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Faculty Working Papers

AN ANALYSIS OF CHANGES IN AGGREGATE STOCK MARKET VOLATILITY

Frank K. Reilly, Professor, Department of Finance
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January, 1979

AN ANALYSIS OF CHANGES IN AGGREGATE STOCK MARKET VOLATILITY*

John M. Wachowicz, Jr.
Frank K. Reilly**

INTRODUCTION

A great deal of research has been done on the return volatility for securities (and portfolios) both in absolute terms and in relation to the aggregate market. This prior research which includes consideration of changes in individual stock return volatility has implicitly assumed that the volatility of the aggregate market is generally stable over time. Only recently has this assumption regarding market volatility been examined. These studies, which will be discussed in detail in section two, have indicated that the variability of the aggregate market is not constant over time but rather has shown major changes. In addition, the studies that have examined market volatility for a recent period have derived conflicting results using somewhat different data. One would anticipate that a change in market return volatility would certainly influence investors' perceptions of future market risk and, therefore, the required risk premium on equity securities. In addition, market volatility has relevance to the whole body of capital asset pricing literature. Specifically, it can be shown that a

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change in market volatility and, hence, the level of market risk will have a significant effect on the slope of the capital market line (CML), the security market line (SML), and the characteristic line for individual securities. Also, aggregate market volatility is an integral part of several composite portfolio performance measures.

Because of the importance of aggregate market volatility to investors risk perception and the whole field of valuation, and the conflicting results of prior studies, our study reconsiders the question of changes in market volatility with a more complete set of data, several additional measures of volatility, and statistical tests of changes in volatility not considered in prior studies. The initial section contains a discussion of the prior studies on market volatility. In the subsequent section we discuss the data series used and the measures of market return. The third section contains an analysis of the characteristics of the distribution of market returns. In the fourth section the alternative market volatility measures are compared and analyzed to determine if there have been significant changes during the period 1926-197[illegible]. Section five contains a summary and conclusions.

PRIOR STUDIES ON MARKET VOLATILITY

Fisher and Lorie Study

The Fisher and Lorie article reports the findings of three studies into the variability of returns on investments in common stocks listed on the New York Stock Exchange (NYSE).[1] In all cases, returns are defined as

[1]Lawrence Fisher and James H. Lorie, "Some Studies of Variability of Returns on Investments in Common Stocks," The Journal of Business, Vol. 43, No. 2 (April, 1970), pp. 99-134.

"wealth ratios" (i.e., the ratio of the value of the investment at the end of the period to the amount invested). The first study examined the frequency distributions of returns from investments in single stocks for 55 specific time periods ranging from one to 40 years, during the period 1926-1965. The second study examined the aggregated distributions of returns from investments in individual common stocks on the NYSE for nonoverlapping periods of equal length from one to 20 years.

The final study examined the variability of returns from investments in portfolios of specified numbers of common stocks on the NYSE. Distributions were found for portfolios of six sizes (i.e., 1, 4, 8, 16, 32, and 128 stocks), and for "all" stocks listed on the NYSE (i.e., the market portfolio). The main objective of the study was to examine the effect of diversification on variability of returns. Those parts of the final study concerning "all" stocks (i.e., the "market portfolio) are of greatest interest. Here the authors provide the first extensive analysis of market variability over time.

The "market" portfolio assumed equal initial investments in all the common stocks listed on the NYSE. Return and variability figures for the "market" portfolio, covering a number of time periods, were presented. Table 1 summarizes the reported market-return data after adjusting the "wealth ratios" to more familiar annual rates of return. The standard deviations and coefficients of variation for market return are "crude" measures of market variability. Notably, these measures indicate that market returns during 1946-1965 were significantly less volatile than during the period 1926-1945.

TABLE 1

SAMPLE MEANS, STANDARD DEVIATIONS, AND COEFFICIENTS OF VARIATION OF ANNUAL RETURN ON THE "MARKET" PORTFOLIO, 1926-1965

Period	Average Annual Rate of Return	Standard Deviation of Annual Rate of Return	Coefficient of Variation of Annual Rate of Return
1926-1945	15.8%	40.0%	2.53
1946-1965	13.8	19.7	1.43
1926-1965	14.8	31.5	2.13

SOURCE: Adapted from Fisher and Lorie, "Some Studies of Variability of Returns on Investments in Common Stocks," Table 5, p. 113.

Officer Study

R. R. Officer examined market-factor variability as measured by the one-year standard deviation of the monthly returns of the market factor.[2] In so doing, he calculated a monthly moving series of standard deviations of returns covering the period 1897-1969. No one index was found to represent adequately the market factor for the entire period; therefore, a number of indexes were needed. The Fisher Arithmetic Index, however, was used for most of the period--namely, February 1926 to June 1963.[3]

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[2] R. R. Officer, "The Variability of the Market Factor of the New York Stock Exchange," The Journal of Business, Vol. 46, No. 3 (July, 1973), pp. 434-453.

[3] The general pattern of the standard deviation time series was probably not materially affected by the index selection. For example, Officer found that the linear relationship between the one-year standard deviation of the 20-stock Dow Jones Index (i.e., the stocks making up the index as of January 1926) and the one-year standard deviation of the Fisher Index had an $r^2 = .96$ over the 1926-68 period.

The major finding of the Officer study is that the decline in variability observed by Fisher and Lorie is better described as a return to "normal" levels of variability after an extended period of abnormally high volatility in the 1930s. (See Fig. 1) The rest of the study examined those factors that may have influenced market-return variability. It was concluded that neither the formation of the SEC, changes in margin requirements, nor the "changing composition" of stocks listed on the NYSE affected the variability of the market-factor over time. Market-factor variability does seem related, however, to business fluctuations as represented by variability of industrial production index relatives. Variability in M2 money supply relatives was related to market-factor variability only around 1929.

Leuthold Study

In contrast to Fisher and Lorie, who examined annual returns, and Officer, who reviewed monthly returns, a study by Leuthold[4] was concerned with market volatility using daily market fluctuations. The proxy used to represent the "market" was the Dow Jones Industrial Index, and the period studied ran from 1897 to September 30, 1975.

The analysis involved what Leuthold calls "high volatility days," which are all days when there was a 2 percent or greater change in the market, up or down, as measured on a close-to-close basis. Leuthold found 1,238

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[4] Steven C. Leuthold, "The Causes (and Cures?) of Market Volatility," The Journal of Portfolio Management, Vol. 2, No. 2 (Winter, 1976), pp. 21-25.

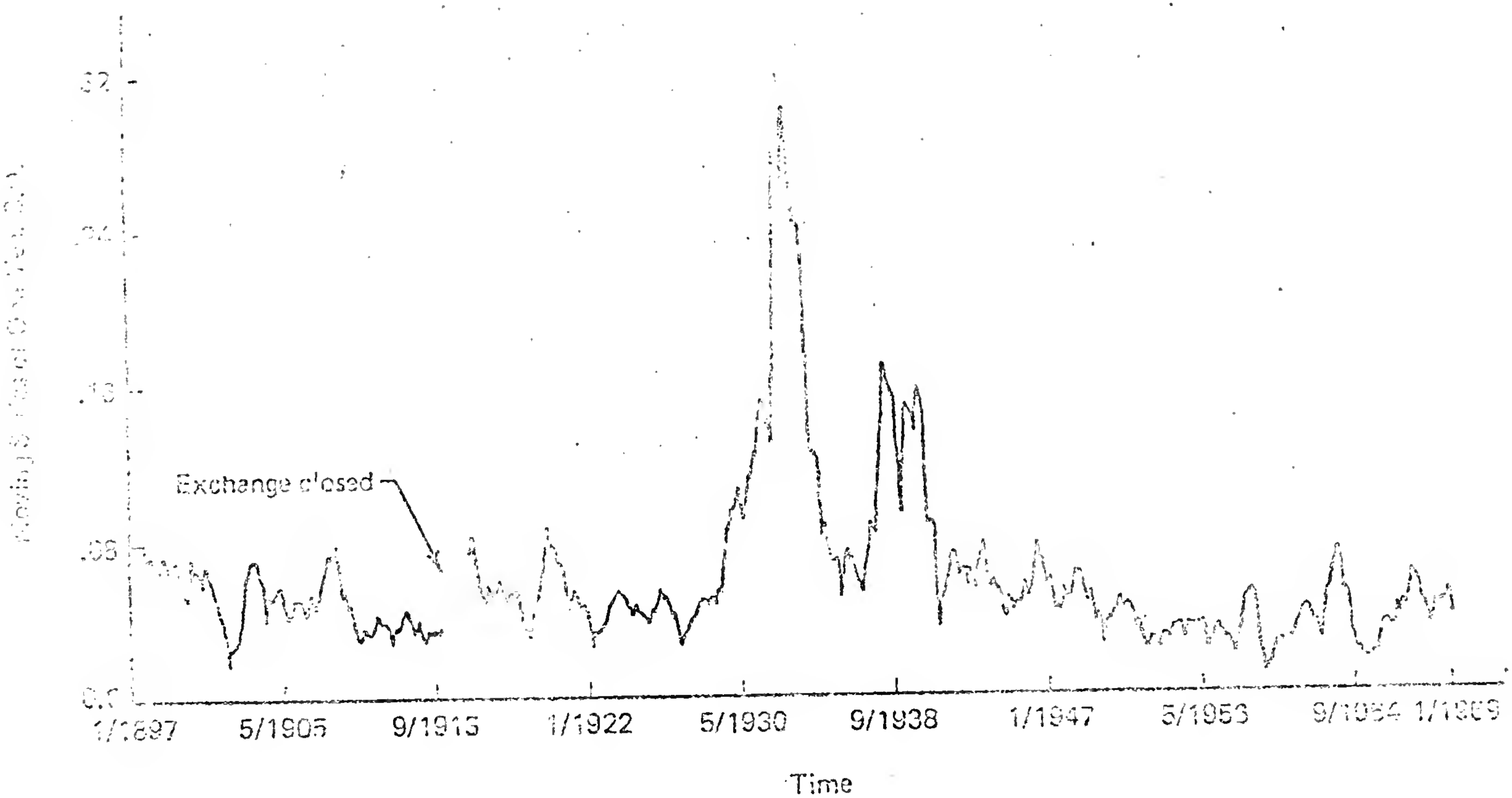

Figure 1. The behavior of the one-year standard deviation of the monthly returns on the market index, 1897-1969.

SOURCE: Reproduced with permission from the University of Chicago Press; R. R. Officer, "The Variability of the Market Factor of the New York Stock Exchange," The Journal of Business, Vol. 46, No. 3 (July, 1973), p. 436.

"high volatility days" over the 1897-1975 period which represents roughly one out of 18 trading days. The analyis involved the determination of the percent of high volatility days during alternative years with an emphasis on the recent period (1973-1975) compared to the total 79 year period and various subperiods: 1897-1925; the 1930s; and 1941-1972.

Based on his study results, Leuthold concludes that sharp day-to-day market swings have recently increased in frequency. This recent market instability is especially dramatic when compared to the "quiet" 1941-1972 period.

The author believes that the reason for the increase in day-to-day volatility is the institutional market of recent years. Although no direct empirical evidence is presented indicating a relationship between volatility and institutional trading, a number of arguments are offered for why one would "expect" institutional trading to cause an increase in volatility. The current authors strongly disagree with this belief that the institutions have caused the increase in stock price volatility. This disagreement is based upon the results of several studies.[5]

Annual data for each of the 79 years included in the Leuthold study were made available at the end of the article. This allows us to re-examine some of the previous two studies' main conclusions using daily instead of monthly or yearly stock price fluctuations.

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[5]Frank K. Reilly, "Institutions on Trial: Not Guilty," Journal of Portfolio Management, Vol. 3, No. 2 (Winter, 1977), pp. 5-10; Frank K. Reilly and John M. Wachowicz, Jr., "More on the Effect of Institutional Trading on Stock Price Volatility," Journal of Portfolio Management, forthcoming; Frank K. Reilly, "Block Trades and Stock Price Volatility," Financial Analysts Journal, forthcoming; and Neil Barkman, "Institutional Investors and the Stock Market," New England Economic Review, Federal Reserve Bank at Boston (November/December, 1977), pp. 60-78.

Fisher and Lorie's study showed an apparent decrease in market volatility from the 1926-1945 period to the 1946-1965 period. Results from an analysis of Leuthold's data are presented in Table 2.

TABLE 2

PERCENT OF YEAR'S TRADING DAYS CONSIDERED "HIGH VOLATILITY DAYS"

Period	Mean	Median	Range
1926-1945	10.9%	4.7%	0.0%-45.0%
1946-1965	1.3	0.4	0.0 - 6.3

SOURCE: Adapted from data presented in Leuthold, "The Causes (and Cures?) of Market Volatility," p. 25.

Daily fluctuations are consistent with Fisher and Lorie's results--the 1946-1965 period appears less volatile than the 1926-1945 period.

Officer's study covered the period 1897-1969. He states that the "... variability of the market factor before the 1930's is similar to that after about 1942."[6] Leuthold's data reveal a somewhat different pattern (see Table 3). The abnormal variability of the 1930s is evident again. However, the analysis of daily market change variability indicates less variability for the recent post-1942 period than for the pre-depression period.

[6] Officer, "The Variability of the Market Factor of the New York Stock Exchange," p. 434.

TABLE 3

PERCENT OF YEAR'S TRADING DAYS CONSIDERED "HIGH VOLATILITY DAYS"

Period	Mean	Median	Range
1897-1928	4.8%	3.4%	0.3%-12.7%
1929-1942	15.0	11.5	1.0 -45.0
1943-1969	1.0	0.4	0.0 - 6.3

SOURCE: Adapted from data presented in Leuthold, "The Causes (and Cures?) of Market Volatility," p. 25.

Logue Study

Logue notes that the popular belief is that securities markets have become more volatile.[7] Reasons suggested for this alleged increase in stock price volatility are: (1) the growth in institutional investor activity; (2) an increase in long-term economic uncertainties; and (3) recent changes in the international economic environment. Since these problems are not unique to the United States, Logue examines the variability of returns in four securities markets--the New York Stock Exchange, the London Stock Exchange, the Toronto Stock Exchange, and the Tokyo Stock Exchange--over the period 1958-1974. For his part, Logue considered what direction stock market volatility might take over the long-term and felt that there were many reasons to believe that market volatility should decline over time.

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[7] Dennis E. Logue, "Are Stock Markets Becoming Riskier?" The Journal of Portfolio Management, Vol. 2, No. 3 (Spring, 1976), pp. 13-19.

The reasons included the fact that the new companies entering the market would add diversity, the aggregate economy has become more diversified over time and also our economy has become more mature and policy-makers should become more adept at controlling fluctuations. It is acknowledged that the Officer results did not support such an expectation.

Annual standard deviations were calculated using monthly return relatives for the four countries. In general, the patterns were similar between the countries and over time. There did not appear to be a decline in volatility in any country over the 1958-1974 period. To test the idea of relative risk, the coefficient of variation for each year was calculated.[8] However, the plot of the coefficients of variation for each country produced patterns similar to those obtained using standard deviations--i.e., the market returns did not appear to have become riskier.

In addition to the variability of nominal stock price relatives, Logue examined the variability of real stock price relatives (actual relatives adjusted for inflation). The time patterns of annual standard deviations of real monthly stock price relatives and coefficients of variation for real returns were similar to the nominal relatives. A subsequent analysis of bond market volatility indicated that only Canada seemed to experience an increase in volatility during the period examined. Finally, Logue examined the variability of inflation in the four countries and contended that only the United States and the United Kingdom experienced an increase in inflation volatility.

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[8]The coefficient of variation was the standard deviation of monthly stock price relatives for a year divided by the average monthly stock price relative for that year. This is considered to be a measure of risk per unit of reward.

In summary, the evidence presented by Logue suggests that neither the stock markets nor the bond markets studied have become riskier in recent years. However, inflation has become more volatile in the United States and the United Kingdom.

Summary of Prior Studies

Fisher and Lorie examined yearly return figures and found a marked decrease in the level of market volatility from the 1926-1945 period to the 1946-1965 period. Officer considered a moving average of one-year standard deviations of monthly returns for the period 1897-1969 and contended that the decline in volatility during 1946-1965 compared to 1926-1945 was really a return to the level of volatility that prevailed prior to the 1930s. Leuthold contended, on the basis of the proportion of trading days with large percent changes, that stock prices in the 1970s had become more volatile than previously. Logue's study seems to contradict Leuthold's contention of increased volatility in the 1970s. Using monthly returns, Logue argues for no change in the level of market volatility from 1958 through 1974.

