. \$210, for $2\frac{1}{2}$ yr., at $7\frac{1}{2}\%$.
17. \$640, for $9\frac{1}{3}$ mo., at 8%.
18. \$1350, from Mar. 1 to Sept. 1, at 10%.
19. \$2080, for 4 mo., at $4\frac{1}{2}\%$.
20. \$1875.35, from July 7 to Jan. 1, at $6\frac{1}{4}\%$.
21. \$70, for 11 mo., at 6%.
22. \$10.50, from Jan. 1 to July 10, at 10%.
23. \$49.50, for 1 yr. 7 mo. 28 da., at 4%.
24. \$112, for 2 yr. 12 da., at 6%.
25. \$129.75, for 2 yr. 17 da., at $5\frac{1}{2}\%$.
26. \$18.50, from Jan. 1, 1808, to Aug. 17, 1887, at 3%.

EXERCISE 235. (WRITTEN.)

Construct 10 examples of your own, find the amount in each, and bring to the class for dictation.

To compute accurate interest.

When interest is to be reckoned on a basis of 365 days to a year, count the exact number of years and days between the dates. Find the interest for years as in the ordinary method. For the days take as many 365ths of 1 year's interest as there are days. Thus,

Find the exact interest on \$240 from March 1, 1885, to July 10, 1885, at 5%. (131 da.)

OPERATION.

$$\begin{array}{r} 12 \\ \hline 240 \times 1 \times 131 \\ \hline 20 \times 365 \end{array} = \$4.31 \text{ Ans.}$$

EXERCISE 236. (WRITTEN.)

Find the exact interest on:

1. \$219, for 25 da., at 7%.
2. \$480, from May 10, 1884, to July 3, 1886, at 6%.
3. \$348, for 73 da., at $6\frac{1}{2}\%$.
4. \$1000, for 219 da., at 4%.
5. \$1220, from March 27 to July 27, at 10%.
6. \$104, from Jan. 9 to Apr. 4, at 12%.
7. \$210, from Apr. 1, 1886, to July 12, 1887, at $7\frac{1}{2}\%$.
8. \$442, for 91 da., at 5%.
9. \$920, from Aug. 17 to Dec. 1, at 8%.
10. \$460, for 75 da., at 10%.
11. \$235, from May 15, 1884, to July 27, 1886, at 4%.
12. \$40, for 40 da., at 12%.

PROBLEMS IN INTEREST.

Analyze by model under Exercise 69:

1. At 7 per cent, \$500 gains \$35 in 1 year; how many years will it take to gain \$105?
2. At 1% \$500 gains \$15 in 3 years; at how many per cent will it gain \$105 in the same time?
3. At 7 per cent, in 3 years \$1 gains 21 cents; how many dollars will it take to gain \$105 at the same rate and time?
4. At 7 per cent, in 3 years \$1 amounts to \$1.21; how many dollars will amount to \$605 at the same rate and time?

In Example 1, by knowing the rate we know the interest for 1 yr. In Example 2, we know, without stating, the interest at 1% for 3 yr. In Example 3, we know the interest of \$1 for 3 yr. at 7%; and in Example 4, the amount of the same. Hence, the examples may be contracted thus:

1. In what time will \$500 gain \$105, at 7%?
2. At what rate will \$500 gain \$105 in 3 yr.?
3. What sum will gain \$105 in 3 yr., at 7%?
4. What sum will amount to \$605 in 3 yr., at 7%?

Observe. { *First apply the conditions of the examples to a unit, or 1, of the things asked for in the answer.*

EXERCISE 237. (ORAL.)

Find:

1. Time in which \$100 will gain \$15, at 6%.
2. Sum that will gain \$20 in 4 years, at 5%.
3. Rate at which \$50 will gain \$1.50 in 6 mo.
4. Sum that will gain \$30 in 3 yr., at 5%.
5. Rate at which \$200 will gain \$25 in $2\frac{1}{2}$ yr.
6. Time in which \$75 will gain \$5, at 4%.
7. Rate at which \$60 will gain \$7.50 in $2\frac{1}{2}$ yr.
8. Time in which \$150 will gain \$21, at 8%.
9. Sum that will gain \$100 in 10 yr., at 10%.
10. Sum that will amount to \$12 in 2 yr., at 10%.
11. Time in which \$1000 will gain \$90, at $4\frac{1}{2}$ %.
12. Rate at which \$800 will gain \$40 in 1 yr. 3 mo.
13. Sum that will gain \$75 in 5 yr., at 5%.
14. Rate at which \$300 will gain \$28 in 2 yr. 4 mo.
15. Time it will take \$700 to amount to \$749, at 7%.
16. Rate at which \$75 gains \$4 in 8 mo.
17. Sum that gains \$200 in 2 yr., at 5%.
18. Rate at which \$450 gains \$72 in 2 yr. 8 mo.
19. At what rate any sum will double itself in 4 yr.; 8 yr.; 10 yr.
20. Time it will take money to double itself, at 5%; at 6%.

EXERCISE 238. (WRITTEN.)

1. Find the time in which \$360 will gain \$97.20, at 6%.
2. In what time will \$900 gain \$84, at 7%; at 8%?
3. What sum will gain \$62.50 in 2 yr. 6 mo., at 5%?
4. Find rate at which \$145 will gain \$5.80 in 6 mo.
5. Rate at which \$240 will gain \$56 in 3 yr. 6 mo.
6. What sum will amount to \$296 in $3\frac{1}{3}$ yr., at 7%?
7. A merchant buys goods for \$700, to be paid in 6 mo.; what sum put at interest to-day at 6% will pay the debt?

The money, which, put at interest at the present time, will amount to a given sum in a given time, is sometimes called the **present worth**; and the difference between the present worth and amount, the **true discount**.

8. How long will it take \$720 to gain \$16.20 at $1\frac{1}{2}$ % a mo.?
9. Find present worth of \$400 due in 4 mo., at $\frac{1}{2}$ % a month.
10. Find true discount of \$390 in 6 mo., at 6%.
11. A man was offered a horse for \$100 cash, or \$104 in 6 mo.; if money is worth 8%, which is the better offer?
12. How long must \$450 be kept at interest, at 8%, to gain what \$700 gains in 2 yr., at 4%?
13. A man owes 3 bills of \$250 each, due in 4, 6, and 9 months respectively; what are the debts worth to-day, at 1% a month?
14. Bought a house for \$7500, payable in 4 mo., and sold it for \$7500 cash; if money is worth $\frac{1}{2}$ % a month, what did I gain?
15. A house that cost \$3400 rents for \$35 a month, what annual rate of interest is received?
16. Find rate at which \$275 will gain \$56.10 in 3 yr. 4 mo. 24 da.
17. Find principal that will gain \$103.95 in 3 yr. 2 mo. 15 da., at $7\frac{1}{2}$ %.
18. What sum of money invested at 6% will give an income of \$100 per month?

19. Find principal that will amount to \$926.06, at 6%, in 3 yr. 7 mo. 21 da.

20. Find time in which \$720 will amount to \$736.20, at $1\frac{1}{2}\%$ a month.

NOTE.—Find the interest first.

21. Find time in which \$125, at 4%, will amount to \$141.50.

22. Find rate at which \$760 will amount to \$926.06 in 3 yr. 7 mo. 21 da.

23. Paid a debt due Apr. 1, 1886, which amounted to \$221.27 June 10, at 6%: find the debt.

24. I loaned my money at 8%, payable quarterly, and received \$125 a quarter. How much did I loan?

25. What principal amounts to \$560.23 in 2 yr. 7 mo. 15 da., at 6%?

26. Borrowed \$90, June 1, 1880, at 7%. Paid it when it amounted to \$100; when did I pay it?

27. Paid \$71.30, at 5%, for the use of \$460 how long?

28. If I owe \$200 payable in 2 mo., \$300 in 3 mo., and \$400 in 4 mo., what should I pay to-day to make the debt good, money being worth $\frac{1}{2}\%$ a month?

29. A carriage for which I paid \$200 cash, I sold for \$210 on 8 mo. credit. Money being worth 6%, what did I gain?

30. Find rate at which \$410 gains \$27.06 in 1 yr. 1 mo. 6 da.

31. Paid in 4 yr. \$210 interest, at 7%. What was the principal?

32. Find time in which \$550 will gain \$102, at 6%.

33. Find difference between the interest and true discount of \$270 for 9 mo., at 8%.

34. Borrowed a sum of money at 6% and lent it again at $7\frac{1}{2}\%$, by which I gained \$35.10 in 3 yr. What was the sum?

35. Find rate at which \$75 will gain \$2 in $\frac{1}{3}$ of a year.

36. Find present value of \$2000, $\frac{1}{2}$ due in 2 mo., $\frac{1}{4}$ in 3 mo., and the remainder in 5 mo., at 6%.

note he writes the words, "on demand" in place of "six months after date."

It is sometimes convenient to pay a note in *parts*, or *installments*. Such payments are called PARTIAL PAYMENTS. They should be written, with their dates, across the back of the note, and are then called *indorsements*.

Suppose the above note to have the following indorsements:

Nov. 8, 1885, received \$250.

Apr. 14, 1886, received \$150.

Write the note on paper and put on the indorsements.

What money was due Nov. 8, 1885?

What was due after the payment of that date?

What was due on the remainder Apr. 14, 1886?

What was still due after the payment of that date?

All payments must first go towards paying interest due. If a payment is not enough to pay the interest, it is counted with the next payment, and its date left out.

SUGGESTION.—The teacher may ask the trustees to purchase a book of note blanks for the practical use of classes. Five of the following notes should be written on the printed blanks.

EXERCISE 240. (WRITTEN.)

Write out the following in proper form on paper, placing the indorsements on the back, and perform. Determine mentally, by inspection, whether a partial payment is too small to be taken out by itself.

1. Date, Jan. 1, 1885. Place, your own town. Face, \$1500. Interest, 6%. Indorsements: Aug. 7, 1885, \$500. Dec. 7, 1885, \$500. What is due Jan. 1, 1886?

2. Face, \$480. Mar. 3, 1884. Interest, 7%. Indorsements: Sept. 3, 1884, \$196.80. Mar. 3, 1885, \$214. Sept. 3, 1885, paid the amount due. Find it.

3. Face, \$1000. July 20, 1884. Interest at 8%. In-

dorsements: Mar. 5, 1885, \$50. July 5, 1885, \$450. What was still due?

4. Face, \$1230. Date, Jan. 1, 1886. Interest at $5\frac{1}{2}\%$. Indorsements: Mar. 1, 1886, \$98. June 7, 1886, \$500. Sept. 20, 1886, \$290. Dec. 10, 1886, \$100. What is due Jan. 1, 1887?

5. Face, \$800. Date, Mar. 1, 1886. Interest at 10%. Indorsements: Aug. 10, 1886, \$200. Sept. 1, 1886, \$50. Jan. 1, 1887, \$15. What was due Mar. 1, 1887?

6. Face, \$365. Date, July 10, 1885. Interest at 6%. Indorsements: Sept. 10, 1885, \$68.65. Nov. 18, 1885, \$103.40. What was still due?

7. Face, \$2500. Date, Aug. 5, 1885. Interest at 7%. Indorsements: Jan. 1, 1886, \$500. March 10, 1886, \$750. Find the sum due Aug. 5, 1886.

8. Face, \$960. Date, June 25, 1886. Interest at $7\frac{1}{2}\%$. Indorsements: Sept. 1, 1886, \$10. Dec. 1, 1886, \$360. Jan. 1, 1887, \$300. What was still due?

9. Face, \$500. Date, Feb. 1, 1884. Interest at 8%. Indorsements: Mar. 1, 1884, \$100. Apr. 1, 1884, \$100. May 1, 1884, \$100. What was due June 1, 1884?

10. Face, \$1200. Date, May 15, 1886. Interest, 6%. Indorsements: Aug. 10, 1886, \$500. Nov. 1, 1886, \$500. What was due Jan. 1, 1887?

EXERCISE 241. (WRITTEN.)

Write 3 notes of your own, put 2 indorsements on each, perform, and bring to the class for dictation.

COMPOUND INTEREST.

Sometimes when a note specifies that interest on it is to be paid yearly, semi-yearly, quarterly, or the like, a special agreement is made that if such interest is not paid when

due it shall be added to the principal, and the amount becomes a new principal for the next period.

This method of computing interest is called **COMPOUND INTEREST**. In many states it is prohibited by law.

Compound the interest at 8% on \$540 for 7 mo. 12 da., payable quarterly.

OPERATION.

\$540	=	Principal.
<u>1.02</u>	=	am't of \$1 for $\frac{1}{4}$ year.
\$550.80	=	" " \$540 for $\frac{1}{4}$ year, or Prin. for 2d quarter.
<u>1.02</u>	=	" " \$1 " $\frac{1}{4}$ "
561.82	=	" " \$550.80 for $\frac{1}{4}$ yr., or Prin. for 3d quarter.
<u>1.009$\frac{1}{3}$</u>	=	" " \$1 for 1 mo. 12 da.
567.06	=	" " \$540 for 7 mo. 12 da., int. computed quarterly.

Find the compound interest above.

EXERCISE 242. (WRITTEN.)

Find the compound interest on:

1. \$1000, for 4 yr., at 6%, payable annually.
2. \$300, for 1 yr. 7 mo., at 8%, payable semi-annually.
3. \$425, for 11 mo., at 4%, payable quarterly.
4. \$250, from Jan. 1, 1886, to Feb. 1, 1887, at 5%, payable semi-annually.
5. \$500, from May 1, 1885, to Aug. 1, 1887, at 6%, payable annually.
6. \$490, for 8 mo., at 8%, payable quarterly.
7. \$1500, from Aug. 1, 1886, to Apr. 10, 1887, at 7%, payable semi-annually.
8. \$275, for 9 mo., payable quarterly, at 6%.
9. \$800, for 2 $\frac{1}{3}$ yr., payable yearly, at 6%.
10. \$1200, for 1 yr. 6 mo. 6 da., at 6%, payable semi-yearly.

Discounting commercial paper.

SACRAMENTO, CAL., Mar. 4, 1885.

\$1500 $\frac{00}{100}$.

Six months after date, I promise to pay to the order of James Kenney, Fifteen Hundred $\frac{00}{100}$ Dollars, value received.

ALLEN PAINE.

Suppose James Kenney carries the above note to the bank, April 4, to get money on it. The bank will deduct from the face a certain per cent, say 1% per month, from April 4 to the date of maturity, and pay him the balance. Find the balance on this note.

This is called **discounting** the note.

Observe. {

1. *The discount is made on the **face** of non-interest bearing notes.*
2. *When the note bears interest the discount is made on the **amount of face and interest at maturity.***

In some of the United States three days, called days of grace, are allowed for the payment of the note after it is actually due, discount being made for the extra time. Days of grace are not allowed in California.

EXERCISE 243. (WRITTEN.)

Write out the following notes on paper and find the sum allowed on each at the bank:

1. Note of \$700, Apr. 10, 1885, payable 4 mo. from date. Discounted at 8%, June 10, 1885.

2. Note of \$850, July 3, 1885, payable 60 days from date. Discounted, Aug. 1, at 1% a month.

3. Note of \$1400, May 19, 1886, bearing interest at 8%, payable 6 mo. from date. Discounted, Aug. 19, 1886, at 8%.

4. Note of \$900, June 1, 1885, bearing interest at 1% per

month, payable 3 months from date. Discounted, July 1, at 1% per month.

5. Note of \$250, Sept. 9, 1881, payable 30 days after date. Discounted, Sept. 9, at 1%.

6. Note of \$1850, May 1, 1885, payable 3 mo. from date. Discounted, July 8, at 1% a month.

7. Note of \$525, Jan. 5, 1886, bearing interest at $\frac{1}{2}\%$ a month, payable 4 mo. from date. Discounted, Feb. 5, at 1% per month.

8. Note of \$300, Dec. 11, 1886, bearing interest at $\frac{3}{4}\%$ a month, payable in 6 mo. Discounted, Mar. 1, 1887, at $1\frac{1}{2}\%$ a month.

9. Note of \$1140, Nov. 28, 1885, bearing interest at 8%, payable 1 yr. from date. Discounted, Jan. 1, at 8%.

10. Note of \$1375, Aug. 5, 1886, payable 3 mo. from date. Discounted, Sept. 1, at 10%.

11. Note, \$735, Jan. 13, 1886, interest at 10%, payable 3 months from date. Discounted, Feb. 25, at 2% per month.

DISCOUNT.

In buying a bill of goods, a discount or discounts are often allowed on the list or marked price of the goods, and a further discount on the result for cash. Thus,

Bought a bill of goods amounting to \$800 at 20 and 5 off, and 5% off for cash.

FIRST OPERATION.

$$\begin{array}{r}
 5)\$800 \quad =\text{marked price of goods.} \\
 \quad 160 \quad =20\% \text{ discount.} \\
 \hline
 20)\$640 \\
 \quad 32 \quad =5\% \text{ discount.} \\
 \hline
 20)\$608 \\
 \quad 30.40 =5\% \text{ off for cash.} \\
 \hline
 \$577.60 =\text{actual cost of the goods.}
 \end{array}$$

SECOND OPERATION.

$$\begin{array}{r} 2 \\ 800 \times 4 \times 19 \times 19 \\ \hline 5 \times 20 \times 20 \end{array} = 577.60$$

Observe. { *Each discount is reckoned by itself and on the sum remaining after the preceding discount.*

EXERCISE 244. (WRITTEN.)

1. Find the actual cost of a bill of goods marked at \$450 at 40% off, and 5% off for cash.

2. Sold a bill of merchandise at 25% off, and 5% off for cash; find the whole discount.

3. Sold a bill of goods marked at \$250 for 30, and 5 off. Was the actual selling price more or less than if a discount of 35% had been made?

4. By getting a discount of 10, and 10 off for cash, I pay \$810 for a bill of goods; what was the list price?

5. Bought furniture to the amount of \$200, on which a discount of 5% was made for cash; what was the cost?

6. For what must I sell goods which were sold me for \$830, list price, at 30, 10, and 5 off, to gain 20%?

7. Paid \$76 for a bill of glass after a deduction of 5%; what was the invoice price?

8. Find the cash value of a bill of cloth amounting to \$425.50 at a discount of 10%, and 5% off for cash.

9. Bought a bill of goods amounting to \$725 on 6 mo. credit, on which a discount of 3% was allowed for cash; what did I pay for the goods?

10. The retail price of a certain book is \$5.50. If I get a discount of 10, and 10 off for cash, what do I pay for the book?

11. I paid \$1.50 for a book after a discount of 25%, and $16\frac{2}{3}\%$ off; what was its marked price?

12. Sold a bill of goods for \$700 on 6 mo. at 15 off, and deducted 4% for cash; what did I receive?

ACCOUNTS.

Every one who receives and spends money should keep a record of receipts and expenses, specifying the date and nature of each transaction.

What does the word "cash" mean? Are greenbacks cash? Bank checks? Postage stamps?

The following is a record of a boy's receipts and expenses:

Jan. 1, 1886, money on hand, \$2.65. Jan. 2, paid 5c. for marbles and 10c. for lead pencil. Jan. 4, paid 25c. for a Speller. Jan. 5, paid 10c. for a bottle of ink. Jan. 6, received 25c. for blacking father's shoes one week. Jan. 7, paid 10c. for a top and 15c. for marbles. Jan. 9, received \$1 for driving cow to pasture and 50c. for milking. Jan. 11, paid 40c. for a Reader and 60c. for an Arithmetic. Jan. 12, sold top for 5c. Jan. 13, paid 10c. for postage stamps. Jan. 14, paid 20c. for candy. Jan. 16, received 10c. for doing errands and paid 5c. for marbles. Jan. 19, lost 10c. Jan. 20, received 40c. for blacking father's shoes. Jan. 21, paid 50c. for a kite. Jan. 25, sold 5 cents' worth of marbles. Jan. 26, received 25c. for clearing the yard. Jan. 27, paid 15c. for setting a broken light of glass. Jan. 29, found 25c. Jan. 30, paid \$1.15 for a Geography.

Obtain paper, rule as below, copy, and fill out the month's items. Find out how much more he received than paid, see if it agrees with the *balance*, then add each column and place the result below.

1886.		CASH.	<i>Rec'd.</i>		<i>Paid.</i>	
			\$	ct.	\$	ct.
Jan.	1	On hand,	2	65		
"	2	Marbles, 5 cents; Lead pencils, 10 cents.				15
"	4	Speller,				25
		Carried forward,	2	65		40

			\$	ct.	\$	ct.
		Brought forward,	2	65		40
Jan.	5	Bottle ink,				10
"	6	Blacking father's shoes,		25		
"	7	Top, 10 cents; Marbles, 15 cents, . .				25
"	9	Driving cow, \$1; Milking, 50 cents, . .	1	50		
"	11	Reader, 40 cents; Arithmetic, 60 cents,			1	00
Jan.	30	Balance,			1	55
			5	50	5	50

What does the "balance" in the above account show? If the amount of money on hand does not agree with balance, what does the difference show? In the account, which column is the larger? Could the other column ever be larger in a "cash" account? *Why?*

The "balance" should be found twice a month, at least, and oftener as the business is larger. Business firms and banks balance their "cash" every day. It is well to write the balance in red ink. *Why?*

Open an account for February with the above balance on hand, and write items of your own. Take care that at no time your "paid" items exceed the "received" items. Balance and bring to the class.

Write out the following "cash" acct. of a teacher, and balance every Saturday:

May 1 (Sat.) 1886, Cash on hand, \$78.80. May 3, Bought 20 cents worth of P. O. stamps. May 4, Paid \$5 borrowed money. May 5, Bought 11 yards cashmere @ \$1.25; pair of shoes \$4.50; 1 doz. hdkfs. \$1.75. May 6, Paid express on package of books 25 cents. May 7, Sent by money order \$2.75 to pay for books, paying 10 cents for the order. May 8 (Sat.), Paid for postal cards 10 cents; stamped envelopes 55 cents; note paper 60 cents.

May 11, Paid 2 weeks' board, to May 15, @ \$4.50. May 13, Paid spool thread 10 cents; bottle mucilage 25 cents. May 15 (Sat.), Carriage hire \$2.50; received \$9.25 for services on Board of Education.

May 18, Paid 2 weeks' board to May 29. May 19, Paid mo. contribution to church \$1.50; gave a poor woman 50 cents. May 20, Paid for sending telegram 75 cents; crackers 25 cents. May 22, Paid 2 mo. subscription to "Daily Herald" @ 65 cents; received mo. salary \$75.

May 24, Deposited in bank \$50. May 26, Paid \$1 for book, 25 cents for "legal cap," 40 cents for ribbon. May 27, Paid \$1.25 for gloves; exchanged a second-hand Reader for a new one worth 60 cents, being allowed 25 cents for the old one, and paid the difference. May 28, Lent a friend \$2. May 29, Paid for pins 10 cents, penknife 50 cents, sheet music 30 cents.

Balance shows my pocket-book 5 cents short, for which I can not account. *Balance.*

Write out the following account:

July 1, 1887, received \$5. July 4, bought 5 flags at 25c. each, 3 bunches of fireworks at 30c. a bunch, 18 yd. bunting at 10c. per yd. July 6, earned 20c. selling papers. July 7, gave a poor woman 10c. *Balance.*

Write out a cash acct. of your own. Begin with \$5 on hand. Have 6 items received, and 8 paid. *Balance.*

What is a debt? A debtor? A credit? A creditor? Why is it necessary to keep an account of our debts and credits? When is a man your debtor? Your creditor? Is John Smith debtor, or creditor, for what we give him? For what he gives us? What, then, does the debtor side of a man's account show? The creditor side? If the debtor side be the larger, what does the balance show? If the creditor side be the larger? If both sides are equal? Explain each item in the following account, which we will suppose to be your account with John Smith:

Dr.		JOHN SMITH.				Cr.			
1886.		\$	ct.	1886.		\$	ct.		
Jan.	1	He owes me . . .	43	65	Jan.	2	Cash	15	00
"	8	2 loads hay . . .	9	50	"	12	Work with team	2	75
"	27	Use of wagon . .		50	"	18	Calf	7	00
"	30	Cash	9	70	"	27	Order on Robert Stewart	28	90
					"	31	Balance	---	---

Copy the above account and complete it. Change it so as to show John Smith's account with you. Write an imaginary continuation of the account during the month of February. Have 5 Dr. items and 5 Cr. items, and have \$5 due John Smith Mar. 1. Have no dates on Sunday.

Observe. { *Any person becomes Dr. for goods or money delivered to him.*
Any person becomes Cr. for goods or money delivered BY him.

The following are the transactions of a farmer with a merchant, S. C. Griggs & Co. Copy as above, writing the account for each party:

1886. Mar. 1, Sold S. C. Griggs & Co. 7 doz. eggs @ 18 cents; 11 rolls butter at 40 cents. Received 10 lb. sugar @ 8 cents; 1 sack salt 25 cents. 3. Delivered them 10 sacks potatoes @ 85 cents. 4. Bought 20 yd. sheeting @ 12½ cents; 12 yd. print @ 10 cents. 6. Sold 12 doz. eggs @ 16 cents; 9 rolls butter @ 40 cents; 10 sacks potatoes @ 80 cents. Bought 4 50-lb. sacks flour @ \$1.12½; 2 lb. tea @ 65 cents; 5 lb. coffee @ 37½ cents. 9. Bought 1 box soap \$1.15; 2 lb. cheese @ 17½ cents. 12. Sold 10 doz. eggs @ 18 cents; 13 rolls butter @ 40 cents; 5 sacks potatoes @ 90 cents. 15. Bought 20 lb. dried apples @ 7 cents; can lard 65 cents; 2 boxes paper collars @ 15 cents; 5 cans apricots @ 30 cents. 18. Sold 2 loads wood @

\$4.50. 22. Bought 1 lamp \$2; 1 pr. boots \$5.50; 1 ham 12 lb. @ 18 cents. 25. Sold 15 doz. eggs @ 20 cents; 8 rolls butter @ 50 cents. Bought suit clothes \$8; 8 lb. sugar 75 cents; 1 10-gallon can kerosene \$1.75; 1 pr. boys' shoes \$3.50; 1 sack oatmeal 50 cents. *Balance.*

Write an imaginary account between the nearest merchant and yourself. Have 8 purchases and 7 sales. Have your prices reasonable and the transactions such as you might make.

Sometimes a person engages in an enterprise, like renting or purchasing grain land, on which he wishes to know his profit or loss over and above interest on the money invested.

The following is an account of the expenses and returns of a barley field. Use the name "Barley field," debit it with all its expenses, including the interest on the value of the land @ 6% for a year, and credit it with all its returns.

Balance, and find the per cent of profit on the land value.

