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Alternative Errors-in-Variables Beta Estimates and Their Implications for Capital Asset Pricing Determination

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Abstract

A triangle relationship among alternative errors-in-variables (EV) methods for estimating daily beta coefficients are investigated in detail. It is shown that the well-known method proposed by Scholes and Williams is not a consistent estimator. Daily data of Dow-Jones thirty is used to demonstrate how these three EV methods can be used to estimate the daily beta coefficients.

I. Introduction

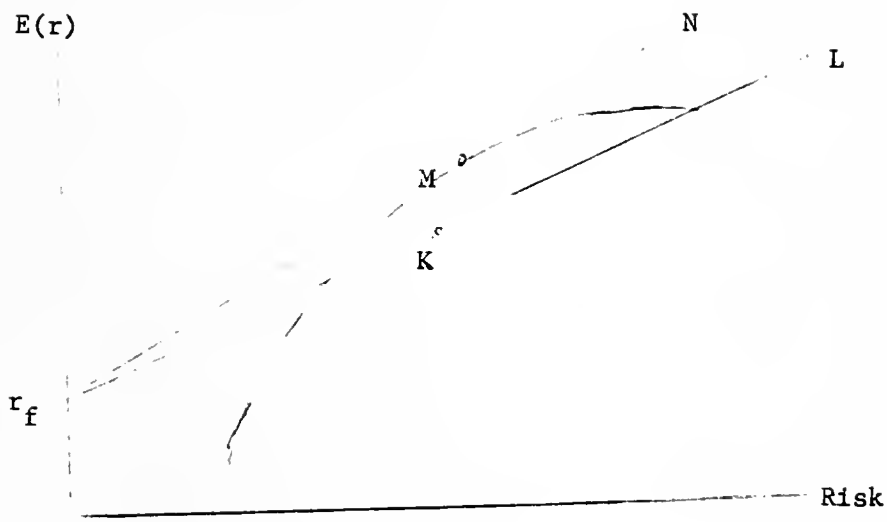
The capital asset pricing model (CAPM) developed by Sharpe (1964) and Lintner (1965) and Mossin (1966) has been received with great enthusiasm by the finance profession, and many attempts have been made to test the validity of the model and the index proxies. Most empirical studies tend to reject the two-parameter asset pricing model.[1] Several errors-in-variables problems are generally used to explain why the CAPM generally fails to pass empirical tests. The main purposes of this paper are to review and integrate several errors-in-variables beta estimates and to re-examine capital asset pricing determination. The second section of this paper reviews some possible problems associated with the traditional CAPM. The third section reviews and integrates errors-in-variables (EV) beta estimates. In the fourth section, daily data of Dow Jones thirty during the period of 1975-1979 are used to show how alternative EV estimates can be used to analyze capital asset pricing determination. Finally, results are summarized and future research efforts will also be outlined.

II. Plausible Explanations for Failure of Testing the CAPM

Levy (1978) proposed a theoretical explanation for the failure of traditional capital asset pricing determination. According to the traditional CAPM, all investors hold in their portfolios all risky assets in the market, and the proportion of each asset held by each investor is the ratio of its total market value to the total market value of all assets. Little attention has been paid to the fact that most individual investors do not diversify their portfolios as implied by the traditional CAPM. Levy quoted a study by Blume and Friend (1975) which shows the average number of securities in a portfolio in 1967 was 3.41. In other words, limited instead of the unlimited diversification should be used to derive the CAPM. Levy theoretically examined the impacts of limited diversification on the beta coefficient. He found that the true risk index of an individual security is somehwere between its variance and the beta coefficient, depending upon how widely the security is held. For securities that are widely held, beta will provide a better explanation of the capital asset pricing behavior; whereas individual variance will be a better indicator of risk and an appropriate factor in the equilibrium pricing determination for securities which are not widely held.

All investors will hold some combination of the market portfolio, M, and the riskless asset bearing interest rate r_f, if all the underlying assumptions of the traditional CAPM hold (see figure 1). Suppose limited diversification prevails because of indivisibility, transaction cost, or some other reason, an investor can hold only a fraction of assets in M, say K. In this case, his possible combination of risky and riskless assets will not be on r_fMN, instead he can only divide his portfolio between some risky portfolio K and riskless asset bearing the rate r_f. Using the return on the market portfolio, R_m, in the pricing determination is incorrect when the investors hold the portfolio K. R_m contains a measurement error when in fact R_k, the return on portfolio K, should have been used. To see

Figure 1



this, let the relationship between the return on a security and the return on the market portfolio under complete diversification be

(1) $$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_t$$

With limited diversification, the proper relationship should be

(2) $$R_{it} = \alpha_i^* + \beta_i^* R_{kt} + \varepsilon_t$$

If we assume that the relationship between R_{kt} and R_{mt} be as follows

(3) $$R_{mt} = R_{kt} + U_t$$

then the commonly used equation (1) can be written as

(4) $$R_{it} = \alpha_i + \beta_i (R_{kt} + U_t) + \varepsilon_t$$

This is different from the true relationship given by equation (2) under the assumption of limited diversification. Using equation (4) in the estimation of the beta coefficient when limited diversification prevails obviously causes problems.

