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## The Aging Phenomenon and Insurance Prices

### Abstract

Insurers typically earn greater profits on policies that have been with the insurer for a number of renewal cycles than on newer business. This tendency is known as the aging phenomenon and is believed to occur on all lines of business. Although the aging phenomenon is common knowledge, no mathematical methods for incorporating this phenomenon into pricing decisions have been documented. This paper sets forth a procedure for determining the maximum acceptable loss ratio on new business that will be profitable for the insurer over its entire renewal cycle by incorporating a discounted cash flow analysis of future profits. The advantages of measuring profitability by cohorts of business; depending on when the policy was originally written, are also demonstrated.



## Section 1 - Introduction

A well known but little documented tendency of property-liability insurance contracts is for the loss ratio on mature business, the book of policies that has been with the insurer for a number of renewal cycles, to exhibit constant improvement. The cause of this tendency, termed the aging phenomenon or seasoning of business, has been addressed by Kunreuther and Pauly [5] and D'Arcy and Doherty [4] and theorized to be the result of the accumulation of private information by the contracting insurer. This information allows the insurer to classify the policyholders properly as valid information about the risk is collected, as opposed to the initial information included in the application and obtained in the initial underwriting screening. Such information could include a verified loss history, as the insurer knows about claims that occur during the coverage period, the condition of the insured property and the degree of cooperation demonstrated by the insured in settling any claims. This insurer also is able to renew policies selectively to weed out the least desirable risks. The remaining policyholders represent a continually improving book of business as more high-risk insureds are properly classified and appropriately charged and the culling process continues to cancel policyholders whose risk profile is higher than the indicated rate level would reflect. For example, a private passenger automobile insured with one at-fault accident may have proven to be such an uncooperative defense witness that the insurer is unwilling to renew the policy even at the classification rate for one accident. As the contracting insurer has an advantage in access to this information,

competition does not work to reduce the premium level on this desirable business in proportion to the improvement in loss experience.

The aging phenomenon is believed to occur for all lines of property-liability insurance, although little published information confirms this belief. Eight insurers have provided the authors with confidential information demonstrating this effect, subject to the condition that they not be identified, and many other insurers have confirmed that the trend also occurs on their business as well. The disparity of record keeping techniques and internal procedures among insurers makes exact measurement of the extent of aging impossible at this point. However, the widespread confirmation of this trend and its importance in pricing and marketing strategy calls for an analysis of the effect of aging on insurance pricing.

The purpose of this paper is to incorporate the aging phenomenon into a pricing model. The initial model is based on fairly simple assumptions in order to demonstrate the effect of aging on prices clearly and to derive numerical results. The assumptions are later altered to reflect more realistic conditions in additional models. Hopefully, individuals with access to their company's databases will be encouraged to generate additional tests of these models.

## Section 2 - Notation

The following notation will be used in the initial model:

P = premium level per policy

E = expenses per policy other than loss adjustment expenses

ER = expense ratio (E/P)

L = losses and loss adjustment expenses (LAE) per policy discounted to correspond to the receipt of premium

LR = loss and loss adjustment expense ratio (discounted) (L/P)

A = aging factor (rate of improvement in losses and loss adjustment expenses per policy as the book of business ages)

W = renewal rate (percentage of current period's policies renewed in the subsequent period)

F = profit per policy on business originally written in the first period

I = risk adjusted interest rate used to discount profits earned in each period

j = subscript to indicate the age of the book of business

In the first period, the insurer writes a book of new business and on that book would earn a profit on each policy of:

$$F_1 = P_1 - E_1 - L_1 \quad (1)$$

This profit is not the traditional underwriting profit, because investment income is reflected by the use of discounted losses and loss adjustment expenses. Also, it is not the traditional operating profit, as the investment income that is reflected is not the amount earned in the current period, as the operating profit represents, but the future investment income that can be attributed to the time lag between the receipt of premium and the payment of claims. This profit can be viewed as a composite profit that reflects both underwriting experience and the time value of money. Determining the proper discount rate to apply to the losses will be discussed more fully in Section 4.

