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Bohm-Hawerk Centenary, I  
Time and Interest: Circulating Versus Fixed Capital

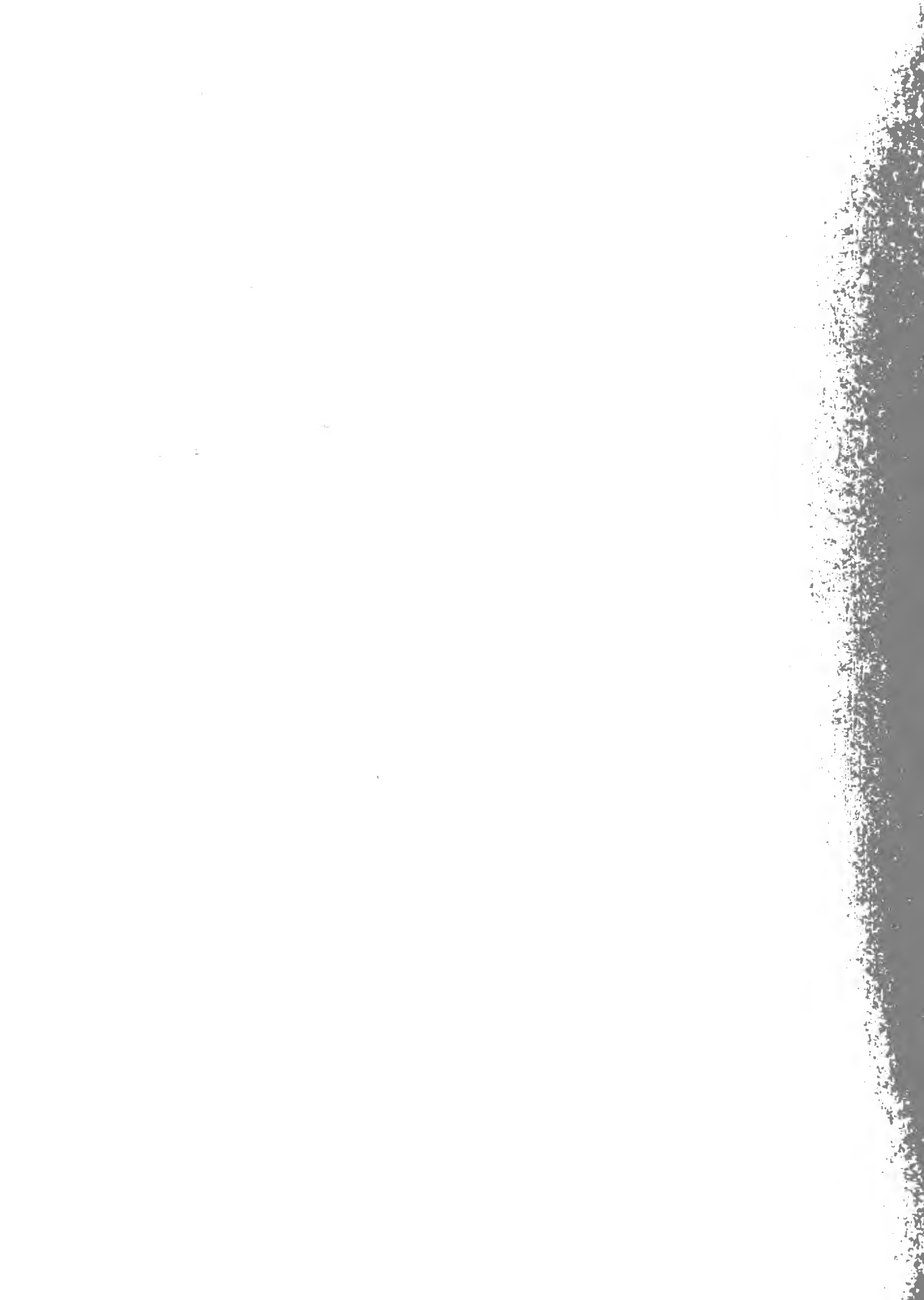
*Hans Brems*

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July 1987

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Böhm-Bawerk Centenary, I  
Time and Interest: Circulating Versus Fixed Capital

Hans Brems, Professor  
Department of Economics



## BÖHM-BAWERK CENTENARY, I

### TIME AND INTEREST: CIRCULATING VERSUS FIXED CAPITAL

Hans Brems

#### ABSTRACT

The Austrian tradition related the length of the time span of capitalist production to the rate of interest. In Böhm-Bawerk's case of circulating capital that time span was the period of production. In the Akerman-Wicksell case of fixed capital the time span was the useful life of durable producers' goods. The two cases were similar: a lower rate of interest would always lengthen the time span of capitalist production. In elementary mathematical form the paper develops the microeconomics and the macroeconomics of the two cases.

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## BÖHM-BAWERK CENTENARY, I

# TIME AND INTEREST: CIRCULATING VERSUS FIXED CAPITAL

Hans Brems

### I. CIRCULATING CAPITAL

#### 1. Circulating Capital

Interest is the price of time, and capitalist production takes time. What it is that takes time differs between circulating and fixed capital.

Since Smith, English classicists made a distinction between circulating and fixed capital. Ricardo [1821 (1951, I: 31)] saw circulating capital as "capital ... employed in the payment of wages, which are expended on food and clothing," and fixed capital as "buildings and machinery [which] are valuable and durable." Their distinction was important to them because they considered circulating capital to be complementary to labor but fixed capital to be a substitute for labor.

What takes time in the case of circulating capital is the maturing of output in slow organic growth in agriculture, cattle raising, forestry,

and winery or in time-consuming construction jobs. Böhm-Bawerk called the lapse of time from initial input to final output "the period of production," and his contribution was to consider the length of that period of production an economic variable: stretching the wage fund over more years would increase physical output per annum per man but also cost more interest. Exactly how far should it be stretched? To that problem Böhm-Bawerk (1889) and the nineteenth-century Wicksell [1893 (1954)] devoted their best efforts and said nothing on fixed capital.

