


UNIVERSITY OF
ILLINOIS LIBRARY
AT URBANA CHAMPAIGN
BOOKSTACKS



Digitized by the Internet Archive
in 2011 with funding from
University of Illinois Urbana-Champaign

<http://www.archive.org/details/airlineoverbooki370brue>

Faculty Working Papers

AIRLINE OVERBOOKING: FIRM BEHAVIOR
UNDER A PENALTY SCHEME

Jan K. Brueckner

#370

College of Commerce and Business Administration
University of Illinois at Urbana-Champaign



FACULTY WORKING PAPERS

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

January 20, 1977

AIRLINE OVERBOOKING: FIRM BEHAVIOR
UNDER A PENALTY SCHEME

Jan K. Brueckner

#370

Airline Overbooking: Firm Behavior
Under a Penalty Scheme

by
Jan K. Brueckner
Department of Economics
University of Illinois, Urbana-Champaign
Urbana, IL 61801

January, 1977

Abstract

This paper investigates firm responses to institution of an explicit penalty for bumping passengers from an overbooked airline flight. It is assumed that this change can be represented by an increase in the implicit fine that airlines are viewed as charging themselves for each bumped passenger under current practice. Monopoly and duopoly models are developed and analysed.

Airline Overbooking: Firm Behavior
Under a Penalty Scheme

by
Jan K. Brueckner*
University of Illinois, Urbana-Champaign

In order to achieve higher rates of capacity utilization, airlines routinely sell or reserve more seats on a given flight than are available. Since passengers holding reservations or tickets frequently do not show up to claim their seats, failure to overbook would result in more empty seats than necessary. Occasionally, however, the number of passengers who show up exceeds the number of seats available, requiring that some travelers be "bumped" from the flight. Under current practice, if the airline cannot arrange for the bumped passenger to arrive at his destination within two hours of his scheduled arrival (within four hours on international flights), the airline must pay up to \$200 of the ticket price.

Between January and September of 1976, Trans World Airlines overbooked 512,800 passengers and bumped 7,800.¹ If these figures can be taken as representative for the entire industry, they indicate that, while overbooking results in a small percentage of overbooked passengers being bumped, the absolute number is far from insignificant. In addition,

*Assistant Professor of Economics. I am indebted to Julian Simon for introducing me to this problem and for several helpful discussions.

¹This information, as well as that in the previous paragraph, was taken from a New York Times article entitled "Airlines to Pay for Overbooking?", which appeared November 7, 1976.

since airlines do not explicitly consider the welfare loss of the bumped passenger in making booking decisions, the level of overbooking is probably non-optimal from society's point of view.

Julian Simon (1968) has developed an auction scheme which would force the airlines to face the true social cost of bumping a passenger as well as identifying the "best" passengers to bump. Under this scheme, when too many people show up for a flight, each would be asked to write down the lowest dollar amount he would be willing to accept to wait for the next flight. The airline would then bump the people with the lowest bids and pay them the amount of their bids. Since people with low time valuations would enter low bids, for a given level of overbooking the airline would automatically minimize the value of passenger time lost in attempting to minimize cost. In addition, the airline would adjust the level of overbooking toward the socially optimal level.

The purpose of this paper is to analyse the airlines' responses to the institution of such a scheme. For simplicity, it is assumed that all individuals have the same time valuation. Three models are developed: the first is a one-flight model, where a monopolistic airline chooses the number of seats sold and the flight capacity, the second is a multi-flight monopoly model, where the choice variable is flight frequency, and the third is a duopoly model where the choice variables again are flight frequency.

The One-Flight Monopoly Model

Suppose there is a monopolistic airline operating a flight at a given hour and that when more passengers show up than can be accommodated,

the excess passengers are bumped. Since no other airline serves the route, the bumped passengers must be flown on the monopolist's subsequent flights. Similarly, people bumped from previous flights will be among the passengers for the given flight. In actuality, the demand for seats on any flight depends on the overbooking policy for all previous flights during the relevant period. While a correct model of the overbooking decision should incorporate this interdependence, the model developed here portrays the airline as ignoring the presence of the bumped passengers for decision-making purposes after they are bumped. A model with interdependence is more complex than the one presented below by several orders of magnitude, and it was felt that the simpler model was a sufficiently close approximation to the more complex one to provide some insight into the problem.