160 acres of land valued at \$70 per acre. Plowing, \$1.30 per A.; sowing, 10 cents per A.; seed, \$1 per A.; harrowing, 25 cents per A.; poisoning squirrels, \$4.50; heading, \$1.75 per A.; thrashing, 10 cents per cental, 2700 centals; sacks, 8 cents each, averaging 135 lb. to a sack; sack twine, \$8; hauling grain to warehouse, 5 cents a sack; sold the lot at the warehouse at \$1.01½ per cental; sold the straw and stubble for \$95.

Do the same with the following Dairy account:

40 cows at \$35 per head.

1886. Jan. 1. Salt, \$1. 5. Rennets, \$1.30; coloring, 50 cents. 11. Wood, \$5. 12. Cheese bandages, \$7.20. 18. Sold 1600 lb. cheese @ 9 cents; freight and commission, 1 cent per lb. 30. Paid 2 men's wages, \$50; board, \$32; pasture for Jan., \$1 per head. Feb. 1. Sold 1400 lb. cheese @ 9½ cents; freight and com., 1 cent per lb. *Balance.*

BALANCE SHEET.

The following "Balance Sheet" is a statement of Luke Smith's debts and credits at the beginning of the year. Copy on the board and explain each item:

1886.		BALANCE SHEET.		<i>Debts.</i>		<i>Credits.</i>	
				\$	ct.	\$	ct.
Jan.	1	Farm and improvements,				758	75
"	"	Household property, . . .				176	50
"	"	Mortgage on farm,	425	85			
"	"	Note payable on demand, . .	56	50			
"	"	John Mason,				43	65
"	"	Wm. Jones,	88	35			
"	"	Chas. Bell's note,				76	50
"	"	Cash,				19	85
"	"	Bank of California,				78	95
"	"	<i>Balance,</i>					

Put into a balance sheet the following statement of Luke Smith's debts and credits Feb. 1, 1886:

Farm, \$472. Improvements, \$326.75. Household property, \$176.50. Mortgage, \$395.25. John Mason owes him \$29.70. He owes Chas. Bell \$18.25 and Thomas Olmstead \$29.85. He has \$28 in money and \$48.25 in the Bank of California.

Compare this with the preceding month and tell whether he has gained or lost. How does he stand with each person Feb. 1, as compared with his standing Jan. 1? If his debts were larger than his credits, how would he settle with his creditors?

Write an account on balance sheet of your own for Luke Smith for March. Leave him in debt.

What is a bank? What use have we for it? How does the bank get pay for taking care of our money?

If you wish to pay a person a debt and have money in

the bank, instead of paying him in money you can write an order on your banker to pay the same.

Such an order upon a bank is called a **check**.

When you deposit money in the bank, the bank gives you a written statement to that effect, called a **certificate of deposit**, and you draw the money on presenting this certificate.

Or the bank will give you a bank account book, and you may draw checks till the money is all drawn out.

[Form of check.]

No. 9.

MERCED, CAL., May 7, 1886.

FIRST NATIONAL BANK

Pay to James Cash or Order, One Hundred Thirteen and $\frac{50}{100}$ Dollars.

\$113.50.

JOHN SIMMS.

Copy the following bank account and explain each item. Write out the checks on paper, with yourself as depositor. Add 10 items and balance with \$75 to your credit in bank.

1886.		BANK OF VENTURA.	<i>Dr.</i>		<i>Cr.</i>	
			\$	ct.	\$	ct.
Jan.	4	Gold	100	00		
"	5	Check I Thos. Cruson			27	85
"	5	Check II Wm. Bell & Co.			28	05
"	9	Silver	46	50		
"	9	Check III Bartlett Bros.,			9	75
"	12	Check IV Self			45	65
"	12	Check on Bank Cal.,	14	25		
"	12	M. Wooley's check on Bank Vent.,	9	05		
			<hr/>		<hr/>	

EXCHANGE.

Suppose you owe A. B. Stanton of New York \$500. To avoid inconvenience and risk of sending the money you may buy of your banker, say D. B. Fairbanks, an order on some New York banker, say S. A. Spring, to pay A. B. Stanton.

Thus:
 \$500.00. *Petaluma, Cal., May 25, 1886.*
 At thirty days ^{5/1886} sight pay to *A. B. Stanton*
 or order, Five ~~Hundred~~ ^{Hundred} Dollars, and charge the
 same, to our account.
D. B. Fairbanks,
 Cashier.
 To *S. A. Spring*
 New York

You send the order to A. B. Stanton: and he, on receiving it, presents it to S. A. Spring. Spring writes acceptance across the face as above, if willing to pay it. At maturity, 30 days from acceptance, Stanton presents the order and receives the money. If he wishes the money before maturity, the banker will discount it for the difference in time.

Such an order is called a **draft**, or **bill of exchange**; and this method of making payments, EXCHANGE.

A draft is always made out in the money of the country on which it is drawn.

Drafts are either "sight" or "time" drafts; that is, payable on presentation, or at a certain specified time after presentation.

Which is the above draft?

The maker of a draft is called the **drawer**; the person to whom addressed, the **drawee**; and the person to whom payable, the **payee**.

Name each in the above draft.

A draft may be transferred, like a note, by indorsement.

If the merchants of New York owe the merchants of San Francisco more than San Francisco merchants owe them, bills of exchange on New York will be plentiful in San Francisco and can be purchased cheaply, or *at a discount*; if the balance is due the other way, bills of exchange on New York will be scarce in San Francisco, and will, therefore, be dear, or *at a premium*.

Time drafts are discounted to the buyer for the time specified. The time discount is understood to be the rate for 1 year, unless otherwise stated.

All discounts or premiums are reckoned as per cent of the face of the draft. The above draft on S. A. Spring has a time discount at 7%; if it be purchased at 1% premium what is paid for it? At 1% discount?

EXERCISE 245. (WRITTEN.)

Write out the following drafts to imaginary payees and drawers, with proper acceptance in red ink. *Write no acceptance on sight drafts.* Why?

1. Find the cost in New Orleans of a draft for \$5000 on New York at 60 days' sight, exchange being $1\frac{1}{2}\%$ premium, interest at 8% per annum.

2. Bought a sight draft on St. Louis, for \$580, at $\frac{1}{2}\%$ discount; what was the cost?

3. I paid \$2481.25 for a sight draft on Chicago, at $\frac{3}{4}\%$ discount; what was the face of the draft?

4. I wish to buy a 60 days draft on London, for £320, exchange at \$4.95 per £, interest at 7%; what will it cost?

5. Paid \$1566.15 for a sight draft on Boston, at $1\frac{1}{2}\%$ discount; what was the face?

6. Paid \$4500 for a draft on New York at 90 days sight, premium $1\frac{1}{2}\%$, interest at 6% per yr.; find the face.

7. Find the cost of a sight draft on Paris for 4000 francs at 1% discount.

8. A sight draft for \$800 cost me \$794; what was the rate of discount?

9. What is the cost of a 10 days sight draft for \$765, at $\frac{3}{4}\%$ premium, time discount 8%?

10. The cost of a 30 days draft for \$800, time discount including grace 6%, was \$799.60; what was the rate of discount or premium?

11. I buy in Sacramento a 45 days draft on Paris for 1000 francs, interest 1% a month, exchange $1\frac{1}{2}\%$ premium; what do I pay?

12. I pay \$162.75 for a draft on Paris at 45 days after date, time discount 1% a month, exchange $1\frac{1}{2}\%$ premium; what is the face of the draft?

13. I buy in Paris a 60 days draft on London for £500, exchange being at 26 francs per £, time discount 5%, what do I pay? Is exchange at a discount or premium?

The payment of small sums at a distance is often effected by means of **postal money orders** or by **bank checks**.

A **money order** is, in effect, a sight draft drawn by the postmaster of the debtor upon the postmaster of the creditor; payable to the creditor, or order.

Name the payer, drawer, and payee.

Money orders are subject to the following charges and regulations :

On orders not exceeding \$10	8 cents.
Over \$10 and not exceeding \$15	10 "
" 15 " " " 30	15 "
" 30 " " " 40	20 "
" 40 " " " 50	25 "
" 50 " " " 60	30 "
" 60 " " " 70	35 "
" 70 " " " 80	40 "
" 80 " " " 100	45 "

A single order may include any amount, to \$100.

Not more than 3 orders may be issued in one day, to the same applicant, payable at the same office, to the same payee.

A money order is negotiable, but subject to one transfer only.

A **check** is, in effect, a sight draft on a bank.

The value of a check as a medium of exchange is, that it passes for money, when certified or signed by the cashier of the bank on which it is drawn, and properly indorsed.

Such a check is called a *certified check*, and is usually cashed by any bank at which it is presented, without discount to one who keeps an account with that bank. To one not keeping an account with that bank, it is customary to discount it at 20 ct. or 25 ct. per \$100, and a like rate is charged the buyer of such a check by a bank with which he does not keep an account.

EXERCISE 246. (ORAL.)

Name the charges on money orders for the following sums, and specify if it takes more than one order for the amount named:

\$2.50	\$20.00	\$219.00	\$.25	\$160.00
25.00	40.00	175.00	200.00	30.50
250.00	125.00	8.50	190.00	40.05
19.90	140.00	3.50	60.00	100.10

EXERCISE 247.

Write sight drafts for the following sums and compute their cost at $\frac{1}{4}\%$ premium:

\$150.00	\$375.00	\$400.50	\$110.00	\$230.75
190.00	75.25	20.00	318.00	500.00

Write a draft for \$325 at 15 days sight, time discount 12%, exchange $\frac{1}{8}\%$ premium. Compute cost.

Write a draft for \$1000, at 10 days sight, exchange $\frac{1}{4}\%$ discount, time discount 9%. Compute cost.

Write a draft for \$725, at 75 days sight, time discount 10%, exchange $\frac{1}{2}\%$ premium. Compute cost.

AVERAGE OF PAYMENTS.

I buy 2 bills of goods Jan. 1 of Mr. A; one of \$300 on 3 mo., and the other of \$250 on 4 mo. If I pay them before they are due, I lose the use of the money for the remainder of the time. If I delay paying them after they are due, Mr. A loses the use of the money for the time. Now, I wish to pay both debts together, without loss to either party.

FULL ANALYSIS.

The use of \$300, 3 mo. = use of \$1 900 mo. (300×3 mo.)

“ “ “ \$250, 4 mo. = “ “ \$1 000 mo. (250×4 mo.)

“ “ “ \$550 $\left\{ \begin{array}{l} 3 \text{ mo.} \\ 4 \text{ mo.} \end{array} \right\}$ = The use of \$1 900 mo. = use

of \$550 $\frac{1}{3 \frac{1}{3}}$ of 1900 mo. = $3 \frac{5}{11}$ mo. $3 \frac{5}{11}$ mo. = 3 mo. 14 da., + Jan. 1 = Apr. 15.

CONTRACTED OPERATION.

$$3 \times 300 = 900 \text{ mo.}$$

$$4 \times 250 = \underline{1000} \text{ mo.}$$

$$550 \quad) \underline{1900} \text{ mo.}$$

$$3 \frac{5}{11} \text{ mo.} = 3 \text{ mo. } 14 \text{ da.} \quad \text{Average Time.}$$

$$\text{Jan. } 1 + 3 \text{ mo. } 14 \text{ da.} = \text{Apr. } 15, \text{ Date of Payment.}$$

EXERCISE 248. (WRITTEN.)

1. I owe \$180 in 5 mo., \$250 in 8 mo., and \$100 in 9 mo. At what date may I pay the whole with no loss?

2. A man owes a note of \$800 payable in 3 mo., and one of \$1000 payable in 4 mo. Find the average time of payment.

3. Bought, Apr. 8, of C. W. Spring & Co., the following bills of goods: \$150 on 3 mo. credit; \$175 on 4 mo. credit; and \$200 on 6 mo. credit. Find the average time and date of payment for all.

4. I owe 2 bills to the same man, one of \$390 due in 16

days, and one of \$475 due in 20 days. In how many days may I pay both together?

5. Find the average date for paying 3 bills due as follows: May 31, \$100; June 18, \$150; July 9, \$200. (Compute each from May 31.)

6. If I borrow \$250 for 8 mo., how long should I lend \$400 to repay me an equal interest?

7. If you lend a friend \$550 for 6 mo., what sum should he lend you for 10 mo., to repay the favor?

8. A man owes a debt of \$1000 on 10 mo., of which he pays $\frac{1}{4}$ in 4 mo. and $\frac{1}{2}$ in 8 mo. When is the remainder due?

9. Carry out the items in the following bill and find when it is due:

SAN FRANCISCO, Mar. 22, 1886.

F. E. ADAMS (Hollister),

Bought of ELLIS, WELLS & Co.

100 yd. broadcloth	@	\$4	on 2 mo.
500 " sheeting	@	16c.	" 3 "
75 pieces fancy goods	@	\$3	" 4 "

10. In bill 3, page 120, assume the purchases to be on 3 months time, and find the average time for payment.

AVERAGE.

Suppose I mix together 2 lb. of tea worth 60 cents a lb., 4 lb. worth 70 cents per lb., and 4 lb. worth 80 cents per lb. What is the weight of the mixture? Its value? Its average value per lb.?

EXERCISE 249.

1. Sold 2 sheep at \$2.50 per head, 3 at \$3 per head, and 10 at \$3.25 per head. What was the average price per head?

2. Mixed 10 centals of wheat worth 90 cents per cental, 8 centals worth 95 cents, and 7 centals worth \$1. What was the value of the mixture per cental?

3. Mixed 45 lb. of sugar at 8 cents per lb., and 30 lb. at $10\frac{1}{2}$ cents per lb. For what must I sell the mixture per lb. to gain 10%?

4. A grocer sold 8 rolls of butter which cost him 40 cents per roll, and 10 rolls that cost him 50 cents per roll, all at 50 cents per roll. What was his average gain per roll?

5. A liquor dealer mixed 50 gal. of liquor worth 35 cents per gal., 50 gal. worth 42 cents per gal., 50 gal. worth 40 cents per gal., and 50 gal. of water. What was the average value per gal. of the mixture?

6. A grocer mixed 12 lb. of sugar worth 6 cents per lb., 9 lb. worth 8 cents per lb., 15 lb. worth 11 cents per lb., and 17 lb. worth 13 cents per lb. What was the value of the mixture per lb.?

7. A confectioner mixed 5 lb. of candy at 40 cents per lb., 7 lb. at 25 cents per lb., 10 lb. at 20 cents per lb., and 2 lb. at 50 cents per lb., selling the mixture at 30 cents per lb. Did he gain or lose, and how much?

Mix 4 kinds of sugar, worth respectively 7, 8, 12, and 13 cents, so that the mixture shall be worth 11 cents per lb.

WORK.		Lb.	PROOF.
Lb.	Gain or loss.		
11	7	1	+4
	8	1	+3
			+7 Total gain.
	12	1	-1
	13	3	-6
		-7 Total loss.	

1	@	7 = 7
1	"	8 = 8
1	"	12 = 12
3	"	13 = 39
6) 66 (11 ct. Av. price.

EXPLANATION.—Taking 1 lb. at 7 ct., the gain is 4 ct.; and 1 lb. at 8 ct., the gain is 3 ct. Total gain, 7 ct.

Taking 3 lb. at 13 ct., the loss is 6 ct., and 1 lb. at 12 ct. makes the total loss 7 ct.

The mixer gains on all goods below the average price, and loses on all above. Any set of numbers which makes his gains and losses equal, is correct. Usually, several correct sets of answers may be found.

Find two more sets of correct answers to the above example. Test each set by the proof given above.

A little skill will always enable the student to balance the gains and losses, using whole numbers. If this be found difficult, make the last number of pounds fractional and multiply the number of pounds of each kind by the denominator of the fraction.

EXERCISE 250. (Answers Variable.)

1. Mix three kinds of tea, worth 55, 60, and 70 cents, to make a mixture worth 65 cents.

2. If 3 lb. of the 55-cent kind is used, how much of each of the others must be used?

3. How much water must be mixed with a cask of wine containing 30 gal. at \$1.50, to reduce the price to \$1?

Sometimes one or more of the quantities may be limited.

4. Claret worth 35 ct., 40 ct., 50 ct., and 56 ct. per gallon, is to be mixed with 20 gallons @ 64 ct., and 14 gallons at 70 ct., to make the mixture worth 52 ct. per gallon. How many gallons of each shall be taken?

POWERS AND ROOTS.

What is the area of a square whose side is 8 inches?

The product of a number by itself is called the **square** of that number. Thus, *64 is the square of 8.*

The number itself is the **square root** of the product. Thus, *8 is the square root of 64.*

The square is indicated thus: $8^2, 9^2, 25^2$.

The square root thus: $\sqrt{64}, \sqrt{81}, \sqrt{625}$; or $64^{\frac{1}{2}}, 81^{\frac{1}{2}}, 625^{\frac{1}{2}}$.

EXERCISE 251. (ORAL.)

Name the results indicated by the signs affixed to the following numbers:

$49^{\frac{1}{2}}$	1^2	10^2	$8100^{\frac{1}{2}}$	$(\frac{2}{3})^2$	$.3^2$
7^2	$1^{\frac{1}{2}}$	20^2	70^2	$(\frac{4}{9})^{\frac{1}{2}}$	$.09^{\frac{1}{2}}$
$\sqrt{25}$	$\sqrt{9}$	$\sqrt{100}$	90^2	$(\frac{3}{5})^2$	$.25^{\frac{1}{2}}$
5^2	4^2	30^2	$\sqrt{3600}$	$(\frac{9}{25})^{\frac{1}{2}}$	$.36^{\frac{1}{2}}$
$64^{\frac{1}{2}}$	2^2	$\sqrt{2500}$	$\sqrt{400}$	$(\frac{49}{64})^{\frac{1}{2}}$	1.1^2
12^2	$\sqrt{16}$	$\sqrt{1600}$	$4900^{\frac{1}{2}}$	$(\frac{7}{8})^2$	$1.21^{\frac{1}{2}}$
9^2	11^2	50^2	100^2	$.1^2$	$.1.2^2$
$\sqrt{121}$	$\sqrt{81}$	$900^{\frac{1}{2}}$	60^2	$0.1^{\frac{1}{2}}$	$1.44^{\frac{1}{2}}$

Refer to method for squaring numbers of two figures on p. 117, and square the numbers from 14 to 19 inclusive by that method.

SQUARE ROOT.

To extract the square root of a number.

The full explanation of the extraction of roots must be left to Algebra. We here give such illustrations as will serve to fix the method in the memory and give a practical explanation of it.

Find the square root of 1024.

FULL OPERATION.	EXPLANATION.—
$ \begin{array}{r} 1024. \underline{(30+2)} \\ \underline{900} \\ 2 \times 30 = 60 \quad \overline{) 124} \\ \underline{120} \\ 4 \\ 2^2 = 4 \\ \underline{0} \end{array} $	$1024 = \text{tens}^2 + 2 \times \text{tens} \times \text{units} + \text{units}^2.$ The largest tens^2 in 1024 is $900 = 30^2$. The remainder $124 = 2 \times \text{tens} \times \text{units} + \text{units}^2$. Since 124 contains $60 \times \text{units}$, units $= 124 \div 60 = 2$ units, with a remainder 4, which is units^2 .
	CONTRACTED. $ \begin{array}{r} 10\overset{2}{2}4. \underline{(32)} \\ \underline{9} \\ 6\overset{2}{2}) \overline{124} \\ \underline{124} \end{array} $

The 0's may be omitted in the operation, and because 60 and 2 are each multiplied by 2, both may be multiplied at once, as shown in the contracted work. In dividing by 6, remember it is 6 tens in 12 tens and not 6 in 124. Omit mentally the right hand dividend figure.

This operation may be extended to any number.

In squaring a number, as 48.6,

$.6^2 =$	$.36$	We see that the square of each figure
$8^2 =$	64	occupies two places. Hence point
$4^2 =$	16	off the number, whose square root

is to be found, into groups of 2 figures each, commencing at the decimal point. Make full groups at the right of the point by annexing a 0 if necessary. You will notice that in finding each figure of the root, you use the group containing its square.

Find the square root of 2361.96.

OPERATION.	
23	61.96 <u>(48.6)</u>
	<u>16</u>
88.)	761.
	<u>704.</u>
96.6)	57.96
	<u>57.96</u>

EXERCISE 252. (WRITTEN.)

- | | | |
|-------------------------|--|---|
| 1. 2401. ^½ | 13. $\sqrt{17\frac{3}{4}}$ | 25. $(\frac{3136}{9801})^{\frac{1}{2}}$ |
| 2. 1.8225 ^¼ | 14. $\sqrt{1040\frac{1}{16}}$ | 26. 1866.24 ^¼ |
| 3. 930.25 ^½ | 15. $\sqrt{424.36}$ | 27. 9312.25 ^½ |
| 4. .1296 ^¼ | 16. $\sqrt{1.0675}$ | 28. 315844. ^¼ |
| 5. 1.225 ^½ | 17. $\sqrt{10575.}$ | 29. 3858. ^½ |
| 6. 7056. ^¼ | 18. $\sqrt{.00625}$ | 30. 226576. ^¼ |
| 7. .8201 ^¼ | 19. $\sqrt{.0625}$ | 31. 28134. ^¼ |
| 8. 384736. ^¼ | 20. $\sqrt{46656.}$ | 32. .120409 ^¼ |
| 9. 349281. ^¼ | 21. $\sqrt{1232136.}$ | 33. 42.025 ^¼ |
| 10. .4096 ^¼ | 22. $\sqrt{163.84}$ | 34. 4.2025 ^¼ |
| 11. 4.096 ^¼ | 23. $\sqrt{6.5536}$ | 35. 516961. ^¼ |
| 12. 11881. ^¼ | 24. $(\frac{369}{2209})^{\frac{1}{2}}$ | 36. 51696.1 ^¼ |

2^½, 8^½, 12^½, 18^½, and 80^½ to 2 decimal places.

PRACTICAL EXPLANATION OF SQUARE ROOT.

Find the square root of 2025.

OPERATION.

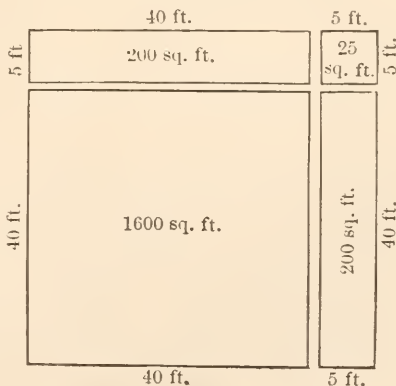
$$\begin{array}{r}
 40^2 = \left| \begin{array}{l} 2025 \mid 40 \div 5 \\ 1600 \\ \hline 425 \\ 5 \\ \hline 85 \end{array} \right. \\
 2 \times 40 = 80 \\
 \quad \quad 5 \\
 \quad \quad 85
 \end{array}$$

EXPLANATION.—Suppose you wish to

lay a square floor containing 2025 sq. ft. You want to know its dimensions. Cut a piece of paper 3 inches square. As near as we can determine by inspection 1600 (40²) sq. ft. is the largest floor. Let your paper represent this

square floor and label as shown in the accompanying figure. There

are still 425 sq. ft. to be built on. By adding strips of the same width to either 2 or 4 sides of a square, we shall preserve the square form. It is easier to add to 2 sides. The strips put on will be of the same length as the square already made, or 40 ft.; making the 2 strips 80 ft. long. Dividing the area 425 sq. ft., which is to be put into these additions, by their length,



gives their width 5 ft. Cut two pieces of paper each 4 in. by $\frac{1}{2}$ in. and lay by the square as shown above, labeling each properly. But a square corner 5 ft. each way must be put on to complete the square. How wide must you cut the paper for this square? The whole length of the 3 additions is 85 ft.; width, 5 ft.; area, 425 sq. ft.

APPLICATION OF SQUARE ROOT.

EXERCISE 253.

To illustrate the following examples, draw figures and label them.

1. Find the side of a square field whose area is 1024 sq. rd.

2. An orange orchard, containing 3364 trees, has the same number of rows that there are trees in a row. How many rows has it?

3. A farmer's ranch, containing 640 acres, is in square form; how many rods around it?

4. I have a garden 66 ft. \times 148 $\frac{1}{2}$ ft. What is the side of a square field equal in area?

~~5.~~ Find the dimensions of a rectangular field containing 3200 sq. rd., and twice as long as broad.

~~6.~~ What is the side of a square field equal in area to a triangular field containing 4096 sq. rd.?

~~7.~~ Mr. A has a field 12 rods square, and Mr. B a square field containing 12 sq. rd. What is the difference in their area?

~~8.~~ How many rods of fence will inclose a square field of 4 acres?

8 Metric. Find the length of a fence which will inclose a square farm of 23 hektares.

~~9.~~ A has a square field of 10 acres; B a rectangular field of 10 acres, 4 times longer than broad. Which field will be the cheaper to fence at \$2.25 a rod?

9 Metric. I have a field 387.5 meters long and 174.8 meters wide. My neighbor has a square field of the same

area. How much more will it cost to inclose my field than my neighbor's at 1.25 francs per meter of fence?

10. If it costs \$425 to fence a field 72 rd. \times 98 rd., what will it cost to fence a square field of the same area?

11. What are the dimensions of the largest possible square table that can be made from a rectangular board 128 in. long and 32 in. wide?

CUBE ROOT.

What is the contents of a cube whose edge is 4 inches?

The product of a number used three times as a factor is called the **cube** of the number.

The number, or factor, is the **cube root** of the product.

Thus, 64 is the cube of 4 ; 4 is the cube root of 64 .

The cube is indicated thus: 4^3 , 3^3 , 5^3 .

The cube root, thus: $\sqrt[3]{64}$, $\sqrt[3]{27}$, $\sqrt[3]{125}$; or $64^{\frac{1}{3}}$, $27^{\frac{1}{3}}$, $125^{\frac{1}{3}}$.

EXERCISE 254.

1^3	$\sqrt[3]{27}$	$(\frac{2}{3})^3$	10^3	$.3^3$
$1^{\frac{1}{3}}$	$\sqrt[3]{64}$	$(\frac{3}{4})^3$	$(\frac{1}{10})^3$	$.4^3$
2^3	5^3	$(\frac{4}{5})^3$	$.1^3$	$0.27^{\frac{1}{3}}$
$8^{\frac{1}{3}}$	$125^{\frac{1}{3}}$	$(\frac{27}{125})^{\frac{1}{3}}$	$1000^{\frac{1}{3}}$	$.008^{\frac{1}{3}}$
3^3	$(\frac{1}{2})^3$	$(\frac{8}{27})^{\frac{1}{3}}$	$.001^{\frac{1}{3}}$	$.064^{\frac{1}{3}}$
4^3	$(\frac{1}{8})^{\frac{1}{3}}$	$(\frac{27}{64})^{\frac{1}{3}}$	$.2^{\frac{1}{3}}$	$(\frac{1}{125})^{\frac{1}{3}}$

Find the cube of 25.

$$25^3 = \left\{ 2 \times 20 \times \frac{20^2}{5^2} \right\} \times \left\{ \begin{array}{l} 20 = \left\{ \begin{array}{l} 20^3 = 8000 = \text{tens}^3 \\ 2 \times 20^2 \times 5 = 4000 = 2 \times \text{tens}^2 \times \text{u.} \\ 20 \times 5^2 = 500 = \text{t}'\text{ns} \times \text{units}^2 \\ 20^2 \times 5 = 2000 = \text{t}'\text{ns}^2 \times \text{units.} \end{array} \right. \\ 5 = \left\{ \begin{array}{l} 2 \times 20 \times 5^2 = 1000 = 2 \times \text{tens} \times \text{u.}^2 \\ 5^3 = 125 = \text{units}^3 \end{array} \right. \end{array} \right.$$

Hence to cube a number of two figures, add tens^3 , $3 \times \text{tens}^2 \times \text{units}$, $3 \times \text{tens} \times \text{units}^2$, and units^3 .

