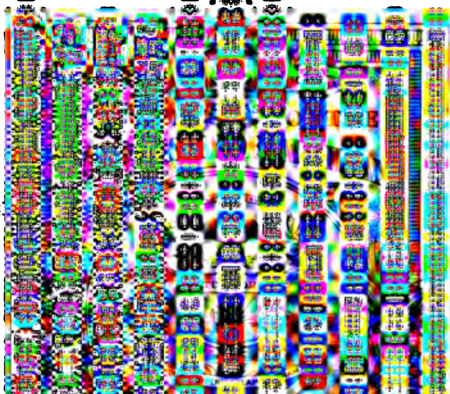

A
NEW and COMPENDIOUS
SYSTEM
OF
OPTICS.

In THREE PARTS, *viz.*

PART I. CATOPTRICS, or the Doctrine of Vision by Rays reflected from *Mirrours*, or polished Surfaces.

PART II. DIOPTRICS, or the Theory of Vision by Rays refracted through *Lenses*, or transparent Substances.

PART III. A Practical Description of a great Number of the most useful Optical *Instruments* and *Machines*, and their Construction shewn from the Theory; *viz.* The EYE, CAMERA OBSCURA, *single* and *double* MICROSCOPES, *Refracting* and *Reflecting* TELESCOPES, PERSPECTIVE GLASSES, the MAGIC LANTHORN, *&c.* The Manner of adapting MICROMETERS to MICROSCOPES and TELESCOPES of the reflecting Sort.

The whole explained, exemplified, and illustrated by a great Variety of Copper-Plate Figures, as big as the Life.

By *BENJAMIN MARTIN.*

Author of the *Philological Library of Literary Arts and Sciences*; and *A Treatise of Logarithms, Common and Logistical, in Theory and Practice.*

Oculi, pars corporis pretiosissima, & qui lucis usu vitam distinguant à morte. PLIN. Nat. Hist. *Lib.* 11. *Cap.* 37.

Nam sic possunt figurari perspicua & specula, ut unum appareat multa —— ut longissimè posita, appareant propinquissima —— ut maxima appareant minima, & alta appareant infima, & oculta videantur manifesta, —— & stellas faceremus apparere quo vellemus. ROG. BACON. Epist.

LONDON:

Printed for JAMES HODGES, at the *Looking-Glass* on *London-Bridge.* MDCCXL.

TO

MARTIN FOLKES, *Esq*;

VICE-PRESIDENT

OF THE

ROYAL SOCIETY, *&c.*

SIR,

THE Reputation You have acquired of a general and profound Knowledge in the Arts and Sciences, as well Literary as Mathematical, and Your particular Taste for *Optics*, and the more delicate Pleaſures ariſing from the various Effects and Inventions of that noble Science, point You out as the moſt proper Patron for my *Syſtem of Optics.* And I preſume the more freely to put my Book out under the Patronage of Your Name, as You have been pleaſed to let me know You thought my former Pieces worth Your Notice and Peruſal. And as You have honoured me with Your Approbation of my new Pocket Microſcopes, I ſhall take the Freedom, e'er long, of preſenting You with ſomething new in the Reflecting-Teleſcope Way, if Succeſs attends my Deſigns. I ſhould think myſelf in nothing happier, than in doing ſomething that You ſhall approve and accept. And if the following Book can be thought to have ſo much Merit,

Merit, my higheſt Ambition will be ſatisfied: And I take this Opportunity of letting You know how much I value Your Judgment and Friendſhip, and am, with all Gratitude and Reſpect,

SIR,

Your moſt obliged, and

Moſt obedient Servant,

BENJ. MARTIN.

THE PREFACE.

AS *the* Eye *among the Organs of the Body, ſo* Optics *among the Sciences, is the moſt delicate, curious, and uſeful; as the Parts and Structure of the Eye ſurpaſs that of moſt other organical Parts, in Point of Mechaniſm and wondrous Contrivance, ſo the Principles and Theorems of* Optics *are of a peculiar Nature, wonderful in their harmonious Origin, and expreſs a whole Science in a Line. Laſtly, As the Eye is that Organ by which*

which we have all our Ideas of the Magnitude, Order, Number, Dispo-sition, Colours, &c. *of Things about us; so* Optics *is that Science which a-lone accounts for the* Reason *and* Man-ner *of such Sensations: And a Man not versed in the* Visual Science, *can no more properly be said to* see Things rationally, *than a Quadruped; and has little better Notions of apparent Magnitudes and Distances of Things, than a blind Man has of Colours.*

THUS Optics *is in itself of the last Importance, and to be well under-stood to discriminate the Man from the Brute, in Regard of the noble* Sense of Seeing. *But this is not all; it is also the* Ground-Work, *or* Fundamen-tal Science, *to many others, as* Perspec-tive Painting, Architecture, Astrono-my, my, Dialling, Surveying, &c. *How lame and imperfect must any of these Arts and Artists be, were they not assisted and succoured by the Principles and Rules of this noble Science?*

AGAIN, If we regard Curiosity, what Science can compare with Optics, *in the whole Encyclopædia? What Gratifications of Sense so exquisite as those of Sight? By* Optics *the Heavens have been revealed to us; the Spots and Feculæ on the Face of the* Sun, *the Horns and waining Phases of* Venus, *the Mountains and Vales in the* Moon, *the Satellites and Belts of* Jupiter, *and* Saturn *with his wondrous Ring; besides innumerable Stars not otherwise to be seen but by the Telescope! What Pleasure, yea, how useful is it, to have Objects thirty or forty Miles*

Miles off, brought within the Distance of one Mile, or half a Mile? Yet this the Telescope *effects. Again, What Discoveries have been made in the World of Miniature, where Objects so very small as otherwise must ever have been unseen and unknown by us, are made conspicuous, and rendered visible in their minuter Parts? Who could ever have thought of the* Animalculæ *in Water, in* Semine, *&c. The Eels in Vinegar and Water; the Pores and Air-Vessels in Wood; the pearly Drops on Leaves of various Plants; the vesicular Substance of Beans, Pease, and all Kind of Pulse; the curious Forms, the particular Structure of Parts, and the rich Colours that adorn most of the invisible Tribes of Animals! Yet all this, and ten thousand times more, is performable by the* Microscope; *an In-*

strument

ſtrument which no reaſonable Man ſhould want, inaſmuch as it ſerves him inſtead of Microſcopic Eyes. *As to the Benefit thoſe receive from this Science, whoſe Eyes require the Aid of* Spectacles, *it is ſo great and ſo general, that it would be meer Impertinence to pretend to expatiate upon it; and many other invaluable Bleſſings reſult from this* Art of improving Sight, *which the Experience of Mankind has long ſince evinced, we can't be without, and not be miſerable.*

YET notwithſtanding what has been ſaid of the exceeding Uſefulneſs of this Science, I am too well convinced but very few underſtand any Thing of it. If you aſk why they do not ſtudy Optics, *they re-interrogate, what Books ſhould we read? If you refer them to Mr*

Mr Molyneaux, *that is too large, and too much perplex'd with algebraical Solutions, and is therefore only fit for Scholars. If you recommend Dr* Gregory's *Elements, the Geometrical Demonstrations of every Proposition deter the Reader, and Mr* Browne's *Supplement thereto involves him in a Labyrinth of analytical Investigations and Solutions, with little Order and Perspicuity, and great want of Schemes. If, lastly, you advise them to read Dr* Smith's Treatise of Optics, *they tell you it is too expensive, and so voluminous, that they cannot pretend to have Time for reading so much upon the Subject, besides that by far the greatest is above their Understanding. These, and such like, are the Objections to the Books extant, and therefore it was judged necessary to draw up a new* System

System of Optics, *which might, if possible, obviate those Objections, and remove the Difficulties that have hitherto discouraged Persons from the Study of so excellent a Science.*

IN Order to this I judged it necessary to dispatch the Theory in as short, yet plain a Manner as possible; this in Catoptrics *I have done from the admirable universal Theorem, invented by the late Mr* Humphrey Ditton; *and in* Dioptrics *I was supplied with that wondrous Theorem, which expresses the whole Science in half a Line, and is one of the many noble Inventions of the justly renowned Dr* Halley. *These two general Theorems I have explained and branched out into all the particular Cases that can arise from different Rays, Mirrours, and Lenses, Distances,*

and

and Positions of Objects; and after that, lest any should not understand the Theory in Species, I have carefully explained, or rather expressed, each Theorem (in Catoptrics *and* Dioptrics) *in* Words at Length, *and so reduced them to Rules, by which any Case may be solved truly, by those who know nothing of* Algebra *or* Geometry. *I have also given Examples (to every Case that required it) in* Numbers, *and illustrated them by Schemes and Figures of* Rays, Mirrours, *and* Lenses, *as large as the Life, which has never been done before that I know of.*

AFTER the Theory, in the Third Part, you have an Account of all the useful Optical Instruments and Machines, whose Nature and Construction, are fully explained from the Theory;

and

and their Uſes exemplified by divers and familiar Examples. In ſhort, whatever I judged curious, new, and worth the Reader's Notice, I have inſerted it all along; but ſtudiouſly avoided all nugatory Remarks, and the Minutiæ *of the Art, on one Hand; and all oſtentatious Subtilties, and uſeleſs Diſquiſitions, that may puzzle but not profit the Reader. My ſole Deſign being to render the Study of this Science as eaſy, delightful, and as general, as poſſible; to effect which, I have done all that is is in my Power; and I can neither do nor ſay any more.*

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CHAP. I.

DEFINITIONS.

I. OPTICS is a Science which teaches the Nature, Properties, and Laws of *Vision*, arising from the Rays of Light, either *reflected* from the Surfaces of Bodies, or *refracted* in passing thro' them, and painting the Images of Objects on the *Retina* on the Bottom of the Eye. Also this Science, in it's most extensive Acceptation, comprehends the whole Doctrine of *Light* and *Colours*, and all the *Phænomena* or Appearances of *visible Objects*. Optics, therefore, consisteth of three Parts, *viz. Catoptrics*, *Dioptrics*, and *Chromatics*.

II. CATOPTRICS is that Part which treats of *Reflex Vision*, or all that relates to the viewing of Objects by Light *reflected* from the Surfaces of Bodies, whether *plain*, *convex*, *concave*, or otherwise; and in Rays *diverging*, *converging*, or *parallel* to each other.

III. Dioptrics treats of the Properties of Light and Viſion, arising from Rays paſſing thro' tranſparent *Media* or Bodies, as *Air*, *Water*, *Glaſs*, *Cryſtal*, *Diamond*, &c.

IV. Chromatics treats of the *Colours* of Light and natural Bodies. Of this Part Sir *Iſaac Newton*'s Optics does almoſt entirely conſiſt.

V. Light is that Property of ſome Bodies by which Objects are rendered *viſible*, or capable of being *ſeen* by the Eye. It conſiſts of very ſmall Particles which iſſue from the luminous Body in ſtrait Lines.

VI. Rays or Beams of Light are thoſe Streams or Emanations of Light, which proceed from the luminous Body, and enlighten or illuminate all Objects ſo that they may be *ſeen*.

VII. The Radiant is that Body or Object which emits, or from which proceed, the Rays of Light under Conſideration at any Time.

VIII. The Species of an Object is the Image or Repreſentation thereof, made by the Rays of Light in the *Focus*, or Place where they unite.

IX. Parallel

IX. Parallel Rays, are ſuch as proceed *equally diſtant* from each other thro' all their Courſe; as thoſe from the *Sun*, and other vaſtly diſtant Objects. See Fig. 1. Plate I.

X. Converging Rays are ſuch as, proceeding from a Body, approach nearer and nearer together in their Progreſs, tending to one certain Point, where they all unite; thus the Rays proceeding from the Object AB to the Point F, are ſaid to *converge* towards that Point. Fig. 2.

XI. Diverging Rays are thoſe, which, proceeding from any Point, as A, do continually recede from each other as they paſs along in their Courſe towards BC. Fig. 3.

XII. The Focus of *Rays* is that Point to which all Converging Rays tend, and in which they unite and interſect each other; as the Point F, Fig. 2. And this is called the *Real Focus*; but

XIII. The Virtual or *imaginary Focus*, is a Point, as F, to which the Rays AB tend, and where they would unite, were they not intercepted by the Obſtacle (ſuppoſe a Mirrour) CD; by which Means they are turn'd aſide, and made to converge in their *Real Focus* F. Fig. 4.

XIV. Reflection of Rays is their *Regress* or *Returning* from the Surface of ſuch Bodies on which they fall, and cannot penetrate or enter. Thus the Ray B C falling on the Surface A D, is reflected or turned back or up again in the Direction CE.

XV. The Plane of *Reflection* is that in which the reflecting Point, or Surface, is ſituated, as A D in Fig. 5. and *a d* in Fig. 6 and 7.

XVI. Mirrours or *Speculums* are thoſe Bodies whoſe Surfaces are ſo very ſmooth, and fine poliſh'd, as to be impervious to the Rays of Light which fall on them, and which therefore they reflect ſo entirely, as to repreſent the *Images of Objects* oppoſed to them. Theſe are generally made of Glaſs poliſh'd on one Side, and quickſilver'd on the other; and are either *plain*, *convex*, or *concave*.

XVII. Plain Mirrours are thoſe whoſe Surfaces are perfect *Planes*, and whoſe Section is a *ſtrait Line*, as A D, Fig. 5. Note, theſe are vulgarly called *Looking-Glaſſes*.

XVIII. Convex Mirrours are ſuch whoſe Surfaces do every Way equally and uniformly riſe above the Plane of their Baſes or loweſt Parts; the Section of which Sort of Mirrour is a *Curve*, either *Circular*, *Elliptical*, *Parabolical*,

Parabolical, or *Hyperbolical.* See Fig. 6. where A D is a *Circular* Section, and the *Mirrour* is the Segment of a *Globe*, or *Spherical* Surface, which are of most common Use. As

XIX. Concave Mirrours are those whose Surfaces sink down with an uniform Hollowness or Curvity, below the upper Parts A D, and whose Section also is a *Curve*, as various as the Convex above; but A D in Fig. 7. is circular, and it's Surface the internal Part of an *hollow Sphere*, as being most in Use.

XX. The Incident *Ray* is that which comes from any Object, and falls on the reflecting Surface as B C; and C E is the *reflected Ray*.

XXI. The Angle of Incidence is that which is contained between the *incident Ray* B C, and a perpendicular to the reflecting Surface in the Point of Reflection F C, *viz.* the Angle B C F. Fig. 5, 6, 7.

XXII. The Angle of Reflection is that contained between the said perpendicular F C, and the reflected Ray C E; *viz.* the Angle F C E. Fig. 5, 6, 7.

XXIII. Refraction of Rays is their being bent or turned out of their first Course, in passing out of one *Medium* into another. Let ADHI be a Body of *Water*, AD it's Surface, C a Point in which a Ray of Light BC (in the Air) begins to enter the same; this Ray, by the greater Density of the Water, will be resisted, and instead of passing strait forwards in it's first Direction to K, it will be bent therefrom, and made to describe the Tract CE, which is called the *refracted Ray*. Let FG be drawn perpendicular to the Surface of the Medium in C, then it is plain the Ray BC, in passing out of a *rarer Medium* (*viz.* of *Air*) into a *denser Medium*, (*viz.* of *Water*) is refracted into a Ray CE, which is nearer to the perpendicular CG, than the *incident Ray*; and, on the contrary, the Ray EC passing out of a *denser* into a *rarer Medium*, will be refracted into CB, which is farther from the perpendicular.

XXIV. The ANGLE BCF is the Angle of *Incidence*, as before; and FCG is the *Angle* of *Refraction*, as being contained between the refracted Ray CE, and the perpendicular CG.

XXV. A LENS is a Medium, generally of *Glass*, of a proper Form to *collect* or *disperse* the Rays of Light which pass through it. Of these there be various Forms, and which, from thence, receive divers Names. As

XXVI. A Plano-Convex, which hath one Side plain, the other Spherical or Convex; as (Fig. 9.) A.

XXVII. A Plano-Concave, *plain* on one Side, and *concave* on the other; as B, Fig. 9.

XXVIII. A Double-Convex, is one *Convex* on both Sides; as C, Fig. 9.

XXIX. A Double-Concave, is one *Concave* on both Sides; as D, Fig. 9.

XXX. A Meniscus, is one *Convex* on one Side, and *Concave* on the other; as E, Fig. 9.

XXXI. The Vertex of a Mirrour or *Lens*, as A B, is the middle Point V, every Way equally diſtant from it's Baſe. Fig. 10.

XXXII. The Axis of a *Mirrour* or *Lens* is the Right Line E D, drawn thrō' the Vertex V, and the Center C, on which it was deſcribed.

XXXIII. The Visual or Optic *Angle*, is that which is contained under the two Right Lines drawn from the extreme Points of an Object to the Eye; thus A E B or C E D

[illegible] ch the
[illegible] at E.

[illegible] double
[illegible] gether
[illegible] ch one
[illegible] oint of
[illegible] L F N
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[illegible]

C H A P.

Fig. 8.

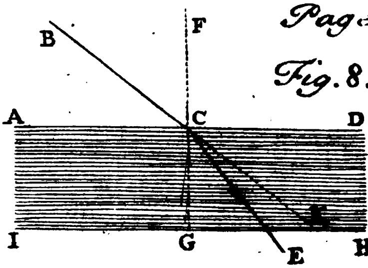


Fig. 9.

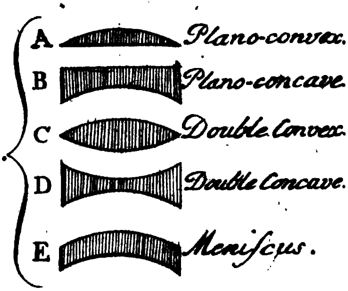


Fig. 10.

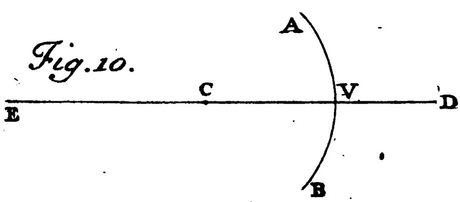


Fig. 11.

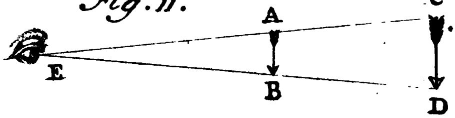
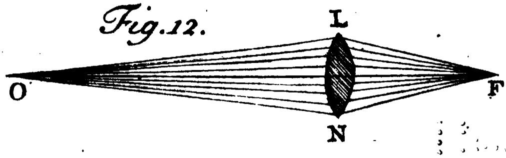


Fig. 12.



CHAP. II.

The Principles of CATOPTRICS *and* DIOPTRICS.

WHAT I shall here propose and lay down as the *Principles of Optics*, are those things on which the following *New Theory*, both of *Catoptrics* and *Dioptrics*, does chiefly depend; and which are in themselves *very evident*, or have been *demonstrated* by Mathematicians, or confirm'd by *Experiments*; or lastly, are such as tho' not *strictly or geometrically true*, may nevertheless be *assumed as such*, without any sensible Error.

PRINCIPLE I.

In very small Angles BCE, *the Sine of the Angle* AD, *the Arch* DB, *and the Tangent* EB, *are very nearly equal to each other.* Plate 2. Fig. 1.

For suppose the Radius CB divided into 100000 equal Parts, the Sine, the Arch, and the Tangent, will each of them be but 29 of these Parts, when the Angle or Arch is but one Minute of a Degree. Or if the Arch be of of one Degree, it's Length will be $1745\frac{3}{10}$, that of the Sine $1745\frac{2}{10}$, and that of the Tangent $1745\frac{5}{10}$, of ſuch Parts. The Difference being ſo extremely ſmall, the Propoſition is evident.

PRINCIPLE II.

In every Triangle, the Sides are proportional to the Sines of their oppoſite Angles.

This is demonſtrated by all the Writers of *Trigonometry*. And therefore, when theſe Angles are very ſmall, *they have the ſame Proportion with their oppoſite Sides*, by Princip. I.

PRINCIPLE III.

In every Plain Triangle B C D, (Fig. 2.) *if the Side* C B *be produced, the outer Angle* A B D *ſhall be equal to the Sum of the Angles at* C *and* D. By *Euclid*'s 32d Prop. of Book I.

PRINCIPLE IV.

Let A B (Fig. 3.) be part of a Circle deſcribed with the Radius C B; and E B an Arch deſcribed with the Radius D B, let C D join the Centers C and D, and draw D E; this ſhall interſect the Arches in G and E, and G E will be the Difference between D E and D B. *Now it is evident the greater* D C *is, the leſs will be* G E, *the Angle* B D E *remaining*

ing the same; also the nearer the Point E *is to* B, *the less is the Difference* GE; *and therefore, when* E *is very near to* B, *the said Difference will be very small, and inconsiderable; and consequently, the Lines* DB *and* DE *may, in such a Case, be assumed equal to each other.*

PRINCIPLE V.

The incident and reflected Ray are both in the same Plane.

PRINCIPLE VI.

The Angle of Incidence BCF *is ever equal to the Angle of Reflection* FCE. (Plate 1. Fig. 5.)

This is evident from Experiment. See the *Philos. Grammar*, Book I. Chap. VI, Note *, and is demonstrated by Dr *Gregory* in his *Elem. of Catop.* Prop. 1.

PRINCIPLE VII.

The incident and refracted Ray are both in the same Plane.

PRINCIPLE VIII.

Refraction out of a rarer Medium into a denser, is made towards the perpendicular; and vice versa *into a rarer Medium. See Defin.* 23.

PRINCIPLE

PRINCIPLE IX.

The Sine AD *of the Angle of Incidence* BCF, *is either accurately or very nearly in a given or constant Ratio, or Proportion to the Sine* HR *of the Angle of Refraction* ECG. Fig. 4.

This Ratio of the Sines, when the Refraction is made out of *Air* into *Water*, is as 4 to 3; that is, AD : HR : : 4 : 3, in Water.

When the Refraction is out of *Air* into *Glass*, the Proportion is as about 17 to 11, or more nearly, as 31 to 20, or as 77 to 50; but for common Use, as 3 to 2; that is, AD : HR : : 3 : 2, in Glass.

If the Refraction be out of Air into Diamond, it is as 5 to 2; and AD : HR : : 5 : 2. For the denser the Medium is, the less will be the Angle and Sine of Refraction.

PRINCIPLE X.

Wherever the Rays of Light, which come from all the Points of any Object, meet again in so many Points after they have been made to converge by Reflection or Refraction, there they will make a Picture of the Object upon any white Body on which they fall.

It

It was necessary to premise this as a *Principle* of *Optics*, tho' the Grounds and Truth of it cannot, till after a few Pages, be made to appear.

PRINCIPLE XI.

An Object seen by reflected or refracted Rays appears in that Place, from whence, after their last Reflection or Refraction, they diverge in falling on the Spectator's Eye.

PRINCIPLE XII.

The apparent Magnitude of an Object is determined or estimated by the Magnitude of the Optic Angle which it subtends, or under which it appears at the Eye of a Spectator.

Thus (in Fig. 11. Plate I.) the Magnitude of the Object AB is estimated or measured by the Quantity of the Angle AEB; so is also the Object CD; but CD is greater than AB; and since they appear under the same Angle, it is evident, the *apparent Magnitudes* of Objects may be equal, when their *true or real Magnitudes* may be unequal in any given Proportion.

CHAP.

CHAP. III.

The Theory of CATOPTRICS.

1. LET GE represent a Portion of a Convex *Speculum* Mirrour; C it's Center; CA or CE the Semidiameter or Radius; let D be a radiant Point in the Axis of the *Speculum*, from whence DA, a Ray of Light proceeding, falls on the Point A, and is reflected into the Direction Af, tending to a Point F, it's *virtual Focus*, in the Axis of the *Speculum* behind the Vertex E. Fig. 5.

2. Let $DE = d$; $CA = CE = r$; $CF = z$; and $FE = f = r - z$; and consequently, $f + z = r = CE$.

3. Suppose the Point A very near to E; then will the Angles at D and C be very small, and consequently will have the same Proportion to each other as their opposite Sides AC and AD have; by Princip. 1, and 2. But $AC = AE$, and DA may be esteemed equal to DE; by Princip. 4. Therefore it will be $ADC : ACD :: CE : DE :: r : d$.

4. Produce CA to I, so shall IA be perpendicular to the Mirrour in the Point of Reflection A; and therefore the Angle $DAI = IAf$, by Princip. 6. But $DAI = dAC$, and

and IAf = CAF, as being Angles at the Vertex of two interfecting Lines, by *Euclid*, Book I. Prop. XV. Therefore dAC = CAF. Again, dAC = ADC + ACD = $r + d$, by Princip. 3. and confequently the Angle CAF $= r + d$.

5. In the Triangle CFA (the Point A being very near E) the Angles at A and C will be very fmall, and will have the fame Proportion as their oppofite Sides FC and FA, by Princip. 1, and 2; that is, the Angle FAC : FCA :: FC : FA : But in this Cafe FA may be efteemed equal to FE, by Princip. 4, and therefore the Angle FAC : FCA :: FC : FE :: $z : f$.

6. But the Angle at C is as DA or DE, that is, as d, by the 3d Step of this Theory; and the Angle FAC as $r + d$, by the 4th Step. Therefore $f : z :: d : d + r$. And by Compofition of Ratio's, $f + z : f :: 2d + r : d$; but $f + z = r$, by Step 2. hereof; and therefore $r : f :: 2d + r : d$; then multiplying the Extremes and Means together, we have $dr = 2df + fr$; dividing this Equation by $2d + r$, there refults this Equation $\frac{dr}{2d + r} = f =$ EF.

7. Therefore in any *Speculum*, when r, or the Radius of it's Curvity, and d, or the Diftance of any Object in the Axis thereof, are known, then f, or the Diftance of the Focus F from the

Vertex E, will alſo be known, or given, for all Rays proceeding from the Point D, and falling on the Mirrour A E on either Side the Axis.

8. This Theorem is applicable to the Caſes of all Kind of Rays, reflected from all Sorts of Mirrours, whether *Convex*, *Concave*, or *Plain*; and is therefore the primary and fundamental Theorem of *Catoptrics*; from which I ſhall now deduce ſuch *particular Theorems* as are ſubſervient to the Solution of all the different Caſes that may ariſe from the threefold Variety of Mirrours and Rays incident upon them. And firſt for

CONVEX MIRROURS.

9. CASE I. Of DIVERGING RAYS. The Ray D A proceeding from a radiant Point D in the Axis D C, at an indefinite Diſtance from the Vertex of the Mirrour E, gives the Diſtance of the Focus F from the Vertex E, according to the fundamental Equation, which therefore I ſhall call the firſt Theorem, *viz.*

$$\frac{dr}{2d+r}=f. \quad \text{Theorem 1.}$$

10. If $d=r$, we have $\frac{dr}{2d+r}=\frac{rr}{3r}=\frac{r}{3}=f$. Theor. 2.

If $d=\frac{1}{2}r$, then $\frac{dr}{2d+r}=\frac{\frac{1}{2}rr}{2r}=\frac{r}{4}=f$. Theor. 3.

If

If $d = \frac{1}{3} r$, then $\frac{dr}{2d+r} = \frac{\frac{1}{3} rr}{\frac{2}{3} r + r} = \frac{r}{5}$ $= f$. Theor. 4.

If $d = \frac{1}{4} r$, then $\frac{dr}{2d+r} = \frac{\frac{1}{4} rr}{\frac{1}{2} r + r} = \frac{r}{6}$ $= f$. Theor. 5.

From hence it is evident, that the Point D and F do both approach the Mirrour in a regular Manner, till at last they coincide at it's Vertex.

11. CASE II. Of PARALLEL RAYS. Fig. 6. If the radiant Point D be supposed to recede from the Mirrour E, to a vast or infinite Distance, the Rays DA, which come from it to the Mirrour, will be *parallel* to the Axis DC, or very nearly so; as is represented Fig. 6. In this Case, therefore, d or DA being *infinite*, the Quantity r as being *finite*, will vanish out of the Equation: For no infinite Quantity can be made *greater or less*, by the Addition or Substraction of any finite Quantity how great soever. Consequently the Equation $\frac{dr}{2d+r} = \frac{dr}{2d} = \frac{r}{2} = f$. Theor. 6.

12. CASE III. Of CONVERGING Rays. As in *Diverging Rays* the Radiant D was always posited in the Axis of the *Speculum*, directly before it's Vertex E, so in *Converging*

 Rays,

Rays, the incident Ray D A will tend towards a Point on the Axis behind, or on the other Side of the Vertex, as at *d*, in Fig. 7, 8, 9, 10. And as the Diſtance D E or *d*, in the former Caſe, had an *affirmative* Sign, *viz.* $+d$; ſo in this preſent Caſe, E *d* being on the contrary Part will have a contrary, that is, a *negative Sign*, thus $-d$; ſo that the fundamental Theorem for Converging Rays will ſtand thus $\frac{-dr}{-2d+r}=f$.

13. In this Theorem, the Dividend being a negative Quantity, *viz.* $-dr$; and the Diviſor $-2d+r$, conſiſting of a negative Part $-2d$, and an affirmative one $+r$; it is evident, when the negative Part exceeds the affirmative, the Quotient *f* will be affected with an affirmative Sign, *viz.* $+f$, as before; but if $-2d$ be leſs than $+r$, the Quotient will be negative, *viz.* $-f$; that is, the incident Ray D A will be ſo reflected as to converge towards a Point *f*, in the Axis before the Mirrour, as it did to a Point F, behind it in the other Caſes.

14. This being premis'd, it is plain the Theorem for Converging Rays will admit of four conſiderable Varieties, *viz.*

If $2d$ exceed r, then $\frac{-dr}{-2d+r}=+f$.

Theor. 7. Fig. 7.

If

If $d = r$, then $\frac{-dr}{-2d+r} = \frac{-rr}{-2r+r}$ $= \frac{-rr}{-r} = r = +f$. Theor. 8. Fig. 8.

If $2d = r$, then $\frac{-dr}{-2d+r} = \frac{-\frac{1}{2}rr}{0}$ $= \infty f$. Theor. 9. Fig. 9.

If $2d$ be leſs than r, $\frac{-dr}{-2d+r} = -f$. Theor. 10. Fig. 10.

Note, $\left\{\begin{matrix} +f \\ \infty f \\ -f \end{matrix}\right\}$ ſignifies an $\left\{\begin{matrix} \text{Affirmative} \\ \text{Infinite} \\ \text{Negative} \end{matrix}\right\}$ Focus.

CONCAVE MIRROURS.

15. As in *Convex Mirrours* the Radius C E lay on the *Right Hand* of the Vertex E, and had an *affirmative* Sign, as $+r$; ſo in *Concave Mirrours*, the Radius C E lying on the other Side the Vertex E, or to the *Left*, muſt have a *negative* Sign of Courſe, or be repreſented by $-r$; and therefore the fundamental Theorem for *Concave Mirrours* will become $\frac{-dr}{2d-r} = f$.

16. CASE

16. CASE I. Of DIVERGING RAYS. The Theorem is as above, and hath alſo four Varieties, *viz.*

If $2d$ be leſs than r, then $\frac{-dr}{2d-r} = +f$. Theor. 11. Fig. 11.

If $2d = r$, then $\frac{-dr}{2d-r} = \frac{-\frac{1}{2}rr}{o} = \infty f$. Theor. 12. Fig. 12.

If $d = r$, then $\frac{-dr}{2d-r} = \frac{-rr}{2r-r} = -r = -f$. Theor. 13. Plate III. Fig. 1.

If $2d$ exceed r, then $\frac{-dr}{2d-r} = -f$. Theor. 14. Fig. 2.

17. CASE II. Of PARALLEL RAYS. In this Caſe, DA or d being infinite, the finite Quantity r will vaniſh where it is found by it ſelf; and the Theorem $\frac{-dr}{2d-r}$ will become $\frac{-dr}{2d} = \frac{-r}{2} = -f$. Theor. 15. Fig. 3.

18. CASE III. Of CONVERGING RAYS. Here again the incident Ray DA will have the

Page, 40.

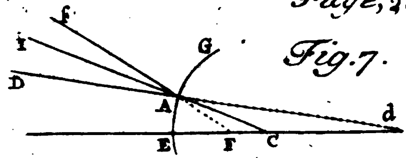


Fig. 7.

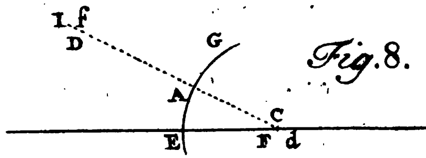


Fig. 8.

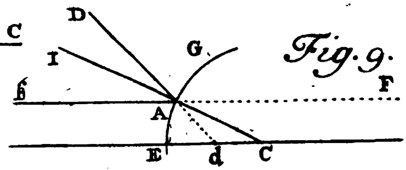


Fig. 9.

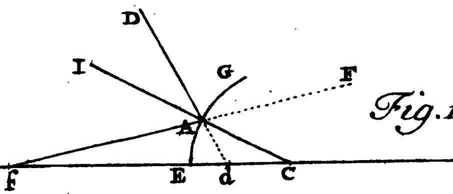


Fig. 10.

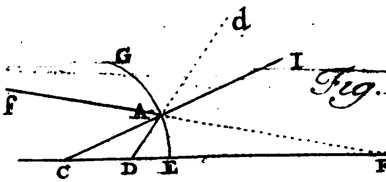


Fig. 11.

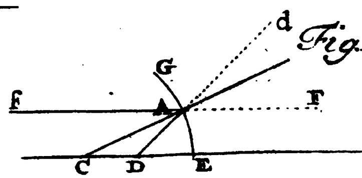
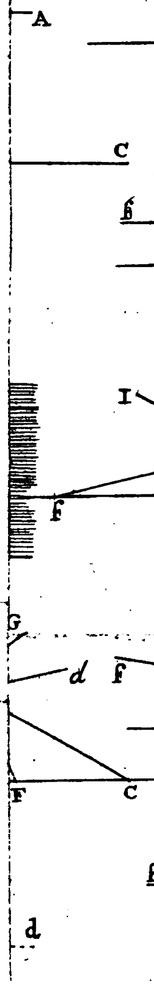


Fig. 12.



the Point of Convergence d on the other Side of the *Speculum*; and the Diſtance Ed or d muſt therefore have a negative Sign, or be $-d$; in this Caſe then, r and d being both *negative*, the Theorem $\frac{-dr}{2d-r}$ will become $\frac{dr}{-2d-r} = f$. Theor. 16. And ſince the Diviſor $-2d-r$ is wholly *negative*, the Quotient or *Focus* f or Ef, will likewiſe be always *negative*, or before the Mirrour G E.

19. If $d=r$, then $\frac{dr}{-2d-r} = \frac{rr}{-3r} = \frac{r}{-3} = -f$. Theor. 17.

If $d=\frac{1}{2}r$, then $\frac{dr}{-2d-r} = \frac{\frac{1}{2}rr}{-2r} = \frac{r}{-4} = -f$. Theor. 18.

If $d=\frac{2}{3}r$, then $\frac{dr}{-2d-r} = \frac{\frac{2}{3}rr}{-\frac{4}{3}r-r} = \frac{r}{-5} = -f$. Theor. 19.

If $d=\frac{1}{4}r$, then $\frac{dr}{-2d-r} = \frac{\frac{1}{4}rr}{\frac{1}{2}r-r} = \frac{r}{-6} = -f$. Theor. 20.

PLAIN MIRROURS.

20. In Regard of *Plain Mirrours*, or Looking-Glasses, the Radius CE $= r$ is *infinite*; for we may consider a small Arch of Circle described with a *vastly great* or *infinite Radius*, as a *Plane*; and therefore the finite Quantity $2d$ vanishing, the Theorem $\frac{dr}{2d+r}$ becomes $\frac{dr}{r} = f$.

21. CASE I. Of DIVERGING RAYS. The Theorem is $\frac{dr}{r} = d = f$. Theor. 21. Fig. 5.

22. CASE II. Of PARALLEL RAYS. In this Case both d and r are infinite, and therefore the fundamental Equation becomes infinite in every Part; and $\frac{dr}{r} = d = f$ infinite, Theor. 22. Fig. 6.

23. CASE III. Of CONVERGING RAYS. Here d or Ed being the Negative, the Equation is $\frac{-dr}{r} = -d = -f$. Theor. 23. Fig. 7.

24. We

24. We have hitherto made Uſe of the *fundamental Equation* for finding the *Focus* of all Sorts of Rays, incident on any Kind of Mirrours: But ſometimes the *Focus* is given, and either the Diſtance of the Object D E, or the Radius of Convexity or Concavity of the Mirrour C E is required for ſome particular Purpoſes; and for either Caſe the ſaid Equation $dr = 2df + fr$ (in Step. 6.) is ſufficient.

25. For ſuppoſe the Radius C E $= r$, and the Focus E F $= f$, were given to find the Diſtance E D $= d$; we have from the ſaid Equation $dr - 2df = rf$, and therefore $\frac{rf}{r - 2f} = d$, Theor. 24, for a *Convex Mirrour*.

26. For a *Concave Mirrour*, the Radius being *Negative*, or $-r$, the Equation will be $2df + dr = fr$; and ſo it will be $\frac{fr}{2f + r} = d$. Theor. 25.

27. In the two laſt Theorems, the Focus is ſuppoſed to be *Affirmative*, or behind the Mirrour; but if we ſuppoſe it Negative, or on the ſame Side with the Radiant D, the Equation, for a *Convex*, will be $dr + 2df = -fr$, and $\frac{-fr}{r + 2f} = d$. Theor. 26.

 28. For

28. For a Concave, the Equation will be $2df - dr = fr$; (for here both f and r are Negative) therefore we have $\frac{fr}{2f - r} = d$. Theor. 27.

29. If the Diſtance of an Object ED $= d$, and the Focus E $= f$ be given to find the Radius of the Convexity or Concavity EC $= r$; the original Equation $dr = 2df + rf$ will be for this Caſe $dr - rf = 2df$; and ſo we have for a Convex $\frac{2df}{d - f} = r$. Theor. 28.

30. For the Radius of a *Concave Mirrour*, the Equation will be $fr - dr = 2df$; and ſo $\frac{2df}{f - d} = r$. Theor. 29.

31. But here again if we ſuppoſe the Focus *Negative*, or before the Mirrour, we ſhall have for a *Convex*, $dr + rf = -2df$; and $\frac{-2df}{d + f} = r$. Theor. 30.

32. And for a *Concave*, the Equation will be $-dr - fr = -2df$; and $\frac{-2df}{-d - f} = r$. Theor. 31.

CHAP. IV.

The Theory of CATOPTRICS *contitinued, for determining the mutual Proportion of the* Object *and it's* Image.

1. LET DE be the Portion of a *Convex Mirrour*, (as Fig. 8.) or of a *Concave*, (as Fig. 9. Plate 3.) C the Center, V the Vertex; O B an Object, and I M it's Image; it is required to find the Proportion between the Object and Image, or the Lines OB and IM.

2. From the Center C let fall on the Radiant or Object, the perpendicular CA; and from the extreme Points of the Objects O, B, draw OC and BC, meeting the Mirrour in the Points D and E; these shall be the Axis in which will be the Focus's of Rays proceeding from the Points O and B.

3. From O let a Ray O V pass to the Vertex of the Mirrour, and make the Angle F V A = O V A, then shall V F be the reflected Ray, by Princip. 6. which tending to the Point I, in the Axis CO, shall there represent the Image of the Point O of the Radiant, by Princip. 10 and 11. Accordingly the Ray B V will be reflected into VG, which

intersecting

interfecting the Axis CB in M, will there reprefent the Image of the Point B in the Radiant. Confequently, all the Points between O and B in the Object will be reprefented between the Points I and M; and therefore IM will be the Image of the Radiant or Object OB.

4. If we fuppofe the Radiant OB very fmall, or at a great Diftance, the Arch or Portion of the Mirrour ED will alfo be very fmall, and not fenfibly different from a right Line, and confequently will be parallel to the Radiant BO, for CA is perpendicular to both BO and ED. Alfo fince the Diftances OD, AV, and BE, are very nearly equal, as being very near each other, it is plain their focal Diftances DI, V*a*, and EM, will alfo be nearly equal; *and therefore the Image* IM *will be very nearly a Right Line, and parallel to the Radiant* OB; *and alfo perpendicular to* CA.

5. The Angle OVA = AVF = *a*VI, and the Angle BVA = AVG = *a*VM, from the Nature of Reflection, Princip. 6: therefore OVA + BVA = *a*VI + *a*VM; that is, the Angle OVB = IVM; that is, *The Radiant* BO *and it's Image* IM *are feen from the Vertex of the Speculum* V *under equal Angles.*

6. The Triangles AVO and *a*VI are equiangular and fimilar, for the Angle OVA = *a*VI, and the Angles at A and *a*, are right ones;

ones; consequently $VA : Va :: AO : aI$. For the same Reason $VA : Va :: AB : aM$; therefore $VA : Va :: OA + AB : Ia + aM :: OB : IM$; that is, *The Distance of the Object is to the Distance of it's Image, from the Vertex* V, *as the Length of the Object to the Length of the Image.*

7. Moreover, since $OA : Ia :: AB : aM$, (for $OA : Ia :: AV : Va :: AB : aM$;) therefore $OA : AB : Ia : aM$; that is, *The Radiant* OB *and it's Image* IM, *are cut in the same Proportion by a Right Line* CA, *drawn from the Center perpendicular to each.*

8. From what has been premised, it is easy to raise a Theorem, *to shew at what Distance any Object ought to be placed to bear any given Proportion to it's Image.* Let the Object be O, it's Image I, and the given Proportion be as $O : I$; then $O : I :: d : f$, (by the Sixth of this) and therefore $\frac{Id}{O} = f$. But the fundamental Theorem for the Focus is $\frac{dr}{2d + r} = f$; consequently $\frac{Id}{O} = \frac{dr}{2d + r}$, and so $2ddI + Idr = drO$, that is, $2dI + Ir = rO$; and $2dI = Or - Ir$; and consequently for *Convex Mirrours*, the Theorem will be $\frac{Or - Ir}{2I} = d$. Theor. 1.

9. In

9. For *Concave Mirrours* where r is negative, or $-r$, the Equation will be $2dI = Ir - Or$, and so $\frac{Ir - Or}{2I} = d$. Theor. 2.

10. For a *Plain Mirrour*, or Looking-Glaſs, where r is infinite, the finite Part of the Equation $2dI$, will vaniſh, or be O; and then $Or - Ir = O$, and $Or = Ir$, that is, $O = I$. Theor. 3.

11. If the Focus be required *negative*, or on the ſame Side of the Glaſs with the Object; the Equation will be for a *Convex*, (f being *negative*) $2dI = -Or - Ir$, and $\frac{-Or - Ir}{2I} = d$. Theor. 4.

12. But for a *Concave*, where r and f are now both negative, the Equation will be $2Id = Or + Ir$, and therefore $\frac{Or + Ir}{2I} = d$. Theor. 5.

13. We have hitherto conſidered the Object and it's Image as *Right* Lines, which have a *ſimple* Proportion to each other; but if the *Object* be a *Superficies*, or a Plane Figure, the Proportion between it and it's Image, will be duplicate of what it was before; for as in Lines, it was $O : I :: d : f$; in Superficies it will be $O : I : d^2 : f^2$; and hence $\frac{\sqrt{Id^2}}{O} = f$

$= dr$

$= \frac{dr}{2d+r}$; and squaring each Side, we have

$\frac{I d^2}{O} = \frac{d^2 r^2}{\overline{2d+r}^2}$, and $I \times \overline{2d+r}^2 = O \times r^2$,

and therefore $\overline{2d+r}^2 = \frac{O}{I} \times r^2$, and putting $\frac{O}{I} = p$, and extracting the square Root we have $2d+r = \sqrt{pr^2}$; and consequently, $d = \frac{\sqrt{pr^2} - r}{2}$. Theor. 6.

14. For a Concave Mirrour, where r is negative, $d = \frac{\sqrt{pr^2} + r}{2}$. Theor. 7.

15. In the same Manner, if the Object and Image are *Solids*, and consequently their Proportion *triplicate*, *viz.* $O : I :: d^3 : f^3$; it is shewn, that, for *Convex* Mirrours, $d = \frac{\sqrt[3]{pr^3} - r}{2}$. Theor. 8.

16. Also, for Concave Mirrours, where it is $-r$, we have $d = \frac{\sqrt[3]{-pr^3} + r}{2}$, Theor. 20.

17. If the Distance d, and the Proportion of the Object and Image O and I, be given the Radius r of the Convexity or Concavity of the Mirrour, for that Proportion is to be had

had from the same Equation, *viz.* $Or = 2Id + Ir$: for $Or - Ir = 2Id$, and $\frac{2Id}{O-I} = r$. Theor. 10.

18. For the Radius of a Concave Speculum, the Equation is $Ir - Or = 2Id$, and $\frac{2Id}{I-O} = r$. Theor. 11.

19. If the Focus be required negative, or before the Mirrour, the Equation for a *Convex* will be $Or + Ir = -2Id$; and therefore $\frac{-2Id}{O+I} = r$. Theor. 12.

