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Anal

BULLETIN
OF THE
NATIONAL RESEARCH COUNCIL

RESEARCH LABORATORIES IN INDUSTRIAL ESTABLISHMENTS OF THE UNITED STATES

Including Consulting Research Laboratories

Originally compiled by
Alfred D. Flinn, Secretary, Engineering Foundation
Revised and enlarged by
Ruth Cobb, Research Information Service

Published by The National Research Council
of
The National Academy of Sciences
Washington, D. C.
1921

Announcement Concerning Publications
of the
National Research Council

The Proceedings of the National Academy of Sciences

has been designated as the official organ of the National Research Council for the publication of accounts of research, committee and other reports, and minutes.

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RESEARCH LABORATORIES IN INDUSTRIAL ESTABLISHMENTS OF THE UNITED STATES

Including Consulting Research Laboratories

Originally compiled by
ALFRED D. FLINN, SECRETARY, ENGINEERING FOUNDATION

Revised and enlarged by
RUTH COBB, RESEARCH INFORMATION SERVICE

CONTENTS

INTRODUCTION

The demand for information concerning industrial research laboratories has indicated such a widespread interest in this subject that it seemed desirable to issue an early revision of the list contained in *Bulletin of the National Research Council,* number 2. The original publication was compiled early in 1920 by Mr. Alfred D. Flinn, Secretary of the Engineering Foundation, with the assistance of Miss Ruth Cobb of the Research Information Service. It contains the names of nearly 300 laboratories in industrial establishments in the United States which had stated in direct correspondence that they were engaged in research. The present publication has revised the original material as of August, 1921, and has added about 250 new names.

As in the original list, all information here given has been obtained directly by correspondence, and statements are based upon information supplied by the laboratories. An endeavor has been made to follow the phraseology of the laboratories wherever possible and to print each name exactly in the style used by the company, with regard for spelling and abbreviations.

No investigation has been made to ascertain the character of any laboratory listed nor the quality of work done. In order to avoid mistakes through misinterpretation, the laboratories were given the opportunity to approve or correct their material after it had been transcribed. During August and September the majority of companies availed themselves of this opportunity.

Three methods were used to collect information about laboratories not originally listed: (a) Forms calling for corrections and additions were distributed widely with the first edition of the list. (b) Special requests for information were sent to over 200 companies believed to maintain research laboratories. These names were obtained through the generous cooperation of The Chemical Catalog Company, Inc., following its recent survey of chemical firms in this country. (c) A press notice of the forthcoming revision was sent to a selected group of technical and trade journals. This notice requested information from directors of research who had not already supplied it.

Of the 300 laboratories originally listed all except seven responded to the appeal for a revision of the first statement. Fourteen other names were dropped because the firms replied that they no longer maintained research laboratories. Eleven had made new connections and appear here under different names.

Laboratories connected with federal, state or municipal governments, or with educational institutions, were from the outset excluded from the inquiry, although frequently they are engaged upon investigations in industrial research. The concerns which are not actually supporting laboratories in their own works have not been included, nor have the associations maintaining fellowships in certain educational institutions. They are to be encouraged, but this compilation is limited to the laboratories themselves, rather than organizations supporting research.

The following information is given for each entry: name and address of company and address of laboratory if different from that of company; name of director of research and number on his staff; chief lines of research work; special equipment, if any, or equipment of unusual character.

In addition to the alphabetical list of laboratories, which carries all the information, there is given a subject classification. This classification combines the headings used in the scientific and commercial classifications of the first edition and thus eliminates some unnecessary duplication. These classifications were devised by members of the National Research Council, the Engineering Foundation and others interested in research work, and are based in part upon the classifications used in *Chemical Abstracts* and *Science Abstracts*. They were revised and combined for this edition with the help of Dr. C. J. West. An alphabetical index of subjects and cross references provides a key to the classification.

A copy of the headings used in the publication was sent to each laboratory listed with the request that it check the subjects under which its name should appear. The suggestions of the laboratories were followed wherever possible. In this way the research activities of the companies by subject are more fairly represented than was possible when the material was all classified by someone unfamiliar with the detailed work of these laboratories.

The geographical classification of the original edition has been dropped and in its place is given an alphabetical list of names and addresses of directors of research in the laboratories included in the bulletin.

Corrections and additional information will be welcomed.

ALPHABETICAL LIST OF LABORATORIES

1. **Abbé Engineering Company,** 50 Church St., New York, N. Y. (Designs pulverizing and grinding machinery.) Laboratory at 230 Java St., Brooklyn, N. Y.

Research staff: H. F. Kleinfeldt and 3 men experienced in machinery.

Research work: Part time of 3 on the solution of problems which involve crushing, grinding, pulverizing, mixing, and sifting machinery.

2. **Abbott Laboratories, The,** Chicago, Ill.

Research staff: A. S. Burdick, 8 chemists and 4 biologists.

Research work: Three-fourths time of 12 on new anesthetics, hypnotics, antiseptics, and other chemical research; animal pathology and bacteriology; pharmacology and investigations of new medicinal preparations.

3. **Abbott, William G., Jr.,** Wilton, N. H. (Research engineer.)

Research staff: W. G. Abbott, Jr., 1 engineer, 1 mechanical expert and 1 chemist (part time).

Research work: Three-fourths time on waste recovery, special machinery and processes for mechanical, electrical, textile and chemical trades.

4. **Acheson Graphite Company,** Niagara Falls, N. Y. (Graphite products, including dry-cell filler, paint pigment, stove polish, pencils, electrodes, crucibles, tubes, muffles, graphite and grease lubricants.)

Research staff: A. M. Williamson and 8 assistants.

Research work: Three-fourths time of 9 on graphite, carbon and lubricants.

5. **Acme White Lead & Color Works,** Detroit, Mich.

Research staff: Clifford D. Halley, 4 chemists and 2 engineers.

Research work: Full time of 7 on paints and varnishes.

Aetna Explosives Company, Inc. See Hercules Powder Co., Emporium Research Laboratory (p. 39).

6. **Allen-Bradley Co.,** 286 Greenfield Ave., Milwaukee, Wis. (Electric controlling apparatus.)

Research staff: Lynde Bradley, 3 chemists and 1 mechanic.

Research work: Full time of 5 on resistance materials and insulation.

Allied Dye & Chemical Corporation. See General Chemical Company (p. 35).

7. **Aluminum Company of America,** Oliver Building, Pittsburgh, Pa. Central Laboratory at New Kensington, Pa. Branch of the Research Bureau at Cleveland Plant of Aluminum Manufactures, Inc.

Research staff: Francis C. Frary and others.

Research work: Aluminum production and utilization.

8. **American Agricultural Chemical Company, The.** Agricultural Service Bureau, 92 State St., Boston, Mass. (Fertilizers.) Chemical laboratory at Carteret, N. J.

Research staff: H. J. Wheeler, 9 agronomists and chemists, superintendent of experiment farm, 1 expert photographer.

Research work: Study of requirements of soils and crops where fertilizers are being introduced or have not been used; study of citrus fruits and other special crops in Florida in connection with various types of soil; experiments and demonstrations with fertilizers in Illinois, Iowa, Minnesota, New Hampshire, Wisconsin and other states.

9. **American Beet Sugar Company,** Denver, Colo. Laboratory at Rocky Ford, Colo.

Research staff: 1 chief, 1 director, 1 agricultural investigator, 1 economic entomologist, 2 factory chemists, and 1 experiment station assistant.

Research work: Full time of 5 on all agricultural phases of sugar beet improvement, including the analysis of irrigation waters and soils, study of rotations, cultural methods, seed breeding, and the investigation of the life histories of economic insect pests.

Equipment: Complete plant pathological and entomological equipment. Greenhouse for propagation and study of various economic phases of plant breeding, control of diseases, and observations on insect pest development and habits.

10. **American Blower Company,** 6004 Russell St., Detroit, Mich.

Research staff: J. A. Watkins and 2 or more assistants.

Research work: Full time of 3 on air propelling mechanisms, air conditioning apparatus, dehydrating or desiccating apparatus, conveying of dust and waste material, heating and ventilating, forced and induced draft for combustion of all kinds of fuels and kindred lines where air movement forms the basis for desired results.

Equipment: All kinds of instruments for measuring the pressure and flow of air, electric dynamometers for determining power expended, electrical measuring instruments, instruments for determination of the purity, density, humidity, temperature and pressure of the atmosphere, etc.

11. **American Brass Company, The,** Waterbury, Conn. Chemical, metallographic and metallurgical laboratory at Waterbury; physical and electrical testing laboratory at Ansonia.

Research staff: William H. Bassett, 3 metallurgists, 2 chemists, 1 physicist and metallographer, 1 metallographer, 2 metallurgical engineers, 1 testing engineer and necessary assistants.

Research work: One-third time of 11 on nature and effect of impurities in copper and its alloys; effects of mechanical working, heat treatment, corrosion and conditions of exposure.

Equipment: Waterbury laboratory: metallographic equipment for study of heat treatment of non-ferrous metals and alloys; Adam Hilger Quartz "D" spectroscope of high sensitiveness; facilities for production of special alloys, corrosion and other special tests. Ansonia: 200,000-pound Olsen, 100,000-pound Riehle and smaller testing machines, covering physical testing of all materials down to very fine wire; fatigue and friction testing apparatus; electrical apparatus for accurate resistance and conductivity tests.

12. **American Can Company,** 120 Broadway, New York, N. Y. Laboratory at 11th Ave. and St. Charles Road, Maywood, Ill.

Research staff: F. F. Fitzgerald, 2 assistant chemists, 1 metallurgist, 2 food technologists, 2 analysts and 4 laboratory assistants.

Research work: One-half time of 12 on cooperative work with packers of food products in investigating chemical changes taking place in food products and their influence upon the preservation of the food, its quality and its wholesomeness. Manufacturing operations, including study of fluxes, white metal alloys, coals, oils and other materials.

Equipment: Special apparatus for analysis of tinplate and solder for tin content; apparatus for investigating tin cans, sealing them, etc.

13. **American Chemical and Manufacturing Corporation**, Cranford, N. J.

Research staff: Harry P. Taber and 1 assistant.

Research work: Part time of 2 on animal and vegetable oils, resins, varnish gums and cellulose esters.

14. **American Chemical Paint Company**, 1126 S. 11th St., Philadelphia, Pa.

Research staff: J. H. Gravell, 2 chemists, 1 engineer and 1 general utility man.

Research work: Full time of 5 on rust-proof paints for iron and steel; scale and rust removal; high temperature paint; methods of preparing metals for painting, enameling and japanning; water-proof and acid-proof barrel linings.

15. **American Cyanamid Company**, 511 Fifth Ave., New York, N. Y. Has three plants and a laboratory at each but research and development work are being centralized at plant nearest New York.

Research staff: W. S. Landis, 5 skilled chemists and several assistants, as a minimum. Usually includes 10 or 15 skilled men being trained for operating positions in new processes.

Research work: Full time of staff on fertilizers, nitrogen fixation, cyanide phosphates, potash, nitrogen compounds and derivatives. Much of the work done in the experimental plants and laboratories is development, rather than true research.

Equipment: Apparatus is of commercial size; frequently a complete small commercial plant is leased for experimental work.

16. **American Diamalt Company**, 419 Plum St., Cincinnati, Ohio. Laboratory at Riverdale, Cincinnati, Ohio.

Research staff: Joseph M. Humble and 5 chemists.

Research work: Half time of 6 on diastatic and malt sugar products in general.

17. **American Hominy Company**, 1857 Gent Ave., Indianapolis, Ind.

Research staff: F. C. Atkinson and 10 to 12 assistants.

Research work: Approximately half time of 12 on corn products.

18. **American Institute of Baking**, 1135 Fullerton Ave., Chicago, Ill.

Research staff: Harry E. Barnard, 2 chemists and 1 technician.

Research work: Full time of 4 on special problems of baking and their investigation from the standpoint of industrial development; sanitation of bakeries.

Equipment: Complete baking equipment.

19. **American Optical Company**, Southbridge, Mass.

Research staff: Charles Sheard, 1 physicist, 1 physicist and physical opticist, 1 physiological opticist, 1 astronomer, 1 general chemist, 1 physical chemist, 1 metallurgist and 7 assistants, including mechanician.

Research work: Full time of 14 on metallurgical research in nonferrous metals, especially on ability of metals and alloys to stand repeated workings. Spectral transmission of glasses, for example, glasses to reflect or absorb infra-red. Optical designing in general, especially designing of scientifically correct ophthalmic lenses; also optical instrument designing. Abrasive material for grinding and polishing glass. Fusing together glasses of different types. Adhesives. Glass strength investigations. Retinal currents due to light stimulation. Relations between radiant energy and the eye. Problems of ocular refraction. Limits of visibility in ultraviolet. Also publishes American Journal of Physiological Optics.

Equipment: Optical measuring apparatus for transmission in the ultraviolet, visible and infra red; Zeiss metallographic outfit.

20. **American Radiator Company,** Buffalo, N. Y. Laboratory at 1807 Elmwood Ave., Buffalo, N. Y.

Research staff: Frank B. Howell, with an average of 11 technicians and helpers.

Research work: Approximately full time of 12 on apparatus for air warming and cooling, involving heating boilers for burning anthracite, bituminous, and lignite coals, coke, gas, oil, etc., for Europe as well as America. Radiators: induction, convection, radiation. Refrigeration.

Equipment: Innumerable brick and steel chimneys of various sizes for determining accurately grate, fuel, ash, heating surface, flue surface and total draft tensions.

21. **American Radio and Research Corporation,** Medford, Mass. (Wireless telegraphs and telephones.)

Research staff: V. Bush, 1 engineer manager and 5 assistants.

Research work: Full time of 7 on phenomena at radio frequencies, and other matters intimately connected with radio telegraphy and telephony. Also investigation of power factor correcting equipment.

Equipment: Apparatus for measurements and research at high frequency, such as arcs, oscillating bulbs, generators and bridges.

22. **American Rolling Mill Co., The,** Middletown, Ohio.

Research staff: Wesley J. Beck, 2 consulting chemical and metallurgical engineers, 1 electrical engineer and assistants, 1 metallurgical engineer and 2 assistants and 1 chemical engineer and 4 assistants with routine chemists.

Research work: Nine-tenths time of staff on corrosion of iron and steel; alloys, paints, magnetic properties of iron and steel.

23. **American Sheet and Tin Plate Company,** 210 Semple St., Pittsburgh, Pa.

Research staff: R. E. Zimmerman, 7 chemical engineers, 2 chemists, 1 physicist and 1 metallurgist.

Research work: Full time of 12 on chemical engineering problems relating to the manufacture of sheet steel, tin plate, and galvanized sheets; metallurgy, metallography and pyrometry as applied to these manufacturing processes.

24. **American Sugar Refining Company, The,** 117 Wall St., New York, N. Y. Service Division.

Research staff: A. V. Fuller and 1 assistant.

Research work: One-half time of 2 on adaptability of various sugar cane products to special purposes; causes of failure in manufacture of sugar products and their remedies, and development of new sugar food products.

Equipment: A trade candy kitchen in conjunction with the laboratory.

American Telephone and Telegraph Company. See Western Electric Company, Incorporated (p. 84).

25. **American Trona Corporation,** Trona, Calif. (Borax, potash, etc.)

Research staff: R. W. Mumford, 2 chemical engineers and 5 chemists.

Research work: Full time of 8 on study of the equilibrium between the chlorides, sulphates, carbonates and borates of sodium and potassium, development of proper evaporation methods for evaporating Searles Lake brine and manufacture of boric acid and borates.

American Vanadium Co. See Vanadium Corporation of America (p. 82).

26. **American Window Glass Co.,** Factory No. 1, Arnold, Pa.

Research staff: L. P. Forman, 4 chemists and 2 ceramists.

Research work: One-third time of 7 on new developments in glass industry, and ceramic work.

Equipment: Pyrometric apparatus; high and low temperature electric furnaces.

27. **American Writing Paper Co.,** Holyoke, Mass., Department of Technical Control.

Research staff: F. C. Clark, director; Ross Campbell, assistant director; L. E. Roberts, in charge of research section; 4 research chemists, 5 chemical engineers, 3 analytical chemists and 1 laboratory helper.

Research work: Full time of 9 on new fibres, new paper-making processes, improvements in present processes; mill experimental work to improve present processes and effect economies in operation.

Equipment: 2 12-pound Noble and Wood beaters, 4 model digesters, special machine for testing tub size. Complete experimental paper mill with 66-inch combination Fourdrinier and cylinder paper machine; small model paper machine producing a sheet of paper 4 inches wide.

28. **Amoskeag Manufacturing Company,** Manchester, N. H. (Textile mills.)

Research staff: William K. Robbins, 3 chemists and 1 laboratory helper.

Research work: Small part time of 4 on waste recovery, dye, bleaching, sizing and testing problems. Semi-commercial scale experiments in plant.

Equipment: Exposure boards for light and weather tests, cloth and yarn breaking machines.

29. **Anaconda Copper Mining Co.,** Anaconda, Mont.

Research staff: F. F. Frick, 9 assistants and 10 to 20 non-technical assistants.

Research work: Full time of 20 to 30 on problems connected with the industry.

30. **Andrews, A. B.**, State Assayer, Lewiston, Me.

Research staff: A. B. Andrews, 2 chemists and 1 engineer.

Research work: Two-thirds time of 3 on paper, ceramics, naval stores and electrical conductivity.

Equipment: Grinding equipment including 1-ton ball mill and digester, beater and calendar for paper.

31. **Ansbacher, A. B., & Company**, 527 Fifth Ave., New York, N. Y. Laboratory at 310 N. 7th St., Brooklyn, N. Y.

Research staff: D. N. Barad and 2 assistant chemists.

Research work: Dry colors and inorganic pigments.

32. **Ansco Company**, Binghampton, N. Y. (Photographic equipment and supplies.)

Research staff: Alfred B. Hitchins and 5 trained men.

Research work: Full time of 6 on photographic work.

Equipment: For photographic emulsions, spectroscopic work, spectro-photography, photometry and photo-micrography, testing of dyes and color filters, polariscopic and refractometric work; high temperature ovens. Experimental laboratory for motion picture work.

33. **Ansul Chemical Company**, Marinette, Wis. (Liquified anhydrous sulphurous acid.)

Research staff: H. V. Higley and 1 chemical engineer.

Research work: Three-fourths time of 2 on relation of anhydrous sulphur dioxide to oils, metals and other materials; development of allied products for manufacture; study of customer's special problems of the use of sulphur dioxide in bleaching, deodorizing, disinfecting, mechanical refrigerating machines and chemical manufacturing.

Equipment: Special apparatus for sulphur dioxide analysis and for plant control work.

34. **Arlington Mills**, Lawrence, Mass. (Worsted textiles.)

Research staff: Hugh Christison, 3 chemists and 3 assistants.

Research work: Problems connected with the manufacture of textiles in the application of dyestuffs.

35. **Armour Fertilizer Works**, 209 W. Jackson Blvd., Chicago, Ill.

Research staff: M. Shoeld, 3 chemists and 2 engineers.

Research work: Full time of 6 men on general research relating to fertilizer industry. (Research work at present interrupted.)

Equipment: Special type electric furnaces; special type fuel fired furnaces.

36. **Armour Glue Works**, 31st Place and Benson St., Chicago, Ill. Laboratory serves also Armour Soap Works, Armour Ammonia Works, Armour Curled Hair Works, and Armour Sandpaper Works.

Research staff: J. R. Powell, 6 chemists, 6 laboratory assistants and 4 helpers.

Research work: Full time of 1 and part time of 2 on investigation of some of the plant processes. Work is principally analytical, for plant control.

37. **Art in Buttons, Incorporated**, Rochester, N. Y.

Research staff: F. W. Ross, chemical research; Richard Stanforth, industrial research, and assistants.

Research work: Full time on problems incident to vegetable ivory button manufacturing.

Associated Factory Mutual Fire Insurance Companies. See Factory Mutual Laboratories (p. 32).

38. **Atlantic Dyestuff Company,** 88 Ames Building, Boston, Mass.

Research staff: A. C. Burrage, Jr., and 3 assistants.

Research work: Part time of 4 on intermediates and dyes.

39. **Atlantic Refining Company, The,** 3144 Passyunk Avenue, Philadelphia, Pa. (Petroleum products.)

Research staff: T. G. Delbridge, 5 chemical engineers, 12 chemists, 1 physicist and 18 assistants. Mechanical and electrical engineering staffs collaborate with laboratory.

Research work: Three-fourths time of 37 on manufacturing methods of petroleum refinery, including study of manufacturing equipment and of equipment for testing.

Equipment: Laboratory-scale petroleum refinery, together with complete equipment for study of petroleum products; large scale manufacturing apparatus in the plant is at disposal of laboratory staff.

Atlas Ball Company. See S. K. F. Industries, Inc. (p. 72).

40. **Atlas Powder Co.,** Wilmington, Del. (Explosives, leather cloth, lacquers and heavy chemicals.) Maintains three laboratories for research.

Research staff: R. L. Hill, Reynolds, Pa., G. C. Given, Stamford, Conn., F. Bonnett, Jr., Landing, N. J., and 30 chemists.

Research work: Full time of 33 on explosives of all kinds, caps, electric detonators, leather cloth, lacquers and miscellaneous chemicals.

Equipment: Designed for experimental work on explosives, leather cloth and lacquers.

41. **Ault & Wiborg Company, The,** Cincinnati, Ohio. (Lithographic and letter press inks, ink varnishes, dry colors and dryers; varnishes, lacquers and enamels; typewriter ribbons and carbon paper; writing fluids, pastes and mucilages; dealers in all lithographic supplies.)

Research staff: Robert W. Hilton and 3 research chemists.

Research work: Full time of 4 on pigments, varnishes, ribbons, carbon paper, lacquers and enamels.

42. **Avri Drug & Chemical Company, Inc.,** 421 Johnston Ave., Jersey City, N. J.

Research staff: L. M. Avstreih and 1 assistant.

Research work: Pharmaceutical, technical and analytical chemistry.

43. **Babcock & Wilcox Co., The,** Bayonne, N. J. (Steam engine boilers.)

Research staff: J. B. Romer and 7 assistants.

Research work: Full time of 2 and part time of 4 on development of refractory materials, embrittlement of steel, aluminum coating on steel and betterment of boiler practice.

Equipment: Furnaces and apparatus for pyrometer and thermometer calibration; 150,000-pound Riehle testing machine; Upton-Lewis torsional and alternate bending machine; Brinell machine; scleroscope; special equipment for refractories research; special equipment for investigation of hydrogen embrittlement in steel.

44. **Babcock Testing Laboratory,** 803 Ridge Road, Lackawanna, N. Y.

Research staff: S. C. Babcock, Bartlett Ramsdell, 1 chemical engineer, 1 chemist and 1 helper.

Research work: One-half time of 5 on driers for paint, varnish and printer's ink trade; by-products utilization, soap, gums, oils and waxes.

Equipment: Small scale unit (200-pound) for production of soap, driers, etc. Destructive distillation equipment.

45. **Baker & Co., Inc.,** Newark, N. J. (Refiners and workers of platinum, gold and silver.)

Research staff: F. Zimmerman, chemical department, and 11 assistants. F. E. Carter, physical department, and 4 assistants.

Research work: Large part of time of 17 on chemical research, and on production and application of precious metal and other alloys.

Equipment: Ajax-Northrup induction furnace, Arsem furnace, metallographic outfit, Brinell hardness machine, Erichsen testing machine, Kelvin bridge, precision potentiometer.

46. **Baker, J. T., Chemical Co.,** Phillipsburg, N. J.

Research staff: Wm. P. Fitzgerald and 3 assistants.

Research work: Full time of 1 on methods of testing reagents, methods of manufacture, etc.

47. **Baldwin Locomotive Works, The,** Philadelphia, Pa.

Research staff: H. V. Wille, 2 chemists and 7 assistants.

Research work: Small part time of 10 on problems connected with the plant.

Equipment: 4 Olsen testing machines up to 600,000 pounds capacity; Brinell machines and scleroscope.

48. **Banks & Craig,** 51 East 42nd St., New York, N. Y. (Consulting Engineers and Chemists.)

Research staff: Henry W. Banks, 3rd, and assistants.

Research work: Food dehydration, food products and processes of food manufacture; organic colloids and engineering problems in connection with water supply, sewage disposal, sanitation, etc.

49. **Barber Asphalt Paving Company, The,** Philadelphia, Pa.

Research staff: Charles N. Forrest and 15 assistants.

Research work: Part time of 16 on application of asphalt and petroleum to commercial purposes.

Equipment: Miniature oil refinery and complete laboratory equipment for chemical and physical testing of bituminous materials.

50. **Barber-Colman Company,** Rockford, Ill. (Small tools, machine tools and textile machinery.)

Research staff: 2 chemists and 1 engineer.

Research work: Approximately one-half time of 3 on improvements on cutting tools, alloy steels and special steels.

Equipment: Complete metallographic equipment.

51. **Barrett Company, The,** 40 Rector St., New York, N. Y. (Coal tar products.) Research Department at the New York office. J. M. Weiss, Manager of Research Department. Research laboratories at Edgewater, N. J. Chemical Department for the manufacture of refined coal tar products at Frankford, Philadelphia. Research on

operating processes also carried on at Frankford. A works laboratory at each of the 35 tar plants.

Research staff (Edgewater laboratory): C. R. Downs, chief chemist, C. G. Stupp, assistant chief chemist, 40 chemists and chemical assistants, 5 engineers and 20 other men. Special products department, under direct control of research department, employs 20 process men and mechanics.

Research work: Full time of 65 on problems in connection with improvement of products or processes, and development of new uses for normal products. General manufacturing department undertakes many experimental engineering problems, for which research department acts in consulting capacity.

Research laboratories occupy 18,000 square feet; adjoining is a 40- x 50-foot building for experimental plant operations. Special products department buildings occupy about 10,000 square feet additional space.

52. **Bausch & Lomb Optical Co.**, Rochester, N. Y. (Lenses and optical instruments.)

Scientific Bureau

Research staff: Hermann Kellner, 15 optical engineers and physicists and 6 laboratory assistants.

Research work: Three-fourths time on development of optical apparatus: ophthalmic optics, microscope optics, photographic and projection apparatus, photometers, spectrometers, glass making problems, etc. One-fourth time on development of manufacturing methods, testing apparatus, etc.

Equipment: Complete equipment of optical instruments.

Chemical Laboratory

Research staff: Frank P. Kolb, 3 chemists and 2 assistants.

Research work: One-half time on emery and rouge washing and grading, grinding and polishing experiments, cements, fillers, glass washing, glass silvering, metal plating.

53. **Beaver Board Companies, The,** Beaver Road, Buffalo, N. Y. (Beaverboard and other wallboards for buildings.)

Research staff: H. F. Gardner, 20 chemists, 2 engineers and 40 inspectors, testers and laboratory assistants.

Research work: One-fourth time of 63 on pulp and board mill, wallboard, asphalt roofing and gypsum products.

54. **Beaver Falls Art Tile Company,** Beaver Falls, Pa.

Research staff: George E. Sladek, 1 ceramic chemist and 2 laboratory assistants.

Research work: Full time of 3 on factory control work, raw material testing, equipment testing and research for the betterment of the product.

Equipment: Miniature plant, microscopic equipment for petrographic work.

Beaver Valley Glass Co. See Fry, H. C., Glass Company (p. 34).

55. **Beckman and Linden Engineering Corporation,** Balboa Building, San Francisco, Calif.

Research staff: J. W. Beckman, H. E. Linden, and a varying number of chemists, physicists and assistants.

Research work: Full time of staff on chemical, electrochemical and organic problems; salts occurring in natural brines; chemistry of barium and strontium salts; electrolytic manufacture of metallic magnesium directly from its oxides; cracking of oils by high-tension discharges.

Equipment: Large motor-generator set for direct-current electrolysis and transformers for high-tension work; 100 kw. electric furnace suitable for experimental purposes.

56. **Beebe Laboratories, Inc.**, 161 3rd St., St. Paul, Minn.

Research staff: W. E. King, 1 expert biological chemist, 1 pharmaceutical chemist, 6 bacteriologists and a number of technical workers and assistants.

Research work: Approximately one-third time of 9 on development of new biological products, therapeutic agents, and modification and direction of processes of manufacture. Studies in bacteriology, immunology, serology, biological and pharmaceutical chemistry.

Equipment: Specially equipped for chemical, bacteriological and serological work, and animal experimentation.

57. **Belden Manufacturing Company**, 23rd St. and Western Ave., Chicago, Ill. (Rubber insulated wires and cables, coil-winding machines, electromagnets and similar products.)

Research staff: J. V. Van Buskirk.

Research work: Problems relating to own industry.

58. **Bennetts' Chemical Laboratory**, 1142 Market St., Tacoma, Wash. (Analytical and consulting chemists, assayers and metallurgists.)

Research staff: B. H. Bennetts, 4 chemists and 1 metallurgist.

Research work: Part time of 6 on concentration of manganese ores of Pacific Coast. Atomizing of copper, zinc and aluminum. Agricultural chemistry.

Equipment: Metal atomizing plant for copper, zinc and aluminum.

59. **Berry Brothers, Inc.**, Detroit, Mich. (Varnishes and paint specialties.)

Research staff: John F. Thomas and 3 chemists.

Research work: One-third time of 4 on paint vehicles, varnishes and shellacs.

60. **Bethlehem Shipbuilding Corporation, Ltd.**, Union Plant, San Francisco, Calif.

Research staff: S. R. Thurston and 1 assistant chemist.

Research work: On improvement of strength and homogeneity of non-ferrous alloys.

Equipment: Low voltage generator, Olsen automatic and autographic universal testing machine of 200,000 pounds capacity; Shore scleroscope, Brinell hardness apparatus.

61. **Betz, Frank S., Company**, Henry and Hoffman Sts., Hammond, Ind. (Electric X-ray apparatus, hospital, surgical and dental supplies.)

Research staff: P. M. Phillips and 1 assistant chemist.

Research work: One-fourth time of 2 chemists on improvement of existing formulas of pharmaceutical products; development of new ideas in drug and toilet preparations; research to improve plant methods. Field covered: Syrups, elixirs, tinctures, fluid extracts, solid extracts, mixtures, ointments, suppositories, compressed coated and hypodermic tablets and lozenges.

62. **Bloede, Victor G., Co.,** Station D, Baltimore, Md. (Chemicals.)

Research staff: Victor G. Bloede, 4 chemists and assistant.

Research work: Full time of 6 on developing adhesive products including dextrines, vegetable glues and special hydrolyzed starch products, and developing special sizings and gums for textile, carpet, wall paper manufacturing purposes, etc.

Equipment: Special dextrinizing and hydrolyzing apparatus and facilities for testing and developing vegetable adhesives and sizings; also machinery for testing out these products on a commercial scale.

63. **Bond Manufacturing Corporation,** Monroe and Fifth Sts., Wilmington, Del. (Bottle seals.)

Research staff: William G. Bond, 1 engineering chemist and 2 engineers.

Research work: Full time of 4 on industrial problems connected with manufacture of composition cork, collapsible tubes and bottle crowns.

64. **Boonton Rubber Manufacturing Company,** Boonton, N. J. (Electrical insulation and molded products.)

Research staff: R. W. Seabury, 1 chemist, 1 electrical engineer and 1 mechanical engineer.

Research work: One-third time of 4 on such problems as non-carbonizing molded insulation for high-tension automobile ignition apparatus; synthetic resins. Development of satisfactory insulation for high frequency, and commercial tests for same.

Equipment: 100,000-volt testing transformer; special apparatus for coating paper and fabric with resins in solution. High frequency phase displacement testing apparatus.

65. **Borromite Co. of America, The,** 105 W. Monroe St., Chicago, Ill. (Water softening systems.) Laboratory at 54 E. 18th St., Chicago, Ill.

Research staff: John A. Montgomery, 3 chemists and 3 engineers.

Research work: Approximately one-half time of 7 on water softening and equipment. A natural zeolite is employed as the water softening medium.

66. **Borrowman, George,** 130 N. Wells St., Chicago, Ill. (Chemist.)

Research staff: George Borrowman and 1 chemist.

Research work: One-half time of 2 on materials of engineering, such as waters, fuels, metals, cements, paints and clays.

Boston Biochemical Laboratory, Inc., The. See Skinner, Sherman & Esselin, Incorporated (p. 72).

67. **Bowker Insecticide Company,** 49 Chambers St., New York, N. Y. (Insecticides and fungicides.) Laboratory at Everett, Mass.

Research staff: Firman Thompson and 1 chemist.

Research work: Full time of 2 on insecticides, fungicides and disinfectants.

68. **Boyer Chemical Laboratory Company,** 940 N. Clark St., Chicago, Ill. (Private label chemical specialties and manufacturing chemists for the wholesale trade.)

Research staff: A. D. Boyer, 1 chemist and 1 laboratory assistant.

Research work: One-third time of 3 on varnishes, oils, waxes, polishing materials, gums, disinfectants, etc.

69. **Brach, E. J., and Sons,** 215 W. Ohio St., Chicago, Ill. (Candies.) Has a laboratory for control and research and a manufacturing laboratory.

Research staff: C. O. Dicken and 3 chemists.

Research work: One-third time of 4 on improvement of analytical methods and problems in manufacture of candy.

70. **Bridgeman-Russell Company,** 1100 W. Superior St., Duluth, Minn.

Research staff: Benjamin F. Eichinger and 2 or more assistants.

Research work: One-half time of 2 on chemical and bacteriological problems of manufacture of all dairy products, including sanitation, standardization and testing new methods of manufacture.

Equipment: Highly perfected equipment for complete chemical and bacteriological analysis of all dairy products, water and food.

71. **Bridgeport Brass Company,** Bridgeport, Conn.

Research staff: W. R. Webster, 1 metallurgist, 1 mechanical engineer, 3 chemists, and 5 assistants.

Research work: Full time of 2 and one-half time of 3 on general problems incidental to manufacture and fabrication of a large variety of alloys. A large amount of research work is done with the cooperation of the operating departments in the factories.

Equipment: Apparatus for testing free cutting qualities of metals.

72. **Brooklyn Union Gas Company, The,** 176 Remsen St., Brooklyn, N. Y.

Research staff: E. C. Uhlig, 2 assistant chemists, 1 chemical engineer, 9 analysts, 3 photometric inspectors, 21 gas testers and 1 photographer.

Research work: Part time of 38 on problems of manufacture and distribution of gas.

Unusual equipment: Apparatus for experimental gasification of oils; photometer for spherical candle power of lamps.

73. **Brown Company,** Portland, Me. Formerly Berlin Mills Company. Mills: Berlin, N. H., and La Tuque, P. Q.; laboratory, Berlin, N. H. (Paper, sulphate and sulphite fiber, chemicals and lumber.)

Research staff: Hugh K. Moore, 28 graduate chemists, 1 mechanical engineer, 2 technical photographers, and 10 chemical assistants.

Research work: Full time of 27 on work including following subjects: improvements in the various mill processes of sulphite and sulphate pulp making; study of commercial electrolytic cells; plant improvements in the hydrogenation of oils and laboratory study of the process; production of liquid chlorine, bleach powder by a continuous process, and acetylene tetrachloride; drying and impregnation of fiber tubes; production of alcohol by fermentation of hydrolyzed wood waste and sulphite waste liquor; properties of SO_2 solutions; study of physical and chemical properties of wood pulp, the beating process, testing of pulp and paper; performance of paper machines; study of lubrication problems; recovery and utilization of paracymene; performance of steam boiler equipment; preservation of wood and pulp; purification of sulphate turpentine; study of color measurement; determination of characteristics of gas-absorption towers; new uses for evaporated sulphite waste liquor.

Equipment: 100,000 K. v. a. transformers and switchboard for electric furnace work; Audiffren-Singrun refrigerating machine; constant temperature and humidity apparatus for pulp and paper testing; high pressure gas compressor.

74. **Brown & Sharpe Mfg. Co.**, Providence, R. I. (Machinery and tools.)

Research work: On gray iron.

75. **Brunswick-Balke-Collender Co., The**, Muskegon, Mich. (Mechanical and hard rubber products.)

Research staff: A. Brill and 3 men.

Research work: One-fourth time of 4 on rubber, glue and woodworking.

76. **Buchanan, C. G., Chemical Company**, Station H, Cincinnati, Ohio. (Case hardening and carbonizing compounds.) Laboratory at Baker Ave., Norwood, Ohio.

Research staff: R. F. Catherman, 1 electrical engineer and 1 chemist.

Research work: Variable amount time of staff on metallic salts, pigments, industrial chemicals and their application in the various industries.

77. **Buckeye Clay Pot Co.**, Bassett and Ontario Sts., Toledo, Ohio. (Fire-clay products.)

Research staff: W. K. Brownlee.

Research work: Two-thirds time of 1 on tests of clay including determinations of dry transverse strength, water of plasticity, linear drying shrinkage, screen analysis for fineness, etc., also melting point, ability to withstand load at high temperatures, porosity, linear burning shrinkages, burned strength, and other properties of burned clay.

Equipment: For making both routine and special tests of clays.

78. **Buffalo Foundry and Machine Co.**, 1543 Fillmore Ave., Buffalo, N. Y. (Vacuum dryers, evaporators and industrial chemical apparatus.)

Research staff: Willard Rother, metallurgical and physical testing department; D. J. Van Marle, chemical department; Charles Lavett, vacuum laboratory and testing departments; 2 assistant chemists and 5 assistant engineers and operators.

Research work: Small part time of 10 on practical experiments on materials furnished by customers to determine in advance what can be done by means of vacuum apparatus.

Equipment: Completely equipped metallurgical, chemical and testing laboratories.

79. **Burdett Manufacturing Company**, St. Johns Court at Fulton St., Chicago, Ill. (Oxygen and hydrogen gas generating apparatus.)

Research staff: J. B. Burdett and 1 chemist.

Research work: Full time of 2 on rates of diffusion of gases, explosive limits of gases, effect of electrolytic action incident to decomposition of water on various materials used in construction (steel, rubber and asbestos) development of special compounds for permanent resistance to such action and to action of comparatively strong alkaline solutions.

Burke Tannery. See International Shoe Co. (p. 44).

80. **Butterworth-Judson Corporation,** Newark, N. J. (Chemicals, intermediates, dyes.)

Research staff: A. Riker, Jr., 6 chemists and 2 helpers.

Research work: Full time of 9 on problems relating to dye manufacture.

Equipment: Particularly adapted for work on intermediates, dyes, acids and heavy chemicals, including semi-commercial scale apparatus.

81. **Byers, A. M., Company,** Pittsburgh, Pa. (Wrought iron pipe, oil well tubing and casing.)

Research staff: James Aston, several metallurgists and chemists, and assistants.

Research work: One-half time of staff on corrosion and protective coatings of iron; development of wrought iron.

Equipment: Apparatus for corrosion tests and for determining physical characteristics. Electric furnace and auxiliary equipment for experimental heats of iron.

82. **Cabot, Samuel, Inc.,** 141 Milk St., Boston, Mass.

Research staff: Samuel Cabot and 1 assistant.

Research work: One-third time of 2 on coal tar distillates, disinfectants, paints, stains, varnishes.

83. **Calco Chemical Company, The,** Bound Brook, N. J.

Research staff: M. L. Crossley and 22 chemists.

Research work: Full time of 23 and part time of plant engineers on intermediates, dyes and pharmaceuticals, including fundamental problems of the reactions involved, development of new processes and plant improvement.

84. **California Fruit Growers Exchange,** Box 518, Corona, Calif.

Research staff: C. P. Wilson and 2 chemists.

Research work: Five-sixths time of 3 on by-products from citrus fruits; chemical problems connected with production, preparation and sale of citrus fruits.

85. **California Ink Company, Inc.,** Camelia and 4th Sts., Berkeley, Calif. (Printing and lithographic inks, varnishes and rollers.)

Research staff: E. T. Frickstad and 3 chemists.

Research work: One-half time of 4 on oil, varnish, dry color, dyes, intermediates and inks.

86. **Calumet & Hecla Mining Company,** Calumet, Mich. Laboratory at Lake Linden, Mich.

Research staff: C. H. Benedict with an average of 4 assistants.

Research work: Hydrometallurgy of copper.

Equipment: Large scale operation in leaching, flotation, etc.

Carbide and Carbon Chemical Corporation. See Union Carbide and Carbon Research Laboratories, Inc. (p. 78).

87. **Carborundum Company, The,** Niagara Falls, N. Y. (Abrasive and refractory materials.)

Research staff: M. L. Hartmann, 15 chemical and electrochemical engineers, 7 assistants with technical experience and 4 non-technical helpers.

Research work: Full time of 27 on problems relating to abrasives and refractories. Development of new products and improvement of present processes. Semi-commercial scale equipment is available in electric furnace laboratory. Especially interested in study and development of the specialized refractory materials. Problems also include those relating to adhesives, rubber, shellac, paper and cloth.

88. **Carnegie Steel Company,** 1054 Frick Annex Building, Pittsburgh, Pa. Central Research Bureau for United States Steel Corporation.

Research staff: J. S. Unger; chemists, physicists, engineers and assistants selected from works staffs as needed.

Research work: At steel plants, covering problems of steel manufacture, properties of refractories and other materials used in steel manufacture, by-products and the testing of finished products, particularly service tests.

89. **Carus Chemical Company,** La Salle, Ill. (Permanganates, manganese salts, titanium salts, saccharine, toluene sulphonamides and their chlorine derivatives, benzoates.)

Research staff: Karl Kleimenhagen and 7 men.

Research work: Three-fourths time of 8 on development of process for producing chemicals manufactured by the company.

90. **Case Research Laboratory,** Auburn, N. Y.

Research staff: Theodore W. Case, 3 technical men and several assistants.

Research work: Full time of at least 4 on problems in light and photo-electricity.

Equipment: Apparatus for photo-electric work.

91. **Caulk, L. D., Company, The,** Milford, Del. (Dental materials.)

Research staff: Arthur W Gray, director physical research; Paul Poetschke, director department of chemistry; D. Anton Zurbrigg, director clinical department.

Research work: Properties and application of materials used in dentistry.

Equipment: Chemical and physical apparatus for determining the properties of dental products of all kinds. Equipment for bacteriological, biological and chemical investigation of dental problems.

92. **Celite Products Company,** Van Nuys Building, Los Angeles, Calif. (Manufacturers and distributors of heat insulating materials, filtering materials and mineral fillers.) Laboratory at Lompoc, Calif.

Research staff: P. A. Boeck, 2 chemical engineers, 1 chemist and 4 assistants.

Research work: Approximately three-fourths time of 10 on filtration of industrial liquids, measurement of thermal insulation and capacity of heat insulating materials and microscopical analysis.

Equipment: Complete equipment for pressure and gravity filtration of industrial liquids, apparatus for the determination of thermal conductivity of insulators and furnace equipment for refractory testing.

93. **Central Dyestuff and Chemical Co.,** Plum Point Lane, Newark, N. J. (Coal tar colors and intermediates.)

Research staff: John Prochazka, 14 chemists and assistants.

Research work: Nine-tenths time of 15 with assistants on dyestuffs, pharmaceuticals, and coal tar intermediates.

Equipment: Separate experimental factory 60x40 with adequate machinery, such as suitable stills, filter presses and autoclaves for small scale manufacture.

94. **Central Scientific Company,** 460 East Ohio St., Chicago, Ill. (Physical, chemical, agricultural and biological apparatus.)

Research staff: Paul E. Klopsteg and 2 assistants.

Research work: Full time of 2 on development of new apparatus and instruments, and improvement in devices already manufactured.

95. **Champion Ignition Company,** Flint, Mich.

Research staff: T. G. McDougal and 2 ceramic engineers.

Research work: Three-fourths time of 3 on perfection of high temperature insulation (electrical); super-refractory furnace linings for own use; continuous high temperature factory processes.

Equipment: Laboratory and factory facilities for operations up to 1800° C. Equipment for measuring electrical leakage up to 900° C.

96. **Champion Porcelain Company,** Detroit, Mich. Formerly Jeffery-Dewitt Co. (Porcelain products.)

Research staff: Frank H. Riddle and 12 assistants.

Research work: One-third time of 13 on ceramic investigations necessary in ignition and high tension porcelain manufacture including development of bodies, methods of testing, manufacturing, etc. Also development of furnaces, special refractories and similar equipment.

Equipment: Electrical equipment for tests of porcelains, for ignition and high tension work, special furnaces for tests of refractories.

97. **Charlotte Chemical Laboratories,** Inc., 606 Trust Building, Charlotte, N. C.

Research staff: F. J. Bartholomew, 1 chemist, 2 chemical engineers.

Research work: Two-thirds time of 6 on development of plant processes.

Equipment: Electric vacuum furnaces. Large capacity grinding units.

98. **Chase Metal Works,** Waterbury, Conn. (Brass, bronze, copper and nickel, silver, rod, wire, sheet and tubing.)

Research staff: Harry George, 3 chemists, 1 electrochemist, 3 metallurgists and 8 assistants.

Research work: One-fifth time of 16 on improvement of properties and methods of manufacture of copper-zinc alloys; also investigation of steels, lacquers, fuels and oils.

Equipment: 100,000-pound Olsen testing machine, 50,000-pound Riehle testing machine, 10,000-1,000-pound Olsen wire testing machine, Brinell machine, Spring tester, scleroscopes; metallographic equipment, electric annealing muffles with electrically controlled thermostats.

99. **Chemical Economy Company,** 1640 N. Spring St., Los Angeles, Calif. (Photographers' chemicals.)

Research staff: C. W. Judd and 4 chemists.

Research work: One-tenth time of 5 on celluloid and by-products and photographic chemicals.

100. **Chemical Products Company,** 44 K St., South Boston, Mass. (Manufacturing chemists.)

Research staff: H. S. Mork, 2 chemists and 1 assistant.

Research work: One-fifth time of 4 on cellulose chemistry.

101. **Chemical Service Laboratories, Inc., The,** W. Conshohocken, Pa. (Analytical, consulting and engineering chemists.)

Research staff: J. Ed. Brewer and 4 assistants.

Research work: One-fourth time of 5 on coal tar, coal tar distillates, fuels, gasworks; raw materials and products.

Equipment: For plant scale experiment.

102. **Chicago Mill and Lumber Company,** Conway Bldg., Chicago, Ill.

Research staff: Don L. Quinn, 2 engineers in forest products, 1 mechanical engineer and 1 chemical engineer.

Research work: Study of designs and mechanical properties of packing boxes, crates and methods of packing; also chemical studies on fibre board construction.

Equipment: 16 ft. revolving drum testing machine which subjects packages to most of the hazards of transportation.

103. **Childs, Charles M., & Co., Inc.,** 41 Summit St., Brooklyn, N. Y. (Paints.)

Research staff: F. D. Heim, 2 chemists and 1 assistant chemist.

Research work: Full time of 4 on production of new color lakes.

Equipment: Special equipment for producing color lakes and machines for coating and polishing paper.

104. **Cleveland Testing Laboratory Co., The,** 511 Superior Building, Cleveland, Ohio.

Research staff: C. A. Black, 2 chemists and assistants as required.

Research work: One-third time of 3 on problems in connection with industrial plants.

105. **Cochrane, H. S. B. W., Corporation,** 17th St. and Allegheny Ave., Philadelphia, Pa., and Earnest, Pa. Formerly Harrison Safety Boiler Works.

Research staff: P. S. Lyon and 5 engineers; J. D. Yoder and 2 chemists.

Research work: Full time of 6 on treatment of boiler feed water; experiments on V-notch weirs and other flow meters; water softening; problems in the development of traps, valves, steam and oil separators, etc.

106. **Coleman & Bell Company, The,** Norwood, Ohio. Successors to National Stain and Reagent Co. (Biological stains and indicators.)

Research staff: A. B. Coleman, W. H. Bell and 1 assistant.

Research work: Approximately full time of 3 on syntheses of chemically pure organic dyestuffs and compounds for use in biology, pathology, botany, and medicine in general; preparation and testing of all kinds of indicators for use in chemistry, biology, etc.; preparation of chemically pure organic compounds and reagents and research upon practical industrial problems in organic chemistry.

Equipment: Complete semi-commercial equipment for the preparation of dyestuffs and facilities for testing chemicals and dyes for use as biological stains and indicators.

107. **Columbia Graphophone Manufacturing Company,** Bridgeport, Conn.

Research staff: W. R. Palmer, general superintendent of engineering.

Research work: General development work in semi-plastics, accoustics, electroplating, material testing and specifications, machine developments, cabinet design and manufacturing methods.

108. **Commercial Testing and Engineering Co.,** 1785 Old Colony Bldg., Chicago, Ill. (Coal analysis and boiler room economies.)

Research staff: Jerome F. Kohout, 3 chemists and 1 engineer.

Research work: Three-tenths time of 5 men on coal problems,—particularly coking low grade coal at high and low temperature; mixing of coals to produce either high coke yield or large recovery of by-products or both. Examination of coal with special reference to proper time, temperature, and pressure conditions in coke oven. Deterioration of coal in storage with reference to its coking properties. Design of furnaces and boilers to meet special conditions of fuel or other requirements.

109. **Commonwealth Edison Company,** 72 West Adams St., Chicago, Ill. (Operator of large electric light and power generating and distributing systems.)

Research staff: Louis A. Ferguson and 6 trained men.

Research work: Part time of 7 on insulation deterioration, potential rises due to switching operations, heat dissipation, electric furnace investigations and storage battery problems.

Equipment: Primary and secondary standardizing instruments, especially for heavy currents; oscillograph and high potential instruments; special generators and transformers; apparatus for dielectric and insulation tests.

110. **Condensite Company of America,** Bloomfield, N. J. (Phenolic condensation products, chlorine substitution products, hydrochloric acid.)

Research staff: W. T. Hutchinson and 1 assistant.

Research work: Three-fourths time of 2 on improvement of products.

111. **Consolidated Gas Company of New York,** 130 E. 15th St., New York, N. Y. Consolidated laboratories at Lawrence Point, Astoria, N. Y.

Research staff: Charles A. Lunn, 5 chemists, 5 chemical engineers, 15 assistant chemists and 6 laboratory assistants.

Research work: Part time of staff on problems consequent to the manufacture and distribution of illuminating gas (coal gas and carburetted water gas).

112. **Consolidated Gas Electric Light and Power Company of Baltimore,** Lexington and Liberty Sts., Baltimore, Md. Laboratory at Spring Gardens Plant, Baltimore, Md.

Research staff: Minor C. K. Jones, 2 chemists and 5 laboratory assistants.

Research work: One-tenth time of 8 on gas purification and general gas manufacture.

Equipment: Complete experimental purifier equipment.

113. **Conwell, E. L., & Co., Inc.,** 2024 Arch St., Philadelphia, Pa. (Engineers, chemists, inspectors.)

Research staff: E. L. Conwell and 3-15 assistants.

Research work: Variable amount of time on cement manufacture; lime products manufacture; uses of cements, limes, etc.; various industries, involving calcination, grinding, etc., and recovery and utilization of waste products.

114. **Cooper Hewitt Electric Company,** 730 Grand St., Hoboken, N. J. (Lamps and rectifiers.)

Research staff: R. D. Mailey and 2 assistants.

Research work: Vapor electric apparatus and applications.

Equipment: Facilities for fabricating clear fused quartz apparatus and methods for fusing (hermetic) clear quartz to all vitreous materials, including metallic leads.

115. **Corning Glass Works,** Corning, N. Y. (Technical glass.)

Research staff: E. C. Sullivan, 3 chemists, 5 physicists and 4 engineers.

Research work: Two-thirds time of 11 on physical properties of glass as related to chemical composition; lens design; furnace design; refractories; manufacturing problems; and new uses for glass.

Equipment: Facilities for high temperature work.

116. **Corn Products Refining Company,** Edgewater, N. J.

Research staff: Christian E. G. Porst, 3 chemical engineers, 4 chemists and 13 helpers and laborers.

Research work: Full time of 21 on problems confined to the industry.

Corona Chemical Co. See Pittsburgh Plate Glass Co. (p. 65).

117. **Cosden & Company,** Tulsa, Okla. (Producers and refiners of petroleum.)

Research staff: Charles K. Francis and about 50 chemists, physicists, engineers and assistants.

Research work: One-third time of about 50 on petroleum and petroleum products, including gas.

Equipment: General chemical and physical equipment for petroleum work.

118. **Cosmos Chemical Co., Inc.,** 709 Berckman St., Plainfield, N. J.

Research staff: Charles Blanc and 3 assistants.

Research work: Organic synthetic compounds for commercial utilization and factory problems.

119-120. **Cramp, William & Sons Ship & Engine Building Co., The,** Philadelphia, Pa.

119. *I. P. Morris Hydraulic Laboratory*

Research staff: F. H. Rogers, 2 engineers, 2 observers and 1 machinist.

Research work: Three-fourths time of 6 in the field of hydraulics and hydrodynamics.

Equipment: Hydraulic testing laboratory designed specially for testing models of hydraulic turbines, centrifugal pumps, spiral pumps, current meters, Pitot tubes, etc. Contains headrace flume, tailrace flume, motor driven pumps, tank for rating current meters and other necessary instruments.

120. *Cramp Chemical Laboratory*

Research staff: N. H. Schwenk and 1 chemist.

Research work: Half-time of 2 on research work along metallurgical lines.

121. **Crane & Co.**, Dalton, Mass. (Paper makers.)

Research staff: C. Frank Sammet.

Research work: Full time of 1 on development of new procedures, novelties and mill problems.

Equipment: Well equipped for research relative to paper manufacture.

122. **Crane Co.** (Metallurgical Department), South Avenue, Bridgeport, Conn., and 836 South Michigan Ave., Chicago, Ill. (Valves, pipes, fittings and other supplies from iron, steel, brass and bronze, for water, gas, and steam work.)

122a. *Bridgeport laboratory*

Research staff: Allen P. Ford, 2 metallurgists, 1 chemist, 3 assistant chemists and 2 helpers.

Research work: Small part time of 9 on problems connected with the industry.

Equipment: Entirely equipped for routine metallurgical work. 100,000-pound tensile testing machine; transverse, torsion and hardness testing machines.

122b. *Chicago laboratory*

Research staff: L. W. Spring, 1 assistant and 12 men, 2 of whom are doing physical and metallographic testing.

Research work: One-tenth time of 14 on problems connected with the industry.

123. **Crompton & Knowles Loom Works**, Worcester, Mass.

Research staff: V. E. Hillman, 2 metallurgists, 1 chemist, 1 librarian and 2 non-technical assistants.

Research work: Full time of 7 on heat treatment of steel; case carburizing and cyanide hardening; quenching mediums; core oils; molding sands; molding methods; blow holes and shrinkage cavities in cast iron; illumination; copper plating metal parts; and work on nonferrous alloys—aluminum, brass, bronze and bearing metals.

124. **Crucible Steel Company of America**, Pittsburgh, Pa.

Research staff: Charles Morris Johnson and 39 chemists and physicists.

Research work: Chemical department, one-fifth time of 8 men. Physical division, four-fifths time of 3 men.

Equipment: 1 Olsen 100,000-pound tensile testing machine, 1 Olsen impact machine, 1 Olsen torsion machine, 1 Olsen new ductility machine for testing the ductility of plates up to one-fourth inch thick, 2 Pittsburgh Instrument Company Brinell testing machines, 2 Shore scleroscopes, 1 O-Z cutmeter tachometer, 1 Brown instrument (critical point machine), Leitz microphotographic outfit and 1 Olsen extensometer.

125. **Cudahy Packing Co., The**, South Side Station, Omaha, Nebr. (Meat packers, etc.) General and research laboratory, Omaha, Nebr. Laboratories also in Chicago, Ill., and Kansas City, Kans.

Research staff: Millard Langfeld, superintendent of laboratories, 5 chemists and 2 workers.

Research work: Gland products, oils and greases, glues, curing meats, etc.

126. **Cumberland Mills,** Cumberland Mills, Me. S. D. Warren Co., Boston, Mass., proprietors. (Pulp and paper.)

Research staff: E. Sutermeister, 2 to 4 chemists and 2 or 3 assistants.

Research work: One-third time of 6 on problems relating to pulp and paper industry. Tests of various woods and fibrous materials; studies on soda and sulphite pulp processes and on solubility, adhesive strength and viscosities of caseins and their solutions and coating mixtures; studies of black ash waste and its possible utilization; studies of rate of absorption of moisture by paper; investigations of the storage conditions for pulp wood; studies on the frothing of coating mixtures; tests of new sizing agents and further studies on rosin sizing. Bleaching studies on sulphite and soda fiber to show effects of variable factors; further applications of a beating test to show relative strength of fibers; investigations relating to manufacture of satin white; studies of defects in papers and of means to overcome them.

Equipment: Apparatus for the manufacture of paper on laboratory scale; complete testing apparatus. Available in mill; 400-pound vertical soda digester; 350-pound beater, and small Fourdrinier paper machine. Apparatus to study foaming of coating mixtures.

127. **Curtiss Aeroplane & Motor Corporation,** Garden City, L. I., N. Y.

Research staff: H. T. Booth, 2 engineers, 1 mechanic and 1 model maker.

Research work: One-half time of 4 on wind tunnel tests of wings, bodies, propellers, etc. Load tests of complete airplanes, performance tests of complete airplanes and miscellaneous investigations along different aeronautical lines.

Equipment: One four-foot wind tunnel in which wind velocities of 75 m. p. h. are obtained. One seven-foot wind tunnel in which wind velocities of 100 m. p. h. are reached.

Curtiss Engineering Corporation, The. See Curtiss Aeroplane & Motor Corporation.

128. **Cutler-Hammer Mfg. Co., The,** Milwaukee, Wis. (Electric controlling devices.)

Research staff: Arthur Simon, 1 physicist, 1 glassblower and mechanical helpers as needed. Has help of Experimental Department with its staff of developing engineers and mechanics.

Research work: Full time of 2 in connection with electrical discharge in gas, particularly evacuated tubes and bearing on control of electric currents.

129. **Davis-Bournonville Company,** Jersey City, N. J. (Welding and cutting apparatus.)

Research staff: Frank J. Napolitan and 1 assistant.

Research work: Large part time of 2 on metallography of oxyacetylene welding, design of new apparatus and development of scope of process.

Equipment: Gas laboratory equipped for measuring flow of gas under high pressures; micro-manometers for measurement of high pressures.

130. **Davis Chemical Products, Inc.,** Springfield, N. J.

Research staff: E. J. Fry, 1 engineer and 1 chemist.

Research work: One-half time of 3 on cellulose esters, nitrocellulose, nitrocellulose solvents and solutions, artificial and imitation leather, coatings, lacquers and films; explosives, commercial and military.

Equipment: Apparatus for testing the physical and chemical properties of films and coatings based on cellulose esters, including viscosity, stability, aging, accelerated life tests and strength; facilities for large scale experiments and demonstrations.

131. **Davison Chemical Company, The,** Baltimore, Md. (Sulphuric acid.)

Research staff: A. E. Marshall and trained research men as required.

Research work: Full time of staff on improvement of manufacturing processes for sulphuric acid and utilization of waste materials.

Equipment: Semi-commercial equipment for development of processes evolved in laboratory.

Dayton Engineering Laboratories Company. See General Motors Research Corporation (p. 35).

132. **Dean Laboratories, Inc.,** 48th St. and Walton Ave., Philadelphia, Pa.

Research staff: J. Atlee Dean, 3 chemists, 3 bacteriologists and 1 technician and clerical helper.

Research work: One-half time of 8 on physiological, pharmaceutical and clinical chemistry; hypodermic preparations, especially the endocrine glands; laboratory reagents such as colloidal gold and microscopic stains.

Equipment: Facilities for rapid and accurate examinations of body fluids.

133. **Dearborn Chemical Company,** McCormick Building, Chicago, Ill. (Scientific boiler feed water treatment.) Laboratories at 1029 W. 35th St., Chicago, Ill.

Research staff: D. K. French, 5 chemists and 5 assistants.

Research work: Small part time of 11 on scientific boiler feed water treatment and chemical control of corrosion.

Equipment: Hess-Ives tintometer and Thurston friction machine; all types of viscosimeters.

134. **Dehls & Stein,** 237 South St., Newark, N. J. (Manufacturing chemists.)

Research staff: L. Stein and 1 chemist.

Research work: One-half time of 2 along lines of fermentology, synthetic essential oils, caramel.

135. **Deister Concentrator Company, The,** 611 High St., Ft. Wayne, Ind. (Concentrating tables for every purpose.)

Research staff: Regular force consists of 1 metallurgical engineer, together with occasional assistance in advisory capacity from other members of the company.

Research work: On gravity or table concentration of various ores sent us for this purpose from all parts of the world; extensive work in the washing of the finer sizes of coal (both anthracite and bituminous) below that usually handled on jigs, etc. This work is done in both small lots and in carload quantities.

Equipment: One 16 by 18-inch Pennsylvania roll crusher, I pair 10-inch corrugated rolls, 1 pair 5½-inch smooth rolls for regrinding, 1 Mitchell vibrating screen, 1 No. 7 Deister-Overstrom diagonal deck coal-washing table, 1 No. 6 Deister-Overstrom diagonal deck table for ore treatment, 1 No. 14 Deister-Overstrom diagonal deck, jr., table, 1 12-foot Dorr thickener, 1 size 4-1 American vacuum filter, 1 Ingersoll-Rand vacuum pump.

136. **DeLaval Separator Co., The,** 165 Broadway, New York, N. Y. (Centrifugal machinery.)

Research staff: A. F. Meston and 1 assistant.

Research work: Full time of 2 on purifying used oils, clarification and separation of commercial products, making of emulsions, clarification of extracts, purifying of crude and fuel oils, application of centrifugal machines to industrial processes, etc.

Equipment: Centrifugal apparatus of all classes.

137. **Dennis, Martin, Company, The,** 859 Summer Avenue, Newark, N. J. (Chrome tannage.)

Research staff: Harold Dennis, 1 chemical engineer and 2 chemists.

Research work: Three-fourths time of 4 on tanning and tanning materials.

138. **Detroit Edison Company, The,** Detroit, Mich. (Operating electric light and power generating stations and distributing systems; central heating stations and distributing systems and illuminating gas plants and distributing systems.)

Research staff: C. F. Hirshfield, 1 engineer, 2 to 8 trained men, and 4 or more assistants.

Research work: Problems in better generation, distribution and utilization of electricity, steam for heating and artificial gas.

139. **Detroit Testing Laboratory, The,** 3726 Woodward Ave., Detroit, Mich. (Analytical consulting and research chemists.)

Research staff: W. P. Putnam, 6 chemists, 1 bacteriologist, 1 chemical engineer and 1 electrical and mechanical engineer, 1 pharmaceutical engineer, 1 foundry engineer, 1 steam engineer and 1 automobile engineer.

Research work: Full time of 1 chemist and 2 engineers on special problems in shale oil development, fertilizer manufacture, metallurgical problems, heat treatment of metals, fuel problems, water purification and ore dressing.

Equipment: 100,000-pound Reihle testing machine, 10,000-pound Olsen testing machine, Weston precision laboratory type instruments, shunts and multipliers for instrument calibration and precision testing, Leeds and Northrup precision type potentiometer and large capacity storage batteries.

140. **Dewey & Almy Chemical Company,** Harvey St., Cambridge, Mass.

Research staff: Bradley Dewey, 1 chemical engineer and 2 chemists.

Research work: One-half time of 4 on adhesives, fluxes, and sealing compounds.

141. **Dextro Products, Inc.,** 25 Illinois St., Buffalo, N. Y. (Soluble starch and dextrin products.)

Research staff: A. D. Fuller and 2 assistants.

Research work: One-fourth time of 3 on hydrolysis of starch, torrification of starch, colloids as related to adhesives and dextrin.

142. **Diamond Chain & Manufacturing Company,** 502 Kentucky Ave., Indianapolis, Ind. (Steel roller and block chains, sprockets, etc.)

Research staff: H. B. Northrup, 1 chief metallurgist and 1 assistant metallurgist.

Research work: Approximately one-half time of 3 on carburizing compounds and carburizing, hardening and drawing of alloy vs. plain carbon steels for chain parts.

143. **Diamond Match Co., The,** Oswego, N. Y.

Research staff: Frederick VanDyke Cruser, 7 chemists and chemical engineers, 1 mechanical engineer and 3 assistants.

Research work: One-half time of 12 on problems connected with match manufacture and its allied branches.

144. **Dicks David Company, Incorporated,** Varick and N. Moore Sts., New York, N. Y. (Dyestuffs and chemicals.) Laboratory at 22d St. and Stewart Ave., Chicago Heights, Ill.

Research staff: H. Philipp, P. H. Condit, W. G. Brunjes, 8 chemists and 4 engineers.

Research work: Small part time of 15 chiefly on triphenylmethane dyestuffs.

145. **Digestive Ferments Co., Detroit, Mich.**

Research staff: Howard T. Graber, director of the chemical laboratory; Henry G. Dunham, director of the bacteriological laboratory, and assistants.

Research work: Two-thirds time of assistants devoted to physiological and proteid chemistry and commercial classification of bacteriology.

Equipment: Apparatus for the electrometric estimation of hydrogen ion concentration. Vitreosil muffle furnace with thermocouple and Brown recording pyrometer for the accurate estimation of ash at definite temperatures. Experimental laboratory vacuum drier, thermocouple and recording thermometer for moisture determinations. Schmidt and Haensch saccharimeter with bichromate cell.

146. **Dill & Collins Co.,** Richmond and Tioga Sts., Philadelphia, Pa. (Paper makers.)

Research staff: Frank H. Mitchell, 2 chemists, 2 chemical engineers and 3 assistants.

Research work: One-half time of 1 chemist to full time of 2 chemists on problems of the paper industry.

147. **Dodge Brothers,** Detroit, Mich. (Automobiles and accessories.)

Research staff: F. E. McCleary, 17 chemists, 25 engineers, physical testers and trouble men.

Research work: Approximately one-tenth time of staff on automobile materials, treatment, application, etc. This covers cast iron, steel, brass and bronze, babbitt, aluminum, wood, rubber, etc.; lubrication, paints and varnishes, baking japans and fuel.

148-150. **Doehler Die-Casting Co.**, Court, Ninth and Huntington Sts., Brooklyn, N. Y. Laboratories also at Smead and Prospect Aves., Toledo, Ohio, and at Chicago, Ill.

148. *Brooklyn Laboratory*

Research staff: Charles Pack, 5 chemists, 6 junior chemists, 1 fuel engineer, 1 steel metallurgist.

Research work: One-fifth time of 14 on problems pertaining directly or indirectly to casting of metals, particularly non-ferrous metals.

149. *Toledo Laboratory*

Research staff: Charles Pack, 1 metallurgist, 1 chemist and 5 junior chemists.

Research work: One-fifth time of 8 on problems pertaining to casting of metals.

150. *Chicago Laboratory*

Research staff: J. C. Fox and 2 chemists.

Research work: One-tenth time of 3 on non-ferrous alloys.

151. **Doherty Research Company**, Empire Division, Bartlesville, Okla.

Research staff: J. P. Fisher, 1 superintendent and 10 engineers.

Research work: Full time of 12 on research problems dealing with production, transportation and refining of petroleum; transportation and distribution of natural gas; conservation of fuel.

152. **Dorite Manufacturing Company, The**, 116 Utah St., San Francisco, Calif. (Stucco, flooring, magnesite.)

Research staff: E. H. Faile and 1 assistant.

Research work: One-half time of 2 on investigation of the best methods for the manufacture of various magnesite products, including stucco and flooring and particularly of the most practical methods in their application and use.

153. **Dorr Company, The**, 101 Park Ave., New York, N. Y. (Engineers.) Testing plant and laboratory at Westport Mill, Westport, Conn.

Research staff: H. A. Linch, 1 analytical chemist, 1 chemical engineer, 1 sanitary engineer, 1 mechanical engineer, 4 assistants. Chemical, metallurgical, sanitary and mechanical engineers from the New York office are available for advice and work as needed.

Research work: Major problems in connection with the production of water-floated materials for pigments, fillers, etc. Concentration and sulphating. Roasting of ores. Washing and classification of abrasives. Studies dealing with the development of mechanical settling and dewatering, classification, continuous agitation and countercurrent washing. Trade waste and sewage treatments.

Equipment: Bins, crushers, grinding mills, classifiers and washers of various types, thickeners, filterers, concentrating tables, flotation machines, mechanical multiple-hearth furnace, electric roasting furnace, etc. Plant fully equipped to work out hydrometallurgical and wet chemical and industrial problems.

154. **Drackett, P. W., & Sons Co., The,** Cincinnati, Ohio. (Manufactures heavy chemicals; distributes Solvay Process Co. alkalis and other heavy chemicals.)

Research staff: K. S. Kersey and 1 assistant.

Research work: Development of products and their uses.

155. **Dunham, H. V.,** 50 E. 41st St., New York, N. Y.

Research staff: H. V. Dunham with from 2 to 6 assistants.

Research work: Full time of staff on food products, oils, including mineral oils and especially developments and improvements in the making and use of milk casein and milk products.

Equipment: Mixing machines, dryers and other semi-industrial equipment.

156-160. **du Pont, E. I., de Nemours & Company,** Wilmington, Del. Chemical Department operates 5 research laboratories in addition to organization at its main office. (Information concerning the entire department is followed by separate accounts of the 5 laboratories.)

Research staff: Charles L. Reese, 200 graduate chemists and engineers, 122 other salaried employees and 200 payroll employees.

Research work: Practically full time of 522 on manufacturing operations of the du Pont Company, including miscellaneous chemicals, dyes and intemediates, explosives, artificial leather, rubber goods, plastics, pyroxylin solutions, lacquers, paint and varnish, including the production of miscellaneous raw materials as mineral acids and nitrate of soda.

156. *Pyralin Laboratory, Arlington, N. J.*

Research staff: E. A. Wilson, 22 graduate chemists and engineers, 13 other salaried employees and 24 payroll employees.

Research work: Practically full time of 59 on pyralin, pyroxylin solutions, and raw materials therefor.

Equipment: Fairly complete line of semi-manufacturing scale equipment for the experimental manufacture of paper, nitrocellulose and pyralin.

157. *Eastern Laboratory, Box 424, Chester, Pa.*

Research staff: C. A. Woodbury, 23 graduate chemists and engineers, 13 other salaried employees and 33 payroll employees.

Research work: Practically full time of 69 on high explosives and raw materials therefor, processes of manufacture, and methods of testing.

Equipment: Very complete facilities for testing properties of explosives.

158. *Experimental Station, Henry Clay, Del.*

Research staff: A. P. Tanberg, 28 graduate chemists and engineers, 30 other salaried employees and 63 payroll employees.

Research work: Practically full time of 121 on smokeless powder, black powder, nitrocellulose, heavy chemicals, paint and varnish, and raw materials therefor. Also miscellaneous organic, inorganic, and biochemical research.

Equipment: For experimental manufacture of propellant powders, constant temperature magazines for stability tests, and storage of smokeless powder, experimental equipment for the manufacture of coated fabrics, ranges for testing small arms powders for velocity, pressure and accuracy.

159. *Jackson Laboratory, Box 525, Wilmington, Del.*

Research staff: Fletcher B. Holmes, 80 graduate chemists and engineers, 28 other salaried employees and 71 payroll employees.

Research work: Practically full time of 179 on dyes and intermediates.

Equipment: Extensive equipment for semi-works operation and investigation of a variety of chemical processes.

160. *Redpath Laboratory, Parlin, N. J.*

Research staff: E. B. Benger, 14 graduate chemists and engineers, 8 other salaried employees and 7 payroll employees.

Research work: Practically full time of 29 on film work.

Equipment: Small scale apparatus for coating films, and equipment for physical and chemical testing of film and photo-chemistry.

161. **Durfee, Winthrop C.**, 516 Atlantic Ave., Boston, Mass. (Consulting and manufacturing chemist.)

Research staff: Winthrop C. Durfee, 5 chemists, 1 physicist and 3 assistants.

Research work: One-half time of 10 on application of dyes and chromium compounds in wool dyeing; chrome tanning.

162. **Duriron Company, Inc., The,** N. Findlay St., Dayton, Ohio. (Acid-proof alloy castings.)

Research staff: P. D. Schenck, 1 metallurgist, 1 chemist, 1 assistant chemist, 1 engineer and 1 laboratory assistant.

Research work: One-fourth time of 6 on chemical corrosion of metals, metallurgical problems, physical properties, etc.; problems relating to the handling of corrosives.

Equipment: Experimental foundry.

163. **Dye Products & Chemical Company, Inc.**, 200 5th Ave., New York, N. Y.

Research staff: C. K. Simon, 1 chemist and 2 assistants.

Research work: Full time of 1 chemist and part time of 2 assistants on problems connected with the manufacture of dyes and intermediates and the improvement of present processes.

164. **Eagle-Picher Lead Company, The,** 208 S. LaSalle St., Chicago, Ill. (Manufacturers, miners and smelters of lead products.) Laboratory at Joplin, Mo.

Research staff: J. H. Calbeck and 4 chemists.

Research work: Full time of 5 on physical and chemical properties of paints and white pigments; storage battery oxides and chemical and metallurgical problems pertaining to the manufacture and uses of the oxides of lead and zinc.

Equipment: Pfund's colorimeter, spectrometer, photometer, microphotographic equipment.

165. **Eastern Finishing Works, Inc.**, Kenyon, R. I.

Research staff: William H. Adams and 2 assistants.

Research work: Part time of 3 on test valuation and general study of waterproofing, dyeing, sizing and mildew resistance in connection with finishing cotton goods.

166. **Eastern Malleable Iron Company**, Naugatuck, Conn. (Castings.)

Research staff: W. R. Bean and 6 assistants.

Research work: Full time of 3 and one-half time of 4 on metallurgical research as applied to composition, annealing and production of malleable iron.

Equipment: Special laboratory muffle annealing furnace, electrically heated, with automatic electric temperature control bath for maintaining indefinitely temperatures up to 2000° F. and also controlling rate of heating and cooling at several rates between 4° F. per hour and 20° F. per hour.

167. **Eastern Manufacturing Company,** Bangor, Me. (Paper.)

Research staff: H. H. Hanson, 5 chemical engineers, 2 chemists, 3 routine chemists, 1 electrical engineer and 1 assistant.

Research work: Full time of 14 on standardization of processes, increasing production, development of by-products and development of improved processes.

Equipment: Small paper beater, apparatus for determining slowness of beater stock, strength of stock in beaters and on finished-paper.

168. **Eastman Kodak Company,** Rochester, N. Y.

Research staff: C. E. K. Mees, 45 chemists, physicists and photographic experts and 60 assistants.

Research work: Full time of 105 on theory of photography, development of new photographic materials and methods, and the study of the theory of manufacturing processes, and the production of synthetic organic chemicals.

Equipment: Sensitometric and lens testing apparatus, physical and colloidal chemical apparatus for use in the study of photographic theory.

169. **Eavenson & Levering Co.,** 3rd and Jackson Sts., Camden, N. J. (Wool scouring and carbonizing.)

Research staff: Chas. E. Mullin, 2 or 3 chemists and assistants.

Research work: Approximately one-half time of staff on textiles, wool particularly; wool scouring, carbonizing and dyeing; utilization of wool waste and refuse such as scouring liquors; wool grease and detergents.

170. **Edison, Thomas A., Laboratory,** Orange, N. J.

Research staff: Thos. A. Edison and about 250 machinists, chemists, physicists, experimenters, designers and draughtsmen.

Research work: Nearly full time of 250 on almost every branch of scientific research.

Equipment: Large scrap heap from which to rob to build other apparatus, and accumulations of every kind of material and chemical so as not to wait.

171. **Eimer & Amend,** Third Ave., 18th to 19th St., New York, N. Y. (Industrial and educational laboratory apparatus, assayers' materials, chemicals and drugs.)

Research staff: O. P. Amend, C. G. Amend, 2 chemists, 4 expert glass blowers and 1 mechanic.

Research work: Organic chemicals and special glass and metal apparatus for scientific investigations.

172. **Electrical Testing Laboratories,** 80th St. and East End Ave., New York, N. Y.

Research staff: Clayton H. Sharp, 1 chief engineer and 7 research men.

Research work: One-tenth time of 9 on dielectric losses; thermal conductivity of heat insulators at high and low temperatures; radiation efficiency of gas heaters; special cases of electrolysis by stray currents; breakdown voltage of sheet insulation.

Equipment: Very complete for electrical standardizing and research, photometry, mechanical measurements, fuel testing, paper and textile testing, thermometer and pyrometer standardization.

173. **Electro Chemical Company, The,** Dayton, Ohio. (Electrolytic cells for producing sodium hypochlorite.)

Research staff: John Gerstle and 1 chemical engineer.

Research work: Two-thirds time of 2 in connection with producing sodium hypochlorite from a sodium chloride solution, principally increasing efficiency of electrolytic cells.

174. **Electrolabs Company, The,** 2635 Penn Ave., Pittsburgh, Pa. (Electrolytic gas specialists.)

Research staff: I. H. Levin, 1 chemist, 1 engineer and 1 physicist.

Research work: Full time of 4 on electrolytic dissociation of water, application of hydrogen to vegetable oil refinement, etc.

Electro Metallurgical Company. See Union Carbide and Carbon Research Laboratories, Inc. (p. 78).

Ellis, Carleton, Laboratories. See Ellis-Foster Company.

175. **Ellis-Foster Company,** 92 Greenwood Ave., Montclair, N. J. (Chemical products and processes.)

Research staff: Carleton Ellis and a variable number of assistants.

Research work: Approximately full time of staff on organic chemistry and ceramics.

176. **Emerson Laboratory,** 145 Chestnut St., Springfield, Mass.

Research staff: H. C. Emerson and 5 chemists.

Research work: One-fourth time of 6 on paper and textile problems.

Empire Gasoline Co. See Doherty Research Company, Empire Division (p. 28).

Empire Tannery. See Gallun, A. F., & Sons Co. (p. 34).

177. **Eppley Laboratory, The,** 12 Sheffield Ave., Newport, R. I. (Physical-chemical laboratory.)

Research staff: Warren C. Vosburgh, 2 chemists and 1 instrument maker.

Research work: One-half time of 4 on cadmium standard cells, physico-chemical apparatus, standards of electromotive force, spectroscopy, theory of solutions from electrical standpoint and thermocouples for precise measurements.

Equipment: Spectroscopes and potentiometers.

178. **Eustis, F. A.,** 131 State St., Boston, Mass. (Metallurgical engineer.)

Research staff: F. A. Eustis.

Research work: Part time of 1 on metallurgical problems connected with copper, sulfur and iron and the purification of smelter smoke.

179. **Factory Mutual Laboratories** under the supervision of Asso-

ciated Factory Mutual Fire Insurance Companies, Inspection Department, 31 Milk St., Boston, Mass.

Research staff: C. W. Mowry, 2 chemists and 8 engineers.

Research work: One-sixth to one-fourth time of 11 on fire-protection engineering problems.

Equipment: Apparatus for chemical, hydraulic and mechanical tests and investigations of fire-protection devices.

180. **Fahy, Frank P.,** 50 Church St., New York, N. Y.

Research staff: Frank P. Fahy.

Research work: Full time of 1 on magnetic-mechanical analysis of iron and steel products.

Equipment: Special magnetic testing devices.

181. **Falls Rubber Company, The,** Cuyahoga Falls, Ohio.

Research staff: G. D. Kratz, 4 chemists and 2 engineers.

Research work: One-half time of 5 and one-fourth time of 2 on the investigation of raw rubbers and the process of vulcanization; new machines and mechanical methods.

Equipment: For the study of problems in the vulcanization of rubber.

182. **Fansteel Products Company, Inc.,** North Chicago, Ill. (Electrical, steel and chemical products.)

Research staff: Clarence W. Balke, 2 chemists, 1 engineer, and 1 assistant.

Research work: One-half time of 5 on rare metals, tungsten, molybdenum, cerium, tantalum and columbium.

183. **Feculose Co. of America,** Ayer, Mass. (Pastes, adhesives, size, etc.)

Research staff: John T. Gibbons and 3 chemists.

Research work: Full time of 4 on starches and starch products.

184. **Federal Phosphorus Company,** Anniston, Ala.

Research staff: J. N. Carothers and 3 chemists.

Research work: Full time of 2 men on plant process for production of phosphoric acid by electric smelting of phosphate rock; production of phosphoric acid salts.

185. **Federal Products Company, The,** 7818 Lockland Ave., Cincinnati, Ohio. (Cologne spirits and denatured alcohol.)

Research staff: J. F. Kraeger and 1 assistant chemist.

Research work: One-half time of 2 on production of ethyl alcohol from materials containing fermentable substances and recovery of valuable by-products from distillery waste.

186. **Firestone Tire & Rubber Company,** Akron, Ohio.

Research staff: E. W. Oldham, director of general laboratory; N. A. Shepard, director of organic research; E. C. Zimmerman, director of physical chemical research and 20 chemists and engineers; J. E. Hale, director of development department, and 12 engineers.

Research work: Full time of 6 on study of vulcanization, physical and chemical properties of vulcanized rubber in conjunction with various accelerators and compounding materials, and problems arising in connection with the manufacture of rubber products.

187. **FitzGerald Laboratories, Inc., The,** Niagara Falls, N. Y.

Research staff: F. A. J. FitzGerald and 3 assistants.

Research work: One-half time of 4 on electric furnaces, refractories and electrometallurgy.

Equipment: For electro-thermal laboratory.

188. **Florida Wood Products Co.,** Jacksonville, Fla. (Phosgene gas.)

Research staff: E. B. Smith and 1 chemist.

Research work: Part time of 2 on development of products of phosgene gas; pharmaceuticals derived from wood products.

Equipment: Special facilities for handling destructive distillation problems, being equipped with iron retorts capacity of 50 pounds to 1500 cubic feet.

189. **Fort Worth Laboratories,** Box 1008, Fort Worth, Texas. (Consulting, analytical chemists and chemical engineers.)

Research staff: F. B. Porter, R. H. Fash, and assistants, 6 chemists and about 8 helpers.

Research work: Small part time on industrial problems as presented, cotton oil refining and boiler water problems.

190. **Foster-Heaton Company,** 27 Badger Ave., Newark, N. J.

Research staff: Edward W. Rhael, 1 chemist and 1 engineer.

Research work: Approximately one-third time of 3 on development of coal tar dyestuffs soluble in oils, fats and waxes.

191. **Frees, H. E., Co., The,** 2528 W. 48th Place, Chicago, Ill. (Brewers and distillers laboratory.)

Research staff: Herman E. Frees, 1 chemist and 1 fermentologist.

Research work: Approximately one-half time of 3 on foods, yeasts, fermentation and beverages.

192. **Fry, H. C., Glass Company,** and **Beaver Valley Glass Co.,** Rochester, Pa.

Research staff: R. F. Brenner and 2 assistants.

Research work: More than one-half time of 3 on new varieties and compositions of glass. This work is carried out first in small crucible meltings and then in regular factory pots.

Equipment: High-temperature gas-fired furnace.

193. **Gallun, A. F., & Sons Co.,** Milwaukee, Wis. (Proprietor, Empire Tannery.)

Research staff: John Arthur Wilson and 7 chemists.

Research work: Approximately four-fifths time of 8 on experimental tanning, pure and applied colloid chemistry, physical chemistry, photomicrography, ultramicroscopy, histology of skin, and special applications of concentration cells.

Equipment: Experimental tannery.

194. **Garfield Aniline Works, Inc.,** Box 196, Passaic, N. J. Laboratory at Garfield, N. J.

Research staff: Arthur F. F. Mothwurf and 6 chemists.

Research work: Full time of 6 on coal tar intermediates, coal tar dyes (azo-colors and triphenylmethane derivatives) and sample dyeing.

195. **General Bakelite Company,** Perth Amboy, N. J. Supplementary laboratory in Yonkers, N. Y.

Research staff: L. H. Baekeland, 2 engineers and 5 chemists.

Research work: Full time of 8, confined almost exclusively to phenol-formaldehyde condensation products, both development and commercial applications.

Equipment: In form of electric ovens, stills, vulcanizers, pebble mills and rubber machinery.

196. **General Chemical Company,** Research Department, 25 Broad St., New York, N. Y.

Research staff: G. P. Adamson and approximately 45 chemists.

Research work: Full time of 46 on improving existing processes of the company, and devising new processes.

General Chemical Company has recently become a part of the Allied Dye & Chemical Corporation and reorganization of its research department is now in progress.

197. **General Electric Company,** Schenectady, N. Y. Laboratories also at Lynn and Pittsfield, Mass., Harrison, N. J. and Cleveland, Ohio.

Research staff: Willis R. Whitney, 2 assistant directors, 50 chemists, 12 physicists, 13 engineers, 50 research assistants, and machinists, glass-blowers, electricians and clerks.

Research work: Full time of staff devising new forms of electric lights and improving existing forms. Development of Coolidge X-ray tube. Invention of new and development of existing forms of electric equipment and apparatus. Study of metals and alloys for electrical uses. Wireless transmission development. Study of insulation. Many fundamental physical and chemical scientific researches also are carried on.

See National Lamp Works of General Electric Company (p. 56).

198. **General Engineering Company, Incorporated, The,** 159 Pierpont St., Salt Lake City, Utah. (Consulting engineers, ore testing.)

Research staff: J. M. Callow, 1 chemist, 2 metallurgical engineers and 2 helpers.

Research work: Full time of 6 on metallurgical and engineering problems, specializing on ore treatment problems.

199. **General Motors Research Corporation,** Box 745, Moraine City, Dayton, Ohio.

Research staff: C. F. Kettering, president and active directing engineer, F. O. Clements, director of research, and 251 employees, divided into specialized groups or departments, made up of chemists, metallurgists, electrical engineers, mechanical and other research engineers, assistants and helpers. (Control division made up of 147 additional employees and manufacturing division, having at the present time 18 members, bring the total number of employees up to 416.)

Research work: Full time of staff on strictly automotive research of interest to General Motors Corporation.

Equipment: Laboratories capable of conversion, upon short notice, into mechanical, chemical or electrical laboratories. Complete shop, foundry and heat treat departments.

200. **General Tire & Rubber Co.,** Akron, Ohio.

Research staff: H. B. Pushee and 2 men.

Research work: One-tenth time of 3 on development of better rubber compounds; rubber accelerators; coefficient of vulcanization.

201. **Gibbs Preserving Company,** 2303 Boston St., Baltimore, Md.

Research staff: David R. Dotterer and 1 assistant.

Research work: Canned goods and jellies.

202. **Gillette Safety Razor Co.,** 47 W. 1st St., Boston, Mass.

Research staff: Henry E. K. Ruppel, 4 chemists, 1 special engineer and technicians.

Research work: Part time of 6 or more on development and improvement of analytical methods; precision measurements; heat treatment of steel: (a) metallographic investigations, (b) practical applications; electro-deposition of metals; abrasives; study of edges with special reference to shaving.

203. **Glass Container Association of America,** 3344 Michigan Ave., Chicago, Ill.

Research staff: A. W. Bitting and 4 assistants.

Research work: Full time of 5 on standardization of glass containers, improved methods of packing glassware for shipment, foods and beverages in glass and improvement in containers and closures.

Equipment: Complete equipment for the preparation and packing of foods in glass and testing bottles, jars and packing materials.

203a. **Glidden Company, The,** Cleveland, Ohio. (Paints, varnishes, enamels, stains, dry colors, insecticides, vegetable oils.)

Research staff: F. M. Beegle, chief chemist, 6 chemists, 2 chemical engineers and a number of physicists. Research committee of 7 members, comprised of the general superintendent and the head of each manufacturing department.

Research work: The greater part of the time of the members of the research committee, as well as that of all the chemists, is spent on research or development work on synthetic gums, treated oils, varnishes, paints, enamels, stains, dry colors, and insecticides.

Equipment: Stacks for oil boiling and varnish making; an electrically heated humidor, the humidity and temperature of which can be controlled and regulated to duplicate the conditions of various manufacturing plants; an oil treating plant and spraying apparatus.

204. **Globe Soap Company, The,** St. Bernard, Ohio.

Research staff: C. P. Long, chemical director, 3 chemists and 2 chemical engineers.

Research work: One-tenth time of 6 on investigation of problems connected with the industry.

205. **Glysyn Corporation, The,** New York, N. Y. Laboratory at Bound Brook, N. J.

Research staff: Harold F. Saunders and 3 chemists.

Research work: Full time of 4 on chlorination processes.

206. **Goodrich, B. F., Company, The,** Akron, Ohio. (Rubber goods of every description.)

Research staff: W. C. Geer, vice-president, in charge of development. Research physical laboratory: 4 physicists and 4 assistants. Engineering and testing laboratory: 3 engineers and 2 engineering assistants. Chemical laboratories: 8 chemists and 3 assistants. Development laboratories: 18 chemical engineers and 26 assistants.

Research work: The entire time of the staff is spent on research and factory control work, although in rubber the factory control is never quite distinguishable from research. The fundamental lines of research are those of compounding ingredients, including the chemical and physical properties of crude rubber, reclaimed rubber, mineral ingredients, and organic chemical individuals, the study of vulcanization, and in particular the main efforts have to do with the physical and chemical design of compositions and articles for particular lines of industrial service.

Equipment: Development laboratory equipped with mills, vulcanizing apparatus, etc.

207. **Goodyear Tire & Rubber Company, The,** Akron, Ohio.

Research staff: Wm. S. Wolfe, development manager, K. B. Kilborn, experimental engineer in charge of machine design, tire design and highways transportation divisions; R. C. Hartong, chief chemist in charge of development service and chemical and physical research; W. E. Shively, chief tire designer, H. E. Morse, manager mechanical goods development and service division; 4 chemical engineers, 3 assistant chemical engineers, 8 research chemists, 5 research physical chemists and physicists, 9 research engineers, 25 technical service, chemical and mechanical engineers, 8 chemical laboratory chemists and assistant chemists, 18 physical laboratory assistants, 8 mechanical goods design engineers, 11 tire design engineers, 6 assistant tire design engineers, 12 compound development chemists, 6 machine design engineers, 12 machine designers, 5 machine design detailers and tracers, 37 machine design workshop machinists, 10 machine design expert template makers, 2 highway transportation engineers. Total employees of department approximately 360.

Research work: Full time of research and development men on mechanism of vulcanization, compounds which affect the rate of vulcanization, development of organic compounds especially adapted to rubber work; application of physical chemistry to study of rubber and compounding materials; physical properties of rubber, and methods of testing and studying them; chemistry of fibrous materials, particularly cotton, and properties of materials used as films or protective agents; industrial processes, such as reclaiming and coagulation of rubber.

208. **Grasselli Chemical Company,** 1300 Guardian Bldg., Cleveland, Ohio. Laboratory at Cleveland mainly for inorganic work. Laboratory also at Grasselli, N. J., for organic work strictly.

Research staff: Henry Howard and a large number of chemists and assistants.

Research work: Full time of staff on problems connected with possible improvements in products at present being manufactured as well as in connection with chemicals, dyes, intermediates, etc., the manufacture of which is being contemplated.

209. **Gray Industrial Laboratories, The,** 961 Frelinghuysen Ave., Newark, N. J.

Research staff: Thomas T. Gray, David Drogin, G. C. Hargrove, E. V. Espenhahn and assistants.

Research work: Full time of 2 on petroleum and its products.

Equipment: Complete semi-commercial oil refining equipment.

210. **Great Western Electro-Chemical Company,** 9 Main St., San Francisco, Calif. (Chlorine products.)

Research staff: Ludwig Rosenstein, 2 chemists and 2 assistants.

Research work: Utilization of chlorine, manufacture of chlorine products, manufacture of caustic and electrolysis of brine.

211. **Great Western Sugar Company, The,** Sugar Building, Denver, Colo.

Research staff: H. W. Dahlberg, 1 chief chemist, 4 chemical engineers, 4 research chemists, 2 mechanics, 1 experimental process man, 3 analysts.

Research work: Full time of 16 on investigations of fundamental principles of processes and practices now in use, examination of proposed new processes and apparatus and study of utilization of byproducts and waste products; production of crude potash, sodium cyanide, ammonium sulphate and certain rare organic chemicals from the Steffen's waste water; refining of crude potash leading to production of carbonate, hydrate, etc.; recovery of organic acids from waste waters.

Equipment: Complete equipment for manufacture of sugar on a small scale under such conditions that special attention may be paid to any stage of the process.

212. **Grosvenor, Wm. M.,** 50 E. 41st St., New York, N. Y. (Consulting chemist and factory engineer.)

Research staff: From 2 to 6.

Research work: Flotation of ores, non-ferrous metallurgy, paper, starch, glues and adhesives, textiles, paper and their finishing, methods of manufacture of organic intermediates, utilization of by- and waste products.

Equipment: Viscosimeters, high speed moving picture equipment, autoclaves up to 1000 lbs. per sq. in.

213. **Gulf Pipe Line Company,** Houston, Tex. (Producers and transporters of petroleum.)

Research staff: F. M. Seibert and 2 trained research men.

Research work: Full time of 3 on methods for production and transportation of oil; special problems on treatment of crude oil emulsions, conservation of oil, gas, etc.

214. **Gurley, W. & L. E.,** 514 Fulton St., Troy, N. Y. (Instruments for civil, mining and hydraulic engineers, and land surveyors.)

Research staff: E. W. Arms, 3 engineers, 3 mechanicians and assistants as needed.

Research work: Practically full time of 7 on investigations for design and manufacture of instruments for civil, mining and hydraulic engineers, such as automatic water stage registers, current meters, hook gages, transits and levels.

Equipment: For testing and calibrating standard precision measures of weight, capacity and length; for investigation of water measurements and for design of instruments for this purpose; automatic water stage registers, current meters and hook gages; special dividing engines for accurate angular and linear graduation; for drawing platinum wire from 0.001- to 0.00002-inch diameter for cross-wire reticles and in research experiments.

215. **Habirshaw Electric Cable Company, Inc.,** Yonkers, N. Y.

Research staff: William A. Del Mar, 3 to 6 engineers, 2 to 6 chemists and 0 to 7 assistants.

Research work: Seven-tenths time of staff on insulating materials and electric cable manufacture.

Equipment: Miniature manufacturing plant for making rubber insulated wire in the laboratory.

216. **Hamersley M'f'g Co., The,** Garfield, N. J. (Waxed papers.)

Research staff: 1 chemical engineer and 5 chemists.

Research work: One-third time of 6 on pulp, paper, and paper mill chemicals.

Equipment: Well equipped for paper mill experiments on semi-commercial scale.

217. **Harbison-Walker Refractories Company,** Farmers Bank Building, Pittsburgh, Pa. (Fire-clay, silica, magnesite and chrome bricks and other refractory products.)

Research staff: R. H. Youngman, 1 to 2 special technical men, 1 chief chemist and 1 or 2 chemists.

Research work: One-half time of staff on problems in connection with refractories.

Equipment: 1 coal and 1 gas-fired test kiln, 1 small ore crusher, 2 Braun planetary pulverizers and 1 hydraulic press of 104 tons capacity.

218. **Harrison Mfg. Co., The,** 55 Union St., Rahway, N. J. (General chemicals and chemical products; thorium nitrate and other rare earth salts and oxides; writing inks.)

Research staff: C. W. Squier.

Research work: Full time of 1 on general lines of research.

Harrison Safety Boiler Works. See Cochrane, H. S. B. W., Corporation (p. 20).

219. **Hayes, Hammond V.,** 84 State St., Boston, Mass. (Consulting engineer.)

Research staff: Hammond V. Hayes, 5 electrical engineers and physicists.

Research work: Full time of 6 on electro-dynamic problems.

Haynes Stellite Co. See Union Carbide and Carbon Research Laboratories, Inc. (p. 78).

220. **Heap, William, & Sons,** Grand Haven, Mich. (Celluloid and china.)

Research staff: H. Stirling Snell and 1 chemist.

Research work: Three-fourths time of 2 on thermoplastics.

221. **Heinrich Laboratories of Applied Chemistry,** 1001 Oxford St., Berkeley, Calif. (formerly Tacoma, Wash.).

Research staff: E. O. Heinrich and 1 chemist.

Research work: Full time of 2 on chemical and photomicrographical problems as applied to criminal investigation.

222-224. **Hercules Powder Co.,** Wilmington, Del. (Explosives.) Laboratories at Kenvil, N. J., Brunswick, Ga., and Emporium, Pa. Executive staff, consisting of G. M. Norman and 6 assistants, supervises work on problems on explosives, mineral acids, nitrogen fixation, pyroxylin solutions, plastics, smokeless powder, and naval stores at three research laboratories.

222. *Experimental station, Kenvil, N. J.*

Research staff: C. F. Bierbauer, 16 graduate chemists and engineers, 7 other salaried employees and 24 payroll employees.

Research work: Approximately full time of 48 on research on high explosives, smokeless powders, plastics, pyroxylin solutions, and naval stores. Some time also devoted to investigations of analytical methods in reference to above.

Equipment: Complete equipment for testing properties of dynamite. Equipment for manufacturing propellant powders, and ranges for testing same for velocity and pressure, semi-works equipment for the manufacture of organic chemicals and plastics.

223. *Naval Stores Laboratory, Brunswick, Ga.*

Research staff: C. M. Sherwood and 3 graduate chemists.

Research work: Seven-tenths time of 4 on problems connected with the manufacture of turpentine, rosins and pine oil, by the steam solvent process.

Equipment: Semi-works scale apparatus duplicates plant process.

224. *Emporium Research Laboratory, Emporium, Pa.*

Research staff: R. B. Smith and 1 assistant chief chemist, 5 assistant chemists and 2 laboratory assistants.

Research work: Full time of 10 on general research and on methods of manufacture of mineral acids and domestic explosives.

Equipment: Semi-commercial scale apparatus for nitration; special equipment for analysis of explosives and for explosive testing.

Hess-Bright Manufacturing Co. See S. K. F. Industries, Inc. (p. 72).

225. **Heyden Chemical Company of America, Inc.,** Garfield, N. J.

Research staff: Robert O. Bengis, 7 chemists, 1 engineer and 1 laboratory assistant.

Research work: Two-fifths time of 10 on medicinal and pharmaceutical chemistry; salicylates and metallic colloids.

Equipment: Equipped semi-commercial plant adjacent to research laboratory.

226. **Hirsch Laboratories, Inc., The,** 50 E. 41st St., New York, N. Y. Laboratory at 593 Irving Ave., Brooklyn, N. Y.

Research staff: Alcan Hirsch and 5 chemists.

Research work: One-half time of 6 on organic chemicals, intermediates, dyestuffs and pharmaceuticals. Metal products; cerium, thorium and molybdenum products.

Equipment: Fully equipped for semi-plant operations. Facilities for duplicating and testing on commercial scale any proposed plant installation or process.

Hirsch, Stein & Company. See United Chemical and Organic Products Co. (p. 79).

227. **Hochstadter Laboratories,** 227 Front St., New York, N. Y. (General chemical analyses and investigations; consultants and technical experts.)

Research staff: Irving Hochstadter, W. B. Stoddard and 2 chemists.

Research work: One-half time of 4 on manufacture and preparation of food and pharmaceutical products with special emphasis on problems relating to pure food regulations and on problems relating to the rare metals, especially "Tungsten" compounds.

228. **Holt Manufacturing Company, The,** Peoria, Ill. (Tractors.)
Research staff: R. M. Hudson, research engineer, and 2 mechanical engineers; F. W. Grotts, inspection and metallurgical engineer, and 2 chemists; 1 expert in microphotography.
Research work: Full time of supervisors and staff on technical, industrial and commercial problems. Industrial research on wage surveys, costs of living, industrial relations and organization problems and principles.
Equipment: Special microphotographic apparatus with grinding and polishing machines; oil distillation apparatus and viscosimeter; electric furnace for experimental heat treating; dynamometer for motor research.

229. **Hood Rubber Company,** Watertown, Mass.
Research staff: Warren E. Glancy, 2 chemists and several routine assistants.
Research work: Small part time of staff on new methods of examination of materials; study of various organic derivatives.
Equipment: Devices and machines for testing rubber, cloth, yarns; large experimental mill room equipped with heavier machinery and heavier testing machines for testing tires (solid, pneumatic, etc.).

230. **Hooker Electrochemical Company,** Niagara Falls, N. Y.
Research staff: T. L. B. Lyster, director of development, A. H. Hooker, technical director, W. J. Marsh, research chemist, 1 research chemist and 4 assistants.
Research work: Full time of 5 on development of new processes and betterment of present processes.
Equipment: Furnace room, annex and industrial laboratory equipped for intermediate scale or development work.

231. **Hoskins Manufacturing Company,** Lawton Ave., at Buchanan, Detroit, Mich. (Electric-furnaces, pyrometers and heating appliances.)
Research staff: W. A. Gatward and 4 engineers.
Research work: Almost full time of 5 on the improvement and production of alloys and allied products.

232. **Houghton, E. F. & Co.,** 240 W. Somerset St., Philadelphia, Pa. (Oils, mechanical leathers and steel heat treating materials.)
Research staff: George W. Pressell, 9 chemists and engineers. Consulting engineers and chemists sometimes employed.
Research work: Research staff is working constantly in producing oils for the industries, mechanical leathers and steel treating materials; also improving methods in the manufacturing industry.

233. **Howard Wheat and Flour Testing Laboratory, The,** Old Colony Building, Minneapolis, Minn. (Comparative baking tests, records and reports, milling tests, chemical and microscopical analyses.)
Research staff: C. H. Briggs and 3 chemists.
Research work: Small part time of 4 on problems connected with causes of peculiar variations of wheats and other cereals when baked into bread or used for other food purposes; efforts to improve methods of separation of wheat proteins; improved methods of quantitative analysis; chemical causes of loaf expansion and effects of various

activating materials in bread making, carried out by cooperation of baking and chemical departments. Some work on distinguishing cereal flours one from another.

Equipment: Moisture testers for grain, haemocytometer, yeast testing apparatus of special design, wheat and grain cleaning and milling department and a baking test department, equipped for handling more than 100 individual tests daily with automatic control of kneading machines, bread raising cabinets, etc.

234. **Hunt, Robert W., and Co.,** 175 W. Jackson Blvd., Chicago, Ill. (Engineers.) Laboratories also at 251 Kearney St., San Francisco, Calif.; 90 West St., New York, N. Y.; Monongahela Bank Bldg., Pittsburgh, Pa.; 905 McGill Bldg., Montreal, Canada; and Syndicate Trust Bldg., St. Louis, Mo.

Research staff: J. H. Campbell and assistant, 12 chemists and 8 engineers.

Research work: Part time of 22 on materials of construction, iron, steel, stone, cement and bitumen.

235. **Hyco Fuel Products Corporation,** 30 Broad St., New York, N. Y. Laboratory at Edgewater, N. J.

Research staff: Allen Rogers, 3 chemists, 1 engineer, 1 draftsman and 5 assistants.

Research work: The plant is built for demonstration and research on oil problems, especially as related to motor fuel.

236. **Hynson, Westcott & Dunning,** 423 N. Charles St., Baltimore, Md. (Bacterial and bio-chemic therapeutic products.) Laboratory at 16 E. Hamilton St., Baltimore, Md.

Research staff: Daniel Base, 2 chemists, 1 pharmacologist and 1 bacteriologist.

Research work: One-half time of 5 on preparation and pharmacological testing of new drugs.

237. **Imperial Belting Company,** Lincoln and Kinzie Sts., Chicago, Ill. (Belting and conveyors.) Laboratory at 400 N. Lincoln St., Chicago, Ill.

Research staff: James A. Millner, 1 engineer and 2 chemists.

Research work: Approximately one-half time of 4 on oils, paints, asphalts and textiles.

238. **Industrial Chemical Institute of Milwaukee,** 200 Pleasant St., Milwaukee, Wis. (Consultants for chemical and engineering problems.)

Research staff: F. M. Dupont, 1 chemical engineer, 1 bacteriologist and 4 chemists.

Research work: Full time of 1 chemist on food, beverage, magnesite, lime, adhesives, antisepticides and general matters.

239. **Industrial Research Corporation,** 1025 Front St., Toledo, Ohio.

Research staff: C. P. Brockway and 2 engineers.

Research work: Full time of 3 on problems related to small machine equipment and small devices in metal.

240. **Industrial Research Laboratories,** 190 N. State St., Chicago, Ill. F. Peter Dengler, Inc., proprietors. (General consulting and research chemists, resource and chemical engineers.)

Research staff: F. Peter Dengler, 5 or 6 chemists and 1 engineer.

Research work: Full time of staff on manufacturing and research problems relative to cement, coal, corn, cotton seed, drugs, dairy, dyes, foods, minerals, paints, paving, petroleum, paper, sewage, soap, steel, sugar, tobacco, water, barley, conservation of waste material and manufacture of non-alcoholic flavoring extracts including vanilla.

Equipment: Commercial equipment for production of coke and by-products and for decolorizing and reclaiming cloth, mill ends, flour bags, sugar bags, all cloth signs and rubber-coated textile materials. Commercial equipment for extracting vegetable alkaloid from tea and coffee. Apparatus for the manufacture of non-alcoholic extracts is being installed on a commercial scale for immediate use.

241. **Industrial Testing Laboratories,** 402 West 23rd St., New York, N. Y.

Research staff: Emil Schlichting, director, H. Winther, chief chemist, and 5 assistant chemists.

Research work: Part time of staff on problems related to beverage, fermentation and food industries.

Equipment: For chemical, biological and microscopical analyses of beverages and foods, their raw materials, by-products, and accessories.

242. **Industrial Works,** Bay City, Mich.

Research staff: R. H. Morgan, metallurgist, J. C. Wheat, development engineer; chemists and assistants as required.

Research work: Heat treatment and properties of metals, properties of other materials, development and control of foundry practice for iron, steel and bronze; welding practice. Development of cranes and accessories to meet needs of users; statistical manufacturing and executive control; standards of production and personnel, standard times and routings.

Equipment: 150,000-pound Riehle testing machine, Shore scleroscope, Berry strain gauges, two proof testing machines of 500,000 pounds and 100,000 pounds capacity, for testing actual parts before assembly.

243. **Ingersoll-Rand Company,** 11 Broadway, New York, N. Y. (Rock drills, etc.)

Research staff: F. W. O'Neil and a number of assistants.

Research work: Full time of some and part time of others on drills, pumps, pneumatic tools, compressors, blowers, condensers and oil engines.

244. **Inland Steel Company,** Indiana Harbor, Ind.,

Research staff: J. C. Dickson, 29 chemists and 5 chemical engineers.

Research work: Full time of 4 and part time of 30 on problems connected with steel industry.

245. **Institute of Industrial Research, The,** 19th and B Sts., N. W., Washington, D. C.

Research staff: Allerton S. Cushman, chemists, physicists and assistants as needed.

Research work: Varying part time of staff on physical testing of cements, rocks, clays, brick, block, iron, steel, wood, rubber, and other materials of construction. In Bitumen Laboratory petroleum and petroleum products, tars and tar products, creosoting oils, asphalts, bituminous emulsions, bituminous aggregates, and all other types of chemical road and paving materials, roofing materials, rubber, etc., are examined and tested. Chemical examinations of rocks, clays, cements, etc., are made and researches conducted on improvements in industrial products and processes and utilization of waste products for road purposes.

Equipment: For cement and bitumen.

246. **International Filter Co.**, 38 S. Dearborn St., Chicago, Ill. (Water softening and filtration plants.) Laboratory at 333 W. 25th Place, Chicago, Ill.

Research staff: 2 to 5 workers.

Research work: Approximately one-half time on materials, methods and processes for purifying liquids.

247. **International Nickel Company, The,** Bayonne, N .J. Successors to The Orford Copper Co.

Research staff: Paul D. Merica, 2 metallurgists, 2 assistant metallurgists, 2 chemists, 2 laboratory assistants, and 1 machinist.

Research work: Metallurgy of copper and nickel; physical properties of nickel and Monel metal; uses of Monel metal and nickel alloys.

Equipment: Laboratory electric furnace equipment; dust and fume sampling apparatus; experimental electroplating plant.

248. **International Shoe Co. (Burke Tannery),** Morganton, N. C.

Research staff: J. S. Rogers, 2 trained assistants, and 1 helper.

Research work: Approximately one-half time of director and part time of assistants on problems in extraction of tanning materials and the tanning and finishing of sole leathers.

Equipment: Some special apparatus for small scale plant experiments.

249. **International Silver Company,** Meriden, Conn.

Research staff: Chas. E. Skidgell and 2 chemists.

Research work: Small part time of 3 on electro-plating.

250. **Interocean Oil Company, The,** East Brooklyn, Baltimore, Md.

Research staff: H. R. Gundlach, 2 chemists and 6 assistants.

Research work: Approximately one-tenth time of 9 on development of refining methods and testing; recovering of waste products, etc.

Equipment: Laboratory scale refinery, also larger scale experimental plant.

251. **James Ore Concentrator Co.,** 35 Runyon St., Newark, N. J.

Research staff: U. S. James, 1 metallurgical engineer and 3 assistants.

Research work: Full time of 3 on ore and coal testing.

252. **Jaques Manufacturing Company,** 16th and Canal Sts., Chicago, Ill. (Manufacturers of K. C. baking powder.)

Research staff: J. R. Chittick and 3 chemists.

Research work: One-third time of 2 on leavening materials.

Jeffrey-Dewitt Co. See Champion Porcelain Company (p. 19).

253. **Johnson & Johnson,** New Brunswick, N. J. (Surgical supplies.)

Research staff: Fred B. Kilmer and 7 assistants.

Research work: One-third time of 8 in research on medical, surgical and hospital supplies (not equipment) and incidentally drugs and commodities used therein.

254. **Kalmus, Comstock & Wescott, Inc.**, 110 Brookline Ave., Boston, Mass. (Consulting, research and operating engineers.)

Research staff: A group of physicists, chemists, metallurgists and chemical engineers of from 20 to 25 in number, directed by Herbert T. Kalmus, Daniel F. Comstock and E. W. Westcott.

Research work: Full time of staff on mechanical, physical, chemical, electrochemical, metallurgical and photographic lines leading to the development of processes, use of waste products, and through the designing, constructing and early operating of plants.

Equipment: Specially designed equipment in the fields of ceramics, abrasives, general chemical engineering, metallurgy, photography, motion pictures, and vegetable oils.

255. **Kellogg Switchboard and Supply Co.**, Adams and Aberdeen Sts., Chicago, Ill.

Research staff: Wilbur J. Anglemyer, 1 electrical engineer and 5 assistants.

Research work: Full time of 6 on testing of materials including analysis, tensile strength tests and magnetic characteristics; checking methods of manufacture and development of special testing instruments and new products.

Equipment: Impregnating apparatus, 5000 and 50,000 volt testing transformers, Burrous permeameter, Rowland dynamometer, G. E. Co. oscillograph, centrifugal extractor, apparatus for testing textile materials and paper and insulation testing equipment.

256. **Keuffel & Esser Co., Hoboken, N. J.** (Drawing materials and mathematical and surveying instruments.)

Research staff: Carl Keuffel, 1 chemist, 2 assistant chemists, 2 optical engineers, and 2 assistants.

Research work: One-half time of 8 on optical glass and various articles manufactured, including design of optical instruments and calculation of optical systems.

Equipment: Special equipment for testing presence of small quantities of iron in silicates, and for physical, chemical and microscopic testing of papers. Optical laboratory equipped for general testing of optical instruments.

257. **Kidde, Walter, & Company, Incorporated**, 140 Cedar St., New York, N. Y. (Engineers and constructors.)

Research staff: Barzillai G. Worth and assistants as necessary.

Research work: Investigation for clients, such as electrolysis of potassium and sodium compounds; electrochemical extraction of oils; chemical salvage systems for tanneries; sanitation of tannery effluent, etc.

258. **Kilbourne & Clark Manufacturing Company**, Seattle, Wash. (Engineers and manufacturers of electrical and radio apparatus.)

Research staff: H. F. Jefferson and 5 men.

Research work: Time of staff as occasion requires, on testing and investigating high-frequency circuits.

Equipment: Wave-meters, decremeters, sphere spark gap (25 cm. sphere) for high voltage tests; condensers, variable and fixed, with air, mica and oil dielectrics; inductances in various forms for high and low voltage; 500-cycle meters for use in connection with audiofrequency circuits in radio work.

Kistler, Lesh & Company. See International Shoe Co. (p. 44).

259. **Klearflax Linen Rug Company,** 63rd and Grand Aves., West, Duluth, Minn. (Linen rugs and carpeting.)

Research staff: Charles F. Goldthwait and variable number of assistants.

Research work: Full time of staff on use of flax fibre and its by-products; humidity, textiles and mechanism of dyeing process.

260. **Kokomo Steel and Wire Co.,** Kokomo, Ind.

Research staff: R. K. Clifford, 2 chemists and 2 assistants.

Research work: One-third time of 5 on standardization of raw materials, specifications and improvement of products in connection with manufacture of open hearth steel, wire and wire products.

Equipment: 100,000-pound Olsen testing machine, Brinell machine, electric furnace for heat treatments, metallographic equipment for grinding, polishing and microphotography.

261. **Kolynos Co., The, New Haven, Conn.** (Dental cream.)

Research staff: L. A. Jenkins, 3 chemists and 2 bacteriologists.

Research work: One-half time of 6 on oral hygiene.

262. **Koppers Company, The,** Pittsburgh, Pa. (Designers and builders of by-product coke and gas plants and apparatus for benzol recovery, tar distillation and gas purification.)

Research staff: F. W. Sperr, Jr., 13 graduate chemists, 1 engineer and 4 assistants.

Research work: Full time of 19 on coal carbonization, gas production, and purification, by-product recovery, secondary treatment of various by-products, general fuel research, refractories, pyrometry, investigation of coal properties.

Equipment: Special apparatus for coal carbonization at high and low temperatures, coal washing, coke research, gas purification by dry and liquid processes, furnaces for investigation of refractory materials at high temperatures, laboratories and experimental plant fully equipped for semi-commercial tests, and plants available for large scale tests in relation to coke and gas manufacture and by-product recovery.

263. **Kraus Research Laboratories, Inc.,** 130 Pearl St., New York, N. Y. (Consulting engineers in refractories.)

Research staff: Charles E. Kraus, 2 ceramists, 2 research engineers and 2 assistants.

Research work: Three-fourths time of 7 on ceramics and refractories.

Equipment: Equipped to make all standard tests on refractory materials, both in raw and finished state.

264. **Krebs Pigment and Chemical Co., The,** Newport, Del.

Research staff: H. W. Fox, 1 chemical engineer, 2 chemists and 2 assistants.

Research work: Full time of 6 on properties of lithopone; efficiency of steps of process.

265. **Kullman, Salz & Co.,** 603 Wells Fargo Building, San Francisco, Calif. (Tanners.)

Research staff: 1 chemist and 1 helper.

Research work: Variable amount of time of 2 on science of tanning.

266. **Laclede-Christy Clay Products Company,** 4600 S. Kingshighway, St. Louis, Mo.

Research staff: C. W. Berry and 1 assistant.

Research work: One-half time of 2 on development of refractories, superior clays for use in paper, graphite crucibles, enamels; unusual basic and neutral refractories, such as high aluminous materials, combinations of alumina and magnesia.

267. **Lakeview Laboratories,** 2 Jersey St., Buffalo, N. Y.

Research staff: A. L. Stevens and 2 assistants.

Research work: Four-fifths time of 3 on wood oils and tars.

268. **Larkin Co.,** 680 Seneca St., Buffalo, N. Y. (Soap.)

Research staff: L. F. Hoyt, 4 chemists and 2 assistants.

Research work: Three-fourths time of 7 on soaps, fats and oils; development along miscellaneous lines of new products for the company.

Equipment: Small experimental plant for producing soap.

269. **Laucks, I. F., Inc.,** 99 Marion St., Seattle, Wash. (Analytical and consulting chemists, assayers and metallurgists.)

Research staff: I. F. Laucks and H. P. Banks, 3 chemists, 3 chemical engineers, 2 agronomists and 3 inspection engineers.

Research work: One-fourth time of 8 on uses for raw materials available in the Orient and adaptation of these materials to American requirements; development of improvements in manufacturers' processes and development of coal by-products.

Equipment: Complete vegetable and fish oil refinery and complete coal by-products plant.

270. **Lee & Wight,** 113 E. Franklin St., Baltimore, Md.

Research staff: Two chemists.

Research work: Part time of 2 on industrial and miscellaneous problems.

271. **Leeds & Northrup Company,** 4901 Stenton Ave., Philadelphia, Pa.

Research staff: Irving B. Smith, 7 trained research workers and 3 mechanicians.

Research work: Full time of 11 on development of apparatus for precise measurements in heat, electricity, magnetism and heat treatment of steel; also for research and control in chemical industries.

Equipment: Apparatus for heat treatment of steel, instruments for precise measurements in heat, electricity and magnetism.

272. **Lehn & Fink, Inc.,** 192 Bloomfield Ave., Bloomfield, N. J. (Antiseptics, disinfectants, drugs, medicines, dentifrice, soaps, fine chemicals.)

Research staff: C. Hinck, 14 chemists and 2 engineers.

Research work: Full time of 17 on organic, biological and pharmaceutical problems.

273. **Lemoine, Pierre, Cie., Inc.,** 294 Pearl St., New York, N. Y. (Essential oils, aromatic chemicals.) Factory Laboratory at L. I. City, N. Y.

Research staff: 2 chemical engineers, 1 analytical and research chemist and 2 associate chemists.

Research work: Part time of 5 on synthetic organic chemicals and essential oils, perfumery oils and raw materials and flavors and flavoring raw materials.

274. **Lennox Chemical Co., The,** 1205 E. 55th St., Cleveland, Ohio. Laboratory at Euclid, Ohio.

Research staff: A. S. Allen and 2 assistants.

Research work: One-half time of 3 on carbonation as related to the soft drink or beverage industry; liquefaction, purification and drying of commercial gases as oxygen, nitrous oxide, and carbon dioxide.

275. **Lewis, F. J., Manufacturing Co.,** 2513 S. Robey St., Chicago, Ill.

Research staff: W. B. Murphy and 2 chemical engineers.

Research work: Part time on coal tar products.

Lewis, Gilman & Moore. See Metals & Chemicals Extraction Corporation (p. 52).

276. **Lilly, Eli, and Company,** Indianapolis, Ind. (Pharmaceutical and biological products.)

Research staff: G. H. A. Clowes, Frank R. Eldred, A. L. Walters and about 40 chemists and pharmacologists.

Research work: Full time of 8 men and half time of 17 directed to development of new therapeutic agents and to broad study of mode of action of drugs from physical, chemical and physiological standpoints.

277. **Lincoln, E. S., Inc.,** 534 Congress St., Portland, Me. (Consulting engineers; electrical laboratories.)

Research staff: E. S. Lincoln and 3 engineers.

Research work: Full time of 4 on electrical problems. Field work a specialty.

Linde Air Products Company, The. See Union Carbide and Carbon Research Laboratories, Inc. (p. 78).

278. **Lindsay Light Company,** 161 E. Grand Ave., Chicago, Ill.

Research staff: H. N. McCoy, 8 chemists and 1 engineer.

Research work: Four-fifths time of 10 on improvements of processes of refining thorium nitrate, cerium compounds, organic preparations such as phenolphthalein and vanillin, preparation of dyes.

279. **Little, Arthur D., Inc.,** 30 Charles River Road, Cambridge 39, Mass. (Chemists, engineers, managers.)

Research staff: Earl P. Stevenson, director, and 8 research chemists cooperating with 10 analytical chemists; 8 engineers, chemical, mechanical, mining; 1 economic geologist; and special staff for valuations and appraisals.

Research work: Full time of 10 on industrial research on lines determined by requirements of clients and on special problems in adhesives, ceramics, utilization of lumbering waste, paper and pulp, textiles, metallurgy, non-metallic minerals, and process developments.

Equipment: Complete experimental paper mill including a 30-inch Fourdrinier machine. Semi-commercial equipment for miscellaneous work.

280. Littlefield Laboratories Co., Seattle, Wash.

Research staff: E. E. Littlefield, 1 electrochemist and electrophysicist, 1 chemist and 1 mechanical engineer.

Research work: Full time of 1 and part time of 3 in chemical, electrical and electrochemical fields; development of special apparatus for initiating and stopping flow of liquids by varying conductivity; electrical treatment of vegetation. Usually done in connection with large industries in the United States and England.

281. Lockhart Laboratories, 33½ Auburn Ave., Atlanta, Ga.

Research staff: L. B. Lockhart.

Research work: Full time of 1 on lubricating oils and greases, special soaps, varnishes, waterproofing, petroleum products, colloids and emulsions.

282. Long, W. H., & Co., Inc., 244 Canal St., New York, N. Y. (Wholesale druggists.)

Research staff: Charles H. Lewis, 2 chemists, 1 assistant and 1 laboratory assistant.

Research work: Drugs, chemicals and dyes.

283. Ludlum Steel Company, Watervliet, N. Y.

Research staff: P. A. E. Armstrong and 4 trained men.

Research work: Full time of 5 on improvement of manufacturing methods for ferro alloys and certain steels, such as magnet steel and non-corrosive steels and methods of chemical analysis of steels and ferro alloys.

284. Lumen Bearing Company, Buffalo, N. Y. (Brass and bronze foundry.)

Research staff: C. H. Bierbaum, metallurgist; B. Woiski, chief chemist, and 2 assistants, and G. F. Comstock, consulting metallurgist.

Research work: Varying portion of time on problems having to do with non-ferrous metallurgy and metallography, chemistry as applied to non-ferrous metals, photomicrography of the non-ferrous metals.

Equipment: 50,000-pound Olsen universal testing machine, Brinell hardness machine, scleroscope, microcharacter.

285. Lunkenheimer Co., The, Cincinnati, Ohio. (Valves, pipe fittings and other metal specialties.)

Research staff: George K. Elliott and 7 assistants.

Research work: Two-fifths time of 8 on metallurgical problems and corrosion. Generation and handling of saturated and super-heated steam; application of arc electric-furnace to production of malleable cast iron, special gray irons, and other high-carbon iron alloys.

286. Lyster Chemical Company, Inc., 61 Broadway, New York, N. Y. Laboratory at Passaic Junction, N. J.

Research staff: William R. Lamar and 2 chemists.

Research work: Full time of 3 on utilization of former waste products in the rectification of wood tar oils for creosote and guaiacol; organic compounds and photographic developers and perfumery chemicals.

287. Maas, A. R., Chemical Company, 308 E. 8th St., Los Angeles, Calif.

Research staff: Arthur R. Maas, 3 analytical chemists, 1 research chemist and 1 chemical engineer.

Research work: Manufacture of sulphites and other products, chiefly those derived from alkali and sulfur dioxide.

Equipment: Absorption towers.

288. **MacAndrews & Forbes Company,** 3d St. and Jefferson Ave., Camden, N. J. (Licorice extract, natural dyestuffs, wallboard and Foamite fire extinguishers.)

Research staff: Percy A. Houseman, 6 chemists and 3 helpers.

Research work: Approximately one-half time of 7 on constituents of licorice root and extract and development of Foamite fire extinguishers.

Equipment: Copper extractors, percolators and vacuum pans of laboratory size and semi-commercial size.

289. **Mallinckrodt Chemical Works,** St. Louis, Mo. (Chemicals for medicinal, photographic, analytical and technical purposes.)

Research staff: W. N. Stull, 22 chemists, 2 chemical engineers and 1 safety engineer.

Research work: Full time of 5, one-half time of 4 and part time of others on improvement in processes of manufacture and methods of analysis.

290. **Manhattan Rubber Mfg. Co., The,** Passaic, N. J. (Mechanical rubber goods.)

Research staff: W. L. Sturtevant, 6 chemists and 6 laboratory assistants.

Research work: One-fourth time of 13 on rubber compounding and vulcanization.

291. **Martin, Glen L., Company, The,** 16800 St. Clair Ave., Cleveland, Ohio. (Builders of airplanes.)

Research staff: Lessiter C. Milburn, 1 metallurgical engineer and 1 chemist.

Research work: One-third time of 3 on new aircraft materials and check of aircraft designs, aircraft performance tests, and general aircraft development, metal construction, etc.

Equipment: Rib testing machine (transverse loading distributed according to any pre-determined ratio). Combined pendulum tension machine and impact test machine, with interchangeable hammers (pendulums) and two ranges of capacity (200 and 1000 pounds).

292. **Martinez Refinery, Shell Co. of California,** Martinez, Calif.

Research staff: A. W. Jurrissen and 2 chemists.

Research work: Varying portion time of 3 on treatment and production of petroleum products.

Equipment: Large scale cracking apparatus and treating plant.

Marvin-Davis Laboratories, Incorporated. See National Biscuit Company (p. 55).

293. **Mathieson Alkali Works (Inc.), The,** Niagara Falls, N. Y.

Research staff: R. E. Gegenheimer, 7 chemists and 4 assistants.

Research work: Full time of 6 on new process development and investigation of problems of electrolytic chlorin and caustic plant operation.

294. **May Chemical Works,** 204 Niagara St., Newark, N. J.

Research staff: Otto B. May and 2 assistants.

Research work: One-half time of 3 on azo-dyes and intermediates.

295. **Maynard, T. Poole,** Atlanta, Ga. (Geological and industrial engineering.)

Research staff: T. Poole Maynard, 1 chemical engineer, 1 mining engineer and 1 civil engineer.

Research work: One-third time on clays, bauxites, fullers earth, refractories, textiles, oil-cloth; recovery of potash from silicates, etc.

296. **M. B. Chemical Co., Inc.,** Johnson City, Tenn.

Research staff: A. J. Buchanan and 2 chemists.

Research work: Large part time of 1 chemist on dyes and intermediates.

297. **McIlhiney, Parker C.,** 50 E. 41st St., New York, N. Y.

Research staff: Parker C. McIlhiney and 2 chemists.

Research work: One-half time of 3 on investigation of paints and varnishes, hydrogenation processes, electrolytic processes, wood distillation processes, shellac and other resins and fats and oils.

298. **McKesson & Robbins, Incorporated,** 55 Berry St., Brooklyn, N. Y. (Drugs and chemicals.) Laboratory at 97 Fulton St., New York, N. Y.

Research staff: E. H. Gane and 2 pharmaceutical chemists.

Research work: Approximately one-half time of 3 on active principles of vegetable drugs, new medicinal compounds and drug standards.

299. **McLaughlin Gormley King Co.,** 1715 Fifth St., S. E., Minneapolis, Minn. (Drugs and herbs.)

Research staff: C. B. Gnadinger and 2 chemists.

Research work: Approximately one-half time of 2 on food products, crude drugs and insecticides.

300. **McNab & Harlin Manufacturing Co.,** 55 John St., New York, N. Y. (Valves, fittings, etc.) Laboratory at 440 Straight St., Paterson, N. J.

Research staff: Ernest G. Jarvis, 1 assistant, 5 chemists, 6 metallurgists and 8 engineers.

Research work: Approximately one-half time of 21 on rare metals and their uses in industrial alloys.

Equipment: Electric laboratory melting furnaces, Hoskins type F. C. 106, miniature rolling mills and all necessary physical testing machines and equipment for testing sheets, rods, wire and castings, and fully equipped metallographic department.

301. **Meigs, Bassett & Slaughter, Inc.,** 210 S. 13th St., Philadelphia, Pa. (Chemical engineers.) Laboratory at Bala, Pa.

Research staff: Harry P. Bassett, 1 chemical engineer and 3 chemists.

Research work: Full time of 5 on paper, paper pulp, plastics, cellulose products, alkali and alkali salts.

302. **Merck & Co.,** 45 Park Place, New York, N. Y. (Chemists.)

Research staff: 2 trained chemists.

Research work: Full time of 2 on problems incident to manufacture of the company's products.

Equipment: Standard equipment for research in connection with manufacture of medicinal, analytical, photographic and technical chemicals.

303. **Merrell, Wm. S., Company, The,** 5th, Pike and Butler Sts., Cincinnati, Ohio. (Manufacturing pharmacists.)

Research staff: 7 chemists, 1 chemical engineer and 4 pharmacists.

Research work: Approximately full time of 3 and part time of 2 on problems of manufacturing pharmaceuticals and pharmaceutical products.

304. **Merrell-Soule Laboratory,** Syracuse, N. Y.

Research staff: R. S. Fleming, 2 chemists and 1 assistant. An engineering department which does much work which might be classified as research.

Research work: Half time of 3 on food problems.

Equipment: Experimental drying plant.

305. **Merrimac Chemical Company,** North Woburn, Mass.

Research staff: Lester A. Pratt and 9 chemists.

Research work: Full time of staff on inorganic and organic research problems.

Equipment: Industrial laboratory for carrying on large scale experiments.

306. **Mesabi Iron Company,** Babbitt, Minn.

Research staff: W. G. Swart, 3 engineers, 2 metallurgists and 1 chemist.

Research work: One-half time of 7 on magnetic separation of ores and sintering.

Equipment: Magnetic cobbers, classifiers and log washers and demagnetizers.

307. **Metakloth Co.,** N. Y., Susq. & Western R. R. and Garibaldi Ave., Lodi, N. J.

Research staff: Herbert B. Fenn.

Research work: Part time of 1 on mildewproofing, fireproofing and waterproofing of cotton, flax and jute fabrics.

Equipment: Apparatus for processing materials under conditions of actual commercial production.

308. **Metals & Chemicals Extraction Corporation,** 1014 Hobart Bldg., San Francisco, Calif. (Heavy chemicals.)

Research staff: L. H. Duschak and 1 chemical engineer.

Research work: Inorganic chemistry, including the manufacture of heavy chemicals, potash, borax, barium compounds and acids.

309. **Metz, H. A., Laboratories, Inc.,** 122 Hudson St., New York, N. Y. Plant and laboratories, 642 Pacific St., Brooklyn, N. Y.

Research staff: A. E. Sherndal, C. N. Myers, C. W. Hooper, G. P. Metz and 4 chemists.

Research work: Studies of chemical, pharmaceutical and medicinal products; technical problems involved in their manufacture; pathological, biological and bacteriological investigations relative to their use.

310. **Meyer, Theodore,** 213 S. 10th St., Philadelphia, Pa.

Research staff: John K. Montgomery and 2 assistants.

Research work: One-fourth time of 3 on antiseptics and insecticides.

Midvale Steel Company, The. See Midvale Steel and Ordnance Company.

311. **Midvale Steel and Ordnance Company,** Nicetown Works, Philadelphia, Pa.

Research staff: A. H. Miller and 7 men.

Research work: One-half time of 8 on investigation of characteristics of iron alloys, such as equilibrium diagrams, physical and magnetic qualities, etc.; also the investigation of new alloys of steel for use in high service purposes.

Equipment: Apparatus for several methods of obtaining critical temperatures, shock testing machines of Charpy and Izod types, Brinell and Shore hardness testing apparatus, magnetic testing apparatus of Koepsel and Burrows and experimental heat-treatment furnaces of both gas and electric types.

312. **Miller Rubber Co., The,** Akron, Ohio. (Tires and other rubber goods.)

Research staff: H. A. Morton and 3 chemists.

Research work: Full time of 4 on rubber and organic chemistry.

Equipment: Scott fabric tester, Curtis & Marbel fabric inspecting apparatus, tire testing apparatus, etc.; compounding laboratory mill and calendar, experimental press, etc.

313. **Milliken, John T., and Co.,** 217 Cedar St., St. Louis, Mo. (Medicines and pharmaceutical products.)

Research staff: Edsel A. Ruddiman and 2 assistants.

Research work: Part time of 3 on medicinal agents.

314. **Milwaukee Coke & Gas Company, The,** 1st National Bank Building, Milwaukee, Wis.

Research staff: George H. Selke and a number of chemists.

Research work: Full time of 1 to increase efficiency of by-product coke plant; includes heating of ovens, and recovery of light oil, ammonia, gas, etc.

315. **Mineral Refining & Chemical Corporation,** Carondelet Station, St. Louis, Mo. (Dry paint pigments.)

Research staff: B. B. McHan and 5 assistants.

Research work: Approximately one-fourth time of 6 on zinc and cadmium hydrometallurgy in its relation to pigment manufacture, and the separation and recovery of the impurities; also barium compounds.

316. **Miner Laboratories, The,** 9 S. Clinton St., Chicago, Ill. (Consulting chemists; pharmaceutical and food problems.)

Research staff: C. S. Miner, 9 chemists and 2 analysts.

Research work: Full time of 4 chemists and part time of 3 chemists on utilization of oat hulls; cause of rancidity of vegetable oils; precooked cereals; yeast manufacture; dehydration of potatoes; also many research problems are handled as a part of consulting service. Supervision of research in molded insulation.

Equipment: Small scale cereal manufacturing equipment.

317. **Minneapolis Steel and Machinery Co.,** 2854 Minnehaha Ave., Minneapolis, Minn. (Tractors, threshers, structural steel work, engines, hoists, etc.)

Research staff: C. S. Moody, 2 engineers and 1 assistant engineer, 3 chemists and 2 assistant chemists; A. W. Scarratt, automotive engineer, 1 engineer and 3 assistants.

Research work: One-fourth time of 14 on materials and construction.

Equipment: Izod impact testing machine, 100,000-pound automatic autographic Olsen testing machine, Brinell hardness machine, small electric furnace for temperature up to 1800 degreee F., Leeds and Northrup potentiometer, Leeds and Northrup optical pyrometer, metallographical equipment and Riehle testing machines, Sprague dynamometer 100 H. P. at 500 R. P. M.

318. **Mojonnier Bros. Co.,** 739 W. Jackson Boulevard, Chicago, Ill. (Scientific dairy apparatus and supplies; milk testing.)

Research staff: Timothy Mojonnier and J. J. Mojonnier, 1 analyst, 3 chemists and 2 chemists and bacteriologists.

Research work: One-tenth time of 8 on scientific control of milk and milk products, particularly in evaporated and condensed plants, ice-cream plants and large dairies. Effect of preservatives on composite milk samples; culture, propagation, etc.

Equipment: Mojonnier Model D Milk Tester, containing rapid cooling desiccators; the Mojonnier Model E Culture Controller for the continual propagation and control of pure lactic cultures; sediment tester, acidity and salt tester.

319. **Monroe Drug Company,** Color Chemical Division, Bottom Road, Quincy, Ill.

Research staff: H. E. Kiefer and 4 assistants.

Research work: Approximately one-fourth time of 5 on direct union colors and intermediates used in their manufacture.

320. **Monsanto Chemical Works,** 1800 South 2nd St., St. Louis, Mo. (Fine and medicinal chemicals, dye intermediates, sulphuric and other technical acids, phenol and other heavy chemicals.)

Research staff: Jules Bebie, 30 chemists, 4 engineers and 1 safety engineer.

Research work: Full time of 5 or 6 chemists on subjects related to synthetic pharmaceuticals and fine chemicals, including intermediates.

Equipment: Semi-commercial scale experimental laboratory.

321. **Morrill, Geo. H., Co.,** Norwood, Mass. (Printing and lithographic inks.)

Research staff: Olney P. Anthony and 3 chemists.

Research work: Full time of 4 on ink research.

Equipment: Dye experimental apparatus.

322. **Morris & Company,** Union Stock Yards, Chicago, Ill. (Packers and provisioners.)

Research staff: J. J. Vollertsen, 3 chemical engineers, 1 chemist and 1 bacteriologist.

Research work: Full time of 6 on industrial investigations of packing house problems and by-products.

323. **Mulford, H. K., Company,** Biological Laboratories, Glenolden, Pa. (Manufacturing and biological chemists.)

Research staff: John Reichel and 9 persons; in addition, dozens of staff and laboratory assistants engage in some research.

Research work: One-third time of 10 and part time of laboratory staff on problems connected with pharmacology, bacteriology, immunology and serology.

Equipment: Specially equipped for dealing with problems relating to pharmaceutical, biological, biological agricultural work and chemistry of soil, and for bacteriological and serological work.

324. **Munn, W. Faitoute,** 518 Main St., E. Orange, N. J.

Research staff: W. Faitoute Munn.

Research work: Nine-tenths time of 1 on electric furnace, color photography and industrial lines in general.

325. **Munning, A. P., & Co.,** Matawan, N. J. (Electroplating and buffing apparatus and supplies.)

Research staff: G. A. Cheney, 1 chemist and 1 consulting mechanical and electrical engineer.

Research work: Approximately one-half time of 1 on problems in connection with the electroplating of metals with the removal of grease and dirt from metal surfaces, the polishing of various surfaces and the compounds required for such polishes.

Equipment: Complete apparatus for electroplating.

326. **Musher and Company, Incorporated,** Baltimore, Md. Formerly The Pompeian Co.

Research staff: Louis M. Roeg and 2 assistant chemists.

Research work: Full time of 3 along general lines of food products with special attention to expression, care and utilization of vegetable oils.

Equipment: Small scale food manufacturing operations, such as expression and filtration of oils.

327. **National Aniline & Chemical Company, Incorporated,** 21 Burling Slip, New York, N. Y. Research laboratories at Buffalo and Marcus Hook, Pa. Dye laboratories at Buffalo and at various sales branches.

Research staff: G. C. Bailey and 9 chemists at Marcus Hook. Varying number of chemists, engineers and other technical men at other laboratories.

Research work: Almost entirely on dyes and intermediates.

Equipment: Semi-commercial scale equipment for testing processes before putting them on a manufacturing basis.

328. **National Association of Corrugated and Fibre Box Manufacturers, The,** 1821 Republic Building, Chicago, Ill.

Research staff: Fred D. Wilson and 1 assistant.

Research work: Full time of 1 on designing and testing corrugated and solid fibre containers to develop the best container for the commodity experimented with.

Equipment: Revolving testing drum for fibre boxes.

329. **National Biscuit Company,** 409 W. Fifteenth St., New York, N. Y. Formerly Marvin-Davis Laboratories, Incorporated.

Research staff: Clarke E. Davis, 4 chemists, 1 engineer, 1 baker and 1 assistant.

Research work: Full time of 8 on food products, their packing and distribution.