To extract the cube root of a number.

Find the cube root of 15625.

FULL OPERATION.

	15625. (20 + 5	EXPLANATION.—15625 =
$20^3 =$	8000	$\text{tens}^3 + 3 \times \text{tens}^2 \times \text{units} +$
$3 \times 20^2 = 1200$	7625	$3 \times \text{tens} \times \text{units}^2 + \text{units}^3.$
	6000	The largest tens^3 in 15625 is
	1625	$8000 = 20^3$. The remainder,
$3 \times 20 \times 5^2 =$	1500	$7625 = \left\{ \begin{matrix} 3 \times \text{tens}^2 \\ 3 \times \text{tens} \times \text{units} \\ \text{units}^2 \end{matrix} \right\} \times \text{units}.$
	125	$3 \times \text{tens}^2 (20^2) = 1200$. This
	125	being the largest part found
$5^3 =$	125	in 7625, dividing 7625 by

CONTRACTED OPERATION.

	15625. (25	
	8	
$3 \times 20^2 = 1200$	7625	nearly 6, but allowance
$3 \times 20 \times 5 = 300$	1525	must be made for the other
$5^2 = 25$	7625	parts in 7625) and 1625
		over. 1625 contains $3 \times$
		$\text{tens} \times \text{units}^2$, or $3 \times 20 \times 5^2$
		$= 1500$, and units^3 , or $5^3 =$
		125.

The 0's may be omitted, as shown in the contracted work; and, instead of multiplying 1200 ($3 \times \text{tens}^2$), 300 ($3 \times \text{tens} \times \text{units}$), and 25 (units^2) by 5 separately, we multiply their sum by 5.

In cubing a number, as 56.8,

$.8^3 =$.512	We see that the cube of each figure
$6^3 =$	216.	occupies 3 places. Hence point
$5^3 =$	125	off the number whose cube root
		is to be taken, into groups of 3 figures each, commencing at
		the decimal point. Each group will be used in finding the
		figure whose cube is in it. Make full periods at the right
		of the decimal by annexing 1 or 2 0's.

84027.672²=?

$4^3 =$	64
$3 \times 40^2 = 4800$	<u>20027</u>
$3 \times 40 \times 3 = 360$	
$3^2 = 9$	
<u>5169</u>	<u>15507</u>
$3 \times 430^2 = 554700$	4520672
$3 \times 430 \times 8 = 10320$	
$8^2 = 64$	
<u>565084</u>	<u>4520672</u>

84027.672'(43.8

EXERCISE 255. (WRITTEN.)

Find the cube root of:

- | | | |
|----------------------|------------------------|----------------------------|
| 1. 195112. | 12. 46656. | 23. .004096 |
| 2. 262.144 | 13. $\frac{729}{4096}$ | 24. 13.824 |
| 3. .830584 | 14. $\frac{343}{512}$ | 25. 970299. |
| 4. $\frac{512}{729}$ | 15. 279726.264 | 26. $3\frac{2}{3}$ |
| 5. 17576. | 16. 54872. | 27. $2\frac{1}{2}$ |
| 6. 175.76 | 17. 12.167 | 28. $15\frac{5}{8}$ |
| 7. $81\frac{5}{11}$ | 18. 1.2167 | 29. $\frac{1000}{1331}$ |
| 8. 166.375 | 19. 91125. | 30. 39.304 |
| 9. 74.088 | 20. 1.728 | 31. 1577635. |
| 10. .117649 | 21. 2197. | 32. 2 to 2 decimal places. |
| 11. 531442. | 22. .005832 | 33. 7 to 2 decimal places. |

PRACTICAL EXPLANATION OF CUBE ROOT.

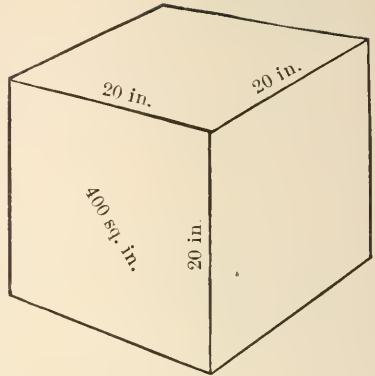
Find the cube root of 10648.

OPERATION.

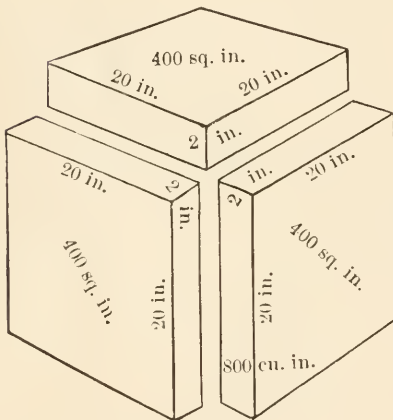
$20^3 =$	10648 <u>20 + 2</u>
$3 \times 20^2 = 1200$	<u>8000</u>
$3 \times 20 \times 2 = 120$	2648
$2^2 = 4$	
<u>1324</u>	<u>2648</u>

EXPLANATION.—Suppose we have to cut a cubical block of stone to contain 10648 cu. in. We wish its dimensions.

The largest cube that can be determined by inspection contains 8000 cu. in., or 20^3 . Its edge will be 20 in. 2648 cu. in. remain to add to the block. Draw on the board a cube similar to the figure here and label it the same.



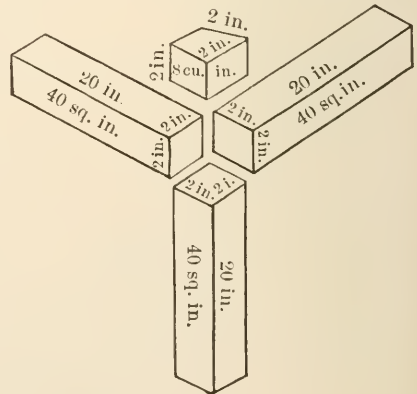
8000 cu. in.



2400 cu. in.

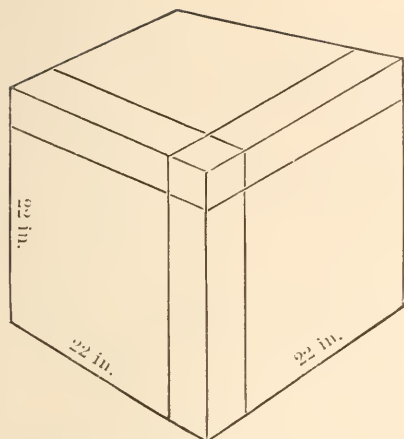
Since a cube has six equal faces, we may cover them all with blocks of equal width and preserve the cubical form; or better, three adjacent faces. These 3 additions will be 20 in. by 20 in. or each have 400 sq. in. in their face, making 1200 sq. in. for the surface of the three. They can contain 2648 cu. in. Therefore, their thickness will be $2648 \div 1200 = 2$ in. with 248 cu. in. over. Draw these additions on the board and label.

Three oblong pieces 20 in. long, 2 in. wide, and 2 in. thick, containing in all 240 cu. in., must be added. Draw and label. Lastly, a small cube, whose edge is 2 in., contents 8 cu. in., must be added. Draw and label. The cube is now complete. The additions contain 2648 cu. in., using all the material.



248 cu. in.

Draw the completed cube representing the additions as shown in the figure on the next page.



10648 cu. in.

The teacher should illustrate each step by the blocks; and extend the work to a second set of additions, making three figures in the root.

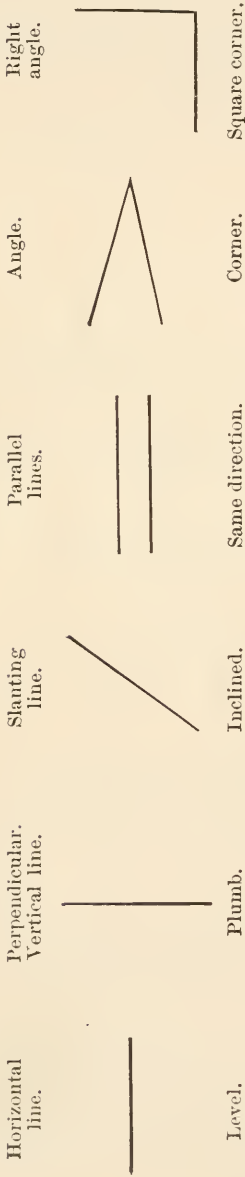
PRACTICAL APPLICATION OF CUBE ROOT.

EXERCISE 256.

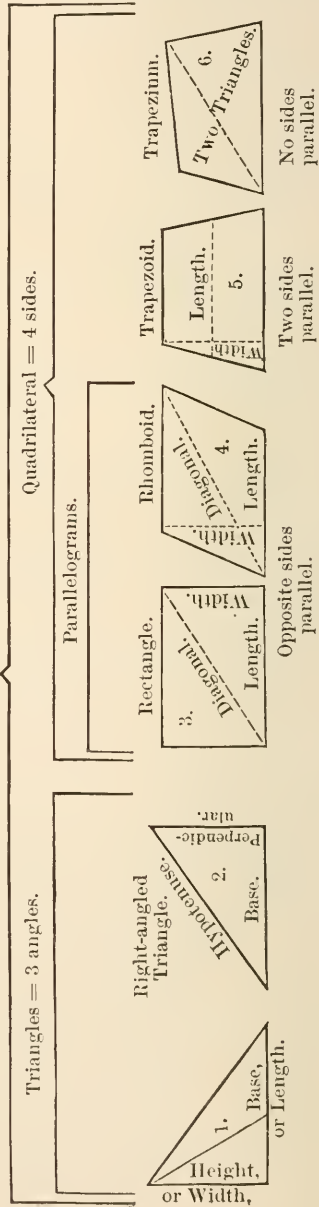
1. Find the dimensions of a cubical box which contains 9261 cu. in.
2. Find the dimensions of a cubical tank which holds 1000 gallons.
3. The area of one of the faces of a cubical box is 576 sq. in. How much will it hold?
4. How many gallons will a tank hold, of cubical form, the area of whose faces is 3750 sq. in.?
5. What is the surface of a cube containing 2744 cu. in.?
6. What are the dimensions of a cubical box containing $\frac{1}{8}$ as much as one whose edge is 4 feet?
7. A certain cubical tank contains 1728 cu. in. What will a tank whose edge is twice this contain?
8. A cubical cistern holds, when full, 4238 kilograms of water. What are its dimensions?
9. The roof of a certain building is 225 meters by 14.2 meters, horizontal dimensions: $2\frac{1}{2}$ centimeters of rain just fill a cubical cistern into which the roof drains. Find the dimensions of the cistern.

MENSURATION.

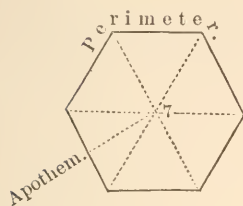
LINES, ANGLES, AND SURFACES.



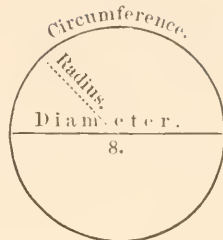
POLYGONS = many angles (3 or more).



Regular Polygon.



Circle.



Observe. $\left\{ \begin{array}{l} \text{In right-angled figures the width and length are} \\ \text{sides of the figure.} \\ \text{In slanting-line figures the width is not a side.} \end{array} \right.$

Draw these figures on your slate until they are familiar.

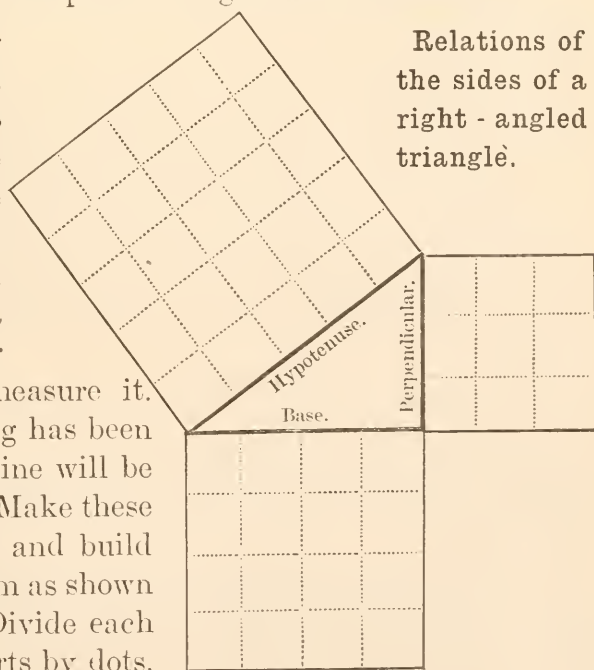
Write definitions of each term given above, from the appearance of the figure and the directions. Write examples of lines, angles, and surfaces similar to the above, that you see in the room or remember. Into what kind of figures does a diagonal divide a rectangle? How does the radius of a circle compare in length with the diameter?

Draw on your slate two lines meeting so as to make a square corner. Have one line 4 in. long; the other, 3 in.

Now draw a line between their free ends and measure it.

If your drawing has been exact the third line will be just 5 in. long. Make these lines very heavy and build squares upon them as shown in this figure. Divide each line into inch parts by dots,

Relations of the sides of a right-angled triangle.



make inch squares by cross lines, and count the number of inch squares in each large square.

Notice how the number in the hypotenuse square corresponds with that in both the other squares put together.

The correspondence found here is always true of right-angled triangles; namely:

The square of the hypotenuse is the sum of the squares of the other sides.

The difference of the squares of the hypotenuse and either side is the square of the other side.

Find, by these laws, what should be the hypotenuse if the base is 8 in. and the perpendicular 6 in. Draw on the board and see if it is true. Try the same with 12 and 9 in.; with 16 and 12 in.

EXERCISE 257. (WRITTEN.)

To illustrate the following examples, draw figures and label them.

1. Perpendicular, 10 ft. Base, 10 ft. Hypotenuse?
2. Base, 15 ft. Hypotenuse, 20 ft. Perpendicular?
3. Perpendicular, 18 ft. Hypotenuse, 25 ft. Base?
4. Distance diagonally across a floor 30×40 ft.?
5. Distance diagonally across a blackboard 8×3 ft.?
6. Length of a ladder to reach the eaves of a building 22 ft. high, the base of the ladder being placed 6 ft. from the building?
7. If you draw the preceding ladder out 3 ft. at the bottom, how high will it reach?
8. What length of rope will reach from the top of a 24-foot pole to the ground on the opposite side of a street 60 feet wide?
9. A rope 250 feet long was stretched from one bank of a river to the top of a pole 65 feet high on the opposite bank; how wide was the river?

10. A tree broken off 14 feet above ground rested on the ground 14 feet from the stump. How tall was the tree?

11. Find the distance from the upper corner to the opposite lower corner of a room $40 \times 30 \times 12$.

12. A pole is held vertical by wires, one of which is 82 feet long, stretched from the top of the pole to the top of a stake 10 feet high and 36 feet from the pole. How high is the pole?

13. What is the length of a path diagonally across a 10-acre square field?

14. Distance from the center of the above field to the center of a side?

15. Diagonal of a cube containing 729 cu. in.?

16. What is the side of a square field whose diagonal is 15 rods? Its area?

17. A ladder 28 feet long placed in a street reaches the top of a building 18 feet high on one side and one 15 feet high on the other. How wide is the street?

18. Two vessels sail from the same point, one north 58 miles, and the other west 72 miles. How far apart are they?

19. Find the longest straight stick you can put into a box $2\frac{1}{2}$ ft. long, $1\frac{1}{2}$ ft. wide, and 12 in. deep.

20. What is the length of the rafters of a building having a gable roof, the building being 36 ft. wide, the eaves 20 ft., and the ridge-pole 30 ft. from the ground?

SURFACE AREAS.

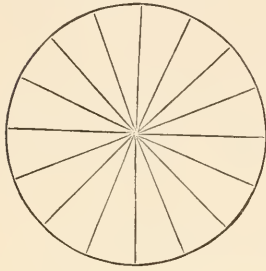
The parallelogram is the basis for computing areas, its area being the length times the width.

All triangles are $\frac{1}{2}$ the size of a parallelogram of equal length and width. Thus,



Figures of equal or parallel sides having more than 4

sides, and irregular figures, are divided into triangles to find their areas. As fig. 6, p. 246, and fig. 7, p. 247.



A circle may be regarded as made up of a very great number of equal triangles having their vertices at the center and their bases forming the circumference. The radius of the circle, therefore, is the uniform width, and the circumference the continuous bases of the triangles.

When the sides of a regular polygon (see p. 247) are very great in number (infinite) the perimeter becomes a circumference and the apothem a radius.

$$\begin{aligned} \text{Area of } \left\{ \begin{array}{l} \text{Parallelograms} = \text{length} \times \text{width.} \\ \text{Triangles} = \frac{1}{2} \text{ length} \left\{ \begin{array}{l} \text{base} \dots \dots \dots \left\{ \dots \dots \dots \left\{ \dots \text{width.} \\ \text{perim.} \dots \dots \dots \left\{ \dots \dots \dots \left\{ \dots \text{apothem.} \\ \text{circum.} \dots \dots \dots \left\{ \dots \dots \dots \left\{ \dots \text{radius.} \end{array} \right. \right. \right. \right. \right. \end{array} \right. \end{array} \right. \\ \text{Circumference} = 3.1416 \left(3\frac{1}{7} \right) \times \text{diameter.} \end{aligned}$$

EXERCISE 258. (WRITTEN.)

To illustrate the following examples, draw figures and label them.

1. A triangular field is 20 rods long and 18 rods wide; what is its area?
2. A field has two parallel sides 25 and 35 rods long, respectively, the distance between them being 13 rd. What is the area of the field?
3. What is the circumference of a circular pond whose radius is 11 rods? Its area?
4. What is the radius of a circle equal in area to a triangle 13×10 ft.?
5. A horse is tied to a stake by a 40-foot rope. What area of ground can he graze over?
6. A circular map of the Eastern Hemisphere is to be 3 feet in diameter. What surface will it cover?

7. A regular 6-sided room has its side 6 feet long and the distance from the center of the room to a side is 5.196 feet. How many sq. yd. of carpet will cover it?

8. The area of a triangular field is 135 sq. rd., and its length 18 rd.; find its width.

9. Find the area of a right-triangle, two of whose sides are equal, and the third is 72 feet.

10. I have 256 sq. ft. of boards. If laid in a floor of triangular form 12 ft. wide, how long will it be?

11. What is the area of a board 18 ft. long, whose ends are respectively 12 in. and 6 in.?

12. What is the distance through a tree that girths 12 ft. 6 in.?

13. The radius of a circle is 10 feet; find the radius of a circle containing 9 times its area; 4 times.

14. A cow is one day tied to the top of a stake 5 ft. high by a rope 20 ft. long; on the next day she is tied to the bottom of the same stake by the same rope. Find the difference in the areas over which she can graze.

15. What will it cost at \$2 a rod to fence a circular plot of land containing 1 acre?

16. Find the cost of a triangular field 72×54 rods at \$125 an acre.

17. At \$85 an acre, and \$1.75 a rod for fencing, what are my expenses in the purchase and fencing of a field having two parallel sides 108 and 144 rods long, respectively, the distance between them being 96 rods, and the other two sides being 97.67 rods each?

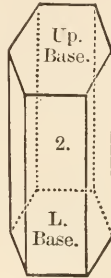
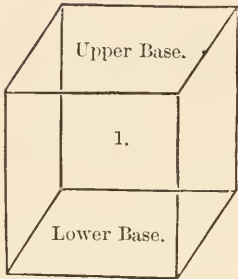
18. A gravel walk around a rectangular grass plot 12 ft. \times 8 ft. is 2 ft. wide; what is its area?

19. How many times will a carriage wheel 4 ft. in diameter turn around in going 1 mile?

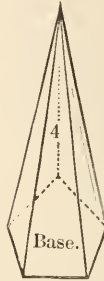
20. A square field contains 31.5 acres; what is the length of its diagonal? What is the circumference of a circular field of the same area?

SOLIDS—SURFACES AND CONTENTS.

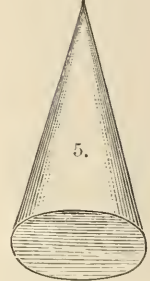
Prisms.



Pyramid.



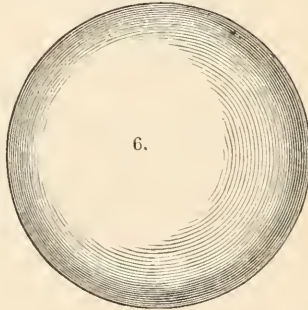
Cone.



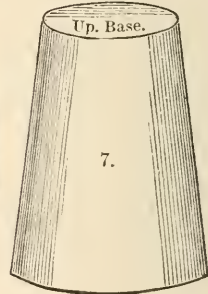
Cylinder.



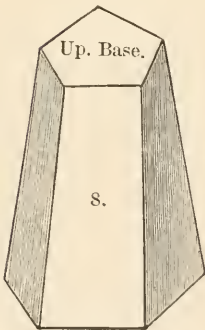
Sphere.



Frustum of Cone.



Frustum of Pyramid.



Observe.

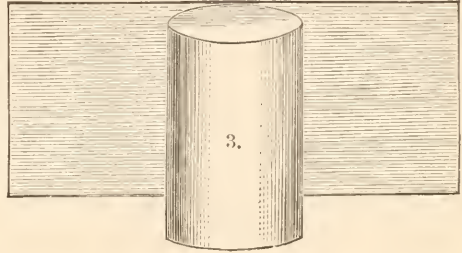
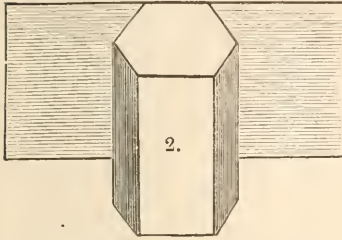
The distance from the top to the bottom, measured by a plumb line, or **actual height** (altitude) of figs. 4, 5, 7, and 8 is shorter than the distance measured down the side.

The distance down the side perpendicularly is called the **slant height**, and is used in finding the areas.

Draw these figures on your slate until they are familiar. Name any surfaces on these that are like the surfaces on p. 246.

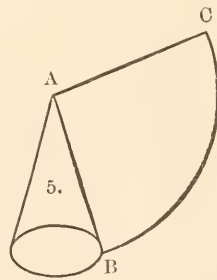
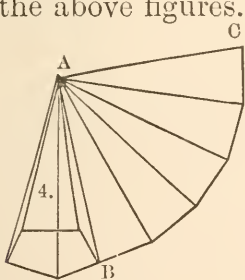
Write examples of these solids that you see in the room or remember. Turn fig. 7 upside down for examples.

The surfaces of all the preceding solids are like the plane surfaces on p. 246. Thus,

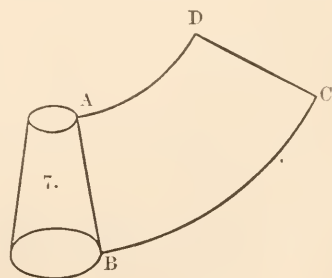
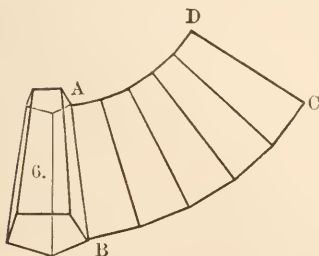


The side surface of a prism or cylinder is seen to be a parallelogram whose length is the distance around, and whose width is the height of the prism or cylinder.

Prove this by cutting a paper rectangle and folding to make the above figures.



The side surface of a regular pyramid or cone is seen to be made up of triangles whose length is the distance around the base and whose height is the slant height of the pyramid or cone.



The side surface of any regular frustum is made up of trapezoids.

Upper and lower bases are circles or polygons.

The surface of a sphere is 4 times that of a circle having an equal diameter.

A rectangular prism (e. g. a room) is the basis for computing the contents of solids. Its contents is the product of three dimensions; *length*, *width*, and *height*; or the area of the *base* (length \times width) multiplied by the *height*.

This gives the law for prisms and cylinders.

A pyramid or cone is found to contain $\frac{1}{3}$ as much as a prism or cylinder of equal base and height; a sphere $\frac{2}{3}$ as much as a cylinder of equal circumference and a height equal to the diameter of the sphere. Hence,

$$\text{Contents of } \left\{ \begin{array}{l} \text{Prism} \\ \text{Cylinder} \end{array} \right\} = \text{Area of base} \times \text{height.}$$

$$\left\{ \begin{array}{l} \text{Pyramid} \\ \text{Cone} \end{array} \right\} = \text{Area of base} \times \frac{1}{3} \text{ height.}$$

$$\text{Sphere} = \text{Great circle} \times \frac{2}{3} \text{ diameter.}$$

$$\text{Frustum} = (\text{Sum of areas of bases} + \text{the square root of their product}) \times \frac{1}{3} \text{ height.}$$

EXERCISE 259. (WRITTEN.)

Draw figures and label them.

1. Find the contents of a box whose length, width, and depth are, respectively, 4 ft., 3 ft., and 2 ft.

2. Find its surface.

3. Find the number of square feet necessary to make a piece of stovepipe $2\frac{1}{2}$ feet long and 5 in. in diameter.

4. Find the amount of tin necessary to make a tin-pail cylindrical in form, 6 in. in diameter and 8 in. deep, without a cover.

5. How many quarts will the pail hold?

6. Find how much water can be put into a tin-pail, 10 in.

deep, like a frustum of a cone in form, whose bottom is 8 in. across, and top 12 in. across.

