Bridge and Structural Design

W. Chase Thomson

Bridge and Structural Design

BY

W. CHASE THOMSON,
M. Can. Soc. C. E.
Assistant Engineer Dominion Bridge Co.,
Montreal, Canada.

NEW YORK:
THE ENGINEERING NEWS PUBLISHING COMPANY.
1905.

PREFACE.

This book has been developed from lectures given by the author during the past five years, under the auspices of the Dominion Bridge Co. His object has been to teach the elements of bridge and structural design in a simple and practical manner. Arts. 1 to 14 inclusive treat of the general principles of design, and are illustrated by numerous examples; while the remaining Articles are examples of typical structures, in which the stresses are analyzed, the members proportioned, and the details carefully worked out.

Both analytical and graphical methods have been employed for obtaining stresses, and the one which seemed best suited for any particular subject has been adopted. But few tables are given, as it was thought unnecessary to repeat information given in any of the rolling mills' hand-books.

Although the book is intended principally for students and draughtsmen, there are parts which may be of interest to practicing bridge designers. Particular attention is here drawn to Art. 17, which treats of the design of a knee-braced mill building; and to Art. 19 which discusses the rivet spacing and web splices in plate girders, in which one-eighth of the web plate is counted on as flange area.

MONTREAL, March 10, 1905.

CONTENTS.

BRIDGE AND STRUCTURAL DESIGN.

By W. CHASE THOMSON, M. Can. Soc. C. E.

ART. 1.—DEFINITIONS.

Mechanics is the study of the effect of force upon matter.

Force is the action of gravity, wind, steam, etc., causing or tending to cause motion.

Matter is any substance whatever, as metal, stone, wood, water, air, etc.

A body is any piece of matter, and its weight is the amount of force which gravity exerts upon it.

Statics is that branch of mechanics which investigates the stresses in a body produced by forces which keep it stationary, as in bridges and buildings.

Dynamics, on the other hand, investigates forces which move the body upon which they act, as in engines and other machinery.

Stress is the effect produced by equal and opposite forces, and is measured in pounds or tons. It is equal to only one of the forces, however. Thus, if two men pull 40 lbs. each at opposite ends of a rope, the stress will not be 80 lbs., but only 40 lbs.; and if a load of 1,000 lbs. rest on top of a column, then the reaction of the foundation on which the column rests will also be 1,000 lbs., but the stress in column will be equal to but one of these forces, viz.: 1,000 lbs. There can be no stress without a reaction which is always equal to the force acting on body.

Tensile Stress or Tension occurs when two opposing forces tend to pull apart the body upon which they act, as in the tie bars and bottom chords of a bridge.

Compressive Stress or Compression occurs when the opposing forces acting on a body tend to compress it, as in the posts and top chords of a bridge.

ART. 2.—THE COMPOSITION AND RESOLUTION OF FORCES.

The resultant of two or more forces is a single force which will produce the same effect as the combined action of those forces.

The components of a force are the several forces which by their combined action would have the same effect as the single force.

Let A B and C B in Fig. 1 represent both the magnitude and direction of two forces in the same plane, acting through the point B. Through A and C lines are drawn parallel to the forces, intersecting in the point D. Then the diagonal B D represents both the magnitude and direction of the resultant of the forces A B and C B. A force of equal magnitude but acting in the opposite direction would balance the original forces, or, in other words, the three forces would be in equilibrium. The figure is called the parallelogram of forces.

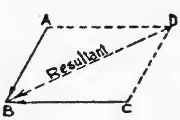


Fig. 1

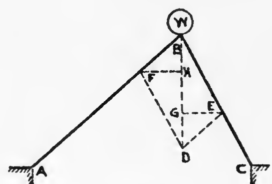


Fig. 2

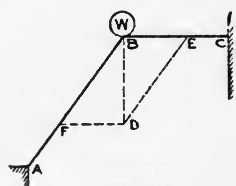


Fig. 3

In Fig. 2 it is required to find the components of the force represented by the load W applied at the apex of the two rafters A B and B C. A vertical line B D is drawn equal to the load W, and from the point D lines are drawn parallel to the rafters. Then D E, parallel to the rafter A B, is the stress in that member, and F D the stress in B C. The horizontal lines F H and G E represent the horizontal thrust of rafters which is the same for both. B H is the vertical thrust of rafter A B, and B G the vertical thrust of rafter B C.

In Fig. 3 one rafter A B is inclined, and the other rafter B C is horizontal. The force W is applied at B. B D is drawn vertically as before equal to the load W, and from the point D lines parallel to A B and B C are drawn. F D the horizontal component of the stress in A B is equal to the stress in B C. B D the vertical component of the stress in A B is equal to the load. In other words, the rafter A B takes the whole vertical load, and B C only horizontal thrust.

If any number of forces in the same plane meet in a point as in

Fig. 4, their resultant may be found by drawing lines end to end equal and parallel to the forces as in Fig. 4a. The closing line represents the magnitude and direction of the resultant. This diagram is called the polygon of forces. The arrow heads represent the direction of the forces and follow each other around the diagram. The resultant, however, always acts towards the last force drawn. A force equal to the resultant but acting in the opposite direction would hold the other forces in equilibrium.

If any number of forces in the same plane and acting through the same point are in equilibrium, their force polygon will form a closed figure, and if the direction of all the forces be known and the magnitude of all but two, these may be found by scale from the force

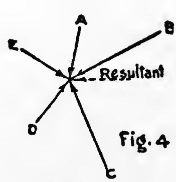


Fig. 4

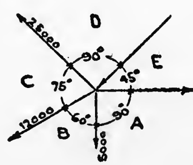


Fig. 5

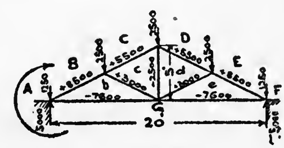


Fig. 6

Fig 4a

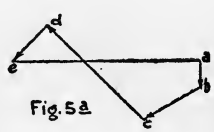


Fig. 5a

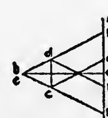


Fig. 6a

polygon. This case is similar to the forces acting at the panel points of a bridge or roof truss, and the principle enables one to find the stresses in all the members of a framed structure by the graphical method.

In Fig. 5 there are five forces acting through the same point which are denoted by the letters between which they lie. This is the usual and undoubtedly the best method of denoting stresses when solving them by the graphical method. The stresses A B = 5,000 lbs., B C = 12,000 lbs. and C D 25,000 lbs. are given, and the direction only of D E and E A.

Fig. 5a is the force polygon which is constructed by drawing lines parallel and equal to the forces A B, B C and C D. From the point d in force polygon a line is drawn parallel to the force D E but of indefinite length; another line is drawn from the point a parallel to E A. The intersection of the two lines in the point e determines the magnitude of the forces D E and E A. D E is about 9,000 lbs. and E A about 35,000 lbs. Any force acting away from the point indicates tension, and any force acting towards it, compression.

ART. 3.—EXAMPLES IN GRAPHICAL STATICS.

Fig. 6 represents a simple roof truss. The span is 20 ft. depth at centre 5 ft., and trusses spaced 10 ft., centre to centre. Assumed load 50 lbs. per square foot of horizontal projection, concentrated at panel points by purlins. The total load on truss will be $20' \times 10' \times 50$ lbs. $= 10{,}000$ lbs. The load at each intermediate panel point $= \dfrac{10{,}000}{4} = 2{,}500$ lbs., and at each end panel point, one-half of this amount $= 1{,}250$ lbs. The end panel loads are carried directly by the walls, and therefore have no effect on the stresses in the truss. Capital letters are used to denote the external forces which consist of loads and the reactions of the end supports; and small letters for the internal forces. The truss members are indicated by the letters between which they lie.

Fig. 6a is the stress diagram. Beginning at the left hand end of truss and going around it in a right handed direction, as indicated by curved arrow, the forces A B, B C, C D, D E and E F are laid off downwards on the vertical load line, Fig. 6a. The next external force is the reaction of the right-hand support equal to one-half the load on span. This force is laid off upwards from F to G as it acts in the opposite direction to the loads. Finally, the reaction of the left-hand support is laid off from G to A the point of beginning. Now, at the left-hand end of truss there are four forces meeting in a point. Two of these forces, the reaction G A and the load A B are known; and two forces, the stresses in rafter Bb and bottom chord bG, are unknown. From the point B on load line, a line is drawn parallel with the rafter Bb, and from G, a line parallel with the bottom chord bG. These two lines intersect in the point b, and determine the stresses Bb and bG. Next, at the first panel point from the end, the stress in bB has just been found, the load BC is known and the stresses in Cc and cb unknown. From the point C on load line, a line is drawn parallel to the rafter Cc, and from the point b in stress diagram, a line parallel to the strut cb. The two intersect in the point e, and determine the stresses in Cc and cb. At the apex of rafters there are now two unknown forces, the stresses in Dd and dc. From the point D on load line, a line is drawn parallel to Dd, and from the point c in stress diagram, a line parallel to dc. The two intersect in the point d and determine the stresses in Dd and dc.

It is unnecessary to proceed any further with the stress diagram, as the stresses in the right-hand end of truss will evidently be the same as those in the left-hand end, but to test the accuracy of the work it is sometimes advisable to go through the whole truss, and if the work has been carefully done the stress diagram will form a closed figure.

Now to know whether a member is in compression or tension, it

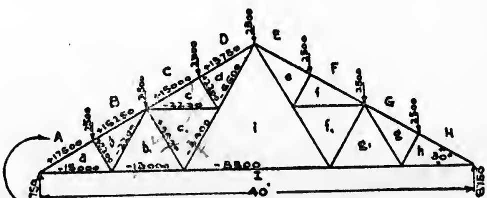


Fig. 7

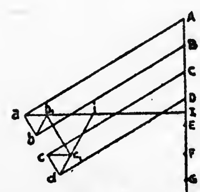


Fig. 7a

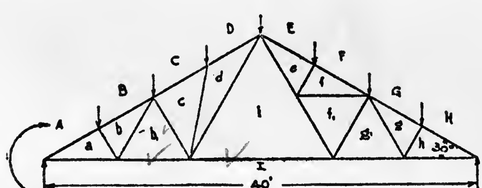


Fig. 8

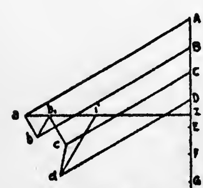


Fig. 8a

is necessary to observe which way the stresses act in the stress diagram. In going around any panel point in the direction in which the external forces have been taken (in this case from left to right) if a force in the stress diagram acts towards the panel point, the member is in compression, and if away from it, in tension. For example: At the apex of rafters, going around the point from left to right, and observing the direction of the forces in the stress diagram, it will be seen that cC acts towards the point, and therefore the stress in rafter cC is compression; the load C D acts towards the point, and the force Dd acts towards it. The next force dc acts away from the point, and so the stress dc is tension.

The external forces and stresses are shown on the truss diagram,

Fig. 6. The sign (+) indicates compression, and the sign (—) tension.

One of the commonest forms of roof trusses is that known as the Fink Truss, so-called from the name of the inventor. Fig. 7 is an example.

The span is 40 ft., the angle of roof with horizontal 30°, the panel load 2,500 lbs. The half-panel loads at the ends have been omitted, as they have no effect on the stresses. Beginning at the left-hand end of truss, as in previous example, the loads A to H are laid off on the load line in Fig. 7a downwards, and the reactions H I and I A upwards to the point of beginning. The stress diagram is then

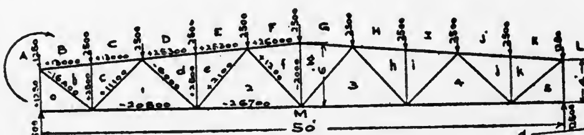


Fig. 9

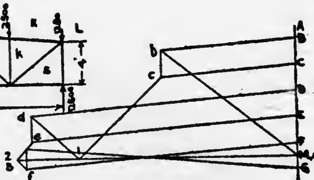


Fig. 9a

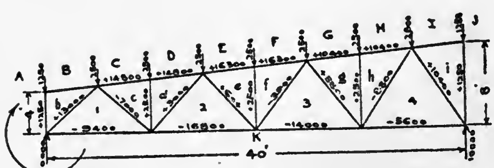


Fig. 10

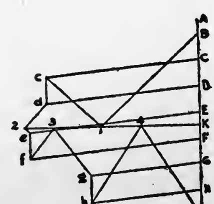


Fig. 10a

proceeded with, beginning at left end. At panel point B C a slight difficulty is encountered, where there are three unknown forces, viz., Cc, cc_1 and c_1 b_1 and the diagram cannot be completed when there are more than two unknown. At the lower end of strut b_1 c_1 the same difficulty is met with. A very nice method of solving this problem is to change some of the web members temporarily as in Fig. 8, from which the stress diagram, Fig. 8a, is obtained, and the stress in the bottom chord member i I. The web members may then be changed back to their original form, and the polygon of forces completed for the panel point at lower end of strut b_1 c_1 where there are now only two unknown forces, viz., b_1 C_1 and c_1 i.

After which the polygon of forces for panel point B C may be drawn. The rest is simple.

When the truss is symmetrical and the panel loads equal, as in the present example, there is no difficulty in constructing the stress diagram, as the points a, b, c, d will always lie in a straight line; but if the panel lengths or the loads are unequal, as is sometimes the case, it will be necessary to use some method for finding an extra force either at the upper or lower end of strut b_1 c_1.

One-half of the stress diagram only has been constructed, as the other half would be exactly the same.

A common form of roof truss is shown in Fig. 9. The span is 50 ft., centre to centre, the depth at ends 4 ft. and at centre 6 ft. The intermediate panel loads are 2,500 lbs. each, and the end panel loads 1,250 lbs. each.

Fig. 9a is the stress diagram which is only constructed for one-half the truss. There is no stress in the end panels of bottom chord 0M and 5M from the vertical loads. These members are required for lateral stability.

Sometimes a roof truss is required to slope in one direction only as in Fig. 10. The stresses are found in the same manner as before, but it is necessary to make the stress diagram for the whole truss.

The span is 40 ft. the depth at one end 4 ft. and at the other end 8 ft. The intermediate panel loads are 2,500 lbs. each, and the end panel loads 1,250 lbs. each.

ART. 4.—THE LEVER AND MOMENTS.

If a force act on a body tending to rotate it about a certain point, it is said to have a moment about that point equal to the amount of the force multiplied by the perpendicular distance from the line of action of force to the said point.

In Fig. 11 the force F acts about the point a with a leverage equal to ab. The point a is called the point of moments, and the distance ab the lever arm.

There may be two or more forces tending to rotate a body about the same point either in the same or in the opposite direction; and if the body is in equilibrium the sum of the left-hand moments must be equal to the sum of the right-hand moments. Fig. 12 represents a beam supported at the point B. The load at A tends to rotate the beam in a left-handed direction about its point of support, and its moment = 3 lbs. × 10′ = 30 ft.-lbs. The load at C tends to rotate the beam in a right-handed direction, and its moment = 5 lbs. × 6′ = 30 ft.-lbs. The moments, therefore, are equal but opposite, so the beams will remain horizontal. A lever may either be straight or bent, but no matter what the actual length of the lever may be the true lever arm is the perpendicular distance from the line of force to the point of moments. Figs. 13 and 14 are examples of bent levers.

Fig. 15 represents a bracket on the side of a wall supporting a load W at the point B. The principle of the lever is here employed

Fig. 11

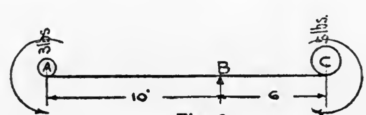


Fig. 12

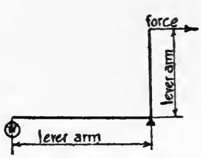


Fig. 13

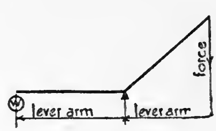


Fig. 14

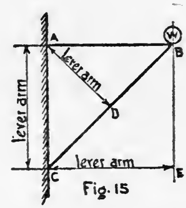


Fig. 15

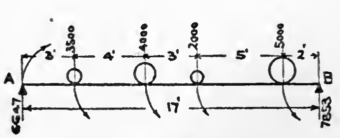


Fig. 16

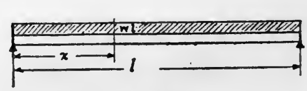


Fig. 17

to determine the stresses. For the stress in A B moments are taken about the point C, then W × lever arm C E = stress in A B × lever arm A C. Therefore stress in A B $= \dfrac{W \times C\,E}{A\,C}$. For the stress in C B moments are taken about the point A. Then W× lever arm A B = stress in C B× lever arm A D. Therefore stress in C B = $\dfrac{W \times A\,B}{A\,D}$.

To determine the reactions for a beam supported at both ends and loaded in any manner, moments of all the loads are taken about one support, and divided by the distance centre to centre of supports.

Example: On the beam A B, Fig. 16, there are four loads placed as shown. For the reaction at A moments are taken about B and divided by the span as follows:

$$\begin{aligned} 5{,}000 \times 2 &= 10{,}000 \\ 2{,}000 \times 7 &= 14{,}000 \\ 4{,}000 \times 10 &= 40{,}000 \\ 3{,}500 \times 14 &= 49{,}000 \\ \hline & 113{,}000 \text{ ft.-lbs.} \div 17' = 6{,}647 \text{ lbs.} \end{aligned}$$

The loads tend to rotate the beam about the point B in a left-hand direction with a moment of 113,000 ft.-lbs. The reaction at A tends to rotate the beam about the point B in a right-handed direction with a moment = 6,647 lbs. × 17′ = 113,000 ft.-lbs. These moments are equal but opposite, and therefore counteract each other, so there is no resultant moment at the support B.

To obtain the reaction B, moments may be taken about A, but as the sum of the reactions must be equal to the sum of the loads, it is only necessary to add together the loads, and subtract the reaction A. The result will be the reaction B, thus:

5,000 + 2,000 + 4,000 + 3,500 — 6,647 = 7,853 = reaction B.

ART. 5.—SHEARING AND BENDING STRESSES IN BEAMS.

A loaded beam is subjected to two kinds of stress, viz.: Shearing and bending. The shearing stress tends to cause the particles of the beam to slide by one another in a vertical plane, as when a plate is cut in a shearing machine. Fig. 17 represents a beam loaded uniformly with a load = w per lineal foot. At each end there are equal and opposite forces acting on the beam, viz., one-half of the load acting downwards, and the reaction of the support acting upwards, each equal to $\frac{w\,l}{2}$. These two forces tend to shear the beam, or cut it crosswise. The shearing force at any point, distant x from one end, is equal to the reaction at that end, less the load on the length x; or, shear at $x = \frac{w\,l}{2} - w\,x$. The bending stresses cause compression on the upper side of the beam and tension on the

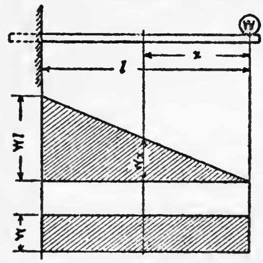


Fig. 18. Beam supported and fixed at one end, and carrying a load W at other end.

Fig. 18a. Moment Diagram.
Max. Moment is at fixed end, $= Wl$
Moment at any point distant x from load $= Wx$.

Fig. 18b. Shear Diagram. Shear at any point $= W$.

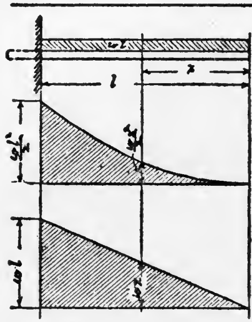


Fig. 19. Beam supported and fixed at one end, and carrying a load w per lineal foot.

Fig. 19a. Moment Diagram.
Max. Moment is at fixed end. $= \frac{wl^2}{2}$
Moment at any point distant x from free end $= \frac{wx^2}{2}$
The curve is a parabola with vertex at free end.

Fig. 19b. Shear Diagram.
Max. Shear is at fixed end. $= wl$.
Shear at any point distant x from free end $= wx$

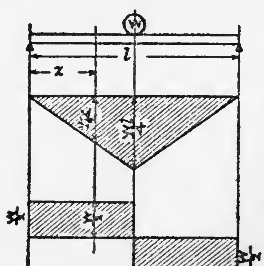


Fig. 20. Beam supported at both end, and carrying a concentrated load W at centre.

Fig. 20a. Moment Diagram.
Max. Moment is at centre. $= \frac{Wl}{4}$.
Moment at any point x between end and centre $= \frac{Wx}{2}$

Fig. 20b Shear Diagram.
Shear at any point $= \frac{W}{2}$.

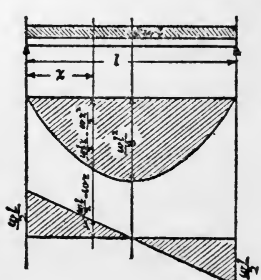


Fig. 21. Beam supported at both ends and carrying a uniform load w per lineal foot.

Fig. 21a. Moment Diagram.
Max. Moment is at centre. $= \frac{wl^2}{8}$.
Moment at any point distant x from support $=$
$\frac{wlx}{2} - \frac{wx^2}{2}$.
The curve is a parabola with vertex at centre.

Fig. 21b. Shear Diagram.
Max. shear at ends. $= \frac{wl}{2}$.
Shear at any point distant x from support. $\frac{wl}{2} - wx$.
Shear at centre $= 0$.

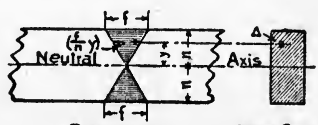
lower side. In the case of an open girder the bending moment is all resisted by the flanges; but, in a solid beam, it is resisted by the entire section.

Bending moments and shears for various cases are illustrated in Figs. 18, 19, 20 and 21.

ART. 6.—MOMENT OF RESISTANCE.

When a beam is loaded transversely, the fibres on one side of the neutral axis are compressed and those on the other side extended, while the fibres in the neutral axis are neither compressed nor extended, and the beam will assume a curved form as in Fig. 22. The extreme outer fibres are stressed most, and the intermediate ones in direct proportion to their distance from the axis.

Fig. 22



ELEVATION CROSS SECTION

Fig. 23

The moment of resistance of a beam is the moment of all the fibre stresses about the neutral axis.

The following is a general method for determining the moment of resistance of a beam of any section:

In Fig. 23. f = stress per square inch on outer fibres.
n = distance in inches from neutral axis to outer fibres.
y = distance in inches from neutral axis to any fibre.
$\triangle$ = a small or elementary area.

Then $\frac{f}{n}$ = stress per square inch on fibres at distance of one inch from neutral axis.

$\frac{f}{n}y$ = stress per square inch on fibres at distance of y from neutral axis.

$\frac{f}{n}y\triangle$ = stress on an element of fibres at distance of y from neutral axis.

$\left(\frac{f}{n}y\triangle\right)y$ = moment of stress on an element of fibres at distance of y from neutral axis.

This last expression taken for all values of y both above and below the neutral axis is the moment of stress (or moment of resistance) of the given section, or

$$M = \Sigma \frac{f}{n} y^2 \triangle = \frac{f}{n} \Sigma y^2 \triangle.$$

The factor ($\Sigma y^2 \triangle$) is called the moment of inertia and is represented by I. It is obtained by multiplying each elementary area by the square of its distance from the neutral axis and taking the sum of the products.

Representing ($\Sigma y^2 \triangle$) by I, then $M = \frac{f}{n} I$ = moment of resistance.

The factor $\left(\frac{I}{n}\right)$ of the moment of resistance is usually represented by R and called the moment of resistance, which is not strictly correct, for the real moment of resistance = Rf.

$\frac{I}{n}$ is sometimes called the section modulus and represented by S.

The outer fibres only of a beam receive the maximum stress per square inch, hence the more metal concentrated in the flange, the greater its resistance to bending.

ART. 7.—MOMENT OF INERTIA.

As stated in Art. 6, the moment of inertia is a factor of the moment of resistance, and is represented by I. The following is an approximate method for finding the moment of inertia of a beam of any section about an axis through its centre of gravity.

The section should be divided into a number of narrow strips parallel with the neutral axis, the area of each strip calculated, and the distance of its centre line from the neutral axis measured. Each area should be multiplied by the square of its distance from the neutral axis, then the sum of these products will be the approximate moment of inertia. The narrower the strips, the more accurate will be the result, but it will always be a trifle too small. To be exact, the moment of inertia of each strip about an axis through its own centre of gravity should be added to the last result—but this is unnecessary in practice.

For the moment of inertia of a rectangle about an axis through its centre of gravity, the section is supposed to be divided into nar-

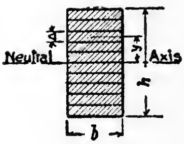


Fig. 24

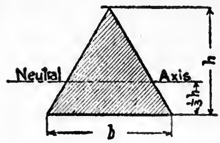


Fig. 25

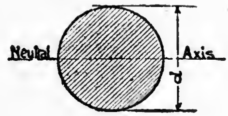


Fig. 26

row strips as shown in Fig. 24. The length of each strip is b, its thickness $= \Delta$, and the distance of its centre from the neutral axis $= y$.

Then the summation of $(b\Delta y^2)=$ the approximate moment of inertia.

By the help of the calculus, the thickness of each strip can be made infinitely small and therefore an *exact* result obtained which is $I=\frac{bh^3}{12}$. In which $b =$ the width of beam and $h =$ the height. This is important.

The moment of inertia of a triangle about an axis through its centre of gravity and parallel with the base, as shown in Fig. 25, is $I=\frac{bh^3}{36}$.

The moment of inertia of a circle about an axis through its centre of gravity, as shown in Fig. 26 is $I = \frac{\pi d^4}{64}$.

The moment of inertia of a compound section about an axis through its centre of gravity is equal to the sum of the moments of inertia of the component parts about axes through their own centres of gravity, plus the areas of the component parts multiplied by the squares of the distances of their centres of gravity from the neutral axis of the whole figure.

ART. 8.—RADIUS OF GYRATION.