In the second period, the insurer would write some new business and some renewal business, but this study will concentrate on the renewal business only since the purpose of this paper is to examine the profitability of one cohort of business as it ages for the insurer. In the second period, the business originally written in the first period would generate a profit of:

$$F_2 = W_1(P_2 - E_2 - L_2) \quad (2)$$

The present value (as of the beginning of the first period) of the second period's profit is:

$$PV(F_2) = W_1(P_2 - E_2 - L_2)/(1 + I) \quad (3)$$

The present value of the third period's profit is, similarly:

$$PV(F_3) = (W_1)(W_2)(P_3 - E_3 - L_3)/(1 + I)^2 \quad (4)$$

The profits and present value of profits can continue to be calculated in this manner until no business is left to renew. Theoretically, this could continue infinitely, although for any personal lines coverage the mortality of the insured would place an upper limit on the number of renewal periods.

### Section 3 - Model 1

In the first model, certain simplifying assumptions are made. First, the premium level per policy in each period is the same ( $P = P_j$  for  $j = 1, n$ ). The insurer does not raise the premium level over time and also does not provide discounts to long-term insureds. Second,

the expenses are constant over time ( $E = E_j$  for  $j = 1, n$ ). The cost of writing new business is the same as renewal business. Next, the proportion of policies renewed each year is the same ( $W = W_j$  for  $j = 1, n$ ). Finally, the improvement in the losses and loss adjustment expenses per policy is constant for each renewal period ( $A = L_{j+1}/L_j$ , for  $j = 1, n-1$ ).

Under these simplifying assumptions, the present value of the profit stream becomes:

$$PV(F) = \sum_{j=1}^{\infty} [(W^{j-1})(P - E - A^{j-1}(L))]/(1 + I)^{j-1} \quad (5)$$

This equation indicates that the insurer is concerned with achieving an adequate profit on a cohort of business over time. New business, although it may not be profitable to the insurer initially, must produce an adequate profit, considering its first term and future renewal cycles with the insurer, in order to justify the insurer's writing it at all. As the losses per policy decline each renewal period, while premiums and expenses are constant, the profitability of each policy renewed increases. However, not all policies are renewed. Some are nonrenewed by the insurer and others at the initiative of the insured. Regardless of cause, fewer policies are renewed each period. Also, as these profits will not be earned until subsequent periods, an appropriate discount factor must be applied to determine the present value of these future profits. Since the premium level is assumed to be a constant, it can be factored out of the equation, leading to a present value of the profit stream per dollar of first period premium or:

$$(PV(F))/P = \sum_{j=1}^{\infty} [(W^{j-1})(1 - ER - A^{j-1}(LR))]/(1 + I)^{j-1} \quad (6)$$

As each policy becomes more profitable in subsequent renewals, two factors act to reduce the impact of these profits on the present value of profitability. First, not all policies are renewed, so the increasingly profitable business is gradually reducing in size. Secondly, the profits are earned in the future and therefore must be discounted to the present value. Thus, the renewal factor,  $W$ , and the interest rate used to discount future cash flows,  $I$ , are included in the profitability analysis.

#### Section 4 - Maximum Initial Loss Ratio

The aging phenomenon encourages insurers to write new business at a loss in order to gain the opportunity to earn future profits on this book of business as it subsequently renews. Competition for new business requires this initial loss ratio to be unprofitable, but the acceptable level of unprofitability on new business is often difficult to determine. In this section the maximum initial discounted loss ratio, termed  $LR^{MAX}$ , is calculated. The term loss ratio will be used for convenience, but this is meant to include loss adjustment expenses.

In this paper, losses and loss adjustment expenses are assumed to be discounted at the appropriate rate of interest back to the time when the premium is written. This adjustment is necessary in order to reflect the time value of money. A number of different approaches have been utilized in practice to account for investment income in insurance pricing. The different approaches are discussed in Cummins

and Harrington [1], D'Arcy [2], D'Arcy and Doherty [3], and Webb [6]. The techniques proposed include the Capital Asset Pricing Model, the Arbitrage Pricing Model, the Option Pricing Model and a Discounted Cash Flow Model. Measuring the appropriate interest rate to use in discounting cash flows based on each of these models has proven to be quite difficult based on available data.

One interest rate that has been proposed to discount cash flows is the one to twelve month U.S. Treasury bill rate, which is termed the risk free interest rate. The expected loss and LAE payout pattern can be discounted based on this rate to determine the actual initial loss ratio in comparison with the maximum loss ratios determined in this paper. One other advantage of using discounted loss ratios is the comparability across coverages and lines. The same discounted loss ratio will apply to fast settling lines such as collision and comprehensive as well as long-tailed lines such as liability, as the investment income component is directly reflected in the discounted loss ratio.

