DAVID W. TAYLOR NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

Bethesda, Maryland 20084

CHALLENGE TO BETTER AGREEMENT BETWEEN THEORETICAL COMPUTATIONS AND MEASUREMENTS IN SHIP HYDRODYNAMICS

THE SIXTH DAVID W. TAYLOR LECTURE

by

Hajime Maruo

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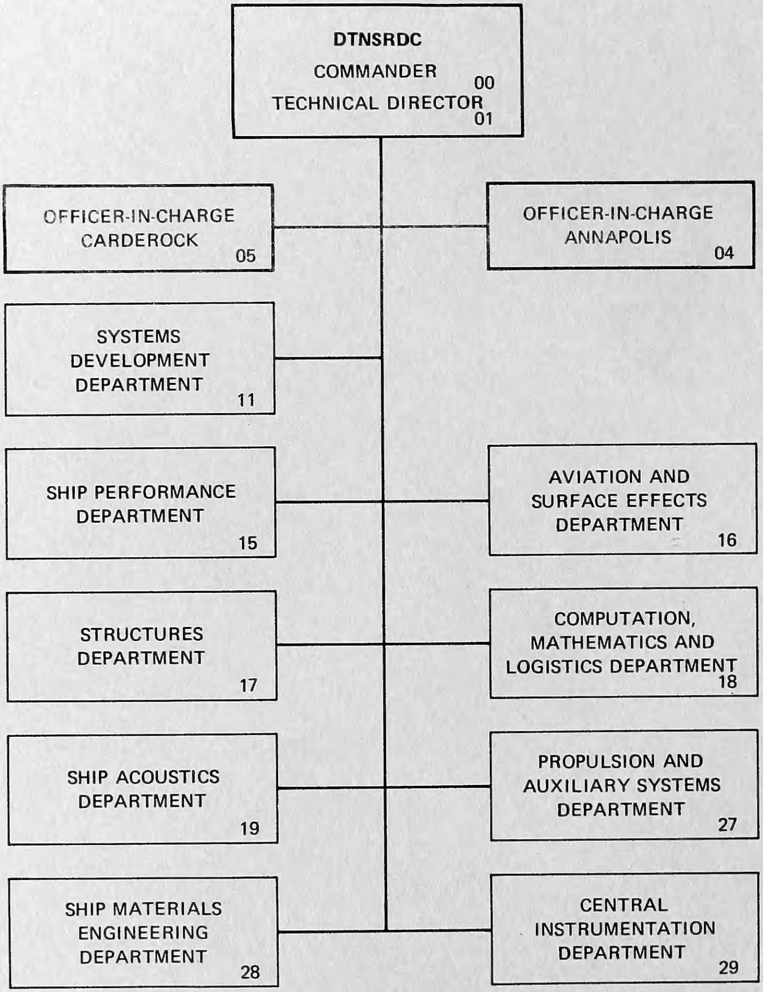

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PREFACE

The David W. Taylor Lectures were conceived to honor our founder in recognition of his many contributions to naval architecture and naval hydrodynamics. Admiral Taylor was a pioneer in the use of hydrodynamic theory and mathematics for the solution of naval problems. He established a tradition of applied scientific research at the "Model Basin" which has been carefully nurtured through the decades and which we treasure and maintain today. It is in this spirit that we have invited Professor Hajime Maruo to be a David W. Taylor Lecturer.

Prof. Maruo was born in Yokohama in 1922. He received his professional education at the University of Tokyo, obtaining his Ph.D in Naval Architecture in 1946. For many years Prof. Maruo has been at the Yokohama National University, where he is Professor in the Faculty of Engineering. He is a leader of ship research in Japan, is an officer of the Japanese Society of Naval Architects, and has been a member of the Resistance Committee of the International Towing Tank Conference as well as other important professional groups. He is an international authority on ship hydrodynamics and is well known for his very significant theoretical and mathematical research on ship wave resistance and ship motions problems.

Prof. Maruo is no stranger to the West. He spent a year at Cambridge University in England doing research under G.I. Taylor, and in 1964 was a Visiting Scientist and Professor at Stevens Institute of Technology. Over the years he has established and maintained close ties with many ship researchers in both the United States and Europe.

FOREWORD

It is a great honor for me to deliver the Sixth David W. Taylor Lecture series and I would like to express my sincere thanks to the David Taylor Naval Ship R&D Center for the kind arrangement which has enabled me to have such an opportunity.

The subject which I am going to talk about is the problem of engineering. It is not the problem of mathematics, although recent progress in technology depends greatly upon mathematical theories, and mathematics has become an indispensable aid to solve engineering problems nowadays.

CHALLENGE TO BETTER AGREEMENT BETWEEN THEORETICAL COMPUTATIONS AND MEASUREMENTS IN SHIP HYDRODYNAMICS

INTRODUCTION

There is a great difference between the idea of engineering and that of mathematics. The substantial importance in engineering is the practical utility. Any theories cannot become useful unless they lead to results which are faithful representations of actual phenomena.

In natural sciences like physics or chemistry, one may be contented with qualitative agreement between theoretical predictions and observations, but in engineering, merely qualitative agreement is not a sufficient condition of the practical usefulness. The agreement should be quantitative within required accuracy. Mathematics, on the other hand, emphasizes logical rationality. If one considers an approximation, the rigorous mathematical idea requires that the simplification should be consistent within itself throughout the approximation. However, we often encounter cases that mathematical rationalism contradicts the practical usefulness. We know quite a few cases that a mathematical theory, with all its rational construction, yields rather unrealistic results when compared with the actual phenomena. On the other hand, there are a number of cases that deviation from the rational formulation can result in much better agreement with measurements. Generally speaking, such inconsistent approaches are not necessarily safe, because their justification is hardly obtained from the purely theoretical point of view. However, the utility of this kind of approximate method can be appreciated in the practical application, since any theory which has failed to give correct predictions is almost useless even though it has a complete logical construction from the mathematical point of view.

The present lecture intends to illustrate how the deviation from the mathematical rationality can improve the agreement with measured results and how the mathematical theory may be revised for practical usefulness. Whether any inconsistent approach can become really useful or not depends greatly upon the engineer's intuition.

The title of the lecture can cover very wide aspects, but I will confine topics only in problems of free surface flow, because it is the problem which seems to be most peculiar to the hydrodynamics of ships. The lecture will deal with four topics, namely:

1. Wave resistance of ships in uniform forward motion,
2. Hydrodynamic forces on oscillating slender ships,
3. Wave pressure on slender ships, and
4. Added resistance of ships in ambient ocean waves.

WAVE RESISTANCE OF SHIPS IN UNIFORM FORWARD MOTION

Short Comments on Thin Ship Theory

The theory of wave resistance is a rather classical problem. It was as early as the end of the last century that Michell[1*] established the theory for thin ships. His theory was already so complete that nothing needed to be added for a first approximation of thin ships. More than eighty years have passed since then; nevertheless the progress in the theory of wave resistance has been comparatively slow.

Michell's thin ship theory is based on the assumption that the beam-to-length ratio is so small that its square can be neglected. There are several examples of comparison of computed wave resistance with measured resistance. Among them are instructive results with a series of models whose breadth is varied systematically.[2] It is indicated that the wave resistance calculated by Michell's formula agrees well with measured results, provided that the beam-to-length ratio is not greather than one fifteenth, as shown in Figure 1. This criterion of beam-to-length ratio is too small for practical hull forms. The beam-to-length ratio of practical hulls is at least one seventh. Consequently, considerable discrepancy appears between theory and experiment in conventional hull forms. This fact does not mean, however, that the thin ship theory is useless for practical purposes. It is known that the Michell thin ship theory has become a very powerful tool for the purpose of designing low resistance hull forms. A direct application of the theory for this purpose is known as the theory of minimum wave resistance.[3] The method is an application of calculus of variations. One can determine the optimum curve of a sectional

*A complete listing of references is given on page 119.

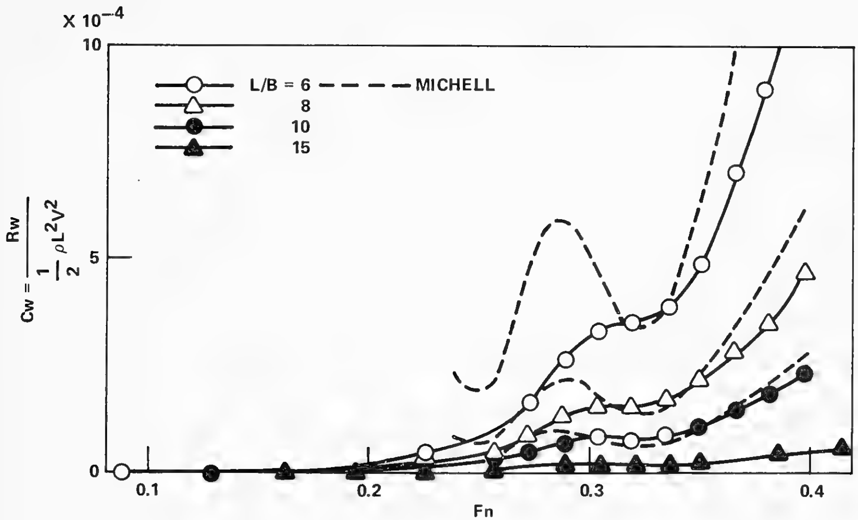

Figure 1 - Comparison of Michell Resistance with Measured Wave-Pattern Resistance (Tsutsumi)

area by which the wave resistance is made minimum under appropriate side conditions. One very interesting fact is that the curve of the sectional area, which is determined by the thin ship theory in such a way that the wave resistance becomes minimum at Froude number 0.25 with the condition of prismatic coefficient being 0.60, is nearly identical with the curve of sectional area of the corresponding Taylor Standard Series[4] (Figure 2). It is recognized that the latter form shows excellent characteristics at medium Froude numbers, and this fact is enough to testify the great genius and intuition of the late Admiral Taylor.

For the purpose of quantitative prediction of wave resistance, on the other hand, Michell's theory is far from useful. Two reasons are considered for the discrepancy of the theory from measured results; one is the effect of viscosity and the other is the effect of finite breadth of the ship. The effect of viscosity upon wave resistance has been studied by several authors,[5] but no reliable theory has been developed yet. As to the effect of finite breadth, on the other hand, several attempts are known

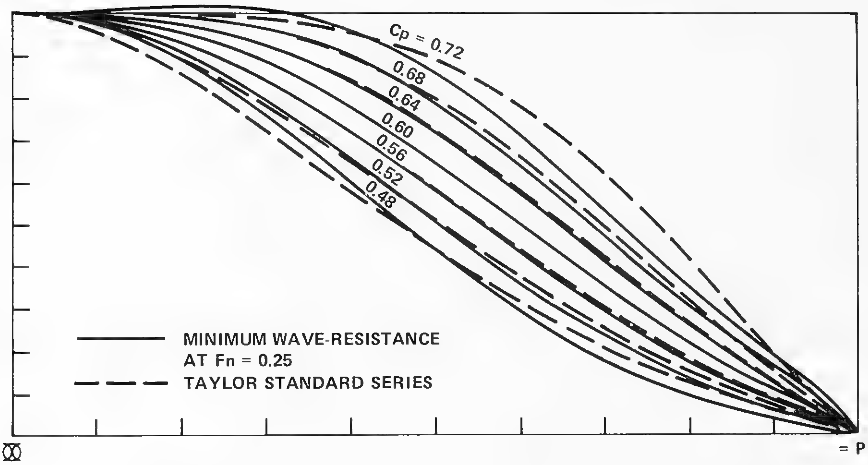

Figure 2 - Curve of Sectional Area of Minimum Wave-Resistance

which intend to find out second order terms with respect to the beam-to-length ratio by means of the successive method starting with Michell's first order solution.[6] However, none of these approaches have been proven successful so the second order theory is not likely to be promising. We can imagine the following fact as the reason for this difficulty. It is known that the first order solution is not a uniformly valid approximation and higher order solutions show more singular behavior. It is suspected that the resulting power series with respect to the beam-to-length ratio which is assumed at the beginning of the perturbation procedure may not converge when infinite terms are taken. That means the perturbation expansion gives an asymptotic series without finite radius of convergence. If it is so, the addition of second order terms does not guarantee any improvement in accuracy of the approximation.

Furthermore, the existence of the continuous solution of exact boundary value problem for the fluid motion around the hull is even doubtful. Observations of the white plume of the wave crest around the ship, may be an indication of the nonexistence of the exact solution.

It is of some interest to observe the numerical result by Guilloton's method,[7] which shows a plausible improvement[8] in agreement with measured wave profile and wave resistance. Unlike the consistent successive approximation, the various conditions involved are not satisfied at the same order of magnitude. This is because the field equation is satisfied only in the first order while boundary conditions are partly satisfied up to the second order.[9] In spite of such inconsistencies, Guilloton's method may be regarded more useful than the consistent second order theory from an engineering point of view.

An Expression for the Solution of Exact Nonlinear Boundary Value Problem

A special feature of the free surface flow is the nonlinearity of boundary conditions. The direct nonlinear analysis is applicable only to the simplest case such as monodirectional waves in a channel. Because of its complexity, the only possibility of an analytical method describing fluid motion around the ship hull is the perturbation analysis. As mentioned in the preceding section, only the first order approximation is useful for practical cases because a higher order approximation cannot improve the result in most cases. The first order solution is usually obtained by the linearization of the free surface condition at the beginning. However, there is a possibility of giving a formal expression for the solution which satisfies the exact nonlinear boundary condition at the free surface. This expression is of no use by itself for the prediction of wave resistance, but it facilitates general discussions of perturbation analysis.

First, we assume an inviscid incompressible fluid with irrotational flow. A Cartesian coordinate system with z-axis vertically upward and the axes of x and y is applied to the fluid on the still water surface. It is convenient to employ dimensionless quantities. The length scales, e.g., x, y, and z, are normalized by a characteristic length, ℓ, which may be taken as the half length of the ship, L/2. If the fluid velocity is normalized by the ship speed U, then the acceleration of gravity is expressed by the dynamic coefficient of gravity $\gamma_0 = g\ell/U^2$, or as a function of the Froude number $\gamma_0 = 1/(2Fn^2)$. Instead of a ship moving in still water, we assume a uniform flow of velocity U in the direction of x which is opposite to the motion of the ship. When a ship hull is introduced at a fixed position in the flow, the flow field is specified by a velocity potential $x + \phi$. The disturbance potential ϕ is harmonic in the space occupied by the fluid. Then the Laplace equation

$$\nabla^2 \phi = 0 \tag{1}$$

is valid outside the hull surface and below the free surface, which is expressed by the equation

$$z = \zeta \tag{2}$$

The fluid velocities are

$$u = 1 + \partial\phi/\partial x, \; v = \partial\phi/\partial y, \; w = \partial\phi/\partial z \tag{3}$$

The fluid boundary is composed of the wetted hull surface, the sea bottom, and the free surface. Since the fluid is assumed nonviscous, the boundary condition on the hull surface is that the fluid velocity is tangential to the hull surface. It is expressed by

$$\frac{\partial\phi}{\partial n} = - \frac{\partial x}{\partial n} \tag{4}$$

where n is the unit outward normal vector into the hull surface. Since the free surface is expressed by the equation $z = \zeta$, where ζ is a function of x and y, the kinematical condition on it is

$$u \frac{\partial\zeta}{\partial x} + v \frac{\partial\zeta}{\partial y} - w = 0 \tag{5}$$

There is a condition of constant pressure which is expressed by the Bernoulli equation.

$$\frac{1}{2}\,(u^2+v^2+w^2-1) + \gamma_0\,\zeta = 0 \tag{6}$$

Then the free surface elevation is given by

$$\zeta = \frac{1}{2\gamma_0}\,(1-u^2-v^2-w^2) \tag{7}$$

Keeping in mind the fact that the boundary conditions are satisfied at the curved surface $z = \zeta$ and that ζ is a function of x and y, the explicit ζ in Equations (5) and (6) is eliminated by the substitution of partial derivatives of Equation (7) with respect to x and y in Equation (5). The result is

$$\frac{1}{2}\left(u\,\frac{\partial}{\partial x} + v\,\frac{\partial}{\partial y} + w\,\frac{\partial}{\partial z}\right)(u^2+v^2+w^2) + \gamma_0\,w = 0 \tag{8}$$

If we substitute the velocities by the expression of Equation (3), the above equation becomes

$$\frac{1}{2}\left(2\,\frac{\partial}{\partial x} + \nabla\phi\cdot\nabla\right)\,|\nabla\phi|^2 + \frac{\partial^2\phi}{\partial x^2} + \gamma_0\,\frac{\partial\phi}{\partial z} = 0 \tag{9}$$

This relation holds on the unknown surface $z = \zeta$, so that the boundary condition is quite nonlinear. The usual way of solution is the perturbation expansion of the boundary condition assuming a small parameter which relates to the shape of the hull surface. The first term of the expansion is the linearized solution. In order to make such a linearized solution valid, a condition of small disturbance is necessary which imposes a restriction on the hull shape. Instead of the application of the perturbation expansion to the boundary condition, let us seek a general expression for the solution of the Laplace equation with the boundary conditions including nonlinear terms. In order to find such an expression, we apply Green's theorem to the velocity potential ϕ and an appropriate Green's function in the space bounded by a closed surface. Figure 3 shows this

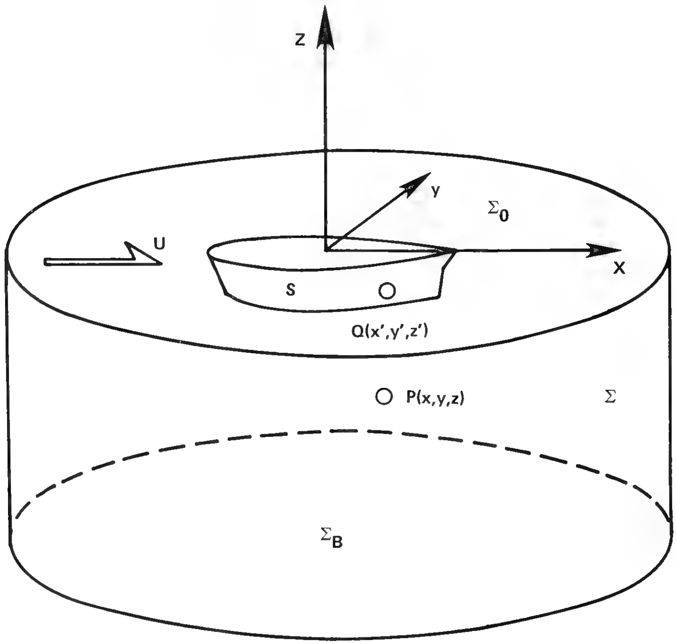

Figure 3 - Coordinate System and Control Surfaces

enclosed surface to encompass the portion of the hull surface in the lower half space S, a large vertical cylinder Σ surrounding the ship, the portion of the horizontal plane z = 0 between S and Σ, Σ_0 and the portion of the sea bottom inside the cylinder Σ_B. Then we have

$$\phi(P) = -\frac{1}{4\pi} \iint_{S+\Sigma+\Sigma_0+\Sigma_B} \left[G(P,Q) \frac{\partial\phi(Q)}{\partial n_Q} - \phi(Q) \frac{\partial G(P,Q)}{\partial n_Q} \; d\, S_Q \right] \qquad (10)$$

where G(P,Q) is Green's function having a simple pole at the point P(x,y,z) = Q(x',y',z'), P is a point inside the enclosed space defined above, Q is a point on the boundary surface and n_Q is the normal to the boundary surface drawn inward at point Q. This expression is valid provided that the analytic continuation of the velocity potential in the region $0 > z > \zeta$ is possible. This is the only assumption for the above formulation. Now we define the Green function in such a way that it satisfies the Laplace equation in the lower half space except a point P = Q, and boundary conditions

$$\left(\frac{\partial^2}{\partial x'^2} + \gamma_0 \frac{\partial}{\partial z'} \right) G(P,Q) = 0 \quad \text{at } z' = 0 \tag{11}$$

$$\frac{\partial}{\partial n} G(P,Q) = 0 \quad \text{on } \Sigma_B \tag{12}$$

It is assumed that the radiation condition

$$\lim_{x \to -\infty} \sqrt{x}\, G(P,Q) = 0 \quad \text{or} \quad \lim_{x' \to \infty} \sqrt{x'}\, G(P,Q) = 0 \tag{13}$$

is satisfied too. Since $\partial\phi/\partial n = 0$ and $\partial G/\partial n = 0$, the contribution by the integral on Σ_B vanishes. If the velocity potential ϕ is assumed to fulfill the radiation condition at infinite distance, the integral on Σ decays out as the radius of the cylinder tends to infinity. Since Σ_0 is a horizontal plane, the normal n is directed vertically downwards. Then the integral on Σ_0 can be transformed as

$$- \iint_{\Sigma_0} \left[G(P,Q) \frac{\partial \phi(Q)}{\partial n_Q} - \phi(Q) \frac{\partial G(P,Q)}{\partial n_Q} \right] d\, S_Q$$

$$= \iint_{\Sigma_0} \left[G(P,Q) \frac{\partial \phi(x',y',z')}{\partial z'} - \phi(x',y',z') \frac{\partial G(P,Q)}{\partial z'} \right]_{z'=0} dx'dy'$$

$$= \iint_{\Sigma_0} \left[G(P,Q) \frac{\partial\phi(x',y',z')}{\partial z'} + \frac{1}{\gamma_0} \frac{\partial^2 G(P,Q)}{\partial x'^2} \phi(x',y',z') \right]_{z'=0} dx'dy'$$

because of the relation of Equation (11). Integrating by parts twice, the second term with respect to x', yields

$$\iint_{\Sigma_0} \left[G(P,Q) \frac{\partial\phi(x',y',z')}{\partial z'} + \frac{1}{\gamma_0} \frac{\partial^2 \phi^2 (x',y',z')}{\partial x'^2} \right]_{z'=0} dx'dy'$$

$$+ \frac{1}{\gamma_0} \int_{L_0} \left[G(P,Q) \frac{\partial\phi(x',y',z')}{\partial x'} - \phi(x',y',z') \frac{\partial G(P,Q)}{\partial x'} \right]_{z'=0} dy'$$

where L_0 is the intersection of S and Σ_0. If

$$\left(\frac{\partial^2\phi}{\partial x^2} + \gamma_0 \frac{\partial\phi}{\partial z} \right)_{z=0} = - \Phi(x,y) \qquad (14)$$

then the velocity potential can be written as

$$\phi(P) = - \frac{1}{4\pi} \iint_S \left[G(P,Q) \frac{\partial\phi(Q)}{\partial n_Q} - \phi(Q) \frac{\partial G(P,Q)}{\partial n_Q} \right] d\,S_Q$$

$$+ \frac{1}{4\pi\gamma_0} \int_{L_0} \left[G(P,Q) \frac{\partial\phi(Q)}{\partial x'} - \phi(Q) \frac{\partial G(P,Q)}{\partial x'} \right]_{z'=0} dy$$

$$- \frac{1}{4\pi\gamma_0} \int_{\Sigma_0} \Phi(x',y') \, G(P,Q) \Big|_{z'=0} dx'dy' \qquad (15)$$

Since the boundary condition on the free surface, Equation (9) can be written as

$$\left(\frac{\partial^2\phi}{\partial x^2} + \gamma_0 \frac{\partial\phi}{\partial z}\right)_{z=0} = -\left[\frac{1}{2}\left(2\frac{\partial}{\partial x} + \nabla\phi\cdot\nabla\right)|\nabla\phi|^2\right]_{z=\zeta}$$

$$- \int_0^\zeta \left(\frac{\partial^3\phi}{\partial x^2 \partial z} + \gamma_0 \frac{\partial^2\phi}{\partial z^2}\right) dz$$

we can solve for the function $\Phi(x,y)$ as

$$\Phi(x,y) = \left[\frac{1}{2}\left(2\frac{\partial}{\partial x} + \nabla\phi\cdot\nabla\right)|\nabla\phi|^2\right]_{z=\zeta} + \int_0^\zeta \left(\frac{\partial^3\phi}{\partial x^2 \partial z} + \gamma_0 \frac{\partial^2\phi}{\partial z^2}\right) dz \tag{16}$$

The first term on the right-hand side of Equation (15) defines the sources and dipoles with their axes in the direction normal to the surface and distributed over the hull surface S. The second term corresponds to a line distribution along the water line of sources and x-directed dipoles, and the third term means the source distribution over the horizontal plane or the still water plane.

