TECHNICAL REPORT

AN ANALYSIS OF ENVIRONMENTAL FACTORS AFFECTING ICE GROWTH

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ABSTRACT

This report describes efforts to evaluate the effects of various oceanographic and meteorological variables on the rate of sea ice growth, in an attempt to ascertain the most important parameters to consider from the forecasting standpoint. Work of Neuman and Stefan, as well as that of Kolesnikov, is utilized in the form of prediction equations. Results are applied to specific Arctic localities with good agreement. It is concluded that the most important factors affecting ice growth are air temperature, snow thickness, and snow density, with several other variables playing a lesser role.

FOREWORD

The increasing importance of defense installations in northern areas has greatly increased the responsibilities of the U.S. Navy in supplying bases in Arctic waters, where sea ice is often an operating obstacle. The Hydrographic Office is charged with the responsibility of developing and testing techniques for observing and forecasting sea ice conditions. Standardized techniques for observing, charting, and reporting sea ice are now in operational use by the Navy, as described in publications issued by the Hydrographic Office. Heretofore, techniques for forecasting the formation, growth, and movement of sea ice have not been published by this Office. This publication describes in detail the factors affecting ice formation and growth.

It is requested that activities receiving this publication forward their comments to the Hydrographic Office.

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A. INTRODUCTION

Forecasting ice thickness can be separated into three problems: (1) determining the time required to reduce the temperature of the water mass to the freezing point by thermohaline convection; (2) forecasting the temperature, wind velocity, cloudiness, humidity, depth, and density of snow fall for the period for which the ice thickness forecast is required; and (3) computing the thickness of the ice accretion which will result from the predicted weather conditions. This study is restricted to the last phase of this problem.

The problem of ice thickness forecasting is one of great complexity, expressible only in terms of a complicated system of differential and integral equations, the solution of which is not possible when the boundary conditions are not simple. The first physicist to present a complete mathematical theory of heat conduction was Joseph Fourier. The application of Fourier's heat conduction equations to the problem of ice formation was first undertaken by Franz Neumann (Weber, 1910) and Stefan (1889). More recently Russian and Norwegian scientists have been active in this field, and it is with the description of the work of A. G. Kolesnikov (1946) and Olav Devik (1931) that this study is chiefly concerned.

After the derivation of a theoretical forecast formula the question arises as to the best practical method of its application. In this connection consideration should be given not only to the facility with which the results of the formula can be obtained but also to the accuracy of the evaluation.

There are two general methods for obtaining the required ice thickness from the formula: (1) by computation for each individual situation, and (2) by taking the required hickness from graphs consisting of parametric curves of ice thickness drawn with temperature and ice thickness as abscissa and ordinate, respectively, and with meteorological factors as parameters. Both of these methods will be derived and explained. To quickly obtain approximate results the graphical method is recommended, but for a more accurate determination in which all the parameters are given individual consideration, the computational method should be employed.

B. INCREASE OF ICE THICKNESS WITH TIME

In approaching the problem of ice growth with time, it is desirable, initially, to formulate an expression in simple terms, that is, in the preliminary steps to neglect the meteorological factors of wind velocity, cloud cover, and humidity and to consider the ice as formed free from the blanketing influence of a snow cover. Initially then, assume a surface of still water lowered by contact with air to some temperature T_0 below the freezing point. There will then be formed a layer of ice whose thickness ξ is a function of the time t. A solution may be reached by equating the amount of heat carried up from the water below the ice sheet plus the heat set free per unit of time (dt) per unit area (as the ice increases in thickness by $d\xi$) to the total amount of heat that flows outward through a unit area of the lower surface of the ice sheet. This heat budget equation must satisfy Fourier's heat transfer equations:

$$\frac{\partial T_1}{\partial t} = \alpha_1 \frac{\partial^2 T_1}{\partial x^2} \quad \text{in the ice } (0 < x < \xi) \qquad (1)$$

$$\text{and} \quad \frac{\partial T_2}{\partial t} = \alpha_2 \frac{\partial^2 T_2}{\partial x^2} \quad \text{in the water } (\xi < x), \qquad (2)$$

where:

T_1 = temperature in the ice,

T_2 = temperature in the water,

$\alpha_1 = \frac{k_1}{c_1 \rho_1}$ is thermal diffusivity in ice,

$\alpha_2 = \frac{k_2}{c_2 \rho_2}$ is thermal diffusivity in water,

t = time

k_1 = thermal conductivity in ice,

k_2 = thermal conductivity in water,

c_1 = specific heat of ice,

c_2 = specific heat of water,

ρ_1 = density of ice, and

ρ_2 = density of water.

The temperature of the boundary surface of ice and water (at $x = \xi$) must always be 0°C (under this simplified formulation) and there will be continual formation of new ice. If the thickness increased by $d\xi$ in time dt, there will be set free for each unit of area an amount of heat

$$Q = L \rho_1 d\xi, \qquad (3)$$

where L is the latent heat of fusion. This heat must escape upward by conduction through the ice, and in addition heat must be carried away from the water below, so that the total amount of heat that flows outward through a unit area of the lower surface of the ice sheet is

$$Q' = k_1 \left(\frac{\partial T_1}{\partial x}\right)_{x=\xi} dt \qquad (4)$$

of this amount the quantity

$$Q'' = k_2 \left(\frac{\partial T_2}{\partial x}\right)_{x=\xi} dt \qquad (5)$$

flows up from the water below. Hence the first boundary condition is

$$\left(k_1 \frac{\partial T_1}{\partial x} - k_2 \frac{\partial T_2}{\partial x}\right)_{x=\xi} = L \rho_1 \frac{\partial \xi}{\partial t}. \tag{6}$$

The other boundary conditions are:

$$T_1 = T_s = C_1 \quad \text{at } x = 0, \tag{7}$$

$$T_1 = T_2 = 0 \quad \text{at } x = \xi, \text{ and} \tag{8}$$

$$T_2 = C_2 \quad \text{at } x = \infty. \tag{9}$$

There are also three other boundary conditions derived from the fact that when $t = 0$, ξ is fixed, while T_1 and T_2 must be given as functions of x. T_1 lies between 0 and ξ while T_2 lies between ξ and ∞. As equation (6), containing the unknown function ξ, is not linear and homogeneous, a solution cannot be reached by the combination of special solutions. The method of solution then will be to find particular integrals of equations (1) and (2) and after modifying them to fit boundary conditions equations (7), (8), and (9) to find under what conditions the solution will satisfy equation (6). This will also determine the initial values of ξ, T_1 and T_2.

Now the function $\Phi(x,\eta)$(the probability integral) is a solution of such differential equations as (1) and (2). Consequently if B_1, D_1, B_2 and D_2, are constants and $\eta \equiv \frac{1}{2}\sqrt{a_1 t}$ and $\eta \equiv \frac{1}{2}\sqrt{a_2 t}$,

$$T_1 = B_1 + D_1 \Phi(x, \eta_1) \tag{10}$$

and

$$T_2 = B_2 + D_2 \Phi(x, \eta_2) \tag{11}$$

are also solutions. Boundary condition (8) means that $\Phi(\xi, \eta_1)$ and $\Phi(\xi, \eta_2)$ must each be constant, which will be true if $\xi = 0$, $\xi = \infty$, or if ξ is proportional to $\sqrt{t}$. The first two of these assumptions are evidently inconsistent with (8). Thus, there remains only the last which may be put into the form

$$\xi = b\sqrt{t} \tag{12}$$

where $\underline{b}$ is a constant to be determined, together with B_1, D_1, B_2, and D_2.

From the properties of $\Phi(x)$ it is known that $\Phi(0) = 0$ and $\Phi(\infty) = 1$. Fitting boundary conditions (7), (8), and (9) in (10) and (11), with the use of (12), the following equations result:

$$B_1 = C_1, \tag{13}$$

$$B_1 + D_1 \Phi\left(\frac{b}{2\sqrt{\alpha_1}}\right) = 0, \tag{14}$$

$$B_2 + D_2 \Phi\left(\frac{b}{2\sqrt{\alpha_2}}\right) = 0, \text{ and} \tag{15}$$

$$B_2 + D_2 = C_2, \tag{16}$$

while (10), (11), and (12) in connection with (6) give

$$\frac{k_1 D_1}{\sqrt{\pi \alpha_1 t}} e^{-\frac{b^2}{4\alpha_1}} - \frac{k_2 D_2}{\sqrt{\pi \alpha_2 t}} e^{-\frac{b^2}{4\alpha_2}} = \frac{L \rho_1 b}{2\sqrt{t}}. \tag{17}$$

Solving equations (13) to (16) for D_1 and D_2 yields

$$D_1 = -\frac{C_1}{\Phi\left(\frac{b}{2\sqrt{\alpha_1}}\right)}, \qquad D_2 = \frac{C_2}{1 - \Phi\left(\frac{b}{2\sqrt{\alpha_2}}\right)}, \tag{18}$$

and substituting these values in (17) finally gives

$$\frac{k_1 C_1 e^{-\frac{b^2}{4\alpha_1}}}{\sqrt{\alpha_1}\,\Phi\left(\frac{b}{2\sqrt{\alpha_1}}\right)} + \frac{k_2 C_2 e^{-\frac{b^2}{4\alpha_2}}}{\sqrt{\alpha_2}\left[1 - \Phi\left(\frac{b}{2\sqrt{\alpha_2}}\right)\right]} = -\frac{\sqrt{\pi}}{2} L \rho_1 b. \tag{19}$$

This derivation is due to Neumann.