This discounted loss ratio,  $LR^{MAX}$  is the level at which the present value of all profits on the cohort of business over its entire renewal cycle is zero. New business written at this discounted loss ratio will generate future profits that will, in present value terms, only offset the initial losses on the cohort of business. Any higher initial discounted loss ratio will generate losses for the insurer. Any lower initial discounted loss ratio will generate profits, in total. Thus,  $LR^{MAX}$  is the upper limit of the discounted loss ratio for new business.

Setting the left-hand side of equation (6) equal to zero and rearranging terms leads to:

$$LR^{MAX} \sum_{j=1}^{\infty} (W^{j-1})(A^{j-1})/(1+I)^{j-1} = (1-ER) \sum_{j=1}^{\infty} (W^{j-1})/(1+I)^{j-1} \quad (7)$$

Each of the terms in the infinite summations,  $WA/(1+I)$  and  $W/(1+I)$ , is between zero and one, since both  $W$  and  $A$  are greater than zero but less than or equal to one and  $I$  is greater than zero. Therefore, equation (7) can be expressed as:

$$LR^{MAX} [1/(1-(WA/(1+I)))] = (1-ER) [1/(1-(W/(1+I)))] \quad (8)$$

or  $LR^{MAX} = [(1+I-WA)/(1+I)][1-ER][(1+I)/(1+I-W)] \quad (9)$

or  $LR^{MAX} = [(1+I-WA)/(1+I-W)][1-ER] \quad (10)$

To illustrate the mathematics of equation (10), the following values will be used:

A (Aging Factor) = 90%

W (Renewal Rate) = 90%

I (Interest Rate) = 10%

ER (Expense Ratio) = 30%

$$LR^{MAX} = [.29/.20][.70] = 1.015 \text{ or } 101.5\%$$

This calculation indicates that if the insurer writes new business at a discounted loss ratio of 101.5%, the initial losses on the business will eventually be exactly offset, in present value terms, by future profits as the policies renew at progressively lower loss ratios. Any higher initial discounted loss ratio will never be, in

total, profitable. Lower initial discounted loss ratios will produce a positive profit, although the adequacy of any particular profit level has not been determined. What is now known is that the insurer should definitely not write new business if the initial discounted loss ratio is in excess of 101.5%.

The first twenty-five years of experience on a cohort of \$1,000,000 of new business is illustrated in Table 1. In the first year of the life of this cohort of business, the insurer incurs a loss of \$315,000 ( $\$1,000,000(1-ER-LR)$ ). In the second year, 90 percent of the initial book of business is renewed, generating a premium volume of \$900,000. The loss ratio improves to 91.4% ( $.9(101.5)$ ), dropping the combined ratio to 121.4%. The composite loss is \$192,600, but the present value of this loss is only \$175,090 ( $192,600/1.1$ ). In subsequent years the premium volume continues to decline, as only 90 percent of the business is renewed each year. The loss ratio also declines with each renewal. In the fifth year the cohort generates a composite profit, but the cumulative value of the composite experience is still negative. By the twenty-fifth year of the cohort, the composite experience is a positive \$1,182,000 (sum of column 5). However, the present value of the composite experience is still a negative \$24,000, as the profits occurring in the later years are discounted over a longer period than the losses of the early years. However, continuing the illustration to infinity would generate a sum of present values that would equal zero, by construction.

Section 5 - Derivatives

The effect on  $LR^{MAX}$  of changes in the parameters, A, W, I and ER, can be determined by taking the partial derivatives of  $LR^{MAX}$  in equation (9) with respect to each value. Equation (9) is used to determine the derivatives rather than equation (10) to simplify the illustration of the effect adding a new business expense factor to the model. (See Section 8.) The derivative with respect to the renewal rate, W, is:

$$\partial LR^{MAX} / \partial W = (1+I)(1+(ER)(A)-A-ER) / (1+I-W)^2 \quad (11)$$

As each of the terms in parentheses is positive, the partial derivative is positive. Thus, an increase in the renewal rate, W, allows the insurer to write at a higher initial discounted loss ratio. Note that in obtaining this derivative, the aging factor is assumed to be independent of the renewal rate. If a higher renewal rate is obtained at the cost of increasing the aging factor, then the relationship between the renewal rate and  $LR^{MAX}$  is not clear cut.

