THE

Capillary = = =

= = Electrometer

IN THEORY AND PRACTICE.

Part I.

BY

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RESEARCHES MADE WITH THE APPARATUS DESCRIBED IN THIS PAPER.

BURCH.—"On a Method of Determining the Value of Rapid Variations of a Difference of Potential by means of the Capillary Electrometer," *Proc.* Roy. Soc., Vol 48.

BURCH.—"The Time Relations of the Capillary Electrometer," *Phil. Trans.*, Vol 183A, p. 81.

BURCH.—"On the Calibration of the Capillary Electrometer," *Proc.* Roy. Soc., Vol. 59, p. 18

BURCH AND VELEY.—"The Variations of the E.M F. of certain Metals in Nitric Acid," *Phil. Trans.*, 182A p 319

SANDERSON AND BURCH.—"On the Localization of the Effect of Injury in Muscle," *Proc.* Physiol. Soc , June 24, 1893.

BURCH AND HILL.—"On d'Arsonval's Physical Theory of the Negative Variation," *Journal of Physiology*, Vol. XVI., p. 319.

GOTCH AND BURCH.—"Electromotive Properties of Malapterurus Electricus," read before the Royal Society, May 7, 1896. Will appear shortly. See *Nature*, May 28, 1896, p. 92.

J. BURDON SANDERSON.—"Electrical Response of Muscle," *Journal of Physiology*, Vol. XVIII., p. 117.

PREFACE.

I HAVE often been asked to describe my method of making a capillary electrometer, and my apparatus for the measurement of rapid changes of E.M.F. My aim in this series of articles, written in October, 1895, has been to do so with sufficient fulness to enable anyone possessed of average skill to construct the instrument.

I have laid most stress on the production and interpretation of the photographic records as illustrating the special characteristics of the capillary electrometer; but it is hardly necessary to point out that it may be used by direct observation with the Wheatstone bridge, or with the potentiometer, instead of a high-resistance galvanometer, and that it may replace the quadrant electrometer in comparing the capacities of condensers. In response to a generally expressed wish, these articles have been reprinted, as a preliminary to the more complete work upon the subject which I hope to write.

GEORGE J. BURCH.

21, Norham Road, Oxford,
August, 1896.

THE

CAPILLARY ELECTROMETER IN THEORY AND PRACTICE.

The capillary electrometer was invented by Lippmann, and described by him in a thesis published in 1875. Being specially adapted for the study of electromotive changes of short duration, it attracted the attention of physiologists. In 1877 Marey employed it in investigating the functions of the electrical organ of the torpedo, and succeeded in obtaining photographic records of its indications. In England Prof. Burdon Sanderson used it first in a research by which he proved that the closure of the leaf of the Venus' Fly Trap is preceded by electrical changes resembling those which accompany the contraction of animal muscles. In a lecture delivered before the Royal Institution in 1882 he exhibited photographs of the excursions of the capillary electrometer produced in this way.

My own connection with the subject dates from the winter of 1886, when I was shown the originals of these photographs and the apparatus which had been in use up to that time. As none of the capillary electrometers were in working order, and there was a difficulty in obtaining fresh ones in England, I offered to make some, and succeeded, after a few trials, on the last day of the year. Finding the whole subject to be of great interest, I undertook also the installation of new photographic apparatus, and since that time have devoted a good deal of my leisure to perfecting the method and collaborating with Prof. Burdon Sanderson in its physiological applications.

It is strange that an instrument so easily constructed and so sensitive should have remained for nearly 20 years practically unknown, save to physiologists. But although the principle of the electrometer had been studied up to a certain point, the practical details essential from the standpoint of the instrument-maker had not been worked out. My object was to find out these details by a systematic investigation of the mechanical and electrical constants involved and to publish them.

After two or three months' work I began to see much greater possibilities. There was a feeling among physiologists that the indications of the capillary electrometer did not represent quantitatively the value of rapid changes of E.M.F., because the meniscus, although it moves very rapidly, requires an appreciable time for its excursion. Accordingly such photographic records as had been obtained were relied on solely to determine the direction of a current and the period, but not the amount, of any change in its intensity. It occurred to me that if the laws of the movement could be determined it might be possible from the photographed curve of an excursion to draw another curve representing the variations of E.M.F. which caused it. I had already nearly sufficient data, and after a few more experiments was able, without using any hypotheses, to show that the velocity of the meniscus at any moment is directly proportional to the distance through which it would have to move before coming to rest under the influence of a constant current equal in intensity to the P.D. between the terminals at that instant. Furthermore, I invented a simple method of measuring the velocity of the meniscus at any point on the photograph of an excursion.

This discovery places the capillary electrometer in a unique position. It becomes available for a class of work that no other instrument can perform. A cycle of electromotive changes can be plotted down within 1 per cent. of the truth, at intervals of $\frac{1}{2000}$th of a second, with the present apparatus, and there is every indication that it might be driven five times as fast. It seems probable that the capillary electrometer may do for the dynamo what the indicator diagram does for the steam engine. In the summer of 1894 I obtained by its means photographs of the excursions produced by telephone currents, in which the characteristic vibrations of the various consonant sounds could be distinguished.

I propose in these articles to describe minutely the method of making an electrometer, and to discuss the relation of its physical constants to the dimensions and form of the capillary. I shall then give full details of the apparatus for producing the photographic records and measuring them, together with a brief account of the applications of the method during the seven years that it has been in use, including among the illustrations the hitherto unpublished photographs of telephone currents which I exhibited at the meeting of the British Association in Oxford. These will be followed by the current-curves of several dynamos recently photographed by myself. I shall conclude with the methods of using the instrument for direct observation without photography.

The principle of the capillary electrometer may be best illustrated by the following experiment. Take a piece of soft glass tubing of about 5mm. or 6mm. diameter, and draw it out in the middle so as to form a capillary of about 0·5mm. bore. Cut it in two, and bend each half into a U-shape, the limb formed by the capillary being shorter than the wide limb. Fill each with pure mercury and support them side by side in a beaker of dilute sulphuric acid of 25 per cent., so that the orifices of both capillaries may be immersed. Expel the air from the capillaries by blowing down the wide limbs of the U-tubes.

The mercury will stand higher in the wide portions of the two tubes than in either of the capillaries, and the difference of level will be greatest in that U-tube which has the narrowest capillary. Now pass a clean platinum wire into each of the wide limbs, and connect these wires for a moment with the derived circuit of a rheocord, or any convenient source of P.D. not exceeding 0·5 volt. The mercury column in the capillary by which the current enters will rise, and that by which it leaves the circuit will be depressed. This altered condition will persist after the circuit is broken, but on joining the two platinum wires together the mercury will instantly return in both tubes to its former level. Lippmann explains this phenomenon by saying that the surface-tension between the mercury and the sulphuric acid is altered by the action of the current, and that this alteration is directly proportional between certain limits to the P.D. between the terminals. In practice it is unnecessary, and would be inconvenient to observe the movement in both tubes, and accordingly one of them is made larger in both limbs than the other, and serves to contain the acid into which the capillary point of the second tube dips. For some purposes, however, the arrangement above described answers better than any other.

How to make a Capillary Electrometer.

In determining the form of the instrument the following considerations must be borne in mind. The pressure of the mercury against the glass must be sufficient to keep the acid from creeping past it to the platinum wires, and the columns should therefore occupy at least 5cm. of the vertical part of the tubes. Provision must be made for washing out the acid and replacing it with fresh, from time to time.

The maximum sensitiveness, and the greatest rapidity of action, are only obtained together when the tube is cut off at a certain part of its length. This is best done after the electrometer has been put together. For these reasons I much prefer those forms in which the capillary is not permanently fixed to the U-tube.

Soft soda-glass answers best. The tubes should be examined with a lens for capillary passages in the walls. These, if large, are not always obliterated in the subsequent manipulation, and they impair the definition under the microscope. The selected pieces must be washed out three times with aqua regia, and then rinsed very thoroughly with *pure* distilled water. If on heating a tube so prepared over a Bunsen burner and sucking air through it, the least taste can be perceived, the cleansing is not sufficient. Finally, the tubes are dried by aspirating air through them. The operation of making the electrometer should be proceeded with at once.

The normal form for ordinary use is shown in Fig. 1. The electrometer tube A is from 4mm. to 6mm. in diameter and 10cm. or 12cm. long from the top to the first bend. The walls should be about 1mm. thick. To make it, cut off about 25cm. of the cleansed tube, and heat it in the middle before the gas blow-pipe till it thickens, and the bore is reduced to one-third of its original diameter. Remove it from the flame, and very gently draw it out, till the centre portion is 1mm. or 1·5mm. in diameter. Fuse the middle of the narrow part in the point of the flame till it separates, sealing up both ends in the act. You have now a pair of tubes tapering at one end to a strong, thick walled capillary, the bore of which is several times larger than that of the finished instrument. The next step is to make the crook. Holding the tube in both hands,

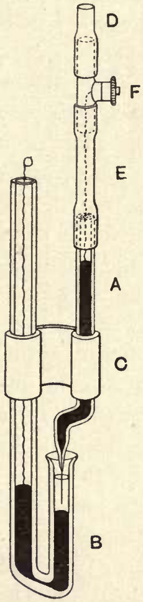


Fig. 1.—Normal Form of Capillary Electrometer.

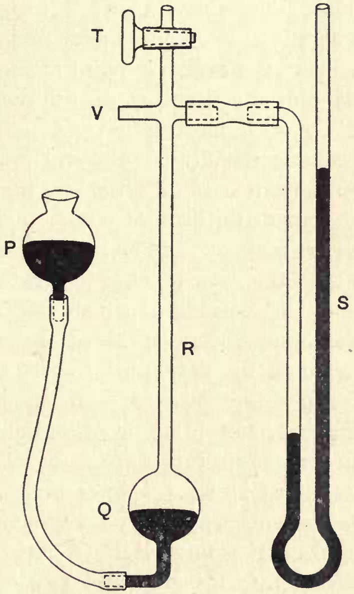


Fig. 2.—The Pressure Apparatus.

heat it just above the tapering part for a distance of 1cm. When it is soft a slight lateral displacement of the hands, the two ends of the tube being kept parallel, forms both bends of the crook in one operation. A little care is necessary to prevent the tube from becoming flattened. Lastly, the upper end of the tube is rounded in the flame, which must not be allowed to blow into it.

The U-tube B has next to be made; this may be of thinner glass than the other. After forming a slight lip on one end and rounding the walls of the other, it is heated in the flame until it thickens and begins to close up. It is then very slightly stretched out and at the same time bent into the shape shown in the figure. The object of making the tube narrow in the bend is to prevent the mercury from flowing too freely from side to side when the instrument is carried about. The clip C is of thin sheet brass. It is about 25mm. wide, and is lined with a strip of morocco leather of the same width. The tubes A and B are gripped firmly by it, but not so tightly as to require undue force to adjust them. It remains to draw the capillary point of the tube A. For this purpose I employ the flame of an ordinary bat's wing burner turned so low as almost to go out. Holding the tube in the left hand and the drawn out end between the finger and thumb of the right hand, I bring the middle of the narrow part into the flame, the heat of which is just sufficient to soften, but not to melt it. Removing it, I suddenly and firmly draw it out, taking care to keep it parallel with the large end of the tube. I then cut it across the middle. For this purpose I use a small three-cornered file with the teeth ground off, hardened by dipping it while red-hot into cold mercury. Nothing but practice will ensure the drawing of a good capillary, but if on examination with the lens it does not appear satisfactory a piece of glass may be fused on and a new point drawn. I have even made a new point to an old electrometer tube without emptying out the mercury. The instrument is now ready for putting together. The tubes are slipped into the clip and twisted round until the end of A comes directly over the mouth of B. Exact parallelism of the capillary part of A with the walls of the U-tube B is secured by slightly bending the brass clip C. Mercury is then poured into A from a capillary pipette freshly made from a piece of perfectly clean glass tubing. The tube B is also filled with mercury to a depth of about 5cm. The mercury must have been recently distilled in vacuo.

It remains to attach the pressure apparatus. This consists of a pair of glass bulbs, P and Q, connected by a thick-walled rubber tube. P is suspended by a cord passing over a small pulley and attached to a bobbin, by turning which it can be raised or lowered. The bulb Q has attached to it the four-way tube R, leading to the pressure gauge S, the stop-cock T, and the tube V, with which the electrometer is connected. The stop-cock is necessary in order that the bulb Q may be easily refilled with air in case of leakage. A piece of pressure tubing, D, about 40cm. long, part of which is shown in Fig. 1, is slipped over the tube V (Fig. 2) and secured with wire.

