

## The Airpolane

By Frederick Bedell

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THE AIRPLANE


## Books by the Author

On Electricity
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On Aerodynamics
Airplane Characteristics
The Air Propeller
The Airplane
THOMAS-MORSE SCOUT

A practical discussion of the principles of airplane flight

BY<br>FREDERICK BEDELL, PH.D.<br>"<br>Author of Airplane Characteristics, The Air Propeller, etc.<br>Member Aero Club of America, Fellow and member Technical Advisory Committee Aerial League of America, Pioneer Member Aeronautical Society of America, Honorary Member Aero Club of Ithaca, Honorary Member Cornell Flying Club, Member and Past Vice-President American Institute of Electrical Engineers, Member American Physical Society, etc.

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The Air Propeller, 1919
The Airplane, 1920
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## PREFACE

The material that forms six of the thirteen chapters of this book, previously published by the author under the titles Airplane Characteristics (ig18, two issues) and The Air Propeller (1919), has been thoroughly revised and in part rewritten. The material for the seven other chapters now appears for the first time and completes the plan of the author to present a well-rounded treatment of the airplane covering the general principles of airplane flight in a manner that is simple and at the same time reasonably complete and accurate.

The author has benefitted by the kindly criticisms of the two preceding works and has been spurred on in his attempt to meet the needs of flyer and designer on the one hand and of the general reader on the other. The interest taken in the work by ex-service men and members of the aeronautic profession, as well as by those whose point of view is scientific or educational rather than practical, leads the author to hope that the book may find a useful field.

The aims of the author are stated in the prefaces to Airplane Characteristics and The Air Propeller. These prefaces are here reproduced as they apply to the present as well as to the earlier publications.

The author desires to express his indebtedness to his students, many of them returned from service, for aid in preparing the book for press and to his friends in the ThomasMorse Aircraft Corporation for valuable suggestions and for looking over some of the proof.

Cornell University, June, 1920.

## PREFACE TO AIRPLANE CHARACTERISTICS

Any contribution to aviation, however small, needs today no justification. The airplane is an accomplished fact and concerning it there is no longer any room for apology or speculation. So far has the art of flying progressed that the principles of flight can in the main be set forth as definite and without surmise, and a collection of the essential elements can now be made that will apply to all airplanes, irrespective of type or structure. Not that there is nothing further to be learned or discovered in aviation-for, far from it, there is ample opportunity for discovery and invention in this direc-tion-but a codification of the well-known ground work can now be made that may be an aid to those who are advancing the art, as well as to those who are learning it.

The introductory discussion in this volume is a contribution to such a codification, which, it is hoped, will prove useful not only to the flyer and designer, for whom the book is primarily intended, but to students and engineers and to others who have only a general interest in aviation. Indeed, so important is the place now taken by the airplane that there are many who desire a knowledge of the principles of its operation.

It is the author's purpose to present the principles of airplane sustentation and stability and the characteristics of an airplane in flight in a way that is direct and simple and at the same time reasonably precise, laying particular stress on that which is vital. Except in minor ways, no claim is made for originality other than in presentation; in fact, the aim has been to include only those things that are essential and are accordingly well known to those versed in the subject. To those not thus well versed, the characteristics of an airplane

## PREFACE

are, however, not so well known as they should be; discussion of the subject is apt to be either superficial and inadequate or involved and complicated. The author has endeavored to give a treatment that is adequate but simple, and without the use of higher mathematics.

Logical sequence, rather than historical development has been kept in mind and no attempt has been made to ascribe. particular features to their inventor or author. Military uses of the airplane, as well as its history, are left to others who may more appropriately discuss such phases of the subject. The author has confined his attention to the principles of airplane flight and has given no discussion of materials of construction-very important, of course, in airplane building-nor of the gas engine, on which there are many specialized treatises.
The author had occasion, as a member of a commission for planning the courses in our Schools of Military Aeronautics, to study carefully the needs of such courses. He has had occasion, also, to conduct several classes in aerodynamics and the principles of flight, made up not only of those with a direct practical interest in flight, as future pilots and designers, but likewise of others with an indirect interest -students in civil and mechanical engineering and physicswho have been interested in the airplane as in any of the mechanisms of our present day civilization. He has noted particularly those parts of the subject that have proved vital and of interest to all,-no matter from what angle the subject is approached.

It is planned that the book-with the added chapters now in preparation-shall be self-contained and complete in its own field, i. e., as an introduction; final practical instruction for flyer or designer must needs be obtained at the flying
field or in the designing room. The author has had flying experience only as passenger and would make no attempt at specific flying instruction.

In view of the present emergency, it is thought best to issue the material contained in this first edition without delay, and to reserve for subsequent publication the material now in preparation (referred to in the Table of Contents) that is needed to complete the work. The author will be glad to have his attention called to any error or obscurity in this presentation and will particularly appreciate constructive suggestions from those practically engaged in airplane operation or development.

> Ithaca, N. Y.,
> July 30 , 1918.

PREFACE TO SECOND ISSUE OF FIRST EDITION
So close upon the heels of the first issue is this second issue that no general revision has been possible. Some corrections and additions are given on the following page. The author thanks those readers who have called his attention to errors and repeats his request for suggestions.

## PREFACE TO THE AIR PROPELLER

It is with some hesitation that the writer adds to the literature of the propeller. He has been lead to do so, however, because many discussions of the subject are unsatisfying and in some cases are not in accord with fact. Indeed, misconceptions of the behavior of a propeller are not uncommonly held even by those who are otherwise wellinformed on aerodynamic subjects.

The author has endeavored to present a brief and simple treatment of the propeller for those who want a practical working knowledge of its characteristics and a general knowledge of its theory. It is believed that the treatment will at the same time serve as a general introduction for those who wish to pursue the subject further and to make a more detailed study of the propeller, either in its theoretical or practical aspects.

As the material herein is soon to be revised and republished in another form, the author would welcome criticism, and would be pleased to have his attention called to any error or obscurity in presentation. He desires to thank Professor S. Noda, Honorary Fellow in Physics, Cornell University, for valuable assistance in the preparation of this book, particularly in the calculation of the characteristics of the propeller.

Ithaca, N. Y.,
August, 1919.

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## CHAPTER I

## SUSTENTATION

The first essential for flight is a sustaining force or sustentation. This is obtained in balloons and in airships by the buoyancy of a light gas which makes the craft as a whole lighter than air and capable of flotation. In an airplane, or in any aircraft heavier than air, this sustentation must be obtained by the reaction of the air upon planes or surfaces which are moving with relation to the air and which force or deflect the air downward. The airplane is the only craft of this kind that has been practically developed and it alone will be discussed in this volume. Other types of flying machines have, however, been contemplated, among which may be mentioned the ornithopter, with wings that flap like those of a bird and sustain the machine by forcing the air downward and at the same time backward; and the helicopter, equipped with propellers which revolve on a vertical shaft and lift the machine by forcing the air directly downward.

A second essential for flight is stability, discussed in later chapters.

The sustentation of an airplane is due to the fact that the air it encounters is deflected downward by the wings; although this downward deflection is but little, it is sufficient to sustain the machine, for new air is being continually encountered and deflected. An understanding of sustenta-
tion will best be gained by a consideration of the pressure exerted by the air upon flat and curved surfaces moving rapidly with relation to it.

## Flat Plane Perpendicular to Air Stream

We will first consider the pressure on a flat plane perpendicular to an air-stream.

## Pressure varies as square of velocity.

If a thin plate or plane, say a book cover or the cover of a cigar box, is held out from a moving automobile - well away from the body and windshield - so that the air-


Fig. I. Pressure on flat surface.
stream strikes the plane perpendicularly to its surface, as in Fig. I, a pressure is felt that is small at low speeds but which increases very rapidly as the speed increases. In fact, if accurate measurements were made, it would be found that when the speed is doubled the pressure becomes four times as great, in other words that the pressure on the plate varies as the square of its velocity through the air.

Furthermore, this same relation would be found to be true if the plane were held still in a wind or in the air from a blower, and so it is seen that the pressure depends not upon the absolute velocity of the plane or of the air but upon their relative velocity, as the air-stream strikes the
plane. This is on account of the great mobility of the air particles, the effect being the same whether the air is stationary or moving. The law of the "square-of-thevelocity," although not a general law that holds in any fluid and for all velocities, may be taken as practically true in air throughout the range of airplane velocities.

## Pressure varies with plane area.

By using planes of different areas, it would be found, likewise, that the total pressure on the plane increases with the plane area and (practically) in direct proportion to the area; for example, an increase of ten per cent. in area is accompanied by an increase of ten per cent. in total pressure on the plane. This relation, however, must be considered as only approximate, particularly when the planes compared differ greatly in shape or area.

## General law.

The total pressure $P$, upon a surface $S$, normal to an air-stream with velocity $V$, may accordingly be expressed by the following law:-

$$
P=K S V^{2} .
$$

Here $K$ is a number or coefficient, the value of which depends upon the units used*. When the area $S$ is given in square feet, the velocity $V$ in miles per hour and the total pressure $P$ in pounds, a practical value for the coefficient $K$, that has been determined experimentally, is 0.003 for air at normal density; that is,

$$
P=0.003 S V^{2} .
$$

The pressure per square foot of surface is thus seen to be

[^0]1.2 pounds at a velocity of 20 miles per hour, 4.8 pounds at a velocity of $40 \mathrm{MPH} .$, 19.2 at $80 \mathrm{MPH} .$, etc.

Metric units.- When $V$ is velocity in meters per second, $S$ the area in square meters and $P$ the pressure in kilograms, the value of $K$ is 0.075 . To get $K$ in English units of pounds, square feet and miles per hour, multiply $K$ for metric units by 0.0408 .

## Variations in $\mathbf{K}$.

The value of $K$ is not strictly a constant. It varies directly with the density of the air, decreasing, therefore, with altitude, as discussed later, and changing somewhat from day to day. It varies, also, with the size and shape of the surface. The values given above ( $K=0.003$ in English


Fig. 2. Square with aspect ratio I ; and rectangle with aspect ratio 6 .
units, $K=0.075$ in metric units) are for a square surface, $0.50 \times 0.50$ meters, in air of normal density. For squares of different sizes, experiments of Eiffel give values for $K$ as follows:-

| Length of side | .15 m. | .375 m. | .500 m. | .707 m. | 1.00m. |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Value of $K$ | .066 | .0716 | .0746 | .0772 | .0789 |

For rectangles of equal area ( $0.225 \mathrm{~m} .^{2}$ ) and with different aspect ratios (the aspect ratio is the length divided by width, see Fig. 2) Eiffel gives:-

| Aspect Ratio | I | 3 | 6 | 10 | 20 | 30 | 50 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Value of $K$ | .066 | .0705 | .0725 | .0755 | .0885 | .092 | .097 |
| $K_{\mathrm{R}} \div K_{\mathrm{S}}$ | I. | 1.07 | 1.10 | 1.145 | 1.34 | 1.40 | 1.47 |

The last line, $K_{\mathrm{R}} \div K_{\mathrm{S}}$, is the value of $K$ for a rectangle
divided by the value of $K$ for a square. The percentage increase in $K$ with increase of aspect ratio, shown in the last line, will hold approximately for rectangles of other areas, for the effect of aspect ratio is almost independent of size of surface.

Somewhat different values hold for circular discs and other shapes. Considerable experimental data must be gathered before the precise value of $K$ can be determined for a surface of any size and shape. Meanwhile the value 0.003 (English system) or 0.075 (metric system) will prove practically useful,


Fig. 3. Turbulent region and rarefaction back of plane.
it being borne in mind that this value is only an approximation and that a closer value should be sought whenever it may be deemed desirable.

## Eddies back of plane.

When a plane encounters an air-stream, there is a compression of the air on the front face of the plane. Immediately back of the plane there is a rarefaction, so that back of the plane air currents are set up that swing around toward its rear surface. These air currents may be felt, by one riding in an open trolley car or in an automobile behind a wind shield, as a wind on the back of the neck.

These air currents are, in a crude way, visualized in Fig. 3.

Back of the plane and near the edge is a turbulent region with complex eddy currents of air. These no doubt have a material effect upon the value of $K$, so that it is not surprising that $K$ has different values for surfaces of different shapes and sizes, for in these the length of edge and the eddy-current effects are not proportional to the area.

## Methods of experimenting.

Experiments on the air resistance of different surfaces or bodies may be made in various ways. The necessary velocity of the body relative to the air may be obtained by dropping it from a suitable tower or other height (employed by Eiffel), by carrying it on a fast moving vehicle, by carrying it at the end of a long rotating arm (employed by Langley), by exposing it to a natural or artificial wind, and finally by carrying it through the air in airplane flight.

The most convenient and approved method now in use is to expose the body to an artificial wind in a wind tunnel, first used by Eiffel* and now used in all aerodynamic laboratories. Air is forced through such a tunnel by means of a powerful fan; the body to be studied is held stationary, being attached to suitable devices for measuring the pressure and, in case of oblique surfaces, for "weighing" the vertical as well as the horizontal component of the pressure.

In such a tunnel can be tested not only surfaces or bodies of various kinds, including wing sections, but even complete airplanes in model size; and it is important to note that the performance of wings and airplanes in flight is found to agree remarkably well with wind-tunnel tests.

[^1]
## Flat Plane Oblique to Air Stream

Let us next consider the action of a flat plane at an oblique angle with the air-stream; such a consideration will show at once the source of the sustaining force in an airplane.

## Lift of an oblique plane.

If a plane be held so as to form an angle $i$ (called the angle of incidence or the angle of attack) with the air-stream or


Fig. 4. Vertical and horizontal components of pressure $P$ on oblique plane.
"relative wind," as in Fig. 4, it will be seen that the total pressure $P$ of the air against the plane has two components:*

A vertical component or lift, commonly designated by the letter $L$, tending to force the plane upward at right angles to the air-stream.

A horizontal component or dynamic resistance (called, wing-resistance or drag,) commonly designated by the letter $D$, tending to force the plane backward in the direction of the air-stream.
These components may be observed by blowing upon a card held in the hand oblique to the air-stream, or by moving the

[^2]hand-slightly inclined-rapidly through water; in the latter case the upward lift may be distinctly felt.

It is the lift that holds an airplane up in flight, i. e., that gives sustentation; for an understanding of the airplane, therefore, a knowledge of what determines the lift is essential. Clearly for horizontal flight the lift must be just equal to the weight of the machine, otherwise the machine will either ascend or descend,-discussed later under climbing and gliding. The problem of horizontal flight is, therefore, the problem of securing a lift equal to the weight. As discussed later, this lift is better obtained from a "cambered" wing (i.e., a wing that is slightly curved from front to back) than from a flat plane.


b

c

$d$

Fig. 4a. Lift is inclined when machine is inclined.
In order to get the lift, the plane or wing must move rapidly through the air. But in order to move the plane or wing through the air it is necessary to overcome the wingresistance and this is done by the thrust from the propeller. To drive the propeller, so as to obtain this thrust, it is neces-

[^3]sary to have power. Obviously it is desirable to have the wing-resistance small, necessitating small thrust and small power, and to have the lift large so that the machine can sustain not only itself but some load in addition. But it is impossible to have the lift without the wing-resistance, and the wing-resistance has aptly been described as the price paid for the lift.

## Variation of lift and wing-resistance with incidence.

Highly important is it to know how the values of lift and wing-resistance vary as the angle of incidence is changed.


Fig. 5. Flow of air past oblique plane, showing turbulent region back of entering edge and the downward deflection of air after leaving the plane.

This information cannot well be derived theoretically on account of the complexity of the problem,-due in part to the eddy currents in the turbulent region back of the entering edge, as indicated in Fig. 5. The information, however, has been found experimentally.

The total pressure $P$, the lift $L$ and the wing-resistance $D$ all vary directly as the area $S$, of the plane or wing, and as the square of the velocity $V$; that is,

Total pressure $=P=K_{P} S V^{2}$;
Vertical component or lift $=L=K_{\mathrm{L}} S V^{2}$;
Horizontal component or wing-resistance $=D=K_{\mathrm{D}} S V^{2}$.

We are particularly interested in the two components $L$ and $D$; also in the two coefficients, $K_{\mathrm{L}}$ called the coefficient of lift and $K_{D}$ called the coefficient of wing-resistance. These coefficients have different values for each angle of incidence. Their values, furthermore, will depend upon the units used.

The values of these coefficients that will give lift and wingresistance in pounds, for different angles of incidence, when $S$


Fig. 6. Coefficient of lift $K_{\mathrm{L}}$ and coefficient of wing-resistance $K_{\mathrm{D}}$ for flat rectangle at different angles of incidence; aspect ratio, 6, i.e., length $\div$ width $=6$, as in Fig. 2.
is in square feet and $V$ is in miles per hour, are shown by the curves in Fig. 6. These are plotted from data by Eiffel* for a flat rectangle with an aspect ratio 6 .

For small values of incidence, it will be seen that the coefficient of lift increases nearly uniformly, in proportion to

[^4]the angle of incidence; if the angle is doubled, the coefficient of lift is ápproximately doubled. After reaching a maximum value (in this case about 0.002) the coefficient decreases somewhat irregularly*, becoming zero at $90^{\circ}$ incidence.

This maximum value is only two-thirds as great as the maximum value obtained by a cambered wing; see Fig. II.

The coefficient of wing-resistance also increases more or less uniformly with incidence and reaches a value of about 0.003 at $90^{\circ}$; but in no case is $K_{\mathrm{D}}$ zero, even at zero incidence.


Fig. 7. Curves showing $L / D$ ratio and $D / L$ ratio for flat rectangle; aspect ratio, 6.

## Lifting efficiency or $\mathrm{L} / \mathrm{D}$ ratio.

The ratio of lift to wing-resistance, is frequently referred to as the lift-over-drag ratio or the $\mathbf{L} / \mathbf{D}$ ratio, or as the lifting efficiency of a plane or wing, and this ratio is of much significance. As it is desirable to have $L$ large and $D$ small,

[^5]it is obviously desirable to have the $L / D$ ratio as large as possible. It is seen that the $L / D$ ratio is equal to $K_{\mathrm{L}} \div K_{\mathrm{D}}$, values for which are obtained from the curves in Fig. 6.

The values of the $L / D$ ratio for a flat rectangle with aspect ratio 6 , thus obtained from Fig. 6 for different angles of incidence, are shown in Fig. 7. The maximum value for $L / D$ is here seen to be a little over 6 at an incidence of $6^{\circ}$,-a rather poor lifting efficiency compared with an $L / D$ ratio 16 or more for a cambered plane.

In Fig. 7, the values are also shown for $D / L$ (see dotted curve); but, as these values approach infinity when $L$ approaches zero, they are not so convenient to use. It is more usual, therefore, to make use of the $L / D$ ratio.

In the case of a theoretical plane with a perfectly smooth surface, the pressure $P$ would be perpendicular to the surface, so that $D / L$ would be the tangent and $L / D$ the cotangent of the angle of incidence; and, at zero incidence, the resistance of the plane would be zero.

But in a practical case, the plate or wing that is used must possess an edge of definite thickness, and this with some skin friction (undoubtedly small) gives a certain resistance, even to a horizontal plate. The resultant pressure $P$, shown by the solid line in Fig. 4, is, therefore, not perpendicular to the surface but is a little back of the perpendicular,-which is shown by the dotted line. Its direction and magnitude can be determined by laying off the two components $L$ and $D$, the resultant pressure being $P=\sqrt{L^{2}+D^{2}}$.

## Center of pressure.

When the plane is perpendicular to the air-stream, i.e., when the angle of incidence is $90^{\circ}$, the center of air pressure on the plane is at the center of the plane. When the plane is
oblique, the center of pressure is found to be in advance of the center of the plane, moving more and more forward toward the entering edge as the incidence decreases. The position of the center of pressure for different angles of incidence is shown by the curve in Fig. 8. The position is somewhat different for different aspect ratios, but in all cases with a flat plane the center of pressure moves forward from the center


Fig. 8. Position of center of pressure on a flat plane, with aspect ratio 6 , for different angles of incidence.
of the plane as the incidence decreases, thus becoming nearer to the front edge of the plane-which is called the entering edge or leading edge-and further from the rear or trailing edge.

## Cambered Wing

## Decreased turbulence when cambered wing is used.

A flat plane encountering an air-stream disrupts the air, the entering edge, and to a lesser extent the trailing edge,
tending to produce air eddies. The turbulence produced by such a plane has been shown in Fig. 5, which as already explained indicates the phenomenon but should not be understood to be an accurate representation of it. It would seem that this turbulence would increase the wing-resistance and that it might decrease the lift; or, put another way, it would seem that, if the turbulence could be eliminated or reduced by any means-for example by giving the wing more or less of a stream-line form,-an increase in lift and decrease in wing-resistance might be obtained. This indeed is the case, such a result being obtained by arching the wing from front to back. A wing thus arched is called a cambered wing and on account of its better performance is always used in airplane construction.

The flow of air past a cambered wing is shown in Fig. 9, which again is merely illustrative and is not exact in detail. The disturbance of the air by the wing is minimized by having the surfaces of the wing, throughout, more or less parallel to the stream-line flow; the wing then enters the air and leaves the deflected air with little disturbance, and eddies are eliminated so far as they can be.

## Creation of lift.

It is to be borne in mind that it is the downward deflection of the air that creates the lift, it being the purpose of the designer to increase this lift and at the same time to decrease the wing-resistance. Lift is obtained to a certain extent by the positive pressure on the lower surface of the wing (indicated by $p$ in Fig. 9) but to a much greater extent by the negative pressure on the upper surface, indicated by $n$; commonly, as much as three-fourths of the total lift is due to this negative pressure. It will be seen that, in a cambered wing as shown in Fig. 9, the upward trend of the upper
surface of the entering edge swings the air-stream upward over the wing before its final deflection downward; this leads to a decrease in pressure on the upper surface and so contributes materially to the lift.

The distribution of pressure on single and multiple planes is shown in a later chapter.

It is the curvature of the upper surface of a wing that is most important,-particularly its dip toward the entering edge, often referred to as the dipping front edge. The curvature of the lower surface is far less important; with a well


Fig. 9. Flow of air past a cambered wing; side view.
formed upper surface, a good wing can be constructed with a perfectly flat lower surface.
An upward turn in the lower surface toward the entering edge, corresponding to the dip in the upper surface and making what is known as "Phillips entry", is not advantageous. The wing shown in Fig. 14, which is accurately drawn to scale, has a more effective entry than the wings sketched in Figs. 9 and ro.

## Incidence, chord, span and area of a cambered wing.

A wing section, or side view, of a cambered wing is shown in Fig. ro. The chord of a cambered wing is a straight line, as shown, tangent to its under surface at front and rear.

The length of chord, or more briefly the chord, is the length of the projection $a b$ of the wing-section upon this line. Similarly, the area of a cambered wing is its area projected on a tangent plane. The span or spread of a wing is the maximum distance from tip to tip. The aspect ratio is the ratio of span to chord. The angle of incidence, or the incidence, of a cambered plane is the angle between its chord and the air-stream or relative air, as shown in the same figure.


Fig. 10. Section of cambered wing, showing angle of incidence between the chord and the air-stream, and the components of pressure.

## Wing structure.

The whole wing structure is often referred to as an aerofoil. Wings are made in many ways; although no standard construction can be shown, the one shown in Fig. ioa is fairly typical. Each wing is commonly built up of two spars, as there shown, running from one end of the wing to the other. Spars may be of I-beam section, double I-beam, box construction, etc. Supported by these spars are the ribs, extending fore and aft, each rib having the exact shape of a wing-section. The top and bottom of each rib is commonly made of strips of spruce, held in place and strengthened by a
thin web, which gives the rib an I-beam cross-section. The web is partly cut away for lightness, as shown in the figure.

The camber of either surface of a wing is the greatest distance between the chord and that surface, and is usually expressed as a fraction of the chord. The camber of the bottom surface indicated in Fig. ioa is about $1 / 40$. Mean camber is the mean between the top camber and the bottom camber.

## Components of pressure.

The pressure $P$ is considered as having its point of application at the point $O$ where it intersects the chord $a b$, as shown


Fig. 10a. Wing structure.
in Fig. 10. This point is called the center of wing pressure and is located (unless the incidence is very small or negative, see Fig. I3) a little in advance of the middle point of the chord.

The pressure $P$ is more or less perpendicular* to the chord. The total pressure $P$ is resolved into its two components, lift $L$ and wing-resistance $D$, which also have their point of application at the center of pressure $O$. The direction of $P$

[^6]can be determined by laying off $L$ and $D$ where these are known; $P=\sqrt{L^{2}+D^{2}}$. In ordinary flight, $D$ is so small


Fig. II. Lift and wing-resistance for cambered wing, U. S. A. 5 . Lift (in pounds) $=K_{\mathrm{L}} S V^{2}$; wing-resistance (in pounds) $=K_{\mathrm{D}} S V^{2}$, where $S$ is in sq. ft. and $V$ is in miles per hour. For wing-section, see Fig. 14.
compared with $L$, as shown in Fig. in, that $P$ and $L$ are practically equal, the difference being less than one per cent.

## Variation of lift and wing-resistance with angle of incidence

 of a cambered plane.Each different wing-section will have its own characteristic curves for lift and wing-resistance, but all such curves have


Fig. 12. Curves for $L / D$ ratio and $D / L$, corresponding to Fig. 11. the same general trend. Fig. II shows curves for the coefficient of lift $K_{\mathrm{L}}$, and the coefficient of wing-resistance $K_{\mathrm{D}}$, for a representative cambered wing, designated as
U. S. A. 5 and shown in Fig. 14. These curves are typical and will serve to illustrate certain general features that are characteristic of practically all cambered wings.

The corresponding values for the $L / D$ ratio (obtained by dividing $K_{\mathrm{L}}$ by $K_{\mathrm{D}}$ ) are given in Fig. 12, the values for $D / L$ being shown by the dotted curve in the same figure.

The numerical values* from which these curves are plotted are given in Table I.; these values will be used in later calculations. The change in these coefficients with the density of the air is discussed in the chapter on Altitude.

Table I. Wing U. S. A. 5

| $\boldsymbol{i}$ | $K_{\mathrm{L}}$ | $K_{\mathrm{D}}$ | $L / D$ | $C . P . \dagger$ |
| ---: | ---: | :---: | ---: | :---: |
| $-4^{\circ}$ | -.000326 | .0001500 | -1.58 | $\ldots$. |
| $-2^{\circ}$ | .000346 | .0000948 | 3.64 | .753 |
| $-\mathbf{1}^{\circ}$ | .000636 | .0000830 | 7.67 | .566 |
| $0^{\circ}$ | .000910 | .0000741 | 12.28 | .498 |
| $\mathbf{1}^{\circ}$ | .001145 | .0000803 | 14.28 | .444 |
| $2^{\circ}$ | .001355 | .0000863 | 15.72 | .415 |
| $3^{\circ}$ | .001565 | .0000966 | 16.21 | .377 |
| $4^{\circ}$ | .001740 | .0001092 | 15.98 | .348 |
| $5^{\circ}$ | .001950 | .0001290 | 15.35 | .337 |
| $8^{\circ}$ | .002470 | .0001830 | 13.52 | .315 |
| $10^{\circ}$ | .002870 | .0002380 | 12.08 | .303 |
| $12^{\circ}$ | .003130 | .0002890 | 10.84 | .300 |
| $13^{\circ}$ | .003240 | .0003290 | 9.84 | .298 |
| $14^{\circ}$ | .003285 | .0003545 | 9.25 | .288 |
| $15^{\circ}$ | .003235 | .0003910 | 8.28 | .292 |
| $16^{\circ}$ | .003205 | .0004210 | 7.63 | .298 |
| $18^{\circ}$ | .003150 | .0006900 | 4.57 | .330 |
| $20^{\circ}$ | .002790 | .0008200 | 3.4 I | .368 |

Distance of the center of wing pressure from the leading edge, expressed as a fractional part of the chord. Shown by curve in Fig. 13.

[^7]
## Characteristic features of a cambered wing.

An inspection of the curves in Figs. if and i2 shows the following:

Lift with negative incidence.-A cambered plane exerts a lift even at a small negative incidence. Zero lift is usually obtained when the incidence is between - $2^{\circ}$ and $-4^{\circ}$ (in Fig. II at $-3^{\circ}$ ) but in extreme cases the incidence may be decreased to - $8^{\circ}$ or $-10^{\circ}$ before zero lift is reached. Although in most cases an airplane flies with a positive incidence, at high velocities it may fly with zero incidence or with a small negative incidence,-but not within two or three degrees of the point of zero lift.

In approaching the point of zero lift it is possible to go too far, so that the machine has insufficient sustention for a horizontal flight and descends in a glide or dive.

Lift is a maximum at about $14^{\circ}$.-As the incidence is increased, the lift increases rather uniformly, $K_{\mathrm{L}}$ reaching a maximum of more than 0.003 at an incidence of $14^{\circ}$ or so, according to the particular wing. The angle of incidence at which $K_{\mathrm{L}}$ is a maximum, ustually between $12^{\circ}$ and $18^{\circ}$, is called the critical angle. Beyond this maximum, which is also known as the burble point, the lift decreases somewhat irregularly and again becomes zero at an incidence of about $90^{\circ}$.

Up to the burble point, i.e., for an incidence of less than $14^{\circ}$ or so, the air-flow past the wing is to a certain extent smooth and without eddies, as shown in Fig. 9. Beyond this point, i.e., for an incidence of $14^{\circ}$ to $90^{\circ}$, there is a confusion of eddies and a turbulence (such as was illustrated in Figs. 3 and 5), accompanied by a decrease in lift and an increase in wing-resistance. ("Burble" means "confuse;" hence the term "burble point.")

Usual range of incidence is about $0^{\circ}$ to $10^{\circ}$. -It is seen that the lift increases more or less uniformly with incidence, from zero lift at - $3^{\circ}$ to maximum lift at $+14^{\circ}$. (It is to be understood that other wing-sections would give slightly different values.) The range for practical flight must be within these limits, without either limit being reached.

The lower limit or zero lift cannot be reached, inasmuch as some lift is necessary for sustentation; it may be approached to a certain extent at high velocities, but too near an approach will cause the machine to glide or dive as already mentioned.

The upper limit or maximum lift is possible but in ordinary flying it, too, is only approached, for as it is approached there is danger of a stall due to increase of wing-resistance, leading to a fall or tail slide. With the decreased velocity which accompanies increased incidence, the stability of the machine becomes less and may vanish entirely; as the power of control depends upon velocity, the recovery of equilibrium when once lost at low speed is difficult. Too great an incidence is a frequent cause of accident.

Exact limits* can not well be set; but, roughly speaking, the range of incidence is between $0^{\circ}$ and $10^{\circ}$, limits which are never greatly exceeded in ordinary flight.
(The angle of incidence may be increased beyond this limit-possibly up to the point of maximum lift $\dagger$-as a machine

[^8]reaches the "ceiling," which is the highest possible altitude a particular machine can attain; also, when slowing down to minimum speed just prior to landing.)

A flat maximum in the lift curve is very desirable, being less dangerous than a sharp maximum in which the lift decreases rapidly after the critical angle is reached. A flat maximum is more readily obtained in a biplane or triplaneparticularly if the planes are staggered-than in a monoplane for the maximum points for the separate planes may not coincide, so that when the separate lifts are added together to get the total lift the maximum point is broadened out.

Wing-resistance is small through usual range of incidence; then increases rapidly.--Wing-resistance, as already mentioned, is in no case zero. It is small throughout the useful range of incidence, gradually increasing throughout this range as the incidence increases. As the incidence is further increased, the wing-resistance increases a little more rapidly until the burble point or point of maximum lift is reached; after which the wing-resistance increases very rapidly,-finally reaching. a value of 0.003 (more or less) at an incidence of $90^{\circ}$.
$L / D$ ratio has a maximum value of about 16 in middle range of incidence.-If the wing-resistance were constant, the $L / D$ ratio would be a maximum when the lift is a maximum. On account of the increase in wing-resistance with incidence, the $L / D$ ratio reaches a well defined maximum before the maximum lift is reached, usually at an incidence between $3^{\circ}$ and $8^{\circ}$. In Fig. 12, this maximum of 16.2 occurs at $+3^{\circ}$. This is a good value for $L / D$. A slightly greater value, 17 or 18 , is obtained by some wings,-at a sacrifice perhaps of some other feature, such as stability or ease of construction.

Comparison with flat plane.-By comparing the curves in Figs. II and 12 for a cambered wing, with those of Figs. 6
and 7 for a flat plane, it is seen that the cambered plane gives more lift (the maximum value being about 50 per cent. more), gives less wing-resistance and a much greater $L / D$ ratio,-nearly three times as great. For these reasons a cambered wing is always used.


Fig. 13. Position of center of pressure on a cambered wing (U.S.A.5.) with aspect ratio 6 , for different angles of incidence.

## Shifting of center of wing pressure with incidence.

The position of the center of pressure for the cambered wing in question, U. S. A. 5, is shown in Fig. 13. All cambered wings show a marked shifting of the center of pressure toward the rear of the plane, when the incidence is small and is decreasing,-a bad feature for stability as discussed in a later chapter.

In this one respect a cambered wing is inferior to a flat plane, in which the center of pressure moves forward when the incidence is decreasing, as shown in Fig. 8.

The shifting of the center of pressure shown in Fig. 13 is much less than is found in the case of many wings, the section of wing here used being in this respect satisfactory.

## Wing-section.

There is no one type of wing that is best. The particular wing, to which all the curves here given refer, is shown in Fig. I4 and is fairly representative. But it will be understood that in different machines different wings may best be


Fig. 14. Section of wing, U.S.A. 5.
used, according to the particular features to be emphasized; high speed in one, large load-carrying capacity in another; stability in one, quickness in manouvering in another, and so forth.

The characteristics of any particular wing-section are shown by means of curves, such as those shown in Figs. ir, 12 and 13 . The usefulness of these curves in determining the problems of flight will be brought out in the following chapters.

## Comments on wing sections.

A few comments on wing sections may here be made, although the reader may find it well to postpone their perusal until he has read some of the subsequent chapters describing various relations and characteristics of an airplane in flight.

Mechanical as well as aerodynamic considerations have to be kept in mind by the designer; there must be room for spars and ribs of adequate strength.

For climbing and for heavy lift, a wing with deep camber (particularly on the upper surface) and large value of $K_{\mathrm{L}}$ should be used. For this purpose a deeply cambered wing is flown at low speed, and at a large angle of incidence as shown in Fig. 15. Such a wing, however, is utterly unsuited for flying at high speed and small incidence, as shown in Fig. 16, on account of its great resistance at small angles of incidence.

For speed, the wing should be flatter with only a little camber, with less lift and with the least possible wingresistance at small incidence and high velocity. It is necessary to sacrifice either speed or lift. A speed wing is sketched in Fig. 17.

Again, both speed and lift may be sacrificed for stability. As has been shown, the center of pressure on a cambered wing, convexed upward, shifts with change of incidence (when the angle of incidence is small) in the wrong direction for stability. If a cambered wing were concave downward-a very bad wing for lift-this shifting of the center of pressure would be in the right direction for stability. The two effects may be combined, in varying proportions, by giving a wing a double curvature, as shown in Fig. 18. In this case, when the machine starts to dive the air strikes, or tends to strike, the reversed curve near the rear of the wing and restores equilibrium; but this means less lift and more resistance.
The characteristics of an aerofoil, although in a general way shown by its section, are best shown by curves for its performance, as in Figs. il, 12 and i3.

A high maximum to the $K_{\mathrm{L}}$ curve is of no advantage in a high speed machine nor in ordinary flight; it gives, however,
the ability to climb to high altitudes and gives a low landing speed, as brought out in the next chapter.

The $L / D$ curve should show a good value, not necessarily a maximum, at about the incidence for which the machine is to be flown; thus, in a high speed machine, it is desirable to have a large value for $L / D$ at a very small incidence. This is only another way of saying that wing-resistance $D$ should be small.


Fig. I5. Wing with high camber, suitable for big lift and slow speed, when flown at large angles of incidence as shown.
Fig. 16. Same wing at small incidence, entirely unsuitable for high speed on account of large wing-resistance due to its camber.
Fig. 17. Flat, stream-lined wing with little camber and small lift; suitable for high speed on account of its very small wing-resistance.
Fig. 18. Wing with reversed curvature toward trailing edge for increased stability. This is paid for by an increase in wing-resistance and a decrease in lift.

In a high speed machine, which is to be flown at small incidence and at small $K_{\mathrm{L}}$, it is desirable to have a curve for $K_{\mathrm{L}}$ that is not too steep as it approaches zero lift; thus, it is desirable to obtain a certain lift, when the plane, let us say, is $3^{\circ}$ or $4^{\circ}$ rather than $1^{\circ}$ or $2^{\circ}$ from the incidence of zero lift. There is then less loss of sustentation, when the machine dips a little as it oscillates about its normal direction of flight.

[^9]
## CHAPTER II

## RELATIONS IN HORIZONTAL FLIGHT

Some interesting relations in regard to flight can be drawn from the expressions given in the preceding pages for the lift and wing-resistance of a cambered plane. In this chapter will be considered the significance of the lift equation, and certain relations between velocity and incidence in horizontal flight that may be derived from it; wing-resistance will be considered in the following chapter.