The partial derivative of  $LR^{MAX}$  with respect to the expense ratio, ER, is:

$$\partial LR^{MAX} / \partial ER = -(1+I-WA) / (1+I-W) \quad (12)$$

As each of the terms in parentheses is positive, the partial derivative is negative. An increase in the expense ratio requires the initial discounted loss ratio to be lower.

The partial derivative of  $LR^{MAX}$  with respect to the interest rate used to discount cash flows, I, is:

$$\partial LR^{MAX} / \partial I = -(W)(1+(ER)(A)-A-ER)/(1+I-W)^2 \quad (13)$$

Again, the terms in parentheses are all positive, so the partial derivative is negative. A higher interest rate lowers the maximum initial discounted loss ratio. This implies that if interest rates were to increase, then the loss ratio on new business should be lowered. However, the loss ratio used in this model is itself discounted, and a higher interest rate would produce a lower discounted loss ratio from the same payout stream. Thus, it is difficult to ascertain the effect of a change in interest rates on conventional, nondiscounted loss ratios. However, the effect on discounted loss ratios is unequivocal. For a coverage that is settled quickly, such as comprehensive or collision, a change in the interest rate used to discount the loss payout pattern would have little effect. In contrast, changing the interest rate for determining present values of composite profits would be significantly affected. For such coverages, the initial loss ratios should decline with increases in interest rates, as future profits will have a smaller impact in offsetting initial losses. This finding contradicts most other studies on the effect of investment income on loss ratios, and is based on viewing profitability on a cohort basis instead of in aggregate.

For example, consider the situation in which there is no lag between the receipt of premium and the payment of claims so the discounted loss ratio is equal to the actual loss ratio and, thus, unaffected by interest rates. Short term policies with a lag in collecting premiums (perhaps from an agent or broker) and in which

insurers can pay losses as soon as they occur (such as automobile collision or comprehensive) may approach such a situation. Paid loss retrospective coverage would also have this behavior. As illustrated in Section 4, for the selected values of the variables, the maximum loss ratio at which the insurer should write new business is 101.5 percent. If interest rates were to increase from 10 percent to 12 percent, then the maximum loss ratio drops to 98.6 percent  $[(1+.12+(.9)(.9))/(1+.12-.9)][1-.3]$ . Since the actual and discounted loss ratios are the same, the insurer has to raise premiums when interest rates rise. This occurs because the future profits on this cohort are discounted at a higher interest rate and, thus, have a reduced impact in offsetting the initial losses incurred on the cohort.

The partial derivative of  $LR^{MAX}$  with respect to the aging factor, A, is:

$$\partial LR^{MAX} / \partial A = -(W)(1-ER)/(1+I-W) \quad (14)$$

This value is also negative as the terms in parentheses are each positive. An increase in the aging factor, that is as it moves closer toward one, reduces the maximum initial discounted loss ratio.

#### Section 6 - Use of $LR^{MAX}$

Once  $LR^{MAX}$  is determined, the insurer sets a premium level that maximizes the profitability of the cohort of business over its lifetime with the insurer. Since  $LR^{MAX}$  indicates the highest initial loss ratio that can be obtained for an insurer to achieve the minimum

acceptable rate of return over the life of the cohort, then the initial premium level must be set so the initial loss ratio is less than or equal to  $LR^{MAX}$ . The premium level that optimizes this long run profitability depends on the elasticity of demand in this region of premium levels.

Elasticity of demand is the relationship between the price level and the quantity of policies sold. Unitary elasticity is defined as the point at which a marginal price increase is exactly offset by an equal decrease in the quantity sold, so that the total revenue remains constant. For example, at an elasticity of one, a 10 percent premium level increase reduces the quantity of policies sold by 9.1 percent, so that the total premiums written do not increase. The insurer collects the same premium income, but with fewer policies each paying a higher premium per policy. The elasticity of demand of greater than one is when an increase in the premium level per policy reduces the quantity of policies sold to a greater extent than the premium increase, so that total revenue declines. Conversely, inelastic demand is the range where the elasticity of demand is less than one, so a premium level increase reduces the quantity of policies sold by a lesser amount, and therefore the total revenue rises.

