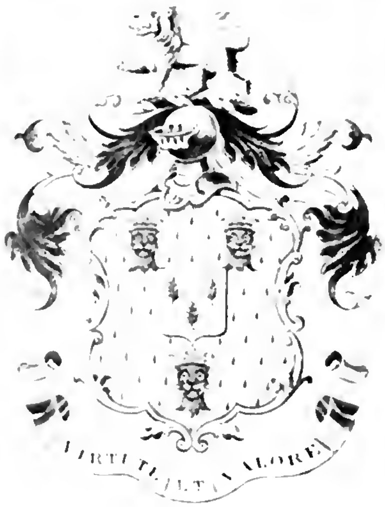


William Noble.

ASTRONOMICAL AND PHYSICAL RESEARCHES

MADE AT

MR. WILSON'S OBSERVATORY,

DARAMONA, WESTMEATH.

DARAMONA.

CONTENTS.

ILLUSTRATIONS.

INTRODUCTION.

IN 1871 a small observatory was built in the garden of Daramona, and provided with a 12-inch equatorial reflecting telescope, by Sir Howard Grubb. There was also attached a small room which could be used for photography; it contained a small transit instrument which I made in the workshop, and a sidereal clock. Except a few photographs of the moon, taken with wet plates, and some experiments on solar radiation with thermo piles, little was done with this instrument. In 1881 I decided to build a new and large observatory, which would be attached to the house.

A 24-inch silver-on-glass mirror, of 10 feet 6 inches focus, was ordered from Sir Howard Grubb, and it was erected on the old mounting. This was not found satisfactory, as the mounting was not stiff or heavy enough to carry the extra weight. In 1892 I decided to have the 24-inch mirror remounted by Sir Howard Grubb, and provided with his best form of driving clock and electrical control. The new mounting has given the greatest satisfaction. The position of the new observatory is, by the Ordnance Survey Map—

Latitude $53^\circ\ 41'\ 12''$ N.
Longitude $0^{h.}\ 29^{m.}\ 58{\cdot}6^{s.}$ W.

In 1889 a physical laboratory was built adjoining the observatory, and a photographic dark room for the development of the astronomical plates. Adjoining this room is also the workshop, which contains a 6-inch Whitworth screw-cutting lathe, a shaping machine, and a drilling machine. These tools are driven by a $1\frac{1}{2}$ N.H.P. Crossley Gas Engine.

Since the erection of the 24-inch mirror, it has been almost entirely used for photographing star clusters and nebulæ. Some enlargements made from the original negatives are published at the end of this volume. They are reproduced by the Collotype process, which unfortunately seems unable to depict the delicate nebulosities which can be seen in the original negatives. It seemed to be of no use giving the accurate position of certain fiducial stars on the enlargements, as, if measures for position have ever to be made, the original negatives would alone furnish the precise data required, measures obtained from paper prints being evidently quite unreliable.

The experiments on the radiation of heat from sun spots were made by means of a large polar heliostat, having a plain silvered mirror of 15 inches in diameter. It is the property of the Royal Society, and the use of it was kindly placed at my disposal by them. Photographs of the sun were taken by a 4-inch refractor at the same time that any measures were made of radiation from the spots, so that the distance of the spot from the sun's limb could afterwards be determined.

Having no assistant to help me in the work of the observatory, much progress would never have been made had my friends—Prof. G. F. Fitzgerald, Dr. A. A. Rambaut, Prof. G. M. Minchin, and Mr. P. L. Gray—not come and kindly aided me in every possible way.

W. E. WILSON.

Experimental Investigations on the Effective Temperature of the Sun, made at Daramona, Streete, Co. Westmeath.

By William E. Wilson, *M.R.I.A., and* P. L. Gray, *B.Sc., A.R.C.S., Lecturer in Physics, Mason College, Birmingham.*

Philosophical Transactions of the Royal Society of London, Vol. 185 (1894) *A.*

The expression "effective temperature of the sun" has by this time obtained a well-defined meaning, and may be taken (as stated by Violle and other physicists) to be that uniform temperature which the sun would have to possess if it had an emissive power equal to unity, at the same time giving out the same amount of radiant energy as at present.

The older estimates of this quantity were little more than guesses, and varied between 1500° C. and 3,000,000° to 5,000,000° C., or more.

The former of these values was given by assuming that Dulong and Petit's formula

$$R = ma^t,$$

where R = intensity of radiation, t = the temperature of the radiating surface, and m and a are constants for any one substance, held up to any limit.

The result given by it is obviously too low, as it is less than even the melting-point of platinum, the vapour of which probably exists in the solar atmosphere, and considerably lower than the temperature which may be obtained in the focus of a large lens.

The higher values were found by using Newton's law, in which radiation is taken as simply proportional to difference of temperature between the radiating body and its surroundings, a law which is proved to hold good only for very small differences.

It would appear, then, that by far the greatest difficulty in estimating the value of the solar temperature arose from ignorance of the law which connects the radiation from a hot body with its temperature, although there are minor difficulties which may still produce uncertainties in the final result.

One thing seems certain, that the temperature of the sun is far higher than any we can produce in our laboratories. This being so, the best that can be done is to make direct determinations of the connection between radiation and temperature within the widest possible limits, find an empirical law to which the observations conform, and trust that no break of continuity may make an extra-polation entirely useless.

So far, the only investigations made in this way appear to be those of LE CHATELIER* and ROSETTI.† LE CHATELIER measured the photometric intensity of the red light from solid bodies heated to different known temperatures, and obtained an empirical law which very fairly expressed his results from 700° to 1800° C.

He then, by passing sunlight through the same piece of red glass, measured the visual intensity of the "red radiation" coming from the sun, and, by applying the law just mentioned, deduced an effective solar temperature of 7600° C., which he admits to be an approximation with a possible error either way of 1000°.

The law he found is expressed thus :

$$I = 10^{6 \cdot 7} T^{-3210 \, T}, \ddagger$$

where I is the photometric intensity, and T the absolute temperature of the radiating body. On plotting the numbers that LE CHATELIER gives for corresponding values of I and T, it will be seen more easily than by mere inspection of the formula that I increases in an enormously rapid ratio as compared with T, which must evidently tend to vitiate the accuracy of the results obtained by extra-polation.

Then, as VIOLLE§ points out, it is probable that the absorption by the red glass decreases as the radiation increases. And in discussing a question in which *total energy* as measured by heat is concerned, it is probably better to deal by experiment with the total energy than with a selected wave-length or a group of wave-lengths.

Still the value thus obtained is sufficiently near those given by the utterly distinct methods of ROSETTI and of ourselves to increase considerably the probability of the approximate accuracy of our results.

ROSETTI attacked the problem in the most direct and complete manner hitherto attempted. He determined a law of radiation which held well up to 2000° C., and found in arbitrary units the heat radiated from an incandescent body at a known high temperature by means of a thermopile and galvanometer. He then measured the heat coming from the sun in the same units, and applied his formula to find the solar temperature, which finally came out at about 10,000° C. The questions of atmospheric absorption and the emissive powers of his incandescent solids were also investigated, and his work will be referred to more than once in the following pages.

* LE CHATELIER, 'Compt. Rend.,' 1892, vol. 114, p. 737.

† ROSETTI, 'Phil. Mag.,' 1879, vol. 8, 5th series, pp. 324, 438, 537.

‡ The negative sign in the exponent is omitted in LE CHATELIER's paper, probably by a mere slip.

§ VIOLLE, 'Compt. Rend.,' 1892, vol. 114, p. 734.

I. General Method and Instruments.

The general idea in this investigation was to endeavour to *balance* the heat of the sun by means of an artificial source of heat at a high known temperature, thus obtaining both directness and simplicity as far as possible. The artificial source of heat was a strip of platinum heated by an electric current; this strip formed part of a modified form of Joly's Meldometer, which is described below, and its temperature could be determined at any moment with a high order of accuracy.

The radiation from a known area of the incandescent strip was balanced against that coming from the sun in a differential radio-micrometer—a modified form of Professor Boys's well-known and excessively delicate instrument.

The essential theory of the method was extremely simple. Knowing the apparent areas of the sun and the artificial source of heat (the latter, of course, being much the greater), and knowing the law connecting radiation and temperature, we can at once find to what point the latter would have to be raised to balance the sun, if these apparent areas were made equal. But this would be the required effective temperature of the sun, if the emissive powers were equal, and both bodies could radiate directly and without intervening absorption on to the receiving surface of the radio-micrometer.

This extreme simplicity, however, cannot be obtained, and correcting factors have to be applied for—

(*a*) Emissive power of the platinum strip;

(*b*) Reflecting power of the glass in the heliostat, which keeps the beam of sunshine in the required position;

(*c*) Terrestrial atmospheric absorption.

Each of these will be discussed in turn, after the instruments used have been described.

The general arrangement of the apparatus is shown in fig 1.

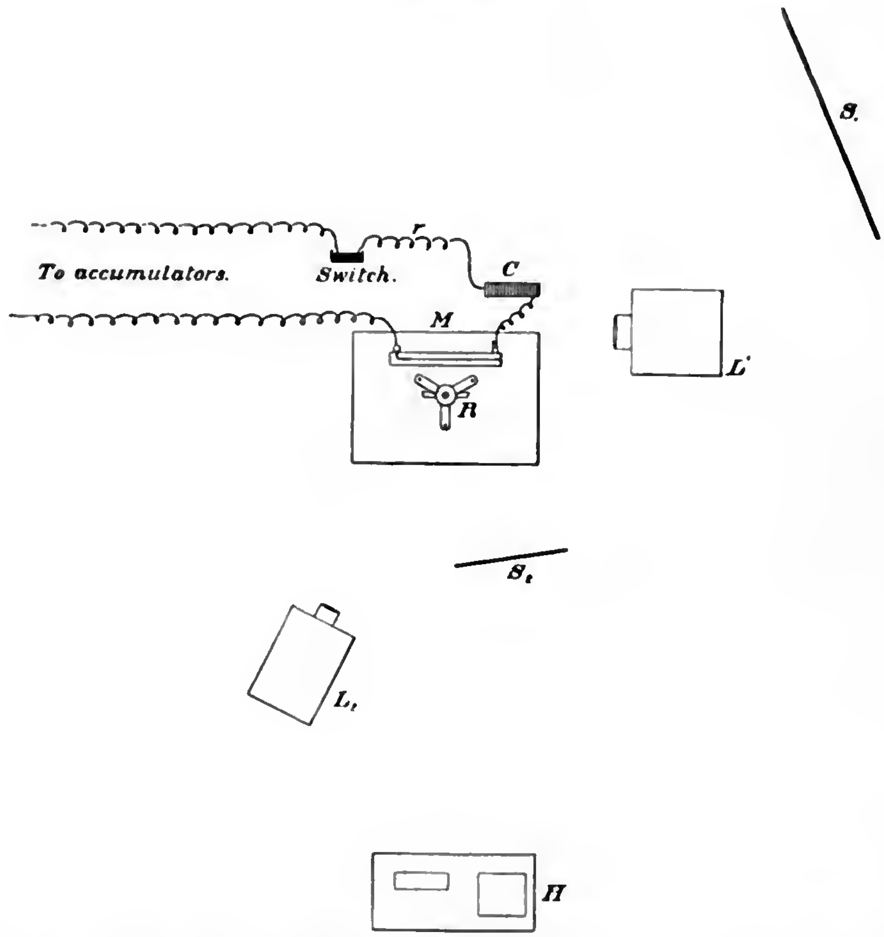
H is the heliostat, which is placed on a window sill outside the laboratory, about 4 metres from the radio-micrometer R, and the meldometer M. The two latter instruments are supported on a table which stands on a concrete pier passing through the floor of the room.

S_1 is the scale of the meldometer, the distance from S_1 to M being about 3 metres. S_2 is the scale of the radio-micrometer, and L_1 and L_2 are the lamps corresponding to the two instruments. C is a variable carbon resistance; r is a platinoid coil; C_1 and the platinum strip in M are in circuit with 26 Epstein accumulator cells, by means of which the strip is heated to any desired temperature.

In an experiment, a beam of sunlight is reflected on to the receiving surface of one circuit—say, the lower—of the radio-micrometer, and the heat from the platinum strip on to that of the higher; the two circuits are arranged so that, under these conditions, the two sources of heat produce turning moments in opposite senses, and the temperature of the platinum is raised until a balance is obtained, indicated by the index spot of light returning to its zero on the scale of the radio-micrometer.

At this same moment the temperature-scale of the meldometer is read, the local time of the observation is noted (to obtain the altitude of the sun), and a reading on the heliostat is made, by which the angle of incidence of the sunlight on the mirror can be calculated.

Fig. 1.



An exactly similar process is then gone through with the sun shining in the upper circuit and the platinum in the lower, and the results of each observation are separately calculated.

Then if R_p = the radiation in our arbitrary units, corresponding to a balancing temperature,

A = the ratio of the total heat to the amount transmitted at the observed altitude of the sun,

b = the ratio of the incident radiation to that reflected from the mirror of the heliostat,

c=the ratio of the apparent areas of the platinum and the sun,

and d=the ratio of the emissivity of bright platinum compared with that of lamp-black,

then R_s, the radiation from the sun outside our atmosphere, will be

$$R_s = R_p \times c \times A \times b \times d.$$

The Meldometer.

The meldometer in its original form was devised by Professor JOLY,* for the purpose of finding the melting-points of minerals, hence its name.† As used by him, it consists of a strip of platinum, on which minute fragments of any mineral can be placed, while any alteration in its length can be determined by means of a micrometer screw which touches a lever connected with one end of the strip.

The strip can be heated by an electric current, and is calibrated by observing the micrometer readings corresponding to the temperatures at which some substances of known melting-points melt.

The first alteration which we made on the original form of instrument was to substitute an optical for a mechanical indication of the expansion of the strip by means of which an alteration in length, due to a rise of 1° C. in temperature, could be detected.

For purposes of calibration it is convenient to place the plane of the strip horizontal, so that the fragment of selected material may rest upon it, and this was the arrangement in our first instrument.

But this introduces the necessity of a mirror at 45° to reflect the heat from the strip into the radio-micrometer—a serious source of error, as no good series of experiments on the reflecting power of speculum metal is to be found, and even if it were, tarnishing of the surface is bound to take place, and make the reflection irregular.

We had, therefore, to solve the problem of keeping our thin strip in a vertical plane, while at the same time supporting fragments of our selected minerals upon it during the calibration experiments. The plan finally adopted was to turn up a very narrow ledge along one edge of the strip, at right angles to the remainder, this ledge serving, with very careful handling, as a support for the mineral fragments. A cross section of the strip was thus L-shaped, but with a very short horizontal arm, thus:

L

* 'Proc. R. Irish Acad.,' vol. 2, 3rd series, 1891, p. 38.

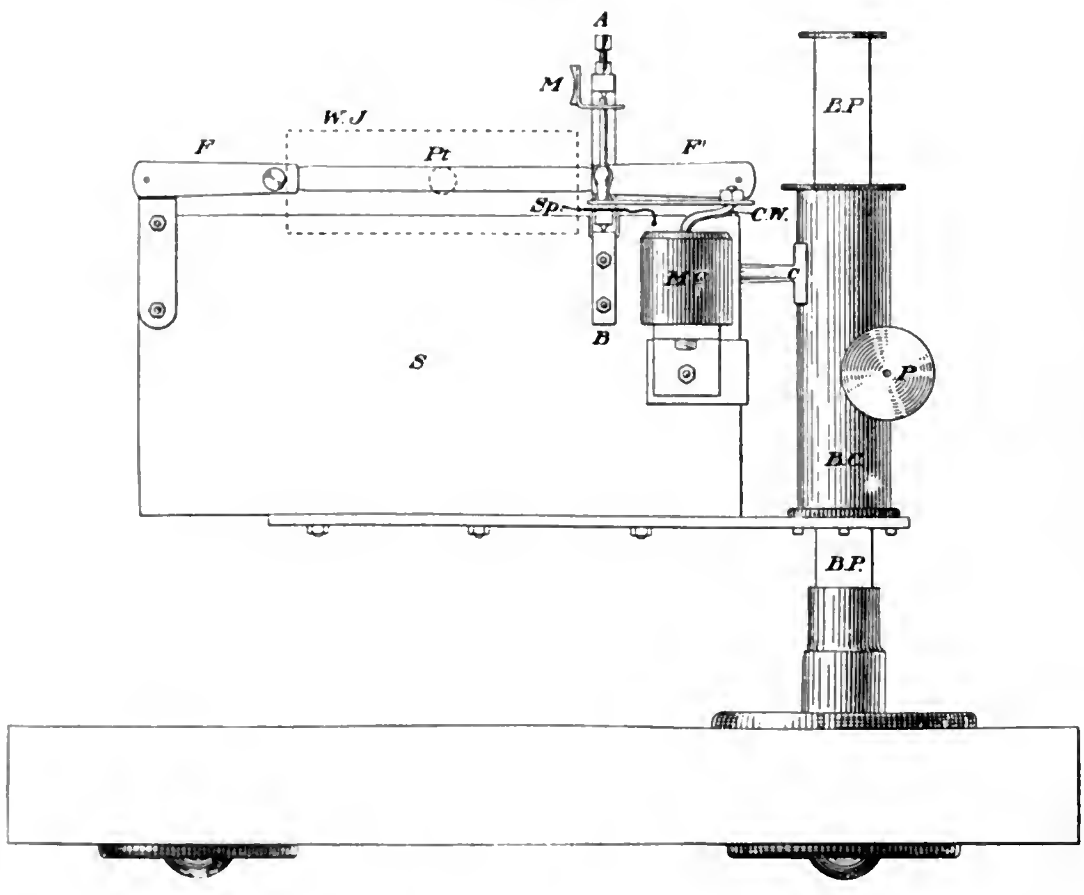
† We have thought it better to retain Professor JOLY's name, although it no longer describes the function of the instrument as used in our work.

The dimensions of the strip were :—

Length	102	millims.
Breadth (including ledge) . .	12	„
Thickness	0·01	millim.

Fig. 2 shows the final form of the instrument with the water-jacket removed. It was made by Messrs. YEATES and SONS, Dublin.

Fig. 2.

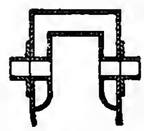


The Meldometer. Scale, about ½.

S is a block of slate, $17\frac{1}{2} \times 9 \times 3$ centims., rigidly fastened to a cylinder of brass, *B.C.*, which can be worked up and down a square brass pillar, *B.P.*, by means of the pinion *P*.

The pillar is screwed firmly to a heavy slate base-plate, on which the radio-micrometer also stands. The platinum strip, *Pt.*, is held between two forceps, of which one, *F*, is fixed, and the other, *F′*, is free to rotate on an axle which is supported between *A* and *B*. In this way the jaws of the forceps, *F′*, which hold the strip between them, can move, when the strip expands, in a small circular arc, which in the experiment is not far from a straight line. *M* is a concave mirror fixed to the axis of rotation; it gives the image of a luminous slit on a straight scale, 3 metres away, and thus indicates an expansion of the strip, as already explained. A piece of stout copper wire, *C. W.*, is connected with the forceps, and dips into a mercury cup, *M.C.*, by means of which a movable electric connexion is maintained with the remainder of the circuit. *Sp.* is a flat spiral spring, which is necessary to keep a slight tension on the strip. A water-jacket of gilded brass (shown in dotted lines) rests on the top of the slate block during an experiment; its shape is shown in fig. 2A, which is a cross-section; its length is a little greater than that of the strip,

Fig. 2A.



Section of Water-jacket.

and in the middle of each of its long sides is a circular hole through either of which the heat of the incandescent platinum passes, the hole not in use being plugged up with a gilt brass cap. The water-jacket serves two purposes: one is that of protecting the glowing platinum from air currents, which would otherwise tend to produce quick variations in its temperature; the other is that of preventing any radiation from the platinum except that which passes through the aperture into the radio-micrometer.

Calibration of the Platinum Strip.

The platinum was obtained from Messrs. JOHNSON, MATTHEY, and Co., Hatton Garden, London, who reduced it in thickness until a convenient current (25 ampères) from the accumulators was able to raise it to full incandescence.

The calibration experiments were performed as follows:—

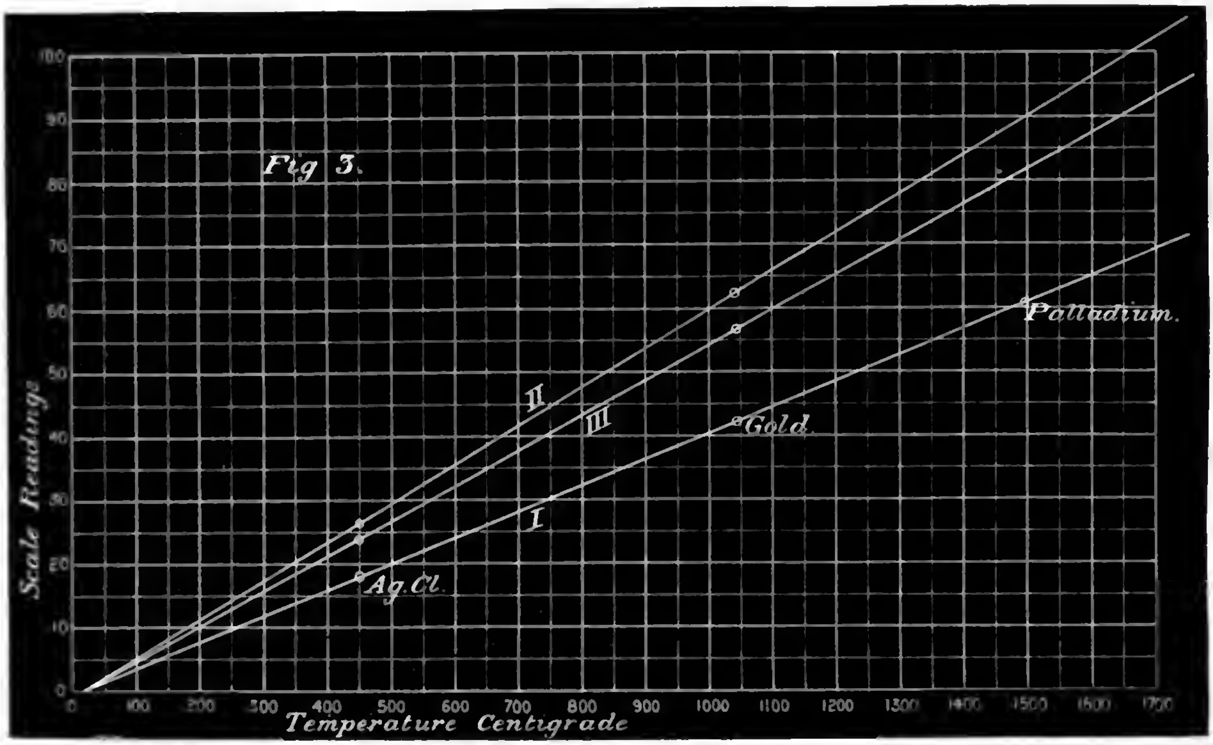
The mirror connected with the strip was turned until the reflected spot of light occupied a convenient position on the scale, which stood at a distance of about 3 metres, and was placed at right angles to the zero position of the index beam of light. A very small fragment of silver chloride (approximately $\frac{1}{12}$ of a milligramme in weight) was then placed on the platinum strip, near the middle of its length, and a low-power microscope was so held in a clamp that the fragment could be plainly seen through an aperture in the water-jacket. The melting point of AgCl is taken as 451° C. (on the authority of CARNELLEY*), at which point the platinum was under a red heat, so that a candle had to be arranged to shine through an open end of the water-jacket, the gilt sides of which reflected the light so well on to the silver

* CARNELLEY "Melting and Boiling-points Tables."

chloride that it stood out with great distinctness against the dark metal in the field of the microscope.

One observer, with his eye at the microscope, then switched on the current, and very slowly raised the temperature of the strip by turning the compressing screw of the carbon resistance, until a sudden definite melting of the fragment took place; at the same moment the second observer took the reading on the scale, which reading then indicates the temperature 451° C.

Fig. 3.



An exactly similar process was gone through, using a minute piece of chemically-pure gold (in weight about $\frac{1}{8}$ of a milligramme), the melting-point of which we took as 1041° C. A curve was then drawn in which the abscissæ are temperatures and the ordinates scale readings. One point on the curve is evidently 0 on the scale at 15° C. (the temperature of the room). The other two points, viz., those corresponding to melting gold and melting AgCl, *lie exactly on a straight line* with this first point. That this coincidence was not mere chance is proved by the fact that we have calibrated three different strips—one in the first meldometer, in which the plane of the strip was horizontal, and two in the second instrument, with the plane of the strip vertical. The straightness of the line in each case is as perfect as it can be drawn with a straight edge.

The figures for the three strips are :

	Melting substance.	Temperature.	Deflection from zero.
		°C.	
1st strip	Ag Cl.	451	18·1
1st strip	Gold	1041	42·0
2nd strip	Ag Cl.	451	26·4
2nd strip	Gold	1041	62·1
3rd strip	Ag Cl.	451	24·2
3rd strip	Gold	1041	56·8

NOTE.—VIOLLE gives the melting-point of gold as 1045° C. CALLENDAR, 'Phil. Mag.,' vol. 33, 1892, gives 1037° C. The mean, 1041° C., of these modern determinations cannot be far from the truth.

The three lines thus given are shown in fig. 3.

In the case of the 1st strip, a piece of palladium was also tried, the melting-point of which is given by VIOLLE as 1500°C.; a deflection of 61 was obtained on the scale, which falls exactly on the line given by the other two substances.

By means of the straight line, corresponding to the particular strip of platinum, therefore, the temperature of the latter may be known with a high degree of accuracy by reading the position of the spot of light on the thermometer scale, on which 1 millim. corresponds to about 2°C.

JOLY,* in his paper, refers to the possibility of a viscous extension of the platinum after being raised to high temperatures; we have proved that this does not take place in our experiments, by noticing that the spot of light returns exactly to zero very soon after the current is cut off, when the platinum has been for some 15 seconds at a temperature of over 1500° C.

The Differential Radio-micrometer.

This instrument is a modification of the single form described by Professor BOYS.† The chief difference consists in a duplication of the circuits, both circuits being supported by the same fibre. The remaining changes consist in an alteration of the position of the magnets, &c., which for our purpose are more conveniently placed vertically instead of horizontally. It was constructed by Messrs. YEATES and SONS, Dublin, and the double circuit by Mr. W. WATSON, B.Sc., of the Royal College of Science, London.

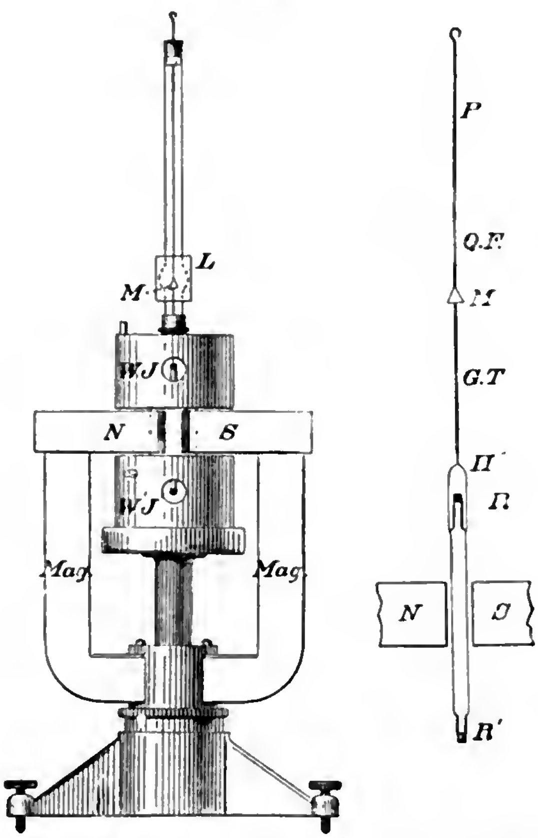
The instrument is shown in elevation in fig. 4, on a scale of about $\frac{1}{4}$, while the circuit is shown *about* $\frac{1}{2}$ *size* on the right of the figure, where R, R' are the two receiving surfaces of blackened copper foil, attached to which are the bars of the alloys. The two pairs of bars are connected by a circuit of fine copper wire, and the whole system is supported by a hoop (H) of similar wire (from which, of course, it is

* JOLY, 'Proc. Roy. Irish Acad.,' 1891, 3rd series, vol. 2, p. 61.

† C. V. BOYS, 'Phil. Trans.,' vol. 180, 1889, A., p. 159.

insulated), to a fine glass tube, *G.T.*, to which is fastened the mirror, *M.* The quartz-fibre suspension, *Q.F.*, is held by the pin, *P*, which passes through a cork, as

Fig. 4.



Scale, about one-fourth. Scale, about one-half.

The Differential Radio-micrometer and Circuit.

shown in the quarter-scale drawing. The weight of the entire system below the pin is about $1\frac{1}{2}$ grains.

In the elevation of the complete instrument, *Mag.* denotes the magnet, *N* and *S* the pole pieces, between which the circuit hangs inside a hollow block of brass, with an iron core as in the ordinary form of the radio-micrometer. *L* is a lens, which, with the small mirror, *M*, forms an image of a luminous slit, on a scale at a distance of about a metre.

W.J. and *W'.J'.* are water-jackets, through which it was found better not to allow the water to circulate. They were kept filled, however, to prevent sudden changes of temperature from affecting the circuits.

The lower water-jacket rests upon a disc of mahogany, which is supported by a brass pillar; the details of the remaining parts of the instrument will be obvious on an inspection of the diagram.

The water-jackets are pierced by tubes, through which the receiving surfaces are visible, and by means of which heat can be allowed to fall upon them. If desired, any or all of the tubes may be stopped by means of corks.

In an experiment, a short tube is inserted in the opening in the water-jacket opposite to the receiving surface, on which the heat from the platinum is to be allowed to fall; the mouth of the tube is partially closed by a stop of polished brass, in which is a circular hole, 4·94 millims. in diameter; the size of the aperture was carefully measured by means of a micrometer gauge. The distance of the aperture from the receiving surface was also carefully measured, and is equal to 60·2 millims.

This gives for the angle subtended by a diameter of the aperture at the receiving surface, 4°·702.*

This number is a constant for any position of the strip, and is equal to the apparent diameter of the disc of glowing platinum as seen from the receiving surface; the distance of the platinum strip, therefore, may be altered without affecting the reading of the radio-micrometer, provided that it be not so great that the angle subtended by its width is less than that subtended by the aperture. In the hole in front of the receiving surface, on which the heat of the sun falls, a brass tube, 8 centims. long, and blackened inside, is inserted to cut off side radiation. A wooden box covers the entire instrument during an experiment, the box containing holes opposite to those in the water-jackets. By this means the instrument is completely protected both from draughts and from accidental radiation from lamps or other sources of heat in the room.

Fig. 5 is from a photograph, showing the radio-micrometer and meldometer in position, with the protecting wooden cover of the former removed.

The Heliostat.

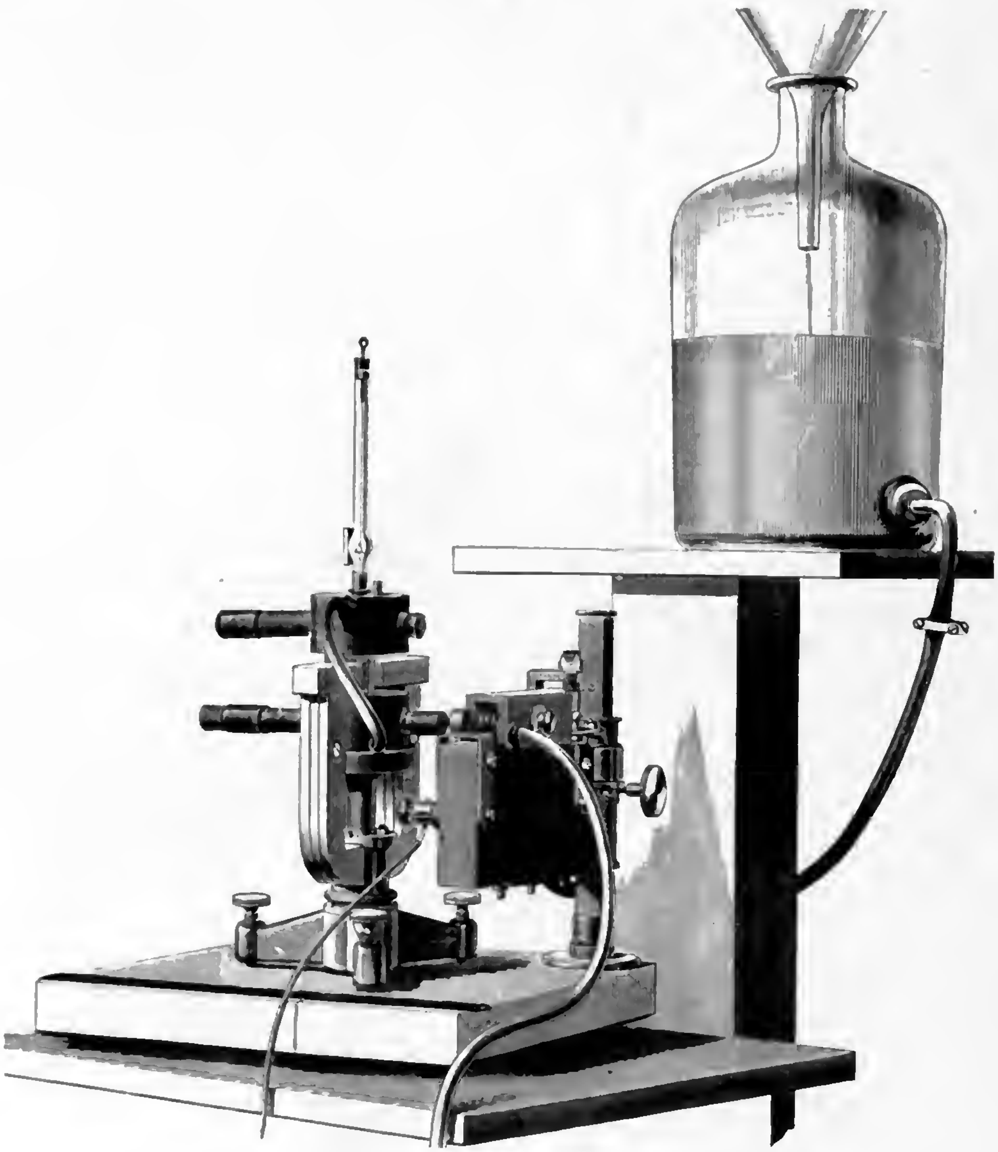
The heliostat used was a single-mirror instrument of Professor G. Johnstone Stoney's design. The mirror was a thick piece of plate glass, with a plane surface carefully figured by Sir Howard Grubb. It was unsilvered, and well blacked at the back, and was of such dimensions that it subtended an angle at the radio-micrometer, when inclined at its usual angle during our experiments, only a little larger than that subtended by the sun. The sunlight from the mirror passed through a small hole in the shutter of the laboratory window, and by this arrangement the heat from the sky round the sun was completely cut off; thus no measurements had to be made, as in Professor Rosetti's work, to obtain the effect of sky radiation.

The use of a single-mirror heliostat was essential, on account of the irregularities produced by polarization in the intensity of the beam reflected from two surfaces, as well as from the difficulty of measuring the two angles of incidence in a two-mirror form.

* See note on p. 31.

The question may arise as to whether it is correct to consider the reflection from the front surface of the heliostat mirror only, or whether multiple reflections from the back surface might not appreciably increase the total amount of heat reaching the radio-micrometer. That the former idea is correct will be evident from the following considerations:—

Fig. 5.



The glass of the mirror was sufficiently thick to clearly separate (at the angles of incidence ordinarily used in our experiments) the image given by the first ordinary reflection from the first given after a "back-reflection," supposing such to exist. We focussed a telescope on the image of the sun in the mirror, but could not discover even a faint ghost of a second image, thus showing that, at least for all wave-lengths in the visible spectrum, there was no regular reflection from the back surface. Even if the black varnish happened to possess a refractive index equal to that of the glass, the virtual effect would merely be a slight thickening of the plate, and it would still hold that all the energy due to what we may call for brevity, the "visible wave-lengths," reaching the back surface, was there absorbed and then diffused in every direction, the amount reaching the radio-micrometer on this account being absolutely negligible.

As for the ultra-red vibrations, it would be unreasonable to suppose that when all the "visible wave-lengths" were absorbed, there should be a rapid change in the nature of the back-reflections, so that a "dark image" might be reflected when no sign of a "light image" was to be found. Moreover, if such a condition could be considered likely, the additional radiation must be extremely small, as we know that by far the greater portion of the heat-energy of the solar radiation is contained within the limits of the visible spectrum.

The point hardly needed further confirmation, but as a check on the curve (fig. 9), obtained from FRESNEL's formula, we made three photometric observations, as mentioned elsewhere (p. 25), which gave points very nearly on the theoretical curve.

ON THE LAW CONNECTING RADIATION AND TEMPERATURE.

We have already mentioned some experiments which have been made in this part of the subject, and seen that it is ignorance of the law which has been the main cause of disagreement in the final estimation of the solar temperature.

ROSETTI's experiments on this point were divided into two parts. He first found the effect on his thermopile of the radiation from a cube filled with water, and afterwards with mercury, at temperatures from about 60° to 300° C. He then found an empirical formula which closely expressed the observed results. The law is expressed thus—

$$y = a\mathrm{T}^2 (\mathrm{T} - \theta) - b (\mathrm{T} - \theta),$$

where

y = the thermal effect of the radiation as given by the deflections on the scale of the thermopile,

T = the absolute temperature of the radiating body,

θ = the absolute temperature of the medium surrounding the body on which the radiation falls;

while

a and b are constants which must be determined from two corresponding values of y and T.

Experiments were then made with the radiating body at higher temperatures, which were obtained either by holding a disc of metal in the flame of a Bunsen burner, or by heating oxychloride of magnesium in the oxyhydrogen flame, preliminary experiments having been made on the emissive power of the various substances at these high temperatures.

Some little doubt must necessarily exist as to the power of knowing exactly what these temperatures actually were; nevertheless the results obtained appear consistent and trustworthy, and the accuracy of the parabolic formula was tested satisfactorily up to a temperature of something like 2000° C.

In our experiments, the heat from the platinum strip was, with our first meldometer, allowed to fall on a mirror of speculum metal at 45°, and thence into the radio-micrometer. The temperature of the platinum was raised step by step, and, at each step, the deflections, both of the temperature scale and of the radio-micrometer, were noted.

Numerous sets of experiments were made, but with some want of uniformity in the results. At first it appeared that STEFAN'S* law of the fourth power expressed the results; then, with additional precautions, ROSETTI'S law appeared to be confirmed. But the want of knowledge as to the reflective power of the speculum metal, with the alterations in the state of its surface, as well as difficulties in throwing the reflection of the glowing platinum fairly into the radio-micrometer, prevented our acceptance of any of these results as beyond suspicion.

With the second meldometer, the need of a mirror was obviated; the differential radio-micrometer was replaced by one of the ordinary single form, perfectly protected against accidental radiations, and, finally, three independent series of experiments gave concordant results which may be very closely expressed by a *fourth power law*.

The radiation is taken as proportional to the deflections on the scale of the radio-micrometer, which was at a distance of about 123 centims.; the extreme angular deflection was about 20°, and up to these limits the proportionality is proved to hold accurately.†

The curve (fig. 6) is calculated from the formula

$$R = a\,(T^4 - T_0^4).$$

where

R = the radiation expressed in scale readings,

T = the absolute temperature of the incandescent platinum.

T_0 = the absolute temperature of the medium surrounding the radio-micrometer (*i.e.*, temperature of the room),

and

a is a constant which was calculated from four points on the experimental curve.

In this case, $\log a = \overline{11}\cdot67868$.

