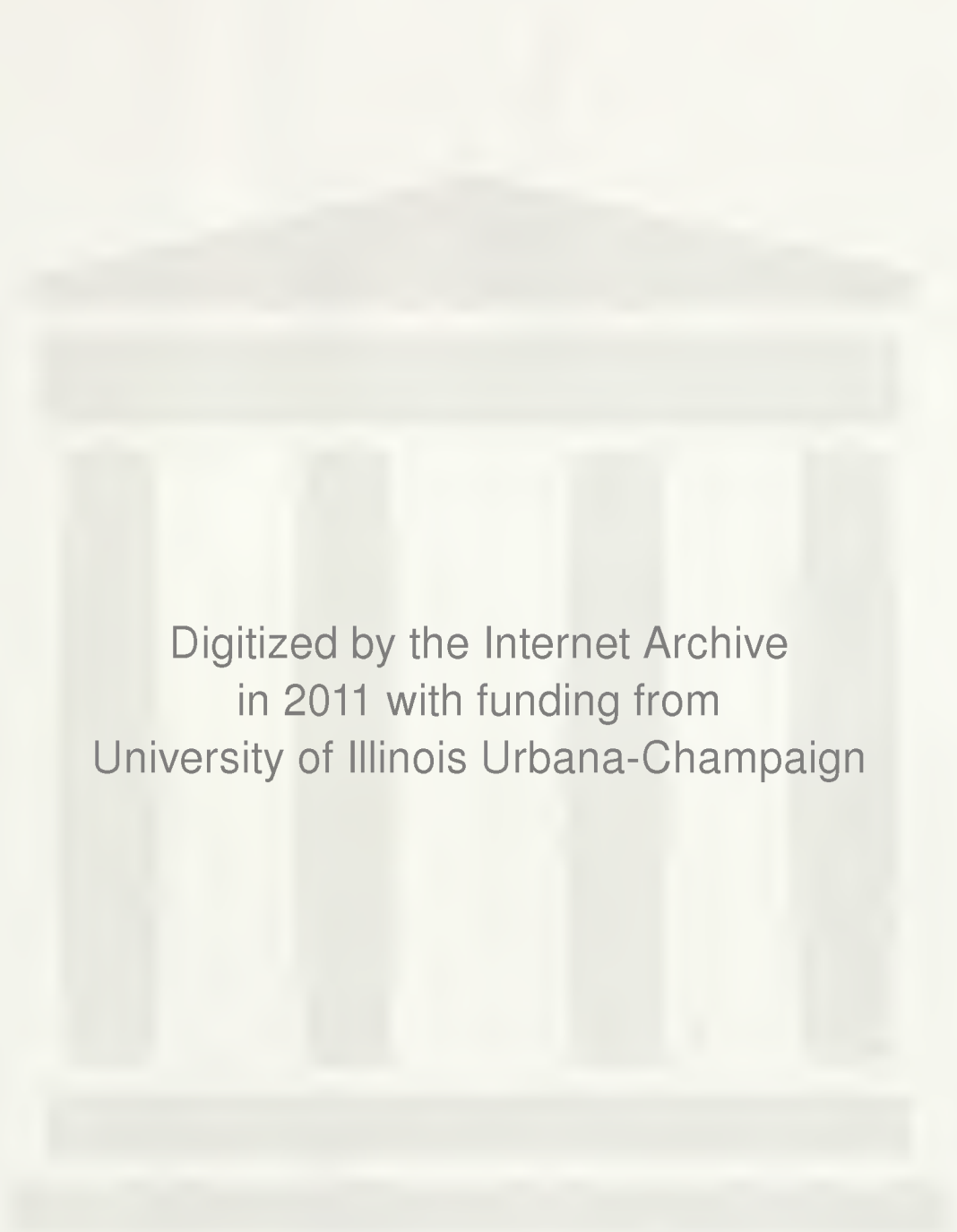




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## Faculty Working Papers

**CAPITAL ASSET PRICING MODEL: SOME ECONOMETRIC TESTS**

**Ali Jahankhani, Assistant Professor of Finance  
Roger P. Bey, University of Missouri-Columbia**

#498

**College of Commerce and Business Administration  
University of Illinois at Urbana-Champaign**



FACULTY WORKING PAPERS

College of Commerce and Business Administration

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July 25, 1978

CAPITAL ASSET PRICING MODEL: SOME ECONOMETRIC TESTS

Ali Jahankhani, Assistant Professor of Finance  
Roger P. Bey, University of Missouri-Columbia

#498

Summary:

This study presents some empirical tests of the Capital Asset Pricing Model (CAPM) using more robust statistical tests. Specifically, the restrictive assumptions of stationarity of beta and independence of error terms in the market model were relaxed. The betas of securities were estimated by the systematic-parameter varying regression technique. This technique does not assume that beta is stationary over time. However, it makes the assumption that beta is changing systematically with the accounting measures of risk. Also, the independence of the error terms (residual returns) was relaxed by estimating the betas of a group of firms in one industry simultaneously.

Our research indicated that there is a linear relationship between risk and return and higher risk is associated with higher average return. These results are consistent with the implications of both Sharpe-Lintner version and Black version of the CAPM. Furthermore, our results did not reject the hypotheses that  $E(Y_0) = \bar{R}_F$  and  $E(Y_1) = \bar{R}_F + \beta(R_M - R_F)$ . Therefore, the empirical results of this study supported all the implications of the Sharpe-Lintner CAPM.

MEMORANDUM FOR THE DIRECTOR, FBI

DATE: 10/15/54

TO: SAC, NEW YORK (100-100000)

FROM: SAC, NEW YORK (100-100000)

SUBJECT:

Reference is made to New York airtel to Bureau dated 10/15/54, captioned as above. The Bureau is requested to advise the New York Office of any information received from other sources which may be pertinent to the above-captioned matter. The New York Office is currently conducting an investigation of the activities of the Communist Party in the New York area. It is noted that the Bureau has advised that the New York Office should be advised of any information received from other sources which may be pertinent to the above-captioned matter. The New York Office is currently conducting an investigation of the activities of the Communist Party in the New York area. It is noted that the Bureau has advised that the New York Office should be advised of any information received from other sources which may be pertinent to the above-captioned matter.



## I. INTRODUCTION

The Capital Asset Pricing Model (CAPM) as developed by Sharpe [42] and Lintner [25] (hereafter the S-L model) and extended by Black [5] has been subjected to extensive empirical testing [6, 8, 13, 15, 17, 32]. Unfortunately, the results of many of the empirical tests are inconsistent with the theoretical CAPM and vary among the empirical studies. The usefulness of the CAPM has been hindered by the lack of strong empirical verification. However, failure of the empirical tests to confirm the theoretical risk-return relationships of the CAPM may be the result of the application of improper or at least inadequate statistical test procedures. Support for our contention that the failure of prior tests of the CAPM to conform to the theoretical CAPM specification is given by Foster [17]. Although he does not address all of the statistical limitations of prior studies, he does show how a value weighted index moves the empirical results in the direction of the S-L model.

The purpose of our research was to remove three major limitations associated with prior tests of the S-L and Black models and to test the models using more appropriate statistical procedures. Our results indicated that the conflicts between the theoretical relationships of the CAPM and the empirical results may have been due to inappropriate statistical tests. We found the S-L model to be a valid description of the risk-return relationship.

The study is separated into five sections. Section I is the introduction. A review of previous CAPM studies and the associated statistical limitations is given in Section II. Section III describes our research methodology. The results of the study are presented in Section IV. Section V contains our conclusions.



