resistor is inserted for R1. Assorted polystyrene or good quality polyester capacitors of lower value are then temporarily connected across the capacitor panel to increase C_x by small increments, while listening on the headphones for a drop in the level of the 1kHz tone as C_x approaches 1μF.

A typical computing capacitor might finally consist of a parallel combination of the following values, 0·68μF, 0·22μF, 0·02μF, and 0·005μF.

If the required value of C_x is exceeded, the note in the headphones will increase in volume when the null point is passed. Allow capacitors to cool off after soldering, and before making a measurement, as heat can cause a temporary or permanent change in capacitance. With the Fig. 5.3 bridge circuit it is possible to detect increments of less than 0·01μF in a nominal 1μF capacitor.

DIFFERENTIAL ANALYSIS WITH UNIT "A"

A second order linear differential equation with constant coefficients has become firmly established as the "classic" introduction to differential analysis on the analogue computer.

The equation describes an oscillatory system with variable damping which can be used to simulate indirectly many physical systems, such as the spring pendulum, a tuned LC circuit, or a servomechanism. Also, the equation is easy to set up on the computer, and does not necessarily demand the use of integrator mode switching.

In general form the equation is,

$$a\frac{d^2y}{dt^2} + b\frac{dy}{dt} + cy = f(t) \qquad \text{(Eq. 5.1)}$$

where a, b, and c are the constant coefficients, y is unknown, and $f(t)$ represents some function of time. Equation 5.1 can be rewritten to suit a particular system by substituting appropriate terms.

Spring pendulum

$$m\frac{d^2y}{dt^2} + \mu\frac{dy}{dt} + ky = f(t) \qquad \text{(Eq. 5.2)}$$

where m is the mass of a weight suspended on a spring of constant k, which is damped by friction μ. The weight is displaced by an amount y when subjected to a force dependent on $f(t)$.

Tuned LC circuit

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = f(t) \qquad \text{(Eq. 5.3)}$$

where L is an inductance tuned by a capacitance C, and damped by a series resistance R. Q is the charge in coulombs on C at any instant of time. The current flowing in the tuned circuit is given by dQ/dt, and $f(t)$ represents an input function.

Servomechanism

$$\frac{d^2\theta_o}{dt^2} + 2\zeta\omega\frac{d\theta_o}{dt} + \omega^2\theta_o = \omega^2\theta_i \qquad \text{(Eq. 5.4)}$$

where θ_o is the angular displacement of the output shaft, ζ the damping factor, ω the angular velocity, and θ_i the angular displacement of the input shaft.

The obvious similarity between the above equations is emphasised when, in Fig. 5.5, it is seen that they all have virtually the same problem layout on the computer. Furthermore, as the computer will allow operation at almost any fraction or multiple of real time, a spring pendulum and a tuned LC circuit can be simulated simultaneously, and interesting electro-mechanical parallels can be seen to exist between the properties of inductance and mass, resistance and friction, and capacitance and elasticity.

The only real difference between the analogous behaviour of a weight on a spring, a servo shaft, and a tuned LC circuit is that the LC combination will normally resonate at a much higher frequency.

PROBLEM EXAMPLE 3. TUNED CIRCUIT ANALYSIS

UNIT "A" will simulate any series tuned circuit by solving Equation 5.2, and will give answers in the form of a.c. meter readings or oscillograms. Tuned circuits resonating in the MHz region are catered for by slowing down the problem to some convenient decadal fraction of real time, so that a simulated circuit on the computer which is, for example, resonating at 300Hz, will serve as a model for a real circuit resonating at 30MHz, with suitable rescaling of L, C, and t.

To initially determine the relative values of L, C, R, voltage V, and current I, without too much paperwork, it is helpful to start with a representative tuned circuit which allows computer operation in real time, at frequencies convenient for display by an a.c. voltmeter or an oscilloscope. 50Hz is a good frequency to employ as a datum because it can be readily obtained from the mains supply, and rounded values of $L = 1$H and $C = 10\mu$F will also offer resonance at 50Hz.

Taking the circuit of Fig. 5.6a as a starting point, from the knowledge that a series tuned circuit will exhibit an impedance equal to R at resonance, the r.m.s. current flow at 50Hz will be E_i/R, or 20mA when $E_i = 2$V r.m.s. and $R = 100$ ohms.

It is necessary to rearrange the basic equation, Equation 5.2, for the computer by dividing through by L, and solving for the second derivative.

$$\frac{d^2Q}{dt^2} = -\frac{R}{L}\frac{dQ}{dt} - \frac{1}{LC} + \frac{f(t)}{L} \qquad \text{(Eq. 5.5)}$$

Substituting known values from Fig. 5.6a,

$$\frac{d^2Q}{dt^2} = \frac{100R}{1H}\frac{dQ}{dt} - \frac{1}{1H \times 10^{-5}C}Q + \frac{f(t)}{1H} \qquad \text{(Eq. 5.6)}$$

$f(t)$ in the present case represents a sine wave input of 2V r.m.s. In other circumstances the input function could be a square wave of amplitude E_{in} and period $2t$.

Equation 5.6 is solved on the computer by successive integration. Looking at the symbolised diagram of Fig. 5.6b, it can be seen that there are two closed-loops, one linking the output of OA1 via CP1 to OA1/Input 1, and the other passing through OA1, OA2, and OA3, via CP2, and thence back to OA1/Input 3. The coefficient of CP1 will be multiplied by the gain factor associated with OA1/Input 1. CP2 coefficient is multiplied by the product of gains OA1/Input 3, OA2, and OA3, i.e. 1,000 × 100 × 1 = 100,000.

d^2Q/dt^2, obtained from the sum of the voltages present at the inputs of OA1, is initially assumed to be present. After one integration OA1 provides an output dQ/dt, and from this all the terms on the right hand side of Equation 5.6 are assembled. So, dQ/dt is multiplied by $R/L = 100$, using CP1 set for a coefficient of 0·1, and is taken back to OA1/Input 1 where it is then added to $f(t)/L = 2$V r.m.s.