According to Levy, to estimate beta given limited diversification is difficult, since one has to know not only the number of assets in each investor's portfolio, but also the size of his investment. As we will show in the next section, the problems caused the measurement error, U_t, might potentially be eliminated (or reduced) by alternative errors-in-variables estimation techniques.

A more serious indictment of testing the CAPM was raised by Roll (1977) when he greeted the finance profession by saying that: "(a) No correct and unambiguous test of the theory has appeared in the literature, and (b) there is practically no possibility that such a test can be accomplished in the future" (1977, pp. 129-30). One of his major conclusions was: "The theory is not testable unless the exact composition of the true market portfolio is known and used in the tests. This implies that the theory is not testable unless all individual assets are included in the sample." In other words, Roll argued that the traditional CAPM cannot be tested unless someone can construct a market index that includes all assets. It should be noted that the measurement error problems associated with incomplete measurement has been previously studied by Roll (1969) and Lee and Jen (1978). No one will deny the fact that current market indices are incomplete and therefore defective in some way, but this does not lead to the conclusions that the quality of the estimated beta coefficient cannot be improved. In other words, admitting that the market indices are imperfect does not preclude the adjustment of the estimator used in the estimation of beta coefficient. Roll's argument is very similar to Levy's in that the return on the market portfolio used in CAPM testing contains measurement errors.

According to Roll, the proper market portfolio should include human capital, real assets and other non-traded assets. It can be inferred from Roll's argument that the absence of human capital and non-traded assets causes measurement errors in the indices commonly used. As showed in the next section, the impacts of this kind of measurement errors on beta estimates can be accounted for by using a proper estimator, and the resulting estimate of beta will still be consistent.

So far two sources of errors in the market indices have been identified. The third source of errors, recognized by Scholes and Williams (1977), may be a problem when daily data are used. This kind of errors arises because many securities are traded infrequently and prices are reported only at distinct and random intervals. As a result, completely accurate calculation of returns over any <u>fixed</u> sequence of periods is impossible. The nonsynchronous trading of securities introduces errors into the market model.

All these kinds of errors can introduce bias into the estimate of the beta coefficient if the estimation technique used is not designed to capture the impacts of measurement errors. Without an unbiased estimate of the beta coefficient, it is difficult, if not impossible, to test the traditional CAPM empirically.

III. EV Beta Estimators - Review and Generalization

III.A. Basic Estimations

By now one should be convinced that measurement errors exist in the estimated returns on the market portfolio. It remains to be shown that the estimation of beta by ordinary least squares (OLS), the commonly used technique, is biased when the types of errors discussed before exist. Before we proceed to suggest alternative estimating techniques, the problems of using OLS is examined first. The OLS estimate of beta coefficient in equation (4) is

$$(5) \qquad \hat{\beta}_i = \frac{\operatorname{cov}(R_{kt}+U_t, R_{it})}{\operatorname{var}(R_{kt}+U_t)}$$

Making the usual assumption that U_t is uncorrelated with any other variable in the model, equation (5) can be reduced to

$$(6) \qquad \hat{\beta}_i = \frac{\operatorname{cov}(R_{kt}, R_{it})}{\operatorname{var}(R_{kt}) + \operatorname{var}(U_t)} = \frac{\beta_i^*}{1 + \dfrac{\operatorname{var}(U_t)}{\operatorname{var}(R_{kt})}} .$$

This proves that the OLS estimate of beta is biased whenever returns on the market portfolio are measured imperfectly due to one of the three sources of errors suggested in the previous section.

Given the existence of errors in the market indices, the OLS estimate is no longer unbiased and alternative estimators must be sought. Three of the estimators designed to allow for measurement errors appear to be good candidates for our purpose here. The first one involves the use of instrumental variables. The second one was suggested by Scholes and Williams (1977). The third approach to solve measurement errors was developed by Karni and Weissman (1974). All three approaches will be used here so that each technique will be checked by the outcome of the others. The results of one technique can be reinforced by the other two.