## II. BACKGROUND

### A. Previous Studies

The most common method of testing the CAPM has been a cross-sectional test. That is, the average returns ( $\bar{R}_i$ ) on a cross-sectional sample of securities over some time period are regressed against each security's estimated beta coefficient ( $\hat{\beta}_i$ ), or

$$\bar{R}_i = \gamma_0 + \gamma_1 \hat{\beta}_i + U_i \quad (1)$$

Beta usually is estimated from the market model

$$R_{it} = \alpha_i + \beta_i R_{mt} + \epsilon_{it} \quad (2)$$

where  $R_{it}$  and  $R_{mt}$  are the returns on security  $i$  and the market portfolio in period  $t$  and are assumed to have a bivariate normal joint distribution and  $\epsilon_{it}$  is assumed to be independent of  $R_{mt}$  and  $\epsilon_{jt}$  for  $i \neq j$ .

Douglas [13] tested the S-L model and found that the average realized returns were positively related to the variance of the returns on securities but not to the covariance of the securities with the market portfolio. Miller and Scholes [32] checked the validity of Douglas's results by examining several of the econometric problems associated with testing the CAPM. They showed that measurement error in  $\hat{\beta}$ , multicollinearity between systematic risk and residual variance, and skewness in the return distributions may have been the cause of Douglas's conflicting results.

Black, Jensen, and Scholes (BJS) [6] carefully designed a study to avoid the statistical problems which existed in Douglas's study. Their results indicated that low risk securities on average have a significantly higher return than predicted by the S-L model. The intercept and slope coefficients varied across subperiods and were not consistent with the S-L model. BJS



concluded that the S-L model does not provide an adequate description of the risk-return relationship of security returns.

BJS then demonstrated that the process generating security returns can be approximated better by a two-factor market model of the following form:

$$R_{it} = (1-\beta_i)R_{ot} + \beta_i R_{mt} + e_{it} \quad (3)$$

where  $R_{ot}$  is the return on the minimum variance portfolio with returns which are uncorrelated with  $R_{mt}$ . BJS tested their two-factor model with equation (1). Their results indicated the relationship between  $\bar{R}_i$  and  $\beta$  is highly linear, but  $\gamma_0$  and  $\gamma_1$  fluctuated from period to period and often were negative. They concluded that their results are consistent with the two-factor model because the returns on a zero-beta portfolio also fluctuate over time.

The results of Fama and MacBeth's [15] research indicated that the relationship between  $\bar{R}$  and  $\beta$  is linear, and  $\beta$  is the only risk measure required to explain the differences in average returns. Their results also indicated that  $E(\gamma_0)$  is substantially greater than  $R_f$ --a condition which is inconsistent with the S-L model.

Foster [17] studied whether relative risk ( $\beta$ ) is a sufficient descriptor of a security's risk. For the period of 1931-74, he found that relative risk explains differences in expected returns of securities. Also his results indicated that after controlling for relative risk, residual risk did not significantly explain differences in expected returns of securities. His methodology was cast in a Fama and MacBeth [15] framework but differed in the procedure used to form portfolios, and he used a value rather than an equally weighted market index.

## B. Limitations

Three major limitations arising from the use of the market model for



estimating  $\beta$  were present in each of the previously cited CAPM studies with the exception of Foster who had two. The market model is based on two major assumptions. The first assumption is that in equation (2)  $\epsilon_{it}$  is independent of  $\epsilon_{jt}$  for  $i \neq j$ . That is, the returns of the securities are assumed to be correlated only through the market portfolio. After the market effect has been removed, the covariances of all pairs of securities are assumed to be zero. If the security returns are dependent on factors other than the market portfolio (e.g., industry factors), the independence assumption does not hold. The empirical results of King [23] and Livingston [27] indicated that the residual industry co-movement of returns on securities is of considerable importance. The assumption of independence is the first major limitation of previous studies.

The second assumption of the market model is that the joint distribution of  $R_i$  and  $R_m$  is stationary over the estimation period which in turn implies  $\beta_i$  is stationary. Stationarity of  $\beta_i$  means that the systematic risk of a security is constant regardless of changes in the operating and financial characteristics of the firm. Research by Hamada [19], Rubinstein [40], Lev [24], and Mandelker [29] indicates that changes in the capital structure (financial leverage), input mix (operating leverage) and mergers all alter the riskiness of a firm.

If the joint distribution of  $R_i$  and  $R_m$  is not stationary, the estimated  $\beta$  from the market model is biased and inefficient. Therefore, since all of the previously cited CAPM studies used the simple market model to estimate  $\beta$ , the empirical results of said studies may have been distorted by the measurement error in  $\beta$ . The assumption of stationarity is the second major limitation of the previous studies.





The third limitation is the use of an equally weighted portfolio of common stocks as a proxy for the market portfolio. Fama [14, pp. 269-71] has argued that there is strong evidence that an equally weighted market index is not an appropriate proxy of the true market portfolio. Furthermore, Fisher [16] found that the standard deviation of returns on a value weighted portfolio of NYSE stocks is only about 80 percent as large as the standard deviation of an equally weighted portfolio of NYSE common stocks. Foster [17] regressed an equally weighted NYSE index on a value weighted NYSE index. The resultant slope coefficient was 1.2665 which was significantly different than zero. The implication is that the equally weighted index is more risky than a value weighted index. Application of an equally weighted portfolio may have caused researchers to reach inappropriate conclusions.

### III. METHODOLOGY

#### A. Sample and Data

The sample consisted of the 207 firms on the annual industrial COMPUSTAT tape with fiscal year ending on December 31 and no missing observations for the period from January 1956 through December 1975. For each firm the values of the following financial variables were obtained: (1) size of the firm (total assets), (2) debt ratio (total debt divided by total assets), (3) degree of operating leverage (operating income plus fixed costs divided by operating income), and (4) the rate of return on common stock (both price appreciation and dividends). Since twenty years of data were used, a total of nineteen rates of return were computed for each firm.

The return on the market portfolio was approximated by the rate of return on Fisher's value weighted index. Since the time horizon in this study was one year, the risk-free rate of interest was approximated by the yield to maturity of a one-year U.S. government bond.

THE HISTORY OF THE UNITED STATES OF AMERICA

CHAPTER I. THE DISCOVERY OF AMERICA

The history of the United States of America begins with the discovery of the continent by Christopher Columbus in 1492. Columbus, an Italian explorer, was sailing for Spain when he reached the island of San Salvador in the Bahamas. This event marked the beginning of European exploration and settlement in North America.

Following Columbus's discovery, other explorers such as Amerigo Vesputi and John Cabot continued to explore the eastern coast of North America. The Spanish and Portuguese established colonies and trading posts along the coast, while the English and French began to settle in the interior.

The early years of settlement were marked by conflict and competition between the different European powers. The English and French fought the Seven Years' War (1756-1763) over control of the continent. The Spanish and Portuguese also fought wars of independence in the Americas.

The United States of America was founded in 1776, following the Declaration of Independence from Great Britain. The new nation was a republic, with a constitution that established a system of checks and balances.

The early years of the United States were marked by westward expansion and the discovery of gold in California. The United States fought the Mexican-American War (1846-1848) and the Civil War (1861-1865).

CHAPTER II. THE EARLY YEARS

1776-1800

The early years of the United States were marked by westward expansion and the discovery of gold in California. The United States fought the Mexican-American War (1846-1848) and the Civil War (1861-1865).

The United States was a young nation, and its people were looking for new opportunities. The discovery of gold in California in 1848 led to a gold rush that attracted thousands of people to the West.

The United States fought the Mexican-American War (1846-1848) over control of the Southwest. The United States won the war and gained a large amount of territory.

The Civil War (1861-1865) was fought over the issue of slavery. The Union won the war, and slavery was abolished. The United States emerged as a more unified nation.

The late 19th century was a time of rapid industrialization and growth. The United States became a world power, and its influence was felt around the globe.