If the elasticity of demand is greater than one at  $LR^{MAX}$ , then the insurer will maximize profits by charging a premium level that is equal to  $1/LR^{MAX}$ . This premium level produces a zero profit (based on the definition of profit explained in Section 2), but the insurer does achieve a risk adjusted rate of return on the business written. This return results from the use of the risk adjusted interest rate to

discount the cash flows from the cohort of business. Any higher premium level decreases total revenue by more than the reduction in losses that occurs from writing a smaller book of business.

If the elasticity of demand is less than one at the premium level that produces an initial loss ratio of  $LR^{MAX}$ , then the insurer maximizes profits by raising the price level until unitary elasticity is reached. Unfortunately, this is difficult to determine in practice, because the elasticity of demand function is not known, but must be estimated. This elasticity is likely to vary by insurer and over time. In fact, the existence of positive profits, those in excess of the minimum risk adjusted required rate of return, would most likely encourage other insurers to compete for this business. New entry would continue to be encouraged until these profits are eliminated. The fear of competition for profitable business is one reason that insurers keep data of the aging phenomenon confidential. Thus, the optimal premium level, the one that maximizes long run profits, cannot be determined exactly. However, the upper limit of the initial loss ratio,  $LR^{MAX}$ , can be determined fairly accurately as this depends only on the expense ratio, renewal ratio, aging factor and the interest rate.

#### Section 7 - Fixed Planning Horizon

The examples above assume the insurer is maximizing the present value of profits derived from a given cohort of business for the current year and all future years. This infinite time horizon, while theoretically valid, may not be acceptable in practice. Insurers may

prefer to determine a premium level that achieves a profit, or at least avoids a loss, over a set period of time. Equations (5) through (10) can be rewritten to deal with a fixed planning horizon (indicated by the subscript n) as follows:

$$PV_n(F) = \sum_{j=1}^n [(W^{j-1})(P - E - A^{j-1}(L))]/(1 + I)^{j-1} \quad (15)$$

$$(PV_n(F))/P = \sum_{j=1}^n [(W^{j-1})(1 - ER - A^{j-1}(LR))]/(1 + I)^{j-1} \quad (16)$$

$$LR_n^{MAX} \sum_{j=1}^n (W^{j-1})(A^{j-1})/(1+I)^{j-1} = (1-ER) \sum_{j=1}^n (W^{j-1})/(1+I)^{j-1} \quad (17)$$

Given a fixed time horizon, this equation reduces to:

$$LR_n^{MAX} [(1-(WA/(1+I))^n)/(1-WA/(1+I))] \\ = (1-ER) [(1-(W/(1+I))^n)/(1-W/(1+I))] \quad (18)$$

or

$$LR_n^{MAX} = [(1+I-WA)/((1+I)^n - (WA)^n)] \\ \cdot [((1+I)^n - W^n)/(1+I-W)] [1-ER] \quad (19)$$

The example calculated earlier is shown in Table 2 for a ten year horizon. The maximum initial loss ratio declines from 101.5 percent for an infinite horizon to 92.2 percent for a ten year horizon.

After ten years the sum of the present value of composite profits is equal to zero. This cohort of business will continue to generate profits in subsequent renewals, but these were ignored in setting

$LR_n^{MAX}$ .

Section 8 - Model 2

The second model is similar to the first, with constant interest rates, renewal rates and aging factors, but the expense ratio is higher on new business than on renewal. This would be the case where new business requires a one time additional expense incurred when the new business is written. The expenses on renewals are all the same. The additional new business expenses per policy will be denoted as X, and the expenses as a percentage of premium denoted as XR. Including this additional new business expense factor revises equation (5) as indicated below:

$$PV(F) = \sum_{j=1}^{\infty} \{[(W^{j-1})(P - E - A^{j-1}(L))]/(1 + I)^{j-1}\} - X \quad (20)$$

The value of X is not discounted because it is incurred when the policies are written, not at a future date. Similarly, equation (6) becomes:

$$(PV(F))/P = \sum_{j=1}^{\infty} \{[(W^{j-1})(1 - ER - A^{j-1}(LR))]/(1 + I)^{j-1}\} - XR \quad (21)$$

The calculation of  $LR^{MAX}$  indicated in equation (9) is revised to:

$$LR^{MAX} = [(1+I-WA)/(1+I)]\{[1-ER][(1+I)/(1+I-W)]-XR\} \quad (22)$$

The example illustrated in Section 4, revised to include an additional new business expense ratio, XR, of 30%, so that total expenses in the first year for the cohort are 60% of premium, yields a value of  $LR^{MAX}$  of 93.6%. The inclusion of a one time new business expense, as would be expected, reduces the maximum initial discounted loss ratio from the previously determined 101.5%.

