INTERIOR BALLISTICS.

A TEXT BOOK

FOR THE USE OF STUDENT OFFICERS AT THE

U. S. ARTILLERY SCHOOL.

BY

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PREFACE TO THE SECOND EDITION.

When in the summer of 1889 it was decided by the Staff of the Artillery School to add to the curriculum a course of interior ballistics, the instructor of ballistics knowing of no text-book on the subject in the English language entirely suited to the needs of the school, employed the time at his disposal before the arrival of the next class of student officers in studying up and arranging a course of instruction upon this subject, so important to the Artillery officer. The text-book then planned was partially completed and printed on the Artillery-School press, and has been tested by two classes of student officers.

In the summer of 1893 the author again had leisure to work on the unfinished text-book, but in the meantime he had found so much of it which admitted of improvement, that with the encouragement of Lieutenant-Colonel Frank, 2d Artillery, the Commandant of the School, it was decided to re-write nearly the entire work as well as to complete it according to the original plan by the addition of the last two Chapters.

With the exception of portions of Chapters IV and V the author claims no originality. He has simply culled from various sources what seemed to him desirable in an elementary text-book, arranged it all systematically, from the same point of view, and with a uniform notation.

ARTILLERY SCHOOL,
February 15th, 1894.

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Sarrau. Recherches sur les effets de la poudre dans les armes, *and* Formules practiques des vitesses et des pressions dans les armes. A translation of these memoirs into English by *Meigs* and *Ingersoll* is given in Vol. X of the Proceedings U. S. Naval Institute.

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Souich. Poudres de Guerre. Balistique intérieure. Paris, 1882.

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Peabody. Thermodynamics of the steam engine. New York, 1889.

Rentgen. The principles of thermodynamics. Translated from the German by Professor *A. Jay Du Boise.* New York, 1889.

CONTENTS.

ERRATA.

Page 40. In Equation (22) the quantity within the braces should be under the radical sign.

Page 64. Upper line. For "less" read more.

Page 85. Middle of page. Supply the variable to the expression under the sign of integration.

Page 85. Bottom of page. For "page 52" read page 74.

Page 115. Equation (7). For $\tau = \frac{N}{N_1}$ read $\tau = \lambda \frac{N}{N_1}$.

INTERIOR BALLISTICS.

CHAPTER I.

Physical and Mechanical Properties of Gunpowder.

Definition and object—Interior ballistics treats of the formation, temperature and expansion of the highly elastic gases into which the powder-charge in the chamber of a gun is converted by combustion, and the work performed by these gases upon the gun, carriage and projectile. Its object is the deduction and discussion of rules and formulas for calculating the velocity—both of translation and of rotation—which the combustion of a given weight of powder of known constituents and quality is able to impart to a projectile, and the reaction of the charge upon the walls of the gun, and upon the carriage.

The discussion of the formulas which will be deduced, will bring out many important questions, such as the proper relation of weight of charge to weight of projectile and length of bore—the best size and shape of powder grains for different guns, and their effect upon the maximum pressure—the velocity of recoil, etc. The most approved formulas for calculating the pressures upon the surface of the bore will be given; but the methods which have been devised for building up the gun so as best to resist these pressures will not be entered upon here as their consideration belongs to another department of the subject.

Early History of the Ballistics of Gunpowder.—For more than five hundred years gunpowder has been used almost exclusively as the propelling agent in fire arms; and though it is apparently about to be superseded (for military purposes at least) by the so called smokeless powders, it has many admirable qualities which the new powders must copy in order to be successful. Gunpowder ignites easily without deflagration, burns quickly, leaves but little residuum and liberates a large quantity of gas at a high temperature. Its effects are regular and sure. Its manufacture is economical, rapid and comparatively safe. Finally, it keeps well in transportation, and indefinitely in properly ventilated magazines. It is on record that experiments made with gunpowder manufactured more than two centuries before, showed that it had lost none of its ballistic qualities.

It is only within the last half of the present century however that the complicated phenomena which take place in the bore of a gun have been clearly apprehended, and we have still much to learn before the subject shall have reached that degree of perfection demanded by modern gunnery.

Robins' Experiments and Deductions.—The celebrated *Benjamin Robins* seems to have been the first investigator who had a tolerably correct idea of the circumstances relating to the action and force of fired gunpowder. In a paper which was read before the Royal Society in 1743 entitled, "New principles of gunnery," *Robins* described among other things some experiments he had made for determining the velocities of musket balls when fired with given charges of powder. These velocities were measured by means of the ballistic pendulum invented by *Robins*, "the idea of which is simply that the ball is discharged into a very large but movable block of wood, whose small velocity, in consequence of that blow, can be easily observed and accurately measured. Then, from this small velocity thus obtained, the large one of the ball is immediately derived from this simple proportion, viz, as the weight of the ball is to the sum of the weights of the ball and the block, so is the observed velocity of the last to a fourth proportional, which is the velocity of the ball sought."[1] The deductions which *Robins* makes from these experiments, so far as they relate to interior ballistics, may be summarized as follows:

(1) Gunpowder fired either in a vacuum or in air produces by its combustion a permanent elastic fluid or air.

(2) The pressure exerted by this fluid is, *cæteris paribus*, directly as its density.

(3) The elasticity of the fluid is increased by the heat it has at the time of explosion.

(4) The temperature of the fluid at the moment of combustion is at least equal to that of red-hot iron.

[1] Hutton's Mathematical Tracts, Vol. 3, p. 210 (Tract 37), London, 1812.

(5) The maximum pressure exerted by the fluid is equal to about 1,000 atmospheres.

(6) The weigth of the permanent elastic fluid disengaged by the combustion is about three-tenths that of the powder, and its volume at ordinary atmospheric temperature and pressure is about 240 times that occupied by the charge.

These deductions, considering the extremely erroneous and often absurd opinions that were entertained by those who thought upon the subject at all in *Robins*' time—and even down to the close of the century—show that *Robins* is well entitled to be called the "father of modern gunnery."

Hutton's Experiments. — Dr. *Charles Hutton*, professor of mathematics in the Royal Military Academy, Woolwich, continued *Robins*' experiments at intervals from 1773 to 1791. He improved and greatly enlarged the ballistic pendulum so that it could receive the impact of 1-pound balls, whereas that used by *Robin's* was adapted for musket balls only. *Hutton's* experiments are given in detail in his thirty-fourth, thirty-fifth, thirty-sixth, and thirty-seventh tracts. They verify most of *Robins*' deductions, but with regard to *Robins*' estimate of the temperature of combustion and the maximum pressure *Hutton* says: "This was merely guessing at the degree of heat in the inflamed fluid, and, consequently, of its first strength, both which in fact are found to be much greater."[1] His own estimate of the temperature is double that of *Robins*', and he places the maximum pressure of fired gunpowder at 2,000 atmospheres. *Hutton* gives a formula for the velocity of a spherical projectile at any point of the bore, upon the assumption that the combustion of the charge is instantaneous and that the expansion of the gas follows *Mariotte's* law —no account being taken of the loss of heat due to work performed—a principle which at that time was unknown.

D'Arcy's Method.—In 1760 the chevalier *D'Arcy* sought to determine the law of pressure of the gas in the bore of a musket by measuring the velocity of the projectile at different points of the bore. This he accomplished by successively shortening the length of the barrel and measuring for each length the velocity of the

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[1] Tracts, vol. 3, p. 211.

bullet by means of a ballistic pendulum. Having obtained from these experiments the velocities of the bullets for several different lengths of travel, the corresponding accelerations could be calculated, and then the pressures, by multiplying the accelrations by the mass. This was the first attempt to determine the law of pressures dynamically.

Rumford's Experiments with fired Gunpowder.—The first attempt to measure directly the pressure of fired gunpowder was made in 1792 by our countryman, the celebrated Count *Rumford.* A most interesting account of his experiments is given in his memoir entitled "Experiments to determine the force of fired gunpowder,"[1] which must be regarded as the most important contribution to interior ballistics which had been made up to that time. The apparatus used by *Rumford* consisted of a small and very strong wrought-iron mortar (or eprouvette), which rested with its axis vertical upon a solid stone foundation. This mortar (or barrel, as *Rumfora* calls it) was 2.78 inches long and 2.82 inches in diameter at its lower extremity and tapered slightly toward the muzzle. The bore (or chamber) was cylindrical, one-fourth of an inch in diameter and 2.13 inches deep. At the center of the bottom of the barrel there was a projection 0.45 inch in diameter and 1.3 inches long, having an axial bore 0.07 inch in diameter connecting with the chamber above, but closed below, forming a sort of vent, but having no opening outside.

By this arrangement the charge could be fired without any loss of gas through the vent by the application of a red-hot ball provided with a hole, into which the projecting vent-tube could be inserted, which latter would thus become in a short time sufficiently heated to ignite the powder. The upper part of the bore or muzzle was closed by a stopper made of compact, well-greased sole leather, which was forced into the bore until its upper surface was flush with the face of the mortar, and upon this was placed the plane surface of a solid hemisphere of hardened steel, whose diameter was 1.16 inches. "Upon this hemisphere the weight made use of for confining the elastic fluid generated from the powder in its combustion reposed. This weight in all the

[1] Philosophical Transactions, London, 1797, p. 222; also The Complete Works of Count Rumford, Boston, 1870, vol. 1, p. 98.

experiments, except those which were made with very small charges of powder, was a piece of ordnance of greater or less dimensions or greater or less weight, according to the force of the charge, placed vertically upon its cascabel upon the steel hemisphere which closed the end of the barrel; and the same piece of ordnance, by having its bore filled by a greater or smaller number of bullets, as the occasion required, was made to serve for several experiments."[1]

As one of the objects of *Rumford's* experiments was to determine the relation between the pressure of the powder gases and their density, he varied the charge, beginning with 1 grain, and for each charge placed a weight, which he judged was about equivalent to the resulting pressure, upon the hemisphere. If, on firing, the weight was lifted sufficiently to allow the gases to escape, it was increased for another equal charge; and this was repeated until a weight was found just sufficient to retain the gaseous products—that is, so that the leathern stopper would not be thrown out of the bore, but only slightly lifted. The density of the powder gases could easily be determined by comparing the weight of the charge with the weight of powder required to completely fill the chamber and vent, which latter was about 25½ grains troy. *Rumford* increased the charges a grain at a time from 1 grain to 18 grains, and from a mean of all the observed pressures he deduced the empirical formula,

$$p = 1.841 x^{1+.0004x},$$

in which p is the pressure in atmospheres and x the density of loading to a scale of 1000—that is, for a full chamber $x = 1000$; for one-half full $x = 500$, and so on. This formula gives 29,178 atmospheres for the maximum pressure—that is, when the powder entirely fills the space in which it is fired. In this case the value of x is 1000, and *Rumford's* pressure formula becomes

$$p = 1.841 \times 1000^{1.4} = 29178$$

Nearly a century later *Noble* and *Abel* (see Chapter III) found by their experiments, which are entirely similar in character to those of Rumford, that the maximum pressure of fired gunpow-

[1] Rumford's Works, vol. 1, p. 121.

der is but 6,554 atmospheres, or 43 tons per square inch; and this result has been accepted by all writers on interior ballistics as being very near the truth. Their formula for the pressure in terms of *Rumford's* x is

$$p = \frac{2.818x}{1 - 0.00057x}$$

in which p and x are defined as before. If in this formula we make $x = 1000$. we have, as already stated,

$$p = \frac{2.818 \times 1000}{1 - 0.57} = 6554$$

For small densities of loading, *Noble* and *Abel's* formula gives greater pressures than *Rumford's* principally because the powder used by the later investigators was the stronger; but as the densities increase this is reversed. With a charge of 18 grains, for which $x = 702$, *Noble* and *Abel's* formula gives a pressure of 3,298 atmospheres, while *Rumford's* gives 8,140 atmospheres. To enable us to understand the cause of this great difference in the results obtained by these eminent savants (which is very instructive) we will go a little into detail. Two experiments were made by *Rumford* with a charge of 18 grains of powder. In the first of these a 24-pounder gun, weighing 8,081 pounds, was placed vertically on its cascabel upon the steel hemisphere closing the muzzle of the barrel. When the charge was fired "the weight was raised with a very sharp report, louder than that of a well-loaded musket." The barrel was again loaded with 18 grains as before, and enough shot were placed in the bore of the 24-pounder gun to increase its weight to 8,700 pounds. Upon firing the powder by applying the red-hot ball "the vent tube of the barrel was burst, the explosion being attended with a very loud report." These experiments were the eighty-fourth and eighty.fifth, and closed the series.

In the eighty-fourth experiment a weight of 8,081 pounds was actually raised by the explosion of 18 grains of powder (about one-fourth the service charge of the Springfield rifle), acting upon a circular area one-quarter of an inch in diameter. To raise this weight under the circumstances would require a pressure of more than 11,200 atmospheres, while, as we have seen, the actual pressure due to this density of loading, according to *Noble* and *Abel's* formula, is but 3,298 atmospheres. Evidently then the weight in this experiment was not raised by mere pressure; but we must attribute a great part of the observed effect (in consequence of the position of the charge at the bottom of the bore), to the energy with which the products of combustion impinged against the leathern stopper, which had only to be raised 0.13 inch (the thickness of the leather) to allow the gases to escape. In *Noble* and *Abel's* experiments there was no such blow from the products of combustion because the apparatus for determining the pressure (crusher gauge) was placed within the charge. Had the leathern stopper in *Rumford's* experiments been an inch or two longer, it is probable that his conclusions would have been entirely different.

Rodman's inventions and experiments.—We have space only to mention the names of *Gay-Lussac*, *Chevreul*, *Graham*, *Piobert*, *Cavalli*, *Mayevski*, *Otto*, *Neumann*, and others, who did original work, of more or less value, for the science of interior ballistics prior to the year 1860. We will, however, dwell a few moments on the important work done by Captain (afterwards General) *T. J. Rodman*, of our own Ordnance Department, between the years 1857 and 1861.[1] The objects of *Rodman's* experiments were: First, to ascertain the pressure exerted upon different points of the bore of a 42-pounder gun in firing under various circumstances. Second, to determine the pressures in the 7-inch, 9-inch, and 11-inch guns when the charges of powder and the weight of projectiles were so proportioned that there should be the same weight of powder behind, and the same weight of metal in front of each square inch of area of cross-section of the bore. Third, to determine the differences in pressure and muzzle velocity due to the variations in the size of the powder grains; and, fourth, to determine the pressures exerted by gunpowder burned in a close vessel for different densities of loading.

For the purpose of carrying out these experiments *Rodman*, instead of using the system of varying weights employed by *Rum-*

[1] Experiments on metal and cannon and qualities of cannon powder, by Captain T. J. Rodman, Boston, 1861.

ford, invented what he called the "indenting apparatus," which has since been extensively used, not only in this country but in all foreign countries, under the name of *Rodman's* pressure (or cutter) gauge; and which is too well known to require a description.

The maximum pressure of gunpowder when exploded in its own space, as determined by *Rodman* by the bursting of shells filled with powder, ranged from 4900 to 12600 atmospheres; the mean of all the experiments giving 8070 atmospheres, or 53 tons per square inch. These results though much nearer the truth than those deduced by *Rumford*, are still about 25 per cent greater than *Noble* and *Abel's* deductions; and this is undoubtedly due to the position of the pressure gauge, which was placed near the exterior surface of the shell, so that when the products of combustion had reached the gauge they had acquired a considerable energy which probably exaggerated the real pressure. The same causes it will be remembered vitiated *Rumford's* experiments. In both cases it was as if a charge of small shot had been fired with great velocity against the leathern stopper in the one case, or the end of the piston of the indenting tool in the other.

General *Rodman* was the first person to suggest the proper shape for powder grains, in order to diminish the initial velocity of emission of gas and to more nearly equalize the pressure in the bore of the gun. For this purpose he employed what he termed a "perforated cake cartridge" composed of discs of compressed powder from 1 to 2 inches thick and of a diameter to fit the bore. *Rodman* demonstrated that such a form of cartridge would relieve the initial strain by exposing a minimum surface at the beginning of combustion, while a greater volume of gas would be evolved from the increasing surfaces of the cylindrical perforations as the space behind the projectlle became greater; and this would tend to distribute the pressure more uniformly along the bore. *Rodman's* experiments with this powder in the 15-inch cast-iron gun which he had recently constructed for the Government—and which is without doubt the most effective and the best smooth-bore gun ever made—fully confirmed his theory; but for many reasons he found it more convenient and equally satisfactory to build up the charge by layers of pierced hexagonal prisms about an inch in diameter fitting closely to one another, instead of having them of the diameter of the bore

The war of the rebellion which was inaugurated while General *Rodman* was in the midst of his discoveries and inventions, put an end forever to his investigations, but his ideas were speedily adopted in Europe, and his "prismatic powder" but slightly modified, is now used by all civilized nations.

Composition of Gunpowder.—Gunpowder is an intimate mechanical mixture of nitre, sulphur and carbon (in the form of charcoal), in certain proportions depending upon the use for which it is intended, and which is called *composition*, (French *dosage*). In order to determine the proper composition to secure the best results it is necessary to understand the role which each of the ingredients performs by its reactions, when fired.

In thermo-chemistry—a science which has been created within the last twenty-five years—chemical compounds are divided, according to the thermal phenomena attending their formation, into two classes, namely: *Exothermous* bodies, whose formation evolves heat; and *endothermous* bodies whose formation is attended with the absorption of heat and consequently whose decomposition disengages more or less heat. Prominent among this latter class of unstable compounds is *nitre* (NO_3K), which is readily decomposed by a moderate heat, setting at liberty the heterogeneous atoms of which it is composed. The potassium and oxygen unite respectively with the sulphur and carbon of the gunpowder forming exothermous compounds, while the nitrogen having but feeble affinities, does not enter into new combinations. In other words the decomposition of the nitre of the gunpowder and the reactions of the various elements form a large volume of gas at a high temperature, and consequently when confined it produces the phenomena of explosion.

Among the binary compounds of nitre and sulphur, or nitre and carbon, which by their reactions produce physical results most nearly analogons to those of fired gunpowder, are the two following:

$$2NO_3K+4S=K_2S+3SO_2+2N$$

and

$$2NO_3K+3C=(K_2O,\ CO_2)+CO_2+CO+2N$$

Of these two compounds the second is the more powerful since there is more heat evolved. But if the powder contained only nitre and carbon, one part of this last element and one part of oxygen would not be fully utilized; for the radical K_2O is not decomposable by the heat produced. It follows therefore that a more energetic explosion would result if to the binary nitro-carbon compound there should be added a sufficient amount of sulphur to take the place of the oxygen of the radical K_2O. This would produce the following reaction:

$$2NO_3K+3C+S=K_2S+3CO_2+2N.$$

The addition of the sulphur produces other advantages also; for as it ignites at a lower temperature than carbon, it increases the inflammability of the compound; and, in addition, favors the preservation of the powder on account of its less affinity for water than the other components, and, also, by giving hardness to the mixture and consequently more body to the grains.

The last equation given above is a *theoretical* formula for the combustion of gunpowder. By introducing the atomic weights of the elements we easily deduce its composition. We find in this way that in 100 parts the theoretical powder would contain by weight

74.815	parts of	nitre.
11.852	" "	sulphur.
13.333	" "	carbon.

This theoretical composition is that which secures the best utilization of the elements. It produces the greatest quantity of gas and heat, and the least fouling matter. From the point of view of the preservation of the powder, this composition also merits the preference; for, the carbon which has the greatest affinity for water of the three ingredients, is only employed in quantity necessary for uniting with the oxygen of the nitre to form carbon dioxide; while the proportion of sulphur just suffices to unite with the potassium of the nitre and to give consistency to the grains.

Influence of Composition upon the Ballistic Effects of Gunpowder.—The theoretical composition is based upon the hypothesis of an absolute purity of the component parts. It is possible, indeed, by refining to eliminate all, or nearly all, of the impurities found in the saltpetre and sulphur of commerce; but the charcoal which, in practice, always supplies the element carbon, contains besides this element, very appreciable quantities of hydrogen, oxygen and salts depending upon the kind of wood employed and the manner of preparing the charcoal.

For any given composition the carbon is the element which has the greatest influence upon the quality of the powder. The brown charcoal which is more porous and richer in gaseous matter than the black charcoal, renders the powder more hygrometric, but also more inflammable. On the other hand, black charcoal assures to the powder a greater density, and the density of the powder is, of all its physical characteristics, that which most modifies its effects in fire arms.

The following table gives the principal compositions actually adopted by different countries:

COUNTRY.		NITRE.	SULPHUR.	CARBON.
FRANCE	Cannon powder, small grain	75	12.5	12.5
	Cannon powder, large grain	75	10	15
	Musket powder	77	8	15
	Sporting powder	78	10	12
	Mining powder	62	20	18
GERMANY	War powder	74	10	16
	Sporting powder	78.5	10	11.5
	Mining powder	66	12.5	21.5
AUSTRIA	War powder	74	10	16
	Sporting powder	75.8	9.7	14.5
	Mining powder	62.2	18.4	19.4
ITALY.	Cannon powder	75	10	15
RUSSIA.	" "	75	10	15
SWITZERLAND.	" "	76	10	14
SPAIN.	" "	75	12.5	12.5
PORTUGAL.	" "	75.7	10.7	13.6
UNITED STATES.	" "	75	10	15
ENGLAND.	" "	75	10	15

Products of Combustion of Gunpowder.—The reactions which take place in the combustion of gunpowder, are much more complicated than is indicated by the theoretical formula. The reason is that this formula does not take account either of the heterogeneous composition of the charcoal, or of the role of the atmospheric air. In reality the phenomena accompanying combustion, whether under small or great pressure, give rise to products much more varied than those deduced by theory. These products are partly gaseous and partly solid; and their relative proportions, as well as their compositions have been made the subject of a great many researches. According to the experiments of *Bunsen* and *Schischkoff* (1857), and of *Noble and Abel* (1874), these products are, (besides potassium sulphate, carbon dioxide and nitrogen, which are given by the theoretical formula), potassium carbonate, potassium monosulphide, oxygen, carbonic oxide, hydrogen, vapor of water, etc., etc. (See *Noble* and *Abel's* Researches, etc.)

The products of combustion vary also with the pressure as has been demonstrated by many experiments. An increase of pressure during combustion produces a more complete decomposition of the powder, from which results a much greater disengagement of heat and less fouling matter.

When a charge of powder is exploded in a cannon, only a small part of the solids and liquids formed remains in the bore; these substances are for the most part volatilized by the intense heat of the explosion and are carried out by the gaseous currents; but it is easy to see that, other things being equal, the amount of fouling matter deposited on the sides of the bore will depend upon the quantity of solid residuum produced by the explosion.

The physical characteristics of gunpowder which have the greatest influence upon its dynamic effects when exploded in a gun, are the following;—the density, size, form and hardness of the grains and their affinity for moisture.

Density of Gunpowder.—By density of gunpowder is meant its specific gravity, or the ratio of the weight of a given quantity to the weight of an equal volume of water at the standard temperature. It is also called *mercurial density*, since it is determined by an apparatus which utilizes the property of mercury of filling the interstices between the grains, without penetrating into the pores. The density varies in different powders according to the pressure to which the grains were subjected during manufacture; and ranges from about 1.5 to 1.9.

Size of Grains.—In order to take account of the influence of the size of the grains upon the mode of action of gunpowder, it is indispensable to analyze the phenomena of *inflammation* and of *combustion.*

Inflammation and Combustion of a grain of Gunpowder.—Inflammation is the spreading of the flame over the free surface of the grain from the point of ignition. Combustion is the propagation of the fire into the interior of the grain.

Ignition is produced by the sudden elevation of the temperature of a small portion of the grain to 300° C., either by contact with an ignited body, by mechanical shock or friction, or by detonation of a fulminate. The velocity of inflammation depends upon the nature of the source of the heat which produces it, also upon the state of the surface of the grain, and upon its density and dryness. A large, dense, round and smooth grain ignites with comparative difficulty. The composition of the powder and time of trituration appear to have but little influence. Brown charcoal makes the powder more inflammable than black charcoal.

The combustion of a grain takes place in successive, concentric layers, and in free air, equal thicknesses are burned in equal times. As the mass of gas disengaged in any given time is proportional to the quantity of powder burned during the same time, and, therefore, proportional to the surface of inflamation, it follows that if the grain is spherical, or nearly so, the emission of gas rapidly decreases from the moment of inflammation of the grain up to the end of its burning.

The velocity of combustion is independent of the section of the grain, and is inversely proportional to its absolute density.

If the grain is angular, the salient parts burn more quickly than the plane or rounded parts, and thus an angular grain soon takes a spheroidal or ovoidal form similar to a water-worn pebble, which it retains, until consumed. It may thus be assumed without appreciable error, that an angular grain is consumed in the same time as its inscribed sphere. Thus all the grains of a charge may be considered as equal grains provided they have the same minimum dimensions; that is to say, if they were made from press-cake of the same uniform thickness.

The velocity of combustion varies with the composition of the powder and with the process of munufacture. It is greater with black charcoal than with brown; also greater the more perfect the trituration, or, in other words, the more homogeneous is the mixture; and finally the velocity of combustion increases with the dryness of the powder.

The velocity of combustion also depends upon the pressure of the medium in which the phenonmena take place. Thus if a grain of powder of perfectly homogeneous constitution be moulded into a cylinder by means of a screw-plug having an orifice for the escape of the gas, it will be found that the smaller the orifice the more quickly will the grain be consumed. If the velocity of combustion did not increase with the pressure only a small portion of the charges of cannon would be consumed during the very short time the projectile is in the bore. Thus, *M. Castan* has demonstrated that, under the variable pressure developed in the larger French naval guns, the powder burns with a mean velocity of 0.32 m. s., while the velocity of combustion in free air is only 0.01 m. s. If this velocity were preserved in the gun there would be for the 24 c. m., gun, for example, but 5 or 6 kgs. of powder consumed during the time indicated, which is but $\frac{1}{5}$ of the normal charge (28 kgs.). It is certain that the enormous pressure developed in modern rifled guns gives velocity of combustion of the grain of from 1.5 m. s., to 2 m. s.

Inflammation and Combustion of a Charge of Gunpowder.— The inflammation of a charge of powder, involves the transmission of the flame from one grain to another. Its velocity depends not only upon the inflammability of the grain, but also, upon the facility with which the gases first generated are able to penetrate the charge; and is therefore, small in pulverized powder.

The velocity of inflammation of a charge of powder depends also upon the pressure to which it is subjected, Thus while it is 2.4 m. s., in a train of powder laid in free air upon a table, it is 8.5 m. s., when the powder is confined in a canvas hose placed in a trough and covered with boards. It is believed that in fire-arms the velocity of inflammation of the charge is as great as 25 and even 30 metres per second, which is equivalent to saying that, practically, it is instantaneous.

The combustion of a charge composed of grains of the same minimum dimensions should, therefore, from what has been said, practically terminate at the same time as each of the grains of which it is composed. The time of combustion of a charge, therefore, increases with the size of the grains and is, in all cases, much longer than the time of inflammation.

If a charge of gunpowder be confined in a close vessel its combustion takes place without noise, and permanent gases and solid matter are produced which can be collected for analysis by opening the vessel, as in the experiments of *Noble* and *Abel*. In this case no work is performed by the gases, and the accompanying phenomena are comparatively simple. But if the combustion take place in a chamber of which one of the walls is capable of moving under the expansion of the gases, which condition is realized in cannon, the resulting phenomena are much more complicated.

Imagine a charge of gunpowder in the chamber of a gun, behind a projectile, to be ignited. All of the grains, supposed to be composed of the same material and of spherical form, will be inflamed practically simultaneously. The first gaseous products will expand into the air-spaces of the chamber, and soon acquire a tension sufficient to overcome the inertia and forcing of the projectile. This latter once in motion, will encounter no resistances in the bore comparable with those which opposed its start, and its velocity will rapidly increase under the continued action of the pressure of the gases.

This pressure will also increase at first; for, though the displacement of the projectile gives a greater space for the expansion of the gases, this will be more than compensated by a more abundant disengagement of gas. But the pressure will soon reach its maximum; for, if, on the one hand, the combustion of the charge is accelerated by the increase of pressure, on the other hand the surfaces of inflammation rapidly diminish, and the increasing velocity of the projectile offers more and more space for the gases to expand in. This velocity itself will soon attain a maximum; for, in addition to the friction which retards the motion of the projectile, the impulsive force decreases by the expansion and cooling of the gases. Therefore the retarding forces rapidly predominate, and if the gun were of sufficient length, the projectile would not leave the bore. Its velocity, starting from zero, passes through a maximum, and if the gun terminated at this point, the projectile would leave the bore with the greatest velocity that the charge of powder was capable of communicating to it, and with a very loud report.

So far charges in general have been considered, only. Take, now, a charge composed of small grains of slight density. The initial surface of inflammation will be very great and the emission of gas correspondingly abundant. The pressure will increase rapidly, and, in consequence, the velocity of combustion. It results from this that the grains which are very small, will be consumed nearly as soon as inflamed, and this before the projectile has had time to be displaced by a sensible amount. Hence all the gases of the charge, disengaged almost instantaneously, will be confined an instant within the chamber; their tension will be very great, and they will exert upon the walls of the gun a sudden and violent force which may rupture the metal, and which in all cases will produce upon the gun and carriage shocks which are destructive to the system and prejudicial to accuracy of fire. On the other hand the projectile will be thrown quickly forward, as by a blow from a hammer, and will attain its maximum velocity almost at the start.

If, on the contrary, the charge is made up of large grains of great density, the total weight of gas emitted will be the same as before; but the mode of emission will be different. The initial surface of inflammation will be less, and the initial tension of the gas not so great. The combustion will take place more slowly, and will be only partially completed when the projectile shall have begun to move. The pressure of the gases will attain a maximum less than in the preceding case, but the pressuse will decrease more slowly. Under the continued action of this pressure, the velocity of the projectile will be rapidly accelerated and at the muzzle will differ but little from that obtained by the fine powder, without producing upon the gun and carriage the destructive effects mentioned above.

This can be made plainer by means of graphical representations of the phenomena under consideration. Thus let the curves *o b c d* for fine-grained powder, and *o b′ c d′* for powder of coarse grains, have each for abscissas the path described by the projectile and for ordinates the corresponding pressures. But it is known from an elementary theorem of mechanics, that for each of these curves, the area comprised between the curve and the axis *OX* represents the sum of the products of the moving force by the displacement, and measures the useful work of the charge up to the point marked by the abscissa *OX*. This useful work is equivalent to the living force of the projectile at this point.

This being admitted, an examination of the two curves suggests the following considerations:

1. The curve *o b′ c d′*, at first below the curve *o b c d*, cuts it at *c* and afterwards remains above it. If the gun were indefinitely long all the work contained potentially in the powder would be utilized and, the charges being equal, the areas *o b c d X* and *o b′ c d′ X* would be equivalent. If on the contrary, the gun has a determinate length *OD*, there is less taken away from the first area than from the second; and therefore for the same weight of charge the quick powder gives an initial velocity greater than the slow powder. By increasing the charge of the slow powder, however, a velocity equal to, and even greater than that given by the quick powder can be obtained, and with a less maximum pressure.

2. Of two powders which, in the same gun, give equivalent areas of useful work, that is unquestionably the better which produces the less maximum pressure: For, the greater the maximum pressure and the sooner it is produced, the greater and more violent is the strain upon the gun. From this it is evident that the study of the maximum ordinate in the curve of pressures, is of the greatest importance; since it is upon its value and position that the requisite strength to be given to the walls of the gun depends.

Reduced Length of the Powder-Chamber.—The powder-chamber is that part of the bore of a gun in which the charge of powder is placed in loading. Its capacity is the volume bounded by the walls of the chamber, the base of the projectile in its initial position for firing, and the face of the breech-block or bottom of the bore. In modern rifled-guns the bore is slightly enlarged to form the chamber; and it will be found convenient to introduce what is called the *reduced length* of the chamber, which is defined as the length of a cylinder whose diameter is the same as the bore and whose volume is the same as the cham ber.

Let

u_0 be the reduced length of the chamber, in feet,

ω the area of cross-section of the bore, in square inches,

C the volume of the chamber in cubic inches,

d the diameter of the bore (calibre) in inches.

Then

$$C = 12 u_0 \omega = 3\pi d^2 u_0.$$

$$\therefore\ u_0 = \frac{C}{3\pi d^2} = 0.106103\,\frac{C}{d^2}. \qquad (1)$$

Gravimetric Density.—The weight of a given volume of gunpowder, not pressed together except by its own weight, evidently varies with the density of the individual grains and with the volume of the interstices between them; and this latter varies with the general form of the grains, or, in other words, with their ability to pack closely, or the reverse. If the weight of unit volume of a standard powder be determined then the ratio of the weight of the same volume of any other powder to this standard weight is called its *gravimetric density*. In France, where the term gravimetric density originated (densité gravimétrique), one litre of the standard powder adopted was found to weigh one kilogramme, and therefore the weight of a litre of any other powder in kilogrammes is its gravimetric density. Reducing to English units we may say that the gravimetric density of a powder is the weight in pounds of a volume of 27.68 cubic inches of the powder not pressed together except by its own weight.

Density of Loading.—The density of loading is defined to be the ratio of the weight of the charge to the weight of a volume of water just sufficient to fill the chamber. In France this is called *densité de chargement;* and in the metric system of units, is the ratio of the weight of a charge in kilogrammes to the volume of the chamber in litres, since a kilogramme is the weight of a litre (or cubic decimetre) of water. With English units the expression for the density of loading may be determined as follows:

Let

Δ be the density of loading,

$\tilde{\omega}$ the weight of the charge in pounds.

Then since there are 61.0254 cubic inches in a litre, and 2.20462 pounds in a kilogramme, it follows that $61.0254 \div 2.20462 = 27.6807$, is the number of cubic inches occupied by one pound of water; and, therefore, $C \div 27.6807$ is the weight of a volume of water equal to the volume of the chamber. Hence by definition,

$$\Delta = \tilde{\omega} \div \frac{C}{27.68} = \frac{27.68\tilde{\omega}}{C} \quad . \quad . \quad . \qquad (2)$$

which gives the relation between the density of loading, weight of charge and volume of chamber, in English units. In metric units the expression for the density of loading reduces to

$$\Delta = \frac{\tilde{\omega}}{C}$$

The density of loading may also be expressed in terms of the absolute density of the powder as follows:

Let

C' be the volume of the charge exclusive of the air-space, in cubic inches.

δ the absolute density of the powder.

Then it follows that

$$\tilde{\omega} = \frac{C'}{27.68}\delta,$$

which substituted in Equation (2), gives

$$\Delta = \frac{C'}{C}\delta \quad . \quad . \quad . \quad . \qquad (3)$$

From this equation the density of loading may be defined as the ratio of the volume of the powder-grains to the volume of the chamber, multiplied by the absolute density ot the powder.

If $C' = C$, that is, if the chamber is filled by a single grain of powder, then $\Delta = \delta$; and this is the superior limit of the density of loading. The inferior limit is, of course, zero, namely, when $C' = 0$. If the density of loading is unity, it follows from Equation (2) that one pound of the powder will occupy 27.68 cubic inches, provided the chamber is filled with the powder grains without forcing, in other words if the gravimetric density is also unity.

