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EDITED BY THE

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ARITHMETIC

ARITHMETIC

DESIGNED FOR THE USE OF SCHOOLS:

TO WHICH IS ADDED A CHAPTER ON

DECIMAL COINAGE.

BY THE

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1876.

THE LATE MASTER OF TRINITY.

Extract from Dr. Whewell's *Work on* 'A Liberal Education,' pp. 158, 159.

As the basis of all real progress in Mathematics, boys ought to acquire a good knowledge of Arithmetic and a habit of performing the common operations of Arithmetic, and of applying the rules in a correct and intelligent manner. This acquirement appears to be often neglected at our most eminent classical schools. Such a neglect is much to be regretted; for the want of this acquirement is a great practical misfortune, and is often severely felt in after-life. Many persons who are supposed to have received the best education which the country affords, are, in all matters of numerical calculation, ignorant and helpless, in a manner which places them, in this respect, far below the members of the middle class, educated as they usually are. Arithmetic is a matter of habit, and can be learnt only by long-continued practice. *For some years of boyhood there ought to be a daily appropriation of time to this object.*

PREFACE.

SINCE this book was first published, some considerable additions have been made to it, besides further modifications, with a view to correcting any defects which experience has from time to time detected, and bringing it up to the requirements of the present day. These have been carried out under my sanction and superintendence, and to my entire satisfaction, by the Rev. J. HUNTER, formerly of the National Society's Training College, Battersea, and chiefly at his suggestion; and I consider that the book has been much improved by them.

I have taken the opportunity, however, of my being in England for a few weeks, to insert some additional pages on the Metric System of Weights and Measures, the principles of which, by a rule of the Council of Education in force in 1872, were required to be taught to all children of Standards V. and VI. in schools under the control of the Government. The rule in question has, however, been since rescinded, as requiring too much from elementary schools, while the use of the Metric System has not yet been rendered compulsory by Act of Parliament. But the general adoption of that System in England is only, it seems plain, a question of time.

J. W. NATAL.

LONDON: *December* 24, 1874.

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ARITHMETIC.

Arithmetic is the science which treats of *numbers* — of the mode of expressing them — of the manner of computing by them — and of the various uses to which they are applied in the practical business of life.

The number *one* is called *unity;* and an *integer*, or *whole number*, is a collection of *ones, unities*, or *units*.

The figures, 1, 2, 3, 4, 5, 6, 7, 8, 9, denote, respectively, the numbers *one, two, three, four, five, six, seven, eight, nine;* the figure 0, called zero or a cypher, expresses *nought* or *nothing*; but by means of these figures, which are called the *ten digits*, or more commonly the *nine digits and zero*, any number whatever can be expressed. This is effected thus:

A figure standing *by itself*, or on the *right hand* of other figures, has its own proper value, expressing so many *units;*

A figure standing in the *second* place from the right is considered to express so many *tens* of units;

In the *third* place, so many *tens of tens*, or *hundreds* of units;

In the *fourth* place, so many *tens of hundreds*, or *thousands* of units, &c., according to the following Table, called the

NUMERATION TABLE.

&c.	Tens of Billions	Billions	Hundreds of T. of Millions	Tens of T. of Millions	Thousands of Millions	Hundreds of Millions	Tens of Millions	Millions	Hundreds of Thousands	Tens of Thousands	Thousands	Hundreds	Tens	Units
7	1	2	8	1	4	3	5	7	1	2	3	4	1	9

and so on to *trillions, quadrillions*, &c. if necessary

Notation is the art of expressing any given number by these figures; *Numeration* the art of reading them, when so expressed.

N.B. Examples in Notation and Numeration may be obtained from those given in Addition and Subtraction.

The Romans used I for 1, V for 5, X for 10, L for 50, C for 100, D or Iↄ for 500, M or CIↄ for 1000.

When any character was followed by one of *less* or *equal* value, the expression denoted the *sum* of their simple values; but when *preceded* by one of *less* value, the *difference;* thus III stood for 3, IV for 4, and VI for 6, XL for 40, and LXX for 70, &c.

Every ↄ annexed to Iↄ, and every C and ↄ joined to CIↄ, increased its value tenfold; thus Iↄↄ stood for 5000, CCIↄↄ for 10,000, &c.

A line drawn over a character increased its value a thousand-fold; thus $\overline{V}$ stood for 5000, $\overline{C}$ for 100,000.

The following *signs* are also made use of in Arithmetic:.

+ (*plus*) shows that the number before which it stands is to be *added;*

− (*minus*) that the number before which it stands is to be *subtracted;*

× (*into*) that the numbers between which it stands are to be *multiplied;*

÷ (*by*) that the number which stands before it is to be *divided* by the one which follows; and

= (*equal*) that the numbers between which it stands are *equal* to each other.

Addition.—When any numbers are taken together, or *added,* the resulting number is called their *sum.*

Ex. *Add*

	94163
	21954
	7812
	593
	35647
	4895
	165064

In order to add whole numbers together, we first place them under one another, with their units-figures in the same vertical line; we then add these figures thus, 5 *and* 7 *are* 12, *and* 3 *are* 15, *and* 2 *are* 17, *and* 4 *are* 21, *and* 3 *are* 24, i. e. 24 *units,* or 2 *tens* and 4 *units;* we set the 4 under the units-figures, to be the units-figure of the result, and *carry* the 2 *tens* to be added to the second or *tens* column; adding this in the same manner, beginning with the 2 carried, thus 2 *and* 9 *are* 11, *and* 4 *are* 15, &c., we find the sum of the column to be 36, i. e. 36 *tens*, or 3 *tens of tens* (i. e. 3 *hundreds*) and 6 *tens;* we set the 6 under the tens-figures, to be the tens-figure of the result, and *carry* the 3 *hundreds* to the *third* or *hundreds* column: pursuing the same course with this, we find the sum of this column to be 40, i. e. 40 *hundreds* or 4 *tens of hundreds* (i. e. 4 *thousands*) and 0 *hundreds;* we set the 0 under the hundreds-figures, to be the hundreds-figure of the result, and *carry* the 4 *thousands*, &c.

N.B. Any sums may be set at pleasure in Addition, and the Answers *proved* by repeating the operation, beginning with the *top* figure of the units column, when the result will be the same, if the sum be worked correctly.

EXAMPLES IN ADDITION.

1.	2.	3.	4.
321413	543123	536123	123456
452734	234512	453215	234561
130421	713145	1234	345612
3718	104234	4231	456123
24561	36142	51234	561234
341323	3451	613254	612345

5.	6.	7.	8.
761284	657890	692387	768453
612874	278679	4956	358428
8719	5798	87958	8796
46759	67843	769378	54937
587999	489567	5790	495
987678	37429	87658	876578

9. *Add together* five hundred and ninety-seven thousand six hundred and eighty-five, forty-nine thousand three hundred and seven, four hundred and nine thousand and sixty-seven, fourteen thousand and nineteen, seven hundred thousand and seventy-four, sixty-five thousand and nine.

10. *Add together* seven hundred and seven thousand four hundred and fifty-nine, ninety-eight thousand and seventy-four, six thousand eight hundred and seven, five hundred thousand three hundred and nine, seven thousand nine hundred and seventy-eight, nine hundred and nine thousand nine hundred and ninety-nine.

11. *Add together* fifty-five millions seven hundred thousand and five, seven hundred millions nine hundred and eight thousand two hundred and five, seventy-six millions fourteen thousand and fifty-nine, eight hundred and seventy-seven millions nine hundred and two thousand and forty-seven, seven millions eight hundred and four thousand five hundred and twelve, five hundred and seventy-five millions eight hundred and one thousand and ninety-nine.

12. *Add together* three hundred and nine millions four hundred and seventeen thousand and eighty-seven, six hundred and seventy-five thousand and forty-nine, seven thousand and ninety-seven millions eight hundred and fourteen thousand three hundred and five, seventy-nine millions five hundred and four thousand and forty-nine, six thousand and seventy-eight millions four hundred and thirty-nine thousand six hundred and forty-seven, seven thousand millions eight hundred and seventy-six thousand four hundred and twenty-nine.

Subtraction.—When one number is taken from another, or *subtracted*, the result is calle l the *remainder* or the *difference.*

Ex. *From* 794327
Take 342814
451513

In order to subtract one whole number from another, we first place the number to be subtracted under the other, with their units-figures in the same line; we then take the units-figure, 4, of the lower number from that of the other, 7, thus 4 *from* 7, 3, i. e. 3 *units*, and we place the 3 under the units-figures, to be the units-figure of the result; then we proceed to the tens-figures, and say, 1 *from* 2, 1, i. e. 1 *ten*, and we set down 1 under the tens-figures; then to the hundreds-figures, and say 8 *from* 3... *I cannot;* but if we take or *borrow* 1 out of the 4 *thousands* (leaving 3 thousands), and treat it as 1 *ten of hundreds*, we shall now have 13 hundreds in the upper line; we can now say 8 *from* 13, 5, i. e. 5 *hundreds*, and we set down 5 as the hundreds-figure of the result: and we have now to take 2 *thousands* from 3 *thousands*, or, which is just the same, but more convenient in practice, instead of supposing the upper figure, 4, *diminished* when we borrow 1, we may suppose the lower corresponding figure, 2, *increased*, i. e. we may *carry* one to it, and say 3 *from* 4, 1, i. e. 1 *thousand*, and so on.

N. B. Any sums may be set at pleasure in Subtraction, and the Answers *proved* by adding the remainder to the *lower* number, when the result will be the *upper*, if the sum be worked correctly.

EXAMPLES IN SUBTRACTION.

1.	765439 343418	2.	697438 635036	3.	758452 418234	4.	543625 492708
5.	683125 492816	6.	712345 538159	7.	564307 479176	8.	702306 475429

9. *From* six hundred and nine thousand seven hundred and one *take* three hundred and ninety-seven thousand and forty-nine.

10. *From* four hundred and fifty thousand and ninety-four *take* ninety-nine thousand nine hundred and nine.

11. *From* seven hundred and eighteen millions fourteen thousand and fifty-seven *take* ninety-seven millions eight hundred and four thousand seven hundred and sixteen.

12. *From* fifty-three thousand millions eighteen thousand and ninety-seven *take* forty thousand five hundred and twenty-eight millions seven hundred and six thousand seven hundred and nine.

Multiplication is the method of finding what number would result from adding several of the *same* numbers together; thus, if we add 6 *sevens* together, the result is

$$7+7+7+7+7+7=42,$$

the same number as that given in the Multiplication-table for the value of 6 *times* 7: and, since the same number is also the sum of 7 *sixes*, or the value of 7 *times* 6, it follows that, when two numbers are multiplied together, it matters not which we take as *multiplier.*

The numbers multiplied in any case are called *factors,* and the result is called the *product.*

Ex. 1.
```
3467
   2
----
6934
```

When the *multiplier*, as in Ex. 1., is not higher than 12, we first set it with the units-figure under that of the *multiplicand;* then we begin to multiply, saying, *twice* 7 *is* 14—*four and carry one*, i. e. we set down the 4 units under the units-figures, and carry the 1, which means 1 *ten*, to be added to the tens; we now proceed, *twice* 6 *is* 12 (i. e. 12 *tens*, since 6 means 6 tens), *and* 1 (i. e. the one carried) *is* 13. . . 3 *and carry* 1, i. e. we set down the 3 *tens*, and carry the 1, which means 1 *ten of tens* or 1 *hundred*, to be added to the hundreds, and so on throughout the line.

Ex. 2
```
   3467....2
    692....8
   -----
   6934
  31203
 20802
 -------
2399164....7
```

When the multiplier, as in Ex. 2., is higher than 12, we first set it under the multiplicand as before, and, having multiplied the upper line by the units-figure, 2, of the lower, as in Ex. 1., we now multiply by the tens-figure, 9, saying 9 *times* 7 *is* 63 (i. e. 63 *tens*, since 9 means 9 tens) . . . 3 *and carry* 6; i. e. we set down the 3 *tens*, and carry the 6 *tens of tens* or *hundreds*, and so on: we now multiply by the hundreds-figure, 6, of the lower line, in the same manner; and then add up the separate lines, when the result is the product required. The 2, 8, and 7, on the right, will be explained presently.

Ex. 3.	37218	Since it is immaterial which number we take as multiplier, it is best always to choose that which is simplest; and if it can be separated into two or more factors each less than 12 (thus $42 = 6 \times 7$), we may multiply separately by each, as in Ex. 3.
	6	
	223308	
	7	
	1563156	

N.B. Any number which can be *separated into factors* is called a *composite* number; any number which cannot be so separated, such as 7, 11, 13, 17, &c., is called a *prime* number.

Ex. 4.	3241	1	If the multiplier ends with one or more *cyphers*, the sum may be worked as in the annexed example, by which many useless cyphers are saved.
	2700	0	
	2268700		
	6482		
	8750700	0	

N. B. Any sums may be set at pleasure in Multiplication, and the Answers proved, either by repeating the operation with the *other* number for multiplier; or by the process of *casting out nines* (for the proof of which see *Algebra*), as follows: add up the figures in the upper number, divide this by 9, and set down the rem^{r}; do the same with the other number; then do the same with the product of these rem^{rs}, and with the product of the two numbers; and if the new rem^{rs} are the *same*, the sum is most probably right; but, if *different*, it is certainly wrong. Thus in Ex. 2., the first pair of rem^{rs} are 2 and 8, and their product 16; the rem^{r} from this is 7, the same as from the Ans^{r}: in Ex. 4., the first pair of rem^{rs} are 1 and 0, and their product is 0; the rem^{r} from this is 0, the same as from the Ans^{r}. *See* NOTE I.

It is desirable that the pupil should be made to apply one or both of these methods to the Examples below given.

EXAMPLES IN MULTIPLICATION.

1. 345673 × 2	2. 457632 × 3	3. 415763 × 4
4. 371281 × 5	5. 635432 × 6	6. 421375 × 7
7. 378914 × 8	8. 476539 × 8	9. 435976 × 9
10. 978564 × 11	11. 496782 × 12	12. 876549 × 12
13. 378125 × 16	14. 456932 × 18	15. 712436 × 24
16. 543817 × 27	17. 593654 × 30	18. 697128 × 36
19. 765438 × 40	20. 596437 × 45	21. 642198 × 60
22. 756328 × 72	23. 814765 × 84	24. 913748 × 96
25. 234915 × 123	26. 704745 × 615	27. 469830 × 369
28. 391525 × 861	29. 1174575 × 2214	30. 3523725 × 2583
31. 1644405 × 7749	32. 231549 × 8856	33. 463098 × 7380
34. 1389294 × 8900	35. 926196 × 7896	36. 2778588 × 9867

Division is the method of finding how often one number is contained in another, i. e. how often one number must be taken to make up another. Hence Division bears the same reference to *Subtraction*, as Multiplication bears to *Addition;* for we might go on subtracting the divisor from the dividend, and then from the 1st rem^r, then from the 2nd rem^r, and so on, until the final rem^r is either *zero*, or is *less* than the divisor itself; and if we counted the number of times we had subtracted it, this would be the result required, or, as it is called, the *quotient*. But the Multiplication-table will enable us much more easily to divide one number by another; thus, since 7 *times* 9 *is* 63, if we divide 63 by 7, we shall have the quotient 9, or if by 9, the quotient 7: and the method of applying it to more difficult cases will be seen by what follows.

Ex. 1. 4) 2379
$594\frac{3}{4}$

When the divisor, as in Ex. 1., is not higher than 12, we first set it in a loop before the dividend; then we take the first figure of the dividend, 2, i. e. 2 *thousands:* but, since 4 will not be contained at all in this, we take then the first *two* figures, 23, i. e. 23 *hundreds*, and say 4 *is in* 23 . . 5 *times and* 3 *over*, and we set down the 5, i. e. 5 *hundreds*, in the quotient, and carry the 3 *hundreds*, or 30 *tens*, to the tens-figure, 7, of the dividend: we have now 37 *tens*, to be divided by 4; we say, therefore, 4 *is in* 37 . . 9 *times, and* 1 *over*, and we set down the 9, i. e. 9 *tens*, in the quotient, and carry the 1 *ten* or 10 *units* to the units-figure, 9, of the dividend: we have now 19 units to be divided by 4; we say, therefore, 4 *is in* 19 4 *times and* 3 *over*, and we set down the 4, i. e 4 units, in the quotient, and place, as is usual, the final rem^r 3 over the divisor with a line between them, as $\frac{3}{4}$ (*three-fourths*), a quantity meaning $3 \div 4$, and called a *fraction*, of which more will be said hereafter.

It appears then that 4 will be contained 594 times in 2379, with 3 over; i. e. we might subtract 4 from 2379 594 times, and have still 3 remaining. This is an example in *Short Division.*

Ex. 2. 42) 379543 ($9036\frac{31}{42}$
378
154
126
283
252
31

When the divisor, as in Ex. 2., is higher than 12, we place it, as before, in a loop before the dividend, and the quotient in a loop after it; and we see that 42 will not be contained in the 3 (i. e. 3 *hundreds of thousands*), nor in the 37 (i. e. 37 *tens of thousands*), but will be contained 9 times in the 379 (i. e. 379 *thousands*); or, which is the same thing, but more convenient in practice, we take the first figure only of the dividend, and say 4 *is in* 37 . . 9 *times;* we set therefore the 9 (i. e. 9 *thousands*) in the quotient, and, multiplying 42 by 9, subtract the product, 378 (i. e. 378 *thousands*) from the dividend; and we have now the rem^r, i. e. 1 *thousand* or 10 *hundreds*, to be carried to the *hundreds:* we take in then the hundreds-figure, 5, of the dividend, and have now 15 *hundreds* to be divided by 42; we say then (42 *is in* 15, *or*) 4 *is in* 1 . . *I cannot;* we set, therefore, 0 (i. e. 0 *hundreds*) in the hundreds place of the quotient, and have now 15 *hundreds*, or 150 *tens*, to be carried to the *tens;* we take in then the tens-figure, 4, of the dividend, and have now 154 *tens* to be divided by 42; we say then 4 *is in* 15 . . 3, and we set the 3, i. e. 3 *tens*, in the quotient, and so on till, at last, we have the final rem^r 31, which we set over the divisor, as a fraction, and have the whole quotient $9036\frac{31}{42}$. This is an example in *Long Division.*

Ex. 3. But when the divisor, as in this case, is made up of two or more factors, less than 12, it is often more convenient to divide by each separately, as follows.

$$\begin{array}{r} 6)\underline{\;379543\;} \\ 7)\underline{\;63257\frac{1}{6}} \\ 9036\frac{31}{42} \end{array}$$

There is here a fraction $\frac{1}{6}$ over in the *first* quotient, and a rem^r $5\frac{1}{6}$ in the *second*, which, according to our previous practice, should be written $\frac{5\frac{1}{6}}{7}$; but such an expression may always be simplified (as will be shown hereafter) by putting the rem^r $5\frac{1}{6}$ in the form $\frac{31}{6}$, (which we obtain by multiplying the 5 by the 6, and adding in the 1); and then multiplying the 6 by the 7, so making $\frac{31}{42}$, the same as the fraction obtained by the other method. *See* NOTE II.

Ex. 4.

$$\begin{array}{r} 39,00)\;7134,53\;(182\frac{3653}{3900} \\ \underline{39} \\ 323 \\ \underline{312} \\ 114 \\ \underline{78} \\ 3653 \end{array}$$

In this Ex. and in all others where there are cyphers at the end of the divisor, the work may be abridged by *marking off*, with a comma, or point, these cyphers, and as many figures also from the right of the dividend; then we proceed, 3 *is in* 7 *twice;* but on trial we should find that 2 would be too large for the first figure in the quotient, (which comes of using 3 for the divisor instead of 39, and this difficulty will sometimes occur, but not so as to embarrass the student, when he gets accustomed to division); we set, therefore, 1 as the first figure in the quotient, and go on, as before, till we have taken down all the figures before the point in the dividend; and then we complete the last rem^r by taking down the two figures cut off, and put it over the divisor as a fraction.

N. B. Any sums may be set at pleasure in Division, and the answers *proved* by either of the methods given in Multiplication; since the product of the *divisor* and *quotient* (if the sum be worked correctly) will give the *dividend*, diminished, however, by the *remainder* (or upper number of the fraction) if any. Thus in Ex. 2., the divisor is 42 and quotient 9036, and the rem^rs from these are 6 and 0; the product of these is 0, and the dividend, diminished by the rem^r 31, is 379512, and the rem^rs from these are 0, 0: in Ex. 4., the divisor is 3900 and the quotient 182, and the rem^rs from these are 3 and 2; the product of these is 6, and the dividend diminished by the rem^r 3653, is 709800, and the rem^rs from these are 6, 6.

The pupil should be required to apply one or other of these methods of proof in the following examples.

EXAMPLES IN DIVISION.

1. 432516 ÷ 2.	2. 351789 ÷ 3.	3. 543756 ÷ 4.
4. 713915 ÷ 5.	5. 385734 ÷ 6.	6. 516824 ÷ 7.
7. 465328 ÷ 8.	8. 395424 ÷ 8.	9. 567035 ÷ 9.
10. 457848 ÷ 11.	11. 716855 ÷ 12.	12. 936571 ÷ 12.

13. 2366745 ÷ 15.	14. 7954326 ÷ 18.	15. 6342576 ÷ 24.
16. 6549372 ÷ 36.	17. 4733491 ÷ 45.	18. 5674331 ÷ 60.
19. 7825687 ÷ 64.	20. 3795469 ÷ 70.	21. 3754329 ÷ 80.
22. 6598769 ÷ 84.	23. 8791605 ÷ 88.	24. 7654325 ÷ 96.

25. 3765897 ÷ 23.	26. 4613578 ÷ 37.	27. 5123495 ÷ 41.
28. 3954371 ÷ 47.	29. 3755123 ÷ 234.	30. 5764123 ÷ 340.
31. 34568135 ÷ 357.	32. 76549139 ÷ 543.	33. 29876533 ÷ 6930.
34. 56854327 ÷ 7323.	35. 95642371 ÷ 8790.	36. 34568795 ÷ 9879.

ANSWERS TO THE PRECEDING EXAMPLES.

ADDITION.

1. 1274170.	2. 1634607.	3. 1659291.	4. 2333331.
5. 3005313.	6. 1537206.	7. 1648127.	8. 2067687.
9. 1835161.		10. 2230626.	
11. 2294129927.		12. 20566726566.	

SUBTRACTION.

1.	422021.	2.	62402.	3.	340218.	4.	50917.
5.	190309.	6.	174186.	7.	85131.	8.	226877.
9.	212652.			10.	350185.		
11.	620209341.			12.	12471311388.		

MULTIPLICATION.

1.	691346.	2.	1372896.	3.	1663052.	4.	1856405.
5.	3812592.	6.	2949625.	7.	3031312.	8.	3812312.
9.	3923784.	10.	10764204.	11.	5961384.	12.	10518588.

13.	6050000.	14.	8224776.	15.	17098464.	16.	14683059.
17.	17809620.	18.	25096608.	19.	30617520.	20.	26839665.
21.	38531880.	22.	54455616.	23.	68440260.	24.	87719808.

25.	28894545.	26.	433418175.	27.	173367270.
28.	337103025.	29.	2600509050.	30.	9101781675.
31.	12742494345.	32.	2050597944.	33.	3417663240.
34.	12364716600.	35.	7313243616.	36.	27416327796.

DIVISION.

1.	216258.	2.	117263.	3.	135939.	4.	142783.
5.	64289.	6.	73832.	7.	58166.	8.	49428.
9.	$63003\frac{8}{9}$.	10.	$41622\frac{6}{11}$.	11.	$59737\frac{11}{12}$.	12.	$78047\frac{7}{12}$.

13.	157783.	14.	441907.	15.	264274.
16.	181927.	17.	$105188\frac{31}{45}$.	18.	$94572\frac{11}{60}$.
19.	$122276\frac{23}{64}$.	20.	$54220\frac{69}{70}$.	21.	$46929\frac{9}{80}$.
22.	$78556\frac{65}{84}$.	23.	$99904\frac{53}{88}$.	24.	$79732\frac{53}{96}$.

25.	$163734\frac{15}{23}$.	26.	$124691\frac{11}{37}$.	27.	$124963\frac{12}{41}$.
28.	$84135\frac{26}{47}$.	29.	$16047\frac{125}{234}$.	30.	$16953\frac{103}{340}$.
31.	$96829\frac{182}{357}$.	32.	$140974\frac{257}{543}$.	33.	$4311\frac{1303}{6930}$.
34.	$7763\frac{5878}{7323}$.	35.	$10880\frac{7171}{8790}$.	36.	$3499\frac{2174}{9879}$.

ARITHMETICAL TABLES.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

I. MONEY.

4 Farthings (*f*) make 1 Penny (*d*)
12 Pence . . . 1 Shilling (*s*)
20 Shillings . . 1 Pound (£)

One farthing, *two* farthings or a half-penny, and *three* farthings, are also denoted by $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$, meaning *one-fourth*, *one-half*, and *three-fourths* respectively of a penny.

A Florin = 2 Shillings.
A Crown . . = 5 Shillings.
A Sovereign = 20 Shillings.
A Guinea . = 21 Shillings.

d.		*s.*	*d.*	*d.*		*s.*	*d.*
12	make	1	0	80	make	6	8
20	. .	1	8	84	. .	7	0
24	. .	2	0	90	. .	7	6
30	. .	2	6	96	. .	8	0
36	. .	3	0	100	. .	8	4
40	. .	3	4	108	. .	9	0
48	. .	4	0	110	. .	9	2
50	. .	4	2	120	. .	10	0
60	. .	5	0	130	. .	10	10
70	. .	5	10	132	. .	11	0
72	. .	6	0	140	. .	11	8

144*d.* make 12*s.*

II. WEIGHTS.

1 AVOIRDUPOIS.

For all Common Goods.

16 Drams make 1 Ounce (*oz*)
16 Ounces . . 1 Pound (*lb*)
14 Pounds . . 1 Stone
28 Pounds . . 1 Quarter
4 Quarters (112 lbs.) 1 Hundredweight (*cwt*)
20 Hundredweight . 1 Ton

2 TROY.

For Gold, Silver, and Jewellery, and in Philosophical Experiments.

24 Grains make 1 Pennyweight (*dwt*)
20 Pennyweights . 1 Ounce
12 Ounces . . . 1 Pound

The lb. Av. contains 7000 grs. Troy.

3 APOTHECARIES.

For mixing and preparing Medical Prescriptions.

20 Grains make 1 Scruple
3 Scruples . . 1 Dram
8 Drams . . . 1 Ounce
12 Ounces . . . . 1 Pound

The gr. oz. and lb. are the same as in Troy Weight.

III. LENGTH.

4 Inches	make	1 Hand
12 Inches	. . .	1 Foot
3 Feet	. . .	1 Yard
6 Feet	. . .	1 Fathom
5½ Yards	. . .	1 Rod, Pole, or Perch
40 Poles (220 yds.)	.	1 Furlong
8 Furlongs (1760 yds.)		1 Mile
3 Miles	. . .	1 League
2¼ Inches	. . .	1 Nail } Cloth M.
4 Nails	. . .	1 Quarter } Cloth M.
4 Quarters	. .	1 Yard } Cloth M.
5 Quarters	. .	1 Ell } Cloth M.

IV. SURFACE.

144 Square Inches	make	1 Square Foot
9 Square Feet	. .	1 Square Yard
30¼ Square Yards	. .	1 Sq. Rod, Pole, or Perch (*P*)
40 Perches	. . .	1 Rood (*R*)
4 Roods (4840 sq. yds.)		1 Acre (*A*)
640 Acres	. . . .	1 Square Mile

V. SOLIDITY.

1728 Cubic Inches	make	1 Cubic Foot
27 Cubic Feet	. .	1 Cubic Yard

VI. CAPACITY.

4 Gills or Noggins	make	1 Pint
2 Pints	. . . .	1 Quart
2 Quarts	. . . .	1 Pottle
4 Quarts	. . . .	1 Gallon
2 Gallons	. . . .	1 Peck
4 Pecks	. . . .	1 Bushel
8 Bushels	. . . .	1 Quarter
5 Quarters	. . . .	1 Load
3 Bushels	. . . .	1 Sack } Coal Meas.
12 Sacks	. . . .	1 Chaldron } Coal Meas.
A Barrel of *Beer*	contains	36 Gallons
A Hogshead	. . .	54 Gallons
A Hogshead of *Wine*	.	63 Gallons
A Pipe	. . . .	2 Hogsheads

VII. TIME.

60 Seconds	make	1 Minute
60 Minutes	. .	1 Hour
24 Hours	. . .	1 Day
7 Days	. . .	1 Week
4 Weeks	. . .	1 Lunar Month
365 Days	. . .	1 Year

Since 52 Weeks, or 13 Lunar Months, contain 364 Days, these are often reckoned as a Year.

The Year is also divided into 12 Months, called *Calendar Months*, which contain unequal numbers of Days—

January	31	May	31	September	30
February	28	June	30	October	31
March	31	July	31	November	30
April	30	August	31	December	31

Of these all contain 31 Days, except February, which has 28, and those mentioned in the following rhymes, which have 30:

Thirty days have September,
April, June, and November.

Every Fourth Year contains 366 Days, and is called Leap-Year; and in such a year February has 29 Days.

CHAPTER I.

ELEMENTARY RULES.

Reduction.

1. This is the name given to the method of converting a quantity expressed in one denomination to another, as from pounds to pence, from ounces to tons, from inches to yards, &c.; thus £3 = 720*d.*, 250880 oz. = 7 tons, 72 in. = 2 yds. &c.

2. *To reduce a quantity to a lower denomination.*

Rule. Multiply the given quantity by the number which shows how many of the *next* lower denomination make one of the higher; and so on, step by step, till we arrive at the proposed lower denomination.

Ex. 1. *Reduce £37 to pence.*

£37
20
740*s.*
12
Ans. 8880*d.*

Here, since £1 contains 20*s.* we first multiply the £37 by 20, to bring them into shillings; and then since 1*s.* contains 12*d.*, we multiply these shillings by 12, to bring them into pence. *See* Note III.

If the given quantity consist of several terms of different denominations, we must add in with each product, as we proceed, the term (if any) of corresponding denomination.

Ex. 2. *Reduce £15 7s. $0\frac{3}{4}$d. to farthings.*

£15 7*s.* $0\frac{3}{4}$*d.*
20
307*s.*
12
3684*d.*
4
14739*f. Ans.*

Here we first reduce £15 to shillings, adding in the 7*s.*; then these shillings to pence; and lastly these pence to farthings, adding in the 3 farthings.

Reduce

Ex. 1.

1. £513 to farthings; and 320 guineas to halfpence.
2. £2000 to halfcrowns; and 2000 guineas to sixpences.
3. £27 10*s.* to pence; and 17*s.* 6½*d.* to farthings.
4. £75 10*s.* 6*d.* to sixpences; and 220 crowns to fourpenny-pieces.
5. £47 10*s.* 11¾*d.* to farthings, and £85 0*s.* 10½*d.* to halfpence.
6. £29 10*s.* 0½*d.* to halfpence; and 1373 halfcrowns to farthings.
7. 23 tons to pounds; and 115 cwt. to ounces.
8. 27 lbs. to drams; and 11 tons to ounces.
9. 3 qrs. 14 oz. to drams; and 47 cwt. 25 lbs. to ounces.
10. 34 cwt. 3 qrs. 11 oz. to drams; and 2 tons 3 qrs. 5 oz. to ounces.
11. 4 tons 15 cwt. 2 qrs. 12 lbs. to lbs.; and 14 cwt. 1 qr. 8 drs. to drams.
12. 15 cwt. 2 lbs. 9 oz. to ounces; and 3 tons 3 qrs. 3 oz. to drams.
13. 16 lbs. Troy to grains; and 105 lbs. Troy to dwts.
14. 27 oz. 10 dwts. to grains; and 3 lbs. 13 dwts. to dwts.
15. 9 oz. 17 dwts. 22 grs. to grains; and 2 lbs. 11 oz. 20 grs. to grains.
16. 7 oz. 19 dwts. to grains; and 3 lbs. 9 oz. 7 grs. to grains.
17. 23 miles 7 fur. to feet; and 2 lea. 2 m. 7 fur. to yards.
18. 3 fur. 135 yds. 4 in. to inches; and 5 fur. 171 yds. 2 ft. to inches.
19. 2 lea. 2 m. 2 fur. 200 yds. to feet; and 5 m. 200 yds. 3 in. to inches.
20. 73 yds. 3 qrs. to nails; and 35 ells 4 qrs. to nails.
21. 54 A. 3 R. to poles; and 17 sq. yds. 8 ft. to inches.
22. 7 A. 12 P. to poles; and 29 sq. yds. to square inches.
23. 13 cub. yds. to feet; and 7 cub. yds. 20 ft. to inches.
24. 23 cub. yds. 1000 in. to inches; and 12 cub. yds. 23 ft. to inches.
25. 137 gals. to pints; and 13 gals. 3 qts. to gills.
26. 17 qrs. to gals.; and 220 bushels to quarts.
27. 3 loads 3 qrs. 3 pks. to gals.; and 2 qrs. 1 gal. to pints.
28. 3 loads 3 bus. to quarts; and 2 qrs. 7 bus. 2 pks. to gallons.
29. 27 years to days; and 3 yrs. 315 d. to minutes.
30. 5 mo. 3 w. 4 d. to hours; and 27 w. 5 d. 15 hrs. to seconds.

3. *To reduce a quantity to a higher denomination.*

RULE. Divide the given quantity by the number which shows how many of the lower denomination make one of the *next* higher; and so on, step by step, till we arrive at the proposed higher denomination.

Ex. 1. *Reduce* 137520 *farthings to shillings.*

4) 137520*f.*
12) 34380*d.*
2865*s.* *Ans.*

Here we first divide the given number of farthings by 4 to bring them into pence, and then we divide these pence by 12 to bring them into shillings.

If there should be a *remainder* after any division, we must set it down as a term of the same denomination as the dividend from which it came.

Ex. 2. *Reduce* 13799 *farthings to pounds.*

```
4) 13799f.
12) 3449d. . . 3f.
2,0) 28,7s. . . 5d.
    £14 7s. 5¾d. Ans.
```

Here, after dividing the given farthings by 4, we have a rem^r 3, which means that in 13799*f.* there are 3449*d.*, and 3*f.* over; we set down therefore the rem^r as 3*f.*, that is, as a term of the same den^n as the dividend from which it came; after dividing the pence by 12, we have a rem^r 5, which we set down, for a similar reason, as 5*d.*; and after dividing the shillings by 20, we have a rem^r 7, which we set down as 7*s.*

N.B. We have divided by 20 by the usual short method, cutting off the last figures of the dividend and divisor.

Reduce **Ex. 2.**

1. 78790236*s.* to guineas; and 150080 sixpences to pounds.
2. 1758960*f.* to crowns; and as many halfpence to halfcrowns.
3. 480144*f.* to sevenshilling-pieces; and 50000*d.* to pounds.
4. 284061*f.* to pounds; and 110012*d.* to pounds.
5. 101010*d.* to guineas; and 123290*f.* to pounds.
6. 350000*f.* to pounds; and 588483 halfpence to guineas.
7. 37568 lbs. to tons; and 108190 drs. to cwt.
8. 2345820 drs. to tons; and 108234 oz. to cwt.
9. 100000 oz. to tons; and 12821 drs. to qrs.
10. 229601 oz. to tons; and 314735 drs. to cwt.
11. 156423 drs. to cwt.; and 1008001 oz. to tons.
12. 237023 oz. to tons; and 371283 drs. to cwt.
13. 13172 grs. to lbs. Troy; and 30066 dwts. to lbs. Troy.
14. 17073 grs. to lbs.; and 12327 grs. to lbs.
15. 108970 grs. to lbs.; and 189081 grs. to lbs.
16. 272821 grs. to lbs. Troy; and 127272 grs. to lbs. Troy.
17. 36090 ft. to miles; and 231031 yds. to leagues.
18. 120835 in. to furlongs; and 378135 ft. to miles.
19. 517900 in. to miles; and 183810 ft. to leagues.
20. 13587 na. to yards; and 181970 na. to ells.
21. 121321 P. to acres; and 33333 sq. inches to yards.
22. 20000 P. to acres; and 20000 sq. inches to yards.
23. 200000 cub. in. to yards; and 138297 cub. in. to yards.
24. 106921 cub. in. to yards; and 180831 cub. in. to yards.
25. 18191 pts. to gallons; and 30983 gills to gallons.

26. 28716 qts. to loads; and 91356 pints to quarters.
27. 89765 pks. to loads; and 56789 pts. to loads.
28. 356187 qts. to loads; and 598712 gals. to quarters.
29. 137819 days to years; and 3561829 sec. to weeks.
30. 235967 hrs. to weeks; and 71871900 sec. to years.

Addition.

4. RULE. Set the quantities to be added under one another, so that terms of the same kind may be in the same column.

Add the numbers in the right-hand column; divide the result by the number of things in this column, which make *one* in the next; set the remainder, if any, under the first column, and carry the quotient to be added to the next; and so on with all the columns.

	£	s.	d.
Ex. 1.	13	0	8
	2	5	6
	23	4	7
	37	8	10
	12	9	7
	0	13	4
	£89	2	6

Here, adding up the pence in the right-hand column, we have 42*d.*; in order to bring this into shillings, we divide by 12, which goes 3 times with 6 over, so that 42*d.* = 3*s.* 6*d.*; we set down the 6*d.* under the first column, and carry the 3*s.* to the next; and so on.

	£	s.	d.
Ex. 2.	22	4	$6\frac{1}{4}$
	0	2	$6\frac{1}{2}$
	36	0	$4\frac{3}{4}$
	7	1	$1\frac{1}{4}$
	£65	8	$6\frac{3}{4}$

Here, adding up the farthings in the right-hand column, we have 7*f.*, which = $1\frac{3}{4}$*d.*; we therefore set down the $\frac{3}{4}$*d.*, and carry 1*d.* to the next column.

Ex. 3.

	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.
1.	3	13	6	2.	14	13	7	3.	65	4	$3\frac{1}{2}$	4.	23	13	$6\frac{1}{4}$
	2	11	9		22	15	9		22	0	$2\frac{1}{4}$		35	17	$0\frac{1}{2}$
	3	17	8		29	11	11		46	15	$7\frac{1}{4}$		35	7	$7\frac{3}{4}$
	2	5	2		82	17	7		73	12	$6\frac{3}{4}$		67	16	$8\frac{1}{2}$
5.	41	16	$8\frac{1}{2}$	6.	36	17	$6\frac{3}{4}$	7.	24	16	$8\frac{1}{2}$	8.	71	17	$2\frac{1}{2}$
	21	10	$7\frac{1}{4}$		14	17	6		51	14	$2\frac{3}{4}$		41	2	$9\frac{1}{4}$
	31	17	$7\frac{3}{4}$		21	12	$7\frac{3}{4}$		11	0	8		54	7	$6\frac{3}{4}$
	24	16	$8\frac{1}{2}$		13	13	$3\frac{1}{2}$		27	1	3		2	11	6
9.	16	5	4	10.	11	13	$3\frac{3}{4}$	11.	42	13	4	12.	76	15	$4\frac{3}{4}$
	35	7	$9\frac{1}{4}$		32	12	$2\frac{1}{2}$		17	6	$8\frac{3}{4}$		32	4	10
	16	10	8		13	13	$3\frac{3}{4}$		90	9	8		21	3	$7\frac{1}{2}$
	42	13	$8\frac{1}{2}$		24	3	0		21	12	$4\frac{1}{2}$		62	18	$4\frac{1}{4}$

	lb.	oz.	dr.
13.	7	3	13
	12	0	9
	23	13	14
	3	15	7

	qr.	lb.	oz.
14.	3	27	15
	1	11	2
	0	21	13
	2	13	14

	cwt.	qr.	lb.
15.	18	2	23
	17	1	19
	15	3	17
	9	2	25

	qr.	lb.	oz.
16.	13	25	7
	4	18	6
	24	17	5
	37	9	14

	qr.	lb.	oz.	dr.
17.	2	15	13	11
	3	5	11	8
	2	27	13	2
	3	17	15	4

	cwt.	qr.	lb.	oz.
18.	27	2	13	4
	32	1	12	15
	28	0	15	12
	32	1	14	3

	tons	cwt.	qr.	lb.
19.	4	17	3	18
	2	3	0	15
	13	9	2	25
	22	18	3	15

	oz.	dwt.	gr.
20.	9	17	23
	4	18	20
	7	5	15
	8	19	4

	lb.	oz.	dwt.
21.	23	8	14
	7	9	19
	37	5	3
	15	7	13

	oz.	dwt.	gr.
22.	7	17	21
	11	5	13
	4	14	20
	10	17	5

	lb.	oz.	dwt.
23.	25	8	14
	37	3	15
	25	9	10
	44	7	11

	lb.	oz.	dwt.	gr.
24.	12	5	13	22
	24	7	19	13
	47	11	17	19
	31	4	11	17

	lb.	oz.	dwt.	gr.
25.	35	3	4	12
	27	8	14	22
	41	9	17	10
	2	3	13	21

	lb.	oz.	dwt.	gr.
26.	27	0	17	22
	5	9	0	23
	17	8	11	13
	22	7	9	15

	dr.	scr.	gr.
27.	5	0	13
	7	2	14
	3	1	17
	6	0	12

	oz	dr.	scr.
28.	11	7	2
	4	3	2
	10	5	0
	9	4	1

	dr.	scr.	gr.
29.	7	1	19
	8	0	1
	11	2	13
	9	1	14

	oz.	dr.	scr.
30.	11	7	2
	10	5	2
	5	2	1
	11	6	2

	yds.	ft.	in.
31.	12	1	11
	22	2	9
	9	0	3
	13	1	4

	fur.	po.	yds.
32.	7	31	$4\frac{1}{2}$
	3	19	$2\frac{1}{2}$
	8	27	3
	4	35	5

	m.	fur.	yds.
33.	5	7	137
	2	4	121
	8	6	213
	3	5	23

	lea.	m.	fur.
34.	7	1	6
	8	2	4
	1	0	5
	9	1	7

	fur.	po.	yds.
35.	5	33	$4\frac{1}{2}$
	7	21	$3\frac{1}{2}$
	2	13	$2\frac{1}{2}$
	6	21	5

	po.	yds.	ft.
36.	27	$4\frac{1}{2}$	2
	35	$3\frac{1}{2}$	1
	24	$4\frac{1}{2}$	0
	13	3	1

	yds.	ft.	in.
37.	5	2	10
	8	1	4
	6	0	7
	9	2	5

	po.	yds.	in.
38.	7	$3\frac{1}{2}$	11
	9	2	10
	5	$1\frac{1}{2}$	8
	6	$2\frac{1}{2}$	6

	po.	yds.	ft.	in.
39.	7	3	1	11
	12	$2\frac{1}{2}$	2	4
	9	4	0	7
	2	$3\frac{1}{2}$	1	9

	m.	fur.	po.	yds.
40.	14	3	17	$2\frac{1}{2}$
	23	5	33	4
	37	1	24	5
	43	7	31	$1\frac{1}{2}$

	m.	fur.	yds.	in.
41.	3	5	137	9
	7	7	77	7
	9	6	203	6
	5	4	156	2

	yds.	qrs.	na.
42.	25	3	2
	37	0	3
	54	1	1
	49	2	3

	yds.	qrs.	na.
43.	183	3	2
	297	0	1
	328	2	3
	169	1	2

	ells	qrs.	na.
44.	79	3	3
	67	4	1
	82	1	3
	98	3	2

	ells	qrs.	na.
45.	35	2	3
	42	4	5
	37	2	2
	25	4	3

	s.yds.	s.ft.	s.in.
46.	20	8	100
	31	7	85
	24	5	34
	37	8	113

	R.	P.	s.yds.
47.	7	33	$20\frac{1}{2}$
	8	13	$14\frac{1}{4}$
	7	25	$2\frac{1}{2}$
	6	17	11

	A.	R.	P.
48.	27	2	31
	35	3	24
	22	1	17
	45	0	29

	A.	R.	P.
49.	27	1	31
	41	2	28
	51	0	19
	42	1	25

	P.	s.yds.	s.ft.	s.in.
50.	2	13	7	85
	3	$20\frac{1}{4}$	8	24
	5	$25\frac{1}{2}$	6	99
	4	$22\frac{3}{4}$	8	37

	A.	R.	P.	s.yds.
51.	35	1	23	$12\frac{1}{2}$
	9	2	15	$27\frac{1}{4}$
	11	1	24	11
	42	0	35	$2\frac{1}{2}$

	R.	P.	s.yds.	s.in.
52.	37	33	$23\frac{1}{4}$	121
	21	25	17	135
	18	17	$20\frac{1}{2}$	102
	25	12	25	97

	c.yds.	c.ft.	c.in.
53.	13	25	872
	22	17	1000
	34	11	1534
	21	8	479

	c.yds.	c.ft.	c.in.
54.	27	22	856
	31	15	979
	24	19	787
	22	6	842

	c.yds.	c.ft.	c.in.
55.	14	20	1431
	32	3	1560
	25	18	937
	22	21	1364

	gal.	qts.	pts.
56.	27	3	1
	31	2	0
	54	1	1
	37	0	1

	gal.	qts.	pts.
57.	17	3	1
	24	2	1
	35	3	0
	25	2	1

	pks.	gal.	qts.
58.	3	1	3
	4	0	2
	5	1	1
	7	1	3

	bus.	pks.	gal.
59.	23	3	1
	31	2	1
	24	0	0
	35	3	1

	qrs.	bus.	pks.
60.	13	3	2
	24	6	1
	37	3	1
	43	5	2

	lds.	qrs.	bus.
61.	13	4	7
	24	3	4
	37	4	0
	43	2	1

	bus.	gal.	qts.
62.	31	1	3
	25	0	2
	41	1	1
	27	1	3

	bus.	pks.	gal.
63.	29	3	1
	37	2	0
	53	3	1
	47	2	1

	gal.	qts.	pts.	gills.
64.	22	3	1	3
	31	2	0	1
	13	3	1	2
	24	3	1	1

	bus.	pks.	gal.	qts.
65.	13	2	1	3
	42	3	1	2
	51	1	0	3
	47	3	1	2

	qrs.	bus.	pks.	gal.
66.	23	3	3	1
	32	4	1	0
	41	6	2	1
	52	2	0	1

	d.	hrs.	min.	sec.
67.	5	13	39	42
	4	22	19	33
	6	20	29	45
	4	17	59	59

	mo.	w.	d.	hrs.
68.	13	3	5	11
	21	2	4	15
	37	3	6	17
	41	2	5	19

	d.	hrs.	min.	sec.
69.	4	11	39	28
	2	13	10	32
	5	21	40	29
	7	23	19	19

	yrs.	d.	hrs.	min.
70.	6	130	23	15
	7	354	10	17
	8	45	22	14
	9	313	13	17

	yrs.	w.	d.	hrs.
71.	14	13	5	23
	22	47	4	3
	35	39	3	18
	21	44	6	15

	yrs.	d.	hrs.	min.
72.	8	244	22	49
	6	315	17	38
	5	223	13	45
	7	129	21	48

Subtraction.

5. Rule. Set the quantity to be subtracted under the other, so that terms of the same kind may be in the same column.

Subtract the right-hand term of the lower line from that of the upper, if possible; if not, subtract it from the number of things in this column, which make *one* of those in the next, and add the upper term to the remainder; place the result under the first column, and carry *one* thing to the lower term of the next; and so on with all the columns.

	£	s.	d.
Ex. 1.	34	17	$9\frac{3}{4}$
	27	8	$4\frac{1}{2}$
	£7	9	$5\frac{1}{4}$

Here, taking $\frac{1}{2}d.$ from $\frac{3}{4}d.$, we have left $\frac{1}{4}d.$ to be set under the farthings; then taking 4*d.* from 9*d.*, we have left 5*d.* to be set under the pence; and so on.

	£	s.	d.
Ex. 2.	19	12	$8\frac{1}{4}$
	16	17	$4\frac{1}{2}$
	£2	15	$3\frac{3}{4}$

Here we cannot take $\frac{1}{2}d.$ from $\frac{1}{4}d.$; we *borrow* therefore 1*d.* from the 8*d.*, and convert it into farthings, thus changing the $8\frac{1}{4}d.$ into $7d. + 1\frac{1}{4}d.$, or 7*d.* 5*f.*; taking, then, the $\frac{1}{2}d.$ or 2*f.* from 5*f.*, we have left 3*f.* or $\frac{3}{4}d.$, to be set under the farthings, and have now to take 4*d.* from 7*d.*, which leaves 3*d.* to be set under the pence.

N. B. In practice, it is best to take the $\frac{1}{2}d.$ *at once* from the 1*d.* borrowed, which leaves $\frac{1}{2}d.$, and add in the $\frac{1}{4}d.$ to this rem^r, which gives $\frac{3}{4}d.$ as before; and also, instead of taking 4*d.* from 7*d.*, we may take 5*d.* from 8*d.*, which will leave the same rem^r 3*d.*, *i. e.* we need not alter the quantity from which we subtract, if we add, or *carry*, one to the quantity subtracted.

Again, as we cannot take 17*s.* from 12*s.*, we borrow £1 from the £19, and thus taking 17*s.* from £1 12*s.* or 32*s.*, we have left 15*s.*, and then, taking £16 from £18, we have left £2. Here, too, it is best to take the 17*s.* *at once* from the £1 borrowed, which leaves 3*s.*, and add to *this* the 12*s.*, which gives 15*s.* as before; also, to *carry* £1 to the £16, making £17, and take *this* from the original £19, which leaves £2 as before.

	£	s.	d.
Ex. 3.	23	6	$0\frac{1}{2}$
	22	18	$11\frac{3}{4}$
	£0	7	$0\frac{3}{4}$

Here, taking $\frac{3}{4}d.$ from 1*d.* borrowed, we have $\frac{1}{4}d.$ left, to which we add the $\frac{1}{2}d.$, making $\frac{3}{4}d.$ to be set down; then carrying 1*d.* to the 11*d.*, we have 12*d.*, which we take from 1*s.* *borrowed*, and have no rem^r; again, carrying 1*s.* to the 18*s.*, we have 19*s.*, which we take from £1 *borrowed*, and have 1*s.* left, to which we add the 6*s.*, making 7*s.* to be set down; and carrying £1 to the £22, we have £23 to subtract, and no rem^r.

Ex. 4.

	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.
1.	23	10	8	2.	45	14	$7\frac{1}{2}$	3.	74	0	$6\frac{3}{4}$	4.	89	15	7
	13	7	5		12	7	$5\frac{1}{4}$		13	8	$4\frac{1}{2}$		74	11	9
5.	93	0	9	6.	24	0	5	7.	132	11	$6\frac{1}{4}$	8.	225	0	0
	37	10	11		15	12	11		129	13	$4\frac{1}{2}$		37	18	$9\frac{3}{4}$
9.	137	13	$0\frac{1}{4}$	10.	234	0	$11\frac{1}{4}$	11.	317	14	$0\frac{1}{2}$	12.	345	0	0
	111	15	$9\frac{3}{4}$		195	18	$10\frac{3}{4}$		239	18	$10\frac{3}{4}$		129	17	$8\frac{3}{4}$
	lbs.	oz.	dr.		qrs.	lbs.	oz.		cwt.	qrs.	lbs.		qrs.	lbs.	oz.
13.	27	11	3	14.	13	3	1	15.	33	0	11	16.	2	23	0
	13	7	1		5	12	14		12	1	24		1	25	9
	qrs.	lbs.	oz.		tons	cwt.	qrs.		cwt.	qrs.	oz.		cwt.	lbs.	oz.
17.	17	11	3	18.	32	1	1	19.	27	1	3	20.	45	0	3
	8	27	15		30	14	3		13	0	7		44	6	13
	oz.	dwt.	gr.		oz.	dwt.	gr.		lbs.	oz	dwt.		oz.	dwt.	gr.
21.	11	19	3	22.	32	7	21	23.	13	7	15	24.	11	0	0
	8	14	17		18	9	22		6	11	18		2	18	22
	oz.	dwt.	gr.		oz.	dwt.	gr.		oz.	dwt.	gr.		oz.	dwt.	gr.
25.	23	0	4	26.	37	0	0	27.	22	2	2	28.	42	0	3
	1	15	20		0	11	13		13	11	11		27	13	21
	dr.	scr.	gr.		oz.	dr.	scr.		lbs.	oz.	dr.		dr.	scr.	gr.
29.	7	1	18	30.	11	0	0	31.	37	7	1	32.	8	0	11
	4	0	19		8	5	2		19	11	2		6	2	15
	yds.	ft.	in.		po.	yds.	ft.		fur.	po.	yds.		m.	fur.	yds.
33.	13	1	7	34.	23	3	1	35.	6	37	2	36.	13	6	123
	11	2	10		13	$4\frac{1}{2}$	2		1	15	$4\frac{1}{2}$		8	7	219
	m.	fur.	po.		fur.	po.	yds.		lea.	m.	fur.		fur.	po.	yds.
37.	24	0	7	38.	6	37	4	39.	37	0	5	40.	7	23	$3\frac{1}{2}$
	11	5	18		5	18	$4\frac{1}{2}$		18	0	7		6	35	5
	po.	yds.	ft.		yds.	ft.	in.		yds.	qrs.	na.		ells	qrs.	na.
41.	23	3	2	42.	23	0	0	43.	17	3	2	44.	24	1	3
	15	$4\frac{1}{2}$	1		15	2	7		13	0	1		19	2	1
	s.yds.	s.ft.	s.in.		P.	s.yds.	s.ft.		R.	P.	s.yds.		A.	R.	P.
45.	13	2	73	46.	22	13	5	47.	3	2	25	48.	37	2	29
	6	8	131		13	$20\frac{1}{4}$	8		2	35	$28\frac{1}{4}$		23	3	35
	A.	R.	P.		R.	P.	s.yds.		R.	s.yds.	s.ft.		s.yds.	s.ft.	s.in.
49.	45	2	35	50.	2	35	20	51.	10	$13\frac{1}{4}$	4	52.	12	2	13
	19	3	39		1	21	$28\frac{1}{4}$		8	10	7		8	7	130

	c.yds.	c.ft.	c.in.		c.yds.	c.ft.	c.in.		c.yds.	c.ft.	c.in.		c.yds.	c.ft.	c.in.
53.	23	13	357	54.	37	2	459	55.	45	24	656	56.	27	13	2
	10	25	1014		7	24	1532		12	19	999		13	23	731

	gals.	qts.	pts.		gals.	qts.	pts.		pks.	gals.	qts.		bus.	pks.	gal.
57.	36	2	0	58.	35	0	1	59.	23	1	0	60.	47	2	0
	33	3	1		29	3	0		19	1	3		28	3	1

	qrs.	bus.	pks.		lds.	qrs.	bus.		bus.	pks.	gals.		lds.	qrs.	bus.
61.	45	3	1	62.	22	3	5	63.	57	1	0	64.	5	1	1
	39	7	2		9	3	7		39	3	1		2	4	5

	hrs.	m.	s.		d.	hrs.	m.		w.	d.	hrs.		mo.	w.	d.
65.	22	39	19	66.	14	17	20	67.	3	5	2	68.	12	2	5
	8	41	30		6	21	35		2	6	13		8	3	6

	yrs.	d.	hrs.		yrs.	w.	d.		yrs.	w.	d.		yrs.	d.	hrs.
69.	32	131	22	70.	27	35	4	71.	45	45	3	72.	26	213	11
	19	300	13		18	47	6		35	1	6		19	231	21

Multiplication.

6. RULE. Set the multiplier under the right-hand term of the multiplicand; multiply this term by it, and find, as before, how many are to be carried to the next term, writing the rem^r under the right-hand term: then multiply the next term, and add in the number carried; and so on.

	£	s.	d.
Ex. 1.	£23	13	5
			4
Ans.	£94	13	8

Here 5*d.* × 4 = 20*d.* = 1*s.* 8*d.*; we set down 8*d.*, and carry 1*s.*:—13*s.* × 4 = 52*s.*, and, adding the 1*s.* carried, we have 53*s.* = £2 13*s.*; we set down 13*s.* and carry £2:—£23 × 4 = £92, and, adding the £2 carried, we have £94.

	£	s.	d.
Ex. 2.	£37	13	8½
			11
Ans.	£414	10	9½

Here 2*f.* × 11 = 22*f.* = 5*d.* 2*f.* or 5½*d.*; we set down ½*d.*, and carry 5*d.*; and so on.

		£	s.	d.			£	s.	d.	
Ex. 5.	1.	23	8	4	× 2	2.	37	13	5¼	× 2
	3.	59	13	7	× 3	4.	48	17	7½	× 3
	5.	78	2	8	× 4	6.	96	15	6½	× 4
	7.	99	17	5	× 5	8.	75	14	2¾	× 5
	9.	171	13	2	× 6	10.	154	11	3¾	× 6
	11.	134	6	9	× 7	12.	161	12	7½	× 7
	13.	165	14	2	× 8	14.	173	18	5¼	× 8
	15.	115	7	9	× 9	16.	135	15	4¾	× 9
	17.	124	5	4	× 10	18.	175	4	9½	× 10
	19.	171	13	11	× 11	20.	183	12	10¾	× 11
	21.	37	0	2¾	× 12	22.	51	10	0½	× 12
	23.	128	17	3	× 12	24.	171	13	5¼	× 12

7. When the multiplier is large, but is composed of two or three factors*, we may multiply separately by each of these.

Ex. 1. *Multiply* £23 11*s.* $4\frac{3}{4}$*d.* by 36.

Since $36 = 6 \times 6$, or $= 4 \times 9$, or $= 3 \times 12$, the sum may stand thus:

£	s.	d.		£	s.	d.		£	s.	d.
23	11	$4\frac{3}{4}$	or	23	11	$4\frac{3}{4}$	or	23	11	$4\frac{3}{4}$
		6				4				3
141	8	$4\frac{1}{2}$		94	5	7		70	14	$2\frac{1}{4}$
		6				9				12
848	10	3 *Ans.*		848	10	3 *Ans.*		848	10	3 *Ans.*

Ex. 2. *Multiply* £17 3*s.* $0\frac{1}{2}$*d.* by 140.

Since $140 = 4 \times 5 \times 7$, the sum may stand thus:

£	s.	d.
17	3	$0\frac{1}{2}$
		4
68	12	2
		5
343	0	10
		7
2401	5	10

Ex. 6.	£	s.	d.			£	s.	d.	
1.	23	17	$5\frac{1}{2}$	× 15	2.	79	14	$10\frac{1}{4}$	× 18
3.	93	8	$3\frac{1}{2}$	× 21	4.	49	12	8	× 28
5.	68	7	$4\frac{3}{4}$	× 35	6.	97	19	$9\frac{1}{2}$	× 48
7.	87	4	$3\frac{1}{2}$	× 64	8.	92	11	10	× 70
9.	37	13	$2\frac{1}{2}$	× 81	10.	42	10	$9\frac{1}{4}$	× 88
11.	98	18	3	× 96	12.	43	12	$5\frac{3}{4}$	× 132
13.	22	10	$8\frac{1}{2}$	× 128	14.	3	15	6	× 176
15.	10	11	$8\frac{1}{4}$	× 270	16.	13	7	$4\frac{3}{4}$	× 275

8. When, however, the multiplier, though large, cannot be broken up into factors, we must proceed as in the first case.

	£	s.	d.
Ex.	23	11	$4\frac{3}{4}$
			37
	£872	1	$7\frac{3}{4}$

Here $3f. \times 37 = 111f. = 27d.\ 3f.$, or $27\frac{3}{4}d.$; we set down $\frac{3}{4}d.$, and carry 27*d.*:—$4d. \times 37 = 148d.$, and, adding the 27*d.*, we have $175d. = 14s.\ 7d.$; we set down 7*d.* and carry 14*s.*; and so on.

* In order to find these, note that any no. is exactly divisible by 5, if it ends in 5 or 0; by 2, 4, 8, if the no. formed by its last *one*, *two*, *three* figs. respectively is div. by 2, 4, 8; by 3 or 9, if the *sum* of its figures is divisible by 3 or 9, respectively; by 11, if the sums of its figs. in *odd* and *even* places, when div. by 11, leave the *same* remr:

Thus 75 and 30 are each divisible by 5, since they end in 5 and 0 respectively; 34 by 2, since 4 is div. by 2; 756 by 4, since 56 is by 4; 1528 by 8, since 528 is by 8; 72908374 by 11, since figs. in *odd* places $=7+9+8+7=31$, and in *even* $=2+0+3+4=9$, and 31 and 9, when div. by 11, leave the same remr, 9.

		£	s.	d.			£	s.	d.
Ex. 7.	1.	43	8	$6\frac{1}{4} \times 19$		2.	47	13	$2\frac{1}{2} \times 23$
	3.	33	15	$8\frac{1}{4} \times 29$		4.	79	16	3×34
	5.	18	15	$2\frac{1}{2} \times 47$		6.	24	14	$3\frac{1}{4} \times 62$
	7.	19	10	$8\frac{1}{2} \times 79$		8.	15	17	$4\frac{3}{4} \times 93$
	9.	23	18	$6\frac{1}{4} \times 106$		10.	16	13	$7\frac{3}{4} \times 139$

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11. 3 qrs. 6 lbs. 13 oz. 15 dr.	× 8	12. 4 tons 13 cwt. 17 lb. 10 oz.	× 9
13. 5 tons 27 cwt. 27 lb. 5 oz.	× 25	14. 9 tons 16 cwt. 1 qr. 5 oz.	× 32
15. 17 cwt. 3 qrs. 15 oz. 7 dr.	× 36	16. 18 tons 3 qrs. 5 lb. 13 drs.	× 45
17. 3 lbs. 8 oz. 15 dwts. 13 grs.	× 49	18. 2 lb. 7 oz. 9 dwts. 22 grs.	× 50
19. 5 fur. 78 yds. 2 ft. 7 in.	× 56	20. 7 fur. 87 yds. 1 ft. 5 in.	× 64
21. 5A. 3R. 27P.	× 70	22. 17A. 1R. 31P.	× 72
23. 3 sq. yds. 8 ft. 131 in.	× 80	24. 17 cub. yds. 21 ft. 57 in.	× 84
25. 87 gals. 3 qts. 1 pt	× 90	26. 37 gals. 2 qts. 1 pt.	× 96
27. 4 qrs. 6 bus. 2 pks.	× 100	28. 3 qrs. 5 bus. 2 pks.	× 108
29. 5 d. 17 h. 39 m. 20 s.	× 120	30. 17 yrs. 110 d. 17 h, 57 s,	× 144

Division.

9. Rule. Set the divisor in a loop to the left of the dividend, and divide the left-hand term by it, setting the quotient under that term: if there be any rem^r, reduce it to the next lower den^n, adding in that term (if any) of the div^d, which is of this lower den^n, and divide the result by the div^r; and so on.

		£	s.	d.
Ex. 1.	3)	38	6	$8\frac{1}{4}$
		£12	15	$6\frac{3}{4}$

Here first we have to divide £38 by 3, whence we get £12 with £2 over: now, as we cannot divide £2 by 3, we reduce it to 40*s.*, and adding in the term 6*s.* in the dividend, we have now to divide 46*s.* by 3:—hence we get 15*s.* with 1*s.* over; and since 1*s.* = 12*d.*, adding in the term 8*d.* in the dividend, we have now to divide 20*d.* by 3:—hence we get 6*d.* with 2*d.* over; and since 2*d.* = 8*f.*, we have lastly to divide 8*f.* + 1*f.*, or 9*f.* by 3, which gives us 3*f.* or $\frac{3}{4}$*d.*

		£	s.	d.
Ex. 2.	8)	376	2	6
		£47	0	$3\frac{3}{4}$

Here the number of pounds is exactly divisible by 8; and since we cannot divide the term, 2*s.*, of the dividend by 8, we reduce it to pence, and adding in the term 6*d.*, we have now to divide 30*d.* by 8; whence we get 3*d.* with rem^r 6*d.*; and since 6*d.* = 24*f.* we divide 24*f.* by 8, and thus have 3*f.* or $\frac{3}{4}$*d.*

		£	s.	d.			£	s.	d.
Ex. 8.	1.	26	15	$3\frac{1}{2}\div 2$	2.		12	14	$3\frac{3}{4}\div 3$
	3.	56	15	$8\div 4$	4.		76	17	$2\frac{1}{4}\div 5$
	5.	84	10	$3\div 6$	6.		90	13	$8\frac{3}{4}\div 7$
	7.	75	7	$6\div 8$	8.		87	16	$8\frac{1}{4}\div 9$
	9.	91	14	$4\frac{1}{2}\div 10$	10.		74	17	$7\frac{1}{4}\div 11$
	11.	57	13	$0\div 12$	12.		87	13	$6\div 12$

10. Division by 10, 100, 1000, &c. is usually performed by *pointing* off *one*, *two*, *three*, &c. figures, respectively, from the right of the dividend.

	£	s.	d.
Ex.	23.15	14	7
	20		
	3.14*s.*		
	12		
	1.75*d.*		
	4		
	3.00*f.*		

Here, dividing 2315 by 100, we have a quotient 23 with rem^r 15; we may *point* off, therefore, the *last two* figures as the rem^r, leaving the rest for the quotient; reducing now this rem^r into shillings, and adding in the term 14*s.*, we have to divide 314*s.* by 100; and since the quotient is 3 with rem^r 14, we may again point off the *last two* figures as the rem^r: and so on.

		£	s.	d.		£	s.	d.
Ex. 9.	1.	176	16	$8\div 10$	2.	30	6	$3\div 10$
	3.	329	1	$3\div 100$	4.	73	12	$11\div 100$
	5.	1511	9	$2\div 1000$	6.	72	18	$4\div 1000$
	7.	645	16	$8\div 10000$	8.	1062	10	$0\div 10000$

11. When the divisor is large, but can be broken up into two or more factors, we may divide separately by each of these.

Ex. 1. *Divide* £3762 3*s.* 6*d.* by 24.

Since $24=4\times 6$, or $=3\times 8$, or $=2\times 12$, the sum may stand thus:

	£	s.	d.		£	s.	d.		£	s.	d.
4)	3762	3	6	or 3)	3762	3	6	or 2)	3762	3	6
6)	940	10	$10\frac{1}{2}$	8)	1254	1	2	12)	1881	1	9
	£156	15	$1\frac{3}{4}$ *Ans.*		£156	15	$1\frac{3}{4}$ *Ans.*		£156	15	$1\frac{3}{4}$ *Ans.*

Ex. 2. *Divide* £40818 15*s.* 0*d.* *by* 1200.

Since $1200=12\times 100$, the sum may stand thus:

	£	s.	d.
12)	40818	15	0
100)	34.01	11	3
	20		
	.31*s.*		
	12		
	3.75*d.*		
	4		
	3.00*f.*		

Here there is no quotient from the shillings, and we have the

Ans. £34 0*s.* $3\frac{3}{4}$*d.*

N.B. In a case where one of the factors of the divisor is 10, 100, &c., it is generally best to divide *last* by that factor.

	£	s.	d.	÷		£	s.	d.	÷
Ex. **10.** 1	702	6	3	÷ 20	2.	187	14	11	÷ 14
3.	275	15	$1\frac{1}{2}$	÷ 18	4.	345	13	4	÷ 40
5.	345	10	5	÷ 25	6.	351	14	8	÷ 32
7.	485	17	6	÷ 120	8.	457	18	4	÷ 400
9.	208	16	9	÷ 36	10.	362	19	$10\frac{1}{2}$	÷ 42
11.	692	10	0	÷ 800	12.	1137	10	0	÷ 2400
13.	347	1	3	÷ 45	14.	457	1	$6\frac{3}{4}$	÷ 63
15.	362	10	0	÷ 6000	16.	1556	5	0	÷ 3600
17.	408	0	9	÷ 54	18.	453	11	$6\frac{3}{4}$	÷ 77
19.	363	18	$2\frac{1}{4}$	÷ 81	20.	473	14	0	÷ 96
21.	386	16	$5\frac{1}{4}$	÷ 99	22.	374	19	3	÷ 108
23.	319	2	9	÷ 132	24.	576	3	0	÷ 144

12. When, however, the divisor, though large, cannot be broken up into factors, we must proceed as in the first case, only setting the quotient in a loop at the right of the dividend, instead of under it.

Ex. *Divide £3715 18s. 9d. by 470.*

Since 470 = 47 × 10, the sum may stand thus:

```
     £    s.  d.  £   s.  d.
47) 3715  18  9 (79   1   3
    329
    ---
     425
     423
     ---
       2
      20
      --
      58 (1
      47
      --
      11
      12
      --
     141 (3
     141
     ---
```

Here the remʳ from the pounds is £2, which we reduce into shillings, adding in the term 18*s.* in the dividend: and so on.

We have now to divide this first quotient by 10:

```
        £   s.  d.
   10) 7.9  1   3
        20
       ----
       18.1
        12
       ----
        1.5
         4
       ----
        2.0
```

Ans. £7 18*s.* $1\frac{1}{2}$*d.*

	£	s.	d.	÷		£	s.	d.	÷
Ex. **11.** 1.	375	13	$5\frac{1}{2}$	÷ 13	2.	289	0	$8\frac{1}{2}$	÷ 17
3.	258	1	8	÷ 190	4.	456	0	$11\frac{1}{4}$	÷ 23
5.	371	2	$9\frac{1}{2}$	÷ 29	6.	513	8	9	÷ 3100
7.	412	0	$2\frac{1}{2}$	÷ 370	8.	712	18	$7\frac{1}{4}$	÷ 41
9.	1375	13	$6\frac{3}{4}$	÷ 123	10.	2559	7	6	÷ 18900
11.	2456	2	11	÷ 3650	12.	2348	11	$4\frac{1}{2}$	÷ 354

13. Hitherto we have had to divide some quantity of money, weight, &c., or, as it is called, some *concrete* quantity, by a simple, or *abstract*, number, that is to say, we have had

to find a certain *part* of such a quantity: thus, to divide £3 7*s*. 6*d*. by 8, is to find the *eighth part* of £3 7*s*. 6*d*.; and here the quotient will also be a *concrete* quantity of the *same kind* as the dividend—as in this case, 8*s*. $5\frac{1}{4}$*d*.

But if we have to divide a concrete quantity by another of the *same kind*, this amounts to finding *how many times* the divisor is contained in the dividend: thus, to divide £3 7*s*. 6*d*. by 16*s*. $10\frac{1}{2}$*d*., is to find how many times 16*s*. $10\frac{1}{2}$*d*. is contained in £3 7*s*. 6*d*.; and here the quotient will be an *abstract* number—as in this case, 4.

The quotient in cases of this latter kind is to be found by reducing the two quantities to the same denomination, and then performing the division.

Ex. 1. *Divide* £3 7*s*. 6*d*. *by* 16*s*. $10\frac{1}{2}$*d*.

Here £3 7 6 = 1620 halfpence }
16 $10\frac{1}{2}$ = 405 halfpence } hence 405) 1620 (4 *Ans.*
1620

Ex. 2. *Divide* 3 tons 2 cwt. 1 qr. 21 lbs. *by* 2 qrs. 7 lbs.

tons cwt. qr. lbs. lbs.
Here 3 2 1 21 = 6993 }
2 7 = 63 } hence 63) 6993 (111 *Ans.*

```
63) 6993 (111
    63
     69
     63
      63
      63
```

Ex. 12.

	£	s.	d.		£	s.	d.
1.	11	7	$9\frac{3}{4}$	÷	1	5	$3\frac{3}{4}$
2.	22	15	$7\frac{1}{2}$	÷	3	15	$11\frac{1}{4}$
3.	102	10	$3\frac{3}{4}$	÷	11	7	$9\frac{3}{4}$
4.	68	6	$10\frac{1}{2}$	÷	2	10	$7\frac{1}{2}$
5.	68	6	$10\frac{1}{2}$	÷	7	11	$10\frac{1}{2}$
6.	205	0	$7\frac{1}{2}$	÷	34	3	$5\frac{1}{4}$
7.	684	7	6	÷	76	0	10
8.	171	1	$10\frac{1}{2}$	÷	57	0	$7\frac{1}{2}$

9. 89 cwt. 22 lb. ÷ 3 cwt. 1 qr. 6 lb.
10. 195 m. 7 fur. ÷ 7 ft. 6 n.
11. 81 cwt. 1 qr. 16 lb. ÷ 1 cwt. 3 qr. 16 lb.
12. 9 lb. 9 oz. 3 dwts. 12 grs. ÷ 5 dwts. 9 grs.
13. 513 m. 4 fur. 23 po. ÷ 17 m. 5 fur. 27 po.
14. 1027 m. 1 fur. 6 po. ÷ 17 m. 5 fur. 27 po.
15. 244 qrs. 3 bus. 1 pk. ÷ 3 qrs. 3 pks.
16. 2366A. 3R. 36P. ÷ 91A. 6P.

---

14. To this head also may be referred certain cases of Reduction, in which we cannot pass directly, step by step, from one den^{n} to another, but must reduce both the given quantity and the proposed to some common lower den^{n}, (it will be best to take the *highest* den^{n} to which they can both

be reduced), and then find by div^n what quantity of the proposed den^n is equivalent to the given quantity.

Ex. *Reduce* £96 16*s*. *to guineas.*

```
     £96 16s.
        20
21) 1936 (92
     189
      46
      42
       4
```

Here, since we know that 21*s*. make a guinea, we first reduce the given sum into shillings, and then divide by 21, to bring these shillings into guineas. The rem^r 4 we set down (Art. 3., Ex. 2.) as 4*s*.

Ans. 92*g*. 4*s*.

Reduce **Ex. 13.**

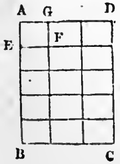
1. 835 guineas to pounds; and 538 pounds to halfguineas.
2. 760 halfcrowns to guineas; and 670 halfguineas to halfcrowns.
3. 325 crowns to halfguineas; and 253 guineas to crowns.
4. 18756 fourpenny-pieces to crowns; and 3700 halfcrowns to fourpenny-pieces.
5. £36 17*s*. 6*d*. to crowns; and £27 5*s*. 4*d*. to sixpences.
6. 100 halfguineas to fourpenny-pieces; and £100 to seven-shilling-pieces.
7. 1 cwt. 2lbs. Av. to Troy weight; and 16 dwts. to Ap. weight.
8. 20 lbs. Av. to Troy weight; and 5 drs. Ap. to Troy weight.
9. 478 ells to yards; and 14 hands to feet.
10. 500 fathoms to yards; and 5 furlongs to fathoms.

---

15. It must be noticed, that we can never *divide* a concrete quantity of one kind by another of a *different* kind, as shillings by ounces, pounds by hours, &c.; since no quantity of shillings will contain ounces, nor of pounds, hours, &c.

Nor can we *multiply* together concrete quantities of *any* kind, whether the same or different: thus, we cannot multiply either shillings by shillings, or shillings by ounces.

16. *Mensuration of rectangular areas.*



Suppose *ABCD* to represent the surface of a table, of which the length *AB* is 5 feet, and the breadth *AD*, 3 feet. Divide then *AB* into 5 equal parts, and *AD* into 3, as in the figure, and through the points of division draw lines parallel to *AB* and *AD*. By this means we shall have divided the whole surface into small figures, such as *AEFG*, all equal to one another; and since $AE =$ one foot, and $AG =$ one foot, it is plain that the surface *AEFG* measures a foot every way, a foot long and a foot broad,—i.e. *AEFG* is a *square foot*, and so are all the other small figures.

Now the number of these figures is $5 \times 3 = 15$, each horizontal row of 3 square feet (the *number* of feet in *AD*) being repeated 5 times (the *number* of feet in *AB*); so that the number of *square feet* in the surface is found by multiplying together the n° of feet in its length and the n° of feet in its breadth.

17. As the same method of proof would apply in any similar case, it appears that the n° of *square* feet in any rectangular surface is found by multiplying together the n° of *linear* feet in its length and breadth; or if we express the length and breadth in *yards, inches*, &c., and multiply them in this form, we shall obtain the n° of *square yards, square inches*, &c. in the surface.

Ex. Find the surface of a floor 17 ft. 8 in. long by 3 yards wide.