## In horizontal flight, weight $=$ lift $=\mathbf{K}_{\mathrm{L}} \mathrm{SV}^{2}$.

We have seen that the lift that supports an airplane is equal to the product of the area of wing $S$, the square of the


WEIGHT
Fig. 19. Lift $=$ weight in horizontal flight.
velocity $V$, and a coefficient of lift $K_{\mathrm{L}}$ that varies with the angle of incidence:

$$
\text { Lift }=L=K_{\mathrm{L}} S V^{2} .
$$

For sustentation in horizontal flight, see Fig. 19, the lift must be just equal to the weight of the machine, including its load; we, accordingly, may write

$$
\text { Weight }=\underset{28}{W}=K_{\mathrm{L}} S V^{2} .
$$

Weight acts downward through the center of gravity or C. G.; lift acts upward through the center of lift or C. L. These two centers are never far apart, although they rarely coincide exactly. When the center of gravity is in front of the center of lift, there is a moment or couple tending to make the machine nose down; when the center of gravity is back of the center of lift, there is a couple tending to make the machine nose up. In neither case does the couple have any affect upon the value of weight and lift (which are equal), although it does affect longitudinal stability discussed later.

## Velocity equals square root of "loading" divided by square root of $\mathrm{K}_{\mathrm{L}}$.

The preceding equation, when transposed, gives the important formula for velocity in horizontal flight,

$$
V=\sqrt{\frac{W}{S K_{\mathrm{L}}}}
$$

This formula, although simple, is quite complete and will bear careful study, for it leads to a number of interesting conclusions, the appreciation of which is essential to a proper understanding of flight.

The ratio $W / S$ is the weight per unit area of wing and is called the loading. It is the loading, rather than weight or area, that affects the value of $V$. The formula shows that the velocity of an airplane depends solely upon the loading and upon $K_{\mathrm{L}}$. Here $W$ is the weight of the loaded machine.

The only possible way for changing the speed of a machine, or for getting different speeds in different machines, is by changing the loading or by changing the value of $K_{\mathrm{L}}$. In practice there are of course limits to both of these changes. For a machine in flight, in which the loading can not be
changed, the only way for changing the speed is by changing the value of $K_{\mathrm{L}}$ by a change in the angle of incidence.

The loading $W / S$ is commonly about 6 lbs . per sq. ft., being less in slow machines and being more ( 8 to io and even 12) in fast machines. The usual limits for $K_{\mathrm{L}}$ are about 0.0008 and 0.0032 .

This gives us at once the speed variation, shown in Fig. 20, for any machine irrespective of wing-section or other features of design except loading. Each curve in Fig. 20 is for a


Fig. 20. Variation of velocity with coefficient of lift for any airplane, when loading is 4,6 (heavy curve), 8 and io.
different loading. The heavy curve shows the change of speed, with $K_{\mathrm{L}}$, for a machine with loading $W / S=6$, this being the plot of the equation $V=\sqrt{6 \div K_{L}}$. The light curves are for loadings 4,8 , and io, an increase in loading raising the curve and thus indicating a greater velocity.

It is seen that velocity increases as $K_{\mathrm{L}}$ decreases; if $K_{\mathrm{L}}$ were zero, $V$ would have to be infinite to create a lift equal to the weight, as is necessary for sustentation.

Speed range.-A machine that has a maximum speed of 100 miles per hour and a minimum speed of 45 miles per hour is said to have a speed range of 55 per cent. The speed range is the difference betweenmaximumandminimum speed,
divided by the maximum speed. The range in speed depends upon the range in the value of $K_{\mathrm{L}}$.

Thus it is seen that, if we take the limits of $K_{\mathrm{L}}$ as 0.0008 and 0.0032 , the speed range for any machine is fifty per cent. The curves in Fig. 20 are drawn solid between these limits. Commonly the speed range is somewhat less than fifty per cent., but in some cases is a little more. (A minimum value of $K_{\mathrm{L}}$ a trifle less than 0.0008 may be attained in some instances; but a greater value, 0.0009 or more, is perhaps a more usual minimum.)

By way of illustration, with limits 0.0008 and 0.0032 for $K_{\mathrm{L}}$, a machine with loading $W / S=$ ro would have speed limits as follows, the speed range being 50 per cent.;

Maximum speed $=\sqrt{10 \div 0.0008}=112 \mathrm{MPH}$.
Minimum speed $=\sqrt{10 \div 0.0032}=56 \mathrm{MPH}$.
As a further illustration, with limits 0.00074 and 0.0034 for $K_{\mathrm{L}}$, the speed range would be about 54 per cent.; thus,

Maximum speed $=\sqrt{10 \div 0.00074}=116$ MPH.
Minimum speed $=\sqrt{10 \div 0.0034}=54 \mathrm{MPH}$.
In this case, speed range $=(116-54) \div 116=0.54$ or 54 per cent.

A greater speed can be attained only by increasing the loading or by decreasing $K_{\mathrm{L}}$. For example, if a speed of 200 MPH. is to be attained: $W / S$ must equal 29.6, if $K_{\mathrm{L}}=$ $0.00074 ; W / S$ must equal 20, if $K_{\mathrm{L}}=0.0005 ; W / S$ must equal ro, if $K_{\mathrm{L}}=0.00025$; etc.

The advantage of a large speed range*-not only a high

[^10]maximum speed for flying but also a low minimum speed for landing-is obvious. To get a sufficiently low landing speed the designer must choose a small loading; see Fig. 20.


Fig. 21. Variation of coefficient of lift with incidence for a particular wing-section; aspect ratio 6 . Velocities are marked for several points, assuming a loading of 6 lbs . per sq. ft.

A given machine has a definite velocity for each angle of incidence, and this is controlled by the elevator.
Since $K_{\mathrm{L}}$ has a different value for each angle of incidence, it is seen that the velocity of a machine varies with incidence
and, for a given loading, velocity depends only* upon incidence. The angle of incidence is controlled by the pilot by means of the elevator, as discussed later in Chapters X and XI.

The variation of $K_{\mathrm{L}}$ with incidence, for a particular wingsection, is shown in Fig. 21, reproduced from Fig. II of the preceding chapter. The numerical values from which the curve is plotted are given in Table I., page 20.

For any given loading, the value of $V$ corresponding to any point on this curve is readily determined. For example, suppose $W / S=6$. For an incidence of $4^{\circ}$, from the curve or table, $K_{\mathrm{L}}=0.00174$; hence, $V=\sqrt{6 \div 0.00174}=58.6$ miles per hour. The values of $V$, determined in this way, are marked for several points on the curve. It is seen that for each incidence, the velocity has one definite value. $\dagger$

[^11]Minimum velocity occurs at the point of maximum $\mathrm{K}_{\mathrm{L}}$, namely at the critical angle of incidence or burble point.
It is seen that the minimum velocity occurs when the value of $K_{\mathrm{L}}$ is a maximum, namely at the critical angle of incidence or burble point. For any other incidence, $K_{\mathrm{L}}$ is less and $V$ is correspondingly greater; in other words, when the angle of incidence is either greater or less than the critical angle, $a$ greater velocity is required in order to produce the lift equal to the weight, the condition necessary for horizontal flight.

## Variation of velocity with incidence.

The variation of velocity with incidence is well shown by the curves in Fig. 22, which correspond to Fig. 21 and refer, therefore, to a particular wing-section,-not to any wingsection as was the case in Fig. 20. Curves for the variation of velocity with incidence for other wing-sections would have much the same general form.

The heavy curve in Fig. 22 shows the velocity for a loading $W / S=6$; the light curves, for loadings of 4 and 8 ,-the greater loading always corresponding to the higher velocity.

It is thus seen that there is a definite minimum velocity which occurs at the critical angle of incidence-in this case $14^{\circ}$ -when $K_{\mathrm{L}}$ is a maximum, as was also shown in Fig. 21. If a machine loses velocity below this minimum, it cannot sustain itself and is said to stall,-the critical angle of incidence being also called the stalling angle.
somewhat raised,-for at large angles the lift of the airplane body becomes effective.

The lift curve for a complete machine may be determined by applying proper corrections to the lift curve of the wing (which is of course the chief factor) when data for these corrections is available; but it is best determined by a wind-tunnel test with a complete model. Some of these corrections are to be considered later in the chapter on Single and Multiple Planes.

The velocity increases on each side of this minimum, with change of incidence, so as to furnish the necessary sustenta-


Fig. 22. Variation of velocity with angle of incidence for particular wing-section, when loading is 4,6 (heavy curve) and 8 lbs . per sq. ft.. These curves correspond to the curve in Fig. 2I.
tion, and would have to be infinite were $K_{\mathrm{L}}$ equal to zero, namely if the incidence were decreased (in this case) to - $3^{\circ}$ or were increased to about $90^{\circ}$; but, as already pointed out, there are limits to the working range of incidence and velocity.

## Usual working range.

Between the limits of $\circ^{\circ}$ and $10^{\circ}$ (corresponding to $K_{\mathrm{L}}=$ 0.00091 and $K_{\mathrm{L}}=0.00287$ ) the curves are drawn as solid lines; these limits would become $-1 / 3^{\circ}$ and $121 / 2^{\circ}$, if the limits of $K_{\mathrm{L}}$ were taken as 0.0008 and 0.0032 as before. (It is understood that precise limits cannot be set.) This shows the usual working range: two or three degrees less incidence woud cause a dive; two or three degrees more incidence
would cause a stall. An inclinometer is commonly used to indicate (usually by a bubble) the inclination of the machine with the horizontal and the advantage of its use is obvious. A stall indicator is less frequently used to display a danger signal when the stalling angle is approached.

## Possibility of changeable wing-area or camber.

For mechanical reasons, wings are made with fixed area and camber. A practical wing with either of these adjustable would do much to advance the art of flying, for it would make possible a great increase in speed range, both by increasing the maximum and decreasing the minimum speed. These improvements have long been considered, the adjustable camber now seeming the more promising of the two.

With adjustable wing-area, the pilot would use large area for low speed and would use small area for high speed.

With adjustable camber, the pilot would use for low speed such camber as gave maximum lift. For high speed he would flatten out the wing and so get less lift without a dangerous reduction in incidence. This flattening of the wing would also bring about a reduction of wing-resistance, a highly important advantage at high speeds.

For the present, however, it is necessary to be content with wings of fixed area and camber.

## Power has no direct effect upon velocity.

It has been shown that the velocity of a machine depends directly upon incidence (ignoring the possibility of a change in wing-area or camber and the effect of altitude), incidence being controlled by the position of the elevator. It may well be asked: What about power? What effect upon velocity has the amount of power supplied by the engine? The
answer is: The power supplied by the engine has no direct effect upon velocity; for velocity is directly controlled by the elevator. Power has a direct effect upon the inclination of the flight path. The effect of power is shown in the following paragraphs.

Let us suppose, for example, a machine is flying with a certain angle of incidence-say, $4^{\circ}$-controlled by the position of the elevator. The velocity of flight is then definite,-58.6 MPH., if we use the data in Fig. 21. At a definite velocity and incidence, the resistance of the airplane (structure as well as wings) is definite-in a certain instance 263 lbs .-to overcome which there must be an exactly equal thrust ( 263 lbs .) requiring the supply of a certain amount of power (in this instance 42 horse power). But this definite amount of power that is required may or may not be supplied dy the engine, for this depends on the throttle. Let us see what happens when the engine does not supply this amount of power.

## Amount of power supplied by engine determines whether machine climbs, glides or flies horizontally; but, if incidence is not changed, does not affect velocity.

If the engine supplies just the right amount of power required to overcome the total air resistance, the machine flies horizontally, as in Fig. 23. If it supplies more power, the machine takes an oblique path upward, as in Fig. 24, the "surplus power" being used against gravity. If the engine supplies less power than is necessary to overcome resistance, the machine takes an oblique path downward, as in Fig. 25, the necessary additional power being in this case supplied by gravity.

Thus, if the power required to overcome the total air resistance is $42 \mathrm{H} . \mathrm{P}$., there are the three cases: when the engine delivers* 42 H.P., flight is horizontal; when it delivers


Fig. 23. Horizontal flight.
more power, 47 H.P. for example, the machine climbs, automatically taking a flight path inclined upward at such an angle that the surplus 5 H.P. is used in overcoming gravity; when the engine delivers 37 H.P., the machine takes an oblique path downward at such angle that 5 H.P. is derived from gravity. The angle of incidence-the angle between the chord and the relative air or flight path-being the same in the three cases, the velocity is substantially the same irrespective of whether the flight path is horizontal or slighly oblique.


Fig. 24. Oblique flight, upward. The same incidence and velocity as in Fig. 23.

It is seen that if power is slightly increased or decreased, by adjustment of throttle, the inclination of the flight-path is changed, but (provided the angle of incidence is not changed) velocity remains unchanged. Indeed, if the power is entirely cut off, the machine takes an oblique

[^12]flight-path downward at a definite gliding angle, while the velocity at small gliding angles remains practically unchanged. These relations will be more precisely discussed in Chapter VII on Power Relations in Flight and in Chapter VIII on Climbing and Gliding.

## When horizontal flight is to be maintained, velocity is changed by a simultaneous adjustment of throttle and elevator.

From the foregoing, it is seen that the one way to change velocity is to change the angle of incidence by means of the elevator; furthermore, if horizontal flight is to be maintained, the throttle must be adjusted at the same time so that the amount of power required for horizontal flight is supplied by the engine,-otherwise the flight path will be oblique. The pilot does not speed up and slow down merely by opening and closing the throttle, as in an automobile. As a matter of


Fig. 25. Oblique flight, downward. The same incidence and velocity as in Figs. 23 and 24.
fact an airplane is, in normal flight, practically a constant speed machine, flying usually at the one velocity corresponding to a certain best angle of incidence for which the machine is designed.

It takes more power to fly at low speed or at high speed than at an intermediate speed. The amount of power required to maintain horizontal flight, as shown in a later chapter, increases very rapidly when the velocity is either
increased or decreased beyond a rather narrow range. Power as well as stability is, accordingly, a factor-in many cases a determining factor-in deciding the range of velocity and the limiting values for the angle of incidence and for $K_{\mathrm{L}}$.

To find out how much power is necessary for horizontal flight, we must first know the thrust required and this is determined by the total resistance that is to be overcome. This will be investigated in the next chapter.

## Conditions of steady flight have been assumed.

The foregoing relation between velocity and incidence is the relation in steady fight, and is modified by the inertia of the machine when velocity is changing; it is assumed, furthermore, that the flight path is not greatly inclined with the horizontal. That is, the relation is modified by the inertia of the machine when velocity is changing, and is also modified when the flight path is far from horizontal, as in a dive.

After a change of elevator or throttle, there is a brief lapse of time before steady conditions of flight along the new flight path are reached. Thus, the immediate effect of opening the throttle is increased speed along the original flight path. The pilot may continue at this increased speed by proper elevator control; or, as already stated, he may use part or all of the added power for climb.

Although the relation between incidence and velocity is direct and simple, a general relation between elevator position and incidence or velocity can not be so simply stated. It is dependent upon several factors, including the setting of the horizontal stabilizer, the amount of slip stream, and the relative positions of the center of thrust and center of
resistance, which will be better understood after the chapter on Longitudinal Stability has been read.

## Flying attitudes.

The position of a machine with respect to its flight path or relative air is called its attitude. In all machines, as usually* constructed, the wings are rigidly fixed to the airplane body. The angle of incidence between the wings and the relative air can, therefore, be changed only by changing the attitude of the whole machine. This the pilot accomplishes by means of the elevator.

There are three attitudes in flight, as illustrated in Fig. 25 bis.

First, there is the normal attitude, in which the airplane


Fig. 25 bis. Three attitudes in horizontal flight.
flies with "even keel;" the airplane body lies in the line of the flight path and offers least resistance. This attitude gives a certain normal angle of incidence between the wingchord and relative air.

Second, there is the "tail-low" attitude, giving a larger angle of incidence; and, thirdly, the "tail-high" attitudegiving a smaller angle of incidence.

[^13]In the tail-low attitude, the air pressure on the body adds to the lift; in the tail-high attitude, it subtracts from it.

A very decided tail high or tail low attitude should be avoided by proper balancing and by making slight adustment in the relative location of surfaces and body by tighting or loosening the proper wires. This process is called tuning $u p$. The setting of the horizontal stabilizer is commonly made adjustable for this purpose.

## CHAPTER III

## RESISTANCE

Resistance is the force that impedes the progress of an airplane through the air, this force being in the same direction as the air-stream, and opposite to the direction of flight, as

[Fig. 26. Resistance is in direction of the air-stream and is overcome by thrust.
shown in Fig. 26. Resistance is overcome by thrust from the propeller, a force in the direction of flight, or nearly* so.

In uniform flight a machine assumes such a velocity and attitude that resistance and thrust are exactly equal.

## Center of resistance.

Thrust is a force forward through the propeller shaft, applied at the center of thrust or C. T. The total resistance of an airplane, the resistance of wings and structure all included, may be considered as a single force backward in the direction of the air-stream applied at the center of resistance or C. R., which may coincide with the center of thrust as in Fig. 26, or may be a little above or below it. When the

[^14]center of resistance is above the center of thrust, there is a tendency for the machine to nose up when power is on, and when the center of resistance is below the center of thrust there is a tendency for it to nose down. This affects longitudinal stability but in no way affects the value of resistance or thrust.

## Wing-resistance and parasite resistance.

Airplane resistance falls under two heads: wing-resistance, due to the wings; and parasite resistance, due to all other parts of the airplane structure. (Parasite resistance is sometimes called "structural resistance" or "head resistance.") The sum of the two is the total resistance or drag; thus,

Total Resist. $=$ Wing-resistance + Parasite Resistance. In English units, resistance is expressed in pounds.

Although expressed by very similar fundamental formulas, each varying as the square of the velocity, wing-resistance and parasite resistance have quite different characteristics:-wing-resistance depends upon incidence, the variation of the coefficient of wing-resistance $K_{\mathrm{D}}$ with incidence having been shown in Fig. in, page 18, of the first chapter; parasite resistance, on the other hand, is practically independent of incidence, except so far as incidence affects velocity.

Whereas parasite resistance always increases as velocity is increased, wing-resistance decreases as velocity is increased until a certain velocity is reached, after which it increases, as brought out in the following discussion.

On account of their different characteristics, wing-resistance and parasite resistance are considered separately.

Wing Resistance

## Fundamental relation.

Wing-resistance is equal to $K_{\mathrm{D}} S V^{2}$, as shown in the first chapter, and is fundamentally determined by this formula. A hasty inspection of the formula, however, might lead to the erroneous conclusion that wing-resistance always increases with velocity. This indeed would be true, if $K_{\mathrm{D}}$ were constant; but in fact, as mentioned above, the changes in the value of $K_{\mathrm{D}}$ with incidence actually cause wing-resistance to decrease through a certain range of velocities, and then to increase at high velocities. This is best shown by some practical calculations and the plotting of curves.

## Practical calculation.

Wing-resistance can be calculated directly from the formula $K_{\mathrm{D}} S V^{2}$. The calculation, however, can be made more readily from the $L / D$ ratio for the particular wing in question. By definition

$$
L / D \text { ratio }=\text { Lift } \div \text { Wing-resistance. }
$$

Since Lift $=$ Weight, in horizontal flight, we may write

$$
\text { Wing-resistance } *=\text { Weight } \div L / D \text { ratio. }
$$

To get the wing resistance, it is merely necessary to divide the known weight by the $L / D$ ratio, the value of this ratio being taken from a table or curve for the particular wing section.

[^15]Example.-For example, the weight of a loaded machine is 2000 lbs. The values for the $L / D$ ratio for the wing used are given by the Table I., page 20, or by the corresponding curve, Fig. 12, page 19. Required to find the wing-resistance and velocity for an incidence of $4^{\circ}$. From the table, $L / D=16$ (approximately); hence

$$
\text { Wing-resistance }=2000 \div \mathrm{r} 6=125 \mathrm{lbs} .
$$

It is seen that every 16 lbs . of weight adds one pound to the wing-resistance and will require one pound more thrust, and a corresponding increase in power, to overcome it.

To determine the velocity corresponding to the wingresistance in the above example, it is necessary to know the loading, for velocity depends upon loading. Thus, if the loading is $W / S=6 \mathrm{lbs}$. per sq. ft., using the formula of the preceding chapter, we have

$$
\text { Velocity }=\sqrt{\frac{W}{S K_{\mathrm{L}}}}=\sqrt{6 \div 0.00174}=58.6 \mathrm{MPH}
$$

It is thus seen that when this particular wing-section is used in a machine weighing 2000 lbs ., with loading 6 lbs . per sq. ft., the wing-resistance is 125 lbs . at a velocity of 58.6 MPH.

## Variation of wing-resistance with velocity.

The variation of wing-resistance with velocity is well shown by the curves in Figs. 27, 28, and 29, the points being calculated, in the manner just described, for different conditions of weight and loading.

It is seen that with increase of velocity (decrease of incidence) wing-resistance always decreases until a certain velocity is reached, after which it again increases. The minimum velocity for any wing-section is obtained at the critical angle of incidence; a greater angle of incidence is
beyond the range of practical flight. The critical angle for the particular wing-section here used is $14^{\circ}$, but the curves have been calculated beyond $14^{\circ}$ in some instances and are shown by dotted lines.


Fig. 27. Variation of wing-resistance with velocity. The three curves show effect of changing weight ( $W=1000,2000$ and 3000 lbs .) when loading is kept constant ( $W / S=6$ ).

As incidence is decreased to $0^{\circ},-r^{\circ},-2^{\circ}$, etc., wing-resistance and velocity both rapidly increase and both would become infinite at the incidence that gives zero lift,-in this case at $-3^{\circ}$.

## Effect of changing weight or loading.

In calculating the curves here shown, it was necessary to know the weight and loading,-inasmuch as wing-resistance $=$ weight $\div L / D$, and velocity $=\sqrt{\text { loading } \div K_{\mathrm{L}}}$. Different curves for the variation of wing-resistance with velocity are accordingly obtained by changing either weight or loading, or both, and in no other way,-it being understood that we are dealing with a particular wing-section flying in


Fig. 28. Variation of wing-resistance with velocity. The three curves show effect of changing loading ( $W / S=4,6$ and 8 ) when weight is kept constant ( $W=2000 \mathrm{lbs}$.).


Fig. 29. Variation of wing-resistance with velocity. The three curves show effect of changing weight ( $W=$ I 333 , 2000 and 2666 lbs .) and loading ( $W / S=4,6$ and 8 ) in proportion, wing-area being constant.
air of constant density. It will be found that all the curves are similar in form and differ only in scale.

There are three cases:
(I) when weight is changed and loading is kept constant;
(2) when loading is changed and weight is kept constant;
(3) when weight and loading are both changed.
(1) Effect of changing weight when loading is kept constant.

In this case machines with greater weight also have greater wing area, the loading remaining constant. Wing-resistance for different machines is then directly proportional to weight. This is shown by the curves in Fig. 27 for three machines of different weight. Any point on these curves, corresponding to a particular incidence and velocity, is merely moved up for a heavier, or down for a lighter, weight machine; the heavier the machine, the greater is the wing-resistance.
(2) Effect of changing loading when weight is kept constant.

In this case weight is constant and wing-area is changed so as to give different loadings. Changing the loading, for a certain weight, changes the velocity corresponding to a certain incidence, but does not change the amount of wingresistance, for that incidence. Hence, as shown by the curves in Fig. 28, a change of loading shifts to right or to left the point corresponding to a particular angle of incidence, the velocity for that incidence being proportional to the square root of the loading.

The loading which gives the least wing-resistance is different at different velocities. Thus, for the case shown by the three curves in Fig. 28 with loading 4,6 and 8, up to about 55 miles per hour the wing-resistance is least for a loading of 4 ; from 55 to 65 miles per hour, for a loading of 6 ; and above 65 miles per hour, for a loading of 8 .

## (3) Effect of changing both weight and loading, wing-area being constant.

This is a combination of the two preceding cases. A point on any of the curves is moved up or down in proportion to weight, and to right or left in proportion to the square root of the loading.

The three curves in Fig. 29 show the effect of changing weight and loading in proportion, wing-area remaining constant; this might be brought about by taking up the same machine at different times with different loads. It is to be noted that, at the same angle of incidence, greater velocity is required to sustain the greater weight; or, at the same velocity, a greater angle of incidence is required.

## Variation of wing-section.

For a given wing-section there are the three possible ways just described for changing wing-resistance,-by changing the weight, the loading or both. If the wing-section is varied, the number of possible variations is infinite. A wing that has high camber in order to secure great lift, also has large resistance, particularly, at small angles of incidence; while, as already mentioned, a flatter wing with less camber and less lift is better adapted for high speed, having small resistance. But there are many intermediate forms and variations that make an interesting field for study.

## Parasite Resistance

## Meaning and importance of parasite resistance.

The wings of an airplane are its first essential, for they create the lift, but in creating lift they at the same time cause a wing-resistance. Wing-resistance, therefore, although not a cause of lift, is seen to be a necessary concomitant being
the price paid for the lift. In a preceding example, it was shown that, in a given case, every sixteen pounds of lift must be paid for by one pound of wing-resistance.

Unfortunately this is not all. In addition to wings an airplane must have other parts-body, landing gear, struts, wires, etc.,-all of which have a resistance; but unlike the wings, these parts do not contribute to the lift. The resistance of these parts is, therefore, with some appropriateness called parasite resistance.

One of the important problems in design is to make this parasite resistance as low as possible, for while small at low


Fig. 30. Air-flow past a cylindrical strut; air eddies and low pressure back of strut cause high resistance.
velocities parasite resistance is very great at high velocities, being perhaps fifty per cent. more than wing-resistance at ordinary maximum flying velocities. In airplane flight, while about two-fifths of the power delivered through the propeller by the engine is used in pushing the wings through the air, three-fifths of the power, approximately, is used up in parasite resistance. It is seen that parasite resistance is the biggest obstacle to high-speed flight.

## Streamline flow.

Fig. 30 shows the flow of air past a cylindrical strut or wire. Behind the strut there is a turbulent space and a partial vacuum, or negative pressure $n$, that tends to suck the strut
along in the direction of the air-stream. A large part of the resistance of the strut to motion through the air is thus due to this region of low pressure behind it.

In Fig. 31, the space behind the cylindrical strut $A$ is partly filled by a piece $B$, so as to reduce the region of tur-


Fig. 31. Cylindrical strut $A$, backed up by a filler $B$ of a form that reduces, but does not entirely eliminate, the air eddies and low pressure back of strut. Resistance is thus reduced.
bulence and low pressure. The resistance to motion through the air is thus greatly decreased. Struts are sometimes made in this manner, a piece $A$ for strength being backed up by a piece $B$, of light material to save weight, and resistance may be reduced in this way.

Although the shape shown in Fig. 3 I is an improvement on the cylindrical strut, it is by no means the best, for it does not


Fig. 32. Strut with air eddies and low pressure back of strut further reduced, by approaching a streamline form.
conform entirely to the streamline flow; there is still a turbulent region $n$, although this is much reduced. The more nearly a body conforms to streamline flow the less is its
resistance, the turbulence and suction back of the body being then reduced to a minimum. Fig. 32 shows a strut more nearly conforming to streamline flow.

Only a little is gained by tapering the front side of a cylinder or strut. Note the blunt breast and tapering tail of a bird, and the shape of a fast swimming fish that can dart through the water with scarcely a ripple.

For low resistance, wheels and body should be enclosed; these, as well as every strut and wire, should be streamlined so far as they can be. It should be remembered that a small cylindrical wire may offer much more resistance than a larger wire that is well streamlined.

## Parasite resistance varies as the square of the velocity.

The law for parasite resistance is summed up in the statement, determined by experiment, that parasite resistance varies as the square of the velocity. This applies not only to the separate parts* but also to an airplane as a whole. Thus, if a certain airplane has a parasite resistance of 64 pounds when flying at 40 miles per hour, it will have a resistance of 256 pounds at 80 miles per hour. In this case the parasite resistance is $0.04 \mathrm{~V}^{2}$.

The curves in Fig. 33 show the values for parasite resistance at different velocities for three cases, $R=0.02 V^{2}, R=0.04 V^{2}$

[^16]and $R=0.06 V^{2}$, these illustrating the variation for a certain range of machines of moderate size.

## Distribution of parasite resistance.

Parasite resistance is roughly distributed about as follows: -body, one-third; wires and struts, one-third; tail and


Fig. 33. Variation of parasiteresistance with velocity; the three curves show $R=0.02 V^{2}, R=0.04 V^{2}$ and $R=0.06 V^{2}$.
landing gear, one-third (about one-sixth for tail and one-sixth for landing gear).

Increase of parasite resistance due to propeller slip stream.
Back of the propeller the air is driven backward in what is called the slip stream or propeller race; in this slip stream
the velocity of the air relative to the airplane is increased, say, 20 or 25 per cent. $V^{2}$ is thus increased about 50 per cent. The parasite resistance of the tail and all parts of thestructure in the slip stream is accordingly increased, let us say, 50 per cent. when the propeller is running. Approximate calculations may be made on this basis. Another approximate method is to consider that the increase of the total parasite resistance due to the propeller slip stream is io per cent. To get accurate results, careful computation would be necessary.

A vast amount of data has been accumulated, and is becoming more and more available, for the resistance of radiators, wheels, struts, wires, etc., of different forms and sizes.

## Detrimental surface.

The structure of an airplane (other than wings) may, so far as parasite resistance is concerned, be replaced by a socalled "equivalent flat plate" or "detrimental surface" $s$, perpendicular to the flight path. In terms of $s$,

Parasite resistance $=k s V^{2}=0.003 s V^{2}$, approximately.
Here $k$ is approximately 0.003 , when $V$ is in miles per hour and $s$ is the detrimental surface in square feet.

The total $k s$ for a complete machine is the sum of the values of $k s$ for each part,-radiators, struts, etc. Obviously every effort is made to make the total $k s$ as small as possible.

The curves shown in Fig. 33 represent the parasite resistance when $k s$ equals $0.02,0.04$ and 0.06 , corresponding to $a$ detrimental surface $s$ equal to $6.6,13.3$ and 20 sq. ft., respectively.

## Note on terminology.

When there is no parasite resistance, as in case of an aerofoil alone, the whole resistance is wing resistance and is
commonly termed drag (formerly drift); the $L / D$ ratio being termed the lift/drag ratio (formerly lift/drift). There is no confusion as to the meaning of drag in this case.

In the case of a complete machine, however, there has been difference in usage, drag being used to refer to wing-resistance, alone, as well as to the total resistance. To avoid possibility of confusion, the term wing-resistance has been used throughout this book, being perfectly clear, although cumbersome.

## Some Conclusions

Three lines for improvement are suggested by the foregoing discusions:-
(1) Reduce parasite resistance.
(2) Reduce weight.
(3) Improve wing-section, so as to get a greater $L / D$ ratio.

Wing-resistance, which is equal to weight $\div L / D$, is reduced by (2) and (3). Most important are (1) and (2), as the improvement that can be made in $L / D$ ratio is probably rather small.

In design every effort should be made to reduce weight and to cut down parasite resistance.

The chief weight is in the engine and the reduction of weight is largely a problem for the engine designer. Reduction in parasite resistance is to be looked for in improved design of structure.

## CFAPTER IV

## THRUST REQUIRED

In the preceding chapter it is explained that the total resistance which impedes an airplane in its flight is equal to the sum of the wing-resistance and the parasite resistance, curves being given to show how each of these component parts of the resistance varies with the velocity of the machine relative to the air. Resistance acts in the direction of the air-stream and is overcome by thrust acting in the opposite direction, as shown in Fig. 26 on page 43. Thrust is obtained from the propeller, as discussed later in Chapter VI, and varies both with engine speed and the velocity at which the airplane is moving through the air.

The amount of thrust necessary to maintain horizontal flight at any given velocity is determined by the airplane resistance at that velocity.

## In horizontal flight thrust is equal to resistance.

Uniform horizontal flight can be maintained only when the thrust* and the opposing resistance are exactly equal. If the two are not equal, the machine assumes an inclined flight path (as explained $\dagger$ in Chapter II). The machine may be restored to its horizontal path by elevatur control and corresponding change of incidence, but in this case the machine assumes a different velocity,-a velocity at which the resistance is just equal to the thrust.

[^17]Thrust and resistance may be made equal in another way, namely, by increasing or decreasing the thrust from the propeller by speeding up or slowing down the engine by throttle control. In any event, whether accomplished by elevator or by throttle control-or by both-the flight path is maintained horizontal by keeping thrust and resistance exactly equal. Inclined flight will be discussed later.

For uniform, horizontal flight, therefore, we may write: Thrust $=$ Total Resistance
$=$ Wing-resistance + Parasite Resistance.

## Variation of thrust with velocity.

Curves showing the variation of wing-resistance and parasite resistance with velocity have already been given. The curve for the variation of total resistance or thrust with velocity is simply found, therefore, by adding the ordinates of the two curves, as in Fig. 34.

It is seen that at low velocities parasite resistance is very small, being much less than wing-resistance and of little consequence. As velocity increases (and angle of incidence decreases) wing-resistance at first decreases and then increases; on the other hand, parasite resistance steadily increases as the square of the velocity so that, except at low velocities, parasite resistance is greater than wing-resistance,-perhaps fifty per cent. greater under usual flying conditions.

## Minimum thrust or resistance.

The total resistance of an airplane has a minimum value at a certain definite velocity and angle of incidence, as shown by the curve in Fig. 34. It will be shown later that this incidence and velocity will give the "best glide,"-the


Fig. 34. Total resistance or thrust is the sum of wing-resistance and parasite resistance. The curves represent case when $W=2000 \mathrm{lbs} . ; W / S=6 ;$ parasite resistance $=0.04 \mathrm{~V}^{2}$.
longest glide and smallest gliding angle when the power is cut off. (See a later chapter, under Gliding, where it is shown that the sine of the gliding angle is equal to $R / W$; hence, when $R$ is a minimum the gliding angle is a minimum.)

## Thrust as affected by weight, loading and parasite resistance.

In the preceding chapter it was shown that for a given wing-section there are three possible changes that affect wing-resistance, namely, change of weight, change of loading and change of both weight and loading. Any increase or decrease in wing-resistance, due to these changes, makes a corresponding increase or decrease in the total resistance or thrust. So, likewise, does any change in parasite resistance.

There are, accordingly, four changes affecting thrust, as shown in the following paragraphs.

## (1) Effect on thrust of change of weight when loading is constant.

Fig. 35 (corresponding to Fig. 27) shows the variation of thrust with velocity of machines of three different weights with loading constant. The greater weight requires the greater thrust, but (on account of the parasite resistance being assumed constant) the increase of thrust is not in proportion to the increase in weight. It will be noted that the points for minimum thrust (corresponding to best glide) for the three curves, do not occur at the same velocity and incidence.

## (2) Effect on thrust of change of loading when weight is

 constant.In this case, wing-area is changed and the weight is constant so as to give different loadings. The effect of this


Fig. 35. Variation of thrust with velocity. The three curves show effect of changing weight ( $W=1000,2000$ and 3000 lbs .) when loading is kept constant ( $W / S=6$ ); parasite resistance $=0.04 \mathrm{~V}^{2}$.
change upon thrust is shown by the three curves in Fig. 36, which corresponds to Fig. 28 in the preceding chapter. Generally speaking, it is desirable to employ a loading that requires the least thrust. It is seen that the best loading depends upon the velocity, a greater loading being best at higher velocities. Thus, for the cases here shown with loadings 4,6 and 8 lbs . per sq. ft., a loading of 4 is best (approximately) up to 55 miles per hour, a loading of 6 up to 65 miles per hour and a loading of 8 for higher velocities.
(3) Effect on thrust of change of weight when loading is changed in proportion to weight, wing-area being constant.
The three curves in Fig. 37 (corresponding to Fig. 29) show the effect upon thrust when the same machine is flown with loads of different weight.

If the same velocity is to be maintained, it is seen from the curves that, when weight is increased, a greater thrust is required and it is necessary to fly at a larger angle of incidence. The larger incidence, which increases the lift, also increases the resistance and so requires greater thrust.

On the other hand, if the machine is to be flown at a certain constant angle of incidence, when the weight is increased it must fly at a higher velocity. Obviously a higher velocity is necessary to support the increased weight.

## (4) Effect on thrust of change of parasite resistance.

When parasite resistance is increased, the total resistance and thrust are correspondingly increased, as shown by the three curves in Fig. 38 (corresponding to Fig. 33).

In all the four cases discussed above and illustrated in Figs. 35-38, the conditions which give the least resistance (best glide) may be noted. The velocity for minimum


Fig. 36. Variation of thrust with velocity. The three curves show effect of changing loading ( $W / S=4,6$ and 8 ) when weight is kept constant ( $W=2000 \mathrm{lbs}$.); parasite resistance $=0.04 V^{2}$.


Fig. 37. Variation of thrust with velocity. The three curves show effect of changing weight ( $W=1333$, 2000 and 2666 lbs .) and loading in proportion ( $W / S$ $=4,6$ and 8 ), wing-area being constant; parasite resistance $=0.04 \mathrm{~V}^{2}$.