The derivatives of  $LR^{MAX}$  under Model 2 can be calculated similarly to those shown for Model 1. The partial derivative with respect to  $W$  remains positive; the partial derivatives with respect to  $ER$  and  $I$  remain negative. The partial derivative with respect to  $A$  becomes:

$$\partial LR^{MAX} / \partial A = -(W) \{ [(1-ER)/(1+I-W)] - [XR/(1+I)] \} \quad (23)$$

If the expression  $[XR/(1+I)]$  exceeded  $[(1-ER)/(1+I-W)]$ , then this derivative would be positive rather than negative. However, for all realistic values of the parameters, this derivative will remain negative.

Additionally, the partial derivative of  $LR^{MAX}$  with respect to  $XR$  can be determined. This value is:

$$\partial LR^{MAX} / \partial XR = -(1+I-WA)/(1+I) \quad (24)$$

This value is negative, as would be expected.

### Section 9 - Model 3

The third model allows for growth (inflation) of expenses and losses per policy, and premium level increases. Letting the value  $G$  stand for the growth factor,  $G_p$  is the growth rate for the premium level,  $G_e$  is the growth rate for expenses and  $G_l$  is the growth rate for losses and loss adjustment expenses. Then equation (5) becomes:

$$PV(F) = \sum_{j=1}^{\infty} [(W^{j-1})(P(1+G_p)^{j-1} - E(1+G_e)^{j-1} - A^{j-1}L(1+G_l)^{j-1})] / (1+I)^{j-1} \quad (25)$$

If the growth rates on premium, expenses and loss and loss adjustment expenses are all equal, in the case where the insurer constantly raises premium levels in line with the growth factor on losses and expenses, then the expense ratio and loss ratio will not be affected by inflation and this model will be similar to Model 1. The value of  $LR^{MAX}$  will not be the same, though, because the premium volume in subsequent years will reduce from the prior year based on the renewal rate,  $W$ , but increase from the prior year based on the inflation rate,  $G_p$ . As an example, if the growth rate in premiums, expenses and losses were a constant 5 percent, then a renewal rate of 90 percent with inflation would generate the same results as a renewal rate of 94.5 percent (.90 times 1.05) without inflation. However, as competing insurers will not be making the same rate level adjustments simultaneously, a higher inflation rate will reduce the renewal rate as policyholders shop for lower premium levels. Large premium increases, even if justified by increases in losses, will discourage some insureds from renewing.

If, at another extreme, the growth rate on premiums and expenses were zero, but the growth rate on losses and LAE were positive, then the addition of the growth factor would work to increase the loss ratio on renewal business, offsetting some or all of the decreasing trend caused by the aging factor. The far more likely situation would be for the three growth rates to be approximately, but not exactly equal. For example, if an insurer were evaluating the profitability of new business in a state that prevented rate increases from fully reflecting increases in loss costs, then this analysis could be .

performed setting  $G_p$  at a value below that of  $G_1$ , and solving for the value of  $LR^{MAX}$  to determine if the business should be written at the allowed rate level. Another situation in which  $G_p$  would be less than  $G_1$  is when the insurer grants discounts to long-term policyholders. Even if general rate level increases were obtained in line with the inflation rate on losses, the discounts would work to hold down the premium level adjustment.

#### Section 10 - Additional Issues

The aging phenomenon raises a number of additional issues not covered in detail in this paper. For example, the expense ratio is treated here as either a constant value over all years or one value for the first year of business with a lower value for all subsequent years. This latter pattern may approximate the expenses for direct writers that pay a straight commission that is lower on renewal business than on new business. Also, one time expenses associated with setting up policy files and computer records would be incurred when the new business is written. However, neither pattern adequately models the expense ratio when the insurer offers a contingent commission to agents that is a function of the loss ratio. In this case, the expenses of renewal business would increase proportionately with the decline in the loss ratio. The exact pattern of the expense ratio over time would depend on the insurer's contingent commission plans.