Here 17 ft. 8 in. = 212 in.*
3 yds. = 108 in.

```
      212
      108
     ----
     1696
    2120
    -----
    22896
```

```
12) 22896
12)  1908
 9)   159
      17 6
```

22896 sq. in. = 17 sq. yds. 6 ft. *Ans.*

Ex. **14.** 1. 37 ft. 2 in. × 2 ft. 9 in. 2. 23 ft. × 3 ft. 5 in.
3. 3 yds. 2 in. × 3 ft. 4. 1 yd. 2 ft. × 1 yd. 1 in.
5. 15 ft. 7 in. × 11 ft. 11 in. 6. 22 ft. 5 in. × 3 yds.

---

7. What is the area of a court, 10 yds. 2 ft. long, and 5 yds. 1 ft. broad?
8. How many sq. yds. of carpet will it take for a room 26 ft. by 32 ft.?
9. What is the surface of a marble slab, whose length is 5 ft. 7 in., and breadth 1 ft. 10 in.?
10. Find the area of a square building, whose side is 46 ft. 8 in.

* It might at first sight appear that we are here multiplying *inches* by *inches*, contrary to the statement in (15); but, in reality, it is only the *numbers* 212 and 108 that we multiply, not the *quantities* 212 in. and 108 in.: so also the resulting product is only the *number* 22896, to which we append *sq. in.*, because we know from the above, that this *is* the number of square inches in the given area. A similar remark applies to all such cases, and to all such expressions as multiplying the *length* by the *breadth*, &c. The Student's attention should be strongly drawn to this.

11. How many square yards of paper will be required for a room 17 ft. long, 12 ft. 7 in. wide, and 8 ft. 5 in. high?

12. How much wainscoting is there in a square room, 18 ft. 3 in. long, and 8 ft. 6 in. high? *See* NOTE IV.

18. Since, by *multiplying* the length and breadth, we get the square area of any rectangular surface, it follows that, by *dividing* the square area by the *length*, we shall get the *breadth*, or, *dividing* it by the *breadth*, we shall get the *length*—taking care to express the quantities concerned, before div^{n}, as quantities of the *same* den^{n}, as, for instance, not dividing *sq. feet* by *inches*, but first bringing them to *sq. inches*, &c.

Ex. What length of paper, that is 2 ft. wide, will be required for a room 14 ft. square, and 10 ft. 4 in. high?

The room being square, the united length of its four sides will be $14 \times 4 = 56$ feet, and their height being 10 ft. 4 in., we shall find the square area of the whole surface of the walls by multiplying these quantities, first reducing them to inches.

Here 56 ft. = 672 in.
10 ft. 4 in. = 124 in.

```
  2688
 1344
 672
 83328 sq. in.
```

The surface of the walls being 83328 sq. in., we have now to divide this by 2 ft. = 24 in., the width of the paper.

```
in.  sq. in.   in.
24) 83328 (3472
    72
    113
     96
     172
     168
       48
       48
```

Ans. 3472 in. = 96 yds. 1 ft. 4 in.

Ex. 15.

1. 5 sq. yds. 6 ft. 15 in. ÷ 18 ft. 7 in.
2. 11 sq. yds. 3 ft. 129 in ÷ 2 ft. 9 in.
3. 8 sq. yds. 6 ft. 84 in. ÷ 5 ft. 9 in.
4. 17 sq. yds. 4 ft. 24 in. ÷ 23 ft.
5. 17 sq. yds. 0 ft. 45 in. ÷ 18 yds. 1 ft. 9 in.
6. 42 sq. yds. 1 ft. 50 in. ÷ 23 ft. 10 in.

---

7. What is the length of a room, whose breadth is 11 ft. 11 in., and which it takes 17 sq. yds. 2 ft. 131 in. of drugget to cover?
8. One side of a rectangular building measures 26 yds. 5 in., and its area contains 683 sq. yds. 2 ft. 25 in.; show that it is square.
9. How many yards of carpeting, 2 ft. 4 in. broad, will it take to cover a room whose dimensions are 26 ft. by 35 ft.?
10. It is found that 288 yds. of paper, 2 ft. 8 in. wide, will cover the walls of a room; how many would be required of paper 2 ft. 3 in. wide?

11. How many yards of matting, 2 ft. 3 in. wide, will be required for a square room, whose side is 18 ft. 9 in.?

12. If the room in (11) be 13 ft. 4 in. high, how many yards of paper 1 ft. 4 in. wide will be required for it?

19. *Mensuration of rectangular solids.*

Suppose we place upon each of the little squares in the preceding figure, a solid (as, for instance, a brick) in the form of a *cubic foot*, that is, measuring a foot every way—a foot long, a foot broad and a foot high—we shall have a layer of such bricks *one* foot high, and containing as many cubic feet as there are square feet in the base; if upon this we pile another similar layer, we shall have the whole solid *two* feet high, and containing *twice* as many cubic feet as there are square feet in the base; and so on; hence the whole n° of cubic feet in any such solid, will be found by taking the product of the n° of feet in height by the n° of square feet in the base, and this last, as in (17), is the product of the n° of feet in length by the n° of feet in breadth.

Hence the n° of *cubic feet* in any rectangular solid or space is found by multiplying together its *length*, *breadth*, and *height* (or *thickness*, as the height would be called when *small*, as, for instance, in the case of a beam of timber), these quantities being all reduced first to the same den^{n}, and their product being of the *same* den^{n}, but in cubic measure.

Ex. Find the solid content of a beam of timber, 30 ft. long, 2 ft. 3 in. wide, and 2 ft. 5 in. thick.

```
                 in.
Here 30 ft. =    360       1728) 281880 (163 cub. ft.
  2 ft. 3 in. =   27             1728
                ----             ----
                2520             10908          27) 163 (6 cub. yds.
                720              10368              162
                ----             -----              ---
                9720 sq. in.     ..5400              1 ft.
  2 ft. 5 in. =   29               5184
                ----               ----
               87480               216 cub. in.
               19440
               -----
```

Ans. 281880 cub. in. = 6 cub. yds. 1 ft. 216 in. by Red^{n}.

20. So also, as before, having given the solid content of any space and any *two* of its three dimensions, we may find the third by dividing the content by the product of these two, reducing all to the same den^n.

Ex. What is the length of a room, whose width is 10 ft. 4 in. and height 10 ft. 6 in.; and which contains 1519 cub. ft. of air?

Here 10 ft. 4 in. = 124 in.
10 ft. 6 in. = 126 in.

```
      744
    1488
    15624 sq in.
```

and 1519 cub. ft. = 2624832 cub. in., hence, performing the div^n, we have

```
  sq. in.    cub. in.      in.
  15624)  2624832  (168
          15624
          106243
           93744
          124992
          124992
```

Ans. 168 in. = 14 ft.

Ex. **16.**

1. 18 ft. 9 in. × 13 ft. 4 in. × 8 ft. 4 in.
2. 3 ft. 9 in. × 6 ft. 8 in. × 2 ft. 1 in.
3. 11 ft. 3 in. × 3 ft. 4 in. × 10 ft. 5 in.
4. 5 yds. × 6 yds. 2 ft. × 4 ft. 2 in.
5. 7 ft. 4 in. × 5 yds. × 8 ft. 3 in.
6. 9 ft. 2 in. × 2 yds. × 6 ft. 8 in.

---

7. How many cubic feet of water can be contained in a vessel with square base, whose side is 3 ft. and height 2 ft. 10 in.?
8. What quantity of timber is there in a beam, whose length is 20 feet, breadth 3 feet, and thickness 2 ft. 6 in.?
9. Find the solid content of a cube, whose side is 7 ft. 5 in.
10. In making a square pond, whose side was 12 yds., there were taken out 336 cub. yards of earth; how deep was it made?
11. What must be the length of a trench, 5 ft. 6 in. deep, and 10 ft. 8 in. wide, that it may contain 7040 cubic feet?
12. The depth of a canal is 7 ft. 3 in., the width 20 ft. 4 in., and the length 10 miles; how many cubic feet of water will it contain?

Miscellaneous Examples. **17.**

1. A sovereign weighs nearly 493 quarter grains; how many lbs. will 1000 sovereigns weigh?

2. In 2551443 seconds, which is the exact length of the lunar month, how many days?

3. What is the cost of 530 lbs. of tea at 3*s.* 7*d.* per lb?

4. Six persons on a journey spend £97 9*s.* 6*d.*; how much is that for each person?

5. The circumference of the Earth contains 131250000 feet; express the same in miles.

6. If 81 oxen are bought for £1779 19*s.* 6*d.*, what is the average price per head?

7. How many letters, paying penny postage, require stamps to the amount of £7947 2*s.* 10*d.*?

8. A pint will contain 9000 barleycorns, and 3 of these, placed end to end, would reach an inch; how many feet would they all reach?

9. How many days would it take to count a million of sovereigns, at the rate of 100 a minute?

10. What is the amount of 42 cwt. of sugar at £2 3*s.* 7*d.* per cwt.?

11. Divide 3587 yds. 9 in. into 27 equal distances.

12. What sum must be divided among 27 men, so that each may receive £14 6*s.* $8\frac{1}{2}$*d.*?

13. How many ducats, each worth 4*s.* 9*d.*, are contained in £231 16*s.*?

14. Divide £1478 12*s.* $9\frac{3}{4}$*d.* into 77 equal portions.

15. How many days in a solar year, which contains 31556928 seconds?

16. A cubic foot of water weighs 1000 ounces; what weight of water is there in a vessel, the length, width, and depth of which are each a yard?

17. The battering ram employed by Titus against the walls of Jerusalem weighed 100000 lbs.; how many tons did it contain?

18. The Calcutta rupee is worth 1*s.* $11\frac{3}{4}$*d.*; what is the value of a lac, which consists of 100000 rupees?

19. Sound travels at the rate of 1140 feet a second; how many miles is a thunder-cloud distant, when the sound follows the flash after 7 seconds?

20. Light travels at the rate of 186040 miles a second; if the Sun's light takes 8 min. 13 sec. in reaching us, what is his distance from the Earth?

21. A cannon-ball travels at the rate of 400 yards a second; how many miles will it go in a quarter of a minute?

22. Find the amount of 200 tons 81 lbs. of iron railing at 7*d.* per lb.

23. Suppose a weekly newspaper, price 3*d.*, has a circulation of 11800; what is the sum realised by its sale in a year?

24. If 2 cwt. 1 lb. cost £116 19*s.* $0\frac{3}{4}$*d.*, what is the cost of 1 lb.?

25. How much silk at 6*s.* 8*d.* a yard may be bought for 20 guineas?

26. To how many persons may £60 15*s.* 6*d.* be distributed, giving £4 13*s.* 6*d.* to each?

27. An Attic drachma was worth $7\frac{3}{4}$*d.*; what was the value of the talent, which contained 6000 drachmæ? and how many minæ did it contain, each worth £3 4*s.* 7*d.*?

28. A Jewish shekel weighed 219 troy grains, and was worth 2*s.* $3\frac{1}{2}$*d.*; what was the weight of a talent, containing 3000 shekels? and the value of 10000 talents?

29. The captains of Israel, after the destruction of Midian, made a free-will offering of 16750 shekels; what sum did this amount to? *See Ex.* 28.

30. How long would a cannon-ball, moving at the rate of 1200 feet a second, be in passing from the Earth to the Moon, 230500 miles?

31. How much is spent in 15 years by a person who spends £825 18*s.* 9*d.* yearly? and how much would he have saved in that time out of an income of £1500?

32. How many pounds weight of bronze are there in a million of pennies, each weighing one-third of an ounce avoird.?

33. A plate of gold cost £161 17*s.* 6*d.*, at £4 7*s.* 6*d.* per ounce; what was its weight?

34. How many patients will an hospital maintain, whose revenue is £5629 10*s.*, when each requires on an average £8 13*s.* 9*d.* per annum?

35. If the duty on brandy, at 10*s.* 5*d.* a gallon, amounted to £26357 5*s.* 10*d.*, on what quantity was it paid?

36. Twenty bricklayers and ten carpenters were employed in building a house, each of the former receiving 27*s.* per week, and each of the latter 29*s.*; what was the amount of their wages in 16 weeks?

37. Two boats start in a race, and one of them gains 5 feet upon the other in every 55 yards; how much will it have gained at the end of half a mile?

38. What is the area of a playground 58 ft. 6 in. long, and 54 ft. 9 in. broad?

39. *A* has £100 4*s.* $11\frac{1}{2}$*d.*, and *B* 64393 farthings; if *A* receive from *B* 11111 farthings, and *B* from *A* £11 11*s.* $11\frac{1}{4}$*d.*, how much will *A* have more than *B*?

40. What is the value of a beam of timber, whose length is 20 ft., breadth 3 ft., and thickness 2 ft., at 3*s.* $8\frac{3}{4}$*d.* per cubic foot?

41. If the length of a cubit was 22 inches, what was the cubic content of the Ark, which was 300 cubits long, 50 broad, and 30 high?

42. A grocer mixes 3 cwt. 24 lbs. of sugar at $6\frac{1}{2}$*d.* per lb. with 2 cwt. 64 lbs. at $4\frac{1}{4}$*d.*; at what price per lb. must he sell the mixture, so as not to lose by the sale?

43. A person gives a five-pound note to pay for lodgings during the month of August, at 2*s.* 8*d.* per night; what sum will be returned to him?

44. Of the three quantities 1347 lbs. avoird., 449 shillings, and £6286, it is required to multiply one quantity by the quotient of the other two.

45. What is the cost of 6 packs of cloth, each containing 6 parcels, each parcel 6 pieces, and each piece 60 yards, at $2\frac{3}{4}$*d.* per yard?

46. A labourer's house-rent is £5 2*s.* 11*d.* a year; what must he lay up weekly to pay it?

47. It is estimated that the average strength of a man is equal to raising 100 lbs. through 1 foot in a second, working 10 hours a day; how many tons will he raise at this rate in the day?

48. In marching, soldiers take 75 steps a minute, in quick marching 108; how far would a regiment advance in 3 hours, the last half-hour at quick march, reckoning each step as 2 ft. 8 in.?

49. If a compositor set up 8500 letters a day, and be paid $5\frac{1}{2}d.$ for every thousand, how much will he earn in a week?

50. Divide £184 11*s.* $2\frac{1}{4}d.$ equally among 39 persons; and, supposing 15 of them to have received their portions, and of the rest only 21 to appear, how much might be given to each of these?

51. A mixture is made of 9 gallons of spirits at 12*s.* 6*d.* per gal., 16 gallons at 18*s.* 9*d.*, and 90 gallons at 22*s.* 3*d.*; what is the value of a gallon of it?

52. A corn-factor buys 2 quarters at 39*s.* per quarter, and 7 bushels at 6*s.* per bushel; at what price per bushel must the whole be sold, so as to gain 23*s.* 9*d.* in all?

53. A side of Lincoln's Inn Square is 770 feet, and of Russell Square 670 feet; how many acres does each contain?

54. What weight of water may be contained in a canal whose depth is 8 feet, width 25 feet, and length 12 miles? *See Ex.* 16.

55. How many yards of carpet, 25 inches wide, will be required to cover a floor that is 19 ft. 7 in. long by 18 ft. 9 in. wide?

56. *A* wished to exchange 50 gallons of brandy, at 21*s.* 9*d.* per gallon, with *B*, for ale at 1*s.* 6*d.* per gallon; how many gallons of ale should he receive?

57. A wall is to be built, 15 yards long, 7 feet high, and 13 inches thick, with a doorway 6 feet high and 4 feet wide; how many bricks will it require, if each, including mortar, occupy 108 cubic inches?

58. Divide £115 10*s.* among 5 men and 6 women, giving to each man thrice as much as to a woman.

59. An equal number of men, women, and boys earned £55 13*s.* in 6 weeks; each man earned 2*s.* 4*d.* a day, each woman 1*s.* 3*d.*, and each boy 10*d.*; how many were there of each?

60. There is a plantation in the form of a hollow square, length externally 252 yards, and depth 16 yards; find the area of the plantation and that of the inner square.

61. Divide £39 into four equal numbers of guineas, half-guineas, crowns, and half-crowns respectively.

62. A clergyman commutes his tithes, valued at £500, for equal quantities of wheat, barley, and oats; how much grain will he receive, supposing the average price of wheat to be 6*s.* 7*d.* a bushel, of barley 3*s.* 11*d.*, and of oats 2*s.* 10*d.*?

63. *A* and *B* go to bed at the same hour daily, but *A* rises at a quarter past 6, and *B* at 8; how much of waking life will *A* have had more than *B* in 40 years, paying attention to the Leap-years?

64. Divide £20 among three persons, so that one may have £3 15*s.* more than each of the others.

65. Divide £550 3*s.* $1\frac{1}{2}$*d.* among 4 men, 6 women, and 8 children, giving to each man double of a woman, and to each woman triple of a child.

66. Divide £2 9*s.* 2*d.* among *A*, *B*, *C*, so that *B* may have 6*s.* 8*d.* more than *A*, and *C*'s share may be double of *B*'s.

67. The circumference of the fore wheel of a carriage being 8 ft. 3 in., and that of the hind wheel 11 ft. 11 in., how many more revolutions would be made by the fore wheel than by the hind wheel in going from Cambridge to London, a distance of 52 miles?

68. In new enclosures, the cost per acre of the *first* crop (wheat) is £6 14*s.* 6*d.*, and the produce 18 bushels at 8*s.*; that of the *second* crop (barley) is £3 16*s.*, and the produce 25 bushels at 4*s.*; and that of the *third* crop (potatoes) is £12 11*s.* 2*d.*, and the produce 100 bags at 3*s.*; deducting one-tenth of the whole produce for tithes, find the result of enclosing 500 acres, in *one* year and in *three.*

CHAPTER II.

GREATEST COMMON MEASURE: LEAST COMMON MULTIPLE.

21. ONE number is said to be a *measure* or a *factor* of another, when it divides it exactly, without remainder.

Thus, 1, 2, 3, 4, 6, 12 are all measures or factors of 12.

Unity, however, is not generally named among the divisors of a number.

22. Any number, which divides without remainder each of two or more numbers, is said to be a *common* measure or *common* factor of those numbers; and, of course, the greatest number which so divides them is their *Greatest Common Measure* (G. C. M.)

Thus 2 is the only common measure of 4 and 6; 3, 5, 15 are, each of them, common measures of 30 and 45, and 15 is their *greatest* common measure; 2, 7, 14 are, each of them, common measures of 14, 42, and 70, and 14 is their *greatest* common measure.

23. *To find the Greatest Common Measure of two numbers.*

RULE. Divide the greater by the less, and the preceding divisor by the remainder, and so on continually, until there is no remainder: the *last divisor* will be the G. C. M. required.

Ex. 1. Find the G. C. M. of 3575 and 125455; and of 279 and 4185.

```
3575) 125455 (35                               279) 4185 (15
      10725                                         279
      -----                                         ----
       18205                                        1395
       17875                                        1395
       -----                                        ----
         330) 3575 (10                                  Ans. 279.
              330
              ---
               275) 330 (1
                    275
                    ---
                     55) 275 (5
                         275
                         ---
        Ans. 55.
```

Ex. 2. Find the G. C. M. of 17 and 36.

```
17) 36 (2
    34
    --
     2) 17 (8
        16
        --
         1) 2 (2
            2
            -
```

Ans. 1.; i. e. the given numbers have no common measure but *unity.*

The *reason* of this Rule can hardly be explained without some knowledge of Algebra in the Student. The Rule itself is here introduced, because it is often useful in reducing Vulgar Fractions to simple forms. *See* NOTE V.

Ex. **18.** Find the G. C. M. of

1.	224 and 336.	2.	348 and 1024.
3.	175 and 2042.	4.	1225 and 625.
5.	2121 and 1313.	6.	429 and 715.
7.	377 and 1131.	8.	2431 and 770.
9.	900 and 3474.	10.	1379 and 2401.
11.	2314 and 3721.	12.	7007 and 7392.
13.	2793 and 2660.	14.	4165 and 686.
15.	5325 and 8307.	16.	3775 and 10000
17.	7056 and 7392.	18.	6327 and 23997.
19.	12321 and 54345.	20.	24720 and 4155.

---

24. One number is said to *contain*, or to be a *multiple of*, another, when it can be divided by it without remainder.

Thus 12 is a multiple of each of 1, 2, 3, 4, 6, 12; and any number is a multiple of each of its measures.

25. A *common* multiple of two or more numbers is one which contains each of them; and, of course, the *least* such number, is their *Least Common Multiple* (L. C. M.).

Thus 6, 12, 18, &c., are all common multiples of 2 and 3; but 6 is their *least* common multiple: 12, 24, 36, 48, &c., are all common multiples of 2, 3, 4, 6, and 12; but 12 is their *least* common multiple.

Of course, a common multiple of any given numbers may be found, by multiplying them all together; thus a common multiple of 6 and 8 is 48, of 4, 6, and 9 is 216. In practice, however, we require the *least* common multiple, especially in preparing Vulgar Fractions for Addition and Subtraction

26. *To find the Least Common Multiple of two or more numbers.*

Rule. Set them in a line, and strike out any that are contained in any of the others. Divide those not struck out by any number that will exactly divide one of them; under any which it *exactly* measures, place the corresponding quotient; under any which it *partially* measures (containing some factor common to it, but not being itself wholly contained in it), place the quotient obtained by dividing it by the common factor; and under any which it does not measure at all, repeat the number itself.

Now treat the new line thus formed, in the same manner as the first; and so on, until all the numbers left in any line have no common measure but unity.

Then the continued product of the numbers in this line and all the divisors is the L. C. M. required of the given numbers.

Obs. It will generally be most convenient to take pretty large numbers, if possible, for divisors; as fewer lines will thus be necessary, especially if such be chosen as contain themselves many simple factors. Thus 12 contains the factors 2, 3, 4, 6, 12, and is therefore, when possible, a very good divisor to be employed.

Ex. 1. Find the L. C. M. of 24, 16, 6, 20, 4, 8, 10, 30, 12, 25.

12) 24 . 16 . ~~6~~ . 20 . ~~4~~ . ~~8~~ . ~~10~~ . 30 . ~~12~~ . 25
~~2~~ . 4 . ~~5~~ . ~~5~~ . 25

Ans. $4 \times 25 \times 12 = 1200$.

The reason of this process may be thus explained.

We are required to find a number, which shall contain 24, 16, 6, 20, 4, 8, 10, 30, 12, and 25. Now if we find a number which contains 24, it will, of course, contain 6, 4, 8, and 12, which are themselves contained in 24. We may therefore strike out 6, 4, 8, and 12; and for a similar reason, 10, which is contained in 20; and we thus reduce the question to finding the L. C. M. of 24, 16, 20, 30, and 25.

Now we choose for divisor, according to the Rule, the number 12, which exactly divides one of these, viz. 24. In order, therefore, that the L. C. M. required may contain 24, it must, of course, contain this number

12, and besides that a factor 2; but we now wish to find what factors *besides* 12 and 2, the L. C. M. must contain, so as to contain all the given numbers. We see then that 12 will also supply the factors 4 of 16, 4 of 20, and 6 of 30; so that the only others besides 12, which must be contained in the required number, are 2 to make up 24, 4 for 16, 5 for 20, 5 for 30, and 25, i.e. the numbers given by our process in the second line; —to which a similar reasoning applies.

Ex. 2. What is the least number that can be divided by each of 14, 16, 40, 50, 25, 8, 64?

```
10) 14 . 16 . 40 . 50 . 25 . 8 . 64
    ------------------------------
     7 .       4 . 5 .          32
```

Ans. 7 × 5 × 32 × 10 = 11200.

Ex. 3. Find the L. C. M. of 27, 24, 6, 15, 5, 9, 126.

```
9) 27 . 24 . 6 . 15 . 5 . 9 . 126
   ------------------------------
2) 3 .  8 .      5 .         14
   ------------------------------
   3 .  4 .      5 .          7
```

Ans. 3 × 4 × 5 × 7 × 9 × 2 = 7560.

Ex. **19.** Find the L. C. M. of

1. 15, 20.
2. 14, 21.
3. 8, 4, 16.
4. 3, 9, 22.
5. 12, 15, 16.
6. 8, 16, 20.
7. 9, 15, 18, 20.
8. 16, 9, 12, 18.
9. 8, 12, 15, 20.
10. 34, 68, 17, 2.
11. 6, 12, 16, 18, 24.
12. 8, 12, 18, 24, 27.
13. 2, 4, 8, 16, 10, 48.
14. 1, 2, 3, 4, 5, 6, 7, 8, 9.
15. 7, 12, 15, 27, 35, 40, 45.
16. 9, 16, 42, 63, 21, 14, 72.
17. 4, 9, 10, 15, 18, 20, 21.
18. 7, 15, 21, 28, 35, 100, 125.
19. 8, 9, 10, 12, 25, 32, 75, 80.
20. 15, 16, 18, 20, 24, 25, 27, 30.

CHAPTER III.

VULGAR FRACTIONS.

27. A *Fraction* is a quantity which represents a part or parts of an integer, or whole.

28. A *Vulgar* (that is, a common) Fraction, in its simplest form, is expressed by means of two numbers placed one over the other, with a line between them.

The lower of these is called the *Denominator*, and shows into how many equal parts the whole is divided; the upper is called the *Numerator*, and shows how many of those parts are taken to form the fraction.

Thus $\frac{3}{4}$ denotes that the whole is divided into four equal parts, and that three of them are taken to form the fraction.

29. A *proper* fraction is one whose numerator is *less* than the denominator, and which is itself therefore *less* than the whole in question; as $\frac{3}{8}$, $\frac{5}{7}$.

An *improper* fraction is one, whose numerator is *equal to* or *greater than* the denominator, and which is itself, therefore, *equal to* or *greater than* the whole in question; as $\frac{8}{8}$, $\frac{11}{7}$.

30. A *mixed* number is one formed of a whole number and a fraction; as $2\frac{2}{3}$, $5\frac{3}{4}$.

A *compound* fraction is a fraction of a fraction; as $\frac{3}{4}$ of $\frac{2}{3}$. $2\frac{1}{2}$ of $\frac{3}{7}$ of $3\frac{1}{4}$.

A *complex* fraction is one in which either the num^r, or den^r or both are fractions; as $\frac{3\frac{1}{2}}{2}$, $\frac{2}{4\frac{2}{3}}$, $\frac{1\frac{1}{3}}{3\frac{3}{4}}$, $\frac{\frac{3}{4}\text{ of }3}{2\frac{1}{2}}$.

31. Every whole number may be considered as a fraction whose den^r is 1; thus 6 is $\frac{6}{1}$.

32. A fraction may be considered as expressing the division of the num^r by the den^r.

Thus $\frac{3}{4}$ expresses $3 \div 4$: for we should obtain the same, whether we divide *one* unit into 4 equal parts, and then take three of these parts, that is, *three-fourths* of the *one* unit; or divide *three* units, each into 4 equal parts, and then take one part out of each four, *i. e.* one-fourth of each unit, and therefore *one-fourth* of the whole *three* units; so that $\frac{3}{4}$ of 1, or $\frac{3}{4}, = \frac{1}{4}$ of 3, or $3 \div 4$.

For instance, $\frac{3}{4}$ of £1, which is 15*s.*, $= \frac{1}{4}$ of £3, which is also 15*s.*

---

33. *To reduce a whole number to a fraction with a given denominator.*

Rule. Multiply the number by the given den^r, and the result will be the *num^r* of the fraction required.

Ex. Reduce 5 to a fraction, with denominator 6.
Since 1 contains 6 sixth parts, ∴ 5 contains 30 sixth parts; or $5 = \frac{30}{6}$.

Ex. **20.** 1. Reduce 8 and 27 to fractions with den^{rs} 5 and 27.
2. Reduce 34 and 135 to fractions with den^{rs} 11 and 17.
3. Reduce 6, 9, 12, 20, to fractions with den^r 15.
4. Reduce 25, 34, 70, 111, to fractions with den^r 34.

---

34. *To reduce a mixed number to an improper fraction.*

Rule. Multiply the *whole number* by the den^r of the fractional part; add the result to the num^r of that part for the new num^r, and retain the same den^r.

Ex. 1. $7\frac{2}{3} = \frac{23}{3}$: for $7 = \frac{21}{3}$ (33); and hence, $7\frac{2}{3} = \frac{21}{3} + \frac{2}{3} = \frac{23}{3}$.
Ex. 2. $1\frac{1}{8} = \frac{9}{8}$. Ex. 3. $5\frac{4}{9} = \frac{49}{9}$.

Ex. **21.** Reduce to improper fractions

1.	$3\frac{5}{7}$.	2.	$10\frac{2}{9}$.	3.	$221\frac{4}{11}$.	4.	$13\frac{15}{17}$.	5.	$32\frac{11}{13}$.
6.	$200\frac{27}{50}$.	7.	$71\frac{11}{12}$.	8.	$115\frac{13}{15}$.	9.	$128\frac{11}{18}$.	10.	$37\frac{15}{37}$.
11.	$200\frac{29}{30}$.	12.	$125\frac{24}{25}$.	13.	$514\frac{5}{16}$.	14.	$101\frac{10}{21}$.	15.	$719\frac{11}{12}$.
16.	$1\frac{113}{115}$.	17.	$17\frac{201}{239}$.	18.	$10\frac{213}{360}$.	19.	$111\frac{100}{111}$.	20.	$85\frac{85}{99}$.

---

35. *To reduce an improper fraction to a whole or mixed number.*

Rule. Divide the num^r by the den^r: the quotient will be a whole number, and the remainder, if any, the num^r of the fractional part of the mixed number required.

Ex. 1. $\frac{45}{9}=5$. Ex. 2. $\frac{113}{15}=7\frac{8}{15}$.

Obs. All improper fractions, occurring in any sum, should (except the contrary be desired) be expressed as whole or mixed numbers.

Ex. **22.** Reduce to whole or mixed numbers

1. $\frac{37}{9}$.	2. $\frac{79}{11}$.	3. $\frac{313}{13}$.	4. $\frac{2990}{23}$.	5. $\frac{1023}{35}$.
6. $\frac{3127}{43}$.	7. $\frac{1210}{55}$.	8. $\frac{2221}{87}$.	9. $\frac{1247}{77}$.	10. $\frac{3136}{95}$.
11. $\frac{3000}{75}$.	12. $\frac{3577}{102}$.	13. $\frac{4148}{117}$.	14. $\frac{4641}{221}$.	15. $\frac{3135}{122}$.
16. $\frac{6000}{375}$.	17. $\frac{5434}{357}$.	18. $\frac{6556}{401}$.	19. $\frac{12321}{200}$.	20. $\frac{23438}{333}$.

---

36. *To multiply a fraction by any whole number* or *integer*, either multiply the numerator, or divide the denominator by it.

Ex. 1. $\frac{2}{15}\times 7=\frac{14}{15}$.

For in each of the fractions, $\frac{2}{15}$ and $\frac{14}{15}$, the whole is divided into 15 equal parts, and 7 times as many of them are taken in the latter case as in the former.

Ex. 2. $\frac{7}{16}\times 4=\frac{7}{4}=1\frac{3}{4}$.

For the whole being divided into 4 times as many equal parts in $\frac{7}{16}$ as it is in $\frac{7}{4}$, each of the parts in the latter is 4 times as great as in the former; and the same number of parts being taken in both cases, the latter fraction is therefore 4 times as great as the former.

Ex. 3. $\frac{3}{5}\times 9=\frac{27}{5}=5\frac{2}{5}$. Ex. 4. $\frac{7}{15}\times 4=\frac{28}{15}=1\frac{13}{15}$.

Ex. 5. $\frac{13}{27}\times 9=\frac{13}{3}=4\frac{1}{3}$. Ex. 6. $\frac{15}{28}\times 7=\frac{15}{4}=3\frac{3}{4}$.

37. Conversely—*To divide a fraction by any integer*, either divide the numerator, or multiply the denominator by it.

Ex. 1. $\frac{12}{13}\div 6=\frac{2}{13}$. Ex. 2. $\frac{15}{28}\div 5=\frac{3}{28}$.

Ex. 3. $\frac{2}{3}\div 5=\frac{2}{15}$. Ex. 4. $\frac{7}{9}+6=\frac{7}{54}$.

Ex. **23.** 1. Multiply $\frac{35}{36}$ by 9, 12, 18, 25; and divide it by 5, 7, 8, 12.
2. Multiply $\frac{125}{144}$ by 7, 8, 9, 16; and divide it by 5, 8, 12, 25.
3. Multiply $\frac{320}{693}$ by the numbers 2, 3, 4, 5, 7.
4. Divide $\frac{320}{693}$ by the numbers 7, 8, 9, 10, 11.

38. If the num^r and den^r of a fraction be *both* multiplied or both divided by the same number, its value will not be altered.

Ex. 1. $\frac{5}{7}=\frac{10}{14}=\frac{15}{21}=\frac{20}{28}=$ &c. Ex. 2. $\frac{36}{48}=\frac{18}{24}=\frac{12}{16}=\frac{9}{12}=$ &c.

For if the *num^r* be *multiplied* by any number, the fraction is *multiplied* by it (36), and if the *den^r* be multiplied by any number, the fraction is divided by it (37); and if any quantity be *both* multiplied and divided by the *same* number, its value is not altered.

Similarly, when the num^r or den^r are both *divided* by the same number.

39. *To reduce a fraction to lower terms.*

Rule. Divide both the num^r and den^r by any common factors they contain.

Ex. 1. $5)\frac{270}{375}=3)\frac{54}{75}=\frac{18}{25}$. Ex. 2. $9)\frac{315}{378}=7)\frac{35}{42}=\frac{5}{6}$.

From (38) it appears that the value of a fraction is not altered by this process.

When a fraction is reduced as much as possible by such division, it is said to be in its *lowest terms*. (See p. 20, *note.*)

Obs. All fractions, occurring in any sum, should (except where the contrary is desired) be expressed in their lowest terms.

Ex. **24.** Reduce to their lowest terms

1. $\frac{324}{720}$.	2. $\frac{720}{864}$.	3. $\frac{324}{396}$.	4. $\frac{1584}{5940}$.	5. $\frac{1296}{1620}$.
6. $\frac{1452}{2178}$.	7. $\frac{495}{1210}$.	8. $\frac{1296}{1728}$.	9. $\frac{1872}{2016}$.	10. $\frac{990}{1935}$.
11. $\frac{3000}{3375}$.	12. $\frac{2592}{3456}$.	13. $\frac{1485}{2160}$.	14. $\frac{864}{3072}$.	15. $\frac{3300}{4235}$.
6. $\frac{6930}{8118}$.	17. $\frac{5544}{6552}$.	18. $\frac{7040}{7992}$.	19. $\frac{11385}{16335}$.	20. $\frac{22176}{23328}$.

---

40. A fraction may be reduced *immediately* to its lowest terms by dividing both its num^r and den^r by their G. C. M.

This process is generally longer than the other, and is therefore, if possible, avoided in practice. It is, however, sometimes, the only way of reducing a fraction, when we are unable to detect by *inspection* the common factors of the num^r and den^r. Thus we should not see, perhaps, that the fraction $\frac{2599}{4181}$ may be reduced to $\frac{23}{37}$ by dividing both its terms by 113, their G. C. M.

Ex. 1. $179)\,\frac{4117}{9487}=\frac{23}{53}$. Ex. 2. $564)\,\frac{13536}{17484}=\frac{24}{31}$.

Ex. **25.** Reduce to their lowest terms

1.	$\frac{321}{749}$.	2.	$\frac{510}{1122}$.	3.	$\frac{299}{529}$.	4.	$\frac{1407}{4422}$.
5.	$\frac{1905}{3175}$.	6.	$\frac{1715}{2695}$.	7.	$\frac{6509}{7889}$.	8.	$\frac{1589}{2270}$.
9.	$\frac{8251}{14718}$.	10.	$\frac{3575}{4719}$.	11.	$\frac{1261}{44232}$.	12.	$\frac{10759}{20405}$.

---

41. We shall now give examples of the application of the foregoing rules to the mult^n and div^n of *concrete* quantities.

Ex. 1.

£	s.	d.
23	13	$9\frac{7}{8}\times 35$
		5
118	9	$1\frac{3}{8}$
		7
829	3	$9\frac{5}{8}$

Here $\frac{7}{8}\times 5=\frac{35}{8}=4\frac{3}{8}$; we set down $\frac{3}{8}d.$, and carry 4*d.*: so also $\frac{3}{8}\times 7=\frac{21}{8}=2\frac{5}{8}$; we set down $\frac{5}{8}d.$, and carry 2*d.*

Ex. 2.

	£	s.	d.
7)	37	14	$8\div 28$
4)	5	7	$9\frac{5}{7}$
	1	6	$11\frac{3}{7}$

Here, in the first div^n, there are 5*d.* over, to be divided by 7, which we set down as $\frac{5}{7}d.$, since (32) $\frac{1}{7}$ of $5d.=\frac{5}{7}$ of 1*d.* We might have brought these 5*d.* to 20*f.* and then, dividing by 7, should have had $2\frac{6}{7}f.$; but as a farthing itself is only a *fraction* of a penny, it is usual, when the result does not come out a clear number of farthings, to express the whole below the pence as a fraction of a penny.

In the second div^n, there is $1\frac{5}{7}d.$ over, or $\frac{12}{7}d.$, which, divided by 4, gives $\frac{3}{7}d.$

Ex. 3.

	£	s.	d.
8)	175	19	$5\frac{3}{4}\div 40$
5)	21	19	$11\frac{7}{32}$
	4	7	$11\frac{27}{32}$

Here, in the first div^n, there is $1\frac{3}{4}d.$ over $=\frac{7}{4}d.$, which, divided by 8, gives $\frac{7}{32}d.$; in the second div^n, there is $4\frac{7}{32}d.$ over $=\frac{135}{32}d.$, which, divided by 5, gives $\frac{27}{32}d.$

```
            £  s.  d.   £   s.  d.
Ex. 4.  13) 54 10  5½  (4   3  10 15/26
            52
            ——
             2
            20
            ——
            50 (3s.
            39        Here there is 7½d. over = 15/2 d, which, divided
            11                                 by 13, gives 15/26 d.
            12
            ————
            137½ (10d.
            130
            ————
              7½
```

```
            £    s.  d.
Ex. 5.  3)  1115 17  8½ ÷ 300
       100) 3.71  19  2 5/6      Here there is 70 5/6 d. over = 425/6 d., which,
              20                  divided by 100, gives 425/600 d. = 17/24 d.
            —————
            14.39
               12
            —————
            4.70 5/6          Ans. £3 14s. 4 17/24 d.
```

Ex. 26.

	£	s.	d.			£	s.	d.			£	s.	d.	
1.	3	17	$4\frac{3}{8}$	× 5.	2.	5	11	$2\frac{4}{5}$	× 7.	3.	4	0	$5\frac{2}{3}$	× 9.
4.	7	8	$11\frac{5}{6}$	× 11.	5.	6	1	$7\frac{9}{10}$	× 15.	6.	8	2	$5\frac{4}{5}$	× 27.
7.	6	17	$4\frac{5}{7}$	× 32.	8.	2	19	$9\frac{7}{8}$	× 44.	9.	4	13	$0\frac{5}{9}$	× 29.
10.	5	3	$4\frac{12}{13}$	× 31.	11.	7	14	$9\frac{2}{35}$	× 37.	12.	6	18	$0\frac{5}{27}$	× 41.

	£	s.	d.			£	s.	d.	
13.	2	0	1	÷ 3.	14.	9	7	$3\frac{1}{4}$	÷ 4.
15.	29	17	8	÷ 5.	16.	72	13	5	÷ 6.
17.	8	13	0	÷ 9.	18.	37	6	2	÷ 10.
19.	73	0	$5\frac{1}{4}$	÷ 8.	20.	29	7	$0\frac{1}{2}$	÷ 7.
21.	69	17	$5\frac{3}{4}$	÷ 9.	22.	53	4	$0\frac{1}{2}$	÷ 12.
23.	124	15	6	÷ 15.	24.	131	11	$8\frac{1}{4}$	÷ 18.
25.	135	14	10	÷ 40.	26.	111	11	$11\frac{1}{4}$	÷ 60.
27.	1275	3	8	÷ 200.	28.	675	13	$6\frac{1}{2}$	÷ 500.
29.	1134	15	10	÷ 1000.	30.	4332	13	$7\frac{3}{4}$	÷ 3000.

---

42. *To reduce a compound fraction to a simple one.*

RULE. Multiply together all the num^{rs} for a new num^{r}, and all the den^{rs} for a new den^{r}.

Ex. 1. $\frac{2}{3}$ of $\frac{4}{5} = \frac{8}{15}$.

For *one-third* of $\frac{4}{5}$ is $\frac{4}{15}$ (37); therefore *two-thirds*, which must be twice as great, is $\frac{8}{15}$ (36).

By similar reasoning, $\frac{4}{5}$ of $\frac{2}{3} = \frac{8}{15} = \frac{2}{3}$ of $\frac{4}{5}$.

Ex. 2. $\frac{3}{4}$ of $5 = \frac{3}{4}$ of $\frac{5}{1} = \frac{15}{4}$.

Mixed numbers must be reduced to improper fractions, before the rule can be applied.

Ex. 3. $2\frac{3}{4}$ of 5 of $3\frac{1}{2} = \frac{11}{4}$ of $\frac{5}{1}$ of $\frac{7}{2} = \frac{385}{8} = 48\frac{1}{8}$.

Compound fractions may often be reduced by striking out factors common to one of the num^{rs} and one of the den^{rs}.

Ex. 4. $\frac{3}{\cancel{4}}$ of $\frac{\overset{2}{\cancel{8}}}{\cancel{9}}$ of $\frac{\overset{3}{\cancel{27}}}{\underset{5}{\cancel{10}}} = \frac{9}{5} = 1\frac{4}{5}$ (35, OBS.).

Ex. **27.** Express as simple fractions

1. $\frac{1}{2}$ of $\frac{5}{6}$ of 4.
2. $\frac{2}{3}$ of $\frac{5}{7}$ of 6.
3. $\frac{3}{4}$ of $\frac{4}{5}$ of 3.
4. $\frac{2}{3}$ of $\frac{1}{4}$ of $3\frac{1}{2}$.
5. $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{5}{8}$.
6. $\frac{2}{5}$ of $3\frac{1}{11}$ of $9\frac{1}{6}$.
7. $\frac{2}{5}$ of $\frac{5}{8}$ of $\frac{3}{10}$.
8. $\frac{2}{7}$ of $\frac{5}{8}$ of $3\frac{1}{2}$.
9. $4\frac{1}{5}$ of $3\frac{3}{4}$ of 10.
10. $2\frac{1}{2}$ of $\frac{3}{5}$ of $7\frac{1}{3}$.
11. $\frac{3}{4}$ of $\frac{5}{8}$ of $7\frac{1}{9}$.
12. $3\frac{1}{7}$ of $1\frac{5}{9}$ of $3\frac{3}{8}$.
13. $\frac{6}{7}$ of $\frac{11}{12}$ of 9 of $6\frac{1}{8}$.
14. $\frac{12}{13}$ of $2\frac{2}{3}$ of $1\frac{1}{25}$ of $1\frac{11}{64}$.
15. $\frac{4}{15}$ of $\frac{9}{21}$ of $\frac{5}{8}$ of 7.
16. $\frac{3}{8}$ of $6\frac{1}{2}$ of $\frac{13}{14}$ of $\frac{7}{26}$.
17. $\frac{3}{11}$ of $1\frac{2}{9}$ of $5\frac{1}{3}$ of $\frac{1}{4}$.
18. $1\frac{1}{2}$ of $2\frac{2}{3}$ of $3\frac{3}{4}$ of $4\frac{4}{5}$.
19. $3\frac{5}{9}$ of $2\frac{1}{4}$ of $\frac{3}{8}$ of $\frac{7}{15}$.
20. $\frac{7}{11}$ of $2\frac{1}{2}$ of $\frac{3}{7}$ of $10\frac{1}{2}$.

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43. *To reduce fractions to a common denominator.*

RULE. Find the L. C. M. of all the den^{rs}, and take this for the common den^{r}: for the new num^{rs} multiply each num^{r} by the number obtained by dividing the common den^{r} by its own den^{r}.

Ex. Reduce $\frac{5}{8}$, $\frac{11}{12}$, $\frac{7}{18}$, to their least common denominator.

The L. C. M. of 8, 12, 18, being 72, we have

$$\frac{5}{8} = \frac{5 \times 9}{72} = \frac{45}{72}, \quad \frac{11}{12} = \frac{11 \times 6}{72} = \frac{66}{72}; \quad \frac{7}{18} = \frac{7 \times 4}{72} = \frac{28}{72},$$

where the factors 9, 6, 4, in the new num^{rs} are obtained by dividing the common den^r 72 by the original den^{rs} 8, 12, 18, respectively.

For, in any one of these fractions, it is plain that its num^r and den^r have *both* been multiplied by the *same* number, viz. that which makes its den^r = 72.

Ex. **28.** Reduce to their least common den^r

1. $\frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{2}{7}$.
2. $\frac{5}{6}, \frac{4}{7}, \frac{4}{5}, \frac{2}{11}$.
3. $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}$.
4. $\frac{1}{8}, \frac{5}{9}, \frac{3}{16}, \frac{13}{18}$.
5. $\frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \frac{31}{32}$.
6. $\frac{5}{6}, \frac{5}{8}, \frac{2}{9}, \frac{13}{24}$.
7. $\frac{7}{16}, \frac{11}{18}, \frac{17}{24}, \frac{19}{36}, \frac{25}{48}$.
8. $\frac{2}{3}, \frac{4}{9}, \frac{16}{27}, \frac{8}{81}, \frac{16}{243}$.
9. $\frac{4}{7}, \frac{3}{10}, \frac{5}{12}, \frac{17}{35}, \frac{4}{63}, \frac{15}{28}$.
10. $\frac{11}{27}, \frac{17}{24}, \frac{5}{6}, \frac{7}{15}, \frac{2}{9}, \frac{35}{36}$.
11. $\frac{3}{5}, \frac{7}{10}, \frac{6}{25}, \frac{11}{30}, \frac{13}{45}, \frac{23}{60}$.
12. $\frac{5}{7}, \frac{11}{12}, \frac{2}{15}, \frac{8}{27}, \frac{9}{35}, \frac{17}{40}$.

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44. *Addition of Fractions.*

RULE. Reduce them (if necessary) to their least common den^r; and take the sum of the num^{rs}, retaining the common den^r.

Ex. 1. $\frac{3}{5}+\frac{1}{5}=\frac{4}{5}$.

For the whole being divided into 5 equal parts, 3 of those parts, together with 1 of those parts, must make 4 such parts.

Ex. 2. $\frac{2}{3}+\frac{3}{4}+\frac{4}{5}=\frac{40+45+48}{60}=\frac{133}{60}=2\frac{13}{60}$.

If any of the given quantities are whole or mixed numbers, it is best to take separately the sum of the integral and fractional parts, and then add the two results together.

Ex. 3. $2\frac{3}{5}+3\frac{9}{10}+5\frac{5}{12}+4$.

Here $\frac{3}{5}+\frac{9}{10}+\frac{5}{12}=\frac{36+54+25}{60}=\frac{115}{60}=1\frac{55}{60}=1\frac{11}{12}$;

$$\therefore\ 2+3+5+4+1\tfrac{11}{12}=15\tfrac{11}{12}.$$

Improper fractions should be reduced to mixed numbers, and compound fractions to simple ones, before the application of this rule.

Ex. 4. $\frac{113}{8}+\frac{3}{5}$ of $\frac{10}{9}+2\frac{3}{4}$ of $2\frac{2}{11}$ of $\frac{5}{8}+5=14\frac{1}{8}+\frac{2}{3}+3\frac{3}{4}+5$.

Here $\frac{1}{8}+\frac{2}{3}+\frac{3}{4}=\frac{3+16+18}{24}=\frac{37}{24}=1\frac{13}{24}$;

$$\therefore\ 14+3+5+1\tfrac{13}{24}=23\tfrac{13}{24}.$$

Ex. **29.** Find the value of

1. $\frac{4}{7}+\frac{2}{7}+\frac{6}{7}+\frac{5}{7}+\frac{3}{7}$.
2. $\frac{1}{2}+\frac{1}{3}+\frac{7}{8}+\frac{5}{12}$.
3. $\frac{1}{2}+\frac{3}{4}+\frac{5}{6}+\frac{7}{9}$.
4. $\frac{13}{18}+\frac{8}{15}+\frac{11}{20}+\frac{13}{30}$.
5. $\frac{2}{3}+\frac{1}{6}+\frac{5}{9}+\frac{11}{12}$.
6. $\frac{7}{8}+\frac{7}{12}+\frac{7}{16}+\frac{7}{18}$.
7. $\frac{3}{10}+\frac{13}{15}+\frac{1}{5}+\frac{4}{9}$.
8. $\frac{11}{70}+\frac{5}{21}+\frac{1}{5}+\frac{17}{42}$.
9. $2\frac{1}{2}+3\frac{1}{3}+4\frac{1}{4}+5$.
10. $3\frac{3}{8}+2\frac{5}{6}+\frac{7}{12}+3\frac{4}{9}$.
11. $2\frac{2}{3}+\frac{3}{5}+4+5\frac{5}{6}$.
12. $1\frac{3}{8}+\frac{1}{6}+\frac{5}{18}+2\frac{1}{12}$.
13. $\frac{2}{27}+11\frac{5}{54}+2\frac{7}{45}+\frac{1}{10}$.
14. $\frac{11}{12}+\frac{14}{15}+\frac{26}{27}+\frac{39}{40}$.
15. $3\frac{1}{42}+\frac{5}{21}+\frac{31}{63}+1\frac{11}{14}$.
16. $17\frac{1}{35}+\frac{3}{7}+\frac{4}{21}+1\frac{7}{15}$.
17. $\frac{2}{7}$ of $18+\frac{3}{5}$ of $1\frac{4}{21}$.
18. $\frac{11}{12}+1\frac{2}{15}+\frac{7}{16}+2\frac{11}{18}+\frac{1}{20}$.
19. $1\frac{15}{16}+2\frac{23}{24}+3\frac{24}{25}+4\frac{29}{30}$.
20. $5\frac{3}{4}+\frac{3}{5}$ of $7\frac{1}{2}+8\frac{3}{10}$.
21. $\frac{2}{3}+7\frac{2}{11}+\frac{4}{5}$ of $\frac{3}{7}$ of $10\frac{1}{2}$.
22. $2\frac{3}{4}$ of $3\frac{2}{3}+\frac{111}{16}+2\frac{4}{5}$ of $4\frac{1}{8}$ of $1\frac{3}{8}+4\frac{2}{3}$ of $\frac{2}{15}$ of $2\frac{1}{8}$ of $1\frac{3}{7}$.

	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.
23.	3	5	$7\frac{3}{8}$	24.	7	5	$8\frac{5}{6}$	25.	3	15	$7\frac{1}{2}$	26.	7	11	$8\frac{3}{4}$
	4	10	$8\frac{1}{2}$		2	13	$3\frac{4}{9}$		5	14	$2\frac{4}{5}$		2	9	$7\frac{5}{12}$
	5	6	$5\frac{5}{6}$		5	11	$4\frac{3}{4}$		7	6	$10\frac{5}{6}$		6	5	$4\frac{7}{16}$
	6	12	$9\frac{3}{4}$		2	8	$5\frac{1}{2}$		8	1	$11\frac{11}{15}$		3	18	$7\frac{9}{24}$
	7	5	$2\frac{7}{12}$		7	17	$3\frac{5}{12}$		2	4	$6\frac{2}{3}$		4	5	$6\frac{7}{8}$
	2	3	$0\frac{2}{3}$		0	10	$4\frac{7}{18}$		1	4	$5\frac{9}{10}$		3	19	$2\frac{1}{2}$

27.	8	5	$7\frac{1}{2}$	28.	7	13	$1\frac{1}{3}$	29.	17	13	$5\frac{1}{3}$	30.	23	2	$6\frac{2}{9}$
	6	1	$2\frac{3}{8}$		2	17	$4\frac{3}{7}$		32	6	$11\frac{1}{4}$		14	1	$5\frac{1}{6}$
	5	17	$8\frac{3}{4}$		5	2	$8\frac{4}{9}$		12	10	$9\frac{5}{6}$		7	8	$11\frac{2}{3}$
	6	4	$2\frac{9}{10}$		6	11	$2\frac{11}{21}$		7	0	$8\frac{1}{2}$		4	9	$5\frac{11}{12}$
	5	1	$7\frac{4}{5}$		4	5	$0\frac{2}{7}$		11	5	$4\frac{1}{8}$		16	4	$2\frac{7}{8}$
	7	12	$6\frac{19}{20}$		6	3	$4\frac{1}{9}$		6	16	$5\frac{7}{12}$		5	4	$3\frac{3}{4}$

45. *Subtraction of Fractions.*

RULE. Reduce them (if necessary) to their least common denr, and take the difference of the numrs, retaining the common denr.

Ex. 1. $\frac{4}{5}-\frac{1}{5}=\frac{3}{5}$.

For the whole being divided into 5 equal parts, and 1 of those parts being taken from 4 of those parts, there will remain 3 such parts.

Ex. 2. $\frac{9}{10}-\frac{7}{15}=\frac{27-14}{30}=\frac{13}{30}$.

If the given quantities are both mixed numbers, or consist of a whole and a mixed number, it is best to take separately the difference of the integral and the fractional parts, and then add the two results together.

Ex. 3. $5\frac{5}{8}-2\frac{1}{2}$.
Here $\frac{5}{8}-\frac{1}{2}=\frac{5-4}{8}=\frac{1}{8}$; $\therefore 5-2+\frac{1}{8}=3\frac{1}{8}$.

Ex. 4. $5\frac{3}{8}-2\frac{1}{2}$.
Here $\frac{3}{8}-\frac{1}{2}=\frac{3-4}{8}=-\frac{1}{8}$; $\therefore 5-2-\frac{1}{8}=3-\frac{1}{8}=2\frac{7}{8}$.

Ex. 5. $6-4\frac{3}{7}=2-\frac{3}{7}=1\frac{4}{7}$.

Improper fractions should be reduced to mixed numbers, and compound fractions to simple ones, before the application of this rule.

Ex. 6. $\frac{1}{5}$ of $2\frac{1}{2}$ of $16-1\frac{3}{7}$ of $5\frac{1}{2}=8-7\frac{6}{7}=7\frac{7}{7}-7\frac{6}{7}=\frac{1}{7}$.

Ex. **30.** Find the value of

1. $\frac{11}{15}-\frac{8}{15}$; $\frac{13}{20}-\frac{7}{20}$; $\frac{8}{15}-\frac{9}{20}$; $\frac{1}{2}-\frac{1}{3}$.
2. $8\frac{3}{4}-1\frac{1}{4}$; $3\frac{3}{4}-2\frac{5}{8}$; $5-2\frac{6}{7}$; $10\frac{3}{5}-\frac{11}{60}$.
3. $1\frac{4}{25}-\frac{3}{4}$; $9-3\frac{4}{25}$; $97\frac{1}{2}-48\frac{5}{6}$; $5\frac{3}{14}-2\frac{10}{21}$.
4. $13\frac{2}{75}-3\frac{8}{15}$; $4\frac{1}{24}-3\frac{1}{16}$; $3\frac{2}{9}-\frac{61}{126}$; $24\frac{1}{24}-21\frac{1}{21}$.
5. $1\frac{8}{25}-\frac{4}{7}$; $17\frac{1}{35}-\frac{4}{21}$; $4\frac{3}{5}-\frac{1}{4}$ of $\frac{2}{3}$; $\frac{9}{10}-\frac{1}{5}$ of $\frac{6}{11}$.
6. $1\frac{2}{3}$ of $2\frac{7}{9}-3\frac{17}{18}$; $5\frac{1}{3}$ of $4\frac{1}{2}-3\frac{1}{4}$ of $3\frac{1}{5}$.
7. $3\frac{1}{4}+4\frac{2}{5}-5\frac{1}{2}+16\frac{5}{8}-7\frac{11}{24}+10-14\frac{5}{6}$.
8. $5\frac{1}{5}-2\frac{5}{6}-3\frac{3}{10}+\frac{13}{2}-16\frac{1}{4}+3\frac{1}{12}+8\frac{1}{9}$.

	£	s.	d.		£	s.	d.		£	s.	d.
9.	13	0	$5\frac{1}{2}$	10.	4	17	$11\frac{3}{4}$	11.	9	0	$0\frac{1}{2}$
	4	17	$6\frac{1}{3}$		3	19	$4\frac{1}{6}$		8	17	$7\frac{2}{9}$
12.	15	0	$3\frac{5}{8}$	13.	7	17	$7\frac{5}{12}$	14.	8	13	$6\frac{3}{5}$
	9	19	$9\frac{11}{12}$		6	19	$9\frac{13}{16}$		4	19	$9\frac{2}{3}$

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46. *Multiplication of Fractions.*

Rule. Multiply the num^{rs} together for the new num^{r}, and the den^{rs} for the new den^{r}.

Ex. 1. $\frac{2}{3}\times\frac{4}{5}=\frac{8}{15}$.

This method is the same as that we should have used to find the value of the compound fraction $\frac{2}{3}$ of $\frac{4}{5}$, or $\frac{4}{5}$ of $\frac{2}{3}$, (42); and we must here observe that the same word 'Multiplication' is used to signify, not merely, in its original sense, and as we have hitherto employed it (when the multiplier was a whole number), the taking a *multiple* of a quantity, i. e. repeating it some number of times, but also (when the multiplier, as here, is fractional) the taking any *part* or *parts* of it; so that 'to *multiply* $\frac{2}{3}$ by $\frac{4}{5}$' is only another way of saying 'to take $\frac{4}{5}$ of $\frac{2}{3}$'; and hence the Rule for the operation is the same in the two cases.

It will be seen, however, that this Rule *includes* the case of Mult^n by whole numbers; thus if we had to find the value of $\frac{3}{4} \times 5$, we might say, $\frac{3}{4} \times 5 = \frac{3}{4} \times \frac{5}{1} = \frac{3 \times 5}{4 \times 1} = \frac{15}{4}$, obviously the same result as we should have obtained by the common rule of Mult^n by whole numbers (36): and it is on this account, viz. that the *general* method of taking any *part* or *parts* of a quantity includes the *particular* case of taking any *multiple* of it, that mathematicians have adopted the name, properly belonging to the latter case only, and applied it also to the former, calling the operation in both cases *multiplication.*

The method, therefore, of Mult^n of Fractions is the same as that for reducing a compound fraction to a simple one; and (as in that case) mixed numbers must be reduced to improper fractions before applying the rule, and the result may be simplified by striking out factors common to num^r and den^r.

Ex. 2. $2\frac{3}{4} \times 3\frac{1}{2} \times 1\frac{3}{5}$ of $\frac{2}{3}$ of $10 = \frac{11}{4} \times \frac{7}{2} \times \frac{8}{5} \times \frac{2}{3} \times \frac{10}{1} = \frac{308}{3} = 102\frac{2}{3}$.

Ex. **31.** Find the value of

1. $\frac{5}{12} \times \frac{9}{16} \times 2\frac{2}{11}$; $2\frac{1}{16} \times \frac{3}{11} \times 1\frac{7}{9}$; $2\frac{5}{11} \times 2\frac{1}{5} \times \frac{5}{33}$.
2. $\frac{11}{25} \times 2\frac{1}{2} \times 100$; $13\frac{1}{3} \times 3\frac{4}{5} \times 1\frac{7}{38}$; $6\frac{3}{4} \times 2\frac{8}{9}$ of 21.
3. $2\frac{1}{2}$ of $3\frac{2}{3} \times 4\frac{3}{4}$ of $1\frac{1}{7}$; $2\frac{1}{5} \times 1\frac{5}{6}$ of $1\frac{2}{13} \times 3\frac{1}{4}$ of $1\frac{5}{11}$.
4. $\frac{1}{2}$ of $\frac{7}{12}$ of $\frac{3}{5} \times \frac{4}{11}$ of $3\frac{1}{7}$; $1\frac{3}{8}$ of $\frac{5}{6} \times \frac{5}{18}$ of $2\frac{1}{22}$ of 8.
5. $\frac{3}{7} \times 1\frac{2}{5}$ of $12\frac{1}{2} \times 2\frac{1}{5}$ of $\frac{3}{44}$; $\frac{2}{3}$ of $1\frac{1}{7} \times 2\frac{2}{5}$ of $4\frac{3}{8}$ of $2\frac{2}{9}$.

47. *Division of Fractions.*

RULE. Invert the divisor, and multiply.

Ex. 1. $\frac{3}{4} \div \frac{5}{7} = \frac{3}{4} \times \frac{7}{5} = \frac{21}{20} = 1\frac{1}{20}$.

Here also the word 'Division' is used in a more general sense than heretofore, to denote the finding that quantity, which, *multiplied* by the divisor, will produce the dividend — the word *multiplied*, being here used in the enlarged sense explained in (46). Hence, in the above Example, where the div^r is $\frac{5}{7}$ and the div^d $\frac{3}{4}$, we must have *quotient* $\times \frac{5}{7} = \frac{3}{4}$: multiply each of these equals by the *same* quantity $\frac{7}{5}$, and the products must be equal; $\therefore$ *quotient* $\times \frac{5}{7} \times \frac{7}{5} = \frac{3}{4} \times \frac{7}{5}$: but $\frac{5}{7} \times \frac{7}{5} = 1$; hence the *quotient* $= \frac{3}{4} \times \frac{7}{5} = 1\frac{1}{20}$, as above.

The quotient thus obtained will have its usual meaning, when the div^r is an integer, i. e. will express how many times the div^d contains the div^r, or what *multiple* the div^d is of the div^r; thus $\frac{3}{4} \div 5 = \frac{3}{4} \div \frac{5}{1} = \frac{3}{4} \times \frac{1}{5} = \frac{3}{20}$, and hence $\frac{3}{4}$ contains $\frac{3}{20}$ *five* times or $= 5 \times \frac{3}{20}$: but when the div^r is a fraction the quotient will express what *part* or *parts* the div^d is of the div^r; thus $\frac{3}{4} \div \frac{5}{7} =$ (as above) $1\frac{1}{20}$, and hence $\frac{3}{4} = 1\frac{1}{20}$ of $\frac{5}{7}$.

Mixed numbers must be reduced to improper fractions, and compound fractions to simple ones, before applying this rule

Ex. 2. $2\frac{2}{3} \div 3\frac{3}{4} = \frac{8}{3} \div \frac{15}{4} = \frac{8}{3} \times \frac{4}{15} = \frac{32}{45}$.

Ex. 3. $(2\frac{2}{3} \text{ of } 3\frac{3}{4}) \div (4\frac{1}{2} \text{ of } \frac{5}{7} \text{ of } \frac{4}{45}) = 10 \div \frac{2}{7} = 10 \times \frac{7}{2} = 35$.

Ex. 4. $\dfrac{\frac{2}{5} \text{ of } 5}{\frac{3}{5} \text{ of } 1\frac{4}{5}} = \dfrac{2}{\frac{27}{25}} = 2 \times \frac{25}{27} = 1\frac{23}{27}$.

Hence it follows that a *complex fraction*, in which both the num^r and den^r may appear as fractions, may be simplified by multiplying together the outside numbers, or *extremes*, for the num^r, and the middle numbers, or *means*, for the den^r.

Ex. 5. $\dfrac{\frac{3}{7}}{\frac{4}{5}} = \dfrac{15}{28}$; $\dfrac{\frac{2}{3}}{3} = \dfrac{\frac{2}{3}}{\frac{3}{1}} = \dfrac{2}{9}$; $\dfrac{2}{2\frac{1}{5}} = \dfrac{\frac{2}{1}}{\frac{11}{5}} = \dfrac{10}{11}$; $\dfrac{2\frac{5}{8}}{3\frac{1}{3}} = \dfrac{\frac{21}{8}}{\frac{10}{3}} = \dfrac{63}{80}$.

So also, in a complex fraction, common factors that appear in either one of the extremes and also in one of the means, may be struck out of both.

Ex. 6. $\dfrac{2\frac{5}{8}}{7}=\dfrac{\frac{\cancel{21}\ 3}{8}}{\frac{\cancel{7}}{1}}=\dfrac{3}{8}$; $\dfrac{5}{3\frac{3}{4}}=\dfrac{\frac{\cancel{5}}{1}}{\frac{\cancel{15}\ 3}{4}}=\dfrac{4}{3}$; $\dfrac{1\frac{7}{10}}{10\frac{1}{5}}=\dfrac{\frac{\cancel{17}}{\cancel{10}\ 2}}{\frac{\cancel{51}\ 3}{\cancel{5}}}=\dfrac{1}{6}$.

Ex. 32. Find the value of

1. $2\div\frac{2}{3}$; $\frac{2}{3}\div\frac{3}{4}$; $2\frac{2}{3}\div1\frac{1}{2}$; $2\frac{1}{12}\div3\frac{1}{3}$; $16\frac{2}{3}\div12\frac{1}{2}$; $\frac{32}{75}\div\frac{8}{15}$.
2. $11\frac{4}{25}\div\frac{3}{5}$; $\frac{7}{9}\div14$; $(\frac{3}{5}$ of $\frac{8}{9})\div(\frac{6}{7}$ of $\frac{3}{4})$; $(4\frac{1}{2}$ of $\frac{5}{27})\div(5\frac{3}{5}$ of $1\frac{3}{7})$.
3. $209\div\frac{1}{5}$ of 20; $(\frac{2}{7}$ of $\frac{7}{8})\div(\frac{3}{4}$ of $\frac{1}{3}$ of 5); $(4\frac{1}{2}$ of $3\frac{1}{3})\div(2\frac{1}{4}$ of $6\frac{1}{4})$.
4. $\dfrac{52}{3\frac{1}{4}}$; $\dfrac{3\frac{3}{4}}{5}$; $\dfrac{\frac{14}{45}}{1\frac{17}{25}}$; $\dfrac{\frac{11}{12}}{7\frac{17}{18}}$.
5. $\dfrac{9\frac{7}{9}}{2\frac{1}{27}}$; $\dfrac{5\frac{3}{11}}{2\frac{7}{11}}$; $\dfrac{8\frac{3}{4}}{5\frac{5}{8}}$; $\dfrac{15\frac{3}{5}}{7\frac{4}{5}}$.
6. $\dfrac{23}{2\frac{2}{3}+\frac{2}{5}}$; $\dfrac{2\frac{1}{3}\text{ of }1\frac{1}{5}}{1\frac{1}{3}\text{ of }1\frac{1}{4}}$; $\dfrac{3\frac{3}{7}\text{ of }2\frac{11}{12}}{\frac{1}{33}\text{ of }8\frac{9}{14}}$; $\dfrac{2\frac{1}{2}+1\frac{2}{3}}{3\frac{2}{3}-2\frac{1}{2}}$; $\dfrac{4\frac{4}{15}\text{ of }2\frac{5}{8}}{5\frac{1}{5}-4\frac{1}{2}}$.

We shall here give examples of the application of the preceding rules to the Mult^n and Div^n of concrete quantities.

Ex. 1. Find the value of $\frac{3}{8}$ of £4.

Since (32) $\frac{3}{8}$ of £4 is the same as $\frac{1}{8}$ of £4 × 3, we first multiply £4 by 3, and then divide the result by 8.

	£4		
	3		
8)	12	0	0
	£1	10	0 *Ans.*

This is the same (46) as to *multiply* £4 by $\frac{3}{8}$.

Ex. 2. Divide 1 ton 13 cwt. 15 lbs. by $1\frac{1}{3}$.

Since $1\frac{1}{3}=\frac{4}{3}$, we have here (47) to multiply by $\frac{3}{4}$. We may do this as in Ex. 1, or (which is often more convenient) by first dividing by 2, which gives $\frac{1}{2}$ of the quantity, and then dividing this half by 2, which gives $\frac{1}{4}$ of it; and adding the two results together, we shall have $\frac{3}{4}$ of it.

	ton	cwt.	qrs.	lbs.
	1	13	0	15
				3
4)	4	19	1	17
	1	4	3	$11\frac{1}{4}$ *Ans.*

	ton	cwt.	qrs.	lbs.
	1	13	0	15
for $\frac{1}{2}$	0	16	2	$7\frac{1}{2}$
for $\frac{1}{4}$	0	8	1	$3\frac{3}{4}$
	1	4	3	$11\frac{1}{4}$ *Ans.*

Compound fractions must be reduced to simple ones before the application of this rule; but, in the case of mixed numbers, it is best to multiply separately for the integral part, and add the result to that obtained by the Rule for the fractional part.

Ex. 3. Multiply £2 10*s.* 4*d.* by $3\frac{5}{12}$.

$$\begin{array}{rrr} \pounds 2 & 10 & 4 \\ & & 3\frac{5}{12} \\ \hline 12)\,12 & 11 & 8 \\ \hline 1 & 0 & 11\frac{2}{3} \\ 7 & 11 & 0 \\ \hline \pounds 8 & 11 & 11\frac{2}{3} \end{array}$$

Sometimes it is convenient to reduce the given quantity to one denomination, before applying the Rule.

Ex. 4. Divide 7*s.* $1\frac{1}{2}$*d.* by $\frac{9}{115}$.

Here 7*s.* $1\frac{1}{2}$*d.* = 342 farthings, which we have to multiply by $\frac{115}{9} = 12\frac{7}{9}$.

$$\begin{array}{r} 342f. \\ 12\frac{7}{9} \\ \hline 9)\,2394 \\ \hline 266 \\ 4104 \\ \hline 4)\,4370 \\ \hline 12)\,1092\frac{1}{2}d. \\ \hline 91s.\ 0\frac{1}{2}d. = \pounds 4\ 11s.\ 0\frac{1}{2}d. \end{array}$$

Ex. **33.** Find the value of

1. $\frac{5}{8}$ of £1; $\frac{13}{20}$ of £5; 6*s.* 8*d.* × $\frac{2}{5}$; $3\frac{3}{4}$ of 2*s.* 6*d.*; $2\frac{8}{9}$ of 21*s.*
2. £3 6 8 × $\frac{7}{10}$; £3 7 5 ÷ $1\frac{1}{2}$; £5 4 $6\frac{1}{4}$ ÷ $1\frac{2}{3}$.
3. £7 6 $8\frac{1}{2}$ × $1\frac{5}{6}$; £8 0 $7\frac{3}{4}$ × $2\frac{3}{4}$; £10 11 $2\frac{1}{4}$ × $3\frac{3}{4}$.
4. £13 15 4 × $4\frac{5}{8}$; £18 17 0 × $4\frac{7}{8}$; £2 10 $6\frac{3}{4}$ × $3\frac{3}{4}$.
5. £30 14 $6\frac{1}{2}$ ÷ $\frac{4}{23}$; £7 13 4 ÷ $\frac{12}{77}$; £4 7 $3\frac{3}{4}$ ÷ $\frac{12}{127}$.
6. $\frac{5}{7}$ of a ton; $\frac{2}{7}$ of a lb. Troy; 3 cwt. 1 qr. ÷ $1\frac{3}{11}$; $11\frac{7}{9}$ of 6*s.* $11\frac{1}{4}$*d.*
7. 2 wk. 3 d. ÷ $\frac{9}{32}$; 3A. 3R. 3P. × $10\frac{5}{12}$; 2*s.* $9\frac{3}{4}$*d.* × $\frac{1}{5}$ of $5\frac{1}{3}$.
8. $\frac{1}{8}$ of $18\frac{1}{3}$*s.*; 1 cwt. 2 qrs. 13 lbs. × $3\frac{11}{24}$; $13\frac{27}{35}$ of £7 5*s.* 10*d.*
9. £1 11*s.* 6*d.* ÷ $\dfrac{1\frac{2}{9}}{3\frac{1}{7}}$; $\dfrac{1\frac{8}{11}}{1\frac{2}{3}}$ of £8 8*s.* $5\frac{1}{4}$*d.*; $\dfrac{3\frac{7}{11}}{4\frac{2}{7}}$ of $\dfrac{10\frac{5}{7}}{7\frac{1}{2}}$ of $\frac{77}{540}$ of 27*s.*
10. 1 m. 5 fur. 91 yds. 2 ft. ÷ $2\frac{7}{8}$ of $1\frac{9}{11}$; £$3\frac{5}{8}$ + $9\frac{3}{16}$*s.* + $5\frac{3}{4}$*d.*
11. £$\frac{3}{5}$ + $\frac{5}{16}$*s.* + $\frac{2}{9}$ of 21*s.*; $\frac{4}{7}$ cwt. + $8\frac{5}{6}$ lbs. + $3\frac{9}{10}$ oz.; 4 d. 5 h. × $1\frac{7}{36}$.
12. $1\frac{7}{9}$ of 10*s.* 6*d.* − $\frac{3}{4}$ of 2*s.* 6*d.* + £$\frac{1}{12}$ − $\frac{1}{14}$ of 21*s.*
13. $\frac{5}{8}$ of 21*s.* + $\frac{5}{8}$ of 5*s.* + $\frac{5}{8}$ of £3 12*s.* 6*d.*

14. $\frac{3}{4}$ of 21*s.* + $\frac{3}{8}$ of 5*s.* + $\frac{3}{5}$ of 7*s.* 6*d.* − $\frac{3}{4}$ of 2*d.*
15. $2\frac{2}{5}$ of $1\frac{3}{4}$ of $8\frac{3}{4}$*d.* + $3\frac{2}{3}$ of $1\frac{10}{11}$ of $\frac{3}{14}$ of $4\frac{1}{2}$*d.*
16. $\frac{2}{7}$ of £15 + $3\frac{2}{7}$ of £1 + $\frac{1}{3}$ of $\frac{5}{7}$ of $\frac{3}{5}$ of £1 + $\frac{2}{3}$ of $\frac{3}{7}$*s.*

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48. *To reduce a given quantity to the fraction of another given quantity.*

RULE. Reduce both to the same denomination; and take the result of the former for the numr, and of the latter for the denr, of the fraction required.

Ex. 1. Reduce 7*s.* 7*d.* to the fraction of £1.
Since 7*s.* 7*d.* = 91*d.*, and £1 = 240*d.*, the fraction required is $\frac{91}{240}$.
For 1*d.* is $\frac{1}{240}$ of £1; and therefore 7*s.* 7*d.*, which = 91*d.*, is $\frac{91}{240}$ of £1.

Any *common* denomination, to which the two quantities may be reduced, would answer the purpose of expressing one of them as the fraction of the other; but if the highest, of which they both admit, be taken, the fraction will be expressed in lower terms.

Ex. 2. Reduce half-a-crown to the fraction of half-a-guinea.

Reducing them to pence, we have the required fraction = $\frac{30}{126}$; but reducing to sixpences we have the same fraction in lower terms, = $\frac{5}{21}$.
Note. $\frac{5}{21}$ expresses what is called the *Ratio* of 2*s.* 6*d.* to 10*s.* 6*d.* (79).

Ex. 34. Reduce

1. 3*s.* 4*d.* to the fr. of £1; 2*s.* $6\frac{1}{2}$*d.* to the fr. of 6*d.*
2. £7 9*s.* 6*d.* to the fr. of £13 4*s.* 6*d.*; 6*s.* $8\frac{1}{2}$*d.* to the fr. of $1\frac{3}{4}$*d.*
3. 3 qrs. 14 lbs. to the fr. of 3 cwt. 1 qr.; 1 ton 4 cwt. to the fr. of 15 cwt. 1 qr. 20 lbs.
4. 3*s.* $7\frac{1}{2}$*d.* to the fr. of £1 3*s.* $4\frac{1}{2}$*d.*; £4 7*s.* $6\frac{3}{4}$*d.* to the fr. of 27*s.*
5. 3 cwt. 2 qrs. 3 lbs. to the fr. of a ton; 14 h. 15 m. to the fr. of $3\frac{1}{2}$ days.
6. 2R. 13P. to the fr. of 3 acres; 14 half-crowns to the fr. of 6*s.* 8*d.*
7. A ton to the fr. of 3 cwt. 3 qrs. 21 lbs.; 30P. 5 yds. to the fr. of 1 fur. 28P.
8. 3 w. 16 m. to the fr. of half-an-hour; 3 qrs. 2 qts. to the fr. of 4 qrs. 3 bus.
9. 8A. 3R. to the fr. of 2A. 32P.; 1 ft. $2\frac{2}{5}$ in. to the fr. of a yard.
10. 7 h. 12 m. to the fr. of a day; £4 12*s.* $1\frac{1}{2}$*d.* to the fr. of £1 9*s.* $3\frac{3}{4}$*d.*

11. 17 lbs. to the fr. of 1 qr. $14\frac{1}{2}$ lbs.; 1 m. 4 fur. to the fr. of 3 yds. 1 ft.
12. 2 sq. yds. 2 ft. 120 in. to the fr. of 3P. $13\frac{1}{4}$ yds. 1 ft. 72 in.; 3 cwt. 14 lbs. to the fr. of 2 ton 2 cwt. 2 qrs.
13. £22 13*s.* $8\frac{1}{4}$*d.* to the fr. of $3\frac{1}{2}$ gs.; £3 16*s.* $6\frac{3}{4}$*d.* to the fr. of £1 3*s.* $5\frac{1}{4}$*d.*
14. 3000 in. to the fr. of 1 fur. 5P.; £2 0*s.* $3\frac{3}{4}$*d.* to the fr. of £1 4*s.* $2\frac{1}{4}$*d.*
15. $1\frac{1}{2}$ guineas to the fr. of £$1\frac{1}{2}$; £11 6*s.* 5*d.* to the fr. of £10 5*s.* 4*d.*
16. $3\frac{3}{4}$ crowns to the fr. of £1 12*s.* $9\frac{3}{4}$*d.*; $2\frac{2}{3}$ half-guineas to the fr. of 10*s.* $11\frac{1}{4}$*d.*

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49. *To reduce a fraction of one given quantity to a fraction of another.*

RULE. Express by (48) the first quantity as a fraction of the second; and the fraction required will then be found by reducing the resulting compound fraction to a simple one.