National Board of Fire Underwriters. See Underwriters' Laboratories (p. 77).

330. **National Canners Association,** 1739 H St. N. W., Washington, D. C.

Research staff: W. D. Bigelow, 4 chemists and 3 bacteriologists.

Research work: Full time of 1 and part time of 1 on study of tin plate from all standpoints; causes of pinholing in tin cans; influence of composition and details of manufacture of steel on service value of tin plate. Full time of 1 on study of heat penetration of canned food; study of various factors affecting penetration of heat to the center of the can; distribution of heat in sterilizing kettles in different systems of management. Full time of 3 on study of microorganisms causing spoilage; isolation of spoilage bacteria and study of their cultural characteristics with special reference to thermal death point; study of habitat of spoilage organisms in canning plants and farms where raw products are grown. Full time of 1 and part time of 1 on study of minor miscellaneous technological qeustions arising from time to time. Some of the most effective work has been done in collaboration with other organizations. For instance, the tin plate investigations are conducted in collaboration with manufacturers of steel, tin plate, and cans.

Equipment: Special canning equipment with laboratory facilities. Experimental small factory scale cannery and canning laboratory.

National Carbon Company. See Union Carbide and Carbon Research Laboratories, Inc. (p. 78).

331. **National Cash Register Company, The,** Dayton, Ohio.

Research staff: A. B. Beaver, 12 chemists, 3 electrical engineers, 6 mechanical engineers and 2 metallurgists.

Research work: Full time of 8 and approximately one-tenth time of others on chemical, mechanical, electrical, metallurgical and manufacturing problems.

Equipment: Special equipment for conducting endurance tests on cash registers.

332. **National Cereal Products Laboratories,** 1731 H St. N. W., Washington, D. C. (Chemical and technical advisors for The National Macaroni Manufacturers' Association and The Alimentary Paste Manufacturers' Association.)

Research staff: B. R. Jacobs and 1 chemist.

Research work: One-fourth time of 2 on standardization of cereal products and raw materials entering into their composition, methods of control in purchasing raw materials and containers for cereal products.

333. **National Gum & Mica Co.,** 12 West End Ave., New York, N. Y.

Research staff: S. Ginsburg, chemist, A. A. Haldenstein, chemical engineer, and 3 assistants.

Research work: Four-fifths time of 5 on adhesives, colloids, gums, starches, colors, sizings, finishings, etc., for paper and textiles.

334. **National Laboratories, The,** 1313 H St. N. W., Washington, D. C.

Research staff: Ivan S. Hocker, 2 chemical engineers, 1 mechanical engineer, 2 chemists, and 1 bacteriologist.

Research work: Gelatine, bacteriological dyes, by-products in acid industries, yeast and fermentation problems, malt extracts and bread improvers, glass, flotation oils and paints, cellulose and paper.

335-339. **National Lamp Works of General Electric Company,** Nela Park, Cleveland, Ohio. Research Department: Edward P. Hyde, director of research, Francis E. Cady, manager, and 5 others. Instrument shop, power plant, lamp shop and library. Renders service to other research and development laboratories.

335. *Nela Research Laboratories*

Laboratory of Pure Science

Research staff: Directorship vacant; 3 physicists, 1 physical-chemist, 1 psychologist, 2 biologists, 5 laboratory assistants, 1 student on Brush Fellowship.

Research work: Full time of 13 on the physics, physiology, and psychology of light, particularly in those phases which pertain to the science of illumination; the production of luminous energy; the laws of radiation; and the effects of luminous and attendant radiation, particularly in connection with its physiological, psychological, biological, and chemical action. Records of researches are presented before scientific and technical societies and are published as contributions to the technical journals.

Laboratory of Applied Science

Research staff: M. Luckiesh, 3 physicists, 1 engineer-physicist, 2 assistant physicists, 1 architect-engineer, 1 architect-designer, 1 lighting assistant, 4 laboratory assistants, 2 clerical workers.

Research work: Full time of 14 on spectrum analysis; light-production; spectrophotometry; photometry; various physical properties and measurements pertaining to glass, metals, etc.; physical, biological, physiological, photo-chemical, and psychological aspects of light utilization; various phases of color.

336. *Lamp Development Laboratory*

Research staff: J. E. Randall, consulting engineer, W. L. Enfield, manager, and 19 men.

Research work: Full time of 21 on development of processes of manufacture of incandescent lamps; investigation of quality of product; design of lamps; development of new types of lamps; investigations of raw materials for use in manufacture of lamps; development work on tungsten wire.

Equipment: Special equipment for use in lamp manufacture built by National Lamp Works shop.

337. *Experimental Engineering Laboratory*

Research staff: Frank M. Dorsey and 33 assistants.

Research work: One-half time of 34 on a variety of problems.

Equipment: Adequate facilities for large-scale experiments, whether on lamp making or chemical and metallurgical processes.

338. *Glass Technology Department*

Research staff: Wm. M. Clark, 7 technical men and 3 experienced practical glassmen.

Research work: One-half time of 4 on development work on glass parts used in connection with the manufacture of incandescent lamps.

Equipment: High temperature furnace equipment both gas and electrically heated. Physical and optical apparatus for determining the physical and optical properties of different glasses.

339. *Engineering Department*

Research staff: S. E. Doane, chief engineer, and 57 electrical engineering graduates.

Research work: One-half time of 15 on determining performance and characteristic data on incandescent lamps and lamp accessories; study of economics of light production; study of methods of light utilization from standpoint of obtaining most satisfactory illumination results.

340. **National Lead Company,** 129 York Street, Brooklyn, N. Y.

Research staff: Gustave W. Thompson, 3 assistants, 7 special investigators and analysts, 2 paint experts, 1 colorist, and necessary assistants.

Research work: Large part of time of 34 on investigations connected with manufacture and utilization of lead products (white lead, lead oxides, alloys, etc.), other paint pigments, linseed oil and other paint vehicles, paint technology, metallurgy of lead and of tin, physical testing and metallography of white metal alloys, microphotography, etc.

Equipment: Apparatus for testing of pigments, oils and metals, including special apparatus for measuring whiteness of pigments; opacity of paint films; fineness of pigments by classification; Howland color photometer; tension and hardness testing machines.

341. **National Lime Association,** 918 G St. N. W., Washington, D. C.

Research staff: M. E. Holmes, E. O. Fippin and 2 assistants. In addition to the resident staff, there are 5 others in university and government laboratories working on fellowships.

Research work: Full time of 3 on properties and uses of lime in the chemical, agricultural and construction fields.

342. **National Malleable Castings Company, The,** 10600 Quincey Ave., Cleveland, Ohio.

Research staff: H. A. Schwartz, 3 metallographers and chemists, 1 physicist, 1 tester of materials and 3 assistants.

Research work: Full time of 9 on properties of ferrous alloys, especially fatigue, alternating and impact stresses and resistance to cutting; equilibrium conditions in non-carbon alloys, particularly in stable system; miscellaneous metallurgical investigations.

Equipment: One 50,000-pound for 6-foot specimens, and one 200,000-pound Olsen 3-screw testing machine; 60,000-inch-pound Olsen torsion machine; Olsen universal efficiency testing machine; Charpy impact machine; Brinell machine, scleroscope; inverted type Bausch & Lomb metallographic microscope; automatic and autographic apparatus for precision heat treatment of metals.

National Stain and Reagent Co. See Coleman & Bell Company, The (p. 20).

343. **National Tube Company,** Frick Building, Pittsburgh, Pa. (Steel and iron tubes and pipes.)

Research staff: F. N. Speller and 6 to 8 men.

Research work: Full time of staff on metallurgical and chemical research work as applied to mill operations and various uses of tubular material by consumers. Considerable portion of time devoted to the problem of corrosion and protection of iron and steel from corrosion.

344. **Naugatuck Chemical Company, The,** Naugatuck, Conn.

Research staff: H. S. Adams, 3 chemists and 5 assistants.

Research work: Full time of 9 on chemicals pertaining to the rubber industry.

Nela Research Laboratories. See National Lamp Works of General Electric Company (p. 56).

345. **Nestlé's Food Company, Incorporated,** 130 William St., New York, N. Y. (Condensed milk.) Laboratory also at Ithaca, N. Y.

Research staff: A. A. Scott, 1 bacteriologist and micologist and 1 assistant; 2 chemists and 2 assistants. F. E. Rice and 1 assistant at Ithaca laboratory.

Research work: Full time of 3 on sweetened condensed and evaporated milk and other products that the company produces or may produce.

Equipment: Experimental equipment for production of condensed and evaporated milk.

346. **Newark Industrial Laboratories,** 96 Academy St., Newark, N. J. (Conduct researches on an experimental as well as on a semi-commercial scale.)

Research staff: Hubert Grunenberg and 3 assistant collegiate chemists.

Research work: Development of synthetic flavoring matters, perfumes, drugs, and dyes.

347. **New England Confectionery Company,** 253 Summer St., Boston, Mass.

Research staff: Edmund Clark and 1 chemist.

Research work: Nine-tenths time of 2 on problems connected with the industry.

348. **New Jersey Zinc Company, The,** 160 Front St., New York, N. Y.

Research staff: J. A. Singmaster, manager of technical department, F. G. Breyer, chief research division, 14 chemists, 8 physicists and 12 assistants.

Research work: Full time of 34 on mechanical and physical investigations connected wth metallurgy of zinc; manufacture and use of zinc oxide in rubber and paint industries; manufacture and utilization of sulphuric acid; production and properties of worked metallic zinc in shapes of strips, sheets, etc.

349. **Newport Company, The,** Pensacola, Fla.

Research staff: R. C. Palmer and 2 assistants.

Research work: Whole time of 1 and one-quarter time of 2 on problems relating to the technical and industrial development of terpenes and terpene products, rosins and rosin products.

Newport Turpentine & Rosin Company of Florida. See Newport Company, The.

350. **New York Quebracho Extract Company, Incorporated,** 80 Maiden Lane, New York, N. Y.

Laboratory at Greene and West Sts., Greenpoint, Brooklyn, N. Y.

Research staff: R. O. Phillips and 4 chemists.

Research work: One-half time of 5 on tannery operation, extract manufacture and various problems in connection with the manufacture and testing of leather.

Equipment: Experimental tannery.

351. **New York Quinine & Chemical Works, Incorporated, The,** 135 William St., New York, N. Y.

Research staff: George L. Schaefer, 7 chemists and 2 engineers.

Research work: Approximately one-half time of 7 chemists on organic products, alkaloids, and medicinal chemicals.

352. **New York Sugar Trade Laboratory, Inc., The,** 79 Wall St., New York, N. Y.

Research staff: C. A. Browne, 5 chemists and 1 helper.

Research work: One-fourth time of 7 on composition and deterioration of sugars; optical and chemical methods of sugar analysis; influence of temperature and other conditions on polarization of sugars; composition and food value of syrups and molasses.

Equipment: Constant temperature laboratory for polarization of sugars.

353. **Niles Tool Works Company, The,** 545 North Third St., Hamilton, Ohio. (Machine tools.)

Research staff: J. W. Bolton, 1 experimental engineer, 2 routine men and labor as desired.

Research work: One-fourth to three-fourths time of 4 on metallurgy of grey iron, especially practical applications of metallography, studies of changes produced by pouring temperatures, section size, etc. Heat treatment, brass and bronze, core oils, etc.

Equipment: Completely equipped laboratory for study of grey iron.

354. **Northwestern Chemical Co., The,** Marietta, Ohio. (Chemical automobile utilities.)

Research staff: A. S. Isaacs and 2 advisors.

Research work: One-half time of 1 on problems incident to automobile trade and news ink trade; cements, polishes, dressings and enamels, printers' ink, oil and carbon black.

355. **Norvell Chemical Corporation, The,** 11 Cliff St., New York, N. Y.

Research staff: 4 chemists.

Research work: One-fourth time of 4 on mercurial products, phosphates, benzoate group, wood distillation derivatives, formaldehyde condensation products, citric and oxalic acid derivatives, aniline derivatives, phosgene condensation products and other pharmaceutical and technical products.

356. **Nowak Chemical Laboratories,** 518 Chemical Building, St. Louis, Mo.

Research staff: C. A. Nowak.

Research work: On flavoring extracts used in soft drink manufacture.

Equipment: Well equipped for brewery and other beverage and food work.

357. **Nulomoline Company, The,** 111 Wall St., New York, N. Y. (Glycerine substitutes.)

Research staff: M. A. Schneller, 1 chemist, 1 confectionery engineer and 1 laboratory assistant.

Research work: Approximately one-half time of 3 on sugar and sugar products.

358. **Ohio Fuel Supply Company, The,** 99 N. Front St., Columbus, Ohio. Laboratory at Utica, Ohio.

Research staff: George T. Koch, 2 chemists, 2 chemical engineers and 2 routine men.

Research work: Approximately three-fourths time of 5 on petroleum, natural gas, gasoline, particularly the manufacture of synthetic chemicals, such as amyl acetate, formaldehyde, formic acid, etc., from the above natural products and absorption processes for gasoline.

359. **Ohio Grease Co., The,** Londonville, Ohio. (Lubricants.)

Research staff: 1 chemist.

Research work: Analysis of oils, fats and greases, such as are required in a grease factory.

360. **Oliver Continuous Filter Co.,** 503 Market St., San Francisco, Calif. Laboratories also at 226 E. 41st St., New York, N. Y., and No. 9 Red Lion Passage, Holborn, London, W. C. I., England.

Research staff: E. L. Oliver in San Francisco, R. Gordon Walker in New York and J. F. Mitchell-Roberts in London, with 3 engineers and 1 chemist available for each laboratory.

Research work: Investigation of methods for increasing efficiency and reducing costs of filtration of all classes of chemical and metallurgical products. No work done on drinking water filtration. Principal products investigated are beet and cane sugar juices and saccharate of lime; lime sludges; wood pulp; sewage; phosphoric acid; cyanide slimes; flotation concentrate; clays of all kinds; dyes, etc.

Equipment: Continuous vacuum filters, small intermittent vacuum filters, various devices for treating filter "cake" during the filter cycle to reduce moisture or increase washing efficiency.

Orford Copper Co., The. See International Nickel Company, The (p. 44).

361. **Package Paper and Supply Corporation,** 150 Birnie Ave., Springfield, Mass. (Waxed papers.)

Research staff: W. M. Bovard, 2 chemists, 1 engineer and 1 assistant.

Research work: Approximately three-tenths time of 5 on wrapping food products, especially for moisture protection, specializing on waxed paper for automatic wrapping machine for wrapping soap, cereals, food products and candy and developing special papers.

362. **Packard Motor Car Company,** Detroit, Mich. Engineering laboratory.

Research staff: L. M. Woolson, 3 engineers and 1 chemist.

Research work: Full time of 5 on problems connected with Liberty motor, motor trucks and automobiles; automobile and truck chassis development.

Equipment: Complete dynamometer equipment for testing truck, car and airplane engines up to 500 H. P. Complete bench testing equipment for all car, truck and airplane accessories. Automotive power plant and accessories.

Page, Carl M. See Riverbank Laboratories (p. 68).

363. **Palatine Aniline and Chemical Corporation,** 81 N. Water St., Poughkeepsie, N. Y. (Dyestuffs and chemicals.)

Research staff: Felix Braude and 2 chemists.

Research work: Full time of 3 on intermediates and dyestuffs.

364. **Palmolive Company, The,** Milwaukee, Wis.
Research staff: V. K. Cassady and 7 assistants.
Research work: Full time of 1 and approximately one-fourth time of 6 on soaps and perfumes.

365. **Pantasote Leather Company, The,** Passaic, N. J.
Research staff: Edgar Josephson.
Research work: Full time of 1 on coatings for textiles, rubber coatings for fabrics, oils, paints, varnishes and all closely related industries.

366. **Parke, Davis & Company,** Detroit, Mich. (Medicinal preparations.)
Research staff: J. M. Francis, chief chemist, Oliver Kamm, chief of chemical research department, E. M. Houghton, chief of medical research department and about 40 chemists, pharmacists, bacteriologists, botanists and pharmacologists.
Research work: Large part time of about 20 is devoted to the improvement in the constitution, or processes of manufacture, of substances now used as medicaments; and in the attempt to discover or produce new therapeutic agents in both pharmaceutical and biologic lines.

Patton Paint Company. See Pittsburgh Plate Glass Co. (p. 65).

367. **Pease Laboratories,** 39 West 38th St., New York, N. Y. (Successors to Lederle Laboratories.)
Research staff: H. D. Pease and a number of chemists, bacteriologists and assistants.
Research work: Small part time of staff along sanitary, chemical and bacteriological lines.

368. **Peerless Color Company,** Bound Brook, N. J.
Research staff: R. W. Cornelison and 2 chemists.
Research work: Part time of 3 on problems dealing directly with the manufacture of dyestuffs.

369. **Peerless Drawn Steel Company, The,** Massillon, Ohio.
Research staff: A. M. LeTellier and 4 assistants.
Research work: Approximately one-half time of 5 on effect of heat treating and cold drawing on all grades of steel and development of the cold drawing of steel.
Equipment: Apparatus for studying chemical and physical properties of steel, including full heat treating department as well as metallography department.

369a. **Peet Bros. Mfg. Co.,** Kansas City, Kans.
Research staff: W. J. Reese and 2 assistants.
Research work: Problems connected with the manufacture of soaps and glycerin.

370. **Penick & Ford, Ltd., Incorporated,** New Orleans, La. (Sugar, cane and corn products.) Laboratory at Marrero, La.
Research staff: F. W. Zerban, 1 chemist, 1 assistant chemist and assistants.
Research work: Full time of 3 or more on manufacture and refining of the products of sugar cane, corn and other saccharine plants.

371. **Pennsylvania Railroad Company, The,** Altoona, Pa.
Research work: Small part time of staff on investigation of cause of failure of steel rails; locomotive design; much work in preparation of specifications for various materials; general field of lubrication;

water treatment and purification; paints and preservatives; heat treatment of metals, etc. Investigation of electrolysis in systems of underground metallic structures; tests and investigations of the construction of various makes of transformers; tests of various makes of primary and secondary battery cells; oscillographic tests for linear and angular velocity, wave forms, etc.; investigations of special cases of electrical troubles; development of an electrical method of measuring the hardness and homogeneity of steel. Tests of locomotives on the road or tests of equipment with special devices; tonnage rating of trains and following up of all experimental appliances which are put into service for test purposes. Methods for determination of elements in plain-carbon steels, alloy steels and non-ferrous alloys used for bearing backs and linings, packing-ring metal for different purposes, etc. Examination of fuels, development of specifications for paint products, lubricating and burning oils, boiler compounds, lacquers, plush, car cleaners, cutting compounds, belt dressing, polishing compounds, hydraulic-jack liquids, fuses, track caps, fire-extinguishing preparations, the recovery of used or wasted products, etc.

Equipment: Six universal tension and compression testing machines, one of 1,000,000, two of 300,000, two of 100,000-pound and one of 150,000-pound capacity; one vibratory endurance spring testing machine of 75,000-pound capacity; one 43-foot and one 57-foot drop-testing machine; two vibrating staybolt testing machines; one Brinell hardness testing machine; one 2000-pound cement testing machine; metallographic equipment.

Apparatus for testing hose: Six rubber stretching machines; one friction test rack for rubber; one hose mounting machine; one vibrating test rack for hose; one continuous test rack for rubber; four tension testing machines for rubber; one stretching machine for rubber insulation; one spring micrometer machine; one vacuum gage testing machine; one arbor press specimen cutter; one hydraulic gage testing machine, capacity 25,000 pounds per square inch; one dead-weight gage testing machine, capacity six gages; one wiggling testing machine for hose; one bumping testing machine for gages; one whipping testing machine for gages; one hydraulic machine for testing gage glasses.

Rubber, air-brake hose and miscellaneous laboratory, machines for air-brake, signal and tank hose, and other miscellaneous tests.

Electrical laboratory, equipment for lamp tests consisting of three photometers, lamp test rack of 1000 lamps capacity, with switchboard, transformers and potential regulator equipment.

372. **Pennsylvania Salt Manufacturing Co.,** Philadelphia, Pa.

Research staff: Director, chief chemist and 3 assistant chemists.

Research work: Problems relating to the manufacture of heavy chemicals.

373. **Permutit Company, The,** 440 Fourth Ave., New York, N. Y. (Water rectification systems.) Factory at Brooklyn, N. Y.

Research staff: T. R. Duggan, 7 chemists and 4 chemical engineers.

Research work: Full time of 3 entirely in connection with water problems and the use and manufacture of artificial zeolites.

374. **Perolin Company of America, The,** 2010 Peoples Gas Bldg., Chicago, Ill. Laboratory at 1112 W. 37th St., Chicago, Ill.

Research staff: E. L. Gross, chemical engineer.

Research work: Protection of metal surfaces against rust and pitting and boiler scale removal and prevention.

Equipment: Beach-Russ vacuum pump and copper retorts.

375. **Pettee, Charles L. W., Laboratories of,** 112 High St., Hartford, Conn. (Analytical and consulting chemist.)

Research staff: C. L. W. Pettee and 1 chemist.

Research work: Three-twentieths time of 2 on recovery and purification of precious metals.

376. **Pfaudler Co., The,** Rochester, N. Y.

Research staff: O. I. Chormann, 1 chemist, 1 metallurgist and 1 helper.

Research work: Three-fourths time of 3 on enamels for steel and cast iron; packings; resistivity of enamels, etc.

377. **Pfister & Vogel Leather Co.,** 447 Virginia St., Milwaukee, Wis. (Tanners and curriers.)

Research staff: Louis E. Levi, 2 research chemists and 7 other chemists.

Research work: Full time of 4 on problems related to leather, glue, hair, gelatine, retarder, bitumen, paints, etc.

378. **Pfizer, Chas., & Co., Inc.,** 81 Maiden Lane, New York, N. Y. (Manufacturing chemists.) Laboratory at 11 Bartlett St., Brooklyn, N. Y.

Research staff: Richard Pasternack, 5 chemists and chemical engineers and 1 engineer.

Research work: Full time of 7 on development of processes and products.

Equipment: Complete laboratory and semi-plant equipment.

379. **Pharma-Chemical Corporation,** 1570 Woolworth Bldg., New York, N. Y. Laboratory at Bayonne, N. J.

Research staff: Eugene A. Markush, 3 chemists and 1 engineer.

Research work: Dyes and pharmaceuticals.

380. **Philadelphia Quartz Company,** Philadelphia, Pa. (Silicate of soda.)

Research staff: James G. Vail, 4 chemists and 1 assistant.

Research work: One-half time of 6 on problems involving application or manufacture of silicate of soda, study of its properties as an adhesive, as an ingredient of acid-proof cement, grinding wheels, soap, asbestos insulating material, coating materials for paper and wooden packages, to prevent the absorption of grease, as an agent in refining of vegetable oils, etc.

Equipment: Crushing and grinding apparatus, two gas-heated furnaces for experiments with fusion, one a small open hearth, and the other a crucible furnace; apparatus for fusion, testing of adhesives, cement, etc., and devices for making the usual commercial tests on paper; small and semi-commercial autoclaves.

381. **Physicians and Surgeons Laboratory,** 605 Paxton Blk., Omaha, Nebr.

Research staff: Theodore M. Agnew, 1 chemist, 1 bacteriologist and 1 pathologist and serologist.

Research work: Variable amount time of 4 on bacteriological, pathological and serological problems.

382. **Pierce-Arrow Motor Car Company, The,** Elmwood Ave., Buffalo, N. Y.

Research staff: J. Miller, metallurgist, and 2 assistants; W. Slaght, experimental engineer and 2 assistants.

Research work: Approximately one-fourth time of 8 on cause of failure of parts, effect of impurities in metals, heat treatment, effect of shocks, alternate stresses and efficiency of engines and transmissions.

Equipment: Olsen testing machine, Avery impact testing machine, Stanton impact testing machine, 150 H. P. electric dynameter and engine test stand.

Pitcairn Varnish Co. See Pittsburgh Plate Glass Co.

383-384. **Pittsburgh Plate Glass Co.,** Milwaukee, Wis. Laboratory also at Newark, N. J.

383. *Patton-Pitcairn Division* (Patton Paint Company and Pitcairn Varnish Company).

Research staff: A. H. Woltersdorf and assistants at Milwaukee; T. R. Collins and 2 assistants at Newark.

Research work: Part time of staff on problems connected with the paint and varnish industry.

384. *Corona Chemical Division* (Corona Chemical Company).

Research staff: C. B. Dickey and assistants.

385. **Pittsburgh Testing Laboratory,** 616 Grant St., Pittsburgh, Pa. Laboratories also in New York, N. Y., Birmingham, Ala., and Cincinnati, Ohio.

Research staff: Jas. O. Handy, director of special investigations, H. H. Craver, manager chemical department, 26 chemists in Pittsburgh, 2 in New York, 3 in Birmingham and 1 in Cincinnati; 3 mechanical and 3 civil engineers.

Research work: Variable amount of time of staff on food and drugs (alcohol substitutes, etc.), oil refining (lubricating oil recovery), corrosion-resisting metals, water purification, metal extraction from ores and refractory materials (basic).

Equipment: Furnaces, special metallographic equipment, coal distillation apparatus (to be installed) and testing machines.

Pompeian Co., The. See Musher and Company, Incorporated (p. 55).

386. **Porro Biological Laboratories,** 625 Puget Sound Bank Bldg., Tacoma, Wash. (Successors to Staniford Laboratories.)

Research staff: Thomas J. Porro and John G. Scott.

Research work: Part time of 2 on chemical, serological and bacteriological problems.

387. **Portage Rubber Co., The,** Barberton, Ohio.

Research staff: R. M. Gage and 2 chemists.

Research work: One-half time of 3 on testing and compounding for rubber goods.

388. **Porter, Horace C.,** 1833 Chestnut St., Philadelphia, Pa. (Consulting chemist and chemical engineer.)

Research staff: Horace C. Porter and 1 assistant.

Research work: Coal carbonization, coking and by-products, "low temperature" carbonization, shale distillation, application of fuels, reduction of wastes, coal storage problems and spontaneous combustion.

Equipment: Coal distillation retort (laboratory scale) and accessories.

389. **Powers-Weightman-Rosengarten Company, The,** 916 Parrish St., Philadelphia, Pa. (Chemists.)

Research staff: George D. Rosengarten and varying number of assistants.

Research work: Variable amount time of staff on improvement of present processes and investigation of new processes.

Prest-O-Lite Co., Inc., The. See Union Carbide and Carbon Research Laboratories, Inc. (p. 78).

390. **Procter & Gamble Co., The, Cincinnati, Ohio.** (Soaps, glycerine, candles, lard substitutes, refined oils, etc.) Laboratory at Ivorydale, Ohio.

Research staff: H. J. Morrison and 12 chemists.

Research work: Improvement of plant processes and products.

Equipment: Complete experimental plants for the various processes.

391. **Providence Gas Company, Incorporated,** Providence, R. I. Manufacturing Department.

Research staff: A. H. Meyer, 1 assistant chemist and 2 minor chemists.

Research work: Small part time of 4 on problems arising in manufacture.

Equipment: Laboratory is complete for gas plant operation.

392. **Pure Oil Company,** Kanawha River Salt and Chemical Division, Charleston, W. Va. Laboratory at Belle, W. Va.

Research staff: W. A. Borror and 1 chemist.

Research work: One-half time of 1 on salt industry, salt brine and development of processes.

393. **Pure Oil Company,** Moore Oil and Refining Company Division, York and McLean Aves., Cincinnati, Ohio.

Research staff: Frank Goodale and 2 assistants.

Research work: Full time of 3 on soaps, greases, polishes, lubricating and soluble oils; textile, boiler and cutting compounds.

394. **Pyrolectric Instrument Company,** 636 E. State St., Trenton, N. J. (Pyrometric and electrical precision instruments.)

Research staff: H. L. Saums, 1 chemist, 1 electrical engineer and 1 mechanical engineer.

Research work: Approximately one-fourth time of 4 on construction and adaptations of electrical instruments; special problems requiring combination of mechanical and electrical development; temperature measurement problems, problems in hydrogen-ion determinations.

395. **Pyro-Non Paint Co., Inc.,** 505 Driggs Ave., Brooklyn, N. Y. (Fire retarding paints and products.)

Research staff: Ernest A. Marx, 1 chemical engineer and 1 chemist.

Research work: One-half time of 3 on technical paints and paint products.

Equipment: Inflammability test apparatus.

396. **Quinn, T. H., & Company**, which includes: Lackawanna, Susquehanna, Vandalia, Tonesta Valley, Keystone, Heinemann, Barclay, Beerston Acetate Co., Smethport Chemical Companies and the Quinn Laboratories Company. General office at Olean, N. Y. Laboratory at E. Smethport, Pa.

Research staff: Edward E. Currier, 3 chemists, 1 engineer and occasional assistance from other specialists.

Research work: Approximately one-third time of 5 on researches on gases from wood, researches on the phenolic constituents of wood oils and tars, formaldehyde and physical properties of charcoals.

Equipment: Destructive distillation plant and formaldehyde plant, both on small scale.

397. **Radiant Dye & Color Works**, 2837 W. 21st St., Brooklyn, N. Y.

Research staff: William Goldstein and 1 chemist.

Research work: Full time of 2 on triphenylmethane dyes and their derivatives.

398. **Radium Company of Colorado, Inc., The**, 18th and Blake Sts., Denver, Colo.

Research staff: W. A. Schlesinger, 12 chemists and 4 engineers.

Research work: Approximately one-fifth time of 17 on radium, uranium and vanadium.

399. **Radium Limited, U. S. A.**, 2 W. 45th St., New York, N. Y. (Radium emanation activators, radium ore, apparatus, etc.)

Research staff: Henry H. Singer, 1 chemist and 2 assistants.

Research work: One-half time of 4 on radium ore, radium, radium emanation, radium luminous material and all other matters affiliated with radium and similar products.

Equipment: Electrometers, fontactoscopes, spinthariscopes, experimental and demonstration outfits and exhibition of rare earth and all kinds of luminous materials and paints.)

400. **Ransom & Randolph Co., The**, 518 Jefferson Ave., Toledo, Ohio.

Research staff: Thomas E. Moore, 1 chemist, 1 mechanical engineer and 2 dentists.

Research work: Three-fourths time of 5 on dental materials.

401. **Raritan Copper Works**, Perth Amboy, N. J. Research Department.

Research staff: S. Skowronski, 3 chemists and 1 physicist.

Research work: Full time of 5 on copper metallurgy, electrolytic refining of copper, and recovery of by-products, gold, silver, platinum, palladium, selenium, tellurium, arsenic, nickel, antimony.

402. **Redlands Fruit Products Company**, Redlands, Calif.

Research staff: H. P. D. Kingsbury and 1 chemist.

Research work: Small part time of 2 on fruit products, for example, bottling orange juice.

403. **Redmanol Chemical Products Co.**, 636 W. 22nd St., Chicago, Ill. (Acid- and heat-proof varnishes and lacquers, synthetic amber, moulding compounds; for electrical insulation and other uses.)

Research staff: L. V. Redman, A. J. Weith and F. P. Brock; 8 chemists and 6 chemical engineers.

Research work: Full time of 6 on electrical insulation from phenol, condensation products and synthetic amber-like resins.

Equipment: Vacuum apparatus, rubber mixing rolls, beater mills, kneading machines, hydraulic presses, stills, dephlegmators and high temperature kilns.

404. **Reliance Aniline & Chemical Co., Incorporated,** Poughkeepsie, N. Y.

Research staff: Philip Kaplan and 1 chemist.

Research work: One-third time of 2 along lines of synthetic dyes.

405. **Remington Arms,** Union Metallic Cartridge Company, Barnum Ave., Bridgeport, Conn. Research Division.

Research staff: 3 chemists, 3 assistant chemists, 1 metallographist, 1 assistant metallographist and pyrometer expert, 2 engineers and 7 micellaneous.

Research work: One-eighth time of 15 on small arms ammunition.

406. **Research Corporation,** 25 W. 43rd St., New York, N. Y. Laboratory at St. Pauls Ave., Jersey City, N. J.

Research work: Problems of converting a work of completed research to commercial or industrial application and use.

Equipment: Apparatus for developing the Cottrell electrical precipitation processes.

407. **Rhode Island Malleable Iron Works,** Hillsgrove, R. I.

Research staff: M. M. Marcus, 1 chemist and 1 engineer.

Research work: Part time of 3 on furnace practice and testing.

Equipment: Commercial air furnaces, annealing furnaces and core ovens.

408. **Richards & Locke,** 69 Massachusetts Ave., Cambridge 39, Mass. (Mining engineers.)

Research staff: Robert H. Richards and Charles E. Locke with from 1 to 3 or 4 engineers and chemists.

Research work: Approximately full time on commercial problems of ore concentration and allied subjects.

Equipment: Full ore testing equipment.

409. **Richardson Company, The,** Lockland, Ohio. Heppes Roofing Division and laboratory at 26th and Lake Sts., Melrose Park, Ill.

Research staff: Robert Holz and 4 chemists.

Research work: One-half time on manufacture of asphalt and roofing products.

410. **Riches, Piver & Co.,** 30 Church St., New York, N. Y. (Chemical and color manufacturers and importers.) Laboratory at Hillside, Elizabeth, N. J.

Research work: Insecticides, fungicides and the raw materials from which they may be made.

411. **Riverbank Laboratories, Geneva, Ill.** (Commercial research and experimental laboratories.)

Research staff: Carl M. Page, several chemists, physicists and other assistants.

Research work: Full time of director and part time of others on physical, chemical and metallurgical problems; rubber.

Equipment: Apparatus for work on phenomena of high-potential discharges and vacuum tubes; includes 16-plate static machine 36-inch diameter, one 18-inch and one 10-inch spark X-ray coils with electrolytic and mercury turbine interrupters, one 20,000-volt alternating current transformer with rotary converter, vacuum tube oven, assortment of special tubes, Gaede mercurial air-pump for high vacuum with a Geryk oil-pump as auxiliary. Large special arc lamps for ultra-violet rays; apparatus for work in molecular transformations of hydrocarbon oils; turbine-driven Sharpless super-centrifuge, with many accessories of own design; small shop for making special apparatus.

412. **Rochester Button Company,** 300 State St., Rochester, N. Y.

Research staff: J. F. Clark, 1 chemist, 2 engineers, 1 designer and 2 assistants.

Research work: Full time of 7 on investigation of plant processes, materials and machinery used in manufacturing buttons.

413. **Rodman Chemical Company,** Verona, Pa. (Case hardening and carbonizing compounds.)

Research staff: Hugh Rodman and 2 assistants.

Research work: Approximately full time of 3 on carburizing of steel, investigation of carbonizing agents, special coking systems, activated carbon and general research upon carbon.

414. **Roessler & Hasslacher Chemical Company, The,** Perth Amboy, N. J.

Research staff: H. R. Carveth, technical director; M. J. Brown, B. S. Lacy, Sterling Temple, E. A. Rykenboer, chief chemists; 10 research chemists with laboratory and engineering assistants.

Research work: Half time on problems connected specifically with manufacture of caustic soda; inorganic and organic chlorine compounds; formaldehyde and its compounds; precious metals used in the arts, principally platinum, gold and silver; ceramic materials, alkali metals, alkali cyanides, peroxides and persalts; metal cyanides; also problems connected with utilization of products cited above in plating; in bleaching and finishing of textiles; in enamelling, rubber accelerators, fumigation.

415. **Royster, F. S., Guano Company,** Norfolk, Va.

Research staff: E. W. Magruder and 3 chemists.

Research work: Small part time of 4 on fertilizer problems entirely, such as cause of hardening of acid phosphate, effects of different materials on each other when mixed, etc.

416. **Rubber Trade Laboratory, The,** 96 Academy St., Newark, N. J. (An advisory organization conducting researches by request in industrial establishments. Laboratory investigations are carried on at this address.)

Research staff: Frederic Dannerth and 4 collegiate chemists.

Research work: Investigations for the industries using rubber and related gums, paints, oils and varnishes. Investigations for the industries which make rubberized and water proof fabrics; coal tar products.

417. **Rumford Chemical Works,** Providence, R. I. (Baking powder,

yeast powder, bread preparation, phosphatic baking acid, acid phosphate, phosphoric acid solutions and similar products.)

Research staff: Augustus H. Fiske, 2 assistant chemists and 5 assistants.

Research work: Equivalent to two-thirds time of 1 on improvement of apparatus for manufacture of phosphoric acid and its salts; improvement of processes of manufacture and of methods of testing products in laboratory.

Equipment: Gas-measuring devices for testing baking powder and specially devised electrolytical apparatus for determination of material by electrolysis.

417a. **Sabine, Wallace Clement, Laboratory,** Riverbank, Geneva, Ill.

Research staff: Paul E. Sabine, 3 physicists and 1 mechanician.

Research work: Full time of staff on transmission and absorption of sound by standard constructions, structural materials; physical characteristics of the ear; absolute measurements in acoustics, special problems in architectural design and acoustics.

Equipment: Sound chamber, calibrated sound sources, apparatus for sound photography, telephonic and other devices for absolute sound measurements.

418. **Saginaw Salt Products Co.,** Saginaw, Mich.

Research staff: John P. Simons and 2 assistants.

Research work: Approximately one-fourth time of 3 on chemical and engineering problems in connection with evaporators, removal of impurities from salt brine, etc.

419. **Sangamo Electric Company,** Springfield, Ill.

Research staff: F. C. Holtz, 1 chemist, 3 electrical engineers, 2 assistants and 2 model makers.

Research work: One-third time of 7 on properties of magnet steels; endurance of material and precious stones used as bearings, paints, varnishes, insulations, brass and steel, development of apparatus employing new principles of operation.

420. **Schaeffer Brothers & Powell Manufacturing Company,** 189 N. Clark St., Chicago, Ill. Laboratory at 102 Barton St., St. Louis, Mo. (Soap, oils, etc.)

Research staff: B. Nichols and 3 assistants.

Research work: One-third time of 4 on vegetable, animal and mineral oil.

421. **Schwarz Laboratories,** 113 Hudson St., New York, N. Y. (Food analyses and research; applied refrigeration; testing of fuels and lubricants.)

Research staff: Robert Schwarz, 5 chemists, 1 biologist, 1 consulting mechanical engineer and 2 assistants.

Research work: One-fifth time of 10 on food and beverage problems, both chemical and biological.

Equipment: Model brewery of 120 gallons capacity.

422. **Scientific Instrument and Electrical Machine Company, The,** 500 S. York and 221 West Coover Sts., Mechanicsburg, Pa.

Research staff: W. W. Strong and 1 or 2 skilled men.

Research work: Practically full time of 3 on ionization of gases, precipitation of fumes, deblooming oil, nitrogen fixation, diamond surfaced glass, smoke and fume recorders and masks, etc.

Equipment: High voltage apparatus, gratings, ultra-violet apparatus.

423. **Scott, Ernest, & Company,** Fall River, Mass. (Engineers; apparatus for saving industrial wastes; vacuum evaporators, vacuum dryers, solvent extraction apparatus, ammonia stills, wood distillation plants.)

Research staff: H. Austin and Robert W. Macgregor, 4 chemical engineers.

Research work: One-tenth time of 6 on vacuum evaporation, vacuum distilling and solvent extraction.

424. **Scovill Manufacturing Company,** Waterbury, Conn. (All varieties of brass, bronze and German silver.)

Research staff: 3 metallurgists, 1 chief chemist and metallurgist with staff of 27 assistants; 2 mechanical engineers, 1 electrical engineer with 3 assistants, 1 plating and finishing expert with 2 assistants.

Research work: About one-tenth time of technical staff is occupied with research problems.

Equipment: Olsen 100,000-pound universal automatic and autographic testing machine, 3-screw type, motor drive, speed 0.025 inch to 6.50 inches a minute; Olsen 50,000-pound universal automatic and autographic testing machine similar to the 100,000-pound machine; Olsen 200,000-pound universal automatic testing machine; Riehle 2,000-pound testing machine, hand drive for tensile tests only; Brinell hardness testing machine. capacity 3,000 kilograms pressure; Olsen and Erichsen sheet metal testers, for ascertaining ductility; Shore scleroscope.

425. **Sears, Roebuck and Co.,** Chicago, Ill. (Diversified manufacturing and mail order business.)

Research staff: G. M. Hobbs, director testing department, C. H. Higgins, head chemical laboratory, Elizabeth Weirick, head textile laboratory, and L. E. Wolgemuth, head mechanical research laboratory; 13 chemists, physicists and engineers.

Research work: Approximately one-fourth time of staff on development of mechanical devices, methods, factory problems and the standardization of merchandise.

Semet-Solvay Company. See Solvay Process Company, The (p. 72).

426. **Seydel Manufacturing Company,** Jersey City, N. J. (Chemicals.)

Research staff: Paul Seydel and 4 to 6 assistants.

Research work: Pharmaceutical and textile chemicals.

427. **Sharp & Dohme,** Baltimore, Md. (Manufacturing chemists.)

Research staff: Herman Engelhardt, 5 research chemists, 1 pharmacologist, 1 pharmacognosist and 10 pharmaceutical chemists.

Research work: One-half time of 5 on pharmaceutical chemistry, crude drugs and synthesis of new compounds.

Skayef Ball Bearing Co. See S. K. F. Industries, Inc.

428. **S. K. F. Industries, Inc.**, New York, N. Y. Research Laboratory, Front St. and Erie Ave., Philadelphia, Pa., also serves Hess-Bright Manufacturing Co., Philadelphia, Pa., Atlas Ball Company, Philadelphia, Pa., and Skayef Ball Bearing Co., Hartford, Conn.

Research staff: Haakon Styri, 4 mechanical engineers, 1 chemist, 2 metallurgists.

Research work: Full time of staff on ball bearing application and endurance fatigue and improvement of material.

429. **Skinner, Sherman & Esselen, Incorporated**, 248 Boylston St., Boston 17, Mass. (Chemists and engineers.)

Research staff: Gustavus J. Esselen, Jr., 9 chemists, 3 engineers and 3 bacteriologists.

Research work: Approximately one-half time of 7 on paper, cellulose and its esters, food and canning industries, industrial bacteriology, adhesives and cement and building materials.

430. **Solvay Process Company, The**, and **Semet-Solvay Company**, Syracuse, N. Y. (Alkali, coke and its by-products.) Do research work also for By-Products Coke Corporation, South Chicago, Ill.

Research staff: The Solvay Process Co., Carl Sundstrom, 10 chemists, 5 chemical assistants, 5 clerks and mechanics. Semet-Solvay Co., A. C. Houghton, 22 chemists, 1 chemical engineer, 2 electro-chemical engineers and 12 chemical assistants and routine men.

Research work: Four-fifths time of 20 and one-half time of 37 on soda ash, caustic soda, bicarbonate of soda, lime and limestone, cement, waste disposal, metal corrosion, new alkali products; potash, indigo, fixation of nitrogen, coal, light oils, causticizing, oxalic acid, sulphonation of benzol, picric acid, salicylic acid, chlorination of toluol, benzaldehyde, benzoic acid, and new products, such as diphenyl oxide, benzyl acetate, benzyl benzoate, aspirin, sodium salicylate and cinnamic acid.

Equipment: Electric, steam and gas ovens and furnaces of nearly all sizes up to 2x3x3 feet, capable of any temperature range up to 1500 degrees C.; temperature measuring equipment ranging from —100 degrees C. to +1750 degrees C.; laboratory kneading and mixing machine.

431. **Souther, Henry, Engineering Co., The**, 11 Laurel St., Hartford, Conn. (Consulting engineers.)

Research staff: J. A. Newlands, F. P. Gilligan, 7 technically trained assistants and 4 others.

Research work: Part time of 6 on oils, waters and greases, ferrous and non-ferrous metals, methods of heat-treatment, electro-plating, foundry practice, boiler water treatment.

Equipment: Pyrometers, furnaces, lead pot for experimental heat treatment; 100,000-pound Olsen physical testing machine, Izod impact tester and White-Souther endurance machines; Emerson bomb calorimeter.

432. **Southern Cotton Oil Company, The**, 120 Broadway, New York, N. Y. Head laboratory at Savannah, Ga.

Research staff: Herbert S. Bailey and 6 or 7 assistants.

Research work: Problems pertaining to the vegetable oil industry such as improved methods of analyses, investigation of catalysers and their preparation, improvements in the methods of refining vegetable oils, investigating and finding new uses for by-products.

433. **Special Chemicals Company,** Highland Park, Ill.

Research staff: Carl Pfanstiehl, Robert S. Black and 3 assistants.

Research work: Rare carbohydrates, amino acids, rare organic biological chemicals and industrial specialties.

Equipment: New Bates variable sensibility half-shade polariscope; use of bacteria as "living chemical reagents."

434. **Speer Carbon Company,** St. Marys, Pa. (Motor and generator brushes.)

Research staff: M. S. May, 2 engineers, 2 chemists and 3 assistants.

Research work: Practically the entire chemical and electrical staff devoted to the development of new products and the improvement of present products.

435-436. **Spencer Lens Company,** Buffalo, N. Y. (Optical instruments, optical glass.) Laboratory also at Hamburg, N. Y., in optical glass factory.

435. *Buffalo Laboratory*

Research staff: Harry G. Ott and 7 trained assistants.

Research work: Half time of 8 on mathematical designing of lens systems; the other half on designing optical instruments and solving the problems of the manufacture of lenses and optical instruments.

436. *Hamburg Laboratory*

Research staff: Donald E. Sharp and 1 trained assistant.

Research work: Full time of 2 on optical glass and problems connected with its manufacture.

437. **Sperry, D. R., & Co.,** Batavia, Ill. (Founders and engineers; makers of filter presses and evaporators.) Sperry Filtration Laboratory.

Research staff: D. R. Sperry.

Research work: One-fourth time of 1 on systematic effort to determine fundamental laws of filtration.

Equipment: Special filter presses.

438. **Sprague, Warner & Company,** 600 West Erie St., Chicago, Ill. (Manufacturers and wholesalers of groceries.)

Research staff: Paul D. Potter and 2 trained chemists.

Research work: One-third time of 3 on problems relating to food.

439. **Spreckels Sugar Company,** 2 Pine St., San Francisco, Calif.

Research staff: R. E. Christie, 1 chief chemist, 3 assistant chemists and 6 bench chemists through operating season of three months; 1 chief chemist and 1 assistant chemist in off season of nine months.

Research work: Equivalent of time of 1 man for nine months on extraction and purification of juices; minimization of sugar losses; reduction of fuel-oil, lime and filter-cloth consumption; recovery of potash soda and ammonia compounds from Steffen waste.

440. **Squibb, E. R., & Sons,** New Brunswick, N. J. (Research and biological laboratories.)

Research staff: John F. Anderson, 6 bacteriologists and 3 chemists.

Research work: One-fourth time of 10 on biological and biochemical problems.

Equipment: For the production, for commercial purposes, of products for theoretical research in the various phases of biological therapeutics.

441. **Stamford Dyewood Company,** Stamford, Conn.

Research staff: Roy H. Wisdom, 1 chemist and 1 engineer.

Research work: One-tenth time of 3 on improvement in manufacture of dyewood extracts and economical methods of use of waste products.

442. **Standard Oil Company (New Jersey),** 26 Broadway, New York, N. Y. Central laboratory at Linden, N. J. Other laboratories at principal plants of the Standard Oil Company in the United States and abroad.

Research staff: Frank A. Howard, manager, C. I. Robinson, chief chemist, C. O. Johns, director research laboratory, N. E. Loomis, director, experimental division.

Research work: Petroleum production, products and refining, natural and artificial gas.

443. **Standard Oil Company of Indiana,** Whiting, Ind.

Research staff: F. M. Rogers, 7 chemists and 6 assistants.

Research work: Full time of 7 on improvement of methods of petroleum refining; development of new products and new processes; study of nature and properties of petroleum products.

Equipment: Fully equipped experimental plant for carrying out refining methods on a scale larger than is possible in the laboratory.

444. **Standard Underground Cable Company,** 26 Washington St., Perth Amboy, N. J.

Research staff: G. D'Eustachio and 2 assistants.

Research work: Approximately half time on insulating material for electrical purposes.

Staniford Laboratories. See Porro Biological Laboratories. (p. 65).

445. **Stewart-Warner Speedometer Corporation,** Chicago, Ill. (Speedometers, tachometers, vacuum gasoline systems, and other automobile accessories.)

Research staff: F. G. Whittington, chief engineer, 1 assistant chief engineer, 1 research engineer, 3 assistant research engineers, 1 electrical engineer, 2 designers and inventors.

Research work: Full time of 5 on investigations of fuel feed systems, speedometers, tachometers, and other automobile equipment.

Equipment: For testing tachometer and speedometer indications at varying temperatures, from —20 to 250° F. Sprague electric, cradle type dynamometer, capacity 50 to 75 h. p. 4000 maximum revolutions per minute; torsion machines; special flux meter for magnetic investigation work.

446. **Stockham Pipe & Fittings Co.,** Birmingham, Ala. (Cast iron fittings.)

Research staff: R. E. Risley.

Research work: Full time of 1 on heat treatment of high speed steel, molding sand selection and treatment and briquetting and remelting cast iron borings.

Equipment: Special equipment for physical testing of molding sand.

447. **Stone & Webster, Incorporated,** 147 Milk St., Boston, Mass. (Engineers, constructors, bankers, operators of public utilities.)

Research staff: 2 chemists, 2 mechanicians.

Research work: Full time of 4 on needs of industrial companies.

448. **Strathmore Paper Company,** Mittineague, Mass.

Research staff: Justus C. Sanborn and 1 assistant chemist.

Research work: One-fifth time of 2 on special paper mill problems.

449. **Structural Materials Research Laboratory, Lewis Institute,** 1951 W. Madison St., Chicago, Ill.

Research staff: Duff A. Abrams in charge of laboratory; J. C. Witt, chief research chemist, and 30 engineers, physicists and chemists.

Research work: Full time of 32 on the properties of concrete and concrete materials, reinforced concrete and related topics. Research is being carried on through a cooperative arrangement between the Lewis Institute and the Portland Cement Association.

Equipment: One 300,000-pound, two 200,000-pound and one 40,000-pound screw-power universal testing machines, 20,000-pound torsion testing machine, 4-unit Deval abrasion machine and standard ball mill for tests of road materials, Ro-Tap sieve shaker for fineness tests of materials, Talbot-Jones rattler for wear tests of concrete, autoclave apparatus for high-pressure steam tests of cement.

450. **Studebaker Corporation, The,** Detroit, Mich. (Automobiles and other vehicles.)

Research staff: E. J. Miles, 2 engineers and 1 mechanic in the dynamometer department; 1 electrical engineer and 1 assistant in the electrical department; 1 chemist in the chemical department, 1 engineer, 1 assistant and a staff of mechanics in the road testing department, 1 engineer on special work.

Research work: One-half to two-thirds time of staff on power output of motors, investigations of electrical appurtenances for automobiles, chemical studies of materials used in manufacture, road testing of automobiles, special problems related to radiators, brakes, oil pumps, fans and other equipment of an automobile.

Equipment: Research laboratory: 3 complete electric dynamometer equipments for motors up to 80-horsepower output; completely equipped for investigations of ignition apparatus, lighting and starting apparatus, storage batteries and all other electrical appurtenances of automobiles; special equipment for investigating oils and grease.

451. **Sun Chemical & Color Co.,** 309 Sussex St., Harrison, N. J. (Dry and pulp colors.)

Research staff: 2 chemists and 1 assistant.

Research work: One-half time of 3 on improving lake and pigment colors.

452. **Swan-Myers Company,** 219 N. Senate Ave., Indianapolis, Ind.

Research staff: Edgar B. Carter, director of biological division, A. D. Thorburn, director of pharmaceutical division, 2 chemists and 5 bacteriologists and biological chemists.

Research work: Approximately one-fourth time of 9 on biological products and organic synthetics used in medicine and pharmaceutical products.

453. **Swenson Evaporator Company,** 945 Monadnock Building, Chicago, Ill. Laboratory at Ann Arbor, Mich.

Research staff: W. L. Badger, 1 chemical engineer, assistants and 1 helper.

Research work: Full time on design of evaporators and other chemical engineering machinery; trial of processes and theoretical research on heat transmission in general.

Equipment: Large specially designed evaporators of all types. Accessory equipment so that processes can be carried out on ton or carload lots of material.

454. **Swift & Company,** Chicago, Ill.

Research staff: William D. Richardson and 9 assistants.

Research work: Full time of 10 on foods and dietetics, meat and meat products, dairy products, oils and fats, soap and soap products, glue and gelatin, fertilizers.

Equipment: Vacuum drying apparatus, agitator pressure tanks, special chill rooms.

455. **Tacony Steel Company,** Philadelphia, Pa.

Research staff: H. A. Baxter and approximately 25 assistants.

Research work: On manufacture and use of special carbon and alloy steels for high duty structural service.

456. **Taggart and Yerxa,** 165 Division St., New Haven, Conn.

Research staff: Arthur F. Taggart, R. B. Yerxa, 3 chemists and 3 engineers.

Research work: Full time of 8 on flotation concentration of ores.

457. **Takamine Laboratory,** Inc., Takamine Bldg., 12 Dutch St., New York, N. Y. (Manufacturing chemists.) Laboratory at Clifton, N. J.

Research staff: Jokichi Takamine, 4 chemists and 1 assistant.

Research work: Full time of 6 on biological, physiological and organic chemistry.

458. **Teeple, John E.,** 50 E. 41st St., New York, N. Y. (Consulting chemist, chemical engineer.)

Research staff: John E. Teeple and 2 to 4 chemists.

Research work: Full time of 2 to 4 on investigations necessary for directing research work in the laboratories of clients.

459. **Telling-Belle Vernon Company, The,** 3825 Cedar Ave., Cleveland, Ohio.

Research staff: W. O. Frohring, 2 bacteriologists and 2 chemists.

Research work: Three-fourths time of 5 on milk and milk products, with large portion of time on ice cream and infant foods.

460. **Thac Industrial Products Corp.,** 58 Middle Rose St., Trenton, N. J.

Research staff: A. I. Appelbaum and 2 assistants.

Research work: Part time of 3 on development of by-products.

461. **Titanium Alloy Manufacturing Co.,** Niagara Falls, N. Y.

Research staff: L. E. Barton, chief chemist, 2 assistant chemists and 1 helper. Physical testing laboratories, G. F. Comstock, metallurgist and 2 metallographists.

Research work: On problems related to the manufacture and use of ferro-carbon titanium and zirconium and zirconium products for ceramic industries.

462. **Titanium Pigment Co., Inc.,** Niagara Falls, N. Y.

Research staff: L. E. Barton, chief chemist, research and technical control of plant, and 4 assistant chemists.

Research work: On manufacture and use of titanium pigments, titanium salts and other titanium products.

463. **T. M. & G. Chemical Co.,** 517 Cortlandt St., Belleville, N. J. (Manufacturing chemists.)

Research staff: O. Ivan Lee and 3 assistants.

Research work: Approximately one-half time of 4 on development of commercial processes for the manufacture of organic chemicals with special reference to intermediates, dyes, and aromatic synthetics for soaps and perfumes; systematic study of the synthesis, separation and purification of secondary and tertiary aromatic amines; chlorination products of aromatic hydrocarbons; and utilization of by-products.

464. **Toch Brothers,** 320 Fifth Ave., New York, N. Y. (Paints, varnishes, colors, enamels; acid, alkali and damp-proof coatings.)

Research staff: Maximilian Toch and 4 to 6 chemists.

Research work: Problems related to water-proofing and protection of Portland cement by integral and surface coating methods; water-proofing of structural materials; anti-corrosive paints and compounds.

465. **Tolhurst Machine Works,** Troy, N. Y. (Specialists in centrifugals: hydro-extractors.)

Research staff: T. A. Bryson, usually 1 engineer and 1 or 2 assistants.

Research work: One-sixth time of 3 on determination of profitable methods of separation (and washing) of liquids from liquids or solids by means of centrifugal force; apparatus for dewatering sewage sludge; separation of foots from oil, recovery of glycerine and salt in soap industry, and improved methods of treating fish and fish oil.

Equipment: Centrifugal machines for filtration, extraction and sedimentation, ranging from small hand-driven, tube and basket centrifuges to higher speed 12 gallons basket capacity centrifugals, with interchangeable baskets of various types for crystalline, granular or fibrous materials, slimes and sludges.

466. **Tower Manufacturing Co., Inc.,** 85 Doremus Ave., Newark, N. J

Research staff: C. P. Harris, 7 chemists and 2 engineers.

Research work: Three-tenths time of 10 on processes for the manufacture of dyes and intermediates.

Equipment: Completely equipped semi-commercial plant.

467. **Ultro Chemical Corporation,** 41 Union Square, New York, N. Y. (Colors and chemicals.) Laboratory at 236 46th St., Brooklyn, N. Y.

Research staff: A. E. Gessler, 1 chemist and 1 assistant.

Research work: Approximately full time of 3 on dry colors and dyestuffs.

468. **Underwriters' Laboratories,** 207 E. Ohio St., Chicago, Ill. Established and maintained by National Board of Fire Underwriters. Departments: Protection, electrical, gases and oils, chemical, casualty. Laboratory also at 25 City Hall Place, New York, N. Y.

Research staff: W. H. Merrill and 50 experts and necessary assistants.

Research work: A variable but large proportion of time of staff on matters affecting performance and classification devices, materials and systems affecting the fire hazard or the personal accident hazard.

468a. **Uniform Adhesive Company, Incorporated,** foot of 39th St., Brooklyn, N. Y.

Research staff: Jerome and Walter Alexander.

Research work: Part time on adhesives, colloids, gums, starches, colors, sizings, finishings, etc., for paper and textiles.

469. **Union Carbide and Carbon Research Laboratories, Inc.,** Thompson Ave. and Manley St., Long Island City, N. Y., a subsidiary of the Union Carbide & Carbon Corporation, New York. Central Research laboratory at Long Island City and branch research and development laboratories at Long Island City and Buffalo, and two at Niagara Falls, N. Y.; two at Cleveland and one at Fremont, Ohio; one each at Indianapolis and Kokomo, Ind., and Clendenin, W. Va.

Research staff: Central laboratory has a staff of over 40, and branches combined, over 30, including chemists, chemical, metallurgical and electrical engineers and physicists.

Research work: Full time of staff on metallurgical and other electric furnace products, calcium carbide, compressed gases, carbon products, dry batteries and storage batteries, flashlights, organic chemicals and equipment for using the above products.

Equipment: Electric furnaces of various types; alloy testing and pyrometric equipment; gas compressing and testing equipment; apparatus for making and testing dry batteries, storage batteries, arc light carbons and brushes for electric motors and generators.

470. **Union Switch & Signal Company,** Swissvale, Pa. (Railway signal equipment.) Materials laboratories are maintained separately under the direction of H. C. Loudenbeck, with 3 chemists.

Research staff: L. O. Grondahl, 2 engineers and 1 assistant in charge of standardizing laboratory.

Research work: Two-thirds time of 4 on development of iron for electro-magnets, heat treatments, methods of test, electrical contacts, insulators, impregnation of coils and of wood.

Equipment: Oscillographs; standardizing equipment for electrical instruments; 50,000-volt insulation testing transformer; Heissler impact testing machine; an experimental impregnating plant, oil heated, with vacuum and pressure pump; and salt spray tester.

471. **United Alloy Steel Corporation,** Canton, Ohio. (Open hearth and electric steels, bars, slabs, billets, blooms, universal plates.)

Research staff: M. H. Schmid, 1 metallurgical engineer, 1 assistant metallurgical engineer, 1 laboratory foreman, 10 assistants and 1 engineer of tests; in the Electric Furnace, 1 chief and 2 recorders; in the Open Hearth Furnace, 1 chief and 8 recorders; in the Rolling Mills, 1 chief and 4 recorders.

Research work: One-half time of 32 on investigations connected with production and use of steel.

Equipment: Heat treatment: 4 Hoskins' electric furnaces, 1 American gas furnace for pieces up to 20 inches length and 5 inches diameter. Physical testing: equipped for tensile, torsion, cold bend, vibratory, Izod, Brinell, scleroscope, staybolt, etc.; also Leeds & Northrup permeameter for determining magnetic permeability of steel and 1 Leeds & Northrup recalescence instrument for determining critical points of steel.

472. **United Chemical and Organic Products Co.,** W. Hammond, Ill. (Successors to Hirsh, Stein & Company.)

Research staff: Jay Bowman and 4 chemists.

Research work: One-half time of 5 on problems arising in connection with plant processes.

Equipment: Semi-manufacturing scale equipment.

473. **United Drug Company,** Boston, Mass.

Research staff: Edward C. Merrill and 10 chemists.

Research work: One-half time of 10, largely on pharmaceutical investigations and research, and independent problems covering miscellaneous subjects.

474. **United Gas Improvement Co., The,** 3101 Passyunk Ave., Philadelphia, Pa.

Research staff: Edward J. Brady and 3 assistants.

Research work: Problems dealing only with the manufacture, purification, measurement and combustion of gas and the development of instruments in connection with the above.

Equipment: Laboratory water gas plant; laboratory blue gas generator; the use of a separate and complete commercial-sized experimental plant available at times; furnaces for refractory testing; high pressure gas equipment; complete physical equipment for high temperatures; high gas pressures; evacuating spectroscopy; electrical standards; radiation measurements; photometry and color.

475. **United Shoe Machinery Corporation,** Boston, Mass. Laboratory at Beverly.

Research staff: Walter Gould Bullard and assistants.

Research work: Examination of raw materials; tests on core oils and compounds, systematic investigation on improvement in antiseptic quality of cutting compounds and on pickling steel bars and plates. Some work on reclamation of waste materials and in attempts to improve methods of manufacturing shoe-factory supplies of all kinds.

476. **United States Bronze Powder Works, Inc.,** Closter, N. J.

Research staff: Everett S. Landman and 2 chemical engineers.

Research work: One-fifth time of 3 on oxidation and reduction of finely divided copper, properties and composition of bronzing liquids, non-tarnishable bronze powders and anti-fouling boat bottom compositions; pulverized copper and alloys for manufacture of electrical brushes.

U. S. Conditioning and Testing Co. See U. S. Testing Co., Inc. (p. 80).

477. **U. S. Food Products Corp.,** Peoria, Ill.

Research staff: J. K. Dale and 2 chemists.

Research work: Full time of 3 on food development problems.

478. **United States Glue Co.,** Milwaukee, Wis.

Research staff: C. R. McKee and 3 trained men.

Research work: One-half time of 4 on improvements in technology in glue and gelatine industry, particularly development of processes to produce glue and gelatine for various specific purposes, such as gelatine with various photographic properties, food gelatine, marshmallow gelatine and special glue.

Equipment: Complete miniature glue and gelatine factory.

479. **U. S. Industrial Alcohol Company,** 27 William St., New York, N. Y. Laboratory at South Baltimore, Md.

Research staff: A. A. Backhaus, 12 chemists, 2 bacteriologists, 10 assistant chemists and 2 chemical engineers.

Research work: Full time of staff on research in connection with the development of alcohol products, utilization of by-products of alcohol manufacture, improvement in the manufacture of alcohol, study of yeasts and bacteria.

480. **United States Metals Refining Co.,** Chrome, N. J.

Research staff: H. D. Greenwood, in charge of chemical department, W. C. Smith in charge of metallurgical department; about 42 assistants.

Research work: Part time of staff on maintaining a high standard in plant metallurgy and discovering new and improved methods.

481. **United States Smelting, Refining & Mining Company,** 55 Congress St., Boston, Mass. (Silver, gold, lead, copper, zinc, iron, arsenic, bismuth, cadmium, and tellurium.) Plants and research laboratories located at various points in the United States and Mexico.

Research staff: Galen H. Clevenger and 20 engineers, chemists and other specialists.

Research work: Full time on metallurgy, industrial chemistry and mining in the development of new processes, improvements in existent processes, investigation of new processes submitted and examination and improvement of products.

Equipment: Thirty-liter-per-hour liquid oxygen machine, equipment for investigating liquid oxygen explosives and for determining the volatilization losses of the precious metals during melting, reducing kiln of 50 tons daily capacity, experimental bag house, and experimental farm for the study of the effect of smelter fume upon growing crops and animal life.

United States Steel Corporation. See Carnegie Steel Company (p. 18).

482. **U. S. Testing Co., Inc.,** 316 Hudson St., New York, N. Y.

Research staff: W. F. Edwards, 5 chemists, 3 engineers and 1 physico-chemist.

Research work: One-half time of 10 on investigations of problems arising in textile and allied industries.

Equipment: Apparatus for investigation of effect of light on dyed textiles.

483. **Universal Aniline Dyes and Chemical Co.,** 11th and Davis Sts., S. Milwaukee, Wis.

Research staff: A. H. Schmidt and 2 assistants.

Research work: Approximately one-half time of 3 on intermediates and dyes.

Equipment: Complete miniature plant equipment.

484. **Upjohn Company, The,** Kalamazoo, Mich. (Fine pharmaceuticals.)

Research staff: Frederick W. Heyl, 4 or 5 chemists, 1 pharmacologist, 1 bacteriologist.

Research work: Part time of 7 on estimation of nitroglycerine; analyses of two Echinacea roots; standardization of commercial papain; some constituents of the roots of Brauneria augustifolia; some constituents of Sunbul root; standardization of the mercurials; Algenta root; some constituents of jambul; analysis of ragweed pollen; chemical examination of the leaves of Adonis vernalis; protein extract of ragweed pollen; yellow coloring substance of ragweed pollen; some constituents of Viburnum prunifolium, stability of Digitalis leaf extracts and infusions; pharmacological action of Adonis vernalis.

485. **Utah Copper Company,** Deseret Bank Bldg., Salt Lake City, Utah. Laboratory at Garfield, Utah.

Research staff: Thomas A. Janney, 6 chemists, 4 engineers and 3 assistants.

Research work: Three-fifths time of 16 on treatment of ores by the flotation process, gravity concentration, lixiviation and related investigations, flotation oils and reagents.

486. **Utah-Idaho Sugar Company,** Salt Lake City, Utah.

Research staff: E. G. Titus, 1 research assistant, 1 agricultural chemist, and 1 laboratory assistant.

Research work: Approximately one-half time of 4 on agricultural problems, beet-seed breeding, crop improvement, seed testing, soil reclamation and analysis, fertilizer experiments, and insect, disease and weed control.

Equipment: Special beet testing machinery, seed germination apparatus.

487. **Utility Color & Chemical Co., The,** 395 Frelinghuysen Ave., Newark, N. J.

Research staff: Joel Taub and 2 assistants.

Research work: One-half time of 3 on development of colors.

488. **Vacuum Oil Company, Incorporated,** 61 Broadway, New York, N. Y. (Refiners of petroleum for all purposes; manufacturers of shipping containers and of products used in the leather industry, etc.) Works and laboratories at Rochester and Olean, N. Y., Paulsboro and Bayonne, N. J.

Research staff: Florus R. Baxter, 3 chemists at Rochester; 1 chemist each at Olean, Paulsboro and Bayonne, also 12 assistants.

Research work: One-fifth time of 7 studying improvements in manufacturing methods; causes of deterioration of oils in service; utilization of by-products, properties of petroleum to determine suitability for specific uses.

Equipment: Fire, steam and vacuum stills, lead lined agitators, fully equipped, wax presses, super-centrifuges, photomicrographic set, apparatus for measurements of specific resistance, di-electric loss and di-electric strength, etc.

489. **Vanadium-Alloys Steel Co., The,** Latrobe, Pa. (High speed, alloy and carbon steels.)

Research staff: James P. Gill and 9 assistants.

Research work: Approximately one-third time of 10 on alloy tool steels, high speed and special steels.

490. **Vanadium Corporation of America,** 120 Broadway, New York, N. Y. Laboratory at Bridgeville, Pa.

Research staff: B. D. Saklatwalla, 5 chemists, 1 chemical engineer and 1 electrochemical engineer.

Research work: One-half time of 8 on metallography of alloy steels, development of metallurgical processes for alloying elements and development of electro-thermic methods of reducing metals.

491. **Van Schaack Brothers Chemical Works, Inc.,** 3358 Avondale Ave., Chicago, Ill. (Amyl acetate, soluble cotton, etc.)

Research staff: R. H. Van Schaack, Jr., and 4 assistants.

Research work: Approximately one-half time of 5 on nitrocellulose solvents.

492. **Ventura Refining Company,** Fillmore, Calif.

Research staff: J. W. Weir and 7 assistants.

Research work: One-seventh time of 8 on petroleum refinery problems.

493. **Vesta Battery Corporation,** 2100 Indiana Ave., Chicago, Ill. (Storage batteries, auto dynamos, etc.)

Research staff: Chester M. Angell, 1 chemist, 1 battery engineer and 1 assistant.

Research work: Approximately one-fourth time of 4 on electrochemistry, practical engineering features and improvement of parts and materials used in manufacture of the lead plate storage battery.

494. **Victor Chemical Works,** Fisher Building, Chicago, Ill. Large laboratory for factory control and general work and two smaller ones for research.

Research staff: L. D. Mathias, 5 chemists and 1 engineer.

Research work: Full time of 6 and one-half time of 2 on problems connected with manufacturing activities.

495. **Wadsworth Watch Case Co., Incorporated, The,** Dayton, Ky.

Research work: Approximately full time of 5 on alloys of precious metals and some of the brasses.

496. **Wahl-Henius Institute, Incorporated,** 1135 Fullerton Ave., Chicago, Ill.

Research staff: Max Henius, 3 experts, 1 chief analytical chemist, 1 chief research chemist, 2 assistant chemists and 3 assistants.

Research work: Full time of chief research chemist and about one-half time of 1 assistant chemist on fermentation and packing-house problems.

Equipment: Apparatus for testing products of fermentation industries and for carrying out experimental work on semi-commercial scale (experimental brewery, bottlery, etc.). Apparatus for testing solid and liquid fuel, and lubricants; differential refractometer (Tornoe's).

497. **Wallace & Tiernan Co., Inc.,** Box 178, Newark, N. J. (Chlorine control apparatus.)

Research staff: C. F. Wallace, G. C. Baker, 3 chemists and 2 engineers.

Research work: Large part time of 7 on chlorine gas control and applications in sterilization, bleaching and other lines; flour bleaching; carburetor laws and mechanical applications; and food products.

Equipment: Carburetor testing outfit complete and chlorine control equipment.

498. **Wallace, Joseph H., & Co.,** 5 Beekman St., New York, N. Y. (Industrial engineers.) Laboratory at Webbs Hill, Stamford, Conn., R. F. D. 29.

Research staff: F. E. Greenwood, 1 consulting engineer and 1 chemist.

Research work: Full time of 3 on cellulose and by-products, pulp, paper, naval stores, etc.

Equipment: Semi-commercial plant for pulp, paper and by-products.

499. **Waltham Watch Company,** Waltham, Mass.

Research staff: F. P. Flagg and 3 chemists.

Research work: Full time of 2 on investigation of the properties of enamel used on watch dials and study of the properties of metals and their relation to watch production.

500. **Warner, William R., & Company, Incorporated,** 113 W. 18th St., New York, N. Y. (Manufacturing pharmaceutists.)

Research staff: Frederick J. Austin, Charles Costa.

Research work: Chemical and pharmaceutical research which has for its object the improvement of products as regards physiological activity, permanence, elegance, etc., together with original work leading to the development of new preparations and new methods of manufacture.

Warren, S. D., Co. See Cumberland Mills (p. 24).

501. **Washburn-Crosby Co.,** Minneapolis, Minn. (Flour mills.)

Research staff: Frank W. Emmons, 3 chemists, 1 specially trained physical laboratory man, 1 expert baker and various assistants.

Research work: Full time of 1 on problems relating to wheat flour.

502. **Wayne Oil Tank and Pump Co.,** Ft. Wayne, Ind. (Tanks, pumps and underground storage outfits.)

Research staff: R. E. Langston, 1 chemist and 2 engineers.

Research work: Approximately three-fourths time of 3 on devising improved methods of handling, storing and using volatile and non-volatile liquids, such as gasoline, paint oil, varnish, lubricating oil, fuel oil, kerosene, etc.; methods of reclaiming used auto oil and purification of used engine oil.

503. **Wedge Mechanical Furnace Company,** 1000 Widener Bldg., Philadelphia, Pa. (Roasting furnaces.) Laboratory at Greenwich Point, Philadelphia, Pa.

Research staff: Carl S. Fogh and a variable number of assistants.

Research work: Full time on roasting ores, concentrates, mattes, mixtures and various materials for smelters and chemical plants.

504. **Weld and Liddell,** 2 Rector St., New York, N. Y. (Consulting engineers.) Laboratory at 961 Frelinghuysen Ave., Newark, N. J.

Research staff: Donald M. Liddell, 3 trained men and 2 untrained assistants.

Research work: Variable amount of time of 6 on stucco, zinc oxide, oil shale and petroleum. Balance of time on research problems of The Gray Industrial Laboratories.

Equipment: Completely equipped for pressure and steam distillations on oil shales or any bituminous or oily products.

505. **Wells, Raymond,** Homer, N. Y. (Chemist and technologist.)

Research staff: Raymond Wells and 2 assistants.

Research work: One-half time of 3 on animal and vegetable oils; fertilizers, soap, candles and glycerine; abattoir by-products; garbage and sewage disposal; lubrication oils and greases; wire mill soaps and drawing compounds; textile soaps and oils and agricultural insecticides and fungicides.

Equipment: Commercial scale equipment for research in oils, etc.

506. **Welsbach Company,** Gloucester, N. J. (Mantles for illuminating gas.)

Research staff: Harlan S. Miner and 6 trained men.

Research work: One-half time of 7 directed especially to economic production of rare earth chemicals, especially thorium and cerium; manufacture of special rare earth salts, nitration of cellulose, production of mesothorium; radio-chemistry.

Equipment: Especially for the study of problems connected with development of incandescent gas mantles.

507. **Western Electric Company, Incorporated,** 463 West St., New York, N. Y., known as the Research Laboratories of the American Telephone and Telegraph Company and the Western Electric Company, conducts research and engineering activities for the Bell Telephone System.

Research staff: F. B. Jewett, chief engineer, E. B. Craft, E. H. Colpitts and W. F. Hendry, assistant chief engineers; heads of functional activities: H. D. Arnold, J. J. Lyng, R. L. Jones, A. F. Dixon, J. W. Harris, L. Keller, H. C. Snook, J. B. Harlow, G. A. Anderegg and H. E. Shreeve who have under their direction approximately 825 research physicists, chemists and engineers, and approximately 750 assistant engineers, draftsmen, etc.

Research work: Full time of 1575 devoted to original investigation and development of new forms and improvement of existing forms of apparatus and equipment for electrical communication. The problems include research in thermionic emission and conduction, vacuum tube performance, microphonic conduction, radio transmission, the physical basis of speech, wave and impulse propagation and the physical and chemical properties of a great variety of materials; the development and design of full-mechanical and semi-mechanical telephone switchboards and systems in preparation for a comprehensive service transformation from the present manually operated system; the development and design of high frequency carrier systems with their associated generators, oscillators, modulators, hybrid coils, repeaters, loading coils, demodulators, amplifiers, and other special apparatus; development of new forms of local and long distance cables, submarine cables, transmitters, receivers, automatic printing telegraph apparatus, lightning arresters, protective fuses, current rectifiers, ringers, ringing systems, precision apparatus for high frequency measurements, marine radio sets, portable radio sets, transmitter life test methods, test methods for transmission efficiency, dry cells, storage cells, farm-light sets, household appliances and numerous problems in the design of keys, cords, plugs, switches, relays, contacts, loading coils, impedance coils, repeaters, transformers, condensers, insulators, lamps, and kindred details of communication apparatus and systems.

A thirteen story building of 400,000 square feet floor area. Physical research laboratory, transmission research laboratory, chemical research laboratory and physical testing laboratory completely equipped with all facilities necessary for this work; also completely equipped shop for the construction of working models and special equipment used in conducting research and development work.

508. **Western Gas Construction Company, The,** 1429 Buchanan St., Ft. Wayne, Ind. (Designers and builders of water, coal and gas apparatus, gas holders and special equipment.)

Research staff: F. Salathe, 2 chemists and 3 engineers.

Research work: One-fourth time of 6 on oils, gas, general organic, mechanical and chemical engineering.

509. **Western Precipitation Company,** 1016 W. Ninth St., Los Angeles, Calif. (Chemical engineers.)

Research staff: H. V. Welch, 1 physicist, 1 engineer, 3 chemists.

Research work: Three-fourths to nine-tenths time of 6 on problems centering around the Cottrell Processes of electrical precipitation.

Equipment: 150,000-volt transformer, 50,000-volt direct current generator, high potential mechanical rectifiers, potash laboratory, digestion and filtration apparatus and special apparatus adapted for study of equilibrium conditions in solutions.

510. **Western Research Corporation, Incorporated,** 514 18th St., Denver, Colo.

Research staff: James M. McClave, 1 chemist and 1 oil chemist.

Research work: One-half time of 3 on investigation of minerals and non minerals, oils and shales; special attention to working out treatment methods and the construction of plants.

511. **Western Sugar Refinery,** Foot 23d St., San Francisco, Calif.

Research staff: S. C. Meredith, 1 chief chemist, 3 engineers and 3 assistant chemists.

Research work: Two-fifths time of 8 on investigations of sugar losses, sugar machinery and materials.

512-513. **Westinghouse Electric & Manufacturing Company,** East Pittsburgh, Pa. (Electrical apparatus of all kinds.) Laboratory also at Essington, Pa.

512. *East Pittsburgh Laboratory*

Research staff, scientific: C. E. Skinner, manager of research department, 10 chemists, 28 physicists, and an operating staff of 23, including plant engineer, office staff, glass blowers, instrument makers, etc.

Research work: Chemical division, organic materials, inorganic chemical research and analytical chemistry; division of physics and metallurgy, magnetic testing and research, magnetic materials, metallurgical preparations, metallurgical testing and research, electrolytic condensers, power condenser research, insulating materials, electrical porcelain, radio bulbs, thermal conductivity and expansivity, resister materials, etc.

Equipment: Electric furnaces and rolls for metallurgical preparations, high vacuum apparatus, special magnetic testing apparatus, thermal conductivity and expansivity apparatus and conductivity of dielectrics.

Research staff, technical: R. P. Jackson, manager of materials and processes department, 8 chemists, 2 physical test men, 10 specialists on materials and their uses and 35 technical and other assistants.

Research work: 5 research laboratories in which work is conducted on technical problems connected with manufacture and testing of raw materials and finished products.

Equipment: High tension testing, special oscillographs and testing machines for determining physical properties.

Standard house: O. B. Riley with staff of about 11 engaged in checking and testing standard instruments and apparatus, chiefly electrical.

513. *Essington Laboratory*

Research staff: A. T. Kasley and 4 assistants.

Research work: Problems connected with heat and power.

514. **Westinghouse Lamp Co.,** Bloomfield, N. J. Engineering and development laboratories under the direction of R. E. Myers with a staff of 85.

Research staff: H. C. Rentschler, 3 physicists, 3 assistant physicists, 2 chemists and 1 assistant chemist.

Research work: Full time of 9 on study of radiation from solids and gases and vapors; also high vacua phenomena.

Equipment: Apparatus for obtaining and measuring high vacua, for producing high potential rectified current and for photometric and optical pyrometer measurements. High frequency electric furnace. Liquid air available at all times. Rare gases for study of their properties and uses are available.

515. **Weston, Byron, Co.,** Dalton, Mass. (Ledger and record paper.)

Research staff: P. W. Codwise and 1 assistant.

Research work: Varying amount time of 2 on problems connected with paper making.

516. **Weston & Sampson, 14 Beacon St.,** Boston, Mass. (Consulting engineers; water sanitation.)

Research staff: Robert S. Weston, 1 chemist and 1 engineer.

Research work: Part time of 3 on water, sewage and sanitation.

517. **Wheeler & Woodruff,** 280 Madison Ave., New York, N. Y.

Research staff: T. L. Wheeler, 2 chemical engineers, 1 mechanical engineer, 1 draftsman, 1 chemist and 1 helper.

Research work: One-half time of 7 on manufacture and regeneration of bone black, manufacture, use and revivification of decolorizing carbons, production and use of gas absorbing carbons, manufacture and treatment of carbon black, refining, deodorizing and hydrogenation of vegetable oils, filtration of water, manufacture of hydrogen, electroplating and electro-chemical problems and corrosion of metals and metal finishes.

Equipment: Semi-commercial size apparatus for study of carbon, etc.

518. **White Tar Company of New Jersey, Inc., The,** 56 Vesey St., New York, N. Y. (Chemicals.) Laboratory at Newark, N. J.

Research staff: Herbert W. Hamilton and 1 assistant.

Research work: Approximately one-third time of 2 on purification of naphthalene and the development of sanitary products.

519. **Whitten, J. O., Company, The,** Cross St., Winchester, Mass. (Gelatines.)

Research staff: G. R. Whitten and 3 assistants.

Research work: One-half time of 4 on treatment of bone and hide preparatory to the manufacture of gelatine.

520. **Wilbur White Chemical Co., The,** 62 Temple St., Owego, N. Y.

Research staff: J. A. Bridgman and 3 chemists.

Research work: One-half time of 4 on new processes for organic intermediates.

521. **Wilckes, Martin, Wilckes Company,** Head of Pine St., Camden, N. J. (Lampblacks, carbonblacks, etc.)

Research staff: A. Malmstrom, 4 chemists and 1 engineer.

Research work: Full time of 1 or 2 chemists on phosphoric acid compounds and baking powders.

522. **Wiley & Company, Inc.,** 904 N. Calvert St., Baltimore, Md. (Analytical and consulting chemists.)

Research staff: Samuel W. Wiley and 7 assistants.

Research work: Full time of 1 and part time of others on problems connected with the fertilizer industry; cellulose and paper; coal, oils and coke; beverages.

523. **Wilson & Co.,** Chicago, Ill. (Packers and provisioners.) Laboratories at Chicago, Ill., Chattanooga, Tenn., Oklahoma City, Okla., and Kansas City, Kansas.

Research staff: L. M. Tolman and 5 assistants.

Research work: One-half time of 6 on problems connected with fermentation, spoilage, etc.; hydrogenation of oils, refining and handling of oils and by-products.

524. **Winchester Repeating Arms Co.,** New Haven, Conn. (Rifles, shotguns, small arms ammunition, fishing tackle, skates, cutlery, flashlights and tools.)

Research staff: J. S. Gravely, 4 research chemists, 2 metallurgists and metallographists, 2 electrochemists and engineers and 8 assistants.

Research work: Three-fifths time of 17 on materials and processes involved in the manufacture of small arms and ammunition, cutlery, tools, hardware and sporting goods, dry batteries, flashlights, etc.

525. **Zinsser & Co.,** Hastings-on-Hudson, N. Y. (Manufacturing chemists.)

Research staff: J. S. Zinsser, 5 chemists, 1 dyer and 1 analyst.

Research work: Full time of 8 on anthraquinone color work.

526. **Zobel, Ernst, Company, Inc.,** 104 2d Ave., Brooklyn, N. Y. (Distillers and manufacturers of pine products and coal tar products; adhesive pitch, etc.)

Research staff: F. C. Zobel and 2 assistants.

Research work: Asphaltum, resin and oil; and coal tar distillate.

INDEX TO SUBJECT CLASSIFICATION OF LABORATORIES

SUBJECT CLASSIFICATION OF LABORATORIES

Abrasives (carborundum, emery, grinding, polishing, sandpaper)

Armour Glue Works
Armour Sandpaper Works
Bausch & Lomb Optical Co.
Carborundum Company, The
Dorr Company, The
Gillette Safety Razor Co.
Kalmus, Comstock & Wescott, Inc.
Maynard, T. Poole
Metals & Chemicals Extraction Corporation
Union Carbide and Carbon Research Laboratories, Inc.

Adhesives (glue, paste, sizing)

Abbott, William G., Jr.
Armour Glue Works
Banks & Craig
Bausch & Lomb Optical Co.
Bloede, Victor G., Co.
Brunswick-Balke-Collender Co.
Carborundum Company, The
Cudahy Packing Co., The
Cumberland Mills
Dewey & Almy Chemical Company
Dextro Products, Inc.
Emerson Laboratory
Feculose Co. of America
Grosvenor, Wm. M.
Little, Arthur D., Inc.
Morris & Company
National Gum & Mica Co.
Pfister & Vogel Leather Co.
Philadelphia Quartz Company
Seydel Manufacturing Company
Skinner, Sherman & Esselen, Incorporated
Swift & Company
Thac Industrial Products Corp.
Uniform Adhesive Company, Incorporated
United Chemical and Organic Products Co.
U. S. Food Products Corp.
United States Glue Co.
Zobel, Ernst, Company, Inc.

Agricultural equipment and engineering (land drainage, threshing machines, tractors)

American Beet Sugar Company
Banks & Craig
Minneapolis Steel and Machinery Co.
Utah-Idaho Sugar Company

Agricultural problems (entomology, genetics, pathology, etc. See also soils and fertilizers)

American Agricultural Chemical Company, The
American Beet Sugar Company
National Lime Association
Utah-Idaho Sugar Company

Air (air-driven machines, air products, compressed air, liquid air, pneumatics)

Abbott, William G., Jr.
Ingersoll-Rand Company
Union Carbide and Carbon Research Laboratories, Inc.

Air conditioning (ventilation)

American Blower Company
American Radiator Company
U. S. Testing Co., Inc.

Aircraft and accessories (see also internal combustion motors)

Curtiss Aeroplane & Motor Corporation
General Motors Research Corporation
Industrial Research Corporation
Martin, Glen L., Company, The
Packard Motor Car Company

Apparatus and instruments, chemical and physical (astronomical instruments, autoclaves, balances, compasses, gages, lenses, microscopes, surveying

instruments, telescopes, transits)

Baldwin Locomotive Works, The
Bausch & Lomb Optical Co.
Brown & Sharpe Mfg. Co.
Central Scientific Company
Corning Glass Works
Cutler-Hammer Mfg. Co., The
Eastman Kodak Company
Eimer & Amend
Electrical Testing Laboratories
Grosvenor, Wm. M.
Gurley, W. & L. E.
Kellogg Switchboard and Supply Co.
Kilbourne & Clark Manufacturing Company
Keuffel & Esser Co.
Mojonnier Bros. Co.
Munn, W. Faitoute
Pyrolectric Instrument Company
Riverbank Laboratories
Sangamo Electric Company
Scientific Instrument and Electrical Machine Company, The
Tolhurst Machine Works
Wallace & Tiernan Co., Inc.
Waltham Watch Company

Automotive vehicles, equipment and accessories (automobiles, tanks, tractors, trucks)

Abbott, William G., Jr.
Boyer Chemical Laboratory Company
Champion Ignition Company
Diamond Chain & Manufacturing Company
Dodge Brothers
Electrical Testing Laboratories
Fansteel Products Company, Inc.
General Motors Research Corporation
Holt Manufacturing Company, The
Industrial Research Corporation
Lunkenheimer Co., The
Minneapolis Steel and Machinery Co.
Northwestern Chemical Co., The
Packard Motor Car Company
Pierce-Arrow Motor Car Company, The
Stewart-Warner Speedometer Corporation
Studebaker Corporation, The
Wallace & Tiernan Co., Inc.

Beverages, non-alcoholic

California Fruit Growers Exchange
Dehls & Stein
Industrial Chemical Institute of Milwaukee
Industrial Testing Laboratories
Lennox Chemical Co., The
Nowak Chemical Laboratories
Schwarz Laboratories
Skinner, Sherman & Esselen, Incorporated
U. S. Food Products Corp.
Wahl-Henius Institute, Incorporated

Biological equipment and supplies

Baker, J. T., Chemical Co.
Beebe Laboratories, Inc.
Central Scientific Company
Coleman & Bell Company, The
Dean Laboratories, Inc.
Digestive Ferments Co.
Eimer & Amend
Lilly, Eli, and Company
Mulford, H. K., Company
Swan-Myers Company

Building materials (asphalt, cement, concrete, lime, marble, road materials, slate. See also iron and steel)

Barber Asphalt Paving Company, The
Beaver Board Companies, The
Borrowman, George
Conwell, W. L., & Co., Inc.
Hunt, Robert W., and Co.
Institute of Industrial Research, The
Interocean Oil Company, The
Lewis, F. J., Manufacturing Co.
Maynard, T. Poole

National Lime Association
Pennsylvania Railroad Company, The
Richardson Company, The
Skinner, Sherman & Esselen, Incorporated
Standard Oil Company (New Jersey)
Structural Materials Research Laboratory
Toch Brothers
Weld and Liddell

By-products from wastes

Abbott, William G., Jr.
Anaconda Copper Mining Co.
California Fruit Growers Exchange
Davison Chemical Company, The
Emerson Laboratory
Federal Products Company, The
Grosvenor, Wm. M.
Harrison Mfg. Co., The
Kidde, Walter, & Company, Incorporated
Koppers Company, The
Lakeview Laboratories
Laucks, I. F., Inc.
Lyster Chemical Company, Inc.
Maynard, T. Poole
Morris & Company
Research Corporation
Scott, Ernest, & Company
Stamford Dyewood Company
Swenson Evaporator Company
Teeple, John E.
Thac Industrial Products Corp.
Vacuum Oil Company, Incorporated
Weld and Liddell
Western Precipitation Company
Wheeler & Woodruff
White Tar Company of New Jersey, Inc., The
Wilson & Co.

Ceramics (bricks, china, glass, magnesite, pottery, porcelain, refractories)

American Window Glass Co.
Anaconda Copper Mining Co.
Andrews, A. B.
Babcock & Wilcox Co., The
Bausch & Lomb Optical Co.
Beaver Falls Art Tile Company
Buckeye Clay Pot Co.
Carborundum Company, The
Celite Products Company
Champion Porcelain Company
Corning Glass Works
Dorite Manufacturing Company, The
Ellis-Foster Company
FitzGerald Laboratories, Inc., The
Fry, H. C., Glass Company
Glass Container Association of America
Harbison-Walker Refractories Company
Kalmus, Comstock & Wescott, Inc.
Keuffel & Esser Co.
Koppers Company, The
Kraus Research Laboratories, Inc.
Laclede-Christy Clay Products Company
Little, Arthur D., Inc.
Maynard, T. Poole
National Laboratories, The
National Lamp Works of General Electric Company
Pfaudler Co., The
Pittsburgh Testing Laboratory
Ransom & Randolph Co., The
Roessler & Hasslacher Chemical Company, The
Spencer Lens Company
Thac Industrial Products Corp.
Titanium Alloy Manufacturing Co.
Union Carbide and Carbon Research Laboratories, Inc.
Waltham Watch Company
Weld and Liddell
Western Gas Construction Company, The

Chemical engineering equipment (agitators, blowers, centrifuges, compressors, concentrators, condensers, dryers, evapora-

tors, filter presses, pulverizers, pumps, separators)

Abbé Engineering Company
Abbott, William G., Jr.
American Blower Company
Anaconda Copper Mining Co.
Andrews, A. B.
Bethlehem Shipbuilding Corporation, Ltd.
Buffalo Foundry and Machine Co.
Cramp, William, & Sons Ship & Engine Building Co., The
Deister Concentrator Company, The
DeLaval Separator Co., The
Dorr Company, The
Ingersoll-Rand Company
International Filter Co.
Oliver Continuous Filter Co.
Scott, Ernest, & Company
Sperry, D. R., & Co.
Swenson Evaporator Company
Tolhurst Machine Works
United States Bronze Powder Works, Inc.
Wayne Oil Tank and Pump Co.
Western Gas Construction Company, The

Chemicals, fine, including solvents

Abbott Laboratories, The
Atlantic Dyestuff Company
Baker, J. T., Chemical Co.
Barrett Company, The
Calco Chemical Company, The
Carus Chemical Company
Central Dyestuff and Chemical Co.
Central Scientific Company
Chemical Economy Company
Chemical Products Company
Coleman & Bell Company, The
Cosmos Chemical Co., Inc.
Dehls & Stein
Digestive Ferments Co.
Eastman Kodak Company
Eppley Laboratory
Federal Products Company, The
Florida Wood Products Co.
General Chemical Company
Harrison Mfg. Co., The
Heyden Chemical Company of America, Inc.
Hynson, Westcott & Dunning
Lakeview Laboratories
Lehn & Fink, Inc.
Lemoine, Pierre, Cie., Inc.
Lindsay Light Company
Long & Co., Inc.
Lyster Chemical Company, Inc.
Mallinckrodt Chemical Works
McKesson & Robbins, Incorporated
McLaughlin Gormley King Co.
Merck & Co.
Monroe Drug Company
Monsanto Chemical Works
Newark Industrial Laboratories
New York Quinine & Chemical Works, Incorporated, The
Norvell Chemical Corporation, The
Ohio Fuel Supply Company, The
Palmolive Company, The
Parke, Davis & Company
Peet Bros. Mfg. Co.
Pfizer, Chas., & Co., Inc.
Pharma-Chemical Corporation
Powers - Weightman - Rosengarten Company, The
Radium Company of Colorado, Inc., The
Radium Limited, U. S. A.
Roessler & Hasslacher Chemical Company, The
Seydel Manufacturing Company
Sharp & Dohme
Special Chemicals Company
Squibb, E. R., & Sons
Thac Industrial Products Corp.
T. M. & G. Chemical Co.
Tower Manufacturing Co., Inc.
Union Carbide and Carbon Research Laboratories, Inc.
U. S. Industrial Alcohol Company
Universal Aniline Dyes and Chemical Co.
Wilbur White Chemical Co., The

Chemicals, heavy (acids, alkalies, fungicides, insecticides, salts)

American Cyanamid Company
American Trona Corporation

Anaconda Copper Mining Co.
Ansul Chemical Company
Armour Ammonia Works
Atlas Powder Co.
Bowker Insecticide Company
Brown Company
Buchanan, C. G., Chemical Company
Butterworth-Judson Corporation
California Fruit Growers Exchange
Carborundum Company, The
Carus Chemical Company
Charlotte Chemical Laboratories, Inc.
Condensite Company of America
Davison Chemical Company, The
Detroit Testing Laboratory, The
Drackett, P. W., & Sons Co., The
du Pont, E. I., de Nemours & Company
Eagle-Picher Lead Company, The
Eastern Manufacturing Company
Federal Phosphorus Company
General Chemical Company
Glidden Company, The
Grasselli Chemical Company
Great Western Electro-Chemical Company
Grosvenor, Wm. M.
Harrison Mfg. Co., The
Hooker Electrochemical Company
Industrial Chemical Institute of Milwaukee
Maas, A. R., Chemical Company
Mallinckrodt Chemical Works
Mathieson Alkali Works, Inc., The
McLaughlin Gormley King Co.
Merrimac Chemical Company
Metals & Chemicals Extraction Corporation
Meyer, Theodore
Monsanto Chemical Works
National Lead Company
Naugatuck Chemical Company, The
New Jersey Zinc Company, The
New York Quinine & Chemical Works, Incorporated, The
Norvell Chemical Corporation, The
Peet Bros. Mfg. Co.
Pennsylvania Salt Manufacturing Co.
Philadelphia Quartz Company
Pittsburgh Plate Glass Co.
Powers - Weightman - Rosengarten Company, The
Pure Oil Company, Kanawha River Salt and Chemical Division
Riches, Piver & Co.
Rodman Chemical Company
Roessler & Hasslacher Chemical Company, The
Saginaw Salt Products Co.
Seydel Manufacturing Company
Solvay Process Company, The
Swenson Evaporator Company
Titanium Pigment Co., Inc.
Union Carbide and Carbon Research Laboratories, Inc.
United Chemical and Organic Products Co.
United States Metals Refining Co.
Utah-Idaho Sugar Company
Victor Chemical Works
Welsbach Company
Wilckes, Martin, Wilckes Company

Chemistry, biological (bacteriology, biology)

Abbott Laboratories, The
American Beet Sugar Company
American Hominy Company
American Institute of Baking
Banks & Craig
Beebe Laboratories, Inc.
Bridgeman-Russell Company
Coleman & Bell Company, The
Dean Laboratories, Inc.
Dearborn Chemical Company
Dehls & Stein
Digestive Ferments Co.
Freed, H. E., Co., The
Gallun, A. F., & Sons Co.
Glass Container Association of America
Hochstadter Laboratories
Industrial Testing Laboratories
Kolynos Co., The
Lehn & Fink, Inc.

Merrell-Soule Laboratory
Metz, H. A., Laboratories, Inc.
Miner Laboratories, The
Morris & Company
Mulford, H. K., Company
National Canners Association
National Laboratories, The
New York Quinine & Chemical Works, Incorporated, The
Parke, Davis & Company
Pease Laboratories
Physicians and Surgeons Laboratory
Porro Biological Laboratories
Schwarz Laboratories
Seydel Manufacturing Company
Skinner, Sherman & Esselen, Incorporated
Special Chemicals Company
Sprague, Warner & Company
Squibb, E. R., & Sons
Swan-Myers Company
Takamine Laboratory, Inc.
Telling-Beele Vernon Company, The
United States Glue Co.
U. S. Industrial Alcohol Company
Upjohn Company, The
Weston & Sampson
White Tar Company of New Jersey, Inc., The
Wilson & Co.

Chemistry, inorganic (carbon, graphite, etc.)

Acheson Graphite Company
American Agricultural Chemical Company, The
American Chemical Paint Company
American Cyanamid Company
American Trona Corporation
Ansbacher, A. B., & Company
Ansul Chemical Company
Atlas Powder Co.
Baker & Co., Inc.
Beaver Falls Art Tile Company
Borrowman, George
Borromite Co. of America, The
Bowker Insecticide Company
Brown Company
Buchanan, C. G., Chemical Company
Burdett Manufacturing Company
Carus Chemical Company
Celite Products Company
Charlotte Chemical Laboratories, Inc.
Chase Metal Works
Childs, Charles M., & Co., Inc.
Condensite Company of America
Dearborn Chemical Company
Detroit Testing Laboratory, The
Diamond Match Co., The
Dorite Manufacturing Company, The
Drackett, P. W., & Sons Co., The
Eagle-Picher Lead Company, The
Eimer & Amend
Emerson Laboratory
FitzGerald Laboratories, Inc.
General Chemical Company
Glysyn Corporation, The
Grasselli Chemical Company
Great Western Electro-Chemical Company
Harrison Mfg. Co., The
Heyden Chemical Company of America, Inc.
Hochstadter Laboratories
Hooker Electrochemical Company
Industrial Chemical Institute of Milwaukee
Jaques Manufacturing Company
Kalmus, Comstock & Wescott, Inc
Laucks, I. F., Inc.
Lee & Wight
Lennox Chemical Co., The
Lindsay Light Company
Maas, A. R., Chemical Company
Mallinckrodt Chemical Works
McNab & Harlin Manufacturing Co.
Merck & Co.
Merrimac Chemical Company
Metals & Chemicals Extraction Corporation
Mineral Refining & Chemical Corporation
Munning, A. P., & Co.

National Aniline & Chemical Company, Incorporated
National Laboratories, The
National Lead Company
National Lime Association
Niles Tool Works Company, The
Northwestern Chemical Co., The
Norvell Chemical Corporation, The
Pennsylvania Salt Manufacturing Co.
Permutit Company, The
Perolin Company of America, The
Pfizer, Chas., & Co., Inc.
Pittsburgh Testing Laboratory
Pure Oil Company, Kanawha River Salt and Chemical Division
Pyro-Non Paint Co., Inc.
Radium Company of Colorado, Inc., The
Radium Limited, U. S. A.
Ransom & Randolph Co., The
Rhode Island Malleable Iron Works
Riches, Piver & Co.
Rodman Chemical Company
Speer Carbon Company
Squibb, E. R., & Sons
Teeple, John E.
Union Carbide and Carbon Research Laboratories, Inc.
United States Bronze Powder Works, Inc.
United States Smelting, Refining & Mining Company
Wadsworth Watch Case Co., Incorporated, The
Waltham Watch Company
Wedge Mechanical Furnace Company
Weld and Liddell
Wheeler & Woodruff
White Tar Company of New Jersey, Inc., The
Wilckes, Martin, Wilckes Company
Wiley & Company, Inc.

Chemistry, mineralogical and geological (quartz, etc.)

Celite Products Company
Charlotte Chemical Laboratories, Inc.
Hirsch Laboratories, Inc., The
Lee & Wight
Little, Arthur D., Inc.
Philadelphia Quartz Company
United States Smelting, Refining & Mining Company
Wedge Mechanical Furnace Company
Western Research Corporation, Incorporated

Chemistry, organic (fermentation, starch, vegetable oils, etc.)

American Beet Sugar Company
American Chemical and Manufacturing Corporation
American Chemical Paint Company
American Cyanamid Company
American Diamalt Company
American Hominy Company
Atlantic Dyestuff Company
Avri Drug & Chemical Company, Inc.
Barrett Company, The
Beckman and Linden Engineering Corporation
Bennetts' Chemical Laboratory
Betz, Frank S., Company
Bloede, Victor G., Co.
Brown Company
Calco Chemical Company, The
California Fruit Growers Exchange
California Ink Company, Inc.
Carus Chemical Company
Central Dyestuff and Chemical Co.
Charlotte Chemical Laboratories, Inc.
Chemical Economy Company
Chemical Products Company
Chemical Service Laboratories, Inc., The
Coleman & Bell Company, The
Corn Products Refining Company
Cosmos Chemical Co., Inc.
Cudahy Packing Co., The
Davis Chemical Products, Inc.

Dearborn Chemical Company
Dehls & Stein
Detroit Testing Laboratory, The
Dewey & Almy Chemical Company
Dextro Products, Inc.
Dicks David Company, Incorporated
Digestive Ferments Co.
du Pont, E. I., de Nemours & Company
Dye Products & Chemical Company, Inc.
Eastman Kodak Company
Eimer & Amend
Ellis-Foster Company
Emerson Laboratory
Feculose Co. of America
Federal Products Company, The
Foster-Heaton Company
Garfield Aniline Works, Inc.
General Bakelite Company
General Chemical Company
Glysyn Corporation, The
Grasselli Chemical Company
Harrison Mfg. Co., The
Heap, William, & Sons
Heyden Chemical Company of America, Inc.
Hirsch Laboratories, Inc., The
Hochstadter Laboratories
Hynson, Westcott & Dunning
Industrial Chemical Institute of Milwaukee
Industrial Testing Laboratories
Lakeview Laboratories
Laucks, I. F., Inc.
Lee & Wight
Lehn & Fink, Inc.
Lemoine, Pierre, Cie., Inc.
Lewis, F. J., Manufacturing Co.
Long & Co., Inc.
Mallinckrodt Chemical Works
May Chemical Works
M. B. Chemical Co., Inc.
McLaughlin Gormley King Co.
Merck & Co.
Metz, H. A., Laboratories, Inc.
Miner Laboratories, The
Monroe Drug Company
Musher and Company, Incorporated
National Aniline & Chemical Company, Incorporated
National Laboratories, The
New York Quebracho Extract Company, Incorporated
New York Quinine & Chemical Works, Incorporated, The
New York Sugar Trade Laboratory, Inc., The
Norvell Chemical Corporation, The
Nulomoline Company, The
Ohio Fuel Supply Company, The
Ohio Grease Co., The
Palmolive Company, The
Pfizer, Chas., & Co., Inc.
Pharma-Chemical Corporation
Pittsburgh Testing Laboratory
Procter & Gamble Co., The
Pure Oil Company, Moore Oil and Refining Company Division
Quinn, T. H., & Company
Radiant Dye & Color Works
Schaeffer Brothers & Powell Manufacturing Company
Schwarz Laboratories
Sears, Roebuck and Co.
Seydel Manufacturing Company
Sharp & Dohme
Skinner, Sherman & Esselen, Incorporated
Southern Cotton Oil Company, The
Special Chemicals Company
Squibb, E. R., & Sons
Stamford Dyewood Company
Standard Oil Company (New Jersey)
Standard Oil Company of Indiana
Swan-Myers Company
Swift & Company
Takamine Laboratory, Inc.
Teeple, John E.
Telling-Belle Vernon Company, The
T. M. & G. Chemical Co.
Tower Manufacturing Co., Inc.
Ultro Chemical Corporation
U. S. Food Products Corp.
U. S. Industrial Alcohol Company
Universal Aniline Dyes and Chemical Co.

Utility Color & Chemical Co., The
Van Schaack Brothers Chemical Works, Inc.
Wallace, Joseph H., & Co.
Wells, Raymond
Western Gas Construction Company, The
Western Sugar Refinery
White Tar Company of New Jersey, Inc., The
Whitten, J. O., Company, The
Wilbur White Chemical Co., The
Zinsser & Co.
Zobel, Ernst, Company, Inc.

Chemistry, pharmaceutical (cosmetics, dentifrice, drugs, disinfectants, medicines)

Abbott Laboratories, The
Avri Drug & Chemical Company, Inc.
Betz, Frank S., Company
Bowker Insecticide Company
Boyer Chemical Laboratory Company
Calco Chemical Company, The
Carus Chemical Company
Caulk, L. D., Company, The
Central Dyestuff and Chemical Co.
Corn Products Refining Company
Cudahy Packing Co., The
Dean Laboratories, Inc.
Heinrich Laboratories of Applied Chemistry
Heyden Chemical Company of America, Inc.
Hirsch Laboratories, Inc., The
Hoehstadter Laboratories
Hynson, Westcott & Dunning
Industrial Chemical Institute of Milwaukee
Industrial Testing Laboratories
Johnson & Johnson
Kolynos Co., The
Lakeview Laboratories
Larkin Co.
Lehn & Fink, Inc.
Lilly, Eli, and Company
Long & Co., Inc.
Lyster Chemical Company, Inc.
MacAndrews & Forbes Company
Mallinckrodt Chemical Works
McKesson & Robbins, Incorporated
McLaughlin Gormley King Co.
Meigs, Bassett & Slaughter, Inc.
Merck & Co.
Merrell, Wm. S., Company, The
Metz, H. A. Laboratories, Inc.
Meyer, Theodore
Milliken, John T., and Co.
Miner Laboratories, The
Monsanto Chemical Works
Mulford, H. K., Company
Newark Industrial Laboratories
New York Quinine & Chemical Works, Incorporated, The
Norvell Chemical Corporation, The
Parke, Davis & Company
Pfizer, Chas., & Co., Inc.
Pharma-Chemical Corporation
Physicians and Surgeons Laboratory
Pittsburgh Testing Laboratory
Sears, Roebuck and Co.
Seydel Manufacturing Company
Sharp & Dohme
Squibb, E. R., & Sons
Standard Oil Company (New Jersey)
Swan-Myers Company
Takamine Laboratory, Inc.
Thac Industrial Products Corp.
Union Carbide and Carbon Research Laboratories, Inc.
United Drug Company
U. S. Food Products Corp.
Upjohn Company, The
Warner, William R., & Company, Incorporated
White Tar Company of New Jersey, Inc., The
Wilckes, Martin, Wilckes Company
Zinsser & Co.

Concentration of ores (see also chemical engineering equipment)

Anaconda Copper Mining Co.

Deister Concentrator Company, The
Dorr Company, The
General Engineering Company, Incorporated, The
Grasselli Chemical Company
Grosvenor, Wm. M.
James Ore Concentrator Co.
Maynard, T. Poole
Mesabi Iron Company
National Laboratories, The
Richards & Locke
Taggart and Yerxa
United States Smelting, Refining & Mining Company
Utah Copper Company
Wedge Mechanical Furnace Company
Western Research Corporation, Incorporated

Consulting research laboratories

Abbott, William G., Jr.
Andrews, A. B.
Babcock Testing Laboratory
Banks & Craig
Beckman and Linden Engineering Corporation
Beebe Laboratories, Inc.
Bennetts' Chemical Laboratory
Borrowman, George
Cabot, Samuel, Inc.
Case Research Laboratory
Chemical Service Laboratories, Inc., The
Cleveland Testing Laboratory Co., The
Commercial Testing and Engineering Co.
Conwell, E. L., & Co., Inc.
Dean Laboratories, Inc.
Detroit Testing Laboratory, The
Dorr Company, The
Dunham, H. V.
Durfee, Winthrop C.
Electrical Testing Laboratories
Electrolabs Company, The
Ellis-Foster Company
Emerson Laboratory
Eppley Laboratory, The
Eustis, F. A.
Fahy, Frank P.
FitzGerald Laboratories, Inc., The
Fort Worth Laboratories
General Engineering Company, Incorporated, The
Gray Industrial Laboratories, The
Grosvenor, Wm. M.
Hayes, Hammond V.
Heinrich Laboratories of Applied Chemistry
Hirsch Laboratories, Inc., The
Hochstadter Laboratories
Howard Wheat and Flour Testing Laboratory, The
Hunt, Robert W., and Co.
Industrial Chemical Institute of Milwaukee
Industrial Research Corporation
Industrial Research Laboratories
Industrial Testing Laboratories
Institute of Industrial Research, The
Kalmus, Comstock & Wescott, Inc.
Kidde, Walter, & Company, Incorporated
Kraus Research Laboratories
Lakeview Laboratories
Laucks, I. F., Inc.
Lee & Wight
Lincoln, E. S., Inc.
Little, Arthur D., Inc.
Littlefield Laboratories Co.
Lockhart Laboratories
Maynard, T. Poole
McIlhiney, Parker C.
Meigs, Bassett & Slaughter, Inc.
Miner Laboratories, The
Munn, W. Faitoute
National Laboratories, The
Newark Industrial Laboratories
New York Sugar Trade Laboratory, Inc., The
Pease Laboratories
Pettee, Charles L. W., Laboratories of
Physicians and Surgeons Laboratory
Pittsburgh Testing Laboratory

Porro Biological Laboratories
Porter, Horace C.
Quinn, T. H., & Company
Research Corporation
Richards & Locke
Riverbank Laboratories
Rubber Trade Laboratory, The
Sabine, Wallace Clement, Laboratory
Schwarz Laboratories
Skinner, Sherman & Esselen, Incorporated
Souther, Henry, Engineering Co., The
Structural Materials Research Laboratory
Taggart and Yerxa
Takamine Laboratory, Inc.
Teeple, John E.
Wahl-Henius Institute, Incorporated
Weld and Liddell
Wells, Raymond
Western Precipitation Company
Western Research Corporation, Incorporated
Weston & Sampson
Wiley & Company, Inc.

Containers, including bottle seals (cans, fiber-board containers, etc.)

American Can Company
Bond Manufacturing Corporation
Chicago Mill and Lumber Company
Dewey & Almy Chemical Company
Glass Container Association of America
Lehn & Fink, Inc.
National Association of Corrugated and Fibre Box Manufacturers, The
National Canners Association
Package Paper and Supply Corporation
Vacuum Oil Company, Incorporated
Wheeler & Woodruff

Dyes, natural and artificial (inks, intermediates, pigments, ribbons)

Amoskeag Manufacturing Company
Arlington Mills
Atlantic Dyestuff Company
Ault & Wiborg Company, The
Banks & Craig
Butterworth-Judson Corporation
Calco Chemical Company, The
California Ink Company, Inc.
Central Dyestuff and Chemical Co.
Coleman & Bell Company, The
Dicks David Company, Incorporated
du Pont, E. I., de Nemours & Company
Durfee, Winthrop C.
Dye Products & Chemical Company, Inc.
Eastern Finishing Works, Inc.
Eavenson & Levering Co.
Emerson Laboratory
Foster-Heaton Company
Garfield Aniline Works, Inc.
Garrison Mfg. Co., The
Grasselli Chemical Company
Grosvenor, Wm. M.
Hirsch Laboratories, Inc., The
Hooker Electrochemical Company
Klearflax Linen Rug Company
Little, Arthur D., Inc.
Lockhart Laboratories
Long & Co., Inc.
MacAndrews & Forbes Company
May Chemical Works
M. B. Chemical Co., Inc.
Merrimac Chemical Company
Monroe Drug Company
Monsanto Chemical Works
Morrill, Geo. H., Co.
National Aniline & Chemical Company, Incorporated
National Laboratories, The
Naugatuck Chemical Company, The
Newark Industrial Laboratories
Northwestern Chemical Co., The
Oliver Continuous Filter Co.

Palatine Aniline and Chemical Corporation
Peerless Color Company
Pharma-Chemical Corporation
Pittsburgh Plate Glass Co.
Radiant Dye & Color Works
Reliance Aniline & Chemical Co., Incorporated
Sears, Roebuck and Co.
Seydel Manufacturing Company
Stamford Dyewood Company
Sun Chemical & Color Co.
T. M. & G. Chemical Co.
Tower Manufacturing Co., Inc.
Ultro Chemical Corporation
U. S. Testing Co., Inc.
Universal Aniline Dyes and Chemical Co.
Utility Color & Chemical Co., The
White Tar Company of New Jersey, Inc., The
Wilbur White Chemical Co., The
Zinsser & Co.

Electrical communication (cable, telegraph, telephone, wireless)

American Radio and Research Corporation
Belden Manufacturing Company
Corning Glass Works
General Electric Company
Hayes, Hammond V.
Industrial Research Corporation
Kellogg Switchboard and Supply Co.
Kilbourne & Clark Manufacturing Company
Munn, W. Faitoute
Western Electric Company, Incorporated

Electrical equipment and instruments (ammeters, lamps, voltmeters, wattmeters)

Abbott, William G., Jr.
Allen-Bradley Co.
American Radio and Research Corporation
Commonwealth Edison Company
Cooper Hewitt Electric Company
Corning Glass Works
Cutler-Hammer Mfg. Co., The
Edison, Thomas A., Laboratory
Electrical Testing Laboratories
Fansteel Products Company, Inc.
General Electric Company
Hoskins Manufacturing Company
Kilbourne & Clark Manufacturing Company
Kellogg Switchboard and Supply Co.
Leeds & Northrup Company
Munn, W. Faitoute
National Lamp Works of General Electric Company
Pyrolectric Instrument Company
Sangamo Electric Company
Scientific Instrument and Electrical Machine Company, The
Speer Carbon Company
Union Carbide and Carbon Research Laboratories, Inc.
Western Electric Company, Incorporated
Westinghouse Electric & Manufacturing Company
Westinghouse Lamp Co.

Electricity, general (economics, utilization)

Belden Manufacturing Company
Cutler-Hammer Mfg. Co., The
Edison, Thomas A., Laboratory
Electrical Testing Laboratories
General Electric Company
Hayes, Hammond V.
Kilbourne & Clark Manufacturing Company
Western Electric Company, Incorporated
Westinghouse Electric & Manufacturing Company

Electric power (conversion, distribution, dynamos, generation, motors, power plants, transmission)

American Radio and Research Corporation
Commonwealth Edison Company

Cutler-Hammer Mfg. Co., The
Detroit Edison Company, The
General Electric Company
General Motors Research Corporation
Imperial Belting Company
Industrial Research Corporation
Lincoln, E. S., Inc.
S. K. F. Industries, Inc.
Union Carbide and Carbon Research Laboratories, Inc.
Vesta Battery Corporation

Electrochemistry (electrochemical processes, electrodes, storage batteries)

Acheson Graphite Company
Anaconda Copper Mining Co.
Andrews, A. B.
Beckman and Linden Engineering Corporation
Carborundum Company, The
Eastern Manufacturing Company
Edison, Thomas A., Laboratory
Electro Chemical Company, The
Electrolabs Company, The
Eppley Laboratory
FitzGerald Laboratories, Inc., The
Grasselli Chemical Company
Great Western Electro-Chemical Company
Grosvenor, Wm. M.
Hirsch Laboratories, Inc., The
Hooker Electrochemical Company
International Silver Company
Kidde, Walter, & Company, Incorporated
Leeds & Northrup Company
Littlefield Laboratories Co.
Mathieson Alkali Works, Inc., The
McIlhiney, Parker C.
National Lamp Works of General Electric Company
Prest-O-Lite Co., Inc., The
Riverbank Laboratories
Speer Carbon Company
Union Carbide and Carbon Research Laboratories, Inc.
United States Smelting, Refining & Mining Company
Vesta Battery Corporation
Weld and Liddell

Electro-plating

Bausch & Lomb Optical Co.
Columbia Graphophone Manufacturing Company
Crompton & Knowles Loom Works
Gillette Safety Razor Co.
Gurley, W. & L. E.
Munn, W. Faitoute
Munning, A. P., & Co.
Sears, Roebuck and Co.
Union Carbide and Carbon Research Laboratories, Inc.
Waltham Watch Company
Wheeler & Woodruff

Explosives and explosions (dynamite, powder, TNT)

Atlas Powder Co.
Barrett Company, The
Davis Chemical Products, Inc.
du Pont, E. I., de Nemours & Company
Grasselli Chemical Company
Hercules Powder Co.
Remington Arms, Union Metallic Cartridge Company
Van Schaack Brothers Chemical Works, Inc.

Fats, fatty oils and soaps

American Chemical and Manufacturing Corporation
Armour Glue Works
Armour Soap Works
Babcock Testing Laboratory
Corn Products Refining Company
Cudahy Packing Co., The
Dunham, H. V.
Electrolabs Company, The
Fort Worth Laboratories
Globe Soap Company, The
Industrial Testing Laboratories
Kalmus, Comstock & Wescott, Inc.
Kidde, Walter, & Company
Larkin Co.

Laucks, I. F., Inc.
Lehn & Fink, Inc.
Lockhart Laboratories
McIlhiney, Parker C.
Miner Laboratories, The
Mojonnier Bros. Co.
Musher and Company, Incorporated
National Lead Company
Ohio Grease Co., The
Palmolive Company, The
Peet Bros. Mfg. Co.
Procter & Gamble Co., The
Pure Oil Company, Moore Oil and Refining Company Division
Riverbank Laboratories
Schaeffer Brothers & Powell Manufacturing Company
Schwarz Laboratories
Seydel Manufacturing Company
Skinner, Sherman & Esselen, Incorporated
Souther, Henry, Engineering Co., The
Southern Cotton Oil Company, The
Swift & Company
Wells, Raymond
Wheeler & Woodruff
Wiley & Company, Inc.

Filtration

Celite Products Company
DeLaval Separator Co., The
Dorr Company, The
International Filter Co.
Oliver Continuous Filter Co.
Sperry, D. R., & Co.

Fire prevention (extinguishers, sprinklers)

Factory Mutual Laboratories
MacAndrews & Forbes Company
Underwriters' Laboratories

Foods (alimentary pastes, bakery, baking powder, biscuit, butter, candy, canning and preserving, cold storage, flavoring extracts, flour, gelatine, meat and meat products, milk, oils, preservatives, wheat, yeast, etc.)

American Can Company
American Hominy Company
American Institute of Baking
Banks & Craig
Brach, E. J., and Sons
Bridgeman-Russell Company
Brown Company
California Fruit Growers Exchange
Carus Chemical Company
Cleveland Testing Laboratory Co., The
Corn Products Refining Company
Cudahy Packing Co., The
Dunham, H. V.
Emerson Laboratory
Forth Worth Laboratories
Frees, H. E., Co., The
Gibbs Preserving Company
Glass Container Association of America
Hochstadter Laboratories
Hooker Electrochemical Company
Howard Wheat and Flour Testing Laboratory, The
Industrial Research Laboratories
Industrial Testing Laboratories
Jaques Manufacturing Company
Lehn & Fink, Inc.
Long & Co., Inc.
McLaughlin Gormley King Co.
Merrell-Soule Laboratory
Miner Laboratories, The
Mojonnier Bros. Co.
Morris & Company
Musher and Company, Incorporated
National Biscuit Company
National Canners Association
National Cereal Products Laboratories
National Laboratories, The
Nestlé's Food Company, Incorporated
Newark Industrial Laboratories
New England Confectionery Company
Nowak Chemical Laboratories

Pease Laboratories
Penick & Ford, Ltd., Incorporated
Pittsburgh Testing Laboratory
Procter & Gamble Co., The
Redlands Fruit Products Company
Rumford Chemical Works
Schwarz Laboratories
Sears, Roebuck and Co.
Seydel Manufacturing Company
Skinner, Sherman & Esselen, Incorporated
Southern Cotton Oil Company, The
Sprague, Warner & Company
Swift & Company
Takamine Laboratory, Inc.
Telling-Belle Vernon Company, The
United Chemical and Organic Products Co.
U. S. Food Products Corp.
United States Glue Co.
Wahl-Henius Institute, Incorporated
Wallace & Tiernan Co., Inc.
Washburn-Crosby Co.
Wheeler & Woodruff
Whitten, J. O., Company, The
Wilckes, Martin, Wilckes Company
Wilson & Co.

Foundry equipment, materials and methods (casting, die casting, moulding)

American Brass Company, The
Crane Co.
Doehler Die-Casting Co.
General Motors Research Corporation
Gurley, W. & L. E.
Lunkenheimer Co., The
Niles Tool Works Company, The
Pettee, Charles L. W., Laboratories of
Rhode Island Malleable Iron Works
Stockham Pipe & Fittings Co.
Union Carbide and Carbon Research Laboratories, Inc.
United States Bronze Powder Works, Inc.

Fuels (alcohol, charcoal, coal, coke, gasoline, kerosene, oil, peat. See also gas, petroleum and wood)

American Can Company
American Radiator Company
Anaconda Copper Mining Co.
Andrews, A. B.
Atlantic Refining Company, The
Babcock & Wilcox Co., The
Barrett Company, The
Bridgeman-Russell Company
Chemical Service Laboratories, Inc., The
Commercial Testing and Engineering Co.
Consolidated Gas Company of New York
Dearborn Chemical Company
Detroit Testing Laboratory, The
Dodge Brothers
Doherty Research Company, Empire Division
Electrical Testing Laboratories
Emerson Laboratory
Federal Products Company, The
General Motors Research Corporation
Gulf Pipe Line Company
Hyco Fuel Products Corporation
Industrial Chemical Institute of Milwaukee
Industrial Testing Laboratories
Interocean Oil Company, The
James Ore Concentrator Co.
Koppers Company, The
Laucks, I. F., Inc.
Lewis, F. J., Manufacturing Co.
Little, Arthur D., Inc.
Lockhart Laboratories
Martinez Refinery, Shell Co. of California
Meigs, Bassett & Slaughter, Inc.
Milwaukee Coke & Gas Company, The
Ohio Fuel Supply Company, The

Porter, Horace C.
Providence Gas Company, Incorporated
Quinn, T. H., & Company
Rhode Island Malleable Iron Works
Rodman Chemical Company
Schwarz Laboratories
Sears, Roebuck and Co.
Souther, Henry, Engineering Co., The
Standard Oil Company (New Jersey)
U. S. Industrial Alcohol Company
United States Smelting, Refining & Mining Company
Wayne Oil Tank and Pump Co.
Western Gas Construction Company, The
Western Research Corporation, Incorporated
Wheeler & Woodruff
Wiley & Company, Inc.

Fuel utilization (boilers, furnaces, gas-producers, radiators, stokers)

American Blower Company
American Radiator Company
Brooklyn Union Gas Company, The
Celite Products Company
Champion Porcelain Company
Cochrane, H. S. B. W., Corporation
Commercial Testing and Engineering Co.
Consolidated Gas Company of New York
Consolidated Gas, Electric Light and Power Company of Baltimore
Doherty Research Company, Empire Division
Hunt, Robert W., and Co.
Kidde, Walter, & Company
Koppers Company, The
Porter, Horace C.
Rhode Island Malleable Iron Works
Western Gas Construction Company, The
Wheeler & Woodruff

Gas, fuel and illuminating, including mantles (acetylene, hydrogen)

Brooklyn Union Gas Company, The
Chemical Service Laboratories, Inc.
Consolidated Gas Company of New York
Consolidated Gas, Electric Light and Power Company of Baltimore
Cosden & Company
Detroit Edison Company, The
Gulf Pipe Line Company
Harrison Mfg. Co., The
Koppers Company, The
Little, Arthur D., Inc.
Milwaukee Coke & Gas Company, The
Ohio Fuel Supply Company, The
Porter, Horace C.
Providence Gas Company, Incorporated
Standard Oil Company (New Jersey)
Union Carbide and Carbon Research Laboratories, Inc.
United Gas Improvement Co., The
Welsbach Company
Western Gas Construction Company, The
Wheeler & Woodruff

Gases, except fuel and illuminating, including generating apparatus (argon, chlorine, helium, neon, nitrogen, oxygen, poisonous gases)

Burdett Manufacturing Company
Electrolabs Company, The
Florida Wood Products Co.
Great Western Electro-Chemical Company
Hooker Electrochemical Company
Lennox Chemical Co., The

Mathieson Alkali Works, Inc., The
Union Carbide and Carbon Research Laboratories, Inc.
Wallace & Tiernan Co., Inc.
Western Gas Construction Company, The

Hair, curled, etc.

Armour Curled Hair Works
Cudahy Packing Co., The
Pfister & Vogel Leather Co.

Heat (calorimetry, pyrometry, thermal physics, thermometry)

Celite Products Company
Commonwealth Edison Company
General Motors Research Corporation
Koppers Company, The
Leeds & Northrup Company
Munn, W. Faitoute
Pyrolectric Instrument Company
Rhode Island Malleable Iron Works
Swenson Evaporator Company
Union Carbide and Carbon Research Laboratories, Inc.
Wahl-Henius Institute, Incorporated

Heating

American Blower Company
American Radiator Company
Cochrane, H. S. B. W., Corporation
Detroit Edison Company, The
Hoskins Manufacturing Company

Hydraulics (waterworks, water power)

Cochrane, H. S. B. W., Corporation
Cramp, William, & Sons Ship & Engine Building Co., The

Illumination, electric, gas and other

Brooklyn Union Gas Company, The
Commonwealth Edison Company
Consolidated Gas Company of New York
Consolidated Gas, Electric Light and Power Company of Baltimore
Cooper Hewitt Electric Company
Corning Glass Works
Harrison Mfg. Co., The
National Lamp Works of General Electric Company
Ohio Fuel Supply Company, The
Providence Gas Company, Incorporated
Union Carbide and Carbon Research Laboratories, Inc.
United Gas Improvement Co., The
Welsbach Company
Westinghouse Electric & Manufacturing Company
Westinghouse Lamp Co.

Insulation, electrical and thermal (cable, cordage, non-conductors, insulated wire)

Allen-Bradley Co.
Belden Manufacturing Company
Boonton Rubber Manufacturing Company
Carborundum Company, The
Celite Products Company
Champion Ignition Company
Condensite Company of America
Electrical Testing Laboratories
General Bakelite Company
Habirshaw Electric Cable Company, Inc.
Kellogg Switchboard and Supply Co.
Kilbourne & Clark Manufacturing Company
Redmanol Chemical Products Co.
Sangamo Electric Company
Standard Underground Cable Company
Vacuum Oil Company, Incorporated

Internal combustion motors (Diesel engines, gasoline engines, motors, oil engines)

Abbott, William G., Jr.
Bethlehem Shipbuilding Corporation, Ltd.
General Motors Research Corporation
Ingersoll-Rand Company
Standard Oil Company (New Jersey)
Studebaker Corporation, The

Iron and steel (cast iron, ferrous alloys, pipe, wrought iron)

American Chemical Paint Company
American Rolling Mill Co., The
American Sheet and Tin Plate Company
Barber-Colman Company
Borrowman, George
Buffalo Foundry and Machine Co.
Byers, A. M., Company
Carnegie Steel Company
Chase Metal Works
Cleveland Testing Laboratory Co., The
Crane Co.
Crompton & Knowles Loom Works
Crucible Steel Company of America
Diamond Chain & Manufacturing Company
Dodge Brothers
Duriron Company, Inc., The
Eastern Malleable Iron Company
Fahy, Frank P.
Fansteel Products Company, Inc.
General Motors Research Corporation
Gillette Safety Razor Co.
Houghton, E. F., & Co.
Hunt, Robert W., and Co.
Industrial Works
Inland Steel Company
Kokomo Steel and Wire Co.
Ludlum Steel Company
Lunkenheimer Co., The
Maynard, T. Poole
McNab & Harlin Manufacturing Co.
Mesabi Iron Company
Midvale Steel and Ordnance Company
Minneapolis Steel and Machinery Co.
National Malleable Castings Company, The
National Tube Company
Niles Tool Works Company, The
Peerless Drawn Steel Company, The
Pettee, Charles L. W., Laboratories of
Pierce-Arrow Motor Car Company, The
Rhode Island Malleable Iron Works
Rodman Chemical Company
Sangamo Electric Company
Stockham Pipe & Fittings Co.
Tacony Steel Company
Titanium Alloy Manufacturing Co.
Union Carbide and Carbon Research Laboratories, Inc.
United Alloy Steel Corporation
United States Smelting, Refining & Mining Company
Vanadium-Alloys Steel Co., The
Vanadium Corporation of America
Waltham Watch Company
Western Gas Construction Company, The

Leather and leather goods (boots, shoes, including machinery, leather substitutes, tanning)

Atlas Powder Co.
Carus Chemical Company
Dennis, Martin, Company, The
Durfee, Winthrop C.
Gallun, A. F., & Sons Co.
Houghton, E. F., & Co.
International Shoe Co.
Kidde, Walter, & Company
Kullman, Salz & Co.
New York Quebracho Extract Company, Incorporated
Pantasote Leather Company, The

Pfister & Vogel Leather Co.
United Shoe Machinery Corporation
Vacuum Oil Company, Incorporated

Light (optical instruments, optics. See also illumination)

American Optical Company
Bausch & Lomb Optical Co.
Case Research Laboratory
Cooper Hewitt Electric Company
Corning Glass Works
Eastman Kodak Company
Gurley, W. & L. E.
Keuffel & Esser Co.
National Lamp Works of General Electric Company
Spencer Lens Company

Liquors, fermented and distilled (alcohol, beer, wine)

Frees, H. E., Co., The
Industrial Chemical Institute of Milwaukee
Industrial Testing Laboratories
National Laboratories, The
Nowak Chemical Laboratories
Wahl-Henius Institute, Incorporated
Wiley & Company, Inc.

Lubricants (carbon, graphite, oil, petroleum)

Acheson Graphite Company
Chase Metal Works
Columbia Graphophone Manufacturing Company
Commercial Testing and Engineering Co.
Dearborn Chemical Company
Dodge Brothers
Doherty Research Company, Empire Division
Gray Industrial Laboratories, The
Industrial Testing Laboratories
Interocean Oil Company, The
Laucks, I. F., Inc.
Lockhart Laboratories
Martinez Refinery, Shell Co. of California
Maynard, T. Poole
Minneapolis Steel and Machinery Co.
Ohio Grease Co., The
Pittsburgh Testing Laboratory
Pure Oil Company, Moore Oil and Refining Company Division
Schwarz Laboratories
S. K. F. Industries, Inc.
Speer Carbon Company
Standard Oil Company (New Jersey)
Union Carbide and Carbon Research Laboratories, Inc.
Vacuum Oil Company, Incorporated
Ventura Refining Company
Wayne Oil Tank and Pump Co.
Weld and Liddell
Wells, Raymond
Western Gas Construction Company, The
Western Research Corporation, Incorporated
Wheeler & Woodruff
Wiley & Company, Inc.

Machine tools and hardware (cutlery, drill-presses, lathes, planers, shapers)

Barber-Colman Company
Brown & Sharpe Mfg. Co.
Niles Tool Works Company, The
Rochester Button Company
Stockham Pipe & Fittings Co.
United Shoe Machinery Corporation
Winchester Repeating Arms Co.

Magnetism

Electrical Testing Laboratories
Kilbourne & Clark Manufacturing Company
Leeds & Northrup Company

Marine engineering (ships)

Cramp, William, & Sons Ship & Engine Building Co., The

Matches

Diamond Match Co., The

Mechanics, general (bearings, ball, roller, etc.)

Minneapolis Steel and Machinery Co.
National Cash Register Company, The
S. K. F. Industries, Inc.

Metal manufactures, miscellaneous (fittings, pipes, valves)

Byers, A. M., Company
Crane Co.
Grasselli Chemical Company
Lunkenheimer Co., The
McNab & Harlin Manufacturing Co.
National Tube Company
Scovill Manufacturing Company
Stockham Pipe & Fittings Co.
Western Gas Construction Company, The
Winchester Repeating Arms Co.

Metallurgy and metallography, including equipment

American Brass Company, The
American Optical Company
American Sheet and Tin Plate Company
Anaconda Copper Mining Co.
Babcock & Wilcox Co., The
Bennetts' Chemical Laboratory
Borrowman, George
Bridgeport Brass Company
Buffalo Foundry and Machine Co.
Byers, A. M., Company
Calumet & Hecla Mining Company
Carnegie Steel Company
Chase Metal Works
Cleveland Testing Laboratory Co., The
Crane Co.
Crompton & Knowles Loom Works
Detroit Testing Laboratory, The
Dodge Brothers
Dorr Company, The
Duriron Company, Inc., The
Eastern Malleable Iron Company
Eustis, F. A.
Fansteel Products Company, The
FitzGerald Laboratories, Inc., The
General Electric Company
General Engineering Company, Incorporated, The
General Motors Research Corporation
Gillette Safety Razor Co.
Hirsch Laboratories, Inc., The
Hoskins Manufacturing Company
Hunt, Robert W., and Co.
Industrial Works
International Nickel Company, The
James Ore Concentrator Co.
Kalmus, Comstock & Wescott, Inc.
Kokomo Steel and Wire Co.
Lumen Bearing Company
Lunkenheimer Co., The
McNab & Harlin Manufacturing Co.
Metals & Chemicals Extraction Corporation
Midvale Steel and Ordnance Company
Minneapolis Steel and Machinery Co.
National Cash Register Company, The
National Lamp Works of General Electric Company
National Lead Company
National Malleable Castings Company, The
Niles Tool Works Company, The
Oliver Continuous Filter Co.
Peerless Drawn Steel Company, The
Pierce-Arrow Motor Car Company, The
Raritan Copper Works
Research Corporation
Rhode Island Malleable Iron Works
Rodman Chemical Company
Scovill Manufacturing Company

Sears, Roebuck and Co.
S. K. F. Industries, Inc.
Souther, Henry, Engineering Co., The
Stewart-Warner Speedometer Corporation
Studebaker Corporation, The
Titanium Alloy Manufacturing Co.
Union Carbide and Carbon Research Laboratories, Inc.
United Alloy Steel Corporation
United States Metals Refining Co.
United States Smelting, Refining & Mining Company
Vanadium Corporation of America
Wadsworth Watch Case Co., Incorporated, The
Waltham Watch Company
Wedge Mechanical Furnace Company
Westinghouse Electric & Manufacturing Company
Wheeler & Woodruff

Military and naval equipment (ammunition, armor, ordnance, small arms, torpedoes)

Abbott, William G., Jr.
Remington Arms, Union Metallic Cartridge Company
Union Carbide and Carbon Research Laboratories, Inc.
Winchester Repeating Arms Co.

Mining, general (testing drills, ropes, tools; ore dressing)

Deister Concentrator Company, The
Doherty Research Company, Empire Division
Dorr Company, The
Ingersoll-Rand Company
James Ore Concentrator Co.
Maynard, T. Poole
National Lead Company
Oliver Continuous Filter Co.
United States Smelting, Refining & Mining Company

Non-ferrous metals (aluminum, bearing metals, brass, bronze, copper, gold, lead, nickel, platinum, silver, tin, titanium, zinc)

Aluminum Company of America
American Brass Company, The
American Can Company
American Sheet and Tin Plate Company
Anaconda Copper Mining Co.
Baker & Co., Inc.
Bethlehem Shipbuilding Corporation, Ltd.
Bridgeport Brass Company
Calumet & Hecla Mining Company
Chase Metal Works
Cramp, William, & Sons Ship & Engine Building Co., The
Crane Co.
Crompton & Knowles Loom Works
Dodge Brothers
Doehler Die-Casting Co.
Eagle-Picher Lead Company, The
Fansteel Products Company, Inc.
General Motors Research Corporation
Glidden Company, The
Grasselli Chemical Company
Grosvenor, Wm. M.
Gurley, W. & L. E.
Hochstadter Laboratories
Industrial Works
International Nickel Company, The
International Silver Company
Lumen Bearing Company
Lunkenheimer Co., The
McNab & Harlin Manufacturing Co.
Metals & Chemicals Extraction Corporation
Mineral Refining & Chemical Corporation
National Canners Association
National Lamp Works of General Electric Company
National Lead Company
New Jersey Zinc Company
Niles Tool Works Company, The
Pettee, Charles L. W., Laboratories of

Radium Company of Colorado, Inc., The
Radium Limited, U. S. A.
Raritan Copper Works
Remington Arms, Union Metallic Cartridge Company
Roessler & Hasslacher Chemical Company, The
Scovill Manufacturing Company
Union Carbide and Carbon Research Laboratories, Inc.
United States Bronze Powder Works, Inc.
United States Metals Refining Co.
United States Smelting, Refining & Mining Company
Vanadium Corporation of America
Wadsworth Watch Case Co., Incorporated, The
Waltham Watch Company
Weld and Liddell

Office equipment (adding machines, cash registers)

Abbott, William G., Jr.
National Cash Register Company, The

Paints, oils and varnishes (dryers, enamels, lacquers, pigments, putty, resins, rust-proofing)

Abbott, William G., Jr.
Acme White Lead & Color Works
American Chemical and Manufacturing Corporation
American Chemical Paint Company
Andrews, A. B.
Ansbacher, A. B., & Company
Atlas Powder Co.
Ault & Wiborg Company, The
Babcock Testing Laboratory
Berry Brothers, Inc.
Borrowman, George
Boyer Chemical Laboratory Company
Buchanan, C. G., Chemical Company
Cabot, Samuel, Inc.
Carus Chemical Company
Chase Metal Works
Childs, Charles M., & Co., Inc.
Chemical Products Company
Condensite Company of America
Davis Chemical Products, Inc.
Dodge Brothers
Drackett, P. W., & Sons Co., The
du Pont, E. I., de Nemours & Company
Eagle-Picher Lead Company, The
Glidden Company, The
Grosvenor, Wm. M.
Hunt, Robert W., and Co.
Imperial Belting Company
Industrial Chemical Institute of Milwaukee
Industrial Testing Laboratories
Krebs Pigment and Chemical Co.
Lakeview Laboratories
Little, Arthur D., Inc.
Lockhart Laboratories
McIlhiney, Parker C.
Mineral Refining & Chemical Corporation
National Laboratories, The
National Lead Company
Newport Company, The
Perolin Company of America, The
Pfister & Vogel Leather Co.
Pittsburgh Plate Glass Co.
Pyro-Non Paint Co., Inc.
Redmanol Chemical Products Co.
Richardson Company, The
Rubber Trade Laboratory, The
Sangamo Electric Company
Sears, Roebuck and Co.
Skinner, Sherman & Esselen, Incorporated
Titanium Pigment Co., Inc.
Toch Brothers
Ultro Chemical Corporation
United States Bronze Powder Works, Inc.
Wayne Oil Tank and Pump Co.
Wells, Raymond
Wheeler & Woodruff
Zobel, Ernst, Company, Inc.