Next let us show that the same potential can be expressed by a distribution of sources only. The velocity potential given by Equation (15) is valid outside the hull surface S. Here let us assume a fictitious velocity potential ϕ^* which is valid inside the surface S and satisfies the linearized boundary condition at z = 0 as

$$\frac{\partial^2\phi^*}{\partial x^2} + \gamma_0 \frac{\partial\phi^*}{\partial z} = 0 \tag{17}$$

Consider a closed surface composed of S and the portion of the plane z = 0 inside S denoted by Σ_0^*. Apply Green's theorem, as before, to ϕ^* and

G(P,Q) in the interior domain of this closed surface. If the point P is outside S, we have

$$\frac{1}{4\pi}\iint_{S+\Sigma_0^*}\left[G(P,Q)\,\frac{\partial\phi^*(Q)}{\partial n_Q^*}-\phi^*(Q)\,\frac{\partial G(P,Q)}{\partial n_Q^*}\right]d\,S_Q=0$$

where n^* is the normal drawn inwards to the domain under consideration. Integrating by parts over Σ_0^*, as before, we find

$$0=\frac{1}{4\pi}\iint_S\left[G(P,Q)\,\frac{\partial\phi^*(Q)}{\partial n_Q^*}-\phi^*(Q)\,\frac{\partial G(P,Q)}{\partial n_Q^*}\right]d\,S_Q$$

$$+\frac{1}{4\pi\gamma_0}\int_{L_0}\left[G(P,Q)\,\frac{\partial\phi^*(Q)}{\partial x'}-\phi^*(Q)\,\frac{\partial G(P,Q)}{\partial x'}\right]_{z'=0}dy' \qquad (18)$$

Subtraction of Equation (18) from Equation (15) yields

$$\phi(P)=-\frac{1}{4\pi}\iint_S\left[G(P,Q)\left\{\frac{\partial\phi(Q)}{\partial n_Q}+\frac{\partial\phi^*(Q)}{\partial n_Q^*}\right\}-\left\{\phi(Q)\,\frac{\partial G(P,Q)}{\partial n_Q}+\phi^*(Q)\,\frac{\partial G(P,Q)}{\partial n_Q^*}\right\}\right]d\,S_Q$$

$$+\frac{1}{4\pi\gamma_0}\int_{L_0}\left[G(P,Q)\left\{\frac{\partial\phi(Q)}{\partial x'}-\frac{\partial\phi^*(Q)}{\partial x'}\right\}-\frac{\partial G(P,Q)}{\partial x'}\,\{\phi(Q)-\phi^*(Q)\}\right]_{z'=0}dy'$$

$$-\frac{1}{4\pi\gamma_0}\iint_{\Sigma_0}\Phi(x',y')\,G(P,Q)\Big|_{z'=0}\,dx'dy' \qquad (19)$$

Now we assume that the fictitious velocity potential is chosen in such a way that it has a value identical with ϕ on S. Then, the relations on S are

$$\phi \frac{\partial G}{\partial n} + \phi^* \frac{\partial G}{\partial n^*} = \phi \frac{\partial G}{\partial n} - \phi^* \frac{\partial G}{\partial n} = 0$$

$$(\phi-\phi^*) \frac{\partial G}{\partial x} = 0$$

Therefore, Equation (19) becomes

$$\phi(P) = -\frac{1}{4\pi} \iint_S G(P,Q) \left\{ \frac{\partial \phi(Q)}{\partial n_Q} + \frac{\partial \phi^*(Q)}{\partial n_Q^*} \right\} d\, S_Q$$

$$+ \frac{1}{4\pi\gamma_0} \int_{L_0} \left[G(P,Q) \left\{ \frac{\partial \phi(Q)}{\partial x'} - \frac{\partial \phi^*(Q)}{\partial x'} \right\} \right]_{z'=0} dy'$$

$$- \frac{1}{4\pi\gamma_0} \iint_{\Sigma_0} \Phi(x',y')\, G(P,Q) \Big|_{z'=0} dx'dy' \qquad (20)$$

Since -G(P,Q) means a source at the point Q, the velocity potential is expressed by the distribution of sources over the hull and the still water surface. The density of the hull surface sources is

$$\sigma = \frac{1}{4\pi} \left(\frac{\partial \phi}{\partial n} + \frac{\partial \phi^*}{\partial n^*} \right)$$

Here we take a plane parallel to x involving the normal and take a length s_1 along the curve of intersection of this plane and the surface S. If α is the angle between the normal and the x axis, we have the relation

$$\frac{\partial \phi}{\partial x} = \cos \alpha \frac{\partial \phi}{\partial n} + \sin \alpha \frac{\partial \phi}{\partial s_1}$$

We have assumed that $\phi = \phi^*$ on S, so that

$$\frac{\partial\phi}{\partial s_1} = \frac{\partial\phi^*}{\partial s_1}$$

Then the line integral is expressed by

$$\int_{L_0} G(P,Q)\left(\frac{\partial\phi}{\partial n} - \frac{\partial\phi^*}{\partial n}\right)\cos\alpha\, dy' = \int_{L_0} G(P,Q)\left(\frac{\partial\phi}{\partial n} + \frac{\partial\phi^*}{\partial n^*}\right) n_x \frac{dy'}{ds}\, ds$$

$$= 4\pi \int_{L_0} \sigma(Q)\; G(P,Q)\; n_x \frac{dy'}{ds}\, ds$$

where s is the length along L_0 and $n_x = \partial x/\partial n$. Then we obtain

$$\phi(P) = - \iint_S \sigma(Q)\; G(P,Q)\; d\, S_Q + \frac{1}{\gamma_0}\int_{L_0} \sigma(Q)\; G(P,Q)\; n_x \frac{dy'}{ds}\, ds$$

$$- \frac{1}{4\pi\gamma_0}\iint_{\Sigma_0} \Phi(x',y')\; G(P,Q)\Big|_{z'=0}\; dx'dy' \qquad (21)$$

If we assume the disturbance velocity is so small that the function $\Phi(x,y)$ is a negligible second order contribution, the velocity potential can be expressed by the source distribution on the hull surface accompanied by the line distribution. This yields

$$\phi(P) = - \iint_S G(P,Q)\; \sigma(Q)\; d\, S_Q + \frac{1}{\gamma_0}\int_{L_0} G(P,Q)\; \sigma(Q)\; n_x \frac{dy'}{ds}\, ds \qquad (22)$$

One can determine the source density $\sigma(Q)$ so as to satisfy the boundary condition on the hull surface. Brard[10] named this kind of boundary value problem the Neumann-Kelvin problem. Several numerical works have been carried out so far.[11] Since the smallness of the disturbance velocity cannot be assumed apriori, other conditions are needed in order to realize it. The simplest case is the thin ship. If the beam of the ship is very small, the inner region, in which the potential ϕ^* is defined, shrinks to a narrow slit and, consequently, $\partial\phi^*/\partial n^*$ becomes higher order with respect to the beam-to-length ratio of the ship. Therefore, $\sigma(Q)$ is determined by $(1/4\pi)\ \partial\phi/\partial n$. The line integral becomes third order and can be omitted. If we write the equation of the hull surface as

$$y = f(x,z)\ \text{sgn}\ y \tag{23}$$

the velocity potential is reduced to

$$\phi(P) = -\frac{1}{2\pi}\iint_{S_c} G(P,Q)\ f_x(x',z')\ dx'dz' \tag{24}$$

where S_c is the center plane of the ship and $f_x = \partial f/\partial x$. This is identical with Michell's potential.

Wave Resistance at Low Speed

Most existing theories of ship waves and wave resistance are based on the linearization of the flow field by a small parameter which specifies the slenderness of the ship hull. Since ship hulls, in practice, are neither so slender nor thin enough to secure the validity of the linearized theory, the agreement between the theoretical prediction and the experimental results is, in general, not satisfactory. The disturbance velocities are not small enough to make their square negligible everywhere on the free surface. Since the inclusion of the nonlinear terms in the free surface condition makes the boundary value problem intractable, some simplification other than the linearization by the beam-to-length ratio as a small parameter is needed to formulate the wave resistance of practical hull forms, expecially full-form ships.

Because the operating speed of ordinary ships of displacement type is in the range of Froude numbers from 0.15 to 0.30, one may regard that usual ships are operating in comparatively low velocity. If the speed of advance is extremely low and the elevation of the free surface is very small, the flow around the hull is comparable to flow with an undisturbed free surface, and is similar to the flow around a double body fixed in a uniform stream. Then the deviation of actual flow from the double body flow is due to the elevation of the free surface. Since the elevation of the free surface depends on the Froude number, we may employ the perturbation expansion of the free surface condition by the Froude number as a small parameter, but it will be found later that this approach is not so simple.

If our purpose is to formulate the wave resistance at low Froude numbers, we can derive an approximate formula directly from a general expression of the wave resistance. The wave resistance is determined by the momentum or energy analysis of the asymptotic expression of the fluid motion at a great distance from the ship.

The Green's function G(P,Q) has an asymptotic expression when the point P is brought to infinite downstream $x \to \infty$. If we assume the case of infinite depth of water, it takes the form

$$G(P,Q) \approx 4\gamma_0 \int_{-\pi/2}^{\pi/2} e^{\gamma_0 (z+z')\sec^2\theta} \sin(\gamma_0 \overline{x-x'} \sec\theta) \cdot \cos(\gamma_0 \overline{y-y'} \sec\theta \tan\theta) \sec^2\theta d\theta \quad (25)$$

The fluid motion there is characterized by the Kochin function.[12] In the case of the distributed Havelock sources[13] of density $\sigma(x,y,z)$ on the surface S, such as the first term of the right-hand side of Equation (22), the Kochin function becomes

$$H(k,\theta) = -\iint_S \sigma(x,y,z) \exp[kz+ik(x\cos\theta + y\sin\theta)]\, dS \quad (26)$$

In the case of Equation (15), on the other hand, the singularities are the surface distribution of sources and dipoles on S, the line distribution of sources and dipoles along L, and the distribution of sources on the horizontal plane Σ_0. Then, the corresponding Kochin function is

$$H(k,\theta) = -\frac{1}{4\pi}\iint_S \left(\frac{\partial\phi}{\partial n} - \phi\frac{\partial}{\partial n}\right)\exp[kz+ik(x\cos\theta + y\sin\theta)]\,dS$$

$$+\frac{1}{4\pi\gamma_0}\int_{L_0}\left(\frac{\partial\phi}{\partial x} - ik\cos\theta\cdot\phi\right)_{z=0}\exp[ik(x\cos\theta + y\sin\theta)]\,dy$$

$$-\frac{1}{4\pi\gamma_0}\iint_{\Sigma_0}\Phi(x,y)\exp[ik(x\cos\theta + y\sin\theta)]\,dxdy \qquad (27)$$

In order to distinguish the above function from the ordinary Kochin function such as Equation (26), it may be called the generalized Kochin function. Keeping in mind the fact that $\Phi(x,y)$ decays out much faster than the fluid velocity on going away from the ship, one can express the wave resistance by Havelock's formula

$$R_w = 8\pi\rho U^2\ell^2\gamma_0^2\int_{-\pi/2}^{\pi/2}|H(\gamma_0\sec^2\theta,\ \theta)|^2\sec^3\theta\,d\theta \qquad (28)$$

Now, let us consider the first approximation for low Froude numbers. It is easily understood by the condition of Equation (5) that the vertical velocity w at the free surface is of the order of Fn^2, since the free surface elevation is $O(Fn^2)$ by Equation (7). Therefore, the zeroth approximation for the velocity potential at low Froude numbers, designated by ϕ_0, is obtained from the condition

$$\partial\phi_0/\partial z = 0 \quad \text{at } z = 0 \qquad (29)$$

This is the disturbance velocity potential for a double body in the uniform flow. Now we write

$$\phi = \phi_0 + \phi_1 \tag{30}$$

and assume $\phi_1 = O(Fn^2)$. This assumption is not self-evident but seems to be legitimate when one considers the case of a vertical cylinder for which $\partial\phi_0/\partial z$ is zero everywhere. Then, the first approximation for the free surface elevation is

$$\zeta_0 = -\frac{1}{\gamma_0}\left[\frac{\partial\phi_0}{\partial x} + \frac{1}{2}\left\{\left(\frac{\partial\phi_0}{\partial x}\right)^2 + \left(\frac{\partial\phi_0}{\partial y}\right)^2\right\}\right]_{z=0} \tag{31}$$

The first approximation for the generalized Kochin function is given by the substitution of ϕ_0 for ϕ in Equation (27)

$$H(k,\theta) = -\frac{1}{4\pi}\iint_S \left(\frac{\partial\phi_0}{\partial n} - \phi_0\frac{\partial}{\partial n}\right)\exp[kz+ik(x\cos\theta + y\sin\theta)]\,dS$$

$$+\frac{1}{4\pi\gamma_0}\int_{L_0}\left(\frac{\partial\phi_0}{\partial x} - ik\cos\theta\cdot\phi_0\right)_{z=0}\exp[ik(x\cos\theta + y\sin\theta)]\,dy$$

$$-\frac{1}{4\pi\gamma_0}\iint_{\Sigma_0}\Phi_0(x,y)\exp[ik(x\cos\theta + y\sin\theta)]\,dxdy \tag{32}$$

where

$$\Phi_0(x,y) = \left[\left(\frac{\partial}{\partial x} + \frac{1}{2}\nabla\phi_0\cdot\nabla\right)\left|\nabla\phi_0\right|^2 + \zeta_0\frac{\partial^2\phi_0}{\partial z^2}\right]_{z=0}$$

$$= \left[\left(\frac{\partial}{\partial x} + \frac{1}{2}\nabla\phi_0\cdot\nabla\right)\left|\nabla\phi_0\right|^2 + \frac{1}{\gamma_0}\left(\frac{\partial^2\phi_0}{\partial x^2} + \frac{\partial^2\phi_0}{\partial y^2}\right)\left(\frac{\partial\phi_0}{\partial x} + \frac{1}{2}\left|\nabla\phi_0\right|^2\right)\right]_{z=0} \qquad (33)$$

The second and third terms on the right-hand side have the factor γ_0^{-1} or Fn^2, while the first term does not, so that one may consider the former two terms are of higher order. This is not the case because, the first and second terms can be transformed into an integral over the plane z = 0. To show this, let us express the double body potential in the form

$$\phi_0(P) = -\frac{1}{4\pi}\iint_{S+S^*}\left[\frac{\partial\phi_0(Q)}{\partial n_Q}\frac{1}{\overline{PQ}} - \phi_0(Q)\frac{\partial}{\partial n_Q}\left(\frac{1}{\overline{PQ}}\right)\right] d\,S_Q \qquad (34)$$

where S^* is the mirror image surface of S in reference to the plane z = 0 and $\overline{PQ}$ is the distance between P and Q. If the point P is inside $S + S^*$, then $\phi_0(P) = 0$. Next we consider the Fourier transform of $\partial^2\phi_0/\partial x^2$ on the plane z = 0. Since $\phi_0 = 0$ inside $S + S^*$, we have, after integrating by parts twice with respect to x

$$\widetilde{\frac{\partial^2\phi_0}{\partial x^2}} = \iint_{\Sigma_0}\left.\frac{\partial^2\phi_0}{\partial x^2}\right)_{z=0} e^{ik(x\cos\theta + y\sin\theta)}\,dxdy$$

$$= -k^2\cos^2\theta\iint_{\Sigma_0}\left.\phi_0\right)_{z=0} e^{ik(x\cos\theta + y\sin\theta)}\,dxdy$$

$$-\int_{L_0}\left(\frac{\partial\phi_0}{\partial x} - ik\cos\theta\cdot\phi_0\right)_{z=0} e^{ik(x\cos\theta + y\sin\theta)}\,dy \qquad (35)$$

The value of ϕ_0 on the plane z = 0, on the other hand, becomes

$$\phi_0\Big)_{z=0} = -\frac{1}{2\pi}\iint_S \left[\frac{\partial\phi_0(Q)}{\partial n_Q}\frac{1}{\overline{PQ}} - \phi_0(Q)\frac{\partial}{\partial n_Q}\left(\frac{1}{\overline{PQ}}\right)\right]_{z=0} d\,S_Q \qquad (36)$$

because of the symmetry. Now we employ the integral representation

$$\frac{1}{\overline{PQ}} = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta \int_0^{\infty} \exp[-k\{|z-z'|+i(x-x')\cos\theta + i(y-y')\sin\theta\}]\, dk \qquad (37)$$

in Equation (34). Then the Fourier transform of $\phi_0)_{z=0}$ becomes

$$\tilde{\phi}_0 = \iint_{-\infty}^{\infty} \phi_0\Big)_{z=0} \exp[ik(x\cos\theta + y\sin\theta)]\, dxdy$$

$$= -\frac{1}{k}\iint_S \left[\frac{\partial\phi_0(Q)}{\partial n_Q} - \phi_0(Q)\frac{\partial}{\partial n_Q}\right]\exp[kz'+ik(x'\cos\theta + y'\sin\theta)]\, d\,S_Q \qquad (38)$$

Combining Equations (35) and (38), we find

$$-\frac{1}{4\pi}\iint_S \left(\frac{\partial\phi_0}{\partial n} - \phi_0\frac{\partial}{\partial n}\right)\exp[kz+ik(x\cos\theta + y\sin\theta)]\, ds$$

$$= \frac{1}{4\pi\gamma_0}\int_{L_0}\left(\frac{\partial\phi_0}{\partial x} - ik\cos\theta\cdot\phi_0\right)_{z=0} \exp[ik(x\cos\theta + y\sin\theta)]\, dy$$

$$= -\frac{1}{4\pi\gamma_0}\widetilde{\frac{\partial^2\phi_0}{\partial x^2}} + \frac{k}{4\pi}\left(1 - \frac{k\cos^2\theta}{\gamma_0}\right)\tilde{\phi}_0 \qquad (39)$$

Insert the above relation in Equation (32) and put $k = \gamma_0 \sec^2 \theta$. If we write the relative flow velocity around the double body as u_0, v_0, w_0, and put $u_0^2 + v_0^2 = q_0^2$, after some reductions, we have the expression as

$$H(\gamma_0 \sec^2\theta, \theta) = -\frac{1}{8\pi\gamma_0} \iint_{\Sigma_0} \left[\frac{\partial}{\partial x} \{u_0(q_0^2-1)\} + \frac{\partial}{\partial y} \{v_0(q_0^2-1)\}\right]_{z=0}$$

$$\cdot \exp[i\gamma_0 \sec \theta(x + y \tan \theta)] \, dxdy$$

$$= \frac{1}{4\pi} \iint_{\Sigma_0} D(x,y) \exp[i\gamma_0 \sec \theta(x + y \tan \theta)] \, dxdy \qquad (40)$$

where the function D(x,y) was defined first by Baba[14] and is written as

$$D(x,y) = \frac{\partial}{\partial x} (\zeta_0 u_0) + \frac{\partial}{\partial y} (\zeta_0 v_0) \qquad (41)$$

We may derive another expression from the source distribution over the hull surface S given by Equation (21). Then the generalized Kochin function becomes

$$H(k,\theta) = - \iint_S \sigma(x,y,z) \exp(kz + ik(x \cos \theta + y \sin \theta)] \, dS$$

$$+ \frac{1}{\gamma_0} \int_{L_0} \sigma(x,y,0) \exp[ik(x \cos \theta + y \sin \theta)] \, n_x \, dy$$

$$- \frac{1}{4\pi\gamma_0} \iint_{\Sigma_0} \Phi(x,y) \exp[ik(x \cos \theta + y \sin \theta)] \, dxdy \qquad (42)$$

The first approximation at low speed is given by the substitution of $\sigma(x,y,z)$ by $\sigma_0(x,y,z)$ and $\Phi(x,y)$ by $\Phi_0(x,y)$, where $\sigma_0(x,y,z)$ is the source distribution of the double body in a uniform flow. The result of this approximation is not identical with that of Equation (32)[15] because the boundary condition on the hull surface is not satisfied by the source distribution $\sigma_0(x,y,z)$.

As a numerical example, wave resistance of Wigley's parabolic model is calculated.[16] Figure 4 shows the result calculated by Equation (28) with the Kochin function defined by Equations (32) or (40) and by (42). The results are compared with the residuary resistance of towing tests and results of wave pattern analysis of the longitudinal cut method as well as the calculation by Michell's formula. A considerable improvement is observed in agreement with the measured results, especially in lower speed

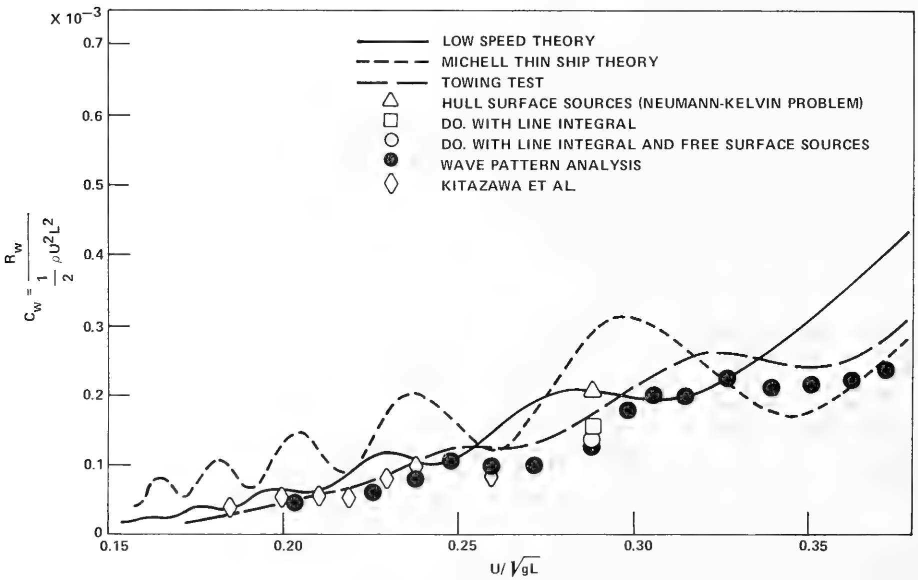

Figure 4 - Computed and Measured Wave-Resistance Coefficient of Wigley Model

by the computation with Equation (32). However, there still exist some discrepancies when the Froude number is greater than 0.20.