Based upon the discussion of these previous studies, a renewed investigation of market volatility should seek to answer a number of questions: (1) What return interval should be studied to best capture market volatility? (2) What measure (or measures) of market volatility should be employed? (3) Has the post-WWII level of market volatility undergone change?

MEASURING MARKET RETURNS

William Sharpe describes the "market portfolio" as the combination of all risky securities existing in the market.[9] Because one cannot directly observe the market portfolio, we will rely on a stock market index serving as a proxy for the market portfolio.

Choice of Stock Market Series

The Standard and Poor's 500 Composite Index (SP500) is used in this study to represent the overall movements of the market. As a market proxy, the SP500 possesses many desirable attributes. First, it is a value-weighted (as opposed to an equal-weighted) index.[10] "Value-weighted indexes have the property to reflect better the macro implications of price movements ..."[11] Therefore, in trying to represent the overall movements of the "market," a value-weighted index seems more appropriate

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[9] William F. Sharpe, Portfolio Theory and Capital Markets (New York: McGraw-Hill Book Company, 1970), p. 82.

[10] In a value-weighted stock index, the stock components are weighted in direct proportion to their contribution to total market value of all stocks in the index.

The formula for the SP500 index is...

$$\text{SP500 Index} = [(P_1Q_1)/(P_0Q_0)](10),$$

where P_1, represents the current market price, P_0 the market price in the base period (1941-1943), Q_1 the number of shares currently outstanding, and Q_0 the number of share outstanding in the base period (1941-1943), subject to adjustment when necessary to offset changes in capitalization. Standard and Poor's Corporation, Standard & Poor's Trade and Securities Statistics: Security Price Index Record, 1978, ed. (New York: Standard and Poor's Corporation, 1978), p. 3.

[11] George M. Frankfurter, "The Effect of 'Market Indexes' on the Ex-post Performance of the Sharpe Portfolio Selection Model," The Journal of Finance, Vol. 31, No. 3 (June, 1976), p. 950.

than an equal-weighted index. Among value-weighted indexes, the SP500 seems an especially good choice because the market value of the 500 stocks used in the index represents 85 to 90 percent of the value of all common stocks listed on the NYSE. In addition, the coverage is broad (425 industrials, 20 railroads, 55 utilities), and historical daily listings enable a review of volatility back to 1928.[12]

Market Return Interval

The prior studies (except Leuthold's) measured volatility in terms of annual or monthly rates of return or price changes (typically defined as ending value minus beginning value divided by the beginning value). Annual as well as monthly changes computed in this manner are not without problems. Specifically, consider a month when the market begins at a given price and subsequently experiences several major declines followed by several days of rising prices and then further declines, but finishes the month at about the same price as at the beginning. If one considers only the beginning and ending values, he would observe that no change had occurred and, therefore, should conclude based on this two observation measure that there was

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[12]"Prior to 1957 the Standard & Poor's daily stock price indexes were based on 90 stocks (50 industrials, 20 rails, and 20 utilities)... The earlier indexes were converted to the base for 1941 to 1943 and were added to the new series, giving a continuous daily record back to 1928. Correlation studies were made by Standard & Poor's at the time to determine the coefficient of correlation between the price index of 90 stocks and the discontinued, broader weekly stock price index of 416 stocks. The study proved that the index of 90 was an accurate measure of the market as a whole." Wilford J. Eiteman, Charles A. Dice, and David K. Eiteman, The Stock Market, 4th ed. (New York: McGraw-Hill Book Company, 1969) p.184.

The SP500 index was changed substantially in August, 1976. The effect was to make the index even broader and more representative of the "market."

little volatility during this period when, in fact, there were major changes within the period. In contrast, envision a market series that experiences a steady movement in one direction in small increments (i.e., small daily changes). In this instance, on the basis of the beginning and ending values, one would observe a very large change and conclude that this was a very volatile period. Obviously, the intraperiod observations show a steady decline with little volatility within the period. The point is, this particular measure, that only looks at the beginning and ending values, ignores a great deal of information regarding what transpired during the period.

In an attempt to explain volatility as opposed to trend, and in the belief that for some investors a holding period of less than a month is a relevant time period for consideration, daily market returns are analyzed in this study. The use of daily figures for studying return and variability characteristics of individual securities is well established.[13] In fact, the U.S. Securities and Exchange Commission, in its publication Statistical Bulletin, defines stock price volatility as "...the extent to which stock prices change on a day-to-day basis."

Precedent for using daily figures to study market returns is also available. Brealey[14] examined the distribution of daily rates of return from the British equity market. And Leuthold, as we have seen, investigated daily market fluctuations.[15]

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[13] See, for example, Eugene F. Fama, "The Behavior of Stock Market Prices," The Journal of Business, Vol. 38, No. 1 (January, 1965), pp. 39-105.

[14] Richard A. Brealey, "The Distribution of Successive Rates of Return from the British Equity Market," Journal of Business Finance, Vol. 2, No. 2 (1970), pp. 29-40.

[15] Leuthold, "The Causes (and Cures?) of Market Volatility," pp. 21-25.

Alternative Market Return Measures

Given a desire to study daily market returns, it is necessary to determine how to measure market return. One alternative is the percentage change in the market index as defined below:

$$R_{mt} = \frac{SP_t - SP_{t-1}}{SP_{t-1}}$$

$$= \frac{\Delta SP_t}{SP_{t-1}}$$

$$= \%\Delta SP_t$$

where R_{mt} = the market return in period t;

SP_t = the value of Standard and Poor's 500 Composite market index at the close of period t;

SP_{t-1} = the value of the index at the close of period t-1;

ΔSP_t = the index change during period t;

$\%\Delta SP_t$ = the percentage change in the index during period t.

The use of changes in the natural logarithm of price as a measure of return is another possibility. This return measure is common in the efficient market literature.[16] Expressing the return on the market index in this fashion, we get...

$$R_{mt} = \ln SP_t - \ln SP_{t-1}$$

$$= \ln (SP_t/SP_{t-1})$$

$$= \ln (1 + \%\Delta SP_t)$$

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[16] For example, see Eugene F. Fama, Lawrence Fisher, Michael C. Jensen, and Richard Roll, "The Adjustment of Stock Prices to New Information," International Economic Review, Vol. 10, No. 1 (February, 1969), pp. 1-21.

Here, return may be considered as a continuously compounded rate of change. Either of the suggested return measure series compensate for potential problems due to trends in means and variances present in series of market levels or absolute changes.[17] A percentage change series will remove or limit the importance of any heterogeneity in variance in the original levels series. Therefore, even though the original series is not stationary, the transformed series should tend to conform to a stationary distribution.[18]

The problem of trying to choose between the two alternative return measures is reduced because of our interest in daily returns which seldom exceed ±10 percent.[19] For market index changes of less than ±10 percent, the change in the natural logarithm of the market index is approximately equal to the percentage change (one-period return).

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[17] Arnold Moore, "A Statistical Analysis of Common Stock Prices," unpublished dissertation, University of Chicago, 1962, pp. 13-15. Moore has shown that the variability of simple price changes for a given stock is an increasing function of the price level of the stock. In a similar fashion, the variability of market index changes is likely to increase with increasing stock market levels.

[18] "Stationarity" is a time-series property. Generally, it means that the characteristics of a stochastic process are invariant with respect to time (i.e., the parameters of the process do not change over time).

[19] For example, in one 15-year period (1/4/60-6/30/75), the largest positive one-day percent change in the SP500 was only +5.02% (5/27/70), while the largest negative one-day percent change was -6.68% (5/28/62). Raymond H. Marcotte, "Analysis of the Impact of Competitive Commission Rates on Aggregate Price Volatility of NYSE Stocks," Securities and Exchange Commission: Economic Staff Paper 75, No. 2 (July, 1975), p. 4.

Because both return measures seem equally satisfactory for studying daily market returns, the percentage change in the market index ($\%\Delta SP_t$) is employed in this study.

Dividend Yield Adjustment

Because the daily dividend yield is small relative to the percentage change in the market index and/or the dividend yield is relatively constant, its omission will not materially affect our conclusions about daily market volatility. Also, attempting to incorporate dividends into a daily market return measure seems to create problems. To begin with, there are no readily available figures on daily SP500 dividend yields. Ying attempted to approximate SP500 daily dividend yields by applying a linear interpolation formula to the quarterly dividend yields.[20] Granger and Morgenstern, however, demonstrated that Ying's adjustments induce autocorrelation into the return series.[21] Therefore, for these reasons no attempt is made to adjust the market return series for dividend yields.

CHARACTERISTICS OF THE MARKET RETURN SERIES DISTRIBUTION

The nature of the market return distribution is important for two main reasons. First, the type of distribution will affect the appropriateness of the various market volatility measures available. Secondly,

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[20] C. Ying, "Stock Market Prices and Volume of Sales," Econometrica, Vol. 34, No. 3 (July, 1966), pp. 676-685.

[21] Clive Granger and Oskar Morgenstern, Predictability of Stock Market Prices (New York: D.C. Heath & Co., 1970), p. 204.

the nature of the distribution determines which statistical tests are suitable for hypothesis testing. If, for example, market returns are normally distributed, only two statistics are needed to completely describe the distribution--the mean and the variance. Either variance or standard deviation, then, would be an appropriate volatility measure. Also, if market returns were normally distributed, the F test could be used to test the null hypothesis that market returns drawn from two different time periods have equal variances.

The work of Fama[22] and Mandelbrot[23] suggest that returns on individual stocks in the United States are distributed according to a stable symmetric distribution with infinite variance (i.e., a non-normal stable Paretian distribution). A similar distribution might best represent market returns. In such a case, the standard deviation would be an inappropriate measure of volatility. Also, statistical tests that assume normality could not be applied to the data.

In this section, the distributions of market returns for selected subperiods within the period 1928-75 are tested for normality. For subperiods where normality can be assumed, classical statistical tests could be used for hypothesis testing. Nonparametric tests for comparisons of dispersion would be used for periods in which normality could not be assumed.

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[22] Fama, "The Behavior of Stock Market Prices," pp. 39-105.

[23] Benoit Mandelbrot, "The Variation of Certain Speculative Prices," The Journal of Business, Vol. 36, No. 4 (October, 1963), pp. 394-419; and Benoit Mandelbrot, "The Variation of Some Other Speculative Prices," The Journal of Business, Vol. 40, No. 4 (October, 1967), pp. 393-413.

Selection of Subperiods

For the purpose of analysis, the entire period 1928-1975 will be broken into subperiods in two different ways. First, every X number of years will be designated as one period (see Table 4). Secondly, each period will be made to correspond to a major "bull" or "bear" market (see Table 5).[24]

General Characteristics of Return Distributions

Information regarding distribution symmetry for the multi-year groupings is contained in Table 6. For fifteen of the thirty--sometimes overlapping--periods, the percentage of returns falling above (or below) the mean differs significantly from fifty percent. In fourteen of these fifteen cases, the percentage of returns falling above the mean is significantly higher than fifty percent. In twenty-five periods the median return was greater than the mean return. However, on average, each period's median return differs from its mean return by only .03501 standard deviations. The distributions for twenty periods show some degree of negative skewness. Every period, however, exhibits a positive Kurtosis statistic--a sign of a peaked (leptokurtic) distribution.

One property of the normal distribution is that a known proportion of observations fall within a given number of standard deviations from the mean. Table 7 compares market returns, expressed in terms of the

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[24]The selection of major "bull" and "bear" markets was made on the basis of whether the SP500 was in a major rising or falling pattern. The monthly cutoff dates used are those reported in: Jerome B. Cohen, Edward D. Zinbarg, and Arthur Zeikel, Investment Analysis and Portfolio Management, 3rd ed., (Homewood, Illinois: Richard D. Irwin, Inc., 1976), p. 505; and James H. Lorie and Mary T. Hamilton, The Stock Market: Theories and Evidence, (Homewood, Illinois: Richard D. Irwin, Inc., 1973), p. 7.

number of standard deviations by which they differed from the mean, to frequencies for the unit normal distribution for selected ranges. A necessary, but not sufficient, condition for a non-normal stable Paretian distribution of market returns would be for Table 7 to reveal an excess of very small and very large returns and a deficiency of medium-sized returns, when compared with the normal distribution. It is, therefore, important to note that each period's distribution exhibits a shortage of medium-sized returns, and in varying degrees, a surplus of extreme returns, relative to the normal distribution.

TABLE 4

SUBPERIOD TO BE ANALYZED:
MULTI-YEAR GROUPINGS (1928-1975)

Two Twenty-Four Year Periods	Four Twelve-Year Periods	Eight Six-Year Periods	Sixteen Three-Year Periods
1928-51 (7055)	1928-39 (3573)	1928-33 (1770)	1928-30 (883)
			1931-33 (887)
		1934-39 (1803)	1934-36 (903)
			1937-39 (900)
	1940-51 (3482)	1940-45 (1789)	1940-42 (904)
			1943-45 (885)
		1946-51 (1693)	1946-48 (847)
			1949-51 (846)
1952-75 (6039)	1952-63 (3039)	1951-57 (1529)	1952-54 (774)
			1955-57 (755)
		1958-63 (1510)	1958-60 (757)
			1961-63 (753)
	1964-75 (3000)	1964-69 (1484)	1964-66 (757)
			1967-69 (727)
		1970-75 (1516)	1970-72 (758)
			1973-75 (758)

Note: The number of daily market returns for each period is enclosed in parentheses.

TABLE 5

SUBPERIOD TO BE ANALYZED:
BULL AND BEAR MARKETS (1928-1975)

Period		Number of Months		Number of Daily Market Returns	
Bull Markets	Bear Markets				
1/28-8/29		20		493	
	9/29-5/32		33		815
6/32-1/34		20		488	
	2/34-2/35		13		323
3/35-1/37		23		579	
	2/37-3/38		14		348
4/38-9/39		18		454	
	10/39-3/42		30		750
4/42-4/46		49		1210	
	5/46-5/49		37		872
6/49-12/52		43		994	
	1/53-8/53		8		169
9/53-6/56		34		712	
	7/56-11/57		17		356
12/57-6/59		19		399	
	7/59-9/60		15		317
10/60-11/61		14		292	
	12/61-9/62		10		209
10/62-12/65		39		819	
	1/66-9/66		9		190
10/66-11/68		26		521	
	12/68-4/70		17		351
5/70-12/72		32		675	
	1/73-11/74		23		484
12/74-12/75		13		274	
	ALL		226		5184
ALL		350		7910	

TABLE 6(a)

RETURN DISTRIBUTION SYMMETRY: MULTI-YEAR GROUPINGS

	Daily Market Returns					
	28-51	52-75	28-39	40-51	52-63	64-75
Percentage of Returns Below the Mean	49.60%	*48.48%	50.35%	*48.82%	*47.91%	*48.40%
Percentage of Returns Above the Mean	50.40%	*51.52%	49.65%	*51.18%	*52.09%	*51.60%
Mean Return	.00015	.00025	.00008	.00022	.00040	.00009
Median Return	.00054	.00043	-0-	.00065	.00070	.00026
Standard Deviation[a]	.01479	.00748	.01899	.00857	.00714	.00782
Skewness[b]	.32187	-.06630	.40058	-1.0196	-.50894	.28505
Kurtosis[c]	11.024	6.4574	6.7222	8.9808	10.960	3.2171
Number of Observations	7055	6039	3573	3482	3039	3000

*Significantly different from 50 percent at the .05 confidence level.

[a] Calculated with (N-1) degrees of freedom.

[b] Normalized 3rd moment about the mean [i.e., $\Sigma(R_m - \overline{R}_m)^3/Ns_m^3$].

[c] Normalized 4th moment about the mean minus three [i.e., $(\Sigma(R_m - \overline{R}_m)^4/Ns_m^4)-3$].

