U.S. Army Coast. Eng. Res. Ctr. CETA

CETA 83-1

Calculation of Wave Shoaling With Dissipation Over Nearshore Sands

by Robert J. Hallermeier



COASTAL ENGINEERING TECHNICAL AID NO. 83-1

MARCH 1983



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SECURITY	CLASSIFICATION	OF THIS PAGE	(When Data Entered)

REPORT DOCUMENTATION	PAGE	READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
CETA 83-1		
4. TITLE (and Subtitle)		5. TYPE OF REPORT & PERIOD COVERED
		Coastal Engineering
CALCULATION OF WAVE SHOALING WIT	Н	Technical Aid
DISSIPATION OVER NEARSHORE SANDS		5. PERFORMING ORG. REPORT NUMBER
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7. AUTHOR(a)		8. CONTRACT OR GRANT NUMBER(0)
Robert J. Hallermeier		DOCUMEN
9. PERFORMING ORGANIZATION NAME AND ADDRESS	and an Agencia of the state of the	10. PROGRAM ELEMENT, PROJECT, TASK
Department of the Army		AREA & WORK UNIT NUMBERS
Coastal Engineering Research Cen		C31551
Kingman Building, Fort Belvoir,	VA 22060	
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE
Department of the Army		March 1983
Coastal Engineering Research Cen		13. NUMBER OF PAGES
Kingman Building, Fort Belvoir, 14. MONITORING AGENCY NAME & ADDRESS(11 differen		21 15. SECURITY CLASS. (of this report)
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		UNCLASSIFIED
		154. DECLASSIFICATION/DOWNGRADING SCHEDULE
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16. DISTRIBUTION STATEMENT (of this Report)		
Approved for public release; dis		
18. SUPPLEMENTARY NOTES		
IS SOFFEEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary an	d identify by block number)	
Agitated sand bed	Nonlinea	r wave propagation
Bottom friction		rbulent flow
Energy dissipation	Wave hei	
Nearshore waves	Wave sho	0
20. ABSTRACT (Continue on reverse stde if necessary and This report provides a simpl wave height changes considering over a strongly agitated bed of ships are from linear monochroma dissipation is that calculated h height. The general effect of a height relatively constant outsi and a calculator program are pro	ified calculatio the energy dissi quartz sand. Al tic wave theory, eight changes de ppreciable energ de the breaker z	pated by rough turbulent flow l elementary wave relation- but one effect of including pend on the absolute wave y loss is to make field wave

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PREFACE

This report provides a calculation procedure for nearshore shoaling of energetic waves outside the breaker zone, including the appreciable effects of energy dissipation over a strongly agitated sand bed. The work reported was conducted under the U.S. Army Coastal Engineering Research Center's (CERC) Numerical Modeling of Shoreline Response to Coastal Structures work unit, Shore Protection and Restoration Research Program, Coastal Engineering Area of Civil Works Research and Development.

The present treatment replaces guidance previously provided in CERC Field Guidance Letter No. 79-04 and CERC Technical Paper No. 80-8 (Grosskopf, 1980). Those publications incorrectly recommend the use of friction coefficients for inert beds in computing nearshore wave shoaling.

The report was written by Dr. Robert J. Hallermeier, Oceanographer, under the general supervision of Mr. R.P. Savage, Chief, Research Division, CERC.

Technical Director of CERC was Dr. Robert W. Whalin, P.E.

Comments on this publication are invited.

Approved or publication in accordance with Public Law 166, 79th Congress, approved 31 July 1945, as supplemented by Public Law 172, 88th Congress, approved 7 November 1963.

Colonel, Corps of Engineers Commander and Director

CONTENTS

	Page CONVERSION FACTORS, U.S. CUSTOMARY TO METRIC (SI)
	SYMBOLS AND DEFINITIONS
I	INTRODUCTION
II	CALCULATION PROCEDURE
III	APPLICATIONS
IV	SUMMARY
	LITERATURE CITED
APPENDIX	CALCULATOR PROGRAM FOR WAVE SHOALING WITH DISSIPATION, 17

Multiply	by	To obtain
inches	25.4	millimeters
	2.54	centimeters
square inches	6.452	square centimeters
cubic inches	16.39	cubic centimeters
feet	30.48	centimeters
	0.3048	meters
square feet	0.0929	square meters
cubic feet	0.0283	cubic meters
yards	0.9144	meters
square yards	0.836	square meters
cubic yards	0.7646	cubic meters
miles	1.6093	kilometers
square miles	259.0	hectares
knots	1.852	kilometers per hour
acres	0.4047	hectares
foot-pounds	1.3558	newton meters
millibars	1.0197×10^{-3}	kilograms per square centimeter
ounces	28.35	grams
pounds	453.6 0.4536	grams kilograms
ton, long	1.0160	metric tons
ton, short	0.9072	metric tons
degrees (angle)	0.01745	radians
Fahrenheit degrees	5/9	Celsius degrees or Kelvins ¹

U.S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

¹To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use formula: C = (5/9) (F -32).

To obtain Kelvin (K) readings, use formula: K = (5/9) (F - 32) + 273.15.