We prefer to set out any author's model in the simplest form which will rigorously deliver his conclusions, i.e., in the case of Böhm-Bawerk that the lower rate of interest of a wealthier economy would lengthen the optimal period of production of circulating capital. As a result, the wealthier economy would enjoy a larger net national product. Our model will need only a single good, i.e., a maturing consumers' good, and only one kind of labor. The economy will be stationary: its available labor force hence its physical output will be stationary. But we shall simulate Böhm-Bawerk by using modern tools such as present-net-worth maximization, the real rate of interest, and compound interest with continuous compounding. We use the following notation.

## 2. Variables

$J \equiv$  present net worth of an endless succession of investments

$L \equiv$  labor employed

$P \equiv$  price of consumers' goods

$S \equiv$  aggregate physical capital stock of producers' goods

$X \equiv$  physical output maturing at the end of an  $y$ -year production run

$y \equiv$  period of production

## 3. Parameters

$\alpha \equiv$  elasticity of physical output per annum per man with respect to  
period of production

$g \equiv$  rate of inflation

$m \equiv$  multiplicative factor in production function (1)

$r \equiv$  nominal rate of interest

$\rho \equiv$  real rate of interest  $\equiv$  nominal rate of interest minus rate of  
inflation

$w \equiv$  money wage rate

4. Microeconomic Equilibrium: An Endless Succession of Production Runs

Let an entrepreneur plan an endless succession of production runs: every  $y$ th year the physical output  $X$  of a production run matures, and a new production run is started.

Define present net worth of such an endless succession as present worth of all its future revenue minus present worth of all its future labor.

5. Present Worth of All Future Revenue

A production run employing  $L$  men and producing every  $y$ th year the physical output  $X$  is producing an average physical output  $X/(Ly)$  per annum per man. Let  $X/(Ly)$  be a rising function of the period of production  $y$  but rising in less than proportion, as Böhm-Bawerk suggested. Let  $\alpha$  be the elasticity of  $X/(Ly)$  with respect to  $y$ , and let us adopt, as Böhm-Bawerk never did, a constant-elasticity production function

$$X/(Ly) = my^\alpha \tag{1}$$

where  $0 < \alpha < 1$ .

Inflation was never mentioned by Böhm-Bawerk but we easily accommodate it to show that the interest rate that mattered was the real one. So let the price of output be inflating at the rate  $g$  per annum:

$$P(t) = e^{g(t - v)} P(v) \quad (2)$$

Occurring every  $y$ th year, revenue is  $P(t)X$  whose present worth is  $e^{-r(t - v)} P(t)X$  or, with (2) inserted and the real rate of interest defined as  $\rho \equiv r - g$ ,

$$e^{-\rho(t - v)} P(v)X \quad (3)$$

Write (3) successively for  $t = v + y, v + 2y, \dots$ . Summing over production runs, find the present worth of an endless succession of future revenue

$$(1 + e^{-\rho y} + e^{-2\rho y} + \dots)e^{-\rho y} P(v)X \quad (4)$$

The parenthesis is an endless geometrical progression with first term 1, common ratio  $e^{-\rho y}$ , and sum  $1/(1 - e^{-\rho y})$ . As a result, using (1), the present worth of the endless succession of future revenue is

$$\frac{e^{-\rho y}}{1 - e^{-\rho y}} P(v)X = \frac{LmP(v)y^{\alpha + 1}}{e^{\rho y} - 1} \quad (5)$$

6. Present Worth of All Future Labor

For circulating capital we shall make no use of Wicksell's [1901 (1934: 172-181)] point-input, point-output scheme but shall adopt a flow-input, point-output scheme and assume our entrepreneur to employ  $L$  men uniformly. Such uniformity, always assumed by Böhm-Bawerk, will rule out reswitching [Samuelson (1966)]. Let the men be employed at a money wage rate inflating at the rate  $g$  per annum:

$$w(t) = e^{g(t - v)} w(v) \quad (6)$$

Occurring continuously, then, labor cost during a small fraction  $dt$  of a year located at time  $t$  is  $Lw(t)dt$ . As seen from time  $v$  the present worth of that is  $e^{-r(t - v)} Lw(t)dt$  or, with (6) inserted,  $e^{-\rho(t - v)} Lw(v)dt$ . The present worth of an endless succession of such future labor cost is