The idea of using instrumental variables is not new to econometricians. All it takes to give us a consistent estimate of beta is to conduct an instrumental variable Z_t which is uncorrelated in the limit with U_t but highly correlated with R_{kt}, then use the following estimator:[2]

(7) $$\hat{\beta}_i^{IV} = \frac{\text{cov}(Z_t R_{it})}{\text{cov}(Z_t R_{mt})}$$

The main difficulty in practice is to come up with the instrumental variables that have the above correlation property. Karni and Weissman (1974) suggested an instrumental variable that can be used to estimate the beta coefficient consistently. Their instrumental variable

(8) $$Z_t = R_{mt-h} + R_{mt-h+1} \cdot \cdot \cdot + R_{mt-1} + R_{mt+1} + \cdot \cdot \cdot + R_{mt+h}$$

can play the role of the instrumental variable. What is not clear in their formulations is how to pick the optimal h which will minimize the variance of the estimator. In this study, h is decided to take on the values from 1 to 10. In other words, 10 instrumental variables are created as follows:

(9)
$$\begin{aligned} Z_{1t} &= R_{mt-1} + R_{mt+1} \\ Z_{2t} &= R_{mt-2} + R_{mt-1} + R_{mt+1} + R_{mt+2} \\ &\cdot \\ &\cdot \\ &\cdot \\ Z_{10t} &= R_{mt-10} + R_{mt-9} + \cdot \cdot \cdot R_{mt-1} + R_{mt+1} + \cdot \cdot \cdot + R_{mt+10} \end{aligned}$$

And the instrumental variable estimator given by equation (7) becomes

(7a) $$\hat{\beta}_i^{IVj} = \frac{\text{cov}(Z_{jt} R_{it})}{\text{cov}(Z_{jt} R_{mt})} \quad j = 1, 2, ..., 10$$

The subscript i here stands for the ith security, whereas the subscript j stands for the jth instrumental variable. So the beta of a security will be estimated using 10 different estimated variables.

In addition to these 10 instrumental variables, an estimation technique for resolving measurement errors suggested by Scholes and Williams (1977) will also be used. They showed that beta coefficient can be estimated consistently by

$$(10) \qquad \hat{\beta}_i^{sw} = \frac{\hat{b}^- + \hat{b} + \hat{b}^+}{1 + 2\hat{\rho}_m}$$

where

$$(10a) \qquad \hat{\rho}_m = \frac{cov(R_m, R_{mt-1})}{\sigma(R_{mt}) \cdot \sigma(R_{mt-1})}$$

$$(10b) \qquad \hat{b}^- = \frac{cov(R_{it}, R_{mt-1})}{var(R_{mt-1})}$$

$$(10c) \qquad \hat{b} = \frac{cov(R_{it}, R_{mt})}{var(R_{mt})}$$

$$(10d) \qquad \hat{b}^+ = \frac{cov(R_{it}, R_{mt+1})}{var(R_{mt+1})}$$

The notation $\sigma(\cdot)$ in the above expressions represents the standard deviation of the variable in question.

The last approach used here to resolve the problem of measurement errors was suggested also by Karni and Weissman (1974). Their method is especially useful when instrumental variables are not available. They proved that the beta coefficient can be consistently estimated by

$$(11) \qquad \hat{\beta}_i^{kw} = \frac{cov(R_{it}, R_{mt}) - cov(\Delta R_{it}, \Delta R_{mt})/2}{var(R_{mt}) - var(\Delta R_{mt})/2}$$

where

$$(11a) \qquad \Delta R_{mt} = R_{mt} - R_{mt-1}$$

and

$$(11b) \qquad \Delta R_{it} = R_{it} - R_{it-1}.$$

III.B. The Relationship Among Three Alter EV Beta Estimators

Following Karni and Weissman (1974), substituting equations (11a) and (11b) into equation (11), we can obtain[3]

$$(11') \qquad \hat{\beta}_i^{kw} = \beta_i^* + \frac{\Sigma_2^{n-1}(R_{m,t-1} + R_{m,t+1})(\varepsilon_t - \beta_i^* U_t) + 0(1)}{2\, \Sigma_2^n R_{m,t} R_{m,t-1} + 0(1)}$$

where 0(1) represents the "end-effects," i.e., terms which correspond to t=1 and t=n.

Substituting $Z_1 = R_{m,t-1} + R_{m,t}$ into equation (7a), we obtain

$$(7b) \qquad \hat{\beta}_i^{IV2} = \beta_i^* + \frac{\Sigma_2^n(\varepsilon_t - \beta_i^* U_t)}{2\, \Sigma_2^n R_{m,t} R_{m,t-1}}$$

By comparing equation (7b) with equation (11'), we can conclude that $\hat{\beta}_i^{kw}$ is approximately equal to $\hat{\beta}_i^{IV2}$.

Now the relationship between $\hat{\beta}_i^{kw}$ and $\hat{\beta}_i^{ws}$ is explored. Substituting equations (11a) and (11b) into equation (11) and rearranging the terms, it can be shown that[4]

$$(11'') \qquad \hat{\beta}_i^{kw} = \frac{\hat{b}^- + \hat{b}^+}{2\hat{\rho}_m}$$

where $\hat{\rho}_m$, $\hat{b}^-$ and $\hat{b}^+$ have been defined in equations (10a), (10b) and (10d) respectively.