Fig. 38. Variation of thrust with velocity. The three curves show effect of changing parasite resistance ( $R=0.02 V^{2}, R=0.04 V^{2}$ and $R=0.06 V^{2}$ ); weight and loading constant ( $W=2000 \mathrm{lbs}$; $W / S=6$ ).
resistance is changed by any change of weight, loading or parasitic resistance. The velocity for minimum resistance is further discussed in connection with Fig. 43a in the next chapter.

## Inclination of propeller thrust.

Although propeller thrust is always in the general direction of airplane flight, only rarely is it precisely in that direction. In fact, as the attitude of a machine changes while flying, the direction of the thrust with respect to the flight path changes. The direction of thrust is along the propeller axis and becomes inclined up or down as the machine noses up or noses down.

In horizontal flight, thrust may then be horizontal, inclined upward or inclined downward, the three cases being illustrated in Figs. 38a, 38b and 38c. The two air forces, resistance $R$ and lift $L$, may in each case be combined into a single force $F$ as shown; while thrust $T$ and weight $W$ may, in a like manner; be combined into a single equal and opposite force $F^{\prime}$. The total thrust may be considered as made up of two components, not shown in the diagrams: a horizontal component $T_{\mathrm{H}}$ along the flight path, equal and opposite to resistance (this component is horizontal in horizontal flight only) and a vertical component, $T_{\mathrm{V}}$, perpendicular to the flight path.

When the thrust is horizontal (in horizontal flight) it is seen, Fig. 38 a , that $T=R$ and $L=W$, there being no vertical component to the thrust.

When the thrust is inclined upward, Fig. 38 b, the vertical component $T_{\mathrm{V}}$ aids the lift in supporting the weight; more weight can be carried with the same lift, or the same weight can be carried with less lift. Thus,

$$
W=L+T_{\mathrm{v}} \quad ; \quad L=W-T_{\mathrm{v}} .
$$

When the thrust is inclined downward, Fig. 38c, the vertical component $T_{\mathrm{V}}$ opposes the lift; either less weight can be supported or more lift must be obtained. Thus,

$$
W=L-T_{\mathrm{v}} \quad ; \quad L=W+T_{\mathrm{v}}
$$

From the foregoing, it is seen that inclining the thrust slightly upward reduces the thrust necessary to support a

given weight. It can be shown* that the ratio of thrust to weight, $T / W$, is a minimum when the thrust is inclined upward at a small angle whose tangent is $R / L$ so that the

[^18]thrust is perpendicular to the total resultant air force $F$. For a larger angle of inclination $T / W$ increases, until $T / W=\mathrm{I}$, when the thrust is vertical as in a helicopter. As an example, if $R / L=0.14$, the ratio of thrust to weight is a minimum when the thrust is inclined upward 8 degrees. (Tangent $8^{\circ}=0.14$. )

Calculation of velocity.-The fundamental aerodynamic equation for lift* is $L=K_{\mathrm{L}} S V^{2}$, whence, by substituting $W \pm T_{\mathrm{V}}$ for $L$, we have for velocity in horizontal $\dagger$ flight

$$
V=\sqrt{\frac{W \pm T_{\mathrm{v}}}{S K_{\mathrm{L}}}} .
$$

When $T_{\mathrm{V}}$ is small, or zero, the formula takes the simpler form $\sqrt{\frac{W}{S K_{\mathrm{L}}}}$, used in calculating the curves in this chapter and elsewhere. It should be borne in mind, however, that $T_{\mathrm{V}}$ is zero only for one particular angle of incidence.

## Airplane thrust compared with traction coefflcient of other vehicles.

The thrust necessary for propelling an airplane is much greater than the "traction coefficient" or "draw-bar-pull" of other vehicles of locomotion. In an airplane, wingresistance alone is, let us say, from 6 per cent. to io per cent.

[^19]of the weight of the machine (corresponding to $L / D=16.6$ to ro). To this is added the parasite resistance, making the total resistance or thrust, in some common types of airplane, from io per cent. to 15 per cent. of the weight; that is $R / W$ $=0.10$ to 0.15 .

The traction coefficient of water craft is less than r per cent. The traction coefficient of trains on level rails is between I and 2 per cent., and of road vehicles on level pavement only a little more. (These values, however, are increased by a grade or bad road surface so as to equal or exceed io or 15 per cent.)

It is seen that the airplane can not compete with other vehicles in regard to low thrust or traction coefficient. It is to be borne in mind, however, that the airplane travels at a much higher velocity than other vehicles and that, for the airplane, the air is a universal right-of-way, extending everywhere, always ready without cost of construction or of maintenance. These advantages make the airplane superior for certain purposes.

The resistance of a dirigible balloon has been reported as between 4 and 6 per cent.

## CHAPTER V

## POWER REQUIRED

In this chapter will be considered the power required to drive an airplane through the air in horizontal flight at different velocities. In subsequent chapters will be considered the power available from the propeller and engine, the relation between the power required and the power available, and the effect of altitude on power.

## Calculation of power required.

When the total resistance to an airplane in flight is known, the thrust that is necessary to overcome the resistance is known, as discussed in the preceding chapter. The power required to produce this thrust can then be readily calculated, for power is proportional to the product of thrust and velocity. Thus, when thrust is expressed in pounds and velocity is expressed in miles per hour,

$$
\text { Horse Power* }=\frac{\text { Thrust } \times \text { Velocity. }}{375}
$$

This means that each pound of resistance requires $\mathrm{I} / 375$ H.P. to overcome it for each mile per hour of velocity.

## Variation of power required with velocity.

In the preceding chapter, the curves in Figs. 34-38 show the variation with velocity of the thrust required for horizontal flight. The corresponding curves for power required, cal-

[^20]

Fig. 39. Power required at different velocities, when $W=2000 \mathrm{lbs} . ; W / \mathrm{S}=6$; parasite resistance $=0.04 \mathrm{~V}^{2}$.
culated from the thrust curves in the manner just described, are shown in Figs. 39-43.

The curve for the power required by an airplane in horizontal flight is seen to be a U-shaped curve having a definite minimum value at a certain velocity and corresponding angle of incidence. It is interesting to note, therefore, that when an airplane is flying at a certain velocity,-namely, the velocity that requires minimum power-more power will be required to fly either slower or faster.

Under usual conditions, all flying is at a higher velocity than that which corresponds to minimum power and is at a smaller angle of incidence. In this usual working range, therefore, in order to go faster in horizontal flight the pilot opens the throttle and decreases the incidence; to go slower, he closes the throttle and increases the incidence.

When flying at a very low velocity, however,-a velocity lower than that corresponding to minimum power,-the conditions of control are reversed, so far as the throttle is concerned. To go slower in horizontal flight, more power is required; the pilot must open the throttle and at the same time increase the angle of incidence. These conditions (flying with low velocity and large angle of incidence and with reversed throttle control) are not usual flying conditions; under such conditions, special care on the part of the pilot is necessary, for a stall is imminent and, furthermore, the airplane loses much of its stability.

The relations between power required, velocity and angle of incidence will be more fully considered later in Chapter VII, after the chapter on Power Available.


Fig. 40. Variation of power required with velocity. The three curves show effect of changing weight ( $W=1000,2000$ and 3000 lbs .) when loading is kept constant $(W / S=6)$; parasite resistance $=0.04 \mathrm{~V}^{2}$


Fig. 41. Variation of power required with velocity. The three curves show effect of changing loading ( $W / S=4,6$ and 8 ) when weight is kept constant ( $W=2000 \mathrm{lbs}$.); parasite resistance $=0.04 \mathrm{~V}^{2}$.


Fig. 42. Variation of power required with velocity. The three curves show effect of changing weight ( $W=1333,2000$ and 2666 lbs .) and loading in proportion ( $W / S=4,6$ and 8 ), wing-area being constant; parasite resistance $=0.04 \mathrm{~V}^{2}$.


Fig. 43. Variation of power required with Jvelocity.
The three curves show effect of changing parasite resistance ( $R=0.02 V^{2}, R=0.04 V^{2}$ and $R=0.06 V^{2}$ ); weight and loading constant ( $W=2000 \mathrm{lbs}$.: $W / S=6$ ).

## Power as affected by weight, loading and parasite resistance.

The three curves in Fig. 40 (corresponding to Figs. 35 and 27) show how power required is affected by change of weight, when loading is constant. For the same velocity, the heavier machine requires more power; for the same power, the heavier machine flies at less velocity and at a greater angle of incidence.

The curves in Fig. 41 (corresponding to Figs. 36 and 28) show the effect on power of change of loading when weight is constant. For each velocity, there is a certain loading that requires least power. At low velocities, a small loading is best; while at higher velocities the least power is required by a greater loading, that is, by a smaller wing-area.

The curves in Fig. 42 (corresponding to Figs. 37 and 29) show the power required when the same machine is flown with three different weights, the loading in each case being in proportion to the weight. For any given angle of incidence, the greater weight requires, to sustain it, a greater velocity and more power.

The change of power in the preceding three cases was brought about by a change of wing-resistance due to a change of weight or loading or both; parasite resistance at each velocity was constant. The curves in Fig. 43 (corresponding to Figs. 38 and 33) show how the power required is increased by an increase of parasite resistance; in this case the angle of incidence and the wing-resistance at each velocity are constant.

Minimum thrust* is shown by the point of tangency on the power curve of a straight line drawn from the origin.

From the first paragraph of this chapter it is seen that

$$
\text { Thrust }=375 \times \frac{\text { Horse Power }}{\text { Velocity }}
$$

Let $a^{\prime \prime}$ be any point on the curve for power required, Fig. 43a. The corresponding thrust required, being proportional


Fig. 43a. Any line oa drawn from the origin intersects the power curve at points of equal thrust, $a^{\prime}$ $a^{\prime \prime}$. Minimum thrust is obtained at $t$, where the line $o b$ is tangent to the power curve.
to horse power divided by velocity, is evidently proportional to the tangent of the angle aoc. It follows that, if a straight line $o a$ is drawn from the origin so as to intersect the power curve at two points, $a^{\prime}$ and $a^{\prime \prime}$, the thrust required at these two points will be equal.

[^21]If the line is swung downward, so that the angle $a o c$ is smaller, the correspoeding thrust becomes smaller. Minimum thrust is required at the point $t$, at which a straight line $o b$ drawn from the origin is tangent to the curve.

It is seen that for minimum thrust $t$, an airplane must fly at a higher velocity and a smaller angle of incidence than for minimum power $p$. As discussed later under Gliding, $t$ also shows the condition for longest glide and minimum gliding angle.

## Minimum expenditure of energy is at point of minimum thrust.

In flying a distance $d$ the total energy expended is $t \times d$, being proportional to the product of force and distance. In flying a given distance, the energy expended is, accordingly, proportional to thrust and is a minimum when thrust is a minimum.

The greatest economy in energy expenditure is obtained, when flying a certain distance, by maintaining such a velocity that the thrust is a minimum. On the other hand, when flying for a certain length of time, the greatest economy is obtained by flying at such a velocity that the power required is a minimum. Whether the greatest economy in fuel consumption is obtained under these conditions will depend not only upon the aerodynamic considerations here considered, but also upon the characteristics of the engine and propeller.

It is seen that a lower or higher velocity, than the one corresponding to $t$, requires a greater expenditure of energy. A lower velocity is generally undesirable; but a somewhat higher velocity, even when paid for by a greater energy consumption, is often worth while.

At a lower velocity than at $t$, a glance at the curve shows a small increase in power gives a large increase in velocity; at $t$ an increase in power is accompanied by a proportionate increase in velocity; while at a higher velocity than at $t$, a large increase in power is necessary to obtain a small increase in velocity. Although it may be worth while to fly at velocities somewhat higher than at the economical point $t$-at some point as $a^{\prime \prime}$, for example,-it is a question whether it is worth while to go to much higher velocities, for the power required increases more and more rapidly as velocity is increased.

The highest total efficiency, as well as aerodynamic efficiency, is obtained at $t$, if engine and propeller are designed for maximum efficiency at this point. Thus far the power plant and the amount of power available have not been considered. These will be next discussed.

## CHAPTER VI

## POWER AVAILABLE FROM THE AIR PROPELLER AND THE AIRPLANE ENGINE

The power required for airplane flight at different velocities depends upon airplane structure, varying as the product of airplane velocity and the total airplane resistance. Neither the source of power nor the amount of power available is involved in the determination of power required, which is quite independent of propeller and engine. Curves for power required are discussed in the preceding chapter.

On the other hand, the power that is available for driving the airplane forward, called power available or thrust horse power, depends entirely upon the air propeller and the airplane engine. The power available is derived directly from the thrust of the propeller, being proportional to the product of propeller thrust and the forward velocity of the airplane; in horse power, it is equal* to the product of thrust in pounds and velocity in miles per hour, divided by 375 . The power necessary for driving the propeller in order to obtain this thrust is supplied by the airplane engine. Although the propeller and engine perform their separate functions, the conditions of operation of the one are dependent upon the operating characteristics of the other, for the power the propeller absorbs must exactly equal the power the engine delivers and the speed of rotation of the one is necessarily the speed of rotation of the other when directly connected.

Although no complete discussion of the airplane engine can be here undertaken, an outline will be given of its chief characteristics that have a bearing on the power available

[^22]in airplane flight. This will be followed by a more complete discussion of the air propeller, its Conditions of Operation, its Characteristics and its Theory.

## The Airplane Engine

The gas engine is the airplane engine in general use on account of light weight.
The internal combustion engine-or the "gas engine" as it is more popularly known-made flight possible; and, although some* of its characteristics are not the best for airplane flight, it meets the important requirement of low weight per horse power. Each year airplane engines have been made of greater power and less weight per horse power, a great advance $\dagger$ being made by the Liberty motor (igi8) which gave 450 H.P. with a weight of only 1.8 lbs . per H.P.

[^23]| Year | 1903 | 1910 | 1914 | 1915 | 1916 | 1917 | 1918 |
| :--- | ---: | ---: | :---: | :---: | :---: | :---: | :---: |
| Horse power | 12 | 54 | 112 | 133 | 185 | 243 | 450 |
| Weight, lbs. | 152 | 309 | 437 | 512 | 570 | 693 | 825 |
| Lbs. per H. P. | 12.7 | 5.7 | 3.9 | 3.8 | 3.1 | 2.8 | 1.8 |

To obtain greater power than can be obtained from a single engine, several engines (and usually as many propellers) are employed. As many as six engines, developing a total of 3000 H . P. have thus been used. The N C 4 seaplane, in the first trans-Atlantic flight (1919), was equipped with four Liberty motors.

Although other types of engine may hereafter be intro-duced,-possibly in huge aircraft requiring many thousand horse power,-the gas engine may be looked upon as the standard type of airplane engine and it only need be here considered.

## Variation of engine power with speed.

The power developed by a gas engine, with throttle wide open or in other constant position, increases in proportion to the number of explosions in a given time and hence in proportion to the number of revolutions per minute, which hereafter will be referred to as the speed $N$. For a considerab'e range of speed this proportionality is quite close, power increasing in direct proportion* to speed; but for higher speeds the power increases less rapidly, and finally a speed is reached at which the power is a maximum and beyond which the power falls off. This falling off in power is due largely to the fact that the fuel-charge received in the cylinders for each explosion is reduced at high speeds on account of the increased friction through ports and passages.

The variation of power with speed for a typical engine $\dagger$ is shown in Fig. 44, in which the solid curve shows the brake horse power when the throttle is wide open. It is seen that power increases very nearly in proportion to speed until, in this case, a maximum of ro6 H.P. is reached at a speed of 1240 R. P. M.

[^24]

Fig. 44. Variation of brake horse power of a typical gas engine with speed. Dotted curves show reduction of power by throttle control.

## Mechanical efficiency.

The entire power developed by the explosions in the cyiinders of a gas engine is called the indicated horse power or I. H. P. Some of this power, however, is wasted in friction and other losses within the engine, so that the useful delivered horse power-called the brake horse power or B. H. P.-is, let us say, io or I 5 per cent. less than the indicated horse power.

The mechanical efficiency of the engine is the ratio of the brake horse power to the indicated horse power; thus when the losses are 10 or 15 per cent., the mechanical efficiency is 90 or 85 per cent. The efficiency of an engine, as well as its power, varies with the speed at which the engine is run. The efficiency is low at low speed and at very high speed (i.e., when the power delivered is low) and is nearly a maximum when power is a maximum. The speed for maximum efficiency is a little lower than the speed for maximum power, but the efficiency remains high (within a few per cent. of its maximum value) for a wide range of speed -a range, let us say, of twenty or thirty per cent. (In Fig. 44, this range extends roughly from the beginning of the word Throttle to the point of maximum power.)

## Range of engine speed.

The best speed for engine operation is no one precise speed but extends through a moderate range of values just below the speed for maximum power. In this range, efficiency and power are both high; beyond this range, however, there is a large falling off both in efficiency and in power.

An engine is often run at a lower speed than this best range, when such lower speed and power is desirable, despite the lower efficiency; but it is rarely run at much higher
speed, on account of increased wear and heating of the engine, as well as decrease in power and efficiency.

The best speed depends upon the design of the engine,size of ports and bore, length of stroke, mass of moving parts, etc. As a usual thing, airplane engines are designed for a lower speed than automobile engines so as to permit of direct connection to the propeller. Structural and other reasons make high propeller speeds undesirable, the usual speeds being between 1200 and 1600 R. P. M., but speeds beyond this range are not uncommon.

A propeller may be geared down, so as to gain the advantage of high engine speed with low propeller speed, but this gearing adds weight, introduces losses and is an added source of trouble.

## Throttle control.

The solid curve for brake horse power in Fig. 44 shows the full power when all adjustments of throttle, spark and carburetor are made so as to give the greatest possible power at each speed. Usually, the pilot controls the power by means of the throttle, less power being obtained by partly closing the throttle. (In some engines, however, the only power control is the ignition switch, by which the power is turned either entirely on or off.)

Dotted curves in the same figure, marked "three-quarters throttle" and "half throttle," show the power obtained by throttling the engine so that, at each speed, the power is three-quarters or one-half the full power at that speed. To obtain precisely the same fraction of full power at each speed would require some adjustment of throttle (with constant throttle, the fractional power being not exactly three-fourths or one-half, or other definite proportion of
full power, at all speeds), but this adjustment would not be great. The curves may, therefore, be considered as illustrating the variation of power with speed for various constant throttle positions; they are sufficiently correct for this purpose, although for exact computation they would require some modification. (These dotted curves practically show, also, the decreased power at higher altitudes, for, as discussed later under Altitude, the decrease in air density causes a decrease in power substantially the same as throttling.)

The curves in Fig. 44 are engine characteristics and show the power delivered by the engine at different speeds and with different amounts of throttle. Before we can determine how much power is available for flight, we must examine the characteristics of the air propeller.

## The Air Propeller

(a) Introductory

The air propeller is mounted on or geared to the engine shaft,-either ahead of the engine as a tractor, or behind it as a pusher. The propeller is usually constructed with two blades, as in Fig. 45, but not uncommonly it is constructed with four and less commonly with three blades when propeller diameter is limited by the space available. The threeblade propeller is structurally difficult. The requisite strength is most readily obtained with two blades, which pass through the hub as one member.

The forward thrust required to overcome airplane resistance in flight is obtained by the propeller driving back a stream of air in a so-called slip-stream; the greater the backward velocity of this slip-stream and the greater its volume, the greater is the forward reaction or propeller thrust.

In the same way that an upward force or lift is obtained from a moving aerofoil because it deflects the air stream downward, a forward force or thrust is obtained from a moving propeller blade because it drives the air stream backward. In each case the force is a reaction obtained by deflecting or driving the air particles in a direction opposite to the force.


Fig. 45. Two-blade propeller.
Although there are various ways of treating the propeller and it is commonly referred to and considered as an air screw, it is most satisfactory to consider the propeller blade-or each element thereof-as an aerofoil, with lift and drag determined by its cross-section and angle of incidence as for any aerofoil. The lift of a propeller blade as an aerofoil determines its thrust as a propeller; its drag as an aerofoil determines the torque necessary to drive it as a propeller, as discussed more fully later.

## Screw definitions; pitch and pitch ratio.

Although the action of a propeller is very different from that of a screw, various terms that originated with the screw are applied to the propeller.

When a screw passes through a solid, the distance it moves forward in one revolution is called the pitch of the screw. This distance divided by the diameter of the screw is called its pitch ratio. The pitch and pitch ratio of a screw may be exactly determined from its dimensions (the distance between threads, and the diameter), the values thus deter-
mined being identical with the values determined by actually driving the screw through a solid or turning a bolt in a nut.

The terms pitch and pitch ratio are applied to a propeller as to a screw. It is found, however, that the effective pitch* of a propeller-the distance the propeller moves forward through the air in one revolution-is not the same as its structural pitch (also called nominal pitch or geometrical pitch) determined from its dimensions as a screw passing through a solid.

In propeller operation it is its effective pitch, rather than its structural pitch, in which we are most interested. As shown later, the effective pitch of a propeller varies with conditions of operation, being sometimes less and sometimes more than the structural pitch which has but one value fixed by its dimensions.

## Definitions of torque horse power, thrust horse power and efficiency.

The power available for propelling an airplane through the air comes directly from the propeller thrust, the amount of power thus furnished, as already pointed out, being proportional to the product of propeller thrust and airplane velocity. The propeller itself, however, does not create this power; it merely transmits power that it receives from the engine, the power it thus receives being proportional to the product of the torque $\dagger$ in the propeller shaft and its speed in revolutions per minute. The speed of the propeller is the speed of the engine, unless reduced by gearing. "Speed,"

[^25]here and elsewhere in this discussion, refers to speed of rotation, often referred to as "revs." or R. P. M., and not to the forward translation referred to as "velocity" or "miles per hour."

Of the three elements for flight,-engine, propeller and airplane,-the propeller is thus seen to be the intermediary or "middle-man," receiving power from the engine and delivering this power in a form available for propelling the airplane. The power the propeller receives is torque horse power and this is the brake horse power of the engine already discussed. The power the propeller delivers is thrust horse power and this is the power available for airplane propulsion.

The efficiency of a propeller is the ratio of thrust horse power to torque horse power. The entire output of the engine would thus be available as thrust horse power for propelling the airplane, if the efficiency of the propeller were 100 per cent. On account of losses, however, the efficiency of a propeller is less than ioo per cent., so that not all of the engine output is thus available. Under best conditions, the efficiency of a propeller may be 80 or 85 per cent., but under working conditions it is usually less; thus, when the brake horse power of the engine is 100 , perhaps only 60 or 70 horse power may be available for propeller thrust.

The air propeller, working in a compressible medium, is more efficient than the marine propeller working in a medium that is practically incompressible. The rarefaction and compression before and behind the blade of a propeller in airas above and below an aerofoil-are factors not found in water. (In water, when a negative pressure created by the relative motion of blade and water exceeds the static pressure of the water, it is not possible for the water to become
rarefied but a discontinuous flow occurs known as cavitation. To avoid this, the blades of a marine propeller are made short and wide, and not long and narrow as in an air propeller.)

A knowledge of how power* and efficiency are affected by different conditions of operation is most essential for the understanding of the propeller. Fortunately these relations, so far as results in operation are concerned are simple.

Let us first see what are the varying conditions of propeller operation. We shall then see what are the characteristics of a propeller under these different conditions of operation, after which we will consider the theory of the propeller that accounts for these characteristics.

## (b) Conditions of Propeller Operation

## V and N , the two variables in propeller operation.

During flight the two variables in propeller operation upon which other quantities depend are its forward velocity $V$ and its speed or revolutions per minute $N$. (The effect of a third variable, the change of air density with altitude, is left for a later discussion.) The dimensions and shape of the propeller itself can only be changed by a change of propeller when not in flight. $\dagger$

It will be found that propeller characteristics depend not only upon the absolute values of $V$ and $N$ but upon their

[^26]relative values as well. The ratio of $V$ to $N$, and the ratio $V / N D$, where $D$ is propeller diameter, have special significance.

The values of $V$ and $N$ are known to the pilot by his airspeed meter and his revolution indicator. They are, furthermore, quantities that he directly controls. For these reasons, propeller characteristics are better understood-by a pilot at least-when expressed in terms of $V$ and $N$ than when expressed in terms of effective pitch, angle of blade incidence or slip, quantities that are only indirectly known and controlled by the pilot. It is well, however, to be able to interpret propeller characteristics when expressed in these various terms, for each has its significance; effective pitch and slip are convenient terms in the comparison of propellers of different diameter, while the angle of incidence the blade makes with the air is useful in propeller theory, as discussed later.

## $\mathrm{V} / \mathrm{N}$, the forward travel per revolution or effective pitch.

When a propeller making $N$ revolutions per minute, is moving forward with a velocity of $V$ ft. per minute, the distance that the propeller moves forward in one revolution is seen to be $V / N$ feet. This distance, in feet or other unit of length, is called the effective pitch of the propeller; it varies with the relative values of $V$ and $N$, under different conditions of operation, but is independent of their absolute values. (Thus, when $V=5000 \mathrm{ft}$. per min. and $N=1000$ R. P. M., the propeller moves forward in one revolution a distance $V / N=5 \mathrm{ft}$., which is the effective pitch of the propeller; when $V=6000$ and $N=1200$, the effective pitch $V / N$ is still equal to 5 ft .)

## $\mathrm{V} / \mathrm{ND}$, the ratio of forward travel per revolution to diameter, or effective pitch ratio.

This forward travel per revolution, when expressed in terms of propeller diameter $D$ (instead of in feet) is $V / N D$ and is called the effective pitch ratio. Thus, in the preceding example, if the propeller has a diameter of 8 feet, $V / N D=$ $5 \div 8=0.625$, which is the effective pitch ratio of the propeller and means that in each revolution the propeller travels forward a distance 0.625 times its diameter. $V / N D$ is a number, independent of units; the units used, however, must be consistent.*

The value of $V / N D$ indicates, to a certain extent, the conditions of propeller operation. Being independent of units, it is more useful for this purpose than $V / N$. Various propeller quantities (for example, efficiency, Figs. 55 and 56) are accordingly plotted in terms of $V / N D$.

Since the peripheral velocity of a propeller is $\pi N D$, it is seen that $V / N D$ is proportional to the ratio of the forward velocity to the peripheral velocity of the propeller tip.

## Dynamic pitch and pitch ratio.

The effective pitch, or forward travel of a propeller per revolution, as just stated varies under different conditions of operation. It will be shown later that, as effective pitch increases, thrust decreases and finally becomes zero. The

[^27]propeller then goes through the air smoothly, as a screw with no slip, without disturbing the air and without imparting velocity to the air particles.

The particular value of effective pitch that gives zero thrust is called the dynamic pitch of the propeller. It is characteristic of each propeller and like a dimension (expressed in feet or other unit of length) can not be changed.

The dynamic pitch ratio is the ratio of the dynamic pitch to the propeller diameter $D$, and is a number independent of units. Otherwise defined, it is the value of $V / N D$ when there is no thrust and no slip.

As an illustration, if a ro ft . propeller creates no thrust when its forward travel per revolution is $V / N=9 \mathrm{ft}$., the dynamic pitch ratio is $V / N D=9 \div 10=0.9$. This is a constant of the propeller and is the same whether $V$ and $N$ be large or small. Practical values for dynamic pitch ratio are between 0.5 and I .5 .

## Slip.

Positive thrust is obtained only when the forward travel per revolution, or the effective pitch, is less* than the dynamic pitch. The difference between the dynamic pitch and the effective pitch, expressed as a percentage of the former, is called the slip. Thus, when a propeller with a dynamic pitch of 8 feet travels forward only 6 feet in one revolution, the slip is 25 per cent., or 0.25 .

When the slip is $s$ per cent., the forward travel per revolution is

$$
V / N=\text { effective pitch }=(\mathrm{I}-s) \text { dynamic pitch. }
$$

[^28]As a ratio in terms of $D$, we have accordingly

$$
V / N D=\text { effective pitch ratio }
$$

$$
=(\mathrm{I}-s) \text { dynamic pitch ratio. }
$$

It is seen that when the slip is zero, the effective pitch becomes equal to the dynamic pitch.

## Dynamic pitch greater than structural pitch.

The dynamic pitch and pitch ratio of a propeller aregreater* than the nominal or structural pitch and pitch ratio. In other words, the actual forward travel of a propeller through the air for no thrust is greater than its travel calculated as a screw passing through a solid, unyielding material. It is for this reason that the screw theory of the propeller is abandoned.

The explanation lies in the fact that the air through which the propeller is passing is a compressible gas and not an unyielding solid. The propeller blade in cutting through the air acts not as a screw but as an aerofoil-as discussed later under propeller theory-and, like any aerofoil, gives rise to a rarefaction $\dagger$ on its upper surface (in front of the propeller) and a condensation on the lower surface (back of propeller). Calculations for a propeller as a screw take no account of this rarefaction and condensation of the air and for this reason calculated or nominal values for pitch and pitch ratio always differ from the actual dynamic values. The result in flight is much the same as though the whole body of air immediately surrounding a propeller were being carried forward with it, so that more than the calculated velocity is necessary in order to get zero thrust and this may serve as a rough explanation.

[^29]Dynamic pitch and pitch ratio can only be determined by experiment, where special facilities are available, whereas nominal pitch or structural pitch and pitch ratio can be determined by measurements described later on the propeller itself. For this reason the values for pitch and pitch ratio usually given are nominal ralues, and are always so understood unless otherwise specified.

The relation between $V$ and $N$ and the significance of slip, pitch and pitch ratio in propeller operation will be brought out more fully in the subsequent discussion of propeller characteristics and propeller theory.

## (c) Propeller Characteristics

The behavior of a propeller under various conditions of operation will be understood by examining the characteristic curves that follow. These are working results, independent of any theory, the performance of the propeller being to many readers of first importance. A discussion of theory will follow, but some may prefer to read the theory before examining the performance.

The conditions of operation depend upon the relation between $V$ and $N$, and this in turn requires first a study of torque horse power. Other characteristics will be studied in turn.

## Torque horse power for different values of N and V .

The torque horse power required to drive a given propeller varies both with its speed $N$ (revolutions per minute) and the forward velocity $V$ (miles per hour) at which it is moving through the air.

For any constant value of $V$, torque horse power increases as the revolutions per minute increase; in other words, it is found, as might be supposed, that more power is required to
turn a propeller fast than to turn it slowly. This is shown by the curves in Fig. 46 for the torque horse power of a particular* propeller at three different velocities, 50 , 100 and 150 miles per hour. By comparing these curves it is seen that torque horse power is less for high than for low velocities.

The decrease in torque horse power as $V$ increases is better shown by the curves in Fig. 47 in which $N$ is constant and $V$ is variable. It takes less power to drive a propeller, at any given number of revolutions per minute, when the propeller has a forward velocity $V$ than when it is stationary. Propeller thrust, as discussed later, is a maximum when the propeller is stationary; torque, also, is then a maximum, as well as torque horse power as shown by the curves.

As $V$ increases, torque horse power continues to decrease and becomes zero when a certain velocity is reached. At higher velocities, torque horse power is negative; the propeller, instead of receiving power from the engine, then drives the engine, receiving power as an air motor from the air. This means that the airplane is descending in a glide or dive and that power is supplied by gravity. The airplane is being retarded by the propeller as by a brake. With $V$ constant, as in Fig. 46, torque horse power is negative when $N$ is less than a certain value; the negative values of torque horse power are not shown in the curves.

Torque horse power is an important element in determining the relation between $N$ and $V$.

[^30]
## Relation between N and V .

The velocity $V$ of an airplane in flight is determined by its angle of incidence, as discussed in Chapter II, and is controlled entirely by the elevator. The revolutions per minute or speed $N$, although controlled by the throttle, depend not only upon engine throttle but also upon airplane velocity $V$. Let us see in what way the speed $N$ is determined.

For any given velocity, for example $V=100$ M P H., we have a curve, as in Fig. 46, showing torque horse power for each value of speed $N$. But torque horse power received by the propeller must equal brake horse power delivered by the engine. The engine and propeller, accordingly, speed up until a speed $N$ is reached at which the propeller absorbs all the power output of the engine, that is, they speed up until torque horse power of the propeller and brake horse power of the engine are exactly equal.

This is made clear in Fig. 48, which shows a curve for engine brake horse power for a particular amount of throttle (reproduced from Fig. 44 with full throttle) and a curve for propeller torque horse power for a particular velocity (reproduced from Fig. 46 for $V=100 \mathrm{M} \mathrm{P} \mathrm{H)} .\mathrm{The} \mathrm{inter-}$ section* of these two curves determines the speed $N$ and also the power, for the particular value of $V$ and particular amount of throttle. The intersection will be shifted as either curve is shifted by control of throttle or change of $V$; or, both curves may be shifted simultaneously with a sort of scissors motion.

[^31]Value of $N$ for different amounts of throttle.-The control of speed and power by throttle, for one value of $V$, is shown in Fig. 49. As the throttle is changed from "full throttle" to " $3 / 4$ throttle" and " $1 / 2$ throttle," the intersection is changed from $N^{\prime}$ to $N^{\prime \prime}$ and $N^{\prime \prime \prime}$, with corresponding change in speed and power.

Value of $N$ for different values of $V$.-The change of speed and power for several different values of $V$, is shown in the same manner in Fig. 50. The speed and power corresponding to velocities of $50,75,100,125$ and 150 M P H., are determined for "full throttle," " $3 / 4$ throttle" or " $1 / 2$ throttle," by the several intersections.

It is seen that the speed $N$ of engine and propeller depends not only upon engine throttle (which is directly controlled by the pilot), but also upon airplane velocity, which is indirectly controlled by the pilot by means of the elevator. For constant throttle, there is a definite speed $N$ corresponding to each velocity $V$; and for constant velocity, there is a definite speed $N$ for each position of the throttle.

The curves shown, Figs. 48, 49 and 50, relate to a particular propeller and engine; for other engines and propellers the general nature of the results would be the same, although numerical results would differ.

## Thrust horse power at varying velocities.

A consideration of the thrust horse power delivered by a propeller under different conditions of operation is obviously of utmost importance, the sole purpose of a propeller being to produce thrust and thrust power. Like other propeller quantities these both vary as $N$ and $V$ are varied. It is, however, most satisfactory to plot their values for varying values of $V$, so that direct comparison can be made between
curves for thrust power (power available) and curves for airplane power required, which are plotted in terms of $V$.

Before discussing thrust power, let us consider propeller thrust upon which thrust power depends.

Propeller thrust.-Thrust depends upon the backward velocity imparted to the air by the propeller, that is, the backward velocity of the slip stream with respect to the surrounding stationary air. The thrust developed by a propeller will, accordingly, vary with the forward velocity $V$ of the propeller and will be a maximum when $V$ is zero, namely, when the airplane is stationary, for the velocity of the slip stream, with respect to the surrounding air, is then a maximum. The thrust, when the airplane is stationary, is called the static thrust.

When the propeller is moving forward with a velocity $V$, the backward velocity of the slip stream (which remains unchanged with respect to the propeller) is less with respect to the surrounding stationary air and the thrust is accordingly less. Thrust decreases as V increases, as shown in Fig. 51.

Thrust continues to decrease as $V$ increases, and finally thrust becomes zero when the propeller has a forward velocity $V$ just equal to the backward velocity of the slip stream relative to the propeller. The velocity of the slip stream, with respect to the stationary air, is then zero and no thrust is created, for no velocity has been imparted to the air by the propeller. Slip is then zero.

The solid curve in Fig. 51 shows the variation of thrust with velocity for a particular propeller when driven at 1200 R.P.M. The dotted curves show the thrust at 1000 and 800 R.P.M. In all cases zero slip corresponds to zero thrust; Ioo per cent. slip corresponds to zero velocity.

Thrust horse power derived from thrust.-Thrust horse power is readily derived from thrust, being equal to the product of thrust and velocity, divided by 375 when thrust is in pounds and velocity is in miles per hour.

Fig. 52 shows curves for thrust and velocity, and a curve for thrust power thus obtained from their product. It is seen that thrust power is zero when $V=0$, at ioo per cent. slip; thrust is then a maximum. It is seen, also, that thrust power is zero when thrust is zero, at zero slip; $V$ then has a certain definite value. These curves are drawn for a constant speed, $N=1200$ R.P.M.

A curve for torque horse power, reproduced from Fig. 47, is shown in Fig. $5_{2}$ for comparison; this makes possible a determination of efficiency, discussed later.

For the case shown in Fig. 52, maximum thrust power is obtained when the slip is 44 per cent.; maximum efficiency, at the point $m$, when the slip is 28.7 per cent.

## Thrust horse power for different values of speed N .

The curves in Fig. 53 show the variation of thrust horse power with velocity for a propeller driven at different speeds $N$. They strikingly show that, for each speed, there is a certain velocity at which the power is a maximum, and that the value of this maximum is greater for greater values of speed $N$.

These are the so-called curves for power available which -when compared with the curves for power requiredhave an important bearing upon power relations in flight. They are the most useful of propeller curves and should be carefully studied so that a picture of them may be kept in mind. Although plotted for a particular propeller, they are typical of the curves for power available for any propeller.

They are independent of the motor used, for any motor (not necessarily a gas engine) may be used provided it has sufficient power to drive the propeller. (Curves for propeller thrust horse power, when a particular engine is used, are shown later in Fig. 57.)