7. How many sq. ft. of tin in the pail described in the last example, without cover?

8. A cylindrical bottle, containing 1 quart of ink, is 3 in. in diameter; how deep is it?

9. A draughtsman puts a map of the world on a globe 12 in. in diameter; what area does it cover?

10. If your ink well is 1 in. across and 1 in. deep how many times can it be filled from a quart bottle?

11. A conical wood pile is 6 ft. high and 12 ft. in diameter at the base; how many cords?

12. How many bushels of oats in a conical pile 2 ft. high and 12 ft. around it at the base?

13. Anticipating rain, the above pile is covered with tent-cloth; how many square yards?

14. Find the depth of a cylindrical tank that holds 20 gallons, and is 18 in. in diameter.

15. If the above tank has a conical top 4 in. high, how many more gallons can be put in?

16. The glass tank of a lamp is spherical in shape and 4 in. in diameter on the inside; how much oil will it hold?

17. If a 5-gallon oil can is 10 in. square on the bottom how deep is it?

18. What is the difference in the number of square feet of lumber necessary to make the sides of a room 16 ft. long, 12 ft. wide, and 10 ft. high, and one of circular floor containing the same area and of the same height?

19. Find the number of cu. ft. inclosed by a barn 60 ft. long, 40 ft. wide, and 20 ft. high, with a pyramidal roof 8 ft. high, all inside measurements.

20. How many cu. ft. of wood are in a log 20 feet long and 14 in. in diameter.

21. The earth's diameter is about 8000 miles; what is its area? Its volume or bulk?

22. At 28 ct. per cu. ft. what is the cost of a stone wall 28 in. thick at the base and 18 in. at the top, 4 ft. high and 36 rd. long?

23. The above wall is laid on a foundation of Portland cement 4 inches wider than the base of the wall, and 8 in. deep. What is the cost of the foundation at 32 ct. a cu. ft.?

24. How many cu. ft. in a regular 8-sided post 10 feet high, the length of one side being 3 in., and the distance through it 7.24 in.?

EXERCISE 260. (ORAL.)

1. Area of a triangular field 10 rd. long and 8 rd. wide?

2. Width of a triangular field containing 1 A., length 20 rd.?

3. Width of a rectangular field, with dimensions as in Example 2?

4. Number of cu. ft. in a conical pile 6 ft. high and 7 ft. across at the base?

5. Area of a square field whose diagonal is 20 rd.?

6. Radius of a circle whose circumference is $6\frac{2}{7}$ ft.?

7. Area of a field having two parallel sides 40 and 30 rd. respectively, and width 10 rd.?

8. How many times will a wheel of $3\frac{1}{2}$ ft. radius turn around in going 4 rods?

9. Length of rafters on a barn whose gable end is 32 ft. wide and the roof 12 ft. high?

10. Number of sq. in. of material to make a 2-ft. length of 7 in. stove pipe, allowing 1 in. for lapping?

11. Cost of fencing the field in Example 3 at \$2 a rod?

12. Height of a pyramid containing 144 cu. in., the area of whose base is 36 sq. in.?

13. Height of a cone of the same measurement as the preceding pyramid?

14. Number of cu. yd. of gravel necessary to cover a walk 3 ft. wide, 54 ft. long, and 3 in. deep?

MISCELLANEOUS PROBLEMS.

EXERCISE 261.

1. Spent $\frac{1}{7}$ of my money for a watch, $\frac{1}{6}$ of the remainder for a chain, $\frac{1}{6}$ of what then remained for a suit of clothes, and $\frac{1}{31}$ of the rest for a pair of shoes, when I had \$150 left; what had I at first?

2. Imported 7 casks of brandy, 30 gal. each, duty \$2 per gal., charges \$27; sold the whole for \$1714.27 $\frac{1}{2}$, gaining 42 $\frac{1}{2}$ % on the whole cost; what was the cost of the liquor per gal. at the foreign port?

3. A man dying leaves in the savings bank for his 16-year old son such a sum of money as shall amount to \$5000 when the son is 21. If the bank adds the interest to the principal every half year, how much money must be left in the bank? Interest at 6%.

4. My agent sells for me 800 bbl. flour at \$4.75, commission 1 $\frac{3}{4}$ %, and buys sugar at 6 $\frac{1}{2}$ cents a lb., commission 2%; what is the whole commission, and how many lb. of sugar do I receive?

5. A rectangular field of 4 $\frac{1}{2}$ A., whose breadth is $\frac{5}{9}$ its length, is surrounded by a close board fence 8 $\frac{1}{2}$ ft. high, with 8-foot posts 5 in. square and 8 ft. apart, and two rows 2-by-4 scantling around the field. If the lumber cost \$440.44, what was the price per M?

6. Find the dimensions of a rectangular field whose length is 3 times its width, and whose area is 327.46 ares.

$$7. \frac{9\frac{1}{10} + 7\frac{1}{8}}{7\frac{7}{8}} \times 41\frac{9}{11} = ?$$

8. Find the cost of carpeting a room 17 ft. by 13 ft. 2 in. with carpet 1 meter wide, at 85 cents a meter, laid lengthwise.

9. Add 5.13875 miles and 25.312 rods, take away 147.3125 yards, and give the result in feet.

10. A horse is tethered by a rope 12.4 meters long; what area can he feed?

11. A land company buys 36 acres of land whose breadth is $\frac{9}{10}$ its length, and divides it into city lots. 3 streets 80 feet wide run lengthwise, and 2 streets 60 ft. wide, crosswise. The lots are 50 by $118\frac{1}{2}$ ft., and are sold at \$220. How much is realized by the sale?

12. Suppose the company to have paid \$500 an acre for the above land, and \$800 in paving, grading, etc.; what is its per cent of profit in the venture?

13. I hire money at 7% to purchase one of these lots, and after the lapse of 15 mo. I sell for \$500; find my profit %.

14. What is the diameter of a circle whose area is 278.54 ares?

15. A grain dealer buys 3000 centals of barley, $\frac{1}{3}$ of which he sells at a gain of 8%, $\frac{1}{4}$ at a gain of 12%, $\frac{1}{5}$ at a gain of 16%, and the remainder at a gain of 20%. Had he sold the whole at a gain of 15%, he would have received \$54 more. Find the cost per cental.

16. A commission merchant sold cotton cloth on $1\frac{3}{8}\%$ commission, and invested the proceeds in cotton on $2\frac{1}{2}\%$ Com. If his commissions amounted to \$241.40, what sum was received for the cloth? Sum given for the cotton?

17. Bought a note for $\frac{2}{3}$ its face, on which a collector obtained 25% more than I paid for it and charged me 5% for collecting. If I realized \$75 by the transaction, what was the face of the note?

18. If 4 men working 10 hr. per day do a piece of work in 60 days, how many men will it take to do twice the work in 40 days, working 8 hr. per day?

~~19.~~ The largest circular path that could be made in a certain square garden was $5\frac{1}{2}$ rods in diameter; what was the area of the garden?

20. What is the base of a triangle whose area is 34.28 ares, and whose altitude is 39.4 meters?

21. Find the area in hektares of a piece of ground 1 mile square.

22. A merchant bought goods for \$3600, marked them at 30% advance, and finally sold them at 10, and 5 off from the marked price for cash. Find his selling price.

23. How large a draft, payable 60 days after sight, can be bought for \$502.25, exchange being 1% and interest 6%?

24. Express as a decimal $\frac{(\frac{2}{3} - \frac{1}{10}) \times (3 + \frac{2}{5})}{(1\frac{1}{2} + \frac{5}{7}) + (3 - 1\frac{2}{3}) \times 5}$.

25. A and B together own 540 acres of land and agree to share it in the proportion of 7 to 11. What number of acres does each receive?

26. Find the surface and solidity of a sphere whose diameter is 3.64 meters.

27. A mechanic agreed to work 80 days on condition that he should receive \$1.75 and board for every day he worked, and pay 75 cents a day for board when idle. His earnings were \$80; how many days did he work?

28. Says A to B, $\frac{3}{4}$ of my age equals $\frac{2}{3}$ of yours. The sum of their ages was 136; find the age of each.

29. A cylindrical tank is 3.8 meters high, and diameter of base 2.8 meters, both inside measurements. How much water will it hold and what is its weight in kilograms?

30. Divide 448 A. 144 sq. rd. of land among A, B, C, and D, so that A shall have $\frac{1}{5}$ of the whole + 4 A. 126 sq. rd.; B $\frac{1}{3}$ of the remainder; C $\frac{1}{3}$ of what then remains; and D the rest.

31. How deep a ditch 3 ft. wide must be dug around a field 5 rods square that the earth removed may raise the surface of the field 6 in.?

32. My garden is 43.6 meters long and 27.9 meters wide. My rain gauge registered 16 centimeters in the late storm. How many kilograms of water fell on my garden?

33. How many hogsheads, of 63 gallons each, will a cylindrical tank, 10 ft. in diameter and 10 ft. deep, hold?

34. What is the value of \$1 in shillings and pence? In francs?

35. A rectangular tank holds 58248 liters of water: two of its inside dimensions are 3.7 meters and 3.42 meters; what is the third dimension?

36. Find the whole cost of 550 yd. Brussels carpeting at \$1.80 a yard, commission for purchasing being $2\frac{1}{4}\%$, draft $\frac{1}{4}\%$, and \$17 freight prepaid.

37. A certain principal at a certain rate amounts to \$750 in 3 yr., and the interest is $\frac{1}{4}$ of the principal. Find the principal and the rate.

38. How much wood in a pile 32.5 meters long, 3.2 meters wide, and 1.8 meters high?

39. Two men dig a ditch for \$53; one man worked $3\frac{1}{2}$ days and dug $14\frac{1}{2}$ rd. a day; the other worked as many days as the first dug rods per day. What did each receive if they shared in proportion to the time worked?

40. A and B furnish capital to engage in business and C does the work for $\frac{1}{3}$ the profit. A contributes \$8000 and B \$10000. They gain \$5400. Find the share of each.

41. If 52 men can dig a trench 355 ft. long, 60 ft. wide, and 8 ft. deep, in 15 days, what is the length of a trench 45 ft. wide and 10 ft. deep, which 45 men can dig in 25 days?

42. At what price must cloth that cost \$3.50 a yard be marked that may fall 20 per cent and still gain 20 per cent on the cost?

43. Bought 8 cd. $6\frac{2}{3}$ cd. ft. of wood at \$7.20 a cord and paid in equal weights of butter and cheese at 20 cents a lb. for butter and 12 cents a lb. for cheese. How many lb. of each were required?

44. Find the surface of a cone whose altitude is 3.8 meters and diameter of base 2.28 meters.

45. Find the prime factors of 729, 336, and 1836.

46. Paid \$2225 for 180 sheep and sold them for \$2675; what should I gain on 1500 sheep at the same rate?

47. Find the g. c. f. of 84, 336, 420, and 504.

48. Write the 38th example with the same values in our measures and work it, giving the result in cords and feet.

49. What is $\frac{3}{4}$ of an acre of land worth, if $\frac{5}{6}$ of an acre is worth \$60?

50. A tank will hold 420 gallons and is $\frac{5}{8}$ full; what part full is it if $87\frac{1}{2}$ gallons be added?

51. Bought 24 T. 4 cwt. 1 qr. 18 lb. of English iron at 3 pence per lb., long ton weight, and sold the same at \$142 per short ton. What did I gain?

52. When rain falls 3 centimeters in depth, how many kilograms have fallen on a garden 73.3 meters long by 38.18 meters wide?

53. \$714.50.

LOS ANGELES, Aug. 28, 1885.

For value received I promise to pay H. Miner, or order, Seven Hundred Fourteen and $\frac{50}{100}$ Dollars, on demand, with interest at 12% per annum.

JAMES TOWLE.

Find the amount Mar. 17, 1886.

54. \$534.00.

LOS ANGELES, Jan. 4, 1886.

Six months after date, I promise to pay B. Caldwell, or order, Five Hundred Thirty-four Dollars, with interest at 8% per annum, value received.

W. P. JOHNSON.

Discounted at a Los Angeles bank Mar. 17, 1886, at 10%. Find the proceeds.

55. When are the hour and minute hands of a clock together next after 12 o'clock?

56. What is the time between 12 and 1 o'clock when the hour and minute hands are equidistant from 12 on opposite sides?

57. I buy a farm for \$5000, to be paid for in 5 payments; interest at 10% payable annually. The payments to be 0, 1, 2, 3, and 4 years from date of purchase. It is so arranged that I pay exactly the same amount of money at each payment. What is the equal payment?

ABBREVIATIONS.

A.	Acre, or acres.	gro.	Gross.
Acc't.	Account.	hdkf.	Handkerchief, or handkerchiefs.
Am't.	Amount.	hhd.	Hogshead, hogsheads.
Anal.	Analysis.	hr.	Hour, or hours.
Ans.	Answer.	in.	Inch, or inches.
Apr.	April.	Jan.	January.
Aug.	August.	l.	Link, or links.
Bal.	Balance.	lb.	Pound, or pounds.
bbl.	Barrel, or barrels.	l. c. m.	Least common multiple.
bu.	Bushel, or bushels.	M.	Meter, meters, one thousand.
bun.	Bundle, or bundles.	Mar.	March.
C.	Cost.	Mdse.	Merchandise.
cd.	Cord, or cords.	mi.	Mile, or miles.
cd. ft.	Cord foot.	min.	Minute, or minutes.
ch.	Chain, or chains.	mo.	Month, or months.
Co.	Company.	Mo.	Monthly.
Com.	Commission.	No.	Number.
Cr.	Credit, or creditor.	Nov.	November.
c., ct.	Cent, or cents.	Oct.	October.
cu. ft.	Cubic foot, or feet.	oz.	Ounce, or ounces.
cu. in.	Cubic inch, or inches.	p.	Page, or pages.
cu. yd.	Cubic yard, or yards.	P. and L.	Profit and loss.
cwt.	Hundredweight.	pk.	Peck, or pecks.
d.	Penny, or pence.	Pop.	Popular.
da.	Day, or days.	pr.	Pair, or pairs.
Dec.	December.	pt.	Pint, or pints.
deg.	Degree, or degrees.	pwt.	Pennyweight, or pennyweights.
do., ditto.	The same.	qr.	Quire, or quires.
doz.	Dozen.	qt.	Quart, or quarts.
Dr.	Debtor.	rd.	Rod, or rods.
far.	Farthing, or farthings.	Rec'd.	Received.
Feb.	February.	rm.	Ream, or reams.
ft.	Foot, or feet.	Sci.	Science.
G.	Gain.	s.	Shilling, or shillings.
gal.	Gallon, or gallons.	scr.	Scruple, or scruples.
g. c. f.	Greatest common factor.	sec.	Second, or seconds.
gi.	Gill, or gills.	Sept.	September.
gr.	Grain, or grains.		
gran.	Granulated.		

S. P.	Selling price.	sq. yd.	Square yard, or yards.
sq. ch.	Square chain, or chains.	T.	Ton, or tons.
sq. ft.	Square foot, or feet.	vol.	Volume, or volumes.
sq. in.	Square inch, or inches.	wt.	Weight.
sq. l.	Square link, or links.	yd.	Yard, or yards.
sq. mi.	Square mile, or miles.	yr.	Year, or years.
sq. rd.	Square rod, or rods.		

SIGNS.

+	Addition.	:	Ratio, or Division.
—	Subtraction.	=	Equals.
×	Multiplication.	\$	Dollars.
÷	Division.	¢	Cents.
()	Parenthesis.	£	Pounds (Eng. money).
%	Per cent.	::	Equals (used in proportion).
.	Decimal Point.	@	At.
“	Account.	√	Square Root.
°	Degree.	“	The same.
'	Minute (circ. measure).	∴	Therefore.
“	Second (circ. measure).		

GLOSSARY.

Some of the terms in the glossary are not employed in the body of the book. Such as are so employed are indicated by the figures in parentheses. These figures refer to the page on which the subject is first noticed.

- Abstract number**, (36), a number used by itself without reference to any particular thing.
- Acceptance**, (229), agreeing to the terms of a draft by writing one's name across the face.
- Account**, (221), a record of debts and credits.
- Accurate interest**, (209), interest for days computed on a basis of 365 days to a year.
- Addition**, (14), the process of putting two or more numbers together into one.
- Ad valorem duty**, (200), a tax on the value of imported goods.
- Aliquot part**, (115), an exact divisor, integral or fractional.
- Alloy**, (168), a mixture of two or more metals; a baser metal which is mixed with a finer, as in money.
- Altitude or actual height**, (252), the shortest distance from the top to the base of a triangle, pyramid, cone, or frustum.
- Amount**, (14), the sum of two or more numbers; (204), in Interest, the sum of principal and interest; also, a sum of money.
- Analysis**, (172), a separation into parts for the special treatment of each.
- Angle**, (246), the difference in direction of two lines that meet.
- Apothem**, (247), the distance from the center of a regular polygon to the central point of any side.
- Arc**, (146), any part of the circumference of a circle.
- Are**, (131), the metric unit of land measure.
- Area**, (129), the number of square units in a surface.
- Arithmetic**, the knowledge of numbers and how to use them.
- Assets**, (178), the actual property of a person, or company.
- Average**, (235), the mean of two or more unequal numbers.
- Avoirdupois**, (142), the weight in common use.
- Axis of the earth**, (147), a straight line joining the two poles.
- Balance**, (221), equality of weights or numbers; the excess of one sum of money over another.
- Balance sheet**, (226), a tabular statement of facts showing the condition of a business.
- Bank**, (226), an establishment for the deposit, exchange, or loan of money.
- Bank discount**, (218), the interest taken by a bank from the face or amount of a note, for paying it before it is due.
- Bankrupt**, one declared by law to be unable to pay his debts.
- Base**, (246), in geometrical figures, the side or face on which they stand; (181), in Percentage, that number of which another is a part or per cent.
- Bill**, (119), a formal statement of goods sold or services rendered.
- Bill of exchange**, (229), a draft.
- Bond**, a written contract for the payment of a sum of money under given conditions.
- Broker**, (190), one who buys and sells for another.
- Brokerage**, (190), a percentage paid to a broker for doing business.
- Cancellation**, (86), the division of dividend and divisor by a common factor.

- Capital**, (178), money or other property by means of which business is done.
- Carat**, (143), a 24th in pure gold of the entire weight of a mixture of gold and baser metals; thus, 18 carats fine means that $\frac{18}{24}$ of the mixture is pure gold.
- Cashier**, (223), one who has charge of the cash and cash transactions of a bank or company.
- Cental**, (142), 100 pounds.
- Centi-**, (127), a prefix in the French metric system meaning $\frac{1}{100}$ of.
- Certificate of deposit**, (227), a written statement by a bank that you have deposited money in it.
- Chain**, (see table, p. 123).
- Check**, (227), an order on a bank for money.
- Chord**, (146), a straight line joining any two points in the circumference of a circle.
- Circle**, (see p. 146).
- Circulate**, (107), the repeating figures in a circulating decimal.
- Circulating decimal**, (107), a decimal fraction in which the same figure, or set of figures, is constantly repeated.
- Circumference**, (146), the bounding line of a circle.
- Column**, (6), a vertical line of numbers.
- Commercial discount**, (218, 219), a deduction on the face of a bill, note, or other writing for money.
- Commission**, (189), a percentage allowed on the value of goods bought or sold, money collected, etc.
- Common denominator**, (76), one common to two or more fractions.
- Common factor**, (65), a whole number exactly contained in each of two or more numbers.
- Common multiple**, (68), a number which exactly contains two or more whole numbers.
- Company**, (194), two or more men uniting in some business or enterprise.
- Complex fraction**, (91), one having a fraction or mixed number in either numerator or denominator, or both.
- Composite number**, (63), one made up of factors.
- Compound interest**, (217), interest on principal and unpaid interest.
- Compound number**, (122), a concrete number expressed in two or more units.
- Compound proportion**, (177), an equality between a simple and a compound ratio.
- Compound ratio**, (177), the indicated product of two or more simple ratios, term by term.
- Concrete number**, (36), one applied to a particular object.
- Cone**, (252), a solid whose base is a circle and summit a point.
- Contents**, (129, 135), the number of units in a surface or solid.
- Corporation**, (201), a body of people authorized by law to do business.
- Credit**, (223), that which one has paid.
- Creditor**, (223), one to whom a debt is owing.
- Cube**, (134), a solid having 6 square faces; (241), the product of a number used 3 times as a factor.
- Cube root**, (241), one of the 3 equal factors of a number.
- Customs**, (200), taxes on imports or exports.
- Cylinder**, (252), a straight solid whose bases are equal and parallel circles.
- Days of grace**, (218), 3 days that a debt may remain unpaid after the time due, allowed by law in many states.
- Debt**, (223), that which one is owing.
- Debtor**, (223), one who owes.
- Deci-**, (127), a prefix in the French metric system meaning $\frac{1}{10}$ of.
- Decimal**, a number so written that each character is tenfold greater at each remove from right to left.

- Decimal fraction**, (102), that part of a number at the right of the decimal point. It is always less than a unit.
- Decimal notation**, (5), the art of writing numbers by the decimal scale, or scale of 10's.
- Decimal point**, (5), a period (.) at the right of units in a decimal.
- Decimal system**, the method of writing numbers in decimals.
- Degree**, (146), 1-360th of a circumference.
- Deka-**, (127), a prefix in the French metric system meaning 10.
- Denominator**, (72), the number below the line in a fraction; corresponds to the divisor in division.
- Diagonal**, (246), a line joining any two corners of a polygon not lying next to each other.
- Diameter of a circle or sphere**, (146), a straight line drawn through the center and terminating in the circumference.
- Difference**, (21), the result obtained by taking one number from another.
- Digits**, the ten symbols of the decimal notation.
- Dimensions**, (254), length and breadth of a surface, or length, breadth, and height of a solid.
- Discount**, (219), a deduction from the face of a debt; (202), in Stocks, the rate the market value is below par.
- Dividend**, (43), in Division, the number to be divided; (72), in fractions, the numerator; (202), in business, the income of a stock company.
- Division**, (43), the process of finding how many times one number contains another; the process of separating a number into equal parts.
- Divisor**, (43), the number by which we divide; (72), in fractions, the nominator.
- Draft**, (229), a written order by one person upon another to pay money to a third.
- Drawee of a draft**, (see p. 229).
- Drawer of a draft**, (see p. 229).
- Duty**, (199), a tax on imports or exports.
- Equation**, a statement of equality between two numbers or sets of numbers.
- Equation of payments**, (233), average of payments.
- Even number**, one having 2 for a factor.
- Exact divisor**, (63), one contained in the dividend an exact number of times; may be integral or fractional.
- Exact interest**, (209), interest for days computed on a basis of 365 days to a year.
- Exchange**, (228), the method of making payments to parties at a distance by drafts.
- Exponent**, (63), a figure placed to the right and above a number, showing how many times the number is to be used as a factor.
- Extremes**, (176), the first and last terms of a proportion.
- Face**, (214), the sum of money mentioned in a business paper.
- Factor**, (35), an integral exact divisor.
- Figures**, (5), the ten symbols of the decimal notation.
- Firm**, (178), the name under which a company does business.
- Formula**, (176), a rule expressed by symbols or figures; a very brief statement.
- Fraction**, (72), an indicated division; one or more equal parts of a unit.
- Franc**, (157), the unit of French money.
- Frustum**, (252), the part of a cone or pyramid left after cutting off the top by a section parallel to the base.
- Gain**, (185), the amount by which the selling price of an article exceeds its cost.
- Grace**, (218), an allowance of 3 days made by some states for the payment of a debt after the set time has expired.

- Gram**, (145), the unit of metric weight equal to 15.43 grains Troy.
- Great circle of a sphere**, (254), one that cuts the sphere into two equal parts.
- Greatest common factor**, (65), the greatest factor common to two or more numbers.
- Greenback**, (221), U. S. currency note, now payable in gold.
- Gross weight**, (200), the weight of packed goods, including the weight of the boxes or other packing material.
- Guarantee**, or guaranty, the warranting by another of the payment of a debt.
- Hekto-**, (127), a prefix in the French metric system meaning 100.
- Horizontal**, (246), parallel to the horizon.
- Hypotenuse**, (245), the longest side of a right-angled triangle.
- Imports**, (199), goods brought into a country.
- Improper fraction**, (73), one whose numerator equals or exceeds its denominator.
- Indorse**, (214), to write on the back of a business paper.
- Indorsement**, (214, 215), any writing on the back of a business paper; as a name or a partial payment.
- Insolvent**, unable to pay debts in full.
- Inspection**, (64), a careful examination; obtaining a result without working the example. By *inspection* I find that 2, 3, 5, and 11, are not factors of 223.
- Installment**, (215), a part payment.
- Insurance**, (194), a security against loss; the value put upon property to be paid in case of loss.
- Integer**, (72), a number of one or more units; that part of a decimal at the left of the decimal point.
- Interest**, (204), money paid for the use of money.
- Kilo-**, (127), a metric prefix meaning 1000.
- Least common denominator**, (76), the smallest denominator common to two or more fractions.
- Least common multiple**, (68), the smallest number that will contain each of several numbers.
- Liabilities**, (178), the debts of a firm or individual.
- Link**, (126), a division of a surveyor's chain, 7.92 inches in length.
- Linear unit**, (128), any line taken as the unit, as the foot, yard, or meter.
- Liter**, (leeter), (141), the unit of metric liquid measure; equal to 1.0567 liquid quarts.
- Long division**, (53), the method of writing the work in division in full.
- Longitude**, (150), distance in degrees east or west of the meridian of Greenwich, Eng.
- Loss**, (185), the amount the cost of an article exceeds the selling price.
- Lowest terms**, (75), when the numerator and denominator of a fraction contain no common factor.
- Market value**, (201), the price of stocks in the market.
- Maturity of a note, draft, or bill**, (214), the date when it becomes due.
- Means**, (176), the second and third terms of a proportion.
- Measuring unit**, (127), a unit in which the quantity measured is expressed.
- Mensuration**, (246), measuring and calculating the contents of surfaces and solids.
- Meter**, (127), the unit of length from which the decimal system of weights and measures is named, equal to 39.37 inches.
- Metric system**, (127), the decimal system of weights and measures.
- Milli-**, (127), a metric prefix meaning $\frac{1}{1000}$ of.
- Miner's inch**, (157), flowing water at the rate of 10.4279 gallons per minute.