The United States fought the Spanish-American War (1898) and acquired territories in the Pacific and Caribbean. The United States became an imperial power.

The early 20th century was a time of progress and innovation. The United States led the world in many areas, including science, technology, and industry.

The United States fought World War I (1914-1918) and emerged as a superpower. The United States played a leading role in the League of Nations and the Treaty of Versailles.

The 1920s and 1930s were a time of economic hardship and social change. The United States fought World War II (1939-1945) and emerged as a superpower.

The United States led the world in the space race and the development of nuclear energy. The United States became a global leader in many areas.

The late 20th century was a time of social and political change. The United States fought the Vietnam War (1955-1975) and the Iran-Iraq War (1980-1988).

The United States led the world in the development of the Internet and other technologies. The United States became a global leader in many areas.

## B. Estimation of Beta

If, as previously discussed,  $\beta$  is nonstationary, estimation of  $\beta$  can be improved by imposing a structure on  $\beta$  which specifies the functional relationship between  $\beta$  and some selected financial variables. That is,  $\beta$  is allowed to vary as the values of the financial variables change. Based on the research of Hamada [19], Rubinstein [40], Perceival [35], and Lev [24] we specified  $\beta$  as a function of financial leverage, operating leverage, and size of the firm. Systematic parameter-variation regression (SPVR) [2] was employed to estimate  $\beta$ .

Financial leverage as an explanatory variable of  $\beta$  is supported by Hamada [19] who linked Modigliani and Miller's [33] capital structure hypothesis with the CAPM and found that theoretically  $\beta$  of a levered firm should vary with the firm's financial leverage. Hamada's empirical results supported his theoretical conclusions. Rubinstein and Perceival concluded that operating leverage affects  $\beta$ . Lev in an empirical study, found operating leverage to be positively related to  $\beta$ . Beaver, Kettle, and Scholes [1] and Breen and Lerner [11] found empirically that a firm's  $\beta$  is negatively related to the firm's size.

The SPVR model we used was:

$$R_{it} = \alpha_i + \beta_{it}R_{mt} + \epsilon_{it} \quad (4)$$

$$\beta_{it} = b_0 + b_1DR_{it} + b_2OL_{it} + b_3S_{it} \quad (5)$$

where  $DR_{it}$  is the debt ratio,  $OL_{it}$  is the operating leverage, and  $S_{it}$  is the size of the  $i^{\text{th}}$  firm in year  $t$ .<sup>1</sup> All other terms are as previously defined.

Substitution of (5) into (4) yields:

$$R_{it} = \alpha_i + b_0R_{mt} + b_1(R_{mt}DR_{it}) + b_2(R_{mt}OL_{it}) + b_3(R_{mt}S_{it}) + \epsilon_{it} \quad (6)$$



All variables in equation (6) are observable. Hence, the coefficients can be estimated. Estimates of  $\beta_{it}$  for each of the nineteen subperiods were obtained by substituting the estimated coefficients ( $b_0, b_1, b_2, b_3$ ) from (6) into (5). If  $b_1, b_2, b_3$  all equal zero, our SPVR reduces to the standard market model. Therefore, the standard market model is an appropriate procedure to estimate  $\beta$  if  $\beta$  is stationary.

If the assumption associated with equation (2) that  $\epsilon_{it}$  is independent of  $\epsilon_{jt}$  for  $i \neq j$  does not hold, the sample error of  $\beta$  will not be minimized. We relaxed the independence assumption and estimated the  $\beta$ 's for all firms in one industry simultaneously by using a seemingly unrelated regression (SUR) technique. The SUR procedure incorporates the interdependence of the  $\epsilon_{it}$ 's and provides estimates of  $\beta$  with smaller sampling errors [45].

The SUR technique was applied as follows. Assume that there are  $nk$  firms in the  $k$ th industry and

$$Y_i = X_i B_i + \epsilon_i \quad (7)$$

is the  $i$ th equation of the  $nk$ -equation system. Equation (7) is the matrix form of (6). In (7)  $Y_i$  is the vector of returns on security  $i$ ,  $X_i$  is a  $T \times 5$  matrix of observations on the explanatory variables,  $\beta_i$  is a  $5 \times 1$  vector of regression coefficients, and  $\epsilon_i$  is a  $T \times 1$  vector of disturbances with zero mean and constant variance. Since there are  $nk$  firms in the industry, the system of equations, of which (7) is one, can be written as:

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ Y_{nk} \end{bmatrix} = \begin{bmatrix} X_1 0 \dots 0 \\ 0 X_2 \dots 0 \\ \cdot \\ \cdot \\ 0 0 \dots X_{nk} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \cdot \\ \cdot \\ B_{nk} \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \cdot \\ \cdot \\ \epsilon_{nk} \end{bmatrix} \quad (8)$$



or more compactly as:

$$Y = XB + \varepsilon. \quad (9)$$

The disturbance vector in (8) is assumed to have the following contemporaneous covariance matrix:

$$\Omega = \text{cov}(\varepsilon) = \begin{bmatrix} \sigma_{11}^I & \sigma_{12}^I & \dots & \sigma_{1nk}^I \\ \sigma_{21}^I & \sigma_{22}^I & \dots & \sigma_{2nk}^I \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{nk1}^I & \sigma_{nk2}^I & \dots & \sigma_{nkn}^I \end{bmatrix} \quad (10)$$

where  $I$  is an identity matrix of order  $T \times T$  ( $T=19$ ) and  $\sigma_{ij} = [\varepsilon_i(t)\varepsilon_j(t)]$ .

It is assumed that the disturbances of each equation are homoscedastic and independently distributed, and the disturbances of different equations are only contemporaneously correlated.

The Aitken general least squares (GLS) estimator of the coefficient vector  $B$  is

$$\hat{B} = [B_1' \ B_2' \ \dots \ B_{nk}']' = (X' \Omega^{-1} X)^{-1} X' \Omega^{-1} Y \quad (11)$$

where  $\hat{B}$  is a  $(nk \times 5) \times 1$  vector of regression coefficients in the system. The covariance matrix of  $\hat{B}$  is

$$\text{cov}(\hat{B}) = (X' \Omega^{-1} X)^{-1}. \quad (12)$$

One difficulty in estimating  $\hat{B}$  and  $\text{cov}(\hat{B})$  is that the  $\Omega$  matrix usually is unknown. However, a consistent estimate of  $\Omega$  can be obtained using the OLS residuals:

$$S = \frac{1}{T} [e_1' e_2' \ \dots \ e_{nk}']' [e_1 e_2 \ \dots \ e_{nk}]$$

where  $e_i = Y_i - X_i B_i$  is the OLS residual vector of the  $i$ th equation. It can be shown that

$$\hat{\Omega}^{-1} = S^{-1} \otimes I$$





where  $\otimes$  represents the Kronecker product. After estimating  $\Omega$ ,  $\hat{B}$  and  $\text{cov}(\hat{\beta})$  were estimated, the beta coefficients in each year were obtained by substituting  $\hat{b}_0$ ,  $\hat{b}_1$ ,  $\hat{b}_2$  and  $\hat{b}_3$  into equation (6).

### C. Measurement Error In $\hat{\beta}$

Since  $\hat{\beta}$  used in the cross-sectioned regressions (1) is not an exact measure of systematic risk, but is a sample estimate subject to error (i.e., sampling errors) the coefficients  $\gamma_0$  and  $\gamma_1$ , in the cross-sectional regression are biased and inconsistent. Johnston ([22], Chapter VI) has shown that the OLS estimate of  $\gamma_1$  is negatively biased and  $\gamma_0$  is positively biased. One approach to reduce the errors in variables is to group the observations. Wald [46], and Richardson and Wu [37] have shown various ways of grouping the observations which may reduce the measurement errors in the explanatory variables. BJS [6], Blume and Friend [8] and Fama and MacBeth [15] used the grouping technique to reduce the effect of measurement errors in  $\hat{\beta}$ . They grouped securities into portfolios and used the returns and betas of the portfolios to test the CAPM.