Reduced Length of the Initial Air-Space.—The initial air-space is that portion of the volume of the chamber not occupied by the grains of the powder-charge; and the reduced length of the air-space is the length of a cylinder whose diameter is the same as the bore and whose volume is equal to the air-space. Denote this by z_0. Then, since C is the volume of the chamber and C' the volume of the powder grains,

$$z_0 = \frac{C - C'}{\omega} = \frac{C}{\omega}\left(1 - \frac{\Delta}{\delta}\right), \quad \left(\text{by Equation (3)}\right);$$

which becomes, by substituting for C its value from Equation (2),

$$z_0 = \frac{27.68\varpi}{\omega}\left\{\frac{1}{\Delta} - \frac{1}{\delta}\right\} \quad . \quad . \quad . \quad . \quad (4)$$

or, in the terms of d,

$$z_0 = 35.2441\,\frac{\varpi}{d^2}\left\{\frac{1}{\Delta} - \frac{1}{\delta}\right\} \quad . \quad . \quad . \quad (5)$$

In Equation (5) z_0 will be in inches since d has been taken in inches. Dividing the second member by 12 to reduce z_0 to feet, and placing this member in a form better suited for logarithmic computation, the equation becomes

$$z_0 = [0.46791]\,\frac{\varpi}{d^2} \cdot \frac{\delta - \Delta}{\delta\Delta} \quad . \quad . \quad . \quad (6)$$

in which the number in square brackets is the *logarithm* of the numerical factor. This equation may also take the form, sometimes useful:

$$z_0 = [0.46791]\,\frac{\varpi}{\delta d^2}\left(\frac{C}{C'} - 1\right) \quad . \quad . \quad . \quad (7)$$

EXAMPLES.

Example 1.—The volume of the chamber of the 3.2-inch steel rifle is 104.24 cubic inches. If the charge is 3½ pounds of powder what is the density of loading? Answer. $\Delta = 0.9294$.

Example 2.—If the gravimetric density of the powder of Ex. 1 be unity, how many pounds will the chamber hold?

Answer. 3.766 pounds.

Example 3.—If the absolute density of the powder of Ex. 1 be 1.75, required the volume of the charge exclusive of interstices.

Answer. $C' = 55.36$ cubic inches.

Example 4.—What is the reduced length of the chamber of the gun of Ex. 1? Answer. $u_0 = 1.0801$ feet.

Example 5.—Compute z_0 with the following data: $\tilde{\omega} = 110$ lbs., $C = 3824$ cubic inches, $d = 8$ inches, $\delta = 1.818$.

Answer. $z_0 = 3.564$ feet.

Example 6.—The capacity of the chamber of the 6-inch Navy B. L. R., Mark III, is 1426 cubic inches. What is the initial air-space when the gun is loaded with 54 pounds of powder of density 1.818? Answer. 603.8 cubic inch.

CHAPTER II.

PROPERTIES OF PERFECT GASES.

Mariotte's Law.—The law of the compression, or expansion, of gases was discovered by *Boyle* in 1662, and afterwards, independently by *Mariotte* in 1679. In England it is generally called "*Boyle's* Law," and on the continent "*Mariotte's* Law." It is enunciated as follows:

The temperature remaining the same, the pressure sustained by a given mass of gas is inversely as the volume it occupies.

As the tension, or elastic force of the gas, is constantly in equilibrium with the pressure it sustains, *Mariotte's* law may also be stated as follows:

The elastic force exerted by a mass of gas of constant temperature, is inversely as the volume it occupies. Or, as the density of a gas is inversely as its volume, we may say, *For the same temperature, the tension of a gas is proportional to its density.* This law has been verified experimentally by *Dulong* and *Arago* with atmospheric air, for pressures up to 27 atmospheres.

Mariotte's law stated symbolically is as follows: Let v be the volume of a given mass of gas (say one pound); and p its tension, or elastic force, measured in pounds per square foot. Then *Mariotte's* law asserts, that if the gas be allowed to expand, v and p will vary in such a way that

$$v\,p = \text{constant},$$

provided the temperature be kept invariable during such expansion. The value of the constant can be determined experimentally. Let v_t and p_t be the measured volume and pressure, respectively, at a temperature t. Then

$$vp = v_t\,p_t$$

For many purposes it is convenient to take the volume of *unit weight* of a gas at 0° C., under the normal atmospheric pressure. Calling the volume and pressure under these conditions v_0 and

p_0, respectively, *Mariotte's* law may be written

$$v\,p = v_0\,p_0.$$

The following are the values of p_0 for different units, with their logarithms:

$p_0 = 10333$ kgs. per m².	log	$= 4.0142264$
" $= 2116.3$ lbs. " ft².	"	$= 3.3255824$
" $= 14.6967$ lbs. " in².	"	$= 1.1672199$

Specific Volume.—v_0 is called the specific volume of a gas; that is, the volume of unit weight of gas at zero temperature and under the normal atmospheric pressure.

Specific Weight.—The specific weight of a gas is the weight of a unit volume at zero temperature and under the normal atmospheric pressure p_0. Therefore, if w_0 is the specific weight of a gas, then

$$w_0 = \frac{1}{v_0}.$$

Law of Gay-Lussac.—*The coefficient of expansion of a gas is the same for all gases; and is sensibly constant for all temperatures and pressures.*

Let, as before, v_0 be the specific volume of a gas, v_t its volume at any temperature t and a the coefficient of expansion; then the variation of volume by *Gay-Lussac's* law will be expressed by the equation

$$v_t - v_0 = a\,t\,v_0.$$

whence,

$$v_t = v_0\,(1 + a\,t).$$

The value of a is approximately $\frac{1}{273}$ of the specific volume for each degree centigrade. The above equation may therefore be written

$$v_t = v_0\left(1 + \frac{t}{273}\right).$$

Characteristic Equation of the Gaseous State.—The last equation, which expresses *Gay-Lussac'z* law, may be combined with *Mariotte's* law, introducing the pressure p_t. The problem may be enunciated as follows: Having given the specific volume of a gas to determine its volume v_t at a temperature t and under the pressure p_t.

Let x be the volume v_t would become at 0° C., under the pressure p_t. Then by *Gay-Lussac's* law

$$v_t = x\,(1 + \alpha t);$$

but by *Mariotte's* law

$$p_t\, x = p_0\, v_0;$$

whence eliminating x,

$$p_t\, v_t = p_0\, v_0 (1 + \alpha t) = \frac{p_0 v_0}{273}(273 + t).$$

Since $\frac{p_0 v_0}{273}$ is constant for any gas, put

$$R = \frac{p_0 v_0}{273};$$

whence

$$p_t\, v_t = R\,(273 + t);$$

or dropping the subscript as no longer necessary,

$$p\,v = R(273 + t).$$

The temperature $(273 + t)$ is called the absolute temperature of the gas. It is the temperature reckoned from a zero placed 273 degrees below the zero of the centigrade scale, or 273 degrees below the freezing point of water. Calling the absolute temperature T there results finally

$$p\,v = R\,T \quad . \quad . \quad . \quad (1)$$

which is called the *characteristic equation of the gaseous state*, and is simply another expression of *Mariotte's* law in which the temperature of the gas is introduced.

Equation (1) expresses the relation existing between the pressure, volume and absolute temperature of a *unit weight* of gas. For any number w units of weight, occupying the same volume the relation evidently becomes

$$p\,v = w\,R\,T \quad . \quad . \quad . \quad (2)$$

For the Fahrenheit scale the expression for R becomes

$$R = \frac{p_0 v_0}{\frac{9}{5} \times 273} = \frac{p_0 v_0}{491.4}$$

A gas supposed to obey *exactly* the law expressed in Equation (1) is called a perfect gas or is said to be theoretically in the perfectly gaseous state. This perfect condition represents an ideal state toward which gases approach more nearly as their state of rarefaction increases. Of all gases, hydrogen approximates most closely to such an hypothetical substance, though at ordinary temperatures the simple gases nitrogen, oxygen and atmospheric air may for practical purposes be considered perfect gases.

Thermal Unit.—The heat required to raise the temperature of unit weight of water at the freezing point, one degree of the thermometer is called a *thermal unit.* There are two thermal units in general use, namely: the British thermal unit (*B. T. U.*) which is the heat required to raise one pound of water from 32° F. to 33° F.; and the French thermal unit (called a *calorie*) which is the heat required to raise one kilogramme of water from 0° C. to 1° C. There is still another thermal unit of frequent use, namely: The heat required to raise one pound of water from 0° C. to 1° C., and which may be designated as the pound-centigrade (*P. C.*) unit.

Mechanical Equivalent of Heat.—The mechanical equivalent of heat is the work equivalent of a thermal unit, and will be designated by E. According to the recent exhaustive experiments of *Rowland* the value of E is 778 foot-pounds for a *B. T. U.* Since a division on the Centigrade scale is $\frac{9}{5}$ of a division on the Fahrenheit scale, we have for a *P. C.* thermal unit, $E = \frac{9}{5} \times 778 =$ 1400.4 foot-pounds. Also, since there are 3.280869 feet in a metre, the value of E for a calorie is

$$\frac{1400.4}{3.280869} = 426.84 \text{ kilogrammetres.}$$

Specific Heat.—The quantity of heat, expressed in thermal units, which must be imparted to a unit weight of a given substance in order to augment its temperature one degree of the thermometer, or the quantity of heat given up by the substance while its temperature is lowered one degree, is called its *specific heat.*

The specific heats of different bodies vary greatly. Thus, if a pound of mercury and a pound of water receive the same quantities of heat the temperature of the mercury will be found to be considerably greater than that of the water. Indeed, it requires about 32 times as much heat to raise the temperature of water 1°

as it does to raise the temperature of mercury by the same amount.

The heat imparted to a body is expended in three different ways:—1st. Augmenting the temperature, which may be called vibration work. 2nd. In doing internal or disgregation work. 3rd. In doing external work or work of expansion. If we could eliminate the two latter, we should get the true specific heat, or the heat necessary to increase the temperature. It is, however, impossible at present to separate the different modes of action of the heat; and, therefore, the specific heats which have been determined differ more or less from the true specific heats. For a perfect gas however, the disgregation work is zero, and in all cases the disgregation work is very small in comparison with the vibration work.

The specific heat of a gas may be determined in two different ways giving results which are of fundamental importance in thermodynamics, namely: specific heat under constant pressure; and specific heat under constant volume.

Specific Heat of Gases under constant pressure.—To fix the ideas suppose a unit weight of gas, confined in a spherical envelope capable of expanding without the expenditure of work, and which allows no heat the gas may have to escape, to be in equilibrium with the constant pressure of the atmosphere. Under these conditions suppose a certain quantity of heat to be applied to the gas, just sufficient to raise its temperature one degree of the thermometer, after it has expanded until equilibrium is again restored. This quantity of heat (designated by c' thermal units) is called the specific heat of the gas under constant pressure.

Specific Heat under constant Volume.—Next repeat the experiment just described, but replacing the elastic envelope, which by hypothesis permitted the gas to expand freely, by a rigid envelope, thus keeping the volume of the gas constant while heat is applied. It will be found now that there will be less heat required than before to raise the temperature of the gas one degree. The amount of heat required in this case is called the *specific heat under constant volume;* and, in terms of the thermal unit employed, is designated by c.

Now, the number of molecules of gas is the same in both experiments; and since the temperatures are equal it follows that the quantity of heat absorbed by the gas, or, what is the same thing, the total *vis viva* of the molecules, is the same in both experiments. But in the experiment made under constant volume, the heat absorbed is necessarily equal to the total heat supplied, namely, c thermal units,—since the envelope is considered impermeable to heat. Consequently in the first experiment there is a loss of heat equal to $c'-c$ thermal units. This lost heat must, therefore, have been expended in pushing aside the atmosphere in expanding; and the work done will be found by multiplying $(c'-c)$ by the mechanical equivalent of heat. That is, for an increase of temperature of one degree of the thermometer,

$$\text{Work of expansion} = (c'-c)\,E.$$

The work of overcoming a constant resistance is equal to the product of the resistance and the length of the path described. In the case of the expanding gas which has just been considered the constant resistance is the atmospheric pressure p_0; and the path described is measured by the increase of volume of the gas. To determine this latter *Gay-Lussac's* law gives for the centigrade scale

$$v_t - v_0 = \frac{t\,v_0}{273},$$

and therefore for an increase of temperature of *one degree* there is an increase of volume equal to $\frac{v_0}{273}$. The mechanical work of expansion for one degree is therefore

$$p_0 \times \frac{v_0}{273} = \frac{v_0 p_0}{273} = R.$$

The quantity R is, then, the external work of expansion performed under atmospheric pressure by unit weight of gas when its temperature is raised one degree of the thermometer. But this work of expansion has already been found to be equal to $(c'-c)\,E$. Therefore there results the important equations

$$(c'-c)\,E = R = \frac{p_0 v_0}{273} \text{ or } \frac{p_0 v_0}{491.4} \quad . \quad . \quad . \quad (3)$$

according as the Centigrade or Fahrenheit scale of temperature is employed.

Numerical Value of R.—The value of R for any particular gas depends upon the units of length and weight taken, the atmospheric pressure, the specific volume of the gas and the scale of temperature adopted. Throughout this work the pound and foot will be taken for the units of weight and length, respectively; and generally the centigrade scale of temperature will be employed. The adopted value of the atmospheric pressure has already been given.

As an example find the numerical value of R for atmospheric air. The specific weight of this gas according to the best authorities is 0.080704 pounds. The specific volume is therefore

$$v_0 = \frac{1}{0.080704} = 12.3909 \text{ ft}^3;$$

and therefore for atmospheric air

$$R = \frac{2116.3 \times 12.3909}{273} = 96.056;$$

that is, for one pound of this gas,

$$p\,v = 96.056\ T:$$

and for w pounds

$$p\,v = 96.056\,w\,T.$$

Law of Dulong and Petit.—*The product of the specific heat of a gas under constant volume by its density is a constant number for all gases.*

By the density of a gas is meant its specific weight expressed in terms of the specific weight of atmospheric air, which is taken as unity. If c_a is the specific heat at constant volume of atmospheric air, and c_1 and d_1 the specific heat at constant volume, and the density, respectively, of any other gas, then in accordance with the law of *Dulong* and *Petit,*

$$c_1\,d_1 = c_a$$

Determination of Specific Heats.—The specific volume and the specific heat at constant pressure of a gas can both be accurately determined by experiment; but the specific heat under constant volume is almost impossible to measure directly. It can, however, be computed for the simple gases by Equation (3), which gives

$$c = c' - \frac{p_0 v_0}{273\,E} \qquad . \qquad . \qquad . \qquad (4)$$

By means of this equation and the direct determination of specific heats under constant pressure, *Regnault* has deduced the following law for the simple gases:

The specific heats under constant pressure and constant volume are independent of the pressure and the volume.

The following table gives the specific volumes, weights, and heats of those gases which approach most nearly to the theoretically perfect gas. In the last column are given the values of R computed by Equation (1) for the centigrade scale. The values of c were computed by Equation (4).

GAS.	SPECIFIC VOLUME. v_0	SPECIFIC WEIGHT. w_0	c'	c	R
Atmospheric Air	12.3909	0.080704	0.23751	0.16892	96.056
Nitrogen	12.7561	0.078394	0.24380	0.17319	98.887
Oxygen	11.2070	0.089230	0.21751	0.15547	86.878
Hydrogen	178.8610	0.005590	3.40900	2.41873	1386.8

The units in this table are the pound and foot. The temperature is supposed to be 0° C., and the barometer to stand at 760 mm = 29.922 inches.

Ratio of Specific Heats.—In the study of interior ballistics the particular values of c' and c for the different gases which are formed by the explosion of gunpowder and other explosives, is of little importance. It suffices generally to know their ratio which is constant for perfect gases, and approximately so for *all* gases at the high temperature of combustion of gunpowder. That this ratio is constant for perfect gases may be shown as follows:—Since

$$R = \frac{p_0 v_0}{273} = \frac{p_0}{273\,w_0} = \frac{p_0}{273\,d\,w_a},$$

in which w_a is the specific weight of atmospheric air, we should have for two gases, distinguished by the subscripts 1 and 2, the relation

$$\frac{R_1}{R_2} = \frac{d_2}{d_1};$$

that is, the values of R for two perfect gases are inversely proportional to their densities. But by the law of *Dulong* and *Petit* we have

$$\frac{c_1}{c_2} = \frac{d_2}{d_1} = \frac{R_1}{R_2} \text{ (as shown above)};$$

and therefore

$$\frac{R_1}{c_1} = \frac{R_2}{c_2} = \text{constant};$$

consequently for any perfect gas the ratio $\frac{R}{c}$ is constant. And therefore from Equation (3)

$$\frac{c'}{c} = 1 + \frac{R}{cE} = \text{constant} = n \text{ (say)}.$$

Computing n by means of atmospheric air we have

$$n = 1 + \frac{96.056}{0.16892 \times 1400.4} = 1.406.$$

Laws Governing Mixtures of Gases.—1. The specific volume of a mixture of two or more gases is the sum of the products of the weights of the gases mixed by their specific volumes. Thus if a_1, a_2, a_3, etc., are the weights of the various gases making up the unit mixture, and v_1, v_2, v_3, etc., the corresponding specific volumes, then the specific volume of the mixture is

$$v_0 = a_1 v_1 + a_2 v_2 + a_3 v_3 + \text{etc.}$$

2. The specific heat (either under constant pressure or constant volume) is the sum of the products of the weights of the gases mixed by their specific heats. Or, as expressed above,

$$c = a_1 c_1 + a_2 c_2 + a_3 c_3 + \text{etc.}$$

As an example of these laws take atmospheric air which is a mixture of oxygen and nitrogen in the ratio by weight of 23 to 77. By the first of these laws its specific volume should be

$$v_0 = .23 \times 11.207 + .77 \times 12.7651 = 12.4067$$

and by the second law its specific heat under constant pressure should be

$$c' = .23 \times 0.21751 + .77 \times 0.2438 = 0.23775$$

and under constant volume,

$$c = .23 \times 0.15547 + .77 \times 0.17319 = 0.16911$$

these results agree practically with experiment. Adv.

Relations between Heat and Work in the Expansion of Gases. The relations which exist between the variations of the volume and pressure of a given weight of gas, and the heat necessary to produce them, may now be determined from Equation (1), as follows: This equation is

$$p v = R T,$$

and contains three variables, viz: p, v and T. If we suppose an element of heat, $d q$, to be applied to the gas, the temperature will generally be augmented by an elementary amount $d T$; and this in three different ways:—1st. We may suppose the volume to increase by the element $d v$ while the pressure remains the same. 2nd. The pressure may increase by $d p$ while the volume remains constant. 3rd. The volume and pressure may both vary at the same time. We will consider each of these cases separately.

1. If, the pressure p remaining the same, the volume v vary by the quantity $d v$, we must determine the corresponding variation of T by differentiating Equation (1). This gives

$$d T = \frac{p d v}{R};$$

and, therefore, the quantity of heat communicated to the gas will be, in thermal units,

$$d q = c' d T = \frac{c' p d v}{R};$$

c' being the specific heat under constant pressure.

2. If, the volume v remaining constant, the pressure vary by $d p$, we shall have, proceeding as before,

$$d q = c d T = \frac{c v d p}{R};$$

c being the specific heat under constant volume.

3. If the volume and pressure vary together, the coresponding element of heat will be the sum of the partial variations given above. That is

$$dq = \frac{1}{R}(c' p\, dv + c\, v\, dp) \quad . \quad . \quad . \quad (5)$$

The differential of Equation (1) is

$$R\, dT = p\, dv + v\, dp \quad . \quad . \quad . \quad (6)$$

whence eliminating dp between Equations (5) and (6) we have

$$dq = c\, dT + \frac{c' - c}{R} p\, dv \quad . \quad . \quad . \quad (7)$$

The quantity $p\, dv$ which enters into the second member of Equation (7) represents the elementary work of the elastic force of the gas while its volume increases by dv. It follows then from Equation (7), that the quantity of heat absorbed by the gas in the elementary transformation is composed of two terms, the first of which is proportional to thc variation of temperature, and the second to the elementary external work. If we suppose that the volume and pressure of the gas vary by finite quantities, we determine the corresponding quantity of heat absorbed by the transformation, by integrating Equation (7). Thus

$$q = \int\left(c\, dT + \frac{c' - c}{R} p\, dv\right).$$

But since for the same gas, c, c' and R are constants, we have

$$q = c\int dT + \frac{c' - c}{R}\int p\, dv.$$

Designating by T_1 and T the initial and final temperatures, and by W the total external work of the elastic force ot the gas, we have

$$q = c(T - T_1) + \frac{c' - c}{R} W \quad . \quad . \quad . \quad (8)$$

or

$$q = c(T_1 - T) + \frac{c' - c}{R} W \quad . \quad . \quad . \quad (8')$$

according as the temperature of the gas increases or decreases during the transformation.

Isothermal Expansion.—If we suppose the initial temperature T_1 to remain constant, that is, that just sufficient heat is imparted to the gas while it expands to maintain its initial temperature, Equation (8) becomes

$$q = \frac{c' - c}{R} W.$$

We see in this case that the quantity of heat absorbed by the gas is proportional to the external work done. The quantity $\frac{R}{c' - c}$ is, therefore, the ratio of the effective work of a unit weight of gas to the quantity of heat absorbed, or the mechanical equivalent of heat. Therefore,

$$E = \frac{R}{c' - c} = \frac{1}{273} \frac{p_0 v_0}{c' - c};$$

a result already established by another method. See Equation (3), The work performed, therefore, by the isothermal expansion of unit-weight of gas is expressed by the equation

$$W = E q = 1400.4\, q \text{ foot-pounds,}$$

where q is expressed in *P. C.* thermal units.

The work of an isothermal expansion may also be expressed in terms of the initial and final volumes or pressures. Thus, substituting in the general expression for the work of expansion,

$$W = \int p\, d\, v,$$

the value of p from Equation (1), and integrating between the limits v_1 and v we have

$$W = R\, T_1 \log_\epsilon \frac{v}{v_1} = p_1 v_1 \log_\epsilon \frac{v}{v_1};$$

or, in common logarithms,

$$W = 2.3026\, p_1 v_1 \log \frac{v}{v_1} \quad . \quad . \quad . \quad (9)$$

where v is the greater volume and v_1 the less.

Since from Equation (1)

$$\frac{v}{v_1} = \frac{p_1}{p},$$

we also have

$$W = 2.3026\, p_1 v_1 \log \frac{p_1}{p}, \quad . \quad . \quad . \quad (9')$$

in which p_1 is the greater tension and p the less.

Equations (9) and (9′) by inverting the fractions, evidently hold good when the initial volume v_1 and initial tension p_1 are changed by compression under constant temperature into the less volume v and greater tension p.

If we denote by A the reciprocal of E, or the *heat equivalent* of work,—that is, the amount of heat which is equivalent to one unit of work, we have for the heat imparted during an isother-expansion

$$\left.\begin{aligned} q = A\ W &= 2.3026\ A\ p_1 v_1 \log \frac{v}{v_1} \\ &= 2.3026\ A\ p_1 v_1 \log \frac{p_1}{p} \end{aligned}\right\} \quad . \quad . \quad . \quad (10)$$

Adiabatic Expansion. Law of Temperatures.—If a gas expands and performs work in an envelope impermeable to heat so that it neither receives nor gives out heat (by radiation or induction) the transformation is said to be adiabatic. In this case the temperature and tension of the gas both diminish and the work performed for the same expansion must be less than for an isothermal expansion. For an adiabatic expansion Equation (8) becomes, since $q = o$, by hypothesis,

$$\begin{aligned} W &= \frac{Rc}{c' - c}\ (T_1 - T) \\ &= \frac{R}{n - 1}\ (T_1 - T) \\ &= c\,E\ (T_1 - T) \quad . \quad . \quad . \quad (11) \end{aligned}$$

which expresses the important fact that in an adiabatic expansion the external work done is proportional to the fall of temperature.

Next consider Equation (7). We shall have in this case, when $d\,q = o$,

$$o = c\ d\ T + \frac{c' - c}{R}\ p\ d\ v;$$

which becomes, after replacing p by its value, $\frac{R\ T}{v}$, derived from the fundamental Equation (1),

$$o = \frac{d\ T}{T} + (n - 1)\ \frac{d\ v}{v};$$

whence by integration

$$T v^{n-1} = \text{constant},$$

an equation which may be written, by taking limits,

$$\frac{T}{T_1} = \left(\frac{v_1}{v}\right)^{n-1} \quad . \quad . \quad . \quad (12)$$

Law of Pressures.—When $d q = o$, Equation (5) becomes

$$o = c' p \, d v + c \, v \, d p,$$

which may be written

$$n \frac{d v}{v} + \frac{d p}{p} = o;$$

whence, integrating,

$$v^n p = \text{constant} = v_1^n p_1$$

where v_1 and p_1 are the initial volume and pressure, respectively. This last equation may be written

$$\left(\frac{v_1}{v}\right)^n = \frac{p}{p_1} \quad . \quad . \quad . \quad . \quad (13)$$

Combining Equations (12) and (13) gives the important relations

$$\left(\frac{v_1}{v}\right)^{n-1} = \frac{T}{T_1} = \left(\frac{p}{p_1}\right)^{\frac{n-1}{n}} \quad . \quad . \quad . \quad (14)$$

By means of these last equations, the work of expansion (Equation (11)) may be expressed in terms either of the initial and terminal volumes, or of the initial and terminal pressures. Thus since

$$T_1 - T = T_1 \left(1 - \frac{T}{T_1}\right),$$

Equation (11) may be written

$$\left.\begin{aligned} W &= c \, E \, T_1 \left\{ 1 - \left(\frac{v_1}{v}\right)^{n-1} \right\} \\ &= c \, E \, T_1 \left\{ 1 - \left(\frac{p}{p_1}\right)^{\frac{n-1}{n}} \right\} \end{aligned}\right\} \quad . \quad . \quad (15)$$

Since $c \, E \, T_1 = \dfrac{p_1 v_1}{n - 1}$, Equation (15) may be written

$$\left.\begin{aligned} W &= \frac{p_1 v_1}{n-1}\left\{1-\left(\frac{v_1}{v}\right)^{n-1}\right\} \\ &= \frac{p_1 v_1}{n-1}\left\{1-\left(\frac{p}{p_1}\right)^{\frac{n-1}{n}}\right\} \end{aligned}\right\} \quad . \quad . \quad . \quad (16)$$

If we assume the ratio of the specific heats for perfect gases to be 1.4, the last equation may be written

$$\left.\begin{aligned} W &= 2.5\, p_1 v_1 \left\{1-\left(\frac{v_1}{v}\right)^{\frac{2}{5}}\right\} \\ &= 2.5\, p_1 v_1 \left\{1-\left(\frac{p}{p_1}\right)^{\frac{2}{7}}\right\} \end{aligned}\right\} \quad . \quad . \quad . \quad (16')$$

Theoretical Work of Fired Gunpowder.—A simple and instructive application of the preceding formulas is found in the theoretical calculation of the work of expansion of the powder gases in the bore of a gun. In this application the combustion of the charge will be considered completed before the projectile has sensibly moved. The vastly more important case of a progressive combustion whereby gases are produced while the projectile is moving along the bore, will be considered later. It will be assumed in this discussion that the gases, on account of their high temperatures, are practically perfect gases; and also, in accordance with the hypothesis of *Bunsen* and *Schischkoff*, that work is produced by the expansion of the permanent gases only, with their proper heat; and that the non-gaseous products, in a liquid state, radiate their heat toward the walls of the gun without sensibly modifying the thermal state of the gases, on account of the feeble absorbing power of these latter. The expansion in this case will be adiabatic.

Let

T_1 be the absolute temperature of combustion of the powder.

c the mean specific heat of the gases under constant volume.

ϖ the weight of the charge.

ε the weight of gas evolved by the combustion of one pound of powder.

w the weight of the projectile.

We have by Equation (15), for $\varepsilon\,\varpi$ unit weights of gas formed,

$$W = E\,c\,T_1\,\varepsilon\,\varpi\left\{1-\left(\frac{v_1}{v}\right)^{n-1}\right\}$$

It remains now to express the ratio $\frac{v_1}{v}$ in terms of the displacement of the projectile. Make a longitudinal section along the axis of the gun. Let A be the initial position of the base of the projectile, and B its position at the end of the time t. The entire weight of gas $\varepsilon\,\tilde{\omega}$ (supposed to be formed before the projectile has moved), occupies an initial volume v_1, which, if we admit with *Noble* and *Abel*, that the solids of combustion occupy sensibly the same volume as the grains of powder from which they were produced, is measured by the reduced length of the initial air space, z_0. The volume v occupied by the gases at the end of the time t, is measured by $C\,B = z_0 + u$, u being the displacement of the projectile. We therefore have

$$\frac{v_1}{v} = \frac{z_0}{z_0 + u};$$

and the expression for the work becomes

$$W = \varepsilon\,\tilde{\omega}\,E\,c\,T_1 \left\{ 1 - \left(\frac{z_0}{z_0 + u}\right)^{n-1} \right\}$$

For smokeless powders which are entirely converted into gas by combustion it is evident that z_0 must be replaced by the reduced length of the chamber u_0.

It will be seen from this last equation that if u increase indefinitely, the limiting value of the work is

$$W = \varepsilon\,\tilde{\omega}\,E\,c\,T_1.$$

With regard to the factor $E\,c\,T_1$ it may be remarked that the specific heat of the gases under constant volume c differs but slightly from the mean specific heat of the entire products of combustion. $E\,c\,T_1$ is, then, the work due to the indefinite expansion of the gases produced by the combustion of unit of weight of the explosive. This ideal maximum work *Sarrau* calls the *potential of the explosive substance.* The factor $c\,T_1$ is called the *heat of combustion from absolute zero temperature.*

Designating the potential of the powder by h, and the heat of combustion by Q, we have

$$h = E\,Q;$$

and the expression for the work becomes

$$W = h \varepsilon \tilde{\omega} \left\{ 1 - \left(\frac{z_0}{z_0 + u} \right)^{n-1} \right\} \quad . \quad . \quad . \quad (17)$$

In order to obtain at once, a first approximation to the velocity of the projectile in terms of the path described in the bore, we will neglect all that relates to the rotation of the projectile, the recoil of the piece, and, also, the proper motion of the products of combustion, all of which absorb a certain part of the work of expansion. We shall then have, simply,

$$W = \tfrac{1}{2} m v^2 \quad . \quad . \quad . \quad (18)$$

v being the velocity of the projectile and m its mass. We thus have

$$\tfrac{1}{2} m v^2 = h \varepsilon \tilde{\omega} \left\{ 1 - \left(\frac{z_0}{z_0 + u} \right)^{n-1} \right\}$$

from which we get

$$v = \left(\frac{2 h \varepsilon \tilde{\omega}}{m} \right)^{\frac{1}{2}} \left\{ 1 - \left(\frac{z_0}{z_0 + u} \right)^{n-1} \right\}^{\frac{1}{2}}$$

This theoretical formula makes the velocity proportional to the square root of the weight of the charge and inversely proportional to the square root of the weight of the projectile; it makes the velocity increase indefinitely with the space passed over by the projectile, and to decrease as z_0 increases, that is to say as the density of loading decreases.

When u is equal to the total length of bore described by the base of the projectile, the value of v becomes the muzzle velocity, V.

We may determine a second approximation to the velocity impressed upon the projectile by the expanding gases, by taking account of the work performed by these gases upon the gun and charge, as well as upon the projectile.

Let

M be the mass of the gun,

U its velocity at the time considered.

The expression for the work will now be

$$W = \tfrac{1}{2} m v^2 + \tfrac{1}{2} M U^2 + K$$

in which K is one-half the *vis viva* of the products of combustion; —neglecting for the present, work of rotation both of the projectile and gun.

We do not know the value of K, but is is evident that one-half the *vis viva* of the products of combustion is less than $\frac{1}{2}\mu v^2$, (designating by μ the mass of the charge); for the mean velocity of translation of these products is evidently less than that of the projectile. We have then

$$K = \tfrac{1}{2}(\theta\mu)v^2$$

in which θ is a fraction less than unity. The expression for the work may, therefore, be written

$$W = \tfrac{1}{2}(m + \theta\mu)v^2 + \tfrac{1}{2} M U^2 \quad . \quad . \quad . \quad (19)$$

We can deduce a second equation between v and U by equating the momenta of the system, projected upon the axis of the gun. In this way we obtain the following equation:—

$$(m + \theta'\mu)v = MU \quad . \quad . \quad . \quad (20)$$

θ' being, like θ, a fraction less than unity.

Eliminating U from Equations (19) and (20), we get for the velocity of the projectile,

$$v^2 = \frac{2W}{m + \theta\mu + \frac{(m + \theta'\mu)^2}{M}}.$$

In this equation

$$\frac{(m + \theta'\mu)^2}{M}$$

is but a small fraction of $m + \theta'\mu$, and can be neglected by the side of $m + \theta\mu$. This gives

$$v^2 = \frac{2W}{m + \theta\mu} \quad . \quad . \quad . \quad (21)$$

This is the same expression as that deduced from Equation (19) by neglecting the term $\frac{1}{2} M U^2$, that is, by neglecting the energy of recoil of the piece; and differs from Equation (18) by the introduction of $\theta\mu$ which takes account of the motion of the products of combustion.

Substituting in Equation (21) the value of W from Equation (17) gives by taking weights instead of masses

$$v^2 = \frac{2gh\varepsilon\tilde{\omega}}{w + \theta\tilde{\omega}}\left\{1 - \left(\frac{z_0}{z_0 + u}\right)^{n-1}\right\}$$

The product $2gh\varepsilon$ is a constant depending upon the nature of the powder employed, and may be designated by H^2. Therefore extracting the square-root of the last equation, and making

$$x = \frac{z_0}{u}$$

it becomes

$$v = H\left(\frac{\tilde{\omega}}{w + \theta\,\tilde{\omega}}\right)^{\frac{1}{2}}\left\{1 - \frac{1}{(1+x)^{n-1}}\right\}^{\frac{1}{2}} \quad . \quad . \quad . \quad (22)$$

There is considerable uncertainty as to the value of θ; and it undoubtedly varies with the energy imparted to the *unburned grains* in the chamber and bore of the gun, as well as to the products of combustion. If the charge is made up as in the Quick disc powders for example, upon which very little energy is expended while burning, we may make approximately $\theta = \frac{1}{3}$.

If we know the velocity v of the projectile we can obtain the corresponding velocity of recoil of the gun by solving Equation (20), which gives

$$U = \frac{v\,(m + \theta'\,\mu)}{M}$$

or, designating the weight of the gun and carriage by w_1 and making $\theta' = \frac{1}{2}$, which is its approximate value, we have finally

$$U = \frac{v\,(w + \frac{1}{2}\tilde{\omega})}{w_1} \quad . \quad . \quad . \quad (23)$$

EXAMPLES.

Example 1.—Determine the volume of 5 pounds of oxygen at a pressure of 50 pounds per square inch by the gauge, and at a temperature of 60° C. Answer. $v = 15.527$ cu. ft.

Example 2.—One pound of atmospheric air confined in a volume of one cubic foot, has a tension of 50,000 pounds per square foot. What is its temperature by the Fahrenheit scale?

Answer. $t = 477°.56$ F.

Example 3.—A gas receiver having the volume of 3 cubic feet contains half a pound of oxygen at 70° F. What is the pressure by the gauge? Answer. 14.876 lbs. per sq. in.

Example 4.—Let δ_1 be the weight of a cubic foot of air when the barometer stands at 30 inches and the detached thermometer at 62° F. Also let δ be the weight of a cubic foot of air when the barometer is at b inches and thermometer at t degrees F. From this data deduce the working expression

$$\frac{\delta_1}{\delta} = \frac{26.43 + 0.0575\,t}{b}$$

Example 5.—If the weight of a cubic metre of dry air is 1.2932 kilos when the barometer stands at 760 m m and thermometer at 0° C., what would the same volume of air weigh with the barometer at 795.6 m m and thermometer at 18°.5 C.

Answer. $w = 1.2679$ kg.

Example 6.—"A spherical balloon 20 feet in diameter is to be inflated with hydrogen at 60° F. when the barometer stands at 30.2 inches, so that gas may not be lost on account of expansion when the balloon has risen till the barometer stands at 19.6 inches, and the temperature falls to 40° F. How many pounds and how many cubic feet of gas are to be run in?"

Answers. $v = 2827.5$ cu. ft.
$w = 15.09$ lbs.

Example 7.—Two pounds of air expand adiabatically from an initial temperature of 60° F. and a pressure by the gauge of 65.3 pounds per square inch to a pressure of 50 pounds per square inch. Determine the initial and terminal volumes, the terminal temperature and the external work done. Compute the work by Equation (15').

Answers. $v_1 = 5.8953$ cubic feet.
$v = 7.1338$ " "
$T = 21°.7$ F.
$W = 10208$ ft-lbs.