Substituting $\hat{\beta}_i^{kw} = \frac{b^- + b^+}{2\hat{\rho}_m}$ into equation (10), we obtain

$$(10') \qquad \hat{\beta}_i^{sw} = \frac{\hat{b} + 2\hat{\rho}_m \hat{\beta}^{kw}}{1 + 2\hat{\rho}_m} = \frac{\hat{b}}{1 + 2\hat{\rho}_m} + \frac{2\hat{\rho}_m}{1 + 2\hat{\rho}_m}\, \hat{\beta}^{kw}$$

Equation (10') implies that $\hat{\beta}^{sw}$ is the weighted average of the OLS estimated beta, $\hat{b}$ and the estimated beta obtained by the EV estimated proposed by Karni and Weissman (1974). The weights are $\frac{1}{1 + 2\hat{\rho}_m}$ and $\frac{2\hat{\rho}m}{1 + 2\hat{\rho}_m}$ respectively. If $\hat{\rho}_m$ approaches to zero, the $\hat{\beta}^{sw}$ approaches to the OLS beta estimates. SW has shown that $\hat{\beta}_i^{sw}$ is an instrumental variable estimator as indicated in equation (7). The instrument used is $Z_t = R_{mt-1} + R_{mt} + R_{mt+1}$.

Since $R_{mt} = R_{kt} + U_t$ (see (3)), it is obvious that the Z_t will not be uncorrelated with the measurement errors U_t as discussed in Section IIIA. And therefore, $\hat{\beta}^{sw}$ is not a consistent estimator for true beta β_i^*. However, both $\hat{\beta}_i^{IV}$ and $\hat{\beta}_i^{kw}$ are consistent estimates for β_i^*. Hence, $\hat{\beta}^{IV}$ and $\hat{\beta}_i^{kw}$ instead of $\hat{\beta}^{sw}$ should be used to estimate β_i^*.

In sum, three EV estimates discussed in this section are interrelated. In the following section, daily data of Dow Jones thirty will be used to show how three alternative EV methods can be used to estimate beta coefficients and draw some implications to capital asset pricing determination.

IV. Empirical Results of Various Errors-in-Variables Estimators

Both $\hat{\beta}^{IV}$ and $\hat{\beta}^{kw}$ are consistent estimators of the beta coefficient. Once the consistent estimate of the beta coefficient is obtained, the validity of the CAPM can be re-examined by running the usual second-pass cross-sectional regressions. In addition, $\hat{\beta}^{sw}$ is also estimated for comparison purposes.

Data used for this study were obtained from the CRSP data tape. Daily returns adjusted for dividends and stock splits covering the five year period from 1975 to 1979 were used. Securities chosen all belong to the Dow Jones 30. The market index used is the valued-weighted Fisher Index.

The results of applying the various errors-in-variables techniques to Dow Jones 30 are given in Table 1. Asymptotic standard errors appear in parentheses below the corresponding estimates.[5] All estimates are statistically significant.

There are four alternative beta estimates listed in Table 1, i.e., (i) OLS estiamtes, (ii) kw estimates, (iii) sw estimates (iv) average IV estimates. Results of Table 1 indicate that the magnitude of OLS beta estimates are similar to those obtained from three EV estimators. By the transaction volume definition of Scholes and Williams (1977) and the size definition defined by Roll (1981) and Reinganum (1982), the thirty securities used in this study belong to volume and large firms, therefore, our findings are consistent with their findings about the relationship between the OLS beta estimates and sw beta estimates for large (or high volume) firms. In addition, this study has also indirectly supported Levy's (1978, 1980) conjecture that the measure errors associated with using R_m as a proxy for estimating betas of large firms (or high volume firms) are negligible.

The second phase of this study involves the following linear regressions

(12a) $\quad \bar{R}_i = f(\hat{\beta}i)$

(12b) $\quad \bar{R}_i = f(\hat{S}_{ei}^2)$

(12c) $\quad \bar{R}_i = f(\hat{\sigma}_i^2)$

(12d) $\quad \bar{R}_i = f(\hat{\beta}_i, \hat{S}_{ei}^2)$

(12e) $\quad \bar{R}_i = f(\hat{\beta}_i, \sigma_i^2)$

where $\overline{R}_i$ is the average daily return on the ith security over the period covered, $\hat{\beta}_i$ is the systematic risk estimated from the time-series regressions given Table 1, $\hat{S}^2_{ei}$ is the residual variance taken from the time-series regressions. and $\hat{\sigma}_i^2$ stands for the estimate of the ith security variance. Equation (12a) is the one often used in the second-pass regression. According to the CAPM, the coefficient of $\hat{\beta}_i$ in equation (12a) is different between the average market return and the risk-free rate. The coefficients associated with $\hat{S}_{ei}^2$ and in equation (12d) should not be significant if the CAPM is indeed valid. Equation (12e) is designed to capture the problem of limited diversification as suggested by Levy (1978). According to Levy, the total risk of a security rather than its systematic risk becomes an important factor in pricing determination when limited diversification prevails. For securities that are not widely held, one will expect the coefficient of $\hat{\sigma}_i^2$ in equation (12e) to dominate the coefficient of $\hat{\beta}_i$.