Petroleum and its products (see also lubricants)

Atlantic Refining Company, The

Babcock Testing Laboratory
Barber Asphalt Paving Company, The
Charlotte Chemical Laboratories, Inc.
Cosden & Company
Doherty Research Company, Empire Division
Dunham, H. V.
Gray Industrial Laboratories
Gulf Pipe Line Company
Institute of Industrial Research, The
Interocean Oil Company, The
Little, Arthur D., Inc.
Lockhart Laboratories
Martinez Refinery, Shell Co. of California
Ohio Fuel Supply Company, The
Richardson Company, The
Schaeffer Brothers & Powell Manufacturing Company
Standard Oil Company (New Jersey)
Standard Oil Company of Indiana
Union Carbide and Carbon Research Laboratories, Inc.
Vacuum Oil Company, Incorporated
Ventura Refining Company
Wayne Oil Tank and Pump Co.
Weld and Liddell
Western Gas Construction Company, The
Western Research Corporation, Incorporated
Wheeler & Woodruff
Zobel, Ernst, Company, Inc.

Phonographs and graphophones

Columbia Graphophone Manufacturing Company
Edison, Thomas A., Laboratory

Photography (cameras, developers, films, moving-picture equipment, plates)

Ansco Company
Chemical Economy Company
Corning Glass Works
Eastman Kodak Company
Grosvenor, Wm. M.
Heinrich Laboratories of Applied Chemistry
Hirsch Laboratories, Inc., The
Kalmus, Comstock & Wescott, Inc.
Munn, W. Faitoute
National Lead Company
United States Glue Co.
Zinsser & Co.

Plastics (bakelite, condensite, redmanol; casting and moulding of plastics)

Abbott, William G., Jr.
Boonton Rubber Manufacturing Company
Champion Ignition Company
Columbia Graphophone Manufacturing Company
Condensite Company of America
du Pont, E. I., de Nemours & Company
General Bakelite Company
Heap, William, & Sons
Meigs, Bassett & Slaughter, Inc.
Redmanol Chemical Products Co.
Rubber Trade Laboratory, The

Properties of engineering materials

American Brass Company, The
Borrowman, George
Carborundum Company, The
Chicago Mill and Lumber Company
Columbia Graphophone Manufacing Company
Electrical Testing Laboratories
General Electric Company
Hunt, Robert W., and Co.
Industrial Works
Institute of Industrial Research, The
Kokomo Steel and Wire Co.
Maynard, T. Poole
National Association of Corrugated and Fibre Box Manufacturers, The

Pennsylvania Railroad Company, The
Pierce-Arrow Motor Car Company, The
Scovill Manufacturing Company
Skinner, Sherman & Esselen, Incorporated
Stewart-Warner Speedometer Corporation
Swenson Evaporator Company
Union Carbide and Carbon Research Laboratories, Inc.
United Shoe Machinery Corporation

Public utilities

Detroit Edison Company, The
Doherty Research Company, Empire Division
Stone & Webster, Incorporated

Pulp and paper (cellulose)

American Writing Paper Co.
Andrews, A. B.
Atlas Powder Co.
Babcock Testing Laboratory
Beaver Board Companies, The
Brown Company
Carborundum Company, The
Chemical Economy Company
Chemical Products Company
Chicago Mill and Lumber Company
Crane & Co.
Cumberland Mills
Davis Chemical Products, Inc.
Dill & Collins Co.
du Pont, E. I., de Nemours & Company
Eastern Manufacturing Company
Eastman Kodak Company
Emerson Laboratory
Glysyn Corporation, The
Grosvenor, Wm. M.
Hammersley M'f'g Co., The
Heap, William, & Sons
Hooker Electrochemical Company
Industrial Testing Laboratories
Little, Arthur D., Inc.
MacAndrews & Forbes Company
Meigs, Bassett & Slaughter, Inc.
Metals & Chemicals Extraction Corporation
Munn, W. Faitoute
National Association of Corrugated and Fibre Box Manufacturers, The
National Laboratories, The
Oliver Continuous Filter Co.
Package Paper and Supply Corporation
Richardson Company, The
Skinner, Sherman & Esselen, Incorporated
Strathmore Paper Company
Van Schaack Brothers Chemical Works, Inc.
Wallace, Joseph H., & Co.
Weston, Byron, Co.
Wiley & Company, Inc.

Railroad equipment (cars, locomotives, signals, etc.)

Baldwin Locomotive Works, The
Hunt, Robert W., and Co.
Industrial Works
Niles Tool Works Company, The
Pennsylvania Railroad Company, The
Union Switch & Signal Company

Razors

Gillette Safety Razor Co.

Refrigeration (artificial ice)

American Radiator Company
Ansul Chemical Company
General Motors Research Corporation
Industrial Research Corporation
Industrial Testing Laboratories
International Filter Co.

Rubber and rubber goods, including other natural gums (gutta-percha)

Abbott, William G., Jr.
Belden Manufacturing Company
Boonton Rubber Manufacturing Company

Brunswick - Balke - Collender Co., The
Carborundum Company, The
Columbia Graphophone Manufacturing Company
Dodge Brothers
Falls Rubber Company, The
Firestone Tire & Rubber Company
General Bakelite Company
General Tire & Rubber Co.
Goodrich, B. F., Company, The
Goodyear Tire & Rubber Company, The
Hood Rubber Company
Manhattan Rubber Mfg. Co., The
Miller Rubber Co., The
Portage Rubber Co., The
Redmanol Chemical Products Co.
Rubber Trade Laboratory, The

Soils and fertilizers (nitrates, phosphates, potash)

American Agricultural Chemical Company, The
American Cyanamid Company
American Trona Corporation
Anaconda Copper Mining Co.
Armour Fertilizer Works
Cudahy Packing Co., The
Detroit Testing Laboratory, The
Grasselli Chemical Company
Maynard, T. Poole
Meigs, Bassett & Slaughter, Inc.
Metals & Chemicals Extraction Corporation
Morris & Company
Royster, F. S., Guano Company
Sears, Roebuck and Co.
Swift & Company
United Chemical and Organic Products Co.
United States Glue Co.
Utah-Idaho Sugar Company
Wiley & Company, Inc.

Sound (acoustics)

Columbia Graphophone Manufacturing Company
Hayes, Hammond V.
Riverbank Laboratories
Sabine, Wallace Clement, Laboratory

Steam power (boilers, economizers, engines, turbines. See also internal combustion motors)

American Radiator Company
Babcock & Wilcox Co., The
Bethlehem Shipbuilding Corporation, Ltd.
Cochrane, H. S. B. W., Corporation
Commercial Testing and Engineering Co.
Detroit Edison Company, The
Ingersoll-Rand Company
Lunkenheimer Co., The
Minneapolis Steel and Machinery Co.

Subatomic phenomena and radioactivity

Radium Company of Colorado, Inc., The
Radium Limited, U. S. A.
Riverbank Laboratories
Welsbach Company

Sugar (sorghums, syrups)

American Beet Sugar Company
American Diamalt Company
American Sugar Refining Company, The
Dehls & Stein
Digestive Ferments Co.
Feculose Co. of America
Great Western Sugar Company, The
Industrial Chemical Institute of Milwaukee
Industrial Testing Laboratories
New York Sugar Trade Laboratory, Inc., The
Nulomoline Company, The
Oliver Continuous Filter Co.
Penick & Ford, Ltd., Incorporated
Schwarz Laboratories
Spreckels Sugar Company
Swenson Evaporator Company

U. S. Food Products Corp.
Utah-Idaho Sugar Company
Western Sugar Refinery

Surgical, dental and hospital equipment and supplies

Caulk, L. D., Company, The
Johnson & Johnson
Lakeview Laboratories
Ransom & Randolph Co., The
Union Carbide and Carbon Research Laboratories, Inc.

Tar and its products

Barrett Company, The
Glysyn Corporation, The
Koppers Company, The
Laucks, I. F., Inc.
Lyster Chemical Company, Inc.
Providence Gas Company, Incorporated
Quinn, T. H., & Company
Rubber Trade Laboratory, The
Union Carbide and Carbon Research Laboratories, Inc.
White Tar Company of New Jersey, Inc., The
Zobel, Ernst, Company, Inc.

Textiles, including machinery (buttons, clothing, cotton and its products, linen, wool; waterproofing)

Abbott, William G., Jr.
Amoskeag Manufacturing Company
Arlington Mills
Art in Buttons
Barber-Colman Company
Chemical Products Company
Crompton & Knowles Loom Works
Durfee, Winthrop C.
Eastern Finishing Works, Inc.
Eavenson & Levering Co.
Emerson Laboratory
Glysyn Corporation, The
Grosvenor, Wm. M.
Imperial Belting Company
Industrial Chemical Institute of Milwaukee
Klearflax Linen Rug Company
Little, Arthur D., Inc.
Maynard, T. Poole
Metakloth Co.
Rochester Button Company
Roessler & Hasslacher Chemical Company, The
Rubber Trade Laboratory, The
Sears, Roebuck and Co.
U. S. Testing Co., Inc.

Water, sewage and sanitation

American Institute of Baking
Babcock & Wilcox Co., The
Banks & Craig
Borromite Co. of America, The
Borrowman, George
Bridgeman-Russell Company
Carus Chemical Company
Cochrane, H. S. B. W., Corporation
Dearborn Chemical Company
Detroit Testing Laboratory, The
Dorr Company, The
Emerson Laboratory
Fort Worth Laboratories
Great Western Electro-Chemical Company
Hochstadter Laboratories
Hooker Electrochemical Company
Industrial Chemical Institute of Milwaukee
Industrial Testing Laboratories
International Filter Co.
Kidde, Walter, & Company, Incorporated
Oliver Continuous Filter Co.
Pease Laboratories
Permutit Company, The
Perolin Company of America, The
Souther, Henry, Engineering Co., The
Wallace & Tiernan Co., Inc.
Wells, Raymond
Weston & Sampson
Wheeler & Woodruff
White Tar Company of New Jersey, Inc., The

Welding, autogenous, gas, electric, forge

- Bethlehem Shipbuilding Corporation, Ltd.
- Davis-Bournonville Company
- Electrolabs Company, The
- Hoskins Manufacturing Company
- Union Carbide and Carbon Research Laboratories, Inc.
- Western Gas Construction Company, The

Wire

- Belden Manufacturing Company
- Hoskins Manufacturing Company
- Kokomo Steel and Wire Co.
- Scovill Manufacturing Company

Wood products, other than cellulose and paper (see also containers)

- Andrews, A. B.
- Babcock Testing Laboratory
- Chicago Mill and Lumber Company
- Florida Wood Products Co.
- Hercules Powder Co.
- Lakeview Laboratories
- Nowak Chemical Laboratories
- Quinn, T. H., & Company
- Rodman Chemical Company
- Wallace, Joseph H., & Co.
- Zobel, Ernst, Company, Inc.

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Graber, Howard T., Digestive Ferments Co., Detroit, Mich.
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Moore, Thomas E., The Ransom & Randolph Co., 518 Jefferson Ave., Toledo, Ohio.
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Morse, H. E., The Goodyear Tire & Rubber Company, Akron, Ohio.
Morton, H. A., The Miller Rubber Co., Akron, Ohio.
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SCIENTIFIC PAPERS PRESENTED BEFORE THE AMERICAN GEOPHYSICAL UNION AT ITS SECOND ANNUAL MEETING

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1922

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BULLETIN
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Vol. 3, Part 2 MARCH, 1922 Number 17

SCIENTIFIC PAPERS
PRESENTED BEFORE THE AMERICAN GEOPHYSICAL UNION AT ITS SECOND ANNUAL MEETING

CONTENTS

Section of Geodesy

Section of Meteorology

Section of Terrestrial Magnetism and Electricity

Section of Physical Oceanography

These papers were presented at the second annual meeting of the American Geophysical Union held at the National Research Council, Washington, D. C., April 18, 19 and 20, 1921. The names of three of the sections (seismology, volcanology and geophysical chemistry) do not appear in the table of contents, as these meetings were devoted to discussion and no scientific papers were presented.

The American Geophysical Union is the Committee on Geophysics of the National Research Council and is the National Committee for the United States of the International Geodetic and Geophysical Union.

MEASUREMENT OF GRAVITY AT SEA

A REVIEW

BY LYMAN J. BRIGGS

METHODS OF DETERMINING GRAVITY AT SEA

The measurement of the acceleration of gravity over the oceans is a matter of interest to the geodesist in the determination of the figure of the geoid and in investigations relating to isostatic compensation. For the requirements of these problems it is desirable that the probable error of the gravity determinations should not exceed one part in 50,000. At first sight this accuracy does not seem to be unattainable, particularly to those who are familiar with the remarkable work that has been done with invariable pendulums on land stations, where the probable error has been reduced to two or three parts in a million. But when we consider that in measurements at sea the vertical acceleration of the ship is imposed upon the gravitational acceleration which we are trying to measure, the difficulty of the problem becomes apparent. For example, in the case of a ship which rises and falls through a height of a meter during a period of 10 seconds, the average vertical acceleration without regard to sign is about 0.004g, or 200 times the permissible probable error of the measurement. Such vertical accelerations of the ship are not uncommon, although the sea is sometimes so smooth that an index point may be set in grazing contact with the surface of a mercury column with almost laboratory precision.

The oscillations of the sensitive element due to the vertical accelerations of the ship may be controlled by damping, but it is doubtful whether the damped system gives the same result as when vertical accelerations are absent. This question cannot be answered from observations at sea, and necessitates laboratory methods of testing gravity apparatus which will be referred to later.

Virtually all the methods that have actually been used for gravity determinations at sea involve and depend upon observations of the length of a mercurial column supported by gas pressure. The pressure p of the gas may be equated to $\rho\, g\, h$ where ρ and h refer to the density and height of the mercury column. Knowing p, ρ and h, the acceleration of gravity g can at once be evaluated. The various methods differ in the manner in which p and h are determined.

Siemens' method: The first actual measurements of the variation of gravity at sea appear to have been made by Sir Wm. Siemens[1] in 1875. He believed that at sea the value of g was diminished by an amount very nearly proportional to the depth of the ocean, and his primary purpose

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[1] C. Wm. Siemens. On determining the depth of the sea without the use of a sounding line. *Phil. Trans.*, 167, 1877, 671-692.

was to develop a sounding apparatus on this principle as an aid to navigation.

His first instrument consisted of a barometer with a large sealed-off air chamber to eliminate the effect of variations in atmospheric pressure (fig. 1). The barometric column included three liquids—mercury, diluted alcohol and juniper oil. The vertical column was expanded into a bulb *b*, at the level where the mercury and alcohol surfaces were in contact, with a second evacuated bulb *d* at the top, which contained the free juniper-oil surface. The readings were made on the position of the alcohol-oil surface *c* in the constricted tube joining the two bulbs. With this device the scale deflection for a given change in g was 300 times that of a simple mercurial column.

This instrument proved unsatisfactory and was abandoned in favor of a second "bathometer," which consisted of a steel spring balance of a peculiar type. A vertical steel tube was fitted with reservoirs at top and bottom, the floor of the lower reservoir consisting of a thin corrugated steel diaphragm. This system contained mercury, the free surface of the mercury being in the upper reservoir, which during measurements was open to the atmosphere. The load on the diaphragm was carried by two long steel spiral springs connected to a yoke beneath the diaphragm and suspended from the upper reservoir. The mercury constituted the load on the balance and at the same time served to damp the oscillations through the action of a constriction in the tube just below the upper reservoir. The change in the load due to a change in g resulted in a vertical displacement of the yoke and was measured by means of a micrometer screw supported from the lower reservoir. The observations required corrections for temperature and atmospheric buoyancy. The instrument was not checked by testing it at two land stations where the relative value of g was accurately known. His results show, however, a remarkable correlation with the depth as obtained by direct soundings, which were made immediately after the bathometer readings. In a series of about 30 observations involving depths up to about 2,500 fathoms, the discrepancy was seldom more than 10 per cent. No corrections were applied for variation in latitude, which ranged from 45° to 49° N.

Hecker's method: The appearance of Helmert's equation for the variation of g with latitude, based on land stations, led Hecker[1] to undertake the task of providing data to test its validity for sea stations as well. In 1901 he began an extended series of gravity measurements at sea which eventually included systematic observations in the Atlantic, Pacific and Indian oceans and in the Black Sea.

In using a barometric column for measuring g, two procedures are available: (1) the air chamber at the base of the column may be sealed,

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[1] For a description of the apparatus employed by Hecker and a summary of his ocean measurements, see Hecker, O., Bestimmung der Schwerkraft auf dem Schwarzen Meere und, an dessen Küste sowie neue Ausgleichung der Schwerkraftmessungen auf dem Atlantischen, Indischen, und Groszen Ozean, *Zentralbur. Internat. Erdmessung*, Veröffentlichungen Berlin, N. F., Nr. 20 (1910).

Fig. 1. Siemens' first apparatus.

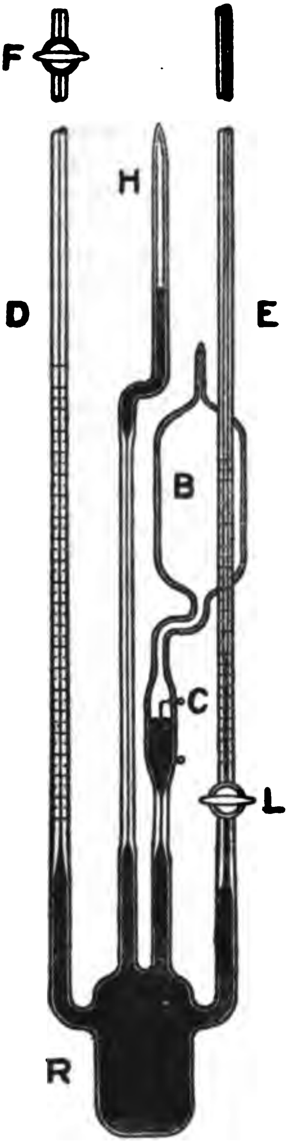

Fig. 2. Duffield's apparatus.

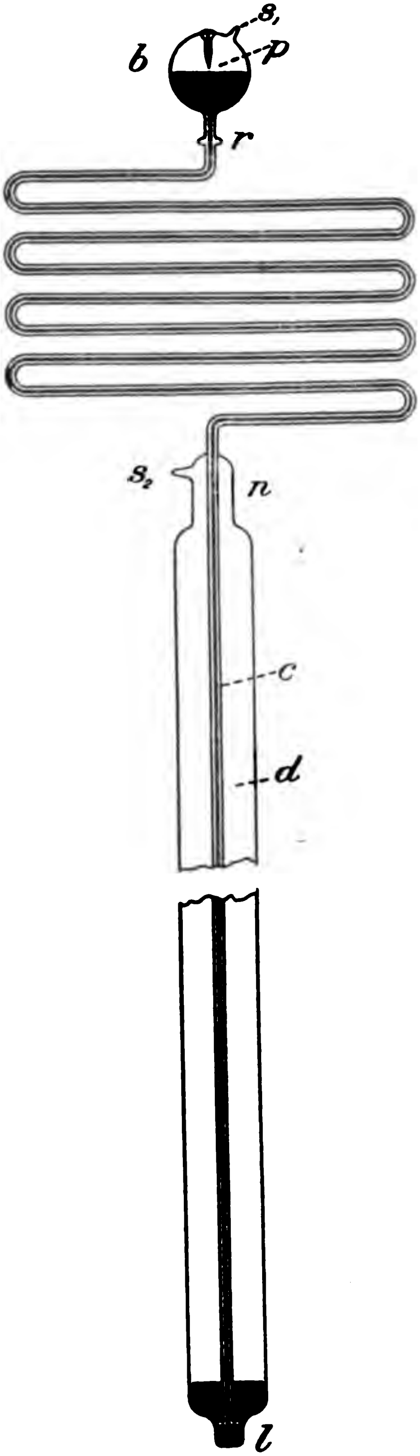

Fig. 3. Briggs' apparatus.

as in Siemens' first instrument, in which event we eliminate variations in pressure due to atmospheric changes, but are left with an instrument which is very sensitive to temperature changes, as it becomes in effect a gas thermometer; or (2) we may leave the air-chamber open to the atmosphere; in this case the temperature effects are greatly reduced, but it is necessary to measure the air-pressure p by some independent means.

Hecker in his pioneer investigations chose the latter procedure and determined the pressure from boiling-point measurements of water referred to vapor-pressure tables. The vapor-pressure of water increases very rapidly with temperature in the neighborhood of the boiling-point,—approximately one twenty-eighth of an atmosphere per degree. Therefore, in order to determine the pressure to 1 part in 50,000 it is necessary to know the temperature interval from freezing-point to boiling-point with an error not greater than 0°.0006 C.

Hecker used mercury thermometers in his boiling-point determinations. He had no means of checking the fundamental intervals of these thermometers at sea, for this determination depends upon accurate barometric pressure measurements which can be obtained only at land stations where the value of g is known. Consequently any departure of the fundamental interval from that determined at land stations enters directly into the boiling-point determinations at sea as a systematic error. Furthermore, the best of mercurial thermometers exhibit variations in the fundamental interval. For example, Waidner and Dickinson[1] found that the fundamental interval of the primary mercurial standards of the Bureau of Standards varied through a range of 0°.015 C. during a ten-day period, which would correspond to a variation of more than one part in 2,000 in the value of g. The probable error of the fundamental interval determinations in Waidner and Dickinson's measurements under favorable laboratory conditions was ±0°.003 C., which may be taken as a measure of the maximum refinement obtainable in barometer-hypsometer measurements aboard ship; and this corresponds to a probable error in the value of g of more than one part in 10,000.

If hypsometer determinations are to be made, resistance thermometry would be preferable to mercurial thermometry, since the resistance thermometer is more sensitive and shows less variation from day to day in the fundamental interval. The steam point of a resistance thermometer can be readily determined under laboratory conditions to 0°.002 C., but whether this accuracy could be obtained on board ship with the galvanometer on an unstable base is questionable.

In my opinion, the barometer-hypsometer method is not the most promising way of attacking the problem of measuring g at sea, because (1) the method involves two operations; (2) the temperature errors in hypsometry lead to errors in the derived value of g which are ten times as great as those produced by equal temperature errors in a closed system; and (3) the motion of the ship relative to the air produces a change in baro-

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[1] *Bull. Bur. Standards*, 3663, Washington, D. C. (1907).

metric pressure below deck. If this relative motion is unsteady from any cause, as for example variable winds, errors may result unless the barometer and hypsometer are read simultaneously. Duffield (1921) on board a destroyer observed pressure effects of this kind as large as one millibar.

The use of a sensitive aneroid barometer has been proposed as a substitute for hypsometric measurements, but here again a double operation is involved. If a spring system could be devised which would be sufficiently sensitive and reliable to measure pressure to the required degree of accuracy, it would be better to employ it directly as a force balance to measure the change in g than to equate the observed pressure to the observed length of a mercury column, for the double operation serves only to increase the probable error of the final result.

Duffield's method: Duffield[1] in 1914 employed the apparatus shown in figure 2 in some preliminary measurements of g during a voyage from Australia to England. The apparatus, which is of the sealed gas-chamber type, possesses a unique and valuable temperature-compensation feature. A constant volume of air is maintained in the bulb B by keeping the mercury always up to the electrical contact at C. The air in the bulb B is under reduced pressure in order to reduce the length of the apparatus. The barometer tube is bent so that H is vertically above C, the length of the column HC being approximately 20 centimeters. The mercury level is kept at C by raising or lowering the mercury in the index tube D. This operation is effected by slowly exhausting or admitting air through F as required. The index tube D is of fine bore and the value of g is calculated from readings upon the level of the mercury in this tube when contact is made with the pointer at C. The side tube E is used only for the purpose of making initial adjustments, and to permit the apparatus to be used for various ranges of temperature.

The reservoir of mercury R is introduced for the purpose of temperature compensation and when the dimensions of the apparatus are suitably chosen the increased pressure of air in the bulb B due to a given rise of temperature is automatically counterbalanced by the rise of the level H, occasioned by the expulsion of mercury from the reservoir. The compensation is perfect at only one temperature, but for small departures the error is small. At sea the apparatus was immersed in a water bath which was hung by cords from the ceiling of the refrigerator room of the ship, and readings on the index tube were taken through a window in the side.

Duffield's apparatus as reconstructed in Australia was over compensated, an increase in temperature of 1° C. necessitating the removal of a thread of mercury 60 mm. long. In other words, a temperature change of this amount resulted in a change in the reading of the index scale corresponding to the computed change in g in going from the equator to 54° N. Lat. Trouble was also experienced from a break in the capillary

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[1] W. G. Duffield. Apparatus for the determination of gravity at sea. *Proc. Roy. Soc. Lond.* (A), 112, 1916, 505-517.

mercurial column in the barometer tube. Owing to these difficulties actual observations were limited to a series of 25 preliminary measurements in the Indian Ocean from Lat. 0° to 16° N. These results show an average deviation with regard to sign of +0.01 cm./sec.^{2} from values computed from Helmert's equation; in other words, a chance distribution of the observations about the line representing Helmert's computed values is indicated. The average deviation of the observations without regard to sign is 0.14 cm./sec.^{2}. The anomalies are, however, not known. If we assume that there are no anomalies, this corresponds to an average error of 1.4×10^{-4}. It is interesting to note in this connection that Schuster computes from the dimensions of Duffield's apparatus a maximum error due to pumping of about 3.6×10^{-4} for vertical motions of the ship of one meter amplitude.

In a recent article Duffield[1] has given a brief description of tests made with apparatus other than his own during his voyage to Australia in 1914. This included instruments of the sealed-cistern barometer type constructed by Prof. Hecker and an aneroid barometer supplied by Sir Horace Darwin. No quantitative results are given.

Schuster[2] has contributed a valuable analysis of the effects of forced vibrations which may be imposed on the mercury in gravity apparatus by the vertical acceleration of the ship. This includes a discussion of (1) a single constricted barometer tube, (2) the oscillations in a complex interconnected system of three tubes as in Duffield's apparatus, and (3) the experimental errors in the latter apparatus as affected by the relative dimensions of the various parts. He emphasizes the importance of the condition that the flow of mercury in the barometer tube and contact tube be such that the difference in level is always that of hydrostatic equilibrium. This condition is fulfilled if the cross-sections of the capillaries are equal and the lengths of the capillaries are inversely as the reduced[3] cross-sections of the tubes at the free surfaces of the mercury.

Briggs' method: Briggs[4] employed apparatus similar to that shown diagramatically in figure 3 for gravity observations during a voyage from Sydney to San Francisco in 1914 and again from New York to San Francisco via Panama in 1915. This apparatus is of the closed-barometer

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[1] The investigation of gravity at sea. *Nature*, **106**, 1921, 732-734.

[2] Arthur Schuster. On the determination of gravity at sea (Note on Dr. Duffield's paper). *Proc. Roy. Soc. Lond.* (A) **112**, 1916, 517-528.

[3] The reduced cross-section a represents the actual cross-section a corrected for the effect of the pressure and volume of the air above the surface of the mercury. Let

V=the original volume of air.
P=the original air pressure.
h=the height of the mercury column equivalent to P.

$$\text{Then } \frac{1}{a_1} = \frac{1}{a} + \frac{h}{V}$$

[4] L. J. Briggs. A new method of measuring the acceleration of gravity at sea. *Proc. Nat. Acad. Sci.* **2**, 1916, 399-407.

type. The mercurial column is contained in the capillary c (bore 0.6–0.7 mm.), the lower end of which opens beneath mercury in the bottom of the gas chamber d. This capillary is sealed to the wall of the gas-chamber where it passes through the upper end. The upper part of the capillary is bent into a flexible zigzag and ends in the spherical bulb b (diameter 2 cm.). The bulb contains a fixed iron point p sealed to the inside of the bulb by means of an inserted platinum wire and extending vertically downward, so that the point is approximately at the center of the bulb. The length of the mercurial colunm is about 74 cm.

The flexible capillary permits a slight vertical movement of the observing bulb with respect to the gas chamber. This movement is determined by a micrometer screw of 1 mm. pitch which controls the motion of a carriage in which the observing bulb is rigidly mounted. The carriage slides on parallel rods mounted on a base which is rigidly cemented to the neck n of the gas chamber, so that the position of the bulb relative to the gas chamber is definitely determined by the screw.

The apparatus is protected by a close-fitting metallic jacket, and is kept at a constant temperature in a bath of melting ice. It is necessary to determine only the position of the upper end of the barometric column. The design of the instrument is such that in setting the index in contact with the mercury surface the enclosed gas is automatically reduced to a constant volume; and since the temperature is constant, all measurements are made at constant pressure. The relative value of g at two stations is thus inversely proportional to the observed length of the column at these stations.

The contact of the index point with the mercury surface can be determined either electrically or by direct observation. Both methods were used. In the latter case the fixed point was observed through a glass tube introduced through the ice, the tube containing a low-power lens. The point was illuminated through a similar tube on the opposite side. If the sea was so rough as to cause pumping of the column, the point was so adjusted that it was in contact half the time as nearly as possible. Since the motion of the ship is not strictly periodic, there is considerable uncertainty connected with such settings.

This instrument possesses the following features which experience has shown are desirable in gravity apparatus: (1) The glass part of the apparatus is hermetically sealed and can be made really gas-tight. There are no stop-cocks, ground joints, or mercury seals. (2) It is necessary to make settings only at the upper end of the barometric column. (3) This permits the complete immersion of the apparatus in an ice-bath, which is the most dependable source of constant temperature for use on shipboard. (4) The apparatus is portable, since at room temperatures the pressure is sufficient to fill the observing bulb with mercury.

Gravity determinations which were made in 1914 on board ship in Wellington harbor and Sydney harbor using observations in San Francisco harbor as a base station, and in 1915 in San Francisco harbor using New

York harbor as a base, show an average departure from pendulum observations of about one part in 50,000. These observations were made under favorable conditions and serve to show the degree of accuracy with which the instruments held their adjustments during the long voyages rather than to provide any indications of the accuracy of the sea observations. In fact, the publication of the sea measurements, which show some rather large anomalies, has been withheld in the hope that apparatus similar to that described in the last section of this paper might be available to determine the errors of the instruments under oscillations approximating sea conditions.

CORRECTIONS FOR THE COURSE AND SPEED OF THE SHIP

Eötvös[1] has shown the necessity of applying a correction for the easterly or westerly motion of the ship, due to the fact that the ship's motion modifies the angular velocity of the apparatus about the earth's axis. The centrifugal force acting on the mercurial column when on board a ship moving east or west is therefore not the same as when the ship is at rest or moving north or south. The correction may be as great as 1 part in 10,000, but can be accurately computed if the course, speed, and approximate latitude of the ship are known. Duffield (1921) observed a change equivalent to 0.1 millibar when the course was altered from east to west when steaming at 22 knots, which corresponds to an apparent change of 1 part in 10,000 in *g*. He does not state where his experiments were made. For Lat. 55°, which represents approximately the mean latitude of the North Sea, the computed change is about 0.19 $cm./sec.^2$, or 2 parts in 10,000 in the value of *g*.

LABORATORY TESTING OF GRAVITY APPARATUS

There seems to be no practical way of determining the accuracy of gravity measurements directly from sea observations. With a ship at one's command, repeated traverses could be made of the same sea station under varying sea and weather conditions; but while this would provide a measure of the accidental errors, it would tell us nothing regarding systematic errors, for the exact value of *g* at the station would not be known. The value of *g* in a long, narrow bay could be closely approximated from pendulum observations on both shores. To satisfy other requirements the bay would have to be sufficiently open and windswept to represent the conditions prevailing in a moderate sea. Such a test seems precluded without the enlistment of government aid.

We can, however, simulate sea conditions at a land station where *g* is accurately known and I wish to emphasize the importance of such tests for all gravity apparatus. Suppose we construct a platform capable of independent reciprocating horizontal translations in two directions at right angles, corresponding in period to the roll and pitch of the ship. Let us mount upon this platform a second one arranged for vertical oscillations,

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[1] See Helmert, *loc. cit.*

large enough to carry the apparatus and the observer. All the reciprocating motions are to be capable of a continuous change in amplitude if desired in order to secure the conditions which arise when the period of the ship differs from that impressed by the waves. We have omitted the motions corresponding to the angular motions of the deck, for structural reasons; for these angular motions would only increase the rotation of the apparatus in its gimbals, and there will always be sufficient movement of the gimbals arising from the horizontal accelerations to simulate disturbances due to friction. With this apparatus it will be possible to investigate the effects of horizontal and vertical accelerations, singly and combined, under conditions where *g* is accurately known and thus obtain a measure of the accidental and systematic errors of the method. Independent rolling and pitching oscillations are probably not required for gravity apparatus, but the equipment proposed has other useful applications, as for example in testing gyroscopic compasses and gyroscopic stabilizers, in which both rolling and pitching accelerations should be included. Plans for such an equipment are now being prepared, and it is hoped that arrangements can soon be made for its construction and installation at the Bureau of Standards, where it will be available for testing all gravity apparatus.

Bureau of Standards,
Washington, D. C.

ISOSTASY

By John F. Hayford

I assume that I am expected to bring forward whatever ideas will, in my opinion, help most at the present time in the development of a study of isostasy by stimulating further thinking along that line. From among the many topics which might be treated, I am selecting three:

(1) I shall make some remarks on the recent paper entitled "The Chemistry of the Earth's Crust," by H. S. Washington.

(2) I shall try to emphasize the desirability of an intensive study of two small areas by observations of gravity and deflections of the vertical.

(3) I shall offer some considerations that lead me to believe that the undertow involved in isostatic readjustment is above, rather than below, the depth of compensation.

CHEMISTRY OF THE EARTH'S CRUST

Dr. Washington, in his paper entitled "The Chemistry of the Earth's Crust,"[1] has set forth a piece of research of much importance to those who are studying isostasy. He has set forth the evidence derived from studies of the densities of igneous rocks. These densities are determined from chemical analyses. In the latter part of the paper he sets forth the correlation which is observable between the density of igneous rocks, on

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[1] *Journal of the Franklin Institute*, 190, December, 1920, 757-815.

the one hand, and, on the other hand, the elevation of that part of the earth's surface under which the rocks lie.

The evidence given in the paper seems to be conclusive in its general features on three points, namely:

(1) That igneous rocks under the oceans are denser than those under the continents;

(2) That igneous rocks under the various continents are less dense the greater the mean elevation of the continent; and

(3) That igneous rocks under different parts of any one continent are less dense the higher is that part of the continent.

All three of these conclusions are in accord with the theory of isostasy and, in Dr. Washington's words, constitute "almost a conclusive proof of the general validity of the theory of isostasy."

Dr. Washington seems to interpret the relations pointed out as being due to original, or early, segregation of the material. I do not question this interpretation so far as (1) is concerned. The relative positions of the oceans and continents are permanent or semi-permanent. It may well be that the oceans are now in their present position because material of such a nature as to form dense rocks was placed early in the positions now occupied by the oceans.

The same interpretation may, however, be questioned, in part, in so far as (2) and (3) are concerned. Are the differences in densities between the different continents referred to in (2) and between different parts of any one continent referred to in (3) due to original or early segregation? Or are these differences due, in part at least, to some response of the material to a change of pressure in such a manner as to bring about a change of density? The continents probably have not had the same relative elevations throughout geologic time as they now have. Certainly, the different parts of any one continent have not, in general, the relative elevations now that they had at various times during the geologic history of the continents. For example, the Appalachian region, referred to by Dr. Washington, in the eastern part of the United States, is now much lower than Utah, Colorado, and Nevada, and the density of the material, as measured by him, is considerably greater under the Appalachian region than under the states named. There was, however, a time during the geologic history of the North American continent when the Utah-Colorado-Nevada region was much lower than the Appalachian region was at that time. Have the relative densities in the two regions changed between that period and the present?

This comment on Dr. Washington's paper is not intended at all as a criticism; it is intended to supplement the paper and to stimulate further thought based on it. All who are interested in isostasy should be very keen to follow the further develpoments along the line indicated by Dr. Washington's paper. I understand that he, himself, will develop the evidence much more fully. The more complete evidence should be examined very carefully, with a view to determining the bearing of this evidence on any theories which may be held in regard to isostasy and the isostatic readjustment.

PROPOSED INTENSIVE STUDY OF SMALL AREAS

We shall take a short time to consider the possible benefits which would follow from an intensive study of two small areas, let us say about 100 miles square, one on rather flat country, such as Louisiana, and another in hilly or mountainous country. The intensive study would be based on closely spaced stations of two kinds, stations at which deflections of the vertical are determined and gravity stations. Let us suppose that observations of these two kinds were made in each of two such areas and then that, by the proper office methods and by the combination of the two kinds of observations, the distribution of the densities beneath each of the two areas was determined with as great accuracy as is possible. What benefits would follow from two such studies? Briefly, I believe that the benefits would be as indicated in the four paragraphs which follow:

1. The studies would bring out the actual advantages and limitations of this line of attack on the problem of determining the distribution of densities beneath the earth's surface. The relative strength or weakness of this line of attack, as compared with the usual line of attack, is not now well known. The usual line of attack is to use deflections of the vertical stations or gravity stations which are widely distributed somewhat uniformly over a very large area.

2. The studies would probably furnish a considerable amount of evidence on the point which has been ably brought out by Dr. William Bowie that the small anomalies in gravity and in deflections of the vertical which remain after correcting for topography and isostatic compensation are closely related in many regions, if not as a rule, to the surface geology of those regions. It is important that the extent to which this is true should be determined. There seems to be no doubt that it is true, in a general way, for large areas. The question is, to what extent is it true for small areas?

3. The studies would give valuable indirect evidence as to the extent to which the present conclusions from the evidence now available are vitiated by local effects which are at present unavoidably assumed, for want of more exact information, to extend half way to the next station, so to speak.

4. The two intensive studies would help to determine whether oil and salt may be located by geodetic measurements. This suggestion has been made at various times by Dr. David White, Chief Geologist of the U. S. Geological Survey. It is important to know whether gravity stations and deflections of the vertical may be used as a divining rod for that purpose with sufficient accuracy. If the accuracy of such a divining rod is so low that its indications are likely to be misleading, then it is not feasible to try to use the observations in this way. On the other hand, if two such intensive studies as are indicated here show that sufficient accuracy is

possible, then certainly the method should be used, in combination possibly with other methods. The two intensive studies might possibly be made in regions in which salt or oil are believed to exist, or are known to exist, in large quantities.

The suggestion that one study should be in rather flat country and the other in rather hilly country is based on two considerations. In very flat country the conclusions reached will be vitiated to a much smaller extent by an error in the assumed surface density of the material than in rough country. On the other hand, in hilly or mountainous country the depth at which any material of abnormal density lies may be determined with greater accuracy than in flat country.

DEPTH OF UNDERTOW

Let us now turn to the question, is the undertow involved in isostatic readjustment above or below the depth of compensation?

Assume that, at some time in the remote geologic past, the North American continent and its various major parts have been almost completely compensated in the isostatic sense. Assume that, in later geologic past time and up to the present time, there have been large amounts of erosion from large portions of the continent and corresponding large amounts of deposition in other parts. Assume that in that period and up to the present, readjustment toward isostatic conditions has been in progress by horizontal transfer of material from beneath the regions of deposition towards the regions of erosion.

It is reasonably certain that, on the whole, these three assumptions are true. The question on which it is desired to concentrate attention is, has the horizontal transfer taken place below or above the depth of compensation?

It is important to secure the correct conclusion on this point, whatever it is, because the apparent correlation of geodetic and geologic evidence, or the apparent contradictions between two lines of evidence, probably depend somewhat intimately upon the conclusion reached. I believe that the horizontal transfer has taken place above the depth of compensation, say within 100 kilometers of the surface, rather than below that depth. I propose to state, very briefly, some of the lines of thought that have led me to that conclusion.

If it is assumed that, under an area of deposition, the material down to the depth of compensation all sinks under the added load and that the horizontal transfer of material occurs below that depth, the case is similar to that of an ice floe. Under each elevation on the upper surface, in this case, there must develop a much larger bump on the lower surface of the floating mass. The conception is that of a crust floating on a relatively plastic stratum. The level of compensation, in this case, is at the lower side of the floating crust. As in this case there must be extensions of the crust downward, below the mountains, the depth of compensation will be variable, being great under high areas and small under low areas. The geodetic evidence, as far as I have been able to examine it, does not seem to be conclusive that there is any such relation between the depth of compensation and the elevation of the surface. This leads me to be skeptical of such theory, which involves a horizontal flow limited mainly to those portions of the earth that are below the depth of compensation.

In general, I am skeptical of any explanations of isostatic readjustment, or of other phenomena in the earth, which involve a relatively plastic stratum in contrast to more rigid material above. In each of the cases in which I have been able to follow the mechanics of the problem to my own satisfaction, I have not been convinced that the resort to the device of introducing a plastic layer into the concept is necessary. So, in the problem now under consideration, which is that of the isostatic readjustment, I do not find it necessary to assume any stratum to be more plastic than the one above, in order to harmonize the observed facts of various kinds.

There is abundant geological evidence of horizontal stresses and strains in the earth's crust. This evidence seems to me to be conclusive. The geologic evidence seems to me to indicate a horizontal transfer of material during isostatic readjustment relatively near the surface, rather than at great depths. If the horizontal transfer involved in the undertow were in a plastic stratum more than 100 kilometers below the surface, certainly the horizontal stresses set up in the surface material would be much less than if the same transfer occurred in less plastic material nearer the surface.

In attempting to determine the mechanics of the isostatic readjustment which apparently takes place when great loads are removed from a region of erosion and equally great loads are added to other regions as deposited material, it is extremely important to keep in mind that material deforms under relatively small differences in the two principal stresses. A very large increase in both the principal stresses is necessary in order to produce deformation if the increases are equal. In an elementary cube of the material, let p_2 be the pressure on the upper and lower faces of the cube. Let p_1 be the pressure on the side faces of the cube. If $p_2=p_1$, the material is under isostatic conditions. Under these conditions both p_2 and p_1 may be increased very largely before appreciable compression is produced in such material as constitutes the earth's crust. On the other hand, if p_2 is increased without changing p_1, then deformation of the cube will be produced for a relatively small difference $p_2—p_1$, corresponding to an added load of a few thousand feet only of material. The cube will be deformed in the sense in which the vertical dimension is decreased and the horizontal dimensions increased. Consider the movements which will take place if the many elementary cubes under a large loaded area are so distorted. Evidently the motions of the material under the margins of such a load will have a horizontal component. This line of thought, followed through to its logical conclusions, and made more definite by careful analysis, will indicate that the horizontal transfer of material occurs largely at moderate depths, certainly at less than 100 kilometers as a rule.

I am perfectly aware that the presentation of the considerations set forth in the last few paragraphs has been too brief for clearness or conclusiveness. The paragraphs have been written as suggestions rather than as demonstrations. They indicate lines of thought which should be followed up carefully if one wishes to reach true conclusions.

I desire to reiterate my opinion,[1] based on such thought as I have been able to give to the subject, that the undertow involved in isostatic readjustment is above the depth of compensation.

THE EARTH-TIDE EXPERIMENT

By Henry G. Gale

I understood Professor Hayford to say that he did not believe that a fluid layer exists beneath the solid crust of the earth. The same conclusion may be drawn from the earth tide experiment which was conducted on the grounds of Yerkes Observatory by Professor Michelson and myself. It seems pretty certain that the earth tides are the same as they would be if the earth were a highly elastic homogeneous solid, both with respect to the phase and amplitude of the earth tides.

The experiment was entirely successful from a physicist's standpoint. The interferometers gave very little trouble. One of them did not require readjustment during the entire year. Two were readjusted to change the width of the fringes, and on one interferometer it was necessary to resilver one of the mirrors. As a source of light we used commercial alternating Cooper-Hewitt lamps, and they proved to be entirely satisfactory. The only serious interruptions were caused by breaks in the electric service due to storms, and occasional short shut-downs by the electric light company which supplied the current. The experiment at Yerkes Observatory was continued for just one year. This is probably long enough to give values of the semi-diurnal and diurnal tides, accurate to a few tenths of one percent. For tides of longer period the experiment should be continued for, say, five years, although possibly three years would be long enough.

The experiment is now being repeated on the grounds of the California Institute of Technology at Pasadena, California. One additional station should be installed, preferably on an island in the Pacific, far from the continental borders. If the three stations should give results in agreement on both the phase and amplitude of the earth tides, I should feel that the problem had been solved. If the three were not in agreement, at least one-half dozen additional stations would be desirable. They should be well scattered in latitude, and with reference to tidal coasts.

The cost of installing a station is not excessive, and one man can easily keep a station in operation and reduce the observations. It would probably be worth while to look for a correlation between the slight changes of

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[1] As early as 1911 I had reached the above conclusion, as indicated by one of the diagrams in an article published in *Science,* 23, No. 841, 199-208, Feb. 10, 1911, entitled "The relations of isostasy to geodesy, geophysics and geology."

level often shown by the apparatus and the approach or passage of the intense barometric lows and highs of large area. It would certainly be worth while to install a specially designed apparatus, similar to that used for the earth tides, to detect and measure the rate of tilting in the surface layers of the earth's strata at especially favorable places. A relation between such rates of tilting and earthquakes might be detected.

THE EÖTVÖS BALANCE

By W. D. Lambert

There is one instrument of use in the study of terrestrial gravity which has not yet been employed in the western hemisphere, though it has to a limited extent in Europe. This instrument is the Eötvös balance. It should be tried, I believe, in making the proposed minute investigation of gravity in a level region of limited extent.[1] To judge by accounts of work done with the balance in Europe it would certainly supplement the pendulum advantageously in the proposed gravity survey and might largely supersede it. This does not by any means signify, however, that the balance is always and everywhere a substitute for the pendulum. This is not the occasion for an exposition of the principles of the instrument, but it may be said that the Eötvös balance connects the results at adjacent points in a limited region with one another in a way that the pendulum cannot well do.

In Europe they evidently believe in its possibilities as an indicator of the existence and location of concealed irregularities and discontinuities in density. Some of these discontinuities may mean strata of geological or commercial interest. At least three attempts have been or are being made to locate such strata by the use of the balance (1) by Dr. Schumann to locate lignite deposits in Austria,[2] (2) by Professor Schweydar for geological purposes in the region about Hamburg, Germany,[3] and (3) an attempt to locate salt deposits in Poland.[4] It will be of interest to know how successful these attempts prove to be.

The Eötvös balance determines certain second derivatives of the gravity potential function. It does not, however, determine the second derivative in the vertical direction. This quantity has to be determined theoretically. This is the most serious deficiency of the instrument and explains why it is of comparatively little use in rough country. Attempts are being made by Berroth, Helmert's assistant, in his last researches on gravity, to devise a means of determining experimentally the second derivative in the vertical direction.[5]

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[1] See p. 13 of this bulletin.

[2] *Akademie der Wissenschaften in Wien:* math. phys. Klasse. Sitzung vom 8 Janner, 1920. Reported in the *Akademische Anzeiger*, Nr. 1.

[3] "Rapport sur les Travaux du Bureau Central de l'Association Geodésique Internationale en 1920," p. 4.

[4] I have seen nothing in print about this third attempt. I have word of it personally from the man proposing to make the observations. [Added in proof: This attempt has not yet been made on account of the lack of instrument.]

[5] Same reference as in second preceding footnote.

THE PROBLEM OF THE EARTH TIDES

By W. D. Lambert

There are two methods of attacking the problem of the elastic properties of the earth, (1) the study of the seismological evidence, on which I shall not touch at all, and (2) the study of earth tides. Even after the best available observational evidence from the earth tides has been obtained, a good deal of hypothesis and interpretation is required before we can say: "The elastic constants of the earth are thus and so." It is not, however, of this that I wish to speak, but rather of the problem of obtaining the true values of the earth tides. This subject is connected with several other geophysical questions and this paper, therefore, falls naturally into three divisions: (1) Earth tides and the long-period oceanic tides; (2) earth tides and the short-period oceanic tides; (3) earth tides and the variation of latitude.

EARTH TIDES AND THE LONG-PERIOD OCEANIC TIDES

The first quantitative evaluation of the earth tides and hence of the elastic properties of the earth came from a discussion of the so-called long-period oceanic tides. The suggestion which initiated the work appears to have come from Lord Kelvin and the method and results are given in Thomson and Tait's "Natural Philosophy,"[1] but the actual discussion of the observations was made by Darwin.[2]

The discussion is based on the assumption that the long-period oceanic tides for an ocean on a perfectly rigid globe may be calculated on the equilibrium theory. This means that it is assumed that the disturbance of equilibrium caused by the tide-producing forces of long period can travel so rapidly through the water and the forces themselves change so slowly that the ocean has time to adjust itself to the forces and that at any given instant the surface of the ocean forms an equipotential surface for the instantaneous field of force.[3] The observed oceanic tide would be the difference between the oceanic tide for a rigid body and the earth tide, and when the first two are known the earth tide may be inferred. From 33 years of observation on both the monthly and the fortnightly tides at 14 different ports Darwin deduced that the observed tides were about two-thirds as large as they would be on a perfectly rigid globe. The earth tide corresponding to the other third implies an effective rigidity of the earth about equal to that of steel.[4] Later methods of attacking the

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[1] Second Edition (1883), § 848.

[2] G. H. Darwin: "Scientific Papers," I, 340. This contains a reproduction of the passage in Thomson and Tait with some changes of notation and unimportant omissions.

[3] This assumption is very evidently incorrect as regards the diurnal and semi-diurnal tides. It was supposed to be at least approximately true for the monthly and fortnightly tides.

[4] The tides at Indian ports, which are more consistent with one another than the others, gave a much higher rigidity.

problem have given results somewhat similar, though in general tending towards somewhat higher values of the rigidity.

Only a few years passed before this result was questioned because of its dependence on the equilibrium theory, and Darwin himself, apparently, was the first to question it.[1] In treating the problem of tides on a globe covered with water—a problem first formulated by Laplace—he discovered solutions which for depths anything like these of our actual oceans gave tidal oscillations of perhaps half the amount deduced from the equilibrium theory. He concludes his article by saying: "Thus it does not seem likely that it will ever be possible to evaluate the effective rigidity of the earth by means of tidal observation." The mathematical treatment of the problem has since been developed by Hough[2] and Goldsborough.[3] The latter has extended the treatment to include basins bounded by two parallels of latitude, a polar basin or one covering the entire globe being special cases. The general result is that for oceanic depths such as we know the monthly and fortnightly tides in such basins differ considerably from what the equilibrium theory gives, being in general considerably smaller.

These results are decidedly puzzling when compared with observation, for the observed tides would then be larger than the computed tides of Darwin, Hough and Goldsborough instead of being smaller by an amount representing the yielding of the earth to the tide-producing forces. Darwin's solution was re-examined critically by the late Lord Rayleigh.[4] He reaches the conclusion that the solution is a very special one that applies, of course, to the ideal water-covered globe postulated, but which has little relation to our actual oceans, interrupted as they are by continental barriers. He says: "If this conclusion be admitted, the theoretical fortnightly tide will not differ materially from its equilibrium value, and Darwin's former calculation as to the earth's rigidity will regain its significance." After a word of caution about possible exceptional conditions he concludes: "In any case I think that observations and reductions of the fortnightly tide should be pursued. Observation is competent to determine not merely the general magnitude of the tide but the law as dependent upon latitude and longitude. Should the observed law conform to that of the equilibrium theory, it would go a long way to verify *à posteriori* the applicability of this theory to the circumstances of the case."

Rayleigh's belief in the legitimacy of calculating the fortnightly tide—and *à fortiori* the monthly tide—from the equilibrium is supported by the opinion of Love[5] and there is still another reason for accepting this idea,

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[1] *Proceedings of the Royal Society of London*, 40 (1886), 337, or "Scientific Papers," I, 366.

[2] *Philosophical Transactions of the Royal Society of London*, 189, 1897, 201, and 191, 1898, 139. Some account of Hough's work is given in Darwin's article on "Tides" in the eleventh edition of the Encyclopædia Britannica, parts of which are given also in Darwin's "Scientific Papers," I, 347.

[3] *Proceedings of the London Mathematical Society*, Vol. for 1914-15, 31 and 207.

[4] *London, Edinburgh and Dublin Philosophical Magazine*, 5, 1903, 136.

[5] "Some Problems in Geodynamics" (Cambridge, England, 1911), 51.

namely, the effect of friction. Hough[1] evaluates the modulus of decay under friction for various types of oscillation. The modulus of decay is a *time* of the same order of magnitude as the period of an oscillation that is slow enough to conform approximately to the equilibrium law. For oscillations most nearly corresponding to the long-period tides and with the laboratory value of the coefficient of viscosity of water he found moduli of the order of magnitude of ten years. But this use of the laboratory value of the viscosity seems to be fallacious. The laboratory value applies to the so-called laminar motion, while the motion in the actual ocean is turbulent. When we call the motion turbulent, we say in effect that we do not understand it very well; but it is known that if we attempt to represent turbulent fluid motion by equations of the same form as are used for laminar motion, then the coefficient of viscosity in the latter, which is the laboratory coefficient, must be replaced by a coefficient of virtual viscosity many times greater. Ekman found for water that the coefficient of virtual viscosity was 15,500 times greater; for air, Taylor found the ratio of the virtual viscosity to the laboratory value to be between 6,000 and 50,000.[2] It is not to be supposed that any one ratio of virtual coefficient to laboratory coefficient would apply under all conditions, but if any such virtual coefficient as is here suggested were used instead of the laboratory value, Hough's modulus of decay would be greatly reduced and the equilibrium theory would apply to tides of comparatively short period, even apart from the effect of continental barriers.

It seems to me, then, that it is pretty safe to assume that the monthly and fortnightly tides conform quite closely to the equilibrium theory and that it would be well worth while to resume the study of these tides for the light that they may throw on the rigidity of the earth. The only discussion of the observations since Darwin's original one is due to Schweydar,[3] in which are discussed observations at 43 ports covering 194 years. There must be available among all the harmonic analyses of tides that have been made since that time a great deal of material still unutilized.

In regard to the utilization of this material and the procurement of new material there are three suggestions I should like to make.

First: In securing the observations care should be taken that the tide gauge has a firm foundation. Tide gauges are frequently located on docks supported by piles, or on made ground. If the cost could be afforded, it would be desirable to place them, if necessary, at some little distance from the water in order to get a good foundation, and to connect the gauge with the water by a large pipe. Incidentally, if a firm foundation

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[1] *Proceedings of the London Mathematical Society*, 28, 1896, 264.

[2] These values are taken from a paper by McEwen: Ocean Temperatures, their relation to Solar Radiation and Oceanic Circulation; miscellaneous Studies of Agriculture, Biology, Semicentennial Publications of the University of California. Other examples of very wide differences between laboratory and field values are also given by McEwen. The work of Ekman and Taylor is found respectively in the *Arkiv for Matematik, Astronomi och Fysik* 2, 1905, 1. and in the *Philosophical Transactions of the Royal Society of London*, 215, Ser. A, 1915, 1.

[3] *Beiträge zur Geophysik*, 9, 1908, 64.

were assured, the tide records would be valuable in studying the secular rising and sinking of the coast. Too many long series of tidal observations that would have been valuable for this purpose in the past have been rendered useless for lack of connection with well-established permanent marks. Schweydar deduced from his discussion of the tides an effective rigidity of the earth rather less than that of steel and suggests that the discrepancy between this result and the higher rigidity obtained from the variation of latitude may be due to a plastic stratum beneath the crust. He seems to have abandoned later his belief in this plastic stratum. Perhaps such a stratum exists, after all, but it may consist simply of the alluvial ground, mud or made land on which some of the tide gauges are situated.

Second: In reducing a year's tidal observations at a place to obtain the long-period tides it is found that the tides are much entangled with one another. To separate them Darwin gives a rather tedious process of successive approximations. It is possible to dispense with these repetitions at the price of a rather heavy piece of preliminary computation that is done once for all series of a given length, such as a year. If many long-period tides are to be reduced, it would be well worth while to do this preliminary work. Furthermore, Harris has suggested that if four consecutive years be taken together, the several long-period tides will separate satisfactorily from one another and the computation will be simpler.

Third: In the final discussion we seek to obtain the earth tides from the difference between the theoretical tides on a rigid globe and the observed tides. Now, even accepting the comparatively simple equilibrium theory, which we are at present supposing to be adequate, the tides on a rigid globe have not been evaluated as accurately as could be wished. The difficulties are the attraction of the water on itself and the presence of the continents. Either difficulty by itself is readily overcome. If there were no continents and the globe were covered with water, a simple factor derived from the fundamental principles of spherical harmonics would take care of the self-attraction of the water. On the other hand, if we disregard the self-attraction of the water, the influence of the continents can be allowed for as Darwin and Turner[1] have done in following Sir William Thomson's (Lord Kelvin's) suggestion.

It is the combination of the two circumstances, neither of which is troublesome by itself, that makes trouble, for the resultant of the two cannot be had by simply superposing two corrections. The total effect cannot be very large, but the modulus of rigidity is rather sensitive to changes in the ratio of the observed tide to the theoretical tide, especially if this ratio be near to unity, so that it seems perhaps quite possible that the modulus of rigidity may be changed as much as thirty or forty percent by the application of the corrections above mentioned.

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[1] *Proceedings of the Royal Society of London*, 40, 1886, 303, or Darwin's "Scientific Papers," I, 328.

The evaluation of this correction of the combined effect of the continents and the self-attraction of the water seems to me a problem worthy of study. No advance seems to have been made since Poincaré's paper,[1] in which he works out a solution very elegant in conception, but one which, as Poincaré himself says, would lead to calculations far too complicated to be practicable even if the shore line of the continents were arbitrarily simplified into a rude approximation to its actual form. I believe, however, that it may be possible to solve the problem numerically by a laborious process of successive approximations, involving the preparation of maps showing the equilibrium tide corrected for the continents alone, and the reading of these maps for many points on the earth, much as contour maps are read to obtain the deflections of the vertical. The labor would be more than an individual investigator would care to undertake, but might well be within the means of an institution. Perhaps the theory of integral equations in its recent developments might afford means of lightening the labor.

EARTH TIDES AND THE SHORT-PERIOD OCEANIC TIDES

In discussing observations of earth tides as a means for obtaining the elastic constants of the earth—observations made either with horizontal pendulums or with Michelson's tube and interferometer—it is important to have a knowledge of the oceanic tides. This statement applies to tides of all periods, but these remarks apply more particularly to tides of short period, i. e., to the diurnal and semidiurnal tides. Tides of longer period have just been discussed.

The shifting mass of tidal water exerts a direct gravitational effect on the horizontal pendulum or on the liquid in the tube, and the direct effect is reinforced by the tilting of the earth's crust under the shifting load of tidal water. The periods of these effects are precisely the periods of the tide-producing forces; hence it is impossible to make an adequate estimate of the yielding to the tidal forces of the solid earth as a whole until the direct and indirect effects of the oceanic tides have been allowed for.

Probably the most satisfactory determination of the earth tides is that of Michelson and Gale,[2] and it is noteworthy that their observations were made at Williams Bay, Wisconsin, which is some 800 miles distance from the ocean. The yielding of the earth deduced from the north-and-south displacements is nearly the same as that deduced from the east-and-west ones and the yielding deduced from the diurnal declinational tide, O_1, is nearly the same as deduced from the principal semidiurnal tide, M_2.

This satisfactory state of affairs no longer obtains with some of the observations taken elsewhere with the horizontal pendulum. Hecker's

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[1] *Journal de Mathématiques Pures et Appliquées*, 2, 1896, 57.

[2] *Journal of Geology*, 22, 1914, or the identical article in the *Astrophysical Journal* for March, 1914. An important correction is given in *Science*, 50, 1919, 327. Definitive results will be found in the *Astrophysical Journal* for December, 1919.

results at Potsdam,[1] which seemed to show a greater yielding of the earth in the meridian than in the prime vertical, have been a standing puzzle; attempts to explain this peculiarity as due to the rotation of the earth have been unsatisfactory.[2]

More recent observations by Schweydar with horizontal pendulums at Freiberg in Saxony[3] show anomalies also. Perhaps these anomalies are due to the inferior accuracy of the horizontal pendulum as compared with Michelson and Gale's apparatus, but it seems to me probable that the uneliminated effects of the oceanic tides may also play a part.[4] This effect has never been calculated, so far as I know, and has been assumed to be so small as to be practically negligible, but, as we shall shortly see, there are reasons for questioning this assumption.

The only serious attempt to allow for the oceanic tides, so far as I know, is due to Prof. Shida[5] of Kyoto University, Japan. He observed with horizontal pendulums for a year at Kamigamo Geophysical Observatory, near Kyoto. In Japan it is, of course, impossible to get far from tide water, and though the tides in the surrounding waters are not particularly large, but rather the contrary, still considerable effect is to be looked for. Harris's cotidal maps[6] were used for the M_2 component. They are based on actual observation for the coast, but necessarily on theory and inference for the open sea. For the O_1 component special maps were drawn from the somewhat meager data available. From the maps the gravitational effects of the tidal water was read off just as the topographic deflection of the vertical may be read. The effect due to the yielding of the earth's crust was also computed on the most plausible assumptions practicable. The actual computation is not unlike that of the deflection of the vertical; the result comes out that the deflection due to the tilting under the tidal load is about twice the direct gravitational pull of the load itself, and the two corrections combined were of the same order of magnitude as the earth tides proper. For example, the observed O_1 tide on the pendulum swinging northwest–southeast came out

$$+0''.00525 \cos t + 0''.00059 \sin t$$

while the total deflection in the same direction due to the ocean tides was computed as

$$+0''.00410 \cos t - 0''.00250 \sin t$$

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[1] "Beobachtungen an Horizontalpendeln über die Deformation des Erdkörpers unter dem Einflusz von Sonne und Mond," Veröffentlichungen des Konigl. Preussischen Geodätischen Institutes n. f. no. 32, Berlin, 1907, and Heft 2, n. f. no. 49, Berlin, 1911.

[2] See Love: "Problems in Geodynamics" (Cambridge, England, 1911), 75.

[3] "Bericht über die Tätigkeit des Zentralbureaus der Internationalen Erdmessung im Jahre 1920," 6.

[4] Orloff's observations at Dorpat (reported in *Astronomische Nachrichten*, 186, 1910, 81) show a peculiarity similar to Hecker's but to a less degree. Dorpat is farther removed than Potsdam from the influence of the large tides in the Atlantic.

[5] "Memoirs of the College of Science and Engineering," Kyoto Imperial University, IV, no. 1 (Nov., 1912).

[6] Manual of Tides, Part IV B (Report of the U. S. Coast and Geodetic Survey for 1904, appendix 5).

where t is the hour angle of the fictitious O_1 tide-producing body of the harmonic analysis. Evidently any inference drawn from the uncorrected earth tides would be quite wide of the mark. The computation was extended to a distance of 40° of great circle (nearly 2,800 statute miles) from the station. This was considered sufficient in view of the meagerness of the tidal data and the approximate nature of the work, but it should be remarked that the zone between 30° and 40° gave in some cases a result equal to more than one-tenth of the whole correction, suggesting the desirability of extending the calculation to even greater distances.

When we consider the great areas of ocean that lie within 40° of any one of the European horizontal-pendulum stations, it seems rash to assume without careful calculation that we may neglect the effects of the oceanic tides on the observed earth tides, and it may even appear desirable to see whether Michelson and Gale's result may not be susceptible of improvement by applying the correction for oceanic tides.

The primary difficulty with calculations of this sort is our lack of knowledge of tides in the open sea. We have Harris's cotidal lines, and it may be said that when these were used for reducing the Kyoto observations the results obtained seem quite satisfactory, thus verifying to a certain extent the theories on which the lines were based. But at best this is theory rather than observation, and Harris himself was as keenly aware as anyone else of the incompleteness of his work and the necessity of verifying it by observation.

The direct observation of tides at sea is a problem beset with difficulties. To observe tides by means of soundings repeated every hour or so at the same point seems impracticable on account of the great depth to be sounded, rendering an accuracy of a foot or less impracticable, and on account of the difficulty of recovering the same point. Pressure gauges in one form or another have been suggested, but the instrument that will sustain the load of a thousand fathoms of water and at the same time be sensitive to variations in that load of a foot or so has not yet been devised.[1]

It has occurred to me, however, that the question of tidal oscillations at sea could be approached somewhat differently, namely, by a study of the horizontal oscillations, that is, the tidal currents. In the open sea these tidal currents would, of course, be small, but not always too small to be detected and studied. Given a good knowledge of the tidal currents, the tidal rise and fall could be inferred with fair certainty. The relatively large tidal currents are to be looked for near the nodal lines of the stationary tidal oscillations, and Harris's theory will indicate plausible places in which to look for such nodal lines.

What has chiefly impressed me with the possibility of measuring tidal currents at sea was the reduction that I made for the late Dr. Harris of

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[1] A recording tide gauge for work at sea invented by M. Favé, a French hydrographic engineer, that is said to have given good results at Dover, England, and in the Thames Estuary, is mentioned in the *Observatory*, 43, Aug., 1920, 279.

observations taken some thirty-five years ago by Lieut. Pillsbury,[1] as he was then, later Rear Admiral Pillsbury. They were not made with the study of tidal currents chiefly in view, but for the exploration of the Gulf Stream. Dr. Harris had them worked over again by more modern methods to see what information about tidal currents could be extracted.[2]

The series were all short, a few days at the most, and some of them did not put in evidence an unquestionable tidal current, but a number of them did. A plotting of these latter showed that the results of the approximate harmonic analyses that were made could not be far from the truth. The velocities found ranged in general from 0.05 to 0.3 knot. It may be of interest to remark that the times and directions of the current were in general agreement with Harris's theory of stationary tidal oscillations. The observations, of course, were made before this theory was formulated, but they were not reduced for tidal purposes till some time after the theory was published, so that they serve as a partial confirmation of it.

If an expedition were sent out to determine tidal currents at sea in somewhat the way here suggested, it would have to occupy one spot for several days, or preferably longer. While the vessel remained on the spot for current observations there would be an excellent opportunity for other kinds of scientific observation, magnetic, geophysical and biological. The intensity of gravity at sea is a great desideratum in geophysics and as soon as adequate apparatus is devised for the purpose observations of gravity should certainly be made in connection with observations of the currents.

EARTH TIDES AND THE VARIATION OF LATITUDE

Just as we may make a harmonic analysis of the readings of a horizontal pendulum or a Michelson tube in order to evaluate the earth tides, so for the same purpose we may make a harmonic analysis of latitude observations. In all three cases we are observing the direction of the vertical or plumb line. In the first two cases the vertical is referred to some mean position determined on the instrument itself and connected with the ground immediately around it, and thus shifting its position as the ground tilts under the influence of the tide-producing forces in the earth and under the load of the oceanic tides. In observations of the latitude the direction of the vertical is referred not to the ground round about the instrument, but to the direction of the earth's axis. The tilting of the ground is allowed for when the level readings are taken and the proper corrections for them applied. This absence of the tilting enables us to get a hold on the problem of the earth tides somewhat different from that afforded by observation with the horizontal pendulum or with the tube and interferometer.

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[1] Appendices to Reports of the U. S. Coast and Geodetic Survey for 1885, 1886, 1887, 1889 and 1890. More especially that for 1890, which contains an account of the apparatus used.

[2] For the tidal data deduced see Harris's Manual of Tides, Part V (U. S. Coast and Geodetic Survey, Report for 1907, Appendix 6), 409–13.

The latitude observations most obviously appropriate for such a harmonic analysis are those of the international latitude service and I believe that they should be systematically discussed in this manner. Some preliminary work of this sort has been done by Shida and Matsuyama,[1] and Shida has proposed to the International Geodetic Association that it undertake the work. Further work on the subject has been done by Przbyllok, whose work has perhaps already been published, though without coming generally to the attention of scientists on this side of the water[2] on account of still unsettled international conditions. I believe the International Geodetic and Geophysical Union should plan to continue the work.

SUMMARY

The observation and reduction of the long-period oceanic tides should not be neglected. The equilibrium theory has not been as fully developed as is desirable and an attempt should be made to allow both for the self-attraction of the water and for the presence of the continents. When this has been done and observation compared with theory, it seems probable that a good value for the long-period earth tides will result. To get good results for the short-period earth tides the oceanic tides of like period must be known and their effects allowed for. One promising means of getting this knowledge seems to be a study of the oceanic tidal currents, which appears to be more feasible than the direct observations of the tides themselves. While the currents were being observed other scientific observations could be made. The earth tides affect the plumb line and their effects must therefore be present in the observations of the International Latitude Service. It is desirable to continue the discussion of these observations in order to throw light on the earth tides.

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[1] "Memoirs of the College of Science and Engineering," Kyoto Imperial University, IV, no. 1, 1912, 277.

[2] "M. le. Prof. Przybyllok a tâché de déterminer plus exactement les constantes de quelques termes périodiques dont les observations du service international des latitudes avaient fait connaitre l'existence; il s'est surtout occupé à déduire des observations astronomiques les constantes de la marée M_2 dans le mouvement de la verticale de la terre considerée comme corps élastique. Les resultats seront publiés sous peu dans les Astron. Nachrichten."—From Rapport sur les Travaux du Bureau Central de l'Association Géodésique Internationale en 1920, p. 3 (dated Jan., 1921).

SOLAR RADIATION AND TERRESTRIAL PHENOMENA

By C. G. Abbot

SOLAR RADIATION

Its Variability and Its Relations to the Atmosphere

For more than fifteen years the Astrophysical Observatory of the Smithsonian Institution has been engaged in making measurements of the radiation of the sun. These measurements have indicated that the sun's emission is variable. The Institution now maintains two stations—one in Arizona and the other in Chile—for observing the solar variability. Telegraphic reports of the results obtained in Chile have been forwarded to Buenos Aires and Rio de Janeiro for the use of the meteorological services of Argentina and Brazil.

The variation of the solar emission is of two kinds—one, of long period, associated with variations in the visible solar phenomena like sunspots, faculæ, prominences, and the like; the other, of short irregular period, apparently depending upon inequalities of radiation in different directions, which, rotating with the sun, produce at the earth the variation just mentioned. This hypothesis is confirmed by the photo-electric cell observations of Guthnick, who found variations of Saturn occurring earlier or later than corresponding ones in solar radiation observed in Chile, depending on the heliocentric longitudes of the earth and Saturn. Higher values of the solar emission occur when the sunspots are most numerous, which gives rise to a paradox, because the temperature of most meteorological stations is lower at sunspot maximum. This negative correlation between solar emission and terrestrial temperature may be due, however, to a variation in terrestrial cloudiness or to other variations in the composition of the terrestrial atmosphere. It has long been known that a close correlation exists between the sunspot numbers and the variations of terrestrial magnetism. The intermediary mechanism producing this relation is not known. However, it has long been suspected to be due to the bombardment of the earth by ions shot out from the sun. If this is the case, these ions may assist in the production of cloudiness and have also influence in the production of ozone in the higher atmosphere, and thus in one or both of these ways operate on terrestrial temperatures.

Observed variations of the sun have hitherto lain within the maximum range of about 12 percent. It is rare that fluctuations exceeding 3 percent occur within a single week or fortnight. Such studies as have been made, notably those of Mr. Clayton of Buenos Aires, indicate that corresponding to these small fluctuations of the sun there may be variations of several degrees in the mean temperature of meteorological stations. Accordingly it is highly desirable to be able to detect with certainty fluctuations of the solar emission of the order of 1 percent. This is a hard requirement, and it is only within the last year that the establishment of the Arizona and Chile stations has warranted the hope that it can be met. Prior to that time, errors of 2 or more percent were probably not infrequent in solar radiation determinations at Mt. Wilson, which until 1918 was the only station in the world where the solar constant observations were being made. Hence we must be required to wait for another decade of years before having a thoroughly satisfactory series of solar radiation measurements to compare with meteorological observations. It would be a very great advantage if two additional solar stations could be equipped in Northern and Southern Africa in the most cloudless and favorable conditions, so that there would be four stations operating under a homogeneous scheme for determining the variability of the sun.

The question arises whether observations of the visible phenomena upon the sun's surface, such as sunspots, faculæ, prominences, or the like, or the observation of terrestrial magnetism, which, as has been said, is closely related to solar phenomena, may furnish some index to solar conditions as valuable as the difficultly obtained determinations of solar radiation. Many statistical comparisons have been published on relations of sunspots and terrestrial phenomena, and to a less degree the other solar appearances have also been correlated thereto. It must be confessed, however, that the result of this enormous amount of work has not been as favorable as would have been hoped. In almost all instances, relations which appeared to hold for a few months or years are reversed in other months or years. Except for the well-known correlation of terrestrial magnetism with the sunspot numbers, there is hardly any other pair of phenomena which would be universally accepted as related. Whether a similar disappointment will attend the proposed studies of solar radiation can not yet be foretold.

Of variable terrestrial influences, the most profoundly active on the solar radiation are the water vapor of the earth's atmosphere and the clouds and haze which are formed from it. Water vapor itself produces powerful absorption bands in the red and infra-red spectrum. Associated with dust, water vapor produces haze which is effective throughout the visible spectrum, and more effective the shorter the wave-length. A great mass of observations of these things has been made by the Smithsonian Institution in connection with its studies of the solar constant of radiation. My colleague, Mr. Fowle, has published a number of papers covering the results of these studies of the subject.

Water vapor itself removes from the direct solar beam which encircles the earth somewhere from 10 to 20 percent of its intensity, depending upon the humidity of the air and other circumstances. The haze may readily produce far greater reduction to the intensity of the solar beam. In addition, we have the clouds. My colleague, Mr. Aldrich, took advantage of the presence of the balloon school near Mt. Wilson to observe from a balloon the reflecting power of the upper surface of the vast oceans of fog which come in from the Pacific and cover the San Gabriel Valley. He found that a continuous sheet of level cloud would reflect away approximately 77 percent of the solar rays. As the earth is on the average about 50 percent cloudy, the great effect of these factors on terrestrial temperatures is obvious.

Cloud measurements are among the most unsatisfactory which are recorded by meteorologists. They depend largely on the personal equation and indeed no really adequate statistical study of them has hitherto been available. The preparation of proper automatic observing apparatus and the study of observations of clouds are highly desirable.

TERRESTRIAL RADIATION

Its Relations to the Atmosphere

When we take up the question of the terrestrial radiation, we deal with another region of wave-lengths from that which is covered by the principal incoming solar rays. The direct rays of the sun and the skylight are almost altogether confined to the region of wave-lengths extending from 0.3 micron to 3 microns. The region of the terrestrial radiation extends from 5 microns to 50 microns. Spectrum measurements have been made through a part of this region by my colleague, Mr. Fowle, who used an artificial source of light and a very long column of air of known humidity and carbon dioxide content. In this way he determined the influence of terrestrial humidity upon the rays as far as 17 microns. Beyond that, from 17 to 50 microns, no adequate studies have been made, and indeed the difficulty of making them is immense. Apparently the water vapor existing in a column of air a quarter of a mile long cuts off all terrestrial rays except in the region from 8 microns to 13 microns. In this region, water vapor is almost perfectly transmissible and in this region occurs, therefore, almost all of the terrestrial radiation which, rising from the earth's surface, escapes to space and tends to cool the earth. No constituents of the air at the earth's surface seem to affect the transmissibility of rays between 9 and 12 microns in wave-length, but the matter is quite different in the upper atmosphere. Mr. Fowle found that a strong band of absorption occurs in the direct solar beam squarely in the middle of this very transmissible region. It appears from some measurements of K. Ångstrom that the cause of this band is ozone. Thus, owing to the accidental position of this powerful absorption band in the middle of the only region where the other atmospheric constituents are almost perfectly transmissible, ozone plays an important part in determining terrestrial temperatures.

A research ought to be undertaken to determine the influence of ozone in this region of the terrestrial spectrum, the variations of its amount in the atmosphere, and, in short, the dependence of the terrestrial temperatures on ozone. This research will be very difficult, owing to the long wave-lengths of the rays involved and owing to the occurrence of ozone high up in the terrestrial atmosphere. The investigation would involve the determination of the dependence of the ozone content of the atmosphere on solar radiation as well as on influencing terrestrial conditions.

Hitherto the measurement of the outgoing terrestrial rays—that is, of the so-called nocturnal radiation—has been very unsatisfactory on account of the lack of a surface which radiates these rays perfectly. Blackened flat surfaces have been used in the instruments employed, but the blackening by means of smoke, lampblack paint, or platinum black are all unsatisfactory because these substances are not full radiators and absorbers for the very long wave-lengths involved. Smoke, for instance, is strongly transmissible beyond 10 microns, and lampblack paint falls off in its absorption very rapidly beyond 15 microns. It is necessary, in order to obtain exact knowledge, to employ some radiating and absorbing instruments which are perfectly radiating and absorbing by reason of their shape; that is to say, which approximate to the so-called absolutely black body.

Hitherto, only one such instrument has been developed, an instrument of which there is yet no published description, namely, the honeycomb pyranometer, or Melikeron, recently invented by Abbot and Aldrich. This instrument consists of 200 deep cells made by fluting a ribbon of thin manganin, the whole presenting a surface comparable to a honeycomb, in which the rays penetrate deeply and are absorbed by repeated reflections. The heat produced by the rays of the sky or outgoing to the sky can be compensated by the introduction of the energy of the electric current. This instrument is but just past its experimental stage, and only a few as yet unpublished measurements have been made with it.

Spectrum observations ought also to be undertaken in the region of wave-lengths from 15 to 50 microns. Rock salt is no longer available in this region, so that some special optical instrument, either a special grating or a special prism to be made of potassium iodide, must be employed.

NEEDED INVESTIGATIONS

It will be seen from these remarks that the most outstanding needs in the investigation of radiation for meteorological purposes are:

First, the continuance of accurate observations of the variation of the sun. These measurements are now going on under the auspices of the Smithsonian Institution in Arizona and Chile, but should preferably be supplemented by the provision of two additional stations, perhaps in North and South Africa, so that variations of the sun could be adequately studied every day in the year. Twenty-seven thousand dollars a year would provide the two stations within two years and maintain them thereafter perpetually.

Second, the painstaking studies of terrestrial cloudiness, its causes and its effect on the incoming of solar radiation.

Third, the study of the quantity and variability of ozone in the upper atmosphere, its dependence on solar and terrestrial conditions, and its influence on terrestrial temperatures.

Fourth, extensive studies with the "black-body" nocturnal radiation instrument, and if possible the development of new instruments of that class.

Fifth, an investigation of the effects of terrestrial atmospheric constituents on rays between the wave-lengths of 15 and 50 microns should be undertaken. This involves the development of special optical means to take the place of rock-salt prism as a dispersing medium, since rock salt is non-transmissible to the rays in question.

RELATIONS BETWEEN SOLAR ACTIVITY AND ITS VARIOUS ASPECTS AND THE PHENOMENA OF TERRESTRIAL WEATHER

By C. F. Marvin

My contribution to this discussion is an appeal for a more careful and consistent recognition of the effects and operations of chance in the study of data which may be employed in investigations of solar and terrestrial relations, periodicities, etc. Weather conditions, atmospheric transmission of radiation, magnetic phenomena, sunspots, observed intensities of radiation, and values of like phenomena are subject to large and irregular accidental variations, due account of which must be taken in reaching conclusions.

SOLAR RADIATION

My point of view is illustrated in an admirable manner by figure 1, which serves to show how weak the observational basis still is to justify the claim that there are important irregular changes from day to day in the intensity of solar radiation.

Discussion of diagram.—The vertical lines of the diagram represent throughout each group 25 observations, counted from the beginning of the group. The points on the zigzag lines are simply consecutive observations without reference to the interval of time between observations.

The chronological sequence of the groups of data is represented by the numbers 1, 2, 3, etc. Only a few observations were made between 1902 and 1907 at Washington at wide intervals. From 1905 to 1918 (groups 2 to 7, inclusive) the observations were made at Mount Wilson, Calif. In all these cases the interval between observations is as nearly as possible one day, although periods of bad weather frequently caused two or three, or more days sometimes, to intervene. As a rule, observations were made only from June to November.

Extreme variability is shown in the observations in group 1. Groups 2, 3, 4, 5, and the first portion of 6, show distinctly a lower order of variability, although occasional extremely high and low values occur infrequently.

Beginning at K, group 6 for 1912, great variability again appears in the consecutive values extending into, although diminishing during, 1913. The high value at K, group 6, marks the arrival at Mount Wilson, Calif., of the dust from the Katmai volcanic eruption.

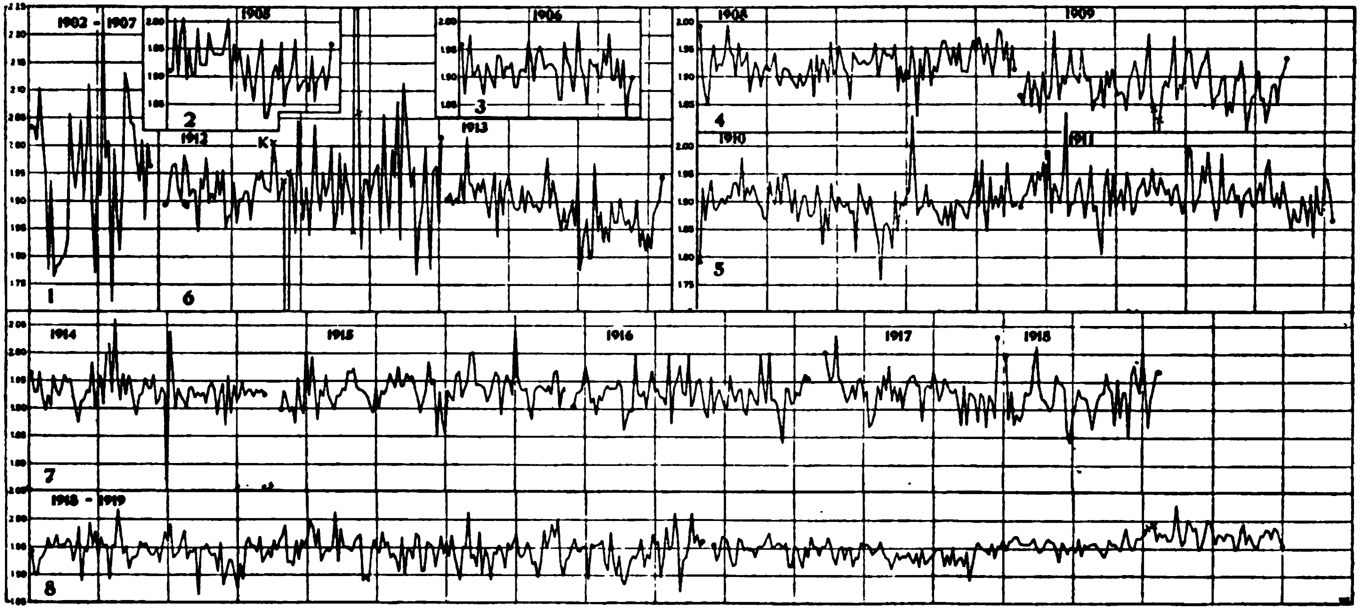

FIG. 1. Diagram of nearly 2000 consecutive observed values of extra-atmospheric intensity of solar radiation, as determined by the Smithsonian Institution from the beginning of observations 1902 to the end of 1919, at stations, Washington, D. C., Mount Wilson, Calif., and Calama, Chile.

From 1914 to 1918, well after the atmosphere had cleared itself of the Katmai dust, the variability became quite similar to observational results at Mount Wilson prior to the Katmai year, 1912.

Group 8 represents the observations at the station at Calama, Chile.

All observations, including the first portion of group 8, were made by the bolographic method.

Finally, the last group of observations, ending December 31, 1919, were made by a new empirical method based on the bolographic method but permitting two or three observations to be made the same day, thus giving a mean average value for a day of higher accuracy.

This diagram tells a very important story with great force and plainness. Great variations in consecutive values of intensity mark the early observations in Washington with imperfect equipment and poor atmospheric conditions.

Observations at Mount Wilson from 1905 to 1912 show far more nearly constant values of radiation until the arrival of the atmospheric dust from the Katmai volcanic eruption, after which day-to-day or consecutive values showed great variations. Everyone probably ascribes these increased variations, not to increased solar activity, but to inaccuracies of measurement due to atmospheric dust. The large variations disappeared with the dust. Furthermore, some increased accuracy (smaller variations) characterized the observations at the station at Calama, Chile, either because of the better instrumentation, greater observational experience, or better observing conditions, or all of these in combination.

Finally, it is most striking that a further marked reduction in day-to-day variability immediately resulted from the introduction in 1919 of the pyranometer method of observation.

The percentage *probable error of a single value* has been carefully computed for each group of observations and the results are shown graphically in figure 2.

Entirely terrestrial causes easily explain the great changes and gradual diminution in variability shown by the observations, the accuracy of which has been wonderfully increased by improvements in instruments, methods, and location of stations. What are now regarded as good observations *for a single day's work* show a probable error as low as 5 or 6 tenths of one percent. This is remarkable precision. This analysis of the whole body of radiation data brings one face to face with the important question:

Is all of this 5 to 6 tenths of one percent of day-to-day variation in solar radiation intensities real error of measurement only? Or is part of it error of measurement and part real solar change? If the latter, what are the respective amounts of each variation?

Some conclusive answer to this question is necessary before inferences and claims of solar and terrestrial correlations can be set up and justified.

It can never be claimed, of course, that single daily values, however carefully made, are perfectly accurate. Probably simultaneous observations at *several* stations is the only answer to this question. Caution is necessary even here, because mere coincidence of variations due absolutely to errors only will come in to affect comparisons at two stations. If, for example, e is the probable variation of, say, a season's work at two perfectly equal stations, then for pure chance relations between values the variation ϵ_m of the mean of the two must be

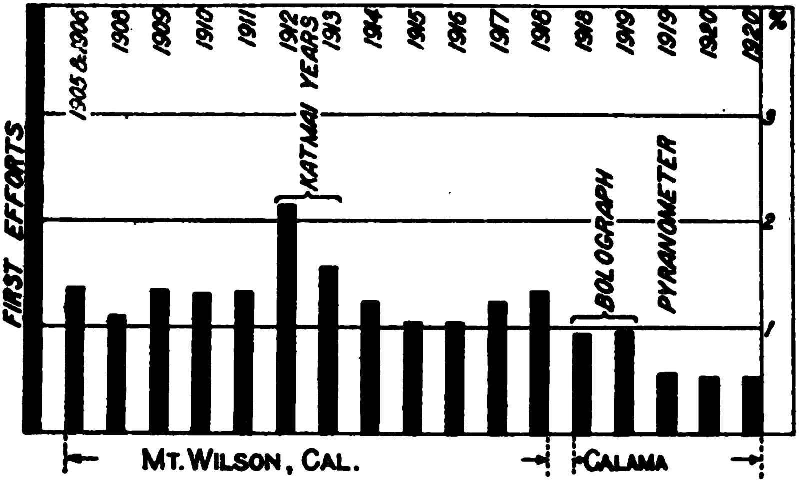

FIG. 2. Height of bars shows probable error of an observation for a single day of the intensity of solar radiation as measured by the Astrophysical Observatory of the Smithsonian Institution at Washington, Mount Wilson, Calif., and Calama, Chile.

$$\epsilon_m = \frac{\epsilon}{\sqrt{2}}$$

This furnishes an interesting test to apply to the simultaneous observations at Arizona and Chile when these are released by the Smithsonian Institution.

TERRESTRIAL DATA

No serious study of any kind dealing with hidden or obscure relations between data subject to large irregular variations should ever be undertaken without a careful application of the principles of probabilities and a consideration of the operation of the elements of chance upon the phenomena under study.

In the discussion of this portion of the subject the author gave the first public account of the mathematical and graphic device which has been designated *The Periodocrite.*

Periodocrite[1] is a word coined from Greek roots signifying a critic, a judge, a decider of periodicities, and is a name applied to a mathematical and graphic method or device which has been developed to aid in the conclusive separation of obscure and hidden cycles and periodicities possessing a real existence from those whose essential features are only such as would result from, and can be explained by, entirely chance combinations of the data employed.

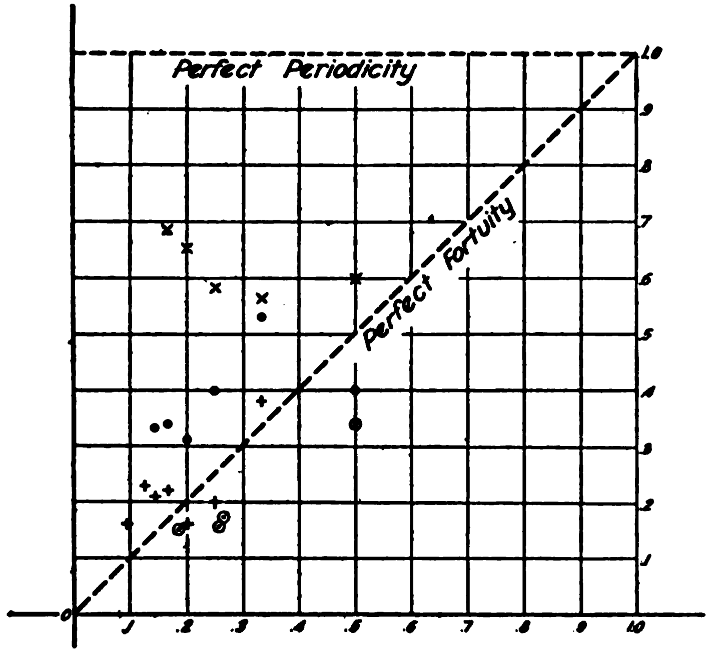

FIG. 3. Rainfall periodocrite: ×, Annual cycle five stations in Iowa, 36-year record; ●, Annual cycle Washington rainfall, 50-year record; +, Annual cycle Boston, Mass., 103-year record, very feebly defined; ⊖, A 15-month sequence Iowa rainfall; other sequences, 15 months, 16 months, one-ninth the variable sunspot period, like the circles, all fall in the class of perfect fortuity.

The periodocrite does not disclose or discover the *length* of suspected periods or cycles. Other methods, such as the harmonic analysis, Schuster's periodogram, or any of the many methods which have been offered for this purpose must first be employed to ascertain the proper length of any suspected cycle.

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[1] Prof. C. F. Talman supplied this name from Κεριοδος, a period + Κριτης, a judge, decider, umpire, from Κρινα, to separate, investigate, judge.

The theory of the periodocrite depends upon the principle that in an entirely fortuitous combination of data the standard deviation, σ_n of a result made up of n individual observations in combination is given by the equation $\sigma_n = \frac{\sigma_0}{\sqrt{n}}$ in which σ_0 is the standard deviation of the original data not in combinations.

Writing $y = \frac{\sigma_n}{\sigma_0}$ and $x = \frac{1}{\sqrt{n}}$ we get

$$y = x$$

which is the equation of a line of perfect fortuity passing through the origin of coordinates (see figure 3). It can also be shown that for perfect periodicity y = constant = 1.

The full account of this aid to scientific investigation of periodicities has been published in the *Monthly Weather Review* for March, 1921, to which the reader is referred for the development of this idea and certain related matters dealing with the discussion of meteorological data briefly outlined in the address.

DAILY METEOROLOGICAL CHARTS OF THE WORLD

By Edward H. Bowie

INTRODUCTION

Investigations in the field of general meteorology have been and continue to be restricted and handicapped by the fact that there are not available to investigators daily charts of the world's weather. It is necessary in investigations in meteorology and particularly in weather forecasting that problems now but imperfectly understood be considered from a world-wide viewpoint, for there can be no doubt that much that we call "weather" is not of local origin, but has its inception in the general actions and reactions that involve at times the atmosphere over an entire hemisphere and possibly both hemispheres. Hence, investigation based on a study of daily synoptic charts for a limited area, such as Europe, the United States of North America, or of India, can lead to but an imperfect understanding of the general physical processes that are in operation to produce our day-to-day weather. Moreover, when it is understood how rapid are the changes in speed, direction of movement and magnitude of areas of high and low barometric pressure, there arises the natural desire to look into the observable facts over a world-wide area in an attempt to determine the causes of them. Meteorology without a world-wide weather map is laboring under difficulties as great, or greater, without realizing it, than astronomy without its star charts. Hence it is contended that many of the important problems of meteorology will not and cannot be solved until there be available daily synoptic charts of the various meteorological elements of, relatively speaking, the entire world. It would redound to our credit if the American Geophysical Union should become instrumental in bringing this about.

NEED FOR WORLD-WIDE CHARTS

There is reason to believe that meteorology has for its goal the making of accurate forecasts of weather, temperature and wind for long periods in advance. Such an attainment would not only mark a distinct advance in the science of meteorology, but in its practical application would be of great economic importance to the life of the nation.

The problem of making such forecasts has for years received consideration not only from meteorologists of good repute—scientifically speaking—but also from others untrained or at least not having a thorough grasp of the question. Many explanations of the frequently marked deviations from the normal that occur in the meteorological elements at any given place have been given publicity. Much attention has been given to the question of cycles, even more has been given to the question of sunspots, and recently attention has been focused on the so-called variations in the solar constant. But all these efforts fail to offer a method that permits the making of long-range forecasts of a sufficient degree of accuracy to have a practical application of any importance.

One naturally inquires why nothing really definite has been evolved that might be useful in long-range forecasting, and the only possible answer that can be made is that the right combination has not yet been found.

My impression is that the answer will probably be had from a prolonged and careful study of world-wide meteorological conditions, and this study is possible only in the event that daily synoptic charts embracing the major part of the world are available. In my opinion all marked deviations from normal weather and temperature are associated with changes in magnitude and position of the so-called "great centers of action." Any material addition to our knowledge of these must come from the study of world charts, for it is only by such a method that we can hope to gain a more comprehensive knowledge of the general atmospheric circulation and the resulting changes in the centers of action. In Supplement No. 1, *Monthly Weather Review,* 1914 (Bowie and Weightman), it is stated that:

> Conspicuously abnormal pressures in the regions of these so-called "centers of action" are related to marked departures from normal weather and temperature conditions in the United States. Some authorities assume that these abnormal distributions of pressure are due to extra-terrestrial and others assert that they are due to terrestrial causes. If it be true that the solar output is a variable quantity,

> it is possible that the solar variations are associated with marked changes in pressure in the "centers of action," and thus may be found a key for defining for considerable periods in advance the general character of coming weather changes for a given region. If on the other hand abnormal pressure distributions occur with an unvarying solar radiation, the causes thereof must be traced to a terrestrial source. The varying effects of the nearly constant radiation on land and water surfaces and on the air under different conditions of temperature, water vapor content, dust content, etc., are sufficient in the minds of some writers to explain these phenomena, i. e., the changes in the position and magnitude of the "centers of action."
>
> Regardless, however, of the cause of abnormalities in the "centers of action" the importance of their relation to the character and paths of storms in the United States is well recognized and therefore should be carefully considered in day-to-day weather forecasting in the United States. To illustrate: Of the centers of action that affect the weather conditions of the United States east of the Rocky Mountains, the subpermanent high over the middle latitudes of the North Atlantic Ocean is perhaps the most influential. When this is well developed and stable, temperatures above the seasonal average are to be expected over the great central valleys and the eastern and southern states, and the areas of high and low barometer crossing the United States will move in high latitudes and pass on to the ocean by way of the St. Lawrence valley. In fact, all prolonged periods of heat in the regions east of the Rocky Mountains occur simultaneously with the abnormal development of this subpermanent high. When, however, it is weak and ill-defined, cool weather prevails over the eastern half of the country.
>
> Again, the variations in the position and magnitude of the elongated subpermanent area of low pressure that normally extends from southeastern Alaska westward to Kamchatka, have a decided influence on the characters of, and courses followed by, storms that cross the United States. If this Aleutian low is north of its normal position, lows will move along our northern border; whereas, if it is south of its normal position, lows will move far south of their normal tracks and stormy weather with great alternations in temperature will occur over the United States.

Perhaps the best examples of unusual winters in the United States are those of 1917-1918 and 1920-1921. In the former, which was one of great severity, the pressure was abnormally high over Alaska and the Aleutian Islands; while in the latter, which will go down in meteorological history as one of the mildest known, the pressure was much below the normal over those regions.

To be able to formulate correct forecasts, a knowledge of the general circulation is fundamental. It is questionable whether a proper understanding of the general circulation can be gained from monthly averages. It certainly is not as stable as the text books would lead us to believe, for there are frequently marked changes in both the surface and upper air flow. It has been customary to think of the general or primary wind circulations of the two hemispheres as separate and distinct, but this view is not tenable. A cursory examination of plate 14, "Bartholomew's Physical Atlas," Meteorology, volume III, shows a tremendous seasonal interflow between the northern and southern hemispheres, indicating that the two systems of general circulation are in a way interlocked. In the winter of the northern hemisphere the air flows normally from the interior of Asia southward over the Indian Ocean, eastern Africa and the East Indian Archipelago on beyond the equator as far south as northern Australia; in the summer of the northern hemisphere a return flow takes place over essentially the same geographical area. No one can say, for the lack of the necessary data, whether these currents do not bring about profound changes from normal weather and temperature conditions over large areas outside the regions where these flows and counterflows are in operation, but it seems logical to suppose that such is the case. Further, the trades and antitrades are probably not fully understood. Certainly these wind systems undergo pronounced changes that are independent of the seasonal changes. In connection with the antitrades, Sir Napier Shaw in a recent number of *Nature* remarks:

> At the same time I may remark that I find it very difficult to grasp the meaning that is intended by "anti-trades." The original convection theory suggested that the anti-trade was the trade returning up aloft above its old path, but, so far as I can understand the situation, the track of the wind from the equator must begin from the east and become southwest by what I will describe as the hurricane track. On the other hand, a southwest wind may be a part of the westerly circulation diverted; the difference of origin of the observed southwesterly wind is of some dynamical importance.

It seems possible that the antitrade may be a northward extension of the southeast trade of the southern hemisphere, which on crossing the equator is turned to the right by the deflective force of the earth's rotation, and being warmer and of less density, overruns the low-lying northeast trade. But this is not definitely known and sufficient data are not at hand to prove or disprove the assertion.

Bjerknes has recently given meteorology the term "polar front," a line of discontinuity separating the polarward from the equatorward flowing winds, and he has urged the collection of daily meteorological observations from larger geographical areas that this "polar front" may be delineated on the weather charts for the aid of the forecaster. He believes this essential to forecasting, for the theory he advances places the origin of cyclones and consequently all marked variations in weather, temperature and wind changes along this line of discontinuity. His presentation of the idea of the "polar front" and its attendant phenomena is worthy of him, and augments the necessity of observations over large geographical areas in weather forecasting.

It will be seen from the foregoing that meteorology must in the near future consider the question of securing observations from every accessible place and assembling them for the construction of daily world-wide weather charts at one or more great world centers for intensive study. First, there must be a skeletonized chart based on observations collected by cable, radio and land lines, and, second, a more nearly perfect and complete chart based on the same observations supplemented by those collected from remote land areas in which cable or radio is not available and from ships at sea. The former chart would serve for day-to-day forecasting; the latter, for study purposes and eventually for long-range or seasonal forecasting.

HISTORY OF THE MOVEMENT FOR WORLD-WIDE CHARTS

The need of a daily synoptic survey of the earth's atmosphere was coincident no doubt with the beginning of synoptic weather charts, which was at approximately the middle of the nineteenth century. We learn that at the first meeting of the International Meteorological Congress, assembled at Vienna in 1873, a proposition was adopted to the effect:

> That it is desirable, with a view to their exchange, that at least one uniform observation, of such a character as to be suitable for the preparation of synoptic charts, be taken and recorded daily and simultaneously throughout the world.

Later, on December 9, 1876, it was announced that the United States, through the Chief Signal Officer, U. S. A., no doubt inspired by the late Prof. Cleveland Abbe, then assistant to the Chief Signal Officer, was undertaking the task of establishing cooperation for the recording and exchange of simultaneous meteorological observations between the United States and the following named countries: Algeria, Austria, Belgium, Great Britain, Denmark, France, Germany, Italy, The Netherlands, Norway, Sweden, Switzerland, Turkey, Greece, Canada, the Hawaiian Islands, Dutch Guiana and Japan. This cooperation extended to naval and merchant vessels of these nations, and thus were secured the simultaneous observations of atmospheric changes over much of the northern hemisphere. Thus came about the "Bulletin of the International Meteorological Observations," published by the Signal Service, U. S. A., for the years 1877–1887—incomplete, it is true, as to world-wide weather maps, but a remarkable contribution which has left its impress on meteorology even until today.

An effort to accomplish the preparation of northern hemisphere weather maps by means of daily telegraphic reports for the purpose of extending the forecast period to cover the general weather of the United States was undertaken by the U. S. Weather Bureau in 1907, and the area covered by such reports grew until the outbreak of the Great War in 1914, when the scheme was unavoidedly interrupted. No other really effective efforts looking to the preparation and publication of even partial world-wide daily synoptic charts are known to me.

PRESENT STATUS OF DAILY SYNOPTIC METEOROLOGICAL CHARTS

Nothing approaching a world-wide daily synoptic chart is prepared and published by the meteorological service of any nation. Instead, every, or nearly every, national meteorological service decides for itself: (a) the scale of the map, (b) the units of measurement, and (c) to a greater or less extent the hours of observation. In addition to the various charts of land observations, charts are also made of the meteorological conditions over one or more of the oceans. Daily weather maps for their respective geographic areas are now prepared and published by the United States, Canada, Mexico, Argentina, Chile, Brazil, Japan, China (Zi-ka-wei Observatory), Australia, India, South Africa, Great Britain, France, Portugal, Belgium, The Netherlands, Norway, Sweden, Denmark, Germany, Austria (?), Russia (?) and others. There is thus available a tremendous mass of valuable data awaiting action that will assemble them into one standard, world-wide weather map which will permit investigations in meteorology to be carried beyond any point now possible.

RECOMMENDATIONS

This matter is believed to be of such importance at the present time that it is contemplated recommending in appropriate form that some action be taken by the Geophysical Union for the accomplishment of the desired objects through international cooperation.

There is no doubt in the minds of those familiar with the present status of meteorology that the carrying out of this proposal will be well worth while, not only from a scientific standpoint but also from the standpoint of service to the general public.

The preparation of the data for charting and the printing or lithographing of the charts for American use could best be done by the U. S. Weather Bureau, but to do this, additional funds must be provided through congressional appropriation.

WORLD AEROLOGY

By Willis Ray Gregg

Aerology may be very simply defined as "the study of the free air";[1] world aerology, as an extension of that study to all parts of the world. By this we mean not only the continental areas, but the seas as well; and not merely sections of a hemisphere, but from pole to pole. It is our purpose to review very briefly (1) what has been and what is being done toward this end; (2) more particularly, to outline what *can at once* and also what *should* later from time to time be undertaken.

PAST AND PRESENT

Methods.—As early as the middle of the 18th century kites were used, by William Wilson at Glasgow University and by Benjamin Franklin at Philadelphia, in making free-air observations. Others followed their example, with more or less success, but it was not until about 1890 that the kite came into general use for this purpose.

So far as known, the first manned balloon ascent in the interests of science was made by Robertson and Lhöest in 1803. During the next 75 years much interesting information as to free-air conditions was obtained by means of numerous similar ascents, among the most notable of which were the classic voyages of Glaisher, Flammarion, de Fonvielle and Tissandier.[2] Unlike the kite, however, the manned balloon has in recent years suffered a decline as a means of aerological exploration, because of the large expense involved and the impossibility of providing satisfactory exposure of the instruments. Although the observations made by these two methods were extremely interesting, yet prior to 1890 they yielded comparatively little of value, owing to their fragmentary character and none too great accuracy.

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[1] See "Meteorological Glossary," British Meteorological Office, M. O. 225 ii, 1918, p. 16. London.

[2] "Travels in the Air," edited by James Glaisher, F. R. S., Philadelphia, 1871.

Since 1890 rapid strides have been made. The kite has been developed from a mere toy into a very efficient means of exploration. With it heights slightly exceeding 7 kilometers have been reached, although the average daily height is a little under 3 kilometers. Recording instruments carried by these kites furnish information as to pressure, temperature, humidity and wind at various heights and their changes from day to day, season to season, and under various types of weather at the earth's surface. Since the kite can be flown only when there is appreciable air movement, its use has in some instances been supplemented by that of a small captive balloon, and thus we have some records in calm weather. Generally speaking, however, the captive balloon has proved to be rather unsatisfactory and its use has been largely discontinued.

For exploring the air to greater heights than can be reached with kites, so-called "sounding" balloons are used. Made of pure rubber, filled with hydrogen and carrying self-recording instruments, these balloons have given us information of great interest and value to heights of 30 kilometers or more. Smaller, so-called "pilot" balloons, because of their comparative cheapness and convenience in handling, have in recent years come into general use for observing wind direction and speed. On clear days, when the wind is not too strong, these balloons can be followed by means of theodolites to heights well above 10 kilometers.

All of these methods have been rather extensively employed in Europe, particularly in England, France and Germany, and in the central and eastern portions of the United States. Some of them have been used to a limited extent also in Canada, Australia, Java and Argentina, as well as on a few expeditions of short duration to different parts of the Atlantic Ocean. In addition, there should be mentioned the great mass of cloud observations, some of which, particularly those during the International Campaign of 1896–97, were accurately and systematically made by means of nephoscopes and theodolites and furnished information, not only as to the heights and other characteristics of the clouds themselves, but also as to wind conditions at various levels.

Results.—Although a considerable amount of data has been gathered by the methods above outlined, it must be confessed that we know even yet comparatively little with reference to what is going on in the atmosphere above the earth's surface. The general state of our knowledge can be briefly summarized as follows:

(a) For parts of Europe and the United States we have well established average monthly, seasonal and annual values of all the meteorological elements from the surface to about the 5-kilometer level. Pressure of course always diminishes with altitude; temperature and humidity do so on the average, except that in the north-central portions of the United States there is a temperature inversion in the lower levels during the winter months; wind velocity increases, sharply in the lowest half kilometer, more gradually above that height; and wind direction is in the mean very nearly westerly at all levels, except in the southern part of the United States, where, during the summer, it is south to east near the surface.

(b) Of conditions between 5 and 25 to 30 kilometers we have rather limited information. We know that the temperature continues to diminish at a fairly uniform rate until a height of 8 to 18 kilometers is reached—this height varying with latitude, season, and sea-level pressure; above this limiting height the temperature ceases to diminish and in fact has a tendency to increase to some extent, at any rate during the summer half of the year. The boundary plane between the lower region of temperature decrease, known as the troposphere, and the upper region of little temperature change, known as the stratosphere, is in general well defined. Clouds do not occur in the stratosphere and winds generally have lower speeds here than in the troposphere. There is some evidence that at still greater heights wind direction changes from westerly to easterly, but data on this point are not conclusive. Other characteristics of the stratosphere are the lower temperature and greater height of its base in low than in high latitudes and during falling than during rising air pressure at the earth's surface; also, its greater height in summer than in winter.

(c) Of the relations found to exist between surface weather and free-air conditions, perhaps none is more significant than that between surface temperature distribution and winds in the upper levels. As is well known, the winds at and very near the surface conform quite closely to the surface pressure gradient, but at greater heights they often depart widely from it. If the temperature is fairly uniform over wide areas, the free-air winds are very nearly parallel to the surface isobars, and show that anti-cyclones and cyclones extend as such to great heights. If, on the other hand, the latitudinal temperature gradients are steep at the surface and also, though to a less extent, in the higher levels, then the surface pressure systems lose their identity at a very low altitude, the isobars opening out on the north side of cyclones and on the south side of anti-cyclones, and the winds veering or backing from those at the surface in conformity with the altered pressure distribution at the higher levels. This relation of free-air winds to surface temperature distribution has not thus far been accorded the attention it deserves. With the development of aviation and the resulting demand for accurate free-air wind forecasts, the significance of this relation must necessarily receive increasing recognition.

The foregoing summary is very sketchy and incomplete, but it will serve as a basis for the consideration of problems which must be attacked in the future, if real progress is to be made.

THE FUTURE

The present age, to a greater extent than any in the past, may be called an "age of projects." More and more mankind is giving heed to Emerson's exhortation, "Hitch your wagon to a star," and perhaps this is an especially appropriate motto for the aerologist to adopt as his own. Of the many ambitious plans that we read and hear about, some undoubtedly will yield negative results only, but it is equally certain that others will contribute very materially to human welfare. And it is better that some should fail than that none should be tried. The projects to be presented here are not visionary, but on the other hand very practical ones, and for the most part they are not difficult to carry out. They will be stated in the order in which it is believed they can be put into execution.

1. *Further study of data already accumulated.*—There is much material now available that has not been summarized and studied in detail or properly applied to the problems of aviation and forecasting, and to the solution of perplexing questions relative to the larger features of atmospheric circulation. One of the first things to be undertaken is the preparation of such a summary. Very few men, outside of the government services, are giving the subject any thought. Those *in* the government services can devote comparatively little time to it, because of other more pressing duties. There are needed at once for this purpose half a dozen well-trained men (well trained both in theory and in field experience) who can give all of their time to this subject for a period of 3 or 4 years. This, then, is a comparatively simple project—one requiring only a small outlay of funds, but giving results of immense value.

2. *Development of new methods of observation.*—As already stated, nearly all observing at the present time is done with nephoscopes, kites, pilot and sounding balloons. All of these methods have well-known limitations, but should be continued. There should be added, if possible, observations with kite balloons and airplanes. Kite balloons, although more expensive than kites and pilot balloons, would furnish data of correspondingly greater value, since they could be used with greater regularity, irrespective of weather conditions. Indeed, their use would be limited only by very high winds, and records could thus be obtained under conditions unfavorable for kites; in other words, by combining the two methods, the atmosphere could be explored up to 3 kilometers practically every day in the year. Like kites, their use would be restricted to regions not frequented by airplanes, because of the danger of fouling with the cable. Their use would be further restricted to places where hydrogen could be obtained. These limitations, however, are no more serious than others under which we now labor and can be overcome at comparatively small expense, when we consider the great value of the results obtained.

Development of suitable apparatus for use in airplanes should be pushed vigorously. As aviation expands, there will necessarily be a large number of places at which regular daily flights will be made at and above the fields for purposes of testing the machines and the training of pilots. Observations during these flights would add little expense and would provide information not otherwise obtainable, such as the thickness of cloud layers, etc. Work along this line has been done in England and France, but thus far to no great extent in this country, because of inadequate appropriations. It will be taken up as soon as funds for the purpose are made available.

3. *Extension of observation stations to all parts of the world.*—This must be done through international cooperation, but the United States can

make a good start by more completely covering its own territory, including Alaska, the Hawaiian and Philippine Islands, etc. In making such an extension, and a further extension later by all other countries, two separate and distinct purposes are to be served: (a) the furnishing of current information of immediate practical value to aviators; (b) the collection of statistical information required to explain the physical causes of various phenomena and, as a necessary consequence, to increase the accuracy of weather forecasting. For the first purpose it is sufficient to establish observing stations having comparatively simple equipment, by means of which the atmosphere may be explored to moderate heights only. It is not essential that temperature and humidity be observed, but it *is* essential that frequent observations be made of wind, cloudiness and visi-

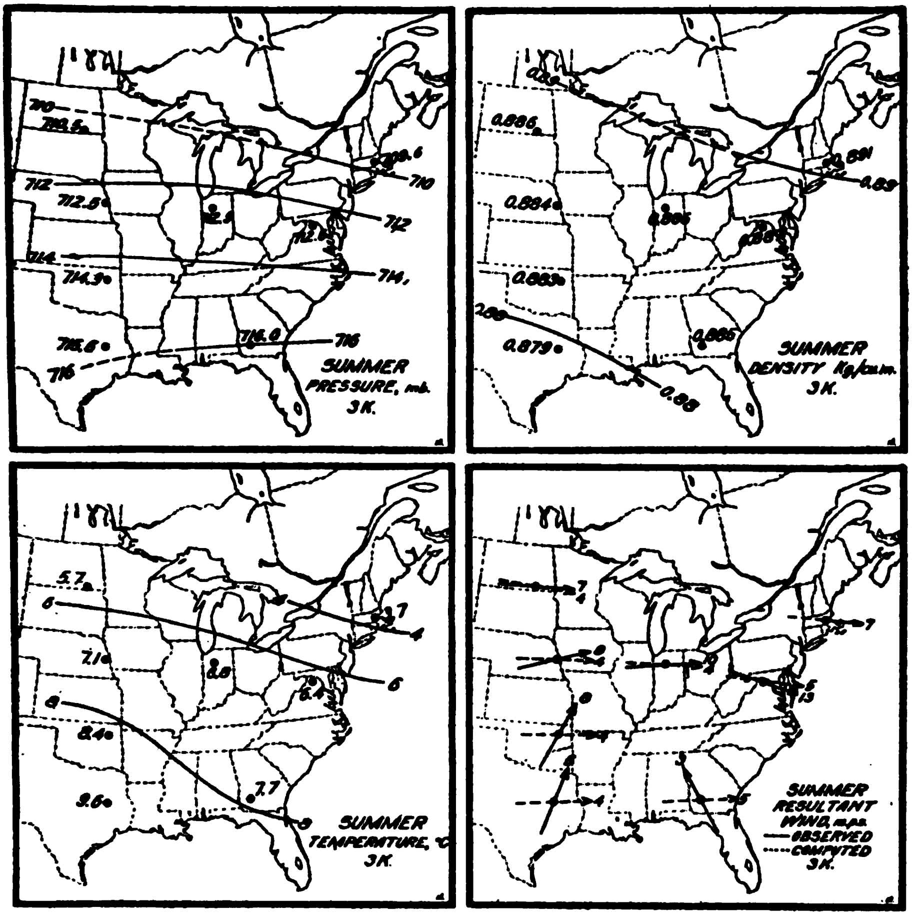

FIG. 4. Average summer values of pressure, temperature, density and resultant wind at the 3-kilometer level.

bility, these being the factors of vital interest to aviators. For the second purpose a much more comprehensive program is necessary. We should have accurate values of temperature and moisture as well as of wind. Observations should extend to as great heights and be as nearly continuous as possible, in order that we may know the diurnal and annual variations throughout the troposphere and much of the stratosphere; the characteristics of the atmosphere under different types of surface pressure and temperature distribution; and latitudinal and longitudinal variations. It is absolutely necessary that these data be collected and carefully studied. Otherwise we shall continue to be bombarded by theories and, worse still, by sweeping conclusions which can hardly stand the test of further light on the subject, but which (and this is the unfortunate feature), being advanced by men of recognized standing, find their way into textbooks as *facts* and thus start the student upon an entirely wrong track. As in all other matters, so in meteorology it is regrettably true that "a little learning is a dangerous thing." Specific references need not be made, but it may be remarked that in the past year or so there have been some particularly glaring instances of the promulgation of theories, based upon incomplete data, and of the more or less universal acceptance of those theories.

As examples of the kind of information needed, figures 4 and 5 are shown. They are based upon all observations thus far made with kites in this country, and give respectively average summer and winter values of pressure, temperature, density and resultant wind at the 3-kilometer level. These and similar charts for other levels, also charts showing relative humidity and vapor pressure, form part of a summary now in preparation, to be known as "An Aerological Survey of the United States." Some of the more prominent features shown in figures 4 and 5 are: (a) the close relation between the latitudinal pressure and temperature gradients; (b) the small latitudinal density gradient, owing to the counterbalancing effects of pressure and temperature, i. e., density varies directly with pressure, inversely with temperature; (c) the slight southward trend of lines of equal values of these elements from the interior to the eastern portions of the country; (d) the close agreement between computed and observed resultant winds in the winter season. The less satisfactory agreement in summer is due to the greater frequency of days with winds too light for kite flying (another argument for the use of kite balloons and airplanes); and (e) the small latitudinal difference in resultant wind speeds, due to the fact that these vary directly with the pressure gradient, but *inversely* with the sine of the latitude.

It is probably not a coincidence, but rather a matter of considerable significance, that the average movement of cyclones in the United States during the winter, as determined by Bowie and Weightman, is 13.4 m.p.s.[1] —a value in striking agreement with the resultant wind shown in figure 5. The agreement is less close in summer, apparently indicating that cyclones

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[1] Types of storms of the United States and their average movements. *Monthly Weather Review*, Supplement no. 1, p. 8.

extend to a greater height in that season than in winter, and this, as already pointed out, is undoubtedly the case.

These figures, however, are not shown with the view of discussing them as such, but rather with that of indicating how important it is that we obtain similar information for all other parts of the world—the sea as well as the land. Obviously, it is impossible to carry out this program at once in its entirety. We must therefore start with the most pressing needs, as follows:

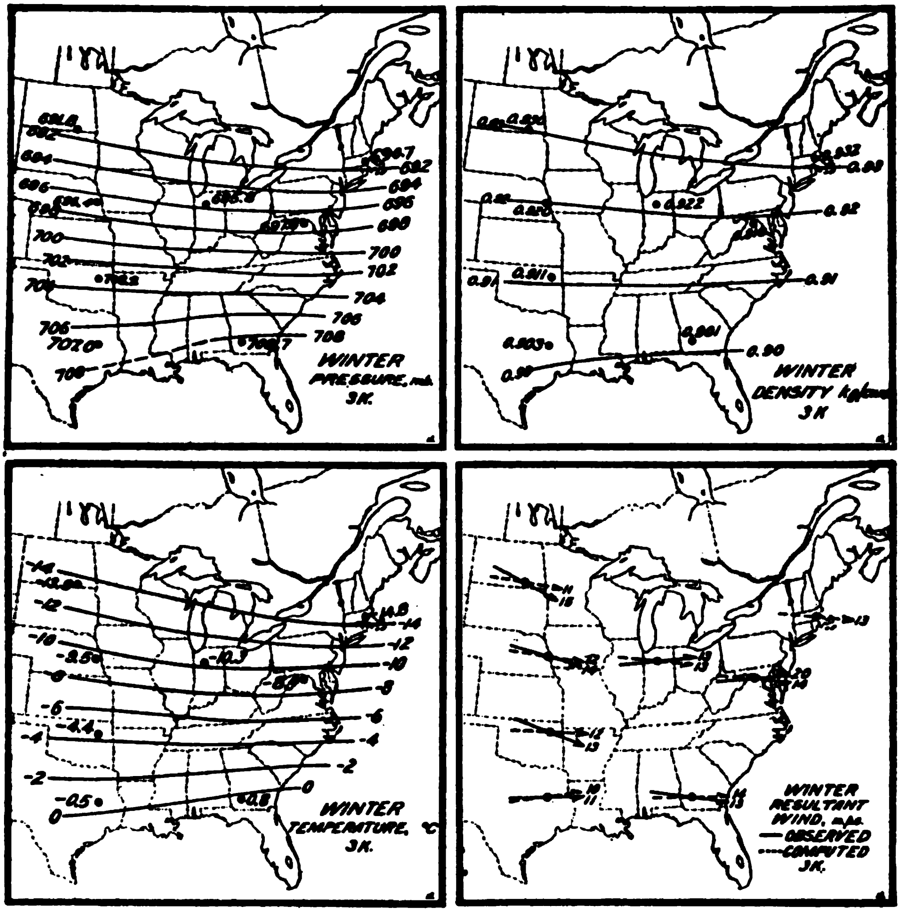

Fig. 5. Average winter values of pressure, temperature, density and resultant wind at the 3-kilometer level.

It is well known that the type of pressure distribution prevailing in the region of Alaska exercises a dominating influence on the weather of the United States. Similar relations are found in other parts of the northern hemisphere and emphasize the importance of having a network of stations, observations from which would make possible the construction of world weather maps—a subject which has already been presented by Major Bowie (see page 36). As indicated by him, such observations would enable the forecaster to follow from day to day the eastward march of the so-called "polar front." [1] There should be a string of stations as far north as possible and, in the southern hemisphere, another as far south as possible. Some of these stations at least should be provided with equipment for free-air exploration, this exploration to include accurate observations of wind and clouds by means of theodolites and nephoscopes. A few should, in addition, be equipped for making measurements of temperature and moisture, as well as wind, to great heights. These few would necessarily have to be not too far removed from sources of supply, but the others, if equipped with radio, could well be located as far north as living conditions would permit.

It has been said that definite meteorological laws will be established only from observations made at sea. These are difficult, perhaps impossible at the present time, to make, but there are numerous small islands where the influences of the land upon the atmosphere are negligible. Data of inestimable value can be obtained by establishing free-air observing stations in Bermuda, the West Indies, the Azores and the islands of the Pacific; also in Central America, where continental effects would be small. We know none too much about the prevailing westerlies, but our knowledge of them is voluminous compared to that of the antitrades. Such a network of stations as I have indicated, especially if operated for a considerable period of time and supplemented by observations from ships at sea, would provide the information now lacking and, in addition, would solve the much discussed and still unsettled questions of the exchange of air between the equator and the poles, the movements of hurricanes, etc.

Aside from the settlement of these theoretical questions, and perhaps more important, is the value of such observations for daily use in forecasting. With the development of radio communication, reports from these stations should be capable of speedy transmission to forecast centers, where they could be charted on upper-air maps, supplementary to the world weather maps, already discussed. Their value to aviators need not be argued. Can anyone doubt their even greater value, with further study, to the forecasting, not only of day-to-day weather, but also of week-to-week, month-to-month and possibly year-to-year changes in weather?

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[1] See V. Bjerknes. The meteorology of the temperate zone and the general atmospheric circulation. *Nature*, June 24, 1920, 522–524.

WORLD DIGEST OF METEOROLOGICAL DATA

By W. J. Humphreys

Meteorological data are gathered for, and serve, many purposes:

They are abundantly used in forecasting the weather of the morrow, but obviously used only once, and hence for this purpose need not be recorded.

They also are collected in the course of special studies, but the completion of each investigation renders useless the preservation of the particular material treated. It is the generalization—the law—that counts, and not the isolated values from which it happened to be deduced.

Finally, they are essential to many studies of interrelations between meteorological elements; to a knowledge of the relation of the weather in one part of the world to that occurring either previously, simultaneously, or subsequently, in others; and to all accurate knowledge of climates and their changes. For each of these purposes it is necessary that meteorological data be indefinitely accumulated, and equally necessary that they be put in manageable form and made widely available.

Now, although fully three-fourths of the surface of the earth is a meteorological blank, the mass of data already accumulated from the remaining one-fourth is so vast and heterogeneous as to be beyond the power of any individual to analyze and study in detail. Furthermore, even approximately complete sets of these data have been assembled in very few places. Hence much of the information contained in this meteorological material certainly is not only unknown, but even beyond the power of individual effort to know.

Therefore it is suggested that a comprehensive digest of all existing meteorological data be made and published. A possible way of accomplishing this great labor is as follows:

1. Let the data to be published (monthly and annual normals and departures therefrom, special phenomena, and what not), the units to be used, the form of publication, and all other details of this kind, be agreed to internationally.
2. Let each country furnish the digest of its own data.
3. Let the digest for each country consist of the individual digests for, and made at, its several meteorological stations.
4. Let some one agency, supplied with adequate funds and personnel, be charged with the duty of assembling sporadic data from countries that have no official meteorological organization; and with the further important duty of editing the entire work.

In this way the proposed vast labor would be divided up between several countries, and further subdivided among many individuals in each country, and the product of the combined effort of the many workers—the digest of all the world's meteorological data—soon made available to every institution that needs it and to every individual who wishes to study it.

There then could be students of world meteorological data, and would be; now there is none—and can not be.

It will be recognized of course that the plea here is for a greater "Réseau Mondial," one covering a larger number of meteorological elements than does that splendid publication, and also extending back to the beginning of meteorological observations. It would both include and supplement the data contained in the present Réseau Mondial, but would not take the latter's place as a convenient annual summary of the more important elements of the world's weather at selected places.

GENERAL ADOPTION OF THE CENTESIMAL SYSTEM OF ANGULAR MEASUREMENT—WITH APPLICATION TO ANEMOMETERS AND NEPHOSCOPES

By Alexander McAdie

Reviewing an article on "Uniformity in Aerographic Notation,"[1] Sir Napier Shaw[2] calls attention to the common usage of the capital letters N.E.S.W. for wind directions, and the established usage in Physics of N for Avogadro's constant, E for Energy, S for Entropy and W for internal work.

The criticism is constructive and suggestive. The question arises: Is it not desirable to follow the lead of navigator and magnetician and use degrees instead of letters to indicate direction of air flow? There are some distinct gains from such a usage for the aerographer or chart maker of the winds. Official weather bureaus record direction on a 45-degree basis; that is, eight directions are given. It has long been felt that such records were not sufficiently detailed. Precision, detail and convenience are gained by the use of the degree.

There is no mechanical difficulty in getting continuous records of wind direction for the entire circle. Many forms of anemoscope give such records. Figure 6 gives such a record sheet based on one used at Blue Hill for 35 years. The eight cardinal directions are noted; but instead of 32 points of the compass, as heretofore, the intervals are at 10 degrees, and thus 36 divisional lines appear instead of the old compass point 11.25. For convenience in computation there is also introduced Greenwich Mean Civil Time, beginning at an hour appropriate for changing records on this coast, noon 75th meridian time being 17 hours Greenwich time. The sheet, however, is adaptable to any station meridian time, by inserting the proper hour in the S.M.T. column. If, however, the centesimal system is to be used, the number of divisional lines is increased to 40, and since there are 400 grads in the circle, each division represents 10 grads or 9 degrees.

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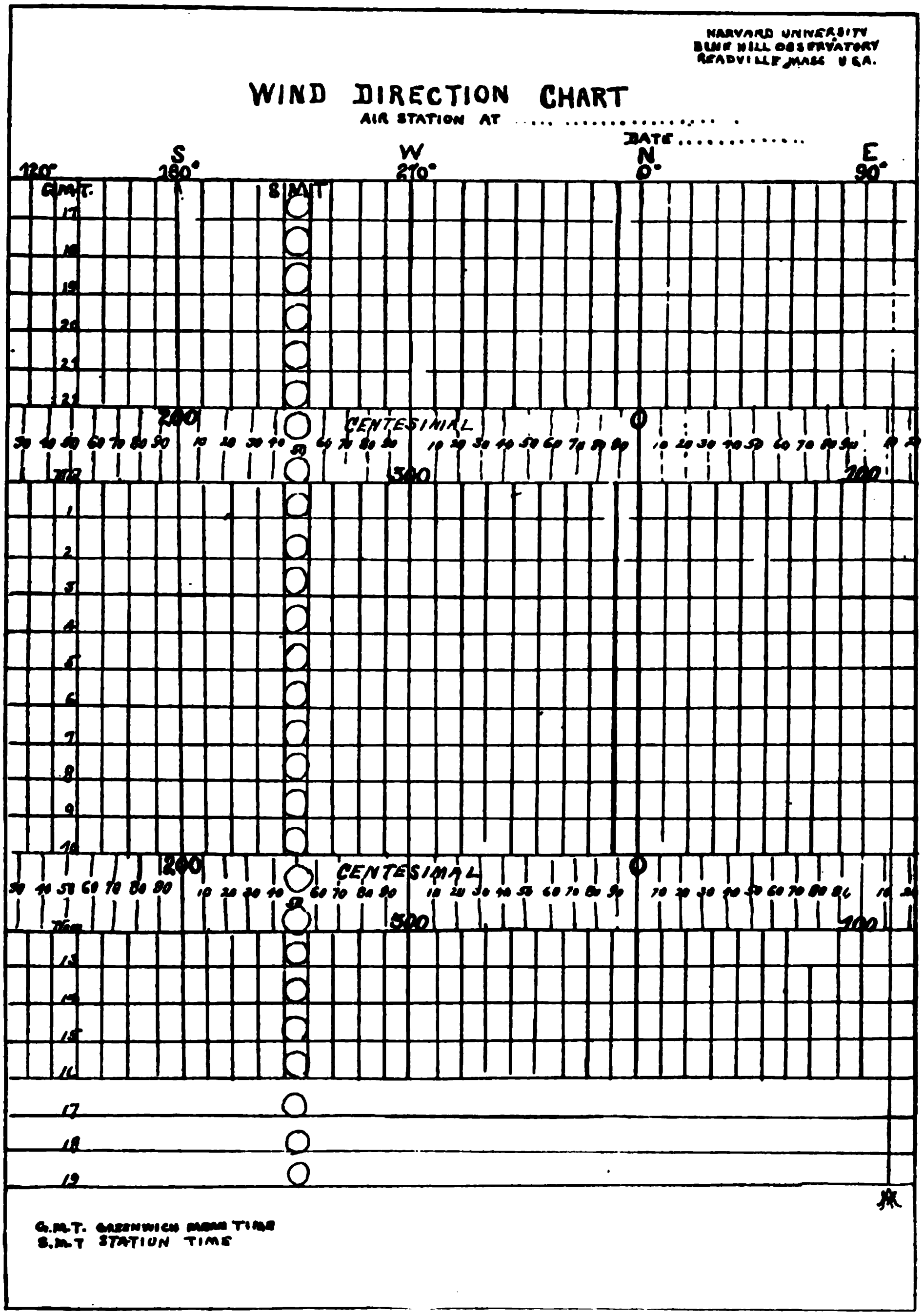
[1] H. A., 83–4, pp. 169–180.

[2] *Nature*, Nov. 4, 1919.



HARVARD UNIVERSITY
BLUE HILL OBSERVATORY
READVILLE, MASS. U.S.A.

WIND DIRECTION CHART

AIR STATION AT ....

DATE ....

G.M.T. GREENWICH MEAN TIME
S.M.T. STATION TIME

Fig. 6. Wind-direction chart.

By the use of such charts, the words westerly, easterly, and other like terms disappear and the flow is more definitely described. Wind vanes, unfortunately, do not fly *with* the wind, but against the wind, the arrowhead pointing into the wind. On the other hand, in all charts of air flow, the arrows fly with the stream.

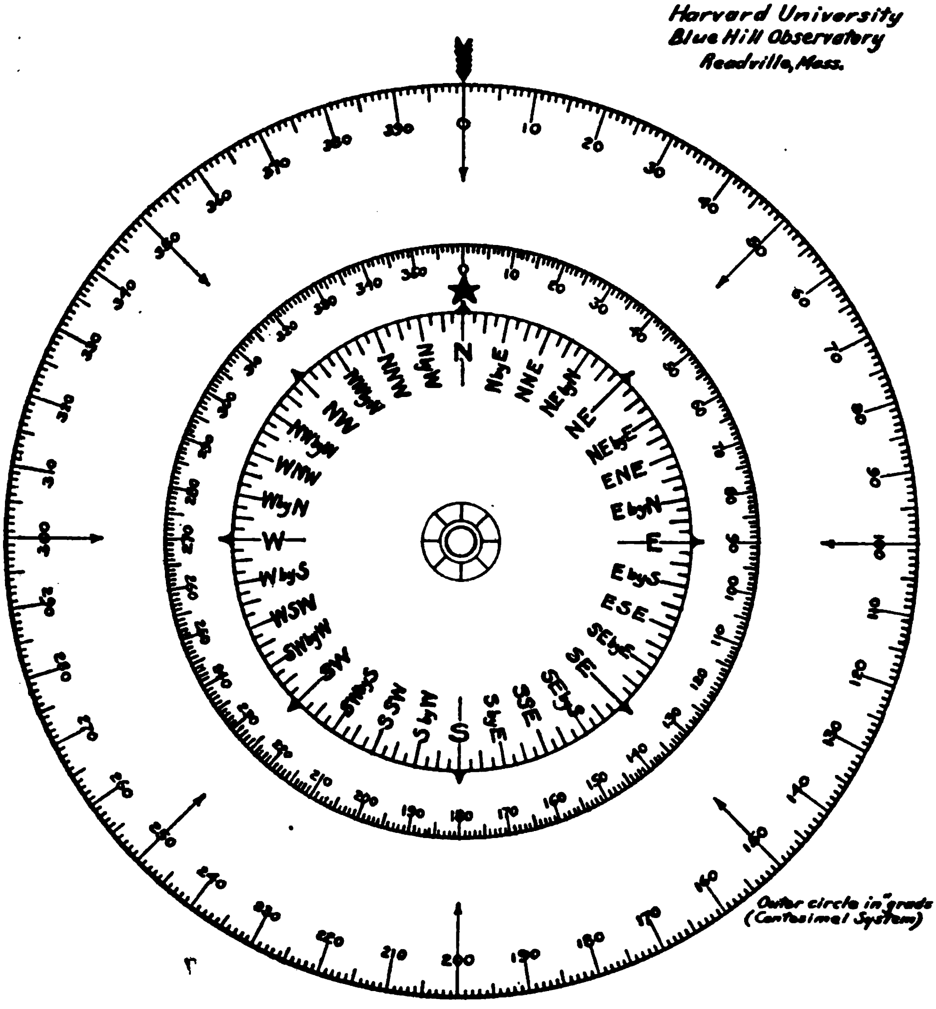


FIG. 7. Wind protractor for use with McAdie nephoscope.

The direction of flow is read to the right, starting from zero, at the north; and thus an east wind is definitely recorded as 100 grads (or 90 degrees), and a south wind as 200 grads (or 180).

The value of the natural sine of 100 grads is 1. The following abridged table gives the sines, cosines, tangents and cotangents of every 10 grads:

grads	sine	cosine	tangent	cotangent
100	1.00000	0.0000	∞	0.0000
90	.9877	0.1546	6.3138	0.1584
80	.9511	0.3090	3.0777	0.3249
70	.8910	0.4540	1.9626	0.5095
60	.8090	0.5878	1.3764	0.7265
50	.7071	0.7071	1.0000	1.0000
40	.5878	0.8090	0.7265	1.3764
30	.4540	0.8910	0.5095	1.9626
20	.3090	0.9511	0.3249	3.0777
10	.1546	0.9877	0.1584	6.3138
0	.0000	1.0000	0.0000	∞

During the World War those of us who were engaged in aerographic work in France found it necessary to use the centesimal system. Since the war, the method has been adopted by the Scandinavian countries.

In nephoscopic determinations, the method has been used with success at Blue Hill. A comparative dial of the compass, the magnetic and the centesimal values is given in figure 7.

A SINE GALVANOMETER FOR DETERMINING IN ABSOLUTE MEASURE THE HORIZONTAL INTENSITY OF THE EARTH'S MAGNETIC FIELD[1]

By S. J. Barnett

A brief historical statement was made with reference to the measurement of the horizontal intensity of the earth's magnetic field by electrical methods, and a general description of sine and tangent galvanometers was given, with the suggestion of an improvement in the latter. Then followed a detailed description of a new sine galvanometer, constructed, with certain exceptions mentioned below, in the workshop of the Department of Terrestrial Magnetism.

The base of the instrument, including the tripod, circles, etc., was taken from one of Wild's theodolites, constructed by Edelmann, and was much improved by the substitution of non-magnetic parts for parts too magnetic, and by the substitution of electrical illumination of the precision circle for daylight illumination by mirrors.

The magnetometer-box is of pure copper, the damping being chiefly electro-magnetic. The magnet-mirror is a fine disc of chrome steel with optically flat and parallel surfaces, being in fact one of the gages made by the Bureau of Standards. The torsion tube and head are similar to those of the C. I. W. magnetometers. A suspension of phosphor-bronze strip with torsional constant about 0.001 is generally used. The telescope is small but powerful; the scale is ruled to thirds of mm., on white pyralin, with all necessary adjustments. The period of the magnet and the damping, which is adjustable, are such that readings require only a few seconds.

The arrangement of coils is approximately that due to Helmholtz. The spool was machined from white Carrara marble impregnated with paraffin at a temperature near its boiling point. The coils were wound under tension in a single layer in spiral grooves cut with a carbon diamond tool. The wire is pure copper, especially prepared in the research laboratory of the General Electric Co. Each coil is wound in two halves and contains 10 turns with a diameter of approximately 30 cm. and a pitch of approximately 2 mm. The two halves start from the same horizontal plane 180 degrees apart, so that the distance between centers of adjacent wires is approximately 1 mm. The axial distance between the centers of the two coils, or the distance between corresponding turns of the spirals, is approximately 15 cm. The insulation resistance between adjacent wires is very high.

The methods of measuring the diameters and axial distances of the spirals were briefly described and some of the results were given in tables

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[1] Abstract of the report presented before the American Geophysical Union, Washington, D. C., April 18, 1921.

and curves, projected on the screen. The magnetic tests, of three kinds, proving the materials to be satisfactory, were also described.

The theory of the instrument, the method of using it, and the calculation of the error in the constant of the coils due to construction, as well as of the other errors introduced in the measurement of the horizontal intensity, were briefly presented.

It was shown that the errors in reading the circle and the telescope scale when sufficiently large angles are used, and the error in the constant of the coil, were quite negligible; and that the only other error necessary to consider, viz, that introduced in the measurement of the current traversing the coils, can also be made entirely negligible. In consequence, the horizontal intensity of the earth's magnetic field can be determined with an error less than 1 part in 10,000, which more than fulfills all necessary requirements.

The instrumental work, done in the shop of the Department, chiefly by Mr. G. H. Jung, instrument-maker, is highly satisfactory.

