OBLIQUE BRIDGES

GEORGE WATSON BUCK

REVISED BY HIS SON

J. H. WATSON BUCK

A PRACTICAL AND THEORETICAL

ESSAY

ON

OBLIQUE BRIDGES

BY

GEORGE WATSON BUCK, M.INST.C.E.

THIRD EDITION

REVISED BY HIS SON

J. H. WATSON BUCK, M.INST.C.E.

AND WITH THE ADDITION OF

DESCRIPTION TO DIAGRAMS FOR FACILITATING THE CONSTRUCTION OF OBLIQUE BRIDGES

BY

W. H. BARLOW, M.INST.C.E.

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PREFACE

TO

THE FIRST EDITION.

The following Essay has been written in compliance with the wishes of several friends, who were aware that, during the execution of that part of the London and Birmingham Railway, extending from London to Tring, which was under the Author's immediate care as resident Engineer, he had availed himself of the opportunity of investigating this important subject. It is an interesting part of Civil Engineering, a knowledge of which is now extensively called for in the construction of the many great undertakings which have received the sanction of the Legislature. In Nicholson's work on Stone-Cutting, published in 1828, the method of constructing oblique arches with spiral courses is briefly explained, and to it we are indebted for the first principles of the art, but it does not enter sufficiently into detail. Having stated thus much, the Author will not hesitate to make use of any of the principles set forth in that work without further acknowledgment; at the same time it is proper to mention, that the matter which may be found common to both, does not extend beyond a small portion of the first and third chapters of this essay.

It has been attempted to enter into every minute particular, perhaps to an extent that some may think unnecessary; however, practical men will be able to appreciate the value of even the smallest degree of information relating to this subject, which is generally allowed to be a difficult one.

The period when oblique arches were first introduced is not known; but some information relative thereto is to be found in the first volume of the Transactions of the Institution of Civil Engineers, and is contained in an account of 'Details of the Construction of a Stone Bridge erected over the Dora Riparia, near Turin, by Chevalier Mosca, Engineer and Architect to the King of Sardinia, &c. &c. Drawn up and communicated by Mr. B. Albano, A. Inst. C.E.,' from which the following is an extract:—

'The nature of the river and the oblique direction of its bed, relative to the axis of the main road at the entrance of the town, were the first difficulties to be surmounted, and the engineer at once conceived the necessity of making a new branch-road through the suburbs, and of constructing a bridge of a single arch. He perceived the impediments and bad effects that an oblique bridge of three small arches would produce, having the piers also oblique to the stream, or even one of a single arch of larger span in a very oblique direction; he felt too that the art, although not of recent origin in Italy,* *does not afford to this day proper means of executing such a work satisfactorily on a very large scale.*'

* 'The art appears to have been known there as early as 1530, when Nicolò, called "Il Tribolo," erected a bridge of this kind over the River Mugnone, near Porta Sangallo, at Florence, on the main road to Bologna.' See Vasari, vol. xi. p. 308, edizione di Milano, 1811.

This extract, with its accompanying note, establishes two facts: namely, the first, that oblique bridges are not of very recent date; and the second, that the art was not, even lately, sufficiently matured to enable an Engineer to venture upon an oblique arch of large dimensions.

This then is a sufficient apology, if any be necessary, for the Author's venturing to contribute anything on the subject. One object aimed at herein has been to determine, by calculation, all the dimensions of every part of a Bridge; they are by this method more expeditiously obtained than from a drawing; the one method is absolutely correct, whilst the other is but an approximation.

In carrying this plan out, some mathematical formulæ have necessarily been introduced; they are extremely simple, and the application of them is given in two examples, worked out at length, which must be amply sufficient for all such as are likely to use this little work. It was not designed for the uneducated workman: the subject cannot be reduced to his level and properly treated at the same time. It is written for the use of Engineers and Architects generally; but principally for those who have the immediate superintendence of public works, and who, being young men, it is to be hoped will not complain of its being too mathematical.

It may be satisfactory to state, that the rules herein given are not speculative, but are such as the Author has been, and now is, in the daily habit of successfully applying to practice.

GEORGE WATSON BUCK.

PREFACE

TO

THE THIRD EDITION.

HAVING frequently been asked by young engineers to explain portions of the second edition of this treatise, I have generally found that the difficulties encountered were chiefly due to inaccuracies in the plates and letterpress; in some instances the diagrams or their lettering not agreeing with the description or formulæ, and in others the letters in the diagrams referred to being misplaced or omitted; several of the formulæ being also incorrectly rendered. I have in consequence been induced to undertake the present revision of this essay, and, in doing so, have endeavoured to simplify the analysis of the developments for obtaining the dimensions of the twisting rules, and have introduced the strictly theoretical formulæ applicable thereto, which I believe to be of increased practical utility, especially in the execution of arches of great obliquity.

Mr. W. H. Barlow's diagrams and description are appended to the present edition, and will be found, as heretofore, most valuable in reducing the labour involved in the calculations for determining the various dimensions required for the construction of Oblique Bridges; to these have been added a synopsis of a paper contributed by Mr. W. H. Barlow to the 'Civil Engineer and Architects' Journal' for 1841, containing a demonstration of the diagram for obtaining the angles of the templates for working the quoin-stones, which will meet the requirements of those who desire to explore the ground upon which the geometrical construction is based.

J. H. WATSON BUCK.

November 1880.

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ESSAY

ON

OBLIQUE BRIDGES.

CHAPTER I.

DESCRIPTIVE GEOMETRY APPLICABLE TO THE FIRST PRINCIPLES.

LET ABCD, Fig. 1, Plate I., be the plan of a semicylinder, the elevation of the end of which is represented by AEB, and suppose it were required to describe on the plan the spiral line enveloping the semicylindrical surface in the length BD, measured parallel to the axis.

Let the semicircle AEB be divided into any convenient number of equal parts, 1, 2, 3, 4, 5, etc., as in the diagram, and let the distance BD be also divided into the same number of equal parts; then if from all the points in the semicircle lines be drawn parallel to the axis of the cylinder, and intersecting the corresponding lines drawn at right angles to the axis from BD, their mutual intersections will be so many points in the projection of the helix on the cylinder.

Further, if BF and DG, drawn at right angles to BD, be made equal to the semicircle AEB, the parallelogram BFGD will be the development of the semicylindric surface ABDC; and the diagonal BG will be the development of the helix B, 1, 2, 3, 4, 5, 6, 7, 8, 9, C.

The diagram No. 1 exhibits the projection of a spiral *line* upon a semicylinder; the purport of fig. 2 is to exhibit the projection of a cylindrical spiral *surface.* The same references are made to the corresponding parts in figures 1 and 2. It is here supposed that the spiral surface is of the breadth NA or BH, and it is only necessary to repeat the steps described in Article 1, and thereby to describe the projection of a spiral line upon a cylinder whose diameter is NH, and length the same as before, BD or HL, and the curved line H *a b c d e f g h i* M, will be the projection of the spiral line round the cylinder, whose diameter is NH and length BD, and the space contained between the two spiral lines thus described is the projection of the spiral *surface.*

Further, if HI and LK, drawn at right angles to HL, be respectively made equal to the semicircle NOH, the parallelogram HIKL will be the development of the semicylindric surface NHLM; and the diagonal HK is the development of the spiral line H *a b c*, etc., M.

In the preceding figures the parallelogram BFGD is the development of the right cylindric surface ABDC, and the parallelogram HIKL is the development of the right cylindric surface NHLM; and BD, or its equal HL, is the length of cylinder necessary for the helix BG to make one semi-turn at the given angle DBG: this distance, BD, we shall call the axial length of the spiral.

Having shown the method of projecting a spiral surface, it is only necessary to surround the cylinder with any number, at pleasure, of such spirals, all being equal and similar, and the entire projection of a screw will be obtained. Thus the axial length BD (see fig. 3, Plate II.) being divided into any number of equal parts,[1] and the projection BC of the spiral in fig. 1 being cut out of cardboard, let its extremities be applied to the corresponding points BC, B′C′, B″C″, on the cylinder in fig. 3, and the spiral lines drawn thereby, the whole projection of the interior of the screw or of the intrados will be obtained.

In a similar manner, if a mould of cardboard be made to the projection of the spiral HM, fig. 2, and applied to the corresponding points HM, H′M′, H″M″, etc., fig. 3, the projection of the exterior of the screw, or extradosal spiral, will be obtained, but which are omitted in this figure to avoid confusion.

It now becomes necessary to explain the method of developing the surface of an oblique section of a cylinder.

Let ACDH, fig. 4, represent a portion of a semi-cylinder whose diameter is AB, cut obliquely at the angle ACB, and let CD or AH be the length of the portion of the cylinder to be developed, it is required to determine the development of the surface ACDH.

[1] It is here divided into ten parts.

Make BF perpendicular to BD and equal to the semicircle AEB.

Divide BEA into any convenient number of equal parts, 1, 2, 3, 4, etc., and its development BF into the same number of equal parts, 1, 2, 3, 4, etc.

Now if lines parallel to the axis of the cylinder be drawn from these points in the semicircle to *a*, *b*, *c*, *d*, etc., on the line AC, and then lines *aa'*, *bb'*, *cc'*, *dd'*, etc., be drawn from *a*, *b*, *c*, *d*, etc., parallel to AB, and meeting lines perpendicular to BF, drawn from the points 1, 2, 3, etc., in BF, their mutual intersections *a'*, *b'*, *c'*, *d'*, etc., will be points in the development of the line of which AC is the projection. Now let a curve be drawn through all the points thus obtained, and CF will be the development of the line AC.

It is obvious that DG, which is the development of HD, must be equal, similar, and parallel to CF, FG being equal and parallel to CD; and CFGD is the development of the surface ACDH.

Now suppose it were required to construct a semicircular arch over a road, the width of which is AB, fig. 5, Plate III., and at the angle ACB. AC will be one side of the arch, and suppose HD to be the other; the width of the bridge or arch being CL. Make BF equal to the semicircle A'EB' as before, join CF and DG. Let the lines CF and DG be divided into a convenient number of equal parts, 1, 2, 3, 4, etc., such that each part shall correspond to the thickness of one course of stone,[1] generally preferring an odd number, in order that there may be one in the centre as a keystone. Now let a line CI be drawn perpendicular to CF and meeting FG produced in I, so shall CI be the development of the spiral, of which the joint CK of the soffit of the arch forms a part. All the other joints are drawn parallel to this first joint; thus, supposing the lines to be drawn from the divisions in CF to those in DG, the lines C4 (1, 5) (2, 6) (3, 7) will represent the joints of the entire courses, and the lines (*a*3) (*b*2) and (*c*1) will represent the short courses which intersect the springing line CD. The whole of the lines in this diagram relate to the under side, or soffit or intrados of the arch. The angle BFC=MCI is technically called the angle of the skewback of the soffit, or the angle of the intrados, it being the angle which the soffit-joints of the courses make with the axis of the cylinder; also CM is the axial length of these courses, MI being at right angles to CM.

The direction of the joints in the development of the extrados is a part of the oblique arch, the proper understanding of which is generally found rather difficult, and without which the positions of the joints in the face of the arch cannot be determined.

Let the development LONW, fig. 6, of the extrados be constructed by the rules hereinbefore given; join LO and WN. Now the axial length CM of the intrados, and

[1] It will simplify the subject to suppose the bridge to be constructed entirely of stone.

PR of the extrados, must be the same for both (see fig. 2); but SO is equal to the semicircle STU, and PQ is the development of the extradosal spiral, as before shown, which is the direction of the coursing joints on the extrados. The distance CP or MR is equal to the thickness of the arch or the breadth of the spiral surface. Let CP, *ad*, *be*, *cf*, etc., be drawn at right angles to CD, then draw *dl*, *ek*, *fi*, *gh*, parallel to PQ, and these lines will be the direction on the extrados of the coursing joints which intersect the springing line or impost LW. Next, make the other side ON of the development exactly equal and similar to LW, for which purpose set off on ON the distance $Oh = Wg$, draw *hn* parallel to PQ, then the distance *nn′* on OL must be divided into the same number of equal parts, 1, 2, 3, 4, etc., into which the line CF in fig. 5 was divided; and the other side WN of the development being treated in the same manner, and lines being drawn parallel to PQ from the points 1, 2, 3, etc., in the line LO, they will be the direction of the coursing joints in the development of the extrados.

It now becomes proper to describe the method of obtaining the elevation of the face of the arch as dependent upon the foregoing principles, which will be no other than the appearance which a screw, if cut at the angle ACB, would present.

Draw the semi-ellipse ADB, fig. 7, Plate IV., such that the semiconjugate DC shall be equal to the radius of the cylinder, and the transverse AB shall equal AC, fig. 5.

Draw the outer semi-ellipse EFG such that the semiconjugate FC=DC+DF (the thickness of the cylinder), and EG=UL, fig. 6. Next (see fig. 5) transfer the distances C*a*, *ab*, *bc*, *cd*, etc., of the joints on the development of the intrados, to B*a*, *ab*, *bc*, *cd*, etc, on BDA, fig. 7, and these points will be the positions of the joints on the soffit of the face of the arch.

In fig. 7, make the distances G*k*, *ka'*, *a'b'*, *b'c'*, *c'd'*, etc., respectively equal to the distances L*k'*, *k'a'*, *a'b'*, *b'c'*, *c'd'*, etc., in fig. 6, and the points *k*, *a'*, *b'*, *c'*, *d'*, etc., in fig. 7, will be the positions of the joints on the extrados on the face of the arch, and B*k*, *aa'*, *bb'*, *cc'*, *dd'*, etc., being joined, will give the directions of the joints in the face of the arch.

It is perhaps necessary here to remark, that the joints B*k*, *aa'*, *bb'*, *cc'*, etc., are not straight lines, but curves, concave on the upper side, that joint situate next to the springing BG being the most so, and the curvature gradually diminishes towards the vertex, where it vanishes altogether. If a third development were made at half the thickness of the cylinder, then a middle series of points for the positions of the joints in the face of the arch would be found precisely in the same way as those points *k*, *a'*, *b'*, *c'*, *d'*, etc., in the extrados were found, and thus three points in the curve would be obtained. But the curvature is too small to be made sensible in a plate of the dimensions to which we are confined in a work like this ; and a much more easy and accurate mode of obtaining it will be given hereafter.

It may now be perceived, that in order to make a correct working-drawing of the face of an oblique arch, a great deal of projection and transference of lines are necessary, rendering it extremely difficult to be accomplished with that precision which is absolutely essential to its utility and to good workmanship. It follows therefore, that if it can be done chiefly by the aid of computation it will be far preferable, and this we shall endeavour to show may be effected.

CHAPTER II.

INVESTIGATION OF FORMULÆ FOR DETERMINING THE DIMENSIONS AND ANGLES.

After having had several drawings of the faces of oblique arches made on a large scale, and projected with great exactitude, on the principles explained in the preceding Chapter, we observed that the following remarkable property exists.

If the lines B*k*, *aa'*, *bb'*, *cc'*, etc., fig. 7, which are the chords of the small curves forming the joints in the face of the arch, be produced, they will all meet in one point O, below the axis of the cylinder ; and this property was found to hold even when the obliquity is so great as to depress the point O out of the cylinder altogether. This discovery admits of an application which greatly facilitates the drawing of the face, and tends to avoid many sources of inaccuracy. We will now investigate the means of determining this point O by computation ; and it will be seen that at the same time we shall obtain formulæ of great practical utility connected with the subject.

Let the radius of the cylinder . . . . . = r
thickness of do. . . . . . . = e
angle of obliquity . . . . . . = θ
circumference of a circle to diameter unity = π

See fig. 5.

$$\text{Then AB} = 2r$$
$$\text{BC} = 2r \,.\, \cot\theta$$
$$\text{BF} = \pi r$$
$$\text{AC} = 2r \,.\, \operatorname{cosec}\theta$$

Tan $\angle$ BFC, or $\text{MCI} = \frac{\text{BC}}{\text{BF}} = \frac{\cot\theta}{\frac{1}{2}\pi} =$ tan of angle of skewback of intrados.

And by similar triangles,

As BC : BF :: BF or MI : CM ;

that is, as $2r.\cot\theta : \pi r :: \pi r : \frac{\pi^2 r}{2\cot\theta} =$ CM the axial length.

Fig. 6. SO or $\text{RQ} = \pi\ (r+e)$

$\frac{\text{RQ}}{\text{CM}} = \frac{\cot\theta}{\frac{1}{2}\pi}\left(\frac{r+e}{r}\right) =$ tan of angle of skewback of extrados.

$\text{PL} = e \,.\, \cot\theta.$

Now the tangent of the small arc Lk', which is cut off from the development of the extrados by the line PQ, will be

$e.\cot\theta \times \frac{\cot\theta}{\frac{1}{2}\pi}\left(\frac{r+e}{r}\right) = \frac{\cot^2\theta}{\frac{1}{2}\pi}\left(\frac{re+e^2}{r}\right) = \text{L}k'$, fig. 6, or G$k$, fig. 7.