We also assume that currently, the airlines can be viewed as charging themselves an implicit penalty for each passenger bumped. They incur a goodwill cost when a passenger is bumped in addition to facing the potential liability for his ticket price (or some fraction of it). It is our belief that the explicit penalty would exceed, on the average, the implicit penalty the airlines can be viewed as charging themselves. De Vany (1974) has estimated that the average value of time spent in air travel is around \$8 per hour. If we include nuisances such as missed connections or the need to rearrange the schedules of people who might be meeting a passenger at his destination, it is easy to imagine the nuisance cost of a two hour delay at \$25 or \$30. It seems that the implicit cost to the airline of bumping a passenger would fall below this range, especially if the frequency with which the airline is obliged to absorb the ticket price is low. Further work is needed to establish the merit of this conjecture.

Central to the problem is the random nature of the arrival at the terminal of passengers with tickets or reservations. Suppose the airline has a subjective probability density for show-ups which is conditional on the number of seats sold or reserved. Let this function be $f(x, S)$, where x is the number of show-ups and S is the number of seats sold or reserved. Clearly, f is zero for $x < 0$ and $x > S$. For analytical convenience, we assume this function is twice differentiable for $x < S$, ruling out distributions with mass points.² The results of the analysis depend on how the density shifts when S changes. The only assumption which yields solutions is that the density shifts to the right without changing its shape by an amount equal to the increase in S . The total differential of the density is

$$f_1(x, S)dx + f_2(x, S)dS. \quad (1)$$

Setting $dx = dS$ and requiring that the height of the density is unchanged after changing x and S , we get

$$f_1(x, S) + f_2(x, S) = 0 \quad (2)$$

for $S \geq 0$ and $0 \leq x \leq S$. Since f is a density, we must have

²If $f(S, S) > 0$, then $f_1(S, S)$ and $f_2(S, S)$ do not exist since $f(x, S) = 0$ for $x > S$. If what follows, we define $f_1(S, S)$ to be the left-hand partial derivative of f with respect to x at $x = S$ and define $f_2(S, S)$ to be the right-hand partial derivative of f with respect to S at $x = S$, which we assume exist and are differentiable.

$$\int_0^S f(x, S) dx = 1, \quad (3)$$

and differentiation with respect to S yields

$$f(S, S) + \int_0^S f_2(x, S) dx = 0. \quad (4)$$

Substituting (2) in (4) we have

$$f(S, S) - \int_0^S f_1(x, S) dx = f(0, S) = 0, \quad (5)$$

which says that the height of the density at $x = 0$ must be zero for all S if (2) and (3) both hold. If (5) were not true, shifting the density to the right would increase the area under it above unity. Differentiating the expected number of show ups with respect to S yields

$$Sf(S, S) + \int_0^S xf_2(x, S) dx = Sf(S, S) - \int_0^S xf_1(x, S) dx. \quad (6)$$

Integrating the second term in (6) by parts yields

$$-xf(x, S) \Big|_0^S + \int_0^S f(x, S) dx,$$

which reduces (6) to unity in view of (3). Hence, the mean number of show-ups increases by the increase in the number of seats sold or reserved when the density shifts according to (2).

Let C be the seat capacity of the flight. The function $\phi(x)$ gives revenues as a function of show-ups:

$$\phi(x) = \begin{cases} p(S)x & 0 \leq x \leq C \\ p(S)C - q(x - C) & C < x \leq S, \end{cases}$$

where $p(S)$ is the ticket price, which depends on the number of seats sold because the airline is a monopolist, and q is the bumping penalty. We assume the airline is risk-neutral, maximizing the expected value of profits from the flight. Letting $K(C)$ represent the cost function for seat capacity, expected profits are

$$\begin{aligned} \pi &= \int_0^S \phi(x) f(x, S) dx - K(C) \\ &= p(S) \int_0^C x f(x, S) dx + p(S)C \int_C^S f(x, S) dx - q \int_C^S (x - C) f(x, S) dx - K(C) \end{aligned} \quad (7)$$