Ex. 1. Reduce $\frac{2}{3}$*s.* to the fraction of £1.

$$1s. = \tfrac{1}{20} \text{ of } £1 : \therefore \tfrac{2}{3}s. = \tfrac{2}{3} \text{ of } £\tfrac{1}{20} = £\tfrac{1}{30}.$$

Ex. 2. Reduce $1\frac{2}{15}$ h. to the fraction of 10 min.

$$1 \text{ h.} = \tfrac{60}{10} \text{ of } 10 \text{ m.} = \tfrac{6}{1} \text{ of } 10 \text{ m.}; \therefore 1\tfrac{2}{15} \text{ h.} = 1\tfrac{2}{15} \text{ of } \tfrac{6}{1} \text{ of } 10 \text{ m.} = 6\tfrac{4}{5} \text{ of } 10 \text{ m.}$$

Ex. 3. Reduce $3\frac{5}{9}$ of £1 0*s.* $9\frac{3}{4}$*d.* to the fraction of £1 10*s.* 10*d.*

$$£1\ 0s.\ 9\tfrac{3}{4}d. = 999f., \text{ and } £1\ 10s.\ 10d. = 1480f.;$$

hence the required fraction $= 3\frac{5}{9}$ of $\frac{999}{1480} = 2\frac{2}{5}$.

Ex. 35. Reduce

1. £$\frac{3}{8}$ to the fr. of a guinea; $1\frac{3}{4}$*s.* to the fr. of £1.
2. $\frac{2}{3}$*d.* to the fr. of 15*s.*; $12\frac{3}{4}$ of 3*s.* 6*d.* to the fr. of £1.
3. $\frac{5}{9}$ of 1*s.* 6*d.* to the fr. of 1*s.*; $\frac{6}{7}$ of a sixpence to the fr. of £1.
4. $3\frac{1}{2}$ of £1 3*s.* 4*d.* to the fr. of £5; $2\frac{2}{3}$ of 17*s.* $6\frac{1}{2}$*d.* to the fr. of 10*s.*
5. $3\frac{1}{7}$ of 1 cwt. 3 qrs. to the fr. of a ton; $3\frac{3}{7}$ d. to the fr. of 3 wks.
6. $1\frac{1}{4}$ of £3 13*s.* 6*d.* to the fr. of 10*s.* 6*d.*; $2\frac{2}{5}$ of £6 to the fr. of £1 13*s.*
7. $2\frac{4}{9}$ of 4 cwt. to the fr. of 3 qrs. 4 lbs.; $4\frac{7}{8}$ crowns to the fr. of 5 gs.
8. $\frac{5}{8}$ lb. Tr. to the fr. of a lb. Av.; $\frac{5}{9}$ po. to the fr. of a fathom.
9. $\frac{3}{8}$ sq. ft. to the fr. of a pole; $12\frac{5}{6}$ of 1 qr. $3\frac{1}{2}$ lbs. to the fr. of 1 ton 2 cwt.
10. $3\frac{1}{2}$ of 2A. 3R. to the fr. of 2R. $2\frac{1}{2}$P.; $1\frac{3}{17}$ of £2 4*s.* $7\frac{1}{2}$*d.* to the fr. of 5*s.*

11. $3\frac{3}{7}$ wks. to the fr. of 1d. $8\frac{5}{8}$hrs.; $2\frac{4}{9}$ of 45 yds. to the fr. of 10 miles.

12. $2\frac{2}{3}$ of 3R. 6P. to the fr. of 1A. 2R. 3P.; $\frac{3}{8}$ of $1\frac{1}{2}$ of 10*s.* $7\frac{1}{2}$*d.* to the fr. of £4 4*s.* $4\frac{1}{2}$*d.*

13. $33\frac{1}{4}$ of 3 qrs. to the fr. of $3\frac{3}{4}$ tons; $3\frac{3}{4}$ of $1\frac{3}{5}$A. to the fr. of 2A. $2\frac{1}{2}$P.

14. $7\frac{1}{5}$ of £2 3*s.* $6\frac{1}{2}$*d.* to the fr. of 7*s.* 6*d.*; $\frac{3}{8}$ of 5*s.* + $\frac{4}{5}$*s.* to the fr. of 21*s.*

15. $4\frac{1}{5}$ of £2 13*s.* $7\frac{3}{4}$*d.* to the fr. of £2 14*s.* $8\frac{1}{4}$*d.*; $1\frac{2}{7}$ of £2 0*s.* $1\frac{1}{4}$*d.* to the fr. of £2 2*s.* $2\frac{1}{4}$*d.*

16. $6\frac{32}{35}$ of £1 10*s.* $5\frac{3}{4}$*d.* to the fr. of £3 3*s.* $0\frac{1}{4}$*d.*; $\frac{7}{9}$ of £1 − $\frac{2}{5}$ of 21*s.* to the fr. of 10*s.* 6*d.*

Miscellaneous Examples. 36.

1. Which is the greatest and which the least of $\frac{2}{13}$, $\frac{7}{45}$, $\frac{3}{20}$?

2. Divide the sum of $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{3}{20}$ by the difference between $\frac{1}{5}$ and $\frac{1}{4}$.

3. What nº added to $\frac{14}{27}$ makes $1\frac{3}{5}$? and what taken from $1\frac{22}{27}$ leaves $\frac{11}{15}$?

4. Which is the greater, $\frac{2}{5}$ of $2\frac{3}{7}$, or $\frac{7}{9}$ of $1\frac{1}{4}$, and by how much?

5. Divide the sum of 10 and $\frac{1}{10}$ by the difference, and also the difference by the sum; and find the sum and difference of the two quotients.

6. Divide the sum of $\frac{3}{5}$ of £3 7*s.* 6*d.*, and $\frac{1}{6}$ of $4\frac{1}{2}$ guineas, by $10\frac{5}{7}$.

7. If I pay away $\frac{1}{2}$ of my money, then $\frac{1}{3}$ of what remains, and then $\frac{1}{4}$ of what still remains, what fraction of the whole will be left?

8. What nº added to $\frac{10}{11}$, $\frac{11}{14}$, $\frac{2}{33}$, $\frac{41}{42}$, will make the sum total 3?

9. What must be the length of a plot of ground, if the breadth be $15\frac{3}{4}$ feet, that its area may contain 46 square yards?

10. Add together the sum, difference, product, and quotient (the greater being divided by the less) of $\frac{3}{4}$ and $\frac{7}{10}$.

11. Find the value of $\frac{3}{4}$ lb. Troy + $\frac{1}{6}$ oz. Troy; and of £$\frac{3}{4}$ − $\frac{3}{4}$*s.*

12. Express $2\frac{5}{9}$ ells as a fr. of a yard; and mult. 3 ft. $7\frac{1}{3}$ in. by $2\frac{1}{2}$ in.

13. Add the sum and difference of $\frac{4}{5}$ of 3 guineas and $\frac{2}{3}$ of £4.

14. Divide $\dfrac{7\,(1\frac{1}{2}\text{ of }\frac{3}{14})}{\frac{1}{6}\left(\frac{3}{3\frac{1}{2}}\text{ of }7\right)}$ by $\dfrac{9}{14}$, and find the value of $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\frac{1}{2\frac{1}{2}}+\frac{1}{3\frac{1}{2}}+\frac{1}{4\frac{1}{2}}}$.

15. To $\frac{4}{15}$ of a dozen add $\frac{13}{24}$ of three hundred, and divide this sum by the difference of $3\frac{3}{4}$ of a hundred and $43\frac{3}{5}$.

16. Multiply the sum of 1, $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$, by the difference of $\frac{4}{15}$ and $\frac{3}{20}$; and divide that product by the double of $21\frac{7}{9}$.

17. Take from 1 its half, third, and twenty-fourth parts; add the product of those parts to the remr; and multiply this sum by $7\frac{11}{19}$.

18. Multiply the sum of $3\frac{2}{3}$, $4\frac{3}{4}$, and $4\frac{4}{5}$, by the difference of $7\frac{6}{7}$ and $5\frac{5}{6}$; and divide the product by the sum of $94\frac{1}{8}$ and $93\frac{1}{9}$.

19. Divide 2 by the sum of $2\frac{2}{3}$, $\frac{4}{5}$, and 4; add $1\frac{2}{3}-\frac{7}{9}$ to the quotient; and multiply the result by the difference of $5\frac{1}{5}$ and $4\frac{1}{2}$.

20. Find the value of $(\frac{1}{2}+\frac{1}{3})\times(1\frac{1}{3}+2\frac{3}{4})\times(2\frac{1}{14}-1\frac{1}{2})\times(3\frac{1}{10}-\frac{3}{7})$; and of $1\frac{3}{4}\div2\frac{1}{2}+5\frac{1}{2}\div3\frac{1}{8}$.

21. A person had $\frac{7}{54}$ of a lottery ticket, which was drawn a prize of of £518 10*s.*: what was the value of his share?

22. Express the sum and difference of £$\frac{5}{12}$ and $\frac{3}{4}$ of a crown as fractions of half-a-sovereign; and find how many times the first contains the second.

23. Multiply $15\frac{5}{8}$*s.* by $109\frac{5}{7}$, and divide £61 4*s.* $7\frac{5}{16}$*d.* by $267\frac{3}{16}$.

24. How often is $\frac{4}{7}$*s.* contained in half-a-crown? and how often is £$\frac{3}{5}$ contained in 24 guineas?

25. If a yard of lace cost £$1\frac{29}{96}$, what will $16\frac{11}{25}$ yards cost?

26. If $\frac{3}{8}$ of a ship be worth £3740, what is the value of the whole?

27. Compare, as fractions of their highest common denomination, the values of $\frac{1}{16}$ of £1, $\frac{1}{20}$ of a guinea, and $\frac{6}{25}$ of a crown.

28. Find the value of $\dfrac{5\frac{5}{8}\div\frac{2}{3}}{(1\frac{1}{5}\text{ of }\frac{5}{9})\div10\frac{1}{3}}\times\dfrac{2}{5}\text{ of }\dfrac{1\frac{1}{2}\text{ of }4\frac{1}{9}}{13\frac{7}{8}\text{ of }5\frac{1}{3}}$.

29. If $\frac{2}{3}$ of an estate be worth £220, find the value of $\frac{3}{11}$ of it.

30. Express in Tr. weight the difference between $\frac{3}{8}$ lb. Tr. and $\frac{3}{8}$ lb. Av.

31. Find the value of $(12\frac{5}{6}-8\frac{3}{4}-1\frac{1}{10}+\frac{8}{15})\times4\frac{1}{2}\times(7\frac{5}{12}-6\frac{1}{2})$, and of $\frac{2}{3}\div1\frac{5}{7}-\frac{5}{8}\div3\frac{2}{11}$.

32. Compare, as fractions of their highest common denomination, the values of $\frac{1}{21}$ of half-a-crown, $\frac{1}{24}$ of 3*s.* 4*d.*, and $\frac{1}{28}$ of 4*s.* $2\frac{1}{2}$*d.*

33. Express, as a fraction of £5, the difference between £$7\frac{4}{5}$ and £$7\times\frac{4}{5}$; and find the value of £$14\frac{14}{15}\div1\frac{10}{11}$.

34. A person owes a guinea to each of 4 creditors: to one he pays $\frac{1}{2}$ of his debt, to another $\frac{3}{4}$, to another $\frac{4}{5}$, and to another $\frac{19}{20}$; what will he still be owing altogether?

35. Express in Troy weight the sum of $3\frac{2}{9}$ lbs. Tr., and $16\frac{1}{3}$ lbs. Av.

36. Find the value of $\dfrac{5\frac{4}{5}-2\frac{1}{8}}{3\frac{3}{4}+\frac{9}{20}}$ of $\dfrac{4\frac{1}{2}+5\frac{19}{25}}{4\frac{1}{20}}$ of $\dfrac{2\frac{3}{5}+1\frac{2}{3}}{7\frac{19}{24}-2\frac{1}{4}}$.

37. If $\frac{3}{16}$ of a ton is worth £4 10*s.*, what is the value of $\frac{1}{5}$ of it?

38. After taking out of a purse $\frac{2}{5}$ of its contents, $\frac{2}{3}$ of the remainder was found to be 13*s.* $5\frac{1}{2}$*d.*; what sum did it contain at first?

39. The dimensions of a room are $29\frac{1}{2}$ ft. by $11\frac{1}{4}$ ft.; what length of carpet, $\frac{5}{8}$ yd. wide, will cover it? and what will be the expense of it, at $3\frac{3}{4}$*s.* per yard?

40. A ship is worth £16000, and a person, possessed of $\frac{5}{16}$ of it, sells $\frac{3}{8}$ of his share; what share has he remaining, and what is it worth?

41. Express 4 bus. 1 pk. 1 gal. 2 qts. as a fr. of a qr.; and reduce 5 cwt. to lbs. Troy.

42. If $\frac{1}{8}$ of a ship be worth £36 10*s.* $7\frac{1}{2}$*d.*, what share will cost £125 5*s.*?

43. Multiply $3\frac{3}{20}$ by $15\frac{5}{7}$, and divide $\dfrac{2}{3\frac{3}{4}}$ by $\dfrac{2\frac{3}{4}}{3}$; and add together the sum and difference of these results.

44. A party having a bill to pay of £12 7*s.* $1\frac{1}{2}$*d.*, one of them pays for himself and three friends the sum of £5 9*s.* 10*d.*; how many were they?

45. Express both in Tr. and Av. weight, $\frac{1}{11}$ lb. Tr. + $\frac{1}{11}$ lb. Av.

46. A pint contains $34\frac{2}{3}$ cubic inches; how many gallons of water will fill a cistern 4 ft. 4 in. long, 2 ft. 8 in. broad, and 1 ft. $1\frac{1}{2}$ in. deep?

47. Add together $1\frac{3}{4}$, $2\frac{2}{3}$, and $3\frac{1}{2}$; multiply this sum by the product of these fractions; subtract from the result the difference of $2\frac{2}{3}$ and $1\frac{1}{2}$; and divide the remainder by the sum of $5\frac{1}{2}$ and $1\frac{1}{3}$ of $3\frac{3}{4}$.

48. How many yards of paper, $\frac{5}{8}$ yd. wide, will be required for the walls of a room that is $20\frac{3}{8}$ ft. long by $11\frac{1}{5}$ ft. wide, and $12\frac{1}{2}$ ft. high? and what will be the cost of it at $2\frac{1}{4}$*d.* a yard?

49. A cubic foot of wood weighing $11\frac{10}{11}$ lbs., what is the weight of a beam 24 ft. long, $2\frac{3}{4}$ ft. wide, and $2\frac{1}{2}$ ft. thick? and what is its value at $3\frac{44}{45}$*s.* per cubic foot?

50. A person dies worth £10000, and leaves $\frac{1}{3}$ of his property to his wife, $\frac{1}{2}$ to his son, and the rest to his daughter. The wife at her death leaves $\frac{2}{5}$ of her legacy to the son, and the rest to the daughter; but the son adds his fortune to his sister's, and gives her $\frac{1}{3}$ of the whole. How much will the sister gain by this? and what fraction will her gain be of the whole?

CHAPTER IV.

DECIMAL FRACTIONS.

50. In common numbers, or *decimal integers*, the actual value of each figure depends upon its position with respect to the place of units, its value in any one position being one-tenth of what it would be, if it stood one place further to the left: thus 3045 denotes 3 *thousands*, 0 *hundreds*, 4 *tens*, and 5 *units*, or $3000+0+40+5$; where we may obtain the actual value of any figure by *multiplying* it by 10, 100, 1000, &c., according as it stands in the 1st, 2nd, 3rd, &c. place to the *left* of the place of units.

Now if we continue the same method of notation to the *right* of the place of units, still reckoning the value of each figure to be *one-tenth* of what it would be, if it stood one place further to the left, we obtain what are called *decimal fractions*, or briefly *decimals*; thus setting, as is usual, a dot, called the *decimal point*, after the unit's place, the number 3.045, &c. will denote 3 *units*, 0 *tenths*, 4 *hundredths*, 5 *thousandths*, &c., or $3+\frac{0}{10}+\frac{4}{100}+\frac{5}{1000}+$ &c.; where we may obtain the actual value of any figure by *dividing* it by 10, 100, 1000, &c., according as it stands in the 1st, 2nd, 3rd, &c. place to the *right* of the place of units.

51. Hence it follows that a decimal may also be defined to be a fraction, whose den^r is 10, or some power* of 10, as 100, 1000, &c., which den^r, however, is not set down, as in vulgar fractions, under the num^r, but expressed by marking off by a point, from the right of the num^r, as many figures as there are cyphers in the den^r, prefixing cyphers to the former, if necessary, to make up the requisite number of figures after the point.

* A power of a number is the product of a number multiplied by itself once or successively. When the number is used as a factor twice, thrice, &c., the product is called the second power, third power, &c., of the n^o.

Thus $\frac{347}{100}=\frac{300+40+7}{100}=3+\frac{4}{10}+\frac{7}{100}=3.47$;

$\frac{13}{1000}=\frac{10+3}{1000}=\frac{1}{100}+\frac{3}{1000}=.013$;

$\frac{2125}{1000}=2.125$, $\frac{119}{10000}=.0119$, $\frac{27}{100000}=.00027$, &c.

52. Conversely, any decimal may be expressed as a vulgar fraction by setting down the figures which compose it as the num^r^, and for the den^r^, 10, 100, 1000, &c. according as there are *one, two, three,* &c. figures after the point. This, in fact, amounts to expressing each figure separately as a vulgar fraction with its own den^r^, and then bringing all these fractions to one common den^r^.

Thus $2.03=2\frac{3}{100}$ or $\frac{203}{100}$; $.379=\frac{3}{10}+\frac{7}{100}+\frac{9}{1000}=\frac{300+70+9}{1000}=\frac{379}{1000}$; $42.037=42\frac{37}{1000}$ or $\frac{42037}{1000}$; $.0029=\frac{29}{10000}$; $15.001=15\frac{1}{1000}$ or $\frac{15001}{1000}$.

Sometimes the resulting fractions admit of reduction to lower terms.

Thus $13.75=13\frac{75}{100}=13\frac{3}{4}$; $23.0625=23\frac{625}{10000}=23\frac{1}{16}$.

53. Any decimal is *multiplied* by 10, 100, 1000, &c. by moving the point *one, two, three,* &c. places to the right, and *divided* by moving it similarly to the left.

Thus

$3.247=\frac{3247}{1000}$; hence $3.247\times10=\frac{3247}{100}=32.47$; $3.247\div10=\frac{3247}{10000}=.3247$,

$3.247\times100=\frac{3247}{10}=324.7$; $3.247\div100=\frac{3247}{100000}=.03247$.

So $.0023\times100=.23$, $2.3\div100=.023$,

$2.3\times1000=2300$, $2.3\div1000=.0023$, &c.

54. It should be carefully noticed, that adding cyphers to the right of a decimal does not alter its value; thus .3, .30, .300, are all equal, representing each of them $\frac{3}{10}$, or as in (52) $\frac{3}{10}$, $\frac{30}{100}$, $\frac{300}{1000}$, respectively; but prefixing cyphers to the *left* of a decimal after the point is equivalent (53) to dividing it by 10, 100, &c.; thus .3, .03 .003, are respectively $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$.

Ex. **37.** Express as decimals

1. $\frac{7}{10}$, $\frac{117}{10}$, $\frac{33}{100}$, $\frac{1015}{1000}$.
2. $\frac{1}{100}$, $\frac{21}{10000}$, $\frac{117}{10000}$, $\frac{3}{10000000}$.
3. 2 tenths + 3 hundredths + 37 millionths.
4. 11 tenths + 11 thousandths + 11 hundred-thousandths.
5. 13 + 3 thousandths + 5 millionths.

6. 101 tenths + 10 thousandths + 101 millionths.

Express as vulgar fractions

7. .037, .0002, .25, .375. 8. .0075, 1.225, .1875, 3.225.

9. .0006875, .0009375, 23.038125.

10. 15.203125, .00234375, 4.0078125.

Multiply and divide

11. .3 by 10 and 1000, .00125 by 100 and 10000, 538.734 by ten thousand.

12. 1.1 by 1000 and 1000000, 11.025 by 1000 and 100000, and 213.012 by a million.

---

55. *Addition and Subtraction.*

RULE. Set down the decimals with their points in the same vertical line, so that units of the same kind may be under one another, filling up the blank places with cyphers; then add or subtract as with common integers, setting the point in the result in the same line with the other points.

Ex. 1. Add together 2.8146, .0938, 8, .875, 31.2788, 4.0087.

```
 2.8146
  .0938
 8.0000
  .8750
31.2788
 4.0087
-------
47.0709
```

Here the figures in the right-hand column represent so many ten-thousandths; so that we have to add together

$$\frac{6+8+0+0+8+7}{10000}=\frac{29}{10000}=\frac{2}{1000}+\frac{9}{10000};$$

we set down therefore the 9 under the column of *ten-thousandths*, and carry the 2 *thousandths* to the next column; and so on.

Ex. 2. Find the difference of 2.418 and 1.2234.

```
2.4180
1.2234
------
1.1946
```

Here we have 4 *ten-thousandths* in the lower line, but none in the upper; we therefore have to borrow one from the 8 in the next column, i. e. we borrow 1 *thousandth* = 10 *ten-thousandths*, from which we take the four *ten-thousandths*, and have 6 remaining; we have now only 7 *thousandths* in the upper line, from which we are to take 3 *thousandths*, or, instead of this (as in former cases of *borrowing* in Subtraction), we may take 4 *thousandths* from 8 *thousandths*; and so on.

Ex. **38.** Find the value of

1. 11.275 + .34132 + .00414 + .0001 + 23.001.
2. 321.4 + 12 + 31.6154 + .01 + 2.214 + 415.62.
3. .001213 + 45.613 + 234 + .0012 + 141.00056.
4. 1.0000123 + 31.1 + 117.154 + 2343.008 + .0002.
5. 32.001 − 12.999; and 3.45 − .00098.
6. 23.1415 − 2.008; and 3.412 − 2.99987.

7. 22.0001 − 2.9999; and 2415.6 − 2414.5987.
8. .001 − .0009987; and 24.004 − .987516.
9. 1.3742 − .03742; and 3.054 − .3054.
10. .0123 − .009087; and 3.33 − 2.98765.

---

56. *Multiplication.*

RULE. Multiply the given decimals as if they were common integers, and mark off in the product as many decimal places as there are in the multiplier and multiplicand together.

Ex. 1. Multiply 1.0025 by 2.5.

```
1.002 5
    2.5
-------
 5012 5
2 0050
-------
2.5062 5  Ans.
```

For $1.0025 \times 2.5 = \frac{10025}{10000} \times \frac{25}{10} = \frac{250625}{100000} = 2.50625.$

Ex. 2. Multiply .0048 by .000012; and 1.005 by .005 × .0064.

```
      .0048                 1.005
    .000012                  .005
-----------             ---------
.0000000576               .005025
                           .0064
                        ---------
                           20100
                          30150
                 ---------------
                 .0000321600 = .0000321 6  Ans.
```

Ex. 39. Find the value of

1. 22.5 × 32.16; and 4.41 × 33.21.
2. .0001 × .001; and 32.1 × 2.31.
3. .0032 × 23.45; and .0002 × 3.01.
4. 22.5 × .0241 × .0024; and .0003 × .01 × 500000.
5. 2.7 × .27 × .027 × 270; and .2 × .04 × .008 × 64000.
6. 1.1 × .011 × 1.01 × .0101; and .013 × 1.6 × .007 × 3.05.

---

57. *Division.*

RULE. If the given divisor is not a whole number, make it so by removing its decimal point altogether, and shift the decimal point of the dividend as many places to the right as there were decimal figures in the divisor; annexing for this purpose decimal cyphers, if necessary, to the dividend.

Then divide as if the given decimals were common integers; and when, in the process of division, the decimal point of the dividend is arrived at, place a decimal point in the quotient.

Decimal cyphers may be annexed to the dividend, to any extent that may be wanted for carrying on the division. (54)

Ex. 1. Divide 277.53 by 12; also .27753 by 12; and 1037 by 305.

```
12)277.5300
    23.1275

12).2775300
   .0231275

305)1037.0(3.4
     915
     1220
     1220
```

Here the divisors are all integral, and the position of the point in the quotient is very simply determined. In the first sum, we take the 12th part of 27 *tens*, which is 2 *tens* and 3 over; then the 12th of 37 *units* is 3 *units* and 1 over; then the 12th of 15 *tenths* is 1 *tenth*, &c.; so that the point in the quotient comes exactly under that of the dividend. In the second sum the 12th of 2 *tenths* is 0 *tenths*; the 12th of 27 *hundredths* is 2 *hundredths*, and 3 over, &c.; and here the student should particularly observe, that when the divisor is a whole number, there will always be a quotient figure, though sometimes, as here, a cypher, for every decimal figure of the dividend.

Ex. 2. Divide .805 by 2.3, .001029 by 1.68, and 1 by .007.

```
2.3).805                    1.68).001029

 23)8.05(.35                168)  .1029(.0006125
    69                            1008
    115                            210
    115                            168
                                    420
.007)1.0000                         336
   7)1000 000                        840
     142.857 &c.                     840
```

In the 1st of these sums the divisor, 2.3, is mult[d] by 10, which removes the point, and the dividend is also mult[d] by 10, by having the point shifted one place to the right. In the 2nd sum the divisor and dividend are mult[d] by 100, and in the 3rd by 1000, to make the divisor integral. In the 3rd sum the quotient will not terminate, but, by annexing cyphers to the dividend, we may continue the quotient as far as we please.

Obs. An integral divisor ending with cyphers may be deprived of the cyphers, if we shift the point of the dividend one place *to the left* for every cypher withdrawn: thus,

$$.45 \div 60 = .045 \div 6$$

A little consideration will enable us often to avoid the trouble of *counting* the decimal places of the dividend and divisor.

Ex. 4. Divide 15.95 by 2.75.

```
2.75) 15.950 (5.8
      13 75
       2 200
       2 200
```

Here, without counting, we may set at once the point after the 5 in the quotient, because it is plain that the divisor, which is a little greater than 2, will go about 5 times in the dividend, which is a little greater than 15.

Ex. 20. Find the value of

1. $15.625 \div 2.5$; and $.015625 \div 25$.
2. $1562.5 \div .00025$; and $1.5625 \div 25000$.
3. $181.3 \div .00037$; and $171.99 \div 27.3$.
4. $9.065 \div .049$; and $.03 \div .001$.
5. $8 \div .002$; and $37.5 \div 7.68$.
6. $15 \div 6.25$; and $17.28 \div .0144$.
7. $.00128 \div 8.192$; and $1708.4592 \div .00024$.
8. $.0002 \div .0163$; and $4 \div .00255$.
9. $11.1 \div 32.76$; and $.0123 \div 3.21$.
10. $2.117 \div .0073$; and $.032 \div 2.137$.

---

58. *To reduce any fraction to a decimal.*

RULE. If the den^r be 10, 100, &c. we may at once express it as a decimal (51): in other cases, if 10, 100 &c. be a *factor* of the den^r, divide the numerator by it as in (53), and then divide the num^r as it now stands by the remaining factor as in (57), and the result will be the decimal required.

Ex. $$\frac{1}{400} = \frac{.01}{4} = .0025\ ;\quad \frac{37}{80} = \frac{3.7}{8} = .4625.$$

59. Sometimes the division will not terminate, but the same figures will be repeated over again continually.

Ex. Reduce $\frac{95}{90}$ or $\frac{9.5}{9}$, $\frac{3}{1100}$ or $\frac{.03}{11}$, and $\frac{4}{7}$, to decimals.

```
9) 9.50000             11) .03000000               7) 4.0000000
   1.05555 &c. = 95/90.    .00272727 &c. = 3/1100.    .5714285 &c. = 4/7.
```

Decimals of this kind, in which the same figures are continually repeated without end, are called *Circulating*, *Repeating*, or *Recurring*, Decimals; and the part repeated is called the *Period* or *Repetend.*

It is usual to express any circulator by writing it down to the end of the first period, and setting dots over the first and last figures of the period; which dots will, of course, be on adjacent figures, when the period consists of only *two* figures, and will coalesce into one dot, when the period consists of only *one* figure.

Thus the above results would be written $1.0\dot{5}$, $.00\dot{2}\dot{7}$, $.\dot{5}7142\dot{8}$.

A *pure* circulator is one in which the period begins immediately after the decimal point; all others are called *mixed.*

Ex. **41.** Reduce to decimals

1. $\frac{2}{50}$; $\frac{13}{250}$; $\frac{42}{8}$; $\frac{1000}{625}$.
2. $\frac{106}{125}$; $11\frac{17}{1250}$; $\frac{4000}{256}$; $5\frac{3}{16}$.
3. $7\frac{13}{64}$; $\frac{17}{128}$; $\frac{1}{6400}$; $11\frac{53}{31250}$.
4. $\frac{1}{512}$; $\frac{1025}{1024}$; $\frac{13}{1600}$; $\frac{7}{5120}$.
5. $\frac{15\frac{1}{2}}{16}$ of $\frac{11}{62\frac{1}{2}}$; $7\frac{1}{2}$ of $\frac{18}{62500}$; $1\frac{2}{19}$ of $1\frac{1}{75}$ of $\frac{2}{7}$.

---

60. Any fraction, to be expressed as a decimal, should first be reduced to its lowest terms; and then, if the den^r contain *only powers* of 2 and 5 as factors, it may be reduced to a *finite* or terminating decimal.

For, in reducing a fraction to a decimal, we set a point after the num^r, and annex cyphers to it, until the den^r will, if possible, exactly divide it. Or, leaving out of consideration the point, (which, it is plain, does not affect the division, but only determines the place of the point in the result), this amounts to *multiplying the num^r by such a power of* 10, *as will make it contain the den^r.* But now, since the fraction is supposed to have been originally in its lowest terms, the den^r can have no factor in common with the original num^r; if, therefore, it be exactly contained in the num^r as it now stands, that is, with the annexed cyphers, it can only be by its being contained in that power of 10, by which the original num^r has been multiplied. But, since 10 contains only the factors 2 and 5, any power of 10 will contain only powers of 2 and 5; and, therefore, the den^r, in order to be contained exactly in some power of 10, must be made up only of powers of 2 and 5 as factors. In this case the division would terminate, and the decimal be *finite;* but not so, if the den^r contain any *other* factors, such as 3, 7, 11, &c., since then no power of 10 whatever would contain the den^r, nor, therefore, would the original num^r, whatever be the number of cyphers annexed, become exactly divisible by it.

61. If the den^r of a fraction, in its lowest terms, contain any other factor than powers of 2 and 5, the fraction may be expressed as a *Circulating* Decimal, where the number of figures in the period will be less than the den^r.

For since, in the division, the figures to be *taken down* are always the *same*, viz. cyphers, it follows that, whenever we have any *former remainder* repeated, we shall also have the same series of figures repeated in the *quotient:* but, if we go far enough, we cannot help having some former remainder repeated; for, all the remainders must, of course, be *less* than the divisor (or den^r), and so the *number* of *different* remainders must be *less* than the den^r itself.

Ex. 1. Reduce $\frac{6}{7}$ to a decimal.

```
7) 6.0 (.857142
   5 6
   ---
    40
    35
    --
     50
     49
     --
      10
       7
      --
      30
      28
      --
       20
       14
       --
        6
```

(The quotient is $.\dot{8}5714\dot{2}$.)

Here we have had in order the remainders 6, 4, 5, 1, 3, 2, which are all there are *less* than the divisor, 7; the next remainder must therefore be one of these again, and accordingly we find it to be 6; now, since the same figure, 0, is *taken down* to it as before, it is plain that the whole series of figures in the quotient will be reproduced in exactly the same order as before.

In the above Example, *all* the possible remainders have occurred, and the period, consequently, consists of as many figures as it possibly could, viz. *one less than the den^r:* this, however, is not usually the case.

Ex. 2. Reduce $\frac{69}{22}=3\frac{3}{22}$ to a decimal.

```
22) 3.0 (.136
    22
    --
     80
     66
     --
     140
     132
     ---
       8
```

Ans. $3.1\dot{3}\dot{6}$.

Sometimes a decimal of very long period may be carried out easily to many places, as in the following example:

Ex. 3. Reduce $\frac{1}{19}$ to a decimal.

```
19) 1.00 (.05263
     95
     --
     50
     38
     --
     120
     114
     ---
      60
      57
      --
       3
```

Hence $\frac{1}{19}=.05263\frac{3}{19}$, $\therefore \frac{3}{19}=.15789\frac{9}{19}$;

and hence $\frac{1}{19}=.0526315789\frac{9}{19}$;

$\therefore \frac{9}{19}=.4736842101\frac{81}{19}=.4736842105\frac{5}{19}$;

and hence $\frac{1}{19}=.05263157894736842105\frac{5}{19}$,

and, by continuing this process, we obviously *double* at every step the number of figures obtained.

This decimal, it will be seen, circulates after the *eighteenth* figure; so that

$$\frac{1}{19}=.\dot{0}5263157894736842\dot{1}.$$

Ex. 42. Reduce to decimals

1. $\frac{13}{9}$; $\frac{103}{180}$; $\frac{129}{55}$; $\frac{17}{1375}$.
2. $\frac{41}{14}$; $\frac{111}{22}$; $\frac{22}{1665}$; $23\frac{52}{333}$.
3. $\frac{89}{9999}$; $\frac{121}{21}$; $17\frac{6401}{49500}$; $\frac{4111}{33300}$.
4. $\frac{135}{3700}$; $\frac{297}{2960}$; $\frac{378}{925}$; $\frac{1139}{55555}$.
5. $\frac{1}{17}$; $\frac{1}{23}$; $\frac{1}{29}$; $\frac{1}{31}$.

---

62. *To reduce a pure circulator to a fraction.*

Since $\frac{1}{9}=.111111$ &c., it follows that $\frac{2}{9}=.2222$ &c., $\frac{5}{9}=.5555$ &c.; so that any pure circulator, having *one* figure in the period may be expressed as a fraction with that figure in the num^r, and 9 in the den^r.

Again,

$\frac{1}{99}=\frac{1}{9}\div 11=.010101$ &c.; hence $\frac{5}{99}=.050505$ &c.; $\frac{23}{99}=.232323$ &c.; so that any pure circulator, having *two* figures in the period, may be expressed as a fraction with those figures in the num^r, and 99 in the den^r.

In like manner, since

$$\frac{1}{999}=\frac{1}{9}\div 111=.001001 \text{ \&c}, \quad \frac{1}{9999}=\frac{1}{9}\div 1111=.0001 \text{ \&c.},$$

and so on, it will follow that *any* pure circulator may be expressed as a fraction with the period itself in the num^r, and in the den^r as many 9's as there are circulating figures.

Thus $.\dot{3}7\dot{8}=\frac{378}{999}=\frac{14}{37}$, $.\dot{0}37\dot{8}=\frac{378}{9999}=\frac{42}{1111}$, $.\dot{0}0037\dot{8}=\frac{378}{999999}=\frac{2}{5291}$.

63. *To reduce a mixed circulator to a fraction.*

If we had a pure circulator with any figures before the point, we might either keep these to form a mixed number, as $3.\dot{4}=3\frac{4}{9}$, $5.\dot{4}\dot{3}=5\frac{43}{99}$; or we might bring the whole at once to an improper fraction, with the same den^r as before, by writing for the num^r all the figures to the end of the first period, subtracting, however, the figures before the point;

thus $3.\dot{4} = \frac{34-3}{9} = \frac{31}{9} = 3\frac{4}{9}$; $5.\dot{4}\dot{3} = \frac{543-5}{99} = \frac{538}{99} = 5\frac{43}{99}$; &c.

The reason of this method may be thus seen:

$$3\frac{4}{9} = \frac{3\times 9+4}{9} = \frac{3\,(10-1)+4}{9} = \frac{30+4-3}{9} = \frac{34-3}{9};$$

$$5\frac{43}{99} = \frac{5\,(100-1)+43}{99} = \frac{543-5}{99}, \text{ \&c.}$$

Now, if the point be not immediately before the period, as in these examples, but moved towards the left, this is equivalent to dividing the decimal by 10, 100, &c., and we must therefore annex to the den^r, as found by the preceding Rule, as many cyphers as there are figures between the point and the first period:

thus $.03\dot{4} = \frac{34-3}{900} = \frac{31}{900}$; $.5\dot{4}\dot{3} = \frac{543-5}{990} = \frac{538}{990} = \frac{269}{495}$.

If there should be any figures of a mixed circulator still left before the point, it will be best to leave these as they are, to form a mixed number:

thus $2.4\dot{6} = 2\frac{46-4}{90} = 2\frac{42}{90} = 2\frac{7}{15}$, the same as $\frac{246-24}{90} = \frac{222}{90} = 2\frac{42}{90}$.

The above results may be thus stated, as a Rule for reducing *any* circulator to a fraction:

Consider only the figures *after* the point; then

For the num^r, write the decimal to the end of the first period, subtracting from it (if any) the figures which do not *circulate;*

For the den^r, write as many 9's as there are figures circulating, followed by as many 0's as there are figures not circulating. *See* NOTE VI.

Ex. **43.** Reduce to fractions

1. $.\dot{3}$; $.\dot{0}\dot{5}$; $.5\dot{4}$; $.\dot{7}2\dot{9}$.
2. $.02\dot{4}$; $.04\dot{3}\dot{2}$; $.00\dot{6}7\dot{5}$; $2.04\dot{3}\dot{2}$.
3. $3.4\dot{1}\dot{8}$; $.044\dot{3}$; $1.1\dot{4}\dot{5}$; $.004\dot{4}\dot{9}$.
4. $4.0\dot{5}3\dot{1}$; $7.6\dot{5}3\dot{1}$; $2.3\dot{4}\dot{5}$; $.093\dot{1}\dot{8}$.
5. $2.\dot{0}90\dot{9}$; $.5\dot{4}95\dot{0}$; $1.0\dot{4}2857\dot{1}$.
6. $2.6\dot{4}2857\dot{1}$; $5.193\dot{1}\dot{8}$; $11.2\dot{8}\dot{7}$.

---

64. It may be noticed that, according to the above rule, the circulator $\dot{9} = \frac{9}{9} = 1$. It is true, we cannot reverse this

operation, and reduce 1 to the decimal .999 &c.; yet it will be evident, by repeating the period, that this decimal really differs from 1 by a quantity so small as to be absolutely insensible: thus

$$1-.9=1-\tfrac{9}{10}=\tfrac{1}{10},\ 1-.99=1-\tfrac{99}{100}=\tfrac{1}{100},\ 1-.999=1-\tfrac{999}{1000}=\tfrac{1}{1000},\ \&c.,$$

where we see that, by repeating the 9's, the difference between 1 and the corresponding decimal becomes less and less, and thus may be made as minute as we please, and will at length become absolutely insensible.

It is in this sense that 1 is said to be the value of the circulator $.\dot{9}$, and, indeed, that *any* vulgar fraction is assigned as the value of *any* circulator; so that, in fact, the equivalent vulgar fraction for any circulating decimal is that to which the value of the decimal will become more and more nearly equal as we repeat its period, and from which it may, by such continued repetition, be made to differ by a fraction as minute as we please, and altogether insensible.

Whenever, therefore, in a decimal we find the figure 9 circulating, we may at once get rid of the period, by adding 1 to the figure preceding it: thus .4999 &c. = .5, the same result as we should obtain by the Rule, since

$$.4\dot{9}=\frac{49-4}{90}=\frac{45}{90}=\frac{5}{10}=.5.$$

65. Arithmetical operations in which circulating decimals are concerned, may often be performed, with sufficient accuracy for all practical purposes, by repeating the period as often as shall seem upon consideration necessary to ensure the result being ***correct*** to some given number of decimal places.

Ex. 1. Add together $13.\dot{5}$, $2.0\dot{2}\dot{5}$, $111.000\dot{4}$, $3.14\dot{1}5\dot{9}$, $2.\dot{0}2\dot{4}$ *correctly* to 6 decimal places.

13.55555555
2.02525252
111.00044444
3.14159159
2.02402402
131.74686812

Here, by carrying out the decimals to 8 places, we ensure the accuracy of the first 6 places; for, although the last two are incorrect, and would be altered, if we carried on our periods farther, yet a little consideration will show us that the sixth and all the preceding figures will not be altered, however often we may repeat the periods.

In such a case it is generally sufficient to carry out the periods to three decimal places more than the number required to be accurate.

Ex. 2. From $1.02\dot{3}4\dot{1}$ take $.62\dot{8}$, *correctly* to 6 decimal places.

$$\begin{array}{r} 1.023413413 \\ .628888888 \\ \hline .394524525 \end{array} \qquad \textit{Ans. } .394524.$$

It is sometimes convenient to reduce the circulators to vulgar fractions, especially for the purpose of multiplying or dividing one circulator by another, in which case the fraction, resulting from the multiplication or division, may be afterwards reproduced in the decimal form.

Ex. 3. $.2\dot{3} \times .\dot{3}\dot{6} = \frac{21}{90} \times \frac{36}{99} = \frac{14}{165} = .0\dot{8}\dot{4}$; $.1\dot{6} \div .00\dot{2}\dot{7} = \frac{15}{90} \div \frac{27}{9900} = \frac{550}{9} = 61.\dot{1}$.

Ex. **44.** Find the value (correct to 7 places of decimals) of

1. $.13\dot{8} + .\dot{1}4285\dot{7} + 2.4\dot{1}\dot{8} + 2.0\dot{6} + 42.\dot{6}\dot{3} + .008\dot{4}9713\dot{3}$.
2. $37.2\dot{3} + .2\dot{6} + 7.\dot{7}\dot{2} + .\dot{2}9\dot{7} + 3.\dot{9}7\dot{3} + 8 + 4.7\dot{5} + 74.0367 + 32.4\dot{1}$.
3. $.\dot{3} - .\dot{0}\dot{9}$; and $.\dot{0}\dot{4} - .00\dot{7}6923\dot{8}$. 4. $7 - 6.\dot{1}4285\dot{7}$; and $.04\dot{2} - .0\dot{3}\dot{6}$.
5. $37.2\dot{3} \times .2\dot{6}$; and $7.\dot{7}\dot{2} \times .\dot{2}9\dot{7}$.
6. $3.\dot{9}7\dot{3} \times 8$; and $74.0367 \times 4.7\dot{5}$.
7. $.\dot{3} \div \dot{0}\dot{9}$; and $.\dot{0}\dot{4} \div .\dot{7}6923\dot{0}$. 8. $7 \div .\dot{1}4285\dot{7}$; and $.042 \div .03\dot{6}$.

---

66. *To find the value of any decimal of a given quantity*

RULE. As in common Reduction, multiply the given decimal by the number of units of the next lower den^n which make one of the given den^n : the integral part (if any) of the result will be so many units of that lower den^n, and the fractional part may now be reduced in the same manner to a lower den^n ; and so on.

Ex. 1. Find the value of £.36875.

$$\begin{array}{r} .36875 \\ 20 \\ \hline 7.37500 \\ 12 \\ \hline 4.50000 \\ 4 \\ \hline 2.00000 \end{array} \qquad \text{or, omitting useless cyphers,} \qquad \begin{array}{r} .36875 \\ 20 \\ \hline 7.37500 \\ 12 \\ \hline 4.500 \\ 4 \\ \hline 2.0 \end{array} \quad \textit{Ans. } 7s.\ 4\tfrac{1}{2}d.$$

If the given quantity be expressed in more than one den^n, it should be reduced to *one*, before applying the Rule.

Ex. 2. Find the value of .07 of £2 10*s.*; and of .7365 of 6*s.* 8*d.*

Here £2 10*s.* = 50*s.*, and 6*s.* 8*d.* = 80*d.*

	.07	.7365
	50	80
	3.50	58.9200
	12	*Ans.* 4*s.* 10.92*d.*
Ans. 3*s.* 6*d.*	6.0	

Ex. 3. Find the value of $.17708\dot{3}$£.

.17708333	Or thus; $.17708\frac{1}{3}$
20	20
3.541666660	$3.54166\frac{2}{3}$
12	12
6.4999992 = 6.5 as in (64).	6.50000
4	4
Ans. 3*s.* $6\frac{1}{2}$*d.* 2.0	2.0

But it is often best to convert a *circulator* entirely to a vulgar fraction in such a case, and so find its value.

Ex. 4. Find the value of $3.2\dot{7}$ of a ton.

Here $3.2\dot{7} = 3\frac{5}{18}$; and $3\frac{5}{18}$ of a ton = 3 tons 5 cwt. 2 qrs. 6 lb. $3\frac{5}{9}$ oz.

Ex. **45.** Find the value of

1. .45 of £1; .68125 of £1; and 2.325 of £1.
2. 32.5 of 5*s.*; 1.85 of 3*s.* 4*d.*; and 2.375 of 13*s.* 4*d.*
3. .13125 of £5; and .001953125 of £40.
4. 3.45 of 5 guineas; and .325 of $1\frac{1}{2}$ ton.
5. 23.42 of a day; and 1.46875 of an acre.
6. 2.74 of 12*s.* 6*d.*; and 22.25 of £2 2*s.* 6*d.*
7. 3.225 of $2\frac{1}{2}$ guineas; and 22.75 of £5 10*s.* 6*d.*
8. 3.03 of 19*s.* 5*d.*; and .0474609375 of £10 13*s.* 4*d.*
9. .176 of 1 fur. 36 p. 2 yds. 5 in.; and .22 of 3 qrs. 15 lbs.
10. .2775 of 1 sq. yd. 3 ft. 72 in.; and 32.156 of 3 m. 330 yds.
11. 2.441 of £32 0*s.* $4\frac{1}{2}$*d.*; and 33.25 of £3 12*s.* $4\frac{1}{4}$*d.*
12. 44.045 of $11\frac{1}{4}$*d.*; and .5*s.* + .7 of a crown + .125£.
13. .634375£ + .025 of 25*s.* + .325 of 30*s.*
14. 8.71875 of 8*d.* + 1.146875 of 6*s.* 8*d.* − .0625 of a guinea.
15. .375 of a guinea + .1875 of a crown + .3 of 7*s.* 6*d.* − .875 of 2*d.*
16. $3.8\dot{3}$ of 4*s.*; and $6.1\dot{5}$ of 2*s.* $9\frac{3}{4}$*d.*

17. $23.\dot{4}\dot{5}$ of 3 m. 5 fur.; and $13.2\dot{7}\dot{5}$ of 5A. 2R.
18. $2.20\dot{7}$ of £3 9*s.* $4\frac{1}{2}$*d.*; and $2.1\dot{4}\dot{5}$ of 5*s.* $8\frac{3}{4}$*d.*
19. $.39791\dot{6}$ of £1; and $.4097\dot{2}$ of a guinea.
20. $.\dot{5}7142\dot{8}$ of a qr.; and $.\dot{2}8571\dot{4}$ of a cwt.

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67. *To reduce a given quantity to the decimal of another given quantity.*

RULE. Begin with the term of lowest denⁿ in the first given quantity, and reduce it to a decimal of the next higher denⁿ; prefix to this decimal the term (if any) of this higher denⁿ, which is found in the first given quantity, and reduce the result to a decimal of the next higher denⁿ; and so on, until we have thus brought it, if possible, to the decimal of the second given quantity.

Ex. 1. Reduce £3 17*s.* $6\frac{3}{4}$*d.* to the decimal of £5.

4) 3.00	Here we first reduce 3*f.* to a decimal of a penny, by
12) 6.7500	dividing by 4; the result is .75, i. e. 3*f.* = .75*d.*, and, pre-
20) 17.562500	fixing the 6*d.*, we have now 6.75*d.*, which we reduce to
5) 3.878125	the decimal of a shilling; and so on.
.775625	*Ans.*

Sometimes, as in common Reduction, we cannot thus pass directly, through different successive denⁿˢ, from the first to the second given quantity; and then it will be necessary to express the first as a fraction of the second, and then to reduce this fraction to a decimal.

Ex. 2. Reduce 2*s.* $9\frac{3}{4}$*d.* to the decimal of 7*s.* $9\frac{3}{4}$*d.*

Here $\dfrac{2s.\ 9\frac{3}{4}d.}{7s.\ 9\frac{3}{4}d.} = \dfrac{135 \text{ farthings}}{375 \text{ farthings}} = \dfrac{9}{25}$

```
25) 9.00 (.36   Ans.
    7 5
    ----
    1 50
    1 50
    ----
```

Ex. **26.** Reduce

1. 9*s.* 6*d.* to the dec. of £1; and 2*s.* $2\frac{1}{4}$*d.* to the dec. of £5.
2. 5*s.* to the dec. of 13*s.* 4*d.*; and 17*s.* 3*d.* to the dec. of 10*s.*
3. £1 2*s.* 6*d.* to the dec. of £1; and 2*s.* $7\frac{1}{2}$*d.* to the dec. of 10*s.*
4. 3*s.* $3\frac{3}{4}$*d.* to the dec. of £1 6*s.* 6*d.*; and £3 4*s.* 2*d.* to the dec. of 2*s.* 4*d.*

5. 6*s*. 6$\frac{3}{4}$*d*. to the dec. of a guinea; and 7*s*. 10$\frac{1}{2}$*d*. to the dec. of £2.
6. 9 oz. 2 dr. to the dec. of a lb.; and 3 fur. 33 yds. to the dec. of a mile.
7. 2 m. 1100 yds. to the dec. of a league; and 12 h. 55′ 21″ to the dec. of a day.
8. 3 qrs. 3 lbs. 1 oz. 7 drs. to the dec. of a cwt.; and 18$\frac{1}{4}$ days to the dec. of a year.
9. 15*s*. 6$\frac{3}{4}$*d*. to the dec. of £4; and 1 cwt. 3 qrs. 7 lbs. to the dec. of 2$\frac{1}{2}$ tons.
10. 3$\frac{3}{4}$ gs. to the dec. of £100; and 4$\frac{1}{2}$ lbs. to the dec. of 3 qrs. 12 lbs.
11. 13*s*. 4*d*. to the dec. of a crown, and 2 tons 4$\frac{1}{2}$ cwt. to the dec. of 1 ton 11$\frac{1}{4}$ cwt.
12. 3$\frac{1}{2}$ in. to the dec. of $\frac{1}{4}$ mile; and 22 guineas to the dec. of £25.
13. 2R. 4P. to the dec of 1R. 5P.; and £2 11*s*. 6$\frac{3}{4}$*d*. to the dec. of £3.
14. 8 sq. ft. 20 in. to the dec. of 12 sq. in.; and 7*s*. 6$\frac{1}{2}$*d*. to the dec. of £1.
15. 2 w. 6$\frac{1}{4}$ d. to the dec. of 4 d. 3 hrs.; and £6 12*s*. 6$\frac{3}{4}$*d*. to the dec. of 1$\frac{1}{2}$ guinea.
16. 3 hrs. 3′ 2$\frac{1}{4}$″ to the dec. of a day; and £24 12*s*. 6$\frac{1}{4}$*d*. to the dec. of £4.

MISCELLANEOUS EXAMPLES IN DECIMAL FRACTIONS.

47.

1. What vulgar fraction is equivalent to the sum of 14.4 and 1.44 divided by the difference?

2. What is the value of .0333 &c. of half-a-crown multiplied by .5?

3. The circumference of a circle = 3.1416 times the diameter; find the radius of the Earth, whose circumference is 24857 miles.

4. If the length of the year be taken at 365$\frac{1}{4}$ days instead of 365.24$\frac{2}{9}$ days, its true value, what will the error amount to in four centuries?

5. Reduce $\frac{7}{256}$ and $\frac{256}{7}$ to decimals; 3.75 and 3.7$\dot{5}$ to vulgar fractions; and multiply .235 by .0021 and 1.2.

6. Reduce 7*s*. 6*d*. to the decimal of £1; find the value of £2.6625; and, if 1 oz. cost .03125£, what will .0625 lbs. cost?

7. Find the value of .6£ + .3125*s*. + .$\dot{2}$ of a guinea.

8. Reduce $\frac{3}{22}$ and 4$\frac{3}{14}$ to decimals; .0$\dot{1}$2$\dot{3}$ to a vulgar fraction; and divide 18.073 by .0341 and 5300.

9. Find the value of .453125£ + 1.1484375*s*. + .71875*d*.

10. Reduce .375£ to the decimal of a guinea; and 1.25 of 3.675£ to the decimal of 10.5*s.*

11. Find the value of $.30069\dot{4}$ of a day; and of $.917897\dot{7}\dot{2}$ of 2A.

12. Find the value of $3\frac{2}{5} + 4\frac{1}{8} + 1\frac{11}{40} + 3\frac{13}{625}$ both by vulgar fractions and by decimals; and show that the two results coincide.

13. Find the value of 1.875 guinea + 1.875 crown + 1.875 of 3.625£.

14. Find the difference between $5\frac{1}{2}$ half-guineas and 3.125£; and reduce the result to the decimal of half-a-crown.

15. Multiply 1*s.* $7\frac{1}{4}$*d.* by 5782.5; and divide £168 5*s.* $4\frac{2}{25}$*d.* by 1.32.

16. The price of $\frac{1}{2}$ an oz. of coffee is $.458\dot{3}$*s.*; what is the value of .0015625 of a ton?

17. Find the difference between 1.6 of 3.4 of 1.125£ and $\frac{1}{5}$ of $3.\dot{6}$ of 9.1125£.

18. Reduce $\frac{17}{256}$ and $\frac{1}{101}$ to decimals; .0675 and $.067\dot{5}$ to vulgar fractions; and find the value of .73125 of £5.

19. If a lb. of sugar cost .0703125 of 8*s.*, what is the value of .0625 cwt.?

20. Add together $\frac{3}{5}$, $\frac{7}{8}$, $\frac{9}{10}$ and $\frac{7}{32}$, both as vulgar fractions and as decimals; and show that the two results coincide.

21. Find the value of 3.5*s.* + 2.9 of 23.375*s.* $-\frac{1.75}{3.5}$ of $16.\dot{6}$*s.*

22. Find the difference between $17.\dot{4}2857\dot{1}$ sq. ft. and $100.\dot{8}$ sq. in.; and between 1.76 cub. yds. and 26.66 cub. ft.

23. Multiply .0235 by 8.03; divide .0625 by 2.5; and find the value of $.843541\dot{6}$ of £5.

24. Multiply 6*s.* $0\frac{3}{4}$*d.* by 85.3125; and divide £10 11*s.* 3*d.* by 29.25.

25. Find the value of $4.\dot{4}$ of a guinea − 3.75 of half-a-crown + $.41\dot{6}$£ $-.\dot{3}5714\dot{2}\dot{8}$ of a guinea.

26. How many yards of matting, $2.\dot{4}$ feet broad, will cover a floor that is $27.\dot{3}$ feet long and $20.1\dot{6}$ feet broad?

27. Find the value of .375 of 5.375£, and of .06328125 of £100; and reduce £2 7*s.* $9\frac{3}{4}$*d.* to the decimal of 10*s.*

28. Find the values of $3.5 + 2.8\dot{3} + .\dot{6} + 1.175$; $11.7\dot{3} - 10.91\dot{6}$; $3.375 \times 1.\dot{6} \times 4.8$; $\frac{3.375}{4.5}$; and find the product of the results.

29. Find the value of 1.2 of 3.5 of 4.375*d.* + $1.8\dot{3}$ of $.95\dot{4}$ of $.\dot{4}2857\dot{1}$ of 4.5*d.*

30. What is the quarter's rent of $22.791\dot{6}$ acres of land, at 3.72£ per annum per acre?

31. Reduce $\frac{7}{64}$ and $\frac{7}{65}$ to decimals; .65 and $.06\dot{5}\dot{1}$ to vulgar fractions; and £2 3*s.* $3\frac{3}{4}$*d.* to the decimal of £4.

32. Find the value of $.\dot{2}8571\dot{4}$ of £30 + $6.\dot{8}5714\dot{2}$£ + $.\dot{6}$ of $.\dot{7}1428\dot{5}$ of .6£ + $1.\dot{3}$ of $.\dot{4}2857\dot{1}$*s.*

33. Reduce $2\frac{5}{8}$ and $\frac{4}{111}$ to decimals; 2.05 and $.20\dot{5}$ to vulgar fractions; and £19 17*s.* $2\frac{1}{4}$*d.* to the decimal of £5.

34. Multiply 1 cwt. 2 qrs. 3 lbs. by 5.125; and divide £3834 0*s.* $5\frac{1}{4}$*d.* by 441.75.

35. If an ounce of gold be worth £$4.\dot{0}09\dot{9}$, what is the value of a bar of gold, weighing $1.68\dot{3}$ lbs?

36. Reduce .6 of £1 + $.\dot{6}$ of 5*s.* 3*d.* + 3.75 of a crown to the decimal of 16*s.*

37. Find what decimal multiplied by 175 will give the sum of $\frac{1}{4}$, $\frac{16}{25}$, $\frac{43}{50}$, and $3\frac{1}{2}$.

38. Multiply .285 by 4.02; divide 2.961 by .007; and find the value of 2.778125 of 6*s.* 8*d.*

39. Reduce $\left(\frac{2.375}{3.1\dot{6}} \text{ of } \frac{4.\dot{4}}{.0625}\right) \div \left(\frac{8.\dot{8}}{7} \text{ of } \frac{4}{5.625}\right)$ to a simple quantity.

40. Multiply £2 16*s.* 10.75*d.* by 144.33; and divide £9753 14*s.* $8\frac{1}{4}$*d.* by 234.5.

41. Find the value of 3.275 of £10; multiply 3.275 by 12.8; and divide .0625 by .00005.

42. Reduce $\frac{11}{512}$ and $\frac{2}{33}$ to decimals; 2.0325 and $.3\dot{4}0\dot{5}$ to vulgar fractions; and 2 lbs. 3 oz. to the decimal of a ton.

43. Reduce 1.75*s.* to the decimal of £1; and $2.\dot{6}$ of £$.87708\dot{3}$ to the decimal of half-a-sovereign.

44. Find the value of $3\frac{37}{43}$ of £3 12*s.* $6\frac{3}{4}$*d.*; and reduce the result to the decimal of £35 0*s.* $3\frac{3}{4}$*d.*

45. Reduce $\frac{2.8 \text{ of } 2.\dot{2}\dot{7}}{1.1\dot{3}\dot{6}} + \frac{4.\dot{4} - 2.8\dot{3}}{1.\dot{6} + 2.\dot{6}2\dot{9}} \text{ of } \frac{6.8 \text{ of } 3}{2.25}$ to a simple quantity.

46. Find the value of $\frac{4}{5}$ of 2.625 guineas; and the difference between 26.5P. and $70\frac{3}{5}$ sq. yds.

47. Find the value of $6.8\dot{3}$ of £$3.867708\dot{3}$ + 5.8 of £$2.411458\dot{3}$ − 4.375 of £1.3.

48. Reduce to a decimal, accurate to 5 places,

$$16\left(\frac{1}{5} - \frac{1}{3 \times 5^3} + \frac{1}{5 \times 5^5} - \frac{1}{7 \times 5^7} + \&c.\right) - \frac{4}{239}.$$ *See* NOTE VII.

49. Find the sum of £1.15 + 2.0625 guineas + .0078125 of 32*s.*; and reduce the result to the decimal of half-a-sovereign.

50. Reduce to a decimal, accurate to 7 places,

$$1 + \frac{1}{1} + \frac{1}{1 \times 2} + \frac{1}{1 \times 2 \times 3} + \frac{1}{1 \times 2 \times 3 \times 4} + \&c.$$ *See* NOTE VII.

CHAPTER V.

PRACTICE.

68. This is an expeditious method of finding the value of any quantity of merchandise, &c. *when the value of a unit of any denomination is given;* as of 456 cwt. at £3 13*s.* 6*d.* per cwt., or of 3 cwt. 3 qrs. 13 oz. at £2 6*s.* 7½*d.* per lb. &c.

69. Case i. *Where the given quantity is expressed in the same denomination as the unit whose value is given.*

Under this head will occur such examples as the following: 36 *cwt.* at £3 10*s.* per *cwt.*, 25 *lbs.* at £2 16*s.* 8*d.* per *lb.*, 37 *oz.* at £5 17*s.* 6*d.* per *oz.*, &c.; where the unit, whose value is *given*, is of the same den^{n} as the quantity whose value is *required.* It is obviously immaterial *what* the unit itself is; that is to say, the values would be the same either of 36 *cwt.* at £3 10*s.* per *cwt.*, or of 36 *lbs.* at £3 10*s.* per *lb.*, or of 36 *oz.* at £3 10*s.* per *oz.*, or (without specifying any unit) of 36 *articles* at £3 10*s.* *each*, or, as it is briefly expressed, of 36 at £3 10*s.*

Ex. 1. *Find the value of* 36 at £3 10*s.*

Here we have, in fact, to multiply £3 10*s.* by 36; let us first then multiply £3 by 36, or, which amounts to the same, multiply £36 by 3, and we shall have £108 as the amount of 36 at £3.

		£36
		3
		108
10*s.*	½	18
	Ans.	£126

Now, instead of multiplying the 10*s.* by 36, we observe that, since 10*s.* is £½, we may take 36 times 10*s.* by taking 36 *half pounds*, which is the same as taking *half* of 36 *pounds* = £18, which we add to the £108, and thus have the whole product of £3 10*s.* × 36.

Ex. 2. *Find the value of* 253 at £2 16*s.* 8*d.*

		£253		
		2		
		506		
10*s.*	$\frac{1}{2}$	126	10	
6*s.* 8*d.*	$\frac{1}{3}$	84	6	8
Ans.		£716	16	8

Here we find, as before, the value of £2 10*s.* × 253: and then, since 6*s.* 8*d.* is £$\frac{1}{3}$, dividing 253 by 3, we have £84 6*s.* 8*d.*, the value of 6*s.* 8*d.* × 253, which we add to the other two lines, and thus have the whole product of £2 16*s.* 8*d.* × 253.

Ex. 48.

		£	s.	d.			£	s.	d.
1.	129 at	6	10	0	2.	343 at	4	6	8
3.	157 at	9	5	0	4.	362 at	7	4	0
5.	271 at	8	3	4	6.	187 at	1	2	6
7.	289 at	11	1	8	8.	495 at	12	11	0
9.	447 at	1	16	8	10.	555 at	4	13	4
11.	361 at	9	11	8	12.	677 at	2	12	6

Ex. 3. *Find the value of* 371 at £5 17*s.* 6*d.*

		£371		
		5		
		1855		
10*s.*	$\frac{1}{2}$	185	10	
5*s.*	$\frac{1}{2}$	92	15	
2*s.* 6*d.*	$\frac{1}{2}$	46	7	6
Ans.		£2179	12	6

Here we find the value of £5 10*s.* × 371 as before: then instead of taking, as we might, 6*s.* 8*d.* as £$\frac{1}{3}$, &c., we may take 5*s.* as $\frac{1}{2}$ of 10*s.*, and so find the value of 5*s.* × 371, by taking half the value of 10*s.* × 371, i. e. half of £185 10*s.*, or £92 15*s.*: in like manner, we may then take 2*s.* 6*d.* as $\frac{1}{2}$ of 5*s.*, and find the value of 2*s.* 6*d.* × 371 by taking half of £92 15*s.*

Ex. 4. *Find the value of* 713 at £4 8*s.* 11$\frac{1}{2}$*d.*

		£713		
		4		
		2852		
5*s.*	$\frac{1}{4}$	178	5	
3*s.* 4*d.*	$\frac{1}{6}$	118	16	8
7$\frac{1}{2}$*d.*	$\frac{1}{8}$	22	5	7$\frac{1}{2}$
Ans.		£3171	7	3$\frac{1}{2}$

Here we find, as in Ex. 2, the value of £4 8*s.* 4*d.* × 713; we then take 7$\frac{1}{2}$*d.* as $\frac{1}{8}$ of 5*s.*, and so divide by 8 the line £178 5*s.*, which is the value of 5*s.* × 713.

Ex. 49.

		£	s.	d.			£	s.	d.
1.	127 at	3	15	0	2.	235 at	5	7	6
3.	339 at	4	12	0	4.	341 at	6	17	6
5.	253 at	7	17	0	6.	457 at	1	18	6
7.	365 at	11	14	6	8.	573 at	7	15	6
9.	285 at	1	6	6	10.	389 at	8	13	6
11.	492 at	6	18	9	12.	297 at	1	16	9

Ex. 5. *Find the value of* 89 at 3*s.* 11$\frac{3}{4}$*d.*

		89*s.*	
		3	
		267	
9*d.*	$\frac{1}{4}$	66	9
2*d.*	$\frac{1}{6}$	14	10
$\frac{3}{4}$*d.*	$\frac{1}{12}$	5	6$\frac{3}{4}$

354*s.* 1$\frac{3}{4}$*d.* = £17 14*s.* 1$\frac{3}{4}$*d.* *Ans.*

Here there are no £ in the given value; but if we multiply 89 by 3 the result will be in *shillings;* then 9*d.* being $\frac{1}{4}$ of 3*s.* we take $\frac{1}{4}$ of 267*s.*; and 2*d.* being $\frac{1}{6}$ of 1*s.* we take $\frac{1}{6}$ of 89*s.*; lastly, $\frac{3}{4}$*d.* being $\frac{1}{12}$ of 9*d.* we take $\frac{1}{12}$ of 66*s.* 9*d.*

Ex. 6. *Find the value of* 111 at 18*s.* 7$\frac{1}{2}$*d.*

		£	s.	d.
		£111		
6*s.* 8*d.*	$\frac{1}{3}$	37		
6*s.* 8*d.*	$\frac{1}{3}$	37		
5*s.*	$\frac{1}{4}$	27	15	
3*d.*	$\frac{1}{20}$	1	7	9
$\frac{1}{2}$*d.*	$\frac{1}{6}$	0	4	7$\frac{1}{2}$
Ans.		£103	7	4$\frac{1}{2}$

We might treat this as the last Ex.; or, to avoid the final reduction, we may begin at once by taking 6*s.* 8*d.* as £$\frac{1}{3}$, &c., drawing a line under the 111, before we divide by 2, since it is not to be added in with the other lines.

		£	s.	d.
£111 × .9 =		£99	18*s.*	0*d.*
6*d.*	$\frac{1}{40}$	2	15	6
1$\frac{1}{2}$*d.*	$\frac{1}{4}$	0	13	10$\frac{1}{2}$
Ans.		£103	7	4$\frac{1}{2}$

Otherwise:—As the n° of shillings in the price is *even*, we can conveniently change it into the decimal of £1, viz., £.9, and multiplying by .9, we may mentally double the decimal of the product for shillings; &c.

Ex. 50.

	s.	d.		s.	d.		s.	d.
1. 227 at	2	1$\frac{1}{4}$	2. 149 at	3	2$\frac{3}{4}$	3. 854 at	4	2$\frac{1}{2}$
4. 356 at	4	9$\frac{3}{4}$	5. 365 at	5	7$\frac{3}{4}$	6. 373 at	7	8$\frac{1}{2}$
7. 177 at	8	11$\frac{1}{2}$	8. 784 at	9	2$\frac{3}{4}$	9. 489 at	11	8$\frac{3}{4}$
10. 193 at	13	5$\frac{1}{2}$	11. 395 at	14	4$\frac{3}{4}$	12. 499 at	17	11$\frac{3}{4}$

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70. It is often convenient to suppose the given value *increased* so as to become an *exact* number of pounds, or shillings, &c. for which we may calculate by common Mult^{n}; then, if we find by Practice the value of the part added to the true value, and subtract it from the other result, we shall get the required amount.

		£	s.	d.
		£253		
		3		
		759		
3*s.* 4*d.*	$\frac{1}{6}$	42	3	4
Ans.		£716	16	8

Thus, in Ex. 2, supposing the given value to be £3, we should multiply £3 by 253; and then taking 3*s.* 4*d.* (the part added to the given value) as £$\frac{1}{6}$, and subtracting the corresponding amount, we have the same result as before.

Similarly, in Ex. 3, we may add 2*s.* 6*d.* to the given value, making it £6; then multiply £6 by 371, and from the result subtract 2*s.* 6*d.* × 371, or $\frac{1}{8}$ of £371.

		£ s. d.		£ s. d.
Ex. 51.	1.	135 at 2 19 $3\frac{1}{2}$	2.	217 at 4 17 $7\frac{3}{4}$
	3.	273 at 3 18 $4\frac{3}{4}$	4.	322 at 7 14 $5\frac{1}{2}$
	5.	289 at 8 $8\frac{1}{2}$	6.	373 at 9 $7\frac{3}{4}$
	7.	431 at 5 17 $11\frac{1}{2}$	8.	397 at 6 15 10
	9.	511 at 7 $10\frac{3}{4}$	10.	623 at 11 $9\frac{1}{4}$
	11.	271 at 6 15 $10\frac{3}{4}$	12.	333 at 5 18 $11\frac{1}{2}$

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71. CASE II. *When the given quantity is not expressed in the same denomination as the unit whose value is given.*

Here we shall have to find the value of 3 cwt. at £2 13*s.* 6*d.* per lb., or of 2 cwt. 3 qrs. 16 lbs. at £3 5*s.* $7\frac{1}{2}$*d.* per cwt., or per qr., or per lb., &c. In all instances, (like the first of these,) where the given quantity can be immediately reduced to the same den^n as the given unit, we may do this, and shall then have only an example nnder Case I. : thus 3 cwt. = 336 lbs., and the value of 336 lbs. at £2 13*s.* 6*d.* per lb. may be found as before. So also, if we can reduce *any part* of the given quantity to the same den^n as the given unit, we may find its value by mult^n ; and, for the rem^r, we may take parts of the given *unit* itself, and proceed as in the following examples.

Ex. 1. *Find the value of 7 cwt. 3 qrs. 11 lbs. at* £2 13*s.* 1*d. per qr.*

The given unit being a qr., we reduce 7 cwt. 3 qrs. to 31 qrs., and find the value of them by multiplying by 31: then to find the value of 11 lbs., we consider that 7 lbs. are $\frac{1}{4}$ qr., and 4 lbs. are $\frac{1}{7}$ qr. ; so that, dividing £2 13*s.* 1*d.* by 4 and 7, and adding up, we have the value of 7 cwt. 3 qrs. 11 lbs.

	£	s.	d.		
	2	13	1 × 31		
			10		
	26	10	10		
			3		
	79	12	6	value of	30 qrs.
	2	13	1	„	1 qr.
7 lbs. = $\frac{1}{4}$		13	$3\frac{1}{4}$	„	7 lbs.
4 lbs. = $\frac{1}{7}$		7	7	„	4 lbs.
Ans.	£83	6	$5\frac{1}{4}$	„	31 qrs. 11 lbs.

Ex. 2. *Find the rent of* 8A. 3R. 10P. *at* £1 17*s*. 8*d*. *per acre.*

Here we find the rent of 8A., as in the last example; and then calculate that of 3R. 10P., and add as before.

	£	*s.*	*d.*		
	1	17	8 × 8		
			8		
	15	1	4	rent of	8A.
2R. = $\frac{1}{2}$		18	10	,,	2R.
1R. = $\frac{1}{2}$		9	5	,,	1R.
10P. = $\frac{1}{4}$		2	$4\frac{1}{4}$	,,	10P.
Ans.	£16	11	$11\frac{1}{4}$	,,	8A. 3R. 10P.

Ex. 3. *Find the rent for* 3 *mo.* 3 *w.* 5 *d. at* £3 13*s*. 6*d*. *per month.*

Here 1 mo. = 4 wks.; and we can take 3 wks. = $\frac{1}{4}$ of 3 mo. &c., as in one of the subjoined forms, or 1 wk. 5 da. = $\frac{1}{7}$ of 3 mo. &c., as in the other.

	£	*s.*	*d.*
	3	13	6
			3
	11	0	6
3 wk. = $\frac{1}{4}$	2	15	$1\frac{1}{2}$
4 da. = $\frac{1}{7}$ m.	0	10	6
1 da. = $\frac{1}{4}$	0	2	$7\frac{1}{2}$
	£14	8	9 *Ans.*

Or thus:	£	*s.*	*d.*
	3	13	6
			3
	11	0	6
1 wk. 5 da. = $\frac{1}{7}$	1	11	6
2 wk. = $\frac{1}{2}$ m.	1	16	9
	£14	8	9

Ex. 52.

1. 6 cwt. 1 qr. 11 lbs. at £2 17*s*. 9*d*. per cwt.
2. 3 cwt. 3 qrs. 5 lbs. at £4 14*s*. per cwt.
3. 9 cwt. 21 lbs. at £5 11*s*. $1\frac{1}{2}$*d*. per cwt.
4. 2 cwt. 4 lbs. 12 oz. at £3 1*s*. per cwt.
5. 3 qrs. 5 lbs. 9 oz. at £2 14*s*. 6*d*. per lb.
6. 2 qrs. 9 lbs. 13 oz. at 15*s*. 9*d*. per lb.
7. 2 qrs. 7 oz. 9 drs. at 18*s*. 6*d*. per lb.
8. 2 cwt. 2 lbs. 2 oz. 12 drs. at £1 3*s*. 9*d*. per lb.
9. 3 cwt. 3 qrs. 27 lbs. 15 oz. 12 drs. at £7 per cwt.
10. 6 oz. 18 dwts. 20 grs. at 7*s*. 9*d*. per oz.
11. 3 lbs. 5 oz. 14 dwts. 12 grs. at 17*s*. 6*d*. per oz.
12. 22 yds. 2 ft. 2 in. at 18*s*. 8*d*. per yard.
13. 13 yds. 1 ft. 7 in. at 9*s*. 4*d*. per foot.
14. 37A. 1R. 28P. at £2 2*s*. per acre.
15. 17A. 3R. 19P. at £5 18*s*. 6*d*. per acre.
16. 21A. 2R. 12 P. at £3 15*s*. 8*d*. per acre.

17. 5 mo. 3 w. 4 d. at 17*s.* 6*d.* per week.
18. 7 mo. 2 w. 5 d. at £2 8*s.* 4*d.* per month.
19. 9 mo. 1 w. 6 d. at £1 2*s.* 9*d.* per week.
20. 6 mo. 3 w. 2 d. at £3 0*s.* 6*d.* per month.

---

72. The method of Practice may be applied, as we have said, to any case where the value of any quantity is sought, that of a unit of any den^{n} being given. It is not, however, necessary (as in the foregoing Examples) that this given value should be the *price* of the unit, &c.; but, whenever any given amount is charged for any reason upon the unit, we may find thus the *corresponding* amount for the given quantity.

Ex. *A bankrupt is able to pay* 12*s.* 6½*d.* *in the* £, *and his debts are* £3600 : *what was his estate worth?*

		3600	
10*s.*	$\frac{1}{2}$	1800	
2*s.* 6*d.*	$\frac{1}{4}$	450	
½*d.*	$\frac{1}{60}$	7	10
Ans.		£2257	10

This means that, for every £ he owes, he can pay only 12*s.* 6½*d.*; here then we have to find the value of 12*s.* 6½*d.* × 3600, which must have been the value of his whole estate.

Miscellaneous Examples. 53.