$$\int_v^{\infty} e^{-\rho(t - v)} Lw(v)dt = \frac{Lw(v)}{\rho} \quad (7)$$

$$\alpha = 0.5$$

$$\rho = 0.04$$

Terms of Transcendental Equation

$$e^{\rho y}(\alpha + 1 - \rho y) - (\alpha + 1) = 0$$

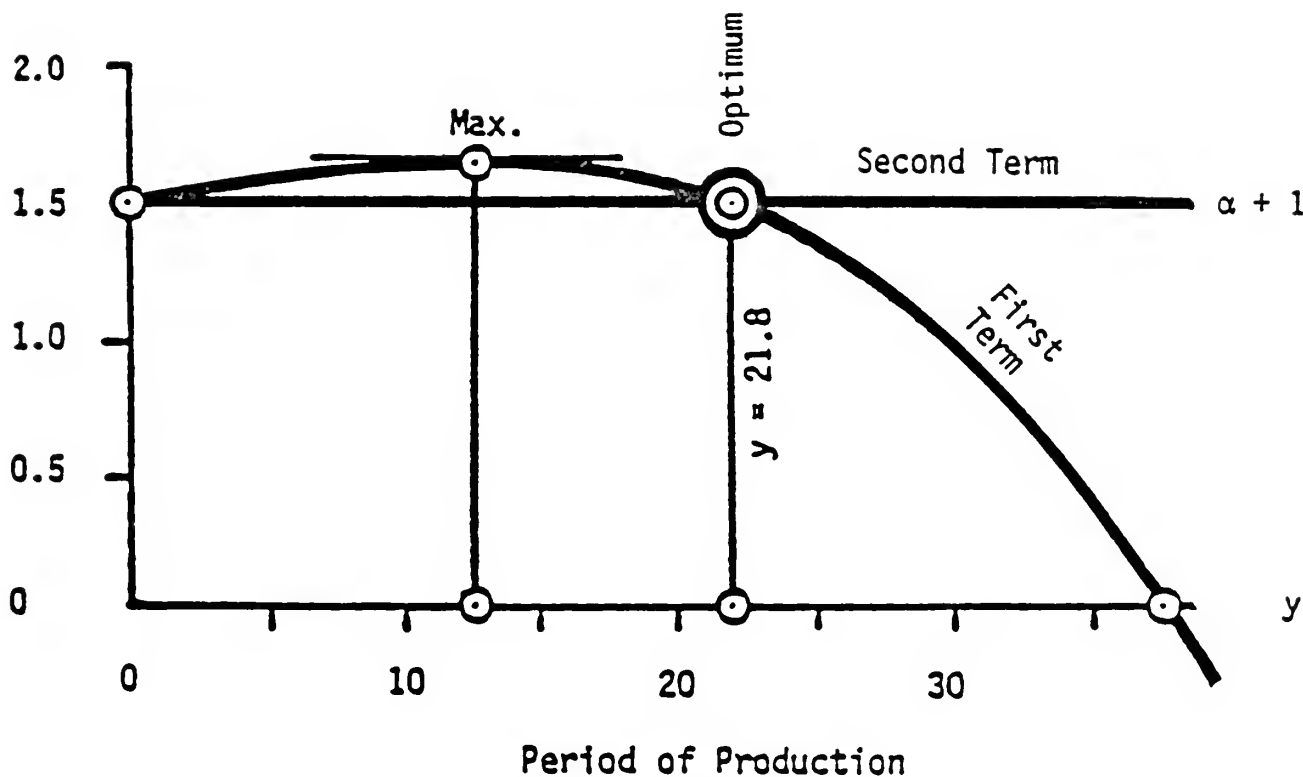


FIGURE 1

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99 Commerce West  
1206 S. Sixth Street  
Champaign, IL 61820

(217) 344-0171

7. Maximizing Present Net Worth of Endless Succession of Production Runs

We defined present net worth of our endless succession as present worth (5) of all its future revenue minus present worth (7) of all its future labor. Call that present net worth  $J(v)$ , insert (5) and (7), and find it to be

$$J(v) = \left[ \frac{mP(v)y^{\alpha + 1}}{e^{\rho y} - 1} - \frac{w(v)}{\rho} \right] L$$

whose second term contains no  $y$ .

Finally maximize present net worth  $J(v)$  of the endless succession of production runs with respect to the length of the period of production  $y$  and write the first-order condition for such a maximum:

$$\frac{dJ(v)}{dy} = \frac{e^{\rho y}(\alpha + 1 - \rho y) - (\alpha + 1)}{(e^{\rho y} - 1)^2} LmP(v)y^{\alpha} = 0$$

which will be zero if the numerator is zero:



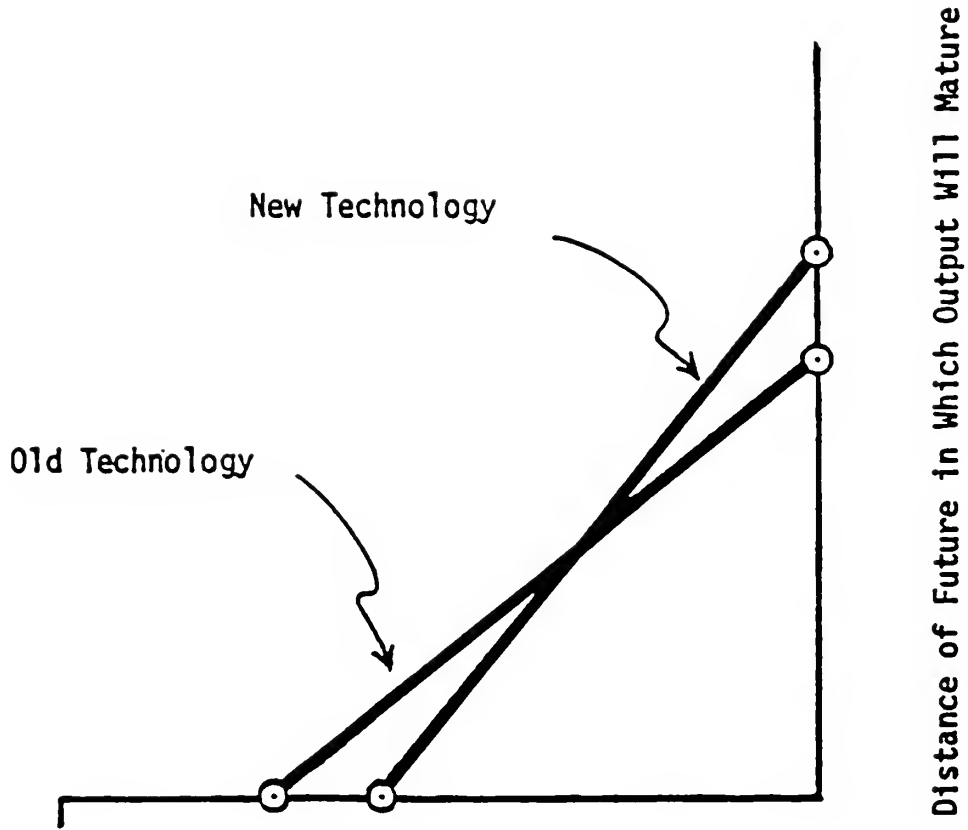
$$e^{\rho y}(\alpha + 1 - \rho y) - (\alpha + 1) = 0 \quad (8)$$

Let us solve our transcendental equation (8) graphically. For, say,  $\alpha = 0.5$  and  $\rho = 0.04$  figure 1 shows its terms. The first term does contain  $y$  and will be a curve starting at the value  $\alpha$  for  $y = 0$ , rising to a maximum for  $y = \alpha/\rho$ , declining to the value zero for  $y = (\alpha + 1)/\rho$ , and being negative thereafter. The second term does not contain  $y$ , hence is a horizontal line. Satisfying (8), curve and line intersect twice, at  $y = 0$  and  $y = 21.8$ . But only the latter intersection satisfies the second-order condition and represents optimal period of production.