For the moment imagine that the airline is unregulated; the regulated case is treated below. Its choice variables are S and C , which it determines by solving the following first-order conditions (the airline will always choose $S > C$, as the formulation of the problem implies):

$$\begin{aligned} \pi_S &= p' \int_0^C x f dx + p \int_0^C x f_2 dx + p' C \int_C^S f dx + p C \int_C^S f_2 dx + p C f(S, S) \\ &\quad - q(S - C) f(S, S) - q \int_C^S (x - C) f_2 dx = 0 \end{aligned} \quad (8)$$

$$\pi_C = (p + q) \int_C^S f dx - K' = 0 \quad (9)$$

Beckmann (1958) analysed a problem related to this one, although his model was considerably more complicated. He derived a simple approximation for his first-order conditions and constructed a numerical example using specific density functions. Our interest lies in analysing how the solution changes when q , the bumping penalty, increases. We have

$$\frac{\partial S}{\partial q} = \frac{1}{A} \begin{vmatrix} -\pi_{Sq} & \pi_{SC} \\ -\pi_{Cq} & \pi_{CC} \end{vmatrix} \quad (10)$$

$$\frac{\partial C}{\partial q} = \frac{1}{A} \begin{vmatrix} \pi_{SS} & -\pi_{Sq} \\ \pi_{CS} & -\pi_{Cq} \end{vmatrix}$$

where A is the determinant of the Hessian of the profit function evaluated at the solution, which is positive by the second order condition. From

(8) and (9), $-\pi_{Cq} = -\int_C^S f dx$, and

$$\begin{aligned} -\pi_{Sq} &= (S - C)f(S, S) + \int_C^S (x - C)f_2 dx \\ &= (S - C)f(S, S) - \int_C^S (x - C)f_1 dx \\ &= \int_C^S f dx, \end{aligned} \quad (11)$$

where (2) and integration by parts have been used. Hence $\partial S/\partial q$ and $\partial C/\partial q$ are proportional to $\pi_{CC} + \pi_{CS}$ and $-(\pi_{SS} + \pi_{SC})$ respectively, the proportionality factor being $k = \int_C^S f dx/A$, which is positive. Now $\pi_{CC} = -(p + q)f(C, S) - K''$ and $\pi_{CS} = p' \int_C^S f dx + (p + q)f(C, S)$, yielding

$$\frac{\partial C}{\partial q} = k[p' \int_C^S f dx - K''] \quad (12)$$

Also, π_{SS} equals

$$\begin{aligned} p'' \left[\int_0^C x f dx + C \int_C^S f dx \right] + 2p' \left[\int_0^C x f_2 dx + C \int_C^S f_2 dx + C f(S, S) \right] \\ + p \left[C \int_C^S f_{22} dx + C f_2(S, S) + \int_0^C x f_{22} dx \right] \quad (13) \\ - q \left[f(S, S) + (S - C) f_2(S, S) + \int_C^S (x - C) f_{22} dx \right]. \end{aligned}$$

Using (2) and integrating by parts, the coefficient of p' in (13) becomes $2 \int_0^C f dx$.

Noting that (2) implies

$$f_{12}(x, S) + f_{22}(x, S) = 0 \quad (14)$$

we have

$$\int_C^S f_{22} dx = - \int_C^S f_{21} dx = c(f_2(C, S) - f_2(S, S))$$

and

$$\int_0^C x f_{22} dx = - \int_0^C x f_{21} dx = -x f_2 \Big|_0^C - \int_0^C f_1 dx = -C f_2(C, S) - f(C, S),$$

where the last two steps use (2) and (5). Hence the coefficient of p in (13) is $-f(C, S)$. Similar manipulations establish that the coefficient of q is $f(C, S)$. So $\pi_{SS} + \pi_{CS}$ equals

$$p'' \left[\int_0^C x f dx + C \int_C^S f dx \right] + 2p' \int_0^C f dx + p' \int_C^S f dx =$$

$$p'' \left[\int_0^C x f dx + C \int_C^S f dx \right] + p' \left[1 + \int_0^C f dx \right], \quad (15)$$

and

$$\frac{\partial S}{\partial q} = k[\alpha_0 p'' + \alpha_1 p'] \quad (16)$$

where α_0 and α_1 represent the terms in brackets in (15), and are both positive.

Suppose for the moment that the airline faces a linear demand curve. This means $\partial S/\partial q < 0$ in view of $p' < 0$. From (12), if $K'' \geq 0$, then $\partial C/\partial q < 0$ as well. However, casual observation suggests that marginal capacity costs are declining in the airline industry, suggesting that $K'' < 0$ and that the sign of $\partial C/\partial q$ is ambiguous.