This transcendental equation (19) can be solved for $\underline{b}$ by plotting the curves

$$y = -\frac{\sqrt{\pi}}{2} L \rho_1 b \quad \text{and} \quad y = f(b),$$

where $f(b)$ represents the left hand side of (19). Then $\underline{b}$ is found as the abscissa of the intersection of the two curves.

Figure 1 shows a graphical solution of (19) by this method, using the following values for the constants involved:

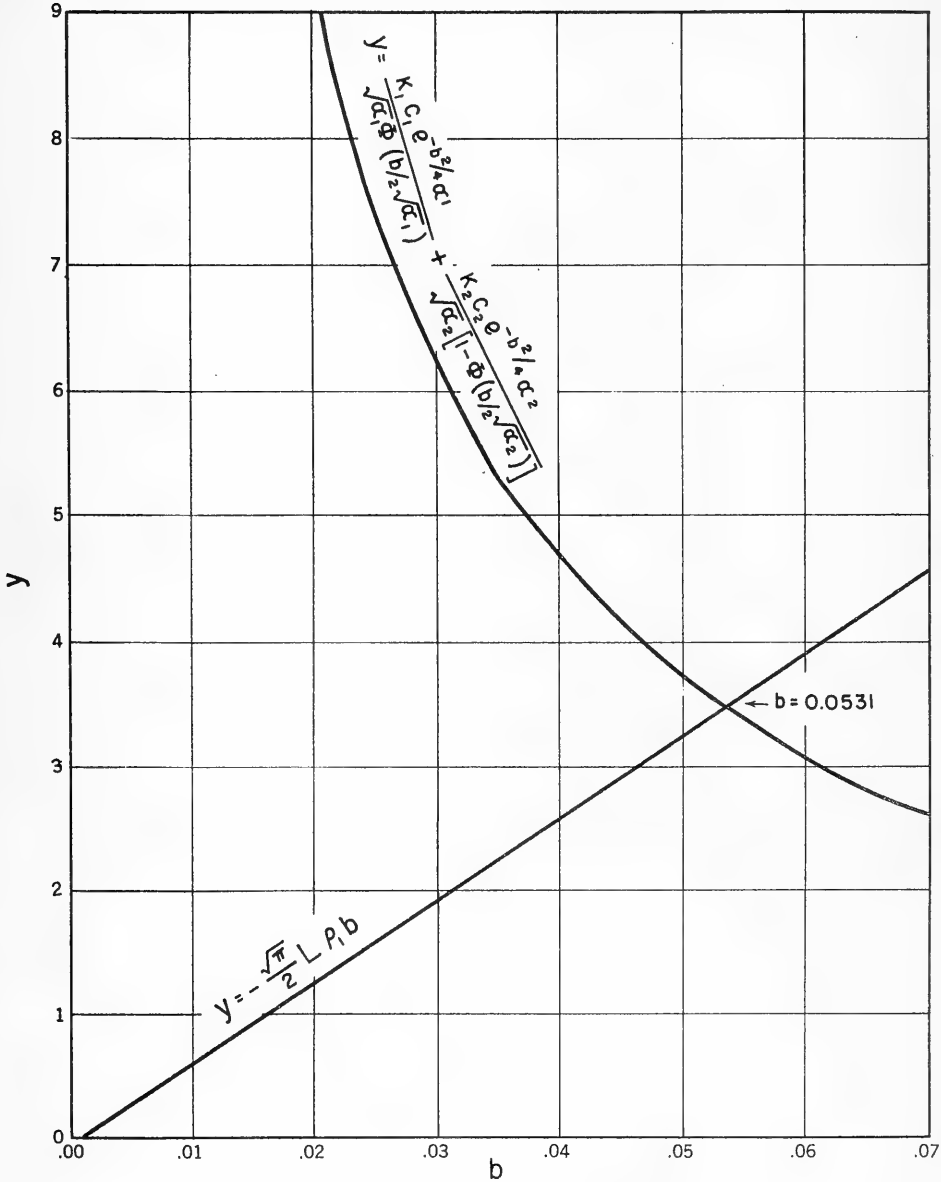

FIG. 1. GRAPHICAL SOLUTION OF EQUATION 19.

$$k_1 = .0053$$
$$c_1 = -20°C.$$
$$\alpha_1 = .0118$$
$$k_2 = .00143$$
$$c_2 = -1°C.$$
$$\alpha_2 = .00143$$
$$L = 80.0$$
$$\rho_1 = 0.92.$$

Using the value of b found from Figure 1, the equation relating the thickness of ice to the time, for an ice surface temperature of -20°C., and water temperature of -1°C., is

$$\xi = 0.0531\sqrt{t} \quad , \tag{20}$$

where ξ = ice thickness in cm. and t = time in seconds. An evaluation of (20) for 14 hours gives an ice thickness of 12 cm. as compared to 8 cm. from the practical forecast curves of Figure 12. The differences may be due to the different values of constants used, i.e. V = 20, N = 0.

Stefan in a similar fashion derived an expression for the ice thickness as a function of time. He made a further simplification by assuming the water temperature to be 0°C. This is merely a special case of Neumann's solution and can be obtained from it by making $C_2 = 0$ in equation (19). To a first approximation Stefan's equation is

$$\xi^2 = -\frac{2 C_1 c_1 \alpha_1 t}{L} \quad , \tag{21}$$

where

ξ = ice thickness,
C_1 = temperature of the ice surface,
L = latent heat of fusion,
c_1 = specific heat of ice,
α_1 = thermal diffusivity of ice, and
t = time.

(α_1 is available from tables but is equal to the thermal conductivity divided by the product of specific heat times density--all available from tables).

Equations (19) and (21) are not suited for practical application, since the effects of initial thickness of ice, thickness and density of snow cover, wind velocity, cloud cover, humidity of the air, and salinity of sea water are neglected, and in addition such a long and tedious computation is not suited for practical use. To take the meteorological conditions into account, it is necessary to form heat budget equations.

Consider first the sunlight and daylight radiation. At a low sun altitude of from 5 to 10 degrees, for example, a water or ice surface will reflect a considerable part of the sun's rays. A fraction will enter the water and be gradually absorbed, while the transmitted radiation will be practically all absorbed by the lower layers of the water body. At a sun altitude of 5° one square centimeter of water surface will receive 0.6 calorie per square centimeter per hour. This is only 0.5 percent of the amount of solar radiation reaching the atmosphere.

Under these conditions and with a clear sky, the diffuse daylight from all parts of the sky will give a greater input, namely 3 cal/cm.^{2} per hour. If the sky is clouded, the diffuse daylight absorbed by a water surface will be less but will still represent 1 cal/cm.^{2}/hr., which is more than the direct radiation would give at the low sun altitude.

These figures of heat gained are small compared to heat losses in winter. But when the sun's altitude increases, the incoming radiation direct and diffused will contribute more and more to the heat absorbed; and in the summer the relation will be reversed, as the heat received will exceed the heat lost.

Restricting the problem to winter conditions, next consider the loss of heat by infrared radiation at a representative air temperature of -10°C. With a clear sky and air temperature -10°C. the heat radiation from a water surface at 0°C. will be about 14 cal/cm.^{2}/hr.

Nearly all surfaces will absorb almost completely infrared radiation of the type radiated from objects of moderate temperature, and with respect to these rays, water is a black body. This is also the case with snow. This means that infrared radiation falling on a water or snow surface is completely absorbed in the uppermost layers in a very thin sheet some hundredths of a millimeter thick. On the other hand the same layer emits infrared radiation of the same wavelengths to the atmosphere.

The next process to consider is the loss of heat by convection. This loss is due to the difference in temperature between the water surface and the air above, but it is also modified by the wind. It is well known that a body is much more rapidly cooled when a wind is blowing than when it is calm. The figures relating to a water surface are: heat loss by convection 2.8 cal/cm.^{2}/hr. with an air temperature of -10°C. and calm air, and heat loss by convection 11.5 cal/cm.^{2}/hr. with a wind of 5 meters per second.

Another heat loss to be considered is the loss caused by evaporation. The evaporation depends first upon the dryness of the air above the water surface, or more exactly upon the vapor pressure, and secondly upon the wind. With an air temperature of -10°C., a vapor pressure of 3.5 millibars, and calm air, the loss by evaporation from a water surface at 0°C. is 1.7 cal./cm.^{2}/hr., but if there is a wind of 5 meters per second it will increase to 7.7 cal./cm.^{2}/hr.

Adding all the heat losses and subtracting the heat gain, the net loss under the first set of meteorological conditions, i.e., air temperature -10°C., vapor pressure 3.5 mb., wind velocity 0 meters/second, cloudiness 0%, sun altitude 5°, and water temperature 0°C., will be 14.7 cal/cm^2/hr. In the second set of meteorological conditions, i.e. air temperature -10°C., vapor pressure 3.5 mb., wind velocity 5 meters/seconds, cloudiness 100% (overcast), sun altitude 5° and water temperature 0°C., the net loss will be 23.1 cal/cm^2/hr. These figures are equivalent to the production per hour of a sheet of ice of thickness 2.0 and 3.2 mm. respectively.

The above briefly outlined analysis forms the basis for the calculation of the actual growth of an sheet ice under conditions existing in nature. The method could be applied directly if the surface of the ice retained the temperature of 0°C. and had the same physical properties as water with regard to heat exchange. The first condition is not true, however, as the surface temperature of the ice will be lower when the ice is thicker, if the air temperature is low.

The flow of heat through the sheet ice takes place in accordance with Fourier's heat conduction equation (1), and the heat budget equations outlined above must be solved in accordance with this equation.