The radius of gyration, which is usually represented by r, is the distance from the neutral axis through the centre of gravity of a section to a point where, if the total area could be concentrated and multiplied by the square of this distance, the result would be the moment of inertia of the section about the same axis; thus $I =$ area $\times r^2$, and therefore $r=\sqrt{\frac{I}{\text{area}}}$. The radius of gyration is used principally in formulæ for the strength of columns.

ART. 9.—FORMULAE RELATING TO BEAMS.

The following notation and relations between bending moments and the various properties of beams, which have already been treated of in Arts. 6, 7 and 8, are here set forth more concisely. The student should familiarize himself with the formulae as they will be referred to frequently in the following pages:

M = bending moment in inch-pounds.
R = moment of resistance, = S. the section modulus.
f = stress per square inch on outer fibres.
I = moment of inertia about axis through centre of gravity of section.
A = area of section.
n = distance from centre of gravity of section to extreme outer fibres.
r = radius of gyration.

Then $M = Rf.$

$$R = \frac{M}{f}$$

$$R = \frac{I}{n}$$

$$f = \frac{M}{R}$$

$$I = Ar^2$$

$$r^2 = \frac{I}{A}$$

$$r = \sqrt{\frac{I}{A}}$$

ART. 10.—EXAMPLES IN THE COMPUTATION OF PROPERTIES OF SIMPLE AND COMPOUND SECTIONS.

Fig. 27 represents a 6 × 3½ × ½ angle. It is divided into two rectangles: one 6″ × .5″ = 3□″, and one 3″ × .5″ = 1.5□″. To find the position of the axis ab through the center of gravity, moments of the areas are taken about the back of the shorter leg as follows;

Areas.		Levers.		Moments.
1.5□″	×	.25″	=	.375
3.0□″	×	3.″	=	9.000
4.5□″				9.375

Then dividing the total moment by the total area, the distance of the centre of gravity from the point of moments is obtained thus: $9.375 \div 4.5 = 2.08''$.

For the position of the axis cd moments are taken about the back of the longer flange, and divided by the area as before.

Areas.	Levers.		Moments.
$3.0\square'' \times$	$.25''$	=	.75
$1.5\square'' \times$	$.2''$	=	3.00
$4.5\square''$			3.75

Then $3.75 \div 4.5 = .833''$, which is the distance from the back of longer leg to the axis cd. The moment of inertia about axis ab will be computed as follows:

$$\text{I for rectangle } (6'' \times .5'') = \frac{bh^3}{12} = \frac{.5 \times 6^3}{12} = 9.00$$

$$\text{I for rectangle } (3'' \times .5'') = \frac{bh^3}{12} = \frac{3 \times .5^3}{12} = .03$$

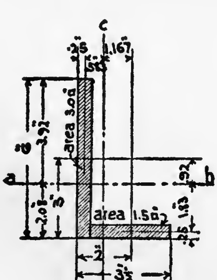
$$3.0\square'' \times .92^2 = 2.54$$

$$1.5\square'' \times 1.83^2 = 5.02$$

$$\text{I } ab = 16.59$$

$$\text{R}ab = \frac{\text{I}}{\text{n}} = \frac{16.59}{3.92} = 4.23, \quad rab = \sqrt{\frac{\text{I}}{\text{A}}} = \sqrt{\frac{16.59}{4.5}} = 1.92.$$

The moment of inertia, moment of resistance and radius of gyration about axis cd are found similarly.



$6 \times 3\frac{1}{2} \times \frac{1}{2}$ L

Fig. 27

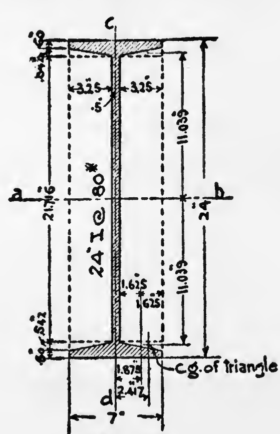


Fig. 28

Fig. 28 represents a 24″ I at 80 lbs. Its moment of inertia about the axis ab is equal to that of the circumscribing rectangle, less the moment of inertia of the voids about the same axis.

Area of rectangle $7'' \times 24'' =$		168.00
Area of 2 rectangles $3.25'' \times 22.716'' = 141.15$		
Area of 4 triangles $\dfrac{3.25 \times .542}{2} =$	3.52	144.67
Area of beam $=$		23.33 sq. in.

MOMENT OF INERTIA ABOUT AXIS *ab*.

For circumscribing rectangle 7×24 $Iab = \dfrac{bh^3}{12} = \dfrac{7 \times 24^3}{12} = 8064.00$

For 2 rectangles 3.25×21.716 $\quad Iab = \dfrac{bh^3}{12} = 2\left(\dfrac{3.25 \times 21.716^3}{12}\right) = 5547.01$

For 4 triangles $\dfrac{3.25 \times .542}{2}$ $\quad Iab = \dfrac{bh^3}{36} = 4\left(\dfrac{3.25 \times .542^3}{36}\right) = \quad .02$

$+$ area of triangles into square of distance from axis $= 3.52 \times 11.039^2 = 428.95 \quad 5975.98$

2088.02

$$I\,ab = 2088.02. \quad R\,ab = \frac{I}{n} = \frac{2088.02}{12} = 174.00.$$

$$r\,ab = \sqrt{\frac{I}{A}} = \sqrt{\frac{2088.02}{23.33}} = 9.46.$$

MOMENT OF INERTIA ABOUT AXIS *cd*.

For circumscribing rectangle 7×24 $Icd = \dfrac{bh^3}{12} = \dfrac{24 \times 7^3}{12} = 686.00$

For 2 rectangles 3.25×21.716 $\quad \dfrac{bh^3}{12} = 2\left(\dfrac{21.716 \times 3.25^3}{12}\right) = 124.24$

$+$ area of rectangles into square of distance from axis $141.15 \times 1.875^2 = 496.14$

For 4 triangles $\dfrac{3.25 \times .542}{2}$ $\quad Icd = \dfrac{bh^3}{36} = 4\left(\dfrac{.542 \times 3.25^3}{36}\right) = 2.07$

$+$ area of triangles into square of distance from axis $= 3.52 \times 2.417^2 = 20.56 \quad 643.01$

42.99

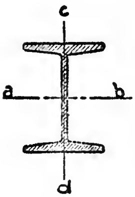
$$I\,cd = 42.99. \quad R\,cd = \frac{I}{n} = \frac{42.99}{3.5} = 12.28.$$

$$r\,cd = \sqrt{\frac{I}{A}} = \sqrt{\frac{42.99}{23.33}} = 1.36$$

It is seldom necessary to work out the properties of angles, beams, channels, etc., as they are to be found in the hand books published by the rolling mills companies.

The following table gives the properties of three special German beams with wide flanges, used principally for columns. They will be referred to again in the following pages:

Fig. 29.—



Description	Area	Wt.	*Iab*	*Icd*	*Rab*	*Rcd*	*rab*	*rcd*
$5\frac{1}{3} \times 4\frac{1}{2} \times \frac{5}{16}$ I	5.25	17.5	23.22	6.41	9.07	2.81	2.12	1.11
$6\frac{1}{8} \times 5\frac{1}{8} \times \frac{3}{8}$ I	7.00	23.3	44.90	11.01	14.72	4.32	2.55	1.25
$8 \times 5\frac{7}{8} \times \frac{5}{16}$ I	8.60	29.0	96.50	15.50	23.89	5.40	3.35	1.34

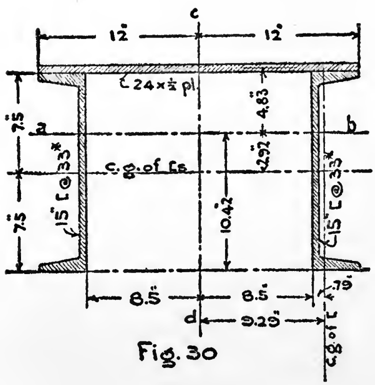


Fig. 30

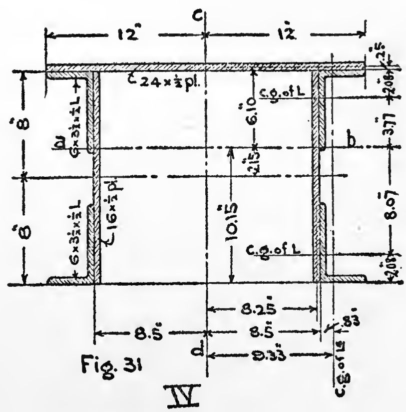


Fig. 31

IV

Fig. 30 represents a chord section composed of

two 15″ [s. @ 33lbs = 19.80
one 24 × ½ plate = 12.00
31.80□″

To find the position of the axis ab through the centre of gravity of the section, the simplest method is to take moments of the areas about the centre line of the channels and divide by the total area. The result will be the distance from the centre line of channels to the axis ab, thus:

	Areas.	Levers.	Moments.
Channels	19.80□″	× 0 =	0
Plate	12.00□″	× 7.75″ =	93
Totals	31.80□″		93

then $\frac{93}{31.80} = 2.92''$, which is the distance from centre line of channel to axis *ab*.

MOMENT OF INERTIA ABOUT AXIS *ab*.

I for $24\times\frac{1}{2}$ plate about its center of gravity $=\frac{bh^3}{12}=\frac{24\times.5^3}{12}$ $=$ .25

Area of plate into square of distance of its centre of gravity from axis *ab* $=12.0\times4.83^2=280.20$

I for 2.15″ [s@33lbs about their centre of gravity (from Carnegie) $=2\times312.6=625.20$

Area of channels into square of distance of their centre of gravity from axis $=19.80\times2.92^2=168.82$

1074.47

$$ab=1074.47.\quad R\ ab=\frac{I}{n}=\frac{1074.47}{10.42}=103.1.$$

$$I\ ab=\sqrt{\frac{I}{A}}=\sqrt{\frac{1074.47}{31.80}}=5.81.$$

MOMENT OF INERTIA ABOUT AXIS *cd*.

I for $24\times\frac{1}{2}$ plate about its centre of gravity $=\frac{bh^3}{12}=\frac{.5\times24^3}{12}=$ 576.00

I for 2.15 [s@33 lbs about their centre of gravity (from Carnegie) $=2\times8.23=$ 16.46

Area of channels into square of distance of their centre of gravity from axis $=19.8\times9.29^2=1708.82$

2301.28

$$I\ cd=2301.28.\quad Rcd=\frac{I}{n}=\frac{2301.28}{12}=191.77.$$

$$r\ cd=\sqrt{\frac{I}{A}}=\sqrt{\frac{2301.28}{31.80}}=8.51$$

Fig. 31 represents a chord section composed of

1 cover plate $24\times\frac{1}{2}$	$=12.00$
2 web plates $16\times\frac{1}{2}$	$=16.00$
4 angles $6\times3\frac{1}{2}\times\frac{1}{2}$	$=18.00$
	46.00□″

To find the position of the axis *ab* moments of the areas are taken about the centre line of the webs thus:

	Areas	Levers	Moments
Web plates and angles	34□″×	0 =	0
Cover plates	12□″×	8.25 =	99
Totals	46□″		99

then $\frac{99}{46}=2.15''$, which is the distance from centre line of web plates to axis *ab*.

The location of centre of gravity of angles is obtained from Carnegie.

MOMENT OF INERTIA ABOUT AXIS *ab*.

I for $24 \times \frac{1}{2}$ cover plate about its centre of gravity $= \frac{bh^3}{12} = \frac{24 \times .5^3}{12} = .25$

I for $2.16 \times \frac{1}{2}$ web plates about their centre of gravity $= \frac{bh^3}{12} = \frac{1 \times 16^3}{12} = 341.34$

I for $4.6 \times 3\frac{1}{2} \times \frac{1}{2}$ angles about their centre of gravity (from Carnegie) $= 4 \times 16.59 = 66.36$

Area of cover plate into square of distance of its centre of gravity from axis $= 12.0 \times 6.10^2 = 446.52$

Area of web plates into square of distance of their centre of gravity from axis $= 16.0 \times 2.15^2 = 73.96$

Area of upper angles into square of distance of their centre of gravity from axis $= 9.0 \times 3.77^2 = 127.92$

Area of lower angles into square of distance of their center of gravity from axis $= 9.0 \times 8.07^2 = 586.12$

1642.47

$$Iab = 1642.47. \quad Rab = \frac{I}{n} = \frac{1642.47}{10.15} = 161.81.$$

$$rab = \sqrt{\frac{I}{A}} = \sqrt{\frac{164}{46}} = 5.98.$$

MOMENT OF INERTIA ABOUT AXIS *cd*.

I for $24 \times \frac{1}{2}$ cover plate about its centre of gravity $= \frac{bh^3}{12} = \frac{.5 \times 24^3}{12} = 576.00$

I for $2.16 \times \frac{1}{2}$ web plates about their centre of gravity $= \frac{bh^3}{12} = \frac{16 \times .5^3}{12} = .37$

I for $4.6 \times 3\frac{1}{2} \times \frac{1}{2}$ angles about their centre of gravity (from Carnegie) $= 4 \times 4.25 = 17.00$

Area of web plates into square of distance of their centre of gravity from axis $= 16.0 \times 8.25^2 = 1089.00$

Area of angles into square of distance of their centre of gravity from axis $= 18.0 \times 9.33^2 = 1566.88$

3249.25

$$I\,cd = 3249.25. \quad R\,cd = \frac{I}{n} = \frac{3249.25}{12} = 270.77$$

$$r\,cd = \sqrt{\frac{I}{A}} = \sqrt{\frac{3249.25}{46}} = 8.40''$$

ART. 11.—EXAMPLES ILLUSTRATING THE METHOD OF DETERMINING THE SIZES OF BEAMS REQUIRED FOR VARIOUS CASES. THE MAXIMUM FIBRE STRESS NOT TO EXCEED 15,000 LBS. PER SQ. IN.

Fig. 32 represents a cantilever beam with a concentrated load of 3,000 lbs. 10 ft. from point of support. $M = 3{,}000 \times 10 = 30{,}000$ ft.-lbs. $30{,}000 \times 12 = 360{,}000$ inch-lbs.

$R = \frac{M}{f} = \frac{360{,}000}{15{,}000} = 24$. Turning to the table of I-beams in Carnegie, it will be found that a 10″ I @ 25 lbs has a moment of resistance (or section modulus) of 24.4. This is the beam required.

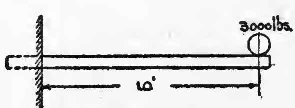


Fig. 32

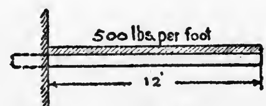


Fig. 33

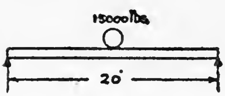


Fig. 34

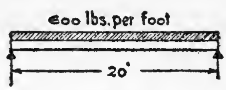


Fig. 35

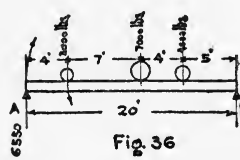


Fig. 36

Fig. 33 represents a cantilever beam, projecting 12 ft. from a wall, and loaded with a uniform load of 500 lbs. per lineal foot.

$M = \frac{500 \times 12^2}{12} = 36{,}000$ ft.-lbs. $36{,}000 \times 12 = 432{,}000$ inch-lbs.

$R = \frac{432{,}000}{15{,}000} = 28.8$. This case requires a 12″ I @ 31.5 lbs. R=36.

Fig. 34 represents a beam of 20-ft. span, with a centre load of 15,000 lbs.

$M = \frac{15{,}000 \times 20}{4} = 75{,}000$ ft. lbs. $75{,}000 \times 12 = 900{,}000$ inch-lbs.

$R = \frac{900{,}000}{15{,}000} = 60$. This case requires a 15″ I @ 45 lbs. R=60.8

Fig. 35 represents a beam of 20 ft. span, with a uniform load of 600 lbs. per foot.

$M = \frac{600 \times 20^2}{8} = 30{,}000$ ft.-lbs. $30{,}000 \times 12 = 360{,}000$ inch-lbs.

$R = \dfrac{360{,}000}{15{,}000} = 24.$ This requires a 10″ I @ 25 lbs. R=24.4.

Fig. 36 represents a beam of 20 ft. span loaded unevenly as shown.

For the reaction at A, moments are taken about B, thus

$$\begin{aligned} 4{,}000 \times 5 &= 20{,}000 \\ 7{,}000 \times 9 &= 63{,}000 \\ 3{,}000 \times 16 &= \underline{48{,}000} \\ &\ 131{,}000 \end{aligned}$$

131,000 ft.lbs. then 131,000÷20=6,550 lbs.=reaction at A.

The maximum bending moment will be under the 7,000 lbs. load. The reaction A acts with a lever arm about this point of 11 ft. tending to cause rotation in a right handed direction, and the load

Fig. 37

Fig. 38

Fig. 39

of 3,000 lbs. acts with a lever arm of 7 ft. about the same point tending to cause rotation in a left handed direction, as indicated by curved arrows. Then M = 6,550 lbs. × 11′ — 3,000 lbs. × 7′ = 51,050 ft.-lbs. 51,050 × 12 = 612,600 inch-lbs. $R = \dfrac{612{,}600}{15{,}000} = 40.8.$ This case requires a 15″ I @ 42 lbs. R = 58.9.

No beam should be used where the span exceeds twenty times the width of the flange unless supported laterally. Floor beams and stringers in buildings are usually stayed at much closer intervals, either by other beams framing into them or directly by the floor.

ART. 12.—COLUMNS AND STRUTS.

A column or strut may fail by crushing, by bending or by both combined. A short column will sustain a greater load than a long one of the same section, and a column with fixed (or square) ends, more than one with hinged (or pin) ends. Columns are usually divided into three classes, as illustrated in Figs. 37, 38 and 39. When overloaded, they will bend as shown.

There are several formulas for proportioning columns, but Rankine's (or Gordon's) is the one in most general use, and agrees closely with experiments. In it, a certain permissible unit stress for direct crushing is assumed, which is reduced for each special case, depending on the length of column, its radius of gyration, and the condition of its ends, whether square or pin.

RANKINE'S FORMULA.

For square ends $$P = \frac{F}{1 + \frac{l^2}{36{,}000\, r^2}}$$

One pin and one square end $$P = \frac{F}{1 + \frac{l^2}{24{,}000\, r^2}}$$

Both pin ends $$P = \frac{F}{1 + \frac{l^2}{18{,}000\, r^2}}$$

In which F = permissible compression per square inch for direct crushing.

P = permissible compression per square inch for columns.

l = length in inches.

r = least radius of gyration in inches.

To find the permissible stress per square inch for a column it is necessary to know its radius of gyration; so some section must be assumed, and if found to be too small or too large it will be necessary to try another. The work is greatly simplified by using the following table, in which F = 12,000 lbs., l = length of column in feet, and r = least radius of gyration in inches. For convenience, the length is taken in feet, and is multiplied by the factor 12 in the formula to reduce it to inches.

PERMISSIBLE COMPRESSION PER SQ. INCH FOR COLUMNS

12,000 LBS. REDUCED BY RANKINE'S FORMULA

Square Ends	Pin and Square Ends	Pin Ends
$\dfrac{12000}{1+\dfrac{(12l)^2}{36000\ r^2}}$	$\dfrac{12000}{1+\dfrac{(12l)^2}{24000\ r^2}}$	$\dfrac{120000}{1+\dfrac{(12l)^2}{18000\ r^2}}$

$\frac{l}{r}$	Square Ends	Square and Pin Ends	Pin Ends	$\frac{l}{r}$	Square Ends	Square and Pin Ends	Pin Ends
2.0	11,820	11,720	11,630	7.6	9,750	8,910	8,210
2.2	11,780	11,660	11,550	7.8	9,650	8,790	8,070
2.4	11,730	11,600	11,470	8.0	9,560	8,670	7,940
2.6	11,680	11,530	11,390	8.2	9,460	8,550	7,800
2.8	11,640	11,460	11,290	8.4	9,360	8,430	7,670
3.0	11,580	11,390	11,190	8.6	9,260	8,310	7,540
3.2	11,530	11,310	11,090	8.8	9,160	8,190	7,410
3.4	11,470	11,220	10,980	9.0	9,060	8,080	7,280
3.6	11,410	11,130	10,870	9.2	8,970	7,960	7,160
3.8	11,350	11,040	10,760	9.4	8,870	7,840	7,030
4.0	11,280	10,950	10,640	9.6	8,770	7,730	6,910
4.2	11,210	10,850	10,520	9.8	8,670	7,610	6,790
4.4	11,140	10,750	10,390	10.0	8,570	7,500	6,670
4.6	11,060	10,650	10,260	10.2	8,480	7,390	6,550
4.8	10,990	10,540	10,130	10.4	8,380	7,280	6,430
5.0	10,910	10,440	10,000	10.6	8,280	7,170	6,320
5.2	10,830	10,330	9,870	10.8	8,180	7,060	6,210
5.4	10,750	10,220	9,730	11.0	8,090	6,950	6,100
5.6	10,660	10,100	9,600	11.2	7,990	6,850	5,990
5.8	10,580	9,990	6,460	11.4	7,900	6,740	5,880
6.0	10,490	9,870	9,320	11.6	7,800	6,640	5,780
6.2	10,400	9,750	9,180	11.8	7,710	6,540	5,680
6.4	10,310	9,630	9,040	12.0	7,620	6,440	5,580
6.6	10,220	9,510	8,900	12,2	7,520	6,340	5,480
6.8	10,130	9,390	8,760	12.4	7,430	6,240	5,380
7.0	10,030	9,270	8,620	12.6	7,340	6,150	5,290
7.2	9,940	9,150	8,480	12.8	7,250	6,060	5,190
7.4	9,840	9,030	8,350	13.0	7,160	5,960	5,100

Supposing it is required to know the permissible stress per square inch on a column 20 feet long, with two pin ends and radius of gyration of 4 inches,

Then $\frac{l}{r}=\frac{20}{4}=5$. In the column headed "Pin Ends," and opposite the number 5.0 in column for $\frac{l}{r}$, will be found 10,000, which is the unit stress required.

The same table may be used for any other value of F (say 15,000), for example: If 10,000 lbs. per square inch be the permissible stress when the factor F = 12,000, then $10,000 \times \frac{15,000}{12,000} = 12,500$ lbs. per square inch is the permissible stress when the factor F = 15,000.

Columns in building are usually figured as though they had pin ends, which is an error on the safe side and compensates to a certain extent for eccentric loading and any slight unevenness in the foundation. The end sections of top chords in pin-connected bridges are usually figured as "square and pin end" columns, and the intermediate sections as square end columns. The posts are usually figured as "pin end" columns whether connected by pins or otherwise. These rules are not always adhered to and one must be guided by the specification under which the structure is to be designed.

ART. 13.—EXAMPLES ILLUSTRATING METHOD OF DESIGNING COLUMNS AND STRUTS.

A column 20 ft. long is required to support a load of 150,000 lbs. Unit stress, 12,000 lbs., reduced for pin ends by Rankine's formula.

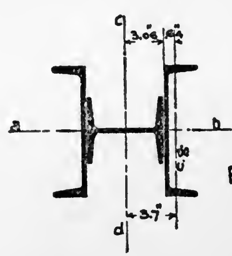


Fig. 40

Fig. 40 represents a trial section consisting of:

1—$6\frac{1}{8} \times 5\frac{1}{8} \times \frac{3}{8}$ I @ 23.3 lbs.	= 7.00	(Table Art. 10.)
2—10″ [s @ 15 lbs.	8.92	(Carnegie)
	15.92 square inches.	

For the least radius of gyration:

I *ab* = (for 2.10″[s. See Carnegie) 66.9×2	= 133.80
(for $6\frac{1}{8} \times 5\frac{1}{8}$ I. See table Art. 10)	11.01
	144.81

$$rab = \sqrt{\frac{I}{A}} = \sqrt{\frac{144.81}{15.90}} = 3.02''$$

I *cd* (for $6\frac{1}{8} \times 5\frac{1}{8}$ I. See table Art. 10) = 44.90
(for 2.10″ [s about c. g. See Carnegie) $2.30 \times 2 =$ 4.60
Area of [s. into sq. of dist. of their c.g. from $cd = 8.92 \times 3.7^2 =$ 122.50

172.00

$$r\,cd = \sqrt{\frac{I}{A}} = \sqrt{\frac{172.00}{15.92}} = 3.28''$$

Thus the least radius of gyration = 3.02″, and $\frac{l}{r} = \frac{20}{3.02} = 6.6$.

The value corresponding to this in table (Art. 12) is 8,900. Then 150,000 ÷ 8,900 = 16.8 sq. inches required. The trial section is slightly too small, but it is only necessary to use heavier channels of the same size. The radius of gyration will remain practically the same. The column should be made of

1—$6\frac{1}{8} \times 5\frac{1}{8} \times \frac{3}{8}$ I @ 23.3 lbs. =	7.00
2—10″ [s @ 20 lbs. =	11.76
	18.76 sq. inches

Columns sustaining comparatively light loads are frequently made of a single I-beam with wide flanges (see table, Art. 10).

A column 16 ft. long is required to sustain a load of 45,000 lbs. Unit stress 12,000 lbs., reduced for pin ends by Rankine's formula.

The area of a $6\frac{1}{8} \times 5\frac{1}{8} \times \frac{3}{8}$ I is 7.0 sq. ins., and the least radius of gyration, 1.25″. $\frac{l}{r} = \frac{16}{1.25} = 12.8$. The value corresponding to this in table (Art. 12) is 5,190 lbs. per sq. in. Therefore the capacity of this column is 5,190 × 7.0 sq. ins = 36,330 lbs., which is too small.

The area of an $8 \times 5\frac{7}{8} \times \frac{5}{16}$ I is 8.6 sq. ins. The least radius of gyration is 1.34″. $\frac{l}{r} = \frac{16}{1.34} = 11.9$. The value corresponding to this in table is 5,630 lbs. per sq. in. Then 5,630 × 8.6 sq. ins. = 48,500 lbs. Therefore this beam is suitable for the purpose.

ART. 14. RIVETS AND RIVETING.