These five equations were run for each of the errors-in-variables techniques, and the results given in Tables 3-15. Table 2 presents the results of the same five regressions using estimates taken from the OLS regression. The t-ratios are given in parentheses below the corresponding estimates. IV1 to IV10 stand for various instrumental variables which correspond to the ones given by equation (9).

Examining Tables 2-15, one will see that the coefficients of equation (12a) are statistically significant in quite a few cases, but the coefficients associated with $\hat{\beta}_i$, except the one derived from the Karni-Weissman technique, have wrong signs. This kind of strange results is not unusual, as Lee (1976) has reported similar results in his study. Moving to equation (12d), the coefficients of $\hat{S}^2_{ei}$ are insignificant in every errors-in-variables method used, which is implied by the CAPM. Judging from results relating to equation (12e), the effects of limited diversification does not exist for these Blue Chip firms. The coefficients of $\hat{\sigma}_{is}$ would have been significant if limited diversification has been true. Securities used in this study are most popular Dow Jones thirty and they are widely held by investors. Hence, one will not expect the total risk to play any role in pricing determination as predicted by the generalized capital asset pricing theory derived by Levy (1978).

V. Summary and Concluding Remarks

In this paper, we have theoretically reviewed different possible sources of measurement errors and have proposed three errors-in-variables techniques to reduce the impacts of these measurement errors on beta estimates. It has theoretically shown that the popular method developed SW is not a consistent estimator. The empirical results seem to be very different from what one would expect. If we go back to arguments raised by Levy (1978), and Scholes and Williams (1977), we can see that the results reported in this paper are consistent with most of their findings. As Levy has pointed out, limited diversification generally affects only securities which are not widely held. According to Scholes and Williams, nonsynchronous trading of securities has greater impacts on securities trading of low volume relative to those of high volume. It is not unreasonable to conclude that measurement errors are not serious in the sample of securities used in this study. One logical extension of this study is therefore to try out the same techniques on securities that are not widely held to examine the size effect and the limited diversification effect. This extension will be undertaken in the near future.

Another contribution of this paper is that the triangle relationship among three EV beta estimates are mathematically analyzed. Implications of these findings to Dimson's (1979) aggregated method of estimating beta coefficients will also be investigated in the future.[6]

Table 1: Estimated Betas of Dow Jones 30 Using OLS and Various Errors-In-Variables Techniques

	OLS		Karni-Weissman	Scholes-Williams	Average Estimated Beta of 10 Instrumental Variables
Allied Chemical	1.275 (.040)	R^2 = .353	1.209 (0.023)	1.255 (0.023)	1.462
Alcoa	1.183 (.041)	R^2 = .314	.934 (0.025)	1.102 (0.033)	1.265
American Brands	.715 (.030)	R^2 = .241	.827 (0.012)	.756 (0.024)	0.771
American Can	.570 (.026)	R^2 = .205	.795 (0.086)	.648 (0.021)	0.571
AT&T	.588 (.018)	R^2 = .359	.532 (0.049)	.570 (0.015)	0.544
Bethlehem	1.252 (.039)	R^2 = .363	.904 (0.024)	1.132 (0.031)	1.225
Chrysler	1.413 (.064)	R^2 = .212	.724 (0.064)	1.174 (0.052)	1.294
DuPont	1.175 (.028)	R^2 = .489	.829 (0.014)	1.056 (0.022)	1.151
Eastman Kodak	1.459 (.033)	R^2 = .516	.659 (0.024)	1.185 (0.027)	1.526
Esmark	1.001 (.040)	R^2 = .255	1.201 (0.021)	1.070 (0.032)	0.945
Exxon	.920 (.025)	R^2 = .432	.853 (0.089)	.898 (0.020)	0.882
General Electric	1.270 (.030)	R^2 = .503	1.077 (0.014)	1.203 (0.024)	1.351
General Foods	.911 (.038)	R^2 = .240	.973 (0.019)	.933 (0.031)	1.005
General Motors	1.051 (.029)	R^2 = .427	.748 (0.013)	.947 (0.023)	0.996
Goodyear	1.054 (.037)	R^2 = .304	.868 (0.021)	.989 (0.030)	0.999
Inco	1.037 (.039)	R^2 = .276	.901 (0.022)	.990 (0.032)	0.987
International Harvester	1.142 (.036)	R^2 = .352	1.138 (0.018)	1.142 (0.029)	1.017
Int'l Paper	1.228 (.035)	R^2 = .347	1.134 (0.018)	1.196 (0.028)	1.208
Johns Manville	1.025 (.046)	R^2 = .218	1.246 (0.027)	1.101 (0.036)	0.889
Minn. Mining	1.136 (.029)	R^2 = .450	1.084 (0.012)	1.116 (0.023)	1.134
Owens Illinois	.895 (.038)	R^2 = .231	.992 (0.019)	.929 (0.031)	0.889
Procter & Gamble	.913 (.026)	R^2 = .399	.801 (0.010)	.874 (0.021)	0.947
Sears	1.158 (.030)	R^2 = .444	.948 (0.014)	1.083 (0.024)	1.189
Standard Calif.	1.111 (.034)	R^2 = .374	1.069 (0.016)	1.099 (0.027)	1.056
Texaco	.943 (.034)	R^2 = .295	.895 (0.016)	.927 (0.027)	0.836
Union Carbide	1.276 (.031)	R^2 = .488	1.041 (0.015)	1.196 (0.024)	1.346
U.S. Steel	1.099 (.036)	R^2 = .341	.876 (0.019)	1.024 (0.029)	1.077
United Tech.	1.117 (.041)	R^2 = .286	1.017 (0.024)	1.087 (0.083)	1.070
Westinghouse	1.337 (.054)	R^2 = .253	1.083 (0.042)	1.249 (0.043)	1.194
Woolworth	1.063 (.044)	R^2 = .239	1.155 (0.026)	1.094 (0.035)	0.998