TABLE 6(b)

RETURN DISTRIBUTION SYMMETRY: MULTI-YEAR GROUPINGS

	Daily Market Returns							
	28-33	34-39	40-45	46-51	52-57	58-63	64-69	70-75
Percentage of Returns Below the Mean	*48.25%	50.31%	48.35%	48.79%	*47.74%	*48.08%	*47.64%	50.20%
Percentage of Returns Above the Mean	*51.75%	49.69%	51.15%	51.21%	*52.26%	*51.92%	*52.36%	49.80%
Mean Return	-.00007	.00023	.00022	.00022	.00037	.00044	.00015	.00003
Median Return	-0-	-0-	.00079	.00057	.00068	.00071	.00035	-0-
Standard Deviation[a]	.02241	.01488	.00840	.00874	.00704	.00724	.00557	.00952
Skewness[b]	.51281	-.03271	-1.0055	-1.0310	-.56746	-.45456	-1.0632	.34547
Kurtosis[c]	5.7413	3.9880	11.504	6.6531	7.6507	13.920	2.1238	2.0146
Number of Observations	1770	1803	1789	1693	1529	1510	1484	1516

*Significantly different from 50 percent at the .05 confidence level.

[a] Calculated with (N-1) degrees of freedom.

[b] Normalized 3rd moment about the mean [i.e., $\Sigma(R_m - \overline{R}_m)^3/Ns_m^3$].

[c] Normalized 4th moment about the mean minus three [i.e., $(\Sigma(R_m - \overline{R}_m)^4/Ns_m^4)-3$].

TABLE 6(c)

RETURN DISTRIBUTION SYMMETRY: MULTI-YEAR GROUPINGS

	Daily Market Returns							
	28-30	31-33	34-36	37-39	40-42	43-45	46-48	49-51
Percentage of Returns Below the Mean	*43.15%	*53.33%	49.50%	49.67%	*44.97%	*44.97%	*45.81%	49.41%
Percentage of Returns Above the Mean	*56.85%	*46.67%	50.50%	50.33%	*55.53%	*55.03%	*54.19%	50.59%
Mean Return	-.00004	-.00010	.00066	-.00020	-.00022	.00067	-.00011	.00056
Median Return	.00155	-.00225	.00091	-0-	-0-	.00091	.00053	.00063
Standard Deviation[a]	.01605	.02732	.01195	.01733	.00983	.00662	.01001	.00725
Skewness[b]	-.71396	.71132	-.27539	.08831	-.93773	-.77879	-.96453	-.95417
Kurtosis[c]	12.296	3.0144	2.9495	3.2769	10.893	3.5111	5.8771	5.2579
Number of Observations	883	887	903	900	904	885	847	846

*Significantly different from 50 percent at the .05 confidence level.

[a] Calculated with (N-1) degrees of freedom.

[b] Normalized 3rd moment about the mean [i.e., $\Sigma(R_m - \bar{R}_m)^3/Ns_m^3$].

[c] Normalized 4th moment about the mean minus three [i.e., $(\Sigma(R_m - \bar{R}_m)^4/Ns_m^4)-3$].

TABLE 6(d)

RETURN DISTRIBUTION SYMMETRY: MULTI-YEAR GROUPINGS

	Daily Market Returns							
	52-54	55-57	58-60	61-63	64-66	67-69	70-72	73-75
Percentage of Returns Below the Mean	*47.42%	48.61%	47.95%	48.07%	*46.24%	47.73%	48.44%	52.37%
Percentage of Returns Above the Mean	*52.58%	51.39%	52.05%	51.93%	*53.76%	52.27%	51.06%	47.63%
Mean Return	.00055	.00018	.00052	.00037	.00010	.00020	.00035	-.00029
Median Return	.00083	.00024	.00085	.00060	.00034	.00039	.00045	-.00092
Standard Deviation[a]	.00541	.00838	.00704	.00743	.00531	.00582	.00725	.01135
Skewness[b]	-.45247	-.51556	-.22234	-.64812	-.28059	.02788	.50346	.32764
Kurtosis[c]	1.6494	6.8457	14.203	13.548	3.6176	.95280	4.2537	.78376
Number of Observations	774	755	757	753	757	727	758	758

*Significantly different from 50 percent at the .05 confidence level.

[a]Calculated with (N-1) degrees of freedom.

[b]Normalized 3rd moment about the mean [i.e., $\Sigma(R_m - \overline{R}_m)^3/Ns_m^3$].

[c]Normalized 4th moment about the mean minus three [i.e., $(\Sigma(R_m - \overline{R}_m)^4/Ns_m^4)-3$].

TABLE 7(a)

COMPARISON OF RETURN DISTRIBUTION WITH NORMAL DISTRIBUTION: MULTI-YEAR GROUPINGS

Intervals in Standard Deviations	Proportion of Observations						
	Normal Distribution	Daily Market Returns (1928-1975)					
		28-51	52-75	28-39	40-51	52-63	64-75
0.0 - 0.5	38.30%	58.43%	49.28%	52.78%	53.04%	48.01%	50.07%
0.5 - 1.0	29.96	23.37	27.65	26.45	25.74	29.19	25.96
1.0 - 1.5	18.38	8.84	12.67	10.11	10.94	13.59	12.37
1.5 - 2.0	8.81	3.90	5.40	5.45	5.71	5.06	6.03
2.0 - 2.5	3.31	2.41	2.48	2.36	2.27	2.14	2.74
2.5 - 3.0	0.97	1.05	1.13	1.09	0.81	0.79	1.40
3.0 - 4.0	0.26	1.23	0.94	1.06	0.83	0.73	1.10
4.0 - 5.0	0.00994	0.36	0.22	0.45	0.20	0.16	0.20
> 5.0	0.00006	0.41	0.23	0.25	0.46	0.33	0.13
Total	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
Number of Observations		7055	6039	3573	3482	3039	3000

TABLE 7(b)

COMPARISON OF RETURN DISTRIBUTION WITH NORMAL DISTRIBUTION: MULTI-YEAR GROUPINGS

Intervals in Standard Deviations	Proportion of Observations: Normal Distribution	Daily Market Returns (1928-1975): 28-33	34-39	40-45	46-51	52-57	58-63	64-69	70-75
0.0 - 0.5	38.30%	54.35%	48.92%	55.17%	51.09%	46.70%	49.60%	45.35%	47.16%
0.5 - 1.0	29.96	24.52	27.23	24.04	26.94	29.04	29.21	29.31	25.47
1.0 - 1.5	18.38	10.57	12.42	10.62	11.57	13.93	13.11	14.02	14.64
1.5 - 2.0	8.81	5.14	6.38	5.47	5.85	5.49	4.77	5.66	7.32
2.0 - 2.5	3.31	2.31	2.39	2.63	2.07	2.62	1.39	3.17	2.97
2.5 - 3.0	0.97	1.08	1.11	0.84	0.77	1.04	0.73	1.01	1.25
3.0 - 4.0	0.26	1.52	1.22	0.56	1.06	0.79	0.53	1.28	0.86
4.0 - 5.0	0.00994	0.23	0.11	0.17	0.30	0.19	0.20	0.13	0.26
> 5.0	0.00006	0.28	0.22	0.50	0.35	0.20	0.46	0.07	0.07
Total	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
Number of Observations		1770	1803	1789	1693	1529	1510	1484	1516

TABLE 7(c)

COMPARISON OF RETURN DISTRIBUTION WITH NORMAL DISTRIBUTION: MULTI-YEAR GROUPINGS

Intervals in Standard Deviations	Proportion of Observations								
	Normal Distribution	Daily Market Returns (1928-1975)							
		28-30	31-33	34-36	37-39	40-42	43-45	46-48	49-51
0.0 - 0.5	38.30%	53.91%	48.82%	46.29%	51.00%	58.19%	50.06%	51.12%	47.64%
0.5 - 1.0	29.96	28.42	25.48	27.91	24.67	22.56	25.65	26.92	27.42
1.0 - 1.5	18.38	8.61	14.65	13.51	12.00	10.07	13.44	12.16	13.71
1.5 - 2.0	8.81	4.42	5.41	6.75	7.22	5.09	5.20	5.08	6.15
2.0 - 2.5	3.31	2.37	2.37	3.44	1.89	1.66	2.71	1.41	2.62
2.5 - 3.0	0.97	0.68	1.92	1.44	1.55	0.88	1.58	1.18	1.16
3.0 - 4.0	0.26	1.02	1.01	0.44	1.34	0.55	1.02	1.30	0.95
4.0 - 5.0	0.00994	0.00	0.23	0.11	0.11	0.34	0.23	0.59	0.11
> 5.0	0.00006	0.57	0.11	0.11	0.22	0.66	0.11	0.24	0.24
Total	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
Number of Observations		883	887	903	900	904	885	847	846

TABLE 7(d)

COMPARISON OF RETURN DISTRIBUTION WITH NORMAL DISTRIBUTION: MULTI-YEAR GROUPINGS

Intervals in Standard Deviations	Proportion of Observations								
	Normal Distribution	Daily Market Returns (1928-1975)							
		52-54	55-57	58-60	61-63	64-66	67-69	70-72	73-75
0.0 - 0.5	38.30%	43.54%	45.83%	46.50%	52.86%	48.88%	41.68%	48.81%	40.90%
0.5 - 1.0	29.96	30.36	30.99	30.65	28.41	29.32	29.16	26.12	30.47
1.0 - 1.5	18.38	14.60	13.11	14.53	11.03	11.63	17.19	13.72	16.76
1.5 - 2.0	8.81	6.20	5.70	5.68	3.98	4.36	6.47	6.47	7.52
2.0 - 2.5	3.31	2.97	1.99	1.72	1.06	2.77	3.30	2.51	2.24
2.5 - 3.0	0.97	0.91	1.19	-0-	1.20	0.93	1.24	1.05	1.32
3.0 - 4.0	0.26	1.16	0.79	0.39	0.66	1.71	0.82	1.06	0.66
4.0 - 5.0	0.00994	0.26	0.14	-0-	0.40	0.27	0.14	0.13	0.13
> 5.0	0.00006	-0-	0.26	0.53	0.40	0.13	-0-	0.13	-0-
Total	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
Number of Observations		774	755	757	753	757	727	758	758

Bull and Bear Market Characteristics. Information regarding the symmetry of market returns for "bull" and "bear" markets is contained in Table 8. In three of the fourteen "bull" market periods, the percentage of returns falling above the mean is significantly higher than fifty percent; and, in one case, the percentage of returns falling below the mean is significantly higher than fifty percent. Similarly, for three out of the thirteen "bear" markets, the percentage of returns falling above the mean is significantly higher than fifty percent.

In ten out of fourteen "bull" market periods, the median return was greater than the mean. But, on average each period's median return differs from its mean by only 0.28 standard deviations. The median return is greater than the mean for eight out of thirteen "bear" market periods. The average difference between each period's mean and median return is again slight--only .055 standard deviations.

Seven out of fourteen "bull" market periods show some degree of negative skewness, while during "bear" market periods there is negative skewness during nine of thirteen periods. With the exception of one "bull" market period, every "bull" and "bear" market period exhibits a positive Kurtosis statistic.

A comparison of the distribution of actual market returns to the expected distribution of returns for a normal distribution during bull and bear markets is contained in Table 9. The results in Table 9 are consistent with the results reported previously in Table 7. Specifically, the actual distribution of returns during bull and bear markets exhibited a clear shortage of mid-size returns, and a surplus of extremely small and large returns, relative to what is expected in a normal distribution.

TABLE 8(a)

RETURN DISTRIBUTION SYMMETRY--BULL MARKETS

	DAILY MARKET RETURNS													
	ALL	1/28-8/29	6/32-1/34	3/35-1/37	4/3[illegible]-9/3[illegible]	4/42-4/46	6/49-12/52	9/53-6/56	12/57-6/59	10/60-11/61	10/62-12/65	10/66-11/68	5/70-12/72	12/74-12/75
entage of rns Below Mean	50.10%	*46.[illegible]5%	*54.71%	49.05%	51.76%	*46.36%	49.09%	*46.91%	46.62%	49.66%	49.94%	49.52%	48.89%	50.36%
entage of rns Above Mean	49.90%	*53.35%	*45.29%	50.95	48.24%	*53.64%	50.91%	*53.09%	53.38%	50.34%	50.06%	50.48%	51.11%	49.64%
Return	.00093	.00122	.00230	.00128	.00109	.00073	.00065	.00101	.00088	.00100	.00062	.00068	.00057	.00098
an Return	.00092	.00193	-0-	.00143	.00077	.00098	.00081	.00146	.00114	.00110	.00064	.00073	.00068	.00093
dard ation [a]	.01089	.00948	.02931	.01025	.01731	.00710	.00676	.00743	.00775	.00521	.00498	.00585	.00720	.01009
ness [b]	.80358	-.70334	.53997	-.41990	.51424	-.73617	-1.0162	-1.1816	-.34045	-.20732	.43702	.18983	.50840	.11245
osis [c]	19.715	1.8703	2.8031	.91514	2.4262	4.2569	6.1266	10.519	18.033	.69926	9.3617	2.0527	4.8449	-.14847
er of rvations	7910	493	488	579	454	1210	994	712	399	292	819	521	675	274

*Significantly different from 50 percent at the .05 confidence level.

[a] Calculated with (N-1) degrees of freedom.

[b] Normalized 3rd moment about the mean [i.e., $\Sigma(R_m-\bar{R}_m)^3 Ns_m^3$].

[c] Normalized 4th moment about the mean minus three [i.e., $(\Sigma(R_m-\bar{R}_m)^4/Ns_m^4)-3$].

TABLE 8(b)

RETURN DISTRIBUTION SYMMETRY--BEAR MARKETS

	Daily Market Returns												
	ALL	9/29-5/32	2/34-2/35	2/37-3/38	10/39-3/42	5/46-5/49	1/53-8/53	7/56-11/57	7/59-9/60	12/61-9/62	1/66-9/66	12/68-4/70	1/73-11/74
Percentage of Returns Below The Mean	*47.59%	49.20%	51.08%	46.84%	*47.47%	*47.13%	50.30%	50.00%	50.16%	46.41%	48.42%	49.00%	52.89%
Percentage of Returns Above The Mean	*52.41%	50.20%	48.92%	53.16%	*52.53%	*52.87%	49.70%	50.00%	49.84%	53.59%	51.58%	51.00%	47.11%
Mean Return	-.00093	-.00214	-.00066	-.00194	-.00059	-.00027	-.00076	-.00030	-.00026	-.00108	-.00097	-.00079	-.00101
Median Return	-.00053	-.00165	-.00105	-.00111	-0-	-0-	.00083	-.00033	-.00034	-.00050	-.00035	-.00065	-.00169
Standard Deviation [a]	.01341	.02285	.01393	.01913	.01030	.00950	.00575	.00799	.00632	.01031	.00720	.00656	.01195
Skewness [b]	-.00995	.37020	-.27761	-.28487	-.88295	-.94125	-.78847	.39418	-.10148	-.76212	-.20492	.19980	.45233
Kurtosis [c]	11.006	4.8578	2.8898	2.8401	10.564	6.4819	1.6337	3.6460	.51672	9.9034	.92349	.35030	1.0410
Number of Observations	5184	815	323	348	750	872	169	356	317	209	190	351	484

* Significantly different from 50 percent at the .05 confidence level.

[a] Calculated with (N-1) degrees of freedom.

[b] Normalized 3rd moment about the mean [i.e., $\Sigma(R_m-\bar{R}_m)^3/Ns_m^3$].

[c] Normalized 4th moment about the mean minus three [i.e., $(\Sigma(R_m-\bar{R}_m)^4/Ns_m^4)-3$].