#### 8. Sensitivity of Optimal Period of Production to Real Rate of Interest

How sensitive is optimal period of production to the real rate of interest  $\rho$ ? Differentiate our first-order condition (8) implicitly with respect to  $\rho$  treating  $y$  as a function of  $\rho$  and find the simple elasticity

$$\frac{\rho}{y} \frac{dy}{d\rho} = -1 \quad (9)$$



Allocation of Available Labor Inputs  
Among Short and Long Periods of Production

FIGURE 2

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or, in English, the elasticity of optimal period of production with respect to the real rate of interest equals minus one: a lower real rate of interest will lengthen the optimal period of production.

## 9. Macroeconomic Equilibrium

Böhm-Bawerk used his microeconomic time-interest relationship to build a macroeconomic equilibrium determining simultaneously the period of production, the rate of interest, and the real wage rate. Capital was circulating capital in the form of a subsistence fund feeding labor for the period of production. The equilibrium rate of interest would be a rate inducing a period of production of exactly the length required to absorb available capital stock and employ available labor force.

Available capital stock was a parameter to Böhm-Bawerk. By assigning alternative values to one's parameters one may watch the effect on one's solution, i.e., engage in comparative statics, and that was exactly what Böhm-Bawerk did in his comparison of a less wealthy and a more wealthy economy. The next step would have been a formal explanation of current available capital stock as the accumulated result of past saving. Such a step would have amounted to growth theory, and dynamics was beyond Böhm-Bawerk's ken.

Intuitively Böhm-Bawerk was aware of the fact that current available capital stock is the accumulated result of past saving. Indeed the first two of his famous three reasons why the interest rate is positive had to do with such saving and the obstacles to it: first, men typically expect to be better off in the future, so why save? Second, men do not feel future wants as intensely as present ones. (The third reason was the superiority of longer periods of production.)

10. Will the Wealthier Economy Enjoy a Larger Physical Net National Product?

A Böhm-Bawerk economy knows no fixed capital, hence no replacement, no capital consumption allowances. Its physical net national product is simply its physical output  $X/y$  of consumers' goods per annum.

Engaging in Böhm-Bawerkian comparative statics we see that a wealthier economy would have a lower rate of interest, hence a longer period of production. The same  $L$  men would be employed but according to our (1) physical output  $X/(Ly)$  per annum per man would be higher. The wealthier economy would then enjoy a larger physical net national product.

Still using simple interest and still assuming all labor inputs to be invested in the same period of production, Wicksell [1893 (1954)] restated Böhm-Bawerk mathematically—but later went beyond him.

## 11. Wicksell's Triangles

Later, Wicksell [1901 (1934)] adopted compound interest with continuous compounding, dropped the assumption that all available labor inputs were invested in the same period of production, and drew his famous triangles, shown in figure 2. Their base showed how a year's available labor inputs were allocated among short and long periods of production, and their height showed in how distant a future output would mature. In other words, the area of Wicksell's triangles would show the size and the time structure of existing capital stock.

Under given technology, thrift would expand the area of a triangle by expanding its base as well as its height, thus leaving less current labor unabsorbed, hence raising the marginal productivity of current labor and generating a Wicksell [1901 (1934: 164)] Effect: "The capitalist saver is thus, fundamentally, the friend of labour ..."

Under given thrift, "so long as no further capital is saved," technological progress might be labor-saving. In that case the area of the triangle would stay the same but with a narrower base and a taller height, thus leaving more current labor unabsorbed, hence reducing the marginal productivity of current labor. Wicksell [1901 (1934: 164)] concluded that "the technical inventor is not infrequently [labor's] enemy."

Three years before his death Wicksell offered yet another, and more fundamental, extension of Böhm-Bawerk's teaching on time and interest, i.e., the extension from circulating to fixed capital, to which we now turn.

## II. FIXED CAPITAL

### 1. Fixed Capital

As we saw, English classicists considered fixed capital to be a substitute for labor. To Ricardo the number of men employed in the construction of the new machines might well be less than the number of men displaced in their operation. Mill [1848 (1923: 94)] agreed that "all increase of fixed capital, when taking place at the expense of circulating, must be, at least temporarily, prejudicial to the interests of the labourers." There the matter of fixed capital rested for a long time. In his general equilibrium of labor and capital Böhm-Bawerk (1889) was silent on fixed capital, and so was, until his old age, Wicksell. With superior mathematical skills Wicksell [1893 (1954)] and Fisher (1907) restated Böhm-Bawerk's general equilibrium of circulating

capital, and Fisher gave us a tool which could equally well have handled fixed and circulating capital, i.e., present-net-worth maximization.

A rigorous treatment of the time-interest relationship for fixed capital had to wait for Wicksell's [1923 (1934)] review of Åkerman (1923). What takes time in the case of fixed capital is the utilization of a durable producers' good over its useful life. Åkerman's and Wicksell's contribution was to consider the length of that useful life an economic variable: better-constructed producers' goods would be longer-lasting, thus delivering their output over more years, but also cost more interest. Exactly how long should they last?