A propeller delivers its maximum power when the slip is, say, 40 to 50 per cent. (in this case about 45 per cent.); it has its maximum efficiency -discussed in a later paragraph -when the slip is, say, 25 to 40 per cent. (in this case about 30 per cent.). There is considerable variation in these values with different propellers, but maximum power always occurs at a greater slip than maximum efficiency. The best range for propeller operation is between the point for maximum power and the point for maximum efficiency.

Thrust horse power for propellers of different diameters D.
Greater thrust horse power can be obtained by increasing $N$ as just shown, but is often better obtained by using a propeller of larger diameter $D$, the greater power in this case being due to the greater volume of the air stream. The curves in Fig. 54 show the power obtained, at different velocities, from propellers of different diameters when driven at the same speed.

An airplane should be designed for as large a propeller as space permits, for a large propeller at moderate speed is (generally speaking) better than a smaller propeller at very high speed; but the larger propeller is objectionable if it necessitates an undue elevation of the center of gravity of the airplane. On account of this limitation and the necessity of having sufficient clearance between the propeller and the ground, the huge propellers satisfactorily used on airships
are not used on airplanes. A clearance as little as io inches has been found sufficient in some types of planes.

Assured of sufficient thrust power delivered by the propeller, we next inquire as to the efficiency of the propeller under different conditions of operation.

## Propeller efficiency for different values of V/ND.

The efficiency of a propeller is equal to thrust horse power delivered divided by torque horse power received. Note the curves for torque power and thrust power in Fig. 52; a comparison of these is very interesting.

It has been found that efficiency depends upon the ratio $V / N$ and not upon the absolute values of $V$ or $N$; if $V$ and $N$ are both changed in the same proportion, efficiency remains unchanged. Furthermore, in comparing propellers of the same design but with different diameters $D$, it is found that efficiency depends not upon $V / N$ but upon $V / N D$, namely, the effective pitch ratio which varies with the slip. Propeller efficiencies are, therefore, plotted for various values of $V / N D$ or for various values of slip. Both scales are shown in Fig. 55.

Referring to Fig. 55, and to Fig. 52 which relates to the same propeller, it is seen that when $V / N D=0$, corresponding to 100 per cent. slip, thrust power is zero and hence propeller efficiency is zero. As $V / N D$ increases, propeller efficiency increases until a maximum efficiency of 80 per cent., or so, is reached. As $V / N D$ is further increased, the efficiency decreases, and again becomes zero when $V / N D$ reaches a certain value (the dynamic pitch ratio of the propeller) corresponding to zero thrust and zero slip. It is seen, from Fig. 55, that this propeller has a dynamic pitch ratio 1.2 , whereas the nominal pitch ratio is 0.9 .

Every propeller has an efficiency curve of the type shown in Fig. 55. It is seen that for a given number of revolutions per minute there is a certain airplane velocity $V$, or for a given airplane velocity $V$ there is a certain number of propeller revolutions $N$, at which the efficiency of a particular propeller is a maximum.
A propeller should, accordingly, be selected* that has high efficiency and power at the operating values of $V$ and $N$. A propeller that is very good for a certain airplane and engine may be very poor for another airplane or engine. In other words, the propeller, engine and plane must fit, so that each will be operating under good conditions.

It is well to operate a propeller at a value of $V / N D$ somewhat lower (rather than higher) than the value of maximum efficiency, in order to obtain greater power. The points for maximum power $\dagger$ and maximum efficiency are marked on the curve. As already stated, maximum efficiency occurs when the slip is, say, 25 to 40 per cent., and maximum power when the slip is, say, 40 to 50 per cent.

A wide range of high efficiency is usually more desirable than a narrower range of slightly higher efficiency.

## Pitch ratio and efficiency.

The efficiency curves for three propellers with different pitch ratios but otherwise similar are shown in Fig. 56.

[^32]The nominal pitch ratios of the three propellers are $0.5,0.7$, and 0.9; the dynamic pitch ratios (shown by the values of $V / N D$ when the efficiency curves fall to zero) are $0.76,0.96$ and r.2, respectively. Which propeller is the best to use is seen to depend upon what is the value of $V / N D$ under working conditions. Thus, when $V / N D$ is less than 0.47 , the propeller with pitch ratio 0.5 is seen to be the most efficient of the three; when $V / N D$ is more than 0.52 , this same propeller is the least efficient.

## Pitch ratio and power.

Pitch ratio does not affect efficiency alone. In Fig. 57 are shown curves of thrust power for the same three propellers, having nominal pitch ratios $0.5,0.7$ and 0.9. It is seen that for maximum power, as well as for maximum efficiency, the value of $V / N D$ must be greater for the propeller of greater pitch. For the three propellers here shown, maximum power is seen to increase with pitch ratio, but this is true only for a limited range of pitch ratio

## Combined engine and propeller characteristics.

We have discussed various propeller characteristics independent of the motor used to drive the propeller, the separate study of engine and propeller being for most purposes preferable. Thus, in Fig. 53 was shown the thrust horse power obtained from a certain propeller driven at specified constant speeds by any motor. This constant speed is obtained by throttle adjustment, when a gas engine is used.

In Fig. 58 are shown curves for thrust horse power for a given propeller driven by a particular gas engine, these curves being not for constant speed as in Fig. 53, but for constant throttle.


Fig. 46. Torque horse power required to drive a particular propeller at different speeds (revolutions per minute) when travelling through the air at 50,100 and $150 \mathrm{M} \mathrm{P} \mathrm{H.}$, level. Propeller diameter, $8^{\prime} 9^{\prime \prime}$; pitch ratio, o.9. Maximum propeller efficiency is at $m$.


Fig. 47. Variation of propeller torque horse power with velocity when propeller is driven at constant speed. Same data as Fig. 46. Maximum efficiency at $m$.


Fig. 48. Speed $N$ and power, of particular engine and propeller, determined by intersection of curves for engine brake horse power and propeller torque horse power.


Fig. 49. Speed $N^{\prime}, N^{\prime \prime}$ and $N^{\prime \prime \prime}$ and corresponding power, for particular engine and propeller, determined for three different amounts of engine throttle, when velocity is 100 MPH .


Fig. 50. Curves for engine brake horse power for different amounts of throttle and propeller torque horse power for different velocities. Intersections determine speed and power for each velocity and amount of throttle. Maximum efficiency at $m$.


Fig. 51. Variation of thrust with velocity, for a particular propeller, when driven at constant speed, Pitch ratio o.9, diameter $8^{\prime} 9{ }^{\prime \prime}$.


Fig. 52. Variation of thrust horse power with velocity for a particular propeller when driven at 1200 R. P. M. Dotted curves show thrust and velocity, thrust power being obtained by multiplying thrust and velocity. (The straight line for velocity might be omitted as it furnishes no additional information.) The upper curve, reproduced from Fig. 47, shows torque horse power for comparison.


Fig. 53. Curves for thrust horse power (usually called powe available) for particular propeller driven at different speeds $N$, by any engine. Maximum propeller efficiency is at $m$.


Fig. 54. Thrust horse power of similar propellers with different diameters, at constant speed $N=1200$ R. P. M. Pitch ratio o.9. Maximum efficiency at $m$.


Fig. 55. Propeller efficiency for different values of $V / N D$. Efficiency is equal to thrust horse power divided by torque horse power; see Fig. 52. Pitch ratio, o.9. Values for $V, N$ or $D$ are not fixed. Dynamic pitch ratio, i.2.


Fig. 56. Efficiency of three similar propellers with different nominal pitch ratios. Dynamic pitch ratio, shown by points of zero slip are 0.76, 0.96 and 1.2.


Fig. 57. Thrust horse power of three similar propellers with different pitch ratios. Point of maximum efficiency is marked by x .


Fig. 58. Combined engine and propeller characteristic. Thrust horse power, or power available, for constant throttle; a particular propeller (pitch ratio o.9, diameter $8^{\prime} 9^{\prime \prime}$ ) driven by a particular engine. Maximum efficiency at $m$. For engine curves, see Fig. 44.

It was shown in Fig. 50 how the brake horse power and speed $N$ are determined for full throttle (or for $3 / 4$ or $1 / 2$ throttle) when $V$ is 50,75 , 100, 125 or 150 M P H. Thrust horse power may be obtained by multiplying brake horse power, as here shown, by efficiency, which is known from Fig. 55 when $V$, $N$ and $D$ are known. The curves in Fig. 58 were thus determined.

The chief propeller characteristics have now been shown, most important being the curves of power available in Fig. 53, useful for direct comparison with curves of power required.

Let us now examine briefly the basis for propeller theory.

## (d) Propeller Theory

The foregoing discussion of the characteristics of a propeller in operation shows working results in simple form; these are the facts that can be easily understood, independent of any theory that may be used to explain them.
The oldest theory of the propeller treats it as a screw, which moves forward as it turns, and upon this conception are based many propeller terms in common use. In an incompressible fluid, as water, the screw theory is fairly satisfactory and the marine propeller is, accordingly, commonly treated as a screw. When, however, the medium is air, which can be compressed and rarefied as any gas and has a density only $1 / 800$ the density of water, the screw theory is far from satisfactory and, as a theory for the air propeller, has been practically abandoned.

## Aerofoil theory of the propeller.

The most satisfactory theory of the air propeller considers the propeller blade as a rotating aerofoil. When the propeller is merely rotating and has no forward velocity, each ele-
ment of the blade moves in a circle, the plane of rotation being perpendicular to the propeller shaft. When the propeller makes $N$ revolutions per minute, the velocity of rotation of any element at a distance $r$ feet from the center of the shaft is $2 \pi r N$ feet per minute.

In flight the propeller has a forward velocity in addition to its rotation, and the path of any element is a cork-screw curve or helix and not a circle. The velocity of any element along this path is then the resultant of its forward velocity $V$ and its velocity of rotation, $2 \pi r N$, as shown later in Fig. 62 .

Fig. 59 shows the plan of a propeller blade and several sections at different distances from the center. The similarity of these sections to the section of an airplane wing and the justification of the aerofoil theory is obvious.

Fig. 60 is an illustrative diagram (not to scale) of one section of a propeller blade, viewed radially, developing more fully the aerofoil theory. Fig. 6r shows this section in relation to the propeller shaft. Each section or "element" of the blade may thus be treated as an aerofoil.

The blade angle or pitch angle (nominal or structural pitch angle) for any particular section is the angle $a$ between its chord and the direction of its rotation in a plane perpendicular to the propeller shaft. As shown in Fig. 59, this angle decreases for the various sections of a blade as we proceed from hub or boss to tip; that is, the blade angle $a$ decreases* as the distance $r$ from the hub increases.

[^33]The angle of incidence $i$, at which any blade element or section attacks the relative air due to its motion, is the angle between the chord of the element and its resultant flight path, as shown in Fig. 60.

When a propeller is rotating, but has no forward velocity, its motion is in the plane of rotation and the velocity of any particular blade element is $2 \pi r N$; the angle of incidence of a blade element is now equal to the pitch angle $a$.

In flight, when there is a forward velocity $V$, the resultant motion or flight path of a blade element is a helix and the resultant velocity of the blade element is the resultant of its forward velocity $V$ and its velocity of rotation $2 \pi r N$, as shown in Fig. 62.

The effective pitch angle, as shown in Figs. 60 and 62 , is the angle $e$ which the resultant flight path makes with the plane of rotation. The effective pitch angle varies with different conditions of operation and increases as $V / N$ increases, the tangent of $e$ being equal to $V / 2 \pi r N$. The effective pitch angle thus determines the forward travel of the propeller per revolution.

As already stated, the angle of incidence of a blade element with the relative wind is the angle $i$ between its chord and the resultant flight path. $O P$, in Fig. 60, shows a short portion of this path. It will be seen that as the forward velocity $V$ increases (relative to $N$ ) the angle of blade incidence decreases.

The effective pitch angle and effective pitch are zero when $V=0$, corresponding to 100 per cent. slip and maximum thrust; the angle of blade incidence is then a maximum and the thrust is a maximum.

Tift and drag.-As in the case of any aerofoil, each blade element has a lift perpendicular to, and a drag or resistance


DIRECTION OF ROTATION

Fig. 59. Plan of propeller blade (viewed in direction of shaft) and sections of blade at. various distances $r$ from center of shaft. Note that as $r$ increases the blade angle decreases.
in the direction of, the flight path. The direction of $L$ and $D$ are indicated in Fig. 60. Their values are

$$
\begin{aligned}
& \text { Lift }=L=K_{\mathrm{L}} S V_{\mathrm{R}}{ }^{2} ; \\
& \text { Drag }=D=K_{\mathrm{D}} S V_{\mathrm{R}}^{2}
\end{aligned}
$$

Here $S$ is the area of the blade element; $V_{\mathrm{R}}$ its resultant velocity along its helical path; $K_{\mathrm{L}}$ and $K_{\mathrm{D}}$ are the usual coefficients which depend upon aerofoil shape and vary with the angle of blade incidence.

Thrust and torque.-The component of force parallel to the shaft of the propeller gives thrust; the component of force in the direction of rotation gives torque. Although thrust results from lift, it is seen that thrust is not precisely equal to lift; nor is torque precisely equal to drag.

The total thrust and torque for the propeller as a whole is the sum of the thrust and torque of the separate blade elements.

Angle of blade incidence.-The angle of incidence $i$ varies during flight from, say, $2^{\circ}$ or $3^{\circ}$ in horizontal flightto, say, $10^{\circ}$ or perhaps $I 2^{\circ}$ in climbing. As the angle of incidence increases (with an increase in slip and decrease in $V / N$ ) the lift, and so the thrust*, increases. The increase in thrust, with increase in slip and decrease in $V / N$, has been shown by the curves in Fig. $5^{1}$. This increase, when climbing, may be obtained by a decrease in $V$ or an increase in $N$, or both.

Negative blade incidence for zero thrust.-As the angle of incidence decreases (with decrease of slip and increase of $V / N)$ the lift, and so the thrust, decreases; but at zero

[^34]\[

$$
\begin{aligned}
& \text { Thrust }=L \cos e-D \sin e \\
& \text { Torque }=(L \sin e+D \cos e) r
\end{aligned}
$$
\]



Fig. 60. Particular section of propeller blade (viewed in a radial direction) at distance of $r$ from center of shaft, showing the behavior of a propeller blade as an aerofoil, attacking the air at an angle of incidence $i$. The angle $e$ is the effective pitch angle.


Fig. 61. Relation of section of propeller blade to propeller shaft; radial view.


Fig. 62. Resultant velocity is the resultant of the velocity of flight $V$, and the velocity of rotation $2 \pi r N$. The angle $e$ is the effective pitch angle.
incidence there is still some lift and thrust. As with any aerofoil there is a certain negative incidence at which there is no lift (as shown in Fig. II, page 18), so with a propeller element there is a certain negative incidence at which there is no thrust. The resultant path of a blade element is then as indicated by the dotted line $O F$ in Fig. 60, in advance of the chord of the blade element, instead of back of it as shown by $O P$ for positive incidence.

The angle between $O F$ (the resultant path for zero thrust) and the direction of rotation $O B$, is the dynamic pitch angle, and the corresponding pitch is the dynamic pitch.

To obtain positive thrust, the effective pitch angle $e$ must be less than the dynamic pitch angle, by an amount dependent upon the slip.*

Angle of blade incidence and thrust at standstill.-The angle of blade incidence increases as the airplane velocity decreases and has its maximum value at standstill when $V=0$. The angle of blade incidence $i$ is then equal to the blade angle $a$.

Since $\tan a=p / \pi$, where $p$ is the structural pitch ratio as shown in a preceding note, the blade angle or value of blade incidence at standstill, corresponding to any value of $p$, can be readily calculated. The following values are thus obtained.

| Pitch ratio, $p \ldots \ldots$ | 0.5 | 1.0 | 1.5 | 2.0 |
| :--- | ---: | ---: | ---: | ---: |
| Blade angle, $a \ldots \ldots$ | $9^{\circ}$ | $177^{1 / 2}{ }^{\circ}$ | $251 / 2^{\circ}$ | $32^{1 / 2}{ }^{\circ}$ |

The thrust of a propeller at standstill depends upon its lift as an aerofoil, and this is a maximum when the incidence is between $12^{\circ}$ and $18^{\circ}$, varying with the shape of the blade or aerofoil (see pages 18 and 21). It is seen, therefore, that to obtain the maximum thrust at standstill,-i. e., the maximum static thrust-a propeller should have a certain pitch ratio,

[^35]between let us say 0.7 and i.o, a larger or smaller pitch ratio giving less aerofoil lift and so less static thrust.

While it is not necessary to have a pitch ratio that will give the maximum static thrust, it is necessary to have a pitch ratio that will give sufficient thrust to get under way and accelerate on a bad field. This is a reason for not employing very high or very low pitch ratios.
$L / D$ ratio for propeller.-The $L / D$ ratio for a propeller section has a maximum value of about 20 , when the blade incidence is, say, $4^{\circ}$ or $5^{\circ}$. It is sought in propeller design and operation to approach a maximum value in this ratio (or rather in the thrust/torque ratio) but there are limitations in structure that make this difficult.

## Blade shape.

The problem of determining the several sections for a propeller blade is much the same as that of determining the section for any aerofoil, although there are differences due to the helical path in one case and the straight path in the other, and consequent differences in the eddies and vortices produced.

The front surface of a propeller blade (the upper surface, considered as an aerofoil in the usual manner) is highly cambered. The amount of this camber and its distribution depend somewhat upon the intended service. The back surface is flat, or nearly so, for it is found that there is little or no gain in shaping this surface. Aspect ratio enters in blade design much as in the design of any aerofoil. To obtain the advantage of high aspect ratio, a long and narrow blade is used for an air propeller. (Not so for a marine propeller.)

The greatest thrust and torque in a propeller blade are obtained toward the tip, the maximum being about $4 / 5$ the way from hub to tip.

Mechanical strength is an important consideration in propeller design. Changes in blade shape that produce little change in aerodynamic efficiency may have a large effect upon strength. In fact in propeller design the question of strength is foremost. Particularly true is this as the hub is approached, for this part of the propeller-on account of its lesser velocity of rotation-contributes little to thrust, so that strength is here practically the entire problem. High polish is given the blade to reduce resistance. The surface is given a waterproof finish to avoid absorption of moisture and consequent warping.

Very important is a proper propeller balance, a slight unbalancing at high velocities of rotation causing forces that soon become destructive.

## Uniform and variable pitch; mean pitch.

When a propeller is constructed with the same pitch at all parts, from hub to tip, the pitch is said to be uniform. When the pitch varies from hub to tip, the pitch is said to be variable. A mean value for pitch is then sometimes given, but the determination of mean pitch is somewhat arbitrary. It has been found that a variable pitch gives no gain in aerodynamic efficiency.

## Scientific basis of aerofoil theory.

The aerofoil theory has put the theory of the propeller on a scientific basis and has cleared up much that was formerly mysterious or, at least, not well understood. Recourse must be made to experiment for fundamental data, but when the theory is understood these experiments may be made systematically and not blindly.

Although a knowledge of the working characteristics of the propeller is all that is needed for many purposes, some knowledge of the theory of the propeller as an aerofoil proves a valuable aid in explaining its behavior under different conditions and in understanding the relations between the various quantities involved in its operation. The foregoing discussion, prepared primarily for this purpose, should also serve as a general introduction to a more detailed study of the propeller, either practical or theoretical.

## CHAPTER VII

## RELATION BETWEEN POWER REQUIRED AND POWER AVAILABLE

For maintaining airplane flight, the power available must be equal to or greater than the power required. The term "power required" is here used with its common meaning of power required for horizontal flight. Power required depends upon airplane weight and resistance and upon the area and shape of wings, as discussed in Chapter V; while power available depends upon the engine and propeller, as just discussed in the preceding pages.

Both power required and power available vary with different conditions of flight. The variation of power required with velocity is shown by the U-shaped curves, Figs. 39-43, pages 71-76. The variation of power available with velocity is shown by the $\Omega$-shaped curves in Figs. 53 and 58, in the preceding chapter.

The relation between power required and power available determines various conditions in flight-the conditions for maximum and minimum velocity, best climb, etc.,-and shows most clearly the essential features in airplane operation, including the effect of throttle and elevator control and their relation to the velocity of the airplane and to the angle of incidence of its wings with the relative air.

## Power available must be equal to or greater than the power required.

In Figs. 63-66, the U-shaped curve $P_{1}$ represents power required and the $\Omega$-shaped curve $P_{2}$ represents the power available, at different velocities. When the power available
is, at every velocity, less than the power required, as in Fig. $6_{3}$, it is obvious that maintained flight is not possible. Furthermore, it is scarcely possible when the power available is just equal to the power required at one particular velocity, as in Fig. 64, for in this case* it would be necessary to maintain this precise velocity without variation; any slight increase or decrease in velocity would cause the airplane to lose altitude.

Again, even though the engine and propeller are capable of delivering a maximum power available greater than the minimum value of the power required, maintained flight is not possible when, as in Fig. 65, the power is available at one range of velocities and is required at a different range of velocities. In this case the airplane is designed for one velocity, the propeller and engine for another; and, at the velocity at which the airplane must fly, the power available is insufficient. The condition shown in Fig. 65 might occur if a propeller with too low a pitch were used.

Practically, therefore, for maintained flight, the power available must be more than the power required over a certain range of velocities, as shown in Fig. 66. The limits of this range are shown by the points $a$ and $b$ at which the power available and the power required are equal. The minimum flying velocity at $a$ and the maximum flying velocity at $b$ are thus determined. See also Figs. 67 and 68.

[^36]Beyond this range maintained flight is impossible, for at a higher or at a lower velocity the power available is less than the power required. Thus, a particular machine might be able to fly in horizontal flight at any velocity between, say, 45 and 100 MPH., but at no higher or lower velocity.

## Surplus power.

Referring to Fig. 66, it is seen that within the range of flying velocities, there is an excess of power available over power required. This excess is called surplus power. It is surplus power that causes a machine to climb, the rate of climb being directly proportional to the amount of surplus power, as discussed in the next chapter.

The surplus power $c c$, Fig. 66, is a maximum at one particular velocity, and this, accordingly, is the velocity for best climb. Maintainance of this velocity depends upon the judgment of the pilot in keeping the airplane in correct flying position. For higher or lower velocities the surplus power is less, and becomes zero when the velocity has its minimum value for sustained flight at $a$ or its maximum value at $b$.

In Fig. 66, the point $p$ is the point of minimum power and $t$ the point of minimum thrust required for horizontal flight as previously discussed. (The dotted line tangent to the power curve at $t$, if prolonged, would intersect the horizontal axis at the origin, not shown in this figure, where $V=0$; see Fig. 43a, page 78.)

The minimum thrust $t$ is always at a higher velocity than the minimum power $p$. No general statement can be made as to the velocity for maximum surplus power; it may be at a higher velocity than $t$ or at a lower velocity than $p$, depending upon the relation between $P_{1}$ and $P_{2}$, or at a velocity between $p$ and $t$ as in Fig. 66. See also Fig. 69.


Fig. 63
Fig. 64


Fig. 65

Relation between power required $P_{1}$ and power available $P_{2}$.

## Power relations for different R.P.M. and for different amounts of throttle.

The relation between power required and power available is more completely shown in Figs. 67 and 68. In each of these figures a curve is given for the power required for horizontal flight at different velocities by a particular assumed airplane. The point of minimum velocity, indicated by $a_{0}$ in each figure, is the absolute minimum velocity and this depends upon the aerodynamic characteristics of the airplane, irrespective of engine and propeller.

The curves for power available are drawn for different R. P. M. in Fig. 67 and for different amounts of throttle in Fig. 68, these curves being determined in themanner described in the preceding chapter.*

In Fig. 67, the limits of velocity are at $a$ and $b$ when the propeller has a speed of r 300 R. P. M., and at $a^{\prime}$ and $b^{\prime}$ when the propeller has a speed of 1200 R. P. M. At these limits the power available is just equal to the power required for horizontal flight. Between these limits there is a surplus power for climbing. Beyond these limits the power available is less than that required for horizontal flight and the airplane must consequently descend.**

Similarly, in Fig. 68, the limits of velocity with full throttle are shown at $a$ and $b$; for $7 / 8$ throttle, at $a^{\prime}$ and $b^{\prime}$; for $3 / 4$ throttle at $a^{\prime \prime}$ and $b^{\prime \prime}$. It is seen that any decrease in power available limits the range between minimum and maximum velocity by increasing the minimum and decreasing the maximum.

[^37]

Fig. 67. Power required for flying a particular airplane at different velocities; points marked show corresponding angles of incidence. Also power available from a particular propeller (any engine) driven at various R.P.M.
$a$ and $a^{\prime}$ are minimum, $b$ and $b^{\prime}$ maximum, velocities at different R.P.M. $a_{0}$ is absolute minimum velocity. $p$ is minimum power, $t$ is minimum thrust, for horizontal flight.

As a matter of fact, the power available when flying near the ground is ordinarily sufficient so that the minimum velocity is the absolute or aerodynamic minimum and is determined by $a_{0}$. From page 29, this minimum velocity is seen to be

$$
V_{\mathrm{MIN}}=\sqrt{\text { loading } / \text { maximum } K_{\mathrm{L}}}
$$

The maximum value of $K_{\mathrm{L}}$ varies but little in machines of a given type; indeed, even in machines of different types the range of variation in the maximum $K_{\mathrm{L}}$ is small,-say, twenty per cent. Loading, on the other hand, may vary several hundred per cent. Minimum velocity, therefore, is chiefly determined by the loading, see Fig. 20, page 30. A small loading indicates a low minimum velocity. Frequently the minimum loading is specified and no other specification of minimum velocity is given.

On the other hand, the maximum velocity is determined by the amount of power available in relation to the power required.

Generally speaking, curves for power available from a propeller are more useful when drawn, as in Fig. 67, so as to be independent of the engine employed,-assuming only that an engine is used that can give the necessary power. From the standpoint of the pilot, however, the curves in Fig. 68, relating to a particular engine as well as a particular propeller, are instructive, for he is most interested in his particular engine and its control.

## Control of power relations in flight.

Two controls operated by the pilot affect the power relations in flight, namely, elevator control and throttle control.

The elevator control determines the angle of incidence, and hence the corresponding values of velocity and power re-


Fig. 68. Power required for a particular airplane; also power available from a particular propeller and engine, with different amounts of throttle.

Minimum velocity is at $a$ (absolute minimum at $a_{o}$ ); maximum velocity is at $b . p$ is a minimum power, $t$ is minimum thrust, for horizontal flight.
quired, as shown by the curves for power required in Figs. 67 and 68. Any adjustment of the horizontal stabilizer* in flight has the same effect as an adjustment of the elevator, and may, accordingly, be considered as elevator control.

The throttle control determines the number of revolutions per minute of the engine and propeller and the amount of power available. If, for example, the pilot keeps the number of revolutions per minute constant at 1400 , 1300 or 1200 , the power available from a particular propeller at different airplane velocities is as shown by the several curves in Fig. 67. Or, if the pilot keeps full throttle, $7 / 8$ throttle or $3 / 4$ throttle, the power available from a particular propeller when driven by a particular engine is shown by the curves in Fig. 68. Under throttle control may be included any engine adjustment that affects the amount of power developed, as carburetor adjustment and the timing of ignition, the curve for full throttle showing the greatest power available when all adjustments are made.

Control by the elevator, for any given position of the throttle, is effected by the pilot almost unconsciously. Thus, when flying horizontally with $2^{\circ}$ incidence and 1200 R. P. M. (Fig. ${ }^{67}$ ), if the pilot increases the R. P. M. to 1300 by opening the throttle, he maintains the flight path horizontal by decreasing the incidence to $0.5^{\circ}$ by a slight and almost imperceptible change in the position of the elevator.

[^38]It is thus seen that for horizontal flight any change in power available (brought about by change of R. P. M., controlled by the throttle) must be accompanied by a like change in power required (brought about by a change in incidence, controlled by the elevator).

If power available and power required are not thus maintained equal, the flight path is no longer horizontal, as discussed more fully in the next chapter. Whenever the power available exceeds the power required, there is surplus power and the machine climbs. Whenever the power available is less than the power required, the machine takes an oblique path downward. In order to climb, a pilot puts on power and by elevator control noses up, thus increasing his incidence and decreasing his velocity, until he has the requisite surplus power $c c$, Fig. 66.

## Normal and reverse control.

In the case illustrated in Figs. 67 and 68, minimum thrust $t$ would be required at a velocity about 70 MPH . and at an incidence about $5^{\circ}$; minimum power $p$ would be required at a lower velocity, about 64 MPH ., and larger incidence, about $7^{\circ}$.

Flying is usually at higher velocity and smaller incidence, than correspond to either $p$ or $t$, being nearer the point $b$ of maximum velocity. In the normal flying range between $p$ and $b$, an increase in velocity involves an increase in the power required; a decrease in velocity involves a decrease in power required.

But at lower velocities than the velocity for minimum power, namely, between $p$ and $a$, in Figs. 67 and 68, this relation is reversed; a decrease in velocity requires more power. This is designated as the range of reverse control.

Flight in the range of reverse control demands more skill on the part of the pilot, not only on account of the reversal of control, but even more on account of the decrease in stability at low velocities, for all airplane stability depends upon velocity. Low velocities, however, are desirable in making a landing. Otherwise, they may well be avoided by the novice.

## Comparison of machines.

The curve for power required depends upon various aerodynamic factors, while the curve for power available depends


Fig. 69. Comparison of power plants. $\quad P_{\mathrm{I}}$ is power required by a particular airplane; $P_{2}$ and $P_{2}{ }^{\prime}$ show power available from two power plants, each with the same maximu.n power. $P_{2}$ gives higher minimum and maximum velocities, $a$ and $b$, while $P_{2}{ }^{\prime}$ gives more surplus power $c^{\prime} c^{\prime}$.
upon the engine and propeller; the shape and position of each curve depends upon design. Particularly important, however, is the relation of the two curves to each other.

For example, in Fig. 69, the power required for a certain machine is $P_{1}$. It may be equipped with an engine and pro-
peller that gives power available $P_{2}$, or with a different power plant that gives power available $P^{\prime}{ }_{2}$. In each case the maximum power available is the same. Which is the more desirable?

With power $P_{2}$ the minimum velocity is at $a$, and the maximum velocity is at $b$; the maximum surplus power is $c c$. With power $P^{\prime}$, minimum and maximum velocities are at $a^{\prime}$ and $b^{\prime}$, both lower than $a$ and $b$, respectively; the surplus power is $c^{\prime} c^{\prime}$. It is seen, therefore, that $P_{2}$ is better for speed while $P^{\prime}$, with more surplus power, is better for climb.
$P_{2}$ and $P^{\prime}{ }_{2}$ might be obtained from the same engine equipped with different propellers, $-P_{2}$ when it is equipped with a high pitch propeller, $P^{\prime}{ }_{2}$ when equipped with a low pitch propeller. An airplane equipped with a variable pitch propeller, can advantageously use a low pitch for climbing and a high pitch for speed. Some airplanes are equipped with two propellers, with different pitch, either of which can be used. Ordinarily, however, a single propeller with fixed pitch is used, and in choosing a propeller it becomes necessary to know for what service the airplane is intended. As discussed later, under Altitude, the greater the altitude at which an airplane flies, the greater should be the pitch of its propeller. This is a further illustration of the desirability of a propeller with variable pitch. A single propeller with fixed pitch entails compromise.

## CHAPTER VIII

## CLIMBING AND GLIDING

At any given velocity, a machine flies horizontally only when the power available is equal to the power required at that velocity; otherwise the machine takes an oblique path upward or downward, according to whether the power available is greater or less than the power required.

## Oblique Path Upward

Climbing angle and rate of climb.
When an airplane takes an oblique path upward it is said to climb. The climbing angle is the angle which the inclined flight path makes with the horizontal, shown in Fig. 70 as the angle $c$.


Fig. 70. Climbing.


Fig. 71. Gliding.

The climbing velocity or rate of climb is the vertical component $V_{\mathrm{c}}$ of the velocity $V$ of the airplane along its flight path as shown in the figure. The relative values of $V$ and $V_{c}$ determine the climbing angle $c$; thus

$$
\sin c=V_{\mathrm{c}} \div V ; \quad V_{\mathrm{c}}=V \sin c
$$

The climbing velocity is equal to the velocity along the flight path multiplied by the sine of the climbing angle.

## Velocity in oblique flight.

Velocity along the flight path in oblique, as well as in horizontal, flight is determined by the formula for lift,

$$
L=K_{\mathrm{L}} S V^{2},
$$

where $L$ is the total lift from all sustaining surfaces. (Compare note on page 33.) In horizontal flight $L=W$ and

$$
V=\sqrt{\frac{W}{S K_{\mathrm{L}}}} .
$$

This expression for velocity is likewise approximately true in oblique flight when the angle of inclination of the flight path is small.

This approximation, while sufficiently accurate for most purposes in connection with climbing, is not justified in gliding and diving, when the angle of inclination may be large. This is discussed later.

More accurately*, $L=W \cos c$, where $c$ is the angle of inclination of the flight path. In case of gliding, this is shown in Fig. 75. It will be seen that, when the angle of inclination is small, it is very nearly true that $L=W$. Thus, when $c=5^{\circ}, L=0.996 \mathrm{~W}$; when $c=10^{\circ}, L=0.985 \mathrm{~W}$; when $c=15^{\circ}, L=0.966 \mathrm{~W}$, etc. Indeed, for small angles this error is probably less than various incidental errors in the calculation.

## Surplus power is used for climbing.

In horizontal flight, the power required for airplane flight is the power necessary to overcome airplane resistance; expressed in horse power $\dagger$ this is $V R / 375$, where $V$ is velocity in miles per hour and $R$ is total airplane resistance in pounds.

[^39]The additional power $P_{\mathrm{c}}$ used in climbing is $V_{\mathrm{c}} W / 375$, where $V_{\mathrm{c}}$ is the vertical velocity in miles per hour and $W$ is the total weight of the machine in pounds.

The total power necessary when climbing is, therefore,

$$
\text { Power }=V R / 375+V_{\mathrm{c}} W / 375
$$

The power used in climbing is the so-called surplus power, i. e, the excess of power available over the power required for horizontal flight.

## Rate of climb is proportional to surplus power divided by weight.

When the weight of a machine is known, its rate of climb may be readily calculated from its surplus power. As shown above, the surplus power is

$$
P_{\mathrm{c}}=W V_{\mathrm{c}} / 375
$$

The climbing velocity is, accordingly,

$$
\begin{aligned}
V_{\mathrm{c}} & =375 P_{\mathrm{c}} \div W, \text { miles per hour, } \\
\text { or, } V_{\mathrm{c}} & =33,000 P_{\mathrm{c}} \div W, \text { feet per minute. }
\end{aligned}
$$

The rate of climb is thus seen to be proportional to the surplus power per pound. A heavier and more powerful motor, by increasing surplus power more rapidly than the total weight, accordingly increases the rate of climb. The surplus power, and the consequent rate of climb, varies with airplane velocity.

## Maximum rate of climb.

The rate of climb is evidently a maximum when the surplus power is a maximum; it is obtained, with the throttle wide open, at a definite airplane velocity and corresponding incidence. The airplane velocity for best climb is often known to the pilot; it may to a certain extent be sensed in flight or determined from the altimeter by noting the rate at which altitude is being gained.

As surplus power is less at high than at low altitudes, the rate of climb becomes less as a machine ascends and finally becomes zero for a given machine at a definite altitude called the "ceiling," beyond which it cannot climb, as discussed in the next chapter.

## Illustrative example.

In Fig. 72, $P_{1}$ represents the power required by a particular airplane, that is, the power required to overcome airplane


VELOCITY, MILES PER HOUR.
Fig. 72. Surplus power $P_{\mathrm{c}}$ is the excess of power available $P_{2}$ over power required $P_{\mathrm{r}}$. Rate of climb is proportional to surplus power.
resistance, or the power required for horizontal flight. $P_{2}$ represents the power available. The excess of $P_{2}$ over $P_{1},-$ as $c c^{\prime}, d d^{\prime}$, etc.,-is the surplus power $P_{c}$ used in climbing; these values for surplus power are plotted below as the curve $P_{\mathrm{c}}$. At the intersections $a$ and $b, P_{1}$ and $P_{2}$ are equal and there is no surplus power; these points determine the mini-
mum and maximum velocities for horizontal flight, in this case 47.5 MPH . and 94 MPH ., respectively.

As the rate of climb is proportional to the surplus power, the curve $P_{c}$ for surplus power may also be taken as a curve for rate of climb,by using a proper scale depending upon the total weight of the airplane. At the right hand of Fig. 72 is shown the scale for rate of climb in feet per minute when $W=2000 \mathrm{lbs}$.

It is seen that the same surplus power, and so the same rate of climb, may be obtained at two different velocities. Thus, the surplus power is 25 H . P. at 53 MPH . and also at 84 MPH ., the rate of climb in each case being

$$
V_{\mathrm{c}}=33000 \times{ }_{25} \div 2000=412.5 \mathrm{ft} . \text { per min. }=4.69 \mathrm{MPH} .
$$

Although the rate of climb is the same, the climbing angle is different in the two cases, for $\sin c=V_{\mathrm{c}} / V$; the higher velocity gives the smaller climbing angle.
Thus, in this case, the climbing angle is $5^{\circ}$ at 53 MPH ., and $3^{\circ}$ at 84 MPH . $\quad\left(4.69 / 53=0.088=\sin 5^{\circ} 3^{\prime} ; 4.69 / 84=\right.$ $0.055^{8}=\sin 3^{\circ}{ }^{\prime} 2^{\prime}$ ).