The mode of completing the circuit is seen in Fig. 1. F is a terminal attached to a piece of brass tube, inside which a platinum wire is soldered. This wire passes down the short piece of rubber tube E nearly to the bottom of the straight part of the tube A, and a similar wire is inserted in the open limb of the tube B. The platinum should be made red-hot before putting it into position, to burn off impurities. All the joints should be made with burnt rubber, save that which connects E with A, for which vaseline may be used, so that it can be easily taken on and off.

A screw clamp fixed to the adjustible stage of a microscope, with its tube horizontal, grasps the tube A firmly just above the clip C, and an insulated support must be provided to take the weight of the rubber tube D so as to prevent any undue strain on the capillary.

When everything is ready the bulb P, Fig. 2, is raised, and mercury flows into Q, expelling some of the air from it, and thus forcing the mercury in the electrometer tube through the capillary. When this is the case a few drops of dilute sulphuric acid of 25 per cent. are poured into the short limb of the tube B, and the pressure-bulb immediately lowered till the flow ceases. The circuit is next completed by connecting the platinum wire in B with the terminal F, after which the pressure may be cautiously increased until the mercury column nearly reaches the tip of the capillary, when the electrometer is ready for testing. The circuit must always be closed before increasing the pressure, otherwise the instrument acquires a charge which prevents the mercury from rising to its proper position in the capillary.

Another form of electrometer, illustrated in Fig. 3, is useful for some kinds of work. It differs mainly in having the point of the capillary directed upwards instead of downwards. The tube B is reduced to a mere collar slipped over the wide part of the U-shaped electrometer tube. A platinum wire twisted round the neck of the capillary passes downwards between it and the tube B, which may be fused on, or secured by cement. Mercury is then poured into A and B, and a few drops of acid are put into B. These electrometers work well for a time, but do not last long if cement is used.

Owing to the curvature of the surface through which the capillary has to be observed, the top and sides of the mercury

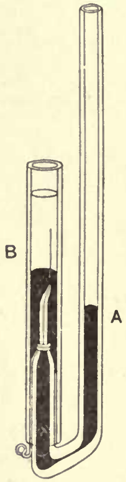


Fig. 3.—Electrometer with Upward Pointing Capillary.

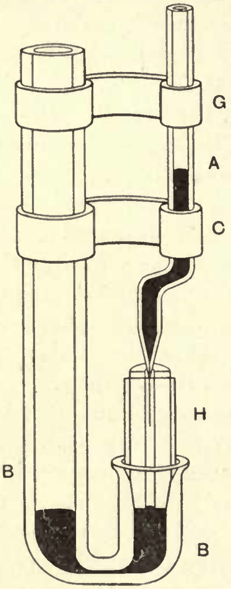


Fig. 4. Projection Electrometer.

column are not in focus simultaneously under the microscope. The definition is greatly improved by placing a thin cover-glass against the tube B, to which a drop of acid causes it to adhere. The acid has the advantage of not drying up, and any other liquid would be likely to creep round into the electrometer and spoil it.

For the production of photographic records by projection more perfect definition is needed, and I accordingly employ the modification shown in Fig. 4, in which nothing but a thin cover-glass comes between the capillary and the objective. The tube A is somewhat longer than the corresponding part of Fig. 1, in order to allow room for two clips, G and C, instead of one. The tube B is considerably larger, and much more strongly lipped. In the shorter limb it carries a piece of very thick-walled burette tubing, H, ground in like a stopper, and projecting about 25mm. The front of this tube is ground away, so that the bore of it is laid open from end to end, thus forming a trough just wide enough to contain the capillary without undue risk of breakage. A piece of thin cover-glass closes the trough in front, and the surface tension of the acid keeps it full of liquid. The clips C and G are clamped firmly by means of screws not shown in the diagram, and they are so arranged that the small adjustments necessary to bring the capillary exactly parallel with the front of the trough can easily be made. This instrument is somewhat fragile, but the definition of the projected image is greatly superior. I am at present engaged upon an improved form, which will need less careful handling.

The excellence of these electrometers depends on the shape and dimensions of the capillary. The following explanation may therefore be found useful. When a glass tube heated in the middle is drawn out, the walls of the central reduced portion are approximately parallel for a certain distance. Between this cylindrical part and the unaltered tube at each end is a portion the bore of which tapers rapidly at first and then more slowly, passing insensibly into the parallel walled capillary. If a tube so drawn out is fixed vertically, and some mercury poured into it, the mercury will be forced by its own weight some little distance down the tapering part, until stopped by the action of its surface-tension. On increasing the pressure with the apparatus described in Fig. 2, the top of the mercury column will be forced lower, but the meniscus at the bottom of the column will move through a much greater distance, owing to the small diameter of the tube at that part. The total increase of pressure on the lower meniscus will therefore be greater than that communicated to it from the pressure apparatus by the additional length of the mercury column in the electrometer tube. As the less tapering parts of the capillary are approached, this effect becomes greater and greater, until a point is reached beyond which the meniscus cannot be pushed without danger of its running down into the cylindrical portion, when the mercury will continue to flow till the tube is empty. To use such a tube for the capillary of an electrometer, it must be cut off at some point short of that at which the mercury begins to run through without further increase of pressure. If the section is made too close to the neutral point, the instrument will be very sensitive, but troublesome to use, on account of the difficulty of stopping the flow when once it has been established. On the other hand, if the capillary is cut off too short, the sensitive part will be sacrificed.

But although any tube thus drawn out can be cut so that it will act as an electrometer, it does not follow that it can be made into a good one. To ensure definiteness of response to a difference of potential it is essential that friction should be avoided. The working portion of the tube must therefore be as short as is consistent with a good shape. For this purpose I draw the tube down to a diameter of about 1mm, keeping the walls fairly thick, and just before putting the instrument together I soften the middle of this part in a very small flame, and draw it out 15mm. or 20mm. Thus the actual capillary is not more than 5mm. to 10mm. long. As the instrument is always used under the microscope, with a power of from 20 to 400 diameters, this length is amply sufficient. The end of the tube should be cut clean across. A splintered orifice impairs the action of the instrument.

Capillaries which are to be used horizontally should be less tapering if equal sensitiveness is desired. Those which point vertically upwards should be of equal bore throughout the working portion. As they can be made of constant sensitiveness and constant capacity for all positions of the meniscus, they are useful for some kinds of work; but the time-relations of the excursion are more complicated, and they are, therefore, not so well fitted for the production of photographic records.

In all downward-pointing instruments, sensitiveness to small differences of potential is gained both by using a smaller capillary, and by making it taper less rapidly. In either case the movement is rendered slower, but from a different cause. With a smaller tube, the internal electrical resistance is greater; but with a capillary of wider bore, made equally sensitive by being more nearly cylindrical, the electrostatic capacity of the instrument is increased, and it may take longer to charge although the resistance is less. Small inequalities in the tube produce a greaterdisturbing effect on the excursions, and the meniscus is more sensitive to mechanical vibrations. There is, therefore, a limit beyond which the diameter of the tube cannot profitably be increased. On the other hand, a very fine tube is liable to get clogged, and although it is very little affected by jars or vibrations, it is not nearly so sensitive to electromotive changes of short duration. Moreover, the pressure-tube has to be wired on to the capillary, and sooner or later leakages occur. Practically, I prefer tubes that require a working pressure of about 20cm. and do not care to go above 30cm. or below 10cm. After a little practice it is easy to draw capillaries of any desired diameter.

When not in use the instrument should be invariably short-circuited. For class work a spring key, held open by the student during an observation, is very useful. Should all the mercury run out of the capillary the electrometer is hopelessly ruined, as it is impossible to get rid of the sulphuric acid which immediately enters. But by having a sufficient quantity of mercury in the upper tube to completely fill the lower tube in case of such an accident, this risk may be obviated. The mercury may then be cautiously sucked back by a negative pressure until it refills the capillary, taking care to stop before any acid enters. Should the supply run short at any time fresh mercury may be introduced with a fine pipette, inserted in the top of the upper tube after disconnecting it from the pressure apparatus.

Sometimes crystals form in the capillary. If these cannot be expelled they may be decomposed by electrolysis, using a P.D. of about 0·5 volt, and making the capillary the cathode. Inasmuch as this process tends to form crystals at the anode, these must be carefully washed out of the U-tube and fresh acid substituted. Old electrometers in which no crystals are visible are often much improved in this way. They must be subjected to the action of the current for an hour or more.

The pressure should be invariably taken off when the instrument is left. The weight of the column of mercury in the tube itself is sufficient to keep the acid from creeping up, and if the meniscus is allowed to stand continually in the working portions of the capillary the mercury is likely to stick whenever it passes that part.

A good instrument should respond easily to $\frac{1}{30000}$th of a volt.

If the meniscus recedes when the electrometer is on short circuit it is a sign of leakage in the pressure tubing. If it "creeps" up or down when the circuit is open there is an E.M.F. somewhere, either due to a dirty contact or to the acid in the electrometer having crept past the mercury to the platinum wires; the latter defect is fatal. If when a charge has been communicated to it and the key is open, the mercury gradually returns to zero, the insulation is defective. Not more than 5 per cent. of the charge should escape in half-an-hour. If made with clean and pure materials an electrometer should remain in good order for several years.

Induction shocks, strong enough to be felt, exert a very deleterious effect; the sudden jar appears to disintegrate the glass where the meniscus touches it, causing the mercury to catch as it passes along the tube, though no mark can be seen. But a step-down transformer with a couple of cells on the primary may be used with impunity. The capillary electrometer is not affected by ordinary magnets, nor by proximity to an arc light, but wires in circuit with it should not be carried near high-tension alternating currents.

Physical Constants of the Capillary Electrometer.

Both from the point of view of the manufacture of the capillary electrometer and of its application as an instrument of research, Prof. Lippmann's Paper left a great deal to be done. The question of the electrical capacity of the instrument and of its resistance, and of the time relations of its excursions, were as yet untouched. I determined to attack these problems systematically by the experimental method.

The point to be first determined was whether any leakage of current could be detected through the instrument itself after the meniscus had ceased to move under the action of a difference of potential. I joined up a high-resistance galvanometer of great sensitiveness, and the electrometer, in series in the derived circuit of a potentiometer. The galvanometer being short-circuited, I made the battery connection, and waited till the meniscus of the electrometer had come to rest in the new position corresponding to the P.D. employed. I then opened the short circuit of the galvanometer. There was no deflection. But on increasing or diminishing the E.M.F. of the derived circuit, there was a temporary current one way or the other while the meniscus was in movement.*

The corresponding problem was this: Does an electrometer which has been charged to a certain P.D. lose its charge when disconnected from the source of electricity? I found that it does not, if the external insulation is good, but that owing to the necessary presence of the dilute sulphuric acid, perfect insulation is not easy to attain. Still, a displacement of the meniscus through 50 divisions of the eye-piece micrometer was maintained for more than an hour with a loss of less than one division. We are led, therefore, to the conclusion that the capillary electrometer, although its circuit is composed entirely of conductors, does not transmit a current, but merely receives a charge as though it were a condenser. But there is no visible dielectric between the mercury and the acid. I have left an instrument for a whole day under the strain of an E.M.F. of half a volt, and not a trace of hydrogen could be detected in the capillary.

The next point was to determine the capacity of the electrometer and its relation to the form of the tube. The following method was adopted. A standard condenser was charged to a known P.D. by means of a potentiometer. It was then suddenly connected with the uncharged electrometer, causing the meniscus to move from zero to a position corresponding to the E.M.F. of the charge communicated to it. The actual value of this E.M.F. was found by switching the potentiometer on to the electrometer circuit, and shifting the rider till the meniscus was adjusted to the observed position. From these data the capacity of the electrometer was calculated by the usual formula. Double break keys were used, and they and

* I found afterwards that Fleischl had tried this in 1879.

the commutators were arranged like the keys of a piano, so that the operations could be conducted with great rapidity.

The results of a large number of experiments with about 40 instruments may be summed up as follows :—

1. The narrower the tube the smaller the capacity. The greater the sensitiveness the greater the capacity.

2. As in most cases the capillary tapers less rapidly towards the tip, and is consequently more sensitive in that part, so also the capacity of most electrometers is somewhat greater when the meniscus is near the tip than when it is farther from it.