TABLE 9(a)

COMPARISON OF RETURN DISTRIBUTION WITH NORMAL DISTRIBUTION--BULL MARKETS

Intervals In Standard Deviations	Normal Distribution	Proportion of Observations													
		Daily Market Returns--(1928-75)													
		All	1/28-6/29	'32-1/34	3/35-1/37	4/38-9/39	4/42-4/46	6/49-12/52	9/53-6/55	12/57-6/59	10/60-11/61	10/62-12/65	10/66-11/68	5/70-12/72	12/74-12/75
0.0 - 0.5	36.30%	[illegible]	46.[illegible]	[illegible]	[illegible]	45.81%	51.74%	47.69%	45.37%	49.37%	42.81%	49.21%	43.57%	49.04%	36.86%
0.5 - 1.0	29.9[illegible]	24.01	25.36	24.1[illegible]	32.93	27.54	[illegible]	28.47	31.60	34.34	27.74	28.57	30.33	27.40	31.75
1.0 - 1.5	18.38	3.71	16.2[illegible]	13.12	15.[illegible]7	14.09	12.48	12.[illegible]7	13.20	10.5[illegible]	14.72	13.31	15.54	12.45	17.5[illegible]
1.5 - 2.0	8.61	7.0[illegible]	7.30	6.35	5.53	7.49	5.12	6.0[illegible]	5.62	3.50	10.62	4.[illegible]	4.50	6.07	[illegible]12
2.0 - 2.5	3.31	[illegible]	1.62	2.09	3.52	2.65	2.89	3.12	1.68	[illegible]	2.74	1.96	3.07	2.8[illegible]	3.55
2.5 - 3.0	0.97	1.12	1.22	1.44	1.21	1.54	1.[illegible]3	1.20	1.41	-0-	1.03	1.46	1.54	0.74	[illegible]
3.0 - 4.0	0.25	0.90	1.21	0.82	1.04	0.66	0.58	0.61	0.70	0.25	-0-	0.37	0.58	1.15	0.36
4.0 - 5.0	0.00[illegible]	0.52	0.41	-0-	-0-	-0-	0.16	0.20	0.28	-0-	0.34	0.24	0.38	0.15	-0-
> 5.0	0.00005	0.49	-0-	0.20	-0-	0.22	0.25	0.20	0.14	1.00	-0-	0.49	-0-	0.15	-
Total	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
Number of Observations		7910	493	488	579	454	1210	954	712	399	292	819	521	675	[illegible]

TABLE 9(b)

COMPARISON OF RETURN DISTRIBUTION WITH NORMAL DISTRIBUTION--BEAR MARKETS

Intervals In Standard Deviations	Normal Distribution	Proportion of Observations												
		Daily Market Returns--(1928-75)												
		ALL	9/29-5/32	2/34-2/35	2/37-3/38	10/39-3/42	5/46-5/49	1/53-8/53	7/56-11/57	7/59-9/60	12/61-9/62	1/66-9/66	12/68-4/70	1/73-11/74
0.0 - 0.5	38.30%	56.66%	48.96%	48.92%	50.86%	58.13%	52.41%	50.89%	48.31%	44.79%	51.20%	45.26%	39.60%	42.98%
0.5 - 1.0	29.96	24.32	28.34	23.53	27.30	23.20	25.34	27.81	26.13	25.87	32.05	29.48	29.06	27.68
1.0 - 1.5	18.38	9.76	13.37	14.24	10.63	10.00	12.39	15.97	13.48	15.46	8.62	11.05	19.94	17.77
1.5 - 2.0	8.81	3.97	4.42	8.67	4.89	4.67	5.04	2.96	7.02	9.46	3.82	7.37	6.56	7.02
2.0 - 2.5	3.31	2.42	2.58	3.40	2.58	1.60	1.84	0.59	2.53	2.84	1.44	4.21	2.85	2.28
2.5 - 3.0	0.97	1.02	0.37	0.62	2.88	0.53	0.80	1.78	1.13	0.95	0.96	1.58	1.14	1.24
3.0 - 4.0	0.26	1.06	1.22	0.31	0.29	0.67	1.49	-0-	1.12	0.63	0.95	1.05	0.85	1.03
4.0 - 5.0	0.00934	0.40	0.37	-0-	0.57	0.67	0.46	-0-	-0-	-0-	0.48	-0-	-0-	-0-
> 5.0	0.00006	0.39	0.37	0.31	-0-	0.53	0.23	-0-	0.28	-0-	0.48	-0-	-0-	-0-
Total	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100,00%	100.00%	100.00%
Number of Observations		5184	815	323	348	750	872	169	356	317	209	190	351	484

In summary, the analysis of the return distributions during alternative market periods ("bull" and "bear" markets) indicated deviations from normality that were very similar to the deviations observed during the annual intervals. In both cases, the distributions were not symmetric and also indicated leptokurtic characteristics.

The Modified Kolmorgorov-Smirnov (K-S) Test for Normality[25]

While an analysis of the information contained in Tables 6 through 9 reveals a number of non-normal aspects to the market return distribution(s), final judgments on normality should rest on statistical testing. The K-S test allows us to check for normality. It determines whether a set of sample values can reasonably be thought to have come from a population having a given theoretical distribution--in this case, a normal distribution. A major disadvantage of the K-S test, however, is that it does not allow us to estimate any of the parameters from the sample data. The population parameters must be specified in advance of testing. Lilliefors has modified the K-S test, however, to allow us to make use of the sample mean and variance.[26] We should want to test...

> H_0: The sample has been drawn from a normal population,

against the alternative hypothesis...

> H_1: The sample has been drawn from a population that is not normal.

---

[25] For a description of the K-S one-sample test and Lilliefors modifications see the Appendix.

[26] Hubert W. Lilliefors, "On the Kolmorgorov-Smirnov Test for Normality with Mean and Variance Unknown," Journal of the American Statistical Association, Vol. 62, No. 318 (June, 1967), pp. 399-402.

The results from tests for normality are presented in Tables 10 and 11. The null hypothesis, that the sample of daily market returns has been drawn from a normal population, is consistently rejected for all but a very few periods.

These results are consistent with the summary information provided in Tables 6 through 9. In general, the return distributions for the various time periods cannot be assumed to have come from normal populations. This finding has import for much of the subsequent analysis. For example, in the next section we consider volatility measures able to deal with fat-tailed non-normal distributions, and also attempt to find a non-parametric test for changes in the level of market volatility because of this non-normality finding.

CHANGES IN THE LEVEL OF MARKET VOLATILITY

As discussed previously, everyone agrees that there was a change in volatility during the 1930's compared to periods before and after. In contrast, there is a difference of opinion regarding the market's most recent level of volatility. Logue reported no change in volatility during the period 1958-1974, while Leuthold contended that the market was more volatile during the period 1973-75. Such differences in results could be caused by the alternative measures of volatility or a difference in the time interval used--i.e., Logue considered monthly data, while Leuthold examined daily price changes. A prior discussion has indicated the problems with using only two observations during a month to measure volatility, while it also seems inappropriate to concentrate on individual large price changes.

TABLE 10(a)

RESULTS FROM K-S TEST FOR NORMALITY--MULTI-YEAR GROUPINGS[a,b]

H_0: The sample of daily market returns has been drawn from a normal population.

H_1: The sample of daily market returns has been drawn from a population that is not normal.

Data \ Periods	1928-51
$D(N)^{1/2}$	8.692
N	7055

Data \ Periods	1928-39	1940-51
$D(N)^{1/2}$	4.835	5.532
N	3573	3482

Data \ Periods	1928-33	1934-39	1940-45	1946-51
$D(N)^{1/2}$	3.854	2.648	4.430	3.543
N	1770	1803	1789	1693

Data \ Periods	1928-30	1931-33	1934-36	1937-39	1940-42	1943-45	1946-48	1949-51
$D(N)^{1/2}$	3.095	1.875	1.637	1.989	3.268	2.707	2.858	1.932
N	883	887	903	900	904	885	847	846

[a]The null hypothesis is rejected at the .01 confidence level for all time periods.

[b]The "studentized range" test was also applied to all periods containing less than one-thousand observations. Results from this test were consistent with results provided by the K-S test.

N = Number of daily market returns in the period.

D = Maximum $|F^*(x) - S_n(x)|$ (see appendix).

TABLE 10(b)

RESULTS FROM K-S TEST FOR NORMALITY--MULTI-YEAR GROUPINGS[a,b]

H_0: The sample of daily market returns has been drawn from a normal population.

H_1: The sample of daily market returns has been drawn from a population that is <u>not</u> normal.

Data \ Periods	1952-75
$D(N)^{1/2}$	4.899
N	6039

Data \ Periods	1952-63	1964-75
$D(N)^{1/2}$	3.[illegible]85	3.583
N	3039	3000

Data \ Periods	1952-57	1958-63	1964-69	1970-75
$D(N)^{1/2}$	2.329	2.791	2.154	2.122
N	1529	1510	1484	1516

Data \ Periods	1952-54	1955-57	1958-60	1961-63	1964-66	1967-69	1970-72	1973-75
$D(N)^{1/2}$	1.249	1.497	1.517	2.565	2.235	*.961	1.691	**.830
N	774	755	757	753	757	727	758	758

[a]The null hypothesis is rejected at the .01 confidence level for all time periods unless otherwise indicated.

[b]The "studentized range" test was also applied to all periods containing less than one-thousand observations. Results from this test were consistent with results provided by the K-S test.

*The null hypothesis is rejected at the .05 confidence level.

**The null hypothesis is accepted at the .05 confidence level.

N = Number of daily market returns in the period.

D = Maximum $|F^*(x) - S_n(x)|$ (see appendix).

TABLE 11

RESULTS FROM K-S TEST FOR NORMALITY--BULL AND BEAR MARKETS[a,b]

H_0: The sample of daily market returns has been drawn from a normal population.

H_1: The sample of daily market returns has been drawn from a population that is not normal.

PERIODS		DATA	
Bull Markets	Bear Markets	$D(N)^{1/2}$	N
1/28-8/29		1.564	493
	9/29-5/32	2.176	815
6/32-1/34		1.555	488
	2/34-2/35	1.308	323
3/35-1/37		1.001*	579
	2/37-3/38	1.579	348
4/28-9/39		1.223	454
	10/39-3/42	2.994	750
4/42-4/46		3.268	1210
	5/46-5/49	2.969	872
6/49-12/52		2.120	994
	1/53-8/53	2.569	169
9/53-6/56		1.608	712
	7/56-11/57	1.209	356
12/57-6/59		9.081	399
	7/59-9/60	0.897*	317
10/60-11/61		0.781**	292
	12/61-9/62	1.658	209
10/62-12/65		2.123	819
	1/66-9/66	0.958*	190
10/66-11/68		0.909*	521
	12/68-4/70	0.468**	351
5/70-12/72		1.689	675
	1/73-11/74	0.947*	484
12/74-12/75		0.513**	274
	ALL	6.979	5184
ALL		9.856	7910

[a]The null hypothesis is rejected at the .01 confidence level for all time periods unless otherwise indicated.

[b]The "studentized range" test was also applied to all periods containing less than one-thousand observations. Results from this test were generally consistent with results provided by the K-S test.

*The null hypothesis is rejected at the .05 confidence level.

**The null hypothesis is accepted at the .05 confidence level.

N = Number of daily market returns in the period.

D = Maximum $|F^*(x) - S_n(x)|$ (see appendix).

A reexamination of market return volatility, based on fluctuations in daily market returns, should confirm the marked difference between pre- and post-WWII levels of volatility. More important, such an analysis should help resolve the conflicting opinions concerning market volatility in the 1970s.

Past statements about changes in the level of market volatility have been based largely on visual inspection of the time series of some variability measure. Although visual inspection is helpful in identifying distinctly different volatility patterns, modest shifts in volatility levels may be overlooked. To discover modest changes in volatility levels and to confirm visual impressions, it is necessary to subject the market volatility time series to statistical testing.

Alternative Market Volatility Measures

There are a number of volatility measures that provide a quantitative appraisal of the dispersion (or variability) within a distribution. Several alternative volatility measures are described in this section. Subsequently these measures are calculated from the return series data. Correlation analyses are performed on the alternative measures for the entire period (and subperiods) to see whether these volatility measures generate comparable results. Based upon the correlation results and the analysis of the characteristics of the market return distribution(s), one measure is selected as a proxy for market return volatility and used in further analysis.

The alternative volatility measures selected for study are as follows:

(a) Standard Deviation (SD)

(b) Semistandard Deviation (SSD)

(c) Mean Absolute Deviation (MAD)

(i) about the Mean (MAD1)

(ii) about the Median (MAD2)

(d) Interquartile Range (IQR)

Standard Deviation (SD)

The standard deviation is the most common of all statistical measures of variability. It is a measure of the spread or dispersion of a series around its mean. Notably, if the return distribution is normal, the SD provides a means of estimating the percentages of observations included within given distances from the mean (e.g., approximately 68 percent of the observations fall within $\bar{R}_m \pm SD$).

Semistandard Deviation (SSD)[27]

The semistandard deviation of market returns (SSD) takes the form...

$$SSD = \frac{\sum_{t=1}^{n} (R^*_{mt} - \bar{R}_m)^2}{n},$$

[27] This measure was employed by Markowitz. Harry M. Markowitz, Portfolio Selection: Efficient Diversification of Investments (New York: John Wiley & Sons, Inc., 1959), Chapter 9. Although the most popular specification of this formula is in terms of deviations from the mean, it is also possible to specify the measure in terms of deviations from other values of interest such as the median, the risk-free rate of return, or simply zero.

where R^*_{mt} equals R_{mt} for $R_{mt} < \overline{R}_m$, and equals $\overline{R}_m$ for $R_{mt} \geq \overline{R}_m$. The semi-standard deviation is a special case of the standard deviation of market returns. Rather than trying to measure the dispersion of the entire distribution, this statistic focuses on the portion of the distribution lying below $\overline{R}_m$. Thus, this statistic can be viewed as a measure of "adverse return" variability, with "adverse return" being defined as any return below the average.

Mean Absolute Deviation (MAD)

One way of determining the dispersion of a series of observations about a given point is to calculate the average distance (ignoring signs) of the observations from the given point. The smaller the average distance about this point, the smaller the dispersion of the observations. The mean absolute deviation of the market return (MAD) is usually defined as...[28]

$$MAD1 = (1/n) \sum_{t=1}^{n} |R_{mt} - \overline{R}_r|,$$

where $|R_{mt} - \overline{R}_m|$ denotes the absolute value of the deviation from the mean,

-OR-

$$MAD2 = (1/r) \sum_{t=1}^{n} |R_{mt} - mdnR_m|,$$

---

[28] The mean absolute deviation was named as a recommended measure of dispersion by the Bank Administration Institute. Bank Administration Institute, Measuring the Investment Performance of Pension Funds for the Purpose of Inter-Fund Comparison (Park Ridge, Illinois: Bank Administration Institute, 1968), p. 30.

where $|R_{mt} - mdnR_m|$ denotes the absolute value of the deviation from the median.[29]

The MAD is, thus, the average distance of returns from a measure of central tendency (e.g., the mean or median). This statistic, like the standard deviation of market returns (SD), considers every obvservation in the return series. However, the MAD does not give the added importance to large deviations that the SD does. Specifically, because the SD squares deviations from the mean, it gives more weight to large deviations than to small ones. Therefore, the SD changes dramatically whenever deviations occur. However, the MAD accords less importance to large deviations than the SD and is, therefore, less erratic (i.e., has less sampling error).[30]

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[29]Sharpe states: "The choice of a measure of dispersion generally implies the use of a corresponding measure of central tendency. For example, if the standard deviation is considered an appropriate measure of dispersion, the arithmetic mean (average) is usually chosen to measure central tendency, since the standard deviation around the mean is less than around any other value. In other words, the mean minimizes the sum of the squared deviations. On the other hand, if the mean absolute deviation is considered an appropriate measure of dispersion, the median is usually chosen to measure central tendency, since the mean absolute deviation around the median is less than that around any other value. In other words, the median minimizes the sum of the absolute deviations. (That is, in fact, the appropriate definition of the median.)" William F. Sharpe, "Mean-Absolute-Deviation Characteristic Lines for Securities and Portfolios," Management Science, Vol. 18, No. 2 (October, 1971), pp. (B-1)-(B-2).

[30]For a comparison of the standard deviation to the mean absolute deviation calculated from samples of increasing size see Fama, "The Behavior of Stock Market Prices," p. 96.

Interquartile Range (IQR)[31]

The interquartile range of market returns (IQR) is another possible variability measure. It is defined as...

$$IQR = (Q_3 - Q_1),$$

where Q_3 is the third quartile and Q_1 is the first quartile.[32] The IQR is, therefore, equal to the range encompassed by the central fifty percent of the return distribution. The IQR is especially useful as a

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[31]Fisher and Lorie applied this measure to security as well as market returns. Fisher and Lorie, "Some Studies of Variability of Returns on Investments in Common Stocks," pp. 99-134.

[32]Quartiles divide a distribution into four "equal" parts; therefore, there are three quartiles, usually designated, Q_1, Q_2, and Q_3. The second quartile, Q_2, divides the distribution in half and is, thus, the same as the median. The first quartile, Q_1, is the value at or below which one-fourth of all the items in the series fall; the third quartile, Q_3, is the value at or below which three-fourths of the items lie.

With ungrouped "ordered" (i.e., low to high) data, all quartiles assume either the value of one of the items or the value halfway between two items. The following rules establish the values for quartiles:

(1) If n/4 is an integer, Q_1, has the value halfway between the n/4th observation and the next observation; if n/4 is not an integer, Q_1, has the value of the observation whose position corresponds to the next higher integer.