Sizes of Rivets used in structural steel work are $\frac{1}{2}''$, $\frac{5}{8}''$, $\frac{3}{4}''$, $\frac{7}{8}''$ and 1″, Those in most general use are $\frac{3}{4}$ and $\frac{7}{8}$. The smaller ones are used only in very light members to avoid cutting out too much section. Rivets larger than $\frac{7}{8}$ are only used when it is impossible to get in enough of a smaller size for lack of space.

Spacing of Rivets.—The distance center to center of rivets should not be less than three times their diameter. In compression members, rivets should not be further apart than sixteen times the thickness of the outside plates, in line of stress, and generally should not exceed six inches. The distance from centre of rivet to end of member should not be less than one and one-half times the diameter of rivet or generally $1\frac{1}{4}$ ins. for $\frac{3}{4}$ rivets, and $1\frac{1}{2}$ ins. for $\frac{7}{8}$ rivets.

Size of Rivet Holes.—In ordinary work the holes are punched 1-16″ larger than rivet, but in more particular work they are punched about $\frac{1}{8}''$ smaller, and reamed after assembling to 1-16″ larger than rivet. Holes cannot be punched in metal of greater thickness than diameter of rivet, and in this case they must be drilled. In tension members, the area of the rivet holes is deducted from the gross area, allowance being made for holes $\frac{1}{8}''$ larger than rivets. In compression members no allowance need be made for rivet holes.

Strength of Rivets.—Rivets may fail by shearing, by crushing (or bearing) or by tension on the heads. Generally it is not considered good practice to use rivets in tension, as their strength in this direction is somewhat uncertain on account of the initial stress in them from cooling, but it is sometimes unavoidable to use rivets in this way.

Permissible Unit Stresses.—In buildings and highway bridges 9,000 lbs. per square inch for shear and 18,000 lbs. per square inch for bearing are usually allowed for shop driven rivets; and for field rivets, 7,500 lbs. shear and 15,000 bearing.

In railway work 7,500 lbs. per square inch shear and 15,000 lbs. per square inch bearing are the usual unit stresses for shop rivets; and 6,000 lbs. shear and 12,000 lbs. bearing for field rivets.

Shearing and Bearing Value of Rivets.—The shearing value of a rivet is equal to the area of its cross section multiplied by the permissible shear per square inch. Thus the shearing value of a $\frac{3}{4}''$ rivet at 7,500 lbs. per square inch $=$ .4418 square inches $\times$ 7,500 lbs. $=$3,310 lbs. The bearing value is equal to the diameter of the rivet multiplied by the thickness of metal on which it bears, multiplied by the permissible bearing per square inch. Thus the bearing value of a $\frac{3}{4}$ rivet on a $\frac{3}{8}''$ plate at 15,000 lbs per square inch $= \frac{3}{4} \times \frac{3}{8} \times 15{,}000$ lbs. $=$ 4,220 lbs.

Rivets may be either in single or double shear. In this first case the joint could fail by the rivets shearing in one plane only, that between the two members joined (see Fig. 41). When rivets are in double shear, they would have to be sheared in two planes, as shown in Fig. 42, before the joint could fail in that manner, and they would have twice the value of rivets in single shear. In most cases, however, when rivets are in double shear, their bearing value is less

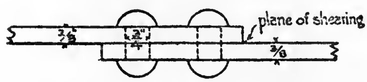


Fig. 41

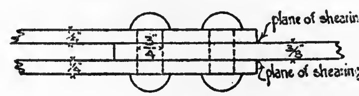


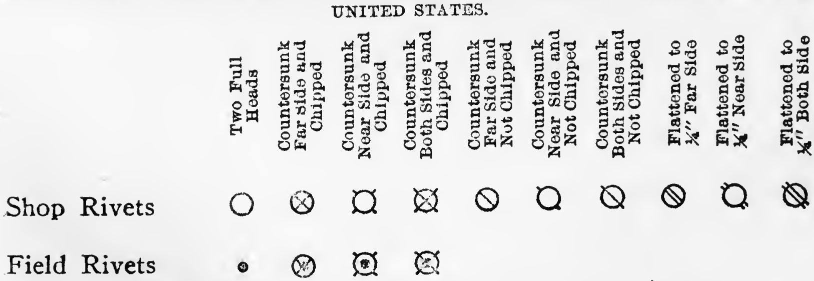
Fig 42

than twice their shearing value, and so the bearing value determines the strength of the joint. In Fig. 41 each rivet is good for 3,310 lbs. in shear and 4,220 lbs. in bearing, so the shearing value governs. In Fig. 42 the rivets are each good for $3,310 \times 2 = 6,620$ lbs. in shear and 4,220 lbs. in bearing, therefore the bearing value determines the strength. If the centre member were $\frac{1}{2}''$ thick the bearing value of each rivet would be $\frac{3}{4} \times \frac{1}{2} \times 15,000 = 5,620$ lbs.

In designing a riveted connection great care must be taken always to use the least value a rivet can have under the circumstances, whether single shear, double shear or bearing,

Tables of shearing and bearing values of rivets may be found in Carnegie's, Pencoyd's and other hand-books.

CONVENTIONAL SIGNS FOR RIVETING IN GENERAL USE IN CANADA AND THE UNITED STATES.



ART. 15. THE COMPLETE DESIGN OF A ROOF TRUSS FOR BUILDING WITH MASONRY OR BRICK WALLS. CAPABLE OF WITHSTANDING WIND PRESSURE.

Data: Width of building out to out of walls 40′.

Thickness of Walls, 1′ 6″.

Span of trusses 38′ 6″ centre to centre of bearings.

Slope of rafters 30°.
Trusses spaced 16′ centres.
Total load, including weight of trusses, roof covering, snow and wind 50 lbs. per square foot of horizontal projection.

Unit Stresses: Tension, 15,000 lbs. per square inch.
Compression, 12,000 lbs. per square inch, reduced by Rankin's formula, the rafters to be considered as columns with square ends, and the compression web members as columns with pin ends, length not to exceed 120 times least radius of gyration.
Rivet shear, 7,500 lbs. per square inch.
Rivet bearing, 15,000 lbs. per square inch.

Total load on truss = 38.5′×16′×50 lbs. = 30,800 lbs., and since the rafters are divided into 8 panels, each panel load = 30,800 ÷ 8 = 3,850 lbs.

Purlins.—Span 16′, load 3,850 lbs. $M = \frac{4{,}850 \times 16}{8} = 7{,}700$ ft.-lbs = 92,400 inch-lbs.

$R = \frac{M}{f} = \frac{92{,}400}{15{,}000} = 6.16$. For intermediate purlins 6″ I s @ 12.25 lbs. will be used, for which R = 7.3. For the end purlins which support only one-half panel load and, at the centre where there are two purlins, 6″ [s @ 8 lbs. will be used.

The truss is of the Fink pattern, and the method of constructing the stress diagram is fully described in Art. 3. The stresses, which are scaled from the stress diagram Fig. 43a, are all written on the diagram of truss, Fig. 43. The required areas of the tension members are obtained directly by dividing the stresses by the permissible unit stress as shown. Then suitable angles are selected from the hand-books of the rolling mills which give the areas for all standard sizes. Allowance must be made for rivet holes: in members subjected to small stress, and connected by one leg only, the area of but one hole need be deducted from each angle; but where angles are connected by both legs it is advisable to allow for two holes in each angle. Angles requiring more than three rivets should, when possible, be connected by both legs. In the present example $\frac{3}{4}''$ rivets will be used, and allowance made for $\frac{7}{8}''$ holes. No angle smaller than $2\frac{1}{2} \times 2 \times \frac{1}{4}$ will be used, and, when necessary to connect both legs, $2\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}$ will be the minimum.

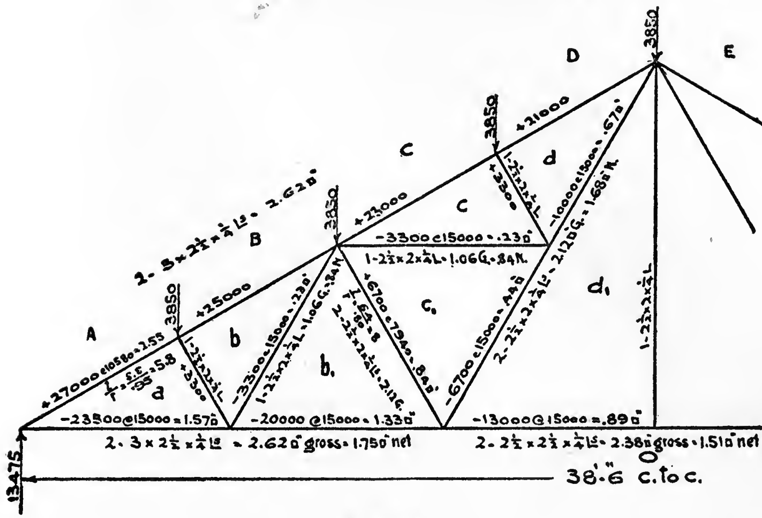


Fig. 43

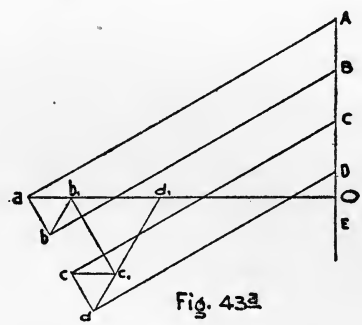


Fig. 43a

Referring to diagram Fig. 43, it will be seen that member *aO* requires 1.57 □′;

then two 3 × 2½ × ¼ Ls = 2.62 □″ gross area
less 4 × ⅞ × ¼ = .87 □″ area of 4 holes

1.75 □″ net area.

In order not to make too many splices, the same angles will be be used for member b_1O.

Member d_1O requires .89□′.

Two 2½×2½ × ¼ Ls = 2.38 □″ gross area
less 4 × ⅞ × ¼ = .87 □″ area of 4 holes

1.51 □″ net area.

Member dd_1 requires .67 □″. Since the stress in this member is small it only requires to be connected by one flange.

$$\begin{array}{lrl} \text{Two } 2\tfrac{1}{2} \times 2 \times \tfrac{1}{4} \text{ Ls} = & 2.12 \;\square'' & \text{gross area} \\ \text{less } 2 \times \tfrac{7}{8} \times \tfrac{1}{4} \quad = & \underline{.44} \;\square'' & \text{area of 2 holes} \\ & 1.68 \;\square'' & \text{net area.} \end{array}$$

The same angles will be used for members c_1d_1.

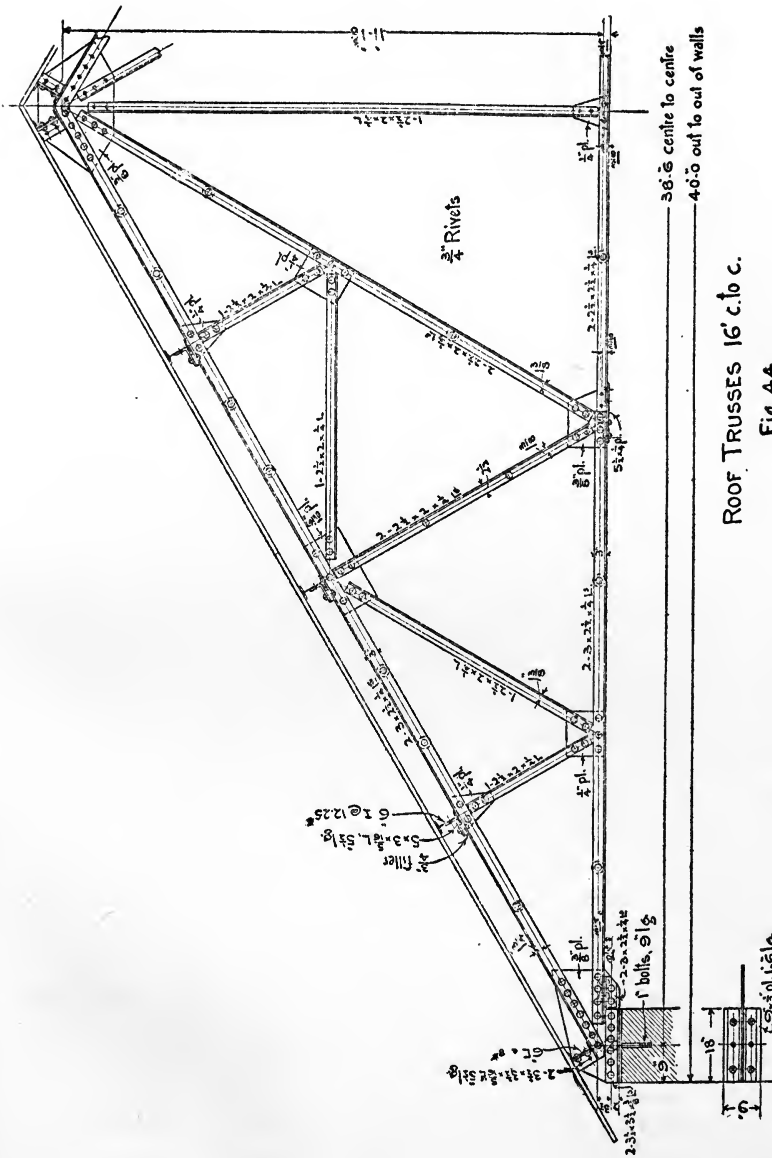
Members bb_1 and cc_1 each require .22 □″.

$$\begin{array}{lrl} \text{One } 2\tfrac{1}{2} \times 2 \times \tfrac{1}{4} \text{ L} = & 1.06 \;\square'' & \text{gross area} \\ \text{less } 1 \times \tfrac{7}{8} \times \tfrac{1}{4} \quad = & \underline{.22} \;\square'' & \text{area of 1 hole} \\ & .84 \;\square'' & \text{net area.} \end{array}$$

Members ab and cd each sustain a compressive stress of 3,300 lbs. and their length is 3.2 feet (about). One $2\frac{1}{2} \times 2 \times \frac{1}{4}$ L is assumed and its least r = .43. $\frac{l}{r} = \frac{3.2}{.43} = 7.4$. By table of compression values (Art. 12), the permissible stress per square inch = 8,350 lbs., then $3{,}300 \div 8{,}350 = .40$ □″ required, and the area of one $2\frac{1}{2} \times 2 \times \frac{1}{4}$ L = 1.06 □″. Rivet holes are not deducted from the area of compression members, as the rivets are supposed to fill the holes completely, and transmit the pressure from one side of the hole to the other.

Member b_1c_1 sustains a compressive stress of 6,700 lbs. and its length is about 6.4 ft. Two $2\frac{1}{2} \times 2 \times \frac{1}{4}$ Ls are assumed, with the longer legs back to back, but separated about ¼″, thus:]⌈, to straddle the connection plates at the ends. Tables of radii of gyration for two angles are given in Carnegie's and Pencoyd's hand books. They are computed for the maximum and minimum thicknesses only, but values for intermediate thicknesses may be interpolated with sufficient accuracy. From these tables the least radius of gyration for two $2\frac{1}{2} \times 2 \times \frac{1}{4}$ Ls as above is found to be .80″. Then $\frac{l}{r} = \frac{6.4}{.80} = 8$ which (in table Art. 12) corresponds to a unit stress of 7,940 lbs. and $6{,}700 \div 7{,}940 = .84$ square inches required. The area of two $2\frac{1}{2} \times 2 \times \frac{1}{4}$ Ls = 2.12 □″ which is more than twice the area required, but a single angle would be too small, because its radius of gyration would be much less.

Member Aa sustains a compressive stress of 27,000 lbs. and its length is about 5.5 feet. Two $3 \times 2\frac{1}{2} \times \frac{1}{4}$ Ls are assumed, with



ROOF TRUSSES 16' c. to c.

Fig. 44

the longer legs back to back, thus ˥˹. The area = 2.62 □″ and the least $r = .95$. Then $\frac{l}{r} = \frac{5.5}{.95} = 5.8$, which corresponds to a permissible unit stress of 10,580 lbs. for square ends, and 27,000 ÷ 10,580 = 2.55 □″ required. Therefore the trial section is suitable, its area being slightly greater than that required.

It is unnecessary to consider the remaining panels of the rafters as the same angles will be used throughout.

At the centre of the truss a small angle hanger is used to prevent the bottom chord from sagging.

Details.—Fig. 44 is a detail drawing. The height at centre is obtained by multiplying one-half the centre to centre span by the tangent of 30°.

At the ends of truss a $\frac{3}{8}$″ plate is used to connect the rafters with the bottom chord. The rivets are in double shear. Referring to table of rivet values in Carnegie or some other hand-book, the shearing value of $\frac{3}{4}$ rivets at 7,500 lbs = 2 × 3,310 = 6,620 lbs., but the bearing value at 15,000 is only 4,220 lbs., which latter must be used.

The number of rivets required in rafter = 27,000 ÷ 4,220 = 7; and the number of rivets in bottom chord = 23,500 ÷ 4,220 = 6. 4 rivets are used in the main angles of bottom chord and 2 rivets in the lock angles. This arrangement not only requires a smaller gusset plate than if all rivets were put in one line, but it distributes the stress much better in the angles.

The bearing plates on the walls must be large enough to distribute the load. If the trusses are to rest directly on a brick wall, the load per square inch should not exceed 100 lbs. Since the total load on truss is 30,800 lbs. the reaction at each end will be 30,800 × $\frac{1}{2}$ = 15,400, and 15,400 ÷ 100 = 154 square inches required in bearing plate. The plate used (9 × 18 = 162 square inches) slightly exceeds this area.

In member *ab* there is a stress of 3,300 lbs. The rivets in this case are in single shear = 3,310 lbs., but the bearing value on ¼″ plate is only 2,810 lbs. Two rivets are sufficient for this member, as well as for *cd*, bb_1 and cc_1.

In member b_1c_1 there is a stress of 6,700 lbs. The rivets are in double shear and their bearing value on $\frac{5}{16}$ plate = 3,520 lbs. each. Two rivets will do here also.

The bottom chord is spliced at panel points near centre of span.

In b_1O the stress = 20,000 lbs. and the value of the connection is as follows:

3 rivets in bearing on	3/8 plate @ 4,220 lbs.	=	12,660
4 rivets in bearing on	5½ × ¼ plate @ 2,810 lbs,	=	11,240
			23,900 lbs.

In d_1O the stress = 13,000 lbs. and the value of connection is:

2 rivets in bearing on	3/8 plate @ 4,220 lbs.	=	8,440
4 rivets in bearing on	5½ × ¼ plate @ 2,810 lbs.	=	11,240
			19,680 lbs.

At apex of rafters there is a 3/8 gusset plate and the rivets are good for 4,220 lbs. each.

The stress in Dd=21,000, then 21,000÷4,220=5 rivets required.

The stress in dd_1 =10,000, then 10,000÷4,220=3 rivets required.

The gusset plates at ends of truss and at apex extend above the rafters. By this arrangement the stresses in them are better distributed.

Purlins.—Although the purlins are designed for vertical loads, for convenience they are set normal to the rafters. If unsupported laterally they would be liable to fail through side bending, but the roof covering is depended on for this contingency.

ART. 16.—ROOF TRUSSES SUPPORTED BY STEEL COLUMNS.

When roof trusses are set on solid brick or masonry walls having sufficient stability to withstand the wind pressure, it is not usually customary to figure the wind stresses in the trusses separately, the vertical load being assumed large enough to cover everything, as in Art. 15. But when supported by steel columns and braced thereto to resist the overturning effect of the wind, it is advisable to treat the wind force and the vertical loads separately. Wind is usually taken at 30 lbs. per square foot acting in a horizontal direction against a vertical plane, and on sloping surfaces it is reduced by the following table of co-efficients which are based on Unwin's experiments.

CO-EFFICIENTS FOR WIND PRESSURE NORMAL TO PLANE OF ROOF.

Angle of Roof	5°	10°	20°	30°	40°	50°	60° to 90°
Co-efficient	.125	.24	.45	.66	.83	.95	1.00

The vertical load consisting of the weight of trusses, roof covering and snow may then be taken at about 35 lbs. per square foot of horizontal projection.

ART. 17.—THE DESIGN OF A KNEE-BRACED MILL BUILDING.

Data: Width of building 40′ 0″ centre to centre of posts.
Height of posts 18′ 0″.
Angle of roof 30°.
Trusses spaced 16′ 0″ centre to centre.
Roof covered with 3″ × 5″ planks on edge.
Sides covered with 3″ tongued and grooved planks, fastened to posts with 3″ railway spikes.
Roof load (dead load and snow) 40 lbs. per sq. ft. of horizontal projection.
Horizontal wind force 30 lbs. per sq. ft.
Wind pressure normal to roof. 30 lbs. × .66 = 20 lbs. per sq. ft.

Unit stresses same as in Art. 15.

Wind Stresses.—The wind pressure on side of building, Fig. 45, is assumed to be concentrated at top of post, at foot of knee brace, and at base of post. The last is neglected, as it has no overturning effect on building.

Horizontal wind force at top of post $= 16' \times \frac{5'}{2} \times 30$ lbs. $= 1,200$ lbs.

" " " " foot of knee brace $= 16' \times \frac{18'}{2} \times 30$ lbs. $= 4,320$ lbs.

Wind pressure on roof = 23′ × 16′ × 20 lbs. = 7,360 lbs., say 7,400 lbs. The intermediate panel loads will then be 7,400 ÷ 4 = 1,850 lbs. each, and the end panel loads 925 lbs. each, as shown on diagram. The resultant of the wind on roof acts at the middle point of rafter. Its vertical component = 6,430 lbs. and its horizontal component = 3,700 lbs.

Reactions.—The horizontal forces are assumed to be resisted equally by both posts, which assumption is undoubtedly accurate enough for all practical purposes.

Horizontal reaction for each post $= (3,700 + 1,200 + 4,320) \times \frac{1}{2} = 4,610$ lbs.

If the posts were free to rotate at their base, the vertical reactions due to the horizontal forces would be obtained by taking moments of these forces about foot of posts and dividing by their distance centre to centre. But the posts are more or less fixed by the dead load of roof and walls, also by the anchor bolts, if properly built into foundations. Consequently there will be a point of no moment somewhere between base of posts and foot of knee brace. This point of no moment, or of contra-flexure, should never be assumed higher than half-way between base of post and knee-brace connection. The existence of a resisting moment at foot of each post changes the vertical reactions from those determined by pure statics. Taking the weight of roof at 20 lbs. per sq. ft. and of the sides at 10 lbs. per sq. ft. the dead load on post is as follows:

$$\begin{aligned} \text{Roof } 16' \times 20' \times 20 \text{ lbs.} &= 6{,}400 \\ \text{Side } 16' \times 18' \times 10 \text{ ``} &= \underline{2{,}880} \\ \text{Total} \ldots\ldots\ldots\ldots &\ 9{,}280 \text{ lbs.} \end{aligned}$$

This force is assumed to act at centre of post which is taken

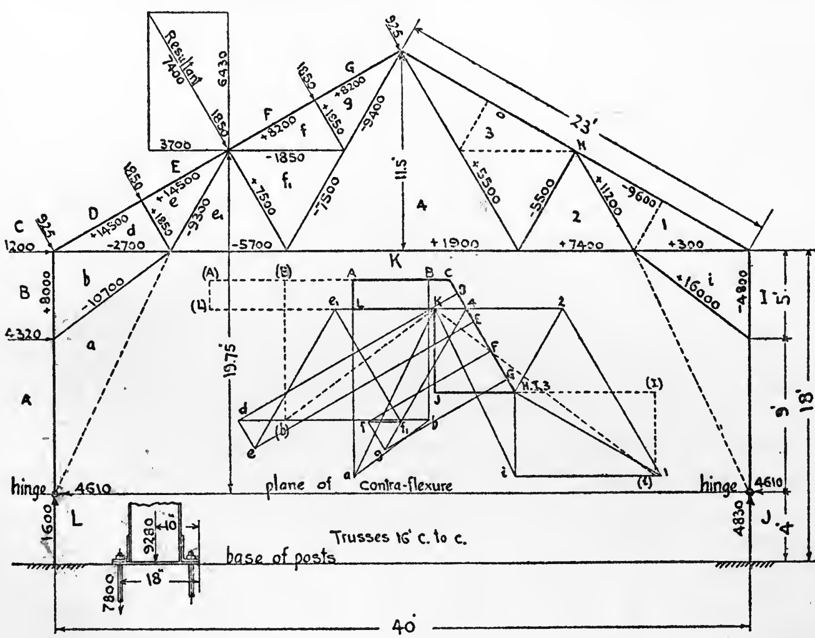


Fig. 45

12″ wide, with a base plate 20″ wide. It will then have a lever arm of 10″ about edge of base. One-inch anchor bolts are assumed, which, allowing for thread, are equivalent to $\frac{13}{16}''$ dia. = .52 sq. in. each. The value of one bolt will be .52 sq. in. × 15,000 lbs. = 7,800 lbs., and it will have a lever arm of 18″ from edge of base.

$$\begin{aligned} \text{Then, moment of resistance at base} = 9{,}280 \text{ lbs.} \times 10'' &= 92{,}800 \\ 7{,}800 \text{ ``} \times 18'' &= \underline{140{,}400} \\ &\ 233{,}200 \text{ in.-lbs.} \end{aligned}$$

The horizontal reaction multiplied by the distance from base of post to point of contra-flexure will be equal to the moment of resistance at base; therefore, distance to point of contra-flexure = $\frac{233,200}{4,610}$ = 50 inches. The plane of contra-flexure will be assumed 4 ft. above base, where the posts are considered to be hinged, as shown.

For the vertical reactions due to horizontal wind forces, moments of these forces will be taken about the hinges.

VERTICAL REACTION L A.

$$
\begin{array}{lrl}
3,700 \times 19.75 & = & 73,075 \\
1,200 \times 14 & = & 16,800 \\
4,320 \times 9 & = & \underline{38,880}
\end{array}
$$

From Horizontal Wind forces = 128,755 ft.-lbs. ÷ 40′ = — 3,220
" Vertical Wind forces = 6,430 × ¾ = + 4,820
+ 1,600 lbs.

VERTICAL REACTION J K.