20. And for a *Concave*, the Equation is $Or + Ir = 2Id$, and $\frac{2Id}{O+I} = r$. Theor. 13.

21. Again, from the Equation $Or = 2Id + Ir$, when the Distance of the Object d, and the Radius of the Mirrour r are known, the Proportion between the Object and Image, *viz*, $O : I$ is also known; for, in Case of a *Convex*, $O : I :: 2d + r : r$. Theor. 14.

22. And for a *Concave*, the Equation being $Or - 2Id = Or$, we have $O : I :: r - 2d : r$. Theor. 15.

23. If the Focus be negative, or $-f$, the Equation for a *Convex* will be $Or = -2Id - Ir$; and so, $O : I :: -2d - r : r$. Theor. 16.

24. For

24. For a Concave Speculum, the Equation being $Or = 2Id - Ir$, we have $O : I :: 2d - r : r$. Theor. 17.

25. These are the principal Theorems in the *Theory of Catoptrics*, and most of them very curious, and of great Use. I shall conclude this Theory with a very notable Observation, *viz. That the musical or harmonical Proportion is the Grounds of this whole Theory:* or, *that the Center* C, *the Focus* F, *the Vertex of the Mirrour* V, *and the Radiant* D, *are harmonical Points, by which the Axis or Line* CD, *is harmonically divided in the Points* D, E, F, C; *so that* $DC : DE :: FC : FE$.

26. For if we take it for granted, we shall have the following Analogy, $DC\ (d+r) : DE\ (d) :: FC\ (r-f) : FE\ (f)$; and therefore $df + rf = dr - df$, and so $2df + rf = dr$; and $\frac{dr}{2d+r} = f$, which is the same as the first fundamental Theory, Chap. III. Art. 6, and this holds equally in *Concave* as in *Convex Mirrours*. This admirable Property of Mirrours in reflecting the Light, was first published by Mr *Ditton*, in N°. 295, of the *Philos. Transactions*.

CHAP. V.

The foregoing Theory of CATOPTRICS *explained and illuſtrated, by familiar Rules and Examples.*

1. I SHALL here explain and illuſtrate the foregoing Theorems in Words at Length, and give an Example, in all the principal ones, in Numbers, ſhewing the Diſtance of the *Focal Point*, real or virtual, of all Sorts of Rays, reflected from all Sorts of Mirrours.

2. I ſhall likewiſe ſhew how every thing happens, by Schemes, as large as the Life; and therefore, in Plate IV, I have given three Schemes of Rays falling on a Convex Mirrour, where you are to obſerve, that A B is the *Mirrour*; C E the *Radius* thereof $1\frac{1}{2}$ Inch; E the Vertex or middle Point thereof; D the radiant Point; and F the Focus to which the Rays tend after Reflexion.

3. Note, the *black Lines* in theſe Schemes denote the Rays of the Sun's Light, as they are ſeen to go to and from the Mirrour in a darkened Chamber; and the *dotted Lines* denote only the Courſe or Tendency of the reflected Rays, were they not intercepted by the Mirrour.

4. In

4. In the following Examples, I ſhall ſuppoſe an Inch divided into *Ten equal Parts*, which Parts are expreſſed by the Numbers I ſhall there uſe, every 10 of which will therefore expreſs the Length of an Inch, and the Remainders will be the *tenth Parts* of an Inch.

For CONVEX MIRROURS. Plate IV.

5. CASE I. For *Diverging Rays*, Scheme 1. Let there be given DE, the Diſtance of the radiant Point from the Mirrour, and CE, the Radius of Convexity, to determine the Diſtance of the Focus FE.

RULE.

Multiply the Diſtance and Radius together, divide that Product by the Sum of the Radius added to twice the Diſtance, the Quotient will be the Diſtance of the Focus required. (*per* Theor. 1. Chap. III.)

EXAMPLE.

Let $\begin{cases} CE = 15, \\ DE = 30, \end{cases}$ and $CE = 15$, $2DE = 60$

Produce 450; Sum 75;

then $75)450(6 = FE$
450

C So

So that in this Caſe, the Focus F of the reflected Rays GH, will be 6 Tenths of an Inch behind the Vertex E.

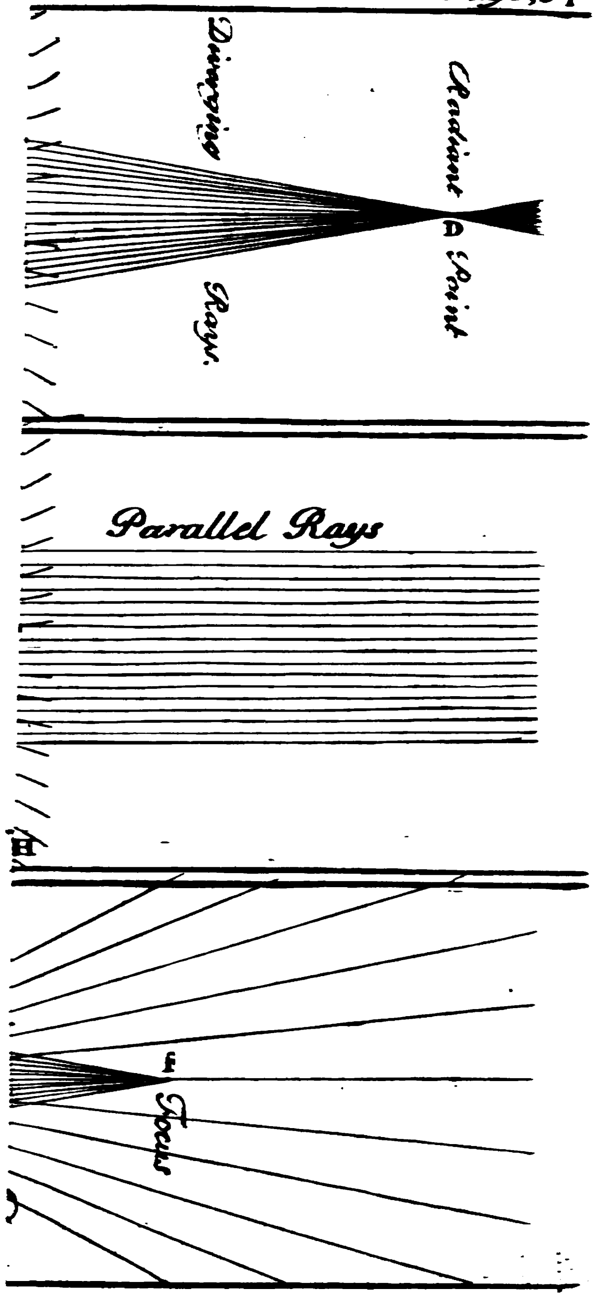
6. If the Diſtance DE be equal to the Radius CE, the Diſtance of the Focus EF will be $\frac{1}{3}$ of the Radius CE, *per* Theor. 2. If DE be $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$ of CE, then ſhall EF be $\frac{1}{4}$, $\frac{1}{5}$, or $\frac{1}{6}$ of CE, till at laſt the Radiant D, and Focus F, both coincide in the Vertex of the Mirrour E. See Theor. 3, 4, 5.

7. CASE II. For *Parallel Rays*, Scheme 2. When Rays fall parallel upon a Convex Mirrour, they are reflected in ſuch a Manner, as to have their focal Diſtance FE juſt equal to half the Radius CE; that is, EF = CF, *per* Theor. 6. Thus, for Example, if CE be 15, then will FE be 7 $\frac{1}{2}$ Tenths of an Inch.

8. CASE III. For *Converging Rays*, Scheme III. In this Caſe, if the Diſtance of the radiant Point (which here is *virtual*, or behind the Mirrour) and the Radius be given, the Rule for finding the Diſtance of the Focus is thus.

RULE.

Multiply the given Diſtance and Radius together, and divide that Product by the Difference between the Diſtance doubled and the Radius;


Diverging Rays
Radiant Point
D
Parallel Rays
F
Focus

Radius; the Quotient is the Diſtance of the Focus required, *per.* Theor. 7.

Note, if the double Diſtance of the Radiant exceed the Radius, the Focus will be behind the Glaſs; if it be leſs than the Radius, the Focus will be before the Glaſs.

9. If the Diſtance be equal to the Radius, the focal Diſtance will alſo be equal to the Radius; that is, thoſe Rays which converge towards the Center C of the Mirrour, will be reflected back again upon themſelves, *per* Theor. 8.

10. If the double Diſtance of the Radiant be equal to the Radius, or the ſaid Diſtance be equal to half the Radius, then will the focal Diſtance be infinite; that is, Rays converging to a Point in the Axis equally diſtant from the Center C, and Vertex E, will be reflected parallel to each other; *per* Theor. 9.

11. Let the double Diſtance of the Radiant be leſs than half the Radius; for Example,

Let $\begin{cases} DE = 15, & \text{and} \quad CE = 15, \\ dE = 5, & \quad\; 2dE = 10, \end{cases}$

Product 75; Diff. = 5;

then 5) 75 (15 = E*f*
75

Here because 2 *d*E is less than CE, the Focus *f* will be before the Mirrour AB; and since *d*E is ⅓ of CE, the focal Distance E*f* is equal to the Radius CE. See Scheme 3.

12. From hence, and a due Consideration of the Properties of the first and third Cases, it is plain they are conversive by the same; the *incident Rays*, *Radiant* and *Focus* in one being but the *reflected Rays*, *Focus* and *Radiant* in the other.

For CONCAVE MIRROURS. Plate V.

13. CASE I. For *Diverging Rays*, Scheme 1. Having given the Distance of the Radiant DE, and Radius CE, the focal Distance is found by the same Rule as in the third Case of a Convex. Only here observe, that if the double Distance of the Radiant be less than the Radius, the Focus will be behind the Mirrour; if greater, it will be before it.

EXAMPLE.

Let DE =	30,	and 2 DE =	60,
CE =	15,	CE =	15,
Product	450;	Diff.	45;

then 45) 450 (10 = FE.
450

. . .

That

That is, the Focus F is 1 Inch before the Mirrour A B.

14. If the Diſtance of the Radiant D E be equal to the Radius C E, then will the focal Diſtance be equal to the Radius alſo, *per* Theorem 13. That is, if an Object be placed in the Center of a Concave *Speculum*, the Image will be reflected upon the Object, or they will ſeem to meet and embrace each other in the Center; which agreeable Phænomenon is eaſily tried by any Mirrour of this Sort.

15. If 2 D E = C E, that is, if the Diſtance of the Radiant be equal to half the Radius, it's Image will be reflected to an infinite Diſtance; for the reflected Rays will be parallel, *per* Theor. 12. Hence, if a luminous Body be placed at the Diſtance of half the Radius from a Concave, it will enlighten Places directly before it at the greateſt Diſtances. And hence appears their Uſe when placed behind a Candle in a Lanthern, and in ſeveral other like Caſes.

16. Case II. For *Parallel Rays*, Scheme 2. In this Caſe the focal Diſtance F E is always equal to half the Radius C E, and before the Mirrour A B; *per* Theor. 15. And ſince the Sun-Beams are parallel among themſelves, if they are received on a Concave Mirrour, they will all be reflected to that Point, and there burn

in proportion to the Quantity of Rays collected by the Mirrour.

17. CASE III. For *Converging Rays*, Scheme 3. The Rule for finding the *Focus* from the Diſtance of the Radiant Point, and the Radius given, is here the ſame as in the firſt Caſe of the Convex; only as there the Focus F was always behind the Mirrour, here it is ever before it.

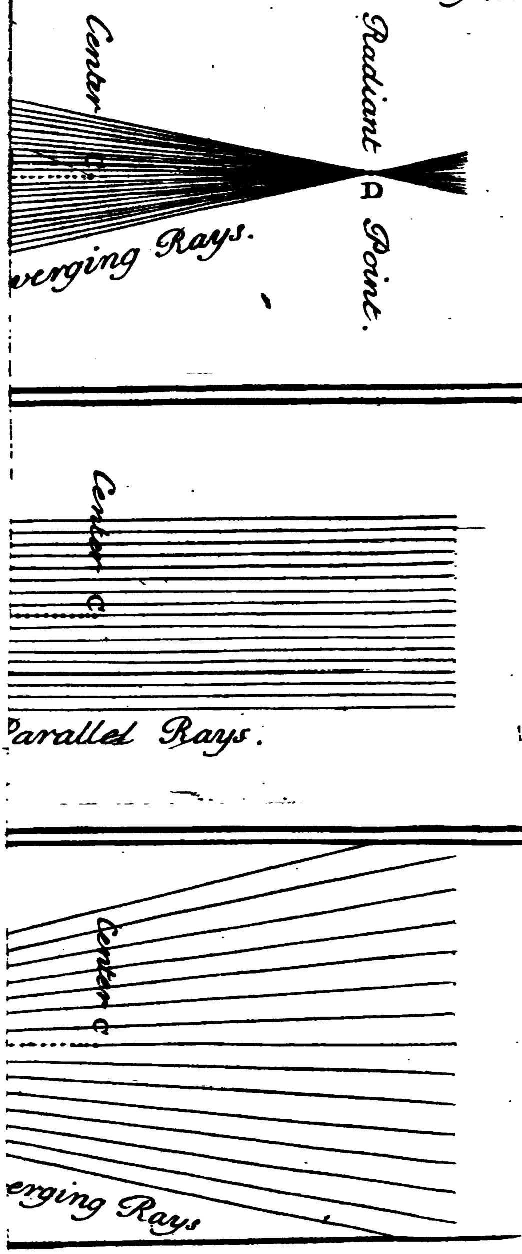
EXAMPLE.

Let $dE = 15$,	and $2dE = 30$,
$CE = 15$,	$CE = 15$,
Product 225;	Sum 45;

then 45) 225 (5 = FE
225

18. If dE be equal to CE, or $\frac{1}{2}$ CE, or $\frac{1}{3}$ CE, or $\frac{1}{4}$ CE, &c. then ſhall the focal Diſtance FE be equal to $\frac{1}{2}$ CE, $\frac{1}{4}$ CE, $\frac{1}{5}$ CE, or $\frac{1}{6}$ CE, &c. As *per* Theor. 17, 18, 19, 20.

19. From what has been hitherto ſaid of the Properties of theſe Mirrours, and from a View of the Schemes, it is plain the Caſes of a *Concave*, are but the *Reverſe* of thoſe of a *Convex* in an inverſe Order. And it is moreover to be obſerved, that in all thoſe Caſes the *Focus*, and *Radiant Point* mutually reſpect each


Center
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Center C
erging Rays

each other, and may be interchangeably taken one for another; that is, in any Caſe, if we take F for the Radiant Point, then will D be the Focus of the reflected Rays.

For PLAIN MIRROURS. Plate VI.

20. CASE I. Of *Diverging Rays*, Scheme 1. In theſe Mirrours there is no Radius to be conſidered, and the Diſtance of the Radiant Point D being given, the focal Diſtance is alſo given, as being equal thereto, *per* Theor. 21. That is, if D be the Point whence Rays proceed, diverging to the Mirrour AB, they will be ſo reflected towards GH, as if proceeded from the Point F, which Point or Focus F, will always be juſt as far behind the Mirrour AB, as the Point D is before it, and alſo on the ſame Side with it. And this is the known Property of a Plain Mirrour or *Looking-Glaſs*.

21. CASE II. Of *Parallel Rays*, Scheme 2. If Parallel Rays fall on a plain Mirrour, they are reflected parallel; and as the *Radiant Point* D is infinitely diſtant, ſo alſo is the *Focus* F, *per* Theor. 22. Hence the Sun being viewed by Reflection in a *Looking-Glaſs*, appears as vaſtly diſtant behind the Glaſs, as he really is before it.

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26. Both theſe Rules regard the Focus behind the Mirrour, if it be before the Mirrour, the Problem will be impoſſible for a *Convex Speculum*, *per* Theor. 26. And for a Concave, the Rule is this.

RULE.

From twice the focal Diſtance take the Radius, and with the Remainder divide the Product of the focal Diſtance into the Radius, the Quotient will be the Diſtance of the Radiant Point, *per* Theor. 32.

27. If the Diſtance of the Radiant and the Focus be given, we may find the Radius of the *Convex Mirrour* by this

RULE.

From the Diſtance of the Radiant take the focal Diſtance, and with the Remainder divide twice the Product of the focal Diſtance into the Diſtance of the Radiant; the Quotient is the Radius of the Convexity required, *per* Theor. 28.

28. The Rule for finding the Radius of Concavity is this:

RULE.

RULE.

From the focal Diſtance take the Diſtance of the Radiant, and with the Remainder divide twice the Product of thoſe Diſtances into each other; the Quotient will be the Radius of Concavity ſought, *per* Theor. 29.

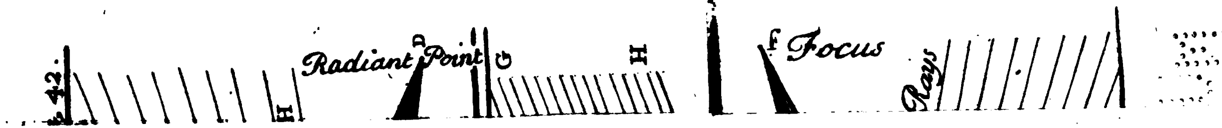
29. If the given Focus be before the Mirrour, the thing is impoſſible for a Convex, *per* Theor. 30; but is ever poſſible for a Concave, and is found by this

RULE.

With the Sum of the Diſtances of Focus and Radiant, divide the double Product of thoſe Diſtances; the Quotient is the Radius of Concavity, *per* Theor. 31.

30. I preſume it is needleſs to give either Examples or Schemes for the Illuſtration of theſe Rules, if thoſe for finding the Focus be well underſtood. I ſhall therefore proceed to the Rules for determining the Diſtances neceſſary for any given Proportion of the Object and Image, in the next Chapter.

CHAP.


42.
H
Radiant Point
D
G
H
F
Focus
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CHAP. VI.

The Rules *for determining the Distance of an* Object, *that shall bear any assigned Proportion to it's* Image.

1. WE have now done with *Radiant Points*, and are come to treat of Lines, Superficies, and Solids, as they appear by Reflection from all Sorts of Mirrours; and first,

Of Convex Mirrours. Plate VII.

2. Let O B be an Object placed before a Convex Mirrour F G, whose Center is C, and Radius C E; the Image of the Object will be I M, as is evident from what has been taught in Chap. IV. Art. 1, 2, 3.

3. Now to find at what Distance the Object O B ought to be placed, that it may bear any assigned Proportion to the Image I M, we have this Rule for *Lines*.

RULE.

From the Length of the Object take the Length of the Image, multiply the Remainder by

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RULE.

Divide the Object by the Image, multiply that Quotient by the Square of Radius; then from the ſquare Root of that Product take the Radius, half the Remainder is the Diſtance ſought, *per* Theor. 6. Chap. IV.

EXAMPLE.

Let the Radius be 20, and the given Proportion of the Object to the Image be as 25 to 1. Here

The Object divided by the Image is	25
Multiply by the Square of Radius	400
Extract the ſquare Root — —	10000(100 the ſquare Root
Subſtract the Radius — — — —	20
The Remainder — — — —	80
The half thereof is — — — —	40

the Diſtance of the Object as required.

5. The Rule is the ſame for a *Solid*, as for a *Superficies*, if inſtead of *Square*, and *Square Root*, we uſe the *Cube* and *Cube Root*, as *per* Theor. 8.

EXAMPLE.

Let the Radius be 20, and the Proportion of the Object to the Image be that of of 125 to 1; to find the Diſtance of the Object in that Caſe.

The Object divided by the Image is	125
Multiplied by the Cube of Radius	8000
Extract the Cube Root of Prod.	1000000 (100 Cube (Root
Subſtract Radius — — —	20
The Remainder — — —	80

Half thereof is — — — — 40, the Diſtance of the Object required.

6. Thus it appears that an Object placed 4 Inches before a Convex Mirrour, of 2 Inches Radius, will

If it be	A Line	be in Proportion to it's Image as	5 to 1
	A Superficies		25 to 1
	A Solid		125 to 1

That is, the Diminutions of ſuch Objects are in the *Simple Square*, and *Cubic* Proportions, at equal Diſtances from the Mirrour, as they ought to be by the Rules of Geometry, which likewiſe proves the Truth of the Rules here laid down.

7. From the 4th Theorem it is evident it is impoſſible to form an Image of any Object before a Convex Mirrour.

8. Alſo it is impoſſible to magnify an Object by a Convex Mirrour, becauſe the Image being

g greater than the Object, cannot be taken
ı it, as the Rule above (in Art. 3.)
ıires. Nor can the Image be ever equal
the Object, but when they both meet or
ncide in the Vertex of the Mirrour.

9. It is evident likewise, if the Object be
rge in respect of the Mirrour, that it's Image
ill not be strait or plain, but curv'd, yet not
ircular or concentric with the Mirrour by
what has been taught in Chap. IV.

10. Again, the Image formed by a Convex Mirrour will be *erect*, as is easily understood from Chap. IV, and is evident from the Scheme of this Plate.

CONCAVE MIRROURS. Plate VIII.

Let the Radius be 20, and the Proportion of the Object to the Image be as 1 to 5, to find at what Distance the Object must be placed; this is the

RULE.

From the Image substract the Object, and multiply the Remainder by Radius; divide that Product by twice the Image, the Quotient will be the Distance of the Object sought, *per* Theor. 2.

EXAMPLE.

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Plate VIII.

RULE.

To the Object add the Image, and multiply that Sum by the Radius, divide this Product by twice the Image, the Quotient will be the Distance of the Object, *per.* Theor. 5.

According to this Rule, any Object may, in this Case, be magnified or diminished in any Proportion we please.

EXAMPLE.

Let the Radius be 20, and the Proportion of the Object to the Image be as 5 to 1, to find the Distance of the Object. See Plate IX.

To the Object — OB = 5,
Add the Image — IM = 1,

The Sum is — — — 6;
Multiply by Radius CE = 20,

The Product is — — — 120;
Then 2 IM = 2) 120 (60 = AE, the Distance of the Object OB required, *viz.* Six Inches.

14. But if IM be supposed the Object, then will OB be the Image, 5 Times larger than it; and the Distance *a* E will be found 12, or one Inch and two Tenths.

15. If the Object and Image are desired equal, the Object must be placed in C, the very Center of the Mirrour; this is easily understood from the Rules, and is visible by Experiment, and is, by the Way, a good Expedient for finding the Center of the Concavity of a Mirrour.

16. The Image will here also be *curved*, and it will be now *inverted*, because the Rays cross each other in the Center, and form it on the other Side; the Reason is plain in the Scheme, and from what has been taught in Chap. IV. of the Theory.

17. It will now be no difficult thing to understand how *Concave Mirrours* make such excellent *Burning-Glasses*. For suppose the Radius of Concavity be 2 Feet, or 24 Inches, and the Diameter of the Mirrour 12 Inches. It has been shewn, that the Image of the Sun would be formed by this Mirrour at the Distance of ½ the Radius, or 12 Inches from the Vertex. See Chap. V. Art. 16. Also the Image being seen under the same Angle, as the Object from the Vertex of the Mirrour (*per* Chap. IV. Art. 5.) will there subtend an Angle of only 32 Minutes. The Diameter therefore of this Image, by the Rules of Trigonometry, will be found about 1 Tenth of an Inch, and the Diameter of the Mirrour being 12 Inches, the Squares of these Diameters will be as 1 to 14400; and consequently the Density of the Sun's Rays in this Image

(where

(where they are all collected) will be to their Density on the Surface of the Glass, as 14400 to 1, and their Heat will be in Proportion. That is, the Heat of the Sun's Ray, in the Focus of such a Glass, will be *fourteen thousand* and *four hundred* Times greater than before; and therefore must needs burn very furiously, and produce very great Effects.

PLAIN MIRROURS. Plate X.

18. In Plain Mirrours, or *Looking-Glasses*, the Image I M will always be equal to the Object O B, at what Distance soever it be placed, *per* Theor. 3. It will also be *erect*, and as far behind the Glass as the Object is before, as hath been already shewn, Chap. V. Art. 20.

19. Hence their excellent Use in representing Objects *every Way like the Life*; but this is too common to be insisted upon. I shall only just observe, that the Mirrour being but at half the Distance of the Image, will compleatly receive an Image of twice it's own Length; and therefore a Man of 6 Foot height may view himself entirely in a Looking-Glass of 3 Feet Length, and half his own Breadth.

20. If the Proportion of the Object and Image be given, and the Distance of the Object from the Mirrour, the Radius of the Convexity may be found for that Purpose by this Rule.

RULE.

Multiply twice the Diſtance of the Object by the Image, divide that Product by the Difference between the Object and Image, the Quotient is the Radius of the Mirrour, *per* Theor. 10.

Note, the Focus is here behind the Glaſs, and it is impoſſible to have it before, *per* Theor. 12.

21. The ſame things given, the Radius of Concavity is found by this

RULE.

Multiply twice the Diſtance by the Image, and divide that Product by the Difference between the Image and Object, the Quotient is the Radius required, *per* Theor. 11.

The Focus here being behind the Mirrour, the Image will be greateſt. But if the Focus be required before the Glaſs, the ſame Rule finds the Radius, if, inſtead of the *Difference*, you take the *Sum* of the Object and Image, *per* Theor. 13.

22. In the laſt place, if the Radius of the Mirrour, and the Diſtance of the Object be given, then may the Proportion between the Object and Image be found for a *Convex*, by this

ANALOGY.

ANALOGY.

As twice the Diſtance added to the Radius, is to the Radius;
So is the Object to the Image, *per* Theor. 14.

23. For a Concave, the Proportion will be found by this

ANALOGY.

As the Radius leſſened by twice the Diſtance, is to the Radius;
So is the Object to the Image, *per* Theor. 15.

24. If the Focus be required before the Glaſs, the Analogy is this.

ANALOGY.

As twice the Diſtance, leſſened by Radius, is to the Radius;
So is the Object to the Image, *per* Theor. 17.

25. I ſhall exemplify this laſt Caſe of a Concave Mirrour, as being the moſt frequent and curious.

Let the Radius be 20, and the Diſtance of an Object from the Mirrour be 130; then twice 130 is 260, from which take Radius 20, there will remain 240; then ſhall the Object be to the Image as 240 to 20, or as

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PART II.

OF

DIOPTRICS.

CHAP. I.

The Theory of DIOPTRICS.

1. IN Plate XI, Fig. 1. Let L N represent a convex Lens, D f it's Axis, D a Radiant Point therein, D A a Ray proceeding from thence to A, a Point in the Surface L B N, C is the Center of the Convexity of that Surface, C G being drawn through the Point A, is a perpendicular to that Surface in the Point A. C A or C B is the Radius, A f is the refracted Ray, and f the Point where it meets the Axis after the first Refraction.

2. Let D B = d, C A = r, E B = t, the Thickness of the Lens; and let the Sine of the Angle of Incidence D A G be called I, and the Sine of the Angle of Refraction C A f, or G A H, be called R.

3. Now since the Point A is supposed to be very near to the Vertex B, D A may be esteemed equal to D B = d, by Princip. 4; and in the Triangle C A D, we shall have A D to A C, as the Angle C to the Angle D, *per* Princip. 1. That is, d : r :: C : D.

4. Also

4. Also $DB + BC = d + r$, will be as the opposite Angle CAD or DAG, for the Sine of both is the same.

5. Then as $I : R :: d + r : \frac{dR + rR}{I}$, which will be as the Angle CAf; this taken from the Angle $ACD = d$, leaves the Angle $AfD = \frac{dI - rR - dR}{I}$.

6. Lastly, As the Angle $f : D :: AD$ or $DB : Af$ or Bf; that is, as $\frac{dI - rR - dR}{I} :: r :: d : \frac{rdI}{dI - rR - dR} = Bf$, the Distance of the Point f, in the Axis after the first Reflection.

7. But since there is a second Surface LEN of the Lens, there must necessarily be a second Refraction of the Ray AD, to some other Point in the Axis, as F; see Fig. 2. Let K be the Center of this second Surface, and KI a perpendicular thereto in the Point a, from whence the said Ray is refracted to F. In this Case, the Refraction being out of a dense into a rarer Medium, the Sine of Incidence will be to that of Refraction, the reverse of what it was before, *viz.* as R to I; that is, the Sine of Iaf is to the Sine of IaF, as R to I.

8. Here let Ka be called *r*, $Ef = d$; and then will be, as $d : r :: K : f$. And Ef + EK

$EK = d + r$; and this will be as the Angle faK, or it's Complement Iaf; therefore $R : I :: d + r : \frac{dI + rI}{R}$, which will express the Angle IaF.

9. Then $IaF - aKF = \frac{dI + rI}{R} - d = \frac{dI + rI - dR}{R} = aKF$. Now as $F : K :: Ka$ or $KB : aF$ or EF; that is, as $\frac{dI + rI - dR}{R} : d :: r : \frac{Rdr}{dI + rI - dR} = EF$.

10. But $Bf - BE = \frac{drI}{dI - rR - dR} - t = d = Ef$, *per* Art. 6.

11. Therefore, putting $I - R = O$, we shall have $d = \frac{drI}{dO - rR} - t = \frac{drI - dOt + rRt}{dO - rR}$.

12. Also $dRr = \frac{drIRr - dotRr + rtRRr}{dO - rR}$.

13. Again $dI + rI - dR = do + rI$; if then we multiply the Equation, Art. 11. by O, and add thereto rI; we shall have $dI + rI - dR = \frac{drIo - doto + rRto + dorI - rRIr}{do - rR}$

14. Then

14. Then $\frac{dRr}{dI+rI-dR}=$

$$\frac{drIRr-dotRr+rtRRr}{drIo-doto+rRto+dorI-rRIr}$$
$=EF$.

15. This laſt Equation may be abridged, by ſubſtituting p for $\frac{R}{O}$, that is, for $\frac{R}{I-R}$, then ſhall we have $\frac{pdrIr-pdotr+rtRrp}{drI-dot+rRt+drI-prIr}$ $=EF$.

16. Laſtly, if now we take $R=pO$, in $pdotr$; and $I-R=O$ in dot; this Equation will be finally reduced to this fundamental Equation for all *Dioptrics*, *viz.*

$$\frac{pdrrI-dtrR+rtrpR}{drI-dtI+dtR+rtR+drI-prrI}$$
$=EF=f$.

17. The Ratio of I to R being in Glaſs as 3 to 2, we ſhall have $\frac{R}{I-R}=\frac{2}{3-2}=2$ $=p$, for a Glaſs Lens.

18. In *Water*, the Ratio of I to R being as 4 to 3, we ſhall have $p=3$; and in Diamond, the

the Ratio being as 5 to 2, p will be $= \frac{2}{3}$. Wherefore, if instead of p, I, and R, we write their Value in Numbers in the foregoing Equation, it will then be suited for a Lens of Glass, *Water*, or *Diamond.* But I shall only regard that of Glass, for which it will stand thus; *viz.* $\frac{6\text{d}\text{r}r - 2\text{d}\text{t}r + 4\text{t}\text{r}r}{3\text{d}\text{r} + 3\text{d}r - \text{d}\text{t} + 2\text{r}\text{t} - 6\text{r}r}$ $= \text{EF} = f.$

19. We shall now adapt this Theorem to the Case of a *Glass Globe*, *Glass Hemispheres*, and all Sorts of *Glass Lenses*, for all Kind of Rays, as follows.

For A Globe.

20. Case I. *Diverging Rays*, Fig. 3. Here $\text{r} = r$, and $\text{t} = 2r$; that is, both the Radius's being equal, and t the Thickness being equal to twice the Radius; the Equation in Art. 18. will be reduced to this Form $\frac{\text{d}r + 4rr}{2\text{d} - r} = f.$ Theor. 1.

21. Case II. Of *Parallel Rays*, Fig. 4. In this Case d being infinite, the Parts of the Equation will vanish where it is not found; consequently it will be $\frac{\text{d}r}{2\text{d}} = \frac{r}{2} = f.$ Theor. 2.

22. Case

22. CASE III. Of *Converging Rays*, Fig. 5. Here *d* being negative, or —d, the Equation will be $\frac{-dr+4rr}{-2d-r}=f$. Theor. 3.

For an HEMISPHERE.

23. CASE I. Of *Diverging Rays*, Fig. 6. If the convex Part be towards the Radiant Point D, then will the Radius *r* be infinite, and t=r; and the Equation will become $\frac{-4dr+4rr}{3d-6r}=f$. Theor. 4.

24. CASE II. Of *Parallel Rays*, Fig. 7. Here d being infinite, we ſhall have $\frac{4dr}{3d}$ $=\frac{4}{3}r=f$. Theor. 5.

25. CASE III. Of *Converging Rays*, Fig. 8. Here d being negative, or —d, the Equation will be $\frac{-4dr+4rr}{-3d-6r}=f$. Theor. 6.

26. If the plain Part of the Hemiſphere be expoſed to the Radiant, there will be for

CASE

CASE I. Of *Diverging Rays*, Fig. 9.

$$\frac{6dr+4rr}{3d-4r}=f. \text{ Theor. 7.}$$

27. CASE II. Of *Parallel Rays*, Fig. 10.

$$\frac{6dr}{3d}=\frac{6}{3}r=2r=f. \text{ Theor. 8.}$$

28. CASE III. Of *Diverging Rays*, Fig. 11.

$$\frac{-6dr+4rr}{-3d-4r}=f. \text{ Theor. 9.}$$

29. Note, if from the focal Diſtance $2r$ in Art. 27, we take $\frac{4}{3}r$ in Art. 24, there will remain $\frac{2}{3}r$, which will be the Difference in turning the Convex and plain Sides of the Hemiſphere towards the Sun-Beams.

FOR DOUBLE CONVEX LENSES.

30. If the Lens be convex on both Sides, and the *Radii* of the Convexities unequal, and the Thickneſs conſidered, we ſhall have for

CASE I. Of *Diverging Rays*, the fundamental Equation it ſelf, *viz.*

$$\frac{6drr-2dtr+4trr}{3dr+3dr-dt+2rt-6rr}=f. \text{ Theor. 10.}$$

31. CASE

31. CASE II. Of *Parallel Rays*, where d is infinite, $\frac{6drr - 2dtr}{3dr + 3dr - dt} = \frac{6rr - 2tr}{3r + 3r - t} = f$. Theor. 11.

32. For *Converging Rays*, where d is negative, or $-d$; we have

$$\frac{-6drr + 2dtr + 4trr}{-3dr - 3dr + dt + 2rt - 6rr} = f.$$

Theor. 12.

33. If the *Thickneſs* of the Lens be neglected, as inconſiderable, which it very well may, and always is in common Uſe; then all thoſe Parts of the Equation where t is found will vaniſh, and it will become for

CASE I. Of *Diverging Rays*, Fig. 12.

$$\frac{2drr}{dr + dr - 2rr} = f.$$ Theor. 13.

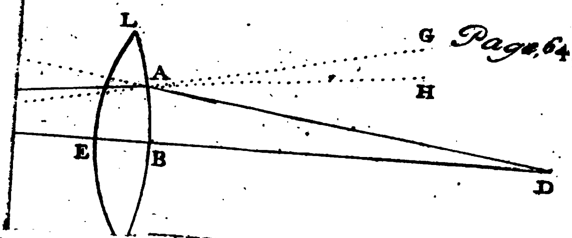
34. CASE II. For *Parallel Rays*, Fig. 13.

$$\frac{2drr}{dr + dr} = \frac{2rr}{r + r} = f.$$ Theor. 14.

35. CASE III. For *Converging Rays*, Fig. 14.

$$\frac{2drr}{dr + dr + 2rr} = f.$$ Theor. 15.

36. If



39. If
Radiant,
be for

Case I. Of *Diverging Rays*, Fig. 18.

$$\frac{6dr - 2dt + 4tr}{3d - 6r} = f.$$ Theor. 19.

 40. Case

Fig. 13.

35. Case III. For *Converging Rays*, Fig. 14.

$$\frac{2\mathrm{dr}r}{\mathrm{dr}+\mathrm{d}r+2\mathrm{r}r}=f. \quad \text{Theor. 15.}$$

36. If

36. If the Radii of the Convexities are equal, that is, r = *r*, and the Thickness of the Lens, or t *, be neglected; then will the Equation be very simple, *viz.*

CASE I. For *Diverging Rays*, Fig. 15. Plate XII. $\frac{dr}{d-r} = f$. Theor. 16.

37. CASE II. For *Parallel Rays*, Fig. 16. $\frac{dr}{d} = r = f$. Theor. 17.

38. CASE III. For *Converging Rays*, Fig. 17. $\frac{dr}{d+r} = f$. Theor. 18.

For a PLANO-CONVEX LENS.

39. If the *Convex Surface* be exposed to the Radiant, *r* being infinite, the Equation will be for

CASE I. Of *Diverging Rays*, Fig. 18. $\frac{6dr - 2dt + 4tr}{3d - 6r} = f$. Theor. 19.

40. CASE II. For *Parallel Rays*, Fig. 19. $\frac{6r - 2t}{3} = 2r - \frac{2}{3}t = f.$ Theor. 20.

41. CASE III. For *Converging Rays*, Fig. 20. $\frac{-6dr + 2dt + 4tr}{-3d - 6r} = f.$ Theor. 21.

42. If the Thickneſs of the Lens t be neglected, as is uſual, the Theorems will be more conciſe, *viz.*

CASE I. For *Diverging Rays*, $\frac{2dr}{d - 2r} = f.$ Theor. 22.

CASE II. For *Parallel Rays*, $\frac{2dr}{d} = 2r = f.$ Theor. 23.

CASE III. For *Converging Rays*, $\frac{2dr}{d + 2r} = f.$ Theor. 24.

43. If the plain Surface be turned to the Radiant, in which Caſe r will be infinite, we ſhall have

CASE

CASE I. For *Diverging Rays*, Fig. 21.

$$\frac{6\mathrm{d}r+4\mathrm{t}r}{3\mathrm{d}+2\mathrm{t}-6r}=f. \text{ Theor. 25.}$$

44. CASE II. For *Parallel Rays*, Fig. 22.

$$\frac{6\mathrm{dr}}{3\mathrm{d}}=2r=f. \text{ Theor. 26.}$$

45. CASE III. For *Converging Rays*, Fig. 23.

$$\frac{-6\mathrm{d}+4\mathrm{t}r}{-3\mathrm{d}+2\mathrm{t}-6r}=f. \text{ Theor. 27.}$$

46. If the Thickneſs, or t, be neglected, we ſhall have

CASE I. For *Diverging Rays*, $\frac{2\mathrm{d}r}{\mathrm{d}-2r}=f.$ Theor. 28.

CASE II. For *Parallel Rays*, $\frac{2\mathrm{d}r}{\mathrm{d}}=2r=f.$ Theor. 29.

CASE III. For *Converging Rays*, $\frac{2\mathrm{d}r}{\mathrm{d}+2r}=f.$ Theor. 30.

47. From the Theorems in Art. 40 and 44, it appears, that the focal Diſtance is greater

 by

by $\frac{2}{3}$ t, when the plain Side of the Lens is turned towards that of the Sun Beams, than when the convex Side is.

For CONCAVE LENSES.

48. If it be a double Concave, whose Radii of Concavities are unequal, and the Thickness considered; each Radius having, in this Case, a negative Sign, the Equation will be for

CASE I. Of *Diverging Rays*,

$$\frac{6\mathrm{dr}r + 2\mathrm{dt}r + 4\mathrm{tr}r}{-3\mathrm{dr} - 3\mathrm{d}r - \mathrm{dt} - 2\mathrm{rt} - 6\mathrm{r}r} = f.$$

Theor. 31.

49. CASE II. Of *Parallel Rays*, $\frac{6\mathrm{r}r + 2\mathrm{t}r}{-3\mathrm{r} - 3r - \mathrm{t}} = f.$ Theor. 32.

50. CASE III. Of *Converging Rays*,

$$\frac{-6\mathrm{dr}r - 2\mathrm{dt}r + 4\mathrm{tr}r}{3\mathrm{dr} + 3\mathrm{d}r + \mathrm{dt} - 2\mathrm{rt} - 6\mathrm{r}r} = f.$$ Theor. 33.

51. If the Thickness of the Lens, or t, be neglected, which in Concaves is most inconsiderable; the Theorems will be much shortened, and stand thus;

CASE

CASE I. For *Diverging Rays*, Fig. 24.

$$\frac{2\mathrm{d}\mathrm{r}r}{-\mathrm{d}\mathrm{r}-\mathrm{d}r-2\mathrm{r}r}=f.$$ Theor. 34.

52. CASE II. For *Parallel Rays*, Fig. 25.

$$\frac{2\mathrm{r}r}{-\mathrm{r}-r}=f.$$ Theor. 35.

53. CASE III. For *Converging Rays*, Fig. 26.

$$\frac{-2\mathrm{d}\mathrm{r}r}{\mathrm{d}\mathrm{r}+\mathrm{d}r-2\mathrm{r}r}=f.$$ Theor. 36.

54. Moreover, if we suppose the Radii of Concavities equal, that is, $\mathrm{r}=r$, we shall have the Theorems still more abridged thus;

CASE I. For *Diverging Rays*, Fig. 27.

$$\frac{\mathrm{d}\mathrm{r}}{-\mathrm{d}-\mathrm{r}}=f.$$ Theor. 37.

55. CASE II. For *Parallel Rays*, Fig. 28.

$$\frac{\mathrm{d}\mathrm{r}}{-\mathrm{d}}=-r=f.$$ Theor. 38.

56. CASE III. For *Converging Rays*, Fig. 29.

$$\frac{-\mathrm{d}\mathrm{r}}{\mathrm{d}-\mathrm{r}}=f.$$ Theor. 39.

Of a PLANO-CONCAVE LENS.

57. If the concave Surface be towards the Radiant, r being infinite, we shall have the Equation for

CASE I. Of *Diverging Rays*, Fig. 30.

$$\frac{-6dr-2dt-4tr}{3d+6r}=f. \quad \text{Theor. 40.}$$

58. CASE II. Of *Parallel Rays*, Fig. 31.

$$\frac{-6r-2t}{3}=-2r-\tfrac{2}{3}t=f. \quad \text{Theor. 41.}$$

59. CASE III. Of *Converging Rays*, Fig. 32.

$$\frac{6dr+2dt-4tr}{-3d+6r}=f. \quad \text{Theor. 42.}$$

60. If the Thickness of the Lens be neglected, the Theorems will be thus abridged.

CASE I. For *Diverging Rays*, $\frac{-2dr}{d+2r}=f.$ Theor. 43.

CASE II. For *Parallel Rays*, $\frac{-2dr}{d}=-2r=f.$ Theor. 44.

CASE

Fig. 24

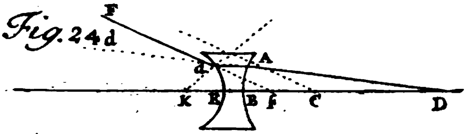


Fig. 25

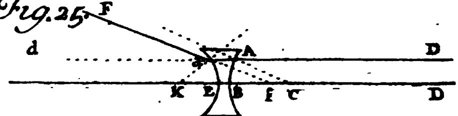


Fig. 26.

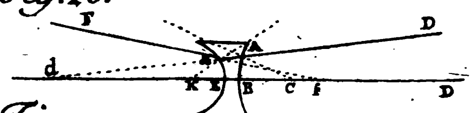


Fig. 27.

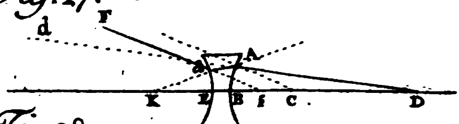


Fig. 30.

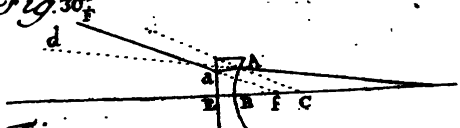


Fig. 31.

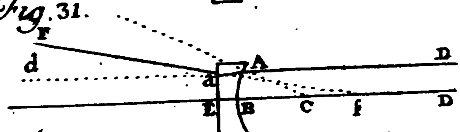
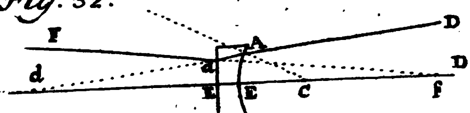


Fig. 32.



CASE III. For *Converging Rays*, $\frac{2dr}{2r-d}=f$. Theor. 45.

61. If the plain Surface be towards the Radiant, r will be infinite; and the Theorems will be for

CASE I. Of *Diverging Rays*, Fig. 33. Plate XIII. $\frac{-6dr-4tr}{3d+2t+6r}=f$. Theor. 46.

62. CASE II. Of *Parallel Rays*, Fig. 34. $\frac{6dr}{-3d}=-2r=f$. Theor. 46.

63. CASE III. Of *Converging Rays*, Fig. 35. $\frac{6dr-4tr}{-3d+2t+6r}=f$. Theor. 47.

64. If the Thickness t be neglected, these Theorems will become for

CASE I. Of *Diverging Rays*, $\frac{-2dr}{d+2r}=f$. Theor. 48.