In order to achieve further improvement, some kind of revision of the formula is desirable. We have employed the double body source distribution in the expression of Equation (42), but it does not satisfy the hull surface boundary condition at finite Froude numbers. Therefore, we try to employ the source distribution by which the hull surface boundary condition is satisfied at the Froude number under consideration. If we assume that the line distribution along the water line and the distribution over the horizontal plane do not make serious effects on the hull surface boundary condition, the density of the source distribution over S is determined by the integral equation

$$n_x = -2\pi\sigma(P) + \iint_S \sigma(Q) \frac{\partial}{\partial n_p} G(P,Q)\, d\, S_Q \tag{43}$$

A numerical method is available to determine the source density $\sigma(P)$. Then the result is substituted in the formula, Equation (42), to calculate the generalized Kochin function, which determines the wave resistance by the formula, Equation (28). A numerical example is given for the case of $\gamma_0 = 6.0$ or Fn = 0.2887. Three kinds of calculations are compared in Figure 4. The first is the calculation by the ordinary Kochin function of the distribution of sources on S only, i.e., only the first term on the right-hand side of Equation (42) is taken. The second is the addition of the line integral, which corresponds with the result of the Neumann-Kelvin solution suggested by Brard. The third example is the inclusion of all terms of Equation (42). Much closer agreement with measured results are obtained by the addition of the third term of Equation (42). Recently Kitazawa[17] et al., presented another approximation. They put the density of hull surface sources as

$$\sigma(P) = \sigma_0(P) + \sigma_1(P) \tag{44}$$

and determined $\sigma_1(P)$ by the integral equation

$$- \sigma_1(P) + \frac{1}{2\pi} \iint_S \sigma_1(Q) \frac{\partial}{\partial n_p} G(P,Q)\, d\, S_Q - \frac{1}{2\pi\gamma_0} \int_{L_0} \sigma_1(Q) \frac{\partial}{\partial n_p} G(P,Q)\, n_x dy'$$

$$= \frac{1}{8\pi^2} \iint_{\Sigma_0} D(x,y) \frac{\partial}{\partial n_p} G(P,Q)\, dx'dy' \tag{45}$$

This is equivalent to the velocity potential

$$\phi(P) = - \iint_S \sigma(Q)\, G(P,Q)\, d\, S_Q + \frac{1}{\gamma_0} \int_{L_0} \sigma(Q)\, G(P,Q)\, n_x dy'$$

$$- \frac{1}{4\pi\gamma_0} \iint_{\Sigma_0} \Phi_0(x',y')\, G(P,Q)\, dx'dy' \tag{46}$$

where the hull surface source density is so determined that the hull surface boundary condition is satisfied. Therefore, the only approximation is the replacement of $\Phi(x,y)$ by $\Phi_0(x,y)$. Numerical results show a plausible agreement with measured results as shown in Figure 4. However, none of these results are regarded as consistent approximations from the rigorous aspect of the perturbation analysis.

Method of Coordinate Transformation

In the preceding section, we assumed the deviation of the flow velocities around the hull from those of double body flow as a small perturbation and formulated the first approximation for the wave resistance. If we want expressions for the flow velocity or wave pattern, however, the perturbation expansion of the velocity potential is needed.

Now let us write the fluid velocity around the hull in the form

$$u = u_0 + u_1,\quad v = v_0 + v_1,\quad w = w_0 + w_1 \tag{47}$$

The first approximation for the free surface elevation is given by Equation (31) or

$$\zeta_0 = -\frac{1}{2\gamma_0}(1-u_0^2-v_0^2)_{z=0} \tag{48}$$

Then the kinematical condition of the free surface, Equation (8), gives

$$w_1 = u_0\frac{\partial\zeta_0}{\partial x} + v_0\frac{\partial\zeta_0}{\partial y} - \zeta_0\frac{\partial w_0}{\partial z} \quad \text{at } z=0 \tag{49}$$

This relation provides the boundary condition at the free surface for the first approximation of the perturbation velocity potential. However, the right-hand side is determined by the double body flow which does not present a wavelike motion, so that the boundary value problem gives the solution which is not wavelike. This result contradicts the actual phenomena. In order to avoid this contradiction, one has to revise the basic assumption. Here we employ the hypothesis which was proposed by Ogilvie.[18] The basic assumption is that the perturbation velocity is wavelike and the wavelength is proportional to the Froude number squared. This means that the differentiation results in the change of order of magnitude by the order of wave number. Secondly, it is assumed that the wavelike nature appears in the first approximation of the perturbation velocities, i.e., u_1, v_1, w_1 which are $O(Fn^2)$, but not in the first approximation of the free surface elevation ζ_0. The second approximation for the surface elevation is given by

$$\zeta_1 = -\frac{1}{\gamma_0}(u_0u_1+v_0v_1)_{z=0} \tag{50}$$

and is $O(Fn^4)$. Since u_1, v_1, w_1 are $O(Fn^2)$ and their differentiation reduces the order by Fn^{-2}, the derivatives of the perturbation velocities are O(1). Inserting the expression, Equation (47) in the free surface condition

$$\frac{1}{2}\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}+w\frac{\partial}{\partial z}\right)(u^2+v^2+w^2)+\gamma_0\, w=0 \tag{51}$$

and collecting terms of O(1), we obtain on the surface $z = \zeta_0$

$$u_0^2\frac{\partial u_1}{\partial x}+u_0\, v_0\left(\frac{\partial v_1}{\partial x}+\frac{\partial u_1}{\partial y}\right)+v_0^2\frac{\partial v_1}{\partial y}+\gamma_0\, w_1$$

$$=-\frac{1}{2}\left(u_0\frac{\partial}{\partial x}+v_0\frac{\partial}{\partial y}\right)(u_0^2+v_0^2)-\gamma_0\,\zeta_0\frac{\partial w_0}{\partial z}$$

$$=\gamma_0\, D(x,y) \tag{52}$$

The function D(x,y) is expressed by Equation (41). If the double body flow velocity is known, the function D(x,y) is a known function. Then Equation (52) gives the boundary condition at the free surface for the perturbation velocity potential ϕ_1 defined by

$$u_1=\frac{\partial\phi_1}{\partial x}\ ,\quad v_1=\frac{\partial\phi_1}{\partial y}\ ,\quad w_1=\frac{\partial\phi_1}{\partial z} \tag{53}$$

as follows.

$$u_0^2\frac{\partial^2\phi_1}{\partial x^2}+2\,u_0\,v_0\frac{\partial^2\phi_1}{\partial x\partial y}+v_0^2\frac{\partial^2\phi_1}{\partial y^2}+\gamma_0\frac{\partial\phi_1}{\partial z}=\gamma_0\, D(x,y) \tag{54}$$

The perturbation velocity potential is harmonic outside the hull surface S and below the curved surface $z = \zeta_0$ on which the boundary condition, Equation (54), is satisfied. Since the boundary condition on the curved surface is not convenient to finding the solution, let us perform the following transformation to the vertical coordinate.

$$z = \zeta_0 + z' \tag{55}$$

After the transformation of the Laplace equation to new coordinates and taking only the first order terms, we find that the Laplace equation does not change, namely

$$\frac{\partial^2\phi_1}{\partial x^2} + \frac{\partial^2\phi_1}{\partial y^2} + \frac{\partial^2\phi_1}{\partial z'^2} = 0 \tag{56}$$

while the boundary condition at the free surface is

$$u_0^2 \frac{\partial^2\phi_1}{\partial x^2} + 2\, u_0\, v_0 \frac{\partial^2\phi_1}{\partial x\partial y} + v_0^2 \frac{\partial^2\phi_1}{\partial y^2} + \gamma_0 \frac{\partial\phi_1}{\partial z'} = \gamma_0\, D(x,y) \tag{57}$$

at $z' = 0$. The homogeneous equation, which is obtained by setting the right-hand side equal to zero, is the boundary condition of free waves on a nonuniform flow field.

$$u_0^2 \frac{\partial^2\phi_1}{\partial x^2} + 2\, u_0\, v_0 \frac{\partial^2\phi_1}{\partial x\partial y} + v_0^2 \frac{\partial^2\phi_1}{\partial y^2} + \gamma_0 \frac{\partial\phi_1}{\partial z'} = 0 \tag{58}$$

Now let us assume the wave potential

$$\phi_1 = A \exp \gamma_0[k(x,y)z'+i\, S(x,y)] \tag{59}$$

On substituting in the Laplace equation and taking terms of the lowest order with respect to γ_0^{-1}, we have the relation

$$\{k(x,y)\}^2 + \left\{\left(\frac{\partial k}{\partial x}\right)^2 + \left(\frac{\partial k}{\partial y}\right)^2\right\} z'^2 - \left(\frac{\partial S}{\partial x}\right)^2 - \left(\frac{\partial S}{\partial y}\right)^2 = 0 \qquad (60)$$

Because of the exponential decay in the z direction, the wave motion is significant only in a surface layer of thickness $0(\gamma_0^{-1})$. Therefore, z'^2 is $0(Fn^4)$ and the second term of the above equation is to be omitted. Then we have

$$k(x,y) = \sqrt{\left(\frac{\partial S}{\partial x}\right)^2 + \left(\frac{\partial S}{\partial y}\right)^2} \qquad (61)$$

that means the local wave number. Substituting Equation (59) in Equation (58) and taking terms of the lowest order, we obtain

$$\left(u_0 \frac{\partial S}{\partial x} + v_0 \frac{\partial S}{\partial y}\right)^2 = k(x,y) \qquad (62)$$

or

$$\left(u_0 \frac{\partial S}{\partial x} + v_0 \frac{\partial S}{\partial y}\right)^2 = \sqrt{\left(\frac{\partial S}{\partial x}\right)^2 + \left(\frac{\partial S}{\partial y}\right)^2} \qquad (63)$$

This is identical with the dispersion relation which has been indicated by Keller.[19] The solution of the above differential equation determines the phase function $S(x,y)$. Keller has proposed a kinematical theory of waves superimposed on the nonuniform flow around the ship by the analogy with the geometrical optics.

Now let us show the possibility of giving an analytical expression for the wave function by a coordinate transformation. To find the solution, we employ the curvilinear coordinates along streamlines and equipotential lines of the double body flow in the plane $z = 0$. Designate the velocity potential of the double body flow by Φ and the stream function on the plane $z = 0$ by $\bar{\Psi}$, the latter of which is slightly different from the stream function in the two-dimensional motion. They are defined by the relation with the velocity of the double body flow as follows.

$$u_0 = \frac{\partial \Phi}{\partial x} = \frac{1}{h}\frac{\partial \overline{\Psi}}{\partial y} \tag{64}$$

$$v_0 = \frac{\partial \Phi}{\partial y} = -\frac{1}{h}\frac{\partial \overline{\Psi}}{\partial x} \tag{65}$$

If the double body is the body of revolution, then h is the radial length. In general cases, it is determined by the partial differential equation

$$u_0 \frac{\partial h}{\partial x} + v_0 \frac{\partial h}{\partial y} + h\left(\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y}\right) = 0 \tag{66}$$

If we change the independent variables of Equation (58) from x,y to $\Phi,\overline{\Psi}$, there is a relation

$$u_0 \frac{\partial \phi_1}{\partial x} + v_0 \frac{\partial \phi_1}{\partial y} = q_0^2 \frac{\partial \phi_1}{\partial \Phi} \tag{67}$$

Because of the fact that differentiation of the perturbation potential reduces the order of magnitude, the order of $\partial\phi_1/\partial\Phi$ and $\partial\phi_1/\partial\overline{\Psi}$ are higher than that of $\partial^2\phi_1/\partial\Phi^2$ by γ_0^{-1}. Differentiating the above equation again and omitting higher order terms, one obtains from Equation (57)

$$q_0^4 \frac{\partial^2 \phi_1}{\partial \Phi^2} + \gamma_0 \frac{\partial \phi_1}{\partial z'} = \gamma_0 \, D(x,y) \tag{68}$$

Next, the independent variables are transformed again to new variables ξ, η, ζ by the following relations.

$$\Phi = \frac{1}{\gamma_0}\int \hat{q}_0^3 \, d\xi \tag{69}$$

$$\bar{\Psi} = \frac{1}{\gamma_0} \int \hat{q}_0^3 \; h \; d\eta \tag{70}$$

$$z' = \frac{1}{\gamma_0} \hat{q}_0^2 \; \zeta \tag{71}$$

where $\hat{q}_0(\xi,\eta) = q_0(x,y)$ and the lower limit of integrals depends on the origin of Φ and $\bar{\Psi}$. If we assume q_0 to be slowly varying, so that $\partial q_0/\partial\Phi$ and $\partial q_0/\partial\bar{\Psi}$ are $O(1)$, we can prove

$$\frac{\partial\phi_1}{\partial\xi} = \frac{q_0^3}{\gamma_0} \frac{\partial\phi_1}{\partial\Phi} + O(\gamma_0^{-2}) \tag{72}$$

Differentiating again by ξ and taking only the term of the lowest order, we have

$$\frac{\partial^2\phi_1}{\partial\xi^2} = \frac{1}{\gamma_0^2} \hat{q}_0^6 \frac{\partial^2\phi_1}{\partial\Phi^2} \tag{73}$$

A similar assumption is applied to $\partial\phi_1/\partial\zeta$ such as

$$\frac{\partial\phi_1}{\partial\zeta} = \frac{1}{\gamma_0} \hat{q}_0^2 \frac{\partial\phi_1}{\partial z'} \tag{74}$$

Multiplying $\hat{q}_0^2/\gamma_0^2$ on both sides of Equation (69) and writing

$$\frac{q_0^2}{\gamma_0} D(x,y) = E(\xi,\eta) \tag{75}$$

we obtain

$$\frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial \phi_1}{\partial \zeta} = E(\xi,\eta) \tag{76}$$

A similar discussion is applied to the transformation of the Laplace equation, and the lowest order term becomes

$$\frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial^2 \phi_1}{\partial \eta^2} + \frac{\partial^2 \phi_1}{\partial \zeta^2} = 0 \tag{77}$$

Thus, the Laplace equation keeps its original form after the transformation. To find the solution which satisfies the inhomogeneous boundary condition of Equation (76), we consider the basic solution which satisfies the boundary condition

$$\frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial \phi_1}{\partial \zeta} = \delta(\xi-\xi',\eta-\eta') \tag{78}$$

where $\delta(\xi-\xi',\eta-\eta')$ is the delta function of two variables. The solution which satisfies the above at $\zeta = 0$ and the radiation condition at infinite distance as well, is given by the function

$$G(\xi,\eta,\zeta;\xi',\eta') = \frac{\text{P.V.}}{4\pi^2} \int_{-\pi}^{\pi} d\theta \int_0^{\infty} \exp[k\zeta+ik\cos\theta(\xi-\xi')+ik\sin\theta(\eta-\eta')] \cdot \frac{dk}{1-k\cos^2\theta}$$

$$+ \frac{1}{2\pi} I_m \int_{-\pi/2}^{\pi/2} \exp[\zeta\sec^2\theta+i\sec\theta\{\xi-\xi'+\tan\theta(\eta-\eta')\}\sec^2\theta d\theta \tag{79}$$

where I_m means the imaginary part is taken. Then the solution of the boundary condition of Equation (76) is given by

$$\phi_1 = \iint_{-\infty}^{\infty} E(\xi',\eta')\ G(\xi,\eta,\zeta;\xi',\eta')\ d\xi' d\eta' \tag{80}$$

The asymptotic form far downstream, i.e., $\xi \to \infty$, is written in the form

$$\phi_1 \approx \frac{1}{\pi} \iint_{-\infty}^{\infty} E(\xi',\eta') d\xi' d\eta' I_m \int_{-\pi/2}^{\pi/2} \exp[\zeta \sec^2 \theta + i \sec \theta \{\xi - \xi' + \tan \theta (\eta - \eta')\}] \cdot \sec^2 \theta\ d\theta \tag{81}$$

Thus, the phase function of the elementary wave defined by Equation (59) is

$$S = \gamma_0^{-1} \sec \theta\ (\xi + \eta \tan \theta)$$

Now let us show that the phase function satisfies the dispersion relation given by Equation (63). There are relations

$$\frac{\partial \xi}{\partial x} = u_0 \frac{\partial \xi}{\partial \Phi} + v_0 \frac{\partial \xi}{\partial \bar{\Psi}}$$

$$\frac{\partial \xi}{\partial \Phi} = \frac{\partial \bar{\Psi}}{\partial \eta} \Bigg/ \frac{\partial(\Phi,\bar{\Psi})}{\partial(\xi,\eta)} = \frac{\gamma_0}{q_0^3}$$

$$\frac{\partial \xi}{\partial \bar{\Psi}} = -\frac{\partial \Phi}{\partial \eta} \Bigg/ \frac{\partial(\Phi,\bar{\Psi})}{\partial(\xi,\eta)} = -\frac{\gamma_0}{q_0^6 h} \int \frac{\partial \hat{q}_0^3}{\partial \eta}\, d\xi$$

Since it can be shown that $\partial \hat{q}_0^3 / \partial \eta = 0(\gamma_0^{-1})$, we have, after deleting higher order terms,

$$\frac{\partial\xi}{\partial x} = \frac{\gamma_0 u_0}{q_0^3}$$

A similar argument is applied to $\partial\xi/\partial y$, $\partial\eta/\partial x$, and $\partial\eta/\partial y$, as

$$\frac{\partial\xi}{\partial y} = \frac{\gamma_0 v_0}{q_0^3}\ , \quad \frac{\partial\eta}{\partial x} = \frac{\gamma_0 v_0}{q_0^3}\ , \quad \frac{\partial\eta}{\partial y} = -\frac{\gamma_0 u_0}{q_0^3}$$

Then we obtain

$$\frac{\partial S}{\partial x} = \frac{\sec\theta}{\gamma_0}\left(\frac{\partial\xi}{\partial x} + \frac{\partial\eta}{\partial x}\tan\theta\right) = \frac{\sec\theta}{q_0^3}(u_0 + v_0\tan\theta)$$

$$\frac{\partial S}{\partial y} = \frac{\sec\theta}{\gamma_0}\left(\frac{\partial\xi}{\partial y} + \frac{\partial\eta}{\partial y}\tan\theta\right) = \frac{\sec\theta}{q_0^3}(v_0 - u_0\tan\theta)$$

It is readily shown that the dispersion relation of Equation (63) is satisfied by the substitution of the above relations.

The amplitude function of the free wave at infinite downstream is expressed by the complex form

$$A(\theta) = \frac{1}{\pi}\iint_{-\infty}^{\infty} E(\xi,\eta)\exp[i\sec\theta(\xi+\eta\tan\theta)]\,d\xi d\eta$$

$$= -\frac{\gamma_0}{\pi}\iint_{-\infty}^{\infty} q_0^{-2}\,D(x,y)\exp[i\sec\theta(\xi+\eta\tan\theta)]\,dxdy \qquad (82)$$

The integral with respect to x, y is carried out over the plane $z = 0$ excluding the interior of the hull.

The second approximation of the free surface elevation is given by

$$\zeta_1 = -\frac{1}{\gamma_0}\left(u_0 \frac{\partial \phi_1}{\partial x} + v_0 \frac{\partial \phi_1}{\partial y}\right)_{z'=0}$$

$$= -\frac{q_0^2}{\gamma_0}\frac{\partial \phi_1}{\partial \Phi}$$

$$= -\frac{1}{q_0}\left.\frac{\partial \phi_1}{\partial \xi}\right)_{\zeta=0} \qquad (83)$$

Therefore, the free wave pattern at far downstream becomes

$$\zeta_1 \approx -\mathrm{Re}\int_{-\pi/2}^{\pi/2} A(\theta)\, e^{i \sec\theta(\xi+\eta\tan\theta)} \sec^3\theta d\theta \qquad (84)$$

because $q_0 \approx 1$ at a great distance.

The wave resistance experienced by the ship is derived from momentum or energy analysis of free waves at a great distance. It is determined by the amplitude function

$$R_w = \frac{\pi}{2}\rho\, U^2\, \ell^2 \int_{-\pi/2}^{\pi/2} |A(\theta)|^2 \sec^3\theta d\theta \qquad (85)$$

In order to calculate $A(\theta)$, we need the inverse transform of Equations (69) and (70). For this purpose, we take curvilinear coordinates along streamlines and equipotential lines along which the length s and t are taken, respectively. It is proved that we have the relations

$$\left.\begin{aligned} d\xi &= \gamma_0 \left\{ \frac{d\Phi}{q_0^3} + 0(\gamma_0^{-1}) \right\} \\ d\eta &= \gamma_0 \left\{ \frac{d\overline{\Psi}}{q_0^3} + 0(\gamma_0^{-1}) \right\} \end{aligned}\right\} \qquad (86)$$

$$d\Phi = q_0 \, ds \qquad d\overline{\Psi} = q_0 \, h \, d \, t \qquad (87)$$

Omitting higher order terms in Equation (86) and integrating, we obtain

$$\xi = \gamma_0 \int \frac{ds}{q_0^2} \, , \; \eta = \gamma_0 \int \frac{dt}{q_0^2} \qquad (88)$$

Since γ_0/q_0^2 corresponds to the local wave number, the new coordinates mean the streamline coordinates with scales vary proportionally to the local wave length. The above result takes account of the distortion of the wave pattern due to the nonuniform base flow velocities near the hull. The boundary condition on the hull surface is satisfied in the case of symmetric flow, because the velocities u and v are tangential to the hull surface at $z' = 0$ and are significant only in a thin layer of thickness $0(\gamma_0^{-1})$ near the free surface. Though the theory developed here looks like a reasonable representation of the actual phenomena, no numerical results have been presented so far. It should be noted that a purely numerical method has been employed in a similar boundary value problem by Dawson[20] and the result shows a plausible agreement with experiments.

HYDRODYNAMIC FORCES ON OSCILLATING SLENDER SHIPS

Application of the Slender Body Theory

As mentioned in the preceding chapter, the possibility of mathematical analysis of the fluid motion around a ship hull depends substantially on the linearization of boundary conditions at the free surface. In the case of ships or other floating bodies making oscillations on the free surface with its average position at a fixed point, indicating no average velocity, the boundary condition at the free surface can be linearized in a simple fashion by assuming that the amplitude of the oscillation is sufficiently small. Therefore, no restriction is imposed on the shape of the body. By the employment of the numerical method, one can calculate hydrodynamic forces on any kind of shapes of floating bodies in principle, and it is known that some numerical results show fairly good agreement with measured values. However, a rational development of the linearized theory becomes too intricate when the steady forward speed is introduced to the oscillating ships. The difficulty in finding a rational solution which is not trivial was first demonstrated by Peters and Stoker.[21] They showed that the hydrodynamic reactions such as the added mass and damping did not appear in the order of approximation of the linearized theory for a thin ship oscillating in the plane of symmetry. There had been extensive works by Haskind[22] and Hanaoka[23] about thin ships in longitudinal oscillations in still water before that time. A full condemnation of these achievements by the reason of inconsistency may be unfair, because the consistent structure of theory breaks down on account of just a single reason of the inclusion of the steady forward speed. If the steady forward motion is introduced to the oscillating ship, the possibility of linearization depends on the hull shape parameter as well. The main difficulty in the oscillating thin ship with forward speed lies in the fact that the disturbance generated by the periodical motion of the ship is weaker than the disturbance due to the forward motion. Consequently, the first order theory would lead to an unrealistic conclusion that no damping to the oscillation could exist. In order to overcome this difficulty, Newman[24] employed two independent parameters, one of which is the oscillation amplitude and the other is the hull shape parameter, namely the beam-to-length ratio of the ship. Although the justification of the damping and added mass of the thin ship is attained by the use of two parameters, more serious difficulty appears when the ship is moving in ambient waves. Cross products between the fluid velocity of incident waves and that due to the steady forward motion appear in the same order of magnitude as that of the fluid motion due to the oscillation, making the free surface condition much more complicated. In this respect, the application of the slender body theory[25] looks more profitable than the thin ship assumption. The assumption of the slender body increases the order of magnitude of the forward motion so that it is higher than that of lateral or vertical motions. Therefore, the effect of the steady forward motion does not appear at the lowest order in the far field expansion with respect to the hull shape parameter. Though the effect of the steady motion may appear in the lowest order in the expansion of the boundary condition in the near field, the first order solution may take a neat form if the wave amplitude and the slenderness ratio of the ship are taken as two independent parameters of the perturbation. However, some numerical computations have revealed another difficulty. This is that the first order theory gives only an unrealistic result.[26] On the other hand, it is widely accepted that the strip theory has been able to present a reasonable agreement with measurement.[27] The strip theory is regarded in one sense as another slender body theory, although it is originally derived by a somewhat intuitive method. In a rigorous sense, it is a rational approximation for a slender ship in oscillations with high frequency without forward speed. Thus, it is eventually known that results may become different if the different choice of magnitude of frequency and forward velocity is taken.[28] Discussions in this connection will be given in the next section.