C. KOLESNIKOV'S EQUATION AND ITS EVALUATION

Kolesnikov has recently (1946) derived an expression for the thickness of ice as a function of time which involves all of the meteorological factors. This was accomplished by setting up heat budget equations involving these parameters and solving in connection with Fourier's heat conduction equation.

The form of Kolesnikov's equation for salt water is

$$\Delta\xi^2 + \left\{ \frac{1.32}{\rho_0^{\,2}}\,\delta + \frac{90}{1.75 \times V_0^{0.656} + 5.23\times10^{-12}\,T_0^{\,3}} + 2\xi_1 \right\}\Delta\xi$$

$$= \frac{9.73}{1 - \frac{S_1}{S_{T_\lambda}} + \frac{T_\lambda}{80}\left(C_2\frac{\rho_2}{\rho_1} - C_1\right)} \int_{t_1}^{t_2} \frac{T_\lambda - \theta}{1 + \frac{C_1}{2K_1}(T_\lambda - \theta)}\,dt \,, \qquad (22)$$

where ξ_1 = initial ice thickness in cm.,

$\Delta\xi$ = increase in ice thickness in cm.,

S_1 = salt content of the ice,

S_{T_λ} = salinity of salt water solution at temperature of freezing,

ρ_1 = density of ice,

$K^1 = K + T_\lambda\left(C_2\frac{\rho_2}{\rho_1} - C_1\right)$,

ρ_2 = density of sea water
δ = snow thickness in cm.,
ρ_0 = snow density,
v_0 = wind velocity in m/sec.,
T_0 = air temperature in °C,
C_1 = specific heat of sea ice,
T_λ = temperature of freezing in °C.,
Θ = equivalent temperature in °C.,
C_2 = specific heat of sea water, and
K = latent heat of crystallization of salt water ice $80(1-\frac{S_1}{S_{T_\lambda}})$

The equivalent temperature Θ is a resultant temperature which takes into account the effects of (1) radiation, (2) convection, (3) evaporation and condensation, (4) wind velocity, (5) humidity, (6) cloud cover, and (7) insolation. It is defined by the following relation

$$\theta = T_0 + \sum \frac{q}{\alpha_c} \tag{22a}$$

where Θ = equivalent temperature in °C.,
T_0 = air temperature in °C.,
q = total heat exchange represented by the seven meteorological factors enumerated above,

$\alpha_c = \alpha_n + \alpha_k$,

$\alpha_n = 5.23 \times 10^{-12} \times T_0^{3}$,
$\alpha_k = 1.75 \times v_0^{0.656} \times 10^{-4}$, and
v_0 = wind velocity in m/sec.

The total heat loss q is determihed as the sum of the individual heat exchanges as follows:

radiation loss = $1.307 \times 10^{-12} \times T_0^{4} (0.255 + 0.322 \times 10^{-.069P_0})(1 - c_0 n_0)$,
convection loss = $1.75 \times v_0^{0.656} \times 10^{-4}$,
evaporation-condensation = $145.4(e_w - e_a)w_a/ 1440$,
insolation: taken from Meteorological Tables,

where
P_0 = vapor pressure in mbs.,
c_0 = cloud coefficient defined as follows:
.86 for Nb, St, and Sc,
.77 for Ac, and
.20 for Cs.,
n_0 = cloud cover in tenths,
e_a = vapor pressure in inches,
e_w = 0.98 e_a, and
w_a = wind velocity in knots.

The combined effect of various meteorological factors on the ice thickness is shown in Figure 2. Approximate average values of $\Sigma q/\alpha_c$ for Arctic latitudes are listed in Table I. These values, in the form of heat gain and loss, are tabulated by months during which ice formation occurs.

TABLE I

Heat Gain or Loss ($\Sigma q/\alpha_c$)
for the Arctic Regions

Month	Heat Loss	Heat Gain	Net Gain (+) or Loss (-)
Nov.	-5.2°C.	0.2°C.	-5.00°C.
Dec.	-5.4°C.	0.16°C.	-5.24°C.
Jan.	-5.9°C.	0.49°C.	-5.41°C.
Feb.	-6.0°C.	1.85°C.	-4.15°C.
Mar.	-6.5°C.	5.95°C.	-0.55°C.
Apr.	-6.6°C.	9.60°C.	+3.00°C.

By the proper choice of parameters and physical constants, the following equation for the practical determination of ice thickness in fresh water is derived:

$$h_1^2 + \left[1.49\frac{\delta}{\rho_0^2} + \frac{100}{1.75 \times v_0^{0.686} + 5.23 \times 10^{-12} \times T_0^3} + 2h_0\right] h_1 \qquad (23)$$

$$= 12\sum \frac{\theta}{1+0.0033\,\theta} ,$$

where
h_1 = increase in ice thickness in cm.,
δ = snow thickness in cm.,
ρ_0 = snow density,
v_0 = wind velocity in knots,
T_0 = air temperature in °C.,
h_0 = initial ice thickness in cm., and
θ = equivalent temperature in °C.

While equation (23) looks complicated and clumsy, it can be evaluated easily by substituting the given meteorological parameters into the numerator and denominator and dividing.

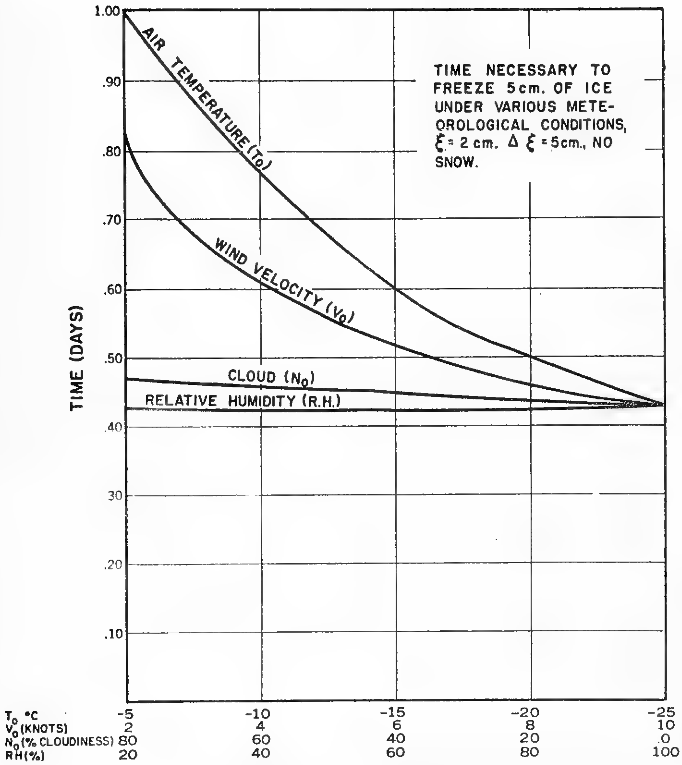

FIG. 2. INFLUENCE OF METEOROLOGICAL CONDITIONS ON THE FREEZING OF ICE.

D. PHYSICAL CONSTANTS USED IN NUMERICAL EVALUATION

In the practical numerical evaluation of Kolesnikov's formula (22) for a given set of meteorological conditions, the following values for the physical constants involved were used.

1. Density of Sea Ice

Utilizing the investigations of Malmgren (1927), which indicate the independence of the ice density from the salinity, the value of the ice density was taken as $\rho_1 = 0.916$.

2. Latent Heat of Fusion

The latent heat of fusion depends upon the salt content of the ice. Malmgren's investigations give the following value for K, the latent heat of fusion:

$$K = 80 \left(1 - \frac{S_1}{S_{T_\lambda}} \right)$$

where S_1 is the salinity of the ice and S_{T_λ} is the salinity of the sea water. The formula indicates that the value of K decreases with increase in salinity and increases with increase of sea water salinity for a given salinity of ice.

3. Specific Heat of Sea Ice (C_1)

The specific heat of sea ice is dependent upon the salinity. However, this dependency is well marked only for small negative temperatures, i.e., in the vicinity of 0°C. For large negative temperatures the dependency of the specific heat upon salinity is greatly decreased. Malmgren's observations indicate that after the initial stages of ice formation, for temperatures between -8° and -14°C. and for salinities of from 4 to 6 ‰, the specific heat varies between 0.57 and 0.88. Therefore, by using small time intervals in the forecast, the specific heat may be considered as a constant.

4. Thermal Conductivity of Sea Ice (λ_1)

As between the one actually observed determination of the thermal conductivity of sea ice by Malmgren of .0051 cal/cm²/sec. and the value of .0045 found by three Russian scientists, Kolesnikov considers this latter value the more reliable.

5. Thermal Conductivity of Snow (λ_0)

Abels' (1892) formula for the thermal conductivity of snow is used:

$$\lambda_0 = 0.0068 \rho_0^2$$

where ρ_o is the density of snow.

6. Convective Heat Loss

For the coefficient of heat loss through convection, Frank's (1929) formula is used:

$$\alpha_k = 1.75 \times V_o^{0.656} \times 10^{-4},$$

where v_o is the wind velocity in m/sec.

7. Density of Sea Water

For purposes of evaluating the formula, sea water density is considered as constant and equal to 1.000 throughout the whole period.

8. Specific Heat of Sea Water

As it varies but little with salinity, the specific heat of sea water (C_2) is considered a constant and equal to 0.975.

9. Effective Radiation of a Black Surface

From Angstrom's formula for the effective radiation of a black body,

$$Re_o = 1.376 \times 10^{-12} \times T_o^4 \left\{0.255 + 0.322 \times 10^{-0.069 P_o}\right\},$$

where T_o = air temperature and
P_o = pressure of water vapor in mb.