Example 8.—Compute the external work of 2 pounds of air at a temperature of 100° C., which expands adiabatically until it doubles its volume. Employ the first of Equations (15).

Answer. $W \doteq 42731$ foot-pounds.

Example 9.—Determine the temperature after expansion in

Example 8, and the ratio of the initial and terminal pressures.

Answers. $t = 09^\circ.68$ C.

$p = 0.3789\, p_1$.

Example 10.—Three cubic feet of air expand adiabatically from an initial temperature of 70° F. and pressure of 85 pounds per square inch (~~by the gauge~~), until external work equal to 8000 foot-pounds has been done. Determine the terminal volume, pressure and temperature, and the weight of air. (Equation (16')).

Answers. $v = 3.768$ cu. ft.

$p = 61.78$ lbs. per sq. in.

$t = 23^\circ.86$ F.

$w = 1.3$ lbs.

NOTE: The *work* computed by Equation (16') differs slightly from that computed by Equation (11). This is because n is taken at 1.4 instead of 1.406 +. See page 30.

Example 11.—In Equation (16) the work of an infinite expansion is finite so long as n is greater than unity; but becomes infinite when $n = 1$. Interpret these results.

CHAPTER III.

NOBLE AND ABEL'S RESEARCHES ON FIRED GUN-POWDER.

Noble and Abel's experiments on the explosion of gunpowder in close vessels were given to the world in two memoirs which were read before the Royal Society in 1874 and 1879, respectively. These experiments have an important bearing upon the subject of interior ballistics, since they furnish the most reliable values we possess of the temperature of combustion of fired gunpowder, the mean specific heat of the products of combustion (solid as well as gaseous), the ratio of solid to gaseous products, and, lastly, what is known as the *force of the powder*,—all of which are important factors in computing the work done by the gases of a charge of gunpowder exploded in the chamber of a gun.

The vessels in which the explosions were produced were of two sizes, the smaller one for moderate charges and for experiments connected with the measurement or analysis of the gases, while in the larger one Captain *Noble* states that he had succeeded in absolutely retaining the products of combustion of a charge of 23 pounds of gunpowder.[1] These vessels consisted of a steel barrel open at both ends, the two open ends being closed by carefully fitted screw plugs (firing plug and crusher plug) furnished with gas checks to prevent any escape of gas past the screw. In the firing plug was a conical hole closed from within by a steel cone which was ground into its place with great exactness, and which when the cylinder was prepared for firing was covered with very fine tissue paper to give it electrical insulation from the rest of the apparatus. The two wires from a *Leclanché* battery were attached, the one to the insulated cone and the other to the firing plug, and were connected within the powder chamber by a fine platinum wire passing through a glass tube filled with mealed

[1] Lecture on Internal Ballistics, by Captain *Noble*, London, 1892, p. 12.

powder. This platinum wire became red-hot when the electric current passed through it, and the charge was thus fired. At the opposite end of the cylinder from the firing plug was the crusher plug fitted with a crusher gauge for determining the pressure of the gases at the moment of explosion. The vessel was also provided with an arrangement for collecting the gases after an explosion, either for analysis, measurement of quantity, or for other purposes.

The difficulties which the experimenters met with in using this apparatus were very great.

"In the first place, the dangerous nature of the experiments rendered the greatest caution necessary, while, as regards the retention of the products, the application of contrivances of well-known efficacy for closing the joints, such as *papier-maché* wads between disks of metal (a method which has been successfully employed with guns), are inadmissable, because the destruction of the closing or cementing material used, by the heat, would obviously affect the composition of the gas. Every operation connected with the preparation of the apparatus for an experiment has to be conducted with the most scrupulous care. Should any of the screws not be perfectly home, so that no appreciable amount of gas can escape, the gases, instantly upon their generation, will either cut a way out for themselves, escaping with the violence of an explosion, or will blow out the part improperly secured, in either case destroying the apparatus.

The effects produced upon the apparatus, when the gas has escaped by cutting a passage for itself, are very curious. If, for example, one of the plugs has not been sufficiently screwed home, so that the products of combustion escape between the male and female threads, the whole of these threads at the point of escape present the appearance of being washed away, the metal having been evidently in a state of fusion, and carried over the surface of the plug by the rush of the highly heated products"[1]

Summary of Results.—The following are the principal conclusions arrived at by *Noble* and *Abel* as the results of their long continued and laborious experiments:

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[1] *Noble* and *Abel*, Researches. Page 15. Artillery School edition.

(*a*) WHEN GUNPOWDER IS FIRED IN A SPACE ENTIRELY CONFINED.

(1) For service powders about 57 per cent by weight of the products of explosion are non-gaseous, and at the moment of explosion are in a liquid state. The principal constituents of this non-gaseous residue are potassium carbonate, potassium sulphate, and potassium sulphide—the first named greatly preponderating.

(2) With the same powders about 43 per cent of the products of explosion are in the form of permanent gases; and these gases, at a temperature of 0° C. and at a barometric pressure of 760 millimeters, occupy about 280 times the volume of the unexploded powder.

(3) A kilogram of dry ordinary service powder, by its explosion, generates about 720 calories of heat.

(4) At the moment of explosion the temperature of the product is about 2200° C.[1]

(5) The mean specific heat of the products of explosion is about 0.31.

(6) The tension varies with the mean density of the products of combustion according to the law given in Equation (2) of this Chapter.

(*b*) WHEN GUNPOWDER IS FIRED IN THE BORE OF A GUN.

(1) The products of explosion, at all events, as far as regards the proportions of the solid and gaseous products, are the same as in the case of powder fired in a close vessel.

(2) The work on the projectile is effected by the elastic force due to the permanent gases.

(3) The reduction of temperature due to the expansion of the permanent gases, is in a great measure compensated by the heat stored up in the liquid residue.

(4) The law connecting the tension of the products of explosion with the volume they occupy is stated in Equation (11).

(5) The work that gunpowder is capable of performing in expanding in a vessel impervious to heat, is given by Equation (14); and the temperature during expansion is given by Equation (12).

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[1] According to *Noble* and *Abel's* latest data the temperature of combustion would seem to be 2475° C. See further on.

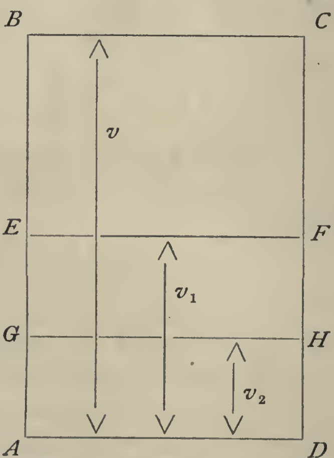
(6) The total theoretic work of gunpowder when indefinitely expanded is about 576 foot-tons per pound of powder.[1]

Pressure in Close Vessels, Deduced from Theoretical considerations.—The expression for the pressure of the gases developed by the combustion of gunpowder in a close vessel is deduced upon the following suppositions:

1st. That a portion of the products of combustion is in a liquid state.

2d. That the pressure due to the permanent gases can only be calculated by deducting the volume of the liquid products from the volume of the vessel.

Upon these hypotheses the expression for the pressure may be deduced as follows:



Let *A B C D* be a section of a close vessel of volume v in which a given charge of powder is exploded.

Let *A E F D* represent the space (v_1) occupied by the charge, and *A G H D* the space (v_2) occupied by the non-gaseous products. Let Δ_1 be the so-called density of the products of combustion,—that is, $\Delta_1 = \frac{v_1}{v}$; and a the ratio of the non-gaseous products to the volume of the charge, or $a = \frac{v_2}{v_1} = \frac{v_2}{\Delta_1 v}$. The gases after explosion will occupy the space $v - v_2 = v - a\Delta_1 v = v(1 - a\Delta_1)$. Let p_1 be the pressure that would be developed if the volume of the vessel were *A E F D* (or v_1). In this case the density of the products of combustion (Δ_1) (the charge remaining the same), would be unity; and the space occupied by the gases would be $v_1 - v_2 = v_1(1 - a) = \Delta_1 v(1 - a)$. Now if p is the pressure when the volume of the vessel is v, we have by *Mariotte's* law (assuming that the temperature is the same for all densities of the products of combustion),

[1] *Noble* and *Abel*, Researches. Pages 110 and 169.

$$p = p_1 \frac{\Delta_1 v(1-a)}{v(1-a\Delta_1)} = (1-a)\,p_1 \frac{\Delta_1}{1-a\Delta_1};$$

or, making

$$f = (1-a)\,p_1,$$

we have

$$p = f\,\frac{\Delta_1}{1-a\Delta_1} \quad . \quad . \quad . \quad (1)$$

The factor f is called the *force of the powder.*

Value of the Ratio a.—Let p_2 and p_3 be the pressures in the same vessel produced by two different charges, and Δ_2 and Δ_3 the corresponding densities of the products of combustion. Then from Equation (1) (assuming f to be the same for all values of Δ_1),

$$p_2 = f\,\frac{\Delta_2}{1-a\Delta_2},$$

and

$$p_3 = f\,\frac{\Delta_3}{1-a\Delta_3};$$

whence by division,

$$\frac{1-a\Delta_3}{1-a\Delta_2} = \frac{p_2\Delta_3}{p_3\Delta_2}.$$

Therefore

$$a = \frac{1}{\Delta_2\Delta_3}\left\{\frac{p_3\Delta_2 - p_2\Delta_3}{p_3 - p_2}\right\},$$

by means of which the mean value of a can be determined when a sufficient number of pressures, corresponding to different values of Δ_1, have been found by experiment. The value of a finally adopted by *Noble* and *Abel* is 0.57.

Determination of the Force of the Powder.—To determine f we have from Equation (1),

$$f = p\,\frac{1-.57\,\Delta_1}{\Delta_1};$$

from which f may be found by means of a single measured pressure corresponding to a given density of the products of combustion. When $\Delta_1 = 1$, that is, when the vessel is completely filled by the charge, p was found to be 43 tons per square inch, and therefore, $f = 43\,(1-.57) = 18.49$ tons or 41417.6 pounds per square inch. Therefore *Noble* and *Abel's* formula for the pressure in a close vessel is, for different densities of the products of combustion,

$$\left.\begin{aligned} p &= 18.49 \frac{\Delta_1}{1-.57\,\Delta_1} \text{ tons per sq. in.} \\ &= 41417.6 \frac{\Delta_1}{1-.57\,\Delta_1} \text{ lbs. per sq. in.} \end{aligned}\right\} \quad . \quad . \quad (2)$$

To transform this equation so that it shall express the pressure in kilos per dm^2 we may employ a simple rule which as it is of frequent use, is here inserted for convenience:

RULE:—To reduce a pressure expressed in tons per square inch to the same pressure expressed in kilos per dm^2, add to the logarithm of the former the constant logarithm 4.1972544 and the sum is the logarithm of the pressure required.

If the pressure to be reduced is in pounds per in^2 then the constant logarithm to be added is 0.8470064.

Applying this rule the expression for the pressure of the products of combustion of a charge of gunpowder fired in a close vessel is found to be

$$p = 291200 \frac{\Delta_1}{1-.57\,\Delta_1}$$

$$\therefore \quad f = 291200 \text{ kilos per dm}^2$$

It will be seen from the definition given to Δ_1 that it is the *density of loading* as defined in Chapter I, when the gravimetric density of the powder is unity,—that is when a kilo of the powder fills a volume of a dm^3; or, what is the same thing, when a pound occupies a volume of 27,68 cubic inches; and in this case, when Δ_1 is unity the charge just fills the receptacle. *Noble* and *Abel* were careful to keep the gravimetric density of the powder they experimented with as near unity as possible.

Interpretation of f.—It will be seen from Equation (1) that the quantity designated by f is the pressure of the gases when

$$\frac{\Delta_1 v}{v(1-\alpha\,\Delta_1)} = 1,$$

that is, when the space occupied by the gases is equal to the volume of the charge, which requires that the vessel should have $1+\alpha$ units of volume. Thus if the kilogramme and litre are the units of weight and volume, respectively, the volume of the vessel must be 1.57 litres in order that the gases may occupy a volume of one litre, and have a tension equal to f. From this f may be defined to be the pressure of the gases of unit weight of powder occupying unit volume at the temperature of combustion T_1.

If ε is the weight of gas furnished by the combustion of unit weight of powder we have from Equation (2), Chapter II,

$$p_1 v_1 = \varepsilon R T_1 ;$$

and if v_1 is the unit of volume, there results

$$p_1 = f = \varepsilon R T_1 \qquad . \qquad . \qquad . \qquad (3)$$

If the pound is the unit of weight the unit of volume is 27.68 cubic inches. In this case the definition of f requires that the volume of the vessel should be $1.57 \times 27.68 = 43.459$ cubic inches.

The value of ε according to *Noble* and *Abel* is 0.43; and therefore the pressure of unit weight of the gases of fired gunpowder at temperature T_1, is

$$\frac{f}{0.43}.$$

From this it follows that the pressure of one pound of the gases of fired gunpowder at temperature of combustion, confined in a volume of 27.68 cubic inches, is

$$\frac{41417.6}{0.43} = 96320 \text{ lbs. per square inch.}$$

Also, the pressure of one pound of the gases of the paragraph immediately preceding, confined in a volume of one cubic foot, is, in pounds per square foot,

$$\frac{96320 \times 27.68}{12} = 222180 \text{ lbs.}$$

If the gravimetric density of the powder be unity, and $\tilde{\omega}$ and v be taken in pounds and cubic inches, respectively, then Equation (1) becomes

$$p = f \frac{27.68\,\tilde{\omega}}{v - 27.68\,a\,\tilde{\omega}} \qquad . \qquad . \qquad . \qquad (4)$$

Solving with reference to $\tilde{\omega}$ and to v gives

$$\tilde{\omega} = \frac{pv}{27.68\,(ap+f)} \quad . \quad . \quad . \quad (5)$$

and

$$v = \frac{27.68\tilde{\omega}(ap+f)}{p} \quad . \quad . \quad . \quad (6)$$

These equations are useful in questions involving the bursting of shells, etc.

Theoretical Determination of the Temperature of Explosion of Gunpowder.—Having determined the value of f, the force of the powder, from the experiments, we can deduce the temperature of explosion by means of the formula

$$T_1 = \frac{273}{\varepsilon p_0 v_0} f$$

According to *Noble* and *Abel's* experiments, if the gravimetric density of the powder is such that a kilogramme occupies one litre, the gases furnished by its combustion will fill a volume of 280 litres at 0°C. under the normal atmospheric pressure of 103.33 kgs. per square decimetre. We therefore have

$$v_0 = \frac{280}{\varepsilon}$$

and

$$p_0 = 103.33$$

whence

$$T_1 = \frac{273 \times 291\,200}{103.33 \times 280} = 2748°\text{C.}$$

This is the absolute temperature of combustion of gunpowder according to *Noble* and *Abel's* latest deductions from their experiments. Subtracting 273° from this temperature we have temperature of explosion = 2475°C. (4487F.).

With regard to the theoretical temperature of explosion the authors remark as follows:

"We have made several experiments with the view of obtaining the temperature of explosion, by ascertaining the effects of the heat developed on platinum. For example, in experiment 78 we introduced into the charge of 'Rifled Large Grain' (R. L. G.), a coil of very fine platinum wire, and also a piece of thin sheet platinum. After the explosion, the sheet platinum was found much heated, but unmelted; but on examination with a microscope there were evident signs of a commencement of fusion on the surface, and a portion of the fine platinum wire was found welded on the sheet. The coil of wire was not to be found, but portions of it were observed welded to the sides of the cylinder.

Now we know that platinum is readily volatilized, when exposed to the hydrogen-blowpipe, at a temperature of about 3200° C.; and therefore, if the temperature of explosion had approached this point, we should have expected the very fine wire to be volatilized; remembering the low specific heat of platinum, we should furthermore have been warranted in expecting more decided signs of fusion in the sheet metal.

Again, in experiments 84, 85 and 86, pieces of platinum wire .03 inch (0.75 millim.) in diameter and 4 inches (100 millims.) long were placed in the cylinder with considerable charges of R. L. G. and F. G. In none of these experiments did the platinum melt, although, as in the case of the sheet platinum, there were signs of fusion on the surface of the wires. In experiment 79 however, in which platinum wire was placed with a corresponding charge of the Spanish powder, the wire was fused, with the exception of a small portion. With this powder, indeed, which is of a very different composition from the English powders, and decidedly more rapidly explosive in its nature, it is quite possible that a somewhat higher heat may have been attained. But, as in one case the platinum wire was nearly fused, and in the others it only showed signs of fusion, the conclusion we draw from the whole of these experiments on the fusion of the platinum, is that the temperature of explosion is higher than the melting-point of that metal, but not greatly so. Now, according to *Deville*, the melting-point of platinum is nearly 2000° C.; and hence we have a strong corroboration of the approximate accuracy of the theoretical temperature of explosion."[1]

Mean Specific Heat of the Products of Combustion.—From Equation (8) Chapter II, we have when $W = o$, that is, when no external work is performed,

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[1] *Noble* and *Abel*, Researches, etc., Page 70.

$$Q = c\,(T_1 - 273)$$

in which Q is the heat of combustion; that is, the quantity of heat that unit of weight of the explosive substance evolves, under constant volume, when the final temperature of the products of combustion is 0°C. From this equation we find

$$c = \frac{Q}{T_1 - 273}$$

The heat of combustion was determined by *Noble* and *Abel* in the following manner:

"A charge of powder was weighed and placed in one of the smaller cylinders, which was kept for some hours in a room of very uniform temperature. When the apparatus was throughout of the same temperature, the thermometer was read, the cylinder closed, and the charge exploded.

Immediately after explosion the cylinder was placed in a calorimeter containing a given weight of water at a measured temperature, the vessel being carefully protected from radiation, and its calorific value in water having been previously determined.

The uniform transmission of heat through the entire volume of water was maintained by agitation of the liquid, and the thermometer was read every five minutes until the maximum was reached. The observations were then continued for an equal time to determine the loss of heat in the calorimeter due to radiation, etc.; the amount so determined was added to the maximum temperature."

In this way the heat of combustion of R. L. G. and F. G. powders was found to be 705 heat-units; that is, the combustion of a unit weight of the powder liberated sufficient heat to raise the temperature of 705 unit-weights of water 1°C. We therefore have

$$c = \frac{705}{2231} = 0.316$$

This result is accepted by *Noble* and *Abel*, and also by *Sarrau*, as a very close approximation to the mean specific heat of the entire products of combustion. If we assume that the mean specific heat of gunpowder of different compositions is constant, we can compute the temperatures of combustion when the heat of combustion has been determined by the calorimeter, by the formula

$$T = \frac{Q}{0.316}$$

in which T is given by the centigrade scale.

Pressure in the Bores of Guns Derived from Theoretical Considerations.—"At an early stage in our researches, when we found, contrary to our expectation, that the elastic pressure deduced from experiments in close vessels did not differ greatly (where the powder might be considered entirely consumed, or nearly so) from those deduced from experiments in the bores of guns themselves, we came to the conclusion that this departure from our expectation was probably due to the heat stored up in the liquid residue. In fact, instead of the expansion of the permanent gases taking place without addition of heat, the residue, in the finely divided state in which it must be on the ignition of the charge, may be considered a source of heat of the most perfect character, and available for compensating the cooling effect due to the expansion of the gases on production of work.

The question, then, that we now have to consider is—What will be the conditions of expansion of the permanent gases when dilating in the bore of a gun and drawing heat, during their expansion, from the non-gaseous portions in a very finely divided state?"[1]

Let $c_{\prime}$ be the specific heat of the non-gaseous portion of the charge, which we can assume without material error, to be constant. We shall then have $c_{\prime}\,dT$ for the elementary quantity of heat yielded to the gases per unit of weight of liquid residue. If there are w_1 units of weight of liquid residue it will yield to the gases $w_1 c_{\prime}\,dT$ units of heat; and if there are w_2 units of weight of gas we shall have in heat-units,

$$dq = -\frac{w_1}{w_2} c_{\prime}\,dT = -\beta c_{\prime}\,dT,$$

in which

$$\beta = \frac{w_1}{w_2};$$

[1] *Noble* and *Abel*, Researches, etc., Page 98.

that is, β is the ratio between the weights of the non-gaseous and gaseous portions of the charge. The negative sign is given to the second member because T decreases while q increases.

Substituting the above value of dq in Equation (7), Chapter II, it becomes

$$-(c+\beta c_{\prime})\, dT = \frac{c'-c}{R}\, p\, dv \quad . \quad . \quad . \quad (7)$$

and this combined with Equation (6), Chapter II, gives, by a slight reduction,

$$-(\beta c_{\prime}+c')\frac{dv}{v} = (\beta c_{\prime}+c)\frac{dp}{p} \quad . \quad . \quad . \quad (8)$$

Since c', c, $c_{\prime}$ and β are, by hypothesis, constant during the expansion, the integration of Equation (8) between the limits v_2 and v_3—the former being the initial volume occupied by the permanent gases and the latter their volume after the projectile has been displaced by a distance u, gives

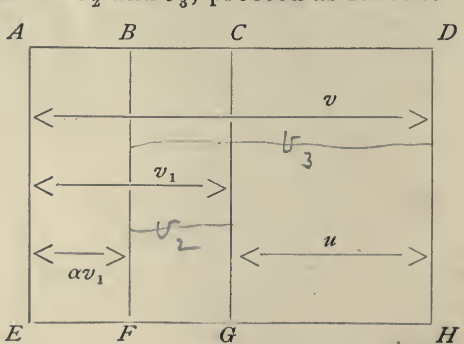
$$p = p_1 \left(\frac{v_2}{v_3}\right)^r \quad . \quad . \quad . \quad (9)$$

in which

$$r = \frac{\beta c_{\prime}+c'}{\beta c_{\prime}+c}$$

Equation (9) it will be seen, becomes identical with Equation (13), Chapter II, when $\beta = o$; that is, when there is no liquid residue.

To introduce v_1 and v, that is the volumes occupied by the charge and the entire volume in rear of the projectile, into Equation (8) in place of v_2 and v_3, proceed as follows: Let $A\ C\ E\ G$



represent the chamber of the gun, which we will suppose filled with powder without compression, and further that one pound of the powder fills a space of 27.68 cubic inches. The gravimetric density and density of loading are each unity; and if v_1 is the volume of the chamber, it follows that

$$v_1 = 27.68\,\tilde{\omega}.$$

Suppose the powder to be entirely consumed before the projectile moves any perceptible distance; and that the non-gaseous products fill the space $A\ B\ E\ F$, whose volume is $a\,v_1$. The gases, therefore, which by their expansion give motion to the projectile will occupy the space $B\ C\ F\ G$ before perceptible motion begins. The volume of the space $B\ C\ F\ G$ is evidently $v_2 = v_1 - av_1 = v_1(1-a)$. Let $D\ H$ be the base of the projectile after it has moved a distance u; and, designating the volume $A\ D\ E\ H$ by v, we evidently have $v_3 = v - a\,v_1$. Substituting these values of v_2 and v_3 in Equation (9) gives

$$p = p_1 \left(\frac{v_1(1-a)}{v - a\,v_1}\right)^r \quad . \quad . \quad . \quad (10)$$

In this equation p_1 is the pressure produced by the combustion of a charge of powder in a close vessel when the density of loading is unity. The values of the constants are given by *Noble* and *Abel* as follows:[1]

$p_1 = 43$ tons per square inch
$a = 0.57$
$\beta = 1.2957$
$c' = 0.2324$
$c = 0.1762$
$c_{,} = 0.45$
$v_1 = 27.68\,\tilde{\omega}$

from which we find $r = 1.074$. Substituting these values in the expression for p it becomes

$$p = 43 \left(\frac{0.43\,v_1}{v - 0.57\,v_1}\right)^{1.074} \quad . \quad . \quad . \quad (11)$$

which gives the pressure in tons per square inch.

[1] Researches, etc. Page 167.

If, as in a close vessel, we let

$$\Delta_1 = \frac{v_1}{v},$$

then

$$p = (0.43)^{1.074} \times 43 \left\{ \frac{\Delta_1}{1 - .57\,\Delta_1} \right\}^{1.074}$$

$$= 17.375 \left\{ \frac{\Delta_1}{1 - .57\,\Delta_1} \right\}^{1.074} \quad . \quad . \quad . \quad (11')$$

Compare this with Equation (2).

REMARKS:—The value of $\beta = 1.2957$, adopted by *Noble and Abel* gives for unit weight, $w_1 = 0.5644$ and $w_2 = 0.4356$, while the values of these quantities adopted in our equations are 0.57 and 0.43, respectively. These last named values would make $\beta = 1.3256$.

Noble and *Abel's* values of the specific heats of the permanent gases of combustion, namely, $c' = 0.2324$ and $c = 0.1762$, make $n = 1.32$; while for perfect gases as was shown in Chapter II, $n = 1.4$ very nearly.

Table of Pressures.—In the following table of pressures the third column gives the pressures in the bore of a gun corresponding to the values of Δ_1 in the first column. They were computed by Equation (11) upon the assumption that the permanent gases in expanding and thereby doing work, borrow heat from the non-gaseous residue; and also that the combustion is complete before the projectile has moved perceptibly; and finally that there is no conduction of heat to the walls of the gun. The tensions in the fifth column were computed by Equation (2) and agree with *Noble* and *Abel's* experiments.

Mean density of products of combustion. Δ_1	Corresponding expansions. $\frac{1}{\Delta_1}$	Tensions calculated by Equation (11).		Tensions in close cylinders, or where gases expand without doing work.	
		Tons per square inch.	Differences.	Tons per square inch.	Differences.
1.00	1.000	43.00	5.01	43.00	4.69
.95	1.053	37.99	4.40	38.31	4.14
.90	1.111	33.59	3.88	34.17	3.68
.85	1.176	29.72	3.44	30.49	3.30
.80	1.250	26.28	3.06	27.19	2.97
.75	1.333	23.22	2.76	24.22	2.68
.70	1.429	20.46	2.48	21.54	2.45
.65	1.539	17.98	2.25	19.09	2.23
.60	1.667	15.73	2.04	16.86	2.05
.55	1.818	13.69	1.86	14.81	1.88
.50	2.000	11.83	1.70	12.93	1.74
.45	2.222	10.13	1.56	11.19	1.61
.40	2.500	8.57	1.43	9.58	1.50
.35	2.857	7.14	1.31	8.08	1.39
.30	3.333	5.83	1.21	6.69	1.30
.25	4.000	4.62	1.11	5.39	1.22
.20	5.000	3.51	1.02	4.17	1.14
.15	6.667	2.49	.93	3.03	1.07
.10	10.000	1.56	.84	1.96	1.01
.05	20.000	.72		.95	

Temperatures of Products of Combustion in Bores of Guns.— The temperature in the bore of a gun during the expansion of the products of combustion, may be determined from Equation (7), which replacing R by its value from Equation (1) Chapter II, becomes

$$\frac{dT}{T} = -\frac{c' - c}{\beta c_1 + c}\frac{dv}{v};$$

whence integrating between the same limits as before, and observing that

$$\frac{c' - c}{\beta c_1 + c} = r - 1,$$

we have

$$T = T_1 \left(\frac{v_2}{v_3}\right)^{r-1}.$$

Replacing v_2 by v_1 $(1-a)$, and v_3 by $v-av_1$, for reasons already given, we have for the absolute temperature of the gases during expansion, the equation

$$T = T_1 \left(\frac{v_1 (1-a)}{v-av_1}\right)^{r-1}.$$

Introducing the density of the products of combustion (Δ_1), and the numerical values of a and r into this last equation, it becomes

$$T = (.43)^{.074} \times T_1 \left\{ \frac{\Delta_1}{1-.57\,\Delta_1} \right\}^{.074}$$

$$= 0.93946\, T_1 \left\{ \frac{\Delta_1}{1-.57\,\Delta_1} \right\}^{.074} \quad . \quad . \quad . \quad (12)$$

The value of T for any given density of the products of combustion (represented by Δ_1) will depend upon their initial temperature, (or absolute temperature of combustion), T_1. Its theoretical value, based upon *Noble* and *Abel's* latest deductions from their experiments, as published in their second memoir, has already been found to be 2748°C. But there are very great difficulties in the way of verifying by experiment the theoretical value of T_1, and Captain *Noble* in his Greenock lecture (February 12th, 1892) takes the absolute temperature of combustion at 2505°C, as deduced in their first memoir. Therefore making

$$T_1 = 2505°\text{C.},$$

the expression for T becomes

$$T = 2353.3 \left\{ \frac{\Delta_1}{1-.57\,\Delta_1} \right\}^{.074} \quad . \quad . \quad . \quad (13)$$

The temperatures in degrees Centigrade and Fahrenheit, calculated from Equation (13) are given in the following table. "It is hardly necessary to point out that the values given in this table are only strictly accurate when the charge is ignited before the projectile is sensibly moved; but in practice the correction due to this cause will not be great."[1]

[1] *Noble* and *Abel.* Page 103.

Mean density of products of combustion. Δ_1	Number of volumes of expansion. $\frac{1}{\Delta_1}$	Temperatures.	
		Centigrade.	Fahrenheit.
1.00	1.0000	2231	4048
.95	1.0526	2210	4010
.90	1.1111	2189	3972
.85	1.1765	2168	3934
.80	1.2500	2147	3897
.75	1.3333	2126	3859
.70	1.4286	2106	3823
.65	1.5385	2085	3785
.60	1.6667	2063	3745
.55	1.8182	2041	3706
.50	2.0000	2018	3664
.45	2.2222	1994	3621
.40	2.5000	1968	3574
.35	2.8571	1940	3524
.30	3.3333	1909	3468
.25	4.0000	1874	3405
.20	5.0000	1834	3333
.15	6.6667	1785	3245
.10	10.0000	1719	3126
.05	20.0000	1615	2939
.00	∞	0	0

Theoretical Work Effected by Gunpowder.—The theoretical work which a charge of gunpowder is capable of effecting during the expansion of its volume from v_1 to any volume v is expressed by the definitne integral

$$W = \int_{v_1}^{v} p\, dv;$$

or, substituting for p its value from Equation (10)

$$W = \int_{v_1}^{v} p_1 \left(\frac{v_1(1-a)}{v - a v_1} \right)^r dv,$$

$$= p_1 v_1^r (1-a)^r \int_{v_1}^{v} \frac{dv}{(v - a v_1)^r};$$

whence, integrating, we have

$$W = \frac{p_1 v_1^r (1-a)^r}{r-1} \left\{ \frac{1}{[v_1(1-a)]^{r-1}} - \frac{1}{(v - a v_1)^{r-1}} \right\}.$$

Multiplying and dividing the second member by $[v_1(1-a)]^{r-1}$, we have

$$W = \frac{p_1 v_1 (1-a)}{(r-1)} \left\{ 1 - \left(\frac{v_1(1-a)}{v - a v_1}\right)^{r-1} \right\}.$$

If, in this last equation, p_1 be expressed in kilogrammes per square decimetre, and v_1 be made unity (one litre), the work will be expressed in decimetre-kilogrammes per kilogramme of powder burned. To express the work in foot-tons per pound of powder burned, we must make $v_1 = 27.68$ cubic inches; and then, since p_1 is given in tons per square inch, divide the result by 12, the number of inches in a foot. Making these substitutions and replacing a and r by their values already given, we have, in foot-tons,

$$W = 576.369 \left\{ 1 - \left(\frac{.43 v_1}{v - .57 v_1}\right)^{r-1} \right\}$$

or, in terms of Δ_1,

$$W = 576.369 \left\{ 1 - 0.93946 \left(\frac{\Delta_1}{1 - .57 \Delta_1}\right)^{.074} \right\} \quad . \quad . \quad (14)$$

Substituting in Equation (14) from Equation (12) we have

$$W = \frac{576.369}{T_1}(T_1 - T)$$

or, since, according to *Noble* and *Abel*,

$$T_1 = 2505°$$

we have

$$W = 0.23008(T_1 - T) \quad . \quad . \quad . \quad (15)$$

which gives the work in terms of the loss of temperature of the products of combustion.

Table 1 gives the work of expansion of the gases of one pound of gunpowder of the normal type and free from moisture, computed by Equation (14). By means of the work given in this table, and by the use of a proper factor of effect determined by experiment, *Noble* and *Abel* consider that the actual work of a given charge of powder upon a projectile may be computed with considerable accuracy. Their method of using this table will be clearly seen by the following extract:

"If we wish to know the maximum work of a given charge, fired in a gun with such capacity of bore that the charge suffered five expansions ($\Delta_1 = 0.2$) during the motion of the projectile in the gun, the density of loading being unity, the table shows us that for every pound in the charge, an energy of 91.4 foot-tons will as a maximum be generated.

If the factor of effect for the powder and gun be known, the above values multiplied by that factor, will give the energy per pound that may be expected to be realized in the projectile.

But it rarely happens, especially with the very large charges used in the most recent guns, that densities of loading so high as unity are employed; and in such cases, from the total energy realizable, must be deducted the energy which the powder would have generated, had it expanded from a density of unity to that actually occupied by the charge. Thus in the example above given, if we suppose the charge instead of a density of loading of unity to have a density of 0.8, we see from Table 1, that from the 91.4 foot-tons above given, there must be subtracted 19.23 foot-tons; leaving 72.17 foot-tons as the maximum energy realizable under the given conditions, per pound of the charge."[1]

To apply these principles practically for *muzzle velocities*, let, as before,

v_1 be the volume occupied by the charge, in cubic inches.
v the total volume of bore and chamber, in cubic inches.
C_1 the volume of the bore.
C the volume of the chamber, in cubic inches.

Then

$$v = C_1 + C;$$

and, if the gravimetric density of the powder be unity,

$$v_1 = 27.68\,\tilde{\omega},$$

where $\tilde{\omega}$ is the weight of the charge in pounds. Therefore the number of volumes of expansion of the products of combustion will be, at the muzzle,

$$\frac{v}{v_1} = \frac{1}{\Delta_1} = \frac{C_1}{27.68\,\tilde{\omega}} + \frac{C}{v_1};$$

[1] *Noble* and *Abel*. Researches. Page 176.

which may be written, if the gravimetric density of the powder be unity,

$$\frac{1}{\Delta_1} = 0.0361263 \frac{C_1}{\tilde{\omega}} + \frac{1}{\Delta}$$

in which Δ is the density of loading as defined in Chapter I.

If the gravimetric density of the powder be not unity, let v_2 be the volume in cubic inches of one pound of powder not pressed together except by its own weight; and let

$$\frac{27.68}{v_2} = m;$$

then we have in all cases,

$$\frac{1}{\Delta_1} = m \left\{ 0.0361263 \frac{C_1}{\tilde{\omega}} + \frac{1}{\Delta} \right\},$$

in which $\frac{1}{\Delta_1}$ is the number of volumes of expansion of the products of combustion.