Fig. 7. $\text{BC} = r \,.\, \operatorname{cosec}\theta$

$\text{BG} = e \,.\, \operatorname{cosec}\theta$

And by similar triangles,

As BG : Gk :: BC : CO,

or as

$$e \,.\, \operatorname{cosec}\theta : \frac{\cot^2\theta}{\frac{1}{2}\pi}\left(\frac{re+e^2}{r}\right) :: r \,.\, \operatorname{cosec}\theta : \frac{\cot^2\theta}{\frac{1}{2}\pi}(r+e) = \text{CO}.$$

If we examine this value of CO, it will be found to be equivalent to $r \times \cot\theta \times \frac{\cot\theta}{\frac{1}{2}\pi} \cdot \frac{r+e}{r}$, that is, it is equal to the radius multiplied by the cotangent of the angle of obliquity of the bridge and by the tangent of the angle of the skewback of the extrados ; and if we call the latter angle ϕ, the expression may be written thus, $CO = r \cot\theta \tan\phi$.

Again, it is obviously equal to $(r+e) \times \cot\theta \times \frac{\cot\theta}{\frac{1}{2}\pi}$, or it is equal to the external radius of the cylinder multiplied by the cotangent of the angle of obliquity of the bridge and by the tangent of the angle of the intrados ; and if we call the latter β, the expression may be written $CO = (r+e) \cot\theta \tan\beta$. These expressions are general, that is, they are applicable to segments as well as to semicircles, and will be found extensively useful, as will be shown hereafter.

The distance CO may be determined geometrically as follows :—in the right-angled triangle ABC (fig. 30, Plate X.), right-angled at B, make $AB = (r + e)$; draw AC, making the angle $ACB = \theta$; next draw BD indefinitely, making the angle $CBD = \beta$, and draw CD perpendicular to BC and intersecting BD in D ; then shall CD=CO (fig. 7).

Now having determined this point, which we shall hereafter, for distinction's sake, call the focus, and the distance CO, the eccentricity of the face, it is obvious that the only development required will be CF in fig. 5 ;

and formulæ for finding the ordinates of this curve will be given in a succeeding part of this Work.

In making a working-drawing on a scale as large as an inch to a foot, it is desirable to show the curvature of the joints in the face of the arch; and this is easily effected by describing a semi-ellipsis for the mean thickness of the arch, and taking $(r+\frac{1}{2}e)$ instead of $(r+e)$ in the second factor of the first expression for the value of the eccentricity CO, by which those intermediate points may be found.

The preceding formulæ, with the exception just mentioned, are applicable to such arches only as are semicylindrical or semicircular in their square or direct section; those for such as are segmental will next be considered.

Fig. 8.—Let AB be the chord of the segment, and AGB the angle of obliquity; make BC equal to the length of the arc APB, and perpendicular to BG; draw GC. Now this line, which corresponds to CF in fig. 5, and which in each is called the spiral of the heading joints, is that which is to be divided into the determinate number of voussoirs, and to which the direction of the coursing joints is to be perpendicular. Draw GH perpendicular to GC until it meets CH produced parallel to BL, and draw HL perpendicular to BL; then GL is the axial length, and GH is the development of the segmental spiral.

Let the radius of the cylinder $= r$
thickness of do . . $= e$
chord of segment . . $= c$

Let the arc of segment . . . $= a$
angle of obliquity . . $= \theta$
3·14159, etc. . . . $= \pi$
Then $BG = c \,.\, \cot\theta$
$BC = a$

$$\frac{BG}{BC} = \frac{c}{a} \cdot \cot\theta = \text{tan of angle of skewback of intrados.}$$

And by similar triangles,

As BG : BC :: BC or LH : GL

That is, as $c \,.\, \cot\theta : a :: a : \dfrac{a^2}{c \,.\, \cot\theta} =$ the axial length of the segmental spiral.

Complete the semicircle DAPBE ; make EF equal to its development $= \pi r$, and draw EK parallel to GH until it meets FK drawn parallel to EI ; now EI is the axial length of the intradosal spiral of the semicircle at the *segmental angle* LGH, and the angle of the intrados LGH and IEK are equal. Produce EF to M, making $EM = \pi\,(r+e)$ or $FM = \pi e$; draw MN parallel to EI, meeting IK produced in N, and draw EN, then IEN = the angle of the coursing joints on the extrados, and EN is the development of the semicircular extradosal spiral at the *segmental* angle.

Again, by similar triangles,

As LH : LG :: IK : IE

Or as $a : \dfrac{a^2}{c \,.\, \cot\theta} :: \pi r = \dfrac{ar\pi}{c \,.\, \cot\theta} =$ the axial length of the semicircular spiral at the segmental angle.

$\dfrac{IN}{IE} = \pi(r+e) \div \dfrac{ar\pi}{c \,.\, \cot\theta} = \dfrac{c \,.\, \cot\theta}{a}\left(\dfrac{r+e}{r}\right) =$ the tan of angle of skewback of extrados. Now if we substitute this value of the tangent of the extradosal angle instead of that before used in the formula for the semicircle, we have

$$e \,.\, \cot\theta \times \frac{c \,.\, \cot\theta}{a}\left(\frac{r+e}{r}\right) = \frac{c \,.\, \cot^2\theta}{a}\left(\frac{re+e^2}{r}\right) = \text{G}k \text{ in fig. 7.}$$

And as

$$e \,.\, \text{cosec}\,\theta : \frac{c \,.\, \cot^2\theta}{a}\left(\frac{re+e^2}{r}\right) :: r \,.\, \text{cosec}\,\theta : \frac{c \,.\, \cot^2\theta}{a}(r+e)$$

$=$CO$=$the eccentricity or focal distance below the *axis* of the cylinder in the oblique segment.

It now becomes proper to observe, that in the actual construction of a bridge it is sometimes, nay generally, the case, that the precise angle of the coursing joints as found by the formula cannot be adhered to with mathematical strictness, and for the following reasons. If it so happens, that when a line drawn at right angles to CF in fig. 5, from the point C, does not exactly coincide with one of the joints in the other face of the arch, or in the line DG, the coursing joint must be made to do so by moving it to the nearest joint, which will consequently alter the angle MCI. Therefore it becomes necessary to determine the exact distance DK, measured on the heading spiral DG, and then to ascertain whether it contains an integer number of divisions, 1, 2, 3, 4, etc., that is, whether the distance DK can be divided by the thickness of one division (which is the thickness of one voussoir) without leaving a remainder.

Let the breadth of the arch CL$=b$, and let $\angle$ BFC$=$ DCK$=\beta$. Then $b \,.\, \text{cosec}\,\theta=CD=$the length of the impost or springing line, and $b \,.\, \text{cosec}\,\theta \sin\beta=$DK. Now

CF being divided into any number of equal parts m, then $\frac{CF}{m}$ = one of those parts, and therefore $DK \div \frac{CF}{m}$ or $\frac{mDK}{CF}$ must be a whole number, in order that the angle DCK may not require alteration; but if DK should not be divisible by $\frac{CF}{m}$, then DK must be either increased or diminished by such a quantity as will make it so; and let this quantity, which will never exceed $\frac{CF}{2m}$, be represented by h, then the side DK will become $DK \pm h$, and the angle DCK must be found for the new triangle CKD, which is now no longer right-angled, but the sides CD and $DK \pm h$ are known, and their included angle CDK remains unaltered, namely equal to $90° - \angle$ BFC. The adjusted angle DCK of the intrados being found by the well-known trigonometrical methods, we proceed to find the corresponding angle of the extrados, and therefore must first find the axial length of the spiral due to the new intradosal angle, which latter angle we will call β_2.

Then $\pi r \cot \beta_2 = CM$, the axial length for the adjusted angle.

And in fig. 6, $\frac{RQ}{CM} = \frac{\pi(r+e)}{\pi r \cot\beta_2} = \frac{r+e}{r} \tan\beta_2 =$ tan of angle of extrados, which let $= \tan \phi$.

The magnitude of the angle DCK, fig. 5, having been altered, a corresponding alteration of the eccentricity CO, fig. 7, takes place, which is easily determined as follows :—

In fig. 6. $PL = e \,.\, \cot\theta$

$Lk = e \,.\, \cot\theta \tan\phi = Gk$, fig. 7.

Then again as before,

As $BG : Gk :: BC : CO$

or as $e \,.\, \mathrm{cosec}\,\theta : e \,.\, \cot\theta \tan\phi :: r \,.\, \mathrm{cosec}\,\theta : r \,.\, \cot\theta \tan\phi$ $= CO$, which is the adjusted focal distance or eccentricity; and this expression is general, and applies to both the segmental and semicircular arch. See page 9.

CHAPTER III.

METHOD OF WORKING THE VOUSSOIRS, ETC.

We will now explain the method of working the voussoirs. Because the depth of a stone, or the breadth of its bed, invariably measures considerably more than its thickness on the soffit, it is the better way to begin by working the former. Now the beds of the voussoirs are portions of the spiral surface BHMC, fig. 2, and therefore consist of what are usually called winding-beds. The mode of obtaining such winding-beds is familiar to workmen, and is done by placing two rules, one of which has its edges parallel and the other diverging, at a determinate distance, and then each is sunk into a draft in the stone until their upper edges are in one plane, when the under edges will be in the intended winding surface or bed: this being done, the superfluous parts of the stone on the other parts of the bed are to be dressed off until a straight-edge applied from one draft to the other, and in contact therewith, being always kept parallel to the soffit, shall in every part of the bed coincide with the surface thereof.

We will now explain the method of determining the dimensions of these rules. The edges of the twisting rule, or winding strip (as the workmen usually call it), are divergent; and let us again refer to fig. 8, which is applicable to both the semicircular and the segmental arc. The intradosal angle is IEK, and the extradosal angle is IEN, and their difference, or KEN, is the angle of the wind or twist of the bed. Further, EK and EN are respectively the secants of the angles IEK and IEN to radius EI, NO drawn perpendicular to EK produced is the tangent of the angle KEN to radius EO, and EO is the cosine of the angle KEN to radius EN. Now any certain distance l, measured upon the coursing joint of the soffit EK, fig. 8, having been fixed upon as convenient for the application of the rules,—the length of which must be equal to e the depth of the voussoir, *neither more nor less*,—the distance between the ends coinciding with the extrados, *at their points of contact with the coursing joint of the extrados* EN, must exceed that on the intrados or soffit, in the ratio EN : EK. Now the adjusted angle of the intrados being β_2, and the angle of the extrados ϕ, the distance between the points of contact of the rules with the coursing joint of the extrados $EN = l \times \frac{\sec \phi}{\sec \beta_2}$, and denoting by l_2 the corresponding distance *upon the line* EO, *also on the extrados of the arch*, and by δ the angle KEN, fig. 8,

$$l_2 = l \times \frac{\sec \phi \cos \delta}{\sec \beta_2}$$

The application of these rules to the bed of a stone is shown by fig. 9, Plate V., where CDEF is the winding-bed of the stone; AB is the parallel rule, and A_2B_2 is the twisting rule: the distance between the ends A and A_2 on the soffit is l, and the distance between the ends B and B_2 is l_2, as found by the formula just given. The difference in the breadths of the two ends of the twisting rule is $l_2 \tan \delta$.

The rules are shown by figs. 10 and 11. Fig. 10 is the parallel rule, and fig. 11 the twisting rule. Their lengths AB, fig. 10, and A_2B_2, fig. 11, must be equal to e the depth of the voussoir. The breadth AC, BD, fig. 10, and A_2E, fig. 11, must be equal (it is usually made three inches), and the other end of the twisting rule B_2G, fig. 11, must be increased by the quantity

$$FG = l_2 \tan \delta$$

A stone worked with the twisting rule on the side of the parallel rule, as shown in fig. 9, will belong to an arch, the obliquity of which is such, that on the supposition of your standing in the axis of the cylinder, and looking through the arch, the acute quoin of the abutment would be on your left hand; and, for the purpose of easy distinction, we usually say such a bridge skews to the left. And here let it be observed, that all the preceding diagrams have reference to a bridge askew or oblique to the left, which uniformity has been adopted to avoid confusion.

In order that the workmen may not be liable to make any mistake in applying these rules with the proper degree of divergence, as well as to obviate the necessity of measuring the distances AA_2 and BB_2, we connect them, when used, by light iron rods, one of which is fastened at one end to one of the rules, and the other rod by one of its ends to the other rule, each rod having a hook of this form, which is made to drop into an eye in the side of the other rule, so that (these rods being made of the proper length) when they are dropped into their respective eyes, the rules must necessarily have the proper degree of divergence or radiation. All the points of attachment should be the same distance from the upper edges of the rules. These rods are represented in fig. 9. It is highly requisite that these directions be well understood and strictly attended to, without which arches of great obliquity cannot be built with precision. If the rules be put into the hands of the workman without the connecting rods, he will generally apply them *parallel* to each other, and this will obviously cause the wind of the bed to be greater than it ought to be; he will not be able to set them in their places in the arch without paring the corners of the beds, and then they will not bear equally on each other.

One bed having been thus worked, the soffit must next be obtained from it, and to effect this, let the following template be made. Prepare two moulds, as shown in fig. 12, where AC is the radius of the cylinder, and DB is its thickness, or the depth of the voussoirs. The stock of the mould AB must be made to fit the curve of the soffit, and may be of any convenient length; BD should be equal to the depth of the voussoir with both edges radiated to the centre. These two moulds should then be framed together, as represented in perspective by fig. 13, which must be done in such a manner that the angle ACB shall equal the angle IKE, fig. 8, which is the complement of the angle of the intrados. This being done, the edges of the two blades BD and CE, fig. 13, will exactly coincide with the spiral bed of the stone which has been already worked by the twisting rules, and the stone being placed with its soffit uppermost, let this template be inverted, and applied with its blades BD and CE to the worked bed, the strip BC, figs. 13 and 14, being at the same time in contact with the soffit DF, fig. 14 ; then let a line be drawn on the stone by the stock AC, fig. 14, and this line so drawn will be at *right angles* to the axis of the cylinder. Let another line be drawn by the side AB, and this line will be *parallel* to the axis of the cylinder. Remove the template, and let chisel drafts be sunk in the soffit on the line CA, to fit the curve of the stock of the template, and also on the line AB (which will be perfectly straight), to fit the side AB, so that when these drafts are sunk to the proper depth, the template on being applied with its blades to the previously worked bed, and the diagonal strip to the arris of the soffit DF, the stock CA and the side AB shall all be exactly in contact in every part at the same time. Two segmental pieces, each of about the length of CA, and alike curved to the radius of the intrados of the cylinder, as shown in fig. 15, should be in readiness, one to apply to the draft AC, and the other upon a line GH, fig. 14, drawn at a convenient distance from and parallel to CA. These segmental pieces must be exactly of the same dimensions, and each must have a centre line marked on it, as at C, fig. 15. They must now be applied thus: one upon the draft CA, and with its centre C coinciding with a line IK, parallel to AB; and the other segment is to be applied upon the line GH, drawn parallel to CA (the more remote the better), and with its centre C also coinciding with the line IK. This second segment should then be sunk in a chisel draft, until its upper edge is out of winding with that of the other segment in the draft CA. This being done, the superfluous stone on the soffit should be dressed off, until a straight-edge applied all over the soffit, parallel to AB, shall coincide with the bottoms of the chisel drafts CA and GH, as well as with every other part of the soffit between them and beyond them, and the soffit will be finished. The other arris LM of the soffit should then be gauged, and knocked off parallel to FD, and then the template should be turned about and applied to the soffit and to the other bed of the stone; but in this case the method is reversed,—that is, instead of obtaining the soffit from the bed, the bed is obtained from the soffit, the template being applied with its diagonal strip CB in contact with the arris LM, and its stock CA, and side AB, in close contact with the soffit. Let drafts be sunk in the second bed of the stone, until the bottoms of them exactly fit the blades of the template; these drafts in the bed will then answer the same purpose for this bed as those did which were obtained by the twisting rules for the first bed, and are to be made a similar use of.

The ends of all the voussoirs, except those which form the exterior face of the arch, are to be worked square on the soffit, as at FL and DM, fig. 14, and the other part of the ends of the stones, technically called the heading joints, must be worked to fit the radiation of the blades of the template, fig. 13. The ends of both beds being marked off from the blades of the template, and then the heading joints worked by a straight-edge applied from one arris to the other, the straight-edge being always applied parallel to the square end FL or DM of the soffit, the heading joints will have the proper wind or twist, such that all the stones will fit, each to each, close together, when put into their places in the arch.

One of the most difficult things to accomplish in the working of the stones for an oblique arch, is to shape those which form the face with such accuracy that they shall not want paring after being set. We will endeavour to show how this may be done, and hope to make it easily understood.

Let the development of the face of the arch, as shown in fig. 17, Plate VI., be laid down upon a platform. This development must be of the full size or of the actual dimensions of the work.[1]

The mode of obtaining it by projection has been shown in fig. 4, and the method of drawing the coursing joints

[1] For a more simple and expeditious mode of obtaining the templates for working the quoin stones, see page 4 of the Appendix.

has been shown in fig. 5. However, it is admitted that to obtain this curve of the full size by actual projection is nearly impossible ; we shall therefore show, a little further on, a method of obtaining it by ordinates, and shall now suppose it done as shown in fig. 17, where CD is a portion of the impost, or springing line, CK is the direction of the first entire coursing joint, making the angle of the intrados DCK, as has been hereinbefore shown, to which all the other joints, 2, 3, 4, 5, etc., are parallel.