Taking from Falkenberg (1928), the emissivity of the snow, <u>a</u>, as 0.995, it follows that

$$aRe_o = 1.307 \times 10^{-12} \times T_o^4 \left\{0.255 + 0.322 \times 10^{-0.069 P_o}\right\}$$

Therefore the heat lost through radiation is

$$\alpha_R = 5.23 \times 10^{-12} \times T_o^3.$$

Devik (1931) found by means of the above formula that for air temperatures between 0 and -20°C. and for relative humidities close to saturation, the magnitude of aRe_o does not vary much. Therefore it can be taken as a constant equal to 30.6×10^{-4}.

10. Effect of Cloudiness on Radiation

This is calculated by means of the following relation:

$$aRe = aRe_o \times (1 - c_o n_o),$$

where c_o is a coefficient taking into account the diminution in the total radiation due to the mean cloudiness, and n_o is the average cloudiness in percentage. Assigned values of c_o are .86 for nimbostratus, stratus and stratocumulus clouds, .77 for altocumulus, and .20 for cirrostratus clouds, from the study of Efimov (1939). The reflection coefficient is taken as a constant equal to 0.65.

E. EFFECT OF CHANGES IN METEOROLOGICAL AND OCEANOGRAPHIC PARAMETERS ON ICE THICKNESS

1. Snow Thickness

The factor having the most pronounced influence on the rate of ice growth is the blanketing effect of a layer of snow on the ice surface. A quantitative graphical evaluation of this effect is presented in Figure 3. The parameters have been given the following values:

Initial ice thickness (ξ_1) = 15 cm.
Snow density (ρ_o) = 0.3
Wind velocity (v_o) = 4.5 m/sec.
Air temperature (T_o) = -20°C.
Equivalent temperature (Θ) = -25°C.

The rate of growth of the ice for snow thickness δ of 0, 10, 20, 30, 40, and 50 cm. is indicated. The following facts are evident from the figure:

The most rapid ice growth takes place with an ice surface free from snow. For this surface the most rapid growth takes place with the lesser ice thicknesses. Here a decreasing rate with increasing ice thickness is evident. Upon reaching a thickness of approximately 50 cm. the rate of growth becomes essentially linear but at a much lower rate than in the initial stages. To add this 50 cm. of ice requires about 18 days with an average temperature of -20°C. From this point on, the rate of growth is essentially linear at the rate of about 10 cm. in 8 days.

A snow layer only 10 cm. thick makes a marked change in the characteristics of the rate of growth. To reach a thickness of 50 cm. now takes about 48 days. The ice thickness increase is essentially linear with time, requiring approximately 8 days to produce an increase of 10 cm. in thickness.

With increasing thickness of snow, the ice thickness increase remains essentially linear but with progressively decreasing slope; with 20 cm. of snow it requires about 14 days to add a thickness of 10 cm.; with 40 cm. of snow about 20 days are required to add 10 cm.; and with 50 cm. of snow about 32 days are needed to add 10 cm. to the ice thickness.

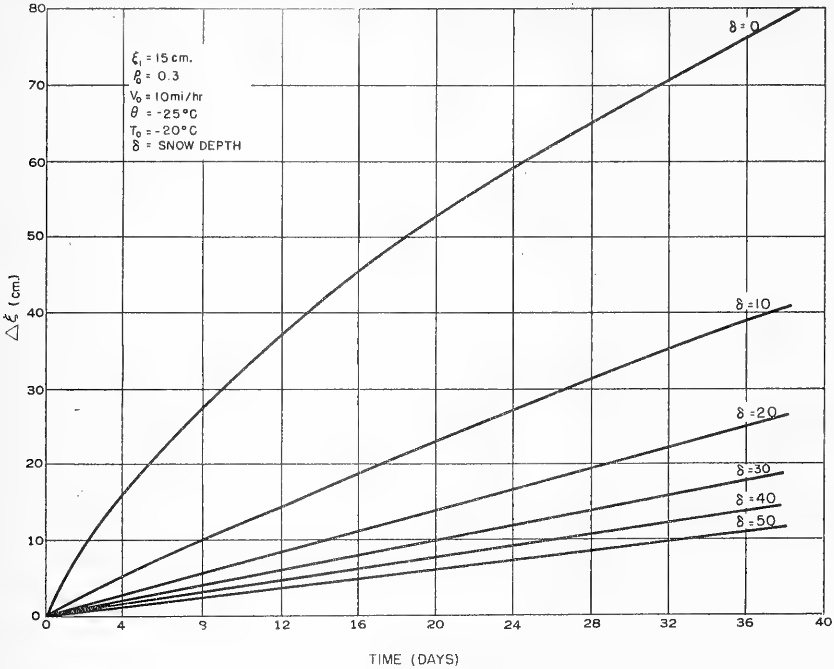

FIG. 3. EFFECT OF SNOW DEPTH ON ICE GROWTH.

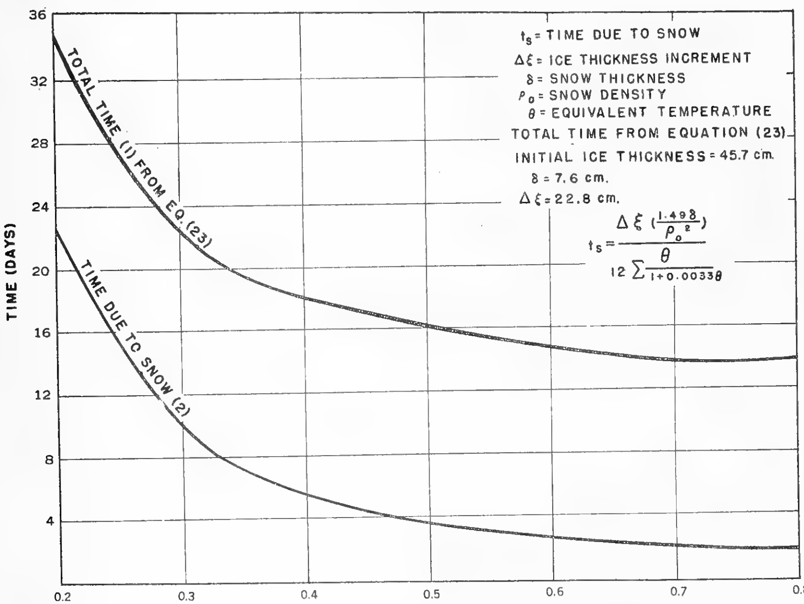

FIG. 4. EFFECT OF SNOW DENSITY ON ICE GROWTH

2. Snow density

The factor next in importance to snow thickness in determining ice thickness growth with time is snow density. In Figure 4 the initial ice thickness is 45.7 cm., the ice increment 22.8 cm., snow thickness 7.6 cm. and snow density varies from 0.2 to 0.8. Curve (1) of Figure 4 shows the time required to add this ice thickness increment of 22.8 cm. with varying snow density. The time diminishes rapidly at a non-linear rate up to a density of 0.5, where it becomes more nearly linear and diminishes slowly up to the maximum density of 0.8. In going from a density of 0.2 to one of 0.5 the time diminishes from about 35 days to 16 days to add on the ice increment of 22.8 cm. When the density changes from 0.5 to 0.8, the time drops only about 2 days, from 16 to 14 days.

Curve (2) shows the amount of time that is added by the snow layer alone. This curve shows that the snow contributes a much higher percentage of the total time at low snow densities than at high densities. At low snow densities the snow layer contributes about 63% of the total time while at high snow densities it contributes only about 10% of the total time.

3. Air and Equivalent Temperatures

The next meteorological factor to consider is $\Theta = T_o + \Sigma\, q/\alpha_c$, which is the sum of the air temperature and the net temperature change due to the heat exchange at the surface of the snow or ice. The quantity $\Sigma\, q/\alpha_c$ is analyzed and derived above. Figure 5 shows the effect of a change in Θ, both for ice formation in salt water (curve 1) (equation 22) and in fresh water (curve 2). Curve 3 indicates the change in time for ice growth in fresh water due to changes in snow thickness. The vertical scale indicates the time in days required to add ice thickness increments of 2 centimeters.

Curve 1 indicates the nonlinear nature of the variation of time with ice thickness for different values of snow thickness on salt water ice. The slope shows a definite increase with increasing thickness of ice. It requires approximately 35 days to add 14 cm. of ice under the indicated conditions with $\Theta = -10°C.$, and approximately 12 days with $\Theta = -30°C.$ Curve 2 indicates the variation for fresh water under the same conditions. It is seen that the growth of ice in fresh water requires slightly less time than in salt water under the same conditions. Curve 3 indicates the variation in time for ice growth in fresh water for different snow thicknesses.