The report was closed with acknowledgments.[1]

Department of Terrestrial Magnetism,
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ACTIVITY OF THE EARTH'S MAGNETISM IN 1915

By D. L. Hazard

At the meeting of the International Commission for Terrestrial Magnetism held at Innsbruck in 1905 a resolution was adopted recommending that magnetic observatories classify each day according to its magnetic character as quiet, moderately disturbed, or severely disturbed, using the notation 0, 1, and 2 for this purpose. This recommendation has been adopted by different observatories, one after another, so that now nearly all of the prominent observatories are sending quarterly reports of the magnetic character of days to the Netherlands Meteorological Institute and that institution is publishing them, thus making the data available for all. While this method of characterization is necessarily rough and influenced by the personal equation of the observer, yet the mean of a large number of estimations (between 35 and 40 at the present time) gives a very good idea of the relative magnetic condition of the whole earth from day to day. It does not, however, give an absolute measure of the daily fluctuations of the earth's magnetism nor does it permit a comparison of conditions in different parts of the earth.

In order to determine quantitatively as well as qualitatively the varia-

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[1] Since the presentation of this report, the constant of the coil has been redetermined by the use of many additional linear measurements, and two series of simultaneous determinations of the horizontal intensity with the sine galvanometer and the C. I. W. standard magnetometer No. 3 have been made, Messrs. Fleming, Fisk, Peters, Ives, and Barnett participating. The results obtained showed a satisfactory agreement between the two different types of instrument. A complete account of the instrument is given in Vol. IV of the "Researches of the Department of Terrestrial Magnetism."

bility of the earth's magnetism as a whole, the late Doctor Bidlingmaier devised a method which takes as a measure of the activity of the earth's magnetism its departure from moment to moment from its normal or undisturbed condition. As we have no means of determining as yet what the normal magnetic condition of the earth is, he adopted as the basis for his computations the mean value for the period under discussion; that is, the activity for a day is based on the momentary departures from the mean value for the day. He found that in determining the activity for the day with reference to the mean, the computation could be separated into two parts, first the departures of the hourly mean from the mean for the day, and second the departures of the individual values in each hour from the mean value for that hour. If it later should become desirable to refer to a base value other than the daily mean it would only be necessary to add a third term, depending on the difference between the daily means and the new base value. In each step the mean of the squares of the departures from the base value is computed and this quantity expressed in γ^2 must be divided by 8π to get the activity expressed in terms of the unit 10^{-10} erg/cm^3; that is, the energy per unit volume.

The regular observatory tabulations contain the data for computing the first part provided the mean ordinate for each hour is tabulated, as is now the established practice. Computation of the second part would ordinarily involve the reading of ordinates at frequent intervals for each hour and the computation of the mean of the squares of the differences from the mean value for the hour, this being the so-called hour-integral used in Bidlingmaier's formula in determining that portion of the activity. The amount of work involved would, of course, be prohibitive and he accordingly simplified the process by reading the ordinates for a limited number of hours and using the relation between the hour-integral and the hourly range as a basis for determining the hour-integral for the remaining hours from the hourly range. When the results were plotted with amplitude (half range) as abscissa and hour-integral as ordinate, it was found that the line joining the plotted points formed a smooth curve of parabolic form. While it was found that the values of hour-integral corresponding to a given hourly range differed considerably among themselves, as would naturally be the case because of the varying character of the fluctuation within the period of an hour, yet it was believed that for most purposes, where the results would be combined to obtain mean values, the relation between hourly range and hour-integral derived from a limited number of hours could safely be used in determining the hour-integral for a long period, as for a year.

The parabolic form of the curve representing the relation between amplitude and hour-integral suggested the probability that a linear relation would be found to exist between the square of the amplitude (or range) and the hour-integral. In fact, this must necessarily be the case if the value of activity is to be independent of the sensitivity of the instruments.

At the request of the International Commission for Terrestrial Magnetism a number of observatories undertook to compute the activity of the earth's magnetism according to this method for each day of the year 1915, the Coast and Geodetic Survey carrying out the work for its magnetic observatory at Cheltenham, Maryland. It was suggested by the International Commission that other observatories might safely accept the relation between hourly range and hour-integral as determined by Bidlingmaier for Wilhelmshaven for the year 1911. It was thought best, however, by several observatories, to re-determine this relation in order to be assured that it did not change from place to place. The results show that while for Wilhelmshaven the hour-integral was equal to 11¼ percent of the square of the range, for Cheltenham the factor was 10 percent and for Seddin, near Potsdam, 8¾ percent, and an investigation by Chree, which included the study of the records of the British Antarctic Expedition, showed that while for ordinary latitudes the variation in the factor was not great, conditions were quite different in very high magnetic latitudes.

When the preparation of this paper was undertaken it was expected that it would be possible to compare the results from several observatories, but it was found that only the Seddin results, in addition to those for Cheltenham, were available in printed form.

The geographic positions and mean values of the magnetic elements for 1915 for these two stations are as follows:

Observatory	Cheltenham	Seddin
Latitude	38° 44′ N	52° 23′ N
Longitude	76 50 W	13 04 E
Declination	6 04 W	8 17 W
Dip	70 47 N	66 25 N
Horizontal intensity	19417γ	18726γ
Vertical intensity	55694γ	42898γ
Total intensity	58982γ	46808γ

It will be seen that the two observatories differ very nearly 90° in longitude, and while Seddin is much farther north than Cheltenham, the magnetic dip and intensity are much greater for Cheltenham, the latter station being nearer the magnetic pole.

The activity has been computed for each hour for *D*, *H*, and *Z* at Cheltenham and for *X*, *Y*, and *Z* at Seddin, and these three are combined to get the total activity. The quantities published are the mean value for each day of the year and the hourly means for each month.

In discussing the results of this method of determining the activity three things must be kept in mind. First, since the activity is based on the *square* of the departure from the mean value, a day of very large disturbance will have an overpowering effect on mean values in which it enters. For example, the activity at Cheltenham for June 17 was 1477 and the total for the other 29 days was only 666. Second, Bidlingmaier's conception of activity is different from the usual idea of activity as represented by a magnetic disturbance. In the latter case we think only of abnormal variations, whereas he includes all variations, whether systematic (as diurnal variation) or abnormal. Third, as part of the variation of the earth's magnetism is a function of local mean time and part is a function of absolute time, the results for different observatories are not strictly homogeneous. For this reason a more satisfactory agreement between the results for different places may be expected if only that part of the activity is considered which is derived from the hour-integral, as this is to a greater extent independent of local mean time. Even then, however, there is some lack of homogeneity, as for example, in the case of the Seddin tabulations the day begins at Greenwich midnight, whereas for Cheltenham it begins at 5^h G.M.T.

The portion of the activity derived from the hour-integral is much smaller than the part depending on the differences between the mean hourly values and the daily mean, only about one-eighth as great on the average for all days and only one-twenty-fifth for the less disturbed days. The fact that the normal diurnal-variation is such a predominant factor in the total activity as derived by Bidlingmaier's method raises the question whether results of greater value would not be obtained if the diurnal variation was eliminated, at least in part, in computing the activity.

A comparison of the daily mean values of total activity for the two observatories for 1915 shows a general agreement, but with considerable difference in detail, largely because of the third point referred to above. The total activity is on the average about 15 percent greater for Cheltenham than for Seddin, as was to be expected on account of its higher magnetic latitude, but for many days and even for some monthly means the Seddin values are greater.

If only the hour-integral activity is used in the comparison, the agreement is much closer, and when the results are smoothed out by taking five-day means the plotted curves for the two places are almost identical in phase. The agreement is almost as good with the international character numbers, both in phase and relative amplitude, and speaks well for that simple method of determining the degree of disturbance.

A comparison of the monthly means with the relative sun-spot numbers for the same periods shows little evidence of systematic agreement, thus, as pointed out by Schmidt,[1] confirming former experience in comparing these numbers with terrestrial phenomena. In this case the lack of agreement is no doubt partly due to the exaggerated effect on the mean activity for a month of a single day of great disturbance.

As to the diurnal variation of the total activity, the predominant feature is a maximum occurring about noon local mean time. At Cheltenham this feature is modified in the months May to August to form a two-peaked summit with maxima about 10^h and 14^h and a considerable depression between. There is little variation in activity during the night hours. The range of activity is greater in summer than in winter, though, as

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[1] *Terrestrial Magnetism*, Sept., 1920.

already pointed out, the effect of a single day of great disturbance is overpowering. The above characteristics can be traced at once to the features of the diurnal variation of the magnetic elements, the two maxima at Cheltenham corresponding to the minimum horizontal intensity before noon and the maximum west declination after noon.

If we consider only the hour-integral activity, we find very little evidence of system in its diurnal variation in the different months, but there seems to be a tendency toward higher values toward the end of the 24 hours. This is more pronounced for Seddin and corresponds to the distribution of disturbed hours arrived at directly. The effect of a few disturbed hours is so great, however, that it is hardly safe to draw definite conclusions from the results of a single year.

Schmidt discusses some other phases of the activity at Seddin in 1915, in the paper referred to above, but time does not permit going into the matter in greater detail here. He also makes some comparisons with the results of simpler methods of determining the activity.

As a result of the activity computations for 1915, I am of the opinion, which is shared by Schmidt and Chree, that equally valuable results can be obtained by other methods that involve much less time and labor than Bidlingmaier's. His method gives undue weight to days of large disturbance in any combination of hourly values, and the introduction of the term depending principally on the diurnal variation of the earth's magnetism prevents a satisfactory comparison of results at different stations.

Division of Terrestrial Magnetism,
U. S. Coast and Geodetic Survey.

ON MEASURES OF THE EARTH'S MAGNETIC AND ELECTRIC ACTIVITY AND CORRELATIONS WITH SOLAR ACTIVITY

By Louis A. Bauer

When attempting to find correlations between manifestations of the sun's activity and those of the earth's magnetic and electric activity, three points require immediate consideration:

(1) What shall be taken as an adequate measure of the sun's activity with respect to such radiations and emanations as are likely to have an effect upon the magnetic and electric fields of the earth?

(2) What shall be taken as an adequate measure of the earth's magnetic activity, or of the earth's electric activity?

(3) What quantities shall be taken as defining the so-called normal or undisturbed condition of the earth's magnetic field, or of the earth's electric field?

With respect to the first question, we have at present at our disposal the sun-spot numbers, sun-spot areas, flocculi areas, prominences, faculæ, and solar-constant values.

For measures of the earth's magnetic activity, as well as of its electric activity, we may use fluctuations in the magnetic and electric quantities, which are more or less periodic in their character, as, for example, the diurnal range or annual range of the magnetic and electric elements. But it is also found that during a magnetic storm and for some time afterwards, the earth's permanent magnetic state, as also possibly its electric state, has been affected. Thus, we have at our disposal both fluctuations about a mean value for a certain interval, and also change in that mean value for a given time. The selection of normal or undisturbed values of any measure taken may be based upon the international list of so-called magnetically-calm or electrically-calm days. Though it must not be overlooked that often the values of the magnetic and electric elements on such days are affected by a peculiar kind of disturbance. In brief it has been found that the magnetic or electric elements on a comparatively undisturbed day are not necessarily normal values. Rather may the values be "normal" which lie intermediate between those for the "quiet" days and those for the days of moderate disturbance.

Every analysis thus far undertaken of any particular magnetic fluctuation indicates that the observed effects are to be ascribed to at least two systems of forces: *E*, an external system consisting most probably of electric currents in the upper regions of the atmosphere; and *I*, an internal system consisting of electric and magnetic systems below the earth's surface. The two systems *E* and *I* are not necessarily related as though *I* were the result of an inductive effect caused by the system *E*. The system *I* would appear rather as a composite system, composed primarily of a direct effect and secondly of an indirect effect which may be related to the fluctuating *E* system. Indications have also been found of the presence of a third system, *C*, consisting of vertical electric currents which apparently pass through the earth's surface, either from the atmosphere or from some internal source. What we observe during a magnetic storm is the combined effect of the three systems, *E*, *I*, and *C*, and this important fact must be borne in mind in endeavoring to find correlations between solar activity and terrestrial activity. It may even happen, as apparently was the case on May 8, 1902, during the eruption of Mont Pelée, that we have a world-wide magnetic fluctuation of internal rather than external origin. Hence, were it feasible, a mathematical analysis should be undertaken first of a magnetic disturbance in order that the effects coming from external sources may be separated from those to be related to internal ones.

The question has also been raised, since at times a magnetic disturbance on the earth apparently precedes some striking manifestation of solar activity, whether there may not be also the possibility of a universe disturbance-system affecting both solar activity and planetary magnetic activity.

As the combined result of my investigations to date, it is found that, in general, the most successful measure of solar activity, of special interest here, is a quantity indicative of the variability of sun-spottedness during a given period. For example, instead of taking the sun-spot numbers (N) direct for comparison with magnetic or electric fluctuations, the range (R) in the sun-spot numbers per month, or the average departure (D) of the daily sun-spot numbers from the monthly mean, irrespective of sign, is taken. The R and D quantities are found to run closely parallel to one another; preference was finally given to the D-measure of solar activity as it utilizes all the sun-spot numbers (N) during a month, whereas, the R-measure depends only on two numbers—the maximum and minimum sun-spot numbers of the month. The annual mean values of R and D are furthermore found to run closely parallel with the N-numbers; the monthly values of R and D, however, generally follow a decidedly different course from the N-numbers and exhibit a closer relationship with the measures of the earth's magnetic, or its electric, activity than do the latter (the N's). Some of these relationships between solar activity, terrestrial magnetism, and terrestrial electricity (earth-currents, atmospheric electricity, and polar lights) are shown in figures 1 and 2 and are summarized below.

The adopted measure of the earth's magnetic activity is a quantity, $w = \epsilon Hv$, where H is the horizontal intensity of the earth's magnetic field at the observing station and v, the observed magnetic variation, or range of the magnetic fluctuation; ϵ is a numerical factor. It may be shown theoretically that this value of w, as a first approximation, is representative of the energy-change which the earth's magnetic field experiences during a magnetic variation.

Under certain assumptions it may also be shown that the R and the D measures of solar activity may be regarded, as a first approximation, as representing an energy-change experienced by the sun during a manifestation of activity.

Obtaining similarly, as just described for the sun-spot numbers, R and D measures of solar activity from the solar-constant values (E), which have been observed under the auspices of the Smithsonian Institution at Calama, Chile, during 1919 and 1920, it is found that these latter measures run much more closely parallel with the R and D measures derived from sun-spottedness than do the numbers E and N.

Connections between sun-spot activity, disturbances of the earth's magnetism, earth-currents, and polar lights have been worked out by various investigators. The present investigation shows that there is a fifth natural phenomenon—atmospheric electricity—by which an interesting and suggestive relationship with solar activity is exhibited. Owing to the many disturbances to which the atmospheric-electric elements are subject, as for example during cloudy and rainy weather, it has been more difficult to establish the existence of definite variations of the chief atmospheric-electric elements during the well-known sun-spot cycle of somewhat over 11 years than in the case of magnetic effects, earth-currents, and polar lights. The new results found are based upon atmospheric-electric data obtained chiefly at four European observatories between 1898 and 1919,

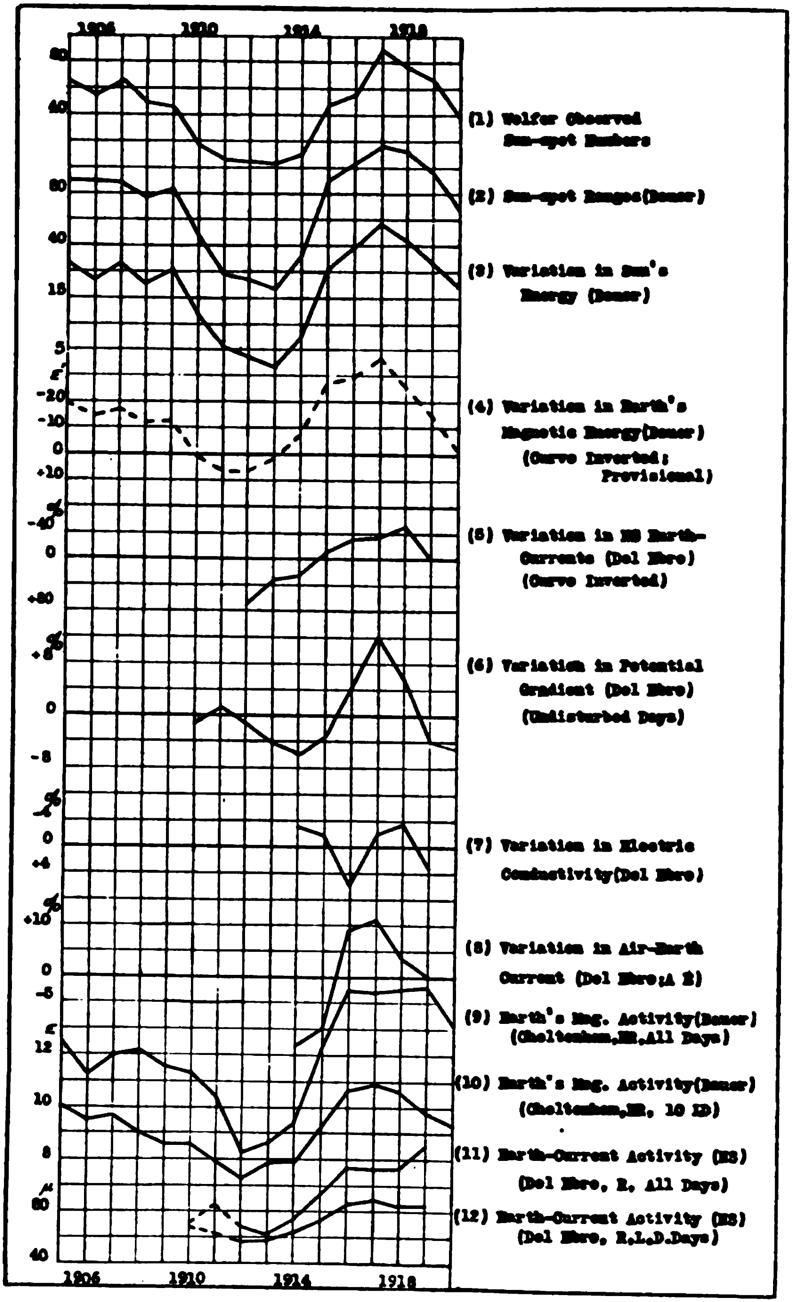

FIG. 1. Variations in solar activity, terrestrial magnetism, atmospheric electricity, and earth-currents during 1905–1920.

the combined data in the case of the potential-gradient thus covering about two sun-spot cycles. Recent observations on board the *Carnegie* also indicate a decrease in the electric potential-gradient since 1917, when sun-spot activity was at a maximum. A half century ago Quételet at Brussels and

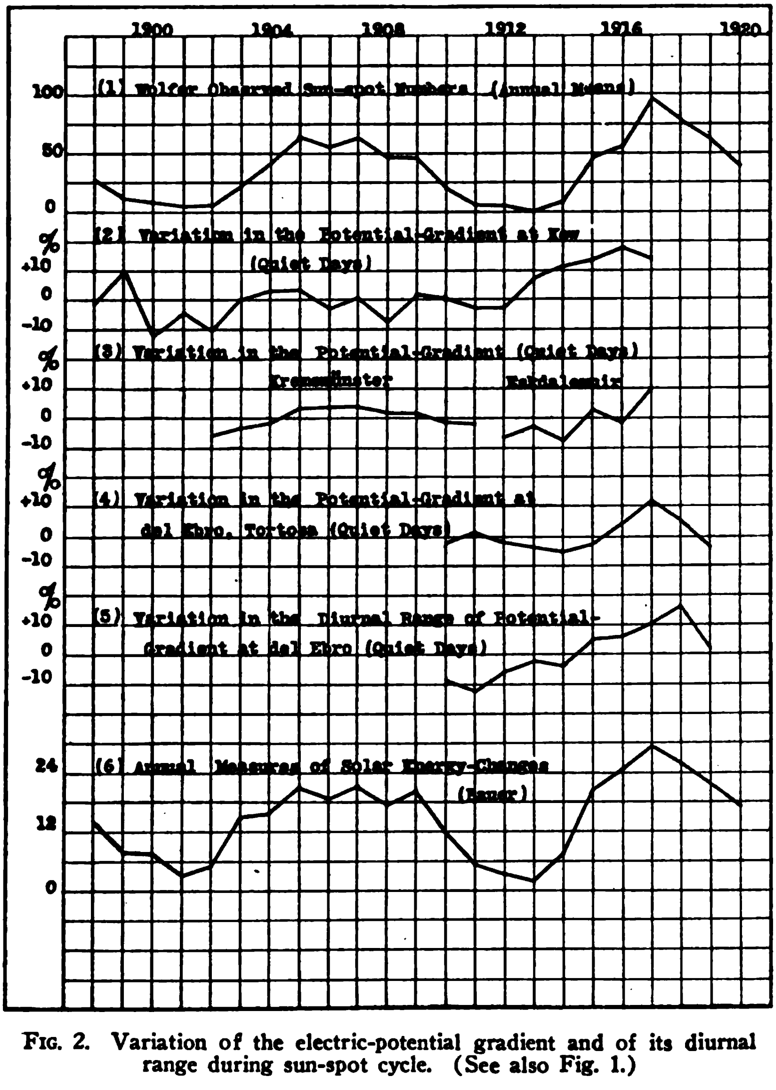

FIG. 2. Variation of the electric-potential gradient and of its diurnal range during sun-spot cycle. (See also Fig. 1.)

Wisclizenus at St. Louis believed that they obtained some definite results showing a variation in the potential-gradient dependent upon sun-spottedness. Owing, however, to the uncertainty of results obtained by the instrumental methods then in use and because of the necessity of thoroughly eliminating the numerous disturbances dependent upon meteorological conditions, these previous results have not been accepted, and so modern treatises on atmospheric electricity omit mention of any possible relationship between atmospheric electricity and solar activity.

More complete publication of the results of the investigations here outlined is made in the 1921 volume of the *Journal of Terrestrial Magnetism and Atmospheric Electricity*, pages 33-68 and 113-115.

CHIEF RESULTS

1. The earth's magnetic energy and average intensity of magnetization, as well as the strength of the normal electric currents circulating in the earth's crust, suffer a diminution during increased solar activity. The electric currents induced in the earth during periods of increased solar activity are in general reversed in direction to the normal currents, the strength of these superposed currents increasing with increased solar activity.

2. The diurnal range of the strength of earth-currents, as in the case of the diurnal range of the earth's magnetic elements, increases with increased solar activity; at time of maximum activity the range, as shown by the observations at the Observatorio del Ebro, Tortosa, Spain, during the period 1910–1919, was about 50 percent higher than at the time of minimum solar activity.

3. The magnetic effect running a concomitant course with the solar-activity cycle is retarded, on the average, about one year so that there is a residual, or an acyclic, effect at the end of the cycle. The actual amount of retardation, in general, increases with intensity of the sun's activity or energy. This lag in the magnetic effect may be accounted for by the fact that the electric currents generated inside the earth during magnetic storms and magnetic variations continue for some time after the apparent cessation or diminution of solar activity, or after the period of the variation experienced. The same lag is shown by polar-light frequencies at times of maximum solar activity.

4. On fine-weather, or electrically-calm, days the atmospheric potential-gradient, or the deduced negative charge on the surface of the earth, increases with increased solar activity, the range in the variation between minimum and maximum solar activity being about 20 percent. The electric conductivity of the atmosphere, on the other hand, shows but little, if any, systematic variation during the solar cycle. Accordingly, since the vertical conduction-current of atmospheric electricity is derived from the product of the potential-gradient and the electric conductivity, it is found that this vertical current also increases in strength with increased solar activity. It would thus appear that atmospheric electricity, like terrestrial magnetism, is controlled by cosmic factors. The results derived here may have an important bearing upon theories of atmospheric electricity.

5. The diurnal range of the electric potential-gradient as deduced from the observations on the electrically-calm days, made at the Observatorio

del Ebro, Tortosa, Spain, 1910–1919, is found to increase with solar activity; the minimum occurred in 1911 and the maximum in 1917, whereas the sun-spot minimum occurred in 1912 and the maximum in 1917. The range between minimum and maximum diurnal range is about 25 percent. (It appears probable that the same fact just stated for the potential-gradient will also be found true for the vertical conduction-current.)

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THE PENETRATING RADIATION AND ITS BEARING UPON THE EARTH'S ELECTRIC FIELD[1]

By W. F. G. Swann

The paper was devoted largely to a description of certain investigations on the penetrating radiation in progress at the University of Minnesota, under the author's direction. It opened with a brief review of the status of our knowledge with regard to the penetrating radiation.

In a hermetically sealed zinc vessel freed from radioactive air, ions are produced at a rate of about 8 or 9 per c.c. per second over the land. According to the computations of A. S. Eve, the normal gamma-ray radiation from the atmosphere is capable of accounting for about 0.06 ion per c.c. per second, while that from the soil will account for 1.6 ion per c.c. per second, making in all 1.7 ions. On account of the secondary ionization resulting from electrons emitted from the walls of the vessel by the primary radiation, this value becomes increased to about 2.5 ions in the case of vessels of the size ordinarily used. If this value be subtracted from the 8.5 ions per c.c. per second found over land, there remains about 6 ions per c.c. per second attributable to causes which are not directly obvious, and this corresponds roughly to the ionization (4 to 6 ions per c.c. per second) observed over the ocean.

The results of several investigators seem to indicate that, in vessels surrounded by thicknesses of water sufficient to absorb practically all gamma-ray radiation of the ordinary type, the ionization does not become reduced greatly below that found over the great oceans, nor does it diminish appreciably with increasing thickness of water, so that, if this ionization is due to an external radiation, this radiation must be of an extremely penetrating type as compared with ordinary gamma-radiation. By using a vessel of ice, in order to be as free as possible from radioactive contamination, McLennan obtained a value as low as 2.6 ions per c.c. per second over Lake Ontario, and inclines to the view that even this small residual is attributable to lack of complete purity of the ice. On the other hand, results obtained by Kolhörster in balloon ascents up to 9.5 kilometers show an increase to about 80 ions per c.c. per second at this altitude, in a her-

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[1] Abstract of the paper presented at the annual meeting of the American Geophysical Union, Washington, D. C., April 18, 1921.

metically sealed, light, tight vessel; moreover, the rate at which the apparent ionization increases with altitude in the neighborhood of 9 kilometers is such as to suggest that at altitudes but slightly greater, the ionization might attain enormous values. The validity of Kolhörster's results has been questioned by C. H. Kunsman in view of possible complications resulting from the effect of the low temperature on the insulating material used; if, however, they should be substantiated, they afford one of the most convincing evidences that one could wish as to the true cosmic nature of part at least of the so-called penetrating radiation. By the employment of precautions such as to prevent all uncertainty as regards leakage, and by the use of small pilot balloons, the writer is at present endeavoring to extend the observations of Kolhörster to greater altitudes.

VARIATION OF RESIDUAL IONIZATION WITH PRESSURE

The study of the variation of the residual ionization with pressure has a very important bearing upon the origin of that ionization. If the residual ionization were due to alpha-rays emitted from the walls of the vessel, it would show practically no increase with pressure in the case of a vessel whose linear dimensions were of an order of magnitude greater than the range of the alpha-particles. Analagous remarks hold for the case of soft beta-rays emitted from the walls. In the case of a radiation of cosmical origin there are three possibilities, viz, (1) a direct ionization by the primary radiation, (2) ionization by slowly moving electrons emitted from the gas by the primary radiation, (3) ionization by rapidly moving electrons emitted from the gas by the primary radiation, the penetrating power of the electrons being such as to enable them to go right across the vessel at atmospheric pressure. A fourth possibility resulting from emission of electrons from the walls of the vessel by the primary rays, is indistinguishable from a corresponding emission resulting from radioactive contamination. Ionization of the first and second type would increase proportionally with the pressure until the pressure attained was so high that the primary radiation itself became appreciably absorbed in passing through the vessel. Effects resulting from the third type of ionization require more detailed consideration.

At each point in the gas there will be a definite value of what we shall call the ionizing intensity I, i.e., the number of ions which would be produced per c.c. per second in an element of gas at *atmospheric pressure* placed at the point in question. For the purpose of this definition, we may suppose the element of gas to be contained in a non-absorbing capsule, since it is of course not implied that the whole of the gas is at atmospheric pressure. The number of ions produced per c.c. per second in the gas at some point O where the pressure is p will be Ip.

Let us consider an element of volume $ds\ dr$, situated at a point P at a distance r from O in some definite direction, dr being element of radius vector, and ds being element of cross section of an elementary cone of solid angle dω drawn from O to P. The contribution of the element of volume to the value of I at O is

$$\Delta I = \frac{pN}{r^2} f(pr)\, \mathrm{d}s\, \mathrm{d}r$$

where, assuming the secondary radiation originating within an element of volume to be proportional to the pressure therein, $pN\, \mathrm{d}s\, \mathrm{d}r$ is the secondary ionization intensity at unit distance from P in the direction PO, on the basis of no absorption, and $f\ (pr)$ is a factor inserted to take account of the absorption in passing from P to O, and also of the variation of the ionizing efficiency along the path of the ionizing agent. By writing $f\ (pr)$ instead of $f\ (r)$, we imply that the diminution of the ionizing intensity per unit distance (apart from spreading) is proportional to the number of molecules per c.c. In terms of the solid angle dω above referred to we have:

$$\Delta I = N f(pr)\, p\, \mathrm{d}r\, \mathrm{d}\omega$$

If we should now increase the pressure from p to p_1, and diminish r to r_1 so that $pr = p_1 r_1$, and dr to dr_1 so that $p\mathrm{d}r = p_1\mathrm{d}r_1$, the new element of volume contained between the radii r_1 and $(r_1 + \mathrm{d}r_1)$ in the cone dω will make the same contribution to the ionizing intensity at O as did the old element of volume comprised between r and $(r + \mathrm{d}r)$. The sum total of all the elements of volume in the vessel, corresponding to the lower pressure will, however, correspond to a sum total of elements of volume which, at the higher pressure, do not fill the vessel. Hence, the actual ionizing intensity at O will be greater at the higher pressure than at the lower pressure. Since q, the number of ions produced per c.c. per second at O, is obtained by multiplying I by the pressure p in atmospheres, we see that the increase in q, per atmosphere increase, should itself increase with the pressure. We may extend this statement so as to include in q the ionization due to the direct action of the external radiation, since this increases proportionally with the pressure. Thus, the actual ionization in the vessel, due to primary and secondary emission from the gas, will be less at one atmosphere than at any higher pressure. If then, the ionization-pressure curve should show a very small increase of ionization per atmosphere increase at high pressures, we know from the above that such increase per atmosphere is nevertheless greater[1] than the portion of the ionization due to primary and secondary action in the gas within the vessel at one atmosphere. We may infer that any greater ionization found at atmospheric pressure is to be attributed to radiation from the walls of the vessel; this radiation, owing to its absorption at the higher pressures, results in a diminishing rate of increase of ionization with pres-

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[1] It would not be quite safe to extend this argument to imply that the ionization here referred to was necessarily greater than the true natural ionization in the open air, since a portion of the ionization in a volume of the external air occupying the space of the vessel would result from secondary radiations originating outside of that volume.

sure. The foregoing discussion has been made for the case where the primary radiation is so penetrating as to be but little absorbed in passing through the gas in the vessel, even at the higher pressures, this being the case which is of interest in discussing the action of a radiation whose degree of penetration is comparable with that attributed to the cosmical penetrating-radiation.

Experiments on the variation of the residual ionization with pressure have been made by several investigators, the most recent being those made under the writer's direction, by Dr. K. M. Downey and by Mr. H. Fruth. The main feature of the experiments of the latter investigators lay in the use of a comparatively large vessel (a sphere one foot in diameter), and the employment of certain special devices to insure freedom from errors due to leakage and lack of constancy of the batteries. Dr. Downey's observations extended in the first instance up to 20 atmospheres, giving over this range a practically perfect linear variation with the pressure and an increase of ionization of about 1.2 ions per c.c. per atmosphere increase. On extending the observations to higher pressures, it was found that the linear relation ceased to hold in the neighborhood of about 27 atmospheres, and the curves finally became parallel to the pressure axis at pressures above 46 atmospheres. If one were to accept this parallelism without reservation, he would be forced to conclude that the portion of the ionization within the vessel which was attributable to the direct or indirect action of the radiation on the gas was immeasurably small.

Mr. Fruth's observations have been made for air, oxygen, and nitrogen up to pressures of 75 atmospheres, and for carbon-dioxide up to its liquefying pressure, with sensibly the same results for all the gases used. While his curves do not attain as complete a parallelism with the pressure axis as do those of Dr. Downey, they correspond to an increase per atmosphere of less than 0.75 ion per c.c. per second at the higher pressures.[1] It is worthy of note that the presence of radium-emanation in the gas would tend to increase the slope of the ionization-pressure curves. The normal emanation-content of the atmosphere is such as to produce about 2.3 ions per c.c. per second. Each additional atmosphere of air would, of course, carry with it its emanation-content, so that the increase per atmosphere for normal air resulting from the emanation-content alone would amount to 2.3 ions per c.c. per second per atmosphere increase. It is therefore necessary to carefully age the air before use. Recalling that the emanation activity dies to half value in 3.85 days, it will be readily seen that in the case of air 3 weeks old the effect of the emanation would become reduced to a negligible amount. In some of Dr. Downey's observations the air was aged for a month before use.

In experiments of this kind it is of the utmost importance to insure complete saturation, and this matter was consequently tested very care-

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[1] Since this was written Mr. Fruth has found complete parallelism for air, oxygen, and carbon dioxide for pressures above 52 atmospheres, when the gases are perfectly dry and dust-free.

fully, the voltages used being considerably higher than those at which experiment showed saturation to have been attained at the higher pressures. One element of uncertainty not usually considered in relation to the attainment of saturation must be here referred to. It pertains to the effect of dust nuclei. Such nuclei could theoretically cause a departure from a saturation which could not be reduced beyond a certain minimum however great the field might be, since increase of the field intensity could not reduce beyond a certain limit the probability of an ion's encounter with a dust nucleus during its passage across the vessel. Departure from saturation due to such a cause would not show up by the failure to attain apparent saturation with increasing field, and it would increase, moreover, with increase of the amount of gas (and consequently of dust nuclei) in the vessel. The comparatively good agreement between the results of Dr. Downey and those of Mr. Fruth, and the agreement of the various results of the latter investigator among themselves, suggest, however, that dust did not play an important rôle in the experiments, particularly when one remembers that the various experiments corresponded to different samples of gas, samples which had undergone, moreover, entirely different treatments. However, it is planned to make a very careful investigation of the effect of dust in this connection; for, if the experiments of Dr. Downey and of Mr. Fruth represent a primary phenomenon, not explicable by subsidiary considerations of this kind, they carry with them the very remarkable conclusion that, of the ionization observed in a vessel at atmospheric pressure and ordinarily attributed to a penetrating radiation, less (and probably considerably less) than one ion per c.c. per second is to be accounted for as having its origin in a direct action of the primary radiation on the gas or in the action of a secondary radiation emitted from the gas by the primary radiation.

EXPERIMENTS ON THE DIRECTION OF THE PENETRATING RADIATION

An important light would be thrown upon the origin of the penetrating radiation if it could be shown to partake of a directive character. Experiments on this matter were originally made by Cook and by Wood, who interposed screens between the apparatus and its surroundings at various orientations. The experiments were inconclusive, but, as far as they went, seemed to indicate that the radiation came equally from all directions. A method of this kind is very seriously affected by lack of constancy of the residual ionization itself during the various experiments between which comparisons are subsequently made, and, for this reason, some experiments were undertaken by Miss J. Herrick under the writer's direction with the object of eliminating the main causes of uncertainty in the earlier experiments of Cook and Wood. The method used by Miss Herrick depends upon the fact that if gamma-rays pass through a thin sheet of metal, the ionizing electrons emitted from the side at which the rays enter differ as regards their number and speed from those which are ejected from the side at which the gamma-radiation leaves. If the penetrating radiation is of a gamma-ray type, it should show similar characteristics. The ratio of the subsequent ionization resulting from the incidence of gamma-rays on a surface to that resulting from the emergence of gamma-rays from the surface depends upon the material of the surface. The apparatus used consisted of two similar cylinders mounted with their axes in the same horizontal line. One semi-circular half of each cylinder was made of lead, and the other half was made of aluminum. The cylinders were provided with central rods which were connected to each other and to the insulated quadrant of an electrometer. By placing potentials of plus fifty and minus fifty volts respectively on the cylinders, the ionization currents in the gas could be caused to feed into the electrometer in such a way as to almost completely compensate. Several precautions to avoid leakage, and errors due to fluctuation in the potentials of the batteries were taken, the details of which it will be unnecessary to describe.

To fix our ideas, suppose that an excess of gamma-radiation comes from above, and that in the case of one of the cylinders the lead half is uppermost. Then, as far as ionization due to the emission of electrons from the wall of the vessel is concerned, the ionization in the vessel in question (or rather the portion of it due to the excess of radiation coming from above) will be due to the emergence radiation from the lead, and the incidence radiation from the aluminum. An alteration of the effect should consequently be produced by rotating the cylinder through 180 degrees, while the other cylinder is left untouched, its function being simply to act as a compensator for the purpose of minimizing effects resulting from an actual variation in the conditions during the experiment. Without here entering into details, it will be seen that it would be possible to obtain a comparison between the radiation coming in different directions and the total radiation entering the vessel in so far as the ionization was due to the electrons emitted from the walls of the vessel and in so far as one assumed the penetrating radiation to partake of the nature of a hard gamma-radiation as regards the difference between the incidence and emergence effects in the case of the metals used.

The first experiments performed by Miss Herrick in the laboratory of the physics building showed marked changes on rotating one of the cylinders, and by plotting a polar diagram representing the ionization due to radiation coming from the various directions it became possible to locate small quantities of radium in different parts of the building. The polar diagram, moreover, showed a hump indicating an excess of radiation coming from above. The apparatus was next moved to the astronomical observatory, where there was no radioactive material, and experiments again gave an indication of an excess radiation from above. In order to be free from all possibility of radioactive contamination, the writer next set the apparatus up in the attic of his house, and carried on observations over a period of 6 weeks, the observations being taken between the same hours each day. The attic of a dwelling house is not the most ideal situation for a quadrant electrometer, but on plotting the results for the various experiments there was again decided evidence of an excess radiation in the downward direction. The magnitude of the excess was such that, when the vessel was turned in the most favorable direction, the ionization was about 9 percent greater than the average, a result in comparatively good agreement with the observations of Miss Herrick made on the university campus.

These experiments are only cited as of a preliminary nature, for there are certain sources of complication which must be removed before a cosmical interpretation may be made of the results. Thus, the potential-gradient in the atmosphere will deposit active material from the atmosphere on the roof of the building in which experiments are made. A simple calculation will show that the amount of such deposition may well be enough to seriously affect experiments of the kind described above. Similar remarks apply to the effect of radioactive material in the soil itself, and that deposited on the surface of the soil by the atmospheric potential-gradient. It is planned to continue the observations under conditions which, it is hoped, will eliminate these causes of uncertainty.

THE EARTH'S PENETRATING RADIATION AND THE ORIGIN OF THE EARTH'S CHARGE

In 1915, the writer proposed a theory of the origin of the earth's charge based on the assumption that high speed electrons were shot into the earth from the atmosphere as a result of a very slight radioactivity of the atmosphere itself, or as a result of the breaking up of the emanation normally in the atmosphere. If one assumes that an electron of sufficiently high speed can have a range as great as 5 kilometers in the atmosphere, it is only necessary to postulate the emission of one such high speed corpuscle per c.c. in the downward direction, each 100 seconds in order to account for the maintenance of the earth's charge. Or, viewed from another standpoint, since we know that about 5 pairs of ions are formed per c.c. per second in the atmosphere, it is only necessary to suppose that in the case of one out of every 500 pairs of ions formed a high speed corpuscle of the above kind is emitted. The theory of passage of electrons through matter is not at all inconsistent with the postulation of great ranges such as those required by the theory; however, in 1917 Swann[1] put forward another theory in which the expulsion of the electrons from the atoms of air is brought about by the penetrating radiation from above, the hard nature of this radiation resulting in its emitting electrons from the air molecules almost exclusively in the downward direction. Under the influence of the electronic bombardment, the earth would charge up until the conduction current back to the atmosphere just sufficed to balance the effect. It appeared that if one assumed as many as 3 corpuscles to be emitted per c.c. per second from the atmosphere, it would only be necessary to postulate a range of penetration of about 9 meters in order to

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[1] *Phys. Rev.*, 9, 555-557, 1917.

account for the maintenance of the earth's charge. As pointed out by the writer at the time, the chief difficulty facing a theory of this kind is the explanation of why the swiftly moving corpuscles do not produce, in the atmosphere, a much greater ionization than is observed. Difficulties of this kind assume a much less formidable aspect, however, when viewed in the light of modern views as to the properties of swiftly moving electrons. In 1918, v. Schweidler independently put forward the second of the above theories, and described an experiment carried out with the object of testing it. The aim of this experiment was the endeavor to observe a charging effect in a thick piece of metal as a result of corpuscles entering it, the piece of metal being surrounded by a shield from which it was insulated. Failure to observe any charging effect caused v. Schweidler to conclude that the replenishment of the earth's charge could not be brought about by a corpuscular radiation of the type discussed. As a matter of fact, the writer had performed an experiment somewhat similar to v. Schweidler's experiment in 1915, in connection with his earlier theory of corpuscular charging. In this experiment an earthed vessel surrounded an insulated hollow cylinder connected to an electrometer. The rate of rise of potential was noted, and a solid copper bar was then inserted in the hollow cylinder, the rate of rise being then again noted. By this device of performing two experiments in which the surfaces exposed were the same, surface effects were eliminated. As in v. Schweidler's experiment, no certain charging effect was observed; and, while this weighed against the former of the theories above referred to, it was not felt that it formed so weighty an argument against the latter, for on that theory it might result that the penetrating radiation would shoot as many electrons out of the bottom of the cylinder as it shot in at the top, except for the absorption of the penetrating radiation itself within the cylinder. In other words, it is the coefficient of absorption of the penetrating radiation in the cylinder rather than the coefficient of absorption of the corpuscles which is of importance. In a recent paper,[1] R. Seeliger discusses the origin of the earth's charge. He considers v. Schweidler's experiment as conclusive evidence against any theory which postulates a corpuscular replenishment at all parts of the earth, but raises the question as to whether a corpuscular replenishment may not take place in certain limited areas, in polar regions for example. It is to be observed that any assumption of this kind invites, in their most serious form, difficulties associated with the ionization which might be expected to result from a passage of the corpuscles through the atmosphere. For a concentration of the corpuscular current in a limited region would result in greatly increased ionization in that region, and on the assumption that a corpuscle produces 50 ions per centimeter of its path and that the coefficient of recombination of ions is 1.6×10^{-6}, it can readily be shown that, unless the area of precipitation were more than one thousandth of the area of the earth, the conductivity produced in the air in the region of precipitation would be so great that, for a potential gradient of 150 volts per meter, one would calculate

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[1] *Annalen der Physik*, 63, 464-481.

for this region alone a total conduction current greater than the corpuscular current. In other words, there is a lower limit to the value which one may assume for the region of precipitation. Thus the avoidance of difficulties concerned with the failure to directly measure a corpuscular current, by relegating that current to regions where experiments have not been made, does not avoid what is perhaps one of the most serious difficulties confronting any corpuscular theory, that of reconciling the comparatively small ionization of the atmosphere with the passage through it of about 1500 high speed corpuscles per square centimeter per second.

THE CONDUCTIVITY OF THE UPPER ATMOSPHERE

The paper concluded with a reference to the importance of a knowledge of the conductivity of the upper atmosphere in relation to the origin of the earth's charge and allied phenomena, and the author described an experiment in progress at the University of Minnesota designed with the object of measuring the distance of the supposed conducting layer by measuring the time taken by wireless waves to reach that layer and return.

RECENT RESULTS DERIVED FROM THE DIURNAL-VARIATION OBSERVATIONS OF THE ATMOSPHERIC-ELECTRIC POTENTIAL-GRADIENT ON BOARD THE *CARNEGIE*[1]

By S. J. Mauchly

The Department of Terrestrial Magnetism, in accordance with its director's plans, has for many years been making not only magnetic but also atmospheric-electric observations aboard its survey vessel, the *Carnegie*. It is thus contributing the chief data for mapping both the earth's magnetic field and its electric field. Furthermore, since 1915 numerous observations have been made aboard the *Carnegie* to determine the nature and magnitude of the changes in the electric condition of the atmosphere which take place during a 24-hour cycle.

For the *potential-gradient* the general procedure in the diurnal-variation observations is to make a set of 20 observations during each of 24 consecutive hours. The observations for such a set require about 20 minutes and their mean value is referred to the mean time for the set. From deductions based on the observations made prior to April, 1916, it appeared that the diurnal variation of the potential-gradient over the oceans probably did not differ much from that which has been found at many land stations; that is, they indicated two rather pronounced maxima and two minima during a 24-hour period.[2] However, very few data were

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[1] Preliminary report presented before the American Geophysical Union, with amplifications.

[2] "Researches of the Department of Terrestrial Magnetism," Vol. III, pp. 416-420, Washington (1917).

available from oceans other than the Pacific, and as pointed out in the report just cited, a large percentage was derived from series of observations which were terminated by the advent of unfavorable weather. It should also be noted in passing that Swann[1] a year later in discussing the results of the observations for the year ending February 20, 1917, states that "the effect of the 12-hour Fourier wave is less important in the present curves than in those already published."

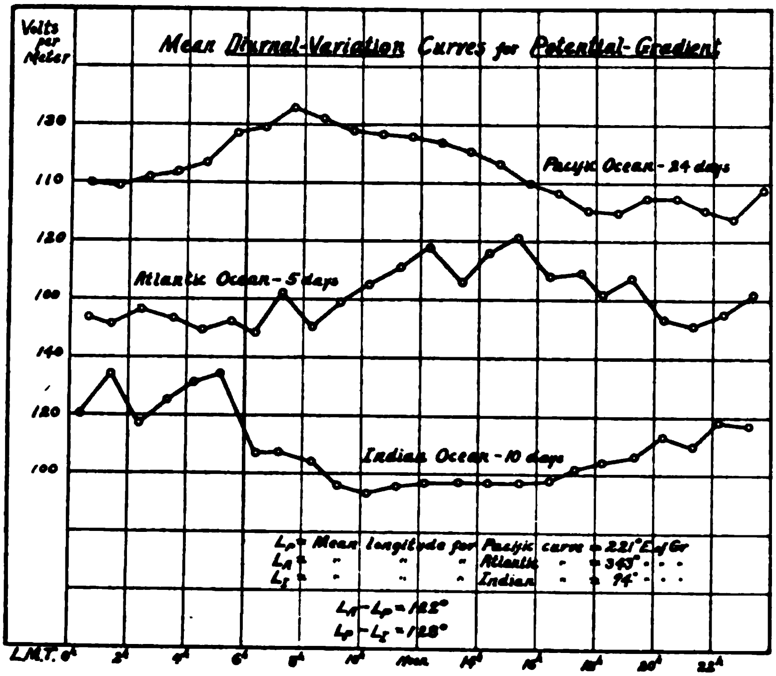

FIG. 3. Diurnal variation of electric potential-gradient on the oceans, plotted according to Local Mean Time.

The largely increased amount of material which has accumulated since 1915 makes it now possible to reject nearly all data corresponding to less than a 24-hour series and still have 45 practically complete 24-hour series available. The data for each series, therefore, correspond to an actually occurring sequence of phenomena, and the mean results are free from the errors which would result from combining the results of partial series of observations.

Of the 45 diurnal-variation series referred to, 30 were made in the Pacific, 5 in the Atlantic, and 10 in the Indian Ocean; the combined data represent about half the earth's surface. The means corresponding to

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[1] W. F. G. Swann. "Supplementary report on atmospheric-electric observations made aboard the *Carnegie* from May 17, 1916, to March 2, 1917," in "Annual Report of the Director of the Department of Terrestrial Magnetism" for the year 1917. Year Book of the Carnegie Institution of Washington, 1917, p. 282.

the separate oceans, as derived from 39 series, are represented in figure 3. They show: (1) That the mean diurnal-variation curves for the Pacific, Atlantic, and Indian oceans are similar in form; (2) that the principal component of the variation consists of a 24-hour wave, and (3) that the times of occurrence of the chief phases of this wave, when referred to local time, differ for the several oceans by amounts which correspond approximately to the differences between the respective mean longitudes, for the several oceans, of all the points at which observations were made.

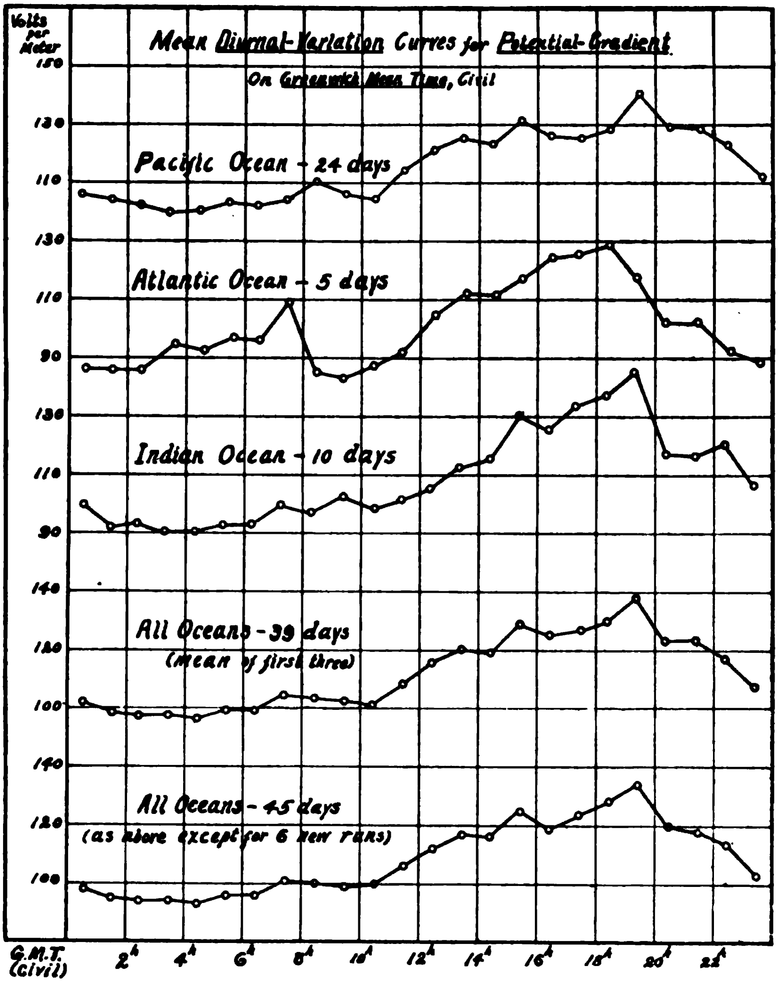

FIG. 4. Diurnal variation of electric potential-gradient on the oceans, plotted according to Greenwich Mean Time.

Since the curves of figure 3 suggest the simultaneous occurrence of maximum (or of minimum) phase over all three oceans, it was decided to refer the results of each series of observations to Greenwich Mean Time (civil), and recompute the means for the separate oceans on this basis. The results are shown in figure 4, together with a curve which includes the data from 6 recent series received from the vessel after the curves in figure 3 had been prepared. The differences between the several curves of figure 4 are of course not to be thought of as representative of separate characteristics, since the smoothness of the respective curves is seen to be closely related to the number of component series.

The curves of figure 4 show a decided similarity to land results for high latitudes and also to many of the winter curves obtained in ordinary latitudes. Indeed, if differences in local mean time are taken into account, it appears that for many land stations at which the single diurnal wave predominates, there is approximate simultaneity as to the time of occurrence of maximum (likewise, of minimum), and this at a time which is in general agreement with what is indicated by the curves of figure 4. For the summer, however, as is well known, most land stations show, in addition to the 24-hour wave, a decided secondary wave which seems to occur in general at about the same local mean time at different stations.

The minimum value of the potential-gradient, according to figure 4, occurs at about 4^h A.M., G.M.T., and in view of the fact that for observatories in western and central Europe the difference between local and Greenwich time is not great, this may account for the fact that various authorities have assumed the occurrence of the principal minimum at about 4^h A.M., local time, to be a rather general characteristic for most stations. It is also significant to note that Mache and v. Schweidler[1] long ago pointed out that the phase angle of the 24-hour wave varied greatly from station to station while the phase angle of the 12-hour wave was approximately the same for nearly all stations. Although the phase angles of the 24-hour Fourier waves for the European stations show among themselves very much greater differences than can be accounted for by the rather small differences in longitude, it must be borne in mind that the results of harmonic analyses are dependent upon local meteorological and cultural, and sometimes topographical and instrumental, factors as well as upon any general characteristics which the potential-gradient may possess.

In the present investigation no account has been taken of possible changes in the characteristics of the diurnal variation with latitude and with time of year, except to ascertain that the preponderance of the 24-hour wave and the approximate progress on a universal-time basis seem to hold throughout the year and for wide ranges of latitude. The present results are, therefore, to be considered as provisional and representing only a general yearly average. In fact, investigations under way show that considerable modification in detail is to be expected as more observational material becomes available. The data from 45 practically

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[1] H. Mache und E. v. Schweidler, "Die Atmosphärische Elektrizität," p. 27, Braunschweig, 1909.

complete series of diurnal-variation observations aboard the *Carnegie*, representing a general distribution over most of the accessible ocean-areas indicate, therefore, as a preliminary result, that *the chief component of the diurnal variation of the potential-gradient over the major portion of the earth (especially the oceans) is a wave of 24-hour period which occurs approximately simultaneously in all localities.*

A fact of considerable interest is that the diurnal-variation curves for the potential-gradient derived from the *Carnegie* observations are very similar to curves which represent the relative frequencies of the Aurora Borealis, as observed at several European stations, and also to curves representing the diurnal distribution of certain classes of magnetic disturbances, when all are referred to the same time-basis. It may also be pointed out that owing to the non-coincidence of the earth's magnetic axis with its axis of rotation, the time of daily potential-gradient maximum, as indicated by the ocean curves, corresponds approximately to the time when the earth's north magnetic pole, for example, is nearest to the sun, while the daily minimum occurs, in a general way, when this pole is farthest from the sun. The actual times of maximum and minimum, however, appear to depend upon the positions of *both* magnetic poles and the fact that their longitude difference is not 180°. These correlations appear to support the assumptions of various investigators that the earth's electric charge and resultant field may be very intimately related to an electric radiation from the sun. The best evidence as to the extent of this support will probably result from a study of the details of the diurnal-variation curves corresponding to various positions of the earth in its orbit. Reductions with this end in view are under way and it is hoped that sufficient data will soon be available to yield some information on this point.

The making of diurnal-variation observations in atmospheric electricity by eye readings is always a burdensome procedure; the carrying on of such work aboard a vessel is not only arduous but also difficult. In this connection the utmost credit is due the several commanders of the *Carnegie*, during her various cruises, and to all the observers who participated in the observational work.

I am indebted to the director, Dr. L. A. Bauer, for his constant interest in and encouragement of the work in hand, and for a suggestion of the possibility of finding in the asymmetry of the earth's magnetic field an explanation of the observed diurnal variation on a universal-time basis. I am also greatly under obligations to the members of the Department of Terrestrial Magnetism who assisted in the reduction of the observational data, especially to Dr. G. R. Wait, both for valuable assistance and helpful suggestions.

The full publication of the observational data and discussion of results will be deferred until after the completion of the present cruise of the *Carnegie*.

Department of Terrestrial Magnetism,
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SUGGESTIONS RELATIVE TO THE APPLICATION OF MATHEMATICAL METHODS TO CERTAIN BASIC PROBLEMS OF DYNAMIC OCEANOGRAPHY

BY G. F. McEWEN

Investigations of the ocean have generally been carried on by geographers and geologists, oftentimes incidentally to those of other divisions of these extensive fields of science. Accordingly, qualitative methods so characteristic of geography and geology have been widely used in oceanographic investigations. Such qualitative methods and the empirical treatment of quantitative field observations have been very suggestive, have stimulated interest, and led to certain broad generalizations that are essentially correct. However, there has been a tendency toward rather loose reasoning and lack of consideration of established quantitative principles of physics, which has resulted in certain erroneous conclusions.

Must we admit that the complexity of such geophysical phenomena renders careful reasoning and quantitative treatment impossible of attainment? Probably many would at first answer yes, but let us consider the matter further before expressing an opinion. Within the past fifty years a few scientists have undertaken, by means of a definite formulation of specific problems, to apply mathematics to ocean data, and thus to contribute to a system of demonstrable principles applicable, in general, to all similar cases; and attention has been increasingly directed to this type of research. Important advances have thus been made, and serious errors in certain former conclusions have been discovered, although, especially in some of the earlier attempts at mathematical applications, significant errors arose from incorrect assumptions and failure to appreciate important attributes of such "field," or natural problems. At first, men accustomed to the problems of laboratory physics attempted to deduce physical laws of the sea from results of laboratory studies, and certain preconceived assumptions regarding oceanic conditions. They also worked under the disadvantage of having very inadequate data. As more accurate and exhaustive field data accumulated, attention was directed more to interpreting and coordinating field observations rather than to depending on the speculative and unsound method of imposing on the sea purely theoretical laws deduced from laboratory researches.

Mohn's pioneer investigation of 1887,[1] based on the deduction of the changes in form of the surface that would give rise to currents actually produced by winds, variation in barometric pressure, and specific gravity, was a great advance beyond earlier attempts at a precise treatment of ocean data, and doubtless contributed greatly to the development of the more satisfactory methods of today. Among the later results thus worked

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[1] Mohn, H. 1887. The Norwegian North Atlantic expedition, 1876–1878: The North Sea, its depths, temperature and circulation. (Christiania, Grondahl), 212 pp., 48 pls.

out are Ekman's[1] hydrodynamical theory of oceanic circulation, which pertains especially to wind-driven currents, and was undertaken at Nansen's[2] suggestion; and Bjerknes's[3] convection theory which pertains especially to the determination of ocean currents due to differences in specify gravity. Later his pupil, Sandström,[4] devised a much more rapid and accurate method of computing such currents.

The Norwegian investigator, Jacobsen,[5] in certain more recent quantitative investigations pertaining to the Atlantic near Denmark, obtained encouraging results by giving special attention to the alternating convective motion of small masses of the water, or to the "mixing phenomena," as he called it. His researches afford strong evidence in support of the idea suggested by earlier qualitative studies, that in lakes and oceans, very small or elementary masses of the water are moving at random in a manner somewhat analogous to the motion of molecules in a gas, except that the direction of motion in large bodies of water is mainly vertical, although the resultant vertical flow may ze zero. This phenomenon of the interchange of small masses of water has been variously referred to as an alternating convective circulation, mixing phenomenon, eddy or vortex motion, and turbulence. Jacobsen's and other recent investigations in this field indicate that this phenomenon is the cause of the processes of diffusion, heat conductivity, and frictional resistance peculiar to oceanic conditions. Comparable values of the "Mischungsintensitat," a coefficient indicating the intensity of the rate of interchange of small water masses, have been deduced independently from the distribution of temperature and salinity, and also from the dynamical treatment of current observations. Thus studies of temperature and salinity distributions may yield appropriate values of the frictional resistance, an essential factor in the dynamical solution of ocean-current problems as well as certain tidal problems. This frictional resistance about which there is so little definite information appears to vary widely with the locality, wind velocity and other factors. It is not a "physical constant" of the *substance,* water.

The precise nature of this mixing motion can not be directly determined, but various reasonable assumptions regarding it can be made, and combined with known fundamental facts into a quantitative theory or generalization, from which deductions can be made, and tested by comparison with observations. Encouraging results already reached appear to justify

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[1] Ekman, V. W. 1905–06. On the influence of the earth's rotation on ocean currents. Arkiv for Matematik, Astronomi och Fysik, 2, 1-53, 1 pl. and 10 figs.

[2] Nansen, Fridtjof. 1902. The Norwegian North Polar Expedition, 1893–1896. Scientific Results, Vol. III, Longmans Green & Co. London, part IX, pages 1–427. 33 pls.

[3] Bjerknes, V. F. K. 1901. Circulation relative zu der Erde. Öfversikt af Kongl. Vet.-Akad. Förhandl., 58, 739–757.

[4] Krümmel, O. 1911. Handbuch der Ozeanogriphic (Stuttgart, Engelhorn), 2, xvi, 766 pp., 182 figs. in text.

[5] Jacobsen, J. P. 1913. Beitrag zur Hydrographie der Dänischen Gewässer. Medd. Komm. Havandersogelser (Hydrografi), 1, no. 94, pp., 14 pls., 17 figs. in text.

further efforts in this direction and point to the possibility of a satisfactory coordination of the various phenomena of conduction, diffusion, and fluid friction by means of a single mathematical theory of the mixing motion. Such an investigation, if successful, would enable one to deduce the temperature distribution in a body of water gaining heat from solar radiation of given intensity, and losing heat by evaporation. Investigations of this simplest case would thus correlate under definite physical principles all of these various thermal phenomena. Again, by so amending such results as to include the effect of a given flow or current on the distribution of temperature determined for the above simplest case, estimates of a current could be made from the difference between the undisturbed and the actual temperature distribution. This has been partly worked out and applied to the determination of the velocity of upwelling in the San Diego region. Thus the temperature disturbance can be quantitatively treated as an effect of a current, without regard to its dynamical causes. Qualitative conclusions relative to ocean currents have long ago been reached from essentially the same general idea, and this fact points to the possibility of such a quantitative theory. Such general quantitative laws of the relation between currents and temperature departures from the undisturbed state might be combined with Bjerknes's dynamical theory, and thus afford a means of deducing answers to more involved questions, such as the following: Given the distribution of solar radiation over the surface of a body of water having given boundaries, and a known initial temperature distribution, to determine the resulting temperature distribution and circulation for any later time. The results of similar determining conditions have not infrequently been either assumed or surmised in order to form a basis for more far-reaching oceanographic conclusions. But it is by the *precise* formulation and sufficiently *accurate* solution of suitable specific problems sufficiently in accord with actual conditions that general laws of oceanic phenomena can be discovered and tested. And the greater the variety of such ideal problems that we are in position to attack, the greater will be our progress in the precise and detailed study of the physics of the ocean. Also, it is to be expected that a satisfactory physical theory, especially of the mixing phenomenon, would greatly aid in the solution of certain problems of sedimentation and ocean chemistry.

One problem of ocean physics, whose simplest special case would be to deduce the vertical temperature distribution in a body of water exposed to solar radiation of approximately uniform intensity over the surface and losing heat by evaporation and conduction from its surface, has received very little attention except of a qualitative or speculative nature. Yet this problem appears to be fundamental in precise oceanographic investigations. Accordingly, the author has for some time attempted to work out a solution, and after trying and rejecting various assumptions, has reached encouraging preliminary results by using certain concepts from statistical mechanics, combined with elementary laws of heat and radiation. It is hoped that these studies will have progressed far enough for publication within a year or two. It is also the intention, after this work on temperatures is in a more finished state, to investigate the problem of diffusion in the sea by similar methods applied first to certain of the numerous salinity determinations made by the Scripps Institution. Problems of the type mentioned in this paper form an extensive and promising field of fundamental importance in oceanography, and demand the attention of all investigators interested in promoting quantitative studies of the sea, but probably only a few will desire to engage actively in their solution.

It has formerly been necessary to make a great deal of use of such scattered data as the investigator could find as a basis for theoretical work. Much has been, and doubtless will be, accomplished in that way. And all original detailed data, as well as summaries and deductions therefrom, should be accessible in some way to investigators, even if publication is not always practicable. But such a procedure has obvious disadvantages, such as insufficient or unknown precision, incomplete data, or lack of significant factors that may impair or greatly restrict the conclusions. Therefore it is also necessary to conduct special programs of observations, designed with reference to particular problems, in order to improve and supplement the above more extensive and preliminary type of work. Thus selection of the locality, season, etc., and the observation of all relevant phenomena affords as nearly as possible a realization of the advantages of the physicist who controls the conditions affecting his laboratory experiments. For example, serial temperatures observed in the central part of a high-pressure area away from currents or land, and where prevailing great depths provide results corresponding to the simplest conditions, would be of great aid in the study of "normal" temperature gradients. Such observations should also be accompanied by observations on the intensity of solar radiation, turbidity, salinity, and evaporation, and should be continued through different seasons and times of day in order to provide the most important kinds of data. Very little of this intensive type of work, carried on with sufficient continuity and completeness, has been done, and it has been restricted to certain portions of small inland seas or inshore regions. Although results thus obtained are valuable in themselves and as a means of interpreting such fragmentary and widespread data as may be available, they can not take the place of similar intensive work at selected stations throughout the ocean. Actual conditions in typical areas of the great ocean must be carefully observed and studied, if any reasonable approach to exhaustive oceanographic investigations is to be realized.

In this paper I have dwelt especially on the deductive treatment of ocean problems, because of the great need of improvement in this aspect of the subject. Although admitting that purely empirical or statistical methods are indispensable in assembling and coordinating various kinds of field data, it seemed desirable to urge the need of progress from such empirical results toward the goal of a complete deductive treatment carried out in accordance with known generalizations of physics.

STATE OF PROGRESS IN CONTINUOUS RECORDING OCEANOGRAPHICAL INSTRUMENTS

By Albert L. Thuras

The modern tendency in physical research is to replace indicating instruments by recording instruments wherever possible. This has been especially true in the science of meteorology where the recent advances have been brought about almost entirely by the remarkable improvements and developments in recording instruments. In the related science of physical oceanography there are practically no recording instruments now in general use, except possibly the tide-gage. If meteorology has been so greatly benefited by such instruments, surely in oceanography, where the changes in the physical properties are so much more regular and therefore more easily interpreted, great advances should be looked for through the addition or substitution of recording instruments.

Heretofore the methods of collecting physical data have been such that no complete knowledge of the physical characteristics of the particular body of water under investigation have been obtainable as the work is progressing. The procedure has been to lay out stations, as intelligently as possible along courses throughout the region of the ocean to be studied which will give the most important information. At these various stations with the use of water bottles and reversing thermometers samples and temperatures of the ocean water are obtained at various depths down as far as the investigations are carried. The thermometers are read as soon as the water bottles are drawn up and samples of the water are stored in bottles which are later chemically measured for salt content in a laboratory on shore. The several disadvantages of this method are apparent: (1) No working knowledge of the ocean water is immediately obtainable and consequently no rearrangement or addition of stations can be made from an examination of the data taken. This is very important especially where our knowledge of the ocean is limited and one wishes to explore the magnitude and extent of the surface and submarine currents. (2) It is impossible to obtain a corroboration of any data where there may be doubt as to the accuracy or reliability of single observations. (3) The data taken are usually inadequate and especially so at those stations where vertical lines of observations pass through various strata of water of different salinity, temperature and density. Curves and cross sections plotted from data taken in these regions are usually a matter of approximation and give very little information as to the mechanism of the mixing of waters of widely different properties. These observations are particularly inadequate in such regions as the southern end of the Grand Banks of Newfoundland where the cold waters of the Labrador Current merge into the warm saline waters of the Gulf Stream.

With the object of improving the technique of the science of physical oceanography, an effort has been made in recent years to develop practical recording instruments which are sufficiently rugged and simple to be used on shipboard. The most important physical properties of the sea of which a continuous record should be made, are temperature, salinity, density, velocity and direction. The first three properties are so related that any one can easily be deduced from a measurement of the other two. The properties most easily measured are temperature and salinity. Salinity is defined as the number of grams of salts in a liter of sea water. From a consideration of the properties of sea water that vary with the salinity, the electrical conductivity seems to be the most susceptible to continuous measurement if the difficulty due to the variation of conductivity with temperature can be overcome. Such a method consists in measuring the ratio of the resistance of sea water in two equal or nearly equal electrolytic cells, one cell containing sea water of a known salinity and the other having the sea water to be measured flowing through it. The ratio is obtained by a Wheatstone bridge, using alternating current to eliminate polarization effects in the cells. A record of the resistance ratios of the two cells is made by an automatic electrical recorder. By immersing the two cells in the same temperature bath almost complete compensation of temperature changes is effected.

A continuous record of the temperature of the ocean is most easily obtained with a platinum resistance thermometer and an automatic regulating Wheatstone bridge quite similar to the continuous salinity recorder. This instrument has been used successfully on shipboard for several years in the region of the Grand Banks of Newfoundland and some very interesting records have been obtained which show the distribution of temperature and thereby indicate the location of ocean currents and also give a knowledge of their boundary conditions which could hardly be obtained by repeated single measurements of temperature.

The continuous recording instruments for temperature and salinity described above give only surface measurements but they could easily be constructed for making measurements below the surface. This would require the use of an insulated cable of 4 or 5 conductors which would be sufficiently strong and flexible. During 1918–19 in connection with submarine listening experiments there were constructed reinforced cables similar to these, which could be repeatedly wound on to and off of a drum and would withstand a weight of 400 to 500 pounds. Data from these instruments gave accurate curves of temperature, salinity and density from the surface down to a depth of probably 500 meters, which is the most interesting part of the ocean dynamically.

R. A. Daly of Harvard University has developed and built a thermograph which can be anchored in very deep water and will give a record of temperature for a period of several days. The instrument has an intermittent mechanism which gives periodic photographs of a mercury column. This instrument was specially designed to withstand very high pressures and it should be especially valuable in studying the small variations of temperature at great depths in the ocean.

The measurement of the movement of waters in the ocean has been quite difficult to perform experimentally. This difficulty has been due chiefly to the non-continuity of measurements and the unknown movements of the vessel from which the measurements are made. Dr. Hans Pettersson of Goteborg, Sweden, has solved this problem by his photographic current meter which will give a continuous record of both direction and velocity for a period of two weeks. This instrument with the use of special anchors and buoys, can be firmly secured at any depth down to 300 meters. The difficult problem which had to be solved in this instrument was the transfer of the motions of a propeller through a water tight case containing the recording apparatus without the addition of friction. This was accomplished by a magnetic coupling.

Dr. Pettersson has also developed densimeters to be used from shore stations which give a record of the movements of the waters of various salinities into and out of the Swedish Fjords. These instruments consist of vessels or cans whose density is equal to the average density of the Fjord waters. As the submarine waves of high salinity come in from the ocean these vessels will rise and a record of their height is automatically recorded. Some interesting theories of submarine waves have resulted from this work and the correlation between the variations of salinity and the abundance of fish in these Fjords is being studied.

This briefly describes the recent developments in recording instruments and in conclusion I wish to suggest the possible application of these instruments to future research in physical oceanography.

A comparison of the yearly observations in the region of the Grand Banks of Newfoundland shows that the volume and strength of the Labrador Current have a decided influence on the course of the Gulf Stream in that vicinity. In some years the Gulf Stream was found almost up to the southern end of the Grand Banks and in other years as far south as the 40th degree of north latitude, a variation of over 100 miles. An accurate knowledge of the volume, velocity and location of these currents from time to time and correlation with meteorological conditions might yield results of great interest.

This information could be obtained by the use of recording instruments in the straits of Florida and across the Gulf Stream before it branches out east of the Grand Banks of Newfoundland.

With continuous salinity and temperature recorders placed on trans-Atlantic vessels a complete record of the variation of temperature salinity and density could be secured across the Atlantic from month to month. These instruments would make measurements at a constant depth below the surface and might throw considerable light on the hydrodynamics of this part of the Atlantic.

It seems to me that the science of physical oceanography has passed the period of exploration and has now reached that stage in its development which calls for a program of research on a large scale with most carefully thought out plans of systematic investigation extended over long periods of time. Results from such an undertaking I believe can be most successfully accomplished by the use of recording instruments.

Western Electric Company,
New York City.

PRESENT STATUS OF RESEARCHES ON MARINE SEDIMENTS IN THE UNITED STATES

By Thomas Wayland Vaughan[1]

INTRODUCTION

The ocean of today stands at the end of a succession of oceans that have existed since land and water were first divided from each other on the earth's surface. This fact, admitted by everyone, needs to be emphasized in order to make clear the transcendent importance of the study of marine sediments. It is possible to measure the depth and the temperature of the waters of the present ocean, to sample its waters from the surface to the bottom of its greatest abysses and examine them chemically, and to measure directly or to infer from measurable factors its currents. It is also possible to study the sediments deposited on the floor of the ocean and around its margins. These and other features of the present ocean can be known by direct processes but over a large part of the earth's surface where there was once sea there is now only land, and the depth, temperature, chemical composition, and currents of bodies of waters no longer existent cannot be measured. That seas once extended over regions now land is known through the record made by the sediments and these sediments supply the fundamental data for recognizing the physical features of the vanished oceans.

Considerable information has already been acquired on modern marine deposits and preliminary maps of parts of the ocean floor have been made. Among the sources of this information are the studies of Bailey and Pourtalès, the classic work of Murray and Renard, Murray, and Murray and Lee, Murray and Philippi, and Philippi, the many papers by Thoulet, several papers by Böggild, including his recently published "Meeresgrundproben der Siboga-Expedition," papers by Walther, and the studies of the shoal-water deposits of Florida and the Bahamas and Murray Island, Australia, with which I have been associated.[2] Of course there are many other authors but I have given the names of those who have done most in areally mapping deposits on the bottom of the sea. The great leaders are Murray and his associates, among whom Philippi is to be reckoned, and Thoulet. It is believed that the characteristics of some deposits and the relations of these deposits to the conditions under which they formed have

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[1] Published by permission of the Director of the U. S. Geological Survey.

[2] K. Andrée is the author of a useful bibliography on literature on marine sediments published between 1841 and 1911. See his article, Uber Sedimentbildung am Meeresboden, Literaturzeichniss: Geolog. Rundschau, 3, 1912, 324-338.

been ascertained with enough accuracy to admit their use in interpreting geological history; but how inadequately some relations are understood is exemplified by the presence of red clay at comparatively shallow depths, 4000 meters, in the enclosed deep basins of the East Indian Archipelago. Böggild says it is necessary to conclude that the capacity of the water to dissolve calcium carbonate is greater in the enclosed basins of the East Indian Archipelago than in the open ocean.[1] Although considerable is known about marine sediments, the information is far below what is needed to understand many important features of sediments in the modern oceans and to supply a basis for interpreting ancient sediments.

RESEARCHES ON MARINE SEDIMENTS IN AMERICA

There is under the Division of Geology and Geography of the National Research Council of the United States a Committee on Sedimentation composed of fourteen members, of which I have the honor to be chairman. This committee is divided into seven subcommittees, as follows: universities and colleges east of the Allegheny Front; universities and colleges between the Allegheny Front and the Rocky Mountains; universities and colleges west of the Rocky Mountains; state geological surveys; chemical and physical researches on sediments; field description of sediments; preparation of a treatise on sedimentation. The report of the committee for the year ending on April 28 has been transmitted to the chairman of the Division of Geology and Geography and is available in mimeographed form to interested persons.

No attempt will be made to give an account of the work of the committee, as that would consume too much time, but it will be said that its scope includes both modern and ancient sediments and both continental and marine deposits. One of the purposes of the committee is to ascertain and to follow all current investigations on sediments within the United States and the machinery for accomplishing this purpose is good.

The U. S. Bureau of Fisheries is trying to arrange for a study of the sediments of the Bay of Maine but the plans have not yet been completed; and an attempt is being made to have the sediments of Chesapeake Bay studied cooperatively by the Bureau of Fisheries, the U. S. Geological Survey, and Johns Hopkins University, but the actual work on the bottom samples has not begun. Prof. G. D. Louderback has been trying to study the bottom deposits collected principally by the Bureau of Fisheries in San Francisco Bay. Some preliminary information on the samples has been published but the investigation has progressed slowly. Two researches with which I have been concerned have had grave difficulties. One of them, the study of the sediments off the mouth of Mississippi River, as representing an area in which great quantities of terrigenous material are being deposited, has come to a standstill with the resignation of Mr. E. W. Shaw from the U. S. Geological Survey. The other study

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[1] Böggild, O. B., Meeresgrundproben der Siboga Expedition: Siboga Expeditie, Mon. 45, p. 11, 1916.

is on the shoal-water deposits of southern Florida and the Bahamas, as representing areas in which very little or no terrigenous material is being deposited, except at the north end of the Florida reef. Fortunately several papers on the Floridian and Bahamian samples themselves and on correlated phenomena, such as papers by Dole and Chambers and Wells on the chemistry of the waters, bacteriological studies by Drew and Kellerman, and temperature records by me, have been published, but a large quantity of data remains unpublished. I am hopeful that within a relatively few months all data already acquired, which include Ekman current meter measurements at about 15 stations, may be prepared for printing.

The researches of F. W. Clarke and W. C. Wheeler on the inorganic constituents of the skeletons of marine organisms is of prime importance but such work needs to be correlated with studies on the bottom deposits themselves. The research on which Clarke is now engaged, the composition of river water discharged into the sea, is also of great value. Wells's researches, such as his published "New determinations of carbon dioxide in water of the Gulf of Mexico" and the studies he is now making on the waters of Chesapeake Bay, are also of much value in understanding problems of sedimentation, but the sediments themselves need to be studied. Richard Field of Brown University is studying some features of modern shoal-water limestones, and E. M. Kindle of the Canadian Geological Survey is conducting important researches on modern limestones; but Kindle may not be credited to the United States.

Of the researches above enumerated, five deal with bottom samples and areal surveys of the sea bottom. The areas are the Bay of Maine, Chesapeake Bay, southern Florida and the Bahamas, the mouth of Mississippi River, and San Francisco Bay. No one of these researches is progressing in a satisfactory way. Furthermore, all these researches deal primarily with shoal-water deposits—there is no work on deep-sea deposits. The only modern deep-sea samples recently described from America are two I described in 1917 from the Tongue of the Ocean, Bahamas. This, it seems to me, is a very poor showing for the United States.

FACTORS THAT RETARD RESEARCHES ON SEDIMENTS

During the period that the United States were participants in the World War, investigations on sediments suffered as did many other kinds of scientific work and our country has not yet finished its readjustment after the conflict. Besides the interruption caused by the war, several competent investigators have been diverted by other duties and a new crop of investigators has not yet ripened.

The interruption of investigations and the diversion of investigators are not the only difficulties in the way of studies of sediments. The subject is one that does not belong exclusively in any one of the sciences as the sciences are currently classified, although those engaged in several kinds of scientific endeavor recognize the value of knowledge of certain aspects of it in the proper performance of some of their work. The engineer, for instance, wishes to understand shore-drift in certain places and the rate of the deposition of sediment in harbors; the student of fisheries wishes to know the relations between bottom material and organisms that may be used as food; the navigator may keep his course through fog and snow by detailed knowledge of the bottom; and the geologist may utilize knowledge of sediments in interpreting some geological formation of economic significance. Of the different kinds of scientific men the geologist is the most broadly concerned, because only by an adequate knowledge of the modern can he understand the ancient deposits and it is part of his work to study the mechanical features and the constituents of sediments, both modern and ancient, though he usually feels that his attention should be directed to past rather than to present history. This is a practical day and students inquire how they can make work on sediments pay. It has been possible to provide funds for some work on sediments but the remuneration is far below that offered by oil companies.

MEANS FOR PROSECUTING RESEARCHES ON SEDIMENTS

In remarks already made I have tried to bring to your attention the present status of researches on marine sediments in this country and I have indicated some factors that I believe retard such investigations. How can the backward condition of researches in this important subject be remedied? I will venture a few suggestions.

My first suggestion is that those interested endeavor to impress upon students the scientific importance of investigations on sediments. This may best be done by the establishment of courses in sedimentation in our universities and the offering of fellowships to graduate students for investigations in the subject. At present courses in sedimentation are given at the universities of Wisconsin, Iowa, and California, and at the University of Iowa a research fellowship is maintained. Courses should be given at more universities and there should be more research fellowships. The Geophysical Union might combine with the divisions of Geology and Geography and of Biology and Agriculture and endeavor to establish two or three more fellowships in sedimentation.

In addition to university work of the kind indicated an institution or institutions in which complicated special studies may be undertaken are needed. An institution comparable to the Geophysical Laboratory of the Carnegie Institution would fulfil the great need but an endowment that will yield an income between $50,000 and $75,000 per year is not easily obtained. However, it is worth striving for. As such an institution does not exist it may be preferable to try to utilize existing institutions by appealing to them and trying to strengthen them. The study of sediments is a fundamental of geology and the U. S. Geological Survey has recognized this and has tried to develop researches on sediments as a part of its work. Furthermore, many geologists, because of their training, are prepared to undertake such investigations. It is, therefore, suggested that those interested in such work make their desires known to the director of the U. S. Geological Survey, that it be pointed out to him how the Geological Survey by doing such work can help science and serve other governmental bureaus, and that he be requested to do as much as the circumstances of the Survey will permit. The Geological Survey has already done enough to place students of sediments under deep obligations to it. If it could study and prepare reports on bottom specimens one of the present difficulties in the way of advance in knowledge of marine bottom deposits would be removed.

Until now it has been possible to obtain larger collections of properly taken bottom samples than it has been possible to study. The U. S. Bureau of Fisheries is fully equipped to collect samples precisely as they should be taken and the heads of that Bureau are anxious to do all they can to aid researches on sediments. Perhaps if provisions could be made to study the samples, the U. S. Coast and Geodetic Survey might make systematic collections. Other than governmental agencies, especially the Department of Marine Biology of the Carnegie Institution, have shown willingness to help in procuring bottom samples for study. The material available for investigation is large in quantity and much of it has been properly collected and is accompanied by all needed data. If these collections could be properly studied and reports on them published, what fine contributions would be made to our knowledge of marine sediments!

CONCLUSION

In conclusion I wish to emphasize the value to science of a proper understanding of the marine sediments in the ocean of today. A proper understanding of these sediments includes knowledge of the depth, temperature, and salinity of the waters above them, the distance from shore to where they were deposited, their relations to currents, and if near land, the relief of the land, its climate, and the rocks composing it. Through such knowledge of modern sediments the criteria for interpreting the sediments of ancient seas are discovered. Having established the needed criteria, the boundaries of the old seas may be traced; the physiography, constitution, and climate of the neighboring lands may be recognized, and the depth, temperature, chemical composition, and currents of the waters of the ancient oceans and the organisms that inhabited them may become known. Modern sediments, though important in understanding what is today, are doubly important because knowledge of them supplies the only key to what would otherwise be an unknown past.

THE INTERVALS THAT SHOULD OBTAIN BETWEEN DEEP-SEA SOUNDINGS TO DISCLOSE THE OROGRAPHY OF THE OCEAN BASINS

By G. W. Littlehales

The intervals between sounding stations must be gauged by the dimensions of the orographical features whose presence it is intended to disclose. Leaving out of consideration details of topography and confining the attention to features of the greatest prominence, inquiry must be made as to the form and dimensions of the slenderest isolated submarine peak that could be raised from the floor of the ocean to a mountainous height and remain standing under the stresses of its own weight and of the superincumbent body of water. For if the spacing of soundings be such as to give indication of the presence of the slenderest form that could stand, then evidence of the presence of any orographical forms that may exist is likely to be afforded. Theoretically, the shape of an isolated submarine peak would be that of a solid of revolution in which the resistance to crushing of any horizontal section is equal to the combined weight of the portion of the formation above that section and of the superincumbent body of water.

Let y denote the radius of any horizontal section and z its distance from the top of the formation. Let K denote the coefficient of resistance to crushing of the material composing the formation; w, the weight of a unit of its volume; and w', the weight of a unit volume of sea water.

Accordingly, $\pi w \int y^2 dz =$ the weight of the formation above any section whose distance from the top is z, $2\pi w' \int y.z.dy - \pi w' \int y^2 dz =$ the weight of the water upon the formation above any section whose distance from the top is z, assuming the top of the formation to reach to the surface, $\pi K y^2 =$ the strength of any section to resist crushing, and

$$\pi w \int y^2 dz + 2\pi w' \int y.z.dy - \pi w' \int y^2 dz = \pi K y^2 \qquad (1)$$

By differentiation, equation (1) becomes

$$\pi w y^2 dz + 2\pi w' y.z.dy - \pi w' y^2.dz = 2\pi K.y.dy \qquad (2)$$

which expresses the condition that the increase of strength of any section in excess of that of the section next above is equal to the sum of the increases of the weight of the formation and the weight of the water upon any section in excess of their combined weight imposed upon the section next above.

Letting S denote the area of any horizontal section whose radius is y, and dS, the differential of S, equation (2) may be written in the following forms:

$$w.S.dz + w'.z.dS - w'S.dz = K.dS$$

$$(w - w')\, S.dz = (K - w'z)\, dS$$

$$\frac{dS}{S} = (w - w')\frac{dz}{K - w'z} = \frac{w - w'}{w'} \cdot \frac{dz}{\frac{K}{w'} - z}$$

By integration, equation (3) becomes

$$\log S = -\frac{w - w'}{w'} \log\left(\frac{K}{w'} - z\right) + C$$

in which C is the constant of integration.

$$\text{Hence } \log\left(\frac{K}{w'} - z\right) = \frac{w'}{w - w'}\, C - \frac{w'}{w - w'} \log S$$

$$\text{or } \frac{K}{w'} - z = \frac{E^{\frac{w'}{w - w'}\, C}}{E^{\left(\frac{w'}{w - w'} \log_e S\right)}}$$

In the absence of knowledge of the value that should be assigned to K, the coefficient of resistance to crushing, this equation has been used in the generalized form,

$$A - z = \frac{B}{E^{\left(\frac{w'}{w - w'} \log_e S\right)}} = \frac{B}{E^{\left(\frac{1.03}{2.67 - 1.03} \log_e S\right)}} = \frac{B}{E^{1.46 \log_{10} S}},$$

to find the equation of their average form from the observed bathymetrical data on Seine Bank in latitude 33° 50' N. and longitude 14° 20' W., Cocos or Keeling Island in latitude 12° 06' S. and longitude 96° 53' E., Enderbury Island in latitude 3° 10' S. and longitude 171° 10' W., Funafuti Atoll in latitude 8° 25' S. and longitude 179° 07' E., Taviuni Bank in latitude 12° 05' S. and longitude 174° 35' W., and the shoal near Midway Island in the North Pacific Ocean in latitude 28° 00' N. and longitude 177° 40' W.

For this purpose the values of z and y, expressed in nautical miles, were inserted in the above equation, and a conditional equation was formed for each pair of coordinates relating to each of the submarine formations. From these conditional equations normal equations were

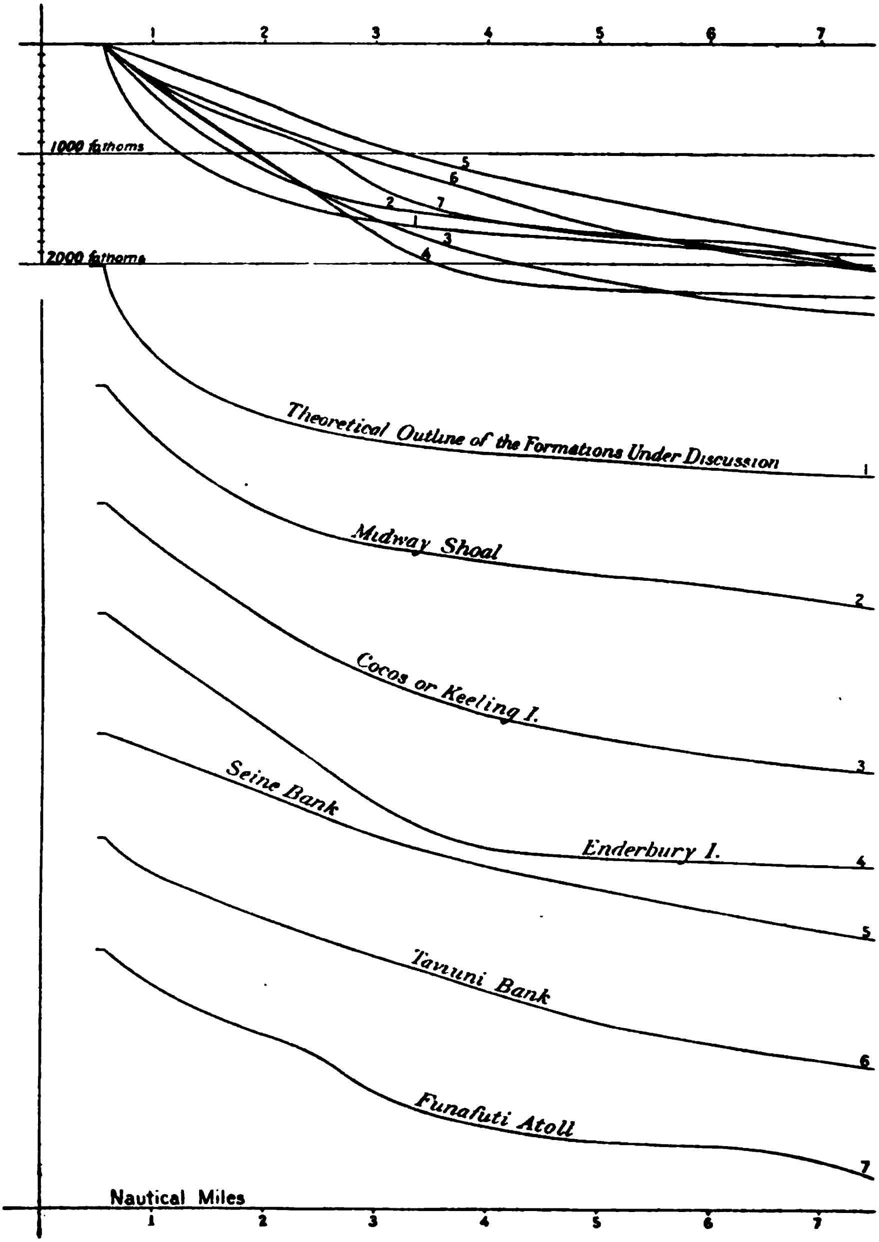

FIG. 1. Profiles of isolated submarine peaks.

found by the method of least squares, which gave the values of the constants A and B. The resulting equation is $1.87 - z = \frac{1.87}{E(1.446 \log_{10} \pi y^2)}$ and the corresponding curve, which by revolution around the vertical axis would generate the average form, is shown in figure 1, together with others which have been plotted for purposes of comparison from measured data. This investigation shows that isolated formations occupying comparatively limited areas at the bottom can and do occur in the ocean depths, and we are able to assign at once the maximum interval that should obtain between deep-sea soundings taken in operations directed toward the development of the orography of the bottom of the sea. An interval of 8 miles coupled with a differential interval of 2 miles would serve for general development, and would prove with certainty the existence or absence of any formation rising close to the surface. Of all the possible ways in which an 8-mile interval could lie with reference to a submerged peak, that which would be most advantageous for a prompt discovery of its existence is the condition in which one end of the interval is at the bottom of the slope and the other near the apex, and that which would be least advantageous is the condition in which the interval is bisected by the position of the apex. In the latter case, there would be nearly equal soundings at both ends, but the soundings at the ends of the adjacent two-mile intervals would in all probability give indication of the slopes.[1]

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[1] Following the presentation of this paper the following suggestion was made by Harry Fielding Reid:

Dr. Littlehales' remarks about the soundings in the oceans bring up a matter that I have had in mind for some time; that is, the value of a detailed sounding of a single deep. We know very little indeed about the shape or conformation of the great ocean deeps; a detailed set of soundings of a particular deep, to bring out not merely the general slope of the bottom, but also details of configuration, would be of great value. If, as seems probable, the great deeps are due to faulting, the soundings should be close enough together to show the existence of fault-scarps. A deep which offers especial facilities for such determinations is the Virgin Islands or Bronson Deep. It is a long east and west trough, lying a little north of Porto Rico, with a recorded sounding of 4,662 fathoms (the greatest depth measured in the Atlantic); although but few soundings have been made in its deeper parts. Its situation is very convenient; San Juan could be used for a base for the western part and St. Thomas for the eastern part.

There are, of course, other parts of the Caribbean region where soundings would be valuable, but I think a detailed sounding of a single deep would yield more valuable results than scattered soundings over a larger area.

NEW METHODS OF OBSERVING WINDS AT FLYING LEVELS OVER THE OCEAN

By Alexander McAdie

Aerography may be defined in a general way as a study of the structure of the atmosphere. There are various ways of obtaining information regarding the flow of air at different levels and the conditions of density, pressure, and temperature of the mixture of air and vapor. Exploration of the upper air has been accomplished by means of close study of the clouds; the establishment of mountain observatories; the ascent of manned balloons; kites and kite balloons; sounding balloons and pilot balloons. To these we propose to add another where measurement is made from the deck of a vessel by employing certain predetermined lapse rates, or rates of fall in temperature with elevation.

The principle in brief is that provided sufficient water vapor is present and condensed as cloud, the height of the level of condensation is a function of the lapse rate. The height can be obtained then from observations of the actual temperature, the temperature of evaporation and the temperature of saturation at sea-level, making proper corrections for surface speed and direction.

It is a little more than twenty years since Teisserenc de Bort at Trappes, and Lawrence Rotch at Blue Hill, close friends and co-laborers, began the systematic sounding of the atmosphere by means of sounding balloons. With the war came a widespread use of pilot balloons. Today, *sondages* are made (or are supposed to be made) at all United States naval air stations, and at many land stations. During the war information regarding the speed and direction of the winds at flying levels thus obtained was of great value—not alone to the airmen, but also to artillerists and gas men.

There is no special difficulty in using sounding balloons or pilot balloons on land; but at sea the sounding balloon is out of the question, owing to difficulty of recovering the record. Pilot balloons, however, can be used; and during the trans-Atlantic flight of the N. C. boats, I obtained fully a hundred observations from sea-level up to 4 or 5 kilometers, while stationed on the U. S. S. *Baltimore* (mine layer).

We can not, however, expect navigating officers of our merchant marine to send up balloons, follow them with theodolites, record the elevations and angles, plot the trajectories and deduce from these the speed in meters per second and the direction of motion for the different levels. Not but that it would pay to do so; for it *will* pay any navigating officer to be posted concerning the structure of the air. It may sometimes mean the safety of the ship. And an intelligent aerographic officer with a moderate outfit of aerographic apparatus on a ship like the *Mauretania* could tell from the upper air movements studied in connection with the surface circulation, the location of the ship with reference to the true centers of gyratory and translatory flow, and could forecast the future path of the storm. A daily weather map or one at more frequent intervals, based on reports received by radio, could be made and used to great advantage with this added knowledge of the upper air conditions.

In cloudy weather the pilot balloon may soon be lost and it is therefore advisable to substitute for the balloon method a method which makes use of clouds, especially lower clouds within 35 degrees of the zenith. Of course in dense fogs, neither method can be used.

The new method makes use of a specially stabilized nephoscope with automatic sighting rods, and an arc with tangent values; also a new type of hygroscope. The combination may be called a marine altoscope.

The nephoscope consists of a black mirror suitably mounted (for details of construction see *Blue Hill Report,* 1910) to permit of motion in azimuth, proper leveling devices, and graduated circle, reading clockwise and in either degrees and tenths or in grads. To this mirror is attached a stabilizing device, suggested by Professor R. W. Wilson of Harvard University. The mirror thus keeps a horizontal position regardless of the ship's motion.

A metal arc or quadrant springs from the plane of the mirror and is graduated in degrees, and also in natural tangents, the reason for which will appear later.

At the free end of the arc a vertical rod is mounted and carries a pantagraph or diamond-shaped rectangle supporting two rods for sighting the cloud. Use is made of the reflection of the line joining cloud and eye, and the second sighting rod forms a straight line prolongation of the line from the center of the mirror to the cloud. The value of this is in fixing the eye, whatever the ship's motion may be. When once set, the eye can be withdrawn or rested for a few seconds and then brought back to the original position without delay or uncertainty. The radials can be provided with sleeves permitting extension.

In observing, first level the instrument. Bring the zero which is also 360° or 400 grads of the horizontal or azimuth circle to the true south point. The circle is graduated clockwise and the true west will therefore be 100 if scale is in grads (90° if in ordinary units).

Since the reflection of the cloud crosses the mirror in the same direction as the cloud is moving, the reading on the azimuthal circle where the cloud image passes off the black mirror will be the direction or angle *from which the wind is blowing.*

The quadrant is now swung into position, making the same angle. With the control screw provided for the purpose of raising or lowering the sighting rods, bring the nearer sighting rod into perfect alignment with the reflection of the other sighting rod. This latter rod joins the cloud point and the center of the mirror. We have now the angular elevation of the cloud from a true horizon. When this angle is 50 grads or 45 degrees, it is plain that the distance the reflection of the cloud moves in the mirror is equal to the height of the intercept corresponding to the height of the cloud; that is, the sine and cosine of the angle are equal, and the natural tangent is unity.

In such a case, we have only to divide the height of the cloud (to be determined later) by the number of seconds to get the rate in meters per second.

If, however, the cloud line does not make an angle of 45, we use directly the value of the tangents. The following condensed table gives these values:

Tangent		*Grads*	*Degree*
.4	=	24	22
.5	=	29	26
.6	=	34	31
.7	=	39	35
.8	=	43	39
.9	=	47	42
1.0	=	50	45
1.1	=	53	48
1.2	=	55	50
1.3	=	58	52
1.4	=	60	54
1.5	=	62	56
1.6	=	64	58
1.7	=	66	59
1.8	=	68	61
1.9	=	69	62
2.0	=	70	63
2.5	=	75	68
3.0	=	80	72
4.0	=	85	76
5.0	=	87	79
6.3	=	90	81
11.4	=	95	85
∞		100	90

One has only to divide the height of the cloud by the arc reading (i. e., tangent value) to get the horizontal distance. This last divided by the number of seconds gives the speed of the cloud in meters per second. We thus have direction and speed of the air at the cloud level, provided the height of the cloud is known.

To get the height we use a special type of psychrometer (McAdie cryoscope). The improvements over the usual psychrometers are:

(1) The amount of air passing over the wet-bulb is under control; i. e., a definite value is given to the wind factor in evaporating the film of water.

(2) The method of wetting the bulb is novel. The old method of using a wick or muslin cloth, bringing a constant supply of water by capillary action, is replaced by a fine metallic mesh shaped to slide over the bulb, easily wetted and containing a known small weight of water, to be evaporated in a given time.

(3) The conversion of vapor pressure into units of force permits the use of a simple equation connecting the actual temperature, evaporation temperature, and saturation temperature.

Of the above factors, the wind velocity is of great importance and must be known definitely if the humidity records are to be regarded as reliable. It may be pointed out that even in official meteorological services at home and abroad the records of relative humidity are open to criticism on the ground of uncertain ventilation. In the best forms of sling and whirling devices no record is kept of the time and number of revolutions.

In the present instrument a definite wind velocity is automatically maintained and the beginning and ending of the movement of the air over the evaporating surface, or what is approximately the same, the movement of the wet-bulb through the air, is definite. The wetted bulb can be swung either vertically or horizontally at any desired speed from 4 to 10 meters per second. The thermometers are carried by a frame which slides on the rod and their distance from the top of the rod or axis of rotation determines the velocity of the equivalent wind. Thus at a distance of 100 centimeters (39.3 in.) the bulb when whirled will travel in one complete revolution 6.283 meters (approximately 20 ft.). It is then only necessary to know the number of rotations and the time to get the speed of the wind. An automatic counter is so connected with the handle that at the completion of every hundred revolutions an alarm bell rings. With a little practice one makes 100 swings per minute.

If desired a watch may be used and the number of seconds counted. The rate mentioned, one hundred per minute, is equivalent to a wind of 10.5 meters per second (23.5 miles per hour).[1]

Now, the rate of evaporation varies as the square root of the wind velocity. Thus the rate at 10.5 m/s is to the rate at 4 m/s as 16 to 10. The hygrometric tables in common use were based on experiments in which the speed of rotation was approximately 4.5 meters per second, although no definite statements are made and there appears to have been no special attention paid to the speed of rotation or the rate of fanning of the wet bulb. Naturally discordant results are obtained by different observers. The speed mentioned (4.5 m/s) is somewhat too low for a good circulation of air, and is indeed below the average wind value at most places. The value of 10 meters seems to be a more representative figure.

In the present instrument the pressure of the water vapor at any temperature ordinarily met with above the freezing point is expressed in units of force, and so far as known this is the first instrument employing these units for water vapor. A kilobar is that pressure which if exerted as force would give an acceleration of one centimeter per second per second to a mass of weight one kilogram. Roughly, it is the pressure given by a wind of 12 meters per second on a plane one meter square and at right angles to the wind. Thus, temperature, pressure and weight are expressed in a uniform, consistent and scientific set of units, namely, the

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[1] In the sling psychrometer used by the Bureau of Mines, if we assume a speed of 100 revolutions per minute the equivalent wind would be about 2.9 m/s (6 miles per hour). There is no counting device and while a higher rate can be obtained, it is difficult to count by the eye more than 120 per minute. In the whirled psychrometer used by the Weather Bureau, the radius of rotation of the bulbs is about the same as in the Bureau of Mines instrument, but a geared handle permits of varying the rate from 175 to 260. The velocity equivalents will vary from 9 to 16 miles per hour, the rate of evaporation in the former being only 75% of that in the latter. McAdie has suggested a simple form of counter for this instrument to standardize the results and has used such a device at Blue Hill Observatory for two years.

kilobar, kilograd, kilogram. These are strictly in accord with the C. G. S. system of units.

To determine relative and absolute humidities, and the temperature of saturation, the so-called dew-point, there is used an equation given by the author in the *Physical Review,* Vol. XIII, No. 4, page 285.

$$p_s = p_e - pC \quad (t-t_e)$$

in which p_s is the pressure of the water vapor at the saturation or dew-point, p_e the pressure of evaporation—that is, the wet-bulb—p the pressure of the atmosphere expressed in kilobars, C a constant, t the temperature of the dry-bulb expressed in kilograds, and t_e the temperature of the wet-bulb.

When the wind velocity exceeds 2 m/s, pC may be written as 0.18; and for purposes of quick calculation we regard it as 20 percent without materially affecting the result.

I stop at this point to read part of a letter just received from Sir Napier Shaw. He says:

> As to the inter-relation of meteorology and oceanography, I think that humidity probably offers the most promising line of attack, if we could be quite certain of getting true humidities on board ship. I suppose that there must be a mathematical expression for the absolute humidity depending upon the air current and the eddy motion which it carries. I could imagine a very useful expedition tracing the increase in absolute humidity down the Trade Wind and ultimately to the West Indies; but it is very difficult to get humidities on board ship because the dry bulb is apt to get wet and the wet bulb to get dry; and both of them to be spoiled by spray. But he will be a great benefactor who will give us a map of the distribution of absolute humidity over the Atlantic Ocean.

Three things in the quotation are important: the suggestion of the map, the expression of belief in the humidity problem as a most promising liaison between meteorology and oceanography, and the remark about the difficulty of getting accurate humidities aboard ship.

Granted, then, that we can get these humidity values at sea with much greater precision by these new instruments, we proceed to use these values in determining the cloud heights.

The temperature of saturation can be obtained without the use of tables, which are always troublesome to use aboard ship, owing to high winds, from the cryoscope, or, if desired, from the accompanying chart (figure 2).

An example will show how this is done.

Let the dry reading be 1063 and the wet, after proper precautions, 1053. The relative humidity is at once shown by the dotted line to be 74, and the dew-point, obtained by running back to left-hand edge of chart parallel to the solid lines, 1046. If the absolute humidity is desired, one has only to follow the 1046 line horizontally to the right-end edge; and one reads 11 grams per cubic meter of space.

We will call 1046 the cloud point or temperature of condensation (heretofore called dew-point, but the new name has some advantages).

What we now want is the difference between the surface temperature

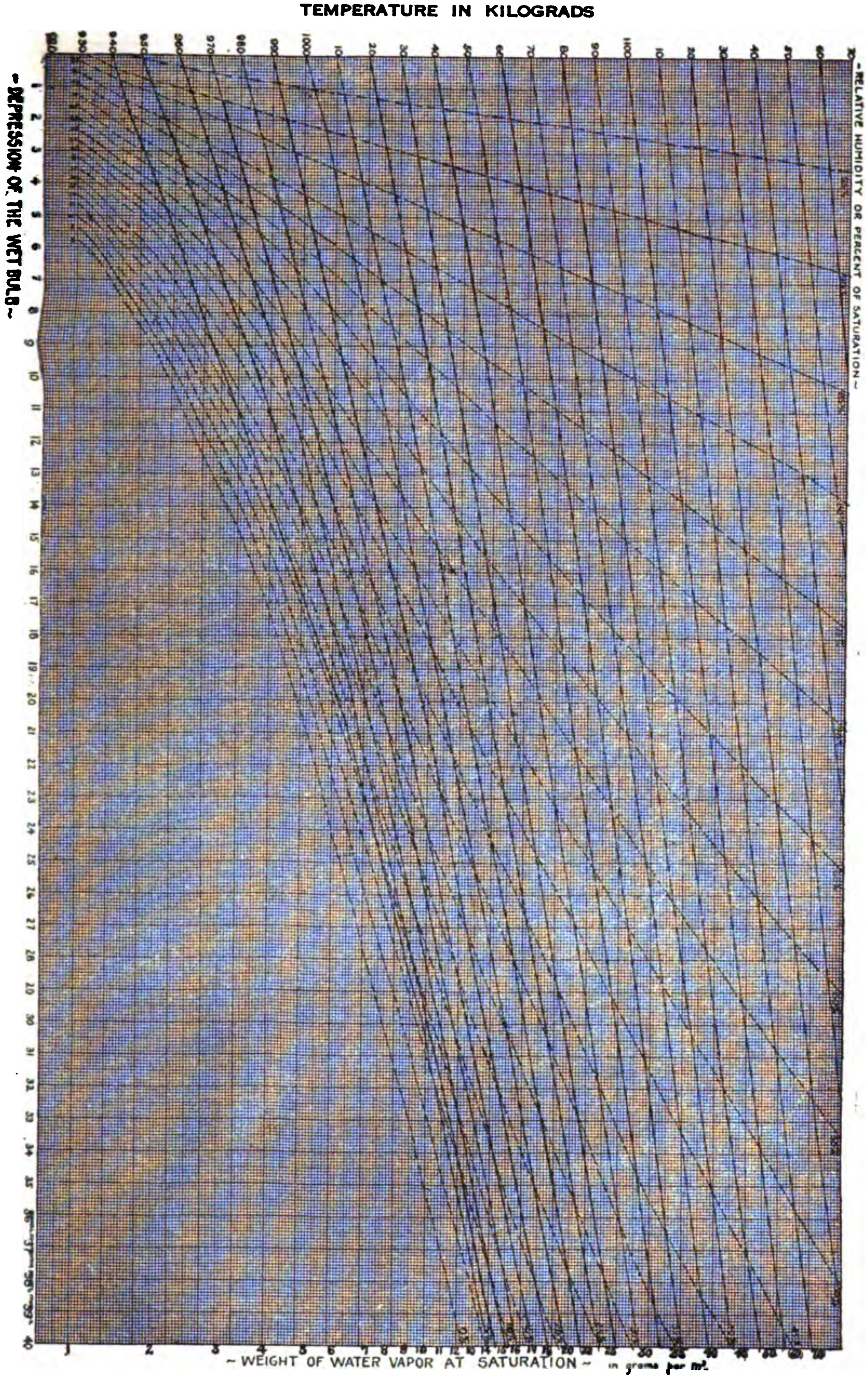

FIG. 2. Absolute and relative humidity

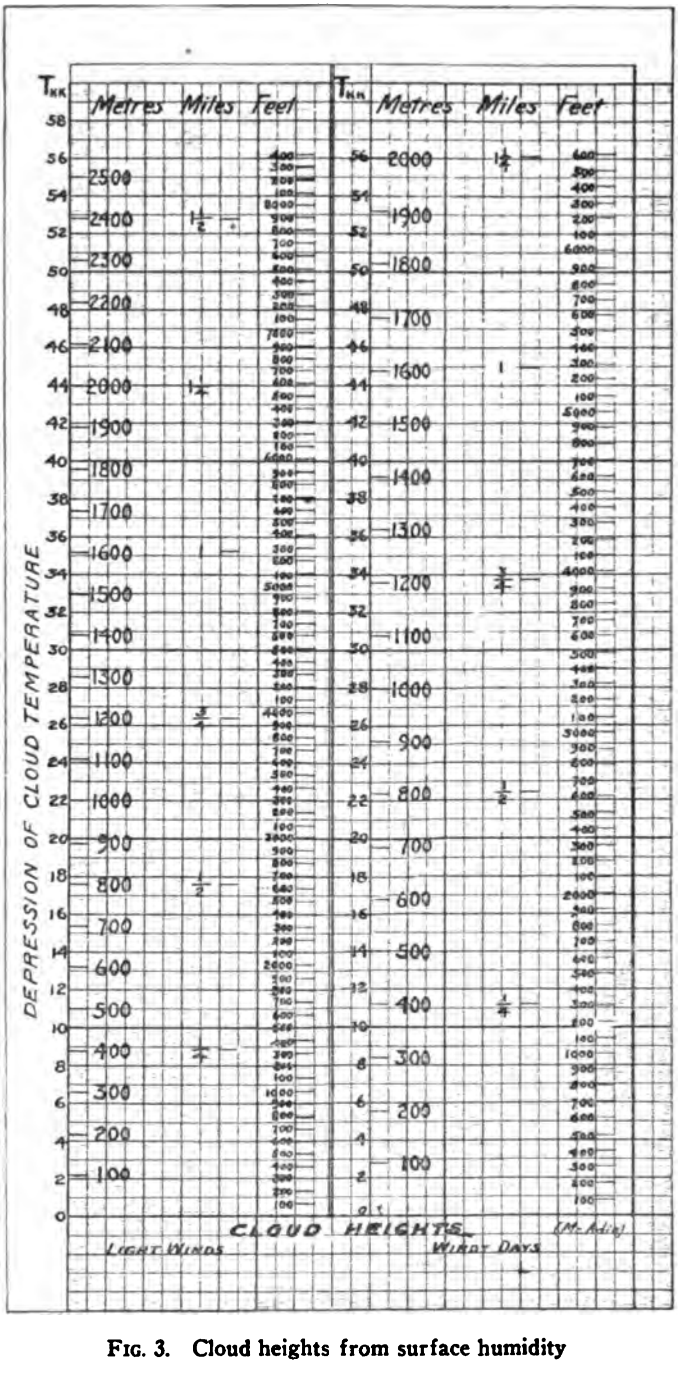

Fig. 3. Cloud heights from surface humidity

and the cloud level; or what may be called the depression of the cloud temperature (see figure 3). 1063 — 1046 = 17. The cloud height opposite 17 is 750 meters for a day of light winds and 600 meters for a windy day. A correction for percentage of saturation and type of structure is desirable.

The height of the cloud being known, the direction and velocity are obtained as described, and the observer can compare these values with the surface values. Nearly always there will be differences. In fair weather there is generally a steady shifting of the wind to a higher value for both speed and direction. At Blue Hill the mean deviation for the 1000-meter level is 7 grads, or 6 degrees to the right. The increase in speed is variable, often 100 percent in the first 500 meters, and we have instances of 200 per cent. On the land we get all sorts of structures, in some of which, such as sea breeze, the depth of the surface flow is shallow and essentially different from the flow above. The values obtained by this nephoscope-cryoscope method are approximately gradient velocities and directions. It is possible to construct a chart when gradient velocity direction and latitude are known, from which the pressure gradient can be deduced; and thus in a rough way the isolated observer could obtain the curvature of the isobar and pressure tendency. In former years this would have meant much; but now, of course, full reports can be obtained by radio and the surface isobars easily drawn.

It only remains to explain the variation in the value of the lapse rate on different days, or rather with different structures.

While the adiabate rate is 35.5 kilograds per 1000 meters, an average rate of cooling of mixed air and vapor is 21 kilograds.

For moist air saturated, a value of 18 may be taken. In windy weather, a fair value is 25 grads.

THE STEERING LINE OF HURRICANES

By Alexander McAdie

As a frontispiece to the "Manual of Meteorology," Part IV, "The Relation of Wind to the Distribution of Barometric Pressure," Sir Napier Shaw gives three storm paths of unusual duration and remarkable recurvature (see figure 4).

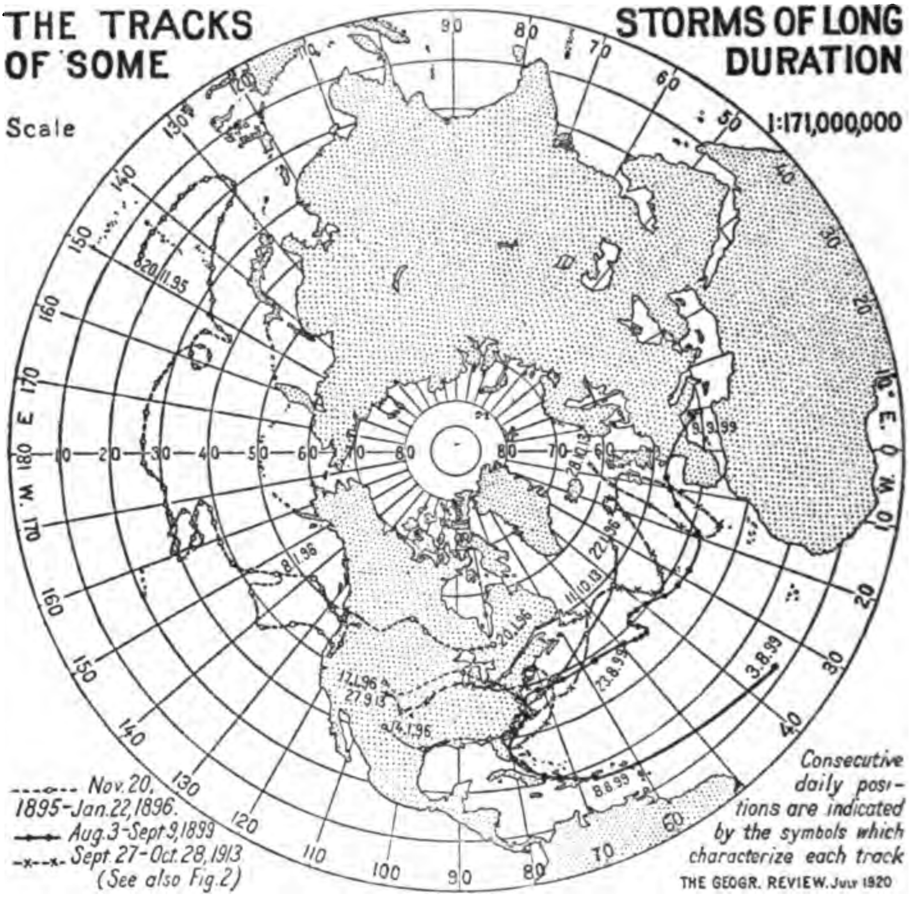

Fig. 4. The tracks of some storms of long duration (after Shaw)

Perhaps the most striking of these is a track of a typhoon or *baguio* charted by McAdie. This storm path was determined by the usual method of connecting pressure minima. The readings were obtained from abstracts of ships' logs, available through the courtesy of the Hydrographic Office. Surface winds and cloud directions were utilized as much as possible.

It was assumed that the minimum pressure and the center of circulation as indicated by surface winds were identical. It is, however, to be remembered that the wind direction as noted on the deck of a moving vessel may need correction. Furthermore the center of a cyclone is not necessarily the center of ascending air; and still further there must be in the convergence of the surface winds a certain distortion due to the travel of the storm.

The storms referred to above are perhaps best described in the words of Sir Napier Shaw (page 119).[1]

> There is evident stability in motion of this character because beginning with examples of whirls lasting for some seconds there is apparently an uninterrupted sequence by way of revolving sandstorms or dust-devils, tornadoes, or whirlwinds, to tropical revolving storms and large cyclonic areas with radii of 10 degrees or more.
>
> The only limit of the series is a revolving air-cap covering the hemisphere or a large part of it. And just as a belt of west wind or a belt of east wind may lie over these [British] Islands for weeks, so the other type of quasi-permanent atmospheric motion, which has always been thought of as a column of air in continuous revolution, may preserve its identity for days or weeks. Through the kindness of Professor McAdie of Blue Hill Observatory, Harvard University, we are enabled to give two notable examples.
>
> The first is that of a tropical revolving storm which started on a westerly track toward the Philippine Islands (where visitations of that kind are known as "Baguios"), turned round toward the north and northeast, crossed the Pacific Ocean and, after some vagaries on the North American continent, continued its journey eastward and crossed the Atlantic in the usual track of cyclonic depressions over that ocean. The whole journey lasted from 20th November, 1895, to 22d January, 1896.
>
> The second is a cyclonic depression of October, 1913, in the outer region of which the tornado was formed which caused so much destruction in South Wales on the 27th of that month.[2] The track of the main depression shows an anomalous path from Canada to the north of the British Isles. [See figures 4, 5, and 6 from the "Geographical Review."]
>
> To these notable examples has been added the long track of cyclonic depression which was figured in the Meteorological Office chart of the North Atlantic and Mediterranean for August, 1904.[3] The cyclone was first noted on 3rd August, 1899, in that part of the North Atlantic Ocean where West Indian hurricanes often take their rise. It moved westward to the West Indies, skirted the coast of Florida and turned eastward over the Gulf Stream. After some hesitation about latitude 40° W. it made for the mouth of the English Channel and, missing that, crossed to the Mediterranean, where it lost itself on 9th September, after a life of thirty-eight days.

In each of the above described storms it is evident that causes other than those developed by the rotating mass of air, operated to retard these storms in their eastward progress.

Let us now trace the path of a West Indian hurricane where the evidence is seemingly more direct.

On the morning of October 15, 1910, this storm was centered between Havana and Key West moving very slowly northward. The maximum wind velocity at the former place was 39.4 m/s (88 miles per hour); on the a. m. of the 14th; and at Key West 26.8 m/s (60 miles per hour).

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[1] See also "Wandering Storms," McArdie, *Geographical Review*, 10, no. 1, July, 1920.

[2] *Geophysical Memoirs*, no. 11. M. O. Publication, no. 22a.

[3] M. O. Publication, no. 149.

FIG. 5. Track of storm of September 27–October 28, 1913.

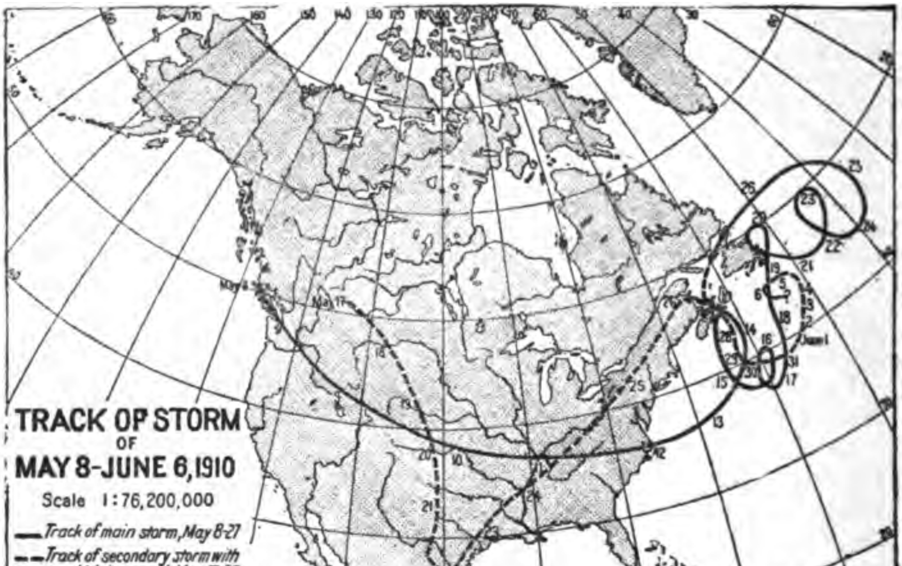

FIG. 6. Track of storm of May 8–June 6, 1910.

The storm's progress northward was checked by a continental area of high pressure moving southward. Thus on the 17th we find the hurricane actually *retrograding* and centering again over Havana. As the continental anticyclone moved east, the hurricane developed a northerly component of motion and on the 18th moved across Florida. It then followed the usual hurricane track passing south of Cape Hatteras on the 20th. The hourly speed increased from 30 kms. to 50 kms. per hour and the direction of motion 40 degrees east of north.

The speed continued to increase averaging 60 kms. per hour and the direction shifted more to the east, approximately 65 degrees east of north, and so at noon of October 21 the center was in the latitude 37 degrees north and 67 degrees west.

Professor Bjerknes has remarked that "anticyclones are born as cyclones die" but the behavior of this and similar storms gives the impression that the path and speed of West Indian hurricanes, off the coast of Florida, are dependent upon the intensity and direction of advancing highs. These in turn may be but the surface expression of an advancing polar front.

Two types of south moving sub-Arctic surges which seem to control the path of hurricanes from the Caribbean Sea to the North Atlantic can be identified. The first of these is a Nichikun high. This is a more appropriate designation of what has heretofore been known as a Labrador high. According to Dr. Klotz[1] there is nowhere else in Canada "so distinct a Pamir or Roof of the World as the neighborhood of Lake Nichikun (in English, Otter Lake)." The lake itself is in latitude 53° N., longitude 71° W., and on the northwest slope of the Height of Land. The drainage is into Hudson Bay. On the south and east the drainage is into the River St. Lawrence. It is this southern slope which concerns us because south moving masses of air pass over the ridge, elevation 730 meters; and being both cold and dry and therefore heavy, fall to sea level in a comparatively short distance, 200 to 600 kilometers.

The other type of sub-Arctic surge is the "Labrador," essentially oceanic.

Both of these tongues may be portions of what Bjerknes has called the polar front. They undoubtedly play an important part in determining the speed and path of storm centers in the North Atlantic States and effectively control the path of tropical storms or hurricanes as they move from the south and change into North Atlantic cyclones.

On the Pilot Chart of the North Atlantic Ocean for October, the path of the hurricane under discussion ends abruptly in the position and on the date given above (Oct. 21). One might in consequence infer that the storm dissipated at sea.

Careful study of pressure conditions shows a depression on the 22d in latitude 35° N. and longitude 60° W. A day later it appears as one of two centers in a large depression extending from New Brunswick to

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[1] In a letter to the writer.

Fig. 7. Hurricane October 13–25, 1910. Lake "low" October 21–24, 1910. (Based on an equal area projection devised by B. J. S. Cahill)

Bermuda. The other center can be traced back to a storm over Lake Superior on October 21. The previous history of this depression, while somewhat obscure, is deserving of study. It appeared as an unexpected abnormal development and invalidated all forecasts made for the Lake Region, Upper Mississippi and Ohio Valleys. Where cold weather, frosts and an absence of precipitation were reasonably anticipated from an advancing high pressure (1030 kb.), there suddenly developed warmer weather with rain. On the face of the map we are unable to connect this low with a more northern slow moving depression of the Alberta type. The weather map of October 20, 1910, will repay study in connection with the steering line of cyclones.

To return to the hurricane and its further history, we have seen that when centered over Florida, there were in juxtaposition two air masses of different origin, one from the tropics with a vapor content of not less than 20 grams per unit volume (one cubic meter of space) and an average northwest speed of one kilometer per hour, while the other air mass was of sub-polar origin, approximately 20 kilograds (5.5 degrees C.) colder, and with an average vapor density of 12 grams per cubic meter. The densities of the two air masses at a pressure of 1 megabar would be approximately 1170 and 1220 grams. Air motion is initiated by difference of pressure rather than difference of density; but it is plain that the south moving air mass would continue to gain momentum and underrun the less dense northbound air. The horizontal pressure gradient was 1 kb./20 km. and hence surface velocities of 30 meters per second or higher would and did occur. The gradient velocities were 23 m/s or higher; and the radii of survature of isobars approximately 100 kilometers.

Figure 7 shows the path of the hurricane from October 13 to 25, and also the path of the lake "low" from October 21 to 24. Other charts show the surface pressure distribution on various dates.

It is much to be regretted that there are no records of winds aloft. When such data shall be available then perhaps definite relations between path, velocity and duration of hurricanes with upper winds will be forthcoming.

Recently it has been claimed by meteorologists of the Bergen (Norway) Institute that the storms of the Northern Hemisphere can be traced back to a "surface of junction of polar and equatorial air." This surface can be detected at the ground as "a line of discontinuity" in surface conditions. In other words, it is the boundary between air masses of different densities, pressures, and vapor content.

Given then a mass of warm moist air moving north of east, under the combined effects of general drift, pressure gradient and rotational deflection, and a second mass of cold dry air moving south, the surface of discontinuity should be detectable as a moving front.

Professor Bjerknes has come to the conclusion from the study of the structure of moving cyclones that a broad belt of rain accompanies the moving (and ascending) warm moist air, and a second smaller rain belt follows, where cold dry air underruns the warm air, that is, along the wind shift or squall line.

A more important point, however, is the discovery through the use of detail maps, that the discontinuity or contrast can be traced from any cyclone to another. As expressed by Bjerknes, cyclones follow each other along a common line of discontinuity like "pearls on a string."

Furthermore this line of discontinuity surrounds the polar regions as a closed circuit. It shows how far the cold air flowing along the ground has penetrated. Shaw describes it as a kind of *polar front line.*[1]

The following substance of the discussion at the Meteorological Office on "new methods of forecasting" may make plain the leading features of Bjerknes's views.[2]

> In the case of a cyclone making progress towards the east, a sector to the south is occupied by a warm current; this warm area on the earth's surface is bounded to the north by the "steering line," to the west by the "squall-line." Bjerknes' generalization is "that these squall-lines and steering lines of all the cyclones of the northern hemisphere are parts of a single line—'the polar front.'" We are to think of two great streams of air, both flowing from the west, the more northerly stream being colder and carrying less moisture. The boundary between these two streams is unstable and its oscillations manifest themselves as cyclones. The warm stream overrides the cold one, which retaliates, so to speak, by turning round and kicking its partner in the back.

Charts 4, 5 and 6 are reproduced through the courtesy of the *Geographical Review,* published by the American Geographical Society, New York City.

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[1] *Nature,* January 24, 1920, p. 524.
[2] *Meteorological Magazine,* November, 1920, p. 213.

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BULLETIN
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THEORIES OF MAGNETISM

Report of the Committee on Theories of Magnetism of the National Research Council

BY

A. P. Wills, S. J. Barnett, L. R. Ingersoll, J. Kunz, S. L. Quimby, E. M. Terry, S. R. Williams

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PREFACE

The present report attempts to sketch in bold outline the evolution and development of magnetic theories from the time of Poisson and Ampere to the present, including some reference to experimental results, particularly in the domain of magnetostriction where theory and experiment are in the greatest need of reconciliation.

It will be noticed that the table of contents for the complete report does not contain any reference to the Hall Effect or allied phenomena. The reason for this omission is that these topics have been assigned by the National Research Council to another committee.

Space limitations have debarred from inclusion in the report some material which appropriately might have found place there. Certain portions of the subjects treated may have been emphasized more than their importance deserves, while others have been unduly slighted. The report, being a composite compilation by contributors so widely separated geographically that close collaboration was not always possible, may lack somewhat in coherency.

But in spite of such deficiencies it is hoped that the report may furnish a perspective of the subject which in its chief outlines is reasonably free from distortion and that the reader may obtain from its perusal a fair idea of the present status of magnetic theory.

The committee is indebted to Professor L. R. Ingersoll of the University of Wisconsin for his contribution on Magneto-optics.

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NATIONAL RESEARCH COUNCIL

Vol. 3, Part 3 AUGUST, 1922 Number 18

THEORIES OF MAGNETISM

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Report of the Committee on Theories of Magnetism of the National Research Council[1]

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MAGNETIC THEORIES PRIOR TO THE DISCOVERY OF THE ELECTRON

By S. L. Quimby
Instructor in Physics, Columbia University

THE BEGINNING OF THE SCIENCE OF MAGNETISM. GILBERT'S "DE MAGNETE"

The science of magnetism, as well as of electricity, began with the researches of William Gilbert (b. 1540, d. 1603). The ancients were acquainted with the fact that amber, when rubbed, attracts light bodies, that the lodestone has the power of attracting iron, and that this power can be communicated to the iron by bringing it near to or stroking it with a lodestone. The latter had been used as a mariners' compass at least since the time of the Crusades. No attempt had been made, however, to order or extend the small amount of available knowledge concerning these phenomena. Rather was it lost in a mass of false doctrine built about it by the medical profession, who were chiefly interested in

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[1] This committee of the Division of Physical Sciences of the National Research Council consists of the following members: A. P. Wills, Columbia University, Chairman; S. J. Barnett, Carnegie Institution; J. Kunz, University of Illinois; S. L. Quimby, Columbia University; E. M. Terry, University of Wisconsin; S. R. Williams, Oberlin College.

utilizing the mysterious property of the lodestone for the curing of disease. Gilbert, himself a physician, dissipated these erroneous notions by clearly separating the medicinal from the magnetic properties of the lodestone, and then proceeded with an exhaustive investigation of the latter.

He differentiated sharply between electrical and magnetic attraction by pointing out the difference in behavior of electrified amber and magnetized iron. He emphasized the dual nature of the magnetic element and examined the effect of the shape of a magnet upon its strength.

After pointing out that the earth is itself a huge magnet, Gilbert investigated the variation and dip of the magnetic needle over its surface and coordinated a vast mass of data which he secured from mariners.

Apart, however, from the intrinsic worth of Gilbert's researches, his work may be regarded as the forerunner of the modern scientific method. His De Magnete (1600)[1] contains the first formulation of natural law based entirely upon the results of experiment. In it Gilbert applied the method which was later set forth with logical precision by Francis Bacon.

No material advance upon the knowledge of magnetic phenomena recorded in Gilbert's book was made for nearly two centuries. During this period developments along different lines were taking place which eventually made possible the great progress in magnetic theory which marks the nineteenth century. One of these was the improvement of methods of experimentation. With the fundamental importance of the experimental method once definitely established advancement along this line reached a point where Coulomb in 1785 was able to prove satisfactorily the inverse square law of magnetic attraction and repulsion.

Another important factor in scientific progress about this time was the rapid growth of mathematical analysis which followed the discovery of the infinitesimal calculus by Newton and Leibniz. Under the inspiration of Laplace, Lagrange, and Legendre, mathematicians, particularly Poisson and Fourier, about the beginning of the nineteenth century began to apply mathematical analysis to physical problems. In 1812 Poisson published a memoir on electrostatics and in 1820 another on the theory of magnetism which remains to the present day a correct mathematical formulation of the phenomenon of magnetic induction.[2]

POISSON'S THEORY OF MAGNETISM

The starting point of Poisson's mathematical theory is Coulomb's law that two magnetic poles attract or repel each other with a force

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[1] English translation by P. Fleury Mottelay; New York, John Wiley and Sons, 1893.

[2] Poisson, "Sur la Théorie du Magnetisme," Mémoires de l'Institut, V (1820), p. 247 and 488.

inversely proportional to the square of their distance apart. As a mechanism for the utilization of this principle he adopted the "two fluid" theory of magnetism which had been previously advanced by Coulomb and others. In accordance with this theory Poisson assumed that all magnetic substances consist of a large number of small particles or magnetic elements containing equal quantities of positive and negative magnetic fluid. These elements are themselves perfect conductors for the fluids, but the spaces between them are impenetrable to the fluids, which cannot be allowed to pass from one element to another. In the unmagnetized state of the body the two fluids are united to form a single neutral fluid. The process of magnetization consists in the separation of the two fluids within the magnetic element, one being displaced in one direction under the action of the magnetizing force and the other in the opposite direction.

In applying Coulomb's law to calculate the interactions between these magnetic elements, Poisson assumed that the force of repulsion exerted by a quantity, q_1, of magnetic fluid upon a quantity, q_2, of the same kind situated a distance r from it, is proportional to

$$\frac{q_1 q_2}{r^2}$$

and is independent of the substance of which the magnetized body is composed.

Using this conception of the phenomenon of magnetization Poisson solved the problem of calculating the magnitude and direction of the resultant force exerted by a magnetized body of any shape upon a unit magnetic pole situated at any point outside the body. He exhibited this force as the negative gradient of a function V, which may be expressed as follows:

$$V = \int_S \frac{1}{r} (\mathbf{I} \cdot \mathbf{n})\, dS - \int_\tau \frac{1}{r} (\text{div } \mathbf{I})\, d\tau,$$

where **n** is a unit normal to an element dS of the surface S bounding a magnetized body of volume τ. I is a vector such that if $\delta\tau$ be any physically small element of volume within the body, then $I\delta\tau$ will be the magnetic moment of that element of volume. It is therefore the "intensity of magnetization" of the substance at a point within $\delta\tau$. The form of the function V shows that the magnetic effect of any magnetized body is the same as that which would be produced by a layer of magnetic fluid of density $\mathbf{I} \cdot \mathbf{n}$ over its surface, together with a distribution of density—div I throughout its volume. These are called "Poisson's equivalent surface and volume distributions of magnetism."

It is evident that for points inside the magnetized body r^{-1} will become infinite for an element of the second integral in Poisson's expression. This difficulty may be removed if we consider the point situated inside a cavity in the medium, small in dimensions, yet very large compared with the dimensions of the elementary magnets themselves.[1] A part of the surface density $I \cdot n$ will be on the wall of the cavity and this part will give rise to a finite force at the point inside it, whose value will depend on the form of the cavity and on the magnetic polarization at the place. If we omit this purely local part of the magnetic force in the cavity, the remaining part, which is that due to the polarized mass as a whole, will be derived from the general volume density div I and surface density $I \cdot n$ just as at an outside point. This latter part arising from the system as a whole, omitting the local term depending on the molecular structure at the point considered, is thus quite definite, and is named the magnetic force H. In this way we arrive at a definition of the magnetic force within a magnetized medium which is consistent with the way it is defined for points external to the substance.

Though the hypotheses regarding the nature of the magnetic element which Poisson adopted have not proved to be correct, the formulæ of magnetostatics which he developed remain valid and useful since they rest upon the experimental fact of induced magnetization and not upon the nature of the mechanism by which this is brought about.

The mathematical labor of developing a complete theory of magnetic induction founded solely upon experimental data was later undertaken by Lord Kelvin. In addition to freeing Poisson's theory from the hypothesis of two magnetic fluids, Kelvin greatly enriched it and simplified the conceptions involved by introducing the terminology which is used today.[2] One such extension in particular should be mentioned here.

Poisson had pointed out that in general the intensity of magnetization in a homogeneous body is a linear vector function of the field intensity, so that in general the specification of I in terms of H would require nine constants depending upon the nature of the substance. If the medium be isotropic as well, these nine constants reduce to one, so that for this case:

$$I = kH.$$

The subsequent researches of Faraday, Plücker and Tyndall having revealed the fact that crystals possess different magnetic properties in different directions, Kelvin extended the theory to a treatment of the

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[1] cf. Larmor, "Aether and Matter," p. 257.

[2] Kelvin, "Reprint of Papers on Electrostatics and Magnetism," XXIV.

problem of magnetic induction in non-isotropic media. He showed[1] that for such media the nine constants introduced by Poission reduce to three, so that:

$$\mathbf{I}=\Phi\cdot\mathbf{H}$$

in which the linear vector function Φ is self-conjugate.

As it has been handed down to us by Poisson, Green, and Kelvin, the mathematical theory of magnetic induction may be regarded as complete. It is inadequate to meet the demands of the modern viewpoint because it is essentially a statistical theory. It deals with the phenomena exhibited by matter in bulk, without attempting to account for the ultimate causes of these phenomena. Just as Thermodynamics preceded Kinetic Theory, so the theory of magnetostatics has preceded a study of the dynamics of the ultimate magnetic particle.

AMPERE'S THEORY OF MAGNETISM

In July 1820 Oersted announced the discovery that a magnetic needle placed near a conductor carrying an electric current tends to assume a position at right angles to the conductor. This discovery inspired Ampere to undertake a series of researches on the relation between current electricity and magnetism which extended over a period of three years and resulted in the publication in 1825 of a memoir on the mathematical theory of electrodynamical phenomena which has been characterized by Maxwell as, "one of the most brilliant achievements of science."[2]

Ampere based his theory of magnetism upon the identity which he established between the magnetic properties of Poisson's "two fluid" magnetic element and a solenoid of molecular dimensions in which an electric current is continually flowing. According to Ampere the molecules of a magnetic substance are perfect conductors about which or within which are flowing perpetually minute currents of electricity.

The process of magnetization consists in changing the orientation of these molecular currents either by changing the plane of the current relative to the molecule, or by turning the molecule as a whole, so that their axes, initially pointing in all directions at random, will tend to align themselves parallel to the magnetizing field. Ampere showed that this sort of magnetic element would suffice to explain not only the phenomena of magnetostatics in accordance with the formulæ deduced by Poisson, but also the laws expressing the mutual actions of magnets and conductors carrying currents, which had been discovered by Biot, Arago, and himself.

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[1] ibid. XXX.

[2] Ampere, "Mémoires de l'Institut." VI (1823), p. 175.

At the time at which Ampere wrote, electromagnetic current induction had not yet been discovered, and therefore he was able to make no hypothesis as to the origin and strength of the molecular currents. Ampere's great contribution to the science of magnetism consisted in showing that all the then known interactions between magnets, and between these and electrical conductors, could be reduced to a single cause.

THE MAGNETIC RESEARCHES OF FARADAY

The fact that an electric current is invariably accompanied by a magnetic field led Faraday to search for a converse effect.[1]

In a paper read before the Royal Society in 1831 he described a series of experiments in which the phenomenon of electromagnetic current induction was discovered. The establishment of this reciprocal relationship between magnetism and current electricity afforded added support to the molecular current hypothesis of Ampere as against the two fluid theory of Poisson. Another discovery by Faraday, however, sufficed to clinch the argument in favor of Ampere's theory and to demonstrate that of Poisson to be untenable.