CASE II. Of *Parallel Rays*, $\frac{-2dr}{d}=-2r=f$. Theor. 49.

CASE III. Of *Converging Rays*, $\frac{2dr}{2r-d}=f$. Theor. 50.

For a MENISCUS.

65. If the Radii of the Surfaces are unequal, and the convex Side exposed to the Radiant, in which Case r will be negative, we shall have the Equation for

CASE I. Of *Diverging Rays*,

$$\frac{-6dr\mathit{r}+2dt\mathit{r}-4tr\mathit{r}}{3dr-3d\mathit{r}-dt+2rt+6r\mathit{r}}=f.$$ Theor. 51.

66. CASE II. Of *Parallel Rays*,

$$\frac{-6r\mathit{r}+2t\mathit{r}}{3r-3\mathit{r}-t}=f.$$ Theor. 52.

67. CASE III. Of *Converging Rays*,

$$\frac{6dr\mathit{r}-2dt\mathit{r}-4tr\mathit{r}}{-3dr+3d\mathit{r}+dt+2rt+6r\mathit{r}}=f.$$ Theor. 53.

68. If the Thickness t be neglected, the Theorems will be for

CASE I. Of *Diverging Rays*, Fig. 36.

$$\frac{-2dr\mathit{r}}{dr-d\mathit{r}+2r\mathit{r}}=f.$$ Theor. 54.

69. CASE

69. CASE II. Of *Parallel Rays*, Fig. 37.

$$\frac{-2\mathrm{r}r}{\mathrm{r}-r}=f. \quad \text{Theor. 55.}$$

70. CASE III. Of *Converging Rays*, Fig. 38.

$$\frac{2\mathrm{dr}r}{\mathrm{dr}-\mathrm{dr}+2\mathrm{r}r}=f. \quad \text{Theor. 56.}$$

71. If the *Radius* of Convexity be equal to the Radius of Concavity, *viz.* $\mathrm{r}=r$; and the Thickneſs t neglected, the Theorems will be for

CASE I. Of *Diverging Rays*, Fig. 39.

$$\frac{-2\mathrm{dr}r}{2\mathrm{r}r}=-\mathrm{d}=f. \quad \text{Theor. 57.}$$

72. CASE II. Of *Parallel Rays*, Fig. 40.

$$\frac{-2\mathrm{rr}}{0}=f. \quad \text{Theor. 58.}$$

73. CASE III. Of *Converging Rays*, Fig. 41.

$$\frac{2\mathrm{d}rr}{2rr}=d=f. \quad \text{Theor. 59.}$$

74. If the concave Side be expoſed to the Radiant, and the Radii of the Surfaces unequal, r being in this Caſe negative, we ſhall have the Equation for

CASE

CASE I. Of *Diverging Rays*,

$$\frac{-6\mathrm{dr}r-2\mathrm{dt}r-4\mathrm{tr}r}{-3\mathrm{dr}+3\mathrm{d}r-\mathrm{dt}-2\mathrm{rt}+6\mathrm{r}r}=f.$$

Theor. 60.

75. CASE II. Of *Parallel Rays*;

$$\frac{-6\mathrm{r}r-2\mathrm{t}r}{-3\mathrm{r}+3r-\mathrm{t}}=f.$$ Theor. 61.

76. CASE III. Of *Converging Rays*,

$$\frac{6\mathrm{dr}r+2\mathrm{dt}r-4\mathrm{tr}r}{3\mathrm{dr}-3\mathrm{d}r+\mathrm{dt}-2\mathrm{rt}+6\mathrm{r}r}=f.$$ Theor. 62.

77. If we neglect the Thickness t, we shall have for

CASE I. Of *Diverging Rays*, $\frac{-2\mathrm{dr}r}{\mathrm{d}r-\mathrm{dr}+2\mathrm{r}r}=f.$ Theor. 63.

78. CASE II. Of *Parallel Rays*, $\frac{-2\mathrm{r}r}{r-\mathrm{r}}=f.$ Theor. 64.

79. CASE III. Of *Converging Rays*,

$$\frac{2\mathrm{dr}r}{\mathrm{dr}-\mathrm{d}r+2\mathrm{r}r}=f.$$ Theor. 65.

80. If

80. If the Radius of Concavity be equal to the Radius of Convexity, and the Thickness t neglected, we have for

CASE I. Of *Diverging Rays*, $\frac{-2\,\mathrm{d}\,\mathrm{r}\,r}{2\,\mathrm{r}\,r} = -\mathrm{d} = f$. Theor. 66.

81. CASE II. Of *Parallel Rays*, $\frac{-2\,\mathrm{r}\,\mathrm{r}}{0} = f$. Theor. 67.

82. CASE III. Of *Converging Rays*, $\frac{2\,\mathrm{d}\,r\,r}{2\,r\,r} = \mathrm{d} = f$. Theor. 68.

For a PLAIN LENS.

83. This being no other than a Piece of common plain Glass, whose Surfaces are parallel Planes, the Radii r and *r* being both infinite, the Equation in Art. 18, will become for

CASE I. Of *Diverging Rays*, Fig. 42. $\frac{6\mathrm{d} + 4\mathrm{t}}{-6} = \mathrm{d} + \frac{2}{3}\mathrm{t} = -f$. Theor. 69.

84. CASE II. Of *Parallel Rays*, Fig. 43. $\frac{6\mathrm{d}}{0} = f$. Theor. 70.

85. CASE

85. CASE III. Of *Converging Rays*, Fig. 44.

$\frac{-6d+4t}{-6} = d - \frac{2}{3}t = f.$ Theor. 71.

86. If the Thickneſs t be neglected, the Theorems will be for

CASE I. Of *Diverging Rays*, $\frac{6d}{-6} = -d = f.$ Theor. 72.

87. CASE II. Of *Parallel Rays*, $\frac{6d}{0} = f.$ Theor. 73.

88. CASE III. Of *Converging Rays*, $\frac{-6d}{-6} = d = f.$ Theor. 74.

89. Thus much for finding the *Foci* of Rays after Refraction. If the Focus be given with either the Diſtance of the Radiant, Radii of the Convexities or Concavities, or the Ratio of Refraction, the reſt may be found by Theorems raiſed from the ſame fundamental Equation in Art. 16, which, if t be rejected as inconſiderable, will ſtand thus, $\frac{pdrr}{dr+dr-prr} = f.$ And when reduced we ſhall have $pdrr = drf + drf - prrf.$

90. If

90. If now f, r, r, p, are given, d will be thus found, $p\mathrm{r}rf = \mathrm{d}\mathrm{r}f + \mathrm{d}rf - p\mathrm{d}\mathrm{r}r$; and therefore it will be $\frac{p\mathrm{r}rf}{\mathrm{r}f + rf - p\mathrm{r}r} = \mathrm{d}$. Theor. 76.

91. If f, r, d, and p be given, r will be found from this Equation $p\mathrm{r}rf + p\mathrm{d}\mathrm{r}r - \mathrm{d}rf = \mathrm{d}\mathrm{r}f$; for then $\frac{\mathrm{d}\mathrm{r}f}{p\mathrm{r}f + p\mathrm{d}\mathrm{r} - \mathrm{d}f} = r$. Theor. 77.

92. If r be required, the Equation will be $p\mathrm{r}rf + p\mathrm{d}\mathrm{r}r - \mathrm{d}\mathrm{r}f = \mathrm{d}rf$; and so $\frac{\mathrm{d}rf}{p\mathrm{r}f + p\mathrm{d}\mathrm{r} - \mathrm{d}f} = r$. Theor. 78.

93. If f, r, r, and d be given, the Equation for p will be $p\mathrm{r}rf + p\mathrm{d}\mathrm{r}r = \mathrm{d}\mathrm{r}f + \mathrm{d}\mathrm{r}f$; and hence we shall have $\frac{\mathrm{d}rf + \mathrm{d}rf}{\mathrm{r}rf + \mathrm{d}\mathrm{r}r} = p$. Theor. 79.

94. Having found p, the Ratio of the Refraction is easily known; for $p = \frac{\mathrm{R}}{\mathrm{I} - \mathrm{R}}$, and therefore $\mathrm{I}p - \mathrm{R}p = \mathrm{R}$, and so $\mathrm{I}p = \mathrm{R} + \mathrm{R}p$; whence $\mathrm{I} : \mathrm{R} :: p + \mathrm{I} : p$.

95. If

... vex of
... ; then
... llows.

... eor. 80.

... r. 81.

... y Con-
... $\frac{rf}{r-f}$

... eor. 83.

... I A P.

CHAP. II.

The Theory of DIOPTRICS *continued, for determining the mutual Proportion of the* Object *and* Image.

1. LET LN be a double equally convex Lens, Plate XIII, AC a it's Axis, OB an Object in a Position perpendicular thereto.

2. From the Extremity of the Object O, suppose a Ray OF fall upon the Lens in such a Point F, that by it's Refraction in the Glass, it be made to pass through the middle Point of the Lens C to G; at G it will be so refracted to I, that the Ray GI shall be parallel to OF. This Ray OCI will be the Axis of all the Rays which fall on the Lens from the Point O, and I will be the Focus where they will all be collected, by what has been before taught.

3. In like Manner BHCKM is the Axis of that Pencil of Rays, which proceed from the Extremity of the Object B, and their *Focus*, suppose at M. Then since all the Points in the Object between O and B, must necessarily

rily have their *Foci* between I and M, IM will consequently be the *Image* of the *Object* OB, *per* Princip. 10 and 11.

4. From the Vertices of the Lens D and E draw the Lines DO, DB, and EI, EM, and produce the incident Ray OF, till it meet the Axis in d, and the refracted Ray IG, till it cuts the Axis in e; now since Od and Ie are both in the same Plane, and parallel to each other, from the Nature of Refractions, the Angle OdA will be equal to the Angle Iea; and since the Points d and e are both within and near the middle of the Lens, it is evident that if the Thickness of the Lens be very small or neglected, the Angle OdA will become equal to ODA, and Iea equal to IEa; and consequently the Angle ODA will be equal to the Angle IEa. In like Manner it is shewn the Angle BDA = MEa. And therefore the whole Angle ODB = MEI. From whence it follows, *That the Image* IM *appears from the Vertex of Emersion* E, *under an Angle equal to that under which the Object appears from the Vertex of Incidence* D.

5. If the Object OB be very small, or at a great Distance, so that the Points O, A, B, are nearly at an equal Distance from the Vertex D; then will their correspondent Points I, a, M, be very nearly in the same Plane; *and the Image* IM *parallel to the Object* OB, *and perpendicular to the Axis* Aa.

6. In

6. In this Case, the Triangles ODA and IEa are similar; for they are equiangular, since the Angle D = E, and A = a, as being both right ones, and therefore O = I; and consequently DA : Ea : AO : Ia; for the same Reason DA : Ea : : AB : aM. But DA : Ea : : OA + AB : Ia + aM : : OB : IM. That is, *The Distance of the Object is to the Distance of the Image, as the Length of the Object to the Length of the Image.*

7. Also AO : AB : : Ia : aM; that is, *The Object and it's Image are divided proportionally by the Axis of the Lens* Aa.

8. Call the Object O, it's Image I; the Distance of the Object and it's Image, d and *f*, as before; then will it be O : I : : d : *f*, and therefore $\frac{Id}{O} = f$. But for the Case of a double and equally convex Lens, whose Thickness is inconsiderable, we before found $f = \frac{dr}{d-r}$, Chap. I. Theor. 16. Therefore $\frac{Id}{O} = \frac{dr}{d-r}$; and so $Idd - Idr = drO$, that is, $Id - Ir = rO$, and $Id = rI + Or$; and consequently $\frac{rI + Or}{I} = d$. Theor. 1.

9. If the Object be a *Surface*, the Image will be so too; and their Proportion duplicate of the former; that is, $O : I : : d^2 : f^2$; and hence

hence $\sqrt{\frac{Id^2}{O}} = f = \frac{dr}{d-r}$; and ſquaring each Side, we have $\frac{Id^2}{O} = \frac{d^2r^2}{\overline{d-r}^2}$, which reſolved is, $I \times \overline{d-r}^2 = O \times r^2$; hence $\overline{d-r}^2 = \frac{O}{I} \times r^2$; put $\frac{O}{I} = p$, then $\overline{d-r}^2 = pr^2$, and $d-r = \sqrt{pr^2}$; and laſtly, $d = \sqrt{pr^2} + r$. Theor. 2.

10. If the Object be a *Solid*, the Image will be ſo; and then it will in like Manner appear (the Proportion being triplicate of the firſt, *viz.* $O : I :: d^3 : f^3$) that $d = \sqrt[3]{pr^3} + r$. Theor. 3.

11. Theſe three Theorems find the Diſtance d, at which an Object (whether *Line*, *Surface*, or *Solid*) muſt be placed that it may bear the ſame Proportion to it's Image, as O does to I, in Caſe we uſe a *double and equally convex Lens.*

12. But the ſame thing is found for any other Lens, as I ſhall ſhew for a Plano-Convex, Double and Plano-Concaves, as follows.

13. In Chap. I. Theor. 22. For a Plano-Convex we had $\frac{2dr}{d-2r} = f$. Therefore $\frac{Id}{O}$

$= 2dr$

$= \frac{2dr}{d-2r}$; and consequently $Id - 2Ir = 2rO$, and $Id = 2rO + 2Ir$; and hence $\frac{2rO + 2Ir}{I} = d$. Theor. 4.

14. If the Object be a Superficies, then $\frac{Id^2}{O} = \frac{4d^2r^2}{\overline{d-2r}^2}$, and $I \times \overline{d-2r}^2 = 4r^2 \times O$; that is, $\overline{d-2r}^2 = 4r^2 \times \frac{O}{I} = 4r^2p$; wherefore $d - 2r = \sqrt{4r^2p}$; and $d = 2r + \sqrt{4r^2p}$. Theor. 5.

15. In the same Manner, for *Solids*, we shall have $d = 2r + \sqrt[3]{8r^3p}$. Theor. 6.

16. For a *double and equally concave Lens*, the Theorem was $\frac{dr}{-d-r} = f$. See Theor. 37. And since in this Case $O : I :: d : -f$; we shall have $\frac{Id}{-O} = f = \frac{dr}{-d-r}$, and so $Id + Ir = rO$; and therefore $Id = Or - rI$; and consequently $d = \frac{Or - rI}{I}$. Theor. 7.

17. For a Superficies it will be $d = \sqrt{r^2 p} - r$. Theor. 8.

18. And for a Solid, $d = \sqrt[3]{p r^3} - r$. Theor. 9.

19. In Case of a *Plano-Concave*, it will be also $\frac{\mathrm{I}d}{-\mathrm{O}} = f = \frac{-2dr}{d+2r}$, (*per* Theor. 43.) and therefore $\mathrm{I}d + 2\mathrm{I}r = 2r\mathrm{O}$; and hence $d = \frac{2r\mathrm{O} - 2r\mathrm{I}}{\mathrm{I}}$. Theor. 10.

20. For a Superficies, $d = \sqrt{4r^2 p} - 2r$. Theor. 11.

21. For a Solid, $d = \sqrt[3]{8r^3 p} - 2r$. Theor. 12.

22. In a *Meniscus*, whose Radii of Convexity and Concavity are unequal, and the convex Side exposed to the Radiant; we have $\frac{\mathrm{I}d}{\mathrm{O}} = \frac{-2drr}{dr - dr + 2rr}$, (*per* Theor. 54.) whence we find $d = \frac{2rr\mathrm{O} + 2rr\mathrm{I}}{r\mathrm{I} - r\mathrm{I}}$. Theor. 13.

23. If

23. If the concave Side be expoſed to the Radiant, we have $d = \frac{2rrO + 2rrI}{rI - rI}$. Theor. 14.

24. If the Radius of Convexity be equal to that of the Concavity, we ſhall have (*per* Theor. 57 and 66.) $\frac{Id}{O} = d$; ſo that $Id = Od$, that is, $I = O$. Theor. 15. Which ſhews, that in this Caſe a Meniſcus Lens can neither magnify nor diminiſh an Object.

25. In all the foregoing Theorems we had Regard only to *affirmative Focus's*; but they may be as eaſily adapted to *negative ones*, if it be required to have the Image on the ſame Side with the Object. Thus for a *double equally convex Lens*, for an affirmative Focus, the Theorem is $\frac{dr}{d - r} = + f$; but for a negative Focus, it will be $\frac{dr}{r - d} = f$; alſo it will be $O : I :: d : -f$; and therefore $\frac{Id}{O} = \frac{dr}{r - d}$; from whence we have $d = \frac{Ir - Or}{I}$. Theor. 16.

26. In this Manner Theorems are raiſed for other Glaſſes, where it is to be obſerved, that the Terms of the Equations are the ſame as for *affirmative Focus's*, the *Signs* only are changed: But this is a Matter not worth inſiſting on, being of little Uſe.

27. If it be required to form an Image equal to the Object, we ſhall have $O = I$; and the Diſtance required will be for the ſeveral Lenſes, as follows.

28. For a double and equally convex Lens, we have $\frac{Ir + Or}{I} = \frac{2Ir}{I} = 2r = d$. Theor. 17.

29. For a *Plano-Convex*, $\frac{2rO + 2Ir}{I} = \frac{4Ir}{I} = 4r = d$. Theor. 18.

30. For a double and equally concave Lens, we have $\frac{Or - Ir}{I} = \frac{O}{I} = d$. Theor. 19. And ſince in this Caſe the Focus is always negative, it is evident the Image can never be equal to the Object at any Diſtance from the Lens, much leſs can it ever exceed it; and therefore

therefore ſuch a Lens can only diminiſh, never magnify an Object.

31. The ſame thing is true, and for the ſame Reaſon, in a Plano-Concave, as will appear from Theor. 10 hereof; and alſo in a Meniſcus from Theor. 13 and 14.

32. If the Diſtance d of the Object, and the Proportion of O to I be given, the Radius r of double and equally convex and concave Lenſes, in this Caſe neceſſary, is found from the Equation in Art. 8, *viz.* $Id = Ir + Or$; and therefore for a *double Convex* $\frac{Id}{I+O} = r$. Theor. 20.

33. For a *Plano-Convex*, the Equation is $Id = 2rO + 2Ir$, and therefore $\frac{Id}{2O+2I} = r$. Theor. 21.

34. For a double and equally concave Lens, the Equation being $Id = Or - Ir$; we have $\frac{Id}{O-I} = r$. Theor. 22.

35. For a Plano-Concave, the Equation is $Id = 2rO - 2Ir$, and ſo $\frac{Id}{2O-2I} = r$. Theor. 23.

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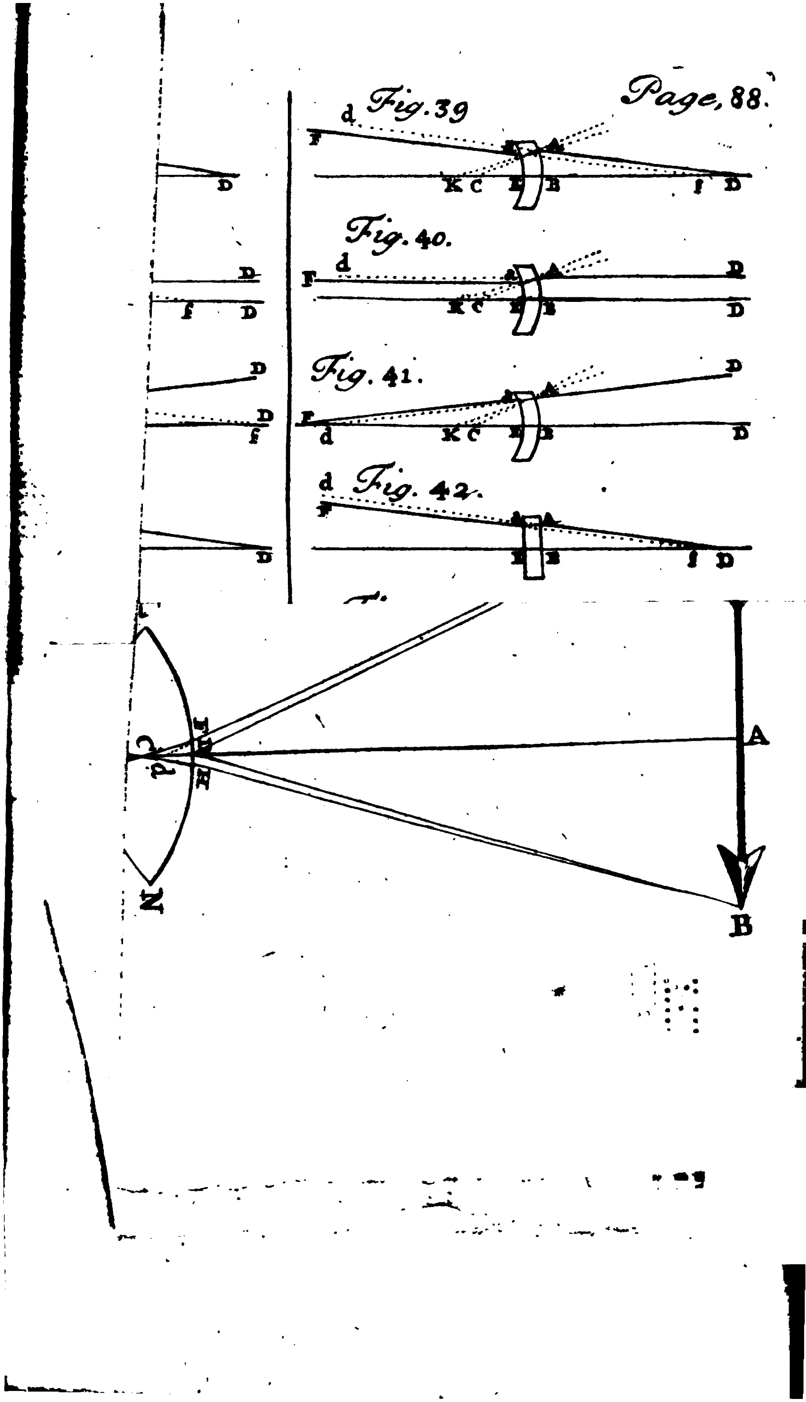
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CHAP. III.

The Theorems relating to GLOBES *and* HEMISPHERES *reduced to practical Rules, and exemplified.*

1. AS in the firſt Part, ſo here I ſhall make uſe of the *Tenths* of an Inch, for the Meaſurements to be made in the enſuing Examples. And firſt, for

A GLOBE.

2. CASE I. For *Diverging Rays*, Plate XIV. Scheme 1. Let there be given the Diſtance of the Radiant D E, and the Radius of the Globe C E, to find the Diſtance of the Focus I F.

RULE.

Multiply the Diſtance of the Radiant by the Radius, and to that Product add four times the Square of the Radius; divide that Sum by the Difference between twice the Diſtance of the Radiant, and the Radius, the Quotient will be the Diſtance of the Focus, *per* Theor. 1. Art. 20. Chap. I.

3. EXAMPLE.

Let DE = 30	CE = 10	2 DE = 60
CE = 10	CE = 10	CE = 10
	Squ. 100	Diff. 50
Product 300	4	
Add 400	400	

Then 50)700(14 = IF, the focal Distance required, *viz.* one Inch, and four Tenths.
50
200
200
...

4. As 2DE is greater, equal to, or less than CE; the Focus F will be affirmative, or behind the Globe, infinite, or negative, or before the Globe.

5. CASE II. Of *Parallel Rays*, Scheme 2. A Glass Globe exposed to Parallel Rays, will collect and converge them to a Point F, which will be just half the Radius of the Globe CI behind it, by Theor. 2. Such a Globe therefore, held in the Sun's Rays, will burn very intensely, if large.

6. CASE III. Of *Converging Rays*. The Focus F is found here by the same Rule as in CASE I; only the *Sum* and *Difference* there must

there must be exchanged for the *Difference* and the *Sum* here. And according as the Product of the Distance of the Radiant Point, into the Radius CE, is greater, equal to, or less than 4 times the Square of the Radius, the Focus will be affirmative, within the Globe, or negative, *per* Theor. 3.

For an HEMISPHERE.

7. CASE I. Of *Diverging Rays.* If the *Convex Part* be towards the Radiant, the Focus will be found by this

RULE.

To the Product of the Distance of the Radiant into the Radius, add the Square of Radius, multiply that Sum by 4: This divide by the Difference between 3 Times the Distance, and 6 Times the Radius, the Quotient will give the Distance of the Focus, *per* Theor. 4.

8. But if the *Plain Side* of the Hemisphere be turn'd towards the Object the Radiant, the Rule will be somewhat alter'd; thus,

RULE.

RULE.

To 6 times the Product of the Diſtance into the Radius, add 4 times the Square of Radius; divide that Sum by the Difference between 3 Times the Diſtance, and 4 Times the Radius; the Quotient is the Diſtance of the Focus required, *per* Theor. 7.

9. EXAMPLE

For the Convex Side towards the Radiant.

Let the Diſtance be — 30
The Radius 10
Product 300
Add the Squ. of Radius 100
Sum 400
Multiply by 4

3 Times the Diſtance is — 90
6 Times the Radius is — — 60
Difference 30

Divide by 30) 1600 (53 = the Diſtance of the Focus, *viz.* $5\frac{3}{10}$ Inches.
150
100
90
10

10. EX-

10. EXAMPLE

Of the Plain Side towards the Radiant.

Let the Di- ſtance be	30	3 Times the Di- ſtance ——	90
The Radius —	10	4 Times the Radius ——	40
Product	300	Difference	50
Multiply by	6		
Product	1800		
Add 4 times the Sq. of R.	400		

Divide by 50) 2200 (44 = the Diſtance of the Focus, *viz.* 4 $\frac{4}{10}$ Inches.

200
—
200
200
—
. . .

11. In this near Situation of the Radiant, the convex Side, expoſed to it, gives the greateſt focal Diſtance, as we ſee by the Examples; but if the Radiant be above 50, or 5 Inches diſtant from the ſame Hemiſphere, then the convex Side will give the ſhorteſt focal Diſtance, as is eaſy to try.

12. CASE II. Of *Parallel Rays*, Scheme 3. If the *Plain Side* be turn'd to the Rays of the Sun; they will be collected at F, at the Diſtance

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CHAP. IV.

The Rules for finding the Focus of a DOUBLE CONVEX LENS.

1. IF the Lens be *unequally Convex*, and the Thickneſs not regarded, then if the Radius of each Surface be known, and the Diſtance of the Radiant from the Lens, the Diſtance of the Focus will be found for

2. CASE I. Of *Diverging Rays*, by the following

RULE.

Multiply twice the Product of the Radii into each other by the Diſtance of the Radiant, this ſhall be the *Dividend*. Then take the Difference between the Product of the Sum of the Radii, multiplied by the Diſtance, and twice the Product of the Radii into each other; by this divide the Dividend above, the Quotient will be the Diſtance of the Focus required, *per* Theor. 13.

3. EX-

3. EXAMPLE.

Let the longest Radius be 50, and the shortest 30; and the Distance of the Radiant 300: Then

Longest Radius — —	50
Shortest — — — —	30
Product — — — —	1500
Multiply by — — —	2
Twice Product — —	3000
Distance — — —	300

```
21000)900000(42.8
      84000
      60000
      42000
     180000
     168000
      12000
```

Longest Radius	50
Shortest — —	30
Sum — — —	80
Distance — —	300
Product —	24000
Subduct — —	3000, twice Prod. of Radii,
Difference —	21000 Divisor.

Thus the focal Distance will be $42 \frac{8}{10}$.

4. CASE II. Of *Parallel Rays*. The Focus of these is found thus.

RULE.

Divide twice the Product of the unequal Radii into each other, by the Sum of the Radii, the Quotient will be the focal Distance required, *per* Theor. 14.

5. EXAMPLE.

Let the Radii be the same as before;

Then the Longest Rad.	50	50
The Shortest	30	30
Product	1500	80 Sum
	2	

80) 3000 (375 = the focal Distance required, *viz.* $37\frac{1}{2}$.
240
600
560
400
400

6. CASE III. Of *Converging Rays*. The Focus of these is found by the Rule of the first Case,

Caſe, if inſtead of the *Difference* we take the *Sum* of thoſe Quantities mentioned in the Diviſor, *per* Theor. 15.

7. If the Lens be *equally Convex*, as are moſt of thoſe in common Uſe, and the Thickneſs neglected as inconſiderable, and cauſing no material Error, then will the Focus of this moſt uſeful Lens be moſt eaſily found by this ſhort and plain Rule, for

8. CASE I. Of *Diverging Rays*, Plate XV. Scheme 1.

RULE.

Multiply the Diſtance of the Radiant by the Radius of the Lens, divide that Product by the Difference between the ſaid Diſtance and Radius, the Quotient is the focal Diſtance required, *per.* Theor. 16.

9. EXAMPLE.

Let the Diſtance DE = 40 —— 40
The Radius — CE = 15 —— 15

The Product —— 600 25 Diff.

Then 25) 600 (24 = FE, the focal Diſtance ſought, *viz.* $2\frac{4}{10}$ Inches.

50
100
100
...

10. And

10. And here it is to be obſerved, that according as the Diſtance of the Radiant D E is *greater*, *equal to*, or *leſs* than the Radius C E, ſo the Focus F will be *affirmative*, *infinite*, or *negative*.

11. CASE II. Of *Parallel Rays*, Scheme 2. In this Caſe the Focus F will be coincident with the Center C; that is, the Rays of the Sun will be collected into a Point F, whoſe Diſtance F E, from the Lens, will be equal to the Radius C E, *per* Theor. 17.

12. Hence a *Convex* Lens becomes a *Burning Glaſs*, whoſe Power is greater or leſs, as it's Surface is larger or ſmaller, if the focal Diſtance be the ſame; or as the ſaid Diſtance is leſſer or greater, if the Quantity of Surface be the ſame.

13. For if the Lenſes be of different Bigneſs, and of the ſame focal Length, their Power of burning will be as the *Squares of their Diameters* directly, for it will be as the Quantity of Rays falling on their Surfaces directly; thus if the Diameter of one Lens be 1 Inch, and of the other 4; the latter will burn 16 times more intenſely than the former.

14. Again, if the Diameters or Surfaces of two Lenſes be the ſame, the Power of burning will be reciprocally, as the Surfaces of the *burning Spots*, which are the Images of the Sun, and theſe are as the Squares of their Diameters; but ſince theſe Diameters are as the

the Diſtances from the Glaſſes, the Power of burning will be reciprocally, as the Squares of the *focal Diſtances.* Thus ſuppoſe two Lenſes of equal Diameters, and the focal Diſtance of one be 1 Inch, but that of the other 4; the Power of burning, in the former, will be 16 times greater than in the latter.

15. The abſolute Power of burning, in theſe Lenſes, is as the Number of Times the burning Spot is contained in the Surface of the Lens, or as the Square of the Diameter of the Spot is contained in the Square of the Diameter of the Lens. Thus ſuppoſe I have a Lens 4 Inches in Diameter, and the burning Spot made thereby I meaſure and find to be one Tenth, *viz.* $\frac{1}{10}$ of an Inch. Then in 4 Inches there being 40 Tenths of an Inch, the Diameter of the *Lens* to that of the *Spot* will be as 40 to 1; and the Squares of theſe Diameters will be as 1600 to 1; that is, the Spot is *one Thouſand ſix hundred Times* leſs than the Surface of the Lens; and the Heat of the Sun's Rays will conſequently be augmented in the ſame Proportion, which therefore will burn very ſtrongly.

16. Note, if the Thickneſs of the Lens be at any Time conſidered, the *focal Diſtance* will be very nearly one *ſixth Part* of the ſaid Thickneſs leſs than the Radius, as appears from the Theory.

17. CASE III. Of *Converging Rays*, Scheme 3. The Rule for finding the Focus of these Rays is as follows.

RULE.

Multiply the Diſtance of the Radiant by the Radius, divide that Product by the Sum of the ſaid Diſtance and Radius; the Quotient will be the Diſtance of the Focus required, *per* Theor. 18.

18. EXAMPLE.

Let the Diſtance d I	= 30	— —	30	
The Radius — CE	= 15	— —	15	
The Product — —	450		45	Sum

Then 45)450(10 = F I, the Diſtance of the Focus required, *viz.* 1 Inch.

45
——
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19. The Focus will in this Caſe be always affirmative, and it's Diſtance leſs than the Radius. As the double and equally Convex Lens is the moſt common and uſeful, ſo all it's Caſes are of the laſt Importance for a due Underſtanding of the Nature, Conſtruction, and Effects, of all Dioptric Machines, as will be ſhewn in the third Part.

CHAP.

CHAP. V.

The Rules for finding the Focus of a PLANO-CONVEX LENS.

1. IF the Thickneſs of a *Plano-Convex Lens* be conſidered, and it be expoſed on it's convex Side to Parallel Rays, as thoſe of the Sun, the Focus will be at the Diſtance of *twice the Radius, wanting* $\frac{2}{3}$ *(two Thirds) of the Thickneſs of the Lens, per* Theor. 20.

2. But if the ſame Lens be expoſed with it's plane Side to Parallel Rays, the Focus will then be preciſely at the Diſtance *of twice the Radius from the Glaſs, per* Theor. 26.

3. If the *Thickneſs* of the Lens be neglected, the Rules for the Focus are the ſame for either of the two Sides towards the Radiant, as appears from the Theorems relating to theſe Caſes.

4. CASE I. Of *Diverging Rays*, Plate XVI. Scheme 1. In this Caſe the Rule for finding the Focus is as follows, *viz.*

RULE.

Divide twice the Product of the Distance of the Radiant into the Radius, by the Difference between the said Distance, and twice the Radius; the Quotient will be the focal Distance required, *per* Theor. 22 and 28.

5. EXAMPLE.

Let the Distance DE = 40 DE = 40
The Radius CE = 8 2CE = 16

The Product 320 Diff. 24
Multiply by — 2

Then 24) 640 ($26\frac{2}{3}$ = IF, the focal Distance required.
48
160
144
16

6. Here again it is evident, from the same Theorems, that as the Distance DE is greater, equal to, or less than twice the Radius, so the Focus will be affirmative, infinite, or negative.

7. CASE II. Of *Parallel Rays*, Scheme 2. If a Plano-Convex be exposed to *Parallel Rays*, as those of the Sun, they will be collected thereby into a Point F, which will be distant

distant from the Glass just twice the Radius C E, or the Diameter of the Sphere, of which the Lens is a Segment; as is evident from Theor. 23 and 29.

8. The focal Distance is here twice as great as in the same Case of a *Double and equally Convex Lens*, See Chap. IV. Art. 11. Consequently, the *Power of Burning* (*cæteris paribus*) will be 4 times greater in the *Double* than in the *Plano*-Convex Lens, *per* Art. 14.

9. CASE III. Of *Converging Rays*, Scheme 3. The Rule for the Focus is the same as above for *Diverging Rays*, if instead of the *Difference* you divide by the *Sum* of the Distance of the Radiant, and twice the Radius, *per* Theor. 24 and 30.

10. EXAMPLE.

Let the Distance DE = 30   DE = 30
The Radius CE = 8   2CE = 16

Product 240   Sum 46
Multiply by — 2

Then 46) 480 (10 = FE, the focal Distance, viz. 1 Inch, and a little more.
46
20

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CHAP. VI.

The Rules for finding the virtual Focus of DOUBLE CONCAVE LENSES.

1. IF the Lens be unequally Concave, and the Thickneſs thereof neglected, (which is much more inconſiderable in theſe than in the Convex Lenſes) then the Rule for finding the Focus for

2. CASE I. Of *Diverging Rays*, is as follows.

RULE.

Multiply twice the Product of the Radii by the Diſtance of the Radiant; divide that Product by the Sum of the Radii multiplied by the Diſtance, and twice the Product of the Radii, the Quotient will be the Diſtance of the Focus, *per* Theor. 34.

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4. CASE II. Of *Parallel Rays.* The Rule r finding the Focus here, is the ſame as for Double unequally Convex Lens, *viz. To ivide twice the Product of the Radii by their um.* See Chap. IV. Art. 4. But the Focus ere is always *negative*, as it is there always *ffirmative.* And conſequently, ſince the Rule the ſame, if the Radii of the Concavities nd Convexities are reſpectively equal, and al- o the Diſtance of the Radiant; the Diſtance of the Focus will alſo be equal from either Lens.

5. CASE III. Of *Converging Rays.* The Rule for finding the Focus here is the very ſame as for *Diverging Rays,* in Chap. IV. Art. 2. Or that above in Art. 2. if inſtead of the *Sum* you take the Difference of the Quantities there mentioned, *per.* Theor. 36.

6. If the Lens be double and equally Concave, and the Thickneſs not conſiderable, the Rule for finding the Focus of *Diverging Rays* is as follows.

7. CASE I. Of *Diverging Rays*, Plate XVI. Scheme 1.

RULE.

Multiply the Diſtance of the Radiant by the Radius; divide that Product by the Sum of

Sum of the ſaid Diſtance and Radius, the Quotient will be the Diſtance of the *virtual* Focus, always negative, *per* Theor. 37.

8. EXAMPLE.

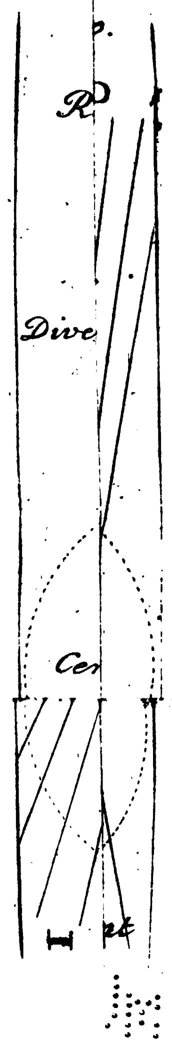
Let the Diſtance DE =	40	——	40
The Radius — CE =	10	——	10
	400		50 Sum

Then 50) 400 (8 = fE, the focal Diſtance.
400
...

9. CASE II. Of *Parallel Rays*, Scheme 2. The virtual Focus of theſe Rays is always at the Diſtance of the Radius from the Lens, as in a Double and equally Convex Lens, *per* Theor. 38. But is here always *negative*, as there it was always *affirmative*.

10. CASE III. Of *Converging Rays*, Scheme 3. The Rule for finding the Focus of theſe Rays is exactly the ſame as that for *Diverging Rays* in a *Double Convex*, Chap. IV. Art. 8. And according as the Diſtance of the Point d, towards which they tend, *viz.* dI is *greater*, *equal to*, or *leſs* than the Radius CI, the Focus will be *virtual* and *negative*, *infinite*, or *real* and *affirmative*, *per* Theor. 39.

11. That is, If Rays converge towards a Point d, as in Scheme 3, beyond the Center C, they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CHAP.

CHAP. VII.

To find the Focus of a PLANO-CONCAVE LENS.

1. IF a Plano-Concave Lens be exposed with it's *Concave Side* to *Parallel Rays*, (*viz.* Rays of the Sun) and the Thickness of the Lens be considered, the virtual Focus will be at the Distance of *twice the Radius, or Diameter of the Sphere, lessened by* ⅓ *of the Thickness of the Lens, per* Theor. 41.

2. The same Lens exposed on the *Plain Side* to Parallel Rays, will have their virtual Focus at just the Distance of *twice the Radius, or Diameter* of the whole Concavity, *per* Theor. 46.

3. If the Thickness of the Lens be neglected, as it always may in this Sort; then the Rules for determining the Foci of all Sorts of Rays are the same, let them fall on which Side of the Lens they will, as is evident from the Theory.

4. CASE I. Of *Diverging Rays*, Plate XVIII. Scheme 1. The Rule for finding the Focus of these Rays by a Plano-Concave is this, *viz.*

RULE.

RULE.

Multiply twice the Radius by the Diſtance of the Radiant, divide that Product by the Sum of the ſaid Diſtance, and twice the Radius; the Quotient is the focal Diſtance, *per* Theor. 43.

5. EXAMPLE.

Let the Diſtance DE = 50 —— 50
Twice the Radius CE = 20 —— 20

1000   70 Sum

Then 70) 1000 (14 = fC, the focal Diſtance required, *viz.* $1\frac{4}{10}$ Inch.
70
300
280
20

6. The Focus in this Caſe is always virtual and negative; for Rays which fall diverging on this Lens are always ſo refracted as to proceed ſtill more diverging.

7. CASE II. Of *Parallel Rays*, Scheme 2. The Focus of theſe Rays will be at juſt twice the Length of the Radius from the Lens, as repreſented in the Scheme, *per* Theor. 44 or 49. That is, it will be diſtant from the Lens

the Diameter of it's Concavity, and will be always virtual and negative. So that it is impossible any concave Lens should be a *burning Glass*; but on the contrary, the Sun's Rays may, by these Lenses, have their Light and Heat lessened in any Proportion whatsoever.

8. CASE III. Of *Converging Rays*, Scheme 3. The Rule for finding the Focus of these Rays is Word for Word the same with that for CASE I, of *Diverging Rays*, in a *Plano-Convex* Lens. See Chap. V. Art. 4. This will be evident by comparing Theor. 45 or 50, with 22 or 28.

9. Here the Focus will be affirmative, infinite, or negative, according as twice the Radius is greater, equal to, or less than the Distance of the Radiant. In this Scheme, the Distance d E is equal to the Radius C E; therefore the focal Distance F E is equal to twice the Radius C E.

10. It is easy by this time for the Reader to observe, that the same Rule which finds the Focus of *Diverging Rays* in a *Convex* Lens, finds the Focus of *Converging Rays* in a *Concave* one; and also that the Rules for finding the Foci of *Diverging* and *Converging Rays*, in any Lens, differ only in the *Sum* or *Difference* of the Quantities in the Divisor.

11. The principal Use of these concave Lenses is in that Sort of Telescopes, which we call


D
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call *Perspective-Glass*, concerning which I shall treat largely in the next Part.

12. It often happens that we have a concave Lens, and know neither the Radius or Focus; to know the focal Distance therefore in such a Case, proceed thus: Take a Piece of Pastboard, &c. and cut in it a rouud Hole; and on another Piece of Pastboard, strike a Circle, whose Diameter is just double the Diameter of the said Hole; then apply the Piece with the Hole in it to the Lens, and hold them in the Sun-Beams, with the other Piece at such Distance behind, that the Light coming through the Hole may spread or diverge, so as to fill the Circle drawn there precisely; then is that Distance equal to the virtual Focus of the Lens; and also to the Radius, if a double Concave; or twice the Radius if a Plano-Concave. Let $GH =$ Diameter of the Hole, $LM =$ that of the Circle; then, since $LM = 2GH$, we have $fK = 2fE$, and therefore $IK = fI = 2CI$, the Radius of the Plano-Concave. Scheme 2. Plate XVIII.

CHAP. VIII.

The Rules for finding the Focus of a Meniscus Lens.

1. IF the Thickneſs of the Lens be not conſidered, the Radii of Convexity and Concavity unequal, and the convex Part expoſed to *Diverging Rays*, we ſhall find the Focus by the following Rule.

2. Case I. Of *Diverging Rays*, Plate XIX. Scheme 1.

RULE.

Multiply twice the Product of the Radius of Convexity into that of Concavity, by the Diſtance of the Radiant; this ſhall be the *Dividend.* Then multiply the Difference between the two Radii by the Diſtance of the Radiant; and to that Product add twice the Product of the Radii; the Sum ſhall be the *Diviſor*, by which divide the Dividend above, the Quotient will be the focal Diſtance required, *per* Theor. 54.

3. EX-

3. EXAMPLE.

Let the Radius of Convexity CE =	15
The Radius of Concavity — KI =	10
Their Product — — — — —	150
Multiply by — — — — — — —	2
	300
Multiply by the Distance — DE =	40
The Dividend — — — — — —	12000
Then from Rad. of Convexity CE =	15
Take the Radius of Concavity KI =	10
The Difference is — — — — —	5
Multiply by — — — — DE =	40
Product — — — — —	200
To that add twice the Prod. of Radii	300
The Sum is the Divisor, *viz.* —	500

Then 500) 12000 (24 = fE, the Distance of the Focus required, *viz.* $2\frac{4}{10}$ Inches.

1000
————
2000
2000
————
....

4. Here the Radius of Convexity *exceeds* that of Concavity; but if they are *equal*, then

will the Diſtance of the Focus be equal to the Diſtance of the Radiant, and *negative* and *virtual*; for the Rays will proceed with the ſame Degree of Divergency as they had when they fell on the Glaſs; for in this Caſe, all that is effected by the convex Surface AEB, is deſtroyed by the equally concave one GIH. See Theor. 57.

5. And univerſally, in the Caſe of Converging Rays, as the Product of the Diſtance of the Radiant into the Radius of Concavity is *leſs*, *equal to*, or *greater* than the Product of the ſaid Diſtance into the Radius of Convexity, added to twice the Product of the Radii into each other, the Focus will be negative, infinite, or affimative.

6. CASE II. Of *Parallel Rays*, Scheme 2. Every thing beſides remaining as before, the Focus of theſe Rays are found by this ſhort

RULE.

Divide twice the Product of the Radii by their Difference, the Quotient will be the focal Diſtance ſought, *per* Theor. 55.