Once the slender body theory is employed, the boundary conditions are expanded by the slenderness ratio. Terms of the lowest order are taken first. One of the features of the slender body theory is the singular perturbation. It makes a difference in the expansion at near field and at far field, and a matching procedure is applied between them. The problem which we are going to discuss is a slender body floating on regular waves and moving with a uniform average speed U in the mean direction of its longitudinal axis. In the most general case, the direction of the forward velocity is different from that of wave propagation. Then the ship undertakes oscillations of six degrees of freedom around its mean position, among which surge, heave, and pitch are longitudinal oscillations and sway, yaw, and roll are lateral oscillations. Since the viscosity effect in the longitudinal oscillations is very little, the potential flow theory seems to offer a fairly accurate prediction, while in the lateral oscillations, effect of viscosity plays an important role, so that we cannot expect any reliable results by theories without taking account of the viscosity. Therefore, we will confine our discussions in the longitudinal oscillations hereafter. The general formulation for the oscillation with six degrees of freedom is described in Appendix A.

On developing the perturbation analysis, we take the ratio δ of wave amplitude to wavelength and the ratio ε of beam-to-length of the ship as basic parameters. Since they are mutually independent, we can expand the velocity potential with respect to δ first. The first term is independent of δ and represents the fluid motion when the ship moves with uniform velocity in still water. The linearized theory takes terms up to the first order with respect to δ. If we are concerned with the ship motion in regular waves, the term which is linear to δ is a simple harmonic and represents the oscillatory part of the velocity potential. The next stage is the expansion of the above portions of the velocity potential, which have been linearized already by δ, by the slenderness ratio ε. There is a term which is independent of ε in the oscillatory potential. It represents the incident waves which may be assumed as simple harmonic too. The other part represents the disturbance by the ship. Consider a relative motion with respect to the coordinates moving with the average forward velocity U, and take the axis of x in the direction opposite to the forward velocity of the ship and the axis of z vertically upwards. Then the velocity potential can be written in the form $Ux + \phi$, and ϕ satisfies the Laplace equation

$$\nabla^2 \phi = 0 \tag{89}$$

in the space occupied by the fluid. The boundary conditions satisfied by the velocity potential are those on the hull surface and on the free surface. If the depth of water is assumed infinite, the condition of fluid motion at infinity, both horizontal and vertical, is that the fluid velocity due to the disturbance by the ship vanishes, and that the fluid motion is just the sum of the uniform flow and the incident wave. The radiation condition at great distance should be considered as well. If the hull surface at each instant is expressed by the equation

$$y = f(x,z,t) \tag{90}$$

the boundary condition on the hull surface becomes

$$\frac{\partial f}{\partial t} + \left(U + \frac{\partial \phi}{\partial x}\right) \frac{\partial f}{\partial x} + \frac{\partial \phi}{\partial z} \frac{\partial f}{\partial z} - \frac{\partial \phi}{\partial y} = 0 \tag{91}$$

Now we consider only the heaving and pitching oscillations because surging motion causes fluid motion of higher order. Designate the vertical displacement of the center of gravity by z_g and the angle of pitch by ψ and take coordinates x_0, y_0, z_0 fixed to the ship. Then the relation between the coordinate systems (x,y,z) and (x_0,y_0,z_0) is

$$\left.\begin{aligned} x_0 &= x \cos \psi - (z-z_g) \sin \psi \\ y_0 &= y \\ z_0 &= x \sin \psi + (z-z_g) \cos \psi \end{aligned}\right\} \; . \tag{92}$$

The equation of the hull surface in reference to the moving coordinates (x_0,y_0,z_0) is independent of time, such as

$$y_0 = f_0(x_0,z_0) \tag{93}$$

It is assumed that z_g and ψ are of the order of δ, and after omitting higher order terms, one can transform the hull boundary condition into the following equation.

$$-\frac{\partial f_0}{\partial x_0} z \dot{\psi} - \frac{\partial f_0}{\partial z_0} (\dot{z}_g - x\dot{\psi}) + \left(U + \frac{\partial \phi}{\partial x}\right)\left(\frac{\partial f_0}{\partial x_0} + \frac{\partial f_0}{\partial z_0}\psi\right)$$

$$+ \frac{\partial \phi}{\partial z}\left(\frac{\partial f_0}{\partial z_0} - \frac{\partial f_0}{\partial x_0}\psi\right) - \frac{\partial \phi}{\partial y} = 0 \tag{94}$$

If we pick up terms which are independent of time, we obtain the boundary condition for the steady part of the velocity potential, denoted by $U\,\phi_0$ as follows.

$$\left(1 + \frac{\partial \phi_0}{\partial x}\right)\frac{\partial f_0}{\partial x_0} + \frac{\partial \phi_0}{\partial z}\frac{\partial f_0}{\partial z_0} - \frac{\partial \phi_0}{\partial y} = 0 \tag{95}$$

Take length n along the outward normal to the hull surface, and designate the direction cosines of the normal as n_x, n_y, n_z. Then the above equation can be written in the form

$$\frac{\partial \phi_0}{\partial n} + n_x = 0 \tag{96}$$

Now let us examine the order of magnitude when the ship is regarded very slender. If n_x is the slope of the hull surface to the longitudinal axis, then its order of magnitude is the slenderness ratio ε. We must keep in mind the fact that the disturbance velocity potential of a slender body is singular along its longitudinal axis, and the differentiation of it in the direction of the normal changes the order of magnitude by ε^{-1}. This fact can be shown also by adopting so-called strained coordinates which measure lengthwise direction and lateral direction by different scales. This procedure is well known and will not be repeated here. As a consequence of this argument, the relation between the order of magnitude of ϕ_0 and that of $\partial\phi_0/\partial n$ is

$$\partial\phi_0/\partial n = \varepsilon^{-1}\, O(\phi_0)$$

Since $n_x = O(\varepsilon)$, the above relation results $\phi_0 = O(\varepsilon^2)$. The oscillatory part of the fluid motion is composed of the incident wave potential ϕ_w and the oscillatory disturbance potential ϕ_1. The former is independent of ε and can be regarded as a given function. In the case of regular waves propagating in the direction making an angle α with the x axis, the velocity potential of the incident wave is expressed by

$$\phi_w = h\sqrt{g/K}\ \exp[Kz-iK(x \cos \alpha+y \sin \alpha)+i\omega t] \tag{97}$$

where h = wave amplitude

K = wave number $2\pi/\lambda$

ω = circular frequency of encounter

The absolute frequency of the wave is

$$\omega_0 = \omega - U\ K \cos \alpha \tag{98}$$

There is a relation between the wave number and the frequency as follows:

$$K = \omega_0^2/g = (\omega-UK \cos \alpha)^2/g \tag{99}$$

The order of magnitude of U and ω or K may not be unity, so that we need to include these quantities in the argument of the order. It can be assumed that the amplitude of the ship's oscillation is of the same order as that of the wave amplitude. The frequency of the oscillation is, however, not the frequency of the wave ω_0 but the frequency of encounter ω. The wave amplitude is of the order of δ/K so that the velocity of the oscillatory motion of the ship has the order $\omega\ \delta\ K^{-1}$. The fluid velocity of the incident wave has, on the other hand, the order $\delta\ K^{-1/2}$, so that they are not necessarily the same order. We have to keep these facts in mind in examining the order of magnitude of the oscillatory part of the velocity potential. Taking the first order terms with respect to δ, we obtain

$$- \frac{\partial f_0}{\partial x_0} z \dot{\psi} - \frac{\partial f_0}{\partial z_0} (\dot{z}_g - x\dot{\psi}) + U \frac{\partial f_0}{\partial z_0} \psi + U \frac{\partial \phi_0}{\partial x} \frac{\partial f_0}{\partial z_0} \psi$$

$$+ \frac{\partial f_0}{\partial x} \frac{\partial}{\partial x} (\phi_1 + \phi_w) + \frac{\partial f_0}{\partial z_0} \frac{\partial}{\partial z} (\phi_1 + \phi_w) - U \frac{\partial \phi_0}{\partial z} \psi$$

$$- \frac{\partial}{\partial y} (\phi_1 + \phi_w) + U(z_g - x\psi) \left(\frac{\partial f_0}{\partial z_0} \frac{\partial^2 \phi_0}{\partial z^2} - \frac{\partial^2 \phi_0}{\partial y \partial z} \right) = 0 \qquad (100)$$

If we assume U and ω are both of the order of unity, ϕ will be of the order of $\delta\ \varepsilon$. The term of the lowest order in the above equation has the order of δ and the equation of the same order is

$$- \frac{\partial f_0}{\partial z_0} (\dot{z}_g - x\dot{\psi}) + U \frac{\partial f_0}{\partial z_0} \psi + \frac{\partial f_0}{\partial z_0} \frac{\partial}{\partial z} (\phi_1 + \phi_w)$$

$$- \frac{\partial}{\partial y} (\phi_1 + \phi_w) + U(z_g - x\psi) \left(\frac{\partial f_0}{\partial z_0} \frac{\partial^2 \phi_0}{\partial z^2} - \frac{\partial^2 \phi_0}{\partial y \partial z} \right) = 0 \qquad (101)$$

If we write the outward normal to the sectional form of the hull in the plane perpendicular to the x axis by n', the above equation takes the form

$$\frac{\partial \phi_1}{\partial n'} = (\dot{z}_g - x\dot{\psi} - U\psi) \frac{\partial z}{\partial n'} - U(z_g - x\psi) \frac{\partial}{\partial n'} \left(\frac{\partial \phi_0}{\partial z} \right) - \frac{\partial \phi_w}{\partial n'} \qquad (102)$$

The right-hand side is regarded as a known function provided that the steady potential ϕ_0 is known. One can divide the periodical potential ϕ_1 into a part determined by the ship's oscillation and a part originated by the diffraction of the ambient wave. The former is the radiation potential ϕ_R for which the boundary condition becomes

$$\frac{\partial \phi_R}{\partial n'} = (\dot{z}_g - x\dot{\psi} - U\psi) \frac{\partial z}{\partial n'} - U(z_g - x\psi) \frac{\partial}{\partial n'} \left(\frac{\partial \phi_0}{\partial z} \right) \tag{103}$$

while the latter is the diffraction potential ϕ_D which has to satisfy the boundary condition

$$\frac{\partial \phi_D}{\partial n'} = - \frac{\partial \phi_w}{\partial n'} \tag{104}$$

Let us consider next the boundary condition at the free surface. If the form of the free surface is given by the equation

$$z = \zeta(x,y,t) \tag{105}$$

the kinematical condition is

$$\frac{\partial \zeta}{\partial t} + \left(U + \frac{\partial \phi}{\partial x} \right) \frac{\partial \zeta}{\partial x} + \frac{\partial \phi}{\partial y} \frac{\partial \zeta}{\partial y} - \frac{\partial \phi}{\partial z} = 0 \tag{106}$$

while the condition of constant pressure is

$$\frac{\partial \phi}{\partial t} + U \frac{\partial \phi}{\partial x} + \frac{1}{2} \left\{ \left(\frac{\partial \phi}{\partial x} \right)^2 + \left(\frac{\partial \phi}{\partial y} \right)^2 + \left(\frac{\partial \phi}{\partial z} \right)^2 \right\} + g \zeta = 0 \tag{107}$$

Eliminating ζ between the above equations, we obtain

$$\left[\frac{\partial}{\partial t} + \left(U + \frac{\partial \phi}{\partial x} \right) \frac{\partial}{\partial x} + \frac{\partial \phi}{\partial y} \frac{\partial}{\partial y} + \frac{\partial \phi}{\partial z} \frac{\partial}{\partial z} \right]$$

$$\cdot \left[\frac{\partial \phi}{\partial t} + U \frac{\partial \phi}{\partial x} + \frac{1}{2} \left\{ \left(\frac{\partial \phi}{\partial x} \right)^2 + \left(\frac{\partial \phi}{\partial y} \right)^2 + \left(\frac{\partial \phi}{\partial z} \right)^2 \right\} + gz \right] = 0 \tag{108}$$

This is the exact nonlinear form of the free surface condition. Let us consider first the near field and examine the order of magnitude of each term. We have written the velocity potential in the form

$$\phi = U\,\phi_0 + \phi_1 + \phi_w \tag{109}$$

We should remember that the order changes by ε^{-1} when differentiating ϕ_0 or ϕ_1 by y or z. Picking up the time-independent part, we obtain

$$\frac{\partial^2\phi_0}{\partial x^2} + 2\,\frac{\partial^2\phi_0}{\partial x\partial y}\,\frac{\partial\phi_0}{\partial y} + \frac{3}{2}\,\frac{\partial^2\phi_0}{\partial y^2}\left(\frac{\partial\phi_0}{\partial y}\right)^2 + \frac{\partial\phi_0}{\partial x}\,\frac{\partial^2\phi_0}{\partial y^2} + \frac{g}{U^2}\,\frac{\partial\phi_0}{\partial z}$$

$$+ 0(\varepsilon^3) = 0 \tag{110}$$

Then the term of the lowest order is $(g/U^2)\ \partial\phi_0/\partial z$ which has the order of ε. Therefore, the free surface condition for the steady potential of the lowest order is

$$\frac{\partial\phi_0}{\partial z} = 0 \quad \text{at } z = 0 \tag{111}$$

that is the condition of double body flow. We cannot proceed to the next step without handling the nonlinear terms in the free surface condition. The periodical part of the free surface condition can be written as

$$\left(\frac{\partial}{\partial t} + U\,\frac{\partial}{\partial x}\right)^2 \phi_1 + g\,\frac{\partial\phi_1}{\partial z} + 2\,U\,\frac{\partial\phi_0}{\partial y}\left(\frac{\partial}{\partial t} + U\,\frac{\partial}{\partial x}\right)\frac{\partial\phi_1}{\partial y}$$

$$+ 2\,U^2\,\frac{\partial^2\phi_0}{\partial x\partial y}\,\frac{\partial\phi_1}{\partial y} + U^2\,\frac{\partial}{\partial y}\left(\frac{\partial\phi_0}{\partial y}\right)^2\,\frac{\partial\phi_1}{\partial y} + U^2\left(\frac{\partial\phi_0}{\partial y}\right)^2\,\frac{\partial^2\phi_1}{\partial y^2}$$

$$- U\,\frac{\partial^2\phi_0}{\partial z^2}\left(\frac{\partial}{\partial t} + U\,\frac{\partial}{\partial x}\right)(\phi_1+\phi_w) + U^2\left\{\frac{\partial\phi_0}{\partial x} + \frac{1}{2}\left(\frac{\partial\phi_0}{\partial y}\right)^2\right\}\frac{\partial^2\phi_1}{\partial z^2}$$

$$+ 0(\delta\varepsilon^2) = 0 \tag{112}$$

If U and ω are of the order of unity, the lowest order term is $O(\delta)$ and the next term is $O(\delta\varepsilon)$. Although these are linear with respect to ϕ_1, if ϕ_0 and ϕ_w are assumed to be known functions, the cross products between derivatives of ϕ_1 and ϕ_0 make the boundary value problem intractable. If only the lowest order is taken, the free surface condition for the radiation potential becomes

$$\frac{\partial \phi_R}{\partial z} = 0 \quad \text{at } z = 0 \tag{113}$$

Thus, the double body condition holds again. The boundary condition for the diffraction potential becomes, on the other hand,

$$\frac{\partial \phi_D}{\partial z} = - U \zeta_w \frac{\partial^2 \phi_0}{\partial z^2} \tag{114}$$

where ζ_w is the wave profile of the incident wave, since we have the relation

$$\frac{\partial \phi_w}{\partial t} + U \frac{\partial \phi_w}{\partial x} + g \zeta_w = 0 \tag{115}$$

Now let us construct the boundary value problem. The field equation is the Laplace equation, but the governing equation in the near field is reduced to the two-dimensional form in the y-z plane because of the singular perturbation. Therefore, the boundary value problem is to find a plane harmonic function with the boundary condition at the hull surface, where the normal velocity is prescribed, and with the condition $\partial\phi_R/\partial z = 0$ at $z = 0$ for the radiation potential. The condition of infinity is left unspecified, yielding an indefiniteness to the solution. Now we assume the solution of the two-dimensional problem of the Laplace equation

$$\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \tag{116}$$

with the boundary condition on the hull surface. We have omitted the term $O(\varepsilon^2)$ in reducing the Laplace equation to the two-dimensional form. Then we can include the term $O(\varepsilon)$ which may be the second order in the perturbation expansion. Next the inner solution for the slender body can be expressed by

$$\phi_N = \phi^{(2D)} + g_1(x) + z\, g_2(x) + y\, g_3(x) \tag{117}$$

where $\phi^{(2D)}$, Reference 20, is the two-dimensional solution of the boundary value problem, and $g_1(x)$, $g_2(x)$, and $g_3(x)$ represent the indefiniteness of the solution which may be determined by the matching procedure with the far field behavior of the fluid motion. In the above solution, we include terms up to the second order, but we have omitted terms of $O(\varepsilon^2)$ in the expansion of the free surface condition, so that the third and fourth terms on the right-hand side of the above expression must be deleted. Then we are obliged to take the expression

$$\phi_N = \phi^{(2D)} + g_1(x) \tag{118}$$

The free surface condition in the far field, on the other hand, takes a different form because the differentiation with respect to y or z does not affect the order of magnitude. Therefore, the leading term of the equation gives simply the linearized boundary condition as follows.

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)^2 \phi + g\,\frac{\partial \phi}{\partial z} = 0 \quad \text{at } z = 0 \tag{119}$$

For the periodical motion with circular frequency ω, we can write the complex form as

$$\phi_1 = \Phi\, e^{i\omega t} \tag{120}$$

and the boundary condition on the free surface becomes

$$- \omega^2 \phi_1 + 2 i U \omega \frac{\partial \phi_1}{\partial x} + U^2 \frac{\partial^2 \phi_1}{\partial x^2} + g \frac{\partial \phi_1}{\partial z} = 0 \tag{121}$$

The Green's function for this boundary condition is

$$G(x,y,z;x',y',z') = \frac{1}{r_1} + \frac{1}{r_2}$$

$$+ \frac{2}{\pi} \int_{-\infty}^{\infty} dm \int_0^{\infty} dn \exp[-|y-y'| \sqrt{m^2+n^2} - i m(x-x')]$$

$$\cdot \{\cos(nz+\varepsilon) \cos(nz'+\varepsilon) - \cos nz \cos nz'\}/\sqrt{m^2+n^2}$$

$$+ \frac{2}{g} \int_{-\infty}^{\infty} dm \exp[(z+z') (mU+\omega)^2/g - |y-y'| \sqrt{m^2-(mU+\omega)^4/g^2}$$

$$-im(x-x')] (mU+\omega)^2/\sqrt{m^2-(mU+\omega)^4/g^2} \tag{122}$$

where

$$\left.\begin{aligned} r_1 &= \sqrt{(x-x')^2+(y-y')^2+(z-z')^2} \\ r_2 &= \sqrt{(x-x')^2+(y-y')^2+(z+z')^2} \end{aligned}\right\} \tag{123}$$

$$\tan \varepsilon = - (mU-\omega)^2/gn$$

If the radical in the last integral becomes imaginary, it has an appropriate sign in accordance with the radiation condition. The outer solution is expressed by means of this Green's function. This can be achieved by the application of Green's theorem in similar fashion to the problem of steady forward motion discussed in the former section. Because of the slender body assumption, the first order solution takes the form in which the singularities on the hull surface shrink to a line distribution of wave sources along the x-axis. Then the far field potential is expressed by

$$\phi_F = - e^{i\omega t} \int m(x') \; G(x,y,z;x',0,0) \; dx' \tag{124}$$

where m(x) is the source density which is determined by the matching procedure between the near field and far field solutions. Thanks to the simple condition $\partial\phi/\partial z = 0$ at $z = 0$, the source density can be determined in a simple way such as

$$m(x) = -\frac{1}{4\pi}\left(i\omega + U\frac{d}{dx}\right)[B(x)\;(zg - x\psi - \zeta_w)] \tag{125}$$

where B(x) is the width at the water plane at each transverse section of the ship. The unknown function $g_1(x)$ is determined by the inner expansion of the far field potential and the final result becomes

$$\phi_N = \phi^{(2D)} - 2\int \frac{d\,m(x')}{dx'} \,\mathrm{sgn}(x-x')\; \ell n(2|x-x'|)\; dx'$$

$$- \int m(x')\; G'(x,0,0;x',0,0)\; dx' \tag{126}$$

where

$$G'(x,y,z;x',y',z') = G(x,y,z;x',y',z') - \frac{1}{r_1} - \frac{1}{r_2} \tag{127}$$

We have assumed that m(x) vanishes at both ends of the ship. One can formulate the forces and moments acting on the ship by the integration of pressure on the hull surface, such as the added mass and damping. Numerical computations are not so simple and may require much computer time, if the forward velocity is taken into account, but they are possible at any rate. There has been some attempt of calculating the added mass and damping of a ship making forced heaving and pitching oscillations during forward motion in still water. However, it has failed to obtain any useful result because the numerical results show very unrealistic features, in spite of a rational appearance of the formulation. This fact will be discussed in another section.

Various Cases of the Order of Magnitude of Frequency Parameter and Froude Number

In the preceding section, we have assumed that the frequency parameter of oscillations and Froude number are both of the order of unity, and it was expected that the theory was valid without any restriction in magnitudes of frequency and forward speed. However, the results were quite disappointing. It was noted, on the other hand, that the different choice in the assumption of the order of magnitude of frequency parameter or Froude number might result in different formulation. The discussion of the order of magnitude in the boundary condition may derive different solutions for different assumptions. Here we will consider cases that the Froude number is not so large, or the frequency of oscillations is not so small.

Now let us begin with the case of low Froude number. The Froude number of conventional merchantile vessels is not much greater than 0.3, so that the speed parameter $U^2/g\ell = \gamma_0^{-1}$ is of the order of 10^{-1}, which may be regarded as a small parameter. Therefore, let us assume that $U/\sqrt{g\ell} = O(\varepsilon^{1/2})$. The order of magnitude of the radiation potential is $\delta\varepsilon$ but the effect of the forward speed appears in the term of the order $\delta\ \varepsilon^{1/2}$. The lowest order term in the free surface condition has the order of δ and the next order is $\delta\varepsilon$. If we take up to the order of $\delta\varepsilon$, the free surface condition in the near field becomes

$$\frac{\partial^2 \phi_1}{\partial t^2} + g\,\frac{\partial \phi_1}{\partial z} + U\,\zeta_w\,\frac{\partial^2 \phi_0}{\partial z^2} = 0 \quad \text{at } z = 0 \tag{128}$$

Therefore, we can take account of the effect of forward speed in the boundary condition on the hull surface without bothering about the quadratic terms in the free surface condition making the boundary value problem intractable. The near field potential for the radiation potential is determined from the two-dimensional boundary value problem with the linearized free surface condition for an oscillating body such as

$$\frac{\partial \phi_R}{\partial z} - \frac{\omega^2}{g} \phi_R = 0 \tag{129}$$

The well known method of solution for the two-dimensional problem can be applied to the determination of the two-dimensional potential $\phi^{(2D)}$. The solution, however, is not identical to that for an oscillating cylinder of infinite length because of the term involving the forward speed. The results from the boundary condition on the hull surface,

$$\frac{\partial \phi_R}{\partial n'} = \{i\omega(z_g - x\psi) - U\psi\} \frac{\partial z}{\partial n'} - U(z_g - x\psi) \frac{\partial}{\partial n'} \left(\frac{\partial \phi_0}{\partial z}\right) \tag{130}$$

The last term on the right-hand side may add some complication. The velocity potential in the near field is then expressed by

$$\phi_N = \phi^{(2D)} + g_1(x) + z\, g_2(x) \tag{131}$$

The term $yg_3(x)$ which appeared in Equation (117) is omitted because of the symmetry of longitudinal oscillations. It is readily shown that there is a relation

$$g_2(x) = \frac{\omega^2}{g} g_1(x) \tag{132}$$

because of the free surface condition. The function $g_1(x)$ is determined by matching with the inner expansion of the far field potential. Since we have assumed that the Froude number is of the order of $\varepsilon^{1/2}$, we expand the far field potential which has been given in the preceding section by U and discard terms of higher order. The result is

$$
\begin{aligned}
g_1(x) = & - 2\int \frac{dm(x')}{dx'} \, \mathrm{sgn}(x-x') \, \ell n(2|x-x'|) \, dx' \\
& + \pi \nu \int m(x') \, [\mathbf{H}_0(\nu|x-x'|) - Y_0(\nu|x-x'|)] \, dx' \\
& + 2\pi i \Omega \int \frac{dm(x')}{dx'} \Big[\mathbf{H}_0(\nu|x-x'|) - Y_0(\nu|x-x'|) \\
& \qquad + \nu|x-x'| \left\{ \frac{2}{\pi} - \mathbf{H}_1(\nu|x-x'|) + Y_1(\nu|x-x'|) \right\} \Big] \, dx' \\
& + 2\pi i \nu (1+4\Omega^2) \int m(x') \, e^{2i\nu\Omega(x-x')} \, H_0^{(2)}(\nu|x-x'|) \, dx' \\
& - 4\pi\nu\Omega \int m(x') \, e^{2i\nu\Omega(x-x')} \, H_0^{(2)}(\nu|x-x'|) \, dx'
\end{aligned}
\tag{133}
$$

where $\nu = \omega^2/g$

$\Omega = \omega U/g$

$\mathbf{H}_0$ = Struve function

$Y_0, Y_1, H_0^{(2)}$ = Bessel functions of the second and third kinds

Because of the term $zg_2(x)$, the boundary value problem becomes a little different from the pure two-dimensional solution for $\phi^{(2D)}$. The most remarkable feature of this case is that the two-dimensional solution is related to the free surface condition

$$
\frac{\partial \phi^{(2D)}}{\partial z} - \nu \, \phi^{(2D)} = 0 \tag{134}
$$

instead of that for the double body flow given in the preceding section. This is the consequence of taking the second term of the perturbation expansion. Though the free surface condition does not involve the forward velocity, the hull boundary condition does.