The temperature of the room being about 15° C. = 288° absolute, then R = 0, $T = T_0 = 288°$, will give a point both on the experimental and the calculated curves.

* STEFAN, 'Wien. Ber.,' vol. 79, (1), 1879, p. 391.

† See p. 18.

It will be noticed at once that at comparatively low temperatures the curve does not accurately express the facts, but that the agreement is very good as the temperature rises. This disagreement has been confirmed by LECONTE STEVENS, whose paper* came under our notice after our experiments were finished and the curve drawn. He concludes that, at comparatively low temperatures, the fourth power law gives too rapid a rate of increase of radiation, which agrees with our observations, but that as the temperature rises this divergence diminishes.

The following table gives the results of the three series of observations, which are also plotted on the curve, fig. 6; in two cases, the difference between the observed and calculated results is so large that some misreading seems likely, otherwise the agreement is very satisfactory :—

TABLE I.

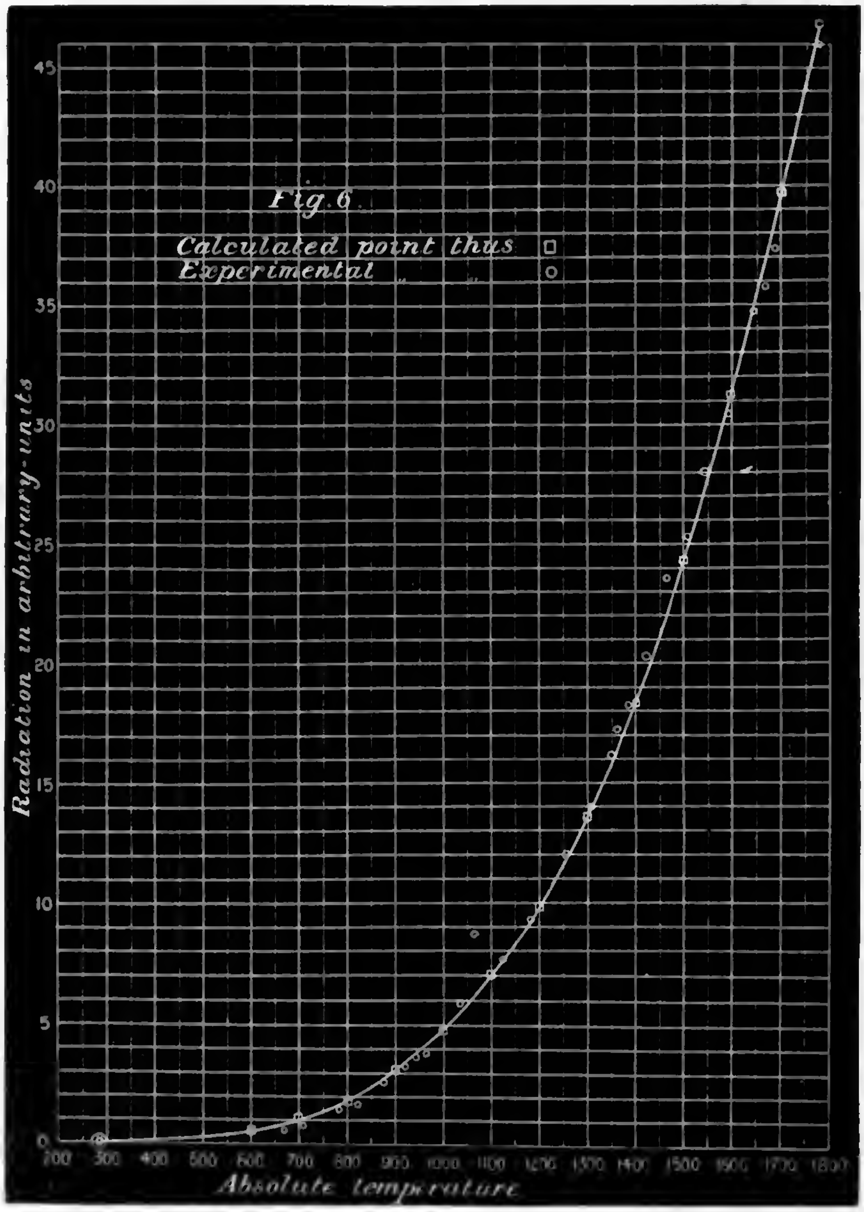
Temperature absolute.	Radiation.		Calculated – observed.
	Observed.	Calculated.	
288°	0	0	0
671	7	9	+ 2
703	9	11	+ 2
788	16	18	+ 2
811	18	20	+ 2
876	26	27	+ 1
915	32	33	+ 1
944	37	37	0
965	39	41	+ 2
1045	59	57	– 2
1125	76	76	0
1181	93	93	0
1253	120	119	– 1
1308	140	140	0
1348	161	158	– 3
1363	172	159	(– 13)
1393	182	180	– 2
1425	202	198	– 4
1466	236	220	(– 16)
1513	253	252	– 1
1547	280	272	– 8
1593	305	306	+ 1
1647	348	348	0
1663	358	360	+ 2
1683	373	380	+ 7
1773	460	462	+ 2
		Mean. . .	$\frac{+24-50}{26} = -\frac{26}{26} = -1$

* 'Amer. Jour. of Science,' vol. 44, 1892, p. 431.

Or, omitting two obviously bad observations, the mean difference between "calculated" and "observed"

$$= \frac{+24 - 21}{26} = \frac{+3}{26} = +0{\cdot}1.$$

Fig. 6.



The latest work on this subject is that of PASCHEN,* who gives full references to the papers of other experimentalists. His method of working is very complicated,

* 'WIEDEMANN'S Annalen,' vol. 49, 1893, p. 50.

and the determination of his high temperature appears to be wanting in certainty. He finally obtains results which do not agree with any formula hitherto given.

The least disagreement is found with an empirical expression given by WEBER,* but PASCHEN'S curve (in which, as in our own, the abscissæ are temperatures, and the ordinates radiation) falls nearly as much below WEBER'S as it rises above STEFAN'S. Taking, as a particular instance, PASCHEN'S observed radiation at 1273° and 1673° (absolute) = 69 and 295 approximately, the fourth power law gives 50 and 148, while WEBER'S gives 76 and 570.

PASCHEN'S results would therefore indicate a much more rapid rise in radiation than that indicated by our fourth power law; in the case just quoted the exponent would be about 5·3.

We are supported, however, in our adoption of the fourth power law, not only by our own and STEFAN'S results, and LECONTE STEVENS' conclusions, but also by some work of SCHNEEBELI,† and in a very interesting way by an investigation of BOLTZMANN'S,‡ who deduces the law from the electro-magnetic theory of light.§

On the whole, therefore, we think, there can be little doubt that, at least in the case of incandescent platinum, the increase of radiation with temperature may be most accurately expressed by the fourth power law, and that the divergent results obtained by different investigators are chiefly due to want of certainty in the determination of high temperatures, and in a less degree to complication of apparatus, with its accompanying accumulation of small errors. In the case of our own experiments, the temperature of the platinum strip is known with a doubt of only some 6° C. at a temperature of 1500° C.; the radiation falls directly on the radio-micrometer, and the proportionality of the deflections of the latter to the radiation falling upon it is strictly demonstrated by experiment. It would seem, therefore, that the results cannot be far from the truth, which conclusion is largely strengthened by the confirmations already mentioned.

It has been generally assumed that the deflections of the spot of light on the scale of the radio-micrometer are proportional to the amounts of radiation falling on the receiving surface of the instrument. In the above experiments the extreme deflection was about 20°, and it therefore seemed necessary to determine by direct experiment whether this proportionality held up to this high limit or not. This was done in the following manner:—

A cube of boiling water was supported at a distance of about 80 centims. from

* H. F. WEBER, 'Berlin Akad. Ber.,' 1888, 2, p. 933.

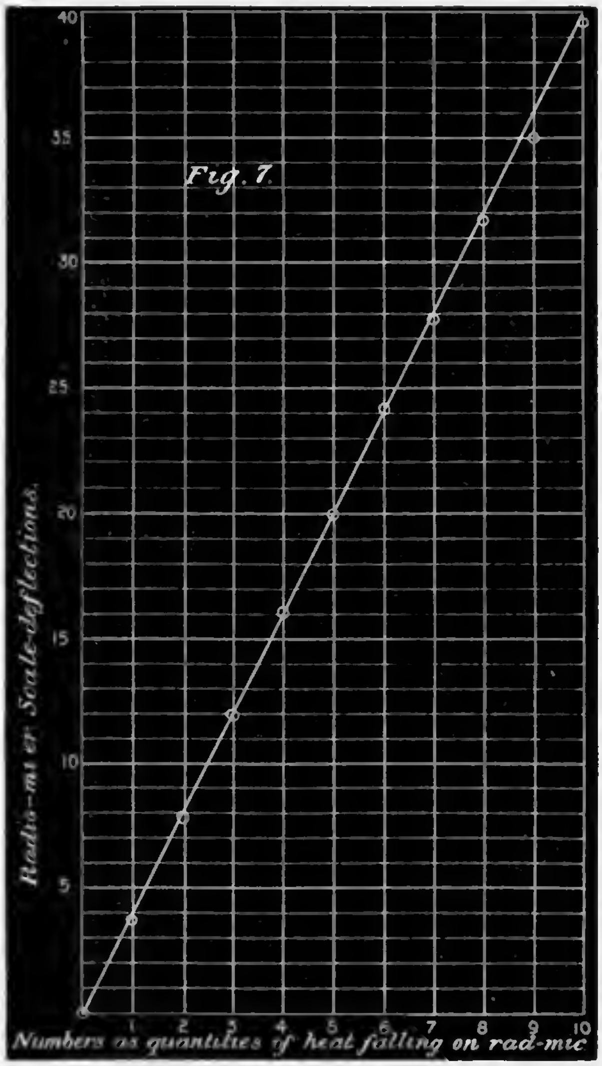
† SCHNEEBELI, 'WIEDEMANN'S Annalen,' 1884, vol. 32, p. 403.

‡ BOLTZMANN, 'WIEDEMANN'S Annalen,' 1884, vol. 32, pp. 31 and 291.

§ [It must be noticed, however, that both STEFAN'S and BOLTZMANN'S results were supposed to apply, strictly speaking, to "pure" radiation from a surface of unit-emissive power, so that the agreement must not be insisted on too strongly. All we can say certainly is that, for the particular results of particular experiments, the fourth power law is found to hold very accurately, and has therefore been adopted.]

the radio-micrometer; between the two a wooden box, 4 inches square in section, was placed to prevent side radiation from disturbing the latter; tin and cardboard screens were also used for the same purpose, until we were assured that the only heat falling on the instrument was that from the lamp-blacked side of the cube, passing through a carefully-cut rectangular aperture, made in cardboard and fixed to the end of the

Fig. 7.



wooden box close to the cube. A horizontal edge of the aperture was divided into ten equal parts, and a wooden screen, with a straight edge, could be placed so as to close the aperture, or to leave any desired fraction of it open. The proportionate area of aperture open, and therefore the proportionate amount of heat falling on the instrument, was then given by the reading of the scale on the horizontal edge of the aperture.

The following Table II. gives the results of two series of experiments. The first column gives the area of aperture, *i.e.*, the quantity of heat falling on the instrument; the second gives the deflection (in centims.) on the scale, in the two series; the third gives the mean; and the fourth gives the deflections calculated by a straight line formula, $y = mx$.

When the observed results are plotted down on curve paper (fig. 7), it will be seen at once that they form as nearly as can be a straight line; and as the extreme deflection in these cases was $21\frac{1}{2}°$, the proportionality of radiation and deflection is strictly demonstrated, up to the greatest value of the latter used in our experiments.

TABLE II.

Quantity of heat.	Deflection.	Mean observed.	Calculated from $y = 3{\cdot}96x$.	Observed − calculated.
0	0·0	0·0	0·0	0·0
1	4·4, 2·9	3·7	4·0	−0·3
2	8·6, 7·2	7·9	7·9	0·0
3	12·5, 11·3	11·9	11·9	0·0
4	16·8, 15·4	16·1	15·8	+0·3
5	20·7, 19·6	19·9	19·8	+0·1
6	24·4, 23·9	24·2	23·8	+0·4
7	27·9, 27·9	27·9	27·7	+0·2
8	31·5, 31·8	31·7	31·7	0·0
9	34·9, 35·3	35·1	35·6	−0·5
10	39·6, 39·6	39·6	39·6	0·0
				Mean $= \frac{+1{\cdot}0 - 0{\cdot}8}{11} = +{\cdot}02$.

It may be noticed here that as the temperature rises, ROSETTI's law becomes more nearly a simple third power law, while ours becomes a simple fourth power law, so that if

R_p = radiation from platinum.
T_p = temperature of platinum,
R_s = radiation from sun,
T_s = temperature of sun.

then

$$\frac{R_s}{R_p} = \frac{T_s^4}{T_p^4}, \quad \text{or} \quad T_s^4 = T_p^4 \times \frac{R_s}{R_p},$$

which gives when $T_s = 6000°$ and thereabouts, a result differing by less than one degree from that obtained by the complete formula $R_s = a(T_s^4 - T_0^4)$.

The simple form gives a great saving of time in calculating out the results of the observations, and we generally adopted it in the course of our work. The only direction in which we can look for an explanation of the great difference between ROSETTI's law and our own, is in that of his method of estimating his high temperatures, which appear to be somewhat uncertain, whereas we can feel confident in the accuracy of our own method to within ± 6° at 1500° C. The chances are that his discs of metal were at a lower temperature than that assumed (but not measured) by him; and if that were so, the differences between his results and ours would be in the direction in which we find it.

THE EMISSIVE POWER OF PLATINUM AT HIGH TEMPERATURES.

SCHLEIERMACHER* and ROSETTI† have made experiments on this subject which at first sight appear to disagree, but on examination confirm one another in an interesting manner. From the curves which SCHLEIERMACHER's results give we obtain the emissions at certain temperatures (1) from polished platinum, (2) from platinum covered with black oxide of copper, which may be assumed as approximately the same as that from a lamp-black surface. The fourth column in the following table gives the ratio of the two emissions :—

Absolute temperature.	Emission.		Ratio $\frac{\text{black}}{\text{bright}}$.
	Plat. (black).	Plat. (bright).	
°			
300	65	12	5·42
400	96	20	4·80
500	147	34	4·32
600	220	52	4·23
700	317	77	4·12
800	445	112	3·97

The figures in the fourth column show a gradual fall in the ratio as the temperature rises. ROSETTI, at an absolute temperature of about 1500°, found for the ratio 100/35 = 2·9, which falls in fairly satisfactorily with a theoretical continuation of SCHLEIERMACHER's results. As it is impossible, with our present arrangement of apparatus, to keep the platinum lamp-blacked at a high temperature, and as the ratio is evidently altering very slowly near the point at which ROSETTI made his determinations, we shall use his ratio in calculating our results, *i.e.*, we shall take

$$\frac{\text{Emission from lamp black}}{\text{Emission from bright platinum}} = \frac{100}{35} = 2{\cdot}9.$$

* 'WIED. Ann.,' 1885, vol. 26, p. 287.

† 'Phil. Mag.,' vol. 8, 1879, p. 445.

The Atmospheric Absorption.

Until LANGLEY* published his "Researches on Solar Heat," the unanimity with which nearly all observers agreed in giving a value of about 21 per cent. to the absorption of light and heat from a radiating body in the zenith, was so striking that there seemed little doubt as to the practical accuracy of this figure. Yet, in every case, since under most favourable conditions the experiments must have been done with a thickness of at least *one* atmosphere, an assumption had to be made as to the effect which would have been produced without this thickness, and Professor LANGLEY showed conclusively that this assumption was not justified by the conditions of the problem.

The formula which had been most generally accepted as expressing the amount of radiation received from a body at different altitudes is

$$q = ab^{\epsilon}$$

where

q = the observed intensity of radiation,

a = the intensity of radiation on unit surface outside the limits of the atmosphere,

b = a "constant," which is the fraction showing the amount of absorption for a body in the zenith; *i.e.*, the "absorption coefficient,"

and

ϵ = the thickness of the atmosphere, the value being taken as unity for a body in the zenith. ϵ is approximately equal to sec ZD. up to a zenith-distance of 60° or 65°.

In the case of the sun, a is the solar constant. One of the mistakes made by the older experimenters was that of assuming the quantity b to be really a constant, which it is not. It is, in fact, a function of two variables, viz., the wave-length of the radiation, and ϵ, the thickness of atmosphere traversed by the radiation. (LANGLEY, in commenting on this fact, seems to have overlooked ROSETTI's work, in which the increase of b with ϵ is clearly and quantitatively stated.)

From the results of his work, LANGLEY obtains 41 per cent. as a probable approximation to the absorption of total radiation for a body in the zenith. His argument may be briefly summarized thus:

The number of wave-lengths in a composite radiation is infinite. Each wave-length may have its own individual coefficient of absorption. The coefficients of absorption will be infinite in number and will vary in value between 0 and unity. As "some sort of adumbration of the complexity of nature's problem and the

* LANGLEY, 'Professional Papers of the Signal Service,' Washington, 1884, and 'Phil. Mag.,' 1884, vol. 18, page 289.

method of his work," he divides the radiant energy before absorption into ten parts A, B, C, . . . J, each having its own coefficient of transmission, a, b, c, . . . j, so that the total radiation outside our atmosphere being

$$A + B + C + D + \&c. \ldots = X,$$

the intensity after passing through unit thickness of air (*i.e.*, $\epsilon = 1$, a zenith observation) will be

$$Aa + Bb + Cc + Dd + \&c. \ldots = M,$$

after passing through two thicknesses ($\epsilon = 2$) will be

$$Aa^2 + Bb^2 + Cc^2 + Dd^2 + \&c. \ldots = N,$$

and so on, assuming that a, b, &c., remain constants for more than one integral value of ϵ, which is not exactly true.

Of course X is unknown from experiment, but M, N, O, &c., can be measured. Then the ratio N/M will give the transmission of the second thickness compared with the first, and $1 - N/M$ the absorption, and similarly with the other series, and these may all agree within close limits. The great mistake lay in assuming that if $\left(1 - \frac{N}{M}\right) = 1 - \left(\frac{O}{N}\right)^{\frac{1}{2}} = 1 - \left(\frac{P}{O}\right)^{\frac{1}{3}}$ *approximately*, then the same ratio held for the first thickness.

By giving values of a, b, c . . . &c. = ·01, ·1, ·2, ·6, ·7, ·7, ·8, ·9, ·9, and 1·0, while A = B = C = &c. = 1, LANGLEY shows that this equality of the ratios is at once destroyed, and holds that this rough division of the whole radiation into parts with varying coefficients of absorption, must give an approximation to the truth. Taking A = B = C = &c. = J = 1, the total outside radiation = 10, while

$$\begin{aligned} Aa + Bb + \ldots Jj &= 5{\cdot}9 = M \\ Aa^2 + Bb^2 + \ldots Jj^2 &= 4{\cdot}65 = N \\ Aa^3 + Bb^3 + \ldots Jj^3 &= 3{\cdot}88 = O, \ \&c. \end{aligned}$$

Then

$$1 - \left(\frac{N}{M}\right) = {\cdot}21,\ 1 - \left(\frac{O}{N}\right)^{\frac{1}{2}} = {\cdot}19,\ 1 - \left(\frac{P}{O}\right)^{\frac{1}{3}} = {\cdot}18,\ \&c.,$$

while

$$1 - \frac{M}{X} = 1 - \frac{5{\cdot}9}{10{\cdot}0} = {\cdot}41,$$

so that instead of 21 per cent. being absorbed in one thickness of atmosphere, it may very well be *double* that absorption taking place.

We now come to an examination of ROSETTI's careful investigation on this point. He does not give the value of the absorption explicitly, but it may be deduced from the figures given by him on p. 546* of his paper already quoted.

* 'Phil. Mag.,' 1879.

From a large number of concordant observations he finally deduces a value of the solar constant = 323 in the scale divisions of his thermo-pile, while in the tables on p. 546 he gives the deflections corresponding to values of ϵ from 1·4 up to 4·8.

We plotted these values on curve paper (fig. 8), and thus found 229 as the corresponding deflection for the sun in the zenith, so that, using the above symbols, X = 323, M = 229. The absorption for one thickness therefore equals

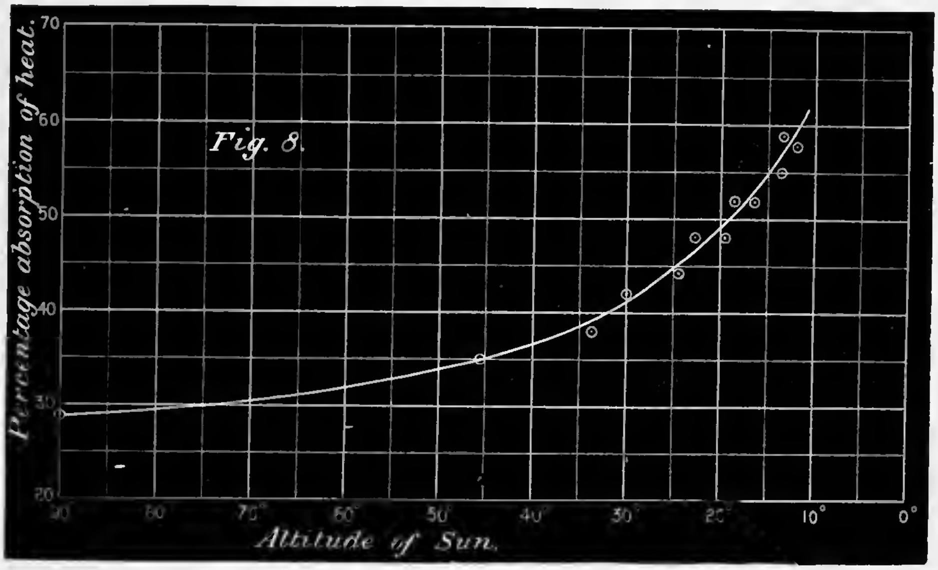
$$1 - \frac{M}{X} = 1 - \frac{229}{323} = 1 - \cdot 71 = \cdot 29.$$

So that 29 per cent. of the total outside radiation is absorbed, and 71 per cent. reaches the earth, with the sun in the zenith.

The ratios corresponding to other values of ϵ were similarly calculated, and the results plotted down, giving the curve (fig. 8), the abscissæ of which are zenith distances and the ordinates percentage absorptions.

The 29 per cent. thus deduced from ROSETTI's results, it will be seen, is considerably greater than the old estimate, which we know to be incorrect, and less than the 41 per cent. of LANGLEY, which is indeed a difference to be, *a priori*, expected for the following reason.

Fig. 8.



We know that by far the greater proportion of the energy (as properly measured by its heating effect) in the solar radiation is confined within narrow limits of wave-length, and that for these wave-lengths atmospheric absorption is less than for the waves of higher refrangibility. The larger transmission coefficients in LANGLEY's calculations should therefore have more weight given to them, and it would be possible to draw up another series with assumed coefficients, by which the 29 per cent. could be reproduced, with the 21 per cent., 19 per cent., &c., following.

The difference then between ROSETTI's and LANGLEY's figures is in a direction which might be expected, and the results deduced from the work of the former may be assumed provisionally as an approximation to the truth.

Climatic conditions in Ireland are such as to entirely prevent a good series of observations on this point; a perfectly clear sky from morning to night, with a fairly constant hygrometric state of the atmosphere, is extremely rare.

ROSETTI, working under the unclouded skies of Northern Italy, was able to make a large number of observations at all hours of the day, with very consistent and apparently reliable results.

We have, therefore, determined to use the correcting factors for atmospheric absorption which have been deduced from his figures, so that whatever doubt may be thrown on the accuracy of his final result will affect ours in a certain proportion.

It is worth noting that YOUNG* gives 30 per cent. as the absorption in the zenith, but without indicating the means by which he arrives at this figure.

THE SOLAR RADIATION.

The general method of making the final experiments has already been described. The necessity for making observations with the sun shining (1) on the upper circuit of the radio-micrometer, (2) on the lower circuit, arises from the unavoidable difference in the constants of the two circuits. No special care has been taken in the construction of the instrument to make the receiving surfaces of equal size, and even if this had been possible, the electrical constants must have differed somewhat. The only way of correcting for these differences is to take independent observations in the manner indicated, and to take the *mean* of the results.

A considerable difference between the figures obtained in the two positions was to be anticipated, and it will be seen that experiment confirms the anticipation.

As we have already pointed out, when a balancing temperature has been obtained, the ratio of the radiation from the sun to that from the platinum is obtained by multiplying together four factors. They are:

(1) The ratio of the apparent area of the sun to that of the platinum, as seen from the receiving surface of the radio-micrometer. The former is obtained from the value of the sun's semi-diameter, as given by the 'Nautical Almanac' for the day of the observation. The latter is a constant, the same "stop" being always used in every position. The angle subtended by a diameter of the stop was 4°·702 †; if σ = angular diameter of the sun at the time of observation, we therefore have :—

$$\frac{\text{area of platinum}}{\text{area of sun}} = \left(\frac{4 \cdot 702}{\sigma}\right)^2.$$

* "The Sun," 'Internat. Sci. Series,' p. 262.

† A new stop was used after Sept. 8th; see p. 31.

(2) The ratio of the incident radiation on the glass mirror of the heliostat to the reflected. This was given by the use of FRESNEL'S formula

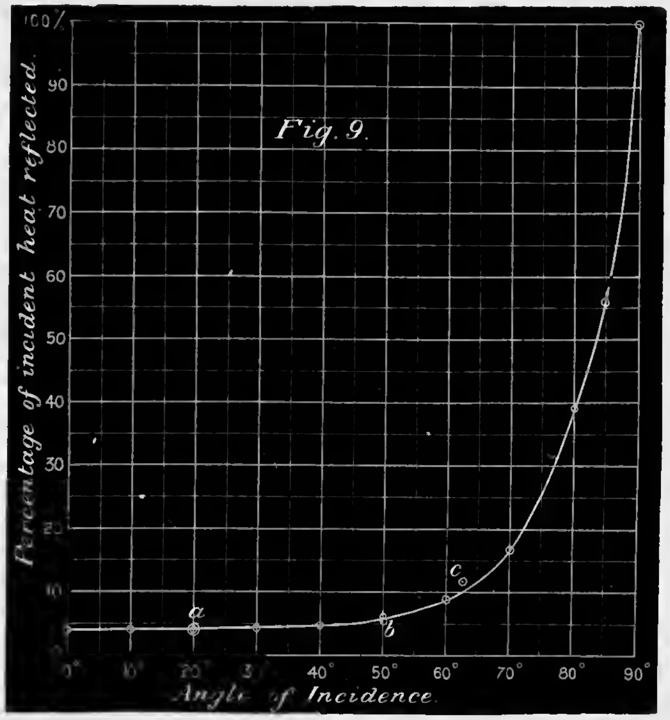
$$\frac{R_i}{R_r} = \frac{1}{2}\frac{\sin^2(i-r)}{\sin^2(i+r)} + \frac{1}{2}\frac{\tan^2(i-r)}{\tan^2(i+r)},$$

where

R_i = intensity of incident radiation,
R_r = " " reflected "
i = angle of incidence,
r = " " refraction, which was obtained by putting $\mu = 1{\cdot}5$ in the ordinary formula, $\sin i = \mu \sin r$.

The values thus obtained for different angles of incidence were plotted down and a smooth curve drawn to give the value at any incidence (fig. 9). In the figure, *a*, *b*,

Fig. 9.



and *c*, are points experimentally determined by photometric measurement as a rough check on the accuracy of the calculations. (It may be noted here that the table given by JAMIN* is erroneous as referring to common light; it is correct for light polarized in the plane of incidence. We mention this, as anyone who took the accuracy of JAMIN'S figures for granted would imagine that our curve was wrong.)

* 'Cours de Physique,' 4th edition, vol. 3, p. 618.

Sir J. CONROY* has shown that the curve drawn from FRESNEL's formula is verified by experiment to within $\frac{1}{2}$ per cent. at the angles of incidence generally used in our observations.

The angle of incidence is obtained at each experiment by finding the distance between the end of a certain steel rod in the heliostat and a collar which slides along it; the angle corresponding to any distance could be found by means of a curve, which it is unnecessary to give here.

(3) The ratio of the radiation outside our atmosphere to the amount which reaches the earth. This is obtained by calculating the altitude from the known declination, hour angle, and latitude, and taking the percentage of absorption from the curve (fig. 8) which we have already discussed.

(4) The ratio of the emissivity of bright platinum to that of a lamp-blacked surface, which, as already mentioned, we take as 35 : 100.

To take a typical case:—

Date, Sept. 4th, 1893. ⊙ Declination = 7°·1 N. ⊙ $\frac{1}{2}$ diameter = 15′·9.

Time, 10^{h} 54^{m}, local. Therefore ⊙ altitude = 41°·8.

Balancing temperature = 1514° absolute.

By curve (fig. 8) absorption = 36 per cent.

Therefore transmission = 64 per cent.

Diameter of ⊙ = 31′·8 = 0°·53.

Therefore

$$\frac{\text{Area of platinum}}{\text{Area of sun}} = \left(\frac{4·702}{·53}\right)^2 = 78·71.$$

Angle of incidence on glass = 61°.

Therefore amount of heat reflected = 9·5 per cent.

Ratio of emissivity of platinum and lamp black = $\frac{35}{100}$.

Therefore the radiation from the sun is

$$78·71 \times \frac{100}{64} \times \frac{100}{9·5} \times \frac{35}{100} = 453·1$$

that of the platinum at a temperature of 1514° absolute.

The temperature of the sun is therefore

$$1514 \times \sqrt[4]{453·1} = 1514 \times 4·614 = 6985° \text{ absolute},$$

according to this single observation.

It was not only necessary to take observations with the sun shining (A) into the lower circuit and (B) into the upper circuit, but on account of possible differences in the state of the surfaces, back and front, of the copper foil receivers, it was essential

* 'Phil. Trans.,' 1889, (A), vol. 180, p 245.

to turn the whole radio-micrometer through an angle of 180°, so that the heat from the platinum should now fall on that side of the receiving surfaces on which previously the sun had shone. The different positions are distinguished as follows :—

Position (1A) = platinum heating upper circuit, and behind the small mirror fixed to the fibre of the radio-micrometer.
,, (1B) = platinum heating lower circuit, and again behind mirror.
,, (2A) = instrument rotated through 180°; platinum in upper circuit *in front* of mirror.
,, (2B) = platinum in lower circuit, again in front of mirror.

The difference between positions 1A and 2A, and between 1B and 2B we should, *a priori*, expect to be small, and the experiments show that this is so, while, as we have already mentioned, the larger differences between the A and B positions were also to be anticipated from unavoidable dissimilarities in the two parts of the combined circuit.

One further point remains to be noticed, viz., that the geometrical mean of the mean temperatures of the A and B positions is not exactly the mean temperature to be deduced from the observations, on account of the curvature of the radiation curve.

To show what difference exists between the geometrical and the true mean, we may take the following numerical example:

Mean balancing temperature in position A = 1600° absolute
,, ,, ,, B = 1300° ,,

Mean balancing temperature = 1443°

Now, to a temperature of 1600° corresponds a radiation of 312 in our arbitrary units; to a temperature of 1300°, a radiation of 136; mean radiation $= \frac{1}{2}(312+136) = 224$. But to a radiation of 224 corresponds a temperature of 1472°, which is 29° higher than the geometrical mean, and the value of $\sqrt[4]{\left(\frac{\text{Radiation of sun}}{\text{Radiation of platinum}}\right)} = 4{\cdot}5$ approximately. That is to say, we must add $29 \times 4{\cdot}5 = 130$ to the mean temperature. A correction of about 100° is therefore to be made on the final mean of all the observations, the separate details of which now follow. Each day's results are given by themselves, with data sufficiently full to allow of any single observation being calculated out.

The date, height of barometer, and notes on the weather are given first; hygrometrical readings are not given, as no useful deductions can be made from them, as ROSETTI points out in his paper.

In the 1st column, the position is noted.
,, 2nd ,, the local time of the observation.

In the 3rd column, the readings on the meldometer scale at the moment of balance.
,, 4th ,, the absolute temperature corresponding to this reading.
,, 5th ,, the sun's altitude.
,, 6th ,, the percentage of transmission of the total solar heat through the earth's atmosphere.
,, 7th ,, the angle of incidence of the sunlight on the mirror of the heliostat.
,, 8th ,, the percentage reflection of the heat in the incident beam.
,, 9th ,, the absolute temperature of the sun as calculated from each single observation.

Date: September 3rd, 1893.
Weather: Passing clouds. Sky, no perceptible haze. Barometer, 30·2 in.

Position.	Local time.	Balance.		Sun's altitude.	Per cent. trans.	Angle of incidence.	Per cent. reflected.	Sun's abs. temp.
		Reading.	Temp. abs.					
	h. m.		°	°		°		°
1A	10 3	72·4	1474	38·0	62·5	57·0	7·6	7176
	10 6	70·5	1447					7044
	10 21	74·3	1513	39·6	63 0	58·8	8·0	7367
	10 23	73·8	1503					7318
	10 40	76·1	1543	41·3	63·8	60·0	8·9	7242
	10 43	76·2	1544					7247
	10 44	76 5	1547					7261
							Mean .	7236
1B	10 29	56·6	1223	40·2	63·5	59·4	8·5	5813
	10 30	56·2	1214					5770
	10 32	56·6	1223					5813
	10 33	56·4	1218					5789
	10 51	58·2	1246	42·1	64·0	60·5	8·9	5826
	10 52	58·6	1250					5845
	10 53	58·5	1249					5841
	0 52	65·0	1361	43·6	64·5	66·5	13·2	5771
	0 54	64·2	1346					5708
							Mean .	5797

NOTE.—New platinum strip put in after these observations were made. Balance readings here refer Calibration-line 2.

Date: September 4th, 1893.

Weather: Hazy clouds, with intervals of light blue sky; Wind S.S.E., moderate.

Position.	Local time.	Balance. Reading.	Balance. Temp. abs.	Sun's altitude.	Per cent. trans.	Angle of incidence.	Per cent. reflected.	Sun's abs. temp.
	h. m.		°	°		°		°
1A	10 54	67·6	1514	41·8	64·0	61·0	9·4	7003
	10 56	67·2	1508	41·8	64·0	61·0	9·4	6975
	10 57	69·0	1539	41·8	64·0	61·0	9·4	7119
	11 35	71·2	1581	43·0	64·3	63·6	11·0	7022
	11 36	71·2	1581	43·0	64·3	63·6	11·0	7022
	11 37	72·0	1594	43·0	64·3	63·6	11·0	7053
							Mean .	7032
1B	11 10	53·2	1254	42·5	64·0	62·2	10·1	5697
	11 27	53·3	1256	42·5	64·0	62·2	10·1	5706
	11 28	53·3	1256	42·5	64·0	62·2	10·1	5706
	11 43	54·0	1268	43·2	64·3	63·8	11·0	5632
	11 51	54·0	1268	43·2	64·3	63·8	11·0	5632
	11 53	54·2	1273	43·2	64·3	63·8	11·0	5654
							Mean .	5671

NOTE.—Balance readings refer to Calibration-line 3.

Date: September 7th, 1893.

Weather: Passing clouds, with intervals of clear blue sky; Wind W., moderate. Barometer, 29.7 in.

Position.	Local time.	Balance.		Sun's altitude.	Per cent. trans.	Angle of incidence.	Per cent. reflected.	Sun's abs. temp.
		Reading.	Temp. abs.					
	h. m.		°	°		°		°
1 A	10 54	67·9	1519	40 6	63·6	61·6	10·0	6930
	10 56	68·2	1525	40·6	63·6	61·6	10·0	6957
	10 57	68·5	1531	40 6	63·6	61·6	10·0	6984
							Mean .	6957
1 B	11 15	53·7	1263	41·4	64·0	62·6	10·5	5683
	11 17	54·2	1273	41·4	64·0	62·6	10·5	5728
	11 18	54·2	1273	41·4	64·0	62·6	10·5	5728
							Mean .	5713
2 A	11 53	75·8	1663	42·2	64·2	65·4	12·4	7173
	11 54	78·0	1703	42·2	64·2	65·4	12·4	7345
	0 5	80·0	1741	42·2	64·2	66·0	13·0	7370
	1 15	81·2	1762	39·6	63·0	67·5	14·2	7381
	1 26	82·2	1778	39·6	63·0	67·5	14·2	7448
							Mean .	7343
2 B	0 42	62·5	1423	41·4	64·0	67·0	14·0	5959
	0 44	62·3	1420	41·4	64·0	67·0	14·0	5946
	0 45	63·0	1421	41·4	64·0	67·0	14·0	5992
	1 58	59·5	1368	36·3	61·9	66·6	13·6	5818
	2 0	59·7	1371	36·3	61·9	66·6	13·6	5831
	2 1	59·0	1359	36·3	61·9	66·6	13·6	5780
							Mean .	5884

Date: September 8th, 1893.

Weather: Generally so cloudy that very few observations were possible. Barometer, 25·9 in.

Position.	Local time.	Balance.		Sun's altitude.	Per cent. trans.	Angle of incidence.	Per cent. reflected.	Sun's abs. temp.
		Reading.	Temp. abs.					
	h. m.		°	°		°		°
2B	11 44	60·0	1378	41·8	63·9	65·0	12·0	5982
	11 45	60·0	1378	41·8	63·9	65·0	12·0	5982
	11 46	60·5	1386	41·8	63·9	65·0	12·0	6016
							Mean .	5993

NOTE.—After the above observations had been made, the aperture through which the radiations from the platinum passed into the radio-micrometer was enlarged, as in some cases the balancing temperature became inconveniently high. The dimensions of the new aperture were:

Diameter = 5·57 millims. Angle subtended = 5°·301.

Date: September 10, 1893.

Weather: Cold N.E. wind, with very slight haze. Barometer, 29·9 in.

Position.	Local time.	Balance.		Sun's altitude.	Per cent. trans.	Angle of incidence.	Per cent. reflected.	Sun's abs. temp.
		Reading.	Temp. abs.					
	h. m.		°	°		°		°
2A	0 8	71·2	1578	41·1	63·9	66·3	13·3	7096
	0 12	71·5	1583	41·1	63·9	66·3	13·3	7119
	0 13	71·6	1585	41·1	63·9	66·3	13·3	7127
	0 33	71·6	1585	40·0	63·4	67·2	14·1	7038
	0 47	71·8	1590	40·0	63·4	67·2	14·1	7060
	0 48	71·5	1583	40·0	63·4	67·2	14·1	7029
	1 5	75·0	1646	39·3	63·0	68·0	15·0	7208
	1 7	74·0	1629	39·3	63·0	68·0	15·0	7133
	1 10	73·0	1612	39·3	63·0	68·0	15·0	7058
	1 15	74·0	1629	39·3	63·0	68·0	15·0	7133
							Mean .	7100
2B	0 20	53·2	1254	41·0	63·8	66·5	13·5	5620
	0 21	52·6	1243	41·0	63·8	66·5	13·5	5571
	0 22	52·8	1245	41·0	63·8	66·5	13·5	5580
	0 55	54·7	1283	39·5	63·1	67·2	14·1	5704
	0 58	55·8	1303	39·5	63·1	67·2	14·1	5792
	0 59	55·0	1289	39·5	63·1	67·2	14·1	5731
	1 22	54·4	1276	38·4	62·7	67·6	14·5	5642
	1 23	54·2	1273	38·4	62·7	67·6	14·5	5629
							Mean .	5659
1B	1 46	55·5	1297	36·2	61·9	67·6	14·5	5754
	1 48	55·0	1289	36·2	61·9	67·6	14·5	5719
	1 49	55·0	1289	36·2	61·9	67·6	14·5	5719
							Mean .	5731
1A	1 53	71·2	1578	35·5	61·4	67·6	14·5	7014
	1 54	71·0	1576	35·5	61·4	67·6	14·5	7005
	1 56	71·0	1576	35·5	61·4	67·6	14·5	7005
							Mean .	7008

MEANS OF DAILY MEANS.