SYMBOLS AND DEFINITIONS

c = L/T	wave celerity
D	median sand grain diameter
d	mean water depth
da	maximum water depth for sand bed agitation by waves
Ē	average energy dissipation rate
f _e	energy dissipation coefficient for rough turbulent flow over strongly agitated sand bed
g	acceleration due to gravity
Ĥ	wave height
H _o '	equivalent wave height in deep water ignoring refraction
Ks	shoaling coefficient in linear wave theory
L	wavelength
$L_o = gT^2/2\pi$	wavelength in deep water
n	ratio of group velocity to wave celerity
P	average wave energy flux
Т	wave period
X	wave propagation distance
ξ	horizontal amplitude of near-bed fluid orbit
ρ	fluid density
Additional s	ubscripts
j	value at location where wave condition is to be predicted
j m	value at location where wave condition is to be predicted value at geometric mean water depth for region of interest

+/- case where d_j is greater or less than d_1

6

CALCULATION OF WAVE SHOALING WITH DISSIPATION OVER NEARSHORE SANDS

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Robert J. Hallermeier

I. INTRODUCTION

As waves propagate toward breaking in shallow water, their attributes are transformed by effects of water depth and bottom features. These nearshore transformations are crucial in the interpretation of wave measurements and in the prediction of sediment transport. Prior to wave breaking, appreciable wave energy can be dissipated by friction between the oscillatory water motion and the nearshore bottom, especially where waves cause strong agitation of bottom sediments. This report considers such frictional dissipation.

Ocean waves can be represented most adequately as distributions of propagating energy with respect to frequency and direction. Near the shore, extremely energetic waves are observed to constitute a somewhat regular wave train approaching along a shore-normal line, with a rather well-defined wave height and period. According to the Shore Protection Manual (SPM) (U.S. Army, Corps of Engineers, Coastal Engineering Research Center, 1977), an idealization of demonstrated value in coastal engineering is to represent real waves by a characteristic height and period, the significant wave condition; this wave representation is utilized here.

This report provides a simple calculation procedure defining changes in significant wave height due to water depth differences combined with bottom friction at agitated sand beds. The procedure uses linear wave theory between the two depths of interest and a computation of energy dissipation rate at an intermediate water depth. Factors in wave transformation ignored here include: wave direction and complex spectra, currents, surface wave breaking, winds, water viscosity, and bottom percolation and elasticity. The technique is meant for application only to energetic field wave conditions in fairly shallow water with straight, parallel depth contours and relatively fine quartz bottom sands.

Section II presents the calculation procedure for wave height changes considering energy dissipation. Section III addresses the application of this procedure and includes two example problems: converting nearshore wave measurements into equivalent wave heights in shallower and in deeper water. The reader is referred to Hallermeier (in preparation, 1983) for a detailed substantiation of elements incorporated in the calculation procedure; that reference reviews the empirical basis for the expression giving energy dissipation coefficient and reports extensive calculated results in clear agreement with multiple wave measurements at the Coastal Engineering Research Center's (CERC) Field Research Facility, Duck, North Carolina, and at other sites.

II. CALCULATION PROCEDURE

Waves are presumed to travel perpendicular to depth contours with propagation described by small-amplitude (linear) wave theory. Required theoretical relations are provided in Section 2.23 of the SPM. Wave and fluid characteristics arising in the wave description are

- d = local mean water depth
- g = acceleration due to gravity
- H = local wave height
- L = local wavelength
- T = wave period
- ρ = fluid density

Wavelength in deep water is $L_0 = (gT^2/2\pi)$ and the dimensionless local wavelength, d/L, is the solution of

$$\frac{d}{L} \tanh\left(\frac{2\pi d}{L}\right) = \frac{d}{L_0}$$
(1)

which is presented in Table C-1 of the SPM. The average wave energy flux per unit crest width is

$$\overline{P} = \frac{1}{8} \rho g H^2 c n$$
(2)

where c = (L/T) is wave celerity and n the ratio of group velocity to wave celerity:

 $n = \frac{1}{2} \left[1 + \frac{4\pi d/L}{\sinh (4\pi d/L)} \right]$ (3)

With no wave refraction, and provided no energy has been added to or removedfrom the wave train, it is convenient to express wave height changes by the factor

$$\frac{H}{H_{0}^{T}} = \frac{1}{\sqrt{2n \tanh(2\pi d/L)}} = K_{S}$$
(4)

where H'_o is equivalent wave height in deep water (ignoring refraction), and K_s the shoaling coefficient. The dimensionless quantities n and K_s are provided for specific values of (d/L_o) in Table C-1 of the SPM. In situations considered here, a nearshore wave measurement at water depth d_1 is to be converted into the corresponding wave condition at another relatively shallow depth, d_j . The wave period is presumed constant during propagation so that (d_1/L_o) and (d_j/L_o) are known, and equation (4) would give the ratio of wave heights as

$$\frac{\mathrm{H}_{j}}{\mathrm{H}_{1}} = \frac{\mathrm{K}_{sj}}{\mathrm{K}_{s1}}$$
(5)

if energy dissipation were to be ignored.

Energy lost from the wave train due to bottom friction is treated by means of a single dissipation calculation at the geometric mean depth for the region of interest

$$d_{\rm m} = \sqrt{d_1 d_j} \tag{6}$$

Average energy dissipation rate at d_m , per unit crest width and per unit length in the propagation direction, is given by

$$\overline{E}_{m} = 0.235 \ \rho \ f_{em} \ (2\pi \ \xi_{m}/T)^{3}$$
(7)

For rough turbulent flow over a strongly agitated bed of quartz sand, the energy dissipation coefficient introduced in equation (7) is

$$f_{em} = \exp\left[-5.882 + 14.57 (D_m/\xi_m)^{0.194}\right]$$
 (8)

Here D_m is median sand grain diameter at d_m and ξ_m is horizontal amplitude of the near-bed fluid excursion arising at d_m without energy dissipation, so that $(2\pi\xi_m/T)$ in equation (7) is peak near-bed fluid velocity. According to linear wave theory,

$$\xi_{\rm m} = \frac{{\rm H}_{\rm m}}{2\,\sinh\,\left(2\pi {\rm d}_{\rm m}/{\rm L}_{\rm m}\right)} \tag{9}$$

where $H_m = (H_1 K_{sm}/K_{s1})$ from equation (5).