Wicksell [1919 (1934: 240)] emphasized the similarity between fixed and circulating capital: "A farmer has to choose between two ploughs, one of which lasts ten years, and the other, equally useful, lasting eleven. If he chooses the more durable (and dearer) plough, he has the benefit of an extra year's service, which, however, ... must ... replace the difference in price between the two ploughs accumulated by the total interest for the eleven years. Similarly, the price of old wine must exceed the price of newly-pressed wine by the interest for the years of storage."

We prefer to set out any author's model in the simplest form which will rigorously deliver his conclusions, i.e., in the case of Åkerman and Wicksell that the lower rate of interest of a wealthier economy

would lengthen the optimal useful life of durable producers' goods. As a result, the wealthier economy would enjoy a larger net national product. Our model will need only two goods, a consumers' good and a durable producers' good, and only two kinds of labor, construction labor and operating labor. The economy will be stationary: its available labor force, physical capital stock, and physical output will be stationary.

We shall greatly simplify things by using Fisher's present-net-worth maximization. Then we need not worry, as <sup>o</sup>Akerman did, about how much construction labor is "left" in a producers' good of a certain age, about how much use value is "left" at that age, or about which particular amortization procedure to adopt. Indeed we need not even refer to amortization!

We shall use the following notation.

## 2. Variables

$a_1$   $\equiv$  labor absorbed in constructing one physical unit of producers' goods

$C$   $\equiv$  aggregate physical consumption

$I$   $\equiv$  aggregate physical gross investment

$J$   $\equiv$  present net worth of an endless succession of investments

$L$   $\equiv$  labor employed



$P \equiv$  price of consumers' goods

$p \equiv$  price of producers' goods

$S \equiv$  aggregate physical capital stock of producers' goods

$u \equiv$  useful life of producers' goods

$X \equiv$  physical output of consumers' goods per annum per producers' good

### 3. Parameters

$a_2 \equiv$  labor absorbed per annum in operating one physical unit of producers' goods

$\beta \equiv$  elasticity of construction labor per producers' good with respect to useful life

$F \equiv$  available labor force

$g \equiv$  rate of inflation

$n \equiv$  multiplicative factor in production function (12)

$r \equiv$  nominal rate of interest

$\rho \equiv$  real rate of interest  $\equiv$  nominal rate of interest minus rate of inflation

$w \equiv$  money wage rate

4. Microeconomic Equilibrium: An Endless Succession of Replacements of Durable Producers' Goods

Let an entrepreneur plan an endless succession of replacements of durable producers' goods: every  $t$ th year a retired producers' good is replaced by a new one physically identical to it.

Define present net worth of such an endless succession as present worth of all its future revenue minus present worth of all its future construction labor minus present worth of all its future operating labor.

5. Present Worth of All Future Revenue

For fixed capital we adopt a point-input, flow-output scheme and assume physical output  $X$  of consumers' goods per annum per producers' good to remain uniform throughout the useful lives of all replacements. Such uniformity, assumed by Wicksell [1923 (1934): 274]], will rule out reswitching [Samuelson (1966)].

Inflation was never mentioned by Wicksell [1923 (1934)] either but we easily accommodate it to show that the interest rate that mattered was the real one. So let the price of output be inflating at the rate  $g$  per annum:

$$P(t) = e^{g(t - v)} P(v) \quad (10)$$

Occurring continuously, then, revenue during a small fraction  $dt$  of a year located at time  $t$  is  $P(t)Xdt$ . As seen from time  $v$  the present worth of that is  $e^{-r(t - v)} P(t)Xdt$  or, with (10) inserted and the real rate of interest defined as  $\rho \equiv r - g$ ,  $e^{-\rho(t - v)} P(v)Xdt$ . The present worth of all such future revenue is then the integral

$$\int_v^{\infty} e^{-\rho(t - v)} P(v)Xdt = \frac{P(v)X}{\rho} \quad (11)$$

#### 6. Present Worth of All Future Construction Labor

Let the scrap value as well as the length of the construction period of producers' goods be negligible. Borrowing a leaf from Ricardo, Wicksell [1923 (1934: 285-288)] revived a distinction lost in later theory, i.e., the distinction between construction ("renewal") labor and operating ("co-operating") labor. Let the construction of durable producers' goods require labor--indeed let us be truly Ricardian and Wicksellian and assume it to require nothing else. Let  $a_1$  be the labor absorbed in constructing one physical unit of producers' goods, and let

$a_1$  be a rising function of useful life  $u$  but rising in less than proportion. Let  $\beta$  be the elasticity of  $a_1$  with respect to  $u$ , and let us adopt, as Wicksell [1923 (1934: 276)] did, a constant-elasticity production function

$$a_1 = nu^\beta \tag{12}$$

where  $0 < \beta < 1$ .

Let producers' goods be priced  $p$  and sold under pure competition and freedom of entry and exit. Then their price will equal their cost of production:

$$p(t) = a_1 w(t) \tag{13}$$

Let the money wage rate be inflating at the rate  $g$  per annum:

$$w(t) = e^{g(t - v)} w(v) \tag{14}$$

Occurring every  $u$ th year, construction-labor cost is  $a_1 w(t)$  whose present worth is  $e^{-r(t - v)} a_1 w(t)$  or, with (14) inserted,

$$e^{-r(t - v)} a_1 w(v) \tag{15}$$

Write (15) successively for  $t = v, v + u, v + 2u, \dots$ . Summing over replacements, find the present worth of an endless succession of future construction labor

$$(1 + e^{-\rho u} + e^{-2\rho u} + \dots)a_1 w(v) \tag{16}$$

The parenthesis is an endless geometrical progression with first term 1, common ratio  $e^{-\rho u}$ , and sum  $1/(1 - e^{-\rho u})$ . As a result the present worth of the endless succession of future construction labor may be written, using (12):

$$\frac{a_1 w(v)}{1 - e^{-\rho u}} = \frac{nu^\beta w(v)}{1 - e^{-\rho u}} \tag{17}$$

### 7. Present Worth of All Future Operating Labor

Throughout its useful life, but regardless of its length, let  $a_2$  be labor absorbed uniformly per annum in operating one physical unit of producers' goods. Such uniformity will rule out reswitching [Samuelson (1966)].