While (12) and (16) do not permit unambiguous statements, the introduction of regulation gives clear-cut results. With regulation, p is fixed at some p^* . The airline now may sell any number of seats between 0 and $S(p^*)$ at price p^* , where $S(p)$ is the inverse demand function. The airline may choose to satisfy the market demand at the regulated price, or it may turn away customers. In the latter case the airline achieves an interior solution on its flat, truncated "demand curve," and the above analysis applies. Since $p' = 0$ at the solution, $\partial S/\partial q = 0$ and $\partial C/\partial q = -K''$. If the airline is turning away customers, then an increase in q (institution of the penalty scheme) will not change the number of seats sold but will increase the seat capacity of the flight as long as marginal costs are decreasing. It follows directly that, if the airline's subjective density f is the true density for show-ups, as it should be given considerable

airline experience in the market, then the expected rate of capacity utilization, $(\int_0^C x dx + C \int_C^S dx)/C$, falls and the expected number of people bumped, $\int_C^S x dx$, falls as well.

If the airline achieves a corner solution for S at $S(p^*)$, then we can treat S as fixed for small changes in q , and we have $\partial C/\partial q = -\pi_{Cq}/\pi_{CC} > 0$, since from above $\pi_{Cq} = \int_C^S dx > 0$ and $\pi_{CC} < 0$ by the second order conditions. Since π_S will be positive in the corner solution situation, and since $\pi_{Sq} = -\int_C^S dx < 0$ from above, increases in q decrease π_S . Eventually π_S may become zero at $S(p^*)$, but then an interior solution obtains and further decreases in q will not change S . Clearly, the results on expected capacity utilization and expected number bumped hold here as well.

If capacity is fixed and $S < S(p^*)$, then $\partial S/\partial q = -\pi_{Sq}/\pi_{SS} < 0$ since $\pi_{SS} < 0$ by the second order condition. However, if the airline meets the demand at p^* , $\partial S/\partial q = 0$, but S may decrease for large changes in q .

We have

Proposition 1: Under the assumptions of the model, a regulated monopolistic airline with decreasing marginal capacity costs will increase flight capacity but will not change the number of seats sold when the bumping penalty increases (when the explicit penalty scheme is introduced). If capacity is fixed and the airline turns away some customers, the number of seats sold decreases when q increases. If the airline meets the market demand, an increase in q may or may not result in a decrease in the number of seats sold.

The Multi-Flight Monopoly Model

In this model, we postulate a demand per period for flights on a monopolized route instead of postulating a demand for each individual flight. Aircraft capacity is fixed and the choice variable is flight

frequency, which is currently unregulated in the U.S. Let the aircraft seat capacity equal one by choice of units and let the cost of operating each flight be α , which does not depend on the number of flights, an assumption which is not crucial in the analysis. The airline divides its passengers equally among flights, an assumption which corresponds to a uniform distribution of desired departure times among passengers and an even spacing of flights by the airline. The revenue function is:

$$\psi(x) = \begin{cases} px & 0 \leq x \leq 1 \\ p - (x - 1)q & 1 < x \leq z \end{cases} \quad (17)$$

where p is the regulated price, z is the number of seats sold or reserved per flight, and x is the number of show-ups. Both z and x are measured in units of aircraft capacity.

First we show that the airline will never turn away customers. Suppose the market demand at p is S seats, but suppose the airline finds it optimal to sell or reserve \tilde{S} seats, $\tilde{S} < S$. Suppose it also chooses t , the number of flights, optimally, selling or reserving \tilde{S}/t seats per flight (we ignore the fact that t is an integer). Then expected profits are

$$\pi = t \left[\int_0^{\tilde{S}/t} \psi(x) f(x, \tilde{S}/t) dx - \alpha \right], \quad (18)$$

since the density f pertains to each flight. If the airline increases \tilde{S} and t by the same proportion, the expression in brackets in (18) is unchanged while t increases. Thus π increases when the number of seats sold or reserved increases above \tilde{S} , indicating \tilde{S} was not optimal. Since this argument holds for any $\tilde{S} < S$, the airline does not turn away customers, selling S/t tickets

Faint, illegible text at the top of the page, possibly a header or introductory paragraph.