From Horizontal Wind forces = + 3,220
" Vertical Wind forces 6,430 × ¼ = + 1,610
+ 4,830 lbs.

Stress Diagram. In order to proceed with the stress diagram without further figuring, imaginary struts are provided, as shown in dotted lines.

The external forces may now be laid off and the stress diagram constructed as follows: Beginning with the force *AB*, the external forces are taken in regular order in going around the frame in a right-handed circular direction, and plotted in the stress diagram, the last force *LA* closing the diagram. These external forces are shown in heavy lines. For wind stresses it is necessary to construct the stress diagram for the whole truss, as the stresses on opposite sides are very different. Beginning at the left hand hinge, there are two known forces *KL* and *LA*, and two unknown forces *Aa* and *aK*. From the point *A* in diagram of external forces a line is drawn parallel with the member *Aa;* and from the point *K*, a line parallel with *aK*, the two lines intersecting in the point *a*. In going around this joint in a right handed direction, and following the forces in the stress diagram, it will be observed that *Aa* acts towards the joint, which indicates that the member is in compression; and that *aK* acts away from it, which indicates tension.

Next, at foot of knee brace, there are now but two unknown forces *Bb* and *ba*. From the point *B* in stress diagram, a line is drawn parallel with the member *Bb;* and from the point *a*, a line parallel with *ba*, the two intersecting in the point *b*. *Bb* acts towards the panel point under consideration, indicating compression; and *ba* towards it also indicating compression. At top of post there are now only two unknown forces, *Dd* and *db*. From point *D* in stress diagram a line is drawn parallel with *Dd;* and from the point *b*, a line parallel with *db*, intersecting in the point *d*. *Dd* acts towards the panel point, indicating compression; and *db* away from it indicating tension. The next joint to be considered is panel point *DE*. From point *E* in stress diagram, a line is drawn parallel with member *Ee;* and from point *d*, a line parallel with member *de*, the two intersecting in point *e*. *Ee* acts towards the joint, indicating compression; and *ed* towards it, indicating compression. At the point where the knee brace connects with the bottom chord, there are now but two unknown forces ee_1 and e_1K. From the point *e* in stress diagram, a line is drawn parallel with member ee_1; and from the point *K*, a line parallel with member e_1K, the two lines intersecting in the point e_1. ee_1 acts away from the panel point, indicating tension; and e_1K also acts away from it, indicating tension. At panel point *EF* there are three unknown forces. Ff, ff_1 and f_1e_1, but the force polygon for this joint may be completed by drawing e_1f_1 of such length that the point f_1 will be half-way between the two parallel lines drawn from the points *F* and *G*. Then from the point f_1 a line is drawn parallel with member ff_1 which intersects the line drawn from the point *F* in the point *f*. *Ff* acts towards the panel point, indicating compression; ff_1 acts away from it, indicating tension; and f_1e_1 acts towards it, indicating compression. The remainder of the stress diagram is quite simple and requires no further explanation. The point *3* happens to coincide with the point H, indicating that there is no stress in member *H3*.

The imaginary struts are now supposed to be removed, and the stress diagram corrected accordingly. The corrections are shown in dotted lines with the points of intersection marked by letters in parentheses. From the point K in stress diagram a dotted line is drawn parallel with the left-hand knee brace, intersecting the line *db* in the point (*b*). At the foot of knee brace, the force *K*(*L*) is required to complete the polygon of forces. This force is supplied by the resistance of post to bending. At the top of post the force BC is increased to $(B)C$. The difference $(B)B$ is equal to the horizontal force at hinge, multiplied by the distance from hinge to foot of knee brace, and divided by the distance from foot of knee brace to top of post $= 4{,}610 \times \frac{9}{5} = 8{,}300$ lbs. The horizontal force $K(L)$ at foot of knee brace is equal to the horizontal force at hinge, plus the force $(B)B$ at top of post $= 4{,}610 + 8{,}300 = 12{,}910$ lbs. At the top of right-hand post, there is also a horizontal force due to the moment of the horizontal force at hinge $= 8{,}300$ lbs., and represented by the dotted line $H(I)$ in stress diagram. At the foot of knee brace the horizontal force to be resisted by the bending value of the post, is the same as for left-hand post, and is represented in the stress diagram by the line $J(I)$.

The wind stresses are all figured on the truss diagram Fig. 45, the sign + indicating compression, and the sign —, tension. If the stress diagram should not close exactly at first, it would be better to work from both ends of truss towards the centre.

The maximum bending moment in the posts is at the point of knee-brace connection, and is equal to the horizontal reaction at hinge multiplied by its distance from this point, $= 4{,}610$ lbs. $\times$ 9 ft. $= 41{,}490$ ft.-lbs. $= 497{,}880$ in.-lbs.

Stresses Due to Vertical Loads.—The total vertical load on truss $= 40' \times 16' \times 40$ lbs. $= 25{,}600$ lbs. The intermediate panel loads $= 25{,}600 \div 8 = 3{,}200$ lbs., and the end panel loads $= 1{,}600$ lbs.

Fig. 46 is a diagram of one-half of the truss, with stress diagram for vertical loads. The stresses due to vertical loads which are marked "V," as well as the maximum wind stresses which are marked "W," are shown on the truss diagram, and those of the same kind added together.

Bending in Rafters.—In addition to direct compression in rafters, there are bending moments due to the loads which, in this case, are uniformly distributed, instead of being concentrated at the panel points by purlins as in Art. 15. For the bending moment in each panel a total load of 60 lbs. per sq. ft. will be assumed. The horizontal length, or span, from one panel point to another $= 5$ ft. Load on span $= 5' \times 16' \times 60$ lbs. $= 4{,}800$ lbs. Bending moment for simple span $= \dfrac{4{,}800 \times 5}{8} = 3{,}000$ ft.-lbs. But these spans are continuous, or fixed at the ends, which reduces the bending moment. The points of maximum moment are at the ends, or panel points, and the moment at these points is equal to two-thirds of the bending moment for a span of the same length but not fixed at the ends, $= 3{,}000 \times 2/3 = 2{,}000$ ft.-lbs. $= 24{,}000$ in.-lbs.

Proportioning of Members.—For the rafters angles must be selected of such section that the maximum stress per sq. in., due to the combination of direct compression and bending, shall not exceed 12,000 lbs. $2 - 5' \times 3 \times \frac{5}{16}$ angles will be assumed with the longer legs vertical. Area $= 4.8$ sq. ins. $R = 3.78$. Then

Max. compression	$\div$ area	$= 36{,}900 \div 4.8$	$=$	7,690
Max. bending	$\div$ R	$= 24{,}000 \div 3.78$	$=$	6,350
				14,040 lbs. per sq. in.

Since the total fibre stress as above is too great, $2 - 5 \times 3 \times 3/8$, Ls will be tried next. Area $= 5.72$ sq. ins. $R = 4.42$, then

$36{,}900 \div 5.72 =$	6,450
$24{,}000 \div 4.42 =$	5,430
	11,880 lbs. per sq. in.

This is satisfactory, and the same angles will be used throughout the rafters to avoid splicing.

In members *de* and *fg* there is a compression stress of 4,650 lbs., and their length is about 3.3 ft. Assuming $1 - 2\frac{1}{2} \times 2 \times \frac{1}{4}$ L $= 1.06$ sq. ins. least $r = .43''$. Then $\frac{1}{r} = \frac{3.3}{.43} = 7.6$, which by table (Art. 12) corresponds to 8,210 lbs. per sq. in. for pin ends, and $4{,}650 \div 8{,}210 = .56$ sq. ins. required. The area provided is nearly double this amount.

In members e_1f_1 the compression $= 13{,}100$ lbs., length $= 6.6$ ft. $2 - 2\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}$ angles will be assumed, area $= 2.38$ sq. ins. least $r = .77$. $\frac{1}{r} = \frac{6.6}{.77} = 8.7$, corresponding to 7,470 lbs. per sq. in. Then $13{,}100 \div 7{,}470 = 1.75$ sq. ins. required.

Members ee_1 will be made of the same section as e_1f_1.

The knee braces must be designed for $-$ 10,700 lbs. or $+$ 1,6000 lbs. Assuming $2 - 3 \times 2\frac{1}{2} \times \frac{1}{4}$ angles with the longer legs back to back, area $= 2.62$ sq. ins. least $r = .95$. $l = 8.3$. $\frac{1}{r} = \frac{8.3}{.95} = 8.7$, corresponding to a unit stress of 7,470 lbs. Then $1{,}6000 \div 7{,}470 = 2.14$ sq. ins. required.

The sections required and those provided for the tension members are all figured on diagram Fig. 46.

The posts must be designed for bending stresses as well as direct

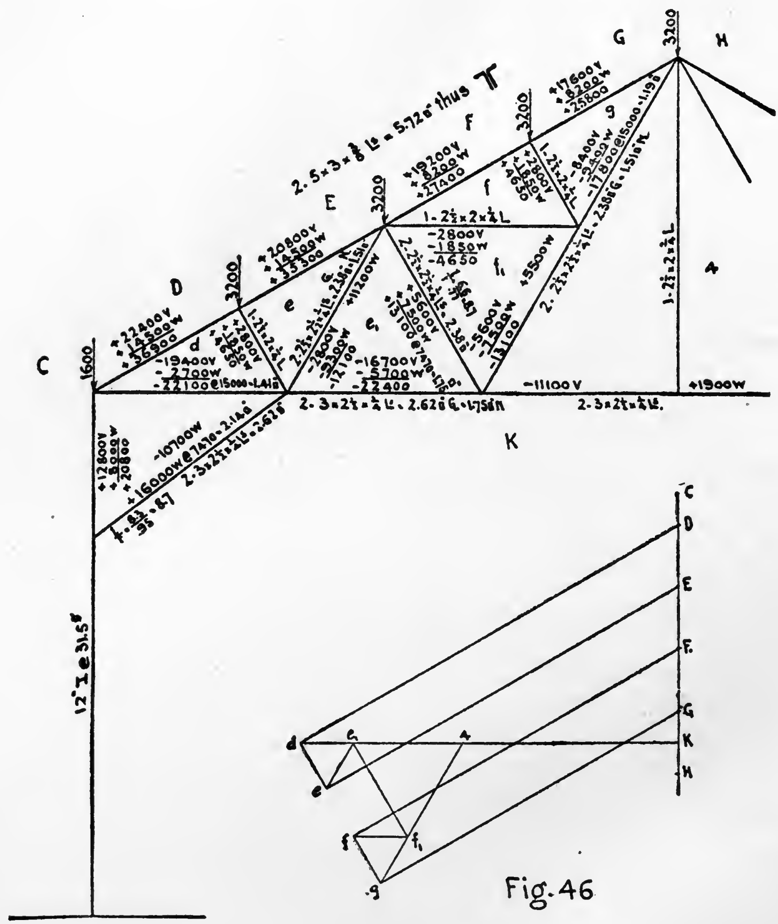


Fig. 46

compression. A 12 in. I @ 31.5 lbs. is assumed. Area = 9.26 sq. ins. R = 36, then

Direct compression ÷ area	= 20,800	lbs. ÷ 9.26	=	2,250
Bending moment ÷ R	= 497,880	in.-lbs. ÷ 36	=	13,830
				16,080 lbs. per sq. in.

The maximum compression as above is greater than that allowed for the other members, but since it is nearly all due to bending from a maximum wind force which the building will rarely, if ever, receive, and as the posts are supported sidewise by the planking, this unit stress is not at all excessive.

Anchorage for Posts.—The value of one anchor bolt has been

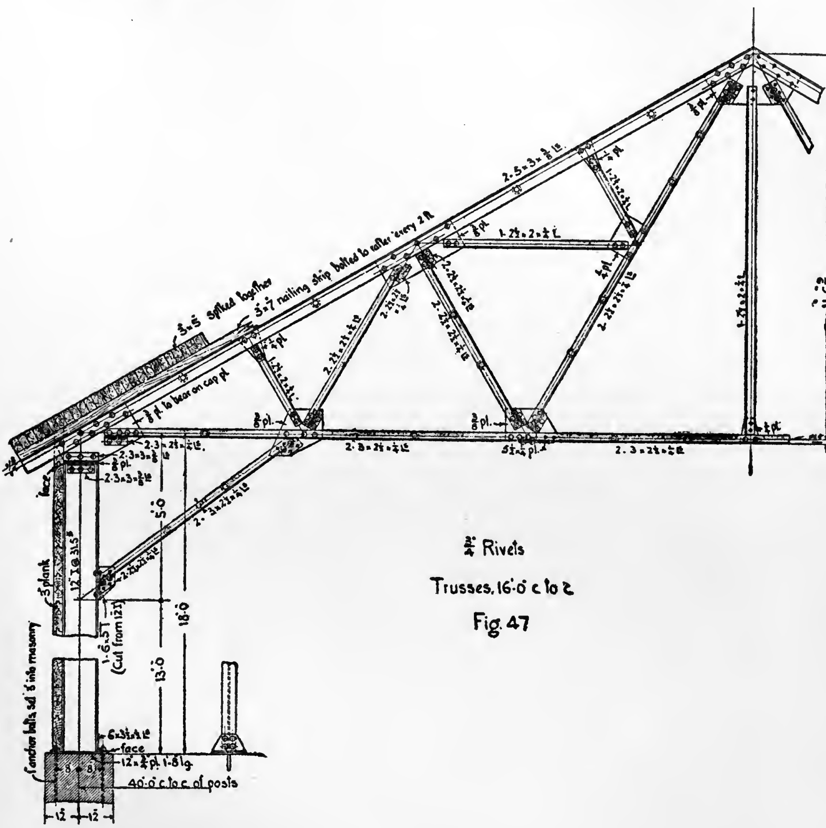


Fig. 47

taken at 7,800 lbs., and it must be let down into the foundation far enough to develop an equal resistance. 100 lbs. per sq. in. is a safe value for the adhesion of cement mortar to iron, and, as the circumference of a one-inch bolt is about 3 inches, the adhesion per lineal inch will be 300 lbs. Then 7,800 ÷ 300 = 26 inches = length of bolt required in foundation. It would be better to extend bolts into foundation somewhat deeper than this, say 36 inches.

The width of foundation wall is assumed to be 24 inches, and its depth 5 ft. In constructing the wall, a break about 24 inches wide should be made at each post. The anchor bolts should then be set in their proper position by means of a template, and the space filled with Portland cement concrete.

The anchor bolt in resisting the bending moment at base of post tends to overturn the wall with a moment equal to the value of one bolt, multiplied by its distance from the further edge of base plate $= 7{,}800$ lbs. $\times$ 18 ins. $= 140{,}400$ in.-lbs. This overturning moment is resisted, partly by the weight of the wall, and partly by the earth filling around it. The resistance of the wall is easily figured, but that of the earth filling is very indefinite and uncertain, so the latter will be neglected in the present case. The weight of wall required is equal to the overturning moment divided by the distance from centre of wall to its edge, $= 140{,}400$ in.-lbs. $\div$ 12 ins. $=$ 11,700 lbs. Taking the weight of masonry at 150 lbs. per cu. ft. the weight of a section of wall one foot long $= 5' \times 2' \times 150$ lbs. $= 1{,}500$ lbs., then $11{,}700 \div 1{,}500 = 7.8$ ft., which is the length of wall required to resist the overturning moment due to anchor bolts. As the posts are 16 ft. apart, it is evident that the wall has ample stability in itself without the assistance of the earth filling.

Fig. 47 is a detail drawing showing one-half of truss and one post. The general method of designing details as explained in Art. 15 also applies to the present example.

ART. 18.—THE DESIGN OF A PLATE GIRDER.

Data: Length, centre to centre of bearings, 50 ft.
Depth, back to back of angles, 5 ft.
Load, 4,000 lbs. per lineal foot.
Tension, 15,000 lbs. per square inch.
Shearing, 7,500 lbs. per square inch for web plates.
Shearing, 10,000 " " " " " rivets.
Bearing, 20,000 " " " " " "
Size of rivets $\frac{3}{4}$-in.

The bending moment at the centre is given by the formula,

$$M = \frac{w\ l^2}{8} = \frac{4{,}000 \times 50^2}{8} = 1{,}250{,}000 \text{ ft.-lbs.}$$

The flange stress at the centre equals the moment divided by the depth, centre to centre, of gravity of flanges. When the flanges have cover plates the centre of gravity is usually near the back of the angles, and sometimes beyond that point, but it is customary to assume the effective depth of such a girder as the distance back to back of angles. As the present example will undoubtedly have cover plates, the effective depth will be 5 ft. Then, the flange stress at centre = 1,250,000 ÷ 5 = 250,000 lbs. and the net area required in bottom flange = 250,000 ÷ 15,000 = 16.67 square inches. When practicable, the area of the angles should be equal to at least one-third of the total flange area.

The following section will suit the case:

	Gross Area.	Rivet Holes.	Net Area.
Two $6 \times 3\frac{1}{2} \times \frac{1}{2}$ Ls=	9.00	$(4 \times \frac{7}{8} \times \frac{1}{2} = 1.75)$	7.25
Two $13 \times \frac{7}{16}$ plates=	11.38	$(4 \times \frac{7}{8} \times \frac{7}{16} = 1.52)$	9.86
	20.38 □		17.11 □

Allowance has been made for two holes in each angle, for, although the holes may not come exactly opposite each other, their stagger will not be great enough to make it allowable to deduct the area of only one hole. When rivets are staggered three inches or more only one hole need be allowed for.

Having decided on the section of the bottom flange, it is customary to make the top flange the same, except in rare cases when the top flange is unsupported laterally. It may then be necessary to make the top flange wider, and to figure it as a column. In the present example, the top flange is supposed to be supported laterally at intervals not exceeding fifteen times its width.

Length of cover plates. In Art. 5 it was seen that the bending moment at any point of a beam supported at both ends and loaded uniformly was represented by ordinates between a parabola, whose vertex was at the centre, and the closing line connecting the points of intersection of the parabola with verticals through the points of support of beam. Now the flange stress at any point; also the required flange area may be represented in the same manner.

Figure 50 represents one half of the girder, which is divided into equal panels of 5 ft. In Fig. 50*a* a parabola is constructed with a base *AB* equal to one-half the span of girder, and height *BC* equal to the area required at centre. To construct the parabola, the line $AD = BC = 16.67$ □″ is divided into the same number of equal parts as the line *AB*, and from the points of division lines are drawn

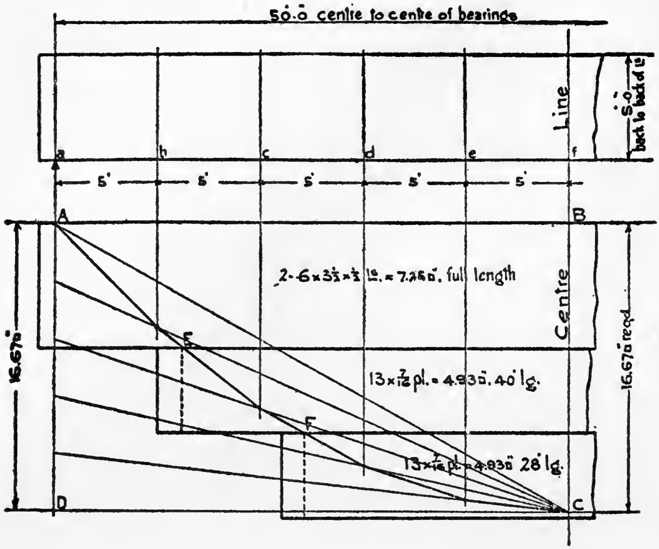


Fig. 50

Fig. 50a

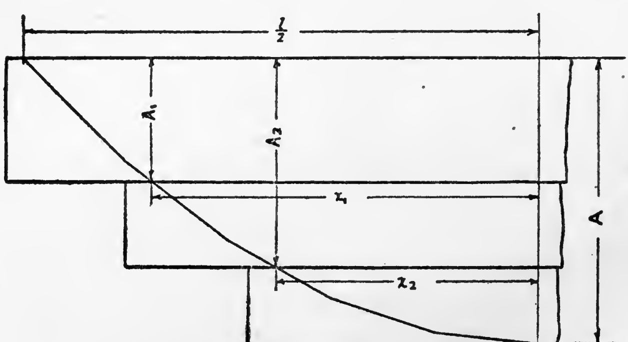


Fig. 51

Fig. 52

to the point *C*, as shown. The intersections of these radial lines with thc verticals through the points of division of the line *AB* are points on the parabola. On this parabola are plotted the areas of the angles and cover plates. The angles extend the full length of the girder. The first cover plate is required to the point *E*, but it is customary to extend it about one foot beyond this point, which will make the length of the plate 40 ft. The second cover plate is required to the point *F*. Adding one foot at each end will make the length of this plate 28 ft.

The following analytical method for determining the proper length of cover plates will give the same results as the above graphical method; but the graphical method is preferable on account of its simplicity, and because with it there is much less chance of errors.

In Fig. 51, A = area required at centre of span.
A_1 = area required at end of 1st cover plate.
A_2 = area required at end of 2d cover plate.
X_1 = distance from centre of span to theoretical end of 1st cover plate.
X_2 = distance from centre of span to theoretical end of 2d cover plate.
l = length of span, centre to centre of bearings.

Then $$X_1=\sqrt{\frac{(A-A_1)\left(\frac{l}{2}\right)^2}{A}} \qquad X_2=\sqrt{\frac{(A-A_2)\left(\frac{l}{2}\right)^2}{A}}$$

In the present example $A=16.67\,\square''$, $A_1=7.25\,\square''$, $A_2=7.25+4.93=12.18\,\square''$, $\frac{l}{2}=25$ ft.

Then $X_1=\sqrt{\frac{(16.67-7.25)\times 25^2}{16.67}}=18.8$ ft. Total length of 1st plate = $(18.8'\times 2)+2'=39.6$ ft., say 40 ft.

$X_2=\frac{(16.67-12.18)\times 25^2}{16.67}=12.9$ ft. Total length of 2nd plate = $(12.9\times 2)+2'=27.8$ ft., say 28 ft.

The web plate must have a sectional area great enough to resist the shear, and it must be thick enough to give sufficient bearing for the rivets in flange angles and end stiffeners.

The maximum shear is at the end and is equal to one-half the load on the span $= 4{,}000 \text{ lbs.} \times \frac{50}{2} = 100{,}000$ lbs. The permissible shear per square inch = 7,500 lbs. then $100{,}000 \div 7{,}500 = 13.3 \square''$ required. A web plate $60'' \times \frac{1}{4}'' = 15 \square''$ is all right for the shear, but may be too thin for rivet bearing, as will be seen presently.

Rivet spacing in flanges. The rivets connecting the web plate with the flange angles are required to transmit the horizontal shearing stress from the web to the flanges; which horizontal shear in any panel is equal to the vertical shear at centre of panel multiplied by its length and divided by the vertical distance centre to centre of rivets. When the load rests directly on the top or bottom flange, the rivets connecting this flange with the web plate are also required to distribute the load. Then the resultant stress on rivets of loaded flange is represented by the hypothenuse of a right angled triangle in which the other two sides represent the horizontal shear and the vertical load. In the present example the load of 4,000 lbs. lineal foot is supposed to be applied to the top flange.

Rivet spacing in vertical legs of top flange angles in panel *ab:*

Vertical shear at centre of panel $= 100{,}000 - 4{,}000 \times 2.5 = 90{,}000$ lbs.

Horizontal shear on rivets $= 90{,}000 \times \frac{60}{56} = 96{,}400$ lbs., then $96{,}400 \div 60'' = 1{,}610$ lbs per lineal inch.

Vertical load on rivets = 4,000 lbs. per lineal foot = 330 lbs. per lineal inch.

Resultant stress on rivets per lineal inch $= \sqrt{1{,}610^2 + 330^2} = 1{,}640$ lbs.

Bearing value of one ¾ rivet on ¼″ plate = 3,750 lbs. Then required spacing $= 3{,}750 \div 1{,}640 = 2.22''$.

As this is somewhat closer than desirable, a $\frac{5}{16}$ web plate will be used in this panel and also in *bc*.

Bearing value of one ¾ rivet on $\frac{5}{16}$ web plate = 4,690 lbs. Then required spacing $= 4{,}690 \div 1{,}640 = 2.86''$.

Rivet spacing in vertical legs of top flange angles in panel *bc*.

Vertical shear at centre of panel $= 100{,}000 - 4{,}000 \times 7.5 = 70{,}000$ lbs.

Horizontal shear on rivets $= 70{,}000 \times \frac{60}{56} = 75{,}000$, then $75{,}000 \div 60'' = 1{,}250$ lbs. per lineal inch.

Resultant stress on rivets per lineal inch $=\sqrt{1{,}250^2+330^2}=1290$ lbs.

Required spacing of rivets in $\frac{5}{16}''$ web plate $= 4{,}690 \div 1{,}290 = 3.65''$.

Rivet spacing in vertical legs of top flange angles in panel *cd*.

Vertical shear at centre of panel $= 100{,}000 - 4{,}000 \times 12.5 = 50{,}000$ lbs.

Horizontal shear on rivets $= 50{,}000 \times \frac{60}{56} = 53{,}600$, then $53{,}600 \div 60'' = 895$ lbs. per lineal inch.

Resultant stress on rivets per lineal inch$=\sqrt{895^2+330^2}=950$ lbs.

Required spacing of rivets in $\frac{1}{4}''$ web plate $=3{,}750 \div 950 = 3.94''$.

Rivet spacing in vertical legs of top flange angles in panel *de*.

Vertical shear at centre of panel $= 100{,}00c - 4{,}000 \times 17.5 = 30{,}000$ lbs.

Horizontal shear on rivets $= 30{,}000 \times \frac{60}{56} = 32{,}500$ lbs., then $32{,}500 \div 60'' = 540$ lbs. per lineal inch.

Resultant stress on rivets per lineal inch$=\sqrt{540^2+330^2}=620$ lbs.

Required spacing of rivets in $\frac{1}{4}''$ web plate$=3{,}750 \div 620 = 6.05'$.

It is unnecessary to proceed further as the maximum spacing should not exceed 6″.