In 1845 while investigating the rotation of the plane of polarization of a beam of light traversing a piece of glass placed in a strong magnetic field, Faraday observed that the glass itself possessed magnetic properties opposite to those of iron and other "magnetic" metals.[2] While a piece of iron would tend to set itself with its greatest length parallel to the field, the glass, if left free to turn, placed itself across the field. Faraday gave the name "diamagnetism" to this new phenomenon, and proceeded to make a thorough examination of the magnetic properties of a vast number of substances, solids, liquids and gases. He definitely established the fact that all substances possess either the diamagnetic or the magnetic property. He even concluded that, "If a man could be suspended, with sufficient delicacy, and placed in the magnetic field, he would point equatorially, for all the substances of which he is formed, including the blood, possess this property."

In accordance with the two fluid theory of magnetism, the elementary magnets of all substances would, when placed in a magnetic field, be polarized in the same direction. Faraday showed, however, that the direction of polarization of diamagnetic bodies in a magnetic field is opposite to that of magnetic bodies in the same field. The two fluid hypothesis, therefore, fails in this respect to account for the facts.

Adopting Ampere's theory, a substance whose molecules were themselves elementary magnets due to the existence of permanent electric

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[1] Faraday, Experimental Researches, I, p. 2. The diamagnetic property of Bismuth had previously been observed by Brugmans.

[2] Faraday, op. cit., III, p. 27.

currents flowing about them, would be magnetic. On the other hand, if no such currents existed initially, then the action of an applied magnetic field might induce such molecular currents, and these, by Faraday's law of current induction, would polarize the molecule magnetically in opposition to the external field: that is, the substance would be diamagnetic.

In 1852 Wilhelm Weber, adopting Ampere's hypothesis and the results of Faraday's researches, developed mathematically a theory which it will be profitable to outline here in some detail, for it laid the foundation for certain of the modern theories of magnetism.

WEBER'S THEORY OF MAGNETISM

Weber starts by assuming that the molecules of a magnetic substance are small permanent magnets whose axes initially point in all directions at random.[1] Let NM (Fig. 1) be such a magnet, which is capable of turning about its center C, under the action of an external field **H**. If the molecule were perfectly free to rotate then the body would be magnetized to saturation by any applied field, however small. This Weber knew was not the case, and he therefore assumed a constraint upon the rotation of the elementary magnets in the form of a molecular magnetic field, **D**, whose direction for each molecule coincides with the initial equilibrium position of its axis, and whose magnitude is constant throughout the body.

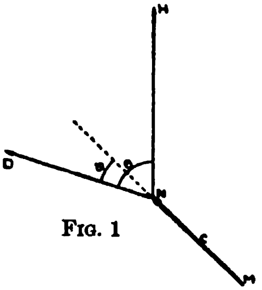

FIG. 1

The magnet will be in equilibrium under the action of the two fields when

$$\tan\phi = \frac{\mathbf{H}\sin\theta}{\mathbf{D} + \mathbf{H}\cos\theta}. \tag{1}$$

If μ denote the magnetic moment of the molecule, its component parallel to **H** is, before the application of the field,

$$\mu\cos\theta$$

which, upon the establishment of the field, becomes

$$\mu\cos(\theta - \phi).$$

Hence the increase in the magnetic moment parallel to **H**, say μ_{H}, due to the presence of the external field is given by:

$$\mu_{\text{H}} = \mu\left\{\cos(\theta - \phi) - \cos\theta\right\}. \tag{2}$$

---

[1] W. Weber, "Über den Zusammenhang der Lehre vom Diamagnetismus mit der Lehre von dem Magnetismus und der Elektricität," *Pogg. Ann.* 87 (1854), p. 145.

Eliminating ϕ between equation (1) and (2) we have for a single molecule:

$$\mu_H = \mu\left\{\frac{H + D\cos\theta}{\sqrt{H^2 + D^2 + 2\,H\,D\cos\theta}} - \cos\theta\right\}$$

This expression must now be summed for all the molecules under consideration. Let there be n molecules per unit volume. Assuming initially a random distribution of the axes in space, the fraction of the molecules whose axes make an angle less than θ with H will evidently be the ratio of the area of the zone cut from a sphere by a cone of semi-angle θ, to the area of the sphere, that is $\frac{1}{2}(1 - \cos\theta)$. The number of molecules whose axes make angles with H lying between θ and $\theta + d\theta$ is, therefore,

$$\frac{n}{2}\sin\theta\, d\theta.$$

The net increase in the magnetic moment per unit volume due to the rotation of all the elementary magnets is, then, given by:

$$I = \frac{n}{2}\int_0^\pi \mu_H \sin\theta\, d\theta.$$

If $H < D$ this integral has the value $I = \frac{2}{3}\mu\, n\frac{H}{D}$.

If $H = D$ " " " " " $I = \frac{2}{3}\mu\, n$.

If $H > D$ " " " " " $I = \mu\, n\left(1 - \frac{1}{3}\frac{D^2}{H^2}\right)$.

If $H = \infty$ " " " " " $I = \mu\, n$.

An examination of these formulæ shows that the intensity of magnetization should increase proportionally to the impressed field until it has reached ⅔ of its maximum value, after which it should approach the latter asymptotically. Weber obtained experimental results for iron in close agreement with this conclusion. His theory, however, is unable to account for residual magnetism, and more accurate experiments have shown that the initial variation of intensity of magnetization with field strength is not linear.

Before proceeding to a discussion of the various modifications which have been suggested to resolve these discrepancies, we will review briefly Weber's theory of diamagnetism.

According to Weber's theory, there exist in the molecules of a diamagnetic substance closed channels in which electricity can flow without resistance. If a magnetic field is established through one of these channels an electric current will be set in motion in it. The magnetic field of this induced current will be opposed to the external field. In the mathematical development of his theory Weber made use of electrodynamical formulæ derived from assumptions regarding the nature of current electricity which have since been abandoned. It will therefore be more profitable to examine the theory in the form in which it was afterwards interpreted by Maxwell.[1]

If L is the coefficient of self induction of a molecular circuit, and M is the coefficient of mutual induction between this circuit and some other circuit, and if, furthermore, i is the current in the molecular circuit, and i′ that in the other circuit, then:

$$\frac{d}{dt}(Li + Mi') = -Ri.$$

But by hypothesis R = O, and we get by integration:

$$Li + Mi' = Li_o,$$

where i_o is thus the initial value of the molecular current.

If the current i′ produces a magnetic field of strength **H** which makes an angle θ with the normal to the plane of the molecular current, then:

$$Mi' = HA \cos \theta,$$

where A is the area of the molecular circuit. Hence:

$$Li + HA \cos \theta = Li_o.$$

Diamagnetic substances differ from magnetic in that in the former there are no permanent molecular currents. Hence for diamagnetic substances $i_o = O$, and we have for the value of the induced current:

$$i = -\frac{HA}{L} \cos \theta.$$

The magnetic moment, μ, of this current is expressed by:

$$\mu = iA = -\frac{HA^2}{L} \cos \theta;$$

and the component of this parallel to **H** by

$$\mu \cos \theta = -\frac{HA^2}{L} \cos^2 \theta.$$

If there are n such molecular currents per unit volume with their axes distributed at random, the number of axes lying between θ and $\theta + d\theta$ will be, as before, $\frac{n}{2} \sin \theta \, d\theta$.

Hence the resultant magnetization per unit volume will be given by:

$$I = \int_0^\pi -\frac{n\,HA^2}{2\,L} \cos^2 \theta \sin \theta \, d\theta$$
$$= -\frac{1}{3}\frac{n\,\mathbf{H}\,A^2}{L},$$

---

[1] Maxwell, Treatise II, §838.

and the diamagnetic susceptibility per unit volume becomes:

$$k = -\frac{1}{3}\frac{n A^2}{L}.$$

It is evident that Weber's theory of diamagnetism offers a satisfactory fundamental explanation of the phenomenon provided that his assumption of the existence of perfectly conducting channels about the molecules be granted. This assumption, however, did not appeal strongly to his contemporaries, as is evident from a remark by Tyndall in the Bakerian Lecture for 1855 that, "This theory, notwithstanding its great beauty, is so extremely artificial, that I imagine the general conviction of its truth cannot be very strong."

The discovery of the electron furnished an adequate mechanism for the verification of Weber's hypothesis, and some of the more recent attempts to explain diamagnetism are nothing more than efforts to fit this mechanism into the fundamental theory which Weber established.

MAXWELL'S MODIFICATION OF WEBER'S THEORY

It has been noted that Weber's theory fails to account for residual magnetism. Maxwell introduced a new assumption designed to remove this deficiency by providing for a permanent alteration in the position of equilibrium of a molecular magnet.[1] He supposed that if the deflection of the magnetic axis of a molecule under the action of a magnetizing field is less than some fixed value β_0, then it will return to its original position on the removal of the deflecting force. If, however, the deflection, β, is greater than β_0, then, when the external field is removed the magnetic axis of the molecule will not return to its initial position but will remain permanently deflected through an angle β-β_0. Incorporating this hypothesis into Weber's theory leaving the remainder of it unchanged, Maxwell obtained theoretical magnetization curves which exhibit the phenomenon of retentivity. But while the main hysteresis loop of a ferromagnetic substance may be roughly accounted for in this way, the modified theory fails to explain the smaller loops which may be superimposed on this by only partially removing the magnetizing field and then reapplying it. Furthermore, a physical justification for the assumption of the critical angle β_0 as well as for the controlling field D of Weber's theory seems to be lacking.

Maxwell made a further extension of Weber's theory by investigating the diamagnetic effect which is, on the hypothesis of molecular currents, sure to be present in all magnetic substances.

In the molecules of such substances the primitive current, i_0, will be diminished by the action of the applied field so that we have, in accordance with the analysis of the previous section,

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[1] Maxwell, op. cit., §444.

$$i = i_0 - \frac{H\,A}{L} \cos\theta.$$

The magnetic moment of the molecule is given by:

$$\mu = iA = i_0A - \frac{HA^2}{L} \cos\theta,$$

and its component parallel to **H** by:

$$\mu \cos\theta = i_0A \cos\theta - \frac{HA^2}{L} \cos^2\theta$$

$$= i_0A \cos\theta \left(1 - \frac{HA^2}{Li_0} \cos\theta\right)$$

If $\frac{HA}{Li_0}$ is small compared with unity, $\mu = i_0A$, and we return to Weber's theory of magnetism. If $\frac{HA}{Li_0}$ is large compared with unity, then $\mu = -\frac{HA^2}{L}\cos^2\theta$, and Weber's theory of diamagnetism follows. It is evident that the greater the value of i_0, the primitive value of the molecular current, the smaller will be the diamagnetic effect. Moreover, a large value of L will bring about the same result. In any event, it follows that the intensity of magnetization should diminish if the impressed field be made sufficiently great. Such an effect has not been observed, but it is evident that it will be very small and the experimental difficulties which must be overcome in order to detect it correspondingly great.

EWING'S THEORY OF RESIDUAL MAGNETISM AND HYSTERESIS

The accurate and extensive researches of H. A. Rowland[1] and others definitely established the inadequacy of existing theories to explain hysteretic phenomena in iron and other ferromagnetic substances. In attacking the problem Ewing discarded the arbitrary postulates regarding the controlling field and angle of permanent set, and endeavored to account for the magnetic behavior of these substances by investigating the effect of the constraint which the molecules exert upon one another by reason of the fact that they are magnets.[2]

Consider, for simplicity, a group consisting of two equivalent molecular magnets, free to rotate about fixed centers. (Fig. 2) In the absence of any disturbing force the two molecules will arrange themselves with their magnetic axes coincident with the line joining their centers. If an external field, **H**, be applied which makes an angle θ with this line the two magnets will each be deflected through an angle ϕ, seeking

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[1] *Phil. Mag.* **46** (1873), p. 140. **48** (1874), p. 321.

[2] Ewing, "Magnetic Induction in Iron and other Metals," p. 287.

a new position of equilibrium for which, evidently,

$$2\,\mathrm{m}\,\mathbf{H}\,\mathrm{r}\sin(\theta-\phi) = \mathrm{m}^2\,\overline{\mathrm{CN}}/\overline{\mathrm{PQ}}^2,$$

where m is the pole strength of the magnets, and 2r is their length.

This position of the molecules corresponds to the initial stage of the magnetization in which there is a small increase in induced magnetism with increasing external field.

When:

$$\frac{d}{d\theta}\left\{2\,\mathbf{H}\,\mathrm{m}\,\mathrm{r}\sin(\theta-\phi)\right\} = \frac{d}{d\theta}\left\{\frac{\mathrm{m}^2\,\overline{\mathrm{CN}}}{\overline{\mathrm{PQ}}^2}\right\}$$

the equilibrium becomes neutral and any further increase in **H** will result in instability. The magnets will then swing violently toward a new position of equilibrium with their axes nearly parallel to **H**. This sudden shift corresponds to the second stage in the magnetization in which a large increase in magnetic moment accompanies a small increase in the magnetizing field.

Any further increase in **H** will not appreciably alter the positions of the molecules and we have the condition of approximate saturation.

It remains only to note that if **H** is now decreased the magnets will not retrace the same path in returning to their original positions. The deflection accompanying a small decrease in **H** will be small until a second state of instability is reached, when they will swing back into positions approximating the initial ones.

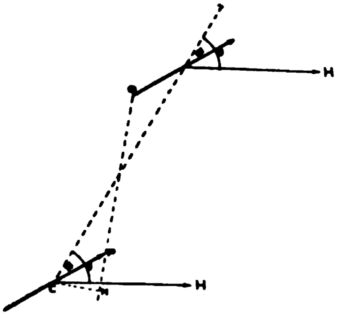

FIG. 2

A single pair of magnets of this sort would give a discontinuous hysteresis loop. If, however, we imagine a large number of such elements with their axes initially distributed at random it is evident that some of these will reach the position of instability earlier than others, and the "magnetization curve" of the aggregate will be a smooth one. "Hysteresis loops" have been obtained experimentally with a group of only twenty-four magnets, which are in perfect qualitative agreement with those observed for iron.

The theoretical retentivity of a substance may be obtained by assuming it to be composed of a large number of groups, with the molecules of each group arranged in some sort of symmetry. This is in agreement with the fact that iron and other magnetic metals are known to be composed of minute crystal matrices of the cubic system irregularly oriented throughout their mass.

It is characteristic of such a cubical formation that the permanent deflection of the molecules must necessarily be either 0°, 90°, or 180°. Referring to Fig. 2, it is clear that if ϕ be the angle of permanent deflection, we have three cases to consider:

(1) Molecules for which θ is less than 45°. These will suffer no permanent deflection. This is because the original lines are more favorably directed than lines at right angles to them. For these molecules $\phi = \theta$.

(2) Molecules for which θ is greater than 45°, and less than 135°. These will be permanently turned through one right angle. In this case $\phi = \theta - 90°$.

(3) Molecules for which θ is greater than 135°. For these molecules $\phi = \theta - 180°$.

If the axes of the molecules are initially directed at random, we have, as before, for the number of molecules whose axes lie between θ and $\theta + \mathrm{d}\,\theta$,

$$\frac{n}{2}\sin\theta\,d\theta,$$

and if the magnetic moment of each molecule is μ, the contribution of these molecules to the net intensity of magnetization will be

$$\frac{\mu n}{2}\sin\theta\cos\theta\,d\theta.$$

Hence the whole residual magnetism will be given by:

$$I = \frac{\mu n}{2}\int_0^{\frac{\pi}{4}}\sin\theta\cos d\theta + \frac{\mu n}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\sin^2\theta\,d\theta$$
$$+\frac{\mu n}{2}\int_{\frac{3\pi}{4}}^{\pi}\sin\theta\cos(\theta-180°)\,d\theta = 0.8927\,\mu n.$$

More recent researches seem to indicate that the behavior of the magnetic elements in crystals is not as simple as Ewing's theory would lead us to believe. The theory is, however, a step in the right direction, for it attacks the problem which is fundamental in the explanation of ferromagnetism, namely, the evaluation of the mutual actions of the elementary magnetic units.

In the preceding review we have not considered the various theories of magnetostriction which belong to the period under consideration. A discussion of these theories will be found in a later section of this report[1].

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[1] P. 225.

PROGRESS IN THE DEVELOPMENT OF THEORIES OF PARA- AND OF DIAMAGNETISM FROM 1900 TO 1920

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CONTENTS

INTRODUCTION

The development of theories of magnetism during the period which the present survey attempts to cover is characterized by successive efforts on the part of theorists to explain magnetic phenomena in terms of the properties of electrons in motion.

Early in the period under review it was found that the assumption of motions of electrons in independent closed orbits in a material body was incompetent to produce a satisfactory explanation of magnetisation in the body. Some type of sub-molar structure of electrons was found to be needed. For convenience we shall designate such a structure a "magneton." The electron theories of magnetism to be reviewed are naturally differentiated through the more or less arbitrary structural properties assumed for the magneton.

Any molecular theory of magnetism is, of course, essentially statistical in character and therefore continually faced with the well known difficulties of statistical mechanics. These difficulties assume rather formidable proportions in a theory which claims a generality sufficient to account for magnetic susceptibilities observed at low temperatures. For it then appears that the theory has to part company with the law of equipartition of energy of classical mechanics and introduce in its place a law of distribution of energy among the magnetons of a body, depending upon some more or less plausible quantum hypothesis.

A primary object of all magnetic investigations on material bodies is, of course, to find out as much as possible concerning the nature of the magneton. So far as we know it cannot be segregated and examined; and our empirical knowledge of the magnetic properties of a material body is of necessity derived from an experimental study of its magnetic quality in bulk. It is the bulk susceptibility which is experimentally determined. This is a statistical quantity, representing the contributions of the statistical units, the magnetons, to the magnetisation of the body in bulk. In the consideration of any molecular theory of magnetism it is therefore necessary to bear in mind that the theory may well stand the test of experiment, and yet the model of the magneton which it assumes be far from a true one, since different types of magnetons might have the same statistical properties.

As far as fundamental physical ideas are concerned the reader of the following report will probably conclude that the interpretation and the extension of old conceptions, those of Ampere and of Weber, rather than the introduction of new ones, save those relating to quantum theories, characterize in general the developments in molecular theories of para- and of diamagnetism during the years from 1900 to 1920.

The development of electron theories of magnetism which began early in the period covered by the present report was stimulated in large measure by the theoretical writings of Professor H. A. Lorentz and of Sir Joseph Larmor. Their results constitute a large part of what is now termed classical electron theory with which the reader is supposed to have some acquaintance.

Kinetic theories of magnetism are of necessity somewhat mathematical in character and the pages of the literature dealing with them are often encumbered with many rather formidable appearing formulæ, which, while oftentimes necessary, operate as a deterrent to the average reader, who is more interested in the physical content of a theory than in the mathematical dress in which it is clothed.

With the object of divesting, so far as possible, the various theories discussed below of the mathematical features which are shared by many of them in common the first three sections have been written. These sections are intended more for reference during the reading of the rest of the report than for continued perusal. The reader who so desires may therefore begin with Section IV, dealing with early attempts at electron theories of magnetism.

For the purposes of the present review it has been found convenient to use a vector notation. That of Gibbs has been adopted.

Vector quantities are printed in the heavy Bookman type—**A**, **B**, **a**, **b** . . and the corresponding scalar values in ordinary type—A, B, a, b . . .

The reader who is unfamiliar with Vector Analysis and who desires to follow those parts of the argument in the text in which vector methods are used will find "Vector Analysis" by J. G. Coffin a very convenient book for reference.

As regards units, for electric and magnetic quantities the Gaussian system is used throughout. For other quantities c. g. s. absolute units are always used. To denote the velocity of light the letter c is used.

I

THE ELECTRIC AND THE MAGNETIC FIELD DUE TO A MOVING ELECTRON

The explanations of magnetisation on the theories of magnetism which we shall notice later are referred back to the electric and magnetic properties of free electrons in motion, or to the corresponding properties of some sort of rotating magneton.

In the present section we shall therefore consider the electric and the magnetic field of a moving electron; and in Section II we shall consider the electric and magnetic properties of rotating magnetons, and also the mechanical moments to which they are subject when placed in an external electric or magnetic field.

The electron, considered as a point charge, will at first be considered to be moving in any arbitrarily assigned manner. The electric and the magnetic force due to the moving electron may be calculated for any field point directly from its retarded scalar- and vector potentials.

Referring to Fig. 1, O represents the origin of a cartesian system of axes fixed in space; Q the position of the electron at the instant under consideration; s the position vector of Q with reference to O; P the field point; r the position vector of P with reference to O; and q a vector drawn from Q to P.

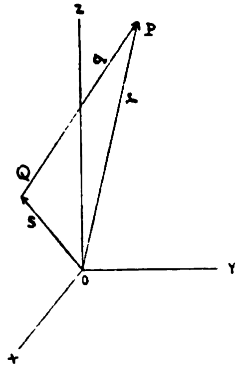

FIG. 1

The cartesian coordinates of Q and P are represented respectively by ξ, η, ζ and x, y, z. From the figure:

$$(1) \qquad q = \sqrt{(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2}.$$

If e be the charge on the electron and v its velocity, then by classical electron theory the scalar- and the vector potentials at the field point P are respectively expressed by:

$$(2) \qquad \Phi = \left[\frac{e}{q\left(1 - \frac{\mathbf{v}\cdot\mathbf{q}}{c\,q}\right)}\right]_{t-\frac{q}{c}} ; \qquad \mathbf{A} = \left[\frac{e\mathbf{v}}{c\,q\left(1 - \frac{\mathbf{v}\cdot\mathbf{q}}{c\,q}\right)}\right]_{t-\frac{q}{c}},$$

where the quantities in square brackets are to be evaluated not at the time at which the electric and the magnetic forces are required but at a time previous by the interval required for radiation to travel from the point Q to the point P, that is at a time t − q/c.

The potentials having been evaluated the electric and the magnetic force at the field point will be given respectively by:

$$\text{(3)} \qquad \mathbf{E} = -\nabla\Phi - \frac{1}{c}\frac{d\mathbf{A}}{dt} \quad ; \quad \mathbf{H} = \text{curl } \mathbf{A}.$$

Upon carrying out the operations here indicated the following expressions for the electric and the magnetic force at the field point P are found:

$$\text{(4)} \quad \mathbf{E} = \frac{e}{c^2}\left[\frac{\dot{\mathbf{v}}}{q}\left(1 - \frac{\mathbf{v}.\mathbf{q}}{cq}\right)^{-2} + \frac{\left(\frac{\mathbf{q}}{q} - \frac{\mathbf{v}}{c}\right)\left(c^2 - v^2 + \dot{\mathbf{v}}.\mathbf{q}\right)}{q^2}\left(1 - \frac{\mathbf{v}.\mathbf{q}}{q}\right)^{-3}\right],$$

$$\text{(5)} \quad \mathbf{H} = \frac{e}{c^2}\left[\frac{\dot{\mathbf{v}}\times\mathbf{q}}{q^2}\left(1 - \frac{\mathbf{v}.\mathbf{q}}{c\,q}\right)^{-2} + \frac{\mathbf{v}\times\mathbf{q}\,(c^2 - v^2 + \dot{\mathbf{v}}.\mathbf{q})}{c\,q^3}\left(1 - \frac{\mathbf{v}.\mathbf{q}}{c\,q}\right)^{-3}\right].$$

The details of the calculation are somewhat involved and may be found in standard treatises dealing with electron theory, e. g., in *The Theory of Electricity* by G. H. Livens, p. 506.

For the cases which will come under our consideration the velocity of the electron may be considered small in comparison with that of light, and the field point may be chosen so that its distance from the electron is small in comparison with the wave length of the radiation emitted by the electron. The general expressions for the scalar and the vector potential given by (2) then reduce to the simple approximate expressions:

$$\text{(6)} \qquad \Phi = \frac{e}{q}, \qquad\qquad \mathbf{A} = \frac{e\mathbf{v}}{cq}.$$

It may be noticed that here the potentials are not retarded.

The corresponding expressions for the electric and the magnetic force due to a moving electron may be obtained directly from (6) by taking the negative gradient of Φ and the curl of **A**. It is thus found that:

$$\text{(7)} \qquad \mathbf{E} = \frac{e}{q^3}\,\mathbf{q},$$

$$\text{(8)} \qquad \mathbf{H} = \frac{e}{cq^3}\,\mathbf{v}\times\mathbf{q}.$$

These approximate equations might have been obtained, of course, from the general expressions (4) and (5) by introducing the restrictions above made.

If the origin O be so chosen that s is small in comparison with r, the quantity $1/q$ in the expressions for the potentials may be developed in a series in which only the first three terms need be retained:

$$\frac{1}{q} = \frac{1}{r}\left\{1 + \frac{\mathbf{r}.\mathbf{s}}{r^2} + \frac{3}{2}\left(\frac{\mathbf{r}.\mathbf{s}}{r^2}\right)^2\right\}.$$

Inserting this expression for $1/q$ in (6) we find:

$$\Phi = \frac{e}{r}\left\{1 + \frac{\mathbf{r}.\mathbf{s}}{r^2} + \frac{3}{2}\left(\frac{\mathbf{r}.\mathbf{s}}{r^2}\right)^2\right\}, \tag{9}$$

$$\mathbf{A} = \frac{e}{cr}\mathbf{v}\Big\{\Big(1 + \frac{\mathbf{r}.\mathbf{s}}{r^2} + \frac{3}{2}\left(\frac{\mathbf{r}.\mathbf{s}}{r^2}\right)^2\Big\}.$$

Taking the negative gradient of Φ and the curl of $\mathbf{A}$ we now obtain the following expressions for the electric force and the magnetic force at the field point:

$$\mathbf{E} = \frac{e}{r^3}\Big\{\Big(1 + \frac{3\mathbf{r}.\mathbf{s}}{r^2}\Big)(\mathbf{r} - \mathbf{s}),\Big\}, \tag{10}$$

$$\mathbf{H} = \frac{e}{cr^3}\mathbf{v}\times\Big\{\Big(1 + \frac{3\mathbf{r}\cdot\mathbf{s}}{r^2}\Big)(\mathbf{r} - \mathbf{s})\Big\}. \tag{11}$$

The mean value of $\mathbf{H}$ for an electron describing a circular orbit with constant speed will later be required. If $\overline{\mathbf{H}}$ denote the mean value of $\mathbf{H}$ for this case, it is easily found from (11), upon observing that $\mathbf{v} = \dot{\mathbf{s}}$, that:

$$\overline{\mathbf{H}} = \frac{e}{2cr^3}\Big(3\,\mathbf{s}\times\dot{\mathbf{s}}\cdot\frac{\mathbf{r}}{r^2}\mathbf{r} - \mathbf{s}\times\dot{\mathbf{s}}\Big). \tag{12}$$

Thus, an electron describing a circular orbit with constant speed is, as far as its mean magnetic field is concerned, equivalent to a small magnet whose moment, $\boldsymbol{\mu}$, is given by:

$$\boldsymbol{\mu} = \frac{e}{2c}\mathbf{s}\times\dot{\mathbf{s}}. \tag{13}$$

This expression can be put in a somewhat simpler form as follows. Let $\boldsymbol{\omega}$ be the angular velocity of the electron about the center of its orbit, then $\dot{\mathbf{s}} = \boldsymbol{\omega}\times\mathbf{s} = \omega\,\mathbf{n}\times\mathbf{s}$, if $\mathbf{n}$ be a unit normal to the plane of the orbit in the direction of $\boldsymbol{\omega}$. We now have:

$$\mathbf{s}\times\dot{\mathbf{s}} = \omega\,\mathbf{s}\times(\mathbf{n}\times\mathbf{s}) = \omega\,\mathbf{s}\cdot\mathbf{s}\,\mathbf{n} = \frac{2}{\tau}S\,\mathbf{n},$$

where τ is the orbital period, and S the orbital area. Then, from (13):

$$\boldsymbol{\mu} = \frac{eS}{c\tau}\mathbf{n}. \tag{14}$$

The moment of the orbit will be subject to change if a magnetic field be created through it. Let $\mathbf{H}$ be the strength of the magnetic field at any instant and $\mathbf{E}$ the corresponding electric force. Supposing the area of the orbit, S, to be very small and its plane invariable, by making use of Stokes' theorem and Maxwell's field equation, curl $\mathbf{E} = -\,d\,\mathbf{H}/c\,d\,t$, we obtain:

$$\int_0^{2\pi s} \mathbf{E}\cdot d\,s = \int_S \mathbf{n}\cdot \operatorname{curl}\mathbf{E}\,d\,S = -\frac{d}{cdt}\int_S \mathbf{n}\cdot\mathbf{H}\,d\,S = -\frac{d}{cdt}(\mathbf{n}\cdot\mathbf{H}\,S).$$

Upon integration the integral on the left gives 2π s E, and hence:

$$2\pi s E = -\frac{d}{cdt}(\mathbf{n}\cdot\mathbf{H}\,S);$$

the expression on the right representing the time rate of decrease of the magnetic flux through the orbit. If $\Delta\,(\mathbf{n}\cdot\mathbf{H}\,S)$ denote the increment of this flux in the orbital time τ, then:

$$(15) \qquad 2\pi s E = -\frac{\Delta\,(\mathbf{n}\cdot\mathbf{H}\,S).}{c\tau}$$

Again, since the moment of the force e $\mathbf{E}$ must equal the time rate of increase of the moment of momentum of the electron in its orbit, we have:

$$s\,e\,E = \frac{d}{d\,t}(m\,s^2\,\omega) = \frac{2\,m\,c}{e}\frac{d\,\mu}{d\,t};$$

consequently, if $\Delta\,\mu$ denote the increment in μ in the orbital time, τ:

$$(16) \qquad s\,e\,E = \frac{2\,m\,c}{e}\,\frac{\Delta\mu}{\tau}.$$

From (15) and (16) it follows that:

$$(17) \qquad \Delta\,\mu = -\frac{e^2}{4\pi m c^2}\,\Delta\,(\mathbf{n}\cdot\mathbf{H}\,S) = -\frac{e^2}{4\pi m c^2}\,\Delta\,(H\,S\cos\theta),$$

where θ is the angle between the directions of $\mathbf{n}$ and $\mathbf{H}$.

Mechanical Action upon a Moving Electron in an External Electromagnetic Field.

If $\mathbf{E}$ and $\mathbf{H}$ now denote respectively the strength of the external electric and magnetic field, from fundamental electron theory we have for the mechanical force, $\mathbf{F}$, upon any electron:

$$(18) \qquad \mathbf{F} = e\mathbf{E} + \frac{e}{c}\,\mathbf{v}\times\mathbf{H};$$

and for the mechanical moment, $\mathbf{N}$, of this force about the origin O,

$$(19) \qquad \mathbf{N} = e\mathbf{s}\times(\mathbf{E}\times\frac{1}{c}\,\mathbf{v}\times\mathbf{H}).$$

II

THE MAGNETON

Since, as mentioned above, the assumption of motions of free electrons in independent orbits is incompetent to lead to a satisfactory explanation of magnetisation, the concept of the magneton made an early appearance in modern theories of magnetism.

The magneton is conceived to be a minute aggregate of positive and negative electrons, possessing certain arbitrarily assigned constitutional or structural properties. We first consider these properties.

Fundamental Assumptions Concerning the Structural Properties of the Magneton.

The algebraic sum of the charges of the electrons in a magneton are assumed to be zero. If the charge on a typical electron be e the structural condition implied by this assumption is expressed by writing:

$$\Sigma e = 0. \tag{1}$$

The distribution of the electrons in the magneton is supposed to be such that the electric moment of the magneton is zero. We now suppose that the typical electron of the magneton is the electron of Section I, and that the origin O coincides with the centroid of the magneton. Then (see Fig. 1, Sect. I) the condition that the electric moment of the magneton shall be zero is expressed by:

$$\Sigma \mathbf{e}\mathbf{s} = \mathbf{i}\Sigma e\xi + \mathbf{j}\Sigma e\eta + \mathbf{k}\Sigma e\zeta = 0, \tag{2}$$

where **i**, **j**, **k** are unit vectors in the directions of the axes (x, y, z) respectively.

It will appear presently that the electric and magnetic properties of the magneton depend in an important way upon the following quantities of the second degree in ξ, η, ζ:

$$P_1 = \Sigma e\xi^2, \quad P_2 = \Sigma e\eta^2, \quad P_3 = \Sigma e\zeta^2, \tag{3}$$

$$D_1 = \Sigma e\eta\zeta, \quad D_2 = \Sigma\zeta\xi, \quad D_3 = \Sigma\xi\eta, \tag{4}$$

$$Q_1 = P_2 + P_3, \quad Q_2 = P_3 + P_1, \quad Q_3 = P_1 + P_2. \tag{5}$$

From the analogy of these quantities with corresponding quantities in mechanics it is appropriate to call the Q's and D's respectively Moments of Inertia of Charge and Products of Inertia of Charge.

The Electric Potential and the Magnetic Potential for a Rotating Magneton.

In the applications of the present theory with which we shall be concerned in our review of theories of magnetism the velocity of any electron will be small in comparison with that of light, and the distance of the field point from the magneton will be small in comparison with the wave length of the radiation emitted by it and yet large in comparison with the dimensions of the magneton.

The appropriate equations for the potentials will therefore be furnished by the equations (9), Sect. I, (for the potentials of a single electron) through summation over all the electrons in the magneton, the origin O being supposed at the center of the magneton. We thus obtain for the electric and the magnetic potential of the magneton respectively:

$$\Phi = \Sigma \frac{e}{r}\left\{1 + \frac{\mathbf{r}.\mathbf{s}}{r^2} + \frac{3}{2}\left(\frac{\mathbf{r}.\mathbf{s}}{r^2}\right)^2\right\}; \tag{6}$$

$$\mathbf{A} = \frac{1}{c}\Sigma \frac{e}{r}\,\mathbf{v}\left\{1 + \frac{\mathbf{r}.\mathbf{s}}{r^2} + \frac{3}{2}\left(\frac{\mathbf{r}.\mathbf{s}}{r^2}\right)^2\right\},$$

where $\mathbf{v}$ is the velocity of an electron, $\mathbf{s}$ and $\mathbf{r}$ the position vectors of the typical electron and the field point, respectively.

Taking account of the structural conditions given by (1) and (2) these expressions reduce to:

$$\Phi = 0, \qquad \mathbf{A} = \frac{1}{c}\Sigma \frac{e}{r}\,\mathbf{v}\left\{\frac{\mathbf{r}.\mathbf{s}}{r^2} + \frac{3}{2}\left(\frac{\mathbf{r}.\mathbf{s}}{r^2}\right)^2\right\}. \tag{7}$$

In general, the approximation will be sufficient if only the first term in the expression for $\mathbf{A}$ be retained; then:

$$\mathbf{A} = \frac{1}{c}\Sigma\, e\,\mathbf{v}\,\frac{\mathbf{r}.\mathbf{s}}{r^3}. \tag{8}$$

The right hand member of this equation may be transformed as follows—noting that $\mathbf{v} = \dot{\mathbf{s}}$ we have identically:

$$\mathbf{v}\,\mathbf{r}.\mathbf{s} = \frac{1}{2}\left\{\mathbf{v}\,\mathbf{r}.\mathbf{s} - \mathbf{s}\,\mathbf{r}.\mathbf{v} + \frac{d}{dt}(\mathbf{s}\,\mathbf{s}.\mathbf{r})\right\} = \frac{1}{2}\mathbf{r} \times (\mathbf{v} \times \mathbf{s}) + \frac{1}{2}\frac{d}{dt}(\mathbf{s}\,\mathbf{s}.\mathbf{r});$$

and, therefore:

$$\mathbf{A} = \frac{\boldsymbol{\mu} \times \mathbf{r}}{r^3} + \frac{1}{2cr^3}\frac{d}{dt}\Sigma\, e\,\mathbf{s}\,\mathbf{s}\cdot\mathbf{r}, \tag{9}$$

where

$$\boldsymbol{\mu} = \frac{1}{2c}\Sigma\, e\,\mathbf{s} \times \mathbf{v}. \tag{10}$$

The Mean Value of the Vector Potential for a Rotating Rigid Magneton.

For a rigid rotating magneton the mean value during one revolution of the second term on the right of (9) will vanish, and if we denote the mean value of $\mathbf{A}$ by $\bar{\mathbf{A}}$ and of $\boldsymbol{\mu}$ by $\bar{\boldsymbol{\mu}}$, then:

$$\bar{\mathbf{A}} = \frac{\bar{\boldsymbol{\mu}} \times \mathbf{r}.}{r^3} \tag{11}$$

From the form of this expression for the mean vector potential it appears that the mean field of a rotating magneton is the same as the field of a small magnet with a moment $\bar{\boldsymbol{\mu}}$; and it may be easily seen that the direction of the vector $\bar{\boldsymbol{\mu}}$ will coincide with that of the axis of rotation of the magneton.

If α, β, γ be the direction cosines of the axis of rotation, and therefore of $\bar{\boldsymbol{\mu}}$, the scalar components of the mean vector potential of the magneton will be given by:

$$
\begin{aligned}
\bar{A}_1 &= \frac{\bar{\mu}}{r^3}(\beta z - \gamma y),\\
\bar{A}_2 &= \frac{\bar{\mu}}{r^3}(\gamma x - \alpha z),\\
\bar{A}_3 &= \frac{\bar{\mu}}{r^3}(\alpha y - \beta x).
\end{aligned}
\tag{12}
$$

The Mean Value of the Magnetic Force Due to a Rotating Rigid Magneton.

Taking the curl of $\bar{\mathbf{A}}$ we find for the mean value, $\bar{\mathbf{H}}$, of the magnetic force:

$$\bar{\mathbf{H}} = \frac{3}{r^5}\bar{\boldsymbol{\mu}}\cdot\mathbf{r}\,\mathbf{r} - \frac{1}{r^3}\bar{\boldsymbol{\mu}}; \tag{13}$$

and the scalar components of this force are easily seen to be given by:

$$
\begin{aligned}
\bar{H}_1 &= 3\frac{\bar{\mu}}{r^5}(\alpha x + \beta y + \gamma z)\,x - \frac{\bar{\mu}\alpha}{r^3},\\
\bar{H}_2 &= 3\frac{\bar{\mu}}{r^5}(\alpha x + \beta y + \gamma z)\,y - \frac{\bar{\mu}\beta}{r^3},\\
\bar{H}_3 &= 3\frac{\bar{\mu}}{r^5}(\alpha x + \beta y + \gamma z)\,z - \frac{\bar{\mu}\gamma}{r^3}.
\end{aligned}
\tag{14}
$$

It appears from these equations that the mean magnetic field is symmetrical to the axis of rotation of the magneton; that the lines of force lie in planes through the axis of rotation; and that the mean field is equivalent to the field of a magnetic doublet whose axis is parallel to the axis of rotation and whose moment is equal to $\bar{\boldsymbol{\mu}}$.

This equivalence, of course, holds only for the mean value of the magnetic field of the magneton, and not for the instantaneous value. For the latter the second term on the right of (9) comes into consideration; and accordingly the instantaneous value of the field will vary with the time, giving rise to radiation, with which, however, we are not here specially concerned.

The Magnetic Moment of a Rotating Rigid Magneton

As has been seen above the mean value of the quantity $\boldsymbol{\mu}$ represents the mean time value of the moment of a rigid rotating magneton; it will therefore be convenient to refer to the quantity $\boldsymbol{\mu}$ itself as the moment of the magneton.

When the magneton is rigid, $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{s}$, and we have from (10):

$$
\begin{aligned}
\boldsymbol{\mu} &= \frac{1}{2c} \Sigma\, e\, \mathbf{s} \times (\boldsymbol{\omega} \times \mathbf{s}) \\
(15) \qquad &= \frac{1}{2c} \Sigma\, e\, (s^2 \boldsymbol{\omega} - \boldsymbol{\omega} \cdot \mathbf{s}\, \mathbf{s}) \\
&= \frac{1}{2c} \Sigma\, e \left\{ (\xi^2 + \eta^2 + \zeta^2)\, \boldsymbol{\omega} - (\omega_1 \xi + \omega_2 \eta + \omega_3 \zeta)\, \mathbf{s}. \right.
\end{aligned}
$$

where ξ, η, ζ are, as usual, the scalar components of $\mathbf{s}$, and ω_1, ω_2, ω_3 are the scalar components of $\boldsymbol{\omega}$, the angular velocity of the magneton.

From the last of these equations it follows, with the aid of (3), (4) and (5), that:

$$
\begin{aligned}
\boldsymbol{\omega} = \frac{1}{2c} \Big\{ &(Q_1 \omega_1 \qquad - D_3 \omega_2 - D_2 \omega_3)\mathbf{i} \\
(16) \qquad &+ (- D_3 \omega_1 + Q_2 \omega_2 - D_1 \omega_3)\mathbf{j} \\
&+ (- D_2 \omega_1 - D_1 \omega_2 + Q_3 \omega_3)\mathbf{k} \Big\}.
\end{aligned}
$$

From this equation it appears that $\boldsymbol{\mu}$ is a self-conjugate linear vector function of $\boldsymbol{\omega}$. In fact, the relation between $\boldsymbol{\mu}$ and $\boldsymbol{\omega}$ is precisely analogous to that of the moment of momentum of a rigid body to its angular velocity of rotation, the Q's in the present case corresponding to the moments of inertia about the axes and the D's to the so-called products of inertia.

The Torque upon a Magneton due to an External Electromagnetic Field.

We now suppose the magneton to be placed in an electromagnetic field which may vary in space and in time. The electric force and the magnetic force of this field will be denoted respectively by $\mathbf{E}$ and $\mathbf{H}$. The torque, $\mathbf{N}$, acting upon the magneton due to the action of this field has now to be found.

With reference to the origin O, this torque, from (19) Sect. I, will be given by:

$$
(17) \qquad \mathbf{N} = \Sigma\, e\, \mathbf{s} \times \left(\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{H}\right),
$$

where the summation is over all the electrons in the magneton.

Since $\mathbf{E}$ and $\mathbf{H}$ may be assumed continuous, they may respectively be developed into the series:

$$\mathbf{E} = \mathbf{E}_0 + (\mathbf{s}.\nabla\mathbf{E})_0 + \ldots\ldots\ldots, \tag{18}$$

$$\mathbf{H} = \mathbf{H}_0 + (\mathbf{s}.\nabla\mathbf{H})_0 + \ldots\ldots\ldots, \tag{19}$$

where the subscripts indicate that the quantities to which they refer are to be evaluated at the point O.

If $\mathbf{N}^e$ and $\mathbf{N}^m$ denote respectively the turning moments upon the magneton due to the external electric and magnetic force, then:

$$\mathbf{N} = \mathbf{N}^e + \mathbf{N}.^m \tag{20}$$

In the evaluation of $\mathbf{N}^e$ attention must be paid to (2), expressing that the total electric polarization of the magneton vanishes. On this account the first term on the right of (18) contributes nothing to the value of $\mathbf{N}.^e$ If furthermore we restrict ourselves to terms of the second order of smallness in the small quantity s, only the second term in the development of $\mathbf{E}$ need be considered and the evaluation of $\mathbf{N}^e$ then gives:

$$\mathbf{N}^e = \Sigma\, e\, \mathbf{s} \times \mathbf{s}\cdot\nabla\,\mathbf{E}, \tag{21}$$

where it is to be understood that the derivations in the factor $\mathbf{s}.\nabla\mathbf{E}$ are to be effected at the point O although the zero subscript is not explicitly carried forward.

In a similar manner the evaluation of $\mathbf{N}^m$ to the same order of approximation gives:

$$\mathbf{N}^m = \frac{1}{c}\Sigma\, e\, \mathbf{s} \times (\mathbf{v} \times \mathbf{H}), \tag{22}$$

where $\mathbf{H}$ is to be taken as the external magnetic force at O.

If the triple vector product in the sum on the right of (22) be expanded and account taken of the perpendicularity of $\mathbf{s}$ and $\mathbf{v}$, it may be seen that (22) transforms into:

$$\mathbf{N}^m = \Sigma\, e\, \mathbf{s}\cdot\mathbf{H}\,\mathbf{v};$$

or, in case the magneton is considered as rigid:

$$\mathbf{N}^m = \frac{1}{c}\,\omega \times \mathbf{c}, \tag{23}$$

where ω is the angular velocity of the magneton about an axis through O and:

$$\mathbf{c} = \Sigma\, e\, \mathbf{s}\cdot\mathbf{H}\,\mathbf{s}. \tag{24}$$

The scalar components of the vector $\mathbf{c}$ with the aid of (3) and (4), remembering that ξ, η, ζ are the scalar components of $\mathbf{s}$, may be expressed as follows:

$$
\begin{aligned}
C_1 &= P_1H_1 + D_3H_2 + D_2H_3,\\
C_2 &= D_3H_1 + P_2H_2 + D_1H_3,\\
C_3 &= D_2H_1 + D_1H_2 + P_3H_3,
\end{aligned}
\tag{25}
$$

showing that c is a self-conjugate linear vector function of H.

Making use of (3) and (4) the scalar components of N^e given by (25) may be expressed by:

$$
\begin{aligned}
N_1{}^e &= D_3\frac{\partial E_3}{\partial x} + P_2\frac{\partial E_3}{\partial y} + D_1\frac{\partial E_3}{\partial z} - D_2\frac{\partial E_2}{\partial x} - D_1\frac{\partial E_2}{\partial y} - P_3\frac{\partial E_2}{\partial z},\\
N_2{}^e &= D_2\frac{\partial E_1}{\partial x} + D_1\frac{\partial E_1}{\partial y} + P_3\frac{\partial E_1}{\partial z} - P_1\frac{\partial E_3}{\partial x} - D_3\frac{\partial E_3}{\partial y} - D_2\frac{\partial E_3}{\partial z},\\
N_3{}^e &= P_1\frac{\partial E_2}{\partial x} + D_3\frac{\partial E_2}{\partial y} + D_2\frac{\partial E_2}{\partial z} - D_3\frac{\partial E_1}{\partial x} - P_2\frac{\partial E_1}{\partial y} - D_1\frac{\partial E_1}{\partial z}.
\end{aligned}
\tag{26}
$$

From (23) with the aid of (25) the corresponding expressions for the scalar components of the turning moment upon the magneton due to the external magnetic field are seen to be given by:

$$
\begin{aligned}
N_1{}^m &= \frac{1}{c}\left\{\omega_2(D_2H_1 + D_1H_2 + P_3H_3) - \omega_3(D_3H_1 + P_2H_2 + D_1H_3)\right\},\\
N_2{}^m &= \frac{1}{c}\left\{\omega_3(P_1H_1 + D_3H_2 + D_2H_3) - \omega_1(D_2H_1 + D_1H_2 + P_3H_3)\right\},\\
N_3{}^m &= \frac{1}{c}\left\{\omega_1(D_3H_1 + P_2H_2 + D_1H_3) - \omega_2(P_1H_1 + D_3H_2 + D_2H_3)\right\}.
\end{aligned}
\tag{27}
$$

Equations for a Rotating Rigid Magneton Referring to its Principal Axes of Charge.

It is always possible to choose three mutually perpendicular axes through the centroid of a magneton such that for them the products of inertia of charge vanish:

$$D_1 = D_2 = D_3 = 0.$$

These axes are called Principal Axes of Charge.

The equations found above for the magnetic moment of a rigid magneton and for the scalar components of the torques upon it due to the action of an external electric and an external magnetic field assume much simpler forms when the axes of reference are Principal Axes of Charge.

Thus, from (16), we have for the magnetic moment of a magneton:

$$\mu = \frac{1}{2c}(Q_1\omega_1\mathbf{i} + Q_2\omega_2\mathbf{j} + Q_3\omega_3\mathbf{k}); \tag{28}$$

and, from (26) and (27), for the scalar components of the torques:

$$
\begin{aligned}
N_1^e &= P_2\frac{\partial E_3}{\partial y} - P_3\frac{\partial E_2}{\partial z},\\
N_2^e &= P_3\frac{\partial E_1}{\partial z} - P_1\frac{\partial E_3}{\partial x},\\
N_3^e &= P_1\frac{\partial E_2}{\partial x} - P_2\frac{\partial E_1}{\partial y}
\end{aligned} \tag{29}
$$

due to an external electric field **E**; and :

$$
\begin{aligned}
N_1^m &= \frac{1}{c}(P_3H_3\omega_2 - P_2H_2\omega_3),\\
N_2^m &= \frac{1}{c}(P_1H_1\omega_3 - P_3H_3\omega_1),\\
N_3^m &= \frac{1}{c}(P_2H_2\omega_1 - P_1H_1\omega_2).
\end{aligned} \tag{30}
$$

due to an external magnetic field **H**.

Equations for Rotating Spherical and Axial Magnetons Referring to Principal Axes.

For the purpose of the present review it is only necessary to consider two special types of magneton, known respectively as the Spherical Magneton and the Axial Magneton.

The Spherical Magneton is defined as rigid and one for which the principal axes of charge and of inertia coincide and for which the principal moments of inertia of charge Q_1, Q_2, Q_3 and the principal moments of inertia, A, B, C, are respectively equal:

$$
\begin{aligned}
&Q_1 = Q_2 = Q_3 = Q \quad \therefore \quad P_1 = P_2 = P_3;\\
&A = B = C = J.
\end{aligned} \tag{31}
$$

The Axial Magneton is defined as rigid and one for which the principal axes of charge and of inertia coincide and for which the principal moments of inertia of charge and the principal moments of inertia are respectively equal for two of its principal axes, say 1 and 2:

$$
\begin{aligned}
&Q_1 = Q_2 = Q, \therefore P_1 = P_2;\\
&A = B = J.
\end{aligned} \tag{32}
$$

For the magnetic moment we have, from (28):

$$\mu = \frac{Q}{2c}\omega \tag{33}$$

for the Spherical Magneton; and:

$$\mu = \frac{1}{2c}(Q\omega_1 \mathbf{i} + Q\omega_2 \mathbf{j} + Q_3\omega_3 \mathbf{k}) \tag{34}$$

for the Axial Magneton.

For the torque due to an external electric field **E** we have, from (29):

$$\begin{aligned} N_1^e &= \frac{Q}{2}\left(\frac{\partial E_3}{\partial y} - \frac{\partial E_2}{\partial z}\right), \\ N_2^e &= \frac{Q}{2}\left(\frac{\partial E_1}{\partial z} - \frac{\partial E_3}{\partial z}\right), \quad \therefore \mathbf{N}^e = \frac{Q}{2}\,\mathrm{curl}\,\mathbf{E}; \\ N_3^e &= \frac{Q}{2}\left(\frac{\partial E_2}{\partial x} - \frac{\partial E_1}{\partial y}\right); \end{aligned} \tag{35}$$

and, from (30), for the torque due to an external magnetic field:

$$\begin{aligned} N_1^m &= \frac{Q}{2c}(\omega_2 H_3 - \omega_3 H_2), \\ N_2^m &= \frac{Q}{2c}(\omega_3 H_1 - \omega_1 H_3), \quad \therefore \mathbf{N}^m = \frac{Q}{2c}\omega \times \mathbf{H}, \\ N_3^m &= \frac{Q}{2c}(\omega_1 H_2 - \omega_2 H_1); \end{aligned} \tag{36}$$

or a Spherical Magneton.

From (29) and (30) the corresponding expressions for an Axial Magneton will be given by:

$$\begin{aligned} N_1^e &= \frac{Q_3}{2}\frac{\partial E_3}{\partial y} - P_3\frac{\partial E_2}{\partial z}, \\ N_2^e &= P_3\,\frac{\partial E_1}{\partial z} - \frac{Q}{2}\frac{\partial E_3}{\partial x}, \\ N_3^e &= \frac{Q_3}{2}\left(\frac{\partial E_2}{\partial x} - \frac{\partial E_1}{\partial y}\right) = \frac{Q_3}{2}\,(\mathrm{curl}\,\mathbf{E})_3, \end{aligned} \tag{37}$$

for the torque due to an external electric field **E**; and:

$$\begin{aligned} N_1^m &= \frac{1}{c}\left(P_3 H_3\omega_2 - \frac{Q_3}{2}H_2\omega_3\right), \\ N_2^m &= \frac{1}{c}\left(\frac{Q_3}{2}H_1\omega_3 - P_3 H_3\omega_1\right), \\ N_3^m &= \frac{Q_3}{2c}(H_2\omega_1 - H_1\omega_2) = \frac{Q_3}{2c}(\omega \times \mathbf{H})_3, \end{aligned} \tag{38}$$

for the torque due to an external magnetic field **H**.

The Rotary Motion of a Rigid Magneton Subject to an External Electromagnetic Field.

We assume the reference axes to coincide at the instant under consideration with the principal axes of inertia of the magneton, for which the moments of inertia are A, B, and C; and also that the principal axes of charge coincide with those of inertia.

By Euler's dynamical equations of motion:

$$(39)\qquad \begin{aligned} A\dot{\omega}_1 - (B - C)\omega_2\omega_3 &= N_1^e + N_1^m, \\ B\dot{\omega}_2 - (C - A)\omega_3\omega_1 &= N_2^e + N_2^m, \\ C\dot{\omega}_3 - (A - B)\omega_1\omega_2 &= N_3^e + N_3^m, \end{aligned}$$

where the N^e and N^m torque components in the general case are given by (29) and (30).

From these equations the rotary motion of the magneton may be theoretically determined when no dissipative forces are assumed.

Special Case of the Spherical Magneton.

In this case we have $A = B = C = J$ and, upon introducing the expressions for the torque components given by (35) and (36), in the equations of motion (39) it appears at once that they are equivalent to the single vector equation:

$$(40)\qquad J\dot{\omega} = \frac{Q}{2}(\text{curl } E + \frac{1}{c}\omega \times H);$$

Since, by virtue of one of Maxwell's field equations, $\text{curl } E = -dH/cdt$, this equation may be written:

$$(41)\qquad J\frac{d\omega}{dt} = -\frac{Q}{2c}\left(\frac{dH}{dt} - \omega \times H\right).$$

This equation assumes a simpler form if the time derivations are taken with respect to the moving space of the magneton instead of fixed space. If $\frac{d'}{dt}$ denote time derivation with respect to the former, then:

$$(42)\qquad \frac{d'\omega}{dt} = \frac{d\omega}{dt}\quad,\quad \frac{d'H}{dt} = \frac{dH}{dt} - \omega \times H,$$

and equation (41) may therefore be written:

$$(43)\qquad J\frac{d'\omega}{dt} = -\frac{Q}{2c}\frac{d'H}{dt}$$

Integration of this equation gives:

$$(44)\qquad \omega = \omega_0 - \frac{Q}{2cJ}H,$$

where ω_0 denotes the value of ω before the application of the external field.

From the last equation it appears that the establishment of an external electromagnetic field brings into existence a rotation of the magneton about an axis parallel to the lines of force of the external field of amount $-\frac{Q}{2cJ}H$ since J is a positive quantity the direction of this rotation will depend upon whether Q is negative or positive.

The magnetic moment of a spherical magneton is given by (33), from which with the aid of (44):

$$\mu = \mu_0 - \frac{Q^2}{4c^2J}H. \tag{45}$$

Therefore the effect of the establishment of the external field upon the moment of the magneton is to bring into existence a component $-Q^2H/4c^2J$ directed parallel to the lines of force of the external magnetic field; since J is a positive quantity the coefficient of H in (45) will be negative.

In the particular case where the external magnetic field remains constant in time the equation of motion (41) for a spherical magneton reduces to:

$$J\dot{\omega} = \frac{Q}{2c}\omega \times H. \tag{46}$$

In accordance with this equation, since $\omega \times H$ is a vector which is perpendicular to ω, the magnitude of ω will remain invariable; but, except in the special case where ω is parallel to H, the direction of the axis of rotation will continually change both in fixed space and in the magneton. The component of ω in the direction of H will not change but the component perpendicular to H will rotate about the direction of H with the constant angular velocity

$$\omega_1 = -\frac{Q}{2cJ}H. \tag{47}$$

The vector ω itself will rotate about an axis parallel to H with this same angular velocity; and the magneton will perform a regular precession about this axis. From (44) and (47):

$$\omega = \omega_0 + \omega_1. \tag{48}$$

The angular velocity ω of the magneton may thus be regarded as the sum of two components; ω_0, representing its angular velocity before the application of the external field, and ω_1, representing an induced angular velocity about the direction of the lines of force, due to the creation of the external field.

The precessional motion of the magneton takes place in a manner similar to that of a symmetrical top in a gravitational field, but with the difference that the applied torque in the present case, $Q\boldsymbol{\omega}\times\mathbf{H}/2c$, is proportional to the angular velocity, while in the case of the top it is independent of the velocity; thus it comes about that the precessional velocity, in the case of the magneton is independent of its angular velocity, while in the case of the top it is inversely proportional to the angular velocity.

Since, by virtue of one of Maxwell's field equations, curl $\mathbf{E} = 0$ for a magnetic field of constant strength, it follows from (35), (36) and (33) that the torque on a spherical magneton in a constant external magnetic field is $\boldsymbol{\mu} \times \mathbf{H}$ or, on account of (45):

$$\boldsymbol{\mu}_0 \times \mathbf{H}. \tag{49}$$

The magneton is thus subject to a couple equal to that which would be experienced by a magnetic needle of moment $\boldsymbol{\mu}_0$ placed in the same magnetic field $\mathbf{H}$. But the motion of the needle would be quite different from that of the magneton, in that the needle would move in a plane containing its axis and parallel to the lines of force, while the magneton, due to its gyroscopic properties, performs a precessional motion about the direction of the lines of force. If either the needle or magneton is to assume a position with axis along the lines of force it is necessary in general that dissipative forces come into play.

Special Case of the Axial Magneton.

For the axial magneton $A = B = J$ and the general equations of rotary motion (39), with the aid of (32) reduce to:

$$\begin{aligned} J\dot{\omega}_1 - (J-C)\omega_2\omega_3 &= \frac{Q_3}{2}\frac{\partial E_3}{\partial y} - P_3\frac{\partial E_2}{\partial z} + \frac{1}{c}\left(P_3\omega_2H_3 - \frac{Q_3}{2}\omega_3H_2\right), \\ J\dot{\omega}_2 - (C-J)\omega_3\omega_1 &= P_3\frac{\partial E_1}{\partial z} - \frac{Q_3}{2}\frac{\partial E_3}{\partial x} + \frac{1}{c}\left(\frac{Q_3}{2}\omega_3H_1 - P_3\omega_1H_3\right), \\ C\dot{\omega}_3 &= \frac{Q_3}{2}\left(\frac{\partial E_2}{\partial x} - \frac{\partial E_1}{\partial y}\right) + \frac{Q_3}{2c}(\omega_1H_2 - \omega_2H_1). \end{aligned} \tag{50}$$

The third of these equations refers to rotation about the axis of the magneton and may be put in the form:

$$C\dot{\omega}_3 = \frac{Q_3}{2}\left\{(\text{curl}\,\mathbf{E})_3 + \frac{1}{c}(\boldsymbol{\omega}\times\mathbf{H})_3\right\}. \tag{51}$$

By virtue of one of Maxwell's field equations:

$$(\text{curl}\,\mathbf{E})_3 = -\frac{1}{c}\frac{dH_3}{dt};$$

so that (51) may be written:

$$\frac{d\omega_3}{dt} = -\frac{Q_3}{2cC}\left(\frac{d\mathbf{H}}{dt} - \omega \times \mathbf{H}\right)_3;$$

or, if the derivations be taken with respect to the moving space of the magneton,

$$\frac{d'\omega_3}{dt} = -\frac{Q_3}{2cC}\frac{d'H_3}{dt}.$$

Integration of this equation gives:

$$\omega_3 = \omega_{03} - \frac{Q_3}{2cC}H_3, \tag{52}$$

where ω_{03} represents the angular velocity of the magneton about its axis before the application of the external field. The external field thus produces a change in the angular velocity about its axis of amount $-Q_3H_3/2cC$. It will also produce changes in the velocities of rotations about two perpendicular equatorial axes the equations of which are the first two of equations (50), assuming no dissipative forces. Owing to radiation due to the disymmetry of structure of the magneton with respect to these axes the motions about them would in course of time be damped out leaving only the motion about its axis.

To the latter there corresponds a magnetic moment which, from (34) and (52), will have for its scalar value:

$$\mu = \mu_{03} - \frac{Q_3^2}{4c^2C}H_3, \tag{53}$$

where

$$\mu_{03} = \frac{Q_3}{2c}\omega_{03}; \tag{54}$$

μ_{03} is the scalar value of the axial component of the moment of the magneton before the application of the external field.

Energy of a Rotating Axial Magneton in a Constant External Magnetic Field.

In what follows the axial magneton will be supposed to consist of a rigid system of negative electrons symmetrically spaced about their centroid and rotating about it, the corresponding positive charge being in the form of a nucleus at their centroid or of a concentric sphere. In this case we may write in equation (34) for the magnetic moment of the magneton:

$$Q = eJ/m \text{ and } Q_3 = eC/m.$$

For the total energy, U, we may write:

$$U = U_1 + U_2 + U_3,$$

where U_1 represents the energy due to the translatory motion of the magneton, U_2 its energy of rotation and U_3 the mutual energy of the magneton and the external field, which, according to the point of view, may be regarded either as kinetic or potential.

If M denote the mass of the magneton and x, y, z the coordinates of its centroid we have for its translatory energy:

$$U_1 = \frac{M}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) \tag{55}$$

In the calculation of the rotatory energy of the magneton we suppose (ξ, η, ζ) to be axes coinciding with its principal axes and therefore fixed in the magneton, A, B, C being its moments of inertia about the axes of ξ, η, ζ respectively; since the magneton is now supposed axial, we put $A = B = J$.

To specify the position of the magneton with reference to the external field and fixed space we use Eulerian angles θ, ψ, ϕ.

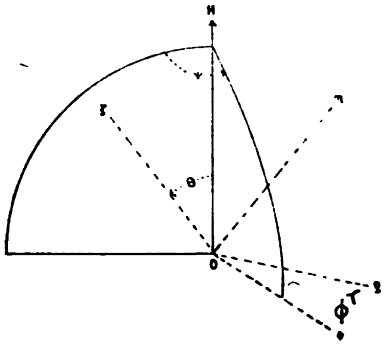

FIG. 2

Referring to Fig. 2, θ is the angle between the positive directions of the external field **H** and the axis ζ; ψ is the longitude of the line of nodes, *on* defined as a line perpendicular to the plane determined by the directions of the field **H** and the ζ-axis; and ϕ is the angle between the line of nodes and the ξ-axis.

If ω_1, ω_2, ω_3 be the scalar components of the angular velocities of the magneton about the axes ξ, η, ζ, respectively, then:

$$\begin{aligned} \omega_1 &= \dot{\psi} \sin\theta \sin\phi + \dot{\theta} \cos\phi, \\ \omega_2 &= \dot{\psi} \sin\theta \cos\phi - \dot{\theta} \sin\phi, \\ \omega_3 &= \dot{\psi} \cos\theta + \dot{\phi}. \end{aligned} \tag{56}$$

We shall therefore have for the energy of rotation of the magneton:

$$U_2 = \frac{J}{2}(\dot{\theta}^2 + \dot{\psi} \sin^2\theta) + \frac{C}{2}(\dot{\phi} + \dot{\psi} \cos\theta)^2. \tag{57}$$

Considering the mutual energy U_3 of the magneton and the external field as kinetic we may write[1]:

$$U_3 = \tfrac{1}{2}\, \mu \cdot \mathbf{H}, \tag{58}$$

---

[1] Cf. R. Gans, *Ann. d. Phys.* 49, p. 154; 1916.

where $\boldsymbol{\mu}$ is the magnetic moment of the magneton. Upon noting that $Q = eJ/m$ and $Q_3 = eC/m$ where m is the mass of a constitutive electron of the magneton, it follows from (34) that:

$$\boldsymbol{\mu} = \frac{e}{2mc}(J\omega_1\mathbf{i} + J\omega_2\mathbf{j} + C\omega_3\mathbf{k}). \tag{59}$$

Upon taking the scalar product of $\mathbf{H}$ with this expression for $\boldsymbol{\mu}$, substituting the expressions for ω_1, ω_2 and ω_3 given by (56) and inserting the resulting expression in (58) we obtain:

$$U_3 = \tfrac{1}{2}\,\boldsymbol{\mu}\cdot\mathbf{H} = \frac{e}{4mc}H\{J\dot{\psi}\sin^2\theta + C(\dot{\phi} + \dot{\psi}\cos\theta)\cos\theta\}. \tag{60}$$

Finally, upon adding the expressions for U_1, U_2 and U_3 given by (55), (57) and (60), we obtain for the total energy of the axial magneton in a constant magnetic field:

$$\begin{aligned} U = {} & \frac{M}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) \\ & + \frac{J}{2}(\dot{\theta}^2 + \dot{\psi}^2\sin^2\theta) + \frac{C}{2}(\dot{\phi} + \dot{\psi}\cos\theta)^2 \\ & + \frac{e}{4mc}H\left\{J\dot{\psi}\sin^2\theta + C(\dot{\phi} + \dot{\psi}\cos\theta)\cos\theta\right\}. \end{aligned} \tag{61}$$

In the writing of the present section the treatment of the subject of the magneton as presented in Abraham's "Theorie der Elektrizität" has been of much assistance.

III

THE DISTRIBUTION FUNCTION IN THEORIES OF PARAMAGNETISM

In kinetic theories of magnetism the problem of the determination of the distribution of the axes of the constitutive magnetons of a body placed in an external magnetic field arises. A knowledge of this distribution is necessary before the contribution of the magnetons to the resultant magnetic moment due to the action of the external magnetic field upon the magnetons can be calculated. For convenience of reference later some results of statistical theory will be considered in the present section.

Let us consider a system, subject to no external field of force, consisting of a large number of like statistical units, the typical one of which is specified as regards its configuration by the generalized coordinates $q_1 \cdots\cdots qr$, subject also to the condition that the total energy of the system is constant. Let the n generalized momenta of the system be denoted by $p_1 \cdots\cdots p_n$.

We suppose the generalized coordinates and the momenta to be subject to statistical variation, through thermal agitation for instance. Then if N be the number of units per unit mass, in accordance with statistical theory, when the system is in a state of equilibrium the probable number of units per unit mass, say dN, which have values of their coordinates and momenta lying respectively within the specified limits

$$q_1 \text{ and } q_1 + dq_1 \ldots\ldots\ldots q_n \text{ and } q_n + dq_n,$$
$$p_1 \text{ and } p_1 + dp_1 \ldots\ldots\ldots p_n \text{ and } p_n + dp_n,$$

will be expressed by the law of distribution:

$$dN = \alpha e^{-2h\epsilon} d\Omega', \tag{1}$$

where

$$d\Omega' = dq_1 \ldots dq_n dp_1 \ldots dp_n,$$

ϵ is the total energy of a unit which is subject to statistical variation expressed in terms of the q's and p's and α and h are constants.

For the determination of the constant α we have the condition:

$$\int \alpha e^{-2h\epsilon} d\Omega' = N, \tag{2}$$

where the integration is to be extended over all possible values of the variables whose differentials appear in the expression for $d\Omega'$.

The function

$$\alpha e^{-2h\epsilon}$$

is called the distribution function for the system of units.

In some cases it may be convenient to introduce new variables in place of some of the generalized momenta. Thus, let us suppose m of the generalized momenta, say $p_1 \dots . p_m$, to be expressed in terms of m new variables, say $r_1 \dots . r_m$, through the equations:

$$
\begin{aligned}
p_1 &= f_1(r_1 \dots . r_m),\\
&\dots\dots\dots\dots\dots\\
&\dots\dots\dots\dots\dots\\
p_m &= f_m(r_1 \dots . r_m).
\end{aligned}
$$

By differentiation:

$$
\begin{aligned}
dp_1 &= \frac{\partial p_1}{\partial r_1}dr_1 + \dots . + \frac{\partial p_1}{\partial r_m}dr_m,\\
&\dots\dots\dots\dots\dots\dots\dots\dots\\
&\dots\dots\dots\dots\dots\dots\dots\dots\\
dp_m &= \frac{\partial p_m}{\partial r_m} + \dots . + \frac{\partial p_m}{\partial r_m}dr_m.
\end{aligned}
$$

From a theorem due to Jacobi:

$$dp_1 \dots . dp_m = \Delta dr_1 \dots . dr_m,$$

where Δ, the modulus of substitution, is given by the determinantal expression:

$$
(3) \qquad \Delta = \begin{vmatrix}
\frac{\partial p_1}{\partial r_1} & \dots\dots & \frac{\partial p_1}{\partial r_m}\\
\dots & \dots\dots & \dots\\
\dots & \dots\dots & \dots\\
\frac{\partial p_m}{\partial r_1} & \dots\dots & \frac{\partial p_m}{\partial r^m}
\end{vmatrix}.
$$

If we write:

$$d\Omega = dq_1 \dots dq_n dr_1 \dots . dr_m dp_{m+1} \dots\dots dp_n,$$

then:

$$(4) \qquad d\Omega' = \Delta d\Omega.$$

The law of distribution (1) is therefore equivalent to:

$$(5) \qquad dN = \alpha e^{-2h\epsilon}\Delta d\Omega,$$

where the energy ϵ is now supposed expressed in terms of $q_1 \dots . q_n$, $r_1 \dots . r_m$ and $p_{m+1} \dots\dots p_n$.

Case of a System of Axial Magnetons in a Constant External Magnetic Field.

It will be assumed for the present that the density of distribution of the magnetons is so small that the molecular field at any given magneton due to the others is neglible. It will also be assumed that the accelerations of the magnetons are so small that their loss of energy by radiation may be neglected. Furthermore the restrictions, whereby $Q=eJ/m$

and $Q_3 = eC/m$, imposed upon the axial magneton in the last part of the preceding section will be supposed to hold.

The total energy of the system may then be considered as constant, since the constant external magnetic field can do no work upon the magnetons, the corresponding mechanical force upon the constitutive electrons of the magnetons being perpendicular to their directions of motion.

We may now take for the total energy of the typical magneton of the system the expression (61) Sect. II:

$$(6)\quad U = \frac{M}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{J}{2}(\dot{\theta}^2 + \dot{\psi}^2 \sin^2\theta) + \frac{C}{2}(\dot{\phi} + \dot{\psi}\cos\theta)^2 + \frac{eH}{4mc}\left\{J\,\dot{\psi}\sin^2\theta + C(\dot{\phi} + \dot{\psi}\cos\theta)\cos\theta\right\},$$

where x, y, z are the coordinates of its centroid and θ, ψ, ϕ its Eulerian coordinates.

From this expression, since the total energy is kinetic, by partial differentiation we obtain for the corresponding generalized momenta, say u, v, w, p, q, r, the following expressions:

$$u = M\dot{x}, \qquad v = M\dot{y}, \qquad w = M\dot{z}, \qquad p = J\dot{\theta},$$

$$(7)\qquad q = J\dot{\psi}\sin^2\theta + C(\dot{\phi} + \dot{\psi}\cos\theta)\cos\theta + \frac{eH}{4mc}(J\sin^2\theta + C\cos^2\theta),$$

$$r = C(\dot{\phi} + \dot{\psi}\cos\theta) + \frac{eH}{4mc}C\cos\theta.$$

The statistical variables of the system are now x, y, z, u, v, w, θ, ψ, ϕ, p, q, and r. But it will prove convenient to replace the momenta p, q, r by new variables P, Q, R, using the following equations of substitution:

$$p = P\cos\phi - Q\sin\phi,$$

$$(8)\quad q = (P\sin\phi + Q\cos\phi)\sin\theta + R\cos\theta + \frac{eH}{4mc}(J\sin^2\theta + C\cos^2\theta),$$

$$r = R + \frac{e}{4mc}H\,C\cos\theta;$$

from which by (3) we find for the modulus of transformation:

$$(9)\qquad \Delta = \begin{vmatrix} \cos\phi, & -\sin\phi, & 0 \\ \sin\theta\sin\phi, & \sin\theta\cos\phi, & \cos\theta \\ 0\ , & 0\ , & 1 \end{vmatrix} = \sin\theta.$$

From (6), with the aid of (7) and (8):

$$(10)\qquad U = \frac{1}{2M}(u^2 + v^2 + w^2) + \frac{P^2 + Q^2}{2J} + \frac{R^2}{2C}$$
$$+ \frac{eH}{4mc}(P \sin \phi \sin \theta + Q \cos \phi \sin \theta + R \cos \theta).$$

We are now dealing with a system of statistical units, the magnetons, which is subject to an external field of magnetic force, and the question arises as to the form of the function ϵ appropriate to this case. Gans, in the paper cited above, has shown this to be equal to this expression for U modified through multiplication of the last term by the factor u; so that:

$$(11)\qquad \epsilon = \frac{1}{2M}(u^2+v^2+w^2) + \frac{P^2+Q^2}{2J} + \frac{R^2}{2C}$$
$$+ \frac{eH}{2MC}(P \sin \varphi \sin \theta + Q \cos \phi \sin \theta + R \cos \theta).$$

In this expression the coordinates x, y, z, ψ do not appear explicitly. Therefore the law of distribution for the remaining statistical variables will be independent of these coordinates. Furthermore the expression involves the statistical variables u, v, w only as a sum of squares and therefore, as a well known result of statistical theory, the law of distribution for the remaining variables will be independent of u, v and w; moreover the constant h in the law of distribution will have the value given by:

$$h = \frac{1}{2kT},$$

where T denotes the absolute temperature and k the gas constant for a single molecule, known as Boltzmann's constant.

Now from the point of view of magnetic theory we shall be concerned only with the law of distribution of the statistical variables θ, ϕ, P, Q, R; and the appropriate expression for ϵ for this case is obtained from (11) by simply ignoring the terms involving u, v and w.

If then dN now denote the number of magnetons per unit mass whose statistical variables, θ, ϕ, P, Q, R, have values which lie within the element of phase $d\Omega$ given by:

$$d\Omega = d\theta\, d\phi\, dP\, dQ\, dR,$$

the equilibrium law of distribution for these variables will be expressed by:

$$(12)\qquad dN = \alpha e^{-\frac{\epsilon}{kT}} \sin \theta\, d\Omega,$$

where

$$(13)\quad \epsilon = \frac{P^2+Q^2}{2J} + \frac{R^2}{2C} + \frac{e}{2mc}H(P \sin\phi \sin\theta + Q\cos\phi\sin\theta + R\cos\theta).$$

We shall have occasion to consider another case in which the number of statistical variables involved is still further reduced. In this case the angular velocity ω_3 of any magneton about its axis of symmetry is considered constant and the same for all magnetons. This requires that the quantity R shall be constant, since:

$$(14)\qquad R = C(\dot{\phi} + \dot{\psi}\cos\theta) = C\omega_3.$$

Consequently R may no longer be considered as a statistical variable and the statistical variables of the present case are therefore θ, ϕ, P, Q. The appropriate expression for the energy function for this case is now required.

From (52) Sect. II, noting that $Q_3/C = e/m$ and that $H_3 = H\cos\theta$:

$$\omega_3 = \omega_{03} - \frac{e}{2mc}H\cos\theta.$$

Since $R = C\omega_3$, we have, with the aid of this expression, for the sum of the terms in (13) involving R:

$$(15)\qquad \frac{R^2}{2C} + R\frac{e}{2mc}H\cos\theta = \tfrac{1}{2}C\omega_{03}^2 - \mu H\cos\theta,$$

where

$$(16)\qquad \mu = \frac{e}{2mc}C\omega_{03};$$

μ denotes the constant scalar value of that portion of the magnetic moment of the magneton which is due to its rotation about its axis of symmetry.

The appropriate expression for the energy function in the present case may now be obtained directly from (11) through elimination of R by means of (15), thus:

$$(17)\qquad U = \frac{1}{2M}(u^2+v^2+w^2) + \frac{P^2+Q^2}{2} + \frac{Cw_{0}{}^2{}_3}{2}$$
$$+ \frac{eH}{4mc}(P\sin\varphi\sin\theta + Q\cos\varphi\sin\theta) - \tfrac{1}{2}\mu H\cos\theta.$$

The modulus of transformation is easily seen to be $\sin\theta$ as before. If now dN denote the number of magnetons per unit mass whose statistical variables, θ, ϕ, P,Q have values which lie within the element of phase $d\Omega$ given by:

$$(18)\qquad d\Omega = d\theta\, d\phi\, dP\, dQ,$$

the equilibrium law of distribution for these variables will be:

$$(19)\qquad dN = \alpha e^{-\frac{\epsilon}{kT}}\sin\theta\, d\Omega,$$

where

$$(20)\qquad \epsilon = \frac{P^2 + Q^2}{2J} + \frac{e}{2mc} H(P \sin\phi \sin\theta + Q \cos\phi \sin\theta) - \mu H \cos\theta$$

the terms involving u, v, w being ignored as before, and likewise the constant term $C\omega^2_{03}/2$.

The constant α in formulas (12) and (19), if required, may be determined in each case from the condition:

$$(21)\qquad \int \alpha e^{-\frac{\epsilon}{kT}} \sin\theta \, d\Omega = N.$$

where the integration in each case is extended over all values of the variables whose differentials appear in the corresponding expression. for $d\Omega$.

The Langevin Distribution Formula.

In the theory of Langevin the magnitude of the magnetic moment of a molecule (magneton) is supposed constant and directed along a polar axis, contributions to its magnetic moment due to its rotations about its equatorial axes being ignored. In effect, the Langevin magneton may therefore be considered as an axial magneton whose rotation about its axis is not subject to statistical variation, and for which the dynamical and magnetic effects due to rotations about its equatorial axes may be taken as negligibly small; the latter condition requires that: $P = Q = 0$.

The law of distribution in Langevin's theory of a paramagnetic gas is simply obtained from (19) by placing $P = Q = 0$ in the expression (20), which involves the disappearance of the coordinate ϕ and deleting $d\phi$, dP, dQ in expression (18) for $d\Omega$.

The Langevin law of distribution is thus found to be:

$$(22)\qquad dN = \alpha e^{a \cos\theta} \sin\theta \, d\theta, \quad \text{where } a \quad \frac{\mu H}{kT}.$$

In accordance with this formula the paramagnetism of a body constituted of Langevin molecules depends simply upon the distribution of the axes of the magnetons with respect to the external field; it is subject, of course, to the restriction of the general theory so far developed that the effects of molecular fields are ignored. This restriction is unimportant in the case of a paramagnetic gas.

For calculation of the magnetisation in Langevin's theory of a paramagnetic gas the spatial mean value of $\cos\theta$, say $\overline{\cos\theta}$, will be required. From (22) it is easily found that:

$$(23)\qquad \overline{\cos\theta} = \coth a - \frac{1}{a}.$$

Modification of Langevin's Distribution Formula Introducing the Magnetic Molecular Field.

A modification of Langevin's distribution formula for a paramagnetic gas, depending upon the consideration of the molecular field due to the magnetons of which the body is supposed constituted, will next be considered.

In specifying the magnetic field at the centroid of a magneton in an isotropic body we may proceed as follows.

Imagine a small sphere of radius s drawn about the centroid, s being the shortest distance between the centroid of the magneton in question and that of its nearest neighbor. Concentric with this sphere imagine a second sphere drawn with radius s′ large in comparison with s but small in comparison with the bulk dimensions of the body.

The magnetic force at the centroid of the typical magneton is then the vector sum of the external force **H**, the force contributed by the part of the body outside of the s′ sphere, which is well known to be $4\pi I/3$ where **I** is the intensity of magnetisation, and a force, **A**, due to the magnetons contained in the zone between the spheres of radii s and s′ respectively.

It is with the determination of the field **A**, called the molecular field, that we are now concerned.

If **F** denote the resultant field, then:

$$(24) \qquad \mathbf{F} = \mathbf{H} + \frac{4\pi}{3}\mathbf{I} + \mathbf{A}$$

$$= \mathbf{K} + \mathbf{A}, \qquad (\mathbf{K} = \mathbf{H} + \frac{4\pi}{3}\,\mathbf{I}).$$

Now it is evident that, as we pass from magneton to magneton in the vicinity of the typical one under consideration, **A** will vary in direction and magnitude.

Let N be the number of magnetons per unit mass at a point P in a paramagnetic body supposed constituted of axial magnetons for which it may be assumed that P and Q are negligible.

In accordance with the fundamental assumption, which closer examination shows to be justified, that all directions of the molecular field **A** are equally probable, for a magneton selected at random that part, say dN_a, of the total number N per unit mass which find themselves in a molecular field **A** whose direction is delimited by a small cone with vertex at P and of solid angle $d\omega$ and for which the magnitude of **A** lies between the limits A and A + dA, will be expressed by:

$$dN_a = N\frac{d\omega}{4\pi}\,w(A)dA,$$

where w(A) is a probability function to be specified later. These magnetons are designated as Group A.

Referring to Fig. 3 we may express $d\omega$ by:

$$d\omega = \sin\gamma\, d\gamma\, d\phi,$$

where γ and ϕ are the co-latitude and the longitude of $d\omega$ with respect to a polar axis in the direction of **K**; also from the figure:

$$A^2 = F^2 + K^2 - 2FK\cos\delta, \tag{25}$$

$$F^2 = A^2 + K^2 + 2AK\cos\gamma \tag{26}$$

where δ is the angle between **K** and **F**.

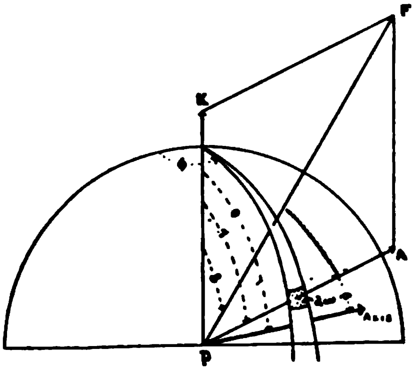

Fig. 3

By differentiation of (26) we find for all magnetons of Group A (for which A is constant):

$$\sin\gamma\, d\gamma = -\frac{FdF}{AK}.$$

We may therefore write:

$$dN_a = -\frac{N}{4\pi K}\frac{w(A)}{A}\, Fd\, A\, dFd\phi. \tag{27}$$

Let θ be the angle which the axis of a typical magneton of Group A makes with the direction of the field **K**; then a number, say dN_b, of the magnetons of Group A will make angles with the direction of **K** which lie within the limits θ and $\theta + d\theta$. These magnetons are designated as of group B. The number dN_b will obviously depend upon the law of distribution of the axes of the magneton in Group A and we may write appropriately:

$$dN_b = dN_a f(\theta) d\theta,$$

where $f(\theta)$ is a function to be determined.

It is evident that the number of magnetons of Group A which make angles with the direction of **K** and which lie within the limits θ and $\theta + d\theta$ will be equal to the number of the same group making angles with the direction of the resultant field F which lie within the limits β and $\beta + d\beta$, if β designate the angle made with F by the axis of a typical magneton of Group A. The latter number is given by the Langevin law of distribution. Consequently:

$$f(\theta)d\theta = \alpha e^{a\cos\beta}\sin\beta\, d\beta, \quad \text{where } a = \frac{\mu F}{kT};$$

and therefore:

$$dN_b = dN_a\, \alpha\, e^{a\cos\beta}\sin\beta\, d\beta;$$

or, upon substitution of the expression for dN_a given above,

$$(28) \qquad dN_b = -\frac{N}{4\pi K}\frac{w(A)}{A} F\alpha\, e^{a\cos\beta}\sin\beta\, dA\, dF\, d\beta\, d\phi.$$

From the condition:

$$\int_0^\pi \alpha\, e^{a\cos\beta}\sin\beta\, d\beta = 1,$$

the value of α is easily found:

$$(29) \qquad \alpha = \frac{1}{2}a\,(\sin h\, a)^{-1}.$$

The spatial mean value of $\cos\beta$, say $\overline{\cos\beta}$, for the magnetons of Group A will be required later. From (28):

$$(30) \qquad \overline{\cos\beta} = \coth a - \frac{1}{a}.$$

The spatial mean value of $\cos\theta$, say $\overline{\cos\theta}$, for all the N magnetons will likewise be required later.

With a view to finding $\overline{\cos\theta}$ we first find an expression for $\cos\theta$ in terms of the distribution variables A, F and β.

Since $\theta = \delta + \beta$, we have:

$$\cos\theta = \cos\delta\cos\beta - \sin\delta\sin\beta;$$

and from (25):

$$\cos\delta = \frac{F^2+K^2-A^2}{2FK}, \qquad \sin\delta = \frac{\sqrt{(2FK)^2-(F^2+K^2-A^2)^2}}{2FK};$$

therefore:

$$\cos\theta = \frac{1}{2FK}\left\{(F^2+K^2-A^2)\cos\beta - \sqrt{(2FK)^2-(F^2+K^2-A^2)^2}\sin\beta\right\}.$$

An expression for $\overline{\cos\theta}$ is obtained by multiplying the right hand member of (28) by this expression for $\cos\theta$, integrating over all values of the distribution variables and dividing by N. It is thus found that:

$$
\begin{aligned}
\overline{\cos\theta} &= \frac{1}{8\pi}\int_0^\infty \frac{w(A)}{A}\,dA\int_{\pm(A-K)}^{A+K}\alpha\,\frac{(F^2+K^2-A^2)dF}{K^2}\int_0^\pi e^{a\cos\beta}\sin\beta\cos\beta\,d\beta\int_0^{2\pi}d\phi \\
(31)\qquad &+\frac{1}{8\pi}\int_0^\infty \frac{w(A)}{A}\,dA\int_{\pm(A-K)}^{A+K}\alpha\,\frac{\sqrt{(2FK)^2-(F^2-K^2-A^2)^2}\,dF}{K^2} \\
&\times\int_0^\pi e^{a\cos\beta}\sin\beta\cos\left(\frac{\pi}{2}-\beta\right)d\beta\int_0^{2\pi}d\phi,
\end{aligned}
$$

where the + sign in the lower limit of the integrals with respect to the variable F is to be taken if $A > K$ and the − sign if $A < K$.

Now the integral

$$\int_0^\pi e^{a\cos\beta}\sin\beta\cos\left(\frac{\pi}{2}-\beta\right)d\beta$$

is proportional to the magnetisation of the magnetons of Group A in a direction perpendicular to that of the resultant field F and this magnetisation must, on grounds of symmetry, vanish.

Consequently from (31), after integration with respect to ϕ and β and the introduction of the value of α from (29), we have finally:

$$(32)\qquad \overline{\cos\theta} = \frac{1}{4}\int_0^\infty \frac{w(A)}{A}\,dA\int_{\pm(A-K)}^{A+K}\left(\coth a-\frac{1}{a}\right)\left(\frac{F^2+K^2-A^2}{K^2}\right)dF,$$

where $\left(a = \frac{\mu F}{kT}\right)$.

No further progress toward the evaluation of $\overline{\cos\theta}$ can be made until the probability function w(A) has been determined.

The statistical problem here presented has been solved by Gans.[1] For the argument the reader is referred to the original paper; it is somewhat lengthy and only the result will be given here.

It is found that:

$$(33)\qquad w(A) = \frac{4\pi A^2}{\sqrt{\pi^3 A_0^3}}\,e^{-\frac{A^2}{A_0^2}},$$

where A_0 is a constant representing the most probable value of the molecular field A.

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[1] Gans: *Ann. d. Phys.* 50, p. 163; 1916.

Under the assumption that there is one magneton per molecule:

$$A_o = \frac{4}{3}\mu\sqrt{\frac{\pi N_o \rho}{M s^3}} \tag{34}$$

where μ is the magnetic moment of a magneton, M the molecular weight, N_o the Loschmidt number, ρ the density and s the nearest distance of approach of two magnetons.

Inserting in (32) the expression for w(A) given by (33) we obtain for the mean value of $\cos\theta$ the following expression:

$$\overline{\cos\theta} = \frac{\pi}{\sqrt{\pi^3 A_o^3}} \int_0^\infty e^{-\frac{A^2}{A_o^2}} A\,dA \int_{\pm(A-K)}^{A+K} \left(\coth a - \frac{1}{a}\right) \frac{F^2 + K^2 - A^2}{K^2})dF, \tag{35}$$

where $\left(a = \frac{\mu F}{kT}\right)$,

and the + sign in the lower limit of the integral involving F is to be used if $A > K$ and the − sign if $A < K$.

The Distribution Function in Quantum Theories of Paramagnetism.

The general law of distribution for the statistical variables of a system of similar units, which is expressed by equation (1) of the present section is a result of classical statistical theory which presupposes that the energy associated with any degree of freedom of a unit is capable of continuous variation.

It will appear, however, in Sect. VII of this review that to arrive at a satisfactory theory of paramagnetism which will account for experimental results at low temperatures it is necessary to replace the assumption that the energy associated with the various degrees of freedom of the rotating magnetons is capable of continuous variation by one which requires the energy to vary in accordance with Planck's quantum relation, $\epsilon = h\nu$.

It becomes necessary, therefore, to modify appropriately the law of distribution furnished by classical statistical mechanics in order to take account of Planck's quantum specifications relating to the energy associated with any degree of freedom of the rotating magnetons.

The problem of quantitization here presented is quite similar to that worked out by Planck in the derivation of his law of black body radiation but is considerably more complicated, as will appear in the discussion given in Sect. VII.

Further consideration of this matter is deferred until that section is reached.

The results obtained in the present section will be of service in connection with the discussion of certain theories of dia- and paramagnetism which will be considered later.

IV

EARLY ATTEMPTS AT ELECTRON THEORIES OF MAGNETISM

At the very beginning of the epoch covered by the present survey the foundations of the modern electron theory of matter were being rapidly laid. Investigations during the closing years of the preceding century furnished strong support to the view that the ultimate structure of matter is essentially electronic in nature.

In particular the assumption of an electronic constitution of matter was found competent to remove many outstanding difficulties encountered by Maxwell's electromagnetic theory in attempts to explain optical phenomena of dispersion.

Impressed with the success of the electron theory in this direction, Professor W. Voigt,[1] in 1902, was led to an investigation having for its object the determination of how far the electronic structure assumed for material bodies in the optical theory of dispersion could be made to serve in the explanation of the phenomena of magnetisation.

About the same time Sir J. J. Thomson[2] engaged in an investigation having the same object in view. His results were in accord with those found by Voigt somewhat earlier.

On account of the importance of the results found by both of these investigators it seems worth while to outline the argument of one of them.

Voigt's Attempt at an Electron Theory of Magnetism.

In the elementary theory of dispersion it is assumed that the molecules of a material body contain a number of electrons which, in the absence of an external electric or magnetic field, are in stable equilibrium, or in orbital motion about equilibrium configurations, under restoring forces of quasi-elastic nature proportional to the displacements of the electrons from their equilibrium positions. In order to account for absorption the assumption is made that a dissipative force acts on each electron proportional to its velocity of displacement. In case the body is subject to an external electric field **E** and an external magnetic field **H** an electron will experience two additional forces: one proportional to the electric field intensity and one proportional to the vector product of its velocity and the magnetic field intensity.

If ξ, η, ζ be the rectangular coordinates of an electron with respect to its equilibrium position as origin its equations of motion will be:

$$
\begin{aligned}
& m\ddot{\xi} = -h\xi - k\xi + eE_1 + \frac{e}{c}(\eta H_3 - \dot{\zeta}H_2); \\
(1)\qquad & m\ddot{\eta} = -h\eta - k\eta + eE_2 + \frac{e}{c}(\dot{\zeta}H_1 - \dot{\xi}H_3), \\
& m\ddot{\zeta} = -h\zeta - k\zeta + eE_3 + \frac{e}{c}(\dot{\xi}H_2 - \dot{\eta}H_1),
\end{aligned}
$$

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[1] W. Voigt: *Ann. d. Phys.*, 9, p. 115; 1902.

[2] J. J. Thomson: *Phil. Mag.* 6, Ser. 6, p. 673; 1903.

where h and k are constants, m the mass of the electron and e its charge. These are the fundamental equations of the elementary electron theory of dispersion, in which, however, the mutual effects of displacements of the electrons are not taken into account.

Professor Voigt introduces at this point the following assumptions:

I. The external electric field shall be zero.

II. The external magnetic field shall be constant and chosen parallel to the z-axis.

III. The dissipative constant h shall be zero, in order to correspond to Ampere's assumption of the existence of molecular currents encountering no resistance.

With these assumptions the solutions of equations (1) are respectively:

$$\begin{aligned} \xi &= a_1 \cos (p_1 t + \alpha_1) + a_2 \cos (p_2 t + \alpha_2), \\ \eta &= a_1 \sin (p_1 t + \alpha_1) - a_2 \sin (p_2 t + \alpha_2), \\ \zeta &= b \sin (pt + \beta), \end{aligned} \tag{2}$$

where a_1, a_2, α_1, α_2, b_1, β, p_1, p_2, p are constants and:

$$p = \sqrt{\frac{k}{m}}, \quad p_1 = p - \frac{eH}{2mc}, \quad p_2 = p + \frac{eH}{2mc}; \tag{3}$$

the values for p_1 and p_2 being approximate, in accordance with the assumption that the square of the natural periodicity p of the electron is large in comparison with the quantity $(eH/2mc)^2$.

As regards the initial conditions, the interval of time required for the establishment of the external magnetic field is supposed to be extremely small and its establishment is supposed to occur in such a way that the effects of the electric field, necessarily present during the period of establishment of the magnetic field, may be ignored.[1] To the order of approximation specified in the previous paragraph it may then be assumed that the configuration and the velocity of an electron is unchanged during the period of establishment of the external magnetic field.

Accordingly, we shall have, from (2), for the initial component displacements of the electron:

$$\begin{aligned} \xi_0 &= a_1 \cos \alpha_1 + a_2 \cos \alpha_2, \\ \eta_0 &= a_1 \sin \alpha_1 - a_2 \sin \alpha_2, \\ \zeta_0 &= b \sin \beta; \end{aligned} \tag{4}$$

and for the component initial velocities:

$$\begin{aligned} \dot{\xi}_0 &= - p_1 a_1 \sin \alpha_1 - p_2 a_2 \sin \alpha_2, \\ \eta_0 &= p_1 a_1 \cos \alpha_1 - p_2 a_2 \cos \alpha_2, \\ \zeta_0 &= pb \cos \beta. \end{aligned} \tag{5}$$

---

[1] It will appear later that the effects thus ignored are of fundamental importance in Langevin's theory of diamagnetism.

We now suppose the electron under consideration to be contained in a small element of volume, $d\tau$, of a material body and that the origin O of our system of coordinates is also contained within the element.

In order to test the magnetisation of the body we shall inquire as to the magnetic force due to this element at a point P on the Z-axis in the neighborhood of the element. We first need to find an expression for the magnetic force at P due to a single electron. It will, in fact, suffice to confine ourselves to the consideration of the z-component of this force.

Denoting OP by D, and supposing D large in comparison with OQ, this component force, to second order approximation in the small quantity ζ/D, from (11), Sect. I, may be expressed by:

$$\frac{e}{cD^3}(\dot\eta\xi - \dot\xi\eta)((1 + \frac{3\zeta}{D}). \tag{6}$$

If Z denote the mean value in time of this expression, we find, with the aid of (2), that:

$$Z = \frac{e}{cD^3}(p_1 a_1^2 - p_2 a_2^2).$$

or, after substituting the values of a_1^2 and a_2^2 obtained from (4) and (5):

$$Z = \frac{e}{cD^3}\left\{(p_1 - p_2)(\xi_o^2 - \eta_o^2) - p_1p_2(\xi_o + \eta_o) + 4pp_1(\dot\eta_o\xi - \dot\xi_o)\eta_o\right\} \tag{7}$$

Noting the values of p_1, p_2 and p given by (3) this equation is seen to reduce to:

$$Z = \frac{e}{cD^3}\left[\dot\xi_o\eta_o - \dot\eta_o\xi_o - \frac{eH}{2cm^2p^2}\left\{\frac{m}{2}(\dot\xi_o^2+\dot\eta_o^2) - \frac{k}{2}(\xi_o^2+\eta_o^2)\right\}\right]. \tag{8}$$

For brevity let:

$$\begin{aligned} Z_o &= \frac{e}{cD^3}(\dot\xi_o\eta_o - \eta_o\dot\xi_o), \\ \Phi_{zo} &= \frac{k}{2}(\xi_o^2 + \eta_o^2), \\ \Psi_{zo} &= \frac{m}{2}(\dot\xi_o^2 + \dot\eta_o^2). \end{aligned} \tag{9}$$

Here, evidently, Z_0 is the value of Z before the application of the magnetic field H; Φ_{zo} is the potential energy of the electron due to its displacement perpendicular to the z-axis at the instant (t = 0) the field is applied; and Ψ_{zo} is its kinetic energy at the same instant due to its motion perpendicular to the z-axis. At any time later the corresponding potential and kinetic energies will be denoted by Φ_z and Ψ_z. Using the abbreviations given by (9) equation (8) may be written:

$$(10) \qquad Z - Z_0 = -\frac{e^2H}{2c^2m^2p^2D^3}(\Psi_{z0} - \Phi_{z0}).$$

This equation expresses the difference between the z-components of the mean value in time of the magnetic force at P due to the motion of the electron at Q before and after the application of the magnetic field. For present purposes what is required is the mean value of $Z - Z_0$ due to the spatial distribution of the electrons in an element of volume $d\tau$ at 0, of which electrons the one considered above is typical; and to find this, the mean value in space of $\Psi_{z0} - \Phi_{z0}$ for the electrons in the element $d\tau$ is required. These electrons are assumed to be originally quite uncoordinated in configuration and motion.

Under no magnetic field the orbit of the typical electron will be elliptical, and the equations of the path of its projected motion on the xy-plane will be:

$$(11) \qquad \xi = \alpha \cos pt, \qquad \eta = \beta \sin pt;$$

so that:

$$\dot{\xi} = -p\alpha \sin pt, \ \dot{\eta} = p\beta \cos pt;$$

and hence:

$$\frac{m}{2}(\dot{\xi}^2 + \dot{\eta}^2) - \frac{k}{2}(\xi^2 + \eta^2) = \frac{k}{2}(\beta^2 - \alpha^2) \cos 2pt.$$

It follows, then, that at the instant the magnetic field is applied:

$$(12) \qquad \Psi_{z0} - \Phi_{z0} = \frac{k}{2}(\beta^2 - \alpha^2) \cos 2pt.$$

Equation (11) is typical for a large number, N, of electrons in the volume element dt. Let dt be the time of description by the typical electron of an element of its orbit of which the projection on the xy-plane is ds. At any instant the probability that the electron will be on this element of its orbit will be dt/T where T is the periodic time in which the electron describes its orbit. The "expectation," then, for the number of the N electrons which will be in the same element of phase in their respective orbits as that defined in position and magnitude by the element ds in the case of the typical electron will be Ndt/T; and hence the mean value, $\bar{q}$, in space of any common quantity, q, associated with each of the N electrons will be expressed by:

$$\bar{q} = \frac{1}{N}\int_0^T q\frac{Ndt}{T} = \frac{1}{T}\int_0^T q\,dt.$$

Therefore, if the left hand member of (12) be taken for q:

$$\bar{\Psi}_{z0} - \bar{\Phi}_{z0} = \frac{1}{T}\int_0^T (\Psi_{z0} - \Phi_{z0})dt.$$

The right hand member of this equation vanishes by virtue of (12), and hence the expression on the left also vanishes. From (10) it now follows that:

$$\bar{Z} = \bar{Z}_0. \tag{13}$$

Consequently, in accordance with the present theory, if the body in question were originally unmagnetised, it would remain so upon the application of a magnetic field.

A medium with the electron structure assumed in the elementary theory of dispersion thus fails to account for either para- or diamagnetism when the electrons are supposed to move in their orbits without dissipation and without collisions. If dissipation be assumed it is necessary to the existence of a steady state that the electrons receive through collisions accessions of energy. The question then arises as to whether under these conditions the magnetisation due to the motion of the electrons will be different with, and without a magnetic field.

As far as the answer to this question is concerned dissipation in a sense may be ignored. For the effect of dissipation on the motion of the electron will be compensated by the continually recurring collisions, which for simplicity are supposed instantaneous. Now in the theory of dispersion the time of description of its orbit by an electron is very small and it is therefore here assumed that an electron will describe its orbit many times between successive collisions.

In the discussion which precedes it was shown that the difference, $Z - Z_0$, between the z-components of the mean value in time of the magnetic force at P due to the motion of the typical electron with and without the magnetic field **H** depends simply upon the difference $\Psi_{z0} - \Phi_{z0}$ of its potential and kinetic energy due to its displacement and motion perpendicular to **H** at the instant the magnetic field is applied. In the case now under consideration, where collisions are taken into account, it is therefore easily seen that the effect of a collision of the typical electron moving in the constant field of strength H is to change the value of $Z - Z_0$ given by (10) to a new value given by:

$$Z - Z_0 = -\frac{e^2H}{2c^2m^2p^2D^3}(\Psi_{z1} - \Phi_{z1}), \tag{14}$$

where Φ_{z1} and Ψ_{z1} are respectively the potential and kinetic energy at an instant just after the collision due to the displacement and motion of the electron perpendicular to **H**.

If the collisions are quite at random, then, in the case of isotropic bodies at any rate, the mean value of Z_0 due to the motions of the electrons in an element of volume $d\tau$ at O must vanish, since magnetisation would require the presence of a magnetic field. Hence to obtain the mean value of Z we have only to ignore Z_0 in (14) and find the mean value in space of the right hand member of this equation. Hence, if n denote the number of electrons per unit volume of the type considered:

$$\overline{Z} = - \frac{e^2 H n d\tau}{2c^2m^2p^2D^3}(\overline{\Psi}_{z1} - \overline{\Phi}_{z1}). \tag{15}$$

Assuming completely uncoordinated configurations and motions of the electrons $\overline{\Phi}_{z1}$ is two thirds of the mean potential energy, and $\overline{\Psi}_{z1}$ is two thirds of the mean kinetic energy of the electrons reckoned for configurations and motions just after collisions. If Φ_1 and Ψ_1 denote respectively the mean potential—and the mean kinetic energy per unit volume for these configurations and motions, then:

$$\overline{Z} = - \frac{e^2 H d\tau}{3c^2m^2p^2D^3}(\Phi_1 - \Psi_1).$$

But this is equal to the magnetic force which would be produced at the point P by a small magnet at O with its axis in the direction OP and with a moment

$$\frac{e^2 H d\tau}{6c^2m^2p^2}(\Phi_1 - \Psi_1).$$

Hence, if M be the magnetic moment per unit volume:

$$M = \frac{e^2 H}{6c^2m^2p^2}(\Phi_1 - \Psi_1); \tag{16}$$

and, if κ be the volume magnetic susceptibility:

$$\kappa = \frac{e^2}{6c^2m^2p^2}(\Phi_1 - \Psi_1). \tag{17}$$

It appears, from the result expressed by equation (17), that with the assumptions of the present argument it is possible to account for both para- and diamagnetism in a medium having the same electronic structure as that which serves so well in the optical theory of dispersion. Moreover, the present theory does not require for the explanation of para- and diamagnetism two essentially different fundamental assumptions, as is the case in the older theories of Ampere and of Weber.

The theory of Voigt leaves open the way to explanation of the experimentally well known variations of magnetic susceptibility with changes in the physical state of the medium, through the variations in the circumstance of collision which such changes of state entail. Our knowledge, however, of what excites and maintains the motion of the electrons is far too scant to enable the theory to predict how any particular medium will behave under the action of a magnetic field.

Sir J. J. Thompson, in his theoretical investigation of the magnetic properties of a material with a molecular structure in which electrons are supposed to be grouped in rings with the electrons in any ring spaced at equal distances around the ring and rotating with a common angular velocity in a plane about an axis through its center, arrived at the result that, unless the electrons were subject to loss of energy through dissipation, the material would show neither dia- nor paramagnetic quality. This is in agreement with the negative result found by Voigt with the method outlined above. In the case for which dissipation is assumed it was found that paramagnetism would result.

The difference between the magnetic properties of electrons describing free orbits with no dissipation, in accordance with the analysis of Thomson, and the constant molecular currents assumed by Ampere, appears from the analysis of Thomson to be due to the fact that in the case of the electrons describing free orbits with no dissipation dia- and paramagnetic effects just cancel each other.

Having been led to the negative result stated above, Thomson, in the same paper (1903), suggested that the magnetic properties of a substance may depend upon the properties of aggregations of large numbers of molecules. In the light of the trend of ideas in the subsequent development of theories of magnetism a quotation is warranted:

"In the case of such aggregations, however, we may easily conceive that the orbits of charged bodies moving within them may not be free, but that in consequence of the forces exerted by the molecules in the aggregate the orbit may be constrained to occupy an invariable position with respect to the aggregate—as if, to take a rough analogy, the orbit was a tube bored through the aggregate, so that the orbit and aggregate move like a rigid body, and in order to deflect the orbit it is necessary to deflect the aggregate. Under these conditions it is easy to see that the orbits would experience forces equivalent on the average to those on a continuous current flowing around the orbit; the aggregate and its orbit would under these forces act like a system of little magnets; and the body would exhibit magnetic properties quite analogous to those possessed by a system of Amperean currents."

There is here a suggestion of a possible modification of the molecular structure assumed in the optical theory of dispersion which might be competent to account for the magnetic properties of material bodies. The direction is indicated along which electron theories of magnetism might naturally develop, while retaining the Amperean conception of a magnetic molecule with currents circulating without resistance within it in orbits which are in rigid connection with the molecule itself.

An important advance in this direction was made by Langevin in 1905.

V

THE THEORY OF LANGEVIN

The electron theory of magnetism proposed by Langevin[1] in 1905 demonstrated that with a suitably conceived magnetic molecule or magneton it is possible to account satisfactorily for both dia- and paramagnetism.

The basic ideas upon which the theory of Langevin rests have been adopted in nearly all theories of magnetism developed since 1905. This theory is therefore reviewed below in some detail.

A magnetic molecule as conceived by Langevin contains a number of electrons of which some are negative and some positive, the algebraic sum of the charges on all the electrons in a molecule being zero. Some of the electrons are supposed to be in orbital motion within the molecule in closed orbits and the planes of the orbits are supposed to maintain, by virtue of internal forces, definite orientations with respect to the molecule as a whole. The arrangement of the orbits may possess such a degree of symmetry that the resultant magnetic moment of the molecule is zero. On the other hand, if the arrangement fail of such symmetry, the magnetic moment of the molecule will have a finite value.

It will appear that the effect of the application of an external magnetic field to a body with a structure of such magnetic molecules is to accelerate the motions of the electrons in their orbits in a sense to produce diamagnetism. In case the magnetic moments of the molecules are not zero there will be superimposed upon this effect another, viz., an orientation of the molecules tending to line up their magnetic axes in the direction of the external field.

In the following brief review of Langevin's celebrated paper of 1905 changes in the notation have been made with the object of making it conform more nearly with that used above and vector methods replace cartesian.

Diamagnetism.

An examination of the properties of the molecular structure assumed by Langevin for diamagnetic isotropic bodies will show how it is competent to account for diamagnetism in such bodies.

We consider a small element of volume of such a body which for generality is supposed to be in motion. The element is supposed to contain a large number of electrons, some of which, at any rate, are in rapid orbital motion about the centroids of the molecules to which they belong. Let O, Fig. 4, be the centroid of these electrons, moving with velocity **v**, and let (x, y, z) be a system of rectangular axes whose directions are fixed in space but whose origin coincides with O at the instant under consideration. Let Q(x, y, z) be the position of a typical electron, C(a, b, c)

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[1] *Ann. de Chim. et de Phys.* Ser. 8, t. V, p. 70; 1905.

the centroid of the molecule to which this electron belongs; and, with reference to O, let $\mathbf{r}$ be the position vector of Q, and $\mathbf{q}$ that of C; while $\mathbf{s}$ is the position vector of Q with reference to C.

Assuming the element to be electrically neutral and unpolarized we have:

$$\Sigma e = 0; \quad \Sigma e\mathbf{s} = 0. \tag{1}$$

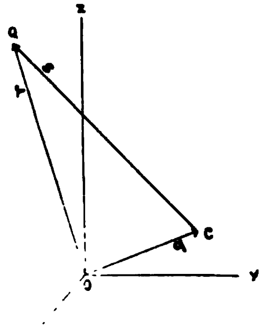

FIG. 4

Since O is the centroid of the element, $\Sigma x = \Sigma y = \Sigma z = 0$; and, since it is isotropic:

$$\begin{aligned} \Sigma xy &= \Sigma yz = \Sigma zx = 0, \\ \Sigma a &= \Sigma b = \Sigma c = 0, \\ \Sigma ab &= \Sigma bc = \Sigma ca = 0. \end{aligned} \tag{2}$$

It then follows, if ξ, η, ζ be the coordinates of Q with reference to C, that:

$$\Sigma\xi = \Sigma\eta = \Sigma\zeta = \Sigma\xi\eta = \Sigma\eta\zeta = \Sigma\eta\xi = 0, \tag{3}$$

where the summations are to be taken over all the electrons in the element.

As far as the mean magnetic field of the electron is concerned the electron at Q, due to its motion with velocity $\dot{\mathbf{s}}$ about the centroid C, is, from (12) Sect. I, equivalent to a small magnet whose moment is

$$\frac{e}{2c}\mathbf{s}\times\dot{\mathbf{s}}; \tag{4}$$

and, if $\mathbf{M}$ be the magnetic moment of the element of volume due to all the electrons of a given type within it, say classical negative electrons, then:

$$\mathbf{M} = \frac{e}{2c}\Sigma\mathbf{s}\times\dot{\mathbf{s}}, \tag{5}$$

where the summation is over all the electrons of the type considered.

By differentiation with respect to the time we find for the time rate of change of this quantity:

$$\dot{\mathbf{M}} = \frac{e}{2c}\Sigma\,\mathbf{s}\times\ddot{\mathbf{s}}. \tag{6}$$

If $\mathbf{F}$ be the force on the typical electron due to the action upon it of the rest of the molecule in which it is situated, $\mathbf{E}$ and $\mathbf{H}$ the electric and magnetic force, respectively, of external origin, we shall have from the equation of motion of the typical electron:

$$m\ddot{\mathbf{s}} = \mathbf{F} + e\mathbf{E} + \frac{e}{c}(\mathbf{v} + \dot{\mathbf{s}})\times\mathbf{H} - m\ddot{\mathbf{q}} - m\dot{\mathbf{v}}, \tag{7}$$

where e is the charge of the electron and m its mass.

Here the quantities **F**, **E**, and **H** all refer to the point at which the typical electron is situated, but, since the element of volume is small, they may be expressed as follows:

$$\begin{aligned} \mathbf{F} &= \mathbf{F}_0 + (\mathbf{s}.\nabla\mathbf{F})_0 + \ldots, \\ \mathbf{E} &= \mathbf{E}_0 + (\mathbf{s}.\nabla\mathbf{E})_0 + \ldots, \\ \mathbf{H} &= \mathbf{H}_0 + (\mathbf{s}.\nabla\mathbf{H})_0 + \ldots. \end{aligned} \tag{8}$$

where the zero subscript indicates that the corresponding quantity is to be evaluated at O, the centroid of the element.

From (6), (8), (2), and (3) we obtain, upon neglecting terms of higher order than the first in the small quantity s, writing

$$\Sigma\xi^2 = \Sigma\eta^2 = \Sigma\zeta^2 = \frac{I}{2}, \tag{9}$$

and dropping the zero subscripts:

$$\dot{\mathbf{M}} = \frac{e}{2mc}\Sigma\, \mathbf{s}\times\mathbf{F} + \frac{e^2}{4mc}\left\{I\left(\text{curl}\,\mathbf{E} + \frac{1}{c}\,\mathbf{v}\,\text{div}\,\mathbf{H} - \frac{1}{c}\mathbf{v}\,\nabla\mathbf{H}\right) - \frac{1}{c}\frac{dI}{dt}\mathbf{H}\right\}, \tag{10}$$

in which the vectors and their space derivatives refer to the point O.

Now, from Maxwell's field equations:

$$\text{div}\,\mathbf{H} = 0; \qquad \text{curl}\,\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t} = \frac{1}{c}\left(\mathbf{v}\,\nabla\,\mathbf{H} - \frac{d\mathbf{H}}{dt}\right);$$

and consequently the preceding equation reduces to:

$$\dot{\mathbf{M}} = \frac{e}{2mc}\Sigma\,\mathbf{s}\times\mathbf{F} - \frac{e^2}{4mc^2}\frac{d}{dt}(I\mathbf{H}). \tag{11}$$

The first term on the right represents a time variation in **M** due to the action of the internal forces of the molecules; this vanishes if, as in Langevin's theory of diamagnetism, each molecule has no initial magnetic moment.

If $\Delta\mathbf{M}$ denote the change in the magnetic moment of the element due to the establishment of the external field within it, then, by integration of the last equation:

$$\Delta\mathbf{M} = -\frac{e^2}{4mc^2}I\mathbf{H}. \tag{12}$$

Owing to the creation of the external field within it the element thus acquires a diamagnetic moment.

It may be noticed that in the expression for $\Delta\mathbf{M}$ the charge of an electron appears as a square. Consequently, if positive as well as negative electrons are in orbital motion within the molecules, they, too, will give rise to diamagnetism in accordance with (12). On account of the large mass of the positive electron, however, the contribution of the positive electrons to diamagnetism would probably be very small in comparison with that of the negative.

For the quantity I we may write nk^2, n being the number of electrons in the element of volume and k^2 the square of the radius of gyration of the mean configuration of the electrons in a molecule with respect to an axis through their centroid. From (12) we then have:

$$\Delta \mathbf{M} = -\frac{ne^2}{4mc^2} k^2 \mathbf{H}. \tag{13}$$

From (14) Sect. I, the mean absolute value of the components of the magnetic moments of the n orbits in the direction of **H**, say $\bar{\mu}_H$, will be given by:

$$\bar{\mu}_H = \frac{eS}{c\tau} \overline{\cos\theta}, \tag{14}$$

where $\overline{\cos\theta}$ denotes the mean value of $\cos\theta$, θ being the angle between n and **H**.

From (13) the change in $\bar{\mu}_H$, say $\Delta\bar{\mu}_H$, due to the creation of the magnetic field **H** will be given by:

$$\Delta\bar{\mu}_H = -\frac{e^2}{4mc^2} k^2 \mathbf{H}. \tag{15}$$

It follows from (14) and (15) that the ratio of $\Delta\bar{\mu}_H$ to $\bar{\mu}_H$ must be very small for all attainable field strengths; in fact less than 10^{-4} if τ be assumed of the order of the period of light vibrations, say 10^{-15} seconds.

If κ be the magnetic susceptibility per unit volume and N the number of electrons per unit volume, then, from (13):

$$\kappa = -\frac{Ne^2}{4mc^2} k^2 = -\frac{\rho}{4}\left(\frac{e}{mc}\right)^2 k^2, \tag{16}$$

where ρ is the mass density of electrons per unit volume.

In accordance with the argument advanced here all substances will possess the diamagnetic property. If the magnetic molecules of any substance possess initial magnetic moment of their own, they will possess paramagnetic as well as diamagnetic quality. If the initial moment be zero, no external action upon the molecule will produce one.

The argument has left out of account any explicit reference to the effect of collisions among the molecules upon the diamagnetic state of the substance. It will be recalled, however, that there has been nothing assumed in the argument to prevent motion of the most general kind of the element of volume containing the ensemble of electrons; and hence, whatever be the motion of the ensemble, its diamagnetic state is at each instant determined simply by its actual configuration with reference to the external magnetic field, and therefore is independent of collisions among the molecules.

Again the argument does not take account of the interior forces of a molecule which may result from the diamagnetic action itself. But it will be seen presently that, in the mean, a diamagnetic modification implies only a change of velocity of an electron in its orbit without deformation of the orbit, and the absence of a mean deformation of the molecule due to a diamagnetic modification implies that the corresponding interior reactions due to it must be negligible.

The fixity of spectral lines lends important support to the view that the intramolecular motions of a substance depend but slightly upon the temperature; the comparatively slow thermal motions can therefore modify but very little the intramolecular motions giving rise to diamagnetism on the present theory. The diamagnetic property is thus practically independent of temperature, according to the experimental law of Curie. There are, however, many exceptions to this law.

An important question is that relating to a possible change in the area of the orbit of an electron due to the action of an external magnetic field. Let f(r) be the central force which holds an electron in its orbit, supposed circular.

In the absence of an external field:

$$m\omega^2 r = f(r), \tag{17}$$

where ω is the angular velocity of an electron in its orbit.

If H_n denote the component of the external field perpendicular to the plane of the orbit, then, after the field is applied:

$$m\,(\omega + \Delta\omega)^2\,(r + \Delta r) + \frac{1}{c} H_n\, e\, \omega\, r = f\,(r + \Delta r),$$

where $\Delta\omega$ and Δr are the variations in ω and r respectively due to the action of the field. Retaining terms of the first order only in the small quantities, $\Delta\omega$ and Δr, we therefore have:

$$f'\,(r)\,\Delta\, r = 2\, m\, r\, \omega\, \Delta\, \omega + m\, \omega^2\, \Delta\, r + \frac{1}{c} H_n\, e\, \omega\, r;$$

and hence:

$$\left\{f'\,(r) - m\,\omega^2\right\} \Delta r = 2\, m\, \omega\, r\, \Delta\, \omega + \frac{1}{c} H_n\, e\, \omega\, r. \tag{18}$$

Now, if τ be the orbital period and S the orbital area, then, using (14) and (17), Sect. I:

$$\frac{\Delta\omega r^2}{2} = \frac{c}{e}\,\Delta\,\frac{eS}{c\tau} = \frac{c}{e}\,\Delta\,\mu = -\,\frac{e}{4\pi mc}\,\Delta\,(H\,S\,\cos\theta) = -\,\frac{er^2}{4mc}\,H_n;$$

and, therefore:

$$r^2 \Delta \omega + 2 \omega r \Delta r = - \frac{H_n e}{2mc} r^2,$$

(19) $$- 4 m \omega^2 \Delta r = 2 m \omega r \Delta \omega + \frac{1}{c} H_n e \omega r,$$

From (18) and (19):

$$\{f'(r) + 3 m \omega^2\} \Delta r = 0.$$

Thus, either:

(a) $$\Delta r = 0; \quad \Delta \omega = - \frac{H_n e}{2mc}.$$

or:

(b) $$f'(r) = - 3 m \omega^2 = - \frac{3f}{r}.$$

If condition (b) is satisfied,

$$\frac{f}{f'} = - \frac{3}{r};$$

and hence:

(20) $$f = \frac{K}{r^3},$$

where K is a constant.

Except, then, in the very special case that the central force holding the electron in its orbit varies inversely as the cube of the radius of the orbit, condition (a) will be satisfied, the effect of the magnetic field being simply to cause a variation of the angular velocity of the electron by the amount $- H_n e/2mc$.

It is evident that the component of the magnetic force in the plane of the orbit will not operate to change the area of the orbit, since the displacements to which it gives rise are perpendicular to the plane of the orbit.

The change of period, giving rise to diamagnetism, in the orbital motions of electrons within the atoms corresponds to the simple Zeeman effect in magneto-optics.

It is of some little interest to compare the formulas (16) found for diamagnetism by Langevin with that which holds for a substance which is constituted of the spherical magnetons discussed in Sect. II.

It was there shown that the effect of establishing an external magnetic field **H** within such a magneton was to change its magnetic moment by an amount:

(21) $$\nu - \nu_0 = - \frac{Q^2}{4mc^2 J} H,$$

where Q is the moment of inertia of charge of the magneton and J its ordinary moment of inertia with respect to an axis through its centroid. If the magneton be constituted of electrons of a single type, of mass m and charge e, symmetrically spaced about a positive nucleus then $Q = ek^2$ and $q = mk^2$ where k is the radius of gyration of the electrons in the magneton.

If, then, κ be the volume magnetic susceptibility of the body constituted of such magnetons it follows from (21) that:

$$\kappa = -\frac{Ne^2}{4mc^2}k^2,$$

where N denotes the number of electrons per unit volume and ρ the mass density of the electrons. This result agrees with that expressed by (16).

Paramagnetism.

A body will exhibit paramagnetic quality in the presence of a magnetic field in addition to the diamagnetism considered above when its magnetic molecules have individually other than zero magnetic moment. The theory appropriate to a paramagnetic gas was developed by Langevin in his paper of 1905. Later this theory was made the basis of a theory of ferromagnetism by Weiss.

In Langevin's theory of a paramagnetic gas the magnitude of the magnetic moment of a molecule is supposed to be invariable under all conditions, the slight diamagnetic changes in its moment being ignored.

It is of interest to examine first, as regards its general nature, the process whereby the paramagnetic state is set up when a gas is subjected to an external magnetic field. At the instant the field is applied the diamagnetic state discussed above will be established immediately. The paramagnetic state, on the other hand, will require an appreciable time for its establishment.

At the instant the magnetic field H is applied a magnetic molecule acquires potential energy with respect to the field of amount

$$-\mathbf{H}\cdot d\boldsymbol{\mu}$$

where $\boldsymbol{\mu}$ is its magnetic moment. This increase in the potential energy of a molecule is derived initially from its kinetic energy of rotation, just as the potential energy of a molecule of a gas subjected to a gravitational field is acquired from its kinetic energy while it is rising in the field. Now the result of this partition of kinetic energy among the various degrees of freedom (translation and rotation) of the molecules is incompatible with thermal equilibrium. It is in the process of the establishment of thermal equilibrium through collisions that paramagnetism makes its appearance. In this process magnetic energy is derived from the energy of thermal agitation of amount

$$-\mathbf{H}\cdot d\boldsymbol{\mu}.$$

If the molecules have no other potential energy relative to their orientation, as in the case of a gas and probably a liquid, in order to maintain the medium at a constant temperature it would be necessary at each instant to furnish to it an amount of heat energy per unit volume equal to $-\mathrm{H}.d\mathrm{I}$, if I denote intensity of magnetisation. In the case of a solid where the molecules have a potential energy of orientation it is only for the case of a closed cycle that a similar conclusion may be drawn.

With the aid of the laws of thermodynamics it is easily shown that the magnetic moment M of a given mass of a paramagnetic substance in an external field of strength H must, in the case of a gas or a liquid, be a function of H/T:

$$\mathrm{M} = f\left(\frac{\mathrm{H}}{\mathrm{T}}\right)$$

where T is the absolute temperature.

For a small reversible modification in which H changes by dH, and T by dT, the heat evolved, say dQ, which depends upon H, will be given by:

$$dQ = \mathrm{H}\,\left(\frac{\partial \mathrm{M}}{\partial \mathrm{H}}\,d\mathrm{H} + \frac{\partial \mathrm{M}}{\partial \mathrm{T}}\,d\mathrm{T}\right).$$

Since the modification is reversible dQ/T must be a perfect differentia and hence:

$$\frac{\partial}{\partial \mathrm{T}}\left(\frac{\mathrm{H}}{\mathrm{T}}\frac{\partial \mathrm{M}}{\partial \mathrm{H}}\right) - \frac{\partial}{\partial \mathrm{H}}\left(\frac{\mathrm{H}}{\mathrm{T}}\frac{\partial \mathrm{M}}{\partial \mathrm{T}}\right) = 0;$$

from which it follows directly that:

$$\mathrm{T}\frac{\partial \mathrm{M}}{\partial \mathrm{T}} + \mathrm{H}\frac{\partial \mathrm{M}}{\partial \mathrm{H}} = 0.$$

The integral of this equation is given by:

$$\mathrm{M} = f\left(\frac{\mathrm{H}}{\mathrm{T}}\right), \tag{22}$$

which is the result it was desired to prove. The argument is readily extended to show that this result will also hold for a solid body, provided its internal energy does not depend appreciably upon H.

Thermodynamics alone will not permit of the determination of the function f. For many substances experiment shows M to be directly proportional to H and this, with the result expressed by (22), if the conditions stated are satisfied, leads to the result:

$$\mathrm{M} = \frac{\mathrm{A}}{\mathrm{T}}\mathrm{H} \tag{23}$$

where A is a constant independent of T.

In the particular case of a paramagnetic gas such as oxygen the form of the function f may easily be determined.

Theory of a Paramagnetic Gas.

In his theory of a paramagnetic gas Langevin assumes each of the magnetic molecules to have a magnetic moment, μ, the magnitude of which is the same for all molecules. The direction of the magnetic axis of the molecule is then that of the vector μ. Effects due to the rotations of a molecule about axes perpendicular to its magnetic axis are ignored. The molecular magnetic field is also ignored, since it will certainly be very small for gases under ordinary conditions.

The appropriate distribution function for this case, as Langevin showed, is given by (22) Sect. III; and from (23) of the same section the mean value, $\overline{\cos\theta}$, of the cosines of the angles made by the magnetic axes of the molecules with the external magnetic field H is expressed by:

$$\overline{\cos\theta} = \coth a - \frac{1}{a}; \qquad \left(a = \frac{\mu H}{kT}\right), \tag{24}$$

where k is Boltzmann's constant and T the absolute temperature.

On grounds of symmetry I, the intensity of magnetisation, must be in the direction of the external field H; and it must be equal in magnitude to the sum of the projections of the moments of the individual molecules in unit volume in this direction. Accordingly I will be given by:

$$\begin{aligned} I &= \mu n \overline{\cos\theta} \\ &= \mu n \left(\coth a - \frac{1}{a}\right) \end{aligned} \tag{25}$$

where n denotes the number of magnetic molecules per unit volume.

From this result it appears that I is a function of H/T as required by thermodynamics and, moreover, owing to the factor n, that it is directly proportional to the pressure of the gas.

When the bracketed expression on the right of the expression for I takes on the value unity I will assume its maximum value, μn. Denoting the maximum value of I by I_0, from (25):

$$I = I_0 \left(\cosh a - \frac{1}{a}\right). \tag{26}$$

The curve in Fig. 5 shows the manner in which I/I_0 varies with a. It will be seen presently that under ordinary conditions of experiment a will be quite small for oxygen; in fact of the order 10^{-2}.

When a is small I/I_0 will vary directly with H. At low temperatures, however, and for powerful fields a may become so large that the relation between I/I_0 and a becomes non-linear.

From (26), by development of coth a in ascending powers of a, neglecting powers of higher order than the first:

$$(27) \qquad I = I_0 \frac{a}{3},$$

with sufficient approximation under ordinary conditions of experiment; and, if κ be the coefficient of volume magnetic susceptibility:

$$(28) \qquad \kappa = I_0 \frac{\mu}{3kT};$$

showing that κ varies inversely with the temperature in accordance with what is known as Curie's law of paramagnetism.

The preceding theory may also be valid for a medium other than a polyatomic gas, such as oxygen, when the energy of rotation of the molecules is known to be a function of the temperature, in accordance with thermodynamic theory. In all such cases it is only necessary that the energy of rotation shall be proportional to the absolute temperature in order that the theory may be applicable; the quantity k only will have to be modified.

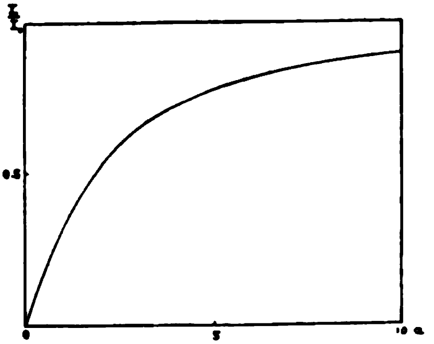

FIG. 5

All magnetic substances for which the mutual actions among the molecules are negligible, such as solutions of paramagnetic salts, should have magnetisation curves exactly similar.

The expression for κ given by (28) may be written:

$$\kappa = \frac{I_0^2}{3p},$$

since $I_0 = n\mu$, and $p = nkT$, p being the pressure of the gas.

At normal pressure, and at the temperature 0°C, Curie found for oxygen:

$$\kappa = 1.43 \times 10^{-7}.$$

It follows that the maximum intensity of magnetisation for oxygen will be given by:

$$I_0 = (3 \times 10^6 \times 1.43 \times 10^{-7})^{\frac{1}{2}} = 0.65.$$

For liquid oxygen, therefore, with a density 500 times greater, a value of $I > 325$ might be expected.

The order of a for oxygen under ordinary conditions of experiment may now be found. We have:

$$a = \frac{I_0 H}{nkT} = \frac{I_0 H}{p}.$$

The value of I found above for oxygen under normal conditions was 0.65. Hence:

$$a = 0.65 \times 10^{-6} H;$$

for a fairly powerful field, H = 10,000 say, and then:

$$a = 0.65 \times 10^{-2}.$$

If it be admitted that the magnetic moment μ for a molecule of oxygen is due to a single electron with a charge equal to that of an atom of hydrogen in electrolysis rotating in a circular orbit of radius r equal to 1.5×10^{-8} cm., the velocity of the electron may be calculated as follows. Since, from (14), Sect. I,

$$\mu = \frac{eS}{c\tau} = \frac{evr}{2c},$$

where S is the area of the orbit, τ the periodic time and v the velocity of the electron, we shall have:

$$I_0 = n\mu = \frac{1}{2c} n\, evr.$$

Now e, being expressed in electrostatic units,

$$\frac{ne}{c} = 0.40;$$

and since under normal conditions, as found above, $I_0 = 0.65$, it follows that:

$$0.65 = 0.40 \times \frac{1.5 \times 10^{-8}}{2} \times v,$$

from which:

$$v = 2 \times 10^8 \text{ cm/sec.}$$

This velocity is of the same order as that which an electron would have in stable circular orbital motion about a positive charge of equal magnitude placed at the center of the orbit. For in this case:

$$\frac{m\,v^2}{r} = \frac{e^2}{r^2}, \quad v^2 = \frac{e^2}{m\,r},$$

from which:

$$v = 10^8 \text{ cms/sec.}$$

It is worthy of note that the resultant magnetic moment of a molecule of oxygen may be accounted for by the orbital motion of a single electron; this would also be true for a molecule of iron, for which the maximum magnetisation per molecule is of the same order as that for oxygen.

In the case of the magnetisation of a paramagnetic gas such as oxygen, we have seen that the kinetic energy of the molecules furnishes per unit volume during the period of rearrangement (which results in the appearance of paramagnetism) an amount of energy

$$-\int \mathbf{H}.d\mathrm{I};$$

so that the energy per unit volume of the medium is augmented by an amount

$$\frac{1}{2}\kappa \mathrm{H}^2.$$

The gas must therefore be heated by an amount which may be calculated as follows.

Suppose the volume to remain constant and let $\Delta\mathrm{T}$ be the rise in temperature due to magnetisation, then:

$$\mathrm{C}\,\Delta\mathrm{T} = \frac{1}{2}\kappa \mathrm{H}^2.$$

C being the specific heat at constant volume. Now, approximately, in C. G. S. units:

$$\mathrm{C} = 10^4; \quad \kappa = 1.43 \times 10^{-7};$$

and, therefore,

$$\Delta\mathrm{T} = 0.8 \times 10^{-11}\mathrm{H}^2.$$

From this result, for $\mathrm{H} = 10{,}000$, $\Delta\mathrm{T} = 10^{-3}\,\mathrm{C}^\circ$; while for $\mathrm{H} = 40{,}000$, $\Delta\mathrm{T} = 10^{-2}\,\mathrm{C}^\circ$. This elevation of temperature would vary directly with the susceptibility κ, and therefore inversely with the absolute temperature.

In concluding this somewhat brief review of Langevin's theory the following remarks may prove to be of interest later.

His theory of paramagnetism is what may be termed an equipartition theory; for it is based on classical statistical theory that leads to equipartition of energy among the statistical coordinates of a system which appear only as the sum of squares in the energy function of a statistical unit.

The property of permanancy is given to the magnetic moments of the molecules; for example, these moments are not subject to variation with temperature.

The effects of intra-molecular forces have been ignored, thus restricting the range of application of the theory to paramagnetic gases.

By ignoring the effects due to rotations of the molecules about axes perpendicular to their magnetic axes they are deprived of gyroscopic properties which, as we shall see, may play an important role in magnetisation.

VI

MODIFICATIONS OF THE THEORY OF LANGEVIN INDEPENDENT OF QUANTA HYPOTHESES

The theory of Langevin, as we have seen, leads in the case of diamagnetism to the result that the diamagnetic susceptibility of all bodies should be independent of the temperature and the field strength; and in the case of paramagnetism to Curie's law, which requires the susceptibility to vary inversely with the absolute temperature.

Now many of the experimental facts found since the time (1905) of publication of Langevin's theory are not in accord with these results. Consequently various attempts at modification of the theory have been made. In the present section we shall consider modifications of the Langevin theory which do not invoke the aid of quantum hypotheses.

Theory of Honda.

Kotaro Honda[1] in 1914 proposed a modification based upon the following two assumptions:

(a)—The magnetic moments of molecules are not constant but depend upon the temperature.

(b)—The molecules exert mutual forces upon one another, the tendency of which is to prevent their lining up in the direction of the external field.

A magnetic molecule in the case of a solid is supposed by Honda to consist in general of an aggregate of a number of actual molecules, such aggregates being subject, however, to the usual laws of thermal molecular motion. In accordance with assumption (a) the form of a molecule is supposed to depend upon the temperature; a change in form involving at the same time a change in the value of the magnetic moment of the molecule. Thus the form of the molecule of a body in the ferromagnetic state is assumed to be spherical, so that it shall not be subject to orientation through thermal impacts. In the ferromagnetic range of temperatures the small mutual forces only will be operative in opposing the tendency of the magnetic molecules to line up with their axes along the direction of the external magnetic field, and in consequence a large magnetisation results in this case. In the passage from the ferromagnetic state to the paramagnetic the magnetic molecule is supposed to pass from the spherical to an elongated form, with the result that a large thermal action opposing the lining up of the molecules becomes operative, and consequently the body passes from the ferro- to the paramagentic state. The energy of deformation of the molecules in this process together with that required by the new degrees of freedom is supposed to account for the heat absorbed in the process of transition.

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[1] K. Honda, Tokio, *Sci. Rep.* 3, p. 171; 1914.

The distribution function proposed by Honda, incorporating the assumptions (a) and (b), is:

$$\alpha\, e^{\frac{\mu_0 f\,(T)\,H}{kT + \phi}\cos\theta},$$

where $\mu_0 f(T)$ represents the magnetic moment of a molecule, μ_0 being the value of this quantity at absolute zero, and ϕ is a constant or a function of the temperature expressing the mutual action of the molecules upon one another. This distribution function of Honda reduces to that of Langevin if $f(T) = \text{const.}$, and $\phi = 0$.

The symbols other than $\mu_0 f(T)$, and ϕ have the same significance as in Langevin's theory.

The function ϕ which expresses the mutual action of the molecules represents an effect which, like thermal action, tends to hinder the lining up of the molecules with their axes in the direction of the external magnetic force, and hence is added to the temperature factor kT. In paramagnetic bodies ϕ is in general small in comparison with kT and only becomes of importance at low temperatures.

The modified distribution function leads to an expression for the magnetic susceptibility which is in good agreement in many cases with the experimental results of K. Onnes and A. Perrier at low temperatures and also with other experimental results at higher temperatures, when appropriate choice of the temperature functions f and ϕ are made.

The theory is also applied with some success to the explanation of the paramagnetic behaviour of ferromagnetic substances at temperatures above the critical temperature.

The functions f and ϕ are not capable of determination from theoretical considerations, and the theory suffers chiefly from this deficiency.

Theories of R. Gans.

In a series of papers beginning in 1910 R. Gans[1,2,3,4] has made successive attempts toward the improvement of theories of dia- and paramagnetism beyond the point reached by Langevin in his paper of 1905.

The progress made by Gans in this connection may perhaps be satisfactorily estimated from a brief review of two of his papers which appeared in 1916, entitled respectively "Theorie des Dia-, Para-, und Metamagnetismus,"[1] and "Uber Paramagnetismus."[4]

In the former of these two papers he considers a material body supposed constituted of axial magnetons. The magneton itself is supposed to consist of a rigid system of classical negative electrons

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[1] R. Gans: *Gott. Nachr.*, p. 197; 1910.

[2] R. Gans: *Gott. Nachr.*, p. 118; 1911.

[3] R. Gans: *Ann. d. Phys.* 49, p. 149; 1916.

[4] R. Gans: *Ann. d. Phys.* 50, p. 163; 1916.

within a uniformly charged positive sphere, the center of which coincides with the centroid of the system of negative electrons. The equatorial moments of inertia of the magneton, A and B, are supposed equal and the polar axis for which C is the moment of inertia is called simply the axis of the magneton.

The angular velocities of rotation of the magneton are supposed so small that the resultant magnetic fields due to these rotations may be considered as linear functions of the corresponding angular velocities; and the accelerations giving rise to radiation to be so small that the energy radiated may be neglected. The magneton system may then be considered as quasi-stationary. Furthermore the inertia mass of a magneton is supposed to be entirely of electromagnetic origin. Finally, the molecular magnetic field is ignored on the present theory.

For a body constituted of magnetons of the type here contemplated, either one or the other of the laws of distribution given respectively by (12) or (19) of Sect. III is applicable, depending upon whether or not the rotations of the magnetons about their individual axes of symmetry are dependent upon or independent of thermal agitation. In the former case the law of distribution leads to a theory of diamagnetism and in the latter to a theory of paramagnetism. We consider the former case first.

From (12) Sect. III, the appropriate law of distribution for this case is:

$$\text{(1)} \qquad dN = \alpha e^{-\frac{\epsilon}{kT}} \sin\theta \, d\Omega,$$

where:

$$\text{(2)} \quad \epsilon = \frac{P^2+Q^2}{2J} + \frac{R^2}{2C} + \frac{e}{2mc}(P\sin\phi\sin\theta + Q\cos\phi\sin\theta + R\cos\theta),$$

and

$$\text{(3)} \qquad d\Omega = d\theta \, d\psi \, d\phi \, dP \, dQ \, dR;$$

the significance of all the quantities here involved is given in Sect. III.