Energy-conserving linear wave shoaling is combined with computed energy dissipation rate into an expression giving a wave height at d_j equivalent to measured wave height at d_1 . This expression is a revised form of equation (2):

$$H_{j\pm}^{2} = \frac{8(P_{1} \pm E_{m}X)}{\rho g c_{j} n_{j}}$$
(10)

where X is the wave propagation distance between the two water depths $(d_1$ and $d_j)$, and the upper [lower] sign is used when d_j is greater [less] than d_1 . The conversion given in equation (10) presumes that computed dissipation rate at d_m can be considered representative of the entire propagation path.

III. APPLICATIONS

Besides the explicitly ignored factors affecting nearshore wave transformations, it is important in applications to consider the requirements stated above on the use of equation (8) for energy dissipation coefficient. The quartz sand bed must be strongly agitated by wave action and near-bed flow must be rough turbulent. Appropriate situations correspond to nearshore field waves with relatively large height and period.

The strength of bed agitation may be judged using an approximate expression (Hallermeier, 1981, eq. 10) giving maximum water depth, d_a , for wave agitation

of a quartz sand bed when viscous effects are negligible:

 $d_a = HT (g/500) D^{0.5}$ (11)

If d_a computed from H_1 , T_1 , and D_m is much larger than the maximum water depth of interest, the requirement for strong bed agitation may be considered satisfied.

To assess whether flow is likely to be rough turbulent, another simple computation can be performed. Incorporating the same approximation for ξ in intermediate water depth ($2\pi d/L$ near unity) as is utilized in equation (11), fundamental results reviewed in Hallermeier (in preparation, 1983) support

$$HT > d \qquad {metric units} \qquad (12)$$

as an approximate criterion for rough turbulent flow at a strongly agitated bed of quartz sand. If equation (12) is true according to H_1 , T_1 , and the maximum water depth of interest, the requirement for rough turbulent near-bed flow may be considered satisfied.

The following example problems demonstrate the use of the present procedure in calculating nearshore wave shoaling with energy dissipation due to a strongly agitated sand bed.

<u>GIVEN</u>: At the CERC Field Research Facility, significant wave height exceeding 3.5 meters was recorded during three 1981 storms by a Waverider buoy located in an 18-meter mean water depth. Wave periods associated with these extremely high waves ranged from 9.3 to 14.0 seconds. Nearshore bathymetry at this site is regularly surveyed to the 9-meter water depth contour, and the wave characteristics at the seaward boundary to the survey region are of interest. The shore-normal distance between the water depths is 1800 meters, and the representative sand size for the intervening bottom is D = 0.12millimeter.

FIND: The wave height at $d_j = 9$ meters corresponding to $H_1 = 3.5$ meters at $d_1 = 18$ meters for: (a) $T_1 = 9.3$ seconds and (b) $T_1 = 14.0$ seconds.

SOLUTION:

(a) For $T_1 = 9.3$ seconds and $d_1 = 18$ meters

$$\frac{d_1}{L_0} = \frac{2\pi d_1}{gT_1^2} = \frac{(2\pi)(18)}{9.81(9.3)^2} = 0.1333$$

Table C-1 in the SPM gives $n_1 = 0.7570$, $(H_1/H_0) = 0.9160 = K_{s1}$ and $d_1/L_1 = 0.1694$, so that $L_1 = 106.3$ meters. With $H_1 = 3.5$ meters and $\rho = 1026$ kilograms per cubic meter for saltwater, equation (2) becomes

$$\bar{P}_1 = \frac{1}{8} \rho g H_1^2 c_1 n_1 = \frac{1}{8} (1026) (9.81) (3.5)^2 \frac{106.3}{9.3} (0.757)$$

= 1.33 • 10⁵ kilogram-meter per second cubed

Since $d_m = \sqrt{d_1 d_j} = \sqrt{(9)(18)} = 12.73$ meters

$$\frac{d_{\rm m}}{L_{\rm o}} = \frac{(2\pi)(12.73)}{9.81(9.3)^2} = 0.09427$$

and Table C-1 gives $(d_m/L_m) = 0.1360$, sinh $(2\pi d_m/L_m) = 0.9621$, $(H_m/H_0^-) = 0.9378 = K_{Sm}$, and $n_m = 0.8199$. Thus, $H_m = (H_1 K_{Sm}/K_{S1}) = (3.5)(0.9378)/(0.9160) = 3.58$ meters and according to equation (9),

$$\xi_{\rm m} = \frac{{\rm H}_{\rm m}}{2\,\sinh\!\left(\frac{2\,\pi{\rm d}_{\rm m}}{{\rm L}_{\rm m}}\right)} = \frac{3.58}{2\,(0.9621)} = 1.86\,\,{\rm meters}$$

With $D_m = 0.12$ millimeter, equation (8) is

$$f_{em} = \exp \left[-5.882 + 14.57 \left(D_m / \xi_m\right)^{0.194}\right]$$
$$= \exp \left[-5.882 + 14.57 \left(1.2 \cdot 10^{-4} / 1.86\right)^{0.194}\right] = 0.0262$$

so that equation (7) becomes

$$\overline{E}_{m} = 0.235 \ \rho \ f_{em} \left(\frac{2\pi\xi_{m}}{T}\right)^{3} = (0.235)(1026)(0.0262) \left[\frac{2\pi(1.86)}{9.3}\right]^{3}$$