Position 1A . .	7236° . .	7 observations.
	7032° . .	6 ,,
	6957° . .	3 ,,
	7008° . .	3 ,,
Mean . .	7058°	
Position 2A . .	7343° . .	5 observations.
	7100° . .	10 ,,
Mean . .	7222°	

Mean of 1A and 2A = 7140.

Position 1B . .	5797° . .	9 observations.
	5671° . .	6 ,,
	5713° . .	3 ,,
Mean . .	5727° . .	
Position 2B . .	5884° . .	6 observations.
	5993° . .	3 ,,
	5659° . .	8 ,,
Mean . .	5845° . .	

Mean of 1B and 2B = 5786
,, 1A and 2A = 7140.

Therefore

$$\text{Mean result} = \sqrt{5786 \times 7140} = 6430° \text{ absolute.}$$

To this 100° must be added for the reason on page 27.

As there must necessarily be errors of observation, and as results on different days give values differing by as much as 300°, chiefly owing, no doubt, to a change in atmospheric conditions, it has been considered unnecessary to go into certain refinements in the calculation such as using the method of least squares. The daily means have also been given equal weights, in spite of differences in the number of observations.

The geometrical instead of the arithmetical mean of the calculated temperature in the A and B positions, is taken for the following reason:

Let

R_s = radiation due to sun falling on *unit area of receiving surface ;*

R_{p1} and R_{p2} = respective radiations due to platinum, also on unit area, when giving heat—(1) to the upper surface ; (2) to the lower. R_s will, of course, be the same in the two positions ;

a_1 = effective area of upper surface ;

a_2 = ,, ,, lower ,,

using the word "effective" to cover any slight difference of absorptive power, &c.

Then, if we suppose, *First*, the radiation due to the sun falling on the upper surface, the lower being sheltered from the platinum, we should have a deflection θ_1, and as deflections may be taken proportional to received radiation, then

$$a_1 \, R_s = m\theta_1$$

where m is a constant.

Secondly, let the radiation from the platinum fall on the lower circuit, the sun being now cut off from the upper ; we shall have

$$a_2 \, R_{p2} = m\theta_2$$

But if both effects are allowed to be produced together, at the moment of balance θ_1 and θ_2 will be equal and opposite, and therefore

$$a_1 \, R_s = a_2 \, R_{p2}.$$

Similarly, with the sun and platinum reversed as regards the upper and lower surfaces, while R_s remains the same, R_p becomes R_{p1}, and we have

$$a_2 \, R_s = a_1 \, R_{p1},$$

which gives immediately

$$R_s = \frac{R_{p1} \, R_{p2}}{R},$$

or

$$R_s = \sqrt{R_{p1} \, R_{p2}},$$

from which the reason for taking the geometrical mean of the corresponding temperatures follows directly.

The final result, therefore, arrived at, is only given to the nearest 100 ; it is

6200° C.

In conclusion, we may point out that this method would probably give excellent results, if a series of observations were undertaken to settle the question of how, or if, the solar temperature varies during a sun-spot cycle. The instrument should, of course, be used in or near the tropics, where atmospheric conditions can be trusted to remain more constant than in this country. Any error in the absolute value obtained might probably be considered constant, so that comparative values from year to year might be trusted to indicate any change.

NOTE, ADDED APRIL 13TH, 1894.

It has been mentioned in the paper that ROSETTI's determination of the amount of the (terrestrial) atmospheric absorption has been used in the calculations of the effective solar temperature. It may be well, however, to give the result obtained by using other estimates of this quantity, which (after the law connecting radiation and temperature) is the most important factor in the final value.

Taking LANGLEY's estimate for zenith absorption, 41 per cent., instead of ROSETTI's, 29 per cent., the respective transmission coefficients being therefore 59 per cent. and 71 per cent., the temperature would be multiplied by $\sqrt[4]{(71/59)}$ approximately; *i.e.*, instead of 6200°, we should obtain

$$6200 \times \sqrt[4]{(71/59)} = 6200 \times 1{\cdot}054 = 6535°\ \text{C}.$$

But a later, and still higher, estimation of the zenith absorption has been made. ÅNGSTRÖM ('WIED. Ann.,' 1890, vol. xxxix., p. 309) has shown that the effect of the carbonic acid gas in the atmosphere is much more important than had hitherto been supposed, and obtains 64 per cent., as against ROSETTI's 30 per cent. and LANGLEY's 41 per cent. This gives 36 per cent. as the transmission coefficient, and, taking this value, the temperature becomes*

$$6200 \times \sqrt[4]{(71/36)} = 6200 \times \sqrt[4]{(2)} \text{ approximately } = 6200 \times 1{\cdot}189 = 7370°.$$

And, to make the case general, if any later investigation shows the zenith transmission coefficient to be X per cent., the effective temperature becomes

$$6200 \times \sqrt[4]{(71/X)}.$$

It may also be of interest to see what effect is produced if absorption in the atmosphere of the sun itself is taken into account. First, considering the falling-off in radiation from the central to the peripheral parts of the sun's disc, from WILSON and RAMBAUT's paper "On the Absorption of Heat in the Sun's Atmosphere" ('Proc. R.I.A.,' 1892, 3rd series, vol. 2, p. 299), we may deduce that, if the absorption were

* The ratio of the zenith-absorptions is practically equal to that of those with a greater thickness of atmosphere, at least down to a zenith-distance of 50°.

everywhere equal to that at the centre, the radiation would be increased by 4/3, and the temperature would become approximately

$$7370 \times \sqrt[4]{(4/3)} = 7370 \times 1{\cdot}074 = 7900^\circ.$$

Secondly, assuming WILSON and RAMBAUT's result for the *total* loss due to absorption in the solar atmosphere—viz., that about one-third of the radiation is cut off—the radiation would be multiplied by 3/2 if the sun's atmosphere were removed, and our estimate of the temperature would have to be multiplied by $\sqrt[4]{(3/2)}$, so that (again taking the highest value given above as being probably nearest the truth) we get finally

$$7900 \times \sqrt[4]{(3/2)} = 7900 \times 1{\cdot}107 = 8740^\circ.$$

We may therefore summarize as follows :—
Effective temperature of the sun, taking

(1) ROSETTI's estimate of loss in the earth's atmosphere .	= 6200° C.
(2) LANGLEY's estimate	= 6500° C.
(3) ÅNGSTRÖM's estimate	= 7400° C.

And finally, considering the probable effect of the sun's own atmosphere, allowing for it by the figures given in WILSON and RAMBAUT's paper already quoted, and using the highest value just obtained, the effective temperature comes out as approximately 8700° C.

NOTE, ADDED JULY 24TH, 1894.

Some investigations by the authors in connection with the temperature of the carbon of the electric arc, which are now in progress, lead to the conclusion that the simple fourth-power law of radiation used above is only an approximation to the truth, closer in the case of bare platinum than in that of blackened, so that the assumption made in the paper that both follow the same law is not strictly correct. The new work will shortly be published, and will probably result in raising by a few hundred degrees the value obtained above. It may be noticed, meanwhile, that the experimental figures given in this paper are sufficient to serve as a basis—whatever law of radiation may be used—from which the solar temperature may be calculated with an accuracy increasing with a growth of more accurate knowledge as to the law of radiation, and the amount of the atmospheric absorption.*

* See note, p. 119.

On the Temperature of the Carbons of the Electric Arc; with a Note on the Temperature of the Sun. Experiments made at Daramona, Streete, Co. Westmeath.

By W. E. Wilson, *M.R.I.A.*, *and* P. L. Gray, *B.Sc.*, *A.R.C.S.*

Communicated by G. Johnstone Stoney, *F.R.S.*

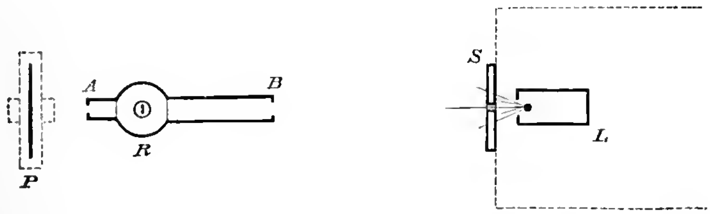
Proceedings of the Royal Society, Vol. 58, 1894.

The temperature of the positive pole of the electric arc, which is now generally believed to be the boiling point of carbon, is usually taken, on the authority of Violle,* as approximately equal to 3500° C. Violle's method of determining it was as follows :—The carbons of the arc were placed horizontally, and the positive pole was so arranged that pieces of its substance could be detached while the arc was passing; these white-hot pieces fell into a calorimeter, and from the amount of heat given up, the temperature was calculated, assuming the specific heat of carbon at this point to have its theoretical value. The method does not seem at first sight a very reliable one, and Violle states that the result is only to be regarded as an approximation.

The method adopted by the authors of this paper is exactly the same as that which they employed last year in their "Experimental Investigations on the Effective Temperature of the Sun,"† in the account of which full descriptions of the apparatus used, &c., are given.

A Brockie-Pell arc lamp was employed in the experiments, the current being obtained from a dynamo worked by a gas engine. It would have been preferable for some reasons to have worked the arc off the 26 Epstein accumulators which we

Fig. 1.



had at our disposal, but the current from these was used in heating the platinum strip, and we did not wish to run the cells off too quickly. Platinoid resistances were inserted in circuit with the arc until it burnt steadily.

The general arrangement of the apparatus is shown diagrammatically in Fig. 1.

* Violle, 'Jour. de Phys.,' 3rd Series, vol. 2, 189, p. 545.

† Wilson and Gray, 'Phil. Trans.,' A, vol. 185, 1894, p. 361.

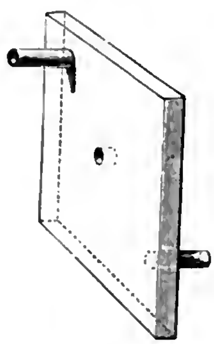
P is the platinum-strip radiator (our modification of JOLY's meldometer), the dotted line representing the water-jacket which is placed over the strip. R is the radio-micrometer; A and B are tubes through which radiation can pass to fall on the receiving surfaces within. The diameter of the aperture at A is accurately known; as also is its distance from the receiving-surface, so that the apparent area of platinum, as seen from the latter, may be calculated.

L is the lamp, which is placed inside a wooden box, lined internally with tin-plate, both wood and metal being pierced with small holes opposite to the arc.

A screen, S, hangs in front of the box, and contains a small, carefully-measured hole, which can be adjusted until the brightest (or any) part of the glowing carbons shines directly into the tube B. The size of the hole in the screen S, and its distance from the radio-micrometer, then give the apparent area of bright carbon as seen from the latter.

The screen S is made of copper, and is really a flat box (fig. 2) provided

Fig. 2.



with an inlet and an outlet tube, so that a continual stream of water from the ordinary house supply could be kept running through it—a precaution necessary from its proximity to the arc. In the experiments, a plentiful stream was kept running through this box, and thence on to the water-jacket round the radiator, the supply being sufficient to prevent any perceptible heating of the screen.

A small hole cut in the side of the wooden box enabled us, with the aid of mirrors, to use a pencil of the light of the arc for reflection from the mirror of the radio-micrometer, thus obviating the necessity of a lime-light, or other bright source, while an incandescent lamp-filament provided us with an extremely sharp band of light on the scale of the radiator. (A larger and better mirror had been affixed to this since its use in our work on the solar temperature, and this mirror, with an incandescent lamp, gives a band of light with edges so sharp on the "temperature scale," that it could, if necessary, be read to the tenth of a millimetre, which is beyond our ordinary requirements.)

The theory of the method is very simple; essentially it is the same as that which applies to the estimation of the effective temperature of the sun, without the complications arising from atmospheric absorption, &c.

In the case of the sun, we can only hope to find (at least at present) the *effective* temperature, as we know little of the radiating power of the photospheric substances, but in the case of the carbons of the arc, we may assume that we are dealing with a "black" surface of approximately unit emissive power.

Let then

T_1 = absolute temperature of bare platinum-strip at balancing point,

e = ratio of emissive power of a black surface to that of the bare metallic surface at this temperature,

A = ratio of the area subtended by the platinum to that subtended by the glowing carbon, at the receiving surface of the radio-micrometer,

and $q = f(T)$ be the "law of radiation" for a "black" surface, where q = quantity of radiation as a function of the absolute temperature T_1 of the radiating surface.

Then the radiation from the carbon is A/e times the intensity of that from a black surface at a temperature T_1.

If the radiation from the platinum at the temperature T_1 be put $= q_p$,

then
$$q_p = f(T_1).$$

Then the radiation from the carbon

$$= \frac{A}{e} \cdot q_p = \frac{A}{e} \cdot f(T_1).$$

And if T_2 = required temperature of the carbon

$$\frac{A}{e} \cdot f(T_1) = f(T_2),$$

whence T_2 may be obtained, when we know (1) the law of radiation, (2) the ratio of the emissive powers of bare and blacked platinum.

We go on to discuss these two points together, as the experiments on the first give us information on the second.

The Law of Radiation and the Ratio of the Emissive Powers.

In our paper already quoted we have given a series of experiments on the radiation from bare platinum at temperatures up to 1600° C. approximately, and we have shown that a simple fourth-power law expresses the results very closely, so that for these experiments the "law of radiation" is $q = a(T^4 - T_0^4)$, where T = absolute temperature of radiating surface, T_0 = temperature of surrounding medium, a = a constant, and q = radiation in arbitrary units. At high temperatures T_0^4 becomes unimportant, and the expression simplifies still further to $q = aT^4$.

Experiments on a blackened surface are difficult to carry out at anything beyond moderately low temperatures; we therefore assumed in our former work that the form of the law was the same for a blackened as for a bright surface, there being good grounds, both theoretical and experimental, for such a belief. Further investigations, however, indicate that this assumption is not correct, as will be seen from the experiments detailed below.

A series of experiments on the radiation from bare platinum was made first, exactly in the same way as those described in our work of last year, that is to say, the radiation from the platinum at different temperatures was allowed to fall on a radio-micrometer of the ordinary form, the sensibility of which was reduced sufficiently to give a readable deflection at the highest temperature used, the deflection as given by the scale-readings being then taken as proportional to radiation. This proportionality has been shown before to be strictly true for deflections up to and greater than those obtained in these experiments.

The platinum-strip was next blackened on one side with black oxide of copper, which was ground very fine, mixed up with methylated spirit, and laid on with a camel's-hair brush; this, when the liquid had dried off, gave a very good, even, dead-black surface, the emissive power of which may be taken as approximately equal to that of an ideal black surface.

Lamp-black, of course, is useless for these experiments, since it burns off at something under 500° C.; it could only be used if the radiator could be placed in a vacuum, or in an atmosphere having no action on the carbon, for which purpose we are having apparatus specially constructed.

At about 900° C. the black oxide of copper begins to suffer a change; its surface becomes somewhat shiny, and an alloy is formed with the platinum; this puts a limit to the temperature at which the radiation may be taken as that of a "black" surface. Our first strip was spoiled in discovering this limiting temperature; the second strip (after calibration, &c., and radiation experiments with the metal bare) was covered on both sides and examined during the progress of the experiments, which was stopped as soon as the black surface showed any signs of change of physical condition; these were not only apparent to the eye, but were almost immediately indicated by a variability of temperature, due to the alteration of emissive power, as the reduction of the oxide crept over the surface of the strip.

Platinum-black would have no advantages in this connection over the copper-oxide, as it reverts to the metallic condition at very nearly the same temperature as that at which the oxide changes in the way mentioned above.

The two series of experiments gave the figures in the following tables :—

The 1st column gives the absolute temperature of the platinum.

" 2nd " " deflection on the scale of the radio-micrometer in millimetres. These numbers then represent radiation in arbitrary units.

" 3rd " " radiation calculated by a formula to be discussed immediately.

" 4th " " differences between the observed and calculated radiation.

TABLE I.—RADIATION OF BARE PLATINUM.

Abs. temp.	Radiation observed.	Radiation calculated.	Obsd. – calc.
637°	7·0	10·2	– 3·2
771	17·5	18·8	– 1·3
804	22·5	22·5	0·0
883	33·0	32·4	+ 0·6
896	36·5	35·5	+ 1·0
979	52·7	51·5	+ 1·2
1008	59·2	58·2	+ 1·0
1033	65·0	64·4	+ 0·6
1044	68·0	67·3	+ 0·7
1049	78·5	78·7	– 0·2
1133	94·5	94·5	0·0
1155	99·5	102·3	– 2·8

TABLE II.—RADIATION OF BLACKED PLATINUM.

Abs. temp.	Radiation observed.	Radiation calculated.	Obsd. – calc.
683°	67	67	0
798	115	115	0
893	166	168	– 2
933	193	195	– 2
957	218	213	+ 5
975	235	227	+ 8
1024	269	268	+ 1
1058	304	299	+ 5
1075	314	316	– 2
1107	349	349	0

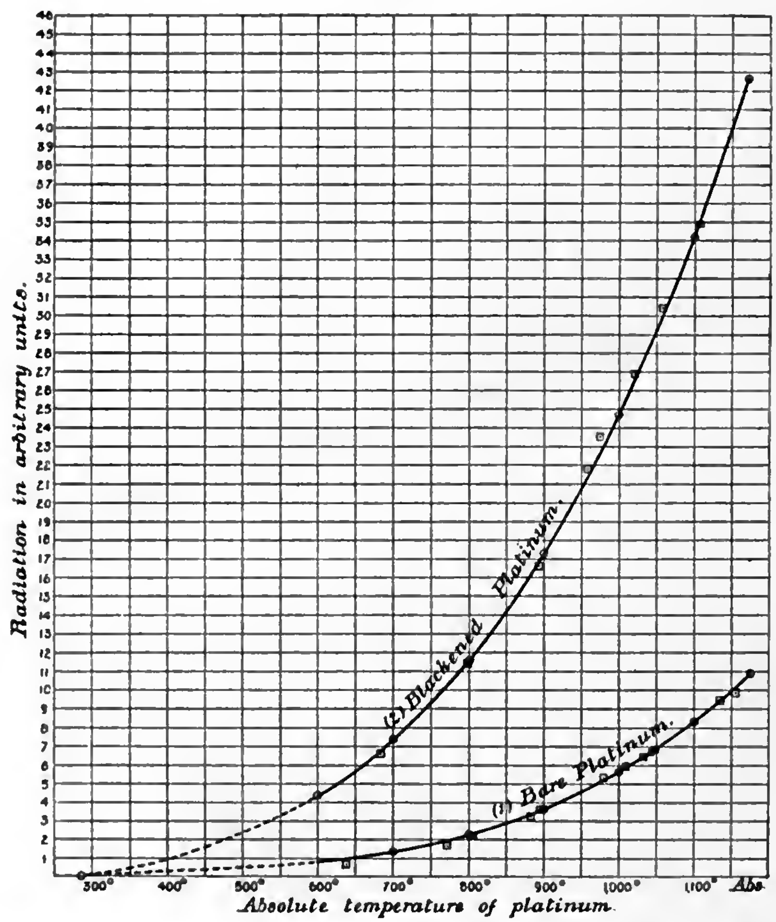
The same results are shown graphically in fig. 3, in which the curve is drawn from the formula through points denoted thus ⊙, while the experimental points are denoted thus ⊡

The curve for bare platinum was taken first, and a simple fourth-power law tried on it; this was found to agree very closely with the observed results throughout the range of the experiments, except at the lowest temperatures, thus confirming our work of last year, *but in the case of blacked platinum the curve is much less steep than that given by a fourth-power law.* In fact, after several trials it was found that the exponent 3·4 in the expression $q = a\mathrm{T}^x$ followed the curve fairly well, but that a formula of the form

$$q = a\,(\mathrm{T}^3 - \mathrm{T}_0^3) + b\,(\mathrm{T}^4 - \mathrm{T}_0^4)$$

would fit both curves, the values of a and b being obtained from the respective experiments in the two cases.

Fig. 3.



Curve showing the relation of radiation to temperature in the case of (1) bare platinum, (2) blacked platinum.

In this expression, as before, q represents quantity of radiation, T the absolute temperature of the radiating surface, T_0 the temperature of its surroundings, while

a and b are constants determined in any case from two experimental points at some distance apart. When these have been calculated, the expression may be simplified by writing $aT_0^3 + bT_0^4 =$ a third constant, c say, so that we have

$$q = aT^3 + bT^4 - c,$$

c being very small and unimportant when any but very low values of T are concerned.

The curves (fig. 3) and figures (Table I. and II.) already given were obtained from expressions of this form; for bare platinum the constants were calculated from

$$\left.\begin{array}{l} T = 804^\circ \text{ abs.}, \; q = 22{\cdot}5 \\ T = 1133 \quad ,, \quad q = 94{\cdot}5 \end{array}\right\}$$

and

from this, the values obtained are

$$\log a_1 = \bar{9}{\cdot}92855$$
$$\log b_1 = \overline{11}{\cdot}81280$$

a being, however, $-$, and b $+$.

T_0 was always about 288 (*i.e.*, the temperature of the room was 15° C.), and in this case, c comes out $= 0{\cdot}24$ (*i.e.* 0·24 mm. on the scale of the radio-micrometer), which is practically negligible.

For blacked platinum the constants were calculated from the experimental points

$$\left.\begin{array}{l} T = 683^\circ \text{ abs.}, \; q = 67{\cdot}0 \\ T = 1107 \quad ,, \quad q = 349{\cdot}0 \end{array}\right\}$$

whence we obtain

$$\log a_2 = \bar{7}{\cdot}22166$$
$$\log b_2 = \overline{11}{\cdot}92915,$$

both a and b being $+$, while $c = 4{\cdot}6$, so that for calculating the radiation, in our arbitrary units, at any temperature, we have

$$q = a_2T^3 + b_2T^4 - 4{\cdot}6 \text{ for blacked platinum,}$$

and

$$q = a_1T^3 + b_1T^4 - 0{\cdot}2 \text{ for bare platinum,}$$

the constants being those given above for blacked and bare platinum respectively.

From the two radiation curves, for bare and blacked platinum respectively, we may obtain the relative values of the emissive powers at different temperatures. That the ratio is not constant has been known for some time;* the table given below will show the nature and extent of the variation.

* SCHLEIERMACHER, 'Wied. Ann.,' vol. 26, 1885, p. 287. Also WILSON and GRAY's paper already quoted, p. 380, in which several references will be found, relating to experiments on this point, and the law of radiation, &c.

The radiation is calculated from the formulæ given on p. 43, for temperatures 600°, 700°......1200° abs.; the fourth column gives the ratio black/bare.

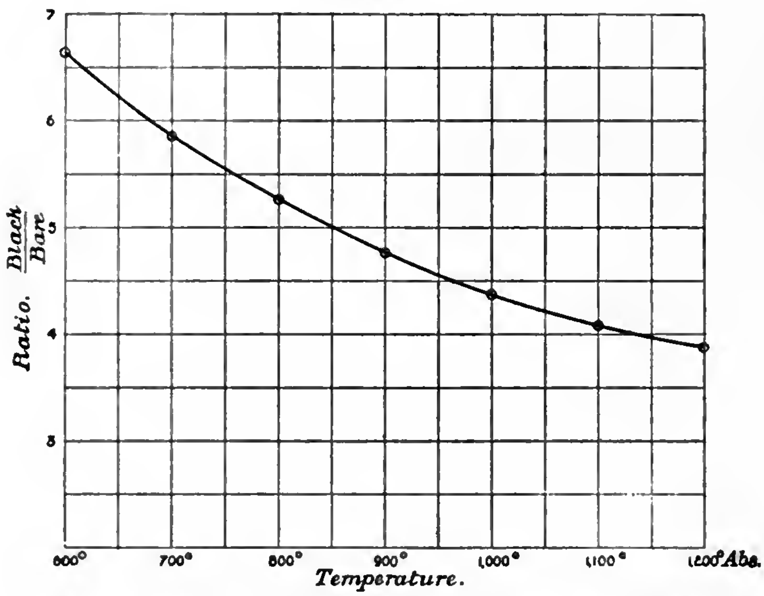
TABLE III.—RATIO OF EMISSIVE POWERS OF BLACK AND BARE PLATINUM.

Abs. temp.	Radiation of black platinum.	Radiation of bare platinum.	Ratio.
600°	42·4	6·4	6·62
700	73·0	12·5	5·84
800	116·1	22·0	5·28
900	173·2	36·2	4·78
1000	247·0	56·3	4·39
1100	341·5	83·6	4·09
1200	459·4	119·9	3·83

The curve (fig. 4) shows the same results graphically.

It will be seen at once that the ratio diminishes as the temperature increases, but less and less rapidly. The observations agree fairly well with the old experi-

Fig. 4.



Curve showing the ratio of the emissive powers of blacked and bare platinum at different temperatures.

ments generally described in the text-books—experiments made at 100° C. and thereabouts—in which the ratio is given as from 10 to 12. They differ slightly from SCHLEIERMACHER's series, the ratio in his being a little lower than in ours at the same temperature.

There is a fair agreement with ROSETTI's result at high temperatures; he obtained 2·9 at about 1500° C., which is very little lower than the value which would be reached by a theoretical continuation of our curve (fig. 4).

The tendency of the curve appears to be to approach a constant value of about 3, but it is impossible to dogmatize on this point, from the physical limitations of the inquiry. The fact that the ratio *does* diminish shows that the physical nature of one or both of the surfaces is different at different temperatures; probably it is the bare platinum which changes, and it is possible that the constant value of the ratio is just attained at the melting-point of the metal. Experiments with a molten surface would be interesting, but very difficult to carry out.

The Balancing Experiments.

We may now pass on to the "balance experiments," from which the temperature of the carbons of the arc may be calculated. The principle of the method has already been described in the earlier part of the paper.

When the arc was shining on the top receiving-surface of the radio-micrometer, and the incandescent platinum on the lower, the position is denoted as Position A, the reverse by Position B.

The angle subtended by the area of incandescent platinum, in both cases, was 5°·301.

The diameter of the hole in the screen (fig. 2), through which the radiation of the arc passed, was 0·337 cm. In position B, its distance from the receiving surface was 57·0 cm.; in position A it was 58·2 cm., giving angular apertures of 0·339° and 0·332° respectively. The screen was close enough to the arc to make it certain that the hole was completely filled with the brightest part of the crater of the + pole. The hole was sufficiently small to form a rough "pin-hole" image of the carbons, by means of which it could be seen during an experiment that the brightest part of the + pole was shining directly into the tube of the radio-micrometer, and so on to the receiving-surface.

In position B, the ratio of the areas of platinum and carbon, as seen from the receiving-surface, was

$$(5{\cdot}301/0{\cdot}339)^2 = 245,$$

and in position A, the ratio was

$$(5{\cdot}301/0{\cdot}332)^2 = 255.$$

The following are the temperatures at which the radiation from the bare platinum balanced that from the hottest part of the + pole:—

Position B.		Position A.	
	715° C.		862° C.
	717		930
	737		924
	720		922
	732		902
	722		933
	722		985
			945
Mean = 724° C. = 997° abs.		Mean = 925° C. = 1198° abs.	

The results in position A are not so concordant as those in position B, but the arc was not quite so steady; the first low reading (862°) was probably taken when the growth of condensed carbon on the – pole was partly shading the receiving surface from the heat of the crater, while the high reading (985° C.) probably corresponds to one of those sudden "bursts" of high temperature which we have frequently observed to take place, although we cannot offer any explanation of them.

The current in the above experiments was about 14 ampères; a few observations were subsequently made with less resistance in circuit, and a current of about 25 ampères; the temperature then appeared to be a little higher than with the smaller current, but the arc in this case was so unsteady as to prevent the observations being made very carefully.

[Later experiments with a higher voltage (110 volts) and a current varying from 10 to 40 ampères indicate an exact equality of temperature, which confirms the usually-accepted view.—April 8th, 1895.]

Working out the results of the two positions separately, we have, for the ratio of emissive powers, at

$$997^\circ \text{ abs.}, \frac{\text{black}}{\text{bare}} = 4{\cdot}4.$$

$$1198 \quad ,, \quad ,, \quad 3{\cdot}85.$$

From the formula (p. 43), for blacked platinum,

$$q = a\mathrm{T}^3 + b\mathrm{T}^4 - 4{\cdot}56,$$

we have for position B, $q = 244{\cdot}48$,
and for ,, A, $q = 456{\cdot}87$,

Therefore we have a radiation from the + pole of the arc, in our arbitrary units, corresponding to

$$244{\cdot}48 \times \frac{245}{4{\cdot}4} = 13613 \text{ in position B},$$

and to

$$456{\cdot}87 \times \frac{255}{3{\cdot}83} = 30420 \text{ in position A},$$

The geometrical mean of these two numbers,

$$\sqrt{13613 \times 30420} = 20350,$$

is the true radiation in our arbitrary units. To find the corresponding temperature, we must substitute this value in the equation on p. 43, *i.e.*,

$$20350 = a\mathrm{T}^3 + b\mathrm{T}^4 - 5.$$

This is most easily solved by trial, T coming out as 3520°.

To this about 100° must be added, owing to the curvature of the radiation curve (for full reason, see WILSON and GRAY's paper already quoted, p. 387), giving approximately.

3600° abs., or 3300° C.

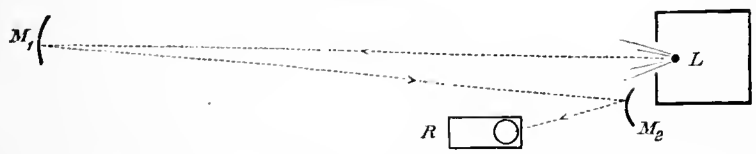
as the temperature of the hottest part of the + pole of the electric arc, a result surprisingly near VIOLLE's estimate, 3500° C.

The Comparative Radiation from Different Parts of the Arc.

After the above experiments had been finished, an attempt was made to obtain comparative values of the radiation, and hence the temperatures, of different parts of the carbons of the arc. For this purpose, a radio-micrometer of the ordinary form was employed, on to the receiving-surface of which radiation could fall through a large pin-hole.

An image of the carbons was then formed by an arrangement shown diagrammatically in the figure (fig. 5), in which L is the arc lamp, inside a lantern with the

Fig. 5



condenser removed, M_1 is a concave mirror, M_2 a convex mirror, both silver-on-glass and R is the radio-micrometer. The reason for not forming a direct image with a lens was the varying transparency of glass for radiation at different temperatures; the mirrors also enabled us to "dilute" the heat considerably, and so obtain convenient direct deflections on the radio-micrometer scale.

The sketch (fig. 6) shows approximately the shape of the image formed, on a scale about two-thirds full size.

Fig. 6.

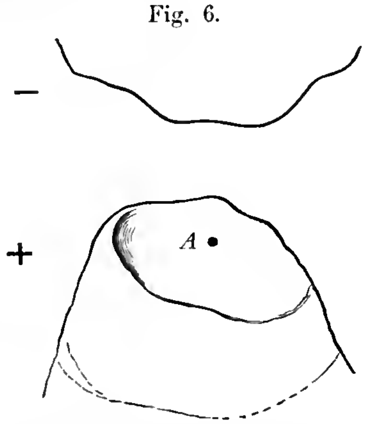


Image of the carbons from a tracing. A represents the size of the aperture by which radiation reached the receiving-surface.

The mirror, M_2, being provided with adjusting screws, it was easy to bring any part of the image, either of the carbons or the pale violet glow of the arc itself, on to the small aperture, the deflection on the scale of the radio-micrometer then giving readings proportional to the radiation from the chosen point.

Magnified to this extent, however, the arc was never steady enough to allow a detailed "mapping-out" of the carbon surfaces with regard to temperature. Even when the light is apparently steady to the eye, the violet arc itself often shifts its position, while the – pole continually alters in shape from the carbon deposited on it, which causes a bulbous excrescence, somewhat as shown in fig. 7, to form gradually.

When this is the case, the arc naturally strikes across from some such position as A to B; B then becomes, as might be expected, much hotter than any other part of the – pole.

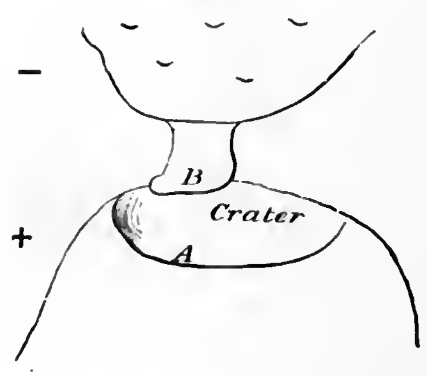
As an example of the kind of difference existing between the two poles, the following figures may be given; they correspond to the hottest obtainable point in the crater of the + pole, and to the hottest point on the – pole, before any excrescence has had time to grow.

The numbers are scale divisions on the scale of the radio-micrometer, and therefore represent the radiation in arbitrary units:—

+ pole. Radiation 60·2 and 67·1
Mean = 63·7
– pole. Radiation = 21·8,

so that the radiation from the hottest part of the + pole was about three times as great as that from the hottest part of the – pole.

Fig. 7.



Taking the temperature of the former as 3300° C., this would give a temperature of about 2350° C. for the latter.

In a case where a "blob" had formed on the − pole, as in fig. 7, the following readings were obtained :—

+ pole. Radiation 57·0, 56·0, 56·6, and 55·5
Mean = 56·3
− pole. Radiation of hottest part = 38·3.

Again, taking 3300° C. for the former, this gives about 2700° C. for the latter, or 350° C. higher than that of the − pole just after the arc is started.

We may say, then, that if the temperature of the crater is about 3300° C., that of the − carbon is ordinarily about 2400° C. in its hotter parts.

As for the temperature of the arc itself, we can say nothing. Allowing the pale violet glow between the poles to fall on the aperture of the radio-micrometer, we obtained deflections of from 1 to 2 per cent. of those obtained when the hottest part of the crater was used, which seems to indicate a comparatively high radiative power for the hot gases which lie between the carbon poles.

Note on the Effective Temperature of the Sun.

In the authors' work on this subject, radiation experiments were made with bare platinum, at temperatures up to 1600° C. approximately, and it was assumed that a formula of the same form as expressed these results would also hold for a blacked surface, while the ratio of the emissive powers at high temperatures was taken on ROSETTI's authority as about 2·9.

The new work, given above, appears to show that the curve for the black surface does not, however, follow a simple fourth-power law so closely as does that for the bare platinum, and that, taking the law as given on p. 43 of the present paper, a correction must be made to the result obtained by the earlier work.

The approximate value of this correction may be obtained by taking the figures given as a typical case on p. 26 of last year's paper, and applying the new law to them.

In this case, the corrected ratio (*i.e.*, the ratio corrected for atmospheric absorption, and for loss by reflection from the glass of the heliostat) of the apparent areas of the bare platinum and the sun was approximately 1295 : 1, and balance was obtained with the platinum at a temperature of 1514° Abs.

Now by the formula on page 43, the radiation of bare platinum at this temperature

$$= a \, . \, 1514^3 + b \, . \, 1514^4 - 0·27 = 311·77$$

a and b having the values given on p. 43.

The radiation from the sun therefore

$$= 1295 \times 311{\cdot}77 = 403450.$$

To find the effective temperature of the sun, we have, therefore, to solve the equation

$$403450 + 4{\cdot}6 = a\text{T}^3 + b\text{T}^4,$$

where a and b now have the values corresponding to the curve for the black surface. This gives T = 7800° Abs., approximately, instead of 7000°, as given by the older method of working.

That is to say, supposing the new formula to be correct, our estimate of the solar temperature would have to be increased by something like 800°.

If, however, the ratio of the emissive powers approaches a constant value, as the figures and curves on p. 44 make possible, the expression for the curve of the black surface would be somewhat altered, in such a direction as to reduce the correction, so that we may say finally that, taking Ångström's estimate of the atmospheric absorption, which gave in our former work an effective solar temperature of 7400° C., its more probable value would now be not very far from 8000° C.

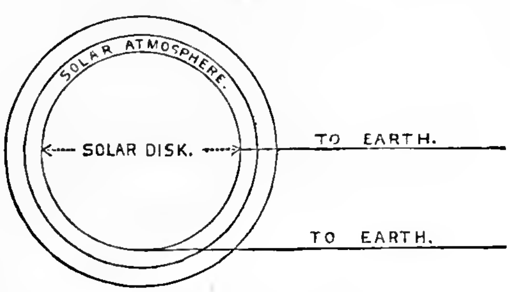
The Absorption of Heat in the Solar Atmosphere.

By W. E. Wilson, *M.R.I.A., and* A. A. Rambaut, *M.A., D.Sc.*

Proceedings of the Royal Irish Academy, 3rd *Ser., Vol II., No.* 2, 1892.

One of the most interesting questions in Solar Physics which awaits solution is—Does the quantity of heat received by the earth from the sun vary from year to year, or is it a constant? Assuming that the internal temperature of the sun remains constant, there are yet two factors, variations in which would cause the amount of solar heat received by the earth to vary. These are—the absorption of heat in the solar atmosphere, and the absorption in the earth's atmosphere. It is with the first of these that we propose to deal in this paper. From the accompanying diagram (fig. 1) it will be seen that the amount of absorption at the edge of the disc of the sun must be much larger than at the centre. This fact has long been known as a matter of observation. Professor Henry of Princeton, in 1845, was the first to discover this, and it has since been confirmed by Secchi, Langley, and many others. These observers all make the amount of heat coming from a

Fig. 1.

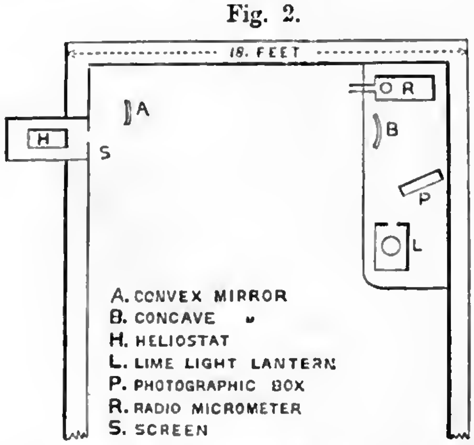


point on the edge of the disc about half that coming from the centre. It is also an interesting subject of inquiry, how much of the sun's total heat is absorbed by his atmosphere. Laplace, making certain assumptions, found that the sun would be 12 times as bright if he was stripped of his atmosphere. Pickering, taking other data, says that the sun without its atmosphere would be $4\frac{2}{3}$ times as bright as it is. Langley and some others have investigated this difficult problem, and whatever the true amount may be, one thing is quite certain, that the sun's radiation is stopped to a considerable extent by his atmosphere. If the solar atmosphere varies in depth it is plain from the diagram that the ratio of the absorptions at the centre and edge of the disc will vary, and if accurate measures were taken from time to time of these quantities, we could determine whether the depth of the solar atmosphere was constant or variable. It is with reference to the solution of this problem that we bring the following researches to your notice.

In 1884 Mr. WILSON had an apparatus made to carry out some experiments in this direction. It consisted of two small thermo-piles which were coupled up in such a way that as long as the corresponding faces of both continued at the same temperature the galvanometer remained at zero. At first, both piles were placed on the edge of the solar image formed by a large Cassagrain reflecting telescope. When the galvanometer was steady at zero one of the piles was moved into the centre of the disc. The deflection of the galvanometer was noted. The experiment was then repeated again and again, each time reversing the order of the piles. These experiments gave 0·52 as the heat from the limb, that from the centre being 1·00. Mr. WILSON soon came to the conclusion that some more accurate means would have to be devised before any final result could be reached. In 1888 Professor C. V. BOYS invented his radio-micrometer. It is an instrument of extraordinary sensibility to radiant heat, and it occurred to Mr. WILSON that it would be an excellent instrument to use in these researches. In the first place it is so sensitive that we can use an enormously large image of the sun and still get plenty of heat to affect the instrument, and secondly it is very prompt and dead beat in its motion. The sensitive surface on which the heat is allowed to fall is only about 2 mm. square, so that a very small portion of the solar surface can be examined at one time. Using an image of the sun of 80 cm. in diameter, the instrument only covers the $\frac{1}{500000}$th part of the entire disc.