The information lost by forming portfolios can be reduced by forming portfolios in such a manner that the range of portfolio betas ( $\beta_p$ ) is as wide as possible. One procedure to assure a wide range of  $\beta_p$ 's is to form portfolios on the basis of securities ranked by  $\beta$ . In this study, in each of the nineteen years, securities were ranked by  $\beta$  in ascending order and twenty portfolios each with nine securities were formed.<sup>2</sup> For example, the first nine securities were allocated to portfolio one, the second nine securities to portfolio two, etc.  $\hat{\beta}_p$  was calculated as:

$$\hat{\beta}_{pt} = \frac{1}{9} \sum_{i=1}^9 \hat{\beta}_{it}$$



and the portfolio return was computed as:

$$R_{pt} = \frac{\sum_{i=1}^9 R_{it}}{9}$$

where  $R_{it}$  is the return on security  $i$  in period  $t$ .  $\hat{\beta}_{pt}$  and  $R_{pt}$  were used to test the CAPM.

#### D. Estimation of the Expected Return-Risk Relation

The risk-return relationship was estimated for each of the nineteen subperiods as:

$$R_{pt} = \gamma_{0t} + \gamma_{1t}\hat{\beta}_{pt} + \gamma_{2t}\hat{\beta}_{pt}^2 + u_{pt}. \quad (13)$$

The estimated  $\gamma_{0t}$ 's,  $\gamma_{1t}$ 's and  $\gamma_{2t}$ 's were used to test the following hypotheses:<sup>3</sup>

- H1: Securities are priced such that the relationship between  $R_p$  and  $\beta_p$  is linear, i.e.,  $E(\gamma_2) = 0$ ;
- H2: There is a positive relationship between  $R_p$  and  $\beta_p$ , i.e.  $E(\gamma_1) > 0$ ;<sup>4</sup>
- H3: The expected value of  $\gamma_0 + \gamma_1$  equals the expected value of  $R_m$ , i.e.,  $E(\gamma_0 + \gamma_1) = \bar{R}_m$ ;<sup>5</sup>
- H4: The expected value of  $\gamma_0$  equals the risk-free rate of interest, i.e.,  $E(\gamma_0) = \bar{R}_f$ ; and
- H5: The expected value of  $\gamma_1$  equals the excess market return, i.e.,  $E(\gamma_1) = \bar{R}_m - \bar{R}_f$ .<sup>6</sup>

As demonstrated by Fama [14], the estimated values of  $\gamma_{0t}$ ,  $\gamma_{1t}$  and  $\gamma_{2t}$  can be interpreted as the returns on portfolios with some special characteristics. For example  $\hat{\gamma}_{2t}$  is an estimate of the return on a minimum variance, zero-beta and zero-investment portfolio with a weighted average of  $\beta^2$  equal to one. According to the CAPM the expected return on this portfolio is zero. Therefore, the linearity hypothesis (H1) is supported if the mean value of  $\gamma_{2t}$ 's is zero.



If H1 is supported then standard errors of  $\gamma_{0t}$  and  $\gamma_{1t}$  can be reduced by estimating equation (14)

$$R_{pt} = \gamma_{0t} + \gamma_{1t} \hat{\beta}_{pt} + u_{pt} \quad (14)$$

rather than equation (13). We used (14) to estimate  $\gamma_{0t}$  and  $\gamma_{1t}$ . The OLS estimate of  $\gamma_{0t}$  is interpreted as the return on a minimum variance portfolio with beta of zero. If the S-L model is correct, we expect that  $E(\gamma_0) = \bar{R}_f$ . Similarly the OLS estimate of  $\gamma_{1t}$  is interpreted as the return on a minimum variance and zero-investment portfolio which has a beta of one. If the market portfolio is efficient or if the capital market is dominated by risk-averse investors, then  $E(\gamma_1) > 0$ . Also according to the S-L model,  $E(\gamma_1)$  should equal  $\bar{R}_m - \bar{R}_f$ .

#### E. Test of the Hypotheses

The five hypotheses and the corresponding test statistics are as follows:

Hypothesis	Test Statistic
H1: $E(\gamma_2) = 0$	$t = \frac{\bar{Y}_2 - 0}{s(\bar{Y}_2)}$
H2: $E(\gamma_1) > 0$	$t = \frac{\bar{Y}_1 - 0}{s(\bar{Y}_1)}$
H3: $E(\gamma_0 + \gamma_1) = \bar{R}_m$	$t = \frac{\bar{Y}_0 + \bar{Y}_1 - \bar{R}_m}{s(\bar{Y}_0 + \bar{Y}_1)}$
H4: $E(\gamma_0) = \bar{R}_f$	$t = \frac{\bar{Y}_0 - \bar{R}_f}{s(\bar{Y}_0)}$
H5: $E(\gamma_1) = \bar{R}_m - \bar{R}_f$	$t = \frac{\bar{Y}_1 - (\bar{R}_m - \bar{R}_f)}{s(\bar{Y}_1)}$



where  $\bar{\gamma}$  is the mean value of the  $\bar{\gamma}$ 's estimated in each of the nineteen subperiods and  $s(\bar{\gamma})$  is the standard deviation of the  $\bar{\gamma}$ 's. Fama and MacBeth [15] calculated  $\bar{\gamma}$  and  $s(\bar{\gamma})$  as:

$$\bar{\gamma} = \sum_{t=1}^T \hat{\gamma}_t / T$$

$$s(\bar{\gamma}) = \frac{s(\hat{\gamma}_t)}{\sqrt{T}}$$

where  $s(\hat{\gamma}_t) = \sum_{t=1}^T (\hat{\gamma}_t - \bar{\gamma})^2 / T$ .

The foregoing assumes that  $\hat{\gamma}_t$  equals the true value of  $\gamma_t$ . A better procedure for calculating  $\bar{\gamma}$  and  $s(\bar{\gamma})$  would take into account the effect of the sampling error of  $\hat{\gamma}_t$ , development of such a procedure follows.

The estimated  $\gamma_t$  may differ from its expected value for two reasons. First, since  $\hat{\gamma}_t$  is a sample estimate it may differ from the true  $\gamma_t$  by a random variable  $u_t$ . From regression analysis we know that  $u_t$  has a zero mean and a standard deviation  $\sigma_{ut}$  which is equal to the sampling error of  $\hat{\gamma}_t$ . Thus  $\hat{\gamma}_t$  can be written as

$$\hat{\gamma}_t = \gamma_t + u_t. \quad (15)$$

Furthermore, the true  $\gamma_t$  in subperiod  $t$  may differ from the expected value of  $\gamma_t$  over the entire period by a random variable  $v_t$ , that is,

$$\gamma_t = \bar{\gamma} + v_t \quad (16)$$

where  $v_t$  has a zero mean and variance  $\sigma_v^2$ . Although  $\sigma_{ut}^2$  may change from one subperiod to another,  $\sigma_v^2$  is constant for all subperiods.