Occurring continuously, then, operating labor cost during a small fraction  $dt$  of a year located at time  $t$  is  $a_2 w(t)dt$ . As seen from time  $v$  the present worth of that is  $e^{-r(t-v)} a_2 w(t)dt$  or, with (14) inserted,  $e^{-\rho(t-v)} a_2 w(v)dt$ , and the present worth of all such future operating labor is

$$\int_v^{\infty} e^{-\rho(t-v)} a_2 w(v) dt = \frac{a_2 w(v)}{\rho} \quad (18)$$

8. Maximizing Present Net Worth of Endless Succession of Replacements

We defined present net worth of our endless succession of replacements as present worth (11) of its future revenue minus present worth (17) of its future construction labor minus present worth (18) of its future operating labor. Call that present net worth  $J(v)$ , insert (11), (17), and (18), and find it to be

$$J(v) = \frac{P(v)X}{\rho} - \frac{nu^{\beta} w(v)}{1 - e^{-\rho u}} - \frac{a_2 w(v)}{\rho}$$

whose first and last terms contain no  $u$ .

$\beta = 0.5$

$\rho = 0.04$

Terms of Transcendental Equation

$$\beta e^{\rho u} - \rho u - \beta = 0$$

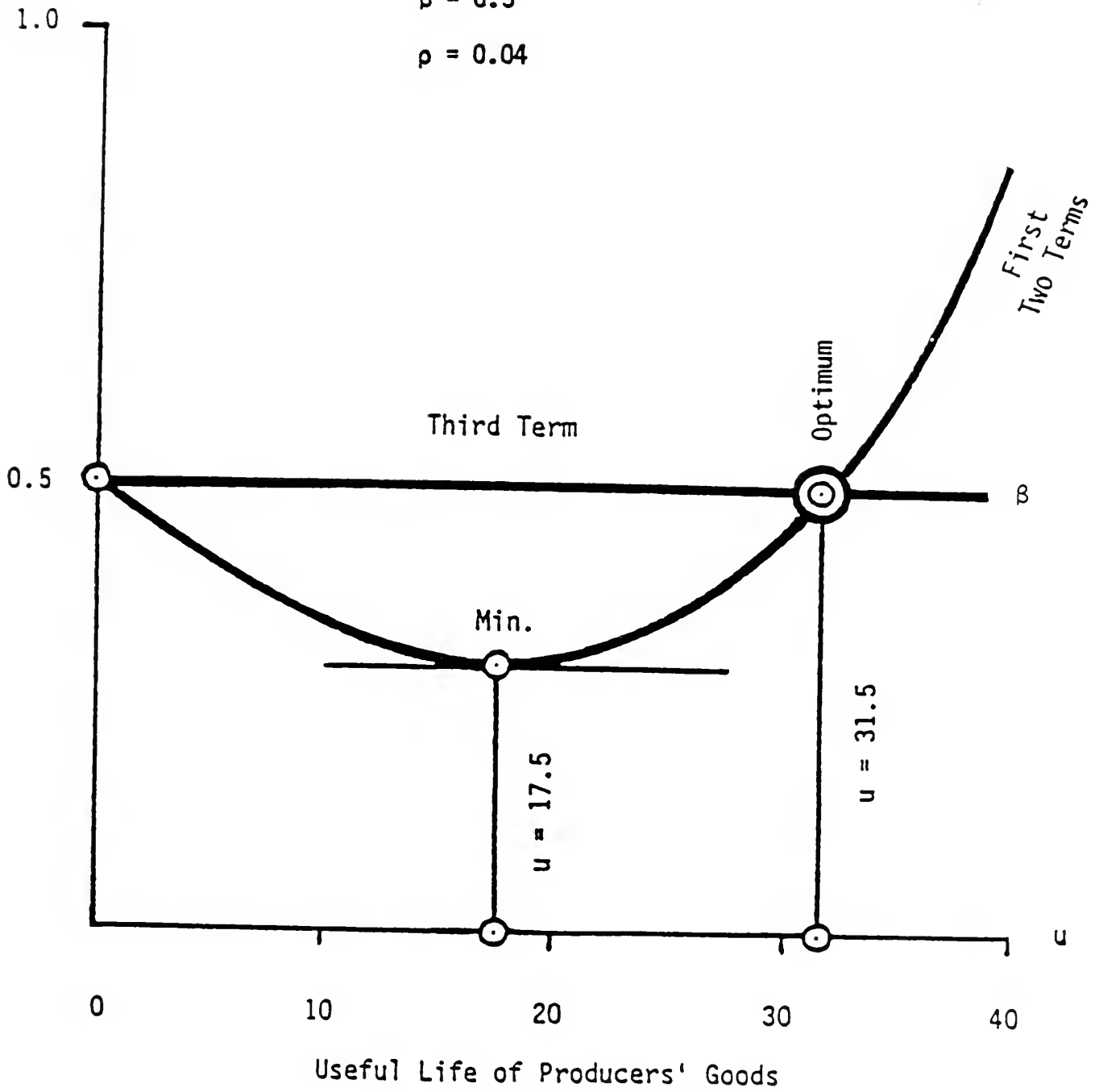


FIGURE 3

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Finally maximize present net worth  $J(v)$  with respect to useful life  $u$  treating the real rate of interest  $\rho$  as a constant and write the first-order transcendental condition for such a maximum:

$$\frac{dJ(v)}{du} = \frac{(\beta + \rho u)e^{-\rho u} - \beta}{(1 - e^{-\rho u})^2} nu^{\beta - 1} w(v) = 0$$