7. EXAMPLE.

Let the Radius of Convexity be CE =	15	15
The Radius of Concavity be KI =	6	6
Product	90	9 Diff.
	2	
Twice the Product	180	

Then 9) 180 (20 = fE, the focal Distance required, *viz.* 2 Inches.
18
‥ 0

8. As the Radius of Convexity is *greater*, *equal to*, or *less* than the Radius of Concavity, the Focus of these Rays will be *negative*, *infinite*, or *affirmative*, as is plain from the Theory. In the last Case therefore, *viz.* when the *Convexity is less than the Concavity*, a *Meniscus* Lens will become a *burning Glass*.

9. CASE III. Of *Converging Rays*, Scheme 3. The Rule for finding the Focus of these Rays is the same as above for diverging ones; and as to the Nature of the Focus, it will be *negative*, *infinite*, or *affirmative*, according as the Product of the Distance and Radius of Convexity is *lesser*, *equal to*, or *greater* than the Product of the Distance into the Radius

of Concavity, added to twice the Product of the Radii into each other, *per* Theor. 56.

10. EXAMPLE.

Let the Radius of Concavity KI =	15 - - - -	15	
The Radius of Convexity CE =	6 - - - -	6	Sub.
Product — — —	90	9	Diff.
Multiply by — —	2	20	= dE
		180	Add
Twice Prod. of Radii	180 - - -	180	
Multiply by the Distance dE =	20	360	Divisor
The Dividend —	3600		

Then 360) 3600 (10 = FI, the focal Distance required, *viz.* 1 Inch, and the Focus affirmative.
360

. . . 0

11. If the Radii of Convexity and Concavity are equal, the Focus of Converging Rays will be at the Distance of the Radiant and affirmative; that is, the Rays in this Case will pass on after Refraction with the same Degree of Convergency as they before had, *per* Theor. 59.

12. When

120

iant

[illegible] *Me-* [illegible] the [illegible] the [illegible] vex [illegible] be [illegible] The [illegible] *ing* [illegible] *ays,* [illegible] 7, [illegible] ve, [illegible] less [illegible] adii [illegible] the [illegible]

[illegible]

[illegible] P.

CHAP. IX.

The Rules which determine the Distance of an Object that shall bear any assigned Proportion to it's Image formed by a CONVEX LENS.

1. IN Plate XX, F G is a double and equally convex Lens, C the Center, C E the Radius, C E A the Axis, O B an Object placed at right Angles therewith, and I M the Image formed by the said Lens. Suppose now it were required to know at what Distance the Object shall be placed, that it may bear the same Proportion to it's Image, as 40 to 10; this is the

RULE.

Add together the Numbers expressing the Object and it's Image, multiply their Sum by the Radius of the Lens; divide that Product by the Image, the Quotient will be the Distance of the Object required, *per* Theor. 1. Chap. II.

2. EXAMPLE I.

The Object is — — 40
The Image — — 10
Their Sum is — — 50
Let the Radius C E = 10

Then 10) 500 (50 = E A, the Distance of the Object, *viz.* 5 Inches.
50
—
000

3. EX-

3. EXAMPLE II.

Suppose I have a double and equally convex Lens, the Radius being 35, and I would have an Image formed in Proportion to the Object, as 15 to 275; Query at what Distance the Object must be placed?

```
The Object is  —  275
The Image is   —   15
                  ----
Their Sum is   —  290
Mult. by the Radius 35
                  ----
                 1450
                 870
                 -----
Divide by 15) 10150 (676, the Distance
              90       of the Object, viz.
              ---       67 ½ Inches, and
              115       a little more.
              105
              ---
               100
                90
               ---
                10
```

4. EXAMPLE III.

Suppose I would have an Image equal to the Object; Query the Distance of the Object? In this Case the Object must be placed at the Distance of twice the Radius of the Lens,

Lens, *per* Theor. 17. And then will the Diſtance of the Image be the ſame alſo, *per* Art. 6, Chap. II. This Problem of *making an Image juſt as large as the Life*, may be found very uſeful to Painters, Deſigners, *&c.* on many Occaſions, as will appear farther on.

5. EXAMPLE IV.

Suppoſe I would have an Object magnified in Proportion 15 to 150, that is, of 1 to 10, by a Lens of $3\frac{1}{2}$ Inches Radius. Then may the Diſtance of the Radiant or Object be found in the Manner as above. Thus.

The Object — —	15
The Image — —	150
Their Sum — —	165
Multiply by Radius	35
	825
	495
Divide by 150)	5775 (38.5, the Diſtance of the Object, *viz.* $3\frac{85}{100}$ Inches
	450
	1275
	1200
	750
	750
	...

6. EX-

6. EXAMPLE V.

Let there be a ſmall double Convex, whoſe Radius is 5, or ½ of an Inch, and let it be required to find at what Diſtance an Object muſt be placed, that it's Image may be 6 times larger than itſelf; proceed as above. Thus.

The Object is —	1
The Image is —	6
Their Sum is — —	7
Multiply by Radius	5

Divide by 6) 35 (5.8, the Diſtance required, *viz.* $\frac{58}{100}$ of an Inch.
30
——
50
48
——
2

7. Theſe Examples are ſufficient to ſhew, that any Object may be magnified or diminiſhed in any given Proportion by a convex Lens, with an affirmative Focus, *viz.* on the other Side of the Lens. That is, the Image will be *leſſer*, *equal to*, or *greater* than the Object, according as the Diſtance of the Object is *greater*, *equal to*, or *leſſer* than twice the Radius of the Lens.

8. The ſame Rule holds good for a *Plano-Convex* Lens, only with this Difference, that the

the Diſtance of the Object will be always twice as great as for a *Double Convex* of the ſame Radius. Thus an Object is *diminiſhed*, *made equal to the Life*, or *magnified*, according as it is placed at a Diſtance *greater*, *equal to*, or *leſs* than 4 times the Radius, or twice the Diameter of the Sphere, of which the Plano-Convex Lens is a Segment. All which is evident from Theor. 4 and 18.

9. If the Image be required on the ſame Side with the Object, the Theorem which finds the Diſtance of the Object for an aſſigned Proportion between it and it's Image, is thus expreſſed in Words.

RULE.

From the Image take the Object, multiply the Difference by the Radius; divide that Product by the Image, the Quotient is the Diſtance required, *per* Theor. 16.

10. From the Rule it is evident the Image muſt, in this Caſe, be always greater than the Object, which therefore cannot be diminiſhed at a negative Focus, nor ever equal to it's Image, but at the Vertex of the Lens.

11. EX-

11. EXAMPLE.

Let there be a double and equally convex Lens, whose Radius is 40, or 4 Inches; and let the Image be to the Object as 50 to 10; it is required to find the Distance of the Object for that Purpose. Proceed thus according to the Rule.

From the Image — 50
Take the Object — 10

The Differerence is 40
Multiply by Radius 40

Divide by 50) 1600 (32 = the Distance of the Object required, *viz.* $3\frac{2}{10}$ Inches.
150
100
100
...

12. The Distance of the Object will always be less than the Radius for a negative Focus; and when it becomes equal to the Radius, the Image is then at an infinite Distance, and infinitely larger than the Object.

13. In this Respect also, every thing is the same in a *Plano-Convex*, but the Distance of the Object; which is, for the same *Data*, always double to that for a double Convex of equal Radii.

14. The

14. The Object hitherto considered is supposed to be a *Line*, or simple Length of any thing; if the Superficies of any Object be considered that will be altered in a *duplicate* Proportion of the Length or Breadth; that is, according to the Square thereof: Thus if the Length of an Object be magnified or diminished 2, 3, 4, 5, *&c.* times, the Superficies of that Object will be magnified or diminished 4, 9, 16, or 25 times, because these Numbers are the *Squares* of the others.

15. But I shall give a Rule that will find at what Distance an Object must be placed, that it's Surface shall bear any assigned Proportion to the Surface of the Image; and is as follows.

RULE.

Divide the Object by the Image, and multiply the Quotient by the Square of the Radius; and to the square Root of that Product add the Radius; the Sum is the Distance of the Object required, *per* Theor. 2.

16. EXAMPLE.

Let there be a double and equally convex Lens, whose Radius is 30, or 3 Inches; and let the Surface of the Object be to that of the Image, as 1000 to 10; to find the Distance at which the Object must be placed for that Purpose. Proceed thus by the Rule.

Image

Image	Object	
10)	1000	(100 Quotient.
	100	900 = Radius Square.
	...0	90000 (300 = Square Root.
		30
		330 = Dist. sought.

17. Hence it appears, that if an Object be placed 33 Inches from the Lens, the Surface of the Image will be an 100 times less than it's own. If this Object were a *Circle* or a *Square*, the *Diameter* of the Circle or *Side* of the Square would be 10 times less in the Image than in the Object at that Distance.

18. By Theor. 3. we have a Rule for the Distance of Objects for any assigned Proportion of the *Solidities* or Bulks; and is as follows.

RULE.

Divide the Object by the Image, multiply the Quotient by the Cube of Radius; to the Square Root of that Product add the Radius, the Sum is the Distance of the Object required.

19. EXAMPLE.

Let the Radius of the Lens be 30, the Solidity or Bulk of the Object be to that of the Image 10000 to 10; the Distance of the Object is found as *per* Rule.

Image Object
10) 10000 (1000 Quotient.
9000 = Cube of Radius.

9000000 (300 Cube Root.
30 = Radius.

330 = the Dist. of the Object.

20. Here again the Distance is the same as before, *viz.* 33 Inches. At that Distance therefore the *Length* of an Object will be diminished 10 times, the *Superficies* an 100 times, and the *Solidity* 1000 times, which is according to the *Simple Square* and *Cubic* Proportion, as it should be by the geometrical Doctrine of Mensuration. But enough of these Matters, which are of more Speculation than Use.

21. When the Distance of an Object, and the Proportion thereof to the Image is given; the Radius of a double and equally convex Lens is found by the following Rule.

RULE.

RULE.

Multiply the Diſtance of the Object by the Image; divide that Product by the Sum of the Object and Image; the Quotient is the Radius ſought, *per* Theor. 20.

22. EXAMPLE.

Let the Diſtance of the Object be 330, the Proportion of the Object to the Image that of 100 to 10, or 10 to 1.

Then multiply the Diſtance 330
By the Image — — — — 10

The Product — — — 3300

Divide by the Sum of the Object and Image } 110) 3300 (30 = the Radius ſought, *viz.* 3 Inches. See Art. 17.
330
\---
...0

23. For a *Plano-Convex* the Radius will be found juſt half as long, becauſe you muſt then divide by twice the Sum of the Object and Image, *per* Theor. 21.

24. If the Diſtance of the Object, and Radius of the ſame Lens be known, the Proportion of the Object to the Image is then known by the following

ANALOGY.

As the Diſtance leſſened by the Radius, is to the Radius;

So is the Object to the Image, *per* Theor. 24.

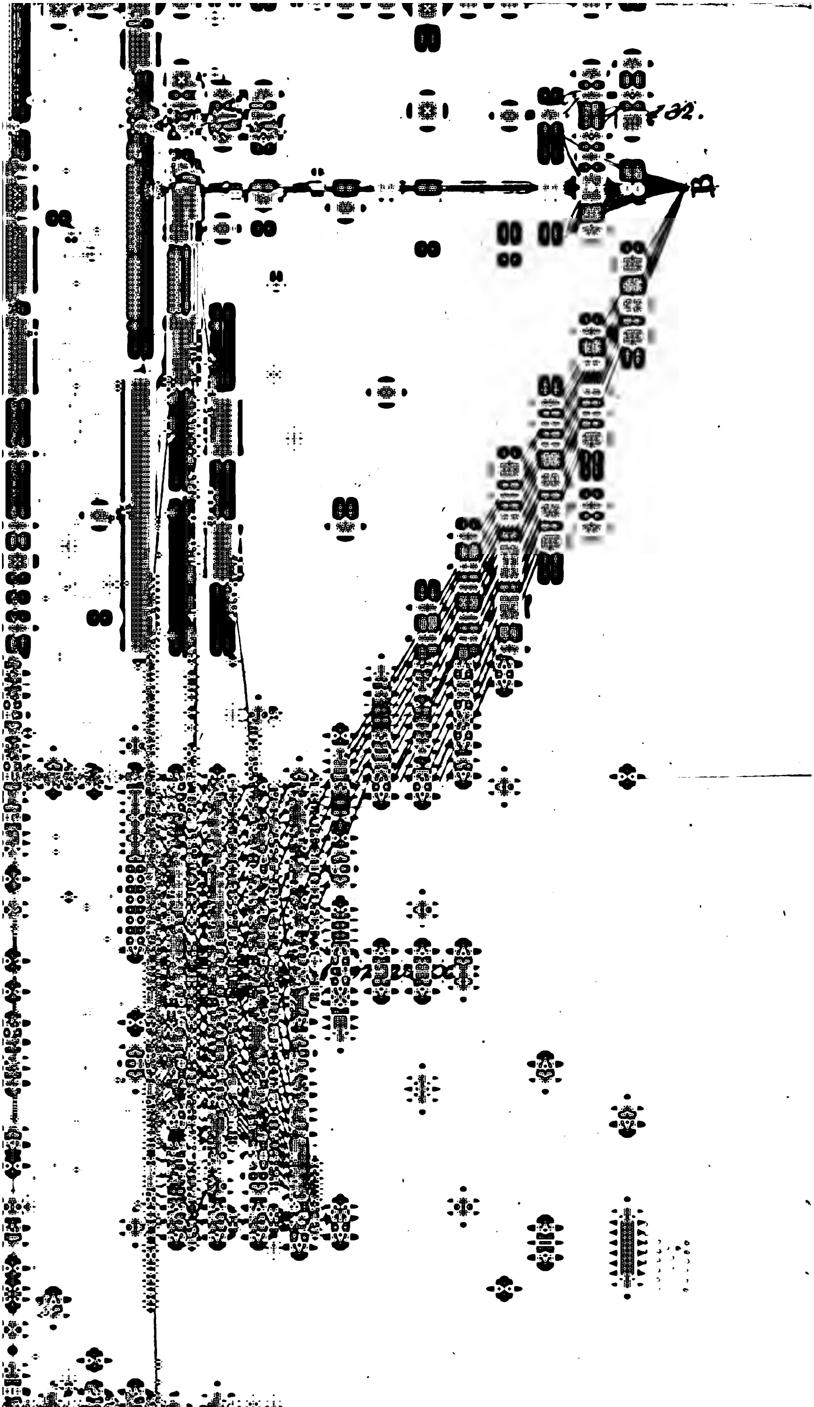
25. EXAMPLE.

Let the Diſtance of the Object be 330, the Radius 30; then the Diſtance leſſened by the Radius is 300; and therefore as 300 is to the Radius 30, ſo is the Object to the Image, but 300 is 10 times greater than 30, therefore the Object is 10 times larger than the Image. For a *Plano-Convex*, inſtead of *Radius* you muſt take the Diameter, or double Radius, *per* Theor. 21.

26. It is evident from the Scheme, that the Object ſeen at an affirmative Focus is *inverted*, but at a negative Focus it will appear in it's proper Poſition; the Reaſon of which was made plain in the Theory, and theſe are the principal Properties of *Convex* Lenſes.

CHAP.

132.



CHAP. X.

The Rules which determine the Diſtance of an Object for any aſſigned Proportion between it and it's Image, formed by a Double-Concave, Plano-Concave, and a Meniſcus Lens.

1. FROM the Theory it appears, that Diverging and Parallel Rays can have no affirmative Focus in a Double or Plano-Concave Lens. Therefore all Objects will have their Images formed at a negative Focus, or on the ſame Side of the Lens with themſelves.

2. The Rule therefore for finding the Diſtance of the Object that ſhall bear an aſſigned Proportion to it's Image formed by a double and equally concave Lens, as that in Plate XXI, is as follows.

RULE.

From the Object ſubſtract the Image, multiply the Remainder by the Radius; divide that Product by the Image, the Quotient is the Diſtance required, *per* Theor. 7.

3. EXAMPLE.

Let it be required to find the Diſtance of an Object that ſhall be in Proportion to it's

Image, as 40 to 10, by an equal and double concave Lens, whose Radius is 10, or 1 Inch.

The Object is — — 40 = OB
Substract the Image 10 = IM

Remainder — — 30
Multiply by Radius 10 = CE

300

Then 10) 300 (30 = AE the Distance
30 sought, *viz.* 3 Inches.

0

4. In a Plano-Concave all other things remaining the same, the Distance will be twice as great, by Theor. 10.

5. If you would find the Superficies or Solidity of an Object directly, the Rules are the same as in Chap. X. Art. 15 and 18, only as there you *added* the Radius to the Square and Cube Root, here you must *substract* it. See Theor. 8 and 9.

6. If the Distance of an Object AE, and the Radius CE of a double and equally concave Lens be known, the Proportion between the Object and Image is also known by this

ANALOGY.

As the Distance added to the Radius, is to the Radius;

So is the Object to the Image.

7. EX-

7. EXAMPLE.

Suppose the Distance AE = 30, and the Radius CE = 10; then will the Object be to the Image, as 30 added to 10, *viz.* 40 to 10, or as 4 to 1; that is, it will be 4 times as large. In Plano-Concaves you must use *twice the Radius*, thus.

8. As the Distance added to twice the Radius, is to twice the Radius;
So is the Object to the Image. Theor. 27.

9. In Double and Plano-Concaves it appears, both from the *Theory* and the *Scheme*, that the *Image* will always be *less* than the Object, on the *same Side* with itself; *erect*, or in the *same Position*; and ever *between the Lens and it's Center* C; till the Distance of the Object become infinite, and then the Image will be formed in the very Center, as that of the Sun, and all vastly distant Objects, is seen to be; and these are the chief Properties of *Concave Lenses*.

10. A MENISCUS Lens, whose Radii of Convexity and Concavity are unequal, and being exposed on the convex Side to the Object, will magnify or diminish an Object in any assignable Degree, at an affirmative Focus; provided the Radius of Concavity exceeds the Radius of Convexity. This is evident from

Theor. 13, which gives the following Rule for the Distance of the Object, *viz.*

RULE.

Multiply the *Sum* of the Object and Image by twice the Product of the Radii; divide that Product by the Difference of the Radii multiplied by the Image, the Quotient is the Distance sought.

11. EXAMPLE I.

Let the Radius of Convexity be 10, the Radius of Concavity 25, and the Proportion of the Object to the Image, as 4 to 1.

Then	25 — — 25		Object 4
	10 — — 10		Image 1
Product	250	15 Diff.	Sum 5
	2		
	500		
	5		

15) 2500 (166 the Distance sought, *viz.*
15 — $16\frac{6}{10}$ Inches.
100
90
100

12. EX-

12. EXAMPLE II.

Let the Object be magnified in the Proportion of 4 to 1, by the same Lens.

Then	25 — — 25			Object 1
	10 — — 10			Image 4
Product	250	15 = Diff.		Sum 5
	2	4 = Image		
	500	60		
	5			

60) 2500 (41 = the Distance required, *viz.* $4\frac{1}{10}$ Inches.
240
100
60
40

13. EXAMPLE III.

Let it be required to have the Object and Image equal by the same Lens. Then 4 times the Product of the Radii divided by their Difference, gives the Distance of the Object, thus;

```
          25 — — 25
          10 — — 10
         ----    ----
Product  250      15
           4
         ----
     15) 1000(66 = the Diſtance, viz. 6 6/10 Inches.
          90
         ----
          100
           90
         ----
           10
```

14. If the concave Side be turned to the Object, and the Image required at an affirmative Focus, the Rule is ſtill the ſame, ſave only that in this Caſe, the Radius of Convexity muſt exceed the Radius of Concavity, as is plain from Theor. 14.

15. If the convex Side be towards the Object, and the Image required at a negative Focus; then you muſt multiply with the *Difference* of the Object and Image inſtead of their Sum; and if the Radius of Convexity be *greateſt*, the Object cannot be *magnified*; if *leaſt*, it can't be diminiſhed.

16. But if the concave Side be towards the Object for a negative Focus, then if the Radius of Concavity be *greateſt*, the Object cannot be *magnified*; nor *diminiſhed*, if it be leaſt.

17. If

17. If the Radii of the Surfaces are equal, the Meniſcus can neither magnify or diminiſh an Object, as appears by Theor. 15. Becauſe, in that Caſe, all that is effected by one Surface is deſtroyed by the other.

18. An Object appears erect at a negative Focus, and *inverted* at an affirmative one. And theſe are the chief Properties of *Meniſcus Lenſes.*

19. A PLAIN LENS, or Piece of Glaſs, both whoſe Surfaces are perfectly plain, will magnify an Object in proportion to it's Thickneſs, according to the following

ANALOGY.

As the Diſtance of the Object, is to the ſaid Diſtance increaſed by two Thirds of the Thickneſs;

So is the Object to it's Image.

20. EXAMPLE.

Suppoſe a Piece of Glaſs be $\frac{3}{10}$ of an Inch thick, then will an Object at one Inch Diſtance be to it's Image ſeen through the Glaſs, as 10 to 12, in Length, in Surfaces as 100 to 144, and in Bulk or Solidity as 1000 to 1728.

PART

PART III.

Of DIOPTRIC Inſtruments *and* Machines.

CHAP. I.

Of the Structure of the Eye; and how Viſion is performed thereby.

1. THE Deſcription of the Eye, I here intend, is rather Optical than Anatomical; as taking Notice of thoſe Parts only which regard *Viſion*, and aſſiſt more or leſs in effecting it.

2. In Plate XXII. Fig. 1. repreſents the Ball or Globe of the Eye, or rather a Section thereof through it's Axis. The Form of the Eye is too well known to need Deſcription; it's Parts are of two Sorts, *viz. Coats* and *Humours*, of which in Order.

3. The *Coats* or *Teguments* of the Eye are the external Parts which contain the Humours, and are in Number 6, *viz.* The *Conjunctiva*, *Sclerotica*, *Cornea*, *Choroides*, *Uvea*, and *Retina*.

4. The

4. The *Tunica Conjunctiva* is common to the Eye-lid, and Ball of the Eye, which it firmly connects together, and makes what is called the *White of the Eye.*

5. The *Sclerotica* is the first or outmost of the *proper Coats* of the Eye represented by AEDB, Fig. 1. This Coat is thick, hard, and smooth, and on the fore-part is transparent where it forms

6. The *Cornea*, or *horny Coat*, because it resembles a thin Piece of transparent *Horn*, and is more protuberant than the rest; represented by AB. By this Part the Light first enters the Eye.

7. The *Choroides* lies under the *Sclerotica*, is much thinner than it, and on the fore-part thereof, between Q and Q is

8. The *Uvea*, sometimes called the *Iris*, which is of various Colours in different Persons; and in the middle has a round Hole, *viz.* IL, called *Pupilla*, or the Pupil of the Eye, by which the Light has admittance to the internal Substance of the Eye.

9. This *Iris* is of a round Form, is composed of two Orders of Fibres, one *Circular*, the other *Strait*, tending towards the Center of the *Pupil*, like Radii towards the Center of a Circle. By means of these Fibres, the Pupil is enlarged or contracted according as greater or lesser Light is required.

10. The *Retina* is not properly a Coat of the Eye, being only a fine Expansion of the

Optic Nerve over the Bottom of the Eye, nearly oppofite to the Pupil. On this delicate Membrane are painted the Images of Objects formed by the *Cryftalline Humour*. It's Reprefentation is at S, S, S.

11. The *Humours* of the Eye are three; the *aqueous* or *watery Humour*, the *Cryftalline*, and the *vitreous* or *glaffy Humour*. The Aqueous Humour hath very much the Appearance of *Water*, it makes the Eye globular on the fore-part, lying immediately under the *Cornea*, and is denoted by QQQ.

12. The *Cryftalline Humour* lies next the *Aqueous* behind the *Uvea*, oppofite to the *Pupil*; reprefented by NOP. In Form it refembles a Double-Convex Lens, being fomewhat more convex on the external, than on the internal Surface. It is connected to, and fufpended by the *Ligamentum Ciliare* MN, MN, on the third Humour, called

13. The *vitreous* or *glaffy Humour*; this is the largeft in Quantity, making the hinder and far greateft Part of the Globe of the Eye; reprefented by RRR. It is contained within a fine Membrane, and over all the fpherical Superficies thereof is fpread the *Retina* in a moft fine and curious Manner, quite to the *Ligamentum Ciliare*.

14. Thefe are conftituent Parts of the Eye; the Ufes whereof, with regard to Vifion, I fhall now more particularly defcribe. But fhall firft premife, *that clear and diftinct Vifion is*

produced

produced only by Parallel Rays, or ſuch as are nearly ſo. For theſe Rays have but one determinate Focus, whereas the Focus of Diverging Rays is as various as the Diſtances of Objects, and therefore can't produce Viſion in a Machine of a fixed and determinate Form and Poſition, as that of the Eye is well known to be.

15. Now in order that Rays, which proceed from near Objects, may be *nearly Parallel,* that they might render ſuch Objects diſtinctly viſible, it was neceſſary that the Aperture of the Eye, by which they were to be received, ſhould be very ſmall, that ſo the Baſe of a *Cone* of Rays, proceeding from any Point in ſuch an Object, being very ſmall in proportion to the Length of the ſaid Cone, that part of the Cone at the Entrance of the Eye might differ very little from a *Cylinder* of Rays; or, which is the ſame thing, their *Divergency* ſhould be ſo very ſmall, that they might differ very little from *Parallel Rays.*

16. To illuſtrate this Matter, let CD, Fig. 2. be the Diameter of the Pupil or Aperture of the human Eye; and O a Point at the Diſtance of 6 Inches draw the Rays OC, OD; and let AC and BD be Parallel Rays. Now it is evident, that becauſe CD is very ſmall in reſpect of CO, the Divergency of the Rays OC, OD, is ſo very ſmall alſo at the Pupil CD, that they almoſt coincide with the Parallel Rays AC, BD, at their Arrival;

 and

and therefore may be eſteemed as ſuch, and will produce diſtinct Viſion.

17. But were the Point O nearer to the Pupil CD, or if the Pupil were larger, in either Caſe, the Rays would fall more diverging on the Eye, and the Images of Objects would be formed at a Point beyond the Bottom of the Eye; and ſo their Pictures on the *Retina* would be very imperfect, and conſequently Viſion would be very indiſtinct and confuſed.

18. Hence it appears why we can never ſee any thing diſtinctly at a leſs Diſtance than about *ſix Inches* by the bare Eye; and alſo why Objects at all conſiderable Diſtances, appear diſtinct and perfect: But all vaſtly diſtant Objects appear both *indiſtinct* and *obſcure.* They appear *indiſtinct*, becauſe their Images in the Bottom of the Eye are ſo *extremely ſmall*, that the Diſtinction of Parts is imperceptible to the Mind it ſelf, all the Parts taken together making as it were but one phyſical Point. Thus if a Man of 6 Foot Stature were to be viewed at the Diſtance of a Mile, his Image on the *Retina* would be but the *thouſandth Part of an Inch* in Length; no wonder then if his Eyes, Noſe, Mouth, &c. appear indiſcernible in a Picture of ſuch extreme *Miniature.*

19. They appear more or leſs *obſcure*, according to their Diſtance or Degree of Light; for Objects at a great Diſtance are ſeen thro'

a greater

a greater Quantity of the Medium, than those at a ſmall one; and therefore the Rays, by a greater Refraction, will be much more effete, and produce a more obſcure Viſion in the former than in the latter Caſe.

20. Again, it is well known, that all Objects appear *bright* or *obſcure*, according to the Degree or Quantity of Light with which they are illuminated; thus diſtant Objects, in a cloudy Day, appear dark and obſcure, whereas, when the Sun ſhines full and ſtrongly upon them they appear clear and bright.

21. I ſhall now obſerve the Method Nature takes in effecting Viſion by the Eye; and how every Part is made ſubſervient to ſo noble a Purpoſe and Service.

22. Suppoſe then CD were parallel Rays, falling on the Eye at AB, Fig. 3. The Aqueous Humour Q being about the ſame Denſity with Water, and of a Convex Surface, by means of the *Cornea* AB, the Rays CD would be made to converge towards a Point F, at the Diſtance of 4 times the Radius of the Convexity of the *Cornea*, if there were no other Medium to prevent it; this is evident from the Theorem in Chap. I. Art. 6. of Part II.

23. But the Point F being beyond the Bottom of the Eye, makes it neceſſary that ſome other Body, of greater Denſity, ſhould be interpoſed in Form of a Convex Lens, to gather them to a Point nearer the Eye, and yet

a little beyond it; and this is effected by the *Crystalline Humour* NN.

24. I said *a little beyond the Eye*, because Nature determining the Eye to be of a globous Form, and for that Purpose having filled all the hinder Part with a Medium RR, of a less Density than the other Medium NN, and of a *Concave Surface* where it received the Rays, the Ray must necessarily be still *more converged* by this Medium, which is the Vitreous or Glassy Humour; and therefore had the *Crystalline Humour* thrown the Rays *just on the Bottom of the Eye*, the *Vitreous Humour* must have converged them to a Point behither it; and so would have made the Vision indistinct and confused.

25. The Density of the *Vitreous Humour* therefore is such, that the Rays are united at the Bottom of the Eye, on the Point S, where they paint the Image of the Object on the *Retina*, and in that Case only, produce *distinct Vision*.

26. Since with Regard to Objects at different Distances, those which are nearest will have their Foci, at a farther Distance from the Crystalline Humour, than those which are farther off; therefore a Power is given to the Eye to alter the Form of the Crystalline Humour, *viz.* To render it more or less Convex, by the Muscular Contraction and Relaxation of the *Ligamentum Ciliare*, to which it is connected, which also must occasion a

greater

greater or lesser Concavity in the *Vitreous Humour*; by which means the *focal Distance* is lessened for near Objects, and enlarged for those farther off, and is so nicely adjustable to the *Retina*, for all Distances of Objects, that their Images are all exactly painted thereon, in a natural or good Constitution of the Eye.

27. In the last Place, it has been observed, that there is a Power of dilating and contracting the Pupil of the Eye, by means of the Muscular Fibres of the *Iris*; on this Account, if Bodies be situated far distant, the Pupil being dilated receives Rays more diverging, and in a greater Quantity, and therefore such Objects appear more *distinct* and *enlightened*.

28. On the other hand, if Objects are situated *very near*, or are *extremely bright*; the Pupil is contracted, and takes in only the least divergent Rays, so that the Objects are more distinct; and, at the same Time, the *extreme Brightness* is diminished, so as not to be offensive to the Eyes.

CHAP. II.

Of the Poſition of the Image in the Eye; of the apparent Magnitude of Objects.

1. HAVING fully explained the Nature of Convex Lenſes, and alſo largely ſhewn that Viſion is performed in the Eye by Refraction through the *Cryſtalline Humour* principally, which bears the Form of ſuch a Lens, it muſt needs be very eaſy to conceive, that the Images of all Objects are formed in an *inverted Poſition* in the Bottom of the Eye.

2. Thus in Fig. 4. Plate XXII. Suppoſe OAB an Object, in an erect Poſition before the Eye CD; then will the Pencil of Rays OEIF, paint the Extremity O in the Point I, and the Pencil BFME, will paint the Extremity B in the Point M; and ſince all the Points between O and B are repreſented between I and M; IM will be the Image of the Object OB. Again, ſince the Axis of the Pencils of Rays croſs each other in the Pupil, which is their common Baſe, the Image muſt neceſſarily

necessarily be painted in an *inverse Position*, in the Bottom of the Eye.

3. Here constantly occurs the Question, *How it comes to pass, since the Images of all Objects are painted in the Eye invertedly, that we see them erect?* To this several Answers are given, but unsatisfactory; I can think of no better way to account for this Paradox, than as follows.

4. If we suppose an Eye viewing the Point I, in the Image in the Direction of the Axis of the Pencil of Rays which paints it there, it will refer it to the Point O in the Object, and there behold, and contemplate it: in like Manner, the Point M would be referred to the Point B, and by a successive Application of the Eye to every Point of the Image, the whole Image I M will be referred to, and considered in the Object OB. If therefore we conceive the *Mind* to be *all Eye*, capable at once of viewing every Point in the Image in it's requisite Directions, it must necessarily refer the Image in the Eye to the Object without it, which also will necessarily cause a Change of the Position; and therefore the Image, though *inverted* in the Eye, will be viewed and contemplated by the Mind in an *erect Position* in the Object.

5. We now proceed to consider the *apparent Magnitude* of Objects; which here shall be that of a *Line*, *viz.* it's Length. In Fig. 5. Let AB be an Object viewed directly by the Eye

Eye QR: From each Extremity A and B, draw the Lines AN and BM, intersecting each other in the Crystalline Humour in I, bisect AB in K, and draw IK; then is the Angle AIK, half the optic Angle AIB, which is the Measure of the *apparent Magnitude* or Length of the Object AB.

6. *Diverse Objects* AB, CD, EF, *whose real Magnitudes are very unequal, may be situated at such Distances from the Eye, as to have their apparent Magnitudes all equal.* For if they are so situated that the Rays AN, BM, shall touch the Extremities of each, they will then all appear under the same optic Angle AIK, which is equal to NIM, which determines the Magnitude of the Image MN, in the Fund of the Eye, the same for them all; and therefore they must all appear of an *equal Magnitude.*

7. *Objects situated at different Distances, direct to the Eye, whose apparent Magnitudes are equal, are to each other, as their Distances from the Eye directly.* Let the Objects be AB and CD, then because the Right-angled Triangles CIL, and AIK, are similar, it will be, as IK : IL :: AK : CL; but AK is half AB, and CL half CD; therefore it will be, as IK : IL :: AB : CD.

8. *Objects of equal Magnitude, situated directly before the Eye at unequal Distances, will appear unequal.* For let AB and GH be two Objects directly before the Eye at different Distances

Diſtances IK and IS; draw the Lines GP and HO croſſing each other in I; then is the optic Angle GIH, manifeſtly greater than the Angle AIB, and the Image OP made by the former greater than the Image MN made by the latter. Therefore the Object GH, is *apparently* greater than the Object AH, tho' it is but equal to it.

9. *Equal Objects ſituated directly before the Eye at unequal Diſtances, have their apparent Magnitudes reciprocally proportional to their Diſtances.* For let AB, GH, be two equal Objects at unequal Diſtances IK, IS, from the Eye produce IG and IH till they interſect AB, each way produced in T and V. Then will TV be the apparent Magnitude of GH, at the Diſtance IK. Since the Triangles ISG and IKT are ſimilar, we ſhall have IS : IK :: SG : KT; but SG is equal to AK; therefore it will be IS : IK :: AK : KT.

10. From what has been ſaid it appears, that there is *no Standard of the true Magnitude of Things*. All that we can be ſenſible of *is the Proportion of Magnitude.* And yet, notwithſtanding the ſenſible Magnitude of things is ever mutable, and varies in proportion to the Diſtance, we ſcarcely ever judge any thing to be ſo great or ſmall as it appears to be, or that there is ſo great a Diſparity in the viſible Magnitude of two equal Bodies at different Diſtances from us.

 11. Thus

[illegible]s.

[illegible] o Men [illegible] before [illegible] od, the [illegible] should, [illegible] apparent [illegible] appear- [illegible] an the [illegible] gh, the [illegible]

[illegible] motions, [illegible] form- [illegible] ions of [illegible] retain [illegible] e ; and [illegible] resented [illegible] and not [illegible]

C H A P.

CHAP. III.

Of the Faults or Defects of Vision, and how they are remedied by Convex and Concave Lenses in Spectacles.

1. IT has been already ſhewn, that in order to effect perfect and diſtinct Viſion, the Images of Objects are to be painted preciſely on the *Retina*, in the Fund of the Eye, and that by Rays which are either parallel, or nearly ſo, in a natural and good Configuration of the Eye.

2. But as it often happens, (for Nature her ſelf is not ever uniform to a Mathematical Nicety) that the Form of the Eye, but principally of the Cryſtalline Humour, is ſuch, that it is either a *little leſs or more Convex than is juſt*; ſo of Conſequence the Focus, or Place where the Images of Objects are formed, will be a little beyond or behither the Bottom of the Eye, which, in either Caſe, will prevent the Perfection of Viſion, and render it indiſtinct and confuſed.

3. This Imperfection of Viſion is, in a great Meaſure, remedied by the Succours of Art: For if, by a vicious Formation of the

Eye,

Eye, the Focus be made to fall on this or on that Side the *Retina*, yet Glaſſes may be formed to ſuch a Degree of Convexity or Concavity, that upon applying them to the Eye, the Focus ſhall be truly adjuſted to the *Retina*, and thereby cauſe diſtinct Viſion.

4. Thus ſuppoſe the *Cornea* C D, (See Fig. 6.) or the Cryſtalline E F, or both, ſhould chance to be *too flat*, either from Nature, or (what is moſt common) from a Deficiency of the Aqueous Humours through Age, in *Preſbytæ*, or *Old Men*, ſo that the Rays which proceed from any Point A, are made to converge at a Point a, beyond the Eye, and thus cauſe a confuſed and imperfect Viſion; I ſay, this Imperfection is cured, in a good Meaſure, by Convex Lenſes in Spectacles.

5. For from what has been ſhewn of Convex Lenſes, it is evident, (1.) That Rays coming from a diſtant Point A, and fall diverging on a Convex Lens G H, are thereby made to proceed *leſs diverging* than before. (2.) Thoſe Rays which are leſs diverging, have their Focus nearer to the Lens, than thoſe which are more ſo. Conſequently the Rays, which proceed from the Point A, are by the Interpoſition of the Convex Lens G H, juſt before the Eye, made to fall *leſs diverging* on the Cryſtalline E F, and therefore will be converged to a Point b, nearer thereto than the Point a; and the Convexity of the Lens G H may be

such, that the Point b shall be precisely on the *Retina*, and so cause *distinct Vision*.

6. For this Reason *Presbytæ*, or old People, always make use of Lenses more or less convex in their Spectacles, as their Eyes are flatter or rounder. And here it is to be observed, *that Objects appear brighter*, as well as more distinct, by means of Convex Lenses; for as much as they bring the Diverging Rays nearer together, some which otherwise would have fallen without the Pupil, will now be brought within the Compass thereof, and so a greater Quantity of Rays entering the Eye, the Object will appear *more bright*.

7. Again, we farther observe, *that Objects seen by Spectacles of Convex Lenses, appear to be more distant than they really are.* For the Rays which come from the Point A, being by the Lens GH made to proceed less diverging, they will appear to come from the Point B, which is farther off, because Rays, as they are more or less divergent at the Eye, come from Objects which are nearer or farther distant from the Eye.

8. Hence also the Reason is evident, why the older Men grow, the more they lose a distinct Vision of near Objects; so that very flat Eyes can see only distant Objects without Confusion: For when in Youth the Eye was so convex, as to form an Image of nigh Objects on the *Retina*, in Age that Convexity of the Eye diminishing, will cause the Images

of

of the ſame Objects to be painted at a Focus farther from or behind the *Retina*; and thoſe Objects only which are at a great Diſtance, will have their focal Diſtance ſhort enough to fall on the *Retina*; and ſo theſe only can produce *diſtinct Viſion*.

9. On the other hand, if the Cornea CD, or Cryſtalline EF, or both, (See Fig. 7.) be more convex than juſt, the Rays which proceed from a Point A, will be made to converge too ſoon; that is, to a Point a, which is between the Cryſtal Lens EF, and the *Retina*; and therefore muſt needs produce a *confuſed Viſion*. A Perſon who hath theſe Eyes, is called *Myops*, *i. e.* Mouſe-ey'd; but he is vulgarly ſaid to be *purblind*, or *ſhort-ſighted*.

10. He is with good Reaſon ſaid to be *ſhort-ſighted*; for ſince all diſtant Objects have the ſhorteſt focal Diſtances, and ſo have their Images formed ſhort of the *Retina*, they muſt needs appear indiſtinct and confuſed. Therefore only thoſe Objects, which are at a ſhort or near Diſtance, can produce *diſtinct Viſion*; and that they do by having a longer focal Diſtance, and ſo reach the *Retina* before their Images are formed.

11. A *Myops* hath his Imperfection of Sight greatly relieved by *Concave Lenſes*. For ſince Rays, which fall divergent on ſuch a Lens, are made to diverge the more by Refraction through it; therefore the Rays which come from

om any diſtant Object A, in paſſing through he Concave Lens GH, are made to fall more liverging on the Eye CD, and ſeem to proceed from a Point B, much nearer the Eye than the real Point A. Since therefore the Radiant Point is brought nearer to the Eye, it's Focus in the Eye will be removed farther from the Lens EF, and ſuch Glaſſes may be choſen as ſhall exactly throw the Focus on the *Retina* at b, and there produce *diſtinct Viſion.*

12. From hence we obſerve, that *Myops*, or purblind People, uſing Spectacles with Concave Lenſes, (1.) Do behold things at *a nearer Diſtance than they are.* (2.) That Objects appear *leſs bright to them*; for the Rays being ſpread wider by the Lens, they cannot enter the Pupil in ſo great a Quantity as otherwiſe, and ſo the Object cannot be ſo much enlightened. (3.) Their *Eyes are amended, and made better by Age*, contrary to what happens to all other People. For as the Fault of their Eyes proceeds from a too great Protuberance of the *Cornea*, and *Cryſtalline Humour*; ſo this and the Aqueous Humour leſſening by Age, diminiſhes the Convexity of the Eyes by Degrees, and renders them capable of viewing diſtant Objects better and better.

CHAP.

CHAP. IV.

Of the Camera Obſcura, *or darkened Chamber, and the Inſtruments pertaining thereto; by which Means the Images, or Pictures of external Objects, are curiouſly painted in their natural Colours and Motions.*

1. THE Words *Camera Obſcura*, literally interpreted, is a *darkened Vault* or *Roof*; and from thence it came (with a little Difference) to ſignify a *Chamber*, or *Box*, or any Place made *dark* for *Optical Experiments*.

2. The *Camera Obſcura* is, tho' a ſimple, yet a very curious and noble Contrivance; in as much as it moſt clearly and naturally explains the Nature and Manner of Viſion in the Eye, and, at the ſame Time, entertains the Spectator with a moſt exquiſite Picture of the Objects without, in their natural Proportions, Colours, and Motion, more vivid and beautiful than the Life it ſelf; filling the Beholder with Delight and Surprize.

3. The Way of making a *dark Chamber* is very eaſy, and not expenſive, and is as follows.

follows. The Chamber or Room propoſed to be darkened, ſhould have Window-Shuts to each Window, which being cloſe ſhut, ſhould be ſo true to the Frame, as to exclude all Light poſſible.

4. In one of the Window-Shuts there is to be a circular Hole cut, about 3 or 4 Inches Diameter, in ſuch a Part thereof, as is judged moſt convenient, and capable of taking in a good View or Proſpect of external Objects.

5. In this Hole is placed an Inſtrument called a *Scioptric Ball*, which hath three Parts, *viz.* A Frame, a Ball, and a Lens. The Frame conſiſteth of two circular Pieces of Wood, made ſpherically hollow through the middle, and ſcrew into each other. In this hollow Part is placed a ſpherical *Ball of Wood*, contained by the two Parts of the Frame ſcrewed together, and is voluble therein more or leſs eaſily, as the two Parts of the Frame are ſcrewed leſs or more tight together. In this Ball is a circular Hole made thro' the middle, which hath a Screw at each End, in which is placed and fixed a Lens, either a *Double* or *Plano-Convex*.

6. The *Scioptric Ball* of this Structure, is a Sort of *Artificial Eye*, which very aptly repreſents the Natural Eye in Form and Office. For (1.) The *Frame* or *Socket* anſwers to the *Orbit* of the *Natural Eye*. (2.) The *Wooden Ball*, which turns every Way in the Frame, reſembles the Globe of the Eye voluble every

Way

Way in it's Orbit. (3.) The *Hole* through the *Wooden Ball*, repreſents the *Pupil* of the Eye. (4.) The *Convex Lens* in the Ball, correſponds to the *Cryſtalline Humour* in the Eye. (5.) The *Dark Chamber* it ſelf, is like to the internal Part of the Eye, which is lined all about, and under the *Retina*, with a Membrane, over all which is ſpread a Mucus of a very black Colour. (6.) The *White Wall*, or Frame of white Paper to receive the Picture of Objects on in a dark Chamber, is the true Repreſentation of the *Retina* in the Eye.

7. So that the Structure of the *Scioptric Ball* and the *Eye* is perfectly ſimilar; and the ſame Agreement will be found in the Offices of every Part of each. For (1.) The Frame of the *Scioptric Ball* is ſcrewed or tacked upon the Hole in the Window-Shut, as the Eye is faſtened within it's Orbit by Muſcles. (2.) The *Ball* is voluble in the Frame every Way, to take in a View of Objects on every Side, as the Eye is in it's Orbit. (3.) The Hole in the Ball is, for the Admiſſion of a competent Quantity of Rays, as is the Pupil in the Eye. (4.) The Lens in the Ball collects the Rays, and unites them at it's focal Diſtance, where it makes a Picture of the external Object placed before it; which is the Office of the Cryſtalline Humour in the Eye. (5.) The white Wall or Paper, held in the Focus of the Lens in the Chamber, is to receive and ſhew the ſaid Picture of Objects to Spectators; as the

Retina

Retina presents to the Mind a View of the Picture made on it by the Eye. (6.) The Chamber is made *dark* to render the Picture visible; as for the same Reason, the internal Part of the Eye is furnished with a *black lining*.