Table 2: Second-pass Regression Using OLS

$\bar{R}_i$ =	γ_0	+ $\gamma_1\hat{\beta}_i$	+ $\gamma_2\hat{S}^2_{ei}$	+ $\gamma_3\hat{\sigma}^2_i$
	0.0008 (3.295)*	-0.0005 (-2.290)*		
	0.0004 (3.274)*		-0.5021 (-0.984)	
	0.0004 (3.415)*			-0.6177 (-1.478)
	0.0008 (3.216)*	-0.0005 (-1.998)	0.0392 (0.070)	
	0.0008 (3.102)*	-0.0005 (-1.654)		0.0007 (0.001)

*significant at 5% level

Table 3: Second-passing Regression Using IV1

$\bar{R}_i$ =	γ_0	+ $\gamma_1\hat{\beta}_i$	+ $\gamma_2\hat{S}^2_{ei}$	$\gamma_3\hat{\sigma}^2_i$
	0.0004 (1.596)	-0.0001 (-0.595)		
	-0.0016 (-1.152)		7.7995 (1.344)	
	0.0004 (3.415)*			-0.6177 (-1.478)
	-0.0030 (-1.194)	0.0003 (0.677)	12.6690 (1.366)	
	0.0005 (1.773)	-0.0000 (-0.093)		-0.6026 (-1.323)

*significant at 5% level.

Table 4: Second-pass Regression Using IV2

$\bar{R}_i$ =	γ_0	+ $\gamma_1\hat{\beta}_i$	+ $\gamma_2\hat{S}^2_{ei}$	$\gamma_3\hat{\sigma}^2_i$
	0.0005 (2.405)*	-0.0002 (-1.216)		
	-0.0014 (-1.287)		6.8975 (1.534)	
	0.0004 (3.415)*			-0.6177 (-1.478)
	-0.0012 (-0.632)	-0.0000 (-0.077)	6.4696 (0.899)	
	0.0006 (2.599)*	-0.0002 (-0.750)		-0.4980 (-1.105)

*significant at 5% level.

Table 5: Second-pass Regression Using IV3

$\bar{R}_i$ =	γ_0	+ $\gamma_1\hat{\beta}_i$	+ $\gamma_2\hat{S}^2_{ei}$	$\gamma_3\hat{\sigma}^2_i$
	0.0004 (2.049)*	-0.0002 (-0.801)		
	-0.0007 (-0.686)		4.0963 (0.940)	
	0.0004 (3.415)*			-0.6177 (-1.478)
	-0.0005 (-0.270)	-0.0000 (-0.120)	3.4471 (0.493)	
	0.0005 (2.285)*	-0.0001 (-0.254)		-0.5711 (-1.253)

*significant at 5% level.

Table 6: Second-pass Regression Using IV4

$\overline{R}_i$ =	γ_0	+ $\gamma_1\hat{\beta}_i$	+ $\gamma_2\hat{S}^2_{ei}$	$\gamma_3\hat{\sigma}^2_i$
	0.0006 (2.452)*	-0.0003 (-1.345)		
	-0.0009 (-0.790)		4.8824 (1.025)	
	0.0004 (3.415)*			-0.6177 (-1.478)
	-0.0010 (-0.398)	-0.0003 (-0.858)	-1.4870 (0.168)	
	0.0006 (2.510)*	-0.0002 (-0.749)		-0.4527 (-1.952)

*significant at 5% level.

Table 7: Second-pass Regression Using IV5

$\overline{R}_i$ =	γ_0	+ $\gamma_1\hat{\beta}_i$	+ $\gamma_2\hat{S}^2_{ei}$	$\gamma_3\hat{\sigma}^2_i$
	0.0007 (3.221)*	-0.0004 (-2.126)*		
	-0.0011 (-0.980)		5.8594 (1.212)	
	0.0004 (3.415)*			-0.6177 (-1.478)
	-0.0027 (1.172)	-0.0007 (-1.896)	-6.982 (-0.851)	
	0.0007 (3.136)*	-0.0004 (-1.504)		-0.1975 (-0.399)

*significant at 5% level.