Let W_2 be the work taken from *Noble* and *Abel's* table (Table 1 of this book), of the gases of one pound of powder for a given value of $\frac{1}{\Delta_1}$, and W_1 the work due to the expansion $\frac{m}{\Delta}$. Also, let F be the factor of effect. Then if we assume that the work of expansion is all expressed in the energy of translation of the projectile, we shall have approximately,

$$\frac{wV^2}{4480\,g} = F\,W\,\tilde{\omega} \quad . \quad . \quad . \quad (16)$$

in which w is the weight of the projectile and

$$W = W_2 - W_1$$

From Equation (16) the muzzle velocity V may be computed when the factor of effect is known; or, we may determine the factor of effect when the muzzle velocity has been measured by a chronograph. These two equations reduced to practical forms are the following:

$$V = 379.57 \sqrt{F W \frac{\tilde{\omega}}{w}} \quad . \quad . \quad . \quad (17)$$

and

$$F = 0.000006941 \frac{V^2 w}{W \tilde{\omega}} \quad . \quad . \quad . \quad (18)$$

As an illustrative application of these formulas to interior ballistics take the following data from *Noble* and *Abel's* second memoir, relative to the English 8-inch gun: It was found by firing a charge of 70 pounds of a certain brand of pebble powder with a projectile weighing 180 pounds, that a muzzle velocity of 1694 foot-seconds was obtained. What was the factor of effect (F) pertaining to this gun and brand of powder? For this particular gun and charge we have $\tilde{\omega} = 70$ pounds, $w = 180$ pounds, $\Delta_1 = 0.1634$, $\Delta = 0.605$ and $m = 1$. In *Noble* and *Abel's* table of work (Table 1 of this treatise), the first column gives values of $\frac{1}{\Delta_1}$, increasing by a common difference, while the second column contains the corresponding values of Δ_1. By a simple interpolation we find for the values of Δ_1 and Δ given above, $W_2 = 99.4$ and $W_1 = 37.6$; whence $W = 61.8$ foot-tons. Substituting these values in Equation (18) we have

$$F = 0.000006941 \frac{180 \times (1694)^2}{70 \times 61.8} = 0.8287.$$

That is, the actual work realized, as expressed and measured by the projectile's energy of translation, as it emerges from the bore, is nearly 83 per cent of the theoretical maximum work which the powder gases are capable of performing, leaving but 17 per cent for the other work done by the gases, namely, the work expended upon the charge, the gun, and carriage, and in giving rotation to the projectile; the work expended in overcoming passive resistances, such as forcing the rotating band into the groove, the subsequent friction as the projectile moves along the bore, and the resistance of the air in front of the projectile; and lastly, the heat communicated to the walls of the gun. It is very difficult to evaluate these non-useful energies, but it is probable that they do not consume more than 17 per cent of the maximum work of the gases. *Longridge* finds by an elaborate calculation that this lost work in a 10-inch B. L. Woolwich gun amounts to 30 per cent of the maximum work;[1] but it is believed that he has greatly overestimated the work required to give motion to the products of combustion. Colonel *Pashkievitsch* makes the

[1] Internal Ballistics. By *Atkinson Longridge*. London, 1889. Chapter V.

lost work rather less than 17 per cent of that expressed in the energy of translation of the projectile.[1]

To test the correctness of Equation (17) for determining muzzle velocities we will apply it to the same gun by means of which the factor of effect was determined, increasing the charge from 70 to 90 pounds, and again to 100 pounds, and compare the computed velocities with those measured with a chronograph. For a charge of 90 pounds of powder we have $\Delta_1 = 0.210$ and $\Delta = 0.780$; whence $W_2 = 89.3$, $W_1 = 20.86$, and $W = 68.44$.

$$\therefore \quad V = 379.57 \sqrt{\frac{.8287 \times 90 \times 68.44}{180}} = 2021 \text{ foot-sec.}$$

The measured velocity with this charge was 2027 foot-seconds. In a similar way we find by the formula that for a charge of 100 pounds $V = 2174$ foot-seconds, while the measured velocity was 2182 foot-seconds. The differences between the computed and observed velocities in these examples are about one-third of one per cent, and are well within the limits of possible error in measuring them.

The factor of effect increases with the caliber of the gun, as is shown by experiment. Thus with the English 10-inch gun fired with charges of 130 and 140 pounds of the pebble powder we have been considering, the factor of effect is 0.855; while with the 11-inch gun, and charge of 235 pounds, the factor of effect is 0.89. With regard to the variation of the factor of effect *Noble* and *Abel* make the following suggestive remarks:

Not only may the factors of effect differ very much with the powders employed, being in this respect dependent upon circumstances, such as the density of the powder, its size of grain, amount of moisture, chemical composition, nature of charcoal used, etc., but they may also vary considerably even with the same powder, if the charges be not fired under precisely the same circumstances. For example: Especially with slow-burning powders the weight of the shot fired exerts a very material influence upon the factor of effect, and the reason is obvious. The slower the shot moves at first, the earlier in its passage along the bore is the charge entirely consumed, and the higher is the energy realized. The same effect, unless modified by other circumstances, is produced when the charge is increased with the same weight of projectile.

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[1] Interior Ballistics. By Colonel *Pashkievitsch*. Translated from the Russian by Captain *Tasker H. Bliss*, U. S. Army. Washington, 1892.

In this case the projectile has to traverse a greater length of bore before the same relief due to expansion is obtained. The higher pressures which consequently rule, react upon the rate of combustion of the powder, and again a somewhat higher energy is obtained.[1]

Observed pressures in the Bores of Guns.—There are two methods for determining experimentally the tension of the gas at various points in the bore of a gun, both of which have been tried more or less successfully. The first method measures the tensions by means of suitable gauges inserted at different points in the walls of the bore. This method was first employed by *Rodman* as stated in Chapter I.

By the second method the velocity of the projectile is measured at different points of the bore, and from these measured velocities the accelerations are calculated, and then the pressures, by multiplying the accelerations by the mass of the projectile. Two methods of determining these velocities have been devised: First, that employed in 1760 by Chevalier *D'Arcy*, an account of which is given in Chapter I. Second, *Noble's* method, which consists in measuring the time at which the projectile passes certain fixed points in the bore, and thence deducing the corresponding velocities.

Noble's Chronoscope.—The almost infinitesimal intervals of time required for the second method are determined by means of a chronoscope invented by Captain *Noble* which is thus described:[2] "In its most recent form it consists of a series of thin discs each 36 inches in circumference, keyed to a shaft which is made to rotate at a very high and uniform velocity, through a train of wheels propelled by a heavy descending weight. The speed with which the circumference of the discs travels is usually about 1200 inches per second; an inch, therefore, represents the 1200th part of a second; and as by means of a vernier we are able to divide the inch into 1000 parts, the instrument is capable of recording less than the one millionth part of a second. The precise rate of the discs is ascertained through an intermediate shaft which by means of a relay, registers its revolutions on a

[1] Researches, etc., Page 178.

[2] *Noble* and *Abel*. Researches. Page 73. Also Captain *Noble's* Greenock lecture. Page 93.

subsidiary chronoscope (each revolution of the shaft corresponding to 200 revolutions of the discs), upon which subsidiary chronoscope a chronometer, also by means of a relay, registers seconds."

"The recording arrangement is as follows:—Each disc is furnished with an induction coil, the primary wire from which is conveyed to any point in the gun where we may wish to record the instant at which the shot passes. There is at each such point a special contrivance by which the shot in passing severs the primary wire, thereby causing a discharge from the secondary, which is connected with the discharger. The spark records itself on the disc by means of paper specially prepared to receive it. When the instrument is in good working order the probable instrumental error of a single observation does not exceed from two to three millionths of a second."

The data employed by *Noble* and *Abel* in their discussion of the phenomena attending the combustion of gunpowder in cannon were chiefly derived from the experiments carried on by the "Committee of Explosives" whose president was Colonel *Younghusband*, F. R. S., using the *Noble* chronoscope above described. Two series of experiments were made by the committee with the 10-inch 18-ton gun the results of which will be discussed in Chapter IV.

EXAMPLES

Example 1.—Trace the locus of Equation (1), p and Δ_1 being the variables. Show that it has an asymptote and give the physical interpretation.

Example 2.—A close spherical shell contains one pound of gunpowder of unit gravimetric density, and nothing else. If immediately after the explosion of the powder, the outward pressure of the gases is just equal to the atmospheric pressure, what is the radius of the shell, and what the density of loading?

Answers. $R = 26.5$ inches.

$\Delta = 0.0003548$

Example 3.—If it require a pressure of 60,000 lbs. per square inch to rupture the walls of a 6-inch B. L. R. steel shell, how many pounds of standard powder would it require, the volume of the cavity being 50 cubic inches? What is the density of loading? Answers. $w = 1.433$ lbs.

$\Delta = 0.79346$

Example 4.—If the density of loading of the spherical cavity of a shell be 0.9, what will be the pressure when exploded? If the diameter of the cavity is 8 inches what is the weight of charge?

Answers. $p = 76542$ lbs. per sq. in.

$\tilde{\omega} = 8.7163$ lbs.

Example 5.—With a certain brand of pebble powder the factor of effect for the 10-inch English 18-ton gun was found to be 0.85. Compute the muzzle velocities of two similar shots weighing 400 pounds each, fired from this gun, one with a charge of 130 pounds ($\Delta_1 = 0.233$, $\Delta = 0.792$); and the other with a charge of 140 lbs. ($\Delta_1 = 0.247$, $\Delta = 0.840$).

Answers. 1610 f. s. and 1698 f. s.

The measured velocities in this case were 1605 f. s. and 1706 f. s. respectively.

Example 6.—For the 8-inch B. L. rifle, model of 1888, we have the following data: $C_1 = 14125$ c. i., $C = 3597$ c. i., $\tilde{\omega} = 125$ lbs., $w = 300$ lbs., $m = 1$, and $V = 1950$ f. s. From this, compute the factor of effect and then the muzzle velocity for a charge of 100 lbs. Answers. $F = 0.71522$.

$V = 1648$ f.s.

79

16

CHAPTER IV.

FORMULAS FOR VELOCITY AND PRESSURE IN THE BORE OF A GUN.

Within the last few years M. *Emile Sarrau*, engineer-in-chief of the French powder factories, has published a series of most important memoirs upon the effects of fired gunpowder in the bore of a gun, in which he has deduced practical formulas by means of which we are able to calculate the muzzle velocity of a projectile and the maximum pressure in the gun with a considerable degree of approximation. In these investigations M. *Sarrau* has for the first time taken into account the *progressive combustion* of the charge under the influence of the *variable pressure* to which it is subjected in the gun, and has thus been able to introduce explicitly into his formulas the characteristic elements of each powder employed, that is, the form and size of grain, and the velocity of combustion in free air. Some of the most important applications of *Sarrau's* methods are those which regard the powder as the principal variable; and by so regarding it he has succeeded in deducing formulas for determining in advance the proper weight of charge, the kind of powder, and the best form and size of grain to bring about desired results.

In his investigations *Sarrau* assumes that the time required to ignite the charge is so short that it can be neglected in comparison with that required for its combustion, an assumption which is undoubtedly correct for the large and dense grains of modern powders. He also assumes that the permanent gases of the products of combustion neither receive heat from the non-gaseous products, nor give off heat to the walls of the gun, but expand adiabatically. This last assumption, which has been proven incorrect by *Noble* and *Abel*, is only adopted provisionally and as a convenient working hypothesis; and in his practical formulas the constants are so determined by experiment as to correct in great measure whatever error there may be in this hypothesis—

in fact, to render these formulas independent of any hypothesis as to the gases receiving or giving off heat, and also as to whether all the powder is consumed before the projectile leaves the bore.

Differential Equation of Motion of a Projectile in the Bore of a Gun.—Let

y be the weight of powder burned in the gun in the time t, counting from the instant the charge is inflamed.

ε the weight of gas formed by the combustion of unit weight of powder; and, therefore,

εy the weight of gas in the bore of the gun at the time t.

T_1 the absolute temperature of combustion, or the absolute temperature of the mass of gas εy if formed in a close vessel, without performing work.

T the actual absolute temperature of the mass of gas εy at the end of the time t, and when the projectile has been moved by the expansion of the gas a distance u.

E the mechanical equivalent of heat.

c the mean specific heat of the gases under constant volume.

Then, since in an adiabatic expansion the external work done is proportional to the fall in temperature, we have, for a mass of gas εy falling from temperature T_1 to temperature T, the relation

$$W = \varepsilon y c E (T_1 - T) \quad . \quad . \quad . \quad (1)$$

Equation (1) is true whether we consider the combustion of the powder as instantaneous or gradual, provided the initial and final temperatures are the same in both cases. In the one case we have a mass of gas εy instantly formed at a temperature T_1, expanding and thereby performing work, until its temperature is reduced to T. In the other case we have a progressive formation of gas at temperature T_1, from zero to mass εy, expanding as formed, and doing work until, as before, the temperature of the mass becomes T. Equation (1) can be transformed by introducing in place of the temperature of the gas, the volume and pressure, upon which its temperature directly depends. We have by *Mariotte's* law for a weight of gas εy occupying a space v at the absolute temperature T, the relation

$$p v = \varepsilon y R T \quad . \quad . \quad . \quad (2)$$

where p is the pressure exerted by the gas upon unit of surface.

Substituting for T in Equation (1) its value from Equation (2) it becomes

$$W = \varepsilon y c E \left\{ T_1 - \frac{pv}{\varepsilon y R} \right\} \quad . \quad . \quad . \quad (3)$$

Making $f = \varepsilon R T_1$, where f is the *force of the powder*, and writing for $\frac{cE}{R}$ its value $\frac{1}{n-1}$, Equation (3) becomes

$$(n-1) W = fy - pv \quad . \quad . \quad . \quad (4)$$

To express the space occupied by the gas at any instant as a function of the distance travelled by the projectile, let

- ω be the area of a right section of the bore.
- u the distance travelled by the projectile from its seat, or initial position.
- z_0 the reduced length of the initial air-space.

Then since, according to *Noble* and *Abel's* experiments, the volume of solid residue left after explosion is equal to the volume of the powder, we evidently have

$$v = \omega (z_0 + u)$$

and this substituted for v in Equation (4) gives

$$(n-1) W = fy - \omega p (z_0 + u) \quad . \quad . \quad . \quad (5)$$

an equation which expresses at each instant of the expansion the relation which exists between the weight of powder burned, the tension of the gas, the distance travelled by the projectile, and the external work performed.

If we assume that the work done by the expansion of the powder gas is measured by the energy of translation imparted to the projectile, we may put

$$W = \frac{w}{2g} v^2 = \frac{w}{2g} \left(\frac{du}{dt} \right)^2$$

in which w is the weight of the projectile.

Also, p being the pressure per unit of area, ωp is the entire pressure on the base of the projectile; and therefore $\frac{g\omega}{w} p$ is the expression for the acceleration. Therefore

$$\omega p = \frac{w}{g} \cdot \frac{d^2 u}{dt^2} \quad . \quad . \quad . \quad (6)$$

Substituting these values of W and ωp in Equation (5), we have finally,

$$(z_0 + u) \frac{d^2 u}{dt^2} + \frac{n-1}{2} \left(\frac{du}{dt} \right)^2 = fg \frac{y}{w} \quad . \quad . \quad . \quad (7)$$

which is *Sarrau's* differential equation of the motion of a projectile in the bore of a gun.

In deducing Equation (7) there were neglected the following energies:

1st. The heat communicated by the gases to the walls of the gun; and also the heat received by the gases (if any) from the solid products.

2d. The work expended on the charge, the gun and carriage, and in giving rotation to the projectile.

3d. The work expended in overcoming passive resistances, such as forcing, friction along the grooves, the resistance of the air, etc.

It will be seen, however, from the form of Equation (7), that all of these neglected energies may be allowed for by giving to f a suitable value; and this without changing the form of the differential equation of motion. The "force of the powder" then, can only be considered as a coefficient in Equation (7) whose value must be determined by experiment.

If v is the velocity of the projectile in the bore of the gun at any instant we have

$$\frac{du}{dt} = v,$$

and

$$\frac{d^2 u}{dt^2} = \frac{d(v^2)}{2\,du};$$

substituting these in Equation (7) it becomes

$$(z_0 + u) \frac{d(v^2)}{du} + (n-1)v^2 = \frac{2fg}{w} y \quad . \quad . \quad . \quad (8)$$

There is great uncertainty as to the proper value to be given to n for the gases of fired gunpowder. As we have seen in Chapter II, the value of this ratio for perfect gases is nearly 1.4; and it has been assumed that at the high temperatures maintained by the powder gases in the bore of a gun they may be regarded as possessing all the properties of perfect gases; and therefore many of the earlier writers on interior ballistics assumed that $n = 1.4$. But recent investigations have shown that this value is too great, but they have not fixed its true value. The value of n derived from *Noble* and *Abel's* experiments is nearly $1\frac{1}{3}$ for the gases of fired gunpowder at or near the temperature of combustion; and this is the value which, for want of a better, will be adopted in what follows. Introducing this value of n into Equation (8) it becomes

$$3(z_0 + u)\frac{d(v^2)}{du} + v^2 = \frac{6fg}{w}y \quad . \quad . \quad . \quad (9)$$

Equation (9) expresses the relation which must exist at any instant between the distance travelled by the projectile and the velocity it has acquired, when a weight of powder y has been burned; and its solution will give the velocity when y is known and f has been determined by experiment. In order to determine the velocity at any point within the bore it will be necessary to know the value of y at that point; and this involves a knowledge of the law of combustion of the grains of which the charge is composed, as a function of the distance travelled by the projectile, which will be considered further on.

If x is the ratio of u to z_0, we have

$$u = x z_0 \quad . \quad . \quad . \quad . \quad (10)$$

and this value of u substituted in Equation (9) gives

$$3(1 + x)\frac{d(v^2)}{dx} + v^2 = \frac{6fg}{w}y \quad . \quad . \quad . \quad (11)$$

Since y is a function of the time it is also a function of u and of x, and may be written

$$y = \varphi(x);$$

and substituting this in Equation (11) it becomes

$$3(1 + x)\frac{d(v^2)}{dx} + v^2 = \frac{6fg}{w}\varphi(x) \quad . \quad . \quad . \quad (12)$$

It appears impossible at present either to determine the actual form of $\varphi(x)$, or to integrate Equation (12) in finite terms even when we give to $\varphi(x)$ the simplest approximate form which the nature of the problem admits of. If we regard y as constant Equation (11) can be solved as follows: Separating the variables we have

$$\frac{d(v)^2}{v^2 - \frac{6fgy}{w}} + \frac{dx}{3(1+x)} = o;$$

from which we get by integration

$$\left\{ v^2 - \frac{6fgy}{w} \right\} (1+x)^{\frac{1}{3}} = C$$

Determining the value of C from the condition that when $x = o$, $v = o$, and solving for v^2, we have

$$v^2 = \frac{6fgy}{w} \left\{ 1 - \frac{1}{(1+x)^{\frac{1}{3}}} \right\} \quad . \quad . \quad . \quad (13)$$

which is the same in form as Equation (22), page 40, otherwise deduced. It follows, therefore, that making y constant in Equation (11) is equivalent to supposing that the combustion of the charge (or that part of it which is burned in the gun) is instantaneous, whereas we know it to be progressive. If all the charge were burned before the projectile left the bore the work of expansion would be the same by either hypothesis provided the final temperature, (that is, the temperature when the projectile is about to leave the muzzle) were the same in both cases. But it is well known that the muzzle tension (and therefore the temperature) of the gases of a slow burning powder is greater *caeteris paribus* than obtains with a quick burning powder, though we may assume that the temperature of combustion is practically the same for both powders. It follows that the work of expansion of a charge of slow burning powder in the bore of a gun is less than that of a quick burning powder of the same weight other things being equal. The hypothesis of an instantaneous combustion of the powder charge gives therefore, by means of Equation (13), too great a muzzle velocity.

It may be however that the *form* of Equation (13) may be employed both for the purpose of calculating muzzle velocities for different guns with different conditions of loading, and also for determining the velocity of the projectile at any point within the bore of the gun. For the first object it would be necessary to determine the value of the coefficient f by means of a measured muzzle velocity, using a typical gun with standard conditions of loading, and employing a charge all of which is burned in the gun. For the second object we should assume that the coefficient f determined as in the first case would give in Equation (13) the velocity of the projectile at any point within the bore, provided the weight of powder burned up to that point, were substituted for y. We know that this would give the correct velocity at two points, namely, at the origin of motion and at the muzzle; but whether this method gives correct velocities at intermediate points must be determined by experiment, which is the final test of nearly all physical formulas.

If we make, for convenience,

$$X_4 = \left\{ 1 - \frac{1}{(1+x)^{\frac{1}{3}}} \right\}^{\frac{1}{2}},$$

Equation (13) may be written

$$v^2 = \frac{6fg}{w} y X_4^2 \quad . \quad . \quad , \quad (13')$$

a monomial form which will be found of use in the sequel.

Muzzle Velocity for Quick Powders.—If we suppose the powder to be all consumed before the projectile leaves the bore, a supposition which is true only for the small grained powders used in small-arms, y will be equal to $\tilde{\omega}$ in Equation (13), and if we designate muzzle velocity by V and the muzzle value of x by x_1, we deduce from Equation (13)

$$V = \sqrt{6fg} \left(\frac{\tilde{\omega}}{w} \right)^{\frac{1}{2}} \left\{ 1 - \frac{1}{(1+x_1)^{\frac{1}{3}}} \right\}^{\frac{1}{2}} \quad . \quad . \quad . \quad (14)$$

With a proper mean value of f determined by experiment Equation (14) should give the muzzle velocity of a projectile with a considerable degree of accuracy when it is certain that all the powder is burned in the gun.

Expression for the Velocity for Slow Powder.—In deducing Equation (14) it was assumed that the charge was composed of quick powder all of which was converted into gas before the projectile had left the gun. With slow powder this assumption cannot be made, and it becomes necessary to take into account the progressive combustion of the charge under the variable pressure to which it is subjected in the bore; in other words to determine a suitable expression for y which shall represent the weight of powder burned in the time t, or when the projectile has travelled the distance u.

If we suppose the entire charge to be ignited at the same instant, an assumption that is nearly realized in the use of the large grained powder employed with heavy guns, the combustion of the charge will be expressed by the same function that expresses the combustion of a single grain. Therefore if $\psi(t)$ is the fraction of a grain burned in the time t, the total weight burned in the same time will be

$$y = \tilde{\omega}\,\psi(t) \quad . \quad . \quad . \quad (15)$$

$\tilde{\omega}$ being the weight of the charge.

Combustion of a Grain of Powder in Free Air.—The form of the function $\psi(t)$, when the combustion takes place in free air, and therefore under a constant pressure, can be expressed as follows for all the various forms of grain used in practice:

$$\psi(t) = \frac{at}{\tau}\left(1 - \lambda\frac{t}{\tau} + \mu\frac{t^2}{\tau^2}\right) \quad . \quad . \quad . \quad (16)$$

in which τ is the time of combustion of the entire grain, and depends upon the nature of the powder, on the size of the grain, upon its density and, generally, upon all the characteristics which have an influence upon the *velocity of combustion;* while a, λ, and μ depend only upon the *form* of the grain, and are purely numerical, retaining the same values as long as the grain in burning remains similar to its original form. It only remains to determine the values of these coefficients for the various forms of grain used in practice. It will be assumed that the grains are homogeneous throughout and of the same density; also that the combustion takes place in successive layers and with uniform velocity.

Spherical Grains.—First take the most simple case, that of a charge made up of equal spherical grains, or grains of a polyedral form. Let

r be the radius of one of the grains,
v the velocity of combustion,
τ the total time of combustion of a grain.

The initial volume of a grain is $\frac{4}{3}\pi r^3$; and at the end of the time t it is reduced to $\frac{4}{3}\pi(r-vt)^3$. Therefore the volume burned at the end of the time t is

$$\tfrac{4}{3}\pi r^3-\tfrac{4}{3}\pi(r-vt)^3,$$

or

$$\tfrac{4}{3}\pi r^3\left\{1-\left(1-\frac{v}{r}t\right)^3\right\}.$$

But we evidently have

$$\tau=\frac{r}{v};$$

whence the expression for the volume burned becomes

$$\tfrac{4}{3}\pi r^3\left\{1-\left(1-\frac{t}{\tau}\right)^3\right\}.$$

Therefore the fraction of a grain burned in the time t, or $\psi(t)$, is

$$\psi(t)=1-\left(1-\frac{t}{\tau}\right)^3=3\frac{t}{\tau}-3\frac{t^2}{\tau^2}+\frac{t^3}{\tau^3}$$

$$\therefore\ \psi(t)=\frac{3t}{\tau}\left\{1-\frac{t}{\tau}+\tfrac{1}{3}\frac{t^2}{\tau^2}\right\}\quad . \quad . \quad . \quad (17)$$

Comparing this with Equation (16), it will be seen that for spherical grains

$$\alpha=3,\ \lambda=1,\ \mu=\tfrac{1}{3}\quad . \quad . \quad . \quad (18)$$

Parallelopiped.—Let α, β, γ be the three dimensions of the grain. Its primitive volume is then $\alpha\beta\gamma$; but at the end of the time t each of its dimensions is diminished by $2vt$; and the remaining volume reduces to

$$(\alpha-2vt)(\beta-2vt)(\gamma-2vt);$$

and the volume burned is

$$\alpha\beta\gamma-(\alpha-2vt)(\beta-2vt)(\gamma-2vt),$$

or, by factoring,

$$\alpha\beta\gamma\left\{1-\left(1-\frac{2vt}{\alpha}\right)\left(1-\frac{2vt}{\beta}\right)\left(1-\frac{2vt}{\gamma}\right)\right\}.$$

If α is taken as the least of the three dimensions, we shall evidently have for the whole time of combustion

$$\tau=\frac{\alpha}{2v},$$

or

$$2v=\frac{\alpha}{\tau};$$

Also let

$$x=\frac{\alpha}{\beta} \text{ and } y=\frac{\alpha}{\gamma}.$$

Making these substitutions in the expression for the volume burned, we have

$$\phi(t)=1-\left(1-\frac{t}{\tau}\right)\left(1-x\frac{t}{\tau}\right)\left(1-y\frac{t}{\tau}\right);$$

which by developing may be put into the form

$$\phi(t)=(1+x+y)\frac{t}{\tau}\left\{1-\frac{x+y+xy}{1+x+y}\cdot\frac{t}{\tau}+\frac{xy}{1+x+y}\cdot\frac{t^2}{\tau^2}\right\} \quad . \quad (19)$$

whence for grains of the form of a parallelopipedon, we have

$$\alpha=1+x+y,\ \lambda=\frac{x+y+xy}{1+x+y},\ \mu=\frac{xy}{1+x+y} \quad . \quad . \quad (20)$$

If the grains are cubical we shall evidently have $x=y=1$, and, therefore,

$$\alpha=3,\ \lambda=1,\ \mu=\tfrac{1}{3},$$

the same as for spherical grains.

Generally the grains of this form have two opposite square faces, in which case x and y are equal and less than unity; and we have for this form of grain (flat grain)

$$\alpha=1+2x,\ \lambda=\frac{2x+x^2}{1+2x},\ \mu=\frac{x^2}{1+2x} \quad . \quad . \quad (21)$$

Cylinder.—Let r be the radius and h the length of the cylinder. Then the primitive volume is

$$\pi r^2 h,$$

and at the end of the time t,

$$\pi(r-vt)^2(h-2vt).$$

The volume burned at the end of the time t is, therefore,

$$\pi r^2 h-\pi(r-vt)^2(h-2vt);$$

and the fraction of the grain burned is

$$\psi(t)=1-\left(\frac{(r-vt)^2}{r^2}\right)\left(\frac{h-2vt}{h}\right)$$

$$=1-\left(1-\frac{2vt}{r}+\frac{v^2t^2}{r^2}\right)\left(1-\frac{2vt}{h}\right).$$

Since in this form of grain h is generally much greater than r, we have for the whole time of combustion

$$\tau=\frac{r}{v};$$

and if we make

$$x=\frac{r}{h}$$

and substitute in the expression for the fraction of a grain burned, we shall have

$$\psi(t)=1-\left(1-\frac{2t}{\tau}+\frac{t^2}{\tau^2}\right)\left(1-2x\frac{t}{\tau}\right)$$

$$=2(1+x)\frac{t}{\tau}-(1+4x)\frac{t^2}{\tau^2}+2x\frac{t^3}{\tau^3} \quad . \quad . \quad (22)$$

$$=2(1+x)\frac{t}{\tau}\left\{1-\frac{1+4x}{2(1+x)}\frac{t}{\tau}+\frac{x}{1+x}\frac{t^2}{\tau^2}\right\}.$$

Therefore for cylindrical grains we have

$$a=2(1+x),\ \lambda=\frac{1+4x}{2(1+x)},\ \mu=\frac{x}{1+x} \quad . \quad . \quad (23)$$

Pierced Cylinder.—This form of grain is that of a cylinder pierced by a concentric canal. Let r and r' be the radii of the external and internal cylinders, respectively, and h the length of the cylinder. Then the primitive volume is

$$\pi(r^2-r'^2)h,$$

and at the end of the time t,

$$\pi[(r-vt)^2-(r'+vt)^2](h-2vt)$$

From this we readily find by developing,

$$\psi(t) = 1 - \left(1 - \frac{2vt}{r-r'}\right)\left(1 - \frac{2vt}{h}\right).$$

If we suppose $r-r'$ to be less than h, we shall have

$$\frac{2v}{r-r'} = \frac{1}{\tau};$$

and if, furthermore, we make

$$x = \frac{r-r'}{h},$$

and, therefore,

$$\frac{2v}{h} = \frac{x}{\tau},$$

we shall have for the function $\psi(t)$

$$\psi(t) = 1 - \left(1 - \frac{t}{\tau}\right)\left(1 - x\frac{t}{\tau}\right)$$

$$= (1+x)\frac{t}{\tau}\left(1 - \frac{1+x}{x}\cdot\frac{t}{\tau}\right); \quad . \quad . \quad . \quad (24)$$

therefore for a pierced cylinder we have

$$a = 1+x, \ \lambda = \frac{x}{1+x}, \ \mu = 0 \quad . \quad . \quad . \quad (25)$$

Values of the Constants a, λ **and** μ **for different Service Powders.**—For all irregular shaped grains, and for sphero-hexagonal, hexagonal, mammoth, cubical and cannon powders the law of burning is approximately that of a sphere; and for these we have $a = 3$, $\lambda = 1$ and $\mu = \frac{1}{3}$.

The pierced prismatic powders, such as the German cocoa and *Dupont's* brown powders, assimilate to the pierced cylinder. For these powders $x = \frac{1}{2}$; and, therefore, from Equation (25),

$$a = \tfrac{3}{2}, \ \lambda = \tfrac{1}{3} \text{ and } \mu = 0.$$

For the square prismatic powder used in our army $x = 0.72$; and, therefore, for this powder, Equation (21) gives

$$a = 2.44, \ \lambda = 0.8, \ \mu = 0.21.$$

Smokeless powders are frequently made into long thread-like grains or cylinders; and since for these forms x is a very small fraction, we have approximately, from Equation (23),

$$a = 2, \ \lambda = \tfrac{1}{2} \text{ and } \mu = 0.$$

It will be seen in all the preceding expressions for $\varphi(t)$ that when λ and μ are zero, the grains give off equal quantities of gas in equal times. Therefore the smaller these factors the more uniform is the generation of gas and the more equable the pressure in the bore of the gun. The best form of grain would make μ zero and λ a small fraction.

Combustion under a Variable Pressure.—The value of y when the combustion takes place in free air, and therefore under constant pressure, may, in accordance with the preceding discussion, take the form

$$y = \tilde{\omega}\varphi(t) = \tilde{\omega}\,\frac{a\,t}{\tau}\left(1 - \lambda\frac{t}{\tau} + \mu\frac{t^2}{\tau^2}\right) \quad . \quad . \quad (26)$$

for all geometrical forms. But when the combustion takes place in a gun this formula is not applicable in its present form; and it becomes necessary to take into account the law of variation of combustion due to the variable pressure developed in the bore. For this purpose it is better to introduce into the expression for y, the length of grain burned in place of the time of burning. Let l_0 be one-half of the mean least dimension of the grains of which the charge is composed, and l the length of grain burned in the time t under a variable pressure p. Then it may be shown, as before, that, when l is burned, the weight of powder consumed is expressed by the equation

$$y = \tilde{\omega}\,\frac{a\,l}{l_0}\left\{1 - \lambda\frac{l}{l_0} + \mu\frac{l^2}{l_0^2}\right\} \quad . \quad . \quad . \quad (27)$$

whatever may be the law of combustion.

Velocity of Combustion under Variable Pressure.[1]—The velocity of combustion under the constant atmospheric pressure p_0 is constant and equal to $\frac{l_0}{\tau}$, while under a variable pressure p, the velocity is $\frac{dl}{dt}$. Therefore, if we assume with *Sarrau* that the velocity of combustion under variable pressure is proportional to the square root of the pressure, we shall have

[1] See paper entitled "Velocities and Pressures in Guns" by Ensign *J. H. Glennan*, U. S. Navy, in Proceedings Naval Institute, vol. XIV, pp. 395-418.

$$\frac{dl}{dt}=\frac{l_0}{\tau}\left(\frac{p}{p_0}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad (28)$$

To express l in terms of u, the distance traveled by the projectile, we have from Equation (6),

$$p=\frac{w}{g\omega}\frac{d^2u}{dt^2},$$

which by Equation (10) becomes

$$p=\frac{wz_0}{g\omega}\frac{d^2x}{dt^2};$$

and this substituted in Equation (28) gives

$$\frac{dl}{dt}=\frac{l_0}{\tau}\left(\frac{wz_0}{g\omega p_0}\right)^{\frac{1}{2}}\left(\frac{d^2x}{dt^2}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad (29)$$

an expression for the velocity of combustion at any instant when we may suppose a weight of powder y to have been burned.

We may replace dt by dx as follows:—We have

$$\frac{d^2x}{dt^2}=\frac{d^2u}{z_0dt^2}=\frac{d(v^2)}{2z_0du}=\frac{d(v^2)}{2z_0^2dx}.$$

$$\therefore \quad \frac{dl}{dt}=\frac{l_0}{\tau}\left(\frac{w}{2g\omega p_0z_0}\right)^{\frac{1}{2}}\left(\frac{d(v^2)}{dx}\right)^{\frac{1}{2}}$$

We also have

$$\frac{dl}{dx}=\frac{dl}{dt}\div\frac{dx}{dt};$$

but

$$\frac{dx}{dt}=\frac{du}{z_0dt}=\frac{v}{z_0}$$

$$\therefore \quad \frac{dl}{dx}=\frac{l_0}{\tau}\left(\frac{wz_0}{2g\omega p_0}\right)^{\frac{1}{2}}\left(\frac{d(v^2)}{dx}\right)^{\frac{1}{2}}\frac{1}{v} \quad . \quad . \quad . \quad (30)$$

Differentiating Equation (13) with reference to x, regarding y as momentarily constant, and reducing,

$$\left(\frac{d(v^2)}{dx}\right)^{\frac{1}{2}}\frac{1}{v}=\frac{1}{\sqrt{3}\sqrt{1+x}\sqrt{(1+x)^{\frac{1}{3}}-1}}=\frac{1}{\sqrt{3}}F(x), \text{ say:—}$$

an equation independent of y.

Substituting this last in Equation (30) and reducing, gives

$$dl = \frac{l_0}{\tau}\left(\frac{wz_0}{6gp_0\omega}\right)^{\frac{1}{2}} F(x)\,dx; \quad . \quad . \quad . \quad (31)$$

whence integrating between the limits o and x, we have

$$l = \frac{l_0}{\tau}\left(\frac{wz_0}{6gp_0\omega}\right)^{\frac{1}{2}} \int_o^x F(x)dx \quad . \quad . \quad . \quad (32)$$

Making the following substitutions:

$$\omega = \frac{\pi d^2}{4},$$

$$K = \frac{1}{\tau}\left(\frac{wz_0}{6gp_0\omega}\right)^{\frac{1}{2}} = \frac{2}{\tau d}\left(\frac{wz_0}{6\pi g p_0}\right)^{\frac{1}{2}},$$

and

$$\int_o^x F(x)\,dx = X_0,$$

Equation (32) becomes

$$\frac{l}{l_0} = KX_0;$$

and this substituted in Equation (27) gives

$$y = \tilde{\omega}\, a\, KX_0 \left\{ 1 - \lambda KX_0 + \mu(KX_0)^2 \right\} \quad . \quad . \quad (33)$$

which gives the weight of powder burned as a function of the distance traveled by the projectile.