In 1888 Mr. WILSON fitted up a heliostat with silvered mirrors which reflected a small beam of sunlight into a dark room. It was received by a concave mirror of 10 feet focus, and a small convex mirror was placed inside of the focus: this formed a fine image of the sun 80 cm. in diameter. In the plane of this image the radio-micrometer was set on a heavy slate shelf. A slice of limelight was allowed to fall

Fig. 2.



on the mirror of the instrument and reflected from that to a scale. The relative position of the various parts of the apparatus is shown in fig. 2.

At first readings were taken with the edge of the sun and centre alternately on the instrument. On account of the very rapid fall in heat near the edge, this plan was not found to give satisfactory results; and at present the best results are got by allowing the image of the sun to transit across the instrument, and by recording the motion of the spot of limelight by means of a falling photographic plate.*

It became evident, from the first photographic curve obtained, that to translate its meaning, corrections would have to be applied for some disturbing causes, such as the damping effect of the air, and the variation in intensity of the source of heat to which the instrument was exposed. Dr. RAMBAUT has investigated these matters, and below will be found an exposition of the methods he has adopted to surmount the difficulties. What we now propose to do is to take frequent curves of absorption from time to time throughout a sun spot cycle of 11 years, and thus try to solve the problem—Whether the sun's atmosphere varies in depth in that time. From a comparison of photographs taken of it in maximum and minimum years, it would appear that the sun's corona varies in form, and also that the spots alter their characters in the same period. Is it not probable that changes are also taking place in the solar atmosphere? If we find that such changes are taking place, as will be shown by the alteration in the ratio of the heat from the limb and centre of the disc, we think it will be quite possible, by an investigation of the co-ordinates of these curves, to determine the change in the value of the solar constant.

EXAMINATION OF THE CURVES.

From the nature of the instruments it is clear that the only forces acting on the system are the directive force of the magnet, the force of torsion, and the resistance of the surrounding medium. If we denote the angle of torsion by ϕ, and the difference in temperature between the two ends of the couple by Θ, we obtain the following differential equation representing the motion :—

$$\frac{d^2\phi}{dt^2} + 2a\frac{d\phi}{dt} + b^2\phi = c\Theta, \tag{1}$$

in which $2a\frac{d\phi}{dt}$ is the resistance of the medium, $b^2\phi$ the torsion, and $c\Theta$ the directive force of the magnet.

Also, if K denotes the intensity of the heat falling on the heated end of the couple, we have the equation

$$\frac{d\Theta}{dt} = kK - l\Theta. \tag{2}$$

* See British Assoc. Report, Cardiff, 1891; Trans. of Section A.

For the following method of obtaining this equation we are indebted to Professor G. F. FITZGERALD :—

If we suppose θ_1 and θ_2 to be the elevation of temperature of the two ends of the couple above that of the surrounding medium, and if we suppose the couple divided into a number of segments ds, taking into account the resistance of the couple, a PELTIER absorption of heat at the hotter, and emission of heat at the colder end, and a THOMSON effect at other parts of the wire, we have, for any segment after the first, an equation of the form

$$\frac{d\theta}{dt} = TC + c\frac{d\theta}{ds} - \rho\theta + rC^2,$$

in which TC is the THOMSON effect, $c\frac{d\theta}{ds}$ the conduction, $\rho\theta$ the radiation, and rC^2 the heating by resistance. At the ends themselves we have a PELTIER, instead of a THOMSON, effect; and for the heated end the equation becomes

$$\frac{d\theta_1}{dt} = kK - P_1C + c\frac{d\theta_1}{ds} - \rho\theta_1 + rC^2,$$

and for the other end

$$\frac{d\theta_2}{dt} = kK + P_2C - c\frac{d\theta_2}{ds} - \rho\theta_2 + rC^2,$$

in which P_1C and P_2C represent the PELTIER effects at the two ends of the couple.

We thus obtain

$$\frac{d\theta_1}{dt} - \frac{d\theta_2}{dt} = kK - (P_1 + P_2)C + c\left(\frac{d\theta_1}{ds} + \frac{d\theta_2}{ds}\right) - \rho(\theta_1 - \theta_2).$$

We may also assume that the terms $\frac{d\theta_1}{ds}$ and $\frac{d\theta_2}{ds}$ are each to a first approximation proportional to the difference of temperature, so that we obtain

$$\frac{d\Theta}{dt} = kK - l\Theta - (P_1 + P_2)C.$$

The last term here is so small that in an investigation like the present we may neglect it so that we reach the result given in equation (2).

If now we eliminate Θ between equations (1) and (2) we obtain the following linear differential equation of the third order

$$\frac{d^3\phi}{dt^3} + (2a + l)\frac{d^2\phi}{dt^2} + (b^2 + 2al)\frac{d\phi}{dt} + b^2l\phi = ckK. \qquad (3)$$

In the method of observation described above the intensity K is a function of the time, so that we obtain the equation

$$\frac{d^3\phi}{dt^3} + (2a + l)\frac{d^2\phi}{dt^2} + (b^2 + 2al)\frac{d\phi}{dt} + b^2l\phi = f(t). \qquad (4)$$

Denoting the roots of the equation $x^2+2ax+b^2=0$ by $-m$ and $-n$, and remarking that

$$e^{-xt}\int e^{xt}f(t)dt=\frac{f(t)}{x}-\frac{f'(t)}{x^2}+\frac{f''(t)}{x^3}\ \&c.,$$

we obtain the solution of equation (4) in the form

$$\phi=Ae^{-lt}+Be^{-mt}+Ce^{-nt}-\frac{1}{(m-n)(n-l)(l-m)}\Big[f(t)\left\{\frac{m-n}{l}+\frac{n-l}{m}+\frac{l-m}{n}\right\}$$
$$-f'(t)\left\{\frac{m-n}{l^2}+\frac{n-l}{m^2}+\frac{l-m}{n^2}\right\}+f''(t)\left\{\frac{m-n}{l^3}+\frac{n-l}{m^3}+\frac{l-m}{n^3}\right\}-\&c.\Big] \quad (5)$$

The general term of this series may be written in the form

$$\frac{(-1)^r}{(m-n)(n-l)(l-m)}\times f^{(r-1)}(t)\times\begin{vmatrix}\frac{1}{l^r}, & \frac{1}{m^r}, & \frac{1}{n^r},\\ l, & m, & n,\\ 1, & 1, & 1,\end{vmatrix},$$

which, being a symmetric function of the roots, can be expressed in terms of the coefficients of the equation. Hence, when the form of $f(t)$ is given, the motion of the couple is determined, and by means of this equation any assumption with regard to the law of variation of the sun's heat from point to point of its disc might be tested.

The question before us at present is, however, the inverse of this, namely, from the curve which represents this equation to determine the corresponding values of $f(t)$.

Now, it is obvious from equation (4) that if we can obtain the values of the constants, a, b, and l, and the quantities

$$\frac{d^3\phi}{dt^3},\ \frac{d^2\phi}{dt^2},\ \frac{d\phi}{dt},\ \text{and}\ \phi$$

at any point along the curve, we can compute the corresponding value of $f(t)$.

In order to determine the constants, we have proceeded in the following manner :—If the couple be mechanically displaced from a position of equilibrium, its motion is represented by the equation

$$\frac{d^2\phi}{dt^2}+2a\frac{d\phi}{dt}+b^2\phi=0.$$

The solution of this equation gives us as the equation of the curve traced by the spot of light

$$\phi=Ae^{-mt}+Be^{-nt},\qquad \text{if } a^2 \text{ is greater than } b^2;$$

or,

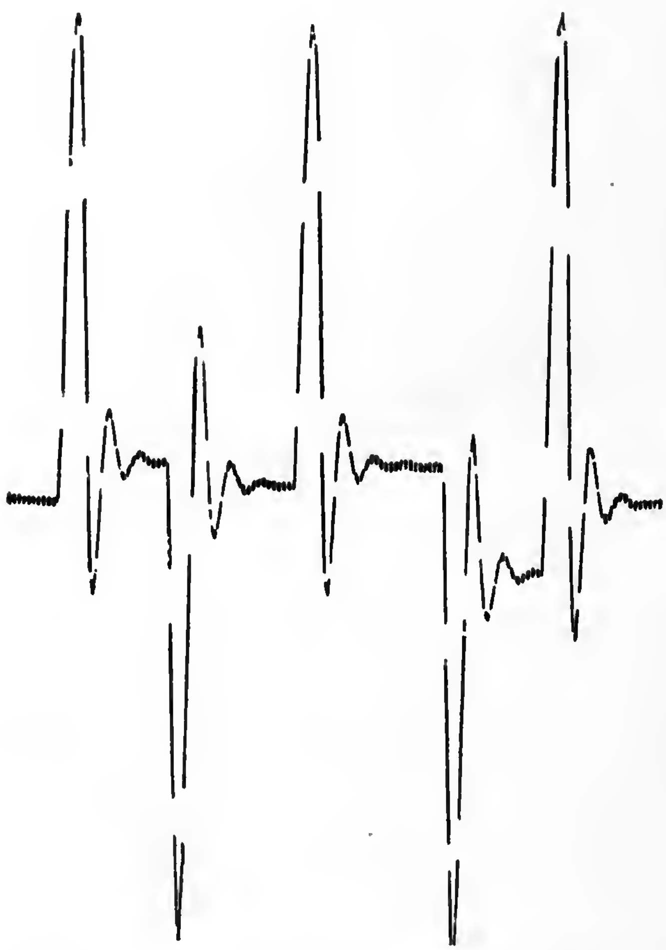
$$\phi=Ae^{-at}\cos(\delta t-\epsilon),\ \text{if } a^2 \text{ is less than } b^2,$$

in which

$$\delta=\sqrt{b^2-a^2}.$$

The first of these gives a continuously increasing value of ϕ—this ordinate only reaching its maximum value after an infinite time. The second equation gives a curve proceeding by a series of continually diminishing waves to the final position of equilibrium. From the form of the curve in fig. 5, which is a reduced copy of a photograph taken on 12th October, 1890, it is easily seen that it is an equation of the second form with which we have to deal in the case before us. In this case the period in which the oscillation takes place will give the value of $\frac{2\pi}{\delta}$, while the logarithmic decrement of the ordinates will give the value of $\frac{a\pi}{\delta}$. We shall thus be in a position to determine both a and δ from which we can immediately deduce the value of b.

Fig. 3.



For this purpose the series of curves shown in fig. 3 were obtained by Mr. WILSON in the following manner:—To the head from which the fibre is suspended he attached a light arm movable between two stops through an angle of about 5°. When the instrument was in equilibrium he then moved the arm rapidly up to one of the stops and back, thus giving the couple an impulse, in consequence of which it continued to oscillate for some twenty seconds or so, while the spot of light traced out the left-hand curve in fig. 3. When equilibrium had been restored he moved the arm in a similar manner up to the other stop and set the couple swinging in the opposite direction, and thus obtained the second curve in the same figure. In this way, by giving alternately right-handed and left-handed impulses to the couple, the five curves in fig. 3 were obtained, from which it is possible to derive the constants a and b.

Now, if ϕ_1 and ϕ_3 are the first two maximum values of the ordinate, and ϕ_2 the first minimum value during the free motion of the couple after an impulse has been imparted to it as above described, and if $\overline{\phi}$ is the final value of ϕ when the position of equilibrium has been attained (which will be of the nature of an index correction to the readings of ϕ), we have clearly

$$\phi_1 = A \cos \epsilon + \overline{\phi},$$

$$\phi_2 = -A \cos \epsilon \,.\, e^{-\frac{a\pi}{\delta}} + \overline{\phi},$$

$$\phi_3 = A \cos \epsilon \,.\, e^{-\frac{2a\pi}{\delta}} + \overline{\phi};$$

whence

$$\phi_1 - \phi_2 = A \cos \epsilon \left(1 + e^{-\frac{a\pi}{\delta}}\right),$$

and

$$\phi_2 - \phi_3 = -A \cos \epsilon \,.\, e^{-\frac{a\pi}{\delta}} \left(1 + e^{-\frac{a\pi}{\delta}}\right);$$

therefore

$$\frac{\phi_3 - \phi_2}{\phi_1 - \phi_2} = e^{-\frac{a\pi}{\delta}}$$

Reading the five curves in fig. 3,* we obtain the following values of ϕ_1, ϕ_2, and ϕ_3, expressed in millimetres :—

	Curve 1.	Curve 2.	Curve 3.	Curve 4.	Curve 5.
ϕ_1	− 33·1 mm.	+ 41·0 mm.	− 33·0 mm.	− 11·5 mm.	− 38·4 mm.
ϕ_2	+ 12·8	− 12·5	+ 12·0	− 33·9	+ 9·1
ϕ_3	− 3·6	+ 5·9	− 4·1	+ 18·2	− 8·1
Resulting value of $e^{-\frac{a\pi}{\delta}}$	0·357	0·344	0·358	0·346	0·362

* The curves in figures 3, 4, 5, and 7 are reduced copies of the original photographs.

The mean of these five determinations gives us 0·3534 as the value of $e^{-\frac{a\pi}{\delta}}$.

We have also determined half the period of a free oscillation from each of the five curves separately with the following results :—

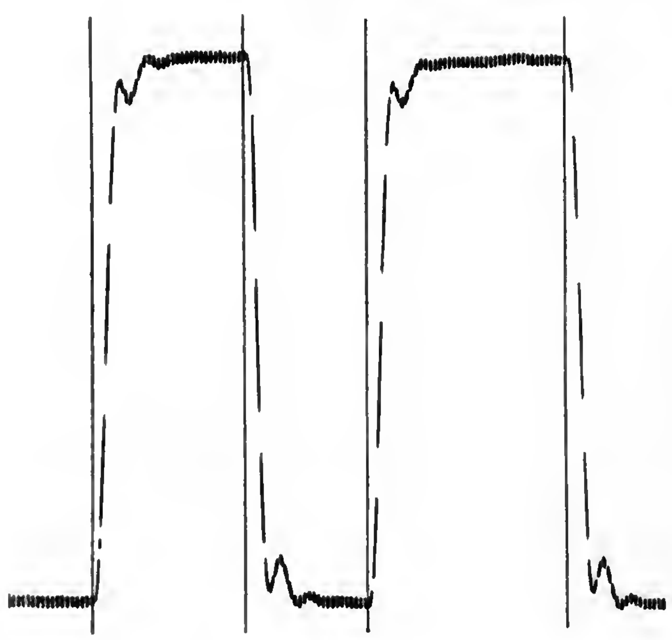
Curve 1	3s·200
„ 2	3 ·200
„ 3	3 ·166
„ 4	3 ·166
„ 5	3 ·233
Mean	3 ·193

We have thus obtained, from these five curves representing the free motion of the system, the following values :—

$$\left.\begin{aligned} \frac{\pi}{\delta} &= 3^{s}\cdot 193 \\ \text{and } e^{-\frac{a\pi}{\delta}} &= 0\cdot 3534 \end{aligned}\right\}.$$

In order to determine the value of the quantity $e^{-\frac{l\pi}{\delta}}$, the instrument was suddenly exposed to the radiation from a LESLIE's cube filled with boiling water until it came to rest, and then the heat was as suddenly cut off, the instant at

Fig. 4.



which the exposure began and ended being indicated by a flash of light which traced a line on the falling plate. The result of this procedure is shown in fig. 4.

Now, equation (5) shows that when the instrument is exposed to a constant source of heat the motion is represented by the equation

$$\phi = Ae^{-at} \cos (\delta t - \epsilon) + Ce^{-lt} + K.$$

When $t = \infty$, or when thermal equilibrium has been established, if $\bar{\phi}$ represent the final value of the ordinate, $\bar{\phi} = K$.

Hence if ϕ_0, ϕ_1, ϕ_2, &c., denote the values of the ordinates at the epochs 0, $\frac{\pi}{\delta}$, $\frac{2\pi}{\delta}$, &c, we shall have

$$\begin{aligned}
\phi_0 - \bar{\phi} &= \quad A \cos \epsilon \qquad\qquad + C \qquad\qquad = \psi_0, \text{ say;} \\
\phi_1 - \bar{\phi} &= -A \cos \epsilon \,.\, e^{-\frac{a\pi}{\delta}} + C \,.\, e^{-\frac{l\pi}{\delta}} = \psi_1, \quad ,, \\
\phi_2 - \bar{\phi} &= \quad A \cos \epsilon \,.\, e^{-\frac{2a\pi}{\delta}} + C \,.\, e^{-\frac{2l\pi}{\delta}} = \psi_2, \quad ,, \\
\phi_3 - \bar{\phi} &= -A \cos \epsilon \,.\, e^{-\frac{3a\pi}{\delta}} + C \,.\, e^{-\frac{3l\pi}{\delta}} = \psi_3, \quad ,, \\
\phi_4 - \bar{\phi} &= \quad A \cos \epsilon \,.\, e^{-\frac{4a\pi}{\delta}} + C \,.\, e^{-\frac{4l\pi}{\delta}} = \psi_4, \text{ \&c.}
\end{aligned}$$

Also, for the motion, when the heat is cut off we shall have an exactly similar series of equations, except that $\bar{\phi}$ will represent the final or zero reading of the ordinate.

In the case before us the values of ψ_4 are so small, in consequence of the rapid damping down of the oscillations, that we had better restrict ourselves to the first four of these equations. If we denote

$$e^{-\frac{a\pi}{\delta}} \text{ by } x, \text{ and } e^{-\frac{l\pi}{\delta}} \text{ by } y,$$

and if we eliminate $A \cos \epsilon$ and C, we obtain from each of the curves the two equations

$$x - y = \frac{\psi_1\psi_2 - \psi_0\psi_3}{\psi_0\psi_2 - \psi_1^2}$$

$$xy = \frac{\psi_2^2 - \psi_1\psi_3}{\psi_0\psi_2 - \psi_1^2}$$

From the four curves in fig. 4 we find the following values of ψ_0, ψ_1, &c., with the resulting values of $x - y$ and xy :—

	ψ_0 mm.	ψ_1 mm.	ψ_2 mm.	ψ_3 mm.	$x - y$	xy
Curve 1, . .	142·6	55·4	8·8	5·5	0·1636	0·1252
,, 2, . .	142·4	65·4	5·6	6·4	0·1566	0·1113
,, 3, . .	142·0	54·1	8·5	5·7	0·2032	0·1373
,, 4, . .	142·4	68·0	5·8	6·7	0·1474	0·1111

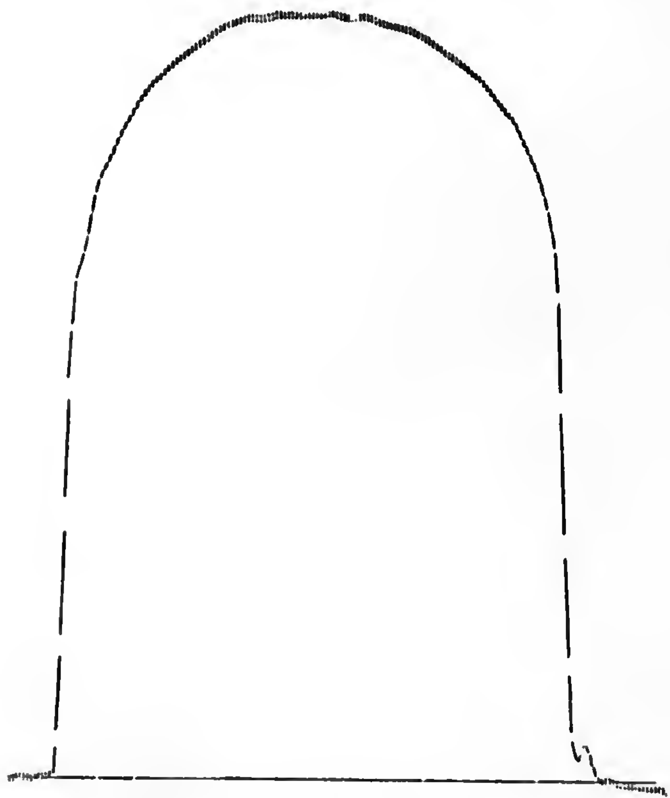
The resulting values of x and y from each of the four curves are, accordingly,

	x	y
Curve 1, ...	0·4449	0·2813
" 2, ...	·4210	·2644
" 3, ...	·4858	·2826
" 4, ...	·4151	·2677
Mean, ...	0·4417	0·2740

These figures seem to show two things:—Firstly, that the damping effect is less when the instrument is heated than when cold, since the value of x just found is greater than that found from the curves of free oscillation.

Secondly, it may be noticed that the values of both x and y are greater as derived from the 1st and 3rd curves than those deduced from the other two curves. The materials are, however, hardly sufficient to decide whether this

Fig. 5.



represents a real difference depending on the direction of the motion or is merely an accidental discrepancy.

We have thus the following results, taking for $e^{\frac{a\pi}{\delta}}$ the value found from the heat curves (the system being then in circumstances more closely analogous to those which take place during a transit of the sun than when cold):—

$$\frac{\pi}{\delta} = 3^{s}\cdot193, \qquad e^{-\frac{a\pi}{\delta}} = 0^{s}\cdot4417, \qquad e^{\frac{l\pi}{\delta}} = 0^{s}\cdot2740.$$

From these we find

$\delta = 0{\cdot}9838$ or $56^\circ{\cdot}37$, $a = 0{\cdot}2558$, $l = 0{\cdot}4054$;

whence $b^2 = 1{\cdot}0333$.

We thus find the coefficients of the differential equation on p. 54:—

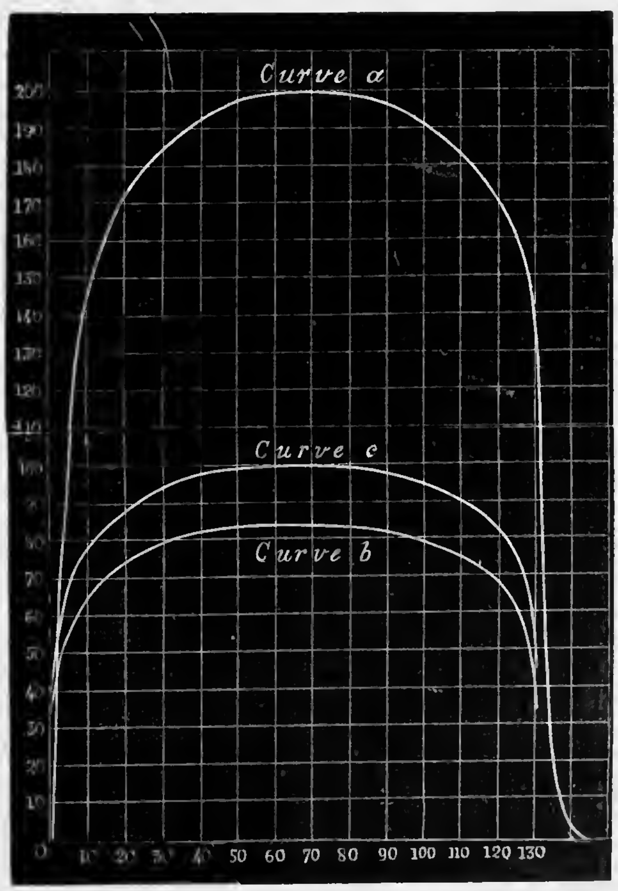
$2a + l = 0{\cdot}9170$, $2al + b^2 = 1{\cdot}2407$, $b^2 l = 0{\cdot}4189$.

Whence $\frac{d^3\phi}{dt^3} + 0{\cdot}9170 \frac{d^2\phi}{dt^2} + 1{\cdot}2407 \frac{d\phi}{dt} + 0{\cdot}4189\, \phi = f(t)$.

If the curve were a smooth curve without points of inflexion, it would be easy to compute the values of the differential coefficients from the differences of the readings of ϕ by means of the usual equation

$$\left(\frac{d}{dt}\right)^n = \{\log(1 + \Delta)\}^n.$$

Fig. 6.



Before applying this formula, however, it will be necessary to smooth the curve by removing the periodic part in the value of ϕ. The periodic terms are all contained in the expression $Ae^{-at}\cos(\delta t-\epsilon)$, and since this value for ϕ is a solution of the equation

$$\frac{d^3\phi}{dt^3}+(2a+l)\frac{d^2\phi}{dt^2}+(2al+b^2)\frac{d\phi}{dt}+b^2l\phi=0,$$

it is clear that the resulting curve is still a solution of the differential equation (4).

Curve *a* in fig. 6 represents the curve when the periodic terms have been removed. This has been done in a graphical manner, first with the eye, and then the curve was still further smoothed down by adjusting the differences in the way described by Sir JOHN HERSCHEL in the Memoirs of the Royal Astronomical Society, vol. v.

We thus obtain the following readings corresponding to each second of time from the beginning of the transit. The values of the successive differential coefficients have also been computed by means of the formula given above, which, when expanded, become

$$\frac{d\phi}{dt}=\Delta\phi-\tfrac{1}{2}\Delta^2\phi+\tfrac{1}{3}\Delta^3\phi-\tfrac{1}{4}\Delta^4\phi+\quad\ldots$$

$$\frac{d^2\phi}{dt^2}=\Delta^2\phi-\Delta^3\phi+\tfrac{11}{12}\Delta^4\phi-\quad\ldots$$

and

$$\frac{d^3\phi}{dt^3}=\Delta^3\phi-\tfrac{3}{2}\Delta^4\phi+\quad\ldots$$

VALUES of ϕ, $\frac{d\phi}{dt}$, $\frac{d^2\phi}{dt^2}$, and $\frac{d^3\phi}{dt^3}$ from fig. 6.

t.	ϕ.	$\frac{d\phi}{dt}$.	$\frac{d^2\phi}{dt^2}$.	$\frac{d^3\phi}{dt^3}$.	t.	ϕ.	$\frac{d\phi}{dt}$.	$\frac{d^2\phi}{dt^2}$.	$\frac{d^3\phi}{dt^3}$.
s.	mm.				s.	mm.			
0	0·0	+31·8	−6·5	+1·5	21	172·5	+1·6	—	—
1	28·8	26·0	5·1	1·1	22	174·1	1·6	—	—
2	52·5	21·5	3·8	0·7	23	175·7	1·6	—	—
3	72·2	18·0	3·2	0·5	24	177·3	1·5	—	—
4	88·7	15·1	2·6	0·3	25	178·8	1·5	—	—
5	102·5	12·6	2·2	0·2	26	180·3	1·2	—	—
6	114·0	10·4	1·8	+0·1	27	181·5	1·1	—	—
7	123·5	8·3	1·2	—	28	182·6	1·0	—	—
8	131·2	7·0	1·0	—	29	183·6	1·0	—	—
9	137·7	5·9	0·8	—	30	184·6	1·0	—	—
10	143·2	5·0	0·7	—	31	185·6	1·0	—	—
11	147·9	4·3	0·5	—	32	186·6	1·1	—	—
12	151·9	3·7	0·4	—	33	187·5	0·9	—	—
13	155·4	3·3	0·4	—	34	188·4	0·9	—	—
14	158·5	2·9	0·4	—	35	189·3	0·8	—	—
15	161·2	2·4	0·3	—	36	190·1	0·7	—	—
16	163·5	2·1	−0·1	—	37	190·8	0·7	—	—
17	165·5	1·9	—	—	38	191·5	0·7	—	—
18	167·4	1·8	—	—	39	192·2	0·6	—	—
19	169·2	1·7	—	—	40	192·8	0·6	—	—
20	170·9	+1·6	—	—	41	193·4	+0·6	—	—

$t.$	$\phi.$	$\frac{d\phi}{dt}.$	$\frac{d^2\phi}{dt^2}.$	$\frac{d^3\phi}{dt^3}.$	$t.$	$\phi.$	$\frac{d\phi}{dt}.$	$\frac{d^2\phi}{dt^2}.$	$\frac{d^3\phi}{dt^3}.$
s.	mm.				s.	mm.			
42	194·0	+0·6	—	—	95	193·8	−0·5	—	—
43	194·6	0·6	—	—	96	193·3	0·6	—	—
44	195·2	0·5	—	—	97	192·7	0·7	—	—
45	195·7	0·5	—	—	98	192·0	0·7	—	—
46	196·2	0·4	—	—	99	191·3	0·7	—	—
47	196·6	0·3	—	—	100	190·6	0·7	—	—
48	196·9	0·3	—	—	101	189·9	0·7	—	—
49	197·2	0·3	—	—	102	189·2	0·8	—	—
50	197·5	0·3	—	—	103	188·4	0·7	—	—
51	197·8	0·2	—	—	104	187·7	0·7	—	—
52	198·0	0·2	—	—	105	187·0	0·7	—	—
53	198·2	0·2	—	—	106	186·2	0·9	—	—
54	198·4	0·2	—	—	107	185·3	0·9	—	—
55	198·6	0·2	—	—	108	184·4	0·9	—	—
56	198·8	0·1	—	—	109	183·5	0·9	—	—
57	198·9	+0·1	—	—	110	182·6	1·0	—	—
58	199·0	0·0	—	—	111	181·6	1·1	—	—
59	199·0	0·0	—	—	112	180·5	1·2	—	—
60	199·0	0·0	—	—	113	179·3	1·2	—	—
61	199·0	+0·1	—	—	114	178·1	1·3	—	—
62	199·1	0·0	—	—	115	176·8	1·3	—	—
63	199·1	0·0	—	—	116	175·5	1·3	—	—
64	199·2	0·0	—	—	117	174·2	1·4	−0·1	—
65	199·2	+0·1	—	—	118	172·8	1·5	0·1	—
66	199·3	+0·1	—	—	119	171·3	1·6	−0·1	—
67	199·4	0·0	—	—	120	169·9	1·8	0·0	—
68	199·4	+0·1	—	—	121	168·0	2·0	−0·2	—
69	199·5	+0·1	—	—	122	166·0	2·1	0·2	—
70	199·6	0·0	—	—	123	163·9	2·2	0·2	—
71	199·6	0·0	—	—	124	161·7	2·3	0·2	−0·1
72	199·6	0·0	—	—	125	159·3	2·6	0·5	0·2
73	199·6	0·0	—	—	126	156·5	3·1	0·7	0·3
74	199·6	0·0	—	—	127	153·0	4·0	1·1	0·4
75	199·6	−0·1	—	—	128	148·4	5·3	1·6	0·5
76	199·5	0·1	—	—	129	142·2	7·2	2·2	0·6
77	199·4	0·1	—	—	130	133·8	9·7	2·9	0·7
78	199·3	0·1	—	—	131	122·5	−13·0	−3·7	−0·8
79	199·2	0·2	—	—	132	81·7	—	—	—
80	199·0	0·2	—	—	133	54·8	—	—	—
81	198·8	0·2	—	—	134	36·7	—	—	—
82	198·6	0·2	—	—	135	24·6	—	—	—
83	198·4	0·3	—	—	136	16·5	—	—	—
84	198·1	0·3	—	—	137	11·1	—	—	—
85	197·8	0·3	—	—	138	7·4	—	—	—
86	197·5	0·3	—	—	139	5·0	—	—	—
87	197·2	0·3	—	—	140	3·3	—	—	—
88	196·9	0·4	—	—	141	2·2	—	—	—
89	196·5	0·4	—	—	142	1·5	—	—	—
90	196·1	0·4	—	—	143	1·0	—	—	—
91	195·7	0·4	—	—	144	0·6	—	—	—
92	195·3	0·5	—	—	145	0·3	—	—	—
93	194·8	0·5	—	—	146	0·0	—	—	—
94	194·3	−0·5	—	—					

It will be observed that from the 5th second to the 125th the values of $\frac{d^3\phi}{dt^3}$ become so small that we may reasonably reject them. Similarly from the 16th to the 116th second the values of $\frac{d^2\phi}{dt^2}$ become too small to be of importance. Between the 130th and 131st second the sun passes off the instrument, and there is near here a point of inflexion on the curve, in consequence of which the method of computing the values of the differential coefficients by means of the differences is inappropriate. This is, of course, of no importance from our present point of view, as this part of the curve merely shows the motion of the couple when in process of cooling.

If now we multiply the values of the differential coefficients given in the above Table by the corresponding coefficients of equation (4) already obtained (p. 61), we find the following quantities:—

t.	$\frac{d^3\phi}{dt^3}$.	$0{\cdot}917\frac{d^2\phi}{dt^2}$.	$1{\cdot}241\frac{d\phi}{dt}$.	$0{\cdot}419\phi$.	$f(t)$.	
s.						
0	+1·5	−6·0	+39·5	0·0	35·0	41·9
1	1·1	4·7	32·3	12·1	40·8	48·8
2	0·7	3·5	26·7	22·0	45·9	54·9
3	0·5	2·9	22·3	30·3	50·2	60·0
4	0·3	2·4	18·7	37·2	53·8	64·4
5	0·2	2·0	15·6	42·9	56·7	67·8
6	+0·1	1·7	12·9	47·8	59·1	70·7
7	—	1·1	10·3	51·7	60·9	72·8
8	—	0·9	8·7	55·0	62·8	75·1
9	—	0·7	7·3	57·7	64·3	76·9
10	—	0·6	6·2	60·0	65·6	78·5
11	—	0·5	5·3	62·0	66·8	79·9
12	—	0·4	4·6	63·6	67·8	81·1
13	—	0·4	4·1	65·1	68·8	82·3
14	—	0·4	3·6	66·4	69·6	83·3
15	—	0·3	3·0	67·5	70·2	84·0
16	—	−0·1	2·6	68·5	71·0	84·9
17	—	—	2·4	69·3	71·7	85·8
18	—	—	2·2	70·1	72·3	86·5
19	—	—	2·1	70·9	73·0	87·3
20	—	—	2·0	71·6	73·6	88·0
21	—	—	2·0	72·3	74·3	88·9
22	—	—	2·0	72·9	74·9	89·6
23	—	—	2·0	73·6	75·6	90·4
24	—	—	1·9	74·3	76·2	91·1
25	—	—	1·9	74·9	76·8	91·9
26	—	—	1·5	75·5	77·0	92·1
27	—	—	1·4	76·0	77·4	92·6
28	—	—	1·2	76·5	77·7	92·9
29	—	—	1·2	76·9	78·1	93·4
30	—	—	1·2	77·3	78·5	93·9
31	—	—	1·2	77·8	79·0	94·5
32	—	—	1·4	78·2	79·6	95·2
33	—	—	1·1	78·6	79·7	95·3
34	—	—	1·1	78·9	80·0	95·7
35	—	—	1·0	79·3	80·3	96·1
36	—	—	0·9	79·7	80·6	96·4
37	—	—	0·9	79·9	80·8	96·7
38	—	—	0·9	80·2	81·1	97·0
39	—	—	0·7	80·5	81·2	97·1
40	—	—	+0·7	80·8	81·5	97·5

t.	$\frac{d^3\phi}{dt^3}$.	$0{\cdot}917\frac{d^2\phi}{dt^2}$.	$1{\cdot}241\frac{d\phi}{dt}$.	$0{\cdot}419\,\phi$.	$f(t)$.	
s.						
41	—	—	+0·7	81·0	81·7	97·7
42	—	—	0·7	81·3	82·0	98·1
43	—	—	0·7	81·5	82·2	98·3
44	—	—	0·6	81·8	82·4	98·6
45	—	—	0·6	82·0	82·6	98·8
46	—	—	0·5	82·2	82·7	98·9
47	—	—	0·4	82·4	82·8	99·0
48	—	—	0·4	82·5	82·9	99·2
49	—	—	0·4	82·6	83·0	99·3
50	—	—	0·4	82·7	83·1	99·4
51	—	—	0·2	82·9	83·1	99·4
52	—	—	0·2	83·0	83·2	99·5
53	—	—	0·2	83·0	83·2	99·5
54	—	—	0·2	83·1	83·3	99·6
55	—	—	0·2	83·2	83·4	99·8
56	—	—	0·1	83·3	83·4	99·8
57	—	—	+0·1	83·3	83·4	99·8
58	—	—	0·0	83·4	83·4	99·8
59	—	—	0·0	83·4	83·4	99·8
60	—	—	0·0	83·4	83·4	99·8
61	—	—	+0·1	83·4	83·5	99·9
62	—	—	0·0	83·4	83·4	99·8
63	—	—	0·0	83·4	83·4	99·8
64	—	—	0·0	83·5	83·5	99·9
65	—	—	+0·1	83·5	83·6	100·0
66	—	—	+0·1	83·5	83·6	100·0
67	—	—	0·0	83·5	83·5	99·9
68	—	—	+0·1	83·5	83·6	100·0
69	—	—	+0·1	83·6	83·7	100·1
70	—	—	0·0	83·6	83·6	100·0
71	—	—	0·0	83·6	83·6	100·0
72	—	—	0·0	83·6	83·6	100·0
73	—	—	0·0	83·6	83·6	100·0
74	—	—	0·0	83·6	83·6	100·0
75	—	—	−0·1	83·6	83·5	99·9
76	—	—	0·1	83·6	83·5	99·9
77	—	—	0·1	83·5	83·4	99·8
78	—	—	0·1	83·5	83·4	99·8
79	—	—	0·2	83·5	83·3	99·7
80	—	—	0·2	83·3	83·1	99·4
81	—	—	0·2	83·3	83·1	99·4
82	—	—	0·2	83·2	83·0	99·2
83	—	—	0·4	83·1	82·7	98·9
84	—	—	0·4	83·0	82·6	98·8
85	—	—	0·4	82·9	82·5	98·7
86	—	—	0·4	82·8	82·4	98·6
87	—	—	0·4	82·6	82·2	98·3
88	—	—	0·5	82·5	82·0	98·1
89	—	—	0·5	82·3	81·8	97·9
90	—	—	0·5	82·2	81·7	97·7
91	—	—	0·5	82·0	81·5	97·5
92	—	—	0·6	81·8	81·2	97·1
93	—	—	0·6	81·6	81·0	96·9
94	—	—	0·6	81·4	80·8	96·7
95	—	—	0·6	81·2	80·6	96·4
96	—	—	0·7	81·0	80·3	96·1
97	—	—	0·9	80·7	79·8	95·5
98	—	—	0·9	80·4	79·5	95·1
99	—	—	−0·9	80·2	79·3	94·9

t.	$\frac{d^3\phi}{dt^3}$.	$0{\cdot}917\frac{d^2\phi}{dt^2}$.	$1{\cdot}241\frac{d\phi}{dt}$.	$0{\cdot}419\phi$.	$f(t)$.	
s.						
100	—	—	- 0·9	79·9	79·0	94·5
101	—	—	0·9	79·6	78·7	94·1
102	—	—	1·0	79·3	78·3	93·7
103	—	—	0·9	78·9	78·0	93·3
104	—	—	0·9	78·6	77·7	92·9
105	—	—	0·9	78·3	77·4	92·6
106	—	—	1·1	78·0	76·9	92·0
107	—	—	1·1	77·6	76·5	91·5
108	—	—	1·1	77·3	76·2	91·1
109	—	—	1·1	76·9	75·8	90·8
110	—	—	1·2	76·5	75·3	90·1
111	—	—	1·4	76·1	74·7	89·4
112	—	—	1·5	75·6	74·1	88·6
113	—	—	1·5	75·1	73·6	88·0
114	—	—	1·6	74·6	73·0	87·3
115	—	—	1·6	74·1	72·5	86·7
116	—	—	1·6	73·5	71·9	86·0
117	—	- 0·1	1·7	73·0	71·2	85·2
118	—	0·1	1·9	72·4	70·4	84·2
119	—	0·1	2·0	71·8	69·7	83·4
120	—	0·1	2·2	71·2	68·9	82·4
121	—	0·2	2·5	70·4	67·7	81·0
122	—	0·2	2·6	69·6	66·8	79·9
123	—	0·2	2·7	68·7	65·8	78·7
124	- 0·1	0·2	2·9	67·8	64·6	77·3
125	0·2	0·5	3·2	66·7	62·8	75·1
126	0·3	0·6	3·8	65·6	60·9	72·8
127	0·4	1·0	5·0	64·1	57·7	69·0
128	0·5	1·5	6·6	62·2	53·6	64·1
129	0·6	2·0	8·9	59·6	48·1	57·5
130	0·7	2·7	12·0	56·1	40·7	48·7
131	- 0·8	- 3·4	- 16·1	51·3	31·0	37·1

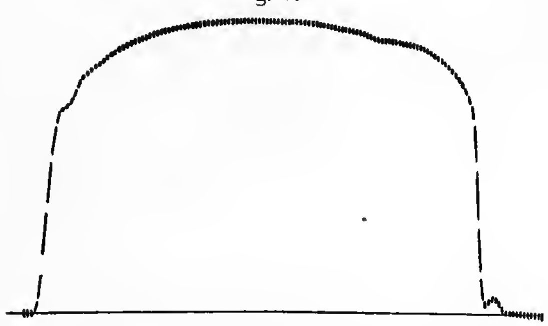
The quantities in the sixth column of this Table represent the sums of those in the four preceding columns, and are taken as the ordinates in curve b, fig. 6, while those in the last column are the same quantities multiplied by 100/83·6 so as to express the intensity of the heat at each point in percentages of that at the centre, and are represented by curve c in the same figure. With these quantities as ordinates the curve c has been laid down. In this reduced curve the ordinates represent the relative intensities of the radiation of heat. The time which the sun's diameter took in transit on the 12th October, 1890, the day on which the photograph was taken, was 130 seconds, but as the sun's image was 800 mm. in diameter, and the diameter of the aperture of the radio-micrometer was 2 mm., the interval between the first and last contact with the limb is longer than this by $\frac{1}{400}$th of the time of transit. The whole time between the first and last contacts is, therefore, 130·3 seconds.