The expression (1) gives the number of magnetons per unit mass whose statistical variables are delimited by the elementary phase domain $d\Omega$. Each of these will contribute to the magnetisation per unit mass an amount $\mathfrak{u}.H/H$, $\mathfrak{u}$ being the magnetic moment of any one of the magnetons of this group.

If M denote the scalar value of the magnetisation per unit mass, then:

$$\text{(4)} \qquad M = N\int \frac{\mathfrak{u}.H}{H} e^{-\frac{\epsilon}{kT}} \sin\theta d\Omega \div \int e^{-\frac{\epsilon}{kT}} \sin\theta d\Omega,$$

where the integration is to be taken over all values of the variables whose differentials occur in $d\Omega$ and is supposed performed after $\boldsymbol{\mu}.\mathbf{H}/H$ is expressed in terms of these variables.

From (60), Sect. II, with the aid of (7) and (8), Sect. III:

$$\frac{\boldsymbol{\mu}.\mathbf{H}}{H} = b\,(P \sin\phi \sin\theta + Q \cos\phi \sin\theta + R \cos\theta), \qquad \left(b = \frac{e}{2mc}\right).$$

If we now let:

$$Z = \int e^{-\frac{\epsilon}{kT}} \sin\theta \, d\Omega, \tag{5}$$

the expression for M may be put in the simple form:

$$M = -\frac{NkT}{H}\frac{d\log Z}{d\log b}. \tag{6}$$

By division of this expression by H we obtain for the susceptibility per unit mass, χ, the following expression:

$$\chi = -\frac{NkT}{H^2}\,\frac{d\log Z}{d\log b}. \tag{7}$$

For the case in which the rotations of the magnetons about their axes of symmetry are supposed independent of thermal agitation the appropriate law of distribution is given by (19), Sect. III:

$$dN = \alpha\, e^{\frac{\epsilon}{kT}} \sin\theta \, d\Omega, \tag{8}$$

where

$$\epsilon = \frac{P^2 + Q^2}{2\,J} + \frac{e}{2mc} H\,(P \sin\phi \sin\theta + Q \cos\phi \sin\theta) - \mu H \cos\theta, \tag{9}$$

$$d\Omega = d\theta\, d\psi\, d\phi\, dP\, dQ. \tag{10}$$

Proceeding in a similar way to that followed in the case just considered the following expression is found for the susceptibility per unit mass:

$$\chi = \frac{NkT}{H^2}\left(\frac{d\log Z'}{(d\log\mu} - \frac{d\log Z'}{d\log b}\right), \tag{11}$$

where

$$Z' = \int e^{-\frac{\epsilon}{kT}} \sin\theta \, d\Omega, \tag{12}$$

ϵ and $d\Omega$ being given by (9) and (10) respectively.

Diamagnetism

Referring to equation [illegible] Z is found after integration with respect to ψ, ϕ, P, Q and R that:

$$(13)\qquad Z = 4\pi^2\sqrt{(2\pi kT)^3 J^2 C}\int_0^\pi e^{-\frac{H^2b^2}{2kT}(J\sin^2\theta - C\cos^2\theta)}\sin\theta\, d\theta.$$

There are now three cases to consider: J > C; J = C; and J < C.

Case I—J > C.

If for brevity we put:

$$(14)\qquad \gamma^2 = \frac{H^2b^2(J - C)}{2kT},\qquad \Phi(\gamma) = \frac{2}{\sqrt{\pi}}\int_0^\gamma e^{-\lambda^2}d\lambda,$$

then from (13):

$$(15)\qquad Z = 4\pi^2\sqrt{\pi}\sqrt{(2\pi kT)^3J^2C}\,\frac{e^{\gamma^2\frac{J}{J-C}}}{\gamma}\Phi(\gamma).$$

Using this value for Z formula (7) furnishes for the magnetic susceptibility per unit mass:

$$\chi = -Nb^2J\left\{1 + \frac{J - C}{2J}\left(\frac{2}{\sqrt{\pi}}\frac{e^{-\gamma^2}}{\gamma\Phi(\gamma)} - \frac{1}{\gamma^2}\right)\right\}.$$

This expression assumes a simpler form if we let:

$$(16)\qquad \Omega(\gamma) = \frac{2}{\sqrt{\pi}\gamma e^{\gamma^2}\Phi(\gamma)} - \frac{1}{\gamma^2} + \frac{2}{3},\qquad h = \frac{3}{2}\frac{1 - C/J}{2 + C/J};$$

whereupon we obtain:

$$(17)\qquad \chi = -Nb^2\frac{2J + C}{3}\{1 + h\Omega(\gamma)\}.$$

The quantity γ defined above is at constant temperature proportional to the field strength, H, while $1/\gamma^2$ at constant field strength is proportional to the absolute temperature, T.

The susceptibility χ, as shown by the expression just found, depends upon the function $\Omega(\gamma)$ which may be calculated with the aid of a table for the probability integral for assumed values of γ and $1/\gamma^2$. The variation of $\Omega(\gamma)$ with γ (proportional to H) and of $\Omega(\gamma)$ with $1/\gamma^2$ (proportional to the absolute temperature) are shown in Fig. 6 and Fig. 7 respectively.

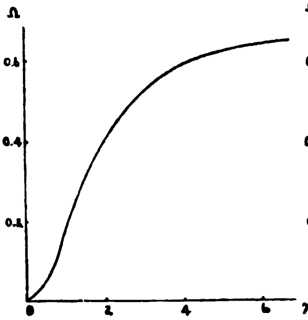

FIG. 6

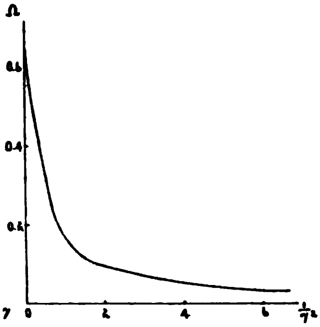

FIG. 7

From formula (17) it is seen that the susceptibility, χ_0, for very weak fields is given by:

$$\chi_0 = -Nb^2 \frac{2J+C}{3};$$

and hence:

$$\frac{\chi - \chi_0}{\chi_0 h} = \Omega(\gamma).$$

In general, then, χ depends upon the field strength and investigation brings out the fact that the curve showing the relation between the susceptibility and field strength is of the type shown in Fig. 8.

In Fig. 9 is shown the type of curve obtained experimentally by

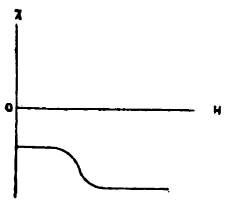

FIG. 8

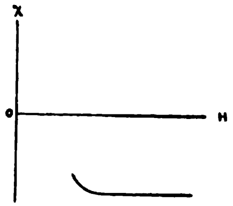

FIG. 9

Honda for many diamagnetic substances. The experiments of Honda were not sufficiently extended in the direction of small field strengths to show whether or not his curves, if continued, would be of the type called for by the present theory.

If the present theory in its main features is correct suitable quantitative measurements of the susceptibility would make possible the derivation of valuable information as to the constitution of the magneton, as regards its size, shape, and moments of inertia.

Case II—$J = C$.

In this case all the principal moments of inertia of the magneton are equal, and hence $h = 0$, and $\gamma = 0$, so that:

$$\chi = \chi_0 = -Nb^2J;$$

the susceptibility is thus independent of both field strength and temperature. This is found experimentally to be the case with many substances.

It is important to remember in connection with this case that although the principal moments of inertia of the magneton are assumed equal, this does not imply that the magneton is to be considered as a geometrical sphere. If this were the case the statistical method would be no longer applicable and the problem would become one of electromagnetism simply.

Case III—$J < C$.

The results found for this case are quite similar to those found for Case I and it is therefore not worth while to consider it in detail.

Moments of Intertia of Diamagnetic Magnetons.

As a result of an extensive series of experiments, H. Isnardi[1] reached the conclusion that diamagnetic susceptibility in general is quite independent of the field strength. If this be so the assumptions of Case II are warranted. The principal moments of inertia of the magneton may then be considered equal and the formula found for the susceptibility for this case may be used.

Upon substituting for b its value e/2mc in this formula we obtain:

$$\chi = -N\frac{e^2}{4m^2c^2}J,$$

where N is the number of magnetons per gram, $e/mc = 1.77 \times 10^7$ electromagnetic units and J the moment of inertia of the magneton.

Assuming one magneton to the atom, if N_0 be Avogadro's number, and A the atomic weight,

$$N_0 = N A = 6.175 \times 10^{23};$$

and we obtain from the formula for χ:

$$J = -2.067 \times 10^{-38} A \chi.$$

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[1] H. Isnardi, Contribución al estudio de las ciencias, Univ. Nacl. de La Plata.—*Ann. d. Phys.* 61, p. 585; 1920.

From the experimental results found by Owen[1] the values of J have been calculated by Gans[2] for various elements with the aid of this formula. These values together with the corresponding values for A and χ are given in:

TABLE I

Element	A	$-\chi\times10^6$	$J\times10^{43}$ in g. cms².	Element	A	$-\chi\times10^6$	$J\times10^{43}$ in g. cms².
Be......	9.1	1.00	1.88	Zr.....	90.6	0.45	8.43
B.......	11.0	0.7	1.59	Ag.....	107.9	0.20	4.46
C(Dia)..	12.0	0.49	1.22	Cd.....	112.4	0.18	4.18
S........	28.3	0.13	0.761	In.....	114.8	0.11	2.61
P.......	31.0	0.90	5.77	Sn(gray)	119.0	0.35 ap.	8.61 ap.
Sa......	32.07	0.49	3.25	Sb.....	120.2	0.82	20.4
Cw.....	63.57	0.085	1.12	Te.....	127.5	0.32	8.43
Zn......	65.37	0.155	2.09	I......	126.9	0.36	9.45
Ga......	69.9	0.24	3.46	Cs.....	132.8	0.10	2.75
Ge......	72.5	0.12	1.80	Aw....	197.2	0.15	6.12
As......	75.0	0.31	4.81	Hg.....	200.0	0.19	7.86
Se......	79.2	0.32	5.24	Tl.....	204.0	0.24	10.1
Br......	79.92	0.40	6.61	Pb.....	207.1	0.12	5.14
Sr......	87.62	0.2 ?	3.62?	Bi.....	208.0	1.40	60.2

It appears that the values for the moments of inertia for the various substances are all of the same order of magnetude. These values are considerably less than those found for paramagnetic substances, as will appear later.

In this connection it should be remarked that Isnardi's conclusion that diamagnetic susceptibilities are in general independent of field strength, is not fully supported by the experiments of Frivold.[3]

Paramagnetism and Metamagnetism.

Formulas (11) and (12) are those required for the explanation of para- and metamagnetism.

From (12), after integration with respect to P, Q, ψ, ϕ, and the substitution of x for cos θ, we obtain:

$$Z' = 8\,\pi^3\,k\,T\,J \int_{-1}^{+1} e^{\alpha^2(1-x^2)}\, e^{a x}\, dx\,, \tag{19}$$

where

$$\alpha^2 = \frac{H^2 b^2 J}{2\,k\,T}\,, \qquad a = \frac{\mu H}{kT}. \tag{20}$$

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[1] M. Owen, *Ann. d. Phys.* 37, p. 664; 1912.

[2] R. Gans, *Ann. d. Phys.* 61, p. 163; 1920.

[3] O. E. Frivold, *Ann. d. Phys.* 57, p. 471; 1918.

The expression (19) after integration with respect to x may be written in the form:

$$Z' = 4\,\pi^3\sqrt{\pi}\,kTJ\,\frac{e^{\alpha^2+\tau^2}}{\alpha}\,\{\Phi(\tau+\alpha) - \Phi(\tau-\alpha)\}, \tag{21}$$

where,

$$\begin{gathered} \tau^2 = \frac{a^2}{4\,\alpha^2} = \frac{\mu^2}{2\,b^2\,kT}, \\ \Phi(\tau \pm \alpha) = \frac{2}{\sqrt{\pi}} \int_0^{\tau\pm\alpha} e^{-\lambda^2} d\lambda. \end{gathered} \tag{22}$$

Observing that $d \log \tau = d \log \mu - d \log b$ and that $d \log \alpha = d \log b$, we obtain from (11) and (20) the following expression for the susceptibility per unit mass:

$$\chi = \frac{Nb^2J}{2\,\alpha^2}\,\Big\{4\,\tau^2 - 2\,\alpha^2 + 1 - \frac{4}{\sqrt{\pi}}\,\frac{e^{-(\tau^2+\alpha^2)}}{\Phi(\tau+\alpha) - \Phi(\tau-\alpha)} (2\,\tau \sinh 2\,\alpha\,\tau + \alpha \cosh 2\,\alpha\,\tau).\Big\} \tag{23}$$

The values of χ divided by the constant N b^2 A for various values of α and τ, given in Table II, were calculated by Gans from formula (23). For a given value of the temperature, T, the quantity τ is constant, from (22); and α is directly proportional to the field strength, from (20).

TABLE II

$\chi + Nb^2A$

α	$\tau = 0$	$\tau = \frac{1}{2}$	$\tau = 1$	$\tau = 2$
0.0	−0.667	−0.500	0.000	+2.00
0.2	−0.670	−0.505	−0.010	+1.92
0.4	−0.681	−0.514	−0.040	+1.69
0.6	−0.698	−0.535	−0.082	+1.348
0.8	−0.719	−0.560	−0.135	+1.093
1.0	−0.746	−0.591	−0.195	+1.900
1.5	−0.8189	−0.682	−0.339	+1.536
2.0	−0.8802	−0.773	−0.488	+1.278
3.0	−0.9444	−0.889	−0.723	−0.143
4.0	−0.9687	−0.937	−0.844	−0.471
5.0	−0.9800	−0.960	−0.900	−0.660
10.0	−0.9950	−0.990	−0.975	−0.915
∞	−1.0000	1.000	−1.000	−1.000

The table shows that, for all values of τ equal to unity or less, χ is negative for all values of α, except that when $\tau = 1$ and $\alpha = 0$, it vanishes; and that for $\tau = 2$, χ may be positive for values of α sufficiently low, and negative for higher values of α; the susceptibility thus depending upon the field strength.

A substance whose susceptibility, as regards sign, depends upon the field strength is called metamagnetic. Weber and Overbeck[1] have observed the phenomenon of metamagnetism in copper-zinc alloys; and Honda has observed it in the element Indium.[2] It is possible, however, that the observed phenomena might have been due to the presence of traces of iron in the specimens.

Another interesting conclusion which may be drawn from the present theory is that, by suitable increase of temperature and field strength, all so-called paramagnetic bodies would become diamagnetic.

The explanation of the curious results called for by the present theory of paramagnetism is to be found in the fact that the theory itself tacitly hypothecates two separable causes operative to produce magnetisation; one tending to produce diamagnetism, and the other paramagnetism.

It is not difficult to see that the cause operating to produce diamagnetism is the rotations, subject to thermal variation, of the magnetons about their equatorial axes of inertia; and that the cause tending to produce paramagnetism is the rotations, not subject to thermal variation, of the magnetons about their axes of symmetry.

The relative strengths of these two operating causes depend, in accordance with the theory, upon the temperature and field strength; and, therefore, according to the values of these two quantities, the one cause or the other may predominate.

In the second[3] of his papers published in 1916, entitled "Uber Paramagnetismus," Gans developed a theory of paramagnetism in which the molecular magnetic field is taken into account, this field having been ignored in his paper on dia-, para-, and metamagnetism just reviewed.

It will be recalled that on the latter theory paramagnetism cannot exist by itself, but always occurs accompanied by diamagnetism, caused by the effects of thermal variations in the rotations of the magnetons about their equatorial axes of inertia; and that, with sufficiently high temperatures and external fields, the diamagnetism due to this cause may predominate over the paramagnetism due to the rotations with constant angular velocity of the magnetons about their axes of symmetry.

For temperatures which are attainable, however, in the case of almost all paramagnetic substances, the paramagnetic effect predominates

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[1] K. Overbeck: *Ann. d. Phys.* 46, p. 677; 1915.

[2] K. Honda: *Ann. d. Phys.* 32, p. 1043; 1910.

[3] l.c.—p. 69, note 4.

strongly over the diamagnetic effect, which may consequently be ignored and each magneton considered to have a constant magnetic moment μ due to its rotation with constant angular velocity about its axis of symmetry; it is assumed that this moment is the same for all magnetons. The magneton thus considered is the equivalent of the magnetic molecule of Langevin.

With the assumptions relating to the magneton here made formula (35), Sect. III, is applicable for the calculation of the magnetic moment per unit mass. This formula gives the mean value, $\overline{\cos\theta}$, of $\cos\theta$, θ being the angle between the direction of the axis of a magneton and the field K, whose relation to the external field H and the intensity of magnetisation, I, is expressed by the equation:

$$(24) \qquad K = H + \frac{4\pi}{3} I,$$

The magnetic moment per unit mass is obtained by multplying $\overline{\cos\theta}$ by the product of the number of magnetons per unit mass, N, and the constant magnetic moment, μ, of a magneton. We thus obtain, from the formula for $\overline{\cos\theta}$ in question, the following expression for the magnetic moment per unit mass:

$$(25) \quad M = N\mu\,\overline{\cos\theta}$$

$$= \frac{\pi\mu N}{\sqrt{\pi^3 A_0{}^2}} \int_0^\infty e^{-\frac{A^2}{A_0{}^2}} A\,dA \int_{\pm(A-K)}^{A+K} (\coth a - \frac{1}{a}) \left(\frac{F^2 + K^2 - A^2}{K^2}\right) dF,$$

where

$$a = \left(\frac{\mu F}{kT}\right).$$

As regards the significance of the symbols, it will be recalled that A is the scalar value of the molecular field, A_0 the most probable value of A, F the scalar value of the resultant magnetic field, k Boltzmann's constant and T the absolute temperature.

From the expression (25) for the magnetisation per unit mass, we now derive an expression for χ, the susceptibility per unit mass.

By definition:

$$\chi = \mathrm{Lt}_{H \doteq 0} \left(\frac{dM}{dH}\right) = \mathrm{Lt}_{K \doteq 0} \left(\frac{dM}{dK}\,\frac{dK}{dH}\right).$$

For isotropic substances, with which the theory is concerned, K and H will be collinear, and from (24) we find:

$$\frac{dK}{dH} = 1 + \frac{4\pi}{3}\frac{dI}{dH},$$

and, since for paramagnetic substances the second term on the right will be very small in comparison with unity, it may be neglected. We may therefore write:

$$\chi = \mathrm{Lt}_{K \doteq 0} \frac{dM}{dK}.$$

In the evaluation of the right hand member of this expression the + sign in the lower limit of the integral involving F in expression (25) for M is to be used, since in the limit K will be less than A. It is found after easy calculation that:

$$(26) \qquad \chi = \frac{2}{3} N\mu \frac{4\pi}{\sqrt{\pi^3} A_0^3} \int_0^\infty \left\{ L(a) + \frac{1}{2} a L'(a) \right\} e^{-\frac{A^2}{A_0^2}} A\, dA,$$

where

$$L(a) = \coth a - \frac{1}{a}; \qquad a = \frac{\mu A}{kT};$$

and $L'(a)$ is the differential coefficient of $L(a)$ with respect to a.

For brevity we now write:

$$(27) \qquad z = \frac{A^2}{A_0^2}, \qquad \tau = \frac{kT}{\mu A_0}, \qquad \chi_0 = \frac{4 N\mu}{3\sqrt{\pi} A_0};$$

by (34) Sec. III:

$$(28) \qquad A_0 = \frac{4}{3}\mu \sqrt{\frac{\pi N_0 \rho}{M s^3}},$$

where N_0 is Loschmidt's number, M the molecular weight, ρ the density and s the smallest distance of approach between two magnetons.

Upon introducing the abbreviations into (26) we finally obtain:

$$(29) \qquad \chi = \chi_0 \int_0^\infty \left\{ L\left(\frac{\sqrt{z}}{\tau}\right) + \frac{1}{2}\frac{\sqrt{z}}{\tau} L'\left(\frac{z}{\tau}\right) \right\} e^{-z}\, dz.$$

This formula implies a dependency of the susceptibility upon the temperature, since τ is proportional to T; and also upon the density, since τ and χ_0 are each inversely proportional to the square root of the density.

For liquids and solids, however, variations of the density with temperature may be disregarded.

For brevity let:

$$\frac{\mu A_0}{k} = \Theta \, ; \tag{30}$$

then, from (27):

$$\tau = \frac{T}{\Theta} \, . \tag{31}$$

Upon introducing the temperature function:

$$\Psi(\tau) = \int_0^\infty \left\{ L\left(\frac{\sqrt{z}}{\tau}\right) + \frac{1}{2}\frac{\sqrt{z}}{\tau} L'\left(\frac{\sqrt{z}}{\tau}\right) \right\} e^{-z} \, dz, \tag{32}$$

we obtain from (29):

$$\frac{\chi}{\chi_0} = \Psi\left(\frac{T}{\Theta}\right) = \Psi(\tau), \tag{33}$$

a formula involving two disposable constants, χ_0 and Θ. This formula implies that, with the exception of gases, all paramagnetic bodies obey a law of corresponding states.

The value of the temperature function $\Psi(\tau)$ is now required. It is convenient to derive expressions for $\Psi(\tau)$ for two cases; viz., when τ is small, and when τ is large. In the first case it is to be understood that τ is not so small as to take the theory out of the equipartition range.

Case 1. τ small.

For details of the calculation the reader may refer to the original paper. The result of the calculation is to show that:

$$\Psi(\tau) = 1 - \frac{\tau\sqrt{\pi}}{2} + \frac{B_2}{2!}(\pi\tau)^4 + \frac{2B_3}{3!}(\pi\tau)^6 + \frac{3B_4}{4!}(\pi\tau)^8 \ldots\ldots ; \tag{34}$$

the B's represent Bernoulli numbers a few of which are:

$$B_1 = \frac{1}{6}, \quad B_2 = \frac{1}{30}, \quad B_3 = \frac{1}{42}, \quad B_4 = \frac{1}{30}.$$

Case 2. τ large.

The details of the calculation are also omitted in this case. It is found that:

$$\Psi(\tau) = \frac{\sqrt{\pi}}{2} \left\{ \frac{3}{1!}\frac{B_1}{\tau} - \frac{5}{2!}\frac{B_2}{\tau^3} + \frac{7}{3!}\frac{B_3}{\tau^5} - \ldots\ldots \right\}. \tag{35}$$

For very high temperatures, terms after the first on the right of (35) may be neglected; it is then found from (33) and (31) that:

$$\chi = \frac{\sqrt{\pi}}{4} \chi_0 \frac{\Theta}{T} = \frac{N\mu^2}{3kT}, \tag{36}$$

which is the Curie-Langevin law for paramagnetism.

This result was to be expected, since at high temperautres the influence of the molecular field upon the magnetons is small in comparison with the disorganizing effects of thermal agitation.

Experimental Test of Theory.

The theory is compared by Gans with experimental determinations of the susceptibility by K. Onnes, Oosterhuis, Perrier and Honda.

For Crystalline Gadolinium Sulphate ($Gd_2(SO_4)_3\ H_2O$), and for Ferric Ammonium Sulphate ($Fe_2SO_4(NH_4)_2SO_4+24H_2O$, the Curie-Langevin law is found to be well obeyed down to the respective temperatures T = 20.1°K, and T = 14.7°K. On the present theory, for these two substances, and in fact for all for which χT is constant, the molecular field A_0 is so small that Θ will also be small, so that T/Θ will still be a large number. The inference is that here the mutual action of the magnetons may be ignored.

The substances listed in Table III, with the values assigned to the disposable constants χ_0 and Θ show, as regards their susceptibilities, agreement with the present theory which leaves little to be desired for temperatures as low as 14.7°K.

TABLE III

Substance	Formula	χ_0	Θ
Crystalline ferrous sulphate......	$FeSO_4.7H_2O$	2212×10^{-6}	12.64
Crystalline manganous sulphate..	$MnSO_4.4H_2O$	4837×10^{-6}	9.90
Water-free ferric sulphate........	$Fe_2(SO_4)_3$	302×10^{-6}	120.00

Molecular constants.—The theory furnishes, with the aid of experimental results for the substances above considered, values for the following constants:

The number of Weiss magnetons per molecule.

The most probable value for the molecular field A.

The smallest distance of approach, s, between two magnetons.

For very high values of T we have, from (36):

$$\frac{N\mu^2}{3k}=\frac{\sqrt{\pi}}{4}\chi_0\Theta. \tag{37}$$

Now, since it has been assumed that each molecule contains only one of the magnetons of the present theory, μN_0 will be equal to the magnetic moment per gram molecule, N_0 being the Loschmidt number with the value 6.175×10^{23}; and $N_0=MN$ where M is the molecular weight. Upon multiplying the preceding expression, (36), by MN; substituting N_0 for MN, and solving the resulting equation for μN_0, we find:

$$\mu N_o = \sqrt{\frac{3}{4}\sqrt{\pi} k N_o M \chi_o \Theta} \tag{38}$$

as the magnetic moment per gram molecule.

If the molecule contain q magnetic atoms, then, in accordance with Weiss, $\mu N_o/q$ is an integer multiple, p, of 1123.5. Thus:

$$1123.5\ p = \sqrt{\frac{3}{4}\sqrt{\pi}\, k N_o M X_o \Theta} \div q, \quad (kN_o = 8.315 \times 10^7). \tag{39}$$

We denote by p′ the nearest whole number to the value for p calculated from this equation.

Weiss usually assumes $q = 1$ for salts, such, for example, as $Fe_2(SO_4)_3$, containing more than one metal atom.

The most probable molecular field is calculated from the second of equations (27) as follows:

$$A_o = \frac{kT}{\mu\tau} = \frac{kN_o\Theta}{\mu N_o} = \frac{kN_o\Theta}{1123.5pq}. \tag{40}$$

The smallest possible distance of approach, s, between two magnetons is obtained from (28):

$$s^3 = \frac{16\pi}{9}\,\frac{\mu^2 N_o \rho}{MA_o^2} = \frac{16\pi}{9}\,\frac{1123.5^2}{N_o}\,\frac{q^2p^2\rho}{MA_o^2}. \tag{41}$$

Using the values of the constants χ_o and Θ given in Table III, the results given in Table IV are obtained for Crystalline Ferrous Sulphate, Crystalline Manganous Sulphate and Water-free Ferric Sulphate.

TABLE IV

Substance	M	ρ	$\chi_o \times 10^6$	Θ	p′	p	$A_o \times 10^{-6}$ in Gauss	$s \times 10^8$ in cm.
$FeSO_4.7H_2O$	278.0	1.90	2212	12.64	26.09	26	0.3587	3.46
$MnSO_4.4H_2O$	223.1	2.11	4387	9.90	29.13	29	0.2516	5.25
$Fe(SO_4)_3 (q = 1)$	399.9	3.10	302.0	120.	35.63	36	2.494	1.22

Remarks—

It will be noticed that the values for p′ do not approximate very closely to integer numbers; and the Weiss magneton theory here fails of any very substantial support. This circumstance is, however, without influence upon the other molecular constants concerned.

The molecular fields are seen to be quite large. Water of crystallization appears to have the effect of decreasing the molecular field, owing probably to increase in the smallest possible distance of approach of neighboring magnetons.

The smallest distance of approach, s, is of the order of one tenth the diameter of a molecule. This may be explained by supposing the magneton excentrically placed in the molecule.

Although the present theory is in good agreement with experiment down to very low temperatures for the substances considered above it breaks down (at very low temperatures) for many others. Gans has therefore proposed a modification based upon a quantum hypothesis. This modification will be considered in Section VII, dealing with quantum theories of magnetism.

Theory of Honda and Okubo.

In a paper entitled "On a Kinetic Theory of Magnetism in General" Honda and Okubo[1] have attempted a modification of Langevin's theory for a paramagnetic gas, in which, effects due to the rotations of a magnetic molecule about axes perpendicular to the magnetic axis are taken into account.

The vector magnetic moment of a molecule is considered as made up of two parts: an axial component in the direction of its axis of rotation, and a transverse component perpendicular to this axis.

In accordance with the argument advanced in the paper cited the axial components of the magnetic moments of the molecules of a body subject to an external magnetic field would, due to the motions of the molecules induced by the field, give rise to paramagnetism; and the transverse components to diamagnetism.

The theory has much in common, as regards its fundamental assumptions, with Gans' theory of dia-, para-, and metamagnetism which has been reviewed in some detail above.

The arguments of Honda and Okubo have been subjected to rather severe criticism by Weaver.[2]

Theory of Oxley.

In an extended series of very interesting papers entitled "On the Influence of Molecular Constitution and Temperature on Magnetic Susceptibility," A. E. Oxley[3] has introduced a modification of Langevin's theory, in which the molecular field plays a leading role in diamagnetic substances, as well as in para,- and ferromagnetic substances.

The theory of Oxley, bringing into prominence, as it does, the molecular field, is analogous in many respects to the theory of ferromagnetism developed by Weiss upon Langevin's theory of a paramagnetic gas as a basis, supplemented by the assumption of the existence within ferromagnetic substances of enormous internal fields.

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[1] Honda and Okubo: *Phy. Rev.* 13, p. 6; 1919.

[2] W. Weaver: *Phy. Rev.* 16, p. 438; 1920.

[3] A. E. Oxley, *Roy. Soc. Phil. Trans.* 214, A, p. 109; 1913–14.—215 A, p. 79; 1914–15.—220 A, p. 247; 1919–20.

It therefore appeared appropriate to treat the work of Oxley and of Weiss together in a separate contribution. This has been done by Professor E. M. Terry in the part of this report dealing with ferromagnetism.[1]

Theory of Frivold.

In a paper entitled "Zur Theorie des Ferro- und Paramagnetismus O. E. Frivold[2] has developed a theory of ferro- and paramagnetism, consisting in a modification of Langevin's theory for a paramagnetic gas, in which the molecular magnetic field is taken into account.

In this theory the elementary magnets or magnetons are identified with the atoms whose centers are supposed fixed at the corners of a cubic space lattice, and capable of rotation about their respective centers.

Statistical theory is applied to this system of magnetons, and results found from which the magnetisation curve may be obtained. Comparison of this curve with the corresponding one which results from the Langevin theory furnishes a measure of the effect of the inter-action of the magnetons, and permits the calculation of the magnetic molecular field.

A more detailed account of this theory is given by Professor Terry in the section of this report referred to above.

While other attempts toward the improvement of Langevin's equipartition theory of magnetism have been made, it is hoped that the consideration of those which have been presented here in more or less detail will serve to enable the reader to form a fair idea of the trend of attempted improvements on this justly celebrated theory.

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[1] cf. p. 154 of this report.

[2] O. E. Frivold, *Ann. d. Phys.* 65, p. 1; 1921, cf. p. 132 of this report.

VII

THEORIES OF PARAMAGNETISM BASED ON QUANTUM HYPOTHESES

In 1911 Nernst[1] showed, in contradiction to the laws of classical statistical mechanics, that the specific heats of polyatomic gases appear to decrease with decreasing temperature. This was confirmed later by the investigations of Scheel and Heuse,[2] and their results ascribed to the behavior of that portion of the specific heat which depends upon the rotation of the molecules.

There then appeared a series of investigations having to do with the rotatory energy of molecules. Of these some were of a theoretical nature in which attempts were made at quantitizing the rotatory energy.

Meanwhile the experimental investigations of Onnes, Oosterhuis, Perrier, du Bois, Honda and Owen on the variation with temperature of the susceptibility of paramagnetic substances gave results which were in opposition to equipartition theories of paramagnetism. The theory of magnetism was thus in a similar dilemma to that in which the theory of specific heats found itself.

Modifications of existing theories of magnetism through the introduction of quantum hypotheses were, of course, in order. The earlier theorists in this field were faced with a fundamental difficulty, shared by some of those working at the improvement of the theory of specific heats, which had its origin in the attempted quantitization of the rotary energy of the molecules.

Poincarè at the Solvay Congress in 1911 called attention to the difficulty as follows:

"Imagine an oscillator with three degrees of freedom, isotropic and capable of vibration in such manner that the periods of vibration are the same with respect to three axes. Thus, for motions parallel to the (x, y, z) axes, let the corresponding energies be respectively $\alpha h\nu$, $\beta h\nu$ and $\gamma h\nu$, where α, β, γ are all integers, h is Planck's constant, and ν, the common frequency. Let the axes now be changed: with respect to the new axes the energies will be $\alpha' h\nu$, $\beta' h\nu$, and $\gamma' h\nu$, where α', β', ν' are integers. This is impossible."

In reply Planck said:

"An hypothesis of quanta for plural degrees of freedom has not yet been formulated, but I believe it to be nowise impossible of achievement."

In 1916 Planck,[3] through the publication of his paper on "Die Physikalische Structur des Phasenraumes," demonstrated the correctness of his view here expressed.

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[1] W. Nernst: *Zeitschr. f. Elektrochem.* 17, p. 615; 1911.

[2] K. Scheel u. W. Heuse: *Berl. Ber.* p. 44; 1913; *Ann. d. Phys.* 40, p. 473; 1913.

[3] M. Planck: *Ann. d. Phys.* 50, p. 385; 1916.

Prior to the publication of Planck's paper writers attempting to improve magnetic theories through the introduction of quanta hypotheses were forced to make such assumptions as seemed plausible, yet not firmly based.

We shall therefore pass over with but brief mention the earlier attempts at quantum theories of paramagnetism.

Theory of Oosterhuis.

Among the first in this field was Oosterhuis[1] who proposed a modification of Langevin's equipartition formula for the susceptibility per unit mass:

$$\chi = \frac{N\mu^2}{3kT},$$

where N is Avogadros's number, μ the magnetic moment of a molecule, and k Boltzmann's constant. Here kT represents the mean energy per degree of freedom of a molecule, and Oosterhuis simply replaces this by the expression

$$\frac{1}{2}\left(\frac{h\nu}{e^{\frac{h\nu}{kT}}-1}+\frac{h\nu}{2}\right),$$

representing the mean energy of rotation of the molecules for one degree of freedom on the quantum hypothesis of Einstein and Stern, which assumes all molecules to rotate, at a given temperature, with the same angular velocity, ν being the common frequency of rotation and h Planck's constant.

Theory of Keesom.

Keesom[2] does not assume with Oosterhuis that all molecules at a given temperature in a substance rotate with a common angular velocity, but considers the motions of molecular rotation to be resolved into a system of standing elastic waves, after the manner of Debye in his theory of specific heats. Owing to the discrete structure of matter, waves with a length shorter than a certain minimum determined by the structure are not possible of existence, and consequently the number of possible frequencies for the standing waves will be finite and all below a certain maximum, ν_m say. The magnetic molecule, as with Oosterhuis, is supposed to have a negligible moment of inertia about its magnetic axis, while its other principal moments of inertia are supposed equal.

The mean rotational energy corresponding to a single degree of freedom is then found to be

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[1] E. Oosterhuis: *Phy. Zeitschr.* 14, p. 682; 1913.

[2] W. H. Keesom: *Phy. Zeitschr.* 15, p. 8; 1914.

$$\frac{3}{2\nu_m^3}\int_0^{\nu_m}\left(\frac{h\nu}{e^{\frac{h\nu}{kT}}-1}+\frac{1}{2}h\nu\right)\nu^2 d\nu;$$

and this expression on Keesom's theory replaces kT in Langevin's formula for the susceptibility of paramagnetic substances.

The theory of Oosterhius shows fairly good agreement with experiment, in fact about as good as that of Keesom, and as it is founded upon far simpler assumptions is to be preferred.

The Theory of Gans.

In his paper "Uber Paramagnetismus,"[1] which appeared in 1916, and which has been reviewed in Section V as far as the part which deals with the equipartition portion of the theory is concerned, Gans proposes a quantum modification, in order to obtain a theory which will be applicable to all paramagnetic substances at very low temperatures.

As was stated in Section V, his equipartition theory is in good agreement with experiments in the case of some substances down to very low temperatures. But susceptibility curves, (χ–T), of observations on Uranium, Magnesium, Aluminium, Molybdenum, Mobium, Tantalum, and Wolfram all show a tendency at some point to become parallel to the T-axis; in fact this tendency in the case of some of these substances is evident at room temperatures; and in the case of Molybdenum and Wolfram at temperatures of 1200°C and 1100°C, respectively.

These experimental results cannot be accounted for on his equipartition theory; and Gans was thus led to modify it through the introduction of a quantum hypothesis relating to the distribution of the rotatory energy of the magnetons. As in the case of his equipartition theory, Gans takes the molecular field into account in his modification.

It is important to remember that the quantum theory of Gans is only applicable for very low temperatures, where by the term low temperatures is meant temperatures at and below which the equipartition theory is no longer valid; thus in the case of Molybdenum and Wolfram temperatures below 1100°C are considered as low temperatures.

At very low temperatures it may safely be assumed that temperature agitation is so slight that the magnetons perform but small vibrations about their positions of equilibrium, which are determined for any magneton, in the absence of an external magnetic field, by the molecular field A at that magneton. In fact the vibration frequency, ν, for the

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[1] l.c.

magneton, and the most probable value of ν, say ν_0, are respectively given by:

$$(1) \qquad \nu^2 = \frac{\mu A,}{4\pi^2 J} \qquad \nu_0^2 = \frac{\mu A_0,}{4\pi^2 J}$$

where J represents the moment of inertia of the magneton about any axis through its centroid perpendicular to its magnetic axis.

The quantum assumption now made is, that the energy distribution for the two degrees of freedom of the magneton about two perpendicular axes in its equatorial plane is the same as that which would obtain if each degree of freedom be treated as though it were that for a simple oscillator with this one degree of freedom.

To give precision, then, to the fundamental assumptions now introduced, it is supposed that the typical magneton with moment μ finds itself in a magnetic field F, and that the temperature is so low as to allow it to perform infinitely small vibrations about its equilibrium position determined by the direction of this field.

Let ϕ_1 and ϕ_2 be the angular displacements of the magneton about two perpendicular axes, then the total energy, ϵ, of the magneton will be given by:

$$(2) \qquad \epsilon = \frac{J}{2}(\dot{\phi}_1^2+\dot{\phi}_2^2) + \frac{\mu F}{2}(\phi_1^2+\phi_2^2) = \frac{\mu F}{2}(C_1^2+C_2^2),$$

where C_1 and C_2 are the maximum amplitudes of ϕ_1 and ϕ_2, respectively.

If β denote the angle which the magnetic axis of the magneton makes with the resultant field F in which it finds itself, then:

$$\cos\beta = 1 - \frac{\beta^2}{2} = 1 - \frac{\phi_1^2+\phi_2^2}{2};$$

and the mean value in time of $\cos\beta$ will therefore be given by:

$$\overline{\cos\beta} = 1 - \frac{\overline{\beta^2}}{2} = 1 - \frac{C_1^2 + C_2^2}{4}$$

and hence, with the aid of (2):

$$\overline{\cos\beta} = 1 - \frac{\epsilon}{2\mu F}.$$

The spatial mean value of this expression over all the N magnetons in a unit mass will be expressed by:

$$\overline{\overline{\cos\beta}} = 1 - \frac{\bar{\epsilon}}{2\mu F},$$

where $\bar{\epsilon}$ is the mean energy of a magneton.

In accordance with the quantum hypothesis made by Gans:

$$\bar{\epsilon} = \frac{2\,h\upsilon}{e^{\frac{h\nu}{kT}} - 1}, \tag{3}$$

the expression on the right being twice the mean energy assigned to each degree of freedom of the magneton, conforming with Planck's original theory of radiation which implies no zero-point energy.

From the last two equations it follows that:

$$\overline{\cos\beta} = 1 - \frac{h\upsilon}{\mu F}\,\frac{1}{e^{\frac{h\nu}{kT}} - 1}. \tag{4}$$

This expression corresponds on the equipartition theory to Formula (30), Sect. III viz.:

$$\overline{\cos\beta} = \coth\frac{\mu F}{kT} - 1\Big/\frac{\mu F}{kT}.$$

The equipartition theory is therefore modified in accordance with Gans's quantum hypothesis by replacing in (25), Sect. VI,

$$\coth\frac{\mu F}{kT} - 1\Big/\frac{\mu F}{kT} \text{ by } 1 - \frac{h\upsilon}{\mu F}\,\frac{1}{e^{\frac{h\nu}{kT}} - 1}.$$

The subsequent development, taking account of the molecular field, is along lines closely analogous to those followed in the equipartition theory. For the details the reader may consult the original paper.

The theory furnishes an expression for the susceptibility which contains three arbitrary constants: χ_0, the susceptibility at absolute zero; θ ($=h\upsilon_0/k$); and Θ ($=\mu A_0/k$).

In the case of Platinum and Water-free Manganous Sulphate, with the values of the disposable constants given below, the theory is found to be in good agreement with experiment:

	χ_0	Θ	θ
Platinum	1.189×10^{-6}	2097.°	50.0°,
Water-free Manganous Sulphate	$670.\times10^{-6}$	84.94°	23.5°.

Molecular constants.—From these experimental results interesting information as to the following molecular constants may be obained:

(a) The most probable vibration frequency, υ_0, of the magnetons in the molecular field.

(b) The equatorial moment of inertia, J, of a magneton.

The most probable vibration frequency, υ_0, for a magneton in the molecular field is given by:

$$\upsilon_0 = \frac{k}{h}\theta.$$

From the second of equations (1) we have for the equatorial moment of inertia of a magneton:

$$J = \frac{\mu A_0}{4\pi^2 \upsilon_0^2},$$

where A_0 may be calculated as in (40), Sect. VI. It is found that for Platinum $A_0 = 1243 \times 10^5$, and for Water-free Manganous Sulphate $A_0 = 2.292 \times 10^5$.

Thus the following values are obtained:

	$\upsilon_0 \times 10^{-12}$	$J \times 10^{40}$
For Platinum....................	1.30	67.7
For Water-free Manganese Sulphate.	0.483	12.4.

Theory of von Weyssenhoff.

Jan von Weyssenhoff,[1] in a paper which appeared in 1916, appears to have been the first to evolve a quantum theory of paramagnetism in which the method operates explicitly with quanta from the beginning. This author avoids the difficulty brought forward by Poincare through the introduction of a simplified model to represent the structure of paramagnetic bodies.

In this simplified model the magnetic molecules (magnetons) are supposed capable of rotation only about axes parallel to a given plane, (the x-y plane), and also perpendicular to their own magnetic axes. The angle between the z-axis and the magnetic axis of a magneton is denoted by θ. The position of a magneton is then uniquely determined by some value of θ between $-\pi$ and π. It may reasonably be expected that such a model will show, as regards its magnetic properties, a behaviour similar to a more general one in which the magneton may turn freely about a fixed point.

An external field of strength H is supposed to act in the direction of the z-axis.

The potential energy, U, of a magneton with magnetic moment μ is expressed by:

$$U = \mu H(1 - \cos\theta) = A^2 \sin^2\frac{\theta}{2}, \qquad \text{where } A^2 = 2\mu H; \tag{7}$$

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[1] J. von Weyssenhoff: *Ann. d. Phys.* 51, p. 285; 1916.

and the kinetic energy, E, by:

$$E = \frac{1}{2} J \dot{\theta}^2 = \frac{\psi^2}{2J}, \qquad \text{where } \psi = J\dot{\theta}, \tag{8}$$

and J is the moment of inertia about the fixed axis of the magneton.

In the present theory the mutual magnetic inter-action of the molecules is not taken into account. Hence when $A=0$ a magneton may turn freely about its fixed axis. For very large values of A all the axes of the magnetons will deviate but little from the direction of the external field H, and they will then behave in a manner quite similar to a system of Planck linear oscillators. For, the total energy of a magneton, ϵ, which in the general case is given by:

$$\epsilon = \frac{1}{2} J\dot{\theta}^2 + A^2 \sin^2 \frac{\theta}{2} = \frac{\psi^2}{2J} + A^2 \sin^2 \frac{\theta}{2} \tag{9}$$

will in this case be expressed by:

$$\epsilon = \frac{1}{2} J\dot{\theta}^2 + \frac{A^2\theta^2}{4} = \frac{\psi^2}{2J} + \frac{A^2\theta^2}{4}, \tag{10}$$

which is an expression identical in type with that for the energy of one of Planck's linear oscillators.

It is now proposed to apply to this model the second quantum theory of Planck, or rather, that portion of it which is termed by him thermodynamic.

To this end it is first necessary to consider the phase domain appropriate to the model. This consists of a strip of the $\theta-\psi$ plane of breadth 2π, parallel to the ψ-axis. Here θ and ψ, already defined above, may be designated respectively as the generalized coordinate θ and the generalized momentum ψ:

$$\psi = \frac{\partial E}{\partial \dot{\theta}} = \frac{\partial}{\partial \dot{\theta}} \left(\frac{1}{2} J\dot{\theta}^2\right) = J\dot{\theta}. \tag{11}$$

The method of Planck now requires the calculation of the magnitude and form of the elementary domains in the $\theta-\psi$ plane of equal probability.

In accordance with Planck's ideas these elementary domains of equal probability must be bounded by curves ϵ = const. For large values of A these curves must be ellipses, as is evident from equation (10).

The magnitude of each of the elementary domains must be the same and equal to Planck's constant, h, since for large values of A the molecules of the model are equivalent to a system of linear oscillators for which, as shown by Planck, the magnitude, h, of an elementary domain is independent of ν, and hence of A.

The family of bounding curves, ϵ = const., for the elementary domains is given by equations of the type:

$$(12) \qquad \frac{\psi^2}{2J} + A^2 \sin^2\frac{\theta}{2} = C^2,$$

where C is a constant for any given curve.

The area bounded by any such curve will be given by:

$$(13) \qquad \int_0^g \psi \, d\theta = 4 \int_0^g \sqrt{2J(C^2 - A^2 \sin^2\frac{\theta}{2})}\, d\theta,$$

the limit of integration, g, depending upon the value of C.

It is now required to find a series of values for C:

$$0, C_1, C_2 \ldots\ldots C_n \ldots\ldots,$$

such that the area of the elementary domain between the (n − 1) st curve and the n'th curve shall be equal to h for all values of n; or, what is the same thing, that the area enclosed by the n'th curve shall be equal to nh.

We have, with the aid of (13):

$$(15) \qquad 4\int_0^g \sqrt{2J}\,\sqrt{C_n^2 - A^2 \sin^2\frac{\theta}{2}}\, d\theta = nh,$$

$$g = \begin{cases} 2 \sin^{-1} C_n/A & \text{for } C_n < A \\ \pi & \text{for } C_n > A. \end{cases}$$

The curves on the θ-ψ plane represented by equation (12) for different values of C are separated into two distinct classes; one class lying within the curve G, shown in Fig. 10, for which the external field H is such that C = A; and the other class lying without this curve. The values of C_n for the first class will all be less than A, while the values of C_n for the second class will all be greater than A. For the requirements of a theory of paramagnetism it will appear presently that only the second class need be considered.

The case when the external field H is such that C_n = A is interesting as representing the case in which the pendulous motion of a magnetic molecule is about to pass into rotary motion. The area of the curve G for this case is easily seen from (13) to be expressed by:

$$(16) \qquad 4A\sqrt{2J}\int_0^\pi \cos\frac{\theta}{2}\, d\theta = 8A\sqrt{2J}.$$

If it were possible to express quite generally C_n as a function of n by means of (15), a formula for the mean energy of the magnetic molecules could be at once derived; also it would be possible to derive an expression for the orientation of the axes of the magnetic molecules as a function of the temperature for a given field strength. Unfortunately, this general procedure is not possible, and the argument has to be restricted to special cases. It will appear, however, that one of these special cases is broad enough to furnish a basis for an explanation of paramagnetism.

For the case in which the external field H is so large that the area of the curve given by (13) for $C_n = A$, viz., $8A\sqrt{2J}$, is much greater than h, all the elementary domains coming into consideration will lie within the curve G, and quite near the origin; and since θ may now replace $\sin\theta$, the bounding curves of the elementary domains will become ellipses, one of which is shown by the dotted line in Fig. 10. This corresponds exactly to the case of Planck's linear oscillators. The attainment of this case, however, would require external fields far greater than can be obtained in practice.

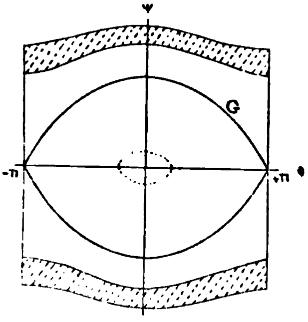

FIG. 10

We now consider the special case in which the external field H is such that the area of the curve G, viz., $8A\sqrt{2J}$ is far smaller than the quantum h.

In this case:

$$H < \frac{h^2}{256\mu J}. \tag{17}$$

As regards order of magnitude, $\mu = 10^{-20}$, $J = 10^{-40}$ and $h = 6.55 \times 10^{-27}$. Hence the order of magnitude of the right hand member of the inequality (17) will be 10^5. This number represents a field considerably greater than any that can be obtained in practice and we may conclude that a theory of paramagnetism may be founded upon this special case.

Now if, for the moment, we consider the external field to be such that $C_1 = A$, then the area of the curve G will be such that $8A\sqrt{2J} = h$; and it follows that the elementary domains coming into consideration in the present case, where $8\pi\sqrt{2J}$ is very small in comparison with h, will all lie outside the curve G. One of these is shown by the shaded area in Fig. 10. The upper limit of the integral in (15) will therefore be π, and the integral itself will therefore be a complete elliptic integral of Legendre.

For the case of paramagnetism we have, therefore:

$$n\,h = 4\sqrt{2J}\int_0^\pi \sqrt{C_n^2 - A^2\sin^2\frac{\theta}{2}}\,d\,\theta. \tag{18}$$

Writing:

$$k_n = \frac{A}{C_n},$$

the integral can be put in the form of a series:

$$\text{(19)} \qquad n\,h = 4\pi\sqrt{2J}\,C_n\left\{1 - \left(\tfrac{1}{2}\right)^2 k_n^2 - \left(\tfrac{1.3}{2.4}\right)^2 \frac{k_n^4}{3} \ldots\right\}.$$

From (19), C_n has now to be found as a function of n and A. We pass over the details of the calculation which may be found on page 301 of the paper under review. The calculation is simplified by the fact that A may be considered as a small quantity. The result shows that:

$$\text{(20)} \qquad C_n = \frac{n\,h}{4\pi\sqrt{2J}}\left(1 + \frac{1}{4}\gamma_n^2 - \frac{1}{64}\gamma_n^4 + \text{—}\right),$$

where

$$\text{(21)} \qquad \gamma_n = \frac{4\pi\sqrt{2J}}{n\,h}.$$

Now let:

N be the total number of magnetic molecules per unit mass;

N ω_n the number of magnetic molecules per unit mass with energies between the limits specified by the boundaries of the n'th elementary domain;

ϵ_n the mean value of the total energy for the N ω_n magnetic molecules. Also let:

$$\text{(22)} \qquad \Psi_n = C_n\sqrt{2J}\sqrt{1 - k_n^2 \sin^2\frac{\theta}{2}}, \qquad \text{where } k_n = \frac{A}{C_n},$$

express the value of Ψ for any point on the n'th boundary curve, obtained from (12).

Then:

$$\text{(23)} \qquad \epsilon_n = \frac{1}{h}\int\!\!\int\left(\frac{\Psi_n}{2J} + A^2\sin^2\frac{\theta}{2}\right)d\theta\,d\psi,$$

where the integration is over the n'th elementary domain. The result of the evaluation of the integral in (23) is to show that:

$$\text{(24)} \qquad \epsilon_n = k\Theta\,(n^2 - n) + \frac{k\Theta}{3} + \frac{A^2}{2} + \frac{\pi^2 J}{h^2}A^4\left(\frac{1}{n-1} - \frac{1}{n}\right),$$

where

$$\text{(25)} \qquad k\Theta = \frac{h^2}{32\pi^2 J}.$$

The constant Θ has the dimensions of temperature.

From here on the calculation follows the lines laid down by Planck in the development of his second radiation formula in which the oscillators are supposed to absorb energy continuously and to emit it in quanta.

The total energy, W, of the N magnetons considered is given by:

$$W = N \sum_{1}^{\infty} \omega_n \epsilon_n, \tag{26}$$

and this being supposed specified, the well known thermodynamic method of Planck[1] leads to the law of distribution of energy:

$$N \omega_n = \alpha e^{-\frac{\epsilon_n}{kT}} = \alpha_1 e^{-\frac{\Theta}{T}(n^2 - n)}, \tag{27}$$

where α_1 is a constant which depends upon A and T but not upon n.

Equation (26) gives the law of distribution of the magnetic molecules as regards their energy, that is, the number of molecules per unit mass with energies lying between the limits specified by the boundaries of the n'th elementary domain.

The results so far found are capable of direct application in the theory of rotatory specific heats, and of paramagnetism. We pass over the part of the paper having to do with the theory of specific heats and consider now the application of the results found to a theory of paramagnetism.

The potential energy of a magneton, from (7), is given by:

$$U = \frac{A^2}{2}(1 - \cos\theta) = A^2 \sin^2 \frac{\theta}{2}. \tag{28}$$

If χ be the magnetic susceptibility per unit mass, then, as on Langevin's theory:

$$\chi = \frac{N\mu}{H} \overline{\cos\theta}, \tag{29}$$

where $\overline{\cos\theta}$ is the spatial mean value of $\cos\theta$, whose value on the present quantum theory will, of course, be different in general from that found on the equipartition theory of Langevin.

From (28), if $\overline{U}$ denote the spatial mean value of U:

$$\overline{\cos\theta} = 1 - \frac{2}{A^2}\overline{U}. \tag{30}$$

Now if U_n denote the mean potential energy of a magneton whose total mean energy, ϵ_n, is specified as being within the boundary limits of the n'th elementary domain (whose area on the ψ-θ plane equals h), then:

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[1] M. Planck: Vorlesungen über die Theorie der Wärmstrahlung—Dritter Abschnitt.

$$U_n = \frac{4}{h}\int_0^{\pi} A^2 \sin^2\frac{\theta}{2}\,(\psi_n - \psi_{n-1})\,d\theta.$$

This equation, after the evaluation of the integral, with the aid of (22), and taking note of (20) and (21), gives:

$$U_1 = \frac{A^2}{2} - \frac{2\pi^2 J A^4}{h^2}, \qquad (n=1),$$

$$U_n = \frac{A^2}{2} - \frac{2\pi^2 J}{h^2} A^4 \left(\frac{1}{n} - \frac{1}{n-1}\right), \qquad (n=2, 3\ldots).$$

Therefore the mean values of $\cos\theta$ in the n domains will be given by:

(31)

$$\overline{(\cos\theta)}_1 = \frac{4\pi^2 J}{h^2} A^2 = \frac{1}{4k\Theta}\mu H, \qquad (n=1),$$

$$\overline{(\cos\theta)}_n = \frac{4\pi^2 J}{h^2} A^2 \left(\frac{1}{n} - \frac{1}{n-1}\right) = -\frac{1}{8k\Theta}\frac{A^2}{n(n-1)} = -\frac{1}{4k\Theta}\frac{\mu H}{n(n-1)}, \qquad (n=2, 3, \ldots).$$

These equations, with the aid of the distribution function given by (27), enable us to derive directly the following expression for $\overline{\cos\theta}$:

(32)

$$\overline{\cos\theta} = \frac{\mu H}{4k\Theta}\,\frac{1 - \sum_2^{\infty}\frac{1}{n\,(n-1)}\,e^{-\frac{\Theta}{T}(n^2-n)}}{\sum_1^{\infty} e^{-\frac{\Theta}{T}(n^2-n)}},$$

From (29) and (32) we obtain the following expression for the magnetic susceptibility per unit mass:

(33)

$$\chi = \frac{8\,N\mu^2}{h^2}\pi^2 J\,\frac{1 - \sum_2^{\infty}\frac{1}{n\,(n-1)}\,e^{-\sigma(n^2-n)}}{\sum_1^{\infty} e^{-\sigma(n^2-n)}},$$

where

(34)

$$\sigma = \frac{\Theta}{T} = \frac{h^2}{32\pi^2 JkT}.$$

From (33) it follows that at sufficiently high temperatures:

$$\chi = \frac{N\mu^2}{2kT},$$

which agrees with the Langevin formula except that there here appears in the denominator a factor 2 instead of a 3, as in the Langevin formula. The model for the molecular structure here adopted allows, however, but one degree of rotary freedom for the magnetic molecule and, if the Langevin calculation be carried out under the assumption of but one degree of freedom for the magnetic molecule, it turns out that the numerical factor in the denominator would be 2 instead of 3. Therefore the author introduces the factor 2/3 on the right of formula (33). The final formula for the magnetic susceptibility then becomes:

$$\chi = \frac{16}{3}\frac{N\mu^2}{h^2}\pi^2 J \frac{1 - \sum\limits_{2}^{\infty} \frac{e^{-\sigma(n^2-n)}}{n(n-1)}}{\sum\limits_{1}^{\infty} e^{-\sigma(n^2-n)}}, \tag{35}$$

This formula gives for the mass susceptibility at absolute zero:

$$\chi_0 = \frac{16}{3}\frac{N\mu^2}{h^2}\pi J. \tag{36}$$

A test of the theory is made through comparison of values of χ, calculated (with appropriate values of the disposable constants χ_0 and Θ) from the experimental values of χ determined by Onnes and Oosterhuis for crystalline- and for water-free manganese sulphate, with results given in Tables (V) and (VI) below.

Theories Based on Planck's Method of Quantitization.

Following the appearance in 1916 of Planck's paper[1] on "Die physikalishe Structur des Phasenraumes," which set forth the procedure to be followed in quantitizing the energy of an oscillator with plural degrees of freedom, the time was ripe for further improvements in the theories of rotatory specific heats and of paramagnetism.

As mentioned above the point had previously been reached in the development of theories in both of these subjects where a method was required for the quantitization of the rotatory energy of a molecule, or magneton, with plural degrees of freedom of rotation.

In theories of magnetism the magneton commonly hypothecated was supposed to have a constant magnetic moment due to its rotation about an axis of symmetry, and to possess dynamic symmetry about axes through its centroid perpendicular to the axis of symmetry; and, since the requirement of constancy for the magnetic moment of the magneton about this axis demands that its motion about it be independent of thermal agitation, only two degrees of freedom were assigned to it.

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[1] l.c., p. 85.

The definite problem up for solution before satisfactory progress could be made was:

To quantitize properly the rotatory energy of a magneton with two degrees of freedom of rotation.

In Planck's quantum theory of radiation the quantum difficulty of Poincare, stated above, does not arise, since the linear oscillator invoked by Planck for the purpose of effecting interchange of energy of different frequencies in black body radiation has but a single degree of freedom. The probability elementary phase domains for a linear oscillator were shown by Planck to be the areas included between consecutive ellipses similar and similarly placed in the ϕ-ψ plane, each area on his quantum hypothesis being equal to the universal constant h; ϕ being the generalized coordinate of the oscillator representing its electric moment and ψ the corresponding generalized momentum, viz., the partial derivative of the kinetic energy of the oscillator with respect to the generalized velocity $\dot{\phi}$.

Now from the viewpoint of Planck the quantum difficulty of Poincare may be stated as that of correctly delimiting the elementary probability domain in the specific problem under consideration. If this delimitation be accomplished, the remaining difficulties are simply those of formal analysis.

In cases where the statistical element or molecule has but a single degree of freedom the proper delimitation of the elementary probability domains is generally a fairly simple matter, as in the case of Planck's linear oscillators, or again, in the case of the constrained motion of the magnetons in the model of molecular structure assumed by v. Weyssenhoff in his theory of paramagnetism.

We shall now notice briefly some quantum theories of paramagnetism based on Planck's method of quantitization.

Theory of Reiche.

Fritz Reiche[1] in 1917 published a very interesting paper entitled "Zur Quantentheorie des Paramagnetismus" in which he generalizes the assumptions of v. Weyssenhoff as regards molecular structure by considering it to be such that each magnetic molecule (magneton with fixed magnetic moment) should be capable of free rotation about a fixed point. The rotation of the magneton about its magnetic axis (axis of symmetry) is supposed independent of thermal agitation and its moment of inertia about any equatorial axes through its centroid, denoted by J, is assumed to be the same for all such axes.

From what has been said above it will be clear that the problem of Reiche differs essentially from that of v. Weyssenhoff only in that

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[1] F. Reiche: *Ann. d. Phys.* 54, p. 401; 1917.

part which has to do with the delimitation of the appropriate elementary phase domains. To go into the details of the analysis whereby this is effected, following the method of Planck, would carry us beyond the scope of the present review and the reader who is interested is referred to the original paper; also to a paper by Adams.[1]

The author finds an expression for the mean value of cos θ for the magnetic molecules, where θ is the angle between the magnetic axis of such a molecule and the direction of the external field H; and then substitutes this in the following expression giving the magnetic susceptibility per unit mass:

$$\chi = \frac{N\mu}{H}\,\overline{\cos\theta},$$

where N is the number of magnetic molecules per unit mass, μ is the magnetic moment of a molecule, and $\overline{\cos\theta}$ is the mean value of cos θ.

It is thus found that:

$$\chi = \frac{N\mu^2}{h^2}\pi^2 J\,\frac{\frac{5}{4}e^{-\sigma}+\frac{1}{3}\sum_{n=2}^{\infty}\frac{e^{-\sigma n^2}}{n(n^2-1)}}{\sum_{1}^{\infty} n\,e^{-\sigma n^2}}, \tag{37}$$

where

$$\sigma = \frac{h^2}{8\pi^2 J k T}. \tag{38}$$

The corresponding expression for χ found by v. Weyssenhoff is given by (35), and it should be noticed that σ in the theory of v. Weyssenhoff has a value equal to one fourth of that given by (38).

For very low temperatures (σ large) formula (37) gives:

$$\chi = \frac{5}{4}\,\frac{N\mu^2}{h^2}\,\pi^2 J; \tag{39}$$

while (35) reduces to:

$$\chi = \frac{16}{3}\,\frac{N\mu^2}{h^2}\,\pi^2 J. \tag{40}$$

For high temperatures (σ small) both (37) and (35) give:

$$\chi = \frac{N\mu^2}{3kT},$$

the equipartition expression of Curie-Langevin.

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[1] E. P. Adams: *Bull. Nat. Res. Coun.* I, 5, p. 301.

Other Theories.

Sophie Rotszajn[1] treated the same problem as that considered by Reiche, using, however, a very different method of analysis. The final formula found for the susceptibility is, as was to be expected, precisely the same as that arrived at by Reiche.

The procedure followed by both Reiche and Rotszajn as regards quanta hypotheses presupposes the validity of what is commonly known as Planck's second theory of radiation, which assumes the absorption of energy by his linear oscillators to be continuous and the emission to be in quanta; and which predicts the existence of a zero-point energy. It will be recalled that on this theory the distribution of energy, for the stationary state, as regards frequency υ, is, if $\bar{\epsilon}$ denote the mean energy of an oscillator of frequency υ, given by:

$$\bar{\epsilon} = h\upsilon\left(\frac{1}{e^{\frac{h\nu}{kT}} - 1} + \frac{1}{2}\right),$$

where h is Planck's and k Boltzmann's constant.

In the first form of Planck's theory, eventually discarded by him:

$$\bar{\epsilon} = \frac{h\upsilon}{e^{\frac{h\nu}{kT}} - 1},$$

and thus does not predict the existence of a zero-point energy.

A. Smekal,[2] in spite of the fact that the second form of Planck's theory is now commonly preferred to the first, thought it worth while to develop a quantum theory of paramagnetism based on the assumptions of the first form of the theory, using the same magneton model as that assumed by Reiche and Rotszajn. He was led to a formula for the susceptibility which shows by no means so good an agreement with the experimental facts as that found by them on the basis of the second form of Planck's theory. His result, then, adds another argument in favor of the second form of the theory and, therefore, for the existence of a zero-point energy.

Comparison of Theories with Experiments.

Of the various quantum theories which have been considered probably the most satisfactory is that of Reiche which, as far as the fundamental assumptions and the final results are concerned, is the same as that of Rotszajn.

The theory proposed by v. Weyssenhoff is also satisfactory from the standpoint of its development from his fundamental assumptions; but

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[1] S. Rotszajn: *Ann. d. Phys.* 57, p. 81; 1918.

[2] A. Smekal: *Ann. d. Phys.* 57, p. 376; 1918.

these are more artificial than those of Reiche and Rotszajn, including as they do the restriction of the movement of the magneton (apart from its rotation about its axis of symmetry) to motion in two dimensions.

The theory of Gans, while based upon an incorrect quantum hypothesis, takes account of the consequences of the presence of the "molecular field" which is ignored on other theories. Comparison of this theory with experiment has already been made (see p. 96 of M. S.)

The theory of Oosterhuis may be taken as representative of those theories, other than that of Gans, based upon quanta hypotheses which were developed before Planck in 1916 published his general method whereby quantitization may be effected in a statistical system whose elements have plural degrees of freedom.

Remarks—

TABLE V

WATER-FREE MANGANESE SULPHATE—$MnSO_4$

Reiche ... $J = 1.99 \times 10^{-40}$;
v. Wey. ... $J = 4.44 \times 10^{-41}$; $\mu = 4.35 \times 10^{-21}$; $\chi_0 = 6.577 \times 10^{-6}$.
Oost. ... $J = .87 \times 10^{-41}$; $\mu = 1.80 \times 10^{-20}$; $\chi_0 = 6.89 \times 10^{-6}$.

T°K	$\chi \times 10^6$ cal. Reiche	$\chi \times 10^6$ cal. v. Wey.	$\chi \times 10^6$ obs.	$\chi \times 10^6$ ca Ost.
14.4	637.9	646	636	628
17.8	614.9	617	627	619
20.1	597.8	594.3	603	603
64.9	315.1	313.4	314.5	325.7
77.4	277.5	276.1	274.8	284.0
169.6	142.7	142.2	144.2	145.4
293.9	86.8	88.9	87.8	86.3

TABLE VI

CRYSTALLINE MANGANESE SULPHATE—$MnSO_4 + 4H_2O$

Reiche ... $J = 3.14 \times 10^{-39}$;
v. Wey.... $J = 1.1 \times 10^{-39}$; $\mu = 3.65 \times 10^{-21}$; $\chi_0 = 7.294 \times 10^{-6}$.
Oost. ... $J = 1.09 \times 10^{-39}$; $\mu = 1.69 \times 10^{-21}$; $\chi_0 = 3.1000 \times 10^{-6}$.

T°K	$\chi \times 10^6$ cal. Reiche	$\chi \times 10^6$ cal. v. Wey.	$\chi \times 10^6$ obs.	$\chi \times 10^6$ cal. Ost.
14.4	1233	1249	1233	1231
17.8	1014	1019	1021	1015
20.1	905	905.8	914	904
64.9	293.2	290.3	292	291
70.5	270.6	267.7	270	268
77.4	247.3	244.1	247	245
169.6	114.5	112.7	111.5	112.6
288.7	67.6	66.5	66.3	66.3

It must be remembered that none of the theories here mentioned takes cognizance of the mutual action of the molecules, except that of Gans, and in this respect is therefore deficient.

Tables V to VIII enable one to judge as to how far the theories are in accord with experiments. In this connection it should be noted that in each of them there are two disposable constants.

TABLE VII

WATER-FREE FERRIC SULPHATE—$Fe_2(SO_4)_3$

Reiche ... $J = 1.40 \times 10^{-40}$; $\mu = 3.42 \times 10^{-21}$; $\chi_0 = 286.4 \times 10^{-6}$.

T°K	$\chi \times 10^6$ cal.	$\chi \times 10^6$ obs.
64.0	177.6	177.1
70.5	167.6	167.3
77.6	156.7	157.2
169.6	85.1	85.6
289.8	53.3	53.3

TABLE VIII

CRYSTALLINE FERRO SULPHATE: $FeSO_4 + 7H_2O$

Reiche ... $J = 2.23 \times 10^{-39}$; $\mu = 2.94 \times 10^{-21}$; $\chi_0 = 3.365 \times 10^{-6}$.

T°K	$\chi \times 10^6$ cal.	$\chi \times 10^6$ obs.
14.7	760.5	756
20.3	568.7	571
64.6	189.8	191
77.3	159.5	160
292.3	42.4	42.4

VIII

DIAMAGNETISM IN METALS DUE TO MOTIONS OF FREE ELECTRONS

In accordance with views on the nature of electric conduction in metals brought forward by Lorentz, Drude and others, there are present in metals large numbers of free electrons which move about among the atoms in a manner similar to that of the molecules of a gas; and, moreover, the thermal properties of metals also lend support to the assumption that free electrons are present in them in large number. Although there are outstanding difficulties in the attempt to ascribe to free electrons many observed electric and thermal properties of metals there is yet strong evidence in favor of this assumption.

If the free electrons are present and moving about in metals like the molecules of a gas, it is evident that in the presence of a magnetic field the free paths of the electrons will be curved, and with a curvature in such sense as to furnish diamagnetic quality to the metal. Superimposed upon the diamagnetism due to the motion of the free electrons there will be, of course, the dia-, and perhaps the paramagnetism, of Langevin.

Erwin Schrodinger[1] in 1912 and H. A. Wilson[2] in 1920 have given theories of the diamagnetism in metals due to the motions of free electrons, arriving at quite similar conclusions by very different methods. For the purposes of the present review it will suffice to outline the argument presented by Schrödinger.

Theory of Schrödinger.

Structural Assumptions—The fundamental assumptions made as regards the structure of a metal are precisely those made by Lorentz in his theory of the motions of electrons in metals.

Two distinct species of particles are supposed to be present in the metal:—

(a) Electrons with mass m and charge e moving freely among the atoms.

(b) The atoms of the metal, some of which carry a charge, while others do not.

The electrons and the atoms are supposed to share in the thermal motion of the metal, the particles of each type in the case of thermal equilibrium having a mean kinetic energy equal to the mean kinetic energy of the motion of translation of a molecule of a gas at the same temperature as that of the metal.

Due to the small mass of an electron as compared with that of an atom the velocities of the atoms are assumed negligible in comparison with those of the electrons.

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[1] E. Schrödinger, *Wien. Ber.* 66, p. 1305; 1912.

[2] H. A. Wilson: *Roy. Soc. Proc. Lond.* 97, p. 321; 1920.

The mutual action among the particles, so far as the electrons are concerned, is supposed to occur through collisions only and as if the colliding particles were perfectly smooth elastic spheres.

Owing to their small size the collisions of the electrons among themselves are ignored, and collisions only of electrons with atoms are considered. Accordingly the mean free paths of the electrons are not determined by their own number and size but by the number and size of the atoms.

The Diamagnetism of Free Electrons.

When such a medium is subjected to the action of a magnetic field the free paths of the free electrons between collisions are no longer straight, but curved, due to the action of the field. The motion of the electrons along these curved free paths must act to produce diamagnetism in the medium.

It is now required to calculate the magnetic moment resulting from the curvature of the free paths under the action of an external magnetic field.

Referring to Fig. 11, dτ is a small element of volume of the medium; ξ, η, ζ the coordinates of an electron at a point Q within dτ with respect to an origin O, also within dτ. P is a point on the z-axis at a distance r from O, large in comparison with the dimensions of dτ. The external magnetic field H is supposed in the direction of the z-axis.

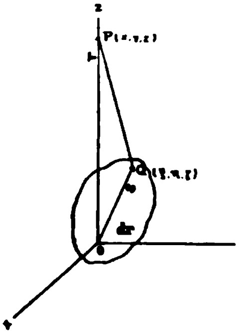

Fig. 11

The magnetic force, say **h**, at P (0. 0. r) due to the typical electron at Q (ξ, η, ζ) moving with velocity **v** (ξ η, ζ) is, from (11), Sect. I, with sufficient approximation expressed by:

$$\mathbf{h} = \frac{e}{cr^3} \mathbf{v} \times (\mathbf{r} - \mathbf{s});$$

r is the position vector of P and **s** that of Q. The scalar z-component of this force is expressed by:

$$h_z = \frac{e}{cr^3} (\eta\dot{\xi} - \xi\dot{\eta}).$$

The expression in brackets is the z-component of twice the areal velocity of the typical electron with respect to O and e/r^3 is constant for all the electrons in the volume element.

The mean value, $\bar{h}_z$, of h_z, is to be found through summation of this expression for h_z over all the electrons in dτ, followed by integration over a sufficiently long time T, and division by T. The order of summation and integration is, of course, indifferent and, since the time integral of the areal velocity of the typical electron is equal to the area swept out by its radius vector, the time integral required is equal to the sum of the areas swept out in the time T on the x-y plane by the projections on this plane of the radii vectorii to all the electrons in the element $d\tau$. If F denote the sum of these areas, then:

$$\bar{h}_3 = \frac{2e}{cr^3}\frac{F}{T}. \tag{1}$$

Calculation of F—The problem is thus reduced to the calculation of F. This requires a knowledge of the law of distribution of the velocities of the electrons.

Before the establishment of the external field Maxwell's law may plausibly be assumed; but with the field present the question arises as to whether this assumption is still plausible. The following considerations show this to be the case.

Following Boltzmann[1] let us consider the case of a mixture of two gases, and let:

ξ, η, ζ be the coordinates of a molecule of the first gas;

$\dot\xi, \dot\eta, \dot\zeta$ be the component velocities of a molecule of the first gas;

X, Y, Z be the component accelerations of a molecule of the first gas, due to the actions of external forces supposed dependent only upon the coordinates, ξ, η, ζ;

m be the mass of a molecule of the first gas;

$f(\xi, \eta, \zeta, \dot\xi, \dot\eta, \dot\zeta)$ be the velocity distribution function for the first gas.

Boltzmann showed that in the case of equilibrium $(\frac{\partial f}{\partial t} = 0)$:

$$f = f_0 e^{-hm(\dot\xi^2+\dot\eta^2+\dot\zeta^2)}, \tag{2}$$

where f_0 and h are such functions of the coordinates ξ, η, ζ that for all values of ξ, η, ζ:

$$\dot\xi\frac{\partial f}{\partial x} + \dot\eta\frac{\partial f}{\partial y} + \dot\zeta\frac{\partial f}{\partial z} + X\frac{\partial f}{\partial \dot\xi} + Y\frac{\partial f}{\partial \dot\eta} + Z\frac{\partial f}{\partial \dot\zeta} = 0. \tag{3}$$

Now it is assumed that this result may be applied to the present case, where the electrons play the role of the first gas and the atoms of the metal that of the second.

The components of the force on an electron due to the external field H, say X, Y, Z, are given by:

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[1] Boltzmann: Gas Theorie, I, pp. 98–134.

$$
\begin{aligned}
X &= \frac{e}{mc}(\dot{\eta} H_3 - \dot{\zeta} H_2),\\
(4) \qquad Y &= \frac{e}{mc}(\dot{\zeta} H_1 - \dot{\xi} H_3),\\
Z &= \frac{e}{mc}(\dot{\xi} H_2 - \dot{\eta} H_1).
\end{aligned}
$$

Now it is noted that a violation of the Boltzmann assumptions is here met with, since X, Y, Z depend upon the velocities. Scrutiny of the Boltzmann proof shows, however, that it is still valid if it be simply assumed that X does not depend upon $\dot{\xi}$, Y does not depend upon $\dot{\eta}$ and Z does not depend upon $\dot{\zeta}$. The above equations show that the X, Y, Z of the present problem are such as to satisfy these conditions.

If now the value of f, from (2), be inserted in (3) we find, with the aid of (4), that the terms in X, Y, Z all vanish and hence that the equation

$$- m(\dot{\xi}^2 + \dot{\eta}^2 + \dot{\zeta}^2)\left(\dot{\xi}\frac{\partial h}{\partial x} + \dot{\eta}\frac{\partial h}{\partial y} + \dot{\zeta}\frac{\partial h}{\partial z}\right) + \dot{\xi}\frac{\partial f_0}{\partial x} + \dot{\eta}\frac{\partial f_0}{\partial y} + \dot{\zeta}\frac{\partial f_0}{\partial z} = 0$$

must be satisfied identically by $\dot{\xi}$, $\dot{\eta}$, $\dot{\zeta}$. But this requires that f_0 and h shall be independent of the coordinates and therefore constant. In this case the distribution function given by (2) is Maxwell's; and the conclusion is reached that the presence of a magnetic field does not alter the distribution of the free electron velocities in a metal.

Proceeding with the calculation of F, let λ_v be the mean free path (Tait's) of an electron moving with velocity v. The probability that an electron moving with the velocity v shall proceed without collision over a path with a length between a and a + da will be[1]

$$(5) \qquad \frac{1}{\lambda_v} e^{-\frac{a}{\lambda_v}} da.$$

It is here assumed that λ_v is independent of the velocity v and that for all electrons:

$$\lambda = \frac{1}{n\pi\delta^2},$$

where n is the number of atoms per unit volume and δ is the radius of an atom.

The number of electrons per unit volume in $d\tau$ with velocities between v and v+dv may be taken to be $\nu_v dv$. Then the number of collisions

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[1] Cf. Jeans—Kin. Th. of Gases—3rd Ed., p. 256.

of such electrons per unit volume in time T will be

$$\frac{vT}{\lambda} \nu_v \, dv.$$

The fraction of these collisions for which the velocities afterward have directions included within the solid angle $d\omega$ and, by virtue of (5), which are such that the colliding electrons after collision shall have free paths of lengths between a and a+da will be

$$\frac{d\omega}{4\pi} \frac{1}{\lambda} e^{-\frac{a}{\lambda}} da.$$

Therefore:

$$\frac{vT}{4\pi\lambda^2} e^{-\frac{a}{\lambda}} \nu_v \, dv \, d\omega \, da \tag{6}$$

will be the number of collisions per unit volume in time T of electrons with velocities between v and v+dv and for which:

(1) the velocities after collision shall be directed within the solid angle $d\omega$.

(2) the free paths after collision shall have lengths between a and a+da.

Such collisions are denoted as of class A.

The volume element $d\tau$ is now supposed subdivided into prismatic columns, $d\Pi$, parallel to the z-axis with sectional dimensions small in comparison with those of $d\tau$, and also small in comparison with all ordinary free paths. The number of collisions of class A within a prism of volume $d\Pi$, by virtue of (6) will be expressed by:

$$\frac{vT}{4\pi\lambda^2} e^{-\frac{a}{\lambda}} \nu_v \, dv \, d\omega \, da \, d\Pi. \tag{7}$$

The areas described on the xy-plane by projections of the radii vectorii of the electrons concerned in these collisions, as they describe their free paths following the collisions, will all be appreciably the same.

The process of the calculation of F now requires the finding of the sum of these areas for electrons of all classes of which class A is typical, followed by integration over all prisms of which $d\Pi$ is typical and, finally, by integration over all velocities of which v is typical.

The area described on the xy-plane by the projection on this plane of the radius vector of a typical electron of class A is found as follows. Referring to Fig. 12, let A be the position with reference to the xy-plane of the electron at the time of a collision, B its position at the time of its next collision. The shaded area bounded by OA, OB and the arc $\overset{\frown}{AB}$ is that required, the arc AB representing the free path of the electron.

From the equations of motion of an electron in a magnetic field, the path $\overset{\frown}{AB}$ is easily shown to be an arc of a circle whose length, b, is given by:

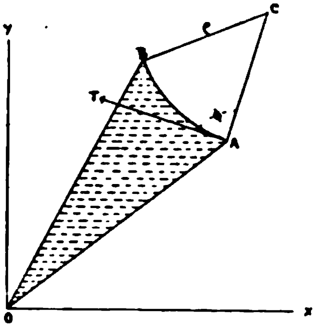

FIG. 12

(8) $$b = a \sin\theta.$$

and whose radius, ρ, is given by:

(9) $$\rho = \frac{mcv \sin\theta}{eH},$$

if θ be the angle between the positive z-axis (direction of H) and that of the axis of the cone corresponding to the solid angle $d\omega$.

If ϕ be the angle between the tangent to the path at A and the radius vector OA, then:

(10) $$\text{Area } O\overset{\frown}{AB} = \text{Area } O\overline{AB} + \text{Area } C\overline{BA} - \text{Area } C\overset{\frown}{BA}.$$

In order to obtain F, the area given by (10) has now to be multiplied by the number of collisions given by (7), and the appropriate integrations made. If we write $d\omega = \sin\theta\, d\theta\, d\phi$, the following expression for F is thus obtained:

$$F = \frac{T}{4\pi\lambda^2}\int_0^{\infty} dv \int^{(d\tau)} d\Pi \int_0^{\infty} da \int_0^{\pi} d\theta \int_0^{2\pi} v\, \nu_v\, e^{-\frac{a}{\lambda}} \Big\{ \overline{OA}\, \rho \sin\Big(\pi - \phi + \frac{b}{2\rho}\Big)$$
$$+ \frac{1}{2}\rho^2 \sin\frac{b}{\rho} + \frac{1}{2}\rho b \Big\}\, d\phi,$$

where b and ρ are given by (8) and (9) as functions of v and θ, and OA depends only upon the position of the prism $d\Pi$.

All the integrations called for with the exception of that with respect to v are easily made and the expression for F reduces to:

$$F = \frac{mc\, T\, d\tau}{3\, e}\int_0^{\infty} \nu_v \left\{ \frac{v^4}{v^2 + \left(\frac{\lambda e H}{mc}\right)^2} - v^2 \right\} dv.$$

Now ν_v the number of electrons per unit volume with velocities between v and v+dv, is given by Maxwell's law:

$$\nu_v = a\, v^2 e^{-h m v^2} dv,$$

where a and h are constants to be determined in terms of the total number n of electrons per unit volume, and the mean squares of their velocities $\overline{v^2}$, so that:

$$a = \frac{4}{\sqrt{\pi}} n (h m)^{\frac{3}{2}},$$

$$h m = \frac{3}{2\overline{v^2}}.$$

The expression last given for F may now be put in the form:

$$F = \frac{4}{3\sqrt{\pi}} \frac{mc\,n\,T\,d\tau\,(hm)^{\frac{3}{2}}}{eH} (J_1 - J_2), \tag{11}$$

where

$$J_1 = \int_0^\infty e^{-hmv^2} \frac{v^6}{v^2 + \left(\frac{\lambda eH}{mc}\right)^2} dv,$$

$$J_2 = \int_0^\infty e^{-hmv^2} v^4 dv = \frac{3\sqrt{\pi}}{8} (hm)^{\frac{5}{2}}.$$

It may be shown that with sufficient approximation[1] that:

$$J_1 = (hm)^{-\frac{5}{2}} \left(\frac{3\sqrt{\pi}}{8} - \frac{\sqrt{\pi}}{4}\alpha^2\right),$$

where

$$\alpha = \sqrt{\frac{3}{2}} \frac{\lambda eH}{mc\sqrt{\overline{v^2}}}. \tag{12}$$

Inserting the expressions found for J_1 and J_2 in (11), the final expression for F is obtained:

$$F = -\frac{1}{3}\frac{e}{mc}\lambda^2 nTH\,(1 - 2\alpha^2)d\tau. \tag{13}$$

The mean field strength, $\overline{h}_3$, at A, due to the motions of the electrons in $d\tau$, is then found with the aid of (1) and (12) to be given by:

$$\overline{h}_3 = -\frac{2}{3}\frac{e^2}{mc^2}\lambda^2 n \frac{Hd\tau}{r^3}(1 - 2\alpha^2).$$

The form of this expression shows that the element of volume $d\tau$ is magnetically equivalent to a doublet of magnetic moment

$$-\frac{1}{3}\frac{e^2}{mc^2}\lambda^2 nH\,(1 - 2\alpha^2)\,d\tau.$$

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[1] Cf. Schrödinger, l.c., p. 1328 and p. 1315.

If κ_f be the magnetic susceptibility per unit volume it follows then that:

$$(14) \qquad \kappa_f = -\frac{1}{3}\frac{e^2}{mc^2}\lambda^2 n\,(1-2\alpha^2).$$

Discussion of Results.

The formula (14), owing to the presence of the term in α^2, shows that in general κ_f depends upon the field strength, but it will appear presently that at ordinary temperatures α^2 will be negligibly small in comparison with unity so that with sufficient approximation we may take:

$$(15) \qquad \kappa_f = -\frac{1}{3}\frac{e^2}{mc^2}\lambda^2 n.$$

Now the values of e and e/mc are well known, and $n\lambda$ and λ may be estimated from electrical conductivity measurements and plausible assumptions concerning the true atomic volume. It is thus possible to calculate approximately the values of κ_f for different metals. This has been done for Bi, Pb, Cu and Ag, chosen with particular reference to the wide range of electrical conductivity exhibited by this series of metals. The results are given in Table VIII together with the susceptibilities observed for these metals. The electrical conductivities σ are expressed in C. G. S. electromagnetic units. The values given all refer to the temperature 18°C.

Table VIII

Metal	$\sigma\times10^5$	$n\lambda\times10^{-16}$	$n\times10^{-22}$	$\lambda\times10^8$	$-\kappa_f\times10^6$ cal.	$-\kappa\times10^6$ obs.
Bi	0.84	0.046	0.8	5.54	2.37	13.7
Pb	4.84	0.267	4.8	5.56	13.8	1.36
Cu	57.2	3.174	52.5	6.04	178.	.076
Ag	61.4	3.405	53.4	6.38	202.	2.10

Comparison of the values calculated for κ_f, the diamagnetic susceptibility due to the free electrons, with the experimental values, κ, shows great differences to exist. It appears then that other sources of magnetisation than that of the free electrons are contributory in an important way to the true magnetic susceptibility. The latter is probably due to the combined effect of:

(a) The diamagnetism of Langevin; due to the induction effect during the establishment of the external field upon the bound circulating electrons within the atom. This effect is independent of the temperature.

(b) The paramagnetism of Langevin; due to the directive action of the external field upon the magnetically polarized atoms or molecules. This effect varies inversely with the absolute temperature.

(c) The diamagnetism of Schrödinger; due to the curvature of the paths of the free electrons under the action of the external field. This effect depends upon the temperature in rather a complicated way.

In the case of good conductors it may happen that the order of the effect (c) is the same as that of the effect (b) in strongly paramagnetic bodies. The dependency of effect (c) upon temperature, which appears through the factor $n\lambda^2$,is, however, by no means so simple as that called for by Curie's law for paramagnetism which makes the susceptibility of paramagnetic bodies vary inversely as the absolute temperature. Therefore in all cases where effects (b) and (c) are in opposition and of the same order of magnitude any simple law of variation of κ with temperature is not to be expected. Hereby is explained the failure of the experimental curves between susceptibility and temperature for metals obtained by Honda, and Owens, to exhibit any simple law of variation of susceptibility with temperature, and in particular why the susceptibilities of metals are so at variance with Curie's law.

The connection between the magnetic susceptibility, κ_f, and the electrical conductivity σ, is obtained through comparison of formula (15) for κ_f, with the following formula for the electrical conductivity obtained by Lorentz on the same constitutive assumptions as those adopted by Schrödinger:

$$\sigma = 2\sqrt{\frac{2}{3\pi}}\frac{e^2}{mc^2}\lambda n \frac{1}{\sqrt{\overline{v^2}}} \tag{16}$$

By division of (16) by (15) we find:

$$\frac{\sigma}{\kappa_f} = -2\sqrt{\frac{6}{\pi}}\frac{1}{\lambda\sqrt{\overline{v^2}}}\,. \tag{17}$$

Under the same conditions λ will not vary greatly from metal to metal, so that at the same temperature κ_f will vary, approximately, directly with σ.

The Dependence of the Susceptibility κ_f upon the Field Strength.

The formula found above (14):

$$\kappa_f = -\frac{1}{3}\frac{e^2}{mc^2}\lambda^2 n\,(1-2\alpha^2)$$

is a closer approximation for κ_f than (15). The approximation is close in either case only when α^2 is small in comparison with unity. Now α is directly proportional to the field strength, since, from (12):

$$\alpha = \sqrt{\frac{3}{2}} \frac{\lambda e H}{mc\sqrt{\overline{v^2}}};$$

and the quantity

$$\frac{mc\sqrt{\overline{v^2}}}{eH}$$

represents the radius of the free path of an electron moving with the velocity $\sqrt{\overline{v^2}}$ perpendicular to the field. If, then, α^2 is to be small in comparison with unity, the mean free path λ must be small in comparison with this radius.

Calculation shows that with an external magnetic field of 5×10^4 gauss, the largest practically obtainable, the order of magnitude of $2\alpha^2$ at 18°C is 10^{-4}. Therefore any effect due to the variation of the external field could hardly be detected. But calculation also shows that at very low temperatures a marked decrease of susceptibility with increasing field strength should be detected.

Note.—Professor Langevin has recently informed the writer of an interesting result found by N. Bohr in his dissertation. In accordance with the argument advanced by him it appears that the free electrons in a metal, subject to Maxwell's Law of distribution for a simple gas, should, on the whole, contribute nothing to its diamagnetic quality, owing to the behaviour of the electrons at the boundary whereby they produce an equal and opposite effect to that of the electrons in the interior. Unfortunately this information reached the writer too late to allow of the incorporation in the report of an outline of Bohr's argument.

THEORIES OF FERROMAGNETISM—INTRINSIC FIELDS

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HISTORICAL STATEMENT

In the early attempts to account for the phenomena of ferromagnetism, two rival theories were offered,—one by Poisson and the other by Weber. Both regarded magnetism as a molecular property, but they differed essentially in this, that while Poisson assumed the molecules possess magnetic properties only when the substance is magnetized, Weber considered that they have constant magnetic moments, and that gross magnetism depends upon alignment. The fact that ferromagnetic bodies all show saturation was taken as evidence in favor of Weber's theory, for it is difficult to see why on the Poisson theory magnetism should not be increased without limit. Again, the effects of vibrations in augmenting susceptibility were readily accounted for, because of the greater freedom thus given to the molecules to fall in line with the magnetizing force. The experiment of Beetz[1] in which he found that iron deposited electrolitically in a magnetic field possesses strong magnetic properties, furnished further evidence in favor of the Weber theory.

The fact that ferromagnetic bodies do not show saturation for very weak fields and the phenomenon of hysteresis are evidences that there must be some form of constraint acting upon the molecular magnets. Weber[2] assumed a restoring force equivalent to that of a constant magnetic field acting upon each molecular magnet in the direction of its axis in the unmagnetized state. This assumption, however, offers no explanation of residual magnetism or of the other phenomena of hysteresis. In attempting to correct this defect in the Weber theory, Maxwell suggested a further assumption based upon the analogy of magnetization to elastic fatigue. He supposed that after a molecule has been deflected from its original position by a magnetizing force, it returns only partly if the deflection exceeds a certain value. While explaining retentivity and some of the other phenomena of hysteresis, this theory fails to account for certain facts observed in repeated magnetization. It was suggested by Wiedemann and others that the deflection of the Weber magnets might be opposed by a frictional resistance which not only opposes alignment, but also holds the molecules in their deflected positions after magnetization. If, however, the molecules were held by friction until the applied force is large enough to start them, the

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[1] *Pogg. Ann.* 140, 1860, p. 107.

[2] *Pogg. Ann.* 88, 1852, p. 167, cf. p. 9 of this report.

susceptibility for very weak fields would be zero, whereas it has initially a small constant value.

THE THEORY OF EWING

In contrast to the arbitrary constraints mentioned above, Ewing proposed the theory that the molecular magnets are entirely free to turn about their centers, and that the only constraints acting are the fields due to neighboring magnets. This idea he developed in great detail and, in fact, laid the foundation for much of the work which has since been carried out. From a mathematical consideration of the simple case of a 2 magnet group acted upon by an external field, he obtained a curve in which the three stages of magnetization are clearly indicated and by an experimental study of a model in which 130 small pivoted magnets were used, he obtained magnetization and hysteresis curves which approximated the observed curves for ferromagnetic bodies with surprising accuracy. He gave a theoretical treatment of the case of a ferromagnetic body made up of rhombic crystals with molecular magnets placed at the corners of their space lattices, where the crystals are placed with all possible orientations. By a statistical method, which has been the basis for the subsequent work of Langevin, Weiss, Honda, and others, he showed that the percentage retentivity should be .8927, and deduced a number of other important results.

THE WEISS MOLECULAR FIELD HYPOTHESIS

Statement of Langevin's Theory.

It was pointed out in a preceding part of this report that Langevin[1] by an application of the method of statistical dynamics, has arrived at an expression for the intensity of magnetization of a paramagnetic gas in terms of the electron theory. For this purpose he supposed that the state of magnetization depends upon two factors only; first, the external field which tends to produce alignment in a given direction, and second the thermal agitation, which acts for disorganization. By an application of the Maxwell-Boltzmann distribution law, in which the number of magnetic molecular axes pointing in a given direction corresponds to the density of a gas, and the angle with the external field to height, he arrived at the following expression for the intensity of magnetization of a paramagnetic gas at a temperature T under the influence of a field H:

$$(1) \qquad \frac{\sigma_m}{\sigma_{m_0}} = \coth a - \frac{1}{a}, \text{ where}$$

$$(2) \qquad a = \frac{\sigma_m}{R} \frac{H}{T}.$$

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[1] Langevin, *Ann. de Chem. et de Phys.*, Ser. 8, 5, 1905, p. 70, cf. p. 55 of this report.

In these equations,

σ_m = Magnetic moment per gram molecule;
σ_{m_0} = Magnetic moment per gram molecule at saturation;
H = External field;
T = Absolute temperature;
RT = Twice the kinetic energy for one degree of freedom of a molecule;
R = Gas constant for a perfect gas referred to the molecular mass ($R = 83.15 \times 10^6$ ergs per degree).

Langevin's equation, plotted as C in Fig. 1, gives the percentage saturation for a paramagnetic gas at any temperature as a function of the applied field. In weak fields, the intensity, σm, is proportional to the field, but the slope becomes less with increasing field and finally approaches assymptotically to the saturation value σmo. By a simple calculation he showed, for the case of oxygen, that a field of 100,000 gauss would be necessary, at ordinary temperatures, to produce an appreciable departure from the linear law.

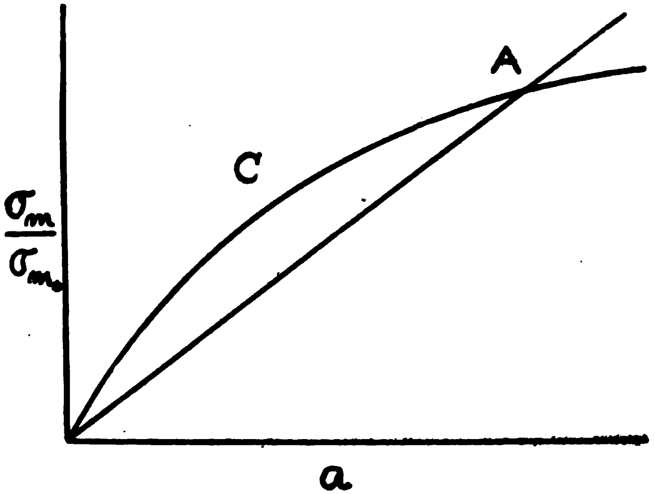

FIG. 1

Langevin showed also that the well known experimental law of Curie, i. e. the inverse proportionality of the susceptibility to the absolute temperature for paramagnetic substances follows directly from his formula. Developing the right hand member of equation (1) in a series, there results:

$$(3) \qquad \frac{\sigma_m}{\sigma_{m_0}} = \frac{a}{3} - \frac{2}{90} a^3 + \frac{4}{45.42} a^5 + \text{———}.$$

Taking, as an approximation which holds over the range of fields experimentally realizable, the first term only in this development, we have:

$$(4) \qquad \frac{\sigma_m}{\sigma_{m_0}} = \frac{a}{3} = \frac{\sigma_{m_0} H}{3\,RT}.$$

Letting $\chi_m = \frac{\sigma_m}{H}$ = the molecular susceptibility, there results:

$$(5) \qquad \chi_m T = \frac{\sigma_{m_0}^2}{3\,R} = C_m.$$

C_m is called the "molecular" constant of Curie, i. e. the proportionality factor when the susceptibility is referred to the gram molecule. Curie's law, as expressed by equation (5) holds for a large number of paramagnetic substances over wide ranges of temperature. Assuming it to hold at absolute zero, σ_{m_0}, the saturation value of the intensity may be determined for a substance by measuring its susceptibility at a known temperature T. This is the hypothesis which has been made by Weiss in his theory of the "Magneton" to be discussed later.

The Molecular Field.

By postulating a "Molecular Field," Weiss[1] has extended the ideas of Langevin to the phenomena of ferromagnetism. In this he was guided by the method which Van der Waals used to develop a kinetic theory of liquids by extending the ideas which Bernoulli had applied to a perfect gas. Just as in the case of a gas, to account for the transition to the liquid state, there must be added to the external pressure an internal one due to the mutual attractions between the molecules, so in the case of a ferromagnetic substance, as it is cooled in a magnetic field from a temperature which has rendered it paramagnetic, the transition to the ferromagnetic state is explained by assuming that, due to the overlapping of the fields of the individual molecules, there comes into existence an internal or molecular field, which added to the external field, accounts for the very large intensity characteristic of this state.

Weiss assumes that the overlapping of the fields of the molecules existing in a given region is equivalent to a uniform field proportional to the intensity of magnetization and directed parallel to it. Thus:

$$H_m = N\,I,$$

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[1] *de Phys.*, 4th Series, Vol. 6, 1907, p. 661. *Arch. des Sciences Phys. et Nat.*, l. 31, 1911, p. 401.

where H_m is the molecular field, I the intensity of magnetization and N, a constant characteristic of the substance. The molecules contributing to this internal field are contained in a definite sphere of action. He assumes, moreover, that the forces due to the magnetic fields are the only ones which act upon the molecules of a ferromagnetic substance and, except for them, the molecules are as free to rotate as in the case of a perfect gas.

Spontaneous Magnetization.

Weiss further supposes that it is not necessary for an external field to be acting in order that the individual parts of a body may be magnetized. On the contrary, he assumes that throughout the body the molecular field alone maintains the intensity of magnetization of the elementary units of volume at a magnitude very near the saturation value for the particular temperature at which the body exists in the same way that a fluid, by virtue of the internal attractive forces, maintains its liquid state in the absence of an external pressure. The volumes throughout which this spontaneous magnetization exists in an uninterrupted manner are very small, limited perhaps to the individual crystals. In a finite body with resultant intensity zero, the directions of magnetization of the individual elements are distributed entirely at random, and the function of the external field, in giving a resultant intensity to the body, is to produce an alignment of the individual group intensities, but not to change their magnitudes. In other words, if one could examine with sufficient minuteness, he would find an unmagnetized body to possess the same intensity as one grossly magnetized in the most powerful fields available.

The magnitude of the spontaneous intensity of magnetization may be obtained in the following manner. Equation (1) gives the value of the intensity of magnetization at any temperature T in terms of the saturation value by means of the auxiliary variable a. It is then merely necessary to replace H in Equation (2) by $H_m = NI$ and substitute in Equation (1). This may be effected most easily by means of a graphical elimination of a between the two equations. In equation (6), I is defined as the magnetic moment per unit volume, while σ_m is the magnetic moment per gram molecule. It is therefore necessary to replace I by its value $\frac{D\,\sigma_m}{m}$, where D is the density of the substance.

Accordingly:

$$\text{(7)} \qquad a = \frac{\sigma_{m_0}}{m}\frac{ND}{RT} \times \sigma_m, \qquad \text{and}$$

$$\text{(8)} \qquad \frac{\sigma_m}{\sigma_{m_0}} = \frac{m}{\sigma_{m_0}^2}\frac{RT}{ND} \times a.$$

The last equation gives the straight line of Fig. 1, which intersects the former curve in two points. It is easy to show that the intersection at O corresponds to a state of unstable equilibrium and that the one at A is the one concerned. Since the parameter a contains T, the spontaneous magnetization as a function of the absolute temperature may be readily deduced.

The Magnitude of the Molecular Field.

Anticipating for the moment what is to be shown presently, it may be stated that the molecular field is very large compared to fields available in the laboratory. However, in the temperature interval between the ferro- and paramagnetic states, there is a small region in which the molecular field is of the same order as realizable fields, and by measurements made in this transition region the constant N of equation (6) may be determined; and from it the value of H_m, the molecular field may be computed.

For this region equation (2) may be written:

$$(9) \qquad a = \frac{\sigma_{m_0}(H_e + NI)}{RT} = \frac{\sigma_{m_0}\left(H_e + \frac{ND}{m}\sigma_m\right)}{RT},$$

where H_e is the external field. At the transformation temperature θ,

$$(10) \qquad \frac{\sigma_m}{\sigma_{m_0}} = \frac{a}{3}, \text{ and } a = \frac{\sigma_{m_0} N D \sigma_m}{R\theta m}.$$

Eliminating *a* from these two equations,

$$(11) \qquad \theta = \frac{\sigma_{m_0} N D.}{3 R m}$$

Combining (11) and (9) and reducing, there results:

$$(12) \qquad \frac{T-\theta}{\theta} = \frac{H_e m.}{\sigma_m N D}$$

Letting $\chi_m = \frac{\sigma_m}{H_e}$, where χ_m is the molecular susceptibility, there results:

$$(13) \qquad (T-\theta)\chi_m = \frac{\theta m}{N D}.$$

This is a modified form of Curie's Law and states that the susceptibility is inversely proportional to the excess of the temperature above the transformation point. This law has been found to hold for this region with very good accuracy and from it the value of N has been deduced.

The following values have thus been obtained:

Substance	N	Hm
Iron................	3,850	6,560,000
Nickel..............	12,700	6,350,000
Magnetite...........	33,200	14,300,000
Cobalt..............	6,180	8,870,000

Experimental Evidence Regarding the Existence of the Molecular Field.

1. *The law of Corresponding States and the Variation of the Saturation Intensity with Temperature.*—As noted above, an unmagnetized body consists of minute crystals all magnetized to the saturation value for that temperature but having their magnetic axes distributed at random. The process of magnetization consists in lining them up, and if we could apply an external field sufficient to produce gross saturation, we should be able to measure the molecular intensity, since it would then be the same as the gross saturation. Further, a study of the variation of the saturation intensity with temperature should furnish a direct test of the concept of the molecular field as given in equations (1) and (6) This test may be facilitated by a general equation applicable to all substances analogous to that for corresponding states in the kinetic theory of gases. Such an equation may be obtained in the following manner.

The slope of the straight line of Fig. 1, is proportional to the temperature T. Accordingly, by giving successive values to T and determining the intersections with the curve C, the law of variation of intensity with temperature may be derived. The limiting case is that in which the straight line coincides with the tangent to the curve at the origin, and corresponds to the temperature θ at which spontaneous ferromagnetism disappears. This transformation temperature may be expressed in terms of the constants of the medium by noting that at θ, equation (1) may be written with sufficient accuracy by using only the first term of the development of equation (3); that is:

$$\frac{\sigma_m}{\sigma_{m_o}} = \frac{a}{3}; \tag{14}$$

Also equation (8) becomes:

$$\frac{\sigma_m}{\sigma_{m_o}} = \frac{a\,m\,R\,\theta}{\sigma_{m_o}^2\,N\,D}. \tag{15}$$

Dividing (15) by (14) there results:

$$\theta = \frac{\sigma_{m_0}^2 \, N D}{3 \, m R}. \qquad (16)$$

Dividing (16) by (8), and simplifying, one obtains:

$$\frac{T}{\theta} = \frac{3}{a} \frac{\sigma_m}{\sigma_{m_0}}. \qquad (17)$$

This equation, together with equation (1) gives the complete law of the thermal variation of spontaneous ferromagnetism, and when expressed in terms of the variables $\frac{T}{\theta}$ and $\frac{\sigma_m}{\sigma_{m_0}}$, is the same for all substances

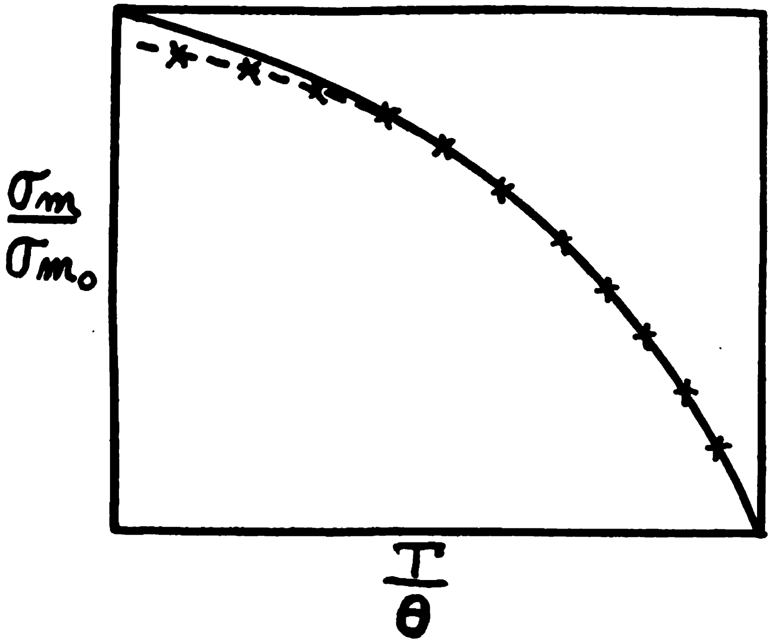

Fig. 2

The full line of Fig. 2, taken from the original paper of Weiss shows the calculated curve, and the crosses, the values obtained for magnetite. The work was carried out in a field of 8300 gauss although previous experiments had shown that this material is practically saturated in a field of 550 gauss. The agreement is satisfactory except in the low temperature region where marked departures occur. For pyrrhotite and the alloy Fe_2 Ni the agreement is more satisfactory than for magnetite; but for iron, nickel and cobalt, the agreement is less satisfactory in that larger systematic departures are found.

2. *The Dependence of Specific Heat upon the Molecular Field.*—If ferromagnetic substances are the seats of molecular fields of the magnitudes stated above, a considerable amount of energy must be supplied as the temperature is raised from absolute zero to the transformation point in order to break up the alignment of the molecular magnets within the crystals. We should expect, then, that in this region, the specific heat would be greater than it would be, if by some means the substance could be deprived of its magnetic properties. This effect should show itself as an additive term to the true specific heat of a corresponding fictitious substance having no magnetic properties. The amount of this additional heat may be computed from the theory of the molecular field in the following manner.

The mutual potential energy E of a group of magnets of moment μ is

$$E = -\frac{1}{2}\Sigma\, \mu\, H \cos\alpha, \tag{18}$$

where H is the resultant field due to the group at the point where an individual magnet is located, and α is the angle between H and this magnet. When the summation is extended to all the magnets contained in a centimeter cube, there results:

$$E = \frac{1}{2} I\, H_m = -\frac{1}{2} N\, I^2, \tag{19}$$

where I is the magnetic moment per unit volume, H_m, the molecular field, and N the constant of equation (6). The negative sign indicates that it is necessary to supply heat to demagnetize the substance. The intensity decreases in a continuous manner from absolute zero to the temperature at which the disappearance of ferromagnetism occurs. Accordingly, the amount of additional heat that must be supplied in raising a ferromagnetic body from a temperature at which the intensity is I, to the Curie point θ, is

$$q = \frac{1}{2}\frac{N}{JD} I^2 = \frac{1}{2J}\frac{H_m I}{D}, \tag{20}$$

where J is the mechanical equivalent of heat and D, the density. The mean specific heat accordingly, is:

$$C_m = \frac{dq}{dT} = \frac{1}{2J}\frac{N}{D}\frac{dI^2}{dT} = \frac{1}{2J} ND \frac{d\sigma^2}{dT} \tag{21}$$

where σ is the magnetic moment per unit mass. This quantity is small at low temperatures, but increases as the temperature is raised and disappears abruptly at θ. At this point, it has the nature, not of a latent heat of allotropic transformation, but of a discontinuity in the true specific heat.

The magnitude of this discontinuity has been calculated by H. A. Lorentz.[1] Developing in a series the theoretical law of the variation of magnetization at saturation as a function of the temperature, he found at θ:

$$\frac{d\,\sigma^2}{dT} = \frac{5}{3}\frac{\sigma_0^2}{\theta}, \tag{22}$$

where σ_0 is the saturation value of σ.

Taking into account the relations:

$$\theta = C\,N\,D, \text{ and} \tag{23}$$

$$\sigma_0 = \frac{3\,R\,C}{m}, \tag{24}$$

where C is the Curie constant referred to unit mass, R the gas constant for a single molecule, and m the molecular mass, he obtained:

$$\Delta C_m = \frac{1}{2\,J}\,\frac{5}{3}\,\frac{3\,R}{m}. \tag{25}$$

On substitution of the numerical values for R and J there results:

$$\Delta C_m = \frac{4.97}{m}. \tag{26}$$

Weiss[2] and his co-workers have tested this theory in a series of experiments extending over a period of several years. In the early work, equation (21) was used as the form in which to make the test and the results seemed to check the theory within the limits of accuracy of the experiment. In the later work, however, where greater care was taken, the check is less satisfactory. The results in which the Lorentz equation (26) was used are summed up in the following table:

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[1] H. A. Lorentz, *Revue Scientifique*, 1912, 50 année, p. 1.

[2] Weiss and Beck, *Journ. de Phys.*, 4th Series, 7, 1908, p. 249. A. Durnas, Zurich Thesis, 1909. Weiss, Piccard and Carrard, *Arch. des Sci. Phys. et Nat.*, 42, 1916, p. 379, also 43, 1917, p. 113, and 43, 1917, p. 199.

TABLE II

Substance	ΔC_m observed	ΔC_m computed	Corresponding mag. molecule
Nickel	.0285	.0282	Ni_3
Magnetite (Artificial)	.0790	.0644	$\frac{1}{3}(Fe_3O_4)$
Magnetite (Natural)	.0735		
Iron (Pure)	.120	.089	Fe
Iron (Swedish)	.124		

3. *Magnetic Properties of Crystals and the Hysteresis Curve.* Weiss[1] and his group have examined a number of iron minerals and found that some of them possess marked magneto-crystalline properties. One of the best examples is Pyrrhotite, a sulphide of iron. These crystals are usually in the form of hexagonal plates bounded at their edges by faces of a hexagonal prism and are deeply striated parallel to the base. If one examines their magnetic properties in planes parallel to the base, he finds that there is one direction in which they are very easily magnetized, while at right angles to this direction it is difficult to produce saturation. Further, in the direction normal to the base, saturation is still more difficult. Weiss found that the fields necessary for saturation in these three directions are 15, 7300, and 150,000 gauss respectively. After an extended examination, he concluded that the complex crystalline structure consists of a juxtaposition of elementary crystals of which the magnetic planes are parallel, that each crystal possesses a direction of easy and difficult magnetization at right angles to each other, and that the crystals are grouped in the magnetic plane with their axes making angles of 60° with each other.

The direction of easy magnetization is further characterized by the fact that the intensity of magnetization can be changed in sense but not in magnitude. For example, if one acts upon a crystal in this direction with a large field, and then gradually reduces it, carrying it through zero to negative values, he finds that the intensity remains constant down to a value of −15 gauss when it suddenly reverses and takes a negative value of equal magnitude. In other words the hysteresis curve is a rectangle with lines parallel to the H axis extending out from the upper right and lower left hand corners. The magnetic properties of Hematite have been studied by Kunz[2] who found it to be similar to pyrrhotite in that it possesses directions of easy and difficult magnetization, though the effect is less marked, and that the coercive field is somewhat larger. It is ferromagnetic in some directions and paramagnetic in others.

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[1] Weiss, *Journ. de Phys.*, 3rd Series, 8, 1899, p. 542.

[2] Kunz, *Arch. des Sci.*, 23, 1907.

Weiss has attempted to explain hysteresis phenomena in pure metals by assuming that their individual crystals possess properties similar to pyrrhotite; that is, directions of easy and difficult magnetization, and that each crystal is magnetized by its own intrinsic molecular field to the saturation value for its existing temperature. In gross matter, in the unmagnetized state, the directions of easy magnetization will be arranged entirely at random. The process of magnetization in a given direction consists then simply in reversing the direction of magnetization of those elementary crystals whose intensities have components opposite to the external field. For a given crystal, this reversal occurs when the component of the external field in the direction of its axis equals H_c, the coercive field.

The form of the hysteresis curve to be expected on the basis of this assumption may be obtained in the following way. Let M be the magnetic moment of each crystal, and N the number of crystals per unit volume. Since the distribution of directions is entirely at random, the end points of the vectors M will be uniformly distributed over the surface of a unit sphere. Let the external field H act in the direction OX of Fig. 3, and let H_e be the magnitude of the coercive field. Elemen-

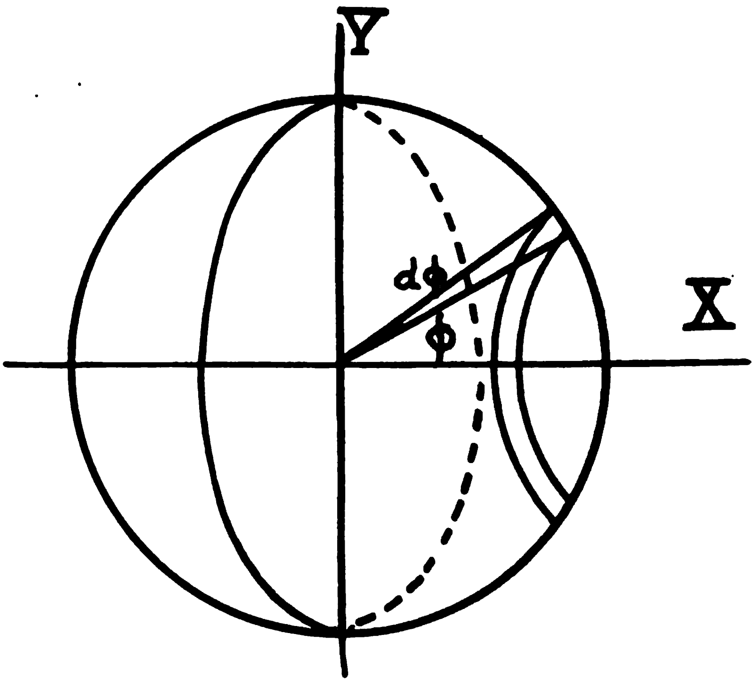


Fig. 3

tary magnets having axes lying within the cone of semi angle ϕ vertical to the one indicated in the figure will be swung into this cone. The angle ϕ is determined by the expression $H \cos \phi = H_c$.

The number of vectors ending in the zone determined by $d\,\phi$ is given by:

$$\frac{2\,\pi\, r^2 \sin \phi\, d\,\phi}{4\,\pi\, r^2} N = \frac{N}{2} \sin \phi\, d\,\phi. \tag{27}$$

The magnetic moment of these magnets in the direction OX is:

$$M_x = M \cos \phi \frac{N}{2} \sin \phi\, d\,\phi. \tag{28}$$

The moment due to all the magnets reversed into the cone is

$$M_x = \int_0^\phi \frac{M\,N}{2} \sin \phi \cos \phi\, d\,\phi \tag{29}$$

$$= \frac{M\,N}{4} \sin^2 \phi = \frac{I_m}{4} \sin^2 \phi.$$

The total magnetic moment due to the magnets in the cone is then:

$$M = \frac{I_m}{2} \sin^2 \phi = \frac{I_m}{2} \left[1 - \left(\frac{H_c}{H}\right)^2\right]. \tag{30}$$

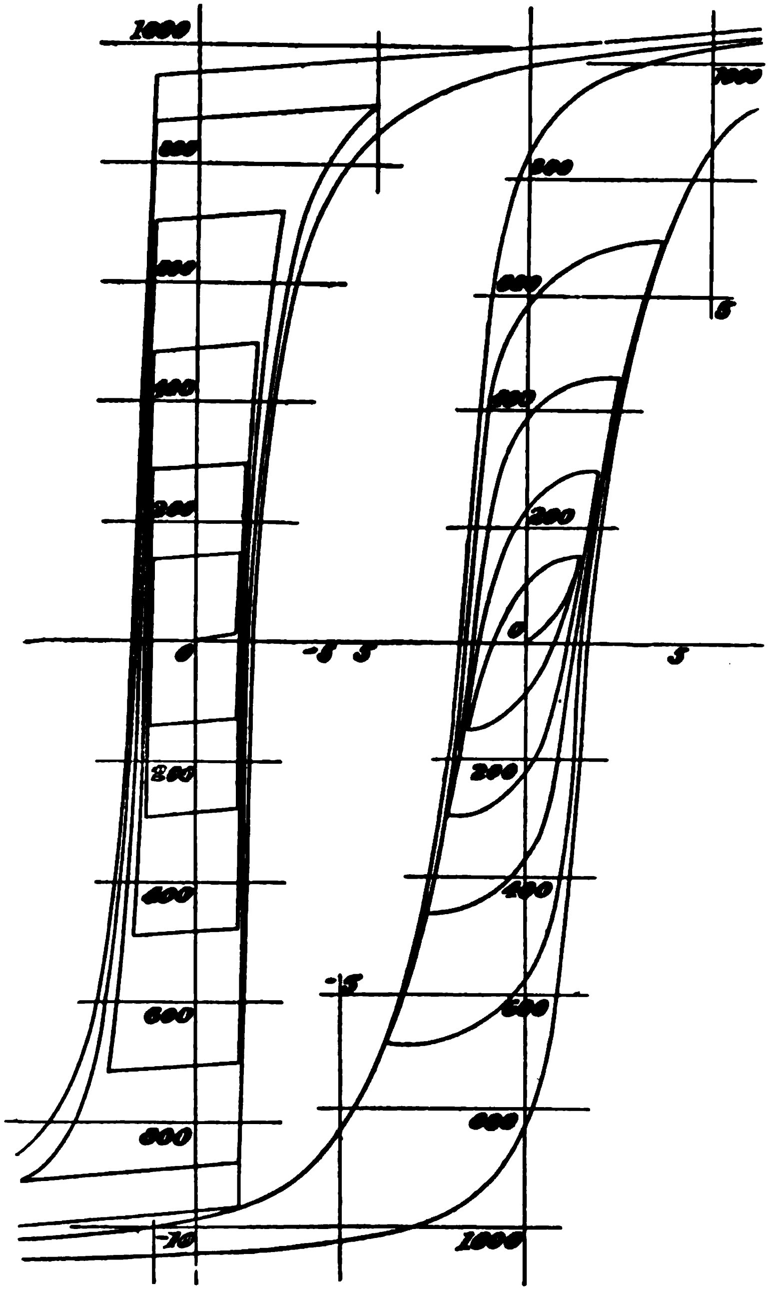
In this discussion it has been assumed that the elementary crystals can be magnetized, only in the direction of easy magnetization, while if they resemble pyrrhotite, they are paramagnetic at right angles to this direction. Weiss has computed the appropriate correction and has matched a set of hysteresis curves taken from the results of Ewing as shown in Fig. 4.

The Elementary Magnets of the Ferromagnetic Substances.

In his original study of a paramagnetic gas, Langevin expressed the intensity of magnetization as the magnetic moment per unit volume instead of per gram molecule as Weiss has done in his later work. For this quantity he used the letter I and his equation was:

$$\frac{I}{I_m} = \cosh a - \frac{1}{a}, \text{ where} \tag{31}$$

$$a = \frac{\mu\, H}{R\, T}; \tag{32}$$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1000

Fig. 4

μ is the magnetic moment for a single molecule and the other quantities have the same meaning as before. In the neighborhood of the transformation temperature θ, these equations become:

$$\frac{I}{I_m}=\frac{a}{3}, \text{ and } a=\frac{\mu H}{RT}, \text{ respectively.} \tag{33}$$

Putting H = N I and eliminating a there results:

$$\mu=\frac{3\,R}{N\,I_m}, \tag{34}$$

an expression by means of which μ may be determined for those substances for which the quantities θ, N, and I_m have been determined. This calculation has been carried out by Kunz.[1] R is the gas constant for a single molecule and may be obtained from the equation:

$$p=N_1\,R\,T, \tag{35}$$

where N_1 is the number of molecules per cc. Substituting the values for the quantities involved, at one atmosphere and 0_o C.,

$$\begin{aligned} p &= 1.01\times10^6 \text{ dynes per cm}^2\\ T &= 273\\ \text{and } N_1 &= 2.7\times10^{19},\\ \text{there results: } R &= 1.36\times10^{-16}. \end{aligned}$$

Taking, for iron, $I_m = 1950$, the value obtained by extrapolating the results of Curie, for N = 3850, the value given by Weiss and Beck,[2] and substituting in equation (34) there results:

$$\mu=4.445\times10^{-20} \text{ absolute electromagnetic units.} \tag{36}$$

As a check on the reasonableness of this result, a calculation of the mass of the hydrogen atom was carried out using it and the known density and molecular weight for hydrogen. Let N_o be the number of molecular magnets per cc. in iron at absolute zero. Then:

$$N_o\mu=1950, \tag{37}$$

$$\text{whence } N_o=\frac{1950}{\mu}=\frac{1950}{4.445\times10^{-20}}=4.386\times10^{22}.$$

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[1] Kunz, *Phys. Rev.*, 30, No. 3, March, 1910.

[2] Weiss and Beck, *Journ. de Phys.*, 7, 1908, p. 249.

If it is assumed that each molecule possesses one elementary magnet of moment μ, then this is also the number of molecules per cc. If m is the mass of one molecule of iron, and D its density, then:

$$N_o m = D = 7.36,$$

$$\text{(38)} \qquad \text{whence } m = \frac{7.36}{4.386 \times 10^{22}} = 1.792 \times 10^{-22} \text{ grams.}$$

If M_H is the mass of the hydrogen molecule and it is assumed that the molecule of iron has two atoms, then:

$$\text{(39)} \qquad M_H = \frac{1.792 \times 10^{-22}}{111.8} = 1.603 \times 10^{-24} \text{ grams.}$$

A recent value of this quantity deduced by Rutherford from radioactive phenomena is:

$$M_H = 1.61 \times 10^{-24} \text{ grams.}$$

If the corresponding calculations are carried out for nickel and cobalt using the best available data the results given in the following table are obtained.

Substance	Im	C	N	NI = HM	$\mu \times 10^{20}$	$M_H \times 10^{-24}$	N
Fe..........	1950	756	3,850	6,540,000	4.445	1.603	2
Ni..........	570	376	12,700	6,350,000	3.65	1.603	6
Co..........	1435	1075	6,180	8,870,000	6.21	1.61	4

It is to be noted that, in order that the computed mass of the hydrogen atom should have the values given, it is necessary to assume that the molecule of nickel has six atoms and that of cobalt four.

The Nature of the Molecular Field.

The hypothesis of the molecular field as introduced by Weiss is a useful concept in the theory of ferromagnetism and has served a number of useful purposes. For example, by adding to the external field the molecular field it is possible to explain many of the complicated phenomena of ferromagnetism by the laws of paramagnetism. It gives a theoretical law for the variation of the saturation value of the intensity with temperature through the ferromagnetic range, and leads to a law for the intensity variation with temperature above the magnetic transformation point. By assuming for the molecular field different values in different directions it is possible to account for many of the complicated phenomena of crystals, and by taking into account the energy associated with the molecular field an explanation for the discontinuity in the specific heat at the transformation point is obtained.

The phenomena of the molecular field, moreover, are not confined to ferromagnetic substances, as there are many instances of its evidence in the case of paramagnetic and diamagnetic substances as well. One may cite, for example, the work of Kammerliegh Onnes and Perrier[1] on the magnetic properties of mixtures of liquid oxygen and nitrogen; that of Kammerliegh Onnes and Oosterhuis[2] on paramagnetic substances at low temperatures; Weiss and Foëx on paramagnetism of crystalline substances, Foëx on concentrated saline substances, and Oxley on diamagnetic substances, to be discussed later. It is true that in many instances, the check is only qualitative and indicates that the theory in its simple form is insufficient, and that the molecular field, instead of being proportional to the intensity of magnetization should be represented by a more complicated function such as:

$$H_m = N_1 I + N_3 I^3 + \ldots\ldots \tag{40}$$

While the hypothesis has thus been useful in explaining many observed facts and directing new lines of investigation one is at once struck by its enormous magnitude and is led to inquire by what means fields of such intensities may be produced. For this purpose one might proceed in the manner employed by H. A. Lorentz for dielectrics and describe a sphere within which there exists a single molecule while on the outside all the other molecules, in their mean effect, play the role of a homogeneous substance. He would then find for the coefficient N of equation (6) the value $\frac{4}{3}\pi$ which falls far short of that experimentally determined on the basis of the theory.

Again, using known data, one might compute on the basis of the inverse square law the necessary distance from a molecular magnet at which the observed molecular field would occur and see whether it leads to values consistent with the known densities of packing of molecules. Take, for example, the molecule of iron which contains eleven magnetons, and suppose that it has a length equal to $.2\times10^{-7}$ cms. the diameter of the atom, and let m be the strength of its magnetic pole. Then since the magnetic moment of the magneton is 16.4×10^{-22}, there results:

$$m\times.2\times10^{-7} = 11\times16.4\times10^{-22},$$
$$\text{or } m = .9\times10^{-12}.$$

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[1] Kam. Onnes and Perrier, *Arch. de Chemie*, 4th Series, 26, Sept., 1913.

[2] Kam. Onnes and Oosterhuis, Comm. Leiden No. 129, p. 132.

The distance from such a pole at which there would exist a field of strength equal to that of the molecular field, 7×10^6, is given by:

$$\frac{m}{r^2}=\frac{.9\times10^{-12}}{r^2}=7\times10^6,$$

$$\text{whence } r=3.6\times10^{-10}\ ;$$

a value much less than the measured distances between molecules. It thus appears that fields of the required magnitude can be obtained neither by superposition of the effects of neighboring molecular magnets using the known average values of the intensity of magnetization, nor by assuming sufficient closeness of packing of the individual molecules.

On the other hand one might enquire whether it is possible to obtain such fields by allowing electrons to rotate with very high velocities in closed orbits about the positive nucleus. For example, suppose an electron having a charge of 1.6×10^{-20} to rotate in a circular orbit of diameter $.2\times10^{-7}$ with a frequency of 10^{15} equal to that of ultra-violet light. The magnetic moment of such a circuit would be:

$$\text{Moment}=1.6\times10^{-20}\times10^{15}\times\pi\times10^{-16}$$

$$=5\times10^{-21},$$

which is equivalent roughly to three magnetons. The strength of the field at the center of the trajectory is:

$$H=\frac{2\cdot\pi\cdot6\cdot10^{-12}\cdot10^{15}\cdot}{\cdot1\cdot10^{-7}}=10^4,$$

which is too small by a factor of 100.

It thus appears that the molecular field can have neither a magnetic nor an electromagnetic origin and must therefore be of a nature different from the ordinary magnetic fields with which we are familiar. Weiss[1] has suggested that the molecular field may be of the same nature as the "magnetizing action of contact" observed by Maurain[2] and others in their study of the magnetic properties of electrolytic iron deposited in a magnetic field. This work will be reviewed briefly. Maurain showed that iron deposited in a field of a few gauss is much more strongly magnetized than that deposited without the field and afterwards subjected to one; also that when the field in which the deposition occurs ex-

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[1] *Annales de Phys.*, 1, 1914, p. 148.

[2] Maurain, *Journ. de Phys.*, 4th Series, 1, 1902, pp. 90 and 151.

ceeds ten or twelve gauss the iron is saturated. This iron maintains its saturation value, practically independent of the field, but suddenly reverses under a coercive force of 20 gauss and the hysteresis curve is practically a rectangle similar to that of Pyrrhotite in its direction of easy magnetization. The saturation values were rather low, however, being only about 840.

Kaufmann and Meyer[1] who repeated the work of Maurain, have confirmed his results regarding the shape of the hysteresis curve and the value of intensity for weak fields, but by using stronger fields they obtained intensities as large as 1100. Schield[2] has also studied iron thus deposited and found an intensity of 980. All of these intensities are considerably less than those for ordinary iron, i. e., 1700, and one is led to suspect that their peculiarities may be due to the presence of a hydride of iron. This seems all the more probable from the fact that many of these peculiarities disappear with time but may be partially restored by making the specimen the cathode of an electrolytic cell. Nevertheless the results obtained have an important bearing on the molecular field theory.

Maurain also found that the first layers of the deposit are different from the later ones in that they are but weakly magnetic. In fact it was only after the deposit had reached a thickness of $80\mu\mu$ that its magnetic moment increased in proportion to its thickness. It thus is evident that there are two fields acting on the molecules at the instant of deposition; first the external field and second, that due to the polarity of the iron already deposited. This latter he called the "magnetic field of contact." He tried opposing these two by reversing the external field after a suitable thickness of deposit had been obtained. It was found that as long as the external field did not exceed the coercive field, usually about 20 gauss, the magnetic moment increased in the direction of the original field for some time in proportion to the thickness and that it was only after the thickness of the new deposit had become comparable to that of the original one that the magnetic moment became zero and finally reversed. The reversal of the polarity of the original deposit took place slowly and could be observed with the magnetometer.

He next studied the dependence of the field of contact upon distance by depositing upon the magnetized cathode suitable layers of neutral metals such as gold, silver, and copper of varying thickness and again depositing iron. With the external field reversed, he found that with a thickness of $38\mu\mu$ of the neutral metal the new layers of iron behaved in the same way as those deposited on an unmagnetized cathode. In other words at this distance the contact field just neutralizes the exter-

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[1] *Phys. Zeitschr.*, 22, 1911, p. 513.

[2] Shield, *Ann. d. Phys.*, 4th Series, 25, 1908, p. 612.

nal field. On the other hand, for very thin layers of neutral metal the contact field is very large compared to the external field. The character of this "field of contact" is as yet unexplained, but it seems probable that it is of the same nature as the "molecular field," and in view of the work of Oxley on diamagnetic substances is worthy of further study.

Theory of Frivold.

As was pointed out above, Weiss concluded that the large molecular fields required by this theory of ferromagnetism could not be of purely magnetic origin but must arise from other magnetic forces. In order to determine to what extent the fields of the individual atomic magnets can account for the molecular field of Weiss, an extended calculation has been carried out by Frivold.[1] For this purpose he assumes that the equilibrium of the elementary magnets is determined not only by the external field and the thermal agitation, as in the Langevin theory, but also by the overlapping of fields of the elementary magnets and treats the problem from the standpoint of statistical mechanics. The calculation is carried out for 2 cases: the unidimensional and the volume distribution.

1. *The Unidimensional Problem:* Elementary magnets of number N are considered to form a long chain and to be free to turn about their midpoints. They are in statistical equilibrium under the influence of their undirected temperature motions, the external fields, and their own mutual field. Let the origin of co-ordinates be located at the middle of the chain, and let θ and ϕ be the usual polar co-ordinates, and let the axis of co-ordinates and the external field coincide with the direction of the chain. If the magnetic moment of an individual magnet is μ, and if their instantaneous positions are given by $\theta_1\phi_1$, $\theta_2\phi_2$, ... $\theta_N\phi_N$, statistical mechanics gives, for the mean magnetic moment of the chain at a temperature T in the direction of the chain, the following expression:

$$(1) \quad M_T = A\int . . . \int \mu \sum_1^N \cos \theta_N \, e^{-\frac{U}{kT}} \, d\,\Omega_1 \, d\,\Omega_2 \; . \; . \; . \; d\,\Omega_N.$$

Here U designates the potential energy of the chain, k the Boltzmann constant ($k = 1.35 \times 10^{-16}$ ergs), and $d\,\Omega$ the solid angle formed by the element of surface $\sin\,\theta d\theta\; d\phi$ on a unit sphere. The integration is to be taken through the 2N variables, $\theta_1\,\phi_1$, $\theta_2\,\phi_2$, ... $\theta_N\,\phi_N$.

The constant A is determined from the following consideration: The probability of a given condition characterized by the fact that the di-

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[1] Frivold, *Ann. der Physik*, **65**, p. 1, 1921.

rection of the axes of the elenentary magnets lie within the solid angles $d\,\Omega_1, d\,\Omega_2, \ldots . d\,\Omega_N$ is

$$A\,e^{-\frac{U}{kT}}d\,\Omega_1\,d\,\Omega_2\,\ldots .\,d\,\Omega_N.$$

Integration of this expression, when the co-ordinates θ and ϕ run throughout the values 1 to N, gives for the probability the value unity. Therefore:

$$A\int\ldots\int e^{-\frac{U}{kT}}\,d\,\Omega_1\,d\,\Omega_2\ldots d\,\Omega_N=1, \tag{2}$$

and

$$M_T=\frac{\int\ldots\int\mu\sum_1^N\cos\theta_N\,e^{-\frac{U}{kT}}\,d\,\Omega_1\,d\,\Omega_2\ldots d\,\Omega_N}{\int\ldots\int e^{-\frac{U}{kT}}\,d\,\Omega_1\,d\,\Omega_2\ldots d\,\Omega_N}. \tag{3}$$

In order to carry out the integrations in equation (3), it is necessary first to determine the potential energy U. This consists of two parts, that due to the mutual potential energy of the elementary magnets, and that due to their positions in the external magnetic field. Calling these U_1 and U_2, respectively, we have:

$$U=U_1+U_2$$

$$=\frac{1}{a^3}\sum_1^{N-1}\left[\left(m_k, m_{k+1}\right)-3\left(m_k,\frac{\bar{a}}{a}\right)\left(m_{k+1}, \frac{\bar{a}}{a}\right)\right]-\sum_1^N\left(m_k, H\right). \tag{4}$$

The expression in brackets is an approximation found to be accurate within 6 per cent.

The mean magnetic moment of an elementary magnet, which consists of such a chain may then be evaluated. Introducing the following abbreviations:

$$(5)\quad\left\{\begin{array}{l}\sum_1^{N-1} f_k=\phi,\text{ where } f_k=\left(m_k, m_{k+1}\right)-3\left(m_k,\frac{\bar{a}}{a}\right)\left(m_{k+1},\frac{\bar{a}}{a}\right);\\[2ex] \sum_1^N g_k=\psi,\text{ where } g_k=\left(\frac{m_k}{\mu},\frac{\bar{H}}{H}\right);\\[2ex] -\frac{1}{a^3kT}=p,\text{ and }\frac{\mu H}{kT}=q;\\[2ex] dS=\frac{d\,\Omega_1}{4\pi}\cdot\frac{d\,\Omega_2}{4\pi}\ldots .\frac{d\,\Omega_N}{4\pi};\end{array}\right.$$

equation (3) becomes:

$$M_T = \frac{\mu \int \ldots \int \psi\, e^{p\phi + q\psi}\, dS}{\int \ldots \int e^{p\phi + q\psi}\, dS} \tag{6}$$

Letting now the integral in the denominator be designated by J, there results:

$$\frac{M_T}{\mu} = \frac{d}{dq} \log J. \tag{7}$$

J may be expanded in a power series in $p\phi$, that is, in powers of

$$\frac{\mu^2}{a^3 k T},$$

and integrated term by term. Thus:

$$J = \int \ldots \int e^{q\psi}\left(1 + p\phi + \frac{1}{2} p^2 \phi^2 + \ldots\right) dS. \tag{8}$$

The approximation, given by the series development, is closer the higher the temperature T. To evaluate A it is necessary to determine the following mean values:

$$\left\{\begin{array}{l} D = \int \ldots \int e^{q\psi}\, dS; \\ D_1 = \int \ldots \int e^{q\psi}\, p\,\phi\, dS; \\ \ldots\ldots\ldots\ldots\ldots\ldots ; \\ \ldots\ldots\ldots\ldots\ldots\ldots \end{array}\right. \tag{9}$$

When these integrations have been carried out, neglecting the quadratic and higher powers in (8), there results for the mean magnetic moment of an elementary magnet in terms of its absolute value, the following expression:

$$\frac{M_T}{N\mu} = \left(\coth q - \frac{1}{q}\right)\left[1 + 4\frac{\mu^2}{a^3 k T}\,\frac{d}{dq}\left(\coth q - \frac{1}{q}\right) + \ldots\right]. \tag{10}$$

If the reciprocal action of one magnet on another is not taken into account, the second term in the square bracket of (10) is zero, and there results the well known Langevin expression. The extent of the deviations from the Langevin expression, brought about by the introduction of the mutual actions, may be seen by substitution of numerical values in (10). If we assume that the chain consists of iron atoms, which, according to Weiss, possess 11 mangetons, each having a moment equal to 16×10^{-22} C. G. S. units, and for "a" assume the value 2×10^{-8} cm., then at a temperature of 300° absolute, since $k=1.35\times10^{-16}$, there results:

$$\frac{4\mu^2}{a^3\,k\,T}=3.7\times10^{-3}\,.$$

In figure (5), curve (1) represents the original Langevin function, while curve (2) is

$$\frac{d}{d\,q}\left(\coth q-\frac{1}{q}\right).$$

This last expression for the values assumed above has a maximum of ⅓ at a field strength of 10^6 gauss. The mean magnetic moment, when the mutual actions are taken into account is represented by curve (3). Since the maximum value of the departure from the Langevin curve is of the order of 10^{-3}, the effect is here greatly exaggerated, and it must be concluded that at this temperature the effect of the mutual actions is quite negligible, and a magnetic body consisting of a chain of magnets with the values given above shows only paramagnetic properties.

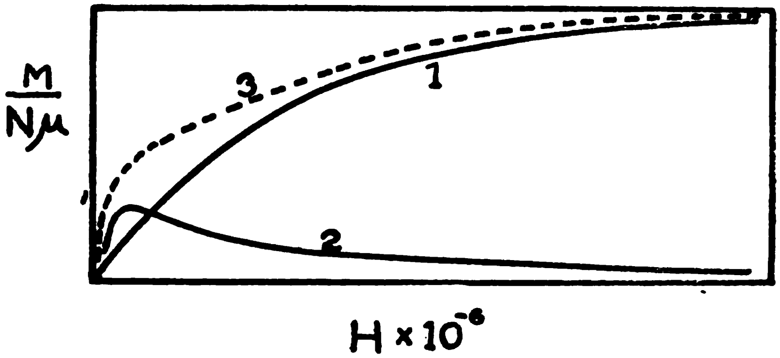

FIG. 5

It is to be noted that in the above development, the integration of the equation (8) was carried out for the first two terms. If the quadratic term is included, the calculation is much more complicated, but the result shows that for external fields of 50,000 gauss, the magnetic moment of the chain is a linear function of H.