= 12.5 kilograms per second cubed

Some conditions must also be computed at $d_i = 9$ meters, where

$$\frac{d_j}{L_0} = \frac{(2\pi) 9}{(9.81)(9.3)^2} = 0.06665$$

so that Table C-1 gives $(H_j/H_0') = 0.9779 = K_{sj}$, $n_j = 0.8688$, and $(d_j/L_j) = 0.1108$ so that $L_j = 81.2$ meters. Finally, because $d_j < d_1$ the lower sign in equation (10) is appropriate, and X = 1800 meters yields

$$H_{j}^{2} = \frac{8(P_{1} - E_{m}X)}{\rho \ g \ c_{j} \ n_{j}} = \frac{8[1.33 \cdot 10^{5} - (12.5)(1800)]}{(1026)(9.81) \ \frac{(81.2)}{9.3}} = 11.58 \text{ square meters}$$

$$H_{j} = 3.40 \text{ meters}$$

From equation (11), maximum water depth for bed agitation is $d_a = H_1T_1(g/5000 D)^{0.5} = (3.5)(9.3)[9.81/(5 \cdot 10^3)(0.12 \cdot 10^{-3})]^{0.5} = 131.6$ meters,

much larger than water depths in the region treated, and the numerical value in metric units of $(H_1T_1) = 32.6$, nearly twice the maximum water depth considered in meters, so that equation (12) indicates rough turbulent flow throughout the region. The calculation procedure is suitable for these conditions, and the effect of bottom friction on wave shoaling is appreciable, in that linear wave theory without dissipation would predict a nearshore wave height of $(H_1 K_{sj}/K_{s1}) = [(3.5)(0.9779)/0.9160] = 3.74$ meters, using equation (5).

(b) For $T_1 = 14.0$ seconds and $d_1 = 18$ meters, $(d_1/L_0) = 0.05882$ so that $n_1 = 0.8833$, $(H_1/H_0) = 0.9963 = K_{S1}$, $(d_1/L_1) = 0.1031$ and $L_1 = 174.6$ meters from Table C-1. Equation (2) with $H_1 = 3.5$ meters gives

$$\overline{P}_1 = \frac{1}{8} (1026)(9.81)(3.5)^2 \frac{(174.6)}{(14.0)} (0.8833)$$

= $1.70 \cdot 10^5$ kilogram-meter per second cubed

At d_m , $(d_m/L_0) = 0.04160$ so that $(d_m/L_m) = 0.08509$, sinh $(2\pi d_m/L_m) = 0.5605$, $(H_m/H_0) = 1.057 = K_{sm}$, and n = 0.9161. Thus, $H_m = (H_1 K_{sm}/K_{s1}) = 3.71$ meters, equation (9) is

$$\xi_{\rm m} = \frac{3.71}{2(0.5605)} = 3.31 \,\,{\rm meters}$$

equation (8) is

$$f_{em} = \exp \{-5.882 + 14.57 (1.2 \cdot 10^{-4}/3.31)^{0.194}\} = 0.0207$$

and equation (7) is

$$\overline{E}_{m} = (0.235)(1026)(0.0207) \left[\frac{2\pi(3.31)}{14.0}\right]^{3}$$

= 16.36 kilograms per second cubed

At dj, $(d_j/L_0) = 0.02941$ so that $(H_j/H_0) = 1.130 = K_{Sj}$, nj = 0.9400, $(d_j/L_j) = 0.0706$ and $L_j = 127.5$ meters. Again, the lower sign in equation (10) is used to give

$$H_j^2 = \frac{8[1.70 \cdot 10^5 - (16.36)(1800)]}{(1026)(9.81) \frac{(127.5)}{(14.0)}} = 13.05 \text{ square meters}$$
$$H_j = 3.61 \text{ meters}$$

Because T is greater here than in part (a), the requirements for rough turbulent flow at a strongly agitated sand bed are clearly satisfied. Although the computation including friction results in H_j slightly larger than H_1 in this case, energy dissipation again has an appreciable effect since linear wave theory would predict a nearshore wave height of $(H_1 K_{sj}/K_{s1}) = 3.97$ meters, from equation (5).

* * * * * * * * * * * * * * EXAMPLE PROBLEM 2 * * * * * * * * * * * * * * * * * *

- <u>GIVEN</u>: A mathematical model is to be used to simulate storm wave effects for water depths shoreward of 9 meters at Nags Head, North Carolina, with the threshold for storm waves taken to be the wave height exceeded 10 percent of the time. The wave climate at this site has been defined by several relatively complete years of data from a pier-mounted gage located in mean water depth of 5.2 meters (Thompson, 1977): wave height exceeded in 10 percent of these measurements is about 1.7 meters and the typical wave period for this wave height is about 8.5 seconds. The shore-normal distance between 5.2- and 9-meter water depth is 600 meters; representative sand size for the intervening bottom is D = 0.20 millimeter.
- FIND: The wave height at $d_j = 9$ meters corresponding to $H_1 = 1.7$ meters and $\overline{T}_1 = 8.5$ seconds at $d_1 = 5.2$ meters.

SOLUTION: For $T_1 = 8.5$ seconds and $d_1 = 5.2$ meters,

$$\frac{d_1}{L_0} = \frac{2\pi d_1}{gT_1^2} = \frac{(2\pi)(5.2)}{(9.81)(8.5)^2} = 0.04610$$

Table C-1 in SPM gives $n_1 = 0.9074$, $(H_1/H_0') = 1.038 = K_{S1}$ and $d_1/L_1 = 0.09002$, so that $L_1 = 57.76$ meters. With $H_1 = 1.7$ meters, equation (2) is

$$\overline{P}_1 = \frac{1}{8} \rho g H_1^2 c_1 n_1 = \frac{1}{8} (1026)(9.81)(1.7)^2 \frac{(57.76)}{(8.5)} (0.9074)$$