Expression for the Velocity.—Substituting the value of y from Equation (33) in Equation (13), and making

$$X_0 \left\{ 1 - \frac{1}{(1+x)^{\frac{1}{3}}} \right\} = X_1,$$

it becomes

$$v^2 = 6gfa\frac{\tilde{\omega}}{w}KX_1 \left\{ 1 - \lambda KX_0 + \mu(KX_0)^2 \right\} \quad . \quad (34)$$

Restoring the value of K and making

$$M = 2\sqrt{6}\left(\frac{g}{\pi p_0}\right)^{\frac{1}{2}},$$

and

$$N = \left(\frac{2}{3\pi g p_0}\right)^{\frac{1}{2}},$$

whence

$$M = 6gN,$$

Equation (34) reduces to

$$v^2 = M\frac{fa\tilde{\omega}}{\tau d}\left(\frac{z_0}{w}\right)^{\frac{1}{2}}X_1\left\{1 - N\frac{\lambda}{\tau d}(wz_0)^{\frac{1}{2}}X_0 + N^2\frac{\mu}{\tau^2 d^2}(wz_0)X_0^2\right\} \quad (35)$$

which is the general expression for the velocity of a projectile in the bore of a gun in terms of the distance traveled and the weight of powder burned.

Expression for the Pressure on the Base of the Projectile.— To deduce an expression for the pressure per unit of surface, on the base of the projectile, we have

$$p = \frac{m}{\omega}\frac{d^2u}{dt^2} = \frac{w}{2g\omega}\frac{d(v^2)}{du} = \frac{2w}{\pi d^2 g z_0}\frac{d(v^2)}{dx};$$

whence, differentiating Equation (35), and reducing,

$$p = \frac{2w}{\pi d^2 g z_0}M\frac{fa\tilde{\omega}}{\tau d}\left(\frac{z_0}{w}\right)^{\frac{1}{2}}\frac{dX_1}{dx}\left\{1 - N\frac{\lambda}{\tau}\frac{(wz_0)^{\frac{1}{2}}}{d}X_0 + N^2\frac{\mu}{\tau^2}\frac{wz_0}{d^2}X_0^2\right\}$$

$$-\frac{2w}{\pi d^2 g z_0}M\frac{fa\tilde{\omega}}{\tau d}\left(\frac{z_0}{w}\right)^{\frac{1}{2}}X_1\left\{N\frac{\lambda}{\tau}\frac{(wz_0)^{\frac{1}{2}}}{d}\frac{dX_0}{dx} - 2N^2\frac{\mu}{\tau^2}\frac{wz_0}{d^2}X_0\frac{dX_0}{dx}\right\} \quad (36)$$

Equations (35) and (36) are the complete expressions for the velocity and pressure upon the base of the projectile, respectively, in terms of the distance traveled by the projectile. The factors M and N are constants independent of the elements of fire; the quantities $\frac{fa}{\tau}$, $\frac{\lambda}{\tau}$ and $\frac{\mu}{\tau^2}$ are constants for the same powder; while $\tilde{\omega}$, w, d and z_0 are constants for the some gun and charge. The only general variables in the second members are, therefore, X_0 and X_1 together with their first derivatives with reference to x. These equations may be simplified by making $\mu = o$, which is its value for pierced grains and for most service powders. We thus have, after reducing,

$$v^2 = M\frac{fa}{\tau}\frac{\tilde{\omega}}{d}\left(\frac{z_0}{w}\right)^{\frac{1}{2}}X_1\left\{1 - N\frac{\lambda}{\tau}\frac{(wz_0)^{\frac{1}{2}}}{d}X_0\right\} \quad . \quad (37)$$

and

$$p = M' \frac{fa}{\tau} \frac{\tilde{\omega}}{d^3} \left(\frac{w}{z_0}\right)^{\frac{1}{2}} X_2 \left\{ 1 - N \frac{\lambda}{\tau} \frac{(wz_0)^{\frac{1}{2}}}{d} X_3 \right\} \quad . \quad . \quad (38)$$

in which M and N are the same as before, while

$$M' = \frac{2M}{\pi g}, \; X_2 = \frac{dX_1}{dx} \text{ and } X_3 = \left\{ X_1 \frac{dX_0}{dx} + X_0 \frac{dX_1}{dx} \right\} \div \frac{dX_1}{dx}.$$

For the same powder and type of gun $\frac{fa}{\tau}$ and $\frac{\lambda}{\tau}$ are constants and may be incorporated with M, N and M', respectively. Making then

$$M_1 = M \frac{fa}{\tau}, \; N_1 = N \frac{\lambda}{\tau} \text{ and } M_2 = M' \frac{fa}{\tau},$$

Equations (37) and (38) may be written

$$v^2 = M_1 \frac{\tilde{\omega}}{d} \left(\frac{z_0}{w}\right)^{\frac{1}{2}} X_1 \left\{ 1 - N_1 \frac{(wz_0)^{\frac{1}{2}}}{d} X_0 \right\} \quad . \quad . \quad (39)$$

and

$$p = M_2 \frac{\tilde{\omega}}{d^3} \left(\frac{w}{z_0}\right)^{\frac{1}{2}} X_2 \left\{ 1 - N_1 \frac{(wz_0)^{\frac{1}{2}}}{d} X_3 \right\} \quad . \quad . \quad (40)$$

in which, in the last equation,

$$M_2 = \frac{2}{\pi g} M_1.$$

In all the above equations the units are the pound and foot. Therefore, the velocity will be in foot-seconds and the pressure upon the base of the projectile in pounds per square foot. As this latter is usually required in pounds per square inch, we must divide the second member of Equation (40) by 144; or what is the same thing compute M_2 by the formula

$$M_2 = \frac{M_1}{72\pi g}.$$

Making $g = 32.16$, which is its mean value for the United States, we have

$$\log M_2 = \log M_1 - 3.86180.$$

We also have for the factors M and N, which are independent of all the elements of fire,

$$\log M = 9.5323975$$
$$\log N = 7.2469302.$$

The quantities X_0, X_1, X_2 and X_3 which enter into the expressions for the velocity and pressure, are all functions of x alone, and may be tabulated with x for the argument. We have,

$$X_0 = \int_0^x \frac{dx}{(1+x)^{\frac{1}{2}} \sqrt{(1+x)^{\frac{1}{3}} - 1}} \quad . \quad . \quad . \quad (41)$$

whence integrating, To integrate – let 1+x = (1+

$$X_0 = 3(1+x)^{\frac{1}{6}} \sqrt{(1+x)^{\frac{1}{3}} - 1} + 3 \log_\epsilon \left\{ (1+x)^{\frac{1}{6}} + \sqrt{(1+x)^{\frac{1}{3}} - 1} \right\}$$

This expression for X_0 may be simplified by means of circular functions. Thus, if we make

$$\sec \varphi = (1+x)^{\frac{1}{6}},$$

and therefore,

$$\tan \varphi = \sqrt{(1+x)^{\frac{1}{3}} - 1},$$

Equation (41) reduces to

$$X_0 = 6 \int_0^\varphi \frac{d\varphi}{\cos^3 \phi}$$

This last definite integral is a well known function of φ and has been extensively used in exterior ballistics, first by *Euler* and afterwards by *Otto, Didion* and others.[1] Symbolizing this function by (φ) we have

$$X_0 = 6(\varphi)$$

and

$$(\varphi) = \tfrac{1}{2} \tan \varphi \sec \varphi + \tfrac{1}{2} \log_\epsilon (\tan \varphi + \sec \varphi).$$

The other functions readily reduce to the following:—

$$\frac{dX_0}{dx} = \cot \varphi \cos^3 \varphi$$

$$X_1 = X_0 \sin^2 \varphi$$

$$X_2 = \sin \varphi \cos^4 \varphi \left(1 + \tfrac{1}{3} X_0 \frac{dX_0}{dx} \right)$$

$$X_3 = X_0 \left(1 + \frac{1}{1 + \frac{1}{3} X_0 \frac{dX_0}{dx}} \right)$$

$$X_4 = \sin \varphi \text{ (See page 52)}$$

[1] See *Otto's* Tables. Paris, 1845. *Didion's* Traité de Balistique. Paris, 1860. *Ingalls'* Handbook of Problems in Direct Fire. New York, 1890, p. 301 and Table IV.

The values of these functions are given in Table II. In the first edition of this work it was assumed that the gases produced by the combustion of a charge of powder might be considered, on account of their high temperature, as perfect gases; and the value of n was therefore taken at 1.4. The table of the functions X_0, etc., computed upon this hypothesis is also here given as Table II′.

Practical Applications.—All the quantities which enter into the second members of Equations (39) and (40) are known or may be easily computed by means of formulas already established. It will readily be seen however that the velocities thus determined must be too great on account of the resistances which were omitted in establishing the differential equation of motion, Equation (7); and which are of such a nature that their evaluation seems to be beyond the power of analysis. This difficulty may be avoided, practically by supposing the force of the powder f to be so diminished that the acceleration produced upon the projectile, free to move, shall be equal to the real acceleration. The value of f will also be still further diminished if we suppose that any part of the heat of the gases is lost by conduction to the walls of the bore. It will be necessary then to determine the force of the powder, or, what amounts to the same thing, the factors M_1 and N_1 in Equation (39), by experiment. Two equations will be needed for this purpose, and these may be obtained by firing the same kind of powder with varying charges and weights of projectiles, in two similarly constructed guns of different calibres and conditions of loading, and measuring the respective muzzle velocities with a chronograph.

Determination of M_1 and N_1.—Let, at the muzzle of the gun,

$$\frac{\tilde{\omega}}{d}\left(\frac{z_0}{w}\right)^{\frac{1}{2}} X_1 = \frac{1}{A}$$

and

$$\frac{(wz_0)^{\frac{1}{2}}}{d} X_0 = B;$$

then Equation (39) becomes, for one of the guns,

$$V_1^2 = \frac{M_1}{A_1}\left\{1 - B_1 N_1\right\}$$

and for the other

$$V_2^2 = \frac{M_1}{A_2}\left\{ 1 - B_2 N_1 \right\};$$

whence

$$M_1 = \frac{A_1 B_2 V_1^2 - A_2 B_1 V_2^2}{B_2 - B_1},$$

and

$$N_1 = \frac{M_1 - A_1 V_1^2}{B_1 M_1} = \frac{M_1 - A_2 V_2^2}{B_2 M_1}.$$

It will readily be seen that the value of M_1 deduced from the above formula will be the more accurate as the divisor $B_2 - B_1$ is the larger; and therefore in selecting the guns and charges care should be taken to have as large a variation of the quantity $\frac{(w z_0)^{\frac{1}{2}}}{d} X_0$ as possible.

The values of M_1 and N_1 can also be determined with the *same* gun and a *single* charge by measuring the velocities of the projectile with a *Noble* chronoscope at two points in the chase. The data in this case will be, in addition to the elements of loading, the two measured velocities v_1 and v_2, and the corresponding distances traveled by the projectile u_1 and u_2.

Example 1.—As a first example of the applications of Equations (39) and (40) we will compute the velocities of a 300-pound projectile at different points in the bore of the English 10-inch (18-ton) gun, propelled by a charge of 70 pounds of pebble powder, and compare the results with the actual velocities measured by means of *Noble's* chronoscope, which are mentioned in Chapter III. The data for the calculations are as follows:

$w = 300$ pounds	$v_1 = 1027$ f. s.
$\tilde{\omega} = 70$ "	$v_2 = 1464.19$ f. s.
$u_1 = 2.26$ feet	$\delta = 1.8$ (assumed)
$u_2 = 8.26$ "	$\Delta = 1$
$d = \frac{5}{6}$ "	

From these data we find by employing the proper formulas, already given, as follows:—

SHORT TRAVEL.	LONG TRAVEL.
$z_0 = 0.9137$ feet	$z_0 = 0.9137$ feet
$x = 2.4734$	$x = 9.0398$

$\log X_0 = 0.6672832$ $\log X_0 = 0.8774895$

$\log X_1 = 0.1983650$ $\log X_1 = 0.6070231$

$\log A_1 = 9.1355059$ $\log A_2 = 8.7268478$

$B_1 = 92.3497$ $B_2 = 149.8449$

From these numbers we find

$$\log M_1 = 5.28319$$
$$\log N_1 = 7.43133 - 10$$
$$\log M_2 = 1.42139;$$

and these substituted in Equations (39) and (40) reduce them to the following:

$$v^2 = [5.94932] X_1 \left\{ 1 - [8.72948 - 10]\, X_0 \right\} \quad . \quad . \quad (42)$$

and

$$p = [4.76218]\, X_2 \left\{ 1 - [8.72948 - 10]\, X_3 \right\} \quad . \quad . \quad (43)$$

in which the numerical coefficients are replaced by their logarithms (in brackets) for convenience of computation. These formulas give the velocities in foot-seconds and the pressures upon the base of the projectile in pounds per square inch. They are extremely simple in comparison with any other formulas that have been suggested; and with the help of Table II the velocity of the projectile at any point within the bore and the corresponding pressure upon its base can be easily computed. It remains now to determine the degree of confidence that can be placed in the velocities and pressures computed by these formulas. For this purpose the following table has been prepared showing the agreement between the theoretical velocities and those deduced by *Noble* and *Abel* from the *time measurements* made by means of the *Noble* chronoscope. The first and second columns are the arguments of the table and need no further explanations. The third and sixth columns are interpolated from *Noble* and *Abel's* Table X;[1] and they give the velocities and pressures, respectively, corresponding to the distances traveled by the projectile, recorded in the second column. The fourth and seventh columns give the velocities and pressures computed by Equations (42) and (43). The fifth column shows at a glance the differences

[1] Researches, page 79.

between the velocities furnished by Equation (42) and those deduced by *Noble* and *Abel* from the experiments with the *Noble* chronoscope. These differences are practically *nil* except during the first six inches of the projectile's travel; and for this short distance the data given by the experiments are hardly sufficient to determine positively which set of velocities is the more accurate. The curve of velocity, (Equation (42)), is concave throughout toward the axis of x, differing in this respect from the curve of velocity deduced by *Noble* and *Abel*, which for a distance of two or three inches from the origin, is convex to the axis of abscissae; but for nineteen-twentieths of the entire distance traveled by the projectile in the bore the two curves of velocity practically coincide.

The curve of pressure (Equation (43)) begins by being concave toward the axis of x, has a maximum ordinate when $x = 0.5$ (nearly) and changes curvature, becoming convex to the axis of abscissae when $x = 1.0$ (nearly). These correspond very closely with *Noble* and *Abel's* deductions.

Table of velocities and pressures in the bore of an English 10-inch, 18-ton gun. Charge 70 pounds of pebble powder of density 1.8. Weight of projectile 300 pounds.

$\frac{u}{z_0}=x$	Distance traveled by projectile in feet. (u)	VELOCITY IN FOOT-SECONDS, ACCORDING TO—			PRESSURE IN POUNDS PER SQUARE INCH, ACCORDING TO—	
		Noble & Abel.	Equation (42).	Diff.	Noble & Abel.	Equation (43).
0.001	0.0009	. .	5.768	. .	. .	3136
.004	.0037	. .	16.002	. .	. .	6204
.007	.0064	. .	24.273	. .	. .	8133
0.01	0.0091	. .	31.633	. .	. .	9644
.1	.09	109	169	+60	20320	25811
.2	.18	224	270	+46	31170	31485
0.3	0.27	317	352	+35	36336	32821
.4	.37	405	420	+15	39215	34656
.5	.46	474	479	+ 5	40224	34703
0.6	0.55	534	532	— 2	39988	34308
.7	.64	569	579	+10	38277	33661
.8	.73	614	622	+ 8	35883	32872
0.9	0.82	657	661	+ 4	33781	32005
1.0	.91	695	696	+ 1	31837	31104
1.1	1.01	732	729	— 3	30027	30193
1.2	1.10	763	760	— 3	28878	29291
1.3	.19	791	789	— 2	27935	28406
1.4	.28	818	816	— 2	27115	27545
1.5	1.37	843	841	— 2	26237	26713
1.6	.46	866	865	— 1	25381	25911
1.7	.55	888	887	— 1	24553	25140
1.8	1.65	911	908	— 3	23764	24400
1.9	.74	930	928	— 2	23072	23689
2.0	.83	949	948	— 1	22355	23009
2.47	2.26	1027	1027	0	19501	19302
3	2.74	1097	1098	+ 1	16981	17580
4	3.65	1173	1203	+30	13819	13917
5	4.57	1258	1281	+23	11601	11322
6	5.48	1323	1342	+19	9943	9390
7	6.40	1378	1390	+12	8640	7903
8	7.31	1424	1430	+ 6	7638	6724
9	8.22	1459	1463	+ 4	6857	5769
9.04	8.26	1464	1464	0	. .	4980
10	9.14	. .	1491	. .	. .	. .
11.081	10.125	1527	1516	—11	. .	. .
X	U	V				

Example 2.—Compute the constants M_1, N_1 and M_2 for German cocoa powder (C_{82}), with the following data derived from firings with the South Boston Navy gun and the 8-inch B. L. R.

South Boston Gun.	8-inch B. L. R.
$\tilde{\omega} = 29.125$ pounds	$\tilde{\omega} = 122$ pounds
$w = 51$ "	$w = 250$ "
$U = 10$ feet	$U = 16.41$ feet
$C = 920$ c. inches	$C = 3824$ c. inches
$d = \frac{1}{2}$ foot	$d = \frac{2}{3}$ foot
$V = 1685$ f. s.	$V = 1999$ f. s.
$\delta = 1.867$	$\delta = 1.867$

The principal results of the computations are as follows:—

$\Delta = 0.8763$	$\Delta = 0.8831$
$z_0 = 1.4388$ feet	$z_0 = 3.3410$ feet
$x = 6.9500$ (muzzle)	$x = 4.9116$ (muzzle)
$\log X_1 = 0.53496$ "	$\log X_1 = 0.43207$ "
$\log X_0 = 0.83690$ "	$\log X_0 = 0.78181$ "
$\log X_2 = 9.50737$ "	$\log X_2 = 9.59458$ "
$\log X_3 = 1.02760$ "	$\log X_3 = 0.96368$ "
$\log A_1 = 8.47483$	$\log A_2 = 8.24250$
$\log B_1 = 2.07072$	$\log B_2 = 2.41882$

From these values of A_1, B_1, A_2, B_2, and the measured muzzle velocities we find

$$M_1 = 96733,$$
$$N_1 = 0.0010597,$$
$$M_2 = 13.298;$$

whence the expression for the velocity and pressure on the base of the projectile, for any similarly constructed gun using the powder of this example are,—

$$v^2 = 96733 \frac{\tilde{\omega}}{d} \left(\frac{z_0}{w}\right)^{\frac{1}{2}} X_1 \left\{ 1 - 0.0010597 \frac{(wz_0)^{\frac{1}{2}}}{d} X_0 \right\}$$

and

$$p = 13.298 \frac{\tilde{\omega}}{d^3} \left(\frac{w}{z_0}\right)^{\frac{1}{2}} X_2 \left\{ 1 - 0.0010597 \frac{(wz_0)^{\frac{1}{2}}}{d} X_3 \right\}$$

For the 8-inch B. L. R. these equations reduce to the following for a charge of 122 pounds, weight of projectile 250 pounds:—

$$v^2 = [6.31100] X_1 \left\{ 1 - [8.66218 - 10] X_0 \right\}$$

and

$$p = [4.67544] X_2 \left\{ 1 - [8.66218 - 10] X_3 \right\}$$

and for the South Boston Gun, with a charge of 29.125 pounds, weight of projectile 51 pounds:—

$$v^2 = [5.97609] X_1 \left\{ 1 - [8.25900 - 10] X_0 \right\}$$

and

$$p = [4.26591] X_2 \left\{ 1 - [8.25900 - 10] X_3 \right\}$$

The following tables were computed by means of the last two sets of formulas:—

Table of velocities and pressures in the bore of the Navy 8-inch B. L. R. Charge 122 pounds of German cocoa powder (C_{82}), density 1.867. Weight of projectile 250 pounds.

$\frac{u}{z_0}$ (x)	u (feet).	Velocity of projectile. (f. s.)	Pressure on base of projectile. (lbs. per in^2.)
0.	0.	0.	0.
0.1	0.334	256.7	21395
0.2	0.668	412.7	26242
0.3	1.002	537.4	28314
0.4	1.336	642.8	29127
0.5	1.670	734.5	29269
0.6	2.005	815.8	29032
0.7	2.339	888.9	28573
0.8	2.673	955.2	27985
0.9	3.007	1015.8	27324
1.0	3.341	1071.6	26627
2.0	6.682	1467.3	20183
3.0	10.023	1709.7	15762
4.0	13.364	1880.2	12744
4.9116	16.410	1999.0	10754
	U	V	

Table of velocities and pressures in the bore of the Navy "South Boston gun". Calibre 6-inches. Charge 29.125 pounds German cocoa powder of density 1.867. Weight of projectile 51 pounds.

$\frac{u}{z_0}$ (x)	u (feet).	Velocity of projectile. (f. s.)	Pressure on base of projectile. (lbs. per in².)
0	0	0	0
0.1	0.144	177.3	8694
0.2	0.288	287.0	10863
0.3	0.432	375.6	11894
0.4	0.576	451.1	12391
0.5	0.719	517.4	12592
0.6	0.863	576.6	12620
0.7	1.007	630.1	12540
0.8	1.151	679.0	12394
0.9	1.294	724.0	12207
1.0	1.438	765.6	11994
2.0	2.878	1068.9	9744
3.0	4.316	1263.6	8057
4.0	5.755	1406.3	6857
5.0	7.190	1518.4	5973
6.0	8.633	1610.6	5296
6.95	10.000	1685.0	4785
	U	V	

The argument of these tables is x or u/z_0, and the corresponding values of u, that is the distances traveled by the projectile, are given in the second columns; while the third and fourth columns contain the velocities and pressures. It will be seen that the maximun pressures upon the bases of the projectiles, given by these tables are about 29269 and 12620 pounds per square inch, respectively. The recorded maximum pressure in the chamber of the 8-inch gun was 34048 pounds.

Example 3.—Determine the equation of the velocity curve of a 100-pound projectile in the bore of the Navy 6-inch B. L. R., fired with a charge of 50 pounds of the powder described in Example 2. Also compute the muzzle velocity.

For this example we have the following data: $\tilde{\omega} = 50$ pounds, $w = 100$ pounds, $U = 11.62$ feet, $C = 1400$ cubic inches and $d = 0.5$ foot. The principal calculations are: $\varDelta = 0.9886$, $\log z_0 = 0.28809$, x (muzzle) $= 5.9856$; and for this value of x, $\log X_0 = 0.81341$ and $\log X_1 = 0.49182$.

Taking the values of M_1 and N_1 as found in Example 2, we have

$$v^2 = [6.12962]\ X_1 \left\{ 1 - [8.47026 - 10]\ X_0 \right\};$$

and the muzzle velocity $V = 1838$ f. s. The measured muzzle velocity was 1836 f. s.

Example 4.—Deduce expressions for velocity and pressure in the bore of a gun, using *Dupont's* black sphero-hexagonal powder of density 1.78, from the following data:—

South Boston Navy Gun.	60–Pdr. Navy B. L. R.
$\tilde{\omega} = 32$ pounds	$\tilde{\omega} = 10$ pounds
$w = 75.4$ "	$w = 46.5$ "
$U = 13.08$ feet	$U = 7.67$ feet
$\Delta = 0.9627$	$\Delta = 0.8987$
$d = \frac{1}{2}$ foot	$d = 0.44$
$V_2 = 2001$ f. s.	$V_1 = 1427$ f. s.

Answers.

$z_0 = 1.24516$ feet		$z_0 = 0.58040$	
$x = 10.505$	(muzzle)	$x = 13.215$	(muzzle)
$\log X_0 = 0.90029$	"	$\log X_0 = 0.93463$	"
$\log X_1 = 0.64617$	"	lag $X_1 = 0.70341$	"
$\log A_2 = 8.43872 - 10$		$\log A_1 = 8.89191 - 10$	
$\log B_2 = 2.18762$		$\log B_1 = 2.00676$	

$$M_1 = 253262$$
$$M_2 = 34.815$$
$$N_1 = 0.0036735$$

These values of M_1, M_2 and N_1 substituted in Equations (39) and (40) will give the expressions for velocity and pressure required for any gun whose elements of loading are known.

For the Sonth Boston gun, for a charge of 32 pounds, and projectile weighing 75.4 pounds, they reduce to the following:

$$v^2 = [6.31868]\ X_1 \left\{ 1 - [8.85241 - 10]\ X_0 \right\}$$

$$p = [4.84108] X_2 \left\{ 1 - [8.85241 - 10]\ X_3 \right\};$$

and for the 64-pounder, charge 10 pounds and projectile weighing 46.5 pounds,

$$v^2 = [5.80825]\, X_1 \left\{ 1 - [8.63722-10]\, X_0 \right\}$$

$$p = [4.56327] X_2 \left\{ 1 - [8.63722-10]\, X_3 \right\}$$

Example 5.—Deduce expressions for the velocity and pressure within the bore of the South Boston gun when loaded with the powder defined in Example 4. Weight of charge 23 pounds; weight of projectile 67 pounds; and density of loading 0.692

Answers

$$v^2 = [6.26301]\, X_1 \left\{ 1 - [8.88886-10]\, X_0 \right\}$$

$$p = [4.60991] X_2 \left\{ 1 - [8.88886-10]\, X_3 \right\}$$

At the muzzle $\log X_0 = 0.85663$ and $\log X_1 = 0.57038$. Whence we find $V = 1739$ f. s. A single measured velocity was 1748 f. s.

Example 6.—Deduce expressions for velocity and pressure in the bore of a gun using the standard Wetteren powder W(13-16), of density 1.765, from the following data given by *Sarrau*:—

19-cm. Gun.	10-cm. Gun.
$\tilde{\omega} = 15$ kgs. $= 33.069$ pounds	$\tilde{\omega} = 3.1$ kgs. $= 6.834$ pounds
$w = 75$ kgs. $= 165.347$ "	$w = 12$ kgs. $= 26.455$ "
$U = 3.29$ m. $= 10.794$ feet	$U = 2.26$ m. $= 7.415$ feet
$d = 19.4$ cm. $= 0.6365$ "	$d = 10$ cm. $= 0.3281$ "
$V_2 = 448$ m. s. $= 1469.7$ f. s.	$V_1 = 485$ m. s. $= 1591.2$ f. s.
$\Delta = 0.870$	$\Delta = 0.957$

Answers.

$z_0 = 0.97035$ feet		$z_0 = 0.61938$	
$x = 11.124$	(muzzle)	$x = 11.972$	(muzzle)
$\log X_0 = 0.90891$	"	$\log X_0 = 0.91992$	"
$\log X_1 = 0.66703$	"	$\log X_1 = 0.67914$	"
$\log A_2 = 8.73938-10$		$\log A_1 = 8.81747-10$	
$\log B_2 = 2.20778$		$\log B_1 = 2.01115$	

$$M_1 = 249744$$
$$M_2 = 34.331$$
$$N_1 = 0.0032561$$

The expressions for the velocity and pressure in the bores of the two guns giving the above results are:—

For the 19-cm. gun,

$$v^2 = [5.99738]\, X_1 \left\{ 1 - [8.81156-10]\, X_0 \right\}$$

$$p = [4.75945]\, X_2 \left\{ 1 - [8.81156-10]\, X_3 \right\}.$$

For the 10-cm. gun,

$$v^2 = [5.90089]\, X_1 \left\{ 1 - [8.60393-10]\, X_0 \right\}$$

$$p = [4.63762]\, X_2 \left\{ 1 - [8.60393-10]\, X_3 \right\}$$

Formula for Maximum Pressure on Base of Projectile.—It will be seen from the tables of pressures computed for the 8-inch B. L. R. and South Boston gun, that the maximum pressure is produced in the former when $x = 0.5$, that is when the projectile has moved about 1.7 feet; and in the latter when $x = 0.5+$, or when the projectile has moved about 0.8 foot. It is evident from general considerations that the point of maximum pressure in a gun is not a fixed point, but that its position varies with the resistance encountered. We may say as a rule that the less the resistance to be overcome by the expanding gases the sooner will they exert their maximum pressure, and also the less will this maximum pressure be. The factor X_2 which enters into the expression for the pressure (Equation (40)), attains its maximum when $x = 0.6$, nearly; but on account of the subtractive term containing X_3 the maximum pressure occurs in all cases when x is somewhat less than 0.6. Making $x = 0.5$, which is probably a fair average value, we have, to determine the travel of the projectile when the pressure is a maximum,

$$z_0 = \frac{C}{\omega}\left(1 - \frac{\Delta}{\delta}\right) = u_0\left(1 - \frac{\Delta}{\delta}\right)$$

in which u_0 is the reduced length of the powder-chamber. Therefore we have for the displacement corresponding to the maximum pressure,

$$u = 0.5\, u_0\left(1 - \frac{\Delta}{\delta}\right) = 0.5 z_0.$$

If, in Equation (40), we make

$$X_2 = 0.725$$

and

$$X_3 = 3.211,$$

we shall obtain an expression for the maximum pressure upon the base of the projectile which is probably as accurate as the methods employed for measuring it. Calling this maximum pressure p_m and substituting for X_2 and X_3 their values given above, Equation (40) becomes

$$p_m = 0.725 M_2 \frac{\tilde{\omega}}{d^3} \left(\frac{w}{z_0}\right)^{\frac{1}{2}} \left\{ 1 - 3.211 N_1 \frac{(wz_0)^{\frac{1}{2}}}{d} \right\} \quad . \quad . \quad (44)$$

in which M_2 and N_1 can be computed from *measured velocities* as already explained.

To eliminate z_0 from this equation we have (see Chapter I),

$$z_0 = \frac{35.244\tilde{\omega}}{d^2} \left\{ \frac{1}{\Delta} - \frac{1}{\delta} \right\}$$

$$= \frac{35.244\tilde{\omega}\delta}{d^2 \Delta^2} \left\{ 1 - \frac{\Delta}{\delta} \right\} \frac{\Delta}{\delta}.$$

If we take the mean density of powder at 1.8, and the mean density of loading at 0.9, both of which values are nearly correct, $\frac{\Delta}{\delta}$ will be $\frac{1}{2}$, and we shall have

$$\left(1 - \frac{\Delta}{\delta}\right) \frac{\Delta}{\delta} = \frac{1}{4}.$$

It is easily seen that the maximum value of this function is $\frac{1}{4}$; and as a function at or near its maximum changes its value very slowly, it is apparent that a small variation in the assumed value of $\frac{\Delta}{\delta}$ will have but a slight effect upon the value of the function

$$\left(1 - \frac{\Delta}{\delta}\right) \frac{\Delta}{\delta};$$

and we therefore have, very nearly,

$$z_0 = \frac{35.244\tilde{\omega}\delta}{4d^2 \Delta^2} = \frac{8.811\tilde{\omega}\delta}{d^2 \Delta^2}.$$

Substituting this value of z_0 in Equation (44) and making (since z_0 and d are both to be expressed in feet),

$$K_0 = \frac{0.725}{72\pi g}\left(\frac{1728}{8.811\delta}\right)^{\frac{1}{2}}$$

and

$$K_1 = 3.211\left(\frac{8.811\delta}{1728}\right)^{\frac{1}{2}},$$

it becomes

$$p_m = K_0 M_1 \Delta \frac{(\tilde{\omega} w)^{\frac{1}{2}}}{d^2}\left\{1 - K_1 N_1 \frac{(\tilde{\omega} w)^{\frac{1}{2}}}{\Delta d^2}\right\} \quad . \quad . \quad (45)$$

in which p_m will be given in pounds per square inch.

If we suppose the mean density of gunpowder to be 1.8 we have the following values of the numerical constants K_0 and K_1:

$$\log K_0 = 7.01716 - 10$$
$$\log K_1 = 9.48802 - 10$$

It is sometimes convenient to have an expression for p_m which is independent of the density of loading; and by means of which the variation of p_m due to a variation of $\tilde{\omega}$ can be readily computed. We have

$$\Delta = \frac{27.68\tilde{\omega}}{C}$$

and this substituted in Equation (45) gives

$$p_m = \frac{K_2 M_1}{C d^2} \tilde{\omega}^{\frac{3}{2}} w^{\frac{1}{2}} \left\{1 - \frac{K_3 N_1 C}{d^2}\left(\frac{w}{\tilde{\omega}}\right)^{\frac{1}{2}}\right\} \quad . \quad . \quad (46)$$

in which

$$\log K_2 = 8.45934 - 10$$

and

$$\log K_3 = 8.04584 - 10.$$

Making

$$K_4 = \frac{K_2 M_1 w^{\frac{1}{2}}}{C d^2}$$

and

$$K_5 = \frac{K_3 N_1 C w^{\frac{1}{2}}}{d^2},$$

Equation (46) becomes

$$p_m = K_4 \tilde{\omega} \left(\tilde{\omega}^{\frac{1}{2}} - K_5\right) \quad . \quad . \quad . \quad (47)$$

From this equation we readily find the variation Δp_m due to a variation $\Delta\tilde{\omega}$ to be

$$\Delta p_m = K_4\left(\tfrac{3}{2}\tilde{\omega}^{\frac{1}{2}} - K_5\right)\Delta\tilde{\omega} \quad . \quad , \quad . \quad (48)$$

In these last two equations K_5 cannot be neglected in comparison with $\sqrt{\tilde{\omega}}$.

Example 7.—Compute the maximum pressure per square inch upon the base, and corresponding travel of the projectile, for each of the guns of Example 2.

Answers.

S. B. gun $\begin{cases} p_m = 12815 \text{ lbs.} \\ u = 0.72 \text{ ft.} \end{cases}$

8-inch gun $\begin{cases} p_m = 29855 \text{ lbs.} \\ u = 1.67 \text{ ft.} \end{cases}$

NOTE—The recorded maximum pressure upon the breech of the 8-inch gun of Example 7 was 34048 pounds per square inch.

Example 8.—Compute the maximum pressure and corresponding travel of projectile with the data of Example 3.

Answers. $p_m = 25514$ lbs.
$u = 0\text{-}14$ ft.

Example 9.—Compute the maximum pressure and corresponding travel of projectile for each of the guns of Example 4.

Answers.

S. B. gun $\begin{cases} p_m = 38342 \text{ lbs.} \\ u = 0.62 \text{ ft.} \end{cases}$

60-pdr. $\begin{cases} p_m = 23439 \text{ lbs.} \\ u = 0.29 \text{ ft.} \end{cases}$

Example 10.—Compute the maximum pressure and corresponding travel of projectile for each of the guns of Example 6.

Answers.

19-cm. gun $\begin{cases} p_m = 32587 \text{ lbs.} \\ u = 0.485 \text{ ft.} \end{cases}$

10-cm. gun $\begin{cases} p_m = 26996 \text{ lbs.} \\ u = 0.31 \text{ ft.} \end{cases}$

Example 11.—Compute the maximum pressure per square inch upon the base of a service projectile in the bore of the 8-inch steel B. L. R., fired with a charge of 125 pounds of the German cocoa powder of Example 2.

We have given: $w = 300$ lbs.; $\tilde{\omega} = 125$ lbs.; $d = \frac{2}{3}$ ft.; $C = 3597$ c. i,; $M_1 = 96733$ and $N_1 = 0.0010597$. Substituting these numbers in Equation (46) we find $p_m = 35948$ lbs.

Example 12.—Compute the muzzle velocity of the projectile of Example 11. Data: $U = 17\frac{1}{9}$ ft. and $\Delta = 0.9619$.

We find: $\log z_0 = 0.46105$; $x_1 = 5.9188$; $\log X_0 = 0.81163$; $\log X_1 = 0.48851$; and $V = 1954$ f. s.

The velocity of this projectile, fired with a charge of 125 lbs. of *U R* brown prismatic powder, as announced by the Ordnance Department, is 1950 f. s.; and the maximum pressure is about the same as computed above. The German cocoa is therefore well adapted for this gun.

Example 13.—The charge for the 10-inch steel B. L. R. is 250 lbs. Supposing the powder to be the German cocoa described in Example 2, what would be the maximum pressure on the base of the projecfile, and the muzzle velocity?

Data: $\tilde{\omega} = 250$ lbs.; $w = 575$ lbs.; $d = \frac{5}{6}$ ft.; $C = 7064$ c. i.; $U = 22.925$ ft.; and $\Delta = 0.9796$. From these numbers we find: $\log z_0 = 0.55176$; $x_1 = 6.435$; $\log X_0 = 0.82483$; $\log X_1 = 0.51293$; $p_m = 44044$ lbs.; and $V = 2140$ f. s.