1. What must be paid to 721 labourers for a week's service, at 17*s.* 4½*d.* each?

2. What would be the amount of 137 tons 12 cwt. of goods, at the rate of £2 4*s.* 10½*d.* per cwt.?

3. Calculate the amount of a salary of 21418 rupees, valued at 2*s.* 1¾*d.* each.

4. A bankrupt's debts are £7357, and he is able to pay 12*s.* 9¾*d.* in the £; what are his effects worth?

5. To how much will a charge of £28 8*s.* 2*d.* per day amount in 365 days?

6. Lodgings at £5 10*s.* 6*d.* per month being occupied for 8 mo. 21 days, how much must be paid for them?

7. What must be given for a gold snuff-box, weighing 5 oz. 9 dwts. 20 grs., at the rate of £4 3*s.* 9*d.* per oz.?

8. What is the dividend on £1710 14*s.* 6*d.*, at 13*s.* 4½*d.* in the £?

9. How many acres will supply 53 horses with hay and oats, if each horse consume annually the produce of 5A. 3R. 26P.?

10. What is the expense of digging a ditch, of which the cubic content is 5755 cubic yards, at the rate of 1*s.* 7¾*d.* per yard?

11. A bankrupt owes £2468, and can pay 15*s.* 6*d.* in the £; what are his effects worth?

12. Find the weight of 1000 pieces of gold coin, each weighing 6 dwt. 7 gr.

13. An officer's pay is 12*s.* 3*d.* per day; what is that in a year?

14. A labourer's pay being 2*s.* $9\frac{3}{4}$*d.* a day, what is the whole pay of 23 men for 25 days?

15. If lodgings let at 13*s.* 6*d.* per week, how much do they let for during 273 days?

16. A merchant bought 182 quarters of wheat at £2 1*s.* 3*d.* per quarter, and retailed the same at £2 18*s.* 4*d.* per quarter; what was his gain, and at what per quarter should he have sold it to have gained exactly 104 guineas?

17. What sum would be required to pay the wages of 377 labourers for a week, at 2*s.* 5*d.* a day each?

18. If a person's estate be worth £1384 16*s.* per ann., and the land-tax be assessed at 2*s.* $9\frac{1}{2}$*d.* in the £, what is his net annual income?

19. An iron bridge consists of 3 arches, the centre one weighing 3046 tons, and the two others 2600 tons each; what is the cost of the iron at £6 13*s.* 6*d.* per ton?

20. What will a room cost in painting, at 1*s.* $7\frac{1}{2}$*d.* per square yard, those height is 10 ft. 3 in., width 16 ft. 6 in., and length 18 ft. 10 in.?

21. An estate of 134A. 3R. 16P. is rented at £2 12*s.* 6*d.* per acre, and afterwards the best pasture, consisting of 51A. 2R. 12P., is let at £3 10*s.* per acre; what will the first tenant still have to make up of his rent?

22. A bankrupt's liabilities are estimated at £3758 17*s.* 6*d.*; what are his assets, if he can pay 13*s.* $7\frac{1}{2}$*d.* in the £?

23. What is the joint value of 5 qu. $3\frac{1}{4}$ bu. of wheat at 7*s.* $4\frac{1}{2}$*d.* per bushel, and 5 qu. $3\frac{1}{4}$ bu. of oats at 4*s.* $2\frac{1}{4}$*d.* per bushel?

24. There were sold three pieces of land, containing $59\frac{1}{2}$A., $76\frac{1}{4}$A., 39A. 12P. respectively: the price of the first piece was £12 7*s.* 10*d.*, of the second £13 15*s.* 9*d.*, and of the third £16 8*s.* 6*d.* per acre; what was given for the whole?

25. What will be the cost of replacing a cistern, to weigh 8 cwt. 2 qrs. 14 lbs., at the rate of £2 0*s.* 6*d.* per cwt., if the plumber allows £1 11*s.* 6*d.* per cwt. for the lead of the old one, which weights 6 cwt. 1 qr. 10 lbs.?

CHAPTER VI.

PROPORTION.

73. THE *Ratio* of one quantity to another is the number which expresses what fraction the former is of the latter, and is therefore obtained, as in (48), by dividing the former by the latter.

Thus the ratio of 108 to 144, or (as it is written) of 108 : 144, is $\frac{108}{144}=\frac{3}{4}$, meaning that 108 is $\frac{3}{4}$ of 144.

The former of the two terms in any ratio is called the *antecedent,* and the latter the *consequent;* and it is plain from the above, that all ratios are equal which may be made to have the same antecedent and consequent by striking common factors out of their two terms.

Thus the ratios of 108 : 144, 36 : 48, 21 : 28, 15 : 20, 3 : 4, &c., are all equal, since each of them is equivalent to the fraction $\frac{3}{4}$; and it will be seen that the first of each of these pairs of quantities is $\frac{3}{4}$ of the second.

74. When two ratios are equal, they are said to form a *Proportion,* and the four terms composing them are called *proportionals,* or are said to be *proportional to one another.*

Thus, since 15 is $\frac{3}{4}$ of 20, and 21 is $\frac{3}{4}$ of 28, and so (as before was said) the ratio of 15 : 20 = the ratio of 21 : 28, these four quantities form a proportion, which is usually expressed thus, 15 : 20 :: 21 : 28, and read *as* 15 *is to* 20 *so is* 21 *to* 28, or 15 *is to* 20 *as* 21 *is to* 28; and here 15 and 21 are the two antecedents, 20 and 28 the two consequents, of the ratios which form this proportion.

N.B. It should be well noticed that the proportion 15 : 20 : : 21 : 28 expresses that 15 *is the same fraction* (proper or improper) *of* 20 *that* 21 *is of* 28.

75. In any proportion, the product of the 1st and 4th terms = the product of the 2nd and 3rd terms, or, as it is commonly said, *the product of the extremes = the product of the means.*

Thus in the proportion 15 : 20::21 : 28, since the two ratios are equal, we have $\frac{15}{20}=\frac{21}{28}$; and, if we multiply each of these equals by 20×28, we get $15 \times 28 = 20 \times 21$, or 1st × 4th = 2nd × 3rd.

76. Conversely, if the product of any two quantities = the product of two others, the four are proportionals, the factors in one product being the *extremes*, and those in the other the *means*, of the proportion.

Thus, since $6 \times 20 = 120 = 8 \times 15$, if we divide each of these equals by 6×8, 6×15, 20×8, 20×15, respectively, we get

$\frac{20}{8}=\frac{15}{6}$, whence 20 : 8::15 : 6	or $\frac{15}{6}=\frac{20}{8}$, whence 15 : 6::20 : 8;
$\frac{20}{15}=\frac{8}{6}$, whence 20 : 15::8 : 6	or $\frac{8}{6}=\frac{20}{15}$, whence 8 : 6::20 : 15;
$\frac{6}{8}=\frac{15}{20}$, whence 6 : 8::15 : 20	or $\frac{15}{20}=\frac{6}{8}$, whence 15 : 20::6 : 8;
$\frac{6}{15}=\frac{8}{20}$, whence 6 : 15::8 : 20	or $\frac{8}{20}=\frac{6}{15}$, whence 8 : 20::6 : 15;

in the first set of which proportions it is seen that the terms of one product, 6 and 20, are the *extremes*, and those of the other product, 8 and 15, the *means*; and vice versâ, in the other set.

77. Hence also it follows, that, if four quantities in any given order are proportionals, they will also be proportionals in any other order, in which the same two terms will *go together*, either as *extremes* or *means*.

Thus, since 6 : 9::10 : 15, it follows by (75) that $6 \times 15 = 9 \times 10$, and therefore by (76) we have also 6 : 10::9 : 15, 10 : 15::6 : 9, &c., in which 6 and 15 still *go together*, either as extremes or as means. We could not have, however, 6 : 15::9 : 10, &c., in which this is not the case.

78. If we have given any three of the four terms of a proportion, we may by means of them easily find the fourth; for since by (75) the 1st × 4th = 2nd × 3rd, we have the

$$1\text{st} = \frac{2\text{nd} \times 3\text{rd}}{4\text{th}}, \text{ the 4th} = \frac{2\text{nd} \times 3\text{rd}}{1\text{st}}, \text{ the 2nd} = \frac{1\text{st} \times 4\text{th}}{3\text{rd}}$$

and the $3\text{rd} = \frac{1\text{st} \times 4\text{th}}{2\text{nd}}$.

Ex. Find the numbers which shall form the 1st and 2nd terms, respectively, of a proportion with the numbers 6, 7, 8.

Here the $1\text{st} = \frac{2\text{nd} \times 3\text{rd}}{4\text{th}} = \frac{6 \times 7}{8} = 5\frac{1}{4}$, and $5\frac{1}{4}$: 6::7 : 8;

the $2\text{nd} = \frac{1\text{st} \times 4\text{th}}{3\text{rd}} = \frac{6 \times 8}{7} = 6\frac{6}{7}$, and 6 : $6\frac{6}{7}$::7 : 8.

Ex. 54. Find numbers which shall form the 1st, 2nd, 3rd, 4th terms, respectively, of a proportion with

1. 2, 3, 4.	2. 3, 4, 5.	3. 4, 5, 6.	4. 5, 6, 7.
5. 2, 5, 7.	6. 4, 5, 8.	7. 2, 7, 9.	8. 5, 7, 7.

---

79. We have hitherto given instances only of the ratios of *abstract* quantities, or *numbers*, to one another; but we may similarly obtain the ratios of *concrete* quantities.

Thus the ratios of £108 : £144, of 9 cwt. : 12 cwt., of 15 gals. : 20 gals., of 39 ft. : 52 ft., are (by 48) respectively $\frac{108}{144}$, $\frac{9}{12}$, $\frac{15}{20}$, $\frac{39}{52}$, each of which reduces to $\frac{3}{4}$; and we say, therefore, that the ratio of £108 : £144 is the same as that of 3 : 4, or $\frac{3}{4}$, meaning that £108 is $\frac{3}{4}$ of £144; and so with the other ratios.

Of course, however, the quantities forming such a ratio must be of the same *kind;* for, otherwise, one of them could not be a fraction of the other.

Thus it would be absurd to speak of the ratio of £108 : 144 cwt., or of 9 cwt. : 12 gals., &c.

So also, though they be of the same kind, we must besides reduce them, as in (48), to the same *denomination*, before we can express the one as a fraction of the other, and so find their ratio.

Thus the ratio of 7*s.* 6*d.* : 4*s.* 2*d.* = the ratio of 90*d.* : 50*d.* $=\frac{90}{50}=\frac{9}{5}$ = 9 : 5.

N.B. Whatever be the nature of the quantities themselves, their ratio is always a mere *abstract number*, expressing, as stated in (73), what fraction the one is of the other.

Thus in the last instance the ratio of 90*d.* : 50*d.* is, as in (48), the *number* $\frac{9}{5}$, not $\frac{9}{5}$*d.*; for it has no reference whatever to the fact, that the given quantities were *pence*, but only to the *magnitude* of the one with respect to the other, i. e. to the fact that the one is $\frac{9}{5}$ of the other; and it would plainly have been just the same, if the ratio had been that of £90 : £50, or of 90 cwt. : 50 cwt., &c.

80. So also, when two such ratios are equal, they form a *proportion;* thus £108 : £144 : : 9 cwt. : 12 cwt.; only here we cannot, as in (77), change the *order* of the terms, except the change be such as still leaves the two ratios *possible.*

Thus, it will be true, as before, that £144 : £108 :: 12 cwt. : 9 cwt, or 9 cwt. : 12 cwt. :: £108 : £144, &c.; but we cannot say that £144 : 12 cwt. :: £108 : 9 cwt., because the two ratios £144 : 12 cwt. and £108 : 9 cwt. are absurd. It would, however, be true, that £144 : £12 :: 108 cwt. : 9 cwt. &c.

81. For the like reason, we cannot exactly say of such a proportion, that the product of the extremes = that of the means; thus it would be absurd to speak of multiplying £144 by 9 cwt., &c.: if, however, we consider only the *numerical* values of the terms, this would be still true: and, having three terms of such a proportion given, we may, by means of their *numerical* values, find as in (78) the *numerical* value of the fourth term, which will be of the same kind and *denomination* as the other term of the ratio to which it belongs.

Thus to find a fourth proportional to £108, £100, 9 cwt., we have its *numerical* value $= \frac{100 \times 9}{108} = 8\frac{1}{3}$, which must be $8\frac{1}{3}$ *cwt.*, since it must be of the same *kind* and *denomination* with 9 *cwt.*, the other term of the ratio to which it belongs; and the proportion will therefore be

£108 : £100 :: 9 cwt. : $8\frac{1}{3}$ cwt.

82. The method above referred to, by which we may find the fourth proportional to three given quantities,—viz. *by multiplying together the 2nd and 3rd, and dividing the product by the 1st*,—is commonly known by the name of the *Rule of Three.*

In practical applications of this Rule, the three given quantities are generally *concrete;* and a very large class of Examples are those, where, the cost of a given quantity of some article being given, we are required to find, either what will be the cost of another given *quantity*, or else what quantity may be bought for another given *cost.* For it is plain that, in any such case, if the first *cost* be double. treble, half, &c. of the second *cost*, the first *quantity* will be double, treble, half, &c. of the second quantity, and, generally, the first cost will be the same fraction of the second cost that the first quantity is of the second quantity;

i.e. the *ratio* of the two costs will be the same as the ratio of the two quantities, or the four will be proportionals, so that we may apply to them the preceding observations.

Ex. 1. *If* 39 *cwt. of sugar cost* £91, *what will be the cost of* 18 *cwt.?*

39 cwt. : 18 cwt. :: £91 : the Ans., whose numerical value we obtain by multiplying £91 by 18, and dividing by 39, without considering these as concrete quantities, and the result 42 will be of the same *kind* as the 3rd term, viz. £s.

```
        18
       ---
       728
        91
       ---
39)1638(£42
    156
    ---
     78
     78
     --
```

Ans. £42.

Ex. 2. *If* £42 *will buy* 18 *cwt. of sugar, what quantity may be had for* £91?

```
£42 : £91 :: 18 cwt.
       18
      ---
      728
       91
      ---
42 { 6) 1638
   { 7)  273
```

Ans. 39 cwt.

Here we have multiplied the 2nd by the 3rd (the least of the two) for convenience, and the result 39 will be of the same kind as the third term, viz. cwt.

Ex. 55.

1. If 12 yards of cloth cost £15, what would 8 yards cost at the same rate?
2. If 46 bu. of wheat cost £16, how many may be bought for £72?
3. What will be the cost of 90 gals. of wine, if 495 gals. cost £396?
4. How many acres of land may be rented for £65, if the rent of 168 acres be £364?
5. If 63 loads of straw can be bought for £180, how many may be had for £100?
6. How much must be given for 25 doz. of wine, at the rate of £176 for 80 doz.?

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83. Since the *Answer* $= \frac{\text{2nd} \times \text{3rd}}{\text{1st}}$, and the value of this fraction is not altered by striking common factors out of its numr and denr, we may sometimes simplify the operation by striking out (before we multiply and divide according to the Rule) a common factor, either from the 1st and 2nd terms, or from the 1st and 3rd terms.

Ex. 3. *If* 275 *reams of paper cost* £158 15*s*., *what would* 990 *reams cost?*

$$275 \text{ rms.} : 990 \text{ rms.} :: £158\ 15s. : \frac{3175s. \times 990}{275}$$

$$\begin{array}{r} 20 \\ \hline 3175s. \end{array}$$

$$\frac{\overset{127}{\cancel{3175}}s. \times 990}{\underset{11}{\cancel{275}}} = 127s. \times 90 = 11430s. = £571\ 10s.$$

Ans.

Here having first, for convenience, reduced the 3rd term to shillings, we have signified the value of the 4th term by a fraction representing the product of the 2nd and 3rd terms divided by the 1st. Then striking out 25 from 3175 and 275, we get 127 and 11; then dividing 990 by the 11, we obtain 127 × 90.

Ex. 4. *If* 14 *tons of bar-iron cost* £106 11*s*. 6*d*., *how much may be had for* 100 *guineas?*

$$£106\ 11s.\ 6d. : 100 \text{ guin.} :: 14 \text{ tons} : \frac{14 \text{ t.} \times 4200}{4263}$$

$$\begin{array}{r} 20 \\ \hline 2131 \\ 2 \\ \hline 4263 \text{ sixp.} \end{array} \qquad \begin{array}{r} 42 \\ \hline 4200 \text{ sixp.} \end{array}$$

$$\frac{14 \times \overset{200}{\cancel{4200}}}{\underset{203}{\cancel{4263}}} = \frac{2 \times 200}{29} = 400 \text{ t.} \div 29$$

$$\begin{array}{r} 29)\ 400\ (13\tfrac{23}{29} \text{ tons. } Ans. \\ 29 \\ \hline 110 \\ 87 \\ \hline 23 \end{array}$$

7. If 385 yards of cloth cost £253, how many may be had for £138?
8. How much cambric may be bought for £45, if 714 yds. cost £85?
9. If 36A. 3R. of land are rented for £84, what should be the rent of 21A. 3R. 20P.?
10. If I pay £18 for 7 cwt. 3 qrs. 14 lbs. of sugar, what would be the cost of 4 cwt. 1 qr. 14 lbs.?
11. How much oats, at £80 15*s*. for 51 quarters, may be bought for £62 14*s*.?
12. If 172 cwt. 2 qrs. 18 lbs. of potatoes cost £94 17*s*. 6*d*., how much must be given for 7 cwt. 3 qrs. 11 lbs.?

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84. The Principles of Proportion may, however, be applied to numberless cases, besides such as we have been hitherto considering; and we must here say a little more of the general nature of what are called *Proportional Quantities*.

We have already seen what is meant by saying that *four* quantities are *proportionals;* but it is common also to speak of *two* quantities being proportional to each other (or *varying as* each other); only here the quantities are used *generally*, whereas the four quantities, in the former case, were *particular* values of such general quantities.

Thus, for example, we say commonly that the *weight* of an article is proportional to, or varies as, the *price;* where the words weight and price are used *generally*, without reference to any particular weights or prices: but by saying this we *mean*, that if we took any two particular weights, and the two corresponding prices, the four would be proportionals: and thus, having given any two weights, and one of the corresponding prices, we might find, by the Rule of Three, the other price; or, having given any two prices, and one of the corresponding weights, we might find the other weight: and this we have been doing in the preceding examples.

85. But now other quantities, considered *generally*, may be similarly proportional to each other; and to these the same principles may be applied. Thus, the rent of a house will vary as the *time* it is occupied, a workman's wages will vary as the *time* he labours, the *distance* run by a coach will vary as the *rate* at which it moves, &c.; in all which cases, if we take any two particular values of the first quantity, and the two corresponding values of the latter, the *four* would be proportionals; so that, if *three* only were given, we could apply the Rule of Three, as before, to find the fourth.

86. Sometimes we can only know from philosophical reasons, that two such general quantities are proportional; as, for instance, that the length of the shadow cast by a vertical rod, at any given hour of the day, varies as the height of the rod; that the velocity acquired by a heavy body in falling varies as the time of motion from rest, &c.; but, in most cases that occur in common practice, it is easy to apply at once the

test of proportionality, as in former examples, viz. by considering whether, by taking any two particular values of one quantity, and the corresponding two of the other, the four would be proportional, i.e. whether the 1st of the former set would be the same fraction of the 2nd, as the 1st of the latter set would be of the 2nd.

This, perhaps, we may do most simply thus: consider if, by *doubling*, *trebling*, &c. any value whatever of the one quantity, the corresponding value of the other quantity would also be *doubled*, *trebled*, &c.;—in which case the above test would be satisfied with these four quantities, and therefore the two given general quantities would be proportional to each other.

87. Hence, when any question is proposed, in which, having given any *two* values of one quantity, and *one* of the corresponding values of the other, we are required to find the other of these values, we must first enquire whether the case *be* one of Proportion. If so, we may proceed to state and solve the sum by the Rule of Three. It will be best, first to set down the 3rd term, which will always be the *single* given term, and of the same kind as the answer (being the antecedent of the ratio in which the answer is the consequent); then the other two terms, (which will always be of the same kind, being two given values of the other quantity,) will form the other ratio—the antecedent, or 1st term, being that value which *corresponds* to the antecedent of the second ratio, or 3rd term.

We may now proceed as before—reducing the 1st and 2nd terms to the same den^{n}—and, if desirable, the 3rd, to any den^{n} we please—striking out common factors (if any) from the 1st and 2nd, or 1st and 3rd—multiplying together the 2nd and 3rd, and dividing by the 1st—when the quotient will give the *Answer*, in the same den^{n} as that in which we have expressed the 3rd term.

Ex. 1. *What is the coach fare for* 130 *miles, if it is* £1 9*s.* 4*d. for* 85 *miles?*

Here 5)85m. : 5)130m. :: £1 9 4

17 26 20

29

12

352

26

2112

704

17) 9152 ($538\frac{6}{17}d.$

85

65

51

142

136

6

Ans. $538\frac{6}{17}d.$ = £2 4 $10\frac{6}{17}$.

Here it is plain that, if we *double* the distance, whatever it may be, the corresponding fare will also be *doubled;* hence the fare varies as the distance, and we proceed as before, setting as the 3rd term the *single* given quantity £1 9*s.* 4*d.*, and, for the terms of the first ratio, the *pair* of given quantities of the same kind, 85m. and 130m., of which we set 85m. *first,* since it is that distance which corresponds to the 3rd term.

Ex. 2. *The rents of a parish amount to* £1750, *and a poor-rate is wanted of* £61 19*s.* 7*d.*; *what is that in the* £?

£1750 : £1 :: £61 19 7

20

1239

12

1750) 14875 (8*d.*

14000

875

Ans. $8\frac{1}{2}d.$ $\frac{875}{1750}=\frac{1}{2}$

Here it is plain that, if we *double* the rent, whatever it may be, the corresponding rate will also be *doubled:* so that the rate varies as the rent. The *single* term is here the whole rate, £61 19*s.* 7*d.*; the terms of the first ratio are the two given rents, £1750 and £1, of which £1750, since it corresponds to the 3rd term, is set first.

This sum in fact amounts merely to one in division; since if £1750 will supply a rate of £61 19*s.* 7*d.*, we may obtain that supplied by £1, by simply dividing this amount by 1750.

Ex. 56.

1. A field of 18 acres is let for £24 18*s.* 6*d.*; what would be the rent of 42 acres at the same rate?

2. If a servant's wages be £25 a year, what should he receive for 87 days' service?

3. If the coach fare for 65 miles be £1 1*s.* 8*d.*, how far ought one to go for £2 18*s.* 8*d.*?

4. If a carding-machine throw off 54 lbs. of wool in 2 hrs. 46 min. 30 sec., in what time will it throw off 24 lbs.?

5. How much land may be rented for £70 10*s.* 6*d.*, if 5 acres are rented for £4 13*s.* 4*d.*?

6. What is the assessment on 20A., if that on 445A. be £14 14*s.* 9$\frac{3}{4}$*d.*?

7. If the tax on a rent of £25 is £2 10*s.*, what will it be on a rent of £10 9*s.* 4$\frac{1}{2}$*d.*?

8. What is the amount of poor-rates to be paid upon £95 10*s.* 9$\frac{1}{2}$*d.*, when £39 11*s.* 8*d.* is levied upon £791 13*s.* 4*d.*?

9. The expenses of the poor in a parish amount to £110 7*s.* 6*d.*, and the whole rent is £2000; how much in the £ must be levied to pay it?

10. What is the tax on a house rented at £65 10*s.* 6*d.*, if that on one rented at 25 guineas be £4 11*s.* 10$\frac{1}{2}$*d.*?

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88. Sometimes we may have two general quantities so depending on each other, that, if we double any value whatever of the one, the corresponding value of the other, instead of being *doubled*, will be *halved*. Thus, if any given number of men would do a piece of work in a certain time, it is plain that *double* that number would do it in *half* the time. In this case the four quantities will still be proportional, but with the terms of the second ratio in *inverted* order; since the 1st value of the former quantity will be the same fraction of the 2nd, that the 2nd of the latter quantity is of the 1st.

The two general quantities are here said to be *inversely* proportional to each other, whereas in the former examples they were *directly* proportional: but the Rule of Three may still be applied, if we take care to *state* the sum rightly, viz. by setting last, as before, the single term, and then setting as the *second* term, or consequent of the first ratio, (instead of, as before, the *first*, or antecedent,) the corresponding value of the other two given ones.

Ex. 1. *A person completed a journey in* 32 *days, travelling* 8 *hrs. a day; how long would he take to do the same, travelling only* 6 *hrs. a day?*

6 hrs. : 8 hrs. : : 32 days
8
6) 256
Ans. 42$\frac{2}{3}$ days.

Here the term 8 hrs. corresponds to the term 32 days, and it is plain that if we *double* the n° of hrs. in each day, the n° of days required will be only *half* of what it was before; so that the n° of hrs. in a day varies *inversely* as the n° of days required. The single or 3rd term is 32 days, and here we put the *corresponding* term, 8 hrs., *second* instead of *first*, as in the former cases.

Or we might reason thus. The whole number of hrs. must be the same in both cases; and therefore $32 \times 8 = 6 \times Ans.$, whence we have the $Ans. = \frac{32 \times 8}{6} = \frac{256}{6} = 42\frac{2}{3}$ days.

Ex. 2. *If* 84 *sheep can be grazed in a field for* 12 *days, how long might* 112 *sheep have been grazed in the same field?*

sheep	sheep	days
112 :	84 : :	12

$$\begin{array}{r} 12 \\ \hline 112)\ 1008\ (9 \text{ days } Ans. \\ 1008 \\ \hline \ldots \end{array}$$

Here it is plain that, if we *double* the nº of sheep, they will be kept for only *half* the time in the same field; so that the nº of sheep varies *inversely* as the nº of days. The single term is 12 days, and we set the *corresponding* term, 84 sheep, *second.*

Or thus: 84 sheep for 12 days consume as much as 84×12 sheep in one day; and we have $84 \times 12 = 112 \times Ans.$ $\therefore Ans. = \frac{84 \times 12}{112} = 9$ days.

Ex. 57.

1. If 100 workmen can do a piece of work in 12 days, how many can do the same in 8 days?

2. If a besieged garrison have 4 months' provisions, at the rate of 18 oz. per man per day, how long would they be able to hold out, if each man were allowed only 12 oz. per day?

3. If I borrowed of a friend £300 for 8 months, for how long a time should I lend him £200 in return?

4. How many men would perform in 168 days a piece of work, which 108 men can perform in 266 days?

5. If a person, travelling 12 hrs. a day, would finish his journey in 3 weeks, how many weeks would he take to do it, if he travelled only 9 hrs. a day at the same rate?

6. If $47\frac{1}{4}$ *shilling* cakes can be made of a quarter of wheat, what will be the price of a cake, if 70 are made of the same quantity of flour?

7. How much land, at 27*s.* per acre, should be given in exchange for 480 acres, at 35*s.* per acre?

8. A besieged fortress has provisions for 3 weeks, at the rate of 14 oz. a day for each man; at what rate per day must the provision be distributed, so that the place may hold out 5 weeks?

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89. We must always be assured, as in the preceding Examples, that the two general quantities concerned in any case *are* proportional to one another, either directly or inversely, and so that the question is one which falls under the

Rules of Proportion. But when satisfied of this, we may relieve ourselves of some of the care required in *stating* the sum, by the following general Rule, which includes both cases, and is that commonly given as the

RULE OF THREE.

Set last the single term, (viz. that which corresponds to the Answer,) and the greater or less of the other two terms second, according as it is seen that the Answer will be greater or less than the third term.

The reason of this is plain; for, if the three quantities *do* form the first three terms of a proportion, the single term must be set 3rd, since it belongs to the ratio of which the Ans. is the other term; and then, as we know that the Ans. will be found by multiplying this term by one, and dividing by the other, of the two remaining terms, it is obvious that, if the Ans. is to be greater than the 3rd term, we should have to multiply by the greater and divide by the less of the two, i. e. *we should have to put the greater of them second*; if less, the *less*.

This explanation, however, is only intended to show that the above Rule will enable us to make the same statement of the sum as we should have done by the proper considerations, and so to get the correct result. It does not at all profess to give the true reason for so stating, which depends upon the foregoing observations. *See* NOTE VIII.

Ex. *If* $10\frac{5}{7}$ *lbs. of salt cost* $1\frac{9}{16}$*s., what will* $3\frac{2}{3}$ *cwt. cost?*

$$10\tfrac{5}{7}\text{ lbs.} : 3\tfrac{2}{3}\times 112\text{ lbs.} :: 1\tfrac{9}{16}s.$$

$$Ans. = \tfrac{25}{16}s. \times \tfrac{11}{3} \times 112 \times \tfrac{7}{75}$$

$$= \frac{11s. \times 7 \times 7}{3 \times 3} = £2\ 19s.\ 10\tfrac{2}{3}d.$$

Here the *single*, or 3rd term, is $1\frac{9}{16}$*s.*; and since the *Ans.* will plainly be *greater* than this, we set the *greater* of the two others in the second place, viz. $3\frac{2}{3}$ cwt. or $3\frac{2}{3}\times 112$ lbs. when reduced to the same denⁿ as the 1st term.

Ex. 58.

1. If 69 lbs. of salt cost 9*s.* $1\frac{1}{4}$*d.*, what will be the cost of 15 lbs.?
2. What is the value of sheep per score, if 311 sell for £585 1*s.* $4\frac{1}{2}$*d.*?
3. A bankrupt owes £4726 10*s.*, and his effects are worth £1181 12*s.* 6*d.*; how much will he be able to pay in the £?
4. If $27\frac{1}{2}$ bushels of potatoes cost £5 4*s.* 6*d.*, what quantity will cost £25 14*s.* 7*d.*?
5. If 39 cwt. 1 qr. 11 lbs. cost £59 6*s.* 6*d.*, what will 13 cwt. cost at the same rate?

6. What weight of sugar may be bought for £374 8*s.*, when the cost of 6 cwt. 2 qrs. is £14 14*s.* 8*d.*?

7. If the tax on £335 7*s.* 6*d.* amount to £58 13*s.* $9\frac{3}{4}$*d.*, what is that in the £?

8. How many gallons of wine, at the rate of £31 16*s.* 4*d.* for 46 gals., may be bought for £117 11*s.* 8*d.*?

9. If 17 cwt. 3 qrs. 14 lbs. of tallow cost £38 2*s.* 8*d.*, how much may be bought for £5 12*s.* 6*d.* at the same rate?

10. If the sixpenny loaf weighs 3 lbs. when wheat is at 6*s.* a bushel, what ought it to weigh when wheat is at 6*s.* 9*d.* a bushel?

11. Suppose there are 12,000,000 sheep fed in this country; what is the value of their wool-produce yearly, if 11 sheep produce 25 lbs. of wool, which is sold at £8 12*s.* per cwt.?

12. From 3 tons 5 cwt. take 1 ton 16 cwt. 3 qrs. 12 oz., and find the value of the remainder at £1 7*s.* 6*d.* for 1 qr. 27 lbs.

13. If a nobleman's rental be £8050 per annum, and the land-tax be charged at the rate of £11 5*s.* per £100, what will be his nett income?

14. If $4\frac{1}{4}$ yards of cloth cost £5 14*s.* $4\frac{1}{2}$*d.*, what would 20 yds. cost?

15. The chain for measuring land is 66 feet long, and divided into 100 links; what is the length of a wall which measures 2456 links?

16. The rateable value of a parish amounts to £1250, and a poor-rate of £27 10*s.* 6*d.* is to be raised; what will a person have to pay whose rents are £525?

17. A wedge of gold, weighing 14 lbs. 3 oz. 8 dwt., is valued at £514 4*s.*; what is the value of an oz.?

18. A bankrupt has assets to the amount of £1020, and debts to the amount of £3225; what will his creditors receive in the £?

19. A bankrupt's effects amounted to £980, which paid his creditors 13*s.* 6*d.* in the £; what did his debts amount to?

20. What is the income corresponding to an income-tax of £13 2*s.* 6*d.*, at the rate of 7 pence in the £?

21. *A* borrowed of *B* £175 5*s.* for 102 days, and afterwards would return the favour by lending *B* the sum of £210 6*s.*; for how long should he lend it?

22. What is the height of a steeple, whose shadow was 148 ft. 4 in., at the same time that the shadow of a staff 6 ft. 4 in. long was 5 ft. 3 in.?

23. A coach goes from London to Liverpool, at the rate of 9 miles an hour, in 24 hours; in what time would the distance be performed on the railroad, at the rate of 32 miles an hour?

24. A besieged town, containing 22400 inhabitants, has provisions to last 3 weeks; how many must be sent away that they may be able to hold out 7 weeks?

25. If a servant receive £$3\frac{1}{2}$ for 20 weeks' service, how many weeks ought he to remain in his place for 12 guineas?

26. If the carriage of $15\frac{1}{2}$ cwt. for 60 miles came to 7*s.* 9*d.*, how far ought $3\frac{1}{4}$ cwt. to be carried for the same money?

27. How much may a person spend in 73 days, if he wishes to lay by every year 50 guineas out of an income of £450?

28. The carriage of a parcel of goods, weighing 1 ton 3 cwt. 2 qrs., cost £2 14*s.*; what will be the charge for 4 other parcels, weighing each 17 cwt. 3 qrs. 7 lbs.?

29. If $3\frac{3}{4}$ shares in a speculation are worth £27 10*s.*, what are $4\frac{5}{8}$ shares worth?

30. If $1\frac{2}{3}$ yard of cotton print cost 2*s.* 6*d.*, what is the cost of $24\frac{1}{2}$ yards?

31. If $1\frac{3}{8}$ cwt. of sugar cost $3\frac{1}{2}$ guineas, what must be given for $17\frac{3}{4}$ lbs.?

32. At 3*s.* $4\frac{1}{2}$*d.* for $4\frac{3}{5}$ lbs., what is the price of $14\frac{3}{8}$ lbs.?

33. If $2\frac{1}{4}$ yards of cotton print cost 1*s.* $10\frac{1}{2}$*d.*, what is the cost of $13\frac{5}{8}$ yards?

34. If $6\frac{2}{3}$ yards be worth 27*s.* $9\frac{1}{2}$*d.*, what quantity is worth 18*s.* $2\frac{55}{64}$*d.*?

35. What is the value of $\frac{3}{7}$ of $\frac{3}{4}$ of a ship, when $\frac{5}{8}$ of the whole is worth £525?

36. If 6336 stones of $3\frac{1}{4}$ ft. length complete a certain quantity of wall, how many similar stones of $2\frac{2}{3}$ ft. length will raise a like quantity?

37. If a ball falling from rest acquire a velocity of $115\frac{1}{5}$ ft. in $3\frac{3}{5}$ seconds, at what rate will it be moving at the end of the first second, and at the end of $4\frac{3}{4}$ seconds?

38. What will 3 cwt. 1 lb. $1\frac{1}{2}$ oz. of merchandise cost, if the cost of $13\frac{2}{5}$ tons be 500 guineas?

39. If $4\frac{5}{9}$ oz. Av. cost $8\frac{31}{32}$*s.*, what will $8\frac{13}{24}$ lbs. cost?

40. If $\frac{1}{192}$ of $\frac{2}{3}$ of $2\frac{1}{2}$ of 40 lbs. of beef cost $1\frac{3}{50}$*d.*, how many lbs. may be bought at the same rate for 6*s.* $7\frac{1}{2}$*d.*?

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90. Suppose it were asked, 'If 9 men can reap 30 acres of wheat in 10 days of 6 hours each, how many men would reap 40 acres *in the same time?*' This would be an instance of common Direct Proportion, and we should have

$$30\text{A.} : 40\text{A.} :: 9 \text{ men} : \frac{40}{30} \times 9 = 12 \text{ men}.$$

But now suppose that, instead of '*in the same time,*' the question had said, 'in 12 days *of the same length.*' Here it is plain that, after finding, as above, the n° of men, 12, who would reap 40A. in 10 days, we must still have another Proportion, to find the n° who will reap the *same* n° of acres in 12 days; thus (the case being here one of Inverse Proportion),

$$12 \text{ days} : 10 \text{ days} :: 12 \text{ men} : \frac{10}{12} \times 12 \text{ men} = 10 \text{ men}.$$

Once more, suppose that, instead of '12 days *of the same length*,' the question had said, '12 days of $7\frac{1}{2}$ hrs. each.' Here, after having found, as above, the n° of men, 10, who will reap the 40A. in 12 days of 6 hrs. each, we must still have a third Proportion, to find the n° who will reap the *same* n° of acres in the *same* n° of days of $7\frac{1}{2}$ hrs. each; thus (the case being here also one of Inverse Proportion),

$$7\tfrac{1}{2} \text{ hrs.} : 6 \text{ hrs.} :: 10 \text{ men} : \frac{6}{7\frac{1}{2}} \times 10 \text{ men} = 8 \text{ men}.$$

91. Now the above is an instance of *Compound* Proportion, whereas the preceding Examples were all instances of *Simple* Proportion; the difference between questions in Simple and Compound Proportion being, that, in the former, we have one general quantity proportional to another; whereas, in the latter, we have one general quantity proportional to *each* of several others, taken *separately*, i. e. supposing that, while we take the two different values of any one of them, the others meanwhile retain the *same* fixed values.

Thus, in the above Proportions, the n° of men is proportional,

in the 1st, to the n° of *acres* (directly) when the n° of *days* continues the *same*, and the n° of *hours* in each day the *same* —

in the 2nd, to the n° of *days* (inversely) when the n° of *acres* continues the *same*, and the n° of *hours* the *same* —

in the 3rd, to the n° of *hours* (inversely) when the n° of *acres* continues the *same*, and the n° of *days* the same.

92. We have seen that, in cases of Simple Proportion, when a single value of one general quantity is given corresponding to one given value of the other, we may find that

corresponding to another given value of the other by the Rule of Three. In like manner, in cases of Compound Proportion, when a single value of the first quantity is given, corresponding to one given set of values of the other quantities, we may find that corresponding to another given set of them, either, as above, by *successive Proportions*, or by what is called the *Double* Rule of Three, which arises from the following consideration. Taking the numerical value of the 1st result in its original form, $\frac{40}{30} \times 9$, we have that of the 2nd, $\frac{10 \times 40}{12 \times 30} \times 9$, and of the 3rd, $\frac{6 \times 10 \times 40}{7\frac{1}{2} \times 12 \times 30} \times 9$, which would, of course, reduce itself to the final answer, 8, i. e. 8 *men:* but now this is the same result as we should get, if we made only one statement, in which we set down the *single* term, 9 men, as usual, *last*, and, for the 1st and 2nd terms, the *products*, respectively, of the numerical values of the 1st and 2nd terms of the three Proportions.

The same will be true in other cases. It is best to set down, one under another, the num. values of the *first ratios* of these Proportions, observing to state them by considering each general quantity *separately*, with reference to that quantity whose single value is in the 3rd term; and then we may multiply these together, (striking out, as before, common factors from the 1st and 2nd, or 1st and 3rd,) and, finally, multiply together the 2nd and 3rd terms of the resulting compound statement, and divide by the first.

Ex. If 5 compositors set up a work of 6 sheets in 8 days, in what time will 6 compositors set up a work of 9 sheets?

Here 8 days is the *single* term, to be set last: now, if we *doubled* the n° of *men* (supposing the *same* n° of sheets), the n° of *days* would be *halved*; hence the n° of days varies inversely as the n° of men, and the corresponding *first ratio* will be 6 men : 5 men. Again, if we *doubled* the n° of *sheets* (supposing the *same* n° of men), the n° of *days* would be *doubled*; hence the n° of days varies directly as the n° of sheets, and the corresponding *first ratio* will be 6 sheets : 9 sheets; we have, therefore, setting down the numerical values of these ratios),

$$\left.\begin{matrix} 6 : 5 \\ 6 : 9 \end{matrix}\right\} :: 8 \text{ da.} : \frac{8 \times 5 \times 9}{6 \times 6} \text{ da.}$$

and now striking out 4 from the dividend, and 2 × 2 from the divisor, we have

$$\frac{\overset{2}{\not{8}} \times 5 \times 9}{\underset{3}{\not{6}} \times \underset{3}{\not{6}}} = 2 \times 5 = 10 \text{ days. } \textit{Ans.}$$

Ex. 59.

1. If 15 pecks of wheat serve 9 persons for 22 days, how long will 20 pecks serve 6 persons?

2. If £33 5*s.* pay 15 labourers for 18 days, how many labourers will £79 16*s.* pay for 24 days?

3. If 27 men can dig $2\frac{1}{4}$ acres in 2 days, how many men can dig 2 acres in 3 days?

4. If 7 horses be kept 20 days for £12, how many may be kept 14 days for £18?

5. If 9 persons spend £147 in 6 months, how many will £130 13*s.* 4*d.* last for 4 months?

6. If 6 horses consume 375 lbs. of oats in 8 days, what quantity will 4 horses consume in 10 days?

7. How much paper is required for 5000 copies of a book of $12\frac{1}{2}$ sheets, if 66 reams are required for 3000 copies of a book of 11 sheets?

8. If 8 men earn £9 wages for 5 days' work, how much would 36 men earn for 24 days' work at the same rate?

9. If £100 will pay the expenses of 5 persons for 22 wks. 6 da., how long would 12 persons be supported by £150 under similar circumstances?

10. If 7 men earn £9 10*s.* 6*d.* in $10\frac{1}{2}$ days, what sum will 28 men earn in $31\frac{1}{2}$ days?

11. If the wages of 25 men amount to £115 in 16 days, how many men must work 24 days to receive £155 5*s.*, the daily wages of the latter being one-half those of the former?

12. If 21 men mow 72 acres of grass in 5 days, how many must be employed to mow 460A. 3R. 8P. in 6 days?

13. If 9 persons spend £120 in 8 months, how much will serve 26 persons for 12 months?

14. If 12 horses in $4\frac{1}{2}$ days plough $10\frac{1}{2}$ acres, how many horses would plough 35 acres in 20 days?

15. If a 3 lb. loaf costs 7*d.* when wheat is at 52*s.* 6*d.* per quarter, what should be the price of wheat when a 2 lb. loaf costs $5\frac{1}{2}$*d.*?

16. If a man travels 65 miles in 3 days, by walking $7\frac{1}{2}$ hours a day, in how many days will he travel 156 miles by walking 8 hours a day?

17. What will be the wages of 15 men for 10 months, when 9 men receive £261 15*s.* for 8 months?

18. If 3 persons are boarded 5 weeks for £17 10*s.*, how long should 14 persons be boarded for 60 guineas?

19. How far should 80 cwt. be carried for £29, if 30 cwt. be carried 17 miles for £5 8*s.* 9*d.*?

20. If 6 men can reap 34 acres of corn in 5 days, how many men will be required to reap 95A. 32P. in $10\frac{1}{2}$ days?

21. If 40 bushels of corn serve 12 horses 37 days, how many days would 195 bushels serve 9 horses?

22. A person completes a journey of 160 miles in 3 days, travelling 11 hours a day; in how many days would he complete 1000 miles, going 15 hours a day at the same rate?

23. If 3 men can reap 7 acres of wheat in 2 days, how long will it take 8 men to reap 20 acres at the same rate?

24. If a ton of turnips will last 25 sheep for a fortnight, how much will be required to supply 40 sheep during the months of January and February in Leap-year?

25. If 6 men can dig a trench, 220 yards long, in $2\frac{1}{2}$ days, by working 8 hours a day, how many will dig a trench, 187 yards long, in $4\frac{1}{4}$ days, working 6 hours a day?

26. If 12 men build 24 rods of wall in 30 days, working 8 hours a day, how many hours a day must 18 men work to build 64 rods in 40 days?

27. If 8 men can plough 84 acres in 12 days of $8\frac{1}{4}$ hours each, how many acres can be ploughed by 20 men in 11 days of $7\frac{4}{5}$ hours each?

28. If 8 men can dig a trench 100 ft. long, 3 ft. broad, and 4 ft. 6 in. deep in 9 hours, how many will be required to dig a trench 80 ft. long, 5 ft. broad, and 2 ft. deep in $5\frac{1}{3}$ hours?

29. If 7 men can erect a certain piece of wall in $20\frac{5}{8}$ days of $9\frac{3}{5}$ hours each, how long would it take 3 men to do $2\frac{3}{4}$ of the same work, reckoning $10\frac{1}{2}$ hours to the day?

30. If 20 men can excavate 185 cubic yards of earth in 9 hours, how many men could do half the work in a fifth of the time?

CHAPTER VII.

MISCELLANEOUS RULES.

93. INTEREST is the consideration paid for the use of money. The *Rate* of Interest is the sum paid for the use of a *certain sum*, generally £100, for a *certain time*, generally one year: thus, if £5 is paid for the use of £100 for one year, the interest is said to be at the rate of 5 *per cent.*

The sum originally lent is called the *Principal;* and the principal, together with its interest for any time, is called the *Amount* for that time.

When interest is only taken for the original principal, it is called *Simple* Interest; but, when at the end of any stated period, as a year, the interest accruing is added to the previous principal, and interest reckoned upon this sum, taken as the principal, for the next year, it is called *Compound* Interest.

94. *To find the Simple Interest on a given sum for a given time at a given rate per cent. per annum.*

RULE. Multiply the principal by the number of years, and by the rate of interest per cent., and divide the result by 100; the quotient will be the interest required.

Ex. 1. *Find the Simple Interest on* £725 *for* 3 *years at* 5 *per cent. per annum.*

£725	
3	
2175	
5	
108.75	
20	
Ans. £108 15*s.*	15.00

For the Int. will be the same, whether we suppose the Principal, £725, repeated *three times* in three successive years, or *three* times in one and the same year; that is, the Int. on £725 for *three* years is the same as the Int. on £2175 for *one* year: and this we find, according to the above definition of Int., by dividing by 100, to see how many *Cents* there are in the sum, and then *taking* 5 *for each*, i. e. multiplying by 5; or, which is the same thing, but more convenient in practice, we first multiply by 5, and then divide by 100.

Ex. 2. *Find the Simple Interest on £212 10s. 4d. for $2\frac{3}{4}$ yrs. at $2\frac{1}{2}$ per cent. per ann.*

	£	s.	d.
	£212	10	4
			$2\frac{3}{4}$
	425	0	8
for $\frac{1}{2}$	106	5	2
for $\frac{1}{4}$	53	2	7
	584	8	5
			$2\frac{1}{2}$
	1168	16	10
for $\frac{1}{2}$	292	4	$2\frac{1}{2}$
	14.61	1	$0\frac{1}{2}$
	20		
	12.21		
	12		
	$2.52\frac{1}{2}$		

Here the rem^r, after dividing by 100, is

$$\frac{52\frac{1}{2}}{100}d. = \frac{105}{200}d. = \frac{21}{40}d.;$$

and, the Int. being £14 12s. $2\frac{21}{40}$d., we have the whole amount £227 2s. $6\frac{21}{40}$d. But it is generally best to represent the whole procedure first symbolically, in order to ascertain whether the calculation may be simplified; thus we have

$$\frac{\text{£}212\ 10s.\ 4d. \times 2\frac{3}{4} \times 2\frac{1}{2}}{100} = \frac{\text{£}212\ 10s.\ 4d. \times 11}{160}$$

so that $\frac{11}{160}$ of the given principal will be the interest.

Ex. **60.** *Find at Simple Interest,*

1. Interest on £500 for 5 yrs. at 5 per cent.
2. Interest on £375 for 3 yrs. at 4 per cent.
3. Amount of £1125 for 4 yrs. at 3 per cent.
4. Amount of £2275 for $3\frac{1}{2}$ yrs. at 5 per cent.
5. Interest on £347 16s. 8d. for 15 yrs. at $4\frac{3}{4}$ per cent.
6. Amount of £2000 for $12\frac{1}{4}$ yrs. at $3\frac{1}{2}$ per cent.
7. Amount of £575 for $8\frac{3}{4}$ yrs. at $3\frac{3}{8}$ per cent.
8. Interest on £325 10s. for 4 yrs. at $5\frac{1}{2}$ per cent.
9. Interest on £500 13s. 4d. for $2\frac{3}{4}$ yrs. at $2\frac{3}{4}$ per cent.
10. Interest on £150 for $3\frac{5}{12}$ yrs. at 4 per cent.

If parts of a year be given, they may be expressed as a fraction of a year.

Thus the Int. for 2 yrs. 3 mo., at any given rate, would be the same as that for $2\frac{1}{4}$ yrs. at the same rate.

But, in practice, more accuracy is generally required; and we must express the given parts of a year in days, and then, finding first the Int. for *one* year, we may find by a proportion the Int. for the given portion of a year.

Ex. 3. *Find the Int. on £325 from March 1, 1871, to May 31, 1874, at 4 per cent. per ann.*

When interest is thus required from one date to another, the day of the first date is to be left out, because it is not until the day following that one day's interest will have accrued. Accordingly, we have here the whole time = 3 yrs. 91 da.

Now, the int. for 1 year is (£325 × 4) ÷ 100 = £13; and for 91 days we have by Proportion—

365 da. : 91 da. :: £13 :	£3	4*s.*	$9\frac{63}{73}$*d.*
Int. for 3 yrs. = £13 × 3 =	39	0	0
Ans. The whole int. is	£42	4	$9\frac{63}{73}$

If the rate of Interest be given in *parts* of a £, they may be expressed as a *fraction* of a £, and the sum treated as before; or we may work for them by the method of Practice.

Ex. 4. *Find the Int. on £500 for 4 yrs., at £5 7s. 6d. per cent.*

$$\frac{£500 \times 4 \times 5\frac{3}{8}}{100} = 5 \times 21\tfrac{1}{2} = £107\ 10s.\ \textit{Ans.}$$

Ex. 5. *Find the Int. of £307 15s. 6d. for 156 days, at £4 14s. 6d. per cent.*

Here it will be best to work throughout by decimals, and to extend them only to so many places as will insure the accuracy of the final result to two or three decimals of a penny. Also we may employ the method of Practice, not only for the rate, but also for the days, 156 da. being = 146 + 10 da. = $\frac{2}{5}$ yr. + 10 da.

	£3.07775	= Principal ÷ 100.
	4	
	12.31100	
10*s.* = $\frac{1}{2}$	1.538875	
4*s.* = $\frac{1}{5}$	.61555	
6*d.* = $\frac{1}{8}$	.076944	
	14.542369	= Int. for 1 yr.
	10	
	365)145.423690	
	.398421	= Int. for 10 da.
73 da. = $\frac{1}{5}$	2.908474	= „ 73 da.
73 da. = $\frac{1}{5}$	2.908474	= „ 73 da.
	£6.215369	= „ 156 da.
	20	
	4.307380	
	12	
Ans. £6 4*s.*	3.688*d.*	

Ex. **61.** *Simple Interest.*

1. Find the amt. on £500 from March 1 to Jan. 10, at $4\frac{5}{8}$ per cent.
2. Find the amt. on £7500 from May 5 to Oct. 27, at $3\frac{1}{8}$ per cent.
3. Find the amt. on £1158 17*s.* 6*d.* for 1 yr. 115 d., at £2 10*s.* per cent.
4. Find the int. on £250 12*s.* 6*d.* from March 26, 1870, to Oct. 31, 1872, at 3 per cent.
5. Find the int. on £3996 15*s.* for 4 yrs. 225 d. at £2 13*s.* 4*d.* per cent.
6. Find the int. on £2755 15*s.* for 3 yrs. 110 d. at £3 2*s.* 6*d.* per cent.

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95. *To find the Compound Interest on a given sum, for a given time, at a given rate per cent. per ann.*

Rule. At the end of each year add the Interest for that year to the Principal at the beginning of it, and this will be the Principal for the *next* year; and so on, till we have found the *final* Principal, or whole *Amount.* *See* Note IX.

Ex. *Find the Compound Interest on £750 for 3 yrs., at 4 per cent. per ann.; and also at $2\frac{1}{2}$ per cent. per ann.*

	£750	First Principal.
$\frac{4}{100}=$	30.00	Int. in 1st year.
	780.00	Second Principal.
$\frac{4}{100}=$	31.20	Int. in 2nd year.
	811.20	Third Principal.
$\frac{4}{100}=$	32.448	Int. in 3rd year.
	£843.648 − 750 = £93 12*s.* $11\frac{13}{25}$*d.* 1*st Ans.*	

	£750	1st Principal.
$\frac{2\frac{1}{2}}{100}=\frac{1}{40}=$	18.75	Int. in 1st year.
	768.75	2nd Principal.
$\frac{1}{40}=$	19.21875	Int. in 2nd year.
	787.96875	3rd Principal.
$\frac{1}{40}=$	19.69921875	Int. in 3rd year.
	807.66796875 − 750 = £57 13*s.* $4\frac{5}{16}$*d.* 2*nd Ans.*	

Ex. **62.**

1. Find the amt. of £95 16*s.* 8*d.*, for 2 yrs., at $2\frac{1}{2}$ per cent. at comp. int.
2. Find the amt. of £50, for 3 yrs., at 5 per cent. at comp. int.
3. Find the difference between the simple and compound interest on £41 13*s.* 4*d.*, for 2 years, at 5 per cent.
4. Find the difference between the simple and compound interest on £365 4*s.* $8\frac{1}{4}$*d.*, for 3 years, at 4 per cent.
5. Find the comp. int. on £225, for 3 years, at $3\frac{3}{4}$ per cent.
6. Find the comp. int. on £300, for 3 years, at $2\frac{2}{3}$ per cent.

96. There are *four* things to be considered in all questions of Interest—the *Principal*, the *Rate of Interest*, the *Time*, and the *Total Interest*, (the *Amount* being only the sum of the first and last of these); and, if any *three* of these be given, we are able to obtain the fourth. Hitherto we have only considered the case which most commonly occurs in practice, viz. that in which the Principal, Rate, and Time are given to find the Interest, (or the Amount); we shall now give an Example of each of the other three cases which may arise in *Simple* Interest—those in Compound Interest being more difficult, and of less frequent occurrence.

I. *When the Principal, Interest (or Amount), and Rate are given to find the Time.*

Ex. In what time will £91 13*s.* 4*d.* amount to £105 6*s.* $0\frac{1}{2}$*d.*, at $4\frac{1}{4}$ per cent. per ann.?

Subtracting the principal from the amount, we have here given the *interest* = £13 12*s.* $8\frac{1}{2}$*d.*; now in *one* year £91 13*s.* 4*d.* produces, at the given rate, $\frac{£91\frac{2}{3} \times 4\frac{1}{4}}{100} = \frac{£11 \times 17}{48}$; we have, therefore,

$$\frac{£11 \times 17}{48} : £13\ 12s.\ 8\tfrac{1}{2}d. :: 1 \text{ year},$$

$$\text{or, } £11 \times 17 : £13\ 12s.\ 8\tfrac{1}{2}d. \times 48 :: 1 \text{ year}.$$

$$\begin{array}{r} 12 \\ \hline 163\ 12\ \ 6 \\ 4 \\ \hline 11)\underline{654\frac{1}{2}} \\ 17)\underline{\ 59\frac{1}{2}} \\ 3\frac{1}{2} \text{ years. } Ans. \end{array}$$

II. *When the Rate, Time, and Interest (or Amount) are given to find the Principal.*

Ex. What sum of money, put out to interest for 4 yrs. at $3\frac{1}{2}$ per cent., will amount to £259 7*s.*?

At the given rate for the given time the interest of £100 would be £$3\frac{1}{2} \times 4$ = £14, and therefore its *amount* £114; we have, therefore,

£114 : £259 7*s.* :: £100 : the *Ans.*,

which we obtain in the usual manner, = £227 10*s.*

III. *When the Principal, Time, and Interest (or Amount) are given to find the Rate.*

Ex. 1. At what rate per cent. will £142 10*s.* amount to £163 13*s.* 11$\frac{1}{4}$*d.* in 4$\frac{1}{4}$ years?

The interest of £142 10*s.* is £21 3*s.* 11$\frac{1}{4}$*d.* in 4$\frac{1}{4}$ years,
∴ for 1 year it is 20349*d.* ÷ 17 = 1197*d.*;
and £142 10*s.* being = 34200*d.*, we have

34200*d.* : £100 :: 1197*d.*
or, 38*d.* : £1 :: 133*d.* : £3$\frac{1}{2}$.
Ans. 3$\frac{1}{2}$ per cent. per ann.

Ex. 2. At what rate per cent. per annum will £5 amount to 5 guineas in 219 days?

In 219 da., or $\frac{3}{5}$ of a year, the int. of £5 is 5*s.*

∴ in 1 year it is 5*s.* ÷ $\frac{3}{5}$ = 8$\frac{1}{3}$*s.*
£5 : £100 :: 8$\frac{1}{3}$*s.* : £8$\frac{1}{3}$.
Ans. 8$\frac{1}{3}$ per cent. per ann.

Ex. **63.** *Simple Interest.*

1. At what rate will the int. on £102 10*s.* amount to £12 13*s.* 8$\frac{1}{4}$*d.* in 2$\frac{1}{4}$ years?
2. What sum will amount to £45 0*s.* 9$\frac{3}{4}$*d.* in 1 year, at 6$\frac{1}{2}$ per cent.?
3. In what time will the int. on £498 16*s.* 8*d.* amount to £10 9*s.* 3$\frac{1}{4}$*d.*, at 6$\frac{1}{8}$ per cent.?
4. At what rate per cent. will the int. on £200, for 146 days, amount to £4 16*s.*?
5. In what time will £732 11*s.* 10*d.* amount to £1709 7*s.* 7$\frac{1}{8}$*d.*, at 5$\frac{1}{3}$ per cent.?
6. What sum must be put out to interest at 4$\frac{2}{3}$ per cent., to become £49 0*s.* 5$\frac{1}{4}$*d.* in 5$\frac{1}{4}$ years?
7. At what rate will the int. on £4127 10*s.* amount to £92 17*s.* 4$\frac{1}{2}$*d.* in a year?
8. What principal will produce £121 15*s.* 5*d.* in 2 yrs. 1 mo., interest at 5$\frac{3}{5}$ per cent.?
9. In what time will £419 amount to £486 4*s.* 3$\frac{1}{2}$*d.*, at 4$\frac{3}{8}$ per cent.?
10. At what rate will £220 12*s.* 6*d.* become £240 4*s.* 8$\frac{2}{3}$*d.* in 3$\frac{1}{3}$ yrs.?
11. What principal in 3 years 73 days will become £10 1*s.* 10$\frac{1}{2}$*d.*, interest at 6$\frac{1}{4}$ per cent.?
12. In what time will the interest on £812 10*s.* 10*d.* amount to £771 18*s.* 3$\frac{1}{2}$*d.*, at 4$\frac{3}{4}$ per cent.?

97. Discount is the sum allowed for the payment of money before it is due.

Thus, if *A* has to pay to *B* £525 at the *end of a year*, and the rate of interest is 5 per cent., he might arrange to discharge his debt by paying him *now* £500, because this sum put out to interest would amount to £525 at the year's end. In this case, therefore, £25 would be the *discount* which *B* would allow to *A*, for paying him the debt at the present time.

The *present value* of a sum, due at some future time, is, therefore, the sum left, when the discount for that time is deducted, (as £500 in the above instance); and may be defined to be that sum which, put out at interest for the time in question, would amount to the sum due at the end of the time; and the *discount* is the difference between the whole sum and its present value, or the interest upon the present value.

98. The most common form in which Discount occurs is in the prepayment of *Bills* or *Notes of Hand*, which are both documents (but differing somewhat in form and character) by which a person engages himself to pay a certain sum, at a certain future time, both named therein. If the *credit* of the party promising payment, or of the party holding the bill, be considered satisfactory, a banker will *discount* it, that is, will pay its *present value* at once, deducting from the whole amount the discount upon it for the time that must elapse before it will become due.

99. In practice, however, it is usual to charge as discount the interest on the *future debt itself;* by which means the present value obtained is evidently *less* than it should equitably be.

Thus, if a banker discounted at 5 per cent. a bill for £525, due at a year's end, he would not calculate what sum (viz. £500) at interest would produce £525 at the year's end, and so deduct the interest (viz. £25) for *this* sum as discount; but he would calculate the interest on the debt, £525, itself (viz. £26 5*s*.), and, deducting *this*, would pay only £498 15*s*. to the holder of the bill as its present value. By this means, since £498 15*s*., with its own interest, would not amount to £525 in a year, the holder is a loser, and the banker gains, as we have seen, the difference of £500 and £498 15*s*., viz. £1 5*s*., by the transaction—being, in fact, *the interest upon the true discount.*

In practice, therefore, questions in discount are reduced merely to questions in Simple Interest; but we shall, here and throughout, give examples in the more correct rule, unless the contrary be expressed.

N.B. In Great Britain and Ireland 3 days, called *Days of Grace*, are always allowed, after the time that a bill is *nominally* due, before it is *legally* due. Thus, if a bill of £250 were drawn on July 10, at 3 months, it would be nominally due on Oct. 10, but legally on Oct. 13; and, if a banker were to discount it on Aug. 20, he would reckon forward to Oct. 13, (the last of these days inclusive,) and, finding the interval to be 54 days, he would reckon the interest on £250 for that time, and, deducting it as discount, would pay the difference as the present value of the bill.

It may be noticed, also, that, if a bill would fall nominally due on the 29th, 30th, or 31st of February, or on the 31st of any month which has only 30 days, it is considered to be nominally due on the *last* day of the month, and therefore *legally* on the 3rd of the following month: and, if any fall *legally* due on *Sunday*, they are paid in Great Britain on the *Saturday*, but in Ireland on the *Monday*.

Ex. 1. What is the discount on £396 17*s*. $5\frac{1}{4}$*d*., due at 9 months, at 4 per cent.?

This example falls under (96), Case II., in Simple Interest; since, therefore, £100 produces in 9 months, at 4 per cent., £3, we have £100, the present value of £103, due at the end of 9 months; and thus we get the proportion,

£103 : £396 17*s*. $5\frac{1}{4}$*d*. :: £100,

which, being solved as usual, gives us the present value £385 6*s*. 3*d*., and therefore the discount, £11 11*s*. $2\frac{1}{4}$*d*.

Ex. 2. What would a banker gain by discounting on Sept. 21 a bill of £318 3*s*., dated July 31, at 4 months, at 5 per cent.?

This bill will be nominally due on Nov. 30, and legally on Dec. 3; and, reckoning from Sept. 21 to Dec. 3, (the last inclusive), we have 73 days. We shall find the interest on £318 3*s*. for 73 days, in the usual manner, to be £3 3*s*. $7\frac{14}{25}$*d*.; and the *present value* of it, i.e. that principal, which at 5 per cent. would become £318 3*s*. in 73 days, we shall find, as in Ex. 1, to be £315, and therefore we have the discount = £3 3*s*.; so that the banker gains upon the whole $7\frac{14}{25}$*d*.

Ex. 64.

1. Find the present value of £284, due at the end of 2 years, at $3\frac{1}{4}$ per cent. per annum.

2. What is the present value of £850, due at the end of 3 years, at $3\frac{1}{2}$ per cent.?

3. Find the discount on £1336 11*s.* 3*d.*, due at the end of $3\frac{1}{2}$ years, at 5 per cent.

4. Required the present value of £151 17*s.* 6*d.*, due at the end of 4 years, at $5\frac{3}{8}$ per cent.

5. What is the discount on £88 2*s.* 5*d.*, due at the end of 5 months, at $4\frac{1}{2}$ per cent.?

6. Find the discount on £210 12*s.* 1*d.*, due at the end of $3\frac{1}{2}$ years, at $4\frac{1}{4}$ per cent.

7. Find the present value of £598 9*s.* 9*d.*, due at the end of 1 year 115 days, at $2\frac{1}{2}$ per cent.

Find the true discount upon the following bills—

	£	*s.*	*d.*	Drawn.	Discounted.
8.	419	12	1	March 6, at 7 months	Sept. 15, at 5 per cent.
9.	457	18	0	Sept. 12, at 5 months	Jan. 13, at 4 per cent.
10.	537	5	2	Feb. 29, at 3 months	April 27, at $3\frac{3}{4}$ per cent.
11.	755	5	9	March 17, at 3 months	May 31, at 6 per cent.
12.	1006	15	6	Aug. 5, at 5 months	Dec. 6, at $3\frac{1}{3}$ per cent.
13.	1337	14	6	May 31, at 4 months	Sept. 3, at 5 per cent.
14.	1846	5	2	Dec. 25, at 2 months	Feb. 8, at 6 per cent.

See NOTE X.

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100. There are other cases of common occurrence in which a *rate per cent.* is charged.

Insurance is a per centage paid for securing property from fire, &c. The charge is regulated by the nature of the property insured, and the hazard to which it is exposed, as laid down in the Tables of the different Insurance Companies. The whole annual payment is called the *Premium*, and the legal document by which the Insurer is secured from loss is called the *Policy of Insurance.*

Life Insurance is a per centage paid for securing the payment of a sum of money upon the death of a person. The charge is regulated by the age and healthiness of the person whose life is assured, at the time the Policy was first taken out, as laid down in the Tables ; and, being thus settled, it is reckoned per cent. upon the whole sum secured—the whole annual payment being called, as before, the *Premium* upon the *Policy of Assurance.*

In each of the above cases the Premium, like Interest, must be *renewed* every year, while the Policy is in force ; but the following charges are, from their nature, paid only once.

Insurance from sea risk is a per centage charged upon the value of a cargo, just as in Fire Insurance.

Commission is a per centage paid to an agent for buying or selling goods.

Brokerage is a smaller per centage of the same nature, paid usually for transacting money concerns.

101. It is usual with tradesmen to allow (what is called) a *discount* of 5 per cent. for *ready-money* payments upon goods purchased, or, (since 5 per cent. is the same as 1 in 20), to allow *a shilling in the pound* upon the account to be paid : thus, for ready-money payment of an account of £7 13*s.* 6*d.*, most tradesmen would allow 7*s.* 6*d.*, (7*s.* for the £7, and 6*d.* for the 10*s.*,) and would be content therefore to receive as full payment £7 6*s.* This, however, differs from the *discount* of which we have before been speaking, since it takes no account of the *time*, at which the debt would otherwise be paid ; but is merely an arrangement to secure to the seller the convenience of a ready-money payment, by giving to the buyer a corresponding advantage.

Ex. 1. What is the sum to be paid for insuring a vessel and cargo, worth £2225, at $3\frac{1}{4}$ per cent.?

£2225
$3\frac{1}{4}$
6675
for $\frac{1}{4}$ | 556 5
72.31 5
20
6.25
12
Ans. £72 6*s.* 3*d.* 3.00

Ex. 2. What is the premium upon a policy of £375 upon a life of 28, the rate being £2 8*s.* 7*d.* per cent. for that age?

Here £375 = $3\frac{3}{4}$ of £100; and the premium is $3\frac{3}{4}$ of

£2 8 7
3
4)7 5 9
1 16 $5\frac{1}{4}$
£9 2 $2\frac{1}{4}$ *Ans.*

Ex. 3. What sum should be insured at 4 per cent., on goods worth £735, that the owner may receive, in case of loss, the value both of goods and premium?

Here, if £100 were insured, it would cover goods to the amount of £96, together with the premium £4; hence we have the proportion

$$£96 : £735 :: £100,$$

whence we get, as usual, the *Ans.*=£765 12*s.* 6*d.*

Ex. 65.

1. What would be the ready-money payment of an amount of £27 13*s.* 6*d.*, discount being allowed at 5 per cent.?

2. What would be the expense of insuring a vessel and cargo, whose value is £2516 10*s.*, at $3\frac{1}{8}$ per cent.?

3. What is the premium on a policy of assurance for £2286 13*s.* 4*d.*, upon the life of a person aged 42, at the rate of £3 10*s.* per cent. for that age?

4. At $4\frac{3}{8}$ per cent., for what sum should goods be insured, which are worth £427 15*s.* 3*d.*, in order that, in case of loss, the owner may recover their value, together with the premium paid?

5. What would be the cash payment of an account of £27 17*s.* 5*d.*, at 5 per cent.?

6. What is the brokerage upon a money transaction of £273 15*s.*, at 3*s.* 4*d.* per cent.?

7. For what sum should a cargo, worth £5263, be insured, at $7\frac{2}{3}$ per cent., so that the owner may recover, in case of loss, the value both of cargo and premium?

8. What is the commission upon £713 6*s.* 8*d.*, at $2\frac{3}{4}$ per cent.?

9. What is the premium of insurance upon £3208 17*s.* 1*d.*, at £2 12*s.* per cent.?

10. What is the premium on a policy of insurance for £1237 10*s.*, upon a life of 21 years, at the rate of £2 2*s.* 4*d.* per cent. for that age?

11. What is the brokerage on £768 2*s.* 6*d.*, at 3*s.* 4*d.* per cent.?

12. For what sum should goods, worth £4384 0*s.* 3*d.*, be insured at £2 6*s.* 8*d.* per cent., that the owner may recover, in case of loss, the value of both goods and premium?

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102. Stock is the name given to Money, lent to some Trading Company, or, more commonly, to our own or some foreign Government, at some given rate of Interest, which is settled at the time the Money is first lent, according to the circumstances then existing.

Thus, if Government were to borrow to the amount of £500,000 at 4 per cent., and *A* had lent £100 of this sum, *A* would be said to have £100, 4 *per cent. stock*, and would receive a document entitling him to receive the Interest (viz. £4) upon this stock from year to year, until Government chose to repay the Principal, and put an end to the debt.

The source from which the Interest is paid is called the 'Public Funds,' being, however, only an imaginary Property, representing the credit of the Country itself, which is pledged to the payment of the debts contracted by its Government; the Interest is paid half-yearly, and the document, entitling the possessor to receive it, may be sold, and transferred from one party to another, just as any other kind of property.

If money would always bring the same amount of Interest, the average price of £100 stock would be always the same, (viz. £100, the price first given for it)—we say the *average* price, because even then the price would evidently be somewhat less immediately *after* the payment of a dividend than it would be immediately *before* it. But not only does this cause affect the price of Stocks, but the continual fluctuations in the value of Money, arising from commercial or political changes or expectations abroad and at home, are constantly disturbing it, even two or three times in the *same* day, according to the news which reach us. The price of stock, then, will *rise* or *fall* according as it seems most likely that Money would fetch elsewhere a *higher* or a *lower* rate of Interest, i. e. would be more *scarce*, and in *demand*, as in prospect of war, or of active speculation, or be lying *upon hand* and *plentiful*, as when trade is looking dull, and there are no means of employing capital.

Thus, if at the time *A* wished to sell his stock, money was elsewhere making 5 per cent., it is plain that no one would give him £100 for the right to receive only 4; but since £80 of common or *sterling* money (as it is called) would now bring £4 interest, he would be able to sell his £100 stock for £80; and the 4 per cents. would be said to be selling at 80.

With this explanation, the mode of treating questions on Stocks will be easily seen from the following Examples.

Ex. 1. If £3500 be invested in the $3\frac{1}{2}$ per cents. at 98, what is the annual income thence derived?

Here $\frac{3500}{98}$ = n° of *cents.* purchased, for each of which £$3\frac{1}{2}$ are paid as interest: hence the whole income $=\frac{3500}{98}\times 3\frac{1}{2}=$ £125.

Ex. 2. The $3\frac{1}{2}$ per cents. are at $99\frac{7}{8}$; how much money must be invested in them to produce an income of £140?

Here $\frac{140}{3\frac{1}{2}}$ = n° of *cents.* required, for each of which £$99\frac{7}{8}$ are paid; hence the whole sum paid $=\frac{140}{3\frac{1}{2}}\times 99\frac{7}{8}=$ £3995.

Ex. 3. If a person were to transfer £29000 stock, from the $3\frac{1}{2}$ per cents. at 99, to the 3 per cents. at $90\frac{5}{8}$, what would be the difference in his income?

Here £29000 in the $3\frac{1}{2}$ per cents. produces $290\times$ £$3\frac{1}{2}=$ £1015 Int., and would be sold out for $290\times 99=$ £28710; this money, invested in the 3 per cents. at $90\frac{5}{8}$, would purchase $\frac{28710}{90\frac{5}{8}}$ cents., and therefore would produce, as Int., $\frac{28710}{90\frac{5}{8}}\times 3=$ £950 8*s.*; and his income, therefore, would be diminished by £64 12*s.*

Ex. 66.

1. The 4 per cents. being at $82\frac{1}{8}$, what must be given for £1000 stock? and what sum would be gained by selling out again at $86\frac{1}{4}$?

2. What income should I get by laying out £1188 in the purchase of 3 per cent. stock at 81?

3. If I lay out £3000 in the 3 per cents. when they are at $84\frac{3}{8}$, what income should I thence derive?

4. A person having £4200 invests it in the $3\frac{1}{4}$ per cents. at 90; find his income.

5. What is the price of stock per cent., when a person can purchase £2766 13*s.* 4*d.* for £2490?

6. What sum must be invested in the 3 per cents. at $94\frac{1}{4}$, to yield an annual income of £500?

7. How much stock at $92\frac{1}{2}$ can be bought for £494, a commission of $\frac{1}{8}$ per cent. being charged on the stock purchased?

8. What is the cost of 850 Bank Annuities at $90\frac{5}{8}$, $\frac{1}{8}$ per cent. being paid for brokerage? And what sum would be lost by selling out at $89\frac{1}{2}$?

9. If I lay out £1000 in the $3\frac{1}{2}$ per cents. at 96, what should I lose by selling out at 95?

10. If a person lays out £4650 in the $3\frac{1}{2}$ per cents. at 93, what will be his loss of property by the stocks falling $\frac{1}{2}$ per cent.?

11. What would be the difference in income, made by the transfer of £5000 stock from the 3 per cents. at 72 to the 4 per cents. at 90?

12. A person transfers £11000 from the 4 per cents. at 92 to the 5 per cents. at 110; what is the difference in his income?

13. What would be the difference in annual income from investing £3450 in the 4 per cents. at 92, and the $3\frac{1}{3}$ per cents. at 69?

14. A person invests £18150 in the 3 per cents. at $90\frac{3}{4}$, and, on their rising to 91, transfers it to the $3\frac{1}{2}$ per cents. at $97\frac{1}{2}$: what increase does he make thereby in his annual income?

15. If I lay out £1110 in the 4 per cents. at $92\frac{1}{2}$, at what price should they be sold to produce a gain of £100?

16. In which is it most advantageous to invest, in the 3 per cents. at $89\frac{1}{2}$, or the $3\frac{1}{2}$ per cents. at $98\frac{1}{2}$?

17. A sum of £3750 was sold out of the 3 per cents. at 95, and put at compound interest for 2 years at 4 per cent.; the amount being laid out in the $3\frac{1}{2}$ per cents. at 104, find the alteration in income.

18. A person has £1000 in the $3\frac{1}{2}$ per cents.; how much must he have also in the 3 per cents. that his whole income may be £200, and what sum would he realise by selling out at $83\frac{5}{8}$ and $77\frac{1}{8}$ respectively?

19. A sum is laid out in the 3 per cents. at $89\frac{3}{8}$, and a half-year's dividend received upon it; the stock being then sold at $94\frac{5}{8}$, and the whole increase of capital being £54, find the original sum laid out.

20. The sum of £1001 was laid out in the 3 per cents. at $89\frac{3}{8}$, and a whole year's dividend having been received upon it, it was sold out; the whole increase of capital being 72 guineas, find at what price it was sold out.

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103. Profit and Loss.—The method of treating questions of this kind will be best learnt from the following Examples.

Ex. 1. If tea be bought at 5*s.* 6*d.* per lb., and sold at 6*s.* 8*d.*, what is the gain per cent.?

Here the gain on the *prime cost*, 5*s.* 6*d.*, is 1*s.* 2*d.*; hence we have

5*s.* 6*d.* : £100 :: 1*s.* 2*d.* : the *Ans.*

which is found by the usual method to be £21 4*s.* $2\frac{10}{11}$*d.*

Ex. 2. If bar-iron, which cost in making £2 1*s.* 8*d.* per cwt., be sold at a loss of $5\frac{3}{8}$ per cent., what price did it fetch per cwt.?

Here bar-iron, which cost £100, would only have sold for £100—£$5\frac{3}{8}$ =£$94\frac{5}{8}$; hence we have

£100 : £2 1*s.* 8*d.* :: £$94\frac{5}{8}$: the *Ans.*

which is found by the usual method to be £1 19*s.* $5\frac{1}{8}$*d.*

Ex. 3. If 5 per cent. be gained by selling 125 yards of cloth for £95, what was the prime cost per yard?