- Minuend**, (21), the number in Subtraction from which another is taken.
- Minus**, (21), less, or diminished by; the name of the sign of subtraction.
- Mixed number**, (73), a whole number and fraction combined.
- Mortgage**, (226), a grant of property to a creditor as security for the payment of a debt.
- Multiple of a number**, (67), a number that contains it an exact number of times.
- Multiplicand**, (34), the number to be multiplied.
- Multiplication**, (34), the process of finding a number of times a given number.
- Multiplier**, (34), the number by which we multiply.
- Myria-**, (127), a metric prefix meaning 10000.
- Negotiable**, (214), can be transferred to another party, as a note or draft.
- Net proceeds**, the sum remaining from a sale after the payment of all expenses.
- Net weight**, (200), the weight of packed goods, not including the weight of cases, or packing material.
- Notation**, (5), writing numbers.
- Note**, (214), a written promise to pay money.
- Number**, (5), one or more units.
- Numeration**, (9), reading of numbers written decimally.
- Numerator**, (72), the number above the line in a fraction; corresponds to the divisor in division.
- Odd number**, not containing two as a factor.
- Oral**, spoken.
- Order**, (224), a written direction to one person to pay money to another.
- Parallel**, (246), having the same direction.
- Parallelogram**, (246), a 4-sided figure whose opposite sides are parallel.
- Partial payment**, (214), payment of a part of a note.
- Partners**, (178), associates in business.
- Partnership**, (178), an association of persons to carry on business together.
- Par value**, (201), nominal of face value.
- Payee**, (229), one in whose favor a draft or check is drawn.
- Payer**, (229), the drawee of a draft or check.
- Per**, by.
- Per cent**, (181), hundredths; literally, by the hundred.
- Percentage**, (181), a number obtained by taking a per cent of another.
- Perch of masonry**, (138, 155), $16\frac{3}{4}$ or $24\frac{3}{4}$ cubic feet, according to custom.
- Perimeter**, (247), the boundary line of a polygon.
- Perpendicular**, (246), at right angles with; vertical.
- Personal property**, (198), movables, including money and stock.
- Plane**, a surface straight in all directions.
- Plus, and, or added to**, (14), name of the sign of Addition.
- Poles of the earth**, points of the surface which have no motion in the daily revolution.
- Policy**, (194), the written contract in insurance.
- Poll tax**, (198), a tax assessed equally upon men without regard to property.
- Polygon**, (246), a plane surface bounded by straight lines.
- Pound**, (157), the unit of English money, \$4.86.
- Power**, (63), the product of a number repeated as a factor.
- Premium**, (194), the percentage paid for a policy of insurance; (202), the rate of the market above par value of stocks.
- Present worth**, (212), the present value of a debt due at a future time.

- Prime factor**, (63), one not made up of other factors.
- Prime number**, (63), one not made up of factors.
- Principal**, (204), money lent at interest.
- Prism**, (252), a solid whose side faces are parallelograms and whose ends are equal parallel polygons.
- Problem**, something to be done.
- Proceeds**, (190), sum left after taking out a discount or commission.
- Product**, (34), the result of multiplying one number by another.
- Profit**, (185), gain.
- Proof**, (18), a test of correctness of work; no arithmetical proof is a perfect test.
- Proper fraction**, (73), one whose numerator is smaller than its denominator.
- Proportion**, (176), two equal ratios.
- Pyramid**, (252), a solid of plane faces whose base is a polygon and summit a point.
- Quadrant**, (146), the fourth part of a circumference; ninety degrees.
- Quadrilateral**, (246), a polygon of four sides.
- Quantity**, anything that can be measured, weighed, or counted.
- Quintal**, (142), 100 pounds, by the long ton table 112 pounds.
- Quotient**, (43), the result of dividing one number by another.
- Radius**, (146), distance from the center to the circumference of a circle.
- Rate of interest or discount**, (181), per cent for a given time.
- Ratio**, (176), the indicated division of one number by another of the same kind.
- Real estate**, (198), lands and houses, immovable property.
- Receipt**, (119), a written acknowledgment of something received.
- Rectangle**, (128), a polygon of four sides and four square corners; a right angled parallelogram.
- Reduction**, (123), changing a number; (74), or fraction in name without changing value.
- Remainder**, (21), number left after taking away one number from another.
- Remittance**, (193), money or an order for money, sent to a distant place.
- Right angle**, (146), a square corner.
- Roman notation**, (12), writing numbers by capital letters of our alphabet.
- Root of a number**, (237), one of its equal factors.
- Rule**, a direction for working problems.
- Section of land**, (129), a mile square.
- Security**, a pledge for the payment of a debt.
- Share**, (201), one of the equal parts into which the capital of a company or corporation is divided.
- Short division**, (50), division in which the result only is written.
- Sign draft**, (229), one payable on presentation.
- Simple number**, (122), a multiple of a single unit; expressed in terms of a single unit.
- Slant height**, (252), the shortest distance down the side of a cone or pyramid.
- Solid**, (134), a body having length, breadth, and thickness.
- Solution**, the process of working a problem, also the answer obtained.
- Specific duty**, (200), a tax on the measure, number, or weight of imported goods.
- Sphere**, (252), a solid body having a uniformly curved surface.
- Square**, (128), a rectangle of equal sides; the product of two equal factors.
- Square root**, (237), one of the two equal factors of a number.
- Standard time**, (154), the time of the meridians of 75° , 90° , 105° , and 120° .

- Statement**, (226), a tabular arrangement of assets and liabilities.
- Stock**, (178, 201), the capital of a firm or company.
- Subtraction**, (21), the process of taking one number from another.
- Subtrahend**, (21), the number to be taken from another.
- Sum**, (14), the result obtained by adding; money.
- Surface**, (128), that which has length and breadth; the outside of a solid.
- Symbol**, a letter or other character used for a member.
- Tangent**, (146), an indefinite straight line touching a curve.
- Tax**, (198), a sum charged by the government or other authority upon property or person.
- Terms of a fraction**, (72), the numerator and denominator.
- Time draft**, (229), one payable a certain specified time after presentation or date.
- Trapezium**, (246), an irregular four sided polygon.
- Trapezoid**, (246), a trapezium with two opposite sides parallel.
- Triangle**, (246), a polygon of three sides.
- Troy weight**, (143), used for gold, silver, precious stones, etc.
- True discount**, (212), the difference between a sum due at a future time and its present value.
- Unit**, (5), a single thing; a collection of several things taken as one.
- Value of a fraction**, (72), the result of dividing the numerator by the denominator.
- Vara**, (154), a Spanish measure of length equal to 2.782 feet.
- Vertex**, (246), the point opposite the base of a cone, pyramid, or triangle.
- Vertical**, (246), at right angles to the horizon.
- Volume of a solid**, the product of its three dimensions; its contents.
- Weight**, (142), the force which draws a body downward.
- Width**, (246), one dimension of a surface or solid.

ANSWERS.

**Exercise 22.**

1. 818.
2. 393.
3. 2756.
4. 7931.
5. 9448.
6. 9135.
7. 3099.
8. 7938.
9. 13872.
10. 6752.
11. 11110.
12. 26700.
13. 18701.
14. 18287.
15. 13945.
16. 10019.
17. 11210.

Exercise 23.

1. 388.
2. 286.
3. 441.
4. 421.
5. 353.
6. 392.
7. 443.
8. 517.
9. 417.
10. 269.
11. 346.
12. 454.
13. 303.
14. 418.
15. 168.
16. 428.
17. 408.
18. 393.
19. 350.
20. 397.
21. 335.

Exercise 24.

1. 7984.
2. 6241.
3. 7226.
4. 8529.
5. 4726.
6. 5532.
7. 13507.

8. 2840.
9. 11792.
10. 16784.
11. 1066197.
12. 1405655.
13. 10674073.
14. 5908144.
15. 13100.
16. 6862.
17. 970.
18. 5528.
19. 8246.
20. 5712.
21. 10042.
22. 5137.
23. 23210.
24. 6354.
25. 1585323.
26. 634650.
27. 432724.

Exercise 25.

1. 1732.
2. 5036.
3. 8082.
4. 30886.
5. 221528.
6. 719733.
7. 75008.
8. 325466.
9. 11094.
10. 9105.
11. 25537.
12. 8879.
13. 144224.
14. 75359.
15. 739557.
16. 41396.
17. 1868.
18. 38729.
19. 226139.
20. 65584.
21. 40596.
22. 76773.
23. 1104692.
24. 419302.
25. 707451.
26. 422395.
27. 58849.
28. 3344048.

29. 14029088.
30. 136815.
31. 658198.
32. 1731895.
33. 584184.
34. 101102016.
35. 20121022.

Exercise 36.

1. 44.
2. 221.
3. 511.
4. 233.
5. 401.
6. 111.
7. 111.
8. 161.
9. 102

Exercise 37.

1. 196.
2. 171.
3. 1476.
4. 70.
5. 531.
6. 479.
7. 6899.
8. 199.
9. 528.

Exercise 39.

1. 188.
2. 74.
3. 1326.
4. 3745.
5. 162.
6. 7209.
7. 410.
8. 481.
9. 8591.
10. 504.
11. 3087.
12. 6174.
13. 6190.
14. 7092.
15. 2898.
16. 1728.
17. 8730.
18. 100.
19. 270.

20. 370.
21. 82.
22. 241.
23. 426.
24. 3852.
25. 8970.
26. 1158.
27. 2211.
28. 718.
29. 2904.
30. 132.
31. 2907.
32. 5778.
33. 8721.
34. 7981.
35. 497.
36. 5229.
37. 1492.
38. 888.
39. 1383.
40. 22.

Exercise 40.

1. 3922.
2. 269.
3. 82.
4. 3153.
5. 953.
6. 161.
7. 3959.
8. 3493.
9. 4573.
10. 4982.
11. 1200.
12. 780.
13. 7198.
14. 304.
15. 389.
16. 707.
17. 1262.
18. 1106.
19. 190.
20. 3131.
21. 70.
22. 4248.
23. 4113.
24. 689.
25. 3219.
26. 3299.
27. 5601.

28. 1709.
29. 493.
30. 344.
31. 924.
32. 943.
33. 2480.
34. 681.
35. 7168.
36. 6626.
37. 4014.
38. 5517.
39. 8493.
40. 8267.
41. 352193.
42. 1884.
43. 19904.
44. 571010.
45. 515136.
46. 497707.
47. 539997.
48. 145204.
49. 99182.
50. 404884.
51. 698238.
52. 356048.
53. 3834791.
54. 965149.
55. 5004054.
56. 397949.
57. 19861.
58. 518058.
59. 3699962.
60. 3490819.

Exercise 41.

1. 4191.
2. 351.
3. 792.
4. 4120.
5. 409.
6. 3722.
7. 6894.
8. 85.
9. 156.
10. 1296.
11. 4318.
12. 135.
13. 80.
14. 2302.
15. 149.
16. 1268.
17. 3161.
18. 6487.
19. 1503.
20. 2976.
21. 350309.
22. 18020.
23. 42290.

24. 394793.
25. 342190.
26. 4190839.
27. 417810.
28. 498197.
29. 4273.
30. 3167.
31. 851.
32. 7283.
33. 34.
34. 205.
35. 5521.
36. 775.
37. 4007.
38. 6990.
39. 370213.
40. 102914.
41. 3492601.
42. 100248.

Exercise 42.

1. 390.
2. 22.
3. 3332.
4. 153.
5. 6886.
6. 1.
7. 405.
8. 8595.
9. 890.
10. 810.
11. 3155.
12. 484.
13. 7466.
14. 4372.
15. 2827.
16. 63.
17. 110.
18. 497.
19. 1031.
20. 202.
21. 166.
22. 3520.
23. 35.
24. 7074.
25. 187.
26. 217.
27. 8783.
28. 1078.
29. 993.
30. 3343.
31. 672.
32. 7654.
33. 4560.
34. 3015.
35. 251.
36. 298.
37. 685.

38. 1219.
39. 4.
40. 364.
41. 3718.
42. 233.
43. 7272.
44. 385.
45. 19.
46. 8981.
47. 1276.
48. 1196.
49. 3541.
50. 870.
51. 7852.
52. 4758.
53. 3213.
54. 449.
55. 496.
56. 883.
57. 1417.
58. 86.
59. 282.
60. 3636.
61. 151.
62. 7190.
63. 303.
64. 101.
65. 8899.
66. 1194.
67. 1114.
68. 3459.
69. 788.
70. 7770.
71. 4676.
72. 3131.
73. 367.
74. 414.
75. 801.
76. 1335.

Exercise 43.

1. 3269.
2. 601.
3. 6868.
4. 7247.
5. 300.
6. 4925.
7. 4903.
8. 1213.
9. 130.
10. 218.
11. 383.
12. 197.
13. 4171.
14. 2917.
15. 1266.
16. 2.
17. 2475.

18. 204.
19. 221.
20. 2426.
21. 198.
22. 368.
23. 2634.
24. 1497.
25. 3347.
26. 3344.
27. 647.
28. 4997.
29. 4200.
30. 299.
31. 378.
32. 161.
33. 5514.
34. 4378.
35. 5866.
36. 1486.
37. 3116.
38. 3612.
39. 183.
40. 155.
41. 302913.
42. 645913.
43. 299400.
44. 546987.
45. 550032.
46. 889.
47. 5106.
48. 346013.
49. 375020.
50. 170214.
51. 4035600.
52. 3691753.
53. 499978.
54. 5099931.
55. 956940.
56. 420042.
57. 309007.
58. 556295.

Exercise 44.

1. 3870.
2. 6267.
3. 4625.
4. 22.
5. 88.
6. 165.
7. 1254.
8. 4183.
9. 2679.
10. 17.
11. 566.
12. 3002.
13. 3.
14. 2697.
15. 3901.

16. 79.
17. 1136.
18. 1488.
19. 496.
20. 3429.
21. 343000.
22. 346513.
23. 3045.
24. 549143.
25. 351119.
26. 29007.
27. 4205814.
28. 343847.
29. 4599953.
30. 4142991.
31. 111035.
32. 247288.
33. 2998.
34. 278.
35. 295.
36. 12.
37. 2458.
38. 3200.
39. 650.
40. 4279.
41. 7002.
42. 313.
43. 43600.
44. 2156.
45. 23901.
46. 514061.
47. 3643013.
48. 667330.

Exercise 47.

1. 3463 A.
2. 1326 trees.
3. \$230.
4. 66 mi.
5. 365 days.
6. \$3081.
7. 5137 people.
8. 63 yr.
9. Mo., 231 mi.
10. 39373 votes.
11. 9141 votes.
12. { \$485, drew;
\$1890, rem.
13. { 91145, both;
58905, more
Chinese.
14. 85 nut trees.
15. \$27.
16. Wash.; 11 yr.
17. { 1401
3052 } cent-
tals.
18. 1782 A. D.

19. 3311 sheep.
20. \$210.
22. 618.
23. \$275.
24. { 2404 mi. to C.
3367 mi. to
N. Y.
25. S. F. to O. 461
mi. further.
27. 19'957 votes.
28. 13181 votes.
29. 7855 votes.
30. \$13.
31. 222.
32. 459.
33. 843.
34. \$1100.
35. 77 marbles.
36. 1519.
38. —
39. \$6100.
40. 581 pupils.
41. 308 girls.
42. 1790.
43. 11699902.
44. { 18199 mi.;
4983 mi.
45. 181 days.
46. 170 ct., one;
120 ct., the
other; 50 ct.
more.
47. \$1487.
48. \$204.
49. 1480
50. 424 centals.
52. 2352 B. C.
53. 14162 ft.
54. 482 mi.
55. \$145.
56. \$505.
57. 303 sh'p; \$873.
58. \$2450.
59. \$11300.
60. 1208 arrests.
61. 153 days.
62. 818 years.
63. \$1975.
64. 2 days.
65. 1848.
66. 30379 ft.
67. 115 ct.
68. 56 l. trees.

Exercise 58.

1. 1448.
2. 1566.
3. 1284.

4. 369.
5. 886.
6. 3555.
7. 2463.
8. 2440.
9. 28488.
10. 16804.
11. 4608.
12. 45055.
13. 28088.
14. 9966.
15. 12696.

Exercise 59.

1. 125.
2. 1673.
3. 3924.
4. 12792.
5. 9444.
6. 10284.
7. 1636.
8. 32200.
9. 216.
10. 5040.
11. 2284.
12. 894.
13. 9171.
14. 1416.
15. 3780.
16. 32064.
17. 1530.
18. 3339.
19. 7104.
20. 10008.
21. 4476.
22. 22955.
23. 3699.
24. 61383.
25. 87849.
26. 20020.
27. 1978.
28. 28068.
29. 25680.
30. 12609.
31. 3282.
32. 57672.
33. 72720.
34. 46900.
35. 11750.
36. 9093.
37. 4420.
38. 15132.
39. 59994.

Exercise 61.

1. 50.
2. 75.

3. 100.
4. 150.
5. 175.
6. 200.
7. 225.
8. 478.
9. 717.
10. 956.
11. 1195.
12. 1434.
13. 1912.
14. 2151.
15. 872.
16. 1308.
17. 1744.
18. 2180.
19. 2616.
20. 3052.
21. 3488.
22. 6396.
23. 9594.
24. 15990.
25. 19188.
26. 22386.
27. 25584.
28. 28782.
29. 14166.
30. 18888.
31. 23610.
32. 28332.
33. 33054.
34. 37776.
35. 42498.
36. 6856.
37. 13712.
38. 17140.
39. 20568.
40. 23996.
41. 27424.
42. 30852.
43. 818.
44. 1227.
45. 2045.
46. 2454.
47. 2863.
48. 3272.
49. 3681.
50. 9200.
51. 13800.
52. 18400.
53. 23000.
54. 27600.
55. 36800.
56. 41400.
57. 72.
58. 108.
59. 144.
60. 180.
61. 252.

62. 288.
 63. 324.
 64. 2016.
 65. 3024.
 66. 4032.
 67. 6048.
 68. 7056.
 69. 8034.
 70. 9072.
 71. 1142.
 72. 1713.
 73. 2855.
 74. 3426.
 75. 3997.
 76. 4568.
 77. 5139.
 78. 593.
 79. 1192.
 80. 1490.
 81. 1788.
 82. 2086.
 83. 2384.
 84. 2682.
 85. 2038.
 86. 3057.
 87. 4076.
 88. 5095.
 89. 6114.
 90. 7133.
 91. 8152.
 92. 472.
 93. 708.
 94. 944.
 95. 1180.
 96. 1652.
 97. 1888.
 98. 2124.
 99. 1512.
 100. 2268.
 101. 3024.
 102. 4536.
 103. 5292.
 104. 6048.
 105. 6804.
 106. 8016.
 107. 12024.
 108. 16032.
 109. 20040.
 110. 24048.
 111. 28056.
 112. 36072.
 113. 2295.
 114. 3000.
 115. 3825.
 116. 4590.
 117. 5355.
 118. 6120.
 119. 6885.
 120. 954.

121. 1431.
 122. 1908.
 123. 2385.
 124. 2862.
 125. 3816.
 126. 4293.
 127. 1776.
 128. 2664.
 129. 3552.
 130. 4440.
 131. 5328.
 132. 6216.
 133. 7992.
 134. 2224.
 135. 3336.
 136. 4448.
 137. 5560.
 138. 6672.
 139. 7784.
 140. 8896.

Exercise 62.

1. 1290.
 2. 15750.
 3. 30000.
 4. 143400.
 5. 1570500.
 6. 2187000.
 7. 214000.
 8. 15022000.
 9. 363900.
 10. 750000.
 11. 540000.

Exercise 63.

1. 13675.
 2. 130733.
 3. 238492.
 4. 1749306.
 5. 2582934.
 6. 1875116.
 7. 223723.
 8. 2516200.
 9. 19392.
 10. 551376.
 11. 180225.
 12. 1722951.
 13. 3143124.
 14. 23054382.
 15. 34040898.
 16. 24712452.
 17. 2948481.
 18. 33161400.
 19. 259524.
 20. 7266672.
 21. 202000.
 22. 1931120.

23. 3522880.
 24. 25839840.
 25. 38153760.
 26. 27698240.
 27. 3304720.
 28. 37168000.
 29. 290880.
 30. 8144640.
 31. 167500.
 32. 1601300.
 33. 2921200.
 34. 21426600.
 35. 31637400.
 36. 22967600.
 37. 2740300.
 38. 30820000.
 39. 241200.
 40. 6753600.
 41. 58750.
 42. 561650.
 43. 1024600.
 44. 7515300.
 45. 11096700.
 46. 8055800.
 47. 961150.
 48. 10810000.
 49. 84600.
 50. 2368800.
 51. 75775.
 52. 724409.
 53. 1321516.
 54. 9393138.
 55. 14312382.
 56. 10390268.
 57. 1239679.
 58. 13942600.
 59. 109116.
 60. 3055248.
 61. 27625.
 62. 264095.
 63. 481780.
 64. 3533790.
 65. 5217810.
 66. 3787940.
 67. 451945.
 68. 5083000.
 69. 39780.
 70. 1113840.
 71. 189150.
 72. 1808274.
 73. 3298776.
 74. 24196068.
 75. 35726652.
 76. 25936248.
 77. 3094494.
 78. 34803600.
 79. 272376.
 80. 7626528.
 81. 249975.

82. 2389761.
 83. 4359564.
 84. 31976802.
 85. 47215278.
 86. 34276572.
 87. 4089591.
 88. 45995400.
 89. 359954.
 90. 10078992.

Exercise 65.

1. \$925.
 2. 768 hr.
 3. \$176.
 4. 95040 ft.
 5. 552 mi.
 6. 9372 trees.
 7. \$1164625.
 8. \$72000.
 9. 22490 lb.
 10. \$900.
 11. \$552.
 12. \$494.
 13. \$6300.

Exercise 74.

1. 1233.
 2. 411.
 3. 4242.
 4. 91.
 5. 91.
 6. 31.
 7. 641.
 8. 532.
 9. 501.
 10. 1001.
 11. 701.
 12. 301.

Exercise 75.

1. 29 $\frac{1}{2}$.
 2. 26.
 3. 19 $\frac{3}{8}$.
 4. 12 $\frac{3}{4}$.
 5. 16 $\frac{3}{8}$.
 6. 17.
 7. 21.
 8. 18 $\frac{3}{8}$.
 9. 39 $\frac{1}{2}$.
 10. 22 $\frac{3}{4}$.
 11. 88 $\frac{3}{8}$.
 12. 72.
 13. 79 $\frac{3}{8}$.
 14. 102.
 15. 101 $\frac{1}{2}$.
 16. 214 $\frac{1}{2}$.
 17. 58 $\frac{1}{2}$.

18. 60.
19. $64\frac{1}{2}$.
20. $758\frac{2}{3}$.
21. $1400\frac{2}{3}$.
22. $888\frac{2}{3}$.
23. 949.
24. $111\frac{2}{3}$.
25. 1432.
26. $813\frac{1}{4}$.
27. 1229.
28. 1175.
29. 1151.
30. $3401\frac{2}{3}$.
31. $145402\frac{2}{3}$.
32. $6492\frac{2}{3}$.
33. 11250.
34. $144701\frac{1}{5}$.
35. $19575\frac{2}{3}$.
36. 9889.
37. $25466\frac{2}{3}$.
38. 7350.
39. $57578\frac{2}{3}$.

Exercise 76.

1. 218104.
2. 109052.
3. $87241\frac{2}{3}$.
4. $72701\frac{1}{4}$.
5. $62315\frac{2}{3}$.
6. 54526.
7. $48467\frac{2}{5}$.
8. 29218.
9. $19478\frac{2}{3}$.
10. 14609.
11. $11687\frac{1}{2}$.
12. $9739\frac{2}{3}$.
13. 8348.
14. $7304\frac{1}{2}$.
15. 45000.
16. 30000.
17. 22500.
18. 18000.
19. 15000.
20. $12857\frac{1}{2}$.
21. 10000.
22. 361753.
23. $241168\frac{2}{3}$.
24. $180876\frac{2}{3}$.
25. $120584\frac{2}{3}$.
26. 103358.
27. 904382.
28. $80389\frac{2}{3}$.
29. 58726.
30. $39150\frac{2}{3}$.
31. 29363.
32. $23490\frac{2}{3}$.
33. $16778\frac{2}{3}$.
34. $14681\frac{1}{2}$.

35. $13050\frac{2}{3}$.
36. $44500\frac{1}{5}$.
37. 29667.
38. $22250\frac{1}{4}$.
39. 17800.
40. $14833\frac{2}{3}$.
41. $12714\frac{1}{2}$.
42. $11125\frac{1}{5}$.
43. 38200.
44. 19100.
45. 15280.
46. $12733\frac{2}{3}$.
47. $10914\frac{1}{4}$.
48. 9550.
49. $8488\frac{2}{3}$.
50. 4900.
51. 3675.
52. 2940.
53. 2450.
54. 2100.
55. $1837\frac{1}{2}$.
56. $1633\frac{2}{3}$.
57. 259103.
58. $172735\frac{1}{3}$.
59. $129551\frac{2}{3}$.
60. $103641\frac{1}{5}$.
61. $86367\frac{1}{3}$.
62. $74029\frac{2}{3}$.
63. $64775\frac{2}{3}$.

Exercise 78.

1. 350 2 rem.
2. 400.
3. 189 16 rem.
4. 50 1 rem.
5. 214 16 rem.
6. 233 12 rem.
7. 266 20 rem.
8. 126 16 rem.
9. 33 11 rem.
10. 143 6 rem.
11. 175 2 rem.
12. 200.
13. 94 36 rem.
14. 25 1 rem.
15. 107 16 rem.
16. 140 2 rem.
17. 160.
18. 75 46 rem.
19. 20 1 rem.
20. 85 46 rem.
21. 116 42 rem.
22. 133 20 rem.
23. 63 16 rem.
24. 16 41 rem.
25. 71 36 rem.
26. 100 2 rem.
27. 114 20 rem.

28. 54 16 rem.
29. 14 21 rem.
30. 61 26 rem.
31. 87 42 rem.
32. 100.
33. 47 36 rem.
34. 12 41 rem.
35. 53 56 rem.
36. 77 72 rem.
37. 88 80 rem.
38. 42 16 rem.
39. 11 11 rem.
40. 47 66 rem.
41. 33 111 rem.
42. 12 58 rem.
43. 47.
44. 40 57 rem.
45. 119 9 rem.
46. 21 211 rem.
47. 8 58 rem.
48. 31 100 rem.
49. 26 257 rem.
50. 79 109 rem.
51. 16 111 rem.
52. 6 58 rem.
53. 23 200 rem.
54. 20 57 rem.
55. 59 209 rem.
56. 13 11 rem.
57. 4 458 rem.
58. 18 400 rem.
59. 16 57 rem.
60. 47 309 rem.
61. 10 511 rem.
62. 4 58 rem.
63. 15 400 rem.
64. 13 257 rem.
65. 39 409 rem.
66. 9 211 rem.
67. 3 358 rem.
68. 13 300 rem.
69. 11 357 rem.
70. 34 9 rem.
71. 8 111 rem.
72. 3 58 rem.
73. 11 600 rem.
74. 10 57 rem.
75. 29 609 rem.
76. 7 211 rem.
77. 2 658 rem.
78. 10 400 rem.
79. 8 857 rem.
80. 26 409 rem.
81. 218 208 rem.
82. 29 436 rem.
83. 45.
84. 361 1506 rem.
85. 58 1452 rem.
86. 145 1208 rem.