= 2.24 • 10⁴ kilogram-meter per second cubed

Since
$$d_m = \sqrt{d_1 d_j} = \sqrt{(5.2)(9)} = 6.84$$
 meters

$$\frac{d_{\rm m}}{L_{\rm o}} = \frac{(2\pi)(6.84)}{(9.81)(8.5)^2} = 0.06064$$

and Table C-1 gives $(d_m/L_m) = 0.1049$, sinh $(2\pi d_m/L_m) = 0.7082$, $(H_m/H_o) = 0.9916 = K_{sm}$, and $n_m = 0.8799$. Thus, $H_m = (H_1 K_{sm}/K_{s1}) = (1.7)(0.9916)/1.038 = 1.624$ meters and according to equation (9),

$$\xi_{\rm m} = \frac{{\rm H}_{\rm m}}{2\,\sinh\left(\frac{2\pi {\rm d}_{\rm m}}{{\rm L}_{\rm m}}\right)} = \frac{1.624}{2(0.7082)} = 1.15 \,\,{\rm meters}$$

With $D_m = 0.20$ millimeter, equation (8) is

 $f_{em} = \exp \{-5.882 + 14.57 (D_m/\xi_m)^{0.194}\}$

= exp { $-5.882 + 14.57 [2 \cdot 10^{-4}/1.15]^{0.194}$ } = 0.0422

$$\overline{E}_{m} = 0.235 \text{ p f}_{em} \left(\frac{2\pi\xi_{m}}{T}\right)^{3} = (0.235)(1026)(0.0422) \left[\frac{2\pi (1.15)}{8.5}\right]^{3}$$

= 6.25 kilograms per second cubed

At $d_1 = 9$ meters,

$$\frac{dj}{L_0} = \frac{(2\pi)(9)}{(9.81)(8.5)^2} = 0.0798$$

so that Table C-1 gives $(H_j/H_o) = 0.9551 = K_{sj}$, $n_j = 0.845$, and $(d_j/L_j) = 0.1230$ so that $L_j = 73.2$ meters. Finally, because $d_j > d_1$ the upper sign in equation (10) is appropriate, and X = 600 meters yields

$$H_{j}^{2} = \frac{8 (P_{1} + E_{m}X)}{\rho g c_{j} n_{j}} = \frac{8 [2.24 \cdot 10^{4} + (6.25)(600)]}{(1026)(9.81) \frac{(73.2)}{(8.5)}} = 2.86 \text{ square meters}$$

 $H_i = 1.69$ meters

To show that the calculation procedure is suitable for these conditions, maximum water depth for bed agitation from equation (11) is

$$d_{a} = H_{1}T_{1} \left[\frac{g}{5000 \text{ D}}\right]^{0.5} = (1.7)(8.5) \left[\frac{9.81}{(5 \cdot 10^{3})(2 \cdot 10^{-4})}\right]^{0.5}$$

= 45.3 meters

much larger than water depths in the region treated, and the numerical value in metric units of $(H_1T_1) = 14.45$, greater than the maximum water depth considered in meters, so that equation (12) indicates rough turbulent flow throughout the region considered. The effect of bottom friction is still appreciable for this relatively low-energy case, in that linear wave theory without dissipation would provide a wave height according to equation (5) of $(H_1 K_{S1}/K_{S1}) = [(1.7)(0.9551)/1.038] = 1.56$ meters at 9-meter water depth.

With linear wave theory, the height change between two water depths depends on wave period. Although only linear theory wave relationships are incorporated in the present calculation procedure, energy dissipation depends both on wave period and wave height (raised to the power of about 2.5). Thus, the calculated results have a nonlinear dependence on wave height: the computed height change between two water depths is affected by the actual value of wave height. This nonlinear aspect implies that these computations are not exactly reversible. Projecting a wave condition offshore without dissipation from a measurement site to d_m can result in a markedly different computed dissipation rate there than if the nominally corresponding waves are projected onshore to d_m . However, the calculation procedure tends to cancel internally this effect of nonlinear height dependence.

Reversing Example Problem 2, using $H_1 = 1.69$ meters at $d_1 = 9$ meters as the initial condition, computed conditions at d_m include $\xi_m = 1.24$ meters, $f_{em} = 0.0406$ and $\overline{E}_m = 7.52$ kilograms per second cubed, but at $d_j = 5.2$ meters the calculated wave height is 1.67 meters, only 1.8 percent less than the near-shore wave height of 1.7 meters originally specified. Using computed final wave heights in Example Problem 1 as input conditions, the reverse calculation procedure gives wave heights in each case only 2 percent less than the specified height of 3.5 meters.

Such slightly irreversible results do not seem too significant for potential applications. However, the Appendix to this report provides a calculator program quickly executing the present procedure, making it convenient to examine results of reverse calculations and to determine a wave condition which appears optimally consistent with that specified.

IV. SUMMARY

The equations and procedures presented here permit calculation of nearshore wave height changes considering the energy dissipated by rough turbulent flow over a strongly agitated bed of quartz sand. All elementary wave relationships are from linear (small-amplitude) wave theory, but one effect of incorporating dissipation is that calculated height changes depend on the actual wave height. Example calculations demonstrate the conversion of a nearshore wave condition into a corresponding wave height in shallower or deeper water; the present procedures are suitable only for field waves of relatively large height and period in fairly shallow water. The general effect of energy dissipation is that nearshore wave height remains more nearly constant outside the breaker zone than linear wave theory would predict.

LITERATURE CITED

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APPENDIX

CALCULATOR PROGRAM FOR WAVE SHOALING WITH DISSIPATION

The following four pages document a calculator program executing the procedure presented in Section II and exemplified in Section III of this report. This program runs in about 120 seconds on a Hewlett-Packard HP-67 Programmable Pocket Calculator, employing metric units, RPN logic, three levels of nested subroutines, 17 address labels, 26 storage registers, and 223 program steps. The program could be converted for use with other calculators having different logic systems but similar features and capacities.