Second block of faint, illegible text, appearing to be the main body of the document.

Final block of faint, illegible text at the bottom of the page, possibly a conclusion or footer.

per flight when t flights are operated.

Replacing \tilde{S} by S in (18) and making a change of variable in the integral, we get

$$\begin{aligned} \pi = \int_0^S \psi(x/t) f(x/t, S/t) dx - \alpha t &= \int_0^t \frac{px}{t} f(x/t, S/t) dx + \int_t^S pf(x/t, S/t) dx \\ &- \int_t^S \frac{q(x-t)}{t} f(x/t, S/t) dx - \alpha t \end{aligned} \quad (19)$$

Setting $\pi_t = 0$ yields

$$\begin{aligned} - \int_0^C px \left(\frac{f_1 x + f_2 S}{t^3} + \frac{f}{t^2} \right) dx - \int_C^S \frac{p}{2} (f_1 x + f_2 S) dx + \int_C^S qx \left(\frac{f_1 x + f_2 S}{t^3} + \frac{f}{t^2} \right) dx \\ - \int_C^S \frac{q}{2} (f_1 x + f_2 S) dx - \alpha = 0 \end{aligned} \quad (20)$$

Since $\partial t / \partial q = -\pi_{tq} / \pi_{tt}$ evaluated at the solution and $\pi_{tt} < 0$ by the second-order condition, the sign of π_{tq} determines the sign of $\partial t / \partial q$. Using (20) and (2),

$$\pi_{tq} = \int_C^Z \left[\frac{q(x-t)(x-S)}{t^3} f_1 + \frac{qx}{t^2} f \right] dx. \quad (21)$$

Integrating by parts repeatedly and cancelling and gathering terms, this reduces to

$$\frac{(t+S)}{t^3} \int_t^S f dx + \frac{(t-2)}{t^3} \int_t^S x f dx. \quad (22)$$

Let us choose a time period long enough so that $t > 2$. Since $\int_t^S x f dx > t \int_t^S f dx$, (22) then exceeds

$$\frac{t + S + (t - 2)t}{t^3} \int_t^S f dx = \frac{t^2 + S - t}{t^3} \int_t^S f dx > 0, \quad (23)$$

since $S > t$. Thus $\pi_{tq} > 0$ and hence $\partial t / \partial q > 0$. We have established

Proposition 2: Under the assumptions of the model, a regulated monopolistic airline will increase its flight frequency when the bumping penalty increases (when the explicit penalty scheme is introduced).

It may also be shown that the expected capacity utilization rate,

$$\int_0^1 x f(x, S/t) dx + \int_1^{S/t} f(x, S/t) dx,$$

and the expected number of passengers bumped,

$$t \int_1^{S/t} x f(x, S/t) dx,$$

both decline with an increase in q .

Multi-Flight Duopoly Models

The general framework for the duopoly problem is the same as that for the multi-flight monopolist: aircraft capacity is fixed and the airline choice variable is flight frequency. The airline with the larger number of flights per period captures a larger share of the market. Before formalizing these ideas, it should be noted that a more convincing rationale for airline myopia about bumped passengers (airlines ignore their presence for decision-making purposes) can be provided in this model. We can imagine that the airline

assumes its bumped passengers go to the other airline for service. Completely eliminating the bumped passengers from the decision problem also requires that each airline believes that the other airline takes care of its own bumped passengers. These assumptions, while logically defensible, are fairly unrealistic, but they reduce the complexity of the problem considerably.

At the regulated price p , the demand per period for seats on the route is S . We assume there exists a function $D(t_1, t_2)$ which implicitly depends on S and which gives the number of seats demanded on an airline when it operates t_1 flights and its competitor operates t_2 flights. Clearly,

$$D(t_1, t_2) + D(t_2, t_1) = S, \quad (24)$$

and setting $t_1 = t_2 = t$ in (24), we have

$$D(t, t) = S/2, \quad (25)$$

which says that when the airlines operate the same number of flights, they divide the market. We assume

$$D(t_1, t_2) > S/2 \text{ for } t_1 > t_2, \quad (26)$$

which says that the airline operating the larger number of flights captures more than half of the market.