8. I shall give an Illustration of this Matter in a curious Scheme, which I have borrowed from Dr *'sGravesande*, (See Fig. 1. Plate XXIII.) where EF represents a darkened Room or Chamber; in one Side thereof IK is made the circular Hole V; in which, on the Inside, is fixed the *Scioptric Ball*; at some considerable Distance is exhibited a Prospect or Landskip of *Houses*, *Trees*, &c. ABCD. The Rays which pass from this Prospect to the Lens V in the Ball, are by it converged to their respective *Foci*, on the opposite Wall or Side of the Chamber GH, where they all together paint a most lively and beautiful Picture of all the Objects in the said Prospect.

9. This is Nature's *Art of Painting*, and it is with Ease observed, how infinitely superior this is to the finest Performance of the *Pencil*. For, (1.) You have here the *Perspective in Perfection*; that is, the just Diminution of Objects proportionate to the Distances, or the Proportion of the Images to the respective apparent Magnitudes of the Objects to an Eye at V. (2.) The *Colouring* is here perfectly *just and natural*; and not only that, but very much heightened, and rendered more

beautiful; thus *green* Objects appear more *intensely green* in the Picture; *yellow*, *red*, *blue*, or *white* Flowers, appear incomparably more beautifully so in the Picture. (3.) The *Lights* and *Shadows* are not only perfectly just, but also greatly heightened, and make the Images appear extremely prominent and natural. (4.) The *Motions* of all the Objects are exactly expressed in the Picture; the Leaves *quiver*, the Boughs *wave*, the Birds *fly*, the People *walk*, the Cart is *drawn*, the Smoke *ascends*, the Clouds *soar*, the Ships *sail*, &c. and all as natural as the Life, and much quicker, as it is performed in a lesser Scene.

10. These are the inimitable Perfections of a Picture drawn by Nature's Hand; in Comparison of which, how *mean*, how *coarse*, how *imperfect*, yea, what sorry daubing is the finest artificial Painting! Select the peculiar Excellencies of the principal Artists, the *just Proportions* of *Raphael*, the *natural Tints* of *Titian*, the *pure Stile* of *Corregio*, the *Decorum* of *Tibaldi*, the *Terrour* of *M. Angelo*, the *Air* of *Guido*, the *Designing* of the *Romans*, the *Shadowing* of the *Venetians*, and the *Colouring* of the *Lombards*, all united would be unable to effect so finished a Piece, in any Branch of their Art, as Nature can exhibit with a *single Lens* only. The *Camera Obscura* is, at the same Time, the Painter's *Aid* and *Reproach*; from hence he receives the best Instructions; from hence he learns his

his Imperfections; here he views what he ſhould do, and knows it is what he cannot do.

11. There is one thing which may be thought an Imperfection in the Picture of a Dark Chamber, and that is, the *inverted Poſition of the Images*; but, ſtrictly ſpeaking, this is not ſo, becauſe Nature has furniſhed us with ſeveral Methods to make the Picture *erect*. For if it be a Sheet or Frame of Paper which receives the Picture, it is but holding it before you, and looking downwards on it, and every thing is right. Or if you ſtand before the Picture, take a *Looking-Glaſs*, and hold it againſt your Breaſt under an acute Angle, and looking therein you will ſee all the Images of the Picture reſtored to their natural or erect Poſition; and not only ſo, but the Reflection of the Mirrour will give it ſuch a Glare or Luſtre, as makes it ſeem very ſurprizing and delightful. This may likewiſe be done by placing a large *Concave Mirrour* before the Picture at ſuch Diſtance, that the Image of the Picture may appear before the Mirrour, which will then be erect and pendant in the Air.

12. There is another Method of erecting the Images on the Picture, and more direct than any of the foregoing, but yet is neither ſo eaſy to be done, nor ſo good if it be done, as it is by them; and that is by placing another *Convex Lens* behind the Paper or Parti-

tion, which receives the firſt Picture at twice the focal Diſtance of the ſaid Lens; if then a Hole be made in the Partition to permit the Rays to paſs on to the ſecond Lens, (which muſt be placed in the Axis of the firſt) there will be a Picture formed thereby, wherein the Images will be *erect*, and as large as in the firſt, but not ſo bright, nor will the Field or Extent of the Picture be conſiderable; and therefore as this Method is ſeldom practicable, ſo it is as little worth while.

13. In making a Dark Chamber, the Glaſs ſhould not have too ſmall nor yet too large a focal Diſtance. For if it be too ſhort, the Images will be very ſmall, and not diſtinct and diſcernable, and will be ſo near the Window, that there will not be Room for a Perſon to ſtand to view it. The Lens ſhould have it's Focus at *three Foot diſtance* at leaſt.

14. On the other hand, the *focal Diſtance* of the Lens ſhould not be *too large*; for if it be larger than the Diſtance of the oppoſite Wall, your Deſign will be fruſtrated, as you can have nothing to receive the Picture upon. And if your Room or Chamber be very large, yet will the Picture be faint, and the Images leſs pleaſing on a double Account. For, (1.) The focal Diſtance being very great, the Images will be proportionally large, and therefore the more faint and obſcure. (2.) The larger the focal Diſtance, the larger the Diameter of the Lens, or the Hole in the *Scioptric*

Ball,

Ball, which therefore will admit too much Light, for the Chamber to be ſufficiently dark for viewing the Picture; and if you make the Hole ſmaller, there will not enter Light enough to make the Images viſible at ſo great a Diſtance.

15. The Focus then ſhould not be at a leſs Diſtance than 3 Feet, nor at a greater than 15 or 20 at fartheſt; and thoſe from 6 to 12 are by much the beſt of any. If it happens that your Lens be too ſhort a focal Diſtance, you may magnify the Images to almoſt what Degree you pleaſe, by viewing the Picture with a large Convex Lens in your Hand; by this means you may ſupply the Want of Lenſes of diſtant Foci, which are very ſcarce and dear; and thus a Lens of 5 Feet may be made to anſwer the End of one of 15 Feet.

16. A Dark Chamber ought never to be attempted *but when the Sun ſhines*; for it is neceſſary the Objects, which are intended to make your Landſkip, ſhould be ſtrongly illuminated by the Sun-Beams; otherwiſe the Picture, which ought to be vivid, bright, and beautiful, will appear obſcure, dull, and of a dirty Hue; as the Objects themſelves would appear by Twilight.

17. Whence it follows, that a *South-Window* is never to be uſed for this Purpoſe; becauſe the Sun can never enlighten the North Side of the Object, which alone can be taken

in by a South Window. Besides, the Sun in this Case would be apt to shine on the Glass, which would make the Picture appear with a false or confused Lustre, and therefore you ought to be very careful always to avoid that thing's happening.

18. An *East Window* will do very well in the *Afternoon*, as a *Western* will for the *Morning*; but none is so good, or will make so noble and glorious a Picture as a *North Window* about *Noon*; for then the Sun being in his Meridian Height, and shining with the greatest Strength and Splendor possible, the Picture made in such a Case will far exceed all others in *Vivacity*, *Beauty*, and *Lustre*.

19. This noble Experiment is not only admirably pleasing and delightful in it self, but also very useful for many Purposes of Business, principally with Respect to *Perspective*, *Painting*, *Designing*, *&c.* For whatsoever is to be *drawn* or *painted*, if it be first exposed to the *Scioptric Ball*, the Perspective thereof will be truly formed, and the Lights and Shadows for every Position, and Action of the Objects, will be represented just as they ought to be in the Images of the Picture. In short, the *Camera Obscura* is the School in which every *Designer* and *Painter* ought to learn the first Rudiments of his Art. He can never, without this dark Education, turn out a bright Proficient. No Instructions can come up to those of Nature; her Lessons are all perfect

fect Patterns and Ensamples, and every one excels in Painting and Drawing so much the more, by how much the truer he copies after them.

20. And another great Convenience is, that the Picture of Objects may be made of any Size you please, either less or greater than the Life, if the Objects be moveable. For if you place the Object farther from the Ball than twice the focal Length of the Lens, the Images will be less than the Object; if they are placed at just twice the focal Length of the Lens, the Images will be just as big as the Life; if they are placed nearer, the Images will be greater than the Life; all which is manifest from the *Theory*, and which I have before observed in the practical Part.

21. Also for immoveable Objects, as those of Houses, Gardens, Fields, Trees, &c. if you have different Lenses, you may form the Picture of so many different Sizes, the shorter Focus making the lesser Picture, and the larger Focus the largest. But these Matters are so obvious, that I need not farther insist on them.

22. But the Use of the *Camera Obscura* does not rest in *Drawing* and *Painting*; but the *Optician* himself is greatly interested therein. By this grand Experiment he demonstrates ocularly the Principles of his Art. For by admitting the Sun-Beams thro' the Hole of the Window-Shut into the darkened Chamber, he can actually shew the Focus of *Parallel Rays* by Reflection from *Concave*

Mirrours, and by Refraction through *Convex Lenſes*, to be juſt as the Theory defines them, by holding thoſe Glaſſes in the ſaid Beams. Thus alſo he ſhews, that the Sun's Rays, after Reflection from *Convex Mirrours*, and after Refraction through *Concave Lenſes*, are ever made to diverge agreeable to the Theory of *Parallel Rays*.

23. Again, by holding a large *Convex Lens* in the Sun's Rays, he can by that means produce *Diverging* and *Converging Rays* in any Degree, and ſo can prove the Truth of all thoſe Properties of *Convex*, *Concave*, and *Meniſcus* Glaſſes, with reſpect to theſe Rays, as the Theory teaches. Alſo the Ratio or Proportion of the *Sines of the Angles of Incidence*, and Refraction in Water, Oil, Glaſs, *&c.* is eaſily proved to be as it is ſtated.

24. The Reaſon and Nature of *Teleſcopes* and *Microſcopes* is demonſtrated to the Senſes hereby; for if the Glaſſes in either be taken out of the Tubes, and fixed at their proper Diſtances, on a ſtrait Piece of Wood, ſo that the Sun Beams fall directly and fully on the Object-Glaſs, you will then ſee the Forms they take in their Courſe thro' the Glaſſes fixed at their proper Diſtances, to be ſuch as the Theory points out, and are neceſſary to anſwer the Purpoſes of theſe Inſtruments.

25. But in making Experiments with the Sun-Beams in a Dark Chamber, ſince thoſe Rays fall with great Obliquity, eſpecially in the

the *Summer-Time*, and ſo the more inconvenient for Uſe, the beſt Way is to fix a ſmall Plain Mirrour to the under Part of the Frame of the *Scioptric Ball*, immediately next the Ball itſelf, by a Hinge, that by moving up and down it may receive and reflect the Sun-Beams in any Direction whatſoever; and as by this Contrivance they may be made *parallel*, it will be found extremely uſeful and expedient on many Occaſions.

26. If the Mirrour be fixed to the Ball it ſelf, then may the Cylinder of Rays be not only caſt in any Direction, but alſo to any Part of the Room, than which nothing can be more uſeful, as I have found in numberleſs Caſes, having made this Addition to one of my *Scioptric Balls*.

27. One great Uſe of the *Camera Obſcura*, is the eaſy Method it ſupplies of *meaſuring the focal Length of any Lens or Mirrour*; which otherwiſe is many Times very difficult to be determined. This Matter is extremely eaſy and certain for *Convex Lenſes* and *Concave Mirrours*, becauſe they have a real and very viſible Focus; and therefore ſetting one End of a Rule on the Lens, the Diſtance of the Focus is ſeen on the Rule in Feet, or Inches, and Decimal Parts of an Inch.

28. But with reſpect to a *Convex Mirrour* or *Concave Lens*, which have no real Focus, it will be eaſy to find the Diſtance of the virtual Focus thus: Deſcribe a Circle, ſuppoſe of

of 3 Inches Diameter, on a Piece of clean white Paper; hold this Paper directly behind the Concave Lens in the Rays, and move it to and from the Lens till the circular bright Spot of the Diverging Rays exactly fill the Circle on the Paper, then measure the Distance of the Paper from the Lens, and also the Diameter of so much of the Lens as is concave; then *multiply the Distance of the Paper by the Diameter of the Concavity, and divide that Product by the Difference between the Diameter of the Circle on the Paper and Concavity of the Lens, the Quotient will be the Distance of the virtual Focus of the Lens.* In the same Manner the said Focus is found for a Convex Mirrour.

29. Another grand Experiment of the Dark Chamber, is *shewing the Spots on the Sun's Disk.* This is easily done, by putting the *Object Glass* of a 10 or 12 Foot Telescope into the *Scioptric Ball*, which turn about till it be filled with the Sun's Rays, then holding a Sheet of white Paper in the Focus of the Lens, you will see a most exceeding fair and bright Image of the Sun formed on the Paper of about an Inch Diameter, in which the Spots on the Sun's Surface will be very distinctly seen, and will afford a very pleasing Spectacle. This Image is rather too bright to be viewed without Offence to the Eyes; and therefore should be viewed through a large Lens of about 6 or 8 Inches focal Distance, which

which will magnify both the Image and it's Spots to a very great Advantage.

30. The Dark Chamber might alſo be made to anſwer the End of a *nocturnal or aërial Teleſcope*; for if an Object Lens, whoſe focal Diſtance is 12, 15, or 20 Feet, were fixed in the Ball, it would give a large Image of the Moon, and a ſmall one of *Venus*, *Jupiter*, and *Saturn*; which may be ſo far magnified by one or more ſmall Lenſes, ſuch as are the Eye-Glaſſes, that the *Phaſes* of the one, and the *Satellites*, *Belts*, and *Rings* of the other may, in all likelihood, be rendered viſible and diſtinct; as I have Reaſon to think from ſome Experiments I have made with Lenſes of a ſhorter focal Diſtance, not having Room to try the larger.

31. Divers *Microſcopical Experiments* are alſo to be exhibited in the *Dark Chamber*, both by *Convex Lenſes* and *Concave Mirrours*. Thus if you fit a Tin or Paſtboard Tube into the Hole of the *Scioptric Ball*, and make an Aperture on each Side to ſlide a Piece of Glaſs through freely, and then either in or upon this Tube you put another, containing two Lenſes properly diſpoſed; this may be moved backward or forward till ſuch times as it gives clear and very large Images on a Sheet of Paper, of ſmall Objects that are ſtuck upon the Slip of Glaſs aforeſaid, which are ſtrongly illuminated by the Sun-Beams paſſing thro' the Ball for that Purpoſe.

32. The

32. The ſame thing is to be done with a *Concave Mirrour*, thus: Let a Cylinder of Rays fall on a large Concave Mirrour, fixed in a Frame for that Purpoſe; then take the Slip or thin Plate of Glaſs, and having put any ſmall Objects thereon, hold it in the incident Rays a little more than the focal Diſtance from the Mirrour; then will you ſee on the oppoſite Wall the Images of thoſe ſmall Objects very large, and exceeding clear and bright among the reflected Rays. This is a moſt eaſy and delightful Experiment, which I have often tried with great Pleaſure to my ſelf and others.

33. The *new and noble Doctrine of Colours*, and the *different Refrangibility of the Sun's Rays*, were the Reſult of Experiments by the *Camera Obſcura*; if you admit a Cylinder of Rays, about ¼ of an Inch Diameter, into the darkened Chamber, and therein hold a triangular *Glaſs Priſm*, you will find, by turning it a little upon it's Axis, the Place where the Rays in paſſing through it will be refracted in a different Degree, ſome more, ſome leſs; and by that means will paint an oblong *Spectrum* or Image of various Colours moſt exceeding ſtrong and vivid; and in the following Order, *Violet*, *Indigo Blue*, *Green*, *Yellow*, *Orange*, and *Red*. This is a moſt ſurprizingly fine and agreeable *Phænomenon*, which, together with divers other Experiments relating thereto,

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CHAP. V.

Of Microſcopes in general; of ſingle Microſcopes in particular, made with a ſmall Lens, Spherule, or Mirrour.

1. THE Word *Microſcope* imports *an Inſtrument for viewing very ſmall Objects.* I have before obſerved, that Nature has ſo formed the *human Eye*, that we can't diſtinctly view an Object at a nearer Diſtance than ſix Inches; and ſince there is an Infinity of Objects, which at that Diſtance appear either as *Points*, or are *wholly imperceptible*, whatſoever Inſtrument or Contrivance will render ſuch minute Objects *viſible and diſtinct*, we properly call a *Microſcope.*

2. It is uſual to ſay that the *Microſcope magnifies Objects ſeen through it*; but this is true only with Regard to the apparent, not the *real* Magnitude of Objects; they indeed appear to be larger with than without the Microſcope, but, in Truth, they are not; and the Reaſon why they appear to be magnified will be eaſy to apprehend, by any Perſon who underſtands what has been delivered concerning the *apparent Magnitude* of Objects in Chap. II, hereof.

3. For

3. For there it was ſhewn, that the *apparent Magnitude* of Objects is meaſured by the Angle which they are ſeen under by the Eye; and farther, that thoſe Angles are reciprocally as the Diſtances from the Eye. If therefore, at the Diſtance of 6 Inches, I can but juſt diſcern an Object, and then by interpoſing a Lens, or other Body, I can come to view that very Object at a nearer Diſtance, the Object will appear to be as much larger through the Lens, than before to the naked Eye, as it's Diſtance from the Lens is leſs than it's Diſtance from the Eye.

4. That this is the Caſe, is evident from Fig. 2. Plate XXIII; where A is a Point in an Object not clearly viſible to the naked Eye, at a leſs Diſtance than A B, becauſe the Rays which proceed from it are too divergent to admit of diſtinct Viſion till they have paſſed that Diſtance; but if the ſame Object be placed in the Focus C of the Lens D, the Rays which proceed from it will become parallel, by paſſing through the ſaid Lens, and therefore the Object is diſtinctly viſible to the Eye E, placed any where before the Lens D. Conſequently it will appear as much larger through the Lens than to the naked Eye, as C D is leſs than A B.

5. If an Object A B be placed in one Focus C of a Lens D E, and the Eye in the other Focus F; (See Fig. 3.) the Eye will ſee juſt ſo much of the Object as is equal to the Diameter

Diameter of the Lens; for the Rays AD and BE, which go from the Object to the Extremities of the Lens D and E, and are united at the Focus F, must necessarily proceed from the Object to the Lens parallel to the Axis FC; and therefore parallel to each other; consequently the Part of the Object AB, seen by the Rays DF, EF, will be equal to the Diameter DE of the said Lens. All which is evident from the Theory.

6. If only the Part de of the Lens be open, then only so much of the Object ab, as is equal thereto, will be perceived by the Eye. Now since AB is equal to DE, or ab to de, therefore the Angle DFE, or dFe, is the Optic Angle under which the Part of the Object AB or ab appears to the Eye at F; and since GF is but ½ FC, therefore the Angle DFE, or dFe, is double to that under which the Part AB or ab would appear to the naked Eye at the Distance FC. That is, the *Eye sees the Object, situated as above, twice as large with the Lens as it would do without it.*

7. *If you would see a Portion of an Object larger than the Lens, your Eye must be placed nearer the Lens than it's Focus.* Let the Lens be DE, (Fig. 4.) it's two *Foci* F and C; in the Focus C let there be an Object AB larger than the *Lens*; suppose the Rays AD, BE, proceed from the Extremities of the Object to those of the Lens, it is evident from the Figure they will be *convergent*, and therefore

therefore will by the Lens be united in a Point K, between the Lens D E, and it's Focus F: If then the Eye be placed at K, it will take into it's View an Object, or Portion of an Object, greater than the Lens D E.

8. Again, let G H be a Portion of an Object A B, leſſer than the Lens D E; draw G D, H E, which will be *Diverging Rays*, and therefore will be united at a Point I, farther diſtant from the Lens than the Focus F: *Hence if an Eye be placed farther from the Lens than it's focal Diſtance, it can never ſee any Object, or Part of an Object, at one View, ſo large as the Lens, but always ſmaller.* And univerſally, *the viſible Part of an Object will be to the Lens, as the focal Diſtance of the Lens, to the Diſtance of the Eye.* All which may be eaſily deduced from the foregoing Theory.

9. Since then it is evident, the Nature of a Convex Lens is ſuch as will render an Object diſtinctly viſible to the Eye, at the Diſtance of it's Focus, the Reaſon why they are uſed as *Microſcopes* is exceeding plain. For ſuppoſe the Diſtance A B (Fig. 2) be 6 Inches, where the naked Eye B, can but juſt perceive the Object A diſtinctly, and let the focal Diſtance C D of the Lens D be $\frac{1}{2}$ an Inch; then ſince C D is but $\frac{1}{12}$ of A B, the *Length* of the Object at C, will appear 12 times as large as at A; if it were a *Surface*, it would be

144 times as great; and the *Solidity* or *Bulk* would be magnified 1728 times.

10. If CD, the focal Diſtance of the Lens D, be but $\frac{1}{4}$ of an Inch, then will that be but $\frac{1}{24}$ of AB = 6 Inches, and ſo the *Length* of Objects will be magnified 24 times; the *Surface* 576 times, and the Solidity 13824 times, for thoſe Numbers are the *Square* and *Cube* of. 24. From whence it appears, that *ſingle Glaſs Lenſes* make very good Microſcopes, which have theſe Advantages, that the Object appears moſt clear, they lie in little Room, may be carried about any where, are to be had for a ſmall Price, and are moſt eaſy to be uſed.

11. The Form of ſuch a Microſcope, which I think moſt convenient, is that in Fig. 5. where AB is a circular Piece of Wood, Ivory, &c. in the middle of which is a ſmall Hole, $\frac{1}{20}$ of an Inch Diameter; upon this Hole is fixed, with a Wire, a ſmall Lens C, whoſe focal Diſtance is CD. At that Diſtance is a Pair of Plyers DE, made of a Watch Spring, and open'd by means of the two little Studs a, e; with theſe you take up any ſmall Object O, and view it with the Eye placed in the other Focus of the Lens at F. And according to the focal Length of the Lens, the Object O will appear more or leſs magnified, as repreſented at IM. If the focal Length be $\frac{1}{2}$ or $\frac{1}{4}$ of an Inch, the *Length*, *Surface*, and *Bulk* of the Object will be magnified

nified, as expressed in Article 9 and 10 hereof. This small Instrument may be put into a Case, and carried about in the Pocket without any Incumbrance. I have made Trial of various Lenses, and find those whose focal Lengths are $\frac{3}{10}$, $\frac{4}{10}$, and $\frac{5}{10}$ of an Inch, the best for common Use.

12. Since the nearer the Eye can approach to an Object, the larger it appears, it is plain a *double and equally Convex Lens* is far preferable to a *Plano-convex Lens*; because if the Sphere or Convexity be the same, the focal Length of the former, is but half as long as of the latter: And since the Double-Convex consist of two Segments of a Sphere, the more an Object is to be magnified, the greater must be the Convexity, and therefore the smaller the Sphere; till at last the *utmost Degree of magnifying* will require that these *Segments* become *Hemispheres*, and consequently the *Lens* will be reduced to a perfect *Spherule*, or very *small Sphere*.

13. With these small *Spherules* extraordinary Degrees of magnifying may be arrived at; for as I have shewn in the Theory, the Focus of *Parallel Rays* is but at $\frac{1}{2}$ the Radius distant from the *Spherule*; therefore if the Radius of the *Spherule* be $\frac{1}{10}$ of an Inch, the Eye will have distinct Vision of an Object by means thereof, at the Distance of a *Radius and half*, *i. e.* $\frac{3}{20}$ of an Inch, which, as it is but the 40th Part of 6 Inches, shews that the

M 2 *Length*

Length of an Object will be magnified 40 times, the Surface 1600 times, and the *Solidity* 64000 times, by ſuch a ſmall Sphere.

14. If the Radius of a *Spherule* be but $\frac{1}{20}$ of an Inch, then will the Eye have diſtinct Viſion of an Object at the Diſtance of $\frac{3}{40}$ of an Inch, which, as it is but the 80th Part of 6 Inches, ſhews the *Length* of Objects will appear 80 times greater, the *Surfaee* 6400 times, and the Bulk 512000 times greater than to the naked Eye at 6 Inches Diſtance.

15. Again, if the Diameter of a Spherule be $\frac{1}{20}$ of an Inch, or the Radius $\frac{1}{40}$, then will the Eye approach the Object ſo near as $\frac{3}{80}$ of an Inch, which is but the 160th Part of 6 Inches; and therefore the Length of Objects will be magnified 160 times, the *Surface* 25600 times, and the Solidity 4096000 times by this Spherule; which is ſo great a Power of magnifying, as ſurpaſſes all human Imagination and Comprehenſion.

16. And yet there are Methods of making Spherules as ſmall, and ſmaller than any above-mentioned. I ſhall mention only two Ways of doing this: The firſt is by breaking a Piece of clear white Glaſs into very ſmall Particles, which are to be taken up by the Point of a fine Needle, and held in the blue Part of the Flame of a Candle, or rather of a Lamp burning with Spirits of Wine; which by means of a Blow-Pipe, will immediately melt the glaſs Particles on the Point of the Needle;

Needle; and being melted they will naturally run into a roundiſh Form, and, by a proper Motion of the Needle, which a little Practice will teach, they may be brought to a true ſpherical Form. And as there are more or leſs Particles on the Needle, the Globule will be greater or ſmaller.

17. The other Way is by melting a Piece of fine Glaſs in a ſmall Crucible, or Bowl of a Tobacco-Pipe, and then by dipping therein the End of a Wire, you may draw out very fine and long Threads of Glaſs, which, when cold, are to be broken into proper Lengths; and one End of ſuch a Thread put into the Flame of a Candle, will immediately melt, and run into a round or globular Form, which, when you think is big enough, is to be taken out, and broke off the Thread.

18. In either of theſe Ways, great Care muſt be uſed not to hold them long in the Flame after they are melted, leſt they are burnt, and thereby rendered opake, and unfit for Uſe. The few that are good, among the many you make, are to be well cleanſed, and let into a very ſmall Hole in a Piece of Braſs, in order for Uſe.

19. In uſing theſe *Spherule Microſcopes*, the Objects are to be placed in one Focus, and the Eye in the other; and ſince the Focus is ſo exceeding near the Glaſs, it is impoſſible to view any but pellucid Bodies; for if any opake Object were to be applied, the Eye

being as it were juſt on the Spherule, would entirely prevent any Light falling on it, and it would be too obſcure to be viewed.

20. It was with theſe Sort of Microſcopes, that the famous *Dutch* Philoſopher Mr *Leeuwenhoek* made ſuch wonderful Diſcoveries; and it muſt be with theſe, if with any, that the *Corpuſcles* or *Atoms*, of which Bodies conſiſt, are to be diſcovered; which the great Sir *Iſaac Newton* thought was poſſible. But the great Difficulty of making very ſmall, and, at the ſame Time, very good ones; their Prejudice to the Eyes in poring very hard and near, the Trouble of poſiting Objects at a due Diſtance, and the very ſmall Part which can be ſeen of any, makes this Sort of Microſcopes very little known or uſed.

21. In Fig. 6. let A B H repreſent a ſmall Globe or Spherule, whoſe Center is C, and Radius C B; alſo let G B be = ½ C B, then will the Point G be the Focus of Parallel Rays paſſing thro' and refracted by the ſaid Spherule; and therefore, if an Object E F be placed in the ſaid Focus G, it may be diſtinctly ſeen by an Eye in the other Focus of the Spherule. Suppoſe now the Spherule be removed, and in the Place of it's Center be placed the Lens L N, whoſe Radius is C G; then will G be alſo the Focus of the ſaid Lens, and the Object E F will be diſtinctly ſuch a Lens alſo. And ſince the Angle E C F, under

under which the Object appears at the Center of the Spherule and Lens, is the ſame, the Object will be equally magnified by them both. But CG, it's Diſtance from the Lens, is three times greater than B G, it's Diſtance from the Spherule; and therefore much more Light will fall on it when viewed by the Lens, than can when viewed by the Spherule; and conſequently in all Caſes, where the Power of magnifying is not required to an extreme, the Uſe of a Lens is much preferable to that of a Spherule.

22. The next Sort of ſingle Microſcopes are thoſe made of *Concave Mirrours*; and the larger the Mirrour, the fitter for the Purpoſe. One 10 Inches Diameter, and 12 Inches focal Length, will do very well. By ſuch a Mirrour, a ſmall Object, may be made to appear very large either behind or before it.

23. The Diſtance at which an Object ſhould be placed, in order to magnify it any propoſed Number of Times, at a Focus behind the Glaſs, is found by the Rule delivered in Art. 11. Chap. VI. Part I. Thus ſuppoſe I would magnify the Diameter of a ſmall Object 10 times, I find by that Rule, that it muſt be placed $10\frac{8}{10}$ Inches before the Mirrour; and when the Diameter, or Length of an Object, is magnified 10 times, the Surface will be magnified an 100 times, and the whole Bulk a 1000 times. This is a very conſiderable Effect, and is attended with this

peculiar Advantage, that not only a ſmall Part of one Object, but the whole of diverſe Objects is here exhibited to the View, which makes it pleaſant and delightful to behold.

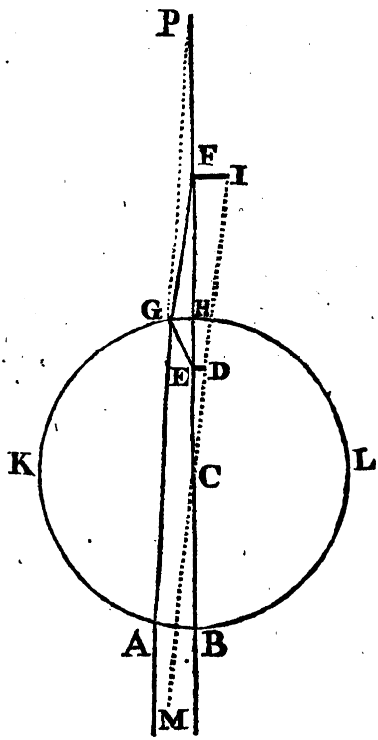
24. The other Way of magnifying Objects at a Focus before the Mirrour, is that already deſcribed in Chap. IV. Art. 32, as being performed in a *dark Chamber*. There remains one other Sort of ſingle Microſcope, which conſiſts of a Glaſs Globe filled with Water, which ſerves for magnifying the ſmall Animalculæ therein very well; this was the Diſcovery of Mr *Stephen Gray*, and is as follows.

25. Let K L be a Globe filled with Water, and A B two parallel Rays falling thereon, of which let B H be the Axis of the Globe, which paſſing through the Center C, will ſuffer no Refraction; the Ray A will be refracted to a Focus F, diſtant from the Globe H, the Diameter thereof, *i. e.* H F = B H, as may be deduced from the Theorem in Art. 16. Chap. I. of *Dioptrics*, by putting I to R, as 4 to 3; which is the Proportion of the Sine of Incidence, to the Sine of Refraction out of Air into Water.

26. Let us now conſider the refracted Ray A G, as reflected from the Concave Surface of the Globe at G, to the Focus E in the Axis. Then will an Object placed at E be ſeen by Reflection in the ſame Manner, as an Object at F will be by Refraction. Through the

Center C, draw MCI, and to the Focuses E, F, draw the perpendicular DE, and IF: Now since unequal Objects IF, DE, are seen under equal Angles ICF, and DCE, at the Center C, the Angles under which equal Objects will be seen at E and F, will be as CF to CE, by what is said in Chap. II. of this Part.

27. Now the Ray A, by it's first Refraction, will tend to a Point P, which will be distant from the Point four times the Radius CB, that is, BP = 4BC, and therefore HP=2CH, as is easy to deduce from the Theorem in Art. 6. Chapter I, of *Dioptrics*. Then put HP = d, and CH = r, we shall find HE = f, by Theor. 17, in Art. 18. Chapter III. of *Catoptrics*, viz.



$\frac{dr}{-2d-r} = f$; for in this Case $d = 2r$, and

therefore

therefore the Theorem becomes $\frac{dr}{-2d-r}$ $= \frac{2rr}{-5r} = \frac{2}{5}r = f = HE$; and so CE $= \frac{3}{5}r$: but CF $= 2r$, therefore CF : CE :: $2r : \frac{3}{5} :: 2 : \frac{3}{5} :: 10 : 3 :: 3\frac{1}{3} : 1$. And consequently an Object, in the Focus of Reflection E within the Globe, will be magnified $3\frac{1}{3}$ times more than it would be, if placed in the Focus of Refraction F, without the Globe.

28. Now suppose the Globule KL were $\frac{1}{5}$ of an Inch Diameter, then would the apparent Magnitude of Objects at F, by Refraction through the Water, be to the apparent Magnitude at 6 Inches from the Eye, as 6 to $\frac{1}{5}$, or as 30 to 1; and therefore the apparent Magnitude at E, by Reflection, will be as $3\frac{1}{8}$ times 30, that is, 100 times magnified,

CHAP.


A
G
C
H
B

CHAP. VI.

ouble or compound Microscopes, viz. hose which consist of two Lenses; and the Manner of computing their Power of magnifying.

IF a Machine be constructed with more than one Lens or Mirrour, proper for agnifying small Objects, it is called a *double compound Microscope*; such as is that repreented in Fig. 1. Plate XXIV.

2. In that Figure d e f is a small Lens, alled the Object-Glass, as being next the Object a c b placed a little below it to be view'd. Suppose the Focus of this small Lens be at O. Then, as is plain from the Theory, if any Object a c b were placed in the Focus O, the Rays which came from any one Point divergent on the Lens, would by it be so refracted, that they would afterwards proceed parallel to one another; or their Focus would have been at an infinite Distance.

3. It has also been shewn, that if an Object a b be placed at any Distance from e C, greater than the focal Distance e o, the Rays d c, f c, which come from any Point c, and fall

fall divergent on the Lens d f, will be ſo refracted, that they will afterwards proceed converging towards, and at laſt unite in a Point C, which will be the Focus of all the Rays proceeding from the Object a b, for the Diſtance c e.

4. Therefore the Rays which proceed from the Points a, c, b, in the Object a b, will be reſpectively repreſented at A, C, B, in the aforeſaid Focus; and conſequently A C B will be the Image of the Object a c b; in an *inverted Poſition*, and is ſo much greater than the Object, as the focal Diſtance e C is greater than the Diſtance of the Object e c. And the leſſer e c is, the greater will Ce be, and the more will the Image A C B be enlarged, or the Object magnified: But, as before obſerved, the Diſtance e c muſt always be greater than the focal Diſtance e o. All which things are likewiſe demonſtrated and exemplified in the Theory.

5. In ſingle Microſcopes, or thoſe of one Lens, we view the Image, or rather the Object itſelf, at an affirmative Focus, or behind the Lens; but in compound Microſcopes it is otherwiſe: For here A B, which is abſolutely the Image of the Object, far diſtant from it at a negative Focus, or on that Side the Lens next the Eye, is the Thing we view, and conſider it now as a new Object.

6. If then any Lens D F be placed juſt at it's focal Diſtance E C, from any Object A B, the

the Rays which come from any Point C, will after Refraction through the Lens, proceed parallel to the Eye at I, and there cauſe diſtinct Viſion: Thus the Object A B, that is, the Image of the Object a b greatly magnified, will be clearly and diſtinctly ſeen thro' a ſecond Lens D F; and if the Eye's Diſtance be leſs, equal to, or greater, than the focal Diſtance E C, then ſuch a Portion of the Image A B will be ſeen as is greater, equal to, or leſs than that Part of the Lens DF, which is open. Note, this Lens D F is called the *Eye-Glaſs*, as being next to the Eye in viewing Objects.

7. Thus is Viſion performed in the Double Microſcope, compoſed of two Glaſſes. I ſhall now ſhew the *Power of magnifying* in ſuch a Microſcope, which will be eaſily apprehended in the following Manner. I have before obſerved, that no Object can be clearly and diſtinctly viewed at a leſs Diſtance than ſix Inches, and by many Eyes not ſo near. But ſix Inches is the Number I ſhall found my Computations upon, becauſe I ſhall then be ſure to be within Compaſs.

8. Firſt, let us ſuppoſe d f the *Object-Glaſs* to be a double and equally Convex Lens, whoſe focal Diſtance e o $= \frac{8}{10}$ of an Inch. Then if an Object a b be placed at the Diſtance e c = 1 Inch, the Image A B will be formed at the Diſtance e C = 4 Inches, as is manifeſt

manifeſt from the Theory, and the Rule exemplified in Chap. X, Art. 1 and 2, Part II.

9. Since then the Diſtance e c is but one Inch, the Angle a e b, which the Object ſubtends at the Lens d e, is 6 times greater than the Angle under which it would appear to an Eye, at 6 Inches Diſtance; and therefore the *Length* of the Object a b, will be magnified 6 times by the Object-Lens only.

10. And ſince the Angles a e b, and A e B, under which the Object and it's Image appear at the Lens d f, are equal, the Image A B to an Eye at e, will appear equal to the Object a b, *i. e. ſix times* larger than the ſaid Object at the Diſtance of *ſix Inches* from the Eye.

11. But ſince the Image A B is rendered viſible, by means of the Eye-Glaſs D F, and is ſeen under the Angle A E B; the apparent Magnitude thereof ſeen by an Eye at E, will be to the ſame to an Eye at e, as the Angle A E B, is to the Angle A e B, or as the Diſtance C e, to the Diſtance C E: But C e is 4 Inches; if then we ſuppoſe C E = 1 Inch. The Image will appear 4 times greater at the Eye-Glaſs, than at the Object-Glaſs; and therefore 4 times 6, that is, 24 times greater than the Object itſelf a b.

12. The *Length* being magnified 24 times, the *Surface* of Objects will be magnified 576 times, and the Solidity or Bulk 13824 times. And the Effect of this double Microſcope is equal

equal to that of a single one of a Lens, whose focal Distance is but ¾ of an Inch, as is evident from Art. 10, of the foregoing Chapter.

13. Again, if the Distance of the Object *e c* be ¾ of an Inch, and the Distance E *e* between the Eye-Glasses remaining the same; then will the Object *a b* appear as much greater by the Eye-Glass *d f*, than to the naked Eye at 6 Inches Distance, as 6 is greater than ¾, *viz.* 8 times. Then also, since C *e* = 4 Inches, and C E = 1, the Object will appear 4 times greater through the Eye-Glass D F, than by the Object-Glass *d f*; and therefore 4 times 8, that is, 32 times greater by both than to the naked Eye. The *Length* being magnified 32 times; the *Surface* will be magnified 1024 times; and the *Bulk* 32768 times, by such an Object-Lens.

14. If you use an Object-Lens that shall give the Distance *e c* = ½ an Inch, then will it magnify the Object *a b* 12 times, and the Eye-Glass D F magnifying that Image A B 4 times more; the Length of the Object will by both be magnified 4 times 12, *viz.* 48 times; and therefore the *Surface* will be magnified 2304 times, and the Solidity 110592 times.

15. In the last place, if you use an Object-Lens, which gives the Distance of the Object *e c* = ¼ of an Inch; then will the Object be magnified by that 24 times, and 4 times more by the Eye-Glass, and therefore by

by both 96 times; and thus the *Surface* will be magnified 9216 times; and the *Solidity* 884736 times, which is to a very great Degree indeed.

16. If you are desirous of knowing what focal Distances eo the 4 several Object-Glasses must have to give the Distance of the Object ec = 1, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$, as above-stated for Calculation, you find them according to the Rules of the Theory, to be $\frac{8}{10}$, $\frac{63}{100}$, $\frac{44}{100}$, $\frac{23}{100}$; supposing those Lenses are double and equally Convex.

17. The *Power of magnifying* is to be varied also by varying the focal Length of the Eye-Glass DF, for the same Object-Glass df; for as now, the focal Length EC being 1 Inch, the Lens DF magnifies the Image AB 4 times, if the said focal Length were but $\frac{3}{4}$ of an Inch, it would magnify the Image AB $5\frac{3}{10}$ times, for then Ce : CE :: $5\frac{3}{10}$: 1. And if CE be but $\frac{1}{2}$ an Inch, then would Ce : CE :: 8 : 1; and so the Lens DE would magnify the Image AB, 8 times.

18. But we can't vary the focal Length of the Eye-Glass DF at pleasure, or to that advantage as we can the Focus of the Object-Glass df, for Reasons hereafter to be given.

19. There is yet another Way, by which the *Power of magnifying* is to be varied or augmented in this Microscope; and that is, by varying the Distance between the two Glasses DF and df, by means of two Tubes sliding one within the other; in the uppermost is fixed

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CHAP. VII.

Of Microſcopes compounded with three Lenſes, viz. *an Object-Lens, and two ocular ones.*

1. WE have ſeen the Nature and Effects of a Microſcope, conſtructed with *one* and *two* Lenſes, let us now contemplate the Nature and Conſtruction of one of *three*, viz. An Object-Lens d f, and two Eye-Glaſſes, one a larger G K, and the other leſſer D F. This broad Lens G K is added (not to increaſe the *magnifying Power*, or *Diſtinctneſs of Viſion*, for that is better effected by one Lens D F, but) to increaſe or *amplify the optic Angle*, or to enlarge the *viſible Area*, or *Circle of Viſion* in the Microſcope. Which it does very conſiderably, as is thus ſhewn. See Fig. 2.

2. Let a b be an Object ſo poſited, that it's Image *A B* may be formed at the Diſtance of e *C*, from the Lens d f; at H ſuppoſe the Lens G K intercept the Rays e K and e G, which it will refract towards ſome Point in the Axis, as O; and conſequently there will be a new and leſſer Image A B formed than before, the Place of which is found by draw-

ing

ing the Lines *A*H, *B*H, from the Extremities of the first Image *AB*, to the Center of the Lens; for they will intersect the Rays KF, GD, in A and B, the Place of the Image AB. For if the Image AB be considered as an Object with regard to the Lens GK, and FK, DG two incident Rays, they will emerge from the Lens in the Direction Ke and Ge, and since CH is less than the focal Distance of the Lens GK, the Rays of every Pencil going from the Object AB, will emerge from the Lens diverging, and tend towards a *virtual Focus C*, where they form the imaginary Image *ACB*. But the Object AB, and it's Image *AB*, are seen under equal Angles *A*H*B*, from the Center of the Lens H; and therefore the Lines *A*H and *B*H determine the Places and Proportions of the Images *AB* and AB. All which is plain from the Theory.

3. Let DF be another Lens placed at it's focal Distance EC, from the Image AB, which then will be distinctly seen through it by an Eye placed any where in the Axis EO. Let Hc be the focal Distance of the Lens GK, draw *ab*, and join *a*H, then will the Angle *a*Hc be the optic Angle under which the half of the Object ac is seen through the Lens GK; and is to the Angle aeC = *a*ec, (under which it appears by the Lens df) as ec is to Hc.

4. Since

4. Since H*a* = Hc, the Point *a* will be the Focus of a Pencil of Rays falling obliquely on the Lens G K, of which *a* H is the Axis, and *a* K the extreme Ray; and ſince all the Rays of the Pencil emerge parallel from the Lens in this Caſe, if K O be drawn parallel to *a*H, O will be the Point in which the Ray e K, after Refraction through the Lens GK, will cut the Axis H O, making the Angle H O K = *a* H c.

5. But the Ray K O, by the Interpoſition of the Lens D F, will ſuffer a ſecond Refraction through it at F; and to find what that will be with the focal Diſtance E C of the Eye-Glaſs D F, ſweep the Arch Ce, and join Ee, then will e be the Focus of an oblique Pencil of Rays falling on the Lens D F, of which e E is the Axis, and e F an extreme Ray. If then you draw F I parallel to e E, it will be the refracted Ray, and I the Point in which it cuts the Axis H O; making the Angle E I F = C E e.

6. From D and F let fall the perpendiculars D *d*, F *f*, on the Image A B; and if there were no other Lens but D F, the Part *df* in the Image is all the Eye could view in the proper Focus L, for *df* = D F. See Art. 5. Chap. V, hereof. But now by means of the Lens G K, the Rays K F and G D, which terminate the Extremities of the Image, are made to fall within the ſame Aperture D F of the Eye-Glaſs, and conſequently the whole

Image

Image AB is seen by the Eye at the Point I, under the Angle EIF.

7. But this Angle EIF is to the proper Angle ELF, as LE is to IE, by Art. 9. Chap. II, of this. Now in order to express this Proportion in Numbers, we must first find HO, for which the Distance eH must be given, which suppose 5 Inches. Then esteeming e as a Radiant Point from whence proceeds the Pencil of Rays GeK, to the Lens GK, whose focal Distance, or Radius of Convexity Hc = 3 Inches. We shall find the Focus O of this Pencil after Refraction, by the Theorem $\frac{dr}{d-r} = y$. See Art. 36. Chap. I, of the Theory of *Dioptrics*.