It will be observed, that the ends of the voussoirs which form the quoin of the soffit of the arch are not rectangular, and this development is for the purpose of obtaining therefrom the exact form of the soffit quoin of each stone, which is done as follows. Suppose the soffit of the stone which occupies the space between the ninth and tenth coursing joints were required. Let a thin, flexible plate of iron, of convenient length, and the breadth of which is equal to the thickness of a course, measured on the soffit, be applied to the lines *ac*, at the ninth joint, and *bd*, at the tenth joint, and then let its end *ab* be cut to fit that portion of the curve which is intercepted by the lines *ac* and *bd*. This being done, apply the iron plate to the soffit of the stone (which is supposed to have been previously worked in every part except the face), and mark thereon the line *ab*, which will be the quoin of the soffit of the stone. The curve in the length *ab* is so exceedingly small, that instead of a flexible plate, it will be sufficiently accurate to make use of a shifting stock, setting the stock to *ac* and the blade to *ab*.

Next, to obtain the direction of the joint in the face of the arch, let one-half of the elevation of the arch, as shown in fig. 18, be laid down of the full dimensions upon a platform, precisely upon the principles which have been before explained. In this figure AC is the half skew span, CD is the radius of the cylinder, and CO is the eccentricity, O being the focus of the face to which the joints radiate. Describe the quadrant BED, draw the tangent AE and radius EC.

Now suppose it to be required to obtain the angle which the joint *ac*, in fig. 17, makes with the corresponding joint *ef*, fig. 18. From *e*, fig. 18, draw *eg*, parallel to AC, and intersecting the arc BED, in *g*. From *g* let fall the perpendicular *gh* upon AC, and from *h* make *hi* perpendicular to EC. With the distance O*g* as radius and centre O, describe the indefinite arc *gk*, and with the distance *hi* as radius, and *e* as a centre, intersect the arc *gk* in *k*. From *k*, through *e*, draw the straight line *kel*. Now the angle *kef* is that which the joint *ef* makes with a line on the soffit, drawn parallel to the axis, or with a horizontal line; but the joint *ac*, fig. 17, is inclined to the axis at the intrados angle KCD, therefore draw the straight line *nem*, fig. 18, such that the angle *kem*=the angle KCD, fig. 17. Next let fall *fp* perpendicular to and intersecting *kel* in *p*, and meeting *nem* in *q*. Draw *qr* indefinitely and parallel to *kel*. With the distance *pf* as radius, and centre *p*, describe the arc *fr*, intersecting *qr* in *r*. With *fe* as radius, and centre *e*, describe the indefinite arc *fut*; and with *qr* as radius and centre *q*, describe the arc *rus*, intersecting *fut* in *u*; from *u* draw *ue*; then will *uem* be the angle which a tangent to the coursing joint makes with the face joint, on the obtuse side of the arch: and *neu* will be the corresponding angle on the acute side of the arch.

But the line *nem* being the tangent to the spiral at the point *e*, the proper allowance for curvature, or departure from the tangent in the length of the voussoir, must be made, which is to be done as follows:—

Suppose it is required to make the length of that part of the mould which is to fit to the coursing joint of the soffit equal to a given distance, as CB in fig. 19, Plate VII., where the angle DCK is the angle of the intrados. Draw AB perpendicular to the impost CD. Now AB is the portion of the cylindric arc which is embraced by the voussoir, whose length is CB, and is the same as the arc AB in fig. 20, and the versed sine A*d*, or its equivalent *e*B, is what we have to determine.

In fig. 19.—Let the angle $DCK=\beta$
$CB=L$

Then $AB=L\sin\beta$.

With radius equal to CD in fig. 18, and centre C in fig. 20, describe an indefinite arc ABD, in which take $AB=L\sin\beta$, and draw B*d* perpendicular to the radius AC, then is A*d*, or *e*B, the distance sought. Now πr being the length of the semicircle, we have by proportion,

As $\pi r : 180^\circ :: L\sin\beta : \angle\, ACB$, which call ρ,

then $r\ \text{versin}\ \rho = Ad$, or eB.

Next make each of the distances *ev* and *ew* in fig. 18, equal to CB in fig. 19, and make *vx* and *wy* respectively equal to A*d*, fig. 20; draw a circular arc[1] through the points *xey*, and then will the angle *xeu* be the form of the obtuse voussoir, and *yeu* of the acute one. These last lines *xey* and *eu* should be permanently and distinctly drawn upon the platform, at each joint, and to these lines the bevel is to be set by which the stones are worked.

Perhaps it may be well to observe, that although a great many lines have been here drawn to obtain one joint, nevertheless some of them are common to all, and in practice it is not necessary to draw all those which are not common, but only to set off the equivalent distances, by which the labour may be abridged, and the work will be more distinct.

It now becomes necessary to show how the development, fig. 17, is to be obtained without projecting it, as shown in Chap. I. fig. 4.

Let A*ab*, fig. 21, be the half of a semicircular arch, the obliquity of which is BDC, and suppose it is required to produce the development D*e*E by means of ordinates obtained by calculation.

Let the arc AB be divided into a convenient number of equal parts, and its development BE also into the same number. Suppose *a* to be one of the divisions of the

[1] This curve is a portion of the projection of the spiral formed by the coursing joint; but this portion of it is too small to make the difference between it and a circular arc appreciable.

arc, and b its corresponding division in the development, such that $Eb = Aa$.

$$\text{Also, let AC the radius} = r$$
$$\angle\ ACa = \epsilon$$
$$\angle\ CDB = \theta$$
$$\angle\ BED = \beta$$
$$Cc = r \sin \epsilon$$
$$cd = r \sin \epsilon \cot \theta = be$$
$$Eb \tan \beta = bf$$
$$be - bf = fe$$

Thus having found a sufficient number of distances fe, corresponding to equal divisions of the arc AB, or to its development BE, and consequently to DE also, let DE be divided into the same number of equal parts (as shown in fig. 17, at e, f, g, h, etc., and e, f', g', h', etc.).

At each of these divisions draw the ordinates fe, fig. 21, making all the angles Dfe each equal to DEB, which is the intradosal angle before treated of, and upon these ordinates set off the distances fe as previously calculated; then the curve DeE being drawn through the points thus obtained, will be the development required.

Only one half of the development is shown, because the ordinates for the second half are equal to those in the first half, but are applied on the contrary side of the line DE.

CHAPTER IV.

APPLICATION OF THE PRECEDING FORMULÆ.

AFTER having proceeded thus far, the subject will be much more easily understood by giving one or two practical examples of the application of the formulæ which have been obtained.

Suppose we have to erect a bridge of thirty-three feet clear or direct span, to carry a railway over a street or turnpike-road, at such an elevation above it, that a semicircle may be applied without inconvenience, and that the angle formed by the intersection of the line of railway with the road is 50° to the right. Suppose also that the external width of the bridge is 31 feet, and that the thickness of the cylinder, or depth of the voussoirs at right angles to the axis, is 2 feet 6 inches. These are the dimensions to which the development fig. 17, and elevation fig. 18, are drawn.

We will arrange the computations and results in the following order :—

EXAMPLE I.

Dimensions of a bridge constructed with a semicircular arch, to carry a railway over a road 33 *feet wide and at an angle of* 50° *to the right.*[1]

	Feet.
r = Radius of cylinder.	16·5

[1] The centre arch of the Watford Viaduct on the London and Birmingham

		Feet.
e=Depth or thickness of ditto . . .		2·5

Now see fig. 5.

CL=External width=b		31·0
AB=Direct span		33·0
AC=Oblique span=$2r$ cosec θ=33 × 1·3054	=	43·078
BC=Obliquity of arch=$2r$ cot θ= × 33·8391	=	27·69
BF=Length of arc = πr = 3·1416 × 16·5 .	=	51·836

$$\tan \angle \text{BFC} = \frac{\cot\theta}{\frac{1}{2}\pi} = \frac{\cdot 8390996}{1\cdot 5708} = \cdot 5341861$$

$= \tan 28°\ 6'\ 37'' = \tan\beta$

FC=Length of heading spiral=πr sec β

=51·836 × 1·1337324	=	58·768

Convenient number of voussoirs will be 47.

Thickness of voussoirs $=\frac{58\cdot 768}{47}$.	=	1·25038
CD=Length of impost=b cosec θ=31 × 1·3054	=	40·4674
DK=Divergence of courses=b cosec θ sin β		
=40·4674 × ·47117	=	19·067

This distance, viz. 19·067, does not correspond to any whole number of voussoirs, but is immediately found on trial to exceed the space occupied by 15 (that is 15 × 1·25038), therefore 15 will be the number of courses intersecting the impost CD. Now the theoretic

Railway is nearly of these dimensions; it is semicircular, the span being 35 feet and at an angle of 55°. The dimensions here given are those of a bridge, the model of which was made and sent by the author to St. Petersburg. It is accurately represented by the drawings given in Plate No. VIII.

angle DCK, fig. 5, must be diminished, to adjust it to the movement of the line CK towards the joint of the 15th voussoir; that is, taking $DK = 15 \times 1{\cdot}25038$, we have

$$\frac{DK}{CD} = \frac{18{\cdot}7557}{40{\cdot}4674} = {\cdot}4634767 = \sin 27°36'42'' = \sin \beta_2 \quad 27°36'42''$$

$$\tan \beta_2 = {\cdot}5230466$$

$$\text{and } \sec \beta_2 = 1{\cdot}1285290$$

and to this angle the axial length, the angle of the extrados, and the eccentricity of the face, must all be adjusted.

CM = The axial length to adjusted angle.

$= \pi r \cot \beta_2 = 51{\cdot}836 \times 1{\cdot}9118755$ . . 99·104

$\frac{RQ}{CM}$ (see fig. 6) $= \tan \Phi = \tan$ of $\angle$ of extrados

$$= \frac{\pi(r+e)}{CM} = \frac{3{\cdot}1416\,(16{\cdot}5 + 2{\cdot}5)}{99{\cdot}104} = {\cdot}6023006$$

= tan . . . . 31° 3′ 38″

and sec Φ = ... 1·1673753

δ = Difference of angles of extrados and intrados = 31° 3′ 38″ − 27° 36′ 42″ . 3° 26′ 56″

tan δ = ... ·0602505

and cos δ = ... ·9981889

CO (fig. 7). Theoretic eccentricity $= \frac{\cot \theta}{\frac{1}{2}\pi}(r+e)$

$$= \frac{({\cdot}8390996)^2 \times (16{\cdot}5 + 2{\cdot}5)}{1{\cdot}5708} = 8{\cdot}5164$$

Ditto, adjusted by formula, $r \cot \theta \tan \Phi$

$= 16{\cdot}5 \times {\cdot}8390996 \times {\cdot}6023006$ . . $= 8{\cdot}3389$

tan Φ may be obtained as follows:—$\tan \beta_2 \frac{r+e}{r} = \tan \Phi$.

It is proper to observe, that the method here made use of to determine the altered angle DCK is only approximative; but in every instance the triangle will be so nearly right-angled, that the approximation is sufficiently accurate.

Having made the preceding calculations and tabulated the results, we are enabled to make all the necessary templates, rules, etc., for the work, as follows :—

THE TWISTING RULES.

The length of the parallel rule, fig. 10, must be equal to the thickness of the cylinder, namely, 2 feet 6 inches, and we will suppose 3 inches broad.

The rule fig. 11, must be 2 feet 6 inches long also, and 3 inches broad at the narrow end, but the breadth B_2G of the other end will depend upon the distance asunder at which it is intended to apply them. We will suppose 3 feet to be a convenient distance measured on the soffit of the stone, as from A to A_2, fig. 9. The distance between their other ends from B to B_2 will be obtained by the formula

$$l_2 = l\frac{\sec\phi\ \cos\delta}{\sec\beta_2} = 36\times\frac{1{\cdot}16737\times{\cdot}99818}{1{\cdot}12853} = 37{\cdot}17 \text{ inches.}$$

Then $l_2 \tan\delta = 37{\cdot}17\times{\cdot}06025 = 2{\cdot}24$ inches $=$ FG, fig. 11; therefore the breadth of this end of the rule must be

$$3 + 2{\cdot}24 = 5{\cdot}24 \text{ inches} = B_2G, \text{ fig. } 11.$$

The correct method of drawing upon the soffit of the stones the lines parallel to the axis, and at right angles thereto, fig. 14, is by the use of a triangular plate of sheet iron, made to the angle of the intrados, which is the angle ABC of the template, as shown in fig. 14. Thus, if an iron plate be desired which shall measure 24 inches on the side AB, the side AC will be $24 \times \tan \beta_2 = 24 \times \cdot 523 = 12\cdot 55$ inches. This plate is shown separately at fig. 16.

The impost upon which the arch rests must be divided into fifteen equal parts, and as many triangular checks (as commonly called) sunk therein, as shown by figs. 22 and 23, Plate VIII. In the former, which is an elevation of the impost parallel to the axis of the cylinder, the small triangles which form the steps or checks are similar to the plate fig. 16, and are marked out thereon after dividing the impost into fifteen equal parts as before mentioned. This is done by placing the said triangular plate with its hypotenuse BC downwards upon the line AA′, fig. 22, and applying in succession its two extremities B and C to each of the divisions, and scoring by the base and perpendicular. The back, or extrados of the impost, should be similarly marked, but with a plate made to the angle of the extrados. This triangular plate, with a base of 24 inches, must have its perpendicular $AC = 24 \times \tan \phi = 24 \times \cdot 6023 = 14\cdot 45$ inches. This difference in the angle of the two plates will produce the proper degree of wind in the bed and cross-joints of the checks of the impost.

The thrust of the arch is parallel to its face, or nearly so; and in order that it may have a proper abutment in that direction, the back of the impost must be worked in vertical steps, the sides of which are respectively parallel, and at right angles, to the face of the arch. When the wall behind the impost is of brick, the width of the step at right angles to the thrust should be made such that the bricks may not require cutting, namely, to correspond to the dimensions of a brick, a brick and a half, etc., as most suitable to the degree of obliquity. The vertical steps here described are exhibited in the plans of the imposts in the designs in Plates Nos. VIII., XI., and XII.

To obtain the direction of the first joint in the face of the arch, which is the top bed of the ends of the impost, viz. BG, fig. 7, let a thin board or iron plate be cut to the required angle $GBk = CBO$, the tangent of which is $\frac{CO}{CB}$.

In the example before us, $CB = \frac{43 \cdot 078}{2} = 21 \cdot 539$, and $\frac{CO}{CB}$ $= \frac{8 \cdot 3389}{21 \cdot 539} = \cdot 3871 = \tan \angle GBk$.

Also $BG = e \,.\, \text{cosec}\, \theta = 2 \cdot 5$ feet $\times\, 1 \cdot 3054 = 3 \cdot 2635$ feet,

and $BG + \tan GBk = Gk$,

that is, $3 \cdot 2635 \times \cdot 3871 = 1 \cdot 263$ feet $= Gk$.

It only remains now to determine the joints and planes of the faces of the voussoirs, the method of doing which has been explained in the preceding chapter; but it is here proper to show how the calculation of the ordinates for the development of the face is performed.

The development fig. 17 is divided into twenty equal parts at *e*, *f*, *g*, *h*, etc.; therefore the half arc, which in this case is a quadrant, is divided into ten equal parts, each being the arc of 9°, and corresponding to the divisions *e*, *f*, *g*, *h*, etc., in one half of the development, fig. 17, and *e*, *f′*, *g′*, *h′*, etc., in the other half.

The following Table exhibits the method of applying the preceding principles, and the use of the formulæ given at the end of the last chapter.

1.	2.	3.	4.	5.	6.
Angles ACa or ε	Log. sin ε	Log. r cot θ sin ε or log. be	Nat. num. be	Values of bf	Ordinates fe or be−bf
°			Feet	Feet	Feet
9	9·1943324	0·3356298	2·1658	1·3845	0·7813
18	9·4899824	0·6312798	4·2783	2·7690	1·5093
27	9·6570468	0·7983442	6·2855	4·1535	2·1320
36	9·7692187	0·9105161	8·1379	5·5380	2·5999
45	9·8494850	0·9907824	9·7900	6·9225	2·8675
54	9·9079576	1·0492550	11·2009	8·3070	2·8939
63	9·9498809	1·0911783	12·3361	9·6915	2·6446
72	9·9782063	1·1195037	13·1675	11·0760	2·0915
81	9·9946199	1·1359173	13·6746	12·4605	1·2141
90	10·0000000	1·1412974	13·8450	13·8450	0·0000

EXPLANATION OF THE ABOVE TABLE.

Column No. 1 contains the different assumed values of the arcs whose ordinates have to be found, and for convenience the decimal division of the quadrant has been adopted.

Column No. 2 contains the corresponding logarithmic sines of the angles in Column No. 1.