Equations (1) to (10) are included with an effective root-finding iteration for wavelength. Values to be specified in metric units for a calculation are: ρ , g, H₁, T₁, d₁, d_j, D_m, and X. The standard value of g is 9.81 meters per second squared, and the value of ρ for seawater may be taken as 1026 kilograms per cubic meter; for freshwater, ρ is about 1000 kilograms per cubic meter, but common situations might not constitute the requisite rough turbulent flow over a strongly agitated bed. Satisfaction of these requirements, related to equations (11) and (12), remains to be considered external to the calculator program.

Program Description

Program Tille NEARSHORE WAVE SHOALING WITH ENERGY DISSIPATION
Contributor's Name ROBERT J. HALLERMEIER
Address RESEARCH DIVISION, U.S. ARMY COASTAL ENGINEERING RESEARCH CENTER
City KINGMAN BUILDING, FORT BELVOIR State VIRGINIA Zip Code 22060
Program Description, Equations, Variables FOR SPECIFIED CONDITIONS, THE PROGRAM COMPUTES
REQUIRED CHARACTERISTICS OF LINEAR GRAVITY WAVES AT THREE WATER
DEPTHS. THE INITIAL WAVE CONDITION AND MATERIAL CHARACTERISTICS
DETERMINE EQUIVALENT WAVE HEIGHT AT THE FINAL WATER DEPTH,
CONSIDERING ENERGY DISSIPATION OVER THE INTERVAL BY ROUGH TURBULEN
FLOW OVER A STRONGLY AGITATED SAND BED.
THE PROGRAM INCORPORATES EQUATIONS (1)-(10) FROM SECTION I.
THE APPROPRIATE SIGN FOR EQUATION (10) IS SELECTED AUTOMATICALLY,
ACCORDING TO WHETHER THE FINAL SITE IS LANDWARD OR SEAWARD OF
THE INITIAL SITE, ITERATIVE SOLUTION FOR NEEDED WAVE LENGTHS BEGINS
WITH FIRST-GUESSES PROVIDED INTERNALLY, AND USES THE EQUATION OF
THE SECANT METHOD FOR ROOT FINDING :
$x_{i+1} = x_i - F(x_i) \int (x_i - x_{i-1})$
$[F(x_{k}) - F(x_{k-1})]$
THESE IDENTITIES FOR HYPERBOLIC FUNCTIONS ARE USED :
$\sinh \chi = \frac{1}{2} \left(e^{\chi} - e^{-\chi} \right); \cosh^2 \chi = 1 + \sinh^2 \chi; \tanh \chi = \sinh \chi / \cosh \chi.$
NOTATION FOR VARIABLES AND CONSTANTS IS STATED ON USER INSTRUCTIONS.
RESULTS FOR SEVERAL EXAMPLES ARE PROVIDED IN SECTION II OF REPORT.
Operating Limits and Warnings SEE TEXT RELATING TO EQUATIONS (11) AND (12); THE
EXPRESSION USED FOR DISSIPATION COEFFICIENT IS VALID ONLY FOR
HIGH-ENERGY NEARSHORE WAVES ON EXPOSED COASTS WITH RELATIVELY
FINE SANDS. LINEAR MONOCHROMATIC WAVES PROPAGATING NORMAL TO
SHORE WITHOUT BREAKING ARE TREATED; THIS IGNORES MANY
FACTORS IN NEARSHORE WAVE TRANSFORMATION (SEE SECTION I).

User Instructions

NEARSHORE WAVE SHOALING WITH ENERGY DISSIPATION I INITIALIZE I= 3 AND SPECIFY 3-DIGIT FIXED-POINT DISPLAY. 2 ENTER QUANTITIES DEFINING SITUATION: REGISTERS 1-5/50-55, C: START

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1.	LOAD PROGRAM INTO CALCULATOR HEMORY AND			
	SPECIFY DISPLAY NEEDED FOR FOUR-DIGIT		FFIX	
	ACCURACY IN ROOT - FINDING PROCEDURE.		DSP 3	
2.	INITIALIZE I-REGISTER USED FOR CONTROL		3	
	OF INTERNAL PROGRAM BRANCHING.		h ST I	
3.	STORE QUANTITIES DEFINING SITUATION, IN			
	MKS UNITS (METERS, KILDGRAMS, SECONDS) :	(MKS)		
	a. PRIMARY VARIABLES			
	INITIAL WAVE HEIGHT, SQUARED	H, ²	STO I	
	WAVE PERIOD	T.	510 2	
	WAVELENGTH IN DEEP WATER	2T,2/21	STO 3	
	INITIAL WATER DEPTH MULTIPLE	zardi	ST0 4	
	FINAL WATER DEPTH MULTIPLE	2m d;	STO 5	
	IT. SECONDARY QUANTITIES (LESS FREQUENT CHANGES)	. 8		
	EXCHANGE PRIMARY SECONDARY STORAGE		F PRS	
	WAVE PROPAGATION DISTANCE	X	0 070	
	SAND GRAIN DIAMETER MULTIPLE	14.57 D.0.194	STOIL	
	NEEDED CONSTANT COEFFICIENT	5.882	STO 2	
	NEEDED CONSTANT EXPONENT	-0.194	STO 3	
	FACTOR IN ENERGY DISSIPATION RATE	0.2350(217)	504	
	FACTOR IN WAVE ENERGY FLUX		505	
	EXCHANGE PRIMARY SECONDARY STORAGE,	pg/8	1259 7	
	EXCHANGE FROMEY SECONDARY STULADE,			
4.				(MKS)
т .	START PROGRAM EXECUTION.			[Cinto]
e				ц.
5.	WAIT FOR STATIONARY DISPLAY OF FINAL ANSWER.			Hi
	EXAMINE OTHER COMPUTED QUANTITIES OF WTEREST,			
6,	E.G., FROM SELONDARY STORAGE REGISTERS :		F PZS	
	EVERGY DISSIPATION COEFFICIENT		RCL 6	Fem
	SPECIFIC ENERGY DISSIPATION RATE		Rel 7	E
	SPECIFIC ENERGY DISSTRATION MATE SPECIFIC INITIAL WAVE ENERGY FLUX		R4L 8	P.
	STECIFIC INITIAL WAVE ENERGY FLUX		259 3	
7.	RETURN TO STEP Z FOR NEW CALCULATION :			
1.	INITIALIZE I AND RE-ENTER ANY QUANTITIES TO			
	BE CHANGED. NOTE THAT AFTER PROGRAM RUNS			
	REGISTER 1 CONTAINS HT NOT H2.			