If zero, the numerator would remain zero if multiplied by  $e^{-\rho u}$ . Such multiplication would permit us to write our first-order condition in its simplest possible transcendental form

$$\beta e^{\rho u} - \rho u - \beta = 0 \tag{19}$$

Let us solve our transcendental equation (19) graphically. For, say,  $\beta = 0.5$  and  $\rho = 0.04$  figure 3 shows its terms. The first two terms contain  $u$  and will be a curve starting at the value  $\beta$  for  $u = 0$ , declining to a minimum for  $u = 17.5$  and rising forever after. The third term does not contain  $u$ , hence is a horizontal line. Satisfying (19), curve and line intersect twice, at  $u = 0$  and  $u = 31.5$ . But only the latter intersection satisfies the second-order condition and represents optimal useful life of producers' goods.



9. Sensitivity of Optimal Useful Life to Real Rate of Interest

How sensitive is optimal useful life to the real rate of interest  $\rho$ ? Differentiate our first-order condition (19) implicitly with respect to  $\rho$  treating  $u$  as a function of  $\rho$  and find the simple elasticity

$$\frac{\rho}{u} \frac{du}{d\rho} = -1 \quad (20)$$

or, in English, the elasticity of optimal useful life with respect to the real rate of interest equals minus one--as found by Wicksell [1923 (1934: 278)]: a lower real rate of interest will lengthen the optimal useful life of producers' goods.

10. Macroeconomic Equilibrium

Wicksell used his microeconomic time-interest relationship to build a macroeconomic equilibrium of full employment as follows.

Labor absorbed in constructing one physical unit of producers' goods was  $a_1$ , and the aggregate physical output per annum of such goods was  $I$ . Consequently aggregate labor employed per annum in construction is

$$L_1 = a_1 I \quad (21)$$

As for operating labor, Wicksell [1923 (1934: 286)] used a Cobb-Douglas function to introduce substitution between operating labor and the producers' goods operated. Instead, we simplify things by using a fixed coefficient between them: labor absorbed per annum in operating one physical unit of producers' goods was  $a_2$ , and the aggregate physical capital stock of such goods is  $S$ . Consequently aggregate labor employed per annum in operation is

$$L_2 = a_2 S \quad (22)$$

Let physical gross investment  $I$  be stationary. With useful life  $u$  physical aggregate capital stock  $S$  of producers' goods will then consist of  $u$  vintages, each of size  $I$ :

$$S = Iu \quad (23)$$

Let there be full employment:

$$F = L_1 + L_2 \quad (24)$$

Insert (21), (22), and (23) into (24) and find the supply of physical producers' good per annum to be

$$I = \frac{F}{a_1 + a_2 u} \quad (25)$$

Once again let us engage in Böhm-Bawerkian comparative statics and compare a less wealthy and a more wealthy economy. The wealthier economy has the lower rate of interest, hence the longer useful life of its producers' goods. How does its greater wealth manifest itself? Simply in a physically larger capital stock: insert (12) into (25), (25) into (23), divide numerator and denominator alike by  $u$ , and find physical capital stock to be

$$S = \frac{F}{nu^{\beta - 1} + a_2} \quad (26)$$

which is always up if  $u$  is up, because  $\beta - 1 < 0$ . So the wealthier economy with its lower rate of interest and longer useful life of its

producers' goods has the larger physical capital stock. Physical capital stock is larger because it is better--better solely in the sense of consisting of more solidly built, hence longer-lasting producers' goods.

11. Will the Wealthier Economy Enjoy a Larger Physical Net National Product?

In our stationary economy the money value of the gross national product is the money value of consumers' goods output CP plus the money value of producers' goods output IP. The latter being pure replacement, the money value of the net national product is CP. Physical net national product, then, is simply aggregate physical consumption C.

What can be said about C? To begin with, each producers' good produced a physical output of consumers' goods X per annum. The aggregate physical capital stock S of such producers' goods will then produce the aggregate physical output of consumers' goods

$$C = SX \qquad (27)$$

expressing aggregate physical consumption C as the multiple S of physical output X of consumers' goods per producers' good. As we just saw, S was always up if u was up.

## 12. Conclusion

We conclude, as Wicksell [1923 (1934: 298-299)] did, that with its larger and longer-lasting physical capital stock the wealthier economy will enjoy a larger physical net national product. We can only agree with Solow (1961):

It is always a comfort to read or re-read Wicksell. No other great economist is so candid, so cheerful about putting all his cards on the table and so unassuming about the strength of his hand. Even so technical a work as his "Mathematical Analysis of Åkerman's Problem" is attractive for this reason as well as for the power and modernity of the capital theory it contains.

In all its "power and modernity" Wicksell's treatment of fixed capital was a sympathetic extension of Böhm-Bawerk emphasizing, as we saw, the similarity between fixed and circulating capital. Later, Böhm-Bawerk would run into rougher weather: Blaug (1978: 553) would call reswitching "the final nail in the coffin of the Austrian theory of capital." We must brave the weather.

REFERENCES

°  
Åkerman, Gustaf, Realkapital and Kaptalzins, Stockholm:  
Centraltryckeriet, 1923.

Blaug, Mark, Economic Theory in Retrospect, third edition, Cambridge:  
Cambridge University Press, 1978.

Böhm-Bawerk, Eugen von, Positive Theorie des Kapitaless, Innsbruck:  
Wagner'sche Universitäts-Buchhandlung, 1889, translated as Positive  
Theory of Capital by William Smart, London and New York: Macmillan,  
1891.

Fisher, Irving, The Rate of Interest, London: Macmillan, 1907.