Substituting (16) into (15) yields:





$$\hat{\gamma}_t = \bar{\gamma} + w_t \quad (17)$$

where  $w_t = u_t + v_t$ . Variables  $u_t$  and  $v_t$  are assumed independent of each other and independent through time. Thus, the variance of  $w_t$  is:

$$\sigma_{wt}^2 = \sigma_{ut}^2 + \sigma_v^2.$$

From (17)  $\bar{\gamma}$  was estimated using the GLS regression technique.<sup>8</sup> The GLS technique was used to estimate  $\bar{\gamma}$  because  $\sigma_{wt}^2$  is changing from one period to another. In (17) the only explanatory variable has a value of 1 in all sub-periods. That is, (17) can be written as:

$$\hat{\gamma}_t = \bar{\gamma} \cdot X + w_t. \quad (18)$$

GLS was used to estimate  $\bar{\gamma}$  as:

$$\hat{\bar{\gamma}} = (\bar{X}'\Omega^{-1}\bar{X})^{-1}\bar{X}'\Omega^{-1}\hat{\gamma}_t \quad (19)$$

where  $X$  is a  $19 \times 1$  vector with all elements equal to 1, and the  $\Omega$  matrix is

$$\Omega = \sigma^2 \theta = \begin{bmatrix} \sigma_{u1}^2 + \sigma_v^2 & 0 & \dots & 0 \\ 0 & \sigma_{u2}^2 + \sigma_v^2 & \dots & 0 \\ \vdots & & & \\ 0 & 0 & & \sigma_{u19}^2 + \sigma_v^2 \end{bmatrix}.$$

The variance of  $\hat{\bar{\gamma}}$  was estimated as:

$$s^2(\hat{\bar{\gamma}}) = (\bar{X}'\Omega^{-1}\bar{X})^{-1}. \quad (20)$$



By assuming that  $w_t$  is normally distributed,  $\hat{\gamma}$  is normally distributed with mean  $\bar{\gamma}$  and variance  $S^2(\hat{\gamma})$ . Given these results, hypotheses H1 through H5 were tested by the previously mentioned t-statistics.

### III. RESULTS

The relationship between  $R_{pt}$  and  $\beta_{pt}$  for 1957, 1965, and 1975 is given in Figures 1-3.<sup>9</sup> As indicated in Figures 1-3 the range, the degree of linearity, and the direction of the relationship between  $R_{pt}$  and  $\beta_{pt}$  varies considerably over time.

Table I contains the OLS estimates of  $\gamma_{0t}$ ,  $\gamma_{1t}$ ,  $\gamma_{2t}$ , their corresponding standard errors, and  $R^2(R_p, \beta_p)$  for each of the nineteen years. The following points can be made from the analysis of Table I.

First, the strength of the risk-return relationship as measured by  $R^2(R_p, \beta_p)$  varies substantially over time and ranges from 0.002 in 1970 to 0.965 in 1975. The mean value of  $R^2(R_p, \beta_p)$  is about 0.47. This result indicates that on average 47 percent of the variability of the returns on portfolios can be explained by their betas.

Second, the variability of  $\gamma_{1t}$  over time was substantially greater than the variability of  $\gamma_{0t}$  and  $\gamma_{2t}$ . This result is consistent with the proposition that the variability of returns on high-beta portfolios (e.g., a portfolio with a beta of one) is greater than the variability of returns on low-beta portfolios (zero-beta portfolios). The results also indicate that the mean value of  $\gamma_{1t}$  is greater than the mean value of  $\hat{\gamma}_{0t}$  and  $\hat{\gamma}_{2t}$ , that is, portfolios with higher betas also had higher mean returns.

---

Insert Table I Here

---



TABLE I

SUMMARY RESULTS OF THE TESTS OF THE CAPM USING OLS TECHNIQUE

	$\hat{Y}_{2t}$	$\hat{Y}_{0t}$	$\hat{Y}_{1t}$	$s(\hat{Y}_{2t})$	$s(\hat{Y}_{0t})$	$s(\hat{Y}_{1t})$	$R_{ft}$	$R_{mt}$	$R^2(R_p, \beta)$
1	0.0337	0.0005	-0.0725	0.0009	0.0015	0.0008	0.0340	-0.1053	0.263
2	0.0949	0.0476	0.4159	0.0008	0.0009	0.0006	0.0250	0.4451	0.948
3	0.0166	0.0115	0.1575	0.0013	0.0016	0.0011	0.0304	0.1294	0.554
4	0.0094	-0.0251	0.0412	0.0012	0.0014	0.0010	0.0502	0.0057	0.080
5	0.0242	0.0410	0.2041	0.0005	0.0004	0.0003	0.0250	0.2712	0.875
6	0.0507	-0.0635	-0.0572	0.0010	0.0009	0.0007	0.0302	-0.0930	0.205
7	0.0053	0.1118	0.0951	0.0008	0.0007	0.0005	0.0295	0.2129	0.484
8	-0.0457	0.1565	0.0378	0.0006	0.0008	0.0005	0.0369	0.1609	0.119
9	0.0212	0.0471	0.1793	0.0007	0.0009	0.0006	0.0393	0.1454	0.740
10	0.0344	-0.0138	-0.0462	0.0008	0.0010	0.0006	0.0470	-0.0887	0.148
11	0.0692	0.0274	0.2939	0.0013	0.0015	0.0010	0.0483	0.2779	0.803
12	-0.0031	0.1255	0.1035	0.0014	0.0017	0.0012	0.0556	0.1360	0.315
13	-0.0016	-0.0576	-0.0440	0.0006	0.0014	0.0009	0.0530	-0.0990	0.103
14	-0.0216	0.0499	0.0034	0.0002	0.0005	0.0003	0.0764	0.0150	0.002
15	-0.0206	0.0245	0.1192	0.0006	0.0007	0.0006	0.0487	0.1559	0.552
16	0.0023	0.0858	0.0739	0.0010	0.0006	0.0005	0.0409	0.1802	0.327
17	0.2153	0.3010	-0.3831	0.0023	0.0043	0.0036	0.0539	-0.1707	0.682
18	-0.0231	-0.0035	-0.2011	0.0009	0.0006	0.0005	0.0701	-0.2701	0.799
19	0.0411	0.0747	0.3982	0.0003	0.0004	0.0003	0.0679	0.3901	0.965