Another choice is the high frequency case. The order of magnitude of the frequency parameter is assumed to be $\varepsilon^{-1/2}$. In the first place, we assume the Froude number still remains in the order of unity. Because of the relation

$$K = \frac{2\pi}{\lambda} = \frac{g}{2\,U^2\cos^2\alpha}\left(1+2\Omega\cos\alpha-\sqrt{1+4\Omega\cos\alpha}\right) \tag{135}$$

the ratio of wavelength to the ship's length is of the order of $\varepsilon^{1/2}$, so that the wave is not extremely short. Since the wave slope is of the order of δ, the wave amplitude is of the order of $\delta\,\varepsilon^{1/2}$. The velocity of the orbital motion of wave is $O(\delta)$, so that the order of ϕ_1 is $\delta\,\varepsilon$. In the boundary condition on the hull surface for the radiation potential, namely

$$\frac{\partial\phi_R}{\partial n} = i\,\omega(z_g-x\psi)\frac{\partial z}{\partial n'} - U\,\psi\,\frac{\partial z}{\partial n'} - U(z_g-x\psi)\frac{\partial}{\partial n'}\left(\frac{\partial\phi_0}{\partial z}\right) \tag{136}$$

the first term is of the order of δ, while other terms which are related to the forward speed are of the order of $\delta\,\varepsilon^{1/2}$. On the other hand, the lowest order of the free surface condition is δ and the next is $\delta\,\varepsilon^{1/2}$. If we take the lowest order terms only in both boundary conditions, the solution is in the case of zero forward speed. We can discuss the effect of the forward speed by taking terms of order $\delta\,\varepsilon^{1/2}$. In this case, the free surface condition for the radiation potential becomes

$$\frac{\partial^2\phi_R}{\partial t^2} + g\frac{\partial\phi_R}{\partial z} + 2\,U\frac{\partial^2\phi_R}{\partial t\partial x} + 2\,U\frac{\partial\phi_0}{\partial y}\frac{\partial^2\phi_R}{\partial t\partial y} - U\frac{\partial^2\phi_0}{\partial z^2}\frac{\partial\phi_R}{\partial t} = 0 \tag{137}$$

Because of the terms except the first and second ones, we cannot solve the boundary value problem in two dimensions by ordinary methods. Ogilvie and Tuck[29] employed a successive method by which the solution for the above boundary condition was derived from the solution for zero forward speed. The solution obtained involves integrals over the plane z = 0. It seems of some interest that the outer solution for the radiation potential takes a degenerated form like

$$\phi_R \approx -\frac{4\pi i}{g}\left\{m(x)+2i\Omega(z-i|y|)\frac{dm(x)}{dx}\right\}\exp[i\omega t+\nu(z-i|y|)] \tag{138}$$

The first term in parentheses means a simple harmonic plane wave propagating outwards in the y direction, while the second term is related to the variation of the source density along the x-axis which means the interference between different sections. An extensive discussion of this case is given by Ogilvie and Tuck and will not be reproduced here.

Next we consider the case of high frequency with low forward speed, that is $\omega = 0(\varepsilon^{-1/2})$, $U = 0(\varepsilon^{1/2})$. This is the case of short waves such that the wavelength is of the same order as that of the breadth of the ship. In this case, the order of magnitude of ϕ_1 is $\delta\,\varepsilon^{3/2}$. The free surface condition becomes

$$\frac{\partial^2\phi_1}{\partial t^2} + g\,\frac{\partial\phi_1}{\partial z} = 0 \tag{139}$$

up to the order of $\delta\,\varepsilon^{1/2}$. The next term is of the order of $\delta\,\varepsilon^{3/2}$, which includes the effect of the forward speed, but has quadratic forms. Therefore, the ordinary linearized treatment can be applied to the first order only, by which the effect of the forward speed cannot be taken into account. The asymptotic form for the outer solution for the radiation potential takes the form

$$\phi_R \approx -\frac{4\pi i}{g}\, e^{i\omega t + \nu(z-i|y|)}\, m(x) \tag{140}$$

that means outward-going plane waves. There are no three-dimensional terms involved and the solution is purely two-dimensional. The strip theory is then valid but without any effect of the forward speed. It should be noted that the diffraction problem cannot be treated in the same way, because the assumption of the slow variation along the x-axis is no longer valid. The diffraction in short waves will be discussed in another section.

Hydrodynamic Forces in Heaving and Pitching

It is widely known that the hydrodynamic forces and moments acting on oscillating ships are predicted by strip theory with fairly good accuracy. However, the reliability of the strip theory is still open to doubt as discrepancies between computed and measured results are observed occasionally. These discrepancies may be attributed to the effect of the forward speed and fluid motion in three dimensions. In the preceding section, we have observed that both the forward speed and the three-dimensionality can be taken into account within a plausible approximation of the perturbation scheme, if the forward speed is of the order of $\varepsilon^{1/2}$ where ε is the beam-to-length ratio.

The hydrodynamic forces are obtained by the integration of pressure over the hull surface. The fluid pressure in the near field is given by

$$-\frac{1}{\rho}(p-p_0) = i\,\omega\,\phi_1 + U\frac{\partial\phi_1}{\partial x} + U\frac{\partial\phi_0^{(2D)}}{\partial y}\cdot\frac{\partial\phi_1^{(2D)}}{\partial y} + U\frac{\partial\phi_0^{(2D)}}{\partial z}\cdot\frac{\partial\phi_1^{(2D)}}{\partial z} \qquad (141)$$

up to the order of $\delta\varepsilon$. The hydrostatic pressure is omitted because it is simply determined by the geometrical relations. Although the third and fourth terms are omitted in the usual linearized theory for oscillating ships, they appear in the same order of magnitude as the second term, so that they have to be retained if one wishes to take account of the effect of the forward speed. The vertical force is given by the integral

$$F_2 = -\iint_S (p-p_0)\,\frac{\partial z}{\partial n}\,dS \qquad (142)$$

Although the quadratic terms in Equation (141) seem to be troublesome in evaluating the above integral, a theorem which has been proven by Tuck[30] becomes a powerful aid. The theorem is proved in Appendix B. Consider a velocity vector $\underline{V}$ of an irrotational motion of an inviscid fluid outside the hull, which satisfies the boundary condition on the hull surface such as

$$\underline{V} \cdot \underline{n} = 0 \tag{143}$$

where $\underline{n}$ is a unit vector along the outward normal to the hull surface. Next we define the vector $\underline{m}$ by the relation

$$\underline{m} = -\frac{\partial \underline{V}}{\partial n} \tag{144}$$

Further we define vectors $\underline{n}^*$ and $\underline{m}^*$ by

$$\left.\begin{aligned} \underline{n}^* &= \underline{r} \times \underline{n} \\ \underline{m}^* &= -\frac{\partial}{\partial n}(\underline{r}\times n) \end{aligned}\right\} \tag{145}$$

where $\underline{r}$ is the position vector (x,y,z). Then, the following relations are valid.

$$\left.\begin{aligned} \iint_S [\underline{m}\phi+\underline{n}(\underline{V}\cdot\nabla\phi)]\, dS &= -\int_{L_0} \underline{n}\,\phi\, w\, ds \\ \iint_S [\underline{m}^*\phi+\underline{n}^*(\underline{V}\cdot\nabla\phi)]\, dS &= -\int_{L_0} \underline{n}^*\,\phi\, w\, ds \end{aligned}\right\} \tag{146}$$

where w is the z-component of $\underline{V}$ and L_0 is the still waterline of the hull. The line integral on the right-hand side is omitted when the body is slender and wall-sided at the water plane. For the vertical force, we take the z-component of the first equation, while for the pitching moment, the y-component of the second equation is taken. In the present case, we put

$$\underline{V} = \left(U,\ U \frac{\partial \phi_0}{\partial y},\ U \frac{\partial \phi_0}{\partial z} \right) \tag{147}$$

Next we define vector functions $\underline{\chi}$, $\underline{\psi}$, $\underline{\chi}^*$, and $\underline{\psi}^*$, which are two-dimensional harmonic functions in the lower half space outside the ship and satisfy the boundary conditions on the hull surface, such as

$$\left. \begin{aligned} \frac{\partial \underline{\chi}}{\partial n} &= \underline{n}, & U \frac{\partial \underline{\psi}}{\partial n} &= \underline{m} \\ \frac{\partial \underline{\chi}^*}{\partial n} &= \underline{n}^*, & U \frac{\partial \underline{\psi}^*}{\partial n} &= \underline{m}^* \end{aligned} \right\} \tag{148}$$

Furthermore, these functions are assumed to satisfy the free surface condition

$$\frac{\partial \phi}{\partial z} - \nu\, \phi = 0 \quad \text{at } z = 0 \tag{149}$$

We have expressed the near-field expression of the radiation potential for a slender ship by

$$\phi_N = \phi^{(2D)} + (1+\nu z)\ g_1(x) \tag{150}$$

Then, the vertical force is written in the form

$$\begin{aligned} F_z &= \rho \int dx \int_{C(x)} i\, \omega\, \phi^{(2D)} \frac{\partial z}{\partial n'}\, dc \\ &+ \rho \int dx \int_{C(x)} U \left\{ \frac{\partial \phi^{(2D)}}{\partial x} + \frac{\partial \phi_0^{(2D)}}{\partial y} \frac{\partial \phi^{(2D)}}{\partial y} + \frac{\partial \phi_0^{(2D)}}{\partial z} \frac{\partial \phi^{(2D)}}{\partial z} \right\} \frac{\partial z}{\partial n'}\, dc \end{aligned} \tag{151}$$

(cont.)

$$+\int dx \, \{i\omega g_1(x)+Ug_1'(x)\} \int_{C(x)} (1+\nu z) \frac{\partial z}{\partial n'} \, dc \tag{151}$$

where C(x) is the contour of each section along which the integral with respect to c is taken. On applying the above mentioned theorem to the second term, we obtain

$$F_z = \rho \int dx \int_{C(x)} i \, \omega \, \phi^{(2D)} \frac{\partial z}{\partial n'} \, dc$$

$$+ \rho \int dx \int_{C(x)} U \frac{\partial}{\partial n'} \left(\frac{\partial \phi_0}{\partial z} \right) \phi^{(2D)} \, dc$$

$$+ \rho \int \{i\omega g_1(x)+Ug_1'(x)\} \, \{-B(x)+\nu S(x)\} \, dx \tag{152}$$

The last term is derived by the application of Gauss' theorem in which B(x) and S(x) are breadth at the waterline and sectional area of each transverse section, respectively. If we write the z-components of $\underset{\sim}{\chi}$ and $\underset{\sim}{\psi}$ by χ_z and ψ_z, respectively, the two-dimensional part of the radiation potential is expressed as

$$\phi^{(2D)} = \{i\omega(z_g-x\psi)-U\psi\} \, \chi_z + U(z_g-x\psi) \, \psi_z \tag{153}$$

Then, the vertical force is written as

$$F_z = - \rho \int dx \, \{\omega^2(z_g-x\psi)+i\omega U\psi\} \int_{C(x)} \chi_z \frac{\partial \chi_z}{\partial n'} \, dc$$

$$+ i \, \rho \, \omega \, U \int dx \, (z_g-x\psi) \int_{C(x)} \psi_z \frac{\partial \chi_z}{\partial n'} \, dc$$

$$- i \, \rho \, \omega \, U \int dx \, (z_g-x\psi) \int_{C(x)} \chi_z \frac{\partial \psi_z}{\partial n'} \, dc$$

$$- \rho \, U^2 \int dx \, (z_g-x\psi) \int_{C(x)} \psi_z \frac{\partial \psi_z}{\partial n'} \, dc \tag{154}$$

(cont.)

$$- \rho \int \{i\omega g_1(x)+Ug_1'(x)\} \{-B(x)+\nu S(x)\} \, dx \tag{154}$$

Because of Green's reciprocal relation, which can apply to x_z and ψ_z, such as

$$\int_{C(x)} \chi_z \frac{\partial \psi_z}{\partial n'} \, dc = \int_{C(x)} \psi_z \frac{\partial \chi_z}{\partial n'} \, dc$$

the second and third terms are cancelled. Therefore, we obtain

$$F_z = - \rho \int dx \; \{\omega^2(z_g - x\psi)+i\omega U\psi\} \int_{C(x)} \chi_z \frac{\partial \chi_z}{\partial n'} \, dc$$

$$- \rho \, U^2 \int dx \; (z_g - x\psi) \int_{C(x)} \psi_z \frac{\partial \psi_z}{\partial n'} \, dc$$

$$+ \rho \int \left[-i\omega\{B(x)-\nu S(x)\} + U \frac{d}{dx} \{B(x)-\nu S(x)\} \right] g_1(x) \, dx \tag{155}$$

The last term is derived by integration by parts. If we assume $U = O(\varepsilon^{1/2})$ and omit the term of $O(\delta\varepsilon^2)$, the second term drops out. A similar expression is obtained for the hydrodynamic moment about the y axis. The first term indicates the result obtained by the strip theory, and the other terms give the effect of the forward speed and the three-dimensional motion.

Numerical Results of Radiation Problem at Zero Forward Speed

Although numerical analysis works when the forward speed and the effect of three-dimensionality are present, no published result of this kind is known so far. There is a rather comprehensive result, on the other hand, for the case of zero forward speed by means of a similar formulation.[31] It is obtained simply by letting U = 0 in the original formula for finite speed with no substantial difference in the method of numerical calculation. The result can well illustrate the effect of three-dimensionality of the fluid motion which is remarkable at lower frequencies.

In the first place, let us put $\phi = e^{i\omega t}\,\Phi$ and express the two-dimensional solution for a heaving cylinder in the form as

$$\Phi^{(2D)} = -a_0\left[\int_0^\infty e^{kz}\frac{\cos ky}{k-K}\,dk - \pi i e^{Kz}\cos Ky\right]$$

$$-\sum_{m=1}^{\infty}\frac{a_{2m}}{(2m-1)!}\left[\frac{\partial^{2m-1}}{\partial z^{2m-1}}\left(\frac{z}{z^2+y^2}\right)+K\frac{\partial^{2m-2}}{\partial z^{2m-2}}\left(\frac{z}{z^2+y^2}\right)\right] \quad (156)$$

It is readily shown that this expression satisfies the free surface condition

$$\frac{\partial\Phi}{\partial z} - K\,\Phi = 0 \quad \text{at } z = 0 \quad (157)$$

where $K = \nu = \omega^2/g$ in the present case. Since the inner expansion of the above function is

$$\Phi^{(2D)} = a_0\,[\ell n Kr+\gamma+Kr\cos\theta\,(1-\ell n Kr-\gamma)+Kr\theta\sin\theta+\pi i]$$

$$+\sum_{m=1}^{\infty}\frac{a_{2m}}{r^{2m}}\left[\cos 2m\theta+\frac{Kr}{2m-1}\cos(2m-1)\theta\right] \quad (158)$$

where we have employed the cylindrical coordinates

$$z = -r\cos\theta, \qquad y = r\sin\theta \quad (159)$$

and γ is Euler's constant, 0.5772157.

In order to make the inner expansion of the far field potential match the above, we employ the expression in the near field as

$$\Phi = \Phi^{(2D)} - a_0(1+Kz)(\gamma+\pi i)$$

$$- \frac{1}{2}(1+Kz)\int_{-\ell}^{\ell} a_0'(x')\ \mathrm{sgn}(x-x')\ \ell n(2K|x-x'|)\ dx'$$

$$+ \frac{\pi}{4}K(1+Kz)\int_{-\ell}^{\ell} a_0(x')\ [\mathbf{H}_0(K|x-x'|)+Y_0(K|x-x'|)$$

$$+2iJ_0(K|x-x'|)]\ dx' \qquad (160)$$

where $x = \pm\ell$ is the x coordinate at each end of the ship. The second term on the right-hand side is added to conform to the expansion of the two-dimensional solution. We can rewrite the above expression as

$$\Phi = \Phi^{(2D)} + \frac{1}{2}(1+Kz)\int_{-\ell}^{\ell} a_0'(x)\ N(K|x-x'|)\ \mathrm{sgn}(x-x')\ dx' \qquad (161)$$

where

$$N(u) = -\gamma - \ell n\ 2\ u + \frac{\pi}{2}\int_0^u \mathbf{H}_0(u')\ du' + \frac{\pi}{2}\int_0^u Y_0(u')\ du' - \pi i$$

$$+ \pi i \int_0^u J_0(u')\ du' \qquad (162)$$

It can be shown that

$$\lim_{K\to\infty} N(K|x-x'|) = 0$$

so that the three-dimensional part vanishes when the frequency becomes infinite, and the fluid motion becomes purely two-dimensional for which the strip theory holds exactly.

Now let us consider the boundary value problem. When a ship is in heaving and pitching oscillations, each transverse section has a vertical velocity $V(x) \cdot e^{i\omega t}$, where $V(x)$ is related to the mode of the oscillation. When the slender body approximation is employed, the boundary condition satisfied by the velocity potential $\Phi\, e^{i\omega t}$ at the surface of the body is

$$\frac{\partial \Phi}{\partial n'} = V(x) \frac{\partial z}{\partial n'} \tag{163}$$

If we introduce the expression for Φ, given by Equation (161), the boundary condition can be written in the form

$$\frac{\partial \Phi^{(2D)}}{\partial n'} = \left[V(x) - \frac{1}{2} K \int_{-\ell}^{\ell} a_0'(x') N(K|x-x'|) \operatorname{sgn}(x-x') dx' \right] \frac{\partial z}{\partial n'} \tag{164}$$

Here we introduce the solution of the two-dimensional problem of a heaving cylinder for which the boundary condition at the body surface is

$$\frac{\partial \Phi^{(2D)}}{\partial n'} = \frac{\partial z}{\partial n'} \tag{165}$$

With this solution, we put the coefficient of the source term as

$$a_0 = A_0 \tag{166}$$

Then the coefficient a_0 for the boundary condition of Equation (164) satisfies the equation

$$a_0(x) = \left[V(x) - \frac{1}{2} K \int_{-\ell}^{\ell} a_0'(x') N(K|x-x'|) \operatorname{sgn}(x-x') dx' \right] A_0(x) \tag{167}$$

The coefficient A_0 can be determined by the method well known in the two-dimensional theory of a heaving cylinder. Then we can determine a_0 by solving the above equation. However, we need not solve the equation exactly. The reason is as follows. If the frequency is very low or to the contrary very high, the integral on the right-hand side can be neglected. So we can put

$$a_0(x) = V(x)\ A_0(x) \tag{168}$$

It may be assumed that the deviation from the above relation at intermediate frequencies is not large. Therefore, we approximate

$$a_0(x) = \left[V(x) - \frac{1}{2} K \int_{-\ell}^{\ell} \{V'(x')A_0(x') + V(x')A_0'(x')\}\ N(K|x-x'|)\,\mathrm{sgn}(x-x')\,dx' \right] \times A_0(x) \tag{169}$$

In order to determine other coefficients, we substitute $V(x)A(x)$ for a_0 in the boundary condition and put

$$W(x) = \frac{1}{2} K \int_{-\ell}^{\ell} \{V'(x')A_0(x') + V(x')A_0'(x')\}\ N(K|x-x'|)\,\mathrm{sgn}(x-x')\,dx' \tag{170}$$

Then the boundary condition becomes

$$\frac{\partial \Phi^{(2D)}}{\partial n'} = [V(x) - W(x)]\ \frac{\partial z}{\partial n'} \tag{171}$$

The solution of the two-dimensional problem with this boundary condition determines other coefficients a_{2m}. We can rewrite the function $a_0(x)$ and $W(x)$ in the following form for numerical purposes.

$$a_0(x) = V(x)\ A_0(x) \left[1- \frac{1}{2} KA_0(x)\{N(K|\ell+x|)+N(K|\ell-x|)\}\right]$$

$$- \frac{1}{2} K^2\ A_0(x) \int_{-\ell}^{\ell} [V(x')A_0(x')-V(x)A_0(x)]\ N'(K|x-x'|)dx' \qquad (172)$$

$$W(x) = \frac{1}{2} K\ V(x)\ A_0(x)\ \{N(K|\ell+x|)+N(K|\ell-x|)\}$$

$$+ \frac{1}{2} K^2 \int_{-\ell}^{\ell} [V(x')A_0(x')-V(x)A_0(x)]\ N'(K|x-x'|)dx' \qquad (173)$$

where

$$N'(K|x-x'|) = - \frac{1}{|x-x'|} + \frac{\pi}{2}\mathbf{H}_0(K|x-x'|) + \frac{\pi}{2} Y_0(K|x-x'|)$$

$$+ \pi i\ J_0(K|x-x'|) \qquad (174)$$

The vertical component of the force acting on the ship by the fluid pressure is given by

$$F_z = - \iint_S \frac{\partial z}{\partial n}\ p\ dS \qquad (175)$$

and the moment about the y-axis is

$$M_y = \iint_S \left(\frac{\partial z}{\partial n}\ x- \frac{\partial x}{\partial n}\ z\right)\ p\ dS \qquad (176)$$

The second term in the parentheses can be omitted because of the slender body assumption. Then the vertical force and the pitching moment on the slender ship are given by

$$\left.\begin{aligned} F_z &= \rho \int_{-\ell}^{\ell} dx \int_{C(x)} \frac{\partial\phi}{\partial t} \frac{\partial z}{\partial n'} ds = \int_{-\ell}^{\ell} f(x) dx \\ M_y &= - \rho \int_{-\ell}^{\ell} x\, dx \int_{C(x)} \frac{\partial\phi}{\partial t} \frac{\partial z}{\partial n'} ds = - \int_{-\ell}^{\ell} f(x)\, x\, dx \end{aligned}\right\} \quad (177)$$

where C(x) is the contour of each transverse section. The function f(x) gives the force per unit length at the section. If we divide the velocity potential in two-dimensional and three-dimensional portions as

$$\phi = e^{i\omega t} \left(\Phi^{(2D)} + \Phi^{(3D)}\right) \quad (178)$$

we can write

$$\begin{aligned} f(x) &= i \rho \omega \int_{C(x)} \Phi^{(2D)} \frac{\partial z}{\partial n'} ds + i \rho \omega \int_{C(x)} \Phi^{(3D)} \frac{\partial z}{\partial n'} ds \\ &= f_1(x) + f_2(x) \end{aligned} \quad (179)$$

Now take the added mass m_z and the damping coefficient N_z for a heaving cylinder of infinite length. Then the two-dimensional portion of the sectional force is expressed by

$$f_1(x) = - (i\omega m_z + N_z) \,[V(x) - W(x)] \quad (180)$$

while the three-dimensional portion has the expression

$$f_2(x) = i \rho \omega S(x) - i \frac{\rho g}{\omega} B(x) W(x) \quad (181)$$

where S(x) is the area and B(x) is the waterline width of each section.