Another issue that arises from the aging phenomenon relates to the subsidy that occurs from profitable old business to unprofitable new

business. The aging phenomenon occurs most likely as the result of relevant information that is not known to the insurer when the policy is first written, but develops over the time while the insurer covers the risk. Thus, poor risks are given a rate that is actually too low to reflect their loss likelihood. However, due to the inability to differentiate accurately the level of risk when the new business is written, the insurer incurs a loss. However, this loss is later recouped by overcharging the business that has been with the insurer for a long period. Many insurers offer some form of discount for long-term policyholders, either to all or to those with no claims, in recognition of this improvement in experience. However, the discounts are not as large as the improvement in loss experience would warrant. Thus, long-term policyholders are subsidizing new business, particularly the poor risks that move from insurer to insurer as their true risk exposure is discovered.

The aging phenomenon also affects the behavior of insurers when the regulatory regime in a state begins to refuse adequate rate increases. Typically, insurers put up with this environment for a considerable period of time before withdrawing from the market. This apparent patience in the face of inadequate rates is actually reflective of the profit potential in the existing book of business. If the insurer withdraws from the market, future renewals on the existing book of business, which are likely to be quite profitable, are no longer possible. If the rate levels are not adequate to justify writing new business, they still may be acceptable for renewal business. Thus, typical reactions involve reducing or eliminating new

business, but continuing to service the existing book of business. Only when the losses on existing business are such that they are unlikely to be offset by future profitability on the book of business does it become economically justifiable to withdraw from the market.

In the models presented in this paper, the renewal rate and the aging factor were assumed to be constant over the life of the cohort. These values likely vary in practice, and this variation can easily be included in the models. The renewal rate is probably lowest on the first few renewals as those insureds least likely to renew, or be renewed, lapse. After a few renewal cycles a constant value, somewhat higher than the early renewal rates, may be achieved. However, for personal lines especially, the renewal rate is likely to decrease substantially after a point as the insured faces mortality risk. In personal lines insurance, the assumption that the insurer can potentially renew policies to infinity is violated.

The constant aging factor used in the current model results in a loss ratio that tends to zero as a book of business ages. Although this convergence would require more renewal cycles than actual policies would experience, this situation represents a potential problem with this model. One way around this continual decline in loss ratios is for the aging factor to increase to one after a certain number of renewals. In this case, no further improvement in the loss ratio would be expected. Little is known about the true behavior of the aging factor over time, as insurers justifiably treat this information as confidential. However, five of the eight insurers providing information on aging maintain data in a form that allows an

analysis of this factor over a short period of time. For each insurer the loss ratios (in some cases including loss adjustment expenses) were provided for a given period broken down by the age of the book of business. The aging factors were calculated from this information by dividing the loss ratio for each renewal cycle by the corresponding ratio for business one period younger. These values are shown in Table 3. For three of these insurers, Firms A, B and E, the aging factor is lower initially and then increases. One insurer, Firm D, has a fairly constant aging factor. The other insurer, Firm C, has an aging factor that gradually decreases. Thus, no consistent pattern emerges from this limited analysis. Hopefully, future research can address this issue. In the meantime, the models can easily be revised to reflect any pattern of aging factors.

Current insurance accounting conventions ignore the cohort concept of profitability and aggregate all business together. Thus, premiums, expenses and losses and LAE do not reflect the experience of an individual cohort of business, but the total company operations. If an insurer has written a constant premium volume for as long a period as policies could conceivably renew, then the total profitability would equal the present value of the profitability of any new business written. However, no such ideal insurer exists. For a typical insurer growing in exposure count, the aggregate profitability will be less than the present value of profitability of a cohort approach. The losses experienced on new business exceed the profitability on renewal business, as the current new business cohort is larger than

the previously written cohorts. Future profitability on current policies is not adequately reflected in aggregate profit statements.

Conversely, for an insurer that is reducing exposure counts, either in total or for a particular state, the aggregate method of accounting overstates the profitability of an individual cohort. The losses on a smaller volume of new business are overweighted by the profits on long-term business. Thus, an insurer could be misled to write additional new business that would actually be unprofitable in the long run due to the apparent profitability of the total book of business. Accounting for individual cohorts could avoid the distortions generated by aggregate accounting.