4. Initial Ice Thickness

The next parameter to consider is that of initial ice thickness (ξ_1). Figure 6 shows that the variation in ice thickness growth with time is linear for constant values of initial ice thickness. Greater values of ice increment result from lower initial ice thicknesses. The relative

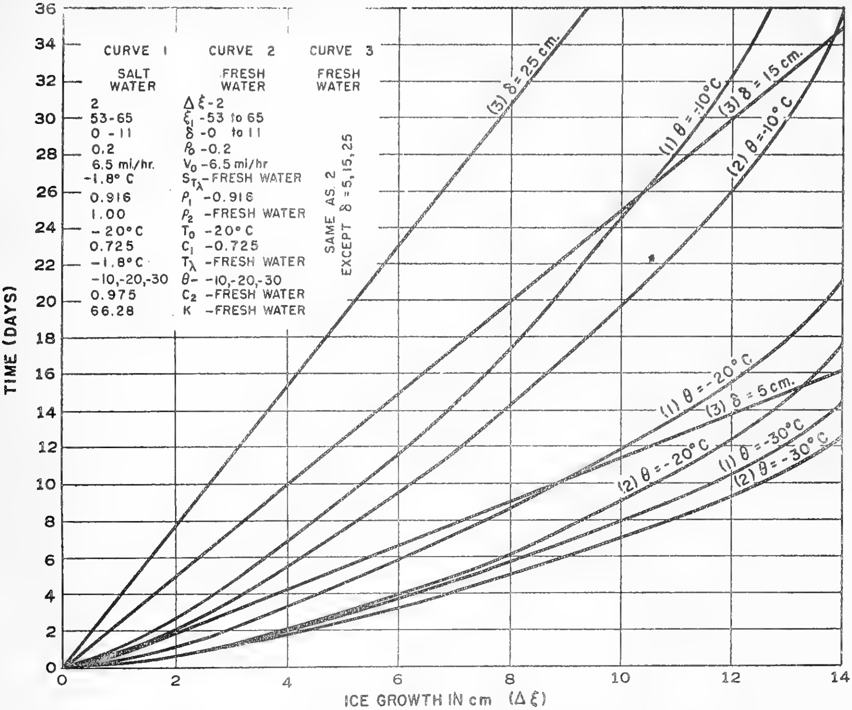

FIG. 5. EFFECT OF EQUIVALENT TEMPERATURE AND SNOW DEPTH ON ICE GROWTH.

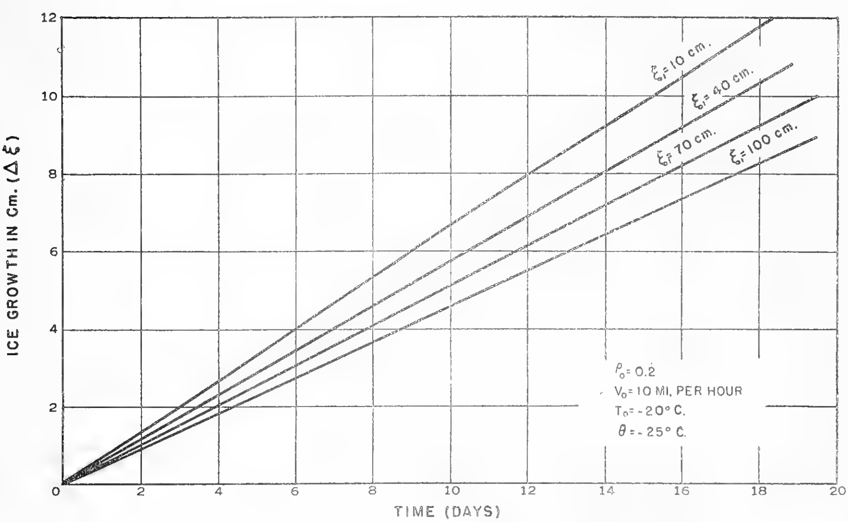

FIG. 6. EFFECT OF INITIAL ICE THICKNESS ON ICE GROWTH.

importance of initial ice thickness and snow cover thickness is clearly shown in Figure 7, where curve 1 indicates the variation in time to freeze 5 cm. of ice for different initial ice thicknesses and curve 2, the time required to freeze the same amount of ice with varying snow thicknesses.

5. Wind

Figure 8 shows the effect of wind velocities of 20, 30, and 40 miles per hour on rate of ice growth for $\Delta\xi = 1$ cm, $\xi_1 = 53$ cm. and $\delta = 10$ cm. Under these conditions a change of 10 miles per hour in wind velocity has very little effect on ice growth. However, for lower values of wind velocity the effect is marked

6. Salinity

The effect of sea water salinity is indicated in Figure 9 Under the conditions indicated on the figure a slightly greater ice thickness accumulates for the length of time in fresh water than accumulates in salt water. However, the two curves do not show much difference in growth rate of ice due to the salinity.

7. Effect of the Heat of the Water Mass

None of the formulas so far considered has attempted to evaluate the effect of the heat of the water mass itself on the growth of the ice sheet. In the presence of warm currents or where the thickness of the layer subject to convectional cooling is great, the effect of the heat of the water mass is appreciable. By taking the heat of the water mass into consideration, a formula can be developed which is identical with Equation 22 with the exception of an additional term which represents the heat of the water mass.

The form of this additional term is as follows:

$$\frac{162\, C_2 \rho_2 (\overline{T}_2 - T_\lambda) H_m}{k' \rho_1 \left[1 + \frac{C_1}{2k'}(T_\lambda - \theta)\right]} \times \sum_{n=1}^{\infty} \frac{\exp\left[-A_2\left(\frac{\pi}{2} \times \frac{2n-1}{H}\right)^2 t_1\right] - \exp\left[-A_2\left(\frac{\pi}{2} \times \frac{2n-1}{H}\right)^2 t_2\right]}{(2n-1)^2} \qquad (24)$$

where: C_2 = specific heat of sea water,
ρ_2 = density of sea water,
$\overline{T}_2$ = average temperature of the layer in °C.,
T_λ = freezing point of sea water in °C.,
H_m = thickness of layer expressed in meters,
ρ_1 = density of ice,
C_1 = specific heat of ice,
Θ = equivalent temperature in °C.,
A_2 = turbulent heat conductivity of sea water,

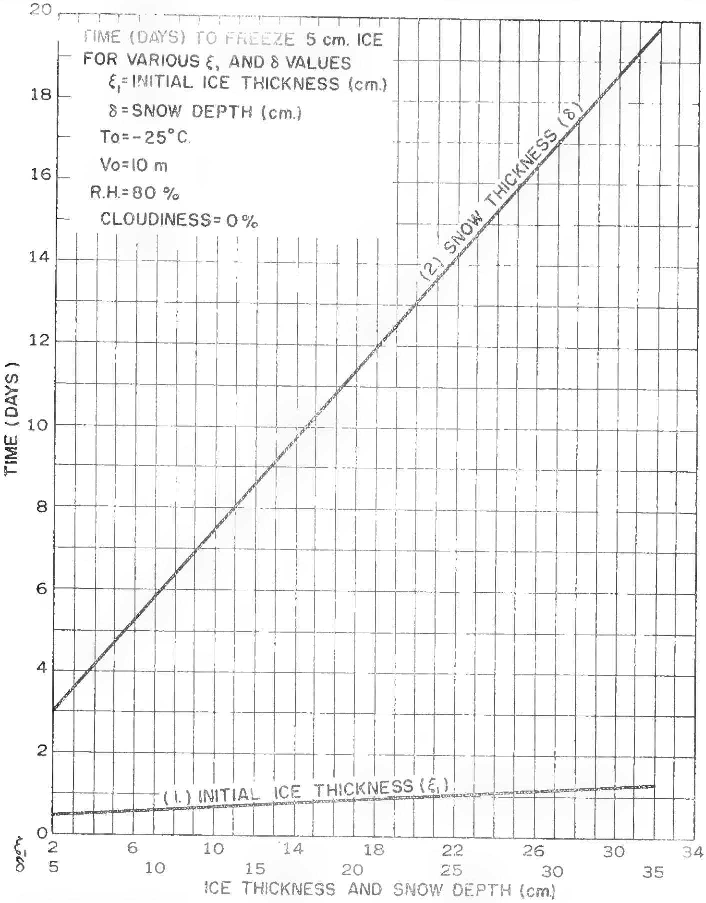

FIG. 7. EFFECT OF SNOW AND ICE THICKNESS ON RATE OF FREEZING

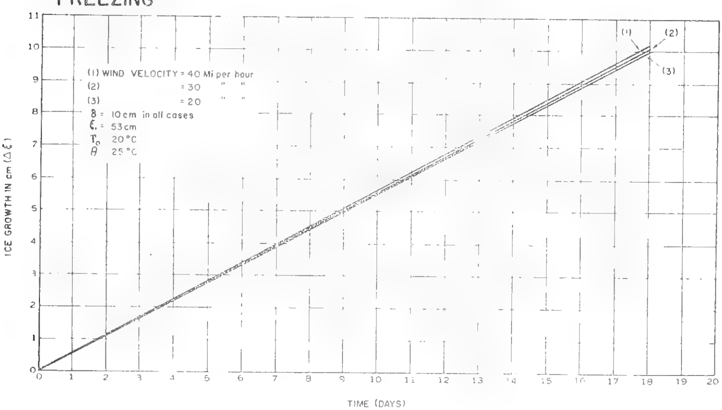

FIG. 8. EFFECT OF WIND VELOCITY ON ICE GROWTH

t = time, and

$$K' = K + T_\lambda \left(C_2 \frac{\rho_2}{\rho_1} - C_1 \right).$$

From this equation, it appears that the larger the depth H to which winter vertical convection reaches, the higher the mean temperature of the layer, and the smaller the coefficient of turbulent heat conductivity, the more important this term becomes. The relative effect of the heat of the water mass is decreased by an increase in the thickness of the snow and ice layers. This effect could be foreseen intuitively, since both the ice and snow layers act as excellent insulators and considerably decrease the flow of heat from the water mass outwards, thus keeping the latter from cooling.