Column No. 3. The numbers in this column are the logarithms of the lines *be* fig. 21, and are obtained by adding $\log r + \log \cot \theta$ to each of the numbers in Column 2. In this case

$$\log r = \log \quad 16{\cdot}5 = 1{\cdot}2174839$$
$$\log \cot \theta = \log \cot 50^\circ = 9{\cdot}9238135$$
$$\log r \cot \theta = 1{\cdot}1412974$$

and this added to each of the numbers in Column 2 produces the numbers in Column No. 3.

Column No. 4 are the natural numbers corresponding to the logarithms in Column No. 3.

Column No. 5. The numbers in this column are obtained in the following easy manner. Because the development BE of the quadrantal arc, fig. 21, is decimally divided, the perpendiculars *bf* will divide BE decimally, and the lines *fg*, drawn from the points of intersection *f*, and parallel to BE, will divide BD decimally also. Now BD is half the obliquity of the arch, which has been before found to be 27·69, therefore BD=13·845, and the other values of *bf* are respectively $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, etc., of 13·845, as will be evident by inspection.

Column No. 6 consists of the differences of the numbers in Columns 4 and 5, and are the values of the ordinates *fe*, which being applied to the line DF, fig. 17, in the manner before described, will afford the means of obtaining the curve of the development of the face of the arch.

This development, in order to be of any practical utility, must be laid down of the full dimensions on a platform, and cannot be obtained accurately except by computation.

Example II.

Dimensions of a bridge, the arch of which is a segment of a circle, erected over a road 30 *feet wide, and crossing it at an angle of* 30°.

		Feet
Radius of cylinder	. . . . .	17·32
Depth of ditto	. . . . .	3·0
External width	. . . . say	31·0
Direct span	. . . . . .	30·0
Oblique span $= c \,.\, \text{cosec}\, \theta = 30 \times 2{\cdot}0$.		$=60{\cdot}0$
Obliquity of arch $= c \,.\, \cot \theta = 30 + 1{\cdot}732$		$=51{\cdot}96$

Length of arc $\frac{15}{17{\cdot}32} = {\cdot}86606508 =$ sine of the half arch = sine 60° 0′ 10″, the whole arc therefore $= 60° \; 0' \; 10'' \times 2 =$ 120° 0′ 20″, the length of which to radius unity, by Hutton's tables = 2·0944436 ∴ $2{\cdot}0944 \times 17{\cdot}32 =$ length of arc . . . . . . $=36{\cdot}275$

Fig. 8. $\tan \angle \, BCG = \frac{c.\cot\theta}{a} = \frac{30 \times 1{\cdot}732}{36{\cdot}275}$

$=1{\cdot}43243 = \tan 55° \; 4' \; 9'' = \beta$.

Length of heading spiral $= a \sec \beta = 36{\cdot}275 \times 1{\cdot}746459$ . . . . $=63{\cdot}352$

	Feet
Convenient number of voussoirs will be 51.	
Thickness of voussoirs $=\frac{63{\cdot}352}{51}=$ (nearly)	1·2422
Length of impost $= b \operatorname{cosec} \theta = 31 \times 2$	=62·0000
Divergence of courses $= b \operatorname{cosec} \theta \sin \beta = 31 \times 2 \times {\cdot}8198438$	=50·8303

This distance corresponds nearly to the thickness of 41 voussoirs, which will be the number intersecting the impost.

Therefore the actual divergence will be

$\frac{41}{51} \times 63{\cdot}352$ . . . . =50·93

and taking this length as the sine of the angle of the intrados to the length of the impost as radius, as in Example I., we have

$\frac{50{\cdot}93}{62} = {\cdot}8214516 = \sin 55^\circ\ 13'\ 49'' = \sin \beta_2$, and

to this angle the axial length, the angle of the extrados, and the focal distance, must be adjusted.

tan of adjusted angle, or $\tan \beta_2 = 1{\cdot}4404351$

secant of do. do. $\sec \beta_2 = 1{\cdot}7535243$

axial length $= a \cot \beta_2 = 36{\cdot}275 \times {\cdot}694234$ =25·183

It may be observed that the tangents of the angles of the intrados and extrados are the developments of the arcs of the intrados and extrados respectively, and, therefore, having found one, the other may be determined by proportion ; thus,

$\frac{r+e}{r} \tan \beta_2 = \tan \phi = \frac{17{\cdot}32 + 3}{17{\cdot}32} \times {\cdot}4404351$

$= 1{\cdot}6899330 = \tan$. $59°23'7''$

secant of do. $= 1{\cdot}9636236$

δ, Difference of angles of intrados and extrados . . . . $= 4°9'18''$

tan of ditto $= {\cdot}0726257$

cos of ditto $= {\cdot}9973717$

CO, Theoretic eccentricity $= \frac{c \cdot \cot^2 \theta}{a}(r+e)$

$= \frac{30 \times (1{\cdot}732^2)}{36{\cdot}275} \times (17{\cdot}32 + 3) \qquad = 50{\cdot}414$

Adjusted ditto $= r \cot \theta \tan \phi = 17{\cdot}32 \times 2 \times 1{\cdot}689933$ . . . . . . $= 50{\cdot}696$

The parallel and twisting rules for working the beds of the voussoirs must be equal to the depth of them, namely, 3 feet long each, and if applied 3 feet 6 inches asunder, measured on the soffit of the voussoir, their other ends must be at the distance $l_2 = l \frac{\sec \phi \cos \delta}{\sec \beta_2} = 42 \times \frac{1{\cdot}963 \times {\cdot}997}{1{\cdot}753}$ $= 46{\cdot}87$ inches, the divergence in this case being $4{\cdot}87$ inches, and the breadth of the twisting rule must be $l_2 \tan \delta = 46{\cdot}87 \times {\cdot}0726 = 3{\cdot}40$ inches greater at one end than at the other.

The angular plate for marking the skewbacks, or checks, and drawing the axis lines on the soffits of the voussoirs, will be as follows :—If the base be 24 inches, the perpendicular will be $24 \times \tan \beta = 24 \times 1{\cdot}44 = 34{\cdot}46$ inches, the greater angle 55° 13′ being that at which the courses rise from the impost.

To obtain the angle of the end of the impost which forms the first joint in the face from which the arch springs, we have

$$\frac{\text{Eccentricity} + \text{radius} - \text{versed sine}}{\text{half oblique span}} = \text{tan of angle of the face of springer, or end of impost.}$$

This is the best practical form to put it in, and the reason for it will be immediately evident on referring to the diagram.

The following Table exhibits the method of computing the Ordinates for obtaining the development of the face of the arch.

1.	2.	3.	4.	5.	6.
Angles A *Ca* or ϵ	Log sin ϵ	Log. r cot θ sin ϵ or log *be*	Nat. num. *be*	Values of *bf*	Ordinates *e* or *be*−*bf*
° 6	9·0192346	0·4963431	3·1357	2·5978	0·5379
12	9·3178789	0·7949874	6·2371	5·1956	1·0415
13	9·4899824	0·9670909	9·2702	7·7934	1·4768
24	9·6093133	1·0864218	12·2017	10·3912	1·8105
30	9·6989700	1·1760785	14·9995	12·9890	2·0105
36	9·7692187	1·2463272	17·6330	15·5868	2·0472
42	9·8255109	1·3026194	20·0733	18·1846	1·8887
48	9·8710735	1·3481820	22·2937	20·7824	1·5113
54	9·9079576	1·3850661	24·2698	23·3802	0·8896
60	9·9375306	1·4146391	25·9800	25·9780	0·0020
60 0′ 10″	9·9375308	1·4146393	25·9800	25·9800	0·0000

EXPLANATION OF THE TABLE.

Column No. 1 contains the different assumed values of the arcs whose ordinates have to be found; and because the whole arc contains 60° 0′ 10″, the even degrees, or 60°, have here been decimally divided, and the proportionate part corresponding to 10″ has been first subtracted from the obliquity 25·98, leaving 25·978, which corresponds to 60°, and which was then decimally divided. This is done merely for the purpose of facilitating the operation.

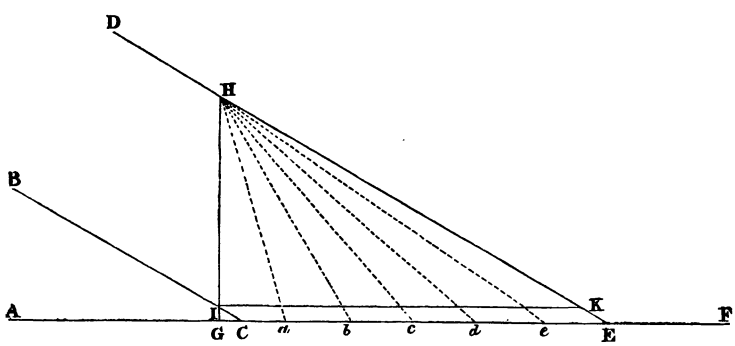
$$\begin{aligned} \text{Here } \log r &= \log 17{\cdot}32 \text{ or } \quad 1{\cdot}2385479 \\ \log \cot \theta &= \log \cot 30^\circ \text{ or } 10{\cdot}2385606 \\ \log r \cot \theta &= \quad 1{\cdot}4771085 \end{aligned}$$

and this added to each of the numbers in Column No. 2, produces the numbers in Column No. 3, everything else being similar to the example before given.

These two examples of the method of making the requisite calculations, we think, will be quite sufficient.

In a bridge of very great obliquity the quoins are not only difficult to preserve, but are unavoidably broken off either in setting, by settlement, or by accidental blows. In order to get rid of this objection, we have practised the following artifice. So much of the acute quoin of the abutment is cut off at right angles to the face of the bridge, as gives to the arch the appearance of having another voussoir. The quantity thus cut off from the acute quoin is gradually diminished to the opposite or obtuse quoin, where the cutting vanishes ; by this contrivance no angle less than a right angle is anywhere presented on the exterior of the work. The new intradosal line thus obtained is a larger elliptic curve, and the effect produced is elegant and pleasing to the eye. It is represented in the plan, elevation, and section of Plates X. and XII. The former is a design for a segmental arch at an angle of 30°, and the latter of a semicircular one at an angle of 25°.

The following is the method which we have practised in determining the quantity to be cut off each voussoir,



and thereby to obtain the outer ellipsis. Let ACB in the annexed figure be the acute quoin of a bridge, and DEF the obtuse quoin : determine from the elevation the distance CG, such that an additional voussoir may make its appearance in the arch at the springing ; then draw GH perpendicular to AC and meeting the opposite impost ED at H ; from I, where GH intersects CB, draw IK parallel to the face CE. Now all that portion of the voussoirs which lies between IK and GE must be cut to form the outer ellipsis, thereby giving the arch what we commonly call a bell-mouth. No part of the arch lying on the contrary side of or within the line IK is to be altered. The point H is at the level of the springing of the arch ; and if we suppose the line GH to revolve about H, as a centre, from I to K, as indicated by the dotted lines H*a*, H*b*, H*c*, H*d*, H*e*, and long enough to reach the face GE in every position ; also always in contact with the soffit of the arch in the line IK, at the same time cutting off the under side of the voussoirs, so that it (that is the line GH) shall touch the lines IK, GE, and the intervening space at the same time, then the bell-mouth will be produced.

By referring to Plates VI., VIII., XI., and XII., it will be observed that in each case the voussoirs, in the elevation, have rusticated steps on the extrados ; these steps form a horizontal bed for the foundation of the spandril walls ; without them the wall immediately over the acute angle would be in danger of sliding off the arch.

CHAPTER V.

MODE OF ERECTION.

THE impost having been prepared, as described in the last chapter, the centre should be set in its place, and well lagged; all the laggings should be fastened down, and should be long enough to project a few inches beyond the face of the arch. The centre should now be marked in the following manner, for the guidance of the masons in setting the voussoirs.

First, the lines for the face, which form the extreme outside, and are perfectly straight on the plan, should be described on the laggings; these are the lines AB, A′B′, fig. 24, Plate VIII. Then bisect each face, and draw a line CC′ joining the middle point of each, and let this line be divided similarly to the impost. All the lines which have now to be put on are spirals, for which purpose a long, broad, thin, flexible straight-edge should be provided. It is usually made of a half-inch deal board, about 12 inches broad and 25 feet long, with one or both edges perfectly straight. Now if the bridge be of such dimensions that the straight-edge will extend from the impost to the crown (that is, half the length of the heading spiral), let the straight-edge be divided into the proper number of voussoirs in the half-arch; if there be a keystone, it is obvious there must be half the keystone, with half the number of voussoirs, on the straight-edge. Then apply it from the first division on the impost, to the first division on the crown, as from A′ to C′, fig. 24, Plate VIII. and draw a line thereby on the laggings with a pencil, at the same time accurately marking with a point, the divisions *a*, *b*, *c*, *d*, *e*, etc., of the voussoirs; let this be done on the heading spiral, AC and A′C′, next to each face. Now begin at the obtuse quoin, and draw a line on the laggings with a straight-edge, from the first joint in the heading spiral, next to the face of the arch, to the bottom of the first division or skewback in the impost. Let the same be done from the second to the second, third to third, etc., until the whole are united; the lines thus drawn are the spiral coursing joints of the voussoirs; some of these lines, *a a′*, *b b′*, *c c′*, are shown projected in plan at fig. 24, and in elevation at *a a′*, *b b′*, *c c′*, etc., fig. 25. If the arch be entirely of stone, these lines will coincide with the bed-joints of the voussoirs; if the arch be wholly or partially of brickwork, they are the lines to which the courses of bricks must be parallel.

In the latter case, in order that the soffit of the arch may look well, the brickwork should have regular half-brick bond throughout; and to enable the bricklayer to do it accurately, it will be necessary to describe on the laggings a convenient number of heading joint spirals, parallel to those first put on (which were divided for the coursing joints). If they be described at the distance of two bricks' lengths asunder all over the laggings, it will be sufficient. Some of these lines are shown intersecting the line CC′ at *u*, *v*, *w*, *x*, *y*, etc.

CHAPTER VI.

PRINCIPLES OF PROJECTION.

A WORK of this nature would be incomplete if the method of accurately projecting the spiral lines in plan, elevation, and section were not given, especially as we are aware that very few are familiar with a neat and correct way of doing it.

When all the lines in a drawing of an oblique arch are accurately delineated, a beautiful effect is produced, but when otherwise, nothing can look more wretched. The curves formed by the projection of the spiral under various angles are extremely delicate, and there is no possibility of drawing them well without moulds made specially for the purpose, which may be cut out of thin wood, or card-board. The mode of obtaining these curves shall be next exhibited.

Let it be required to make the moulds for a drawing of the bridge in Example I.

THE MOULDS FOR THE PLAN.

Fig. 26, Plate IX. Describe the semicircle ABC with a radius equal to that of the intrados. Divide each quadrant into the same number of equal parts, say ten, and numbered as in the figure.

Make the angle ADC=the angle of obliquity of the bridge=θ in the formulæ. Produce CD to E, making DE=the axial length as found by calculation. Make AF parallel and equal to CE, an l GF=DE. Let the axial length GF be divided into the same number of equal parts as the semicircle, namely twenty, and the mutual intersections of the lines drawn from the semicircle ABC, parallel to AF, and those drawn from the divisions of the axial length GF, and perpendicular thereto, will give points in the horizontal projection of the spiral of the coursing joint DF, as has been before more particularly explained in Chapter I.

Next, let the obliquity CD be divided into twenty equal parts, and draw perpendiculars to CD from each of these divisions, and the mutual intersections of these perpendiculars with the lines drawn from the semicircle ABC will give points in the curve AD, which is the horizontal projection of the spiral of the heading joints of the voussoirs; both these spirals belong to the *plan* of the *soffit*, or intrados; when similar ones for the extrados are required, they can of course be had by the application of the same rules which have been before given.

THE MOULDS FOR THE ELEVATION OF THE FACE OF THE ARCH.

Fig. 26. Two moulds are required for this elevation, namely, one for the vertical projection of the coursing

joint DF, and the other for the vertical projection of the heading joint DA. The vertical projection of these curves is parallel to the line ADH, to which perpendiculars must be drawn from each of the twenty points in the curves by which they were traced out, and then, because each of these points must have the same altitude in every vertical section, being only seen under different angles, the ordinates 1, 2, 3, 4, etc., in these curves are the sines 1, 2, 3, 4, etc., in the semicircle ABC, as will be obvious from an inspection of the lines drawn to the equal semicircle of the same radius standing on the line AD.

MOULDS FOR THE VERTICAL SECTION THROUGH THE CROWN OF THE ARCH PARALLEL TO THE AXIS OF THE CYLINDER.

Fig. 26. Two moulds are required for this elevation, namely, one for the projection of the coursing joint DF, standing on the axial length GF as its base, and one for the heading joint DA, standing on the obliquity AG or CG as its base; the ordinates of these curves are also drawn perpendicular to their bases from the same, twenty points in each, by which their projections on the plan were obtained: each ordinate respectively is the sine of the corresponding arc in the semicircle, as will be immediately apparent by inspecting the diagram. Further reference than this cannot be required for elucidation.