Here, if the cloth had sold for £105, the prime cost would have been £100; therefore the selling price per yd. being $\frac{95 \times 20}{125}$*s.*, we have

£105 : $\frac{95 \times 20}{125}$*s.* :: £100 : $\frac{304}{21}$*s.* = 14*s.* $5\frac{5}{7}$*d.* *Ans.*

Ex. 4. If 4 per cent. be lost by selling linen at 2*s.* 9*d.* a yard, at what price must it be sold to gain 10 per cent.?

Here, cloth which would have cost £100 would have been sold for £96 at the first price, and for £110 at the second; we have, therefore,

£96 : 2*s.* 9*d.* :: £110 : second price = 3*s.* $1\frac{13}{16}$*d.*

Ex. 67.

1. How must nutmegs, which cost 18*s.* 9*d.* per lb., be sold, so as to gain 16 per cent.?
2. If tea be bought at 2*s.* 11*d.* per lb., and sold at 3*s.* 7*d.*, what is the gain per cent.?
3. A merchant, by selling sugar at £1 16*s.* 6*d.* per cwt., loses 18 per cent.; what was his prime cost?
4. If cheese, which was bought at £3 4*s.* 7*d.* per cwt., be sold at £3 12*s.* 4*d.*, what is the gain per cent.?
5. If iron, raised at an expense of £4 5*s.* $3\frac{1}{13}$*d.* per ton, be sold at £4 19*s.* 9*d.*, what is the gain per cent.?
6. If I buy 2048 yards of linen at 3*s.* $2\frac{1}{2}$*d.* per yard, and sell the whole for £359 6*s.* 8*d.*; required the whole gain and the gain per cent.
7. If hemp cost £48 7*s.* 6*d.* per ton, and be sold at £43 per ton, how much per cent. is lost, and how much is lost in the sale of 39 tons, 3 cwt.?
8. If 64 ells of lace cost £115, at what price per yard must it be sold, so as to gain 18 per cent.?
9. A plumber sold 96 cwt. of lead for £109 2*s.* 6*d.*, and gained at the rate of $12\frac{1}{2}$ per cent.; what did it cost him per cwt.?
10. On the sale of 112 yards of silk velvet at 14*s.* 9*d.* per yard, a

merchant loses £10 14*s.* 8*d.*; find the prime cost of the whole, and the loss per cent.

11. If teas at 2*s.* 9*d.*, 3*s.* 3*d.*, and 2*s.* 4*d.* be mixed in equal quantities, and the mixture sold at £16 16*s.* per cwt., what will be the gain or loss per cent.?

12. A person has $\frac{1}{7}$th of a ship, worth £6600, and insured for $91\frac{1}{4}$ per cent. of its real value; what damage would he sustain in case of its being lost?

13. What was the cost of printing 500 copies of a book, which was sold for 5*s.*, if the expense of sale was 34 per cent., and the author's profit £37 15*s.* upon the whole?

14. If $5\frac{1}{2}$ per cent. be gained by selling butter at £5 5*s.* 6*d.* per cwt., what will be the gain per cent. by selling it at 1*s.* 3*d.* per lb.?

15. If 8 per cent. be gained by selling 218 yards of cloth for £92 13*s.*, at what price per yard must it be sold, so as to gain 17 per cent.?

16. A person buys 50 reams of paper, which he thought to sell at £1 2*s.* 6*d.* per ream, making 8 per cent. profit on the prime cost; but, 5 reams being damaged, what did he gain or lose per cent. by selling the remainder at the same rate?

17. A person buys 4 cwt. of goods for £15, intending to gain 12 per cent. by the sale; but, a guinea's worth (at this calculation) being damaged, at what price should he sell per cwt., to gain as much upon his whole outlay as he intended?

18. Bought 236 yards of cambric at 7*s.* $10\frac{1}{2}$*d.* per yard, and sold one-fourth at 10*s.* 3*d.*, one-third at 8*s.* 6*d.*, and the remainder at 7*s.* per yard; what was the gain or loss per cent. upon the whole outlay?

19. If eggs be bought at the rate of 5 a penny, how many should be sold for 7*d.*, to gain 40 per cent.?

20. A person purchases pins, 18 in a row, and sells them, 11 in a row, at the same price; how much is his gain per cent. on his outlay?

There are various examples depending upon the following Rule, the method of treating which will be best explained in the instances below given.

104. PROPORTIONAL PARTS.—*To divide a given quantity into parts which shall have to each other given ratios.*

RULE. Form fractions whose common den^r is the *sum* of the numbers expressing the ratios, and the num^rs the separate numbers themselves; and take these fractions of the given quantity: they will be the parts required.

Ex. 1. Divide 75 into two parts which shall have the ratio of 2 : 3.

Here the fractions are $\frac{2}{5}$ and $\frac{3}{5}$, and the parts required are $\frac{2}{5}$ of 75 = 30, and $\frac{3}{5}$ of 75 = 45, which are plainly in the given ratio.

The reason of the Rule is evident, since the sum of the num^{rs} makes up the den^{r}, and therefore the sum of the fractions makes up *unity*, i. e. the sum of the parts makes up the *whole* of the number; while the parts themselves, having a common den^{r}, are in the ratio of their num^{rs}.

Ex. 2. Gunpowder is composed of 76 parts of nitre, 14 of charcoal, and 10 of sulphur: how much of each of these will be required for a cwt. of powder?

Here the fractions are $\frac{76}{100}=\frac{19}{25}$, $\frac{14}{100}=\frac{7}{50}$, $\frac{10}{100}=\frac{1}{10}$, and the parts are 3 q. $1\frac{3}{25}$lbs., $15\frac{17}{25}$lbs., and $11\frac{1}{5}$lbs. respectively.

Ex. 3. Divide £1000 among *A*, *B*, *C*, so that *A* may have half as much again as *B*, and *B* a third as much again as *C*.

Here, representing *C*'s part by 1, *B*'s is $1\frac{1}{3}$, and *A*'s $1\frac{1}{3}+\frac{1}{2}$ of $1\frac{1}{3}=2$; and, therefore, the parts are to be as the numbers 2, $1\frac{1}{3}$, 1, or 6, 4, 3. Hence the fractions will be $\frac{6}{13}$, $\frac{4}{13}$, $\frac{3}{13}$; and the parts required £461 10*s.* $9\frac{3}{13}$*d.*, £307 13*s.* $10\frac{2}{13}$*d.*, £230 15*s.* $4\frac{8}{13}$*d.*

N.B. — It will be found most convenient, where there are many fractions with the same den^{r}, to find the part corresponding to that den^{r} with num^{r} *unity*, and then multiply this successively by the num^{rs} of the different fractions; thus we should find $\frac{1}{13}$ of £1000, and then multiply this by 6, 4, 3, respectively.

Ex. 4. *A*, *B*, and *C* form a joint capital for conducting a business, of which *A* contributes £500, *B* £650, and *C* £700. At the end of a year the profits are £555; what share should each receive?

Their shares should evidently be in the ratio of their contributions of capital, i. e. in the ratio of 500, 650, 700, or of 10, 13, 14; hence the fractions are $\frac{10}{37}$, $\frac{13}{37}$, $\frac{14}{37}$, and since $\frac{1}{37}$ of £555 = £15, we have the shares required £150, £195, £210.

Ex. 5. *A* begins business with a capital of £800, and, at the end of 3 months, takes *B* into partnership, with a capital of £1000; at the end of another 6 months they divide their profits, £330; what should each receive?

Here *A* contributes £800 for 9 months, and *B* £1000 for 6 months; and the interest of £800 for 9 months = interest of £800 × 9 for 1 mo., and the interest of £1000 for 6 months = interest of £1000 × 6 for 1 month; hence the value of *A*'s and *B*'s outlay may be represented by the products 800 × 9 and 1000 × 6, or 7200 and 6000 respectively, and their shares of the profits must be in this ratio = that of 6 : 5; hence *A*'s share = $\frac{6}{11}$ of £330 = £180, and *B*'s share = $\frac{5}{11}$ of £330 = £150.

N.B.—It appears, as in the above Ex., that the *values* of sums employed in business, &c., for different times are proportional to the products of the sums by the times, or rather of their numerical values, the sums being expressed in the same den^n, and so also the times.

Ex. 6. *A* and *B* enter into partnership, *A* contributing £500 and *B* £300; at the end of 9 months they take in *C* as partner, who brings into the concern a capital of £1000. The profits, £2000, being divided at the end of another 9 months, what shares did they each receive?

Here, as in Ex. 5, at the end of 18 months, the shares of capital supplied by *A*, *B*, *C*, respectively, may be measured by the numbers 500×18, 300×18, 1000×9, or 5, 3, 5 respectively: hence the fractions will be $\frac{5}{13}$, $\frac{3}{13}$, $\frac{5}{13}$; and since $\frac{1}{13}$ of £2000 = £153 16*s.* $11\frac{1}{13}$*d.*, their shares of profit will be £769 4*s.* $7\frac{5}{13}$*d.*, £461 10*s.* $9\frac{3}{13}$*d.*, £769 4*s.* $7\frac{5}{13}$*d.*, respectively.

Ex. 68.

1. Divide 1065 into parts, which shall be to each other in the ratio of 3, 5, 7; and also into parts which shall be in the ratio of $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$.

2. *A*, *B*, and *C* engage in trade, investing capital to the amount of £128, £176, £192 respectively: their profits amount to £279; what were their shares of it?

3. How much copper and tin will be required to cast a cannon weighing 16 cwt. 3 qrs. 11 lbs., gun-metal being composed of 100 parts of copper and 11 of tin?

4. Divide £153 among five persons in the proportion of the fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$.

5. Divide 1400 into parts, which shall have the same ratio to one another as the cubes of the first four natural numbers.

6. Pure water is composed of 2 gases, oxygen and hydrogen, in the proportion of 88.9 to 11.1; what weight of each is there in a cubic foot (1000 oz.) of water?

7. Divide £300 among three persons, so that the first shall have twice as much as the second, and the third twice as much as the other two together.

8. *A* works regularly 9 hours a day; *B* remains idle the first two days of the week, and works $6\frac{1}{4}$, $8\frac{1}{2}$, $10\frac{3}{4}$, 12 hours, respectively, on the other four; what sum should each receive out of £11 12*s.* $6\frac{3}{4}$*d.* at the month's end?

9. The standard silver coin of this realm is made of 37 parts of pure silver and 3 of copper, and a lb. Troy of this metal yields 66 shillings; what weight of pure silver is there in 20*s.*?

10. In England, gunpowder is made of 75 parts of nitre, 10 of sulphur, and 15 of charcoal; in France, of 77 of nitre, 9 of sulphur, and 14

of charcoal: if half a ton of each be mixed, what weight of nitre, sulphur, and charcoal, will there be in the compound?

11. The standard gold coin of this realm is made of gold, 22 carats fine, and a lb. Troy of this metal yields $46\frac{29}{40}$ sovereigns; what weight of pure gold is there in 100 sovereigns?

12. If 4 oz. of gold, 17 carats fine [*see Appendix*], are mixed with 3 oz., 13 carats fine, how much fine gold will there be in a gold ornament made of the compound, and weighing $3\frac{1}{2}$ oz.?

13. *A* and *B* engage in trade, their capitals being in the ratio of 4 : 5; and, at the end of three months, they withdrew respectively $\frac{2}{3}$ and $\frac{3}{4}$ of their capitals: how should they divide their whole gain, £335, at the end of the year?

14. *A*, *B*, *C* join their capitals, which are in the proportion of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$; at the end of 4 months *A* withdraws $\frac{1}{2}$ of his capital, and at the end of 9 months more they divide their profits, £284; what should each receive?

15. *A* and *B* rent a pasture for £75; *A* puts in 80 sheep and *B* 100, but at the end of 6 months they each dispose of half their stock, and allow *C* to put in 50 sheep to feed; what should *A*, *B*, *C*, severally pay towards the rent at the year's end?

16. Four parcels of gold, weighing respectively 10, 4, 2, and 4 oz., and of 13, 12, 11, and 10 carats fineness, being mixed, what was the fineness of the compound?

17. If the preceding be reduced by refining to 16 oz., what will be the fineness of the mass? or if its fineness, when reduced, be 16 carats, what will be the reduced weight?

18. If 8 oz. of gold, 10 carats fine, and 2 oz., 11 carats fine, be mixed with 6 oz. of unknown fineness, and that of the mixture be 12 carats, what was the unknown fineness?

19. *A*, *B*, *C*, are sent to empty a cistern, by means of two pumps of the same bore. *A* and *B* go to work first, making 37 and 40 strokes respectively a minute; but, after 5 minutes, they make each 5 strokes less a minute, and, after 10 minutes more, *A* gives way to *C*, who works at the rate of 30 strokes a minute. The cistern is emptied in 22 minutes altogether, and the men are paid 12*s.* 7*d.* for their labour. What should each receive?

20. *A* and *B* are partners, having each embarked £500 in their business. At the end of 3 months they gained £300, when *A* withdraws £200, and *B* at the same time advances £200. At the end of the next 3 months, they gained £780, when *A* again withdraws £200, and *B* at the same time advances £200. At the end of the year they separated, dividing their property, which by losses during the last 6 months was reduced to £400. What should *A* and *B* each receive?

105. Chain Rule.—When a comparison of several successive quantities is made by stating how many of the second are equivalent to a given number of the first, how many of the third are equivalent to a given number of the second, and so forth, and it is required to find how many of the last are equivalent to a given number of the first, the answer is conveniently found by the *Chain Rule.* The following is an example:

What is the value of 20 lbs. of bacon, if 15 lbs. of bacon be equal in value to 14 lbs. of cheese, and 35 lbs. of cheese equal to 46 lbs. of pork, if pork be worth 6*s.* 3*d.* per stone of 8 lbs.?

In applying the Chain Rule to this question, we first set down the direct demand—*How many pence* = 20 *lbs. of bacon?*—which may be written briefly thus: *? pence* = 20 *lbs. bacon*; then we set down a given quantity of bacon as equivalent to a given quantity of something else: thus, 15 *lbs. bacon* = 14 *lbs. cheese*; then another given quantity of cheese as equivalent to something else: thus, 35 *lbs. cheese* = 46 *lbs. pork*; then another given quantity of pork as equivalent to something else: thus, 8 *lbs. pork* = 75 *pence.* These equations should be placed in successive lines, as follows:

? pence = 20 lbs. bacon,
if 15 lbs. bacon = 14 lbs. cheese,
35 lbs. cheese = 46 lbs. pork,
8 lbs. pork = 75 pence;

where it may be observed that the first and last quantities in the statement are of like denomination, viz. *pence*, and that the second side of an equation is always of the same kind and denomination as the first side of the next equation. The answer for the term of demand (*? pence*) will now be found by dividing the continued product of the right-hand numbers by that of the left-hand numbers. Thus:

$$\frac{\overset{5}{\cancel{20}} \times \overset{2}{\cancel{14}} \times 46 \times \overset{15}{\cancel{75}}}{\cancel{15} \times \underset{7}{\cancel{35}} \times \underset{2}{\cancel{8}}} = 5 \times 46 = 230d. = 19s.\ 2d.\ \textit{Ans.}$$

The reason of the equating and calculating processes will be evident if we employ unity to express the antecedent of each condition; thus:

? pence = 20 lbs. bacon,
if 1 lb. bacon = $\frac{14}{15}$ lb. cheese,
1 lb. cheese = $\frac{46}{35}$ lb. pork,
1 lb. pork = $\frac{75}{8}$ pence;

for now it is obvious that 20 lbs. bacon $= \frac{14}{15} \times 20$ lbs. cheese $= \frac{46}{35} \times \frac{14}{15} \times 20$ lbs. pork $= \frac{75}{8} \times \frac{46}{35} \times \frac{14}{15} \times 20$ pence.

The most important application of the Chain Rule belongs to what is called *Arbitration of Exchange.*—*See* NOTE XI.

Ex. 69.

1. If 10 first-class labourers do as much work per hour as 12 second-class, 14 second-class as much as 16 third-class, 18 third-class as much as 21 fourth-class, what number of the first class corresponds to 8 of the fourth?

2. When $94\frac{1}{2}$ Dutch florins is the exchange for 100 Austrian florins, and 16 sovereigns are given for $193\frac{1}{2}$ Dutch florins, how many Austrian florins should be given for 28 sovereigns?

3. How many lbs. of tea are equivalent to $10\frac{1}{2}$ lbs. of butter, when 5 lbs. of tea are equivalent to 14 of coffee, 9 of coffee to 20 of sugar, 10 of sugar to 6 of cheese, and 10 of cheese to 9 of butter?

4. If 8 sacks of flour be equal in value to 13 loads of straw, 3 sacks of flour to 10 sacks of potatoes, 27 sacks of potatoes to 26 cwt. of rice, and 18 bushels of oats to 5 cwt. of rice, how many loads of straw are worth as much as 10 bushels of oats?

5. If 16 pears be equal in price to 25 apples, and 18 oranges equal to 12 pears, and 20 lemons equal to 27 oranges, and lemons cost $13\frac{1}{2}d.$ a dozen, what is the cost of 15 apples?

6. How many yards of velvet are equal in value to 60 of muslin, when 25 of muslin are equal to 16 of calico, 21 of calico to 13 of flannel, 40 of flannel to 27 of linen, $58\frac{1}{2}$ of linen to 28 of silk, and 47 of silk to 35 of velvet?

7. How many pounds sterling will be the value of 1000 rupees, when 15 rupees are worth 7 American dollars, 5 dollars worth 26 francs, and 101 francs worth £4?

8. If 4 quarters of oats be worth 3 quarters of barley, 14 quarters of barley worth 11 quarters of wheat, 27 quarters of wheat worth 32 bags of rice, 24 bags of rice worth 67 sacks of potatoes, and 2 sacks of potatoes weigh 3 cwt., what quantity of potatoes is equivalent to 63 *bushels* of oats?

9. When $\frac{1}{4}$ of a lb. of tea is equal in value to $\frac{1}{7}$ of a stone of mutton, and $\frac{5}{9}$ of a stone of mutton equal to 3 lbs. of coffee, and $\frac{1}{8}$ of a lb. of coffee equal to $\frac{1}{5}$ of a lb. of beef, how many lbs. of beef are equivalent to 20 lbs. of tea?

10. If an ounce troy of standard silver, of which 37 in 40 parts of the whole are fine, be worth $5s.\ 1\frac{1}{2}d.$, and copper worth 5 guineas per cwt., what is the ratio of the value of fine silver to that of copper?

106. Square Root.—The square root of a given number is that number which, when multiplied by itself, produces the given number. Thus, the square root of 49 is 7, because $7 \times 7 = 49$.

The sign of the square root is $\surd$, a corrupted form of the initial letter of the Latin word *radix*, root; thus we write $\sqrt{49} = 7$.

Few numbers, comparatively, are *perfect* squares; as may be seen by the intervals of the numbers 1, 4, 9, 16, 25, 36, 49, 64, 81, which are the squares, respectively, of 1, 2, 3, 4, 5, 6, 7, 8, 9, and which indicate that every perfect square must have 1, 4, 5, 6, or 9, as its last significant figure.

107. Now, as the square root of 49 is 7, because $7 \times 7 = 49$, so the square root of 186624 is 432, because $432 \times 432 = 186624$; but, while simply from recollection of the ordinary Multiplication Table it is easy to tell what is the square root of 49, a process somewhat complex is requisite to extract from 186624 the square root of that high number.

We proceed to exemplify the method of extracting the square root of a large number, referring for *proof* of the method to the chapter on Involution and Evolution in Colenso's *Algebra*.

Ex. 1. Extract the square roots of 186624, 77841, 9659664.

```
      1̇86̇62̇4̇(432          7̇78̇41̇(279            9̇65̇96̇64̇(3108
      16                  4                       9
                                                 
  83)266             47)378                  61) 65
      249                329                      61
                                                 
 862) 1724          549) 4941              6208) 49664
      1724               4941                    49664
```

Here we first place a dot over the last figure, and then over every *second* figure, reckoning from it; by which means the number will be divided into *periods*, as they are called, consisting each of two figures, except the first, which (when the number of figures in the given number is odd) will evidently consist of only one figure.

We then take the nearest square n° not greater than the first period: this is 16 in the first of the above instances, and we set its square root, 4, as the first figure in the root; we then subtract its square, 16, and bring down the next period, 66.

We now set the double of the first figure in the root, 8, in a loop, as divisor, to the left of the rem^r, regarding it, however, as standing for 80, not for 8, since we shall presently have to set another figure after it. Dividing the rem^r by this div^r, 80, we set the quotient, 3, as the second figure both in the root and also in the div^r: then, multiplying the 83 by 3, we subtract the product, and take down the remaining period, 24.

To form the next div^r, we double the last figure of the preceding one, making 86, which (as before) we regard as 860, and proceed exactly in the same manner: and if finally, as here, we find there is *no* rem^r, we may conclude that we have found the exact square root.

In the 2nd instance, notice (i) that the second rem^r, 49, is greater than the div^r, 47; this may sometimes happen, but no difficulty can arise from it, as it would be found that, if instead of 7 we took 8 for the second figure, the subtrahend would be 384, which is too large: And (ii), that the last figure, 7, of the first div^r, being doubled in order to make the second div^r, and thus becoming 14, causes 1 to be added to the preceding figure, 4, which now becomes 5.

In the 3rd instance, we have an intermediate cypher in the root.

Ex. 2. Extract the square roots of 1000, 2, 1.6, .002.

√1000.00 = 31.6&c.	√2 = 1.41&c.	√1.60 = 1.26&c.	√.0020 = .0447&c.
9	1	1	16
61) 100	24) 100	22) 60	84) 400
61	96	44	336
626) 3900	281) 400	246) 1600	887) 6400
3756	281	1476	6209
144	119	124	191

In the 1st instance, we find there is a rem^r, 39, when we have made use of the last period of the given number, 1000; but we may continue the operation as long as we please in such a case, by setting a decimal point after the given number, and annexing cyphers as decimal places; and for every period of two cyphers thus formed we shall obtain a decimal figure in the root.

The same is true of the 2nd instance, except that we have not taken the trouble to set down the extra cyphers at the end of the given number, though we have taken them down as required, and set the decimal point in the root.

In the 3rd instance, it is to be noticed that the first dot must always be placed on the last figure of the *integral* part of any number, i.e. on the one next before the decimal point, and then on every second figure on each side of it. Of course, in the 4th instance, the figure next before the decimal point, though not expressed, is 0. And, in both these, we have had to annex one cypher to the original number, to complete its points.

In all such cases the square root can never be exactly obtained; but by annexing cyphers, it may be ascertained to as many places of decimals as we please. Such roots are called *irrational* or *surds.*

Ex. 3. Extract the square roots of $\frac{169}{289}$, $\frac{37}{64}$, $\frac{7}{12}$, and $\frac{5}{7}$.

(i.) $\frac{169}{289}=\frac{13\times 13}{17\times 17}$, $\therefore \sqrt{\frac{169}{289}}=\frac{\sqrt{169}}{\sqrt{289}}=\frac{13}{17}$.

(ii.) $\sqrt{\frac{37}{64}}=\frac{\sqrt{37}}{\sqrt{64}}=\frac{\sqrt{37}}{8}=\frac{6.082762+}{8}=.760345+.$

Or, $\sqrt{\frac{37}{64}}=\sqrt{.578125}=.760345+.$

(iii.) $\sqrt{\frac{7}{12}}=\sqrt{\frac{21}{36}}=\frac{\sqrt{21}}{6}=\frac{4.58257+}{6}=.76376+.$

Or, $\sqrt{\frac{7}{12}}=\sqrt{.583333..}=.76376+.$

(iv.) $\sqrt{\frac{5}{7}}=\sqrt{\frac{35}{49}}=\frac{\sqrt{35}}{7}=\frac{5.91608-}{7}=.845154.$

Or, $\sqrt{\frac{5}{7}}=\sqrt{.71428571+}=.845154.$

In the 1st instance, the given fraction is a perfect square, and its root is found by extracting separately the roots of num^r and den^r. Observe that the square root of a proper fraction is always necessarily greater than the fraction.

In the 2nd instance, the den^r only is a perfect square, and we may either proceed as in the 1st instance, or reduce the given fraction to a decimal, and then seek the root.

In the 3rd instance, neither num^r nor den^r is an exact square, but if we multiply both by 3, we shall have the latter an exact square, and may then proceed as in the 1st instance. Otherwise, we may first reduce $\frac{7}{12}$ to a decimal.

In the 4th instance, we proceed as in the 3rd.

Ex. 70.

Extract the square roots of—

1. 5329 and 8836.
2. 34225 and 137641.
3. 531441 and 350164.

4. 95481 and 249001.
5. 348100 and 6512490000.
6. 37491129 and 16949689.
7. 3534400 and 65561409.
8. 99960004 and 24088464.
9. 119550669121 and 368451428004.
10. 8, 20, and 363.
11. 35120 and 8837.
12. 134909.29 and 650506.7716.
13. 6663.114 and 27.773.
14. .225 and 51.12965025.
15. .012012 and .00158404.
16. .000082355625 and .021.
17. $\frac{529}{5329}$, $\frac{18}{49}$, and $\frac{129}{400}$.
18. $\frac{7}{72}$, $\frac{14}{243}$, and $3\frac{37}{196}$.
19. $287\frac{5}{8}$, $6136\frac{1}{9}$, and $367\frac{2}{7}$.
20. $\frac{1}{303}$, $\frac{.00144}{.155}$, and $3-\frac{4}{5-\frac{6}{7}}$.
21. $1\frac{2}{3}$ of $(4\frac{2}{3}+5\frac{3}{4})$, and $1+\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}$.
22. How many links in length is a square field containing 8 ac. 2 ro. 9 po.?
23. Find the length of a square having the same area as a rectangle 43 ft. 5 in. long and 34 ft. 7 in. broad.
24. What sum of money must be divided among *A*, *B*, *C*, so that *A* may have 6*s.* and *C* 9*s.* $4\frac{1}{2}$*d.*, and that *B* may have as much per cent. more than *A* as *C* has more than *B*?

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108. Cube Root.—The cube root of a given number is that number which when multiplied by its square produces the given number. Thus, using $\sqrt[3]{}$ as the sign of the cube root, we have $\sqrt[3]{512}=8$, because $8\times8\times8=512$.

The first nine numbers are the respective cube roots of 1, 8, 27, 64, 125, 216, 343, 512, and 729.

The method of extracting the cube root of a large number is much more complex than that required for the square root, as will appear from the following example. A proof of the method will be found in the chapter on Involution and Evolution in Colenso's *Algebra*.

Ex. Extract the cube root of 80677568161.

		$\sqrt[3]{80677568161} = 4321$
		64
123	4800	16677
	369	
	5169	15507
1292	554700	1170568
	2584	
	557284	1114568
12961	55987200	56000161
	12961	
	56000161	56000161

Here we first divide the number into periods by placing a dot over the last figure, and then over every *third* figure beginning from it. Then we take the nearest cube n° not greater than the first period, 80; this is 64, and we set its cube root, 4, as the first figure in the root; then, subtracting its cube, 64, we bring down the next period, 677. We now set the *triple* of the first figure of the root, 12, at some distance to the left of the remr; (there is 123 in the sum, but the 3 will be accounted for by and by;) then we multiply this triple by the first figure of the root, and place the product, 48, between 12 and the remr, annexing two cyphers to it.

We now divide the remr by this 4800, and set the quotient, 3, as the second figure in the root, and also after the 12, making 123: now we multiply this 123 by 3, the second figure in the root, set the product, 369, under 4800, add them up, multiply the sum, 5169, by the second figure in the root and subtract the product, 15507. We bring down the next period, 568, and have now to form the two quantities to the left of it. The first is obtained by tripling the last figure, 3, of 123, which gives 129 (the final 2 in 1292 will be accounted for when the next figure in the root is found); and the other quantity, 5547, is found by adding 9, the square of the second figure in the root, to the two preceding middle lines $\frac{369}{5169}$. We now add two cyphers, and repeat the whole process described in this paragraph.

The remarks made above with respect to *surd* square-roots apply also to cube-roots: thus, .01, 24.1 would be pointed for the cube-root $.01\dot{0}$, $2\dot{4}.10\dot{0}$.

Ex. 71.

Find the cube roots of—

1. 185193 and 405224.
2. 21952 and 6859000.
3. 4330747 and 35287552.
4. 94818816 and 959530803000.

5. 529475129 and 111423515328.

6. 261775532773 and 176369715712.

7. 357759791.299 and .050243409.

8. 50000 and 527.71.

9. 8047 and 5678.9.

10. $\frac{5}{6}$ and $30\frac{1}{4}$.

11. A box is 3 ft. 5 in. long, 1 ft. 8 in. wide, and 14 inches deep. Required the edge of a cubical box of the same capacity.

12. The volumes of spheres are to one another as the cubes of their diameters. If, therefore, the Sun be $1\frac{1}{4}$ million times as large as the Earth, and the Earth's diameter be 7912 miles, how many miles will the Sun's diameter measure?

MISCELLANEOUS EXAMPLES.

1. The circumference of a coach-wheel being $16\frac{1}{2}$ ft., how often will it turn round between London and Oxford, a distance of 59 miles?

2. If a person's estate produce £400 a year, and the land-tax be assessed at 2*s.* 9*d.* in the pound, what is his net annual income?

3. Reduce $\frac{4158}{10395}$ to its lowest terms, and £1 15*s.* 6*d.* to the fraction of a guinea; find the value of $\frac{3}{28}$ of half-a-guinea, and add together $\frac{1}{3}$, $\frac{2}{5}$ of $\frac{10}{21}$, $1\frac{2}{7}$, and $3 \div 2\frac{2}{5}$.

4. Divide $21\frac{1}{2}$ guineas equally among 12 men.

5. What is the rent of 145A. 1R. 32P. of land, at £10 5*s.* 3*d.* per acre?

6. The produce of a farm one year was 150 quarters, which were sold at 58*s.* a qu.; in the next year the price of wheat fell to 48*s.*, but the crop, being plentiful, produced on the sale the same amount as before: of how many quarters did the second crop consist?

7. A straight plank is $3\frac{1}{2}$ in. thick, and $6\frac{1}{4}$ in. broad; what length must be cut off so as to contain $6\frac{1}{4}$ cubic feet of timber?

8. A person holding 50 shares in the London and North-Western Railway, sells out at 170; what income would he have by buying into the $3\frac{1}{2}$ per cents. at $93\frac{1}{3}$?

9. If 5 lbs. of tea be worth 12 lbs. of coffee, and 7 lbs. of coffee worth 20 lbs. of sugar, and 14 lbs. of sugar worth 7*s.* $1\frac{3}{4}$*d.*, what is the worth of 9 lbs. of tea?

10. A common pasture containing 54A. 3R. $35\frac{1}{2}$P., another containing 39A. $13\frac{3}{4}$P., and a third containing $54\frac{1}{2}$A., are to be divided into 60 equal parts, after deducting from the whole 11A. $2\frac{3}{8}$R. for tithes; of how much does one part consist?

11. Find the square root of 370881, and the side of a square containing 7367 sq. ft. 52 in.

12. If the produce of wheat be tenfold of the seed, how many quarters can be obtained from one grain in 10 years, supposing there to be 7580 grains in a pint?

13. If I lose $1\frac{1}{4}$*d.* in 3*s.* 4*d.*, how much do I lose per cent.?

14. In the centigrade thermometer the freezing point is zero, and the boiling point is 100°; in Fahrenheit's the freezing point is 32°, and the boiling point is 212°; what degree C. corresponds to 68 F.?

15. How much water must be added to a cask, containing 40 gallons of spirits at 13*s.* 8*d.*, to reduce the price to 10*s.* 6*d.*?

16. A bill for £100 has six months to run, and the holder has it discounted at 5 per cent., and receives £97 10*s.*; how much less than his due does he receive?

17. Find the value of $\frac{2}{9}$ of a guinea; reduce 2*s.* $3\frac{1}{2}$*d.* to the fraction of a pound, and 1 hr. $7\frac{1}{2}$ min. to the fraction of 1 da. 6 hrs.

18. A person invested £1000 in the 3 per cents. at $90\frac{5}{8}$; but the price rising to $91\frac{1}{4}$, he sold out, and invested the proceeds in the $3\frac{1}{2}$ per cents. at $97\frac{1}{3}$: find the increase in his income.

19. Find the square root, and also the cube root, of $95951\frac{161}{625}$.

20. A general levies a contribution of £870 on four villages, containing 250, 300, 400, and 500 inhabitants respectively; what must they each pay?

21. *A* can do a piece of work in 10 days, which *B* could do in 13; in what time would they do it together?

22. A stationer sold quills at 11*s.* a thousand, by which he cleared $\frac{3}{8}$ of the money; what would he clear per cent. by selling them at 13*s.* 6*d.* a thousand?

23. Reduce $\frac{3872}{92807}$, $17\frac{5}{12}+\frac{4}{15}+144\frac{11}{21}$, $2\frac{13}{35}-\frac{17}{25}$, $\frac{3}{4}$ of $\frac{6}{7}\times\frac{4}{15}$ of $\frac{11}{18}$ of $\frac{21}{23}$, and $6347\div2\frac{3}{4}$, to their simplest forms.

24. Divide the value of 79 florins between *A* and *B*, giving *A* half-a-crown more than *B*.

25. Three persons rent a piece of land for £60 10*s.*; *A* puts in 5 sheep for $4\frac{1}{2}$ months, *B*, 8 sheep for 5 months, and *C*, 9 sheep for $6\frac{1}{2}$ months: what must each pay of the rent?

26. What is the present worth of £75, due 15 months hence, at 5 per cent.?

27. If *A* can do a piece of work in 10 days, and *A* and *B* can do it together in 7 days, in what time would *B* alone do it?

28. Find the cube root of 133354510.

29. Divide £16 0*s.* 10*d.* among 4 persons in the proportion of the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$.

30. Divide 1037 into two parts, which shall have to one another the ratio of the sum of 7.625 and 5.375 to their difference.

31. A cistern has two pipes, by one of which it may be filled in 40 min., and by the other in 50 min.; it has also a discharging pipe, by which it may be emptied in 25 min. If all these three were open together, in what time would the cistern be filled?

32. There is a number which, when divided by $\frac{2}{3}$ of $\frac{4}{5}$ of $1\frac{1}{2}$, will produce 1; find its square.

33. If a person lend me 1296 guineas for 125 days, how long should I lend him £1620 to requite the favour?

34. Find the square roots of 9.21677 and 921677.

35. If 6 men will dig a trench, 15 yds. long and 4 broad, in 3 days

of 9 hours each, in how many days of 8 hours each will 8 men dig a trench 20 yds. long and 7 broad?

36. Reduce 13*s*. $7\frac{1}{2}d$. to the decimal of a pound, and $\frac{3}{7}$ of 1*s*. $5\frac{1}{2}d$. to the fraction of half-a-crown; divide 1001 by 390625, .1001 by .000390625, and 10.01 by 390.625.

37. The cost price of a book is 3*s*. 9*d*.; if the expense of sale be 6 per cent. upon this, and the profit 24 per cent., what would be the retail price?

38. If the Sun moves through 360° in 365 days 5 hrs. 48 m., how many minutes and seconds will he pass through in a day?

39. Divide £15 among 10 men, 13 women, and 25 children; each man to receive twice as much as each woman, and each child half as much as each woman.

40. There is a fraction which, when multiplied by the cube of $1\frac{1}{2}$, and divided by the square root of $1\frac{7}{9}$, produces $\frac{3}{8}$; find it.

41. A floor, 24 ft. 4 in. broad and 96 ft. 6 in. long, is to be laid at $1\frac{1}{2}d$. per square foot; find the cost.

42. *A* sells to *B* $\frac{7}{8}$ of $\frac{1}{3}$ of $\frac{4}{5}$ of 30 sheep for $\frac{3}{14}$ of $\frac{9}{26}$ of $\frac{8}{9}$ of £210; what was the average price of each sheep?

43. The estate of a bankrupt, £21000, is to be divided among four creditors, whose debts are, *A*'s to *B*'s as 2 : 3, *B*'s to *C*'s as 4 : 5, *C*'s to *D*'s as 6 : 7; what must each receive?

44. A cubic foot of water weighs 63 lbs.; what is the weight of water in a vessel 1 ft. deep, 16 ft. 7 in. long, and 8 ft. 4 in. wide?

45. The profits of a mine for one year amounted to £3296 13*s*. $5\frac{1}{4}d$., and a person holding 14 shares received for his dividend the sum of £1025 12*s*. $7\frac{1}{2}d$.; how many shares were there in all?

46. If the price of gold be 4 guineas an oz., what is the cost of a gold ornament weighing 3 oz., of which 18 parts out of 24 are pure gold; allowing 3*s*. 4*d*. per oz. for the value of alloy, and 25 per cent. upon the whole for expense of workmanship?

47. Find the square roots of .064 and $26.\dot{1}2345679\dot{0}$.

48. What is the price of a piece of timber, of which the length, breadth, and thickness are respectively 23 ft. 9 in., 2 ft. 4 in., and 2 ft., at $9\frac{1}{2}d$. per solid foot?

49. If 90 degrees correspond to 100 French grades, how many degrees and how many grades are there in the sum of 36.45 degrees, and 36.45 grades?

50. A man can reap $302\frac{1}{2}$ square yards in one hour; in what time will 3 such men reap $2\frac{7}{9}$ acres?

51. A farmer gave for a horse a bill of £156, due 8 months hence, at $4\frac{1}{2}$ per cent., and sold him at once for £180; required his gain per cent.

52. *A* can do a piece of work in 3 days, *B* can do thrice as much in

8 days, and *C* five times as much in 12 days: in what time would they do it together?

53. If a tradesman marks his goods 20 per cent. above the cash price, what ready money would he take for an article marked 26*s.*?

54. If 6 men can earn £20 in 21 days, when the days are 12 hrs. long, how much can 4 men earn in 35 days, when the days are 10 hrs. long?

55. If 45 bricks will pave a square yard, how many will be wanted for a space 34 ft. long and 14 ft. wide, allowing for a path, 2 feet wide, all round?

56. Reduce $3\frac{1}{2}$*s.* to the decimal of $\frac{5}{11}$ of a guinea; and find the values of .232 of a cwt., and $4.01\dot{7}\dot{1}$ of a mile.

57. A gentleman had 5 sons, to whom he left £3750 in cash, and two bills of £151 each, due at the end of two and three months respectively; the eldest son had by the will $\frac{1}{4}$ of the property, and, taking charge of the whole, he paid the others their shares, which were equal, in cash. What would these be, reckoning interest at 4 per cent.?

58. Find the sq. root of 39.0625, and the cube root of 2116.874304.

59. What is the annual interest on £76978, bought into the Danish $3\frac{1}{2}$ per cents. at 77? and what sum would be gained by selling out at $77\frac{7}{8}$?

60. It is desired to cut off an acre of land from a field $15\frac{1}{2}$P. in breadth; what length must be taken?

61. Express a degree ($69\frac{1}{22}$ miles) in metres, when 32 metres are equal to 35 yds.

62. At $9\frac{3}{4}$*d.* per sq. yd. what is the cost of painting a room which is 24 yds. round, and 10 ft. 4 in. in height?

63. Find the difference between $\sqrt{\frac{2}{3}}$ and $\sqrt[3]{\frac{2}{3}}$.

64. What is the alteration in income made by transferring £10000 from the 3 per cents. at 92 to the 4 per cents. at 110?

65. Divide $4\frac{1}{3}$ into two parts, one of them to be $4\frac{1}{3}$ times the other.

66. A plate of gold, 3 in. square and $\frac{1}{8}$ in. thick, is extended by hammering so as to cover a surface of 7 sq. yds.; find its present thickness.

67. I bought 171 gallons of brandy in bond for £79 3*s.* 4*d.*, and on taking it out paid duty equal to $112\frac{1}{2}$ per cent. of the bonded value; what was the duty per gallon?

68. Compare the interest on £350 at $4\frac{3}{8}$ per cent., with the interest on £450 at $3\frac{3}{8}$ per cent., for one year.

69. The day's journey in Turkey being 10 hours, of $4\frac{1}{2}$ English miles each, and the proportion of an English to a Roman mile being 12 : 11 nearly, how many Roman miles are there in 13 days' journey in Turkey?

70. A drawing-room, 36 ft. 10 in. long and 23 ft. 2 in. wide, is surrounded with a cornice $3\frac{1}{2}$ in. wide, the gilding of which cost £4 11*s.* $10\frac{1}{2}$*d.*; how much was that per square foot?

71. A steward receives for his landlord £1987 of rent, and disburses one-fifth; he pays his landlord £195 12*s.*, and the remainder is invested in an estate at 30 years' purchase: find the rent of the estate.

72. Reduce $\frac{7}{13}$ of half-a-crown to the fraction of half-a-guinea, and 6*s.* $3\frac{3}{4}$*d.* to the decimal of a £; find also the value of $\frac{2}{5}$ of $\frac{3}{4}$ of £6666 13*s.* 4*d.*

73. What is the yearly interest on £1127 bought into the 4 per cents. at 92?

74. Find the value of £1368 7*s.* 5*d.* sterling in dollars and cents, a dollar being equal to 100 cents, and to 4*s.* 4*d.* English money.

75. A sum of £333 3*s.* $3\frac{3}{4}$*d.* is to be divided among 4 persons, whose shares are to be in proportion as 1, 2, 3, 4; find the share of each.

76. The circumference of the Earth in the lat. of London is 15120 miles; find the distance between two successive meridians of longitude, and the space passed over by the Sun in his apparent daily motion in a minute.

77. If a person accepts £247 1*s.* 8*d.* as present payment of £252 0*s.* 6*d.* due four months hence, at what rate per cent. does he allow discount?

78. Divide 13*s.* $1\frac{1}{2}$*d.* into six parts, each succeeding part to be $6\frac{1}{2}$*d.* more than each preceding.

79. How much stock, at $93\frac{1}{4}$ per cent., can be purchased for £540, a commission of $\frac{1}{8}$th per cent. being charged on the stock purchased?

80. If either 5 oxen or 7 horses will eat up the grass of a close in 87 days, in what time will 2 oxen and 3 horses eat up the same?

81. The sum of £3 13*s.* 6*d.* is to be divided among 21 men, 21 women, and 21 children, so that a woman may have as much as two children, and a man as much as a woman and a child; what will each man, woman, and child receive?

82. *A* sells to *B* $\frac{1}{8}$ of $\frac{3}{4}$ of $\frac{7}{19}$ of a package of tea, which weighs $\frac{3}{7}$ of $\frac{1}{2}$ of 1 cwt. 21 lbs. at 3*s.* 6*d.* per lb.; what did it come to?

83. How many revolutions will a carriage-wheel, whose diameter is a yard, make in a mile, the ratio of the diameter to the circumference being 1 : 3.14159?

84. A cistern can be filled by two pipes, *A* and *B*, in 4 min. and 5 min. respectively, and emptied by *C* in $2\frac{2}{5}$ min. *A* is opened for 2 min., and then *A* and *B* together for 1 min. more, when *C* is also opened. In what time would the cistern, which now contains 361 gals., be full? and how many gallons would have passed through *A* and *B* respectively?

85. What is the yearly interest on £27225, bought into the $3\frac{1}{4}$ per cents. at $97\frac{1}{2}$?

86. Express in its simplest form $\frac{15}{16}-\frac{14}{15}+\frac{13}{14}-\frac{11}{12}$; and add together $\frac{3}{8}$ of a guinea, $\frac{3}{16}$ of a crown, and $\frac{3}{10}$ of 7*s.* 6*d.*, and reduce the result to the decimal of 16*s.*

87. Find the simple interest on £325 16*s.* 8*d.*, for 5 months, at $4\frac{1}{2}$ per cent.

88. If 18 men eat 16*s.* worth of bread in 3 days, when wheat is at 54*s.*, what value of bread will 45 men eat in 27 days, when wheat is at 45*s.*?

89. What length of paper, $22\frac{1}{2}$ in. broad, will be used for a room 21 ft. $9\frac{1}{2}$ in. long, 15 ft. 7 in. broad, and 8 ft. $1\frac{1}{2}$ in. in height? and what will it cost at 1*s.* 3*d.* a yard?

90. Find the value of 36.42 tons of coal, at 17*s.* $7\frac{1}{4}$*d.* per ton; and the difference between $\frac{5}{6}\times\frac{9}{10}\times\frac{17}{18}$ of 10*s.*, and $\frac{1}{7}$ of $\frac{2}{3}$ of £3 11*s.* 9*d.*

91. The 3 per cents. are at $85\frac{1}{8}$; what price should the $3\frac{1}{2}$ per cents. bear, that an investment may be made with equal advantage into either stock? And what income would be derived by so investing £5000?

92. A farm lets for £92 per annum: the tenant pays for 2 years' occupation, with interest accumulating at 5 per cent.; the landlord pays $\frac{1}{4}$ the amount for repairs of house, $\frac{1}{3}$ of this for repairs of barn, and £2 3*s.* 4*d.* for other expenses: find the balance.

93. What will be the cost of painting a room at $9\frac{1}{2}$*d.* per square yard, if the sides are each 19 ft. $10\frac{1}{4}$ in., the ends 16 ft. $1\frac{3}{4}$ in., and the height 10 ft. 3 in.?

94. Express $1618\frac{1}{2}$ Eng. miles in degrees (a degree $=69\frac{1}{22}$ miles): find the values of $\frac{5}{7}$ of £2 7*s.* $8\frac{1}{4}$*d.*, and of $\frac{3}{10}$ of £1 6*s.* 8*d.*, and reduce their difference to the decimal of £20.

95. Twenty-six wedges of gold, weighing in all 33 lb. 3 oz. 7 dwt. 4 gr., are to be coined into sovereigns: find the weight of each wedge, and the number of sovereigns coined from the whole, at the rate of $3\frac{143}{160}$ sovereigns per oz.

96. How many feet in 150 must a road 10798 feet long rise, to be carried from a plain to a hill 463 feet in perpendicular height?

97. A gentleman selling a mortgage of £4410, for which he received 5 per cent. interest, bought into the $3\frac{1}{2}$ per cents. Bank Stock at 70; after receiving the interest for 5 years, on the stocks rising to 75, he sold out. What was his gain upon the whole transaction, over what he would have received had he continued the mortgage?

98. What is the present worth of £325 16*s.* 8*d.*, due at the end of 5 months, at $4\frac{1}{2}$ per cent.?

99. Find the square roots of $6242\frac{1}{4}$ and 1438.237, and the cube roots of .000328509 and 27054.036008.

100. If 40 men in $7\frac{3}{7}$ days can dig 3 rectangular fields, each 150 yds.

by 130; how long will 37 men be digging 5 fields, each $129\frac{1}{2}$ yds. by 90?

101. If 3 men, 5 women, or 8 children, could do a quantity of work in $26\frac{1}{2}$ hours, in what time will 2 men, 3 women, and 4 children complete it?

102. A person, leaving Paddington at 13 minutes before 2, P.M., travels the first 162 miles at 27 miles an hour, the next 121 miles at $9\frac{1}{2}$ miles an hour, and the last 27 miles at 8 miles an hour: when will he reach his destination, Penzance?

103. How many square yards are there in a parade, 864 ft. 3 in. long and 62 ft. 6 in. broad?

104. *A* met two beggars, *B* and *C*, and, having in his pocket $(3\frac{7}{11} \div 4\frac{2}{7})$ of $(10\frac{5}{7} \div 7\frac{1}{2})$ of $\frac{77}{540}$ of a moidore (27*s.*), gave *B* $\frac{1}{7}$ of $\frac{3}{4}$ of that sum, and *C* $\frac{2}{5}$ of the remainder; what did each receive?

105. What is the present worth of £1147 10*s.*, due 3 years hence, at $4\frac{1}{2}$ per cent. simple interest?

106. *A* and *B* entered into partnership: *A* put into stock at first £2000, and at the end of 8 months £1000 more; *B* put in at first £750, and at the end of 4 months £3000, but took out £1300 at the end of 3 months more. At the year's end they had gained £1635; what should each receive?

107. Allowing that $44\frac{1}{2}$ guineas weighed a lb. Troy, when 32 half-pennies weighed a lb. Av., and observing that a lb. Av. contains 7000 gr. Troy, what was the difference in grains between the weights of a guinea and half-penny?

108. How much stock must be bought at 88 per cent., in order that, by selling out when the stocks are at 90, 20 guineas may be gained?

109. A bankrupt pays $3\frac{1}{2}$*d.* in the pound, and the total of his payments amounts to £154; what was his debt?

110. A person has £18752, for which he is receiving $3\frac{1}{2}$ per cent., but spends annually £27 more than the whole original interest; what has he at the end of 3 years?

111. If £100 be placed at interest at 5 per cent., and the interest be added to the principal every 20 years, in how many years will it amount to £1000?

112. The prime cost of a 50-gall. cask of wine is £25, and 10 gall. are lost by leakage; at what price per gall. must the remainder be sold, so as to gain 10 per cent. on the whole original cost?

113. To do a certain piece of work *A* by himself would require 16 hours, *B* 18, *C* 20. Suppose that after *A* and *B* working together for 5 hours, and then *B* and *C* for 3 hours, the remainder of the work is left for *C* to finish, in what time would he finish it?

114. If the carriage of 60 cwt. for 20 miles cost £$14\frac{1}{2}$, what can I have carried 30 miles for £$5\frac{7}{16}$?

115. Find the side of a square whose area equals 14 sq. ft. 11 in.

116. *A* and *B* engage in a speculation, and dividing the proceeds of it, *A* took £57 18*s*., and *B* £29 14*s*., as their respective portions; what sum did each lay out, it being known that *A* paid £7 16*s*. 8*d*. more than *B*?

117. A person had £2950 in the Danish 3 per cents., at $75\frac{1}{4}$, which he transferred to the Russian 5 per cents., at $110\frac{5}{8}$; required the alteration in his income.

118. Extract the square root of .009059 and of $464\frac{52}{81}$, and the cube root of $.57\dot{8}7\dot{0}\dot{3}$.

119. If 7 oxen are worth 64 sheep, and 3 sheep cost £5 12*s*., what must be given for 100 oxen?

120. A person buys teas at 3*s*. and 4*s*. the lb., and mixes them in the proportion of 4 : 7; what will he gain per cent. by selling at 3*s*. 9*d*. per lb.?

121. Find the difference between the simple and compound interest on £150 in 3 years, at $4\frac{1}{2}$ per cent.

122. If 5 men can reap a field, in length 800 ft. and breadth 700 ft., in $3\frac{1}{2}$ days of 14 hours each; in how many days of 12 hours each will 7 men reap a field of 1800 ft. by 960 ft.?

123. Three soldiers, *A*, *B*, and *C*, divide 770 cartridges in the following manner: as often as *A* takes 4, *B* takes 3; and as often as *A* takes 6, *C* takes 7: how many will each have?

124. If £190 in 2 years gain £12 interest, what principal will gain £6 15*s*. in $4\frac{1}{2}$ months?

125. A person desires to exchange 25 Spanish £100 bonds, and £800, $3\frac{1}{2}$ per cent. Stock, for 3 per cent. Consols; the prices of these securities being 48, 99, $93\frac{3}{8}$ respectively, what quantity of Consols can he obtain?

126. A person buys three estates of 56, 67, and 71 acres, and gives £81 3*s*. 6*d*., £92 4*s*. 8*d*., and £109 3*s*. 2*d*. an acre for them respectively; what should they produce annually to pay 15 per cent. upon his whole outlay?

127. If a beam which is 10 in. wide, 8 in. deep, and 5 ft. 6 in. long, weigh 8 cwt. 1 qr., find the length of another beam, the end of which is a square foot, which shall weigh a ton.

128. *A* and *B* have 18*s*. and 12*s*. respectively; and if *A* give *B* $2\frac{3}{8} \div 4\frac{3}{4}$ of the difference of $2\frac{3}{13} \div 13\frac{5}{13}$ of their respective sums, and $\frac{1}{7}$ of $2\frac{1}{2}$ of *A*'s present sum be added to $\frac{11}{15}$ of $\frac{1}{2}$ of *B*'s, *C*'s money will be $1\frac{1}{2}$ of this sum: find it.

129. What is the expense of carpeting a room, 28 ft. long and 19 ft. wide, with carpet $\frac{3}{4}$ yd. wide, at 5*s*. 9*d*. a yard?

130. A person transfers £2500 sterling from the $3\frac{1}{2}$ per cents. at 99, to the 3 per cents. at $86\frac{5}{8}$; what is the difference in his income?

131. Multiply £2 16*s.* 10.75*d.* by 144.33, and divide £9753 14*s.* $8\frac{1}{4}$*d.* by 234.5.

132. What would be the purchase-money for an estate producing a rental of £3228 3*s.* 4*d.*, at the rate of $8\frac{3}{4}$ per cent.?

133. What will be the expense of glazing a hall-window containing 60 squares, each 1 ft. 3 in. long, and $11\frac{1}{4}$ in. wide, at 1*s.* 10*d.* per sq. ft.?

134. A lb. of tea and 4 lbs. of sugar cost 5*s.*; but if sugar were to rise 50 per cent., and tea 10 per cent., they would cost 6*s.* 2*d.* Required the prices of tea and sugar per lb.

135. If I buy 14 sheep for £39 6*s.* $5\frac{1}{2}$*d.*, and sell 6 of them at 36*s.* each, for what must the remainder be sold that I may gain 4 per cent. on the whole?

136. The weights of equal quantities of lead and cork are as 11.324 and .24; and 60 cubic inches of lead, with 54 of cork, weigh as much as $1538\frac{2}{3}$ of fir: what number represents proportionally the weight of fir?

137. By selling an article for 10*s.*, the seller loses 5 per cent.; what will be the loss or gain when sold for 12*s.* 6*d.*, and what was its prime cost?

138. *A* puts out to interest £2000 at 4 per cent.; he spends annually £75, and adds the remainder of his dividend to his stock: what is he worth at the end of 5 years?

139. A country containing 711117 inhabitants increases to 732666; find the increase per cent.

140. If 12 men can complete a piece of work in 15 days, working 6 hrs. a day, how many can do it in $85\frac{1}{2}$ days working $12\frac{12}{19}$ hrs. a day?

141. A bankrupt has good debts to the amount of £456 18*s.* 1*d.*, and the following bad debts, £360 7*s.* 10*d.*, £120 13*s.*, and £19 18*s.*, for which he receives respectively 4, 5, and 9 shillings in the £; his own liabilities amounted to £3408 12*s.*: how much can he pay in the £?

142. *A* had £2 13*s.*, and *B*, when he had paid *A* $6\frac{2}{7} \div 1\frac{2}{9}$ of £1 11*s.* 6*d.*, found that he had remaining $\frac{1}{43}$ of the sum which *A* now had: what had *B* at first?

143. Find the sq. roots of .0026009 and .0002404, and the cube root of $\frac{8}{9}$.

144. A rectangular cistern, of which the length is $13\frac{3}{8}$ ft. and the breadth 6 ft., contains $294\frac{1}{4}$ cubic feet of water; what is the depth of the cistern, and what is the weight of water when one cubic inch weighs 252.5 grains?

145. At what rate per cent. of simple interest will £1 become a guinea in 5 years?

146. How much will a broker, who charges 5 per cent. discount, give for a bill for £600 due at 2 months?

147. Riding a journey of 27 miles into town, I meet the coach which left town at the same moment that I started from hence (viz. 7 o'clock), at the 15th mile-stone from town. Supposing that it travels 10 miles an hour, find the hour when we meet, and the time when (proceeding at the same rate as before) I shall reach London?

148. If 12 casks are carried 18 miles for £16 when the carriage is at 1*s.* 3*d.*, how far ought they to be carried for £72 when the carriage is at 10*d.*?

149. Add together $\frac{3}{5}$ of $\frac{8}{9}$ of a guinea, $\frac{4}{9}$ of a pound, and $3\frac{5}{11}$ of 14*s.* 8*d.*; and reduce the sum of $1 \div 3\frac{1}{2}$ of half-a-guinea and $3 \div 3\frac{3}{4}$ of 15*s.* 6*d.* to the decimal of a pound.

150. What number of lbs. of tobacco, at the same number of pence per lb., amounts to £16 10*s.* 9*d.*?

151. A manufacturer employs 50 men and 35 boys, who work respectively 12 and 8 hours a day during 5 days of the week, and half-time the other day; each man receives 6*d.*, and each boy 2*d.*, an hour. What is the whole amount of wages for a year?

152. A man buys 27 sheep for £30, and sells 12 of them, so that he loses 3 per cent. in the sale; at what price per sheep must he sell the remainder, so that he may gain $2\frac{1}{2}$ per cent. on the whole purchase?

153. Two persons buy respectively, with the same sums, into the 3 and $3\frac{1}{2}$ per cents., and get the same amount of interest; the 3 per cents. being at ·75, at what are the $3\frac{1}{2}$ per cents.?

154. Find the present worth and discount on £226 1*s.* 11*d.*, due 7 months hence, at $4\frac{3}{4}$ per cent.

155. Three tons of merchandise cost £26 15*s.* 5*d.*; at how much per cwt. must it be sold so as to gain 20 per cent.?

156. Divide $3\frac{1}{2}$ guineas among 6 persons, so that their shares may be in the proportion of the reciprocals of the first 6 units.

157. Divide 999 into three parts, so that 6 times the first, 7 times the second, and 11 times the third may be equal.

158. Half the trees in an orchard are apple trees, a fourth pear trees, a sixth plum trees, and there are besides 50 cherry trees; how many trees are there altogether?

159. A banker borrows money at $3\frac{1}{2}$ per cent., and pays the interest at the end of the year: he lends it out at 5 per cent., but receives the interest half-yearly, and by this means gains £200 a year: how much does he borrow?

160. By selling tea at 2*s.* 8*d.* a pound, a grocer clears $\frac{1}{8}$th of his outlay; he then raises the price to 3*s.*: what does he clear per cent. upon his outlay at the latter price?

161. How much tea, at 2*s.* $4\frac{1}{2}$*d.*, must I give for 28 lbs. of sugar, at $4\frac{3}{4}$*d.*, so as to gain 5 per cent. by the exchange?

162. Reduce $\frac{729}{1917}$ to its lowest terms, and $\frac{1}{256}$ to a decimal; and add together $2\frac{3}{4}$, $3\frac{7}{10}$, $\frac{1}{25}$, and $1\frac{7}{8}$; and divide $2\frac{3}{5}$ of $1\frac{2}{3}$ of $1\frac{5}{6}$ by $7\frac{19}{24}$.

163. If 54.32 cub. in. of gold be as heavy as 101.36 cub. in. of silver, how many oz. of silver are equal in bulk to $226\frac{1}{4}$ oz. of gold?

164. What is the present worth of £131 12*s.* 6*d.*, payable in $\frac{1}{4}$ of a year, at 5 per cent.?

165. The length of a street is 937 ft. 6 in., and its breadth 66 ft. 8 in.; find the cost of paving it at $8\frac{1}{2}$*d.* per square yard.

166. If 100 men, in 6 days of 10 hours each, can dig a trench 200 yards long, 3 wide, and 2 deep, in how many days of 8 hours long will 180 men dig a trench of 360 yards long, 4 wide, and 3 deep?

167. A person spending annually £240, saves £2 15*s.* of it quarterly by ready payment; what is the rate of discount? and if he by this means makes an increase of $20\frac{5}{8}$ per cent. upon his annual saving, what was his annual income?

168. A certain sum of money was divided among three persons, *A*, *B*, *C*. Suppose that *A*'s share was £264 12*s.*, and *C*'s £2 8*s.*, and that *A*'s share contained the value of *B*'s as often as *B*'s share contained *C*'s; what must the whole amount have been?

169. Add together $3\frac{5}{8}$ of $2\frac{1}{5}$ of $7\frac{11}{20}$ of a £, $9\frac{3}{7}$ of $3\frac{8}{9}$ of a shilling, and $8\frac{1}{4}$ of $4\frac{1}{8}$ of a penny, and divide the sum by $\frac{11}{12}$ of $\frac{5}{14}$ of $\frac{3}{8}$ of $3\frac{1}{2}$*d.*

170. Extract the square roots of 2.054 and of 42.03361; and the cube roots of 15.438249 and 629.422793.

171. If 6000 lbs. of iron are cast off at a foundry in 24 hours, how many tons weight will be cast off in 308 days, supposing them to work 16 hours each day? and if the price of iron be £3 3*s.* per ton, what will be the gain per cent. upon the annual expenditure, supposing it to be £20 per week of 6 days?

172. How must wine, which cost 15*s.* per gall., be sold, so as to gain $21\frac{1}{4}$ per cent.? and how so as to lose the same?

173. The value of a pound of gold is 14 times that of a pound of silver, and the weights of equal quantities of gold and silver are in the ratio of 19 to 10; find the value of a bar of silver equal in bulk to £1750 worth of gold.

174. *A*, *B*, and *C*, together, can dig an acre of land in $7\frac{1}{8}$ days. *A* digs 32 perches in 5 days, and *B* 54 perches in 7 days. Find the three lowest integral numbers expressing the comparative powers of these men; and the time in which *C* digs $17\frac{2}{3}$ perches.

175. What is the price of a silver cup weighing 1 lb. 10 oz. 12 dwt. 6 grs., worth 5*s.* an ounce?

176. Divide the cube root of $\dfrac{5345344}{116603}$ by the square root of 260100.

177. Reduce 2 w. 2 d. $19\frac{1}{5}$ hrs. to the fraction of a month, and $\frac{13}{16}$ of a shilling + $\frac{5}{6}$ of half-a-crown + $\frac{11}{15}$ of a guinea to the decimal of a £.

178. A fast train leaves Bristol for London, a distance of 120 miles, at 2 o'clock, and travels at the rate of 25 miles per hour; at what time must a luggage train, which travels at the rate of 15 miles in 50 minutes, have left, so as not to be overtaken by the fast train?

179. Find the commission on £126 at $\frac{5}{8}$ per cent., and reduce the answer to the decimal of £1 11*s.* 6*d.*

180. If, by selling fine Irish cloth at 5*s.* per yard, I gain 8 per cent., what will be my rate of profit if I sell at 6*s.* 4*d.* per ell?

181. Add together the cube roots of .007301384 and 32768, and multiply the result by the square root of $72\frac{1}{4}$.

182. What ready money will discharge a debt of £528 9*s.*, due 4 months hence, at $4\frac{7}{8}$ per cent.?

183. Find the least common multiple of 64,720,960; and find what decimal 17 yds. 1 ft. 6 in. is of a mile, and what fraction of 3*s.* 6*d.* is $\frac{5}{9}$ of $\frac{15}{13}$ of 2*s.* 6*d.*?

184. The 3 per cent. stock is at $98\frac{3}{8}$, and the $3\frac{1}{2}$ per cents. at $106\frac{1}{4}$; into which is it most advantageous to buy?

185. £1000 is to be divided among *A*, *B*, and *C*, so that for every £3 given to *A*, *B* is to receive £5 and *C* £8; what sum had they each?

186. Reduce $4\frac{97}{875}$ lbs. Av. to Troy weight, and 3 cwt. 34 lbs. 2 oz. to the decimal of a ton; and .0975, .63, .5243, to their equivalent fractions.

187. The quantity of copper ore sold at Truro on a certain day was 3696 tons (of 21 cwt. each), and the produce $6\frac{7}{8}$ per cent.; find the quantity of fine copper obtained from it in common weight.

188. A rectangular parish, 6 fur. long and 4 fur. broad, is enclosed; a belt of plantation, 200 ft. wide, is carried the whole way round; a main road, 60 ft. wide, runs across the land in the direction of its length, and a cross road, 41 ft. wide, in the direction of its breadth: how many acres of field were there?

189. If the sixpenny loaf weigh $5\frac{1}{2}$ lbs. when wheat is at $5\frac{3}{4}$*s.* per bushel, what must be paid for $52\frac{1}{4}$ lbs. of bread when wheat is at 8*s.* 6*d.* per bushel?

190. Find the present value of £273 0*s.* 9*d.*, due 3 months hence, at $4\frac{1}{2}$ per cent., and the compound interest on £105 in 3 years, at $3\frac{1}{2}$ per cent.

191. A body of 7300 troops is formed of four battalions, so that $\frac{1}{2}$ of the first, $\frac{2}{3}$ of the second, $\frac{3}{4}$ of the third, and $\frac{4}{5}$ of the fourth, are all composed of the same number of men; how many were there in each?

192. Among the Jews the *coin* mina (or pound) was worth 50 shekels of silver, each weighing 219 grs.; the *weight* mina, when of gold,

weighed 100 shekels, when of silver, 60; what were the values of these minæ, rating gold at £4 and silver at 5*s.* an ounce?

193. A father left to the elder of his two sons $\frac{13}{25}$ of his estate, and $\frac{13}{25}$ of the remainder to the younger, and the residue to his widow; find their respective legacies, it being found that the elder son received £1690 more than the younger.

194. Divide 240 into two parts, such that $\frac{1}{4}$ of one added to $\frac{1}{10}$ of the other shall equal 36.

195. If 193 Russian versts be equal to 205.9 French kilomètres, and 1552.94 kilomètres equal to 964.9 English miles, how many miles are equal to 100 versts?

196. If the rent of 2 acres for $\frac{3}{4}$ of a year be £1 3*s.* 3*d.*, what will be the rent of 547 acres for a half year?

197. If I buy 3 per cents. at $78\frac{3}{8}$, and $3\frac{1}{2}$ at $95\frac{3}{16}$, which is the best investment? If I had invested £6962 19*s.* $3\frac{3}{4}$*d.* in each, and the former rose and the latter fell $\frac{1}{16}$, how much should I lose or gain?

198. If 3 men can mow 7 acres of grass in 5 days of 9 hours each, in how many days of 8 hours each will 5 men mow $17\frac{1}{2}$ acres?

199. Add together $3\frac{2}{15}$, $2\frac{5}{12}$, $\frac{1}{9}$, and $\frac{4}{45}$; find the difference of $3\frac{4}{27}$ and $2\frac{5}{9}$, and divide $3\frac{4}{27}$ by $2\frac{5}{9}$.

200. Five thousand copies are issued of a 6*s.* book: the cost of printing is 1*s.* per copy, of binding 4*d.*, and of carriage, advertising, &c., 2*d.*: the publisher disposes of them to the retail bookseller, charging 25 copies as 24, and 30 per cent. less than the selling price, and upon the whole receipts takes 10 per cent. commission for himself: what are the gains respectively of author, publisher, and bookseller on this edition?

201. Find the square root of $\frac{169}{813}$, and the cube root of 352045.367981.

202. Find the discount on £1294 10*s.* for $1\frac{3}{4}$ year, and the interest on the discount for the same time, at $4\frac{1}{2}$ per cent.

203. Divide 100 guineas into an equal number of guineas, half-guineas, crowns, half-crowns, shillings, and sixpences, and reduce the remainder to a fraction of a pound.

204. A person has £3500 to lay out; the 3 per cents. are at $82\frac{1}{2}$, and the $3\frac{1}{2}$ at 96: what would be his income from each?

205. How many inches are there in the diagonal of a cub. ft., and how many square inches in a superficies made by a plane through two opposite edges?

206. A merchant employs £700 in trade, and at the end of 3 years takes another into partnership, who advances £1900. At the end of 4 years from this time they have gained £500; how ought this to be divided between them?

207. If 24 pioneers, in $2\frac{1}{2}$ days of $12\frac{1}{2}$ hours long, can dig a trench

139.75 yds. long, $4\frac{1}{2}$ yds. wide, and $2\frac{1}{2}$ yds. deep, what length of trench will 90 pioneers dig in $4\frac{1}{5}$ days of $9\frac{2}{3}$ hours long, the trench being $4\frac{7}{8}$ yds. wide and $3\frac{1}{5}$ yds. deep?

208. What is the discount on £257 8*s.* $8\frac{1}{4}$*d.*, paid 210 days before due, at $4\frac{1}{2}$ per cent.?

209. What is the cost of papering a room 15 ft. long, 12 ft. wide, and 10 ft. high, with paper 30 in. broad, at $7\frac{1}{2}$*d.* per yard?

210. The sum of £925 was so divided among *A*, *B*, *C*, and *D*, that *B*'s portion was equal to $\frac{11}{12}$ of *A*'s, *C*'s was equal to $\frac{3}{8}$ of *B*'s, and *D*'s was half as much as *B*'s and *C*'s together: what did each receive?

211. A draper bought 5 pieces of silk, each 52 yards, at 4*s.* $3\frac{1}{4}$*d.* per yard, and sold the whole so as to gain as much as $16\frac{1}{4}$ yards were sold for; what was the selling price per yard?

212. £100 stock, in the 3 per cents., is sold for £91 15*s.*; how much can be bought for £540, allowing, for commission, $\frac{1}{8}$ per cent. upon the stock bought?

213. A gentleman's income is £896 13*s.* 4*d.* per ann.; he gives to the poor quarterly £13 10*s.*, and lays up 200 guineas at the year's end: how much does he spend in 6 days?

214. A grocer buys 13 lbs. of tea at 2*s.* 3*d.*, 16 lbs. at 2*s.* 5*d.*, and 18 lbs. at 3*s.* 3*d.*, and mixes them: at what rate per lb. must he sell the mixture so as to gain on the whole $17\frac{1}{2}$ per cent.?

215. What is the present worth of £2035 15*s.*, due in 2 yrs. $5\frac{1}{2}$ mo., at $4\frac{1}{2}$ per cent.?

216. What is the expense of paving a rectangular court-yard, whose length is 63 ft., and breadth 45 ft., it being paved with pebbles at 1*s.* 9*d.* per sq. yard, except a foot-path, which runs the whole length, 5 ft. 3 in. broad, and is paved with flag-stones at 3*s.* per square yard?

217. *A* and *B* can do a piece of work alone in 12 and 16 days respectively; they work together at it for 3 days, when *A* leaves it, but *B* continues, and after 2 days is joined by *C*, and they finish it together in 3 days; in what time would *C* do it alone?

218. Find the value of $13\frac{53}{4480}$ of 2 cwt. 2 qrs.; and of $\frac{57}{55}$ of £8 8*s.* $5\frac{1}{4}$*d.*

219. *A* can mow $2\frac{1}{2}$ acres of grass in $6\frac{2}{3}$ hours, and *B* $2\frac{1}{6}$ acres in $5\frac{1}{3}$ hours: they mow together a field of 10 acres; in what time will they do it, and how many acres will each mow?

220. In making gold thread for embroidery, a cylinder of silver weighing 360 oz. Av. is cased with one of gold weighing 6 oz.; and this mass is drawn through a series of circular holes, continually diminishing in diameter, until it becomes so thin that 202 feet in length weigh one dram: what is now the length of the thread?

221. The gross weight of the Chinese silver, brought home in

January 1842, was 143639 lbs., and the mint-refiner undertook to pay all expenses of refining on being allowed $3\frac{1}{2}$ grs. of gold (less 10 per cent.) on every pound weight gross of silver: what sum did this amount to, at £4 1*s.* 3*d.* per oz.?

222. The weight of gold extracted from the above was 2530 oz. 1 dwt. 17 grs.; what was its value at the same rate?

223. What would be the interest on £256 5*s.* 9*d.*, at $4\frac{1}{2}$ per cent., for 4 yrs. $5\frac{1}{2}$ mo.? and what would be the compound interest on £1040, at 4 per cent., for 3 years?

APPENDIX.

The choice of the number 10, as the base or *radix*, as it is called, upon which the *decimal* system or *scale* of Notation depends, common as it is to so many nations, barbarous as well as civilised, may be conceived to have had its foundation in the natural practice of counting on the fingers, whence the term *digit;* but we might have taken any other number for base, and, having characters for zero and all the figures less than the base, we might express any number whatsoever in such a scale. (*See Alg. Notation.*)

The admirable method of notation by the use of the *nine digits and zero* is of extreme antiquity; and though called the *Arabic* method, (because first introduced into Europe through the Moors in Spain about the 11th century, though it was not till about the 14th that it superseded the old Roman system,) was certainly known to the Hindoos long before the rise of Arabian science, and even by them ascribed, for its excellence and the remoteness of its origin, to the direct revelation of the Divine Being. It seems to have been traced with some probability to the regions of Thibet.

The system of the Greeks was almost identical with that of the Hebrews, or Phœnicians: that of the Romans, though very simple, was singularly cumbrous and inconvenient; and it is a striking proof of their extreme indifference to any advances in scientific matters, that they so pertinaciously retained it, notwithstanding their acquaintance with the far more perfect and comprehensive notation of the Greeks.

The *figures* now in use are derived from the old Arabic, though much modified and corrupted by the course of time.

When numbers are used with reference to the things numbered, as when we say 3 *apples*, 4 *pens*, 5 *shillings*, they are said to be concrete numbers; when used without such reference, merely to indicate a certain number of units of the same kind, as when we say simply 3, 4, 5, they are called *abstract* numbers.

The concrete quantities, required in ordinary calculations, are those which are necessary to express *Money*, *Weight*, *Space*, and *Time*. In the *Tables* will be found the most common of these quantities; but we shall here make a few additional remarks about them, and explain the *Standards*, which are used in each of these classes.

The standard *gold* coin of this realm is made of a metal, of which 22 parts in 24 are *pure gold*, and 2 parts *alloy*, a mixture of silver and copper. From a lb. Troy of this metal are coined $46\frac{29}{40}$ sovereigns = £46 14*s*. 6*d*.; so that the Mint price per oz. of *standard* gold = $\frac{1}{12}$ of £46 14*s*. 6*d*. = £3 17*s*. $10\frac{1}{2}$*d*.; and since there are 11 oz. of *pure* gold in 12 oz. of *standard*, we shall have (neglecting the value of the alloy) the value per oz. of *pure* gold at the Mint = $\frac{1}{11}$ of £46 14*s*. 6*d*. = £4 4*s*. $11\frac{5}{11}$*d*.

The standard *silver* coin is made of a metal, of which 37 parts in 40 are *pure* silver, and 3 parts *alloy* (copper). From a lb. Troy of this metal are coined 66*s*., so that the Mint price per oz. of standard silver is 5*s*. 6*d*.: and since there are $\frac{37}{40}$ of an oz. of pure silver in this, the value per oz. of *pure* silver at the Mint is $\frac{40}{37}$ of 5*s*. 6*d*. = 5*s*. $11\frac{13}{37}$*d*.

From a lb. Av. of copper are coined 24 pence: but this is not a legal tender for more than 12*d*., nor is the *silver* coinage for more than 40*s*., the *gold* coinage being the standard of the realm.

The following coins are noticeable, occurring often in ancient documents:—

Groat = 4*d*., Tester = 6*d*., Noble = 6*s*. 8*d*., Angel = 10*s*., Merk = 13*s*. 4*d*., Carolus = 23*s*., Jacobus = 25*s*., Moidore = 27*s*.

Great inconvenience having been long felt in this country, from the want of uniformity in the systems of weights and measures, which were in use in different parts of it, an Act of Parliament was passed in 1824, and came into operation on Jan. 1, 1826, by which certain weights and measures, therein specified, were declared to be the only lawful ones in this realm, under the title of *Imperial Weights and Measures*.

It was settled by this Act—

1. That a certain yard measure made by an order of Parliament in 1760, (by comparison with the yards then in common use,) should be henceforward the *Imperial Yard*, and the Standard of *Length* for the kingdom: and that in case this Standard should be lost or injured, it might be recovered from the knowledge of the fact, that the length of a pendulum, oscillating in a second, *in vacuo*, in the latitude of London, and at the level of the sea, (which can always be accurately obtained by certain scientific processes,) was 39.13929 inches (or *twelfth* parts) of this yard;

2. That the half of a double-pound Troy, made at the same time, should be the *Imperial Pound Troy*, and the Standard of *Weight;* and that of the 5760 grains, which this lb. contains, the lb. Av. should contain 7000: and that in case this Standard should be lost or injured, it might be recovered from the knowledge of the fact, that a *cubic inch* of distilled water, at the temperature of 62° Fahrenheit, and when the barometer is at 30°, weighs 252.458 grains;

3. That the *Imperial Gallon*, and Standard of *Capacity*, should contain 277.274 cubic inches, (the *inch* being above defined,) which size was selected from its being nearly that of the gallons already in use, and from the fact that 10 lbs. Av. of distilled water, weighed in air, at a temperature of 62°, and when the barometer is at 30°, will just fill this space.

The name *Troy Weight* has been derived from Troyes, a city of France, where great fairs were once held, and to which it was introduced, about the time of the Crusades, from Cairo in Egypt; but it has also been derived from the monkish name for London, *Troynovant*, from Trinovantum. The name *Avoirdupois* is probably derived from the old Norman, *avoirs*, goods and chattels, and *pois*, weight.