87. 19 1436 rem.
88. 30.
89. 241 506 rem.
90. 39 452 rem.
91. 109 208 rem.
92. 14 2436 rem.
93. 22 2000 rem.
94. 180 3506 rem.
95. 29 1452 rem.
96. 87 1208 rem.
97. 11 3436 rem.
98. 18.
99. 144 3506 rem.
100. 23 2452 rem.
101. 72 4208 rem.
102. 9 4436 rem.
103. 15.
104. 120 3506 rem.
105. 19 3452 rem.
106. 63 2208 rem.
107. 8 2436 rem.
108. 12 6000 rem.
109. 103 2506 rem.
110. 16 5452 rem.
111. 54 4208 rem.
112. 7 2436 rem.
113. 11 2000 rem.
114. 90 3506 rem.
115. 14 5452 rem.
116. 48 4208 rem.
117. 6 4436 rem.
118. 10.
119. 80 3506 rem.
120. 13 452 rem.

Exercise 80.

1. 110 21 rem.
2. 83 37 rem.
3. 67 12 rem.
4. 73 87 rem.
5. 65 76 rem.
6. 127 34 rem.
7. 77 43 rem.
8. 70 1 rem.
9. 82 33 rem.
10. 97 12 rem.
11. 41 39 rem.
12. 31 40 rem.
13. 25 33 rem.
14. 27 82 rem.
15. 24 32 rem.
16. 48 10 rem.
17. 29 22 rem.
18. 26 40 rem.
19. 31 9 rem.
20. 36 46 rem.
21. 159 19 rem.
22. 120 40 rem.

23. 96 88 rem.	82. 9275 56 rem.	141. 8 6348 rem.	27. { 1425 ct.
24. 106 72 rem.	83. 7458 80 rem.	142. 23 1902 rem.	28. { 150 ct.
25. 94 94 rem.	84. 8221 58 rem.	143. 6 2036 rem.	29. \$5.
26. 184 16 rem.	85. 7308 14 rem.	144. 7 2037 rem.	30. 400 ct.
27. 111 76 rem.	86. 14186 20 rem.	145. 2 10818 rem.	31. 124 oranges.
28. 101 7 rem.	87. 8613 14 rem.	146. 18 254 rem.	32. 175 bbl.
29. 118 78 rem.	88. 7779 59 rem.	147. 47 1926 rem.	33. { 46375 lb.
30. 140 20 rem.	89. 9158 24 rem.	148. 12 4652 rem.	34. { 288 mi. a da.
31. 136 33 rem.	90. 10798 40 rem.	149. 14 4654 rem.	35. { 12 mi. an hr.
32. 103 23 rem.	91. 1990 42 rem.	150. 4 22216 rem.	36. 1435 yd.
33. 83 6 rem.	92. 1505 62 rem.	151. 13 4358 rem.	37. \$700.
34. 91 49 rem.	93. 1210 82 rem.	152. 36 513 rem.	38. \$84.
35. 81 38 rem.	94. 1334 60 rem.	153. 9 4401 rem.	39. 714.
36. 157 50 rem.	95. 1186 38 rem.	154. 11 374 rem.	40. 65 mi.
37. 95 77 rem.	96. 2302 50 rem.	155. 3 17574 rem.	41. 61 mi.
38. 86 59 rem.	97. 1398 20 rem.		42. 40. 10.
39. 101 78 rem.	98. 1262 86 rem.		43. 25.
40. 120 17 rem.	99. 1486 58 rem.		44. \$31835.
41. 403 32 rem.	100. 1753 1 rem.		45. { \$657.
42. 305 19 rem.	101. 111 645 rem.		46. { 1314 da.
43. 245 44 rem.	102. 154 297 rem.		47. 830 $\frac{2}{3}$ $\frac{6}{11}$ bales.
44. 270 49 rem.	103. 185 386 rem.		48. Train, 480 mi.
45. 240 49 rem.	104. 174 261 rem.		49. \$1781.
46. 406 43 rem.	105. 125 501 rem.		50. { \$6080.
47. 283 37 rem.	106. 207 198 rem.		51. { \$960.
48. 256 1 rem.	107. 381 228 rem.		52. 60 da.
49. 301 30 rem.	108. 211 381 rem.		53. 55 sacks.
50. 355 24 rem.	109. 689 120 rem.		54. 105 ct.
51. 733 21 rem.	110. 16 4057 rem.		55. 112 trees.
52. 552 32 rem.	111. 95 780 rem.		56. 144 da.
53. 446 96 rem.	112. 132 368 rem.		57. 375 boxes.
54. 4956 80 rem.	113. 159 239 rem.		58. { 63000 or'g's.
55. 4406 14 rem.	114. 149 410 rem.		59. { 5250 doz.
56. 8553 5 rem.	115. 107 644 rem.		60. \$630.
57. 5192 80 rem.	116. 178 38 rem.		61. 32 da.
58. 4670 38 rem.	117. 327 209 rem.		62. \$100.
59. 5521 49 rem.	118. 181 380 rem.		63. { 15 watches.
60. 6510 38 rem.	119. 592 32 rem.		64. { 5 rings.
61. 990 26 rem.	120. 14 2074 rem.		65. 32 mo.
62. 749 14 rem.	121. 18 372 rem.		66. \$2928.
63. 602 42 rem.	122. 25 300 rem.		67. \$2.
64. 664 4 rem.	123. 30 330 rem.		68. \$8820.
65. 500 26 rem.	124. 28 420 rem.		69. { \$4166 $\frac{8}{15}$.
66. 1145 41 rem.	125. 20 540 rem.		70. { \$20000.
67. 695 56 rem.	126. 34 114 rem.		71. \$14.
68. 628 32 rem.	127. 63 21 rem.		72. \$17.
69. 739 55 rem.	128. 35.		73. \$104.
70. 872 12 rem.	129. 113 123 rem.		74. \$655.
71. 1525 25 rem.	130. 2 4082 rem.		75. \$2805.
72. 1153 66 rem.	131. 651 10 rem.		76. 24 watches.
73. 927 81 rem.	132. 899 382 rem.		77. \$418.
74. 1022 64 rem.	133. 1081 407 rem.		78. \$50.
75. 909 9 rem.	134. 1016 46 rem.		79. \$2.
76. 1764 36 rem.	135. 731 658 rem.		80. \$2280.
77. 1071 36 rem.	136. 1207 403 rem.		81. \$240.
78. 967 69 rem.	137. 2224 14 rem.		82. 14553 cu. in.
79. 1139 19 rem.	138. 1233 346 rem.		83. \$315.
80. 1343 19 rem.	139. 4017 13 rem.		84. \$180.
81. 12262 48 rem.	140. 97 3233 rem.		85. \$75.

Exercise 81.

1. 25 cows.
2. 31 da.
3. 11 mo.
4. 24 hr.
5. 93 A.
6. 35 mi.
7. 960 A.
8. 346 chests.
9. 12 mo.
10. 11 ponies.

Exercise 82.

1. 2250 yd.
2. 216 T.
3. \$21280.
4. 4800 rd.
5. \$341.
6. 107 $\frac{2}{13}$ T.
7. 3525 lb.
8. 81 $\frac{1}{8}$ bales.
9. 14 $\frac{3}{8}$ bags.
10. 1800 eggs.
11. 407 $\frac{17}{4}$ da.
12. 8760 hr.
13. \$23800.
14. 141 calves.
15. 3 mi.
16. \$1925.
17. \$12624550.
18. 9098 $\frac{2}{37}$ sacks.
19. 124 $\frac{98}{7}$ cents.
20. \$91.25.
21. \$200.
22. { \$4.20.
23. { 42 loaves.
24. \$1570.
25. \$19.
26. 45 ct.

- 79. \$4.
- 80. 24 marbles.
- 81. 70 ct.
- 82. 45 yr.
- 83. 225 lb.
- 84. 155 girls.
- 85. 42 rd.
- 86. 29½ steps.
- 87. 33 da.
- 88. \$15.

Exercise 86.

- 1. 6.
- 2. 11.
- 3. 24.
- 4. 45.
- 5. 9.
- 6. 5.
- 7. 24.
- 8. None.
- 9. 3.
- 10. 4.
- 11. 15.
- 12. 36.
- 13. 35.
- 14. 9.
- 15. 11.
- 16. 20.
- 17. 12.
- 18. 15.
- 19. 36.
- 20. 13.
- 21. 12.
- 22. 7.
- 23. 7.
- 24. 41.
- 25. 55.
- 26. 3.
- 27. 13.
- 28. 30.
- 29. 33.
- 30. 15.
- 31. 12.
- 32. 17.
- 33. 7.
- 34. 2.
- 35. 14.
- 36. 10.

Exercise 88.

- 1. 135.
- 2. 9.
- 3. 26.
- 4. 1001.
- 5. 725.
- 6. 29.
- 7. 3.

- 8. 3.
- 9. 15.
- 10. 102.
- 11. 18.
- 12. 3.
- 13. 6.
- 14. 16.
- 15. 9.
- 16. 47.
- 17. 31.
- 18. 71.
- 19. 9.
- 20. 13.
- 21. 29.
- 22. 3.
- 23. 3.
- 24. 3.
- 25. 2.
- 26. None.

Exercise 89.

- 1. 7.
- 2. 21.
- 3. 41.
- 4. 13.
- 5. 53.
- 6. 31.

Exercise 90.

- 1. 90.
- 2. 504.
- 3. 40.
- 4. 60.
- 5. 504.
- 6. 300.
- 7. 120.
- 8. 924.
- 9. 840.
- 10. 108.
- 11. 210.
- 12. 60.
- 13. 72.
- 14. 240.
- 15. 420.
- 16. 750.
- 17. 180.
- 18. 360.
- 19. 4752.
- 20. 462.
- 21. 5544.
- 22. 4200.
- 23. 630.
- 24. 840.
- 25. 105.
- 26. 570.
- 27. 108.
- 28. 390.

- 29. 660.
- 30. 672.
- 31. 156.
- 32. 208.
- 33. 3450.
- 34. 99.
- 35. 312.
- 36. 600.
- 37. 1344.
- 38. 612.
- 39. 1530.

Exercise 92.

- 1. 945.
- 2. 1183.
- 3. 1014.
- 4. 15708.
- 5. 12936.
- 6. 2583.
- 7. 1176.
- 8. 1116.
- 9. 105.
- 10. 84.
- 11. 80.
- 12. 300.
- 13. 180.
- 14. 108.
- 15. 32.
- 16. 567.
- 17. 448.
- 18. 132.
- 19. 23391.
- 20. 33033.
- 21. 609.
- 22. 32250.
- 23. 38178.
- 24. 77499.
- 25. 5922.

Exercise 94.

- 1. 3 ft.
- 2. 8 ft.
- 3. 60 mi.
- 4. { 120 ft.
7 lots.
- 5. { 80 min.
400 rd.
320 rd.
- 6. 120 qt.
- 7. { 105 lb.
\$10 B.
\$18 W.
- 8. 840 ct.
- 9. 120 ct.
- 10. 28 each.
- 11. 252 nuts.
- 12. 240 marbles.

- 13. 120 yd.
- 14. 340.
- 15. 15.
- 16. { \$300.
\$1200.
- 17. { 28 pupils.
7 cards.
- 18. 154 ct.
- 19. { 3 each.
17 children.

Exercise 98.

- 1. $\frac{551}{12}, \frac{729}{10}$.
- 2. $\frac{988}{9}, \frac{307}{12}$.
- 3. $\frac{1104}{19}, \frac{723}{38}$.
- 4. $\frac{1127}{8}, \frac{341}{24}$.
- 5. $\frac{178}{13}, \frac{141}{8}$.
- 6. $\frac{1283}{15}, \frac{637}{10}$.
- 7. $\frac{347}{7}, \frac{293}{14}$.
- 8. $\frac{721}{3}, \frac{157}{15}$.
- 9. $\frac{243}{10}, \frac{437}{24}$.
- 10. $\frac{521}{3}, \frac{641}{6}$.
- 11. $\frac{631}{8}, \frac{599}{12}$.
- 12. $\frac{509}{50}, \frac{387}{20}$.

Exercise 101.

- 1. $\frac{1}{2}$.
- 2. $\frac{23}{113}$.
- 3. $\frac{27}{181}$.
- 4. $\frac{3}{11}$.
- 5. $\frac{1}{6}$.
- 6. $\frac{3}{7}$.
- 7. $\frac{43}{250}$.
- 8. $\frac{79}{196}$.
- 9. $\frac{37}{65}$.
- 10. $\frac{5}{7}$.
- 11. $\frac{21}{101}$.
- 12. $\frac{337}{367}$.
- 13. $\frac{41}{3}$.
- 14. $\frac{13}{45}$.
- 15. $\frac{79}{109}$.
- 16. $\frac{7}{18}$.
- 17. $\frac{9}{16}$.
- 18. $\frac{3}{4}$.
- 19. $\frac{4}{7}$.
- 20. $\frac{3}{17}$.
- 21. $\frac{5}{9}$.
- 22. $\frac{1}{2}$.
- 23. $\frac{5}{12}$.
- 24. $\frac{17}{19}$.
- 25. $\frac{4}{5}$.
- 26. $\frac{113}{133}$.

27. $\frac{1}{5}$.
28. $\frac{1}{4}$.
29. $\frac{7}{9}$.
30. $\frac{4}{5}$.
31. $\frac{3}{5}$.
32. $\frac{11}{28}$.
33. $\frac{1}{4}$.
34. $\frac{1}{2}$.
35. $\frac{3}{5}$.
36. $\frac{5}{8}$.
37. $\frac{3}{8}$.
38. $\frac{3}{5}$.
39. $\frac{7}{11}$.
40. $\frac{2}{7}$.
41. $\frac{3}{7}$.
42. $\frac{7}{9}$.
43. $\frac{1}{2}$.
44. $\frac{2}{7}$.
45. $\frac{6}{13}$.
46. $\frac{1}{11}$.
47. $\frac{3}{4}$.
48. $\frac{3}{11}$.
49. $\frac{10}{21}$.
50. $\frac{5}{6}$.
51. $\frac{6}{13}$.
52. $\frac{17}{25}$.
53. $\frac{2}{7}$.
54. $\frac{11}{21}$.
55. $\frac{1}{7}$.
56. $\frac{3}{4}$.
57. $\frac{1}{2}$.
58. $\frac{4}{5}$.
59. $\frac{7}{9}$.
60. $\frac{1}{4}$.
61. $\frac{3}{7}$.
62. $\frac{1}{25}$.
63. $\frac{5}{16}$.
64. $\frac{2}{5}$.
65. $\frac{19}{20}$.
66. $\frac{9}{14}$.
67. $\frac{1}{3}$.
68. $\frac{1}{2}$.
69. $\frac{4}{5}$.
70. $\frac{9}{16}$.

Exercise 104.

1. $\frac{35}{40}, \frac{24}{40}, \frac{150}{40}$.
2. $\frac{280}{17}, \frac{17}{17}, \frac{13}{17}$.
3. $\frac{18}{2}, \frac{49}{72}, \frac{153}{72}$.
4. $\frac{66}{88}, \frac{180}{88}, \frac{33}{88}$.

5. $\frac{20}{40}, \frac{135}{40}, \frac{1016}{40}$.
6. $\frac{123}{120}, \frac{104}{120}, \frac{96}{120}$.
7. $\frac{6}{9}, \frac{275}{9}, \frac{7}{9}$.
8. $\frac{18}{11}, \frac{11}{11}, \frac{561}{11}$.
9. $\frac{25}{100}, \frac{1170}{100}, \frac{12}{100}$.
10. $\frac{77}{350}, \frac{150}{350}, \frac{224}{350}$.
11. $\frac{15}{48}, \frac{16}{48}, \frac{15}{48}$.
12. $\frac{31}{84}, \frac{244}{84}, \frac{1596}{84}$.
13. $\frac{802}{260}, \frac{250}{260}, \frac{247}{260}$.
14. $\frac{56}{72}, \frac{2664}{72}, \frac{45}{72}$.
15. $\frac{40}{56}, \frac{36}{56}, \frac{36}{56}$.
16. $\frac{33}{42}, \frac{14}{42}, \frac{21}{42}$.
17. $\frac{7540}{319}, \frac{511}{319}, \frac{203}{319}$.
18. $\frac{80}{112}, \frac{64}{112}, \frac{49}{112}$.
19. $\frac{1040}{56}, \frac{203}{56}, \frac{1}{56}$.
20. $\frac{112}{16}, \frac{9}{16}, \frac{12}{16}$.
21. $\frac{10}{20}, \frac{15}{20}, \frac{342}{20}$.

Exercise 107.

1. $\frac{123}{11}$.
2. $\frac{9}{11}$.
3. $\frac{119}{24}$.
4. $\frac{5}{8}$.
5. 1.
6. $\frac{123}{11}$.
7. $\frac{2}{11}$.
8. $\frac{2}{11}$.
9. $\frac{117}{24}$.
10. $\frac{2}{11}$.
11. $\frac{316}{11}$.
12. $\frac{31}{11}$.
13. $\frac{295}{35}$.
14. $\frac{113}{11}$.
15. $\frac{12}{11}$.
16. $\frac{47}{11}$.
17. $\frac{17}{11}$.
18. $\frac{17}{11}$.
19. $\frac{19}{24}$.
20. $\frac{3}{11}$.
21. $\frac{2}{11}$.
22. $\frac{13}{11}$.
23. $\frac{29}{11}$.
24. $\frac{1}{11}$.
25. $\frac{1}{24}$.
26. $\frac{1}{11}$.
27. $\frac{3}{11}$.
28. $\frac{3}{11}$.
29. $\frac{1}{11}$.
30. $\frac{1}{11}$.
31. 0.
32. $\frac{17}{11}$.

Exercise 109.

1. $\frac{9}{10}$.
2. $\frac{221}{17}$.
3. $\frac{92}{17}$.

4. $\frac{315}{38}$.
5. $\frac{2911}{40}$.
6. $\frac{1517}{24}$.
7. $\frac{187}{9}$.
8. $\frac{77}{17}$.
9. $\frac{61}{17}$.
10. $\frac{2010}{35}$.
11. $\frac{2734}{35}$.
12. $\frac{6911}{4}$.
13. $\frac{4259}{60}$.
14. $\frac{7822}{9}$.
15. $\frac{35}{56}$.
16. $\frac{11}{11}$.
17. $\frac{41175}{110}$.
18. $\frac{33}{11}$.
19. $\frac{29}{11}$.
20. $\frac{21}{16}$.
21. $\frac{103}{20}$.

Exercise 111.

1. $45\frac{7}{36}, 72\frac{8}{45}$.
2. $109\frac{1}{18}, 24\frac{3}{16}$.
3. $57\frac{131}{342}, 181\frac{94}{42}$.
4. $140\frac{1}{2}, 132\frac{2}{5}$.
5. $123\frac{27}{34}, 166\frac{5}{2}$.
6. $847\frac{3}{90}, 62\frac{4}{15}$.
7. $481\frac{97}{126}, 20\frac{2}{3}$.
8. $239\frac{1}{10}, 9\frac{6}{5}$.
9. $14\frac{67}{44}, 17\frac{3}{2}$.
10. $103\frac{43}{3}, 103\frac{1}{3}$.
11. $78\frac{1}{2}, 49\frac{3}{5}$.
12. $91\frac{93}{25}, 181\frac{1}{3}$.

Exercise 114.

4. \$111 $\frac{1}{4}$.
5. $\frac{59}{5}$.
6. $\frac{110}{67}$.
7. 137 $\frac{17}{33}$ rd.
8. 154 $\frac{11}{16}$ lb.
9. 65 $\frac{1}{30}$ hr.
10. 85 $\frac{39}{60}$ mi.
11. 57 $\frac{13}{2}$ yd.
12. 48 $\frac{1}{4}$ ft.
13. $\frac{1}{5}$.
14. 162 $\frac{99}{25}$ cd.
15. 406 $\frac{152}{105}$ mi.
16. 63 $\frac{63}{5}$ mi.
17. 286 $\frac{93}{100}$ A.
18. $\frac{2}{1}$.
19. 63 $\frac{3}{4}$ ft.
20. \$124 $\frac{9}{10}$.
21. 95 $\frac{7}{15}$ hr.

Exercise 116.

1. 505 $\frac{1}{2}$, 801 $\frac{9}{10}$.
2. 1207 $\frac{1}{5}$, 281 $\frac{1}{12}$.

3. 639 $\frac{3}{10}$, 2091 $\frac{1}{3}$.
4. 1549 $\frac{1}{8}$, 156 $\frac{7}{24}$.
5. 178, 229 $\frac{1}{4}$.
6. 1111 $\frac{14}{15}$, 828 $\frac{1}{10}$.
7. 644 $\frac{3}{15}$, 272 $\frac{1}{14}$.
8. 3124 $\frac{1}{3}$, 136 $\frac{1}{15}$.
9. 258 $\frac{3}{10}$, 309 $\frac{1}{2}$.
10. 1771 $\frac{1}{10}$, 1816 $\frac{1}{4}$.
11. 1340 $\frac{1}{8}$, 848 $\frac{7}{15}$.
12. 173 $\frac{3}{50}$, 3281 $\frac{19}{20}$.

Exercise 119.

1. \$193 $\frac{3}{4}$.
2. \$49 $\frac{2}{5}$.
3. \$10480.
4. 378 gal.
5. \$192 $\frac{1}{2}$.
6. 39 $\frac{17}{2}$ mi.
7. 221 $\frac{3}{5}$ hr.
8. 74 $\frac{3}{5}$ pp.
9. \$8746.
10. 440 yd.

Exercise 122.

1. 10.
2. 19 $\frac{1}{3}$.
3. 20 $\frac{2}{3}$.
4. 15 $\frac{1}{3}$.
5. 12.
6. 6.
7. 0.
8. 16.
9. 11 $\frac{1}{3}$.
10. 18.
11. 9 $\frac{1}{3}$.
12. 7 $\frac{1}{3}$.
13. 18 $\frac{6}{7}$.
14. 15 $\frac{6}{7}$.
15. 12.
16. 8 $\frac{1}{7}$.
17. 3.
18. 6.
19. 10 $\frac{2}{3}$.
20. 13 $\frac{5}{7}$.
21. 19 $\frac{2}{7}$.
22. 24.
23. 16 $\frac{2}{3}$.
24. 18 $\frac{2}{3}$.
25. 12 $\frac{2}{3}$.
26. 19 $\frac{1}{3}$.
27. 22 $\frac{5}{6}$.
28. 44 $\frac{1}{3}$.
29. 40.
30. 31 $\frac{1}{9}$.
31. 10 $\frac{5}{9}$.
32. 3 $\frac{3}{5}$.
33. 14 $\frac{1}{3}$.
34. 45.
35. 51 $\frac{2}{3}$.

36. 25.
37. 30.
38. $35\frac{5}{8}$.
39. $11\frac{3}{8}$.
40. $6\frac{2}{8}$.
41. $40\frac{5}{8}$.
42. $44\frac{1}{8}$.
43. $51\frac{3}{8}$.
44. $20\frac{3}{8}$.
45. $27\frac{1}{8}$.
46. 15.
47. 35.
48. $42\frac{1}{2}$.

Exercise 125.

1. $\frac{5}{8}$.
2. $\frac{1}{15}$.
3. $\frac{5}{8}$.
4. $\frac{1}{125}$.
5. $\frac{1}{3}$.
6. $\frac{1}{5}$.
7. $\frac{1}{5}$.
8. $\frac{1}{5}$.
9. $\frac{2}{3}$.
10. $\frac{2}{3}$.
11. $\frac{3}{4}$.
12. $\frac{4}{10}$.
13. $\frac{2}{9}$.
14. $\frac{9}{1}$.
15. $\frac{1}{5}$.
16. $\frac{1}{40}$.
17. $\frac{9}{15}$.
18. $\frac{1}{4}$.

Exercise 126.

1. $31\frac{1}{2}$.
2. $21\frac{1}{4}$.
3. $85\frac{3}{4}$.
4. $67\frac{1}{2}$.
5. $61\frac{1}{2}$.
6. $64\frac{3}{4}$.
7. $70\frac{1}{2}$.
8. $48\frac{1}{2}$.
9. $64\frac{1}{2}$.
10. $99\frac{1}{2}$.
11. $25\frac{1}{2}$.
12. $52\frac{1}{2}$.
13. $83\frac{3}{4}$.
14. $62\frac{1}{2}$.
15. $12\frac{1}{4}$.
16. $92\frac{1}{2}$.
17. $3317\frac{13}{16}$.
18. $2808\frac{13}{16}$.
19. $1105\frac{13}{16}$.
20. $2001\frac{13}{16}$.
21. $2411\frac{1}{2}$.
22. $5448\frac{1}{10}$.
23. $1037\frac{1}{5}$.

24. $2515\frac{2}{3}$.
25. $276\frac{6}{8}$.
26. $11132\frac{1}{10}$.
27. $39371\frac{1}{10}$.
28. $196\frac{983}{1000}$.

Exercise 128.

1. $\$44\frac{1}{10}$.
2. $424\frac{2}{3}$ mi.
3. $\$61\frac{1}{4}$.
4. $\$2\frac{7}{11}$.
5. $268\frac{3}{4}$ ct.
6. $2305\frac{2}{8}$ lb.
7. $\$40\frac{3}{8}$.
8. $\$57\frac{7}{8}$.
9. $1491\frac{3}{10}$ mi.
10. $264\frac{1}{10}$ mi.

Exercise 134.

1. $\frac{1\frac{1}{2}}{\frac{2}{3}}$.
2. $\frac{5\frac{2}{3}}{\frac{5}{8}}$.
3. $\frac{7\frac{2}{3}}{\frac{2}{3}}$.
4. $\frac{3\frac{2}{3}}{\frac{3}{10}}$.
5. $\frac{2\frac{1}{4}}{\frac{1}{4}}$.
6. $\frac{3\frac{0}{1}}{\frac{3}{1}}$.
7. $\frac{1\frac{3}{10}}{\frac{2}{5}}$.
8. $\frac{100}{100}$.
9. $\frac{1}{1}$.
10. $\frac{1}{2}$.
11. $\frac{1}{2}$.
12. $\frac{1}{2}$.
13. $\frac{1}{2}$.
14. $\frac{1}{5}$.
15. $\frac{1}{16}$.
16. $\frac{2}{3}$.

Exercise 135.

1. 11 chairs.
2. 70 $\frac{1}{2}$ da.
3. $8\frac{2}{4}$ yr.
4. 13 dresses.
5. 13 dresses.
6. 4 sons.
7. 27 div.
8. 9 steps.
9. $5\frac{1}{2}$ da.
10. $12\frac{1}{2}$ cu. ft.

Exercise 136.

1. $\frac{7}{5}$.
2. 3.
3. $5\frac{1}{4}$.
4. $2\frac{1}{3}$.
5. 5.
6. $\frac{4}{10}$.
7. $1\frac{1}{4}$.

8. $1\frac{3}{4}$.
9. 8.
10. 17.
11. $2\frac{1}{2}$.
12. 5.
13. $\frac{1}{10}$.
14. $\frac{1}{10}$.
15. $\frac{1}{10}$.
16. $\frac{1}{10}$.
17. $\frac{1}{10}$.
18. $\frac{1}{10}$.
19. $\frac{1}{10}$.