3. The capacities of the instruments examined ranged from 0·1 microfarad to 30·0 microfarads—from 0·5 to 2·0 giving the best results.

4. The electrical capacity as thus measured is that of the part of the capillary at which the meniscus comes to rest under the influence of the charge, and may be greater or less than that of the point from which it started to move.

5. With this proviso, the capacity is entirely independent of the potential difference of the condenser charge, which may be as great as can be safely employed, or as small as can be accurately measured.

6. The capacity is the same for any given part of the tube, whether the zero be above it, so that the condenser charge causes the mercury to fall; or below it, so that a similar charge in the reverse direction causes the mercury to rise, so long as it comes to rest when charged at the same part of the tube in both cases.

7. The capacity can be measured equally well by causing the charged electrometer to share its electricity with an uncharged condenser of known capacity.

8. Some capillaries are equally sensitive through a considerable portion of their length. In such cases the capacity is constant throughout the same part.

9. The capacity of a newly made electrometer usually, though not always, increases during the first few days. After about a week it becomes constant. I am inclined to ascribe this to the presence of air or gas condensed on the surface of the glass.

It is evident from these experiments that the capillary electrometer may be regarded for all practical purposes as a condenser possessing a definite electrical capacity for each position of the meniscus, and that this capacity is independent of the potential difference of the charge, which is a function of the change of pressure necessary to keep the mercury stationary in that position.

The change of level of the end of the mercury column produced by putting it in connection with a constant current is due to the flow of a measurable quantity of electricity. It also involves the performance of a definite amount of work, namely, in lifting the column of mercury from one position to the other, in altering the shape and area of the meniscus, and in overcoming friction. Evidence of the effect of friction can be observed in electrometers of which the tube is not perfectly clean, but otherwise I was unable to detect it.* The shape of the meniscus while at rest in any given portion of the tube is not visibly different under the action of a difference of potential in either direction, its outline projected on a screen with a magnification of 250 diameters remaining unaltered. The difference of section area in the working portion of the capillary of a sensitive electrometer is also small compared with the variation of the length of the mercury column. Putting P for the total pressure, l for the distance through which the meniscus is moved, and r for the radius of the tube, we may say that, approximately,

$$\pi r^2 l \mathrm{P} = \alpha \mathrm{C} \mathrm{V},$$

where V is a very small difference of potential and C is the capacity of the tube at the point where the meniscus stops, and α is a constant depending on the instrument used. But the working pressure varies inversely as the radius of the capillary. Writing therefore $\frac{1}{\mathrm{P}} = r$, and omitting constants, we find that $\frac{l}{\mathrm{P}}$ is proportional to the work done, and proportional also to the quantity of the charge C V. I determined this ratio for a large number of instruments, and found it to be fairly constant, in spite of the great differences in sensi-

* Fluid friction affects the velocity, but not the extent of an excursion.

tiveness, diameter and electrical capacity of the capillaries employed.

Resistance.

The resistance of the circuit has not the same effect upon the capillary electrometer as upon a galvanometer. It simply diminishes the rapidity of the movement without in the least reducing its extent—unless, of course, the instrument is partly short-circuited by defective insulation. This being so, it is impossible to measure the resistance of the electrometer by the ordinary methods.

When a difference of potential is communicated to the instrument, the meniscus moves with a gradually-diminishing rapidity to its new position of rest, which is attained only after the lapse of an appreciable, but indefinite, time. The time required to fully charge the instrument cannot be measured. But the time of half-charge—*i.e.*, when the P.D. between the terminals of the electrometer is half that of the source, is well defined, and easy to measure. I effected it, in the first place, by means of a rheotome arranged so as to close the circuit of a potentiometer, and break it again after a known fraction of a second. The duration of the closure was varied until the excursion produced by it was equivalent to that given by a constant P.D. of half the strength. I then added a resistance of 100,000 ohms to the electrometer circuit and found that a longer closure was necessary to produce the same excursion, namely, that of the half-charge. The total quantity of electricity being the same in each case, it was possible from these data to calculate the "equivalent resistance" of the electrometer. This varied, in the different instruments, from 90,000 ohms to 700,000 ohms. It is convenient in practice to know the "equivalent resistance" of an electrometer, especially when high resistances have to be used or a shunt introduced. But the actual resistance varies from point to point of the tube, and necessarily so, since it is due, to a considerable extent, to the low conductivity of the dilute acid contained in the portion of the capillary not occupied by the mercury. But it is not wholly due to this. I found by a series of careful measurements of the time of half-charge, with the zero position of the mercury at different distances from the tip of the tube, that the "equivalent resistance" does not vary so rapidly as it should do if it depended solely on the resistance of the sulphuric acid and the mercury. Some other cause—probably mechanical—conditions the velocity of the movement, so that the equivalent resistance at any given part of a capillary would be expressed by the formula

$$R = r\,(L + l),$$

where l = the distance of the meniscus at any moment from the tip of the tube, and L is a constant of the particular instrument employed.

With an electrometer adapted for the production of photographic records, the determination of the equivalent resistance for any given position of the capillary is extremely simple. Two normal excursions are photographed, one with say 100,000 ohms in circuit, and the other with no resistance, or if the electrometer is affected by overshooting, with a resistance of 10,000 ohms. The angle of inclination of each curve at corresponding points is then ascertained in a way that will be subsequently described, by measuring their respective subnormals. Let the subnormal to the first curve be N, and let that to the same part of the second be N′,

then $$N - N' : N' :: 100{,}000 \text{ ohms} : R,$$

where R = the equivalent resistance of the electrometer at that point. This method has been used by me in the Physiological Laboratory, Oxford, ever since July, 1890.

Production of Photographic Records.

The capillary electrometer is essentially the poor man's instrument, costing, even including the price of a microscope, less than any other form of electrical measuring instrument of equal delicacy. It is also by far the easiest to make. But it has certain disadvantages. It is fragile and easily spoilt by careless usage, and unless employed for zero methods, requires careful calibration against a standard instrument. It has, however, one property which outweighs all these disadvantages, and in virtue of which it stands alone among the implements of research. The rapidity with which it responds to all changes of potential is so great that nothing but photography can render all its movements visible. It is accordingly this branch of the subject to which I have devoted a great deal of my time. The form of the instrument is such as to render the production of photographic records of its excursions extremely easy. The magnified image of the mercury column is projected on to a narrow vertical slit, behind which a sensitive plate travels with a uniform horizontal motion. It thus forms a continuous band of shadow, varying in height according as the position of the meniscus changes. But however sharply the top of the mercury column may be focussed on the screen, a sudden to-and-fro excursion of the meniscus will produce a blur instead of a sharp-edged shadow, unless the plate moves through a space greater than the width of the slit during its occurrence. And if the slit is made very narrow, not only is the illumination feeble, but diffraction comes into play and spoils

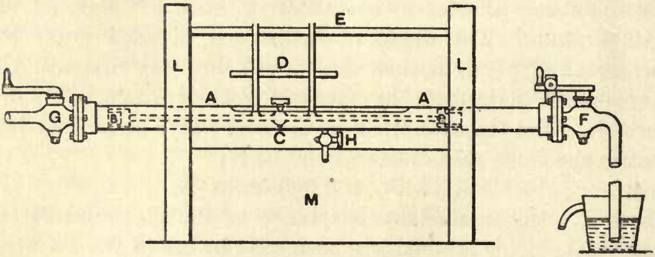


FIG. 5.—Hydraulic Motor.

the definition. My first improvement consisted in the use of a cylindrical lens of 25mm. focus placed about 25cm. behind the slit. This had two advantages. It concentrated the light from a much wider slit into a mere line, and it increased the working distance between the sensitive plate and the apparatus. But it necessitated the adoption of a more perfect machine for propelling the plate. Driving clocks were found to be useless, and I therefore designed the hydraulic motor illustrated in Fig. 5.

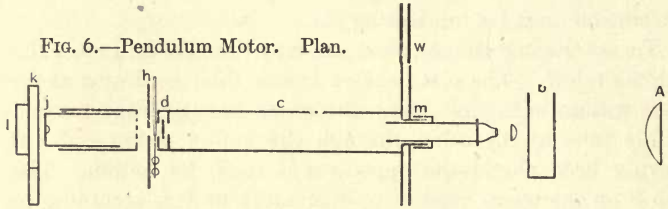
A is a brass tube 3cm. in diameter and 110cm. long. It has a slot about 1cm. wide and 45cm. long cut in the middle portion. It is flanged at each end, and provided with taps, F and G. The piston B is tubular, and has a cup leather packing at each end, and a tap C in the middle. To this piston B is attached the plate holder D, which is guided at its upper part by the brass rod E held parallel to the tube A by the wooden framework of the apparatus L, L, M. The tap G is connected with the water supply, and a waste-pipe is led from F to the nearest sink. The lower end of the waste-pipe must be kept constantly under water to prevent the entrance of air. For the same reason the supply pipe must be fitted with an air trap if the water is taken direct from the main. The tap H serves to empty the central portion of the tube A of any water that may leak past the piston heads and of the surplus oil used for lubricating it.

To use the instrument, close the taps F and G and open the middle tap C. The plate-holder D can then be drawn as far as it will go to the left hand, the water passing from one end of the tube to the other through the hollow piston rod. C having been closed the apparatus is ready for action. The tap F on the waste pipe is opened more or less according to the speed at which the plate is require to travel. When all is ready, the tap G is suddenly turned full on. The pressure of the water upon one end of the piston is resisted by the friction of the flow through the tap F, and the result is a very steady motion of the plate-carrier from left to right. The power of the instrument is considerable, and the rate is constant after the first 2cms. The speed may be varied from 0·3cm. to 5cm. per second, by adjusting the tap F, the long handle of which comes up against a set screw.

This machine answered admirably. Electrical phenomena lasting from 4sec. to 2min. were recorded by its means, and the resulting photographs were perfectly sharp and even. But we had not realised with what great rapidity the capillary electrometer can respond to changes of potential.

While examining with a lens one of the physiological photographs which will be described later on, I noticed a well-marked notch in the commencement of the curve, showing that two impulses had been given to the mercury in rapid succession, and that it had responded first to one and then to the other. It was known that two changes of potential difference had actually taken place in the circuit; but the interval between them could not possibly be greater than 0·01sec. This indicated that the instrument was capable of acting much more rapidly than we supposed. I accordingly designed a simple experimental arrangement for trying higher velocities. It was in the form of a balanced pendulum, and carried the plate in a circular arc instead of a straight line. This was at first considered a disadvantage; but shortly afterwards I discovered that, owing to a peculiar property of polar curves, it was in reality far better for our purpose, as will be seen from the description of the mode of measuring the photographs. The principle was, therefore, retained, and

Fig. 6.—Pendulum Motor. Plan.



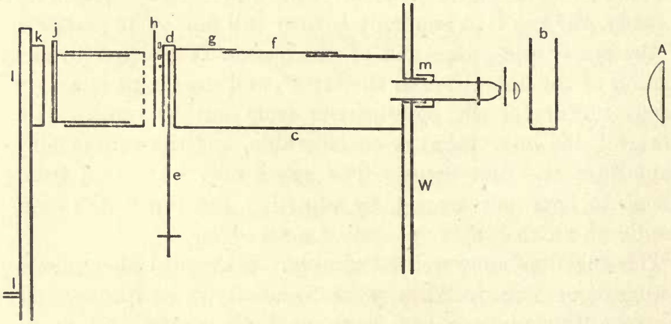


Fig. 7.—Pendulum Motor. Side Elevation.

subsequent alterations have been simply improvements of the details of the apparatus.

The Pendulum Apparatus.

The present arrangement of the photographic apparatus is shown diagrammatically in plan in Fig. 6 and in elevation in Fig. 7. A is an ordinary 5-inch condenser, in the focus of which is placed an arc lamp on an adjustable stand. A tank, *b*, filled with water or alum solution, serves to intercept some of the heat rays. Sometimes a second tank containing ammoniosulphate of copper is also used. This effectually stops the heat, and does not appreciably diminish the actinic power of the light, but makes focussing rather difficult. Next comes the microscope, *m*, with its condenser, and the electrometer. This part of the apparatus is situated in a window in the wall of the dark room. The microscope, which is fixed to the solid stone, occupies half the window, and the other half is closed by a door made perfectly light-tight. The electrometer is supported on a stand provided with vertical and horizontal rack work motion and two tilting adjustments for setting the capillary vertical and at right angles to the optic axis. The tube of the microscope is fitted with a light-tight joint where it passes through the partition W, which it does not touch.