It is probable that a grain of wheat was the element of *weight* in former days, and a grain of barley (barleycorn) the element of *length.*

The *pennyweight* was so called as being the weight of the silver penny then in use.

The words *ounce* and *inch* are both derived from the Latin *uncia*, or *twelfth* part, of a pound and foot respectively.

The following weights and measures are noticeable, besides those given in the Tables.

Carat (of Diamond)	$=3\frac{1}{6}$ grs.	Line	$=\frac{1}{12}$ in.
Carat (of Gold or Silver)	=240 grs.	Barleycorn	$=\frac{1}{3}$ in.
Firkin (of Butter)	=56 lbs.	Span	=9 in.
Fodder (of Lead)	$=19\frac{1}{2}$ cwt.	Cubit	=18 in.
Great Pound (of Silk)	=24 oz.	Pace	=5 ft.
Pack (of Wool)	=240 lbs.	Degree	$=69\frac{1}{9}$ miles
Yard (of Land)	=30 acres	Flemish Ell	=3 qrs.
Hide (of Land)	=100 acres	French Ell	=6 qrs.

Firkin (of Beer)	=9 gals.	Anker (Wine or Spirits)	=10 gals.
Kilderkin	=18 gals.	Runlet	=18 gals.
Barrel	=36 gals.	Tierce	=42 gals.
Hogshead	=54 gals.	Hogshead	=63 gals.
Butt	=108 gals.	Puncheon=2 Tierces	=84 gals.
Tun	=2 butts	Pipe=2 Hogsheads	=126 gals.

Since there are 24 carats in a lb. of gold, the fineness of gold is often expressed by saying that it is so many *carats fine*, meaning so many parts out of 24; thus our *standard* gold is 22 carats fine, and jewellers' gold (as marked on the stamp of a watch) is 18 carats fine.

In measuring land, surveyors use a *chain*, called Gunter's chain, which is 22 yards long, and divided into 100 links; and 10 square chains, or 100,000 square links, make an acre.

In FRANCE, the standard of linear measure is the *metre*, which is one ten-millionth part of the Terrestrial arc from the Equator to the Pole=39.371 inches; and their other measures are all decimal parts or multiples of this: thus the *decimetre*=3.9371 in., *centimetre*=.39371 in., *millimetre*=.03937 in., &c., and so the *decametre* =393.71 in., and similarly for the *hectometre* (hecatometre), *kilometre* (chiliometre), *myriometre*, &c.

The standard of weight is the *Gramme*=weight of a cubic centimetre of distilled water=15.4340 grs.; and this is likewise subdivided and multiplied into the *decigramme*, *centigramme*, *kilogramme*, &c.

The standard of capacity is the *litre* = 61.028 cub. inches, that of superficial measure, the *are* = 119.6046 sq. yds., that of solid measure, the *stere* = 35.317 cub. ft. — all of which may be subdivided and multiplied as before.

The Greek unit of linear measure was the ποῦς = 12.135 inches. The principal Attic measures of length were

δάκτυλος	($\frac{1}{16}\pi$) = $\frac{3}{4}$ in. nearly.	πλέθρον	(100π) = $101\frac{1}{8}$ ft.
πῆχυς	($1\frac{1}{2}\pi$) = $1\frac{1}{2}$ ft. or $\frac{1}{2}$ yd.	στάδιον	(600π) = $606\frac{3}{4}$ ft.
ὀργυία	(6π) = 6 ft. or a fathom.	δίαυλος	(1200π) = $1213\frac{1}{2}$ ft.

It will be found that there are very nearly $8\frac{3}{4}$ stadia in a mile. The Persian *parasang* was 30 *stadia*, rather more than a *league*.

The principal square measures were the square ποῦς and πλέθρον, which latter contained 4 ἄρουραι, and was a little less than a *rood*.

The Roman unit of length was the *pes* = 11.6456 inches.

Their other ordinary measures were the *digitus* ($\frac{1}{16}$ *pes*), *uncia* ($\frac{1}{12}$*p.*), *palmus* ($\frac{1}{4}$*p.*), *palmipes* ($1\frac{1}{4}$*p.*), *cubitus* ($1\frac{1}{2}$*p.*), *gradus* ($2\frac{1}{2}$*p.*), *passus* (5*p.*), *milliarium* or *mille passuum* (5000*p.* = 1618 *yds.*).

Their principal square measure was the *jugerum* (240*p.* by 120) = 28800 *pedes quadrati*, or $\frac{5}{8}$ acre, nearly.

For rough calculations, the ποῦς and *pes* may each be considered to be equivalent to a *foot* English.

The Greek and Roman systems of *money* were naturally founded upon those of *weight*, the denominations of money and weight being identical.

The Attic unit of weight and money was the *drachma*, which, as a weight, was equivalent to $66\frac{1}{2}$ grs.; and this weight of silver being worth $9\frac{3}{4}$*d.*, this was the value of the silver coin, *drachma*. Their other coins (all in silver) were as follows —

6 *obols* (ὀβολοί) made 1 *drachma* (δραχμή)
100 *drachmæ* . . . 1 *mina* (μνᾶ)
60 *minæ* . . . . 1 *talent* (τάλαντον);

so that the obol was worth about $1\frac{1}{2}$*d.*, the mina £4 1*s.* 3*d.*, the talent £243 15*s.*

Besides these, there were the *diobolus*, *triobolus*, *didrachm*, *tetradrachm* (or *stater*), &c., whose values are explained by their names.

In later times, the value of the drachma as a coin corresponded to the Roman *denarius* = $8\frac{1}{2}$*d*

The Roman unit of weight was the *libra*, or pound, = 5204 grs., that is, nearly $\frac{3}{4}$ lb. Av., or very nearly $\frac{9}{10}$ lb. Troy. This weight of the metal *æs* or *bronze* (a mixture of copper and tin) formed originally the coin *as*, or pound; but the weight of the coin was subsequently reduced in the proportion of 8 : 5.

The *as* or *libra* was divided into 12 *unciæ*, i. e. *twelfth-parts;* and the following names were given to the different multiples of an uncia.

$1\frac{1}{2}$	*unc.*	(sesqui-uncia)	*sescunx*	7	*unc.*		*septunx*
2	...	($\frac{1}{6}$ lb.)	*sextans*	8	...	($\frac{2}{3}$ lb.)	*bes*
3	...	($\frac{1}{4}$ lb.)	*ter-uncius* or *quadrans*	9	...	($\frac{3}{4}$ lb.)	*dodrans*
4	...	($\frac{1}{3}$ lb.)	*triens*	10	...	($\frac{5}{6}$ lb.)	*dextans*
5	...		*quincunx*	11	...		*deunx*
6	...	($\frac{1}{2}$ lb. = *semi-as*)	*semis*	12	...		*libra* or *as*

The name *bes* is supposed to be formed from *des* (as bis from δίς), and this from *de-triens* (*desit triens*), meaning an *as* wanting a *triens* or third; just as *dodrans*, *dextans*, *deunx*, are formed from *de-quadrans*, *de-sextans*, *de-uncia*.

It should be observed that the word *uncia*, or ounce, means simply a *twelfth-part;* and therefore the above terms *sescunx*, *sextans*, &c. were used by the Romans, as so many fractions, for subdivisions of other units, as well as of the *as:* thus, we have had above the *uncia* of length = $\frac{1}{12}$ *pes*, and see also below among the measures of capacity.

The *uncia* of weight = 434 grs. = very nearly an *ounce* Av.

The Romans had also a silver coinage, consisting of the *denarius* and its parts. These were the *denarius*, worth $8\frac{1}{2}$*d.*, and equivalent (as its name denotes) to 10 ancient *ases* or 16 later ones; the *quinarius* (5 ancient *ases*) = $4\frac{1}{4}$*d.*, called also *victoriatus*, from the image of Victory upon it; the *sestertius* (i. e. *semis tertius nummus*, or a coin worth $2\frac{1}{2}$, viz. ancient *ases*) = $2\frac{1}{8}$*d.*; *libella* = $\frac{1}{10}$ *den.*, *sembella* (*semi-libella*) = $\frac{1}{20}$ *den.*, *teruncius* = $\frac{1}{40}$ *den.* = (as above) $\frac{1}{4}$ ancient *as* or $\frac{2}{5}$ later *as* = $\frac{1}{5}$*d.*, nearly.

For rough calculations we may reckon the *as* at $\frac{1}{2}$*d.*, *sestertius* 2*d.*, *denarius* $8\frac{1}{2}$*d.* The *sum* of 1000 *sestertii* was called a *sestertium* = £8 17*s* 1*d.*, but there was no *coin* for this amount.

The Greek ξεστής = Roman *sextarius*, may be conveniently taken as the unit of capacity, being equivalent to (.9911 or) just *one pint*

English. The *sextarius* was so called as being $\frac{1}{6}$ of the *congius*, and contained 12 κύαθοι, *cyathi;* and the multiples of the *cyathus* had the same names among the Romans as those of the *uncia*, or ounce of weight: thus, 2 *cyathi* was a *sextans*, or $\frac{1}{6}$ of a *sextarius*, &c.

The Greeks had also the κοτύλη $= \frac{1}{2}$ *sext.* $= \frac{1}{2}$ pt., χοῖνιξ $= 1\frac{1}{2}$ pt., χοῦς $= 4$ χοίνικες $= 3$ qts., μετρητής $= 9$ gals., μέδιμνος $= 12$ gals.; and the ἕκτος and ἡμίεκτος were the *sixth* and *twelfth* parts of the *medimnus*. The Romans had, beside the *cyathus* and *sextarius*, the *hemina* $= \frac{1}{2}$ pt., *congius* $= 6$ *sext.* $= 3$ qts., *modius* $= 2$ gals., *urna* $=$ 3 gals., *amphora* $= 6$ gals.

A *Solar Day* is the interval between two successive transits of the Sun over the meridian of any place; but, from several causes, this interval is continually varying, though slightly, in duration. If, however, we take the *mean* of many observations, we shall get the length of the *Mean Solar Day*, and this is the Standard unit for the measurement of *Time* in ordinary life: though Astronomers have another unit in common use.

The *Solar Year* is the interval between the Sun's leaving and returning to a certain fixed point in his apparent orbit round the Earth (the *Ecliptic*), and is accurately determined by Astronomers to contain 365.242218 mean solar days $=365$ days, 5 hrs., 48 min., $47\frac{1}{2}$ sec. nearly. Hence the common, or *Civil*, Year, which contains only 365 days, is somewhat shorter than the Solar, or True, Year; and this error, being nearly $\frac{1}{4}$ of a day, would accumulate, if not corrected, so as to produce at length a complete confusion in the times at which the seasons would return, and we should have Summer, sometimes in July, sometimes in December.

Julius Cæsar first corrected this; and, supposing, in the then state of Science, that the Solar Year contained exactly 365 days, 6 hrs. $= 365.25$ days, he ordered that every *fourth* year should contain 366 days instead of 365. But this correction was really too great by .007782 of a day, since the Solar Year contained only 365.242218 days; and in 400 years this error amounted to $400 \times .007782 = 3.1128$ days; and hence it happened that the vernal equinox, which fell, in A.D. 325, at the Council of Nice, on March 21, fell in A.D. 1582 on March 11. Pope Gregory, in consequence, caused 10 days to be omitted in that year, making Oct. 15 to follow Oct. 4, so that the vernal equinox fell next year again on March 21; and, to prevent the recurrence of this error, he ordered that in every succeeding cycle of 400 years, 3 of the leap years should be omitted, viz. those which complete a century, when the number of *hundreds* is not divisible by 4; thus, 1600, 2000 are leap years, but not 1700, 1800, 1900, &c.

The Gregorian correction was introduced in England in 1752. when it had become necessary to omit 11 days of the current year; and the Calendar thus rectified is called the *New Style*, the Julian reckoning (which is still retained in Russia) being the *Old* Style.

This correction is too great on the other side by .000282 of a day, but the error only amounts to a day in 4000 years.

N.B.—Until A.D. 1752, the New Year's day in England for all official records was the 25th March: hence, we often find, in works relating to an earlier period, a double date given, as 1703-4, whenever the event referred to occurred during the month of January, February, or March, up to March 25—the former indicating the year according to the old, and the latter, according to the modern, reckoning.

DECIMAL COINAGE.

1. It may be desirable to say here a few words upon the subject of a *Decimal Coinage*, which has been for some time under the consideration of the Government, has been recommended for adoption by a Committee of the House of Commons, and is likely, therefore, before long, to be introduced in England, as it has been already in France and in the United States of America.

2. Two systems of decimal coinage have been proposed, and each has met with warm supporters,—the one based upon the *penny* or *farthing*, the chief coin of the poorer classes, as the unit of reference, the other upon the *pound sterling* or *sovereign*, the chief coin of the wealthier classes. Each of these systems has its own peculiar advantages and disadvantages, which we shall proceed briefly to explain. Of the two, the advantages of the latter, based upon the pound sterling, seem to be upon the whole the greatest; and as it has been specially recommended by the House of Commons' Committee, it is probably that which will be ultimately sanctioned by Act of Parliament, perhaps, with some modification of its details, as, for instance, in the names at present proposed for the new coins.

3. I. One system of decimal coinage takes the *farthing* for its unit of reference, and its money-table would be somewhat as follows:—

10 *Farthings*	make 1 *Doit*	=	10*f.* = $2\frac{1}{2}$*d.*
10 *Doits*	make 1 *Florin*	=	100*f.* = 2*s.* 1*d.*
10 *Florins*	make 1 *Pound*	=	1000*f.* = 20*s.* 10*d.*

The coins required for use in this system would be the following:—

Copper—farthing, halfpenny, and penny, as now;
Silver —*doit* ($2\frac{1}{2}$*d.*), *groat* (5*d.*), *shilling* ($12\frac{1}{2}$*d.*), *florin* (25*d.*);
Gold —*half-pound* (125*d.*), *pound* (250*d.*).

It might also be convenient to have a *dollar* or *double-florin* (50*d.*) in silver, and a crown ($62\frac{1}{2}$*d.*) in gold, so that five dollars, or four crowns, would go to make the pound. The difference in size between the doit and the groat, being much greater than that existing between the present 3*d.* and 4*d.* pieces would allow very well of their being both coined in silver.

4. The advantages of this system are the following:—

(1.) All coins now in use would be still available; and thus, while the banks would be collecting the old coins, and gradually withdrawing them from circulation, business might be carried on as usual with the old shilling, florin, and pound. This would prevent, no doubt, much confusion at first, especially among the poorer classes.

(2.) The farthing, halfpenny, and penny, would be permanently retained, and the price of food, the rate of wages, &c., being generally fixed by the penny, much inconvenience would be saved by this means to the mass of the population.

(3.) No change need be made in the penny postage, the penny-stamp, the tolls for turnpikes, bridges, &c., nor in any fixed payment whatever, as now existing.

5. The disadvantages of this system are the following:—

(1.) The present pound sterling, which is the usual unit of reference in all great questions of national and commercial finance, would be ultimately displaced altogether.

(2.) The accounts of bankers, merchants, &c., kept during past years according to the old coinage, or the sums of money mentioned in statistical or other records could not be immediately compared with corresponding entries under the new system, nor without the trouble of reducing them in each case to their equivalent expressions in the new coinage.

(3.) The process of reduction from the old coinage to the new, though easy on this system, is much more easy on the other system of decimal coinage, as will presently appear.

6. We may here complete what we have to say on this system, by explaining the process of reduction from the old coinage into the new.

To reduce a Sum of Money from the present Coinage into the new Decimal Coinage (Penny System).

Since one old pound contains 960 farthings,
and one new pound contains 1000 farthings,
it follows that if a denote the number of old pounds, and b the number of new pounds, in the same given sum of money, then

$$960a = 1000b^*, \text{ or } b = \tfrac{96}{100}\,a = (1 - \tfrac{4}{100})\,a = a - .04a.$$

Hence we may find the number (b) of new pounds, corresponding to any given number (a) of old pounds, by subtracting from a the quantity $.04a$, which we obtain by merely multiplying a by 4, and moving the decimal point in the result two places to the left, or otherwise by deducting 4 per cent. from the amount.

Ex. 1. Reduce £765 from the old Coinage into the new (Penny) Coinage.

$$\begin{aligned} \text{Here } a &= 765.00 \\ .04a &= 30.60 \\ b &= \overline{734.40} = 734 \textit{ Pounds } 4 \textit{ Florins} \text{ (new Coinage).} \end{aligned}$$

Since 1 shilling $= (\tfrac{1}{20} = \tfrac{5}{100} =)$ .05 of a pound, any number of shillings in the given sum may be expressed at once as a decimal of a pound, by merely multiplying by 5,

* For if we denote that sum by S when reduced into farthings,

$$\frac{S}{960} = a, \text{ and } \frac{S}{1000} = b,$$

$$\therefore\ S = 960a = 1000b.$$

and setting the product to fill the two places of figures immediately after the point.

Ex. 2. Reduce £343 17*s.* into the new (Penny) Coinage.

Here $a = 343.850$

$.04a = \underline{\ 13.754}$

$b = 339.096$ = 330 *Pounds*, 0 *Florins*, 9 *Doits*, 6 *f.* (new Coinage).

If there are any odd pence in the given sum, these have only to be reduced to farthings, and added in as *thousandths* of a *pound.*

Ex. 3. Reduce £409 11*s.* $8\frac{1}{2}d.$ into the new (Penny) Coinage.

Here $a = 409.550$

$.04a = \underline{\ 16.382}$

393.168

$8\frac{1}{2}d. = \underline{\ \ \ \ 34}$

$b = 393.202$ = 393 *Pounds*, 2 *Fl.*, 2 *f.* (new Coinage).

7. The converse process of reduction from the *new* coinage into the *old* would be performed as usual.

Ex. 1. £734.4 (new) = 4)734400*f.*
12)183600*d.*
20)15300*s.*
765£ (old).

Ex. 2. £330.096 (new) = 4)330096*f.*
12)82524*d.*
20)6877*s.*
343£ 17*s.* (old).

Ex. 3. £393.202 (new) = 4)393202*f.*
12)$98300\frac{1}{2}d.$
20)8191*s.* $8\frac{1}{2}d.$
409£ 11*s.* $8\frac{1}{2}d.$ (old).

8. II. The other system of decimal coinage takes the *pound sterling*, or *sovereign*, for its unit of reference, and its money-table would be somewhat as follows:—the *mil* being the $\frac{1}{1000}$ of a pound sterling $= \frac{6}{25}$ of a penny $= \frac{24}{25}$ of a farthing.

10 *Mils* make 1 *Cent.* $= \frac{1}{100}$ £ $= 2\frac{2}{5}d.$
10 *Cents* make 1 *Florin* $= \frac{1}{10}$ £ = 2*s.*
10 *Florins* make 1 *Pound sterling* = 20*s.*

The coins required for use in this system would be the following: —

Copper—*mil* ($\frac{6}{25}d.$), *two-mils* or *double* ($\frac{12}{25}d.$), *five-mils* or *doit* ($1\frac{1}{5}d.$);

Silver—*cent* ($2\frac{2}{5}d.$), *two-cents* or *groat* ($4\frac{4}{5}d.$), *five-cents* or *shilling* (12*d.*), *florin* (2*s.*).

Gold—*half-sovereign* (10*s.*), sovereign (20*s.*).

It might also be convenient to have a *dollar* or *double-florin* (4*s.*) in silver, and a *crown* (5*s.*) in silver or gold.

9. The disadvantages of the system are the following: —

(1.) It would abolish the coins most in use with the poor, namely, the farthing, halfpenny, penny, and 3*d.*, 4*d.*, and 6*d.* pieces, leaving them only the shilling, and coins of larger value. The *sixpence*, indeed, might still be used for a time, as it is exactly equivalent to 25 mils; but it would ultimately be withdrawn from circulation.

(2.) It would be impossible to pay *exactly* in the new coinage a sum in the old coinage which contained (besides pounds and shillings) any number of *pence*, except it were *six*-pence. For $1d. = 4\frac{1}{6}$ mils, $2d. = 8\frac{1}{3}$ mils, &c.

(3.) Hence also it would be necessary that, wherever a rate of 1*d.* is now levied for any purpose, a change should be made, and either 4 mils or 5 mils charged instead. Where large sums are raised by such a rate, this would produce a very considerable difference in the amount so obtained.

To take, for instance, the case of the penny postage: if 4 mils be charged instead of $1d. = 4\frac{1}{6}$ mils, the *loss* to the government upon every penny would be $\frac{1}{6}$ mil, and upon a million of pounds $240000000 \times \frac{1}{6}$ mils $= 40,000,000$ mils $=$ £40,000; whereas, if 5 mils be charged instead of 1*d.*, the *gain* to the government would be $\frac{5}{6}$ mil upon every penny, or, upon a million of pounds, £200,000.

The same would be true of *tolls* taken for turnpikes, bridges, &c., which are usually rated at 1*d.*, 2*d.*, 3*d.*, 4*d.*, &c., and the difficulty of coming to a satisfactory arrangement in such cases would be much greater than in that of a government impost. For, in the latter case, it is the government, that is, the nation itself, which would be the gainer or loser by the loss or gain of the public in paying the *tax;* whereas, in the former, the loss or gain of the public would occasion a corresponding gain or loss to the private individuals or companies who might be the proprietors of the *tolls.*

10. Notwithstanding the above disadvantages, the recommendations of this system are so great, (1) from its not abolishing the *shilling, florin, crown, half-sovereign,* and *sovereign;* (2) from its allowing old accounts to be compared at sight with those of the present day, without the trouble of reduction; (3) from the facility with which a sum may be converted on this system from the old coinage into the new; that there is little reason to doubt its being ultimately adopted, if our present system is exchanged for any other.

11. *To reduce a Sum of Money from the present Coinage into the new Decimal Coinage (Pound System).*

Here the number of *pounds* remains unchanged; the *shillings,* if any, may (as before) be expressed as a decimal of a pound by multiplying by $\frac{1}{20}$ or .05; and, since 1*d.*$=4\frac{1}{6}$ mils, if the *pence* be converted into farthings, the number of farthings will give the number of equivalent mils, except that 1 mil must be added whenever the number of pence is 6*d.*, or above it. If special accuracy be required, then 1 mil should be added for any number of odd pence between 3*d.* and 9*d.*, and 2 mils for any number of odd pence above 9*d.*; by which arrangement the loss and gain upon the fractional parts of a mil, when there are several sums of money concerned, would in the long run be fairly balanced.

Ex. Reduce £409 11*s.* $8\frac{1}{2}$*d.* from the old Coinage into the new (Pound) Coinage.

Here £409 11*s.* 0*d.* = £409.550
$8\frac{1}{2}$*d.* 35

Ans. £409.585 = £409 5 fl. (85 cents, or) 8 cents 5 mils.

12. The converse operation would be performed as usual.

£409.585
20
11.700
12
8.40
4
1.60 *Ans.* £409 11*s.* $8\frac{1}{2}$*d.* nearly.

13. We may exemplify the application of this system in one or two instances.

Ex. 1. Multiply £37 17*s.* $4\frac{1}{2}$*d.* by 43.

Old Coinage.

£37	17	$4\frac{1}{2}$
		10
£378	13	9
		4
£1514	15	0
113	12	$1\frac{1}{2}$
£1628	7	$1\frac{1}{2}$ = £1628.356 (new coinage).

New Coinage.

£37 17 $4\frac{1}{2}$ = £37.850
19
37.869
43
113607
151476
£1628.367

N.B.—The difference in these two results arises from the fact that in the one we have expressed $1\frac{1}{2}$*d.* by 6 mils, instead of $6\frac{1}{4}$ mils, its true value, and in the other we have expressed $4\frac{1}{2}$*d.* by 19 mils, instead of $18\frac{3}{4}$ mils, its true value. The second error of $\frac{1}{4}$ mil when multiplied by 43 produces an error of $10\frac{3}{4}$ mils, which added to the first error of $\frac{1}{4}$ mil makes up the whole difference of 11 mils.

Ex. 2. Find the value of 5 cwt. 3 qrs. 14 lbs. at £14 9*s.* 8*d.* per cwt.

£14 9 8 =		£14.483
		5
		72.415
2 qrs.	$\frac{1}{2}$	7.2415
1 qr.	$\frac{1}{2}$	3.62075
14 lbs.	$\frac{1}{2}$	1.810375
		£85.087625 = £85.088 (nearly) = £85 0*fl.* 88*f.*

14. It would be of little use to pursue this subject any further at present, while the whole matter is yet under consideration, and the details of the measure, to be hereafter proposed to Parliament, are by no means fixed.

THE METRIC SYSTEM.

15. Besides the Decimal Coinage, there is also a Decimal System of Weights and Measures, commonly called the French or Metric System, which has been adopted by nearly all the Continental nations of Western Europe,* and will probably at no very distant day be established also in England. The first step indeed to such establishment had been already taken, when the Council of Education required in their Code of Regulations (1871) † that a chart of the Metric System should be hung conspicuously on the walls of all schools under Government inspection, and that in all such schools children in Standards V and VI should know the principles of the Metric System, and be able to explain the advantages to be gained from the uniformity in the method of forming multiples and sub-multiples of the unit.‡

* The Metric System has been adopted in France, Holland, Belgium, Greece, Spain, Portugal, Italy, Roumania, the North German Confederation, Wurtemberg, Bavaria, Baden, and also by Chili, Equator, Uruguay, Brazil, the Argentine Confederation, New Granada, Peru, Venezuela, and partially or in substance in Norway, Canada, British India, and the United States; while a Decimal System of Weights and Measures, differing only from the Metric System in the unit chosen as the base of the System, exists by law in Austria and Switzerland.

† But this rule is not at present (1874) in force.

‡ In 1864 the Metric Act of Parliament (27 & 28 Vict. c. 117) was passed, which provides that, 'Notwithstanding anything contained in any Act of Parliament to the contrary, no contract or dealing shall be deemed to be invalid or open to objection on the ground that the weights or measures expressed or referred to in such contract or dealing are weights or measures of the Metric System, or on the ground that decimal subdivisions of legal weights and measures, whether Metric or otherwise, are used in such contract or dealing.' In other words, this Act permitted the use of the Metric System. And yet, 'by a strange

16. The advantages in question are obvious. Thus in Avoirdupois Weight 16 drams make 1 ounce, 16 ounces make 1 pound, 28 pounds make 1 quarter, 4 quarters make 1 hundred-weight, 20 hundred-weight make 1 ton, where the numbers, indicating the multiples of the unit of the next lower denomination which make one of the higher, are respectively 16, 16, 28, 4, 20 ; and so in Troy Weight they are 24, 20, 12, in Apothecaries' Weight, 20, 3, 8, 12 ; and the same irregularity prevails in the Tables of Measures. But in the Metric System the number is always the same, *viz.* 10, so that *ten times* the unit of the next lower denomination makes always one of the higher—except a slight modification in Square Measure, as shown below. By this means all laborious multiplications and divisions are avoided, such as are required under the old system, *e.g.* for reducing ounces to tons, or miles to inches. And arithmetical operations of all kinds are so much simplified in practice by the use of the Metric System that (to use the words of Prof. LEONE LEVI, *Metric System*, p. vi), 'Here is a tool which offers facilities for saving one-half of the time in arithmetical education, and one-fourth, or one-third, of the time spent in all the transactions which include calculations of weights and measures.' Being, moreover, so generally employed on the Continent, it is very desirable, with a view to international communication, that it should be as soon as practicable adopted also in England. And, in fact, it is already used exclusively in some popular scientific class-books, and a knowledge of it is required by Examiners in Physics and Chemistry.

inconsistency, as the law now stands, whilst the restriction is removed against contracting in terms of the Metric System, any person using such weights and measures for the purpose of buying and selling in shops and other places subject to the visits of Inspectors of Weights and Measures, or having them in his possession, is liable to have them seized and to conviction and forfeiture.' Prof. LEONE LEVI, *Theory and Practice of the Metric System*, p. 6.

17. The Metric System is so called from the French word *mètre* (derived from the Greek *metron*, 'measure'), the name given to a line of a certain length (39·37 inches, rather more than a yard), which was fixed upon in 1799 by the French Legislature as the standard unit of linear measure, and which was at that time supposed to be the ten-millionth part of the distance from the Equator to the Pole. It has been since found, however, that the measurement of the Earth's circumference then made was not quite correct. And, consequently, the Metre, as originally determined by that measurement, is really an arbitrary length, like the English imperial yard.

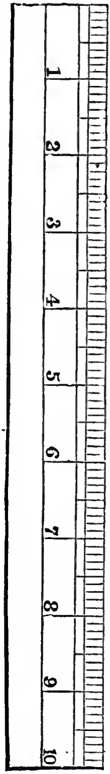
18. The Metric System has four principal units, all depending on the metre.

1. The Metre (39·37 *inches*) is the unit of measures of length.

2. The Are (120 *square yards*), the square of ten metres, is the unit of measures of surface.

3. The Litre (61 *cubic inches*), the cube of the tenth of a metre, is the unit of measures of capacity.

4. The Gram ($15\frac{1}{2}$ *grains*) is the unit of measures of weight, and is the weight in vacuo of so much water at its greatest density as would fill the cube of the hundredth part of a metre.



Actual length of a decimetre = the tenth of a metre.

19. The standard Metre is a platinum bar, and the standard Kilogram (p. 161) a platinum cylinder, which are preserved carefully in the Hôtel des Archives at Paris. Exact copies of them are deposited at the Conservatoire

des Arts et Métiers, and are used to verify the metric standards for foreign countries. But England possesses two platinum copies of the standard Metre, deposited with the Royal Society in London, and a platinum copy of the standard Kilogram, deposited at the Standard Department. Besides these, brass copies of the Metre, Kilogram, and Litre, have been carefully made, and presented by the French to the British Government, and are now deposited at the office of the Warden of the Standards.

20. Each unit has its decimal multiples and submultiples, as follows :—

	Length	Surface	Capacity	Weight
1000	kilometre	...	kilolitre	kilogram
100	hectometre	hectare	hectolitre	hectogram
10	dekametre	...	dekalitre	dekagram
1	METRE	ARE	LITRE	GRAM
·1 $(=\frac{1}{10})$	decimetre	...	decilitre	decigram
·01 $(=\frac{1}{100})$	centimetre	centiare	centilitre	centigram
·001 $(=\frac{1}{1000})$	millimetre	...	millilitre	milligram

21. The following are the tables of measures employed in the Metric System, with their respective *units*.

I. *Measures of Length* or *Linear Measure*.

The unit of Linear Measure is the *Metre*=39·37 inches, or 3·28 feet, or 1·09 yard (more correctly 39·3708 *in.* =3·2809 *ft.*=1·0936 *yds.*).

10 millimetres	make	1 centimetre.
10 centimetres	,,	1 decimetre.
10 decimetres	,,	1 metre.
10 metres	,,	1 dekametre.
10 dekametres	,,	1 hectometre.
10 hectometres	,,	1 kilometre.
10 kilometres	,,	1 myriometre.

Hence, in order to reduce from one denomination to another, the French arithmetician merely throws the decimal point one or more places to the right or left as the case may require. Thus 98765·4321 *metres*=98765432·1 *millim.* =9·87654321 *myriom.*; whereas under the English system, in order to reduce 987654321 inches to leagues, we should have to divide by 12, 3, $5\frac{1}{2}$, 40, 8, 3, successively, a very laborious process.

N.B. The *dekametre* (10 *m.* or 100 *decim.*=32·8 *ft.* or 10·9 *yds.*) is used as a chain in surveying, and is divided into 50 links, each containing 2 *decim.*

The *kilometre* (1000 *m.*=1093·6 *yds.*) is nearly 5 furlongs (1100 *yds.*), so that 8 *kilom.*=5 *miles* nearly.

The *myriometre* (10 *kilometres* or 10,000 *m.*)=50 furlongs, or $6\frac{1}{4}$ miles nearly (more nearly=10936 *yds.* or $6\frac{1}{5}$ miles).

II. *Measures of Surface* or *Square Measure.*

The unit of Square Measure is the *Are* or square dekametre, that is a square of which the side is a dekametre=10 metres, and which therefore contains (p. 26) 100 square metres=119·6 square yards.

100 centiares (square metres)	make	1 are.
100 ares (=10,000 square metres)	,,	1 hectare.

III. *Measures of Solidity* or *Cubic Measure.*

The unit of Cubic Measure is the *Stere* or cubic metre =61027 cubic inches, or 35·3166 cubic feet, or 1·30802 cubic yard, nearly.

10 decisteres	make	1 stere.
10 steres	,,	1 dekastere.

N.B. These measures are chiefly used for wood and carpentry.

IV. *Measures of Weight.*

The unit of Weight is the *Gram*, which is the weight in vacuo of 1 cubic centimetre of distilled water at its greatest density, viz. at the temperature of 4° of the centigrade thermometer=15·43234 grains or $15\frac{1}{2}$ grains, nearly.

10 milligrams	make	1 centigram.
10 centigrams	„	1 decigram.
10 decigrams	„	1 gram.
10 grams	„	1 dekagram.
10 dekagrams	„	1 hectogram.
10 hectograms	„	1 kilogram.
10 kilograms	„	1 myriogram.

N.B. The kilogram or kilo, as it is often called, $=15432{\cdot}34$ grains $=2\frac{1}{5}$ lbs. Av. (15,400 grains) nearly, is the weight usually employed on Continental railways; and the half-kilo ($=1\frac{1}{10}$ lb. Av.) is also generally used as a weight on the Continent.

The centner $=50$ kilos. $=771{,}617$ grains $=110\frac{1}{4}$ lbs. Av. (771,725 grains) $=1$ cwt. (112 lbs.) nearly.

The quintal $=10$ myriogr. or 100 kilos. $=220\frac{1}{2}$ lbs. Av. $=2$ cwt. nearly.

The millier or tonne $=10$ quintals or 1,000 kilos $=2205$ lbs., or 20 cwt., or 1 ton, nearly.

V. *Measures of Capacity.*

The unit of Capacity is the *Litre* or cubic decimetre $=61{\cdot}027$ cubic inches $=1{\cdot}76$ pint.

10 centilitres	make	1 decilitre.
10 decilitres	„	1 litre.
10 litres	„	1 dekalitre.
10 dekalitres	„	1 hectolitre.
10 hectolitres	„	1 kilolitre.

N.B. The hectolitre $=100$ litres $=176$ pints $=22$ gallons, or $2\frac{3}{4}$ bushels, nearly.

22. Since 1 decimetre $=10$ centimetres, therefore (p. 28) a cubic decimetre or litre $=1000$ cubic centimetres. Hence the weight in vacuo of a litre of distilled water at its greatest density is the weight of 1000 cubic centimetres of such water, or 1000 grams, that is to say, the weight of a litre of such water is 1 kilogram.

In like manner, since 1 metre=10 decimetres, therefore the weight of a cubic metre of such water is that of 1000 cubic decimetres, *viz.* 1000 kilos or 1 millier. Thus a mass of rock 4 metres long, 3 metres wide, and 2 metres deep, would contain ($4\times3\times2=$) 24 cubic metres, and fill 24 kilolitres; and as this quantity of water would weigh 24 milliers, the weight of the mass in question would be found at once by multiplying this weight by the number which expresses the specific gravity of the rock compared with water.

23. A metric quantity may be read in various ways, in terms of one denomination or of more than one, at pleasure. Thus 35·703 *metres* may be read as 35 *metres* 7 *decim.* 3 *millim.*, or as 3·5703 *dekam.*, or as 357 *decim.* 3 *millim.*, or as ·035703 *kilom.*

But, in writing a metric quantity from dictation, it is necessary sometimes to insert cyphers, as in the following examples:—

Thirteen kilometres, seven grams=13·007 kilometres or 13007 grams;

Seven hectolitres three centilitres=7·0003 hectolitres or 700·03 litres;

Seven hectares six ares five centiares=706·05 ares or 7·0605 hectares.

But it should be noted carefully that in Square Measure such an expression as 5·7 *sq. m.* means—not 5 *sq. metres* 7 *sq. decim.*, but—5·7 ($=5\frac{7}{10}$) *sq. metres*=5 *sq. metres* 70 *sq. decim.* (since 1 *sq. metre*=100 *sq. decim.*). Similarly in Cubic Measure 5·07 *cub. m.* means $5\frac{7}{100}$ *cub. metres* =5 *cub. metres* 70 *cub. decim.* And conversely, since 1 *sq. metre*=100 *sq. decim.* and 1 *cub. metre*=1000 *cub. decim.*, therefore 9 *sq. metres* 5 *sq. decim.*=9·05 *sq. m.*, and, in like manner, 8 *cub. metres* 91 *cub. decim.*=8·091 *cub. m.*

24. Since the metre=1·09 yard or $1\frac{1}{10}$ yard, nearly, and the half-kilo=$1\frac{1}{10}$ lb. Av., nearly, it follows, that when goods are sold by the metre or half-kilo, the prices should be 10 per cent. higher than when they are sold by the yard or pound respectively. In like manner since the centner (50 kilos.)=$110\frac{1}{4}$ *lbs.*, which is less than a hundred-weight (112 *lbs.*) by $1\frac{3}{4}$ *lb.*=$\frac{1}{64}$ *cwt.*, the prices of goods, when sold by the centner or millier (20 centners), should be $\frac{1}{64}$ less than when sold by the hundred-weight or ton (20 *cwt.*) respectively, which amounts to a reduction of $2\frac{3}{4}$*d.* in the £.

25. The metre, half-kilo, centner, and millier, might be called the *metric yard*, *metric pound*, *metric hundred-weight*, *metric ton*, respectively. And the following names, corresponding to the names of English measures, are given by Prof. Levi, *Metric System*, p. 64.

Metric	league (half-myriometre)	=	3·1 miles.
,,	mile (kilometre)	=	1094 yards.
,,	furlong (double-hectometre)	=	219 ,,
,,	chain (double-dekametre)	=	21·9 ,,
,,	pole (half-dekametre)	=	5·5 ,,
,,	fathom (double-metre)	=	6·56 feet.
,,	cubit (half-metre)	=	1·6 ,,
,,	hand (decimetre)	=	3·9 inches.

26. The following is a table of approximate equivalents in the English and Metric Systems, where great accuracy is not required (Prof. Galbraith, as quoted by Prof. Levi, *Metric System*, p. 49).

Length.

1 metre =3 feet 3 inches 3 eighths.
64 metres=70 yards.

Linear, Square, and Cubic Measure.

10 metres =11 yards.
10 sq. metres =12 sq. yards.
10 cub. metres=13 cub. yards.

Land Measure.

1 are	= 4	perches.
10 ares	= 1	rood.
1 hectare	= 2½	acres.

Weight.

1 kilogram	= 2⅕	lbs. Av.
30 grams	= 17	drams Av.

Liquid and Dry Measure.

4½ litres	= 1	gallon.
1 hectolitre	= 22	gallons.

27. The following table gives a more accurate list of the equivalents of the principal metric measures in terms of English measures, and *vice versâ.*

Measures of Length.

Millimetre	=	·03937	*inch.*
Centimetre	=	·3937	,,
Decimetre	=	3·937	*inches.*
	or	·32809	*foot.*
Metre	=	39·37079	*inches.*
	or	3·28089	*feet.*
	or	1·09363	*yard.*
Dekametre	=	10·93633	*yards.*
	or	1·98842	*pole.*
Inch	=	·0254	*metre.*
Foot	=	·30479	,,
Yard	=	·91438	,,
Pole	=	5·0291	*metres.*
Chain (4 *p.*)	=	20·1164	,,
Furlong (10*p.*)	=	201·1644	,,
Mile	=	1609·3149	,,
	or	1·6093	*kilom.*

Measures of Surface.

Square decimetre	=	15·50059	*square inch.*
Square metre	=	1·19603	*square yard.*
	or	10·76429	*square feet.*

Hectare	=	2·47114	*acres.*
Are	=	·02471	*acre.*
Square inch	=	6·45137	*square centimetres.*
Square foot	=	9·28997	*square decimetres.*
Square yard	=	·8361	*sq. metre (centiare).*
Square pole	=	·2529	*are.*
Rood	=	10·11678	*ares.*
Acre	=	40·4671	,,
	or	·40467	*hektares.*

MEASURES OF SOLIDITY.

Cubic decimetre	=	61·02705	*cubic inches.*
Cubic metre	=	35·31658	*cubic feet.*
	or	1·30802	*cubic yard.*
Cubic inch	=	16·38618	*cubic centimetres.*
Cubic foot	=	28·3153	*cubic decimetres.*
Cubic yard	=	·7645	*cubic metre.*

MEASURES OF WEIGHT.

Gram	=	·56438	*dr.*
	or	·03527	*ounce Avoirdupois.*
	or	15·43234	*grains.*
	or	·64301	*dwt.*
Hectogram	=	3·52739	*ounces Avoirdupois.*
	or	3·21507	*ounces Troy.*
Kilogram	=	35·2739	*ounces Avoirdupois.*
	or	2·2046	*pounds Avoirdupois.*
	or	2·6792	*pounds Troy.*
	or	·01968	*hundred-weight.*
Millier	=	·98420	*ton.*
Grain	=	·0648	*gram.*
Pennyweight	=	1·55517	,,
Ounce Troy	=	31·1035	*grams.*
Pound Troy	=	373·24195	,,
Dram	=	1·77184	*gram.*
Ounce Avoirdupois	=	28·34954	*grams.*
Pound Avoirdupois	=	453·59265	,,
Stone (14 *lbs.*)	=	6·3503	*kilometres.*
Quarter (28 *lbs.*)	=	12·70059	,,

Hundred-weight	=	50·80238	*kilometres.*
Ton	=	1016·0475	„
	or	10·160475	*quintals.*
	or	1·0160475	*millier.*

MEASURES OF CAPACITY.

Centilitre	=	·07043	*gill.*	*Pint*	=	·56755	*litre.*
Decilitre	=	·17607	*pint.*	*Quart*	=	1·13510	„
Litre	=	1·7607	„	*Gallon*	=	4·54041	*litres.*
	or	·88038	*quart.*	*Bushel*	=	3·63233	*dekalitres.*
	or	·22009	*gal.*		or	36·3233	*litres.*
Dekalitre	=	2·20096	„	*Quarter*	=	2·90586	*hectolitres.*
Hectolitre	=	22·0096	„		or	29·0586	*dekalitres.*
	or	2·751208	*bus.*		or	290·586	*litres.*
	or	·343901	*gr.*				

NOTES AND EXAMINATION-PAPERS

ON

ARITHMETIC.

NOTES.

NOTE I.

Casting out the Nines, as a method of Proof for Multiplication, depends on the two following considerations:—

(i.) Any n° divided by 9 leaves the same remainder that would be left if the sum of its digits were divided by 9.

Thus, $687 \div 9$ leaves 3; and $(6+8+7) \div 9$ leaves 3.

(ii.) If each of two no^s be divided by any n°, say 9, and the product of their remainders be taken, this product divided by 9 will leave the same remainder that would be left if the product of the two no^s were divided by 9.

Thus, $1547 \div 9$ leaves 8, and $687 \div 9$ leaves 3; then, $(8 \times 3) \div 9$ leaves 6, and $(1547 \times 687) \div 9$ also leaves 6.

The *first* of these considerations will appear just from the following illustration.

10, or 100, or 1000, or any other power of 10, is an exact n° of *nines* + 1; therefore,

80	is an exact n° of	*nines*	+ 8,
600	is	ditto	+ 6,
680	is	ditto	+ 6 + 8,
687	is	ditto	+ 6 + 8 + 7;

so that $687 \div 9$ leaves the same remainder as $(6+8+7) \div 9$.

It is evident, then, that to ascertain what remainder would be left after dividing any n° by 9, we need only sum the digits of the n°, and cast out 9 as often as it arises in the addition.

The *second* consideration may be illustrated by the following example;—

Since $1547 = 171$ *nines* $+ 8$,
and $687 = 76$ *nines* $+ 3$,
therefore, 1547×687 is equal to

(171 *nines* + 8) × 76 *nines*, [which gives an exact n° of *nines*]
+ (171 *nines* + 8) × 3; [which gives an exact n° of *nines* + 8 × 3];

evidently, therefore, the whole product is an exact n° of *nines* + 8 × 3, or + 24, or + 6; the 6 being obtained by adding the digits 2 and 4.

Note II.

When a divisor is composed of two or more factors, and the quotient is found by using those factors successively, the remainders after the several divisions may be converted into the full remainder in the manner employed in the following example:—

Divide 39711 by 35, or by 5 × 7.

5)39711		Or, 7)39711	
7) 7942...1	5 × 4 + 1	5) 5673...0	7 × 3 + 0
1134...4	= 21 *rem.*	1134...3	= 21 *rem.*

Quotient, $1134\frac{21}{35}$, or $1134\frac{3}{5}$.

Dividing by 5 first, the successive remainders are 1 and 4; or, dividing by 7 first, they are 0 and 3; and to find the entire remainder, we multiply the first divisor by the second remainder, and to the product add the first remainder.

The reason of this procedure may be shown thus:—

We are required to find how many *thirty-fives* are contained in 39711 *units*. Dividing first by 5 *units* we find that 39711 is = 7942 *fives* + 1 *unit;* and then dividing the *fives* by 7 we find them = 1134 *thirty-fives* + 4 *fives;* so that 37911 *units* are equal to

1134 *thirty-fives* + 4 *fives* + 1 *unit,* = 1134 *thirty-fives* + 21 *units;*
= 1134 *thirty-fives* + $\frac{21}{35}$ of 35; = $1134\frac{21}{35}$ *thirty-fives.*

In the second form of the division we have 0 as the first remainder: in such instances, the second remainder placed over the second divisor gives the fractional part of the quotient in a simpler form.

Note III.

Strictly, in reducing £37 to shillings, we multiply—not £37 by 20, which would produce £740, but 37 by 20; the reasoning is that £37 contains 20 times as many shillings as pounds.

Note IV.

The multiplication of dimensions is frequently performed by what is called the method of *Duodecimals*, which subdivides both square feet and cubic feet into denominations called *primes, seconds, thirds,* &c.; 12

superficial primes being = a square foot, 12 cubic primes = a cubic foot, and, in both cases, 12 seconds = a prime, 12 thirds = a second, &c.

Primes, seconds, &c., are marked thus,

$$15 \text{ sq. ft. } 7' \ 10'' \ 5'''; \quad 15 \text{ cub. ft. } 7' \ 10'' \ 5'''.$$

In the first of these expressions the *seconds* evidently are *square inches*, for they are 144*ths* of a square foot; and if to these we add the 7 primes, or twelfths of a sq. foot, = 84 *one-hundred-and-forty-fourths* of a sq. foot, we have 94 sq. inches, and the whole expression is equivalent to 15 sq. ft. $94\frac{5}{12}$ sq. in.

In the second of the expressions the *thirds* are evidently *cubic inches*, for they are 1728*ths* of a cubic foot, and if to these we add the 7 primes and 10 seconds, which are $= \frac{7}{12} + \frac{10}{144} = \frac{1008}{1728} + \frac{120}{1728}$ of a cubic foot, we have 1128 + 5 cubic inches, and the whole = 15 cub. ft. 1133 cub. in.

Suppose, now, it is required to find by duodecimal multiplication the area of a rectangular surface, 37 ft. 7 in. by 5 ft. 9 in.

ft.	pr.	
37	7	
5	9	
187	11	
28	2	3
216 sq. ft.	1′	3′

Here, since 37 ft. 7 in. = $37\frac{7}{12}$ ft., if the rectangle were 1 ft. broad the area would be $37\frac{7}{12}$ sq. ft., or 37 sq. ft. 7′; then, as the breadth is $5\frac{9}{12}$ ft., we multiply 37 sq. ft. 7′ by 5 units 9 twelfths, as follows:—Placing the greater dimension over the less, we first multiply 37 sq. ft. 7′ by 5, then we multiply the same quantity by 9 considered as *twelfths*, and by setting the remainder, arising from a twelfth of 9 times 7, one place to the right of inches, and carrying 5 to the next product, of which in like manner we take a twelfth, we shall evidently have 9 *twelfths* of 37 sq. ft. 7′ = 28 sq. ft. 2 twelfths of a sq. ft. 3 twelfths of a twelfth of a sq. ft.

The entire product is 216 sq. ft. 1 prime 3 seconds.

If 12*ths* of an inch, commonly called *parts*, occur in either of the factors, the duodecimal multiplication is performed in the same way.

Let it be required to multiply 28 ft. 9 in. 6 pts. by 11 in. 9 pts.

ft.	pr.	sec.		
28	9	6		
0	11	9		
26	4	8	6	
1	9	7	1	6
28 sq. ft.	2′	3″	7‴	6⁗

It should be observed that the annexed process, which is conducted in the same way as the preceding one, is equivalent to finding first $\frac{11}{12}$ of the multiplicand, then $\frac{9}{144}$ or $\frac{1}{16}$ of it, and that we do not really multiply one concrete quantity by another, which would be absurd.

NOTE V.

For a demonstrative arithmetical example of the process of finding the greatest common measure of two numbers, *see* Hunter's *Art of Teaching Arithmetic*, p. 64. A very slight acquaintance with Algebra will enable the student to understand the following illustration of the general Rule for finding the G.C.M.

Let it be required to determine the G.C.M. of 1275 and 561.

The G.C.M. of 1275 and 561 evidently cannot exceed 561, and must be $=561\div$some factor of 561. Let x denote that factor. Therefore, the G.C.M. of the proposed nos will be $\frac{561}{x}$, when x has the least value that allows $\frac{561}{x}$ to measure 1275.

We have to find, then, the least value of x making $1275\div\frac{561}{x}$, or $\frac{1275x}{561}$ a whole no.

Now, $\frac{1275x}{561}=2x+\frac{153}{561}$ of x; so that $\frac{153}{561}$ of x is a whole no.

Put $\frac{153}{561}$ of $x=\text{A}$; $\therefore x=\frac{561}{153}$ of $\text{A},=3\,\text{A}+\frac{102}{153}$ of A;

$\therefore\ \frac{102}{153}$ of $\text{A}=$ a whole no, which we may call B;

$\therefore\ \text{A}=\frac{153}{102}$ of $\text{B}=\text{B}+\frac{51}{102}$ of B;

similarly, $\frac{51}{102}$ of $\text{B}=\text{C}$; $\therefore\ \text{B}=\frac{102}{51}$ of $\text{C},=2\,\text{C}$ exactly.

Now, we should get $\text{B}=$ a whole no, whatever whole no we might choose for the value of C; but we must take $\text{C}=1$, the lowest whole no, that we may obtain the lowest integral value of x.

Hence, $\frac{1275x}{561}=\frac{1275}{561}$ of $\frac{561}{153}$ of $\frac{153}{102}$ of $\frac{102}{51}$ of $1=\frac{1275}{51}$;

$\therefore\ \frac{x}{561}=\frac{1}{51}$, or, $\frac{561}{x}=51$, the G.C.M. required.

From the above analysis, then, it appears that the G.C.M. of two nos is obtained by dividing the greater by the less, then the less by the remainder, and so on as prescribed by the Rule.

To determine the G.C.M. of three nos, find that of two of them, and then that of the result and the third number. Thus, the G.C.M. of 12528, 16182, and 13804, will be found $=58$; for that of the first two nos is 522, and that of 522 and 13804 is 58.

To find the G.C.M. of fractional quantities, as, for example, of $8\frac{3}{4}$ and

$19\frac{5}{6}$, express them as fractions having a common denominator, then find the G.C.M. of the numerators, and under it write the common denominator. The result for the supposed example will be $\frac{7}{12}$, which is contained 15 times in the first n° and 34 times in the second.

NOTE VI.

For the conversion of a mixed circulating decimal to a vulgar fraction, the following rule is self-demonstrating:—Multiply the given decimal by 10, or 100, or 1000, &c., according as there are one, two, three, &c., decimal places before the circulating period; express the result as a mixed fraction, and then divide it by the 10, or 100, &c., previously used as a multiplier, which will evidently restore the value of the given expression.

Thus, to convert $.03\dot{4}$ and $.27\dot{3}\dot{4}5$ to vulgar fractions:—

(i.) $.03\dot{4} \times 100 = 3.\dot{4} = 3\frac{4}{9}$;

$$\text{and } 3\frac{4}{9} \times \frac{1}{100} = \frac{31}{900}.$$

(ii.) $.273\dot{4}\dot{5} \times 1000 = 273.\dot{4}\dot{5} = 273\frac{45}{99} = 273\frac{5}{11}$;

$$\text{and } 273\frac{5}{11} \times \frac{1}{1000} = \frac{3008}{11000} = \frac{376}{1375}.$$

What is further included in the usual Rule has reference to an easy method of multiplying by the denominators 9, 99, 999, &c.

$$\text{Thus, } 273\frac{45}{99} \times \frac{1}{1000} \text{ being} = \frac{273 \times 99 + 45}{99 \times 1000},$$

$$\text{and } 273 \times 99 \text{ being} = 273 \times (100 - 1),$$

$$\text{we have } \frac{27300 - 273 + 45}{99000} = \frac{27345 - 273}{99000}$$

$$= \frac{27072}{99000} = \frac{3008}{11000} = \frac{376}{1375}.$$

NOTE VII.

The series proposed for calculation in Ex. **47**, 48, is one by which the ratio of the circumference of a circle to its diameter may be approximately computed. *See* Colenso's *Plane Trigonometry*, Part II. p. 7. The result signifies that the circumference of any circle is nearly 3.14159 times the diameter.

The series proposed for calculation in Ex. **47**, 50, is that whereby what is called the base of the Napierian system of Logarithms is approximately computed. *See* Colenso's *Plane Trigonometry*, Part I. p. 121, or Hunter's *Treatise on Logarithms*, p. 55. The result signifies that the Napierian Logarithm of any given number is that power of 2.7182818 which when calculated produces the given number.

Note VIII.

Questions in Proportion can always be worked independently of the artificial Rule of *stating*, and though sometimes not so conveniently, yet always in a more satisfactory way as regards simplicity of demonstration. It will appear from the following examples that a knowledge of the first principles or fundamental rules of Arithmetic is sufficient for the solution of all problems in the Rule of Three.

(1) If 15 lbs. of salt cost 1*s.* 6*d.*, what cost 25 lbs.?

Cost of 15 lbs. = 18*d.*

" 1 lb. = $\frac{1}{15}$ of 18*d.*

" 25 lbs. = $\frac{25}{15}$ of 18*d.*

$$\frac{18d. \times 25}{15} = 6d. \times 5 = 2s.\ 6d.\ \textit{Ans.}$$

(2) If 25 lbs. of salt cost 2*s.* 6*d.*, what quantity cost 1*s.* 6*d.*?

No. of lbs. for 30*d.* = 25 lbs.

" 1*d.* = $\frac{1}{30}$ of 25 lbs.

" 18*d.* = $\frac{18}{30}$ of 25 lbs.

$$\frac{25\text{ lbs.} \times 18}{30} = 5\text{ lbs.} \times 3 = 15\text{ lbs. } \textit{Ans.}$$

(3) What is the coach fare for 130 miles at the rate of £1 9*s.* 4*d.* for 85 miles?

Fare for 85 miles = $29\frac{1}{3}$*s.*

" 1 mile = $\frac{1}{85}$ of $29\frac{1}{3}$*s.*

" 130 miles = $\frac{130}{85}$ of $29\frac{1}{3}$*s.*

$$\frac{88s. \times 130}{3 \times 85} = \frac{88s. \times 26}{51} = 44s.\ 10\tfrac{6}{17}d.\ \textit{Ans.}$$

(4) If 112 sheep were grazed in a field for 9 days, how long might 84 sheep have been grazed in the same field?

Time that 112 sh. were grazed = 9 da.

" 1 sh. might be grazed = 112 times 9 da.

" 84 sh. " " = $\frac{1}{84}$ of 112 times 9 da.

$$\frac{9\text{ da.} \times 112}{84} = 3\text{ da.} \times 4 = 12\text{ da. } \textit{Ans.}$$

(5) A person completed a journey in 32 days, travelling 8 hours a day; how long would he have taken to do the same, travelling only 6 hours a day?

No. of days at 8 hrs. a day = 32 da.

" at 1 hr. a day = 8 times 32 da.

" at 6 hrs. a day = $\frac{1}{6}$ of 8 times 32 da.

$$\frac{32\text{ da.} \times 8}{6} = \frac{128}{3} = 42\tfrac{2}{3}\text{ da. } \textit{Ans.}$$

(6) Three partners with a joint stock of £1026 11*s*. 6*d*. gain £287 6*s*.; what share of the gain falls to one of the partners whose stock is £365 17*s*.?

$$\textit{Gain} \text{ on } £1036\ 11s.\ 6d. \text{ (or 41463 sixp.)} = 5746s.$$

$$\text{,, on 1 sixp.} = \frac{1}{41463} \text{ of } 5746s.$$

$$\text{,, on } £365\ 17s.\ 0d. \text{ (or 14634 sixp.)}$$

$$= \frac{14634}{41463} \text{ of } 5746s.$$

$$\frac{5746s. \times 14634}{41463} = \frac{5746s. \times 1626}{4607} = £101\ 8s.\ \textit{Ans.}$$

(7) If $10\frac{5}{7}$ lbs. of sugar cost $4\frac{11}{16}s.$, what will $3\frac{2}{3}$ cwt. cost?

$$\textit{Cost} \text{ of } 10\tfrac{5}{7} \text{ lbs.} = 4\tfrac{11}{16}s.$$

$$\text{,, of 1 lb.} = \frac{7}{75} \text{ of } \frac{75}{16}s. = \frac{1}{16} \text{ of } 7s.$$

$$\text{,, of } 112 \times 3\tfrac{2}{3} \text{ lbs.} = \frac{112 \times 3\frac{2}{3}}{16} \text{ of } 7s.$$

$$\frac{7s. \times 112 \times 11}{16 \times 3} = \frac{49s. \times 11}{3} = £8\ 19s.\ 8d.\ \textit{Ans.}$$

Note IX.

In calculating the amount of any sum of money, by compound interest, for any n° of years, at 4 per cent. per annum, we add to the original principal $\frac{4}{100}$ of itself to obtain the 2nd principal, then to this principal we add $\frac{4}{100}$ of itself to obtain the 3rd principal, and so on.

Now, adding to any n° $\frac{4}{100}$ of itself is the same as multiplying it by $1\frac{4}{100}$, or by 1.04; and accordingly, the amount of £750 for 3 years, at 4 per cent. per annum., comp. int. might be found thus:—

$$£750 \times 1.04 \times 1.04 \times 1.04,$$
$$= £750 \times 1.04^3 = £750 \times 1.124864,$$
$$= £843.648.$$

Similarly, the amount of £750 for 4 yrs. at 5 per cent. would be $£750 \times 1.05^4$. And, generally, to find the amount of £P, by comp. interest, for any n° of years, at any annual rate, we may first add a hundredth of the rate to 1, then raise the sum to that power which is denoted by the n° of years, and then multiply by P.

Suppose that, in this way, we have to find the compound interest of £95 6*s*. 8*d*., for 3 yrs., at 5 per cent. per ann., payable half-yearly:— the rate is here intended to denote $2\frac{1}{2}$ per cent. per half-year, for 6 half-years.

We have accordingly to find the 6th power of 1.025; and this we could obtain at once from compound interest Tables; or we could very easily calculate it from a Table of Logarithms. The simplest form of the arithmetical process is as follows; the divisor 40 determining the interest in each case, because $2\frac{1}{2}$ is $\frac{1}{40}$ of 100.

40) 1.025	Amt. of £1 for		1	hf. yr.
.025625				
40) 1.050625	Do.	„	2	do.
.0262656				
40) 1.0768906	Do.	„	3	do.
.0269223				
40) 1.1038129	Do.	„	4	do.
.0275953				
40) 1.1314082	Do.	„	5	do.
.0282852				
1.1596934	Do.	„	6	do.

Hence the compound *interest* of £1, at the end of the 3rd year, is £.1596934; which multiplied by $95\frac{1}{3}$ gives the comp. int. of £95 6*s.* 8*d.* =£15.2241, or £15 4*s.* 5.78*d.* *Ans.*

Now, suppose it is required to find what principal at $2\frac{1}{2}$ per cent. per annum, comp. int., will in 6 yrs. amount to £110 3*s.* 5*d.*: that is, what is the present worth, by comp. int., of £110 3*s.* 5*d.* payable in 6 yrs.:—we have

$$1.025^6 \times P = 110.170833;$$

$$\therefore\ 110.170833 \div 1.1596934 = £95.\ \textit{Ans.}$$

Again; let it be required to find at what rate of comp. int. £95 will amount to £110 3*s.* 5*d.* in 6 yrs.:—

110.170833 ÷ 95 = 1.1596934, the 6th root of which may be found by logarithms = 1.025; or, $\sqrt{1.1596934} = 1.0768906$, the cube root of which is 1.025. Hence the rate is $2\frac{1}{2}$ per cent. *Ans.*

Lastly; to find in what time £95 will amount to £110.170833, at $2\frac{1}{2}$ per cent. per ann., comp. int.:—Here we should ascertain by logarithms what power of 1.025 is equal to 1.1596934; but when the time is an exact n° of years, as in this instance, it would be found by raising 1.025 through consecutive powers till the required amount of £1 is found equal to the 6th power, denoting the time to be 6 yrs.

Note X.

A Rule called Equation of Payments is introduced in some treatises on Arithmetic. It teaches how to ascertain the single time at which two or more debts, due at different times, might be discharged by one payment of the sum of the debts. It is merely a particular application of the principle of Discount; and it is given in two forms, according to true discount and mercantile discount, respectively.

Examp. I owe £1085; of which £651 is due 5 months hence, and £434 is due 8 months hence; how many months hence would one payment of £1085 discharge both debts, reckoning the use of money worth 5 per cent. per annum?

We compare the several sums by means of their present values, considering that the discount on £651 for 5 months added to the discount on £434 for 8 months, should be equal to the discount on £1085 for the time sought.

Now, according to *Mercantile Discount*, we have

$$\frac{5}{1200} \text{ of } £3255 = \text{int. of } £651 \text{ for 5 months;}$$

$$\text{and } \frac{5}{1200} \text{ of } £3472 = \text{int. of } £434 \text{ for 8 months;}$$

$$\therefore \frac{5}{1200} \text{ of } £6727 = \text{int. of } £1085 \text{ for } 6{\cdot}2 \text{ months. } \textit{Ans.}$$

because $6727 \div 1085 = 6{\cdot}2$.

This method is evidently independent of the rate of interest; and hence, for equating terms of payment according to mercantile discount, we have the following

Ordinary Rule. Multiply the several debts by their times in any uniform denomination, and divide the sum of the products by the sum of the debts.

Thus, the above process is reduced to the following:—

$$\begin{array}{rl} 651 \times 5 = & 3255 \\ 434 \times 8 = & 3472 \\ \hline 1085 \quad) & 6727 \\ & 6\tfrac{1}{5} \text{ months. } \textit{Ans.} \end{array}$$

The meaning of which is, that as the int. of £651 for 5 months is that of £3255 for a month, and the int. of £434 for 8 months is that of £3472 for a month, so the int. of £6727 for a month is that of £1085 for $6\frac{1}{5}$ months.

But secondly, according to *True Discount*, we have

$$\frac{5}{12} \text{ of } £5, \text{ or } £2\tfrac{1}{12} = \text{disc. on } £102\tfrac{1}{12} \text{ for 5 mths.}$$

$$\text{or } \frac{1}{49} = \text{disc. on } 1;$$

$$\frac{8}{12} \text{ of } £5, \text{ or } £3\tfrac{1}{3} = \text{disc. on } £103\tfrac{1}{3} \text{ for 8 mths,}$$

$$\text{or } \frac{1}{31} = \text{disc. on } 1;$$

$\therefore \frac{651}{49} = £13\frac{2}{7}$ is the disc. on £651 for 5 mths.

$\frac{434}{31} = \ \ 14$ is the disc. on 434 for 8 mths.

$£27\frac{2}{7}$ is the disc. on £1085 for the time sought.

We have to find, therefore, in what time $£1057\frac{5}{7}$ would produce $£27\frac{2}{7}$ interest, or £7404 would produce £191.

$$\left.\begin{array}{r} 7404 : 100 \\ 5 : 191 \end{array}\right\} :: 12 \text{ mo.} : 6\tfrac{118}{617} \text{ mo. } \textit{Ans.}$$

This answer, equal to about 6.19 months, is a little less than 6.2, the answer found according to mercantile discount; but as the method of true discount is much more laborious than the other, and in most practical questions gives a result very little less than the other, it is generally sufficient, as it is more convenient, to follow the ordinary rule.

The Rule for equating according to true discount may be given as follows:—

Find for each of the debts the discount that would reduce it to its true present value; then find the time for which the sum of the discounts would be the true discount on the sum of the debts.

For a discussion of the principle of Equation of Payments, *see* Hunter's *Art of Teaching Arithmetic*, p. 79.

Note XI.

In Paper IX. will be found a variety of Questions relating to the comparison of the money of different countries. This subject is frequently treated in books on Arithmetic under a special Rule called *Exchange.*

The *Par* of Exchange is the intrinsic value of the coin of one country as compared with a fixed sum of the money of another. The *Course* of Exchange is the variable sum of the money of one country actually given for a fixed sum of the money of another.

Thus, France exchanges with England a variable number of francs, averaging about 25.30, for the pound sterling; for the actual *Course* of Exchange, being dependent on the course of trade, is in almost continual fluctuation. Moreover, as in England gold is the adopted standard of value, and France has a silver standard;—as also the values of gold and silver are not always in the same proportion, and each metal has not always the same value in both countries,—the *Par* itself is not invariable.

Arbitration of Exchange is the estimation of the rate of Exchange implied in the purchase of indirect Bills of Exchange, Bullion, Coins, &c., in one country, as compared with their sale in another.

Thus, to find what arbitrated rate of Exchange is established between

London and Paris by bills on Vienna bought in London at 10 florins 1 kreutzer per £ sterling, and sold in Paris at 254 francs per 100 florins; a florin being = 60 kreutzers:—

Here we have given £1 = 601 kreutzers, and 600 kreutzers = 25.4 francs; $\therefore$ 1 kr. $= \frac{25.4}{600}$ fr.,

and 601 kr. = 25.4 fr. $\times 1\frac{1}{600}$ = 25.44 fr. per £. *Ans.*

Again; to find what arbitrated rate is established between London and Paris by the purchase of gold in London at 77*s.* $10\frac{1}{2}$*d.* per ounce standard, and the sale of it in Paris at 4 per mille premium: an ounce Troy being = 31.1 grammes, and 1000 grammes of English standard gold being worth 3151 francs:—

Here we have 311 grammes = 10 oz., or 1 gramme $= \frac{10}{311}$ oz.;

$$\therefore 1000 \text{ grammes} = \frac{10000}{311} \text{ oz.},$$

1000 grammes bought in London for $77\frac{7}{8}s. \times \frac{10000}{311}$;

1000 grammes sold in Paris for 3151 frs. × 1.004;

$\frac{6230000}{8 \times 311}s. : 20s. :: 3163.6$ frs. : 25.27 frs. *nearly. Ans.*

EXAMINATION-PAPERS.

Paper I.

Questions on the Introductory Pages.

1. (*a*) Explain the principle by which the decimal system of notation is made capable of expressing any number whatever.

(*b*) Distinguish between the arts of Notation and Numeration.

2. Add *Thirteen thousand thirteen hundred and thirteen* to *Seventeen thousand seventeen hundred and seventeen.*

3. Subtraction may be performed (*a*) for the purpose of diminishing a quantity by taking away some quantity it contains, or (*b*) for the purpose of comparing two quantities as to their absolute magnitudes. Give properly distinctive names for the results in these two cases.

4. (*a*) If two numbers be equally increased, how is their difference affected? A father is 3 score and 5 years old, and his son is 37; what is the difference of their ages? and what will be the difference of their ages 10 years hence? (*b*) Apply these considerations to explain the process of borrowing *ten* and carrying *one* in subtraction.

5. (*a*) What name is given to two or more numbers connected by multiplication? (*b*) Show how six *sevens* are equal in amount to 7 *sixes.* (*c*) Show why multiplying successively by 6 and 7 gives the same result as multiplying by 42.

6. What are the methods commonly used for proving the accuracy of multiplication? How might division (if the pupil understood that process) be used as a trial of correctness in multiplication?

7. (*a*) Divide 27564 by 21 in two ways:—resolving 21, first, into successive divisors 7 and 3, and secondly, into successive divisors 3 and 7. (*b*) Explain by reference to your work the usual process of finding the full remainder by means of the two partial remainders.

Paper II.

Questions on Articles 1 *to* 20.

1. In reducing £7 to shillings what multiplier, strictly considered, do we employ? Explain.

2. In dividing a concrete quantity by an abstract number, as for example in finding the 8th part of £3 7*s.* 6*d.* (Colenso, p. 24), which of the expressions is properly the quotient? and why?

3. How would you reduce crowns to guineas? florins to crowns? sovereigns to guineas? yards to English ells? lbs. Avoirdupois to lbs. Troy?

4. (*a*) Under what conditions may one concrete quantity be added to another? subtracted from another? divided by another?

(*b*) Why cannot one concrete quantity be multiplied by another?

5. (*a*) How is the square measure of a rectangular surface found from its length and breadth? If the length be 5 feet, and breadth 4 feet, is the area = 5 ft. × 4 ft.? Explain.

(*b*) How is the width of a rectangular space found when the length and area are given?

6. (*a*) How is the cubic measure of a rectangular solid found from its length, breadth, and height? Suppose the dimensions are 8, 6, and 2 feet:—explain the process of finding the solidity.

(*b*) How is the height or the thickness of a rectangular solid found, when its cubic content and its length and breadth are given?

Paper III.

Questions for Illustration of Ex. **17.**

1. (*a*) A man's yearly income is known:—How would you find the sum he must spend weekly, so as to lay by a given sum at the year's end?

(*b*) Given, a man's daily income and his yearly expenditure:—How do we find his weekly saving?

2. The sum of 3 crowns, 3 florins, and 3 pence, is equal to 3 times the sum of a crown, a florin, and a penny, that is, 3 times 85*d.*—Apply this consideration to the solution of Exs. 59, 61, and 62, in Set **17.**

3. If £342 is to be multiplied by 242, and the product divided by 11, 8, and 4, successively, the effect of the whole may be symbolically expressed thus, $\frac{£342 \times 242}{11 \times 8 \times 4}$, which, by cancelling, becomes $\frac{£171 \times 22}{8 \times 2}$ and by further cancelling becomes $\frac{£171 \times 11}{8} = \frac{£1881}{8} = £235$ 2*s.* 6*d.* —Apply this mode of treatment to the solution of Exs. 45, 55, and 67, in Set **17.**

4. How do you find the average value per yard of a quantity of goods, consisting of 20 yards at 12*s.* 6*d.* and 35 yards at 9*s.* 10*d.*?—Would the result be affected by the alteration of taking one-fifth of each

of the given quantities, making them together = 11 yards?—Solve Exs. 42 and 51, in Set **17.**

5 In Ex. 63, Set **17**, show that the result equals $\frac{1\frac{3}{4} \times 40}{24}$ years, or $\frac{70}{24}$, or $\frac{35}{12}$, of a year, $= 2\frac{11}{12}$ years; and explain the following process:—

$$\begin{array}{rrr} & 12)365 \text{ da.} & 6 \text{ hrs.} \\ & 30 & 10\frac{1}{2} \\ \hline \textit{Ans. } 2 \text{ yrs.} & 334 \text{ da.} & 19\frac{1}{2} \text{ hrs.} \end{array}$$

6. (*a*) Reduce 4 men 7 boys to an equivalent number of boys, supposing a man equivalent to 3 boys.

(*b*) Reduce 7 men 12 women 5 children to an equivalent number of children, supposing 2 women equivalent to a man, and 3 children equivalent to a woman.

(*c*) Apply the above species of reduction to the solution of Exs. 58 and 65, in Set **17.**

7. (*a*) If the number 365 is to be divided into four parts, three of them equal, and the fourth 95 less than each of the others; how many times the first part would make 365 + 95?

Apply a similar mode of inquiry in the solution of Ex. 64, Set **17.**

(*b*) If the sum of two numbers is 135 and their difference is 95, show how each number may be found.

Divide a sovereign between Harry and George, giving George 20*d.* less than Harry.

8. Explain the following method of solving the latter part of Ex. 68, in Set **17** ; and find the first answer similarly :—

3 yrs. profit on 500 ac. @ £4 2*s.* 4*d.* =	£2058	6	8
Tithes = produce of 50 ac. @ £27 4*s.* 0*d.* =	1360	0	0
2*nd Ans.* Gain in the three years	£698	6*s.*	8*d.*

Paper IV.

Questions on Chapters II, III, and IV.

1. What is meant by a common measure of two or more numbers? How is their G.C.M. ascertained?

2. What is meant by a multiple of a number? How do you find the L.C.M. of two or more numbers?

3. Show that the product of two numbers divided by their G.C.M. gives their L.C.M.

4. Find that the G.C.M. of 11310, 12354, and 64090, is 58.

5. How do you find the G.C.M. of numbers all or partly fractional? Find the G.C.M. of $26\frac{1}{4}$, $28\frac{7}{8}$, and $29\frac{1}{6} = \frac{7}{24}$.

6. How do you find the L.C.M. of numbers all or partly fractional? Find the L.C.M. of $10\frac{1}{2}$, $6\frac{7}{8}$, and $4\frac{9}{10} = 4042\frac{1}{2}$.

7. What is a fraction? Is 3 farthings an integral or a fractional quantity? Define a concrete fraction.

8. What arithmetical operation is signified by the line separating the terms of a fraction? What is an improper fraction, and how is it reduced to a proper form?

9. What rule of fractions is anticipated in reducing a mixed fraction to an improper one?

10. Why is it necessary that fractions should be of one common denominator for addition or subtraction?

11. (*a*) Show that multiplying the numerator of a fraction is equivalent to dividing the denominator, and that dividing the numerator is equivalent to multiplying the denominator.

(*b*) Hence show that the value of a fraction is not changed by multiplying or dividing both its terms by any one number.

12. What name is given to a fractional expression of the form $\frac{3}{5}$ of $\frac{7}{8}$? Which quantity is thus denoted to be a multiplier of the other?

13. (*a*) Prove the rules for multiplication and division of fractions: exemplify with $\frac{3}{5}$ and $\frac{7}{8}$.

(*b*) What does multiplication by a fraction strictly mean?

14. Explain the meaning of such a fraction as $\dfrac{£1\ 11s.\ 8d.}{£2\ 17s.}$

15. (*a*) A certain quantity, *A*, is given:—If it be $\frac{3}{5}$ of another quantity *B*, how would you find *B*? If it be half as much again as *B*, how would you find *B*?

(*b*) A number increased by its 5th part amounts to 30: how would you find the number?

(*c*) A number diminished by its 5th part becomes 24: how would you find the number?

16. (*a*) Distinguish between decimal and vulgar fractions. What is the special utility of decimal fractions?

(*b*) Compare the metrical, or French, scale of lineal measure with the English.

17. (*a*) State and prove the rule for pointing in multiplication of decimals. (*b*) How do you determine the local values of the quotient figures in division of decimals?

18. (*a*) What are circulating decimals? (*b*) Distinguish those vulgar fractions that are convertible into terminating decimals; and show that all others are convertible into recurring decimals.

Paper V.

Supplementary Questions in Reduction of Measures.

1. Reduce 22870062 square inches to acres, &c.

```
     ┌12)22870062
144 ┤
     └12) 1905838... 6 ┐
          9) 158819...10 ┘ 126 in.

30¼)      17646... 5 ft.
 4           4

121  ┌ 11)70584 qr. yds.
     └ 11) 6416 .. 8 ┐ 41 qr. yds. = 10 yds. 2 ft.  36 in.
        40)583... 3  ┘                          5   126
           4) 14...23 po.
```

Ans. 3 ac. 2 ro. 23 po. 10 yds. 8 ft. 18 in.

2. Reduce the preceding result to square inches.

```
3 ac. 2 ro. 23 po. 10 yds. 8 ft. 18 in.
4
14      ro.
40
583     po.
30¼
145¾
17500
17645¾  yds.
9
158819¾  ft.
12
1905837
12
22870062  in. Ans.
```

¶ 3. Reduce 1254492 sq. in. to sq. poles, &c.