Exercise 138.

1. 150.
2. 176.
3. 428.
4. 405.
5. 1296.
6. 600.
7. 1440.
8. 413.
9. 273.
10. 1001.

Exercise 140.

1. \$45.
2. $\frac{1}{5}$.
3. 625 sheep.
4. \$1890.
5. \$18450.
6. \$8610.
7. $\frac{5}{9}$.
8. $\$44\frac{1}{2}$.
9. $\$56\frac{2}{3}$.
10. \$15360.
11. \$3840.
12. $\frac{1}{3}$.
13. $\frac{1}{8}$.
14. 240 A.
15. 2000 sheep.
16. \$3.
17. $\frac{7}{11}$.
18. \$525.
19. $\frac{2}{3}$.
20. 14300.

Exercise 142.

1. 6 yd.
2. $29\frac{1}{4}$ A., \$1176 $\frac{3}{4}$.
3. \$31 $\frac{1}{4}$.
4. 3.
5. \$3948 $\frac{3}{4}$.
6. 16 lb.
7. \$3150.
8. 61 coats.

9. \$2.12 $\frac{1}{24}$.
10. 10 lb.
11. 76 A.
12. $\frac{3}{8}$.
13. \$29 $\frac{1}{2}$.
14. 19 rd.
15. 1700 yd.
16. \$490 $\frac{3}{10}$.
17. 8 lots.
18. $12\frac{1}{2}$ yd.
19. $20\frac{1}{4}$ ct.
20. $18\frac{1}{10}$ T.
21. \$1 $\frac{1}{5}$.
22. $\frac{1}{3}$.
23. 171 cattle.
24. { \$1302 A.
\$1953 B.
\$1519 C.
25. { 10 suits.
2 vests.
26. $6\frac{2}{7}$ da.
27. 11 cents.
28. 720 hogs.
29. \$246 $\frac{2}{3}$.
30. $\$90\frac{4}{5}$.
31. $\frac{1}{3}$.
32. \$72 $\frac{1}{2}$.
33. A, by $1\frac{1}{2}$ mi.
{ All 8 da.
17 $\frac{1}{2}$ A, 24 B,
40 C.
35. \$80.
36. $\frac{3}{8}$.
37. $\frac{2}{3}$ yd.
38. 52 sheep.
39. 4 children.
40. \$76.
41. \$15.
42. \$140.
43. { \$432 A.
\$1296 B.
44. \$170 $\frac{1}{4}$.
45. 122.
46. \$78 on all.
47. $85\frac{9}{18}$.
48. { \$630 A.
\$1200 B.
\$945 C.
49. 85 ct.
50. { \$3080 A.
\$5100 B.
\$7140 C.
51. \$144 $\frac{3}{4}$.
52. \$403 $\frac{3}{4}$.
53. 800 cents.
54. { $13\frac{7}{8}$ sum.
 $\frac{2}{7}$ dif.
 $\frac{1}{10}$ prod.
55. 56 lb.

56. \$3.
 57. \$200.
 58. \$145 $\frac{1}{4}$.
 59. $\frac{1}{2}$.
 60. { \$100 A.
 \$200 B.
 61. 1692 $\frac{1}{8}$ yd.
 62. 94 cwt.
 63. 4 $\frac{7}{10}$ T.
 64. 8706 hr.
 65. \$90 $\frac{1}{4}$.
 66. { \$14.25 rec'd.
 \$1.50.
 67. { 50 mi. to S. J.
 483 " to L. A.
 68. { \$2.05 1st.
 \$6.15 2d.
 \$4.10 3d.
 69. 93 $\frac{3}{4}$ mi.
 70. 1 $\frac{3}{4}$ mi.
 71. 11 bbl.
 72. 6 collars.
 73. \$99 $\frac{7}{10}$.
 74. \$16.
 75. \$3.50.
 76. \$3 $\frac{1}{3}$.
 77. { 2 $\frac{5}{8}$ mi. R.
 3 mi. W.
 78. \$228 $\frac{3}{4}$.
 79. 9 boxes.
 80. { \$16 $\frac{1}{5}$ Fred.
 \$8 Frank.

Exercise 154.

1. 1'03.52207.
 2. 9212.12079.
 3. 23347.257307.
 4. 7703.1697.
 5. 1603.178.
 6. 34407.
 7. 4489.9514.
 8. 12515.2574.
 9. 300.244187.
 10. 67.28521.
 11. 688.1695.
 12. 27231.6401.
 13. 52.872.
 14. 85.05435.
 15. 4041.1615.
 16. 11.62.
 17. 83.668.
 18. 320.39.
 19. 927.155.
 20. 32.1.
 21. 12.71.
 22. 9.6585.

Exercise 155.

1. 75.015.

2. 772.0686.
 3. 857.03592.
 4. 857.13709.
 5. 452.13.
 6. 778.36.
 7. 12.9296.
 8. 164.3405.
 9. 27134.6793
 10. 81.0972.

Exercise 158.

1. 5646.0196.
 2. 9.3925.
 3. 9553.18334.
 4. 5.89849.
 5. 526.50598.
 6. 9.8786558.
 7. 104437.086.
 8. .683774.
 9. 9.3925.
 10. .015325.
 11. 16.391375.
 12. .0098125.
 13. .875875.
 14. .01643375.
 15. 173.7375.
 16. .0011375.
 17. 112397.62534.
 18. 11155.4977534.
 19. 9.71118448.
 20. 38159121.
 21. 53109.36631.
 22. 10330.50018.
 23. 267.5593924.
 24. 1024657.893262
 25. 67.28549.
 26. 6.6781049.
 27. .00581528.
 28. .000228435.
 29. 31.793285.
 30. 6.18423.
 31. .1601714.
 32. 613.399157.
 33. 49.5443949.
 34. 140.1421021.
 35. 1201.0782784.
 36. 190182.24225.
 37. 33'8.2649.
 38. 3686.480798.
 39. 491239749.
 40. .987987.
 41. .929584929.
 42. 2.629439441.
 43. 22.535430464.
 44. 3568.3258725.
 45. 63.197629.
 46. 69.16820758.
 47. .0092169.3729.

48. .01853727.
 49. 114120.93327.
 50. 1403.93799.
 51. 4378283.6829.
 52. 5684788.293.
 53. 407281.42407.
 54. 65521.2759.
 55. 41198.0259.
 56. 14037.99.
 57. .74717643.
 58. .00919191.
 59. 28.6656461.
 60. 37.219637.
 61. 2.6665663.
 62. .4289831.
 63. .2697331.
 64. .09191.

Exercise 160.

1. 43.5.
 2. 34.
 3. .98.
 4. .5184.
 5. .512.
 6. 42.66 $\frac{2}{3}$.
 19. 456.
 20. 414.

Exercise 162.

1. 3757.
 2. 6.25.
 3. 6556.55.
 4. 3.925.
 5. 350.35.
 6. 6.5735.
 7. 69495.
 8. .455.
 9. 9022.5263+.
 10. 895.4884+.
 11. .779+.
 12. .0306+.
 13. 4263.263+.
 14. 829.263+.
 15. 21.477+.
 16. 82252.6526+.
 17. .02162+.
 18. .0611+.
 19. .5241+.
 20. 83.0022+.
 21. 1.47+.
 22. 1.6089+.
 23. .000214+.
 24. .00041+.
 25. 81.294+.
 26. 1+.
 27. 3118.882+.
 28. 4049.574+.

29. 290.128+.
 30. 46.6742+.
 31. 29.3475+.
 32. 10.

Exercise 168.

1. 151.3575 bales.
 2. 299.25 lb.
 3. \$3339.
 4. .25.
 5. \$4.50.
 6. 81.12 A.
 7. 12 books.
 8. 74.25 mi.
 9. 11110.50 $\frac{1}{2}$ cu.
 in.
 10. \$407.
 11. 16.5 rd.
 12. 187.46 mi.
 13. 31.24 $\frac{1}{2}$ mi.
 14. 1415.425 A.
 15. 16 pr.
 16. 7276.5 cu. in.
 17. 121 rd.
 18. 240.43+ turns.
 19. \$17.50.
 20. 21 cd.
 21. 30.76 A.
 22. \$1538.
 23. \$100.
 24. .21 $\frac{2}{3}$.
 25. 63 cd.
 26. \$477.
 27. 64 times.
 28. 336.6 rails.
 29. 22.4482 in.
 30. 127.5 gal.
 31. \$164.40.
 32. 127.4 mi.
 33. 11 hr.
 34. 150.7968 sec.
 35. 25132.8 mi.

Exercise 172.

1. \$25.12 $\frac{1}{2}$.
 2. \$26.49.
 3. \$4.87 $\frac{1}{4}$.
 4. \$18.
 5. \$59.50.
 6. \$100.
 7. \$7.55.
 8. \$6.05.
 9. \$6.28 $\frac{5}{8}$.

Exercise 174.

30. { 15887 ft.
 5295.6 $\frac{2}{3}$ yd.

31. { 110 in.
3.05 $\frac{2}{3}$ yd.
32. { 5315 ft.
1.0066+ mi.
33. { 768 in.
64 ft.
34. { 462 in.
38.5 ft.
35. { 63810 in.
1772.5 yd.
36. { 17160 ft.
3.25 mi.
37. { 10596 ft.
2.0038+ mi.
38. { 693 in.
3.5 rd.
39. { 8360 yd.
4.75 mi.
40. { 15983 $\frac{1}{4}$ ft.
5327.75 yd.
41. 1485 in.
43. $\frac{2}{3}$ mi.
44. $83\frac{1}{3}$ mi.
45. 120 rd.
46. .1553 + rd.
47. .2047 + mi.
48. 723 —.
49. .1324 + mi.
50. 40 $\frac{1}{2}$ + rd.
51. $\frac{63}{128}$ mi.
52. $\frac{2187}{4224}$ mi.
53. $\frac{19}{54}$ mi.
54. $\frac{1921}{15840}$ mi.

Exercise 177.

9. 5 yd. 2 ft. 2.3 in.
10. 38 yd. 2 ft. 2.8 in.
11. 3.59 + mi.
12. 199566.11 + me.

Exercise 178.

18. { 39204 sq. in.
43500 sq. ft.
17. 98027 sq. ft.
20. 130380 sq. ft.
21. 929331 $\frac{1}{4}$ sq. yd.
22. 24 $\frac{5}{16}$ sq. ft.
23. 685 sq. ft. 166 sq. in.
24. 28 sq. rd. 25 sq. yd. 8 sq. ft.
25. 10 A. 13 sq. yd.
27. \$13457.46 +.
27. $\frac{7}{8}$.

Exercise 179.

10. 64000000 sq. l.
12. 12839 sq. ch.
14. 64110625 sq. l.
15. 8 A. 4 sq. ch. 4 sq. rd. 90 sq. l.
16. 253 A. 7 sq. ch. 6 sq. rd. 146 sq. l.
17. 617 A. 2 sq. ch. 2 sq. rd.
18. 1 sq. mi. 345 A. 7 sq. ch.
19. 470 A. 8 sq. ch.
20. $\frac{3}{4}$.
21. { $\frac{1}{2}$ A. lost.
 $\frac{2}{3}$ cultivat'd
8 ch.
22. { 6.89 A.

Exercise 180.

3. 43217.68 sq. m.
5. 6477.732 ares.

Exercise 181.

1. 39 $\frac{2}{3}$ yd.
2. Each 46 $\frac{2}{3}$ yd.
3. { 25 $\frac{2}{3}$ yd. cross
25 yd. length
4. \$64.58 $\frac{1}{2}$.
5. \$103.12 $\frac{1}{2}$.
6. \$32.74 $\frac{1}{2}$.
7. 1200 $\frac{1}{4}$ sq. yd.
8. 1447 sq. yd.
9. 110 $\frac{2}{3}$ sq. yd.
10. \$34.75.
11. \$9.50.
12. \$30.25.
13. \$8.19.
14. 10.8 M.
15. 180 bricks.
16. 2240 sq. ft.
17. \$43.66 $\frac{2}{3}$.
18. 60 tiles.

Exercise 183.

1. { 2940 cu. ft.
108 $\frac{8}{9}$ cu. yd.
2. 15 $\frac{3}{4}$ cd.
12. 6 cu. ft.
16. 625536 cu. in.
17. 426829 cu. in.
18. 3 cu. yd. 11 cu. ft. 752 cu. in.
19. 29408 cu. in.
20. 44 cu. yd.

21. 1440 cu. in.
22. 4725 lb.
23. 762048 cu. in.
24. 10 $\frac{3}{5}$ ft.
25. 12 cu. yd. 19 cu. ft.
26. 11 $\frac{3}{8}$ cd.
27. \$25.41.
28. 9 $\frac{1}{2}$ cu. yd.
29. 2930 $\frac{2}{3}$ cu. yd.
30. \$3005.86.
31. .75 cu. yd.
32. .125 cu. ft.
33. $\frac{1}{2}$ cu. yd.
34. 34992 cu. in.
35. 26 cu. ft. 561.6 cu. in.
36. 48 cu. ft.

Exercise 184.

3. 7.866 cd.
4. 6.75648 cd.

Exercise 185.

1. \$511.64.
2. 57309 bricks.
3. \$832.61.
4. 240 perches.
5. 1280 perches.
6. 87018 $\frac{3}{4}$ bricks.
7. { 18 $\frac{3}{4}$ ft.
\$14.40.
9. 162 $\frac{2}{3}$ ft.
10. \$16.68 $\frac{1}{3}$.
11. \$15.12.
12. 540 ft.
13. \$62.98.
14. 8 ft.
15. 240 ft.
16. 280 ft.
17. 000 ft.
18. 3 0 ft.
19. 384 ft.
20. 13 $\frac{7}{8}$ ft.
21. 712 $\frac{1}{2}$ ft.

Exercise 186.

13. 257 pt.
14. 636 qt.
15. 252 pt.
16. 12 $\frac{1}{2}$ pt.
17. 19 pt
18. 66 qt.
19. 255 pt.
20. 62 bbl. 11 gal.
21. 37 bbl. 29 gal. 3 qt. 1 pt.

22. 75 bbl. 23 gal. 1 qt.
23. 313 bbl. 13 gal. 1 qt. 1 pt.
24. 214 bbl. 13 gal.
25. 128 bottles.
26. \$27.05.
27. \$59.05.
28. \$40.80.
29. { 3000 cans.
\$525.00.
30. $\frac{11}{84}$ bbl.
31. 23 gal. 2 qt. 1 pt.
32. $\frac{5}{8}$.
33. $\frac{11}{16}$.
34. .0238+ bbl.

Exercise 187.

4. 5605.6 li. or kg.
5. { 1481.28 gal.
12332.32 lb.

Exercise 189.

39. $\frac{7}{16}$ lb.
40. .2025 lb.
41. 19 $\frac{1}{2}$ pwt.
43. .375 lb.
44. $\frac{643}{6400}$ T.
45. .15025 T.
46. 8 cents 33 lb. 5 $\frac{1}{8}$ oz.
47. 1 cental 50 lb.
48. $\frac{28}{5}$.
49. $\frac{13}{660}$.
50. .875 T.

Exercise 190.

- { 1 lb. 1 oz. 18 pwt. 15.912 gr. Troy.
3. { 15 oz. 125.412 gr. Av.
4. 74625.837 li.
5. 119.2 li.
6. 150937.5 li.

Exercise 191.

9. 106526".
10. 2 32' 9" 25".
11. 16° 52' 30".
12. $\frac{1}{22500}$ quadr't.
13. $\frac{1}{85}$.
14. .98 circumfer.
15. $\frac{3}{8}$.
16. .3763+.

17. $\frac{1}{2}$.
18. 7' 30".

Exercise 192.

10. 13 hr. 20 min.
10 sec.
11. 7 min. 45 sec.
12. \$312.
13. 18925 sec.
14. 10 7052 min.
15. 97863853 sec.
16. 1 yr. 317 da.
11 hr. 49
min. 39 sec.
17. 33 da. 2 hr. 35
min.
18. 4 da. 22 hr. 42
min. 9 sec.
19. 109 da. 9 hr.
40 min.
20. 99 da. 3 hr. 59
min. 43 sec.
21. 530 min.
22. 13 hr. 42 min.
19 sec.
23. 31 da. 16 hr.
25 min.
24. 202 hr.
25. 304 da. 4 hr.
26. $\frac{67}{10}$ da.
27. $\frac{1}{2}$.
28. $\left\{ \begin{array}{l} 1 \text{ hr. } 48 \text{ min.} \\ 4 \text{ da. } 9 \text{ hr.} \\ 137 \text{ da. } 23 \text{ hr.} \\ 16 \text{ min. } 48 \\ \text{sec.} \end{array} \right.$
29. $\frac{11}{3}$ week.
30. 211 da. 16 hr.
48 min.
31. 125.
32. 355 da. 21 hr.
33. 45 da. 15 hr.
34. $\frac{3}{4}$ mo.
35. $\frac{803}{210000}$ mo.
36. $\frac{11}{16}$ week.
37. $\frac{7}{32}$ da.
38. 4 yr. 239 da.
1 hr. 48 min.
39. 446 + week.
40. 333 +.
41. .0125.

Exercise 193.

8. 36° 16' 30".
9. 21° 4'.
10. 48° 37'.
11. 78° 18'.

12. 62° 3'.
13. 102° 41' 30".
14. 131° 26' 45".
15. 1 hr. 57 min.
17 sec.
16. 1 hr. 4 min.
19 $\frac{1}{5}$ sec.
17. 3 hr. 57 min.
56 $\frac{1}{75}$ sec.
18. 3 hr. 13 min.
36 $\frac{1}{5}$ sec.
19. 7 hr. 51 min.
42 sec.
20. 35 min. 181 $\frac{1}{5}$
sec.
21. 29 min. 32 $\frac{1}{5}$ se.
22. 11 hr. 19 min.
48 $\frac{2}{5}$ sec.
23. 8 hr. 18 min.
58 $\frac{7}{15}$ sec. P.M.
24. 6 hr. 30 min.
9 $\frac{3}{5}$ sec. A. M.
next da.
25. 1 hr. 51 min.
50 $\frac{8}{15}$ sec. A.M.
next da.
26. $\left\{ \begin{array}{l} 168^\circ 56' \text{ W.} \\ 62^\circ 41' \text{ W.} \\ 81^\circ 26' \text{ W.} \\ 159^\circ 49' \text{ E.} \end{array} \right.$
27. 46° 28' E.
28. 152° 20' 22" E.
29. 108° 56' 30" E.
30. 133° 54' 38" W.
31. 84° 41' W.
32. 3° 4' W.
33. New York.
34. 72° 35' 7 $\frac{1}{2}$ " W.
35. 13 min. 36 $\frac{1}{5}$
sec. past 6
A. M. same
da.
36. 64° 45' 3" W.
37. 60° 57' W.

Exercise 194.

1. 1 mi. 125 rd. 3
yd. 2 ft.
2. 22 yd. 2 ft. 10
in.
3. 19 rd. 1 yd. 2
ft. 2 in.
4. 74 mi. 96 rd.
2 yd. 3 in.
5. 80 rd. 4 yd. 3
in.
6. 46 mi. 252 rd.
2 yd. 8 in.

7. 103 mi. 41 rd.
5 yd. 3 in.
8. 3 yd. 1 ft. 4 in.
9. 3 yd. 1 ft.
10. 240 rd. 3 yd. 1
ft. 6 in.
11. 3 mi. 60 ch. 1
rd. 23 l.
12. 109 A. 154 sq.
rd. 3 sq. yd.
6 sq. ft. 72
sq. in.
13. 80 sq. rd. 5 sq.
ft.
14. 96 sq. rd. 14
sq. yd. 1 sq.
ft.
15. 19 cd. 3 cd. ft.
13 cu. ft.
16. 281 cu. yd.
22 $\frac{7}{10}$ cu. ft.
17. $\left\{ \begin{array}{l} 49 \text{ gal. } 1 \text{ qt.} \\ \$19.70. \end{array} \right.$
18. $\left\{ \begin{array}{l} 211\frac{1}{2} \text{ bu.} \\ \$846. \end{array} \right.$
19. $\left\{ \begin{array}{l} 83\frac{1}{2} \text{ boxes.} \\ \$10.44. \end{array} \right.$
20. \$19.51 $\frac{1}{4}$.
21. 5 gross 11 doz.
3.
22. 6353 sheets.
23. $\left\{ \begin{array}{l} £23 \text{ 16s.} \\ \$115.67. \end{array} \right.$
24. \$3.17 $\frac{1}{2}$.
25. 60 T. 7 cwt.
44 lb.
26. 241 T. 6 cwt.
51 lb. 4 oz.
27. 11 lb. 9 oz. 18
pwt.

Exercise 195.

1. 47 A. 89 sq. rd.
2. 3 mi. 234 rd.
5 yd. 6 in.
3. 12 A. 7.26 $\frac{3}{4}$ sq.
ch.
4. 19 cu. yd. 14
cu. ft. 1464
cu. in.
5. 2 gal. 2 qt. 1 pt.
6. $\left\{ \begin{array}{l} \$14.44\frac{1}{2}. \\ 8 \text{ gal. } 1 \text{ qt.} \end{array} \right.$
7. 1 cwt. 59 lb.
10 oz.
8. 3.25 Troy oz.
9. 93 lb. 15 oz.

10. 2 oz. 7 pwt. 6
gr.
11. 1 lb. 2 oz. 6 dr.
13 gr.
12. 2 yrs. 2 mo. 1
wk. 4 da. 4
hr.
13. 2 mo. 1 wk. 2
da. 15 hr. 40
min. 31 sec.
14. 10 da.
15. 1 wk. 6 da. 5
hr. 17 min.
16.8 sec.
16. \$322.18.
17. 193 rd. 5 $\frac{1}{2}$ yd.
18. 345600 sec.
19. 20 yr. 1 mo. 1
wk. 1 da. 4
hr. 35 min.
37 $\frac{1}{5}$ sec.
20. 6 T. 7 cwt.
18 $\frac{9}{16}$ lb.
21. 4 $\frac{3}{4}$ sq. yd.
22. £9 2s. 8d. 3
far.
23. 21° 36' 40".
24. 5 lb. 3 oz. 10
pwt.
25. 10s. 7 $\frac{1}{2}$ d.
26. 2070 sq. ft.

Exercise 196.

1. 45 mi. 258 rd.
2 yd. 2 ft. 3
in.
2. 6 mi. 79 ch. 3
rd. 11 l.
3. 10 A. 148 sq.
rd. 4 sq. yd.
2 sq. ft. 6
sq. in.
4. 1275 cd. 46 cu.
ft. 1214 cu.
in.
5. 136 bbl. 22 gal.
3 qt.
6. 63 lb. 2 oz. 4
pwt.
7. 1027 T. 12 cen-
tals 56 lb. 14
oz.
8. 608 yr. 9 mo.
1 wk. 2 da.
10 hr. 14
min. 24 sec.
9. 70 rd. 4 yd. 4
in.

10. \$623.51.
 11. 273 mi. 105 rd.
 1 yd. 1 ft. 6 in.
 12. { 18 bbl. 26 gal.
 2 qt.
 \$1187.
 13. The wine, \$992
 14. 56 A. 111 sq. rd. 5 sq. yd. 7 sq. ft. 3³/₈ sq. in.
 15. 4 oz. 9 pwt. 6 gr.

Exercise 197.

1. 1 mi. 8 rd. 4 yd. 7⁵/₈ in.
 2. 11 ch. 1 rd. 20²/₃ l.
 3. 18 sq. rd. 16 sq. yd. 8 sq. ft. 141¹/₈ sq. in.
 4. 3 cd. 59 cu. ft. 145¹¹/₈ cu. in.
 5. 1 bbl. 8 gal. 1³/₁₅ pt.
 6. 3 lb. 9 oz. 11 pwt. 23¹/₅ gr.
 7. 91 lb. 9³/₅ oz.
 8. 1 yr. 5 mo. 2 wk. 3 da. 11 hr. 59 min. 14²/₁₇ sec.
 9. 70¹⁰/₁₇ lb.
 10. 6 lb. 4¹⁷/₅ oz.
 11. 371 A. 6 sq. ch. 2 sq. rd. 40³/₇ sq. l.
 12. 3 rd. 2 yd. 2 ft. 31⁹/₂₁ in.
 13. 162 cu. ft. 432 cu. in.
 14. 46 cups.
 15. 352 rails.

Exercise 198.

1. 8 mi. 203 rd. 4 yd. 7 in.
 2. 1202 cu. yd. 18 cu. ft.
 2. *Metric*, 1201 cu. yd.
 3. 570 cwt. 63¹/₂ lb.
 4. 56⁸/₁₇ mi.
 4. *Metric*, 90396.97 meters.

5. 5 cu. ft. 700 cu. in.
 5. *Metric*, cu. me. 152686385.
 6. 120²/₃ sq. ft.
 7. 4014489600 sq. in.
 7. *Metric*, hekt. 259.1093.
 8. 2³/₁₅ cd.
 8. *Metric*, steres, 7.846544.
 9. 6 rd.
 10. 73 spoons.
 11. 6 yr. 10 mo. 1 wk. 3 da. 12 hr. 24 min. 18 sec.

12. \$135.
 13. \$400.95.
 14. \$2.88¹/₂.
 15. .125 mi.
 16. 225.28 centals.
 17. \$386.29.
 17. *Metric*, same.
 18. 27231³/₁₀ mi.
 18. *Metric*, met. 587420.78.
 19. 1 lb. 3 oz. 8 pwt. 21 gr.
 20. \$574.42.
 20. *Metric*, \$573.96.
 21. 2680 cu. yd. 20 cu. ft.
 22. 6 qt. 1¹/₂ pt.
 23. 90.75 cu. ft.
 24. 48²/₅ lots.
 25. \$22.96.
 25. *Metric*, \$7.17¹/₂.
 26. 1 da. 1 hr. 43 min. 30 sec.
 27. 158 A. 31 sq. rd.
 28. \$18.28.
 29. 1120 times.
 30. 22 sq. yd. 4 sq. ft. 82 sq. in.

31. 56 yd.
 32. \$21.40.
 33. 278¹/₃ cu. ft.
 34. 138 sq. rd. 27 sq. yd. 2 sq. ft. 99 sq. in.
 35. 7 hr. 57 min. 18 sec. A. M.
 36. 11 hr. 37 min. 20 sec. A. M.
 37. 375.

38. 200 rd. 6 ft. 4 in.
 38. *Metric*, 1007.76 meters.
 39. 3³/₄ ft.
 40. \$10.41.
 41. 60 yd.
 41. *Metric*, 53.99 meters.
 42. 23⁵/₈ yd.
 43. \$1015.
 44. \$.91.
 45. { \$209.04.
 28²/₅ yd.
 46. ⁵/₆ yd.
 47. 57 rd. 9 ft. 10³/₈ in.