In Fig. 72, a horizontal line (as $P_{\mathrm{c}}={ }_{25}$, or $P_{\mathrm{c}}=36$ ) intersects the curve for $P_{\mathrm{c}}$ at two points that have the same rate of climb. A diagonal line, as oh drawn from the origin, intersects the curves for $P_{\mathrm{c}}$ at two points that have the same climbing angle, for $P_{\mathrm{c}} / V$ has the same value in each case. Thus, as shown above, at 53 MPH ., when $P_{\mathrm{c}}=25$, the climbing angle is $5^{\circ}$; the rate of climb being 412.5 feet per min. At 76.3 MPH., when $P_{c}=36$, the climbing angle is also $5^{\circ}$; the rate of climb being 594 feet per min. The maximum climbing angle is determined by the point were the line og drawn from the origin is tangent to $P_{\mathrm{c}}$, for $P_{\mathrm{c}} / V$ is then a maximum.

It is thus seen that for every low velocity, it is possible to select a higher velocity that will give the same climbing angle
with greater rate of climb, or a still higher velocity that will give a smaller climbing angle with the same rate of climb.

For a given rate of climb, the high velocity with small climbing angle is much safer than the low velocity with large climbing angle. The low velocity gives greater chance for stalling and should generally be avoided, but may be necessary in certain maneuvers and in avoiding trees or like obstructions.

There is a wide difference in the climbing ability of different machines. One may have little surplus power and leave the ground slowly; Fig. 72 may be taken as an illustration of such a machine. Another, with ample surplus power, may shoot up with a vertical velocity of 25 or 30 MPH . An altitude of 10,000 feet has been attained in 4 minutes, 52 seconds.

The climbing angle and rate of climb throughout this discussion relate to steady climbing and not to short spurts. After acquiring sufficient momentum, a machine may for a moment climb at a much greater angle than its steady climbing angle. This is called zooming. It is often possible to zoom over an obstruction, the velocity lost thereby being subsequently regained by levelling out or even descending a little.

## Procedure in starting and climbing.

In leaving the ground a machine must climb. The ordinary procedure in starting is as follows:

The engine is started and as soon as the airplane is released, the pilot by use of the rudder keeps a straight course with full power* into the wind, keeping on the ground until a velocity well above the minimum flying velocity is reached. After the first few feet the pilot pushes the control forward, thus

[^40]depressing the elevator, which raises the tail of the machine from the ground; he then gradually moves the control back and holds the machine rolling on the ground in normal horizontal flying attitude, until flying velocity is reached. The control is pushed forward if necessary at any time to prevent


Fig. 73. Operation in starting and climbing. The pilot at first starts to "taxi" (1). Then, opening the throttle and passing immediately through (2) and (3) by depressing the elevator and thus raising the tail of the machine, he increases velocity while "rolling" (4). He "takes off" or leaves the ground $(5,6)$ by raising the elevator when sufficient velocity is attained, climbs (7) and straightens out $(8,9)$ when the desired altitude is reached.
the machine from leaving the ground too soon. The wings are kept level by lateral control.

When well above the minimum flying velocity, the control is pulled back a little so that the machine noses up and leaves the ground, ascending steadily at a small climbing angle and high velocity, until a considerable altitude is reached.

Too large a climbing angle will result in a stall, with loss of velocity and loss of sustentation. Although it is possible for
an experienced pilot, by a slight backward movement of the control, to leave the ground at minimum flying velocity, this should be avoided as it gives no margin of safety in case of trouble. It is well to know in advance the airplane velocity for best climb. Safety lies in a straight course into the wind, high velocity and small climbing angle. Ability to start in a cross wind is acquired with experience. Turns* near the ground, particularly at low velocities, are to be avoided. Some rules of the Air Service, given at the end of this chapter, may be here noted.

As pointed out elsewhere, some machines are balanced so as to climb when the control is in neutral, thus requiring little attention after leaving the ground until considerable altitude is reached.

The usual procedure in starting and climbing is illustrated in Fig. 73.

## Oblique Path Downward with Power On

When there is a deficit of power instead of a surplus, a machine takes an oblique path downward.
When the power available is less than the power required for horizontal flight, there is a deficit of power instead of a surplus, and $P_{\mathrm{c}}$ is negative. The vertical velocity $V_{\mathrm{c}}$ in this case is negative, that is, it is a rate of descent and not of climb. The machine takes an oblique path downward, the flight path making an angle $c$ with the horizontal, as shown in Fig. 71. As before, $\sin c=V_{\mathrm{c}} / V$.

[^41]For example, as shown in Fig. 72, suppose 118 horse power is required for horizontal flight at 103 MPH., while at this velocity only 93 horse power is available. The deficit, shown as $f^{\prime}$, is $P_{\mathrm{c}}=-25$. The rate of descent, when $W=2000 \mathrm{lbs}$., is $V_{\mathrm{c}}=-33000 \times 25 \div 2000=412.5 \mathrm{ft}$. per min. or 4.69 MPH. The angle of descent $c$ is between $2^{\circ}$ and $3^{\circ}(4.69 /$ ro3 $=\sin 2^{\circ} 37^{\prime}$ ). It is seen that negative values of $V_{\mathrm{c}}$ and $c$ are calculated in the same way as positive.

In the foregoing example, of the 118 horse power required to overcome air resistance, 93 horse power is supplied by the power plant and 25 horse power is supplied by gravity. Gravity is used as a source of power.

## Gliding and Diving

Gliding is a special case of oblique flight downward in which all the power is supplied by gravity, the power from the power plant being cut off and the angle of inclination of the flight path being small. Gliding becomes diving when the inclination of the flight path is large. There is no sharp line between gliding and diving; the same laws apply to both.

Gliding angle is determined by the ratio of resistance to weight.
In gliding, the power supplied by gravity $V_{\mathrm{c}} W / 375$ is equal to the power $V R / 375$ used in overcoming air resistance, for there is no other source of power or expenditure of power. Hence,

$$
\begin{aligned}
& V R=V_{\mathrm{c}} W \\
& V_{\mathrm{c}} / V=R / W=\sin c .
\end{aligned}
$$

The gliding angle is thus determined by the ratio of resistance to weight; for an airplane of given weight the gliding angle is determined solely by its resistance. The value of the
resistance depends upon the angle of incidence and corresponding velocity.

For example, a machine which weighs 2000 lbs . has a resistance of $3331 / 3 \mathrm{lbs}$., when flying at a certain velocity and angle of incidence. The gliding angle is $c=9^{\circ} 36^{\prime}$. $(R / W$ $=0.166=\sin 9^{\circ} 36^{\prime}$.) It will be shown later that changing the weight of a certain machine by changing its load will cause no change in the gliding angle $c$, for the change in weight will cause a proportional change in resistance so that $R / W$ remains constant.

## Length of glide or gliding range.

The gliding range depends upon the gliding angle. Let $d$ be the length of glide, measured along the flight path; and let $h$ be the vertical distance through which the airplane descends. The length of glide is

$$
d=h / \sin c=h \times W / R
$$

Thus, in the preceding example, $\sin c=\mathrm{I} / 6$; hence, $d=$ 6 h . That is, if the resistance of an airplane is $\mathrm{I} / 6$ of its weight, the airplane will glide six times its initial altitude from the ground. Starting to glide at an altitude* of one mile, such a machine would glide 6 miles, measured along the oblique flight path. The horizontal distance along the ground, equal to $h / \tan c$, would be somewhat less.

As a practical illustration, if the machine just referred to were a seaplane flying at an altitude of one mile, it should never fly at a greater distance inland than 5 or 6 miles, in order to insure a safe landing in case of trouble. The gliding range with some machines is 10 or more times the initial

[^42]altitude. A pilot should always keep in mind the possibility of making a forced landing in case of engine trouble.

## Condition for best glide.

For a machine of given weight, the gliding range is a maximum and the gliding angle a minimum when the resistance $R$ is a minimum.

It has been shown in Chapter IV that the resistance of any given machine varies with the velocity at which it is flown and


Fig. 74. Minimum reistance and best glide is at $t$ where $o b$ is tangent to the curve of power required. Minimum power and minimum rate of descent is at $p$. Any two points as $a^{\prime} a^{\prime \prime}$, on a line oa drawn from the origin, have equal resistance and the same gliding angle.
has a minimum value for a particular velocity and corresponding incidence, as shown in Fig. 34, page 59. This is the velocity and incidence giving the minimum gliding angle and maximum gliding range,-and is commonly called the condition for best glide. Note also the curves in Figs. 35-38.

The point for minimum resistance and for best glide is also shown in Fig. 74 at the point $t$, where the line $o b$ is tangent to the curve for power required. Fig. 74 is reproduced from Fig. 43a, page 78, where it is shown that $t$ is the point for minimum resistance (often referred to as minimum thrust) and minimum expenditure of energy.

In Fig. 74, any line as oa cuts the curve for power required at two points $a^{\prime} a^{\prime \prime}$ of equal resistance and equal glide,-


Fig. 75. When power is off, there is no thrust; weight is equal to the resultant of resistance and lift. The tangent of the gliding angle $c$ is $R / L$; the sine is $R / W$.
each of which gives a greater gliding angle and smaller gliding range than the point of best glide $t$.

The point $t$, where the energy expended is a minimum, shows the condition for gliding the greatest distance; but the point $p$, where the power expended is a minimum, shows the condition for gliding the longest time,* i.e., the condition for

[^43]minimum rate of descent, for the rate of descent $V_{c}$ is proportional to power. (Power $=V_{\mathrm{c}} W / 375=V R / 375$.)

## Weight balanced by resultant of resistance and lift.

In horizontal flight weight is balanced by and is equal to lift; resistance is balanced by and is equal to propeller thrust. When thrust is zero, as in gliding, an airplane takes an oblique flight path at such an angle that the resultant of resistance and lift is equal and opposite to weight, as shown in Fig. 75.

It is no longer true that $W=L$. It is seen from the figure that

$$
L=W \cos c ; R=W \sin c ; R / L=\tan c
$$

## Gliding or diving angle determined by the tangent relation.

The aerodynamic formulas for lift and resistance are
(土) $L=K_{\mathrm{L}} S V^{2}$;
(2) $R=K_{\mathrm{D}} S V^{2}+k s V^{2}$.

Equation (2) is a statement of the fact that resistance consists of two parts, wing-resistance and parasite resistance, as discussed in Chapters III and IV; see page 55.

The gliding or diving angle $c$ is accordingly determined from ( 1 ) and (2) by the relation:

$$
\text { (3) } \tan c=R / L=\frac{K_{\mathrm{D}} S+k s}{K_{\mathrm{L}} S} \text {. }
$$

The angle $c$ will change as the values of $K_{\mathrm{L}}$ and $K_{\mathrm{D}}$ (and possibly $k$ ) are changed by a change in the angle of incidence; but the gliding angle, as shown by the tangent relation, is independent of weight.* Increase of weight will increase

[^44]velocity, as shown below; but, provided the angle of incidence is not changed, a change of weight will not change the gliding angle.

## Velocity in gliding.

The velocity in gliding is determined from equation ( I ) above; thus,

$$
\begin{gathered}
W \cos c=L=K_{\mathrm{L}} S V^{2} \\
\text { (4) } \quad V=\sqrt{\frac{L}{K_{\mathrm{L}} S}}=\sqrt{\frac{W \cos c}{K_{\mathrm{L}} S}}
\end{gathered}
$$

For a given angle of incidence, $K_{\mathrm{L}}, S$ and $c$ are constant; velocity varies as the square root of weight.

## Velocity in diving.

When the angle $c$ is greater than $45^{\circ}$, velocity is best determined from equation (2); thus,

$$
\begin{aligned}
& W \sin c=R=K_{\mathrm{D}} S V^{2}+k s V^{2} \\
& \text { (5) } \quad V=\sqrt{\frac{W \sin c}{K_{\mathrm{D}} S+k s}}
\end{aligned}
$$

## Gliding and diving diagram.

Every airplane has a certain gliding angle and velocity, corresponding to each angle of incidence, as shown in Fig. 76 for an assumed airplane weighing 2000 lbs . Thus*, as shown

[^45]$$
V=\sqrt{\frac{W \cos c}{K_{L} S}}=\sqrt{\frac{2000 \times 0.85}{0.115}}=122 .
$$

The values of gliding angle and $V$ are thus calculated for each angle of incidence. For an incidence of $-3^{\circ}$, when $K_{\mathrm{L}}=0$ and $K_{\mathrm{D}}=0.000$ I I 4 , the velocity is calculated from equation (5); thus,

$$
V=\sqrt{\frac{W \sin c}{K_{D} S+k s}}=\sqrt{\frac{2000}{0.0408+0.04}}=160 .
$$

in the figure, when the angle of incidence is $-2^{\circ}$ the gliding angle is $32^{\circ}$ and the velocity 122 MPH. When the angle of incidence is $-\mathrm{I}^{\circ}$, the gliding angle is $18^{\circ}$ and the velocity 94 MPH. When the angle of incidence is $0^{\circ}$, the gliding angle


Fig. 76. Gliding and diving diagram, showing the gliding angle and velocity corresponding to each angle of incidence for a particular airplane. Change of weight or air density changes the scale for velocity but does not change the gliding angle.
is $12^{\circ}$ and the velocity 80 MPH ., and so on. Each point on the curve in Fig. 76 is thus determined by laying off the velocity $V$ at the proper gliding angle, for each angle of incidence.

The gliding angle is a minimum at $t$, where the line $a$ is tangent to the curve. This is the point of minimum resistance, which has been designated by $t$ in preceding figures.

The rate of descent is $V_{c}$, the vertical component of velocity. The rate of descent is a minimum at $p$, the uppermost point on the curve. This is the point of minimum power, designated by $p$ in preceding figures.

It will be observed that when the gliding angle is slightly larger than the minimum, there are two velocities at which the airplane can glide, a low velocity and a high velocity. When gliding at too low a velocity an airplane loses stability and gets out of control; stability and controllability increase with velocity.

It is interesting to note that a small change in the angle of incidence from $-2^{\circ}$ to $-3^{\circ}$ (the angle at which $K_{\mathrm{L}}=0$ ) increases the gliding angle from $32^{\circ}$ to a vertical dive of $90^{\circ}$.

Effect of change of weight.-Change of weight merely changes the scale in Fig. 76, for as shown above it changes velocity but does not change gliding angle. An increase in $W$ from 2000 to 3000 lbs . would cause $V$ to be increased by a factor $\sqrt{3 / 2}=\mathrm{r} .22$; the scale for $V$ in the drawing, now approximately 50 miles per inch, would become 6 r miles per inch. The gliding angle in every case would remain unchanged.

Effect of change of alt:tude.-Change of density with altitude would likewise change the scale for $V$ but would not change the gliding angle; $V \propto \sqrt{\mathrm{I} / \delta}$, as shown in the next chapter.

## Landing.

Most important is it for a flyer to know how to land; otherwise his flying knowledge is futile. The possibility of a forced landing should always be kept in mind and a pilot should endeavor to keep within gliding distance of a suitable landing place if possible.

The pilot may approach* the landing field with either a rapid or a gradual descent, with power entirely off or with engine partly throttled or running intermittently, as he judges to be best considering the initial altitude, the distance to be covered, the direction and velocity of the wind, etc. It is well to put power on occasionally in order to keep the manifold clear so that the engine will not choke if it becomes necessary to put on full throttle suddenly. Turns and spirals are made when the distance is short, but a turn should not be made near the ground as there is danger of the low wing striking the ground.

The pilot plans the approach so that when three or four hundred feet from the ground he can make a moderate glide straight to the landing place, directly into the wind if possible, with the two wings level. He should not lose velocity and sustentation by gliding too flat,-i.e., he should not "stall down"; nor should he gain excessive velocity by too steep a dive when near the ground. The relative velocity with the ground is the least when a landing is made against the wind. Landing with the wind or in a cross wind should be avoided; but such landings may be accomplished by an experienced pilot and are sometimes necessary. $\dagger$

Making a landing is an art and much is left to the judgment of the pilot as to the condition of the field, length of run, etc. As he approaches the ground with a moderate glide, he levels

[^46]off more and more, so that the gliding angle and velocity* become less and the rate of descent $V_{c}$ becomes less, as shown by the curve $A B$ in Fig. 77. When a few feet from the ground the flight path becomes practically horizontal and the vertical velocity $V_{c}$ becomes zero. As the machine thus glides horizontally it loses velocity and, as it begins to settle, the pilot slowly draws back the control stick, thus checking the velocity of the machine and lowering its tail before touching the ground.


Fig. 77. In landing, the pilot levels off after a moderate glide, as shown by the curve $A B$, the vertical velocity being reduced as much as possible before the machine touches the ground. Curve $A C$ shows the flight path when the pilot levels off too high; curve $A D$ shows the flight path when he levels off too low. The vertical scale is exaggerated in the drawing.

If a pilot finds he is about to make a bad landing, he should put on all power, gain altitude and try again.

Curve $A C$ in Fig. 77 shows a landing in which the pilot has leveled off too high; as the machine loses velocity, it loses sustentation and finally "pancakes" and descends to the ground with a vertical velocity $V_{c}$ that may be disasterous. This may be avoided by opening the throttle and regaining

[^47]speed before striking the ground. It is a common fault to glide too flat when nearing the ground.

Curve $A D$ shows a landing in which the pilot has leveled off too low; then, in his effort to avoid a crash he has over-controlled his machine so that it rises a little instead of levelling off flat. Over-control after a steep glide is likely to occur, for the velocity acquired during the glide renders the controls over-sensitive.

A three-point landing, in which the tail skid touches the ground at the same time as the two wheels, is considered a good landing under most conditions. Landing with tail low, so that the tail skid touches the ground first, may throw the forward part of the machine down violently; while landing with tail high, so that the wheels first touch, may cause the machine to pitch forward and overturn. Good landings, with tail high and with tail low, are, however, made; much depends upon the nature of the field and the skill of the pilot. When taxying to the hangar after landing, rapid turns should be avoided so as to keep the wings from striking the ground.

## Rules of the Air Service.

Among the rules of the Air Service are the following:
I. Speed always means control. Loss of speed means loss of control. If motor starts to miss while getting out of field always nose plane down to gain flying speed before trying any maneuvers. Most crashes are caused by trying to turn close to the ground without sufficient flying speed.
2. After flight has begun if conditions arise that make flying hazardous, land as soon as possible.
3. Do not trust any altitude instruments. Learn to judge altitude, especially on landings. Barometric conditions may change in a crosscountry flight, so that even a barometer that is functioning properly may
read an incorrect altitude. Moreover, the altitude of the landing place may be different from that of the starting point.
4. At all times keep machine in such position, in reference to suitable ground, that a landing can be effected at any time.
5. Do not cut across bows of other machines when making your first turn.
6. No vertical banks, steep climbing turns or zooming will be done under 300 feet.
7. All acrobacy such as loops, wing-overs, eights, rolls, half-rolls and spins must be completed at not less than 1,500 feet.
8. Come out of steep side-slips and spirals at not less than 300 feet.
9. At no time will "hedge-hopping" be tolerated.
10. Whenever possible, landings and take offs will be directly into the wind.
11. No spins on back or tail slides will be indulged in as they put unnecessary strain on the machine.
12. All machines will land in a straight glide from 500 feet.
13. To go off the ground in a side wind, be sure to allow the machine to have flying speed before attempting to arise; then turn slightly into the wind, gain a safe altitude and then level out before attempting to turn and go with the wind.
14. If machine slides in, use more rudder or take off some of your bank or combine both.
15. If flying against the wind and you wish to turn and fly with the wind, do not make a sharp turn close to the ground.
16. In gliding for a landing, if gliding flat at a high altitude, increase the angle of the glide and store up speed when approaching the ground. If gliding flat and you wish to make a turn, increase the angle of glide and allow the machine to pick up speed, then make the turn. Glide steep rather than flat. Increase glide for a turn.
17. Motors have been known to stop during a long glide on account of running same throttled down too low. If pilot wishes the use of motor for landing, open throttle at intervals during the glide.
18. In coming down with excess speed, level out and allow machine to skim along close to ground. Do not attempt to force machine on ground with more than flying speed; the result is bouncing and ricocheting.

## CHAPTER IX

## AIRPLANE PERFORMANCE AT DIFFERENT ALTITUDES

The density of air decreases with altitude. The sustentation of an airplane and its resistance in flight both depend upon the reaction of its surfaces against the air; and, as this reaction varies with air density, it follows that the aerodynamic performance at high and at low altitudes is materially different. It will be understood, however, that, although there is a difference in the values of various quantities, there is no difference in the principles, for these hold at all altitudes.

Aerodynamically there are advantages as well as disadvantages in flight at high altitude, for, although there is a decrease in sustentation in air of less density, there is a corresponding decrease in resistance. The decrease in sustentation makes a high velocity more desirable, while decrease in resistance makes a high velocity more readily obtainable; aerodynamically, the higher the altitude the higher should be the velocity.

Aerodynamic performance, however, is not the only question. The gas engine depends upon the oxygen of the air for part of its fuel supply, and the decrease in engine power at high altitudes due to the decrease in oxygen, is, unfortunately, a most serious consideration. Very desirable would be an engine developing full power* at all altitudes.

The various factors affecting airplane performance at different altitudes will be separately considered.

[^48]
## Density of air.

Air at sea level weighs about 0.075 pounds per cubic foot; its density is, accordingly, said to be 0.075 . As a matter of fact the density of the air changes with weather conditions, but 0.075 is often taken as a standard* value for calculating


Fig. 78. Decrease of air density with altitude.
airplane performance. One cubic foot of water weighs as much as 833 cu . ft. of air, namely, 62.4 lbs .

Although air is light compared with water and other substances, it has a weight which is not inconsiderable. The weight of the air in a room $20^{\prime} \times 20^{\prime}$ with a $10^{\prime}$ height of ceiling is 300 lbs ., a greater weight than a man can carry. In a class room the weight of the air may be 500 or 1000 lbs .; in a certain lecture room it is 12000 lbs .

## Relative air density, $\delta$.

In many cases we are concerned with relative rather than absolute density, the density of air at ground level being

[^49]taken as I .00 . The relative density, $\delta$, of the air at any altitude is the ratio of the density of the air at that altitude to the density of the air at ground level. The relative density at any altitude varies with weather conditions. Standard values of relative density used by the Air Service are given in the accompanying table.* These values are also shown by the curve in Fig. 78.

## Table

Relative Density of Air at Different Altitudes

| Altitude Feet | Relative Density | Altitude Feet | Relative Density |
| :---: | :---: | :---: | :---: |
| $\bigcirc$ | 1.000 |  |  |
| 1,000 | . 972 | 16,000 | . 603 |
| 2,000 | . 944 | 17,000 | . 582 |
| 3,000 | .917 | 18.000 | . 562 |
| 4,000 | . 890 | 19,000 | . 543 |
| 5,000 | . 863 | 20,000 | . 524 |
| 6,000 | . 837 | 21,000 | . 505 |
| 7,000 | .8II | 22,000 | . 487 |
| 8,000 | . 786 | 23,000 | . 470 |
| 9,000 | .761 | 24,000 | . 453 |
| 10,000 | . 737 | 25,000 | . 437 |
| 11,000 | . 713 | 26,000 | . 42 I |
| 12,000 | . 690 | 27,000 | . 406 |
| 13,000 | . 667 | 28,000 | . 391 |
| 14,000 | . 645 | 29,000 | .376 |
| 15,000 | . 624 | 30,000 | .362 |

[^50]
## Lift and velocity, as affected by air density.

Sustentation is in direct proportion to relative air density $\delta$; thus,

$$
L=\delta K_{\mathrm{L}} S V^{2}
$$

In horizontal flight, lift is equal to weight and may be taken as constant for a particular machine; $S$ is also constant. To get the necessary sustentation at high altitudes, it is evident that either $K_{\mathrm{L}}$ or $V$ must be increased as $\delta$ decreases.

When a machine flies at different altitudes at the same velocity, the coefficient of lift $K_{\mathrm{L}}$ must be increased (by inincreasing the angle of incidence) as the density decreases; $\delta K_{\mathrm{L}}$ is constant and hence $K_{\mathrm{L}} \propto \mathrm{I} / \delta$.

When a machine flies at different altitudes with the same angle of incidence, $K_{\mathrm{L}}$ remains unchanged and the velocity $V$ must be increased as the density decreases; $\delta V^{2}$ is constant and hence $V \propto \sqrt{\mathrm{r} / \delta}$. So far as sustentation is concerned, velocity should increase with altitude. Let us see whether other considerations point to the same conclusion.

## At constant incidence, resistance remains constant when $\delta$ decreases and $V$ increases.

The $L / D$ ratio, at any given angle of incidence, is independent of density; for

$$
\begin{aligned}
& L=\delta K_{\mathrm{L}} S V^{2}, \text { and } D=\delta K_{\mathrm{D}} S V^{2} . \\
& L / D=K_{\mathrm{L}} / K_{\mathrm{D}} .
\end{aligned}
$$

Since, for constant angle of incidence, $\delta V^{2}$ is constant, the wing-resistance $D$ is constant and is independent of density. Parasite resistance, $\delta k s V^{2}$, is also constant, at constant angle of incidence; for, here as before, $\delta V^{2}$ is constant.
It, therefore, follows that the total resistance $R$-the sum of wing and parasite resistance-is constant at constant angle
of incidence. It is to be kept in mind, however, that at constant incidence $V \propto \sqrt{\mathrm{I} / \delta}$; the velocity necessary for sustentation is greater at higher altitudes but resistance remains the same.

When incidence changes and velocity is constant, the total resistance at high altitudes may be either greater or less than


Fig. 79. Variation of airplane resistance with velocity at different altitudes. For any particular angle of incidence, resistance is the same at all altitudes; velocity varies inversely as the square root of the air density.
at ground level; in the usual flying range it is very much less, as will be seen by the curves in Fig. 79.

The effect of change of density upon resistance is shown by the curves for total airplane resistance when $\delta=\mathrm{I}$ (ground level), $\delta=0.5$ (altitude approximately 21000 ft .) and $\delta=0.375$ (altitude approximately 29000 ft .). As $\delta$ decreases, any point on the curves (for example, the point corresponding to an incidence of $2^{\circ}$ ) is merely shifted to
the right, since $V$ varies as $\sqrt{I / \delta}$. It is interesting to note that the minimum resistance has the same value at all altitudes and is obtained at the same angle of incidence but at different velocities.
Gliding.-Since the sine of the gliding angle equals $R / W$, it follows that, at constant incidence, the gliding angle is the same at all altitudes; the gliding velocity varies as $\sqrt{\mathrm{I} / \delta}$. The minimum gliding angle is the same at all altitudes.


Fig. 80. Curve for power required at ground level $(\delta=1)$ and at an altitude of 21000 ft . $(\delta=0.5)$. Power required and velocity both vary as $\sqrt{\overline{\mathrm{I}} / \delta}$, for a given angle of incidence.

## Power required at different altitudes.

Horse power required equals $R V / 375$ and, for a constant angle of incidence, varies as $V$ or as $\sqrt{\mathrm{I} / \delta}$, for $R$ is constant, as shown above.

Fig. 80 shows curves for power required when $\delta=1$ (ground level) and when $\delta=0.5$ (altitude approximately

21000 ft .). For each angle of incidence the ordinate as well as the abscissa is proportional to $\sqrt{\mathrm{I} / \delta}$; the decrease in $\delta$ from I to 0.5 increases the velocity and likewise the power required, for any particular angle of incidence, by a factor $\sqrt{2}$. Thus, in the case shown, the velocity is 100 MPH. and the power required is $100 \mathrm{H} . \mathrm{P}$. at ground level when the angle of incidence is $2^{\circ}$; at an altitude of $2 \mathrm{I}, 000 \mathrm{ft}$. (with the same angle of incidence) the velocity is 14 I MPH. and the required horse power is 14 I H.P.

Corresponding points, for the same incidence at different altitudes, lie on a straight line, as od, drawn from the origin. The points of minimum thrust, $t$ and $t^{\prime}$, in a like manner lie on the tangent line oc.

## Power available at different altitudes.

The decrease in air density at high altitudes has much the same effect upon a gas engine as a decrease in air supply by throttling. The indicated* horse power is reduced, approximately, in proportion to the amount of air supply. With the usual gas engine, when the relative density is $\delta=0.5$ (at an altitude of about $2 \mathrm{I}, 000 \mathrm{ft}$.) the air supply and the indicated $\dagger$ horse power are only half as much as at ground

[^51]level. Although this is not precise, and a variation is produced by the type of carburetor and by different secondary effects, it is sufficiently correct for the purposes of the present discussion.*

If the mechanical efficiency of the engine were constant, the brake horse power which the engine delivers at different altitudes would be in proportion to the indicated horse power and so in proportion to the density. As a matter of fact the mechanical efficiency is less at high altitudes, for the constant friction loss is a greater percentage of the power when the power is less.

Brake horse power, accordingly, decreases more rapidly than indicated horse power and density, as altitude is acquired; but, as an approximation, we may assume that B.H.P. varies as I.H.P.. namely, in proportion to density.

## Curves for brake horse power.

The solid curves in Fig. 8i show the brake horse power of a particular engine at ground level, and at altitudes of 9500 ft . and 21000 ft ., assuming that brake horse power varies in direct proportion to $\delta$. It will be seen that the curves so

[^52]drawn for $\delta=1, \delta=3 / 4$, and $\delta=1 / 2$ correspond to curves for full throttle, $3 / 4$ throttle and $1 / 2$ throttle, already discussed; see Fig. 44, page 84. On account of the effect of losses, as explained above, the actual B.H.P. for $\delta=3 / 4$ and $\delta=1 / 2$ would be somewhat less, as indicated by the dotted curves.

The interesting conclusion is that the power delivered by a gas engine decreases with altitude, approximately in propor-


Fig. 81. Brake horse power at different altitudes. Power is less at high altitudes and varies approximately as the air density. In the ordinary engine the R.P.M. for maximum power is practically the same at all altitudes.
tion to density. Furthermore, it is seen from the curves that the engine speed for maximum power is practically the same at all altitudes.

It has already been shown to be desirable, aerodynamically, to increase velocity at high altitudes. In other words, we want to increase $V$ while holding the engine speed $N$ constant. But propeller efficiency depends upon the ratio $V / N$
and is a maximum for a certain value of $V / N$, as was shown in Chapter VI. It is important, therefore, to ascertain whether this relation is affected by change in air density.

## Operation of a propeller at different densities.

At any given value of $V / N$, -i.e., when the angle of incidence of the propeller blade is constant-propeller thrust and propeller torque vary in direct proportion to the relative air density, $\delta$, in the same way that the lift $L$ and drag $D$ of an aerofoil vary. Hence, thrust horse power (which is proportional to the product of thrust and $V$ ) and torque horse power (which is proportional to the product of torque and $N$ ) both vary directly as $\delta$, when an airplane is flown at different altitudes with $V$ and $N$ constant.

It follows that the ratio of thrust horse power to torque horse power-i.e., the efficiency of a propeller-isindependent of $\delta$, provided the ratio $V / N$ is constant. This it true even if $V$ and $N$ change provided they change in proportion, for any increase in thrust horse power, due to increase in $V$, is accompanied by a like increase in torque horse power, due to increase in $N$. Likewise, when $V$ and $N$ decrease, thrust horse power and torque horse power both decrease in proportion.

The efficiency of a propeller varies, therefore, as $V / N$, irrespective of density; the curves for propeller efficiency, Figs. 55 and 56, page 114, hold for all altitudes.

This means that, if we are to maintain a practically constant engine speed $N$ as required by the engine, with a given propeller we must fly at a practically constant velocity $V$ at all altitudes, if the propeller is to be operated near its maximum efficiency. If we fly at a higher velocity at higher altitudes, as is aerodynamically advisable, it is impossible to
maintain a given engine speed $N$ and to operate a given propeller near its maximum efficiency at all altitudes.

This, unfortunately, takes away much of the aerodynamic gain in flight at high altitudes and at high velocities; the aerodynamic gain is paid for not only by loss in engine power but also by lower propeller efficiency.

The desirability of an engine capable of delivering full power at all altitudes has already been pointed out. The


Fig. 82. Power available, or thrust horse power, from a given engine and propeller at different altitudes.
advantage of a propeller operating near maximum efficiency under all conditions-i.e., for different values of $V / N$-is likewise obvious. A glance at Fig. 56 shows how this advantage may be gained by the use of a propeller of adjustable pitch or by the substitution of one propeller for another in flight. The advantage of changing propeller pitch in flight, according to whether one desires high velocity or ability to climb, has already been pointed out (page 138 ); but the
mechanical difficulties of changing pitch have, in the past, offset the aerodynamic advantages. These difficulties are, however, being overcome.

## Curves for thrust horse power available at different altitudes.

Curves for thrust horse power available from a particular engine and propeller at different altitudes, on the assumption that brake horse power varies as $\delta$, are shown in Fig. 82.


Fig. 83. Power required and power available at different altitudes (density $\delta=1,7 / 8,3 / 4$ and $1 / 2$ ). Minimum velocity at $a, a^{\prime}, a^{\prime \prime}$; maximum velocity at $b, b^{\prime}, b^{\prime \prime}$; minimum thrust at $t, t, t^{\prime \prime}, t^{\prime \prime \prime}$.

Comparison of power required and power available at different altitudes.
In Fig. 83 are drawn curves for power required and for power available at different altitudes, corresponding to $\delta=\mathrm{I}, \delta=7 / 8, \delta=3 / 4$ and $\delta=1 / 2$, these being similar to the curves in Figs. 80 and 82.

The points of minimum thrust, $t, t^{\prime}, t^{\prime \prime}, t^{\prime \prime \prime}$ lie on the line oc tangent to the curves for power required. The line od locates the points on the curves for power required corresponding to an incidence of $2^{\circ}$; in a like manner the line oe locates points for an incidence of $\mathrm{I}^{\circ}$, as explained in connection with Fig. 80.

Minimum velocity is at $a$ when $\delta=1$; at $a^{\prime}$ when $\delta=7 / 8$, etc. Maximum velocity is at $b$ when $\delta=1$; at $b^{\prime}$ when $\delta=7 / 8$, etc. It is seen that as $\delta$ decreases with increase in altitude, the range between the minimum and maximum velocities becomes less.

At higher altitudes the surplus power also becomes less. When an altitude is reached at which $\delta$ has a certain value, the points of minimum and maximum velocity coincide, the curves for power required and power available being tangent, as in Fig. 64, page 129. (For the case shown in Fig. 83, this would be for a value of $\delta$ more than $1 / 2$ and less than $3 / 4$.)

This defines the "ceiling," the highest altitude the machine can attain. At the ceiling, the surplus power is zero; horizontal flight is possibly at one velocity only, the machine losing altitude at any other velocity.

## Rate of climb decreases with altitude.

In climbing, the rate of climb becomes less and less as a machine gains altitude, for the surplus power is less.*

[^53]Fig. 84 shows the actual climbing performance of a certain machine which reached its ceiling of approximately 21000 ft . in about 100 minutes.* A machine is usually said to have reached its ceiling when its rate of climb becomes less than roo ft. per minute.


Fig. 84. Typical climbing curve.
The machine whose climbing performance is shown in Fig. 84 was not a fast climber; it reached an altitude of $10,000 \mathrm{ft}$. in about 20 minutes. This altitude is reached by better

\footnotetext{
*The climbing performance of a machine reaching a ceiling of 28,000 ft . in 70 minutes is here given by way of further illustration.

| Altitude | $\begin{gathered} \text { Minutes } \\ 0 \end{gathered}$ | Rate of Climb | R. P. M. $\mathbf{1}, 800$ | Air Speed |
| :---: | :---: | :---: | :---: | :---: |
| 2,000 | 1.I | 1,795 | 1,905 | 85 |
| 5,000 | 2.9 | 1,530 | 1,935 | 85 |
| 10,000 | 6.75 | 1,095 | 1,960 | 83 |
| 15,000 | 12.6 | 650 | 1,960 | 81 |
| 20,000 | 22.4 | 340 |  | 78 |
| 25,000 | 44.6 | 170 |  | 70 |
| 28,000 | 69.8 | 75 |  | 63 |

In this case the R. P. M. are seen to increase with altitude; in many cases they decrease slightly. Rate of climb is given in feet per min.; air speed, in miles per hr.
climbers in 8 or 10 minutes, and it has been reached in slightly less than 5 minutes.

A heavy machine may have a ceiling of only a few thousand feet; other light machines, built for high altitudes, may climb to 30,000 or $35,000 \mathrm{ft}$. An altitude* of $32,450 \mathrm{ft}$. was reached by R. Rohlfs, Sept., 1919, Still higher altitudes are likely to be attained.

As shown by the curves in Fig. 42, page 75, weight is an important factor in determining power required, and so in determining the surplus power of a machine and hence its ceiling.