8. For here $d = eH = 5$, $r = Hc = 3$, and $y = HO$, to be found thus; $\frac{dr}{d-r} = \frac{5 \times 3}{5-3} = \frac{15}{2} = 7.5 = y$. Or thus by Analogy, $d-r$ (ec) : d (eH) :: r (Hc) : y (HO) $= 7\frac{1}{2}$ Inches. Having thus found HO, suppose the Distance between the two Glasses HE = 2 Inches, then will EO = $5\frac{1}{2}$ Inches; and if the Radius of Convexity EC, of the Eye-Glass DF be 1 Inch; we shall find EI, by the Theorem in Art. 38. Chap. I,

 of

of *Catoptrics*, *viz.* $\frac{dr}{d+r} = y$; for here $d =$ EO = 5.5, $r =$ EC = 1, and $y =$ EI; therefore $\frac{dr}{d+r} = \frac{5.5}{6.5} = 0.85$ very nearly. Or by Analogy, as $d + r$ (= CO = 6.5) : d (= EO = 5.5) : : r (= EL = 1) : y (= EI = 0.85, *fere*).

9. Therefore the optic Angle DLF proper to the Lens DF alone, is to the amplified Angle DIF, as 0.85 to 1, or as 85 to 100; the visible Areas therefore will be as the Squares of these Numbers, *viz.* as 7225 to 10000, or as 7 to 10 nearly; which Augmentation of the optic Angle by the Addition of the Lens GK is very considerable, and renders the Pleasure of viewing Objects proportionably greater. Now this Angle DIF is greater or smaller, as the Distance between the Lenses HE is greater or smaller; for as the Lens DF approaches to the Lens GK, the Distance EO approaches nearer to an Equality with CO, or EI with EL, that is, the Angles I and L become nearer equal; and the contrary, as DF is placed farther from GK.

10. The Degree or *Power of magnifying* in this given Combination of Lenses, is computable in the following Manner. Suppose I can see the Line ac distinctly at the Distance of 6 Inches, but by means of the Lens df I can see it at the Distance eC, under the Angle

Angle a e C. Again, since a e C = *a* e c, and H c is the focal Length of the Lens G K, the Object a C will appear to an Eye at H, under the Angle *a* H c, which is to the Angle *a* e c, (or a e C) as e c to c H. Lastly, the Angle under which it appears by the Eye-Glass D F is E I F = C E e, but this Angle C E e is to the Angle *a* H c = C O e, as C O to C E. Therefore the first Ratio of magnifying, *i. e.* by the Lens d f is 6 : C e, or $\frac{6}{Ce}$; the second Ratio, by the Lens G K is e c : c H, or $\frac{ec}{cH}$; therefore the Appearance of a C at H, is to that at the naked Eye, as $\frac{6}{ec} \times \frac{ec}{cH}$. The third Ratio is by the Lens D F, in regard of the last, as C O : C E, or $\frac{CO}{CE}$; and consequently an Object a c will appear under an Angle D I F, which is to that under which it appears to the naked Eye at 6 Inches Distance as $\frac{CO}{CE} \times \frac{ec}{cH} \times \frac{6}{eC}$ to 1.

11. To give an Example, let C e = ½ an Inch; c H = 3, C E = 1; and the Distance e H = 5; then will c e = 2, and C O = 6.5, things remaining as in Art. 9. Then

$\frac{CO}{CE} \times \frac{ec}{cH} \times \frac{6}{eC} = \frac{6.5}{1} \times \frac{2}{3} \times \frac{6}{0.5} = \frac{78}{1.5} =$ 52. Therefore the apparent Magnitude thro' the Microſcope, is to that by the naked Eye, as 52 to 1, for a *Line*, as 2704 to 1, for a *Surface*; and as 14068 to 1, for a *Solid*.

12. The Lens GK being removed, all other things remaining the ſame, the magnifying Power will be greater than before; for it will be as $\frac{Ce}{CE} \times \frac{6}{ce} = \frac{6}{1} \times \frac{6}{0.5} = \frac{36}{0.5}$ $= 72$, above a third greater than 52. And by removing GK we are obliged to change the Object-Lens df to keep the Diſtances the ſame; yet if df were retained, the Diſtance ce would be ſo very little enlarged to bring the Image *AB*, to the Situation AB, that if the Lens DF be the ſame, and at the ſame Diſtance from df, the magnifying Power will ſtill be greater much by itſelf, than in Combination with the Lens GK.

13. If inſtead of $Hc = 3$, we take it $= 4$ Inches; then the optic Angle DIF will be diminiſhed, all other things remaining the ſame, as in Art. 11. For now $r = Hc = 4$, and $d = He = 5$, and $d - r = ce = 1$; therefore $\frac{dr}{d-r} = \frac{5 \times 4}{1} = 20 = y$, or HO; and ſo $CO = 19$, and $EO = 18$. Therefore to find EI we have $d = EO = 18$, and

and $r = EL = 1$, and $d + r = CO = 19$; and then $\frac{dr}{d+r} = \frac{18 \times 1}{19} = 0.95$ nearly, which in this Caſe is the Length of EI, whereas before it was but 0.85; (Art. 8.) and conſequently the Angle DIF is now diminiſhed in the Ratio of 95 to 85, or nearly as 9 to 8.

14. But the magnifying Power will be increaſed; for $\frac{CO}{CE} \times \frac{ce}{cH} \times \frac{6}{eC} = \frac{19}{1} \times \frac{1}{4} \times \frac{6}{0.5} = \frac{114}{2} = 57$; whereas before (Art. 11.) it was but 52; but ſince this Increaſe of the magnifying Power is ſmall, and the Diminution of the optic Angle very conſiderable: (Art. 13.) a Lens GK, whoſe focal Diſtance is 3 Inches, is preferable to one of 4 Inches.

15. Again, if we make the focal Diſtance of the Lens GK equal to 2 Inches, other things being the ſame; we ſhall have $\frac{dr}{d-r} = \frac{5 \times 2}{3} = \frac{10}{3} = 3.33 = HO$; and then $EO = 1.33 = d$; and ſo $\frac{dr}{d+r} = \frac{1.33}{2.33} = 0.56 = EI$; and therefore the Angle DIF : DLF :: 1 : 0.56 :: 100 : 56, that is, the optic

optic Angle at I, is almoſt as big again as the proper Angle at L. But notwithſtanding this Advantage, ſince the magnifying Power is diminiſhed, (it being only $\frac{CO}{CE} \times \frac{ce}{cH} \times \frac{6}{eC} = \frac{2.5}{1} \times \frac{3}{2} \times \frac{6}{0.5} = 42$.) and the great Convexity of the Lens G K, diſtorting and colouring the Object about the Extremities, the focal Length of three Inches is preferable to this of 2 Inches; and conſequently to any other focal Length of G K, in this Conſtruction of the Microſcope.

16. In this Combination of Eye-Glaſſes, the Effect of the Microſcope is $\frac{CO}{CE} \times \frac{ce}{cH} \times \frac{6}{Ce}$ to 1; but in a Microſcope with a ſingle Eye-Glaſs DF, whoſe focal Diſtance is *CE*, and the Diſtance of the Image from the Object-Lens df, is *Ce*, hath it's Effect in the Ratio of $\frac{Ce}{CE} \times \frac{6}{ec}$ to 1. If now we ſuppoſe an Object a b, equally diſtant from the Object-Lens d f, when diſtinctly ſeen in both, then according as $\frac{CO}{CE} \times \frac{ce}{cH}$ is *leſſer*, *equal*

to,

to, or *greater* than $\frac{Ce}{CE}$, the Power of magnifying, in the former Microſcope, will be *leſſer*, *equal to*, or *greater*, than the ſame in the latter, or that with a ſingle Eye-Glaſs.

17. If the focal Diſtance of the Eye-Glaſs DF, be the ſame in both Microſcopes, their magnifying Effects will be in the Ratio of $CO \times \frac{ce}{cH}$ to Ce. And if $ce = cH$, then will their Effects be in the Ratio of $\frac{CO}{CE}$ to $\frac{Ce}{CE}$. If then we would have them magnify equally in this Caſe, $CE : EC :: CO : Ce$, is the Analogy.

18. If the Microſcope hath the Dimenſions of Art. 11, the Aperture of the Lens GK being given, and the Diſtance between the Lenſes HE, the Aperture of the Lens DF is given alſo; for $HO : EO :: GK : DF$. Suppoſe $GK = 1.5$, then $HO = 7.5 : EO = 5.5 :: GK = 1.5 : DF = 1.07$. The half of which $EF = 0.53$; whence we may alſo find the Quantity of the optic Angle DIF: for $EI = 0.85$, by Art. 13. Wherefore we have this Analogy, As $IE = 0.85$ is to $EF = 0.53$, ſo is the Tangent of Radius

45°.

45°, to the Tangent of 27° : 45′ ferè, the double of which, *viz.* 55° : 30′, is the Quantity of the Angle DIF, in this Constitution of the Microscope.

CHAP. VIII.

Of the apparent Position of the Objects, and the visible Area, or Field of View, in a Microscope of Convex Eye-Glasses.

PLATE XXIV. Fig. 3.

1. WE have hitherto been considering the Structure or Disposition of Glasses, and the Power of magnifying in Microscopes; the next thing to be taken Notice of is the *apparent Position* of the Object, and the *Area*, or *Field of View*, therein.

2. In order thereto, let N, O, P, be Cones of Rays coming from the Object-Lens, and painting in their Focus the Image ACB. This Image will be in a Position *inverted*, or contrary to that of the Object, as is evident from Fig. 1 and 2, of this Plate; the Reason whereof has been fully explained before.

3. The

3. The Image ACB we are now to consider as an Object viewed through a Convex Lens DF, by the Eye *ABQR*. This Object, as being ſeen in the Microſcope, muſt be alſo conceived to be in the Focus of the Lens DF, which is the Eye-Glaſs; for in this Caſe only, the Rays AD, A*g*, A*h*, alſo C*n*, C*o*, C*p*, and the Rays BF, B*m*, B*l*, which come from the ſingle Points A, C, B, will fall parallel on the Pupil QR, and if they do not come parallel, thoſe Points of the Object will not be diſtinctly ſeen, as has been ſhewn in Chap. I, of this Part.

4. Now from the Points A, G, H, I, C, K, L, M, B, ſuppoſe parallel Rays proceed to the Glaſs DF, in the Points D, *g*, *h*, *i*, *n*, *o*, *p*, *k*, *l*, *m*, F, then ſhall theſe be all united in the Focus at E, in which we will ſuppoſe the Pupil of the Eye to be placed where it will then view as much of the Object AB, as is equal to the Diameter of the Eye-Glaſs DF; as was demonſtrated in Chap. V, Art. 5.

5. The Eye being poſited in the Focus of the Glaſs, will by it's Humours converge or unite the Rays which enter parallel on the *Retina*, in the Bottom thereof; and therefore the Rays AD, A*g*, A*h*, proceeding from the Lens parallel to the Eye, will by it be united on the *Retina* at *A*, and there repreſent the Point A of the Object ACB. After the ſame Manner we conceive the Points B and C in the Object, repreſented at *B* and *C* in

in the Bottom of the Eye; and all the intermediate Points G, H, I, K, L, M, after having passed the Lens, will traverse in the Point E, and proceed to g, h, i, k, l, m, where they will be represented on the *Retina*.

6. Consequently *ACB* on the *Retina*, will be the Image of the Object A B C, that is, it will be the second Image of the small Object a b, under the Microscope, of which A B is the first enlarged Image. And since the Image A C B, is contrarily posited to the Image A C B, it will be rightly posited with the Object itself, or the Object a b, and it's secondary Image *AB*, have both the same Position, that is, their similar Extremities a, *A*, and b, *B*, lie towards the same Parts.

7. Now it has been shewn, that when an Object appears *right* or *erect*, it's Image in the Bottom of the Eye will be *inverted*, or have a Position *contrary* to that of the Object; (See Chap. II, Art. 1, 2, 3, 4.) and therefore when the Image of an Object in the Eye is the same with that of the Object itself, the said Object cannot appear *right* or *erect*, but *inverted* or *contrarily posited* to what it truly is. And such therefore is the Position of an Object, as it appears through a *double Microscope*.

8. The Position of the Object, in a single Microscope, is not altered; for let D F be the magnifying Lens, and A B the Object in one Focus, and Q R the Pupil of the Eye in the

the other; then, as hath been ſhewn, the Poſition of the Image *A C B*, in the Bottom of the Eye is *inverted*, or contrary to that of the Object; in which Caſe it will appear *erect*, or in the ſame Poſition with the Object.

9. The Object A B remaining in the Focus of the Lens D F, if another Lens be interpoſed between the Eye and the ſaid Lens D F, it will not alter the apparent Poſition of the Object, from which it is by the ſingle Lens D F alone. For all the Effect this will have, is only making the Converging Rays F E, D E, converge the ſooner, and ſo ſhorten the focal Diſtance o E; but there will ſtill be but one Point E, in which Parallel Rays, which come from one Part of the Object croſs thoſe which come from the contrary Part; and conſequently there can be but one Inverſion of the Object A B, and therefore the apparent Poſition of the Object muſt be the ſame as before, whether in the double or ſingle Microſcope.

10. It is very eaſy however to rectify this inverted Poſition of the Object in the double Microſcope, by the Addition of two more Lenſes, as will be ſhewn when we come to treat of Teleſcopes; and then it may be aſked, Why is not this Addition commonly made to the Microſcope, as it is to the Teleſcope? I anſwer, firſt, It is not material in ſmall Objects how they appear ſituated; for inſtance, there can be no more Advantage in ſeeing the

Head

Head of a Mite lie towards the Right-Hand, than towards the Left. Secondly, the Object ſeen through 4 Glaſſes will not be ſo bright and lively as through only 2 or 3 Glaſſes. Thirdly, the Diſpoſition of three Eye-Glaſſes, that ſhall erect the Object, will occaſion the Microſcope to be enlarged, and will very much alter the preſent commodious and beautiful Form thereof for the worſe. Not to mention, that four Glaſſes will come dearer than two or there.

11. The inverted Image may indeed be made to appear right by two Eye-Glaſſes only: for if the lowermoſt D F be removed a little farther from the Image AB, than it's focal Diſtance, it will form a ſecond Image on the Side, which may be viewed by a third Lens, placed at it's focal Diſtance from that ſecondary Image. But unleſs the Lens DF be extremely convex, the ſecondary Focus will be at a greater Diſtance than will be proper for this Machine; beſides that, the Object will be very much coloured and diſtorted; and, in ſhort, any Method of rectifying the Object will prove a Remedy much worſe than the Diſeaſe.

12. Concerning the *viſible Area*, or what is commonly called the *Field of View* in a Microſcope, you are to underſtand the following Particulars relating thereto. If the Image A B be in one Focus of the Eye-Glaſs D F, and the Eye Q R in the other, the Eye

will

will see just so much of the said Image, as is equal to the Glass DF, either in Diameter or Surface; that is, in such a Case the *Area*, or *Field of View*, is equal to the Area of the Glass DF, or that Part of it which is open. This is evident from Chap. V, Art. 5, of this Part.

13. But since, in this Case, the Pupil of the Eye is posited in that Point E, where Rays coming from the Extremities of the open Part of the Lens as DE, FE, intersect each other, it is evident the Eye takes in then the greatest Area possible; and therefore an Object or Image AB, which is greater than the open Part of the Lens can never be all seen at once, but one that is less than the Aperture of the Lens may.

14. If the Eye be moved from the Focus E, either nearer to or farther from the Glass DF, the visible Area will be diminished; and you will not discern so much of the Image as before at E: For if it be moved nearer to the Glass in the Axis EC, it will soon arrive to a Part of the Cone DEF, whose Diameter is greater than that of the Pupil, and consequently the external Rays DE, and FE, will fall without the Pupil, and so the Points of the Image A and B (whence they proceed) cannot be seen.

15. If it approach still nigher, it will leave the Rays *g*E, *m*E, and then the Parts AG

and BM in the Image, will not be ſeen. In like manner if it approach ſo near, that the Rays *h* E, *l* E, are excluded, the Parts AH and BL will diſappear; and ſo the Image will vaniſh more and more from the Sight, till the Eye comes to be upon the Glaſs, and then it will perceive no more of the Image, than is equal to the Pupil of the Eye. And it is eaſy to apprehend, that the ſame things will happen if the Eye be removed farther off in the Cone *A* E *B*.

16. The Eye by being moved tranſverſely through the Cone of Rays DEF or *A* E *B*, will ſucceſſively view all the Parts of the Image AB; for ſuppoſe it enter the Cone DEF, on the Side DE, it will ſucceſſively take in the Rays DE, *g* E, *h* E, *&c.* which, at the ſame Time, render viſible the Points D, *g*, *h*, *&c.* in a direct Order. But if the Eye enters the Cone *B* E *A*, on the Side *B* E, in paſſing through it, it will indeed ſucceſſively view all the Parts of the Image AB, but in an inverſe Order, *viz.* from B towards A.

17. I would add one thing for the Sake of thoſe who make Microſcopes, and that is, that the Focus E, or Vertex of the Cone DEF, ſhould ariſe ſo far above the Top of the Microſcope, that when it is applied to view, it may be capable of receiving and adjuſting the ſaid Focus E to the Pupil, for

then

210.

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CHAP.

CHAP. IX.

Of the Nature and Effects of a compound Microscope, with a double Concave Eye-Glass.

1. THE Construction of this Microscope is represented in Fig. 1, of Plate XXV. Where ab is the Object, and df the Object-Lens, as before. But the several Pencils of Rays adAfa, bdBfb, &c. which, in the Microscope with a Convex Eye-Glass, formed the Image ACB, are in this Case, by the Interposition of the double Concave DEF, made to diverge still more towards the Parts K and L; according to what has been taught of a double Concave Lens in the Theory.

2. It has been also shewn, that if any Rays dEf fall converging on a double Concave DEF, placed at the Distance of the Radius from the Focus C of those Rays or Point, towards which they converge; then those Rays will be so refracted by the said Concave, as to become *parallel* afterwards, and so fit to produce *distinct Vision.* The like is to be understood of the Rays dDf, and dFf; and therefore,

3. A

3. A double Concave DEF, placed at the Distance of the Radius below the Focus ACB, of the Rays proceeding from the Lens d f, will equally cause a Parallelism of those Rays by Refraction, with a double Convex GHI, placed at the Distance of the Radius above it. And consequently A o, the distinct Vision of the Object a b, whence these Rays proceed, will be produced by a double Concave thus posited, as by the double Convex Eye-Glass.

4. Let us suppose the Rays a d, b f, after they are refracted by the Lens d f, pass parallel to the Concave DF; then because dD, and fF, are parallel, they will be so refracted into DK and FL, that those Rays DK, FL, being produced below the Concave, shall meet in a Point *B* in the Axis, which is the Center of Concavity; from what has been shewn in the Doctrine of Concaves. This Point *B* therefore will be the Focus in which the Image *ABC*, of the Object a c b, will be formed, and rendered visible to the Eye.

5. The same things supposed, let A*C*, *A*B, be drawn thro' the Vertex of the Concave E, from each Extremity of the Image *AC*; and because the Distance E*B*, EC, are equal, therefore the *visible Image ABC*, will be equal to the *virtual Image* ACB; or it will appear under an Angle *A*E*C*, equal to the Angle AEB. Wherefore,

6. Since the virtual Image AB, subtends the Angle AeB, under which the Object a b

 appears

appears by the Lens df; and the Angle AEB, under which it appears by the Concave DF, the Object will be magnified in the Proportion of those Angles; that is, the Object ab will be to the Image *A*B, (or *A C*) as the Distance EC (or E*B*) to the Distance Ee. See Chap. II. Art. 9, of this Part.

7. Hence it follows, that if the Radius of an equally Concave Eye-Glass DF, be equal to the Radius of an equally Convex Eye-Glass GI, the Power of magnifying will be the same in both with the same Object-Lens *d*f: For since E*B* or EC is equal to CH, by supposition; the Angle *A*E*C*, or AEB, will be equal to the Angle AHB. But since the optic Angles are equal from either Glass, the Power of magnifying must be so too.

8. Since the Pencils of Rays bF and aD, tend towards the same Parts, where the Extremities of the Object lie from whence they proceed, after they are refracted by the Concave DF, the Eye will perceive the Position of the Image, to be the same with that of the Object, and not *inverted*, as by a Convex Eye-Glass, wherein the Image is seen by Rays which cross or intersect each other before they enter the Eye.

9. Since the Rays, after they are refracted through the Concave DF, pass on more diverging, it is easy to understand, that an Eye placed any where over the said Glass, can perceive only that Part of the Object, whence those

those Rays which enter the Pupil do proceed; thus, if the Eye receive the Rays F L, it will see the Part b; if the Rays D K, it will see the other extreme a, but if the Eye take in the middle Rays E C, the middle Part C of the Object will only be visible.

10. Hence it appears that the visible Area, or Field of View in this Microscope, is in Proportion to the Magnitude of the Pupil of the Eye and the Density: For as the Pupil is greater or lesser, so a greater or lesser Quantity of Rays will enter it; and consequently the visible Area, or Part of the Object, will be proportionably greater or less.

11. Again, in Regard of the Density of Rays, it is evident the Pupil, placed where they are most dense, will collect most Rays, and therefore see the largest Part of the Object possible, and that is at the Concave Surface of the Lens itself. For as the Rays diverge from thence, they grow more rare, and so the higher the Eye is raised above the Glass D F, the lesser Quantity of Rays it will collect, and consequently the more will the visible Area of the Object be diminished.

12. In order to compute the visible Area of the Object by this Microscope, let P P be the Diameter of the Pupil of the Eye, (Fig. 2.) applied as near the Concave D F as possible, d f the Object-Lens, and A B the Object. From P, P, thro' the Center e, of the Lens d f, draw the Lines P a, P b, these will

cut the Object in the Points a and b; and ſo a b will be the Diameter of the viſible Area of the Object A B. Suppoſe the Object, Lenſes, and Pupil, all parallel to one another; it will be, *As the Diſtance of the Concave* D F, *from the Lens* d f, *is to the Diſtance of the Object* A B; *ſo is the Diameter of the Pupil* P P, *to the Diameter of the viſible Area* a b.

13. And therefore, for Example, If the Diſtance of the Concave be 3 Inches, and that of the Object $\frac{1}{2}$ an Inch, and the Diameter of the Pupil $\frac{3}{20}$ of an Inch; the Analogy will be, As $3 : \frac{1}{2} :: \frac{3}{20} : \frac{3}{120}$ = the Diameter a b, of the viſible Area of the Object. Now, as in this Caſe, this Diameter is 6 times leſs than that of the Pupil, the whole circular viſible Area, will be but one 36th Part ſo big as the Pupil of the Eye. If the Power of magnifying were alſo 6 times, then the apparent or viſible Area in the Image, would be juſt equal to the Pupil of the Eye.

14. Since theſe Microſcopes are capable of exhibiting but ſo very ſmall a Part of the Object, they are never uſed; in as much as the Convex Glaſſes magnify as much, and take in a vaſtly larger Area, which gives a Pleaſure always in Proportion; for the more we can view, at once, the more we are delighted with the View. Indeed there is nothing to be ſaid for the Uſe of Concaves in Microſcopes, but the ſhortening of the Inſtrument: For ſuppoſing the Eye-Glaſſes of equal focal Lengths,

that

that is, EC = CH, it is plain from the Figure, and what has been taught, that the Eye in view with the Concave, is nearer to the Object-Lens df, than it is in viewing with the Convex, by 3 times the focal Length of the said Eye-Glasses.

CHAP. X.

Of Cata-dioptric or Reflecting Microscopes.

1. THESE Microscopes differ from the common Sort, principally in this; that whereas they consist *wholly of Lenses*, and perform all their Effects by *Refraction*, these are constructed with *one or more Speculums and Lenses conjointly*, and perform their Effects partly by *Reflection*, and partly by *Refraction*. Of these there are two Sorts, *viz.*

2. The first Sort is that represented in Fig. 3. Plate XXV. Where instead of the Lens def, there is placed a small Speculum *def*; the Object *acb* being placed above it, at a little greater Distance than the Focus *g*, has it's Image ACB formed by Reflection, as in the other Case it was by Refraction thro' the Lens df. Now if we suppose the focal Distance of the Object-Speculum *def*, and Lens

Lens def the same, the Effect of the Microscope will, in other Respects, be the same also.

3. For, (1.) The Distance of the Object *ab* above the Speculum, will be equal to the Distance of the Object ab below the Lens, in order that the Image may be formed at the same Distance Ce. (2.) *The Position of the Object will be inverted*; for all the Rays flowing from the Point *a*, will be reflected by the Speculum to the Point A, in the same Manner as if they came by Refraction through the Lens from the Point a; thus the Part *b* in the Object, will be reflected to the Focus B in the Image, which therefore is inverted. (3.) *The Power of magnifying will also be the same in both.* For since the Image A B, and the Object *a b*, are seen under equal Angles, from the Vertex e of the Speculum, the Triangles *aeb* and A*e*B will be similar; and therefore AB : *ab* : : C*e* : *ce*; but in the other it is, AB : ab : : Ce : ce. But the latter Ratio of these Analogies are the same in both, and consequently the first are so too.

4. This Microscope is not so easy to manage as the common Sort; for Vision, by Reflection, as it is much more perfect, so it is far more difficult than that by Refraction. Nature sells her best Commodities at very great Prices generally. Nor is this Microscope so useful, for any but very small or transparent Objects; for the Object being between

between the Speculum and Image, would, if it were large and opake, prevent a due Reflection.

5. To make this Microſcope anſwer well, the following Things muſt be obſerved. (1.) The focal Diſtance of the Speculum ought to be 1 ½ or 2 Inches. (2.) The Diameter or Aperture *df*, ½ Inch. (3.) The ſmall Speculum ſhould be placed in the Center of a larger one, inclined to the Bottom of an open Tube in an Angle of 45 Degrees, that ſo the Object may be illuminated by the Light reflected therefrom. (4.) In the Place where the Image A B is formed, ſhould be placed a Diaphragm of Wood, as O P, with a Hole QR, of ſuch Size as to exclude all the imperfect Margin of the Object, or Field of View, and exhibit only the moſt perfect and diſtinct Part thereof. (5.) The Eye-Glaſs D F may be 1 ½ Inch focal Diſtance, and ſhould be 10 or 12 Inches diſtant from the Speculum *df*. (6.) Laſtly, the Aperture or Hole in the Eye, Plate, or Piece at Y, ſhould be juſt as big in Diameter, as the principal Pencil C E, *viz.* equal to *mn*, which is eaſy to define; for ſince the Triangles *d*C*f* and *m*C*n* are ſimilar; it will be as C*e* : CE : : *df* : *mn*, which therefore is known.

6. The ſecond Sort of Reflecting Microſcopes, is that of Fig. 4. Plate XXV; whoſe Performance is by *two Reflections, and one Refraction*; and is the Invention of Dr *Smith*, Profeſſor

Profeſſor of Aſtronomy, &c. at *Cambridge:* the Theory and Conſtruction of which follow.

7. AD is a large *Concave Speculum*, and ad a ſmall *Convex one*; each perforated in the middle with the Holes BC, bc: Both theſe are Segments of the ſame Sphere, or ground on Tools of an equal Radius, *viz.* of 2 Inches, that ſo the focal Diſtance of each Speculum may be juſt one Inch.

8. Theſe two Specula are placed at the Diſtance of about 1 ½ Inch from each other, that ſo an Object OPQ being placed a little below the nether Speculum might be between the Focus F, and Center E, of the larger Speculum. Things thus conditioned, the Rays PA, PD, which flow from the Point P in the Object, on the Speculum AD, will be reflected towards a Focus *p*, where an Image *opq* would be formed, if the Rays were not intercepted by the Convex Speculum ad, and the Point *p* being nearer than it's Focus f, the Rays Aa, Dd, which tend or converge towards it, will be reflected to a Focus P, where the laſt Image *OPQ* will be formed, to be viewed through the Eye-Glaſs G, by the Eye at I.

9. Such being the Structure of this Microſcope, the Effects thereof may be explained as follows. Becauſe the Object and Image is ſeen from the Vertex V of the Concave Speculum, under the ſame or equal Angles OVQ and *o*V*q*; therefore (ſuppoſing the Speculum

ad

a d away) the Object O Q would be to it's Image *oq*, as V P to V *p*; and so the Object is not much magnified by the large Concave, which is not the Design of it, but to give the Rays a proper Degree of Convergence on the lesser or Convex Speculum a d, for the Purpose of magnifying.

10. For the Image *opq*, is now to be considered as a virtual Object to the real Image, formed in the Focus *P*, by Rays reflected from the Convex Speculum a d. And since by the Theory, the Object and Image appear under equal Angles at the Center of any Speculum, if from e, the Center of the Speculum a d, be drawn the Rays e *q* *Q*, and e o O; then shall *OP* *Q* be the last magnified Image of the Object O P Q. Which seen by the Eye I, through the Eye-Glass G, is to the Object O P Q, seen by the Eye at the Distance of six Inches, in the compound Ratio of

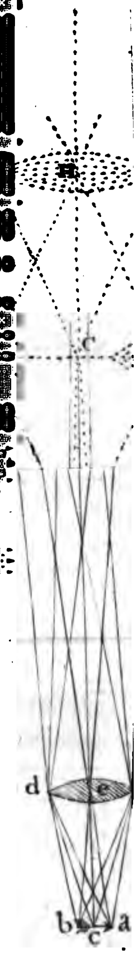
$$\frac{6 \times Vp \times eP}{VP \times ep \times PG}$$

11. For, (1.) The Object O P Q is seen by the naked Eye, at the Distance of 6 Inches, under an Angle, which is to the Angle O V Q, under which it is seen from the Vertex of the Speculum A D, as V P to 6; and so the first Ratio of magnifying is $\frac{6}{VP}$. (2.) The Image *opq* is to be considered now as an Object, and the Angle *o* V *q*, under which it is seen

from

from the Vertex of the Mirrour A D, is to the Angle *o* e *q*, under which it appears from the Center of the Mirrour a d, as e*p* to *p* V, and so the second Ratio of magnifying is $\frac{Vp}{ep}$. (3.) The Image *OPQ* appears under an Angle, at the Eye-Glass G, which is to the former Angle Q e *O*, as e*Q* to *P*G, and therefore the third Ratio of magnifying is $\frac{eP}{PG}$. And so the whole Power of magnifying is expressed by a Composition of these Ratios, *viz.* $\frac{6}{VP} \times \frac{Vp}{ep} \times \frac{eP}{PG}$.

12. To exemplify this, suppose the Distance of the Mirrours V*v* = 1.6, and that *v*P = 0.1143, so that VP = 1,7143; then will the first Ratio be $\frac{6}{1,7143}$. Then having the Distance VP of the Object, that of the Image V*p* will be found = 2,4; by Theor. 14, of the Theory. And therefore we have e*p* = 1.2; and so the second Ratio will be $\frac{Vp}{ep} = \frac{2,4}{1,2} = 2$. Lastly, having the Distance of the Object *vp* from the Convex Mirrour = 0,8; we shall find the Distance of the Image *v P* = 4, by Theor. 10, of *Catoptrics*; and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ıerefore the third Ratio is $\frac{eP}{PG} = \frac{4}{0,5}$ suppose the focal Distance *P* G, of the Glass G, to be ½ an Inch. So that the e Power of magnifying will be $\frac{6}{1,7143} \times \frac{4}{0,5} = 56$; that is, the Object will magnified 56 times by this Microscope.

13. But the Author of this Microscope ɔposes the Object to be at 8 Inches from e naked Eye, for distinct Vision; and also at by such a Combination of Speculums, ıe Confusion arising from an Aberration of ays is so far avoided, that an Eye-Glass G may be used, whose focal Distance *P* G is not greater than 0,18, or 0,15 of an Inch, and that then the Microscope will magnify 300 times. They that will, may see a great deal concerning the Structure and Dimension of Parts, with all the necessary Cautions for Workmen in fitting up such a Microscope, in the Author's Remarks on his Treatise of Optics, from Page 87 to 97.

14. This Microscope being so near allied to the Reflecting Telescope in it's Nature, that what might be here further said with Respect to the Aberration of Rays, &c. will be referred to those Chapters, wherein the Theory of these Telescopes is more perfectly considered.

CHAP.

CHAP. XI.

Of Telescopes in general; and of the common Dioptric Telescope in particular.

1. THE *Telescope* is the next principal and most useful optical Instrument. The Word is of *Greek* Derivation, and signifies, *The Perfection of the Sight or View.* For as our Sight, at best, extends not far with Distinctness, so most Bodies situated on the Earth, and all those in the Heavens, are so remote from the Eye, that unless it were assisted with a proper Instrument, it would be uncapable of a nice, distinct, and compleat View, and so we could never be able to form any just, regular, or adequate Ideas of those Objects, or their Parts, Forms, Colours, Magnitudes, *&c.*

2. But, by means of the Telescope, remote Objects are made to seem near, small apparent Magnitudes are enlarged, confused Objects are rendered distinct, and the invisible and obscure Parts of very distant things, are brought out to Sight, and rendered clear to the View; which therefore it greatly *perfects*, and merits the Name it bears with the greatest Propriety.

3. Of

3. Of Telescopes there are two Kinds, according to the two different Sorts of Vision, *viz.* by *Refraction* through Lenses, and *Reflection* from Mirrours. The Telescope therefore which is constructed with Lenses, and performs it's Effects wholly by refracted Light, is called a *Dioptric Telescope*; and is that in common Use. But the other Kind of Telescope performs it's Effects partly by *Reflection*, and partly by *Refraction*, and therefore is composed of *Mirrours* and *Lenses* jointly; on which Account it is called a *Cata-dioptric Telescope*, or, most commonly, a *Reflecting Telescope*.

4. The essential Parts of a *Dioptric Telescope* are two Lenses, *AB* and EY; (See Fig. 1. Plate XXVI.) of which *AB* is called the Object-Glass, and EY the Eye-Glass, for the Reasons those were, in the Microscope, so called. These two Glasses are so combined in a Tube, that the Focus of each is coincident in the same Point F between them.

5. This being understood, let OB be a vastly distant Object; the Rays then which come from it to the Object-Glass will be nearly parallel. Suppose two of these Rays be O*A*, and B*B*, these Rays in passing through the Glass *AB*, will be made to converge to it's focal Point F, where they intersect each other, and pass on to the Eye-Glass EY; but F being also the Focus of the Glass EY, these

in passing it will be again made parallel, and so entering the Eye, *will produce distinct Vision of the distant Object* O B, according to the *Theory*.

6. The Eye sees it also *magnified*; for O B being vastly distant, the Length of the Telescope is inconsiderable in Regard of it; and therefore supposing the Eye viewed from the Center of the Object-Glass C, it would see it under the Angle O C B; let O C and B C be produced to the Focus of the Glass, they will there limit the Image I M, of the Object formed in the said Focus. From the Extremities of the Image I M, let the two parallel Rays proceed to the ocular Lens E Y, these will be converged in it's Focus D, and the Eye will there see the Image under the Angle E D Y; all which is evident from the Principles before laid down.

7. Therefore the apparent Magnitude of the Object, seen by the naked Eye, is to that which is viewed in the Telescope, as the Magnitude of the Angle O C B, or I C M, is to that of the Angle E D Y, or I G M: (for since G D = G F, and I M = E Y, the Angle E D Y = I G M) But the Angle I G M is to the Angle I C M, as C F to F G. (by Art. 9. Chap. II. of this Part.) Therefore the Magnitude of the Object by the naked Eye, is to that by the Telescope, as C F to F G; that is, *as the focal Length of the Object-Glass, to the focal Length of the Eye-Glass.*

8. The

8. The Object will also appear inverted by this Telescope, in the Focus I M, the Reason of which has been explained in the Construction of the Microscope with a single Eye-Glass, and is evident from the Scheme itself. And, in short, they who understand the Doctrine or Theory of the Microscope, must needs know that of the Telescope, because the Analogy of Construction in both is so very much the same.

9. As to the Power of magnifying by these Telescopes, it appears from the foregoing Proportion, (Art. 7.) to be capable of vast Augmentation; because the focal Length of the Object-Glass C F may be vastly increased, in Regard of the focal Length of the Eye-Glass F G, in the lengthening of the Instrument. Thus, if C F = 36 Inches, and F G = 1 Inch, this Instrument will magnify the Diameter of Objects 36 times, their Surfaces 1296 times, and their Solidity 46656 times.

10. If now you would enlarge the Telescope, and chuse an Object-Glass 10 Feet Focus, or 120 Inches, the focal Length of an Eye-Glass of about 2 Inches, will be sufficient; and then will the Diameter of Objects be magnified 60 times, their Surfaces 3600 times, &c. which is vastly more than the former, and yet the Object nearly as distinct.

11. Again, if you would use an Object-Glass of 40 Feet focal Length, you fit thereto an ocular Lens of 3 ½ Inches; and then

will ſuch a Tube magnify the Diameters of Bodies (with the ſame Diſtinctneſs) near 140 times. If, laſtly, the Length of the Focus in the Object-Glaſs be 100 Feet, or 1200 Inches, then an Eye-Glaſs of 5 ½ or 6 Inches Focus will be ſufficient; and ſuch an one will magnify the Diameter of Objects 200 times; the Surfaces 40000 times; and the Solidity 8000000 times; which, though it is ſo prodigious an Effect, has been exceeded by Teleſcopes of this Sort made by ſome Artiſts; particularly that of Mr *Huygens*, which was 120 Feet long.

12. But then theſe very long Teleſcopes magnifying ſo much, are not fit for viewing Objects on the Earth, becauſe they have not Light enough when thus magnified, to render their Images viſible. For ſuppoſe of two Teleſcopes, one magnifies 10 times as much as the other, then in the focal Space or Area, there will be 10 times leſs in Diameter (and conſequently 100 times leſs in Surface) of the Object repreſented by the greater Magnifier, than by the leſſer; but the Light on the Surface of the Object being the ſame, it is plain the Quantity of Light in the *focal Area*, (of the ſame Dimenſions in each Tube) will be 100 times leſs in the great Teleſcope, than in the ſmall one; and therefore a Teleſcope above 6 or 8 Foot long, is not fit for terreſtrial Uſes.

13. What has been ſaid about the Illumination of the Image in the Teleſcope, ſuppoſes the Aperture of the Object-Lens to be the ſame in both the Tubes; but when the Tube is to be enlarged, it will bear a greater Aperture alſo, and will therefore admit of more Light; thus if a Tube of 4 Feet may have an Aperture of 1 Inch, a Tube of 30 Feet will admit of an Aperture of 3 Inches. But the Light entering by theſe Apertures, will be as the Squares of their Diameters; that is, as 1 to 9; or 9 times more Light will enter the Aperture of the greater Lens, than that of the leſſer; but then, ſince the Image of Objects, by the greater Lens, is to that formed by the leſſer, as 30 to 4 in *Diameter*; they will be in Surface, as 900 to 16, or as $55\frac{1}{4}$ to 1; and therefore the Light on the ſame Area of the Image, will be 55 times leſs in the greater Tube, than in the leſſer one; therefore the Light in theſe two Teleſcopes, will be as 9 to 55 nearly; that is, above 6 times greater upon the Image in the leſſer, than in the larger one.

14. Since the *Power of magnifying*, is the Proportion of the focal Length of the Object and Eye-Glaſſes, and this Proportion being to be varied in any Degree, in any Length of a Teleſcope, it may ſeem ſtrange that a ſhort Teleſcope will not anſwer that Purpoſe, as well as a long one. But they who thus think, muſt conſider, that if the Power of magnifying be

 augmented,

augmented, the Length of the Telescope being the same, it is necessary to diminish the focal Length of the Eye-Glass in the same Proportion; but this can't be done by reason of the great Confusion of Colouring and Distortion of the Image, on Account of the too great Convexity of the ocular Lens in such a Case.

15. For suppose in a Telescope of 3 Feet, the Power of magnifying be as $\frac{1}{36}$, (See Art. 9.) if you would augment this Degree of magnifying 5 times, (*viz.* makes it as $\frac{1}{180}$) then must you have an Eye-Glass not above , of an Inch focal Length, and this would be so very small as to admit of an Aperture not bigger than the Pupil of the Eye, nor would the Aperture of the Object-Glass admit of Light enough to illuminate the Object. On both which Accounts, the Design and Use of a Convex ocular Lens would be frustrated.

16. But suppose you require it to magnify but *twice* as much as before, *viz.* that the Power be as 7 ½; even this can't be done conveniently for the same Length of the Tube. For, (1.) The Lens being but ½ an Inch focal Length, will not admit an Aperture of above $\frac{2}{10}$ of an Inch conveniently, to view the Object distinctly, which is still too small. (2.) The Aperture of the Object-Glass will then be too small, to admit a requisite Degree of Light. (3.) This Aperture of the Object-Lens can't be enlarged, because there would be

be then too much Light for the Power magnifying, which would produce Confusedness in the Image. (4.) Neither must we attempt to enlarge the Aperture of the ocular Lens; for then the Rays would fall too far from the Axis of the Lens, to be sent parallel to the Eye, and therefore would prevent *distinct Vision.*

17. On these, and some other Accounts, it is evident, that if we would increase the Power of magnifying, we must necessarily increase the Length of the Telescope, by using an Object-Lens of a greater focal Length; for it principally depends on that, the focal Length of the ocular Lens being very little augmented. As for Example, if the Power of magnifying in any Telescope be known, or made a Standard, that Power, in any other Telescope, may be augmented to any Degree you please, in the following Method.

18. The Eye-Glass must have the focal Distance such a Number of Inches, as is expressed by the proposed Degree of magnifying; and the same Number squared, and multiplied by the focal Distance of the Object-Lens of the lesser Telescope, will give the focal Length of that required.

19. Thus if the Telescope, in Art. 9, be the given one, or Standard, whose Power is $\frac{1}{36}$; and you would increase this Power *once and an half*, or make it $\frac{1}{54}$, then must the Eye-Glass of the enlarged Telescope be $1\frac{1}{2}$

 Inches;

Inches; the Square of which is 2 ¼, which multiplied by 3, is 6 ¾ Feet for the focal Length of the Object-Glass in the Telescope required. Again, if it be required the Telescope should magnify *twice* as much, then must the focal Length of the ocular Lens be 2 Inches; the Squares whereof, 4 multiplied by 3, gives 12 Feet for the Distance of the Focus in the Object-Lens. And thus a Telescope that shall magnify 3 times as much, shall have an Eye-Glass of 3 Inches focal Length, and an Object-Glass of 27 Feet. And universally in Telescopes of this Sort, *The focal Distances of the Object-Glasses, should be always nearly in Proportion to the Squares of the focal Distances of the Eye-Glasses.*

20. But (as I have before observed) when Telescopes of this Sort are made of a great Length, they are not fit for viewing terrestrial Objects, but are appropriated to celestial Observations: But for such Purposes the Glasses are not managable in Tubes, which are either too slight and apt to bend, or too bulky, heavy, and unwieldy, if made strong in Wood: And therefore the Object-Glass is generally fixed on the Top of an high Pole, *&c.* and an Eye-Glass so adjusted thereto, that their Axis exactly coincide, and thus they view the heavenly Bodies which pass along the Ecliptic. This Form is called an *Aërial Telescope.*

21. For

21. For Tubes in Teleſcopes, as they anſwer the Purpoſe of a *Dark Chamber* to repreſent the Images of Objects in, have here no Uſe in that Reſpect; for the Night being the Time for Aſtronomical Obſervations with the Teleſcope, it is generally dark enough of itſelf, without a Tube, and the celeſtial Objects being very bright, make very clear, diſtinct, and bright Images, let the Teleſcopes magnify as much as they will.

22. Tho' the brighter the Object, the leſs the Aperture of the Object-Glaſs ſhould be; thus in viewing the Sun, or Venus, we are obliged to uſe a ſmaller Aperture than for the Moon, Mars, Jupiter, or Saturn, and their Satellites. And tho' (as was ſhewn Art. 8.) in this Teleſcope with one Eye-Glaſs, the Object is inverted, yet this is not heeded by Aſtronomers, it is all one to their Purpoſes which Part of the Sun, Moon, or Planets, appears Eaſt or Weſt, North or South.

23. But in viewing terreſtrial Bodies, this Inverſion of the Object can't be admitted; it would be unnatural and diſpleaſing to ſet the Earth above, the Sky beneath; Houſes, Trees, *&c.* turned Topſy-turvy, *&c.* To remedy this Inconvenience therefore, there are added two other Eye-Glaſſes, L N and R S, (See Plate XXVI. Fig. 2.) which erect the Object, and ſo make all things appear in their natural Poſitions.

24. In

24. In order to effect this, the first of these two Glasses, *viz.* L N, is placed at twice it's focal Distance from the former Eye-Glass E Y, (for they are all three supposed to be of the same Sphere) and the other R S, at the same Distance from it. Now the Pencils of Rays being made to intersect each other, by the first Glass E Y, in it's Focus D, and at the same Time, the Rays of each Pencil made parallel; these Pencils will be rendered parallel by the second Glass L N, and their Rays made to converge in it's Focus *F*; where they now represent the Image of the Object erect, in the Points *I*, *F*, *M*. The Pencils proceeding parallel from hence to the third Eye-Glass R S, are by it collected together in it's Focus P, where their Rays again made parallel, render this erect Image distinctly visible? All which is evident from the Scheme, and the foregoing Theory.