Program Listing I

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
OC 1	FLBLC	31 25 13	BRANCH TO COMPUTATIONS		RCLO	34 00	1
	GTO 3	22 03	FOR SUCCESSIVE DEPTHS		h RTN	35 22	1
	PLELO	31 25 00	DECREMENT I- REGISTER		FLOLD	31 25 14	FINITE - DIFFERENCE
	FDSZ	31 33		060	EEX	43	APPROXIMATION FOR
	GTO(1)	22 24	1		CHS	42	
	h RTN	35 22	1		2	02	FIRST DERIVATIVE IN
	FLBL3	31 25 03	INITIAL WATER DEPTH.		F %	31 82	SECANT METHOD.
	RCL 4	34 04	INITIAL WATER DEFIN.		STOC	33 13	
	STO E	33 15			2	02	
010	RCL Z	34 02	1		÷	51	
	F GSB E	31 22 15	1			51	
	FGSB A	31 22 11					
	FGSBB				STO A	33.11	
				870	STOO	33 00	
	L RTN	35 22	-	10	FGSB5	31 22 05	
	FLOL Z	31 25 02	FINAL WATER DEPTH.		STO D	33 14	
h	RCL 5	34 05			RCLA	34 11	
	STOE	33 15			RCLC	34 13	
	RCL Z	. 34 02			+	61	
	FGSBE	31 22 15			STO O	33 00	
020	FGSBA	31 22 11			FGSB5	31 22 05	
1	h RTN	35 22			STO B	33 12	
	FLBLI	31 25 01	INTERMEDIATE DEPTH:		RCLD	34 14	
	RCL 4	34 04	EQUATION (6)		-	51	
	RCL5	34 05		060	RCLC	34 13	1
	×	71			÷	81	
	FIX	31 54			h PTN	35 22	
	STOE	33 15			FLBL 5	31 25 05	L DEFINITION :
	RCL O	34 00			RCLE	34 15	
	FGSBE	31 22 15			RCLO	34 00	EQUATION (1).
030	9 GSB Fa	32 22 11			+	31	
	L RTN				yer		
			·		ge	32 52	
	FLBLE	31 25 15	ITERATIVE ROOT FINDER		ENTERA	41	
	FGSBD	31 22 14	USING SECANT METHOD.	090	h 1/2	_35_62	
	RCLB	34 12		090	-	51	
	GTO 4	22 04			2	02	
	FLBL6	31 25 06			÷	81	
	RCLO	34 00			STO 6	33 06	
	FG58 5	31 22 05			922	32 54	
	STO B	33 12			<u><u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u></u></u>	01	
040	FLBLA	31 25 04			+	61	
	RCLA	34 11			FJX	31 54	
	RELO	34 00			STO 7	33 07	
	STO A	33 11			RCL 6	34 06	
	-	51		100	+	81	
	RELD	34 14			h 1/2	35 62	
	RCLB	34 12			RELO	34 00	
	STO D	33 14			+	81	
	-	51			RELE	34 15	
	÷	81			×	71	
050	*	71			RCL3	34 03	
	510-0	33 51 00			n 1/2	35 62	
	RCL O	34 00			RELE		
	÷	81			X	34 15	
	FRND	31 24		110	×	51	
	F # # O				h RTN	35 22	
						35 22	
	GTO 6	22.06		TTOC	RIS	341	
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۹. (_; L, ; L	; Lul H^2; Ho2; H	$\frac{1}{12}^{2}T_{1}$	3gT,2/21 4211d,	27Tdi	6 sinh	coch	8 m; m; m; m ⁹ 5m
50	101		102 ICA 21	-8	S6 Fern		58 - 59
X	14.57 Dm	5.882	-0.194 0.235p(2m)	°pg18	Ferr	ST Em	P, (cm), (cm);
^ Li-1	8	F(L;)	CAL	^D F(L;-	.)	2md,; 2md;; 2m	INDEX
~ `						-1)	\$1