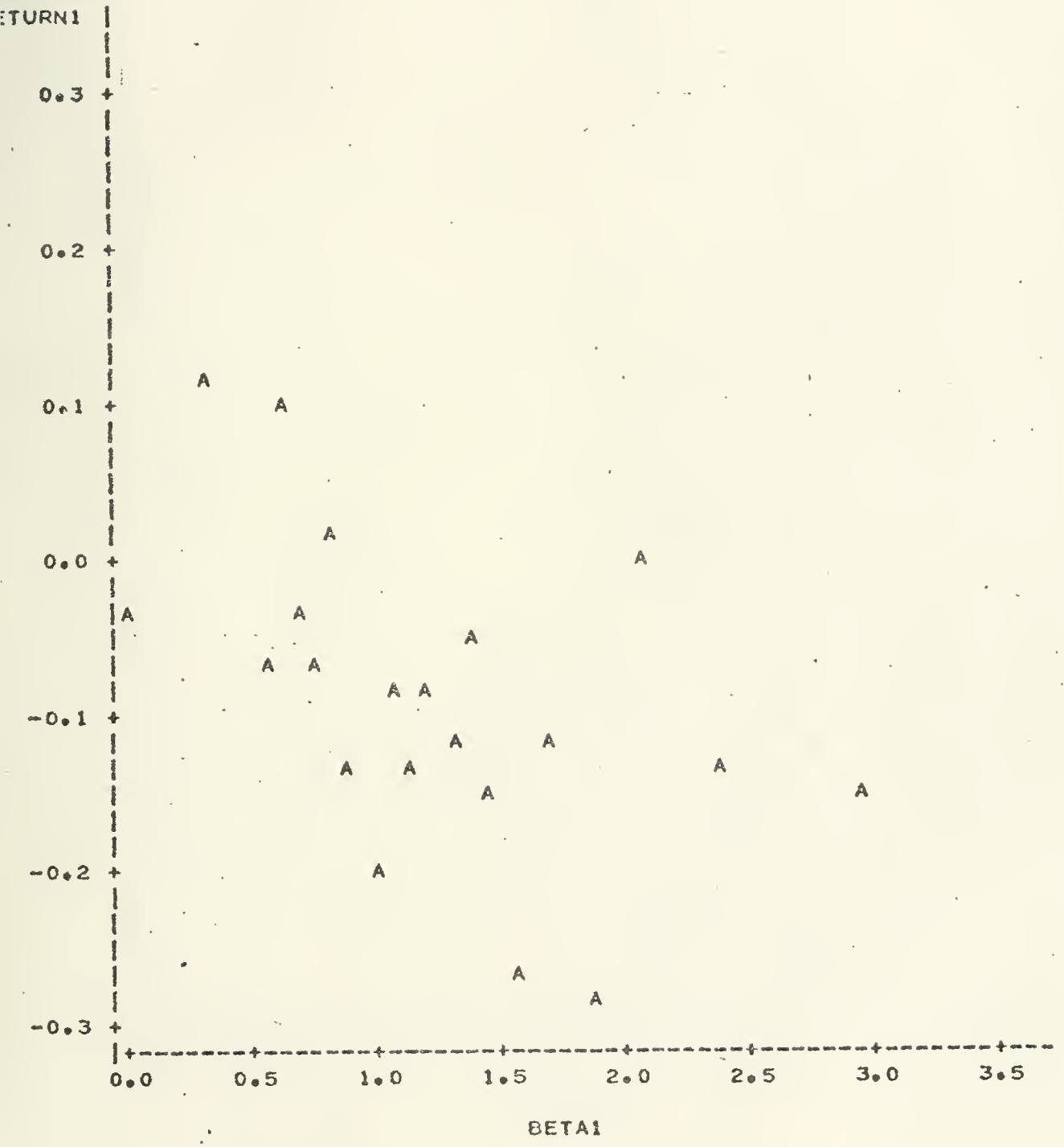


Figure 1

Empirical Relationship Between Return and Beta in 1957





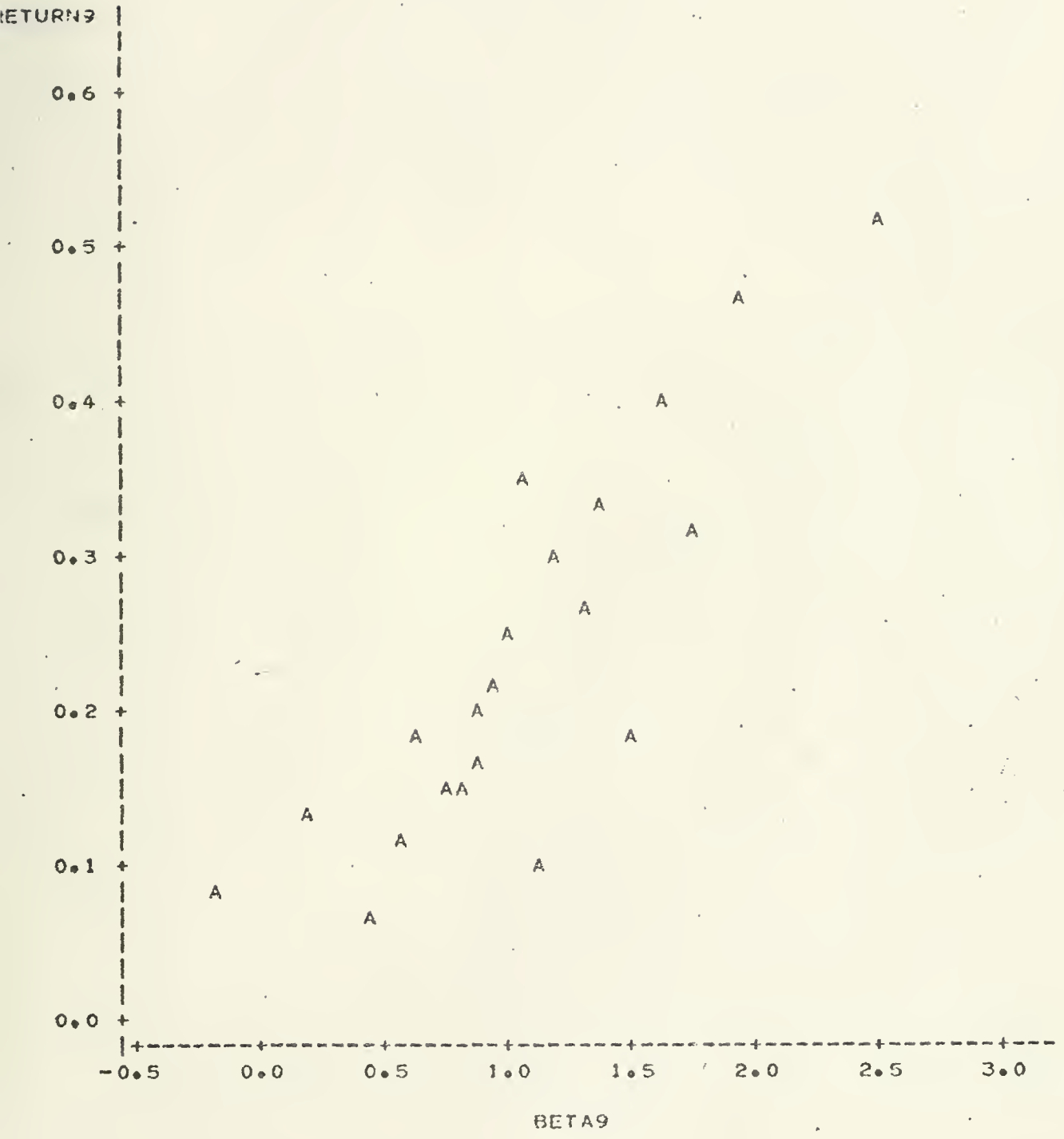


Figure 2

Empirical Relationship Between Return and Beta in 1965



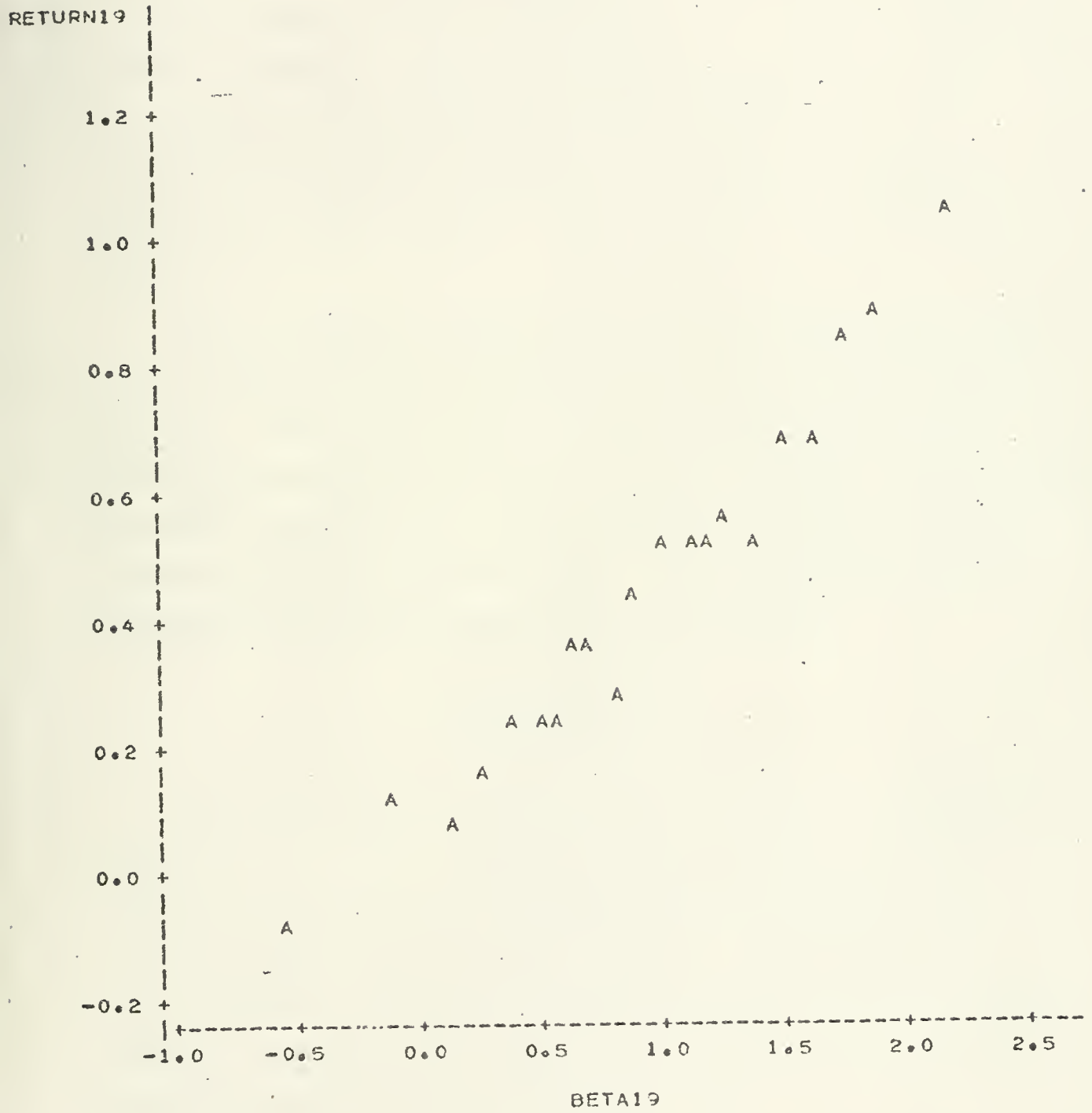


Figure 3

Empirical Relationship Between Return and Beta in 1975



Third, the results indicate that in some periods  $\hat{\gamma}_{2t}$  was substantially different from zero. This may indicate that in these periods the risk-return relation was nonlinear. However, as discussed in the previous section,  $\hat{\gamma}_{2t}$  is the return on a portfolio and may vary from one year to another. The linearity hypothesis should be rejected if the expected value of  $\hat{\gamma}_{2t}$  is significantly different from zero. Similar arguments can be made about  $\hat{\gamma}_{0t}$  and  $\hat{\gamma}_{1t}$ .

Table II contains the estimated expected value of  $\hat{\gamma}_{0t}$ ,  $\hat{\gamma}_{1t}$ ,  $\hat{\gamma}_{2t}$ , their standard errors, and their corresponding t-statistics for testing the five hypotheses. Table II also contains the mean values of  $R_{ft}$  and  $R_{mt}$  which were used to test hypotheses H4 and H5. The five hypotheses were divided into two groups. The first group contained hypotheses H1, H2, and H3 which are relevant to both the S-L and Black models. The second group consisted of hypotheses H4 and H5 which pertained only to the S-L model.

-----  
 Insert Table II Here  
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The t-statistic for H1 (see Table II) is 1.54 which is statistically non-significant for  $\alpha = 0.05$ . Therefore, our empirical results support the hypothesis that the risk-return relation is linear. Therefore, we conclude that expected return is a linear function of beta. The linearity between  $\bar{R}$  and  $\beta$  also implies that the market portfolio is a minimum-variance portfolio. The results of the linearity hypothesis agree with those of BJS [6] Blume and Friend [8], and Fama and MacBeth [15].

The second hypothesis, H2, states that higher expected risk is associated with higher expected return (i.e.,  $\bar{\gamma}_1 > 0$ ). Based on a t-statistic (see Table II) we were unable to reject H2 for  $\alpha = 0.05$ . Thus this result agrees



TABLE II

## RESULTS OF THE FIVE HYPOTHESES

---

	<u>OLS Estimate</u>
$\bar{Y}_0$	0.0466
$\bar{Y}_1$	0.0712
$\bar{Y}_2$	0.0237
$\bar{Y}_0 + \bar{Y}_1$	0.1190
$s(\bar{Y}_0)$	0.0222
$s(\bar{Y}_1)$	0.0458
$s(\bar{Y}_2)$	0.0154
$s(\bar{Y}_0 + \bar{Y}_1)$	0.0373
t(H1)	1.54
t(H2)	1.56
t(H3)	-0.44
t(H4)	0.02
t(H5)	-0.41
$\bar{R}_m$	0.1355
$\bar{R}_f$	0.0461

---





with the hypothesis that the capital markets are dominated with risk-averse investors who require compensation for bearing risk. The results also imply that the slope of the security market line is positive and hence the market portfolio is located on the positively sloped segment of the minimum-variance boundary. That is, our results support the hypothesis that the market portfolio is an efficient portfolio.