Inside the dark room the apparatus is attached to the floor, and is only connected with the window by a creased paper band, which keeps out the light without transmitting mechanical vibrations. The rays pass first into a narrow box 50cm. long, with a sliding lid, *f*, and a pane of ruby glass, *g*, at the farther end. By opening the lid, and placing a piece of paper in the path of the light, the movements of the mercury can be observed, and the ruby glass allows the operator to watch the meniscus up to the very moment of the exposure. At the end of the box *c* are the two false slits *d*, and between them is the shutter *e*. This shutter works on a pivot underneath the apparatus, and is connected with the same handle that releases the pendulum *l*. Immediately beyond the false slits are the tuning fork and the signal, which will be separately described, and close behind them is the true slit *h*. This is contained in a separate frame, so that it can be removed entirely during focussing and replaced exactly in the centre of the image of the mercury column. The width of the slit can be adjusted to suit different velocities of the plate. Behind the slit comes the cylindrical lens *j*. This is of 25mm. focus and is placed about 25cm. from the slit. It is provided with an adjustment for focussing, and is fixed to a slide so that it can be drawn up quite out of the path of the light. The plate carrier, and focussing screen *k*, are fixed to the top of the pendulum *l*.

Fig. 8 is a diagram of the apparatus for propelling the plate, seen from behind. It consists of two parts, viz., the pendulum and the automatic key for determining the instant at which an experiment shall commence. The pendulum A is a wooden bar 125cm. long by 12cm. wide, which swings by an axis at its centre on bearings fixed to the table. To avoid friction the shaft is very small, but it is supported on each side of both bearings by a solid bracket bolted to A. The bearing nearest the pendulum has no cap. The shaft rests upon it, and the weight is so arranged that the other end of the shaft, in which a V-shaped groove is turned, presses upwards into a semi-circular notch in the edge of a brass plate. Thus the pendulum can be easily unshipped, and at the same time the plane of its movement is always at a fixed distance from the cylindrical lens. C is the plate carrier. It is made to hold an ordinary dark slide. S is a ground-glass focussing-screen. At the lower end of the pendulum is a movable weight, B. This has to be fixed so that when the dark slide is in place with a plate in it and the shutter drawn up, the pendulum will rest indifferently in any position. The arm D carries a weight, E, attached to it by a cord. In the actual instrument the weight is replaced by a board, hinged to the table at one end and loaded more or less heavily according as a slow or high speed is required.

The action is as follows: the pendulum is held in the position shown in the figure by the latch G. When this is raised the weight E causes the pendulum to swing over with increasing velocity until it rests on the table, just before the sensitive plate comes into the line of light. The acceleration now ceases, and the pendulum being in neutral equilibrium continues to move with a constant angular velocity until caught and held by the catch H. To counteract the slight retardation due to friction, a small weight, fixed to the bar D, is adjusted until the vibrations of a tuning fork, recorded on a long negative, measure the same at each end of it. For fast rates, up to 150cm. per second, a weight of 2 kilos. is employed. For slow speeds the bar A is loaded with equal weights at both ends, and the weight E is reduced until it is only just sufficient to act with certainty.

The apparatus is capable of two classes of work, viz., it can record a short portion of a series of repeated movements of the image, as in investigating dynamo currents, or the vibrations of a spring; or it can register the effect of a single excitation of a muscle, or of an induced current. For the latter class of work it is necessary to have means of determining with great accuracy the exact instant at which the experiment commences. This is done by the arrangement shown to the left hand in Fig. 8. M is a bar attached to the pendulum A. It

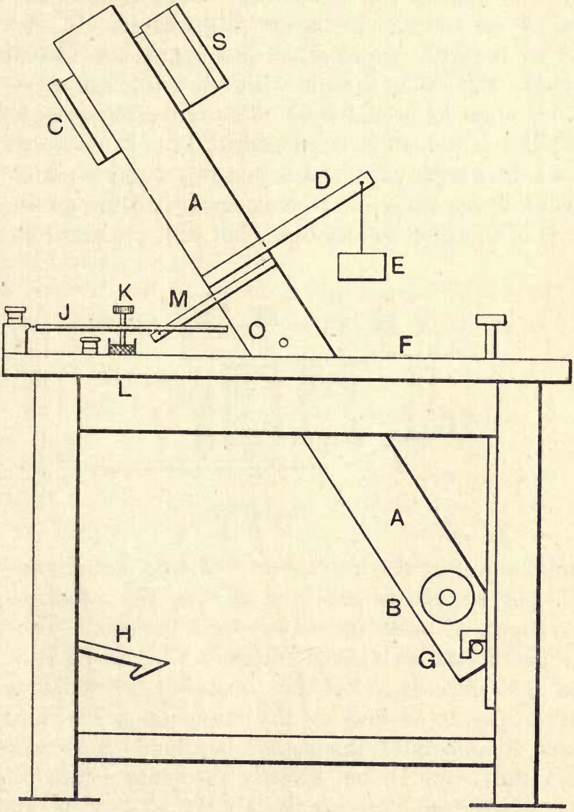


Fig. 8.—Pendulum Motor. End Elevation.

carries a pin on which rests the light arm J of the automatic key. This is furnished with a contact screw, K, dipping into a cup of mercury, L. Terminals are provided, by which this key is connected with a battery, and with the signal key shown on a larger scale in Fig. 9. It is essential that the screw K should be connected with the terminal by a flexible wire, and not through any rubbing contact. By adjusting K, and then moving the pendulum slowly across by hand, this key may be made to break contact when the plate reaches any desired position, with an error of not more than 1mm. It only serves, however, to actuate the signal key, which records the exact instant of the stimulus in the act of giving it. In Fig. 9, A and B are the coils of a small electromagnet, the terminals of which, M_1, M_2, are in circuit with the automatic key. The armature C is connected by a piece of watch spring with a brass pillar on which is the terminal P_2. It rests upon the pole of A by a knife edge, and is just kept from touching B by the point of the set-screw D, connected with the terminal P_1. The end of C, which is very light, but stiff, projects somewhat

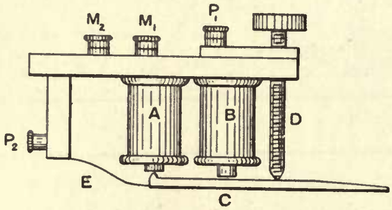


Fig. 9.—Signal Key.

beyond the rest of the instrument. A stop, not shown in the figure, prevents the spring from moving the armature more than a couple of millimetres away from the pole. The signal key is fixed in the position indicated in Figs. 6 and 7, between *h*, and *d*, so that the end of the armature crosses the slit near the upper part. As long as the automatic key is closed, the magnets A and B of the signal key hold the armature C against the screw D, but directly the plate comes into the requisite position, the circuit M_1 M_2 is broken and the armature springs back, breaking the circuit P_1 P_2 and recording the exact instant of so doing by the sudden drop of its shadow upon the sensitive plate.

It will be noticed that in this key also there are no rubbing contacts, and only one place at which the circuit can be broken. I sometimes use a modification in which a spring key is struck open by a small hammer connected with an electromagnet. But the principle is the same—the shadow of the key itself is recorded on the plate.

It remains to describe the time-marker. This is simply a standard fork making 500 double vibrations per second. It is fixed horizontally so that the lower prong projects across the slit at its upper part, leaving a mere line of light 1mm. long above it. The prongs are forced apart by a cam so as to allow a block attached to one end of a pivotted lever to be placed between them. On pulling the lever the fork is released, and vibrates with sufficient amplitude for 15 or 20 seconds.

To illustrate the working of the machine I will describe the process of photographing a "normal excursion," *i.e.*, the excursion produced by placing the uncharged electrometer suddenly in connection with a constant source of E.M.F. of known intensity, *e.g.*, 0·01 volts.

The derived circuit of a potentiometer is put in series with the electrometer terminals, and a short-circuit across them is connected with the terminals P_1 and P_2 of the signal key. The arc lamp having been switched on, the lid *f* (Fig. 7) is slid back and a piece of paper placed in the path of the light. The pressure-bulb is then raised until the meniscus appears in the capillary in the proper position. The light-tight door in the window of the dark room is kept open while the electrometer stand is adjusted so that the image of the mercury column appears on the shutter in the middle of the false slit. The shutter *e* is then opened, the true slit *h* removed, the cylindrical lens *j* drawn up out of the way, and the pendulum placed so that the magnified image falls on the focussing screen S (Fig. 8). The top of the meniscus is then focussed as sharply as possible, with the aid of a lens. A long rod, not shown in the figure, passes through the partition W to the fine adjustment of the microscope. Two pins on the end of this rod engage in two *large* holes in the head of the screw, which can thus be adjusted with great nicety. When the focussing is finished the rod is drawn back clear of the screw. The next step is to replace the slit *h* exactly in the centre line of the mercury column. Then the cylindrical lens is pushed down into its place, and focussed until the image of the slit appears upon the screen as a mere line of light.

It remains to test the circuit. The rider of the potentiometer is first lifted off the wire so as to break the electrometer circuit. Placing his finger and thumb on the terminals of the electrometer, the operator rubs first one and then the other. The meniscus ought to move up and down freely, remaining up or down if the fingers are removed, and returning to zero sharply when the key is closed. But when the rider is on the potentiometer, it ought not to be possible to cause the mercury to move by rubbing the terminals, because the circuit is no longer open.

A dark slide containing a plate is now put into the plate-holder and the pendulum pushed back into the position shown in Fig. 8. This action closes the automatic key, so that on raising the armature C of the signal key (Fig 9) it will be held by the magnet. The short-circuit of the potentiometer being now closed, no charge should enter the electrometer, and accordingly there should be no movement of the meniscus on opening the electrometer key. It remains to set the tuning fork by placing the block between its prongs. The light-tight door is closed, the sliding lid shut to, and the shutter of the dark slide drawn up. The tuning fork is made to sound, and a second later the handle of the latch G (Fig. 8) is raised. This opens the exposing shutter and at the same time releases the pendulum, which swings over. The automatic key releases the signal key, and this in its turn breaks the short circuit of the potentiometer, the current from which flows into the electrometer and charges it, the shadow of the moving meniscus forming a curved band upon the sensitive plate.

As soon as the operator hears the clash of the pendulum against the catch H, he lowers the handle, thus closing the exposing shutter, and removes the sensitive plate.

After the first adjusment very little attention is needed, save an occasional glance at the focussing screen, and plates can be exposed almost as quickly as in an ordinary camera.

It should be pointed out that the actual form of the apparatus was determined by the position in which it had to be erected. I do not confine myself to the pendulum motion. I have also made a portable camera in which the sensitive plate is attached to a disc which for very high speeds is made to revolve continuously. It is provided with an arrangement for giving the exposure as soon as the requisite velocity is attained. Details of this machine will be communicated hereafter.

Measurement of the Potential Difference indicated by Rapid Movements of the Mercury of the Capillary Electrometer.

In all other electrical instruments measurements are effected by noting the amount of a completed deflection. It was a new thing to attempt the estimation of an E.M.F. by determining the initial velocity of the excursion caused by it. I had long believed that this would be possible in the case of the ballistic galvanometer, and accordingly set myself to determine the time relations of the capillary electrometer with a view of putting the method into practice. While the photographic apparatus was being made I had studied the physical constants of the instrument, and was now ready to attack the more important problem. I knew the conditions to be complex. A mechanical effect was produced by an electric cause, and the rate of movement might be determined by either or by both. In the former case the flow of liquid might be governed by Torricelli's law or by Poiseuille's, or it might take place according to some intermediate formula. But if electrical causes predominated, then there were the possible variations of sensitiveness and capacity, and the inevitable alteration of resistance, as the mercury approached the end of the tube, to take into account. It would have been futile to seek hypotheses. I therefore made experiments.

(1) Dilution of the acid beyond a certain point lessened the rapidity of an excursion without diminishing the extent of it. This afforded additional evidence of the effect of electrical resistance.

(2) Shortening the capillary, so as to reduce the length of the acid column above the mercury, made the instrument act more quickly. But here both frictional and electrical resistance were lessened.

(3) The shape of the tapering part of the tube as well as of the orifice, had a marked influence on the rapidity of the excursions. This was clearly a mechanical effect.