4. Reduce 1 ac. 3 ro. 39 po. 14 yd. 5 ft. to sq. inches.

5. Reduce 123456789 sq. inches to acres, &c.

6. Reduce 2 ac. 3ro. 13 po. 14 yd. 5 ft. 100 in. to sq. inches.

7. Reduce 9532482 sq. inches to acres, &c.

8. Reduce 2 ro. 22 po. 14¼ yd. to sq. feet.

9. Express 22 sq. po. 2 yd. 4 ft. 72 in. in the denomination of sq. yards.

10. An imperial gallon measures 277.274 cubic inches; how many gallons would a vessel contain of which the capacity is 196½ cub. feet?

11. The length of a wall, according to the French metrical system, is 9 metres 4 decimetres 8 centimetres; reduce this to English feet, the length of the metre being 39.371 inches.

12. Reduce 13 feet to metres.

13. How many decametres correspond to 1760 yards?

14. A chain 66 feet long is divided into 100 equal parts called links. Reduce an acre to square links.

15. A rod of brickwork, viz. a square pole, or $272\frac{1}{4}$ square feet, has a standard thickness of a brick and a half:—If a piece of brickwork be 48 feet long and 22 feet high, and $2\frac{1}{2}$ bricks thick, to how many rods of standard thickness is it equivalent?

Paper VI.

Questions on Ratio. (See *Art.* 73.)

1. If the ratio of L to M is 5 : 8, and that of M to N is 6 : 7; what is the simplest form of the ratio of L to N?

Here L is $\frac{5}{8}$ of M, and M is $\frac{6}{7}$ of N;

$\therefore$ L is $\frac{5}{8}$ of $\frac{6}{7}$ of $N = \frac{15}{28}$ of N. *Ans.*

Or, L is to N as 15 : 28. *Ans.*

2. M buys 15 cows and 130 sheep for a certain sum, and N buys 9 cows and 175 sheep, at the same rates as M, for the same sum. Compare the values of a sheep and a cow.

Since N has 6 cows *fewer* than M,
but has 45 sheep *more* than M,
and both persons pay the same amount,
it is evident that 6 cows are worth 45 sheep,
or 1 sheep worth $\frac{6}{45}$, or $\frac{2}{15}$, of a cow,
or the values of a sheep and a cow are as 2 : 15. *Ans.*

3. One vessel contains a mixture of 16 pints of brandy and 5 of water; another contains 24 pints of brandy with 11 of water. Compare the strengths of the two mixtures.

1st mixture 21 pints, 16 of which are brandy,
2nd „ 35 „ 24 „ „

$\therefore$ the strengths are $\frac{16}{21}$ and $\frac{24}{35}$,

or as $\frac{2}{3}$ to $\frac{3}{5}$, or as 10 : 9. *Ans.*

¶ 4. A boat whose speed was $9\frac{3}{4}$ miles an hour sailed from *A* to *B*, a distance of 65 miles; and a second boat, which left *A* $2\frac{1}{4}$ hours after the first, arrived at *B* 5 minutes before the first. Compare the rates of sailing.

5. *A* and *B* buy oranges at 10 for a shilling; *A* retails them at 9 for a shilling, and *B* at 17*d.* for a dozen. Compare their gains on selling the same number of oranges.

6. If *A's* rate of profit is $\frac{4}{9}$ of *B's*, and for every guinea that *B* gains *C* gains a sovereign, compare the profits of *A* and *C*.

7. A sum of money is so divided among Roger, Henry, William, and Thomas, that R. gets 3*d.* as often as H. gets $2\frac{1}{2}d.$, H. gets 3*d.* as often as W. gets $4\frac{1}{2}d.$, and W. gets 4*d.* as often as T. gets $3\frac{1}{2}d.$ Find the direct proportion of the four shares.

8. If 3 men and 11 boys, working together, can do 5 times as much work per hour as a man and a boy together, compare the work of a boy with that of a man.

9. One vessel *M* contains a mixture of 27 gallons of wine and 11 of spirits; another vessel *N* contains a mixture of 43 gallons of wine and 14 of spirits. Compare the strengths of the two mixtures, supposing the strength of spirits to be three times that of wine.

Paper VII.

Questions on Averages.

1. In a school register of daily attendance the numbers for a certain week were—Monday 83, Tuesday 80, Wednesday 75, Thursday 80, Friday 77, Saturday 72. What was the average daily attendance?

2. A tradesman's receipts of money in one week were—Mon. $33/10\frac{1}{2}$, Tues. 26/6, Wednes. *nothing*, Thurs. $10/8\frac{1}{2}$, Fri. $43/11\frac{1}{2}$, Saturday 30/10. What was the average daily receipt?

3. The quantities of maize raised in the United States, in three successive years, were—494618200, 421953000, and 417899000 bushels. What, in British currency, was the value of the average yearly produce, rating it at 25 cents per bushel, and reckoning the dollar of 100 cents to be worth 4*s.*?

4. Required the mean of the following observations of temperature:—$41^\circ\ 29'$, $41^\circ\ 27\frac{1}{2}'$, $39^\circ\ 13'$, $41^\circ\ 33'$, $37^\circ\ 47\frac{1}{2}'$, $44^\circ\ 28'$, and $40^\circ\ 13'$.

5. If 3 quarts of stout at 9*d.* a quart are mixed with 10 pints of ale at $2\frac{1}{2}d.$ a pint, what is the worth of a pint of the mixture?

6. At a competitive examination there were 4 candidates at the age of 19, 3 at 20, 2 at 21, and 3 at 23. Find the average age.

7. How many square feet are in a regularly tapering plank 10 ft. 6 in. long, the width being 9 inches at one end and 7 inches at the other?

8. The average of twenty-one results is 61, that of the first eight being 64, and of the next eleven 59. Required the average of the last two.

9. Three quantities of tea, at 3/8, 4/2, and 4/4 per lb., respectively, make a mixture of 136 lbs., there being 5 lbs. more of the first kind than of the second, and 6 lbs. more of the third than of the first and second together. What is the worth of the mixture per lb.?

10. The average of ten results was $17\frac{1}{2}$; that of the first three was $16\frac{1}{4}$, and of the next four $16\frac{1}{2}$; the eighth was 3 less than the ninth, and 4 less than the tenth. What was the last result?

11. If 9 gallons of spirits at 18/6 are mingled with 7 gallons at 21/, how much water must be added to reduce the value to 16/6 a gallon?

Paper VIII.

Questions on the Relation between Time and Power.

1. *M* can do a piece of work in 20 days of 7 hours, and *N* can do it in 14 days of 8 hours. For how many hours a day should *M* and *N* be engaged together, that the work may be done in 10 days?

M does 1 measure of work per hour;
140 such measures = the whole work.
N can do, per hour, the 112th of the whole,
viz. $140 \div 112$, or $1\frac{1}{4}$ measures;
$\therefore$ *M* and *N* together do $2\frac{1}{4}$ meas. per hour;
or the whole work in $140 \div 2\frac{1}{4} = 62\frac{2}{9}$ hrs.
$62\frac{2}{9}$ hrs. $= 10$ days, is $6\frac{2}{9}$ hrs. a day. *Ans.*

2. A cistern is filled by two pipes, *A* and *B*, in 20 and 24 minutes respectively, and is emptied by a tap *C* in 30 minutes. What part of it will be filled in 15 minutes, if *A*, *B*, and *C* are all turned on together?

If *A* runs 1 measure per minute, 20 measures would fill the cistern; then *B* would run, per minute, the 24th of 20, viz. $\frac{5}{6}$ of a measure, and *C* the 30th of 20, viz. $\frac{2}{3}$ of a measure; and *A*, *B*, and *C* being all opened, the cistern would gain $1 + \frac{5}{6} - \frac{2}{3}$, or $1\frac{1}{6}$ meas. per minute, and in 15 min. would gain
$1\frac{1}{6} \times 15 = 17\frac{1}{2}$ meas.,
which is $17\frac{1}{2}$ *twentieths* $= \frac{7}{8}$ of the cistern. *Ans.*

3. *F* and *G* together reap a field in $8\frac{3}{4}$ days, and *F* alone can reap as much in $3\frac{1}{2}$ days as *G* can do in 5. In what time could each by himself reap the field?

F in 1 day does 1 measure, G $\frac{1}{5}$ of $3\frac{1}{2}$ meas. $=\frac{7}{10}$ meas.

$\therefore$ the whole work is $1\frac{7}{10}\times 8\frac{3}{4}=14\frac{7}{8}$ meas.

$$\left.\begin{array}{l} 14\frac{7}{8}\div 1 = 14\frac{7}{8} \text{ da. by } F \text{ alone} \\ 14\frac{7}{8}\div\frac{7}{10}=21\frac{1}{4} \text{ da. by } G \text{ alone,} \end{array}\right\} Ans.$$

4. *Y* and *Z* began together a piece of work which they could have done singly in 34 and 38 days, respectively. *Y* continued till the work was finished; but *Z* had left him 4 days before its completion. In what time was the work done?

Y did 1 measure per day, and the whole work was 34 measures; so that *Z* did, per day, the 38th of $34=\frac{17}{19}$ of a measure.

Now, if *Z* had continued the whole time of *Y*, 4 times $\frac{17}{19}$, or $3\frac{11}{19}$ *extra* measures of work would have been done, viz. $37\frac{11}{19}$ meas. by both agents in *Y's* time; therefore

$$37\tfrac{11}{19}\div 1\tfrac{17}{19}=714\div 36=19\tfrac{5}{6} \text{ da. } Ans.$$

5. A cistern has two supplying pipes, *A* and *B*, and a tap *C*. When the cistern is empty, *A* and *B* are turned on, and it is filled in 4 hours; then *B* is shut and *C* turned on, and the cistern is quite emptied in 40 hours; when, lastly, *A* is shut and *B* turned on, and in 60 hours afterwards the cistern is again filled. In what time could the cistern be filled by each of the pipes *A* and *B*, singly?

A and *B* together supply 1 measure per hour, and the whole content of the cistern is 4 measures.

B runs, per hour, more than *C*, $\frac{1}{60}$ of the 4 meas.

C runs, per hour, more than *A*, $\frac{1}{40}$ of the 4 meas.

$\therefore$ *B* runs $\frac{4}{60}+\frac{4}{40}$, or $\frac{1}{6}$ meas. per hour more than *A*.

$\therefore$ *A* and *B* together, in 1 hour, run $\frac{1}{6}$ meas. more than *A* runs in 2 hours;

but *A* and *B* together run 1 measure per hour;

$\therefore$ *A* runs $1-\frac{1}{6}$, or $\frac{5}{6}$ meas. in 2 hours,

B runs $2-\frac{5}{6}$, or $\frac{7}{6}$ meas. in 2 hours;

$$\left.\begin{array}{l}\frac{5}{6}\text{ meas. : 4 meas. :: 2 hrs. : }9\frac{3}{5}\text{ hrs. by }A,\\ \frac{7}{6}\text{ meas. : 4 meas. :: 2 hrs. : }6\frac{6}{7}\text{ hrs. by }B,\end{array}\right\}\textit{Ans.}$$

¶ 6. *A* can do a piece of work in 25 days, *B* can do it in 20 days, and *C* in 24. The three work together for 2 days, and then *A* and *B* leave; but *C* continues, and, after $8\frac{3}{5}$ days, is rejoined by *A*, who brings *D* along with him, and these three finish the remainder of the work in 3 days more. In what time would *D* alone have done the whole work?

7. A piece of work can be done by *A* and *B* together in 14 hours, or by *B* and *C* in $10\frac{1}{2}$ hours, or by *A* and *C* in 12 hours. In what time could each person do it by himself?

8. To complete a certain work, *B* would take twice as long as *A* and *C* together, and *C* thrice as long as *A* and *B* together; and *A*, *B*, and *C*, by their united exertions can do it in 5 days. In what time could each do it by himself?

9. *A* can do a piece of work in 10 days, *B* in 9, *C* in 12. They all begin it together; but only *C* continues till the work is finished,—*A* leaving it $3\frac{3}{5}$ days, and *B* $2\frac{2}{5}$ days before its completion. In what time is it performed?

10. A cistern has two pipes, *A* and *B*, which singly could fill it in 9 hours and 10 hours, respectively. It has also two taps, *C* and *D*, which singly could empty it in 12 hours and 8 hours, respectively. Suppose that when the cistern stands half-full of water, *A* and *D* are turned on for 3 hours; that then *B* is also turned on for the next 2 hours; that then *A* and *D* are turned off, and *C* is turned on for the next 8 hours; after which all are shut, and the cistern is found to contain 95 gallons more than its half content:—Find the content of the cistern. Find also how much per hour the cistern would lose or gain, if all the pipes were set open at once.

Paper IX.

Questions on Exchange. (See *Note* XI.)

1. Reduce 396 dollars 53 cents American to British money, at 4*s.* 6*d.* per dollar.

2. Convert 1206.70 American dollars into French money, at 5 francs 45 centimes per dollar.

3. Reduce £3758 16*s.* 6*d.* to francs, at 25.35 francs per £.

4. Find the value, in British money, of goods sold for 7889 francs 90 centimes,—exchange, 24 fr. $41\frac{1}{2}$ cts. per £.

5. What in English money is the value of the franc, at the exchange of 25.57 francs per £ sterling?

6. How many pence per milree (=1000 rees) is the exchange between Portugal and Britain, when £823 5*s.* 6*d.* worth of wine costs 3161 milrees 375 rees?

7. If, when the course of exchange between England and Spain is $38\frac{1}{2}d.$ per dollar of 20 reals, a merchant in Liverpool draws a bill of £354 16*s.* 3*d.* on Madrid, how many dollars and reals will pay the draft?

8. What is the arbitrated rate of exchange between London and Lisbon, when bills on Paris, bought in London at 25.65 francs per £, are sold in Lisbon at 525 rees per 3 francs?

9. If 11.65 Dutch florins are given for 24.89 francs, 383 florins for 437 marks Hambro', and $68\frac{1}{4}$ marks for 32 silver rubles of Petersburgh; how many francs should be given for 932 silver rubles?

10. Reckoning a Roman scudo worth $5\frac{2}{5}$ francs, and a shilling worth $1\frac{1}{4}$ franc, what amount of discount do I allow by accepting £10 in exchange for 45 scudi and 12 francs? And if I were to allow 4 per cent. discount, how many francs along with 50 scudi should I give for £12?

11. A merchant in London owes to one in Amsterdam 350.75 florins, which must be remitted through Paris. The quotations being, for London on Paris 25 francs 30 cents. per £, and for Amsterdam on Paris $45\frac{1}{2}$ florins per 100 francs, the London merchant delays remitting till the rates are 25.45 francs per £, and 11 florins per 24 francs. What does he gain or lose by the delay?

12. £1000 sterling is due from London to Portugal, when the exchange is $61\frac{1}{2}d.$ per milree. Whether is it better, for Portugal, to draw directly on London, or circuitously, at an expense of $1\frac{1}{2}$ per cent., through Holland and France;—exchange between Britain and Holland 11.90 florins per £ sterling, between Holland and France 10 florins for 21 francs, and between France and Portugal 480 rees for 3 francs?

13. When English money bears a premium of 5 per cent. in America, how much sterling should be given for 750 dollars, each worth 4*s.* 6*d.* at par?

14. A rupee contains 16 annas each 12 pice:—Find, in French money, the annual interest, at $3\frac{1}{2}$ per cent., on 5217 rup. 3 an. 6 pi., exchange 2.63 francs per rupee.

15. If goods bought in London at a guinea be exported to New York, at how many dollars should they be sold there, in order to cover all expenses; estimating the export charges to be $7\frac{1}{2}$ per cent., and the sale charges 5 per cent.; the course of exchange being 6 per cent. premium for bills on London?

16. At what price in Company's rupees (each = 16 annas) was indigo purchased in Calcutta, if the sale of it in London at 5*s.* per lb. yielded a profit of 20 per cent.; the shipping charges in Calcutta being 6 per cent., sale charges in London 9 per cent., and loss of weight $1\frac{1}{4}$ per cent.:—exchange 25*d.* per rupee?

17. Given—that 1 ounce Troy equals 31.1 grammes; that 10 grammes of French standard gold are worth 31 francs; and that the worth of a given weight of English standard gold is to that of the same weight of French standard as 3151 to 3100:—

(i.) To what number of Troy ounces of English standard gold is the franc equivalent, and what is the fixed number of francs equivalent to £1?—the English mint price for standard gold being 77*s.* $10\frac{1}{2}$*d.* per ounce.

(ii.) How many francs are equivalent to £1, when gold purchased in London at 77*s.* $10\frac{1}{2}$*d.* is sold in Paris at $14\frac{1}{2}$ per mille (i.e. per 1000) premium on the fixed price? and how many, when gold is at 1 per mille discount?

(iii.) Find that the results are correctly stated in the following newspaper reports; and give the percentage results more nearly:—

a. The premium of gold at Paris is $7\frac{1}{2}$ per mille, which, at the English mint price of £3 17*s.* $10\frac{1}{2}$*d.* per ounce for standard gold, gives exchange $25.35\frac{3}{4}$; and the exchange at Paris on London, at short,* being $25.33\frac{1}{2}$, it follows that gold is about 0.09 per cent. dearer in Paris than in London.

b. The quotation of gold at Paris is about $\frac{1}{2}$ per mille premium, and the short exchange on London is $25.27\frac{1}{2}$. On comparing these rates with the English mint price of £3 17*s.* $10\frac{1}{2}$*d.* per ounce for standard gold, it appears that gold is nearly 4–10ths per cent. dearer in London than in Paris.

Paper X.

Questions on the uniform consumption of uniformly growing produce.

1. Suppose that in a meadow of 20 acres the grass grows at a uniform rate, and that 133 oxen could consume the whole of the grass in 13 days, or that 28 of the oxen could eat up 5 acres of it in 16 days; how many of the oxen could eat up 4 acres of it in 14 days?

133 ox. to 20 ac. is $26\frac{3}{5}$ ox. to 4 ac.
28 ox. to 5 ac. is $22\frac{2}{5}$ ox. to 4 ac.

* That is, by bills payable at short sight, as 3 days' sight, and therefore immediately worth their amount in cash.

$$\begin{array}{l} \phantom{22.4\text{ ox.} : 26.6\text{ ox.} :: 13\text{ da.} :\ } 16 \text{ da.} \\ 22.4\text{ ox.} : 26.6\text{ ox.} :: 13\text{ da.} : \underline{15\tfrac{7}{16}}\text{ da.} \end{array}$$

3 days' growth eaten by 22.4 ox. in $\frac{9}{16}$ da.

$$\frac{9}{16}\text{ da.} : 16\text{ da.} :: 3\text{ da. growth} : 85\tfrac{1}{3}\text{ da. growth.}$$

$$\underline{16}$$

$\therefore$ the original grass is $= 69\frac{1}{3}$ da. growth.

$$\begin{array}{ll} 69\frac{1}{3} & 69\frac{1}{3} \\ \underline{16} & \underline{14} \\ 85\frac{1}{3}\text{ da. growth} : & 83\frac{1}{3}\text{ da. growth} \\ 14\text{ da.} & : 16\text{ da.} \end{array} \Bigg\} :: \overset{\text{ox}}{22.4} : \overset{\text{ox.}}{25}.\ \textit{Ans.}$$

Note. In explanation of the above form of solution, it may be observed that as the orig. grass + 13 da. growth of the 4 acres is eaten by 26.6 ox. in 13 da.

$\therefore$ orig. grass + 13 da. growth is eaten by 22.4 ox. in $15\frac{7}{16}$ da.
but, orig. grass + 16 da. growth is eaten by 22.4 ox. in 16 da.
$\therefore$ 3 da. growth is eaten by 22.4 ox. in $\frac{9}{16}$ da.,
which amounts to $85\frac{1}{3}$ da. growth in the whole 16 da.;

so that the quantity of grass in the meadow at first must have been $69\frac{1}{3}$ days' growth; and we have now given, orig. grass + 16 da. growth eaten by 22.4 ox. in 16 da., to find how many ox. would eat orig. grass + 14 da. growth in 14 da.

For another manner of solving problems of this kind *see* Hunter's *Art of Teaching Arithmetic*, p. 105, and *Examination Questions on* 'Colenso's Algebra,' p. 62.

2. If 133 oxen consume the grass of a meadow in 13 days, and 112 of the oxen could consume the grass of the same meadow in 16 days,—the grass growing uniformly; in what time could 125 of the oxen do it?

Here, as in the preceding solution, the original grass will be found $= 69\frac{1}{3}$ days' growth; and now, $16 + 69\frac{1}{3}$ da. growth being eaten by 112 oxen in 16 da., *the time is required* in which 125 oxen would eat *what grows in the required time* $+ 69\frac{1}{3}$ da. growth.

$$\left.\begin{array}{r} 112 : 125\text{ ox.} \\ 16 : \quad 1\text{ da.} \end{array}\right\} :: 85\tfrac{1}{3}\text{ da. growth} : 5\tfrac{20}{21}\text{ da. growth.}$$

or, 125 oxen eat $5\frac{20}{21}$ da. growth in 1 day,

$$\underline{1}$$

thus consuming $4\frac{20}{21}$ da. growth of the orig. grass per day, or the whole in $69\frac{1}{3} \div 4\frac{20}{21} = 14$ da. *Ans.*

¶ 3. If 29 oxen would eat up a field of grass in 7 weeks, or 25 oxen would eat up the same field in 9 weeks,—the grass growing uniformly; how many oxen would do it in 6 weeks?

4. Suppose that a tank receives a regular and continual supply of

water, and that, when it contains a certain quantity, 12 equal taps being set open would empty it in $7\frac{1}{3}$ minutes, or 7 of the same taps would empty it in 16 minutes; how many of the taps would empty it in 50 minutes?

5. Suppose that in a certain meadow the grass is of uniform quality and growth, and that 20 oxen would exhaust the grass in $12\frac{3}{4}$ days, or 21 oxen would do so in 12 days; in what time would 26 oxen do it?

6. I find that I can engage 15 workmen for 11 weeks, or 31 workmen for 5 weeks, at uniform wages, and in either case pay the wages exactly by means of the interest now accumulated on a certain sum of money and that which will arise during the particular period of engagement:—For how long could I engage 9 workmen on the same principle?

7. If 23 oxen consume 8 acres of pasture in 26 days, and 25 oxen consume 7 acres of the same in 20 days,—the grass growing uniformly; how many acres of it would 33 oxen consume in $5\frac{7}{9}$ days?

8. Suppose that 17 oxen in 30 days, or 19 oxen in 24 days, could consume a field of uniformly growing pasture; find what number of oxen, diminished by the removal of 4 at the end of 6 days, would eat up the same field in 8 days.

9. In a field in which grass grows uniformly, suppose that 31 oxen can consume $8\frac{3}{4}$ acres in $\frac{3}{4}$ of the time in which 15 oxen would consume $5\frac{1}{4}$ acres, and that 22 oxen would require 3 days longer to consume $7\frac{1}{2}$ acres than 20 oxen would require for $6\frac{1}{4}$ acres:—In what time would the 31 oxen eat up the $8\frac{3}{4}$ acres?

10. An empty cistern has two supplying pipes A and B, and two taps C and D A would fill the cistern in $42\frac{1}{2}$ minutes, and B in 46 minutes; and D can carry off per minute half as much again as C. After A and B, running together, have supplied a certain quantity, C is allowed to run with them, and takes 51 minutes to empty the cistern; but had D been turned on along with C, the two would have taken only $5\frac{3}{4}$ minutes to empty it. In what time would the cistern have been emptied if D had been turned on instead of C? and how much of the cistern was filled when C was set open?

Paper XI.

Questions similar to Concluding Misc. Examp. 134 *&* 194.

1. A certain number is divided into two parts, such that 10 times the first added to 18 times the second gives 15 times the entire number; what fraction of the whole is each of the parts?

Questions of this kind closely resemble *Examp.* 2 in Paper VI., and may be solved similarly; thus, since we have

10 times the first part and 18 times the second together equal to 15 times the first and 15 times the second, it is evident that (15 − 10) times the first compensates or equals (18 − 15) times the second; i.e. 5 of the 1*st* = 3 of the 2*nd*; or, 1 of the 1*st* = $\frac{3}{5}$ of the 2*nd*; or, 1*st* : 2*nd* :: 3 : 5; so that the parts are $\frac{3}{8}$ and $\frac{5}{8}$ of the whole.

Otherwise.

10 times the 1*st* with 18 times the 2*nd* = 15 times both;
10 10 10

∴ 8 times the 2*nd* = 5 times both;

or, the 2*nd* is $\frac{5}{8}$ of the whole

and the 1*st* is $\frac{3}{8}$ of do.

2. Divide the quantity 520 into two parts, such that 118 times one part added to 128 times the other shall give 63700.

Here, we have 63700 [÷520] = $122\frac{1}{2}$ times the entire no.

∴ 118 times the 1*st* with 128 times the 2*nd* = $122\frac{1}{2}$ times both;

∴ 10 times the 2*nd* = $4\frac{1}{2}$ times both;

or, the 2*nd* is $\frac{9}{20}$ of the whole, = 234 } *Ans.*
and the 1*st* is $\frac{11}{20}$ of do. = 286 }

3. A person borrows £618 in two separate sums, at the respective rates of $3\frac{1}{2}$ and 5 per cent. per annum; and he repays the two loans at the end of 10 months, with interest amounting to £22 10*s*. Required the amount of each loan.

The respective interests are

$\frac{5}{6}$ of $\frac{3\frac{1}{2}}{100}$ of 1*st* loan, and $\frac{5}{6}$ of $\frac{5}{100}$ of 2*nd*;

and these together are equal to $\frac{22\frac{1}{2}}{618}$ or $\frac{15}{412}$ of both loans;

i.e. $\frac{7}{240}$ of 1*st* with $\frac{10}{240}$ of 2*nd* = $\frac{15}{412}$ of both;

∴ $\frac{3}{240}$ of 2*nd* = $\left(\frac{15}{412} - \frac{7}{240}\right)$ of both

$= \frac{716}{412 \times 240}$ of £618.

∴ 2*nd* = $\frac{716}{412 \times 3}$ of £618 = £358 } *Ans.*
1*st* = £260 }

¶ 4. Sold 449 yards of cloth, part at 12*s.* a yard, and the remainder at 17*s.*, and for the whole received £315 13*s.* How many yards were sold at each rate?

5. A woman sold $7\frac{1}{2}$ dozen apples for 6*s.* 2*d.*, some at the rate of 3 for $2\frac{1}{2}d.$, and the rest at 8 for $6\frac{1}{2}d.$ How many were sold at each rate?

6. I gave 8*s.* for a basket of oranges and lemons, buying the former at the rate of 2 for 3*d.*, and the latter at 5 for 4*d.* I then sold all at the uniform rate of 5 for 6*d.*, and gained $6\frac{1}{4}$ per cent. How many had I of each kind?

7. 12 lbs. of tea and 25 lbs. of coffee together cost £4 6*s.* 8*d.*; but if tea were to rise $2\frac{1}{2}$ per cent. and coffee to fall $4\frac{1}{2}$ per cent., the same quantities would cost £4 5*s.* 11*d.* Required the prices of tea and coffee per lb.

8. If the increase in the number of male and female criminals be 1.8 per cent., while the decrease in the number of males alone is 4.6 per cent. and the increase in the number of females is 9.8; compare the antecedent numbers of male and female criminals.

Paper XII.

Questions on Involution and Evolution.

1. Simplify the expression $\frac{3}{7}$ of $\frac{300}{\sqrt{5}} \times \sqrt{\frac{2}{3}}$.

To remove surd denominators, multiply the numerator and denominator of the second fraction by $\sqrt{5}$, and those of the third fraction by $\sqrt{3}$, which gives

$$\frac{3}{7} \text{ of } \frac{300 \times \sqrt{5}}{5} \times \frac{\sqrt{6}}{3} = \frac{60}{7} \sqrt{30}. \text{ *Ans.*}$$

2. Which is the greater quantity, $\sqrt{2}$ or $\sqrt[3]{3}$?

$$2^{\frac{1}{2}} \text{ and } 3^{\frac{1}{3}} = 2^{\frac{3}{6}} \text{ and } 3^{\frac{2}{6}} = 8^{\frac{1}{6}} \text{ and } 9^{\frac{1}{6}};$$

$$\therefore \sqrt[3]{3} \text{ is the greater.}$$

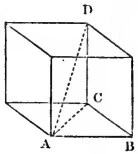
3. Find the diagonal of a rectangular space, 792 feet long and 406 feet broad.

The length and breadth form with the diagonal a right-angled triangle, of which the two perpendicular sides are given, to find the third or longest side. Now, in every right-angled triangle, the sum of the squares of the perpendicular sides is equal to the square of the longest side; therefore,

$$792^2 + 406^2 = 792100, \text{ square of diag.}$$

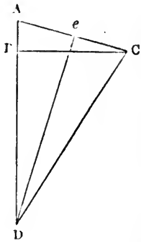
$$\sqrt{792100} = 890 \text{ ft., the diagonal. *Ans.*}$$

4. Show that the length of the edge of a cube multiplied by $\sqrt{3}$ gives the diagonal of the cube.



If AB and BC, edges of a cube, be each represented by 1, then the square of AC, the diagonal of a superficial side, is evidently $1^2+1^2=2$, and the square of the cube's diagonal AD is $=AC^2+CD^2=2+1=3$; therefore $AD=\sqrt{3}$ when the edge of the cube is 1; or, by similar triangles, the diagonal of every cube is the product of the length of the edge by $\sqrt{3}$.

5. The tip of a reed was 8 inches above the surface of a lake; but, forced by the wind, it gradually advanced, and was submerged at a distance of 28 in. Find the depth of the water.



Let AD, =DC, represent the reed; BC the lake's surface; BD the depth.

Given $AB=8$, $BC=28$, to find BD.

The right-angled triangles ABC, A*e*D, having the acute angle A common to both, are similar; hence, DA : A*e* :: CA : AB; or, since A*e* is $\frac{1}{2}$ AC, $\therefore \frac{2\,DA}{CA}=\frac{CA}{AB}$; or, $2\,DA\times AB$ $=CA^2=8^2+28^2=848$; or, $DA\times 16=848$; or $DA=53$ inches. Hence $BD=53-8=45$ inches. *Ans.*

6. What quantity is $\frac{20}{23}$ of its reciprocal?

The quantity ÷ its reciprocal is $=\frac{20}{23}$; but the quotient of any quantity ÷ its reciprocal is the square of that quantity;

$\therefore \sqrt{\frac{20}{23}}$, or $\frac{2}{23}$ of $\sqrt{115}=.9325$. *Ans.*

¶ 7. A square space contains 1056 sq. yards: Express the length of its side as the decimal of $\frac{1}{11}$ of a mile.

8. Find the side of a square field containing 2 ac. 3 ro. 17 po. 30 yds.

9. A square space contains 38 sq. poles 6 yds. 4 ft. 72 in.; find the length of its side.

10. A rectangular field is 190 yds. long and 123 yds. wide; find the side of a square field of half the area; find also the length of a field twice as large as the first, and twice as long as it is broad.

11. Show that $10\div\sqrt{2}$ is $=5\times\sqrt{2}$.

12. Multiply $\sqrt{112}$ by $\sqrt{175}$.

13. If the perpendicular sides of a right-angled triangle are 13.02 and 5.2 feet, what is the third side?

14. If the town *A* is 72 miles west of *B* and 135 south of *C*, what is the distance from *B* to *C*?

15. Which is the greater of the two quantities $\sqrt[3]{9}$ and $\sqrt[4]{19}$? and which of the two $\sqrt{3}$ and $\sqrt[5]{15}$?

16. If the diagonal of a rectangular surface is 3.4061 inches, and the length 3.406 inches, what is the width?

17. The diagonal of a square is 353.55; find the length of its side.

18. The members of a party being solicited for contributions to a charitable object, each person gave a number of half-pennies equal to the number of members, and thus made up a sum total of 12*s.* $0\frac{1}{2}$*d.* What sum was contributed by each?

19. Suppose the top of a straight ladder, $18\frac{1}{4}$ feet long, to rest against a building at the height of $13\frac{3}{4}$ feet from the ground; at what horizontal distance from the bottom of the building is the foot of the ladder placed?

20. The edge of a cube is 250; what is its diagonal?

21. Find the edge, and also the surface, of a cube of wood, the diagonal of which is 3 ft. 9 in.

22. Of what sum of money is £28 the same fraction that the sum itself is of 60 guineas?

23. If the compound interest of £250 for 2 years be £20 8*s.*, what is the rate per cent. per annum?

24. The capacity of a cistern is 478.4 gallons:—Required (*a*) the length equal to the breadth of a cistern of the same capacity $2\frac{1}{2}$ feet deep; and (*b*) the breadth equal to twice the depth of a cistern of the same capacity 6 feet long:—a gallon being = 277.274 cub. inches.

25. What fraction of $(\sqrt{4050} \times .00\dot{2} \div .2\dot{0} + \sqrt{1458}) \div \sqrt{.02}$ is

$$\sqrt{(6.00\dot{8} \div .3042)} + \sqrt{(116.6 \times .04\dot{6})}?$$

26. *A* can excavate 14.2884 cubic yards per day; how many can *B* do per day, if *A* could do *B*'s daily quantity in $\frac{11}{12}$ of the time that *B* would take to do *A*'s daily quantity?

27. The original cost of a pipe of port is £55, and it is sold to *A* at a certain loss per cent.; then *A* sells it to *B* at the same losing rate; but *B* sells it to *C*, at a profit of 12 per cent., for the original cost. What was the loss per cent. at which *A* and *B* sold the wine?

Paper XIII.

Supplementary Miscellaneous Questions. [A.]

1. What is the greatest unit of time with which 15 ho. 12 min. and 1 da. 3 hr. 33 min. can be both represented by integers?

2. How many times can .0087 be subtracted from 2.291, and what will the remainder be?

3. What is the greatest number by which 2500 and 3300 can be divided, so as to leave remainders 4 and 36, respectively?

4. Define Proportion.—Can the quantities 2 yds. 2 ft. $10\frac{1}{4}$ in., £24 3*s*., £12 11*s*. $6\frac{3}{4}$*d*., and 5 yds. 2 ft., be formed into a proportion? Give the reason.

5. State the distinction (i) between simple and compound division, (ii) between simple and compound proportion, and (iii) between simple and compound interest.

6. Distinguish mercantile from true discount; and show that the difference between the interest and the true discount on the same sum is the interest of the discount.

7. Find by duodecimal multiplication the product of 13 ft. 5 in. 7 pts. by 3 ft. 5 in.

8. Multiply, by the method of duodecimals, 29 ft. 7 in. by 9 ft. 8 in. 6 pts.

9. Express the results of the two preceding questions in square feet, square inches, and a fraction of a square inch.

10. Find, by duodecimal multiplication, that the product of 26 ft. 8 in. by 5 in. 9 pts. is 12 sq. ft. $9' \; 4''$; and calculate by Practice the value of the latter quantity at 15*s*. $9\frac{1}{2}$*d*. per square foot.

11. What two quantities have for their sum 9 guineas and 9 shillings, and for their difference 10 crowns and 10 pence?

12. *A* offers to *B* 6 cwt. 2 qrs. 7 lbs. of sugar, worth 38*s*. per cwt., for 24 yds. of cloth, worth 8*s*. $3\frac{3}{4}$*d*. per yard. How much per cent. would *B* gain or lose by accepting the offer?

13. If one man can plough a quarter of an acre in 2 hrs. 23 min., and another can do it in 2 hrs. 34 min., what fraction of an acre could they together plough in an hour?

14. What sum of money increased by $\frac{2}{5}$ of $\frac{4}{5}$ of $\frac{7}{8}$ of itself amounts to 3*s*. 4*d*.?

15. What decimal fraction diminished by .037 of itself becomes .6955?

16. Show that the amount of £7 for 3 years, at 5 per cent. per annum, compound interest, is $= £7 \times 1.05^3$.

17. If $3\frac{3}{4}$ per cent. is lost by selling steel nibs at 3*s*. 6*d*. a gross, how much would be gained or lost per cent. by selling them at 2*s*. $6\frac{5}{9}$*d*. a hundred?

18. A fruiterer by selling apples at the rate of 8 for $6\frac{1}{2}$*d*. gains 17 per cent.; at what rate should he sell them per dozen to gain 20 per cent.?

19. If by selling cloth at 28*s*. 6*d*. for 5 yards my gain would be $6\frac{2}{3}$ per cent., what should I gain or lose per cent. by selling it at 37*s*. 6*d*. for 7 yards?

20. The population of a town is 3370; what was its population a year ago, if in the interval there has been an increase of about 2.65 per cent.?

21. The amounts £210 and £155 are payable 2 years and 5 years hence, respectively; assign the mean period, or equated time, at the end of which, according to mercantile discount, these two amounts might be paid at once?

22. The sum of £434 is due as follows:—$\frac{1}{3}$ of it in 4 months, $\frac{1}{5}$ in 5 months, and the remainder in 7 months. Find the equated time for one payment of £434, according to mercantile discount.

23. Find the value of

$$\frac{\frac{1}{2}-\frac{2}{5}}{\frac{37}{6}-1\frac{3}{7}\text{ of }4\frac{1}{5}} \text{ of } \frac{\text{£1 } 11s.\ 8d.}{\text{£2 } 17s.} \text{ of } \frac{142\text{ yds. } 0.8\text{ ft.}}{2\text{ yds. } 1.7\text{ ft.}} \text{ of 13 days 3 hrs.}$$

Invent a question to which the last three factors in this expression may be the answer; and show how they are so.

24. Divide 99 into four parts, so that the first shall contain 3 for every 4 in the third and every 5 in the fourth, and so that $\frac{1}{3}$ of the second may be $\frac{1}{8}$ of the sum of all the rest.

25. Divide 8*s*. among *A*, *B*, *C*, so that *A* may receive 8*d*. as often as *B* receives 3*d*., and *B* may receive 5*d*. as often as *C* receives 3*d*.

26. Express in lowest terms the product of

$$1+\frac{1}{9}+\frac{1}{25}+\frac{1}{49}+\frac{1}{81} \text{ and } \frac{1}{11}-\frac{3}{59}+\frac{1}{181}.$$

27. The sum of $7\frac{1}{2}$*d*. was divided among *A*, *B*, *C*, in such proportion that *A* received $1\frac{1}{2}$*d*. more than *C*, and *B* $2\frac{1}{4}$*d*. less than *C*: Suppose a sovereign had been divided among them in the same proportion, what would each have received?

28. What half-yearly dividend is derived from an investment of £1000 in the 3 per cents. at $87\frac{3}{8}$, after deducting for income-tax 7*d*. in the £?

29. What interest does a person obtain for his money, who invests in the $3\frac{1}{2}$ per cents. at 91?

30. How many acres, roods, &c. are equal to $\frac{3}{4}$ of $\frac{1\frac{3}{7}}{1\frac{4}{5}}$ of $\frac{8.5}{2.25}$ of

$\frac{1.\dot{3}}{.02\dot{6}}$ of $\frac{£3\ 5s.}{14s.\ 3d.}$ of $\frac{1 \text{ lb. } 4 \text{ oz. } 17 \text{ dwt. } 12 \text{ grs.}}{2 \text{ lbs. } 2 \text{ oz. (avoird.)}}$ of $\frac{77 \text{ da. } 4 \text{ ho. } 30 \text{ m.}}{6 \text{ da. } 12 \text{ ho.}}$ of 518 sq. ft. 28 in.?

31. Find the true discount on £100 10*s.* 10*d.* payable in 4 years, interest being at $3\frac{1}{2}$ per cent. per annum.

32. What sum of money improved by simple interest, at $3\frac{1}{2}$ per cent. per annum, for half a year, will amount to £14 16*s.*?

33. What would be the true present worth of £294 2*s.* 6*d.*, for $3\frac{3}{29}$ years, reckoning simple interest at the yearly rate of $4.02\dot{7}$ guineas per £100?

34. If the simple interest of £$162.87\dot{1}$ for 148 days were £$2.81\dot{4}\dot{2}$ what would be the rate per cent. per annum?

Paper XIV.

Supplementary Miscellaneous Questions. [B.]

1. Two numbers have for their greatest common measure 537 and for their least common multiple 18795. What must the greater n° be, if the less is = 105 times $\frac{2\frac{6}{7}}{4\frac{5}{6}}$ of $\frac{363.37}{8.4}$?

2. The circumference of the fore wheel of a carriage is $6\frac{7}{8}$ feet, and that of the hind wheel is $12\frac{5}{6}$ feet. How many feet must the carriage pass over before both wheels shall have made a complete number of revolutions?

3. The diameter of the fore wheel of a carriage is $\frac{5}{9}$ of that of the hind wheel, and the former makes 528 revolutions in passing over $\frac{3}{4}$ of a mile. How many revolutions does the hind wheel make in passing over a mile? and what is the circumference of each wheel?

4. In what proportion must water be mingled with spirits worth 10*s.* 6*d.* a gallon, to reduce the value to 9*s.* 11*d.* per gallon?

5. How much ore must one raise, that on losing $\frac{17}{40}$ in roasting and $\frac{8}{19}$ of the residue in smelting, there may result 506 tons of pure metal?

6. £225 9*s.* is due in 48 days, and £599 8*s.* in 26 days:—What sum paid at present would discharge both these debts? and how many days would be the equated time for one payment of the £824 17*s.*?—interest being reckoned at 5 per cent.

7. A cubic foot of water weighs 1000 oz. avoirdupois; a pipe whose bore is $3\frac{1}{2}$ square inches discharges 252 lbs. per minute; find the velocity per hour of the issuing water.

8. If when corn is 15*s.* 9*d.* a quarter, and hay $5\frac{1}{2}$*d.* per stone, 7 horses can be kept 8 days for £4 1*s.* 3*d.*; how many weeks can 16 horses be kept for £95, when corn is 2*s.* a bushel, and hay 70*s.* a ton, supposing that 126 lbs. of hay are consumed with 1 bushel of corn?

9. An analysis of the Board of Trade returns for 1861, respecting shipwrecked lives, gave the following results:—Saved by life-boats, $13\frac{1}{2}$ per cent.; by rocket and mortar apparatus, 8 per cent.; by ships' boats, &c., 62 per cent.; by individual exertion $\frac{1}{2}$ per cent.: lost, 16 per cent. Determine the number of lives saved, by the several means enumerated, corresponding to an excess of 2619 rescues by ships' boats over those by life-boats.

10. Find two decimal fractions together equal to $\frac{1}{15}$, and such that one may be $\frac{1}{15}$ of the other.

11. A stationer by selling quills at a guinea a thousand, gained $\frac{2}{7}$ of what they cost him. What was the prime cost?

12. A ring weighs 1 dwt. 4 grs., and is worth £1 2*s.* If 1050 of such rings be packed in a box weighing $3\frac{1}{2}$ lbs., what would it cost to convey them 144 miles, at the rate of 5*s.* per ton per mile, insurance being demanded at the rate of $\frac{1}{8}$ per cent.?

13. A monolith of red granite in the Isle of Mull is said to be about 108 feet in length, and to have an average transverse section of 113 square feet. If shaped for an obelisk, it would probably lose one-third of its bulk, and then weigh about 600 tons. Determine the number of cubic yards in such an obelisk, and the weight in pounds of a cubic foot of granite.

14. Show that, in comparing the rates of two locomotive bodies, A and B, if the distance passed over per unit of time by A is $\frac{3}{4}$ of that by B, then A's time per unit of distance is $\frac{4}{3}$ of B's.

15. A has 38 florins and a sovereign; B has 61 half-sovereigns and 11 florins. What sum transferred by B to A would make B have exactly 6 times as much money as A?

16. The difference of two numbers is $477\frac{2}{11}$, and one of them is to the other as $\frac{3}{7}$ of $2\frac{3}{8}$ of 1.53 is to $5\frac{11}{13} \times 4\frac{1}{4}$. Find the two numbers.

17. With what capital did a tradesman commence business, if at the end of 12 months his nett gain amounted to £210 14*s.*; a certain portion only of that gain being accounted trade profit, the remainder, viz. 5 shillings for every 9 shillings of the trade profit, being legal interest of capital?

18. The sum of £100 has been accumulating at compound interest

for 125 years at 3 per cent.: the amount is now invested in 3 per cent. consols at 95. What will be the annual income therefrom?

N. B. $1.03^{50}=4.383906$; and only four places of decimals need be retained in the result.

19. If the discount on £567 be £34 14*s.* $3\frac{3}{7}$*d.*, simple interest being reckoned at $4\frac{1}{2}$ per cent., when is the sum due?

20. A narrow rectangular field, *ABCD*, has its length *AB* 160 yds. and breadth *BC* $31\frac{3}{7}$ yards. To what point *E* in the side *AB* must a straight line from *C* be drawn, so that *AECD* may contain an acre?

21. A person invests £6200 in the 3 per cents. at $89\frac{1}{8}$, and pays income-tax 10*d.* in the pound; on the stock rising to 92 he sells out, and invests the proceeds in £50 railway shares which yield an annual dividend of $3\frac{1}{2}$ per cent., clear of income-tax. Find the alteration in his income.

22. Certain railway shares pay an annual dividend of £3 10*s.* A person having bought 12 shares, at such a price that they yielded $5\frac{5}{9}$ per cent. on his investment, sold them when the price had risen £5, and invested the proceeds in $3\frac{1}{4}$ per cent. stock at 85. Find the alteration in his income.

23. What fraction of $\sqrt[3]{.0135}$ is $\sqrt[3]{.004}$.

24. From $\frac{1}{27}$ of $\sqrt[3]{5.92}$ subtract $\frac{1}{67}$ of $\sqrt[3]{61.77}$.

Paper XV.

Supplementary Miscellaneous Questions. [C.]

1. A corn merchant having bought 1300 quarters of wheat, sold one-fifth of it at a profit of 5 per cent., one-third at a profit of 8 per cent., and the remainder at a profit of 12 per cent.; but had he sold all at a profit of 10 per cent., his gain would have been £16 13*s.* 8*d.* more. What did the wheat cost him?

$$1-\frac{1}{5}-\frac{1}{3}=\frac{7}{15} \text{ sold at 12 p. c. profit.}$$

∴ the several quantities are as 3, 5, and 7.

$$\begin{aligned} £3 \times 1.05 &= £3.15 \\ 5 \times 1.08 &= 5.40 \\ \underline{7} \times 1.12 &= \underline{7.84} \\ &16.39 \\ 15 \times 1.10 &= \underline{16.50} \\ &.11 \end{aligned}$$

That is, on every £15 of the whole prime cost the gain would have been £.11 more; hence,

£.11 : £16 13*s.* 8*d.* :: £15 : £2275. *Ans.*

2. The gross receipts of a railway company in a certain year are apportioned thus:—40 per cent. to pay the working expenses, 54 per cent. to give the shareholders a dividend at the rate of $3\frac{1}{2}$ per cent. on their shares; and the remainder, £28350, is reserved. Find the paid-up capital of the company.

$100 - 40 - 54 = 6$ p. c. of gross receipts is reserved.

∴ 6 : 54 :: £28350 : £255150 amt. of dividends.

$3\frac{1}{2}$: 255150 :: £100 : £7290000. *Ans.*

3. What is the exact time between 5 and 6 o'clock when the hour and minute hands of a watch should be at right angles to each other? and what, when they should be coincident?

Call the hour hand *H*, and the minute hand *M*. At 5 o'clock, *H* is 5 twelfths of the circumference in advance of *M*; and it is required to find at what time after 5 o'clock the interval between *H* and *M* will be 3 twelfths.

Now, as $(5-3)$ twelfths and $(5+3)$ twelfths are both proper fractions, there will be two occurrences of the interval.

In the first instance, *M* has to gain 2 twelfths on *H*, and in the second instance 8 twelfths; and, as *M* goes 12 times as fast as *H*, and gains 11 twelfths of the circumference per hour, we have

11 tw. : 2 tw. :: 60 min. : $10\frac{10}{11}$ min. past 5;

11 tw. : 8 tw. :: 60 min. : $43\frac{7}{11}$ min. past 5;

which are the times when the hands intercept a fourth of the circumference, or are at right angles.

Similarly, to find when the hands are coincident is to find when *M* will have gained 5 twelfths of the circumf. on *H*.

11 tw. : 5 tw. :: 60 min. : $27\frac{3}{11}$ min. past 5;

which is the time when *H* and *M* point in one direction.

Note. The third answer might have been found thus:

$(10\frac{10}{11} + 43\frac{7}{11}) \div 2 = 27\frac{3}{11}$ min. past 5.

4. At what rate must I sell sherry that cost me 40*s.* a dozen, if I am to gain on every £100 of outlay the selling price of 5 dozen?

£100 ÷ £2 = 50 dozen bought for £100;

and I am to sell $(50-5)$ or 45 dozen for the prime cost of 50 dozen, viz. for £100;

∴ £100 ÷ 45 = 44*s.* $5\frac{1}{3}$*d.* per doz. *Ans.*

5. *A*'s present age is to *B*'s as 9 to 7; and 34 years ago the proportion was 5 to 2. Find the present age of each.

In solving such problems it is borne in mind that the *difference* of the ages of two persons is *always the same*, though the *ratio* of the ages is *always varying*.

Here, then, we have A's present age to B's as $9 : 7$; and 9 is $4\frac{1}{2}$ times $(9-7)$. Similarly, A's former age was to B's as $5 : 2$; and 5 is $1\frac{2}{3}$ times $(5-2)$.

Therefore, A's present age is $4\frac{1}{2}$ times the difference of A's and B's ages; and his former age was $1\frac{2}{3}$ times the same difference; so that we have

$$A\text{'s former age} = \frac{1\frac{2}{3}}{4\frac{1}{2}} \text{ or } \frac{10}{27}, \text{ of his present age;}$$

$$\therefore \frac{17}{27} \text{ of } A\text{'s present age} = 34$$

$$\left.\begin{array}{ll}\therefore A\text{'s present age} & = 54, \\ B\text{'s } \frac{7}{9} \text{ of } 54, & = 42.\end{array}\right\} Ans.$$

6. A boatman rows 5 miles with the tide in the time he would take to row 3 miles against it; but if the hourly velocity of the current were $\frac{1}{2}$ a mile more, he would move twice as rapidly with the tide as against it. What is his power of rowing in still water?

If 5 represent his rate with the tide, then 3 represents his rate against the tide, and the average of these, viz. $\frac{1}{2}(5+3)$, or 4, represents his rate in still water; also $5-4$, or $4-3$, viz. 1, represents the velocity of the current, $=\frac{1}{4}$ of his rate in still water.

Again, if 2 be his rate with the tide, and 1 his rate against it, then $\frac{1}{2}(2+1)$, or $1\frac{1}{2}$, is his rate in still water; also $2-1\frac{1}{2}$, or $1\frac{1}{2}-1$, viz. $\frac{1}{2}$, is the velocity of the current, $=\frac{1}{3}$ of his rate in still water.

$\therefore \frac{1}{3} - \frac{1}{4}$, or $\frac{1}{12}$ of his rate in still water is $=\frac{1}{2}$ a mile per hour; and hence his rate in still water is $\frac{1}{2}$ a mile $\times 12 = 6$ mi. an hour. *Ans.*

7. A contractor engages what he considers a sufficient number of men to execute a piece of work in 84 days; but he ascertains that three of his men do, respectively, $\frac{1}{6}$, $\frac{1}{7}$, and $\frac{1}{9}$, less than an average day's work, and two others $\frac{1}{8}$ and $\frac{1}{10}$ more; and in order to complete the work in the 14 weeks, he procures the help of 17 additional men for the 84th day. How much less or more than an average day's work on the part of these 17 men is required?

Here, instead of 5 men working with ordinary ability, during the 84 days, there are

$$\frac{5}{6}+\frac{6}{7}+\frac{8}{9}+\frac{9}{8}+\frac{11}{10}=4\tfrac{2027}{2520} \text{ ordinary men;}$$

so that the deficiency to be made up is equal to the work of 1 ordinary workman for 84 times $\frac{493}{2520}$ da.

$$= 1 \text{ ordinary workman for } \frac{493}{30} \text{ days,}$$

$$=17 \text{ ordinary workmen for } \frac{29}{30} \text{ of a day,}$$

or, 17 men each doing $\frac{1}{30}$ less than an average day's work. *Ans.*

8. A farmer gave for a horse a bill of £73 due in 1 month, and sold him at once for a bill of £87 at 4 months. Required the farmer's gain per cent, reckoning interest at $4\frac{1}{2}$ per cent.

$$100\tfrac{3}{8} : 100 :: £73 : £\frac{800}{11}, \text{ Pres. Worth of £73;}$$

$$101\tfrac{1}{2} : 100 :: £87 : £\frac{600}{7}, \quad \text{Do.} \quad \text{of £87;}$$

$$\therefore \frac{800}{11} : \frac{600}{7} :: 100 : 100 \times \frac{6}{7} \times \frac{11}{8} = 117\tfrac{6}{7};$$

or, $17\frac{6}{7}$ per cent. gain. *Ans.*

9. Divide the number 237 into three parts such that 3 times the first may be equal to 5 times the second and to 8 times the third.

Since 5 times the 2nd = 3 times the 1st,

$$\therefore \text{ the 2nd} = \frac{3}{5} \text{ of the 1st;}$$

$$\text{similarly, the 3rd} = \frac{5}{8} \text{ of the 2nd;}$$

$$\text{and the three parts are as } 1, \frac{3}{5}, \text{ and } \frac{5}{8} \text{ of } \frac{3}{5},$$

or, as 40, 24, and 15;

$$\left.\begin{aligned} \therefore \frac{40}{79} \text{ of } 237 &= 120, \text{ the 1st,} \\ \frac{24}{79} \text{ of } 237 &= 72, \text{ the 2nd,} \\ \frac{15}{79} \text{ of } 237 &= 45, \text{ the 3rd.} \end{aligned}\right\} \textit{Ans.}$$

10. Divide £5433 18*s.* into three sums, such that their amounts by compound interest at 5 per cent. per annum, for 20, 23, and 27 years, respectively, shall be equal.

$$\text{The 1st} \times 1.05^{20} = \text{the 3rd} \times 1.05^{27},$$
$$\therefore \text{ the 1st} = \text{the 3rd} \times 1.05^{7};$$
$$\text{The 2nd} \times 1.05^{23} = \text{the 3rd} \times 1.05^{27},$$
$$\therefore \text{ the 2nd} = \text{the 3rd} \times 1.05^{4}.$$

Thus, the three required parts of the given sum will be as 1.05^7, 1.05^4, and 1; or, as 1.4071, 1.2155, and 1; or as 14071, 12155, and 10000;

accordingly, the 36226th part of the given sum, viz. 3*s.*, multiplied by these proportional numbers gives £2100 13*s.*, £1823 5*s.*, and £1500. *Ans.*

¶ 11. Suppose 9 men or 15 women to earn 25*s.* a day at reaping, when they work $9\frac{2}{11}$ hours a day; how many men with 4 women would earn 35*s.* a day at the same employment, if the duration of daily work were an eighth less than in the former case?

12. Thirteen horses do the same work as twenty ponies, and 12 horses can just draw a certain load on level ground; how many ponies along with 5 horses could draw a load $\frac{3}{7}$ as heavy up a gradual slope which makes the traction more laborious by $\frac{1}{8}$ for ascent and $\frac{1}{10}$ for roughness?

13. What must a person have invested in the 3 per cents. at $90\frac{5}{8}$, if a transfer of $\frac{3}{5}$ of his capital to the 4 per cents. at 115 would increase his income by £7?

14. Suppose that from an official return of the arrivals of oxen, calves, sheep, pigs, and horses, in the port of London, from the continent, in a certain week, it appears that there were 3 times as many sheep as oxen, that the number of pigs was $13\frac{1}{3}$ per cent. of the number of sheep, that for every 28 pigs there were 25 calves, that the horses were $\frac{1}{10}$ per cent. of the whole, and that the horses and oxen together were 3587:—What was the number of oxen?

15. A merchant has three qualities of whisky, viz. at 18*s.*, 16*s.*, and 15*s.* a gallon, and in quantities, respectively, as 3, 4, 5; and with these he mingles such a quantity of water as makes the average value 15*s.* 6*d.* a gallon. How much per cent. of the mixture is water?

16. Suppose that 15 men would be necessary to excavate 966 cubic yards in 8 days of $10\frac{1}{2}$ hours each:—How many men did a contractor engage for 12 days of $7\frac{1}{2}$ hours, to excavate 575 cubic yards, if he found it requisite to engage 4 additional men during the last 4 days, in order to complete the work in the 12 days?

17. I bought 128 yards of cloth for £100, and am now obliged to sell it at a loss of as much money as I shall receive for a dozen yards. At what do I sell it per yard?

18. I bought paper at the rate of 3*s.* $7\frac{1}{2}$*d.* for 5 quires, and sold it so as to gain as much on the cost of 32 quires as 3 quires were sold for. At what rate did I sell it per quire?

19. I gave 3 sovereigns for two dozen of wine, at different rates per dozen; and by selling the cheaper kind at a profit of 15 per cent., and the dearer at a loss of 8 per cent., I obtained a uniform price for both. What did each dozen cost me?

20. *F* and *G* are partners in trade; *F* contributes $\frac{2}{5}$ of the joint capital for $10\frac{1}{2}$ months, and *G* receives $\frac{5}{8}$ of the gain. Required *G*'s period of investment.

21. At what time between 11 and 12 o'clock will the hour and minute hands of a clock make with each other an angle intercepting 27 of the minute divisions?

22. A merchant buys two pipes of wine, one for £112, one for £120, and he also buys a third pipe; on mixing the three, he sells his wine at 50*s.* per dozen, gaining 25 per cent. on his outlay; what was the price of the third pipe?—The nº of dozens in a pipe is 56.

23. My age is 62, and my son's age 30; how long ago was my age 5 times that of my son? and how many years hence (if we are both alive) will my age be a third of 5 times his age?

24. My age was 24 when my eldest son was born, and when I attain to twice my present age he will be 8 times as old as he is now. What is his age?

25. A boatman rowing against the tide passes a body floating with the tide, and in 9 minutes afterwards is a mile distant from it; in 35 minutes more he rows $2\frac{1}{2}$ miles, and then returns. At what rate per hour does he return, supposing the tide to flow uniformly in one direction?

26. A corn merchant bought 121 quarters of wheat, and he sells it so as to gain $17\frac{1}{2}$ per cent. on 26 quarters, and 13 per cent. on the remaining quantity, having previously tried to sell the whole at a uniform advance of 15 per cent., which would have brought him £4 5*s.* more than he actually received. What did the wheat cost him per quarter?

27. A watch that gains 24 seconds per hour is set to right time at a quarter to 5 P.M. What will be the right time between 8 and 9 o'clock the same evening, when the hour and minute hands of the watch point in exactly opposite directions?

28. Of the whole cost of constructing a railway, $\frac{5}{7}$ is held in shares, and the remainder, £400000, was borrowed on mortgage at 5 per cent. Find what amount of gross annual receipts,—of which 40 per cent. will be required for the working expenses of the line, and 8 per cent. for a reserve fund,—will yield to the shareholders a dividend of $4\frac{1}{2}$ per cent. on their investments?

29. A dealer buys 18 cwt. 3 qrs. at 1*s.* 3*d.* a lb., which, to obtain a fair profit, he should retail at $8\frac{1}{3}$ per cent. above cost price. But, while he professes to sell at the rate of 3 lbs. for 3*s.* 10*d.*, he serves his customers, to his own advantage, with a false balance, in which 10 lb. weighs $10\frac{1}{2}$ lb., and at the same time he uses a false lb. of 6860 grains. How much does he make beyond the fair profit?

30. I have this day paid £2180, being repayment, with interest, of two loans, both contracted by me at one time, viz. of £1163 borrowed at 4 per cent. per annum, and £994 at $4\frac{1}{2}$ per cent. How long is it since the sums were borrowed?

31. A person borrowed £272 6*s.* 6*d.* at 5 per cent. per annum, and repaid the loan by yearly instalments of £100, that sum including the year's interest; how much of the debt was discharged in 3 years?

32. What must be the gross rental of an estate, so that, after deducting 7*d.* in the £ income-tax, and $4\frac{1}{2}$ per cent. on the remainder for expenses of collecting, there may be left a nett rental of £1000?

33. I sold an amount of railway stock at 104, and invested the proceeds in the 3 per cents at 91; I then sold out the 3 per cent. stock at 95, and re-purchasing the railway stock at 105, I found myself a gainer of £50 by the whole transaction. Required the amount of railway stock.

34. The interest on a certain sum of money for 2 years is £71 16*s.* $7\frac{1}{2}$*d.*, and the discount on the same sum, for the same time, is £63 17*s.*, simple interest being reckoned in both cases. Find the rate per cent. per annum, and the sum.

35. At what rate per cent. per annum, compound interest, would a sum of money in 2 years amount to the same as at $3\frac{1}{2}$ per cent. per annum simple interest?

36. If a publisher, in selling a book for cash, rates it at 25 per cent. below publishing price, and then charges for 13 copies as 12, how long credit could he allow, so that, on the principle of true discount at 4 per cent. per annum, the sum to be received for a book should be just 29 per cent. below publishing price?

37. The external length, breadth and height of a rectangular wooden

closed box are 18, 10, and 6, inches, respectively, and the thickness of the wood is half an inch. When the box is empty it weighs 15 lbs., and when filled with sand, 100 lbs. Compare the weights of equal bulks of wood and sand.

38. I bought goods at 23*s.* 9*d.* with 4 months' credit, and sold them forthwith at 25*s.* 6*d.* with such allowance of credit as made my gain $6\frac{2}{3}$ per cent. How long credit did I give, reckoning interest at 4 per cent. per annum?

39. If I am allowed $1\frac{1}{4}$ per cent. discount on an amount charged to me for goods, and give my acceptance at five months for the nett sum; and if by selling the goods forthwith for a bill of £162 12*s.* 2*d.*, payable in 7 months, my present gain is $11\frac{1}{9}$ per cent.; what is the amount originally charged to me, interest being reckoned at 5 per cent. per annum?

40. The present income of a railway company would justify a dividend of 4 per cent., if there were no preference shares; but as £200000 of the stock consists of such shares, which are guaranteed 5 per cent. per annum, the ordinary shareholders receive only $3\frac{1}{2}$ per cent. What is the whole amount of stock?

41. A man bought a house, which cost him 4 per cent. upon the purchase money to put into repair; it then stood empty for a year, during which time he reckoned he was losing 5 per cent. upon his total outlay. He then sold it again for £1192, by which means he gained 10 per cent. upon the original purchase-money. What did he give for the house?

42. (*a*) Show that if 5 times A, 6 times B, and $7\frac{1}{2}$ times C, are equal quantities, then A, B, and C are in the proportion of $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{2}{15}$.

(*b*) What is meant by the reciprocal of a number? What fraction divided by its reciprocal gives a quotient equal to $\frac{153}{272}$?

43. Divide 33 cwt. 2 qr 22 lb. into three such parts that 6 times the first, 9 times the second, and 10 times the third may be equal amounts.

44. Divide £36 8*s.* into four parts such that their simple interests for 4, 6, 7, and 10 months, and at 3, 4, 5, and 6 per cent. per annum, respectively, shall be all equal.

45. Divide £3010 into three sums, so that if the first be put out at simple interest for 3 years at 4 per cent., the second for 5 years at 3 per cent., and the third for 2 years at $2\frac{1}{2}$ per cent., the amount of the second shall be double that of the first, and the amount of the third treble that of the second.

46. By the sale of goods which cost me £3 19*s.* 2*d.* I lost a sum

equal to $5\frac{5}{9}$ per cent. of the proceeds; and by the sale of another quantity which cost me £5 I gained a sum equal to $31\frac{3}{7}$ per cent. of the proceeds. What did I gain per cent. on the whole?

47. If 9 oxen are kept for the same money as 7 horses (for any given time), and a team of oxen are $\frac{1}{5}$ as long again in ploughing 97 acres as the same number of horses are in ploughing 90 acres, and a field costs as much whether ploughed by oxen or horses, viz. £7 5*s.* 6*d.*; the same men being required in both cases, and being paid by the time, what is due to them?

48. If 28 men can excavate 750 cubic yards in 4 days, working $6\frac{3}{4}$ hours a day; what uniform length of day will 24 men require, to excavate 615 cubic yards in $3\frac{1}{2}$ days, supposing that any 5 of the latter party can do as much in 4 hours as any 6 of the former can do in $3\frac{1}{2}$ hours, and that 2 men will be withdrawn from the latter party after $2\frac{1}{2}$ days' work?

49. In a certain manufactory, 158 men of ordinary ability, and working the same number of hours each day, execute a certain piece of work in a week; but if the abilities of 2 of them had been, respectively, $\frac{1}{7}$ and $\frac{1}{9}$ less than ordinary, and the abilities of 2 others $\frac{3}{5}$ and $\frac{3}{8}$ more, the work could have been finished $\frac{23}{83}$ of an hour sooner. How many hours a day did the men work?

50. The interval between the firing of two guns, at a railway station, was 6 minutes, and a passenger in a train, approaching the station at a uniform rate, heard the second report 5 min. 51 sec. after hearing the first. Now, suppose the sound of the train's approach to have become audible at the station when the train was 2 miles off, how soon after that did the train pass the station,—sound travelling 1125 feet per second?

ANSWERS TO THE EXAMPLES.

1.

1. 492480; 161280.
2. 16000; 84000.
3. 6600; 842.
4. 3021; 3300.
5. 45647; 40821.
6. 14161; 164760.
7. 51520; 206080.
8. 6912; 394240.
9. 21728; 84624.
10. 996528; 73029.
11. 10708; 408584.
12. 26921; 1741872
13. 92160; 25200.
14. 13200; 733.
15. 4750; 16820.
16. 3816; 21607.
17. 126060; 15620.
18. 28624; 45780.
19. 44160; 324003.
20. 1180; 716.
21. 8760; 23184.
22. 1132; 37584.
23. 351; 361152.
24. 1074088; 599616.
25. 1096; 440.
26. 1088; 7040.
27. 1158; 1032.
28. 3936; 188.
29. 9855; 2030400.
30. 3960; 16815600.

2.

1. 3751916; 3752.
2. 7329; 29316.
3. 1429; £208 6*s.* 8*d.*
4. £295 17*s.* 11$\frac{1}{4}$*d.*; £458 7*s.* 8*d.*
5. 400*g.* 17*s.* 6*d.*; £128 8*s.* 6$\frac{1}{2}$*d.*
6. £364 11*s.* 8*d.*; 1167*g.* 13*s.* 1$\frac{1}{2}$*d*
7. 16 tons 15 cwt. 1 qr. 20 lbs.; 3 cwt. 3 qrs. 2 lbs. 9 oz. 14 drs.
8. 4 tons 1 cwt. 3 qrs. 7 lbs. 5 oz. 12 drs.; 60 cwt. 1 qr. 16 lbs. 10 oz.
9. 2 tons 15 cwt. 3 qrs. 6 lbs.; 1 qr. 22 lbs. 1 oz. 5 drs.
10. 6 tons 8 cwt. 14 lbs. 1 oz.; 10 cwt. 3 qrs. 25 lbs. 6 oz. 15 drs.
11. 5 cwt. 1 qr. 23 lbs. 7 drs.; 28 tons 2 cwt. 2 qrs. 1 oz.
12. 6 tons 12 cwt. 1 qr. 1 lb. 15 oz.; 12 cwt. 3 qrs. 22 lbs. 5 oz. 3 drs.
13. 2 lbs. 3 oz. 8 dwts. 20 grs.; 125 lbs. 3 oz. 6 dwts.
14. 2 lbs. 11 oz. 11 dwts. 9 grs.; 2 lbs. 1 oz. 13 dwts. 15 grs.
15. 18 lbs. 11 oz. 10 grs.; 32 lbs. 9 oz. 18 dwts. 9 grs.
16. 47 lbs. 4 oz. 7 dwts. 13 grs.; 22 lbs. 1 oz. 3 dwts.
17. 6 m. 6 fur. 150 yds.; 43 lea. 2 m. 2 fur. 31 yds.
18. 15 fur. 56 yds. 1 ft. 7 in.; 71 m. 4 fur. 205 yds.
19. 8 m. 1 fur. 86 yds. 4 in.; 11 lea. 1 m. 6 fur. 110 yds.
20. 849 yds. 3 na.; 9098 ells 2 qrs. 2 na.
21. 758 A. 1 R. 1 P.; 25 sq. yds. 6 ft. 69 in.
22. 125 A.; 15 sq. yds. 3 ft. 128 in.
23. 4 cub. yds. 7 ft. 1280 in.; 2 cub. yds. 26 ft. 57 in.
24. 2 cub. yds. 7 ft. 1513 in.; 3 cub. yds. 23 ft. 1119 in.

25. 2273 gals. 3 qts. 1 pt.; 968 gals. 1 pt. 3 gills.
26. 22 lds. 2 qrs. 1 bus. 1 pk. 1 gal.; 178 qrs. 3 bus. 1 pk. 1 gal. 2 qts.
27. 561 lds. 1 bus. 1 pk.; 22 lds. 7 bus. 1 pk. 2 qts. 1 pt.
28. 278 lds. 1 qr. 2 bus. 3 pks. 3 qts.; 9354 qrs. 7 bus.
29. 377 yrs. 214 days; 5 w. 6 d. 5 hrs. 23 m. 49 s.
30. 1404 w. 3 d. 23 h.; 2 yrs. 101 d. 20 h. 25 m.

3.

£ s. d.	£ s. d.	£ s. d.	£ s. d.
1. 12 8 1	2. 149 18 10	3. 207 12 7¾	4. 162 14 11
5. 120 1 8	6. 87 1 0	7. 114 12 10¼	8. 169 19 0½
9. 110 17 5¾	10. 82 1 10	11. 172 2 1¼	12. 193 2 2½
lbs. oz. dr.	qrs. lbs. oz.	cwt. qrs. lbs.	qrs. lbs. oz.
13. 47 1 11	14. 8 18 12	15. 61 3 0	16. 80 15 0
qr. lb. oz. dr.	cwt. qr. lb. oz.	tons cwt. qr. lb.	
17. 12 11 5 9	18. 120 2 0 2	19. 43 9 2 17	
oz. dwt. gr.	lb. oz. dwt.	oz. dwt. gr.	lb. oz. dwt.
20. 31 1 14	21. 84 7 9	22. 34 15 11	23. 133 5 10
lb. oz. dwt. gr.	lb. oz. dwt. gr.	lb. oz. dwt. gr.	
24. 116 6 2 23	25. 107 1 10 17	26. 73 2 0 1	
dr. scr. gr.	oz. dr. scr.	dr. scr. gr.	oz. dr. scr.
27. 22 2 16	28. 36 4 2	29. 37 0 7	30. 39 6 1
yds. ft. in.	fur. po. yds.	m. fur. yds.	lea. m. fur.
31. 58 0 3	32. 24 34 4	33. 21 0 54	34. 27 0 6
fur. po. yds.	po. yd. ft.	yds. ft. in.	po. yds. ft. in.
35. 22 10 4½	36. 102 0 1	37. 30 1 2	38. 28 4 2 11
po. yds. ft. in.	m. fur. po. yds.	m. fur. yds. ft.	
39. 32 4 0 7	40. 119 2 27 2	41. 27 0 133 2	
yds. qrs. na.	yds. qrs. na.	ells qrs. na.	ells qrs. na.
42. 167 0 1	43. 984 0 0	44. 328 3 1	45. 142 0 1
s.yds. s.ft. s.in.	R. P. s.yds.	A. R. P.	A. R. P.
46. 115 3 44	47. 30 9 18	48. 131 0 21	49. 162 2 23
P. s.yds. s.ft. s.in.	A. R. P. s.yds.	A. P. s.yds. ft. in.	
50. 16 24 3 101	51. 98 2 18 23	52. 103 9 25¼ 3 23	
c.yds. c.ft. c.in.	c.yds. c.ft. c.in.	c.yds. c.ft. c.in.	
53. 92 9 429	54. 106 10 8	55. 95 11 108	
gals. qts. pt.	gals. qts. pt.	pks. gal. qt.	bus. pk. gal.
56. 150 3 1	57. 103 3 1	58. 21 1 1	59. 115 1 1
qrs. bus. pks.	lds. qrs. bus.	bus. gal. qt.	bus. pks. gal.
60. 119 2 2	61. 119 4 4	62. 124 5 1	63. 168 3 1
gal. qt. pt. gills	bus. pks. gal. qts.	qrs. bus. pks. gal.	
64. 93 1 0 3	65. 155 3 1 2	66. 150 0 3 1	
d. h. m. s.	mo. w. d. h.	d. h. m. s.	
67. 22 2 28 59	68. 115 1 1 14	69. 20 21 49 48	
y. d. h. m.	y. w. d. h.	y. d. h. m.	
70. 32 114 21 3	71. 94 41 6 11	72. 28 184 4 0	

4.

£ s. d.	£ s. d.	£ s. d.	£ s. d.
1. 10 3 3	2. 33 7 $2\frac{1}{4}$	3. 60 12 $2\frac{1}{4}$	4. 15 3 10
5. 55 9 10	6. 8 7 6	7. 2 18 $1\frac{3}{4}$	8. 187 1 $2\frac{1}{4}$
9. 25 17 $2\frac{1}{2}$	10. 38 2 $0\frac{1}{2}$	11. 77 15 $1\frac{3}{4}$	12. 215 2 $3\frac{1}{4}$
lbs. oz. drs.	qrs. lbs. oz.	cwt. qrs. lbs.	qrs. lbs. oz.
13. 14 4 2	14. 7 18 3	15. 20 2 15	16. 0 25 7
qrs. lbs. oz.	ton cwt. qrs.	cwt. lbs. oz.	qrs. lbs. oz.
17. 8 11 4	18. 1 6 2	19. 14 27 12	20. 3 21 6
oz. dwt. gr.	oz. dwt. gr.	lbs. oz. dwt.	oz. dwt. gr.
21. 3 4 10	22. 13 17 23	23. 6 7 17	24. 8 1 2
oz. dwt. gr.	oz. dwt. gr.	oz. dwt. gr.	oz. dwt. gr.
25. 21 4 8	26. 36 8 11	27. 8 10 15	28. 14 6 6
dr. scr. gr.	oz. dr. scr.	lbs. oz. dr.	dr. scr. gr.
29. 3 0 19	30. 2 2 1	31. 17 7 7	32. 1 0 16
yd. ft. in.	po. yds. ft.	fur. po. yds.	m. fur. yds.
33. 1 1 9	34. 9 3 2	35. 5 21 3	36. 4 6 124
m. fur. po.	fur. po. yds.	lea. m. fur.	fur. po. yds.
37. 12 2 29	38. 1 18 5	39. 18 2 6	40. 0 27 4
po. yds. ft.	yds. ft. in.	yds. qrs. na.	ells qrs. na.
41. 7 4 1	42. 7 0 5	43. 4 3 1	44. 4 4 2
s.yds. s.ft. s.in.	P. s.yds. s.ft.	R. P. s.yds.	A. R. P.
45. 6 2 86	46. 8 22 6	47. 0 6 27	48. 13 2 34
A. R. P.	R. P. s.yds.	R. s.yds. s.ft.	s.yds. s.ft. s.in.
49. 25 2 36	50. 1 13 22	51. 2 $2\frac{1}{4}$ 6	52. 3 3 27
c.yds. c.ft. c.in.	c.yds. c.ft. c.in.	c.yds. c.ft. c.in.	c.yds. c.ft. c.in.
53. 12 14 1071	54. 29 4 655	55. 33 4 1385	56. 13 16 999
gals. qts. pt.	gals. qt. pt.	pks. gal. qt.	bus. pks. gal.
57. 2 2 1	58. 5 1 1	59. 3 1 1	60. 18 2 1
qrs. bus. pks.	lds. qrs. bus.	bus. pk. gal.	lds. qr. bus.
61. 5 3 3	62. 12 4 6	63. 17 1 1	64. 2 1 4
hrs. m. s.	d. hrs. m.	w. d. hrs.	mo. w. d.
65. 13 57 49	66. 7 19 45	67. 0 5 13	68. 3 2 6
yrs. d. hrs.	yrs. w. d.	yrs. w. d.	yrs. d. hrs.
69. 12 196 9	70. 8 39 5	71. 10 43 4	72. 6 346 14

5.