(2) If n/2 is an integer, Q_2, has the value halfway between the n/2th observation and the next observation; if n/2 is not an integer, Q_2, has the value of the observation whose position corresponds to the next higher integer.

(3) If 3n/4 is an integer, Q_3, has the value halfway between the 3n/4th observation and the next observation; if 3n/4 is not an integer, Q_3, has the value of the observation whose position corresponds to the next higher integer.

The above-listed decision rules used to determine the values for quartiles are found in: Anderson, T. W., and Sclove, Stanley L., Introductory Statistical Analysis, (Boston: Houghton Mifflin Company, 1974), pp. 72-75; Chao, Lincoln L., Statistics: Methods and Analyses, (New York: McGraw-Hill Book Company, 1969), p. 90; and Richmond, Samuel B., Principles of Statistical Analysis, (New York: The Ronald Press Company, 1957), pp. 181 and 184.

dispersion measure with open ended distributions because it is not sensitive to extreme values, but is still able to indicate shifts in the relative number of large daily returns.

Relative Market Volatility Measures

The volatility measures just described are all absolute measures of dispersion. For comparing dispersions of distributions having distinctly different means, relative measures of dispersion are often suggested. We need only to divide each of our absolute measures of dispersion by an appropriate measure of central tendency to create a relative measure of dispersion. For example, the SD divided by $\overline{R}_m$ produces the coefficient of variation of market returns.

Although there is an advantage to calculating relative measures of dispersion when samples being compared have widely different means (or medians), it is unlikely that samples of daily market returns would possess such widely different values of central tendency. Also, our measures are somewhat normalized already because we have been dealing with rates of return rather than absolute dollar returns. Therefore, absolute measures of dispersion should prove adequate for studying daily market return volatility. In addition, when dealing with daily returns, a sample's mean (or median) return will probably be close to zero, or zero. Relative dispersion measures calculated for such samples would produce extremely large and/or infinite values which would be useless. Therefore, because absolute measures of dispersion are felt to be adequate for capturing daily return volatility and because relative measures of dispersion would result in division by zero in many cases, only absolute measures of dispersion are considered.

Time Series Plots of Volatility Measures

Figures 2(a) through 2(e) provide time-series plots for all monthly volatility measures, calculated from daily percent changes in the SP500 Index. All five plots appear very much alike. Time-series plots for quarterly and half-yearly measures also show similar patterns. Therefore, for illustrative purposes, only the MAD2 and IQR quarterly and half-yearly measures are presented in Figures 2(f) through 2(i). The twin peaks of high volatility for the depression years (1931-33 and 1938-39) stand out in every figure. The relatively low level of post-WWII volatility "appears" to be broken only during the period 1973-75.

Correlation Analysis of Volatility Measures

The pattern of similarity shown among the time-series plots for the various measures of volatility is verified by correlation analysis. Table 12 lists the various correlation analyses that were performed. Tables 13 and 14 present the results from these analyses. All the measures of market return volatility are highly correlated over time, regardless of the time period selected. All correlations are significant at the .01 confidence level.

Tests for Changes in Market Volatility

F Test for Variance Differences. The F test might be appropriate for testing whether the market return variances for two subperiods were equal. However, this test, concerning population variances, is "...strictly true only for normal parent populations."[33] Since, for

[33] Jan Kmenta, Elements of Econometrics (New York: The Macmillan Company, 1971), p. 148.

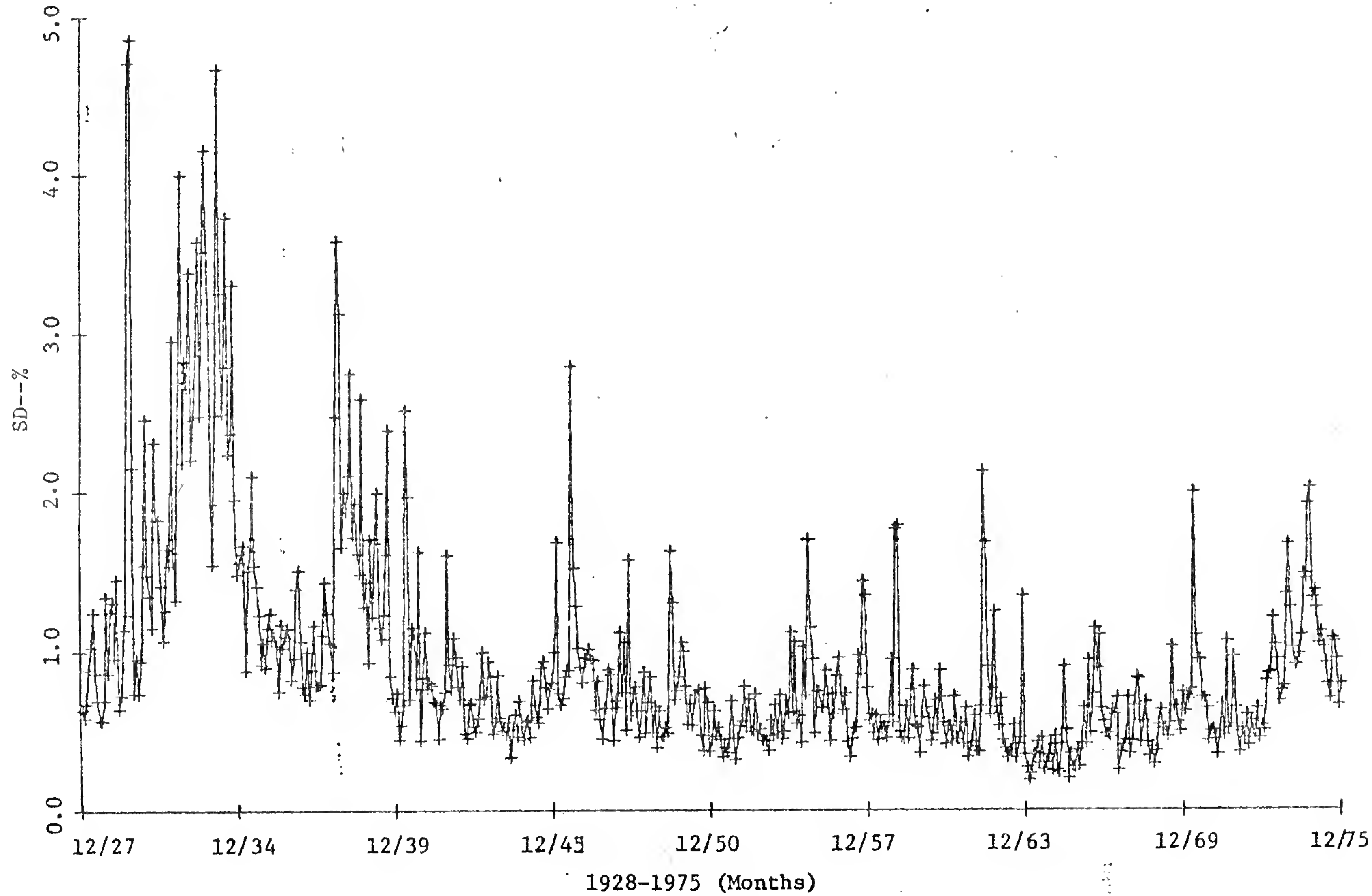

FIGURE 2(a): SD Over Time (by Months)

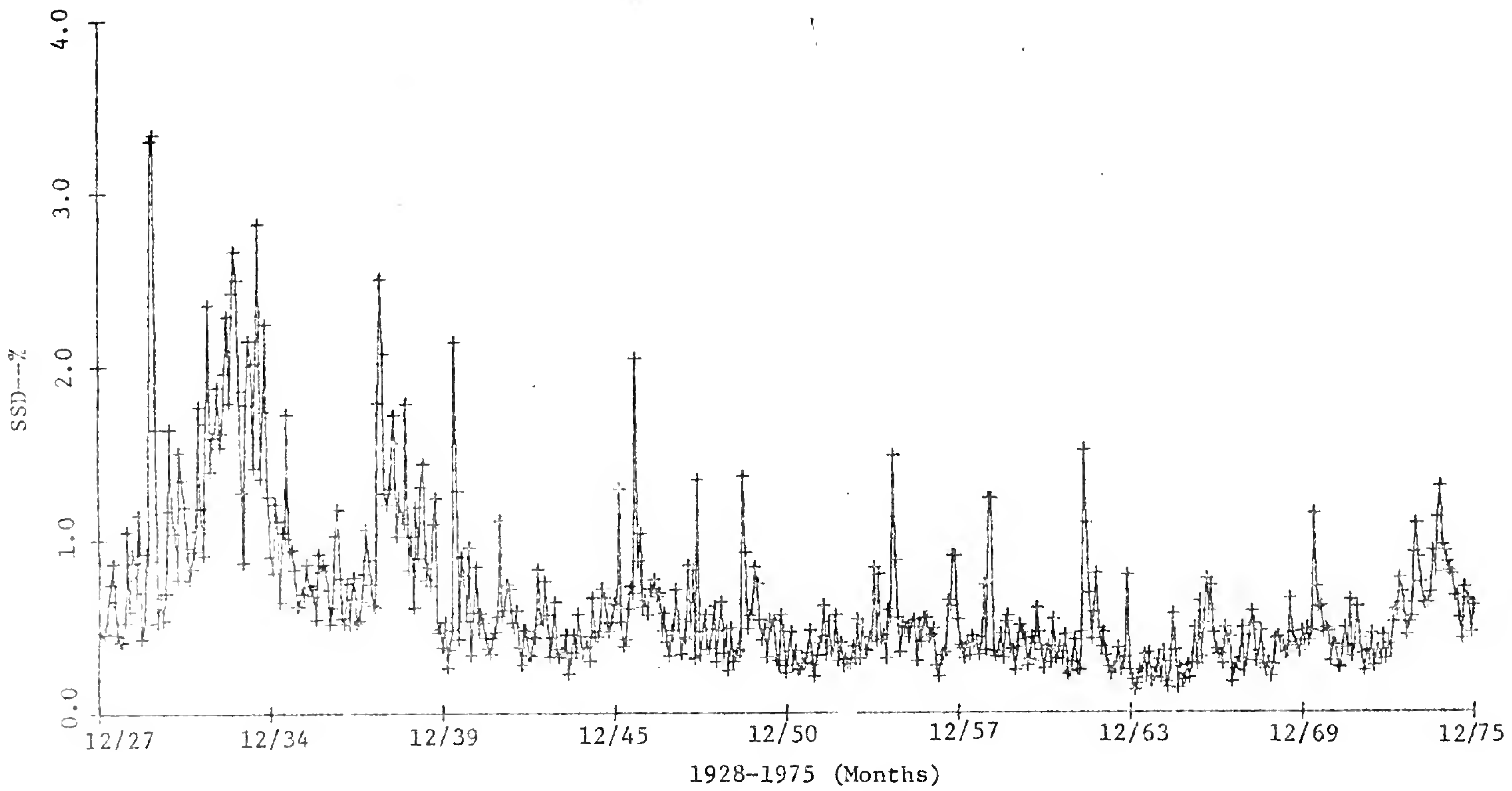

FIGURE 2(b): SSD Over Time (by Months)

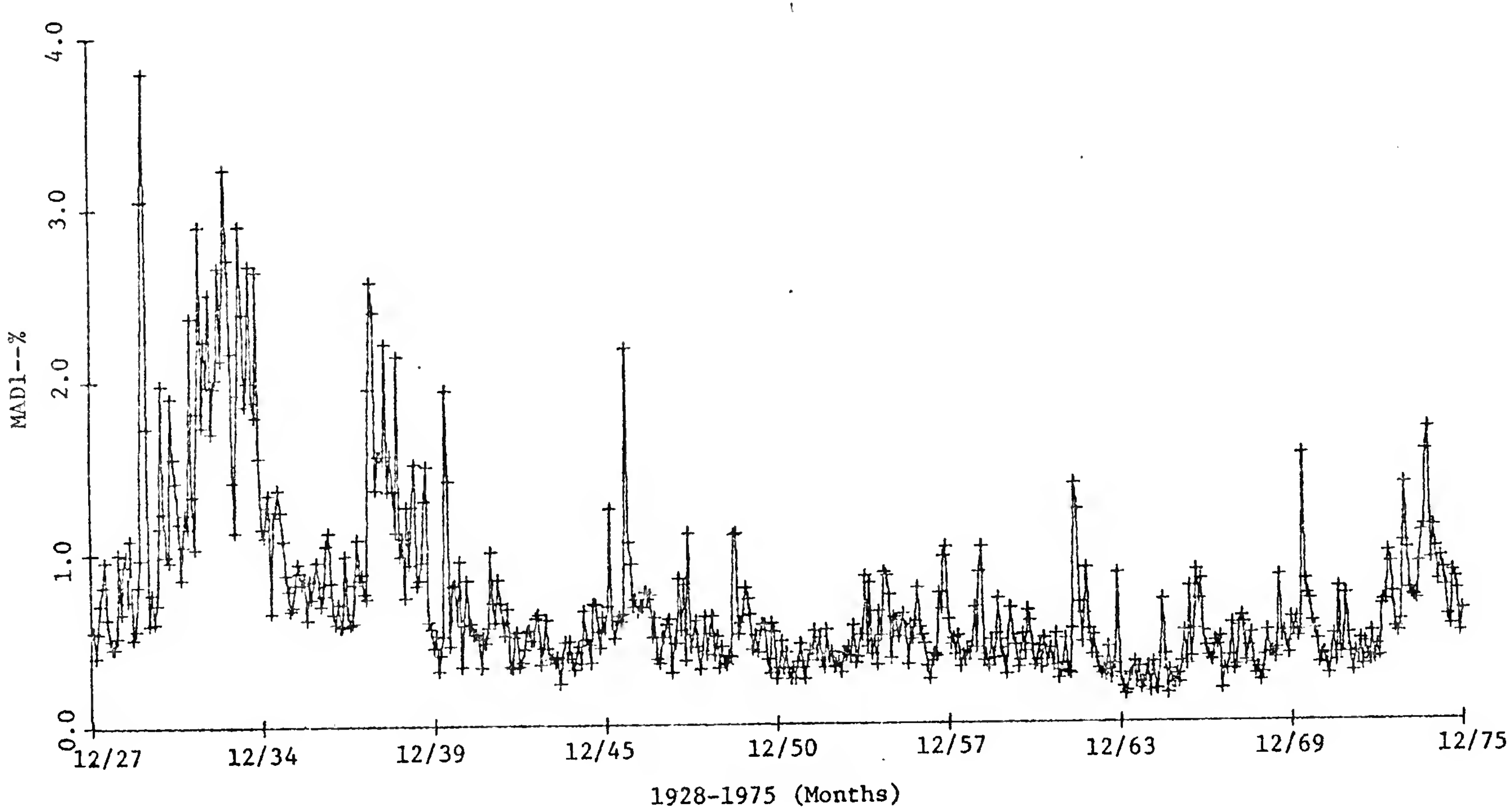

FIGURE 2(c): MAD1 Over Time (by Months)