In pure heaving, we put $V(x) = i\omega z_g$, and we can write

$$f(x)e^{i\omega t} = -(m_z+m_z')(-\omega^2 z_g) - (N_z+N_z')\, i\,\omega\, z_g \tag{182}$$

where m_z' and N_z' indicate the three-dimensional effect to the added mass and damping coefficient, respectively. In the case of pure pitching, we put $V(x) = -i\omega x\psi$ and we can write

$$f(x)e^{i\omega t} = -(m_z x+m_z'')\,\omega^2\,\psi + (N_z x+N_z'')\, i\,\omega\,\psi \tag{183}$$

Here we put

$$\left.\begin{array}{ll} a^* = m_z + m_z' & b^* = N_z + N_z' \\ d^* = m_z\, x + m_z'' & e^* = N_z\, x + N_z'' \end{array}\right\} \tag{184}$$

and define the following integrals.

$$\left.\begin{array}{ll} a = \int_{-\ell}^{\ell} a^*\, dx & b = \int_{-\ell}^{\ell} b^*\, dx \\ d = \int_{-\ell}^{\ell} d^*\, dx & e = \int_{-\ell}^{\ell} e^*\, dx \\ A = \int_{-\ell}^{\ell} d^*\, x\, dx & B = \int_{-\ell}^{\ell} e^*\, x\, dx \\ D = \int_{-\ell}^{\ell} a^*\, x\, dx & E = \int_{-\ell}^{\ell} b^*\, x\, dx \end{array}\right\} \tag{185}$$

These constants give hydrodynamic coefficients in the coupled equation of heaving and pitching as follows.

$$\left.\begin{aligned} (a+\rho\nabla)\,\ddot{z}_g + b\,\dot{z}_g + c\,z_g - d\ddot{\psi} - e\dot{\psi} - g\psi &= F \\ (A+k_{yy}^2\rho\nabla)\,\ddot{\psi} + B\dot{\psi} + C\psi - D\,\ddot{z}_g - E\,\dot{z}_g - G\,z_g &= M \end{aligned}\right\} \tag{186}$$

where ∇ is the displacement volume and k_{yy} is the radius of gyration. On account of Haskind's relation, we have the relation

$$D = d \qquad E = e \tag{187}$$

For numerical example, the hull form of Series 60, $C_B = 0.7$ is employed because reliable data of model experiments have been available for comparison.

The coefficients a^*, b^*, d^*, e^*, and also a, b, A, B, d, and e are calculated. They are compared with results computed by means of the strip theory and the measured results, as shown in Figures 5 through 7. One can

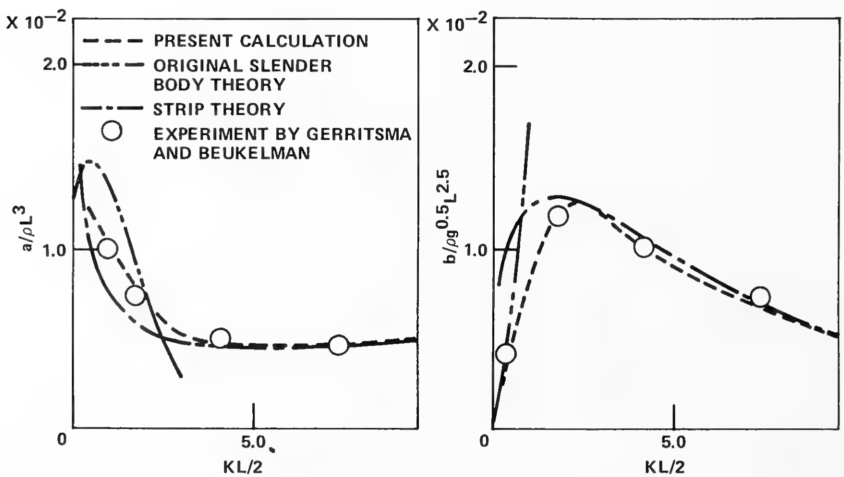

Figure 5 - Hydrodynamic Coefficients a and b

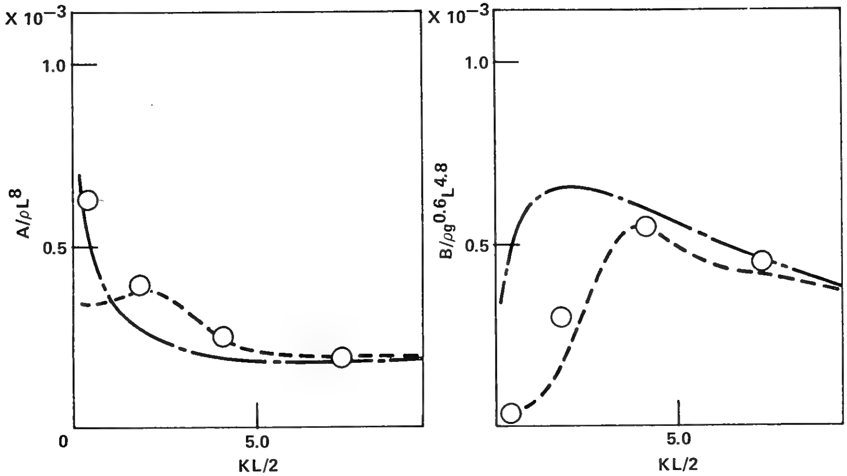

Figure 6 - Hydrodynamic Coefficients A and B

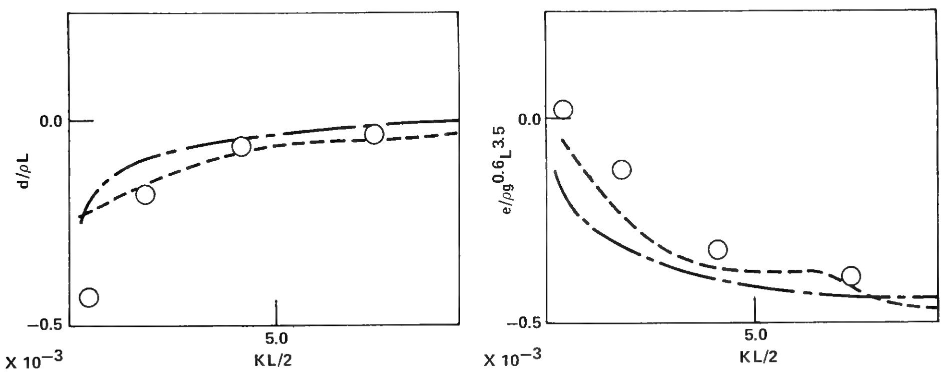

Figure 7 - Hydrodynamic Coefficients d and e

observe a remarkable improvement in agreement with measured results by the calculation including the three-dimensional effect, especially at lower frequencies. One of the reasons of deviation with the results by the strip theory may be attributed to the logarithmic term; ℓnKr, in the two-dimensional solution which makes the added mass infinite at zero frequency limit. The three-dimensional part of the velocity potential has a term which cancels the above logarithmic singularity.

The application of the simple slender body theory, which has been described in an earlier section, presents such an unrealistic result as negative value of added mass and infinite increase in damping coefficient at higher frequencies. Computed results for the source term $a_0(x)$ are shown in Figure 8. The result by the simple slender body theory which determines the near-field solution by the condition for a double body shows much deviation from other theories. This fact may be the main reason of the ill behavior of the slender body calculation. The fair agreement between strip theory and the present calculation suggests that the source term can be determined by the two-dimensional calculation without regard for the three-dimensional effect, or $a_0(x) = V(x)A_0(x)$.

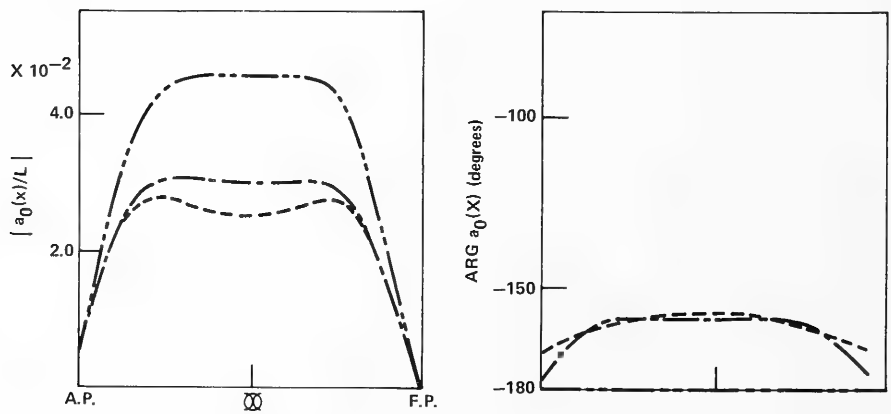

Figure 8 - Source Distribution

As explained in the preceding section, the present theory is a consequence of the inclusion of the second term in the slender body expansion. Nevertheless it is by no means a second order theory, but still a first order theory. The formal perturbation procedure, taking successive approximations, starting from the lowest order term and proceeding to the second order approximation, never leads to the same result as the above. This fact is a peculiar feature of the asymptotic expansion of the singular solution such as the present problem.

WAVE PRESSURE ON SLENDER SHIPS

Boundary Value Problem for the Diffraction Potential

Although the strip theory is employed in the usual practice of predicting wave exciting forces on ships, the diffraction problem does not admit the use of the strip theory in longitudinal waves. Although the strip theory is an acceptable approximation in high frequencies for the radiation problem, the short wavelength associated with the high frequency invalidates the condition of slow variation along the ship's axis. In the case of long waves, on the other hand, the frequency becomes low and the three-dimensional effect comes in as in the radiation problem.

Now we consider first the case of a ship with forward speed in long waves, so that we assume U and ω are both of order of unity. The boundary value problem for the diffraction potential has been given previously, but its solution needs some contrivance. The boundary value problem in the near field is the two-dimensional Laplace equation

$$\frac{\partial^2 \phi_D}{\partial y^2} + \frac{\partial^2 \phi_D}{\partial z^2} = 0 \tag{188}$$

with the hull surface boundary condition of Equation (104)

$$\frac{\partial \phi_D}{\partial n'} = - \frac{\partial \phi_W}{\partial n'} \tag{189}$$

and the free surface condition of Equation (114)

$$\frac{\partial \phi_D}{\partial z} = - U \zeta_w \frac{\partial^2 \phi_0}{\partial z^2} \quad \text{at } z = 0 \tag{190}$$

If the incident wave propagates along the direction of x, ζ_w is constant in the transverse plane. Then the function

$$\phi' = \phi_D + U \zeta_w \frac{\partial \phi_0}{\partial z} \tag{191}$$

is a plane harmonic function in the lower half space. The boundary condition for it is

$$\frac{\partial \phi'}{\partial n'} = - \frac{\partial \phi_w}{\partial n'} + U \zeta_w \frac{\partial}{\partial n'} \left(\frac{\partial \phi_0}{\partial z} \right) \tag{192}$$

on the hull surface, and

$$\frac{\partial \phi'}{\partial z} = 0 \quad \text{at } z = 0 \tag{193}$$

If we take only the first order terms, we can write the hull surface boundary condition as

$$\frac{\partial \phi'}{\partial n'} = - \frac{\partial \phi_w}{\partial z} \frac{\partial z}{\partial n'} + U \zeta_w \frac{\partial}{\partial n'} \left(\frac{\partial \phi_0}{\partial z} \right) \tag{194}$$

or

$$\frac{\partial \phi'}{\partial n'} = - i \omega \zeta_w \frac{\partial z}{\partial n'} + U \zeta_w \frac{\partial}{\partial n'} \left(\frac{\partial \phi_0}{\partial z} \right) \tag{195}$$

This is the same form as the hull boundary condition of the radiation potential for heaving, because the latter is

$$\frac{\partial \phi_R}{\partial n'} = i\,\omega\, z_g \frac{\partial z}{\partial n'} - U\, z_g \frac{\partial}{\partial n'}\left(\frac{\partial \phi_0}{\partial z}\right) \tag{196}$$

Therefore, the diffraction is taken into account in the hull boundary condition by replacing the vertical movement of the section by the relative displacement to the surface of the incident wave. This relation holds in the case of $U = 0(\varepsilon^{1/2})$ too. Then we can take up to the next term, so that the relation is to be modified as

$$\frac{\partial \phi'}{\partial n'} = -\, i\,\omega\, \zeta_w (1+Kz) \frac{\partial z}{\partial n'} + U\, \zeta_w \frac{\partial}{\partial n'}\left(\frac{\partial \phi_0}{\partial z}\right) \tag{197}$$

The added term Kz comes from the exponential factor e^{Kz} in the incident wave potential and indicates the Smith correction.

The boundary value problem for the diffraction potential is now reduced to a similar form to that for the radiation problem. The field equation for the potential ϕ' is the two-dimensional Laplace equation

$$\frac{\partial^2 \phi'}{\partial y^2} + \frac{\partial^2 \phi'}{\partial z^2} = 0 \tag{198}$$

with the hull boundary condition given above. The free surface condition at $z = 0$ is

$$\left.\begin{aligned} &\frac{\partial \phi'}{\partial z} = 0 && \text{when } U = 0(1), && \omega = 0(1) \\ \text{or} \\ &\frac{\partial \phi'}{\partial z} - \frac{\omega^2}{g}\phi' = 0 && \text{when } U = 0(\varepsilon^{1/2}), && \omega = 0(1) \end{aligned}\right\} \tag{199}$$

Therefore, the inner solution for the diffraction potential in the case of $\omega = O(1)$ is expressed by the form

$$\phi_0 = \phi'^{(2D)} - U \zeta_w \frac{\partial \phi_0}{\partial z} + \left(1 + \frac{\omega^2}{g} z\right) g(x) \tag{200}$$

where $\phi'^{(2D)}$ is the solution of the two-dimensional problem with the above mentioned boundary conditions. The factor $\omega^2 z/g$ in the third term on the right-hand side can be added only when $U = O(\varepsilon^{1/2})$ and should be omitted in the case of $U = O(1)$. Another case to be considered is $\omega = O(\varepsilon^{-1/2})$. This is the short wave case, but some complication appears if we apply the slender body theory. The basic idea of the slender body is that the field equation in the near field can be reduced to the two-dimensional Laplace equation. However, the short wavelength hinders the above possibility. In the case of $\omega = O(\varepsilon^{-1/2})$ with $U = O(1)$, the ratio of the wavelength to the ship's length is $\lambda/\ell = O(\varepsilon^{1/2})$ and the variation of the flow field along the x-axis is related to the wavelength. If the order of the diffraction potential is $\delta\ \varepsilon$, the order of magnitude of each term in the Laplace equation in the near field is

$$\frac{\partial^2 \phi_D}{\partial x^2} + \frac{\partial^2 \phi_D}{\partial y^2} + \frac{\partial^2 \phi_D}{\partial z^2} = 0$$

$$(\delta) \qquad (\delta\varepsilon^{-1}) \qquad (\delta\varepsilon^{-1})$$

Therefore, omitting the term of higher order than ε, we get a two-dimensional Laplace equation. However, the omitted term in the long wave case has been of higher order, ε^2. Therefore, the validity of the two-dimensional equation becomes much weaker in comparison with the long wave case. This may damage the accuracy appreciably. The free surface condition in this case is

$$\frac{\partial^2 \phi_D}{\partial t^2} + g \frac{\partial \phi_D}{\partial z} + 2 U \frac{\partial^2 \phi_D}{\partial t \partial x} + 2 U \frac{\partial \phi_0}{\partial y} \frac{\partial^2 \phi_D}{\partial t \partial y} - \frac{\partial^2 \phi_0}{\partial z^2} \frac{\partial \phi_D}{\partial t} + g U \zeta_w \frac{\partial^2 \phi_0}{\partial z^2} = 0 \quad \text{at } z = 0 \tag{201}$$

if terms of order $\delta\ \varepsilon^{1/2}$ are retained. The terms involving the effect of steady forward potential prevents the straightforward solution, so that this case should not be used for the practical purpose of prediction.

If the forward speed is low, namely $U = O(\varepsilon^{1/2})$, the ratio of wave length to ship's length is of the order of ε. Since each term in the three-dimensional Laplace equation has the same order of magnitude, its two-dimensional version is no longer valid. Therefore, the strip theory is not applicable to the diffraction problem in the longitudinal waves. An alternative method for the diffraction problem in short waves will be discussed later.

Wave Pressure and Hydrodynamic Forces

As was mentioned before, the diffraction problem requires not only the integrated total force, but some local quantities such as wave pressure at each point on the hull surface and the distribution of forces along the x-axis. If we write

$$\phi = \phi_D + \phi_W \tag{202}$$

the periodical pressure on the hull surface is given by

$$-\frac{1}{\rho}(p-p_0) = i\ \omega\ \phi + U\frac{\partial\phi}{\partial x} + U\frac{\partial\phi_0}{\partial y}\frac{\partial\phi}{\partial y} + U\frac{\partial\phi_0}{\partial z}\frac{\partial\phi}{\partial z} \tag{203}$$

up to the order of $\delta\ \varepsilon$, if U and ω are both of the order of unity. Although the third and fourth terms are omitted in the usual theory, they appear in the same order as the first and second terms, so they have to be taken into account if the effect of forward speed is considered. The vertical force is given by the same way as in the radiation problem, so Tuck's theorem can be applied again. The vertical force is given by

$$\begin{aligned} F_z &= \rho\iint_S\left(i\omega\phi+U\frac{\partial\phi}{\partial x}+U\frac{\partial\phi_0}{\partial y}\frac{\partial\phi}{\partial y}+U\frac{\partial\phi_0}{\partial z}\frac{\partial\phi}{\partial z}\right)\frac{\partial z}{\partial n}\,dS \\ &= \rho\iint_S\left(i\omega\phi\frac{\partial\chi_z}{\partial n}-U\phi\frac{\partial\psi_z}{\partial n}\right)dS \end{aligned} \tag{204}$$

At a great distance from the ship, the diffraction potential ϕ_D and the incident wave potential ϕ_w satisfy the linearized free surface condition

$$\left(i\omega+U\frac{\partial}{\partial x}\right)^2 \phi + g\frac{\partial\phi}{\partial z} = 0 \quad \text{at } z = 0 \tag{205}$$

However, the boundary condition for ϕ_D at $z = 0$ changes near the ship as was pointed out before. Instead of ϕ_D, we can define a function ϕ_D' which satisfies the above free surface condition even in the near field. We assume that ϕ_D' coincides with ϕ_D in the far field, while it coincides with ϕ' in the near field. Then in the near field, we can put

$$\begin{aligned}\phi &= \phi_D + \phi_w \\ &= \phi_D' - U\,\zeta_w \frac{\partial\phi_0}{\partial z} + \phi_w\end{aligned} \tag{206}$$

Since the boundary condition on the hull surface is

$$\frac{\partial\phi}{\partial n} = 0$$

the boundary condition for ϕ_D' becomes

$$\frac{\partial\phi_D'}{\partial n} = U\,\zeta_w \frac{\partial}{\partial n}\left(\frac{\partial\phi_0}{\partial z}\right) - \frac{\partial\phi_w}{\partial n} \tag{207}$$

Next we assume the auxiliary functions χ_z and ψ_z satisfy the boundary condition

$$\left(i\omega-U\frac{\partial}{\partial x}\right)^2 \phi + g\frac{\partial\phi}{\partial z} = 0 \quad \text{at } z = 0 \tag{208}$$

and evaluate the integrals

$$\iint_S \phi \frac{\partial \chi_z}{\partial n} dS \quad \text{and} \iint_S \phi \frac{\partial \psi_z}{\partial n} dS$$

We have the relation which is derived by Green's identity.

$$\iint_S \left\{ (\phi_D' + \phi_w) \frac{\partial \chi_z}{\partial n} - \chi_z \frac{\partial}{\partial n} (\phi_D' + \phi_w) \right\} dS$$

$$= \iint_\Sigma \left\{ (\phi_D' + \phi_w) \frac{\partial \chi_z}{\partial n} - \chi_z \frac{\partial}{\partial n} (\phi_D' + \phi_w) \right\} dS$$

where Σ is a vertical cylinder of large radius surrounding the ship. We have omitted a line integral term which can be assumed small because of the slender body assumption. Because of the boundary condition on the hull surface, there is a relation

$$\iint_S \chi_z \frac{\partial}{\partial n} (\phi_D' + \phi_w) \; dS = \iint_S U \zeta_w \frac{\partial}{\partial n} \left(\frac{\partial \phi_0}{\partial z} \right) \chi_z \; dS$$

while the radiation condition satisfied by ϕ_D' and χ_z gives

$$\iint_\Sigma \left(\phi_D' \frac{\partial x_z}{\partial n} - \chi_z \frac{\partial \phi_D'}{\partial n} \right) dS = 0$$

Therefore, we have the relation

$$\iint_S (\phi_D' + \phi_w) \frac{\partial x_z}{\partial n} dS = \iint_S U \zeta_w \frac{\partial}{\partial n} \left(\frac{\partial \phi_0}{\partial z} \right) \chi_z \; dS$$

$$+ \iint_\Sigma \left(\phi_w \frac{\partial x_z}{\partial n} - \chi_z \frac{\partial \phi_w}{\partial n} \right) dS$$

Therefore, also

$$\iint_S \phi \frac{\partial \chi_z}{\partial n} dS = \iint_S \left\{ U \zeta_w \frac{\partial}{\partial n}\left(\frac{\partial \phi_0}{\partial z}\right) \chi_z - U \zeta_w \frac{\partial \phi_0}{\partial z} \frac{\partial \chi_z}{\partial n} \right\} dS$$

$$+ \iint_\Sigma \left(\phi_w \frac{\partial \chi_z}{\partial n} - \chi_z \frac{\partial \phi_w}{\partial n} \right) dS \qquad (209)$$

The same relations hold when x_z is replaced by ψ_z. Then the vertical force is expressed by

$$F_z = \rho \iint_S \left\{ U \zeta_w \frac{\partial}{\partial n}\left(\frac{\partial \phi_0}{\partial z}\right) (i\omega\chi_z - U \psi_z) \right.$$

$$\left. - U \zeta_w \frac{\partial \phi_D}{\partial z} \frac{\partial}{\partial n} (i\omega\chi_z - U \psi_z) \right\} dS$$

$$+ \rho \iint_\Sigma \left\{ \phi_w \frac{\partial}{\partial n} (i\omega\chi_z - U \psi_z) - \frac{\partial \phi_w}{\partial n} (i\omega\chi_z - U \psi_z) \right\} dS \qquad (210)$$

If there is no forward speed, we have

$$F_z = i \rho \omega \iint_\Sigma \left(\phi_w \frac{\partial \chi_z}{\partial n} - \chi_z \frac{\partial \phi_w}{\partial n} \right) dS \qquad (211)$$

This relation is called the Haskind relation. The functions χ_z and ψ_z are related to the solution of the radiation problem. Therefore, the above relation enables the wave excitation to be evaluated without solving the diffraction problem. Similar relations are obtained for pitching moment too.