	£	s.	d.		£	s.	d.		£	s.	d.
1.	46	16	8	2.	75	6	$10\frac{1}{2}$	3.	179	6	9
4.	146	12	$10\frac{1}{2}$	5.	312	10	8	6.	387	2	2
7.	499	7	1	8.	378	11	$1\frac{3}{4}$	9.	1029	19	0
10.	927	7	$10\frac{1}{2}$	11.	940	7	3	12.	1131	8	$4\frac{1}{2}$
13.	1325	13	4	14.	1391	7	6	15.	1038	9	9
16.	1221	18	$6\frac{3}{4}$	17.	1242	13	4	18.	1752	7	11
19.	1888	13	1	20.	2020	1	$10\frac{1}{4}$	21.	444	2	9
22.	618	0	6	23.	1546	7	0	24.	2060	1	3

6.

	£	s.	d.		£	s.	d.		£	s.	d.
1.	358	1	$10\frac{1}{2}$	2.	1435	7	$4\frac{1}{2}$	3.	1961	14	$1\frac{1}{2}$
4.	1389	14	8	5.	2392	18	$10\frac{1}{4}$	6.	4703	10	0
7.	5581	13	4	8.	6481	8	4	9.	3050	9	$10\frac{1}{2}$
10.	3743	7	10	11.	9495	12	0	12.	5758	7	3
13.	2884	10	8	14.	664	8	0	15.	2857	15	$7\frac{1}{2}$
				16.	3676	13	$10\frac{1}{4}$				

7.

1.	825	1	$10\frac{3}{4}$	2.	1096	3	$9\frac{1}{2}$	3.	979	14	$11\frac{1}{4}$
4.	2713	12	6	5.	881	14	$9\frac{1}{2}$	6.	1532	4	$9\frac{1}{2}$
7.	1543	5	$11\frac{1}{2}$	8.	1475	17	$9\frac{3}{4}$	9.	2536	3	$2\frac{1}{2}$
				10.	2318	16	$9\frac{1}{4}$				

11. 6 cwt. 1 qr. 26 lb. 15 oz. 8 dr.
12. 41 tons 18 cwt. 1 qr. 18 lb. 10 oz.
13. 159 tons 1 cwt. 10 lb. 13 oz.
14. 314 tons 10 lbs.
15. 31 tons 19 cwt. 1 qr. 6lb. 11 oz. 12 dr.
16. 811 tons 15 cwt. 3 qrs. 3 lb. 4 oz. 9 dr.
17. 182 lb. 10 oz. 1 dwt. 13 gr.
18. 131 lb. 2 oz. 15 dwt. 20 gr.
19. 12 lea. 1 m. 4 fur. 16 yds. 8 in.
20. 19 lea. 2 m. 1 fur. 98 yds. 8 in.
21. 414 A. 1 R. 10 P.
22. 1255 A. 3 R. 32 P.
23. 319 sq. yds. 1 ft. 112 in.
24. 1493 cub. yds. 11 ft. 1332 in.
25. 7908 gals. 3 qts.
26. 3612 gals.
27. 96 lds. 1 qr. 2 bus.
28. 79 lds. 3 qrs. 2 bus.
29. 1 yr. 323 d. 6 h. 40 m.
30. 2491 yrs. 247 d. 2 h. 16 m. 48 s.

8.

	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.
1.	13	7	$7\frac{3}{4}$	2.	4	4	$9\frac{1}{4}$	3.	14	3	11	4.	15	7	$5\frac{1}{4}$
5.	14	1	$8\frac{1}{2}$	6.	12	19	$1\frac{1}{4}$	7.	9	8	$5\frac{1}{4}$	8.	9	15	$2\frac{1}{4}$
9.	9	3	$5\frac{1}{4}$	10.	6	16	$1\frac{3}{4}$	11.	4	16	1	12.	7	6	$1\frac{1}{2}$

9.

1.	17	13	8	2.	3	0	$7\frac{1}{2}$	3.	3	5	$9\frac{3}{4}$	4.	0	14	$8\frac{3}{4}$
5.	1	10	$2\frac{3}{4}$	6.	0	1	$5\frac{1}{2}$	7.	0	1	$3\frac{1}{2}$	8.	0	2	$1\frac{1}{2}$

10.

1.	35	2	$3\frac{3}{4}$	2.	13	8	$2\frac{1}{2}$	3.	15	6	$4\frac{3}{4}$	4.	8	12	10
5.	13	16	5	6.	10	19	10	7.	4	0	$11\frac{3}{4}$	8.	1	2	$10\frac{3}{4}$
9.	5	16	$0\frac{1}{4}$	10.	8	12	$10\frac{1}{4}$	11.	0	17	$3\frac{3}{4}$	12.	0	9	$5\frac{3}{4}$
13.	7	14	3	14.	7	5	$1\frac{1}{4}$	15.	0	1	$2\frac{1}{2}$	16.	0	8	$7\frac{3}{4}$
17.		11	$1\frac{1}{2}$	18.	5	17	$9\frac{3}{4}$	19.	4	9	$10\frac{1}{4}$	20.	4	18	$8\frac{1}{4}$
21.	3	18	$1\frac{3}{4}$	22.	3	9	$5\frac{1}{4}$	23.	2	8	$4\frac{1}{4}$	24.	4	0	$0\frac{1}{4}$

11.

	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.
1.	28	17	$11\frac{1}{2}$	2.	17	0	$0\frac{1}{2}$	3.	1	7	2	4.	19	16	$6\frac{3}{4}$
5.	12	15	$11\frac{1}{2}$	6.	0	3	$3\frac{3}{4}$	7.	1	2	$3\frac{1}{4}$	8.	17	7	$9\frac{1}{4}$
9.	11	3	$8\frac{1}{4}$	10.	0	2	$8\frac{1}{2}$	11.	0	13	$5\frac{1}{2}$	12.	6	12	$8\frac{1}{4}$

12.

1.	9.	2.	6.	3.	9.	4.	27.
5.	9.	6.	6.	7.	9.	8.	3.
9.	27.	10.	137896.	11.	43.	12.	436.
13.	29.	14.	58.	15.	79.	16.	26.

13.

1. 876 & 15*s.*; 1024 & 8*s.*
2. 90 & 10*s.*; 2814.
3. 154 & 8*s.*; 1062 & 3*s.*
4. 1250 & 2*s.*; 27750.
5. 147 & 2*s.* 6*d.*; 1090 & 4*d.*
6. 3150; 285 & 5*s.*
7. 138 lb. 6 oz. 10 dwt.; 6 dr. 1 scr. 4 gr.
8. 24 lb. 3 oz. 13 dwt. 8 gr.; 12 dwt. 12 gr.
9. 597 & 2 qr.; 4 & 8 in.
10. 1000; 550.

14.

	s.y.	s.f.	s.in.		s.y.	s.f.	s.in.		s.y.	s.f.	s.in.
1.	11	3	30	2.	8	6	84	3.	3	0	72
4.	1	6	60	5.	20	5	101	6.	22	3	108
7.	56	8	0	8.	92	4	0	9.	1	1	34
10.	241	8	112	11.	55	2	142	12.	68	8	72

15.

1. 2 ft. 9 in.
2. 12 yds. 1 ft. 5 in.
3. 4 yds. 1 ft. 8 in.
4. 2 yds. 10 in.
5. 2 ft. 9 in.
6. 5 yds. 11 in.
7. 13 ft. 1 in.
8. The other side is 26 yds. 5 in.
9. 130.
10. 341 yds. 1 ft.
11. 52 yds. 3 in.
12. 250.

16.

	c.yds.	ft.	in.		c.yds.	ft.	in.		c.yds.	ft.	in.
1.	77	4	576	2.	1	25	144	3.	14	12	1080
4.	46	8	0	5.	33	16	864	6.	13	15	1152
7.	0	25	864	8.	5	15	0	9.	15	2	1673
10.	7 ft.			11.	120 ft.			12.	7783600 c. ft.		

17.

1. 21 lbs. 4 oz. 15 dwt.
2. 29 da. 12 hrs. 44 min. 3 sec.
3. £94 19*s.* 2*d.*
4. £16 4*s.* 11*d.*
5. 24857 mi. 1680 yds.
6. £21 19*s.* 6*d.*
7. 1907314.
8. 250 ft.
9. 6 da. 22 hrs. 40 min.
10. £91 10*s.* 6*d.*
11. 132 yds. 2 ft. 7 in.
12. £387 1*s.* $1\frac{1}{2}$*d.*
13. 976 ducats.
14. £19 4*s.* $0\frac{3}{4}$*d.*
15. 365 da. 5 hrs. 48 min. 48 sec.
16. 15 cwt. 7 lbs. 8 oz.
17. 44 tons 12 cwt. 3 qrs. 12 lbs.
18. £9895 16*s.* 8*d.*
19. 1 mi. 4 fur. 20 yds.
20. 91717720 mi.
21. 3 mi. 3 fur. 60 yds.
22. £13069 0*s.* 7*d.*
23. £7670.
24. 10*s.* $4\frac{3}{4}$*d.*
25. 63 yds.
26. 13.
27. £193 15*s.*; 60 minæ.
28. 114 lbs. 15 dwt.; £3437500.
29. £1919 5*s.* 5*d.*
30. 11 da. 17 hrs. 43 min. 20 sec.
31. £12389 1*s.* 3*d.*; £10110 18*s.* 9*d.*
32. $20833\frac{1}{3}$ lbs.
33. 37 oz.
34. 648.
35. 50606 gal.
36. £664.
37. 26 yds. 2 ft.
38. 355 sq. yds. 7 ft. 126 in.
39. £33 2*s.* $6\frac{1}{4}$*d.*
40. £22 7*s.* 6*d.*
41. 102700 cub. yds. 16 ft. 1152 in.
42. $5\frac{1}{2}$*d.*
43. 17*s.* 4*d.*
44. 168 tons $7\frac{1}{2}$ cwt.
45. £148 10*s.*
46. 1*s.* $11\frac{3}{4}$*d.*
47. 1607 tons 2 cwt. 3 qrs. 12 lbs.
48. 7 mi. 2 fur. 120 yds.
49. 23*s.* $4\frac{1}{2}$*d.*
50. £4 14*s.* $7\frac{3}{4}$*d.*; £5 8*s.* 2*d.*
51. 21*s.*
52. 6*s.* 3*d.*
53. 13 ac. 2957 sq. yds. 7 ft.; 10 ac. 1477 sq. yds. 7 ft.
54. 353571 tons 8 cwt. 2 qrs. 8 lbs.
55. $58\frac{3}{4}$ yds.
56. 725 gal.
57. 5044.
58. A man, £16 10*s.*; a woman, £5 10*s.*
59. 7.
60. 3 ac. 584 sq. yds.; 10 ac.
61. 20.
62. 750 bu.
63. 2 yrs. 334 da. 19 hrs. 30 min.
64. £9 3*s.* 4*d.*; £5 8*s.* 4*d.*; £5 8*s.* 4*d.*
65. A man, £66 0*s.* $4\frac{1}{2}$*d.*; a woman, £33 0*s.* $2\frac{1}{4}$*d.*; a child, £11 0*s.* $0\frac{3}{4}$*d.*
66. *A*, 7*s.* $3\frac{1}{2}$*d.*; *B*, 13*s.* $11\frac{1}{2}$*d.*; *C*, 27*s.* 11*d.*
67. 10240.
68. Loss in one year, £122 10*s.*; gain in three years, £698 6*s.* 8*d.*

18.

1. 112.	2. 4.	3. 1.	4. 25	5. 101.
6. 143.	7. 377.	8. 11.	9. 18.	10. 7.
11. 1.	12. 77.	13. 133.	14. 49.	15. 213.
16. 25.	17. 336.	18. 57.	19. 3.	20. 15.

19.

1. 60.	2. 42.	3. 16.	4. 198.	5. 240.
6. 80.	7. 180.	8. 144.	9. 120.	10. 68.
11. 144.	12. 216.	13. 240.	14. 2520.	15. 7560.
16. 1008.	17. 1260.	18. 10500.	19. 7200.	20. 10800.

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20. 1. $\frac{40}{5}, \frac{135}{5}; \frac{216}{27}, \frac{729}{27}$. 2. $\frac{374}{11}, \frac{1485}{11}; \frac{578}{17}, \frac{2295}{17}$.

3. $\frac{90}{15}, \frac{135}{15}; \frac{180}{15}, \frac{300}{15}$. 4. $\frac{850}{34}, \frac{1156}{34}; \frac{2380}{34}, \frac{3774}{34}$.

21.

1. $\frac{26}{7}$.	2. $\frac{92}{9}$.	3. $\frac{2435}{11}$.	4. $\frac{236}{17}$.	5. $\frac{427}{13}$.
6. $\frac{10027}{50}$.	7. $\frac{863}{12}$.	8. $\frac{1738}{15}$.	9. $\frac{2315}{18}$.	10. $\frac{1384}{37}$.
11. $\frac{6029}{30}$.	12. $\frac{3149}{25}$.	13. $\frac{8229}{16}$.	14. $\frac{2131}{21}$.	15. $\frac{8639}{12}$.
16. $\frac{228}{115}$.	17. $\frac{4264}{239}$.	18. $\frac{3813}{360}$.	19. $\frac{12421}{111}$.	20. $\frac{8500}{99}$.

22.

1. $4\frac{1}{9}$.	2. $7\frac{2}{11}$.	3. $24\frac{1}{13}$.	4. 130.	5. $29\frac{8}{35}$.
6. $72\frac{31}{43}$.	7. 22.	8. $25\frac{46}{87}$.	9. $16\frac{15}{77}$.	10. $33\frac{1}{95}$.
11. 40.	12. $35\frac{7}{102}$.	13. $35\frac{53}{117}$.	14. 21.	15. $25\frac{85}{122}$.
16. 16.	17. $15\frac{79}{357}$.	18. $16\frac{140}{401}$.	19. $61\frac{121}{200}$.	20. $70\frac{128}{333}$.

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23. 1. $\frac{35}{4}, \frac{35}{3}, \frac{35}{2}, \frac{875}{36}; \frac{7}{36}, \frac{5}{36}, \frac{35}{288}, \frac{35}{432}$.

2. $\frac{875}{144}, \frac{125}{18}, \frac{125}{16}, \frac{125}{9}; \frac{25}{144}, \frac{125}{1152}, \frac{125}{1728}, \frac{5}{144}$.

3. $\frac{640}{693}, \frac{320}{231}, \frac{1280}{693}, \frac{1600}{693}, \frac{320}{99}$.

4. $\frac{320}{4851}, \frac{40}{693}, \frac{320}{6237}, \frac{32}{693}, \frac{320}{7623}$.

24.

1. $\frac{9}{20}$.	2. $\frac{5}{6}$.	3. $\frac{9}{11}$.	4. $\frac{4}{15}$.	5. $\frac{4}{5}$.
6. $\frac{2}{3}$.	7. $\frac{9}{22}$.	8. $\frac{3}{4}$.	9. $\frac{13}{14}$.	10. $\frac{22}{43}$.
11. $\frac{8}{9}$.	12. $\frac{3}{4}$.	13. $\frac{11}{16}$.	14. $\frac{9}{32}$.	15. $\frac{60}{77}$.
16. $\frac{35}{41}$.	17. $\frac{11}{13}$.	18. $\frac{20}{21}$.	19. $\frac{23}{33}$.	20. $\frac{77}{81}$.

L 2

25.

1.	$\frac{3}{7}$.	2.	$\frac{5}{11}$.	3.	$\frac{13}{23}$.	4.	$\frac{7}{22}$.
5.	$\frac{3}{5}$.	6.	$\frac{7}{11}$.	7.	$\frac{282}{343}$.	8.	$\frac{7}{10}$.
9.	$\frac{37}{66}$.	10.	$\frac{25}{33}$.	11.	$\frac{13}{456}$.	12.	$\frac{29}{55}$.

26.

	£	s.	d.		£	s.	d.		£	s.	d.
1.	19	6	$9\frac{7}{8}$.	2.	38	18	$7\frac{3}{5}$.	3.	36	4	3.
4.	81	18	$10\frac{1}{6}$.	5.	91	4	$10\frac{1}{2}$.	6.	219	7	$0\frac{3}{5}$.
7.	219	16	$6\frac{6}{7}$.	8.	131	12	$2\frac{1}{2}$.	9.	134	18	$4\frac{1}{9}$.
10.	160	5	$8\frac{8}{13}$.	11.	286	5	$11\frac{4}{35}$.	12.	282	18	$7\frac{16}{27}$.

13.	0	13	$4\frac{1}{3}$.	14.	2	6	$9\frac{13}{16}$.	15.	5	19	$6\frac{2}{5}$.
16.	12	2	$2\frac{5}{6}$.	17.	0	19	$2\frac{2}{3}$.	18.	3	14	$7\frac{2}{5}$.
19.	9	2	$6\frac{21}{32}$.	20.	4	3	$10\frac{5}{14}$.	21.	7	15	$3\frac{11}{36}$.
22.	4	8	$8\frac{1}{24}$.	23.	8	6	$4\frac{2}{5}$.	24.	7	6	$2\frac{11}{24}$.
25.	3	7	$10\frac{9}{20}$.	26.	1	17	$2\frac{31}{80}$.	27.	6	7	$6\frac{11}{50}$.
28.	1	7	$0\frac{13}{40}$.	29.	1	2	$8\frac{7}{20}$.	30.	1	8	$10\frac{59}{96}$.

27.

1.	$1\frac{2}{3}$.	2.	$2\frac{6}{7}$.	3.	$1\frac{4}{5}$.	4.	$\frac{7}{12}$.	5.	$\frac{1}{4}$.
6.	$11\frac{1}{3}$.	7.	$\frac{3}{40}$.	8.	$\frac{5}{8}$.	9.	$157\frac{1}{2}$.	10.	11.
11.	$3\frac{1}{3}$.	12.	$16\frac{1}{2}$.	13.	$43\frac{5}{16}$.	14.	3.	15.	$\frac{1}{2}$.
16.	$\frac{39}{64}$.	17.	$\frac{4}{9}$.	18.	72.	19.	$1\frac{2}{5}$.	20.	$7\frac{7}{44}$.

28.

1. $\frac{105,\ 140,\ 126,\ 60}{210}$.
2. $\frac{1925,\ 1320,\ 1848,\ 420}{2310}$.
3. $\frac{16,\ 18,\ 20,\ 21}{24}$.
4. $\frac{18,\ 80,\ 27,\ 104}{144}$.
5. $\frac{24,\ 28,\ 30,\ 31}{32}$.
6. $\frac{60,\ 45,\ 16,\ 39}{72}$.
7. $\frac{63,\ 88,\ 102,\ 76,\ 75}{144}$.
8. $\frac{162,\ 108,\ 144,\ 24,\ 16}{243}$.
9. $\frac{720,\ 378,\ 525,\ 612,\ 80,\ 675}{1260}$.
10. $\frac{440,\ 765,\ 900,\ 504,\ 240,\ 1050}{1080}$.
11. $\frac{5\ 0,\ 630,\ 216,\ 330,\ 260,\ 345}{900}$.
12. $\frac{5400,\ 6930,\ 1008,\ 2240,\ 1944,\ 3213}{7560}$.

29.

1. $2\frac{6}{7}$. 2. $2\frac{1}{8}$. 3. $2\frac{31}{33}$. 4. $2\frac{43}{180}$. 5. $2\frac{11}{36}$.
6. $2\frac{41}{144}$. 7. $1\frac{73}{90}$. 8. 1. 9. $15\frac{1}{12}$. 10. $10\frac{17}{72}$.
11. $13\frac{1}{10}$. 12. $3\frac{65}{72}$. 13. $13\frac{19}{45}$. 14. $3\frac{851}{1080}$. 15. $5\frac{34}{63}$.
16. $19\frac{4}{35}$. 17. $5\frac{6}{7}$. 18. $5\frac{107}{720}$. 19. $13\frac{329}{400}$. 20. $18\frac{11}{20}$.
21. $11\frac{74}{165}$. 22. $34\frac{1139}{1440}$.

	£	s.	d.		£	s.	d.		£	s.	d.
23.	29	3	$10\frac{17}{24}$.	24.	26	6	$6\frac{1}{3}$.	25.	28	7	$9\frac{13}{30}$.
26.	28	10	$1\frac{17}{48}$.	27.	39	3	$0\frac{11}{40}$.	28.	32	12	$9\frac{8}{63}$.
29.	87	13	$8\frac{5}{8}$.					30.	70	10	$11\frac{43}{72}$.

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30. 1. $\frac{1}{5}$; $\frac{3}{10}$; $\frac{1}{12}$; $\frac{1}{6}$. 2. $2\frac{1}{2}$; $1\frac{1}{8}$; $2\frac{1}{7}$; $10\frac{5}{12}$.
3. $\frac{41}{100}$; $5\frac{21}{25}$; $48\frac{2}{3}$; $2\frac{31}{42}$. 4. $9\frac{37}{75}$; $\frac{47}{48}$; $2\frac{31}{42}$; $2\frac{167}{168}$.
5. $\frac{131}{175}$; $16\frac{88}{105}$; $4\frac{13}{30}$; $\frac{87}{110}$. 6. $\frac{37}{54}$; $13\frac{3}{5}$.
7. $6\frac{29}{60}$. 8. $\frac{23}{45}$. 9. £8 2 $11\frac{1}{6}$.
10. 18*s.* $7\frac{7}{12}$*d.* 11. 2*s.* $5\frac{5}{18}$*d.* 12. £5 0 $5\frac{17}{24}$.
13. 17*s.* $9\frac{29}{48}$*d.* 14. £3 13 $8\frac{14}{15}$.

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31. 1. $\frac{45}{88}$; 1; $\frac{3}{4}$. 2. $78\frac{4}{7}$; 60; $409\frac{1}{2}$.
3. $49\frac{16}{21}$; 22. 4. $\frac{1}{5}$; $5\frac{5}{24}$. 5. $1\frac{1}{8}$; $17\frac{7}{9}$.

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32. 1. 3; $\frac{8}{9}$; $1\frac{7}{9}$; $\frac{5}{8}$; $1\frac{1}{3}$; $\frac{4}{5}$. 2. $18\frac{3}{5}$; $\frac{1}{18}$; $\frac{112}{135}$; $\frac{5}{48}$.
3. $52\frac{1}{4}$; $\frac{1}{5}$; $1\frac{1}{15}$. 4. 16; $\frac{3}{4}$; $\frac{5}{27}$; $\frac{3}{26}$.
5. $4\frac{4}{5}$; 2; $1\frac{5}{9}$; 2. 6. $7\frac{1}{2}$; $2\frac{2}{5}$; $38\frac{2}{11}$; $3\frac{4}{7}$; 16.

33.

1. 12*s.* 6*d.*; £3 5*s.*; 2*s.* 8*d.*; 9*s.* $4\frac{1}{2}$*d*; £3 0*s.* 8*d.*
2. £2 6*s.* 8*d.*; £2 4*s.* $11\frac{1}{3}$*d.*; £3 2*s.* $8\frac{11}{20}$*d.*
3. £13 8*s.* $11\frac{7}{12}$*d.*; £22 1*s.* $9\frac{5}{16}$*d.*; £39 11*s.* $11\frac{7}{16}$*d.*
4. £63 13*s.* 5*d.*; £91 17*s.* $10\frac{1}{2}$*d.*; £9 9*s.* $7\frac{5}{16}$*d.*
5. £176 13*s.* $7\frac{3}{8}$*d.*; £49 3*s.* $10\frac{2}{3}$*d.*; £46 4*s.* $0\frac{11}{16}$*d.*
6. 14 cwt. 1 qr. 4 lbs.; 3 oz. 8 dwts. $13\frac{5}{7}$ grs.; 2 cwt. 2 qrs. 6 lbs.; £4 1*s.* $8\frac{1}{2}$*d.*
7. 8 w. 4 d. 10 h. 40 m.; 39 A. 1 R. $1\frac{1}{4}$ P; 3*s.*
8. 2*s.* $3\frac{1}{2}$*d.*; 5 cwt. 2 qrs. $9\frac{23}{24}$ lbs.; £100 8*s.* 4*d.*
9. £4 1*s.*; £8 14*s.* $6\frac{3}{4}$*d.*; 4*s.* 8*d.*
10. 2 fur. 124 yds. 2 ft.; £4 2*s.* 2*d.*

11. 16*s.* $11\frac{3}{4}$*d.*; 2 qrs. 17 lbs. $1\frac{7}{30}$ oz.; 5 d. 38 m. 20 sec.
12. 16*s.* $11\frac{1}{2}$*d.* 13. £3 1*s.* $6\frac{3}{4}$*d.* 14. £1 2*s.*
15. 3*s.* $7\frac{1}{2}$*d.* 16. £7 17*s.* $5\frac{1}{7}$*d.*

34.

1. $\frac{1}{6}$; $5\frac{1}{12}$.	2. $\frac{13}{23}$; 46.	3. $\frac{7}{26}$; $1\frac{5}{9}$.	4. $\frac{29}{187}$; $3\frac{35}{144}$.
5. $\frac{79}{448}$; $\frac{19}{112}$.	6. $\frac{31}{160}$; $5\frac{1}{4}$.	7. $5\frac{5}{63}$; $\frac{5}{11}$.	8. $1008\frac{8}{15}$; $\frac{11}{16}$.
9. $3\frac{43}{44}$; $\frac{2}{5}$.	10. $\frac{3}{10}$; $3\frac{1}{7}$.	11. $\frac{2}{5}$; 792.	12. $\frac{1}{45}$; $\frac{5}{68}$.
13. $6\frac{29}{168}$; $3\frac{4}{15}$.	14. $\frac{100}{297}$; $1\frac{2}{3}$.	15. $1\frac{1}{20}$; $1\frac{23}{224}$.	16. $\frac{4}{7}$; $2\frac{14}{25}$.

35.

1. $\frac{5}{14}$; $\frac{7}{80}$.	2. $\frac{1}{270}$; $2\frac{37}{160}$.	3. $\frac{5}{6}$; $\frac{3}{140}$.	4. $\frac{49}{60}$; $4\frac{61}{90}$.
5. $\frac{11}{40}$; $\frac{8}{49}$.	6. $8\frac{3}{4}$; $8\frac{8}{11}$.	7. $12\frac{4}{9}$; $\frac{13}{56}$.	8. $\frac{18}{35}$; $1\frac{19}{36}$.
9. $\frac{1}{726}$; $\frac{21}{128}$.	10. $18\frac{2}{3}$; $10\frac{1}{2}$.	11. $17\frac{19}{29}$; $\frac{1}{160}$.	12. $1\frac{31}{81}$; $\frac{17}{240}$.
13. $\frac{133}{400}$; $2\frac{42}{43}$.	14. $41\frac{4}{5}$; $\frac{107}{840}$.	15. $4\frac{3}{25}$; $1\frac{2}{9}$.	16. $3\frac{43}{125}$; $\frac{92}{135}$.

36.

1. $\frac{7}{45}$ greatest, $\frac{3}{20}$ least. 2. $14\frac{2}{3}$. 3. $1\frac{11}{135}$.
4. $\frac{7}{9}$ of $1\frac{1}{4}$, by $\frac{1}{1260}$. 5 $\frac{20002,\ 400}{9999}$. 6. 5*s.* 3*d.*
7. $\frac{1}{4}$. 8. $\frac{62}{231}$. 9. $26\frac{2}{7}$ ft. 10. $3\frac{27}{280}$.
11. 9 oz. 3 dwt. 8 gr.; 14*s.* 3*d.* 12. $3\frac{7}{36}$; $108\frac{1}{3}$ sq. in.
13. £5 6*s.* 8*d.* 14. $3\frac{1}{2}$; $1\frac{17}{88}$. 15. $\frac{1}{2}$.
16. $\frac{1}{128}$. 17. 1. 18. $\frac{1}{7}$. 19. $\frac{583}{720}$.
20. $5\frac{7}{36}$; $2\frac{23}{50}$. 21. £67 4*s.* $3\frac{1}{9}$*d.* 22. $1\frac{5}{24}$; $\frac{11}{24}$; $2\frac{7}{11}$.
23. £85 14*s.* $3\frac{3}{7}$*d.*; 4*s.* 7*d.* 24. $4\frac{3}{8}$; 42. 25. £21 8*s.* $1\frac{1}{2}$*d.*
26. £9973 6*s.* 8*d.* **27.** $\frac{25,\ 21,\ 24}{20}$. 28. $4\frac{23}{64}$. 29. £90.
30. 19 dwt. 9 gr. 31. $14\frac{81}{160}$; $\frac{97}{504}$. 32. $\frac{240,\ 280,\ 303}{84}$.
33. $\frac{11}{25}$; £7 16*s.* $5\frac{1}{3}$*d.* 34. 21*s.* 35. 23 lbs. 17 dwt. $5\frac{1}{3}$ gr.
36. $1\frac{53}{75}$. 37. £4 16*s.* 38. £1 13*s.* $7\frac{3}{4}$*d.*
39. 59 yds.; £11 1*s.* 3*d.* 40. $\frac{25}{128}$; £3125. 41. $\frac{71}{128}$; $680\frac{5}{9}$ lbs.
42. $\frac{3}{7}$. 43. 99. 44. 9.
45. 2 oz. 8 dwts. 8 gr. Troy; 2 oz. $10\frac{74}{175}$ dr. Av. 46. 81.
47. $12\frac{11}{54}$. 48. $140\frac{1}{3}$ yds.; £1 6*s.* $3\frac{3}{4}$*d.*
49. 17 cwt. 2 qrs. 5 lbs.; £32 16*s.* 4*d.* 50. £333 6*s.* 8*d.*; $\frac{1}{30}$.

37.

1. .7, 11.7, .33, 1.015.
2. .01, .0021, .0117, .0000003.
3. .230037.
4. 1.11111.
5. 13.003005.
6. 10.110101.
7. $\frac{37}{1000}, \frac{1}{5000}, \frac{1}{4}, \frac{3}{8}$.
8. $\frac{3}{400}, 1\frac{9}{40}, \frac{3}{16}, 3\frac{9}{40}$.
9. $\frac{11}{16000}, \frac{3}{3200}, 23\frac{61}{1600}$.
10. $15\frac{13}{64}, \frac{3}{1280}, 4\frac{1}{128}$.
11. 3, 300; .03, .0003.
 .125, 12.5; .0000125, .000000125; 5387340, .0538734.
12. 1100, 1100000; .0011, .0000011; 11025, 1102500; .011025, .00011025; 213012000; .000213012.

38.

1. 34.62156.
2. 782.8594.
3. 420.615973.
4. 2492.2622123.
5. 19.002 : 3.44902.
6. 21.1335 : .41213.
7. 19.0002 : 1.0013.
8. .0000013 : 23.016484.
9. 1.33678 : 2.7486.
10. .003213 : .34235.

39.

1. 723.6 : 146.4561.
2. .0000001 : 74.151.
3. .07504 : .000602.
4. .0013014 : 1.5.
5. 5.31441 : 4.096.
6. .0001234321 : .00044408.

40.

1. 6.25 : .000625.
2. 6250000 : .0000625.
3. 490000 : 6.3.
4. 185 : 30.
5. 4000 : 4.8828125.
6. 2.4 : 1200.
7. .00015625 : 7118580.
8. .0122699 &c. : 1568.627 &c.
9. .3388278 &c. : .00383177 &c.
10. 290 : .014974 &c.

41.

1. .04 : .052 : 5.25 : 1.6.
2. .848 : 11.0136 : 15.625 : 5.1875.
3. 7.203125 : .1328125 : .00015625 : 11.001696.
4. .001953125 : 1.0009765625; .008125; .0013671875.
5. .1705; .00216; .32.

42.

1. $1.\dot{4}$: $.57\dot{2}$: $2.\dot{3}4\dot{5}$: $.012\dot{3}\dot{6}$.
2. $2.\dot{9}28571\dot{4}$: $5.0\dot{4}\dot{5}$: $.0\dot{1}3\dot{2}$: $23.\dot{1}5\dot{6}$.
3. $.\dot{0}08\dot{9}$: $5.\dot{7}6190\dot{4}$: $17.12\dot{9}\dot{3}\dot{1}$: $.12\dot{3}4\dot{5}$.
4. $.03\dot{6}4\dot{8}$: $.1003\dot{3}7\dot{8}$: $.40\dot{8}6\dot{4}$: $.0\dot{2}050\dot{2}$.
5. $.\dot{0}58823529411764\dot{7}$.
 $.\dot{0}43478260869565217391\dot{3}$.
 $.\dot{0}34482758620689655172413793\dot{1}$.
 $.\dot{0}3225806451612\dot{9}$.

43.

1. $\frac{1}{3} : \frac{5}{99} : \frac{6}{11} : \frac{27}{37}$.
2. $\frac{4}{165} : \frac{8}{185} : \frac{1}{148} : 2\frac{107}{2475}$.
3. $3\frac{23}{55} : \frac{133}{3000} : 1\frac{8}{55} : \frac{89}{19800}$.
4. $4\frac{59}{1110}$; $7\frac{145}{222} : 2\frac{19}{55}$; $\frac{41}{440}$.
5. $2\frac{101}{1110} : \frac{111}{202} : 1\frac{3}{70}$.
6. $2\frac{9}{14}$; $5\frac{17}{88}$; $11\frac{19}{66}$.

44.

1. $47.411\dot{4}5528\dot{6}$.
2. $168.7023\dot{9}1145\dot{6}$.
3. $.\dot{2}\dot{4} : .0327116$.
4. $.\dot{8}5714\dot{2} : .00\dot{5}\dot{8}$.
5. $9.92\dot{8} : 2.\dot{2}9\dot{7}$.
6. $31.\dot{7}9\dot{1} : 352.08564$.
7. $3.\dot{6} : .05\dot{2}$.
8. $49 : 1.1\dot{4}\dot{5}$.

45.

1. 9*s.* : 13*s.* $7\frac{1}{2}$*d.* : £2 6*s.* 6*d.*
2. £8 2*s.* 6*d.* : 6*s.* 2*d.* : £1 11*s.* 8*d.*
3. 13*s.* $1\frac{1}{2}$*d.* : 1*s.* $6\frac{3}{4}$*d.*
4. £18 2*s.* 3*d.* : 9 cwt. 3 qrs.
5. 23 d. 10 h. 4 m. 48 sec. : 1 A. 1 R. 35 P.
6. £1 14*s.* 3*d.* : £47 5*s.* $7\frac{1}{2}$*d.*
7. £8 9*s.* $3\frac{3}{4}$*d.* : £125 13*s.* $10\frac{1}{2}$*d.*
8. £1 11*s.* $6\frac{3}{4}$*d.* : 10*s.* $1\frac{1}{2}$*d.*
9. 13 P. 2 yds. 1 ft. 4 in. : 21 lbs. 12 oz. 7.68 drs.
10. 3 sq. ft. $67\frac{1}{2}$ in. : 102 m. 875 yds. 5.76 in.
11. £78 3*s.* 1.8645*d.* : £120 5*s.* 9.3125*d.*
12. £2 1*s.* 3.50625*d.* : 6*s.* 6*d.*
13. £1 3*s.* $0\frac{3}{4}$*d.*
14. 12*s.* $1\frac{3}{4}$*d.*
15. 10*s.* 11*d.*
16. 15*s.* 4*d.* : 17*s.* $3\frac{3}{4}$*d.*
17. 85 m. 7 p. $1\frac{1}{2}$ yd. : 73 A. 2 P. $20\frac{1}{6}$ yds.
18. £7 13*s.* $1\frac{39}{40}$*d.* : 12*s.* $3\frac{1}{2}$*d.*
19. 7*s.* $11\frac{1}{2}$*d.* : 8*s.* $7\frac{1}{4}$*d.*
20. 16 lbs. : 1 qr. 4 lbs.

46.

1. .475 : .021875.
2. .375 : 1.725.
3. 1.125 : .2625.
4. .125 ; 27.5.
5. .3125 ; .196875.
6. .5703125 ; .39375.
7. .875 ; .5384375.
8. .777587890625 ; .05.
9. .19453125 ; .03625.
10. .039375 ; .046875.
11. $2.\dot{6}$; 1.424.
12. $.000220\dot{9}\dot{5}$; .924.
13. $1.8\dot{6}$; .859375.
14. $97.\dot{6}$; $.37708\dot{3}$.
15. $4.\dot{9}\dot{0}$; $4.208\dot{3}$.
16. .127109375 ; $6.15651041\dot{6}$.

47.

1. $1\frac{2}{9}$.
2. $\frac{1}{2}$*d.*
3. 3956 miles nearly.
4. $3\frac{1}{9}$ days.
5. .02734375, $36.\dot{5}7142\dot{8}$; $3\frac{3}{4}$, $3\frac{34}{45}$, .0004935, .282.
6. .375 ; £2 13*s.* 3*d.* ; $7\frac{1}{2}$*d.*
7. 16*s.* $11\frac{3}{4}$*d.*
8. $.1\dot{3}\dot{6}$, $4.2\dot{1}4285\dot{7}$; $\frac{61}{4950}$; 530, .00341.
9. 10*s.* $3\frac{1}{4}$*d.*
10. $.3\dot{5}7142\dot{8}$; 8.75.
11. 7 n. 13 m. ; 1 A. 3 R. 13 P. 22 yds

12. $11\frac{513}{625}=11.8208$.
13. £9 4*s.* $8\frac{1}{4}d$.
14. 4*s.* 9*d.* = 1.9 of 2*s.* 6*d.*
15. £463 16*s.* $1\frac{1}{8}d$; £127 9*s.* 6*d.*
16. £2 11*s.* 4*d.*
17. 11*s.* 3*d.*
18. .06640625, $.0\dot{0}9\dot{9}$; $\frac{27}{400}$, $\frac{76}{1125}$; £3 13*s.* $1\frac{1}{2}d$.
19. 3*s.* $11\frac{1}{4}d$.
20. $2\frac{19}{32}=2.59375$.
21. £3 2*s.* $11\frac{9}{20}d$.
22. 16 ft. $104\frac{52}{63}$ in.; 20 ft. $1486\frac{2}{25}$ in.
23. .18988 ; .025 ; £4 4*s.* $4\frac{1}{4}d$.
24. £25 17*s.* $2\frac{31}{64}d$.; 7*s.* $2\frac{2}{3}d$.
25. £4 4*s.* $9\frac{1}{2}d$.
26. $75\frac{1}{6}$ yds.
27. £2 0*s.* $3\frac{3}{4}d$.; £6 6*s.* $6\frac{3}{4}d$.; 4.78125.
28. 8.175 ; $.81\dot{6}$; 27 ; .75 ; 135.1940625.
29. 1*s.* $9\frac{3}{4}d$.
30. £21 3*s.* $11\frac{1}{10}d$.
31. .109375, $.1\dot{0}7692\dot{3}$; $\frac{13}{20}$, $\frac{43}{660}$; .54140625.
32. £15 14*s.* $10\frac{2}{7}d$.
33. 2.625, $.0\dot{3}\dot{6}$, $2\frac{1}{20}$, $\frac{37}{180}$; 3.971875.
34. 7 cwt. 3 qrs. $8\frac{3}{8}$ lbs.; £8 13*s.* 7*d.*
35. £81.
36. 2.140625.
37. .03.
38. 1.1457 ; 423 ; 18*s.* $6\frac{1}{4}d$.
39. 59.0625.
40. £410 11*s.* $9\frac{123}{400}d$.; £41 11*s.* $10\frac{1}{2}d$.
41. £32 15*s.*; 41.92 ; 1250.
42. .021484375, $.\dot{0}\dot{6}$; $2\frac{13}{400}$, $\frac{63}{185}$; .0009765625.
43. .0875 ; $4.6\dot{7}$.
44. £14 0*s.* $1\frac{1}{2}d$. ; .4.
45. 9.
46. £2 4*s.* $1\frac{1}{5}d$.; 24 P. 5.025 sq. yds.
47. £34 14*s.* $6\frac{19}{24}d$.
48. 3.14159.
49. £3 6*s.* $6\frac{3}{4}d$.; 6.65625.
50. 2.7182818.

48.

	£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*
1.	838	10	0	2.	1486	6	8	3.	1452	5	0	4.	2606	8	0
5.	2213	3	4	6.	210	7	6	7.	3203	1	8	8.	6212	5	0
9.	819	10	0	10.	2590	0	0	11.	3459	11	8	12.	1777	2	6

49.

1.	476	5	0	2.	1263	2	6	3.	1559	8	0	4.	2344	7	6
5.	1986	1	0	6.	879	14	6	7.	4279	12	6	8.	4455	1	6
9.	377	12	6	10.	3374	11	6	11.	3413	5	0	12.	545	14	9

50.

1.	23	17	$7\frac{3}{4}$	2.	24	1	$1\frac{3}{4}$	3.	179	13	11	4.	85	13	3
5.	103	0	$8\frac{3}{4}$	6.	143	15	$2\frac{1}{2}$	7.	79	5	$7\frac{1}{2}$	8.	361	15	8
9.	286	15	$6\frac{3}{4}$	10.	129	17	$5\frac{1}{2}$	11.	284	6	$4\frac{1}{4}$	12.	448	11	$7\frac{1}{4}$

51.

	£	s.	d.		£	s.	d.		£	s.	d.
1.	400	4	$4\frac{1}{2}$	2.	1059	9	$1\frac{3}{4}$	3.	1070	2	$0\frac{3}{4}$
4.	2486	15	7	5.	125	16	$8\frac{1}{2}$	6.	179	17	$10\frac{3}{4}$
7.	2542	0	$0\frac{1}{2}$	8.	2696	5	10	9.	201	14	$9\frac{1}{4}$
10.	366	13	$2\frac{3}{4}$	11.	1841	7	$9\frac{1}{4}$	12.	1980	13	$1\frac{1}{2}$

52.

	£	s.	d.		£	s.	d.		£	s.	d.
1.	18	6	$7\frac{5}{16}$	2.	17	16	$8\frac{5}{14}$	3.	51	0	$11\frac{17}{32}$
4.	6	4	$7\frac{5}{112}$	5.	244	1	$1\frac{7}{8}$	6.	51	16	$6\frac{9}{16}$
7.	52	4	$8\frac{119}{128}$	8.	268	11	$6\frac{63}{64}$	9.	27	19	$11\frac{49}{64}$
10.	2	13	$9\frac{23}{40}$	11.	36	10	$2\frac{1}{4}$	12.	21	4	$1\frac{7}{9}$
13.	18	18	$9\frac{1}{3}$	14.	78	11	$10\frac{1}{5}$	15.	105	17	$5\frac{23}{80}$
16.	81	12	$6\frac{1}{10}$	17.	20	12	6	18.	18	11	$1\frac{4}{7}$
19.	43	1	3					20.	20	12	$8\frac{5}{14}$

53.

1. £626 7*s.* $4\frac{1}{2}$*d.*
2. £6174 16*s.*
3. £2619 16*s.* $11\frac{1}{2}$*d.*
4. £4713 1*s.* $6\frac{3}{4}$*d.*
5. £10369 0*s.* 10*d.*
6. £48 6*s.* $10\frac{1}{2}$*d.*
7. £22 19*s.* $11\frac{1}{8}$*d.*
8. £1144 0*s.* $11\frac{29}{80}$*d.*
9. 313A. 1R. 18P.
10. £473 11*s.* $9\frac{1}{4}$*d.*
11. £1912 14*s.*
12. 26 lbs. 2 oz. 11 dwts. 16 grs.
13. £223 11*s.* 3*d.*
14. £80 17*s.* $2\frac{1}{4}$*d.*
15. £26 6*s.* 6*d.*
16. £155 9*s.* 2*d.*; £2 13*s.* 3*d.*
17. £273 6*s.* 6*d.*
18. £1191 10*s.* $1\frac{1}{5}$*d.*
19. £55042 1*s.*
20. £6 10*s.* $9\frac{7}{18}$*d.*
21. £173 9*s.* $4\frac{1}{2}$*d.*
22. £2560 14*s.* $8\frac{1}{16}$*d.*
23. £25 0*s.* $0\frac{15}{16}$*d.*
24. £2430 8*s.* $1\frac{9}{10}$*d.*
25. £7 9*s.* $7\frac{1}{2}$*d.*

54.

1. $1\frac{1}{2}$, $2\frac{2}{3}$, $2\frac{2}{3}$, 6.
2. $2\frac{2}{5}$, $3\frac{3}{4}$, $3\frac{3}{4}$, $6\frac{2}{3}$.
3. $3\frac{1}{3}$, $4\frac{4}{5}$, $4\frac{4}{5}$, $7\frac{1}{2}$.
4. $4\frac{2}{7}$, $5\frac{5}{6}$, $5\frac{5}{6}$, $8\frac{2}{5}$.
5. $1\frac{3}{7}$, $2\frac{4}{5}$, $2\frac{4}{5}$, $17\frac{1}{2}$.
6. $2\frac{1}{2}$, $6\frac{2}{5}$, $6\frac{2}{5}$, 10.
7. $1\frac{5}{9}$, $2\frac{4}{7}$, $2\frac{4}{7}$, $31\frac{1}{2}$.
8. 5, 5, 5, $9\frac{4}{5}$.

55.

1. £10.
2. 207.
3. £72.
4. 30.
5. 35.
6. £55.
7. 210.
8. 378 yds.
9. £50.
10. £10.
11. $39\frac{3}{5}$ qu.
12. £4 6*s.* 3*d.*

56.

1. £58 3*s*. 2*d*. 2. £5 19*s*. $2\frac{19}{73}$*d*. 3. 176 m. 4. 1 h. 14 m.
5. 75A. 2R. 10P. 6. 13*s*. 3*d*. 7. £1 0*s*. $11\frac{1}{4}$*d*. 8. £4 15*s*. $6\frac{19}{40}$*d*.
9. 1*s*. $1\frac{49}{200}$*d*. 10. £11 9*s*. $4\frac{1}{20}$*d*.

57.

1. 150. 2. 6 mo. 3. 12 mo. 4. 171.
5. 4. 6. $8\frac{1}{10}$*d*. 7. $622\frac{2}{9}$A. 8. $8\frac{2}{5}$ oz.

58.

1. 1*s*. $11\frac{3}{4}$*d*. 2. £37 12*s*. 6*d*. 3. 5*s*. 4. $135\frac{5}{12}$ bu.
5. £19 12*s*. 6. 165 cwt. $19\frac{13}{17}$ lbs. 7. 3*s*. 6*d*. 8. 170.
9. 2 cwt. 2 qrs. 15 lbs. 5 oz. 10. 2 lbs. $10\frac{2}{3}$ oz.
11. £2094155 16*s*. $10\frac{46}{77}$*d*. 12. £79 1*s*. $7\frac{1}{2}$*d*.
13. £7144 7*s*. 6*d*. 14. £26 18*s*. $2\frac{14}{17}$*d*. 15. $540\frac{8}{25}$ yds.
16. £11 11*s*. $2\frac{13}{25}$*d*. 17. £3. 18. 6*s*. $3\frac{39}{43}$*d*.
19. £1451 17*s*. $0\frac{4}{9}$*d*. 20. £450. 21. 85 days.
22. 178 ft. $11\frac{19}{63}$ in. 23. $6\frac{3}{4}$ hrs. 24. 12800. 25. 72.
26. $286\frac{2}{13}$ m. 27. £79 10*s*. 28. £8 3*s*. $8\frac{32}{47}$*d*.
29. £33 18*s*. 4*d*. 30. £1 16*s*. 9*d*. 31. 8*s*. $5\frac{29}{44}$*d*.
32. 10*s*. $6\frac{9}{16}$*d*. 33. 11*s*. $4\frac{1}{4}$*d*. 34. $4\frac{3}{8}$ yds.
35. £270. 36. 7722. 37. 32 ft.; 152 ft.
38. £5 17*s*. $11\frac{5}{128}$*d*. 39. £13 9*s*. $0\frac{3}{4}$*d*. 40. $26\frac{1}{24}$ lbs.

59.

1. 44 da. 2. 27. 3. 16. 4. 15. 5. 12.
6. $312\frac{1}{2}$ lbs. 7. 125 rms. 8. £194 8*s*. 9. 14 wks. 2 da.
10. £114 6*s*. 11. 45. 12. 112. 13. £520 14. 9.
15. 61*s*. $10\frac{1}{2}$*d*. 16. $6\frac{3}{4}$ da. 17. £545 6*s*. 3*d*. 18. 3 wks. 6 da.
19. 34 mi. 20. 8. 21. $240\frac{1}{2}$. 22. $13\frac{3}{4}$.
23. $2\frac{1}{7}$ days. 24. 6 tons 17 cwt. 16 lbs. 25. 4.
26. $10\frac{2}{3}$ hrs. 27. 182. 28. 8. 29. 121 days. 30. 50.

60.

1. £125. 2. £45. 3. £1260. 4. £2673 2*s*. 6*d*.
5. £247 16*s*. $7\frac{1}{2}$*d*. 6. £2857 10*s*. 7. £744 16*s*. $1\frac{1}{8}$*d*.
8. £71 12*s*. $2\frac{2}{5}$*d*. 9. £37 17*s*. $3\frac{1}{10}$*d*. 10. £20 10*s*.

61.

	£	s.	d.		£	s.	d.		£	s.	d.
1.	519	19	$1\frac{53}{73}$	2.	7612	7	$5\frac{13}{73}$	3.	1196	19	6
4.	19	10	$11\frac{7}{10}$	5.	492	0	$4\frac{4}{5}$	6.	284	6	$1\frac{1}{8}$

62.

1.	100	13	$8\frac{3}{8}$	2.	57	17	$7\frac{1}{2}$	3.	0	2	1
4.	1	15	$6\frac{9}{25}$	5.	26	5	$5\frac{169}{256}$	6.	24	12	$10\frac{362}{375}$

63.

1. $5\frac{1}{2}$. 2. £42 5*s.* 10*d.* 3. 125 days. 4. 6.
5. 25 yrs. 6. £39 7*s.* 6*d.* 7. $2\frac{1}{4}$. 8. £1043 15*s.*
9. $3\frac{2}{3}$ yrs. 10. $2\frac{2}{3}$. 11. £8 8*s.* $2\frac{3}{4}$*d.* 12. 20 yrs.

64.

	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.
1.	266	13	4	2.	769	4	$7\frac{5}{13}$	3.	199	1	3	4.	125	0	0
5.	1	12	$5\frac{40}{163}$	6.	27	5	5	7.	579	8	9	8.	1	7	6
9.	1	13	0	10.	1	18	6	11.	2	9	6	12.	3	0	6
13.	5	9	6	14.	6	1	0								

65.

	£	s.	d.		£	s.	d.		£	s.	d.
1.	26	5	$9\frac{9}{10}$	2.	78	12	$9\frac{3}{4}$	3.	80	0	8
4.	447	6	8	5.	26	9	$6\frac{11}{20}$	6.	0	9	$1\frac{1}{2}$
7.	5700	0	0	8.	19	12	4	9.	83	8	$7\frac{1}{4}$
10.	26	3	$10\frac{1}{2}$	11.	1	5	$7\frac{1}{4}$	12.	4488	15	0

66.

1. £821 5*s.*; £41 5*s.* 2. £44. 3. £106 13*s.* 4*d.*
4. £151 13*s.* 4*d.* 5. £90. 6. £15708 6*s.* 8*d.*
7. £533 6*s.* 8*d.* 8. £771 7*s.* 6*d.*; £10 12*s.* 6*d.*
9. £10 8*s.* 4*d.* 10. £25. 11. Increase of £10.
12. Increase of £20. 13. £16 13*s.* 4*d.* 14. £53 6*s.* 8*d.*
15. $100\frac{5}{6}$. 16. The $3\frac{1}{2}$ per cents. 17. £17 3*s.* 6*d.*
18. £5500; £836 5*s.*; £4241 17*s.* 6*d.* 19. £715. 20. $93\frac{1}{8}$.

67.

1. £1 1*s.* 9*d.* 2. $22\frac{6}{7}$ per cent. 3. £2 4*s.* $6\frac{6}{41}$*d.*
4. 12 per cent. 5. 17 per cent. 6. £30 16*s.*; $9\frac{3}{8}$.
7. $11\frac{1}{9}$; £210 8*s.* $7\frac{1}{2}$*d.* 8. £1 13*s.* $11\frac{1}{10}$*d.* 9. £1 0*s.* $2\frac{1}{2}$*d.*
10. £93 6*s.* 8*d.*; $11\frac{1}{2}$ per cent. 11. 8 p. c. gain. 12. £82 10*s.*
13. £44 15*s.* 14. 40. 15. 9*s.* $2\frac{1}{2}$*d.* 16. $2\frac{4}{5}$ p. c. loss.
17. £4 9*s.* $7\frac{1}{5}$*d.* 18. $5\frac{5}{9}$ p. c. gain. 19. 25. 20. $63\frac{7}{11}$.

68.

1. 213, 355, 497; 525, 315, 225. 2. £72, £99, £108.
3. C, 15 cwt. 0 qrs. 20 lbs.; T. 1 cwt. 2 qrs. 19 lbs.
4. £46 13*s.* 4*d.*, £35, £28, £23 6*s.* 8*d.*, £20. 5. 14, 112, 378, 896.
6. O. 889 oz.; H. 111 oz. 7. £66 13*s.* 4*d.*; £33 6*s.* 8*d.*; £200.
8. £6 17*s.* 3*d.*; £4 15*s.* $3\frac{3}{4}$*d.* 9. 3 oz. 7 dwt. $6\frac{6}{11}$ grs.
10. N. $1702\frac{2}{5}$ lbs.; S. $212\frac{4}{5}$ lbs.; C. $324\frac{4}{5}$ lbs.
11. 1 lb. 11 oz. 10 dwt. $20\frac{100}{623}$ grs. 12. 2 oz. 4 dwt. 14 grs.
13. £160, £175. 14. £102; £104; £78.
15. £28 2*s.* 6*d.*; £35 3*s.* $1\frac{1}{2}$*d.*; £11 14*s.* $4\frac{1}{2}$*d.*
16. 12 carats. 17. 15 carats: 15 oz. 18. 15 carats.
19. 4*s.* $2\frac{1}{2}$*d.*; 6*s.* $7\frac{1}{2}$*d.*; 1*s.* 9*d.* 20. £100; £300.

69.

1. 5. 2. $358\frac{1}{3}$. 3. 3. lbs. 2 oz. 4. $1\frac{13}{32}$. 5. A shilling.
6. $5\frac{169}{235}$. 7. £96.10561. 8. 15 sacks $59\frac{1}{2}$ lbs.
9. $98\frac{26}{35}$. 10. 86.186 : 1.

70.

1. 73; 94. 2. 185; 371.
3. 729; 592. 4. 309; 499.
5. 590; 80700. 6. 6123; 4117.
7. 1880; 8097. 8. 9998; 4908.
9. 345761; 607002.
10. 2.828427 + ; 4.472136 − ; 19.052559 −.
11. 187.403308 + ; 94.005319 −. 12. 367.3; 806.54.
13. 81.6279 − ; 5.270009 +. 14. .47434 + ; 7.1505.
15. .1096 − ; .0398. 16. .009075; .144914 −.
17. $\frac{23}{73}$; .6060915 + ; .56789 +. 18. .3118048 − ; .2400274 + ; $1\frac{11}{14}$.
19. 16.9595 + ; $78\frac{1}{3}$; 19.1647 −.
20. .0574485 − ; .096386 + ; 1.426353 −. 21. $4\frac{1}{6}$; 1.103026.
22. 925 links. 23. 38 ft. 9 in. *nearly*. 24. 22*s.* $10\frac{1}{2}$*d.*

71.

1. 57; 74. 2. 28; 190.
3. 163; 328. 4. 456; 9870.
5. 809; 4812. 6. 6397; 5608.
7. 7099; .369. 8. 36.8403 + ; 8.081 +.
9. 20.03909 + ; 17.84109 +. 10. .941036 + ; 3.1158 +.
11. 1 ft. 10.624 + in. 12. 852300 miles.

MISCELLANEOUS.

1. 18880. 2. £345. 3. $\frac{2}{5}$; $1\frac{29}{42}$; 1*s.* $1\frac{1}{2}$*d.*; $3\frac{5}{84}$.
4. £1 17*s.* $7\frac{1}{2}$*d.* 5. £1492 13*s.* $7\frac{7}{20}$*d.* 6. $181\frac{1}{4}$ qu.
7. $41\frac{1}{7}$ ft. 8. £318 15*s.* 9. 31*s.* 6*d.*
10. 2A. 1R. $5\frac{19}{80}$P. 11. 609; 85 ft. 10 in. 12. $2576\frac{517}{758}$ qrs.
13. $3\frac{1}{8}$. 14. 20. 15. $12\frac{4}{63}$. 16. 1*s.* $2\frac{20}{41}$*d.*
17. 4*s.* 8*d.*; $\frac{11}{96}$; $\frac{3}{80}$. 18. £3 2*s.* $0\frac{24}{29}$*d.* 19. 309.76; 45.78082 –.
20. £150, £180, £240, £300. 21. $5\frac{15}{23}$ days. 22. $96\frac{4}{11}$.
23. $\frac{32}{767}$; $162\frac{29}{140}$; $1\frac{121}{175}$; $\frac{11}{115}$; 2308. 24. *A*, 80*s.* 3*d.*; *B*, 77*s.* 9*d.*
25. £11 5*s.*, £20, £29 5*s.* 26. £70 11*s.* $9\frac{3}{17}$*d.* 27. $23\frac{1}{3}$ days.
28. 510.9 –. 29. £6 5*s.*; £4 3*s.* 4*d.*; £3 2*s.* 6*d.*; £2 10*s.*
30. 884, 153. 31. 3 h. 20 min. 32. $\frac{16}{25}$. 33. 105 da.
34. 3.035913 +; 960.040103 +. 35. $5\frac{29}{32}$ da., or 5 da. $7\frac{1}{4}$ hrs.
36. .68125; $\frac{1}{4}$; .00256256, 256.256, .0256256.
37. 4*s.* $10\frac{1}{2}$*d.* 38. 59 min. $8\frac{14708}{43829}$ sec.
39. 13*s.* $2\frac{22}{91}$*d.*, 6*s.* $7\frac{11}{91}$*d.*, 3*s.* $3\frac{51}{91}$*d.* 40. $\frac{4}{27}$. 41. £14 13*s.* $6\frac{1}{4}$*d.*
42. £1 19*s.* $6\frac{66}{91}$*d.* 43. £3200, £4800, £6000, £7000.
44. 3 tons 17 cwt. 2 qrs. $26\frac{1}{4}$ lbs. 45. 45. 46. £11 19*s.* $4\frac{1}{2}$*d.*
47. .25298 &c.; $5\frac{1}{9}$. 48. £4 7*s.* $8\frac{11}{12}$*d.*
49. $69\frac{51}{200}$ degrees = $76\frac{19}{20}$ grades. 50. $14\frac{22}{27}$ hrs. 51. $18\frac{11}{13}$.
52. $\frac{8}{9}$ of a da. 53. 21*s.* 8*d.* 54. £18 10*s.* $4\frac{4}{9}$*d.* 55. 1500.
56. .36; 25 lbs. 15 oz. 11.904 drs.; 4 miles $30\frac{2}{9}$ yds.
57. £759 5*s.* $7\frac{73}{101}$*d.* 58. 6.25; 12.84. 59. £3499; £874 15*s.*
60. $10\frac{10}{31}$P. 61. 111104. 62. £3 7*s.* 2*d.* 63. .057 &c.
64. £34 10*s.* $10\frac{10}{11}$*d.* 65. $3\frac{25}{48}$; $\frac{13}{16}$. 66. $\frac{1}{8064}$ in.
67. 10*s.* 5*d.* 68. 245 : 243. 69. $638\frac{2}{11}$. 70. 2*s.* $7\frac{1}{2}$*d.*
71. £49 9*s.* 4*d.* 72. $\frac{5}{39}$; .315625; £2000. 73. £49.
74. 6315 *dollars* $55\frac{10}{13}$ *cents.*
75. £33 6*s.* $3\frac{39}{40}$*d.*, £66 12*s.* $7\frac{19}{20}$*d.*, £99 18*s.* $11\frac{37}{40}$*d.*, £133 5*s.* $3\frac{9}{10}$*d.*
76. 42 m.; $10\frac{1}{2}$ m. 77. 6 per cent. 78. 10*d.*, 1*s.* $4\frac{1}{2}$*d.*, 1*s.* 11*d.*, &c.
79. $578\frac{26}{83}$. 80. 105. 81. 1*s.* 9*d.*, 1*s.* 2*d.*, and 7*d.*
82. 3*s.* $5\frac{11}{32}$*d.* 83. 560.22 &c. 84. $1\frac{1}{2}$ min.; $427\frac{1}{2}$, 190.
85. £907 10*s.* 86. $\frac{9}{560}$; .69140625. 87. £6 2*s.* $2\frac{1}{4}$*d.*
88. £15. 89. 107 yds. 2 ft. 11 in.; £6 14*s.* $11\frac{7}{12}$*d.*
90. £32 1*s.* $1\frac{29}{40}$*d.*; 3*d.* 91. $99\frac{5}{16}$; £176 4*s.* $2\frac{170}{227}$*d.* 92. £123 11*s.* 4*d.*
93. £3 4*s.* 11*d.* 94. $23\frac{670}{1519}$; £1 14*s.* $0\frac{3}{4}$*d.*; 8*s.*; .06515625.
95. 1 lb. 3 oz. 7 dwt. $4\frac{10}{13}$ grs.; $1555\frac{29}{19200}$. 96. $6\frac{2331}{5399}$.
97. £315. 98. £319 16*s.* $8\frac{120}{163}$*d.*
99. 79.0079 +; 37.9241 –; .069; 30.02. 100. 8 days. 101. 15 hrs.
102. 6 min. $17\frac{7}{19}$ sec. A.M. 103. $6001\frac{53}{72}$ yds.
104. *B*, 6*d.*; *C*, 2*s.* 6*d.* 105. £1011 0*s.* $3\frac{39}{227}$*d.*

106. £840, £795. 107. $89\frac{111}{356}$. 108. £1050. 109. £10560.
110. £18668 2*s.* $7\frac{331}{500}$*d.* 111. 65. 112. 13*s.* 9*d.*
113. 1 hr. $51\frac{2}{3}$ min. 114. 15 cwt. 115. 3 ft. 9.02221 &c. in.
116. *A*, £16 1*s.* 8*d.*; *B*, £8 5*s.* 117. £11 16*s.* 8*d.*
118. .095178 + ; $21\frac{5}{9}$; $\frac{5}{6}$. 119. £1706 13*s.* 4*d.* 120. $3\frac{1}{8}$.
121. 18*s.* $5\frac{1961}{2000}$*d.* 122. 9 days. 123. *A*, 264; *B*, 198; *C*, 308.
124. £300. 125. $2133\frac{1}{3}$. 126. £2771 7*s.* $0\frac{3}{10}$*d.*
127. 7 ft. $4\frac{8}{9}$ in. 128. 16*s.* 3*d.* 129. £22 13*s.* $2\frac{2}{9}$*d.*
130. £1 8*s.* $6\frac{6}{7}$*d.* 131. £410 11*s.* $9\frac{123}{400}$*d.*; £41 11*s.* $10\frac{1}{2}$*d.*
132. £36893 6*s.* 8*d.* 133. £6 8*s.* $10\frac{7}{8}$*d.*
134. 3*s.* 4*d.*; 5*d.* 135. £3 15*s.* $2\frac{7}{8}$*d.*
136. .45. 137. $18\frac{3}{4}$ per cent.; 10*s.* $6\frac{6}{19}$*d.* cost price.
138. £2027 1*s.* $7\frac{9173}{15625}$*d.* 139. $3\frac{1}{33}$. 140. 1. 141. 3*s.* 4*d.*
142. £8 7*s.* 143. .05099902 − ; .0155048 + ; .9615 −.
144. $3\frac{2}{3}$ ft.; 8 tons 3 cwt. 3 qrs. $1\frac{4}{175}$ lbs. 145. 1 per cent.
146. £595 0*s.* $9\frac{111}{121}$*d.* 147. 8 hrs. 30 min.; 10 hrs. $22\frac{1}{2}$ min.
148. $121\frac{1}{2}$. 149. £3 10*s.* $9\frac{1}{15}$*d.*; .77. 150. 63.
151. £4957 6*s.* 8*d.* 152. £1 3*s.* $9\frac{1}{15}$*d.* 153. $87\frac{1}{2}$.
154. £220, £6 1*s.* 11*d.* 155. 10*s.* $8\frac{1}{2}$*d.*
156. 30*s.*, 15*s.*, 10*s.*, 7*s.* 6*d.*, 6*s.*, 5*s.* 157. 415.8, 356.4, 226.8.
158. 600. 159. £12800. 160. $26\frac{9}{16}$. 161. $4\frac{4}{9}$ lbs.
162. $\frac{27}{71}$, .00390625, $8\frac{73}{200}$, $1\frac{1}{51}$. 163. $121\frac{1}{4}$. 164. £130.
165. £245 18*s.* $11\frac{7}{9}$*d.* 166. 15. 167. $4\frac{7}{12}$, £293 6*s.* 8*d.*
168. £292 4*s.* 169. £62 3*s.* $8\frac{117}{160}$*d.*, 34733.92.
170. 1.4$\dot{3}$, 6.48$\dot{3}$; 2.49, 8.57. 171. 550 tons, $68\frac{3}{4}$.
172. 18*s.* $2\frac{1}{4}$*d.*, 11*s.* $9\frac{3}{4}$*d.* 173. £65 15*s.* $9\frac{9}{19}$*d.*
174. 6384, 7695, 8321; $2\frac{37}{314}$ da. 175. £5 13*s.* $0\frac{3}{4}$*d.*
176. $\frac{2}{285}$. 177. $\frac{3}{5}$, .914791$\dot{6}$. 178. 12 hrs. 8 min.
179. .5. 180. $9\frac{11}{25}$.
181. 273.649. 182. £520. 183. 2880, .0099431$\dot{8}$, $\frac{125}{273}$.
184. The $3\frac{1}{2}$ per cents. 185. £187 10*s.*, £312 10*s.*, £500.
186. 4 lbs. 11 oz. 19 dwts., .165234375, $\frac{39}{400}$, $\frac{7}{11}$, $\frac{97}{185}$.
187. 266 tons, $16\frac{1}{10}$ cwt. 188. 176A. 540 sq. yds.
189. 7*s.* $0\frac{6}{23}$*d.* 190. £270, £11 8*s.* $3\frac{13809}{20000}$*d.*
191. 2400, 1800, 1600, 1500.
192. £5 14*s.* $0\frac{3}{4}$*d.*, £182 10*s.*, £6 16*s.* $10\frac{1}{2}$*d.*
193. £3250, £1560, £1440. 194. 80 and 160. 195. 66.286.
196. £211 19*s.* 3*d.* 197. The 3 per cents.; 19*s.* $7\frac{3}{8}$*d.* 198. $8\frac{7}{16}$.
199. $5\frac{3}{4}$, $\frac{16}{27}$, $1\frac{16}{69}$. 200. £532 4*s.*, £100 16*s.*, £492.
201. .45593 − ; 70.61. 202. £94 10*s.*, £7 8*s.* $10\frac{1}{20}$*d.* 203. 51, $1\frac{29}{40}$.
204. £127 5*s.* $5\frac{5}{11}$*d.*, £127 12*s.* 1*d.* 205. 20.7846 &c., 203.646 &c.
206. £196, £304. 207. $491\frac{1}{160}$. 208. £6 9*s.* $11\frac{1}{4}$*d.*
209. £2 5*s.* 210. £320, £293 6*s.* 8*d.*, £110, £201 13*s.* 4*d.*

211. 4*s.* $6\frac{2}{3}$*d.* 212. $587\frac{37}{49}$. 213. £10 8*s.* 214. 3*s.* $1\frac{37}{40}$*d.*
215. £1832 19*s.* $6\frac{822}{1777}$*d.* 216. £29 17*s.* $2\frac{1}{4}$*d.* 217. 12 days.
218. 1 ton 12 cwt. 2 qrs. 3 lb. 5 oz.; £8 14*s.* $6\frac{3}{4}$*d.*
219. 12 hrs. 48 min.; $4\frac{4}{5}$, $5\frac{1}{5}$. 220. 224 miles 64 yds.
221. £3829 8*s.* $9\frac{21}{128}$*d.* 222. £10278 9*s.* $5\frac{9}{32}$*d.*
223. £51 8*s.* $4\frac{389}{1600}$*d.*, £129 17*s.* $2\frac{34}{625}$*d.*

EXAMINATION-PAPERS.

Paper V.

3. 31 sq. po. 30 yd. 2 ft. 4. 12524940 in.
5. 19 ac. 2 ro. 29 po. 2 yd. 5 ft. 81 in. 6. 17778376 in.
7. 1 ac. 2 ro. 3 po. 4 yd. 5 ft. 6 in. 8. $27897\frac{3}{4}$ ft.
9. 668 sq. yds. 10. 1224.6 gall. 11. 31.103 ft.
12. 3.962 met. 13. 160.93 decam. 14. 100000. 15. $6\frac{46}{99}$.

Paper VI.

4. 13 : 20. 5. 8 : 13. 6. 7 : 15.
7. 96 : 80 : 120 : 105. 8. 1 : 3 or $\frac{1}{3}$. 9. *M* to *N* as 18 : 17.

Paper VII.

1. $77\frac{5}{6}$. 2. $24/3\frac{3}{4}$. 3. £22241170. 4. 40° 53′.
5. $3\frac{1}{4}$*d.* 6. $20\frac{7}{12}$. 7. 7 sq. ft. 8. 60.
9. $4/1\frac{1}{2}$. 10. $21\frac{3}{4}$. 11. 3 gall.

Paper VIII.

6. $22\frac{1}{2}$ da. 7. *A*, $33\frac{3}{5}$ hrs.; *B*, 24 hrs.; *C*, $18\frac{2}{3}$ hrs.
8. *A* 12, *B* 15, *C* 20 da. 9. $5\frac{3}{5}$ da.
10. 360 gall.; 1 gall. per hr. gained.

Paper IX.

1. £89 4*s.* $4\frac{1}{2}$*d.* 2. 6576 fr. $51\frac{1}{2}$ cts. 3. 95286.21 fr.
4. £323 3*s.* $1\frac{17}{19}$*d.* 5. 9.386*d.* 6. $62\frac{1}{2}$*d. nearly.*
7. 2211 dol. $16\frac{4}{11}$ re. 8. $53\frac{1}{2}$*d.* per milree, *nearly.* 9. 3722.07 fr.
10. 4*s.*; $42\frac{1}{2}$ francs. 11. Gains 8*s.* *nearly.*
12. Circuitously, by 35.985 milrees. 13. £160 14*s.* $3\frac{3}{7}$*d.*

14. 480 fr. $24\frac{1}{2}$ cents. 15. 5 doll. $59\frac{3}{4}$ cents.
16. 1 rupee 11.13 annas per lb. 17. (i.) .0102045 oz.; 25.17 francs.
17. (ii.) 25 fr. $53\frac{1}{2}$ cts.; 25 fr. $14\frac{1}{2}$ cts. 17. (iii.) *a*. .088 p. c. dearer.
17. (iii.) *b*. .367 p. c. dearer.

Paper X.

3. 32 oxen. 4. 4. 5. $9\frac{3}{11}$ da. 6. 20 wks.
7. 3 ac. 8. 40 oxen. 9. 21 days.
10. 14.076 min.; $\frac{1593}{2930}$ of the cist.

Paper XI.

4. 264 at 12*s.* &c. 5. 42 and 48. 6. 40 or.; 45 lem.
7. T. 3*s.* 9*d.*, C. 1*s.* 8*d.* 8. 5 : 4.

Paper XII.

7. .2031. 8. 21 po. $2\frac{1}{5}$ yd. 9. 6 po. 1 yd.
10. 108.097 yd.; $305\frac{3}{4}$ yd. 12. 140. 13. 14.02 ft.
14. 153 mi. 15. $\sqrt[4]{19}$; $\sqrt{3}$. 16. .0261 in.
17. 250. 18. $8\frac{1}{2}d.$ 19. 12 ft. 20. 433 *nearly*.
21. 2 ft. 2 in. *nearly*; $28\frac{1}{8}$ sq. ft. 22. £42. 23. 4.
24. 5.5413 ft.; 5.058 ft. 25. $\frac{2}{31}$.
26. 13.6801 cub. yds. 27. 5.51 p. c. *nearly*.

Paper XIII.

1. 57 min. 2. 263 times; .0029 rem. 3. 192.
7. 46 sq. ft. 0′ 0″ 11‴. 8. 287 sq. ft. 2′ 5″ 6‴.
9. 46 sq. ft. $0\frac{11}{12}$ in.; 287 sq. ft. $29\frac{1}{2}$ in. 10. £10 1*s.* $9\frac{7}{18}d.$
11. £6 4*s.* 5*d.*, £3 13*s.* 7*d.* 12. Gain 25 p. c. 13. $\frac{405}{2002}$ ac.
14. 2*s.* $7\frac{1}{4}d.$ 15. $.7\dot{2}$. 17. $\frac{5}{6}$ per cent. gained.
18. 10*d.* 19. *Nothing.* 20. 3283.
21. 3 yrs. 100 da. 22. $5\frac{3}{5}$ mths. 23. Value = $242\frac{1}{4}$ da.
24. 18, 27, 24, 30. 25. *A* 5*s.*, *B* 1*s.* $10\frac{1}{2}d.$, *C* 1*s.* $1\frac{1}{2}d.$
26. $\frac{17}{315}$. 27. *A* 11*s.* 4*d.*, *B* 1*s.* 4*d.*, *C* 7*s.* 4*d.*
28. £16 13*s.* 4*d.* 29. Nearly £3 16*s.* 11*d.* p. c.
30. 40 ac. 9 po. 10 yds. $32\frac{4}{7}$ in. 31. £12 6*s.* $11\frac{1}{3}d.$
32. £14 10*s.* $10\frac{10}{11}d.$ 33. £260. 34. $4\frac{23}{88}$.

Paper XIV.

1. 3759. 2. $192\frac{1}{2}$ ft. 3. $391\frac{1}{3}$ rev.; $7\frac{1}{2}$ and $13\frac{1}{2}$ ft. circumf.
4. 1 gall. water to 17 spirits. 5. 1520 tons.
6. £821 5*s.*; 32 days. 7. 1 mile, $1557\frac{3}{4}$ yds. *nearly.*
8. 12 weeks. 9. 729, 432, 3348, 27.
10. $.0041\dot{6}$ and .0625. 11. 16*s.* 4*d.* 12. 31*s.* $4\frac{1}{5}d.$
13. $301\frac{1}{3}$ c. yds.; 165.19 lbs. 15. 4 florins. 16. $31\frac{10}{11}$ and $509\frac{1}{11}$.
17. £1505. 18. £127 1*s.* 5*d.*
19. $1\frac{31}{69}$ yr., or 1 yr. 164 da. 20. 12 yards from *B.*
21. £24 increase. 22. £10 16*s.* decrease.
23. $\frac{2}{3}$. 24. .008.

Paper XV.

11. 12 men. 12. 2 pon. 13. £6947 18*s.* 4*d.* 14. 3570.
15. 3.627 p. c. 16. 7 men. 17. 14*s.* $3\frac{3}{7}d.$ 18. $9\frac{3}{5}d.$
19. 26*s.* 8*d.*, 33*s.* 4*d.* 20. $11\frac{2}{3}$ mths.
21. At 24 min. and at $30\frac{6}{11}$ min. past 11. 22. £104.
23. 22 yrs. ago; 18 yrs. hence. 24. 4.
25. $9\frac{1}{21}$ mi. an hour. 26. 68*s.*
27. $9\frac{6}{11}$ min. past 8. 28. £125000.
29. £1 11*s.* 3*d.* 30. 92 days.
31. *The whole.* 32. £1078 11*s.* 7*d.* *nearly.*
33. £1400. 34. $6\frac{1}{4}$ p. c.; £574 13*s.*
35. 3.4408. 36. $7\frac{2}{3}$ mths. 37. 3 : 7.
38. 6 mths. 39. £147. 40. £600000.
41. £1000. 42. *b.* $\frac{3}{4}$. 43. 14 cwt. 3 qr. 13 lb., &c.
44. £17 16*s.* $4\frac{4}{11}d.$, £8 18*s.* $2\frac{2}{11}d.$ &c.
45. £322, £627 4*s.*, £2060 16*s.* 46. $23\frac{11}{43}$ p. c.
47. For ploughing the field with oxen, £4 7*s.* 9*d.*; For ploughing it with horses, £3 18*s.* $9\frac{3}{4}d.$
48. $7\frac{1}{5}$ hours. 49. $10\frac{1}{6}$ hrs. 50. 6 min. 6.08 sec.

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