(4) The next step was to ascertain whether there was any delay between the communication of a P.D. to the

electrometer, and the commencement of the movement of the meniscus.

This was done by photographing a normal excursion in the manner already described, although not with that apparatus, which had not then been constructed. An examination of the photograph showed that the fall of the signal was synchronous with the commencement of the excursion.

The signal key was then arranged to actuate a rheotome, by which short currents of known duration were thrown into the electrometer. The photographs obtained showed that with such differences of potential as would be employed in practice, the mercury ceases to move directly the source of electro-motive force is withdrawn, *i.e.*, the instrument is practically dead-beat.

It has been generally stated that it is perfectly dead-beat, but this is only true of "slow" instruments. A strong condenser charge communicated to a quick electrometer such as I employ, will in some cases cause the meniscus to overshoot its final position of rest by as much as 10 per cent. of the excursion, if there is no resistance in circuit. But the insertion of 50,000 ohms makes it dead-beat, without seriously reducing the rapidity of the action. Overshooting is more marked with the condenser charge than with rheotome experiments. The legitimate conclusion to which these observations tend is that in practice *the meniscus commences to move the instant a potential difference is communicated to it, and stops directly it is withdrawn.* Moreover, the movement commences quite suddenly. No trace can be detected, even with a lens, of gradual acceleration. The inference follows that "*the velocity of the meniscus at any moment must be some function of the accelerating force at that moment,*" *i.e.*, it is independent of any previous motion and contributes nothing to the velocity with which it moves during the next interval.

It remained to determine the form of this function. This I did in two ways, with electrometers differing as widely as possible.

For "slow" instruments I employed the method of direct observation with the rheotome already referred to. The potentiometer was arranged so as to give with 150mm. of wire a total excursion of 126 divisions of the eyepiece micrometer. The rheotome was then switched on to the circuit, and the period of closure adjusted until it produced an excursion of exactly half the previous extent—namely, 63 divisions. This required a closure of 0·414sec. The rider of the potentiometer was then shifted to 50mm. This gave, with the constant current, an excursion of 42 scale divisions, and with the rheotome 21 divisions. With the rider at 25mm. the total excursion was 21 divisions, and that given by the rheotome 10·5 divisions. The experiments were repeated with the direction of the currents reversed and with other electrometers, and the results were similar.

There is one simple function that fulfils these conditions.

Let y = the distance through which the meniscus has to move before reaching its position of rest under the action of a constant current.

Let x = indefinite time required to complete the excursion. Then, if x and y be so related that

$$\log y = -c\,x,$$

we shall have

$$\log\left(\frac{y}{2}\right) = \log 2 - c\,x\,;$$

that is to say, the time occupied in tracing the *first half* of a curve having x and y for its co-ordinates will be independent of the initial values of both x and y.

I had not expected to find so simple an expression for the time relations of the excursion, and it still remained to be seen whether in the more rapid electrometers used for photography the same relation would hold good. I accordingly took a number of photographs of normal excursions, using first the hydraulic motor, and afterwards the more perfect pendulum machine.

The results are embodied in my Paper, "On the Time Relations of the Capillary Electrometer," published in the *Philosophical Transactions*, Vol. 183A, p. 81.

Briefly, it may be stated that although the complete expression for the time relations of the excursion is very complicated, it is easy to select instruments in which the two principal sources of error neutralise each other, and that these instruments fulfil the following laws* :—

* Since my results were published, Prof. Einthoven, of Leyden, has arrived independently at the same conclusions, and I desire to acknowledge his courteous recognition of my claims to priority in the matter.—G. J. B.

1. The distance of the meniscus from its zero position at any instant is proportional to the P.D. of the charge contained in it at that instant.

2. The velocity of the meniscus at any instant is proportional to the difference between the E.M.F. of the external circuit at that instant, and the E.M.F. of the charge already contained in the electrometer.

3. In all cases in which the action is dead beat, the algebraic sum of the P.D. represented by the distance through which the meniscus has already moved, and that indicated by the velocity with which it is still moving, equals the total E.M.F. acting upon the electrometer at that instant.

This law holds good during the most rapid changes in the direction and intensity of a current.

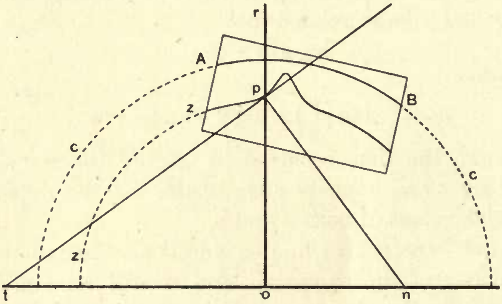


Fig. 10.—Diagram to Explain the Theory of the Subnormal.

I now come to the most important part of my discovery, viz., the method of analysing the photographic records and expressing in volts the variations of E.M.F. indicated by the electrometer.

I shall here deal only with the method as it relates to the curves obtained with the pendulum machine, because with these the process is more simple and more accurate. The principle of the operation is shown in Fig. 10, and the measuring machine itself will be shown in Figs. 12, 13 and 14.

Since the plate is carried through a circular arc, and the capillary coincides with a line passing through the centre of motion, the curve given by the excursion is most naturally expressed in polar co-ordinates. In Fig. 10, *o* is the pole and

$o\,r$ the radius vector, the plate A B being supposed to move from right to left. The position of a point p upon the curve is given by the length $o\,p$ and the angle $q\,o\,p=\theta$, through which the plate has moved since the arbitrarily selected initial point q passed the fixed line $o\,r$. The dotted line $c\,c$ is called the reference circle. It corresponds to the upper limit of the slit, and being sharply defined on the negative is used to measure from. The dotted line $z\,z$ is the circle of zero-potential on which the point q is situated.

At the point p the excursion has already commenced. It is required to find the E.M.F. acting on the electrometer at that instant, and two operations are necessary.

(a) Measure the distance through which the meniscus has already risen. This is done by turning the plate about the centre o till the initial point q comes under the line $o\,r$. Let the distance between q and the reference circle be $c\,q$ millimetres. Measure $c\,p$ in the same way. Then

$$c\,q - c\,p = \Delta\, r,$$

where $\Delta\, r$ is the distance through which the meniscus has already risen in millimetres.

But each millimetre of rise or fall of the meniscus corresponds to a certain P.D. easily ascertained once for all by calibrating the electrometer. Let k be the factor which turns scale-readings into volts. Then the portion of the E.M.F. indicated by the rise that has taken place,

$$v = k\,\Delta\, r \text{ volt.}$$

(b) The remainder of the E.M.F. is indicated by the velocity with which the meniscus was moving at the point p. As the angular velocity of the plate is constant, the time t is directly proportional to θ, and the velocity of the movement of the meniscus is the differential $d\,r/d\,\theta$. But this, by a well-known property of polar curves, can be easily ascertained. Draw $p\,t$ tangent to the curve at the point p, and draw $p\,n$, the normal, at right angles to the line $p\,t$. Through the pole o draw a line at right angles to $o\,r$, cutting the normal at n. The line $o\,n$ is called the polar subnormal, and

$$o\,n = d\,r/d\,\theta = \text{N}.$$

In other words the length of the subnormal N is directly proportional to that portion of the E.M.F. indicated by the velocity with which the meniscus is moving at the point *p*. And if *l* be the factor which turns subnormal scale-readings into volts, we have for the total E.M.F. at the point *p*

$$V = k \Delta r + l N.$$

It is necessary to observe the signs of both terms of this expression. At the apex of the curve, where the tangent is horizontal, the sub-normal is reduced to zero, and then only is the total E.M.F. directly proportional to the extent of the excursion. But on the descending portion of the curve the sub-normal passes to the left side of the pole, and the formula becomes

$$V = k \Delta r - l N,$$

and before the meniscus has descended $\frac{1}{8}$mm. the opposite P.D. indicated by its downward velocity may be greatly in excess of that corresponding to its position. Should this state of the circuit continue the meniscus would descend below the zero line, when the formula would be $V = -K \Delta r - l N$.

It remains to describe in what way the factor *l*, which turns the scale readings of the sub-normal into volts, is ascertained. A "normal excursion," *i.e.*, that given by the action of a constant P.D. of known value, is photographed. The method of doing this has been described. The resistance must be approximately the same as that in circuit when the curves to be analysed were taken. But as 1,000 ohms more or less makes very little difference, one photograph will serve for a large number of experiments. The curve of such an excursion is shown in Fig 11.

The most accurate measurements of the sub-normal are obtained when it makes an angle of about 45deg. with the radius vector *o r*. It is also easier to find the tangent to the curve at a point some little distance from its commencement. Selecting a convenient point, I measure its vertical distance from the zero line. Deducting the known value of this from the E.M.F. given by the potentiometer, the remainder is the E.M.F. corresponding to the velocity of the meniscus at that point, and therefore also to the length of the polar sub-normal N,

whence

$$l = \frac{V - k \Delta r}{N}.*$$

* Δr is measured in millimetres N in centimetres.

In the electrometer at present in use 15cm. on the subnormal = 0·02 volt, so that $l = 0{\cdot}1\dot{3}$. And an excursion of 10mm. is given by 0·013 volt, so that $k = 0{\cdot}0013$.

The measuring machine shown in plan and elevation in Figs. 12 and 13 enables these operations to be rapidly and accurately performed. The negative is clamped to a carrier B, pivoted at O upon the high table A, in the top of which a hole is cut so as to allow the photograph to be examined by transmitted light. Two screws, C and D, serve to adjust it to a position corresponding exactly to that which it occupied on the pendulum machine. The staff E is two metres long, and corresponds to the fixed line *t o n* of the diagram (Fig. 10). It is graduated in centimetres, and the values of the subnormal

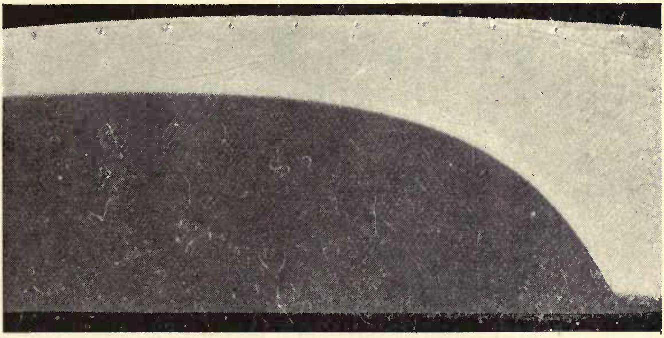


Fig. 11.—A Normal Excursion of the Capillary Electrometer given by communicating to it a known constant P.D.

are read off upon it. F is a brass plate, projecting sufficiently high above the front edge of the table to support a fine thread, attached to the pivot O, and stretched by a weight, just clear of the surface of the negative. This thread corresponds to the radius vector *o r* of Fig. 10. But it is only used in adjusting the apparatus. The line which actually represents the radius vector is the image of a very fine copper wire G, stretched across an adjustable frame about a foot from the floor. There is a mirror underneath the wire G, and the lens H—an ordinary lantern objective—serves to focus the wire upon the negative.

In using the machine, the thread is laid in a notch in F, exactly at right angles to the centre of the staff E. The wire

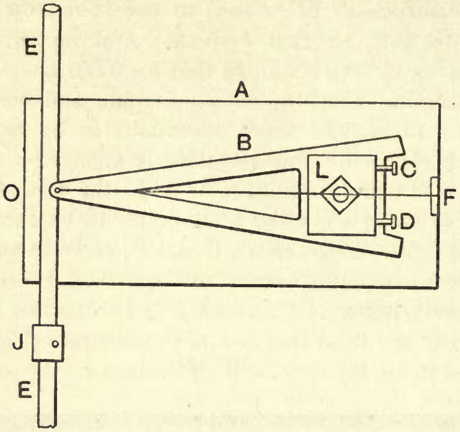


Fig. 12.—The Measuring Apparatus. Plan.

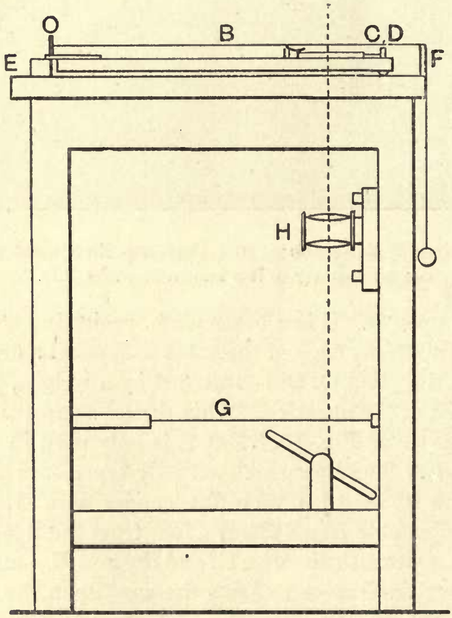


Fig. 13.—The Measuring Apparatus. Elevation.