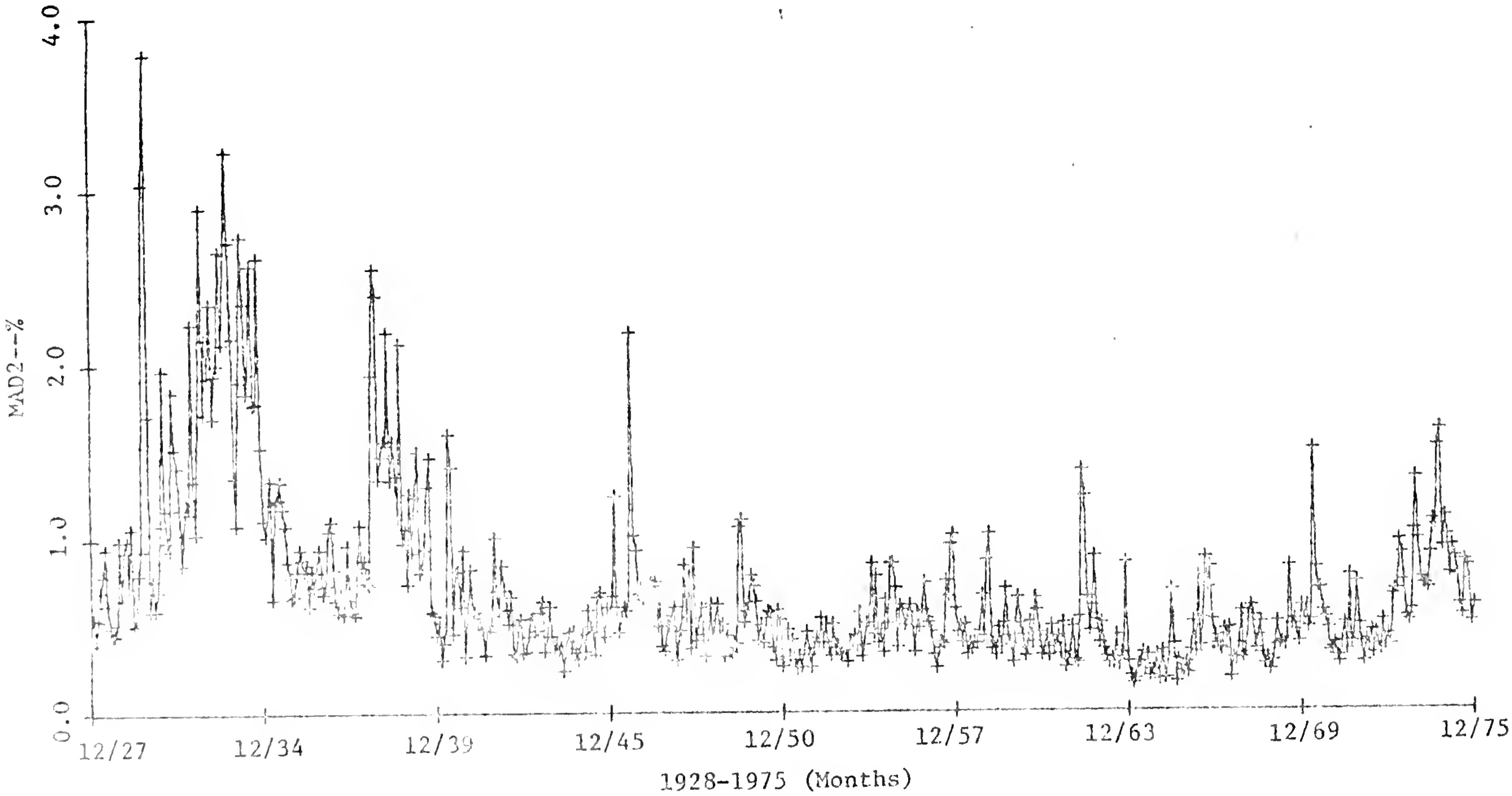

FIGURE 2(d): MAD2 Over Time (by Months)

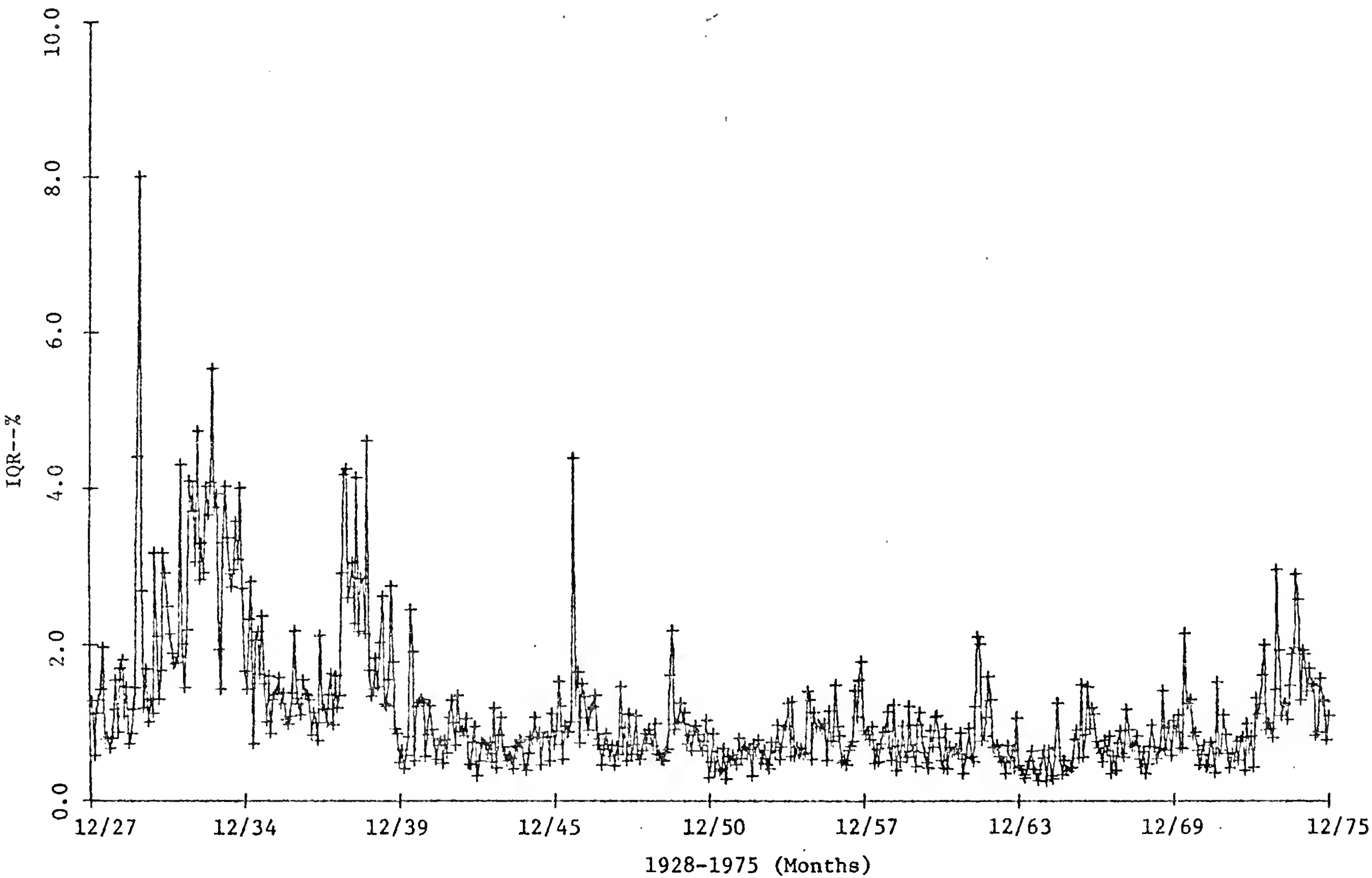

FIGURE 2(e): IQR Over Time (by Months)

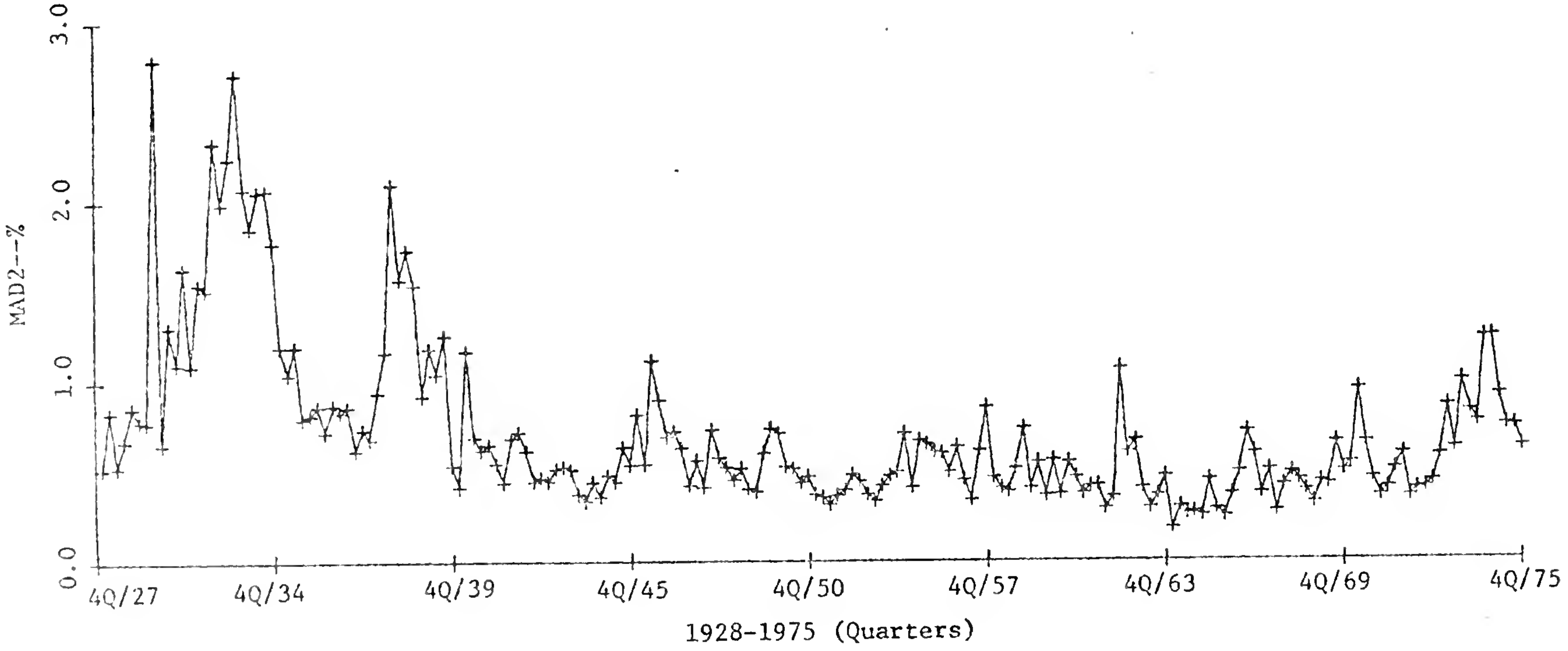

FIGURE 2(f):MAD2 Over Time (by Quarters)

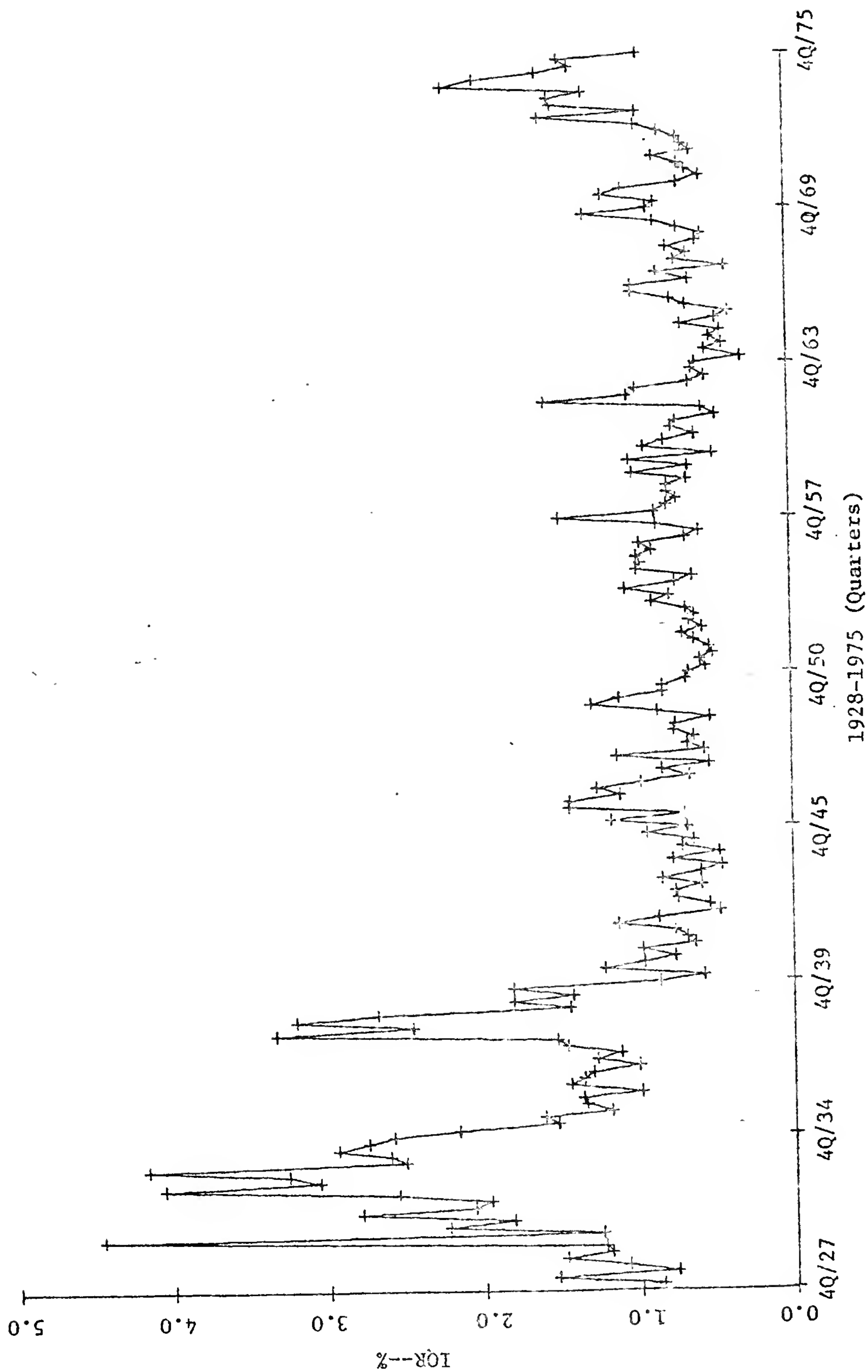

FIGURE 2(g): IQR Over Time (by Quarters)

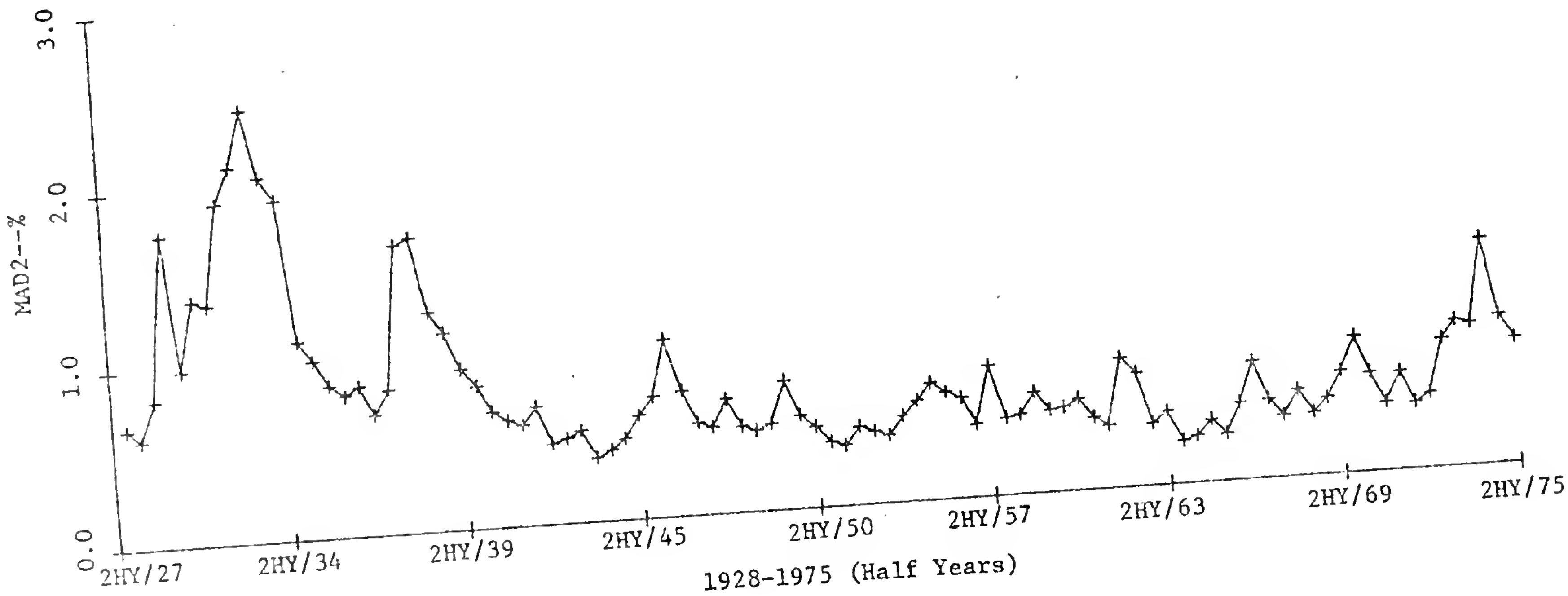

FIGURE 2(h): MAD2 Over Time (by Half Years)