			rrogram					
STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMM	ENTS
	FLBLA	31 25 11	CONPUTATION OF (MC)	170	X	71		
	FGSB7	31 22 07		170	RCL 7	<u>34 07</u> 81		
	RCLB	34 08			STO +1	33 81 01		
	RCLO.	34 00			RCL I	34 01		
	RCL2	34 02			FIX	31 54		
	÷	81			2	02		
120	FPZS	31 42			÷	81		
	5109	33 09			RCL 6	34 06		
	FPZS	31 42			÷	81		
	ARCI	35 34			5709	33 09		
	2	02		180	FPZS	31 42		
	-	51			RCL 3	34 03		
	F 7 = 0	31 51			h mt RCLUI	35 63		
	GTO O	22 00			KCL	34 01		
	h RTN	35 22	<i>c</i>		RCL Z	וד 34 סב		
130	FLBLT		COMPUTATION OF M :		RCL Z	54 02		
	RCL 6	34 06	EQUATION (3),	-	API	32 52		
	RCL 7	34 01			9 ex 570 6	33 06		
·	RCLO	34 00			RCL 4	34 04		
	×	71		190	×	71		
	h 1/2	35 62			FPZS	31 42		
	RCLE	34 15			RCL9	34 09		
	×	71			RCL 2	34 02		
	1	01			÷	81		
	+	61			3	03		
140	2	02			h m²	35.63		
	÷	81			UX VX	71		
	STO B	33 08			FPZS	31 42		
	L RTN	35.22		. 200	STO 7	33 07		
	FLBL B	31 25 12	COMPUTATION OF PI, KS		FPZS	31 42		
	RCLI	34 01	EQUATIONS (2), (4).		RCL 5 RCL 4	34 05		
	FP25 RCL9	31 4Z 34 09			F PZS	31 42		
	X	71				32 81		
-	RCL 5	34 05			TTO B	22 08		
150	×	71			9 2 > 4 GTO 8 GTO 9	22 09		
	STO B	33 08			FLBLB	31 25 08		
	FPZS	31 42			RCL 7	34 07		
	RCL 6	34 06			CHS	42		
	RCL7	34 07		210	GTO Fe	22 31 15		
	÷	81			FLBL9	31 25 09		
	RCL 8	34 08			RCLT	34 07		
	×	11			g LBL Fe	32 25 15		
	2	02			RCLO	34 00		
160	STO XI	33 71 01			RCL B	34 08		
	GTO O	22.00			+	51 00		
	h RTN	35 22			RCL S	34 05		
		32 25 11	COMPUTATIONS FOR		+	81		
	9 LBL Fa FGSB7 RCL B	31 22 07	INTERNEDIATE DEPTH	220	RCL9	34 09		
	RCL 8	34 08			÷	.81		
	2	02	EQUATIONS (5), (7)-(0)	'	FVZ	31 54		
	×	11	WITH SIGN CHOICE,		PPZS	31 42		
	RCL 6	34 06	LABELS	1	h RTN	35 22	SET STATUS	
A	185	- K			FLAGS			
mc EG	N. BP. K.	EQNS. C STAN		T FINDER		FLAGS	TRIG	DISP
adm ca	LCS. D	- ° -	- d e+	I; EQN.	- 1		DEG 🕺	FIX DA
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Program Listing II

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Hallermeter, Robert J.	Hallermeler, Robert J.
Calculation of wave shoaling with dissipation over nearshore sands by Robert J. HalteneaterTork Belvoir, Va. : U.S. Arwy. Corps of Engineer Coastal Engineering Research Center ; Springfield, Va. : available from NTIS, 1983. [21] p. : ill. : 28 cmCoastal engineering technical aid ; no. [21] p. : ill. : 28 cmCoastal engineering technical aid ; no. [22] wore title. "More title. "Nore title. "Nore 1983." Report provides a calculation procedure for nearshore shaaling of Report provides a calculation over a strongly agitated bed of quartic and. Example computation over a strongly agitated bed of quartic stretgeic waves outside the breaker zone. including the apreciable effects of energy dissipation over a strongly agitated bed of quartic and. Example computations and a calculation. 3. Energy dissipation. 4. Nearshore waves. 5. Monlinear wave propagation. 6. dave height. (U.S.). III. Seties.	Calculation of wave shoaling with dissipation over nearshore sands / by Robert J. HallermeiterFort Belvoir, va. 10.5. way, Corps of Engineers, Coastal Engineering Research Center; Springfield, Va. : available from NTIS, 1983. [21] p. : ill.; 28 cm(Coastal engineering technical aid ; no. [21] p. : ill.; 28 cm(Coastal engineering technical aid ; no. 83-1) Cover title. "March 1993." "March 1993." Report provides the breaker zone, including the appreciable effects of energy dissipation over a strongly agitated bed of quartz sand. Example computations and a calculator program are provided. 1. Attracted sand bed. 2. Bortom friction. 3. Energy dissipation 4. Nearehore waves. 5. Nonlinear wave propagation. 6. Wave height. 7. Wave sholling. 1. Title. II. Coastal Engineering Research Center (U.S.). fill. Sertes.
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Calculation of wave shoaling with dissipation over nearshore sands (b) by Robert J. HalteneterFort Belvoir, Va. : U.S. Arwy, Corps of Engineers, Coastal Engineering Research Center ; Springfield, Va. : available from NTTS, 1935. [21] p. : 111. ; 28 cm(Coastal engineering technical aid ; no. [21] p. : 111. ; 28 cm(Coastal engineering technical aid ; no. [22] varten 1983. March 1983. Report provides a calculation procedure for nearshore shoaling of Report provides a calculation over a strongly agitated bed of quartic and. Example computations and a calculator program are provided. 1. Agitated sand bed. 2. Bottom friction. 5. Mare height. Areashore waves. 5. Montinear wave propagation. 6. Wave height. (U.S.). TIL. Setes.	Calculation of wave shoaling with dissipation over nearshore sands by Robert J. HallermeitPort Belvoir, Va. 1.5. Arwy, Corps of Engineers Coastal Engineering Research Center; Springfield, Va. : available from NTIS, 1983 [21] p. :111. ; 28 cm(Coastal engineering technical aid ; no. (21] p. :111. ; 28 cm(Coastal engineering technical aid ; no. (20 er title. March 1983." Report provides a calculation procedure for nearshore shoaling of energetic waves outside the breaker zone, including the appreciable effects of energy dissipation over a strongly aglisted bed of quartic sand. Example computations and a calculator program are provided. 1. Agtracted sand bed. 2. Bontinear wave propagation. 6. Wave height. 7. Wave shoaling. 1. Title. II. Coastal Engineering Research Center (U.S.). III. Sertes.
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