48. ³/₅ cd.
 49. \$12.25.
 50. ¹/₅.
 51. { Lots 45x145 ft.
 Gain \$1300.

Exercise 199.

22. 256 lb.
 23. 39 books.
 24. \$54.62¹/₂.
 25. 64 lb.
 26. \$10.50.
 27. \$81.
 28. 152 lb.
 29. \$732.
 30. \$1230.60.
 31. 45⁵/₇ yd.
 32. 14 chickens.
 33. \$141.
 34. 512 bags.
 35. 28 tons.
 36. \$601.
 37. \$22.50.
 38. \$63.
 39. 43 lb.
 40. 20 sheep.
 41. \$765.
 42. \$403.
 43. 114⁷/₉ rolls.
 44. \$664.
 45. \$605.
 46. \$1859.26.
 47. \$6283.75.
 48. \$591.36.
 49. \$18345.55.
 50. \$307.90.
 51. 76 cwt.
 52. \$188.58.
 53. \$592.54.
 54. 1333¹/₃ lb.

55. \$27.67.
 56. \$248²/₂₅.
 57. \$15.52.
 58. \$124.
 59. \$250.
 60. 50 eagles.
 61. 200 half dollars.
 62. 100 quarter dollars.
 63. 500 dimes.
 64. 297.6 half dollars.
 65. 368¹/₃ pieces.

Exercise 200.

1. \$75.
 2. 7 days.
 3. \$189.
 4. 517 mi.
 5. 105 T.
 6. 27 A.
 7. 40 chairs.
 8. \$25.
 9. 9 men.
 10. 24 men.
 11. \$142.80.
 12. \$12.50.
 13. 11²/₃ da.
 14. \$891.
 15. 22 cows.
 16. 7¹/₈ yd.
 17. \$562.50.
 18. \$360.
 19. \$72.
 20. 90 horses.
 21. 18 da.
 22. 4000 lb.
 23. 20¹/₄ yd.
 24. \$20.
 25. 24 da.
 26. 9 brooms.
 27. 5 men.
 28. 1 da.
 29. \$56.
 30. \$3.15.
 31. 10 ft. 2¹/₂ in.
 32. 10 ft.
 33. 14 da.
 34. 32 gal.
 35. \$238.
 36. \$148.
 37. 87¹/₂ doz.
 38. 239 lb.
 39. 3000 sacks.
 40. 84 weeks.
 41. 21 sacks.

Exercise 201.

1. { A. \$1150.
B. \$690.
2. { A. \$245.
B. \$392.
3. { 1st \$44.
2d \$52.
4. { 1st \$4000.
2d \$2000.
A. \$1612.
B. \$2015.
C. \$2418.
6. \$45, \$60, \$90.
A. \$66 $\frac{2}{3}$.
B. \$1000.
7. { C. \$1200.
D. \$1333 $\frac{1}{3}$.
E. \$1800.
8. 83 $\frac{1}{3}$ ct., \$1100.
A. 54 T.
B. 75 T.
C. 120 T.
10. { A. \$131.
B. \$393.
C. \$262.
11. { A. \$1800.
B. \$600.
C. \$1200.
12. { 1st \$357 $\frac{1}{2}$.
2d \$642 $\frac{1}{2}$.
13. { \$171.60.
\$257.40.
432.
14. { 576.
720.
A. \$990.
B. \$750.
C. \$1350.

Exercise 209.

1. \$2500.
2. \$432.
3. \$37.50.
4. 20%.
5. \$66.
6. \$4500.
7. \$1728.
8. $\frac{1}{5}$ %.
9. \$4555.
10. \$6.
11. 9 $\frac{1}{5}$ %.
12. \$52.
13. \$673.30.
14. \$450.
15. \$1091.70.
16. 12 $\frac{1}{2}$ %.
17. \$10.

18. \$27.
19. 16 $\frac{2}{3}$ %.
20. \$625.

Exercise 212.

1. \$99.
\$250 board.
\$125 cloth-
ing.
\$450 inci-
dentals.
2. {
3. \$300.
4. $\frac{3}{5}$; 15%.
5. $\frac{1}{2}$; 12 $\frac{1}{2}$ %.
6. \$5.
7. \$40.
8. 14 $\frac{2}{3}$ %.
9. \$40000.
10. \$900.

Exercise 214.

1. \$25.
2. \$9.90.
3. 12 $\frac{1}{2}$ %.
4. \$4.50.
5. { \$96 loss.
\$3744 S. P.
6. 20%.
7. 33 $\frac{1}{3}$ %.
8. \$1.20.
9. \$1.50.
10. 33 $\frac{1}{3}$ %.
11. 12%.
12. \$1.37 $\frac{1}{2}$.
13. { \$49 C.
\$42 S. P.
14. 20 ct.
15. 8 $\frac{1}{2}$ ct.
16. 11%.
17. 80 ct.
18. 11 $\frac{1}{10}$ % loss.
19. 24 $\frac{1}{3}$ % gain.
20. 2 $\frac{1}{9}$ % loss.
21. 90 ct., \$1.14,
\$1.32.
22. 33 $\frac{1}{3}$ %.
23. 10 ct.
24. \$15.13.
25. \$37.
26. \$82.50.
27. { \$14.40.
6 $\frac{1}{4}$ % loss.
28. { \$1.94 $\frac{1}{2}$.
16 $\frac{2}{3}$ %.
29. \$1.50.
30. 58 $\frac{1}{3}$ %.

Exercise 216.

1. { \$112.50 loss.
\$1387.50 S. P.
2. { \$20000 cost.
\$20500 S. P.
3. { \$13 $\frac{1}{3}$ loss.
\$1333 $\frac{1}{3}$ cost.
4. { 3 $\frac{3}{5}$ % loss.
\$1925 S. P.
5. { \$85.25 gain.
\$1170.25 S. P.
6. 28 $\frac{8}{10}$ % gain.
7. { \$2100 cost.
\$2247 S. P.
8. { \$2502 cost.
\$417 loss.
9. 20% loss.
10. { \$1.23 $\frac{1}{3}$ loss.
\$19.73 $\frac{1}{3}$ cost.
11. { \$1350 cost.
\$1395 S. P.
12. \$1690 S. P.
13. \$750 cost.
14. \$654 $\frac{6}{11}$ cost.
15. { \$138 loss.
\$782 S. P.
16. { \$6300 cost.
\$5460 S. P.
17. 6 $\frac{1}{5}$ % gain.
18. 14 $\frac{2}{3}$ % gain.
19. { \$980 gain.
\$7980 S. P.
20. { \$51 cost.
\$76.50 S. P.
21. 1 $\frac{2}{3}$ % rate.
22. { \$3 loss.
\$12. S. P.
23. \$18 cost.
24. \$1000 cost.

Exercise 218.

1. { \$21 com.
\$379 net.
2. 10 $\frac{2}{3}$ %.
3. \$536.50.
4. { \$3000 value.
\$120 com.
5. \$4.99 $\frac{1}{5}$ (\$5.00.)
6. 30 ct.
7. \$315.
8. 300 bales.
9. { \$2905.
9 $\frac{1}{4}$ ¢. per lb.
10. 800 vol.
11. \$2920.
12. { \$80 com.
\$3420 net.
13. 2 $\frac{1}{2}$ %.

14. 3 $\frac{1}{3}$ %.
15. \$4100.
16. 14 $\frac{2}{3}$ %.
17. { \$262.50 com.
\$3937.50 val.
18. \$2100.
19. \$1.59 $\frac{3}{8}$.
20. 79500 lb.
21. { 9000 lb.
81 $\frac{1}{2}$ ct. per lb.
22. 1000 lb.
23. \$432.
24. { \$506.25.
23750 lb.
25. 2%.

Exercise 220.

1. { 2 $\frac{1}{2}$ % rate.
\$6435 pro.
2. \$8000 cost.
3. { \$5463.41 C.
\$136.59 com.
4. { \$900 S. P.
\$886.50 pro.
5. { \$174.64 com.
\$8557.36 pro.
6. 2 $\frac{1}{5}$ % rate.
7. 3% rate.
8. 6% rate.
9. 6 $\frac{2}{3}$ % rate.
10. { \$2140 cost.
\$42.80 com.
11. 1 $\frac{1}{5}$ % rate.
12. \$1756 S. P.
13. 2.8% rate.
14. $\frac{1}{5}$ % rate.
15. \$1188 cost.
16. \$5 com.

Exercise 222.

1. { \$44 prem.
\$5456 loss.
2. \$4000.
3. $\frac{3}{4}$ %.
4. \$1980.
5. \$3750.
6. 2%.
7. { \$100 prem.
\$7500 val.
8. 1 $\frac{3}{4}$ %.
9. \$6756.
10. \$6.34.
11. 1 $\frac{1}{4}$ %.
12. First.
13. \$2184.
14. \$58.32.
15. 1 $\frac{3}{8}$ %.

16. \$3168.
17. \$500.
18. \$5.25.
19. \$2116.80.
20. $11\frac{1}{5}\%$.

Exercise 225.

1. $\frac{1}{2}\%$.
2. \$30.50.
3. \$2900.
4. \$40.50.
5. 45 ct.
6. \$500000.
7. \$1000000.
8. $\frac{1}{3}$ mills.
9. $\frac{1}{5}\%$.
10. \$7.
11. $1\frac{1}{2}\%$.
12. \$86.
13. \$873.
14. \$12.86 $\frac{2}{3}$.
15. \$600000.
16. 12% .

Exercise 227.

1. \$43.
2. \$375.
3. \$221.76.
4. \$4448.30.
5. \$813.50.
6. \$3806.
7. \$1084.50.
8. \$3.
9. \$750.
10. \$320.
11. \$54.
12. \$18.80.
13. \$18.

Exercise 228.

2. \$1022.50.
3. $3\frac{3}{4}\frac{2}{3}\%$.
4. $\left\{ \begin{array}{l} 80 \text{ sh.} \\ \$8169. \end{array} \right.$
5. $\left\{ \begin{array}{l} 80 \text{ sh.} \\ \$4800. \end{array} \right.$
6. $4\frac{16}{8}\frac{7}{2}\%$.
7. F. N. $\frac{1}{10} + \%$.
8. $\left\{ \begin{array}{l} 160 \text{ sh.} \\ \$13120 \text{ cost.} \end{array} \right.$
9. Nearly $21\frac{1}{3}\%$.
10. Each 5% .
11. 16 sh.
12. \$376.87 $\frac{1}{2}$.
13. 11% .
14. \$251.25.

15. 40 sh.
16. $\left\{ \begin{array}{l} 95 \text{ sh.} \\ \$6317.50. \end{array} \right.$
17. Or. Nav.
18. 12 sh.
19. 10 sh.

Exercise 231.

1. $\left\{ \begin{array}{l} \$11.91. \\ \$191.41. \end{array} \right.$
2. $\left\{ \begin{array}{l} \$57.20. \\ \$382.20. \end{array} \right.$
3. $\left\{ \begin{array}{l} \$146.19. \\ \$904.94. \end{array} \right.$
4. $\left\{ \begin{array}{l} \$142.03. \\ \$1166.28. \end{array} \right.$
5. $\left\{ \begin{array}{l} \$7.21. \\ \$591.71. \end{array} \right.$
6. $\left\{ \begin{array}{l} \$55.77. \\ \$781.61. \end{array} \right.$
7. $\left\{ \begin{array}{l} \$83.02. \\ \$470.97. \end{array} \right.$
8. $\left\{ \begin{array}{l} \$17. \\ \$42.37. \end{array} \right.$

Exercise 232.

1. \$6; \$7.50; \$9.
2. \$43.54; \$60.95; \$69.66.
3. \$57.46.
4. \$94.07.
5. \$109.69.
6. \$59.48.
7. \$83.77.
8. \$11.23.
9. \$14.65.
10. \$16.92.
11. \$180.24.
12. \$154.

Exercise 238.

1. 4 yr. 6 mo.
2. $\left\{ \begin{array}{l} 1 \text{ yr. 4 mo.} \\ 1 \text{ yr. 2 mo.} \end{array} \right.$
3. \$500.
4. 8% .
5. $6\frac{2}{3}\%$.
6. \$240.
7. \$679.61.
8. $1\frac{1}{2}$ mo.
9. \$392.16.
10. \$11.36.
11. Offers equal.
12. 1 yr. 6 mo. 20 da.
13. \$705.59.

14. \$147.06.
15. $12\frac{5}{17}\%$.
16. 6% .
17. \$432.
18. \$20000.
19. \$760.
20. $1\frac{1}{2}$ mo.
21. 3 yr. 3 mo. 18 da.
22. 6% .
23. \$218.75.
24. \$6250.
25. \$484.
26. Jan. 2, 1882.
27. 3 yr. 1 mo. 6 da.
28. \$885.75.
29. \$1.92.
30. 6% .
31. \$750.
32. 3 yr. 1 mo. 3 da.
33. 92 ct.
34. \$780.
35. 8% .
36. \$1970.50.

Exercise 240.

1. \$577.38.
2. \$99.88.
3. \$576.67.
4. \$285.99.
5. \$603.49.
6. \$200.
7. \$1386.78.
8. \$325.08.
9. \$209.45.
10. \$228.95.

Exercise 242.

1. \$262.48.
2. \$39.71.
3. \$15.80.
4. \$13.75.
5. \$70.23.
6. \$26.60.
7. \$73.33.
8. \$12.56.
9. \$116.86.
10. \$112.58.

Exercise 243.

1. \$690.67.
2. \$840.93.
3. \$1426.88.
4. \$908.46.
5. \$247.50.

6. \$1835.82.
7. \$519.44.
8. \$297.83.
9. \$1141.73.
10. \$1350.56.
11. \$738.31.

Exercise 244.

1. \$256.50.
2. $28\frac{3}{4}\%$.
3. \$3.75 more.
4. \$1000.
5. \$190.
6. \$593.11.
7. \$80.
8. \$363.80.
9. \$703.25.
10. \$4.45.
11. \$2.40.
12. \$571.20.

Exercise 245.

1. \$5008.33.
2. \$577.10.
4. \$1565.52.
6. \$4500.
7. \$736.56.
8. $\frac{3}{4}\%$.
9. \$769.04.
10. $\frac{1}{2}\%$ prem.
11. \$186.
12. 875 francs.

Exercise 248.

1. 7 mo. 5 da.
2. 3 mo. 17 da.
3. $\left\{ \begin{array}{l} 4 \text{ mo. 14 da.} \\ \text{Aug. 22.} \end{array} \right.$
4. 18 da.
5. June 23.
6. 5 mo.
7. \$330.
8. 20 mo.
9. June 14, 1886.
10. Oct. 15, 1884.

Exercise 249.

1. \$3.10.
2. $94\frac{2}{3}$ ct.
3. $9\frac{9}{10}$ ct.
4. $9\frac{1}{4}\%$.
5. $20\frac{1}{4}$ ct.
6. 10 ct.
7. 45 ct. gain.

Exercise 253.

1. 32 rd.

2. 58 rows.
3. 1280 rd.
4. 99 ft.
5. 80x40 rd.
6. 64 rd.
7. 8.54 rd.
8. 101.2 rd.
9. A's, \$90.
10. \$420.
11. 64 in. sq.

Exercise 256.

1. 21 in.
2. 61.3+ in.
3. 8 cu. ft.
4. 67.64 gal.
5. 1176 sq. in.
6. 3.17 ft.
7. 13824 cu. in.
8. 1.6 met.
9. 79.875 cu. met.

Exercise 257.

1. 14.14 ft.
2. 13.23 ft.
3. 17.35 ft.
4. 50 ft.
5. 8.54 ft.
6. 22.8 ft.
7. 20.95 ft.
8. 64.62 ft.
9. 241.4 ft.
10. 33.8 ft.
11. 51.42 ft.
12. 83.67 ft.
13. 56.57 rd.
14. 20 rd.
15. 15.59 in.
16. { 10.6 rd.
112.5 sq. rd.
17. 45.08 ft.
18. 92.45 mi.
19. 3 ft. .93 in.
20. 20.59 ft.

Exercise 258.

1. 180 sq. rd.
2. 390 sq. rd.
3. { 69.12 rd.
380.13 sq. rd.
4. 4.55 ft.
5. 18.46 sq. rd.
6. 7 sq. ft. +
7. 10.4 sq. yd.
8. 15 rd.
9. 1293 sq. ft.
10. 42 ft. 8 in.
11. 13½ sq. ft.
12. 47½ in.
13. 30 ft.; 20 ft.
14. 78.54 sq. ft.
15. \$89.68.
16. \$1518.75.
17. \$7208.85.
18. 93 sq. ft.
19. 420+ times.
20. { 100.399 rd.
dia.
251.6 rd. cir.

Exercise 259.

1. 24 cu. ft.
2. 52 sq. ft.
3. 3.27¼ sq. ft.
4. 179.07 sq. in.
5. 3.91 qt.
6. 795.87½ cu. in.
7. 2.53 sq. ft.
8. 8.17 in.
9. 3.1416 sq. ft.
10. 73½+ times.
11. 1 cd. 98½ cu. ft.
12. 6.14 bu.
13. 1.85 sq. yd.
14. 18.16 in.
15. 1.47 gal.
16. .58 qt.
17. 11.55 in.
18. 68.80 sq. ft.
19. 54400 cu. ft.
20. 21.38 cu. ft.

21. { 201032400 sq.
mi.
268083200000
cu. mi.
22. \$1275.12.
23. \$337.92.
24. 3+ cu. ft.

Exercise 261.

1. \$220.50.
2. \$3.60.
3. \$3720.47.
4. { \$139.71 com.
56312 lb.
5. \$20.
6. { 104.48 me-
ters wide.
313.4+ me-
ters long.
7. 86½.
8. \$17.65.
9. 27108.31 ft.
10. 4.83 ares.
11. \$42240.
12. 124⅔%.
13. 109% nearly.
14. 188.32 meters.
15. \$1.
16. { \$6035.
\$5793.60.
17. \$600.
18. 15 men.
19. 30½ sq. rd.
20. 174+ meters.
21. 259+hektares
22. \$4001.40.
23. 502.25.
24. .2169+.
25. { 210 A.
330 B.
26. { 41.62 sq.met.
25.25 cu.met.
27. 56 da.
28. 64 A.; 72 B.
29. { 23396 liters.
23396 kilo-
grams.

30. { 60 A. 144 sq.
rd. A.
77 A. 96 sq.
rd. B.
103 A. 74⅔
sq. rd. C.
206 A. 149½
sq. rd. D.
31. 3 316 ft.
32. 194630.4 kilo-
grams.
33. 93.26 hhd.
34. { 4s. 1.38d.
5.376 fr.
35. 4.6 meters.
36. \$1031.85.
37. { \$600.
8⅓%.
38. 187.2 steres.
39. { \$10.31.
\$42.69.
40. { \$1600 A.
\$2000 B.
\$1800 C.
41. 546⅔ ft.
42. \$5.25.
43. 198 lb.
44. 18.29 sq. me-
ters.
36.
3, 7, 24.
22, 33, 17.
46. \$3750.
47. 84.
48. 51 cd. 77 cu. ft.
49. \$54.
50. ⅝.
51. \$556.10.
52. 83957.8 kilo-
grams.
53. \$761.42.
54. \$538.85.
55. 5⅕ min. past
1 o'clock.
56. 4⅘ min. be-
fore 1 o'clock.
57. \$1199.08.

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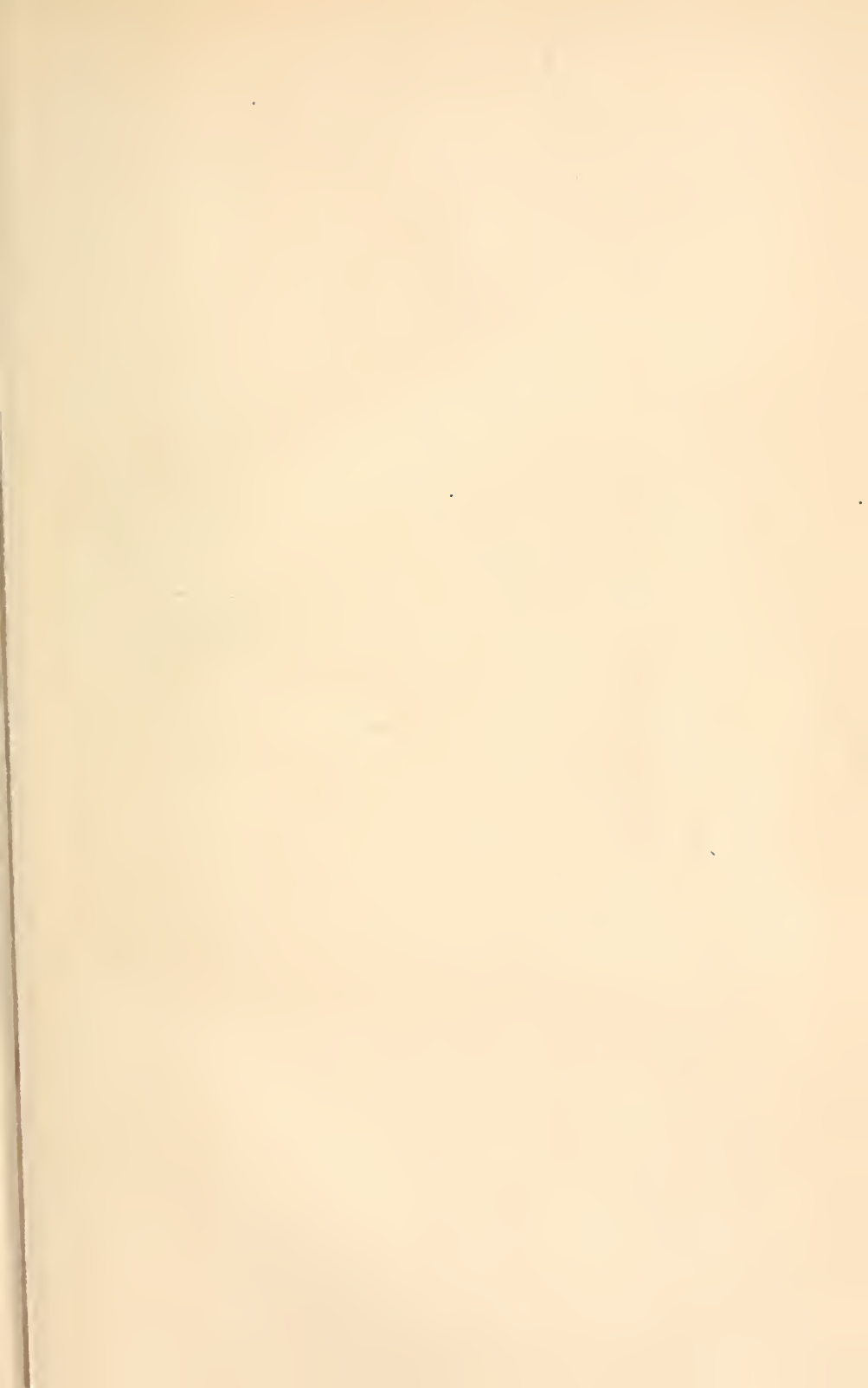
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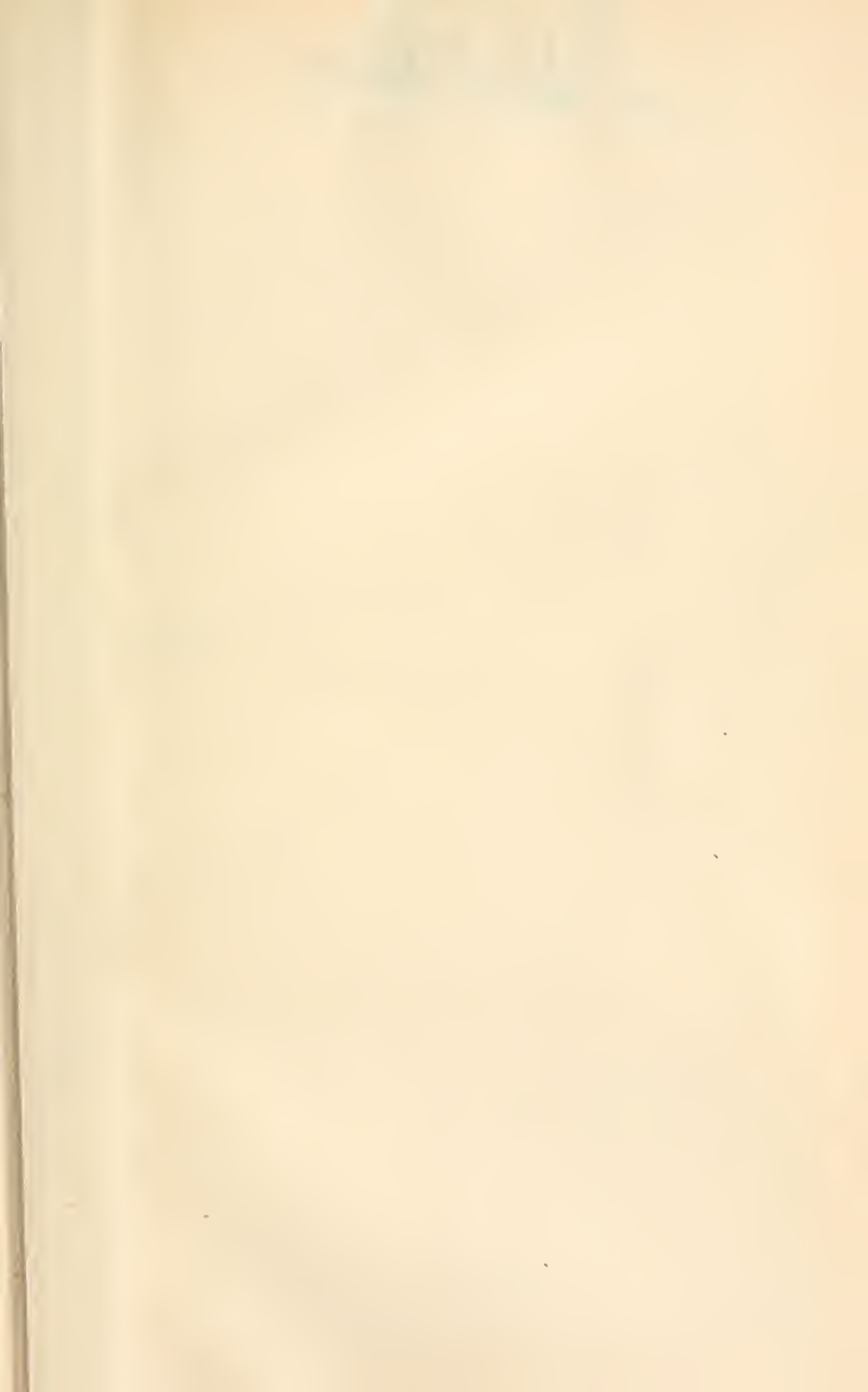
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