G is set so that its image on the plate, as seen through a lens, coincides with the thread, which is then removed. The plate has now to be centred. This is done by adjusting the screws C and D, till, on traversing the plate carrier from side to side, the reference circle on the negative runs truly against the image of a pin fixed by the side of the wire G.

To find the zeros of time and of E.M.F. that part of the record of the signal key at which it drops suddenly is brought up to the radius vector, the image of G, and a mark is made with a needle point where it cuts the serrated record of the tuning fork. The distance $c\,q$ along the radius vector between the reference circle and the electrometer curve is measured with a glass millimetre scale. It is entered in the note book under the

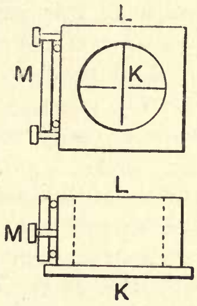


Fig. 14.—Plan and Elevation of Mirror-Block.

column R, against the entry $t_0 = 0$ of the first column. The carrier is then shifted until the earliest point at which any change in the level of the meniscus can be detected comes under the radius vector. The time t_1 at which this took place is noted. From this point the analysis of the curve commences. The carrier is moved on through a suitable time interval, usually less than 0·001sec., so as to make the next value of t a whole number. The distance $c\,p$ is entered in the column $R - r$, and the difference $c\,q - c\,p$ in a third column $\Delta\, r$. Then the subnormal is measured by the following method.

The block L, shown in plan and elevation in Fig. 14, has upon its under surface a glass plate, k, on which are ruled two lines at right angles to each other. One of these, the tangent, is continuous, but the other, the normal, is broken where it crosses the tangent. On the side of the block, set exactly parallel with the tangent line, is the mirror M, and on the mirror, in a line with the end of the normal, is a pointed index. To measure a subnormal, the block L is placed upon the negative and adjusted so that the three lines, tangent, normal and radius vector, may intersect at the point where the radius vector line crosses and the tangent touches the curve. Then, passing to the back of the staff E, the observer moves the slider J upon it until the pin of the slider and its image in the mirror M coincide simultaneously with the point of the index on the mirror. In this way the length of the subnormal can be measured within about 1 per cent. The values so obtained are entered under the column N.

The process is repeated for a number of points on the curve. Where the changes of P.D. are very sudden the time intervals may be less than 0·001sec., and any changes of direction separately noted. Finally, the results are calculated out by the formula already given, and plotted in graphic form. All measurements are made with a lens magnifying 10 or 12 diameters.*

The relative importance of the two components of the indicated E.M.F. varies according to the character of the experiment. Slow changes, *e g.*, from 0 to 0·1 volt in a tenth of a second, are shown mainly by the change of level of the meniscus, but the same P.D. established in the course of the hundredth part of a second would produce a comparatively small rise with large values of the subnormal. That the method is reliable was proved by applying it to the estimation of known P.D.'s, which were caused to act for 0·01sec. on the electrometer.

It must be borne in mind that the value in volts of the subnormal scale-readings depends on the velocity of the plate during the observation. It is, therefore, essential that the rate of the pendulum should be the same as when the normal excursion was taken from which this value was calculated. The velocity of the plate, with a given load on the pendulum, is so nearly constant that the error resulting from it may be neglected; but for different loads, or for greater accuracy, the

* Detailed examples of the method of analysis will be given at the conclusion of the series.

values of the subnormal may be either calculated or ascertained by measuring a specially photographed normal excursion.

Each photographic record represents the integration of the charges received by the electrometer during its production. And as the electrometer throughout that portion of its range is generally of constant capacity, the curve represents also the integration of the quantities of current which have flowed through the circuit, if there have been no changes of resistance. The derived curve, obtained by this method of analysis, shows the changes of E.M.F. also on the supposition that there have been no material changes of resistance. But as the equivalent resistance of the electrometer circuit is seldom under 100,000 ohms, such variations must be considerable to affect the result.

It may be asked, How does this method distinguish between the natural return of the meniscus to its zero position by discharging itself through the experimental circuit, and the reversal of any given movement by a difference of potential in the opposite direction? In such a case, if the E.M.F. indicated by the subnormals is exactly equal and opposite in sign to that corresponding to the distance of the meniscus from the position of zero potential, the electrometer is simply discharging through the circuit, and the E.M.F. calculated by the formula is zero. If there is an excess in either direction, that excess corresponds to the E.M.F. acting at the time. The curve of discharge and the curve of charge have precisely the same time-relations.

I have already explained the method of finding the value of the subnormal when the resistance in circuit is not the same as that used in photographing the normal excursion.

Application of the Capillary Electrometer to the Study of Telephone Currents and Dynamos.

In the spring of 1887 I photographed the result of blowing a whistle near a telephone connected with the electrometer. It was found that the currents so produced were of sufficient intensity to electrolyse the acid in the capillary, and the shadows of the gas bubbles were very evident upon the plate. Electrolysis is also brought about by a single induction shock whenever the resulting E.M.F. exceeds a certain limit, and in such cases, of which Fig. 15, photographed June 21, 1892, is an example, no measurable interval can be detected between the establishment of the P.D. and the evolution of the gas. I am investigating the bearing of this fact on Kohlrausch's method of measuring resistance. It had been long known that a good electrometer would respond visibly to certain sounds spoken into the telephone, but no attempt had been made to photograph these excursions.

In the summer of 1893 I made one of the assistants sing a high falsetto note at a distance of about two feet from a telephone in circuit with the electrometer. The result is shown in Fig. 16, which was the last of several photographs taken in succession. His voice was beginning to be fatigued, and the sound wavered. It will be noticed that there is a periodic change in the inclination of the serrations.* This is absent in the earlier negatives taken when the voice was steady. The pitch of the note, as judged by comparing it with the sound of the tuning fork, corresponded exactly with the number of vibrations recorded, viz., 650 double changes of E.M.F. per second. Had the photograph showed simply a serrated band, it might have been ascribed to a mere vibration set up in the meniscus under the action of an alternating current; but this evidence that the electrometer could record changes in the quality of a tone determined me to make further experiments on the subject.

No opportunity offered till the spring of 1894. I then found that an ordinary telephone did not give currents of sufficient intensity to record more than the pitch of the voice, unless the mouth was placed quite close to the diaphragm, and in that case the disturbing influence of air currents made the results irregular. I therefore tried the microphone, using a contact between two carbon buttons resting on a diaphragm of thin vulcanite. The microphone circuit was connected with the primary of an induction coil, the secondary of which

* These details were too small for reproduction by the method used in Fig. 15. An enlarged image of the curve was thrown on a sheet of Bristol-board, carefully outlined, and reduced by photography to 1½ times the size of the original. I have measured the E.M.F. of these serrations, and found it to vary between 0·01 and 0·02 volt. In another experiment of the same kind the maximum E.M.F. was 0·06 volt. This is much greater than is commonly supposed in the case of telephone currents.

was in circuit with the electrometer. This arrangement was so very sensitive that I could do nothing with it. I succeeded once in photographing the sound of the letter "r"; but in most cases there was an irregular vibration running through the records. The cause was discovered by placing a telephone in the circuit, when the sound of the engine and dynamo, which supplied the arc lamp, was plainly heard. The apparatus was fixed to the wall of the room, and this transmitted the sound. Another reason induced me to abandon the microphone. The slightest change of the adjustment would completely alter the clang-tint of a sound reproduced

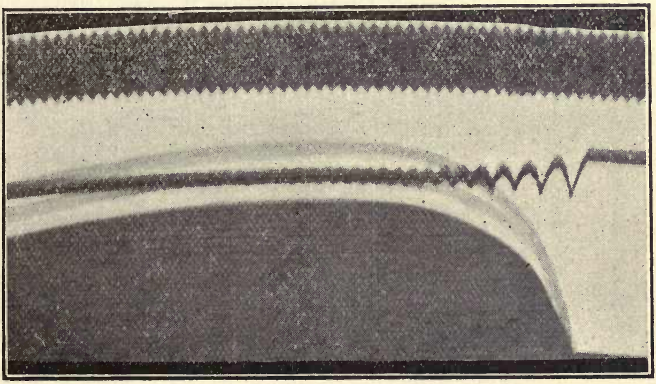


Fig. 15.—Effect of a Single Induction Shock strong enough to cause electrolysis. The upper serrated line is produced by a tuning fork making 500 double vibrations per second. The narrow band beneath it is the gnomon of the signal key, the fall of which breaks the primary circuit of the induction coil. The curve at the lower part of the plate is the shadow of the meniscus, and the faint streaks above it are produced by bubbles of hydrogen. This photograph shows that the interval between the fall of the signal and the evolution of gas is too small to be measured even with a plate moving 80cm. per second.

by it. For instance, the ticking of a watch placed near the microphone could be made to resemble that of a clock by merely adjusting the weight upon the carbons. And although the sound was always recognisable and even characteristic, it was impossible to doubt that in one case the deeper vibrations were reinforced and the upper tones suppressed, while in the

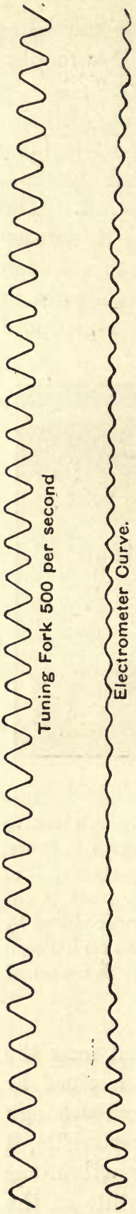


Fig. 16.—Outline of Excursions produced by singing a high tenor note (650 double vibrations per second) near a Telephone. Voice fatigued. Curves show the wavering of the sound.

other case the lower tones were absent. Then I tried a similar experiment with a number of telephones, and found that each one reinforced certain notes more powerfully than others, and that the effect depended on the shape and size of the diaphragm, and was constant for each instrument. This accounts for the fact that electrolysis is produced in the capillary of an electrometer when certain notes are sounded near a telephone connected with it, while other notes, although much louder, produce no such effect. I decided, therefore, in order to obtain more faithful records of the vibrations of the voice in speech, to join up all my telephones in series. This plan had also another advantage, namely, that of increasing the E.M.F. of the currents. For the capillary electrometer differs from the high-resistance galvanometer in requiring much less current but a higher E.M.F. to give good readings. I used five telephones of three several types—each one, as it chanced, sensitive to a different fundamental tone. These were placed close together in a convenient position for receiving the sound of the voice. To shield the electrometer from the effect of accidental noises two mercury keys were attached to the pendulum machine, one to short-circuit the electrometer until the plate had begun to move, and the other to perform a similar function directly it had passed the slit.