All of these projections of the spiral are curves of sines differently proportioned; those last described are known as harmonic curves;[1] and the ordinates being equidistant, it is obvious that the curves may be obtained without projection, by dividing their bases equally, and setting off the ordinates from a table of sines. Their bases are known (axes more correctly) by the formulæ before given. Formulæ for determining by calculation the unequal distances of the ordinates of the curves on the oblique line ADH might be given; but as it is not likely that draftsmen would amuse themselves by using them, they are not inserted.

It is worthy of observation that the vertical projection of the curve F 10′ G might be obtained from the horizontal projection F 10 D; for if the latter were divided in the middle at 10, and joined by bringing the ends D and F together, the curve F 10′ G would be the result; and the same holds good with reference to the curves A 10 D and A 10′ G.

The curves projected in this diagram are those by which the drawings of the bridge given as the first example were made; and in order to be more explicit, the mode of applying them is as follows:—

THE LINES OF THE PLAN.

The lines of direction of the coursing joints, which are supposed to be marked upon the laggings, as shown in fig.

[1] See Leslie's 'Geometry of Curve Lines,' p. 416.

24, Plate VIII., are all parts of the curve F 10 D, fig. 26. They are described by a mould, one end of which, as F, is always in contact with one of the divisions of the impost, or of the impost produced towards I, fig. 24, whilst the other end D, fig. 26, is always in contact with the corresponding division of the opposite impost, and whilst in this position, so much of the spiral as may be intercepted by the two faces AB, A′B′ of the bridge is drawn in, and then the mould is similarly applied to the next division and so on.

The curves which are the projections of the heading joints, and which are those crossing the coursing joints everywhere at right angles, are drawn in by a mould cut out to the curve A 10 D. One end is always applied in contact with one impost, and the other at the corresponding division on the other impost, wherever a cross joint may be required to be described.

THE LINES IN THE ELEVATION OF THE FACE.

The lines of the coursing joints in the elevation of the face are described by a mould cut to the curve which stands upon the base DH, fig. 26, and one end D, being applied to one of the divisions of the impost (or of the impost produced), as shown in fig. 25, Plate VIII., so much of the curve as is comprehended between the projections of the two faces of the bridge must be drawn in; and this is to be repeated until all are completed.

The joints at right angles to these are, in a similar manner, drawn by means of a mould cut to the curve which stands on the base DA, which must also be applied to the impost.

THE LINES IN THE SECTIONAL ELEVATION THROUGH THE AXIS.

The lines of the coursing joints are described with the mould F 10′ G, fig. 26, the base FG being applied upon the impost, with one end of the mould at one of the intersections of the impost, or of the impost produced, as shown at *d*, *e*, *f*, *g*, *h*, *i*, *k*, *l*, *m*, etc., fig. 22, Plate VIII.

The heading joints which are at right angles to these are described by the mould G 10′ A, fig. 26, the base GA being always applied upon the impost, and one of its extremities at the proper intersection therewith. These few concise directions we think must be quite sufficient.

CHAPTER VII.

FURTHER INVESTIGATION AND CONCLUDING OBSERVATIONS.

It will naturally be asked to what extent of obliquity is it safe or practicable to construct an arch on the principles herein given. We will attempt a solution of this question, or at least to throw some light upon it. In order to do this, let us suppose a bridge formed with an oblique semicircular arch, constructed in every respect on the foregoing principles. It is required to determine at what altitude above the level of the axis of the cylinder the thrust of the arch will be perpendicular to the bed of the voussoirs. It will be observed that the angle which the coursing joints make with the horizon, at the level of the axis, always exceeds that which the bed joint of the first voussoir in the face of the arch makes with the horizon; or, in other words, the intradosal angle is always greater than the angle GB*k*, fig. 7. It also exceeds it by such a quantity that the bed of the voussoir at the springing of the arch is not at right angles to the face of the arch, and consequently does not present an abutment perpendicular to the thrust, which is essential to the perfect stability of the bridge.

Let BAC, fig. 27, Plate X., be the plan of the acute angle of the abutment of an oblique semicircular arch, BA being the face and AC the soffit, and let CAC′, fig. 28, be the elevation of the corresponding part of the intrados, the angle CAC′ being that which the coursing joint on the soffit makes with the horizon and with the axis of the cylinder: also let BAB′, fig. 29, be the angle which the bed joint of the first voussoir in the face of the arch makes with the horizon. Now, in order that the first voussoir resting on AB′, fig. 29, may not have any tendency to move laterally, either to or from the line AB, fig. 27, and that the direction of the line of descent upon the plane of the joint B′A, fig. 29, may be parallel to the plane of the face of the arch, or to BA, fig. 27, a line BC drawn upon the bed of the voussoir, at right angles to BA, must be horizontal.

Let AC, fig. 28, be made equal to AC, fig. 27; and AB, fig. 29, equal to AB, fig. 27; and let the perpendiculars CC′, fig. 28, and BB′, fig. 29, be drawn: now it is manifest that in order to have BC, fig. 27, horizontal, CC′, fig. 28, and BB′, fig. 29, which are the two extremities of BC, fig. 27, must be equal.

Now BB′, fig. 29 = tan BAB′ to radius AB, which we shall consider the unit: then resuming our former notation,

$$\tan \text{BAB}' = \frac{\text{CO}}{\text{AC}} = \frac{\cot^2 \theta}{\frac{1}{2}\pi}.r+e \div r \text{ cosec } \theta = \frac{\cot^2 \theta \sin \theta}{\frac{1}{2}\pi}.\frac{r+e}{r}.$$

Also $\frac{\cot \theta}{\frac{1}{2}\pi} = \tan \text{CAC}'$, fig. 28.

But AB, fig. 27, being radius, AC is secant,

$$\text{and } \frac{\cot\theta \sec\theta}{\frac{1}{2}\pi} = \frac{\operatorname{cosec}\theta}{\frac{1}{2}\pi} = CC', \text{ fig. 28.}$$

These values of the perpendiculars BB′, fig. 29, and CC′, fig. 28, are those which obtain at the level of the axis of the cylinder, where, except in cases of extreme obliquity, the latter is invariably the greater, and it remains to determine the altitude of the point above the axis where the two will become equal, and let the angle at which this takes place be τ. Now it may be shown that the tangent of the angle which the tangent to the intradosal spiral makes with the horizon, diminishes as $\cos\tau$, therefore the tangent of the internal angle at the point sought will be $\frac{\operatorname{cosec}\theta \cos\tau}{\frac{1}{2}\pi}$.

Next, let E, fig. 29, be the position of the point sought in the elliptic face.

Draw EF parallel to AC, CGDH being one-half of the semicircle; then $GCD = \tau$, $FC = r \sin\tau$, $DF = r\cos\tau$, $EF = r\cos\tau \operatorname{cosec}\theta$, and $CO = \frac{\cot^2\theta}{\frac{1}{2}\pi} \cdot r + e$

$$\text{Then } \tan IEK = \frac{CO+FC}{EF} = \frac{\left(\frac{\cot^2\theta}{\frac{1}{2}\pi} \cdot r + e\right) + \left(r \sin\tau\right)}{r\cos\tau \operatorname{cosec}\theta};$$

and equating these values of the internal and external angles, we have

$$\frac{\operatorname{cosec}\theta\cos\tau}{\frac{1}{2}\pi} = \frac{\left(\frac{\cot^2\theta}{\frac{1}{2}\pi} \cdot r + e\right) + \left(r \sin\tau\right)}{r\cos\tau \operatorname{cosec}\theta}:$$

Whence

$$\sin\tau = \sqrt{\left\{\left(1-\frac{r+e}{r}\cos^2\theta\right)+\left(\frac{\pi}{4}\sin^2\theta\right)^2\right\}} - \frac{\pi}{4}\sin^2\theta. \quad (1)$$

The value of sin τ, as given by this expression, evidently diminishes as θ diminishes ; and consequently, the greater the obliquity of the bridge, the more nearly will the point at which the thrust is parallel to the face approximate to the level of the axis of the cylinder.

If we make $\sin\tau = 0$ in this equation, we shall obtain the conditions which must exist when the thrust is parallel to the face at the level of the axis ; and this will be found to take place

$$\text{when } \sec\theta = \sqrt{\frac{r+e}{r}} \quad . \quad . \quad . \quad (2)$$

$$\text{or when } e = r(\sec^2\theta - 1) \quad . \quad (3)$$

Now it is observed that AC, fig. 27, is sec θ to radius AB, and consequently must be equal to $\sqrt{\frac{r+e}{r}}$ in order that BC may be horizontal ; and it is evident that the smaller θ is taken the more nearly will it be so.

Now let us find an expression for the distance AD of the point D from A, fig. 27, to which a horizontal line BD may be drawn when C is higher than B. Let AD, fig. 28, be equal to AD, fig. 27, and draw the perpendicular DD′; then DD′, fig. 28, must be equal to BB′, fig. 29.

Let the distance $\text{AD} = x$

Then . . . $\text{DD}' = \frac{x\cot\theta}{\frac{1}{2}\pi}$

$$\text{But BB}'=\frac{\cot^2\theta\sin\theta}{\frac{1}{2}\pi}\cdot\frac{r+e}{r}$$

Putting these equal, we have

$$\frac{x\cot\theta}{\frac{1}{2}\pi}=\frac{\cot^2\theta\sin\theta}{\frac{1}{2}\pi}\cdot\frac{r+e}{r}$$

from which we obtain

$$x=\cos\theta.\frac{r+e}{r} \quad . \quad . \quad . \quad (4)$$

Here it may be observed that if the value of x were only $\cos\theta$, the line BD, fig. 27, would be perpendicular to AC, or the voussoirs would slide down parallel to AC, and consequently would not have any tendency to press towards the centre of the cylinder; but as the value of $\frac{r+e}{r}$ is always greater than unity, the angle ADB will always be less than a right angle.

Having pursued the subject thus far analytically, we shall compute the value of τ for several different angles. Previous to doing so it will be necessary to assign some particular value to $\frac{r+e}{r}$ in Equation 1.

That which we have adopted is as follows:—e is made equal to the twentieth part of the oblique span, and is therefore equal $\frac{2r\operatorname{cosec}\theta}{20}=\frac{r}{10}\operatorname{cosec}\theta$, or radius being unity it is simply $\frac{\operatorname{cosec}\theta}{10}$. Then, making use of this value of e in each case we have

When $\theta=65°$ then	$\tau=27°\ 17'$
$\theta=55°$	$\tau=25°\ 13'$
$\theta=45°$	$\tau=21°\ 47'$
$\theta=35°$	$\tau=15°\ 38'$
$\theta=25°\ 40'$	$\tau=\ 0\ \ 0$

It will be observed that the last angle is given 25° 40′, at which the point E, fig. 29, descends to the level of the axis of the cylinder, and the whole semicircle is safe. This value of θ was obtained as follows:—

By Equation 3, $e=r\ (\sec^2\theta-1)$ or simply $(\sec^2\theta-1)$ to radius unity: and in the preceding calculations we have taken $e-\frac{\operatorname{cosec}\theta}{10}$; equating these we have

$$\frac{\operatorname{cosec}\theta}{10}=\sec^2\theta-1 \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

If we express secant and cosecant in terms of tangent and radius, and substitute their values so expressed in Equation 5, we have

$$\frac{\sqrt{(1+\tan^2\theta)}}{10\tan\theta}=\tan^2\theta \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

which reduces to

$$\tan^6\theta-\cdot01\tan^2\theta-\cdot01=0 \qquad (7)$$

which gives $\tan\theta=\cdot48051=\tan 25°\ 40'$.

It is obvious that θ will vary as e varies, and if e had been taken equal to $\frac{\operatorname{cosec}\theta}{11}$, then θ would have been 25° very nearly.

This then is the natural limit of obliquity to which an arch can, with scientific propriety, be constructed, when composed of an entire semicircle on the direct or square span, and the angle of the coursing joints is such a function of θ as that assigned to it by the principles explained in Chapter II.

It is very remarkable that hitherto most engineers have considered 45° as the greatest degree of obliquity at which it is safe to construct an arch, and it is probable that such an opinion has been formed in consequence of their having practically found the difficulty of making semicircular arches safe at that angle, and concluded that because the difficulty increases from 90° to 45°, it would continue to increase from 45° downwards, whereas the direct contrary is the case, as is evident from the preceding investigation.

When we say the contrary is the case, it is on the supposition that the joints of the voussoirs are *permitted* to radiate to the point which we have called the focus: but it is remarkable that nearly all practitioners endeavour to *prevent* this, and work the beds of the voussoirs, forming the face of the arch, such that the joints shall be at right angles to a tangent to a curve at that point, according to the custom hitherto practised by engineers and architects in all countries. This method of radiating the joints is also given by Nicholson in his work on stone-cutting; but by adopting it, the difficulty and insecurity of the work is greatly augmented, an effect which is conclusive as to the propriety of abandoning it.

When the joints are in the position which is the natural result of an oblique section of a helix, or all converging to one point, the arch which is semicircular on the square, and consequently a semi-ellipse on the oblique span, possesses the properties of a segment of a circle, and the joints are nearly at right angles to the line of pressure. The arch thus radiated very much resembles that given in Mr. Seaward's able work on the rebuilding of London Bridge, in which he has satisfactorily shown the customary mode of radiating an ellipse to be unscientific and a source of instability.

It is worthy of observation, that the stability of the lower part of the oblique arch depends upon the relative value of the internal and external angles; and that the former varies as $\cot\,\theta$, and the latter as $\cot^2\theta\,\sin\,\theta$; but $\cot^2\theta\,\sin\,\theta = \cot\,\theta\,\cos\,\theta$; therefore these angles vary as $1 : \cos\,\theta$, and consequently they approach to a ratio of equality as θ diminishes.

Further, $\cot\,\theta$ being greater than $\cot^2\theta$ when θ is greater than 45°, and the converse when θ is less than 45°; and because when $\theta = 45°$ the internal angle is 32° 29′, which nearly approaches the angle of repose of the voussoirs, we say for these reasons, an obliquity of 45° is about the most unsafe angle to which a *semicircular* arch can be applied. In the viaduct which carries the London and Birmingham Railway over the turnpike-road at Watford, the oblique arch is semicircular, at an angle of 55° and $\tau = 24°\ 52'$, which corresponds to the height of the seventh voussoir above the springing of the arch; up to this level the arch is secured by iron dowels and bolts 12 feet long, laid into the work behind. We had not then obtained a formula for the value of τ, but it was found mechanically, by permitting a ball to roll freely down the bed of each voussoir, beginning with the lowest, until its path of descent was observed to be parallel to the face of the arch, or to BA in fig. 27. This experiment was made on the model of the arch.

It should be remembered, that if that part of the semicircle which falls below D, fig. 29, is omitted in the construction of an arch, as it ought to be, and it is wished that the thrust of the arch shall be exactly in the plane of the face, then the internal angle must be taken of the same value as that found for the semicircle, and not be determined by the segmental formula given in Chapter II.

In Equation 1 the value of τ is deduced from that of the intradosal angle which has been assigned to arches which are semicircular on the direct span, or when the tangent of this angle $= \frac{\cot \theta}{\frac{1}{2}\pi}$; but the tangent of this angle in arches which are segmental has been shown to be $\frac{c}{a} \cot \theta$: now if we substitute the latter for the former and pursue the same steps as before, we obtain the following equation :—

$$\frac{c}{x} \operatorname{cosec} \theta \cos \tau = \frac{\frac{c}{a}(r+e) \cot^2 \theta + r \sin \tau}{r \cos \tau \operatorname{cosec} \theta}$$

$$\text{Whence } \sin \tau = \sqrt{\left\{\left(1 - \frac{r+e}{r}\cos^2\theta\right) + \left(\frac{a}{2c}\sin^2\theta\right)^2\right\}} - \frac{a}{2c}\sin^2\theta \quad . \quad . \quad . \quad (8)$$

It is to be observed that the only difference between this last equation and Equation 1, is found in the coefficient of $\sin^2\theta$, which is $\frac{a}{2c}$ instead of $\frac{\pi}{4}$; but when the segment becomes a semicircle $\frac{a}{2c} = \frac{\pi}{4}$. This last equation is therefore general, and applicable to both forms.

Some persons may be led to think these niceties may be dispensed with; the more especially as many bridges are erected without the knowledge or application of scientific principles. This is to be accounted for on the supposition that works which stand are copied, whilst those which fail are not. And further, we know that it is actually difficult to build an arch of small dimensions, which will not stand; but where unusual dimensions and proportions are involved, it is essential to proceed upon scientific principles in order to ensure success.

We have recorded in the first volume of the 'Transactions of the Institution of Civil Engineers,' page 185, that the Chevalier Mosca dare not venture to erect an oblique arch of 147 feet span, although he was aware that smaller ones had been built. But we do not hesitate to say that it is perfectly practicable and safe to erect an oblique arch of any dimensions, limited only by the same law which limits the extent of rectangular arches, namely the strength of the materials.

Throughout this work we have treated of such oblique arches only as are cylindrical upon their right section, being of opinion that such only ought to be made use of.