[^54]
## CHAPTER X

## SINGLE AND MULTIPLE PLANES*

Sustentation is obtained in a monoplane from a single plane or sustaining surface, in a biplane from two and in a triplane from three such surfaces; more than three planes are rarely used. The term multiplane is used to designate any machine with more than one plane. The choice of type, as to the number of planes, is determined chiefly by practical questions of design, for the aerodynamic principles are the same regardless of the number of sustaining surfaces. So also whether the propeller is located in front** as in a tractor or behind as in a pusher machine, and whether several propellers are used or one, are questions of design and not of principle. The same is true of various other features of airplane structure. Generally speaking, it is the aim of the designer to combine a practical construction with aerodynamic advantages.

The reader will understand that the essential elements that determine sustentation, thrust, power, stability, etc., as outlined in this volume, pertain to all types of airplanes with only such minor modification as may be necessary in the application of these general principles to particular types.

A complete description of various types of machines will not be undertaken here, but certain facts and relations that

[^55]bear upon airplane structure and operation-some of which are touched upon in other chapters-will be presented.

## Lift is largely obtained from the upper surface of a wing.

As stated in the first chapter, lift is largely obtained from the negative pressure resulting from the rarefaction of the air on the upper surface of a wing, caused by its motion relative to the air. The production of lift by the passage of air over a surface may be shown by a simple experiment with a sheet of paper held just below the mouth, as shown in position $A$ Fig. 85. As air is blown from the mouth, the sheet rises from position $A$ to position $B$, due to a rarefaction above the sheet as the air passes over it. This illustrates the chief cause of lift in an airplane. It will be understood that the lift is produced by the


Fig. 85. Sheet is raised from A to B by blowing. relative motion of the surface and the air, regardless of whether it is the surface or the air that moves.

The amount of lift thus produced by rarefaction $n$ the upper surface of an aerofoil depends a great deal upon its shape. The entering edge and upper surface of the aerofoil are so designed that the airstream swings upover the aerofoil, as shown in Fig. 9 on page 1 $_{5}$, so as to increase the rarefaction and hence the lift. The shape of the upper surface of a wing is thus highly important, whereas the shape of the lower surface is of less consequence; indeed, a flat under surface is almost as good as any. The airstream as it leaves the wing is always deflected downward.

## Distribution of wing pressure.

The distribution of pressure over the upper and lower surfaces of a particular wing is shown in Fig. 86. This is the distribution over the central section of the wing, at an angle of incidence of $6^{\circ}$. The dis-


Fig. 86. Distribution of pressure on a single wing. Negative pressure on the upper surface produces more lift than the positive pressure on the lower surface. tribution over other sections and for other angles of incidence, while different, is more or less of the same general character*. Wings of different designs of course have different distributions of pressure but the type of distribution is as shown in the figure. Near the wing tips, the leakage of air around the ends decreases both the positive pressure beneath the wing and the negative pressure above it. A raked end $\dagger$ to the wing and a decrease of incidence near the wing tip both tend to lessen this air leakage.

It is seen: first, that the negative pressure above a wing is much greater than the positive pressure underneath; and, second, that the pressure is more on the forward part $\ddagger$ of the wing than on the rear part.

[^56]The entering edge and front part of the wing are most effective in producing lift; the rear part of the wing contributes but little.

Although the back part of the wing has little effect upon lift, it has considerable effect upon wing-resistance, and by a proper shaping of the back part of a wing wing-resistance may be materially reduced. The back part of the wing, also, has a direct effect upon the inherent longitudinal stability, but this is of minor importance for stability is chiefly taken care of by the tail.

## Aspect ratio.

The aspect ratio or aspect of an aerofoil is the ratio of its span, measured from wing tip to wing tip, to its chord. Since, as shown in the preceding paragraph, the forward part of the wing is most effective in producing lift, it follows that a wing with high aspect ratio-in which the span is great compared with the chord-has more lift than a wing of the same area with low aspect ratio, for the wing with high aspect ratio has a longer entering edge. The wing with high aspect ratio encounters more new air and so gets more lift. Furthermore, the loss of lift due to the leakage of air around the ends of the wing becomes relatively less as the aspect ratio increases.

It follows, therefore, that a wing is best aerodynamically when it has a large span and small chord, that is when it has a large aspect ratio. For structural reasons, however, the span and aspect ratio cannot well be increased beyond a certain reasonable amount. Aspect ratios from 5 to 7 (approximately the aspect of birds) are common and little is gained by using much higher values. An aspect ratio of 6 is usually taken as a standard for comparing data for various wing sections.

The effect of change of aspect ratio upon lift and $L / D$ ratio, for a certain wing, is shown in the following table:

## Correction for Aspect Ratio

| Aspect ratio | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $L$ | 0.62 | 0.73 | 0.85 | 0.95 | 1.00 | 1.04 | 1.08 |
| $L / D$ | 0.54 | 0.64 | 0.77 | 0.90 | 1.00 | 1.10 | 1.16 |

For an aspect ratio 6, the value of $L$ is taken as unity; the value of $L / D$ is also taken as unity. Relative values for other aspect ratios are shown in the several columns. $L$ and $L / D$, determined for an aspect ratio 6 , may be multiplied by the correction factors in the table to obtain the proper values for any other aspect ratio. Thus, for an aspect ratio 5 , the lift is 95 per cent. and the $L / D$ ratio is 90 per cent. of the corresponding values for an aspect ratio 6 .

It is seen that little is gained in going to a higher aspect than 5,6 or 7 ; on the other hand there is a considerable loss, both in $L$ and $L / D$, in going to a lower aspect. The preceding table is for an incidence of $6^{\circ}$. Somewhat different values are obtained for other angles and for other wing sections.

A high aspect ratio is desirable for a large lift; but for small lift, high speed and handiness in maneuvering a low aspect ratio is more suitable.

From the standpoint of longitudinal stability, a short chord (with a correspondingly high aspect ratio) is better than a long chord, for the travel of the center of pressure-a certain percentage of the chord-is then less. The shifting of the center of pressure, as incidence changes, has already been shown in Fig. 13, page 24.

## Biplanes and triplanes.

Wing area is usually better obtained in a biplane or triplane than in a monoplane, particularly when large wing
area is required. This is for structural reasons. A single plane is structurally weak unless well braced, for a spar can be no thicker than the wing itself; a single plane can not well be built when a certain span is exceeded. When two or three planes are used, one above another, as in a biplane or triplane, the planes themselves form members of a truss and a strong structure is thus obtained; but this gain in strength is accompanied by a loss in aerodynamic efficiency.

On account of these structural advantages the biplane or triplane is most generally used, not only for large loadcarrying machines but for speed machines as well. Compared with a monoplane, a multiplane machine may have a shorter chord as well as a lesser span; with a shorter chord there is less travel to the center of wing pressure and hence there is greater stability.

On the other hand a monoplane gives more unobstructed vision to the pilot or observer and has a certain simplicity. Furthermore, for the same aspect ratio, a monoplane has a greater lift per unit area than a multiplane. This is due to the fact that the lift of multiplanes is diminished by the interference between planes, discussed in a following paragraph.

In a monoplane it is desirable to have the loading as high as the structure will permit and to have the velocity high, so as to require as small a wing area as possible; there are, however, structural limits to the possible loading.

The monoplane wing is usually strengthened by bracing; but it is to be kept in mind that all struts and wires, in monoplanes as well as in multiplanes, contribute materially to parasite resistance

## Interference between planes of a multiplane.

A multiplane, although structurally better than a monoplane, is aerodynamically inferior on account of the interference between planes.

The rarefaction or negative pressure on the top surface of a lower plane is partially neutra-


Fig. 87. Distribution of pressure on a biplane. Negative pressure on upper surface of lower wing is much reduced by interference with the upper wing. lized by the compression or positive pressure on the under side of the plane above, so that the total lift is thus reduced. Obviously, the same air can not be both rarefied and compressed at the same time.

In a biplane, about 55 per cent. of the lift is contributed by the upper plane and about 45 per cent. by the lower.

Fig. 87 shows the distribution of pressure on the two planes of a biplane. It will be noted that the negative pressure on the top surface of the lower plane is much less than the corresponding negative pressure on the top surface of the upper plane. This of course means a decrease in lift. This decrease may be made less (I) by increasing the gap between the planes; (2) by giving the planes a stagger so that the upper plane is in advance of the lower, as in Fig. 88; and (3) by increasing the aspect ratio and thus decreasing the chord.

## Biplane correction.

On account of the interference between the planes of a biplane, both lift and $L / D$ ratio are reduced. This reduction becomes less as the gap is increased, as shown by the following table from National Physical Laboratory data for an incidence of $6^{\circ}$; little is gained, however, by making the gap greater than the chord.

## Correction for Biplane Interference

| Gap | 0.4 | 0.8 | 1.0 | 1.2 | 1.6 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $L$ | 0.6 I | 0.76 | 0.8 r | 0.86 | 0.89 |
| $L / D$ | 0.75 | 0.79 | 0.8 r | 0.84 | 0.88 |

The first line gives the gap in terms of the chord, which is taken as unity.

To obtain the value of $L$ or $L / D$ for a biplane of any given gap, the corresponding values for a single plane are to be multiplied by the number given in the table; thus, if the gap is r .2 times the chord, a biplane has 86 per cent. as much lift and 84 per cent. as much $L / D$ as a monoplane, provided the biplane has the same aspect ratio and has no stagger.

A biplane may readily be given a somewhat larger aspect ratio than a monoplane (with the same area and span a biplane has twice the aspect ratio) and the increase of lift with aspect ratio* may thus partially or wholly offset the decrease due to biplane interference.

Interference between the planes of a biplane may be reduced by placing the upper plane slightly in advance of the lower-i.e., by a positive stagger; in this way $L$ may be increased as much as 8 per cent., with little effect upon $L / D$.

Example.-As an illustration, suppose we are given $K_{\mathrm{L}}=$ 0.00195 for a single plane, aspect ratio 6 , at $5^{\circ}$ incidence,

[^57]as given in the Table I , on page 20. To get $K_{\mathrm{L}}$ for a staggered biplane, aspect ratio $=7$, gap $=1.2:$-multiply 0.00195 by 1.04 to correct for aspect ratio, by 0.86 for biplane interference, by 1.08 for stagger; hence, $K_{\mathrm{L}}=0.00188$. In this case, $K_{\mathrm{L}}$ for the biplane is 3.5 per cent. less than $K_{\mathrm{L}}$ for the monoplane.

## Biplane construction.

Of various forms of machines, the biplane tractor is one


Fig. 88. Biplane construction.
of the most common. A good illustration of a machine of this type is shown in the frontispiece. Biplane construction is illustrated in Fig. 88.

The distance between the two planes of a biplane is called the gap; this is usually measured as the shortest distance
between the two chords (see glossary) but is sometimes taken as the vertical distance.

Stagger is determined by measuring the distance between the entering edge of the lower plane and a plumb line dropped from the entering edge of the upper plane; this distance may be expressed in inches but is better expressed as a percentage of the chord. Stagger is positive or negative according to whether the lower plane is behind or in advance of the upper plane. (Negative stagger is not common, but is sometimes used to obtain greater range of upward vision.)

As already mentioned (page 23), a biplane -particularly when the planes are staggered-has a flatter burble point than a single plane. The stalling angle is thus reached less suddenly.

Longitudinal stability may be increased by giving the lower or rear plane of a staggered biplane slightly less* incidence than the upper or forward plane, the effect on stability being the same as giving the tail less incidence than the main plane. This is more fully discussed in the chapter on Longitudinal Stability.

Wing structure has been illustrated in Fig. roa, page 17. The upper and lower wings of a biplane are held apart by compression members or struts (see Fig. 88) fastened to the spars of the two wings. Usually, the truss formed by the two wings is strengthened by wire tension members or stays. Stays that are under tension in flight are termed flying wires; other stays, under tension when the machine is landing or has landed, are termed landing wires.

[^58]The understrticture for carrying the weight of a machine when resting or running on the ground is termed the landing gear or undercarriage and is usually equipped with wheels (see frontispiece). The tail is commonly supported by a tail skid.

Very commonly, the lower plane of a biplane is given less span than the upper; see sketch on outside cover. The two planes may differ not only in size, but in shape and structure.

## Controls.

The controls of an airplane, illustrated in Figs. 89, 90 and 9I, are three:
(r) The rudder, for directional or horizontal control, i.e., for keeping a straight course and for turning to left or right. The rudder on an airplane acts in the same manner as the rudder on a boat.
(2) The elevator, for longitudinal or vertical control, i.e., for keeping the flight path horizontal and for inclining it upward or downward. The elevator acts as a rudder having a horizontal axis.
(3) The ailerons, for lateral control, i.e., for keeping the two wings on the same level and for rolling so that one wing goes up and the other down.

## Rudder.

The rudder is usually operated by a foot-bar, as shown in Fig. 89, a turn to the right being made by pressing the right foot forward. (The wires in some machines are crossed, so that a turn to the right is accomplished by pressing the left foot forward; but such machines are now uncommon.)

## Elevator.

The elevator is usually operated by pushing the control stick or control column (Fig. 90) forward, to lower the elevator so that the machine noses down; or backward, to raise


Fig. 89. Operation of rudder.
the elevator so that the machine noses up The elevator is usually in two parts, one part being on each side of the rudder as shown in the frontispiece.

When a machine is banked the rudder aids in vertical control and the elevator in horizontal control. When the


Fig. 90. Stick control for operating elevator. To nose down, push stick forward thus lowering elevator. To nose up, pull stick back thus raising elevator.
machine is banked more than $45^{\circ}$ the functions of the elevator and rudder thus become interchanged.

## Ailerons.

The control surfaces for lateral control are called ailerons; these are hinged surfaces, one (or more) on each side of the machine, and are commonly placed back of the wing near the
wing tip*, as shown in Fig. 91. In a biplane, the ailerons are attached to both upper and lower wings, or to the upper wing only, as shown in Fig. 88. When an aileron is turned up, the air strikes its upper surface, tending to force that side of the machine down; when an aileron is turned down, the air strikes its lower surface, tending to force that side of the machine up. The ailerons are so connected that when the right one is turned up the left one is turned down, and vice versa.

The operation of ailerons is shown in Fig. 91. When the control stick is pushed to left or right, the corresponding side


Fig. 91. Operation of ailerons for lateral control. Turning wheel or moving stick to one side causes wing to descend on that side.
of the machine is depressed by its aileron which is turned up; the other side of the machine is raised by its aileron which is turned down.

Instead of wing-tip ailerons, just described, interplane ailerons, placed midway between the upper and lower plane of a biplane are sometimes used, but interplane ailerons greatly increase resistance. The operation is the same for interplane as for wing-tip ailerons; see Fig. 110, page 223.

[^59]
## Wheel Control.

Instead of the control stick just described, a control wheel as in an automobile, called a Dep. or Deperdussin control, is frequently used; the wheel is turned to the left to depress the left side of the machine and to the right to depress the right side. A control wheel is shown, in diagram, at the left of Fig. 9r. (As in the case of the rudder, ailerons are sometimes connected so as to be operated by a motion opposite to the one described.)

## Control surfaces.

Control surfaces for rudder, elevator or aileron are not uncommonly double convex, as in Fig. 92, but (in the case of elevator and aileron) such surfaces are often given a section

somewhat like a wing, to add to the effectiveness of their operation or to increase stability.

Balanced control.-A small control surface may easily be turned up or down, left or right, when it is swung from its front edge, as the aileron in Fig. 88. Large control surfaces, however, (rudder, elevator or ailerons) are frequently swung from an axis placed back from the front edge, as at $b$ in Fig. 92, so as to require less force in their operation. Such control surfaces are said to be balanced. Note the balanced rudder in Fig. 113, page 230.

## Stability.

Controls are used for directing an airplane and for maintaining it in equilibrium, i.e., for giving it stability. Their action will be more fully brought out in the following chapters on longitudinal, lateral and directional stability.

## CHAPTER XI

## LONGITUDINAL STABILITY

Before discussing longitudinal stability let us consider the question of stability in general.

## Stability in General

## Stable, unstable and neutral equilibrium.

In order to fly, an airplane must have not only sustentation, as discussed in the first chapter, but it must also have a certain inherent*stability, i.e., it must be in stable equilibrium, so that it will automatically return to its normal position whenever it is displaced therefrom. To be sure this return may be effected in part by control by the pilot, but to depend upon such control entirely, as would be necessary if the machine itself were unstable, would indeed be precarious and would make flight a feat similar to balancing on a tight rope.

Equilibrium may be stable, unstable or neutral. A rocking chair is (within limits) in stable equilibrium because any slight displacement gives rise to a restoring force tending to bring it back to its normal position. An egg balanced on end is in equilibrium, but its equilibrium is unstable because any slight displacement will cause it to upset. A ball on a level floor is in neutral equilibrium; for, when it is displaced, it remains in

[^60]its new position, as there is no force tending either to restore it or to displace it further.
A body, as an airplane, moving with uniform motion in a definite attitude with respect to the surrounding medium, is in stable equilibrium when any displacement sets up forces that restore the original state of motion. On the other hand, its equilibrium is unstable when any displacement is accompanied by forces which cause it to depart further from its original state of motion. When it tends neither to deviate further from nor to return to its former condition of motion, when acted upon by a slight disturbance, its equilibrium is neutral.

A stable airplane, after any temporary disturbance, always returns to its original attitude, which is determined by the positions of its controls and the power delivered by the engine and propeller. With controls and power unchanged, an airplane is in equilibrium for one angle of incidence only.

## Conditions for stability.

There are two general conditions for equilibrium. To these a third is added in order that the equilibrium shall be stable:-

First. The resultant of all forces acting upon the body in question must equal zero; or, otherwise stated, the sum of the vertical forces must equal zero and the sum of the horizontal forces must equal zero.

Second. The sum of all moments tending to produce rotation of the body must equal zero.

Third. There must be a restoring moment whenever the body is displaced.

The first condition has been fully dealt with in the preceding chapters and needs no further discussion. Thus, it has
been shown that, for horizontal flight, Lift $=$ Weight (sum of vertical forces equals zero), and Thrust $=$ Resistance (sum of horizontal forces equals zero). This condition is illustrated in Fig. 96. In gliding, it has been shown (Fig. 75, page 150 ) that weight is equal to the resultant of lift and resistance, i.e., the resultant of all forces acting is zero.

The second and third conditions in regard to moments remain for consideration.

Moments.-An airplane rotates about its center of gravity, and every force acting upon it that does not pass through the center of gravity tends to produce rotation. The tendency of a force to produce rotation is measured by the moment of the force about the center of gravity, i.e., the product of the force and the perpendicular distance from the center of gravity to the line of action of the force.

When two equal forces act along parallel lines in opposite directions, they are said to produce a couple; the moment of a couple is a measure of its tendency to produce rotation and is equal to the product of either of the two forces and the perpendicular distance between them. See Figs. 97 and 98; also foot note, page 207.

## Three stabilities of an airplane.

Stability of an airplanemay beconsidered under three heads:
Longitudinal or fore-and-aft stability, about a "pitching" axis which passes through the center of gravity and is perpendicular to the plane of symmetry of the machine.

Lateral stability, about a "rolling" axis, which passes through the center of gravity and is more or less in the line of flight.

Directional or "weather-cock" stability, about a "turning" or "vertical" axis, often referred to as a "yawing" axis, which
passes through the center of gravity and is more or less vertical.

These stabilities are not independent, for rolling produces turning and turning produces rolling, and either may cause an airplane to change its longitudinal attitude, i.e., to nose up or to nose down.

## Amount of stability that is desirable depends upon intended use.

The amount of stability that a machine should possess depends to a large extent upon the use for which it is intended. A high degree of stability decreases the senstiveness of control and, although advantageous in many ways, is not desirable when quick maneuvers are to be made.

Indeed, for such maneuvers, lateral and directional stability may well be quite small or practically nil, control by the pilot being depended upon to preserve lateral equilibrium. For cross-country transport, on the other hand, such small stability would be quite inappropriate and would put upon the pilot too great a burden of control. In no case, however, can longitudinal stability be safely so reduced.

## Distribution of weight.

Concentration of weight near the center of gravity gives sensitiveness of control and is generally desirable. When weight is concentrated oscillations are quickly damped out but a machine is more sensitive to gusts; it is not desirable for a machine to be so sensitive that it follows every puff.

Distributing the weight gives less sensitive control; oscillations have a greater period and are less readily set up, but thev are less easily damped and persist longer.

Weight may be distributed longitudinally and at the same time remain concentrated laterally and vertically. Such a
distribution of the weight fore-and-aft will increase the moment of inertia about the pitching and the turning axes; the oscillations about these axes will thus become slower and the control less sensitive.

## Damping of oscillations.

Oscillations are damped by large surfaces placed at a distance from the axis about which the oscillations takes place. The damping may be so great that no oscillations take place, the return of the airplane to its original attitude after displacement being dead beat.

## Effect of velocity upon control and stability.

The control of an airplane and likewise its stability are dependent upon air pressure on the airplane surfaces and are therefore dependent upon airplane velocity. Loss of velocity means loss of control and loss of stability. When velocity is decreased to a certain amount, the controls are somewhat "wobbly" and, as the velocity is further decreased, become entirely ineffective.

## Many factors affect stability.

There are many factors that affect airplane stability and these factors have different relative importance in different machines. Some factors of general significance will be discussed, but it should be kept in mind that in certain types of machines factors not here mentioned may assume importance. The effects here discussed are, however, typical and should give the reader a general view of the subject as a whole.

## Longitudinal Stability

A single lifting surface is unstable.
In order to secure large lift and relatively small resistance, a cambered plane is always used as a lifting surface. But a
cambered plane is in itself unstable. This is on account of the fact that the center of pressure on such a plane shifts forward when the incidence is increased and backward when the incidence is decreased, as already discussed; see Fig. 13, page 24. Any slight departure from the normal incidence, caused by a slight nosing up or a slight nosing down of the machine, is immediately augmented, on account of this shifting of the center of pressure. If the machine noses up a little, it tends to nose up more and more; if it noses down a little, it tends to nose down more and more and finally to dive. It is seen, therefore, that a single plane of suitable shape for flying is longitudinally unstable. It is true that stability might be secured by using a single flat plane, in which case the center of pressure shifts backward when the incidence is increased and forward when the incidence is decreased, as shown in Fig. 8, page 13. Stability might also be secured by using a wing of pronounced double curvature, but it would not be desirable to obtain stability in either of these ways on account of the great sacrifice in lift and the relatively large wingresistance.

## Stability is secured by an additional plane or tail.

The practical method for obtaining longitudinal stability is by adding a second plane or tail, usually placed behind* the main plane.

The tail plane may be ( 1 ) neutral, in which case the air exerts no pressure upon it either up or down; it may be (2) lifting, when the air pressure upon it is upward; or it may be

[^61](3) depressing, when the air pressure is downward. A depressing tail is often referred to as a negative tail.

A lifting tail gives least stability; a depressing tail gives most stability and possesses other advantages discussed later.

The type of tail to be used for a particular machine depends upon the location of the center of gravity of the machine with respect to the line of pressure on the main plane.

## Positive and negative pitching moments.

When the line of resultant pressure on a plane passes through the center of gravity of the machine, the pressure produces no turning moment; whereas, if the line of pressure passes behind or in front of the center of gravity, there arises a momentcalled the pitching moment-tending to produce rotation about the pitching axis, so that the machine noses up or down.

This moment is said to be positive when it tends to make the machine nose up and negative when it tends to make the machine dive or nose down. The moment is equal to the product of the pressure and its lever arm, which in this case is the perpendicular distance from the center of gravity to the line of pressure.

This is true for any plane, whether it be main plane or tail; see Figs. 93-95. For a machine to be in equilibrium the sum of the positive moments must equal the sum of the negative moments so that no rotation will be produced.

Simple Case. Longitudinal Stability When Thrust Passes Through the Center of Gravity

To obtain ease in control, a machine is usually so constructed that the line of thrust passes near or through the center of gravity. For simplicity we shall first consider the longitudinal stability of a machine in which the thrust passes directly through the center of gravity.

As the machine changes attitude, thrust and weight in this case continue to act through the center of gravity; these forces, therefore, produce no restoring or upsetting moment and do not affect stability.

Furthermore, for simplicity, we shall represent the machine as consisting merely of two planes, a main plane and a tail, as shown in Figs. 93-95, The forces affecting stability may thus be reduced to two: a total pressure $P$ on the main plane (the resultant of lift and resistance); and a total pressure $p$ on the tail plane (also the resultant of lift and resistance).

For equilibrium, the total pitching moment must be zero. Any moment due to the main plane must, therefore, be balanced by an equal and opposite moment produced by the tail. Furthermore, for stability, when the machine is displaced the resultant moment must no longer be zero but must be a restoring moment to bring the machine back to its original attitude. The function of the tail is to produce this restoring moment when the machine is displaced.

There are three cases to be considered, according to whether the line of resultant pressure on the forward plane passes through the center of gravity of the machine (necessitating a neutral tail), ahead of the center of gravity (requiring a lifting tail) or back of the center of gravity (necessitating a depressing tail). These cases are discussed in the following paragraphs. The effect is most clearly shown in the diagrams by representing an airplane simply as a single forward plane and a single rear plane or tail. It is to be understood, however, that a plane may be multiple, as in a biplane or triplane, and may be cambered, although shown in the diagrams as a single flat surface.

Case I. Neutral tail. Line of resultant pressure on forward (main) plane passes through center of gravity.
Let us first consider the case in which the line of resultant pressure $P$ on the forward (main) plane passes through the center of gravity $G$, when the machine is in normal flying attitude, as shown in Fig. 93. As $P$ passes through $G$, the forward plane causes no turning moment.

Condition for equilibrium. - Inasmuch as the forward plane exerts no turning moment, in order that the total moments


Fig. 93. Case I. Center of gravity, $G$, is in line of pressure, $P$, of forward (main) plane. The tail plane must be neutral and in the rear.
shall be zero, for equilibrium, the rear plane likewise must cause no turning moment. This is accomplished by setting the rear plane in the air-stream in the position of zero lift. All the lifting is done by the forward plane, which is, therefore, the main plane; as the rear plane exerts no lift it serves only as a tail.

Condition for stability.-When the machine is in its normal flying position, the rear plane or tail is neutral and gives rise to no pitching moment. When, however, this machine is displaced from its normal position, the positive or negative lift of the tail creates a restoring moment. Thus, when the machine starts to nose up, it is restored to its normal position
by the positive lift of the tail; and when the machine starts to nose down, it is restored to its position by the negative lift of the tail. The machine is, therefore, in stable equilibrium.

The magnitude of this restoring moment depends upon the pressure on the tail (which in turn depends upon its area and its incidence and velocity in relation to the air-stream) and upon the perpendicular distance between the line of this pressure and the center of gravity of the machine. On account of its long lever arm, the restoring moment of the tail may be made of considerable magnitude; it must always exceed any upsetting moment of the main plane which may be produced by the line of pressure of the main plane shifting forward or backward with change of incidence.

## Relative wind for the tail.

The air-stream, as it strikes the main wings of an airplane, is in the direction of the flight path, but as it leaves the wings it is deflected downward as shown in Fig. 9, page 15. The relative wind for the tail is, accordingly, not in the same direction as the relative wind for the main wings; for zero lift, therefore, a tail is inclined slightly downward toward the rear.

Furthermore, the velocity of the relative wind for the tail is greater than for the main wings, being increased perhaps 20 or 25 per cent. by the back draught or slip stream from the propeller.

Case II. Lifting tail. Line of resultant pressure on forward plane passes ahead of center of gravity.
The line of resultant pressure $P$ on the main plane, instead of passing through the center of gravity $G$, as in the case just discussed, may pass either ahead of or behind $G$.

When $P$ passes ahead of $G$, as in Fig. 94, it gives rise to a positive moment $P \cdot x$ tending to make the machine nose up, $x$ being the perpendicular distance of $G$ from the line of pressure $P$.

Condition for equilibrium.-For equilibrium the positive moment $P . x$ of the forward plane must be balanced by a nega-


Fig. 94. Case II. Center of gravity, $G$, is behind line of pressure, $P$, of forward plane. Rear plane must be lifting, and is usually (but not necessarily) smaller than the forward plane.
tive moment p.y of the rear plane. Both planes must be lifting planes as the center of gravity is located between* the two.

If the two planes have equal lift, $G$ is located midway between them; or $x=y$, when $P=p$, for $P \cdot x=p \cdot y$. When the lifts are not equal, $G$ is nearer the plane with the greatest lift, which is usually (but not necessarily) the forward plane; $x<y$, when $P>p$.

[^62]Condition for stability.-Great stability can not be obtained with a lifting tail. When the machine noses up, the lift and turning moment of both planes increase; when the machine noses down, they both decrease. Should the moments of the two planes increase or decrease at exactly the same rate, there would be no difference between the two and hence no resultant restoring moment.

To obtain a restoring moment, p.y must be greater than $P \cdot x$ when the machine noses up and less than $P \cdot x$ when the machine noses down. This is accomplished by making the incidence of the rear plane less than the incidence of the forward plane. In this case, when the machine noses up the lift and negative moment of the rear plane increase more* rapidly than the lift and positivemoment of the forward plane; there is thus a resultant negative moment that restores the machine to its original position. On the other hand, when the machine noses down the lift and negative moment of the rear plane decrease more rapidly than the lift and positive moment of the forward plane, so that there is now a resultant positive moment that restores the machine.

It is thus seen that, with a lifting tail, stability is due merely to a small difference in the rate of change of lift on main plane and tail when these are set so that the tail has a smaller incidence than the main plane. With a depressing tail, on the other hand, the restoring moment when the machine is displaced from its normal attitude is more pronounced, for, as shown later, when P.x increases $p \cdot y$ decreases (and vice versa), so that the effect is additive rather than differential.

In all cases, the stabilizing effect of the tail must be greater than any tendency toward instability due to shifting of the

[^63]center of pressure on the main plane. This is readily taken care of by a powerful tail,-i.e., a large tail surface placed well in the rear.

Case III. Depressing tail. Line of resultant pressure on forward (main) plane passes back of the center of gravity.
The line of resultant pressure $P$ on the main plane passes behind the center of gravity $G$, at a perpendicular distance $x$, as shown in Fig. 95.

Condition for equilibrium. -The pressure $P$ on the main


Fig. 95. Case III. Center of gravity, $G$, is ahead of line of pressure, $P$, of forward (main) plane. The tail plane must be depressing and in the rear.
plane gives a pitching moment $P \cdot x$ that tends to cause the machine to nose down. For equilibrium, this must be balanced by an equal and opposite tail moment $p \cdot y$ that tends to cause the machine to nose up. This is obtained by a depressing tail, for when the tail moves down the nose of the machine moves up.

Condition for stability.-It will be seen that the equilibrium is stable. When the machine starts to nose up, the lift and moment of the forward plane increase while the lift and
moment of the tail decrease. A restoring moment is thus created both by the main plane and by the tail, the total restoring moment being the sum of the two,-and not the difference as in the case of the lifting tail. When the machine starts to nose down, the lift and moment of the forward plane decrease, and of the tail increase. Here also there is an additive restoring moment.

## Decalage; longitudinal dihedral angle, or longitudinal V.

In all cases of stability,-whether with neutral, lifting, or depressing tail-it is seen that the main plane and tail are inclined with respect to each other, this inclination being sometimes referred to as decalage.* It is seen, furthermore, that the two planes make a dihedral $\dagger$ angle, or $\mathbf{V}$, pointing downward.

The angle between the two planes is most pronounced with a depressing tail, which as has been shown gives the greatest stability, and least pronounced with a lifting tail which gives the least stability. If the dihedral angle were inverted, making a $\Lambda$ pointing upward, the machine would be unstable.

## Relative advantages of lifting and depressing tail.

The only advantage in a lifting tail is that it adds to the total lift of the machine, but this addition is small and is usually not worth while in view of the greater stability and other advantages of a depressing tail.

A depressing tail is not only more stable than a lifting tail but it helps in the control of the machine in two important ways. First, it tends to get a machine out from a nose dive,

[^64]whereas a lifting tail makes it difficult to get out of one. Second, with a depressing tail the slip stream from the propeller tends to make the machine nose up when power is thrown on and to nose down to a proper gliding angle when power is thrown off. A depressing tail thus helps the operator in control; a lifting tail hinders.

A tail, therefore, is usually either depressing or neutral and is rarely lifting.

## Horizontal stabilizer and elevator.

The function of a tail is two-fold,-to give stability and to give control. The entire tail may be movable, so that control is obtained by changing the incidence of the whole tail plane, or it may be partly fixed and partly movable.

When the whole tail plane is movable, undue responsibility is placed on the pilot, continuous effort on his part being required in order to maintain the proper flying attitude.

Usually the tail consists of two parts: a forward fixed part, called the horizontal stabilizer, of proper dimensions and incidence to insure stability without effort on the part of the pilot; and a rear movable part, called the elevator, which is hinged to the stabilizer and is moved up or down by the pilot in his control of the machine. Thisis shownin thefrontispiece.

The horizontal stabilizer may be set so that for certain flying conditions (for example, horizontal flight or climbing) the elevator is neutral and the machine keeps its desired attitude without effort on the part of the pilot.

The setting of the horizontal stabilizer is attended to while rigging the airplane on the ground and should always be looked to when "tuning up" a machine for flight. Frequently the horizontal stabilizer is so fixed that its adjustment may be readily changed during flight, so that the
machine automatically maintains whatever attitude the pilot may desire.

An improper adjustment of the stabilizer tends to make a machine travel "nose heavy" or "tail heavy." This may be occasioned by a warping of the fusilage, by improper tension of various wires, etc. Thus, a slight displacement of the upper wing of a biplane, perhaps a drifting back due to stretching of wires, may cause a considerable lack of balance of the machine. A machine may become nose heavy or tail heavy due to a change in the load-as by dropping bombs-in flight, and in this case an adjustable stabilizer proves useful in maintaining the proper attitude.

## Tail causes machine to lie on its flight path.

Horizontal flight can only be maintained when resistance is balanced by an equal thrust. If power is cut off so that thrust is zero, resistance tends to slow down the machine so that, with decreased sustentation, it settles and thus takes an oblique path downward. Temporarily the angle of incidence is thus increased. The stabilizing action of the tail (a negative diving moment, or-in case of a depressing tail-a decrease in the positive moment of the tail) now comes into play and makes the machine nose down until it has its normal* incidence with its flight path. It may, therefore, be said that a machine lies on its flight path due to the stabilizing action of the tail. The ultimate gliding angle and gliding velocity are under the control of the pilot by operation of the elevator. (See gliding diagram,-Fig. 75, page 150 ).

Similarly, when surplus power inclines the flight path upward, the tail noses the machine up.

It is seen, therefore, that the action of a tail is to cause a machine to follow its flight path, with constant attitude, as

[^65]an arrow follows its course. This is the primary effect; there are various secondary effects.

## General Case

In the preceding discussion it is shown that for stability a tail is necessary, that in practically all cases the tail must be in the rear and that stability is greater when the tail is depressing than when it is lifting.

It is also shown that, on account of the stabilizing action of the tail, a machine tends to maintain its attitude with respect to the flight path and to nose up or down as the flight path becomes inclined up or down, that is, an airplane tends to lie on its flight path.

For simplicity these conclusions were derived by assuming the airplane to consist of a single main plane and a tail, and by assuming further that the thrust passes through the center of gravity and has no turning moment; but the conclusions just stated are equally true when the airplane structure is not thus limited and when the thrust passes either above or below the center of gravity.

An airplane, as a matter of fact, does not consist merely of two planes. The wings themselves are commonly biplane or triplane; and, furthermore, in addition to wing and control surfaces, there are many surfaces-body, struts, undercarriage, etc. Upon each of these surfaces there is an air pressure which may be considered as a single force or resolved into lift and resistance components.

Four forces under consideration: lift, weight, thrust and resistance.

In the case already discussed, in which it was possible to leave thrust out of consideration, it was simpler to consider the air pressures as consisting of two pressures, $P$ on the main plane and $p$ on the tail. There are advantages in com-
bining the lift components of all air pressures into a resultant or total lift acting through the center* of lift, and in combining the resistance components into a resultant or total resistance passing through the center of resistance. The total resistance includes the resistance of all the structure. The total lift is practically the sum of the lift of the wings and horizontal stabilizer, the lift on other surfaces being small and commonly neglected.


Fig. 96. In horizontal flight, lift $=$ weight and thrust $=$ resistance. Forces are here shown as concurrent, all passing through the center of gravity. See also Figs. 97 and 98.

There are thus four forces affecting stability: a weight $W$ and a total lift $L$ equal to $W$; a total resistance $R$ and a thrust $\dagger T$ equal to $R$.

## Concurrent and non-concurrent forces.

When all four forces act through the center of gravity, as in Fig. 96, the forces are said to be concurrent or to have coincident centers.

[^66]In general the forces are not concurrent. Thrust may be below or above the center of gravity; and it may be above or below resistance, as shown in Fig. 97. Likewise, lift may be ahead of or back of the center of gravity, as shown in Fig 98.

There are three caes of equilibrium,* $(a),(b)$ and (c) as shown in Figs. 97 and 98.
(a) Thrust in line with resistance; requires total lift to pass through center of gravity.