Rivets in flange plates. The 1st flange plate requires a sufficient number of rivets between its end and the end of the 2d flange plate to transmit to it its full proportion of the flange stress. The distance from the end of the 1st plate to the end of the 2d plate is 6 ft. = 72 inches. The net area of the plate = 4.93 square inches; then $4.93 \times 15{,}000 = 73{,}950$ lbs., = its proportion of the flange stress. The value of one $\frac{3}{4}$ rivet in single shear = 4,420 lbs. Then $73{,}950 \div 4{,}420 = 16$ rivets required in 72″. Since there are two lines of rivets in the plate, the required longitudinal spacing $= 72'' \div 8 = 9''$. But the pitch should not exceed 16 times the thickness of the plate, nor should it be more than 6″. The 2d flange plate requires the same number of rivets between its end and the centre of the span = 14 ft.

The rivet spacing on top and bottom flanges will be made alike and as uniform as possible in order to simplify the template work. In the vertical legs of angles the spacing will be $2\frac{1}{2}''$ from *a* to *b*, 3″ from *b* to *d* and 6″ from *d* to *f*. In the flange plates the spacing will be 6″ throughout, except at splice, and the rivets will be staggered with those in the vertical legs of flange angles.

Web splices. A $60 \times \frac{5}{16}$ web plate will be used from *a* to *c*, and a $60 \times \frac{1}{4}$ web plate from *c* to *f*. The plates will be spliced at *c* and *f*.

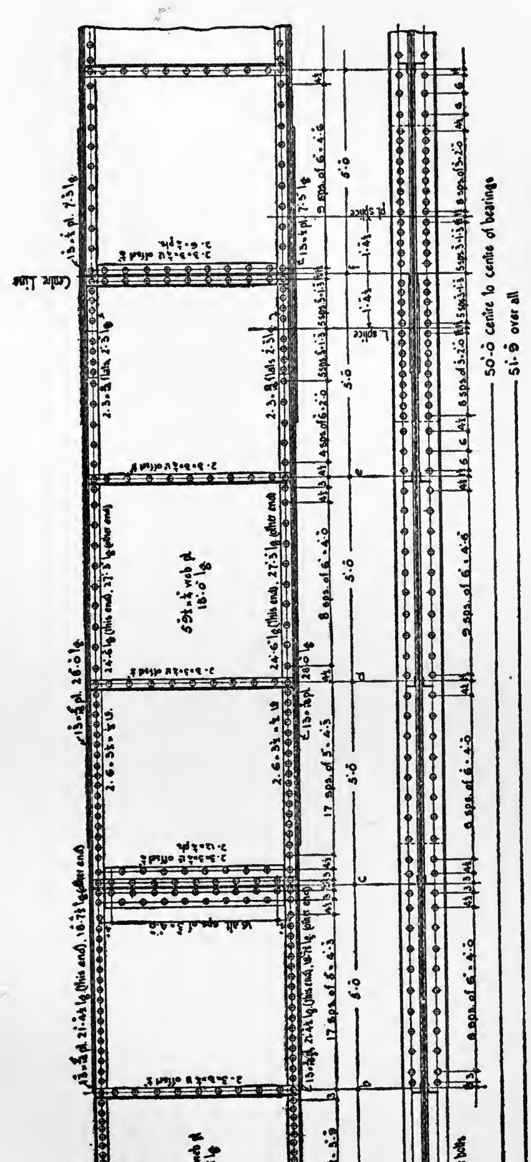
Web splice at *c*. The shear at this point = 60,000 lbs. The rivets, although in double shear, have bearing on one side of the splice of only $\frac{1}{4}''$. The value of one $\frac{3}{4}$ rivet, bearing on $\frac{1}{4}''$ plate = 3,750 lbs. Then, the number of rivets required side of splice adjacent to $\frac{1}{4}''$ web plate = 60,000 ÷ 3,750 = 16. Two $12'' \times \frac{1}{4}''$ splice plates will be used, and the rivets staggered as shown in detail, Fig. 53. The same number of rivets are used on side of splice adjacent to $^5/_{16}''$ web plate for symmetry and to simplify the template work.

Web splice at *f*. There is no shear at this point. Two $6'' \times \frac{1}{4}''$ splice plates will be used with a single line of rivets about 6″ apart, each side of joint.

End Stiffeners. The duty of the end stiffeners is to transfer the shear from the web plate to the abutments. They act as columns, but as the ratio of their length to their radius of gyration is small, the unit stress of 12,000 lbs. per square inch may be used without reduction by formula. The end shear or reaction = 100,000 lbs., then 100,000 ÷ 12,000 = 8.33 square inches required in end stiffeners. Four $5 \times 3 \times {}^5/_{16}$ Ls = 9.60 square inches will be used, placed as shown in Fig. 52, which arrangement is best suited to distribute the load over the bearing plate. The rivets are in bearing on $^5/_{16}$ plate, then the value of one $\frac{3}{4}$ rivet = 4,690 lbs., and the number of rivets required = 100,000 ÷ 4,690 = 21, or 11 rivets in each pair of angles. Between the end stiffeners and the web plate $3'' \times \frac{1}{2}''$ fillers will be used to avoid offsetting the angles, and thus obtain a better fit. These stiffeners and fillers should fit against the bottom flange angles perfectly.

Intermediate Stiffeners. In case of a heavy concentrated load at any point of the girder, stiffeners should be proportioned in the same manner as the end stiffeners to carry this load and distribute it into the web plate; otherwise, the intermediate stiffeners are simply to prevent the web from buckling, and are usually spaced about as far apart as the depth of girder. There is no scientific method of proportioning them, and no generally accepted rule. In the present case two $3 \times 3 \times \frac{1}{4}$ Ls will be used. At the top and bottom they will be offset, or crimped, to fit over the flange angles.

Bearing Plates. If the girder rest on a solid stone at each end, the bearing pressure may be 300 lbs. per square inch, but if it be supported on brickwork, the bearing pressure should not exceed



¾" Rivets

PLATE GIRDER
Fig. 53

100 lbs. per square inch. In the present case, the former condition will be assumed; then the required area of bearing plate = 100,000 ÷ 300 = 333 square inches. A 16″ × $\frac{3}{4}$ × 21″ plate will be used; its area = 336 square inches.

Flange Splice. Angles and flange plates may be obtained up to sixty or seventy feet or even longer; but, if the girder is to be made from stock lengths, it may be necessary to splice the angles and the first cover plate. For the purpose of illustration, these will be spliced near the centre of the span as shown in Fig. 53. The net area of plate = 4.93 square inches, then 4.93 × 15,000 = 73,950 lbs. = value of plate. The single shearing value of one $\frac{3}{4}$ rivet = 4,420 lbs. then 73,950 ÷ 4,420 = 17 rivets required on each side of splice. The drawing shows 18 rivets. The net area of the angles = 7.25 square inches, then 7.25 × 15,000 = 108,750 lbs. = value of angles, and 108,750 ÷ 4,420 = 25 rivets required. The drawing shows 18 rivets in the 6″ legs of angles, and 4 rivets in the $3\frac{1}{2}$″ legs, which latter are in double shear and may be counted as 8 rivets. The total number of rivets in angle splice is then 18 + 8 = 26. It should be noted that there are about twice as many rivets in the 6″ legs as in the $3\frac{1}{2}$″ legs. The splice plates should have at least as much section as the pieces spliced. A 13 × $\frac{1}{2}$ plate is used; its net area, making allowance for two $\frac{7}{8}$ holes = 5.62 square inches. In addition to this, the vertical legs of the angles are covered by two 3 × $^9/_{16}$ flats of net area = 2.39 square inches. Then 5.62 + 2.39 = 8.01 square inches in splice material for angles, which is somewhat greater than the net area of the angles. A common error is to put a long string of rivets in a narrow splice plate, but it will readily be understood that it is useless to use more rivets in a splice plate than are required to develop its full strength. Here, the two 3 × $^9/_{16}$ flats = 2.39 square inches net area, and 2.39 × 15,000 lbs. = 38,500 lbs. In these flats are 4 rivets in full double shear which are equivalent to 8 rivets in single shear, then 4,420 × 8 = 35,360 lbs. = value of rivets in splice plates.

ART. 19.—PLATE GIRDER WITH ONE-EIGHTH OF WEB PLATE COMPUTED AS FLANGE AREA.

In the foregoing example, Art. 18, it has been assumed that the bending moment is resisted entirely by the flanges, and that the web plate takes shear only. This is not strictly correct, but is in conformity with many specifications. It seems to be better practice, however, to make due allowance for the resistance of the web plate in designing the flanges.

The moment of resistance of web plate $= \frac{bh^2}{6} = \frac{bh}{6}h = \frac{A^w}{6}h.$

In which b = thickness of web plate.
h = height " " "
A^w = area " " "

Making allowance for a vertical line of one-inch holes, 4-inch centres, the net moment of resistance of web plate $= \frac{A^w}{8}h.$

The moment of resistance of the flanges $= A^f h.$
In which A^f = area of one flange.
h = depth of girder centre to centre of gravity of flanges, and assumed to be equal to the height of web plate when flange plates are used.

Then, total moment of resistance of girder =

$$\left(\frac{A^w}{8}h\right) + \left(A^f h\right) = \left(\frac{A^w}{8} + A^f\right)h.$$

Therefore, one-eighth of the area of web plate may be computed as flange area. The following is an alternative design for the girder, using the same shears and moments as before. A $\frac{5}{16}''$ web plate will be used throughout.

Flange area required at centre = 16.67 sq. ins.
Flange material provided =

$\frac{1}{8}$ of $59\frac{1}{2} \times \frac{5}{16}$ web plate		=	2.32
2 — $6 \times 3\frac{1}{2} \times \frac{7}{16}$ Ls	= 7.94 gross (less 4, $\frac{7}{8}$ holes)	=	6.41
2 — $13 \times \frac{3}{8}$ plates	= 9.76 gross (less 4, $\frac{7}{8}$ holes)	=	8.44
			17.17 sq. ins. net.

The first cover plate requires to be 37 ft. long, and the second cover plate 26 ft. long.

Rivet spacing in vertical legs of flange angles.

The longitudinal shear per lineal inch, at any point on the rivet line, is equal to the vertical shear at the point, divided by the distance, in inches, centre to centre of the rivets in the vertical legs of the top and bottom flange angles; and the amount of this shear to be transferred by the rivets to the flanges is proportioned to

$$\frac{\text{Area of one flange at the point.}}{\text{Area of one flange} + \frac{1}{8}\text{ of web plate.}}$$

Rivet spacing in panel *a b*.

Shear at centre of panel = 90,000 lbs.

Distance centre to centre of rivets in vertical legs of top and bottom flange angles = 56 inches.

Net area of one flange at this point	= 6.41
$\frac{1}{8}$ of $59\frac{1}{2} \times \frac{5}{16}$ web plate	= 2.32
	6.73 sq. ins.

Vertical load on rivets = 330 lbs. per lineal inch.

Bearing value of one $\frac{3}{4}$ rivet on $\frac{5}{16}$ plate = 4,690 lbs.

Then, longitudinal shear per lineal inch at rivet line $= \dfrac{90{,}000}{56} = 1{,}610$ lbs.

longitudinal shear per lineal inch on rivets $= 1{,}610 \times \dfrac{6.41}{8.73} = 1{,}180$ lbs.

resultant stress on rivets per lineal inch $= \sqrt{1{,}180^2 + 330^2} = 1{,}220$ lbs.

required spacing of rivets in top flange $= \dfrac{4{,}690}{1{,}220} = 3.84$ inches.

The required rivet spacing in the remaining panels is found similarly. The required rivet spacing in flange plates is determined as before.

Web Splices.—Since one-eighth of the web plate has been computed as flange area, the splices must be capable of resisting the full amount of bending moment attributed to the web, as well as the vertical shear at the point.

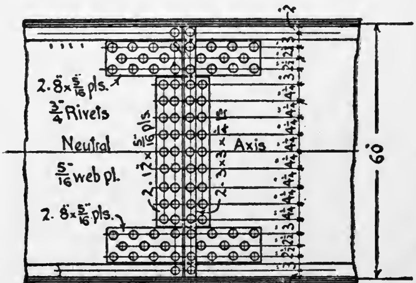


Fig. 54.

Bending value of web plate = 2.32 sq. ins. × 15,000 lbs. × 60 ins. = 2,088,000 in.-lbs.

The moment of resistance of the splice must be equal to or greater than that of the web.

Horizontal splice plates 8 inches wide will be used adjacent to the flange angles, and vertical splice plates, 12 inches wide between them, as shown in Fig. 54.

The number and spacing of the rivets will first be assumed, and then their value investigated.

The maximum bearing value of one $\frac{3}{4}$ rivet on $\frac{5}{16}$ web = 4,690 lbs., but its value in resisting bending moment when located in neutral axis of girder is zero.

The distance from neutral axis to top or bottom of girder = 30 inches. Then the value of one rivet, one inch from neutral axis = $\frac{4,690}{30}$ = 156 lbs. The value of any rivet will be equal to 156 lbs., multiplied by its distance from the neutral axis, and its moment of resistance will be equal to 156 lbs., multiplied by the square of this distance. Then taking all the rivets in splice plates on one side of the joint, both above and below the neutral axis, their moment of resistance is as follows:

$$\begin{array}{llll}
4 \text{ rivets} \times 156 \text{ lbs.} \times 4\tfrac{1}{4}^2 & = & 11,270 & \\
4 \quad " \quad \times \quad " \quad " \times 8\tfrac{1}{2}^2 & = & 45,080 & \\
4 \quad " \quad \times \quad " \quad " \times 12\tfrac{3}{4}^2 & = & 101,430 & \\
4 \quad " \quad \times \quad " \quad " \times 17^2 & = & 180,320 & \\
 & & \overline{} & 338,100 \text{ in.-lbs.} = \text{moment of resistance of rivets in vertical plates.}
\end{array}$$

$$\begin{array}{llll}
8 \text{ rivets} \times 156 \text{ lbs.} \times 20^2 & = & 499,200 & \\
6 \quad " \quad \times \quad " \quad " \times 22\tfrac{1}{2}^2 & = & 473,800 & \\
8 \quad " \quad \times \quad " \quad " \times 25^2 & = & 780,000 & \\
 & & \overline{} & 1,753,000 \text{ in.-lbs.} = \text{moment of resistance of rivets in horizontal plates.} \\
 & & & \overline{2,091,100} \text{ in.-lbs.} = \text{total moment of resistance of rivets in splices.}
\end{array}$$

The net area of horizontal splice plates must be such that their moment of resistance shall be as great as that of the rivets in same, viz., 1,753,000 in.-lbs. The permissible tension for the flanges of the girder = 15,000 lbs. per sq. in. 30 inches from neutral axis; then, at centre of horizontal splice plates, $22\frac{1}{2}$ ins. from neutral axis, the permissible tension = $15,000 \times \frac{22.5}{30} = 11,250$ lbs. per square inch. And

$$11,250 \times \text{net area of plates} \times 22.5 \text{ inches} = 1,753,000 \text{ in.-lbs.}$$

Therefore, net area of plates $= \frac{1,753,000}{11,250 \times 22.5} = 6.9$ sq. ins.

for upper and lower plates together; or 3.5 sq. in. for one pair of plates. Two $8 \times \frac{5}{16}$ plates will be used. Their net area, allowing for two $\frac{7}{8}$ holes in each, = 3.9 sq. in.

ART. 20.—DESIGN FOR A WARREN GIRDER HIGHWAY BRIDGE.

(Figs. 55 and 56.)

Data: Length, centre to centre, of bearings, 50′ 0″. 4 panels of 12′ 6″.

Depth, centre to centre of chords, 6′ 0″.

Roadway 16′ 0″ clear. Width, centre to centre of trusses, 17′ 0″.

Dead load (wooden stringers and floor planking)...	250
Dead load (steel)................................	150
Total (pounds per lineal foot)..................	400

Live load, 80 lbs. per square foot of roadway. Then 80 lbs. × 16′ = 1,280 lbs. per lineal foot.

Tension, 15,000 lbs. per square inch.

Compression, 1,200 lbs. per square inch, reduced by Rankine's formula. Top chords to be considered as columns with square ends, and web members as columns with pin ends. No compression member shall have a length exceeding 120 times its least radius of gyration.

Rivet shearing, 7,500 lbs. per square inch.

Rivet bearing, 15,000 lbs. per square inch.

In determining the dead load, as above, the floor is supposed to consist of 3″ plank, laid on 3 × 12 joists about 2′ centres, and estimated to weigh 3 lbs. per foot (board measure). The weight of steel per lineal foot is given by the formula 2 × L + 50 in which L = length of span in feet. Then (2 × 50′) + 50 = 150 lbs. per lineal foot.

The dead load per panel for one truss $= \frac{400}{2} \times 12.5' = 2{,}500$ lbs.

The live load per panel for one truss $= \frac{1{,}280}{2} \times 12.5' = 8{,}000$ lbs.

These loads are supposed to be concentrated at the lower panel points, *c*, *e* and *g*.

The length of the diagonal members $= \sqrt{6^2 + 6.25^2} = 8.67'$.

The stress in any diagonal is equal to the shear in the panel in which it is situated, multiplied by the length of diagonal and divided by depth of truss; and the shear in any panel is equal to the end reaction, minus any loads between this end and the panel considered.

The stress in any chord section is equal to the bending moment at the point where the diagonals in the panel intersect the opposite chord, divided by the depth of truss.

The dead load stresses will be considered first.

The reaction at either end is equal to one-half the load on span, or one and one-half panel loads, $=2{,}500\times 1\frac{1}{2}=3{,}750$ lbs.

The shear in panel *ac* is equal to the reaction at *a*, and the stresses in members *aB* and *Bc* which lie in this panel will be equal but of opposite kind. Thus *aB* will be in compression and *Bc* in tension.

The shear in panel *ce* is equal to the reaction at *a*, minus the load at *c*, and the stresses in *cD* and *De* are also equal but opposite.

For the stress in *ac* moments are taken about the point *B*, which is distant 6.25′ horizontally from *a*. The moment at this point is equal to the reaction at *a* multiplied by its distance from *B*.

For the stress in *ce* moments are taken about the point *D*, which is distant 18.75′ horizontally from *a*. The moment is equal to the reaction at *a*, multiplied by its distance from *D*, minus the load at *c*, multiplied by its horizontal distance from *D*.

For the stress in *BD* moments are taken about the point *c*, distant 12.5′ from *a*, and the moment is equal to the reaction at *a*, multiplied by this distance.

For the stress in *DF* moments are taken about the point *e*, distant 25′ from *a*. The moment at this point is equal to the reaction at *a* multiplied by its distance from *e*, less the load at *c* multiplied by its distance from *e*.

With the above explanation the following table of stresses will readily be understood :

DEAD LOAD STRESSES.

Shear in panel	$ac=3750-\quad 0$	$=3750$
" "	$ce=3750-2500$	$=1250$
Moment at B	$=3750\times 6.25'$	$=23427$
" "	$D=3750\times 18.75'-2500\times 6.25'$	$=54687$
" "	$c=3750\times 12.5'$	$=46875$
" "	$e=3750\times 25.0'-2500\times 12.5'$	$=62400$

Stress in	$aB\ Bc=3750\times\frac{8.67}{6}=$	5420
" "	$cD\ De=1250\times\frac{8.67}{6}=$	1810
" "	$ac=23437\times\frac{1}{6}=$	3910
" "	$ce=54687\times$ " $=$	9110
" "	$BD=46875\times$ " $=$	7810
" "	$DF=62400\times$ " $=$	10400

The diagonals in the end panels as well as the top and bottom chords throughout, will receive their maximum live load stresses when the bridge is fully loaded; but the maximum stresses in the intermediate diagonals is caused by unsymmetrical loading; thus *cD* will receive its greatest compression, and *De* its greatest tension with live loads at *e* and *g* only; while *eF* will receive its greatest compression, and *Fg* its greatest tension with live load at *g* only.

LIVE LOAD REACTIONS.

Reaction a.	Bridge fully loaded	$=8000\times 1\frac{1}{2}=12{,}000$ lbs.
"	Loads at e and g only	$=8000\times \frac{3}{4}=6{,}000$ lbs.
"	Load at g only	$=8000\times \frac{1}{4}=2{,}000$ lbs.

LIVE LOAD STRESSES.

Shear in panel $ac=12{,}000 \quad = 12{,}000$

" " $ce=6{,}000 \quad = 6{,}000$

" " $eg=2{,}000 \quad = 2{,}000$

Moment at $B=12{,}000\times 6.25' = 75{,}000$

" " $D=12{,}000\times 18.75'-8{,}000\times 6.25'=175{,}000$

" " $c=12{,}000\times 12.5' =150{,}000$

" " $e=12{,}000\times 25.0'-8{,}000\times 12.5=200{,}000$

Stress in $aB\ Bc=12{,}000\times\frac{8.67}{6}=17{,}340$

" " $cD\ De=6{,}000\times\frac{8.67}{6}=8{,}670$

" " $eF\ Fg=2{,}000\times\frac{8.67}{6}=2{,}890$

" " $ac =75{,}000\times\frac{1}{6}=12{,}500$

" " $ce =175{,}000\times\frac{1}{6}=29{,}170$

" " $BD =150{,}000\times\frac{1}{6}=25{,}000$

" " $DF =200{,}000\times\frac{1}{6}=33{,}330$

In Fig. 57, which represents one-half of the span, the dead load and live load stresses are summarized. The compression of $+2{,}890$ lbs. in member eF, due to live load at g, is shown on member De, for this latter member would receive the same stress with live load at c only. Since the dead load stress in De is -1810, the resultant compression will be $+2890-1810=+1080$ as shown. Eight-tenths of this latter amount, which is called a counter stress, is added to the maximum tensile stress. In the same manner the tension of $-2{,}890$ lbs. in member Fg is shown on the corresponding member cD.

The required area for the tension members is obtained directly by dividing the total stress by the unit stress of 15,000 lbs. For the net area allowance has been made for two seven-eighths holes in each angle.

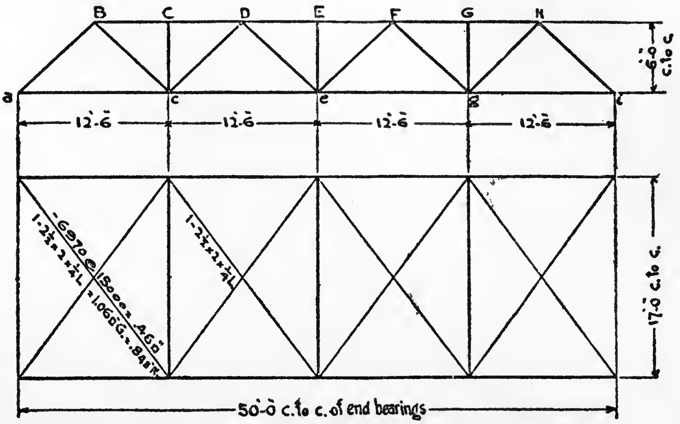
The top chord is supported horizontally at intervals of 12′ 6″ by means of the vertical members which are braced to the floor beams; and it is supported vertically at intervals of 6′ 3″. Thus it requires greater stiffness horizontally than vertically. Two $4\times 3\times\frac{5}{16}$ Ls will be assumed, with the shorter legs back to back as shown. The least radius of gyration $=.86''$, then $\frac{1}{r}=\frac{6.25}{.86}=7.3$, which (by table, Art. 12) corresponds to 10,000 lbs. per square inch for square ends. This unit stress will apply to the whole top chord. Two $4\times 3\times\frac{5}{16}$ Ls will be used for BD and two $4\times 3\times\frac{3}{8}$ Ls for DF.

Assuming the same section for end posts as top chords, $\frac{1}{r}=\frac{8.67}{.86}$

= 10.1, which corresponds to a unit stress of 6,600 lbs. per square inch for pin ends. Two $4\times 3\times \frac{5}{16}$ Ls will be used. For member *cD* two $3\times 2\frac{1}{2}\times \frac{1}{4}$ Ls are assumed, with the 3″ legs back to back. $\frac{1}{r}=\frac{8.67}{.95}=9.1$, then allowable unit stress $=7{,}300$ lbs. It will be

Fig. 55

Fig. 56



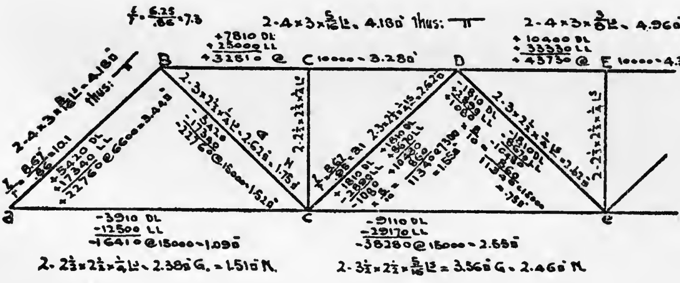


Fig. 57

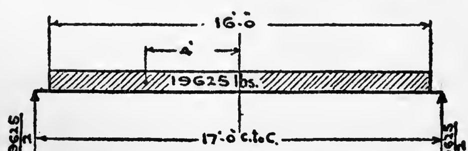


Fig. 58

found that the area of two $3\times 2\frac{1}{2}=\frac{1}{4}$ Ls is considerably greater than required for the stress, but if smaller angles were used the length would exceed 120 times the least radius of gyration.

For member *De* the same section will be used, as this is also a compression member under certain conditions of loading, as explained on page 56.

The verticals *Cc Ee* have no direct stress; their duty is to stiffen the top chord. For these members two $2\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}$ Ls will be used.

Floor Beams, Fig. 58. The dead load consists of the floor, which was assumed to weigh 250 lbs. per linl. ft., plus the weight of the beam itself. Then

$$\begin{aligned} \text{Dead load} &= 250 \times 12.5' + 500 = 3{,}625 \\ \text{Live load} &= 1{,}280 \times 12.5' \qquad = \underline{16{,}000} \\ & \qquad\qquad \text{Total, } 19{,}625 \text{ lbs.} \end{aligned}$$

The load extends over sixteen feet of the floor beam, which is the width of roadway; but the effective length for computing the moment is the distance centre to centre of trusses $=17$ ft. The reaction at each end $=\frac{19{,}625}{2}$. The moment at the centre is equal to the reaction multiplied by one-half the span, less the portion of the load on one side of the centre, multiplied by the distance to its centre of gravity.