25. Notwithstanding it is usual to use three Eye-Glasses, as before explained; yet two will erect the Object, and keep the same Magnitude of the Image. For suppose the middle Lens L N, taken quite away, if the first Lens E Y, be placed at D, which is double it's focal Distance from the Image I M; it will, at the same Distance D *F* on the other Side, form a secondary Image *I M*, equal to the primary Image I M, and also in a contrary Position, (by Theor. 17. Chap. II. of

of *Dioptrics*) that is, it will be just the same in Magnitude, Position, and Place, as it was by two Lenses.

CHAP. XII.

Of Galileo's *Telescope, or Prospective Glass.*

1. THIS is a Telescope composed of a Convex Object-Lens *AB*, and a Concave ocular Lens EY. (see Fig. 3.) It is the first Form we have of a Telescope, and was invented, and applied to Use, by that celebrated *Italian* Philosopher *Galileo*, who lived in the Beginning of the last Century.

2. The Theory of this Telescope, is entirely the same with that of a Microscope, with a Concave ocular Lens, in Chap. IX, already explained. For this Telescope represents Objects *distinct and erect*, and magnifies them in Proportion of the *focal Length of the Object-Glass, to the focal Length of the Eye-Glass*; and the Eye the *nearer it is applied to the Lens* EY, *the larger Area it takes in*; and still greater there *in Proportion to the Largeness of the Pupil of the Eye*; the Reason of all which has been there explained, and is evident

evident in comparing the Figure of this Telescope and that Microscope together.

3. Now though the *visible Area*, or *Field of View*, in this Telescope, depends on the Pupil of the Eye, yet it is of much greater Service here than in the Microscope; because in that the small invisible Parts of near Bodies were to be magnified in a very large Image, of which the Pupil can take in but a small unsatisfactory Part; whereas in the Telescope, the Parts of large distant Bodies are to be rendered distinct in a small Image, in the Focus of the Object-Lens, which Image the Pupil of the Eye can generally take entirely in, and thereby view the whole Object, or Picture of Objects, at once, which is sufficient Satisfaction.

4. I shall make this easy by an Example. Fig. 4. Let *AB* be the Object-Lens of this Telescope, 4 Feet focal Length, and E Y the ocular concave Lens. Let O B be an Object to be viewed, whose Height is 40 Feet, and Distance *A* O 3 Miles, or 15840 Feet. Now since it was shewn in the Theory, that the Object and it's Image are as their Distances from the Lens *AB*, say, As the Distance *A* O 15840, is to B O 40, so is the focal Distance of the Lens *AB* 4, to $\frac{160}{15840}$ of a Foot for the Length of the Image I M, which is about $\frac{1}{10}$ of an Inch, the Pupil of the Eye being larger than that, will take in the whole Image I M, and so be able to view the whole Object

Object O B, at once, with great Pleasure and Satisfaction.

5. If the focal Length of the Concave be 1 Inch, then will the Object be magnified in the Proportion of 48 to 1, that is, it will appear 48 times higher, and therefore 48 times nigher also than to the naked Eye; that is, a Person, by the Help of the Telescope, will be as well able to view the Object O B at 3 Miles Distance, as he would by the naked Eye, at the Distance of 330 Feet; which is but the 48th Part of 3 Miles.

6. On this Account they are of great Use to People for viewing of terrestrial Objects; but still of much greater Use for viewing of the heavenly Bodies. For an Object-Lens of 5 Feet Focus, will bear an Eye-Glass of but 1 Inch; and such a Telescope will magnify the Diameters of the Planets 60 times, and their Surfaces 3600 times, which is sufficiently enough to render the Spots in the Sun, the Horns of Venus, the Satellites of Jupiter, and Ring of Saturn visible, if well managed; the Truth of this I know very well by Experience.

7. Again, the *Field of View* being so small in these Tubes, is not so much to be insisted on here, where the Images of the Planets are vastly less than the *visible Area*, except of the Sun and Moon; Jupiter, and all his Satellites, are many Times to be seen at one View, in these Telescopes; though it must be acknowledged, that there is some Difficulty in finding

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CHAP.

CHAP. XIII.

Of the manifold Uſes of a Dioptric Teleſcope, viz. *in ſhewing the Spots of the Sun, Eclipſes,* &c. *in a dark Chamber; in Surveying and Leveling; in Meaſuring the Diſtances of Objects at one Station; and divers Aſtronomical Purpoſes.*

1. THE Uſes of the Dioptric Teleſcope are many, and of vaſt Importance to the Sciences and civil Uſes of Life, ſome of the principal of which I ſhall conciſely point out and deſcribe in this Chapter.

2. And, firſt, *to view the Spots in the Sun through this Teleſcope.* Fit the Teleſcope to a Focus for viewing the celeſtial Bodies, and then take a circular Piece of plain and clear Glaſs, and hold it a little above the Point of the ſteady Flame of a Candle, till it be uniformly blackened over with the Smoke; then applying this ſmoked Glaſs before the Eye-Glaſs next the Eye, and you will be able to view the ſplendid Face of the Sun without Offence to the Eyes, and with Pleaſure ſee the

the various Spots, &c. dispersed here and there upon it.

3. But the best Way of viewing the Spots in the Sun, is by fixing the End of a Telescope of about 4 or 5 Feet long, into a Scioptric Ball, fixed to the Hole PQ of a Window-Shut HK, in a darkened Chamber. (See Fig. 5.) In this Case one Eye-Glass EY is sufficient, whose Focus let be F, and let IM be the Image of the Sun, formed in the Tube by the Object-Glass *AB*, in the moveable Ball. Now it has been shewn, that in viewing Objects with the Eye through the Tubes, the Image IM must always be exactly in the Focus F of the Eye-Glass, to produce distinct Vision.

4. For then the Rays EO, YO, proceeding from any Point O in the Image, would have passed parallel after Refraction through the Eye-Glass EY. But now, for this Purpose, the Eye-Glass is to be drawn a little farther from the Image than is it's focal Distance, and then a Pencil of Rays EOY, proceeding from the Point O, will be made by the Eye-Glass to converge to a Focus R, according to Theor. 16, of Chap. I, of *Dioptrics*. In this Focus R therefore, the Image IOM will be formed anew, and in an *erect Position*, in the Image *I* R *M*.

5. The Diameter *IM* of this secondary Image, is to the Diameter of the former IM, as the Distance GR, to the Distance GO, by the

the Theory. Thus if G O be 1 $\frac{1}{4}$ of an Inch, G R will be 5 Inches, and so the Diameter of the Image *I M*, to the Diameter I M, as 5 to 1 $\frac{1}{4}$. If the Focus G F be 1 Inch, and it's Distance from the Image F O be $\frac{1}{10}$ of an Inch; then will G R be 11 Inches, *viz.* 10 times as large as G O, and therefore *I M* will be 10 times as large as I M, and consequently the Surface of the secondary Image will be 100 times as large as that in the Tube I M. And therefore a Sheet of Paper held in that Focus R, will receive the vast Image *I M* of the Sun's Face, and the Spots therein will be very large and distinct. I have sometimes exhibited the larger Spots this Way, near an Inch in Diameter. The Moon may likewise be shewn this Way; but her Light is too faint to make the dark Parts appear well. But an Eclipse of the Sun is this Way, the best of any, shewn to an Advantage. This is a most agreeable Sight.

5. Suppose then E L C and Y M C, two Rays proceeding from the extreme Limbs of the Sun's Disk, and passing through the Center C of the Object-Lens; they will define the Extremities of the Image I M, and falling on the Glass E Y, will be converged to the Point D, where crossing each other, they proceed to limit the Extremities of the Image *I M*. Now *I D M* is the Angle under which this Image is seen at the Distance D R, by the Eye, and is equal to the Angle E D Y: But

the Angle EDY is a little larger than it would be, if the Image IM were in the Focus F: (becauſe GC is then ſomewhat ſhorter, and conſequently GD ſomewhat longer) Therefore the Image *IM*, at the Diſtance DR, is ſeen ſomewhat larger than the Sun appears in the Teleſcope adapted for diſtinct Viſion.

6. In *Surveying*, the Uſe of the Teleſcope is admirable; the Buſineſs here is to take the angular Diſtances between diſtant Objects on a Plane truly horizontal; or elſe the angular Elevation, or Depreſſion of Objects, above or below the Plane of the Horizon; in order to obtain either of theſe Sorts of Angles to a requiſite Degree of Exactneſs, it is neceſſary the Surveyor ſhould have as clear and diſtinct a View, as poſſible, of the Objects, or *Station Staves*, which he fixes up for his Purpoſe, that he may more certainly determine the Point or Part of the Object viewed, which exactly correſponds to the Line or Edge of the movable Arm (or *Alidade*) on the Theodolite, which cuts off the Degrees and Minutes contained in the required Angle.

7. Now as theſe Objects are generally at too great a Diſtance, for the Surveyor to be able to diſcern ſo well with the naked Eye, he takes the Aſſiſtance of the Teleſcope, and thus obtains his Ends compleatly on two Accounts, *viz.* (1.) His having a more perfect Sight of the Object, by the magnifying Power of the Glaſs; and, (2.) His being able to determine

determine the aforeſaid Point in the Object by means of the Croſs-Hairs in the Focus of the Teleſcope.

8. That he is aſſiſted greatly, in the Sight of his Objects, by means of the Teleſcope, is evident; for ſuppoſe the Teleſcope were but 13 Inches long, it will admit of an Object-Lens of 12 Inches, and an ocular Lens of 1 Inch, and will therefore magnify the Diameter of Objects 12 times; that is, it will make the Objects appear 12 times nearer to the Surveyor than they are, and conſequently he can view them 12 times more diſtinctly than by the naked Eye.

9. Again, in the ſecond Place, by means of the *Croſs-Hairs* in the Focus of the Teleſcope, he is able to determine exactly that *Point* or *Line* in the Object, which is in the Plane that paſſes through the Center of the Theodolite, and the Degree and Minute in the Limb of the Circle, which is the Quantity of the Angle required. For theſe Croſs-Hairs, or Wire, being ſo placed in the Focus, as that one of them is parallel to the Horizon, and the other perpendicular thereto, and their Point of Interſection being exactly in the Axis of the Tube; it follows, that the Point in the Object, which anſwers the Point of the Interſection of the Wires, or the Line in the Object, which is covered by the vertical Wire in the Tube, is the Point or Line required; and which alone can give the Angle

with Certainty, and this is the very Grounds and Foundation of all *Surveying* and *Levelling.*

10. There is another Use to be made of the Dioptric Telescope, very singular in it's Nature; and that is, *to measure the Distance of an Object at one Station.* This is the great *Desideratum* of Geometry, which furnishes Methods enough to do it at *two or more* Stations, but none at *one.* Yet this, in Theory, is perfected by the Telescope, and in a good Degree to be ascertained in Practice.

11. The Theorem for this Purpose is obtained from Theor. 16, Chap. I, of *Dioptrics,* *viz.* $\frac{dr}{d-r} = y$. For hence we have $dr = dy - yr$, and then $dy - dr = yr$, whence we have this Analogy, $y - : r :: y : d$; that is, in Words, *As the Difference between the focal Distance of the Object, and Radius of the Object-Lens is to the said Radius; so is the focal Distance of the Object, to it's true Distance from the Object-Lens.* But the three first Terms of this Analogy are known, or may be known very easily; and therefore the fourth Term or Distance of the Object is given.

12. For the Radius of the Object-Glass is known by holding it in the Sun, and measuring the Distance of the Sun's Image therefrom, when brightest and most distinct, for that will be the Radius of the Lens, as was shewn

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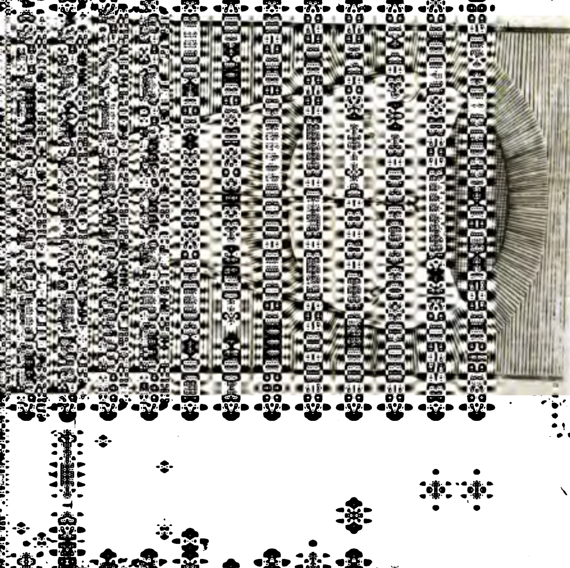


ſhewn in the Theory. Then for the focal Diſtance of the Object in the Tube, you are firſt to get a pretty good Sight of the Object, and then ſlide in the Drawer till the Object begins to appear obſcure, and mark that Place on the Tube preciſely; again, draw out the Tube till the Object begins again to be obſcured, and then make another mark on the Tube at the End of the Drawer, as before. Laſtly, take the middle Point between theſe two Marks, and that will be the Point where the Image of the Object will be formed moſt diſtinct, which you are to meaſure nicely from the Object-Lens, and you have the Data required. An Example will render this Matter eaſy.

13. Let *AB* be the Object-Lens, EY the ocular Lens, EC the Radius of the Lens *AB*, and C*F* the focal Diſtance of the Object OB, whoſe Diſtance QC, from the Glaſs *AB*, is to be meaſured. (See Fig. 1. Plate XXVII.) Now ſuppoſe CF = 48 Inches, or 4 Feet; and you find, by the Method above, that C*F* is 50 Inches; then F*F* is 2 Inches, and the Analogy is, As F*F* = 2, is to CF = 48; ſo is C*F* = 50, to CQ = 1200 Inches, or 100 Feet; and ſo far is the Object OB, from the Lens *AB*. Again, ſuppoſe you find C*F* = 49 Inches, then will F*F* = 1 Inch; and the Analogy is 1 : 48 : : 49 : 2352 = QC, or 196 Feet. If C*F* be found 48 ¼ Inches; then ¼ : 48 : : 48 ¼ : 9264 = QC, or 772

 Feet.

Feet. So that this Telescope will measure only small Distances.

14. But suppose *AB* a Lens, whose Radius, or solar Focus CF = 12 Feet, or 144 Inches; and you find by the above Method, (in Art. 12.) that C*F* is 146 Inches; then will F*F* = 2 Inches; and then by the Analogy, As 2 : 144 : : 146 : 21024 Feet = QC, or 1752 Feet, the Distance of the Object. If with this large Telescope you look at an Object OB, just 100 Feet distant from the Lens *AB*, it will give C*F* = 163 $\frac{6}{10}$ Inches, and therefore F*F* will be 19 $\frac{1}{2}$ Inches. On the contrary, suppose in viewing an Object OB, you find F*F* but $\frac{1}{10}$ of an Inch, this will give the Distance of the Object CQ = 17292 Feet, or 3 $\frac{1}{3}$ Miles nearly.

15. Since then the Difference F*F*, between the Radius and focal Distance of the Object, is so considerable as 2 Inches in a Tube of 4 Feet, and 18 in one of 12 Feet; it is easy to contrive Methods for shewing the Distance of nigh Objects by the former, and of very distant ones by the latter, by Inspection only. For it is but well adjusting or drawing a spiral Line round the Drawer, or Tube, thro' the *two Inch Space* in the small Telescope, and by Calculation graduate it for every 100 Feet, and the intermediate Inches, and you will no sooner view an Object, but at the same Time see it's Distance upon the Tube.

16. But

16. But in regard of the larger Object-Lens, it might be a better Way to fix it in a Scioptric Ball, in a darkened Chamber; and at the focal Distance of 12 Feet, on a Table or Stand MN, fix up an Instrument consisting of two Planes ABCD, EHKL, and a Screw OF, which is fixed into the Plane AC, and moves the Plane EK over the Space FG, which here represents the Difference between the Radius and focal Distance of the Object, *viz.* 18 Inches. If now on the Plane AC, be described Concentric Circles, or (which would be better) a Spiral from the Center to the Circumference, and graduated by a Calculation made for every 100 Feet, beginning from the Center O, for the Measures next to G, and proceeding towards the Circumference for the Parts towards F, which, because they will run very near together, will have the larger Space to be defined in.

17. These Things being done, it will be easy by the Screw to move the Plane EK backwards and forwards, on the Space GF, till the Image *F* appears most plain and distinct, and then the Index OP (fixed to the Screw) will shew the Distance of the Object on the graduated Spiral. The larger the Plane AC is, the more exactly will the Distance of remote Objects be shewn thereon; if it be a Foot square, it may do pretty well, but not less. Note, the best Way to get the true Place of the Plane EK, will be that in Art. 12.

Having juſt hinted theſe things, I ſhall leave it to the Invention of the Artiſt to vary Circumſtances.

18. In the Buſineſs of *Aſtronomy*, ſcarce any thing is done but by the Aid of the Teleſcope; for which Purpoſes it is applied in various Shapes, and compounded with ſuch a Number of Inſtruments, that to give an Account of them all, would fill a Volume; I ſhall therefore content myſelf with giving of one Inſtrument, which is of a general Nature, and by which the Conſtruction of others will be eaſily underſtood. See Fig. 3.

19. This Inſtrument conſiſts of a Teleſcope A B, whoſe Object-Glaſs is B, and Eye-Glaſs A, in whoſe Focus is a Ring of Croſs-Hairs C; this Tube is fixed to the Side of a Quadrant E F G, ſo that the Axis of the Tube, and Side of the Quadrant E F, are exactly as poſſible at Right Angles, or perpendicular to each other. The Quadrant is ſuſpended by, and movable about a Center-Pin D, paſſing thro' it's Center, and fixed into the ſquare Pilaſter D H, which ſupports it very ſteadily. On this Pilaſter at O, is fixed a ſmall Plate N O, called a *Nonius*, from the Name of the Inventor; whoſe Uſe I ſhall next ſhew.

20. Fig. 4. Let A B be a Portion of the graduated Limb of the Quadrant, and E D the *Nonius* Plate by it; whoſe Index is D C. The Artifice of this Contrivance conſiſts in this; the whole Length of the *Nonius* is divided

divided into 12 equal Parts, which are made equal to 13 of the equal Divisions on the Limb of the Quadrant A B; and therefore as $12 : 13 :: 1 : 1\frac{1}{12}$; that is, one of the equal Parts of the *Nonius* exceeds one of equal Parts (*viz.* a *Degree*) on the Limb, by a 12th Part of the latter, that is, by 5 Minutes.

21. Suppose now you have taken a Sight of an Object, and then observe the Index DC of the *Nonius*, at some Point between the 17th and 18th Degree, as in the Figure; the *Nonius* will tell you the Minutes above 17 Degrees thus: Look to see where any Division of the *Nonius* stands exactly against a Division on the Limb, and you will perceive in the present Case, that it is at the 6th Division of the *Nonius*; then, since 1 Division gives 5 Minutes, (Art. 20.) 6 Divisions will give 30 Minutes, which shews that the Index points to $17^{\circ} : 30''$ on the Limb. And hence you see the Reason also, why they are numbered from the Right-Hand to the Left in the *Nonius*.

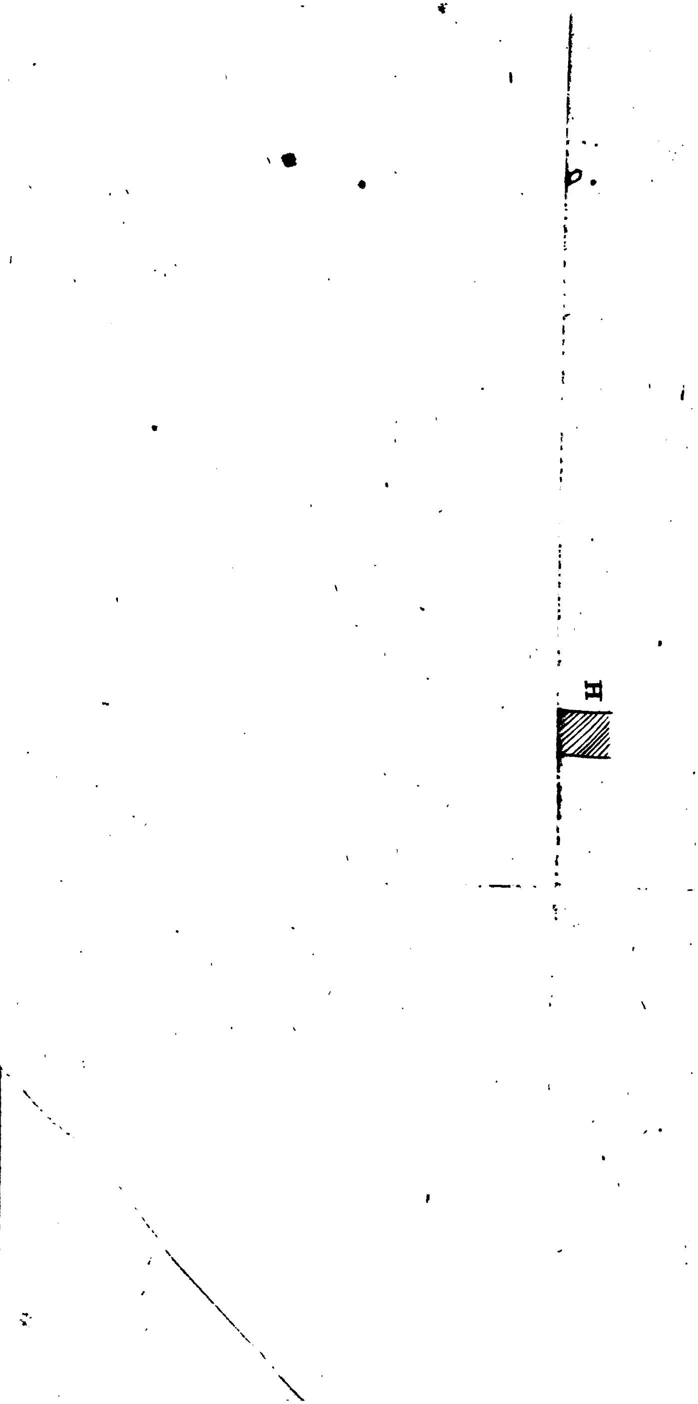
22. If the Quadrant were sufficiently large, not only the *Minutes* of a Degree, but the *Seconds* of a Minute might be shewn by this excellent Contrivance; as in that called the *Mural Arch*, in the Royal Observatory at *Greenwich*, which is an Iron Quadrant with a Brass Limb, most exactly graduated, and fixed into the Side of a Free-Stone-Wall, which is in the Plane of the Meridian; the Length

Length of the Radius is 8 Feet. On the Center, and by the Limb of this Quadrant, moves a large Telescope, which carries a *Nonius* upon the Edge of the Limb; which shews the Angles in Degrees, Minutes, and Seconds.

23. By the Telescope applied in such a Manner, divers Astronomical Problems and Positions are determined and defined to a great Degree of Nicety and Exactness. Thus suppose you would know the Altitude of a Star, at any Instance, direct the Telescope to it, and bring the Star on the Intersection of the Cross-Hairs, and the *Nonius* will shew the Degrees and Minutes of it's Altitude: Thus the Altitude of the Sun is to be found for many Purposes, as of *Dialing*, *&c.* And thus the Altitude of the *North Star*, and consequently the *Latitude of the Place* will be very truly known.

24. Again, the Axis of the Telescope being in the Plane of the Meridian, and the vertical Hair; you may observe the Difference of Time, in which any of the heavenly Bodies pass over the Hair or Meridian, and turning into Degrees you will thereby have the *Difference of the Right Ascension* of those Bodies. Note, 15 Degrees of the Equator, answers to one Hour of Time, and 15 Minutes of a Degree to 1 Minute of Time; and therefore 1 Degree to every 4 Minutes of Time.

25. But



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25. By this Means alſo, the Quantity of a Siderial Day is to be determined; for fixing the Teleſcope preciſely in the Plane of the Meridian, direct it to any fixed Star, ſo that it may come upon the Interſection of the Croſs-Hairs, and thus let it remain till the next Night, and then again obſerve when the Star comes to the *vertical Hair*; the Time between theſe two Moments is the Length of the Siderial Day; and is eaſy to be meaſured by a good Pendulum Clock.

26. When the vertical Hair in the Teleſcope is nicely in the Plane of the Meridian, it is eaſy then to find the *apparent Time*, and from thence the *true Time*. For by a good Pendulum Clock obſerve the Moments of Time, when the vertical Hair touches the two extreme Edges of the Sun's Diſk. The middle Moment between theſe, is the *apparent Time* of *Noon*, and if the Hand be then at XII, the Clock is true for the apparent Time; if not, it's Diſtance from it, on this Side or that, ſhew how much the Clock is too ſlow or too faſt; and having the *apparent Time*, the *true Time* is found by a common Equation Table.

27. There are great Numbers of other Uſes to be made of a Teleſcope thus mounted, furniſhed, and adjuſted to a larger Quadrant, eſpecially when a *Micrometer* is fitted in it, which the Reader may ſee in Treatiſes of Aſtronomy, &c. but cannot expect in this Compendium.

CHAP.

CHAP. XIV.

Of the Colours of the Sun's Light; of the different Refrangibility and Reflexibility thereof; and the Imperfection of Dioptric Telescopes arising from thence.

1. IN the Theory we considered a Pencil of the Sun's Rays reflected from Mirrours, or refracted thro' Lenses to one *single Point* in the Axis thereof; but that, strictly speaking, is not the Case in Nature; though nothing to the contrary was understood or taught in Optics, till Sir *Isaac Newton*'s Time. That great Man was the first, who discovered that a Beam of the Sun's Light, when reflected or refracted so as to be made converging, did yet not converge or tend all of them to one sole Point or Part of the Axis; but were so severed, and differently inclined by the Medium, that some Rays tended towards one Point, and some towards another.

2. He not only discovered, that the Sun's Light was differently *refrangible* and *reflexible*, but also at the same Time, that it was within certain Limitations, and in a certain Order and

and Proportion; and that Rays of each several Degree of Refrangibility, were of a different Colour from each other when separated; and accordingly that Bodies were tinged with Colours thereby.

3. So much of this new Doctrine, as is necessary to be known in Optics, or the Construction of Instruments, I shall here relate from his excellent Book on that Subject; and give the Experiments by which he made these Discoveries. The first whereof is as follows: Let A B C represent the transverse Section of a Prism, G F a Beam or Cylinder of the Sun's Light coming thro' the Hole H, in the Window-Shut of a darkened Chamber, and falling on the said Prism. (Fig. 1. Plate XXVIII.)

4. Now this Beam G F, will be refracted out of it's natural Course E T, in passing thro' the Prism, into some other D Q; nor will it now appear in a round cylindric Form D Q S E, as it would were it in every Part equally refrangible; but in a broad, dilated, diverging Form D X Y E. The Beam thus new modified, being received on a Sheet of Paper L M, at a proper Distance, will not now, as before, be white, but will paint a various coloured *Spectrum* or Image X Y, of an oblong Form. If the refracting Angle of the Prism A C B, be 64 Degrees, and the Distance of the Paper from the Prism about 18 Feet,

the

the Length of the Image X Y will be about 10 Inches, and the Breadth 2 Inches.

5. Now it is evident, that ſince ſome Part of the Beam D X, is refracted much farther out of it's natural Courſe E T, than ſome other Part of the Beam, as E Y, the Rays towards D X have a much greater Diſpoſition to be refracted, than thoſe towards E Y; and that this Diſpoſition does ariſe from the naturally different Qualities of thoſe Rays is manifeſt, ſince the refracting Angle, or Power of the Priſm, is the ſame in Regard of the ſuperior Part of the Beam, as the inferior. Thus the Inequality of Refractions, or the *different Refrangibility* of the Rays of Light, in equal Incidences, is plainly proved by this Experiment.

6. The Rays of Light are variouſly coloured, as they differ in Refrangibility; thus the moſt refrangible Rays D V, are *Violet*; the leaſt refrangible E R, are *Red*; the Order of Colours, thro' the whole Image, being as follows, *Violet*, *Indigo*, *Blue*, *Green*, *Yellow*, *Orange*, *Red*, as repreſented in the Figure. This is the Reaſon why Rays, falling near the Edges of a Lens, are differently refracted, and conſequently paint or tinge the Object with Colours; which makes it neceſſary to exclude ſuch Rays as much as poſſible.

7. For that a Lens, as A B, Fig. 2, has the ſame Property as the Priſm *a c b*, in refracting Rays, is evident from this Experiment,

ment, *viz.* Let O be an Object of a *Violet* Colour, the Ray DA proceeding thence to the Lens, and falling on the Surface A*a*B, at A, will be refracted from the perpendicular CD, to the Point E; but were the Object O red, it's Focus would be found at F, much farther from the Lens than E; and therefore the red Ray AF, will not be so much refracted from the perpendicular by the Lens, as the Violet coloured Ray AE.

8. But the best Experiment of this Sort, is to take a Slip of Pastboard, Card, *&c.* as ABEF, (Fig. 3.) and paint one half ABCD Red, the other half CF Violet or Indigo, and tying black Threads a-cross it, set it near the Flame of the Candle G; then take a Lens HI, and holding a Sheet of white Paper behind it, move it backwards and forward upon the Edge of a graduated Ruler, till you see the black Threads most distinctly in the Image; and you will find the Focus of the Violet Part *fc*, much nearer than that of the Red *ac*; which plainly shews, that Bodies of different Colours can never be shewn without some Degree of Confusion, by Refraction thro' Lenses.

9. The same Author also found that the Rays of the Sun's Light were *differently reflexible*, or that some Rays of a Beam of Light were reflected farther from the perpendicular to the reflecting Surface than others; and also that those Rays, which are most refrangible,

refrangible, are alſo moſt reflexible. All which he confirmed by the following Experiment.

10. In Plate XXIX. Fig. 1. F M is a Beam of Light propagated into the dark Chamber, through the Hole F, of the Window-Shut E G. A B C and B C D are two Priſms tied together in the Form of a Parallelopepid, their Sides B C and C B being contiguous, and A C and B D parallel. H I K is a third Priſm by which the ſaid Beam is refracted at O, to the white Paper at V G; for ſince the two Priſms A C B and B C D, are placed in ſuch a Manner, it is eaſy to conceive, that the Effects of the firſt Priſm are reverſed or deſtroyed by the ſecond, and ſo the Beam will paſs through both to the third Priſm, without any Alteration in the Nature of it's Light.

11. The Beam therefore is refracted at O, into the different coloured Rays O V, O I, O B, O G, *&c.* Now upon turning the Parallelopepid A C D B about it's Axis, in the Order of the Letters A, C, D, B, when the contiguous Planes B C and C B become ſufficiently oblique to the incident Beam F M, the ſaid Beam will begin to be reflected at the Point M; and there will vaniſh totally out of the refracted Light V G, firſt of all the moſt refracted Rays O V, or Violet, (the other remaining as before) then the Rays I O, B O, G O, *&c.* ſucceſſively to the leaſt refracted or red. Hence it is very manifeſt, that the

Plane

Plate, XXVIII.

Page, 256.

Plane BC, did first of all reflect the most refrangible Rays, and the rest in Order to the least refrangible ones, which were reflected last.

12. This he farther illustrated by adding a fourth Prism XYZ, to receive the reflected Beam MN, and to refract it upon the Paper at *vg*. For then the Light NO, which in the fourth Prism is more refracted, will become fuller and stronger when the Light OV vanishes at V; and so as all the rest vanishes at I, B, G, *&c.* the coloured Light at *i*, *b*, *g*, *&c.* becomes increased successively, that is, the Colours at *i*, *b*, *g*, are increased, while those at *v*, *i*, *b*, receive no farther Increase. And as the trajected Beam MO, is always of such a Colour, as ought to result from the Mixture of Colours at VG; so the reflected Beam is always of that Colour as results from the Colours at *vg*.

13. Those Rays which have a peculiar Degree of Refrangibility, and are tinged with a proper, simple Colour, are called *Homogeneal Light*, and all others *Heterogeneal Light*. Let AC be a Ray of heterogeneal Light, falling upon a Medium of Glass OPQ, in the Point C. This Ray will be separated by the Medium into all it's homogeneal Rays, of which CD is that of the greatest Refraction, and CE of the least. (See Fig. 2.) The Sines of these Refractions are GD and EF, and of the incident Ray AB. Sir *Isaac Newton*

has ſhewn that AB is to GD, as 50 to 78, and to FE, as 50 to 77. (See his *Optics*, page 73.) Make GI and FH equal to AB; then will ID and HE expreſs very nearly the greateſt and leaſt Degree of Refrangibility, and are to each other, as 28 to 27. So that ID exceeds HE by a $\frac{1}{28}$ of ID; and therefore we may take this Difference of the greateſt and leaſt Refraction to be a $27\frac{1}{2}$ Part of the mean Refraction.

14. Let AB be a Plano-convex Lens, (Fig. 3.) Q it's Center of Convexity; QG it's Axis produced; DA an heterogeneal Ray parallel to the Axis, which by the Lens at the Point A, is refracted into all it's homogeneal Rays, of which AL is that of the leaſt Refraction, AK that of the greateſt, and AO that of a mean Refraction. Suppoſe in like manner another Ray EB, refracted into BN, BM, and BO. Theſe Rays interſect the Axis in the Points F, O, G. And thoſe of greateſt and leaſt Refraction interſect each other in H and I. And therefore HI is the Diameter of the Circle, in which the Rays of every Degree of Refraction will be found.

15. Draw DAP parallel to the Axis QG, and HP perpendicular thereto. Then will PH, PO, PI, be the Sines of the *leaſt*, *mean*, and *greateſt Refractions*, or of the Angles HAP, OAP, and IAP, or of the Angles equal to them, AGC, AOC, AFC. But the Sines of the Angles AGC and AOC

are

are as the Sides AO and AG, or as CO and CG nearly (by Princip. II). Thus the Sines of the Angles AOC and AFC, are as CF and CO; consequently PH, PO, and PI, are as CF, CO, and CG, which therefore may represent the least, mean, and greatest Refractions.

16. Now since FG is a 28th Part of CG, (by Art. 13.) OG will be a 56th Part of CG: But since the Triangles HOG and ACG are similar, OG : CG :: OH : AC :: HI : AB :: 1 : 56; that is, the Diameter of the Circle HI, into which all the Rays which come parallel from any Point of an Object, are collected by a Plano-convex Object-Lens, is the 56th Part of the Diameter of the Aperture of the said Object-Lens.

17. What this Diameter HI is, in the Focus of reflected Rays, may be thus shewn. Let AB be a Concave Speculum, whose Center is Q; DA a Ray parallel to the Axis QY, and reflected most of all towards F, suppose O the Focus of Rays of a mean Reflection, and draw OI perpendicular to the Axis, which will be the Semidiameter of the circular Space into which Rays of every Sort will be reflected from the Point A.

18. Since F will always fall between O and C, the Point A may be taken at such a Distance from the Vertex C, that FA may be equal to FQ: In which Case, on F describe the Semicircle QAY, cutting the Axis in Y;

then shall OF be equal to ½ CY. For OF = QF — QO, and so 2 OF = 2 QF — 2 QO = QY — OC = CY; and therefore OF = ½ CY. But CY is the Excess of the Secant QY, above the Radius QA. Consequently when the Angle AQC is given, CY is known, and therefore it is half OF; and then since OF : FC : : OI : AC, the Ratio of IO to AC, the Semi-aperture of the Glass will be known.

19. For Example. Suppose the focal Length OC of the Speculum AB be 12 Inches, and the Angle AQC = 2° 3″; whence the Arch AC will be very near ½ an Inch. Then if the Radius QA or QC = 100000, the Secant QY will be 100095; therefore CY = $\frac{95}{1000000}$, the half whereof $\frac{47.5}{1000000}$ = FO. Since then FO is the $\frac{47.5}{1000000}$ Part of QC, it will be the $\frac{95}{100000}$ Part of OC; and therefore IO will be the same Part of AC, which is near 20 times less than a $\frac{1}{56}$ Part, as in the Convex Lens.

20. If each individual Point of an Object were to affect but single or individual Points on the *Retina*, the Picture there made would be absolutely *perfect*, and the Idea conveyed to the Mind, or *Vision*, would also be perfectly *distinct*; but since it has been shewn, that Rays coming from any single Point in an Object, and collected by a Lens, Mirrour, or the Eye, do not represent that Point in a Point, but circular Space of the Picture; and since

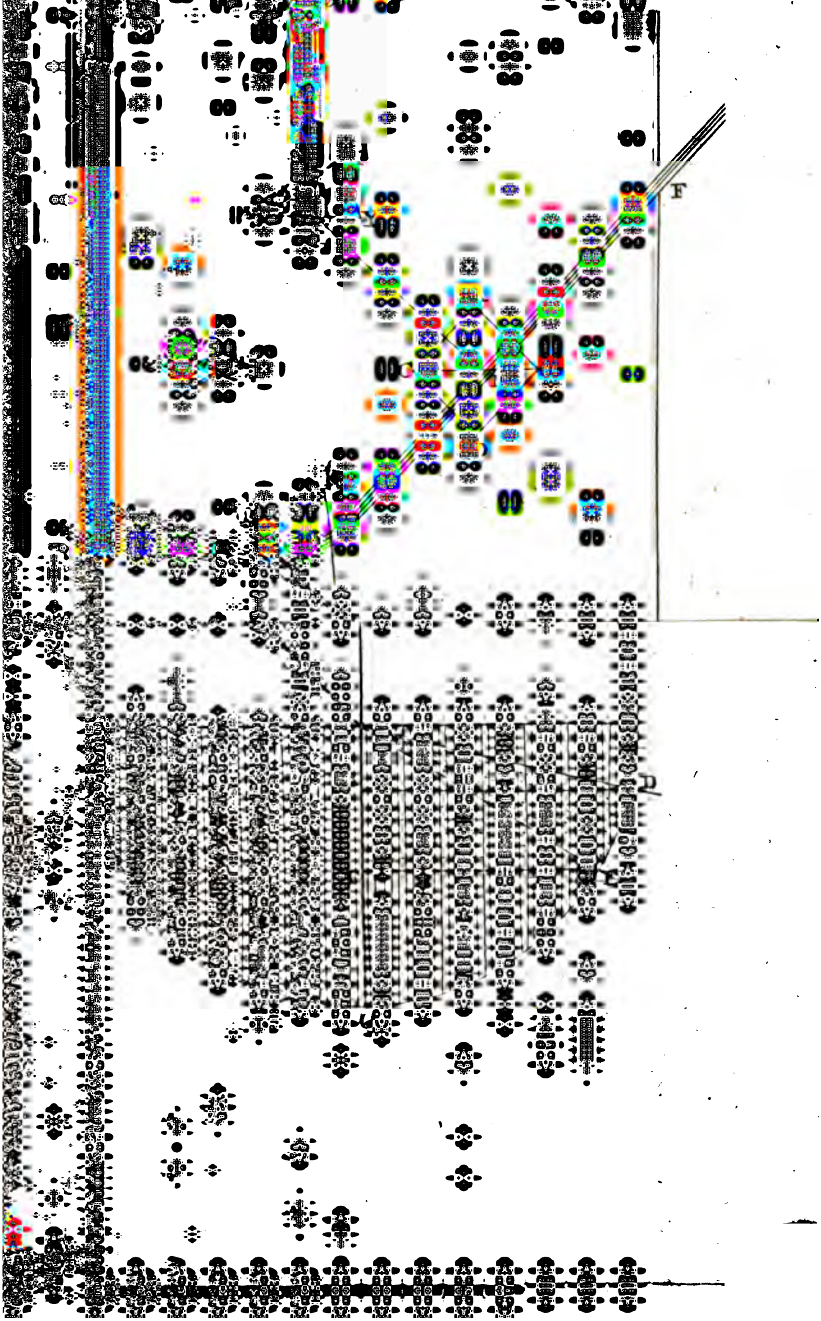


Fig. 260.

ſince every Point of the Picture may be the Center of ſuch a circular Space, and this circular Space will admit of a Mixture of as many others, as there are Points within the Diſtance of a Semidiameter; and, laſtly, ſince the very central Point of this Space will be affected or covered with all thoſe Circles, whoſe Centers fall within it's own Circumference: I ſay, from all theſe Conſiderations it is very manifeſt, the Picture muſt be extremely confuſed and indiſtinct, and conſequently the Viſion will be ſo too; and that in Proportion to the Area of the Circle, Confuſion is painted in the Picture on the *Retina* of the Eye.

21. Now ſince the Areas of theſe Circles, formed by reflected Rays, are incomparably leſs than thoſe formed by refracted Rays, it is evident, the Viſion by reflecting Inſtruments muſt far exceed that by refracting ones, and conſequently that reflecting Teleſcopes are vaſtly preferable to refracting ones in their Effects, as well as Conveniency of Form; to the Theory of which I now proceed.

CHAP. XV.

Of the Cata-dioptric or Reflecting Telescope; it's Theory, Construction, and Use.

1. THE imperfect Vision, by Dioptric Telescopes, of a great magnifying Power, unless very long, which then were very cumbersome and unmanageable, gave the Opticians Occasion to meditate and contrive some Method of shortening, and making this excellent Instrument of a more commodious Form, and of more general Use.

2. This, from the Nature of the Science, they were soon convinced, was to be effected by *Reflection*; and therefore several Persons, in several Parts, began to contrive and make *Reflecting Telescopes*, some one Way and some another. The first Hint and Figure of any thing in this Kind was by Dr *James Gregory*, a *Scotchman*, in his Book called *Optica promota*, pag. 93, 94.

3. The Form and Manner of the *Gregorian* Telescope was as follows. In Fig. 2. let A B E F be a Tube open at the End A F, towards the Object; at the other End he placed

placed a parabolic Concave Speculum B E, with an Hole C D perforated in it's Vertex; a little beyond the Focus *e* of this, he placed another ſmall Concave G, (on the Foot G H) of an Elliptic Form; at the End of the great Tube B E, he ſcrewed in a ſmall Tube C D K I, containing an ocular Lens of a conoid Form on one Side, and plain on the other.

4. Now ſuppoſe *a*, *b*, two parallel Rays, falling on the Speculum B E, in *c*, *d*, from thence they are reflected to, and collected in, it's Focus *e*, where the Image is formed and inverted; this Point *e* is alſo one Focus of the Elliptic Speculum G, and therefore the Rays coming thence, and falling on G, are by it reflected to the other Focus thereof, in the little Tube at *f*. Here again the Image is reformed, very large, and erect, which being alſo in the Focus of the Plano-conoid Lens I K, is ſeen by parallel Rays very clear and diſtinct.

5. Such was the Form and Theory of the original *Gregorian* Tube, but the Doctor, being deficient in Mechanics, never brought it to Perfection, but propoſed it for others to execute. He had alſo ſpherical Speculums of Metal, but could not uſe them for want of a good Poliſh. And thus nothing was done in the Teleſcope of this new Invention, till about the Year 1666, when Sir *Iſaac Newton* thought proper to alter a little the Conſtruc-

tion of the *Gregorian* Tube, and inſtead of placing the Eye-Glaſs at the End, he put it into the Side of the Tube, as in Fig. 3.

6. Where A B E F is the Tube, B E the Object ſperical Concave, which reflect the parallel Rays *a*, *b*, to a ſmall plain Speculum G, placed a little within the Focus of B E, and ſo as to throw the converging Rays directly to the Side of the Tube, upon a ſmall Eye-Glaſs I, placed in a Hole therein; and the Situation of the Speculum G was ſuch, that the Focus of Parallel Rays from B E, was made to fall exactly on the Focus of the Eye-Glaſs in *e*; and conſequently the Eye muſt then have diſtinct Viſion by Parallel Rays. This was the Structure of the *Newtonian* Tube.

7. A few Years after one Monſieur *Caſſegrain* publiſhed a Deſcription of a Reflecting Teleſcope, as his own Invention; but ſince it was entirely of the Form of that in Fig. 4, where the only Difference between it and the *Gregorian* Teleſcope is, that the ſmall Speculum G is in this a *Convex*, and in Dr *Gregory*'s a *Concave*. And therefore this of Monſieur *Caſſegrain*'s ſeems to be only that of Dr *Gregory*'s diſguiſed.

8. About this Time alſo Dr *Hook* contrived a Teleſcope of this Kind; the Form thereof was that of Fig. 5, wherein you ſee it differs nothing from the *Gregorian* Teleſcope, but the placing of the Eye-Glaſs I, in the Hole

of

of the great Concave B E. But whether Dr *Hook* inſiſted on this as his own Invention I cannot ſay, though he ſeems to do ſo in a Letter he wrote about it to ſome Lord.