The value of p_m is too great for safety; and either a slower powder should be adopted, or the charge should be diminished, or the capacity of the chamber increased.

Example 14.—Determine the charge of German cocoa powder which would make $p_m = 35000$ lbs. in the 10-inch steel B. L. R., and the corresponding muzzle velocity.

Substituting given numbers in Equation (47) it becomes

$$\tilde{\omega}\left(\tilde{\omega}^{\frac{1}{2}} - 2.8725\right) = 2570.5,$$

which by a few trials gives $\tilde{\omega} = 216.86$ lbs., weight of charge required.

For the muzzle velocity we find $\Delta = 0.8496$; $\log z_0 = 0.63813$; $x_1 = 5.2745$; $\log X_0 = 0.79327$; $\log X_1 = 0.45397$; and $V = 1940$ f.s.

The actual muzzle velocity obtained with this gun, using a charge of 250 pounds of *V U* brown prismatic powder, is 1975 f.s. The German cocoa powder is therefore too quick for this gun with its present capacity of chamber.

Example 15.—What capacity of chamber in the 10-inch steel B. L. R. would make $p_m = 35000$ lbs., with a charge of 250 lbs. of German cocoa powder; and what would be the muzzle velocity?

Making

$$K_6 = \frac{K_2 \, M_1 \, \tilde{\omega}^{\frac{3}{2}} \, w^{\frac{1}{2}}}{d^2}$$

and

$$K_7 = \frac{K_3 N_1}{d^2}\left(\frac{w}{\tilde{\omega}}\right)^{\frac{1}{2}},$$

Equation (46) may be written

$$p_m = \frac{K_6}{C}(1 - K_7 C);$$

whence

$$C = \frac{K_6}{p_m + K_6 K_7} \quad . \quad . \quad . \quad (49)$$

Substituting known numbers in Equation (49) we find $C=$ 8491 c. i.

For the muzzle velocity we have $\Delta = 0.8151$; $\log z_0 = 0.70547$; $x_1 = 4.5169$; $\log X_0 = 0.76825$; $\log X_1 = 0.40580$; and $V = 2036$ f. s.

It may be inferred from these examples that the same kind of powder can be safely used in all our heavy sea-coast guns provided the volumes of the chambers are suitably proportioned.

Expression for the First Power of the Velocity.—When $\mu = \frac{1}{3}$, as is the case with spherical, sphero-hexagonal and cubical grains, the second member of Equation (35) is very nearly a complete square. Extracting the square root, retaining but two terms in the second member, we have

$$v = M^{\frac{1}{2}}\left(\frac{fa}{\tau}\right)^{\frac{1}{2}}\left(\frac{\tilde{\omega}}{d}\right)^{\frac{1}{2}}\left(\frac{z_0}{w}\right)^{\frac{1}{4}} X_1^{\frac{1}{2}}\left\{1 - \tfrac{1}{2}N\frac{\lambda}{\tau}\,\frac{(wz_0)^{\frac{1}{2}}}{d}X_0\right\} \quad . \quad (50)$$

in which M and N are independent of the elements of fire. The values of these constants are given on page 84

If we make

$$M^{\frac{1}{2}}\left(\frac{fa}{\tau}\right)^{\frac{1}{2}} = M_1^{\frac{1}{2}} = F_1$$

and

$$\tfrac{1}{2}N\,\frac{\lambda}{\tau} = \tfrac{1}{2}N_1 = G_1,$$

Equation (50) becomes

$$v = F_1 \left(\frac{\tilde{\omega}}{d}\right)^{\frac{1}{2}} \left(\frac{z_0}{w}\right)^{\frac{1}{4}} X_2^{\frac{1}{2}} \left\{ 1 - G_1 \frac{(wz_0)}{d} X_0 \right\} \quad . \quad . \quad (51)$$

in which F_1 and G_1 are to be determined by experimental firing as already explained for M_1 and N_1.

If we make

$$\left(\frac{\tilde{\omega}}{d}\right)^{\frac{1}{2}} \left(\frac{z_0}{w}\right)^{\frac{1}{4}} X_1^{\frac{1}{2}} = \frac{1}{A}$$

and

$$\frac{(wz_0)^{\frac{1}{2}}}{d} X_0 = B$$

we shall have for the determination of F_1 and G_1 the equations

$$F_1 = \frac{A_1 B_2 V_1 - A_2 B_1 V_2}{B_2 - B_1}$$

and

$$G_1 = \frac{F_1 - A_1 V_1}{B_1 F_1} = \frac{F_1 - A_2 V_2}{B_2 F_1}.$$

The values of F_1 and G_1 and the numbers employed in their calculation are smaller and more easily handled when we use V than when V^2 is employed; otherwise there is very little difference in the labor involved in the use of Equations (39) and (51).

This latter formula is slightly more accurate than the former for charges made up of spherical and kindred grains for which μ does not vanish. But the difference in this respect is of no practical importance in computing v and p.

Sarrau's Binomial Formula for Muzzle Velocity.—Substituting in Equation (50) for z_0 its approximate value in feet, already given, and combining the constants into two new constants F and G, we have

$$v = F \left(\frac{fa}{\tau}\right)^{\frac{1}{2}} \frac{\tilde{\omega}^{\frac{3}{4}} \delta^{\frac{1}{4}}}{\Delta^{\frac{1}{2}} d w^{\frac{1}{4}}} X_1^{\frac{1}{2}} \left\{ 1 - G \frac{\lambda}{\tau} \frac{\tilde{\omega}^{\frac{1}{2}} \delta^{\frac{1}{2}} w^{\frac{1}{2}}}{\Delta d^2} X_0 \right\} \quad . \quad (52)$$

If we require an equation for muzzle velocity only, we can substitute for X_0 and X_1 in Equation (52) certain functions of U/z_0 (upon which their values directly depend), U being the distance traveled by the projectile from its initial position up to the muzzle. *Sarrau* as the result of his investigations, makes[1] at the muzzle,

$$X_0 = Q_1 \left(\frac{U}{z_0}\right)^{\frac{1}{2}} = Q_2 \frac{\Delta\, d\, U^{\frac{1}{2}}}{\tilde{\omega}^{\frac{1}{2}}\ \delta^{\frac{1}{2}}}.$$

We also have

$$X_1 = X_0 \left\{ 1 - \frac{1}{(1+x)^{\frac{1}{3}}} \right\};$$

and since for modern guns the term within the braces varies (at the muzzle) very nearly as $\left(\frac{U}{z_0}\right)^{\frac{1}{4}}$, we may write

$$X_1^{\frac{1}{2}} = Q_3 \left(\frac{U}{z_0}\right)^{\frac{3}{8}} = Q_4 \frac{U^{\frac{3}{8}}\ \Delta^{\frac{3}{4}}\ d^{\frac{3}{4}}}{\tilde{\omega}^{\frac{3}{8}}\ \delta^{\frac{3}{8}}};$$

in which Q_1, Q_2, Q_3 and Q_4 are numerical constants independent of the elements of fire.

Substituting these values of $X_1^{\frac{1}{2}}$ and X_0 in Equation (52), and designating by A the product of all the constants outside the brackets, including $\delta^{\frac{1}{8}}$, and by B the product of all the constants within the brackets, the equation becomes, for muzzle velocity,

$$V = A \left(\frac{fa}{\tau}\right)^{\frac{1}{2}} (\tilde{\omega} U)^{\frac{3}{8}} \left(\frac{\Delta}{w\, d}\right)^{\frac{1}{4}} \left\{ 1 - B \frac{\lambda}{\tau} \frac{(Uw)^{\frac{1}{2}}}{d} \right\} \quad . \quad (53)$$

which is *Sarrau's* binomial formula for muzzle velocity.

For the same kind of powder we shonld have by making

$$A \left(\frac{fa}{\tau}\right)^{\frac{1}{2}} = A_1$$

and

$$B \frac{\lambda}{\tau} = B_1,$$

$$V = A_1 (\tilde{\omega}\ U)^{\frac{3}{8}} \left(\frac{\Delta}{w\, d}\right)^{\frac{1}{4}} \left\{ 1 - B_1 \frac{(Uw)^{\frac{1}{2}}}{a} \right\} \quad . \quad . \quad (54)$$

[1] Proceedings of the Naval Institute. Vol. X. Page 137.

The constants A_1 and B_1 in Equation (54) are usually determined by firing the same kind of powder in two similarly constructed guns of different calibres and conditions of loading, and measuring the respective muzzle velocities with a chronoscope. Then if we distinguish the guns by subscripts, and make

$$(\tilde{\omega} U)^{\frac{3}{8}}\left(\frac{\Delta}{w\,d}\right)^{\frac{1}{4}} = \frac{1}{X}$$

and

$$\frac{(U w)^{\frac{1}{2}}}{d} = Y$$

we shall have

$$A_1 = \frac{X_1 Y_2 V_1 - X_2 Y_1 V_2}{Y_2 - Y_1}$$

and

$$B_1 = \frac{A_1 - X_1 V_1}{A_1 Y_1} = \frac{A_1 - X_2 V_2}{A_2 Y_2}.$$

All the variables composing the second member of Equation (54) are independent with the exception of $\tilde{\omega}$ and Δ which are functions of each other. We have in fact

$$\Delta = \frac{27.68\tilde{\omega}}{C},$$

which substituted in Equation (54) gives

$$V = A_1 \tilde{\omega}^{\frac{5}{8}} U^{\frac{3}{8}} \left(\frac{27.68}{w\,d\,C}\right)^{\frac{1}{4}} \left\{ 1 - B_1 \frac{(U w)^{\frac{1}{2}}}{d} \right\} \quad . \quad . \quad (55)$$

Equation (55) shows that the muzzle velocity is proportional to the $\frac{5}{8}$ power of the charge, and inversely proportional to the $\frac{1}{4}$ power of the capacity of the powder-chamber, both of which relations were established by *Hélie* from the experiments made by the commission of *Gavre* with Navy guns and powders; and these results justify the values given by *Sarrau* to X_0 and $X_1^{\frac{1}{2}}$.

For the same gun and powder-charge Equation (54) has sometimes been written

$$v = A' u^{\frac{3}{8}} \left(1 - B' u^{\frac{1}{2}}\right),$$

in which

$$A' = A_1 \tilde{\omega}^{\frac{3}{8}} \left(\frac{\Delta}{w\,d}\right)^{\frac{1}{4}},$$

and

$$B' = B_1 \frac{w^{\frac{1}{2}}}{d};$$

and in this form has been used to compute the velocity, and (by differentiation) the pressure upon the base of the projectile at different points within the bore, u being considered an independent variable, while in fact, it is a constant. It is unnecessary to say that the results arrived at are at the best but rough approximations, except in the neighborhood of the muzzle.

Sarrau's Monomial Formula for Muzzle Velocity.—To transform Equation (50) into an equivalent monomial expression for muzzle velocity we must express the function

$$X_1^{\frac{1}{2}} \left\{ 1 - \tfrac{1}{2} N \frac{\lambda}{\tau} \frac{(wz_0)^{\frac{1}{2}}}{d} X_0 \right\}$$

in terms of properly chosen powers of the variables. It is easily seen that this function imcreases with u/z_0 (upon which the values of X_0 and X_1 depend); and decreases as $\frac{\lambda}{\tau} \frac{(wz_0)^{\frac{1}{2}}}{d}$ increases. We may therefore regard it as proportional to a positive power γ of the first of these variables and to a negative power γ' of the second.[1]

According to the exhaustive experiments made at Gâvre it was found that for a very quick powder $\gamma = \frac{1}{8}$ and $\gamma' = \frac{1}{2}$; and for a very slow powder $\gamma = \frac{1}{4}$ and $\gamma' = \frac{1}{4}$. Taking a mean of these (which *Sarrau* considers an approximation applicable to the normal conditions of practice), we have $\gamma = \frac{3}{16}$ and $\gamma' = \frac{3}{8}$. We therefore have

$$X_1^{\frac{1}{2}} \left\{ 1 - \tfrac{1}{2} N \frac{\lambda}{\tau} \frac{(wz_0)^{\frac{1}{2}}}{d} X_0 \right\} = B \left(\frac{\tau}{\lambda} \right)^{\frac{3}{8}} \left(\frac{d}{(wz_0)^{\frac{1}{2}}} \right)^{\frac{3}{8}} \left(\frac{U}{z_0} \right)^{\frac{3}{16}}$$

in which B is a numerical factor independent of the elements of loading. Substituting this in Equation (50), replacing z_0 by its approximate value in feet, viz:

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[1] *Sarrau*. Proceedings U. S. Naval Institute. Vol. X. Page 121.

$$z_0 = \frac{8.811\tilde{\omega}\delta}{1728 d^2 \Delta^2},$$

and combining constants $\left(\text{including } \delta^{\frac{1}{8}}\right)$, we have

$$V = H \frac{\tilde{\omega}^{\frac{3}{8}} \Delta^{\frac{1}{4}} U^{\frac{3}{16}} d^{\frac{1}{8}}}{w^{\frac{7}{16}}}, \quad . \quad . \quad . \quad (56)$$

in which

$$H = B\left(\frac{fa}{\tau}\right)^{\frac{1}{2}}\left(\frac{\tau}{\lambda}\right)^{\frac{3}{8}} \quad . \quad . \quad . \quad . \quad (57)$$

For very quick powder for which $\gamma = \frac{1}{8}$ and $\gamma' = \frac{1}{2}$, the expression for V becomes

$$V = H \frac{\tilde{\omega}^{\frac{3}{8}} \Delta^{\frac{1}{4}} U^{\frac{1}{8}} d^{\frac{1}{4}}}{w^{\frac{1}{2}}} \quad . \quad . \quad . \quad (58)$$

in which

$$H = B\left(\frac{fa}{\tau}\right)^{\frac{1}{2}}\left(\frac{\tau}{\lambda}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad (59)$$

We may also employ Equation (14) for a very quick powder which may obviously be written

$$V = \sqrt{6fg}\left(\frac{\tilde{\omega}}{w}\right)^{\frac{1}{2}} X_4 \quad . \quad . \quad . \quad . \quad (60)$$

Sarrau's Formula for the Maximum Pressure on the Base of the Projectile.—*Sarrau* assumes that, as the maximum pressure occurs when the projectile has moved but a short distance, the second term within the braces of Equation (45) may be omitted. Therefore replacing M_1 by its value $M\frac{fa}{\tau}$ and writing K for $K_0 M$ we have

$$p_m = K \frac{fa}{\tau} \frac{\Delta\sqrt{\tilde{\omega}w}}{d^2} \quad . \quad . \quad . \quad (61)$$

which is *Sarrau's* formula for the maximum pressure on the base of the projectile.

CHAPTER V.

CHARACTERISTICS OF POWDER.

Before proceeding to discuss the influence of what *Sarrau* calls the "characteristics of a powder" upon the maximum pressure and the muzzle velocity, we will consider what effect the *shape* of the grains of which the charge is composed has upon the velocity of emission of gas during the different stages of burning. This is necessary to enable us to decide upon the best form of grain,—that is upon the form of grain which, without diminishing the muzzle velocity, will give the least maximum pressure upon the walls of the gun.

Velocity of Emission.—The velocity of emission is the rate of combustion of a grain of powder, or of a charge composed of similar grains, at any instant; or, it may be defined as the ratio of the weight of the fraction of a grain consumed in any element of time to this element.

Let

S be the surface of the ignited grain at any instant,
v the velocity of combustion,
δ the absolute density of the grain,
η the velocity of emission.

The volume burned in the element of time dt, will be $S\,v\,dt$, and the corresponding weight (in metric units), $Sv\,\delta\,dt$. Therefore the rate of combustion, or velocity of emission will be

$$\eta = \frac{Sv\,\delta\,dt}{dt} = Sv\,\delta \qquad . \qquad . \qquad . \qquad (1)$$

The velocity of emission (and therefore the disengagement of gas) varies then as the surface of the grain, the velocity of combustion and the absolute density. In free air, that is, under constant pressure, it was found experimentally by *Piobert* that the velocity of combustion varies inversely as the density, and therefore in this case, the product $v\delta$ is constant. We therefore have in free air

$$\eta = mS,$$

in which m is constant. That is, the velocity of emission in free air is proportional to the burning surface; but this surface varies at a rate depending upon the form of the grain.

We may deduce another and more convenient expression for the velocity of emission of a grain of powder, or of a charge of powder composed of similar grains, in free air, by differentiating Equation (15) Chapter IV. This gives

$$\frac{dy}{dt} = \eta = \tilde{\omega}\frac{d\psi(t)}{dt} \quad . \quad . \quad : \quad (2)$$

Velocity of Emission for a Spherical Grain.—By differentiating Equation (17), Chapter IV, we obtain

$$\eta = \frac{3\tilde{\omega}}{\tau}\left(1 - \frac{t}{\tau}\right)^2.$$

When $t = o$ we have

$$\eta_0 = \frac{3\tilde{\omega}}{\tau};$$

and when $t = \tau$,

$$\eta_\tau = o;$$

the velocity of emission for a spherical grain therefore, continually decreases to its inferior value which is zero.

Velocity of Emission for a Grain in the Form of a Parallelopipedon.—We have in this case from Equations (19) and (20), Chapter IV,

$$\psi(t) = \frac{at}{\tau}\left\{1 - \lambda\frac{t}{\tau} + \mu\frac{t^2}{\tau^2}\right\}$$

in which

$$a = 1 + x + y,\ \lambda = \frac{x + y + xy}{1 + x + y},\ \mu = \frac{xy}{1 + x + y};$$

and

$$x = \frac{a}{\beta},\ y = \frac{a}{\gamma},$$

a being the least dimension of the grain and β, γ the other two dimensions. Therefore differentiating with respect to the variable t we have

$$\eta = \frac{a\tilde{\omega}}{\tau}\left\{1 - 2\lambda\frac{t}{\tau} + 3\mu\frac{t^2}{\tau^2}\right\}.$$

Making $t = o$, we have

$$\eta_0 = \frac{a\tilde{\omega}}{\tau} = \tilde{\omega}\,\frac{1+x+y}{\tau};$$

and when $t=\tau$

$$\eta_\tau = \frac{a\tilde{\omega}}{\tau}(1-2\lambda+3\mu) = \tilde{\omega}\,\frac{(1-x)(1-y)}{\tau}.$$

Generally the grains of this form have two opposite square faces, in which case x and y are equal and less than unity; and we have for this form of grain (flat grain)

$$\eta_0 = \tilde{\omega}\,\frac{1+2x}{\tau};\ \text{and}\ \eta_\tau = \tilde{\omega}\,\frac{(1-x)^2}{\tau}.$$

For a cubical grain we have $x=y=1$; and therefore for a cubical grain

$$\eta_0 = \frac{3\tilde{\omega}}{\tau}\ \text{and}\ \eta_\tau = o.$$

It will readily be seen from what precedes that the velocity of emission at the beginning of combustion is less, and at the end greater, for a flat grain than for a cubical or spherical grain, from which it appears that the former is more *progressive* than the latter,

Velocity of Emission for a Pierced Cylinder.—For a pierced cylinder we have from Equations (24) and (25), Chapter IV,.

$$\phi(t) = \frac{at}{\tau}\left(1-\lambda\frac{t}{\tau}\right),$$

and

$$a = 1+x,\ \lambda = \frac{x}{1+x},\ x = \frac{r-r'}{h};$$

whence as before,

$$\eta = \frac{a\tilde{\omega}}{\tau}\left(1-2\lambda\frac{t}{\tau}\right) = \frac{\tilde{\omega}}{\tau}\left(1+x-2x\frac{t}{\tau}\right)$$

and

$$\eta_0 = \frac{\tilde{\omega}}{\tau}(1+x),\ \eta_\tau = \frac{\tilde{\omega}}{\tau}(1-x).$$

It follows from these results that for the same value of x, the pierced cylindrical grain is much more progressive than the flat grain. In all these forms of grain however uniformity of emission, which is the great *desideratum*, is far from being attained.

For the army prismatic powder (flat grain) for example, $x = 0.72$; and therefore $\eta_0 = \frac{2.44}{\tau}\tilde{\omega}$ and $\eta_\tau = \frac{0.0784}{\tau}\tilde{\omega}$. That is, for this grain, the velocity of emission at the beginning of combustion is more than thirty times what it is at the end. For the German cocoa powder (pierced prismatic, assimilating to the pierced cylinder) $x = \frac{1}{2}$; and for this we have $\eta_0 = \frac{3\tilde{\omega}}{2\tau}$ and $\eta_\tau = \frac{\tilde{\omega}}{2\tau}$, which gives

$$\frac{\eta_0}{\eta_\tau} = 3$$

This form is therefore a great advance on both the flat and cubical grains.

Disc Powders.—"A still greater uniformity of emission can be obtained by the use of the so-called disc powders, that is to say by powders pressed into thin discs of uniform thickness, and of the same diameter as the gun chamber, so assembled together as to permit the simultaneous ignition between adjacent discs. In this case the surface being nearly constant, the velocities of emission in free air would be nearly uniform and the time of total combustion dependent on the thickness of the discs."[1]

Mr. *Quick* of the English Navy has recently patented a powder of this description which bids fair to supersede all other forms for heavy artillery, and which he calls the "Quick cake powder." The following description of this form of powder is taken from a very instructive paper on this subject, by Mr. Quick, and published in the proceedings of the U. S. Naval Institute, Vol. XV, page 414.

"Alternate quadrants of the cake are in relief, so that when two cakes are placed together, the raised quadrants of the one fit into the sunk ones of the other, and the two cakes are in a manner locked. Moreover, the quadrants are dished, or somewhat hollowed out towards the center to permit the flame readily reaching the whole surface of the cakes. A cartridge is formed from a number of these cakes placed together, which may be

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[1] *Longridge's* Internal Ballistics. Page 77.

united by means of a waterproof cement, which is highly combustible, and which leaves only a very small amount of residuum, namely, about a quarter of one per cent.''

Some of the advantages realized by using this kind of powder are thus stated by Mr. *Quick:* ''The disposition of the air spacing here adopted permits the ready flow of the gases to the base of the projectile, which is not the case when powder of the ordinary form is employed. With the latter, when long cartridges are ignited from their rear ends, the gas pressure then generated ruptures the powder in front, crushing it probably into very small fragments—whereby the velocity of combustion is increased—thus causing excessive local or so-called 'wave' pressures, which may injure the gun.

Large air spacing is an expedient adopted for the purpose of mitigating the evils arising from powder improperly ignited and improperly consumed. This large air spacing has always largely diminished the efficiency of the powder, fewer tons of energy being obtained per pound of powder than when the air spacing is small. But as this large air spacing enabled heavier charges to be used without dangerously high pressures being generated, it had a certain amount of utility, although it necessarily added to the weight of the gun and to the size and weight of the breech mechanism.

Now with this form of perforated cake powder we can use a much higher gravimetric density of charge, get a larger amount of energy per pound of powder, and obtain a higher velocity with lower pressure, whilst at the same time we can reduce the diameter of the breech of the gun and reduce the size and weight of the breech mechanism.

With the Quick cake powder the cartridges will be truly cylindrical, so that there can be no cutting or straining of the cartridge cases. The cakes being locked together by the clutch-like surface, there can be no attrition of the surfaces to generate dust, even if the cakes be not cemented together. At a very trifling expense, however, the whole of the cakes forming a charge or section of a charge may be cemented into one rigid mass, and the whole rendered waterproof and airtight. The front end of the cartridge may be protected by a shield or wad of non-inflammable material, so that any small amount of burning residuum would be swept forward by it out of the way of the powder, and thus premature explosion, even if fire remained in the gun, would be prevented. It is needless to say that the cylindrical form of the cartridge would greatly faciliate storage, handling and loading."

Formula for the Velocity of Emission in the Bore of a Gun.— The weight of powder consumed in the bore of a gun in terms of the length of grain burned, is given by Equation (27) Chapter IV, namely:

$$y = \tilde{\omega} \frac{al}{l_0} \left\{ 1 - \lambda \frac{l}{l_0} + \mu \frac{l^2}{l_0^2} \right\},$$

in which l_0 is one-half of the least dimension of the grain and l the length of grain burned in the time t under a variable pressure p. Taking the derivative with respect to the time we have

$$\frac{dy}{dt} = \eta = \frac{a\tilde{\omega}}{l_0} \left\{ 1 - 2\lambda \frac{l}{l_0} + 3\mu \frac{l^2}{l_0^2} \right\} \frac{dl}{dt}.$$

But we have, Equation (28), Chapter IV,

$$\frac{dl}{dt} = \frac{l_0}{\tau} \left(\frac{p}{p_0} \right)^{\frac{1}{2}};$$

whence

$$\eta = \frac{a\tilde{\omega}}{\tau} \left(\frac{p}{p_0} \right)^{\frac{1}{2}} \left\{ 1 - 2\tau \frac{l}{l_0} + 3\mu \frac{l^2}{l_0^2} \right\} \quad . \quad . \quad (3)$$

From Equation (3) we find when $l = o$, and $l = l_0$, that is, at the beginning and ending of combustion,

$$\eta_0 = \frac{a\tilde{\omega}}{\tau} \left(\frac{p}{p_0} \right)^{\frac{1}{2}}$$

and

$$\eta_\tau = \frac{a\tilde{\omega}}{\tau} \left(\frac{p}{p_0} \right)^{\frac{1}{2}} \left\{ 1 - 2\lambda + 3\mu \right\},$$

equations which are the same as those deduced for constant atmospheric pressure, multiplied by the square-root of the pressure in atmospheres.

By dividing the first of these last two equations by the second we have

$$\frac{\eta_0}{\eta_\tau} = \frac{1}{1 - 2\lambda + 3\mu} \quad . \quad . \quad . \quad (4)$$

From this equation it follows that the velocity of emission is uniform for any grain whose form is such that $\lambda = \mu = o$; or for which

$$\mu = \tfrac{2}{3}\lambda;$$

and the smaller λ and μ are, the more uniform is the emission of gas. There is no known form of grain which will make $\eta_\tau = \eta_0$; but a thin disc burning on both sides affords a good approximation.

It may be shown as in Chapter IV that for a grain of this form we have

$$\phi(t) = a\frac{t}{\tau}\left(1 - \lambda\frac{t}{\tau} + \mu\frac{t^2}{\tau^2}\right),$$

in which

$$a = 1 + x,\ \lambda = \frac{4x + x^2}{4(1 + x)},\ \mu = \frac{x^2}{4(1 + x)}$$

and

$$x = \frac{h}{r},$$

h being the thickness of the disc and r the radius.

From these equations we find

$$\frac{\eta_0}{\eta_\tau} = \frac{2(1 + x)}{2(1 - x) + x^2}$$

If we suppose the thickness of the disc to be one-eighth its radius, we shall have

$$\frac{\eta_0}{\eta_\tau} = 1.27.$$

Determination of the Least Dimension of a Grain of Powder.— To determine the value of τ (or the time of burning of a grain of powder in free air) experimentally, requires that the least dimension of the grain should be known. For the pierced prismatic, and some cubical grains, which are carefully moulded of uniform size this is simply a matter of direct measurement of a single grain. But for pebble, sphero-hexagonal, mammoth, square prismatic (or flat) grains, which depend for their least dimensions upon the thickness of the press cake of which they are made, and which varies more or less, the mean thickness (or least dimension) can be most easily calculated from the density and the number of grains in a pound of the powder.

Spherical and Allied Grains.—Let N be the number of spherical grains supposed to be of the same size, in a pound of powder, r their radius; and δ their density.

Then since 27.68 is the number of cubic inches in a pound of water, we evidently have the relation

$$\tfrac{4}{3}\pi r^3 \delta N = 27.68$$

whence

$$r = \frac{1.8766}{(\delta N)^{\frac{1}{3}}};$$

or, by logarithms,

$$\log r = 0.27336 - \tfrac{1}{3}\log(\delta N) \qquad . \qquad . \qquad . \qquad (5)$$

Irregular Grains.—When the grains are not exactly spherical or of uniform size, as is the case with the irregular shaped grains of cannon powder, and also with pebble, mammoth, sphero-hexagonal, etc., powders, the above formula will give the radius of a spherical grain whose weight is the mean weight of the irregular grains in question.

Grains in the Form of a Parallelopipedon.—Let α, β, γ be the three dimensions of the grain, of which α is the least; and put

$$x = \frac{\alpha}{\beta} \text{ and } y = \frac{\alpha}{\gamma}.$$

The volume of the grain is therefore $\alpha\beta\gamma = \dfrac{\alpha^3}{xy}$. Therefore

$$\frac{\alpha^3 \delta N}{xy} = 27.68;$$

whence

$$\alpha = \left(\frac{27.68\, xy}{\delta N}\right)^{\frac{1}{3}}$$

For a flat grain with a square base we have $x = y$; and in this case the expression for α becomes

$$\alpha = \left(\frac{27.68\, x^2}{\delta N}\right)^{\frac{1}{3}};$$

or, by logarithms,

$$\log \alpha = 0.48072 + \tfrac{2}{3}\log x - \tfrac{1}{3}\log(\delta N) \qquad . \qquad . \qquad (6)$$

Time of Burning of a Grain in Free Air.—From the definition of N_1 (page 84) we have

$$\tau = \frac{N}{N_1}\lambda \quad . \quad . \quad . \quad (7)$$

in which N is independent of the elements of fire (page 82), λ depends upon the form of the grains, and N_1 is determined by experimental firing as fully explained and illustrated in Chapter IV.

Value of the Factor f.—From the definition of M_1 (page 84) and the value of τ given by Equation (7) we find

$$f = \frac{N}{M}\frac{M_1}{N_1}\frac{\lambda}{a} = [7.71453 - 10]\frac{\lambda}{a}\frac{M_1}{N_1} \quad . \quad . \quad (8)$$

Weight of Powder Burned in the Gun.—We have from the definitions of K and N (page 82),

$$K = \frac{N}{\tau}\frac{\sqrt{wz_0}}{d} = \frac{N_1}{\lambda}\frac{\sqrt{wz_0}}{d} \quad . \quad . \quad . \quad (9)$$

by means of which, when N_1 has been determined by two firings, K can be computed for the kind of powder employed, and then substituting in Equation (33), page 82, we find the value of y for any travel of the projectile, X_0 being taken from Table II for the given value of x, or u/z_0.

Example 1.—Compute τ and f for the German cocoa powder of Example 2, Chapter IV; and determine the equation for y for each of the guns.

For this powder $a = \frac{3}{2}$, $\lambda = \frac{1}{3}$ $\mu = o$, $M_1 = 96733$ and $N_1 = 0.0010597$. From these data we find, by means of the equations given above, $\tau = 0.555$ seconds, $f = 105120$;—and for the South Boston gun,

$$y = [8.91220 - 10]\, X_0 \left\{ 1 - [8.25900 - 10]\, X_0 \right\} \tilde{\omega};$$

and for the 8-inch B. L. R.,

$$y = [9.31539 - 10] X_0 \left\{ 1 - [8.66218 - 10] X_0 \right\} \tilde{\omega}.$$

From these last formulas we find that at the muzzle 90 per cent. of the charge of the 8-inch gun, and 49 per cent. of the charge of the 6-inch gun, had been burned. From these calculations it is evident that this powder is well adapted for the larger gun; but that the 6-inch gun with its light projectile requires a quicker powder.

Example 2.—Compute τ and f for the pebble powder of Example 1, Chapter IV; and determine the equation for y.

For this powder $a = 3$, $\lambda = 1$ and $\mu = \frac{1}{3}$. The values of M_1 and N_1 given on page 88 were determined by neglecting the term containing μ in Equation (35), page 83, and therefore have not the accuracy which the method and formulas admit of. This is of no practical consequence in computing the values of v and p in the bore of the gun, since by the manner of determining the constants M_1 and N_1, *when these are used together*, the error of each corrects in great measure that of the other. But since the value of τ depends upon that of N_1 alone, it is better to deduce this factor from Equation (51) which takes account of μ. Employing the formulas given on page 101 we find

$$\log M_1 = 5.29922$$
$$\log N_1 = 7.51000 - 10$$

from which are deduced as in Example 1, $\tau = 0''.506$ and $f = 106320$. The equation for y becomes

$$y = [9.28527]X_0 \left\{ 1 - [8.80815]X_0 + [7.13918]X_0^2 \right\} \tilde{\omega}$$

At the muzzle $\log X_0 = 0.90833$, and the corresponding value of y is

$$y = 0.89\,\tilde{\omega};$$

that is, 89 per cent. of the charge was burned in the gun.

Example 3.—Compute τ and f for the sphero-hexagonal powder of Example 4, Chapter IV; and determine the equation for y for each gun.

The formulas found on page 101 give

$$\log M_1 = 5.44509$$
$$\log N_1 = 7.68378 - 10,$$

from which follow $\tau = 0''.366$ and $f = 99706$. For the South Boston gun,

$$y = [9.44823]X_0 \left\{ 1 - [8.97111]X_0 + [7.46510]\,X_0^2 \right\} \tilde{\omega};$$

whence at the muzzle, $y = 0.9832\,\tilde{\omega}$.

For the 64-pounder,

$$y = [9.23304]X_0 \left\{ 1 - [8.75592]X_0 + [7.03472]\,X_0^2 \right\} \tilde{\omega};$$

whence at the muzzle, $y = 0.8677\,\tilde{\omega}$.

Example 4.—How much of the charge was burned in the gun of Example 3, Chapter IV?

Answer 70 per cent.

Example 5.—Compute τ and f for the Wetteren powder of Example 6, Chapter IV; and determine the equation for y for each gun.

For this powder, which is square prismatic, $x = 0.786$; and therefore by Equation (21), $a = 2.572$, $\lambda = 0.851$ and $\mu = 0.24$. Therefore employing the formulas on page 101 we have

$$\log M_1 = 5.42991$$
$$\log N_1 = 7.62000 - 10$$

from which we get $\tau = 0''.360$ and $f = 110690$. For the 19-cm gun,

$$y = [9.39920] X_0 \left\{ 1 - [8.91886] X_0 + [7.35807] X_0{}^2 \right\} \tilde{\omega};$$

whence at the muzzle, $y = 0.9703\,\tilde{\omega}$.

For the 10-cm gun,

$$y = [9.19157] X_0 \left\{ 1 - [8.71123] X_0 + [6.94281] X_0{}^2 \right\} \tilde{\omega};$$

whence at the muzzle, $y = 0.8182\,\tilde{\omega}$.

From these examples, and many others not here given, it is shown conclusively that the value of f increases *caeteris paribus*, with the calibre of the gun. That is, a greater per centage of the force of the powder is expressed in the energy of the projectile for large calibres than for small. This agrees with the deductions of *Noble and Abel* with regard to the factor of effect. See page 64.

Determination of the Characteristics of a Powder by means of the Maximum Pressure.—It frequently happens that but one gun is available for the determination of the characteristics of a powder. In this case if the maximum pressure upon the base of the projectile is measured by means of a pressure gauge placed in the chamber of the gun, and also the muzzle velocity of the projectile by a chronograph, M_1 and N_1 can be computed as follows: From Equation (44), Chapter IV, we have

$$p_m = a_1 M_1 \frac{\tilde{\omega}}{d^3}\left(\frac{w}{z_0}\right)^{\frac{1}{2}} \left\{ 1 - 3.211\, N_1 \frac{(wz_0)^{\frac{1}{2}}}{a} \right\} \quad . \quad . \quad (10)$$

in which

$$a_1 = \frac{0.725}{72\pi g},$$

and

$$\log a_1 = 5.99854 - 10.$$

Also, from Equation (39), Chapter IV,

$$\frac{a_1 w V^2}{d^2 z_0 X_1} = a_1 M_1 \frac{\tilde{\omega}}{d^3}\left(\frac{w}{z_0}\right)^{\frac{1}{2}} \left\{ 1 - N_1 \frac{(wz_0)^{\frac{1}{2}}}{d} X_0 \right\} \quad . \quad . \quad (11)$$

Dividing Equation (11) by Equation (10), making

$$m = \frac{a_1 w V^2}{d^2 p_m z_0 X_1},$$

and solving with reference to N_1 we have

$$N_1 = \frac{(1-m)d}{(X_0 - 3.211\, m)\,(wz_0)^{\frac{1}{2}}} \quad . \quad : \quad . \quad (12)$$

Having found N_1 by Equation (12), M_1 can be most easily computed by Equation (39), Chapter IV, which for muzzle velocities is

$$V^2 = M_1 \frac{\tilde{\omega}}{d}\left(\frac{z_0}{w}\right)^{\frac{1}{2}} X_1 \left\{ 1 - N_1 \frac{(wz_0)^{\frac{1}{2}}}{d} X_0 \right\} \quad . \quad . \quad (13)$$

Example 6.—Determine the values of f and τ for a brown prismatic powder of density 1.867 for the 12-inch B. L. R., (1891) so that with a charge of 520 pounds and a projectile weighing 1000 pounds, a muzzle velocity of 2100 f. s. may be obtained with a maximum pressure on the base of the projectile of 33300 pounds per square inch.