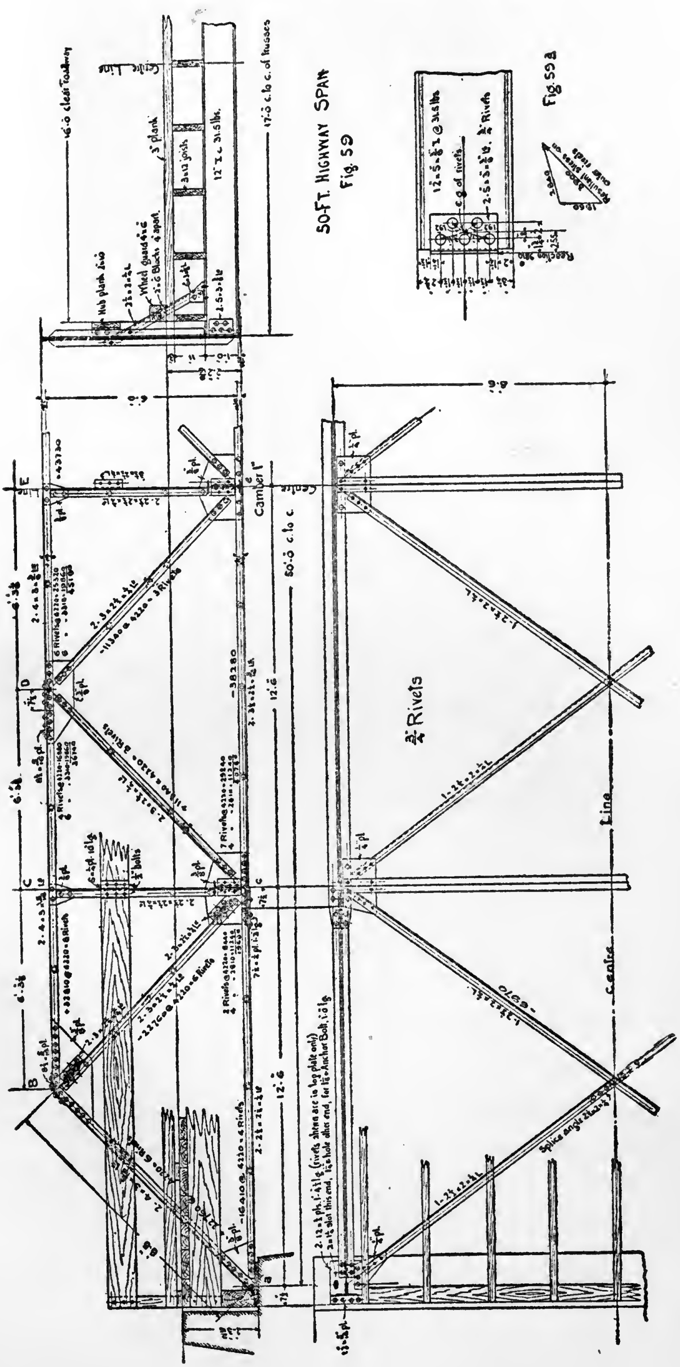
Floor beam moment $=\left(\frac{19{,}625}{2} \times 8.5'\right)-\left(\frac{19{,}625}{2} \times 4'\right)=\frac{19{,}625}{2} \times (8.5-4)=44{,}150$ foot-lbs.

44,150 foot-lbs. $\times 12=529{,}800$ inch-lbs. Then $529{,}800 \div 15{,}000 = 35.3 =$ R required.

A 12″ I @ 31.5 lbs. will be used. Its R=36.

Laterals. The lateral system, Fig. 56, is a horizontal truss of 50 ft. span, and 17 ft. deep. There are 4 panels of 12′ 6″ each. Length of diagonals $= \sqrt{12.5^2 + 17^2} = 21.1'$. The wind pressure is taken at 300 lbs. per lineal foot of bridge, then a panel load $= 300 \times 12.5 = 3{,}750$ lbs. As in the vertical trusses, there is a panel load at each point *c*, *e*, and *g*, and half panel loads at *a* and *i*, which latter do not affect the stresses. The reactions as well as the shear in the end panel $= 3{,}750 \times 1\frac{1}{2} = 5{,}620$ lbs. The stress in the end diagonals $= 5{,}620 \times \frac{21.1}{17} = 6{,}790$ lbs., and $6{,}790 \div 15{,}000 = .46$ square inches required. One $2\frac{1}{2} \times 2 \times \frac{1}{4}$ L will be used. Its net area, allowing for one $\frac{7}{8}''$ hole $= .84$ square inches. The same section will also be used for the next diagonal.

Details, Fig. 59. Taking the shearing value of rivets at 7,500 lbs.


50-FT. HIGHWAY SPAN
Fig. 59
Fig. 59a
¾" Rivets
Centre Line
Camber 1"
50'-0" c. to c.
3" plank
3×12 joists
Hub plank 3×10
Wheel guard 4×6
17'-0" c. to c. of trusses
16'-0" clear roadway
Centre Line
Splice angle 2½×2×¼

per square inch, and the bearing value at 15,000 lbs. per square inch, the value of one $\frac{3}{4}''$ rivet in single shear = 3,310 lbs., bearing on $\frac{3}{8}''$ plate = 4,220 lbs. and bearing on $\frac{1}{4}''$ plate = 2,810 lbs. The number of rivets required are clearly shown on the drawing, and it is only necessary to explain one or two points.

The bearing plate at *a* requires to be large enough so that the pressure on the masonry shall not exceed 300 lbs. per square inch. The dead load reaction = 3,750 lbs. and the live load reaction = 12,000 lbs., making a total of 15,750 lbs. Then, 15,750 ÷ 300 = 50 square inches required. The plate used, which is $12'' \times 16\frac{1}{2}'' =$ 198 square inches, is much larger than necessary for bearing on the masonry. It also requires to be large enough for the anchor bolt and to make connections with the bottom chord angle and the laterals. At *B* the hip cover plate is added to give more lateral stiffness at this point. The bottom chord splice at *c* is formed partly with the gusset plate, and partly with the plate on the bottom. The net area through the splice should not be less than that required in the member *a c*, viz., 1.09 square inches. It is evident that the whole width of the gusset plate cannot be relied on, as the pull is all on one edge, and if the plate were to begin to fail at the edge, a piece of it would soon be torn off. It is only safe to figure on a width of plate equal to twice the rivet gauge in the angle, $= 2\frac{3}{4}''$, and from this width should be deducted the diameter of the rivet hole, $\frac{7}{8}''$. Then the net area $= (2.75'' - .875') \times \frac{3}{8}'' = .70$ square inches. The net area of the bottom plate making allowance for two $\frac{7}{8}''$ holes $= (7.5 - 1.75) \times \frac{1}{4} = 1.44$ square inches. Then the net area through splice $= .70 + 1.44 = 2.14$ square inches, which is considerably greater than required. The value of the rivets at left end of splice should be at least equal to the stress in *a c* = 16,410 lbs. Then

2 rivets bearing on $\frac{3}{8}''$ plate at 4,220 lbs. =	8,440
4 rivets bearing on $\frac{1}{4}''$ plate at 2,810 lbs. =	11,240
Total	19,680 lbs.

The value of the rivets at right end of splice should be equal to or greater than the stress *c e* = 38,280 lbs. Then,

7 rivets bearing on $\frac{3}{8}''$ plate at 4,220 lbs. =	29,540
4 rivets bearing on $\frac{1}{4}''$ plate at 2,810 lbs. =	11,240
Total	40,780 lbs.

The splice in top chord at *D* is designed similarly.

Floor beam connection, Fig. 59a. The reaction or end shear is equal to one-half the total load on floor beam = 9,810 lbs. The rivets connecting the end angles with the truss are in direct single shear, therefore 9,810 ÷ 3,310 = 3 rivets required, whereas there are 4 rivets provided. In addition to the vertical load on the rivets connecting the end angles with the web of beam, these rivets are required to resist a bending moment equal to the reaction multiplied by the distance from back of angles to the centre of gravity of rivets. The greatest stress, due to bending, is on the rivets farthest from their centre of gravity, and is equal to the bending moment, divided by the moment of resistance of rivets. The direction of this stress is perpendicular to a line drawn through the centre of outer rivet, and the centre of gravity of the system. The centre of gravity of rivets and their distances from this point are shown in Fig. 59a.

The bending moment = 9,810 lbs. × 2.55″ = 25,000 inch-lbs.

The polar moment of inertia is obtained by multiplying each rivet by the square of its distance from the centre of gravity as follows:

$$\begin{aligned} I = 1 \text{ rivet} \times .8''^2 &= .6 \\ 2 \text{ rivets} \times 1.92^2 &= 7.4 \\ 2 \text{ rivets} \times 3.11^2 &= \underline{19.4} \\ & 27.4 \end{aligned}$$

The moment of resistance is obtained by dividing the above result by the distance from the centre of gravity to the farthest rivet, thus:

$$R = \frac{I}{n} = \frac{27.4}{3.11} = 8.8.$$

Then the stress on the outer rivets from bending = 25,000 inch-lbs. ÷ 8.8 = 2,840 lbs. The vertical load on each rivet is equal to the reaction divided by the total number of rivets = 9,810 lbs. ÷ 5 = 1,960 lbs. The resultant stress = 3,900 lbs. and is obtained graphically as shown. It is less than the bearing value of a $\frac{3}{4}''$ rivet on the web of beam which is $\frac{3}{8}''$ thick, and consequently the connection is satisfactory.

Camber. Bridge trusses are constructed with a slight arch called camber. This adds nothing to their strength, and is intended principally to offset the deflection due to the dead and live loads. The camber is obtained by making the top chord slightly longer than the bottom, and increasing the length of the diagonal members in proportion. The following rule is taken from Trautwine:

$$i = \frac{8\ d\ c}{s}$$

In which i = total increased length of top chord.
d = depth of truss.
c = camber at centre.
s = span.

All in feet or all in inches.

In the present example a camber of 1″ is assumed, $d = 6' = 72''$,

$s = 50' = 600''$, then $i = \dfrac{8 \times 72 \times 1}{600} = .96''$, say 1″. Since there are four panels in bridge, each top chord panel must be increased $\frac{1}{4}''$, or each half panel $\frac{1}{8}''$. The lengths of *BC*, *CD*, *DE* will then be $6'\ 3'' + \frac{1}{8}''$. To obtain the lengths of diagonals, the mean between the top and bottom half-panel lengths is combined with depth of truss thus: length of diagonals $= \sqrt{(6'\ 3\tfrac{1}{16}'')^2 + (6'\ 0'')^2} = 8'\ 8''$.

ART. 21.—DESIGN FOR SKEW WARREN GIRDER HIGHWAY BRIDGE.

(Fig. 60.)

Data: Length, centre to centre of bearings, 72′ 0″. 4 panels of 15′ 0″ and 1 panel of 12′ 0″.

Depth, centre to centre of chords, 7′ 6″.

Roadway 16′ 0″ clear. Trusses, 17′ 6″ centre to centre.

Dead load (wooden stringers and floor planking)...	250
" " (steel = 2 L + 50 = (2 × 72′) + 50 = 194) say	200
Total (pounds per lineal foot).....................	450

Live load for trusses, 75 lbs. per square foot of roadway = 1,200 lbs. per lineal foot.

Live load for floor beams 100 lbs. per square foot of roadway.

Horizontal wind force, 300 lbs. per lineal foot, one-half of which to be treated as live load.

Unit Stresses: Tension, 15,000 lbs. per square inch.
Compression, 12,000 lbs. per square inch, reduced by Rankine's formula (Art. 12). Top chords to be considered as columns with square ends, and web members as columns with pin ends. No compression member shall have a length exceeding 120 times its least radius of gyration.
Rivet shearing, 7,500 lbs. per square inch.
Rivet bearing, 15,000 lbs. per square inch.

PANEL LOADS FOR ONE TRUSS.

For panel points *c*, *e* and *g* dead load $= \frac{450}{2} \times 15 = 3375$ lbs.

" " " " live load $= \frac{1200}{2} \times 15 = 9000$ lbs.

" " *i* dead load $= \frac{450}{2} \times \frac{15+12}{2} = 3000$ lbs.

" " *i* live load $= \frac{1200}{2} \times \frac{15+12}{2} = 8100$ lbs.

Length of regular diagonal members $= \sqrt{7.5^2 + 7.5^2} = 10.61'$.

Length of diagonals is *iJ* and *Jk* $= \sqrt{7.5^2 + 6^2} = 9.60'$.

The dead load reaction at *a* is equal to the moments of the panel loads about *k*, and divided by the length of span *ak*.

$$= \frac{\{(3{,}000 \times 12)\} + \{3{,}375 \times (27 + 42 + 57)\}}{72} = 6{,}405 \text{ lbs.}$$

The dead reaction at *k* is equal to the sum of the panel loads, less the reaction at $a = 3{,}000 + (3{,}375 \times 3) - 6{,}405 = 6{,}720$ lbs.

In the following table of dead load stresses, the same general method employed in Art. 20 is observed, but both ends of the truss are considered.

The shear in panel *ac* is equal to the reaction at *a*.
" " *ce* " " " *a*, minus the panel load at *c*.
" " *eg* " " " *k* " " " *g* & *i*.
" " *gi* " " " *k* " " " *i*.
" " *ik* " " " *k*.

The bending moments at *B*, *D*, *F*, *c* and *e* are obtained by taking moments of the reaction a about these points, and deducting the moments of the panel loads between these points and a. For the bending moments at H, J, g and i moments of the reaction k are taken about these points, and the moments of the panel loads between these points and k are deducted.

DEAD LOAD STRESSES.

Shear in ac	$=6,405-0$	$=6,405$
" " ce	$=6,405-3,375$	$=3,030$
" " eg	$=6,720-(3,375+3,000)$	$=345$
" " gi	$=6,720-3,000$	$=3,720$
" " ik	$=6,720-0$	$=6,720$
Moment at B	$=6,405\times 7.5$	$=48,000$
" " D	$=6,405\times 22.5-3,375\times 7.5$	$=118,800$
" " F	$=6,405\times 37.5-3,375\times(7.5+22.5)$	$=138,950$
" " H	$=6,720\times 19.5-3,000\times 7.5$	$=108,550$
" " J	$=6,720\times 6$	$=40,300$
" " c	$=6,405\times 15$	$=96,000$
" " e	$=6,405\times 30-3,375\times 15$	$=141,500$
" " g	$=6,720\times 27-3,000\times 15$	$=136,450$
" " i	$=6,720\times 12$	$=80,650$

Stress in $aB\ Bc$	$=6,405\times\frac{10.61}{7.5}$	$=9,000$
" $cD\ De$	$=3,030\times$ "	$=3,200$
" $eF\ Fg$	$=345\times$ "	$=500$
" $gH\ Hi$	$=3,720\times$ "	$=5,250$
" $iJ\ Jk$	$=6,720\times\frac{9.60}{7.5}$	$=8,600$
" ac	$=48,000\times\frac{1}{7.5}$	$=6,400$
" ce	$=118,800\times$ "	$=15,800$
" eg	$=138,950\times$ "	$=18,500$
" gi	$=108,550\times$ "	$=14,400$
" ik	$=40,300\times$ "	$=5,400$
" BD	$=96,000\times$ "	$=12,800$
" DF	$=141,500\times$ "	$=18,800$
" FH	$=136,450\times$ "	$=18,200$
" HJ	$=80,650\times$ "	$=10,700$

The maximum live load stresses in top and bottom chords, and in diagonals aB, Bc, iJ and Jk will occur when the bridge is fully loaded. The maximum compression in cD and tension in De, with loads at e, g and i only. The maximum compression in eF and tension in Fg with loads at g and i only. The maximum tension in eF and compression Fg with loads at c and e only. The maximum tension in gH and compression in Hi with loads at c, e and g only. In computing the live load reaction a, moments are taken about k; and in computing the reactions k, moments are taken about a, except for case of bridge fully loaded, when reaction k is equal to the total load on span, less the reaction a.

LIVE LOAD REACTIONS.

Reaction *a*.	Bridge fully loaded	$=\dfrac{[8100\times 12]+[9000\times(27+42+57)]}{72}$	$=17,100$
" *a*.	Loads at *e*, *g* and *i*	$=\dfrac{[8100\times 12]+[9000\times(27+42)]}{72}$	$=9,970$
" *a*.	Loads at *g* and *i*	$=\dfrac{(8100\times 12)+(9000\times 27)}{72}$	$=4,725$
" *k*.	Bridge fully loaded	$=8100+(9000\times 3)-17100$	$=18,000$
" *k*.	Loads at *c*, *e* and *g*	$=\dfrac{9000\times(15+30+45)}{72}$	$=11,250$
" *k*.	Loads at *c* and *e*	$=\dfrac{9000\times(15+30)}{72}$	$=5,625$

LIVE LOAD STRESSES.

Shear in $ac=17,100$	$=17,100$
" " $ce=9,970$	$=9,970$
" " $eg=4,725$	$=4,725$
" " $eg=5,625$	$=5,625$
" " $gi=11,250$	$=11,250$
" " $ik=18,000$	$=18,000$
Moment at $B=17,100\times 7.5$	$=128,000$
" " $D=17,100\times 22.5-9,000\times 7.5$	$=317,500$
" " $F=17,100\times 37.5-9,000\times(7.5+22.5)$	$=371,250$
" " $H=18,000\times 19.5-8,100\times 7.5$	$=290,250$
" " $J=18,000\times 6$	$=108,000$
" " $c=17,100\times 15$	$=256,500$
" " $e=17,100\times 30-9,000\times 15$	$=378,000$
" " $g=18,000\times 27-8,100\times 15$	$=364,500$
" " $i=18,000\times 12$	$=216,000$

Stress in $aBBc$	$=17,100\times$	$\dfrac{10.61}{7.5}$	$=24,100$
" $cDDe$	$=9,970\times$	"	$=14,050$
" $eFFg$	$=4,725\times$	"	$=6,650$
" $eFFg$	$=5,625\times$	"	$=7,950$
" $gHHi$	$=11,250\times$	"	$=15,850$
" $iJJk$	$=18,000\times$	$\dfrac{9.60}{7.5}$	$=23,050$
" ac	$=128,000\times$	$\dfrac{1}{7.5}$	$=17,050$
" ce	$=317,500\times$	"	$=42,300$
" eg	$=371,250\times$	"	$=49,400$
" gi	$=290,250\times$	"	$=38,700$
" ik	$=108,000\times$	"	$=14,400$
" BD	$=256,500\times$	"	$=34,200$
" DF	$=378,000\times$	"	$=50,400$
" FH	$=364,500\times$	"	$=48,500$
" HJ	$=216,000\times$	"	$=28,800$

Since the truss is unsymmetrical, it is necessary to make a complete stress diagram, Fig. 61. The various members are proportioned in the same manner as in previous example of 50-ft. span. As the present span is much longer than the last, the vertical legs of angles are turned out as shown to give greater stiffness horizon-

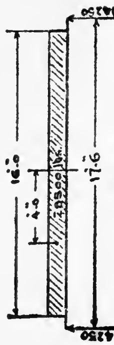


Fig. 62

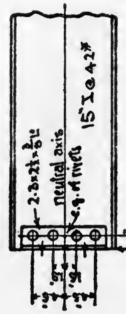


Fig. 63

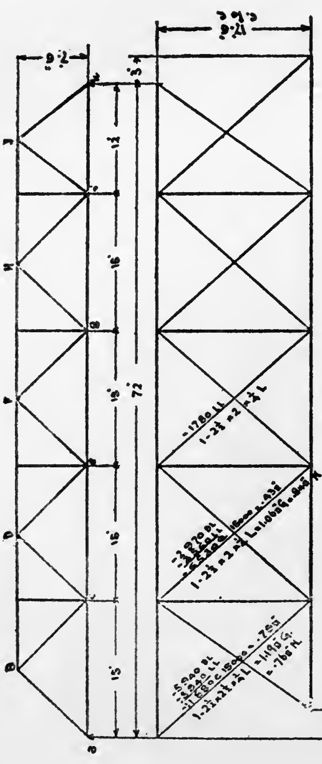


Fig. 60.

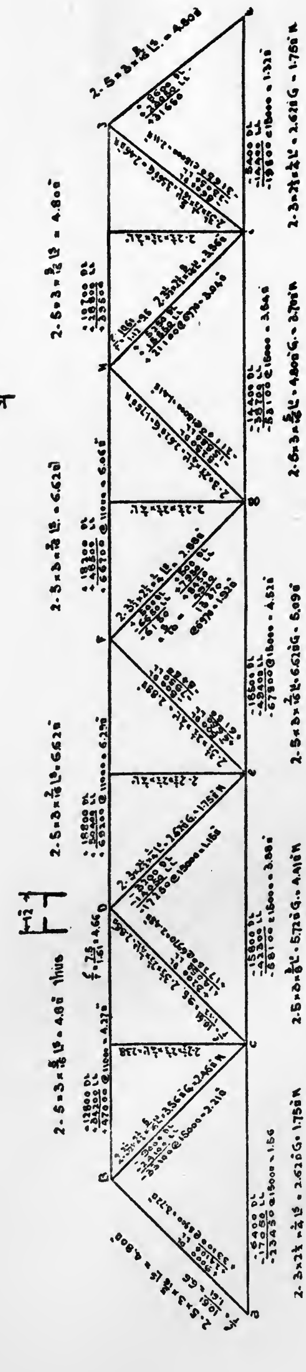


Fig. 61

tally. This will necessitate double gusset plates, and all angles will require to be latticed.

Floor Beams, Fig. 62:

Dead load (floor) = 250 lbs. × 15′ =	3,750	
Dead load (beam) = say	750	
		4,500
Live load = 16′ × 15′ × 100 lbs. =		24,000
Total		28,500 lbs.

Reaction $= \dfrac{28{,}500}{2} = 14{,}250$ lbs. Moments at centre = $(14{,}250 \times 8.75) - (14{,}250 \times 4) = 14{,}250 \times (8.74 - 4) = 67{,}700$ ft.-lbs. $67{,}700 \times 12 = 812{,}400$ inch-lbs. Then, $812{,}400 \div 15{,}000 = 54.16 =$ R required.

A 15″ I at 42 lbs. will be used. R = 58.9.

Laterals, Fig. 60. It will be sufficiently accurate to assume that the horizontal truss consists of five equal panels of 15′ 0″ each, and 17′ 6″ deep. The length of the diagonals $= \sqrt{15^2 + 17'\,6''^2} = 23.04$ ft. One-half the wind force is to be treated as a stationary or dead load, and one-half as a moving or live load, then

Panel dead load = 150 lbs. × 15′ = 2,250 lbs.
" live " = 150 lbs. × 15′ = 2,250 lbs.

The maximum shear in panel *a c* is when the live load covers the span.

The maximum shear in panel *c e* is when the live load is at *e*, *g* and *i* only.

The maximum shear in panel *e g* is when the live load is at *g* and *i* only.

	DEAD LOAD STRESSES.		LIVE LOAD STRESSES.	
1st lateral	$= 2{,}250 \times 2 \times \dfrac{23.04}{17.5}$	= 5,940	$2{,}250 \times 2 \times \dfrac{23.04}{17.5}$	= 5,940
2d "	$= 2{,}750 \times 1 \times$ "	= 2,970	$2{,}250 \times \frac{6}{5} \times$ "	= 3,560
3rd "	=	= 0	$2{,}250 \times \frac{3}{5} \times$ "	= 1,780

The total stresses, the areas required and provided are shown in Fig. 60.

Floor beam connection, Fig. 63. The reaction = 14,250 lbs. and the rivets connecting the end angles with the truss are in direct shear; therefore, 14,250 ÷ 3,310 = 5 rivets required, whereas the drawing shows 6 rivets. In addition to the vertical load on rivets connecting the end angles with web of beam, these rivets are required to resist a bending moment equal to the reaction mulitplied by the distance from back of angle to centre line of rivets. The greatest stress, due to bending, is on the rivets farthest from their centre of gravity and is equal to the bending moment, divided by the moment of resistance of rivets. The direction of this stress is horizontal. The centre of gravity of rivets and their distances from this point are shown in Fig. 63.

The bending moment = 14,250 lbs. × 1.75″ = 24,900 in.-lbs.

The moment of inertia of rivets is obtained by multiplying each rivet by the square of its distance from the center of gravity, as follows:

$$\begin{aligned} I = 2 \text{ rivets} \times 1.5^2 &= 4.5 \\ 2 \text{ rivets} \times 4.5^2 &= \underline{40.5} \\ & 45.0 \end{aligned}$$

The moment of resistance is equal to the above result divided by the distance from centre of gravity to farthest rivet, thus:

$$R = \frac{45}{4.5} = 10.$$

Stress on outer rivets from bending = 24,900 in.-lbs. ÷ 10 = 2,490 lbs.

Stress on outer rivets from direct load = 14,250 lbs. ÷ 4 = 3,560 lbs.

Resultant stress on outer rivets = $\sqrt{2{,}490^2 + 3{,}560^2} = 4{,}340$ lbs.

The web of beam is $\frac{13}{32}''$ thick, therefore, the bearing value of one $\frac{3}{4}$ rivet = $\frac{13}{32}'' \times \frac{3}{4} \times 15{,}000$ lbs. = 4,570, which is greater than the maximum stress on rivets.

The other details, Fig. 64, are self-explanatory.

ART. 22.—DESIGN FOR A PIN-CONNECTED PRATT TRUSS HIGHWAY SPAN. (FIG. 65.)

Data: Length, 120′ 0″, centre to centre of end pins. 8 panels of 15′ 0″.

Depth, 20′ 0″.

Roadway, 16′ 0″ clear. Trusses, 17′ 0″, centre to centre.