A rationale for the existence of the function D can be provided by imagining that the "visibility" of the airline and hence demand for its tickets depends on flight frequency. As above, we assume that the airline allocates its customers equally among flights. It is not possible to justify the existence of the D function by postulating a distribution of desired departure times among passengers, with each customer dealing with the airline that has a departure closest to his desired time. In this situation both the number and spacing of flights of both airlines determine the market share of each airline. This is really a complex problem in spatial competition, where analysis of equilibrium is prohibitively difficult.

Assuming both airlines have the same flight cost α and the same subjective density f , expected profits when airlines 1 and 2 operate t_1 and t_2 flights respectively are, using (19),

$$\begin{aligned}\pi^1 &= \int_0^{D(t_1, t_2)} \psi(x/t_1) f(x/t_1, D(t_1, t_2)/t_1) dx - \alpha t_1 \\ \pi^2 &= \int_0^{D(t_2, t_1)} \psi(x/t_2) f(x/t_2, D(t_2, t_1)/t_2) dx - \alpha t_2\end{aligned}\tag{27}$$

More generally, profits can be written $\pi^1 = \pi(t_1, D(t_1, t_2))$ and $\pi^2 = \pi(t_2, D(t_2, t_1))$. The Cournot or Nash duopoly equilibrium is characterized by the solution to the following system:

$$\frac{\partial \pi^1}{\partial t_1} = \pi_1(t_1, D(t_1, t_2)) + \pi_2(t_1, D(t_1, t_2))D_1(t_1, t_2) = 0 \quad (28)$$

$$\frac{\partial \pi^2}{\partial t_2} = \pi_1(t_2, D(t_2, t_1)) + \pi_2(t_2, D(t_2, t_1))D_1(t_2, t_1) = 0$$

By symmetry, $t_1 = t_2 = t$ is a solution for some t , which is found by setting $t_1 = t_2$ and solving either equation in (27). We have been unable to rule out solutions where $t_1 \neq t_2$. Analysing the sensitivity of the symmetric solution to variations in q requires totally differentiating

$$\pi_1(t, S/2) + \pi_2(t, S/2)D_1(t, t)$$

with respect to t and q , a calculation which yields ambiguous results.

Although we are not able to do comparative statics for the Nash equilibrium, another appealing behavioral concept yields immediate results. If it is assumed that each airline believes that its competitor will exactly match its own flight frequency, then in view of (25), the profits of each airline are

$$\pi = \int_0^{S/2} \psi(x/t)f(x/t, S/2t)dx - at. \quad (29)$$

Each airline optimizes over t , believing its competitor will choose the same number of flights. It should be noted that unless the airlines are identical, there is no equilibrium in this model. If their subjective densities or flight costs differ, then the solution results in different flight frequencies, violating the behavioral premise of the model. Since

(29) is formally identical to (19), the comparative statics results are identical to those for the monopoly case. We have

Proposition 3: Under the assumptions of the model, two identical regulated airlines in a duopoly market where each believes the other mimics its own flight frequency choice will operate the same number of flights and will both increase flight frequency when the bumping penalty increases (when the explicit penalty scheme is instituted).

As before, the expected rate of capacity utilization and the expected number of passengers bumped both fall when q increases.

Conclusion

The results in this paper conform to intuition. When the penalty for bumping a passenger increases, airlines reduce the likelihood that passengers will be bumped by increasing the number of flights in the multi-flight models or by increasing capacity, or reducing seats sold when capacity is fixed, in the one-flight model: Further work could be directed toward attempting to increase the generality of the change in the f density when its upper limit increases and toward eliminating the myopia assumption concerning bumped passengers. That much improvement is possible in these areas seems doubtful, however.

If one disagrees with the conjecture that instituting the explicit penalty scheme is equivalent to increasing the bumping penalty, believing instead that it would amount to a decrease in the penalty, then the results are exactly reversed: capacity or flight frequency decrease, and the expected rate of capacity utilization and the expected number of passengers bumped both increase with the institution of the explicit penalty.

References

- Beckmann, M. "Decision and Team Problems in Airline Reservations." Econometrica, 26, 1 (January 1958).
- De Vany, A. "The Revealed Value of Time in Air Travel." Review of Economics and Statistics, LVI, 1 (February 1974).
- Simon, J. "An Almost Practical Solution to Airline Overbooking." Journal of Transport Economics and Policy, II, 2 (May 1968).

UNIVERSITY OF ILLINOIS-URBANA



3 0112 060296560