9. To theſe ſeveral Forms, I ſhall add one that I contrived for my own Uſe, which is ſomewhat of the *Newtonian* Structure, but of a perpendicular Poſition, as repreſented in Fig. 6, A B E F is the Tube, in which there is an Opening or Aperture O P, in the upper Part; againſt this Hole, within the Tube, is placed a large plane Speculum G H, at half a Right Angle, with the Axis or Sides of the Tubes, with an Hole C D perforated thro' the middle thereof. The Parallel Rays *a b*, falling on the inclined Plane G H, are reflected perpendicularly and parallel on the great Concave B E, in the Bottom of the Tube; from thence they are reflected converging to a Focus *e*, through the Hole of the Plane C D, which being alſo the Focus of the Eye-Glaſs I K, the Eye will perceive the Object very much magnified and diſtinct.

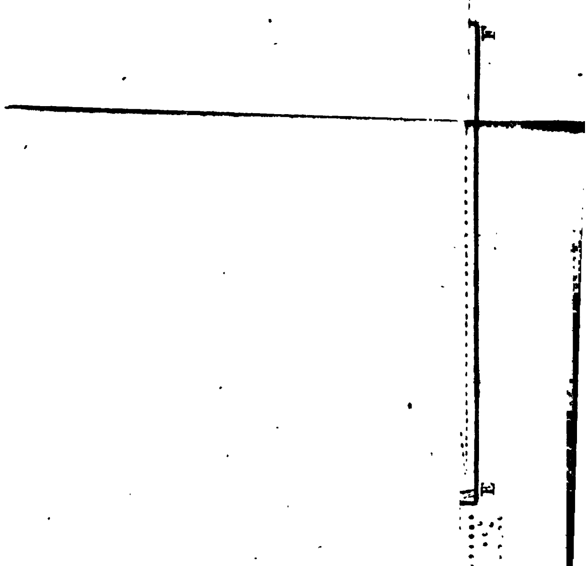
10. Now though the Forms and Structure of theſe ſeveral Tubes are different, yet their Effects all flow from one Principle, *viz. The ſuperior Perfection of reflex Viſion.* For they all perform their Effects either by one Reflection, as Fig. 3 and 6; or elſe by two, as Fig. 2, 4, and 5. And an Image formed by reflected Rays is ſo very perfect and diſtinct, that it will bear viewing with an Eye-Glaſs

of

of a much shorter focal Distance, than when it is painted by refracted Rays; for in that Case, if it be magnified too much, the Confusion of the Picture will be rendered sensible, and the distinct Vision of the Object entirely impeded.

11. I shall first consider the Effects of the *Gregorian* Telescope by two Reflections. In Plate XXXI. Fig. 1. A B E F is the Tube containing the great Concave B E, and little one G; let *a*, *b*, be two Parallel Rays coming from a vastly distant Object O B, and falling on the Concave in the Points *c*, *d*; from hence they will be reflected to the Focus *e*, where they form an Image I M inverted. Let *f* be the Focus of the small Concave G, and since the Image I M is a little farther distant from the Concave G than it's Center, the Rays come from it with such a Degree of Divergency, as to be reflected by the little Concave to another Focus *k*, and there crossing again form another Image *I M*, which will be now in an erect Position; and being supposed in the Focus also of the Eye-Glass K L, the Eye will see a Part of it very distinct, and greatly magnified. This has been in every Part so particularly demonstrated, in the Theory and Practice of *Catoptrics*, that I need not here again repeat it.

12. Now as to the *Power of magnifying*, let *h* Q and *g* Q be two Rays coming from the Extremities of a distant Object, and meeting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ing in the Center Q of the Speculum BE; these Rays will terminate the Image IM, because the Object and it's Image are both seen under the same Angle gQb, to an Eye at the Center or Vertex of a Speculum, by Art. 5. Chap. IV, of *Catoptrics*. Now this first Image IM may be considered as an Object, with Respect to the small Speculum G, whose Image *IM* is formed in the Focus *k*, and terminated by the Rays *I*G and *M*G, drawn through the Extremities of the Object IM. This secondary Image *IM*, is that which is viewed by the Eye-Glass KL, under the Angle *I*N*M*, supposing it could be all seen at once. And therefore the Object is to the Image in this Telescope, as the Angle gQb, to the Angle *I*N*M*.

13. But these Angles are in the compound Ratio of Q*e* to *e*G, and of *k*G to *k*N, for the Angle IQM is to IGM, as *e*G to *e*Q; and *I*G*M* (= IGM) is *I*N*M*, as *k*N to *k*G; and therefore $\frac{IGM}{IQM} \times \frac{INM}{IGM} \left(= \frac{INM}{IQM}\right) = \frac{Qe}{eG} \times \frac{kG}{kN}$. That is, the Angle *I*N*M*, is to the Angle IQM or gQb, as Q*e* × *k*G is to *e*G × *k*N. All which is demonstrated in Chap. II, of this Part.

14. Hence as Q*e*, the focal Distance of the Object Speculum BE, is increased, or *k*N, the

the focal Diſtance of the Eye-Glaſs is diminiſhed, the magnifying Power of the Teleſcope will be increaſed. And it is evident, that the ſame things are at the ſame Time ſhewn for *Caſſegrain's* Teleſcope, Fig. 2. where the ſmall Speculum G is a Convex, and the Image I M only *virtual*, or behind it, and the Image *I M inverted*, other things being the ſame.

15. If the focal Diſtance Q*e*, of the Object-Speculum B E, and G*f* of the ſmall one G be given, and you have determined the Point *k*, where the ſecond Image is to be formed, the Diſtance G*e*, of the firſt Image from the Speculum G, may then be found, and a Theorem raiſed for calculating the magnifying Power of the Inſtrument. For having the focal Diſtance Q*e*, and the Diſtance Q*k*, there is known *ke*; then put $ke = X$, $Ge = d$, and $Gf = \frac{1}{2}r$; and we ſhall have $X + d = Gk = \frac{dr}{2d - r}$; by Theor. 14. Chap. III, of *Catoptrics*.

16. Then — —	1	$X + d = \frac{dr}{2d - r}$
Therefore — — —	2	$\left\{ \begin{array}{l} 2dX + 2dd - Xr \\ - dr = dr. \end{array} \right.$
Add *dr*, — — —	3	$\left\{ \begin{array}{l} 2dX + 2dd - Xr \\ = 2dr. \end{array} \right.$
And by Tranſpoſ.—	4	$\left\{ \begin{array}{l} 2dX + 2dd - 2dr \\ = Xr. \end{array} \right.$

Divide

Divide by 2 — —	5	$\begin{cases} dX + dd - dr \\ = \frac{1}{2}Xr. \end{cases}$
Put — — — —	6	$X - r = 2Z.$
Then — — — —	7	$dd + 2Zd = \frac{1}{2}Xr.$
Compare the Squ.—	8	$\begin{cases} dd + 2Zd + ZZ \\ = \frac{1}{2}Xr + ZZ. \end{cases}$
Extract the Root —	9	$d + Z = \sqrt{\frac{1}{2}Xr + ZZ}.$
Therefore — — —	10	$\begin{cases} d = \sqrt{\frac{1}{2}Xr + ZZ} \\ - Z, \text{ Theorem.} \end{cases}$

17. Having thus found $d =$ Distance of the first Image from the small Mirrour G; the magnifying Power of the Tube may easily be computed as follows. Let the focal Distance of the great Mirrour BE be 6 Inches, that of the lesser 1 Inch; and let it be proposed to have the second Image, or Point k, at 1 Inch, before the Mirrour BE; then will $X = ke = 5$, $r = 2$, and $X - r = 2Z = 3$, and so $Z = 1.5$. Whence by the Theorem, (Art. 16.) we shall find $d = 1.192$. See the Operation.

$$\frac{1}{2}Xr = 5$$
$$ZZ = 2.25$$

$$\frac{1}{2}Xr + ZZ = 7.25 \quad (2.692 = \sqrt{\frac{1}{2}Xr + ZZ}.$$
$$4 \qquad 1.5 = Z.$$

$$46)\ 325 \qquad 1.192 = d = Ge.$$
$$276$$

$$529)\ .4900$$
$$4761$$

$$.13900$$

18. Having

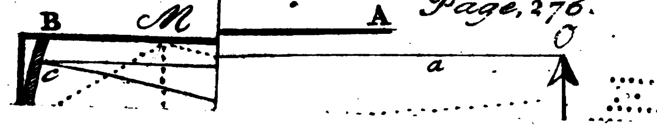
18. Having $Ge = 1.192$, and $Qe = 6$, we have $\frac{Qe}{Ge} = \frac{6}{1.192}$, which is the first Part of the magnifying Power. Then if the focal Distance Nk of the Eye-Glass be $1\frac{1}{4}$ of an Inch, we have $\frac{Gk}{Nk} = \frac{6.192}{1.25}$. And therefore $\frac{Qe}{Ge} \times \frac{Gk}{Nk} = \frac{6}{1.192} \times \frac{6.192}{1.25} = \frac{37.152}{1.49}$ $= 25$ very nearly. Therefore an Eye-Glass KL, whose focal Distance is $Nk = 1\frac{1}{4}$ of an Inch, will magnify 25 times, and if Nk be but $\frac{1}{2}$ Inch, it will magnify the Diameters of Objects above 50 times.

19. There is one Case when the Power of magnifying will be, as the *Square of the focal Distance of the great Mirrour, to the Rectangle under the focal Distances of the smaller Concave, and the Eye-Lens.* And that is, when $Qk = fe$; for then the said Power will be as $\frac{Qe \times Qe}{Gf \times Nk}$. Thus suppose $Qe = 6$ Inches, and $Gf = Nk = 1$ Inch, such a Telescope will magnify 36 times; for $\frac{Qe \times Qe}{Gf \times Nk} = \frac{6 \times 6}{1 \times 1}$ $= 36$.

20. If Qe, the focal Distance of the greater Concave BE, be 9 Inches, and that of the lesser

Plate XXXI.

Fig 1.



Page 276.

lesser G be 1½, and of the Eye-Glass 1 Inch; then if $Qk = fe$, we shall have $\frac{9 \times 9}{1.5 \times 1} = 54$, the Number of Times such a Telescope will magnify. If the focal Length of the Eye-Glass be but 1 Inch, the Power will be $\frac{9 \times 9}{1 \times 1} = 81$; for so many times will it magnify the Diameters of Objects.

21. If $Qe = 12$ Inches, $Gf = 2$, and $Nk = 1.5$; then if $Qk = fe$, we have $\frac{12 \times 12}{2 \times 1.5} = \frac{144}{3} = 48$, the Number of Times the Diameters of Objects will be magnified; but if $Gf = 1.5$, and $Nk = 1$ Inch, then the Power will be $\frac{12 \times 12}{1.5 \times 1} = 96$ times. If the Image *IM* be required at any other Distance before or behind the great Concave Q, the Power of magnifying may be found as in Art. 16 and 17.

22. If the Telescope be larger, *viz.* 1½ Foot, or 2, 3, 4, 5, 6, *&c.* Feet long, there are two Eye-Glasses applied, as in Fig. 3. Where the Image *IM* is projected to some Distance Qk, behind the great Mirrour BE, and by the Interposition of the first Eye-Glass WX, it is contracted into RS, by Refraction of the Rays W*M*, X*I* into WN, XO, at the

the ſecond Eye-Glaſs YZ, which again refracts them to the Point P, where the Eye views the Object in it's laſt Image, under the Angle NPQ. The Effects of a Combination of two Eye-Glaſſes have been already explained in Chap. VII, of the Microſcope, where the Power of magnifying, the Amplification and Quantity of the optic Angle NPO, are conſidered in Compariſon with the ſame in a ſingle Eye-Glaſs.

23. For if the Nature and Conſtruction of this Teleſcope be well obſerved, we ſhall find it of a *Teleſcope and Microſcope conjoined in one, or compounded in one Machine.* For, (1.) We are to conſider the Object Mirrour BE, forming the Image IM of diſtant Objects, in it's Focus e, and the ſmall Speculum G, placed at it's focal Diſtance from the Image, will give diſtinct Viſion thereof by Parallel Rays to any Eye at Q, which is the Function of an Eye-Glaſs, and magnifies the Object in Proportion of their focal Lengths Q*e* and *e*G; which is all that is performed in a common Teleſcope. But, (2.) If the Image IM be now looked upon as a Microſcopic Object, it is removed a little from the Focus *f* of the ſmall Speculum, ſo far as to cauſe a large Image thereof *IM*, to be formed near the Eye, which is the Effect of an Object-Glaſs in a Microſcope; this Image *IM* is diſtinctly viewed in the Focus of an Eye-Glaſs or two,

which

which is all that is performed in a common Microſcope.

24. Therefore ſince the Reflecting Teleſcope is nothing but a common Teleſcope and Microſcope combined together, and it's Effect compounded of the Effects of thoſe two Inſtruments; it is evident the Nature or Theory of this admirable Inſtrument will be eaſy to underſtand, when thoſe of the ſimple Teleſcope and Microſcope, before explained, are well conſidered, and underſtood.

25. I ſhall conclude this Account of the Reflecting Teleſcope with an Obſervation or two, neceſſary for thoſe that ſhall undertake to make them; and, firſt, with Regard to the Hole CD, in the great Speculum BE, it's Diameter ſhould be equal to that of the ſmaller Speculum G; for if it be leſs, no more parallel Rays (which make the principal Pencil *cyxd*) can be reflected, than if it were equal to *xy*, and ſo it can anſwer no Purpoſe; but may do Harm, in contracting the viſible Area within too narrow Limits.

26. Again, it muſt not be bigger than the Mirrour G, becauſe ſome parallel Rays will then be loſt, and thoſe of moſt Conſequence too, as being neareſt the Center; if any Part of the great Mirrour can be ſpared, it muſt be on the Extremity. Now the Breadth of the little Mirrour *xy*, is eaſy to be determined by the focal Diſtances Q*e* and *e*G, and Aperture of the great Mirrour *cd*; for

Qe : eG : : $dcxy$. Suppose QE = 6, eG = 1.2, and cd = 1.5; then $\frac{1.2 \times 1.5}{6} = 0.3$ of an Inch, and therefore xy may be a little more than $\frac{3}{10}$ of an Inch. If Qe = 9, eG = 1.5, and cd = 2; then $\frac{1.5 \times 2}{9} = 0.34$ = xy. If Qe = 12, eG = 2, and cd = 2; then $\frac{2 \times 2}{12} = 0.14 = xy$, which may be then made about ½ an Inch. For the Diameter of the Mirrour should be a little bigger than the Pencil of Rays, which it receives.

27. The next thing to be regarded is the small Hole at P, in the End of the Eye-Piece, which must be nicely adjusted to the Size of the Cylinder of Rays, emerging from the principal Pencil in the nearest Lens YZ. For if it be bigger, it will permit the foreign Light of the Sky, *&c.* to enter the Eye, which cannot be suffered; for the Eye must receive nothing but what comes from the Surface of the small Mirrour G. If the Hole be smaller than the Diameter of the Cylinder Prs, then some of the necessary Light of the principal Pencil will be excluded, and the Object rendered more obscure.

28. If the Eye-Glass YZ were alone, and l the Focus thereof, the Diameter of the Hole would

would be found by this Analogy; As $Gl : lr :: xy : rs$ = the Diameter of the Cylinder or Hole. Suppose $Gl = 8$, $lr = 1$, and $xy = 0.3$; then $\frac{1 \times 0.3}{8} = 0.04 = rs$, nearly; therefore the Hole must not exceed $\frac{1}{25}$ of an Inch in Diameter. Again, suppose $Gl = 16$, $lr = 1.4$, and $xy = 0.5$; then $\frac{1.4 \times 0.5}{16} = 0.044 = rs$; and therefore in a Telescope of this Length, the Hole at P must not exceed $\frac{11}{250}$ or $\frac{1}{25}$ of an Inch in Diameter. And this will be the Size of the said Hole, if the Image RS be formed in the middle between two Eye-Glasses WX, and YZ. For since $lr = ln$, no will be equal to rs; and it is $Gl : lm :: xy : no = rs$, as before.

29. It is usual to place a Plate TV, in the Focus l of the Eye-Glass YZ, with a Hole in the middle of such a Diameter RO, as will circumscribe the Image, and limit the Angle of Vision, in such a Manner, as to exhibit only so much of the visible Area, as appears distinct, and exclude the confused, coloured, and contorted Part thereof; and all the collateral and superfluous Rays, which enter by the Sides of the little Mirrour, or are reflected from it's Margin. The Diameter of this Hole RS is various, according to the various Dimensions of the Telescope, and

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C H A P.

CHAP. XVI.

Of Micrometers, and the Method of fitting them to Microſcopes and Teleſcopes.

1. THE Word *Micrometer* is *Greek*, and ſignifies an Inſtrument *to meaſure ſmall Objects*, as thoſe which are the Subjects of our View through the Microſcope or Teleſcope; and therefore any Contrivance in either of theſe, or any other Ways applied, by which we can meaſure ſuch ſmall Objects, whoſe Dimenſions cannot be taken by a common Rule, is called a Micrometer. Of theſe there are ſeveral Kinds, ſome of which have been applied to the Teleſcope, but none fixed in a Microſcope before that which I make and ſell under the Title of the *Pocket reflecting Microſcope with a Micrometer.*

2. Thoſe which I make and uſe in the Microſcope are of two Sorts, which I ſhall now deſcribe. The firſt conſiſteth of a circular Piece of Glaſs A B, on the middle Part of which are ſtrait parallel Lines *c d e f*, drawn with the fine Point of a Diamond, (in an Inſtrument made for that Purpoſe) of which

40 are contained in an Inch, or the Interval between two Lines is precisely the 40th Part of an Inch. See Fig. 1. Plate XXXII.

3. The Intervals of these Lines, tho' scarce discernable to the naked Eye, are very distinct, and appear very large through a Lens, whose focal Distance is but an Inch, larger than the *Tenth of an Inch* to the naked Eye. And therefore, if it be placed in the Focus of the Eye-Glass of the Microscope, the Image of the Object will be painted upon those Lines, and the Parts thereof may be compared with the Intervals, and their true Magnitude or Dimensions thereby very nearly known.

4. Let A B be the Focus of the Eye-Glass D F, (Fig. 2.) in this Focus I either place another Lens for amplifying the Optic Angle, or else the Glass Micrometer for measuring small Objects; let this Micrometer be represented by the Circle G H I K, and the parallel Lines by L M N O, all magnified by the Eye-Glass D F. Let the Edge of a Ruler *g h*, graduated into Inches and Tenths, be laid under the Object-Glass d f, and a b, two of those Tenths which will be represented by A C, C B, in the Focus. Again, let each Tenth a c, and c b, be divided into four equal Parts, these Divisions will be shewn by the dotted Lines between A C and C B in the Image, and will appear very large.

5. Now

5. Now you are to consider, that so many Times as the Distance Ce, is greater than CE, so many Times AB is bigger than ab; and consequently, so many Intervals of 40th Parts in the Glass Micrometer, will be equal to one such Interval in the Image AB. If Ce : CE :: 4 : 1, then four such Intervals in the Micrometer are equal to one in the Image; if Ce : CE :: 5 : 1, or 6 : 1; then will five or six of those in the Micrometer correspond to one of those in the Image; and therefore by an Observation of this Kind it will be easy to know, at any Time, what the Ratio of Ce to CE is, and consequently how much the Microscope magnifies an Object.

6. With this Micrometer it will be very easy to measure any minute Object very exactly; for the Image being cast upon it, it will be easy to judge what Proportion the Diameter of the Object, or Part to be measured, bears to that of an Interval between two Lines of the Micrometer, and from thence to determine it in Parts of an Inch. Thus, if I observe the Width of an Object to be just the same with that of an Interval, I know it is the 40th Part of an Inch; if half the Width, one 80th Part; if $\frac{1}{4}$ of the Width, it will be one 160th Part of an Inch.

7. Thus if an Object cover a 5th Part of an Interval, it will be but the 200th Part of an Inch in Diameter; if I find it is in Length equal to one Interval, or a 40th Part of an

Inch, the Superficies of that Object will be $\frac{1}{200} \times \frac{1}{40} = \frac{1}{8000}$, or one 8000th Part of a ſquare Inch. And thus the Length, Superficies, and Solidity of any minute Object, or Part thereof, may be known, and meaſured to a ſufficient Degree of Exactneſs by this Glaſs Micrometer, which is ſo eaſy to be underſtood, that I need ſay no more of it.

8. The ſecond Sort of Micrometer is more artificial, and, if well underſtood, would be of very great Uſe where the utmoſt Exactneſs is required. It conſiſts of a Screw and Nut; (ſee Fig. 3.) the Nut is fixed into the Side of the Microſcope at G, at the Focus of the Eye-Glaſs D F; on the external Part or Face of the Nut is a graduated Circle, repreſented in the Fig. N T, in the Center of which the Screw H O moves, the Hand H Q pointing to the Diviſions of the Circle.

9. The Screw H O, that I uſe in my Pocket Microſcope, has 50 Threads in an Inch preciſely; and the Circle N T is divided into 20 equal Parts, and numbered as in the Figure. Now ſince one Turn or Revolution of the Hand moves the End of the Screw O, over a Space equal to the $\frac{1}{50}$ of an Inch, the Motion of the Hand over one of the Diviſions of the Circle, will cauſe the Point O to move over the $\frac{1}{20}$ of $\frac{1}{50}$ of an Inch, that is, over the $\frac{1}{1000}$ Part of an Inch.

10. Now when the lower Eye-Glaſs G K is taken away, the Image A B, of any Object

a b,

a b, is projected into the Place of the Screw, or Focus of the Glass D F, and so the Screw H O will appear to lie or move upon the Surface of the Image; and as the Screw will appear very large, so it's Motion will be very visible over the least Part of the Image. Suppose now the Image A B, be 4 times larger than the Object a b; then, since the whole Revolution of the Screw moves the Point O over one 50th of an Inch in the Image, it will be but one 4th of one 50th in the Object a b; that is, it will measure a 200th Part of an Inch in the Object. Again, since the Motion of the Hand over one Division in the graduated Circle, measures the 1000th Part of an Inch in the Image, it will measure but a 4000th Part of an Inch in the Object.

11. And thus if the Image A B be 5, 6, 8, or 10 times greater than the Object a b, one Revolution of the Hand will accordingly measure the 250th, 300th, 400, or 500th Part of an Inch in the Object; and the Motion over one Division of the Circle will be over one 5000th, 6000, 8000th, or 10000th Part of an Inch in the Object. And thus knowing the Measure of one Revolution, or of one Division, the Measure for any Number of Revolutions, or Divisions in the Circle, is also known. For Instance;

12. Let A B be to a b, as 8 to 1; and suppose in measuring the Length or Breadth of an Object, your Index makes four Revolu-

tions,

tions, and stands at the 13th Division of the Circle. Then since in such a Microscope, one Revolution is the 400th Part of an Inch, four Revolutions will be the 100th Part of an Inch in an Object a b, and, again, since one Division is the 8000th Part, 13 Divisions will be $\frac{13}{8000} = 0.0016$ of an Inch; but $0.01 + 0.0016 = 0.0116$ of an Inch; that is, the Length or Breadth of such a Part of the Object was $\frac{116}{10000}$ Part of an Inch.

13. Or thus; 4 Revolutions make 80 Divisions on the Circle, which, with the odd 13, make 93 Divisions in all; then $\frac{93}{8000} = 0.0116$, the Decimal Part of an Inch, as before, for the Length of the Object, or Part measured. In Practice, the best Way is to determine the Proportion of Ce to CE, or of AB to ab; and then to form a Table, shewing, at Sight, the Measure in Parts of an Inch, answering to all the Revolutions and Divisions thereof, as you judge there will be Occasion for; which is easy to be done by those who have any Skill in such Matters.

14. Note, whenever you go to measure an Object, set the Index precisely at 10, or the Beginning of the Divisions; also let the Screw stand out from the Plate somewhat farther than you judge to be the Length to be measured; and, lastly, with one Hand turn the inner Tube of the Microscope, and with the other move the Object a b, so that the Part to be measured may be brought very nicely to

touch the ſharp Point O of the Micrometer; then turning the Index about, you will eaſily perceive when the ſaid Point is juſt on the other Extremity of the Part, then counting the Revolutions and Degrees, you turn them into Parts of an Inch, as before taught.

15. In Teleſcopes of the refracting Sort, Micrometers have had a long ſtanding. In thoſe of great Lengths, a very curious Machine of this Kind has been uſed to meaſure ſmall Angles; for in the Teleſcope, it is the Angles themſelves, not their Subtenſes, as in the Microſcope, that are meaſured by the Micrometer. And to know the Angle which any Body or Object ſubtends, being of little Uſe, except in the heavenly Bodies; the Micrometer in the Teleſcope is only applied to meaſure the Angles ſubtended by the Diameters of the Sun, Moon, and Planets, *&c.*

16. But ſince long Teleſcopes are going out of Uſe, and ſince the Micrometer abovementioned is a very compound and expenſive thing, and it's Deſcription would be very tedious, I chuſe to paſs it by, and give an Account of a more ſimple one, which contains in it the true Nature and Eſſentials of a Micrometer; conſiſting only of two Screws in a Piece of Wood, Ring of Braſs, *&c.* fixed to that Part of the Tube of the Teleſcope, which is the common Focus of the Object and Eye-Glaſs. See Plate XXXIII. Fig. 1.

17. Let AB and DE be the Object and Eye-Glass of a Telescope; and KG, HL, the two Screws in their common Focus. Now if the focal Length of the Object-Glass CF be known, and also the Number of Threads in the Screws which are equal to an Inch, then will it be easy to compute the Quantity of an Angle measured by one Revolution, or Part thereof, of the Screw. Thus suppose CF = 10 Feet, or 120 Inches, and that 40 Threads of the Screw make an Inch precisely; again, let GH be the greatest Opening of the Screws, and therefore GCH, the greatest Angle that can be measured in this Telescope.

18. Then if GH = 1 Inch, HF will be ½ an Inch, and therefore in the Right Angle Triangle FCH, there being given the Side CF and FH, we shall find the Angle FCH by this Analogy.

As the focal Length CF = 120 =	2.0791812
Is to the Side - - - FH = 0.5 =	9.6989700
So is the Radius - - 45° : 00′ =	10.0000000
To the Tangent of the Angle FCH = 14′ 20″ =	7.6197988

19. Now the Screw in passing over FH will make 20 Turns or Revolutions, and since 14′ : 20″ = 860′; therefore say, As 20^R : 860″ : : 1^R : 43″; that is, an Angle of

of 43″ is meaſured by one Revolution of the Screw. If the Index, or Hand of this Screw, moves over a Circle on the Face of the Micrometer, divided into 10 equal Parts, and theſe ſubdivided into 10 others each; then every tenth Diviſion will meaſure 4″ : 18‴; and every hundredth Part $25'''\frac{8}{10}$. And if the focal Diſtance C F be greater, the Angles meaſured will be ſmaller in Proportion.

20. If it happens that no Number of Threads are exactly contained in an Inch, or any known Meaſure, there are ſeveral Ways whereby the Angle meaſured by a Revolution, or Part thereof, may eaſily be computed. Thus ſuppoſe A B, C D, were two Lines drawn parallel to each other, on a Wall at a ſufficient Diſtance, and E F their Diſtance; then if the Teleſcope be directed to the Line E F, ſo that the Axis thereof K L, be perpendicular thereto; and the Diſtances E F, K L, preciſely meaſured, the Angle E K F will be found by Trigonometry, as above, and therefore it is equal to *f* K *e*. Then through the Teleſcope view the Image *f e*, and adjuſt the Ends of the Screws to the Extremities thereof very nicely, and count how many Revolutions and Parts of a Revolution are made before the Screws meet. Then ſay, As the Number of Revolutions is to the whole Angle *f* K *e*, ſo is one Revolution to the Angle it meaſures. Fig. 2.

21. Or

21. Or thus, suppose E F represents a Portion of the Equinoctial in the Heavens, and let E be a Star in or near it; then having directed the Telescope to the Star, and set the Ends of the Screws at the Distance of a certain Number of Revolutions, bring the Star to touch one of them, and observe, by a good Pendulum Clock, the Interval of Time which the Star takes up in passing to the other, then turn that Time into the *Minutes*, *Seconds*, and *Thirds*, of an Arch, and they will be the Measure of the Angle of that Aperture of the Screws.

22. Or, if the Star be at any considerable Distance from the Equinoctial, the Interval of Time observed in the Star's Transit over the Opening of the Screws, must be lessened in the Ratio of the following Analogy, *viz. As the Radius to the Sine of the Star's Distance from the Pole, so is the Interval of Time observed, to the Time required*; which turned into Minutes and Seconds, gives the Angle of the Aperture of the Screws.

23. In the same Manner you find the Angle for any Aperture of the Screws in a Reflecting Telescope; and consequently a Micrometer may be fitted in these, as well as in the other Sort. I shall conclude with an easy, practical Method of finding the Power of magnifying in a Reflecting Telescope, which is as follows.

24. In

24. In a Tube of 6 Inches Length, fix at one End a Piece of Glaſs, on which are drawn ſome parallel and equi-diſtant Lines, and at the other End a Piece of Paſtboard or Wood, with a very ſmall Hole in the middle; with this view ſome diſtant Object, whoſe apparent Diameter will juſt equal the Interval of two of thoſe parallel Lines. Then put the ſame Piece of Glaſs into the Focus of the Eye-Glaſs of the Teleſcope, and viewing the ſame Object again through the Teleſcope, obſerve very nicely, how many of thoſe magnified Intervals the magnified Diameter of the Object now equals; then may the Power of magnifying be eaſily known.

25. For ſuppoſe the focal Diſtance of the Eye-Glaſs be 1 Inch, the Intervals of the Lines will be magnified 6 times; again, ſuppoſe the Diameter of the Object through the Teleſcope appears equal to 8 of thoſe Intervals, it is evident the Teleſcope magnifies 6 times 8, or 48 times.

CHAP.

CHAP. XVII.

An Optical Inſtrument for meaſuring the Angle of Viſion, or eſtimating the apparent Magnitude of Bodies; alſo for viewing Perſpective Prints, Pictures, &c.

1. THE Inſtrument for theſe Purpoſes is that of Fig. 3. Plate XXXIII. conſiſting of a Tube ACBD, containing two Convex Lenſes AB and CD, of an equal focal Diſtance *ib* or *hn*, and placed at twice that Diſtance from each other; exactly in the midſt between them, and therefore in the Focus of each, is placed the Glaſs Micrometer (before deſcribed) EF; upon which the Image *gh*, of any diſtant Object GH is formed by the Object-Glaſs CD.

2. This Image being alſo in the Focus of the Eye-Glaſs AB, is ſeen diſtinctly by the Eye placed in the other Focus *m*; the Rays *gl* and *hk* being parallel, *gh* is equal to *kl*; and therefore the Angle $kml = hig = GiH$; and therefore the Image, formed in this Inſtrument, is ſeen under the ſame or an equal Angle, as the Object ſubtends to the naked Eye;

Eye; And conſequently the Object is neither magnified nor diminiſhed thereby.

3. Now ſuppoſe the Lines on the Micrometer EF, are 40 to an Inch; (as in that I uſe) and the focal Diſtance of the Glaſſes be one Inch; then the Angle which the Interval between two of thoſe Lines on the Micrometer ſubtends to the Eye, is of $1^{\circ}:26'$; ſo two will ſubtend an Angle of $2^{\circ}:52'$; and three an Angle of $4^{\circ}:18'$; and ſo on: the Angles and their Subtenſes being very nearly proportional, when ſo ſmall. *Hence the optic Angle, or Angle of Viſion, under which any Object at any Diſtance is ſeen by the Eye, is immediately known by this Inſtrument.*

4. *The apparent Magnitudes of all Objects are hereby eaſily eſtimated and compared*; for as theſe are always as the Images formed on the Micrometer, ſo the Lines or Spaces on that will readily ſhew the comparative Magnitude of them, and conſequently of the Objects themſelves in Appearance; for the Object whoſe Image meaſures two Lines is apparently twice as big as one that meaſures in it's Image but one Line, and but $\frac{2}{3}$ as big as another, whoſe Image meaſures three Lines.

5. By this Inſtrument *you alſo may meaſure very nearly the Diſtance of Objects*; thus, ſuppoſe you obſerve an Object, whoſe Image meaſures two Lines or Intervals, and you go back till it meaſures but one, or forward till it meaſures four, in either Caſe, the Space or

Length you went, is equal to the Diſtance of the Object from the Place where you firſt obſerved it.

6. Again, *the Diſtances of Objects being known, their Heights are meaſured pretty exactly by this Inſtrument.* For the Proportion is As the focal Diſtance of the Eye-Glaſs to the Length of the Image, ſo is the Diſtance of an Object to it's Height; that is, ig : gh :: iG : GH.

7. *This Inſtrument is very uſeful in drawing the Out-Lines of any Landſcape or Object you would delineate.* For by means of a circular Piece of Glaſs, with ſmall Squares drawn thereon, and put in the Place of the Micrometer EF, you may repreſent the Object or Picture in the ſame juſt and natural Proportion and Diſpoſition on any larger Squares, as you ſee in the Image on the ſmall ones; ſo will the Perſpective of the Piece be perfect.

8. *This ſmall Machine is extremely uſeful for viewing perſpective Prints, Views, and Pictures,* &c. For as when you view Nature, it gives you a beautiful Projection thereof on the perſpective Plane; ſo, on the contrary, when you view a Print, Picture, or any Piece in perſpective, it reſolves it all into Nature, and gives you the ſame Ideas of the Poſitions and Diſtances of Objects in the Print, as you would have by viewing the things themſelves in Nature. It gives to Pictures ſuch a natural and ſurprizing *Relievo,* as make the Life itſelf be there. If it be a Face, the Cheeks are

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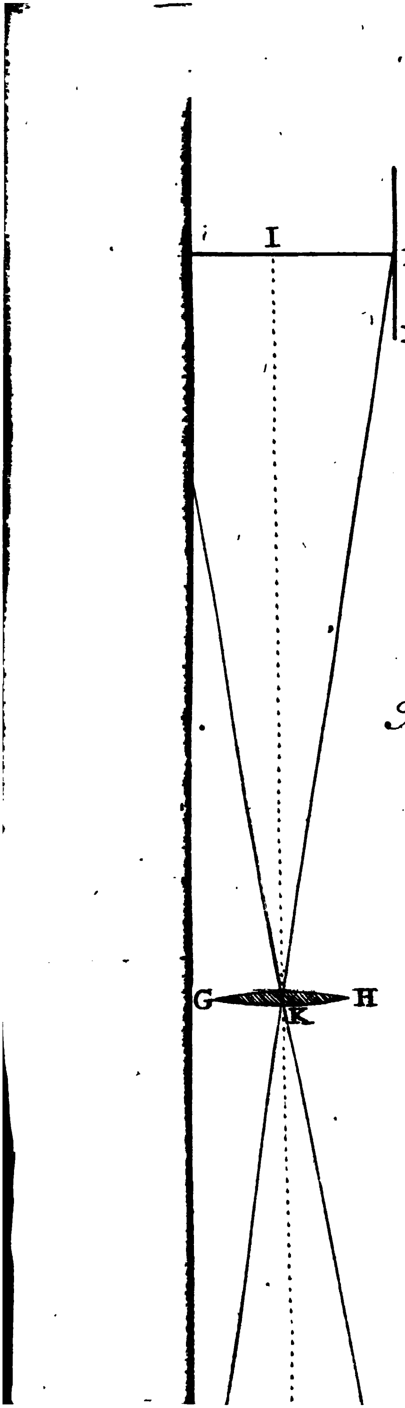

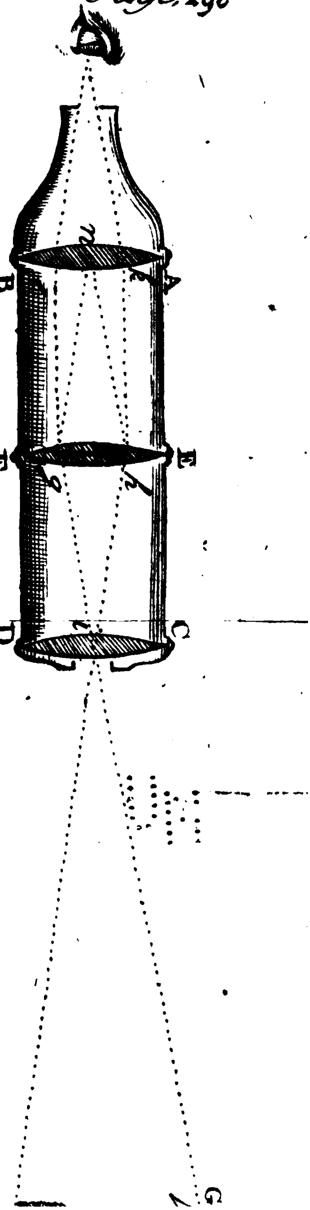

Fig. 3.

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C H A P.

CHAP. XVIII.

Of the Magic Lanthorn, and Opera Glaſs.

1. THE *Magic Lanthorn* is an optical Machine, ſaid to be the Invention of Mr —— *Kircher*, in order to magnify ſmall Objects in a dark Room; and has been ſince uſed rather to ſurprize and amuſe ignorant People, and for the Sake of Lucre, than for any other Purpoſe, and thence it had it's common Name: It has been alſo called *Lanterna Megalographica*, from it's Property of *magnifying ſmall Objects.*

2. The Conſtruction and Theory of this Inſtrument are very eaſy to be underſtood, and are as follows. In a darkened Room ABCD, is placed a dark Lanthorn EFGH, in the Side of which FH is fixed a round Tube KLNM, within which ſlides another Tube OP, ſo that the whole may be lengthened or ſhortened as Occaſion requires. In the inmoſt End of the firſt Tube is placed a large Plano-convex Lens KL, and towards the external Part of the other another double Convex Lens ST. In the firſt Tube there is a Contrivance for paſſing through it a ſmall Frame or Plane of Glaſs, on which are painted divers

divers ſmall Objects in tranſparent Colours, as at QR. See Plate XXXIV. Fig. 1.

3. In this Lanthorn is a Lamp or Candle I, which by means of the great Convex KL before, and a large Concave Speculum XY behind, does very ſtrongly illuminate the Object QR. If now the Lens ST be moved a little farther from the Object QR than is it's focal Diſtance, it will form a Repreſentation of the ſame at a great Diſtance on the oppoſite Wall, in a large Image as VW; which will be as much larger than the Object QR, as the Diſtance ZV is greater than ZR. All which is evident from the Theory of a Convex Lens.

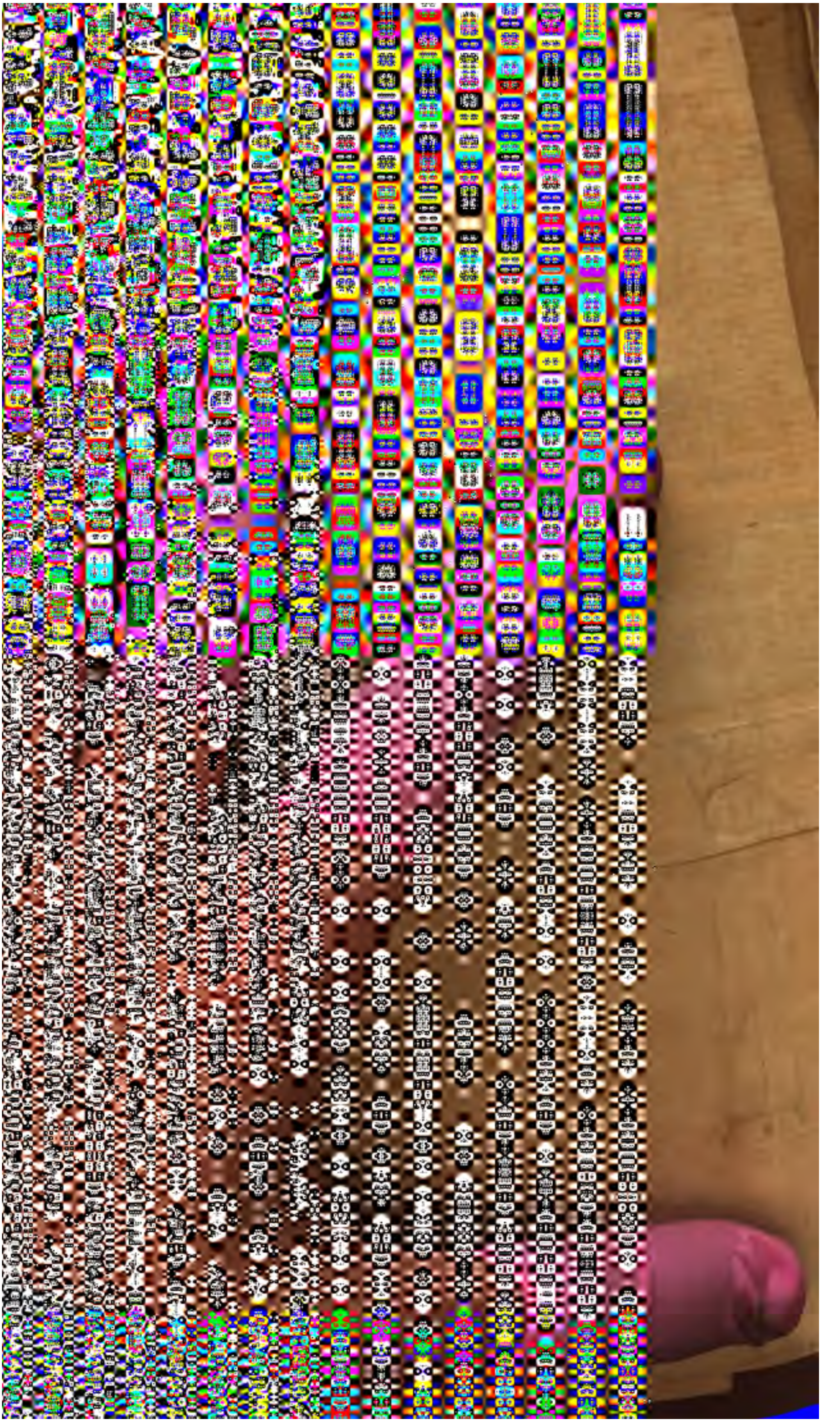
4. As the Tube OP is moved farther out of, or into the Tube MN, the Image VW will be ſmaller or larger, according to the Diſtance of the oppoſite Wall. And tho' thoſe Objects are generally ſome humerous, ridiculous, or frightful Figures, to divert or ſcare the Spectators; yet, I believe, this Machine might be applied to more uſeful Purpoſes, in magnifying the tranſparent Parts of Animal and Vegetable Subſtances, as Wings of Flies, Membranes, *&c.* eſpecially if enlightened by the Sun-Beams in a darkened Chamber, as I have many Times experimented.

5. But enough of this Inſtrument. That which I ſhall next deſcribe is called an *Opera Glaſs*, from it's being uſed by Gentlemen in Play-houſes, and ſometimes a *Diagonal Perſpective*, from it's Conſtruction, which is as follows.

follows. ABCD is the wooden Tube about 4 Inches long; EF, GH, are two Holes on each Side, exactly against the middle of IK a Plane Mirrour, which reflects the Rays falling upon it to the Convex LM, thro' which they are refracted to the Concave Eye-Glass NO, whence they emerge parallel to the Eye at the Hole *rs*, in the End of the Tube. Fig. 2.

6. Let PaQ be an Object to be viewed, from whence proceed the Rays P*c*, a*b*, and Q*d*, these Rays being reflected by the plane Mirrour IK, will shew the Object in the Direction *cp*, *ba*, *dq*, in the Image *pq*, equal to the Object PQ, and as far behind the Mirrour IK, as the Object is before it; the said Mirrour being placed so as to make an Angle of 45 Degrees, or half a Right Angle with the Sides of the Tube, all which is evident from the Theory of a Plain Mirrour, heretofore explained.

7. Therefore considering *pq* as an Object, the Case of this perspective Glass is reduced to that of a common refracting Telescope of *Galileo*'s Form, whose Object-Glass is LM, and Eye-Glass NO. And since in viewing Objects near at hand, no magnifying of them is necessary, the focal Distances of both the Glasses may be nearly equal; or if that of LM be three Inches, and that of NO be one Inch, the Distance between them will be but two Inches, and the Object will be magnified



nified three times, which is enough to anſwer the Deſign of this Glaſs.

8. If the Object be very near, as XY, it is viewed through a Hole *xy*, at the other End of the Tube AB, without an Eye-Glaſs, the upper Part of the Mirrour being poliſhed for that Purpoſe, as well as the under. This Tube unſcrews near the Object-Glaſs LM, for taking out and cleanſing the Glaſſes and Mirrour. I preſume enough is ſaid to explain the Nature and Conſtruction of this common and well known Inſtrument.

9. The peculiar Artifice of which is, to view a Perſon at a ſmall Diſtance in ſuch a Manner, that no one ſhall know who it is that is ken'd at, though they know your Deſign; and that on a double Account, *viz.* becauſe the Inſtrument points towards another Quarter than that in which the Perſon is; and becauſe there being a Hole on each Side, it is impoſſible to know on which Hand the Object is ſituated which you are viewing. The Poſition of the Object will be erect thro' a Concave Eye-Glaſs. And Objects ſituated high or low will eaſily be found, by turning the Inſtrument round one Way and the other about it's Axis.

F I N I S.