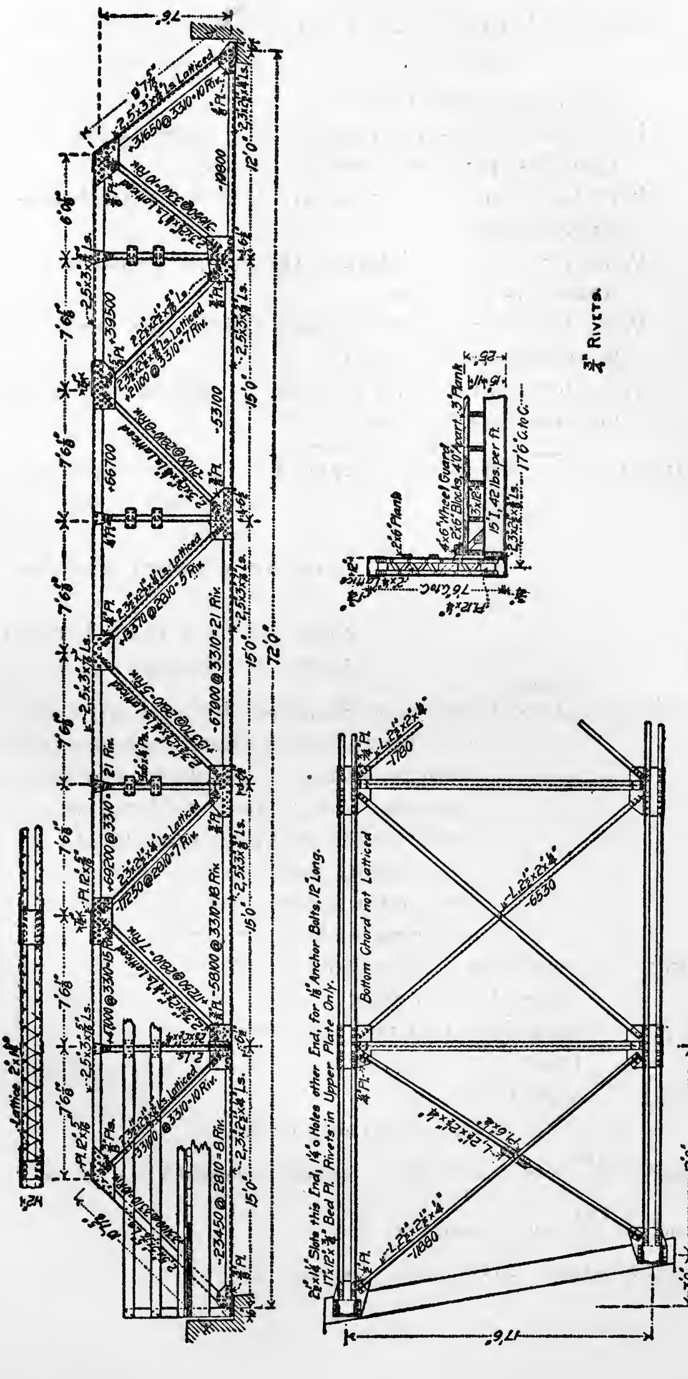

72'0"
Lattice 2x1/4"
Bottom Chord not Latticed
2 1/2"x1 1/4" Slots this End, 1 1/4"o Holes other End, for 1 1/8" Anchor Bolts, 12" Long.
17x12x3/4" Bed Pl. Rivets in Upper Plate Only.
4x6" Wheel Guard
2x6" Blocks, 4'0" apart. 3" Plank
15" I, 42 lbs. per ft.
17'6" C. to C.
7'6" C. to C.
3/4" Rivets.
17'6"
12'0"
3'0"
-6530
-1180
-1180
-53100
-67000@3310=21 Riv.
+66700
+39500
-19800

Fig. 64

Dead load (steel) = 2 × L + 50 = ...290
" " (floor) = ...250

Total, per lineal foot = .540 lbs.

Live load, 75 lbs. per square foot. Then 75 lbs. × 16′ = 1,200 lbs. per lineal foot for trusses.

Live load, 100 lbs. per square foot for floor beams and hip verticals.

Wind force for top laterals, 150 lbs. per lineal foot, to be treated as dead load.

Wind force for bottom laterals, 150 lbs. per lineal foot, to be treated as dead load.

Wind force for bottom laterals, 150 lbs. per lineal foot to be treated as live load.

		STEEL	IRON	
Unit stresses:	Tension.	15,000	12,000	for bottom chords, main diagonals and floor beams.
	"		9,000	for counters and hip verticals.
	"		8,000	for floor beam hangers.
	"		15,000	for laterals.

	STEEL	
Compression,	12,000	reduced by Rankine's formula (Art. 12). No member to have a length exceding 120 times its least radius of gyration; end and intermediate posts to be considered as columns with two pin ends; end sections of top chord as columns with one pin and one square end; intermediate sections of top chord as columns with square ends.
Shearing,	10,000	for pins and shop rivets.
"	7,500	for field rivets.
Bearing,	20,000	for pins and shop rivets.
"	15,000	for field rivets.
Bending,	22,500	for pins.

PANEL LOADS FOR ONE TRUSS.

Dead Load $= \frac{540}{2} \times 15' = 4{,}000$ **lbs.** **Length of diagonals** $= \sqrt{15^2+20^2} = 25'$.

Live Load $= \frac{1200}{2} \times 15' = 9{,}000$ **lbs.**

Dead Load Reaction $= 4000 \times 3\frac{1}{2} = 14{,}000$ **lbs.**

DEAD LOAD STRESSES.

Shear in panel *ab*=14,000 =14,000	Stress in *aB* = 14,000×$\frac{25}{20}$= 17,500
" " *bc*=14,000— 4,000=10,000	" *Bc* = 10,000×$\frac{25}{20}$= 12,500
" " *cd*=14,000— 8,000= 6,000	" *Cc* = 6,000× 1 = 6,000
" " *de*=14,000—12,000= 2,000	" *Cd* = 6,000×$\frac{25}{20}$= 7,500
" " *ef*=14,000—16,000=—2,000	" *Dd* = 2,000× 1 = 2,000
Moment at *B* =14,000×15 =210,000	" *De* = 2,000×$\frac{25}{20}$= 2,500
" " *c&C*=14,000×30—	" *Ee* = 2,000× 1 =—2,000
4,000×15=360,000	" *Ef* = 2,000×$\frac{25}{20}$=+2,500
" " *d&D*=14,000×45—	" *abc* =210,000×$\frac{1}{20}$= 10,500
4,000×(15+30)=450,000	" *BC&cd*=360,000×$\frac{1}{20}$= 18,000
" " *e* =14,000×60—	" *CD&de*=450,000×$\frac{1}{20}$= 22,500
4,000×(15+30+45)=480,000	" *DE* =480,000×$\frac{1}{20}$= 24,000

For the stress in *a b c* moments are taken about panel point *B*. For the stress in *B C* moments are taken about *c*, and for the stress *c d* moments are taken about *C*. But since *c* and *C* are the same distance horizontally from *a*, the stresses in *B C* and *c d* are equal. Likewise the stresses *C D* and *d e* are equal.

LIVE LOAD REACTIONS.

Bridge fully loaded 9,000×3½=31,500 lbs. for maximum stresses in chords and end posts.

Loads at *c, d, e, f, g* and *h*, 9,000×$\frac{21}{8}$=23,600 lbs. for maximum stress in *Bc*.

Loads at *d, e, f, g* and *h*, 9,000×$\frac{15}{8}$=16,900 lbs. for maximum stresses in *Cc* and *Cd*.

Loads at *e, f, g* and *h*, 9,000×$\frac{10}{8}$=11,250 lbs. for maximum stresses in *Dd* and *De*.

Loads at *f, g* and *h*, 9,000×$\frac{6}{8}$=6,750 lbs. for maximum stresses in *Ee* and *Ef*.

LIVE LOAD STRESSES.

Shear in panel *ab* = 31,500	Stress in *aB* = 31,500×$\frac{25}{20}$=39,400
" " *bc* = 23,600	" *Bc* = 23,600×$\frac{25}{20}$=29,500
" " *cd* = 16,900	" *Cc* = 16,900× 1 =16,900
" " *de* = 11,250	" *Cd* = 16,900×$\frac{25}{20}$=21,100
" " *ef* = 6,750	" *Dd* = 11,250× 1 =11,250
Moment at *B* 31,500×15 = 472,500	" *De* = 11,250×$\frac{25}{20}$=14,050
" " *c&C* 31,500×30—	" *Ee* = 6,750× 1 = 6,750
9,000×15= 810,000	" *Ef* = 6,750×$\frac{25}{20}$= 8,450
" " *d&D* 31,500×45—	" *abc* = 472,500×$\frac{1}{20}$=23,625
9,000×(15+30)=1,010,000	" *BC cd*= 810,000× "=40,500
" " *e* 31,500×60—	" *CD de*=1,010,000× "=50,500
9,000×(15+30+45)=1,075,000	" *DE* =1,075,000× "=53,750

Hip Vertical *Bb*. The live load for the hip vertical is taken at

100 lbs. per square foot of roadway = 1,600 lbs. per lineal foot of bridge. The stress in this member will then be $\frac{1,600}{2} \times 15' =$ 12,000 lbs.

The stresses and material are shown in Fig. 65. Square iron bars are used for all tension members requiring less than four square inches, and flat steel bars for all others. The square bars have loop ends which must be welded, and steel is not suitable on this account, as it is difficult to weld it satisfactorily. The heads on the flat steel bars are upset and forged or pressed into dies without welding. The intermediate posts have a much greater area than is required for the stress, but with smaller channels the length would exceed 120 times the least radius of gyration, and the lighter weight of the 6″ channels has a web of only .20 inch, which is entirely too thin.

The area of top chords is also rather large, but smaller channels are unadvisable. The end posts will be considered in connection with portal bracing.

Top Laterals, Fig. 65. The top lateral truss consists of six 15-ft. panels, 17.25 ft. deep. Diagonal $= \sqrt{15^2 + 17.25^2} = 22.8$ ft. Panel load = 150 lbs. × 15 ft. = 2,250 lbs.

STRESSES.

1st lateral $= 2250 \times 2\frac{1}{2} \times \frac{22.8}{17.25} = 7500$ lbs.

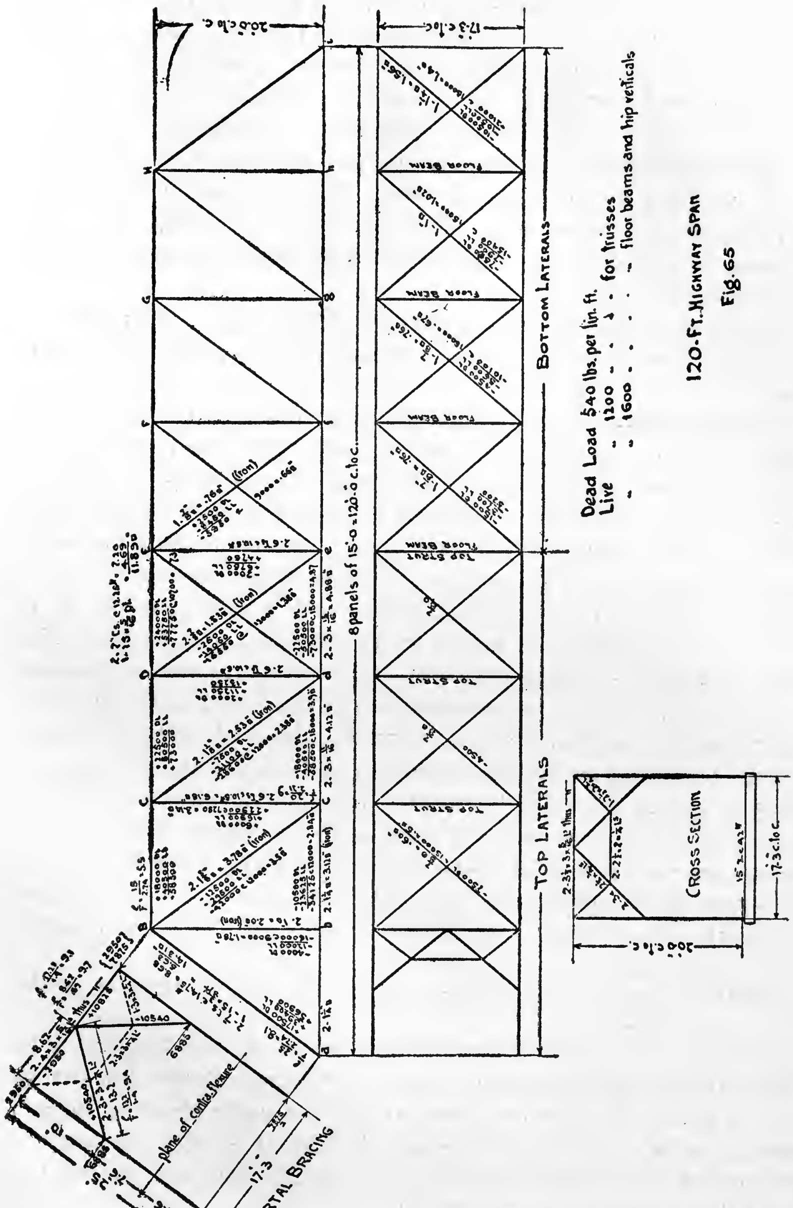
2nd " $= 2250 \times 1\frac{1}{2} \times \frac{22.8}{17.25} = 4500$ lbs.

8rd " $= 2250 \times \frac{1}{2} \times \frac{22.8}{17.25} = 1500$ lbs.

Bottom Laterals. The bottom lateral truss consists of eight 15 ft. panels 17.25 ft. deep. Diagonal $= \sqrt{15^2 + 17.25^2} = 22.8$ ft. Dead panel load = live panel load = 150 lbs. × 15 ft. = 2250 lbs.

DEAD LOAD STRESSES.	LIVE LOAD STRESSES.
1st lateral $= 2,250 \times 3\frac{1}{2} \times \frac{22.8}{17.25} = 10,500$	$2,250 \times 3\frac{1}{2} \times \frac{22.8}{17.25} = 10,500$
2nd " $= 2,250 \times 2\frac{1}{2} \times$ " $= 7,500$	$2,250 \times \frac{21}{8} \times$ " $= 7,900$
3rd " $= 2,250 \times 1\frac{1}{2} \times$ " $= 4,500$	$2,250 \times \frac{15}{8} \times$ " $= 5,600$
4th " $= 2,250 \times \frac{1}{2} \times$ " $= 1,500$	$2,250 \times \frac{10}{8} \times$ " $= 3,700$

Portal Strut and End Posts, Fig. 65. It is assumed that the wind force, which is equal to three and one-half top lateral panel loads, is



120-FT. HIGHWAY SPAN

Fig. 65

applied at the top of portal strut, and that it is resisted equally at the foot of both end posts. It is also assumed that the posts, although hinged at the bottom in plane of trusses, are fixed in the plane of the portal, and that the plane of contra-flexure is midway between the foot of posts and the lower extremities of portal struts. Then in figuring the portal stresses the ends of ports may be considered to lie in this plane of contra-flexure. The problem of finding the stresses in a portal strut is similar to that of finding the wind stresses in a roof truss supported on and braced to steel columns as explained in Art 17.

The force applied at top of portal $= 2{,}250 \times 3\frac{1}{2} = 7{,}875$ lbs.

The horizontal reaction at foot of each post $= 7{,}875 \times \frac{1}{2} = 3{,}935$ lbs.

Since the plane of contra-flexure is assumed to be midway between the foot of posts and connection of knee braces, the moments at these points will each be equal to 3,935 lbs. $\times$ 7.5′ $=$ 29,500 ft.-lbs. The moment at foot of post is resisted by the direct thrust in post acting with a lever arm equal to one-half the width of bearing plates. The moment at knee brace connection is resisted by a force at top of post acting with a lever arm of 10 ft. Therefore, this force $=$ 29,500 ft.-lbs. $\div$ 10′ $=$ 2,950 lbs. This force of 2,950 lbs. induces tension of the same amount on leeward side of top strut, and compression on the windward side, but in the latter case the applied force of 7,875 lbs. should be added. Therefore the total compression on windward side of top strut $= 2{,}950 + 7{,}875 = 10{,}825$ lbs. The horizontal force at the lower end of knee braces is equal to the induced force at top of posts, plus the horizontal reaction at foot of posts $= 2{,}950 + 3{,}935 = 6{,}885$ lbs.; and the stress in the knee braces is equal to this latter force, multiplied by length of knee brace and divided by one-half the width of portal $= 6{,}885 \times \dfrac{13.2}{8.62} = 10{,}540$ lbs. This stress will be tension on the windward side of portal and compression on the leeward side. There are no stresses in the members shown in dotted lines; they help to stiffen the main members, and give a more pleasing appearance to portal. As the stresses are all light, it is only necessary to proportion the members so that their length shall not exceed 120 times their least radius of gyration.

The posts should be proportioned so that the maximum fibre stress resulting from the combined action of the dead load, live load and wind force shall not exceed by more than 25% the permissible unit strees for dead and live loads only.

The direct stress in post as shown in Fig. 60 = 56,900 lbs.

The bending moment as above = 29,500 × 12 = 354,000 inch-lbs.

The area of the section assumed = 14.31 square inches, and its moment of resistance about an axis perpendicular to the cover plate = 53.8.

Then: Direct stress ÷ area =	56,900 lbs. ÷ 14.31 =	3,970
Bending moment ÷ R =	354,000 in.-lbs. ÷ 53.8 =	6,580
Total,............................... per square inch.		10,550 lbs.

The permissible unit stress = 7,870 + 25% = 9,835 lbs. per sq. inch. Thus the post is stressed slightly too much to fulfill the conditions, but the assumed wind force is probably much greater than the actual.

The intermediate top struts, Fig. 65, are in this case made similar to the portal struts. They are not proportioned for any definite stress, but nevertheless they contribute to the stiffness of the bridge, and relieve the portal struts and end posts from a portion of their wind stresses.

Floor Beam. Span, 17.25 ft.

Dead load (floor) = 250 × 15′ =	3,750	
(beam)	650	
		4,400
Live load = 1,600 × 15 =		24,000

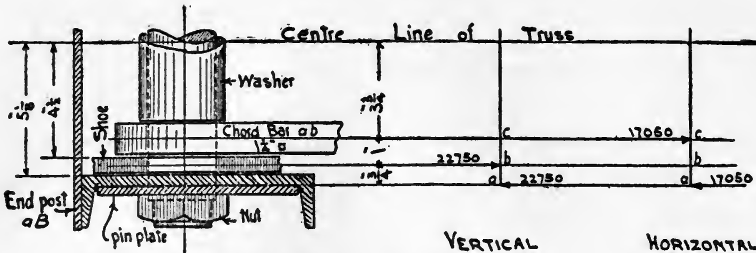
Total (distributed over a length of 16 ft.) = 28,400 lbs.

Reaction = 28,400 × ½ = 14,200 lbs.

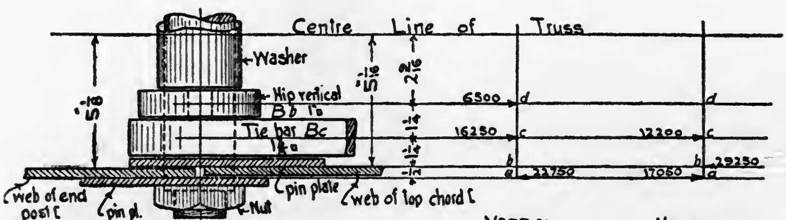
Taking moments about the center, M = 14,200 × 8,625 — 14,200 × 4 = 65,675 ft.-lbs., 65,675 × 12 = 788,100 inch-lbs., 788,100 ÷ 15,000 = 52.7 = R.

A 15″ I @ 42 lbs. will be used. R = 58.9.

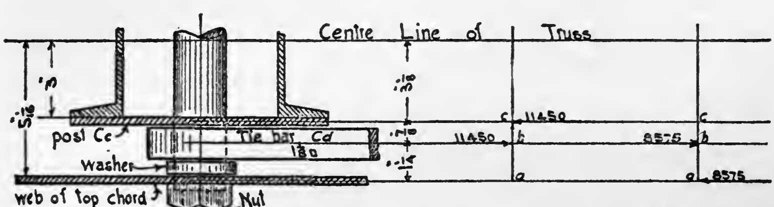
Pin Moments. The pins in the bottom chord and the one at the hip usually receive their maximum moment when the bridge is fully



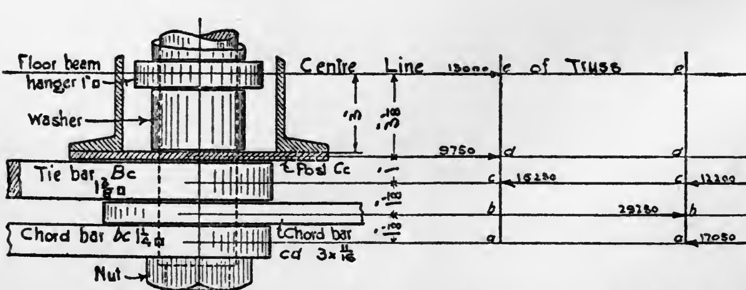
HORIZONTAL SECTION PIN a Fig. 66



HORIZONTAL SECTION PIN B Fig. 67



HORIZONTAL SECTION PIN C Fig. 68



HORIZONTAL SECTION PIN c Fig. 69

loaded, but the other pins in top chord will be subjected to their maximum moment, simultaneously with the maximum stress in main diagonals connecting thereto. Before figuring the pin moments, it is necessary first to find the stresses in all the members which are connected by pin for the condition of loading giving the maximum moment on pin. The stresses in the diagonal members should be resolved into their vertical and horizontal components, and the vertical and horizontal moments figured separately. The resultant moment at any point will be equal to the hypothenuse of a right angled triangle whose vertical and horizontal sides are equal respectively to the vertical and horizontal moment on pin. Having found the maximum moment, the size of pin required is obtained from a table similar to that in Carnegie p. 183, giving the maximum bending moment allowable on pins of various sizes. The values are obtained by multiplying the R of pin by the allowable stress per square inch on outer fibres—in this case 22,500 lbs. A pin may be large enough for the bending moment and yet too small for bearing; so care must be taken to ensure that the bearing of all members on the pin does not exceed the allowable amount, which, in the present example has been taken at 20,000 lbs. per square inch.

Pin a, Fig. 66. The vertical component of the stress in end post is equal to this stress multiplied by depth of truss and divided by length of post $= 56{,}900 \times \frac{20}{25} = 45{,}500$. The horizontal component is equal to stress in post multiplied by length of panel and divided by length of post $= 56{,}900 \times \frac{15}{25} = 34{,}100$ lbs.

In order to determine the distances between the members, the joint is sketched to a large scale as shown. It is only necessary to consider the forces on one side of the centre line of truss, which are as follows: on the shoe, an upward force equal to one-half the vertical component of stress in end post = 22,750 lbs.; on end post, a downward force of 22,750 lbs., and a horizontal force acting towards the left of 17,050 lbs.; and on the chord bar, a horizontal force to the right of 17,050 lbs. The sum of the forces acting in one direction must always equal the sum of the forces acting in the opposite direction. The vertical and horizontal forces are shown plotted on separate diagrams. Then:

Vertical Moments.		Horizontal Moments.	
At b = 22,750 × ¾″ =	17,050	At b = 17,050 × ¾″ =	12,800
At c =	17,050	At c = 17,050 × 1¾″ =	29,850

Resultant Moment.

$$\text{At c} = \sqrt{17{,}050^2 + 29{,}850^2} = 34{,}400 \text{ inch-lbs.}$$

Now, from table of pin moments in column for 22,500 lbs. fibre stress, it will be found that a 2½″ pin has a value of 34,500 inch-lbs.

As stated above, the pins in the bottom chord and at the hip usually receive their maximum moment when the bridge is fully loaded. It will therefore be necessary to figure the stresses in the web members for this condition of loading before proceeding further. The dead and live loads may be taken together and the total stresses found in exactly the same manner as for dead load only. The total panel load = 4,000 + 9,000 = 13,000 lbs. Reaction = 13,000 lbs. × 3½ panel = 45,500 lbs.

DEAD AND LIVE LOAD STRESSES IN WEB MEMBERS; BRIDGE FULLY LOADED.

Shear in panel *ab*=45,500— 0=45,500	Stress in aB = 45,500 × $\frac{25}{20}$ = 56,900
" " *bc*=45,500—13,000=32,500	" Bc = 32,500 × $\frac{25}{20}$ = 40,600
" " *cd*=45,500—26,000=19,500	" Cc = 19,500 × 1 = 19,500
" " *de*=45,500—39,000= 6,500	" Cd = 19,500 × $\frac{25}{20}$ = 24,400
	" Dd = 6,500 × 1 = 6,500
	" De = 6,500 × $\frac{25}{20}$ = 8,100

For the pin moments the stress in the hip vertical and in all floor beam hangers will be taken as an ordinary panel load, viz.: 13,000 lbs.

Pin B. When the bridge is fully loaded, the vertical and horizontal components of the stresses in the various members connected by this pin are as follows:

Members.	Vertical Components.	Horizontal Components.
End Post aB	56,900×$\frac{20}{25}$=45,500	56,900×$\frac{15}{25}$=34,100
Tie Bar Bc	40,600×$\frac{20}{25}$=32,500	40,600×$\frac{15}{25}$=24,400
Hip Vertical Bb	13,000	0
Top Chord Bc	0	58,500

One-half of the above vertical and horizontal forces are plotted in separate diagrams, Fig. 67.

Vertical Moments.		Horizontal Moments.	
At b=22,750×½″	=11,375	At b=17,050×½	=8,525
At c=22,750×1¾	=39,800	At c=17,050×1¾—29,500×1¼	=6,800
At d=22,750× 3—16,250×1¼	=47,950	At d	=6,800

Resultant Moment.

$$\text{At } d=\sqrt{47{,}950^2+6{,}800^2}=48{,}500.$$

Consequently, a $2\frac{7}{8}''$ pin which has a value of 52,500 inch-lbs. is the size required.