When thrust passes through the center of resistance, as in Fig. 97 (a), there is no resistance-thrust couple. Hence, for


Fig. 97. Relation between thrust and resistance.
(a) Thrust in line with resistance. Thrust-resistance couple is zero.
(b) Thrust is below resistance. Thrust-resistance couple is positive.
(c) Thrust is above resistance. Thrust-resistance couple is negative.

The thrust-resistance couple must be balanced by an equal and opposite lift-weight couple. See Fig. 98.
equilibrium there must be no lift-weight couple, and lift must pass through the center of gravity $G$, as in Fig. 98 (a).
(b) Thrust lower than resistance; requires lift back of weight.

When the center of thrust is lower than the center of resistance, as in Figs. 97 (b), there is a positive couple that

[^67]tends to make the machine nose up. This is equal to the product of thrust or resistance and the distance between them and is independent* of the height of the center of gravity. (The height of the center of gravity has no effect upon equilibrium so long as steady conditions are maintained and there is no force due to acceleration. When equilibrium is dis-

(a)

(b)

(c)

Fig. 98. Relation between lift and weight.
(a) Lift passes through $G$. Lift-weight couple is zero.
(b) Lift is back of $G$. Life-weight couple is negative.
(c) Lift is ahead of $G$. Lift-weight couple is positive.

The lift-weight couple must be balanced by an equal and opposite thrust-resistance couple. See Fig. 97.
turbed, the height of the center of gravity assumes importance; see a later paragraph.)

For equilibrium, the postive thrust-resistance couple must be balanced by an equal and opposite negative couple, obtained by having lift back of weight, as in Fig. 98 (b).
*The moment of $T$ at a distance $t$ from the center $G$ is $T t$. The moment of $R$ at a distance $r$ from the center $G$ is $R \cdot r$. The resultant moment or couple is $T \cdot t+R \cdot r=T(t+r)=R(t+r)$; it is the product of $T$ (or $R$ ) and the distance between $T$ and $R$, independent of the center $G$.

In the design of a machine, the center of lift can be shifted forward or back by shifting the positions of the wings; also by adjusting the horizontal stabilizer. The center of lift may be shifted back a small amount by sweeping back the wing tips. The position of the center of gravity can be changed by shifting the load, changing the location of seats, fuel tank, etc.

The center of resistance can be changed by raising or lowering the under-carriage, by stream-lining certain parts of the structure, and by other changes. The center of thrust can be changed only by raising or lowering the propeller shaft.

## (c) Thrust higher than resistance; requires lift ahead of weight.

When the center of thrust is higher than the center of resistance, as in Fig. 97 (c), the thrust-resistance couple is negative and tends to make the machine nose down or dive. For equilibrium, this negative couple must be balanced by an equal lift-weight couple that is positive and tends to make the machine nose up. This is accomplished by placing lift ahead of weight, as shown in Fig. 98 (c).

## Distance between L and W is small compared with distance between R and T .

As lift and weight are much larger than thrust and resistance, the distance between them is correspondingly smaller, in order that the couples may balance. Thus, suppose $L$ (or $W$ ) is six times $T$ (or $R$ ); then, in order that the liftweight couple shall equal the thrust-resistance couple, the distance between $L$ and $W$ must be one-sixth the distance between $T$ and $R$.

## Large couples undesirable.

Although balanced couples give equilibrium, whether they be large or small, it is desirable that they be small in order to avoid undue stresses in the structure. Thrust, therefore, should be not far from resistance (a little below or above) and lift should be still closer to weight (a little behind or in advance of $G$ ). More commonly thrust is below resistance and lift is back of weight.

## Changing conditions for equilibrium.

For equilibrium in uniform flight the positive and negative turning moments are balanced in the manner already described. When changes occur in any of the forces and in their moments, there is a re-adjustment until a new state of equilibrium is reached. Thrust may be changed by change of throttle, weight by consumption of fuel or dropping of load, while resistance and lift may be changed by change of incidence and velocity brought about by elevator control.

In the new state of equilibrium, the flight path may be horizontal or inclined, but in any case the positive and negative turning moments must be equal. As already stated, the machine tends to lie on its flight path, due to the stabilizing action of the tail. This is automatic to a certain extent, but is supplemented by the control of the pilot.

## Height of center of gravity with respect to thrust.

In uniform flight, thrust is equal to resistance. If thrust differs from resistance, the machine changes its attitude and velocity until resistance is just equal to thrust.

Any unbalanced thrust $T^{\prime}$, equal to $T-R$, acts through the center of thrust, either forward or backward according to whether $T$ is greater or less than $R$, thus giving to the
machine a positive or negative acceleration. The unbalanced thrust $T^{\prime}$ and the acceleration $a$ are both temporary.

The temporary unbalanced thrust $T^{\prime}$ gives a turning moment about $G$, which continues until thrust and resistance become equal, and flight is again uniform. The direction of this turning moment depends upon whether thrust passes above or below $G$. When thrust passes through $G$, a sudden change in the amount of thrust-brought about by opening or closing the throttle-will in itself cause no tendency for the machine to nose up or down, although ultimately the machine takes its new position of equilibrium as already described. When thrust does not pass through $G$, any change of throttle immediately tends to cause the machine to nose up or down, and this may make control by the pilot easier or more difficult, according to whether it tends to swing the machine towards or away from its new position of equilibrium.

Thrust below center of gravity is desirable.-When the center of thrust is below $G$, an increase in thrust due to opening the throttle gives a positive moment that temporarily causes the machine to nose up; a decrease in thrust due to closing the throttle gives a negative moment that temporarily causes the machine to nose down. This is an aid to the pilot in maintaining the proper flying attitude. When power is cut off, the machine noses down and tends to assume the proper gliding angle; when normal power is again put on, the machine tends to resume its horizontal flight path and, when excess power is put on, the machine tends to climb.

Aircraft can usually be designed so that thrust passes through $G$ or a little below it; it should not be so far below that the tendency to nose up or down is too great.

Thrust above center of gravity is undesirable.-When thrust passes above $G$, excess thrust has a negative moment which
tends to make the machine nose down when the throttle is opened and to nose up when the throttle is closed. This tendency, although temporary, is bad, for it requires decided elevator control on the part of the pilot to counteract it. Unless thus controlled, a machine may nose up and stall when power is cut off.

The effect may be offset to a certain extent by the action of the propeller slip stream on a depressing tail; see page 202.

Placing the center of gravity below the thrust can usually be avoided, but becomes necessary in some types of machines, for example, in flying boats with heavy hulls and high thrust.

## Stability shown by curve of moments.

The stability characteristics of a machine are well shown in a particular case by the values of its pitching moment in different attitudes. If the numerical values of the resultant moment at some particular velocity, are calculated for various angles of incidence,-with a given setting of wings, horizontal stabilizer and elevator-the degree of stability of the machine becomes known. The results are well shown by curves as in Figs. 99 and ioo.

For equilibrium the pitching moment must be zero at some particular angle of incidence. For stability, the pitching moment must be positive for smaller angles of incidence (tending to make the machine nose up) and negative for larger angles of incidence (tending to make the machine nose down) so that there will always be a restoring moment whenever the machine becomes displaced.

Curve 1 , in Fig. 99, represents the pitching moment of a machine for which there is no position of equilibrium, for the pitching moment is never zero. Curve 2 , shows equilibrium at $a$, but the machine is unstable for the pitching moment
is always positive, tending to make it nose up. It is true that, if the machine starts to nose down there is a restoring moment, but if it starts to nose up it continues to nose up more and more.


Fig. 99. Curves of pitching moment that show instability.
Curve 3 shows equilibrium at b; but the machine is decidedly unstable, for the negative moment at small angles


Fig. 100. Curve of pitching moment that shows stability.
of incidence and the positive moment at large angles is always an upsetting moment.

Stability requires that the pitching moment be positive for small and negative for large angles of incidence, as shown by
the curve in Fig. roo. The degree of stability is shown by the rate of increase in the restoring moment with displacement, i.e., by the slope of the curve of the pitching moment.

Preferably the restoring moment should be small for a small displacement, so that the machine may rise and fall easily without being too stiff, and should rapidly increase when the displacement becomes large. The angle of incidence for zero moment should be the normal angle of incidence at which the $L / D$ ratio has about its maximum value,say, $5^{\circ}$ or $6^{\circ}$.

The curve for moments in Fig. roo shows equilibrium at $4^{\circ}$. As the machine oscillates up or down there is seen to be a small restoring moment. If the machine starts to dive, the restoring moment rapidly increases when the machine noses down more than a few degrees. This is highly desirable and tends to prevent a nose dive. For large angles of incidence, on the other hand, there is not such necessity of a large restoring moment; if there is a reasonable restoring moment, the rest may be left to control by the pilot.

Longitudinal stability is essential for safe flight. Lateral and directional stability, while important, are not so vital. These will be next discussed.

## CHAPTER XII

## LATERAL STABILITY

Lateral stability of an airplane is stability about a fore-andaft axis, called the "rolling" axis, which passes through the center of gravity and either coincides with the line of flight or is slightly displaced therefrom. Its position with respect to the machine is fixed; its position with respect to the flight path varies as the angle of incidence varies.

## Rolling axis.

The rolling axis lies in the plane of symmetry of the machine and along its principle axis of inertia, i.e., the axis


Fig. IoI. Three positions of rolling axis.
along which the weight is most distributed and about which the moment of inertia is a minimum. This is the axis about which a machine rolls most readily. As shown in Fig. ior, the rolling axis may be either:

A neutral axis, when it coincides exactly with the line of flight;

A raised axis, when its forward end is raised above the line of flight (the common case); or,
A lowered axis, when its forward end is lowered below the line of flight, which is not a common case.
The line of flight is always understood to be the path of the center of gravity.

Stability requires that a restoring moment be set up whenever the machine is displaced from its normal position. If the machine is rolled over to one side, with one wing raised and the other lowered, there must be a rolling moment tending to roll the machine back until the two wings are again on the same level. Although for steadiness in normal flight a certain positive lateral stability is desirable, for quick maneuvers a less positive or even an indifferent stability becomes advantageous. But in no case is it desirable to have a negative stability, which would tend to overturn the machine when once displaced from its normal position.

## Neutral axis.

A machine with a neutral axis has, from symmetry, a neutral or indifferent stability, inasmuch as the angle of incidence at which the air strikes the various surfaces (including keel and other surfaces as well as wings) and hence the pressure, remain unchanged, irrespective of any displacement of the machine about its rolling axis. The displacement, therefore, causes neither a restoring nor an upsetting moment.

When a machine rolls, however, it tends to take an oblique path downward on account of decreased lift. The flight path being thus lowered, a neutral rolling axis becomes temporarily a raised axis, which gives a certain positive stability as discussed later.

## Inclined axis.

The conditions for stability in a machine in which the rolling axis is inclined depend upon whether the axis is raised or lowered and upon the disposition of keel surfaces and upon the shape of the wings.

## Keel surface as affecting lateral stability.

An important element in lateral stability is the keel surface which includes all surfaces that can be seen in a "side view" of the machine, that is, when the machine is viewed in a direction perpendicular to the plane of symmetry. The keel surface of body, struts and other structure may in itself be sufficient, but it is frequently supplemented by a vertical stabilizer or keel so as to give a keel surface of desired size and location,-i.e., either a high keel above the rolling axis or (rarely) a low keel below the rolling axis, as may be required for stability.

Small keel surfaces are sometimes placed on top of the wings, when this position suits the design of the machine, or


Fig. 102. Stable.
between the wings of a biplane. The keel or vertical stabilizer is, however, more commonly in the rear of the machine, with the rudder hinged to its back edge, as shown in Fig. in 2 of the next chapter.

Raised axis; high keel gives stability, low keel gives insta-bility.-Place a card on a stick or wire, so as to make a flagshaped model as in Fig. raz; and view it from in front along the line of flight; let the stick represent the rolling axis and the surface of the card represent the keel surface. When the model is thus viewed from the front (from the left in the illustration), it will be seen that with a raised axis a high keel surface is stable, for when displaced by rolling, one side of the
keel plane is exposed to the wind (in the model, is exposed to view) and the pressure on this side is in a direction to restore the plane to its normal position. Likewise, by means of a model as Fig. Io3, it will be seen that with a raised axis a low keel is unstable.

The restoring moment is seen to depend upon the position of the keel surface with respect to the rolling axis, and to be independent of its position with respect to the center of gravity,-that is, it may be above or below the center of gravity, in front of or behind it, without affecting lateral stability. (The location of the keel surface with respect to

the center of gravity has, however, a direct effect on directional stability, discussed later, and on stability in gusts.)

Lowered axis; low keel gives stability, high keel gives instability. -In a like manner, it may be seen that, with a lowered rolling axis as shown in Figs. 104 and ro5, a low keel gives stability and a high keel gives instability.

As the center of pressure on the keel surface is never very far from the rolling axis, the restoring moment is correspondingly small. A longer lever arm and hence a larger moment would be obtained if the keel surface were located well out on the wings, and some machines have keel surfaces so placed. The same effect, however, is practically obtained by turning the wings up so as to form a dihedral angle, as discussed below.

## Wing shape, as affecting lateral stability.

Lateral stability is materially affected if the two wings, instead of lying in a straight line, when viewed from in front, are turned up so as to form a dihedral angle (or V) as in Figs.


Figs. 106 and 107. Dihedral angle for monoplane and biplane; front view.

106 and i07, or are made to retreat as in Fig. 108, or are given a raked end as shown in the same figure.

The dihedral angle (called "lateral" or "transverse" dihedral to distinguish it from the "longitudinal" dihedral angle between the main plane and tail) may be caused or


Fig. Io8. Showing retreating or swept-back wings; showing also raked ends to wings. Top view.
noticably increased by flexure of the wings in flight. The dihedral angle often amounts to several degrees, but it may be so small (perhaps one degree or less) as to be scarcely perceptible to the eye.

## Model to show effect of wing shape.

The effect of wing shape upon lateral stability depends upon the position of the rolling axis and this is best seen by inspection of a model, readily made from a small rectangular board, as in Fig. iog.

Different types of wings, made of card board or tin, can be inserted in the slits 1,2 or 3 to obtain different angles of


Fig. 109. Model. Wings of different forms are inserted in the slits 1,2 and 3 and viewed from the front (from the left in the illustration) the line of sight being in the direction of the air stream.
incidence. The board can be rotated about the pegs $N N$ for a neutral axis, about $R R$ for a raised axis and about $L L$ for a lowered axis. (Other pegs may be used to give other angles to these axes.)
When the model is viewed from in front along the flight path, an increase in incidence of left or right wing is shown by an apparent increase in its area; a decrease in incidence of either wing by an apparent decrease in its area. The change in incidence of the raised and of the lowered wing of an airplane as it rolls is thus easily seen. It will be noted that when a machine with an inclined axis rolls, the incidence of one wing is increased and the other wing
decreased. Let us see under what conditions this gives stability.

## Lateral stability as affected by dihedral angle.

By means of a model as just described the following conclusions can be readily verified. There are four possible cases.
(1) Dihedral angle (V) and raised axis, gives stability, because, when the machine is displaced by rolling:

Raised wing has a smaller angle of incidence and hence less lift;
Lowered wing has a greater angle of incidence and hence more lift.
This makes a restoring moment and it can be shown that this moment is greater as the angle of incidence at which the machine is flying is greater. See note in next paragraph. Case ( I ) is the method actually employed for obtaining lateral stability. Cases (2), (3) and (4), although not found in practice, are, however, of interest.
(2) Dihedral angle (V) and lowered axis, gives instability, because, when machine is displaced by rolling:

Raised wing has a larger angle of incidence and hence more lift;
Lowered wing has smaller angle of incidence and hence less lift.
This makes an upsetting moment, which can be shown to be greater as the incidence* is greater.

In a like manner, it may be seen that:

[^68](3) Inverted dihedral ( $\mathbf{\Lambda}$ ) and raised axis, gives instability.
(4) Inverted dihedral ( $\boldsymbol{\Lambda}$ ) and lowered axis, gives stability.

With the inverted dihedral, the upsetting or restoring moment is less as incidence is greater. The inverted dihedral is practically never used, although machines have been so made and flown. It may be noted that a gull, when it lowers its head as it flies near the water in search of fish, also droops its wings so as to make an inverted dihedral; in this way stability is secured with what is now a lowered axis. As the head is raised again for normal flight and the rolling axis changes from a lowered axis to a neutral and then to a raised axis, the inverted dihedral ( $\boldsymbol{\Lambda}$ ) may be seen to disappear and an upright dihedral $(V)$ to take its place.

A dihedral angle, or an inverted dihedral angle, gives a tendency for a machine to roll when struck by a side gust. Too large a dihedral angle is, accordingly, undesirable; but other means may be used for increasing lateral stability.

## Straight wings.

With straight wings, that is when there is no dihedral, the incidence on both wings is always the same. Irrespective of whether the axis is raised or lowered, there is no difference in the incidence of the two wings when the machine rolls and hence no restoring or upsetting moment due to this cause.

A raised axis, however, has a small tendency toward stability, because when the machine rolls the lowered wing moves forward and its center of pressure (which is always ahead of the middle of a plane) now moves toward the wing tip, thus increasing the lever arm of the lift on this wing; the raised wing on the other hand moves backward and its center of pressure moves towards the body of the machine, thus decreasing its lever arm. There is thus a restoring moment,
obtained with straight wings and a raised axis; it is much less, however, than the restoring moment obtained by using a dihedral angle.

In a similar way, a lowered axis with straight wings tends toward instability.

## Retreating wings and wings with raked ends.

Retreating or swept-back wings, with a raised rolling axis, give lateral stability, for (as may be seen by inserting such wings in the model, Fig. ro9), when the machine rolls, the descending wing moves forward and enters the air more squarely, with what is practically an increased aspect ratio, so as to attack more air and get more lift, thus restoring the machine to its position of equilibrium.

In a like manner it can be shown by the model that, with a raised axis, lateral stability is increased if the ends of the wings are raked, i.e., if the trailing edge is longer than the entering edge.

With a lowered axis, retreating wings and wings with raked ends would be unstable.

Retreating wings and wings with raked ends are thus seen to have the same effect as a dihedral angle upon lateral stability, but with the advantage that in a side gust they create no tendency for a machine to roll. It will be shown later that all three devices-dihedral angle, retreating wings and raked ends-give directional stability. But they all have the disadvantage of reducing the so-called lifting efficiency, or $L / D$ ratio, i.e., there is a decrease in the amount of lift for a given wing-resistance.

## Effect of velocity on lateral stability.

All stability, depending upon the pressure of relative air upon surfaces, increases as the velocity increases.

## Control.

Lateral control might well be obtained by shifting the center of gravity to left or right, but this is not done. It has been obtained by warping (i.e., distorting) the planes, but lateral control is now generally obtained by means of auxiliary planes or ailerons which may be independent of the main planes (i.e., between the two planes of a biplane as in Fig. iro, although this is not common) or attached to the wings as wing flaps, as in Fig. iri. Wing-flap ailerons are the usual means for obtaining lateral control. To roll the machine, the pilot simultaneously turns the aileron on one


Fig. iro. Aileron independent of main wings; not so common as the wing-flap aileron.
wing down and the other aileron up, thus giving more lift to one wing so that it rises and less lift to the other so that it descends. This movement of the ailerons is commonly effected by pushing the control stick, or by turning the control wheel, to left or to right, as illustrated in Fig. 91, page 186. The term "warping" is frequently used to include this control by ailerons.

## Banking usually aids in turning.

A machine is said to be banked when it is keeled over on a turn, as a bicycle rider leans inward on a curve.

When a machine is banked, the keel surface back of the center of gravity tends to turn the machine toward the
lower wing and so aids in making a turn. This is due to the fact that a machine descends and side slips toward the lower wing when banked, on account of the decrease in the vertical component of the lift. (See Fig. II5 in the next chapter.) There is, accordingly, an upward and side pressure on the keel, that swings the tail up and out. In other words when a machine is banked it tends to nose down and to turn* in toward the lower wing.

There is another smaller effect that usually tends to turn the machine out toward the upper wing, as follows. A machine is banked on a turn by elevating one wing and depressing the other, this being accomplished by manipulat-


Fig. III. Section of wing with wing flap or aileron. In (a) the wing tip and aileron normally have positive incidence (the usual construction); in (b) they are up-turned and have negative incidence (a rare construction).
ing the ailerons in the manner described; but this manipulation of the ailerons will itself tend to turn the machine to one side, if the wing-resistance is thereby increased on that side and decreased on the other side. With the usual form of wing tip and aileron, this tendency is in a direction to oppose rather than aid in making a turn.

[^69]If the raised wing has its resistance increased when its aileron is turned down, and the lowered wing has its resistance decreased when its aileron is turned up (as is the case with most wing sections, as $a$ in Fig. IIr), the raised wing will be retarded and will tend to turn the machine toward the higher (outer) wing. Such a turn* is not desirable but is prevented by the rudder or by the pressure on the keel surface discussed above.

A turn toward the lower (inner) wing would be aided by having the wing tips (including ailerons) somewhat upturned so as to have a negative incidence in normal flight; see $b$ in Fig. in . The resistance of the raised wing would then be decreased when its aileron is swung down, and the resistance of the lower wing increased as its aileron is swung up, so that the lower wing would be retarded and would aid in making a turn toward the lower (inner) wing. The negative wing tip and aileron, although advantageous for the reasons just described, mean a sacrifice for they give less lift and greater wing-resistance, and their use is rare.

Some machines are made to depend entirely upon banking as a means for turning, no rudder being provided. Conversely, as discussed in the next chapter, turning produces banking and in some machines the rudder has been the only means for banking, no ailerons or similar devices being provided. Although it is in many ways desirable to have a machine thus turn in automatically when it is banked, some prefer to have the control left entirely to the pilot, the machine having no tendency to turn either in or out.

There is room for difference of opinion as to how great an extent banking and turning should be automatically depend-

[^70]ent upon each other and to what extent their control should depend upon the pilot.

## Propeller torque.

In a machine with one propeller, as the propeller rotates in one direction there is a tendency (when the power is on) for the whole machine to rotate* in the opposite direction. This may be easily corrected for in the control by the pilot, or automatically by a difference in the lift of the two wings, as described below. When flying, any correction is made by the pilot unconsciously. When starting, however, the correction may be noticeable, for the amount of correction changes as the engine accelerates; furthermore it is particularly important while near the ground to keep both wings even. When two propellers are used, rotating in opposite directions, the effects of propeller torque are neutralized.

## Automatic correction for propeller torque.

The correction for the torque of the propeller when power is on is often made automatically by a lack of symmetry in the two wings so that one wing has more lift than the other. This is sometimes done by a droop and rise (a droop near the end of one wing and a rise near the end of the other) and sometimes by a wash out on one wing (a progressive decrease in incidence from body to wing tip) and a wash in on the other wing (a progressive increase of incidence from body to wing tip.)

[^71]Any such lack of symmetry, however, gives a tendency for the machine to rotate when power is off. In horizontal flight this may be corrected for by the controls, but in diving it may make a spin that can not be controlled; for this reason lack of symmetry is undesirable. A machine that is unsymmetrical so as to correct for propeller torque is the easiest to fly in ordinary flight with power on, while a symmetrical machine is the better when power is off. The designer has to make a choice between the two.

The gyroscopic effect of the propeller and other revolving parts is discussed in the next chapter.

## CHAPTER XIII

## DIRECTIONAL STABILITY

When an airplane swings off from its course, to left or right, it is said to turn or yaw. Directional stability is the stability that keeps a machine on its course, that is it restores the machine to its course whenever it yaws. The vertical or yawing axis passes through the center of gravity of the machine, lies in the plane of symmetry and is more or less perpendicular to the flight path.

This stability is similar to that of a weathercock and depends upon having the center of the keel surface back of the yawing axis, thus insuring a restoring moment whenever the machine departs from its course. It is to be remembered that the keel surface is all the surface seen from the side, including structure as well as auxiliary keels or fins. In some machines enough directional stability is obtained by the keel surface of the body itself, but this is usually supplemented by the addition of a small keel or vertical stabilizer in the rear. If the keel center is too far aft, side gusts will cause the machine to yaw too much.

A machine should fly straight on its flight path; but it will fail to do so and will proceed crab-fashion if there is unequal resistance on the two sides. This might be caused by unequal incidence of the two wings, distorted surface or cambre, lack of symmetry in the tail, wrong alignment of body or fin or anything that might act as a rudder, for example the setting of struts or stream-line wires so as not to lie true in the line of flight,-points to be looked at in "tuning up" a machine.

## Dihedral angle and retreating wings.

Although the keel surface is the chief element in directional
stability, the wings may contribute. Directional stability is in all cases aided by retreating wings and by wings with a transverse dihedral angle, on account of the greater resistance of the wing which advances when the machine swings. its course; this is independent of the location of the rolling axis. The same is true of wings with raked ends (i.e., with trailing edge longer than entering edge). As pointed out in the preceding chapter (see Figs. 106, iо7 and ro8) these forms of wings likewise tend toward lateral stability, provided there is a raised rolling axis.


Fig. II2. Vertical stabilizer (v) and rudder ( $r$ ).
On the other hand an inverted dihedral or advancing wing tips, forms of wings which are not used, would in all cases tend toward directional instability; with a lowered axis, these forms would, however, give lateral stability, as already explained.

## Turning.

Turning is the deflection of the flight path to left or right. Rotation of the machine about its vertical axis, although it usually accompanies turning, is not in itself sufficient. Although turning might be effected by other means,-as by shifting the center of gravity, extending a panel on one wing
to increase its resistance, etc.-it is usually effected by a rudder at the rear of the machine. This is often hinged on a vertical fin or stabilizer (already referred to in connection with lateral stability) which forms part of the keel surface; such a rudder is shown in Fig. iı2. A balanced rudder, as in Fig. II3, reduces the force necessary for control and for this reason is commonly used on large machines. The rudder is usually operated by the pilot's feet, as illustrated in Fig. 89, page 185.


Fig. II3. Balanced rudder ( $r$ ).

## Turning by rudder.

A rudder alone without a keel would be ineffective, for the machine when rotated by the rudder would tend to skid along its original flight path, as does a toboggan on smooth ice. Turning control like all control depends also on a certain velocity, as in watercraft which require "steerage way" in order to answer the helm.

The relation between keel and rudder in turning is shown in Fig. II4. When the rudder is turned to one side, the pressure on the rudder causes the whole machine to rotate about its vertical axis until the rudder moment ( $p$ times its lever arm) is balanced by the keel moment. (the keel pressure $P$ times its lever arm); the lever arm in each case is the perpendicular distance of the force $p$, or $P$, from the vertical
axis passing through the center of gravity, shown as $G$ in the figure.

It is clear that, when the two moments are equal, the force $P$, with the shorter lever arm, is greater than $p$. The resultant of the forces $P$ and $p$ is a force $R, *$ deflecting the machine from its original flight path.

This deflecting force:
Increases as $P$ increases (increasing, for a given keel moment, as the keel surface is greater and its distance from $G$ is less);

Increases as $p$ decreases (increasing, for a given rudder moment, as the rudder surface is smaller and further back.)
A rudder is most effective, therefore, when it is placed far back, and the keel surface is placed near the center of gravity.

## Secondary effects.

There are important secondary effects on turning, the principal ones (see Fig. 114) being:
(a) Turning causes banking, for two reasons: (I) The outer wing having the higher velocity and greater lift tends to rise and the inner wing tends to descend on a turn; (2) The pressure on the keel surface on a turn tends to keel the machine over in the same direction as in (1), provided (as is

[^72]usually the case) the keel center is above the rolling axis. (In some machines, as mentioned in the preceding chapter, the rudder has been the only means for banking, no ailerons or similar devices being provided.)
(b) Turning causes increase of resistance and loss of velocity due to the fact that the pressures $P$ and $p$ on keel and rudder each have a backward component.


Fig. II4. Action of keel in turning.
(c) Turning causes a decrease in lift and a tendency to descend, i.e., a tendency to stall, for two reasons: (r) On account of loss of velocity described in $b$, the pressure on the wings, and hence the lift, is decreased; (2) On account of banking described in $a$, the vertical component of the lift is
decreased, see Fig. II5, becoming zero when a machine is banked ninety degrees.
(d) Machine may nose up* on a turn and thus have a further tendency to stall, if the pressures $P$ and $p$ on keel and rudder are higher than $G$, on account of the backward component of these pressures. Stalling may end in a tail slide.

The tendency to stall on a turn may be overcome, if necessary, by maintaining velocity either by putting on more power or by nosing down a little by means of the elevator. Loss of velocity is to be avoided. Obviously, to attempt to climb on a turn is dangerous.


Fig. II5. Horizontal and vertical components of lift when banking. Note that lift is not vertical.

## Side slipping and skidding.

If a machine is banked too much for a particular turn, it will slip in and down, on account of the horizontal component there is to the lift (see Fig. i15) and the decrease in the vertical component. This may result in a nose dive.

If a machine is not banked enough, it will skid out and (in some cases, due to the inertia of the machine) $u p$, this being

[^73]likely to happen on sharp turns and at high velocity. This may end in a stall, as the relative wind strikes the machine less from the front and more from the side and so gives less support to the machine on account of its decreased forward velocity.

## Banking on a turn.

With the proper banking, the centripetal force towards the center of the turn due to banking must equal the apparent centrifugal force away from the center. There being no skidding or side slipping, the pilot will feel no side wind on either cheek. He will feel a pressure holding him to his seat with no pressure to left or right. Strings tied to guy-wires blow straight back and not at an angle. If rolling is indicated by an inclinometer* like a level (arched upward), placed across the machine, the bubble remains central. In skidding or side slipping, the machine leaves the bubble behind; the pilot ought to keep in mind that the control should follow the bubble. It is a good plan to start banking just before beginning a turn.

The pilot instinctively gets the proper "feel" of a turn, as does the rider of a bicycle, without a study of moments and couples. The bicycle rider, however, usually learns by taking a few spills,-but this the air pilot can not afford to do.

## Turning by banking.

When the wings are inclined, whatever the cause, the lift on the wings has not only a vertical component but also a

[^74]horizontal sideways component, as shown in Fig. II5, which tends to move the machine horizontally toward the side of the machine that is down. The flight path is thus deflected. This becomes more pronounced in machines with large keel surface. By placing large keel surfaces, both forward and aft, certain machines are turned entirely by banking and are provided with no rudder.

## Gryoscopic action.

The propeller and revolving parts of the engine form a gyroscope, so that a sudden turn of the machine sideways will cause it to pitch or rear. Similarly any sudden pitching or rearing will cause the machine to turn to one side; for, when a sudden force is applied perpendicular to the axis of a gyroscope, the axis swings sideways at right angles to that force. The direction of this effect will depend upon the direction of rotation* of the revolving parts, and so may be opposite in different machines. This effect will be but small when controls are not jerked suddenly; indeed they should not be operated suddenly on account of the severe stresses produced.

## Relative values of factors affecting stability are different in different machines.

In this and the two preceding chapters a qualitative discussion is given of some of the factors that affect stability. It will be realized that the relative importance of the several factors will differ widely in different machines, that a factor negligible in one may assume importance in another, while

[^75]certain characteristics of machines of one type may be entirely absent from machines of a different type. For example, in a certain maneuver one machine may automatically nose up or turn to one side; another machine in exactly the same maneuver may nose down or turn to the opposite side. General conclusions that apply to all machines are, accordingly, difficult to make. A fuller discussion, involving a study of all the factors in various types of machines, would be beyond the scope of this book. It is believed, however, that the simple analysis of certain factors here given should give the reader a clear idea of the general nature of the problem, after which he might well proceed to a detailed numerical study if interested in design.


## APPENDIX

## GLOSSARY*

Aerofoil: A winglike structure, flat or curved, designed to obtain reaction upon its surface from the air through which it moves.

Aeroplane: See Airplane.
Aileron: A movable auxiliary surface used to produce a rolling moment about the fore-and-aft axis.

Aircraft: Any form of craft designed for the navigation of the air-airplanes, balloons, dirigibles, helicopters, kites, kite balloons, ornithopters, gliders, etc.

Airplane: A form of aircraft heavier than air which has wing surfaces for support in the air, with stabilizing surfaces, rudders for steering, and power plant for propulsion through the air. This term is commonly used in a more restricted sense to refer to air-planes fitted with landing gear suited to operation from the land. If the landing gear is suited to operation from the water, the term "seaplane" is used. (See definition.)

Pusher.-A type of airplane with the propeller in the rear of the engine.
Tractor.-A type of airplane with the propeller in front of the engine.

Air-speed meter: An instrument designed to measure the speed of an aircraft with reference to the air.

Altimeter: An aneroid mounted on an aircraft to indicate continuously its height above the surface of the earth.

Anemometer: Any instrument for measuring the velocity of the wind.

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## Angle:

Of attack or of incidence of an aerofoil.-The acute angle between the direction of the relative wind and the chord of an aerofoil; i.e., the angle between the chord of an aerofoil and its motion relative to the air. (This definition may be extended to any body having an axis.)
Critical.-The angle of attack at which the lift-curve has its first maximum; sometimes referred to as the "burble point." (If the "lift curve" has more than one maximum, this refers to the first one.)
Gliding.-The angle the flight path makes with the horizontal when flying in still air under the influence of gravity alone, i.e., without power from the engine.
Appendix: The hose at the bottom of a balloon used for inflation. In the case of a spherical balloon it also serves for equalization of pressure.
Aspect ratio: The ratio of span to chord of an aerofoil.
Aviator: The operator or pilot of heavier-than-air craft. This term is applied regardless of the sex of the operator.
Axes of an aircraft: Three fixed lines of reference; usually centroidal and mutually rectangular.

The principal longitudinal axis in the plane of symmetry, usually parallel to the axis of the propeller, is called the fore and aft axis (or longitudinal axis); the axis perpendicular to this in the plane of symmetry is called the vertical axis; and the third axis, perpendicular to the other two, is called the transverse axis (or lateral axis). In mathematical discussions the first of these axes, drawn from front to rear, is called the X axis; the second, drawn upward, the Z axis; and the third, forming a "left-handed" system, the Y axis.

Balancing flaps: See Aileron.
Ballonet: A small balloon within the interior of a balloon or dirigible for the purpose of controlling the ascent or descent, and for maintaining pressure on the outer envelope so as to prevent deformation. The ballonet is kept inflated
with air at the required pressure, under the control of a blower and valves.

Balloon: A form of aircraft comprising a gas bag and a basket. The support in the air results from the buoyancy of the air displaced by the gas bag, the form of which is maintained by the pressure of a contained gas lighter than air.

Barrage.-A small spherical captive balloon, raised as a protection against attacks by airplanes.
Captice.-A balloon restrained from free flight by means of a cable attaching it to the earth.
Kite.-An elongated form of captive balloon, fitted with tail appendages to keep it headed into the wind, and deriving increased lift due to its axis being inclined to the wind.
Pilot.-A small spherical balloon sent up to show the direction of the wind.
Sounding.-A small spherical balloon sent aloft, without passengers, but with registering meteorological instruments.

Balloon bed: A mooring place on the ground for a captive balloon.

Balloon cloth: The cloth, usually cotton, of which balloon fabrics are made.
Balloon fabric: The finished material, usually rubberized, of which balloon envelopes are made.

Bank: To incline an airplane laterally-i.e., to roll it about the fore and aft axis. Right bank is to incline the airplane with the right wing down. Also used as a noun to describe the position of an airplane when its lateral axis is inclined to the horizontal.

Barograph: An instrument used to record variations in barometric pressure. In aeronautics the charts on which the records are made indicate altitudes directly instead of barometric pressures.

Basket: The car suspended beneath a balloon, for passengers, ballast, etc.
Biplane: A form of airplane in which the main supporting surface is divided into two parts, one above the other.
Body of an airplane: The structure which contains the power plant, fuel, passengers, etc.
Bonnet: The appliance, having the form of a parasol, which protects the valve of a spherical balloon against rain.
Bridle: The system of attachment of cable to a balloon, including lines to the suspension band.
Bullseyes: Small rings of wood, metal, etc., forming part of balloon rigging, used for connection or adjustment of ropes.
Burble point: See Angle, critical.
Cabane: A pyramidal framework upon the wing of an airplane, to which stays, etc., are secured.
Camber: The convexity or rise of the curve of an aerofoil from its chord, usually expressed as the ratio of the maximum departure of the curve from the chord to the length of the chord. "Top camber" refers to the top surface of an aerofoil, and "bottom camber" to the bottom surface; "mean camber" is the mean of these two.

Capacity: See Load.
The cubic contents of a balloon.
Center: Of pressure of an aerofoil.-The point in the plane of the chords of an aerofoil, prolonged if necessary, through which at any given attitude the line of action of the resultant air force passes. (This definition may be extended to any body.)

## Chord:

Of an aerofoil section.-A right line tangent at the front and rear to the under curve of an aerofoil section.
Length.-The length of the chord is the length of the projection of the aerofoil section on the chord.