We thus obtain the following results :—

D.	*t.*	*H.*	$\frac{1}{2}(H+H')$.	*H'.*	*t.*	*D.*
	s.				s.	
0	65·15	100·0	100·0	100·0	65·15	0
10	58·65	99·8	99·9	100·0	71·65	10
20	52·15	99·5	99·6	99·8	78·15	20
25	48·90	99·3	99·3	99·3	81·40	25
30	45·65	98·9	98·8	98·7	84·65	30
40	39·15	97·2	97·3	97·4	91·15	40
50	32·65	95·3	95·3	95·3	97·65	50
60	26·15	92·2	92·5	92·8	104·15	60
70	19·65	87·8	88·7	89·6	110·65	70
75	16·40	85·3	86·3	87·4	113·90	75
80	13·15	82·5	83·9	85·3	117·15	80
90	6·65	72·0	74·9	77·8	123·65	90
95	3·40	61·8	65·6	69·4	126·90	95
98	1·45	51·5	55·0	58·5	128·85	98
100	0·15	42·9	45·1	47·3	130·15	100

In the first and last columns is given the distance of a point from the sun's centre taking its radius as 100. The second and sixth contain the values of *t*. In the third and fifth are given the corresponding values of the heat radiated, the mean of which will be found in the fourth column. These values, down to $D=75$, agree in a remarkable manner with those obtained by LANGLEY, using the bolometer, and beyond that are in as close agreement with this distinguished observer's results as could well be expected in view of the fact that they are based on a single photograph.

Fig. 7.



This close agreement seems to show that the method is a most reliable one, and the ease and rapidity with which the curve may be obtained especially adapt it to the purpose of keeping a constant watch on the sun, so that any variation in the heat-absorbing power of the sun's atmosphere may be detected. For this purpose a series of weekly photographs would probably supply all the material necessary. Or if it were found advisable to take photographs every day that the sun was seen, even this would not put too severe a strain on the observing resources of an observatory.

It will be of interest to find how far the above results are borne out by another photograph shown in fig. 7. In this photograph, which is on a smaller scale than the last, the sun did not pass centrally across the aperture, but we have the means of determining what chord of the sun was actually observed by the length of time occupied in transit.

If we treat this curve similarly to the last, smoothing it down in the manner already explained, we obtain the following values of ϕ and its differential coefficients :—

VALUES of ϕ, $\frac{d\phi}{dt}$, &c., from curve *a*, fig. 8.

t.	ϕ.	$\frac{d\phi}{dt}$.	$\frac{d^2\phi}{dt^2}$.	$\frac{d^3\phi}{dt^3}$.	t.	ϕ.	$\frac{d\phi}{dt}$.	$\frac{d^2\phi}{dt^2}$.	$\frac{d^3\phi}{dt^3}$.
s.					s.				
1	3·3	+ 15·4	− 5·0	+ 3·0	45	79·6	+ 0.1	—	—
2	16·5	11·4	2·9	1·3	46	79·7	0·2	—	—
3	26·6	9·0	1·8	0·3	47	79·9	0.1	—	—
4	34·7	7·3	1·5	0·2	48	80·0	0·1	—	—
5	41·3	5·9	1·0	+ 0·1	49	80·1	0·1	—	—
6	46·7	4·9	0·9	—	50	80·2	0·1	—	—
7	51·4	4·0	0·8	—	51	80·3	0·1	—	—
8	54·9	3·3	0·7	—	52	80·4	+ 0·1	—	—
9	57·9	2·6	0·6	—	53	80·5	0·0	—	—
10	60·3	2·1	0·5	—	54	80·5	0·0	—	—
11	62·2	1·6	0·3	—	55	80·5	+ 0·1	—	—
12	63·7	1·4	0·2	—	56	80·6	+ 0·1	—	—
13	65·0	1·2	0·1	—	57	80·7	0·0	—	—
14	66·2	1·1	− 0·1	—	58	80·7	0·0	—	—
15	67·3	1·0	—	—	59	80·7	0·0	—	—
16	68·3	0·8	—	—	60	80·7	0·0	—	—
17	69·1	0·8	—	—	61	80·7	0·0	—	—
18	69·9	0·8	—	—	62	80·7	0·0	—	—
19	70·7	0·7	—	—	63	80·7	− 0·1	—	—
20	71·4	0·7	—	—	64	80·6	− 0·1	—	—
21	72·1	0·7	—	—	65	80·5	0·0	—	—
22	72·8	0·6	—	—	66	80·5	− 0·1	—	—
23	73·4	0·6	—	—	67	80·4	0·0	—	—
24	73·9	0·5	—	—	68	80·4	− 0·1	—	—
25	74·4	0·5	—	—	69	80·3	0·1	—	—
26	74·9	0.5	—	—	70	80·2	0·1	—	—
27	75·3	0·4	—	—	71	80·1	0·1	—	—
28	75·7	0·4	—	—	72	80·0	0·1	—	—
29	76·0	0·3	—	—	73	79·9	0·1	—	—
30	76·3	0·3	—	—	74	79·8	0·1	—	—
31	76·7	0·4	—	—	75	79·7	0·1	—	—
32	77·0	0·3	—	—	76	79·6	0·1	—	—
33	77·3	0·3	—	—	77	79·5	0·2	—	—
34	77·6	0·3	—	—	78	79·3	0·2	—	—
35	77·9	0·3	—	—	79	79·1	0·2	—	—
36	78·2	0·2	—	—	80	78·9	0·2	—	—
37	78·4	0·2	—	—	81	78·7	0·2	—	—
38	78·6	0·2	—	—	82	78·5	0·2	—	—
39	78·8	0·1	—	—	83	78·3	0·2	—	—
40	78·9	0·1	—	—	84	78·1	0·2	—	—
41	79·0	0·2	—	—	85	77·9	0·2	—	—
42	79·2	0·1	—	—	86	77·7	0·2	—	—
43	79·3	0·2	—	—	87	77·5	0·3	—	—
44	79·5	+ 0·1	—	—	88	77·2	− 0·3	—	—

t.	ϕ.	$\frac{d\phi}{dt}$.	$\frac{d^2\phi}{dt^2}$.	$\frac{d^3\phi}{dt^3}$.	t.	ϕ.	$\frac{d\phi}{dt}$.	$\frac{d^2\phi}{dt^2}$.	$\frac{d^3\phi}{dt^3}$.
s.					s.				
89	76·9	−0·3	—	—	110	67·6	−0.9	—	—
90	76·6	0·3	—	—	111	66·7	1·1	—	—
91	76·4	0·2	—	—	112	65·6	1·2	−0·1	—
92	76·1	0·3	—	—	113	64·3	1·5	0·2	—
93	75·9	0·2	—	—	114	62·6	2·0	0·6	−0·2
94	75·7	0·2	—	—	115	60·2	2·9	1·1	0·5
95	75·5	0·3	—	—	116	56.5	4·7	2·4	1·5
96	75·2	0·4	—	—	117	50·2	8·3	4·9	3·3
97	74·8	0·4	—	—	118	38·4	−16·2	−10·6	−6·5
98	74·4	0·3	—	—	119	21·9	—	—	—
99	74·1	0·4	—	—	120	13·7	—	—	—
100	73·7	0·4	—	—	121	9·2	—	—	—
101	73·3	0·5	—	—	122	6·3	—	—	—
102	72·8	0·5	—	—	123	4·4	—	—	—
103	72·3	0·5	—	—	124	3·1	—	—	—
104	71·8	0 5	—	—	125	2·0	—	—	—
105	71·3	0·6	—	—	126	1·1	—	—	—
106	70·7	0·7	—	—	127	0·5	—	—	—
107	70·0	0·7	—	—	128	0·2	—	—	—
108	69.3	0·8	—	—	129	0·0	—	—	—
109	68·5	−0·9	—	—					

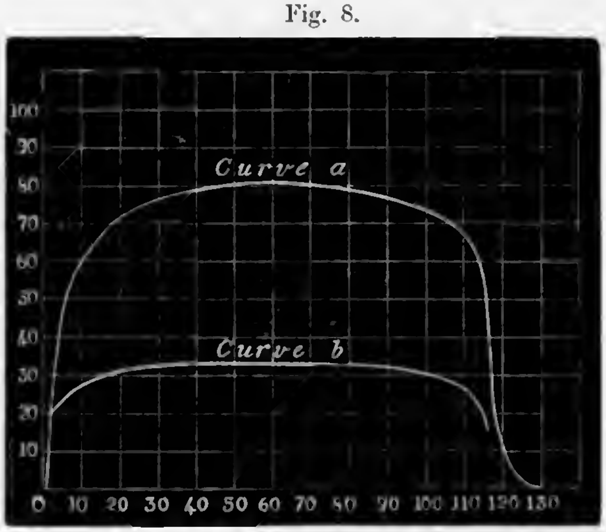
We thus obtain the following Table :—

t.	$\frac{d^3\phi}{dt^3}$.	$0{\cdot}917\,\frac{d^2\phi}{dt^2}$.	$1{\cdot}241\,\frac{d\phi}{dt}$.	$0{\cdot}419\,\phi$.	$f(t)$.	t.	$\frac{d^3\phi}{dt^3}$.	$0{\cdot}917\,\frac{d^2\phi}{dt^2}$.	$1{\cdot}241\,\frac{d\phi}{dt}$.	$0{\cdot}419\,\phi$.	$f(t)$.
s.						s.					
1	+3·0	−4·6	+19·1	+1·4	18·9	27	—	—	+0·5	+31·6	32·1
2	1·3	2·7	14·1	6·9	19·6	28	—	—	0·5	31·7	32·2
3	0·3	1·7	11·2	11·1	20·9	29	—	—	0·4	31·8	32·2
4	0·2	1·4	9·1	14·5	22·4	30	—	—	0·4	32·0	32·4
5	+0·1	0·9	7·3	17·3	23·8	31	—	—	0·5	32·1	32·6
6	—	0·8	6·1	19·6	24·9	32	—	—	0·4	32·3	32·7
7	—	0·8	5·0	21·5	25·7	33	—	—	0·4	32·4	32·8
8	—	0·6	4·1	23·0	26·5	34	—	—	0·4	32·5	32·9
9	—	0·6	3·2	24·3	26·9	35	—	—	0·4	32·6	33·0
10	—	0·5	2·6	25·3	27·4	36	—	—	0·2	32·8	33·0
11	—	0·3	2·0	26·1	27·8	37	—	—	0·2	32·8	33·0
12	—	0·2	1·7	26·7	28·2	38	—	—	0·2	32·9	33·1
13	—	0·1	1·5	27·2	28·6	39	—	—	0·1	33·0	33·1
14	—	−0·1	1·4	27·7	29·0	40	—	—	0·1	33·1	33·2
15	—	—	1·2	28·2	29·4	41	—	—	0·2	33·1	33·3
16	—	—	1·0	28·6	29·6	42	—	—	0·1	33·2	33·3
17	—	—	1·0	29·0	30·0	43	—	—	0·2	33·2	33·4
18	—	—	1·0	29·3	30·3	44	—	—	0·1	33·3	33·4
19	—	—	0·9	29·6	30·5	45	—	—	0·1	33·4	33·5
20	—	—	0·9	29·9	30·8	46	—	—	0·2	33·4	33·6
21	—	—	0·9	30·2	31.1	47	—	—	0·1	33·5	33·6
22	—	—	0·7	30·5	31·2	48	—	—	0·1	33·5	33·6
23	—	—	0·7	30·8	31·5	49	—	—	0·1	33·6	33·7
24	—	—	0·6	31·1	31·7	50	—	—	0·1	33·6	33·7
25	—	—	0·6	31·2	31·8	51	—	—	0·1	33·6	33·7
26	—	—	+0·6	+31·4	32·0	52	—	—	+0·1	+33·7	33·8

t.	$\frac{d^3\phi}{dt^3}$.	$0·917\,\frac{d^2\phi}{dt^2}$.	$1·241\,\frac{d\phi}{dt}$.	$0·419\,\phi$.	$f(t)$.	t.	$\frac{d^3\phi}{dt^3}$.	$0·917\,\frac{d^2\phi}{dt^2}$.	$1·241\,\frac{d\phi}{dt}$.	$0·419\,\phi$.	$f(t)$.
s.						s.					
53	—	—	0·0	+ 33·7	33·7	86	—	—	− 0·2	+ 32·6	32·4
54	—	—	0·0	33·7	33·7	87	—	—	0·4	32·5	32·1
55	—	—	+ 0·1	33·7	33·8	88	—	—	0·4	32·3	31·9
56	—	—	+ 0·1	33·8	33·9	89	—	—	0·4	32·2	31·8
57	—	—	0·0	33·8	33·8	90	—	—	0·4	32·1	31·7
58	—	—	0·0	33·8	33·8	91	—	—	0·2	32·0	31·8
59	—	—	0·0	33·8	33·8	92	—	—	0·4	31·9	31·5
60	—	—	0·0	33·8	33·8	93	—	—	0·2	31·8	31·6
61	—	—	0·0	33·8	33·8	94	—	—	0·2	31·7	31·5
62	—	—	0·0	33·8	33·8	95	—	—	0·4	31·6	31·2
63	—	—	− 0·1	33·8	33·7	96	—	—	0·5	31·5	31·0
64	—	—	− 0·1	33·8	33·7	97	—	—	0·5	31·3	30·8
65	—	—	0·0	33·7	33·7	98	—	—	0·4	31·2	30·8
66	—	—	− 0·1	33·7	33·6	99	—	—	0·5	31·0	30·5
67	—	—	0·0	33·7	33·7	100	—	—	0·5	30·9	30·4
68	—	—	− 0·1	33·7	33·6	101	—	—	0·6	30·7	30·1
69	—	—	0·1	33·6	33·5	102	—	—	0·6	30·5	29·9
70	—	—	0·1	33 6	33·5	103	—	—	0·6	30·3	29·7
71	—	—	0·1	33·6	33·5	104	—	—	0·6	30·1	29·5
72	—	—	0·1	33·5	33·4	105	—	—	0·7	29·9	29·2
73	—	—	0·1	33·5	33·4	106	—	—	0·9	29·6	28·7
74	—	—	0·1	33·4	33·3	107	—	—	0·9	29·3	28·4
75	—	—	0·1	33·4	33·3	108	—	—	1·0	29·0	28·0
76	—	—	0·1	33·4	33·3	109	—	—	1·1	28·7	27·6
77	—	—	0·2	33·3	33·1	110	—	—	1·1	28·3	27·2
78	—	—	0·2	33·2	33·0	111	—	—	1·4	27·9	26·5
79	—	—	0·2	33·1	32·9	112	—	− 0·1	1·5	27·5	25·9
80	—	—	0·2	33·1	32·9	113	—	0·2	1·9	26·9	24·8
81	—	—	0·2	33·0	32·8	114	− 0·2	0·6	2 5	26·2	22·9
82	—	—	0·2	32·9	32·7	115	0·5	1·0	3·6	25·2	20·1
83	—	—	0·2	32·8	32·6	116	1·5	2·2	5·8	23·7	14·2
84	—	—	0·2	32·7	32·5	117	3·3	4·5	10·3	21·0	2·9
85	—	—	− 0·2	+ 32·6	32·4	118	− 6·5	− 10·0	− 20·1	+ 16·1	—

These results are exhibited in the curve *b*, fig. 8.

Fig. 8.



Now we see from the way the curve in fig. 7 begins to rise that the sun must have come on the instrument at $0^s·7$ q.p., and comparing the curve with that obtained on 12th October, it appears to have left the instrument at $116^{s}\cdot1$ q.p. We have thus $115^{s}\cdot4$ as the time of transit of the chord observed. If d be the distance of the middle point of this chord from the centre of the sun, expressed in seconds, we have (since the sun's diameter was 128^{s})

$$d = \sqrt{(64)^2 - (57\cdot7)^2} = 27^{s}\cdot69.$$

Also if D is as before the distance of any point on the chord from the centre of the sun expressed in percentages of the sun's radius, since the middle of the transit took place at $58^{s}\cdot4$, we have

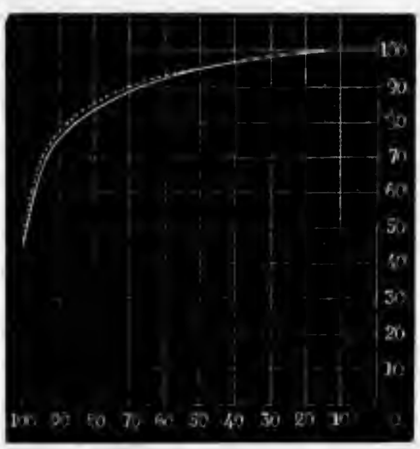
$$t = 58\cdot4 \pm \sqrt{4096\, D^2 - 766\cdot74},$$

by means of which we can find the times corresponding to various values of D for comparison with the results previously obtained. We thus get the following results :—

D.	t_1.	t_2.	H_1.	H_2.	H' Mean.	2·86 H.
50	42·4	74·4	33·3	33·3	33·3	95·3
60	31·8	85·0	32·7	32·4	32·5	92·9
70	23·2	93·6	31·5	31·5	31·5	90·1
75	19·2	97·6	30·6	30·8	30·7	87·8
80	15·3	101·5	29·5	30·3	29·9	85·5
90	7·9	108·9	26·4	27·6	27·0	77·2
95	4·3	112·5	22·8	24·3	23·5	67·2
98	2·1	114·7	19·7	20·9	20·3	58·1
100	0·7	116·1	[18·5]	[13·3]	[15·9]	[45·5]

These results are exhibited graphically in fig. 9, where they are represented by the dotted curve. The continuous curve in the same figure represents the results already obtained from the photograph of 12th October.

Fig. 9.



Considering the comparatively small scale on which the curve in fig. 7 was taken, and the degree of uncertainty attaching to the exact length of the chord observed, these results seem to agree fairly well. The discrepancies that remain, however, show that it would be well to determine for each photograph the exact instant at which the sun enters and leaves the instrument, which can easily be done automatically by putting a reflector behind the couple so as to reflect a beam of sunlight on to the plate all the time the sun is in transit.

Absorption in the Sun's Atmosphere.

Although the results given on page 67 are of course only provisional, it will be of interest to calculate from them the amount of heat intercepted by the sun's atmosphere.

In the tenth book of the *Mécanique Céleste*, No. 12, Laplace shows that the law of extinction of the sun's rays in passing through an atmosphere is

$$\frac{dv}{v} = -\frac{H d\theta}{\sin\theta},$$

where v is the intensity of the ray, $d\theta$ the element of the refraction, θ the zenith distance of the radiating body, and H a constant. Hence if V be the intensity of the beam before entering the atmosphere, we have

$$\log\frac{V}{v} = \frac{H\delta\theta}{\sin\theta},$$

where $\delta\theta$ is the whole refraction. This formula is also employed by Forbes in his Memoir "On the Extinction of the Solar Rays in passing through the Atmosphere," in the 'Philosophical Transactions of the Royal Society,' 1842. In applying this to the absorption of heat in the sun's own atmosphere as Laplace has done in the next section to that referred to above, we assume that the absorption in its atmosphere obeys *a law similar to the terrestrial absorption, differing only in the constant.* In this case θ will represent the angular distance from the sun's centre.

If v_0 is the intensity of the heat observed at the centre, since for small zenith distances we may put $\delta\theta = a \tan\theta$, we have

$$\log\frac{V}{v_0} = Ha,$$

and consequently

$$\log\frac{v}{v_0} = H\left(a - \frac{\delta\theta}{\sin\theta}\right),$$

and, therefore,

$$\log\frac{v}{v_0} = \left(1 - \frac{\delta\theta}{a\sin\theta}\right)\log\frac{V}{v_0},$$

from which $\frac{V}{v_0}$ may be calculated.

In this we have assumed that V, the intensity of the heat before absorption, is the same at all points of the disc. LAPLACE, on the other hand, considers that if the sun's atmosphere were removed it would be found to radiate more heat from the regions near the limb than from the centre. He says: "Une portion du disque du soleil transportée, par la rotation de cet astre, du centre vers les bords du disque, doit y paraître avec une lumière d'autant plus vive qu'elle est aperçue sous un plus petit angle; car il est naturel de penser que chaque point de la surface du soleil renvoie une lumière égale dans tous les sens." He accordingly assumes that the intensity varies as sec θ. It is, however, more natural to suppose that the sun's surface would behave similarly to other radiating surfaces, in which the inclination of the surface will not increase the intensity, and this is borne out to some extent by the following Table in which we give the values of $\frac{V}{v_0}$ calculated from the observed values of $\frac{v}{v_0}$ on both hypotheses for every tenth degree of θ:—

θ	$\frac{V}{v_0}$: $V = V_0 \sec \theta$.	$\frac{V}{v_0}$: $V = V_0$.	D.
10°	3·425	1·178	17·36
20	3·518	1·325	34·20
30	3·461	1·365	50·00
40	3·259	1·362	64·28
50	2·798	1·324	76·60
60	2·561	1·277	86·60
70	2·128	1·223	93·97
80	1·647	1·147	98·48
90	—	1·023	100·00

This Table shows that down to $D=94$, or for the most reliable part of the curve, the different values obtained on the hypothesis of a uniform radiation agree fairly well together, while even beyond this they compare favourably with those in the second column of the Table. The gradual rise and fall in these values as θ increases seem to show that the law we have assumed does not exactly represent the radiation, and these results might be brought better together by assuming that, if V_0 is the intensity at the centre before absorption, $V = V_0 (1-b \sin 2\theta)^{\sec \theta - 1}$, in which $b=0·322$, or better still, if we take $V = V_0 [1-b \sin (2\theta+a)]^{\sec \theta - 1}$, where $b=0·235$, and $a=20°$. Such a law as this would seem to indicate a radiating stratum of limited extent whose temperature increases towards the centre, so that the colder layers on the outside would absorb some of the intenser radiation coming from the interior. It would, however, be unsafe on such slender foundation to adopt such an artificial law of radiation as that represented by the equation just given, and we have preferred, for the present at least, to assume that $V = V_0$ all over the disc.

If we take the mean of the results in the third column of the Table, omitting the last, which rests on observations at the limb, we find $\frac{V}{v_0} = 1{\cdot}275$. We thus see that a vertical passage through the sun's atmosphere diminishes the intensity of the heat by about $\frac{1}{5}$th of its amount, while at the limb nearly $\frac{2}{3}$rds of it is lost.

In order to calculate how much the total heat is reduced we observe that on the assumption of uniform radiation

$$\frac{v}{v_0} = \left(\frac{V_0}{v_0}\right)^{1 - \frac{\delta\theta}{a \sin\theta}}.$$

Also the total radiation is, if we put the radius of the sun equal to unity,

$$2\pi \int_0^{\frac{\pi}{2}} \sin\theta \cos\theta \, d\theta . v.$$

We may also assume $\delta\theta = a \tan\theta$, which will represent the refraction very nearly except for values of θ corresponding to points very close to the limb from which but a very small proportion of the heat comes. If now we put $v_0 = 1$, $x = \cos\theta$, and $e^f = V$, this expression becomes

$$2\pi \int_0^1 e^{f\left(1 - \frac{1}{x}\right)} x dx = 2\pi e^f \int_0^1 e^{-\frac{f}{x}} x dx.$$

But since the total heat which we should receive if there were no absorption is πe^f, we find as the proportion (R) of the heat penetrating the atmosphere

$$R = 2 \int_0^1 e^{-\frac{f}{x}} x dx.$$

The expression found by LAPLACE on the same assumption as before with regard to the local intensity before absorption is $\int_0^1 e^{-\frac{f}{x}} dx$, which he shows can be reduced to the continued fraction,

$$\cfrac{qe^{-f}}{1 + \cfrac{2q}{1 + \cfrac{q}{1 + \cfrac{3q}{1 + \cfrac{2q}{1 + \cfrac{4q}{1 + \cfrac{3q}{1 + \&c.,}}}}}}}$$

in which $q = \frac{1}{f}$.

But integrating by parts we find

$$2\int e^{-\frac{f}{x}}xdx = e^{-\frac{f}{x}}x^2 - f\int e^{-\frac{f}{x}}dx.$$

Hence

$$R = e^{-f} - \cfrac{e^{-f}}{1+\cfrac{2q}{1+\cfrac{q}{1+\cfrac{3q}{1+\cfrac{2q}{1+\&c.}}}}}$$

We have already found $e^f = 1{\cdot}275$, whence we obtain $f = 0{\cdot}2429$, $q = 4{\cdot}1169$, and $e^{-f} = 0{\cdot}7843$, and substituting these values in the expression for R we find that it lies between 0·668 and 0·623, or that more than one-third of the sun's heat is intercepted by his atmosphere.

In conclusion, we may observe that if photographs are from time to time taken in the manner here described, and are all reduced in an exactly similar manner, even though the assumptions made with regard to the laws governing the radiation at the sun's surface do not *exactly* represent the real state of affairs [and that they are not in error to any great extent seems clear from the agreement found in the values of $\frac{V}{v_0}$ or e^f], still any variation in the resulting values of R would represent a real change in the absorbing power of the sun's atmosphere, and would thus enable us to detect an alteration in the state of the solar surface which would be wholly masked in direct observations by the varying conditions of the earth's atmosphere under which the observations would necessarily be conducted.

On the Effect of Pressure of the Surrounding Gas on the Temperature of the Crater of an Electric Arc Light.

Preliminary Notes of Observations made at Daramona, Streete, Co. Westmeath.

By W. E. WILSON.

Proceedings of the Royal Society, Vol. 58, 1895.

Communicated by Professor FITZGERALD, *F.R.S.*

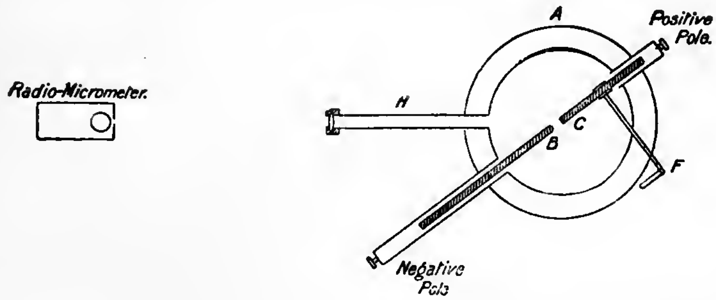
OF late years it has often been assumed that the temperature of the crater forming the positive pole of the electric arc is that of the boiling of carbon. The most modern determinations give this point as about 3300°—3500° C.

Solar physicists have thought that the photosphere of the sun consists of a layer of clouds formed of particles of solid carbon. As the temperature of these clouds is certainly not below 8000° C., it seems very difficult to explain how carbon can be boiling in the arc at 3500° and yet remain in the solid form in the sun at 8000°. Pressure in the solar atmosphere seemed to be the most likely cause of this, and yet, from other physical reasons, this seemed not probable.

In order to investigate whether increased pressure in the gas surrounding an electric arc would raise the temperature of the crater, I had an apparatus made by the Cambridge Instrument Company. It consists of a strong cast-iron box, which was tested by hydraulic pressure to 2000 lbs. per square inch. In the following plan, A is the box, B and C are the two carbon poles enclosed in steel tubes. The negative carbon was kept in position against a copper ring by a spiral spring behind it. The positive carbon was hand-fed by a friction roller, which was moved by a handle, F, outside the box. A steel tube, H, was screwed into the box at such an angle that, by looking down it, we could see well into the crater of the positive pole. The end of this tube is closed by a glass lens, which formed an image of the crater at a distance of 80 cm.

A Boys's radio-micrometer, with its aperture reduced to about 2 mm. diameter, was so placed on the pier in the laboratory that the image of the crater fell on its small aperture. The instrument thus gave deflections proportional to the radiation coming from the crater. The current was supplied from a battery of accumulators, giving an E.M.F. of 110 volts. Suitable resistances of platinoid wire were put in the circuit, so that the current could be varied from 40 to 10 ampères. An ammeter was also in circuit, and the poles of the arc were connected to a voltmeter.

The gas used was nitrogen, and the pressure was got by connecting the box by a copper pipe with the valve of a 20-foot steel cylinder filled with the gas at a pressure of 120 atmospheres. A T-joint on the copper pipe was connected with a BOURDON pressure-gauge, which showed the pressure in the box at any moment.



The method of experimenting was first to start the arc with the pressure in the box at that of the atmosphere. The image of the brightest part of the crater was thrown on the aperture of the radio-micrometer, and a series of observations taken of the deflections of the instrument. The pressure was then gradually increased and the maximum deflections observed. As the pressure rises the resistance of the arc increases, and, in order to keep the same current flowing, the resistance in the circuit was reduced. It soon became evident that, even with moderate pressures of about 5 atmospheres, the temperature of the crater had fallen. This was not only shown by the reduction in the deflections of the radio-micrometer, but also by the fall in brilliancy of the image of the crater to the eye. The pressure was then increased to about 20 atmospheres, and the brilliancy of the crater fell to a dull red colour. These experiments were repeated several times, and always with the same results.

I then tried the effect of reducing the pressure in the box by means of an air-pump, but as some of the glands in the box were only intended for an internal pressure, I found it impossible to get a good vacuum; yet by keeping the pump at work, and thus getting a moderate vacuum, I found the radiation of the crater to be much greater than at the atmospheric pressure.

The temperature of the crater seemed very sensitive to any sudden diminution of pressure in the gas. If the blow-off valve was suddenly opened, the brilliancy of the crater fell so much that it became nearly invisible. When the box was being exhausted by the air-pump, although the temperature of the crater was rising as the vacuum improved, yet at each stroke of the pump the eye could see a distinct falling-off of brilliancy in the image.

It was thought that the diminution of brilliancy might be due to smoke inside the box, but on looking through the window everything was seen sharply defined, also the gas as it issued from the blow-off was perfectly clear. The arc was also kept burning for some time in the box at the atmospheric pressure, but the image remained quite clear, and the inside of the box seemed quite free of smoke.

From these experiments it would seem as if the temperature of the crater, like that of a filament in a incandescent lamp, depends on how much it is cooled by the surrounding atmosphere, and not on its being the temperature at which the vapour of carbon has the same pressure as the surrounding atmosphere. That carbon volatilizes in some form at comparatively low temperatures seems likely, from the way in which the carbon of incandescent lamp filaments is transferred to the glass. The pressure of the vapour of carbon in the arc may consequently be very small; and, further, it would seem that the supposition of high pressures in the solar photosphere, which has been referred to in the beginning of this paper, is not borne out by these experiments, and that carbon may exist there in the solid form at very high temperatures although the pressures are comparatively low.

The experiments on high pressures were conducted on several occasions. On the last occasion, in addition to repeated former experiments, the experiments on reduced pressures were performed, and I then had the great advantage of the presence, advice, and assistance of my friends Professor MINCHIN and Professor G. F. FITZGERALD. The later series of experiments entirely confirmed my former ones.

On the Effect of Pressure in the Surrounding Gas on the Temperature of the Crater of an Electric Arc. Correction of Results in former Paper.

By W. E. Wilson, *F.R.S.*, *and* G. F. Fitzgerald, *F.R.S.*

Proceedings of the Royal Society, Vol. 60, 1896.

In May, 1895, a preliminary paper by one of the authors was read at the Royal Society, in which is described the apparatus used for these experiments, and the results which were then obtained.

The primary object of this research was to determine, if possible, whether the temperature of the crater in the positive carbon varies when the pressure in the surrounding gas is changed.

It has been suggested that the temperature of the crater is that of boiling carbon. The most modern determinations give this temperature of the crater as about 3300°—3500° C.*

If this is the true boiling point of carbon, it is then clear that solar physicists must find some other substance than solid carbon particles to form the photospheric clouds in the sun, as the temperature of this layer is most probably not below 8000° C., † unless, indeed, the pressure in the solar atmosphere is sufficient to raise the boiling point of carbon to about this temperature (see p. 76). It is in order to throw some light on this subject that these experiments were undertaken.

The gas used in our first experiments was nitrogen, and we found that the radiation from the crater fell off in a most remarkable manner whenever the pressure was raised in the box surrounding the arc. This falling-off was not due to any very large extent to visible cloud or smoke, and the crater seemed so much reduced in temperature as to glow with only a red heat. This seemed to show that the temperature of the crater depends on how much it is cooled by the surrounding gas, and not on its being the temperature at which the vapour of carbon has the same pressure as the surrounding atmosphere.

It was found that we were limited to pressures not exceeding about 20 atmos., as at this pressure we could not withdraw the negative carbon sufficiently to see into the crater without the arc breaking. We were then only able to obtain a current from a battery of accumulators which had an E.M.F. of 110 volts. Since

* Wilson and Gray, 'Roy. Soc. Proc.,' vol. 58; Violle, 'Journ. de Phys.,' 3rd series, vol. 2, 1893, p. 545.

† Wilson and Gray, 'Phil. Trans.,' A, vol. 185, 1894.

then we obtained a Crompton dynamo which could give 300 volts and 15 ampères, and which was driven by a turbine.

From the great difficulty of obtaining a sufficient quantity of pure nitrogen under pressure, we obtained a 20 ft. cylinder of air compressed to 120 atmos. With this we tried a series of experiments, and these at first seemed to corroborate our former ones, in which we used nitrogen, but we found that at any rate some of the radiation, and possibly a great deal of it, was cut off by the formation of what appeared to be red fumes of NO_2. We found no absorption from this cause so long as the pressure was nearly atmospheric, but at about 100 lbs. pressure this gas was formed with great rapidity, and undoubtedly cut off a great deal of the radiation. We easily confirmed our belief in the presence of this gas by its well-known absorption spectrum.

Lest heat dissociation might cause an apparent increase in the amount of NO_2, we tried heating some of this gas in a flask. We observed that when hot the brown fumes became golden yellow, and the absorption bands nearly disappeared, so that the heating could *not* have been the cause of the apparently enormous production of NO_2 at high pressure.

We next tried whether oxygen blown into the arc would burn up the carbons, but found it did not do so to any serious extent, and so tried the arc in a compressed atmosphere of this gas.

The arc burned very nicely indeed in the oxygen, the carbons keeping a good shape, and a very steady crater. The oxygen was, however, so contaminated with nitrogen that at high pressure enormous quantities of NO_2 were again formed, so that we could not proceed further with the radiation experiments. The arc was a bright blue bead, about the size of a pea, and the spectrum was a beautiful banded one.

From these results we concluded that the reduction of radiation, and red-hot appearance of the crater in the former experiments in nitrogen, were due to its being contaminated with oxygen, and to the large quantities of NO_2 which were formed by the arc when under pressure.

We next tried the arc in hydrogen. The gas was obtained as pure, but contained hydrocarbons as an impurity, possibly from having been compressed into a cylinder which had previously been charged with coal-gas.

The arc in hydrogen at atmospheric pressures was a long, thin flame, that moved as far up the carbons as possible; especially on the negative carbon it walked up a cm. along the cone. It went so far that it fused the copper ring that held the negative carbon, and we had to replace it by an iron wire lashing. It was very unsteady, and trees of soot and a deposit of hard graphitic carbon formed on this positive carbon as if there were electrolysis of the hydrocarbon, and carbon were electro-negative compared with hydrogen. This growth took place all round the crater, while there was no tendency for anything to grow on the negative carbon.

The arc was only 5—6 mm. wide, and sometimes over 2 cm. long. There was a green outer flame, with a bright red line not a mm. wide down the middle of it. Where it impinged on the negative carbon there was a bright red flame from the middle of the bright spot on the carbon. The outer greenish part seemed to give much the same spectrum as the green cone in a Bunsen burner, while the red flame and line was undoubtedly glowing hydrogen. As we saw the C and F hydrogen lines very distinctly, the red C line being dazzlingly bright and not nearly so wide as in a coil spark at atmospheric pressure whenever the image of the red part of the arc was thrown on the slit of the spectroscope, the appearance was quite like that of a solar prominence.

The end of the positive carbon was pitted into a number of craters, as the arc was very unsteady, and when the pressure was raised it was almost impossible to keep an arc going, partly because the arc broke when it was elongated the least bit, and partly because a complete lantern of soot trees grew all round the crater, and seemed to short-circuit the arc from time to time.

The arc being very unsteady, no satisfactory reading of the voltage and current was possible. At from 60 to 80 lbs. pressure the voltage varied from 60 to 80, and the ampères kept continually varying from 15 to 20. At 40 lbs. with 20 ampères the volts varied from 50 to 60. The crater was not well developed, so that the radiation observation, even at low pressures, was not very satisfactory, while at high pressures the arc was too short to see into the crater at all, and the lantern of soot trees hid a considerable length, 3 or 4 mm. of the negative carbon besides. The radio-micrometer gave 440 divisions with a good arc in air, and 380 with a moderately good crater in hydrogen. But this difference is no greater than would often occur with a good and moderately good crater, so that there is not any proof of a difference of temperature due to cooling power of hydrogen. These experiments showed us that it was quite hopeless to get any measures of radiation under pressure with hydrogen.

We finally tried an atmosphere of carbon dioxide. We used a cylinder of liquid CO_2, which was connected to our arc box by a copper tube and stop valve. The arc burned fairly well in this gas, and, except for the difficulty of getting a sufficiently long arc at pressures above 150 lbs., some pretty satisfactory measures of radiation were obtained. We found that whenever the pressure was suddenly reduced, there was a fog formed in the box, which cut off the light enormously. Also by looking down the steel tube, which is closed at its end by a lens, we could see powerful convection currents in the gas which scattered a lot of light. At high pressure the refraction due to these currents prevented any sort of an image of the crater being formed while the pressure was varying. While the pressure was steady a good image could be formed. This tube is nearly 3 ft. in length, and only $\frac{1}{2}$ in. in bore, and it would naturally take time for the gas to settle down throughout its length. We propose to have this tube removed, and the aperture in the box closed by a strong piece of plain glass, and to form an image of the carbons by a lens placed at a suitable distance outside. This we expect will remove the difficulty arising from these convection currents.