For H3 the t-statistic (see Table II) is statistically nonsignificant for  $\alpha = 0.05$ . Therefore, the expected value of  $\gamma_0 + \gamma_1$  is not significantly different from the value predicted by both S-L and Black models. In summary, the empirical results of H1, H2, and H3 support three important implications of the S-L and Black models. These are: (1) the risk and return relationship for portfolios of securities is linear, (2) there is a positive relationship between risk and return, and (3)  $E(\gamma_0 + \gamma_1) = \bar{R}_m$ .

One assumption of the S-L model is that unrestricted borrowing and lending can occur at the risk-free rate of interest. The testable implication of this assumption is that the expected return on the portfolio with returns uncorrelated with the returns on the market portfolio is  $R_{ft}$ . As previously stated,  $\hat{\gamma}_{0t}$  is an estimate of the return on a zero-beta portfolio. Thus, H4 is equivalent to the hypothesis that  $\bar{\gamma}_0 = \bar{R}_f$ . The t-statistic for H4 is very small, 0.02, indicating strong support for this hypothesis. The average value of the risk-free rate of interest over the nineteen years is 4.61 percent which compares very favorably with  $\hat{\gamma}_0$  of 4.66 percent.

Our results also support H5 which states that  $\bar{\gamma}_1 = \bar{R}_m - \bar{R}_f$ . The t-statistic is -0.41, which is statistically nonsignificant for  $\alpha = 0.05$ . Therefore, H5 cannot be rejected. The correlation coefficient of 0.91 between  $\gamma_{1t}$  and  $R_{mt} - R_{ft}$  indicated a strong association. Thus the time series analysis



provided further support for the S-L model.

Unlike Fama and MacBeth [15], Blume and Friend [8], and BJS [6], who concluded that the S-L model was not supported empirically, we found empirical support for the S-L model. The conflicts between our results and prior studies may be due to the differences in the statistical techniques applied and/or to the differences in the proxy used for the market portfolio. Since the statistical techniques used to estimate the beta values were found to be superior to the simple market model (which assumes beta to be stationary), we conclude that their negative conclusions of the S-L hypotheses may be due to biases in their statistical techniques. Furthermore, it also can be shown that their negative conclusions about the S-L hypotheses may be due to the use of an inadequate proxy for the market portfolio.

In all of the prior CAPM studies (except Foster), an equally weighted portfolio of NYSE stocks was used as a proxy for the market portfolio. Fisher [16] found that the standard deviation of the returns on an equally weighted portfolio E of NYSE stocks is 25 percent larger than the standard deviation of returns on a weighted-value portfolio W of NYSE stocks. This means that as shown in Figure 4 portfolio E is located on the right hand side of portfolio W on the efficient boundary.