Pin C, Fig. 68. The condition for maximum moment on this pin is when the bridge is loaded from *d* to *h*. The stresses *C c* and *C d* are as shown on stress diagram Fig. 65. Since the top chord is continuous at this point, it is only the difference of the stresses in *B C* and *C D* which need be considered, and this is evidently equal to the horizontal component of the stress in *C d*.

The vertical and horizontal components of stresses in members are as follows:

Members.	Vertical Components.	Horizontal Components.
Top chord *CD*	0	$28{,}600\times\frac{15}{25}=17{,}150$
Tie bar *Cd*	$28{,}600\times\frac{20}{25}=22{,}900$	$28{,}600\times\frac{15}{25}=17{,}150$
Post *Cc*	22,900	0

Vertical Moments.		Horizontal Moments.	
At b=	0	At b=8,575×1¼	=10,700
At c=11,450×⅞	=10,000	At c	10,700

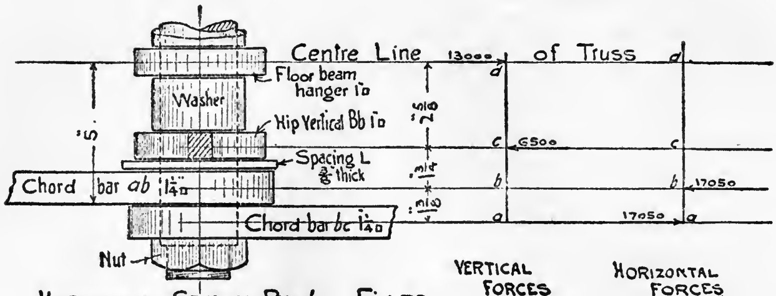
Resultant Moment.

$$\text{At } c=\sqrt{10{,}000^2+10{,}700^2}=14{,}600 \text{ inch-lbs.}$$

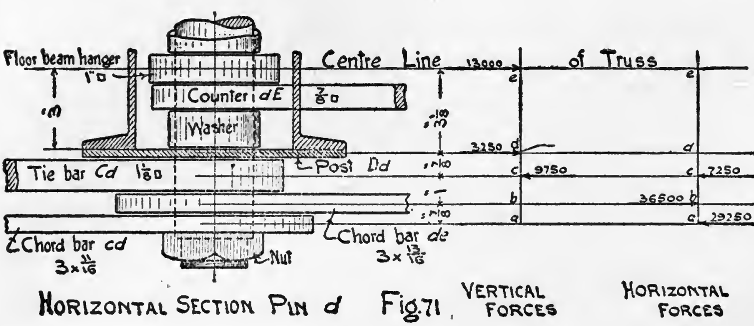
A $2''$ pin which has a bending value of 17,700 inch-lbs. is the size required. This size may also be used at *D* and *E*.

Pin c, Fig. 69. When the bridge is fully loaded, the vertical and horizontal components of the stresses in the various members connected by this pin are as follows:

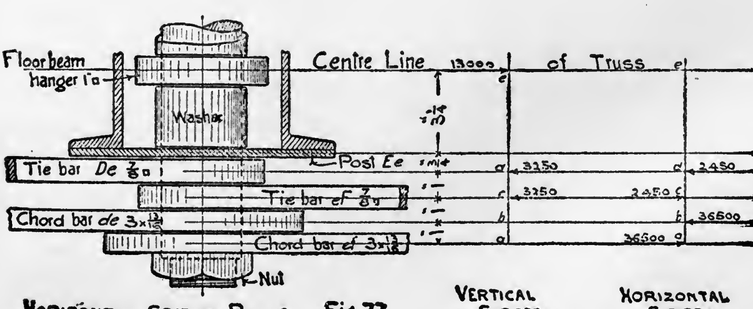
Members.	Vertical Components.	Horizontal Components.
Chord bar *bc*	0	34,100
Chord bar *cd*	0	58,500
Tie bar *Bc*	$40{,}600\times\frac{20}{25}=32{,}500$	$40{,}600\times\frac{15}{25}=24{,}400$
Post *Cc*	19,500	0
Floor beam hanger	13,000	0



HORIZONTAL SECTION PIN *b* Fig. 70



HORIZONTAL SECTION PIN *d* Fig. 71



HORIZONTAL SECTION PIN *e* Fig. 72

Vertical Moments.		Horizontal Moments.	
At b=	0	At b=17,050×1⅛	=19,200
At c=	0	At c=17,050×2¼—29,250×1⅛=	5,600
At d=16,250×1	16,250	At d=	5,600
At e=16,250=4⅛—9,750×3⅛=36,500		At e=	5,600

Resultant Moment.

$$\text{At e} = \sqrt{36.500^2 + 5,600^2} = 37.000.$$

Thus a $2\frac{5}{8}$ pin which has a bending value of 40,000 in.-lbs. is the size required.

Pin b, Fig. 70. When the bridge is fully loaded, the vertical and horizontal components of the stresses in the various members connected by this pin are as follows:

Members.	Vertical Components.	Horizontal Components.
Chord bar *ab*	0	34,100
Chord bar *bc*	0	34,100
Hip vertical *Bb*	13,000	0
Floor beam hanger	13,000	0

The chord bars *a b* and *b c* should be spaced so that they will pull in as nearly a straight line from *a* to *c* as possible. By reference to Fig. 66, the distance from center line of truss to center line of chord bar will be found to be $3\frac{3}{4}''$; and in Fig. 69, the center of chord bar *b c* is $6\frac{3}{8}''$ from center line of truss. The average of these distances is about 5″, which should be the distance to the centre of the two chord bars at *b*, as shown in Fig. 70. The object of the spacing angle between the hip vertical and chord bar *a b* is to hold the top of floor beam at the proper distance below pin.

Vertical Moments.		Horizontal Moments.	
At b=	0	At b=17,050×1⅜=23,500	
At c=	0	At c	=23,500
At d=6,500×2⅝=17,000		At d	=23,500

Resultant Moment.

$$\text{At d} = \sqrt{17,000^2 + 28,500^2} = 29,000.$$

A 2½″ pin which has a bending value of 34,500 inch-lbs. is the size required.

Pin d, Fig. 71. When bridge is fully loaded, the vertical and horizontal components of the stresses in the various members connected by this pin are as follows:

Member.	Vertical Component.	Horizontal Component.
Chord bar *cd*	0	58,500
Chord bar *de*	0	73,000
Tie bar *Cd*	$24,400 \times \frac{20}{25} = 19,500$	$24,400 \times \frac{15}{25} = 14,500$
Post *Dd*	6,500	0
Counter *dE*	0	0
Floor beam hanger	13,000	0

Vertical Moments.		Horizontal Moments.	
At b=	0	At b=$29,250 \times \frac{7}{8}$	=25,600
At c=	0	At c=$29,250 \times 1\frac{7}{8} - 36,500 \times 1$	=18,500
At d=$9,750 \times \frac{7}{8}$=	8,500	At d=	18,500
At e=$9,750 \times 4 - 3,250 \times 3\frac{1}{8}$	=28,800	At e=	18,500

Resultant Moment.

$$\text{At e} = \sqrt{28,800^2 + 18,500^2} = 34,200.$$

A $2\frac{1}{2}''$ pin which has a bending value of 34,500 inch-lbs. is the size required.

Pin e, Fig. 72. When the bridge is fully loaded the vertical and horizontal components of the stresses in the various members connected by this pin are as follows:

Members.	Vertical Components.	Horizontal Components.
Chord bar *ef*	0	73,000
Chord bar *de*	0	73,000
Tie bar *eF*	$8,100 \times \frac{20}{25} = 6,500$	$8,100 \times \frac{15}{25} = 4,900$
Tie bar *De*	$8,100 \times \frac{20}{25} = 6,500$	$8,100 \times \frac{15}{25} = 4,900$
Post *Ee*	0	0
Floor beam hanger	13,000	0

Vertical Moments.		Horizontal Moments.	
At b=	0	At b=$36,500 \times 1$=	36,500
At c=	0	At c=$36,500 \times 2 - 36,500 \times 1$=	36,500
At d=$3,250 \times 1$=	3,250	At d=$36,500 \times 3 - 36,500 \times 2 + 2,450 \times 1$=	38,900
At e=$3,250 \times 5 + 3,250 \times 4$=	29,250	At e=	38,900

Resultant Moment.

$$\text{At e} = \sqrt{29,250^2 + 38,900^2} = 48,700.$$

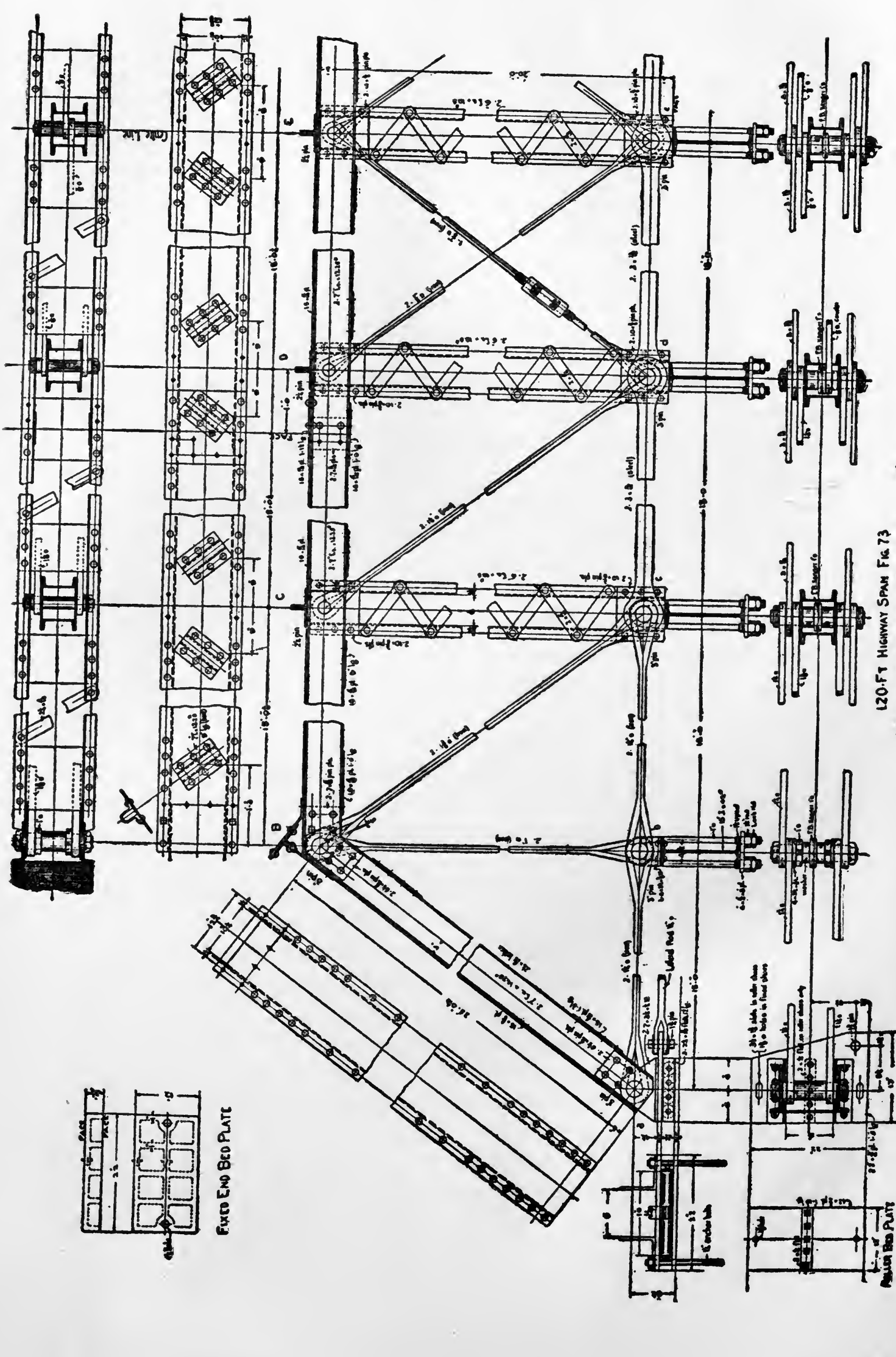
A $2\frac{7}{8}''$ pin which has a bending value of 52,500 inch-lbs. is the size required.

The size of pins required at the various panel points, as determined above, are as follows: $2\frac{7}{8}''$ at B and e; $2\frac{5}{8}''$ at c; $2\frac{1}{2}''$ at a, b and d; and $2''$ at C, D and E. But, in order to simplify the shop work as much as possible, $3''$ pins will be used throughout, except $2\frac{1}{4}''$ pins at C, D and E.

The pins in the top chords and end posts should be placed as near the centre of gravity of the section as practicable. In the present example this centre of gravity will be found to be a little more than one inch above the centre line of channels, but it would be inconvenient to set the pins at this height as there would not be space enough for some of the bar heads, so they are located one-half inch above centre line of channels or three inches below top flange. This arrangement gives just room enough for the largest bars at panel point B, where the space required equals one-half the diameter of pin, plus the thickness of bars, $= 1\frac{1}{2}' + 1\frac{3}{8}'' = 2\frac{7}{8}''$, leaving $\frac{1}{8}''$ clearance between bars and cover plate.

Shoes, Rollers and Bed Plates. Fig. 73. The total load on the shoes is equal to the vertical component of stress in end post $= 56{,}900 \times \frac{20}{25} = 45{,}500$ lbs. Then, since the permissible bearing for pins $= 20{,}000$ lbs. per square inch, $45{,}500 \div 20{,}000 = 2.27$ square inches required for shoe standards. Two $7 \times 3\frac{1}{2} \times \frac{1}{2}$ Ls have been used, and the bearing area of pin on these $= \frac{1}{2}'' \times 3'' \times 2 = 3$ square inches. This is somewhat more than required, but it is well to make these members quite stiff. It is difficult to determine the exact thickness to make the shoe plates and bed plates, and this matter is usually left to the judgment of the designer. In the present case these have been made $\frac{3}{4}''$ thick. The area of the bed plates should be great enough so that the pressure on the masonry will not exceed 300 lbs. per square inch. They must also be large enough to accommodate the rollers and the anchor bolts. The bearing area required $= 45{,}500 \div 300 = 152$ square inches. The area of bed plates used $= 12'' \times 22'' = 264$ square inches.

The rollers should be designed so that the pressure on them per lineal inch shall not exceed $600\sqrt{\text{diameter}}$. Assuming $2\frac{1}{2}''$ rollers, the permissible bearing $= 600\sqrt{2.5''} = 950$ lbs., then $45{,}500 \div 950 = 48$ lineal inches required. There are 4 rollers and the effective length of each $= 16'' - 2\frac{1}{4}'' = 13\frac{3}{4}''$. Then $4 \times 13\frac{3}{4} = 55$ lineal inches provided. The rollers are turned down at center to pass over guide strips on shoe and bed plates, and the ends are turned down to enter holes in the $2\frac{1}{4} \times \frac{5}{16}$ spacing bars, as shown. The ends of the outer rollers are made long enough to insert cotter



120-FT HIGHWAY SPAN FIG. 73

pins, but the ends of the intermediate rollers just pass through the spacing bars.

The shoe plates are extended, as shown, to provide connections for the end laterals, which are made with forked eyes.

The fixed end bed plates are made of cast iron, and their height is equal to the diameter of rollers plus the thickness of roller beds $= 2\frac{1}{2} + \frac{3}{4} = 3\frac{1}{4}''$.

The anchor bolts should have sufficient cross section to resist the total shear from the assumed wind force in plane of bottom chords, plus one-half of the wind force on portal strut. In the table of lateral stresses the dead and live load shears in each panel each $= 2{,}250 \times 3\frac{1}{2} = 7{,}875$ lbs., and the wind force at top of portal also equals 7,875 lbs. Then the total force to be resisted by anchor bolts $= 7{,}875 \times 2\frac{1}{2} = 19{,}700$. The area of two $1\frac{1}{4}''$ bolts $= 2.45$ square inches, and their shearing value $= 2.45 \times 10{,}000$ lbs. $= 24{,}500$ lbs.

End Post and Top Chord. The stress in end post $= 56{,}900$ lbs., and the permissible pin bearing 20,000 lbs. per square inch, then $56{,}900 \div 20{,}000 = 2.84$ sq. inches required. The diameter of pins $= 3''$, therefore thickness of bearing required $= 2.84 \div 3 = .95''$. The thickness of the webs of the $7''$ [s @ 14.75 lbs. $= 7/16''$. In addition, $5/16''$ pin plates are used, making the total thickness of metal $(\frac{7}{16} + \frac{5}{16}) \times 2 = 1\frac{1}{2}''$. The amount of bearing on the pin plates will be in the ratio of their thickness to the total thickness of bearing $= 56{,}900 \times \dfrac{\frac{5}{8}}{1\frac{1}{2}} = 23{,}700$. There should be sufficient rivets in pin plates to meet this stress. The single shearing value of one $\frac{3}{4}''$ rivet $= 4{,}420$ lbs., then $23{,}700 \div 4{,}420 = 6$ rivets required; 8 rivets are shown—4 in each plate. At the upper end of posts, the pin plates extend beyond the post, forming what are called jaw plates. The pin plates on top chord at this point also extend beyond pin, but are on the inside of channels, as shown.

The stress in top chord $BC = 58{,}500$ lbs., then $58{,}500 \div 20{,}000 = 2.92$ square inches bearing required; and $2.92 \div 3 = .97'' =$ thickness of bearing required. The webs of the $7''$ [s at 12.25 lbs. are $5/16''$ thick, and $5/16''$ pin plates are used, making the total thickness of bearing $(5/16 + 5/16) \times 2 = 1\frac{1}{4}''$. The amount of bearing on pin plates will then be $58{,}500 \times \frac{1}{2} = 29{,}250$ lbs. The number of $\frac{3}{4}''$ rivets required $= 29{,}250 \div 4{,}420 = 7$. Between the end post and top chord a space of $\frac{1}{8}''$ is left to ensure that the whole bearing will come on pin.

At panel point C the top chord is continuous, and it is only necessary to provide sufficient bearing for the horizontal component of the stress in tie bar $C\,d$, which is equal to $28{,}600 \times \frac{15}{25} = 17{,}150$ lbs. Then $17{,}150 \div 20{,}000 = .85$ square inches required, and $.85 \div 2.25 = .38'' =$ thickness of bearing required. The webs of the two channels $= \frac{5}{8}''$, thus no pin plates are required.

The top chord is spliced 1′ 0″ from panel point D. Since this is a faced joint, the stresses are transmitted directly by the abutting surfaces, and the splice plates are required only to hold the members in line.

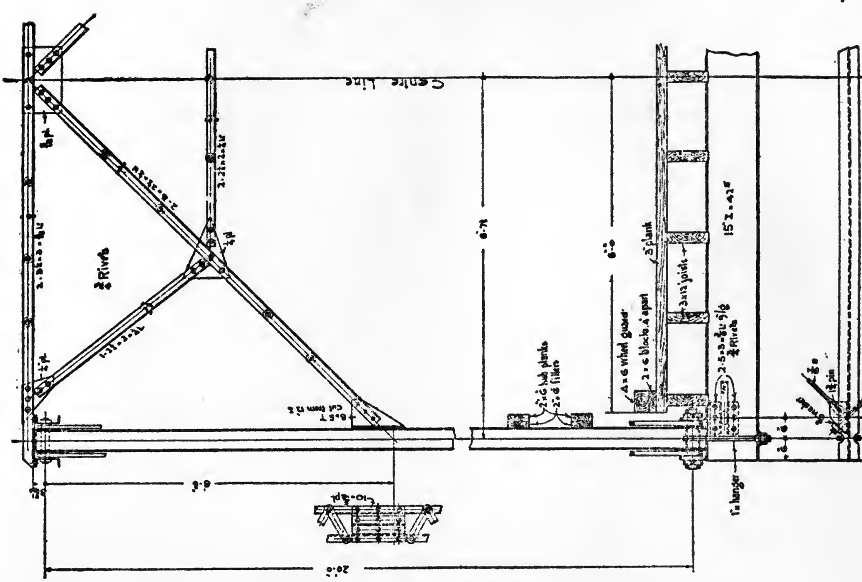
The bent channels on the top chord are lateral connections. The lateral rods, which are $\frac{7}{8}''$ diameter, are upset at the ends to $1\frac{1}{4}''$, and threaded for standard nuts. Plate washers, $\frac{3}{8}''$ thick, are used to screw up against the ends of bent channels.

Intermediate Posts. The stress in post $Cc = 22{,}900$ lbs., then $22{,}900 \div 20{,}000 = 1.14$ square inches bearing area required on pins. The diameter of the upper pin is $2\frac{1}{4}''$, therefore $1.14 \div 2.25 = .50 =$ thickness of bearing required. Two $\frac{3}{8}''$ pin plates are used at top and bottom of all posts. The posts must be wide enough to permit the floor beam hangers to pass between the channels as shown. The number of $\frac{5}{8}''$ rivets required in pin plates $= 22{,}900 \div 3{,}070 = 8$, whereas there are 12 rivets provided.

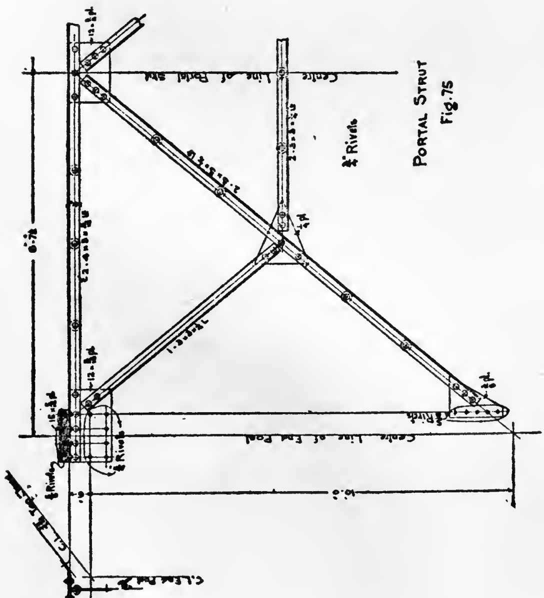
Bars. The loop ends of the square iron bars are made by bending the ends of the bar around a pin and welding these ends to the body of bar. The distance from centre of pin to crotch should be $2\frac{1}{2}$ times the diameter of pin. The heads of the flat steel bars are made by upsetting the ends of the bars and forging in dies of the required size. The heads are made round and of such diameter as to give an area through centre line of pin hole 50% greater than that of body of bar. Thus the width of bars $= 3''$, diameter of pins $= 3''$ then $3'' + 3'' + 1\frac{1}{2}'' = 7\frac{1}{2}'' =$ diameter of heads.

Pins are turned down or shouldered at the ends and provided with chambered nuts to ensure a full bearing for the outer members.

Floor Beams. The end reaction of floor beams $= 16{,}000$ lbs. and the unit stress for floor beam hangers $= 8{,}000$ lbs. per square inch, therefore $16{,}000 \div 8{,}000 = 2$ square inches required in hangers. The hangers are made of 1″ square iron, bent to fit over pins and upset to $1\frac{1}{2}''$ round at ends and threaded for nuts as shown in Fig.



CROSS SECTION 'C'
Fig. 74



PORTAL STRUT
Fig. 75

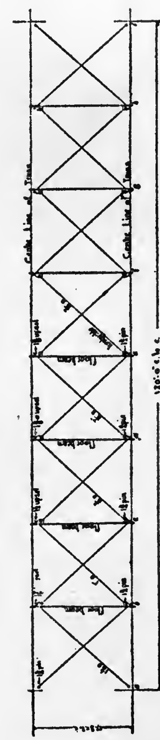


DIAGRAM OF BOTTOM LATERALS
Fig 76.

73. The main nuts are of the same thickness as diameter of thread, and the lock nuts are 1″ thick. The top and bottom flanges of the beams are notched as shown in Fig. 74, and on the bottom there is a washer plate $4'' \times \frac{3}{4} \times 8''$. The floor beams are screwed up firmly against the ends of the posts, which extend below the pins at *c*, *d* and *e* far enough to clear the largest bar heads. At panel point *b*, where the vertical members are rods, spacing angles are used to hold the beam at the same distance below the pins.

Bottom Laterals. The end laterals which are $1\frac{1}{4}''$ square are forked at one end to connect with the shoe plate as shown in Fig. 73. At the opposite end they are upset to $1\frac{7}{8}''$ round and threaded for standard nuts. These upset ends pass through holes in the web of floor beam *b*, and between the lug angles which are riveted to the web of floor beam, and cut at right angles to centre line of lateral rods as shown in Fig. 74. The pins connecting laterals with shoe plates may be considered as girders supported at both ends and having a concentrated load at the center equal to the stress in lateral. The distance centre to centre of forks is about 2″, then

$$M = \frac{Wl}{4} = \frac{21.000 \times 2}{4} = 10{,}500 \text{ inch-lbs., which requires a } 1\tfrac{3}{4}'' \text{ pin.}$$

For uniformity this size will be used throughout.

The laterals in the 2d panel, which are 1″ square, have plain loop eyes at one end to connect with the lug angles on floor beam *b*. At the opposite end they are upset to $1\frac{1}{2}''$ round, and pass through holes in the web of the floor beam *c*, and between the lug angles riveted thereto.

The laterals in the 3d panel, which are $\frac{7}{8}''$ square, have plain loop eyes to connect with the lug angles on floor beam *c*, and are upset to $1\frac{3}{8}''$ round at opposite end and pass through holes in web of floor beam *d*, and between the lug angles riveted thereto.

In one panel adjacent to the centre of the span there will be a pair of laterals $\frac{7}{8}''$ square with loop eyes at both ends. These rods are made in two parts and provided with turnbuckles for adjustment.

The lateral connections on the floor beams should be placed as near the top flange and as close to the hangers as possible.

The arrangement of the bottom laterals is shown in Fig. 76.

The portal and intermediate struts shown in Figs. 74 and 75 need no further description.