Vowels and sibillants are easy to photograph, because they can be sounded continuously from the moment the handle is raised until the click of the pendulum against the catch warns the operator that the plate has passed the slit. But consonants are more difficult to catch. I was obliged to use a slower rate, and to repeat the sounds as fast as possible. Thus the characteristic vibrations of the letter "p" are given by the syllables "Pop-op-op-op," and those of "d" and "k" by "Dod-od-od" and "Kok-ok-ok" respectively. It will be observed that the vowel is the same in all three cases. This was done purposely for the sake of uniformity, although the plate did not move fast enough to bring out the vibrations due to the vowel. The accompanying illustrations, Fig. 17 and Fig. 18, show a marked difference between the labial and

Fig. 17.—The Syllables "pop-op-op" Spoken near a Battery of Five Telephones in series. Plate moving more slowly. (Enlarged).*

Fig. 18.—The Syllables "dod-od-od" spoken in the same way. (Enlarged).*

dental consonants. The guttural sound gave a curve different from either, but unfortunately the two examples I obtained were not well adapted for reproduction. The vowel "ē" in "me" was easily distinguishable from the short sound of "ŏ" used in the experiments on the consonants, and

* Figs. 17 and 18 are simplified drawings, more or less diagrammatic, of the original negatives, showing only the principal excursions and the general shape of the curves. The peculiar flat-topped groups in Fig. 17 are characteristic of the labials, and the more or less lenticular series of fine oscillations in Fig. 18 always result from the dental checks. The intervals between the fine lines correspond to the pitch of the voice. The excursions extend a good deal farther both upwards and downwards than is here represented, and many of them have a hazy outline, indicating that they are in reality groups of excursions, which on a more rapidly moving plate would be separately visible.

it is here reproduced (Fig. 19). It will be observed that the plate was driven much faster than in the two previous cases, and that the negative has not been enlarged as those were. While it was still wet, fine serrations could be distinguished with a lens, and, I have no doubt, that by using a still higher velocity these could be rendered visible

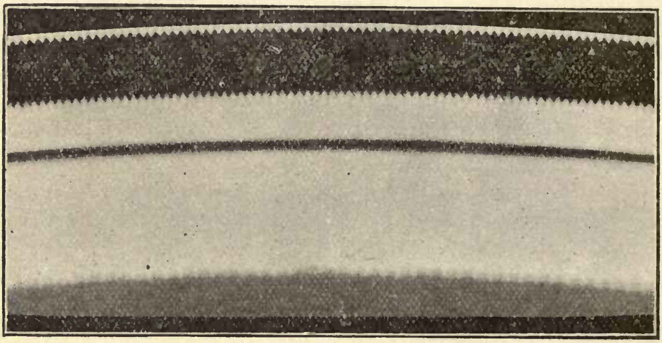


FIG. 19.—The vowel "ē" as in "mē."

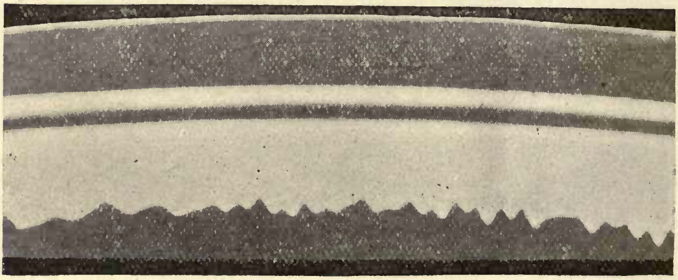


FIG. 20.—The Letter "r" Trilled with the Tongue, about 2ft. from a Microphone. Plate moving very rapidly, shows about two-thirds of the complete cycle of vibration, as ascertained from other photographs with slowly-moving plates.

to the eye. The letter "r," strongly trilled, was taken on the slow rate with the battery of telephones, and on the fast rate with the microphone, the result of the latter experiment being given in Fig. 20. The tuning fork was not sounded in this case, as it would have affected the microphone, but the velocity of the plate was between two and three metres per second. By comparing it with the telephone records, it was evident that the microphone photograph represents about two-thirds of a very complex series of vibrations, the repetition of which constitutes the characteristic sound of the trilled lingual "r."

I propose with a special apparatus to continue the investigation. Meanwhile, these experiments bid fair to lead to results of practical as well as scientific importance. Now that the familiar use of the telephone inclines us to regard its shortcomings in a more critical spirit, few persons would deny that it caricatures rather than reproduces the tones of the voice. The reason is not far to seek. We are dealing with an instrument capable of responding to a large number, but not all, of the vibrations concerned in speech-sounds. Moreover it reproduces them, not with the same, but with different relative intensity. As in the Jew's harp we reinforce by resonance certain harmonics of a fundamental tone, so in the telephone—and also in the phonograph—we bring out the harmonics of the diaphragm. These experiments suggest the possibility of improving the telephone by the use of several diaphragms of different dimensions. They have an interest also to the student of phonetics. The curves obtained by my method differ from those of Prof. McKendrick and Prof. Hermann in the relative amplitude of the slow and rapid vibrations. In the present stage of the investigation it is not possible to say which is nearer the truth, but, unquestionably, in all such researches it will be necessary to take into account the characteristics of the recording apparatus to a greater degree than has been done hitherto.

But the main interest to myself is the proof they afford of the extraordinary rapidity of the response of the capillary electrometer to alternating currents. Evidently the limit of its capabilities has not yet been reached, and these results, even though they should have no further application, imply that confidence may be placed in the photographic records of less rapid changes.

The success of the experiments with the telephone induced me to photograph some dynamo currents. The facilities for such work are not great in a laboratory devoted to physiology, but I was able to do sufficient to convince myself of the possibility of using the capillary electrometer for this purpose.

In Fig. 21 are shown the curves given by a small inductor-alternator with a frequency of 400 per second. Such curves are, of course, too small to be analysed individually, but the maximum E.M.F. of each could be easily measured. But the electrometer would stand currents a good deal stronger; and even without putting any greater pressure upon it, by increasing the velocity of the plate, which in this particular instance was only 60cm. per second, a curve could easily be produced on which eight or ten determinations of P.D. could be made for each period, as in the case of the photograph of microphone currents reproduced in Fig. 20.

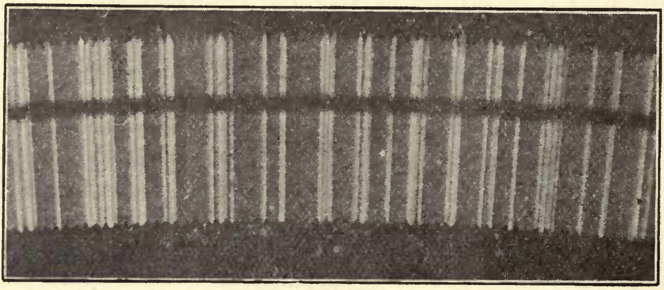


Fig. 21.—Curves given by a small Inductor Alternator making 400 complete periods per second.

The next example, Fig. 22, is taken from a very poor photograph, but an exceedingly interesting one. It was taken after altering the adjustments of the inductor-alternator so as to throw it a little out of truth. The change was very slight, but it is recorded with unmistakable plainness upon the plate, a periodic rise and fall of potential, corresponding to a complete revolution of the armature-shaft, being superposed upon the more rapid alternations due to the passage of the pole-pieces past the magnets. The importance of this result can hardly be over-estimated. It confirms the evidence afforded by the experiments with the telephone, particularly that in which the wavering of the voice when fatigued was recorded

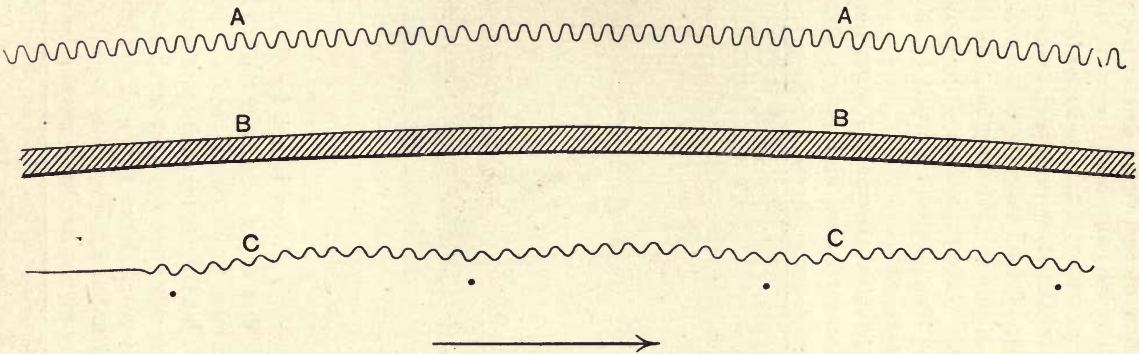


Fig. 22.

Enlarged Reproduction of Curves given by the Inductor Alternator with the Armature Shaft set so as not to run true. Shows a rise and fall of potential corresponding to each revolution, in addition to the rapid alternations due to the passage of the pole-pieces past the magnets.

A is the time record given by a tuning-fork making 500 double vibrations per second. B is the shadow of the signal-key. C is the outline of the electrometer curve. The dots mark the completion of each revolution.

(Fig. 16). But whereas in that case the degree of correspondence between the variation of the sound and of the photographed curve could only be judged by memory, in this photograph of alternating currents it can be tested by observing that the number of serrations in each period corresponds precisely with the number of field magnets in the cycle.* I am continuing my experiments in this direction, and will for the present confine myself to the expression of my conviction that this use of the capillary electrometer will be of great service in dynamo investigations.

Fig. 23 shows the current curves of the same inductor-alternator driven at a slower speed, and giving about 30

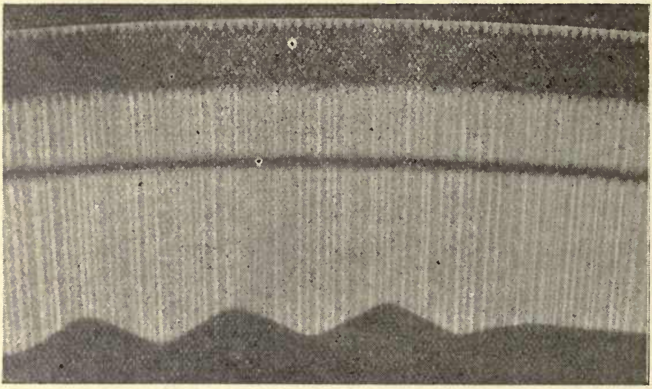


Fig. 23.—Inductor Alternator running slowly and making 30 periods per second.

double alternations per second. Were it not for the imperfection of the photograph, due to the spitting of the arc lamp, which was a very poor one, these curves could easily be measured and the E.M.F. determined for each two-thousandth part of a second. The electrometer was kept on short circuit until after the plate had reached the slit, and the interval between the fall of the signal and the commencement of the curve is due to the delay of the auxiliary magnet, by which

* Fig. 16 illustrates the interference of two vibrations of nearly equal period, but Fig. 22 is the result of compounding two cycles of P.D., one slow and the other rapid.

the key was opened and the current thrown in. It should be noticed that the *average level of the mercury in the capillary is not altered* by the action of an alternating current not too powerful for the instrument to bear with safety.

This is not the case with the next photograph (Fig. 24), taken with an ordinary lecture-table dynamo fitted with a Siemens shuttle armature, and driven by a water motor. The current was sent through a non-inductive resistance, and a derivation led to the electrometer, which was kept on short circuit until the plate had reached the slit. The result shows the effect of suddenly throwing in a pulsating direct

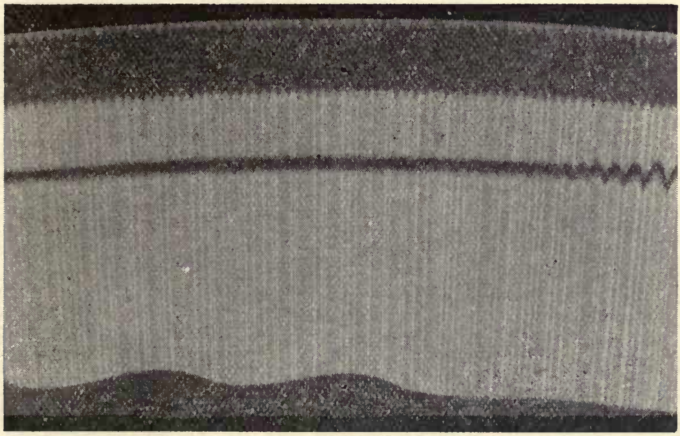


FIG 24.—Curves given by a Shunt wound Dynamo with Siemens Shuttle Armature, making about 10 revolutions per second.

current. The curves look very much like those of the alternator, but a closer inspection shows that the descents are not so steep as the ascents for the first few periods, and that consequently the meniscus *mounts*.

In Fig. 25, in which the plate was driven more slowly, this is shown better. The mercury column rises steadily through a distance depending on the average P.D. between the ends of the derived circuit, and continues oscillating about this new level. At first sight this might seem to indicate that the E.M.F. never falls to zero, but on analysing the curves by the method I have described it is found that at a certain point on the descent of each the downward velocity of the meniscus indicates an E.M.F. exactly equal and opposite to the P.D. corresponding to the height of the mercury above the zero line, showing that the current actually falls to zero twice during each revolution of the armature.

It remains to describe briefly a method of observation which I had in view, but which Prof. Wedensky had quite independently discovered and perfected before I put my own idea into execution.