The result of all these experiments so far is that it would require more evidence than we have been able to get to affirm that either the temperature of the crater of the arc is raised or lowered by pressure. We got some very concordant observations, which showed the temperature to be lowered with pressure, and in which at the time we could see no evidence of absorption by fog, but then, at other times, there was undoubtedly absorption from this cause. We certainly got no evidence that there is any appreciable increase of temperature. When the arc was started in the gas at a low pressure and then the pressure was raised, the radiation at the low pressure was greater than at a high pressure; but when the arc was started first in the gas at high pressure and then the pressure reduced, the radiation was rather higher in the gas at high pressure. From all this we concluded that the greater part of the differences we were observing were due to the absorption of the light in the long tube already mentioned, which increased the longer the arc was kept burning, and was probably greater at high than at low pressures. The best observations were made with variations of pressure from 15 up to 100 lbs. per sq. in., and there seems very little evidence of much change of radiation with this change of from 1 up to between 6 and 7 atmos.

The whole question is surrounded with great difficulty. If the carbon be really in equilibrium with its own vapour at the temperature of the crater and at the pressure of the surrounding atmosphere, some relation must exist between the change in pressure and change in temperature of the crater. If we knew the latent heat of volatilization of carbon, we should be able to calculate the change of temperature from the well-known thermodynamic formula

$$\frac{\delta T}{T} = \frac{\Delta v}{\lambda} . \delta p$$

Δv can certainly be approximately determined on the supposition that the absolute temperature of the crater is fifteen times the absolute temperature of the freezing point, *i.e.*, 3800. We thus get for gaseous carbon $\Delta v = 10^4$, *q.p.*, at this temperature. For 1 atmos. $\delta p = 10^6$, *q.p.*, so that

$$\frac{\delta T}{T} = \frac{10^{10}}{\lambda} .$$

Hence, unless the latent heat of carbon be enormously great compared with that of other substances, $\delta T/T$ will be considerable. If λ be as great as the latent heat of vaporization of carbon given by TROUTON's law, *i.e.*, about 4000 calories, or

$16 \cdot 8 \times 10^{10}$ ergs, $\delta T/T$ would be about $\frac{1}{17}$, and δT would be nearly 220° C. for each atmosphere, and a change of pressure of about 18 atmos. would raise the temperature of the crater to that estimated for the sun. The corresponding increase of radiation would be very great, for the radiation varies, at least approximately, as the fourth power of the absolute temperature. This would lead one to expect that the radiation would be nearly doubled for each 4 atmos. added. Such an increase as this certainly does not take place, so that we may conclude that either the temperature of the crater is not that of boiling carbon, or else that the latent heat of volatilization of carbon is very considerably greater than that calculated from TROUTON'S law. Even though this latent heat were as great as the heat of combustion of C to CO_2, *i.e.*, 7770, there would be an increase of about 70 per cent. in the radiation for an increased pressure of 6 atmos. Such an enormous latent heat is unprecedented, and yet our experiments would, almost certainly, have shown such an increased radiation as this. So far, therefore, the experiments throw considerable doubt on the probability that it is the boiling point of carbon that determines the temperature of the crater. It might be questioned whether there is energy enough in the current to do all this work, but upon an extravagant estimate of the amount of carbon volatilized in the crater, it appears that there is more than a hundred times as much energy supplied by the current as would be required for volatilizing the carbon, even though its latent heat were as great as the heat of combustion of C into CO_2.

There is another considerable difficulty in the theory of the temperature of the crater being that of boiling carbon arising from the slowness of evaporation. The crater on mercury is dark, but then it volatilizes with immense rapidity, and, the supply of energy by the current being more than 100 times that required merely for evaporation, there seems very little reason why even a considerable difference in latent heat should make any sensible difference in the rate of evaporation of mercury and carbon, especially as, at the same temperature, the diffusion of carbon vapour is nearly three times as fast as that of mercury vapour and the temperature immensely higher.

We would, in conclusion, call attention to a cause of opacity in the solar atmosphere that is illustrated by the effect of convection currents in the long tube we were observing at high pressures; these convection currents behaved just like snow, or any other finely divided transparent body immersed in another of different refractive index. Light trying to get through is reflected backwards and forwards in every direction, until most of it gets back by the way it came. The consequence was that even the electric arc light was unable to penetrate the tube at high pressure, when these convection currents were active. The only light that came out of the tube was the feeble light outside, which was returned to us by reflection at the surfaces of these convection currents. In a similar manner we conceive that any part of the solar atmosphere which is at a high pressure, and where convection currents, or currents of different kinds of materials, are active, would reflect back to the sun any radiations coming from below, and reflect to us only the feeble radiations coming from interplanetary space. In his paper on "The Physical Constitution of the Sun and Stars" ('Roy. Soc. Proc.,' No. 105, 1868), Dr. STONEY called attention to an action of this kind that might be due to clouds of transparent material, like clouds of water on the earth, but in view of the high solar temperature it seems improbable that any body, except, perhaps, carbon, could exist in any condition other than the gaseous state in the solar atmosphere; so that it seems more probable that sun-spots are due, at least partly, to reflection by convection streams of gas, rather than by clouds of transparent solid or liquid particles.

A Method of recording the Transits of Stars by Photography.

By W. E. Wilson.

Monthly Notices of Royal Astronomical Society, Vol. 50, *No.* 2.

I wish to bring before the notice of the Society a method by which the transits of stars can be recorded by photography, and the personal errors eliminated. If a sensitive photographic plate is placed in the focus of a transit instrument, close behind the wires, and the image of a star of suitable magnitude allowed to transit across it, the result is a straight black line on developing the plate. If instead of having the plate fixed, we have it so arranged that it can be given a small up and down motion each second, the result on the plate is a broken line, thus ‾_‾_‾_‾_‾_, the breaks on which are equal to seconds of time. The motion is given to the plate by an electro-magnet driven by a current sent by the observatory clock. During or after the transit, a light from a small electric lamp is allowed to fall through the object-glass on the plate for a few seconds. This gives an impression of the wires superposed on the star transit. With a rough apparatus I find the time of transit can be recorded to $\frac{1}{4}$ second, and I believe with some care the time could be taken to a very small fraction of a second.

I used for these experiments the 4-inch finder of my 2-foot reflector. The object-glass was not corrected for photographic rays, and the star trail on the plate was therefore not as fine as it would be with a suitable objective.

December 12*th*, 1889.

Note.—This method was thought of and put into use at the Georgetown College Observatory at the same time as here. Since then in the hands of Rev. F. Hagen, S.J., it has given very good results.

W. E. Wilson, *February*, 1899.

A New Photographic Photometer for determining Star Magnitudes.

By W. E. Wilson, *F.R.A.S.*

Monthly Notices of the Royal Astronomical Society, Vol. LII., No. 3.

I would like to bring before the notice of the Society the design of an instrument which I think will be of use in stellar photography, and especially in determining photographic magnitude of stars.

The instrument consists of a photographic plate and holder (6½ in. × 1 in.), moving in a slide in the direction of its greatest length. A spiral spring tends to pull the holder to one end of the slide, and a simple electro-magnetic escapement each time the magnet is excited allows the spring to advance the plate and holder $\frac{1}{10}$ inch. The entire apparatus screws into the eye-end of a photographic telescope.

A star whose magnitude is to be determined is focussed close to the end of the photo plate, and an exposure of say 100^{s} given. The magnet is then excited for a moment by the current from a contact-maker, driven by a clock; the plate moves forward suddenly $\frac{1}{10}$ inch, and a second exposure is given, which lasts only 63^{s}. Again the plate moves forward to give a third exposure of $39^{s}\cdot8$, and the exposures are thus continued in the above ratio until they are reduced to 1^{s}. The telescope is then set on a standard star, such as *Polaris*. The holder is moved back to its original position, and *Polaris* is placed $\frac{1}{10}$ inch below the first exposure of star No. 1. The same series of exposures are then given, and the plate developed. The result will be like this:—

The relative number of images of the two stars will give their magnitudes to 0·5. The times of exposures will vary as the number whose log. is 0·2, but there is no reason why they should not be made to give 0·1 magnitudes.

The contacts are made by a wooden disc, revolving uniformly by the driving clock of the equatorial. On its edge are brass pins, which are placed so as to pass under a wiper at the correct intervals. The entire process is automatic once the star is set in its right place. Each plate will hold ten sets of exposures.

The instrument will also be of use for determining the actinic value of the sky before taking a stellar photograph. In this case, by taking a series of *Polaris*, and finding thus at what exposure it fails to record itself, the exposure necessary to record a star of another magnitude will be known.

Also to determine the value of wire screens in front of the O.G., a series can be taken with and without the screen, and the necessary value found.

I hope to exhibit some negatives taken with the instrument shortly before the Society.

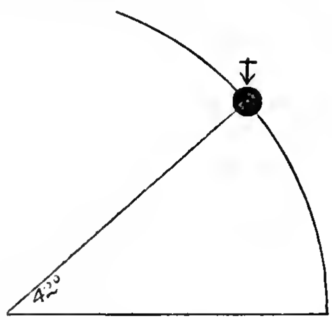
1892, *January* 3.

Notes on the Transit of Venus, as observed at Streete, Co. Westmeath.

By W. E. Wilson, *F.R.A.S.*

Scientific Proceedings of the Royal Dublin Society, 1882.

The 6th was cloudless throughout. The error of the observatory mean time clock was got by a transit of the sun at noon. The 24-inch reflector was stopped down to 12 inches, and a polarizing eye-piece used. The 4-inch finder was also used with a power of 75. The first contact was observed at $1^h\ 31^m\ 54^s$ Streete mean time. I was not quite sure of the exact position-angle that contact would take place at, so that I am sure the above time is a few seconds late. The time I calculated for first contact was $1^h\ 31^m\ 14^s$ S.M.T. At $1^h\ 38^m$ the entire disc of Venus could be seen against the corona. The planet seemed much darker than the corona, and was surrounded by a thin ring of silvery light. This ring was much brighter at the place marked in the following diagram by ‡. I suppose it was caused by a bank of clouds in the planet's atmosphere.



Internal contact occurred at $1^h\ 52^m\ 14^s$ S.M.T., and is, I believe, very close to the calculated time. There was no appearance of "black drop." The two cusps of light were almost quite sharp, and gradually closed up until they touched. As soon as Venus was well on the disc I mounted the spectroscope, and made a careful search for any absorption lines in the planet's atmosphere. Nothing of any certainty was observed. I thought some of the lines near D were thickened a little at their base, but I would not place much reliance on the observation. The sun by this time was getting very low, and the image of the planet was unsteady. If the atmosphere of Venus is about the same depth as the earth's, it will easily be seen that it would be an exceedingly thin ring round the planet, and by placing the planet over half the slit it would be most difficult to see the short absorption lines. My telescope being a Cassagranian, and giving a large image of the planet, would have a better chance than the Newtonian or Refractor. During the forenoon the sun was observed on the chance of seeing a transit of a satellite of Venus, with a negative result.

The Thermal Radiation from Sun Spots. Preliminary Notes of Observations made at Daramona, Streete, Co. Westmeath, 1893.

By W. E. Wilson, *M.R.I.A.*

Proceedings of the Royal Society, Vol. 55, 1894.

Communicated by G. Johnstone Stoney, *F.R.S.*

These observations were made by means of a large heliostat, lent by the Royal Society, and a Boys's radio-micrometer. The heliostat consists of a plane silver-on-glass mirror of 15 in. aperture. It is mounted equatorially, and driven by a clock. When in use, it is adjusted to reflect the sunlight to the north pole, and, as long as the driving clock is kept in motion, the beam of light remains fixed in that position. In the track of this beam, and about 12 ft. from the plane mirror, is mounted a concave silver-on-glass mirror of 9 in. aperture, and about 13 ft. focus. Its axis points to the south pole, so that the cone of rays formed by it strikes the centre of the plane mirror, and a short distance inside the focus. A small plane mirror mounted on the end of an arm is then so placed as to intercept the cone of rays, and reflect it horizontally into the observatory window; an achromatic lens enlarges the solar image which is formed on a screen in the room to 4 ft. in diameter.

Behind this screen, and standing on a pier of concrete, is mounted the radio-micrometer. The aperture through which radiant heat reaches the sensitive thermo-couple is a round hole drilled through a thick sheet of brass, and is only 1 mm. in diameter. A white cardboard screen is placed in front of the brass one to cut off heat from falling on the latter, and is provided with a hole slightly larger. A beam of limelight is thrown on the mirror of the radio-micrometer, and reflected on to the scale in the usual way. The diagonal mirror of the heliostat is provided with slow motions in two directions, which are moved by long rods and hook joints inside the observatory. Thus any part of the sun's disc can be placed on the small aperture of the radio-micrometer, and the driving clock will then keep it there.

The observations are taken in the following manner:—A small screen is placed over the aperture of the radio-micrometer, and the zero position of the spot of light on the scale noted. The screen is then removed, and the umbra of a sun spot placed on the aperture. The reading is then taken and entered in column *u*. The image is then moved, so that a part in the neighbourhood of the spot, but at *the same distance from the centre of the solar disc,* is placed on the aperture. This reading is entered in column N. Finally, a reading is taken at the centre of the disc, and entered in column C. The throws of the instrument are then got by subtracting the figures in columns *u*, N, and C from the zero. The deflections of the instrument have been experimentally proved to be *strictly proportional* to the amount of radiant heat

HELIOSTAT.

falling on the thermo-couple. The following is a typical observation taken August 7, 1893, of a large sun spot then visible. The *umbra* of this spot measured 0·8 in. across on the screen, so that the aperture of the radio-micrometer was only covering about $\frac{1}{400}$ of the apparent area of the umbra.

Zero.	*u*.	N.	*u* − *z*.	N − *z*.
15·8	17·1	20·4	1·3	4·6
15·6	16·9	20·2	1·3	4·6
15·5	16·8	19·9	1·3	4·4
15·3	16·7	19·8	1·4	4·5
15·2	16·6	19·6	1·4	4·4
15·1	16 4	19·5	1·3	4·4
14·9	16·1	19·4	1·2	4·5
		Means ...	1·31	4·49

The ratio $\frac{\text{umbra of spot}}{\text{neighbouring photosphere}} = \frac{1·31}{4·49} = 0·292.$

Five concordant readings gave a mean deflection of 4·57 for the centre of the sun, which gives for the ratio $\frac{\text{umbra}}{\text{centre}} = 0·287.$

The spot was at a distance from the centre of the disc of about 0·4 of the radius.

As the radiation from the photosphere falls off from the centre to the edge of the disc, it seemed an interesting point to determine if any change in the ratio of *u*/C would take place as a spot was carried across the disc by the sun's rotation. If the spot is, as is generally thought, a depression, the absorption of heat ought to increase as it is carried towards the limb, on account of the increased depth in the solar atmosphere through which the radiation would have to pass. On the other hand, if the spot was floating *above* the absorbing atmosphere the radiation from it would remain constant in any position on the solar disc.

The following is the value of the heat radiation from the photosphere taken along a radius of the sun, where 0 = centre and 100 the limb. The radiation R equals 100 at the centre.*

D.	R.	D.	R.	D.	R.
0	100·0	40	97·2	80	82·5
10	99·8	50	95·3	90	72·0
20	99·5	60	92·2	95	61·8
25	99·3	70	87·8	98	51·5
30	98·9	75	85·3	100	42·9

* "The Absorption of Heat in the Solar Atmosphere," by W. E. Wilson and A. A. Rambaut, 'Proceedings of the Royal Irish Academy,' 3rd series, vol. 2, No. 2.

It will be seen by the following observations of spots, taken from August 5 to November 9, that there is distinct evidence that the radiation from the spot does not fall off as rapidly when near the limb as the neighbouring photosphere; in fact, the ratio u/C remains nearly constant, whereas the ratio u/N gets nearer unity as the spot approaches the limb. The spot observed on the 22nd of October is a good example, as the same spot was observed again on the 26th, 29th, and on the 30th, when it had reached within a distance, D, of 95 from the centre. It will be seen that on these four dates the ratio u/C was respectively 0·338, 0·360, 0·313, 0·356, whereas the ratio u/N was 0·349, 0·410, 0·706, 0·783.

LANGLEY,* in 1874 and 1875, measured the radiation from sun spots. He used a thermo-pile and galvanometer, and obtained as the mean of his results a ratio of $0·54 \pm 0·05$.

His method was first to take a reading in the neighbourhood of the spot, but between it and the centre of the disc. He then took a reading in the umbra, and, finally, a third reading in the neighbourhood between the spot and the edge of the sun.

Date.	$\frac{u}{C}$.	$\frac{u}{N}$.	D.
1893.			
Aug. 5	0·370	0·427	60
7	0·287	0·292	40
8	0·286	0·323	50
8	0·339	0·377	40
8	0·418	0·512	90
14	0·364	0·373	50
19	0·368	0·375	50
Sept. 2	0·309	0·309	10
3	0·298	0·298	10
4	0·420	0·450	30
4	0·430	0·446	30
7	0·287	0·355	85
Oct. 1	0·398	0·401	30
1	0·489	0·570	80
22	0·338	0·349	52
26	0·360	0·410	40
29	0·313	0·706	90
30	0·356	0·783	95
Nov. 8	0·365	0·800	97
9	0·339	0·848	85

The mean of the two photospheric readings he used as a divisor for the umbral reading. He then says, "The decrement of heat as we approach the limb is, though not exactly, yet so very nearly, in the same ratio for photosphere and spots, that no correction is needed on this account for the present observations."

* 'Monthly Notices,' vol. 37, No. 1.

If LANGLEY failed, through want of instrumental means, to notice the difference between the absorption in a spot and the photosphere near the limb, his method would make his umbral readings too high. The mean of twenty observations here equals 0·356, against LANGLEY's 0·54. This is a serious difference, and, I think, can only be accounted for either by the use of superior instrumental means, or by a possible variation in the radiation of spots in different years of the sun spot cycle.

It is difficult to see how *too low* a value for umbral radiation could be got, whereas too high a one might be found by want of definition and trembling in the image, so that some of the penumbral radiation would reach the thermo-couple.

The Thermal Radiation from Sun Spots. Observations made at Daramona, Streete, Co. Westmeath.

By W. E. Wilson, *F.R.A.S.*

Monthly Notices of the Royal Astronomical Society, Vol. LV., No. 8, 1895.

In 1894, March, Dr. Johnstone Stoney kindly communicated to the Royal Society some observations of mine on the above subject.* In this preliminary note a full description is given of the instruments and of the method of observations. It is therefore unnecessary to again describe them, but merely to mention that a large image of the sun is formed by means of a heliostat and projected on to a screen in the observatory. Behind this screen, and standing on a pier of concrete, is placed a Boys radio-micrometer. A small pinhole in the screen admits radiant heat into the radio-micrometer. By placing any part of the solar image over the pinhole we can measure the relative radiation of this part. Two long rods and handles come into the observatory from the heliostat, and by turning these any desired part of the solar image can be accurately placed over the pinhole aperture. The driving clock of the heliostat then keeps it in this position while an observation is being made. The standard to which all the values of radiation from spots, &c., are compared is the value on an arbitrary scale of the deflection of the radio-micrometer when the centre of the solar image is allowed to fall into the pinhole aperture in the screen.

In taking an observation, the zero position of the spot of light on the scale when the pinhole is closed is first noted, and its value entered in column Z. The umbra of a spot is then placed, by means of the slow-motion handles, on the pinhole, which is then opened. The amount of deflection produced is then entered in column U. The image is then moved so that a portion near the spot, but at the same distance from the limb, falls into the pinhole. The deflection from this is entered in column N. And finally the solar image is placed centrally, and the radiation from the centre of the disc is entered in column C.

When the values in column Z are subtracted from those in U, N, and C, we get comparative values of the radiation from the umbra, the neighbourhood, and the centre of the sun.

In a former paper † it has been shown that the deflections of the radio-micrometer are, at any rate up to 20°, strictly proportional to the amount of radiant energy falling on the instrument. We are therefore authorized in taking these values as equal to the amount of radiant energy reaching us from the different parts of the sun. It has long been known that the radiation from the limb of the sun is much less than from the centre, and that this is probably caused by a layer of what we may call smoke, which lies over the photosphere. It seemed to be an interesting point to investigate whether the radiation from a sun spot falls off when near the

* 'Proceedings of the Royal Society,' vol. 55.

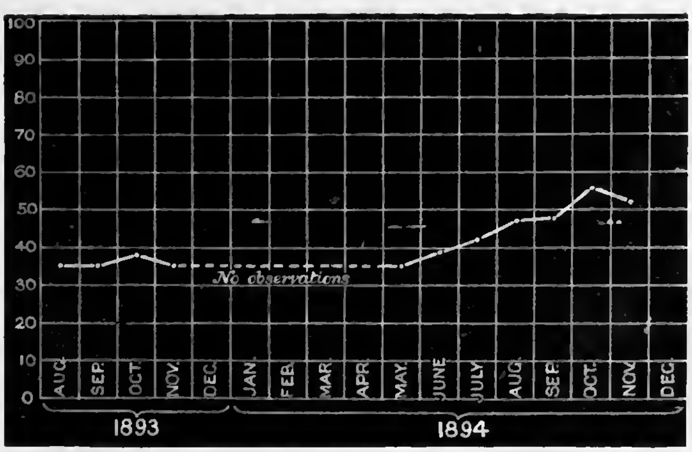
† "Effective Temperature of the Sun," Wilson and Gray, 'Phil. Trans. vol. 185 A.

limb in the same manner as a point in the photosphere, and since the commencement of these observations this point has been investigated. In the preliminary note published in the spring of 1894 I showed from observations then recorded that the radiation from the umbra of spots does *not* suffer absorption when near the limb, in the same manner as a point on the photosphere. These observations have been continued during the summer and autumn of last year and confirm the earlier ones.

In 1874 and 1875 LANGLEY,* by means of a thermo-pile and galvanometer, measured the radiation from sun spots. The mean value that he gives for the umbral radiation is 0·54. Instead of taking the radiation from a point in the centre of the disc as his standard, LANGLEY took two points on the photosphere, the first situated between the spot and the centre, and the second between the spot and the limb. The mean of these two photospheric readings he then used as a divisor for the umbral reading. LANGLEY seems to have been unable to observe that the radiation from the umbra of a spot is nearly constant for different distances from the centre of the disc. He says, "The decrement of heat as we approach the limb is, though not exactly, yet so very nearly, in the same ratio for photosphere and spots, that no correction is needed on this account." An inspection of the values of $\frac{U}{N}$ in Table I. clearly shows that the radiation from the umbra of a spot, as its distance from the centre increases, becomes closer to the value of the radiation from the neighbouring photosphere, whereas the values of $\frac{U}{C}$ remain nearly constant. As LANGLEY's observations were taken near a solar minimum year and mine at a solar maximum, it is just possible that the radiation from spots may vary during this period. Future observations taken throughout a complete sun-spot cycle may show that the spots are formed during the solar minimum at a lower level, and thus suffer more absorption near this limb than they do during a solar maximum.

I have collected the monthly means of my observations from August 1893 to Nov. 1894, and plotted out the values of $\frac{U}{C}$ in the following curve :—

Fig. 1.

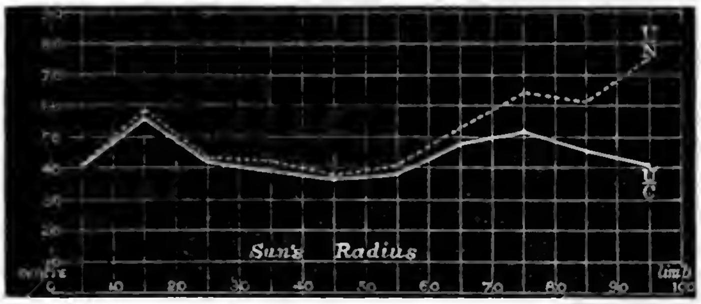


Radiation from an equal area of the photosphere at the centre of the disc = 100.

* 'Monthly Notices,' vol. 37, No. 1.

It will be seen that the umbral radiation is considerably lower during 1893, and since May 1894 the values of $\frac{U}{C}$ have increased from 0·35 to over 0·50.

Fig. 2.



Radiation from an equal area of the photosphere at the centre of the disc = 100.

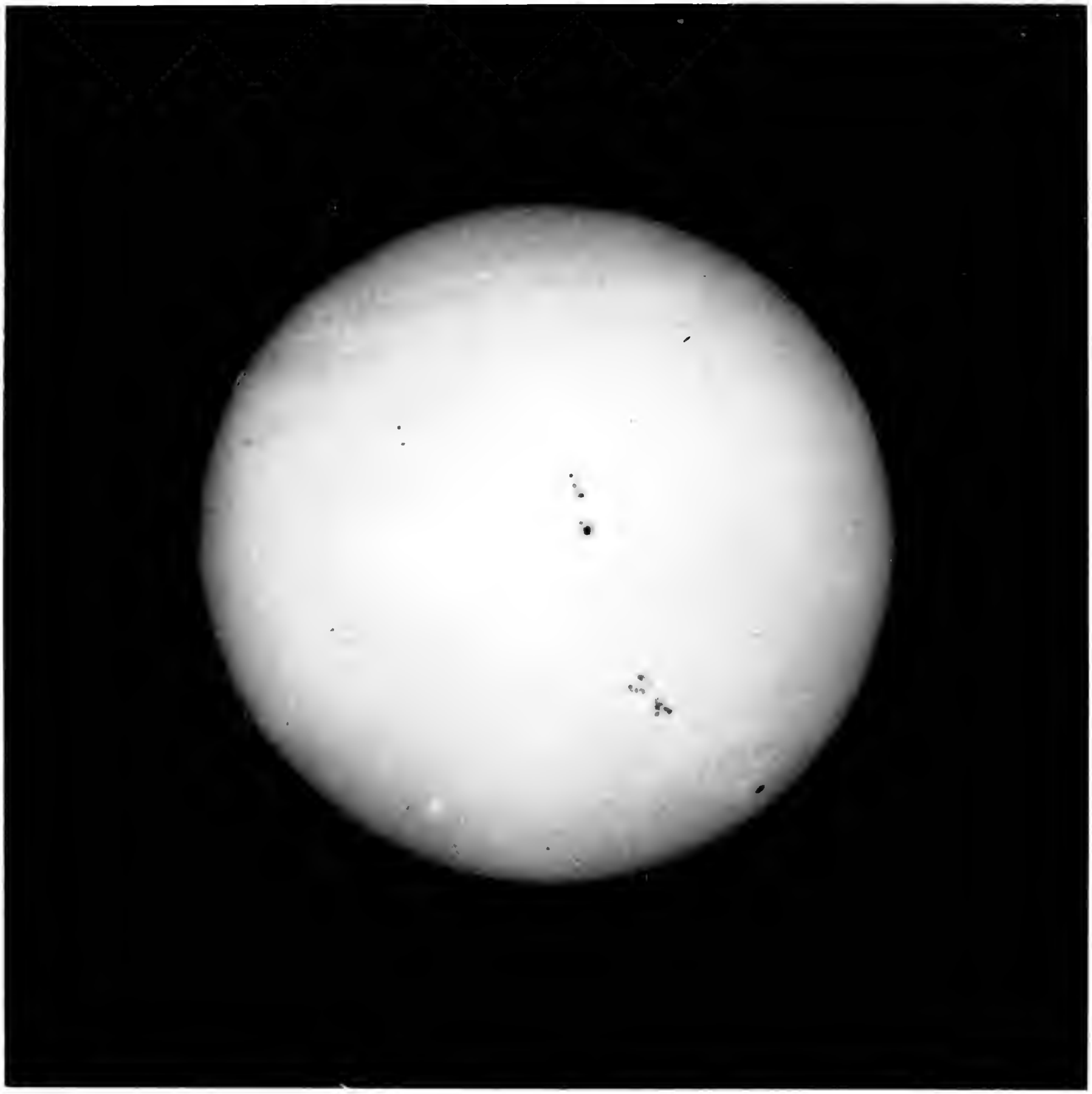
In fig. 2 I have collected the values of $\frac{U}{C}$ and $\frac{U}{N}$ according to distance along the sun's radius. The continuous curve gives the values of $\frac{U}{C}$, while the dotted one gives those of $\frac{U}{N}$, and shows how the latter departs from the former as the limb is approached.

Since I published my first observations my attention has been drawn to a paper on this subject by Professor FROST, of Dartmouth College, U.S.A. He there points out the same fact as I have—that the light from umbra of spots does not suffer absorption when they are near the sun's limb in the same manner as a point in the photosphere. He also records some observations in which he found the radiation from the umbra greater than that from the neighbouring photosphere when the spot was close to the limb. I have never been able to observe this. The greatest value for $\frac{U}{N}$ that I have recorded is 0·83.

In Table I. are given the ratios of the radiation from the umbra (U), the neighbouring photosphere (N), and the centre (C). In column D is given the distance of the spot from the sun's centre, in hundredths of the radius. Where observations are bracketed together it signifies that it is the same spot that has been observed on different days. Unfortunately, in this climate we are very seldom able to observe a spot for many days in succession.

TABLE I.

Date.	$\frac{U}{C}$.	$\frac{U}{N}$.	D.	Remarks.
1893.				
Aug. 5	0·37	0·43	60	
7	0·29	0·29	40	
8	0·29	0·32	50	
8	0·34	0·38	40	
8	0·42	0·51	90	
14	0·36	0·37	50	



Date.	$\frac{U}{C}$.	$\frac{U}{N}$.	D	Remarks.
1893.				
Aug. 19	0·37	0·37	50	
Sept. 2	0·31	0·31	10	
3	0·30	0·30	10	
4	0·42	0·45	30	
4	0·43	0·45	30	
7	0·29	0·35	85	
Oct. 1	0·40	0·40	30	
1	0·49	0·57	80	
22	0·34	0·35	52	
26	0·36	0·41	40	
29	0·32	0·71	90	
30	0·36	0·78	95	
30	0·58	0·65	...	No real nucleus.
Nov. 8	0·36	0·80	97	
9	0·34	0·85	92	
1894.				
May 30	0·34	0·34	...	Spot A.
30	0·32	0·34	57	Spot B.
31	0·33	0·34	45	Spot A.
31	0·39	0·40	42	Spot B.
June 6	0·38	0·54	86	
30	0·41	0·61	89	Spot A.
30	0·62	0·66	...	Spot B. No definite umbra.
July 1	0·43	0·76	97	Spot A. Spot B nearly filled up.
21	0·61	0·64	60	Sky hazy.
21	0·46	0·52	58	
30	0·42	0·47	65	
30	0·38	0·78	94	
30	0·39	0·67	92	
Aug. 6	0·49	0·51	41	
22	0·46	0·70	96	
Sept. 8	0·42	0·58	74	
9	0·47	0·51	61	
10	0·43	0·45	41	
13	0·47	0·47	...	Spot nearly central.
14	0·45	0·50	53	Same spot as observed on 8th 9th, 10th, 13th.
29	0·58	0·66	83	
30	0·55	0·61	67	
Oct. 4	0·60	0·66	72	
4	0·56	0·59	19	
4	0·55	0·64	84	
7	0·54	0·64	...	Clouds
19	0·53	0·83	96	
Nov. 5	0·51	0·52	57	
5	0·61	0·61	5	
5	0·49	0·71	84	
8	0·42	0·83	90	
8	0·54	0·69	72	
8	0·58	0·60	40	
30	0·48	0·80	94	
30	0·50	0·78	83	

The Electrical Measurement of Starlight. Observations made at the Observatory of Daramona House, Co. Westmeath, in April, 1895. Preliminary Report.

By G. M. Minchin, M.A.

Communicated by Professor Fitzgerald, F.R.S.

Proceedings of the Royal Society, Vol. 58.

The method employed in these experiments for measuring the intensity of the light which reaches the earth from the stars and planets consists in the determination of the electromotive force generated by such light in certain photo-electric cells, the square of this electromotive force being proportional to the energy of the incident light.

It will, then, be well to describe first the nature and construction of these cells.

The Photo-electric Cells.

In these cells the surface on which the incident light is received is formed by depositing a thin layer of selenium on a surface of clean aluminium, and immersing the sensitive layer in a glass cell filled with œnanthol.

The mode of formation of the sensitive surface is as follows :—

Fig. 1.

Take a tube, AB, of soft glass, the diameter of the bore of which is 1 mm., or smaller if desired ; take a short length, AL, of aluminium wire, which nearly fits the tube, and to one end, L, of this piece of aluminium attach a platinum wire, LP, which emerges from the end, B, of the glass tube, the contact at L being made by boring a fine hole through the aluminium and pinching the two metals together ; then, in the flame of a Bunsen burner or a blow-pipe, melt the glass well round the aluminium, until the glass fits round the aluminium as tightly as possible. The contact of the glass and the aluminium should be perfect, or, at least, liquid-tight, and, unfortunately, it has been found hitherto impossible to realize this condition. If this condition could be attained, the photo-electric cells would remain constant in their action for a very long time, if not, indeed, permanently. At present, owing to this want of liquid-tightness, about four weeks seems to be the limit of constancy

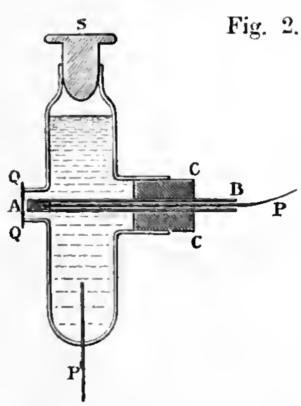
(There is also another condition essential to constancy, which will be presently mentioned.)

The next step is to grind the end A of the tube on fine sand or emery-paper until a flat surface is formed by the end of the aluminium wire and the wall of the glass tube, the end of the aluminium wire being then scraped clean.

Now place the tube AB, with the end A uppermost, between two nearly vertical plates of asbestos, the end A just appearing beyond the edges of the plates of asbestos ; on the middle of the aluminium wire at A place a very small piece of selenium (about the size of a very small pinhead); heat the asbestos by means of a spirit lamp or a Bunsen flame until the selenium melts on the end A of the tube. Care must be taken to keep the flame away from the selenium, so that the latter melts in virtue of the heat of the aluminium wire. The selenium now lies as a very black little liquid globule on the end of the tube, and it must be spread uniformly over the end of the tube by means of a heated glass rod. The layer of selenium should not be a thick one. The flame being removed, allow the selenium to cool into a hard black layer. When it reaches this condition, apply the heat again, as before, until the black surface changes into one with a uniform brownish-grey colour, the heat being continued after this with great care until the selenium is on the point of melting again into a black liquid. On the first sign of this latter change, instantly remove the heat and blow over the surface of the selenium. This will at once stop the tendency to melt, and the surface will then be in its most sensitive state. There should be no glossy streaks on the surface ; if there are, it must be heated over again and the whole process repeated. Screen the tube from light and allow it to cool for a few minutes ; it will then be ready to put into the cell with œnanthol.

The œnanthol cell is a small glass tube, represented in fig. 2. It consists of a glass tube about 3 cm. long, and nearly a centimetre in diameter, with two short glass tubes fitted into it on opposite sides ; one of these is ground flat, and

Fig. 2.



has a thin quartz window, QQ, cemented to it with gelatine and acetic acid, or glue and glycerine, or any cement that will withstand the action of œnanthol ; the other is tightly closed by a cork, CC, through which passes the glass tube, AB, which contains the aluminium and platinum wires above described. The cell is closed at one end by a ground glass stopper, S, and through the other end passes a platinum wire P′, sealed in. The two poles of the cell are the platinum wires P and P′.

The light of a star is destined to shine through the quartz window QQ, on the centre of the sensitive surface A, which is placed in focus of a telescope, or rather a little behind this focus, so that the light of the star may cover the whole of the selenium area.

The covering of the whole area A by the light is essential for the following reasons:—

The seat of the electromotive force is the surface of contact of the liquid and the selenium, the selenium receiving a positive and the liquid a negative charge. If, now, P is connected with one pole of an electrometer and P′ with the other, and if there is any portion of the selenium surface which is not exposed to the light (and consequently not the seat of an E.M.F.), this inert portion will act simply as a conductor conveying a portion of the positive charge to the wrong pole of the electrometer, and thus giving a diminished effect.

The truth of this is easily verified with any kind of photo-electric cell, *e.g.*, one formed of a sensitive tinfoil surface divided into two portions which can be metallically joined together outside the cell or kept separate. If while the portions are joined, one is exposed to, and the other screened from, incident light, the E.M.F. indicated is much less than it is when both are exposed, or when one alone is exposed while the other is disconnected from it. (A description of such tin-foil cell will be found in the 'Philosophical Magazine' and in the 'Proceedings of the Physical Society.')

This fact now enables us to see the importance of preventing the liquid from entering the glass tube AB, which contains the conducting wire P, for it is clear that, when the light is incident at A, the liquid which has crept into the tube round the aluminium wire will convey a portion of the negative charge imparted to the liquid in the cell to the wrong pole of the electrometer, and will thus diminish the effect of the light.

The capillary entrance of the liquid into the tube AB may, of course, be prevented by sealing into the tube a *platinum* instead of an *aluminium* wire, and coating the end of it with the selenium layer. But unfortunately platinum is not so good a base for the selenium as is aluminium, owing, almost certainly, to the fact that selenium enters into chemical composition with platinum, while it does not do so with aluminium, or with some other metals which, possibly, may yet be used.

The entrance of the liquid could also be prevented by using a platinum wire instead of an aluminium one, and then coating the end of the platinum wire at A with a deposit of aluminium; but although this is doubtless possible, success in the attempt has not yet been attained.

Air-tightness is another essential condition of the constancy of these photo-electric cells, for it is found that in cells which are not quite air-tight the resistance of the œnanthol increases very much after a few weeks, probably owing to the oxidation of the liquid by the air; and this great increase of resistance promotes sluggishness in the response of the cell to the action of light.

An examination of the seleno-aluminium cell with the various portions of the spectrum of lime-light shows that the cell is sensitive to all the rays from the end of the red and below it, to beyond the violet, the maximum E.M.F. being produced in the yellow: but the magnitude of the E.M.F. does not vary very greatly until the violet is reached. In this respect the seleno-aluminium cell differs from all other photo-electric cells, for the sensitiveness of most of the latter is almost wholly confined to the blue. It may be mentioned, however, that a cell formed by immersing clean silver plates in a solution of eosine gives electromotive forces of opposite signs for the red and the blue rays.

The energy incident on a photo-electric cell has been found to be proportional to the square of the electromotive force generated. If one candle held at a certain distance from the cell gives a difference, E, of potential between the poles, P, P′, two candles close together will be found to give a difference $E\sqrt{2}$. Or, if a candle be tried at different distances from the cell, the difference of potential will be found to vary inversely as the distance.

Intrinsic Energies of Stars.

If I is the total amount of energy radiated into space in any time by a star at the distance r from the earth, the quantity received on any given surface on the earth will be proportional to I/r^2; and if E is the electromotive force which this generates in a given cell we have

$$I/r^2 = kE^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1),$$

where k is some constant. Hence if I′ is the intrinsic energy of another star at the distance r', and E′ the corresponding E.M.F.,

$$I'/r'^2 = kE'^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2),$$

from which we have

$$\frac{I}{I'} = \frac{E^2 r^2}{E'^2 r'^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

Hence, if the parallaxes of the two stars are known, say p and p' respectively, we have

$$\frac{I}{I'} = \left(\frac{Ep'}{E'p}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

When it is desired to compare the energy of a star with that of the sun, we must know the area of the sensitive layer, A, fig. 1, of selenium in the cell. Let this be a, and let A be the area of the aperture of the telescope.

Then, since it is not desirable to concentrate on the selenium the amount of solar light which falls on the large area A, we must turn the cell to the sun without the aid of the telescope. Let E be the E.M.F. observed, S the intrinsic energy of the sun, and r the distance of the sun from the earth. Then

$$a\frac{S}{r^2} = kE^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (5),$$

while, for any star whose distance is R, giving an E.M.F. equal to e,

$$A\frac{I}{R^2} = ke^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (6).$$

$$\therefore \frac{I}{S} = \frac{a}{A}\left(\frac{eR}{Er}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (7).$$

As the electromotive force produced by the light of the sun falling directly on the cell is probably too large, it will be desirable to diminish its intensity by taking it through a small measured aperture and placing the cell at a known distance behind.