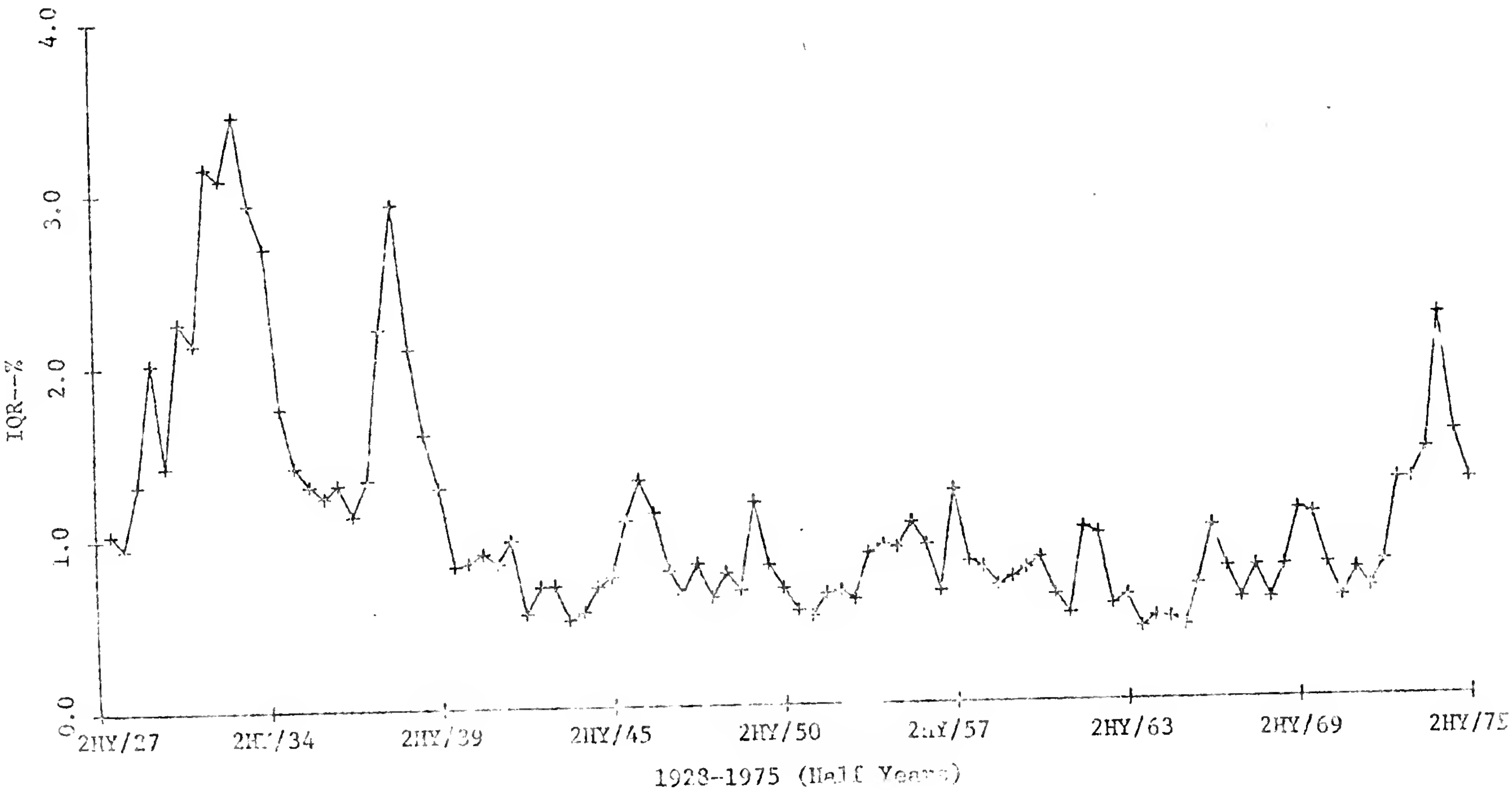

FIGURE 2(i): IQR Over Time (Half Years)

TABLE 12

LISTING OF TWENTY-THREE CORRELATION MATRICES
CALCULATED IN STUDYING THE RELATIONSHIPS BETWEEN ALTERNATIVE
MEASURES OF MARKET RETURN VOLATILITY[a]

Correlation Coefficients Between Alternative Measures of Market Return Volatility For the Period ...

Monthly Measures

1) 1928-75 (576 Months);

2) 1928-51 (288 Months);
3) 1952-75 (288 Months);

4) 1928-39 (144 Months);
5) 1940-51 (144 Months);
6) 1952-63 (144 Months);
7) 1964-75 (144 Months);

8) Bull Markets (350 Months);
9) Bear Markets (226 Months);

Quarterly Measures

10) 1928-75 (192 Quarters);

11) 1928-51 (96 Quarters);
12) 1952-75 (96 Quarters);

13) 1928-39 (48 Quarters);
14) 1940-51 (48 Quarters);
15) 1952-63 (48 Quarters);
16) 1964-75 (48 Quarters);

Half-yearly Measures

17) 1928-75 (96 Half-years);

18) 1928-51 (48 Half-years);
19) 1952-75 (48 Half-years);

20) 1928-39 (24 Half-years);
21) 1940-51 (24 Half-years);
22) 1952-63 (24 Half-years);
23) 1964-75 (24 Half-years);

[a]Monthly, quarterly, and half-yearly volatility measures are calculated from daily percent changes in the SP500.

TABLE 14(a)

PEARSON PRODUCT MOMENT (r) AND SPEARMAN RANK (rs) CORRELATIONS BETWEEN ALTERNATIVE MEASURES OF MARKET RETURN VOLATILITY FOR SEVEN MULTI-YEAR PERIODS[a,b,c]

		SD		SSD		MAD1		MAD2	
		r	rs	r	rs	r	rs	r	rs
	a	.983	.986						
	b	.980	.984						
	c	.981	.982						
SSD	d	.975	.981						
	e	.975	.960						
	f	.977	.975						
	g	.988	.987						
	a	.991	.990	.976	.975				
	b	.991	.990	.976	.973				
	c	.975	.985	.942	.968				
MAD1	d	.989	.992	.972	.975				
	e	.984	.976	.964	.936				
	f	.964	.975	.926	.949				
	g	.994	.991	.982	.982				
	a	.990	.989	.975	.974	.999	.999		
	b	.990	.989	.975	.972	.999	.999		
	c	.976	.984	.943	.966	.999	.999		
MAD2	d	.987	.991	.974	.975	.999	.999		
	e	.979	.975	.948	.933	.994	.998		
	f	.964	.975	.926	.945	.999	.998		
	g	.994	.991	.983	.981	.999	.999		
	a	.931	.894	.913	.880	.962	.935	.964	.936
	b	.930	.914	.913	.898	.961	.948	.963	.950
	c	.854	.847	.809	.835	.932	.903	.931	.904
IQR	d	.909	.912	.891	.892	.948	.942	.949	.945
	e	.854	.779	.831	.754	.909	.861	.924	.867
	f	.727	.737	.670	.709	.864	.831	.864	.831
	g	.941	.931	.932	.930	.961	.956	.961	.956

[a]All correlations are significant at the .01 confidence level.

[b]Monthly volatility measures are calculated from daily percent changes in the SP500 Index.

[c]KEY:
a = 1928-75 (576 Months)
b = 1928-51 (288 Months)
c = 1952-75 (288 Months)
d = 1928-39 (144 Months)
e = 1940-51 (144 Months)
f = 1952-63 (144 Months)
g = 1964-75 (144 Months)

TABLE 13

PEARSON PRODUCT MOMENT (r) AND SPEARMAN RANK (rs) CORRELATIONS BETWEEN ALTERNATIVE MEASURES OF MARKET RETURN VOLATILITY FOR ...

BULL MARKET PERIODS 1928-75 (350 MONTHS)[a,b]

	SD		SSD		MAD1		MAD2	
	r	rs	r	rs	r	rs	r	rs
SSD	.981	.985						
MAD1	.990	.989	.976	.973				
MAD2	.988	.988	.976	.970	.999	.999		
IQR	.927	.882	.910	.866	.962	.926	.964	.928

BEAR MARKET PERIODS 1928-75 (226 MONTHS)[a,b]

	SD		SSD		MAD1		MAD2	
	r	rs	r	rs	r	rs	r	rs
SSD	.985	.985						
MAD1	.991	.989	.975	.975				
MAD2	.991	.989	.974	.975	.999	.999		
IQR	.934	.905	.911	.890	.963	.944	.964	.945

[a]All correlations are significant at the .01 confidence level.

[b]Monthly volatility measures are calculated from daily percent changes in the SP500 Index.

TABLE 14(b)

PEARSON PRODUCT MOMENT (r) AND SPEARMAN RANK (rs) CORRELATIONS BETWEEN ALTERNATIVE MEASURES OF MARKET RETURN VOLATILITY FOR SEVEN MULTI-YEAR PERIODS[a,b,c]

		SD		SSD		MAD1		MAD2	
		r	rs	r	rs	r	rs	r	rs
SSD	a	.988	.991						
	b	.985	.988						
	c	.986	.988						
	d	.980	.981						
	e	.984	.979						
	f	.987	.979						
	g	.991	.992						
MAD1	a	.991	.988	.977	.977				
	b	.992	.985	.977	.970				
	c	.971	.989	.942	.977				
	d	.992	.988	.977	.972				
	e	.973	.954	.947	.938				
	f	.952	.983	.926	.967				
	g	.995	.997	.991	.990				
MAD2	a	.991	.988	.977	.976	.999	.999		
	b	.991	.985	.977	.968	.999	.999		
	c	.972	.989	.944	.977	.999	.999		
	d	.991	.938	.978	.973	.999	.999		
	e	.969	.955	.939	.934	.999	.999		
	f	.952	.983	.926	.967	.999	.999		
	g	.995	.997	.991	.990	.999	.999		
IQR	a	.947	.895	.928	.880	.975	.942	.976	.942
	b	.950	.926	.931	.906	.976	.963	.977	.963
	c	.859	.875	.818	.862	.945	.922	.943	.922
	d	.943	.943	.929	.929	.969	.969	.969	.969
	e	.754	.742	.715	.738	.862	.851	.872	.846
	f	.725	.772	.675	.752	.879	.843	.879	.845
	g	.942	.953	.945	.944	.967	.964	.966	.964

[a]All correlations are significant at the .01 confidence level.

[b]Quarterly volatility measures are calculated from daily percent changes in the SP500 Index.

[c]KEY:
a = 1928-75 (192 Quarters)
b = 1928-51 (96 Quarters)
c = 1952-75 (96 Quarters)
d = 1928-39 (48 Quarters)
e = 1940-51 (48 Quarters)
f = 1952-63 (48 Quarters)
g = 1964-75 (48 Quarters)

TABLE 14(c)

PEARSON PRODUCT MOMENT (r) AND SPEARMAN RANK (rs) CORRELATIONS BETWEEN ALTERNATIVE MEASURES OF MARKET RETURN VOLATILITY FOR SEVEN MULTI-YEAR PERIODS[a,b,c]

		SD		SSD		MAD1		MAD2	
		r	rs	r	rs	r	rs	r	rs
SSD	a	.989	.990						
	b	.986	.985						
	c	.987	.989						
	d	.979	[illegible]						
	e	.985	.978						
	f	.988	.970						
	g	.995	.987						
MAD1	a	.990	.985	.977	[illegible]				
	b	.990	[illegible]	.968	.966				
	c	.965	.974	.935	.969				
	d	.988	.988	.959	.957				
	e	.958	[illegible]	.921	[illegible]				
	f	.930	[illegible]	[illegible]	.953				
	g	.995	[illegible]	[illegible]	.990				
MAD2	a	[illegible]	[illegible]	[illegible]	.970	[illegible]	.999		
	b	.969	[illegible]	[illegible]	[illegible]	.999	.999		
	c	[illegible]	[illegible]	[illegible]	.969	.999	.999		
	d	[illegible]	[illegible]	[illegible]	[illegible]	.999	.999		
	e	[illegible]	[illegible]	[illegible]	.914	.999	.998		
	f	.929	[illegible]	[illegible]	.956	.999	.997		
	g	[illegible]	.988	[illegible]	.990	.999	.999		
IQR	a	[illegible]	[illegible]	[illegible]	[illegible]	.978	.957	.978	.958
	b	[illegible]	[illegible]	[illegible]	[illegible]	.978	.973	.979	.975
	c	[illegible]	[illegible]	[illegible]	[illegible]	.962	.952	.962	.950
	d	[illegible]	[illegible]	[illegible]	[illegible]	.966	.963	.967	.959
	e	[illegible]	[illegible]	[illegible]	.753	872	.902	.881	.892
	f	[illegible]	[illegible]	[illegible]	[illegible]	.908	.934	.908	.922
	g	[illegible]	[illegible]	[illegible]	[illegible]	.975	.952	.975	.953

[a]All correlations [illegible] at the [illegible]01 confidence level.

[b]Half-yearly [illegible] calculated from daily percent changes in the SP500 Index.

[c]KEY:
- a = 1928-75 [illegible]
- b = 1928-51 (48 Half years)
- c = [illegible]
- d = [illegible]
- e = 1940-51 [illegible]
- f = 1952-63 (24 half years)
- g = 1964-75 (24 Half years)

almost every subperiod, the normality assumption cannot be justified, it will be necessary to employ a nonparametric test.

The Siegel-Tukey Test.[34] The Siegel-Tukey test is a nonparametric procedure designed to test the null hypothesis that two independent samples come from the same population, against the alternative hypothesis that the samples come from populations differing in variability. This test does not require any assumption that the distribution(s) from which the samples were drawn is normal, or any other specific shape.

The test is sensitive to differences in variability when the "location" parameters of the populations are equal or nearly equal, but, is relatively insensitive to an alternative hypothesis when the two populations differ mainly in "location". Thus, the Siegel-Tukey test is a good procedure for judging whether two samples came from populations with different dispersions--if the "location" parameters of the populations are approximately equal. To avoid the potential problem of unequal "location" parameters, one distribution, from each pair of sample distributions tested, was always shifted until its median value was coincident with the median value of the other sample. The Siegel-Tukey test was then performed on these samples. Therefore, any differences in dispersion would not be confused with differences in "location".

The application of the Siegel-Tukey test can be more formally stated as follows:

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[34] For a complete description of the Siegel-Tukey test see Appendix B.

H_0: The market returns for two sample periods come from populations with equal dispersions.

H_1: The market returns for two sample periods come from populations with significantly different dispersions.

Significance Level = .05

Critical Value = 1.96

Test Statistic = $|Z|$ (See Appendix B)

if $|Z| \leq 1.96$, accept H_0;

if $|Z| > 1.96$, reject H_0.

Results from applying the Siegel-Tukey test to various paired periods are presented in Tables 15 and 16. Table 15 is concerned with adjacent three-year periods. Only for two pairs of periods--(1958-60 and 1961-63) and (1967-69 and 1970-72)--can the null hypothesis be accepted at the .05 confidence level. For many adjacent pairings, there appears to be a slight, yet significant, difference in dispersions.

Table 16 compares the 1973-75 period with other three-year periods. A number of important findings result from these comparisons:

1) The period 1973-75 has significantly more dispersion than every other three-year period beginning with 1940-42.

2) The 1973-75 level of dispersion is surpassed only by the twin peaks of the great depression--(1931-33) and (1937-39).

3) The 1973-75 level of dispersion is not significantly different from those levels attained in 1928-30 and 1934-36.

TABLE 15

RESULTS FROM SIEGEL-TUKEY TESTS--ADJACENT THREE-YEAR PERIODS

H_0: The market returns for two sample periods come from populations with equal dispersions.

H_1: The market returns for two sample periods come from populations with significantly different dispersions.

Periods	$\lvert Z \rvert$	Accept/Reject H_0 at the .05 Confidence Level
1928-30 and 1931-33*	13.034	Reject
1931-33* and 1934-36	14.975	Reject
1934-36 and 1937-39*	5.321	Reject
1937-39* and 1940-42	13.293	Reject
1940-42* and 1943-45	3.833	Reject
1943-45 and 1946-48*	5.833	Reject
1946-48* and 1949-51	3.104	Reject
1949-51* and 1952-54	4.465	Reject
1952-54 and 1955-57*	6.705	Reject
1955-57* and 1958-60	3.552	Reject
1958-60 and 1961-63	1.404	Accept
1961-63* and 1964-66	4.643	Reject
1964-66 and 1967-69*	5.017	Reject
1967-69 and 1970-72	1.694	Accept
1970-72 and 1973-75*	10.727	Reject

*Indicates the period having the larger dispersion for those paired samples where H_0 is rejected.