Wave Pressure in Short Waves

When the order of magnitude of the frequency parameter $\omega\sqrt{\ell/g}$ is $\varepsilon^{-1/2}$, the variation of the diffraction potential along the x-axis is no longer small. Therefore, the slender body assumption, in which the flow field varies slowly along the axis of the body, does not hold, and the strip theory is by no means applicable. We need not worry about this fact, as far as the total force or moment is concerned, because the total force becomes very small and unimportant in short waves. However, we need another theory if we intend to discuss more local phenomena such as pressure on the surface of the body.

In the case of a long ship in a heading with the propagation of short waves, one may think of an infinitely long cylinder placed in waves with its axis parallel to the propagation of the wave. However, it has been shown by Ursell[32] that there is no steady state solution in this case. The fluid motion is highly three-dimensional and cannot be replaced by the two-dimensional solution. However, there is an acceptable assumption which can reduce the boundary value problem to a much more simplified form. That is, the variation of the diffraction potential in the longitudinal axis of the ship deviates from the sinusoidal variation very slightly. Here, we consider the case of $\omega = O(\varepsilon^{-1/2})$.

As mentioned before, it is not easy to formulate the linearized solution of the boundary value problem in the case of $\omega = O(\varepsilon^{-1/2})$ and $U = O(1)$. If $U = O(\varepsilon^{1/2})$, the effect of the forward speed appears only in the higher order term. Therefore, we will confine our discussion in the case without forward speed. The velocity potential of the incident wave which propagates in the positive x-direction can be expressed by

$$\phi_w = c\ h \exp(Kz-iKx+i\omega t) \tag{212}$$

where $c = g/\omega$ is the phase velocity of the wave. Now let us express the diffraction potential in the form

$$\phi_D = \psi(x,y,z) \exp(i\omega t-iKx) \tag{213}$$

According to the assumption stated above, the function $\psi(x,y,z)$ varies very slowly in the x-direction. In order to facilitate the analysis, we take the Fourier transform of ϕ_D such as

$$\tilde{\phi}_D = \int_{-\infty}^{\infty} \phi_D \, e^{ikx} \, dx = e^{i\omega t} \int_{-\infty}^{\infty} \psi \, e^{ix(k-K)} \, dx$$

$$= e^{i\omega t} \, \tilde{\psi}(k-K,y,z) \tag{214}$$

where $\tilde{\psi}$ is the Fourier transform of ψ. The diffraction potential satisfies the Laplace equation

$$\frac{\partial^2 \phi_D}{\partial x^2} + \frac{\partial^2 \phi_D}{\partial y^2} + \frac{\partial^2 \phi_D}{\partial z^2} = 0 \tag{215}$$

and its Fourier transform is

$$- k^2 \, \tilde{\phi}_D + \frac{\partial^2 \tilde{\phi}_D}{\partial y^2} + \frac{\partial^2 \tilde{\phi}_D}{\partial z^2} = 0 \tag{216}$$

Substituting the above mentioned expression for the diffraction potential, Equation (214), shifting the first term of Equation (216) to the right-hand side and putting $k - K = k'$, we obtain

$$\frac{\partial^2 \tilde{\psi}}{\partial y^2} + \frac{\partial^2 \tilde{\psi}}{\partial z^2} = (k'+K)^2 \, \tilde{\psi} \tag{217}$$

The basic assumption of the slow variation of ψ suggests that, if $\psi = 0(1)$ then $\tilde{\psi}(k',y,z) = 0(1/k')$. Therefore, k' must be $0(1)$ if $\tilde{\phi}_D = 0(1)$. Since

$K = O(\varepsilon^{-1})$ and the singular perturbation in the near field suggests $\partial/\partial y = O(\varepsilon^{-1})$, $\partial/\partial z = O(\varepsilon^{-1})$, we obtain for the lowest order, the Helmholtz equation

$$\frac{\partial^2 \tilde{\psi}}{\partial y^2} + \frac{\partial^2 \tilde{\psi}}{\partial z^2} = K^2 \tilde{\psi} \tag{218}$$

or, by the Fourier inversion,

$$\frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = K^2 \psi \tag{219}$$

The omitted terms are of the order higher by ε. If we employ the cylindrical coordinates r, θ,

$$z = - r \cos \theta, \quad y = r \sin \theta \tag{220}$$

particular solutions of the differential equation are

$$\left. \begin{matrix} I_n(Kr) \\ \\ K_n(Kr) \end{matrix} \right\} \left\{ \begin{matrix} \cos n\theta \\ \\ \sin n\theta \end{matrix} \right. \tag{221}$$

where I_n and K_n are Bessel functions of imaginary arguments.
Since the forward velocity is absent in the boundary value problem, the boundary condition on the free surface for the diffraction potential applies to the function ψ too.

$$\frac{\partial \psi}{\partial z} - K\psi = 0 \quad \text{at } z = 0 \tag{222}$$

One can construct a solution which satisfies the above equation and vanishes at infinity by combining the basic solutions of Equation (221) as follows.

$$\Psi_n = K_{2n-2}(Kr) \cos(2n-2)\,\theta + 2\,K_{2n-1}(Kr) \cos(2n-1)\,\theta$$

$$+ K_{2n}(Kr) \cos 2\,n\,\theta \tag{223}$$

However, this type of function does not form a complete set and we cannot express the diffraction potential which satisfies the body boundary condition by means of the linear combination of these functions. In order to find the missing term, we consider a distribution of wave sources with density $1/2\ \sigma(x)\ e^{i\omega t - iKx}$ along the x-axis. Taking the Fourier transform of $\sigma(x)$ such as

$$\tilde{\sigma}(k) = \int_{-\infty}^{\infty} \sigma(x)\ e^{ikx}\ dx \tag{224}$$

we have the velocity potential of the type $e^{i\omega t}\ \chi$, where

$$\chi = -\ iKe^{Kz} \int_{-K}^{K} \frac{\exp[i\alpha x - i\sqrt{K^2-\alpha^2}\,|y|]}{\sqrt{K^2-\alpha^2}}\ \tilde{\sigma}(-\alpha-K)\ d\alpha$$

$$+ K\ e^{Kz} \left\{ \int_{-\infty}^{-K} + \int_{K}^{\infty} \right\} \frac{\exp[i\alpha x - \sqrt{\alpha^2-K^2}\,|y|]}{\sqrt{\alpha^2-K^2}}\ \tilde{\sigma}(-\alpha-K)\ d\alpha$$

$$+ \frac{1}{\pi} \int_{-\infty}^{\infty} e^{i\alpha x}\ \tilde{\sigma}(-\alpha-K)\,d\alpha \int_{|\alpha|}^{\infty} \frac{\sqrt{u^2-\alpha^2}\ \cos\sqrt{u^2-\alpha^2}\ z + K\ \sin\sqrt{u^2-\alpha^2}}{u^2 - \alpha^2 + K^2}\ e^{-u|y|}\ du \tag{225}$$

Now we change the variables of integration to

$$\alpha + K = p \qquad \sqrt{u^2-\alpha^2} = Kv \tag{226}$$

and assume the slow variation of $\sigma(x)$, such as

$$\tilde{\sigma}(k) = O(1/k)$$

and $Ky = O(1)$, $Kz = O(1)$, because the wavelength is comparable with the breadth of the body. Then the asymptotic expression of the above integrals for large K leads to the following expression.

$$x = \sqrt{2\pi K}\, e^{Kz-iKx-i\,\pi/4} \int_{-\ell}^{x} \frac{\sigma'(\xi)}{\sqrt{x-\xi}}\, d\xi$$

$$+ \sigma(x)\, e^{-iKx} \left\{ 2 \int_{0}^{\infty} \frac{v\cos(Kzv) + \sin(Kzv)}{(v^2+1)\sqrt{v^2+1}}\, e^{-K|y|\sqrt{v^2+1}}\, v dv - \left(2\pi K|y| + i\,\frac{\pi}{2}\right) e^{Kz} \right\} \tag{227}$$

where $x = -\ell$ is the forward end of the body. The second term is the two-dimensional part of the potential, while the first term gives the three-dimensional effect. Because of the presence of the factor $\sqrt{K}$ in the first term, the order of the second term is higher by $\varepsilon^{1/2}$. If we take the lowest order term only, we can omit the second term. Then the boundary value problem becomes extremely simple. The boundary condition on the hull surface is

$$\frac{\partial\phi_D}{\partial n'} = -\frac{\partial\phi_w}{\partial n'} \tag{228}$$

If we write

$$n'_y = \frac{\partial y}{\partial n'} \qquad n'_z = \frac{\partial z}{\partial n'} \tag{229}$$

the boundary condition can be written as

$$n'_y \frac{\partial\psi}{\partial y} + n'_z \frac{\partial\psi}{\partial z} = - K\ c\ h\ n'_z\ e^{Kz} \tag{230}$$

It is known that the above condition can be satisfied by putting

$$\psi = e^{iKx}\ \chi \tag{231}$$

and the density of the wave source is determined by such a simple equation as

$$\sqrt{2\pi K}\ e^{-i\pi/4} \int_{-h}^{x} \frac{\sigma(\xi)}{\sqrt{x-\xi}}\, d\xi + c\ h = 0 \tag{232}$$

This is Abel's integral equation, the solution of which is

$$\sigma(x) = \frac{c\ h\ e^{i\pi/4}}{\sqrt{2\pi^3 K(x+\ell)}} \tag{233}$$

This approximation was actually given by Faltinsen[33] who calculated the pressure and force acting on a body with semicircular cross section. However, the solution shows the density of the source being infinite at the forward end of the body which results in an infinite pressure there. Such things never happen in actual phenomena. In order to eliminate this difficulty, one has to retain other terms in the expression of ψ other than the lowest order. In this case, the function ψ is expressed by a linear combination of χ and Ψ_n as

$$\psi = \sqrt{\pi K}\ e^{Kz-i\pi/4} \int_{-\infty}^{x} \frac{\sigma(\xi)}{\sqrt{x-\xi}}\, d\xi$$

$$+ \sigma(x) \left(\Psi_0 - i\,\frac{\pi}{2}\, e^{Kz} + \sum_{n=1}^{\infty} p_n\, \overline{\Psi}_n\right) \tag{234}$$

where

$$\Psi_0 = 2 \int_0^\infty \frac{v \cos(Kzv) + \sin(Kzv)}{(v^2+1)\sqrt{v^2+1}} e^{-K|y|\sqrt{v^2+1}} v dv$$

$$- 2 \pi K|y| e^{Kz} \quad (235)$$

The coefficients p_n can be determined independently from $\sigma(x)$ in the boundary condition, while $\sigma(x)$ should satisfy an integral equation of the Volterra type of the second kind. The solution of this integral equation never presents a singular behavior. Numerical results for the pressure distribution on a body with semicircular cross section are compared with measurements as well as the results of the lowest order approximation and those by the strip theory.[34] Fairly good agreement with measured results is obtained by the present calculation as shown in Figures 9 and 10, while

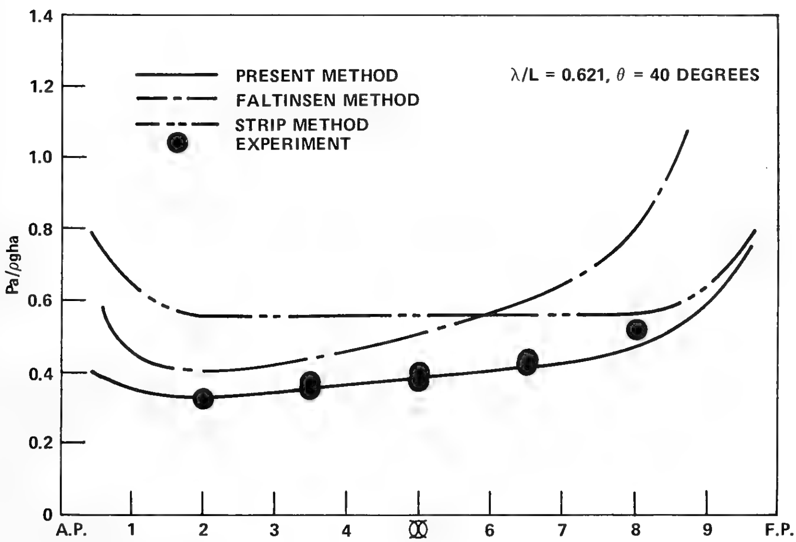

Figure 9 - Distribution of Wave Pressure

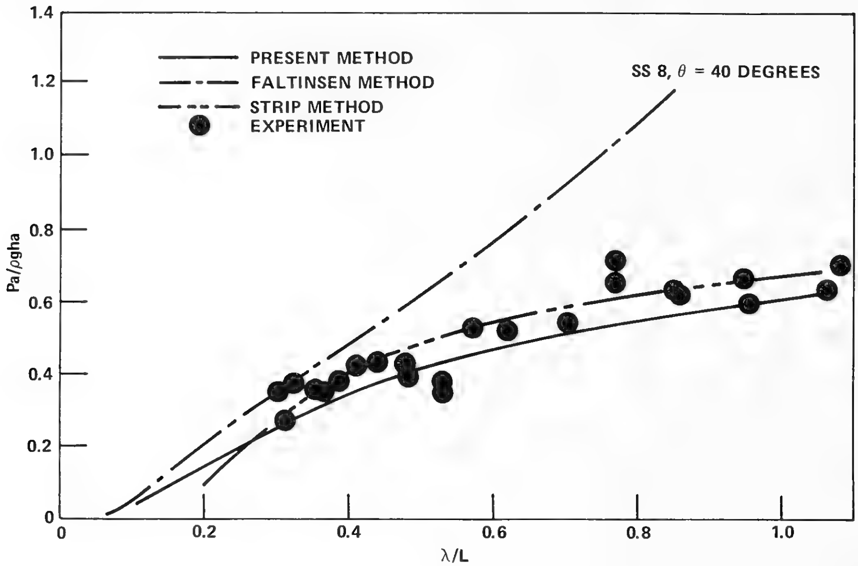

Figure 10 - Wave Pressure at Square Station Number 8

remarkable discrepancies are observed in the results by the lowest order approximation and those by the strip theory. It should be noted that the present solution can never be derived by the successive method starting from the lowest order approximation. This then is another example which illustrates the importance of the inclusion of terms next to the lowest order in the asymptotic expansion.

ADDED RESISTANCE IN WAVES

Formulation of the Steady Drift Force in Waves

When a ship floating on water encounters incident waves, the hydrodynamic pressure on the hull surface is periodical which yields periodical forces and moments acting horizontally on the ship. They are called wave excitations or wave loads.

However, the ship experiences another kind of horizontal force which is independent of time. Such a force is usually called the steady drift force by the wave. When the ship is travelling among ambient waves, the steady drift force results in an increase of resistance to the forward motion of the ship. This is called the added resistance in waves. In some cases, the added resistance attains an amount even larger than the resistance when the ship travels in calm water. It is generally regarded that the added resistance should occupy a major part of the sea margin which should be considered when the power of ships in the service condition is predicted from model test results. This problem looks to be more complicated when compared with the problem of resistance in calm water or periodical hydrodynamic forces in oscillations, because the steady drift force originates from the periodical action of waves. The model test in waves is difficult also in respect to achieving accurate measurements. It needs much time and labor. A reliable method of prediction by theoretical formula has been expected for a long time. From the theoretical side, this problem looks difficult because the steady drift force belongs to the second order forces. However, the problem of added resistance is now regarded as one of the most successful applications of hydrodynamic theories in the practical field of ship building.

Since the drift force is a kind of second order phenomena, one may think of the necessity of the second order solution of the boundary value problem, but it is not the case. The steady force can be calculated from the pressure integral on the hull surface. In doing so, however, we have to pick up terms of the second order without exception. This is quite a difficult task, because we need solutions up to the second order completely. Fortunately we have another method which does not need the second order solutions. That is the momentum analysis by assuming a large surface surrounding the ship at a great distance as a momentum control surface. The steady horizontal force is evaluated by the time average of the momentum flux across the control surface. Its principle can be easily understood by considering a two-dimensional case. Here, we consider a cylindrical body of uniform cross section which is floating on the surface of water and a train of regular waves comes in the direction perpendicular to the axis of the body. The incident wave is partly transmitted beyond the body but partly reflected, generating reflected waves which propagate in the direction opposite to the propagation of the incident wave. At a great distance from the body, there exist regular sinusoidal waves. On the weather side, there are incident waves and reflected waves, while on the lee side, there are transmitted waves. If we assume vertical planes on both sides at a great distance from the body, the momentum flux across these planes can be evaluated simply by the expression for regular waves. If we write the amplitude of the incident wave by h, that of reflected wave by h_R, and that of transmitted wave by h_T, there is a relation for the energy conservation law

$$h^2 = h_R^2 + h_T^2 \tag{236}$$

Now we apply the momentum principle to the fluid between the vertical surfaces and take the time average for one period of the wave. The result shows the steady horizontal force D in the direction of the propagation of the incident wave experienced by the floating body, the amount of which is given by a simple relation[35]

$$D = \frac{1}{2} \rho \, g \, h_R^2 \tag{237}$$

Since the first order forces are periodic, the steady force is the second order force. Nevertheless, it is calculated by the first order solution of the diffraction problem. A similar analysis can be applied to the three-dimensional problem even when the forward speed is present.

Instead of the actual case in which the ship penetrates into waves with a velocity U, we consider a ship floating on a uniform stream with a train of regular waves. A constant horizontal force is assumed in such a way that keeps the average position of the center of gravity of the ship at a fixed point. The velocity potential of the fluid motion in this case is expressed by $Ux + \phi$ as before. Consider a vertical circular cylinder of large radius with its axis through the origin of the coordinates. Taking the immersed part of the surface of the ship S, the cylindrical surface Σ, and the portion of the free surface Σ_0, inside the surface Σ, as the momentum control surfaces, we apply the momentum principle to the fluid contained in the space bounded by these surfaces and consider the fact that there is no flux across S and Σ_0 and the pressure is constant on Σ_0. Then the rate of change of momentum $\underline{M}$ of the fluid enclosed by these surfaces is

$$\frac{d\underline{M}}{dt} = \iint_S p\,\underline{n}\,dS + \iint_\Sigma p\,\underline{n}\,dS + \rho \iint_\Sigma \nabla\,\Phi(\underline{n}\cdot\nabla\Phi)\,dS \qquad (238)$$

$$+ \text{(gravitational force)}$$

where $\underline{n}$ is the unit normal to each surface directing inward to the fluid under consideration, and we have put

$$\Phi = Ux + \phi \qquad (239)$$

The force acting on the ship is the integral pressure on S. Taking the x-component, the force in the x-direction is given by

$$F_x = -\iint_S p\,n_x\,dS$$

$$= -\frac{dM_x}{dt} + \iint_\Sigma p\,n_x\,dS + \rho\iint_\Sigma \left(U + \frac{\partial\phi}{\partial x}\right)(U\,n_x + \underline{n}\cdot\nabla\phi)\,dS \qquad (240)$$

where subscript x means the x-component of each vector.
If the time average is taken, dM_x/dt vanishes because of the periodicity

of the fluid motion. Because of the periodic motion of the body, average flux across the control surface must be zero. Therefore,

$$\overline{\iint_{\Sigma} (U\, n_x + \underline{n} \cdot \nabla\phi)\, dS} = 0 \tag{241}$$

where the bar means the time-averaged during one period.
The average force in the x-direction yields the resistance to the forward motion of the ship.

$$D = \bar{F}_x = \overline{\iint_{\Sigma} p\, n_x\, dS} + \rho\, \overline{\iint_{\Sigma} \frac{\partial\phi}{\partial x} (U\, n_x + \underline{n} \cdot \nabla\phi)\, dS} \tag{242}$$

Taking cylindrical coordinates R and θ,

$$x = R \cos\theta, \qquad y = R \sin\theta \tag{243}$$

making use of the pressure equation

$$\frac{p - p_0}{\rho} = -\frac{\partial\phi}{\partial t} - \frac{1}{2} (\nabla\phi)^2 - U \frac{\partial\phi}{\partial x} - gz \tag{244}$$

and designating the elevation of the free surface by ζ, we obtain

$$D = \rho\, \overline{\int_0^{2\pi} R\, d\theta \int_{-\infty}^{\zeta} \left[\frac{\partial\phi}{\partial x} \frac{\partial\phi}{\partial R} - \left\{ \frac{\partial\phi}{\partial t} + \frac{1}{2} (\nabla\phi)^2 + gz \right\} \cos\theta \right] dz} \tag{245}$$

Because of the periodic motion, we can put

$$\overline{\int_0^{2\pi} R\, d\theta \int_{-\infty}^{0} \left(\frac{\partial\phi}{\partial t} + gz \right) \cos\theta\, dz} = 0$$

Then we have the relation

$$D = \rho \int_0^{2\pi} R \, d\theta \, \overline{\int_{-\infty}^{\zeta} \left[\frac{\partial\phi}{\partial x}\frac{\partial\phi}{\partial R} - \frac{1}{2}(\nabla\phi)^2 \cos\theta \right] dz}$$

$$+ \rho \int_0^{2\pi} R \, d\theta \, \overline{\int_0^{\zeta} \frac{\partial\phi}{\partial t} \cos\theta \, dz} - \frac{1}{2}\rho g \int_0^{2\pi} \overline{\zeta^2} \, R \cos\theta \, d\theta \qquad (246)$$

where the free surface elevation becomes

$$\zeta = -\frac{1}{g}\left[\frac{\partial\phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2 + U\frac{\partial\phi}{\partial x} \right]_{z=\zeta} \qquad (247)$$

Thus, the lowest order terms are quadratic functions of the disturbance velocity potential. A similar formulation is applied to the y-component of the steady force, giving

$$F_y = \rho \int_0^{2\pi} R \, d\theta \, \overline{\int_{-\infty}^{\zeta} \left[\frac{\partial\phi}{\partial y}\frac{\partial\phi}{\partial R} - \left\{ \frac{\partial\phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2 + gz \right\} \sin\theta \right] dz} \qquad (248)$$

On developing the formulation, a relation of the energy conservation is utilized as well. The energy contained in the space bounded by the control surface is

$$E = \rho \iiint \left[\frac{1}{2}(\nabla\Phi)^2 + gz \right] dV \qquad (249)$$

If the control surface is moving with normal velocity v_n directed inward, the rate of change of the energy is given by

$$\frac{dE}{dt} = \frac{\rho}{2} \iiint \frac{\partial}{\partial t}(\nabla\Phi)^2 \, dV - \rho \iint \left[\frac{1}{2}(\nabla\Phi)^2 + gz \right] v_n \, dS \qquad (250)$$

Since we have

$$\frac{1}{2}\iiint \frac{\partial}{\partial t} (\nabla\Phi)^2 \, dV = \iiint (\nabla\Phi) \left(\nabla \frac{\partial \phi}{\partial t}\right) dV$$

$$= - \iint \frac{\partial \Phi}{\partial t} \frac{\partial \Phi}{\partial n} \, dS$$

by Green's theorem and

$$\frac{1}{2} (\nabla\Phi)^2 + gz = \frac{\partial \Phi}{\partial t} - \frac{1}{\rho} (p-p_0)$$

from the pressure equation, one can write

$$\frac{dE}{\partial t} = - \rho \iint \left[\frac{\partial \Phi}{\partial n} \frac{\partial \Phi}{\partial t} + \left(\frac{\partial \Phi}{\partial t} - \frac{p}{\rho}\right) v_n\right] dS$$

$$v_n = - \frac{\partial \Phi}{\partial n}$$

on S and Σ_0 by the boundary condition, while $v_n = 0$ on Σ and $p = 0$ on Σ_0. Then the rate of change of energy becomes

$$\frac{dE}{dt} = \iint_S p \, v_n \, dS - \rho \iint_\Sigma \frac{\partial \Phi}{\partial n} \frac{\partial \Phi}{\partial t} \, dS \tag{251}$$