At low temperatures, on the other hand, the conclusions are quite different. For small values of T, a number of simplifications in equation (8) may be introduced and the equation corresponding to (10) is found to be:

$$\frac{M_T}{N\mu} = \frac{d}{d\,q}\log J = 1 - \frac{1}{2m}\left[1 + 2\frac{n^2}{m^2} + \;.\;\;.\;\;.\;\;.\;\;.\right], \tag{11}$$

where $n = -\frac{p\,\mu^2}{2}$;

$$m = 2\frac{\mu^2}{a^2\,k\,T} + \frac{\mu\,H}{2\,k\,T}.$$

The results for four values of T are shown in figure (6). Since the approximations do not hold for extremely weak fields, the curves are

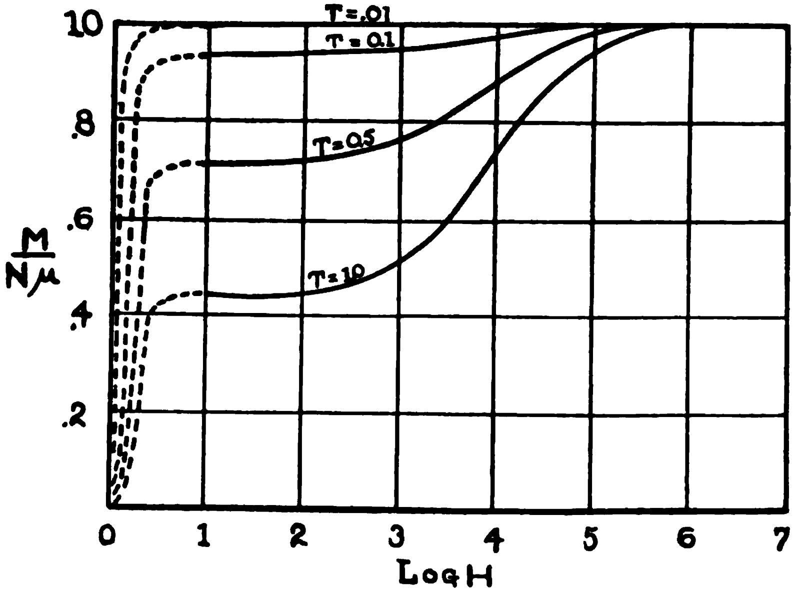

FIG. 6

shown dotted in this region. The chain shows, therefore, at low temperatures, properties characteristic of ferromagnetic substances, but it goes over into the paramagnetic state for temperatures of a few degrees absolute.

2. *The Three Dimensional Problem.* The calculation for the case of the space lattice is carried out in a manner similar to that of the unidimensional problem. The elementary magnets are regarded as located at the corners of a cubical space lattice and turn about their midpoints. The expression for J in equation (8) is evaluated as before but is complicated by the fact that double summations must be made. If, as a first approximation, the expression corresponding to D and D_1 of equation (9) are evaluated, there results the well-known Langevin equation:

$$\frac{M}{N\mu} = \coth q - \frac{1}{q}. \tag{12}$$

If, however, the quadratic term of equation (8) is included, an expression, in which the mutual actions appear, is obtained. Two cases have been studied—the ordinary cubical space lattice and the centered cubic. For the former, Frivold obtains:

$$\frac{M}{N\mu} = \frac{1}{3}\frac{\mu H}{k T}\left[\left\{1 - \frac{6.27}{4}\left(\frac{\mu^2}{a^3 k T}\right)^2 + \ .\ .\ .\right\} + \left(\frac{\mu H}{k T}\right)^2 \left\{\frac{2.56}{4}\left(\frac{\mu^2}{a^3 k T}\right)^2 - \frac{1}{15} + \ .\ .\ .\ .\right.\right], \tag{13}$$

and for the latter:

$$\frac{M}{N\mu} = \frac{1}{3}\frac{\mu H}{k T}\left[\left\{1 - \frac{8.88}{4}\left(\frac{\mu^2}{a^3 k T}\right)^2 + \ .\ .\ .\right\} + \left(\frac{\mu H}{k T}\right)^2 \left.\frac{3.22}{4}\left(\frac{\mu^2}{a^3 k T}\right)^2 - \frac{1}{15}\right\} + \ .\ .\ .\ .\right]. \tag{14}$$

A comparison of these equations shows that the numerical factors in the two cases are of the same order and, consequently, the difference in the arrangement of the atoms plays no important role. Accordingly, in the following discussion only the former case will be considered. If the mutual action is left out of account, equation (13) gives for the case, in which the external field is relatively small, the well-known Langevin equation for small fields:

$$\frac{M_T}{N\mu} = \frac{1}{3}\frac{\mu H}{k T}, \tag{15}$$

and for the initial permeability:

$$(16) \qquad \frac{M_T}{H} = \frac{1}{3}\frac{\mu^2}{k\,T}\,N.$$

If, however, the mutual action is taken into account, equation (16) becomes:

$$(17) \qquad \frac{M_T}{H} = \frac{1}{3}\frac{\mu^2}{k\,T}N\left[1 - \frac{6.27}{4}\left(\frac{\mu^2}{a^3\,k\,T}\right)^2 + \;.\;\;.\;\;.\;\;.\;\;.\right],$$

which indicates that the effect of the mutual action is to reduce the initial permeability.

A study of equation (13) shows that for external fields of such magnitude that q^2 comes into consideration, a temperature transformation point is evidenced. For example, the magnetization curve (13) lies above, coincides with, or is below the curve of equation (17), according as the sign of the coefficient

$$\left(\frac{\mu\,H}{k\,T}\right)^3$$

is positive, zero, or negative, or, in other words, are equal as T is greater than, equal to, or less than

$$\frac{\mu^2}{a^3k}\sqrt{\frac{4}{2.56\times 15}}\;.$$

The magnitude of these departures is, however, very small, as may be seen by the substitution of the generally accepted numerical values for iron. For example, putting $\mu = 1.76\times 10^{-20}$ C. G. S. units, $a = 2.86 \times 10^{-8}$ cms., and $T = 390°$ absolute,

$$\left(\frac{\mu^2}{a^2\,k\,T}\right)^2 \cong 10^{-7}.$$

From this it must be concluded that at ordinary temperatures, the influence of the mutual actions upon the magnetization curve for the case of the hypothetical magnetic substance we have here considered, is negligible. The reason for this is the small value for the moment of the elementary magnet. It is of interest to consider the case in which the atoms contain, in addition to the elementary magnets, electric dipoles, the electric moments of which are of the order of those found for SO_2 e. g. 10^{-18}; that is, 100 times larger than the magnetic moments for iron atoms. The external magnetic field then starts the lining up process, whereby the internal electric and magnetic fields are brought into play. Frivold has carried through the calculation in this case, also, and obtained the following expression:

$$(18)\quad \frac{M_T}{N\mu}=\frac{1}{3}\frac{\mu_m H}{k T}\left[\left\{1-\frac{6.27}{4}\left(\frac{\mu_e^2}{a^3 k T}\right)^2+\ .\ .\ .\ .\right\}\right.$$
$$\left.+\left(\frac{\mu_m H}{k T}\right)^2\left\{\frac{2.56}{4}\left(\frac{\mu_e^2}{a^3 k T}\right)^2-\frac{1}{15}\right\}+\ .\ .\ .\ .\right].$$

where the subscripts m and e refer to magnetic and electric moments, respectively. Much larger departures from the case in which mutual actions are neglected are thus obtained, for here

$$\frac{\mu_e^2}{a^3 k T}\cong 1.$$

Unfortunately, the integral of equation (8), upon which the entire treatment rests, is developed in powers of p ϕ, i. e. of

$$\frac{\mu_e^2}{a^3 k T},$$

and the series is convergent only when this expression is less than unity. The question as to whether ferromagnetism can thus be explained by the assumption of electrical dipoles, is still left open, but interesting possibilities are here suggested.

Theory of Gans.

A theory of ferromagnetism has been developed by Gans in which he has attempted to take into account the effect of molecular structure upon magnetic properties, and to make more precise the ideas concerning the nature of the molecular field than was done by Weiss. He supposes that an elementary magnet or "Magneton" is an electrified body of revolution rotating rapidly about its axis of figure. An elementary complex consists of a group of such magnetons, distributed according to the laws of probability throughout a space which has the form of an ellipsoid, the three axes of which are unequal. The magnetons are free to move about within the complex in the same manner as the molecules of a gas. A ferromagnetic crystal is built up of such complexes placed at the intersections of a space lattice with their corresponding axes parallel.

By applying the laws of statistical mechanics to a system of such complexes, relations are obtained between magnetic and thermal quantities, similar in form to those of Weiss, but which are somewhat more comprehensive. From stability considerations, he is able to deduce the hysteresis curve to obtain a relation between coercivity and temperature, and to determine the number of magnetons per unit volume and the magnetic moment per magneton.

The chief difference between the Weiss theory and that of Gans, briefly stated, is as follows: In the former, the exciting field, acting at a definite point within a ferromagnetic body, is composed of two parts, the external field H, and the molecular field N M, where M is the intensity of magnetization and N is a constant characteristic of the substance. In the Gans theory, the exciting field consists of three parts, the external field H, the "structure field," due to the gross magnetization of the body, which, by a treatment similar to that of Lorentz for dielectrics, is found to be

$$\frac{4}{3}\pi M,$$

and a molecular field A due solely to the action of the magnetons of the particular complex in which the magneton under consideration is located. It is assumed that each direction for the molecular field A is equally probable, and that its magnitude is independent of direction. The molecular field, on account of the different directions which it assumes, has a tendency for disorganization and acts, therefore, in the same sense as the thermal agitation. At high temperatures, the action of the molecular field may be neglected in comparison to the thermal agitation, while at low temperatures, thermal agitation may be neglected in comparison to the molecular field.

The equation for the magnetization curve for a ferromagnetic substance may be deduced in the following manner:[1] The magnetic moment of the magnetons of a particular group has, from symmetry considerations, the direction of the resultant field F, which is obtained by vectorial addition of K and A, where K is the sum of the external and structure fields, and is equal to

$$H+\frac{4}{3}\pi M$$

and A is the molecular field. This is shown in Fig. 3, section III of the report on kinetic theories of dia- and paramagnetism by Dr. Wills.

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[1] cf. p. 46 of this report.

When the component of this magnetic moment is taken in the direction of the external field and the summation extended to all the groups included within a unit volume of the substance, its intensity of magnetization is obtained. It is shown by equation (32) of the above reference, that the average value of $\cos\theta$, where θ is the angle between the axis of a magneton and the external field, is:

$$(1) \qquad \overline{\cos\theta} = \frac{1}{4\,K^2}\int_0^\infty \frac{\overline{W}\,(A)\,d\,A}{A}\int_{\pm(A-K)}^{(A+K)} (\coth a - \frac{1}{a})(F^2+K^2-A^2)\,d\,F.$$

where $\overline{W}\,(A)\,dA$ denotes the probability that the molecular field A lies between the limits A and A+d A, and

$$a = \frac{\mu\,F}{k\,T} = \gamma\,F.$$

In carrying out this integration, the positive sign of the lower limit should be used for $K<A$ and the negative for $K>A$.

Since it is assumed that the molecular field is constant in magnitude and that all directions are equally probable, equation (1) reduces to:

$$(2) \qquad \overline{\cos\theta} = \frac{1}{4\,K^2}\int_{\pm(A-K)}^{(A+K)} (\coth a - \frac{1}{a})(F^1+K^2-A^2)\,d\,F.$$

The magnetic moment per unit volume will, therefore, be given by:

$$(3) \qquad M = N\,\mu\,\overline{\cos\theta} = \frac{M_0}{4\,A\,K^2}\int_{\pm(A-K)}^{(A+K)} (\coth a - \frac{1}{a})(F^2+K^2-A^2)\,d\,F,$$

where M_0 equals $N\,\mu$. Since

$$K = H + \frac{4}{3}\,\pi\,M,$$

this relation, together with equation (3), gives M as a function of H, and the magnetization curve may accordingly be deduced. It may be shown that for $A=0$, equation (3) reduces to that of Langevin.

To apply this formula to a ferromagnetic body and to see how the phenomenon of hysteresis is concluded, let us think of a ferromagnetic crystal having a rhombic space lattice such as pyrrhotite, with ellipsoidal elementary complexes situated at the intersections with corresponding axes parallel. Let the magnetization and field strength at points within the complexes be designated by M^1 and H^1, respectively, and let M and H refer to the corresponding quantities at points within the crystal but outside the complexes. We may then write:

$$H_x' = H_x + N_1' M_x,$$

$$H_y' = H_y + N_2' M_y,$$

$$H_z' = H_z + N_3' M_z,$$

where N_1', N'_2, N_3' are constants depending upon the structure. Further:

$$M = n\,V\,M'$$

where n is the number of elementary complex per unit volume and V is the volume of a single complex.

The quantity K of equation (3) is defined by:

$$K = H' + \frac{4}{3}\pi M';$$

but may be expressed in terms of H and M by the following relations:

$$K_x = H_x + N_1 M_x,$$

$$K_y = H_y + N_2 M_y,$$

$$K_z = H_z + N_3 M_z,$$

where

$$N_1 = N_1' + \frac{4\pi}{3nV}; \quad N_2 = N_2' + \frac{4\pi}{3nV}; \quad N_3 = N_3' + \frac{4\pi}{3nV}.$$

K may be regarded as the directive part of the total force acting on the magnetons of a complex and is made up of the external field H within the crystal and another field having components $N_1 M_x$, $N_2 M_y$, $N_3 M$ which depends essentially upon the form and arrangement of the elementary complexes and which may appropriately be named the " Structure " field.

The magnetization curve, i.e. the M, H curve for a crystal in the direction of one of the axes of symmetry, e.g. the X axis, may be obtained from the M K curve by a shearing process as follows:

Let the dotted curve of Fig. 7 be that given by equation 3, and let S S′ be the shearing line inclined to the O M axis by an angle such that $\tan\alpha = N_1$.

If P is a point on the M K curve, then P′ is the corresponding point on the M H curve, where P P′=QR. The shearing angle α depends upon the structure constant N. Two cases are to be considered, i.e. $\alpha < \beta$, and $\alpha > \beta$, where β is the angle between the tangent to the M K curve at the origin and the O M axis. In the first case the M H curve lies entirely in the first quadrant; but in the second it follows the path O C D of Fig. 8. From stability considerations it may be shown that for $\alpha < \beta$ the magnetons of the elementary complexes are in stable equilibrium throughout the entire range of field strengths and that the substance is paramagnetic. On the other hand, when $\alpha > \beta$, between the fields designated by the vertical tangents at the points C C′, the equili-

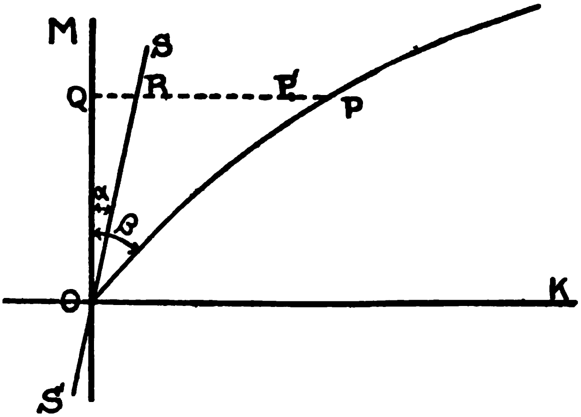

FIG. 7

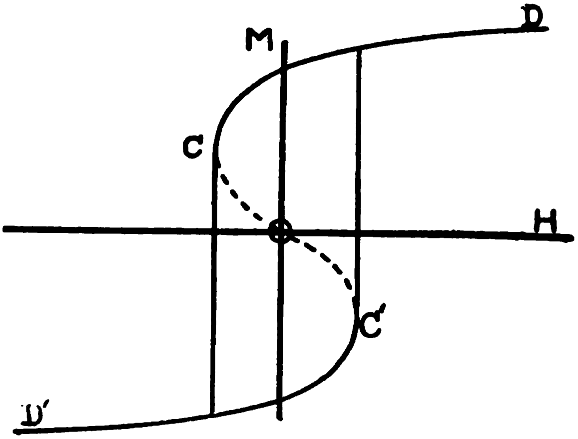

FIG. 8

brium is labile. The substance is then ferromagnetic and exhibits the properties of hysteresis, as indicated by the curve.

By developing the integrand of equation (3) in a power series and making certain approximations to simplify the integrations, Gans has deduced a number of important relations between magnetic properties and temperature. For example, he has deduced equations connecting retentivity and temperature, coercive force and temperature, and obtained a relation between susceptibility and temperature for a ferromagnetic substance above the Curie point. The first of these is substantially the same as obtained by Weiss and is in good agreement with the observations of Weiss[1] and Foëx for magnetite but not for iron or nickle. The second and third relations are in good agreement with the results of Terry[2] for iron, nickel, and cobalt at high temperatures.

The Theory of Honda and Okûbo.

In contrast to the theory of Weiss, in which molecular fields of the order of several million gauss are assumed to be acting, an attempt has been made by Honda and Okûbo,[3] following the ideas of Ewing, to deduce the curves of magnetization and hysteresis and to explain the

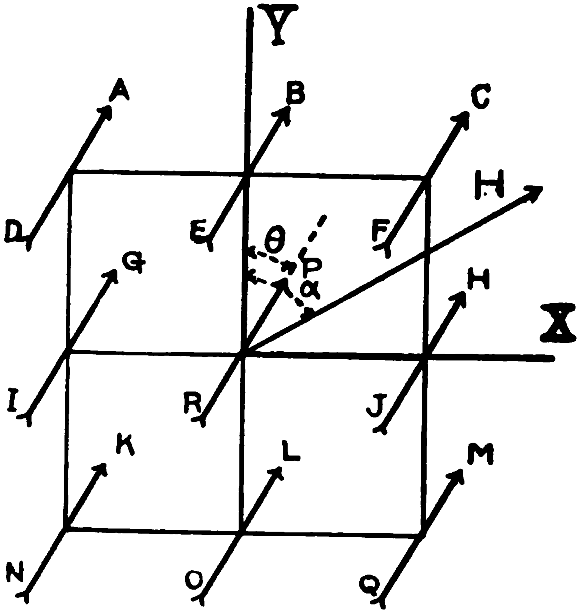

Fig. 9

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[1] Weiss and Foëx, *Arch. des Sci. Phys. et Nat.*, 31, p. 4, 1911.

[2] Terry, *Phys. Rev.* 33, No. 2, 1910 and 60, N. S. No. 5, p. 394, 1917.

[3] *Science Reports Tohoku Univ.* 5, No. 3, p. 153, 1916.

properties of crystals by taking account of the mutual actions of magnetic molecules whose poles act according to the law of inverse squares. For this purpose they have considered a Ewing model of nine coplanar magnets placed at the corners of a square space lattice as shown in Fig. 9. Although the real problem is three dimensional, a study of the two dimensional case is sufficient to indicate the degree of success to be expected from such a theory. If no external field acts, the elementary magnets take positions of stable equilibrium parallel to one of the sides of the space elattice. Under the action of a field, however, the group turn as a whole toward the direction of the field and takes an equilibrium position determined by it and the mutual actions of the group.

To make the problem definite, let the origin of coordinates be at the center of the magnet P R and let an external field H act in a direction α with respect to the Y axis, and suppose the elementary magnets to be turned through an angle θ in consequence. Let 2a, 2r and m be the sides of the space lattice, the length of an elementary magnet, and the pole strength respectively, and put $\frac{a}{r} = k$.

The position of equilibrium of one of the magnets such as P R may be determined by equating the torque due to the external field to that of the 16 remaining poles of the group. The restoring torque is obviously a function of 4θ since the magnets of a square space lattice are in equilibrium when they stand end to end; the equilibrium being stable when they are parallel to one of the axes of coordinates, and unstable when parallel to the diagonals. The analysis shows that the equilibrium condition may be written:

(1) $$H \sin (\alpha - \theta) = A \sin 4\theta,$$

where $A = \frac{m}{r^2} \phi (K)$ is a quantity depending upon the strength of the elementary magnets and their particular arrangement within the group.

The intensity of magnetization I in the direction of the applied field is:

(2) $$I = 2\, m\, r\, n \cos (\alpha - \theta) = I_o \cos (\alpha - \theta),$$

where n is the number of elementary magnets and I_0 the saturation value of the intensity of magnetization.

(3) Using $$\frac{I}{I_0} = i, \text{ and } \frac{H}{A} = h,$$

as " reduced " values of the intensity and field respectively, we have the relations:

(4) $$i = \cos (\alpha - \theta),$$

(5) and $$h \sin (\alpha-\theta) = \sin 4\theta,$$

as the equations defining the magnetization of a simple complex. If h and α are given, equation (5) gives the value of θ, and this, when substituted in equation (4), gives the value of i. Equation (5) is, however, of the eighth degree in $\sin\theta$ or $\cos\theta$ and must therefore be solved by an indirect process. It is necessary, first to point out the way in which an elementary complex behaves when acted upon by external fields of various magnitudes in different directions. As indicated by equation (1) the restoring torque on each magnet due to the mutual actions of the group is a function of period $\frac{\pi}{4}$. It is a maximum for angles of $\frac{\pi}{8}$ with the sides, and reverses sign at angles of $\frac{\pi}{4}$. Let us suppose that the magnets are originally parallel to Y and that a field h acts at an angle α and rotates them through θ. Four cases present themselves.

Case 1. $0<\alpha<\frac{\pi}{4}$. The component magnetization in the direction of h starts with the value $i = \cos\theta$, increases continuously with h and becomes unity for $h = \infty$.

Case 2. $\frac{\pi}{4}<\alpha<\frac{\pi}{2}$. The magnetization increases continuously with h until the deflecting torque exceeds the restoring torque, when the magnets jump to a new position of equilibrium between h and X. This new position is the same as though the magnets had remained in their original positions and α were changed to $\alpha-\frac{\pi}{2}$. There results a discontinuous increase in i. For angles α in this octant, the jump occurs for values of θ in excess of $\frac{\pi}{8}$. With further increase in h, i increases continuously to unity as h approaches infinity.

Case 3. $\frac{\pi}{2}<\alpha<\frac{3\pi}{4}$. The magnetization increases continuously with h until the restoring torque is exceeded by the deflecting torque when a discontinuity occurs, and the magnetization follows the same course as though α were replaced by $\alpha-\frac{\pi}{2}$. This case is similar to case 2, except that in the new equilibrium position π is greater than $\frac{\pi}{2}$.

Case 4. $\frac{3\pi}{4}<\alpha<\pi$. The magnetization up to the discontinuity is the same as in the above cases. The discontinuity, however, may be of two types. For directions of h somewhat greater than $\frac{3\pi}{4}$ the torque is greater for the maximum near $\frac{\pi}{8}$ than for the one near $\frac{5\pi}{8}$; the magnets jump to a position somewhat less than $\frac{5\pi}{8}$ and the subsequent magnetization takes place as though α were replaced by $\alpha-\frac{\pi}{2}$ as in case 3. If, on the other hand, α lies in the neighborhood of π, the torque is greater near $\frac{5\pi}{8}$ than $\frac{\pi}{8}$ and the magnets jump to a position between h and the negative Y axis, and the subsequent magnetization takes place as though α were replaced by $\alpha-\pi$.

The field h_m at which the discontinuity occurs, may be obtained in the following way. Since

$$\text{(6)} \qquad h=\frac{\sin 4}{\sin(\alpha-\theta)},$$

the value of θ for which h is a maximum is given by:

$$\text{(7)} \qquad \frac{dh}{d\theta}=\frac{5\sin(\alpha+3\theta)+3\sin(\alpha-5\theta)}{2\sin^2(\alpha-\theta)}.$$

Calling this θ_0, there results:

$$\text{(8)} \qquad 5\sin(\alpha-3\theta_0)+3\sin(\alpha-5\theta_0)=0.$$

The field h_m is obtained by solving this equation for θ_0 and substituting in (6).

We are now in a position to study the magnetization of a ferromagnetic mass consisting of a large number of elementary complexes with their space lattices distributed uniformly in all directions in a plane. Let N be the number of complexes, and d N the number whose axes make with a certain direction an angle between α and $\alpha+d\alpha$ when no external field is acting. Then:

$$\text{(9)} \qquad dN=\frac{N}{2\pi}\,d\alpha.$$

Let M be the moment of a complex, α the angle between its initial direction and that of the external field h, and let it be turned through an angle θ by the action of this field. In the direction of the field its component is M cos $(\alpha-\theta)$. The magnetization due to all the complexes is:

$$I=2\int_0^\pi \frac{M\,N}{2\,\pi}\cos(\alpha-\theta)\,d\,\alpha=\frac{I_0}{\pi}\int_0^\pi \cos(\alpha-\theta)\,d\,\alpha, \tag{10}$$

where $I_0=M\,N$ is the saturation value of the magnetization. The reduced magnetization i is given by:

$$i=\frac{1}{\pi}\int_0^\pi \cos(\alpha-\theta)\,d\,\alpha. \tag{11}$$

This, together with equation (80) furnishes the solution to the problem of finding the equation for the magnetization curve. That is, for a given value of h, θ may be found from (6) in terms of α, and this value when substituted in (11) gives i. Owing to the discontinuities in θ discussed above, it is, however, necessary to consider the problem for large and small values of h separately.

When h is small, θ is also small, and we may put $\sin 4\,\theta=4\,\theta$. Equation (80) then becomes:

$$h\,(\sin\alpha-\theta\cos\alpha)=4\,\theta; \tag{12}$$

whence

$$\theta=\frac{h\sin\alpha}{4+h\cos\alpha}.$$

Substituting in equation (11) there results:

$$\begin{aligned} i&=\frac{1}{\pi}\int_0^\pi(\cos\alpha+\theta\sin\alpha)\,d\,\alpha\\ &=\frac{1}{\pi}\int_0^\pi\left(\cos\alpha+\frac{h\sin^2\alpha}{4+h\cos\alpha}\right)d\,\alpha \qquad (13)\\ &=\frac{1}{\pi}\int_0^\pi\cos\alpha\,d\,\alpha+\frac{h}{4\pi}\int_0^\pi\sin^2\alpha\,\left(1+\frac{h}{4}\cos\alpha\right)^{-1}d\,\alpha. \end{aligned}$$

The first integral vanishes and the second, when expanded in a power series and integrated, gives

$$i=\frac{h}{4}\left(\frac{1}{2}+\frac{h^2}{8.4^2}+\frac{\pi}{16}\,\frac{h^4}{4^4}+\ldots\right)$$

$$=.125\,h+.00195\,h^3+.00007\,h^5+\ldots\ldots \quad (14)$$

The intensity i is here expressed as an odd function of h and is nearly linear in the neighborhood of the origin with an upward concavity which increases with h. It approximates well the experimentally determined curves.

The solution for large values of h is complicated by the abrupt changes in the value of θ. Further, the angle at which these discontinuities occur depends upon both the external field and the orientation of the complex. It is therefore necessary, in evaluating (11) to divide the integration interval into several parts. For a given h, the critical angles may be determined from equations (8) and (6), a study of which shows that, for reduced fields slightly in excess of unity, there will be three such angles, giving four integration intervals. Calling these angles α_1, α_2, and α_3 we have :

$$\int_0^{\pi}=\int_0^{\alpha_1}+\int_{\alpha_1}^{\alpha_2}+\int_{\alpha_2}^{\alpha_3}+\int_{\alpha_3}^{\pi} \quad (15)$$

For the complexes lying within the intervals of the first and fourth integrals, the magnets remain stable since the torque due to the external field does not exceed the restoring torque. For the complexes of the second integral, the magnets make jumps of $\frac{\pi}{2}$ as explained in cases 2 and 3 above, and the integration limits must be changed from α_1 and α_2, to $\alpha_1-\frac{\pi}{2}$ and $\alpha_2-\frac{\pi}{2}$, respectively. For the third integral, the magnets lie beyond the first and second positions of stable equilibrium, and jump by an angle π. The limits accordingly must be changed to $\alpha_2-\pi$ and $\alpha_3-\pi$, respectively.

The integrand of equation (11) contains θ and the evaluation can be effected more easily in terms of this variable than of α. The elimination of α may be made as follows: Differentiating (6) with respect to θ, there results:

$$h\cos(\alpha-\theta)\left(\frac{d\alpha}{d\theta}-1\right)=4\cos 4\,\theta;$$

whence

$$\frac{d\alpha}{d\theta}=\frac{4\cos 4\,\theta}{h\cos(\alpha-\theta)}+1.$$

(16) Since $\cos(\alpha-\theta) = +\frac{1}{h}\sqrt{h^2-\sin^2 4\theta}$,

$$i = \frac{1}{\pi}\int\left\{\frac{4\cos 4\theta}{h} + \frac{1}{h}\sqrt{h^2-\sin^2 4\theta}\right\}d\,\theta.$$

The new integration limits corresponding to α_1, α_2, and α_3 for given values of h may be obtained by substituting these values successively in equation (6) and solving for θ. When this has been done, there results:

$$\int_0^{\pi} = \int_{\theta_1}^{\theta_1'} + \int_{\theta_2}^{\theta_2'} + \int_{\theta_3}^{\theta_3'} + \int_{\theta_4}^{\theta_4'}. \tag{17}$$

From equation (90):

$$i = \frac{1}{\pi h}\left\{\sin 4\theta\Big]_{\theta}^{\theta'} \pm \frac{1}{\pi}\int_{\theta}^{\theta'}\sqrt{1-\frac{1}{h^2}\sin^2 4\theta\, d\theta}\right\}. \tag{18}$$

The integral in this equation may be written:

$$\frac{1}{\pi}\int_0^{\theta'}\sqrt{1-k^2\sin^2 4\theta}\,d\theta - \int_0^{\theta}\sqrt{1-k^2\sin^2 4\theta}\,d\theta, \tag{19}$$

where $k^2 = \frac{1}{h^2}$. These are elliptic integrals of the second kind with modulus k, and may be written:

$$\frac{1}{4\pi}\{E(k, 4\theta') - E(k, 4\theta)\}.$$

Expanding E as a power series in k and determining the appropriate limits of θ in equation (17) from equations (8) and (6), Honda and Okubo have computed the intensities corresponding to four different values of h. The results are given in Table I and plotted in Figure 10 which is seen to possess, in a marked degree, the characteristics of the experimentally determined magnetization curve for a ferromagnetic substance.

TABLE I

1.0	......	......	......	....	0.183
1.5	0.301	0.436	0.035	−0.096	0.677
2.0	0.286	0.448	0.127	−0.045	0.816
5	0.289	0.475	0.172	−0.031	0.875
0	0.241	0.483	0.196	−0.011	0.909

The residual magnetism to be expected on the basis of this theory may be obtained as follows: When h has been made infinite the magnets of all the complexes having orientations between $\pm\frac{\pi}{4}$ and $\pm\frac{3\pi}{4}$ take new positions of equilibrium corresponding to discontinuous rotations of $\frac{\pi}{2}$ with respect to their initial positions, while those lying between $\pm\frac{3\pi}{4}$ and π jump by π. When the field is reduced to zero, all the magnets then behave in the same manner as those lying between zero and $\frac{\pi}{4}$ which return reversibly to their original positions. The residual magnetism R is then given by:

$$(20)\qquad R=2\int_0^{\frac{\pi}{4}} M\cos\theta\, dN,\text{ where } dN=\frac{2N}{\pi}d\theta$$

$$=\frac{4I_0}{\pi}\int_0^{\frac{\pi}{4}}\cos\theta\, d\theta=\frac{4I_0}{\pi\sqrt{2}};$$

and the reduced residual magnetism r is:

$$r=\frac{R}{I_0}=.8927.$$

The portion of the hysteresis curve lying between the retentivity point and maximum induction may be deduced by considering that the magnetization process in this interval takes place reversibly and that all the complexes have initial orientations lying between $\pm\frac{\pi}{4}$ with respect to the direction of the field. The law of magnetization is then given by the equations:

$$i=\frac{4}{\pi}\int_0^{\frac{\pi}{4}}\cos(\alpha-\theta)\, d\alpha,\text{ and } h=\frac{\sin 4\theta}{\sin(\alpha-\theta)}.$$

(21) For h small we have:

$$i=\frac{1}{\pi}\int_0^{\frac{\pi}{4}}(h+4\cos\alpha)\left(1+\frac{h}{4}\cos\alpha\right)^{-1}d\alpha$$

$$=.8927+.047\,h-.083\,h^2\ldots\ldots$$

For larger value of h, equation (19) must be used where the proper limits of intergation are obtained from equations (8) and (6). The portion of the hysteresis curve for negative values of h is obtained by assuming that the case is equivalent to the magnetization by a positive field of a group of complexes whose initial magnetic directions are uniformly distributed between the angles $\frac{3\pi}{4}$ and $\frac{5\pi}{4}$.

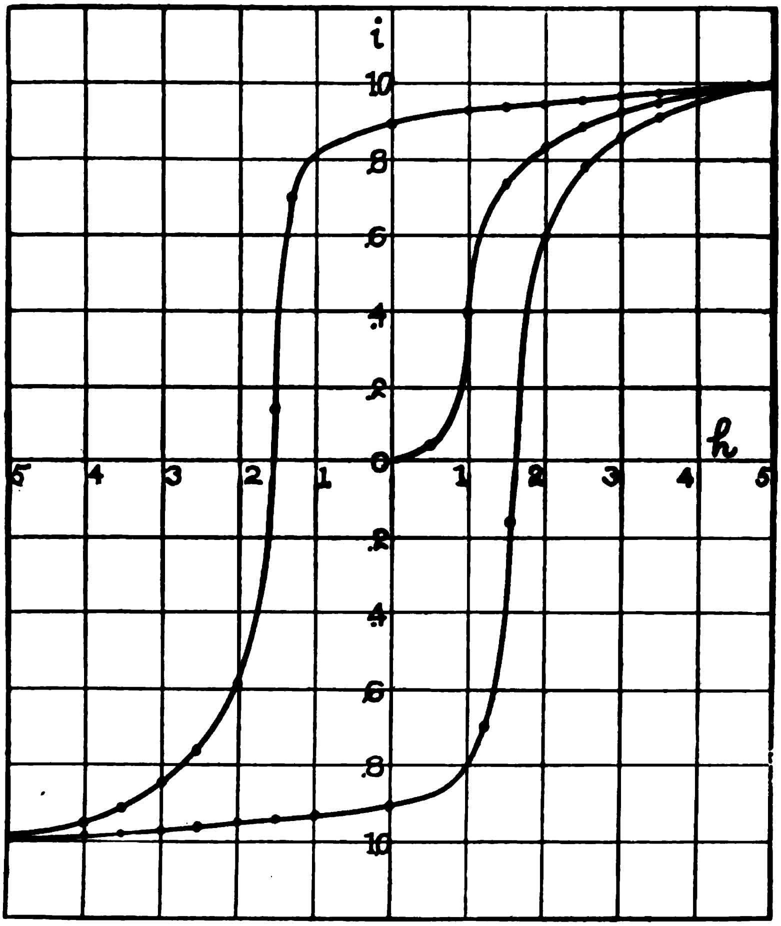

Fig. 10

The results of calculations are shown in Table II and plotted in Figure 10.

TABLE II

h	i	h	i
+0.0	1.000	−1.0	0.815
3.5	0.973	−1.5	0.015
3.0	0.962	−2.0	−0.584
2.5	0.956	−2.5	−0.786
2.0	0.944	−3.0	−0.847
1.5	0.932	−5.0	−0.981
1.0	0.922	$-\infty$	−1.000
0.0	0.893		

The similarity between these curves and the curves of experiment is striking. The most important departure is probably the large value of the retentivity. For the curves here deduced the remanence is 89 per cent, while in practice one seldom finds a value greater than 60 per cent. This discrepancy is probably due to the fact that in this theory no account is taken of thermal agitation. Hysteresis phenomena are assumed to take place only when the molecular magnets turn abruptly through angles of $\frac{\pi}{2}$ or π, otherwise the processes are reversible. The energy losses due to hysteresis must be accounted for by the kinetic energy acquired by the magnets during these jumps which is then dissipated by friction, radiation or some other process.

Honda and Okûbo have extended their study to the case of magnetic crystals. For this purpose, the only change it is necessary to make for those of the rectangular system, such as Magnetitie and Hematite is that all the elementary groups are oriented in the same direction instead of at random as in the case discussed above. For Pyrrhotite, a hexagonal space lattice must be used for which F (θ) has a period of $\frac{\pi}{6}$. By this means they have deduced the results of Weiss, Quittner and Kunz on these crystals with the same degree of accuracy as was obtained in the case of ordinary ferromagnetic substances.

The Mean Molecular Field of Diamagnetic Substances.

In the resume of the Weiss theory it was pointed out that many of the phenomena of ferromagnetism may be explained in terms of the laws of paramagnetism by the introduction of an internal or molecular field due to the presence of surrounding molecules. Langevin has indicated that the origin of the magnetic properties of both para- and diamagnetic substances is to be found in the rotation without damping of electrons in closed orbits about the positive nuclei. If the arrangement of the orbits possesses complete symmetry, the resultant magnetic moment and hence the field at distances large compared to molecular magnitudes is zero, and the substance is diamagnetic. If, on the other hand, there is a lack of symmetry in the orbital arrangement, the field at a distance is not zero, and the substance is paramagnetic. The pondermotive action of repulsion exhibited by diamagnetic substances when introduced into a magnetic field is accounted for by assuming changes in the electronic orbits in accordance with the ordinary laws of induced currents in a manner analogous to the explanation of the Zeeman effect given by Lorentz.

In his theory of diamagnetism, Langevin has considered the effect of the external field only and has not taken into account the action of neighboring molecules when the substance is polarized. The fact that the Zeeman effect and the rotation of the plane of polarization, both closely related to diamagnetism are, in the case of ferromagnetic substances, proportional to the intensity of magnetization and not to the applied external field would indicate that in diamagnetism also, the susceptibility should be a function of the state of polarization. Inasmuch as the forces of diamagnetic repulsion are small and the susceptibility is in general independent of the temperature, the existence of an internal or molecular field would be difficult to prove. Nevertheless with a change in aggregation, such as accompanies the transition from the liquid to the crystalline state, one should expect, if such a field exists, a measureable change in susceptibility, due to the distortion of the electronic orbits caused by the effects of the magnetic fields resulting from the new state of polarization.

Oxley[1] has investigated a large number of diamagnetic substances and has found that with few exceptions there is a decrease in diamagnetic susceptibility of about 5 per cent. when the substance passes from the liquid to the crystalline state. On the theoretical side he has ex tended the method of Langevin by the introduction of an internal field depending upon the polarization to account for this discontinuity at the transition. This extension to the theory is as follows:

Instead of assuming, as Langevin did, that the force acting on any electron of a rotating group, is simply e E, where E is the electric field strength and e the charge on the electron, he assumes, with Lorentz, that it is given by

$$e\big(E+f\ (P)\big)$$

where P is the electric polarization of the medium and f (P) a function which characterizes the grouping of the molecules for a given substance. The crystalline state may be regarded as isotropic to a first approximation since the crystals will have all possible orientations. The effect

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[1] Oxley, *Phil. Trans. Roy. Soc.*, 214, p. 109, 1914; 215, p. 79, 1914; 220, p. 247, 1920. *Proc. Roy. Soc. A.*, 95, p. 58, 1918.

due to the modification of the internal motions of an atom or molecule by the process of crystallation will be taken into account by a change in the value of f (P). Following the theory of Langevin, let (a, b, c,) be the coordinates of the center of gravity of a molecule and (x, y, z,) those of a particular electron. Also let $(\xi, \eta, \zeta) = (x-a, y-b, z-c)$ be the coordinates of an electron with respect to the center of gravity of the molecule in which it is situated. Since the medium is homogeneous and isotropic,

$$\Sigma\, \xi = \Sigma\, \eta = \Sigma \zeta = \Sigma\, \xi\, \eta = \Sigma\, \eta\, \zeta = \Sigma\, \zeta\, \xi = 0. \tag{1}$$

The sectorial velocity of an electron with reference to the center of gravity of the molecule will have a component along o z given by:

$$\Omega_z = \frac{1}{2}\,(\xi\, \dot{n} - n\, \dot{\xi}), \tag{2}$$

and the component of the magnetic moment of the molecule along this axis is then:

$$M_z = \Sigma\, e\, \Omega_z. \tag{3}$$

From (2) and (3) there results:

$$\dot{M}_z = \frac{e}{2}\,\Sigma\,(\xi\, \dot{n} - n\, \dot{\xi}). \tag{4}$$

Let X, Y, Z, be the components of the internal forces determined by the configuration of the molecules which act upon the electron of mass m and let E and H be the total electric and magnetic fields respectively. If the origin moves with a velocity having components u, v, w, then the equations of motion are:

$$\left.\begin{aligned} m\, \ddot{\xi} &= X + e\,[E_x + f\,(P_x)] + e\, H_z\,(v + \dot{y}) - e\, H_y\,(w + \dot{z}) - m\, \ddot{a} - m\, \dot{u}; \\ m\, \ddot{\eta} &= Y + e\,[E_y + f\,(P_y)] + e\, H_x\,(w + \dot{z}) - e\, H_z\,(u + \dot{x}) - m\, \ddot{b} - m\, \dot{v}. \end{aligned}\right. \tag{5}$$

These equations differ from those given by Langevin only in the addition of the term f (P). Because of the small dimensions of the elementary system considered, the electric force and the polarization will be nearly constant throughout its extent, and, designating their values at the center of the system by E_0 and f (P_o) respectively, we may, by expanding and neglecting powers higher than the first, write:

$$\text{(6)} \qquad E_x = E_{ox} + \left(\frac{\partial E_x}{\partial x}\right)_o x + \left(\frac{\partial E_x}{\partial y}\right)_o y + \left(\frac{\partial E_x}{\partial z}\right)_o z;$$

$$\text{(7)} \qquad f\,(P_x) = f\,(P_{ox}) + \left[\frac{\partial f\,(P_x)}{\partial x}\right]_o x + \left[\frac{\partial f\,(P_x)}{\partial y}\right]_o y + \left[\frac{\partial f\,(P_x)}{\partial z}\right]_o .z$$

Calculating M from the above equations, there results:

$$\begin{aligned} \dot{M}_z = \frac{e^2}{4m}\Bigg[& A\left\{\left(\frac{\partial E_x}{\partial x}\right)_o - \left(\frac{\partial E_x}{\partial y}\right)_o\right\} + w\,A\left\{\left(\frac{\partial H_x}{\partial x}\right)_o + \left(\frac{\partial H_y}{\partial y}\right)_o\right\} \\ \text{(8)} \qquad & - A\left\{u\left(\frac{\partial H_z}{\partial x}\right)_o + v\left(\frac{\partial H_z}{\partial y}\right)_o - H_{oz}\frac{dA}{dt}\right\}\Bigg] \\ & + \frac{e^2}{4m}\left[A\left\{\left(\frac{\partial f\,(P_y)}{\partial x}\right)_o - \left(\frac{\partial f\,(P_x)}{\partial y}\right)_o\right\}\right] + \frac{e^2}{2m}\Sigma\,(\xi\,Y - \eta\,X), \end{aligned}$$

where $\frac{A}{2} = \Sigma\,\xi^2 = \Sigma\,{}^2\eta = \Sigma\,\zeta^2$. The last term of (8) is zero provided each molecule has no initial moment as Langevin's theory requires. Dropping the subscript 0 and using the electromagnetic field equations:

$$\text{(9)} \qquad \text{curl } E = \dot{H} -, \text{ and div } H = 0,$$

their results:

$$\text{(10)} \qquad \dot{M}_z = -\frac{e^2}{4m}\frac{d}{dt}(H_z A) + \frac{e^2}{4m}\left\{A\left[\frac{\partial f\,(P_y)}{\partial x} - \frac{\partial f\,(P_x)}{\partial y}\right]\right\}.$$

Intergrating from the time 0 (H = 0) to τ ($H = H_z$) their results:

$$\text{(11)} \qquad \Delta M_z = -\frac{e^2}{4m}H_z A + \frac{e^2 A}{4m}\int_o^t \left[\frac{\partial}{\partial x}f\,(P_y) - \frac{\partial}{\partial y}f\,(P_x)\right]dt,$$

where $\Delta\,M_z$ is the magnetic moment produced in the molecule by the change in field which occurred during the interval. The second term depends upon the molecular configuration of the substance and implies a modification of the electron circuits which will change their self inductance. Any such change of self inductance may be represented by a small change in the intensity of the applied magnetic field, and we may then write:

$$\text{(12)} \qquad f\,(P) = a\,P,$$

where "a" characterizes the grouping of the molecules. Accordingly:

$$(13) \qquad \frac{\partial}{\partial x} f\,(P_y) - \frac{\partial}{\partial y} f\,(P_x) = a\left(\frac{\partial P_y}{\partial x} - \frac{\partial P_x}{\partial y}\right) = -a\frac{\partial}{\partial t}(\delta H_z),$$

where a (δH_z) is the elementary change in the external field during a small interval of time τ.
Therefore:

$$(14) \qquad \Delta M_z = -\frac{e^2}{4m} H_z A - \frac{e^2 A a}{4m}\int_0^{\tau} \frac{\partial}{\partial t}(\delta H_z)\,dt$$

$$= -\frac{e^2 H_z A}{4m}\left[1 + \frac{a\Delta H_z}{H_z}\right].$$

The term q $\Delta\, H_z$ is the total variation of H_z caused by the distortion of the electron orbits. If N is the number of molecules per gram, the specific susceptibility may be written:

$$(15) \qquad \chi = \frac{N\,\Delta\,M_z}{H_z} = -\frac{N\,e^2\,A}{4m}\left[1 + a\,\frac{\Delta\,H_z}{H_z}\right].$$

An expression is thus obtained in which the susceptibility is shown to depend by means of the quantity "a" upon the state of polarization of the substance, and the term a $\Delta\, H_z$ is the molecular field produced thereby. If $a = 0$, (15) reduces to the expression originally obtained by Langevin. Calling a_l and a_c the polarization constants for the liquid and crystalline states respectively, the variation of χ on crystallization may be written:

$$(16) \qquad \frac{\delta\chi}{\chi} = (a_c - a_l)\,\frac{\Delta H_z}{H_z}.$$

It has been shown by Larmor[1] that a_l is of the order $\frac{1}{3}$ for most liquids. The value of a_c is large but its exact determination in any particular case is difficult since it depends upon the actual distribution of the molecules about which we know relatively little. It is possible, however, to obtain an approximate value of its magnitude from the work of Chaudier[2] on the change of magnetic rotatory power with change of state. He has shown that a_c must be at least of the order 10^2 and is probably larger. $\frac{\Delta\,H_z}{H_z}$ is accordingly of the magnitude of $5x10^{-4}$.

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[1] *Phil. Trans. Roy. Soc.*, 1897, A, p. 213.
Comptes. Rend., 156, p. 1008, 1913.

A comparison of the molecular field for diamagnetic substances with that of ferromagnetic substances according to the Weiss theory may be made as follows: For a supercooled liquid, we may write:

$$\chi_l = -\frac{N e^2 A}{4 m}; \tag{17}$$

while for crystals at the same temperature χ_c is given by equation (15). Hence:

$$\chi_c = \chi_l \left(1 + a_c \frac{\Delta H_s}{H_s};\right), \tag{18}$$

$$\text{whence } H_s \chi_c = \chi_l (H_s + a_c \Delta H_s). \tag{19}$$

The term $a_c \Delta H_s$ is the mean molecular field of the diamagnetic crystals. Since in equation (6) of the Weiss theory, the molecular field constant, which we will here designate as N^*, is taken as the proportionalty factor beween molecular field and intensity, while in equation (19), $a_c \Delta H_s$ is itself the molecular field, it is necessary to compare N^* with a_c' where the latter is defined by equation:

$$a_c \Delta H_s = a_c' N \Delta M_s \rho \tag{20}$$

in which ρ, the density of the substance, is approximately unity for the crystals investigated by Oxley. By using the first relation of equation (15) and putting $\frac{\Delta H_s}{H_s} = 5 \times 10^{-4}$ there results:

$$a_c' = \frac{5 \times 10^{-4} a_c}{\chi}. \tag{21}$$

Assuming $\chi = 5 \times 10^{-7}$, and $a_c = 10^2$, a_c' is found to be of the order 10^5, which is of the same order as the values of N^* given by Weiss and Beck.

The Local Molecular Field.

In the above discussion of the mean molecular field, it was pointed out that the change of susceptibility which accompanies the transition from the liquid to the crystalline state can be satisfactorily interpreted in
f a mean molecular field appreciable only in the crystalline state,
ay be represented magnetically by a term $a_c \Delta H_s$. The nature
eld was not specified further than to say that it is of such a
le as to produce within the crystal a distortion or polarization

equivalent to that actually produced by the molecular forces of the molecules of the crystalline structure.

On the theory of magnetism developed by Langevin a diamagnetic molecule contains oppositely spinning systems of electrons which counter balance each other at distances large compared to molecular dimensions, but which nevertheless produce fields close to the molecules which may be very large. Each molecule of a crystal is accordingly subjected to the intense magnetic fields of its neighbors and the resulting distortion in the electron orbits may account for the shifting of an absorption band when a liquid crystallizes, and the natural double refraction of crystals. The direction of this local field will alternate as we pass from molecule to molecule through the space lattice, and is distinguished from the mean molecular field in that it exists whether an external field is acting or not. The forces, due to these mutual magnetic actions, are responsible for the rigidity of crystals and the existence of plane of cleavage.

To obtain an idea of the intensity of the local molecular field, we assume that it is of such a magnitude as to produce a change in susceptibility of the order of that actually observed in the crystallization experiments.

From the theory of Langevin, we have:

$$\frac{\Delta M}{M} = -\frac{H \tau e}{4 \pi m}; \tag{22}$$

where ΔM is the change in the magnetic moment of an electron orbit of moment M by the application of the field H; τ, the period of an electron, and $\frac{e}{m}$ the ratio of the charge to its mass. From equation (22) we have:

$$\frac{\Delta M_1}{M_1} = \frac{H \tau e}{4 \pi m}, \tag{23}$$

and

$$\frac{\Delta M_c}{M_c} = \frac{H \tau e}{4 \pi m},$$

where the subscript 1 and c refer to the liquid and crystalline states respectively. In passing from the liquid to the crystalline state the alteration of (M_1) produced by the local molecular field H_c is $\Delta M_1'$, where:

$$\frac{\Delta M_1'}{M_1} = -\frac{e \tau H_c}{4 \pi m}, \tag{24}$$

and

$$M_c = M_1 \pm \Delta M_1'. \tag{25}$$

Although H_c alternates as we pass from molecule to molecule, the sign of $\Delta M_1'$ will remain the same, for when H_c' changes sign, M_1 reverses also so that every molecule suffers the same distortion due to the local molecular field. The double sign implies that the arrangement of molecules due to their particular kind of packing will be such that in some cases $\delta\chi$ is positive and in others, negative. From equation (24) and (25) we find that:

$$M_c = M_1\left(1 \pm \frac{e\tau_1 H_c}{4\pi m}\right); \tag{26}$$

also that:

$$\frac{\Delta M_c - \Delta M_1}{\Delta M_1} = \frac{\tau_c}{\tau_1}\left(\frac{1 \pm e\tau_1 H_c}{4\pi m}\right) - 1. \tag{27}$$

The electrons which give rise to diamagnetism also produce the Zeeman effect, a slight change in frequency being responsible for both phenomena. We may, therefore, write $\tau_c = \tau_1 \pm \delta\tau$, where $\delta\tau$ is the change in period produced by the local molecular field H_c when crystallization sets in. From equation (15) it follows that

$$\chi_c = \frac{N n}{H} \Delta M_c,\ \chi_1 = \frac{N n}{H} \Delta M_1, \text{ and} \tag{28}$$

$$\delta\chi = \chi_c - \chi = \frac{N n}{H} (\Delta M_c - \Delta M_1),$$

where n is the number of electrons per molecule and N the number of molecules per gram. The change of period $\delta\tau$ is defined by:

$$\frac{\delta\tau}{\tau} = \frac{e\,\tau_1 H_c}{4\pi m}. \tag{29}$$

From equations (27), (28), and (29) it follows that:

$$\frac{\delta\chi}{\chi} = \left(1 \pm \frac{e\,\tau_1 H_c}{4\pi m}\right)\left(1 \pm \frac{e\,\tau_1 H_c}{4\pi m}\right) - 1. \tag{30}$$

This equation gives the order of magnitude of the local field H_c in terms of the percentage change in χ on crystallization. In all the substances investigated this change amounts to a few per cent. Hence:

$$\frac{1}{100}=\frac{e^2\,\tau^2\,H_e^2}{16\,\pi^2\,m^2}.$$

Taking $\tau_1=10^{-15}$ seconds, and $\frac{e}{m}=2.\times 10^7$, we get:

$$H_e=6\times 10^7 \text{ gauss.}$$

We have no data at present as to how far an absorption line is shifted when a substance passes from the liquid to the crystalline state, but such evidence would be a direct test of the magnitude of H_e. On the other hand, it is known that the magnetic double refraction induced in a liquid is proportional to the square of the external field. If we assume that this law holds up to fields of the order 10^7, we should expect on the basis of the local field idea for a crystal, a double refraction about 40,000 times as great as the largest values induced in a liquid. This is about the ratio of the double refractions of nitrobenzene subjected to a field of 3×10^4 gausses and the natural double refraction of quartz. The fact that most uniaxial crystals have a double refraction comparable to that of quartz, and hence, a magnitude much greater than that induced in liquids by fields available in the laboratory would support the idea that the intrinsic molecular field, if interpreted magnetically, must be of an order high compared to 3×10^4. These fields are even larger than those observed for ferromagnetic substances when interpreted according to the Weiss theory.

The Stresses and Energy Associated with the Molecular Field.

If there exists a molecular field of the order deduced in the previous sections, then the forces associated with the diamagnetic crystalline structure must be very large and the potential energy of the crystalline state will be considerable. It should, therefore, be possible to give a rough check on the value of the local molecular field from a consideration of the latent heat of fusion of crystals. If $|i|$ is the local magnetic moment which in conjunction with the local field H_c, binds one molecule to another in the crystalline structure, and if all the elementary systems are independent, then the energy possessed by one gram of the substance in virtue of a particular crystalline grouping, may be written:

$$E=\frac{L}{2\rho_0}\,n\,\Big|\,i\,\Big|;\ H_c=\frac{1}{2\rho_0}a_e'\,I_2; \tag{31}$$

where n is the number of molecules per cc., ρ the density, and $I = n\,|i|$ the aggregate of the local intensity of magnetization per cc. Here a_e' is the constant of the local molecular field as used above. The local molecular field $H_e = a_e' I$ has been shown to be of the order 10^7, and since a_e' is of order 10^5, it follows that I is of order 10^2. Hence, the energy per gram given by equation (31) is of order of 10^9, the thermal equivalent of which is approximately 25 calories. This represents the energy necessary to destroy the crystalline structure, that is, the latent heat of fusion. It is of the right order of magnitude since a large number of diamagnetic crystalline substances have latent heats ranging from 21 for aniline to 44 for acetic acid. It is also the order of magnitude of the latent heat of transformation of iron from the ferro-to the paramagnetic state as found by Weiss and Beck. It is obvious that until we know the arrangement within the crystalline structure the value of a_e must necesssarily be merely an approximation; but the fact that it agrees even as regards the order of magnitude is good evidence for the existence of such local molecular fields and intensities as have been assumed.

Molecular Field and Tensile Strength.

Whatever may be the nature of the forces which hold the molecules of a liquid together, we have in addition to them, on crystallization, those of the intrinsic local field. If it is assumed that the only additional forces binding molecules together on crystallization are those due to their magnetic fields, then it should be possible to predict their tensile strengths from considerations of their local fields and intensities of magnetization. The potential energy associated with each unit volume of a crystalline substance in addition to that when in the liquid form will be

$$\frac{1}{2} H_e\, I.$$

This is then a measure of the mechanical stress which binds the molecules together and determines the rigidity of the substance. In a previous section it has been shown that for diamagnetic substances I is of the order of 10^2 and since H_e is of order 10^9 it follows that the tensile strength should be of the order $.5 \times 10^9$ dynes per squares centimeter. That this is of the order experimentally determined in some cases may be seen by comparing with glass $1.1 - 1.5 \times 10^9$, quartz 10×10^9, lead $.16 \times 10^9$, etc. Moreover if one uses the corresponding values of intensity and molecular field for ferromagnetic substances as determined by Weiss, he obtains the following values for tensile strength: iron, 5.5×10^9, nickel 1.4×10^9 and cobalt 4.4×10^9 which compare favorably with the observed values.

It may then be concluded that the stresses due to the local molecular field give a satisfactory interpretation of ultimate tensile strength of crystalline media for both dia-and ferromagnetic substances.

The Change of Density on Crystallization Interpreted as a Magnetostriction Effect of the Molecular Field.

It has been shown by Larmor[1] that the potential energy per gram of a diamagnetic liquid, the molecules of which have a small mutual influence, is

$$\frac{1}{2\rho_x}\left[K_1 H^2+\lambda K_1^2 H^2\right], \tag{32}$$

where K_1 is the susceptibility per unit volume and λ is a constant approximately equal to $\frac{1}{3}$. If now a liquid is subjected to a magnetic field a change of volume occurs such that the internal pressure is reduced by an amount equal to the potential energy per unit volume of the magnetic field. Since K_1 is of the order of -7×10^{-7}, the second term of (32) is negligible compared to the first, and if C is the compressibility of the liquid, the change in volume due to the field may be written:

$$\delta V=\frac{1}{2\rho} C K_1 H^2: \tag{33}$$

a relation which has been verified by Quincke for fields up to 50,000 Gauss. If it is assumed that this law holds for fields of the order of the local molecular fields, i.e. 10^7 gauss, then the change of volume on crystallization may be computed by replacing the first term of equation (32) by the second expression of equation (31). There results then:

$$\delta V=\frac{1}{2} C a_0' I^2. \tag{34}$$

From considerations involving the determination of the quantities a_0' and I from internal stresses accompanying the change of freezing point with pressure, Oxley deduced for the substances listed below the following values:

$$a_0'=2.5\times10^4, \text{ and } I=400.$$

Since C for these substances is of the order $.8\times10^{-10}$, there results:

$$\delta V=\frac{1}{2} .8\times10^{-10}\times2.5\times10^4\times16\times10^{10}=.16 \text{ cc.}$$

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[1] Larmor, *Proc. Roy. Soc. A.*, 52, p. 63, 1892.

The following are observed values of δV for a few substances.

Substances	δV
Benzene	.10
Naphthalene	.14
Benzophenone	.19
Formic acid	.10
Di-phenylamine	.10

The calculated values agree as well as could be expected with the observed values, since, for a_e' and I, we know the orders of magnitude only, since they are unknown functions of the molecular structure and the space lattice which are different for each substance.

THEORIES OF MAGNETIC CRYSTALS AND THE MAGNETON

By J. Kunz

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The ferromagnetic crystals, which have been investigated so far, are pyrhotite $Fe_7 S_8$, apparently hexagonal; magnetite $Fe_3 O_4$, of the cubical system; iron crystals of the cubical system; and hematite, $Fe_2 O_3$, rhombohedric and hemihedric. The majority of investigations are due to P. Weiss and his coworkers.

The simplest phenomena are offered by Pyrhotite, which has first been investigated by P. Weiss,[1] and whose studies were continued by J. Kunz[2] and by M. Ziegler.[3]

The methods of investigation are essentially the same in all measurements; they have been partly introduced and widely perfected by P. Weiss and his students: they are either methods of deflection, or ballistic methods. The three dimensional problem is reduced to a two dimensional one by cutting thin plates from a crystal, parallel to a certain crystal surface. These plates, in horizontal or vertical position, moveable round about a vertical axis, are placed in a magnetic field of given direction and magnitude. If the plate is placed horizontally, the deviation D gives the component I_n of magnetization perpendicular to the direction of the horizontal magnetic field H, according to the formula:

$$D = V \times H \times I_n,$$

where V is the volume of the plate. If the magnetic field is turned round about the crystal plate, we find readily the normal component of magnetization for the various directions of the crystal plate.

In order to determine the component I_p parallel to the field, we suspend the same plate in a vertical position, so that the field falls in the surface of the plate, which is at rest, R. If we now rotate the field by a small angle α to the right or to the left, the plate will be subject to a moment of force

$$D = I_p V.\ H \sin(\alpha - \beta).$$

The plate itself rotates by a small angle β. If, moreover, the plate has a component of magnetization I_3 perpendicular to the plane of the

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[1] P. Weiss, Les propriétés magnétiques de la pyrrhotine. *Journal de physique*, 1905, p. 469.

[2] P. Weiss and J. Kunz, *J. d. Phy.*, 1905, p. 847.

[3] Max Ziegler. Kristall Magnetische Eigenschaften des Pyrrhotins. Dissertation Zürich, 1915.

plate, it will make a contribution $I_3 \cos(\alpha-\beta)$ to the moment. In order to reduce this part to a minimum, we chose the plates as thin as possible. In the case of the normal pyrhotite the magnetic plane facilitates essentially the measurements. Morever, the demagnetizing action of the plates can be neglected in many cases, so that the external field may be used directly as magnetizing field. Because of the corrections β and I_3 this method is cumbersome and is often replaced by the ballistic method. A primary coil produces a uniform magnetic field in which is placed a secondary coil, S, connected with a ballistic galvanometer. A ballistic deflection arises when the crystalline plate is introduced or withdrawn from the secondary coil, expressed by:

$$edt = G\, I_{p}, V,$$

where G is a constant, e the induced e.m.f. At the same time, with the normal component, we can determine the hysteresis of rotation, by turning the field first in one, and then in the opposite direction round about the plate suspended in a horizontal plane. The apparatus required has been perfected and described by Weiss and his students (for instance, in the thesis of V. Quittner and Karl Beck.)

We proceed to the results obtained with the various crystals, among which the normal pyrhotite is distinguished by the possession of a magnetic plane and rather simple magnetic properties.

PYRHOTITE

The chemical composition corresponds approximately to $Fe_7\,S_8$; it crystallizes apparently in the hexagonal system, and its magnetic properties correspond at most to the rhombic system.

A. Streng[1] made in 1882 the important discovery of the magnetic plane of the pyrhotite, at least for the permanent magnetism. These measurements were made complete by Abt[2] and later by the detailed measurements of Weiss, and Weiss and Kunz. We must distinguish between two types of pyrhotite: the crystals from Morro-Velho in Brazil, without cleavage, and with uneven fracture. The magnetic properties are very simple. Weiss calls these crystals normal pyrhotites. The abnormal pyrhotites are widely spread; leaf-like; with badly defined magnetic properties; and with great thermomagnetic irregularities, especially with respect to hysteresis.

The plane of base of the normal pyrhotite is the magnetic plane, in which the crystal is much easier magnetizable than in the perpendicular direction. The magnetic properties repeat themselves three times in

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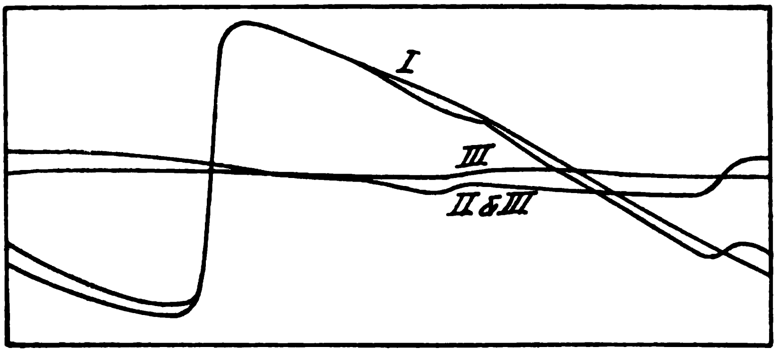
[1] A. Streng. *Neues Jahrbuch der Mineralogie*, 1, p. 185, 1882.

[2] Abt. *Wiedemann's Annalen*, 1896, p. 135.

angular distances of 60° in the magnetic plane, but in various magnitudes. It looks as if the crystals were made up of three elementary crystals (crystal components) placed side by side so that the magnetic planes are parallel to each other and that the directions of easy magnetization are inclined mutually by 60°. In order to obtain the properties of the simple or elementary crystal, we have to correct the measurements by a graphical method of successive approximation. We chose such samples in which one of the components predominates strongly. For the purified crystal a curve arises of rhombic symmetry, where every elementary crystal plate shows a distinguished direction, in which saturation is reached by very weak fields; while in the perpendicular direction up to 13400 Gauss are required for saturation to take place.

MEASUREMENTS AT ORDINARY TEMPERATURES

Fig. 1 gives the curves of the couple in the magnetic plane for 5550 Gauss. I represents the component I_n of magnetization perpendicular to the field. Fig. 2 gives I_n for the various directions and different fields. Fig. 3 gives the corresponding components I_p parallel to the field. I_n passes twice through zero in the interval from 0....180°, while I_p in the same interval shows only maxima and minima; this is a common property of the two components for all plates of all crystals. It is easy to construct the resultant I by means of the two components. The result is shown in Fig. 4. If the end point of the vector H covers the whole magnetic plane, the end point of I, the resultant magnetization, remains within a certain circle which Weiss called the circle of magnetization. If the vector H of the field rotates with constant velocity round about the point O, then,



1. Principal Component. (100%).
2. Second Component. (14.1%).
3. Third Component. (2.5%).

Fig. 1

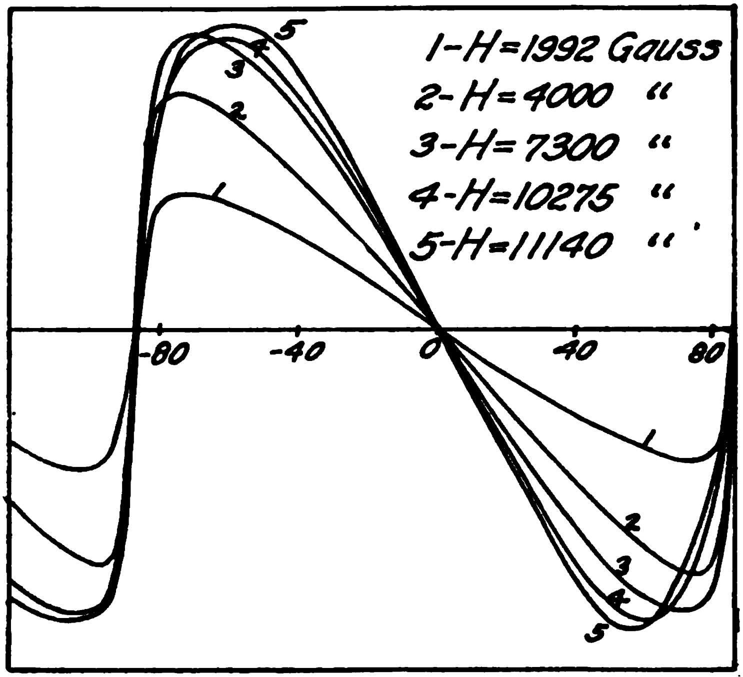


FIG. 2

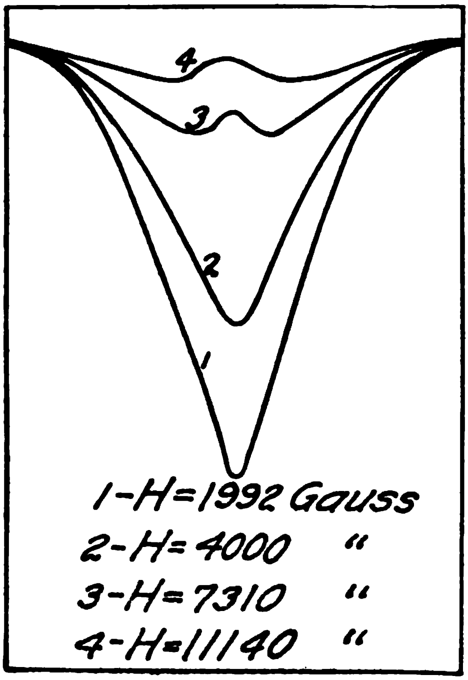
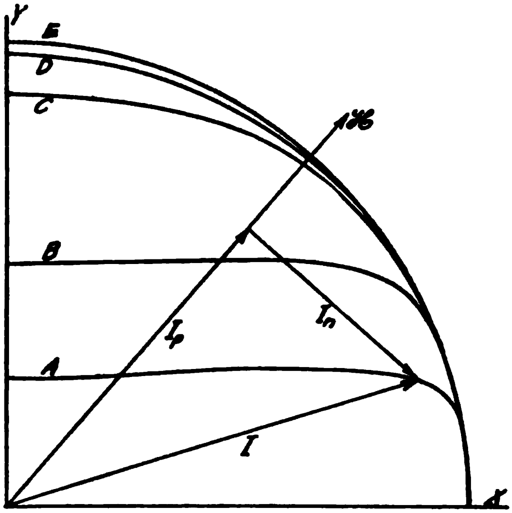


FIG. 3

starting from the direction of easy magnetization Ox, the vector I of magnetization follows at first very slowly the field; its end point remains on the circle of magnetization until H has nearly reached the direction Oy of difficult magnetization. Then I leaves the circle of magnetization and curves rapidly on a flat curve behind the field, in order to reach it in the direction Oy. The larger the magnetic field, the more the curve of magnetization will approach the circle of saturation. For sufficiently high fields (13400 Gauss) the circle of saturation will be described by I with sufficient approximation. (Between 30° and 60° deviations of about 1% occur). P. Weiss assumes that for an infinitely large, perfectly homogeneous crystal in the direction of easy magnetization Ox saturation is reached even in the weakest magnetic fields; in the other two principal directions the same would occur, if there would not exist an internal demagnetizing field of magnitude N I, where N is a constant coefficient.



A−H = 1992 Gauss, B−H = 4000 Gauss
C−H = 7310 Gauss, D−H = 10275 Gauss
E−H = 11140 Gauss
FIG. 4

HYPOTHESIS OF WEISS

Intrinsic Molecular Field H_i

In order to represent the properties so far described of the normal pyrhotite to a first approximation, P. Weiss makes the following assumption: in the directions of the three principal axes of the crystal there exists an intrinsic molecular field proportional to I in that direction and proportional to a certain coefficient having a special value for each axis. With respect to the sum of the external and the molecular magnetic fields H the crystal behaves like an isotropic medium. Let X, Y, Z be the three perpendicular principal axes of the crystal, H, the external field, with the components H_x, H_y, H_z, the intensity of magnetization I with the components I_x, I_y, I_z, the constant coefficients of the molecular field N_1, N_2, N_3 respectively; then the components of the molecular field are equal to:

$$H_{mx} = N_1 I_x;\ H_{my} = N_2 I_y,\ H_{mz} = N_3 I_z.$$

In general the molecular field has not the direction of I, except in the direction of the three axes. The resultant components of the magnetic force are equal to:

$$H_x + N_1 I_x;\ H_y + N_2 I_y,\ H_z + N_3 I_z.$$

If in a certain direction the resultant magnetic force coincides with the resultant intensity I of magnetization, the following equations will hold:

$$\text{(1)} \qquad \frac{H_x+N_1 I_x}{I_x}=\frac{H_y+N_2 I_y}{I_y}=\frac{H_z+N_3 I_z}{I_z}=n,$$

where n is the reciprocal value of the susceptibility of the crystal which is isotropic with respect to the total field. It follows immediately:

$$\text{(2)} \qquad I_x=\frac{H_x}{n-N_1},\quad I_y=\frac{H_y}{n-N_2},\quad I_z=\frac{H_z}{n-N_3}$$

$\frac{1}{n-N_1}, \frac{1}{n-N_2}, \frac{1}{n-N_3}$ are the susceptibilities in the direction of the three axes with respect to the external field alone. N_1, N_2, N_3 are considered as constants, while n must be considered as function of the sum of the external and the internal field; for sufficiently weak fields n is constant; therefore the curves of magnetization, according to (2), in the direction of the 3 axes for small fields, are straight lines through the origin of the system of coordinates; the curve of saturation $I=I_m$ is a line parallel to the axis of H, and one straight line goes over into the other by a certain curve. If the magnetization is restricted to the plane xy, then we have:

$$\text{(3)} \qquad \frac{H_x+N_1 I_x}{I_x}=\frac{H_y+N_2 I_y}{I_y};$$

or, considering Fig. 5:

$$\frac{H\cos\alpha+I N_1\cos\varphi}{I\cos\varphi}=\frac{H\sin\alpha+N_2 I\sin\varphi}{I\sin\varphi};$$

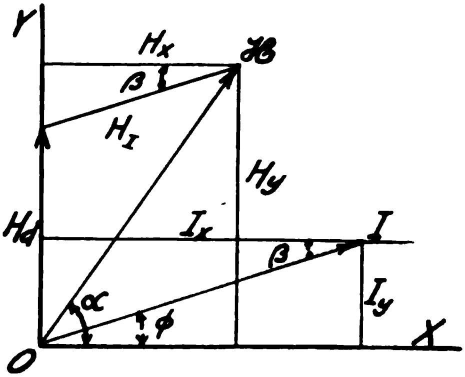


FIG. 5

or

$$I\,H\sin(\alpha-\varphi)=(N_1-N_2)\,I^2\sin\varphi\cos\varphi:$$

or

$$H\sin(\alpha-\varphi)=N\,I\sin\varphi\cos\varphi$$

if we put $N_1-N_2=N$. The independence of the coefficient N_1-N_2 of the magnetic field can be tested in the following way according to Weiss.

$$H_d = H_y - H_x \tan \beta = H_y - H_x \frac{I_y}{I_x} = I_y\left(\frac{H_y}{I_y} - \frac{H_x}{I_x}\right),$$

or, by means of (3):

$$H_d = (N_1 - N_2)\, I_y.$$

Hence I_y as function of H_d is a straight line, passing through the origin. For saturation we have:

$$I_m\,(N_1 - N_2) = 7200.$$

In large fields the agreement is good; in weak fields deviations from the straight lines occur, which are not yet explained. Neglecting these small deviations we may state: the crystal destroys a component H_d of the field proportional to I_y; the remaining component H_I is proportional to the magnetization (and parallel) to I.

If for smallest fields saturation shall be obtained in the direction Ox of easy magnetization, then $\frac{1}{n_1 - N}$ must be equal to ∞, or $n - N_1 = 0$. In the other two principal directions the same would be true, if it were not for a demagnetizing field N I. The curve of magnetization in the direction O x should be a straight line parallel to the axis H; in the direction O y a straight line inclined toward H_1, and the deviations may be explained by a lack of homogeneity of the crystal. This points to the necessity in these magnetic measurements of testing at first the crystals by the usual crystallographic methods for purity and homogeneity. A physicist and a crystallographer ought to cooperate in these investigations. The approximate truth of the theory can be tested by the moment of force D. Here also small deviations between theory and experiment, amounting to about 3 per cent. occur.

P. Weiss has given the following interpretation of the law $H \sin(\alpha - \varphi) = (N_1 - N_2)\, I \sin \varphi \cos \varphi$. In a state of equilibrium the molecular magnets shall be distributed in parallel straight lines within the magnetic plane, so that the crystal presents saturation in the direction O x, even without an external field. If now under an angle α the field H acts, the magnets will turn away from the direction O x and assume a new position of equilibrium, given through the angle φ. We assume that the adjacent magnets act in such a way upon each other that there results upon a pole in a magnetic force $A\,\mu \cos \varphi$ in a horizontal direction, and a force $B\,\mu \sin \varphi$ in a vertical direction. Then the resultant x component of the magnetizing force will be:

$$H_x = H \cos a + A\,\mu \cos \varphi,$$

and:

$$H_y = H \sin \alpha - B\,\mu \sin \varphi.$$

But the equilibrium requires:

$$H_x \sin \varphi = H_y \cos \varphi;$$

or:

$$(H \cos \alpha + A\,\mu \cos \varphi) \sin \varphi = \cos \varphi\,(H \sin \alpha - B\,\mu \sin \varphi);$$

or:

$$H \sin(\alpha - \varphi) = N\,I \sin \varphi \cos \varphi$$

when we put:

$$(A+B)\,\mu = N\,I. \quad \text{(Fig. 6)}$$

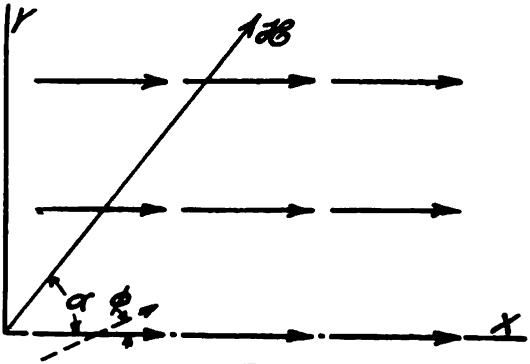


Fig. 6

HYSTERESIS

We consider at first the ordinary alternating hysteresis in the direction O x of easy magnetization. According to the molecular scheme in the direction O x saturation occurs even without a field; if then the external field begins to act in this direction in increasing magnitude, the magnetization I remains constant and will be represented by the straight line AB of Fig. 7. If now the field decreases, I remains unchanged until H assumes a negative value—H_c; then the molecular magnets swing round suddenly in the opposite direction and the end point of the vector I goes from C into C′ and moves then upon the straight line C′ B′. If we reverse the direction of the field, the magnetization passes through the points B′ C′ A′ A B. The loop of hysteresis is therefore the right angular surface A C C′ A′. Per unit volume and per cycle the energy dissipated is equal to:

$$4\,H_c\,I_m = 4.15.4.47 = 2900 \text{ ergs};\ H_c = 15.4,$$

while in the same crystal the demagnetizing field in the direction O y was 730 Gauss. The connection between these two quantities has not yet been found theoretically. The experimental curve approaches the theoretical one to a certain degree. It is similar to the hysteresis loops of ordinary iron. The distance between two points in the same height remains nearly constant and equal to 2 H_c. We proceed now to the hysteresis of rotation. If we turn the magnetic field round the crystal plate in one and then in the opposite sense, it will describe a curve with a loop, indicated in Fig. 8. In the neighborhood of M, which corresponds to the direction of easy magnetization (O x in Fig. 9) there is no hysteresis in a wide range of angles, which corresponds about to the arc AB of Fig. 9. We shall assume with P. Weiss that on the arc A B

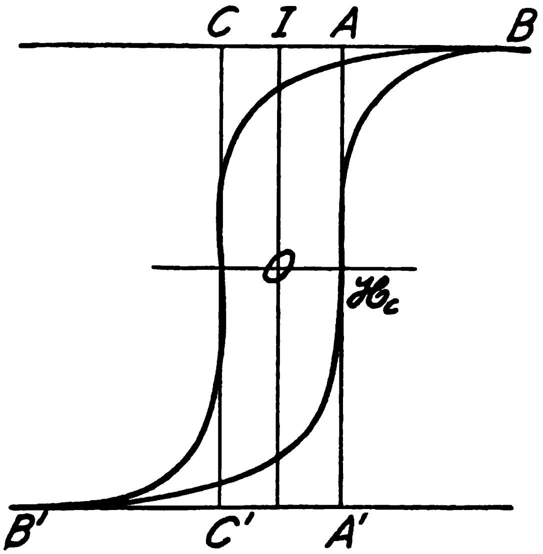


FIG. 7

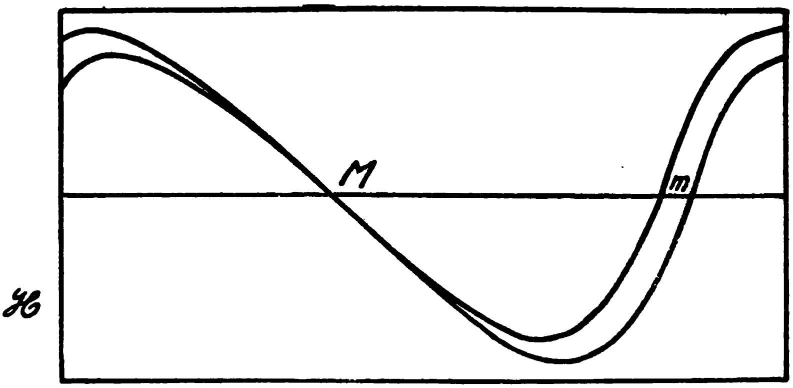


FIG. 8

of the circle of saturation there is no hysteresis, but that hysteresis appears, when in an irreversible process the magnetization passes from B through C to D. The hysteresis of rotation ought to disappear in very strong fields, when the magnetization describes the circle of saturation. For the direction O x the coercive field is H_e, for the parallel direction B D we shall assume $H_e' = H_e \frac{I_m - I_y,}{I_m}$ also a constant coercive force. If $I_y = I_m$, then $H_e' = 0$ and H_e' ought to diminish with I_y. As a matter of fact the hysteresis does not disappear for $I_y = I_m$ on a straight line. Moreover, the law depends on the manner in which the moments of force are measured because of viscosity. The energy dissipated per cycle and per unit volume would be equal to $4 \times H_e'$ times the length of the cord C B= $\sqrt{I_m^2 - I_y^2}$ $E = 4\,H_e^1 \sqrt{I_m^2 - I_y^2}$. But here experience agrees even worse with the theory of Weiss than before and in the case of abnormal pyrhotite crystals the phenomena are much more complicated.

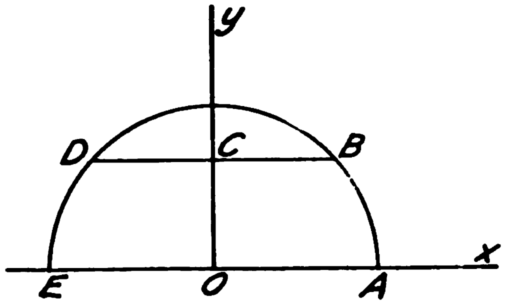


FIG. 9

INFLUENCE OF TEMPERATURE

At first the influence of the temperature on I_m, $n - N_1$, $n - N_2$, $n - N_3$ of the normal elementary crystals of pyrhotite has to be investigated. This has been done by Ziegler, by means of the method of couples and by the following considerations:

$$D = V\ H\ I_n = V\ H\ I \sin(\alpha - \varphi) = V\ I^2\ (N_1 - N_2) \sin\varphi \cos\varphi.$$

We consider two adjacent positions of H and I. In the first position both vectors shall coincide with the axis x, in the second position they shall be removed by an infinitesimal amount. Then putting the last two expressions equal to each other, we obtain:

$$H_x\,\Delta\,(\alpha - \varphi) = I_x\,\Delta\varphi\,(N_1 - N_2) = H_x\,(\Delta\alpha - \Delta\,\varphi);$$

or

$$\left(\frac{\Delta\,\varphi}{\Delta\,\alpha}\right)_x = \frac{H_x}{(N_1 - N_2)\,I_x + H_x}; \text{ for } v = 1$$

$$\Delta\,D_x = I_x^2\,(N_1 - N_2)\,\Delta\,\varphi,$$

hence

$$\mathrm{Lim}\ \left(\frac{\Delta D}{\Delta \alpha}\right)_x=\left(\frac{d D}{d \alpha}\right)_x=\frac{(N_1-N_2)\ I_x^2 H_x}{(N_1-N_2)\, I_x+H_x}.$$

As saturation in the direction x is obtained by fields of 2000 Gauss, we can write for the experiments made I_m instead of I_x and obtain:

$$\left(\frac{d D}{d \alpha}\right)_x=\frac{(N_1-N_2)\ I_m^2 H_x}{(N_1-N_2)\, I_m+H_x},$$

and in the direction y:

$$\left(\frac{d D}{d \alpha}\right)_y=\frac{H_y^2}{n-N_1};$$

moreover, for the maximum couple:

$$D_m=\frac{v\, I_m^2}{2}(N_1-N_2).$$

The experiments which required great skill, showed that N_1-N_2 is independent of the temperature, and that $n-N_1$ remains constant nearly up to the Curie point, and then probably increases rapidly; $n-N_2$ increases in the neighborhood of the Curie piont, I_s decreases, or the magnetic plane becomes more pronounced at higher temperatures. I_m has a distinct maximum at about 160° of the absolute scale. In the neighborhood of about 320° I_m vanishes rather abruptly. These features do not correspond to the theory which P. Weiss has given of the curve I_m, T. If normal crystals are heated under the simultaneous influence of a magnetic field up to 330°, the magnetic plane remains unchanged and the ratio of the three crystal components remains also the same.

THE ABNORMAL CRYSTALS OF PYRHOTITE

While the magnetic properties of the normal pyrhotite are comparatively simple, those of the abnormal pyrhotite are very manifold and have not yet been interpreted theoretically. Already in weak fields the magnetization is not restricted to the magnetic plane, but reaches considerable amounts in the perpendicular direction, neither does the law of the demagnetizing field $H_d=(N_1-N_2)\ I_y$ hold. The phenomena of hysteresis are much more developed in the abnormal than in the normal crystals. Hysteresis appears not only in the direction of difficult magnetization, but also of easy magnetization, which even increases with increasing field, sometimes even with the number of cycles described. If we determine the couple as function of rising and falling temperature under the influence of a constant field, the resultant curves show even thermic hysteresis. After the heating and cooling of a crystal in the magnetic field, the couple does not return to its original value. After the first heating the couple often appears to be increased, in the ratio 1:3 or even 1:4, 5. It may also decrease.

CRYSTALLINE GROUPS

The curves of the couples which result when the magnetic field is turned round the horizontal plate show, as in the case of the normal pyrhotite, in general three groups or components of crystals in various ratios. In a given case the original composition was given by the ratios: 66.3:25.6:8.1. After heating and cooling under the simultaneous action of a field of 2000 Gauss, which acted in various directions, we found the ratios: 46.9:29.4:23.7. It looks as if the elementary magnets under the action of high temperature and of the magnetic field distribute themselves in different groups among the three components. If the crystal is heated without a magnetic field, the three components are about equally strong, this is also true if a field is applied and rotated during the cooling process. The new grouping requires a small interval of time, i.e., more than 2 seconds and less than 4 minutes. The hysteresis in the "magnetic plane", gives another method of determining the ratios of the three components.

While in the normal pyrhotite all phenomena, even hysteresis, are reversible in a thermal sense, i.e., nothing appears similar to the hardening and tempering of steel; in the abnormal pyrhotites slow or rapid cooling affects the curves of the couples as well as those of the hysteresis. As in the case of steel rapid cooling increases the hysteresis.

If we start from a state of equal distribution of the three components of the crystal and heat it up to 350° and then cool it while at the same time a field of 500 Gauss acts in the direction of one of the three maxima of magnetization, then this maximum acquires a dominating influence and the two other maxima remain equal to each other but remain inferior to the first maximum. With increasing field the uneven distribution of the three components becomes more conspicuous. It is not yet known whether the directing influence of the field tends towards a limit or not. According as a field of about 5000 Gauss coincides with one or the other of the three maxima, that component will be increased at the expense of the two adjacent components of crystallization, which according to Fig. 10 amount to about 50 per cent. of the principal component. The three maxima play the same role not only with respect to the couples but also to the hysteresis. The varying ratio of the three components might also be explained by the assumption that the magnetic state of the three crystal groups is an independent variable. The experimental answer to this question was not decisive.

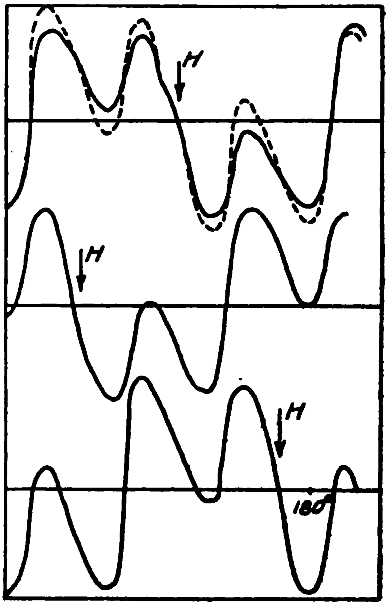


FIG. 10

We have given here a somewhat detailed description of the magnetic properties of normal and abnormal pyrhotite; of the first, because it is the only crystal whose properties we understand approximately owing to the theory of Weiss; of the second, because it shows a large variety of phenomena which are unexplained problems. In his theory Weiss has considered the mutual action of the elementary magnets in a summary way by introducing the molecular magnetic field as proportional to the intensity of magnetization. The dynamics of crystal structure has to be applied to the magnetic phenomena.

In all other magnetic crystals we have not yet a sufficient theory. We shall only point out a few of the manifold phenomena, as it would be useless to give a full description of the large variety of all magnetic properties known in these substances. For a fuller account of the experimental facts we refer the reader to the original researches.

MAGNETITE

In an investigation of the year 1896, P. Weiss showed that magnetite, though a crystal of the cubical system, has certain directions in which the magnetization is either too weak or too strong according to the cubical system. The theory of the magnetic ellipsoid of W. Thomson rests on the assumption that the intensity of magnetization is proportional to the magnetic field. But as magnetite is ferromagnetic, where that proportionality does not exist, the magnetic theory of Thomson does not hold. The investigation of Weiss was continued by his student Quittner.[1]

SYMMETRY

The magnetic properties of many, if not all, samples of magnetite crystal deviate more or less from those of the cubical symmetry. If a

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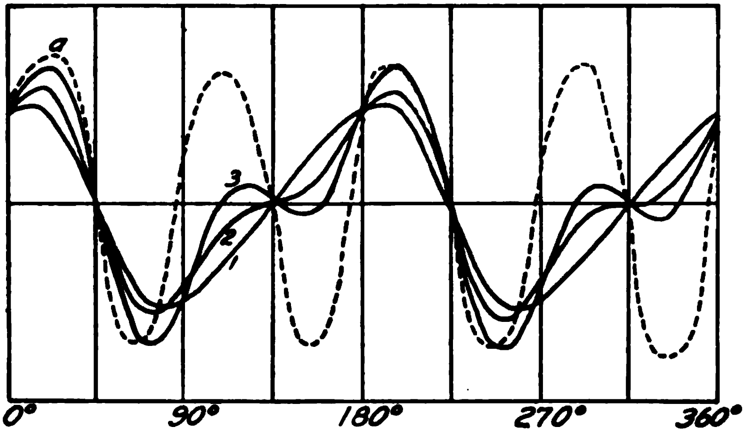
[1] V. Quittner. Die magnetischen Eigenschaften des Magnetites. Thesis, Zürich, 1908 und Annalen der Physik, 1909, N. F. 30.

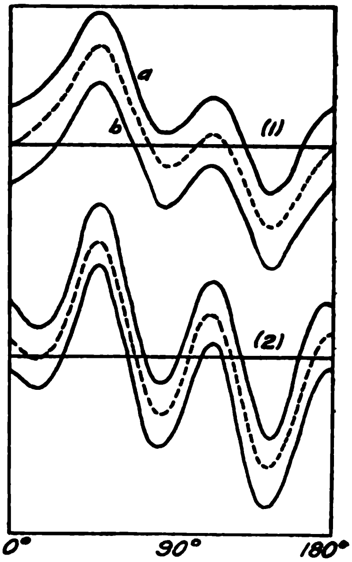
thin plate parallel to the cubical surface is cut out of the crystal, then according to the cubical symmetry and the quarternary axes the magnetic properties should repeat themselves four times within 360°. One might expect a curve of the type (a) of Fig. 11. Instead of this curve we obtain different curves of less symmetry according to the intensity of the field. The axes are at 0°, 90°, 180°, 270°, the diagonals of the cubical face pass through 45°, 135°, 225°, 315°. Instead of four waves the first curve obtained with a field of 57.3 Gauss shows only two identical waves. With a small deviation the phenomenon repeats itself from 180° to 180°. In the second curve we observe an inflexion in the diagonal at 135°, which increases with increasing field, in order to form a new wave which reaches an amplitude, at 300 to 400 Gauss, almost as large as the first wave. The greater the magnetic force, the more magnetite approaches the cubical symmetry. The two principal axes of the cubical face are never quite equivalent and the question arises as to the magnetic behavior of the third axis. In order to answer this question, a plate parallel to the octahedron has to be cut out of the crystal and to be tested magnetically; this face contains no principal axis, but makes equal angles with all three axes. If the symmetry is cubical, three equal waves must appear within 180°; if it is quadratic, two equal waves, the third different; and if it is rhombic, the three waves are different, one or two of them may even disappear. In Fig. 12a (63.0 Gauss) only two different waves appear; for a stronger field (78.7 Gauss) three waves with different amplitudes appear, and these three waves become similar to each other only for much stronger fields; while for very weak fields only one wave appears. According to these magnetic properties



1 – H = 57.3 Gauss 2 – H = 78.4 Gauss 3 – H = 94.5 Gauss

FIG. 11

magnetite has at most the symmetry of the rhombic system. The curves a and b of Fig. 12 correspond to the motion in one and the opposite direction of the magnetic field, their distance is a measure of the hysteresis of rotation of I_n. The middle curve is the curve of magnetization. The degree of deviation from the cubic symmetry is very different for the different crystals, plates even have been found which showed almost no deviation, but they had a very pronounced viscosity. If certain plates parallel to each other are cut out of the same crystal, they show as a rule identical properties, but in exceptional cases their magnetic properties vary. These phenomena again show the necessity of testing the crystals by etching methods before they are used for magnetic measurements.



1 – H = 63.0 Gauss
2 – H = 78.7 Gauss

Fig. 12

A large number of measurements of the normal and parallel components of magnetization in thin plates cut parallel to the faces of the cube and of the octahedron have been made. For very weak fields the material is practically isotropic, and seems to approach isotropism for very strong fields; the normal component I_n approaches zero in very strong fields. Strange reversals in the maxima of the curves occur; they appear for certain fields on the projections of the axes, for other fields on the sides of the triangles. On the whole the phenomena are very complicated.

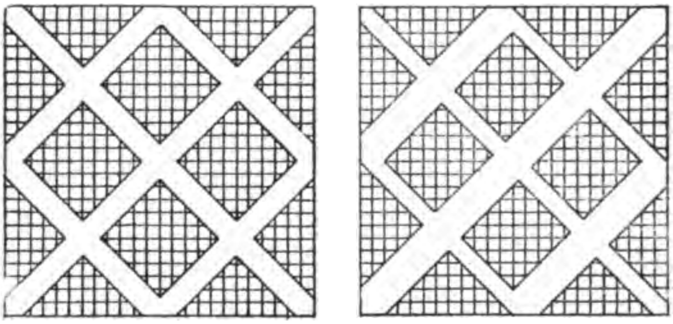


Fig. 13

V. Quittner has discussed several theories of molecular structure of magnetite and has reached the following conclusion: Magnetite consists of equal parts of three elements with magnetic planes simliar to pyrhotite; the magnetic planes of these three elements are perpendicular to each other. The molecules are arranged so as to form three systems of planes at equal distance from each other. This ideal magnetite is cut by four systems of intermediate unmagnetic planes, which are parallel to the faces of the octahedron. If these four planes (of cleavage) are equal to each other then the crystal presents cubical symmetry, if they are different, the symmetry has a lower degree. The Figs. 13 illustrate the structure assigned to magnetite by Quittner.

IRON CRYSTALS

Iron crystals have been prepared in the following way by de Freudenreich: Castings of the Goldschmidt process were made in a crucible imbedded in a heap of sand and slowly cooled down. The resulting pieces of iron, of the size of a fist, were broken and showed very beautiful crystal surfaces up to 2 cm. of length, which were identified as cubical surfaces. The chemical impurities amounted to about 2 per cent., of which silicon was the predominating part. The magnetic investigation was carried out in the laboratory of P. Weiss[1] according to the methods of the couples (I_n) and of induction (I_p). In the results Beck gives the components of magnetization per unit mass $\sigma = \frac{I}{d}$, where d is the density. A number of plates were subject to magnetic measurements. They were cut parallel to the faces of the cube, the octahedron and the rhomboidal dodecahedron.

Figs. 14, 15 and 16 give the components of magnetization σ_p parallel to the magnetic field of three plates cut parallel to these three faces for different field strength. Fig. 17 gives the components of magnetization σ_n perpendicular to the field for a cubical plate, and Fig. 18 gives the resultant magnetization in various directions for three different fields. We consider at first the curves of the Fig. 14. In weak fields σ_p is nearly constant in the various directions, but in H the demagnetizing action of the plate is not considered. If the field increases characteristic differences appear in the different directions; minima appear in the diagonals; maxima in the quarternary axes; and the differences are most pronounced in middle fields of about 392 Gauss, where they amount to about 12 per cent. of the average value. These differences disappear again with increasing fields. The maximum value of σ_p measured was 208.3 for a field of 4090 Gauss. If we compare these curves with the corresponding curves of magnetite, we find that the axes play the opposite role in the two cases; maxima are found in the direc-

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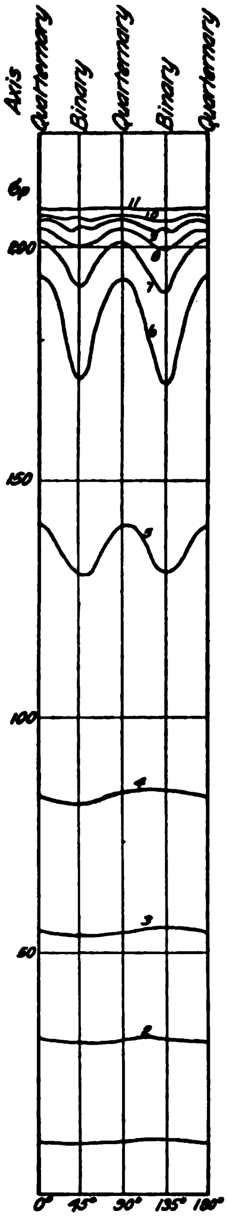
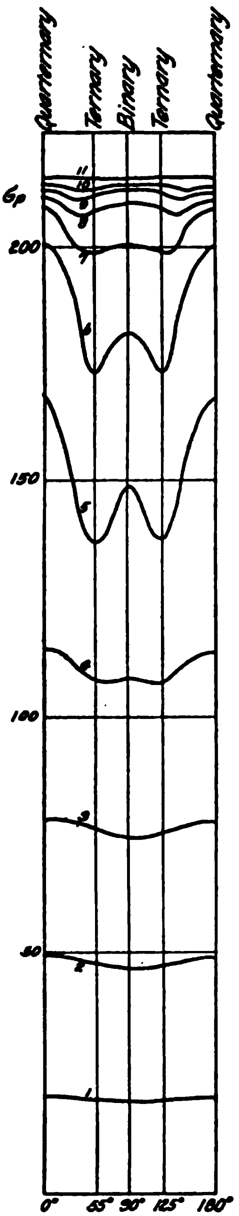
[1] Karl Beck. Das magnetische Verhalten von Eisenkristallen bei gewöhnlicher Temperatur. Dissertation Zürich, 1918.



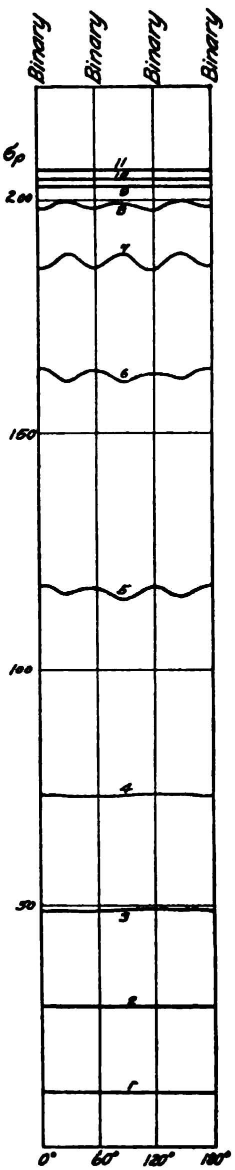
σ_p, V$_2$ Face of Cube

FIG. 14



σ_p, VII, Face of Rhomboidal Dodecahedron

FIG. 15

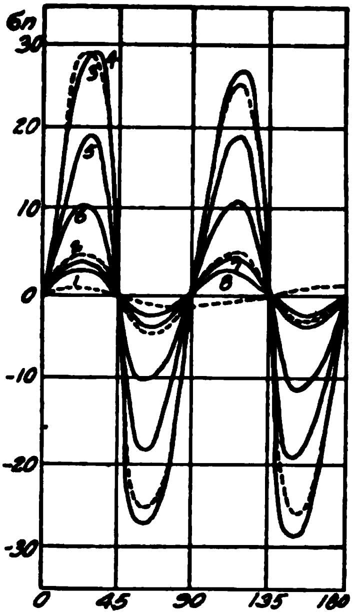


σ_p, VIII, Face of Octahedron

FIG. 16

tion of the diagonals of magnetite. In the case of iron, too, between 1500 and 1000 Gauss, small secondary maxima appear in the direction of the diagonals.

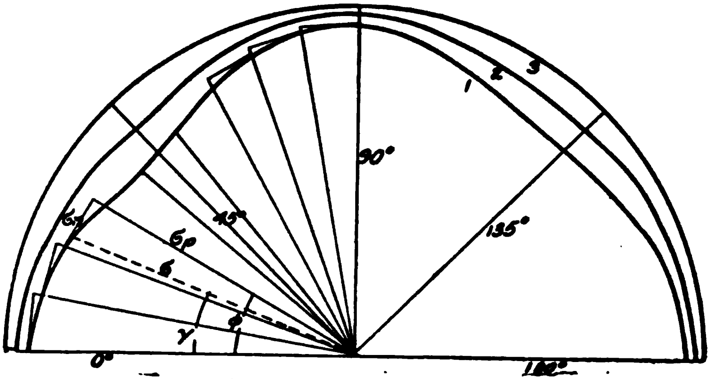
The symmetry required by the cubical system, is not rigorously satisfied by the magnetic properties of iron crystals, but the deviations are very small, at all events much smaller than in the case of magnetite. The component σ_n of magnetization normal on the field shows in a range of 180° (Fig. 17) for middle and large fields as function of the azimuth φ of (H); a curve with four zeros and two positive and two negative nearly equal maxima. The zeros correspond to the direction of the diagonals. While σ_p remains always positive, σ_n becomes once positive and once negative in every quadrant. This happens because in the neighborhood of the diagonals σ rotates more quickly than H. The amplitudes of the curves of σ_n are very small in small fields; increase in middle fields rapidly to a maximum; and decrease again with increasing fields. The maximum of σ_n is about 20 per cent. of the maximum of σ_p. Fig. 18 contains the resultant σ from σ_p of Fig. 14 and σ_n of Fig. 17. Between these curves and the corresponding curves of pyrrhotite characteristic differences appear. In the first place, there is no special direction of easy magnetization, the circle of saturation is reached here in all directions only for an infinitely strong field. Moreover, the sudden swinging round of the elementary magnets across the direction of difficult magnetization does not occur in iron, at all events this phenomenon takes place in the direction of the diagonals to a small extent. By means of the given curves the curves of magnetization in any given direction can be drawn.



1 − H = 71 Gauss
2 − H = 189 Gauss
3 − H = 392 Gauss
4 − H = 595 Gauss
5 − H = 1110 Gauss
6 − H = 2050 Gauss
7 − H = 6000 Gauss
8 − H = 9420 Gauss

Fig. 17

We proceed next to plates cut parallel to the face of the rhomboidal-dodecahedron. The parallel component of magnetization σ_p of a favorable plate is given in Fig. 15. The plate contains at 0° and 180° a quarternary axis, at 90° one binary axis and at 55° and 125° a ternary axis. In middle fields we see again as in the cubical face the difference between the principal or quarternary axes and the binary axes or diagonals. Moreover, the directions of the ternary axes are weaker magnetically than the other two axes so that σ_p offers a maximum in the binary axes between the two ternary axes. As in Fig. 14, so in Fig. 15, the differences of σ_p disappear in the various directions for very weak and very strong fields. The normal components σ_n show again four zeros within 180° in the direction of the axes at 0°, 55°, 90°, 125° and 180°. The two maxima, to the right and to the left of the binary axis at 90° are smaller than the other two maxima. The amplitudes of the curves reach a maximum in middle fields and decrease with increasing fields.



1—$\vec{H}$=392 Gauss, 2—H=595 Gauss, 3—H=∞

FIG. 18

We consider finally the face of the octahedron. The corresponding curves show partly the same features as in previous faces, partly simpler properties. Here we find only three binary axes at an angle of 60°. Fluctuations of the magnetization must therefore repeat themselves three times within 180°. This becomes evident from the curves of Fig. 16. The fluctuations disappear again in weak and strong fields. For H = 189 and 392 Gauss σ_p has a maximum in the direction of the binary axes, but an inversion of the fluctuation appears already for H = 595 where the maxima appear in the binary axes; but the variations are very small, and are apparently only of second order of magnitude. σ_n assumes only very small values in all fields; it describes three simisoidal curves within 180°. The amplitude of the waves of σ_n reaches a small maximum already in small fields and vanishes in stronger fields almost completely. The resultant σ in a polar system of coordinates is given by a curve very little deviating from a circle. The phenomena of hysteresis remind us of those of the curves of magnetization. The energy dissipated by hysteresis increases at first with increasing field, reaches a maximum and decreases again.

The magnetic properties of iron crystals on the whole are quite different from those of pyrhotite. There is no magnetic plane and the existence of an intrinsic molecular field is not established.

HEMATITE

The magnetic properties of this crystal have been investigated by Westmann, Bavink, Abt, Kunz and Smith.[1] Crystals have been used of different origin, which may be very different magnetically. They are found on the Vesuvius, in Dagnaska in Hungary, in Ouropreto in Brazil, in Schabri in the Ural mountains, in Caveradi in Graubünden, Switzerland; beautiful crystals come from Elba and from Siebenbürgen. Kunz found that the crystals can be divided into two groups, similar to those of pyrhotite, normal and abnormal hematites. Common to both is a magnetic plane, as in the case of pyrrhotite, which coincides with the plane of base. Perpendicular to this plane hematite is paramagnetic without hysteresis.

The crystalline plates, parallel to the base plane, often arranged in table-like groups, from the Vesuvius, show a regular magnetic behavior. They seem to possess only one component of crystallization and have a small hysteresis both for alternating and rotating fields. At a temperature of 650° under the simultaneous action of a magnetic field, they undergo no change.

The other group of crystals from Caveradi in Graubünden, from Elba and Siebenbürgen, seem to be composed of several elementary magnets, which are inclined by 60° to each other, and appear in various ratios. They have such large hysteresis that often the hysteresis predominates over the intensity of magnetization. Under the simultaneous influence of high temperature and a magnetic field, the magnetic structure of the crystal changes, as in the abnormal crystals of pyrhotite.

Group I; Abnormal Crystals.

At first the component of magnetization I_n normal to the magnetic field was measured by means of the couples and a great variety of phenomena was found. In the weakest fields of 138 Gauss the crystals of Siebenbürgen and Granbünden showed for I_n a pure sine curve (Fig.

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[1] J. Westmann. Upsala. Math. och Naturw. II, 1897.

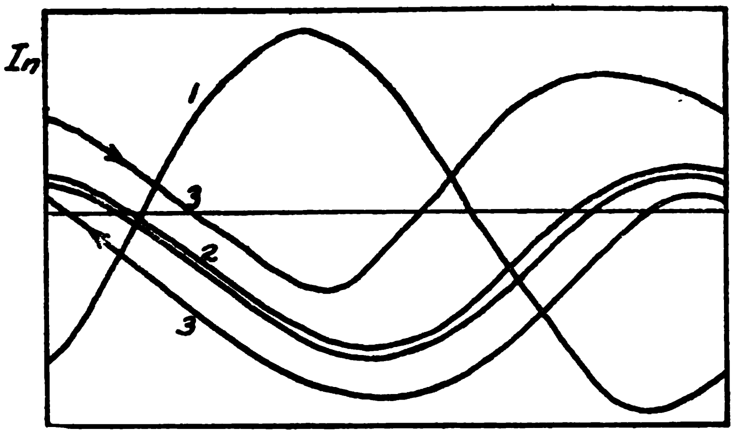
B. Bavink. Dissertation Göttingen, *Neues Jahrbuch für Mineralogie*, Bd. XIX, 377, 1904.

A. Abt. Über die mag. Eigenschaften des Hematites. *Ann. der Physik*, N. F. 68, p. 658, 1899.

J. Kunz. Über die magnetischen Eigenshaften des Hematites. *Neues Jahrbuch der Mineralogie*, I Bd. p. 62, 1907.

T. T. Smith. The magnetic properties of hematite. *Physical Review*, Vol. VIII, p. 721, 1916.

19, 1). The magnetization is like that of a permanent magnet. For H = 339 Gauss in the second curve of Fig. 19 a deviation from the sine curve appears with hysteresis of rotation, which under H = 700 Gauss reaches a considerable amount; while curve 3 of magnetization still preserves the same uniform character as in weaker fields. But at H = 1400 Gauss in the interval of 180° three equal maxima and minima appear, and the hysteresis assumes values so large, that it is the principal phenomenon, little influenced by the variations of magnetization. The band between the curves a and b of Fig. 20 corresponds to the hysteresis of rotation. With increasing fields the fluctuations of the curves of magnetization increase (Fig. 21 for H = 11300 Gauss) at the same time the maxima of magnetization are displaced towards each other for the forward and backward motion of the magnetic field. In other crystals the phenomena become still more complex, for instance, the three equal waves of the last figure may be very different.



1 – H = 138 Gauss, 2 – H = 339 Gauss, 3 – H = 700 Gauss

FIG. 19

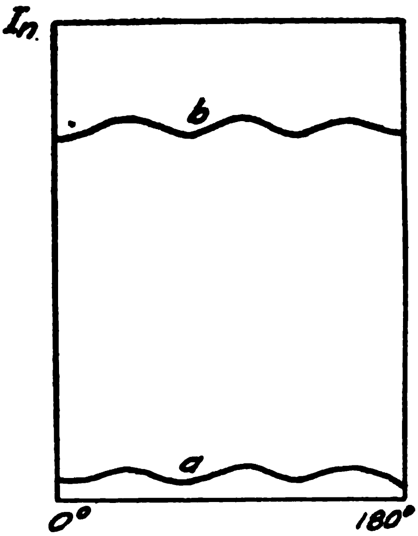


FIG. 20

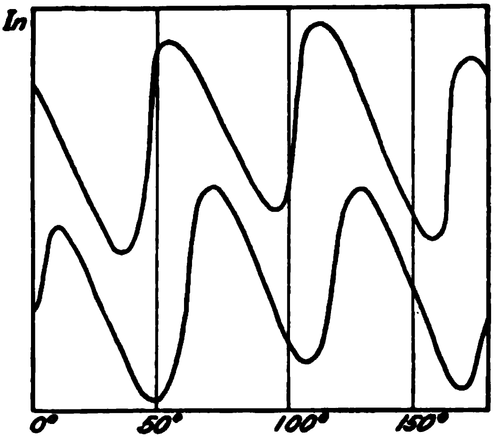
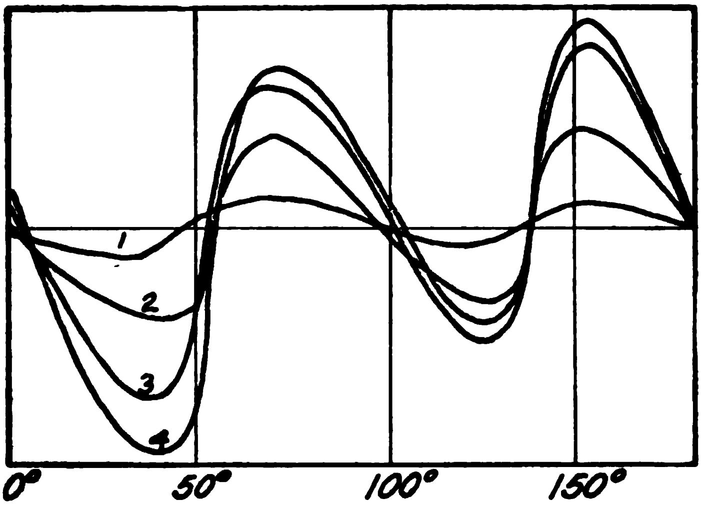


FIG. 21

Group II; Apparently Simple Crystals.

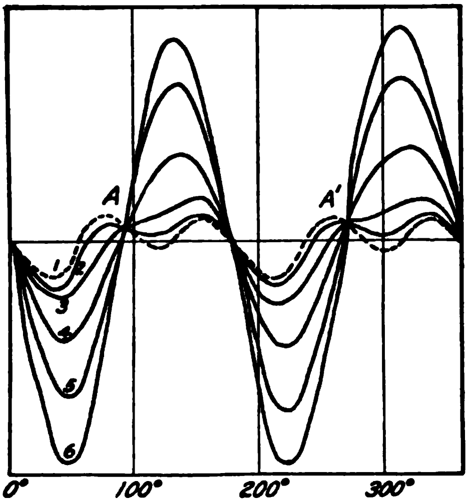
These crystals are found on the Vesuvius. They form thin black leaves of high luster, often arranged in crystallin aggregates of higher order. The normal components I_n of magnetization for different fields are given by the curves of Fig. 22 and Fig. 23. For fields varying from 48.5 to 3500 Gauss the phenomenon repeats itself from 180° to 180°.



1—H=48.5 Gauss, 2—H=161.6 Gauss
3—H=421 Gauss, 4—H=862 Gauss

FIG. 22

Two maxima and two minima appear, most pronounced for H = 862 Gauss. Above 6000 Gauss one maximum and one minimum disappears, and dissymmetries make themselves felt. Surprising are also the points of intersection of the curves A and A′ above the axis of abscissas. The curves of Fig. 23 are essentially different from those of pyrhotite. The amplitudes of the corresponding curves of pyrhotite reach a maximum with increasing field and these curves show very distinctly the directions of easy and of difficult magnetization. These features do not appear in the curves of hematite. The resultant intensity of magnetization increases with increasing field, reaches a maximum and decreases again in higher fields. The maximum value of I is only about 3.6 absolute units.



1 – H = 862 Gauss, 2 – H = 1775 Gauss
3 – H = 3850 Gauss, 4 – H = 7172 Gauss
5 – H = 10750 Gauss, 6 – H = 13340 Gauss

FIG. 23

The influence of temperature on normal and abnormal hematite crystals is the same as on the corresponding crystals of pyrhotite

ON THE THEORY OF FERROMAGNETIC CRYSTALS

Besides the theory of P. Weiss[1] for pyrhotite and K. Beck for iron, we owe theoretical studies to K. Honda and J. Okûbo[2] and to O. E. Frivold.[3]

K. Honda calculates at first the forces X and Y exerted on a magnetic pole P by eight adjacent magnets arranged in a plane according to the cubical system, he neglects the action of the remaining magnets as insignificant. Let 2_a, Fig. 24, be the sides of the space lattice; 2r and m the length and the pole strength of the elementary magnets, respectively. One side of the space lattice we take as the x axis, the other side as the axis of y. Denoting by X and Y the sum of the com-

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[1] P. Weiss. Le travail d'aimantation des cristaux. *J. d. Phy.* III, p. 194, 1904.

[2] K. Honda. *Science Reports* 5, p. 153, 1916.

[3] O. E Frivold, *Ann. d. Physik*, Bd. 65, p. 1, 1921.

ponents of the magnetic forces exerted by the neighbors on the central magnet in the directions x and y, we find as the condition of equilibrium:

$$m\ H \sin(\alpha-\theta) = -Y \sin\theta + X \cos\theta$$

Without the magnetic field, the magnets point all in the positive y direction; if a magnetic field under an angle α with respect to the y axis is applied, the magnets will turn through an angle θ and assume a position of equilibrium given by the last equation.

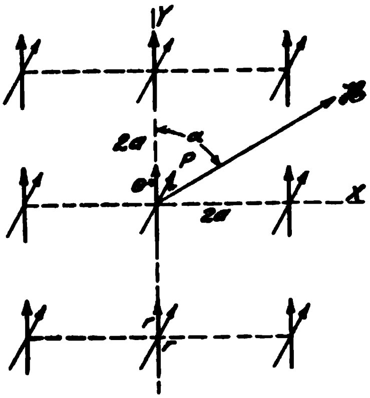


FIG. 24

Putting $k=\frac{a}{r}$, $p=\frac{2k}{1+k^2}$ and $q=\frac{2k}{1+2k^2}$ we have $a=kr$; $0<p<1$; $0<q<\frac{2}{3}$. By simple trigonometry we calculate X and Y and expand the expressions in powers of p and q and we get:

$$\begin{aligned}H\sin(\alpha-\theta) = \frac{m\,k}{r^3}\,\frac{3}{2}\cdot\frac{5}{2}\cdot\frac{7}{2}\sin 4\theta\Bigg\{&\frac{p^3}{4(1+k^2)^{\frac{3}{2}}}\left(\frac{1}{3!}+\frac{9}{2}\frac{p^2}{5!}+\ldots\right)\\ &-\frac{q^3}{(1+2k^2)^{\frac{3}{2}}}\left(\frac{1}{3!}+\frac{9}{2}\frac{q^2}{5!}+\frac{9}{2}\cdot\frac{11}{2}\frac{q^4}{7!}+\ldots\ldots\right)\\ &-\frac{9}{2}\cdot\frac{11}{2}\left[\frac{p^7}{4(1+k^2)^{\frac{3}{2}}}\left(\frac{1}{7!}+\frac{1}{2}\cdot\frac{13}{2}\frac{p^2}{9!}+\ldots\ldots\right)\right.\\ &\left.+\frac{q^7}{(1+k^2)^{\frac{3}{2}}}\left(\frac{1}{7!}+\frac{13}{2}\frac{q^2}{9!}+\ldots\ldots\right)\right]\sin^2 2\theta\Bigg\}\text{ or:}\end{aligned}$$

$$H\sin(\alpha-\theta) = F(\theta) = \frac{m}{r^3}\,f(\theta),$$

where $f(\theta)$ contains k only as a parameter. $F(\theta)$ is a periodic function of θ, having $\frac{\pi}{2}$ as its period. $F(\theta)$ may generally be written in the form

$$F(\theta) = A\sin 4\theta,$$

where $$A=\frac{3\cdot5\cdot7}{8}\,\frac{m}{r^3}\left\{\frac{8}{357}\,\varphi(k) - \varphi'(k)\sin^2 2\theta+\ldots\right\},$$

and φ, φ' are functions of k only. For small values of k, φ' (k) becomes very small in comparison with φ (k), and the form of F (θ) approaches a sine curve.

By an elementary complex Honda means a very minute crystal, in which all elementary magnets are arranged in a definite space lattice. A piece of iron consists then of a large group of such elementary complexes, whose axes point in all directions. We shall at first study the magnetization of such an elementary complex of the cubical system. The component of magnetization I in the direction of the applied field is obviously:

$$I = 2\,m\,r\,n\cos(\alpha-\theta) = I_0\cos(\alpha-\theta),$$

where n is the number of elementary magnets and I_0 the saturation value of the intensity of magnetization. Denoting $\frac{I}{I_0}$ by i, we have

$$i = \cos(\alpha-\theta) \tag{1}$$

moreover:

$$H\sin(\alpha-\theta) = A\sin 4\,\theta$$

or putting $\frac{H}{A} = h$, we get:

$$h\sin(\alpha-\theta) = \sin 4\,\theta \tag{2}$$

A depends on the properties of a particular substance so also I_0. But if we use the reduced i and h instead of the actual intensity of magnetization and of the field, the relations (1) and (2) apply for all ferromagnetic substances belonging to the regular system. If h and α are given, equation (2) gives θ and equation (1) the value of i. Honda considers therefore equations (1) and (2) as the laws of magnetization. These two equations can be solved by a graphical method. In Fig. 25 four curves representing the relation between i and h are given, in which the angle α was taken at 30°, 70°, 120° and 170°, respectively. These curves give the intensity of magnetization in the direction of the respective field, when the magnitude of the latter is so varied that it is always in equilibrium with the internal resisting force sin 4 θ. In the curve for $\alpha = 30°$, the point a corresponds to the value of cos 30°; as h increases, θ becomes greater, but always less than α, and therefore $i = \cos(30-\theta)$ steadily increases, tending assymptotically to the value of $i = 1$ with $h = \infty$. In the curve for $\alpha = 70°$, the point b corresponds to the value of cos 70°; as h increases from O, θ and therefore sin 4 θ also increases. Since, however, the latter quantity reaches a maximum at $\theta = \frac{\pi}{8}$, h must be diminished from a certain value of θ upward, if the magnetization is to be effected reversibly. With $\theta = \frac{\pi}{4}$ the resisting force $\sin 4\theta$ vanishes and therefore h must be diminished to zero, with a further increase of θ, sign 4θ changes sign and therefore h must be applied in an opposite direction, if the magnetization is to be made reversibly. If θ approaches to 70°, h becomes ∞ in the limit and the magnetization tends assymptotically to unity. The curve for $\alpha = 120°$, which begins at the point c on the negative side of i, passes through a maximum and a minimum of h and coincides with the curve for $\alpha = 30°$ as the value of i increases. The curve for $\alpha = 170°$ beginning at a point d on the negative side of i, passes through two maxima and one minimum, of h with the increase of i and approaches assymptotically to the line $i = 1$.

From this figure we draw the following inferences:

Case I. $0 < \alpha < \frac{\pi}{4}$. With the increase of the field, the magnets are turned more and more toward the direction of the field, and coincide in direction with the field, when it becomes infinite. If the field is gradually diminished, the magnets return to their original position.

Case II. $\frac{\pi}{4} < \alpha < \frac{\pi}{2}$. By gradually increasing the field, the magnets are turned toward the direction of the field, till the internal resisting force attains a maximum. From this position, a further reversible rotation of the magnets can only be effected by diminishing the field beyond zero. With a negative infinite value of the field, the magnets are brought in the direction of the field. If the field be reduced before the resistng force reaches a maximum, the magnets return to their original position; if, however, it exceeds the critical value, the magnets do not return to their original position with the withdrawal of the field, but take another position of stable equilibrium, different from the original by $\frac{\pi}{2}$. Hence in this case the magnetization is irreversible. Similar conclusions can be drawn from curves c and d of Fig. 25.

So far we have only considered the magnetization of a single complex, but we are now able to study the magnetization of a mass of ferromagnetic substance, such as iron, which consists of a great number of such elementary complexes with their magnetic axes uniformly distributed in all directions. Honda considers a complex in the form of a sphere, and puts the action of the adjacent elementary crystals equal to $\frac{4}{3}\pi I$, where I is assumed uniform Equation (2) has to be replaced by (3).

$$(\mathrm{H}+\frac{4}{3}\pi\,\mathrm{I})\sin(\alpha-\theta)=\mathrm{A}\sin 4\theta. \tag{3}$$

With these assumptions Honda gets a curve of magnetization and a hysteresis loop which resemble closely the experimental curves. He proceeds then to the theory of magnetization of magnetite. It has been shown by W. Voigt [Gött. Nachrichten, (1900) 331] and Wallerant (Comptes rendus 1901, p. 630) that a crystal of the cubical system is not isotropic with respect to the magnetic properties, if the permeability is a function of the magnetic force.

We shall consider a crystal of magnetite acted upon by a field parallel to one of the faces of the cube. In the case of magnetite stable orientations of the molecular magnets are the three positive and three negative directions parallel to the sides of the cube, but the directions of the diagonals of the faces of the cube are the orientations of unstable equilibrium. Hence, when the space lattice is quite regular, and the

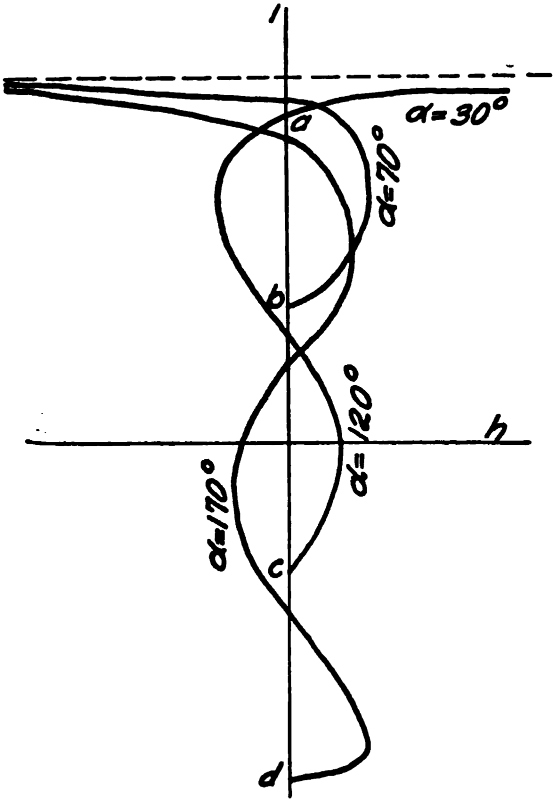


FIG. 25

thermal agitation is zero, the magnets in each group will take one of the six directions of stable equilibrium in the absence of a magnetizing field.

For the calculation of the magnetization we take the two sides of the plane lattice as x and y axes, and consider two pairs of all magnets in the direction of positive and negative x as well as y axes and the remaining pair in the direction perpendicular to the xy-plane, the total effect being thus zero. If the magnetizing field be applied in a direction making an angle α with the x axis, the intensity of magnetization is given by:

$$i=\frac{1}{6}\left\{\cos(\alpha-\theta_1)+\cos(\pi-\alpha-\theta_2)+\cos\left(\frac{\pi}{2}-\alpha-\theta_3\right)\right.$$
$$\left.+\cos\left(\frac{\pi}{2}+\alpha-\theta_4\right)+2\cos\left(\frac{\pi}{2}-\theta_5\right)\right\};$$

or

$$i=\frac{1}{6}\left\{\cos(\alpha-\theta_1)-\cos(\alpha+\theta_2)+\sin(\alpha+\theta_3)+\sin(\theta_4-\alpha)+2\sin\theta_5\right\},$$

where α and θ are related by the equations:

$$h=\frac{\sin 4\,\theta_1}{\sin(\alpha-\theta_1)}=\frac{\sin 4\,\theta_2}{\sin(\alpha+\theta_2)}=\frac{\sin 4\,\theta_3}{\sin(\alpha+\theta_3)}=\frac{\sin 4\,\theta_4}{\sin(\theta_4-\alpha)}=\frac{\sin 4\,\theta_5}{\cos\theta_5}$$

In the calculation we must take into account the abrupt change of rotation of the magnets in passing through the critical values of θ. If h is very large or very small, it is readily seen that i is independent of α, that is, the crystal behaves as an isotropic substance. Honda proceeds to calculate the magnetization in the direction of the x axis and of the diagonal of the plane-lattice and finds a qualitative agreement between the experimental curves of magnetization by Quittner and his theoretical curves. For moderate fields of $H=100$ up to $H=500$ Gauss, the magnetization along the diagonal is much stronger than along the direction of the crystallographic axis.

Let us next keep the field constant and change its direction from 0 to π, starting from the direction of the x axis. At $\alpha=0,\ \frac{\pi}{4},\ \frac{\pi}{2},\ \frac{3}{4}\pi,\ \pi$, the direction of the magnetization coincides with that of the field, so that there is no component perpendicular to the field. The results are graphically given in Fig. 26, the dotted lines represent the change of the curves as affected by some want of regularity in the space lattice and by thermal agitation. If we compare these curves of magnetization with the empirical curves of Quittner we find not more than a qualitative agreement, while several phenomena, for instance the reversals in the experimental curves, are not explained. Honda calculates also the two components of magnetization parallel and perpendicular to the field in plates parallel to the face of the dodecahedron and of the octahedron. The perpendicular components agree better with the experimental curves than the parallel components, but there are marked differences between the empirical and the theoretical curves.

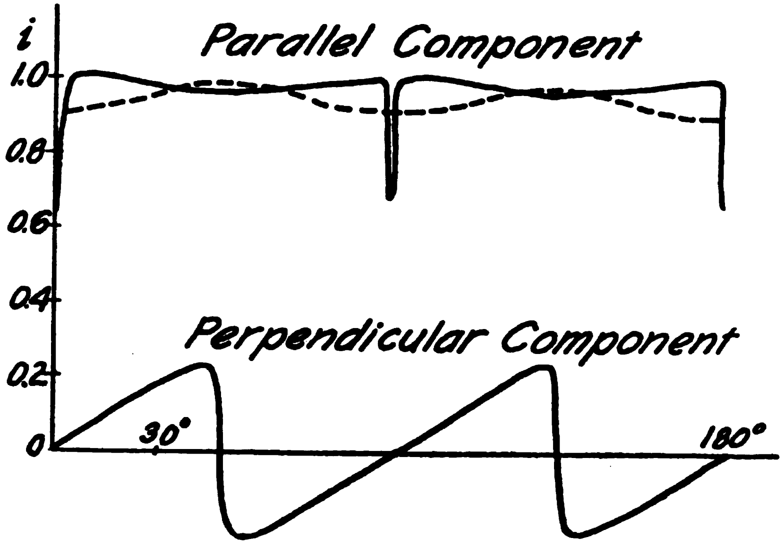


FIG. 26

Honda gives a theory of the magnetic properties of pyrhotite, assuming the hexagonal system. But its magnetic properties correspond at most to the rhombic system, in spite of its hexagonal appearance. Honda considers also the case treated by Weiss, in which each of the magnets is acted on only by its neighbors in the same row. The condition of equilibrium is expressed by the equation:

$$H \sin(\alpha-\theta) = F(\theta),$$

where

$$F(\theta) = A \sin 2\theta\, [\varphi(k) - \varphi_1(k) \sin^2 \theta],$$

while Weiss obtained the relation:

$$H \sin(\alpha-\theta) = A \sin 2\theta.$$

Honda points out that the assumption of Weiss, that the angle which the internal force makes with the direction of the stable equilibrium, is equal to the angle of deflection of these magnets, is not generally true. The theoretical curves, however, obtained by Honda do not closely resemble the empirical curves. The directions of easy and of difficult magnetizations are perpendicular to each other in the magnetic plane, a fact which does not agree with the hexagonal system.

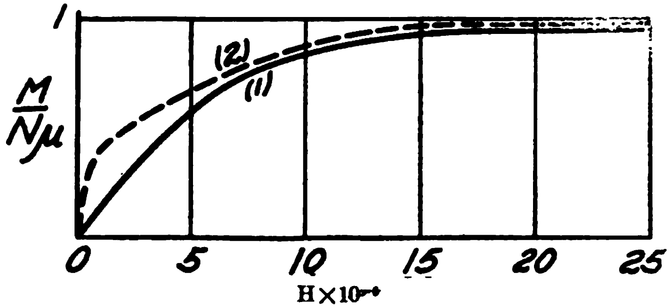


Fig. 27

It should be mentioned that W. H. Bragg and M. Nishikawa investigated the diffraction of Roentgen rays through magnetite (Phil. Mag. 30, p. 305, 1915); and that A. W. Hull (Physical Review, 9, p. 84, and 10, p. 661, 1917), investigated the iron crystal, which is characterized by a centered cubic lattice, whose unit is a cube with an atom in each corner, and one at the center of the cube.

O. E. Frivold considers at first elementary magnets, arranged along a straight line, which are in statistical equilibrium under the influence of the thermal agitation, their mutual action, and the external field. This theory is therefore a straight forward extension of Langevin's theory. The result is indicated in Fig. 27. Curve (1) is that of Langevin, curve (2) contains the correction of (1) due to the mutual action of the magnets, strongly magnified, because the ordinates of the curve of Langevin should be increased only by 1/1000 part. The mutual action is of course most marked at the lowest temperatures, where saturation might be reached even in weak magnetic fields.

The problem for three dimensions is carried out in a way analogous to that of one dimension. The elementary magnets are taken in the corners of a cubic space lattice, and for a space centered cubical lattice. The result of the calculation is given by the following two equations for the two space lattices respectively:

$$\frac{M_T}{N\mu} = \frac{1}{3}\frac{\mu H}{k T}\left[\left\{1 - \frac{6.27}{4}\left(\frac{\mu^2}{a^3 k T}\right)^2 + \ldots\ldots\right\}\right.$$

$$+\left(\frac{\mu H}{k T}\right)^2 \left\{\frac{2.56}{4}\left(\frac{\mu^2}{a^9 k T}\right)^2 - \frac{1}{15}\right\}\Bigg];$$

and:

$$\frac{M_T}{N \mu} = \frac{1}{3}\frac{\mu H}{k T}\Bigg[\left\{1 - \frac{8.88}{4}\left(\frac{\mu^2}{a^3 k T}\right)^2 + \ldots\ldots\right\}$$

$$+\left(\frac{\mu H}{k T}\right)^2 \left\{\frac{3.22}{4}\left(\frac{\mu^2}{a^3 k T}\right) - \frac{1}{15}\right\}\Bigg].$$

Langevin's formula is simply:

$$\frac{M_T}{N \mu} = \frac{1}{3}\frac{\mu H}{k T},$$

a is the distance between two successive magnets. Even here the mutual action plays a very insignificant role, especially in higher temperatures. The mutual influence appears larger, when the electrical forces of dipoles are taken into account. The field of ferromagnetic crystals is full of experimental and theoretical problems.

MAGNETONS

The conceptions of magnetons are due to Ampere and Wilhelm Weber, but magnetons as special physical realities were introduced into the optical sciences by Walter Ritz for the explanation of the Balmer series of the hydrogen spectrum. If an electron describes a circular orbit perpendicular to a magnetic field H, with velocity v, it is acted upon by a force $F = e\,v\,H$; this force is balanced by the centrifugal force $m\,v^2/r$, where r is the radius of the circle. $v = \omega r = 2\pi n r$, n being the frequency. We have therefore $e\,v\,H = m\,v^2/r$, or $e\,H/m = v/r = 2\pi n$.

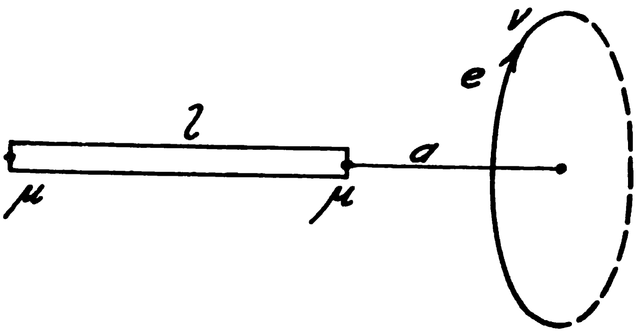


FIG. 28

The frequency is proportional to the field. Ritz assumed the existence of elementary magnets like thin, short iron rods in all hydrogen atoms, of length l and pole strength μ. In a distance a from μ the magnetic force $H_1=\frac{\mu}{a^2}$, while $H_2=\frac{\mu}{(a+)^2 l}$, or the resultant magnetic force $H=H_1-H_2=\mu\left(\frac{1}{a^2}-\frac{1}{(a+l)^2}\right)$, and the frequency of the electron:

$$n_1=\frac{e\,H}{m\,2\,\pi}=\frac{e\,\mu}{m\,2\,\pi}\left(\frac{1}{a^2}-\frac{1}{(a+l)^2}\right).$$

If we assume two magnetons joined together in the same line the resultant magnetic force will be equal to:

$$H=\frac{\mu}{a^2}-\frac{\mu}{(a+2l)^2},$$

and the frequency n_2:

$$n_2=\frac{e\,\mu}{2\,\pi\,m}\left(\frac{1}{a^2}-\frac{1}{(a+2l)^2}\right).$$

For 3 magnetons we would obtain:

$$n_3=\frac{2\,\mu}{2\,\pi\,m}\left(\frac{1}{a^2}-\frac{1}{(a+3l)^2}\right), \text{ etc.}$$

To every line of the spectrum corresponds a system of magnetons. This explanation of the Balmer series has been replaced by the theory of Bohr.

More important than this magneton of Ritz, which has only historical interest, is the magneton of P. Weiss, which is based on a large number of physical measurements on paramagnetic substances. As references we mention: P. Weiss: Physikalische Zeitschrift 12, p. 935, 1911; P. Weiss: L'etat actuel de la question du magneton, Bibliothique Universelle 35, p. 406, 1913. A comprehensive review has recently been given by B. Cabrera: Magneto-Chimie, in Journal de Chimie physique, 16, p. 442, 1918, with a complete bibliography.

DEFINITIONS

Let M = the magnetic moment per gram atom at the temperature T.

M_0 = the same at the absolute zero.

k = susceptibility per unit volume.

d = density, m = the molecular weight of the substance.

Then for the molecular susceptibility, χ_m:

$$\chi_m = \frac{k}{d} m = \frac{M}{H}, \text{ where } H = \text{magnetic field.}$$

For paramagnetic gases we have according to Langevin:

$$\frac{M}{M_0} = \operatorname{ctgh} a - \frac{1}{a} \text{ where } a = \frac{M_0 H}{R\,T}.$$

For gases we can write with sufficient approximation:

$$\coth a - \frac{1}{a} = \frac{a}{3},$$

therefore:

$$M = M_0 \frac{a}{3} = \frac{M_0^2 H}{3\,RT}, \text{ or } \frac{M\,T}{H} = \frac{M_0^2}{3\,R};\ R = 8.315 \cdot 10^7.$$

$$\chi_m\, T = \frac{M_0^2}{3\,R}.$$

But Curie found:

$$\chi_m\, T = C_m = \text{constant.} \quad \text{(Curie's constant).}$$

$$C_m = \frac{M_0^2}{3\,R}, \text{ or } M_0 = \sqrt{3\,R\,C_m}.$$

The only paramagnetic gases are O_2 and NO for which Weiss, and Weiss and Piccard, give the following values:

	$\chi_m . 10^6$	C_m	M_0	n
O_2......	3381.8	0.9937	15745	14.014
NO.......	1400.3	0.4132	10156	9.039
O_2......	3449.6	1.0107	15920	14.12
NO.......	1461	0.4281	10330	9.20

According to Weiss the magnetic moment per gram atom or the magneton is 1123.5, deduced from magnetic measurements of Fe, Ni, Co and Fe_3O_4 at low temperatures. If we divide 15745 by 1123.5 we obtain the number of magnetons n = 14.014 per molecule of oxygen. According to Weiss all the paramagnetic moments are integral multiples of the magneton 1123.5. Unfortunately Weiss, Bauer and Piccard (Comptes rendus T 167, p. 484, 1918), found for the number of magnetons in oxygen 14.12 and in NO 9.20, numbers which deviate considerably from integers.