Taking a resultant temperature of −20°C., the freezing point of sea water (T_λ) as −1.8°C., the average temperature of the layer $\bar{T}_2$ as 0.5°C. and the turbulent heat conductivity of sea water (A_2) as 8, the following expression results:

$$2.76\, H_m \sum_{n=1}^{\infty} \frac{e^{-8.47 \times 10^3 \left(\frac{\xi}{H}\right)^2 (2n-1)^2}}{(2n-1)^2} . \qquad (25)$$

The evaluation of the infinite series of (25) is not particularly difficult, as it is rarely necessary to carry the computation to more than 3 or 4 terms to secure the required degree of accuracy. Carrying out the evaluation of equation (24) by using the parameter values as in equation (25) and multiplying by $\Delta \xi$ and dividing by the right hand side of equation (22) evaluated for an air temperature of −20°C. yields the results shown in Figure 10, where the horizontal scale is in days of freezing time added for an air temperature of −20°C., and the vertical scale is ice thickness in cm. Each isoline of mean water temperature indicates the freezing time added by mean temperatures ranging from −1.5° to 0.5°C. in the water layer, which is assumed to be 100 m. thick, and the freezing point of sea water T_λ is taken as −1.8°C. The figure shows that the heat of the water mass adds to the freezing time least when the water temperature is lowest. At a mean temperature of −1.5°C. the heat of the water mass increases the time for freezing 10 cm. of ice from an initial thickness of 15 cm. by about 1.1 days, and to the time for adding a thickness of 70 cm. to an initial thickness of 15 cm. by about 7.6 days. The variation between these two points is essentially linear. At the warmest mean water temperature of 0.5°C., the heat of the water mass adds about 10 days for the 10 cm. addition (to the initial thickness of 15 cm.) and about 54 days for the 70 cm. increment of ice thickness. At this water temperature the variation is nonlinear, showing a slower rate of time increase at high ice thicknesses than at lower ones. Figure 11 shows the total freezing time for ice thickness increments varying from 10 to 70 cm., including the time added by the heat of

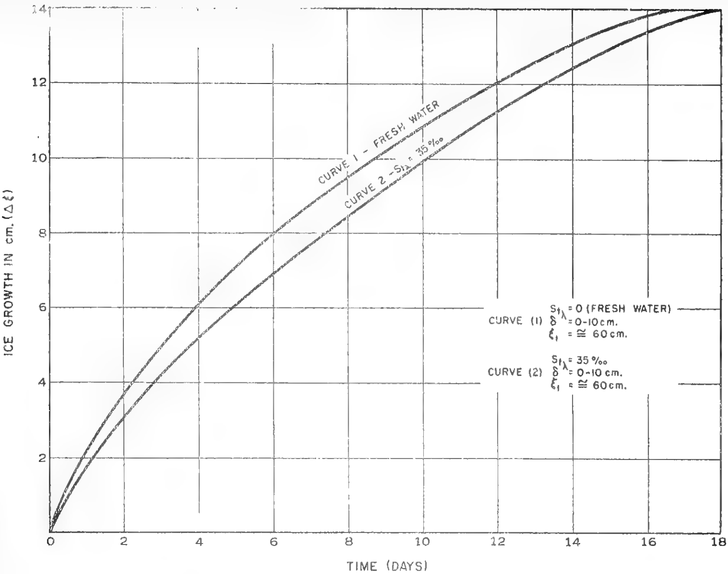

FIG. 9. EFFECT OF SALINITY ON RATE OF FREEZING.

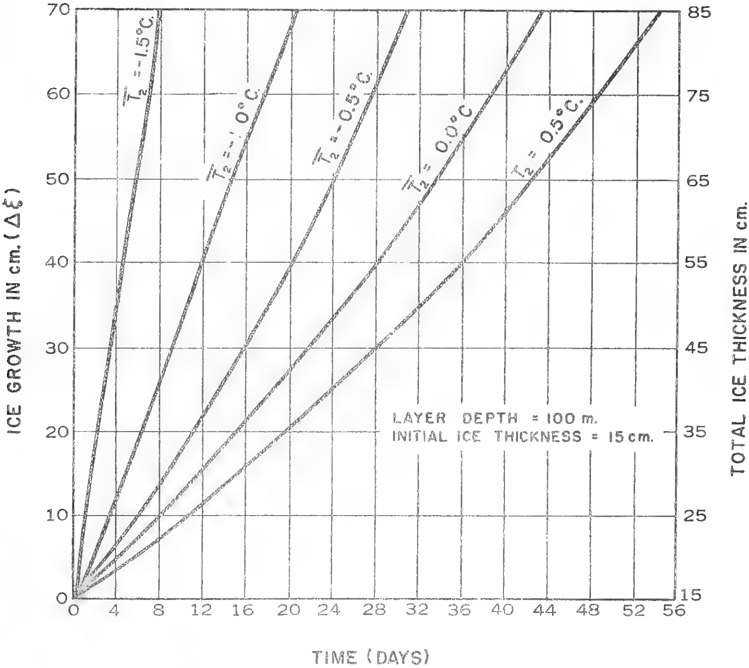

FIG. 10. EFFECT OF HEAT OF WATER MASS ON RATE OF FREEZING.

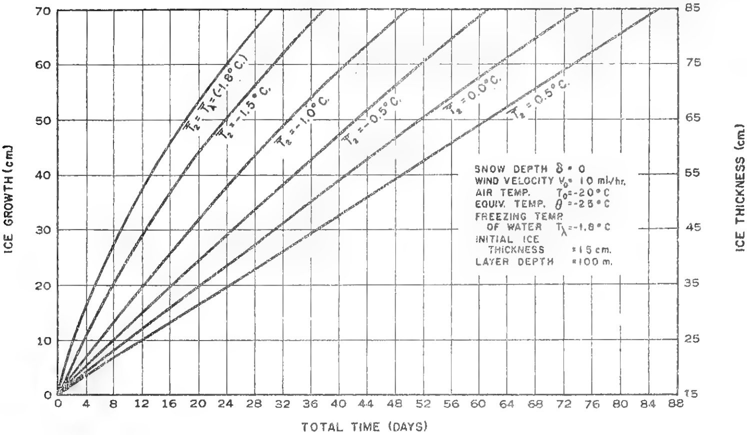

FIG. 11. EFFECT OF WATER TEMPERATURE ON THE GROWTH OF ICE

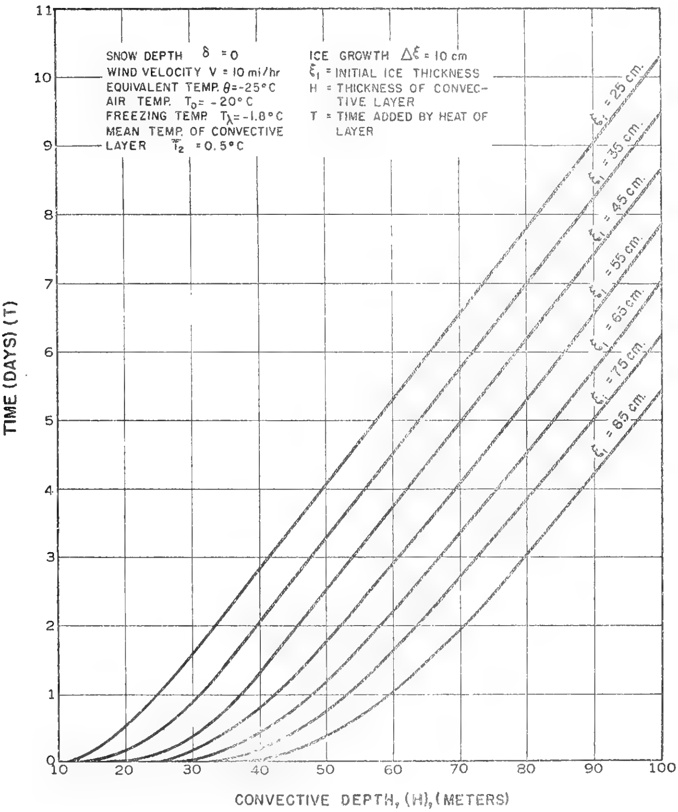

FIG. 12. EFFECT OF THICKNESS OF MIXED LAYER ON GROWTH OF ICE.

the water layer. This figure indicates, that at a mean water temperature of -1.5°C. and for an ice thickness increment of 70 cm., the water mass adds only approximately 18% of the total time, while at a mean water temperature of 0.5°C. the heat of the water mass adds about 63% of the total time.

Figure 12 shows the increase in the freezing time for 10 cm. of ice for differing initial thicknesses ranging from 25 to 85 cm., when the water layer increases in depth from 10 to 100 meters. At an initial thickness of 25 cm. and a layer depth of 10 m., no appreciable time is added to the time needed to freeze 10 cm. of ice when the meteorological conditions are as indicated on the diagram. However, when the warm layer depth is increased to 20 m., the time increases by about 0.5 day. The variation from this point to a layer depth of 100 m. is essentially linear. At the 100 m. layer depth the time added for an initial thickness of 25 cm. is about 10.2 days. All of these values are computed for a mean layer temperature of 0.5°C. At the upper limit of initial ice thickness, 85 cm. the increase of freezing time added by a layer depth of 40 meters is practically zero; at 100-meter depth it is about 5.2 days.

F. PRACTICAL ICE THICKNESS FORECASTING

The above analysis is mainly concerned with the development of theoretical formulas expressing ice thickness as a function of time. In order to take up the problem of the practical prediction of ice thickness there are two main avenues of approach, (1) the graphical method, and (2) the computational method.