Data: $\tilde{\omega} = 520$ lbs., $w = 1000$ lbs., $U = 30.41$ ft., $d = 1$ ft., $C =$ 13582 cubic inches, $V = 2100$ f. s., and $p_m = 33300$ lbs. From these numbers we find: $\Delta = 1.0598$, $\log z_0 = 0.63617$, $x_1 =$ 7.0282, $\log X_0 = 0.83864$, $\log X_1 = 0.53816$, $m = 0.89351$, $\log M_1$ $= 4.66796$, and $\log N_1 = 6.63970 - 10$. Since for prismatic powder $a = 1.5$, $\lambda = \frac{1}{3}$ and $\mu = o$, we find $f = 122910$ and $\tau = 1''.349$.

The German cocoa powder of Example 2, Chapter IV, would then answer for this gun provided the grains were made $1.349 \div 0.555 = 2.4$ times as large in all their dimensions.

Example 7.—Determine the values of f and τ with the data and conditions of Example 6, except that (*a*) $\tilde{\omega}$ is diminished to 500 lbs.; and (*b*) $\tilde{\omega} = 400$ lbs.

We find: (*a*) $\Delta = 1.019$, $\log z_0 = 0.65759$, $x_1 = 6.69006$, $\log X_0 = 0.83100$, $\log X_1 = 0.52418$, $\log M_1 = 4.70233$ and $\log N_1 = 6.68938 - 10$, $f = 118650$ and $\tau = 1.203$.

(*b*) $\Delta = 0.81522$, $\log z_0 = 0.75112$, $x_1 = 5.3939$, $\log X_0 = 0.79685$, $\log X_1 = 0.46076$, $\log M_1 = 4.87640$, $\log N_1 = 6.83872 - 10$, $f = 125610$ and $\tau = 0.8533$.

Influence of the Characteristics of a Powder upon the Muzzle Velocity and Maximum Pressure.—The characterics of a powder are the quantities f, τ, a, λ and μ and were so called by *Sarrau* because they characterize the physical qualities of the powder. Of these factors f depends principally upon the composition of the powder; and in the same gun is practically constant for all powders. It increases with the calibre of the gun; and for this reason its value determined for one calibre cannot be depended upon for any other calibre. The factor τ, the time of combustion of a grain of powder in free air, depends upon the minimum dimension of the grain and upon its density. The factors a, λ and μ depend exclusively upon the form of the grain, and for the carefully prepared powders employed with our sea-coast guns their values can be determined with great precision. *Sarrau* considers that μ may be regarded as zero in all cases.

In discussing the influence of the characteristics of a powder upon the muzzle velocity and maximum pressure it will be convenient as well as sufficient in most cases, to employ *Sarrau's* binomial formula for muzzle velocity which is

$$V = A\left(\frac{fa}{\tau}\right)^{\frac{1}{2}}(\tilde{\omega}U)^{\frac{3}{8}}\left(\frac{\Delta}{w\,d}\right)^{\frac{1}{4}}\left\{1 - B\frac{\lambda}{\tau}\cdot\frac{(Uw)^{\frac{1}{2}}}{d}\right\} \quad . \quad (14)$$

and in connection with it his empirical monomial formula for the maximum pressure upon the breech, namely

$$P_m = K_0\frac{fa}{\tau}\,\Delta\,\frac{\tilde{\omega}^{\frac{3}{4}}\,w^{\frac{1}{4}}}{d^2} \quad . \quad . \quad . \quad (15)$$

both of which formulas as experience has shown, give excellent results. The coefficients A, B and K_0 in these formulas are constants and are independent of all the elements of fire; but, unlike the corresponding coefficients in Equations (37), (38) and (45), Chapter IV, cannot be computed directly but must be determined by experimental firing. As the factors f and τ also require to be determined in the same way there are five unknown quantities to be established. The method followed by *Sarrau* was to adopt a standard powder of well defined form of grain for which a and λ are known, and for this powder assume both f and τ to be unity, and let their real values whatever they may be, go into the coefficients A and B, to be determined by experimental firing. *Sarrau* adopted as the standard the Wetteren powder [13—16], having a flat grain with a square base, for which (see Example 5), $a = 2.572$ and $\lambda = 0.851$. With these values of the characteristics and the firing data with the 19-cm and 10-cm French guns (see Example 6, page 95), we readily find, employing the formulas on page 104,

$$A = 502.63 \qquad\qquad B = 0.0058891$$
$$\log A = 2.70125 \qquad\qquad \log B = 7.77005 - 10$$

These values of A and B are considered invariable for all guns and powders. The formula for muzzle velocity becomes then for all guns and powders:

$$V = [2.70125] \left(\frac{fa}{\tau}\right)^{\frac{1}{2}} (\tilde{\omega}\, U)^{\frac{3}{8}} \left(\frac{\Delta}{wd}\right)^{\frac{1}{4}} \left\{ 1 - [7.77005] \frac{\lambda}{\tau} \frac{(w\, U)^{\frac{1}{2}}}{d} \right\} \qquad (16)$$

One firing, employing a pressure gauge in the face of the breech block, determines K_0, at least approximately. *Sarrau* found for the standard powders $\log K_0 = 4.25092$. We therefore have universally

$$P_m = [4.25092] \frac{fa}{\tau} \Delta \frac{\tilde{\omega}^{\frac{3}{4}} w^{\frac{1}{4}}}{d^2} \quad . \quad . \quad . \quad (17)$$

It will be seen that the velocity formula consists of two terms the first of which represents the ideal case in which the form of grain is such that the velocity of emission of gas in free air is uniform (see page 113). The second term which is subtractive, depends for its value, so far as the form of grain is concerned, upon the value of λ; and it is evident that the smaller this term, that is, the more uniform the emission of gas, the greater will be the muzzle velocity.

If we consider two powders, fired in the same gun or in two guns of the same calibre, whose characteristics are, respectively, f, τ, a, λ and f', τ', a', λ', it will be seen from Equation (17) that the maximum pressure will be the same for the two powders if

$$\frac{fa}{\tau} = \frac{f'a'}{\tau'}.$$

The first term of the velocity formula will also remain constant for these two powders, but not the second term unless, in addition,

$$\frac{\lambda}{\tau} = \frac{\lambda'}{\tau'}$$

If $\frac{\lambda'}{\tau'} < \frac{\lambda}{\tau}$ for example, then the quantity within the braces of Equation (14) will be greater for the powder whose characteristics are λ' and τ', than for the other; and consequently the muzzle velocity will be greater for the former with the same maximum pressure.

Influence of the Force of the Powder.—Suppose the form of the grain to be the same for two powders, so that $a' = a$, $\lambda' = \lambda$; and suppose the force of the powders to be proportional to the values of f. Then if the maximum pressures are to be the same we must have by Equation (17),

$$\frac{f}{\tau} = \frac{f'}{\tau'}: \text{ or } \frac{f\tau'}{f'\tau} = 1.$$

The subtractive terms in the two resulting velocity formulas will be equal if

$$\frac{1}{\tau} = \frac{1}{\tau'}; \text{ or } \tau = \tau'.$$

If $\tau' > \tau$, which necessitates that $f' > f$ in order to maintain the equality of pressures, the muzzle velocity will be increased. That is, if the time of burning of the grains be lengthened either by increasing their size or density, the muzzle velocity can be increased by employing a stronger powder without adding to the maximum pressure.

On this point *Sarrau* remarks: "It seems impossible to vary the 'force of the powder' of the nitrate of potassium class of powders by varying the proportions. When they are varied, the heat of combustion and the volume of gas vary considerably; but experiment shows that the product of these two quantities, which is an approximate measure of the force, varies very little. We obtain much more force by using the picrates; which, with equal weights, evolve more heat and more gas than ordinary powder. This essential condition was probably not realized in certain circumstances in which picric powder was observed to exert a dangerous energy. The foregoing considerations indicate that they may be less destructive than ordinary powder, since, as they can give a greater velocity with the same pressure, they can probably give a lower pressure with the same velocity."[1]

Influence of the Form and Dimensions of the Grain.—Suppose we use in the same gun, or in two guns of the same calibre, two powders of the same composition and therefore equally strong, but differing in the form and dimensions of their grains. Putting $f = f'$, the conditions of equality of pressure gives

$$\frac{a}{\tau} = \frac{a'}{\tau'}.$$

The muzzle velocities will be the same if

$$\frac{\lambda}{\tau} = \frac{\lambda'}{\tau'};$$

or, what is the same thing, taking into account the equality of pressures, if

$$\frac{\lambda}{a} = \frac{\lambda'}{a'}; \text{ or } \frac{\lambda' a}{\lambda a'} = 1.$$

If

$$\frac{\lambda'}{a'} < \frac{\lambda}{a}$$

or, what is the same thing, if

$$\frac{a\lambda'}{a'\lambda} < 1,$$

the muzzle velocity will be increased while the maximum pressure will remain unaltered.

[1] Proceedings U. S. Naval Institute. Vol. X. Page 117.

For the spherical and cubical grains we have (pages 76 and 77),

$$a = 3, \ \lambda = 1;$$

and for the pierced cylindrical grain (page 79),

$$a' = 1 + x, \ \lambda' = \frac{x}{1 + x}.$$

Therefore

$$\frac{a\lambda'}{a'\lambda} = \frac{3x}{(1 + x)^2}.$$

The second member of this last equation is less than unity for all values of x, and has a maximum value of $\frac{3}{4}$ when $x = 1$. Consequently by using a pierced cylindrical grain whose height (h) is equal to its thickness ($r - r'$), the subtractive term corresponding to spherical or cubical grains may be diminished by one-fourth of its value and the muzzle velocity thereby increased without adding to the maximum pressure. The gain in velocity increases as x diminishes. If, for example, we make $x = \frac{1}{2}$, that is, make the cylindrical perforated grain so that $h = 2(r - r')$, or, what is the same thing, double the diameter of the perforation, then the ratio above given becomes two-thirds: and thus the subtractive term in the velocity formula is diminished by one-third of itself.

For grains of the form of a parallelopipedon we have (page 77)

$$a' = 1 + x + y, \ \lambda = \frac{x + y + xy}{1 + x + y}.$$

If the base of the grain is square and the minimum dimension is perpendicular to its base then $x = y$, and

$$a' = 1 + 2x, \ \lambda' = \frac{2x + x^2}{1 + 2x},$$

so that we have for this grain as compared with a spherical grain

$$\frac{a\lambda'}{a'\lambda} = \frac{3x(2 + x)}{(1 + 2x)^2}.$$

This ratio diminishes by a variation of x from 1 to 0, but quite slowly. When the side of the base is twice the thickness of the grain, then $x = \frac{1}{2}$, and the above ratio becomes fifteen-sixteenths, so that by the use of such grains the subtractive term in the velocity formula is diminished by one-sixteenth of itself; but whether the muzzle velocity of the projectile would thereby be increased is by no means evident, since we do not know what effect the neglected term containing μ has upon the muzzle velocity. It will not do to push these empirical formulas too far.

CHAPTER VI.

NON-USEFUL ENERGY.

Efficiency of a Gun considered as a Machine.—It is evident from all that precedes that a gun is simply a thermodynamic machine intended to perform certain external work which manifests itself in the energy of translation and of rotation of the projectile. Its efficiency is measured, like that of any other machine, by the ratio of the useful to the total work performed; that is, by the ratio of the energy impressed upon the projectile as it emerges from the bore to the entire work done by the heat disengaged by the combustion of the charge. If in Equation (14), page 60, we make $\Delta_1 = o$, which implies an infinite expansion, we have

$$W = 576.369 \text{ foot-tons},$$

which is the maximum theoretical work stored up in one pound of gunpowder, according to *Noble* and *Abel's* hypothesis of a constant equilibrium between the temperatures of the gases and solid products of decomposition, in a gun of infinite length.

The total energy expressed in the projectile is

$$E = \frac{wV^2}{4480g}\left\{ 1 + \left(\frac{\pi k}{nr}\right)^2 \right\} \quad . \quad . \quad . \quad (1)$$

in which w is the weight of the projectile, V its muzzle velocity, n the length of twist of the gun measured in calibres, r the radius of the body of the projectile in feet, and k its radius of gyration.[1] Therefore the coefficient of useful effect of a gun fired with a charge of ϖ pounds of gunpowder is

$$\psi = \frac{wV^2}{4480 \times 576.369 g\varpi}\left\{ 1 + \left(\frac{\pi k}{nr}\right)^2 \right\} \quad . \quad . \quad . \quad (2)$$

By means of Equation (2) and the given conditions of loading we can easily compute the coefficient of useful effect for any gun; and it will be found in all cases that this coefficient is very small.

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[1] See *Ingalls'* Handbook of Problems in Direct Fire. Problem XX.

For example, for the 8-inch B. L. R. (steel), for which $w = 300$ lbs., $\tilde{\omega} = 125$ lbs., and $V = 1950$ f. s., we find (omitting the second term within the brace, which is very small in comparison with unity), $\psi = 0.1099$; while for the Springfield rifle $\psi = 0.1398$.

The following are the principal causes which reduce the useful work of a charge of powder: In the first place it hardly ever happens that all of the charge is burned in the gun. In fact, in the majority of cases, particularly with heavy guns, the charge is designedly greater than can be consumed before the projectile leaves the bore. Besides this the work of expansion of the gases actually formed in the gun is far from ceasing when the projectile leaves the bore. As is well known the pressure of the gas on the base of the projectile at the muzzle is often as great as 10,000 pounds per square inch; and all of this high-grade energy is wasted upon the atmosphere. This energy, which is necessarily sacrificed by the limitations imposed upon the length of the gun. is the principal cause of its small degree of efficiency regarded as a machine. On page 63 it is shown that if we regard the theoretical work done by the expansion of the gas *within the bore* as the total work, then the coefficient of useful effect for the 10-inch gun is as high as 0.8287; and still greater for guns of larger calibre.

In the second place, although but a small fraction of a second elapses between the ignition of the charge and the rush of the projectile from the muzzle, still on account of the enormous difference between the temperature of the gas and that of the walls of the gun a large amount of heat is lost by conduction to the latter; and particularly so in the case of small-arms which have relatively a very great cooling surface.

Finally a considerable amount of work actually performed by the gas is non-useful work—such as the deformation of the band, which results from forcing; friction along the grooves; the work expended in moving the gun and carriage and the unburned charge; the resistance of the air, etc. All this non-useful work was ignored in deducing the expressions for velocity and pressure in Chapter IV; but as was there stated, this only resulted in changing the "force of the powder" into a simple coefficient whose value for any kind of powder and type of gun must be determined by experiment. It is interesting and instructive however, to determine as far as possibie the amount of non-useful work due to each one of the above mentioned causes separately and then combine then into a whole.

The Loss of Heat through the Walls of the Gun.—As is well known a breech-loading small-arm may be fired so rapidly that the barrel cannot be touched by the naked hand without being burned; and even the temperature of a field gun becomes considerably increased after a few rounds.

Experiments made by the Count de Saint-Robert.[1]—The first attempt to determine by experiment the amount of heat actually communicated by the gases of a charge of powder to the gun in which it was fired, was made by *Saint-Robert* at Turin, in the year 1865. He employed rifled muskets loaded with $4\frac{1}{2}$ grams of musket powder. In the first of the series of experiments the muskets were loaded in the usual manner; in the second the bullet was placed near the muzzle; in the third series the muskets were loaded with powder only. The muskets were wrapped with several layers of white woolen material, and every precaution was taken to prevent the loss of heat by radiation and conduction. Having determined the temperature of the barrels, a series of shots was fired, as far as possible under identical conditions; then each of the barrels was filled with mercury of a determined temperature and the muzzle was closed by a gutta-percha cork. The maximum temperature reached by the mercury was carefully noted, and knowing the weight of mercury in the barrel, the specific heats, respectively, of the barrel and of mercury, the quantity of heat absorbed by the barrel could be determined by calculation. This quantity of heat per kilogram of powder, was shown to be as follows:

1	For the barrel loaded in the usual manner	220 calories.
2	For the barrel with the bullet near the muzzle	244 "
3	For the barrel loaded with a blank charge	222 "

Noble and *Abel* found (see page 52) that the heat of combustion Q is 705 calories; and it is therefore seen from these figures that in firing a musket ball in the ordinary manner a little less than one-

[1] Traité de Thermodynamique. Turin, 1865.

third of the full quantity of heat which is given off by the powder charge is absorbed by the walls of the barrel. In firing with blank charges there occurs a slightly greater heating of the barrel than in firing with the ordinary charge; and there is a very perceptibly greater heating when the bullet is placed near the muzzle. The cause of the greater heating of the barrel in firing with a blank charge than with the service charge is that in the first case no heat is expended in communicating motion to the projectile and therefore a greater quantity is available for heating the barrel. The difference between the quantities of heat transferred to the barrel in these two cases would be much more marked were it not for the fact that the gases remain in contact with the barrel much longer with a service charge than with a blank charge.

When the bullet is placed near the muzzle the powder gases impinge against the bullet, in which case their motion is transformed into heat, and consequently under such conditions there is observed a greater heating of the barrel.

In England *Noble* and *Abel* experimented with a 12-pounder B. L. gun. In the first experiment nine rounds were fired with a charge of $1\frac{3}{4}$ pounds and a projectile weighing nearly 12 pounds. Prior to the rounds being fired, arrangements had been made for placing the gun, whenever the series should be concluded, in a vessel containing a given weight of water; and before the experiment was commenced the gun and water were brought to the same temperature, and that temperature carefully determined. After the firing the gun was placed in the water, and the rise of temperature due to the nine rounds determined.

The second experiment was made with five rounds of $1\frac{1}{2}$ lbs. of the same powder with the same weight of projectile. The authors consider that the heat communicated to this gun as determined by their experiments is not far from 100 calories per unit weight of powder burned, or about $\frac{1}{7}$ of the entire amount of heat given off. They also consider that the loss of heat suffered by the gases in the 10-inch gun is about 25 calories, or $\frac{1}{28}$ of the entire amount of heat, or about $3\frac{1}{2}$ per cent.

Influence of the Rifling.[1]—As a result of the rifling of a gun a part of the work of expansion of the powder gases is absorbed in overcoming the resistance of the grooves to the motion of translation of the projectile, and also in communicating rotation to it.

The resistance of the grooves to the motion of translation arises: (1) From the fact that the guiding parts (base rings, etc.) are forced into the grooves, and (2) from the components of the pressure of the grooves and of friction acting in a direction contrary to the motion of translation of the projectile.

We will now consider the influence of the pressure of the grooves on the motion of translation of the projectile. Take a cross-section of the bore and suppose for simplicity that there are only two grooves, opposite to each other, and let the prolongation of the bearing surface pass through the axis of the bore, as is practically realized in the latest systems. Let M represent the point of application of the pressure of the bearing surface of the upper groove. We will take three coördinate axes: one axis x is coincident with the axis of the bore, while the others (y and z) are in the plane of the cross-section of the bore and perpendicular to each other. Let the axis y be directed along OM, and z in a direction perpendicular to OM. The pressure at M whatever its direction may be, can be replaced by the following three mutually perpendicular components: The first, perpendicular to the axis of the bore, and consequently, acting along the radius prolonged to M; the second, lying in the plane tangent to the surface of the bore (normal to the radius OM) and acting along the normal to the groove; the third lying in this same plane and tangent to the groove.

The first of these components will be destroyed by the similar component on the opposite groove and does not enter into the equation of motion of the projectile. The second component, which is the pressure against the bearing surface of the groove, we will designate by R. The third component being in the tangent to the groove, represents the friction on the guiding side of the groove, and consequently may be designated by fR in which

[1] Interior Ballistics by Colonel *Pashkievitsch*. Translated from the Russian by Captain *Tasker H. Bliss*. This work has, by permission, been freely used in preparing this chapter.

f is the coefficient of friction. The forces R and fR give the following components along the axes x and z:

	axis of x	axis of z
Force R	$-R\sin\theta$	$R\cos\theta$
Force fR	$-fR\cos\theta$	$-fR\sin\theta$.

The positive direction of the axis x is toward the muzzle; that of z in the direction of the force R; and θ is the angle which the groove makes with the axis x. The full component for the upper groove is

On axis x . . . . $-R(\sin\theta + f\cos\theta)$
On axis z . . . . $R(\cos\theta - f\sin\theta)$.

For the lower groove the component along the axis x has the same value and sign as the upper one; while the component along the axis z has the same value but the opposite sign.

Besides these forces the projectile is also subjected to the pressure of the powder gases on its base, acting along the axis of x in the positive direction, which force call P. If we replace the grooves by the forces enumerated above we may consider the projectile a free solid body and apply to it *Euler's* equations. These equations are six in number; but as is readily seen they reduce to two in the case under consideration, namely: an equation of motion of translation along the axis of x; and of rotation about the same axis, or what is the same thing, the axis of the projectile. The first equation is

$$M\frac{d^2x}{dt^2} = P - 2R(\sin\theta + f\cos) \quad . \quad . \quad (3)$$

and the second

$$Mk^2\frac{d\omega}{dt} = 2rR(\cos\theta - f\sin\theta) \quad . \quad . \quad . \quad (4)$$

in which r is the radius of the projectile, ω the angular velocity of the projectile about its axis, and k the radius of gyration of the projectile.

If we designate by h the length of twist of the groove at the section of the bore under consideration, in feet, and by v the corresponding velocity of the projectile, we shall have[1]

[1] *Ingalls'* Handbook. Page 190.

$$\omega = \frac{2\pi v}{h};$$

whence, differentiating with reference to t,

$$\frac{d\omega}{dt} = 2\pi \left[\frac{1}{h} \frac{dv}{dt} - \frac{v}{h^2} \frac{dh}{du} \cdot \frac{du}{dt} \right]$$

u being the path described by the projectile; consequently

$$\frac{d\omega}{dt} = \frac{2\pi}{h} \left\{ \frac{dv}{dt} - \frac{v^2}{h} \cdot \frac{dh}{du} \right\} \quad . \quad . \quad . \quad (5)$$

For a uniform twist we evidently have

$$\frac{dh}{du} = o;$$

and, therefore,

$$\frac{d\omega}{dt} = \frac{2\pi}{h} \frac{dv}{dt} \quad . \quad . \quad . \quad (6)$$

For a groove with an increasing twist the quantity dh/du depends for its value upon the changes in the twist, and is a determinate function of u. Since for such a groove h decreases as u increases, dh/du is negative and the second term of Equation (5) is positive. Therefore making

$$\frac{2\pi}{h^2} \frac{dh}{du} = \xi(u)$$

Equation (5) becomes

$$\frac{d\omega}{dt} = \frac{2\pi}{h} \frac{dv}{dt} + v^2 \xi(u) \quad . \quad . \quad . \quad (7)$$

Substituting the value of $d\omega/dt$ from Equation (7) in Equation (4) it becomes

$$Mk^2 \left(\frac{2\pi}{h} \frac{dv}{dt} + v^2 \xi(u) \right) = 2rR \left(\cos\theta - f \sin\theta \right)$$

whence

$$R = \frac{Mk^2}{2r} \frac{\frac{2\pi}{h} \cdot \frac{dv}{dt} + v^2 \xi(u)}{\cos\theta - f \sin\theta}$$

Substituting this expression for R in Equation (3) we have

$$M\frac{d^2x}{dt^2}=P-\frac{Mk^2}{r}\,\frac{\sin\theta+f\cos\theta}{\cos\theta-f\sin\theta}\left\{\frac{2\pi}{h}\,\frac{dv}{dt}+v^2\xi(u)\right\}$$

or, since

$$\frac{d^2x}{dt^2}=\frac{dv}{dt},$$

$$\frac{dv}{dt}\left\{M+\frac{Mk^2}{r}.\,\frac{\sin\theta+f\cos\theta}{\cos\theta-f\sin\theta}.\,\frac{2\pi}{h}\right\}$$
$$=P-\frac{Mk^2}{r}.\,\frac{\sin\theta+f\cos\theta}{\cos\theta-f\sin\theta}\,v^2\,\xi(u)\quad . \quad (8)$$

Since for a uniform twist $\xi(u)=o$, this last equation, in this case, reduces to

$$\frac{dv}{dt}\left\{M+\frac{Mk^2}{r}.\,\frac{\sin\theta+f\cos\theta}{\cos\theta-f\sin\theta}.\,\frac{2\pi}{h}\right\}=P\quad . \quad . \quad (9)$$

If the bore were smooth the equation of motion of the projectile would be

$$M\frac{dv}{dt}=P;$$

from which it appears that the influence of the grooves upon the motion of the projectile for a constant twist is equivalent to increasing the mass of the projectile by the quantity

$$\frac{Mk^2}{r}.\,\frac{\sin\theta+f\cos\theta}{\cos\theta-f\sin\theta}.\,\frac{2\pi}{h}$$

The value of this supplemental term can be easily calculated for any given projectile and gun when the coefficient of friction f is known. Captain *Noble*,[1] as the result of very careful experiments made by him with 12-cm quick-firing guns, makes $f=0.2$; and this value will be adopted in what follows. For cored shot we have very nearly[2]

$$k^2=0.5\,r^2.$$

Further, if we suppose the twist to be one turn in 25 calibres, we have

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[1] "On the energy absorbed by friction in the bores of rifled guns". See Ordnance Construction Note. No. 60.

[2] *Ingalls'* Handbook. Page 189.

$$\tan\theta = \frac{2\pi r}{h} = \frac{\pi}{25};$$

whence $\theta = 7° 9' 45''$. Making these substitutions, the supplemental term reduces to 0.021 M. Then the retarding effect upon the motion of the projectile of a groove of constant twist of 25 calibres is equivalent to an increase of the mass of the projectile by its $\frac{2}{100}$ part, approximately.

To determine the influence, approximately, of such an increase in the mass of the projectile on the values, respectively, of the muzzle velocity, the maximum pressure on the base of the projectile and the maximum pressure on the breech-block, we will make use of Equations (53) and (61) of Chapter IV, and (15) of Chapter V. It will be observed that in the formula for the muzzle velocity the weight w of the projectile occurs both outside and inside the brackets. For simplicity we will reject the term containing w within the brackets, and since this term is negative its rejection slightly exaggerates the influence of the rifling on the loss of velocity of the projectile. Reduced to their simplest forms for our present purpose these formulas become, respectively,

$$V = \frac{A_1}{w^{\frac{1}{4}}}$$

$$p_m = A_2\, w^{\frac{1}{2}}$$

$$P_m = A_3 w^{\frac{1}{4}}$$

whence we easily find by taking logarithms and differentiating,

$$\Delta V = -\tfrac{1}{4} \times \tfrac{2}{100}\, V$$

$$\Delta p_m = \tfrac{1}{2} \times \tfrac{2}{100}\, p_m$$

$$\Delta P_m = \tfrac{1}{4} \times \tfrac{2}{100}\, P_m$$

As an illustration, suppose that for a smooth-bore gun we had

$V = 1500$ f. s., $p_m = 2000$ atmos., $P_m = 2200$ atmos.,

then if we rifle the gun with a constant twist of 25 calibres length we shall produce:

A diminution in V of $7\frac{1}{2}$ f. s.

An increase in p_m of 20 atmos,

An increase in P_m of 11 atmos.

This illustration shows that the friction of the rifling exercises but a small influence upon the initial velocity of the projectile and of the pressure of the gases within the bore.

Energy Expended in giving Rotation to the Projectile.—If E_ω is the energy of rotation and E_0 the energy of translation of a projectile at the muzzle of the gun then it has been shown in another part of the course[1] that

$$E_\omega = \mu \left(\frac{\pi}{n} \right)^2 E_0,$$

in which

$$\mu = \frac{k^2}{r^2}$$

and n is the length of twist in calibres. Making $\mu = \frac{1}{2}$ and $n = 25$, we have

$$E_\omega = 0.0079\, E_0.$$

From this it is evident that in communicating motion of rotation to the projectile there is used only a very insignificant part of the work of the gases which gives motion of translation to the projectile, and this part of the work of the powder gases may be safely neglected.

Energy Expended in giving Motion to Gun and Carriage.—Equation (23), page 40 gives the velocity of recoil of the gun and carriage. The energy of recoil is, therefore,

$$\frac{w_1 U^2}{2g} = \frac{v^2(w + \frac{1}{2}\tilde{\omega})^2}{2gw_1} = E_1$$

while the projectile's energy of translation is

$$\frac{wv^2}{2g} = E_0$$

We therefore have

$$\frac{E_1}{E_0} = \frac{w}{w_1} \left(1 + \frac{\tilde{\omega}}{2w} \right)^2$$

Usually

$$w_1 = 100\, w;\ \tilde{\omega} = \tfrac{1}{3} w.$$

Consequently

$$E_1 = 0.013\, E_0$$

[1] *Ingalls'* Handbook. Page 201.

Whence we see that in communicating motion to the gun and carriage there is employed but a very small part of the energy communicated by the powder gases to the projectile.

Energy Expended in giving Motion to the Products of Combustion.—In this investigation it will be assumed that the gun is motionless; that is, that the velocity of recoil may be neglected in comparison either with the velocity of the projectile or with the mean velocity of the products of combustion. It will also be assumed that the whole charge is burned before the projectile has moved from its seat, and that the particles of the products of combustion have only a motion of translation parallel to the axis of the bore. As a consequence of the motion of translation of the projectile it is evident that the density of the powder gas in those strata which lie near the projectile is less than in the strata which lie nearer the bottom of the bore. In this respect the column of gas in the act of expanding acts like a compressed spiral spring fixed at one end and doing work by moving a weight at the free end. The density of any elementary stratum of gas, normal to the axis of the bore, is some decreasing function of its distance from the breech-block, where the density is greatest. But in view of the fact that the law of density is unknown it will be further assumed that at any given instant the density of the gas is uniform over the whole extent of space from the breech-block to the projectile; and consequently that its variations are dependent only on the time.

As a consequence of these assumptions it will now be shown that the velocities of the elementary strata of gas normal to the axis of the bore, at any given instant of time are proportional to their distances from the bottom of the bore. Designate by δ the density of the products of combustion at a certain time t when the projectile is at the distance u from the bottom of the bore and when it has, as has also the stratum of gas which lies adjacent to it, a velocity v'; let u_0 be the initial distance of the projectile (when $t = o$), and δ_0 the corresponding density of the products of combustion.

Imagine within the column of gas in the bore of the gun at the time t a certain stratum normal to the axis and at a distance ξ from the bottom of the bore, and having a thickness $d\xi$; let the initial distance of this stratum be ξ_0. Since by the hypotheses that have been made, the mass of gas which was at first included between the bottom of the bore and the cross-section at a distance ξ_0, is the same as that which after the time t had expanded to a distance ξ, we shall have, designating by S the area of a cross-section of the bore

$$S \xi_0 \delta_0 = S \xi \delta$$

or

$$\xi_0 \delta_0 = \xi \delta$$

In a similar manner we have

$$u_0 \delta_0 = u \delta.$$

From these two expressions we get, eliminating δ_0 and δ,

$$\xi_0 u = u_0 \xi \quad . \quad . \quad . \quad (10)$$

Designating the velocity of the stratum $d\xi$ by v'', it is evident that

$$v'' = \frac{d\xi}{dt};$$

and in a similar manner

$$v' = \frac{du}{dt}.$$

The differential of Equation (10) with reference to t is

$$\xi_0 \frac{du}{dt} = u_0 \frac{d\xi}{dt};$$

and therefore

$$\xi_0 v' = u_0 v'';$$

or, by Equation (10),

$$\frac{v''}{v'} = \frac{\xi}{u} \quad . \quad . \quad . \quad (11)$$

and therefore under the assumptions that have been made the velocities of the elementary strata are directly proportional to their distances from the bottom of the bore.

The volume of the elementary stratum $d\xi$ is

$$S\, d\xi$$

its mass

$$S \delta d \xi,$$

and its energy

$$\tfrac{1}{2} S \delta v''^2 d\xi = \tfrac{1}{2} S \delta v'^2 \frac{\xi^2}{u^2} d\xi, \text{ by Equation (11).}$$

The energy E_2 of the whole charge is found by integrating this expression between the limits $\xi = o$ and $\xi = u$. In this manner we find

$$E_2 = \tfrac{1}{6} S \delta u v'^2;$$

and since $S \delta u$ is the mass of the charge, we have at the muzzle

$$E_2 = \frac{\tilde{\omega}}{6g} V^2.$$

The energy of translation of the projectile as it emerges from the gun is

$$E_0 = \frac{w V^2}{2g};$$

and, therefore,

$$\frac{E_2}{E_0} = \tfrac{1}{3} \frac{\tilde{\omega}}{w}$$

Usually $\tilde{\omega}/w$ does not exceed $\frac{1}{3}$; and consequently,

$$E_2 = \tfrac{1}{9} E_0;$$

that is, in communicating motion to the products of combustion there is employed approximately not more than 10 per cent. of the energy communicated to the projectile.

Summary of Non-useful Work performed by the Powder Gas. From the foregoing discussions it will be seen that of the various non-useful energies considered the two most important are, the cooling of the gas by conduction to the walls of the gun, and the communication of motion of translation to the products of combustion. They all amount to about 18 per cent. of the energy impressed upon the projectile of the 10-inch English gun, which agrees very fairly with the "factor of effect" for this gun deduced by *Noble* and *Abel's* method. See page 63. For smaller guns the wasted energies would be more than this, principally on account of the greater quantity of heat lost to the walls of the gun; and for larger guns the loss of energy would be less than 18 per cent. for the opposite reason.

Force of the Powder.—If the "force of the powder" could be determined by direct experiment, then comparing it with the value of the factor f determined by experimental firing, as already explained, would give at once the amount of work of the powder gas not expended upon the projectile. With regard to the force of the powder M. *Sarrau* remarks. "The force of the powder is the element whose determination offers the greatest difficulties; there is no experiment which gives its value with certainty. In the want of more precise data, we present the theoretical values which we derived with M. *Roux* from the experimental determinations made at the Dépôt Centrale des Poudres et Salpêtres. These values were deduced by the equation

$$f = \frac{p_0 v_0 T_1}{273},$$

from the temperatures of combustion and the volume of the permanent gas.

KIND OF POWDER.	PROPORTIONS.			Temp. of comb.	Vol of gas.	Force of the powder.
	Nitre	Sulphur	Charcoal	T_1	v_0	f
Fine sport'g powder	78	10	12	4654	234	412000
Cannon powder	75	12.5	12.5	4360	261	431000
Small-arm powder	74	10.5	15.5	4231	280	448000
Commercial powder	72	13	15	4052	281	430000
Blasting powder	62	20	18	3372	307	392000

The units taken are the kilogram and decimetre. Consequently, according to the definition of the force of a powder, the figures in the last column of the table express in kilograms the pressure per square decimetre of the permanent gases of a kilogram of powder, occupying, at the temperature of combustion, the volume of one litre.