## Clinometer: See Inclinometer.

Concentration ring: A hoop to which are attached the ropes suspending the basket.
Controls: A general term applying to the means provided for operating the devices used to control speed, direction of flight, and attitude of an aircraft
Control column: The vertical lever by means of which certain of the principal controls are operated, usually those for pitching and rolling.
Crow's foot: A system of diverging short ropes for distributing the pull of a single rope.
Decalage: The angle between the chords of the principal and the tail planes of a monopolane. The same term may be applied to the corresponding angle between the direction of the chord or chords of a biplane and the direction of a tail plane. (This angle is also sometimes known as the longitudinal V of the two planes.)
Dihedral in an airplane: The angle included at the intersection of the imaginary surfaces containing the chords of the right and left wings (continued to the plane of symmetry if necessary). This angle is measured in a plane perpendicular to that intersection. The measure of the dihedral is taken as $90^{\circ}$ minus one-half of this angle as defined.

The dihedral of the upper wing may and frequently does differ from that of the lower wing in a biplane.
Dirigible: A form of balloon, the outer envelope of which is of elongated form, provided with a propelling system, car, rudders, and stabilizing surfaces.

Nonrigid.-A dirigible whose form is maintained by the pressure of the contained gas assisted by the carsuspension system.
Rigid.-A dirigible whose form is maintained by a rigid structure contained within the envelope.
Semirigid.-A dirigible whose form is maintained by means of a rigid keel and by gas pressure.

Diving rudder: See Elevator.
Dope: A general term applied to the material used in treating the cloth surface of airplane members and balloons to increase strength, produce tautness, and act as a filler to maintain air-tightness; it usually has a cellulose base.

Drag: The component parallel to the relative wind of the total force on an aircraft due to the air through which it moves.

That part of the drag due to the wings is called "wing resistance" (formerly called "drift"); that due to the rest of the airplane is called "parasite resistance" (formerly called "head resistance").
Drift: See Drag. Also used as synonymous with "leeway," $q . v$.
Drift meter: An instrument for the measurement of the angular deviation of an aircraft from a set course, due to cross winds.

Drip cloth: A Curtain around the equator of a balloon, which prevents rain from dripping into the basket.
Elevator: A hinged surface for controlling the longitudinal attitude of an aircraft; i.e., its rotation about the transverse axis.

Empennage: See Tail.
Entering edge: The foremost edge of an aerofoil or propeller blade.
Envelope: The portion of the balloon or dirigible which contains the gas.
Equator: The largest horizontal circle of a spherical balloon.

Fins: Small fixed aerofoils attached to different parts of aircraft, in order to promote stability; for example, tail fins, skid fins, etc. Fins are often adjustable. They may be either horizontal or vertical.

Flight path: The path of the center of gravity of an aircraft with reference to the earth.

Float: That portion of the landing gear of an aircraft which provides buoyancy when it is resting on the surface of the water.

Fuselage: See Body.
Gap: The shortest distance between the planes of the chords of the upper and lower wings of a biplane.
Gas bag: See Envelope.
Glide: To fly without engine power.
Glider: A form of aircraft similar to an airplane, but without any power plant.

When utilized in variable winds it makes use of the soaring principles of flight and is sometimes called a soaring machine.

Gore: One of the segments of fabric composing the envelope.
Ground cloth: Canvas placed on the ground to protect a balloon.

Guide rope: The long trailing rope attached to a spherical balloon or dirigible, to serve as a brake and as a variable ballast.

Guy: A rope, chain, wire, or rod attached to an object to guide or steady it, such as guys to wing, tail, or landing gear.
Hangar: A shed for housing balloons or airplanes.
Helicopter: A form of aircraft whose support in the air is derived from the vertical thrust of propellers.
Horn: A short arm fastened to a movable part of an airplane, serving as a lever-arm, e. g., aileron-horn, rudderhorn, elevator-horn.

Inclinometer: An instrument for measuring the angle made by any axis of an aircraft with the horizontal, often called a clinometer.

Inspection window: A small transparent window in the envelope of a balloon or in the wing of an airplane to allow inspection of the interior

Kite: A form of aircraft without other propelling means than the towline pull, whose support is derived from the force of the wind moving past its surface.
Landing gear: The understructure of an aircraft designed to carry the load when resting on or running on the surface of the land or water.

Leading edge: See Entering edge.
Leeway: The angular deviation from a set course over the earth, due to cross currents of wind, also called drift; hence, "drift meter."

Lift: The component perpendicular to the relative wind, in a vertical plane, of the force on an aerofoil due to the air pressure caused by motion through the air.
Lift bracing: See Stay.
LoAD:
Dead.-The structure, power plant, and essential accessories of an aircraft.
Full.-The maximum weight which an aircraft can support in flight; the "gross weight."
Useful.-The excess of the full load over the dead-weight of the aircraft itself, i.e., over the weight of its structure, power plant, and essential accessories. (These last must be specified.)

Loading: See Wing, loading.
Lobes: Bags at the stern of an elongated balloon designed to give it directional stability.
Longeron: See Longitudinal.
Longitudinal: A fore-and-aft member of the framing of an air-plane body, or of the floats, usually continuous across a number of points of support.
Monoplane: A form of airplane whose main supporting surface is a single wing, extending equally on each sideof the body.

Mooring band: The band of tape over the top of a balloon to which are attached the mooring ropes.

Nacelle: See Body. Limited to pushers.
Net: A rigging made of ropes and twine on spherical balloons, which supports the entire load carried.
Ornithopter: A form of aircraft deriving its support and propelling force from flapping wings.
Panel: The unit piece of fabric of which the enevelope is made.

Parachute: An apparatus, made like an umbrella, used to retard the descent of a falling body.
Patch system: A system of construction in which patches (or adhesive flaps) are used in place of the suspension band.

Permeability. The measure of the loss of gas by diffusion through the intact balloon fabric.

Pitot tube: A tube with an end open square to the fluid stream, used as a detector of an impact pressure. It is usually associated with a coaxial tube surrounding it, having perforations normal to the axis for indicating static pressure; or there is such a tube placed near it and parallel to it, with a closed conical end and having perforations in its side. The velocity of the fluid can be determined from the difference between the impact pressure and the static pressure, as read by a suitable gauge. This instrument is often used to determine the velocity of an aircraft through the air.

Pontoons: See Float.
Pusher: See Airplane.
Pylon: A mast or pillar serving as a marker of a course.
Race of a propeller: See Slip stream.
Relative wind: The motion of the air with reference to a moving body. Its direction and velocity, therefore, are found by adding two vectors, one being the velocity of the air with reference to the earth, the other being equal and opposite to the velocity of the body with reference to the earth.

RIP CORD: The rope running from the rip panel of a balloon to the basket, the pulling of which causes immediate deflation.

Rip panel: A strip in the upper part of a balloon which is torn off when immediate deflation is desired.

RUDDER: A hinged or pivoted surface, usually more or less flat or stream lined, used for the purpose of controlling the attitude of an aircraft about its "vertical" axis, i. e., for controlling its lateral movement.

Rudder bar.-The foot bar by means of which the rudder is operated.

Seaplane: A particular form of airplane in which the landing gear is suited to operation from the water.

Serpent: A short, heavy guide ${ }^{\text {Frope. }}$
Side slipping: Sliding downward and inward when making a turn; due to excessive banking. It is the opposite of skidding.

Skidding: Sliding sideways away from the center of the turn in flight. It is usually caused by insufficient banking in a turn, and is the opposite of side slipping.

Skids: Long wooden or metal runners designed to prevent nosing of a land machine when landing or to prevent dropping into holes or ditches in rough ground. Generally designed to function should the landing gear collapse or fail to act.

Slip stream or propeller race: The stream of air driven aft by the propeller and with a velocity relative to the airplane greater than that of the surrounding body of still air.

Soaring machine: See Glider.
Span or spread: The maximum distance laterally from tip to tip of an airplane wing, or the lateral dimension of an aerofoil.

Stability: A quality in virtue of which an airplane in flight tends to return to its previous attitude after a slight disturbance.

Directional.-Stability with reference to the vertical axis.
Dynamical.-The quality of an aircraft in flight which causes it to return to a condition of equilibrium after its attitude has been changed by meeting some disturbance, e. g., a gust. This return to equilibrium is due to two factors; first, the inherent righting moments of the structure; second, the damping of the oscillations by the tail, etc.
Inherent.-Stability of an aircraft due to the disposition and arrangement of its fixed parts-i.e., that property which causes it to return to its normal attitude of flight without the use of the controls.
Lateral.-Stability with reference to the longitudinal (or fore and aft) axis.
Longitudinal.-Stability with reference to the lateral axis.
Statical.-In wind tunnel experiments it is found that there is a definite angle of attack such that for a greater angle or a less one the righting moments are in such a sense as to tend to make the attitude return to this angle. This holds true for a certain range of angles on each side of this definite angle; and the machine is said to possess "statical stability" through this range.
Stabilizer: Any device designed to steady the motion of aircraft.

Stagger: The amount of advance of the entering edge of the upper wing of a biplane over that of the lower, expressed as percentage of gap; it is considered positive when the upper surface is forward.
Stalling: A term describing the condition of an airplane which from any cause has lost the relative speed necessary for control.

Statoscope: An instrument to detect the existence of a small rate of ascent or descent, principally used in ballooning.

Stay: A wire, rope, or the like used as a tie piece to hold parts together, or to contribute stiffness; for example, the stays of the wing and body trussing.
Step: A break in the form of the bottom of a float.
Stream-line flow: A term in hydromechanics to describe the condition of continuous flow of a fluid, as distinguished from eddying flow.
Stream-line shape: A shape intended to avoid eddying and to preserve stream-line flow.

Strut: A compression member of a truss frame; for instance, the vertical members of the wing truss of a biplane.

SUSPENSION BAND: The band around a balloon to which are attached the basket and the main bridle suspensions.

SUSPENSION BAR: The bar used for the concentration of basket suspension ropes in captive balloons.

SWEEP BACK: The horizontal angle between the lateral axis of an airplane and the entering edge of the main planes.

TAIL: The rear portion of an aircraft, to which are usually attached rudders, elevators, stabilizers, and fins.

Tail cups: The steadying device attached at the rear of certain types of elongated captive balloons.

Thimble: An elongated metal eye spliced in the end of a rope or cable.

Tractor: See Airplane.
Trailing edge: The rearmost edge of an aerofoil or propeller blade.

Triplane: A form of airplane whose main supporting surface is divided into three parts, superimposed.

Truss: The framing by which the wing loads are transmitted to the body; comprises struts, stays, and spars.

Undercarriage: See Landing gear.
Warp: To change the form of the wing by twisting it.
Wash out: A permanent warp of an aerofoil such that the angle of attack decreases toward the wing tips.
Weight: Gross. See Load, full.
Wings: The main supporting surfaces of an airplane.
Wing flap: See Aileron.
Wing loading: The weight carried per unit area of supporting surface.
Wing mast: The mast structure projecting above the wing, to which the top load wires are attached.

Wing rib: A fore-and-aft member of the wing structure used to support the covering and to give the wing section its form.
Wing spar or wing beam: A transverse member of the wing structure.
Yaw: To swing off the course about the vertical axis. Angle of.-The temporary angular deviation of the fore-and-aft axis from the course.

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[^0]:    *Thus, when $P$ is expressed in ounces per square foot, the numerical value of $P$, and so of $K$, is 16 times as great as when $P$ is expressed in pounds per square foot.

[^1]:    *Eiffel's experiments are beautifully described in his book "The Resistance of the Air and Aviation," translated into English by J. C. Hunsaker.

[^2]:    *These two components are horizontal and vertical only in normal horizontal flight. More accurately defined, lift is the component of

[^3]:    total pressure that is perpendicular to the air-stream and that lies in the plane of symmetry of the machine; this plane is vertical as long as the machine does not roll. Lift is accordingly vertical in normal flight, but becomes inclined when the machine is inclined, as shown in Fig. 4 a. Wing-resistance is, in all cases, the component of total pressure that is in the direction of the air-stream.

    It is recommended by the U.S. Advisory Committee on Aeronautics that the word "drift," sometimes used as a designation for wingresistance, be abandoned; see "drift" in Glossary, Appendix I. But the initial letter $D$, in such common use, may well be retained as a symbol for drag or wing-resistance; likewise, $K_{\mathrm{D}}$, formerly called the coefficient of drift, may be retained as the coefficient of drag or wingresistance. See Note on Terminology p. 53.

[^4]:    *"Resistance of the Air," p. 122; size of rectangle $90 \times 15 \mathrm{~cm}$.

[^5]:    *After $20^{\circ}, K_{\mathrm{L}}$ increases slightly, having by Eiffel's data the same value at $30^{\circ}$ as at $15^{\circ}$. The incidence at which the maximum occurs, and its value are different for different aspect ratios; but the curves are all of the type here shown.

[^6]:    *For a certain angle of incidence (when cotangent $i=L / D$ ), $P$ coincides exactly with the perpendicular to the chord. For a smaller angle of incidence, $P$ is back of the perpendicular, due to the relatively large value of $D$ compared with $L$. For a larger angle of incidence, $P$ is in advance of the perpendicular, due to the relatively larger value of $L$.

[^7]:    *From tests made by Captains E. S. Gorrell and H. S. Martin, abstracted by A. Kelmin and T. H. Huff; published in Aviation, Vol. II., p. 256 , 1917. These tests were made on a model, $18^{\prime \prime} \times 3^{\prime \prime}$, made of brass; density of standard air: 0.07608 lbs . per cu. ft.; wind velocity, 30 MPH .

[^8]:    *The question of power is taken up later. There is, for each machine, a certain angle of incidence-well within these limits-at which the power required is a minimum. If the incidence is either increased or decreased, there is a very great increase in the amount of power required.
    $\dagger$ Although the main wing-surfaces may thus in some cases be at their maximum lift, the surfaces used for lateral control, called ailerons discussed later, should always be well below maximum lift. Otherwise they would become inoperativc, for latcral control depends upon increasing the lift on one aileron and decreasing the lift on the other; if the ailerons were at maximum lift, any change would decrease the lift on both and would not give the desired control.

[^9]:    Note. The development of the subject in the chapters immediately following depends so directly upon Chapter I, that the general description of the airplane, its structure and control is deferred until a subsequent chapter on Single and Multiple Planes. This description may, however, be read at this point to advantage.

    Lift, resistance, velocity and power are all directly affected by the density of the air. A discussion of this is also deferred; see a later chapter on Altitude.

[^10]:    *Some early machines had a very small speed range, let us say from a minimum of 35 to maximum of 50 miles per hour, giving a speed range of 15 miles per hour. A gust from behind of more than 15 miles per hour would reduce the relative air speed below the requisite 35 miles per hour necessary for sustentation, so that the machine had no support. This was one cause for the so-called holes in the air.

[^11]:    *The density of the air is here assumed to be uniform; see a later chapter on change of density and $K_{\mathrm{L}}$ with altitude.
    $\dagger$ Modification in complete machine.-The lift curve for a wing is modified in a complete machine by whatever lift there is (either positive or negative) on other parts of the machine-body, tail and other surfacesand, in a biplane or triplane, by an interference between the planes that reduces the lift. This reduction is less when the gap between the planes is large and when they are given considerable stagger than when the gap is small and there is no stagger. Lift likewise increases with aspect ratio, the ratio of wing-span to chord. For the curves here shown the aspect ratio is assumed to be 6 , the usual standard value.

    For simplicity a detailed consideration is not given here of these features, for they in no way affect the general character of the conclusions, although they do affect the precise numerical values. Proper corrections have to be applied when exact numerical values are to be obtained.

    The net result of these corrections usually shifts each point on the lift curve a little to the right, say a degree or so, so that a lift $K_{\mathrm{L}}=$ o.oor 74 and velocity 58.6 MPH . shown by the curve at $4^{\circ}$ would, for a complete machine, be at say $5^{\circ}$, and so on for other points, the shift being slightly different for different points. So far as lift and velocity are concerned, the effect of these corrections is merely to change the angle of incidence at which a particular value of lift and of velocity occur. Put in a different way, at small angles (perhaps through the working range) the lift curve is somewhat lowered and at large angles is

[^12]:    *The power available for producing thrust, delivered through the propeller, is here referred to. The question of propeller efficiency is not here taken into consideration.

[^13]:    *An adjustable wing would permit a change of incidence without a change of attitude. An airplane so equipped could fly, under changing conditions, with even keel; the thrust could be maintained in a constant direction with respect to the flight path. The objections to an adjustable wing are structural; it may develop that these objections can be overcome.

[^14]:    *When thrust is inclined with reference to the flight path, resistance is overcome by the component of thrust in the direction of the flight path. The small vertical component, when upward, supports part of the weight so that correspondingly less lift is required of the wing for sustentation; when downward, correspondingly more lift is required.

[^15]:    *Variation of wing-resistance with incidence.-The formula given above may also be written:-

    Wing-resistance $=$ Weight $\times D / L$ ratio.
    Wing-resistance is, therefore, proportional to $D / L$. A curve giving the values for wing-resistance for different angles of incidence would be similar to the $D / L$ curve, Fig. 12, page 19, the only difference between the two curves being the scale.

[^16]:    *There is an appreciable departure from the square law when the skin resistance of a body is large compared with the direct impact or dynamic resistance-the more so when the surface is rough and the fore-and-aft dimension of the body is large in proportion to the diameterbut practically it is simplest to neglect this and to assume the square law as strictly true. For only a small range in velocity, any error in this is inappreciable; for a large range, it may be necessary to take different values of the coefficient at different velocities.

    If the square law were exact, the best stream-lining for one speed would be the best for all speeds. This, however, is found to be not always so; thus, a strut of one section may on test prove best at one speed, a strut of another section at some other speed.

[^17]:    *When thrust is inclined with respect to the flight path, resistance is balanced by a component of thrust in line with resistance, as shown in a later paragraph.
    $\dagger$ See also Chapters VII and VIII.

[^18]:    *In the triangle formed by $W, F^{\prime}$ and $T$, when $W$ and the angle between $F^{\prime}$ and $W$ have given values, $T$ is evidently a minimum when perpendicular to $F^{\prime}$ and $F$. The angle between $T$ and the horizontal is then equal to the angle between $F$ and $L$, the tangent of which is $R / L$.

[^19]:    *The wing surfaces give most of the lift, but other surfaces may contribute; see note on "Modification in complete Machine," page 33.
    $\dagger$ In oblique flight, weight is sustained by the vertical component of lift, plus or minus the vertical components of thrust and resistance; $W=$ $L_{\mathrm{V}} \pm T_{\mathrm{v}} \pm R_{\mathrm{v}}$. The last two terms usually have opposite signs. For practical purposes, when the flight path does not depart far from the horizontal, $W=L$ and $V=\sqrt{\frac{W}{S K_{\mathrm{L}}}}$.

[^20]:    *By definition,
    Horse Power $=\frac{\mathrm{ft} . \mathrm{lbs} . \text { per min. }}{33000}$
    $=\frac{1 \mathrm{bs} . \times \text { miles per hr. }}{33000} \times \frac{5280}{60}=\frac{\text { lbs. } \times \text { miles per hr. }}{375}$.

[^21]:    *As thrust and resistance are equal in horizontal flight, minimum thrust is here equivalent to minimum resistance.

[^22]:    *See page 70.

[^23]:    *Particularly undesirable is the decrease in the power developed by a gas engine at high altitudes, the power developed being in direct proportion to the density of the air, as discussed in a later chapter. At an altitude of about 20,000 feet, only half as much power is developed as near the ground. To obviate this decrease in power with altitude has been the aim of inventors. Methods for accomplishing this have been devised but are not in general use.
    $\dagger$ Size and weight of engines.-The original motor of the Wright Brothers, used in the first airplane flight in 1903, gave 12 horse power, with a weight of 12.7 lbs . per horse power. The development of the airplane engine since then, until the 1918 Liberty motor, is shown by the following table. The numbers, except in the first and last column, are average values of principal engines for each year.

[^24]:    *Power variesastorque $\times$ speed. When power varies in direct proportion to speed, torque is constant. The torque in a gas engine is nearly constant through the working range. As the power curve falls off from a straight line, the torque decreases.
    †One hundred H. P. Gnome Monosoupape Motor. This particular motor is no longer made. Its performance, however, may be taken as typical.

[^25]:    *Called also "experimental pitch."
    $\dagger$ Torque is a turning moment equal to the product $F r$ of a force $F$ and the perpendicular distance $r$ from the axis of rotation to the line of action of the force. If $F$ is in pounds and $r$ in feet, torque horse power is $2 \pi N \mathrm{Fr} / 33000$, for one horse power is 33000 ft . lbs. per min.

[^26]:    *Power is of first importance. It makes little difference how efficient a propeller is, if it does not have enough power to do its intended work. A small desk fan, even were it ioo per cent. efficient, would not serve for propelling an airplane so well as a propeller with adequate power having an efficiency of only 50 per cent. Adequate power being assured, conditions of operation that give high efficiency should be sought.
    $\dagger$ Adjustable propellers, in which the pitch can be changed during flight, have been used but have not been widely introduced. See pp. 138, 169 .

[^27]:    *If $N$ is expressed in R. P. M. and $D$ in feet, $V$ must be expressed in ft . per minute. For example, an 8 ft . propeller makes 1200 R. P. M. When $V / N D=0.5, V=0.5 \times 1200 \times 8=4800 \mathrm{ft}$. per min. ( 54.5 MPH.); when $V / N D=1, V=9600 \mathrm{ft}$. per min. ( 109 MPH .). The value of $V / N D$ is frequently in this range ( 0.5 to 1.0 ), but these values are given for illustration and not as limits.

    If $N$ were revolutions per sec. and $D$ meters, $V$ must be meters per second; $V / N D$ would be unchanged, being independent of units.

[^28]:    *When the travel is greater than the dynamic pitch and the slip is negative (as in diving) a negative thrust is developed, the propeller then acting as a brake; see Fig. 5I.

[^29]:    *Forty-eight propellers discussed by Durand in Report No. 14, referred to later, have values of dynamic pitch between I.17 and I. 54 times the nominal pitch.
    $\dagger$ See Fig. 86, p. 176.

[^30]:    *The curves in Figs. 46 (and subsequent curves, unless otherwise stated) relate to a propeller $8^{\prime} 9^{\prime \prime}$ in diameter, with nominal pitch ratio 0.9. The curves have been plotted from calculations based upon experimental data for Propeller No. 3, as given by W. F. Durand in Report No. 14, National Advisory Committee for Aeronautics, 1917. All curves relate to ground level, air density $=0.0789$. The point $m$ on any curve is the point of maximum propeller efficiency.

[^31]:    *Curves may be drawn for engine and propeller torque, instead of engine and propeller power, determining $N$ by their intersection in the same manner. Curves for propeller torque, at different speeds $N$, are continuously rising curves, somewhat like the curves for torque horse power in Fig. 46. Curves for engine torque at different speeds are nearly horizontal, through a certain range of speed, dropping sharply at higher speeds.

[^32]:    *The same care in selection has to be taken in case of a marine propeller. Take as an illustration two tug boats, with identical hulls and engines, but different propellers. One boat, with propeller that gives full power when travelling at high velocity, far outstrips the other in the race to an incoming steamer, but when it comes to pulling a load it is inferior to the other boat equipped with a propeller tha gives full power when travelling at. low velocity.
    $\dagger$ The point for maximum power depends upon $D$ and pitch ratio. The point is marked here for $D=8^{\prime} 9^{\prime \prime}$ and pitch ratio $=0.9$.

[^33]:    *Determination of pitch from propeller measurement.-The tangent of $a$ is equal to the structural pitch divided by $2 \pi r$. The structural pitch of a propeller is, accordingly, equal to $2 \pi r \tan a$ and can be determined by measuring the pitch angle at any distance $r$ from the hub.

    The structural pitch ratio $p$ is $\pi \tan a$, being equal to the strictural pitch divided by $2 r$. We have, likewise, $\tan a=p / \pi$.

[^34]:    *Thrust is equal to the component of $L$ parallel to the shaft, less the component of $D$ in that direction. Torque is the sum of the components of $L$ and $D$ in the plane of rotation. Thus,

[^35]:    *Slip varies as the difference between the tangents of the two angles.

[^36]:    *This condition, with the curves $P_{\mathrm{x}}$ and $P_{z}$ tangent as shown in Fig. 64, occurs when a machine attains its maximum altitude, called the ceiling, discussed later under Altitude. Near the ground the relations between $P_{1}$ and $P_{2}$ are as shown in Fig. 66, but the curves $P_{\mathrm{r}}$ and $P_{2}$ shift, as altitude is increased, so as to give less surplus power and at the ceiling the two curves touch each other at one point only, showing that there is no surplus power. The point of tangency of the curves $P_{x}$ and $P_{2}$ is not necessarily at the minimum point of $P_{1}$ and the maximum point of $P_{2}$ (as in Fig. 64); but may be as in Fig. 65, if the curves $P_{x}$ and $P_{2}$ were slightly shifted so as to be tangent.

[^37]:    *See Figs. 53 and 58, pages 113 and 115.
    **When the velocity is less than that corresponding to $a_{0}$, the descent may be in an oblique path, or it may be a stall leading to a fall.

[^38]:    *The setting of the horizontal stabilizer in some machines can be made only on the ground, in other machines it may be made in flight; but in any case the change in setting is only occasional, whereas the control of the elevator is continuous. The horizontal stabilizer may be set so that a machine will climb or so that it will fly horizontally, with the elevator control in a neutral position, thus requiring less effort on the part of the pilot either when climbing or when flying horizontally, as the case may be.

[^39]:    *This holds when thrust is in line with resistance; in this case $W$ is the hypotenuse of a right triangle of which the other two sides are $L$ and $T-R$. In any case weight is the sum of the vertical components of $L, R$ and $T$, with proper sign; thus, $W=L_{\mathrm{v}}+T_{\mathrm{v}}+R_{\mathrm{v}}$.
    $\dagger$ See definition of horse power, page 70.

[^40]:    *Some engines will not stand full throttle until a certain altitude is reached.

[^41]:    *In making a turn with a low powered machine it is advisable to nose down, especially if near the ground or if the pilot is inexperienced, in order to maintain velocity and avoid the possibility of a stall or loss of control. There is a tendency for a machine to lose velocity on a turn due to increase of resistance, as explained in a later chapter on Directional Stability.

[^42]:    *When flying at the same angle of incidence, $V_{\mathrm{c}}$ and $V$ are greater at high than at low altitudes; but the ratios $V_{\mathrm{c}} / V$ and $R / W$ are constant and the gliding angle remains unchanged. This is discussed further in the next chapter.

[^43]:    *A machine weighing $W$ lbs. at height $d \mathrm{ft}$., has $W d \mathrm{ft}$. lbs. of energy due to its position. It will take the longest time to expend this energy when the rate of its expenditure (power) is a minimum.

[^44]:    *In the expression $\sin c=R / W$, the gliding angle apparently varies with $W$; it would seem that increasing $W$ would give a smaller gliding angle. This would be true if $R$ were independent of $W$. But $R$ varies as $V^{2}$ and $V^{2}$ varies as $W$; so that $R$ varies as $W$ and $R / W$ is not changed by increasing the weight of a particular machine.

[^45]:    *Assume $W=2000 \mathrm{lbs}$.; wing-area $S=3331 / 3 \mathrm{sq} . \mathrm{ft} . ; k s=0.04$; $K_{\mathrm{L}}$ and $K_{\mathrm{D}}$, as on page 20. For $-2^{\circ}, K_{\mathrm{L}}=0.000346, K_{\mathrm{L}} S=0.115$; $K_{\mathrm{D}}=0.0000948, K_{\mathrm{D}} S=0.0316 ; K_{\mathrm{D}} S+k s=0.0716$. The tangent of the gliding angle is $0.0716 / 0.115=0.62$, and the gliding angle is $32^{\circ}$. The velocity is

[^46]:    *A steep descent may be made by diving, or, when the pilot is familiar with his machine, by stalling. In either case the pilot should pull out into a moderate glide before acquiiing too great velocity or approaching too near the ground.
    $\dagger$ A cross wind tends to carry the machine sidewise as it touches the ground thus breaking the undercarriage. By side-slipping into the wind the pilot may counteract this tendency, but it requires experience and judgment.

[^47]:    *Reversible pitch propeller.-A propeller with reversible pitch may be used as a brake when making a landing. With pitch reversed, as power is applied the negative propeller thrust retards the machine, the amount of retardation being controlled by adjusting the pitch and by changing the throttle, as in some motor boats.

[^48]:    *This result is partially obtained by supercharging, i.e., using mechanical means for increasing the density of the air that is supplied to the motor.

[^49]:    *Other values are also used as standard: $0.07608,0.0762,0.0789$, etc. See the following footnote; also pages 20 and 97.

[^50]:    *In this table, at ground level the standard density is 0.0762 lbs . per $\mathrm{cu} . \mathrm{ft}$.; pressure, 29.92 inches of mercury; temperature $15^{\circ} \mathrm{C}$. At an altitude of $30,000 \mathrm{ft}$., standard pressure is 9.26 inches of mercury; temperature $-27^{\circ} \mathrm{C}$. The scale on the right hand of Fig. 78, has been drawn for convenience with 0.075 corresponding to $\delta=\mathbf{I}$.

[^51]:    *The entire power developed by the explosion of gas in the cylinders. See paragraphs on The Airplane Engine in Chapter VI, pages 82-87.
    $\dagger$ Decrease in B. H. P. with altitude is greater than decrease in I. H. P. on account of losses.-Consider, for example, an engine in which the losses are ıо H. P., delivering $100 \mathrm{~B} . \mathrm{H} . \mathrm{P}$. at ground level. The I. H. P. is ino. When the relative density is $\delta=0.5$ (alt. $2 \mathrm{I}, 000 \mathrm{ft}$.), I. H. P. $=0.5 \times$ $110=55 ;$ B. H. P. $=55-10=45$.

    It is thus seen that, for a relative density 0.5 , when the indicated horse power is reduced to 50 per cent., the brake horse power is reduced to 45 per cent. of its ground level value.

[^52]:    If the losses were zero, the B.H.P. would be reduced the same as the I.H.P., i.e., in this case to 50 per cent. If the losses were 20 H . P., the B. H. P. in the preceding example for $\delta=0.5$ would be only $40 \%$ of ground value. (The I. H. P. $=120$ at ground, 60 when $\delta=0.5$; B. H. P. $=120-20=100$ at ground, $60-20=40$, when $\delta=0.5$ ).

    The question of engine efficiency, although important at ground level, assumes greater importance at high altitudes; starting with a given B. H. P. at the ground, the less efficient the engine the more rapid will be the loss in B. H. P. as higher altitudes are reached.
    *Tests on airplane engines made in an air chamber (Report No. 45, National Advisory Committee for Aeronautics) show that brake horse power is 0.71 when the density of the air corresponds to an altitude of 10,000 ; 0.49 for 20,000 ; 0.325 for 30,000 , the power at ground level being i.oo. It is seen that these values are a little less than the values of $\delta$ of corresponding altitudes given in the preceding table.

[^53]:    *This is practically true in usual machines, which are designed so as to have adequate surplus power for leaving the ground at "take off." If a machine is designed so that, at a certain altitude, the minimum power required and maximum power available occur at the same velocity, it may have a little more surplus power at this altitude than at some lower altitudes. Furthermore, some engines can not be operated on full throttle until a certain altitude is reached.

[^54]:    *This is the true altitude corrected for temperature. Without this correction, the altitude indicated was $34,690^{\prime}$.

[^55]:    *This chapter, with its brief review of some of the essential elements of airplane structure, may well be read in connection with Chapter I, or at such other time as suits the reader.
    ${ }^{* *}$ In favor of a tractor it may be remarked that in case of a crash it is better for the pilot to have the heavy engine in front of him rather than behind. Furthermore, on a tractor a broken propeller, being well away from the machine, will cause less damage than on a pusher.

[^56]:    *In some cases there is a slight negative pressure on the underside of the wing near the trailing edge. Likewise, when a machine tends to dive there may be a positive pressure on the upper side near the trailing edge, particularly when this edge has an upward turn, giving the wing a reversed curvative, as in Fig. 18 on page 27.
    $\dagger$ See Fig. 108, page 218.
    $\ddagger$ The center of pressure is about one third the way back from the entering edge, its position shifting as the angle of incidence is changed, as shown in Fig. 13 on page 24.

[^57]:    *See table, Correction for Aspect Ratio, on page 178.

[^58]:    *The decalage, i.e., the difference in incidence between two planes, is in this case said to be positive. A negative decalage is generally undesirable on account of decreased stability, but it is said in some cases to aid in increasing speed range. For "decalage," referring to incidence between main plane and tail, see page 201.

[^59]:    *The aileron is usually incorporated as part of the wing, as shown in Fig. 91 and in the frontispiece.

[^60]:    *Mechanical Stabilizers.-Only mention can be made of gyroscopic and other mechanical stabilizers which maintain a machine in equilibrium by automatically operating the control surfaces whenever equilibrium is disturbed. The movement of a mass due to the forces of inertia, the swing of a pendulum, the flow of mercury in a containing vessel, the pressure of the dir on a small surface balanced by springs, etc., may be used to actuate the control mechanism. Gyroscopic control has, however, been most successful.

[^61]:    *In practically all cases the tail is placed behind. It will be shown later that a neutral tail or a depressing tail must be placed behind the main plane in order to give stability, while a lifting tail may be placed either in front or behind. Two tails have been used, one in front and one behind. A front tail is exceedingly sensitive.

[^62]:    *With $G$ under or back of the rear plane, a machine can not bestable. In this case the rear plane would be the main lifting plane. The forward plane must be a neutral or depressing tail, and this would increase any displacement of the machine instead of restoring it to its original position.

[^63]:    *It will be seen by the Table on page 20 that the percentage increase in lift, when the incidence is increased one degree, is greater for small than for large angles of incidence.

[^64]:    *The term "decalage" is likewise used to designate the difference in incidence of the two planes of a bi-plane; see note, page 183.
    $\dagger$ Called the longitudinal dihedral angle, to distinguish it from the lateral dihedral angle, discussed in the next chapter.

[^65]:    *A machine may oscillate before reaching steady flight conditions.

[^66]:    *The centers of lift, resistance and thrust are not points but are the lines along which the several forces act. Thus, the center of thrust is the line of the propeller shaft; the centers of lift and of resistance are the lines along which the resultant lift and resultant resistance act, respectively. The center of gravity, only, is a point.
    $\dagger$ It is here assumed that thrust is in line with resistance. When thrust is inclined upward it has a vertical component that supports part of the weight. See page 67.

[^67]:    *Another method considering three forces, $W, T$ and $P$.-In the following pages equilibrium is studied by resolving all forces into four, namely, $L, W, T$ and $R$. If $L$ and $R$ are combined into a single force $P$, namely, the total air force on the whole machine, there are three forces, $W, T$ and $P$. $W$ passes through $G$ and has no turning moment. If $t$ and $x$ are the perpendicular distances, respectively, of $T$ and $P$ from $G$, the moments $T . t$ and $P . x$ must be equal and opposite. $T$ is not necessarily in line with $R$, but may be inclined. This is a very good method of analysis but will not be followed here.

[^68]:    *This is true for small angles of incidence only, i.e., for the range employed in flight in which the lift increases with incidence. Were the incidence increased beyond, say, $14^{\circ}$ or $15^{\circ}$, the lift and the restoring moment would decrease.

[^69]:    *This effect is often used when taxying as an aid to the rudder in making a turn, the control stick being pushed to the right (instead of the left, as is usual) in making a left hand turn. The resistance of the right wing is thus decreased and the resistance of the left wing is increased. The velocity is so low that there is not enough lift on either wing to incline the machine laterally.

[^70]:    *A tendency for a machine to nose $u p$ on a turn is described under Secondary Effects (d), page 233, in the next chapter. See also footnote.

[^71]:    *Right hand engine.-A propeller or engine is said to have right hand rotation when the rotation is clockwise as viewed along the shaft in the direction of flight. Right hand rotation of the propeller tends to cause left hand rotation of the machine as a whole (depressing the left wing and elevating the right) and this tends to turn the machine to the left.

[^72]:    *The letter $R$ here stands for resultant and not for resistance.
    Strictly speaking, $R$ is not the resultant of $P$ and $p$, although for the present purpose it may be so called. More correctly, $P$ and $p$ may each be replaced by a couple and by a force acting at $G$, these two forces being shown in the figure by light dotted lines. The two couples thus formed are equal and opposite and so cancel each other. This leaves the two forces acting at $G$ with the resultant $R$.

    The two couples thus cancelled do not affect the motion of the airplane as a whole; they do, however, enter into strength computations.

[^73]:    *This tendency for a machine to nose up when it turns may conflict with the tendency for a machine to nose down when it banks (page 224) for a machine commonly banks and turns simultaneously. Which tendency will predominate depends upon the design of the machine, and the execution of the maneuver,-i.e., the velocity at which the turn is made, the sharpness of the turn, the amount of banking, etc.

[^74]:    *Inclinometers.-Various forms of inclinometers may be grouped under two heads: (1) absolute, that show departure from the vertical; and (2) relative, that show departure from the proper banking. But a pilot should never depend upon any instrument. In an open machine the wind on the face is the most reliable guide.

[^75]:    *With a right hand engine (see note, page 226) gyroscopic action tends to force the nose down when turned suddenly to the right (up when turned to the left); and tends to force the nose to the right when suddenly nosed up (left when nosed down).

[^76]:    *From Report No. 15, on "Nomenclature for Aeronautics," by the National Advisory Committee for Aeronautics.

[^77]:    *Prepared by A. S. Walker.