That many oblique arches are erected which have an elliptical direct section, we are well aware, but consider them deficient in stability, much more difficult to execute, and consequently more expensive, especially in masonry; and as far as we have been able to investigate the subject, they do not admit of the application of simple formulæ, similar to those we have established for oblique cylindrical arches. We do not think any combination of circumstances likely to arise in which it would be imperative on the engineer to erect an oblique elliptical arch, and for these reasons have rejected them altogether; nevertheless we shall be much gratified if anyone would take up the investigation of that subject and convince us of our error, if it be one.

ADDENDUM.

Although not strictly appertaining to the subject of the preceding Essay, the author thinks it not altogether out of place to give the following Table of Trigonometrical Equivalents, which he constructed for his own use, whilst engaged in the investigation of the foregoing and many other engineering subjects. It is in a novel form, and will be found extremely useful in those analytical operations in which the arithmetic of sines is made use of.

The quotients and products of the quantities arranged horizontally and vertically will be found at their mutual intersections.

TRIGONOMETRICAL EQUIVALENTS.

	DIVIDED BY						MULTIPLIED BY					
	sin	cosec	tan	cot	sec	cos	sin	cosec	tan	cot	sec	cos
sin	1	$\sin^2$	cos	sin tan	sin cos	tan	$\sin^2$	rad	$\frac{\sin}{\cot}$	cos	tan	$\frac{\sin}{\sec}$
cosec	cosec^2	1	cosec cot	sec	cot	cosec sec	rad	cosec^2	sec	$\frac{\text{cosec}}{\tan}$	$\frac{\text{cosec}}{\cos}$	cot
tan	sec	tan sin	1	$\tan^2$	sin	tan sec	$\frac{\sin}{\cot}$	sec	$\tan^2$	rad	$\frac{\tan}{\cos}$	sin
cot	$\frac{\text{cosec}}{\tan}$	cos	$\cot^2$	1	$\frac{\cos}{\tan}$	cosec	cos	$\frac{\text{cosec}}{\tan}$	rad	$\cot^2$	cosec	$\frac{\cot}{\sec}$
sec	sec cosec } $\frac{\text{cosec}}{\cos}$	tan	cosec	sec tan	1	$\sec^2$	tan	$\frac{\text{cosec}}{\cos}$	$\frac{\tan}{\cos}$	cosec	$\sec^2$	rad
cos	cot	$\frac{\sin}{\sec}$ } sin cos	$\frac{\cot}{\sec}$	sin	$\cos^2$	1	$\frac{\sin}{\sec}$	cot	sin	$\frac{\cot}{\sec}$	rad	$\cos^2$
unity	cosec	sin	cot	tan	cos	sec						

DESCRIPTION TO DIAGRAMS FOR FACILITATING THE CONSTRUCTION OF OBLIQUE BRIDGES.

By W. H. BARLOW, M.Inst.C.E.

Although the principles on which Oblique Bridges should be constructed have been fully investigated, there is yet wanting a ready means of applying them to practice. In Mr. Buck's work, which justly claims pre-eminence, both as to its theoretical as well as its practical exposition of the subject, the dimensions of the several parts are obtained by means of trigonometrical formulæ, and it is from this circumstance placed beyond the reach of many who are called upon to superintend the detail of such structures; on the other hand, the geometrical constructions given by other authors involve a degree of complication which few have the time or the patience to go through. The intention of the Skew Bridge Diagrams is to remove these difficulties. They are so made that, by a simple construction, multipliers are found from which all the requisite dimensions may be obtained for forming the templates and working the stones of bridges of every angle of obliquity between 30° and 90°.

The method by which the several dimensions are arrived at will be seen to be nothing more than that of trigonometrical formulæ, denuded of their mathematical symbols; the sines, tangents, etc., being obtained by measurement on a scale which admits of being read to the third place of decimals.

The twisting rules, and the angles between the joint-lines in the face and soffit, are obtained from the theoretical angle of intrados, and not from the adjusted angle; much complication being thus saved, and no perceptible inaccuracy incurred. The skewbacks at the springing are the only parts which are perceptibly affected by a slight variation in the angle of the intrados, and in these the corrected dimensions are obtained.

To facilitate the process, the lengths of the lines *l c* and *l h* (see diagram) are given in figures, in columns D and F, opposite the respective angles; and the divisions on the right-hand side of the diagram enable the lengths of the horizontal and vertical lines to be read off without applying the scale.

DIRECTIONS FOR USING THE DIAGRAM.

The data assumed to be known are the span, the rise, the thickness of the arch, the width, and the angle of obliquity. The dimensions, where not otherwise stated, are to be taken in feet and decimals.

Divide the rise of the arch by the span, and in column C find the number thus obtained; the number opposite thereto in column B, multiplied by the span, will be the radius of the arch.

In the vertical column E find the angle of obliquity, and find the point at which the horizontal line opposite this angle intersects the vertical line opposite the number obtained by dividing the rise by the span. Call this point *p*. The position of this point being found, all the dimensions of the bridge may be determined as follows:—

IF THE ARCH BE ENTIRELY OF BRICK.

Join *p c*, and where it intersects the circular arc at *e*, draw the vertical line *e g*. Read or measure off by the scale the distance *e g* and *l c*, then

$$\text{Span} \times lc = \text{oblique span},$$
$$\text{Width} \times lc = \text{length of impost};$$

and assuming 3 inches to be the thickness of a course of bricks, $\frac{3}{g\,e}$ = length of check on impost. The length of check thus obtained may either be adjusted so that each extremity of the impost coincides with the extremity of a check, or retaining the computed length of check, they may be so placed on the impost that the springing shall take place at the same elevation on both sides of the arch. After which, if the courses are properly gauged on the laggings of the centre, and the course-lines drawn down to their respective checks, no mistake can arise in laying the bricks.

IF THE ARCH BE ENTIRELY OF STONE OR HAVE STONE FACINGS.

Join *p c*, and where it intersects the circular arc at *e*, draw the vertical line *e g*, and produce it to *d*, making *g e* to *g d* as the radius of the arch is to the radius added to the thickness ; through *d* draw *b d* at right angles to *p c*. Join *d c* and produce it to *f*, and from *i*, where it intersects the circular arc, draw the vertical line *i k*. Through *p* draw *p l* horizontal, intersecting the tangent line *h l* in *l*, and join *l c*.

Then measuring off with the scale the lines

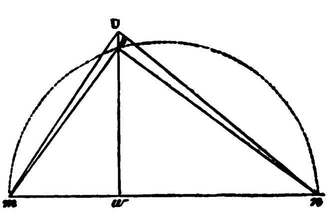
p c
b e

$b\,d$
$l\,h$ given in column F.
$l\,c$ given in column D.
$h\,f$, $g\,e$, $k\,i$ } These may be read off by the divisions.

The span $\times$ $l\,c$ oblique span.
„ width $\times$ $l\,c$ length of impost.
„ span $\times$ $p\,c$ = length of heading spiral.

Having fixed on the number of courses, the length of heading spiral divided by this number will be the correct thickness of the courses, and the thickness divided by $g\,e$ = the approximate length of check on impost, from which the number of checks will be found, and the length of the impost divided by this number will give the correct length of check on impost.

To mark the checks on the springing stone, it is convenient to use triangular templates of sheet-iron or wood.



The template for the intrados will be a right-angled triangle, having its side $m\,n$ = the length of check on the impost, and the side $m\,t$ = the thickness of the courses; consequently the third side is readily found by construction. The template for marking the checks on the extrados will be found from that of the intrados as follows. Let fall the perpendicular tw, and produce it to U, making wt to wU as the radius is to the radius added to the thickness of the arch; join mU and nU, and mnU is the template for the extrados. Also the angles $m\,n\,t$ and $m\,n$U are the corrected angles of intrados and extrados.

TWISTING RULES.

Let D be the distance in inches at which the twisting rules are to be placed apart at the intrados, the length of the rules being made the exact thickness of the arch or depth of voussoirs.

Then D × be = the divergence or additional distance in inches of the rules at the extrados.

And D × bd = the difference in width of the upper ends of the rules in inches, the lower ends being equal, and one rule parallel.

The radius of the curve of the spiral line of the intrados will be $\frac{\text{radius}}{(ge)^2}$.

In the elevation of the oblique face, the joint lines radiate from a point below the centre of the cylinder; this distance (called by Mr. Buck the eccentricity) is radius × lh × fh.

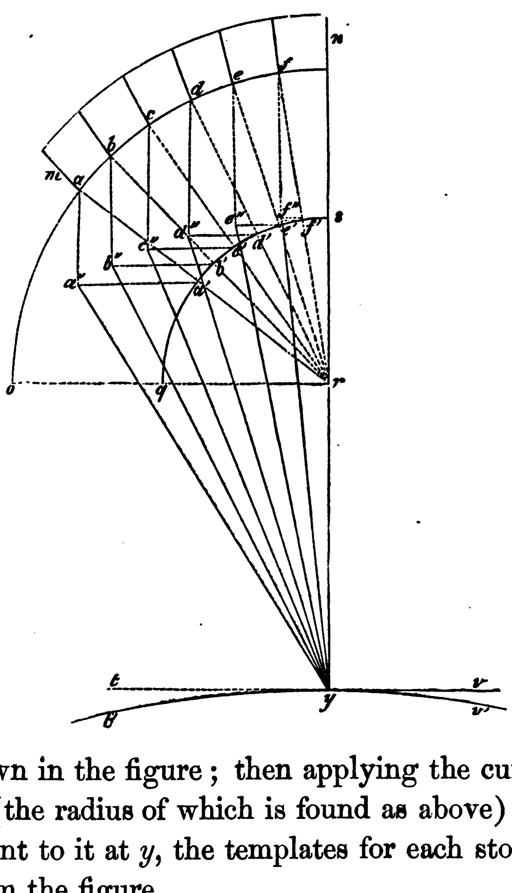
TEMPLATES FOR WORKING THE QUOIN-STONES IN THE FACE OF THE ARCH.

To find the angles made between the joint-lines in the face and soffit.

Let mn[1] be the elevation of the half arch on the square, and or its radius. Make rs = radius × ki, and with this distance describe the arc qs. Produce nr to y, making ry = radius × $\frac{ki}{lh \times fh}$.

[1] The demonstration of this construction is given in the 'Civil Engineer and Architect's Journal,' for 1841, page 364, and it has been used in practice on the Manchester and Birmingham Railway.

From the joints $a\ b\ c\ d\ e\ f$ draw lines to the centre r, intersecting the arc $q\,s$ in $a'\ b'\ c'\ d'\ e'\ f'$. From the points $a'\ b'\ c'\ d'\ e'\ f'$ draw horizontal lines, and from the points $a\ b\ c\ d\ e\ f$ draw vertical lines, intersecting each other in $a''\ b''\ c''\ d''\ e''\ f''$, and from these last points draw lines to



y, as shown in the figure; then applying the curve of the intrados (the radius of which is found as above) so that tv is a tangent to it at y, the templates for each stone will be found from the figure.

The templates for the soffit may be obtained by a development of the spiral curve on a scale sufficiently large to measure from, or from the course-lines and the face-line as drawn on the laggings of the centre at the time of erecting, the latter being the readiest for the workmen.

EXAMPLE.

The lines *pc*, *dg*, etc., drawn on the diagram, show the construction for obtaining the dimensions for a bridge with a stone arch, of which the angle of obliquity is 50°, the span 30 feet, rise 7·5 feet, thickness of arch 2 feet, and the width of the bridge 20 feet.

Here the rise divided by the span $=\frac{7\cdot5}{30}=\cdot250$. Finding this number in column C, the number opposite thereto in column B is ·625:

Then $\cdot625 \times 30 = 18\cdot75$ ft. = radius of arch.

The point *p* having been found, and the construction made as above directed, the lengths of the several lines required will be found by measurement to be as follows:

$p\,c = 1\cdot430$
$b\,e = \cdot037$
$b\,d = \cdot051$
$l\,h = \cdot839$ given in column F.
$l\,c = 1\cdot305$ given in column D.
$h\,f = \cdot802$ } These may be read off by the divisions.
$g\,e = \cdot586$ }
$k\,i = \cdot624$

Then span $\times\, l\,c = 30 \times 1\cdot305 = 39\cdot15$ ft. = oblique span.

" width $\times\, l\,c = 20 \times 1\cdot305 = 26\cdot10$ ft. = length of impost.

" span $\times\, p\,c = 30 \times 1\cdot430 = 42\cdot90$ ft. = length of heading spiral.

A convenient number of courses would be 33;

then $\frac{42\cdot9}{33} = 1\cdot3$ ft. = correct thickness of courses, and $\frac{1 \cdot 3}{\cdot 586} = 2 \cdot 22$ ft. = approximate length of check,

from which it is found that the number of checks will be 12.

and $\frac{26 \cdot 10}{12} = 2 \cdot 175$ ft. = correct length of check.

TWISTING RULES.

Let the distance at which the twisting rules are to be placed apart at the intrados be 30 inches ;

Then $\text{D} \times b\,e = 30 \times \cdot 037 = 1 \cdot 11$ in. = the divergence, or additional distance, of the rules at the extrados ;

And $\text{D} \times b\,d = 30 \times \cdot 051 = 1 \cdot 53$ in. = the difference in width of the upper ends of the rules or winding strips, the lower ends being equal, and one rule parallel.

$\frac{\text{radius}}{(g\,e)^2} = \frac{18 \cdot 75}{\cdot 586 \times \cdot 586} = 54 \cdot 6$ ft. radius of curve of spiral line of intrados.

Radius $\times\, l\,h \times h\,f = 18 \cdot 75 \times \cdot 839 \times \cdot 802 = 12 \cdot 62$ ft. = eccentricity.

TEMPLATES FOR THE QUOIN-STONES IN THE FACE OF THE ARCH.

To construct the figure for obtaining the angles in this case, we have

$$o\,r = \text{radius} = 18 \cdot 75$$

$$r\,s = \text{radius} \times k\,i = 18 \cdot 75 \times \cdot 624 = 11 \cdot 7$$

$$\text{and } r\,y = \frac{\text{radius} \times k\,i}{l h \times h f} = \frac{18 \cdot 75 \times \cdot 624}{\cdot 839 \times \cdot 802} = 17 \cdot 38.$$

THE END.

Spottiswoode & Co., Printers, New-street Square, and Parliament-street.

DIAGRAMS FOR FACILITATING THE CONSTRUCTION OF OBLIQUE BRIDGES.

By W. H. BARLOW, M.Inst.C.E.

DEMONSTRATION OF THE DIAGRAMS OF ANGLES OF FACE AND COURSING JOINTS.

Let $A\,B$, fig. 1, be the elevation of a skew arch, square to the axis, let c be the centre and a the point where any face joint $a\,m$ meets the intrados. Let $C\,D\,E$, fig. 2, be plan of the same, a' being position in plan of a in fig. 1.

Let a vertical plane $P\,P$, perpendicular to the axis of the cylinder, pass through a, intersecting the spiral surface $C\,f\,a'\,m'$ in the line $a'\,l'$. From l' draw $l'\,m'$ tangent to the spiral line of the extrados, meeting $D\,E$ in m'. Because $P\,P$ is perpendicular to the axis of the cylinder, the line $l'\,a'$ will be a straight line, at right angles to the tangent $l'\,m'$; the triangle $a'\,l'\,m'$ is therefore the projection of a right-angled triangle; in the short distance $l'\,m'$, the tangent nearly coincides with the spiral surface, hence the angle $l'\,a'\,m'$ will closely approximate that formed between the chord of the curved face joint and the line $l\,a$, which is the angle required to be found.

In fig. 1, let $l\,a$ be a projection in elevation of the line $l'\,a'$; join $a\,c$ and produce indefinitely; from o' where the axis intersects $D\,E$ (face) draw $o'\,u'$, parallel to $l'\,m'$, meeting $P\,P$ in u', produce $r\,o'$ to c'.

In fig. 1, on produced line $a\,c$, make $l\,a$ to $a\,u$, as $l'\,a'$ is to $a'\,u'$; through u draw $u\,o$ at right angles to $a\,u$, meeting Bc

produced, in *o*; through *l* draw *l m* at right angles to *l a*, meeting *o a* produced, at *m*, then will *l a*, *a u* be in the same straight line (from the properties of spiral surfaces), and the (real) triangle *a o u* will be a right-angled triangle, similar to and in the same plane with *a′ m′ l′*, having the

FIG. 1.

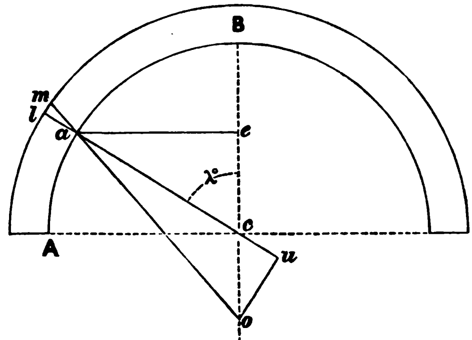
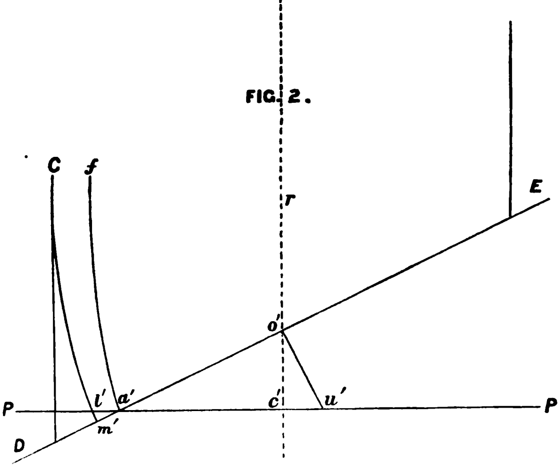


FIG. 2.



angle *o′ a′ u′* equal to the angle *l′ a′ m′*, also *o a* is the hypothenuse, and is drawn from the point *a* to the point of convergence of the face joints. *o′ u′* is also the hypothenuse of the triangle *o′ u′ c′*, and (real) angle *c′ o′ u′* equals the angle of extrados.