TABLE 16

RESULTS FROM SIEGEL-TUKEY TESTS--1973-75 VS. OTHER THREE-YEAR PERIODS

H_0: The market returns for two sample periods come from populations with equal dispersions.

H_1: The market returns for two sample periods come from populations with significantly different dispersions.

Periods	$\lvert Z \rvert$	Accept/Reject H_0 at the .05 Confidence level
1973-75 and 1928-30	0.395	Accept
1973-75 and 1931-33 *	13.816	Reject
1973-75 and 1934-36	0.447	Accept
1973-75 and 1937-39 *	4.770	Reject
1973-75* and 1940-42	9.336	Reject
1973-75* and 1943-45	13.173	Reject
1973-75* and 1946-48	7.208	Reject
1973-75* and 1949-51	10.845	Reject
1973-75* and 1952-54	14.492	Reject
1973-75* and 1955-57	8.290	Reject
1973-75* and 1958-60	11.754	Reject
1973-75* and 1961-63	12.458	Reject
1973-75* and 1964-66	16.228	Reject
1973-75* and 1967-69	12.713	Reject
1973-75* and 1970-72	10.727	Reject

* Indicates the period having the larger dispersion for those paired samples where H_0 is rejected.

It appears that this empirical evidence tends to confirm Leuthold's observation of a more volatile stock market in 1973-75, and casts doubt on Logue's findings of no change in market volatility during the recent period.

SUMMARY, CONCLUSIONS, AND IMPLICATIONS

Summary

The purpose of this study has been to define, measure, and study changes in the level of market volatility over time. Aggregate market volatility was defined as the *ex post* variability in market rates of return. It was pointed out that changes in the level of market volatility could influence the expected market return, the risk/return relationship of all individual securities, and an individual security's "beta."

A review of four prior studies on market volatility indicated agreement that market returns during the pre-World War II period were significantly *more* volatile than during the 1946-1970 period. There was disagreement, however, on whether aggregate stock price volatility had increased in the 1970's.

The Standard and Poor's 500 Composite Index (SP500) was selected as the proxy for the market portfolio. This is a value-weighted index, of broad coverage, that provides a historical daily listing back to 1928. In order to better capture *volatility* as opposed to *trend*, *daily* returns (calculated as percent changes in the index) were employed.

The characteristics of the market return series distribution were studied in detail, because the type of distribution affects the appropriateness of the alternative market volatility measures, and because the nature of the distribution determines which statistical tests are suitable for hypothesis testing. For the purpose of analysis, the 1928-1975 time span was broken into three-year, six-year, twelve-year, and twenty-four-year subperiods. Also, additional subperiods were formed to correspond to major "bull" and "bear" markets. Based upon studies of return distribution symmetry, comparisons of return distributions with normal distributions, and Kolmorgorov-Smirnov (K-S) tests for normality, it was concluded that return distributions for almost all time periods do <u>not</u> come from normal populations. All the return distributions exhibited signs of peakedness and fat-tails relative to normal distributions. Because of these results it was necessary to select volatility measures able to deal with fat-tailed non-normal distributions, and employ non-parametric tests for analyzing changes in the level of market volatility.

Five different volatility measures--standard deviation (SD), semi-standard deviation (SSD), mean absolute deviation about the mean (MAD1), mean absolute deviation about the median (MAD2), and interquartile range (IQR)--were employed in studying the return-series data. All five measures were calculated on monthly, quarterly, and half-yearly bases from daily percent changes in the SP500 Index. All fifteen time series plots showed twin peaks of significantly higher volatility during the depression years (1929-1939). Also, the relatively low level of post-World War II volatility "appeared" to be broken only during the period 1973-1975. Correlation analysis verified the patterns of similarity shown among time-series plots for the various volatility measures.

The Siegel-Tukey test was used to test the null hypothesis that the market returns for the two sample periods come from populations with equal dispersions, against the alternative hypothesis that the two samples come from populations with significantly different dispersions. Results from applying the Siegel-Tukey test after adjusting median values to paired adjacent three-year periods revealed that for most pairings, there was a slight, yet significant (at the .05 confidence level), difference in dispersion. When the 1973-1975 period was singled out for comparison with other three-year periods because of its "seemingly" high level of volatility, a number of important findings resulted: (1) the period 1973-1975 showed significantly more dispersion than every other three-year period beginning with 1940-1942; (2) the 1973-1975 level of dispersion was surpassed only by the twin peaks of the great depression--1931-1933 and 1937-1939; and (3) the 1973-1975 level of dispersion was not significantly different from the level of volatility attained in 1928-1930 and 1934-1936.

Conclusions

The results of this study indicate two major conclusions. One is that daily market return distributions do not appear to come from normal populations. For the various time periods studied, all the return distributions exhibited signs of peakedness and fat-tails relative to normal distributions. The second conclusion relates to the primary focus of the study. Specifically, the results provided strong evidence that aggregate market volatility has not been constant over time, but rather has experienced major changes. The twin peaks of the great depression--1931-1933 and 1937-1939--showed the highest levels of volatility. The recent 1973-1975 period, however, showed significantly more volatility than every other three-year period beginning with 1940-1942.

Implications

The empirical results and conclusions of this study have implications for those individuals concerned with: (1) the form of the daily market return distribution(s); (2) the effects of changes in the level of market volatility. Because the return distributions are peaked and have fat-tails relative to normal distributions, any search for a better distribution with which to describe daily market returns should focus on those distributions that are leptokurtic relative to normal distributions.

Application of the normality assumption to daily market returns would certainly provide a less than exact description of reality. However, it is notable that five different volatility measures--with varying abilities to deal with non-parametric distributions--still exhibited high positive correlations over time. Thus, the choice of a volatility measure for daily returns may not be so sensitive to the actual underlying distribution's characteristics as might have been feared.

Changes in the level of market volatility are important because as discussed in the introduction to the paper, such a change in volatility could influence expected return on the market portfolio, the risk/return relationship for all securities, and an individual security's "beta." Now that periods of differing volatility have been identified, the question of how the market return and the slope of the SML react to a change in the level of market volatility can be addressed empirically.

The widespread use of the "market model" may make it necessary to determine what effect a change in the level of market volatility has on a security's "beta." The "market model" specifies that security returns are a linear function of a general "market" factor. Empirical analysis of the "market model" is possible from a time series, least-squares regression of the following form:

$$R_{it} = a_i + b_i R_{mt} + e_{it}$$

where R_{it} = the *ex post* return on security i in period t;

R_{mt} = the *ex post* return on the market factor in period t;

e_{it} = the error term in period t;

a_i, b_i = the intercept and slope associated with the linear relationship.

Stability of "beta" over the sample period is assumed when one empirically determines a security's "beta." Therefore, a regression analysis made over periods of dissimilar levels of market volatility might not be appropriate if "betas" change with changing levels of market volatility.

APPENDIX A

Kolmorgorov-Smirnov (K-S) Test for Normality With Mean and Variance Unknown (as developed by Lilliefors)[1]

The K-S one-sample test is a test of goodness-of-fit (i.e., it is concerned with the degree of agreement between the distribution of a set of sample observations and some specified theoretical distribution). It determines whether the sample observations can reasonably be thought to have come from a population having the theoretical distribution.

The K-S test is an alternative to the chi-square test. For samples of any size, it often appears to be a more powerful[2] test than the chi-squared test. Unfortunately, when certain parameters of the theoretical distribution are estimated from the sample, the K-S test no longer applies--or, at least, does not apply using the commonly tabulated critical values. If the test is used in this case, the results will be extremely "conservative" (i.e., the probability of a Type I error will be smaller than as given by tables of the K-S statistic).[3]

Lilliefors presents a table for use with the K-S statistic when testing that a set of observations are from a normal population but the mean and

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[1]This section is based on: Hubert W. Lilliefors, "On the Kolmorgorov-Smirnov Test for Normality With Mean and Variance Unknown," Journal of the American Statistical Association, Vol. 62, No. 318 (June, 1967), pp. 399-402; Lindgren and McElrath, Introduction to Probability and Statistics, pp. 151-153; and Siegel, Nonparametric Statistics for the Behavioral Sciences, pp. 47-52.

[2]The "power" of a test is defined as the probability of rejecting the null hypothesis when it is, in fact, false. Thus, power equals (1-probability of Type II error).

[3]Lilliefors, op. cit., p. 399.

variance are not specified.[4] It is the K-S test as modified by Lilliefors that will be described below.

Objective

The test involves specifying the cumulative normal distribution function $F^*(x)$, with $\mu = \bar{X}$ and $\sigma^2 = s^2$, and comparing that with a sample cumulative distribution function of size n, $S_n(x)$. At some point, these two distributions will show maximum divergence. The size of this divergence (or difference) is determined. The test seeks to determine whether a difference of the observed size would be likely to occur if the observations were really a random sample from the normal distribution.

Method

For testing the null hypothesis,

> H_0: The sample has been drawn from a normal population with mean and variance unknown,

against the alternative hypothesis,

> H_1: The sample has not been drawn from a normal population with mean and variance unknown,

we make use of the idea that if the null hypothesis is true, for every value of x, the difference between $F^*(x)$ and $S_n(x)$ is expected to be small and within the limits of random errors. The test focuses on the largest of the differences, regardless of sign. The statistic used is the maximum absolute deviation of $F^*(x)$ from $S_n(x)$, D:

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[4]Lilliefors, op. cit., p. 400.

$$D = \text{maximum } |F^*(x) - S_n(x)|,$$

where $F^*(x)$ = the proportion of observations expected to have values equal to or less than x;

$S_n(x)$ = the number of observations equal to or less than x, divided by the total sample size.

Critical values for D were obtained by Monte Carlo calculation. For each value of n, one thousand or more samples were drawn and the distribution of D was estimated. Lilliefors presents his results in table form for small sample sizes, for various preselected significance levels. For large values of n, he provides asymptotic formulae.[5]

The test itself is defined as follows:

if $D \leq$ some critical value, accept H_0;

if $D >$ some critical value, reject H_0.

---

[5]For large sample sizes (over 30), Lilliefors' asymptotic formulae are as follows:

Level of Significance for $D = \text{maximum } \|F^*(x) - S_n(x)\|$				
.20	.15	.10	.05	.01
$(.736)(N)^{-1/2}$	$(.768)(N)^{-1/2}$	$(.805)(N)^{-1/2}$	$(.886)(N)^{-1/2}$	$(1.031)(N)^{-1/2}$

APPENDIX B

Siegel-Tukey Test[1]

Objective

The Siegel-Tukey test is a nonparametric procedure for testing the null hypothesis that two independent samples come from the same population, against the alternative hypothesis that the samples come from populations differing in variability or "spread."

Method

To illustrate the method, we must refer to an example. Assume the following observations come from two samples:

Observations from Sample a:	5	14	0	15	8	8
Observations from Sample b:	12	6	3	10	10	11

The observations are first combined into a single series, in order of increasing size, retaining their identification as a's or b's.

Observations:	0	3	5	6	8	8	10	10	11	12	14	15
Sample:	a	b	a	b	a	a	b	b	b	b	a	a

(When ties occur between two observations from the same sample, as in the above series, the order in which we arrange the observations does not matter. Ties across samples, however, pose a problem. The suggested method for handling tied observations will be explained below.)

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[1]This section is based on Sidney Siegel and John W. Tukey, "A Non-parametric Sum of Ranks Procedure for Relative Spread in Unpaired Samples," Journal of the American Statistical Association, Vol. 55, No. 291 (September, 1960), pp. 429-445.

Ranks 1 to 12 are assigned in the ordered observations to a manner that attaches low ranks to extreme observations and high ranks to central observations. We assign ranks to the lowest number of the sequence, rank 1 to 3 to the two highest members in the sequence, ranks 4 and 5 to the next two lowest, etc. (If the total number of observations is odd, the middle observation is dropped in order that the highest assigned rank will be even.) The ranking procedure is illustrated below:

Observations:	0	3	5	6	8	8	10	10	11	12	14	15
Sample:	a	b	a	b	a	a	b	b	b	b	a	a
Rank:	1	4	5	8	9	12	11	10	7	6	3	2

> Assigning the ranks in this way puts the lower ranks at the extremes in the ordered sequence and the higher ranks in the middle of the sequence. If the null hypothesis were true, the observations from the two populations would tend to be well mixed, so that the mean rank assigned to one of the samples would tend to equal the mean rank assigned to the other sample. If on the other hand, the alternative hypothesis were true, we would expect more of the observations from the population with greater spread to be near the extremes of the ordered sequence and, therefore, to be assigned the lower ranks, and we would expect more of the observations from the less variable population to be near the middle of the sequence, and, therefore, to be assigned the higher ranks. Thus, we would expect the mean rank assigned to the observations from the more variable population to be considerably smaller than the mean rank assigned to the observations from the less variable population.[2]

Since the total sum of ranks is fixed, we may work with the sum of ranks for either group. If the two groups are of different size, it is usual to choose the sum of ranks for the smaller group. In the example presented above, the sum of ranks for the a's is $R_a = 32$.

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[2] Siegel and Tukey, op. cit., pp. 430-431.

Procedure for Large Samples

Suppose two samples are drawn from identical populations and ranked jointly in the manner described above. Let R_1 equal the sum of the ranks for the smaller sample, n_1 equal the size of the smaller sample, and n_2 equal the size of the larger sample. Then the sum of $N = n_1 + n_2$ ranks is $(1/2)(N)(N+1)$ and the sum of their squares is $(1/6)(N)(N+1)(2N+1)$. Therefore, the mean and variance of the N ranks are:

$$\mu = (1/2)(N+1)$$
$$\sigma^2 = (1/12)(N^2-1).$$

If n_1 and n_2 are not too small, the means, (R_1/n_1), of samples of size n_1 randomly drawn without replacement from the N ranks will be approximately normally distributed with mean equal to μ and variance equal to:

$$VAR(R_1/n_1) = (\sigma^2/n_1)\ [(N-n_1)/(N-1)]$$

and therefore

$$Z = \frac{(R_1/n_1) - \mu}{[VAR(R_1/n_1)]^{1/2}}$$

will be approximately normally distributed with zero mean and unit variance.

After making some substitutions, and correcting for continuity, we have,

$$Z = \frac{2R_1 - n_1(n_1+n_2+1) \pm 1}{[(n_1)(n_2/3)(n_1+n_2+1)]^{1/2}}$$

where we choose that sign for $\pm$ 1 in the numerator which makes the magnitude of Z smaller.

Treatment for Ties

If two tied observations come from the same sample, treating them as arranged in either order does not affect the value of R_1. No adjustments for such ties are needed in calculating R_1.

If two observations from different samples are tied, "breaking" the tie in the two possible ways would lead to different values of R_1. We should therefore assign average ranks to the tied observations.

If both samples are at least moderately large, we need only correct the variance of R_1 and continue to use the modified Z which results. The denominator of Z would now become:

$$[(n_1)(n_2/3)(n_1+n_2+1) - 4[(n_1)(n_2)/[(n_1+n_2)(n_1+n_2-1)]]\ (S_1-S_2)]^{1/2},$$

where S_1 is the sum of squares of the ranks (not averaged) of the tied observations, and S_2 is the sum of squares of the averaged ranks of the tied observations.

The treatment for tied observations can be illustrated by reference to our earlier example. Tied pairs of observations were ranked 9 and 12, and 10 and 11. We could replace these by the corresponding average ranks--10.5 and 10.5, and again 10.5 and 10.5. Then we would have,

$$S_1 = (9)^2 + (12)^2 + (10)^2 + (11)^2 = 446$$

$$S_2 = (10.5)^2 + (10.5)^2 + (10.5)^2 + (10.5)^2 = 441.$$

It is suggested that if we are going to use average ranks for ties coming from different samples, we should be consistent and use average ranks for <u>all</u> ties.

Test Formulation

The null hypothesis can be stated formally as,

H_0: The observations from two samples come from populations with the same dispersion,

and the alternative as,

H_1: The observations from two samples come from populations not having the same dispersion.

The test itself would be:

if $|Z| \leq$ some critical value, accept H_0;

if $|Z| >$ some critical value, reject H_0.