Mill, John St., Principles of Political Economy with Some of Their  
Applications to Social Philosophy, London: J. W. Parker, 1848,  
Longmans, Green, 1923.

Ricardo, David, Principles of Political Economy and Taxation, in Sraffa,  
Piero and Dobb, Maurice H. (eds.), The Works and Correspondence of  
David Ricardo, vol. I, Cambridge: Cambridge University Press, 1951.

Samuelson, Paul Anthony, "A Summing Up," Quarterly Journal of Economics,  
November 1966, 80, 568-583.

Solow, Robert M., "Notes Toward a Wicksellian Model of Distributive  
Shares," in Lutz, F. A., and Hague, D. C., The Theory of Capital,  
New York: St. Martin's Press, 1961, 245-265.

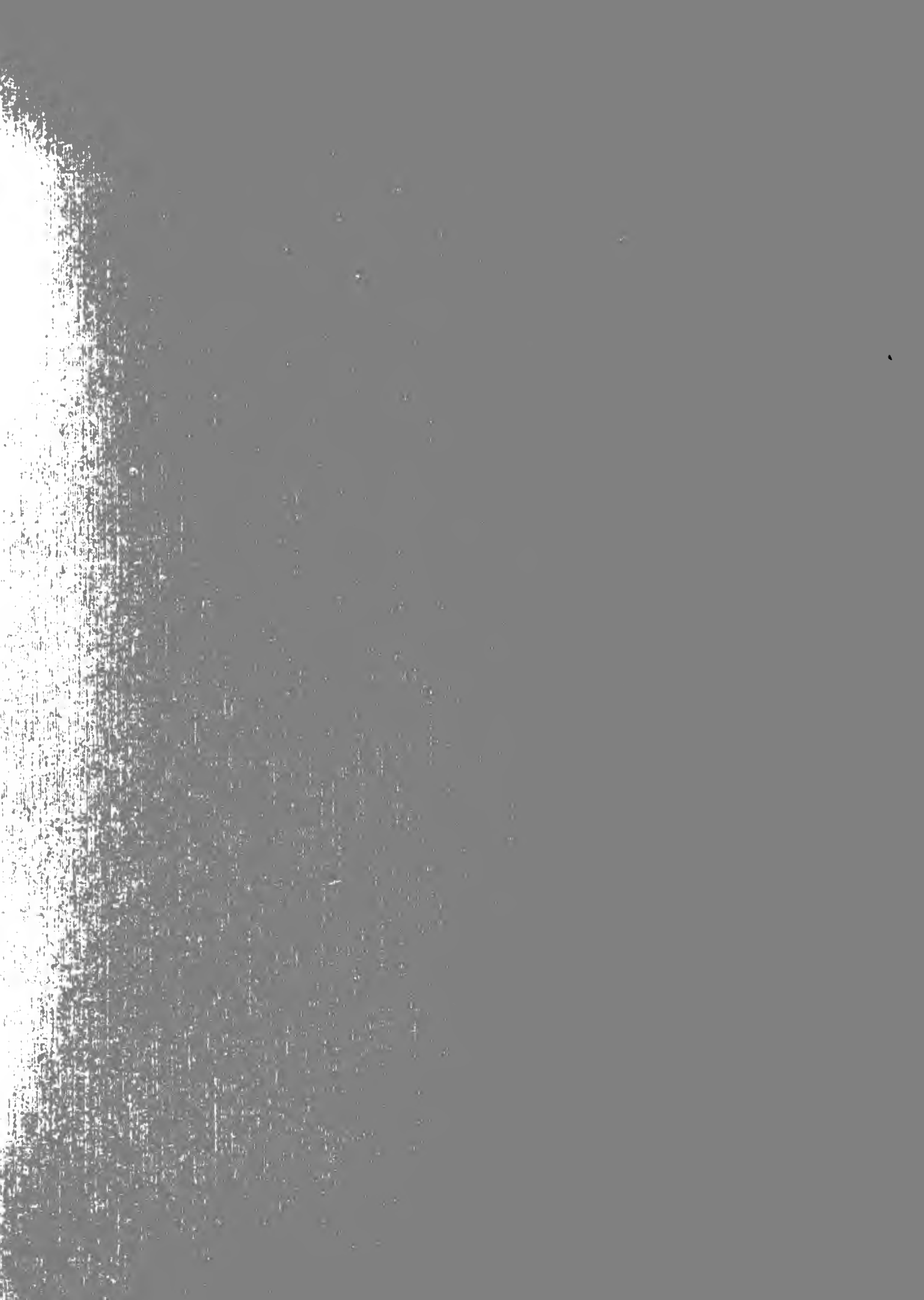
Wicksell, Knut, Ueber Wert, Kapital und Rente, Jena: G. Fischer, 1893,  
translated by S. H. Frohwein as Value, Capital and Rent, London:  
Allen & Unwin, 1954, reprinted 1970.

\_\_\_\_\_, Föreläsningar i nationalekonomi, I, Lund: Gleerup, 1901,  
translated by E. Classen and edited by Lionel Robbins as Lectures  
on Political Economy, I, London: Routledge & Kegan Paul, 1934,  
reprinted 1967.

\_\_\_\_\_, "Professor Cassels nationalekonomiska system," Ekonomisk  
Tidskrift, 1919, 21, 195-226, translated by Solomon Adler as  
Appendix 1, "Professor Cassel's System of Economics," to Wicksell's  
Lectures on Political Economy, I, London: Routledge & Kegan Paul,  
1934, reprinted 1967.

\_\_\_\_\_, "Realkapital och kapitalränta," Ekonomisk Tidskrift, 1923,  
25, 145-180, translated by Solomon Adler as Appendix 2, "Real  
Capital and Interest," to Wicksell's Lectures on Political Economy,  
I, London: Routledge & Kegan Paul, 1934, reprinted 1967.





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