From Figure 4 it is apparent that the estimated intercept will be larger when portfolio E is used than when portfolio W is used. Thus, as expected the estimated intercept in all the previous studies was greater than  $R_f$ . In this study, a value-weighted portfolio was used as a proxy for the market portfolio and the estimated value of the intercept was very close to  $R_f$ . In his book Fama [14] examined the inadequacy of the equally-weighted



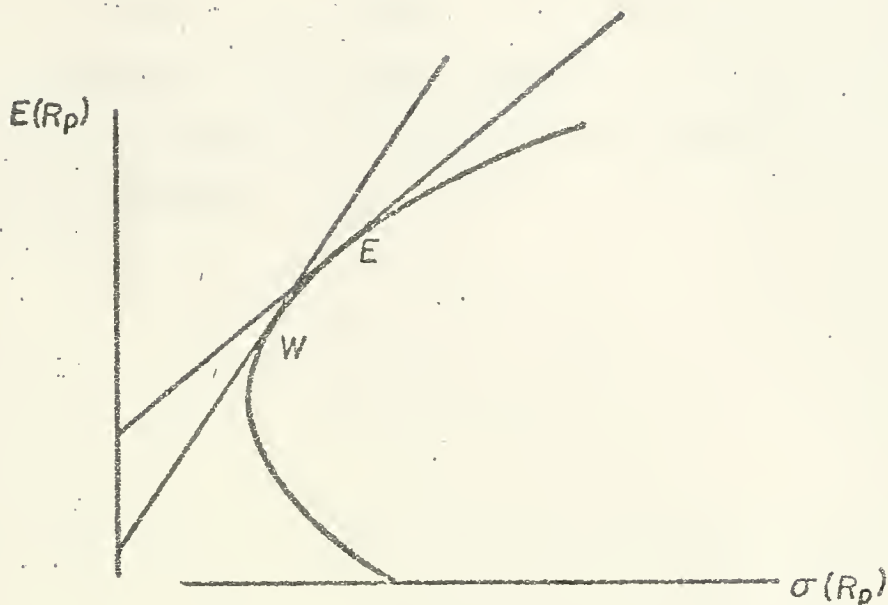


FIGURE 4

Positive Bias in the Estimated Intercept  
when Portfolio E is Used as a Proxy for  
the Market Portfolio

portfolio and concluded that the tests of the Sharpe-Lintner hypotheses reported by Fama and MacBeth, BJS, and Friend and Blume are inappropriate.

#### IV. CONCLUSIONS

In summary, the empirical results did not provide any evidence against the five hypotheses. Our research indicated that there is a linear relationship between risk and return. Furthermore, we could not reject the hypothesis that higher risk is associated with higher return. These results are fairly consistent with the results of previous studies by BJS [6], Fama and MacBeth [15], and Blume and Friend [8]. However, the results of this study with respect to the Sharpe-Lintner hypotheses are in conflict with the results obtained by Friend and Blume, BJS, and Fama and MacBeth. The findings of this study



indicate that the Sharpe-Lintner CAPM is a valid description of the risk-return relation. The conflicts in the results between our study and previous studies may be due to the differences in the statistical techniques or the market portfolio proxies employed.





## NOTES

1. A limitation of this approach is that beta is assumed to vary systematically with the changes in the financial variables. It is more realistic to assume that in addition to systematic variation beta has some stochastic or random variations over time. The addition of the stochastic term, however, makes it more difficult to estimate the covariance matrix of the composite error term. Furthermore, prior information about the mean and variance-covariance of the stochastic terms is needed to estimate the parameters of equation (6). Because of the small sample size (19 observations) and complexity of the estimation technique, the stochastic variation of the beta was not considered. As argued by Belsley [2] the results will be satisfactory if beta is significantly related to the financial variables.
2. Prior to forming portfolios, securities with  $\beta$  greater than 4 or less than -2 in any subperiod were considered to be outliers and were omitted. Deleting the outliers reduced the sample from 207 to 180.
3. The quadratic function was used to represent nonlinear risk-return relations because it provides considerable flexibility in approximating many nonlinear functions. The quadratic form also was used by Fama and MacBeth [15] to test for linearity. The linearity hypothesis is equivalent to the proposition that the market portfolio is a minimum-variance portfolio.
4. Hypothesis H2 implies that the market portfolio is on the positively-sloped segment of the minimum-variance frontier, and hence, it implies that the market portfolio is an efficient portfolio. Furthermore, since  $\gamma_1$  is the slope of equation (1), hypothesis H2 indicates that higher risk is associated with higher expected return. Hypothesis H2 also may be interpreted to imply that the capital market is dominated by risk-averse investors.
5. The hypothesis that  $E(\gamma_0 + \gamma_1) = E(R_m)$  has not been tested in previous studies which tested the CAPM. This hypothesis is a complement to the linearity hypothesis. Like the linearity hypothesis, this hypothesis holds if the market portfolio is a minimum-variance portfolio.
6. Unlike hypotheses H1, H2 and H3, which are relevant to both variants of the CAPM, i.e., the Sharpe-Lintner CAPM and Black's two-factor CAPM, hypotheses H4 and H5 were tested only to determine whether the process that generates the security returns is consistent with the S-L model.
7. See Appendix A for the estimation of  $\sigma_v^2$ .
8. GLS estimate of  $\bar{\gamma}$  is equivalent to the weighted average of  $\gamma_t$ 's where the weights are proportional to the inverse of the variance of the composite error terms. The tests also were repeated using the simple average of  $\gamma_t$ 's. The outcomes of the tests of the five hypotheses were the same under both procedures.
9. The figures included are typical of the relationship between  $R_{pt}$  and  $\beta_{pt}$ . To conserve space the remaining sixteen years were not included.



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## APPENDIX A

### Estimation of $\sigma_v^2$

In this appendix, the procedure for estimating the variance of the true  $\gamma_t$  over time is explained. In section III, it was shown that:

$$\hat{\gamma}_t = \bar{\gamma} + w_t$$

where  $w_t = u_t + v_t$

where  $u_t$  represents the deviation of  $\hat{\gamma}_t$  from its true value and  $v_t$  represents the deviation of the true  $\gamma_t$  from its expected value,  $\bar{\gamma}$ , over the nineteen subperiods. It was explained that the variance of  $u_t$  can be obtained from the regression analysis. In contrast to  $u_t$ , whose variance is known, the variance of  $v_t$  is unknown because the true value of  $\gamma_t$  can not be observed. However, the variance of  $v_t$  was estimated by using the following iterative technique.

Step one, estimate  $\bar{\gamma}$  from equation (21), assuming that  $\sigma_v^2 = 0$ .

When  $\sigma_v^2 = 0$ ,  $\Omega$  matrix can be written as:

$$\Omega = \sigma^2 \theta = \begin{bmatrix} \sigma_{u1}^2 & 0 & \dots & 0 \\ 0 & \sigma_{u2}^2 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & \sigma_{u19}^2 \end{bmatrix}$$

Step two, estimate the variance of  $v_t$  by using the following formula:

$$\sigma_v^2 = [\hat{\gamma}_t - X' \hat{\bar{\gamma}}] \theta^{-1} [\hat{\gamma}_t - X' \hat{\bar{\gamma}}]$$

where  $\hat{\bar{\gamma}}$  is the estimated value of  $\bar{\gamma}$ . Step three, substitute  $\hat{\sigma}_v^2$  into



$$\sigma_{wt}^2 = \sigma_{ut}^2 + \sigma_v^2$$

and form the covariance matrix  $\Omega$ . Step four, estimate  $\bar{\gamma}$  using the new  $\Omega$  matrix. Repeat steps two through four until  $\hat{\sigma}_v^2$  converges to a constant value. The final estimate of  $\hat{\sigma}_v^2$  was used to form the  $\Omega$  matrix and hence, to estimate  $\bar{\gamma}$  and its standard error.















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