Let the angle $CDE=\theta$; $c' o' u'=\phi$; $a c B=\lambda$; $a c$ $=$radius$=$R. Draw $a e$ at right angles to $B c$, then :—

$a e=$R sin $\lambda=a' c'$; $o' c'=$R sin λ cot θ; $o' u'$ (real) $=$R sin λ cot θ sec ϕ;

$o u=$R sin λ cot θ tan ϕ; $u c=$R sin λ cot θ tan ϕ cot λ $=$R cos λ cot θ tan ϕ.

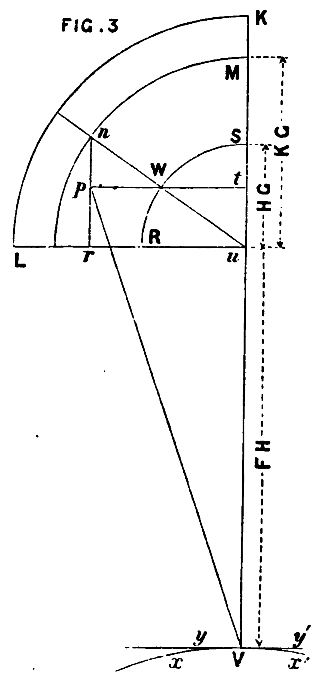
$a u=$R$+\{$R cos λ cot θ tan $\phi\}$. $\therefore$ Tan $u' a' o'$ (real)$=$

$$\frac{\text{R} \sin \lambda \cot \theta \sec \phi.}{\text{R} + \{\text{R} \cos \lambda \cot \theta \tan \phi\}.}$$

Let R cot θ sec $\phi=a$; and R cot θ tan $\phi=b$.

$$\text{Tan } l' a' m' = \tan u' a' o' = \frac{a \sin \lambda}{b \cos \lambda + \text{R}}.$$

Hence if the arch is semicircular, the expression for the angle $l' a' m'$, at the springing, is tan l' $a' m'=$cot θ sec ϕ. $c o=$ $o u$ cosec $\lambda=$R sin λ cot θ tan ϕ cosec $\lambda=$R cot θ tan ϕ.



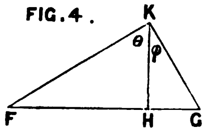
The formula $\frac{a \sin \lambda}{b \cos \lambda + \text{R}}$ leads to another construction for finding the angles of face and coursing joints.

Let $L M$, fig. 3, be an elevation of half the arch on the square, $n u$ being the radius.

From K in the straight line K H (fig. 4), draw K F and K G making H K F$=\theta$, and H K G$=\phi$: make K G$=n u$ (fig. 3)$=$radius, and through G, draw G F at right angles to K H ; upon the vertical line M u (fig. 3) produced, set off u V$=$F H and from u with distance H G, describe the arc R W S, draw $p t$ parallel to L u, and from n draw $n r$ parallel to

M u, intersecting $p\,t$ in p. Join p V, then p V t is the required angle, for the face and coursing joints correspond-

FIG. 4.



ing to the point n; for let F H (fig. 4)= r', R H= r' cot θ; K G= r' cot θ sec. ϕ; H G= r' cot θ tan ϕ; tu= r' cot θ tan ϕ cos λ; u V= r'; t V= $r' + \{r'$ cot θ tan ϕ cos $\lambda\}$; $p\,t = r\,u = r'$ cot θ sec ϕ sin λ (=K G sin λ). Hence $\frac{p\,t}{t\,\text{V}} = \frac{a \sin \lambda}{b \cos \lambda + r}$.

$y\,y'$ being drawn at right angles to M V and $x\,x'$ being the curve of the intradosal spiral (the osculating circle is practically correct enough) applied, so that $y\,y'$ is a tangent to it at V, p V x and p V x' will be the angles for the templates, for the bed joints at n; corresponding to the acute and obtuse sides of the arch.

From the properties of spirals the osculating circle will be found from the following expression—

Let R=radius of the arch on the square.

Let β=angle of intradosal spiral.

Let r=radius of osculating circle,

then $r = \frac{\text{R}}{\sin^2 \beta}$.

FIG. 2

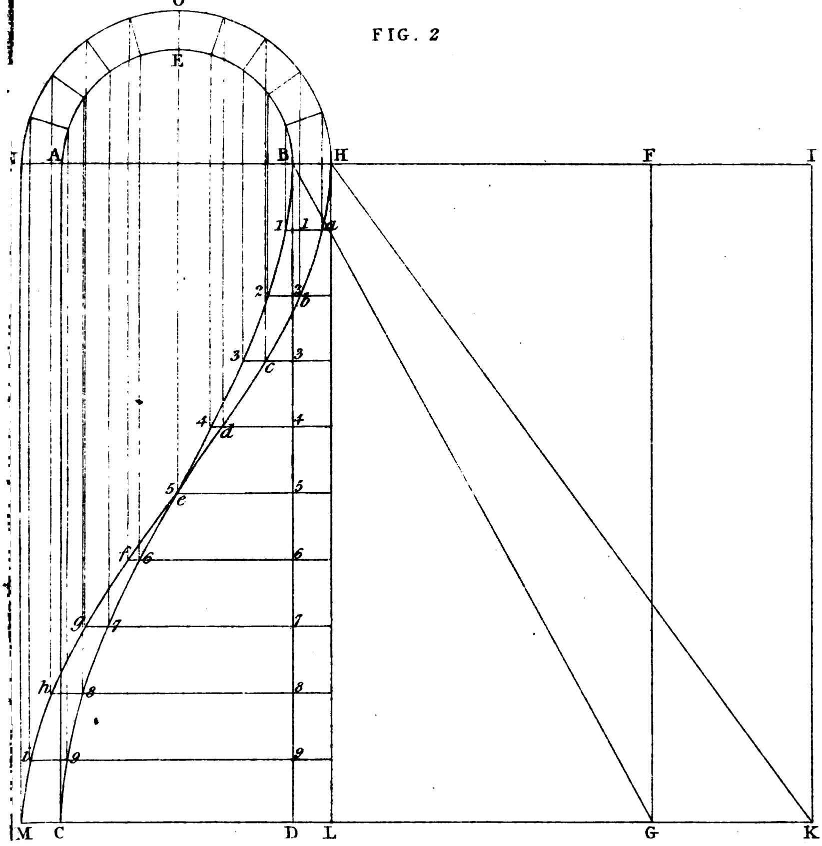


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FIG. 4

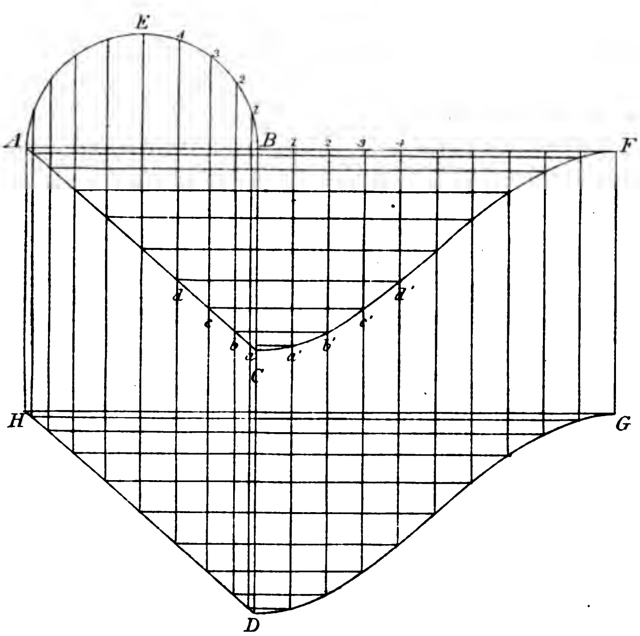

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Plate III.

FIG. 6

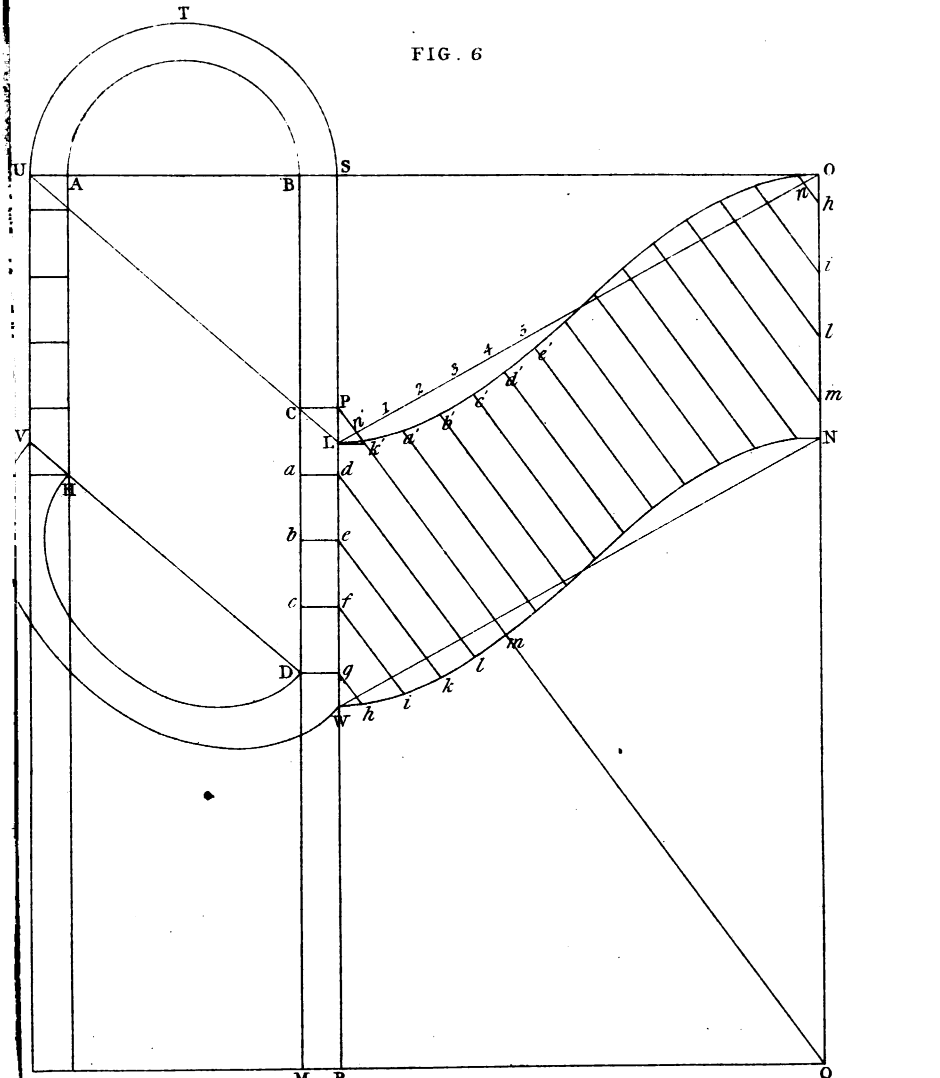

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Plate IV.

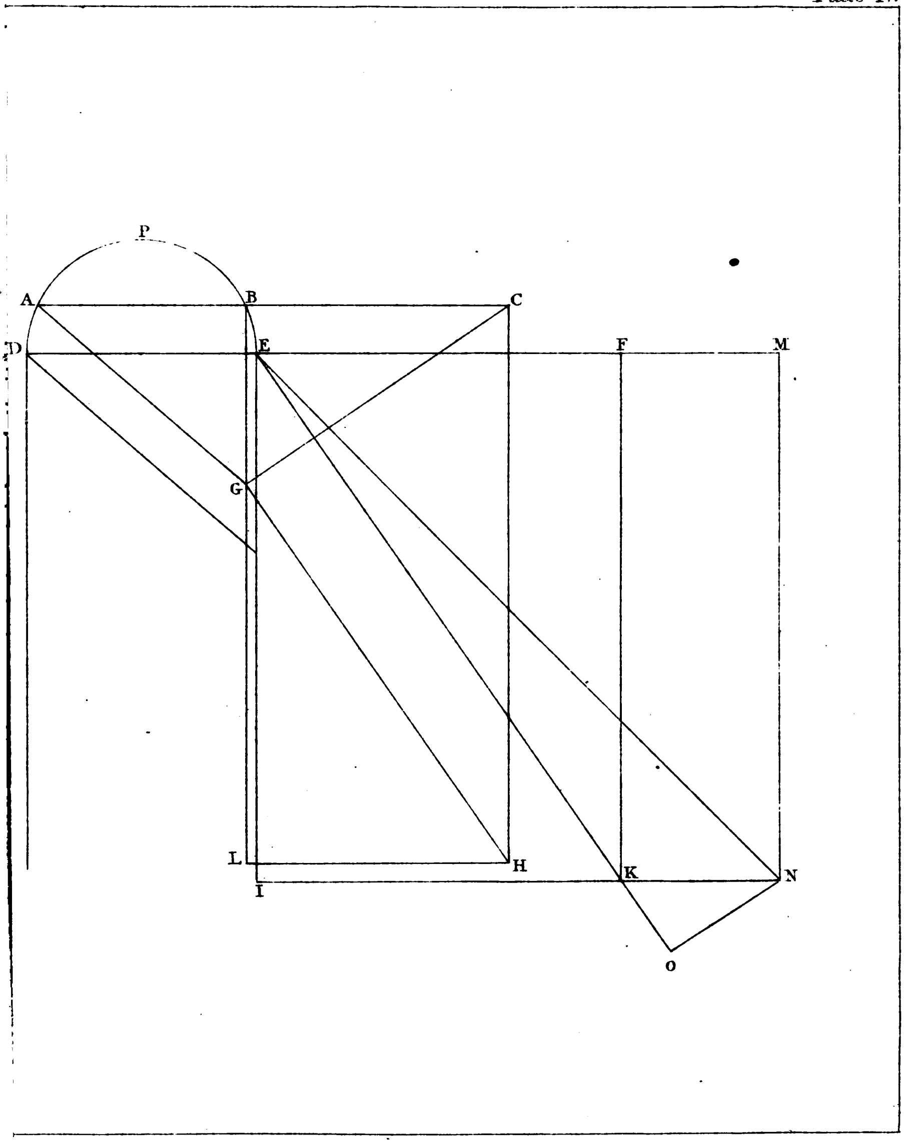

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Plate V.