Table 8: Second-pass Regression Using IV6

$\bar{R}_i$ =	γ_0	+ $\gamma_1\hat{\beta}_i$	+ $\gamma_2\hat{S}^2_{ei}$	$\gamma_3\hat{\sigma}^2_i$
	0.0007 (3.076)*	-0.0004 (1.994)		
	-0.0013 (-1.098)		6.6536 (1.321)	
	0.0004 (3.415)*			-0.6177 (-1.478)
	-0.0020 (0.809)	-0.0006 (-0.519)	-4.6203 (-1.522)	
	0.0007 (2.996)*	-0.0004 (-1.363)		-0.2430 (-0.491)

*significant at 5% level.

Table 9: Second-pass Regression Using IV7

$\bar{R}_i$ =	γ_0	+ $\gamma_1\hat{\beta}_i$	+ $\gamma_2\hat{S}^2_{ei}$	$\gamma_3\hat{\sigma}^2_i$
	0.0007 (3.195)*	-0.0004 (-2.108)*		
	-0.0015 (-1.176)		7.3164 (1.389)	
	0.0004 (3.415)*			-0.6177 (-1.478)
	-0.0019 (0.783)	-0.0006 (-1.588)	-4.3145 (-0.482)	
	0.0007 (3.101)*	-0.0004 (-1.478)		-0.1943 (-0.389)

*significant at 5% level.

Table 10: Second-pass Regression Using IV8

$\bar{R}_i$	=	γ_0	+	$\gamma_1\hat{\beta}_i$	+	$\gamma_2\hat{S}^2_{ei}$	$\gamma_3\hat{\sigma}^2_i$
		0.0008 (3.459)*		-0.0005 (-2.365)*			
		-0.0014 (-1.081)				6.9457 (1.289)	
		0.0004 (3.415)*					-0.6177 (-1.478)
		-0.0031 (1.297)		-0.0008 (-2.155)*		-8.5358 (-0.971)	
		0.0008 (3.360)*		-0.0005 (-1.762)			-0.1106 (-0.223)

*significant at 5% level.

Table 11: Second-pass Regression Using IV9

$\bar{R}_i$	=	γ_0	+	$\gamma_1\hat{\beta}_i$	+	$\gamma_2\hat{S}^2_{ei}$	$\gamma_3\hat{\sigma}^2_i$
		0.0008 (3.550)*		-0.0005 (-2.438)*			
		-0.0014 (-1.087)				6.983 (1.295)	
		0.0004 (3.415)*					-0.6177 (-1.478)
		0.0029 (1.269)		-0.0007 (-2.203)*		-7.7122 (-0.921)	
		0.0008 (3.444)*		-0.0005 (-1.837)			-0.0499 (-0.099)

*significant at 5% level.

Table 12: Second-pass Regression Using IV10

$\bar{R}_i$ =	γ_0	+ $\gamma_1\hat{\beta}_i$	+ $\gamma_2\hat{S}^2_{ei}$	$\gamma_3\hat{\sigma}^2_i$
	0.0008 (3.369)*	-0.0005 (-2.285)*		
	-0.0014 (-1.056)		6.9332 (1.260)	
	0.0004 (3.415)*			-0.6177 (-1.478)
	0.0027 (1.143)	-0.0007 (-2.016)*	-7.1097 (-0.817)	
	0.0008 (3.242)*	-0.0005 (-1.657)		-0.0802 (-0.154)

*significant at 5% level.

Table 13: Second-pass Regression Using Karni-Weismann Technique

$\bar{R}_i$ =	γ_0	+ $\gamma_1\hat{\beta}_i$	+ $\gamma_2\hat{S}^2_{ei}$	$\gamma_3\hat{\sigma}^2_i$
	-0.0002 (-0.790)	0.0005 (1.760)		
	0.0004 (3.947)*		-0.7385 (-1.626)	
	0.0004 (3.415)*			-0.6177 (-1.478)
	-0.0001 (-0.407)	-0.0006 (2.064)*	-0.8446 (-1.951)	
	-0.0001 (-0.473)	0.0007 (2.360)*		-0.8618 (-2.148)*

*significant at 5% level.

Table 14: Second-pass Regression Using Scholes-Williams Technique

$\bar{R}_i$ =	γ_0	+ $\gamma_1\hat{\beta}_i$	+ $\gamma_2\hat{S}^2_{ei}$	$\gamma_3\hat{\sigma}^2_i$
	0.0006 (1.678)	-0.0003 (-0.895)		
	0.0004 (3.318)*		-0.5187 (-1.024)	
	0.0004 (3.415)*			-0.6177 (-1.478)
	0.0005 (1.566)	-0.0002 (-0.522)	-0.3995 (-0.711)	
	0.0005 (1.357)	-0.0000 (-0.077)		-0.5944 (-1.141)

*significant at 5% level.