The Electrometer employed.

The instrument employed for measuring the electromotive forces generated by the light of different stars is a quadrant electrometer differing from the forms in ordinary use in having its quadrants made of aluminium, two of these being supported on brass pillars connected with the case of the electrometer and always earthed, while the other two are supported on pillars of melted quartz. The quadrant box is about 2 cm. high and 5 in diameter; the needle is of thin aluminium foil, cut into the peculiar shape figured in CLERK MAXWELL'S 'Electricity and Magnetism,' and is suspended by a quartz fibre about 9 cm. long. The needle and quadrants are surrounded by a thick metal case, and the instrument is both air-tight and induction-tight.

It had been intended to use with the electrometer an air condenser consisting chiefly of two gilt brass plates, each about 15 cm. in diameter, to multiply the potentials indicated by the electrometer; but at present there are difficulties in the way of its employment, and the measures made on this occasion were made by the electrometer alone.

Both instruments were constructed with the aid of the Government Grant administered by a Committee of the Royal Society and were made by Mr. PAUL of Hatton Garden.

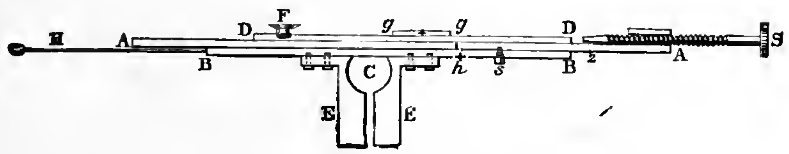
The Telescope.

This was Mr. WILSON's 2-feet reflector, which was at first used as a Cassagrain instrument, and subsequently as a Newtonian, the cell-carrier being in each case fixed to the telescope in place of the eye-piece.

The Cell-Carrier.

Fig. 3 represents the cell-carrier in plan.

Fig. 3.



A thin plate of brass, AA, with a circular hole of about ·75 cm. diameter in the centre of its vertical face (not represented in the figure) is fitted with a screw, S, at one end. This screw pushes forward another thin brass plate, DD, which moves backwards and forwards in a grooved space in the plate AA. The plate DD has likewise a hole in its centre, and is moved in one direction by S and in the opposite direction by the finger applied to a screw F, attached to DD near one end. A small thin brass plate, *gg*, is attached to DD, and can move up and down (*i.e.*, at right angles to the plane of the figure) in a grooved space in DD by means of a screw which is not represented in the figure. This plate *gg* has also a hole in its centre over which is cemented a thin circular glass plate carrying crossed spider lines, represented in the figure by +. The point of intersection of these cross-lines is capable of being brought into any desired position by means of the horizontal motions of the plate DD and the vertical motions of *gg*.

To the face of AA opposite to that on which DD moves is attached a thin brass plate, BB, which moves in a grooved space in AA. Through BB passes a screw, *s*, which penetrates a short distance into a special groove in BB which is terminated at the points marked 1 and 2. These points are, therefore, stops to determine the extreme positions of the sliding plate BB on the fixed plate AA.

The plate BB has a circular hole of about ·75 cm. in diameter in its middle, and also another, *h*, a little to the side. Over *h* is cemented a thin plate of glass on which can be marked two cross-lines marked + in the figure, or a dot.

An ebonite block with a cylindrical hole, C, into which fits the cell represented in fig. 2 is screwed to BB just over the central hole in BB.

The plate BB is moved backwards and forwards by a projecting handle, H.

The fixed plate AA is screwed to a stout cylindrical tube about two inches long (not represented), and this tube fits on the telescope instead of the usual eye-piece. AA can be adjusted, if necessary, to various positions relative to this tube, *i.e.*, relative to the telescope.

To set the apparatus for a star, the procedure is this: move the plate BB (and with it of course the cell) by the handle H until it is stopped by the stop 2; move the intersection of the cross-lines on *gg* by means of the screw S and the screw which moves *gg* vertically until this point of intersection is exactly opposite the centre of the sensitive surface of the cell (which is, of course, visible through all the holes in the plates); bring back BB by means of H until it is stopped by the stop 1; the glass plate covering the hole *h* is now visible opposite *gg*—or rather through *h* we can see the intersection of the cross-lines on *gg*; mark on the glass at *h* with a fine brush point a dot exactly opposite the intersection of the cross-lines of *gg*; remove *gg* out of the field of view by the vertical motion of the screw attached to it and to the plate DD—this being done in order that the light of the star may not have to pass through the glass plate on *gg*. This completes the adjustment.

To use the instrument with a star, keep BB stopped by the stop 1, so that the mark or dot at *h* is in position to receive the image of the star. Suppose, then, that by the adjustment of AA relatively to the telescope the image of the star falls exactly on this mark. Now by means of the handle H move BB until it is stopped by the stop 2. This brings the centre of the sensitive surface into the position occupied by the mark at *h*, *i.e.*, the image of the star is now falling on the sensitive surface of the cell.

When we desire to throw off the star, we can do so by moving BB with the handle H until it is stopped by the stop 1; but it is better to effect this by moving the telescope itself in declination, without going near the cell, until the star is out of the field, as indicated by the finder; in this latter way the cell suffers no disturbing effects of temperature, &c.

Connection of Cell with Electrometer.

One pole (the insensitive) of the cell was connected with earth by a wire attached to a gas-pipe, while the other (that of the sensitive surface) was attached to a fine uncovered copper wire, carefully insulated throughout; which passed down through a shellac plug in the floor of the observation room to the electrometer in the room below.

The readings of the electrometer deflections caused by the light of the stars were made in the lower room by Professor FITZGERALD, while Mr. WILSON and I attended to matters upstairs. But in this part of the work the services of my two colleagues were of very much greater use than mine.

The Observations.

Regulus was the first star taken, on the night of the 11th April, and only two observations of the deflection on the electrometer scale produced by its light were made before proceeding to an examination of Arcturus for comparison. It is thought desirable to show in tabular form a few of the results obtained. A LECLANCHÉ cell produced on the scale a deflection of 530 mm., the scale being about 7 feet from the electrometer.

Every photo-electric cell of the type previously described and of maximum sensitiveness has a certain native or disturbing E.M.F., which is always opposed to the E.M.F. generated by light. In this case it was represented by 11·2 mm.

Experiment.	Scale reading, star off cell.	Scale reading, star on cell.	Scale reading, star off cell.	Difference from mean.
1	150	146·5	151	4·0
2	151	146·0	·150	4·5

The second column of this table contains the number on the scale at which the spot stood when the cell was in the dark, *i.e.*, when the only E.M.F. in it was its disturbing E.M.F.; the third column contains the number to which the spot moved when the light of the star was allowed to fall on the sensitive surface in the cell.

It will thus be seen that the E.M.F. due to Regulus in these two experiments was represented by the number 4·25, or about 4·25/530 of a LECLANCHÉ.

Arcturus when tried gave the following results, after a few preliminary observations (which were tried in the case of Regulus also, and which are rendered necessary after the disturbances caused in shifting the telescope, &c.) :—

Experiment.	Reading, star off.	Reading, star on.	Reading, star off.	Difference from mean.
1	135·5	128·0	135·5	7·5
2	135·5	Clouds	came	up.
3	145·0	138·0	143	6·0
4	143·0	Clouds	came	up.
5	136·0	137·0	Found star	off the wires.
6	139·0	132·5	139	6·5
7	139·0	130·0	135	7·0
8	135·0	127·0	134	7·5

These observations were somewhat irregular, because the sky was not quite clear during most of the observations, but was getting clearer as they proceeded.

The last was probably the best observation, and in that case Arcturus produced a voltage of about 0·02.

An experiment was made to try whether, if the cell were exposed to the sky in the neighbourhood of the star, but not to the star itself, any effect was produced, and a deflection of about 0·5 mm. in the direction *opposed* to that of the deflection caused by light was constantly observed.

If we take this effect into account, we have as the deflections due to

Regulus ....4·75
Arcturus ....8·00

Assuming now the latest determinations of the parallaxes of these stars to be

Regulus..................0·093″
Arcturus ....0·018″

we have from equation (4) the ratio of their respective intrinsic energies, I, I′, the result

$$I'/I = 75{\cdot}72,$$

showing that Arcturus radiates into space about $75\frac{3}{4}$ times as much energy as Regulus in a given time.

The telescope was next turned to the star η Boötis, with the result :—

Experiment.	Reading, star off.	Reading, star on.	Reading, star off.	Difference from mean.
1	133·5	133	133·5	0·5
2	133·5	133	133·5	0·5

If the radiation to the sky is added, the star's deflection amounts to 1 mm. *i.e.*, a voltage of 0·0028.

This observation is, however, recorded merely for the purpose of showing that a comparatively faint star is able to give an unmistakable E.M.F.

The telescope was next turned on Saturn, whose image was so large that the ends of his rings were probably off the sensitive surface in the cell. The observation was :—

Experiment.	Reading, star off.	Reading, star on.	Reading, star off.	Difference from mean.
1	129	125·5	129	3·5
2	129	125·0	129	4·0
3	129	122·5	126	3·75

Between the second and third of these experiments the cell had been slightly disturbed, so that the last is not very satisfactory, although it gives the mean result.

The glare from the moon was now very distinctly apparent, and its effect seemed to be a deflection of about 0·5 mm., which must be deducted from Saturn's effect. This latter would, then, be represented by about 3·25 mm., or a voltage of 0·009.

The next observations were made on the night of Friday, the 12th, and on this night the atmosphere was very hazy, although many stars were visible. It is not necessary to enter into the details of observations on such a night; nevertheless, it may be interesting to see the effects which were observed.

For the following bodies the mean deflections were as tabulated :—

Jupiter	6·33 mm.
α Cygni	0·50 „
Vega	2·26 „
Arcturus	2·50 „
Regulus	1·10 „

A standard candle placed at a distance of 9 feet from the cell produced a deflection of 11 mm.

Nothing of a quantitative nature can be deduced from these results, owing to the presence (and unequal distribution) of haze, the effect of which was to make Vega produce a deflection less than that of Arcturus.

On the night of the 14th the sky was clearer, and while Arcturus gave a very fairly constant deflection of 4·5 mm., Regulus gave 1·85 mm., ε Boötis 0·32 mm., α Coronæ 0·6 mm., and β Herculis something less than 0·2 mm.

The night of the 15th was, for a short time, much clearer, and during this time the following more reliable measures were made :—

Arcturus	8·2 mm.	(mean of 4 observations)
Saturn	5·6 „	(„ 4 „)
Vega	11·5 „	(„ 2 „)
Candle at 9 feet . . .	10·0 „	
LECLANCHÉ cell . .	513·0 „	

It would be interesting to compare the intrinsic energies of Arcturus and Vega, but the parallax of Vega seems to be almost more uncertain than that of Arcturus. It is given in YOUNG'S 'Astronomy' as 0·16, while Miss CLERKE gives ELKIN'S value as "0·034?" If the first of these is taken, with the value 0·018 for Arcturus, equation (4) gives the intrinsic energy of Arcturus equal to 38·2 times that of Vega; but if the second is taken, this number becomes only 1·8.

Comparison with the Photometric Method.

In the ordinary method of comparison of "magnitudes," if B and B′ are the brightnesses of two stars whose magnitudes are m and m', respectively, we have, by definition,

$$\log \frac{B}{B'} = \tfrac{4}{10}(m'-m) \quad . \quad . \quad . \quad . \quad . \quad (5).$$

Now, taking Arcturus and Regulus as of the magnitudes 0·3 and m', respectively, and the electromotive forces of their lights as 8 and 4·75 (determination of April 11, previously cited), we have

$$\log \left(\frac{8}{4\cdot 75}\right)^2 = \tfrac{4}{10}(m'-0\cdot 3),$$

$$\therefore m' = 1\cdot 43.$$

The magnitude of Regulus, as a matter of fact, is variously cited as from 1·42 to 1·7 ; thus the amount of correspondence between the photo-electric and photometric methods is seen.

Concluding Remarks.

Among the few bright stars which we found available was Procyon, and even this star offered an opportunity for observation during a very limited time, owing to mechanical hindrances in the observatory. The stars in the Great Bear shone brilliantly, and, under favourable circumstances, their light could have been easily measured. The constellation was, however, so nearly vertical, that the aperture in the roof of the observatory was not sufficiently wide to suit the aperture of the telescope, and hence no observation of any of these stars was attempted.

On one night observations of Procyon and Regulus were taken. The readings were much smaller than had been anticipated from the great sensitiveness of the cell and electrometer. When these observations were completed, the cell was exposed to a candle at 9 feet, and the effect was so small, that it was evident that some accidental circumstance was intervening. The cell was, therefore, taken down from the telescope and examined, with the result that we found an opaque movable portion of the cell holder covering a portion of the sensitive surface in the cell. This was at once removed, and then the candle, Arcturus, and Vega gave the large deflections quoted in the observations of the 15th. It was, however, then too late to get Procyon again. But the observations which had been made with this star and Regulus, while the partial obstruction of the cell remained, gave the mean of their deflections as

Regulus	1·27 mm.
Procyon	1·90 ,,

Now, although the accidental obstruction renders this comparison unsatisfactory, it is remarkable that these numbers accord fairly well with the "magnitudes" of the two stars, as given by Miss CLERKE ('System of the Stars,' Appendix, Table III.). Thus the "magnitude" of Regulus is given as 1·4, and that of Procyon as 0·5

Now, in equation (5), if we put $B/B' = (190/127)^2$, and assume the magnitude of Regulus=1·4, while that of Procyon $= m$, we have

$$5 \log \tfrac{190}{127} = 1{\cdot}4 - m,$$
$$\therefore m = 0{\cdot}53,$$

which is a rather close coincidence.

On the same data equation (4) gives the result

Intrinsic energy of Regulus = 3·6 × intrinsic energy of Procyon.

It is hoped that these measurements will be resumed about the end of next September, at Daramona, by the same observers; and in the meantime, some improvements will be effected in the cell-holder which will facilitate observation. An endeavour will also be made to improve the cell itself in the directions indicated at the beginning of this report.

The experiments prove conclusively that there is little difficulty in obtaining fairly accurate measurements of the light of stars of the first and second "magnitudes," even without the employment of a multiplying condenser or a larger telescope. A telescope with an aperture of 5 or 6 feet would certainly annex a very great number of stars to the list.

It is right to put on record the fact that the first photo-electric observations of planets and stars were made by Mr. MONCK, in Dublin, in the year 1892, in conjunction with Professor FITZGERALD. My cells were at that time much less sensitive than the present ones; and, for reasons set forth in this paper, their sensitiveness fell off after about six hours. The liquid in those cells was acetone, and the aluminium on which the selenium was deposited was not insulated from the liquid. Nevertheless, Mr. MONCK and Professor FITZGERALD were able to observe electromotive forces due to the light of Venus, Jupiter, and, I think, Mars. Mr. MONCK'S telescope is a refractor of 9 ins. aperture, so that large results were not to be expected. These observers were not quite certain whether Vega and Capella produced measurable effects or not; but their observations were much interfered with by draughts of air, and other things, in their observatory.

The Electrical Measurement of Starlight. Observations made at the Observatory of Daramona House, Co. Westmeath, in January, 1896. *Second Report.*

By G. M. Minchin, *M.A., F.R.S., Professor of Mathematics, Cooper's Hill College.*

Proceedings of the Royal Society, Vol. 59.

In the 'Proceedings of the Royal Society,' vol. 58, pp. 142, &c., will be found the first account of the measurement of stellar radiation by means of the electromotive force generated by the action of starlight on my photo-electric cells. The observations were resumed in the beginning of January, 1896, in Mr. W. E. Wilson's observatory by the same three observers—namely, Mr. Wilson, Professor Fitzgerald, and myself.

Our appliances were substantially the same as before; but the photo-electric cells had in the meantime been rendered markedly more sensitive by the prevention of the ascent of the liquid (œnanthol) into the glass tube containing the aluminium wire, the end of which was coated with the selenium layer on which the light was received.

This ascent of the liquid was prevented by simply painting the circle of contact of the wire and tube with liquid gelatine, and allowing the gelatine ring thus formed to dry. As gelatine is not dissolved by œnanthol, the liquid was in this way kept out of contact with the aluminium wire, and the result was a great increase in the sensitiveness and lasting power of the cell.

It is to be regretted that out of the fourteen nights at our disposal for work, only *one* proved favourable for observations; but even on this night we were obliged to omit observations on such bright stars as Sirius, Rigel, and Capella, owing to the persistence of haze in the atmosphere.

The aim of our work on this occasion was to measure the candle power of the light of each of the stars in Orion and the Great Bear, together with that of as many others as spare time would allow us to observe.

As one cell may not be exactly the same in sensitiveness as another, I have set down in the following table the value of the light of each star in terms of that of a standard candle held at a distance of 10 ft. from the sensitive surface in the cell; but the value of the light in volts is easily deduced from the data.

The electrometer employed was that previously used and described; and it continued to give with remarkable constancy a deflection of 630 mm. on a scale distant 7 ft. from the mirror for one Leclanché cell, or, say, 432 mm. for 1 volt.

One observation was attempted for Rigel; but when a deflection of 8 mm. had been attained, the light was cut off by clouds, and no further opportunity occurred.

OBSERVATIONS MADE IN JANUARY, 1896.

Source of light.	Deflection on scale, in millimetres.	Value of light, in terms of standard candle.
Candle, at 10 feet	18·70	—
Jupiter	61·20	3·272
Betelgeuse	12·80	0·685
ζ Orionis (lowest in belt)	3·17	0·170
ε Orionis (middle in belt)	3·29	0·175
Aldebaran	5·21	0·279
η Ursæ Majoris (last in tail)	5·07	0·271
Procyon	4·87	0·261
α Cygni	4·90	0·262
Pole Star	3·10	0·166
Pointer next Pole	2·20	0·117
Ursæ Minoris	2·44	0·130

With regard to the last column in this table, be it understood that it gives *the ratio of the E.M.F. produced by the light of the star as concentrated in a telescope of 2 ft. aperture to the E.M.F. produced by the light of the candle falling directly on the cell from a distance of 10 ft.*

In order to determine the candle value of any star, we must, of course, take into account the area of the telescope aperture, the area of the sensitive selenium surface in the cell, the distance of the star from the earth, and the distance of the candle from the cell.

Thus, if I = intrinsic brightness of a star (*i.e.*, the total amount of energy radiated in all directions by the star per unit time)

i = intrinsic brightness of candle,
A = diameter of aperture of telescope,
a = diameter of selenium surface,
R = distance of star,
r = distance of candle,
e = deflection on electrometer scale due to star,
E = deflection on electrometer scale due to candle,

we have

$$\frac{I}{i} = \left(\frac{a \,.\, R \,.\, e}{A \,.\, r \,.\, E}\right)^2.$$

For example, selecting the star Procyon in the above list as that whose parallax is best known, we find (*see* Miss CLERKE's 'System of the Stars') the value

0·266 second for the parallax. Also the diameter, a, of the sensitive surface was about $\frac{1}{20}$ in., while A was 2 ft., so that $a/A = 1/480$; hence we have (*q.p.*)—

$$I/i = 516 \times 10^{24},$$

that is, Procyon is equivalent to about 516 billions of billions of standard candles.

We cannot help being struck by the enormous power of Betelgeuse as indicated in the table. The deflection observed was the mean of *five*, all very fairly close. Now the only parallax recorded for this star, so far as I can ascertain, is a *negative* one! The star is, therefore, immeasurably distant, and yet the E.M.F. of its light is very great; hence it must be possessed of tremendous energy.

We hope that our next observation will be taken with still better appliances, as Mr. WILSON has suggested a great improvement in the cell-carrying apparatus, which will enable us to see simply and with certainty when the light of the star is falling exactly on the sensitive selenium surface.

On the Proper Motion of Stars in the Dumbbell Nebula.

By Professor Arthur A. Rambaut, *M.A., Sc.D., and* W. E. Wilson, *F.R.A.S.*

Monthly Notices of the Astronomical Society, Vol. LIV., No. 5, 1894.

Most photographs of the Dumbbell Nebula in *Vulpecula* (M. 27, N.G.C. 6853) represent the brighter portion of it as being of a sort of hour-glass form or approximately the figure which would be formed by two opposite sectors of a circle of about 60° each. The stars, too, which are found in the nebula are not distributed wholly without relation to the form of the latter, for it is a remarkable fact that exactly at the common centre of the two sectors, or the neck of the hour-glass, a comparatively conspicuous star is placed, while the extremities of each of the bounding arcs are approximately indicated by stellar points. These stars are are respectively *d, a, m, o,* and *c* in the figure.

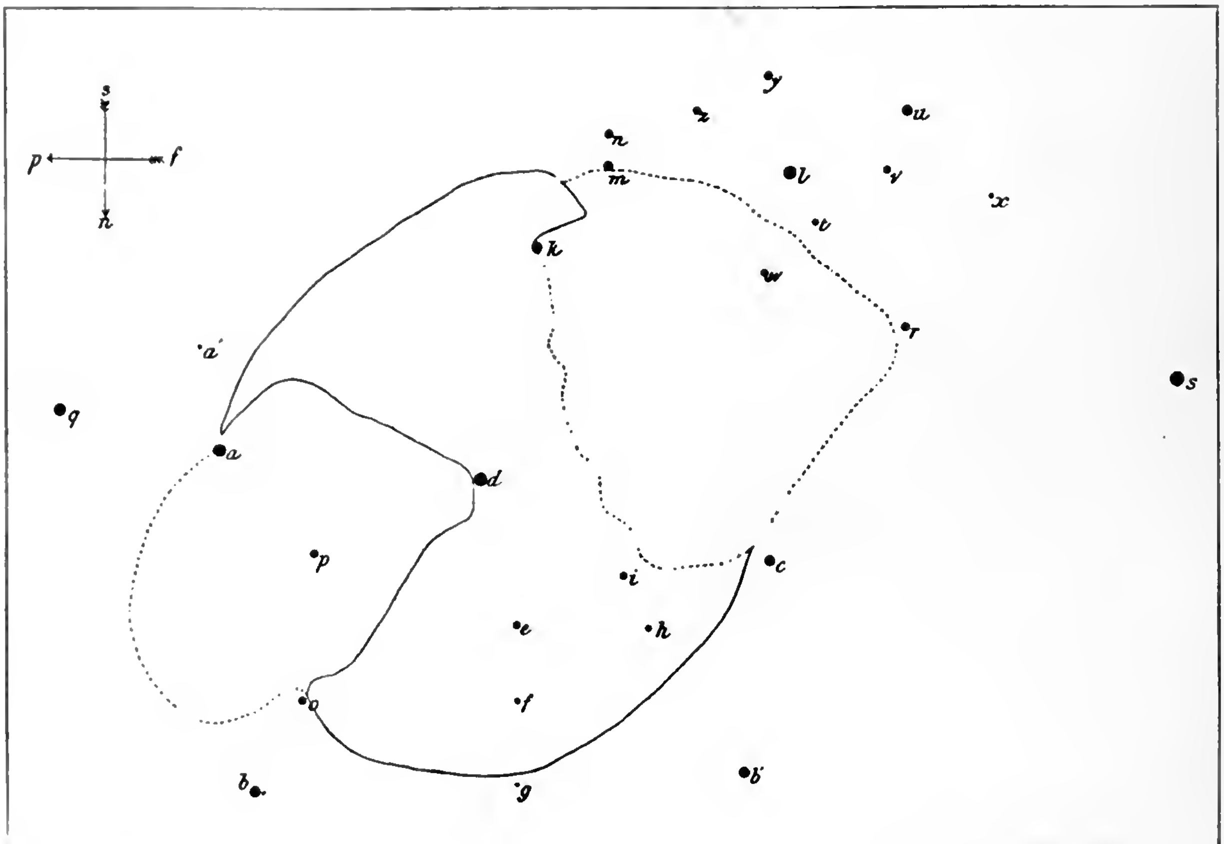
In the ‘Philosophical Transactions of the Royal Society,’ vol. 151, Part III., the Earl of Rosse publishes a note on the positions of twenty-six stars as determined at the Poulkova Observatory by M. Otto Struve, in which the latter expresses his conviction that the probable error in the co-ordinates will be considerably less than one second of arc. As some of these stars lie outside the region over which the nebula extends on the photographs or that which it occupies on the Parsonstown drawing it appears an interesting inquiry whether those stars apparently related to the nebula indicate any proper motion, as compared with those lying outside it, in the forty-two years which have elapsed since M. Struve's measures were made.

In order to investigate this question we measured two photographs taken with the 2-foot equatorial reflector belonging to Mr. Wilson on 1893, September 3 and 8, with exposures of 27^{m} and 2^{h} respectively.

The plates were examined in the measuring microscope of the Dunsink Observatory, in which the co-ordinates are determined on two screws at right angles to each other. The plates were placed in position in the microscope by means of the relative positions determined by M. Struve for the stars *a* and *s* of his list, so that each screw was parallel to one of the axes of co-ordinates employed by that astronomer. The scale value of the photograph, too, was determined from the value of $\Delta\alpha \cos \delta$ given by him for the same stars, which, depending as it does on a larger number of observations than the other measures, is the best determined interval in the group, besides being very nearly the longest. Consequently any error affecting its length will be felt to a smaller extent in the measures of any other interval.

Our co-ordinates are thus directly comparable with those of M. STRUVE, our axis on the plate being, by the above process, set in the direction of the parallel passing through *a* at the date of his observations. The correction for refraction has not been applied, as it was found that in no case would its effect amount to a tenth of a second of arc, and the measures, though probably correct to within two or three tenths of a second, could hardly be relied on to a higher decree of accuracy.

A comparison of our results with those obtained by Mr. BINDON STONEY, using the 6-foot reflector belonging to the Earl of ROSSE, and with those of M. STRUVE, is



given in Table II. The former cannot be depended on with any high degree of precision on account of a difficulty in determining the parallel arising from the altazimuth form of the great reflector. They are, however, given in the table, as in several cases they point to the existence of a proper motion in the same direction as is deduced from a comparison of our measures with those of M. STRUVE.

It will be seen from the columns headed $X_s - X_w$ and $Y_s - Y_w$ that in most cases there are discrepancies which can hardly be attributed to errors of observation. These would indicate in the case of the stars *p*, *d*, *k*, *i*, *b'*, and *y* proper motions amounting in forty-two years to 4″·6, 3″·9, 5″·7, 6″·0, 5″·2, and 5″·3 respectively.

It does not seem, however, that there is any obvious relation between the position which a star bears with regard to the nebula and the amount or direction of its proper motion, but further observations will be necessary to decide this point.

In the accompanying plate the positions of the stars are plotted down from our measures, and the form of the nebula is indicated in outline to facilitate the identification of the stars.

TABLE I.—*Co-ordinates of Stars in the Dumbbell Nebula.*

Name.	X	Y	Name.	X	Y	Name.	X	Y
q	− 94″·4	− 20″·6	*z*	+ 276″·6	− 197″·1	*n*	+ 223″·3	− 183″·7
	94·4	20·6		275·9	196·8		222·8	184·1
	− 94·4	− 20·6		+ 276·3	− 197·0		223·1	− 183·9
a′	− 9·9	− 58·1	*b′*	302·8	+ 187·5	*m*	223·6	− 163·7
	9·2	56·8		303·4	186·8		222·7	164·2
	− 9·6	− 57·5		303·1	+ 187·2		223·2	− 163·9
b	+ 21·9	+ 199·1	*w*	316·2	− 102·7	*i*	232·8	+ 73·6
	22·7	199·2		315·8	101·2		233·1	75·0
	+ 22·3	+ 199·1		316·0	− 102·0		232·9	+ 74·3
o	47·5	+ 145·5	*c*	317·6	+ 63·9	*v*	386·9	− 162·3
	49·0	145·6		317·2	64·3		387·2	161·5
	48·2	+ 145·5		317·4	+ 64·1		387·0	− 161·9
p	56·4	+ 59·6	*y*	317·2	− 216·5	*r*	396·6	− 72·8
	56·9	60·6		317·4	217·1		396·6	72·5
	56·6	+ 60·1		317·3	− 216·8		396·6	− 72 7
d	151·7	+ 16·9	*l*	328·5	− 160·5	*u*	398·9	− 198·0
	151·3	16·3		328·2	160·2		398·9	198·2
	151·5	+ 16·6		328·4	− 160·4		398·9	− 198·1
e	169·8	+ 99·9	*t*	345·3	− 132·2	*x*	442·3	− 146·7
	169·5	100·5		344·4	131·3		442·7	145·6
	169·7	+ 100·2		344·9	− 131·8		442·5	− 146·2
g	171·9	+ 192·8	*f*	171·4	+ 146·9	*s*	555·8	− 41·7
	172·3	193·0		170·8	146·6		555·8	42·2
	172·1	+ 192·9		171·1	+ 146·8		+ 555·8	− 41·9
h	247·1	+ 102·6	*k*	183·0	− 116·7			
	246·7	104·4		182·5	117·3			
	+ 246·9	+ 103·5		+ 182·8	− 117·0			

TABLE II.—*Comparison of Measures.*

Wilson.			*Rosse.*				*Struve.*				
Name.	X_w	Y_w	X_r	$X_r - X_w$	Y_r	$Y_r - Y_w$	X_s	$X_s - X_w$	Y_s	$Y_s - Y_w$	
	″	″	″	″	″	″	″	″	″	″	
q	− 94·4	− 20·6	...	...	...	...	− 97·6	−3·2	− 20·9	−0·3	
a′	− 9·6	− 57·5	−10·9	−1·3	− 59·8	− 2·3	− 12·3	−2·7	− 57·2	+0·3	v. v. f.
b	+ 22·3	+199·1	+23·0	+0·7	+199·7	+ 0 6	+ 21·6	−0·7	+199·7	+0·6	
o	48·2	+145·5	46·8	−1·4	+149·5	+ 4·0	+ 49·6	+1·4	+144·0	−1·5	
p	56·6	+ 60·1	48·1	−8·5	+ 61·3	+ 1·2	+ 60·6	+4·0	+ 57·9	−2·2	Ill defined.
d	151·5	+ 16·6	154·4	+2·9	+ 17·5	+ 0·9	+152·5	+1·0	+ 12·8	−3·8	= *a* in mag.
c	169·7	+100·2	172·8	+3·1	+ 98·6	− 1·6	+172·5	+2·8	+ 98·8	−1·4	
g	172·1	+192·9	173·9	+1·8	+193·2	+ 0·3	+174·3	+2·2	+191·5	−1·4	v. v. f.
f	171·1	+146·8	172·3	+1·2	+144·5	− 2·3	+174·6	+3·5	+142·2	−4·6	
k	182·8	−117·0	187·4	+4·6	−121·3	− 4·3	+186·8	+4·0	−121·0	−4·0	
n	223·1	−183·9	...	...	...	...	+224·9	+1·8	−184·7	−0·8	
m	223·2	−163·9	...	...	...	...	+225·0	+1·8	−164·3	−0·4	
i	232·9	+ 74·3	235·8	+2·9	+ 70·8	− 3·5	+238·2	+5·3	+ 71·5	−2·8	>*h*.
h	246·9	+103·5	246·2	−0·7	+100·5	− 3·0	+251·1	+4·2	+101·7	−1·8	v. nebulous.
z	276·3	−197·0	...	...	...	...	+279·6	+3·3	−199·7	−2·7	
b′	303·1	+187·2	303·6	+0·5	+183·4	− 3·8	+308·3	+5·2	+186·3	−0·9	
w	316·0	−102·0	323·5	+7·5	−115·0	−13·0	+317·7	+1·7	−102·0	0·0	
c	317·4	+ 64·1	325·3	+7·9	+ 63·1	− 1·0	+321·1	+3·7	+ 62·8	−1·3	
y	317·3	−216·8	...	...	...	...	+321·8	+4·5	−219·5	−2·7	
l	328·4	−160·4	...	...	...	...	+332·4	+4·0	−162·2	−1·8	
t	344·9	−131·8	...	...	...	...	+347·8	+2·9	−128·9	+2·9	Ill defined.
v	387·0	−161·9	...	...	...	...	+390·7	+3·7	−161·7	+0·2	
r	396·6	− 72·7	+398·2	+1·6	− 91·6	−18·9	+396·9	+0·3	− 74·0	−1·3	
n	398·9	−198·1	...	...	...	...	+402·4	+3·5	−200·8	−2·7	
x	442·5	−146·2	...	...	...	...	+442·0	−0·5	−148·5	−2·3	v. v. f.
s	+555·8	− 41·9	...	...	...	...	+555·8	0·0	− 42·0	−0·1	

Astrophysical Journal, August, 1899.

Radiation from a Perfect Radiator.

By W. E. Wilson.

Although Kirchhoff introduced the conception of a perfectly "black" body in the deductions of his well-known law connecting the emissive and absorptive power of a body in regard to radiant heat, he seems not to have investigated the subject experimentally, but points out that the law of radiation for a truly "black body" must necessarily be of a simple character.*

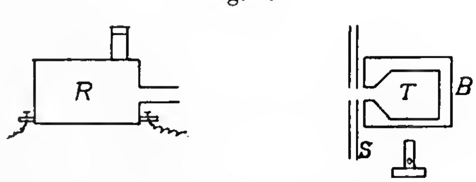
During a conversation with Mr. Lanchester in the autumn of 1895, he pointed out to me that he thought, if we took a hot enclosure into which there was only a small aperture and measured the radiation passing out through this aperture, that the internal walls of the enclosure would behave as a perfect radiator, whether they were a bright metallic surface or coated with lampblack or any other substance.

About the same time Ch. E. St. John also pointed out that in a heated enclosure, such as a fire-clay furnace, metals raised to a red heat appeared of almost equal brightness whether their surfaces were polished or blackened with oxide.

As all investigations up to this time on the laws of radiation were made with the assumption that lampblack was a perfectly black body and therefore a perfect radiator, it seemed of interest to compare the radiation from it with that coming from a hot enclosure with a small aperture, and which would evidently behave as a perfect radiator.

We procured a half-gallon tin *T*, and soldered it by the neck into a large biscuit box *B*. Some water was placed in the biscuit box and kept boiling with a Bunsen burner so that the tin enclosure was completely surrounded with steam at 100° C.

Fig. 1.



A Boys' radio-micrometer *R* was mounted in front of the aperture, and suitable screens *S* were interposed so as to cut off all radiation except that coming from the enclosure through the aperture. The outside of the biscuit box near the aperture was coated with lampblack, and by slightly moving the box we could allow this blackened surface to radiate to the radio-micrometer instead of the enclosure.

* G. Kirchhoff, 'Pogg. Ann.,' 109, 292, 1860.

The temperature of this blackened surface of the biscuit box must have been very nearly the same as that of the enclosure, but we were astonished to find that if we represented the radiation from the enclosure as 100, the radiation from an equal area of the blackened surface was only about 60.

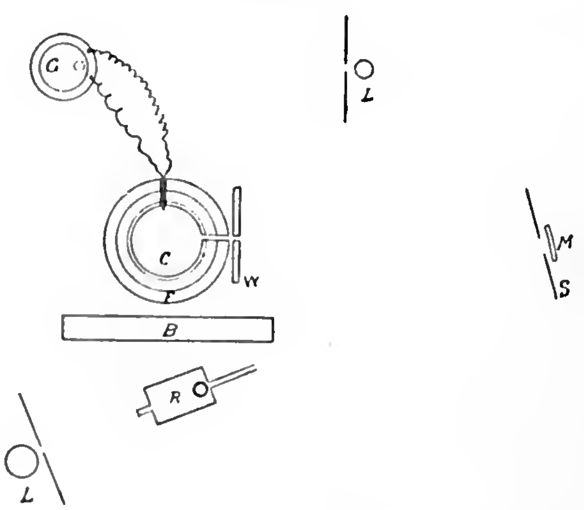
The result of this rough experiment was so interesting that I determined to investigate the law connecting the *total* radiation and temperature of such a theoretically perfect radiator.

An enclosure was formed of a large plumbago crucible with a cover of the same material. This stood in a Fletcher's gas furnace, and could be raised to any desired temperature.

A hole was bored through the walls of the furnace and also through one side of the crucible. This hole was lined with a porcelain tube through which could be seen the interior of the crucible.

A second porcelain tube also passed into the crucible, and was used to carry a thermo-electric junction, made of pure platinum and platinum-rhodium. The current from this was measured by a D'Arsonval galvanometer of low resistance, and its calibrating curve, which was practically a straight line, was obtained by inserting the junction in steam at 100° C., in pure lead freezing, and in pure gold freezing. The radio-micrometer was used to measure the radiation coming from the enclosure, but, as this instrument was so sensitive as to give a considerable deflection before the crucible was even red hot, some means had to be devised to reduce the sensibility of the instrument by a known amount as the temperature of the crucible was raised.

Fig. 2.



Instead of allowing the radiation to fall directly on to the radio-micrometer it was received by a concave silver-on-glass mirror, and this formed an image of the aperture of the hot enclosure on the thermo-couple of the radio-micrometer. In front of this mirror were placed a set of stops of known area, and by changing them the intensity of the image of the aperture and thus the deflections of the radio-micrometer could be altered.

A hole was bored right through the radio-micrometer and provided with a low power positive eyepiece; by looking through this I could see an image of the aperture and also the thermo-couple hanging in front of it. By this means I could be sure that the image of the hot aperture always completely covered the thermo-couple of the radio-micrometer.

In front of the hot aperture was placed the copper screen through which a current of cold water was kept flowing. This screen was provided with a hole about 2 mm. in diameter through which the radiation passed from the enclosure to the mirror and then to the radio-micrometer. The hole was of such a size that the inside of the porcelain tube could not be seen from the radio-micrometer, but only a small area of the inside of the hot enclosure.

Observations were made by raising the temperature of the enclosure to about 1,200° C.; the gas was then cut off, and the furnace and crucible were allowed to cool very slowly. Readings were then taken at frequent intervals of the deflections of the radio-micrometer, and simultaneously the temperature of the enclosure.

TABLE I.

T° absolute.	Radiation.	T° absolute.	Radiation.	T° absolute.	Radiation.	T° absolute.	Radiation.
1,317	1,300	988	360	787	155	630	60
1,266	1,080	963	350	761	133	614	52
1,206	920	945	310	751	127	593	46
1,193	824	921	284	726	112	582	43
1,161	736	901	262	708	99	551	33
1,105	560	875	230	700	94	541	32
1,075	460	855	208	676	81	522	24
1,050	456	826	194	663	74	491	19
1,022	400	805	172	645	65	466	16
1,007	436						

In Table I. the values of a set of readings thus obtained are given, and in fig. 3 the logarithms of these values are plotted. The straight line drawn on the chart represents a fourth power curve, and it will be seen that the observations lie very close to it, and thus seem to confirm STEFAN's* law of radiation where $R = aT^4$.

In another set of readings the observations did not lie quite so close to this curve, but seemed to conform better with $R = aT^{4\cdot3}$.

While this investigation was being carried on, LUMMER and PRINGSHEIM† were also working at the same object, and their very carefully carried out experiments seem to confirm mine, and give also nearly a fourth power law.