The figures of this table confirm what has already been said concerning the equality of the force of powders which have been differently made."[1]

In applying the above formula for f, p_0 is the mean atmospheric pressure, and is equal to 103.33 kilograms per square

[1] Proceedings of the U. S. Naval Institute. Vol. X. Page 119.

decimetre; v_0 is the volume occupied by the gases of one kilograme of powder at 0° C., and T_1 the absolute temperature of combustion. Therefore for cannon powder we have

$$f = \frac{103.33 \times 261 \times 4360}{273} = 431000 \text{ kgs.}$$

as in the table. To reduce this to our units (foot and pound) we have only to multiply by the number of feet in one decimetre. Thus

$$f = 0.32809 \times 431000 = 141400 \text{ lbs.}$$

If this be taken as the true value of f, then about 25 per cent. of the force of the powder was consumed in doing work other than that of imparting velocity to the 300-pound projectile fired from the 10-inch English gun. See page 116.

CHAPTER VII.

PRESSURE ON THE LANDS FOR DIFFERENT SYSTEMS OF RIFLING.

Advantages of Rifling.—The superiority of an oblong over a spherical projectile is two-fold. In the first place the oblong projectile has an ogival or pointed head which meets with less resistance from the air; and in the second place owing to the length of the projectile it has a greater mass than a spherical projectile of the same calibre, with which to overcome the resistance of the air. For example it may be shown that a solid spherical cast-iron shot having the same power to overcome the resistance of the air as an 8-inch oblong projectile weighing 180 pounds, both moving with the same velocity, must have a diameter of 29.65 inches and weigh 3567 pounds. The most obvious advantages flowing from the greater ballistic efficiency of oblong over spherical projectiles are: 1st, a higher striking velocity which in conjunction with their pointed heads and great sectional density gives them a vast superiority in penetrating power; and, 2d, a much flatter trajectory which increases the probability of their hitting the target. Experimental firing has demonstrated that the mean error of a rifled gun at ordinary ranges is about one-third that of a smooth-bore.

These are very substantial advantages; but to secure them it is essential that the oblong projectile should keep point foremost in its flight through the air; for otherwise it would have neither range, accuracy nor penetration. The only way to secure steadiness of flight and keep the axis of an oblong projectile constantly in the tangent of the trajectory it is describing is to give it a high rotary velocity about this axis, which is accomplished by rifling, as it is called; that is, by cutting spiral grooves in the surface of the bore into which the copper band near the base of the projectile is forced by the expansion of the powder gases as soon as motion of translation begins.

The Developed Groove. Uniform Twist.—When the twist of a rifled gun is such that the grooves make a constant angle at all points with the axis of the bore the twist is said to be uniform. The developed groove in this case is a right line AC making with the longitudinal element of the surface of the bore AB the constant angle BAC, which call β. The tangent of this angle is the ratio of BC to AB. If we suppose AB to be the longitudinal element passing through the beginning of the groove A near the bottom of the bore, and that the projectile travels n calibres while making one revolution about its axis, and if we make $AB = nd$, then $BC = \pi d$ (the circumference of the projectile); and therefore,

$$\tan \beta = \frac{BC}{AB} = \frac{\pi d}{nd} = \frac{\pi}{n} \quad . \quad . \quad . \quad (1)$$

that is, in a uniform twist, the angle made by a groove at any point with the axis of the bore, is

$$\beta = \tan^{-1}\left(\frac{\pi}{n}\right)$$

The twist of rifling in this case is said to be one turn in n calibres. With a uniform twist the maximum pressure on the lands in giving rotation to the projectile (as will presently be shown) occurs at the point of maximum acceleration or maximum pressure of the powder gas on the base of the projectile, which, as we know, is near the bottom of the bore; and from there the pressure continually and rapidly decreases to the muzzle where it is not more than one-fourth of its maximum value. In order to make the pressure upon the lands more uniform recourse is had to an increasing twist,—that is, the angle which the groove makes with the axis of the bore instead of being constant as in a uniform twist is a variable angle. At the beginning of the rifling at the breech end of the gun this angle is either zero, or very small, and increases in value until at or near the muzzle it becomes equal to β, its value for a uniform twist. It will thus be seen that at the point of maximum acceleration the twist is quite small and goes on increasing as the acceleration diminishes up to the muzzle. It is clear that the developed groove in this case is a curve,—generally one of the family of curves called parabolas,—though this class of curves is not necessarily the best for this purpose.

Expression for Pressure on the Lands.—Before attempting to decide upon the best system of rifling it will be necessary to deduce an expression for the pressure on the lands. For this purpose we will make use of Equations (3) and (4) of Chapter VI. The expression for a variable angular velocity is

$$\omega = \frac{d\varphi}{dt}$$

where φ is the angle turned through by the projectile; and therefore

$$\frac{d\omega}{dt} = \frac{d^2\varphi}{dt^2}.$$

If x and y are the rectangular coordinates of the developed groove with the origin at the beginning of the groove and the axis of x parallel to the axis of the bore, x will be the distance traveled by the shot and y will be equal to $r\varphi$. Therefore

$$\frac{d\omega}{dt} = \frac{1}{r}\frac{d^2y}{dt^2}.$$

Equations (3) and (4), Chapter VI, become, therefore,

$$M\frac{d^2x}{dt^2} = P - 2R(\sin\theta + f\cos\theta) \quad . \quad . \quad . \quad (2)$$

and

$$M\frac{k^2}{r^2}\frac{d^2y}{dt^2} = 2R(\cos\theta - f\sin\theta);$$

or, making for simplicity,

$$\frac{k^2}{r^2} = \mu,$$

$$M\mu\frac{d^2y}{dt^2} = 2R(\cos\theta - f\sin\theta) \quad . \quad . \quad . \quad (3)$$

Before these equations can be used for determining R we must eliminate dt; and this we can do by means of the equation of the developed groove. Let

$$y = f(x)$$

be this equation. Therefore, employing the usual notation,

$$\frac{dy}{dx} = f'(x) = \tan\theta \quad . \quad . \quad . \quad . \quad (4)$$

and

$$\frac{d^2y}{dx^2} = f''(x).$$

Also since

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = f'(x)\frac{dx}{dt},$$

we have

$$\frac{d^2y}{dt^2} = f''(x)\left(\frac{dx}{dt}\right)^2 + f'(x)\frac{d^2x}{dt^2}$$

$$= v^2 f''(x) + \tan\theta\,\frac{d^2x}{dt^2}$$

Substituting this value of $\frac{d^2y}{dt^2}$, and the value of $\frac{d^2x}{dt^2}$ from Equation (2), in Equation (3) and solving for R we have by making a few slight reductions,

$$2R = \frac{\mu \sec\theta \left\{ P\tan\theta + Mv^2 f''(x) \right\}}{1 - \tan\theta \left\{ f - \mu(f + \tan\theta) \right\}} \quad . \quad . \quad . \quad (5)$$

In using this equation $f''(x)$ and $\tan\theta$ are obtained from the equation of the developed groove, μ and M from the projectile, v and P from the velocity and pressure equations deduced as in Chapter IV, while f is determined by experiment. The resulting value of $2R$ will be the sum of the rotation pressures on all the lands.

For uniform twist $f''(x)$ is zero and θ becomes constant and equal to β. Its value is given by Equation (1). Making these substitutions the expression for $2R$ becomes for uniform twist,

$$2R = \frac{\mu \tan\beta \sec\beta}{1 - f\tan\beta + \mu\tan\beta(f + \tan\beta)} \cdot P \quad . \quad . \quad (6)$$

In the second member of Equation (6) the factor P, that is, the pressure on the base of the projectile, is the only variable; and therefore R is directly proportional to this pressure, and is a maximum when P is a maximum. In Equation (5) there are four variables in the second member, namely, P, v, θ and $f''(x)$;

and it is not obvious on simple inspection where the point of maximum rotation pressure is located. It will be shown however by examples that for increasing twist this point is much nearer the muzzle than when the twist is uniform.

Application to our Sea-coast Guns.—In all our sea-coast guns the twist begins with one turn in 50 calibres and increases to one turn in 25 calibres at 20 inches from the muzzle where it becomes uniform. The developed curve is a semi-cubical parabola whose general equation is

$$y + b = p(x+a)^{\frac{3}{2}} \qquad . \qquad . \qquad . \qquad (7)$$

The axis of x is parallel to the axis of the bore and the origin is at the beginning of the groove. The coordinates of the vertex of the parabola are $-a$ and $-b$. Differentiating Equation (7) we have

$$\frac{dy}{dx} = \tfrac{3}{2}p(x+a)^{\frac{1}{2}} = \tan\theta \qquad . \qquad . \qquad . \qquad (8)$$

When $x = o$, that is, at the origin, suppose the rifling starts with a twist of one turn in m calibres; and that near the muzzle the twist becomes uniform with one turn in n calibres ($m > n$). Therefore at the origin we have from Equations (1) and (8)

$$\frac{3p\sqrt{a}}{2} = \frac{\pi}{m};$$

and near the muzzle where $x = u_1$ (say),

$$\frac{3p(u_1+a)^{\frac{1}{2}}}{2} = \frac{\pi}{n}$$

From these two equations we easily find

$$a = \frac{u_1}{\left(\frac{m}{n}\right)^2 - 1} \qquad . \qquad . \qquad . \qquad (9)$$

and

$$p = \frac{2\pi}{3m\sqrt{a}} = \frac{2\pi}{3n\sqrt{u_1+a}} \qquad . \qquad . \qquad . \qquad (10)$$

As the curve passes through the origin we have $y = o$ when $x = o$; which gives

$$b = p\, a^{\frac{3}{2}} = \frac{2\pi a}{3m} \quad . \quad . \quad . \quad (11)$$

We also have by differentiating Equation (8)

$$f''(x) = \frac{3p}{4(x+a)^{\frac{1}{2}}} \quad . \quad : \quad . \quad (12)$$

Application to the 10-inch B. L. R.—The 10-inch B. L. steel rifle, model of 1888, has 60 grooves which beginning at 20.1 inches from the the bottom of the bore with a twist of 1 in 50, increase to 1 in 25 at 20 inches from the muzzle, and from thence continue uniform. We therefore have $m = 50$, $n = 25$; and, since the length of the bore is 22.925 feet, we also have $u_1 = 19.583$ feet. We next find the values of the constants a, b and p of the developed grooves, by means of Equations (9), (11) and (10) as follows:

$$a = \frac{19.583}{3} = 6.528 \text{ feet,}$$

$$b = \frac{6.528\pi}{75} = 0.27344 \text{ feet,}$$

$$p = \frac{2\pi}{3 \times 50 \sqrt{6.528}} \quad \therefore \ \log p = 8.21471 - 10.$$

The equation of the developed groove (changing x to u to indicate travel of projectile) is, therefore,

$$y + 0.27344 = [8.21471 - 10]\left(u + 6.528\right)^{\frac{3}{2}} \quad . \quad . \quad (13)$$

in which y is given in feet. We also have from Equation (8)

$$\tan \theta = [8.39080 - 10]\left(u + 6.528\right)^{\frac{1}{2}};$$

Making $u = o$ in this last equation we find

$$\theta = 3^\circ\ 35'\ 42''$$

which is the inclination with which the groove starts. At 20 inches from the muzzle where the twist becomes uniform (and which is therefore a point of discontinuity on the developed groove) we have $u = 19.583$; and at this point

$$\theta = 7^\circ\ 9'\ 45'',$$

and retains this value to the muzzle.

From Equation (12) we find

$$f''(x) = \frac{3p}{4(u+a)^{\frac{1}{2}}} = \frac{[8.08977-10]}{(u+6.528)^{\frac{1}{2}}}$$

This function decreases from the origin to the point of discontinuity; and beyond this point we have

$$f''(x) = 0.$$

If we put

$$K = \frac{\mu \sec \theta}{1 - \tan \theta \left\{ f - \mu(f + \tan \theta) \right\}},$$

Equation (5) becomes

$$2R = K \left\{ P \tan \theta + M v^2 f''(x) \right\} \quad . \quad . \quad . \quad (14)$$

For given values of μ and f, the factor K increases slowly as θ increases. If we make, as in Chapter VI, $\mu = 0.5$ and $f = 0.2$, we have for the 10-inch B. L. R. at the beginning of motion, $K = 0.5032$; and at 20 inches from the muzzle where the twist becomes uniform, $K = 0.5064$. We might, therefore, take for K the arithmetical mean of these two values and write Equation (14)

$$2R = 0.5048 \left\{ P \tan \theta + M v^2 f''(x) \right\}$$

which is correct enough for most practical purposes for all our sea-coast guns.

Another Form of Increasing Twist.—If we make a and b zero in Equation (7), it becomes

$$y = p x^{\frac{3}{2}} \quad . \quad . \quad . \quad (15)$$

which is the equation of the developed groove when the vertex of the semi-cubical parabola is at the origin. In this case the twist increases from zero at the origin to one turn in n calibres at or near the muzzle. If we suppose the 10-inch B. L. R. to be rifled with this kind of groove, we shall find by employing the methods already given

$$p = \frac{2\pi}{75\sqrt{19.583}} \qquad \therefore \ \log p = [8.27718-10]$$

$$\tan \theta = [8.45327-10]\sqrt{u}$$

$$f''(x) = \frac{[8.15224-10]}{\sqrt{u}}$$

In these formulas, as before, the inclination of the grooves is 7° 9′ 45″ at 20 inches from the muzzle, where the twist becomes uniform; and between this point and the muzzle $f''(x) = o$.

Uniform Twist.—If we suppose the 10-inch B. L. R. to be rifled throughout with a uniform twist of one turn in 25 calibres we have $\beta = 7° 9' 45''$; and employing the values of μ and f already given, Equation (6) reduces to the simple form

$$2R = [8.80362-10]P \quad . \quad . \quad . \quad (16)$$

Pressure and Velocity in the Bore due to a Service Charge fired in the 10-inch B. L. R.—The next step is the deduction of formulas for the pressure on the base of the projectile at any travel in the bore, and its corresponding velocity. We can then form a table which will give a tolerably correct idea of the pressure upon the lands as the projectile moves toward the muzzle, for each of the three systems of rifling which have been considered. For the 10-inch B. L. R. we have the following data: $\tilde{\omega} = 250$ lbs., $w = 575$ lbs., $d = \frac{5}{6}$ ft., $C = 7064$ c. i., $U = 22.925$ ft., and $V = 1975$ f. s. We will assume that the density of the powder δ is 1.82, and that the grains are of the pierced prismatic form, so that $\alpha = \frac{3}{2}$, $\lambda = \frac{1}{3}$ and $\mu = o$. We will further assume that the maximum pressure in the powder-chamber is 37000 lbs. per square inch; and that reducing this 10 per cent. gives the maximum pressure on the base of the projectile. Therefore $p_m = 33300$ lbs. per square inch. We now have all the data necessary for computing N_1 and M_1 by means of Equations (12) and (13), page 118.

The principal results of the calculations are the following: $\Delta = 0.97964$, $\log z_0 = 0.53918$, $x_1 = 6.6242$; and for this value of x_1, $\log X_0 = 0.82938$, $\log X_1 = 0.52127$, $\log X_2 = 9.51986-10$, $\log X_3 = 1.01886$; and $m = 0.84102$. From these numbers we find $\log N_1 = 6.86521-10$ and $\log M_1 = 4.83668$; and then the expressions for the velocity of the projectile at any point within the bore and the corresponding pressure on its base, namely:

$$v^2 = [6.20356]X_1 \left\{ 1 - [8.59381 - 10]X_0 \right\} \quad . \quad . \quad (17)$$

$$p = [4.72060]X_2 \left\{ 1 - [8.59381 - 10]X_3 \right\} \quad . \quad . \quad (18)$$

The quantities X_0, X_1, X_2, X_3 it will be remembered, are functions of x (or u/z_0); and are to be taken from Table II for the different assumed values of the argument x.

It will be convenient to change Equation (18) so that it will give the entire pressure on the base of the projectile P; and to avoid large numbers, we will adopt the ton as the unit of weight. We therefore have

$$P = \frac{\pi r^2}{2240} p;$$

and Equation (18) becomes

$$P = [3.26544]X_2 \left\{ 1 - [8.59381 - 10]X_3 \right\} \quad . \quad . \quad (18')$$

Finally the mass of the projectile expressed in tons is

$$M = \frac{575}{2240g} \qquad \therefore \log M = 7.90210 - 10.$$

We have now all the formulas and data necessary for computing the pressures on the lands of the 10-inch B. L. R. fired with the service charge, for the three principal systems of rifling. These calculations are given in the table following:

Pressures on lands required to produce rotation of shot in the 10-inch B. L. R. for different systems of rifling. Charge 250 lbs. Projectile 575 lbs. Muzzle velocity 1975 f. s. Maximum pressure on base of projectile 33300 lbs. per square inch.

x	Travel of shot. Feet.	Velocity of shot. f. s.	Pressure on base of shot. Tons.	Pressure on Lands. Tons.		
				Uniform twist.	Increasing twist Eq. (15)	Increasing twist. Eq. (13).
0.0	0.	0.0	0.0	0.0	0.0	0.0
.1	0.3461	227.7	841.1	53.5	12.0	27.0
.2	0.6922	366.7	1036.4	65.9	21.5	36.9
0.3	1.0382	478.1	1122.4	71.4	29.0	42.3
.4	1.3843	572.4	1158.3	73.7	35.3	46.1
.5	1.7304	654.7	1167.4	74.3	40.9	48.9
0.6	2 0765	727.8	1161.1	73.9	44.8	51.1
.7	2.4226	793.6	1145.6	72.9	48.5	52.9
.8	2.7686	853.4	1124.7	71.6	51.6	54.3
0.9	3.1147	908.2	1100.7	70.0	54.3	55.5
1.0	3.4608	958.7	1075.0	68.4	56.7	56.5
1.1	3.8069	1005.6	1048.5	66.7	58.7	57.3
1.2	4.1530	1049.3	1021.9	65.0	60.5	58.1
1.3	4.4991	1090.2	995.5	63.3	62.1	58.7
1.4	4.8452	1128.6	969.7	61.7	63.5	59.2
1.5	5.1912	1164.7	944.5	60.1	64.7	59.7
1.6	5.5372	1198.9	920.1	58.5	65.8	60.1
1.7	5.8833	1231.4	896.4	57.0	66.7	60.5
1.8	6.2294	1262.1	873.6	55.6	67.5	60.8
1.9	6.5755	1291.4	851.7	54.2	68.2	61.1
2.0	6.9216	1319.4	830.5	52.8	69.0	61.3
3.0	10.3824	1543.4	659.4	42.0	72.7	62.4
4.0	13.8432	1702.8	541.4	34.4	73.5	62.3
5.0	17.3040	1824.9	456.4	29.0	73.2	61.6
5.6586	19.5833	1891.6	412.4	26.2	72.6	61.0
6.0000	20.7648	1922.8	392.4	25.0	25.0	25.0
6.6242	22.9250	1975.0	359.9	22.9	22.9	22.9
	U	*V*				

The last three columns of this table show that the maximum pressure on the lands is greater for uniform twist than for either form of increasing twist; but the difference between these maxima is not very great. Moreover the maximum pressure for uniform twist occurs at the trunnions where its torsional effect upon the gun,—so far as deranging the aim is concerned—is a minimum; while the position of the maximum pressure upon the lands for either form of increasing twist is well down the chase. It is difficult to see any superiority of an increasing twist over a uniform twist, especially in view of the fact demonstrated by Captain *Noble's* experiments, that the energy expended in giving rotation to the projectile with rifling having an increasing twist is nearly twice as great as with a uniform twist.

Time of Burning of a Grain in Free Air. Least Dimension of Grain.—Before leaving this part of the subject we will attempt to determine the least dimension of the grain of gunpowder suitable for the 10-inch B. L. R. By Equation (7), page 115, together with the value of N_1 already deduced, we find the time of burning of a grain for this gun, in free air, to be 0.8 seconds. If we knew the thickness of grain burned in a second in the atmosphere, that is, under atmospheric pressure increased by the pressure of the gases as they are constantly being formed on the surfaces of the burning grains, we could determine the least dimension of the pierced prismatic grains of which we have assumed the charge to be made up. But this has never been determined satisfactorily by experiment. If e is the least dimension of the grain and v' the velocity of combustion in inches per second, we shall have for grains of the same density

$$\tau = \frac{e}{2v'}.$$

Judging from several computed values of τ the velocity of combustion of a grain of powder is in the neighborhood of one-half inch per second; and it therefore follows that the least dimension of a grain of gunpowder in inches, is equal to the time of burning of the grain in free air expressed in seconds. If this be admitted then the least dimension of a pierced prismatic grain of gunpowder suitable for the 10-inch B. L. R. is eight-tenths of an inch.

Force of the Powder.—The factor f computed by Equation (8), page 115, and the values of M_1 and N_1 already determined, is 107840. This differs but slightly from the value of f computed for pebble powder and the 10-inch English gun. We may therefore assume that the composition of a powder suitable for the 10-inch B. L. R. is that of the ordinary service gunpowder.

The forces which produce rotation are by Equation (3), $2R \cos \theta (1 - f \tan \theta)$, acting at the end of an arm r, and through distances equal to $r\varphi$ or y. We might therefore draw curves whose ordinates would represent the force $2R \cos \theta (1 - f \tan \theta)$ and the abscissas the corresponding values of y computed by means of Equations (13) and (15) for the increasing twists considered. For a uniform twist the force producing rotation is equal to $2R \cos \beta \ (1 - f \tan \beta)$; while the value of y is determined by the equation

$$y = u \tan \beta = \frac{\pi u}{n}$$

The areas under these curves represent the work of rotation; and since the same final angular velocity is given in each case they are equal.

Mean Value of the Pressure acting on the base-ring necessary to produce any Muzzle Energy of Rotation.—Let P_1 be a constant pressure which acting at the end of the arm r through the distance $r\varphi$ (or y) will produce a rotational energy

$$E_\omega = \frac{M}{2} (k\omega)^2$$

We therefore have

$$P_1 y = \frac{M}{2} (k\omega)^2$$

$$\therefore \ P_1 = \frac{M}{2y} (k\omega)^2$$

At the muzzle we have

$$\omega = \frac{\pi V}{nr}$$

and

$$y = u' \tan \beta = \frac{\pi u'}{n}$$

and therefore

$$P_1 = \mu \frac{M V^2}{2u'} \tan \beta$$

For the 10-inch B. L. R., $\mu = \frac{1}{2}$, $M = 0.0079818$, $V = 1975$, $u' = 19.583$ and $\beta = 7° \ 9' \ 45''$. We therefore have

$$P_1 = 49.83 \text{ tons.}$$

Mean Value of the Pressure acting on the Base of the Projectile necessary to produce any Muzzle Energy of Translation.—Let P_2 be a constant pressure which acting upon the base of the projectile during its passage through the bore will give to the projectile the muzzle energy

$$E_0 = \frac{MV^2}{2}.$$

We therefore have

$$P_2 U = \frac{MV^2}{2}$$

$$\therefore P_2 = \frac{MV^2}{2U}$$

For the 10-inch B. L. R., $U = 22.925$ ft. Whence $P_2 = 679.03$ tons $= 19367$ lbs. per square inch.

Conclusion. Application to Smokeless Powders.—In deducing the formulas for velocity and pressure for given charges of gunpowder from the differential Equation (4), page 71, it was assumed that z_0 was constant for the same charge and gun. In other words we availed ourselves of the fact, first established by *Noble* and *Abel*, that the solid or non-gaseous products developed by the explosion of a charge of gunpowder in the bore of a gun, occupy practically the same volume as the charge of powder itself; from which it follows that the volume occupied by the gas at any instant when the projectile has traveled a distance u is

$$\text{vol.} = \omega (z_0 + u)$$

in which ω is the area of a right cross-section of the bore and z_0 the reduced length of the initial air-space and, in accordance with what has been said, a constant. This principle, in connection with the assumption made by *Sarrau*, that the velocity of combustion of a grain of powder under a variable pressure is proportional to the square root of the pressure, led directly to formulas (35) and (36), page 83, which give, respectively, the velocity and pressure at any point within the bore of a gun, when the constants M_1 and N_1 have been determined by two measured velocities. These formulas give typical velocity and pressure curves, and in all cases give concordant results. For instructive examples see the tables on pages 90, 92. 93 and 148, which will well repay careful attention.

For smokeless powders, which are said to be entirely converted into gas by combustion, z_0 will not remain constant, but will increase from its initial value; and if the powder is all consumed in the gun, will finally become equal to the reduced length of the powder-chamber u_0. It is believed however that for these powders Equations (39) and (40), page 81, and especially Equations (12) and (13), page 118, will give good results. There is not however sufficient data available to test the correctness of this belief.

Table I.—Giving for value of v up to $v = 50$, the total work that dry gunpowder of the W. A. standard is capable of performing in the bore of a gun, in foot-tons per lb. of powder burned.[1]

Number of volumes of expansion.	Correspondi'g density of products of combustion.	Total work per lb. burned in foot-tons.	Difference.	Number of volumes of expansion.	Correspondi'g density of products of combustion.	Total work per lb. burned in foot-tons.	Difference.
1.00	1 000	. . .	. . .	1.32	.758	23.246	1.113
1.01	.990	.980	.980	1.34	.746	24.324	1.078
1.02	.980	1.936	.956	1.36	.735	25.371	1.047
1.03	.971	2.870	.934	1.38	.725	26.389	1.018
1.04	.962	3.782	.912	1.40	.714	27.380	.991
1.05	.952	4.674	.892	1.42	.704	28.348	.968
1.06	.943	5.547	.873	1.44	.694	29.291	.943
1.07	.935	6.399	.852	1.46	.685	30.211	.920
1.08	.926	7.234	.835	1.48	.676	31.109	.898
1.09	.917	8.051	.817	1.50	.667	31.986	.877
1.10	.909	8.852	.810	1.52	.658	32.843	.857
1.11	.901	9.637	.785	1.54	.649	33.681	.838
1.12	.893	10.406	.769	1.56	.641	34.500	.819
1.13	.885	11.160	.754	1.58	.633	35.301	.801
1.14	.877	11.899	.739	1.60	.625	36.086	.785
1.15	.870	12.625	.726	1.62	.617	36.855	.769
1.16	.862	13.338	.713	1.64	.610	37.608	.753
1.17	.855	14.038	.700	1.66	.602	38.346	.738
1.18	.847	14.725	.687	1.68	.595	39.069	.723
1.19	.840	15.400	.675	1.70	.588	39.778	.709
1.20	.833	16.063	.663	1.72	.581	40.474	.696
1.21	.826	16.716	.653	1.74	.575	41.156	.682
1.22	.820	17.359	.643	1.76	.568	41.827	.671
1.23	.813	17.992	.633	1.78	.562	42.486	.659
1.24	.806	18.614	.622	1.80	.555	43.133	.647
1.25	.800	19.226	.612	1.82	.549	43.769	.636
1.26	.794	19.828	.602	1.84	.543	44.394	.625
1.27	.787	20.420	.592	1.86	.537	45.009	.615
1.28	.781	21.001	.581	1.88	.532	45.614	.605
1.29	.775	21.572	.571	1.90	.526	46.209	.595
1.30	.769	22.133	.561	1.92	.521	46.795	.586

[1] From *Noble* and *Abel's* Researches on Fired Gunpowder.

Table I.—Continued.

Number of volumes of expansion.	Correspondi'g density of products of combustion.	Total work per lb. burned in foot-tons.	Difference.	Number of volumes of expansion.	Correspondi'g density of products of combustion.	Total work per lb. burned in foot-tons.	Difference.
1.94	.515	47.372	.577	3.55	.282	76.940	.625
1.96	.510	47.940	.568	3.60	.278	77.553	.613
1.98	.505	48.499	.559	3.65	.274	78.156	.603
2.00	.500	49.050	.551	3.70	.270	78.749	.593
2.05	.488	50.383	1.333	3.75	.266	79.332	.583
2.10	.476	51.673	1.290	3.80	.263	79.905	.573
2.15	.465	52.922	1.249	3.85	.260	80.469	.564
2.20	.454	54.132	1.210	3.90	.256	81.024	.555
2.25	.444	55.304	1.172	3.95	.253	81.570	.546
2.30	.435	56.439	1.135	4.00	.250	82.107	.537
2.35	.425	57.539	1.100	4.10	.244	83.157	1.050
2.40	.417	58.605	1.066	4.20	.238	84.176	1.019
2.45	.408	59.639	1.034	4.30	.232	85.166	.990
2.50	.400	60.642	1.003	4.40	.227	86.128	.962
2.55	.392	61.616	.974	4.50	.222	87.064	.936
2.60	.384	62.563	.947	4.60	.217	87.975	.911
2.65	.377	63.486	.923	4.70	.213	88.861	.886
2.70	.370	64.385	.899	4.80	.208	89.724	.863
2.75	.363	65.262	.877	4.90	.204	90.565	.841
2.80	.357	66.119	.857	5.00	.200	91.385	.820
2.85	.351	66.955	.836	5.10	.196	92.186	.801
2.90	.345	67.771	.816	5.20	.192	92.968	.782
2.95	.339	68.568	.797	5.30	.188	93.732	.764
3.00	.333	69.347	.779	5.40	.185	94.479	.747
3.05	.328	70.109	.762	5.50	.182	95.210	.731
3.10	.322	70.854	.745	5.60	.178	95.925	.715
3.15	.317	71.584	.731	5.70	.175	96.625	.700
3.20	.312	72.301	.716	5.80	.172	97.310	.685
3.25	.308	73.002	.701	5.90	.169	97.981	.671
3.30	.303	73.690	.688	6.00	.166	98.638	.657
3.35	.298	74.365	.675	6.10	.164	99.282	.644
3.40	.294	75.027	.662	6.20	.161	99.915	.633
3.45	.290	75.677	.650	6.30	.159	100.536	.621
3.50	.286	76.315	.638	6.40	.156	101.145	.609

Table I.—Continued.

Number of volumes of expansion.	Correspondi'g density of products of combustion.	Total work per lb. burned in foot-tons.	Difference.	Number of volumes of expansion.	Correspondi'g density of products of combustion.	Total work per lb. burned in foot-tons.	Difference.
6.50	.154	101.744	.599	9.90	.101	117.395	.366
6.60	.151	102.333	.589	10	.100	117.757	.362
6.70	.149	102.912	.579	11	.091	121.165	3.408
6.80	.147	103.480	.568	12	.083	124.239	3.074
6.90	.145	104.038	.558	13	.077	127.036	2.797
7.00	.143	104.586	.548	14	.071	129.602	2.566
7.10	.141	105.125	.539	15	.066	131.970	2.368
7.20	.139	105.655	.530	16	.062	134.168	2.198
7.30	.137	106.176	.521	17	.059	136.218	2.050
7.40	.135	106.688	.512	18	.055	138.138	1.920
7.50	.133	107.192	.504	19	.052	139.944	1.806
7.60	.131	107.688	.496	20	.050	141.647	1.703
7.70	.130	108.177	.489	21	.047	143.258	1.611
7.80	.128	108.659	.482	22	.045	144.788	1.530
7.90	.126	109.133	.474	23	.043	146.242	1.454
8.00	.125	109.600	.467	24	.042	147.629	1.387
8.10	.123	110.060	.460	25	.040	148.953	1.324
8.20	.122	110.514	.454	30	.033	154.800	5.847
8.30	.120	110.962	.448	35	.028	159.667	4.867
8.40	.119	111.404	.442	40	.025	163.828	4.161
8.50	.117	111.840	.436	45	.022	167.456	3.628
8.60	.116	112.270	.430	50	.020	170.671	3.215
8.70	.115	112.695	.425				
8.80	.114	113.114	.419				
8.90	.112	113.528	.414				
9.00	.111	113.937	.409				
9.10	.110	114.341	.404				
9.20	.109	114.739	.398				
9.30	.108	115.133	.394				
9.40	.106	115.521	.388				
9.50	.105	115.905	.384				
9.60	.104	116.284	.379				
9.70	.103	116.659	.375				
9.80	.102	117.029	.370				

TABLE II.

x.	log X_0.	log X_1.	log X_2.	log X_3.	log X_4.
0.001	9.03899	5.56162	8.73764	9.16405	8.26132
0.010	9.53911	7.05911	9.23296	9.66437	8.76000
0.1	0.03494	8.53009	9.68493	0.16295	9.24757
0.2	0.18111	8.95170	9.78653	0.31194	9.38529
0.3	0.26509	9.18802	.82962	.39851	.46147
0.4	0.32372	9.34942	.85051	.45956	.51285
0.5	0.36855	9.47036	9.86028	0.50663	9.55091
0.6	.40469	9.56610	.86371	.54488	.58070
0.7	.43489	9.64471	.86325	.57705	.60491
0.8	0.46075	9.71100	9.86027	0.60479	9.62512
0.9	.48334	9.76802	.85562	.62913	.64234
1	.50334	9.81784	.84984	.65081	.65725
1.1	0.52128	9.86193	9.84329	0.67034	9.67032
1.2	.53752	.90136	.83623	.68809	.68192
1.3	.55234	.93693	.82882	.70436	.69229
1.4	0.56597	9.96926	9.82119	0.71936	9.70165
1.5	.57856	.99884	.81343	.73328	.71014
1.6	.59026	0.02605	.80561	.74625	.71789
1.7	0.60119	0.05122	9.79777	0.75840	9.72501
1.8	.61143	.07459	.78996	.76981	.73158
1.9	.62106	.09638	.78219	.78057	.73766
2	0.63015	0.11678	9.77449	0.79075	9.74331
3	.70032	0.26858	.70304	.87009	.78412
4	.74836	0.36662	.64225	.92527	.80913
5	0.78469	0.43759	9.59029	0.96700	9.82645
6	.81379	.49253	.54521	1.00074	.83937
7	.83801	.53698	.50549	1.02890	.84948
8	0.85873	0.57411	9.47004	1.05304	9.85769
9	.87682	.60585	.43809	1.07413	.86452
10	.89284	.63349	.40901	1.09283	.87032
11	0.90723	0.65790	9.38233	1.10963	9.87534
12	.92027	.67972	.35770	1.12485	.87972
13	.93219	.69941	.33484	1.13877	.88361
14	0.94317	0.71734	9.31350	1.15159	9.88708
15	.95334	.73377	.29351	1.16346	.89021

TABLE II′

x	log X_0	log X_1	log X_2	log X_3	log X_4
0.1	9.99512	8.56806	9.72185	0.12329	9.28647
0.2	0.14107	8.98823	9.82102	.27218	9.42358
0.3	.22484	9.22322	9.86189	.35870	9.49919
0.4	.28328	9.38339	9.88073	.41968	9.55006
0.5	.32792	9.50319	9.88861	.46670	9.58764
0.6	.36389	9.59787	9.89028	.50488	9.61699
0.7	.39391	9.67549	9.88817	.53699	9.64079
0.8	.41962	9.74085	9.88364	.56467	9.66061
0.9	.44205	9.79699	9.87754	.58896	9.67747
1.0	.46190	9.84597	9.87038	.61058	9.69203
1.1	.47970	9.88928	9.86254	.63006	9.70479
1.2	.49580	9.92795	9.85424	.64776	9.71608
1.3	.51049	9.96281	9.84566	.66398	9.72616
1.4	.52398	9.99445	9.83689	.67892	9.73524
1.5	.53645	0.02338	9.82807	.69280	9.74347
1.6	.54803	0.04997	9.81923	.70572	9.75097
1.7	.55884	.07454	9.81036	.71781	9.75785
1.8	.56896	.09733	9.80164	.72919	9.76419
1.9	.57848	.11857	9.79297	.73990	9.77004
2 0	.58746	.13843	9.78439	.75004	9.77548
3.0	.65665	.28570	9.70563	.82894	9.81453
4.0	.70388	.38030	9.63932	.88354	9.83820
5.0	.73952	.44849	9.58296	.92510	9.85448
6.0	.76803	.50110	9.53422	.95850	9.86653
7.0	.79170	.54354	9.49138	.98633	9.87592
8.0	.81193	.57891	9.45324	1.01018	9.88349
9.0	.82955	.60906	9.41886	1.03096	9.88976
10.0	.84515	.63527	9.38762	1.04947	9.89506
11.0	.85914	.65840	9.35904	1.06591	9.89963
12.0	.87182	.67906	9.33264	1.08088	9.90362
13.0	.88340	.69767	9.30818	1.09456	9.90714
14.0	.89405	.71457	9.28535	1.10713	9.91026
15.0	.90392	.73008	9.26397	1.11878	9.91308