The graphical method consists in the utilization of a diagram showing the ice thickness as a function of time and the meteorological parameters. For this purpose the diagrams constituting Figure 13 are suitable. In these diagrams the ice thickness is on the vertical scale, the air temperature on the horizontal scale, and the wind velocity lines indicate the growth curves for varying velocities. The figure is divided into three parts for cloud coverages of 0, 50 and 100% and shows ice growth over a period of 14 hours. Now, for example, if it is required to know the thickness of ice which will result from a temperature of -15°C., wind velocity of 5 m/sec., cloud cover 0%, for a period of 14 hours, the required value taken from Figure 13a is 6.0 cm. If the cloud cover had been 50% instead of 0, Figure 13a would give the required thickness as 5.5. cm., and if the cloud cover were 100%, the thickness taken from Figure 13a would be 4.8 cm. To determine the ice thickness for a longer period than 14 hours it is only necessary to know the average value of the parameters over each 14-hour period and make a cumulative sum of the thickness for integral multiplies of 14 hours plus the fractional part. This diagram, however, has a definite weakness in that it does not take into account the snow cover or the initial thickness of the ice, both of which are important factors in determining the rate at which ice is formed. This is a weakness inherent in all diagrams of this type, because of the fact that it is impossible to include all of the important parameters on the diagram. For this reason,

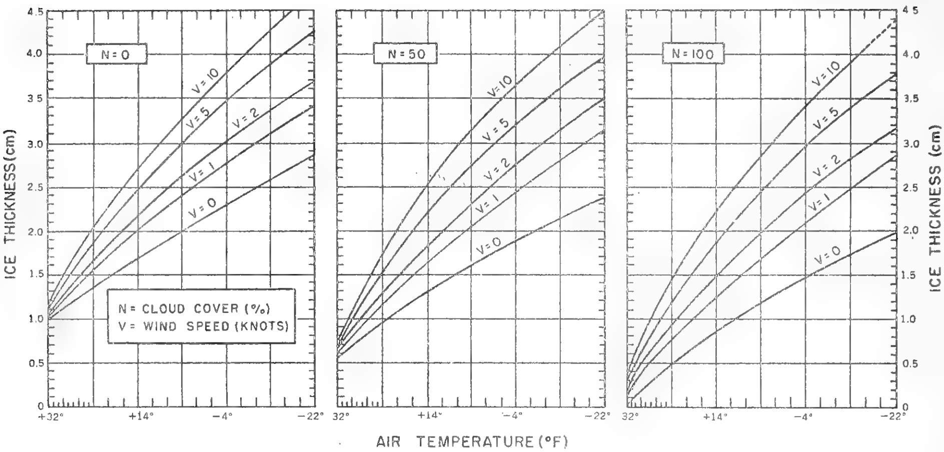

FIG. 13. EFFECT OF CLOUDINESS AND WIND SPEED ON GROWTH OF ICE (FREEZING TIME 14 HOURS)

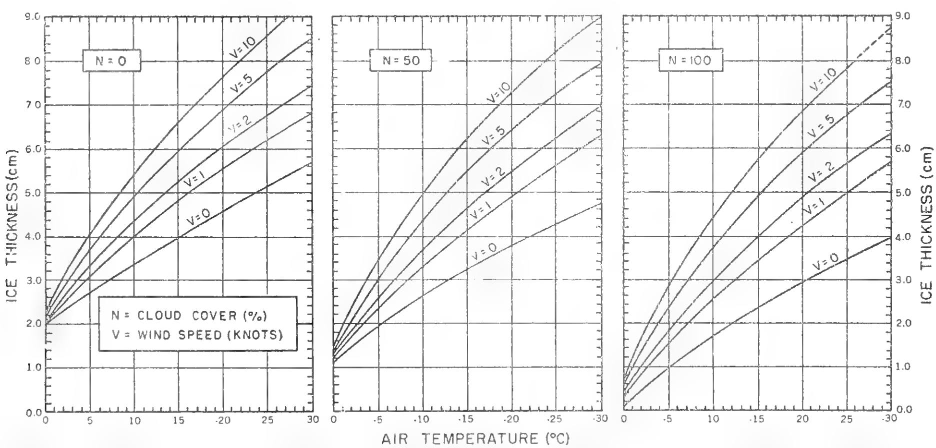

FIG. 13a. EFFECT OF CLOUDINESS AND WIND SPEED ON GROWTH OF ICE (FREEZING TIME 14 HOURS)

then, where greater accuracy is required than is available in Figure 13a, recourse must be made to computation. This is not a difficult process, however, using equation (22). As pointed out previously, it is only necessary to evaluate first the left and then the right sides of the equation by means of the given meteorological and oceanographic factors, and then divide the left by the right side of the equation to find the time expressed in units of a 24-hour day.

That this process yields accurate results can be shown by the following examples: The observed growth of the ice at Archangel averaged over a 12-year period is shown in Figure 14 and the computed growth for the same period in the same figure. The close agreement is immediately evident. It is to be noted that the meteorological parameters used in the computation of Figure 9 are the same as the observed values and that the air temperature was -20°C. and the effective temperature -25°C.

Similarly, the observed ice thickness growth observed on the Denmark Expedition from November 1, 1906, to February 8, 1907, is shown in Figure 15 and the ice growth computed by equation 22 is shown on the same figure. Complete meteorological data including humidity, cloud cover and type of cloud, and snow density were not available from the records, but probable, reasonable values were used which resulted in good agreement with the observed ice thickness data.

More recent ice observation data is that for Padloping Island showing the observed and computed ice growth for January 1949 and January 1950. The observed ice thickness data for this station is shown in curve 1 on Figures 16 and 17. This station is located off Baffin Island, Davis Strait at about 67°N. The weather data for this station were obtained from the Padloping Island Ice Observer's Log, and the snow and ice data from the weather summary. The computed ice growth is indicated in curve 2. The agreement between the observed and computed ice thickness is quite close. The weather, snow and ice data from which the computations were made are indicated on the figures.

G. EMPIRICAL FORMULAS FOR ICE GROWTH PREDICTION

Perhaps one of the best known empirical formulas for ice accretion as it varies with time is that of Zubov (1938). It is of the form:

$$\xi^2 + 50\xi = 8 \sum T_0 \quad , \tag{26}$$

where ξ = ice thickness and
T_0 = air temperature.

This expression was obtained by the use of observed data from a particular location with particular average values of the various parameters involved.

Comparing the results obtained from the use of this equation with observed results obtained at Archangel, averaged for a 12-year period, the

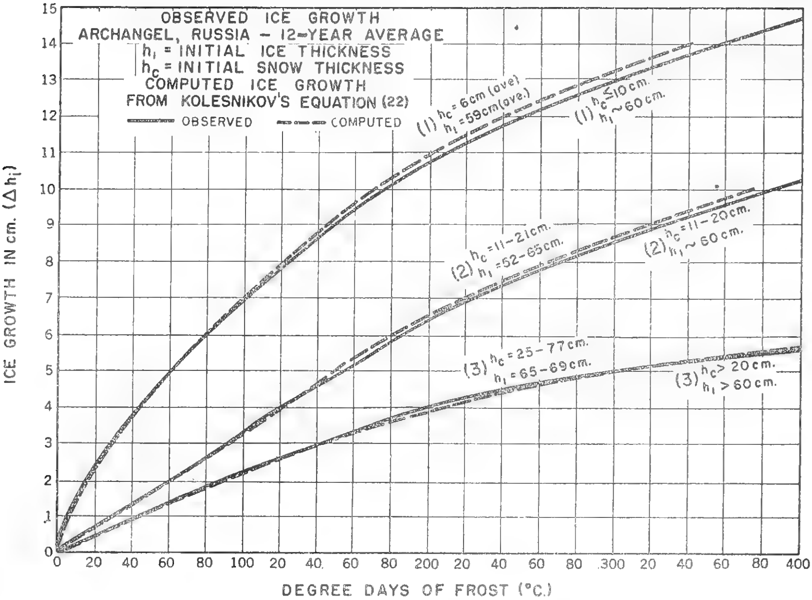

FIG. 14. OBSERVED AND COMPUTED ICE GROWTH AT ARCHANGEL, U.S.S.R.

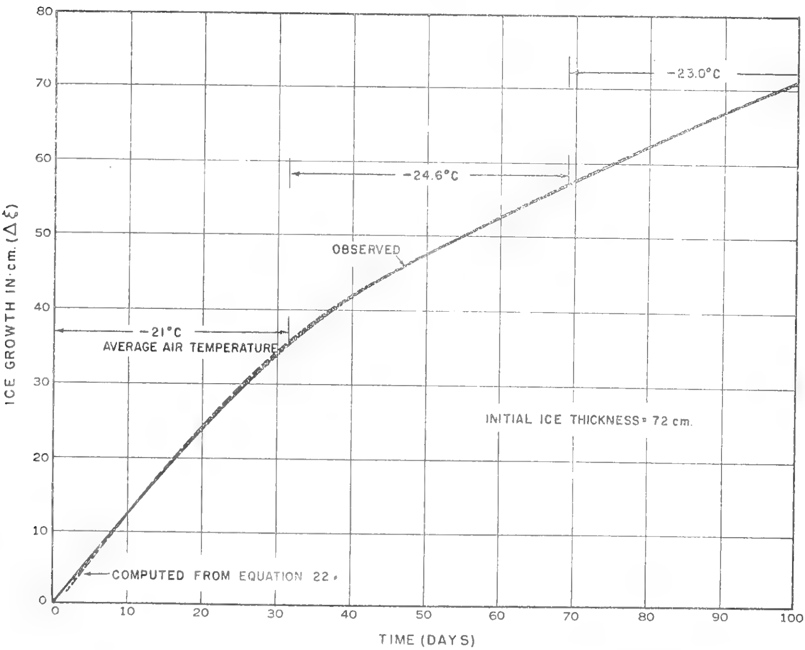

FIG. 15. OBSERVED AND COMPUTED ICE GROWTH, DENMARK EXPEDITION RECORDS, NOV. 1, 1906, TO FEB. 8, 1907.