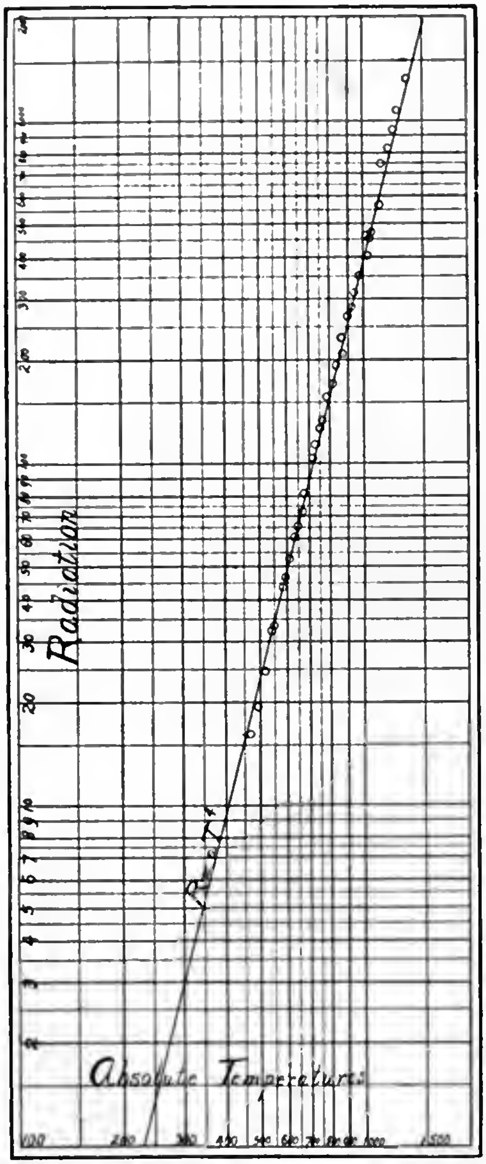
* I. STEFAN, 'Wien. Ber.' (2), 79, 391–428, 1879.

† O. LUMMER und E. PRINGSHEIM, "Die Strahlung eines 'schwarzen' Körpers zwischen 100° und 1300° C.," 'WIED. Ann.,' 54, 1897.

It is only by knowing the true law of radiation that we can possibly measure directly the temperature of the Sun, and therefore any advancement in our knowledge on this subject is of the greatest importance.

In our investigation here in 1893 on the Effective Temperature of the Sun* we had first to study the law of radiation from a platinum strip which was raised to any

Fig. 3.



desired temperature by an electric current, and the radiation from which then balanced the radiation coming from the Sun; the balancing instrument being a duplex Boys' radio-micrometer especially designed for this work.

* Wilson and Gray, "Effective Temperature of the Sun," 'Phil. Trans. Royal Society,' 185, 1894.

In order to cut off all radiation from the incandescent platinum strip except that coming from a known area, the strip was covered with a water-jacket with a small hole through which the radiation passed into the radio-micrometer. The interior walls of this jacket were highly polished and plated with gold.

Since this investigation was made it has been pointed out that if we have a hot body inside an enclosure the inside walls of which are perfect reflectors, and allow heat to pass out through an aperture in the walls, the hot body will behave as if it was a perfect radiator. Now we assumed that the radiation from our platinum strip was only $\frac{35}{100}$ of that from a perfect radiator,* whereas our strip as mentioned was probably behaving very nearly as a perfect radiator. The fact that the law of radiation which we then found from this strip was a fourth power one, and the same as I have since found from a perfect radiator, seems also to indicate that the strip was behaving as a perfectly black body. If this surmise is correct we must clearly multiply the value we obtained of the solar temperature by $\sqrt[4]{\frac{100}{35}} = 1{\cdot}30$. Therefore the effective solar temperature would be 8700° C. $\times\, 1{\cdot}30 = 11300^\circ$ C. An experimental investigation is now being made to clear up this point.

* Schleiermacher, 'Wied. Ann.,' 26, 287, 1885; Rosetti, 'Phil. Mag.,' 8, 445, 1879.

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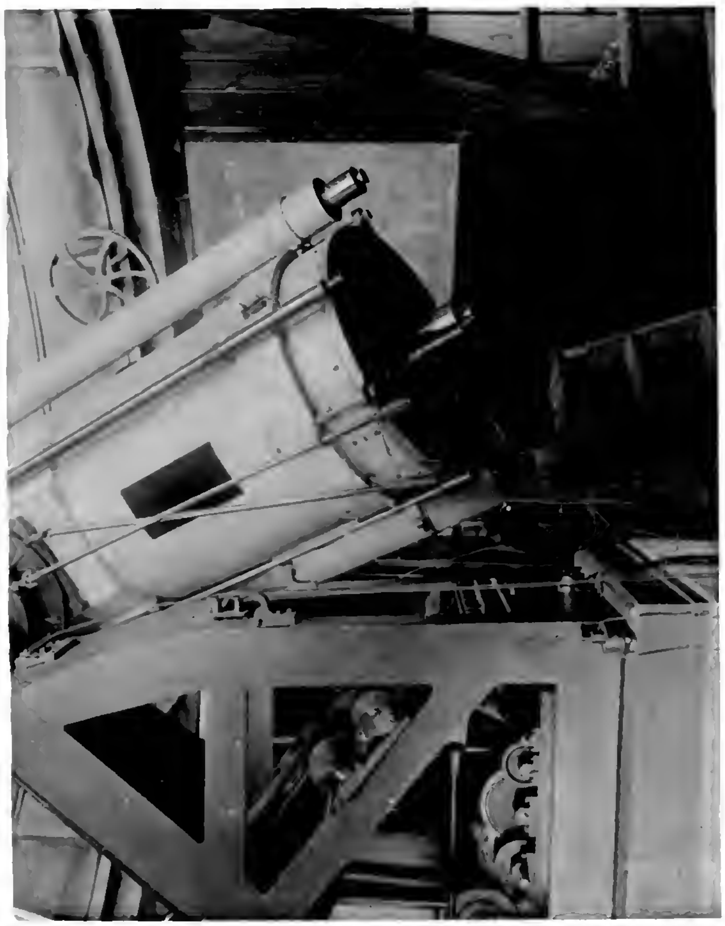
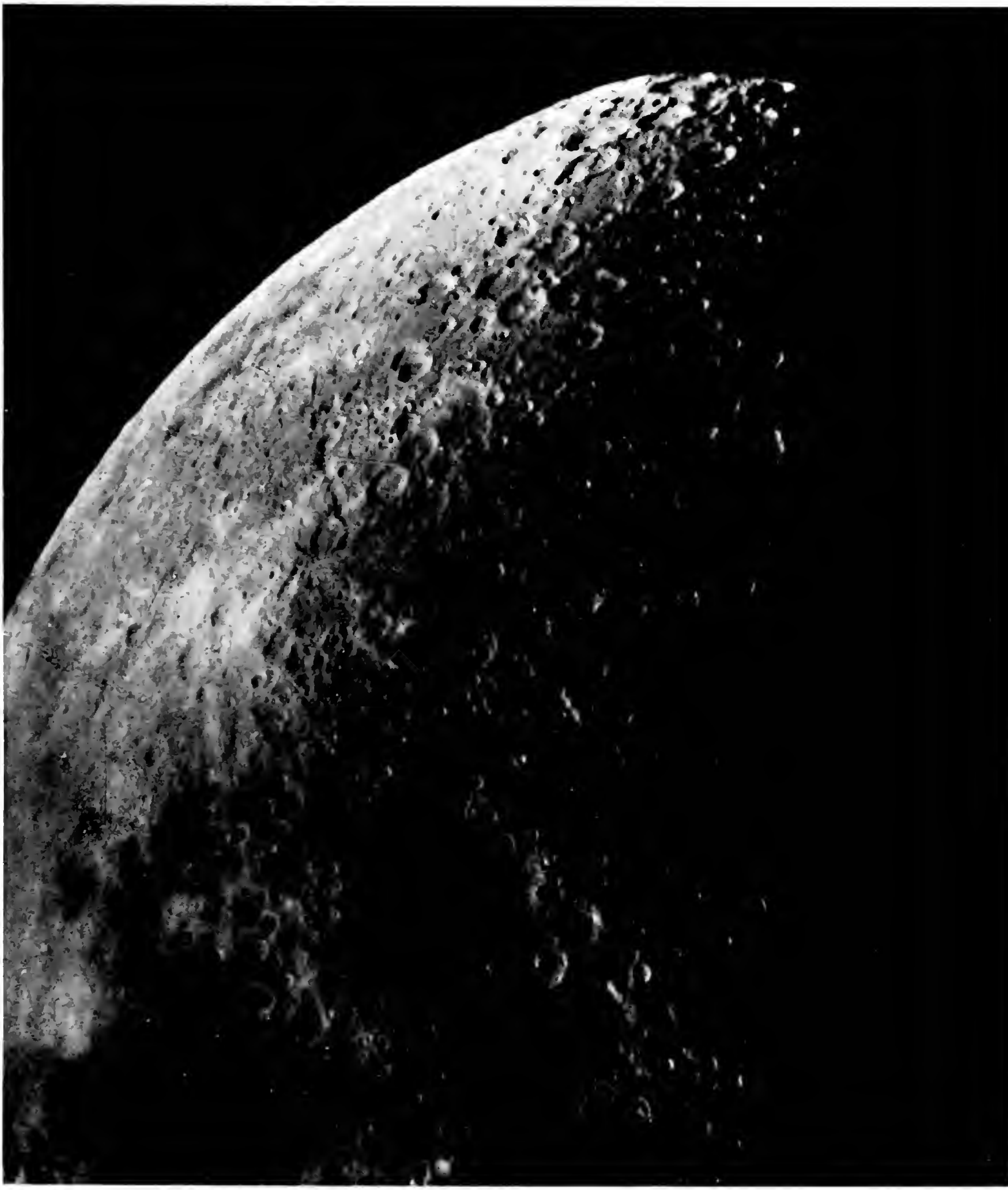
Reflecting Telescope in Daramona Observatory.



The Moon.

This photograph was taken in 1894. Instead of taking the photograph in the primary focus the telescope was used as a Cassegrain, which then gives an image of the moon of about 4·5 inches in diameter. The exposure was about 2^s. Unfortunately, the hole in the great mirror will only allow about half the image to pass through.

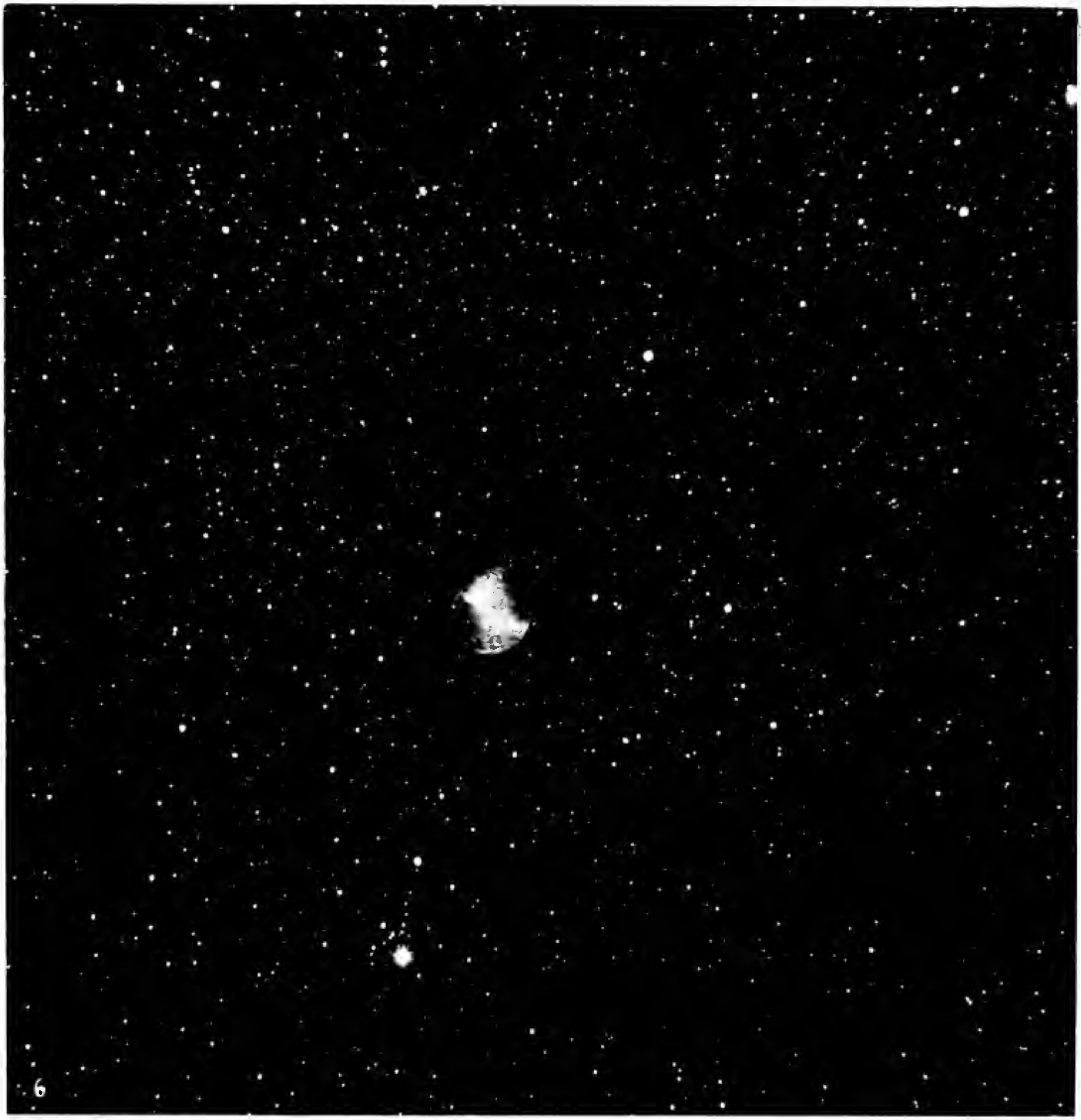


"Dumb-bell" Nebula in Vulpecula.

R. A. $19^{h.}$ $55^{m.}$ $17^{s.}$. Dec. N. 22° 26·8′.

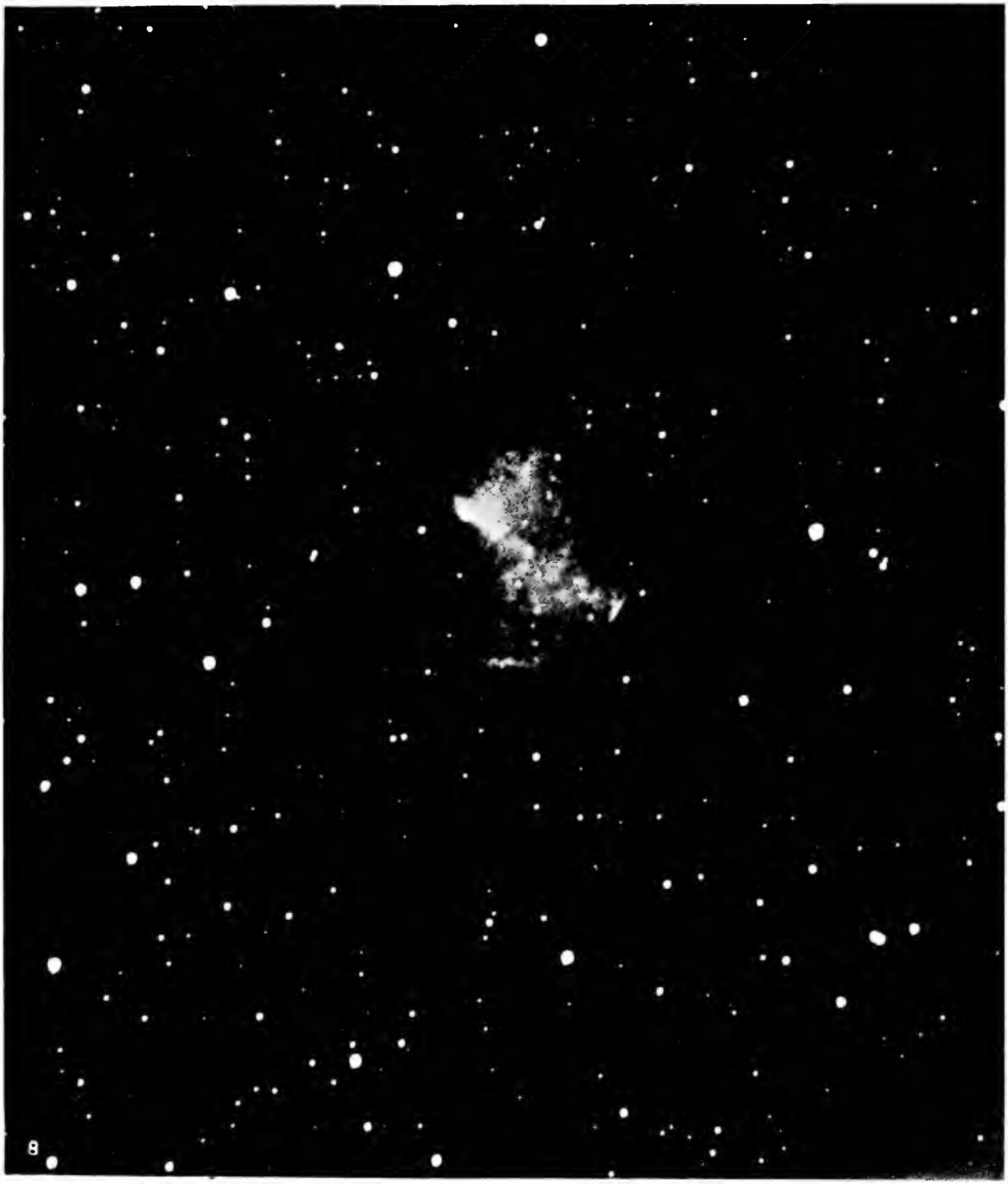
This photograph was taken on the 6th of August, 1894, with an exposure of the plate for $1^{h.}$

This photograph shows the brighter parts of the nebula well, but the fainter parts which lie S.F. and N.P., and which make the entire structure of an elliptical shape, are lost in the reproduction.



THE "DUMB-BELL" NEBULA.

This photograph is an enlargement to ten times linear from the same negative as the last, No. 6. It shows more detail of the brighter parts.

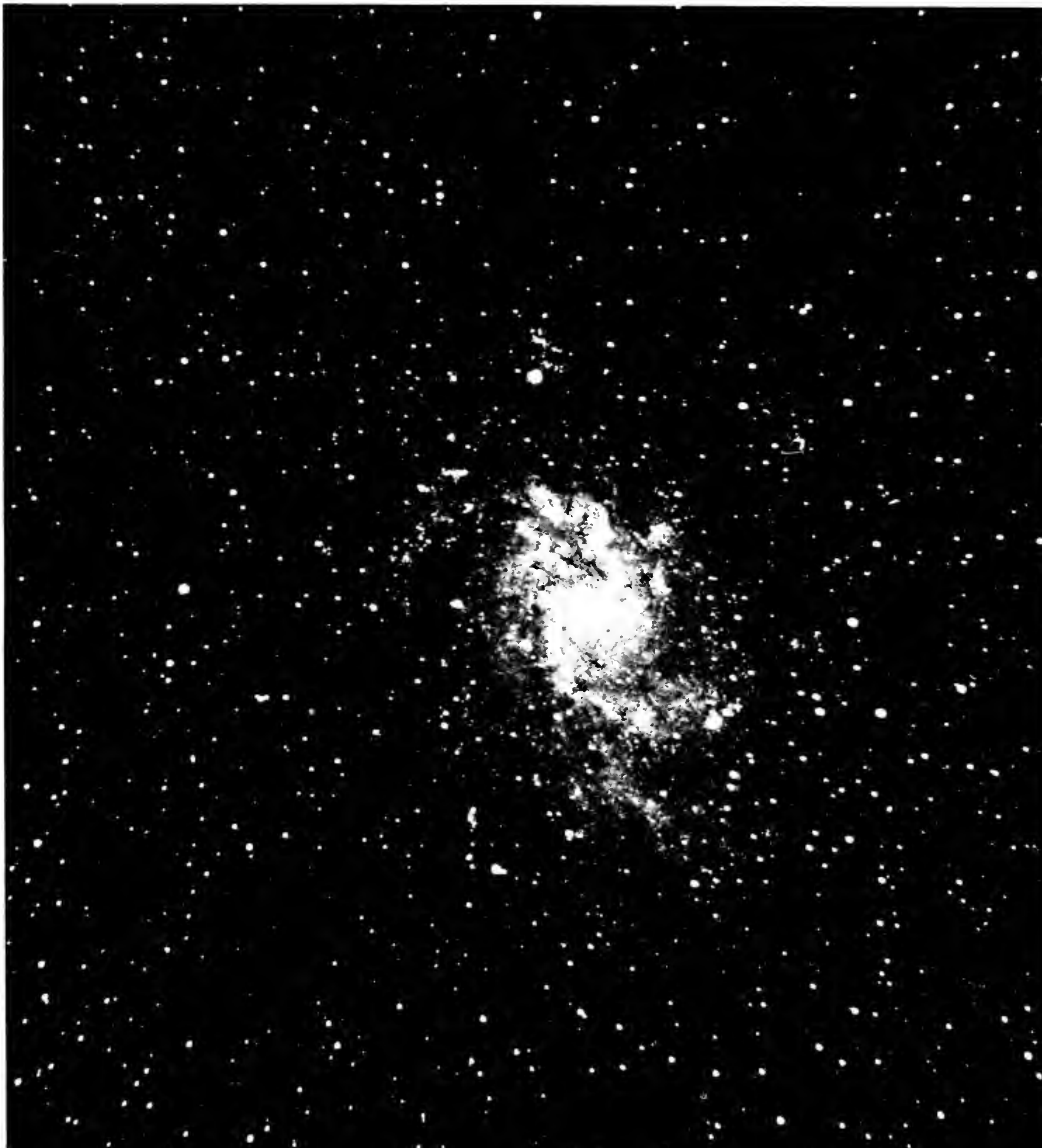

8

Spiral Nebula M 33 Trianguli.

R. A. $1^{h.}$ $27^{m.}$ Dec. N. 30° $4'$.

This photograph was taken in October, 1898, with an exposure of the plate for $1^{h.}$ It shows the spiral structure, and also a marked tendency to break up into knots of more dense nebulosity.

Note.—The stars in the N.F. side are not quite round, and have tails. This was caused by the plate during enlargement not being placed quite parallel with the negative, in which the stars are quite circular. The defect was not noticed until after reproduction.



Spiral Nebula M 51 Canum Venaticorum.

R. A. $13^{h.}$ $25^{m.}$ $39^{s.}$ Dec. N. 47° 42·6′.

This photograph was taken on the 6th of March, 1897, with an exposure of the plate for $1^{h.}$ $30^{m.}$

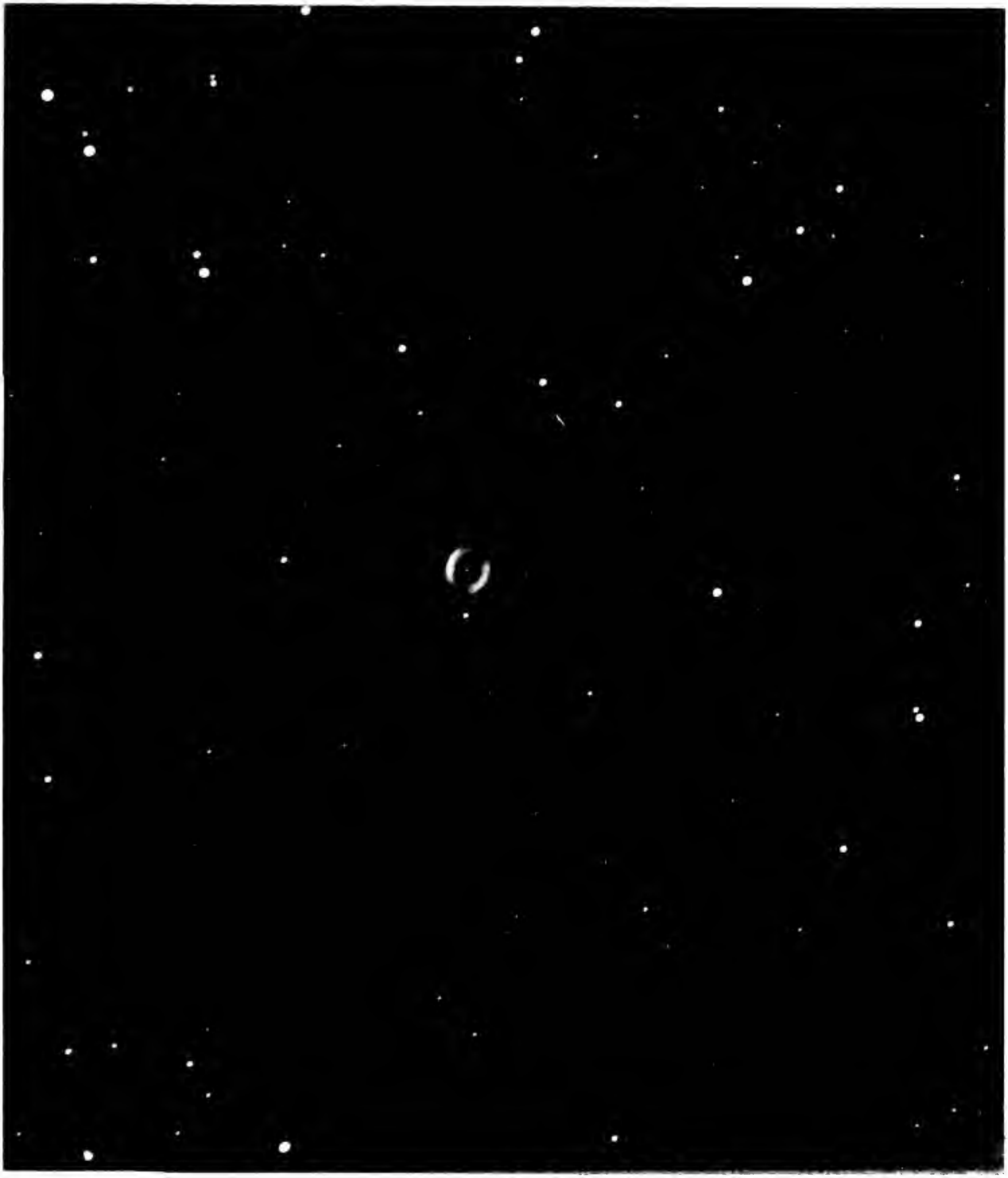
The print here reproduced is enlarged 10 times linear, and shows that the numerous convolutions have an apparent tendency to form into denser knots, which again have cometary tails which are curved like a plume away from the central nucleus. In the original negative both nuclei can be seen to be stellar.

Sir J. Herschel, Mr. Lassell, and Lord Rosse have described and drawn this nebula. The latter, using his great 6-feet reflector, depicts it fairly like the photograph, but even with the use of this great instrument he was evidently unable to see the numerous details here photographed.

Ring Nebula 57 M Lyræ.

R. A. $18^{h.}\ 49^{m.}\ 28^{s.}$ Dec. N. $32^{\circ}\ 53{\cdot}6'$.

This photograph was taken on September 9th, 1894, with an exposure of the plate for $20^{m.}$ Another plate, which got $1^{h.}$ exposure, shows the centre of the ring to be filled with nebulosity, but shows none of the appendages branching into space which Lord Rosse has figured in his drawings. The ring with the long exposure seems to assume a more square outline than in the photograph here shown. Holden, with the great Washington refractor, draws the nebula very like the photograph, but without the central star. Roberts mentions that in some of his photographs this star is very faint, which makes it very probable that it is variable. Huggins finds that the spectrum of the nebula shows it to be luminous gas.



The Great Nebula in Orion.

R. A. $5^{h.}$ $30^{m.}$ Dec. S. 4° 27′.

This photograph was taken on January 23rd, 1897, with an exposure of the plate for $40^{m.}$

7

Nebula H.V. 14. Cygni.

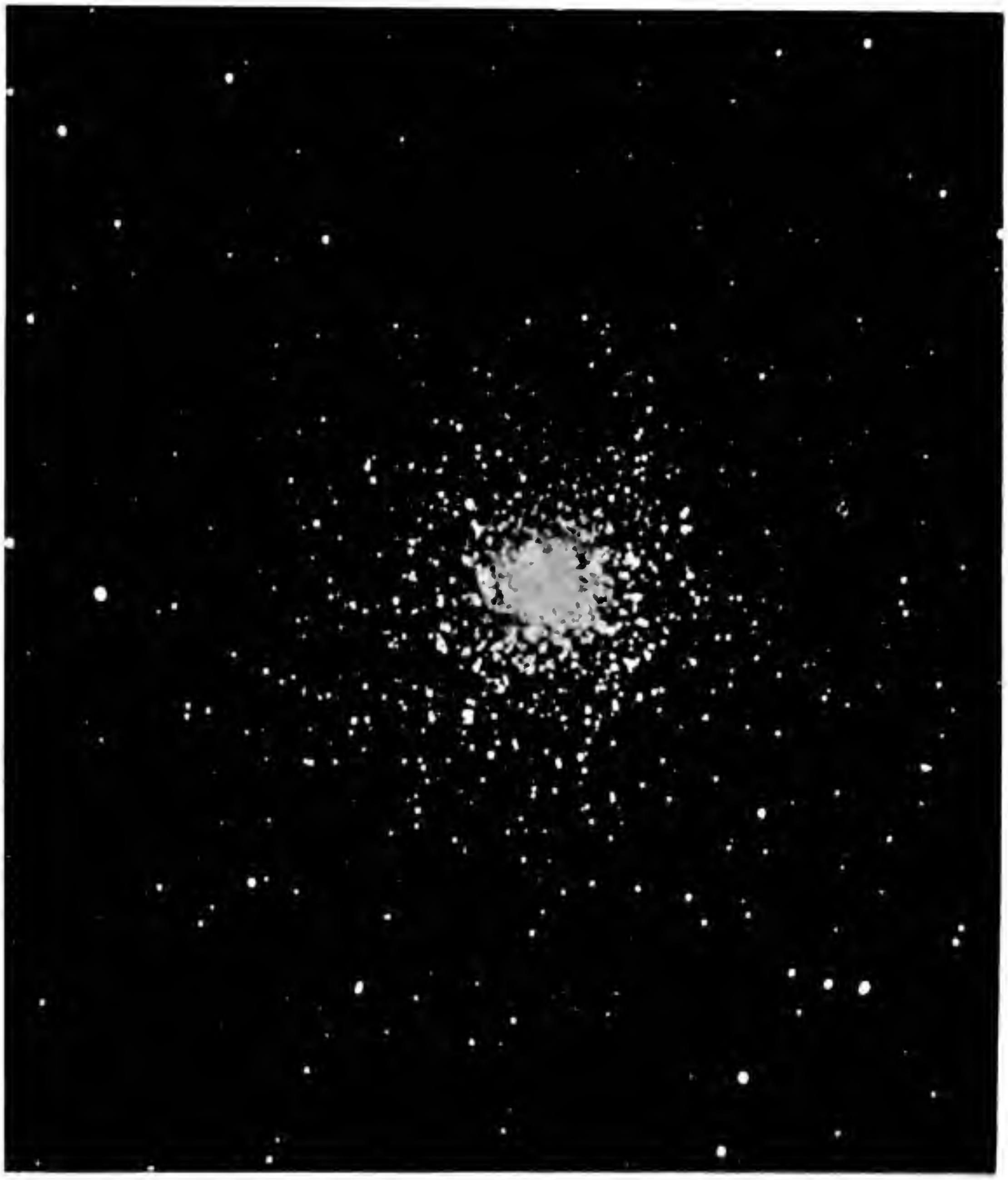
This photograph was taken on October 7th, 1899, with an exposure of the plate for $1^{h.}$ $30^{m.}$. It is an extremely faint object, and even when examined visually with the 24-inch reflector only the brighter parts are visible. There are some curious thin streaks of nebulosity visible in the photograph. The most conspicuous one passes through a rather brighter star in the S.P. quadrant. There is also a curious tendency throughout the entire nebula to form patches in which the general trend are parallel to each other.

Cluster M 13 Herculis.

R. A. $16^{h.}$ $37^{m.}$ Dec. N. 36° 41′.

This photograph was taken on the 5th of August, 1894, with an exposure of the plate for $1^{h.}$

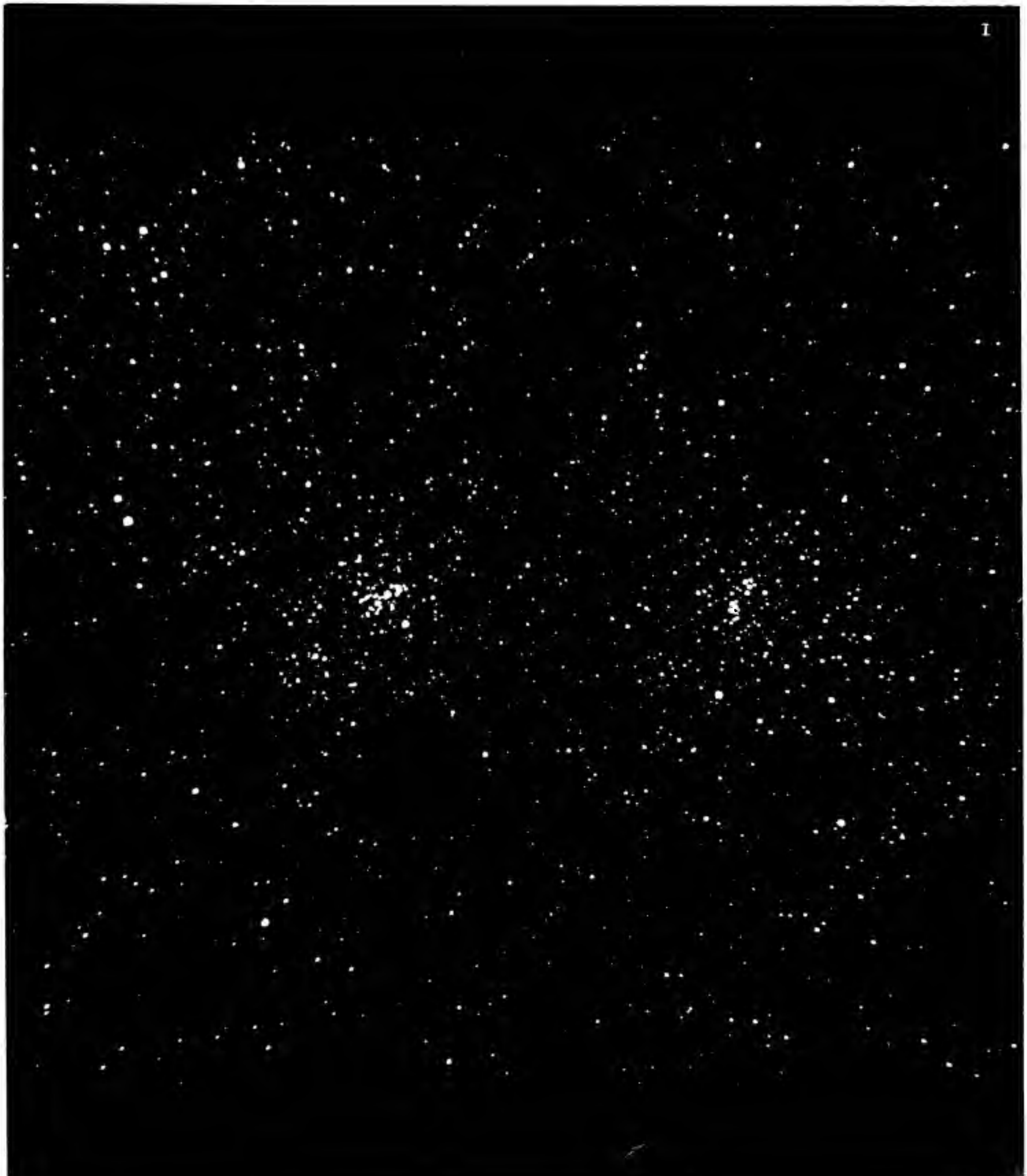
The photograph shows clearly the dark lanes observed by Lord Rosse and others, which meet in the S.F. quadrant forming a figure like the letter **Y**. The stars of the cluster are from the 10th to 15th magnitude.



Clusters $\underset{o}{H}$ VI 33, 34 Persei.

R. A. $2^{h.}$ $12^{m.}$ $2^{s.}$ and $2^{h.}$ $15^{m.}$ $23^{s.}$ Dec. N. 56° 41·3′ and 56° 39·2′.

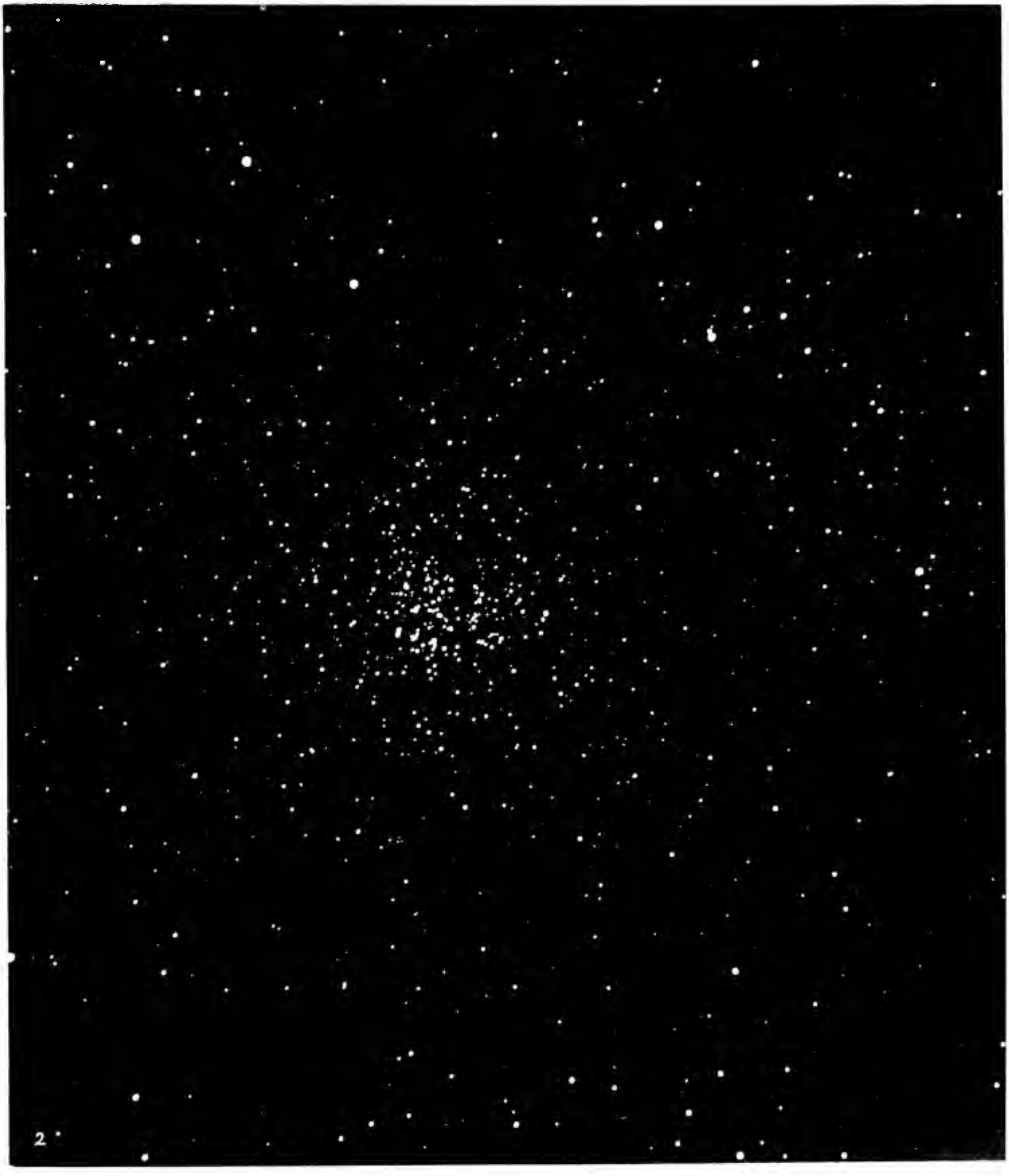
This photograph was taken on October 9th, 1898, with an exposure of the plate for only $15^{m.}$


I

Cluster M 37 Aurigæ.

R. A. $5^{h.}\ 44^{m.}$ Dec. N. $32^{\circ}\ 32'$.

This photograph was taken on the 25th February, 1895, with an exposure of the plate for $40^{m.}$



2