It is not always convenient, or, indeed, possible to use the photographic method. Prof. Wedensky has supplied us

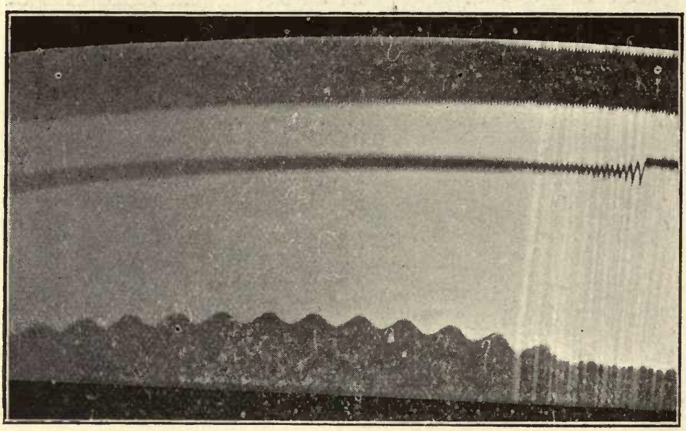


Fig. 25.—Curves given by a Shunt-wound Dynamo with Siemens Shuttle Armature, making about 10 revolutions per second, taken on a slowly moving plate. Note the rise of the meniscus, which does not occur when an alternating current is used.

with a means of observing the phases of a rapidly recurring cycle of electromotive changes without recording them. His apparatus, which was designed for physiological work, is equally applicable to the study of dynamo currents. It is an adaptation of the familiar principle of the zoetrope. The microscope used for observing the capillary electrometer is illuminated by a series of flashes, nearly, but not quite synchronous with the cycle of electromotive changes under observation. Thus, although the meniscus is in rapid motion, the observer sees it only during a definite small period of each excursion. The stroboscopic disc, by which the illumination is interrupted, may be either driven at a slightly slower speed, or fitted with an arrangement by which it can be given a definite amount of lag, variable at will. In this way, by a series of separate observations, the electrometer curve for the entire cycle can be plotted down. But it must not be forgotten that in this case, as with the photographic records, the distance through which the mercury is observed to have moved above or below its zero-point is *not* proportional to the actual P.D. between the terminals at the instant of each flash. To ascertain the true variation of P.D. with Prof. Wedensky's apparatus, it would be necessary to plot a curve from a large number of observations, and then analyse it in the manner described in this Paper. And the analysis of a plotted curve is neither easy nor satisfactory. Still, I have no doubt that the stroboscopic method will prove of service in cases where photography may not be practicable.

In conclusion, I would recommend the capillary electrometer to the notice of electricians for two reasons. It is the cheapest form of electrometer that can be made. When it has received the attention it merits, I have no doubt it will become as familiar an instrument as the thermometer. The construction is so simple that even if it should be broken a new one can be made, unless the operator is very unfortunate, in less than an hour. It is essentially the instrument for those who have to think twice before spending a shilling. But it has qualities of its own distinct from those of any other form of electrometer or galvanometer, and these will make it a valuable addition to the most perfectly appointed laboratory. I have described the method by which these special properties may be utilised, and some of the apparatus I have designed for the purpose during the nine years I have worked with the instrument, in order if possible to bring it into general use. For the same reason I have refrained from patenting any part of it, preferring to present my discovery and invention to the scientific public. The same method may be applied not only to the capillary electrometer, but to any form of electrometer or galvanometer, or thermometer, or other measuring instrument, the movement of which is either dead beat or governed by a known law. The motion of the recording surface may be rectilinear or circular. The surface itself may be flat or developable. It may be propelled with uniform velocity or with uniform acceleration by any of the devices used for such purposes and known to scientists. The motion of the plate may be continuous, and the exposure may be given by opening and closing the shutter during the passage of the whole or a part of the sensitive surface. Several methods are available for determining with the requisite accuracy the exact moment of the experiment. For slow speeds diffused daylight or a lamp will serve, while for higher velocities the limelight, or magnesium, or the electric arc or sunlight must be employed; in short, the methods and apparatus by which labour is lessened and improved results obtained in other branches of photography are applicable to this. From the records so obtained the derived curve may be calculated or drawn either in the way I have described, or by means of any known device for differentiating curves, and for certain purposes the same process may be repeated on the derived curve itself. The apparatus has already done good service in physiology, and it now remains to extend its application to other departments of science.

Note added July 2, 1896.

The following detailed analyses of the photographed curves of two dynamos, one giving an alternating and the other a pulsating current, will illustrate both the practice of the method and its applicability to the study of motors and generators. By far the greater number of curves I have measured relate to the physiology of muscle and nerve. Most of them are as yet unpublished, but several are discussed by Prof. Burdon Sanderson in the *Journal* of Physiology, Vol. XVIII., pp. 117-159.

The portion given includes a complete period. The photograph is similar in appearance to Fig. 23, which was taken on the same day, viz., August 15, 1895. Owing to the spitting of the arc lamp, the negative is ribbed, making accurate measurement impossible. In this, as in the next example, Δ R is measured from the position of zero-potential. With the alternating current Δ R changes sign, as does also N, although not simultaneously.

No. 52, B.—Analysis of a Portion of the Curve given by the Inductor Alternator, while making 24 periods per second, with the armature-shaft set as in Fig. 22 so as not to run true. The values of N+*k* Δ R *in the last column are directly proportional to the E M.F.*

T.	Δ R.	*k* Δ R.	N.	N+*k* Δ R.
0	0	0	0	0
68	+10·0	+2·0	− 1·0	+ 1·0
70	+ 8·5	+1·7	−12·0	−10·3
72	+ 7·5	+1·5	−18·0	−16·5
74	+ 5·5	+1·1	−19·8	−18·7
76	+ 3·5	+0·7	−19·2	−18·5
78	+ 1·5	+0·3	−18·7	−18·4
80	− 0·5	−0·1	−17·5	−17·6
82	− 2·0	−0·4	−15·5	−15·9
84	− 3·0	−0·6	−11·5	−12·1
86	− 3·8	−0·8	− 8·5	− 9·3
88	− 4·5	−0·9	− 3·0	− 3·9
90	− 4·5	−0·9	+ 2·0	+ 1·1
92	− 3·8	−0·8	+15·0	+14·2
94	− 2·5	−0·5	+15·4	+14·9
96	− 1·0	−0·2	+16·0	+15·8
98	+ 0·3	+0·1	+16·0	+16·1
100	+ 1·5	+0·3	+16·5	+16·8
102	+ 3·5	+0·7	+16·0	+16·7
104	+ 4·8	+1·0	+15·5	+16·5
106	+ 6·0	+1·2	+14·5	+15·7
108	+ 7·5	+1·5	+ 7·5	+ 9·0
110	+ 8·0	+1·6	+ 2·0	+ 3·6
112	+ 7·5	+1·5	−12·0	−10·5

The curve of which the analysis is given on page 52, was a duplicate of that reproduced in Fig. 24, and was photographed on August 15, 1895. The electrometer employed for these dynamo experiments was not the same as that referred to in the earlier part of the Paper, so that different constants are used in the calculation. The nominal E.M.F. of the dynamo was about 4 volts. The current was sent through a non-inductive resistance, from which a derivation led to the electrometer. As the exact value of this derivation was not ascertained, I have not reduced the subnormal readings to volts, but roughly 1cm. on the subnormal =0·25 volt at the brushes.

In both cases the first column shows the time, T, in thousandths of a second, from an arbitrary zero.

No. 54, B.—Analysis of Curves given by a Shunt-wound Dynamo with Siemens Shuttle Armature, making about 700 revolutions per minute, and generating a pulsating current.

T.	Δ R.	k Δ R.	N.	N + k Δ R.
0	35·0	7·00	0	+ 7·00
2	35·0	7·00	+ 2·3	+ 9·30
4	35·4	7·08	+ 3·7	+10·78
6	35·9	7·18	+ 6·7	+13·88
8	36·8	7·36	+10·5	+17·86
10	37·8	7·56	+14·0	+21·56
12	39·3	7·86	+14·7	+22·56
14	40·6	8·12	+13·5	+21·62
16	41·4	8·28	+ 9·5	+17·78
18	42·3	8·46	+ 7·0	+15·46
20	42·8	8·56	+ 4·7	+13·26
22	43·2	8·64	+ 2·5	+11·14
24	43·4	8·68	0·0	+ 8·68
26	42·9	8·58	− 2·5	+ 6·08
28	42·3	8·46	− 7·0	+ 1·46
30	41·0	8·20	− 8·2	+ 0·0
32	40·3	8·06	− 8·0	+ 0·06
34	39·9	7·98	− 7·5	+ 0·48
36	38·9	7·78	− 6·6	+ 1·18
38	38·4	7·68	− 5·0	+ 2·68
40	38·0	7·60	− 4·0	+ 3·60
42	37·9	7·58	− 2·0	+ 5·58
44	37·9	7·58	0·0	+ 7·58
46	38·0	7·60	+ 3·0	+10·60
48	38·5	7·70	+ 5·6	+13·30
50	39·2	7·84	+12·0	+19·84
52	40·4	8·03	+15·4	+23·48
54	41·9	8·38	+16·5	+24·88
56	43·5	8·70	+12·6	+21·30
58	43·9	8·78	+ 8·5	+17·28
60	44·6	8·92	+ 3·5	+12·42
62	45·0	9·00	0·0	+ 9·00
64	44·9	8·98	− 1·5	+ 7·48
66	44·7	8·94	− 2·7	+ 6·24
68	44·4	8·88	− 4·0	+ 4·88
70	43·9	8·78	− 6·3	+ 2·48
72	43·3	8·66	− 8·0	+ 0·66
74	42·5	8·50	− 8·5	+ 0·0
76	41·6	8·32	− 7·5	+ 0·82
78	40·9	8·18	− 6·7	+ 1·48
80	40·4	8·08	− 5·5	+ 2·58
82	39·9	7·98	− 4·5	+ 3·48
84	39·4	7·88	− 2·2	+ 5·68
86	39·1	7·82	− 0·2	+ 7·62
88	39·4	7·88	+ 3·0	+10·88

N.B.—Here Δ R (and k Δ R) remains continually positive, but fluctuates in value, while N not only fluctuates, but changes sign.

In the second column, Δ R, the distance of the meniscus from the position of zero-potential, is given in units of a scale of which 50 divisions = 14·1mm. on the photograph. This scale, which was ruled on glass, was fixed close to the wire G of the measuring apparatus (Fig. 13). The zero of its image was made to coincide with the position of zero-potential on the plate, and the value of Δ R was read off upon it as each successive point on the curve was brought over the wire for the determination of the subnormal. I find this method more accurate and more convenient than that previously described.

The third column shows the value of Δ R in terms of the subnormal. The constant multiplier ($k = 0{\cdot}2$) is found by taking the ratio of the difference between any two values of Δ R and the difference between the corresponding values of N upon the normal excursion (Fig. 11). This ratio is constant if the electrometer is a good one.

The fourth column gives the subnormals N at the points measured, reckoned as positive if the meniscus is rising and negative if it is falling.

The last column is directly proportional to the E.M.F., and shows how it falls to zero twice in each revolution of the armature, although the photographed curve remains continually above zero. It is interesting to note in No. 54 B, the way in which the negative value of the subnormal balances the positive E.M.F. indicated by the position of the meniscus on both occasions, and how the actual zero lags behind the zero of the subnormal. On examining the machine I found that the armature was not centrally situated between the field-magnets, and that its poles were not symmetrical, so that once in each revolution it came very near to one of the horns. Something of this kind is indicated by the curve of the E.M.F., which is more peak-shaped in one half than in the other. On plotting this curve I find that the areas of the two halves corresponding to the two faces of the armature coils are in the proportion of 195 to 206. This divergence from equality may be due to the fact that the brushes are not exactly 180deg. apart, and the two halves of the commutator are not quite equal. The smaller area belongs to the steeper curve.

I am at present engaged upon a modification of this method. The sensitive plate is mounted as a disc upon the armature shaft, or a shaft revolving synchronously with it, and the image of the capillary is projected upon it, as in my pendulum machine, the whole being enclosed in a camera with an automatic arrangement for exposing the plate at any given time during a single complete revolution. I propose to employ various well-known ways of connecting the whole or part of the armature coils with the electrometer circuit, with a view of obtaining polar diagrams of E.M.F. by analysing the photographic records.

Note.—If it is intended to analyse the curves, the development should be stopped as soon as the high lights show on the back of the film. The definition is then perfectly sharp under a magnification of 10 diameters, but the negative is too "thin" for reproduction.