SOLUTIONS

For dilute solutions the same paramagnetic law has been assumed as in gases. But the solvent is not without influence, in general. There are cases where the rule of Wiedemann holds and cases where it does not hold. The rule of Wiedemann states that in the solutions the susceptibility χ obeys the law of mixtures. Let the concentration of a salt be C_a, its susceptibility χ_a, the susceptibility of the solvent χ_s, its concentration $1-C_a$. Then the rule of Wiedemann will be expressed by the equation:

$$\chi = C_a\,\chi_a + (1-C_a)\,\chi_s.$$

Moreover in a compound like $FeSO_4$ the negative group SO_4 will have an influence on the magnetic moment of the salt. In the inorganic compounds we know very little of this influence, but in the organic compounds of related constitution Pascal found that in many cases the molecular diamagnetism of a compound is equal to the sum of the diamagnetism of the atoms plus an additive constant λ or in symbols $\chi_m = \Sigma_{na}\,\chi_a + \lambda$; λ is characteristic for the molecular constitution. $\lambda = o$ only in the saturated carbon-hydrogen compounds $C_n\,H_{2n+2}$, so that the molecular susceptibility is purely additive. For all other compounds of the aliphatic series λ is positive, for the aromatic compounds λ is negative. Much more complicated are the relations in the oxygen compounds, for instance O_2 and N O are paramagnetic, while C O and $H_2\,O$ are diamagnetic. Still more complicated are the properties of the inorganic compounds, where we know as yet no additive or analogous law. Nevertheless Weiss assumes that the diamagnetic atoms maintain their diamagnetism in the compounds with paramagnetic atoms. In order to obtain the pure paramagnetism in the paramagnetic salts, he uses the following atomic and molecular coefficients:

	$-\chi_a \times 10^6$	$-\chi_a \times 10^6$	$-\chi_m \times 10^6$
H........	3.05	Fl 12	SO_4 38 5
C........	6.25	CCl 21	NO_3 19.0
O........	4.8	Br 32	NH_3 15.0
S........	15.6	I 46.5	CN 11.25
Se........	24.	N_a 4	H_2O 13.5
Te........	39.	K 11	
P........	27.4	Hg 35	

Using these corrections Weiss deduced from the measurements of Pascal the following molecular moments and number of magnetons. Neglecting the fourth and seventh example we obtain almost as good integers as Weiss, by replacing the decimal point one cipher to the left. Unfortunately, Weiss had used for the susceptibility of water $-7.5 \cdot 10^{-7}$

Substance	M_o	$n = \frac{M_o}{1123.5}$	n' = integer
K and NH_4 Ferricyanide....	11700	10.41	10
Fe and NH_4 Pyrophosphate..	24600	21.69	22
Fe and NH_4 citrate.........	24680	21.96	22
Na ferripyrophosphate......	27100	24.04	24
Na ferrimetaphosphate......	31500	28.03	28
Ferrichloride..............	31390	27.93	28
Ferrisulphate..............	33800	30.09	30
K-ferrometaphosphate......	29200	25.99	26
Na-ferrooxalate............	30480	27.11	27
Na-ferropyrophosphate......	31370	27.91	28
Ferrosulphate..............	31120	27.69	28

instead of $-7.2 \cdot 10^{-7}$. This means a considerable error so that these first numbers of Weiss have only historical interest. But they gave rise to a careful study of the solutions of paramagnetic substances, which revealed a number of interesting phenomena, and led in special cases to integral numbers of magnetons. Not all solutions follow the rule of Wiedemann, the susceptibility of the substance dissolved is often a function of the concentration. The salts of Ni follow the law of Wiedemann and give rise to integral numbers of n. Weiss et Bruins, and Cabrera, Moles and Guzman give the following results:

	C_m	n
$Ni\,Cl_2$	1.300	16.03
$Ni\,(NO_3)_2$	1.299	16.02
$Ni\,(SO_4)$	1.306	16.07

More recently Theodorides (Archives de Geneve 3, 1921) finds for $NiCl_2$, in the temperature interval 0° – 125°, n = 16.03; and between 150°–300°, n = 16.92.

Similar are the results of the salts of chromium, $Cr\ (N\,O_3)_3$ n = 18.99, and $Cr_2\ (S\,O_4)_3$, n = 18.99. On the contrary the salts of iron (valence 2) do not seem to give a whole number of magnetons but there exists a lack of agreement among the measurements, as can be seen from the following table:

	C_m	n
$FefSO_4\ 7\ H_2O$	3.400	25.9
	3.589	26.6
$FefSO_4$	3.385	25.9
	3.551	26.51
$Fe\,Cl_2\ 4\ H_2O$	3.478	26.2
$Fe\,Cl_2$	3.349	25.7

The measurements of solid salts and their solutions do not always agree. Approximately the number of magnetons in ferro-salts is 26. In the three valent iron salts in solution Wiedemann's rule does not hold. Here the influence of hydrolysis appears. Cabrera and Moles give the following curves for the variation of C_m as function of the concentration.

These measurements seem to indicate horizontal asymptotes corresponding to 29 magnetons for $Fe\ Cl_3$ and 27 magnetons for $Fe_2\ (SO_4)_3$.

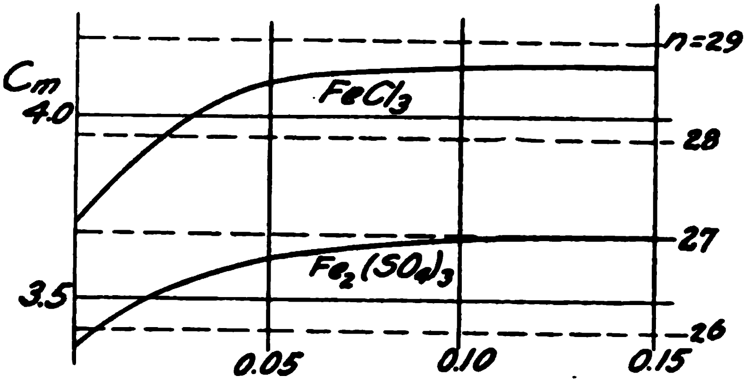
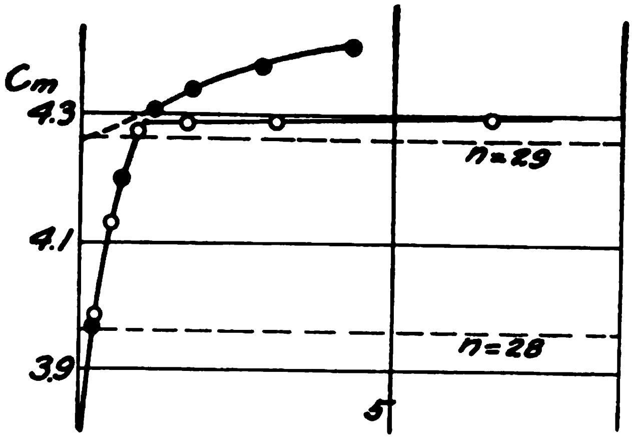


Fig. 29

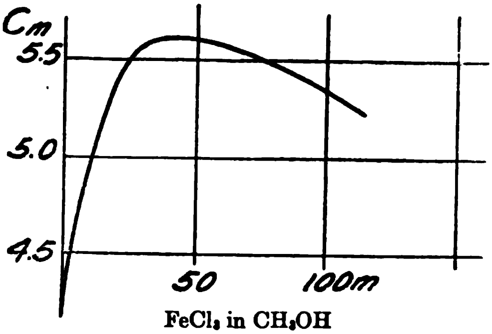
If these curves depend on hydrolysis, they must undergo a change by the addition of the cathion H_+, which will oppose hydrolysis and increase



○–$FeCl_3$+HCl ●–$FeCl_3$+HNO_3

Fig. 30

the magnetic susceptibility. These curves (Fig 30) show the influence of increasing quantities of HCl and HNO_3 in a solution of $Fe\ Cl_3$, whose concentration was 0.00838 gr. per cm³. The curves consist of two branches. In the first part C_m increases rapidly by the addition of small quantities of acid. In the following branch the curve rises much more slowly. The first branch of the curve corresponds to decreasing hydrolysis. If we continue the second branch until it cuts the axis of ordinates, we obtain a value of C_m for zero hydrolysis. This value corresponds to 29 magnetons, but unfortunately the limiting values of C_m for $Fe\ Cl_3$ are not quite the same when the hydrolysis is reduced by HCl and by HNO_3. Moreover, we obtain for infinite dilution for $Fe\ Cl_3$ and $Fe\ (NO_3)_3$ 27 magnetons; for $Fe_2\ (SO_4)_3$ 26 magnetons. If in complex salts the paramagnetic atoms form a part of the anions, they lose sometimes their paramagnetic character; for instance, $K_4\ Fe\ Cy_4$ is diamagnetic.

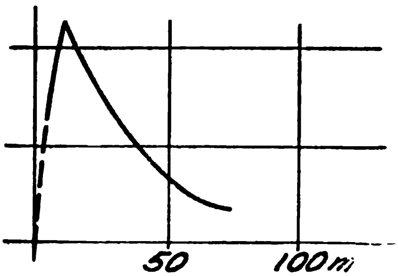


$FeCl_3$ in CH_3OH

FIG. 31

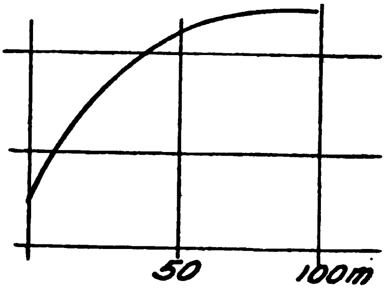
Interesting phenomena appear if paramagnetic iron salts are dissolved in organic solvents, which have been dried carefully.

They are represented by 3 curves (Figs. 31, 31A, 31B) taken from Cabrera's review. Sharp maxima appear especially in the solution of



$FeCl_3$ in $(C_2H_5)_2O$

FIG. 31A



$FeCl_3$ in HCOOH

FIG. 31B

$Fe\ Cl_3$ in $(C_2H_5)_2O$ and in C_2H_5OH. If we continue the curves until they cut the axis of ordinates such values of C_m will sometimes appear as will lead to integral numbers n.

The salts of manganese, cobalt and copper have in general not given integers n, though in some limiting cases, where the hydrolysis is total or zero, approximately whole numbers of magnetons appear. That salts of the diamagnetic copper are paramagnetic is in itself interesting.

PARAMAGNETIC SUBSTANCES IN THE SOLID STATE

P. Weiss extended the theory of paramagnetic gases to solid substances and found from the molecular susceptibilities of a series of compounds the following approximately integral numbers of magnetons.

	n	n_1
$Fe\ Cl_3$	28.83	2.88
$Fe\ Cl_3\ 2NH_4\ ClH_2O$	26.99	2.70
$Fe\ F_3\ 3NH_4\ F$	28.94	2.89
$Fe\ F_3\ 2NH_4\ FH_2\ O$	29.19	2.919
Ferriacetylacetonate	21.23	2.12
1/3 $M\ n_3\ O_4$	17.97	1.797
$Cr\ Cl_3$	20.04	2.004
Cobaltacetylacetonate	21.12	2.11
1/2 $Cr\ (NH_3)_6\ Cr\ (C_2\ O_4)_3\ 3H_2\ O$	20.16	2.0126
1/2 $Cr\ (NH_3)_4\ C_2O_4\ (Cr\ NH_3)_2\ (C_2\ O_4)_2$	20.16	2.0126

If we again replace the decimal point one cypher to the left, we obtain even better integral numbers n, than Weiss (n). In these salts the magneton would be 10 times larger than Weiss' magneton. The last table contains older measurements. Newer measurements of K. Onnes have shown that often paramagnetic salts with crystal water follow Curie's law in a wide range of temperature. In most other cases, however, this law must be replaced by the following form:

$$\chi\ (T+T_1) = \text{Constant},$$

where T_1 is a constant (positive or negative). When liquid oxygen and nitrogen are mixed together, then T_1 depends on the concentration of oxygen: it approaches zero as the dilution becomes infinite, i.e., if the molecules of liquid oxygen are separated sufficiently, Curie's law holds even for dilute solutions. Gans, Keesom and Lenz have given interpretations of the modified Law of Curie. Even in these cases Cabrera has deduced an integral number of magnetons, for instance, from the measurements by E. H. Williams of the oxides of the rare earths. For Didymium he found as many as 52 magnetons.

FERROMAGNETIC METALS AT LOW TEMPERATURES

The magnetism of ferromagnetic bodies increases even below the temperature of liquid air with decreasing temperature, and reaches the absolute saturation at the absolute zero:

$$I_0 = M.N.$$

P. Weiss and K. Onnes have measured at 14° abs. the magnetic moments per gram atom M. given in the following table.

	M_0	n
Nickel.......	3370	3.00
Cobalt.......	10042	8.94
Iron.........	12360	11.002
Magnetite....	7417	

The magneton per gram atom of Weiss 1123.5 is the common divisor of iron and nickel. Magnetite does not agree, nor did cobalt because of the extraordinary hardness of this element, which makes the measurement of the intensity of magnetization I impossible at these low temperatures. O. Bloch however has subjected the Ni-Co alloys to a systematic study and has measured I of alloys containing up to 70 per cent. Co, down to liquid air temperature. The absolute saturation is proportional to the content of cobalt. By extrapolation Bloch found for M_0 10042 corresponding to 8.94 or 9 magnetons. The alloys of Ni-Fe and of Co-Fe show a different behavior. If we represent I as a function of the concentration, we obtain 2 straight lines which intersect each other in a point corresponding to the compounds Fe_2 Ni and Fe_2Co. Continuing these straight lines until they cut the axes, we find the following values of M_0 and n:

	M_0	n		M_0	n
Ni	3450	3.07	Co	10080	8.973
Fe	12450	11.09	Fe	12355	10.997
				11232	9.998
Fe_2Ni	34390	30.6	Fe_2Co	40544	36.087

In the iron-cobalt alloys the number of magnetons changes from 11 to 10. Interesting in itself is the fact that the iron cobalt compound Fe_2Co is the strongest magnetic substance known until now: stronger than pure iron and pure cobalt. How the 36 magnetons distribute themselves among the iron and the cobalt is unknown. In the neighborhood of the absolute zero Ni contains 3, Fe 11 or 10, and Co.9 magnetons.

MAGNETIC PROPERTIES AT THE CURIE POINT

Finally, the magnetic properties of the Curie point offer a possibility of determining the magnetic moment per gram atom, and the number of magnetons. At that temperature, the ferromagnetism disappears, but there remains afterwards a small interval of temperature, in which considerable magnetism may appear through simultaneous action of an external and an internal field. Now for ferromagnetism an internal magnetic field is characteristic:

$$H_m = N\,I, \text{ or } H = N_1\,M,$$

and Weiss assumed that a ferromagnetic body behaves like a gas, whose molecules are acted on by an internal and an external field. Then we have:

$$M = \frac{M_o^2}{3\,R\,T}(H + N_1\,M);$$

$$\frac{M}{H} = \frac{M_o^2}{3\,R\,T}(1 + \frac{N_1\,M}{H})\;;$$

or

(1) $$\frac{M}{H}\,(1 - \frac{M_o^2\,N_1}{3\,R\,T}) = \frac{M_o^2}{3\,R\,T}.$$

We had:

$$M = M_o\,\frac{a}{3};\; a = \frac{M_0\,H}{R\,T}.$$

At the Curie point we have $T = \Theta$, and $H = N_1\,M$, hence:

$$M = \frac{M_o^2\,N_1\,M}{3\,R\,\Theta}\;;\text{ or } \qquad \Theta = \frac{M_o^2\,N_1}{3\,R}.$$

Substituting this expression in (1) we obtain:

$$\chi_m\left(1 - \frac{\Theta}{T}\right) = \frac{M_o^2}{3\,R\,T} \text{ or } \chi_m\left(T - \Theta\right) - \frac{M_o^2}{3\,R} = C_m.$$

This last equation was confirmed by P. Weiss and his coworkers for many ferromagnetic substances. The temperature curve of magnetization, however, above Θ is not uniform, but shows different discontinuities, which separate probably various modifications of iron. In iron the modifications β and γ are well known, but according to the magnetic measurements the range β must be divided in 2 parts, at 828°, into β_1 and β_2. The same holds for Ni. Some of the results obtained are recorded in the following table due to Cabrera.

		C_m	n	*n*
Iron β_1....	Fe_3	6.639 6.536 6.526	36.24 35.94 35.91	36.03
Iron β_2.....	Fe_3	4.587 4.560 4.599 4.646	30.12 30.02 30.15 30.30	30.15
Iron β_3.....	Fe_3	18.306 18.580	60.15 60.60	60.37
Iron γ.....		6.605	36.13	
Ni β_1.......	Ni	0.3258 0.3234 0.3261 0.3264	8.03 7.99 8.04 8.05	8.03
Ni β_2.......	Ni	0.4033 0.4032 0.4109 0.4033	8.96 8.93 9.03 8.96	8.97

The measurements of Honda and Takagi, and of Terry, do not agree with those of Weiss, and the previous authors do not find the magneton. Above the Curie point Θ, magnetite behaves strangely: its curve $\left(\frac{1}{\chi}, T\right)$ consisting of 5 straight lines, to which correspond magnetons in the ratios: 4 : 5 : 6 : 8 : 10. It looks as if at certain temperatures the magnetic moment of the molecules changes in definite amounts so that the number of magnetons increases with increasing temperature. Concluding the report on the magneton of Weiss we shall collect in a table the number of magnetons per atom, which have been found by the different methods indicated.

Method	Fe n	Ni n	Valence
Ferromagnetism...............	11	3	
at low temperature............	10[1]		
	12.0	8.03	
Curie point: Fe................	10	8.97	
	20		
Fe.....................	18		
paramagnetic solutions.........	26	16.03	2
	27 29	....	3
Solid paramagnetic substances...	29		
	27		
	21		

[1] In cobalt alloys.

The statement holds in general that the number of magnetons per atom in paramagnetic compounds of an element is larger than in the metallic ferromagnetic compounds. No relations between Weiss' magneton and other properties of the atom are known. All these numbers of the last table raise new questions. While there is a large amount of experimental evidence in favor of this magneton, its existence cannot yet be considered as established; deviations from integral numbers are not infrequent: for instance, Ph. Theodorides investigated in the solid state the following salts: M_nSO_4, $COSO_4$, $Fe_2(SO_4)_3$, $CoCl_2$, $NiCl_2$, M_nCl_2, M_nO, from 25° to 25° in an interval of 600°. The sulphates have a negative molecular field (T_1 = positive); the chlorides a positive, (T_1 = negative). The sulphates and the chlorides of Co and Ni give integral numbers of magnetons, while M_nCl_2 and M_nO give fractional numbers n, i.e. no magnetons.

The magnetic susceptibilities of nickel and cobalt chloride solutions have been measured recently by Miss Laura Brant.[1] The susceptibilities of the salts have been computed from the susceptibilities of the solutions by application of the Wiedemann law, and the susceptibilities of the metals obtained by extending the Wiedemann relation to the salts. The molecular susceptibility of nickel was 0.004423, and of cobalt 0.01036. These values give 16 magnetons for the nickel atoms and 24.5 magnetons for the cobalt atoms.

ELEMENTARY MAGNETIC MOMENTS BY J. KUNZ

Before I became acquainted with Weiss' magneton, I applied in the year 1910 the equation $\Theta = \frac{M_0^2 N_1}{3R}$ or the equivalent equation $m = \frac{3r\Theta}{NI_s}$, to the ferromagnetic substances Fe, Ni, Co, F_3O_4 and Heusler alloys. $r = 1.36 \cdot 10^{-16}$; Θ = Curie point; N = constant of the internal magnetic field in the equation $H_m = NI$; I_s = absolute intensity of saturation; m = magnetic moment of the molecular magnet or the magneton. If for instance the magnetic moment of iron is m, and if there are Z magnetons per unit volume at temperature 0°, then:

$$Z.m = I_s$$

This equation yields Z. If the mass connected with one magneton is equal to μ_{Fe}, and δ the density at the absolute zero, then $Z\mu_{Fe} = \delta$; if we assume μ_{Fe} to be the molecule of iron and μ_H the mass of an atom of hydrogen, then:

$$\frac{\mu_{Fe}}{112} = \mu_H.$$

---

[1] *Physical Review.* Vol. 17, p 678, 1921.

(The absolute values of the moments of the elementary magnets of iron, nickel and magnetite, *Physical Review*, Vol. XXX, p. 359, 1910). I have assumed that the temperature law of corresponding states holds down to the absolute zero; an assumption which has been shown to be not correct. From μ_H we find the elementary charge e of the electron by the relation $\frac{e}{\mu_H}=9654$. The magnetic moments obtained and the values of e are collected in the following table:

	$m \cdot 10^{20}$	$e \cdot 10^{20}$
Fe..................	5.15	1.60
Fe_3O_4	2.02	0.90
Ni..................	3.65	1.54
Co..................	66.21	1.56
Heusler alloy No. 1....	3.55	1.54
Heusler alloy No. 2....	4.23	2.04

The average value of e is $1.53 \cdot 10^{-20}$ instead of $1.59 \cdot 10^{-20}$. It had to be assumed, however, that the elementary magnet of iron contains 2, of Co, 4, and of Ni, 6, atoms. We have to consider that the ratio of the densities of nickel and iron is nearly the same as the ratio of the atomic weights, the number of atoms per unit volume is therefore nearly the same in both metals, but the magnetic moment of nickel is only about 20 per cent. smaller than that of iron. We should therefore expect the intensity of magnetization of nickel to be only about 20 per cent. smaller than that of iron, while in reality the magnetization of iron is about 3.5 times stronger than that of nickel. The numbers given involve wide extrapolations and cannot claim high accuracy. Moreover the modern theory of crystal lattice does not agree with these older assumptions.

THE MAGNETON OF BOHR

The atom model of Bohr has been so successful in the explanation of the line spectrum of hydrogen, that one might expect it would lead also to an explanation of the magnetic properties. In the original theory of quanta by Planck, an oscillator could only absorb and emit whole multiples of the quantum of energy $E=h\nu$, while according to Bohr the quantum relation appears in two different ways in the atom. In the first place the electron falls, while emitting light, from an outer into an inner stationary orbit, so that the equation, $E_a-E_e=h\nu$, holds. In the second place, for the stationary orbits, the moment of momentum, $p=\frac{n\,h}{2\,\pi}$, holds (here n is an integer). If the electron moves on a circle with radius a, then:

$$p=m\,a\,v,$$

where m is the mass of the electron, and v the velocity $\mu = a\, 2\, \pi\, \nu$, or:

$$p = m\, a^2\, 2\, \pi\, \nu.$$

On the other hand the magnetic moment, M_1, of such a revolving electron is equal to

$$i\, A = e\, \nu\, \pi\, a^2 = \frac{p e}{2 m} = \frac{n h e}{4 \pi m}$$

when $n = 1$, $M_1 = \frac{h\, e}{4\, \pi\, m}$. This is the magneton of Bohr. If we multiply M_1 with Loschmidt's number, $L = 6.06 \cdot 10^{23}$, then we obtain the magnetic moment, M, per gram atom:

$$M = M_1\, L = \frac{h\, e\, L}{4\, \pi\, m} = 5584;\ M_1 = 9.21 \cdot 10^{-21}.$$

$5 \times 1123.5 = 5617.5$; i.e., Bohr's magneton is about 5 times larger than Weiss' magneton. The magnetic moments per gram atom ought to be integral multiples of 5584. We can not yet test this formula, as measurements of monatomic paramagnetic gases are not yet available. Perhaps the vapors of alkali metals are paramagnetic. Measurements of these vapors would be very valuable. Bohr's magneton is only too large in the case of nickel at lowest temperatures; in all other cases so far known the paramagnetic elements and their compounds contain more than 5 Weiss magnetons. If we proceed to the diatomic gases, then Bohr's older theory gives us no satisfactory idea on the structure of molecules. According to Bohr's model the molecule of hydrogen ought to be paramagnetic while this gas is diamagnetic. We can not, therefore, directly extend the above result to molecules. If we apply it nevertheless to the only known paramagnetic gases, oxygen, and NO, then we obtain according to Weiss, Bauer and Piccard (Comptes rendus T 167, p. 484, 1918) the following numbers for the susceptibilities at 20° C:

$$O_2: \chi_{20} = 1.077 \cdot 10^{-4};$$

$$N\,O: \chi_{20} = 0.487 \cdot 10^{-4}.$$

The magnetic moments per molecule are:

$$M_{O_2} = 1.587 \cdot 10^4,$$

and

$$M_{NO} = 1.033 \cdot 10^4.$$

Hence, according to Weiss, oxygen would contain 14.12 magnetons, and NO 9.2 magnetons: not integral numbers. According to Bohr we obtain, by division with 5584, for oxygen 2.86 and for NO 1.86 magnetons: numbers which are not satisfactory. Now, W. Pauli Jr. (Physikalische Zeitschrift 21, p. 615–617, 1920) has suggested that the formula of Langevin requires a correction, if the paths have to be quantified, i.e., if the angle θ between the axis of the magnetic moment and the direction of the magnetic field can not assume continuously all values but has to be restricted so that $\cos\theta$ can only assume the rational values

$$\cos\theta = \pm\frac{k}{n},\ k = 1, 2, 3\ldots\ldots n,$$

where n is the number of quanta of the moment of momentum, or the number of magnetons. Then he obtains instead of $M_0 = \sqrt{3\,R\,C_m}$, the corrected formula:

$$M_0 = \sqrt{\frac{3\,R\,C_m}{1/2\,(n+1)\,(2\,n+1)}}.$$

For n = 1,

$$M_0 = \sqrt{3\,R\,C_m}\cdot\frac{1}{\sqrt{3}}.$$

For NO we obtain by means of this formula:

$$M_0 = 10330 : 1{,}732 = 5960,$$

and

$$\frac{5960}{5584} = 1.066.$$

For n = 2 we obtain:

$$M_0 = \sqrt{3\,R\,C_m}\,\frac{1}{\sqrt{\frac{15}{2}}}.$$

This gives for oxygen:

$$M_0 = 15870 : 2\cdot 739 = 5800$$

and,

$$\frac{5800}{5584} = 1.04,$$

or 4 per cent. deviation from the assumption that oxygen contains two Bohr magnetons. For NO the deviation amounts to 7 per cent. from the assumption that this gas contains one Bohr magneton. Pauli has assumed that the axis of the magnetic moment is perpendicular to the axis of figure and that for the spatial quantification only the moment of momentum of the electrons and not the total moment of momentum has to be used. The magneton thus determined, in spite of the deviations, is very probable, as the number of magnetons is restricted to one and two.

THE MAGNETON OF PARSON

No theory explains so far why the electron, moving on a circular or elliptic orbit, about a nucleus, does not lose the energy by radiation, or why the orbit remains stationary or free from radiation. In order to eliminate this question, Parson assumed that the electron is at the same time a magneton, i.e., the electron consists of a linear circular ring in which the electricity moves with the velocity of light. It is a closed circuit without resistance and without radiation. It is almost the conception by Ampere of stationary currents in the atoms, which give rise to the magnetic properties. Such a circuit, at rest or moving with constant velocity, loses no energy through radiation. Parson assumes the radius $r = 1.5 \cdot 10^{-9}$ cm. The moment of this magneton is equal to

$$A\,i = M = \pi r^2 \frac{e}{T};$$

$$T = \frac{2\pi r}{c};$$

$$M = \frac{\pi r^2 c e}{2\pi r} = \frac{r e c}{2} = 3.5 \cdot 10^{-19} \text{ e.m.u.},$$

while that of Bohr is equal to $9.21 \cdot 10^{-21}$ and that of Weiss equal to $1.85 \cdot 10^{-21}$. As Parsons' magneton is 38 times greater than the magneton of Bohr, which almost corresponds to the magnetic moment of NO, Parsons' magneton appears almost impossible for magnetic reasons. In order to eliminate this difficulty Parson assumes that this is the greatest magnetic moment which an atom may assume, and that the moment of most atoms will be smaller, because the different magnetons of an atom neutralize each other. Moreover, in the molecules with several atoms in the solid and liquid state the magnetons of the various atoms will oppose each other.

This magneton of Parson can not be discovered in the devation of cathode rays through its magnetic moment. We might expect to increase the concentration of electrons in a conductor connected to earth, by the application of a magnetic field. The magnetic work required to bring up the magneton, would be equal to H M, and the electric work gained equal to

$$\frac{V e}{2} = H M;$$

$$V = \frac{2 H M}{e} = 4.5 \cdot 10^{-4} \text{ volts},$$

for H = 1000 Gauss. A theory of spectral lines, Roentgen spectra, fine structure, Stark effect, is not based upon this conception of the electron. Parson discusses especially chemical questions. The chemical forces are intimately connected with the magnetic and electrostatic effects of the magneton. According to Parson the magnetons or electrons are imbedded in a sphere of positive electricity which is uniformly distributed, and in which special elastic forces are in equilibrium with the repelling forces of the positive particles. The positive electricity is compressible and may condense round about the magnetons and weaken their electrostatic effects. With these conceptions Parsons tries to build up a qualitative theory of the chemical compounds and of the natural system of the elements. An idea, which has been emphasized later in the considerations of Kossel, Lewis, and Langmuir, appears here at first, namely, the idea that the most perfect form of symmetry is the distribution of 8 magnetons in the corners of a cube; which gives a high degree of stability with a minimum of magnetic energy. Parson has also built a model with 8 coils, in which are made visible the mutual positions of 8 magnetons. Figure 32 shows the upper 4 coils, which are just equal to the lower 4 coils, so that the model is diamagnetic. These models correspond to the inert gases, which indeed are diamagnetic. Parson, in contradiction with modern evidence—assumed already in in helium 8 magnetons, then the number of magnetons increases almost regularly by one unit. Li has therefore 9 magnetons of

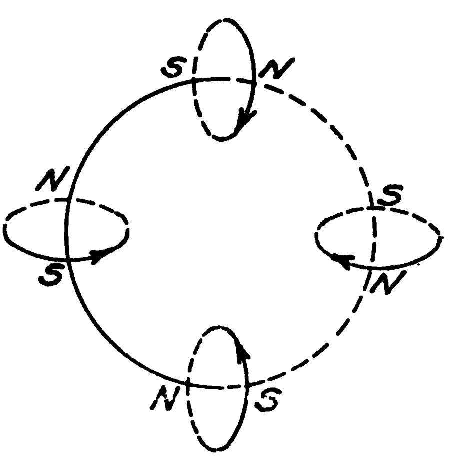


FIG. 32

which only one has a free magnetic moment, the element is therefore paramagnetic. The hydrogen atom contains one magneton and is therefore paramagnetic in the free state, while the molecule is diamagnetic so that the two magnetons neutralize each other. For iron, nickel, and cobalt, Parson assumes 32 magnetons while the atomic numbers are

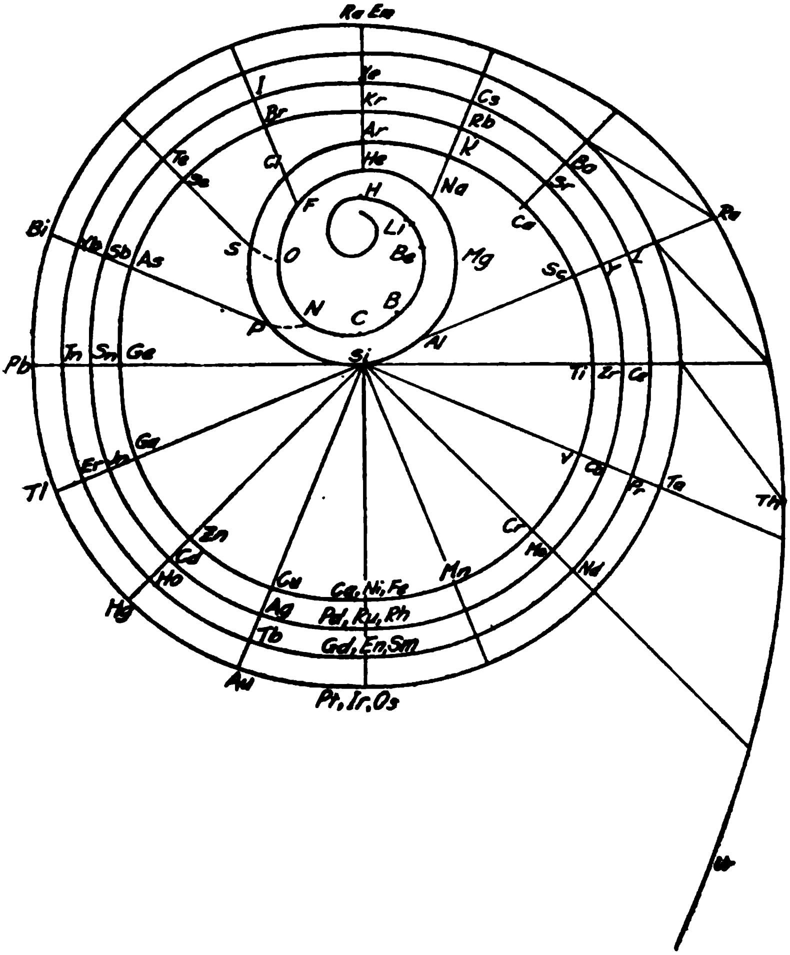


Fig. 33

26, 27, 28 respectively. Parson does not explain why these elements, which fall in the group 8 with the inert gases, are so strongly magnetic, nor does he give a sufficient explanation of the periodic change of diamagnetic and paramagnetic properties of the chemical elements. In connection with the theory of cubes Parson makes the interesting observation: the compounds HF, H_2O, H_3N, H_4C, are perfect cubes and are diamagnetic as well as the salts of Li, Na, K, Rb, Cs with F, Cl, Br, I, and the analogous compounds: CaO, SrO, BaO, $NaNO_3$, Na_2SO_4, KNO_3, K_2SO_4.

In connection with Parson's magneton a few remarks may be made on the relation between magnetism and chemistry. The experimental facts are legion, the theory is hardly started. It is well known that a certain periodicity exists between atomic weights or atomic numbers and the magnetic susceptibilities of the elements. The curve representing this relation is of an irregular character representing seven distinct maxima, among which that of the iron group is by far the predominating one. If the sign only of the magnetic properties is taken into account, one gets the best representation perhaps by the method of the helix due to B. K. Emerson, which is given in Fig. 33.

The strongly magnetic groups appear on a diameter, where we find Fe, Ni, Co, then Pd, Ru, Rh, then Gd, Eu, Sm, then Pt, Ir, Os. To the right of the diameter D all elements are paramagnetic or ferromagnetic; to the left of D all elements are diamagnetic with the exception of tin and oxygen. Tin in some temperature interval is paramagnetic, in another interval diamagnetic. Oxygen is surprisingly paramagnetic. Its regular diamagnetic properties appear only in some of the organic and inorganic compounds.

Moving on the spiral from iron to the right, we meet Mn and Cr, elements which are paramagnetic, but whose strongly magnetic properties appear only in some of their alloys such as the Heusler alloys, manganese-antimony, manganese-tin, Cr_5O_9. On the right hand side from the ferromagnetic elements, there are paramagnetic elements; on the left hand side the diamagnetic elements.

The literature on the magnetic properties of chemical compounds is very rich in facts, which however are not yet correlated by a theory. It is very surprising that some iron compounds are diamagnetic. The rules discovered by Pascal for organic compounds have already been mentioned.

MAGNETOSTRICTION AND ITS BEARING ON MAGNETIC THEORIES

By S. R. Williams

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Under the title of magnetostriction may be classified those mutual relations which exist between magnetic and mechanical deformations of *ferromagnetic* substances. In general, a magnetic field causes a change in dimensions of such bodies while reciprocally mechanical deformations produce changes in the magnetic properties. Among such phenomena may be listed:

1. Joule Effect. (Change in length due to a magnetic field.)
2. Wiedemann Effect. (Twist due to the superposition of longitudinal and circular magnetic fields.)
3. Longitudinal Currents due to a twist superimposed on a longitudinal field.
4. Longitudinal Magnetization due to a twist superimposed on a circular field.
5. Volume Change due to a magnetic field.
6. Villari Effect. (Change in magnetic properties due to a mechanical stress.)
7. Production of Sound due to a magnetic field.
8. Change of Resistance due to a magnetic field.
9. Effect of Magnetic field on Thermo-electric phenomena.

It should be emphasized that all of these phenomena entail a definite relation between mechanical and magnetic characteristics. To investigate these inter-relations is one of the most important studies of magnetic phenomena which may be undertaken, for not only is there wrapped up in such a study the possibility of judging mechanical properties of substances by their magnetic behavior, which is important for the industries, but for the particular subject in hand, magnetostriction has a very significant contribution to make to a comprehensive magnetic theory. Thus far magnetic theories have avoided trying to explain the magnetostrictive phenomena and as a consequence the newer investigations along this line must have a very definite outlook on magnetic theories in general. A comprehensive theory of magnetism must explain the entire range of magnetic phenomena. A glance at the list of effects given above will demonstrate what a real theory of magnetism must explain. In the interest of clarity a short description of each of the above experimental facts will be given. To a large extent references will be given to the more recent researches in which a bibliography of earlier papers may usually be found.

1. If a rod of iron or steel is subjected to a magnetic field which may be varied continuously from zero upwards it will be found that the rod first increases in length and after a certain field strength is attained it begins to shorten and becomes shorter than in its virgin state. At large field strengths there appears to be no change in length and the curve showing the changes in length becomes asymtotic to the field axis. This is known as the Joule[1] effect. It varies in different ferromagnetic substances and the characteristic changes in length for different materials are shown in Fig. 1. Temperature, tension, extraneous magnetic fields,[2] etc., all have an influence on the changes in length which occur due to a magnetic field.

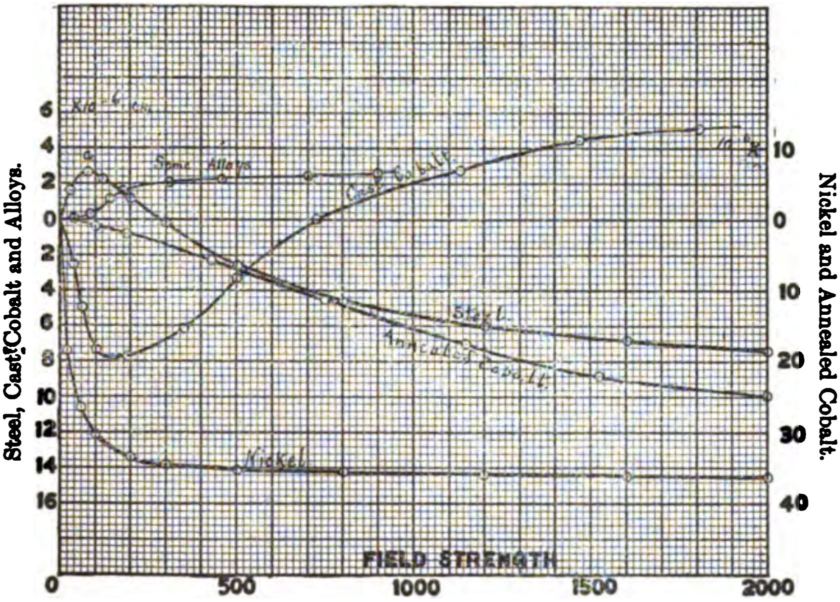

Fig. 1

2. Wiedemann[3] found that if a rod of ferromagnetic substance is magnetized longitudinally and then simultaneously a current is sent along the rod producing a circular magnetic field, the superposition of these two magnetic fields causes the two ends of the rod to rotate in opposite directions. This is often spoken of as a special case of the Joule effect in that changes in length occurring along the helical direction of the resultant magnetic field produces a twist. This experiment[4]

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[1] Joule, *Phil. Mag.* 30, 1847, 76 and 225; Bidwell, *Proc. Roy. Soc.*, 55, 1894, 228; 56, 1894, 94; Williams, *Phys. Rev.*, 34, 1912, 258.

[2] Williams, *Phys. Rev.*, 10, 1917, 133.

[3] Wiedemann, *Elektricität*, 3, 689.

[4] Williams, *Phys. Rev.*, 32, 1911, 281.

as usually carried out maintains either the longitudinal or the circular magnetic field constant while one or the other is varied in some continuous manner from zero upwards. Inasmuch as this procedure causes the direction of the resultant field to vary continuously there is a wide departure from a true Joule effect in which the direction of the field is always constant. A comparison with the Joule effect should be made only when both the longitudinal and the circular fields are varied together so as to keep the resultant field direction constant.

3. When a ferromagnetic rod is magnetized longitudinally in a solenoid, a twist imparted to the rod establishes a circular magnetic field in the rod which may be detected by the electric current which is transiently set up along the length of the rod while the rod is being twisted.

4. Magnetize a rod circularly by running a current along the same and if a twist is imparted to the rod in this state a longitudinal magnetization will be produced in the rod which may be detected by the current induced in the surrounding solenoid. Both of the special phenomena enumerated in (3) and (4) are reciprocal relations accompanying the Wiedemann[1] effect. In (2), (3) and (4) there are three factors which go to make up the effects; a longitudinal magnetization; a circular magnetization; and a twist. By establishing any two the third will be produced.

5. Joule,[2] the first to observe changes in length magnetically, also observed that while changes in length were occurring, variations in dimensions transversely were also taking place. That is, if the rod increased in length, the dimensions at right angles to the length decreased. This led to the question as to whether the transverse and longitudinal changes just compensated each other and gave no change in volume. Joule was not able to confirm the idea that there was a change in volume accompanying magnetization because of the smallness of the effect, but later on an effect was confirmed by Cantone[3] in the case of nickel. Nagaoka and Honda[4] are the ones who have really established beyond a doubt the presence of this effect in all ferromagnetic bodies. In all of the magnetostrictive phenomena each substance has its own peculiar effect. In this instance cobalt is just the opposite of iron, viz., iron has its volume increased by magnetization while cobalt is decreased. There is also a very definite relation between the effect of magnetization on the change of volume and the change in intensity of magnetization due to hydrostatic pressure. They appear to be reciprocal relations.

6. Thomson[5] has pointed out that there are certain reciprocal relations in magnetism where if the changes in length were known it might be

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[1] Wiedemann, *Elek.*, 3, 692.

[2] Joule, loc. cit.

[3] Cantone, *Rendiconti d. R. Accad. d. Lincei*, 6, 1890, 252.

[4] Nagaoka and Honda, *Phil. Mag.*, 46, 1898, 261; 4, 1902, 45.

[5] Thomson, Applications of Dynamics to Phys. and Chem., p. 47 et seq., 1888.

predicted with certainty the effect which a longitudinal pull or compression would produce in that same specimen in the way of changing its magnetic properties. This latter phenomena is known as the Villari[1] effect. If a ferromagnetic rod shows an increase in length due to a magnetic field, that same rod will show an increase in magnetization when stretched or a decrease in magnetization when compressed longitudinally. If the rod shortens in a magnetic field a corresponding Villari effect ensues. For substances which show an increase in magnetization for weak fields and a decrease for strong fields, there is a certain critical field strength where the intensity is the same whether the rod is stretched or not. This is known as the Villari reversal point. Substances showing a Villari reversal also show a Joule reversal such as Bidwell demonstrated in iron. The question as to whether the Villari reversal occurs in nickel[2] seems now to be pretty well settled in favor of the negative.

7. The magnetostrictive effects have a relationship to the tones which are emitted by a rod when placed in an alternating magnetic field. It is a complicated phenomena because if a rod is placed in a periodically varying magnetic field, the alternating changes in length will not only give rise to vibrations in the rod, but there is danger of not getting the rod placed symmetrically in the field and this will give alternating pulls and thrusts on the rod and so set up vibrations. Bachmetjew's[3] work on strongly stretched rods, which gave no tones, would indicate that magnetostriction was the cause of many of the tones produced. Honda and Shimizu[4] have carried out some interesting experiments on this subject which indicate very definitely that the tones emitted by a rod in an alternating magnetic field are largely produced by the change in length. Maurain[5] observed the frequency of tuning forks in a magnetic field and in various azimuths and found a change in the period. This is a field which needs further investigation, but changes in dimensions of the fork must effect some change in the period. Warburg[6] and St. Meyer[7] have also shown that mechanical vibrations affect the magnetic properties of ferromagnetic substances. This is really a special case of the Villari effect.

8. A change in resistance due to a magnetic field is a complicated phenomenon. It seems to be pretty well accepted that it is primarily related to the Hall effect. However, with change in dimensious which occur, there should be some change in resistance and if in the process of

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[1] Villari, *Pogg. Ann.*, 126, 1868, 87.

[2] Ewing and Cowan, *Phil. Trans.*, 179, 1888, A. 325: Heydweiler, *Wied. Ann.*, 52, 1894, 462; 15, 1904, 415: Honda and Shimizu, *Ann. d. Phys.*, 14, 1904, 791; 15, 1904, 855: Williams, *Phys. Rev.*, 10, 1917, 129.

[3] Bachmetjew, *Rep. d. Phys.*, 26, 1890, 137.

[4] Honda and Shimizu, *Phil. Mag.*, 4, 1902, 645.

[5] Maurain, C. R. 121, 1895, 248.

[6] Warburg, *Pogg. Ann.*, 139, 1870, 499.

[7] Meyer, Boltzmann Festschrift, P., 68, 1904.

magnetization an orientation of oblate spheroidal particles exists, a change in resistance similar to that found on rotating spheroidal particles in an electrolyte[1] may be expected, and would augment the change in resistance more than the mere change in dimensions. For the most part measurements of the changes in resistance which occur in magnetic fields have been carried out with powerful electromagnets. It hardly seems possible to carry out such experiments without getting some real distortions in the specimens.

9. Thermocouples may be made out of substances which are the same chemically, but not physically.[2] If magnetized and unmagnetized iron are used as the elements, a P. D., ca. 1/22 volts, as maximum exists. Bachmetjew[3] in applying a tension to the iron wire thus magnetized found that the direction of the P. D. could be reversed, thus placing a partial cause of the phenomenon at the door of magnetostriction. The results of those who have studied the behavior of thermocouples in a magnetic field indicate a marked similarity between the character of the curves showing the change in E.M.F. with magnetic field and the Joule effect.

The above phenomena must be explained by a comprehensive magnetic theory. What must be the character of that theory? Preeminently it must be a mechanical theory. On the one hand there are plain straightforward mechanical effects due to a magnetic field; while on the other, by impressing on the same substance a distortion of any character whatever, a very distinct change in the magnetic qualities obtains. As Burrows[4] expresses it, "Experimental evidence seems to point to the conclusion that there is one and only one set of mechanical characteristics corresponding to a given set of magnetic characteristics, and conversely there is one and only one set of magnetic characteristics corresponding to a given set of mechanical characteristics."

Poynting and Thomson[5] have called attention to the fact that the magnetostrictive effects are yet to be explained on the molecular hypothesis. They state, "It would obviously require some further assumption as to molecular grouping or as to molecular dimensions in different directions." This point will be discussed in greater detail later on.

If a catalog of magnetic phenomena is made it will be seen how important a role magnetostriction plays in studying magnetic theories, for magnetostriction occupies a large section of the list of magnetic effects. Not only will the following table show how the magnetostrictive effects bulk up among other magnetic phenomena but it will be useful in orienting one's thinking in the field of magnetism.

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[1] Williams, *Phys. Rev.*, 2, 1913, 241.

[2] Williams, *Science*, 40, 1914, 606.

[3] Bachmetjew, *Wied. Ann.*, 43, 1891, 723.

[4] Burrows. *Bul. Bur. Stands.*, 173, 1916, 13.

[5] Poynting and Thomson, *Elec. and Mag.*, p. 201, 1914.

I. Induction Effects.
    1. Relation between field strength and magnetic induction, permeability, susceptibility, coercive force, retentivity, hysteresis, etc.
    2. Dia-, para- and ferromagnetism.
    3. Terrestrial magnetism.
    4. Alternating currents.
    5. Inductive effects as influenced by temperature, mechanical strains, ageing, extraneous fields, etc.
    6. Relation between susceptibility and chemical properties.

II. Mechanical Effects.
  (a) Reaction effects between magnetic fields.
    1. Attraction and repulsion of magnetic poles.
    2. Motion of electric conductors, (solids, liquids and gases), carrying currents when placed in a magnetic field.
    3. Hall effect and its reciprocal relations.
  (b) Magnetostrictive effects.
    1. Joule effect. Its reciprocal relations.
    2. Villari effect.
    3. Wiedemann effect. Its reciprocal relations.
    4. Volume change. Its reciprocal relations.
    5. Change in resistance due to a magnetic field.
    6. Production of sound.
    7. Piezo- and Pyromagnetism.
    8. Magnecrystallic action.
    9. Effect of magnetic field on thermo electric phenomena.
    10. Barkhausen Effect

III Magneto-optical Effects.
    1. Faraday effect.
    2. Kerr effect.
    3. Zeemann effect.
    4. Magnetic double refraction.

Naturally one might question some points in this classification. Certainly changes would be made if more were known about the subject. Whatever the arrangement of subjects a complete magnetic theory must explain all of the above phenomena. This is a real task.

Following the suggestions of Ewing in his theories it seems to be pretty well conceded that induction phenomena are to be explained by the orientation of something within the ferromagnetic substance which we may call the elementary magnet. If the rotation of the elementary magnets due to an external field explains ferromagnetism then one may properly ask if the rotation of the elementary magnets might not also explain the magnetostrictive effects, since these effects appear in ferromagnetic substances. Ewing's theory and model have been quite successful in the general field of magnetism. Why should they not be effective also in magnetostriction? This leads to an emphasis of the suggestion[1] that if dimensions in different directions be ascribed to the elementary magnets then orientation of such a group of magnets would give rise to changes in dimensions such as are found in magnetostriction. It would appear that some such picture must be made of what happens in a ferromagnetic substance when it is magnetized or else a line of argument such as that suggested by Borelius[2] and others must be followed, viz., that there are two effects present and their combination gives rise to the results which we obtain in magnetostrictive phenomena. Ewing's work and that of his followers have been on the basis of a specific model which could be set up and tested in the laboratory.

Another class of theorists is that in which mathematical formulae have been set up for the mechanical stresses produced by magnetic fields. Maxwell[3] deduced the first one which was followed by one of a more general character by Helmholtz[4] in which he emphasized the terms arising from the change in density of the medium, a fluid being the medium he had particularly in mind. In 1884 Kirchhoff[5] gave a formula which was even more general than those preceding him which included terms dependent upon the elongation as well as those dependent upon changes of density. Kirchhoff's theory really confirms the idea from a mathematical standpoint that when a substance, at least a ferromagnetic substance, is subjected to a magnetic field, strains are set up in the body thus placed and give rise to anisotropic susceptibility.

The equations which Kirchhoff set up show the relations which exist between the intensity of magnetization and the field strength when this relation is modified by the effects of the strains set up in the magnetized medium. The equations which define these coefficients are the following:

$$I_x = \{k - k'(\lambda_x + \lambda_y + \lambda_z) - k''\lambda_x\} H_x:$$

$$I_y = \{k - k'(\lambda_x + \lambda_y + \lambda_z) - k''\lambda_y\} H_y;$$

$$I_z = \{k - k'(\lambda_x + \lambda_y + \lambda_z) - k''\lambda_z\} H_z.$$

The ordinary relation between I and H is that $I = k\,H$. In the above equations it will be noticed that the terms within the brackets are of

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[1] Williams, *Phys. Rev.*, Abstract, Feb., 1911; *Phys. Rev.*, 34, 1912, 40. *Phys. Rev.*, 35, 1912, 282; Poynting and Thomson, *Elec. and Mag.*, p. 201, 1914.

[2] Borelius, *Ann. d. Phys.*, 58, 1919, 489.

[3] Maxwell, *Elec. and Mag.*, p. 257, 2nd. Ed.

[4] Helmholtz, *Wied. Ann.*, 13, 1881, 385.

[5] Kirchhoff, *Sitzber. d. K. Akad. d. Wiss. zu Berlin*, p. 47, 1884.

the character of k in the simple equation. If there were no strains set up then k′ and k″ would be zero and the equations of Kirchhoff would be the usual relation between I and H. k′ is the coefficient of magnetization introduced because of the changes in volume altering the magnetization of the specimen. This is the term which Helmholtz introduced in his equations and which he thought applied more particularly to liquid media. k″ is the coefficient which changes the intensity of magnetization because of the change in length which has occurred in the material parallel to the direction of the magnetic field. Kirchhoff assumed that in applying his formulae, solid elastic media, free from hysteresis and time lag, were dealt with and that they were initially isotropic. Sano[1] has extended the theory of Kirchhoff somewhat by making all of the coefficients, k, k′ and k″, some function of the field applied.

When Kirchhoff developed his mathematical theory of magnetostriction there was little or no data on the values of k′ and k″ because these had to be determined from the changes of volume and of length which occur in ferromagnetic substances when subjected to a magnetid field. Particularly the change in volume due to a magnetic field had not been definitely determined. It is to Nagaoka and Honda[2] and their pupils that we are indebted for the careful, painstaking work which has put the theory of Kirchhoff to such rigid tests. Nor should the elaborate researches of Cantone[3] in calculating the coefficients of Kirchhoff, k′ and k″, from the experimental values of the change of volume and of length in nickel by magnetization be forgotten. Nagaoka and Honda extended the work of Cantone to various other ferromagnetic substances. One cannot read the various papers of these two indefatigable investigators without feeling how thorough-going their work is. From the theory of Kirchhoff, Cantone calculated and found the change in length and in volume of ovoids to be of the following values respectively:

$$\text{(a)}\quad \frac{\delta l}{l}=\frac{H^2}{E\,(1+2\,\theta)}\left\{\frac{4\,\pi\,k^2}{3}\,(1+\theta)+\frac{k-k'}{2}-\frac{k''}{2}\,(1+2\,\theta)\right\};$$

$$\text{(b)}\quad \frac{\delta v}{v}=\frac{H^2}{K\,(1+3\,\theta)}\left\{\pi\,k^2+3\,\frac{(k-k')}{4}-\frac{k''}{4}\right\};$$

where E is Young's modulus, K the rigidity, and θ a constant defined by the relation;

$$\frac{E}{2}\left(\frac{1+2\,\theta}{1+3\,\theta}\right)=K.$$

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[1] Sano, *Phys. Rev.*, 14, 1902, 158.

[2] Nagaoka and Honda, *Phil. Mag.*, 46, 1898, 261; 49, 1900, 329; 4, 1902, 45.

[3] Cantone, *Mem. d. R. Accad. dei Lincei*, 6, 1890, 487.

In an analogous way Nagaoka and Honda calculated these same changes for long wires or rods when placed in a uniform magnetizing field and found them to be:

$$\text{(c)}\quad \frac{\delta l}{l}=\frac{H^2}{E}\left\{2\pi k^2-\frac{k''}{2}+\frac{k-k'}{2(1+2\theta)}\right\};$$

$$\text{(d)}\quad \frac{\delta v}{v}=\frac{H^2}{2K(1+3\theta)}\left\{2\pi k^2+\frac{3}{2}(k-k')-\frac{k''}{2}\right\};$$

The results of Cantone and Nagaoka and Honda show the volume change for the ovoid and the extended rod to be the same while the length change is less for the ovoid than for the rod or wire. Inasmuch as magnetic fields beget mechanical strains and vice versa, mechanical strains produce changes in the magnetic properties of ferromagnetic substances and these effects are reciprocal, Nagaoka and Honda calculated both k′ and k″ from the changes in magnetization due to increase in volume by hydrostatic pressure σ, viz.,

$$\text{(e)}\qquad \delta I=-H\left(k'+\frac{1}{3}k''\right)\sigma,$$

and also from the change in susceptibility due to longitudinal stretching, λ, of a long rod:

$$\text{(f)}\qquad \delta k=\left\{k'\frac{E}{K}-3\left(k'+\frac{1}{3}k''\right)\right\}\lambda.$$

From the ovoid Nagaoka and Honda got:

$$\text{(A)}\qquad \begin{cases} k'=\dfrac{p(1+2\theta)-q}{2(1+3\theta)}, \\[2ex] k''=\dfrac{3q-p}{2(1+3\theta)}; \end{cases}$$

while from a long rod:

$$\text{(B)}\qquad \begin{cases} k'=k+\dfrac{E^2}{2KH^2}(\lambda-(1+2\theta)\sigma); \\[2ex] k''=4\pi k^2-\dfrac{E^2}{2KH^2}(3\lambda-\sigma); \end{cases}$$

where p and q are defined in terms of known quantities and are used here for brevity's sake. From changes in magnetization and susceptibility

$$\text{(C)} \qquad \begin{cases} k' = \left(\delta k - \dfrac{3\lambda\,\delta I}{\sigma H}\right)\dfrac{K}{\lambda E}; \\ k'' = -3\left(\dfrac{\delta I}{\sigma H} + k'\right). \end{cases}$$

Young's modulus and coefficent of rigidity were determined in the usual way. Nagaoka and Honda "found wide quantitative divergences between the results of experiment and calculations, though in nearly all cases there was agreement as to quality."[1] It is with "imperfect success" that mechanical deformations due to magnetic fields may thus be explained by a theory of magnetic stress and vice versa. One has only to turn to the careful work of Nagaoka and Honda[2] to see how far Kirchhoff's theory comes from explaining the facts as they exist. They assert that Kirchhoff's theory is a "rough approximation and will perhaps only hold when the strain is infinitely small. We cannot, therefore, expect that such a theory can explain the relations between the strains caused by magnetization and the effects of stress on magnetization in all their qualitative and quantitative details." According to Kirchhoff the change in magnetization is made proportional to the strain; experimental data shows k, k′ and k″ as functions of the strain.

In the case of the change of magnetization produced by the elongation λ of the wire, Nagaoka and Honda calculated the various values of δI and obtained results which agree qualitatively with those obtained experimentally in iron and nickel and yet the values of the term $(3k' + k'')$ used in the calculation varied 50 per cent. from the observed values for nickel at low fields.

Computing the change of magnetization, δI, due to a decrease of volume, $-\sigma$, the agreement with experiment was fairly good for nickel, but there was a big discrepancy for iron and steel.

They applied the theory of Kirchhoff to the Wiedemann effect and found here a better concurrence between theory and experiment for nickel than they did for iron. The curves showing the calculated and observed values for the Wiedemann effect in iron would indicate a difference for some field strengths greater than 50 per cent. While the correspondence between calculated and observed values, qualitatively, are not to be minimized, yet it must be conceded by all that a theory which gives variations of over 50 per cent. between calculated and ob-

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[1] *Encyl. Brit.*, p. 340, 11th, Ed.

[2] Nagaoka and Honda, *Phil. Mag.*, 46, 1898, 277; 49, 1900, 336; 4, 1902, 66.

served data leaves much to be desired in the way of coordinating the phenomena. As Nagaoka and Honda said, "The present state of the theory of magnetostriction may perhaps be compared with that stage in history of the theory of magnetism when the intensity of magnetization was supposed to be simply proportional to the magnetizing force. In fact the theory is still in its infancy, so that there are ample grounds for further development and research."

The correlation between magnetostriction and other magnetic and physical properties must be extended and in all cases should be carried out as far as possible on the same specimens, as there are no two samples exactly alike either magnetically or mechanically. Comparisons between the results of workers using specimens different from those of others are of limited value.

THEORIES OF MAGNETOSTRICTION.

By S. L. Quimby
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When a body is placed in a magnetic field the matter in every element of volume experiences certain forces due to the action of the field upon the magnetic particles of the body. In addition the fòrces between neighboring molecules may undergo considerable change as a result of the molecular re-orientation which accompanies magnetization. Both of these effects contribute to produce a deformation of the body which is known as "Magnetostriction."

The problem of deriving a theoretical relationship between the strength of the magnetizing field and the consequent strain of the magnetic medium evidently resolves itself into an evaluation of these forces. This done, the strains may be calculated by the ordinary procedure of the theory of elasticity.

We shall first review the theories of magnetostriction of Maxwell and von Helmholtz. These investigators concerned themselves solely with an endeavor to evaluate the forcive per unit volume acting on the medium as a whole due to the external magnetic field. Their results, therefore, will not include the intrinsic stresses arising from the mutual actions of neighboring molecules.

Both Maxwell and von Helmholtz sought, at the outset, expressions for the potential of the desired field forces. The point of departure of the two theories exhibits itself at once in the quite different values obtained for this quantity.

Maxwell's Theory of Stress in a Magnetized Medium.[1]

If a volume element, $d\tau$, of a magnetic medium be assumed to contain a large number of elementary magnetic bipoles, then by calculating the work done in bringing these bipoles from a position where the field is zero into a field of strength $\mathbf{H}$ the magnetic potential energy of the medium in $d\tau$ may readily be shown to be:[2]

$$(1) \qquad dW = -\mathbf{H}\cdot\mathbf{I}\,d\tau.$$

We seek, now, an expression for the potential function of the mechanical force on $d\tau$. Let the matter in $d\tau$ be shifted from a place where

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[1] Maxwell, Treatise, II, § 639 et seq.
[2] ibid., § 389.

the field is $\mathbf{H}$ to one where it is $\mathbf{H}+\delta\mathbf{H}$. The consequent change in potential energy will be:

$$\delta W = -\mathbf{H}\cdot\delta\mathbf{I}\,\delta\tau - \mathbf{I}\cdot\delta\mathbf{H}\,\delta\tau. \tag{2}$$

The first term on the right hand side of equation 2 evidently represents a decrease in potential energy arising from a change in the internal configuration of the elementary magnets in $d\tau$. It measures the work done against intermolecular forces of other than magnetic types opposing the change in polarization and is stored up as internal energy of the medium of a purely elastic or thermal character. This part of the potential energy clearly has nothing to do with the mechanical forces acting on the medium as a whole.[1]

The second term of the right hand member of equation 2 is the change in potential energy which would have occurred had the magnetization of the element been held rigid, so that no work could be done internally. It therefore represents the work of the mechanical bodily forces on $d\tau$, and:

$$W' = -\int_{\tau} d\tau \int_{0}^{\mathbf{H}} \mathbf{I}\cdot d\mathbf{H} \tag{3}$$

is the required potential function of these forces.

From equation (3) we may write at once for the external forcive per unit volume on the magnetized medium:

$$\mathbf{F} = \nabla_{\mathbf{H}}\,(\mathbf{I}\cdot\mathbf{H}), \tag{4}$$

where $\nabla_{\mathbf{H}}$ indicates that ∇ operates upon $\mathbf{H}$ alone.

If the medium carries an electric current whose density is $\mathbf{J}$ there will be an additional electromagnetic force $\mathbf{J}\times\mathbf{B}$ on this current,[2] so that the total force per unit volume is given by:

$$\mathbf{F} = \nabla_{\mathbf{H}}\,(\mathbf{I}\cdot\mathbf{H}) + \mathbf{J}\times\mathbf{B}. \tag{5}$$

Using the relationships:

$$4\pi\mathbf{J} = \nabla\times\mathbf{H};$$

$$\nabla\cdot\mathbf{B} = 0;$$

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[1] On the subject of energy in a magnetized medium see Livens, "The Theory of Electricity," Ch. VI and Ch. XI; Larmor, "The Electrodynamic and Thermal Relations of Energy of Magnetisation," *Proc. Roy. Soc.* 71 (1903), p. 229; Langevin, "Magnetisme et Theorie des Electrons," *Ann. de Chim. et de Phys.* 5–6 (1905), p. 105.

[2] Cf., however, p. 61 infra.

we may obtain the X-component of **F** in the form:

$$(6)\quad \mathbf{F}_x = \frac{1}{4\pi}\left\{\frac{d}{d_x}(\mathbf{H}_x\,\mathbf{B}_x - \frac{1}{2}\,\mathbf{H}^2) + \frac{d}{d_y}(\mathbf{H}_x\,\mathbf{B}_y) + \frac{d}{d_z}(\mathbf{H}_x\,\mathbf{B}_z)\right\},$$

with similar expressions for $\mathbf{F}_y$ and $\mathbf{F}_z$.

This forcive Maxwell identified with a stress composed of:

1. A hydrostatic pressure equal to $\frac{1}{8\pi}\mathbf{H}^2$.

2. A tension along the line bisecting the angle, ϵ, between the directions of the magnetic force and the magnetic induction equal to $\frac{1}{4\pi}$ B H $\cos^2 \epsilon$.

3. A pressure along the line bisecting the exterior angle between these directions equal to $\frac{1}{4\pi}$ B H $\sin^2 \epsilon$.

4. A couple tending to turn every element of the substance in the plane of the two directions from the direction of magnetic induction to the direction of magnetic force equal to $\frac{1}{4\pi}$ B H $\sin 2\epsilon$.

Thus we may imagine every element of volume of a magnetized body to be strained under the action of the magnetizing field as though it were acted upon by Maxwell's distribution of stress. In addition to these strains there will be others arising from the intrinsic magnetic stresses before mentioned.

The Theory of Magnetostriction of von Helmholtz and Kirchoff.

Von Helmholtz first attacks the problem of investigating the stresses which arise in a dielectric placed in an electric field when it is in electrical equilibrium. Unlike Maxwell, von Helmholtz based his development upon the presupposition that the expression for the *total* potential energy per unit volume in an electrostatic field would constitute the potential function of the mechanical bodily force on unit volume of the dielectric medium.[1]

The electric field is assumed to arise from a continuous volume distribution of charge of density ρ. The work done in bringing this charge up gradually in the presence of the dielectric is given by either of the following expressions:

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[1] von Helmholtz, *Pogg. Ann.* 13 (1881), p. 385. *Wiss. Abh.*, 1, p. 798.

$$W = \int \frac{K}{8\pi} E^2 \, d\tau,$$

or:

$$W = \int \frac{1}{2} \rho \varphi \, d\tau,$$

where $E = -\nabla \varphi$ and K is the dielectric constant of the medium.

Neither of these expression satisfies the imposed condition of electrical equilibrium, i.e., $\frac{\partial W}{\partial \varphi} \neq 0$. By combining the two forms we obtain:

$$W = \int \left\{ \rho \varphi - \frac{K}{8\pi} E^2 \right\} d\tau, \tag{7}$$

whose variation with respect to φ is null.

This expression, von Helmholtz uses as the potential function of the mechanical forces acting on the medium as a whole.

By variation of W as given by equation (7) von Helmholtz obtains a forcive which he identifies with Maxwell's electric stress.[1] His method of performing this variation has, however, been criticized by Larmor[2] and Livens,[3] who show that in its correct analytical form von Helmholtz's theory involves the existence of a bodily forcive on the elements of the free ether, which could not, therefore, be in equilibrium.

The source of this discrepancy has already been pointed out. In von Helmholtz's theory no distinction is drawn between the two fundamental constituents, ether and polarizable matter, of the field, which is regarded as consisting of a single uniform medium capable of transmitting the electric and magnetic actions in the same manner as an ordinary elastic solid transmits mechanical forces. His theory would be valid if there were only one medium under consideration, of which W is the energy function. But in fact we have to deal with the ether with its stress and the polarized matter with its reacting mechanical forces, and there is no means of disentangling from a single energy function such as that of von Helmholtz the portions of the energy associated with these different effects.

The formulae of von Helmholtz, which are equivalent to Maxwell's electric stress and quite different from his magnetic stress, are now ap-

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[1] Maxwell, op. cit., I, p. 159.

[2] Larmor, "A Dynamical Theory of the Electric and Luminiferous Medium," *Phil. Trans.*, A 190 (1897), p. 280.

[3] Livens, *Phil. Mag.*, 32 (1916), p. 162.

plied by him and by Kirchoff[1] to account for the mechanical forcive in a magnetized medium. Hertz,[2] using a method similar to that of von Helmholtz, arrives at expressions identical with his. These writers include in their formulae subsidiary terms involving the change in K and μ arising from the strains in the medium. More recently Sano[3] has extended the method to include the variation of μ with the intensity of magnetization and has given formulae applicable to crystalline media.[4] A general form of the theory of magnetic stress based upon the method of energy has been developed by Cohn[5] and further elaborated by Gans[6] and Kolacek.[7] This theory is, however, subject to the same criticism as that directed by Larmor at von Helmholtz's procedure. This criticism seems to have been entirely overlooked by the majority of writers on magnetostriction, with the result that a great part of the theoretical work on the subject is fundamentally at fault.

Larmor's Application of the Energy Principle.

Larmor[8] has indicated the way in which the complete results sought by Kirchoff may be obtained by a different application of the energy principle. The forcive of von Helmholtz's and Kirchoff's theory is now regarded as that which would result if the magnetostrictive deformations arising from the action of the field on the body were prevented by a constraint.

The procedure may be illustrated by an investigation of the change in intrinsic length of a bar of magnetic material, caused by its introduction into a magnetic field. Clamp the bar to its natural length when at a great distance; then introduce it into the magnetic field so as to lie along the lines of force; then unclamp it in such a way that it may do as much work as possible in pushing away resistances to its magnetic elongation; finally remove the unclamped bar to a great distance. If this cycle is performed at uniform temperature, it follows from Carnot's principle that there can be no resultant work done in it. Now the work done by the magnetic forces in introducing the bar is:

$$\int I\,dH;$$

that is:

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[1] Kirchoff, *Wied. Ann.* 24 (1885), p. 52; 25 (1885), p. 601.

[2] Hertz, *Wied. Ann.* 41 (1890): "Electric Waves," pp. 259–268.

[3] Sano, *Phys. Rev.* 13 (1902), p. 158.

[4] *Phys. Zeit.* 3 (1902), p. 401.

[5] Cohn, "Das Electromagnetische Feld," p. 510.

[6] Gans, *Ann. d. Phys.* 13 (1904), p. 634; *Encyc. der Math. Wissensch.*, 15.

[7] Kolacek, *Ann. d. Phys.* 13 (1904), p. 1.

[8] Larmor, loc. cit., p. 283.

$$\int\left(\kappa+Q\frac{d\kappa}{dQ}+I\frac{d\kappa}{dI}\right)H\,dH$$

per unit volume, where κ is the magnetic susceptibility which is presumably a function of the internal longitudinal pressure Q in the bar and of its intensity of magnetization I. The work done in unclamping it is $\frac{1}{2}$ Q_1 l_1 per unit volume, where l_1 is the intrinsic magnetic elongation and Q_1 is the pressure corresponding to the strength H_1 of the part of the field in which it is unclamped. This is on the assumption that the bar is so long that there are no free magnetic poles near together which would diminish Q by their mutual attraction. The work done per unit volume by the magnetic forces during the removal of the bar is:

$$-\int\left(\kappa+I\frac{d\kappa}{dI}\right)H\,dH.$$

The resultant work in the cycle being null, we have:

$$\frac{d\kappa}{dQ}\int Q\,H\,dH=-\frac{1}{2}\,Q_1\,l_1=-\frac{1}{2}\,\frac{Q_1^2}{M}$$

where M is Young's elastic modulus. This can only be satisfied if Q is of the form $\lambda\,H^2$, where λ is a constant, and it then gives:

$$\frac{d\kappa}{d\,Q}=-\frac{2\lambda}{M};$$

and the elongation:

$$l=-\frac{1}{2}\frac{d\kappa}{dQ}\,H^2;$$

while the corresponding stress:

$$Q=-\frac{1}{2}\,\frac{d\kappa}{d\,l}\,H^2.$$

The effect of the variation of the elastic coefficients with magnetization may be included by a similar analysis.[1]

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[1] Larmor, loc. cit., p. 299.

This method has been utilized by Heydweiller[1] and Houstoun,[2] who obtain relationships similar to those of Larmor between the elastic and magnetic constants of a stretched wire. Their formulae are applied to experimental data secured by Rensing[3] and Nagaoka and Honda[4], but the agreement between theory and experiment is far from satisfactory.

The Investigations of J. J. Thomson

J. J. Thomson[5] has made use of Hamilton's principle to obtain expressions for the reciprocal relationships observed to exist between strain and magnetization. The first step in the deduction is to set up the Lagrangian function for a magnetized medium in terms of suitable magnetic and strain coordinates. For the former we may take the magnetic field **H** and the intensity of magnetization **I**. If the components parallel to X, Y and Z of the displacement of a volume element of the medium are α, β and γ, the resulting strains will be:

$$e = \frac{d\alpha}{dx} \qquad f = \frac{d\beta}{dy} \qquad g = \frac{d\gamma}{dz}$$

$$a = \frac{d\gamma}{dy} + \frac{d\beta}{dz} \qquad b = \frac{d\alpha}{dz} + \frac{d\gamma}{dx} \qquad c = \frac{d\beta}{dx} + \frac{d\alpha}{dy}$$

Considering the case of a cylindrical bar of homogeneous isotropic material whose axis coincides with the axis of X, the Lagrangian function per unit volume is now written:

$$(8) \quad l = \frac{1}{2} A\, I^2 + H\, I - \frac{1}{2} m\, (e^2+f^2+g^2) - \frac{1}{2} n\, (e^2+f^2+g^2-2ef-2eg-2fg),$$

where n is the coefficient of rigidity and $m-n/3$ the bulk modulus. "A" is a function defined by the equation:

$$\frac{1}{\kappa} = -\frac{d}{dI^2}(A\, I^2).$$

This gives

$$\frac{1}{2} A\, I^2 = -\int H\, dI;$$

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[1] *Ann. d. Phys.* 12 (1903), p. 602.

[2] *Phil. Mag.* 21 (1911), p. 78.

[3] *Ann. d. Phys.* 14 (1904), p. 363.

[4] *Phil. Mag.* 46 (1898), p. 260.

[5] J. J. Thomson, "Applications of Dynamics to Physics and Chemistry," p. 47.

so that the expression for the magnetic energy of equation (8) is seen to be in accordance with that used by Maxwell.

Now by Hamilton's principle:

$$\partial \int L\,dt = 0;$$

where

$$L = \int_{\tau} l\,d\tau.$$

Equating to zero the variation caused by a small displacement in the material we get for each coordinate an equation of the type:

$$\frac{dL}{dx} - \frac{d}{dx}\left(\frac{dL}{de}\right) = 0 \tag{9}$$

inside the body, and:

$$\frac{dL}{de} = 0$$

at the boundary. The first term of (9) represents the mechanical forcive on the medium which is equivalent to Maxwell's magnetic stress. We may assume that the strains arising from this term are known. If we now let e, f, and g, be the strains due to the second term alone we can derive expressions relating the intensity of magnetization with the state of strain in the medium. These appear in the form:

$$n\frac{de}{dI^2} = \left(1 - H\frac{d\kappa}{dI}\right)\left(\frac{m}{3m-n}\,\frac{1}{\kappa I}\,\frac{dI}{de} - \frac{m-n}{3m-n}\,\frac{1}{\kappa I}\,\frac{dI}{df}\right); \tag{10}$$

$$n\frac{df}{dI^2} = \left(1 - H\frac{d\kappa}{dI}\right)\left(-\frac{m-n}{3m-n}\,\frac{1}{\kappa I}\,\frac{dI}{de} + \frac{m+n}{3m-n}\,\frac{1}{\kappa I}\,\frac{dI}{df}\right).$$

The sign of the effects now under review will evidently depend, for a given specimen, upon the coefficients dI/de and dI/df. Also, as these effects are superimposed upon the strains arising from Maxwell's stress and in many cases are much larger than the latter, the resultant deformation may be either an extension or a contraction.

A study of the experimental data bearing upon magnetostriction is reserved for a later section, but it may be remarked that Thomson's equations are adequate to account qualitatively for all the observed magnetostrictive effects, including the reciprocal relationship between torsion and magnetization. So far as the writer is aware no satisfactory attempt has been made to apply them quantitatively.

This completes our review of the applications of the energy principle to problems of magnetostriction. Properly used, it is adequate to give us certain correct relations between the statistical elastic and magnetic coordinates of the molecular system. It cannot, however, furnish us with any idea of the nature of the mechanism which produces the phenomenon. A complete theory of magnetostriction must start with the magnetic forces exerted on the moving electrons within the molecule, and from these, in combination with known intermolecular forces of other nature, calculate the resulting changes in the molecular configuration. The orderly array presented by the structure of crystals probably offers the most promising field for effort in this direction.

In the following investigation, due to Larmor, it will be shown that the aggregate forcive on the moving electrons within a body arising from the presence of an impressed magnetic field may be represented by Maxwell's stress.

Maxwell's Stress on the Electron Theory.[1]

Considering first the electrons which are in orbital motion within the molecules, it is shown elsewhere in this report that the force of magnetic origin acting on an electron is:

$$\mathbf{F} = e\,(\mathbf{v} \times \mathbf{B}),$$

where v is the velocity of the electron and e its charge.

The average value of this over the orbit of a single electron is given by:

$$\overline{\mathbf{F}} = \frac{e}{T}\int_T \left(\mathbf{v} \times \mathbf{B}\right) dt$$

$$= i\int d\,\mathbf{r} \times \mathbf{B}.$$

We may replace the line integral by a surface integral over a cap bounded by the orbit, whence we get:

$$\overline{\mathbf{F}} = i\int_S [\nabla_B\,(\mathbf{B}\cdot\mathbf{n}) - (\nabla\cdot\mathbf{B})\mathbf{n}]\,dS$$

$$= i\int_S \nabla_B\,(\mathbf{B}\cdot\mathbf{n})\,dS,$$

since $\nabla\cdot\mathbf{B}=0$ always. If we sum expressions of this type for all the electrons in an element of volume, assuming that $\nabla\,(\mathbf{B}\cdot\mathbf{n})$ is constant

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[1] Cf. Larmor, *Phil. Trans.*, A 186 (1895), p. 735.

over the orbit of a single electron, we find for the force per unit volume on the electron orbits:

$$\mathbf{F} = \nabla_B (\mathbf{B} \cdot \mathbf{I}).$$

If, in addition, there is a conduction current of density $\mathbf{J}'$ traversing the volume element there will be an additional force $\mathbf{J}' \times \mathbf{B}$ giving a total forcive per unit volume on the medium of:

$$(11) \qquad \mathbf{F} = \nabla_B (\mathbf{B} \cdot \mathbf{I}) + \mathbf{J}' \times \mathbf{B}.$$

It is to be observed that this forcive is not wholly external to the volume element under consideration. A portion of it equal to $\mathbf{J}' \times 4\pi \mathbf{I}$ represents the actions of the electrons furnishing the magnetism upon the free electrons in the element. Similarly the part $\nabla_B (4\pi \mathbf{I} \cdot \mathbf{I})$ represents the local reactions between the various electron orbits. In a calculation of the mechanical force on the medium as a whole these locally compensated forces should be omitted. This procedure is generally adopted with regard to the latter of the two. The part $\mathbf{J}' \times 4\pi \mathbf{I}$ has, however, with curious inconsistency, been allowed to remain.[1]

The X-component of the force given by equation (11) may be expressed in the form:

$$(12) \quad F_x = \frac{1}{4\pi}\left\{\frac{d}{dx}\left[H_x B_x - \left(\frac{1}{2}H^2 + 8\pi^2 I^2\right)\right] + \frac{d}{dy}\left(H_x B_y\right)\right. + \frac{dz}{d}\left(H_x B_z\right) \quad + \frac{1}{4\pi} B \times \dot{E}.$$

Neglecting the ethereal displacement current[2] the stress system equivalent to this forcive is seen to be identical with that of Maxwel except for the addition of the term $2\pi \cdot I^2$ to the hydrostatic pressure constituent. This term represents the stress arising from the mutual actions of the polarized molecules. The intrinsic stress due to the interaction of the molecular and finite currents is not so easily separated out, though it is implictly contained in the above as well as in the Maxwellian representative stress system. For this reason neither of them expresses accurately the actual mechanical bodily force of extraneous origin on a volume element of a magnetized medium carrying finite electric currents.

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[1] Cf. Livens, "The Theory of Electricity," p. 589: Larmor, *Phil. Trans.*, A 190 (1897), p. 251.

[2] The retention of this term leads to second order effects only. Cf. Livens, op. cit., p. 593.

THE ANGULAR MOMENTUM OF THE ELEMENTARY MAGNET

By S. J. Barnett

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1. Ampère's assumption that the elementary magnet, or magneton, is a permanent whirl of electricity, and Weber's assumption that electricity in general, and that of Ampère's whirls in particular, has mass require together that the elementary magnet should possess angular momentum, unless it is constituted of both positive and negative electricities rotating in opposite directions. In this case a finite magnetic moment *might* be accompanied with no angular momentum. If the magneton has angular momentum, it must exhibit the dynamical properties of a gyroscope. Furthermore, if all the magnetons in a magnetized body or magnet have angular momentum in the same direction, or if the angular momentum in one direction is preponderant, the whole magnet must behave like a gyroscope. Similarly, a coil of wire traversed by an electric current consisting of a stream of one kind of electrons only, or with the linear momentum of one kind preponderant, must have angular momentum.

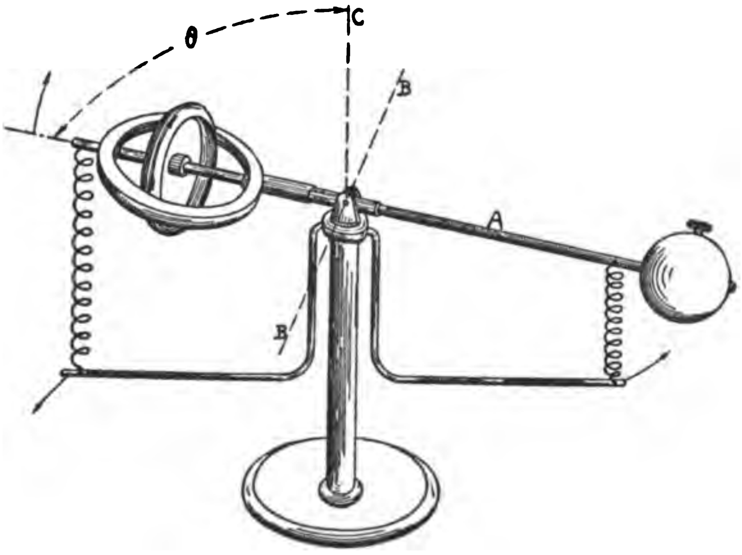

Fig. 1

2. In Fig. 1 is shown a modification of a common type of gyroscope, whose wheel, pivoted in a ring, can be rotated rapidly about its axis A. Except for the action of two springs, the ring and the axis A are free

to move in altitude about a horizontal axis B, the axis A making an angle θ with the vertical C; while the axis B, together with the wheel and the framework supporting it and the springs, can be rotated about the vertical axis C. If the wheel is spun about the axis A, and the instrument then rotated about the vertical C slowly, so that the centrifugal torque is negligible, the wheel tips up or down so as to make the direction of its rotation coincide more nearly with the direction of the impressed rotation about C. The greater the rotary speed about C the greater is the tip of the wheel; it would tip until the axes A and C became coincident if it were not for the springs (centrifugal torque being still supposed negligible).

3. If in apparatus of the kind just described we replace the wheel, or the wheel and its supporting framework complete, by a magnet, coil of wire traversed by an electric current, or electromagnet, with its axis along the axis AA and its center of mass on the axis B, the body, if it possesses angular momentum, should behave like the gyroscope of section 2. The centrifugal torque will not, however, in general be negligible, especially as great speeds about the vertical will be necessary to make the gyroscopic effects appreciable when the angular momentum due to the magnetons or electric current is small. (Mechanical disturbances due to the two springs may be made negligible). The centrifugal torque may be altered at will, or made to vanish by adjusting suitable weights attached to the body along an axis CC intersecting A and B normally. See Fig. 2.

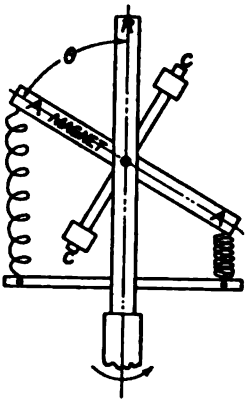

FIG. 2

4. The first attempt to detect by direct experiment the angular momentum of electricity flowing in a coil of wire, or the angular momentum associated with the Ampèreian currents in magnetic matter, was made by Maxwell[1] in 1861, with apparatus somewhat similar to that of Fig. 2, but with the springs removed. Maxwell did not use a permanent magnet, but a symmetrical coil of wire traversed by an electric current and provided at will with a core of iron.

Let A, B, C denote the moments of inertia of the magnet (or coil) about its own axis, the horizontal axis B, and the central axis normal to the two, respectively.

Let θ denote the angle between the axis A of the magnet and the vertical C, Ω the impressed angular velocity about the vertical, J the

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[1] Maxwell's Electricity and Magnetism, § 575.

total angular momentum under investigation, and β the angle between J and the axis A. See Fig. 3.

Let us suppose that under the action of the springs, producing a torque T in the direction of increase of θ, the angular velocity Ω and the angle θ are maintained constant. J can be resolved into two rectangular components: one parallel to the axis of the impressed rotation, J cos $(\theta-\beta)$, which is constant; and one perpendicular to this axis, J sin $(\theta-\beta)$, which has the constant rate of change Ω J sin $(\theta-\beta)$. By the second law of motion this is equal to the torque T; thus:

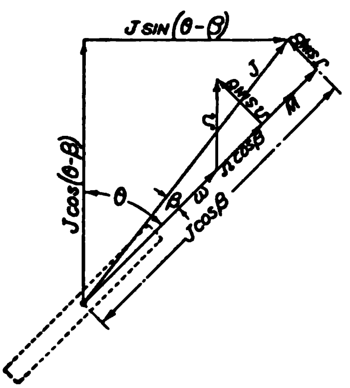

FIG. 3

(1) $$T=\Omega J \sin(\theta-\beta).$$

Now

(2) $$J\cos\beta=A\,\Omega\cos\theta+M,$$

and

(3) $$J\sin\beta=C\,\Omega\sin\theta.$$

Hence:

(4) $$T=(A-C)\,\Omega^2\sin\theta\cos\theta+M\,\Omega\sin\theta.$$

If C is somewhat greater than A the applied torque T necessary to maintain the motion constant will vanish and equilibrium (under the action of the centrifugal torque) will be stable with the springs removed when

(5) $$\cos\theta=\frac{M}{(C-A)\,\Omega}.$$

By means of two nuts acting on screws the axis CC was adjusted to be a principal axis with C just exceeding A, so as to make the instrument very sensitive. On account of disturbances due to the earth's magnetic field the results were very rough, but no change in θ with reversal of M or Ω could be detected even when an iron core was inserted in the coil.

Maxwell concludes that if a magnet contains matter in motion the angular momentum of the rotation must be very small in comparison with any quantity which we can measure.

By calculating M as the product of the constant R, determined below, and the magnetic moment of the magnet, and by taking account of the fact that equation (5) holds only if the horizontal axis about which rotation is possible passes exactly through the center of gravity of the magnet, W. J. de Haas and G. L. de Haas-Lorentz[1] have shown that a change in θ would hardly be perceptible even in very favorable conditions.

By making $A-C$ very small and θ very nearly 90°, and measuring T and Ω, M could also be determined from equation (4).

5. Let M now denote the angular momentum of a magneton, μ its magnetic moment, and

$$R=\frac{M}{\mu} \tag{6}$$

the ratio of the first to the second.

Let us assume that in the experiments of section 4 we have a permanent magnet symmetrically magnetized about the geometric axis, and that the magnetons are all alike. If θ denotes the angle between the axis of a magneton and the intensity of magnetization I in its neighorhood, we have

$$I=\Sigma\,\mu\cos\theta, \tag{7}$$

the summation extending over the unit of volume. The internal angular momentum j per unit volume will then be:

$$j=\Sigma\,M\cos\theta=R\,\Sigma\,\mu\cos\theta=R\,I. \tag{8}$$

Thus if I denotes the mean intensity of magnetization along the axis of the magnet, V its volume, and I V its magnetic moment, its total internal angular momentum will be R I V. Thus R can be determined from (4) and (5).

6. Maxwell's experiment was an experiment on the angular momentum of a gross magnet, and it does not seem to have occurred to him to make an experiment in which every one of the countless multitude of magnetons in a magnetic body should simultaneously replace his magnet, and to measure the total change in the orientation of all the magnetons by a magnetic method.

The first experiment based upon this idea appears to be one made nearly a third of a century ago by John Perry[2], who tried, but without

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[1] W. J. de Hass and G. L. de Hass-Lorentz, *K. Akad. Amsterdam*. Proc. 19, 1916, p. 248.

[2] John Perry, Spinning Tops, October, 1890, footnote, p. 112.

success, to detect a change in the magnetization of an iron rod produced by rotating it about its axis. In 1912 and 1915, respectively, Schuster[1] and Einstein and de Haas[2] had the same idea. In the meantime, in 1909, the same idea occurred to the author[3], who, with the help of L. J. H. Barnett, then began experiments which were first successful in 1914. These proved to be the first successful experiments in this whole field.[4]

The qualitative theory of the phenomenon with which these experiments are concerned is as follows. When the body of which it is a part is set into rotation about any axis, the magneton, if it has angular momentum, will behave much like the wheel of section 2, and will thus change its orientation so as to make its direction of rotation coincide more nearly with the direction of impressed rotation; the coincidence will finally become exact if not prevented by the action of the rest of the body.

In an ordinary ferromagnetic body in the usual state with which we are familiar only a slight change of orientation can occur on account of the forces due to adjacent molecules, which perform the function of the springs in the case of our gyroscope. The rotation causes each molecule to contribute a minute angular momentum, and thus also a minute magnetic moment, parallel to the axis of impressed rotation; and thus the body, whose magnetons originally pointed in all directions equally, becomes magnetized along the axis of impressed rotation.

If the revolving electrons, or rotating magnetons, are all positive, the body will thus become magnetized in the direction in which it would be magnetized by an electric current flowing around it in the direction of the angular velocity imparted to it. If they are all negative, or if the action on the negative magnetons is preponderant, it will be magnetized in the opposite direction. This is what actually happens.

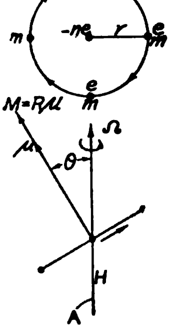

FIG. 4

7. To develop a quantitative theory[5], assume the magneton to consist of a symmetrical electrical system rotating with angular velocity ω, magnetic moment μ, and angular momentum $M = R\ \mu$ about the axis of symmetry, the electrical charge in rotation being all of one sign. Fig. 4 illustrates a special case.

The vectors representing M and μ are in the same direction or opposite directions according as the moving charge is positive or negative.

Let A denote the moment of inertia of the magneton about its axis of rotation, and suppose $B = C$ the

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[1] A. Schuster, *Phys. Soc. Lond. Proc.* 24, 1911–12, p. 121.
[2] A. Einstein and W. J. de Haas, *Verh. d. D. Phys. Ges.* 17, 1915, p. 152.
[3] S. J. Barnett, *Science*, 30, 1909, p. 413
[4] S. J. Barnett, *Phys. Rev.* 6, 1915, p. 239.
[5] See S. J. Barnett, l. c. ante and Einstein and de Haas, l. c. ante.

(mean) moment of inertia about any central axis normal to the axis of symmetry.

If now the body of which this magneton is a part is set into rotation with angular velocity Ω about an axis C, the magneton, behaving like the wheel of a gyroscope, will strive, as it were, to take up a position with its axis of revolution coincident with that of the impressed rotation; but it will be prevented from turning so far by a torque T due to the action of the rest of the body and brought into existence by the displacement. In a minute time kinetic equilibrium will be reached, and the axis of the magneton will then continuously trace out a cone making a constant angle θ with a line through its center parallel to the axis C of the impressed rotation. When this state has been reached T will be given by equation (4) above, which may be written:

$$\text{(9)} \qquad T = \left\{ M\,\Omega + (A - C)\,\Omega^2 \cos\theta \right\} \sin\theta.$$

Now imagine the body, instead of being rotated, to be placed in a uniform magnetic field whose intensity H is directed along the previour axis of rotation, and consider a magneton whose magnetic axis, aftes displacement by the field, makes the angle θ with H. The magneton would keep on turning under the action of the field until its axis coincided with H, but is prevented from doing so by the torque T′ upon it due to the action of the rest of the body and brought into existence by the displacement. This torque is well known to be:

$$\text{(10)} \qquad T' = \mu\,H \sin\theta.$$

To find, therefore, the magnetic intensity which would produce the same effect on the orientation of the magneton as would be produced by rotating the body at the angular velocity Ω, all we have to do is to equate T and T′. This gives:

$$\text{(11)} \qquad \mu\,H \sin\theta = \left\{ M\,\Omega + (A - C)\,\Omega^2 \cos\theta \right\} \sin\theta;$$

or:

$$\text{(12)} \qquad H = \frac{M\,\Omega}{\mu}\left(1 + \frac{(A - C)\,\Omega}{A\,\omega} \cos\theta\right).$$

The values of Ω experimentally attainable are so small in comparison with any possible values of Ω in the case of any magneton of probable type that the last term is negligible. Hence we have for any magneton in the body, whatever its orientation, with all sufficient exactness:

$$H = \frac{M\,\Omega}{\mu} = R\,\Omega = 2\,\pi\,R\,N, \tag{13}$$

where N is the impressed angular velocity in revolutions per second.

From what precedes it follows that *if all the magnetons in a body are alike*, rotating it at an angular velocity of N r.p.s. will produce the same intensity of magnetization in it as placing it in a field of strength $2\,\pi\,R\,N$ Gausses.

If the magnetons in a body are of two kinds, positive and negative, with constants R_1 and R_2, rotating the body will have the same effect as if a magnetic intensity $H_1 = R_1\,\Omega$ were applied to the positive magnetons and an intensity $H_2 = R_2\,\Omega$ were applied to the negative magnetons. If the effect on the negative magnetons is preponderant, the rotation will thus produce an intensity of magnetization in the direction of H_2 but of magnitude less than that which would be produced by the intensity $R_2\,\Omega$ if all the magnetons were negative.

8. The ratio R will now be determined for three types of magnetons. (1) Suppose the magneton (Fig. 4), to consist of n similar electrons, all positive or all negative, with total charge n e revolving in a circular orbit of radius r, with constant angular velocity ω (and areal velocity $\alpha = 1/2\,\omega\,r^2$) about a much more massive, and fixed, nucleus with charge $-n\,e$. If the radius of the orbit is great in comparison with that of an electron, no appreciable error in calculating the angular momentum will be made if we assume the (electromagnetic) mass and the charge of each electron both to be concentrated. In this case, if m denotes the mass of one electron, we have

$$\mu = n\,e\,\alpha; \quad A = n\,m\,r^2, \tag{14}$$

$$M = A\,\omega = n\,m\,r^2\,\omega = 2\,n\,m\,\alpha, \tag{15}$$

and

$$R = \frac{M}{\mu} = 2\,\frac{m}{e}. \tag{16}$$

If the distance between successive electrons in the orbit is great in comparison with the diameter of an electron, m will be practically identical with the mass of the free electron. Otherwise it will be greater. If the diameter of the orbit is reduced, the electromagnetic field being restricted more nearly to the space between each electron and the nucleus, the angular momentum will become smaller, as suggested to me by Dr. Tolman.

(2) Voigt[1] has examined the behavior in a magnetic field of magnetic elements, or magnetons, consisting of homogeneous uniformly charged solids in rotation. No account is taken of the electromagnetic origin of the mass, but the mass density is taken as everywhere proportional to the electric density. For this type of magneton it is easy to show that, as in the case of an electron ring,

$$R = 2\frac{m}{e}. \tag{17}$$

(3) Abraham[2] has considered the behavior in a magnetic field of a spherical electron in rotation and uniformly charged either over the surface or throughout the volume, and has calculated the angular momentum on the assumption that the mass and momentum are purely electromagnetic.

The electron, or magneton, masses, m_s and m_v for surface and volume charge, respectively, are

$$m_s = \frac{2}{3}\frac{e^2}{a} \text{ and } m_v = \frac{4}{5}\frac{e^2}{a}, \tag{18}$$

if e denotes the charge of the electron and a its radius.

For the angular velocity ω radians per second the corresponding angular momenta are

$$M_s = \frac{1}{3} m_s a^2 \omega \text{ and } M_v = \frac{1}{7} m_v a^2 \omega; \tag{19}$$

and the corresponding magnetic moments are

$$\mu_s = \frac{1}{3} e a^2 \omega \text{ and } \mu_v = \frac{1}{5} e a^2 \omega. \tag{20}$$

Thus the corresponding values of the ratio $R = \frac{M}{\mu}$ are

$$R_s = \frac{m_s}{e} \text{ and } R_v = \frac{5}{7}\frac{m_v}{e}. \tag{21}$$

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[1] W. Voigt, *Ann. der Phys.* 9, 1902, p. 130.

[2] M. Abraham, *Ann. der Phys.* 10, 1903, pp. 151, 169, 171.

The first of these is just one-half the value for an electron orbit, and the second somewhat smaller.

9. In very weak fields ferromagnetic bodies all receive magnetic moments proportional to the intensities of the fields applied. Similarly, since $R\,\Omega$ is equivalent to a very small value of H for even the greatest speeds practicable, these bodies must be magnetized by rotation proportionally to the speed.

If, however, in either case we start with a ferromagnetic substance not in or near the neutral state but at the steep portion of the magnetization curve, the application of either a small increment of magnetic intensity or of a small speed may be sufficient to produce a considerable and irreversible change in magnetization. Proportionality exists only for elastic displacements.

10. An interesting case of the theory of magnetization by rotation is its application to a single magneton or a swarm of magnetons, whose behavior in an ordinary magnetic field has been considered by Voigt[1] and Abraham.[2]

Consider first the case of Voigt's magneton, a solid of revolution with electric and mass densities proportional, placed in a uniform magnetic field with intensity H, and permanently endowed with an angular velocity ω about its unique axis greater than H/R ($=H\div 2m/e$, on Voigt's assumption).

In this case, in accordance with classical electromagnetic theory, the rotation proceeds undamped about the unique axis, while it is damped about the other (equal) axes, and the action of the field on the magneton is as follows: When the field is applied, precession of the magneton's axis about the direction of the field, accompanied by nutation, begins, with angular velocity H/R. The nutation is damped out by dissipation or radiation, and the precession is retarded for the same reason. Hence the direction of the axis of the magneton gradually approaches coincidence with the direction of the field, when it is in stable equilibrium. During this process the velocity of rotation diminishes slightly, the motion being affected as in the case of the electricity in Weber's molecular grooves.

If there are N such magnetons in the unit of volume, and if the demagnetizing and molecular fields and the upsetting effect of collisions are negligible, all the magnetons will ultimately become oriented with their axes in the direction of the magnetic field. In this case the moment of unit volume will be

$$I=\frac{N\,A}{R}\left(\omega-\frac{H}{R}\right); \tag{22}$$

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[1] W. Voigt, l. c. ante.

[2] M. Abraham, l. c. ante.

or

$$I = \frac{N A e}{2 m}\left(\omega - \frac{e H}{2 m}\right), \tag{23}$$

where A is the moment of inertia of the magneton about the unique axis of permanent rotation.

The first and principal term is entirely independent of H. The change of orientation is produced by the magnetic field, but only the time taken to arrive at the steady state is affected by its magnitude. The second term is a diamagnetic term, and arises from the fact that owing to the change of extraneous flux through the magneton during the process of its orientation its velocity is decreased, just as in the case of the Weber-Langevin theory.

In this case we have, except for the small diamagnetic term, which vanishes with the intensity, saturation for even the weakest fields; and we have less nearly complete saturation for stronger fields.

When collisions are not absent, a magneton's axis will be repeatedly deflected in its approach toward coincidence with the direction of the field, and the intensity of magnetization will not reach saturation; but it will increase with the field strength, being greater for a given field strength the greater the mean time between collisions and the weaker the molecular and demagnetizing fields. Increase of temperature, shortening this time between collisions, and increasing their violence, will, if the magnetons remain unchanged, thus diminish the magnetization for a given field strength.

The precessional process described above is doubtless similar in a general way to the process by which in every case in paramagnetic and ferromagnetic substances the magnetons are aligned more or less completely with the magnetic field.

The behavior of Abraham's spherical magneton is very similar to that of Voigt's magneton. Although the moments of inertia about three rectangular axes are identical, motion about any axis normal to the unique axis of permanent rotation changes the magnetic field and thus radiates or dissipates energy, while rotation about the unique axis leaves the field fixed and unaltered.

The angular velocity of the precession of the magneton's axis about the direction of the field is given for both magnetons by $\Omega = H/R$, which becomes in the case of Abraham's magneton

$$\Omega_s = H \div \frac{m_s}{e} \tag{24}$$

for surface charge, and

$$\Omega_v = H \div \frac{5}{7}\frac{m_v}{e} \tag{25}$$

for volume charge. Abraham has referred to this difference as a possible mode of descriminating between the validities of the two hypotheses as to the distribution of the charge.

If the magnetons are subjected to an angular velocity Ω instead of to a magnetic field with intensity H, we have, when the effects of collisions and the molecular and demagnetizing fields are negligible,

$$I = N\, A/R \cdot (\omega + \Omega). \tag{26}$$

The first and only important term is independent of Ω. Here the orientation is produced by the velocity impressed, but only the time taken to reach the steady state is affected by its magnitude.

The second term, here added to the first, corresponds to Voigt's diamagnetic term. We have, except for the small second term, saturation for even very small values of Ω.

If collisions are not absent, or if the magnetic fields of adjacent molecules and the demagnetizing field become appreciable, the intensity of magnetization will not reach saturation; but it will increase with Ω, being greater for a given value of Ω the greater the mean interval between collisions, the less their violence, and the weaker the field.[1]

11. In studying magnetization by rotation experimentally and thereby determining the ratio R in both sign and magnitude two general types of methods may be used, viz., methods of electromagnetic induction and magnetometer methods.

In all methods the substance under investigation is in the form of an approximately cylindrical rod, which is mounted with its axis horizontal and in the magnetic prime vertical in a region in which the earth's magnetic field has been neutralized.

In the methods of electromagnetic induction the intrinsic magnetic intensity of rotation is determined by comparing the change of flux through the rod under investigation, produced by rotation about its axis at measured speed, with the change of flux produced through the same rod by the application, parallel to the axis of rotation, of a uniform magnetic field of known intensity. The changes of flux are proportional to the intensities, if small.

In the only method of this kind which has hitherto been used the changes of flux are measured ballistically, with a galvanometer of the type which has come to be known as a fluxmeter, a coil of wire surrounding the rod being in the fluxmeter circuit.

In another method,[2] which has not yet been used, the rod may be

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[1] See S. J. Barnett, *Nature* vol. 107, p. 8, 1921.

[2] Suggested by an investigation of Tolman's in a different field.

oscillated about its axis instead of being set into continuous unidirectional rotation; and the alternating electromotive force thus developed in the surrounding coil of wire may be compared with the electromotive force produced by an alternating field of known intensity, an amplifier being used to increase the sensibility if necessary.

In the magnetometer method, an astatic magnetometer is mounted with the center of its lower magnet system in the polar axis, or, preferably, in the equatorial plane, of the rod, or rotor, under investigation. The magnetometer deflection produced by reversing the direction of rotation of the rotor, driven at a measured speed, is compared with the deflection produced by reversing a known magnetic field in the rotor parallel to its axis. The deflections are proportional to the changes in the magnetic moments in the two cases, and these are proportional to the intrinsic intensity of rotation and the standardizing field intensity.

12. In section 5 it has been shown that if but one type of magneton is present the angular momentum of the magnetons per unit volume in a magnetized substance is

$$j = R\,I; \qquad \text{(8) bis}$$

an equation first developed by O. W. Richardson[1] for the special case of electron-orbit magnetons, for which $R = 2\frac{m}{e}$.

If the frame holding the gyroscope of Fig. 1 is forcibly turned about the axis B, thus altering the component of the wheel's angular momentum about the vertical, there will be a reaction in the frame producing an equal change of momentum in the opposite direction.

In the same way the change in the angular momentum of the magnetons produced by a change in the magnetization will be accompanied by an equal and opposite change of momentum. Probably the reaction measured by the rate of increase of this momentum will be upon the remaining part of the molecules of which the magnetons are constituents; that is, the total reaction will be exerted upon the rod. But this is not certain; a part of the reaction may be upon the magnetizing coil. The probability of this, however, is small; for the rotation of a symmetrical coil about its axis in no way affects its magnetic field.[2] If the rod is magnetized from a neutral state the total angular momentum acquired by rod and coil together will be:

$$-jV = -R\,IV$$

where V is the volume of the rod.

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[1] O. W. Richardson, *Phys. Rev.* 26, 1908, p. 248. For discussions of the case in which more than one kind of magneton is present. Richardson's paper and also that by J. Q. Stewart (l. c. infra) may be consulted.

[2] A second order effect on the field, which in general occurs, is here neglected.

If the magnetons are *all positive*, the vector representing the angular momentum of the reaction will be opposite to that representing the intensity of magnetization; if they are all negative, the two vectors will have the same direction.

If two types of magneton are present, we shall have:

$$j = j_1 + j_2 = R_1 \Sigma \mu_1 \cos \theta_1 + R_2 \Sigma \mu_2 \cos \theta_2 = R_1 I_1 + R_2 I_2. \tag{27}$$

13. Two general methods of investigating this effect experimentally have been tried. In the ballistic method, proposed and tried by O. W. Richardson[1], to whom the idea of rotation by magnetization is due, a cylindrical rod of the substance under investigation is suspended with its axis vertical by an elastic fibre, and the total angular momentum imparted to the rod is determined from the throw occurring when a known change in the rod's magnetization is produced by altering a vertical magnetic field.

In the method of resonance, first applied to this problem by Einstein and de Haas,[2] a system similar to that just mentioned, which has a definite frequency n_0 of vibration about its vertical axis, is magnetized by an alternating field whose frequency n is variable over a range including the frequency of the vibrating system. The angle α through which the cylinder vibrates is determined as a function of the frequency of the impressed field. From the maximum value α_m of the angle at resonance, the maximum magnetic moment A of the rod, which is kept saturated during nearly the whole of each half cycle, or from α_m, α, A, $n - n_0$, and the moment of inertia of the rod about its axis, the magnitude of R can be determined. And from the phase relation between the deflection of the rod and the magnetizing current, the sign of R can be determined.

A modification of this method, in which the magnetizing solenoid is wound directly on the rod itself and in which the reversals of the current are produced by the motion of the electromagnet thus formed, has been used by de Haas.[3] It has the advantage of doing away with two of the experimental difficulties, though it introduces others; it has also the great theoretical advantage that the total torque upon both rod and solenoid is measured. Only a few rough experiments have hitherto been made by this method.

Another modification has been used for a lecture experiment by Einstein,[4] but only rough experiments have been made. In this method

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[1] O. W. Richardson, l. c. ante.

[2] A. Einstein and W. J. de Haas, l. c. ante.

[3] W. J. de Haas, *K. Akad. Amsterdam Proc.* 18, 1916, p 1280; *verh. d. D., Phys. Ges.* 18, 1916, p. 423.

[4] A. Einstein, *Verh. d. D. Phys. Ges* 18, 1916, p. 173.

an ingenious device is used for applying the magnetizing field for only a small fraction of the period, and thus greatly reducing certain extraneous disturbances.

14. As stated in section 6, the first successful experiments in the field to which this article is devoted were made by L. J. H. Barnett and myself in 1914,[1] in November and December of which year they were described before the Ohio Academy of Sciences and the American Physical Society. These were experiments on the magnetization of cold rolled steel by rotation and were made by the fluxmeter method of electromagnetic induction. They confirmed equation (14) as to proportionality of the intrinsic intensity of rotation with speed, showed that the role of negative electricity in magnetism was at least preponderant, and gave for $2\pi R$, on the assumption of only one type of magneton, the value -3.6×10^{-7}, or, for R, about one-half of $\frac{2m}{e}$ for the negative electron in slow motion, viz., $-1.13\times10^{-7}=-7.1\times10^{-7}/2\pi$.

In the early part of 1915 we completed another and more thorough investigation of steel by the same method, with considerable improvements. This gave for $2\pi R$ the value $-3.1_5\times10^{-7}$.

In 1917[2] we completed an investigation of steel, cobalt, and nickel by a magnetometer method, and obtained values of R which were, as before, all negative, and whose means were intermediate between the values previously obtained for steel and twice those values.

We have since[3] made by improved magnetometer methods very extensive and conclusive experiments on steel, soft iron, cobalt, nickel, and Heusler alloy, and have obtained for all these substances values of R similar to those of 1914 and 1915 for steel, viz., the value of $\frac{m}{e}$ for the negative electron in slow motion, within the limits of the experimental error. In the course of this work we have discovered a systematic error in the work of 1917, which made the values of R too high.

These experiments must be taken to prove in a direct and conclusive way, on the basis of classical dynamics alone, (1) that Ampèreian currents, or molecular or electronic currents of electricity in orbital revolution or in rotation, exist in the ferromagnetic substances; (2) that the role of negative electricity in ferromagnetism is at least preponderant; (3) that the magnetons have inertia and angular momentum, so that

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[1] S. J. Barnett, l. c. ante.

[2] S. J. Barnett, *Phys. Rev.* 10, 1917, p. 7.

[3] For brief descriptions of the earlier part of the more recent work see S. J. Barnett, *Proc. Phil. Soc. Washington* for Oct. 9, 1920 (*Jour. Wash. Acad. Sci.* 11, 1921, p. 162); and S. J. Barnett and L. J. H. Barnett, *Proc. American Phys. Soc.* for Dec. 1920 (*Phys Rev.* 17. 1921, p. 404). For the latest work see *Proc. American Phys. Soc.* for April, 1922.

each behaves like a minute gyroscope. Furthermore, if we admit the classical theory of radiation, these experiments, together with the exitence of permanent magnetism, prove (4) that the arrangement of the electricity in the magnetons is continuous or Saturnian, rather than planetary. Finally (5) the value obtained for R shows either that negative magnetons, such as that of Abraham, with a value of R much less than that for an orbital ring are responsible for magnetism, or else that positive electrons or magnetons, whose rotation produces an opposite effect, are also involved. It is suggestive that the value of R for the superficially charged electron of Abraham and Lorentz (section 8, (3)), when rotating as a magneton, is equal to those found by experiment on the assumption that all the magnetons in a substance are alike. The investigation has established, within the limits of the experimental error, a new and apparently universal constant in magnetism.

15. The first experiments to yield any results on rotation by magnetization, the effect converse to that discovered and described in 1914, were made early in 1915 by A. Einstein and W. J. de Haas,[1] by the first method of resonance described in section 13. On the assumption of a single type of magneton, and the assumption that all the reaction to the magneton momentum goes to the rod, they obtained from a few experiments on soft iron the magnitude $2\frac{m}{e}$, or $7.1\times10^{-7}/2\pi$, for R; but, as was soon pointed out by Lorentz,[2] the sign of the effect was not determined with any certainty.

In September, 1915, de Haas,[3] by another resonance method referred to in section 13, obtained from a few experiments about the same magnitude of R, viz., $2\frac{m}{e}$, and determined the sign of the effect, making R negative, as Einstein and de Haas had originally claimed. These are the only experiments on rotation by magnetization yet made in which the magnetizing coil was wound on the rod.

In 1916 Einstein's resonance method yielded about the same magnitude of R.[4]

The first thorough experimental investigation in the field of rotation by magnetization was completed in 1917 by J. Q. Stewart[5], who used an ingenious modification of the ballistic method first attempted by Richardson, and studied the angular momentum produced in small rods by annulling their residual magnetism. Stewart investigated

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[1] A. Einstein and W. J. de Haas, l. c. ante.

[2] See A. Einstein, *Verh. d. D. Phys. Ges.* 17, 1915, p. 203; also W. J. de Haas, l. c. ante.

[3] W. J. de Haas, l.c. ante, *K. Akad. Amsterdam*, 18, 1916.

[4] A. Einstein, l. c. ante, *Verh. d. D. Phys. Ges.* 18, 1916, p. 173.

[5] J. Q. Stewart, *Phys. Rev.* 11, 1918, p. 100.

many small rods of iron and nickel, and, on the assumptions already referred to, found R negative and equal to about one-half of $2\frac{m}{e}$ for both iron and nickel.

In 1919 Emil Beck[1] described a thorough investigation in which he repeated with great improvements the work of Einstein and de Haas and extended it to nickel. For R he found 53 per cent of $2\frac{m}{e}$ in the case of iron, and 57 per cent in the case of nickel.

In 1919 G. Arvidsson[2] made a similar investigation on iron, and for two specimens obtained a value of R about equal to one-half of $2\frac{m}{e}$.

If we admit the validity of the assumption that the reaction to the angular impulse given to the magnetons on magnetizing a rod is applied to the rod itself, the results of Stewart, Beck and Arvidsson confirm in a striking way those obtained by the method of magnetization by rotation, into the theory of which no assumptions appear to enter except such as can be justified completely. Otherwise they must be taken simply as a proof that the assumed relation between the momenta is at least approximately true, and not as affording evidence with respect to R, independent of the experiments on magnetization by rotation.

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[1] Emil Beck, *Ann. der. Phys.* 60, 1919, p. 109.

[2] G. Arvidsson, *Phys. Zeit.*, 21, 1920, p. 88.

MAGNETO-OPTICS

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Introduction. The connection between magneto-optic theory,[1] as it stands today, and current theories of magnetism is not as intimate as might be expected. The explanation of this lies, perhaps, in the fact that—speaking in general terms—the most acceptable magneto-optic theory concerns itself chiefly with the changes produced by the magnetic field in the *period* of vibrating electrons (which are either emitting or absorbing light), the matter of *orientation* of the orbits being of secondary importance, while in theories of magnetism—at least ferro-magnetism—the reverse is the case. In spite of this difference in point of view, however, magneto-optic phenomena furnish a very interesting sidelight on the general subject of magnetism and one whose importance cannot be overlooked.

The discovery by Faraday[2] in 1845 of the rotation of the plane of polarization of light produced by transmission through a block of heavy glass in a magnetic field was the first connecting link found between the fields of magnetism and light. This was followed some thirty years later by Kerr's discovery[3] of the phenomena accompanying reflection at the polished poleface of a magnet. A still more striking phenomenon, however, was that of the magnetic resolution of spectral lines brought to light by Zeeman[4] in 1896.

These three classes, known respectively, as the Faraday, Kerr and Zeeman effects, constitute, together with certain magnetic double-refraction effects, the phenomena of magneto-optics. While more or less closely connected in theory, we shall, for purposes of clearness, consider them separately.

THE FARADAY EFFECT

Chief Characteristics. Magnetic rotation is a property common to (probably) all isotropic media. While analogous to natural rotation

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[1] General references. W. Voigt. Art. "Magnetooptik," pp. 393–710 in Graetz, "Handb. d. Elekt. u. Magn." Bd. IV. Leipzig 1915. The most comprehensive work on the subject. Very extensive list of references up to 1913. Largely supercedes the author's earlier "Magneto-u. Elektrooptik" (Leipzig, 1908).

H. A. Lorentz, "Theorie der Magneto-optischen Phänomena," Encyclopadie der Math. Wiss. 5, 3, Heft 2 (Leipzig, 1909).

P. Zeeman, "Researches in Magneto-optics" London, 1913. Contains complete bibliography of Zeeman Effect up to 1913. Also (p. XI) an excellent list of general references on magneto-optics.

[2] M. Faraday, "Exper. Researches 19, No. 2146 ff.

[3] J. Kerr, *Phil. Mag.* (5) 3, 321, 1871.

[4] P. Zeeman, *Phil. Mag.* (5) 43, 236, 1897.

it differs from it in that the absolute direction in which the plane is turned is independent of whether the light is passing from the north pole to the south, or vice versa; hence reflecting the light back on itself produces a double effect instead of annulling the rotation, as would be the case for natural activity. The rotation may also be doubled by reversal of the magnetic field—a fact which is generally taken advantage of in its measurement. The majority of substances produce a positive rotation, i.e., in the direction of the current producing the field, and this rotation is proportional (Verdet's Constant) to the field strength, save in the case of strongly magnetic substances where it varies (Kundt's Constant) as the intensity of magnetization. Any time lag of the effect, if it exists at all, is exceedingly minute.[1] Magnetic rotation measurements are made with polarimetric apparatus of various kinds dependent on the spectral region to be investigated. In the visible a polarimeter with tri-field Lippich[2] system is probably the best. In the ultra-violet, photographic methods such as used by Macaluso and Corbino[3] or by Landau[4] are in general required, although some work has also been done with fluorescent oculars. In the infra-red the methods are radiometric, with selenium mirrors for polarizers as used by Meyer,[5] or a system involving double-image prisms as developed by Ingersoll.[6]

Magnetic Rotatory Dispersion. With the exception of iron, nickel and cobalt, in which the rotation shows an increase for longer wavelengths, the effect in most substances is roughly proportional to the inverse square of the wavelength, that is for a spectral region which does not contain an absorption band in which absorption and rotation are dependent on the same set of electrons. Near such bands we must distinguish two separate cases, in the first of which the rotation increases (and has the same sign) on each side of the absorption band, while in the other it shows a decrease on one side (short wavelength) and an increase on the other, of the usual "anomalous dispersion" type. Sodium vapor, investigated by Macaluso and Corbino[3] and especially by Wood,[7] is a good example of the first case, while the results of Elias[8] on solutions of a salt of praseodymium may probably be taken as conclusive evidence for the existence of the second case.

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[1] O. Lodge, *Chem. News.* 59, 191; Eng. 47, 250, 1889.

[2] F. Lippich. *Zeit. f. Instrum.* 14, 326, 1894.

[3] D. Macaluso and O. M. Corbino, *C. R.* 127, 548, 1898.

[4] St. Landau, *Phys. Zeit.* 9, 417, 1908.

[5] U. Meyer, *Ann. d. Phys.* 30, 607, 1909.

[6] L. R. Ingersoll, *Phys. Rev.* 23, 489, 1906; *Phil. Mag.* (6) 11, 41, 1906; 18, 74, 1909.

[7] R. W. Wood, *Phil. Mag.* (6) 10, 408, 1905; 12, 329, 499, 1906; 14, 145, 1907; R. W. Wood and H. W. Springsteen, *Phys. Rev.* 21, 41, 1905; R. W. Wood and F. E. Hackett, *Astrophys. Jour.* 10, 339, 1909.

[8] G. J. Elias, *Ann. d. Phys.* 35, 298, 1911.

Of the many formulas which have been suggested for expressing magnetic rotatory dispersion, perhaps the most satisfactory is of the type:

$$R=\frac{1}{\mu}\left\{\frac{A}{\lambda^2}+\frac{B\lambda^2}{(\lambda^2-\lambda_1^2)^2}+\ldots\right\}$$

for an absorption band at wavelength λ_1. This, or formulas of a similar type have been found to represent the rotatory dispersion of carbon bisulphide and of certain optically isotropic crystals over considerable spectral ranges. It will be noted that near an absorption band this formula gives a dispersion of the first type mentioned above.

Wiedemann's Law. The proportionality of magnetic and natural rotation for different wavelengths was enunciated as a law by G. Wiedemann,[1] as a result of experiments on turpentine. It is a law to which, however, there are very many exceptions.

Theories of Magnetic Rotation. That any theory of magnetic rotation may start from the Fresnel point of view, i.e., the resolution of a plane vibration into two circular ones which travel with different velocities in the medium, has been amply established by the experiments of Righi,[2] Becquerel[3] and Brace.[4]

This does not, however, lead very far towards the ultimate explanation of a phenomenon which is essentially electronic in character. Drude[5] has worked out at some length two theories, founded respectively on the hypotheses of molecular currents and of the Hall effect. In the first, the rotating electrons whose motion is either induced or modified by the magnetic field are supposed to be acted on by the light waves so that their centers of rotation are set in vibration. Treating them then, more or less as rigid bodies, he arrives at an expression for magnetic rotation. While useful in explaining magnetism and diamagnetism and therefore a natural hypothesis on which to found a theory of magnetic rotation, this assumption of molecular currents leads to equations which call for rotations of opposite sign on the two sides of an absorption band and this has scant experimental verification. Voigt[6] criticises the standpoint of this theory as well as that of the somewhat similar Langevin molecule in that they involve some disagreements in the matter of the Zeeman effect.

In the second hypothesis the electrons set in motion by the light waves are acted on by the magnetic field with forces proportional to

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[1] G. Wiedemann, *Pogg. Ann.* 82, 215, 1851.

[2] A. Righi, *N. Cim.* (3) 3, 212, 1878.

[3] H. Becquerel, *C. R.* 88, 334, 1879.

[4] D. B. Brace, *Wied. Ann.* 26, 576, 1885; *Phil. Mag.* (6) 1, 464, 1901.

[5] "Optik," p. 407 ff. Leipsig, 1912.

[6] *Loc. cit.* p. 661.

their velocities. This is shown to result in an expression for magnetic rotation of the type given above, which gives a rotation of the same sign on each side of an absorption band as is the case for sodium vapor.

With certain simplifying assumptions this expression can be shown to be in agreement with one involving the refractive index μ, of the type:

$$R = K \frac{e}{m} \lambda \frac{d\mu}{d\lambda},$$

which is similar to that of H. Becquerel.[1] Siertsema[2] has applied this formula to a series of transparent substances and the calculated values of e/m are in fair agreement with the accepted one. Voigt[3] deduces a similar formula; he also by an ingenious application of the inverse Zeeman effect (see later) arrives at an explanation of the rotation phenomena which is particularly satisfactory for the immediate neighborhood of an absorption band. Magnetic rotation theories (for gases) on the basis of the Bohr molecule have also been worked out by Sommerfeld[4] and by Pauer,[5] while certain electronic considerations have been exhaustively treated in a theory by S. S. Richardson.[6]

THE ZEEMAN EFFECT

Chief Characteristics. Using the word in its widest sense we may include under the Zeeman Effect any changes in light emission or absorption produced by placing the emitting or absorbing body in a magnetic field. In a narrower sense, however, we understand by this term the magnetic resolution of sharp spectral emission or absorption lines into two, three or more components. If a line source—flame, spark or vacuum tube—is placed in a strong magnetic field the lines are split into components which, while only slightly separated, may be easily observed and studied with modern high-resolving-power spectroscopic apparatus, such as the Michelson echelon grating, Lummer-Gehrcke plate, interferometer or speculum grating of good size.

This resolution is accompanied by characteristic polarization phenomena. In the simplest case a single line, when the vision is at right angles to the lines of force (transversal effect), is separated by the magnetic field into three, of which the central one retains its original frequency and is plane polarized with electric vector parallel to the field, while the components on either side are polarized in a plane at right angles to this. On looking parallel to the field, however, (longitudinal

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[1] H. Becquerel, *C. R.* 125, 697, 1897.

[2] L. H. Siertsema, *Amst. Proc.* 5, 413; Leiden Com. No. 82. 1902.

[3] *Loc. cit.* p. 561.

[4] A. Sommerfeld, *Ann. d. Phys.* 53, 497, 1917.

[5] E. Pauer, *Ann. d. Phys.* 56, 261, 1918.

[6] S. S. Richardson, *Phil. Mag.* (6), 31, 232, 454, 1916.

effect) the central component is lacking while the two side ones are, respectively, right and left-handed circularly polarized.

As might be anticipated from Kirchhoff's law, an exactly similar effect holds in the case of absorbing gases and vapors. A dark line such as may be observed in the continuous spectrum when white light traverses an absorbing vapor (e. g. flame containing a metallic salt, between the poles of an electromagnet) is resolved into three (or more) when the field is excited. On examination it is found that the absorption is limited entirely to the states of polarization described above. This, which is known as the *Inverse* Zeeman Effect, has an intimate bearing on the theory of the Faraday Effect and on magnetic double-refraction. It is also the basis of the fundamentally important researches of Hale[1] and his co-workers in Solar magneto-optics.

While the simple triplet, as described, is the commonest type of resolution it is by no means the only one. The sextette is only second to it in frequency of occurrence and there are other cases of separation into from four to fifteen or more components, while a very few lines remain unaffected by the field. Preston has stated as a rule that for any given element all lines belonging to the same spectral series exhibit resolutions of the same type and, if drawn to the scale of frequencies, of the same magnitude. Also, corresponding lines of the different elements behave in the same way. Runge and Ritz have also developed rules applying particularly to the more complicated types of resolution.

Theory. The Lorentz theory of the Zeeman Effect, reduced to its simplest terms is this: Any electronic vibration may be resolved into three axial components, of which the Z vibration, parallel to the lines of force, will not be affected by the field. The other two are each resolvable into a right and left circular orbital motion in the X Y plane, which may be combined into one right and one left circular. The effect will be exactly the same as if the light emitted by the source came from three groups of electrons vibrating in these three simple ways.

Now looking in the direction of the field a negative electron moving in a right-handed circular path will experience an electromagnetic pull towards the center of its orbit which will shorten its period of revolution, while the reverse effect will take place for the other direction of rotation. As seen in the Z direction, then, the (longitudinal) effect will be confined to these two opposite circular polarizations. Viewed transversely, however, the circular motions appear "edge on" as linear vibrations perpendicular to the field, while the central or unshifted component is polarized at right angles to these.

Putting the matter now in mathematical form, we may write down the equations of motion of an electron of mass m whose coordinates

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[1] G. E. Hale, Mount Wilson Sol. Obs. Contr. No. 30 and many subsequent papers.

are ξ, η, ζ relative to its mean position, as:

$$m\frac{d^2\xi}{dt^2}=-k\xi;\quad m\frac{d^2\eta}{dt^2}=-k\eta;\quad m\frac{d^2\zeta}{dt^2}=-k\zeta,$$

where k is the coefficient of the "quasielastic" forces. That the restoring force must be proportional to the displacement follows from the fact that the frequency of vibration is independent of the amplitude. Following the usual methods of integration these equations give as a frequency (expressed in terms of an angular velocity):

$$\omega_0=\sqrt{k/m.}$$

Now in the presence of a field H these equations must be modified to:

$$m\frac{d^2\xi}{dt^2}=-k\xi+\frac{eH}{c}\frac{d\eta}{dt};\quad m\frac{d^2\eta}{dt^2}=-k\eta-\frac{eH}{c}\frac{d\xi}{dt}\quad m\frac{d^2\zeta}{dt^2}=-k\zeta;$$

the last being unchanged by the field.

Combining the first two we eventually get as solutions:

$$\omega_+^2=\omega_0^2-\frac{e\omega_+H}{cm};\quad \omega_-^2=\omega_0^2+\frac{e\omega_-H}{cm};$$

or, to a very close approximation:

$$\omega_\pm=\omega_0\pm q;$$

where

$$q=-\frac{eH}{2cm}.$$

Now since

$$\omega=\frac{2\pi c}{\lambda},$$

and

$$\delta\omega\,(=q),\ =-2\pi c\frac{\delta\lambda}{\lambda^2};$$

we have:

$$\frac{\delta\lambda}{\lambda^2}=\frac{e}{m}\,\frac{H}{4\pi c^2};$$

or, putting e in electromagnetic units:

$$\frac{\delta\lambda}{\lambda^2} = \frac{e}{m}\frac{H}{4\pi c}.$$

The separation $\delta\lambda$ is, therefore, proportional to the square of the wavelength and its measurement allows a calculation of the ratio $\frac{e}{m}$, which was first determined by Zeeman in this way.

The preceding theory is adequate in explaining only the simplest case of the Zeeman phenomena, i.e., the normal triplet, or separation which gives a value of $\frac{e}{m}$ in agreement with the accepted one as determined by other means. Extensions of the theory require in general specific assumptions as to the nature of the atoms or molecules. By assuming molecular magnets much like Weiss' magnetons Ritz has been able to represent satisfactorily many of the complicated resolutions, but his theory has been severely critized by Voigt.[1] J. J. Thomson's model atom was thought at first sight to possess possibilities for the interpretation of quartets, quintets, etc., but now appears to yield only the simple doublet and triplet of the elementary theory.

The Bohr atom does not, at first sight, lend itself very readily to the explanation of the Zeeman effect. However, Debye,[2] Bohr,[3] Lorentz, and Sommerfeld[4] have put forth a quantum explanation of this phenomenon, which, while necessarily more complicated than the simple Lorentz theory, has an advantage in that it accounts not only for the simple but also for some of the more complicated effects as well. It is of interest to note that while the use of the Bohr model in explaining the Stark electro-optic effect gives resultant equations in which the quantum constant *h*, explicitly occurs, this constant is eliminated in the final equations for the corresponding magneto-optic (Zeeman) effect, and the results are consequently in agreement with those of the simple Lorentz theory.

Lastly, Lorentz[5] has built up a general system of equations founded on the supposition of magnetically isotropic atoms, which Voigt[6] has modified, abandoning the assumption and making use of coupled electrons. Voigt's equations, while necessarily complicated and lacking in

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[1] W. Voigt, *Ann. d. Phys.* (4) 36, 873, 1911.

[2] P. Debye, *Phys. Zeit.* 17, 507, 1916.

[3] N. Bohr, *Phil. Mag.* 27, 506, 1914.

[4] A. Sommerfeld "Atombau u. Spektrallinien", p. 416 ff. Braunschweig, 1921.

[5] H. A. Lorentz, *Wied. Ann.* 63, 278, 1897.

[6] *Loc. cit.* p. 592.

clearness, are, with a suitable choice of constants, capable of explaining all the resolutions observed.

Voigt's Theory of Magnetic Rotation. On the basis of the inverse Zeeman Effect—longitudinal case—a single absorption band is resolved into two, slightly separated in wavelength, of right and left circular polarization. Now resonance absorption, such as occurs in a flame with metallic vapor, has a characteristic effect on the index of refraction, which shows, in the neighborhood of the absorption band, "anomalous dispersion." Such curves are illustrated, on a frequency scale, in Fig. 1. Remembering that on the Fresnel basis magnetic rotation is due to the difference in velocity of the right and left circular components of a plane polarized beam (rotation $R = \pi n \left(\frac{1}{v_2} - \frac{1}{v_1}\right)$ radians per unit length) we may conclude that such rotation for any particular wavelength (or frequency, n) will be proportional to the ***difference of the ordinates*** of the two curves, as shown in the lower figure. The rotation should have the same sign on each side of the absorption band and the ***opposite*** sign within the band. Also, as is seen by drawing the figure for a larger magnetic resolution, the rotation within the band, after a certain field intensity has been reached should ***decrease with further***

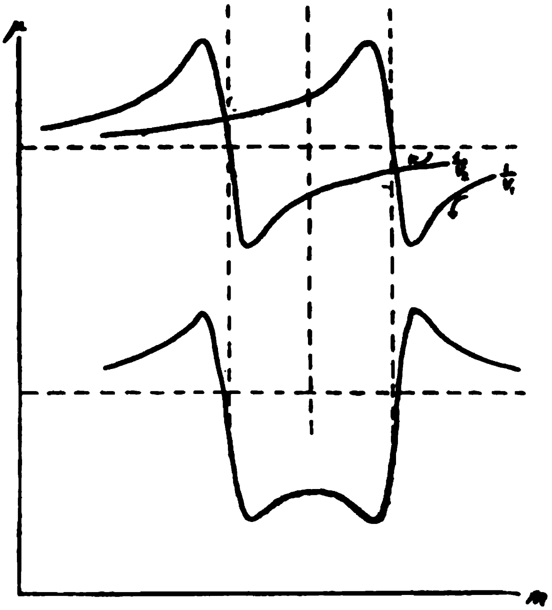

FIG. 1

increase of field. These predictions have all been amply verified in experiments with sodium flames by Zeeman,[1] Hansen,[2] and others.

Magnetic Double-Refraction. If the preceding reasoning is applied to the transverse case, we may expect a condition of affairs given by Fig. 2. Here the absorption band is resolved by the field into a triplet with plane-polarized components. At any particular frequency, then, we shall find waves of the two planes of polarization having different refractive indices and this means double-refraction. The magnitude of the effect is indicated by the dotted line. Experiments by Zeeman and Geest[3] on sodium vapor, and particularly those by Voigt and Hansen[4] on that of lithium, verify in a most satisfactory manner the existence of this new magneto-optic phenomenon.

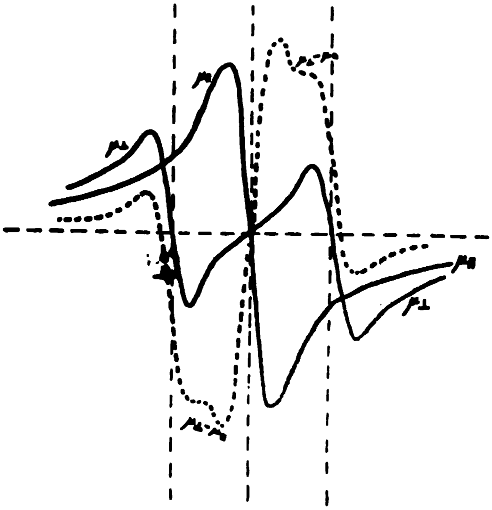

FIG. 2

There are, however, some cases of magnetic double-refraction, such as that discovered in pure liquids by Cotton and Mouton,[5] which cannot be explained by Voigt's theory. According to this theory magnetic double-refraction and rotation are directly related and one ought to be calculable from the other, whereas, the observed double-refraction for liquids exceeds the calculated a thousand times. In a statistical-mechanics application of his hypothesis of molecular orientation, Langevin[6] deduces formulae which fit this latter case. According to

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[1] P. Zeeman, *Amst. Proc.* 5, 41, 1902.

[2] H. M. Hansen, *Ann. d. Phys.* 43, 169, 1914.

[3] P. Zeeman and J. Geest, *Amst. Proc.* 7, 435, 1904.

[4] W. Voigt and H. M. Hansen, *Phys. Zeit.* 13, 217, 1912.

[5] A. Cotton and H. Mouton, *Ann. Chim. Phys.* (8), 11, 145, 289, 1907.

[6] P. Langevin, *C. R.* 151, 475, 1910.

his theory magnetic rotation and double-refraction must be ascribed to essentially different causes. The first being allied to magnetic resolution and diamagnetism is to be explained through modification of electronic paths by the magnetic field, the other by re-orientation of the molecular axes. While this theory will apply to liquids, Voigt's is eminently the most satisfactory for metallic vapors. It is indeed quite possible that in the general case both of these factors are operative.

THE KERR EFFECT

General Characteristics. There may be distinguished three different types of phenomena—collectively referred to as the "Kerr Effect," although Kerr himself was able to observe only the first two—which arise when a surface of iron, nickel or cobalt on which is incident plane polarized light, is subjected to a strong magnetic field. The three types correspond to the three possible cases of magnetization, viz., with the lines of force normal to the reflecting surface (polar case), or lying in the surface and respectively parallel (meridional case), or perpendicular (equatorial case) to the plane of incidence of the light.

The phenomena in general are somewhat complicated, inasmuch as they are superposed on the optical effects which accompany the reflection of light from the unmagnetized metallic surface. In the simplest and best known case—polar magnetization and normal incidence—the effect is practically nothing but a very small rotation of the plane of polarization. For large angles of incidence, however, or for the other types of magnetization an accompanying ellipticity, which is very slight in the normal polar case, becomes prominent. The experimental side has been worked out by Kerr, Righi,[1] DuBois[2] and others. (For additional references see Voigt, loc. cit., p. 671 ff.) An interesting point in this connection is the fact that Heusler's magnetic alloy shows no appreciable Kerr effect.

Theory. Inasmuch as reflection of light involves a certain amount of penetration into the material, any theory of the Kerr phenomena must involve the (Faraday) effect resulting from the penetration, as well as the general boundary conditions for the passage of a vibration from one body to another. That the first consideration alone is insufficient will be gained from the fact that in the polar case—normal incidence—the Kerr effect is just opposite in sign to the Faraday rotation due to penetration.

In a continuation of the same type of reasoning as he uses in explaining magnetic rotation on the basis of the inverse Zeeman effect, Voigt

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[1] A. Righi, *Ann. Chim. Phys.* (6), 4, 435, 1885; 9, 65, 117, 1886.

[2] H. E. J. G. Du Bois, *Wied. Ann.* 39, 25, 1890.

has worked out a theory of the Kerr phenomena from the modern electronic view point. Applying the general boundary conditions of electromagnetism which express the continuity of the electric and magnetic field components parallel to the surface and taking care to satisfy the energy principle he arrives at a certain complex parameter of the metal, Q, on which the effect uniquely depends. This constant, which involves the vibration periods of free and bound electrons and is proportional to the inner field or magnetization, has been computed, using as a basis Ingersoll's results[1] on the equatorial case, by Snow[2] and by Voigt.[3] The latter concludes that for steel and cobalt the magneto-optic behavior in the wave length region around 2μ is conditioned principally by conducting electrons. Calculation of the internal fields leads to a value of the order of 10^7 and this seems to warrant the conclusion that in the ferromagnetic metals, there exist molecules or molecular complexes with very strong magnetic polarity, which through the external field maintain a tendency to parallelism. The closed internal field computed on this basis by Weiss agrees in order of magnitude with the above value deduced from magneto-optic considerations.

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[1] L. R. Ingersoll, *Phys. Rev.* 35, 312, 1912.

[2] C. Snow, *Phys. Rev.* (2), 2, 29, 1913.

[3] W. Voigt, *Phys. Zeit.* 16, 299, 1915.

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