#### Section 11 - Summary

The aging phenomenon represents an additional dimension of the insurance equation that is often overlooked in pricing. Insurers should maintain records by policy cohort in order to determine optimal pricing levels. The use of aggregate statistics can mislead an insurer about the true profitability of a book of business. The effect of higher interest rates on acceptable loss ratios for new business is complex, since a higher rate lowers the discounted loss ratio while reducing the impact of future profitability. However, for short tailed lines, a higher interest rate clearly lowers the acceptable new business loss ratio if aging is considered. This result contrasts with the effect assumed if aging is ignored.

Much additional work remains to be done on aging and many issues must be solved before accurate determinations of acceptable new

business loss ratios can be made. The models developed in this paper represent simplifications of the actual aging phenomenon in order to perform initial tests of the effect of aging on insurance pricing. More information must be collected to determine if renewal and aging rates are constant, as assumed in these models, or change over time. Another key issue is the appropriate risk adjusted interest rate, both for the loss payout patterns of each year as well as for future years' profitability. Accurate statistics on aging must be compiled in order to facilitate more in depth research. Hopefully, this paper will inspire more insurers to collect this information and apply these techniques.

Table 1

Illustration of Cohort Experience by Age  
 Infinite Horizon  
 Initial Premiums Written: \$1,000,000  
 (000) omitted

Year	Premium Volume	Expense Ratio (%)	Discounted Expense Ratio (%)	Composite Profit/ (Loss)	Present Value	Cumulative Present Value
1	1000	30	101.5	(315)	(315)	(315)
2	900	30	91.4	(193)	(175)	(490)
3	810	30	82.2	(99)	(82)	(572)
4	729	30	74.0	(29)	(22)	(594)
5	656	30	66.6	22	15	(579)
6	590	30	59.9	60	37	(542)
7	531	30	53.9	85	48	(494)
8	478	30	48.5	103	53	(441)
9	430	30	43.7	113	53	(388)
10	387	30	39.3	119	50	(338)
11	349	30	35.4	121	47	(291)
12	314	30	31.9	120	42	(249)
13	282	30	28.7	116	37	(212)
14	254	30	25.8	112	33	(179)
15	229	30	23.2	107	28	(151)
16	206	30	20.9	101	24	(127)
17	185	30	18.8	95	21	(106)
18	167	30	16.9	89	18	(88)
19	150	30	15.2	82	15	(73)
20	135	30	13.7	76	12	(61)
21	122	30	12.3	70	10	(51)
22	109	30	11.1	64	9	(42)
23	99	30	10.0	59	7	(35)
24	89	30	9.0	54	6	(29)
25	80	30	8.1	50	5	(24)
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Table 2

Illustration of Cohort Experience by Age  
Ten Year Horizon

Initial Premiums Written: \$1,000,000  
(000) omitted

Year	Premium Volume	Expense Ratio (%)	Discounted Loss Ratio (%)	Composite Profit/ (Loss)	Present Value	Cumulative Present Value
1	1000	30	92.17	(222)	(222)	(222)
2	900	30	82.95	(117)	(106)	(328)
3	810	30	74.66	(38)	(31)	(359)
4	729	30	67.19	20	15	(344)
5	656	30	60.47	63	43	(301)
6	590	30	54.43	92	57	(244)
7	531	30	48.98	112	63	(181)
8	478	30	44.08	124	64	(117)
9	430	30	39.68	130	61	(56)
10	387	30	35.71	133	56	0
Total				297	0	

Table 3

Aging Factors by Renewal Cycle

Renewal Cycle	Firm A	Firm B*	Firm C	Firm D	Firm E
1	.90	.80	.89	.95	.86
2	.85	.99	.87	.95	.95
3	1.02	.99	.85	.94	-
4	.89	.91	.83	.96	-
5	.93	1.02	-	.95	-
6	.92	.94	-	.97	-
7	1.08	.96	-	.96	-
8	.95	.98	-	-	-
9	.93	1.06	-	-	-
10	1.01	-	-	-	-
11	.94	-	-	-	-

The aging factors are calculated by dividing the loss ratio (which may include loss adjustment expenses, depending on the company) for business of a given age by the same ratio for business one policy term previously. For example, the aging factor for the first renewal cycle is determined by dividing the loss ratio of policies that have been renewed one time by the new business loss ratio.

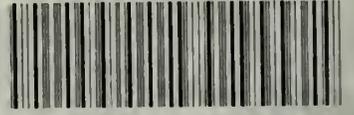
\*Firm B maintained data on six month renewal cycles. All the other firms maintained data only on an annual basis.

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