Footnotes

[1]For some examples of testing CAPM empirically, see Blume and Friend (1973), and Black, Jensen and Scholes (1972), Roll (1977, 1978), Roll and Ross (1980).

[2]For a description of instrument variables and their asymptotic properties, see Johnston (1972), pp. 278-291.

[3]See equation (3.2) in Karni and Weissman (1974)

[4]See Appendix A for the derivation.

[5]The formulas for computing the various asymptotic standard errors are given in the above mentioned references.

[6]Both Dimson's (1979) and Sholes and Williams' (1977) beta estimates have been used by Reinganum (1982) to test Ross's conjecture on the firm size effect.

References

F. Black, M. C. Jensen, and M. Scholes, "The Capital Asset Model: Some Empirical Tests," in Michael C. Jensen, ed., Studies in the Theory of Capital Markets, New York, 1972.

M. Blume and I. Friend, "A New Look at the Capital Asset Pricing Model," Journal of Finance, March 1973, pp. 19-33.

M. Blume and I. Friend, "The Asset Structure of Individual Portfolios and Some Implications for Utility Functions," Journal of Finance, May 1975, pp. 585-603.

E. Dimson, "Risk Measurement When Shares are Subject to Infrequent Trading," Journal of Financial Economics, 7 June 1979, pp. 197-226.

J. Johnston, Econometric Methods, McGraw-Hill, New York, 1972.

E. Karni and I. Weissman, "A Consistent Estimator of the Slope in a Regression Model with Errors in the Variables," Journal of the American Statistical Association, March 1974, pp. 211-213.

C. F. Lee, "Performance Measure, Systematic Risk, and Errors-in-Variables Estimation Method," Journal of Economics and Business, 1976-1977, pp. 122-127.

C. F. Lee and Frank C. Jen, "Effects of Measurement Errors on Systematic Risk and Performance Measure of a Portfolio," Journal of Financial and Quantitative Analysis, 1978, 8, pp. 299-312.

H. Levy, "Equilibrium in an Imperfect Market: A Constraint of the Number of Securities in the Portfolio," American Economic Review, September 1978, pp. 643-58.

H. Levy, "The CAPM and Beta in an Imperfect Market," Journal of Portfolio Management, Winter 1980, pp. 5-11.

J. Lintner, "Security Prices, Risk, and Maximal Gains from Diversifications," Journal of Finance, December 1965, pp. 587-616.

J. Mossin, "Equilibrium in a Capital Asset Market," Econometrica, 1966, 34, pp. 768-783.

B. M. Reinganum, "A Direct Test of Roll's Conjecture on the Firm Size Effect," Journal of Finance, 1982, 37, pp. 27-35.

R. Roll, "A Possible Explanation of the Small Firm Size Effect," Journal of Finance, 1981, 36, pp.

R. Roll, "A Critique of the Asset Pricing Theory's Tests; Part I: On Past and Potential Testability of the Theory," Journal of Financial Economics, March 1977, pp. 129-176.

R. Roll, "Bias in Fitting the Sharpe Model to Time Series Data," Journal of Financial and Quantitative Analysis, 4, pp. 274-289.

R. Roll and S. Ross, "An Empirical Investigation of the Arbitrage Pricing Theory," Journal of Finance, 35, pp. 1073-1103.

M. Scholes and J. Williams, "Estimating Betas from Nonsynchronous Data," Journal of Financial Economics, December 1977, pp. 309-327.

W. Sharpe, "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," Journal of Finance, September 1964, pp. 425-442.

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Appendix A: Derivation of Equation (11')

$$\hat{\beta}_i^{kw} = \frac{\mathrm{Cov}(R_{it}, R_{mt}) - \mathrm{Cov}(\Delta R_{it}, \Delta R_{mt})/2}{\mathrm{Var}(R_{mt}) - \mathrm{Var}(\Delta R_{mt})/2}$$

$$= \frac{\mathrm{Cov}(R_{it}, R_{mt}) - \mathrm{Cov}(R_{it-1}, R_{mt-1}) + \mathrm{Cov}(R_{it-1}, R_{mt}) + \mathrm{Cov}(R_{it}, R_{mt-1})}{\mathrm{Var}(R_{mt}) - \mathrm{Var}\ R_{mt-1} + 2\mathrm{Cov}(R_{mt}, R_{mt-1})}$$

if $\mathrm{Cov}(R_{it}, R_{mt}) = \mathrm{Cov}(R_{it-1}, R_{mt-1})$ and $\mathrm{Var}(R_{mt}) = \mathrm{Var}(R_{mt-1})$ then

$$\hat{\beta}_i^{kw} = \frac{\mathrm{Cov}(R_{it-1}, R_{mt}) + \mathrm{Cov}(R_{it}, R_{mt-1})}{2\ \mathrm{Cov}(R_{mt}, R_{mt-1})}$$

$$= \frac{\beta_i^- + \beta_i^+}{2\rho_m}$$