FIG. 1

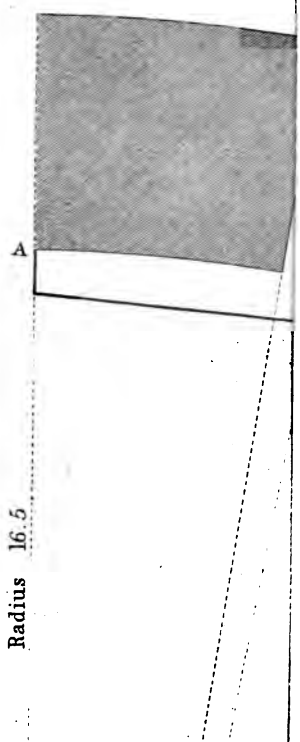


FIG. 15

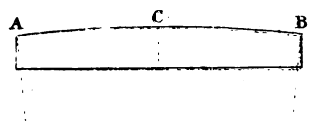
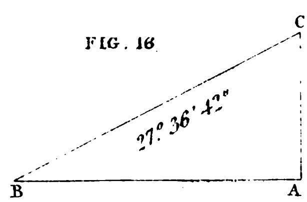


FIG. 16



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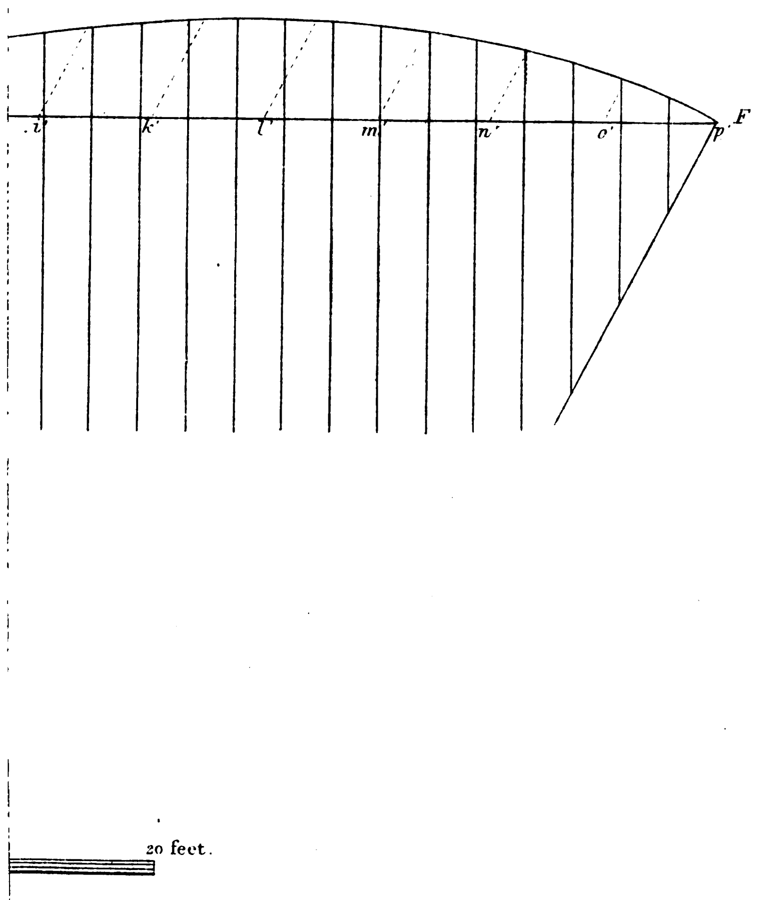
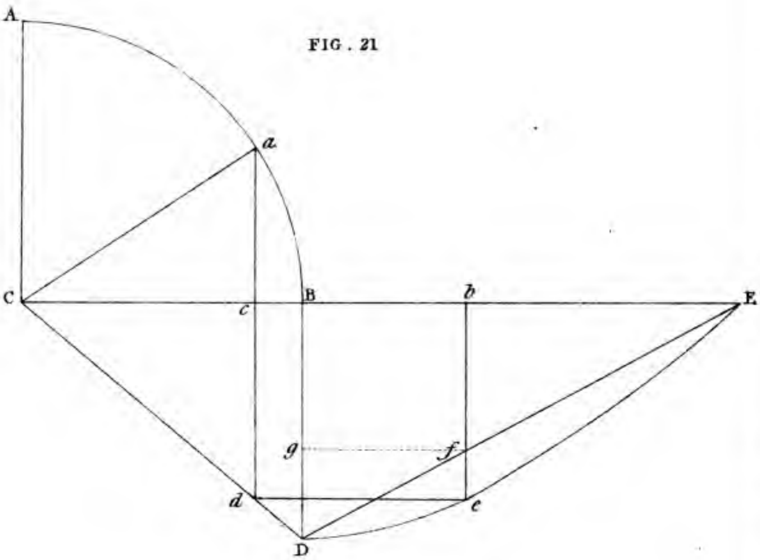
C

Plate VI.



J. W. Lowry sculp.

FIG. 21

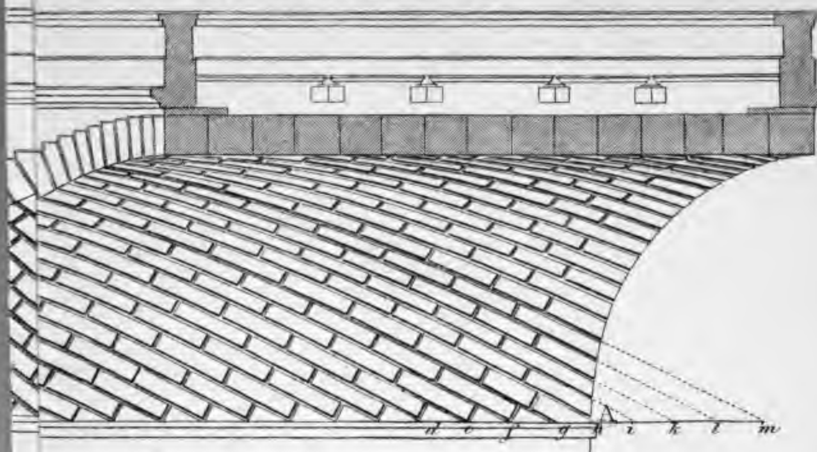


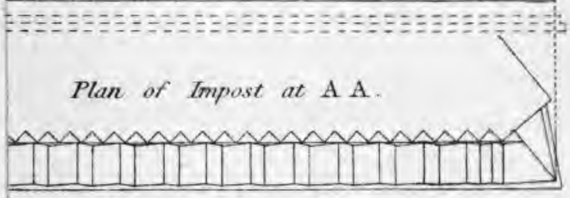
S. Bellin sc.

Stationers' Hall Court, London.

FIG. 22.

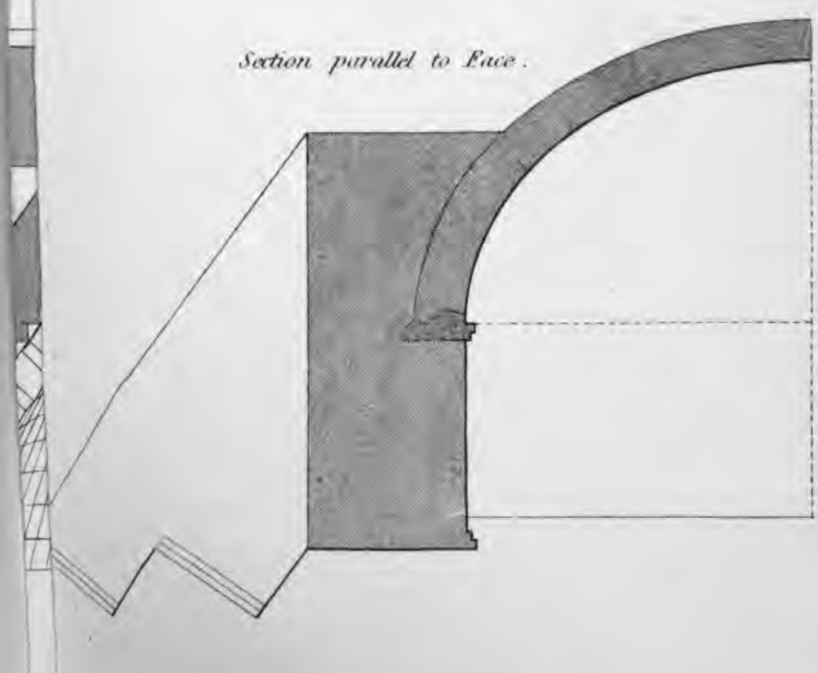
Sectional Elevation through the Axis.





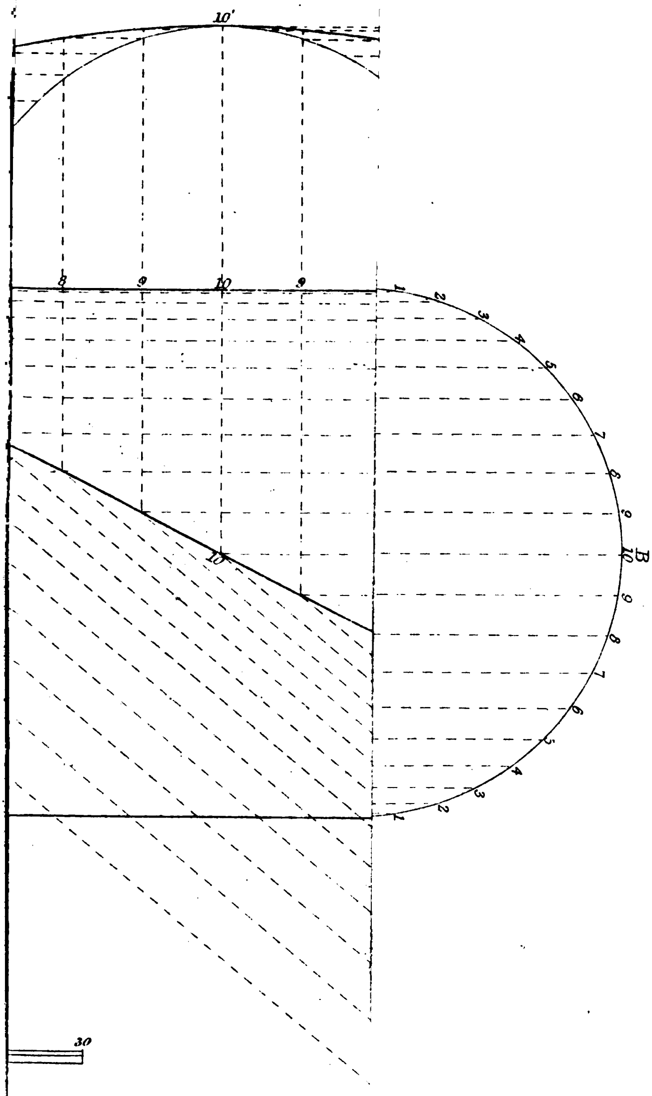
Plan of Impost at A A.

Section parallel to Face.



J. W. Lowry sculp.

Plate IX.



J. W. Lowry sculp.

Cros

FIG. 28

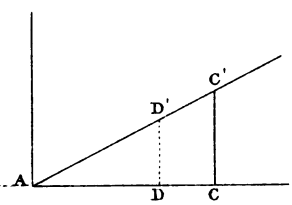
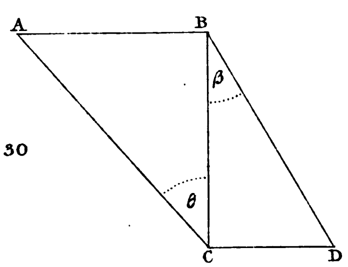


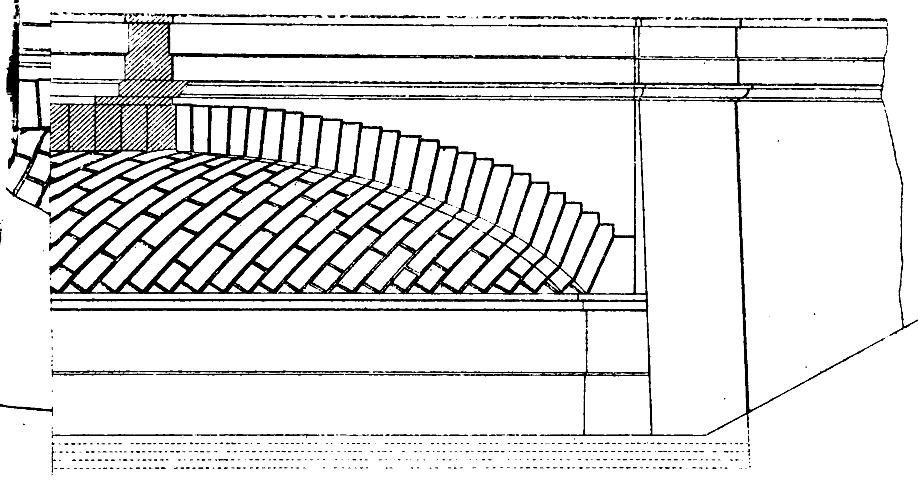
FIG. 30



J. Bellin sc.

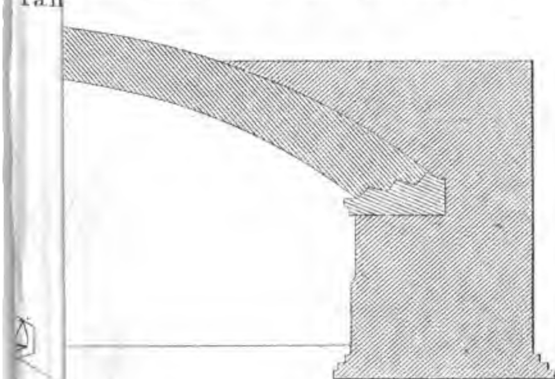
7. Stationers' Hall Court, London.

n through the Axis

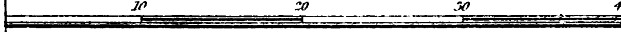


ction parallel to Face of Arch

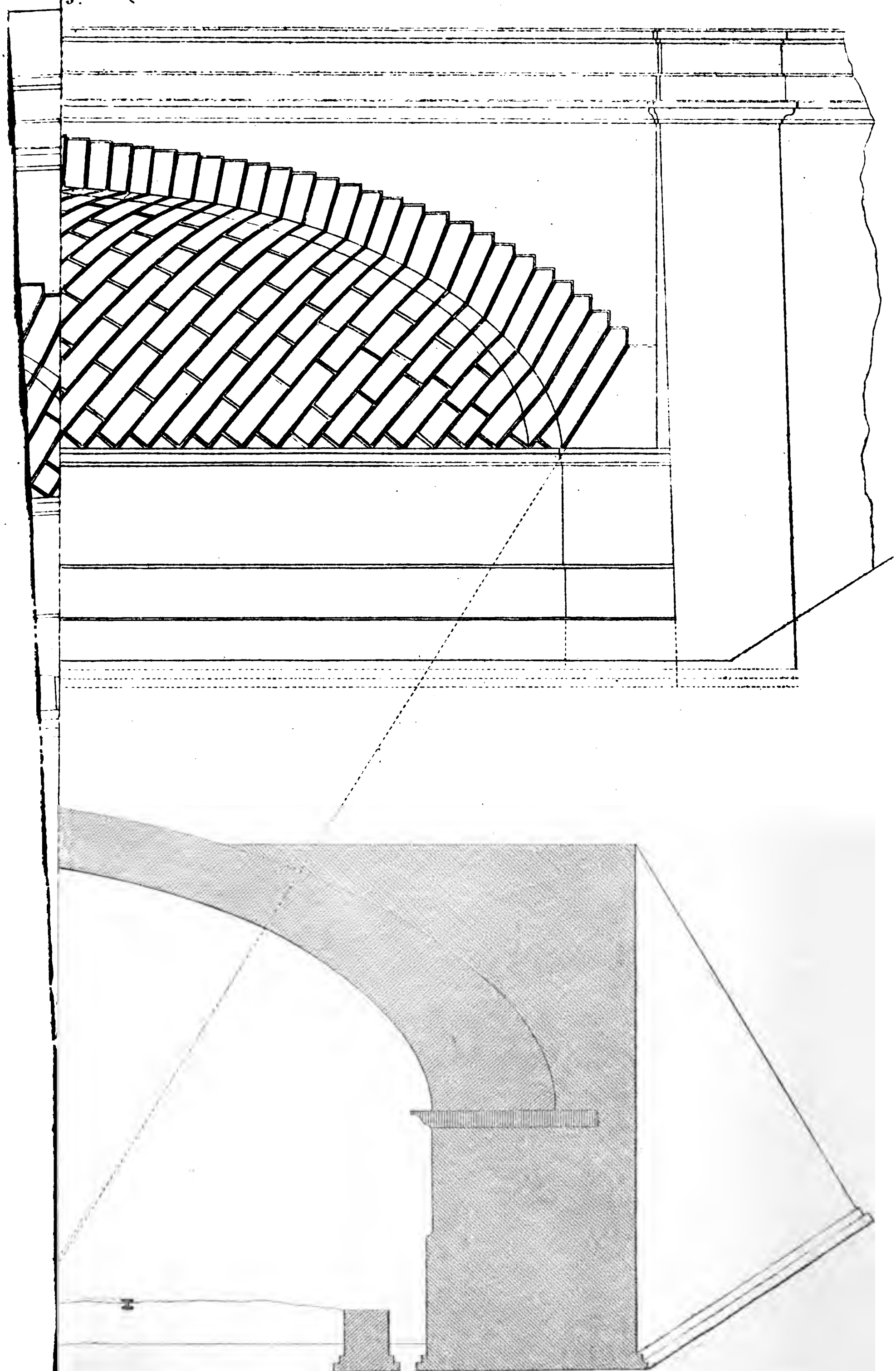
Plan



Scale



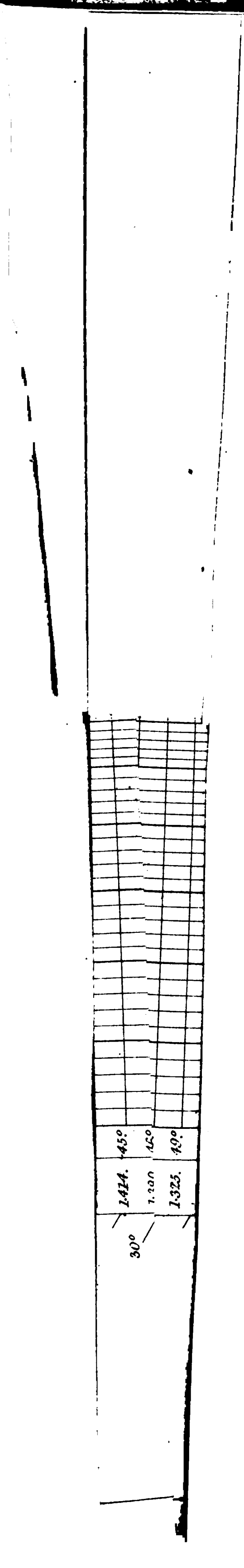
S. Bellin sc.



Section parallel to Face.

J. W. Lowry sculp.

Crosby Lockwood and C... ...


45°
49°
1.424.
1.200
1.325.
30°