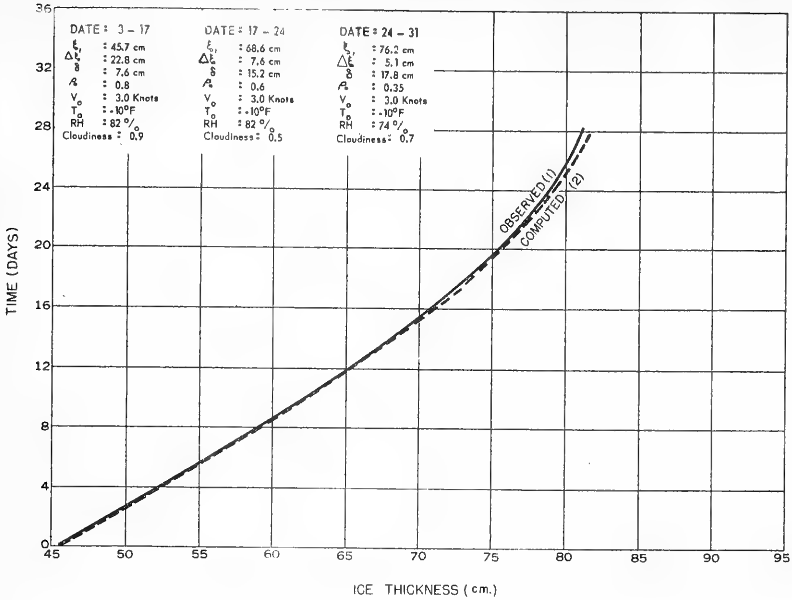

FIG.16. OBSERVED AND COMPUTED ICE GROWTH AT PADLOPING ISLAND, N.W.T., JANUARY 1949.

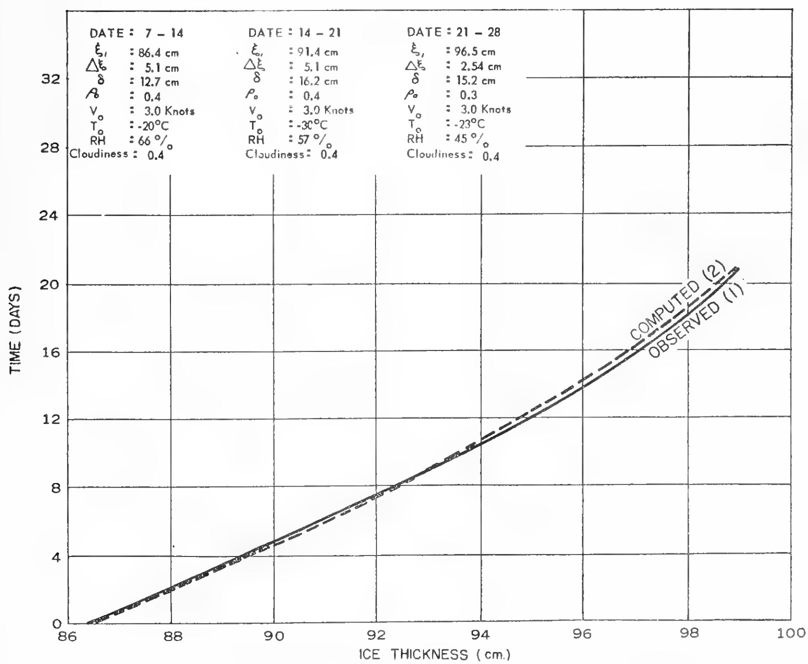

FIG.17. OBSERVED AND COMPUTED ICE GROWTH AT PADLOPING ISLAND, N.W.T., JANUARY 1950.

curves of Figure 18 were obtained. Curve 1 shows the observed time required for the accretion of ice. Curve 2 shows the ice accretion computed by evaluating Zubov's formula with the first constant determined so that the initial value of time was equal to the observed value. Curve 3 is the same when the constant is determined to make the final value the same as the final observed value. Curve 4 is found by using an average of the two constants in curves 2 and 3. A second-degree polynomial of the form of Zubov's equation can be determined by a statistical analysis of the observations. An equation of this type will yield a curve with a comparatively close fit to the observed data. However, in all these cases the value of the constants depends upon the average value of the parameters involved. Thus, this expression of Zubov's as with other empirical relations of this form, is valid only for observations obtained under similar conditions to those for which they are derived and is not generally applicable to all locations and all meteorological and oceanographic conditions.

Another empirical equation of this type is that of Barnes (1928), which is of the form:

$$\xi^2 + 2\xi = \frac{2\lambda_1}{K^1 \rho_1} \Delta T_0 \times t \,, \tag{27}$$

where ξ = ice thickness,

λ_1 = conductivity of ice,

$K^1 = K + T_\lambda \left(C_2 \frac{\rho_2}{\rho_1} - C_1 \right)$,

k = latent heat of crystallization,

T_λ = freezing point of sea water,

c_1 = specific heat of ice,

c_2 = specific heat of sea water,

ρ_1 = density of ice,

ρ_2 = density of sea water,

ΔT_0 = difference in temperature between the top and bottom of the ice, and

t = time.

This expression evaluates theoretically the constant which Zubov secures empirically. No account, however, is taken of snow thickness, which is of paramount importance in determining the rate of accretion of ice thickness.

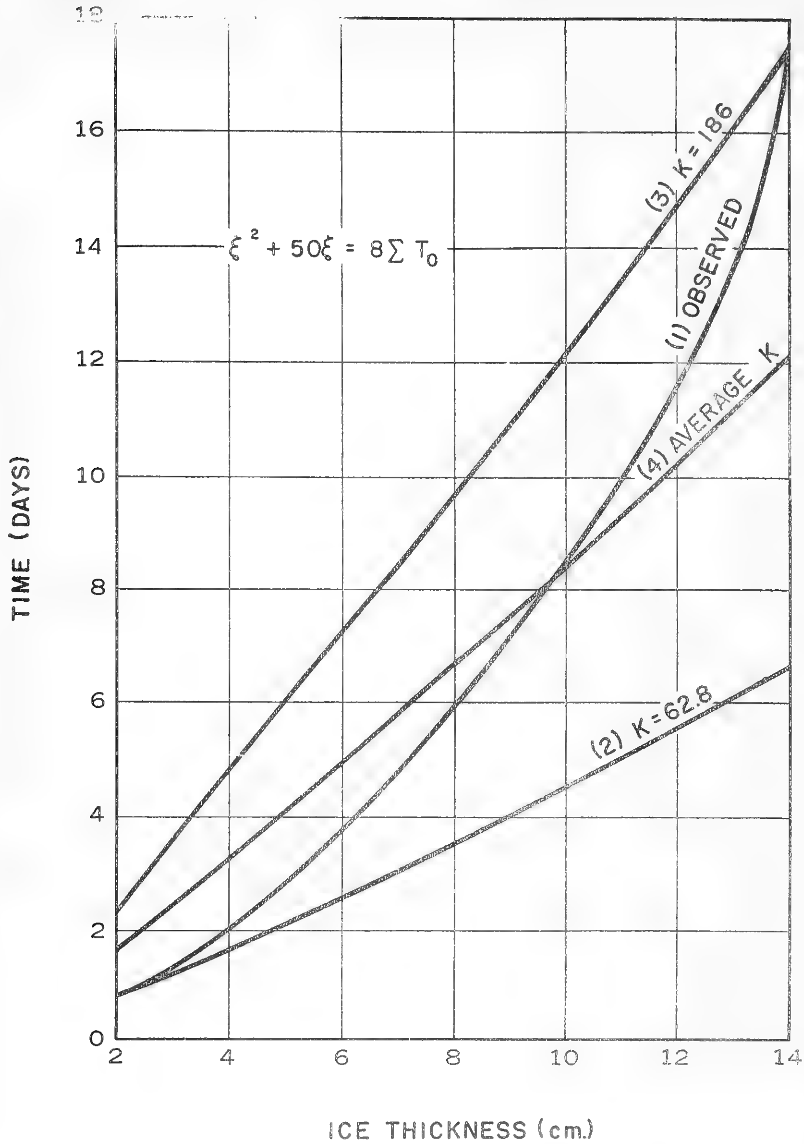

FIG. 18. EMPIRICAL FORMULAE OF ZUBOV'S TYPE COMPARED WITH OBSERVED ICE GROWTH AT ARCHANGEL, U.S.S.R.

H. CONCLUSION

From the foregoing discussion it is evident that for the degree of accuracy necessary under ordinary conditions, the use of equation (23) with a value of taken from Table I is adequate for forecasting ice thickness when there is a snow cover on the ice surface and when the ice has considerable initial thickness. With no snow cover and neglecting the initial ice thickness, the graphical method of Figure 13 furnishes an adequate solution. Only for a greater degree of accuracy is it necessary to make an individual computation of Θ taking account the particular coefficients of radiation, convection, evaporation, condensation, humidity, and reflection.

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Discusses experimental evidence of effect of environmental factors on growth of fresh- and salt-water ice. An equation relating these factors developed by Kolesnikov is evaluated and the results applied to forecasts in specific areas in USSR and Baffin Bay.

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2. Sea Ice Forecasting
3. Sea Ice Potential
4. Ice - Environmental Factors
5. Ice - USSR
6. Ice - Baffin Bay
7. Heat
8. Convection

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iii. H. O. TR-7

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