If the time average is taken, the first term on the right-hand side gives the work done by the ship. The ship is floating freely on the free surface and no external force exists except the constant towing force and the gravitational force which keep the average position of the ship fixed in space. Therefore, no work is done nor is there any dissipation of energy, because the viscosity is neglected. Therefore,

$$\overline{\iint_S p \, v_n \, dS} = 0 \tag{252}$$

Owing to the periodicity of the motion

$$\overline{\frac{dE}{dt}} = 0 \tag{253}$$

we have the relation

$$\overline{\iint_\Sigma \frac{\partial\Phi}{\partial n}\frac{\partial\Phi}{\partial t}\, dS} = 0$$

or

$$\int_D^{2\pi} R\, d\theta \; \overline{\int_{-\infty}^{\zeta} \frac{\partial\phi}{\partial t}\left(\frac{\partial\phi}{\partial R} + U\cos\theta\right) dz} = 0 \tag{254}$$

The velocity potential is the sum of the incident wave, the steady forward motion, and the periodical disturbances, so we can write

$$\phi = \phi_w + U\,\phi_0 + \phi_1 \tag{255}$$

Now we consider that the direction of the wave propagation which makes an angle α with the direction of the forward motion of the ship, namely the negative direction of x. The incident wave potential is

$$\phi_w = c\; h\; \exp(Kz + iKx\cos\alpha + iKy\sin\alpha + i\omega t) \tag{256}$$

The contribution of ϕ_0 to D is the wave resistance in calm water. The periodical disturbance potential has, on the other hand, an asymptotic expression

$$\phi_1 = -\,2\,i\,e^{i\omega t}\left[\int_{-\pi/2}^{\theta-\pi/2} - \int_{\pi/2}^{\theta+\pi/2}\right] H(a_1,\alpha)\,\frac{a_1\,\exp[a_1 z + i a_1 R\cos(\alpha-\theta)]}{\sqrt{1-4\Omega\cos\alpha}}\,d\alpha$$

(257)
(cont.)

$$+ 2 i e^{i\omega t} \int_{\theta-\pi/2}^{\theta+\pi/2} H(a_2,\alpha) \frac{a_2 \exp[a_2 z + i a_2 R \cos(\alpha-\theta)]}{\sqrt{1-4\Omega \cos \alpha}} d\alpha$$

$$+ O(R^{-1}) \tag{257}$$

where

$$\left.\begin{matrix} a_1 \\ a_2 \end{matrix}\right\} = \frac{g}{U^2} \frac{1 - 2 \Omega \cos \alpha \pm \sqrt{1-4\Omega \cos \alpha}}{2 \cos^2 \alpha} \tag{258}$$

The interval $-\alpha_0 < \alpha < \alpha_0$ must be omitted from the integral where

$$\alpha_0 = 0 \quad \text{for } \Omega \leq 1/4$$

$$\alpha_0 = \cos^{-1} (1/4\Omega) \text{ for } \Omega > 1/4$$

The function $H(a_i,\alpha)$ is the Kochin function which is determined by singularities representing the hull. Inserting the above expressions for the velocity potential in the integral giving the force in x-direction, and taking the time average of the lowest order term, we obtain an expression for the mean resistance to the forward motion. It is composed of the wave resistance in the calm water and the added resistance due to incident waves. Let us define the latter by ΔR. The derivation of the final formula is rather lengthy and will not be reproduced here. Only the result is shown as follows.[36]

$$\Delta R = 2 \pi \rho \left[\int_{-\pi/2}^{-\alpha_0} + \int_{\alpha_0}^{\pi/2} - \int_{\pi/2}^{3\pi/2}\right] |H(a_1,\theta)|^2 \frac{a_1(a_1 \cos \theta - K \cos \alpha)}{\sqrt{1-4\Omega \cos \theta}} d\theta$$

$$+ 2 \pi \rho \int_{\alpha_0}^{2\pi-\alpha_0} |H(a_2,\theta)|^2 \frac{a_2(a_2 \cos \theta - K \cos \alpha)}{\sqrt{1-4\Omega \cos \theta}} d\theta \tag{259}$$

where α is replaced by θ in a_1 and a_2. Note that this formula is valid when the energy relation mentioned before holds. If there is another kind of energy dissipation such as viscosity, the above relation needs some alteration. In order to facilitate numerical work, the integration variable is changed to

$$m = K_0 \frac{1 - 2\,\Omega \cos\theta \pm \sqrt{1-4\Omega\cos\theta}}{2\cos\theta} \tag{260}$$

where $K_0 = g/U$. Then we obtain

$$\Delta R = 4\,\pi\,\rho \left[-\int_{-\infty}^{-K_1} + \int_{K_2}^{\infty} \right] \frac{(m+K_0\Omega)^2 (m-K\cos\alpha)}{\sqrt{(m+K_0\Omega)^4 - K_0^2 m^2}} \, |H^*(m)|^2 \, dm \tag{261}$$

where

$$H^*(m) = H(a_i, \theta) \qquad i = 1,\ 2 \tag{262}$$

$$K_1 = \frac{1}{2} K_0 (1+2\Omega+\sqrt{1+4\Omega}), \; K_2 = \frac{1}{2} K_0 (1+2\Omega-\sqrt{1+4\Omega}) \tag{263}$$

If $\Omega < 1/4$, the interval

$$\frac{1}{2} K_0 (1-2\Omega-\sqrt{1-4\Omega}) < m < \frac{1}{2} K_0 (1-2\Omega+\sqrt{1-4\Omega})$$

must be omitted from the integral.

Simplification of the Formula

At a great distance from the ship, the disturbance by the ship, including both radiation and diffraction, is represented by a combination of wave sources and y-directed wave dipoles distributed along the x-axis.

The former represents the longitudinal disturbance such as heaving and pitching, and the latter represents the lateral disturbance such as swaying, yawing, and rolling. Now let us designate densities of these wave sources and wave dipoles by $m(x)e^{i\omega t}$ and $\mu(x)e^{i\omega t}$. Then the Kochin function is expressed as

$$H(a_i,\theta) = \int \{m(x)+ia_i \sin\theta\mu(x)\} e^{i a_i x \cos\theta} dx \tag{264}$$

Then we put

$$\left.\begin{aligned} H_1(a_i,\theta) &= \int m(x) e^{i a_i x \cos\theta} dx \\ H_2(a_i,\theta) &= \int \mu(x) e^{i a_i x \cos\theta} dx \end{aligned}\right\} \tag{265}$$

On integrating with respect to θ the integrals in Equation (264), odd functions of θ vanish. Therefore, we can replace $|H(a_i,\theta)|^2$ by

$$|H_1(a_i,\theta)|^2 + a_i^2 \sin^2\theta \, |H_2(a_i,\theta)|^2$$

Changing the integration variable from θ to m, we obtain

$$\Delta R = 4\pi\rho \left[-\int_{-\infty}^{-K_1} + \int_{-K_2}^{\infty} \right] \frac{(m+K_0\Omega)^2(m-K\cos\alpha)}{\sqrt{(m+K_0\Omega)^4-K_0^2m^2}} |H^*(m)|^2 \, dm \tag{266}$$

where

$$|H^*(m)| = |H_1^*(m)|^2 + \frac{(m+K_0\Omega)^4 - K_0^2 m^2}{K_0^2} |H_2^*(m)|^2 \tag{267}$$

and

$$\left.\begin{aligned} H_1^*(m) &= \int m(x)\, e^{i\,m\,x}\, dx \\ H_2^*(m) &= \int \mu(x)\, e^{i\,m\,x}\, dx \end{aligned}\right\} \tag{268}$$

One can evaluate the above integral provided that the densities of sources and dipoles are known.

Considerable data of computation of added resistance in head sea waves by means of the above formula have been reported.[37] As for the case of oblique seas, numerical results so far obtained are rather infrequent because of the complication of ship motions. Unfortunately the above formula has failed to strike the fancy of engineers of a practical mind, because the integral in the infinite interval necessitates much calculation. It will be found that the formula can be simplified if high frequencies such as $\omega = O(\varepsilon^{-1/2})$ are assumed.[38] If we assume first $U = O(\varepsilon^{1/2})$ and $\omega = O(\varepsilon^{-1/2})$, then $K_0 = O(\varepsilon^{-1})$ and $\Omega = O(1)$. Omitting terms of $O(\varepsilon)$, the formula of Equation (266) is reduced to

$$\Delta R = 4\,\pi\,\rho \int_{-\infty}^{\infty} (m-K\cos\alpha) \left[|H_1^*(m)|^2 + \frac{\omega^4}{g^2} |H_2^*(m)|^2 \right] dm \tag{269}$$

Substitute Equation (268) in the above and apply the Fourier integral theorem. Now we assume that $m(x)$ and $\mu(x)$ vanish at both ends of the ship. Then, by integration by parts, we have

$$\int m(x)\, e^{i\,m\,x}\, m\, dx = -\,i \int \frac{dm(x)}{dx}\, e^{i\,m\,x}\, dx$$

The added resistance due to the longitudinal disturbance is then represented by

$$\begin{aligned} \Delta R_1 &= 4\,\pi\,\rho \int_{-\infty}^{\infty} (m-K\cos\alpha)\,|H_1^*(m)|^2\, dm \\ &= -\,8\,\pi^2\,\rho\, \mathrm{Re} \int \left[K\cos\alpha\,|m(x)|^2 + i\bar{m}(x)\,\frac{dm(x)}{dx} \right] dx \end{aligned} \tag{270}$$

where $\bar{m}(x)$ is the complex conjugate of m(x). Here we write

$$\arg m(x) = \delta_1(x) \tag{271}$$

Then we obtain

$$\Delta R_1 = -8\pi^2 \rho \int |m(x)|^2 \left\{ K \cos \alpha + \frac{d\delta_1(x)}{dx} \right\} dx \tag{272}$$

A similar reduction is applied to the portion by the lateral disturbance. In consequence, the added resistance is given by

$$\Delta R = -8\pi^2 \rho \int \left[|m(x)|^2 \left\{ K \cos \alpha + \frac{d\delta_1(x)}{dx} \right\} + \frac{\omega^4}{g^2} |\mu(x)|^2 \left\{ K \cos \alpha + \frac{d\delta_2(x)}{dx} \right\} \right] dx \tag{273}$$

where

$$\arg \mu(x) = \delta_2(x) \tag{274}$$

It can be readily shown, that the expression for ΔR_1 is valid in the case of $U = 0(1)$ and $\omega = 0(\varepsilon^{-1/2})$ too, but the expression for the added resistance due to lateral disturbances takes another form which is more complicated and spoils the practical utility to some extent. A similar simplification can be applied to the lateral drift force, and the final result is

$$D_y = 8\pi^2 \rho \int \left[2 \frac{\omega^4}{g^2} |m(x)| |\mu(x)| \sin\{\delta_1(x) - \delta_2(x)\} - K \sin \alpha \left\{ |m(x)|^2 + \frac{\omega^4}{g^2} |\mu(x)|^2 \right\} \right] dx \tag{275}$$

The densities of sources and dipoles are determined from the inner solution at the near field which has been discussed previously. They are identical with the source term or dipole term in the solution for two-dimensional cylinders in the corresponding motion of each transverse section. However, the existence of the forward velocity results in a slight change of the boundary condition at the hull surface. If we consider the case of head seas, the oscillation of the ship is heaving and pitching. The effect of surging is omitted because of the higher order. Then the boundary condition on the hull surface is

$$\frac{\partial \phi_1}{\partial n'} = (\dot{z}_g - x\dot{\psi} - U\psi)\ n'_z - U(z_g - x\psi) \frac{\partial}{\partial n'}\left(\frac{\partial \phi_0}{\partial z}\right) - \frac{\partial \phi_w}{\partial n'} \tag{276}$$

where $n'_z = \partial z/\partial n'$, n' being the outward normal to the contour of the transverse section. The first term comes from the relative velocity of the section and the last term means the relative velocity of the orbital motion of the incident wave. However, on account of the second term, the boundary condition is not determined by the relative velocity of the section only. If we assume U and ω are both of the order of unity, the inner solution is associated with the free surface condition such as

$$\frac{\partial \phi_1}{\partial z} = - U\ \zeta_w \frac{\partial^2 \phi_0}{\partial z^2} \quad \text{at } z = 0 \tag{277}$$

The source density in this case can be determined in a manner similar to the radiation problem.

$$m(x) = - \frac{1}{4\pi}\left(i\omega + U \frac{d}{dx}\right)\{B(x)(zg - x\psi - \zeta_w)\} \tag{278}$$

In the case of $U = 0(\varepsilon^{1/2})$ and $\omega = 0(1)$, however, we have to solve the two-dimensional problem with the free surface condition

$$\frac{\partial \phi_1}{\partial z} - \frac{\omega^2}{g}\phi_1 = - U\ \zeta_w \frac{\partial^2 \phi_0}{\partial z^2} \tag{279}$$

Putting

$$\phi' = \phi_1 + U\,\zeta_w \frac{\partial \phi_0}{\partial z} \tag{280}$$

we have the boundary condition at the free surface

$$\frac{\partial \phi'}{\partial z} - \frac{\omega^2}{g}\,\phi' = 0 \tag{281}$$

while the boundary condition on the hull surface can be written as

$$\frac{\partial \phi'}{\partial n'} = n'_z \left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x}\right)(z_g - x\psi - \zeta_w) - U(z_g - x\psi - \zeta_w)\,\frac{\partial}{\partial n'}\left(\frac{\partial \phi_0}{\partial z}\right) \tag{282}$$

If we omit the second term, we get the usual form of boundary conditions for a heaving cylinder with vertical velocity

$$V = \left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x}\right)(z_g - x\psi - \zeta_w) \tag{283}$$

The amplitude of the vertical oscillation is then

$$Z_A = |V|/\omega \tag{284}$$

If we write the amplitude of waves generated by the cylinder as ζ_A and define a two-dimensional pulsating source of strength, $\sigma\, e^{i\omega t}$, which generates the radiating wave of amplitude ζ_A, there is the relation for the amplitude ratio $\bar{A}$ as

$$\bar{A} = \frac{\zeta_A}{Z_A} = \frac{2\pi\sigma\omega}{gZ_A} \tag{285}$$

We have the relation between the density of the three-dimensional source and that of the two-dimensional source as

$$\sigma = 2m \tag{286}$$

Therefore, we obtain

$$|m(x)| = \frac{g\bar{A}Z_A}{4\pi\omega} = \frac{g\bar{A}|V|}{4\pi\omega^2} \tag{287}$$

The determination of the phase of m(x) needs some consideration. Choose the vertical velocity of the cylinder expressed by

$$V = \frac{2g\zeta_A}{\pi\omega B} (A_0 \cos \omega t + B_0 \sin \omega t) \tag{288}$$

in such a way that the source term has the phase like sin ωt. If we write the vertical movement of the cylinder as

$$Z = Z_A \, e^{i\omega t + i\beta} \tag{289}$$

the vertical velocity becomes

$$V = \omega \, Z_A \, e^{i\omega t + i\beta + i\pi/2} \tag{290}$$

Then the phase angle of the source is given by

$$\delta_1(x) = \alpha + \beta \tag{291}$$

where

$$\tan \alpha = B_0/A_0 \tag{292}$$

The amplitude ratio of the radiating wave is, on the other hand, given by

$$\bar{A} = \frac{\zeta_A}{Z_A} = \frac{\pi \omega^2 B}{2g\sqrt{A_0^2+B_0^2}} \tag{293}$$

and the absolute density of the source is given by

$$|m(x)| = \frac{g\bar{A}|V|}{4 \pi \omega^2} = \frac{B(x)|V|}{8\sqrt{A_0^2+B_0^2}} \tag{294}$$.

The coefficients A_0 and B_0 are determined by the standard calculation of the two-dimensional problem of a heaving cylinder. The density and phase of dipoles which represent lateral oscillations can be determined in a similar way. Since the function $U \zeta_w \partial\phi_0/\partial z$ does not include the source term nor horizontal dipole, the density of sources and dipoles of ϕ' is identical with that of ϕ_1. It is not easy to give a general formulation for the effect of the second term of Equation (282), but it can be expressed by a form of a correction term to the vertical velocity in the case of semi-circular cross section, since the hull boundary condition in this case is given by the form

$$\frac{\partial\phi'}{\partial n'} = \frac{n'_z}{B(x)}\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\{B(x)(z_g - x\psi - \zeta_w)\} \tag{295}$$

If we assume the above relation in arbitrary cross section, we can put the vertical velocity of each section in the form

$$V = \frac{1}{B(x)}\left(i\omega + U\frac{\partial}{\partial x}\right)\{B(x)(z_g - x\psi - \zeta_w)\} \tag{296}$$

A similar approximation can be applied to the lateral oscillation.

Numerical Examples for Added Resistance

In the previous sections, formulae for calculating the added resistance and steady side drift force are presented. The formula, which is originally a three-dimensional type, has been simplified to a great extent by assuming the frequency high enough. As a numerical example, the model of Series 60 and $C_B = 0.7$ is employed. Here we consider three kinds of calculation methods in carrying out the numerical work.

Method 1: Assume $U = O(1)$ and $\omega = O(1)$.

The added resistance is calculated by a three-dimensional formula. The boundary condition at the free surface in near field is

$$\frac{\partial \phi_1}{\partial z} + U \zeta_w \frac{\partial^2 \phi_0}{\partial z^2} = 0 \quad \text{at } z = 0 \tag{297}$$

The density of the source distribution in head sea waves takes the form

$$m(x) = -\frac{1}{4\pi}\left(i\omega + U\frac{d}{dx}\right)\{B(x)\,(z_g - x\psi - \zeta_w)\} \tag{298}$$

Making use of this source distribution in Equation (265) or (268) to calculate the Kochin function, a consistent approximation in the present case is obtained. However, the result of numerical computation yields enormous values of added resistance which is hardly compatible with measured results. The present formulation assumes that the disturbance by the ship hull is represented by a distribution of singularities along the axis of the ship which is taken on the undisturbed free surface. However, the above mentioned results indicate that the singularities on the free surface generate too strong disturbance. In order to avoid this difficulty, we assume the singularity distribution a little below the free surface. As a mean depth of the singularities, let us take the mean depth of disturbance given by γT where γ is the vertical prismatic coefficient and T is the draft of the ship. Then the Kochin function is expressed as

$$H_1^*(m) = e^{-m\gamma T} \int m(x)\, e^{imx}\, dx \tag{299}$$

Method 2: Assume $U = O(\varepsilon^{1/2})$ and $\omega = O(1)$.

The boundary condition at the free surface in near field is

$$\frac{\partial\phi_1}{\partial z} - \frac{\omega^2}{g}\phi_1 + U\,\zeta_w\,\frac{\partial^2\phi_0}{\partial z^2} = 0 \quad \text{at } z = 0 \tag{300}$$

The source density is determined from the two-dimensional solution of a heaving cylinder. A consistent approximation for the added resistance is obtained by the three-dimensional formula, Equation (266).

Method 3: Assume $U = O(1)$ or $O(\varepsilon^{1/2})$ and $\omega = O(\varepsilon^{-1/2})$.

The boundary condition at the free surface is the same as for Equation (300) in Method 2. The source density is determined from the two-dimensional solution of a heaving cylinder with the hull boundary condition of Equation (296) which includes the effect of the forward speed. The added resistance is calculated by the simplified formula of Equation (272), but the solution is inconsistent in the order of approximation.

Results of computations of the added resistance of the Series 60 model by means of various methods at several Froude numbers are illustrated in Figures 11 through 13. Results of experiments are also shown for comparison. It is observed that the best agreement between computations and measurements is obtained by the calculation by means of the simplest formula, Method 3, while more consistent methods can provide only less accurate predictions.

CONCLUDING REMARKS

Readers may already have recognized that the four topics discussed in this report have been arranged in the order of simplicity of their physical phenomena. However, what is beyond our expectation for us to find out is the fact that the seemingly simplest problem such as the wave resistance in the uniform motion presents the greatest difficulty in attaining satisfactory agreement between theoretical computations and measured results, while theoretical predictions for much more complex phenomena, such as the

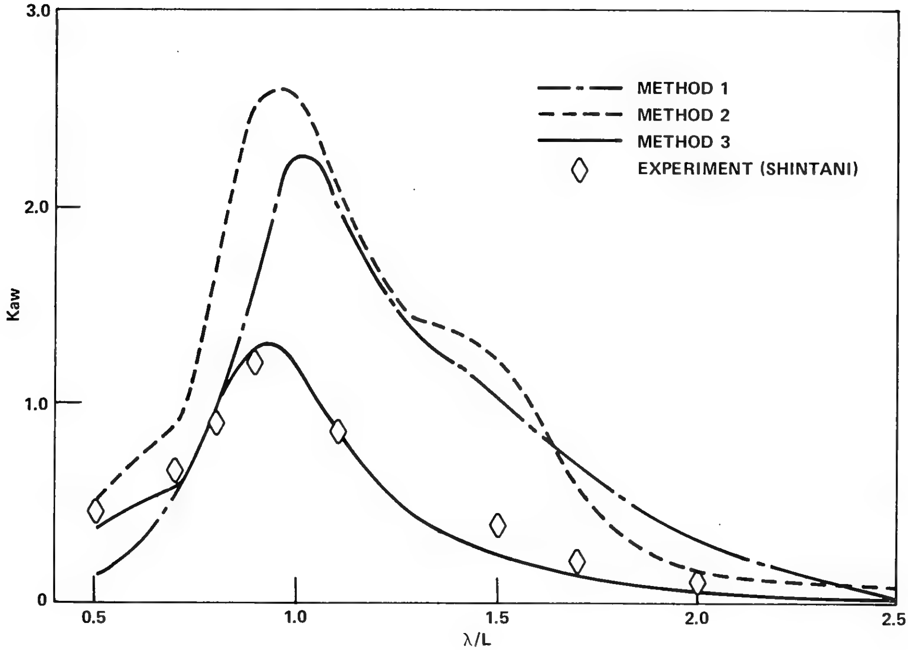

Figure 11 - Added Resistance Coefficient of Series 60 Model for C_b = 0.7 and f_n = 0.1

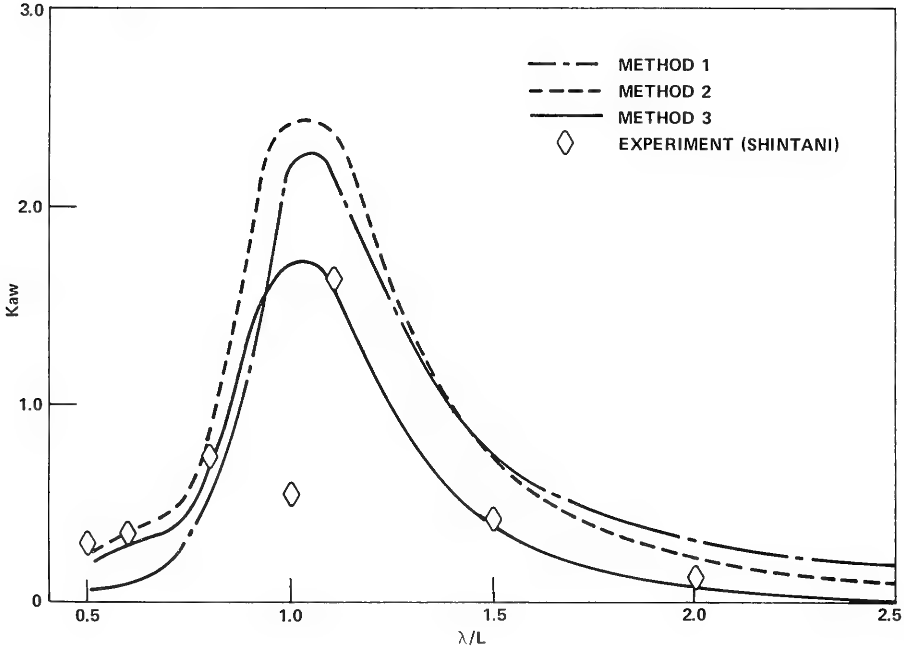

Figure 12 - Added Resistance Coefficient of Series 60
Model for C_b = 0.7 and f_n = 0.15

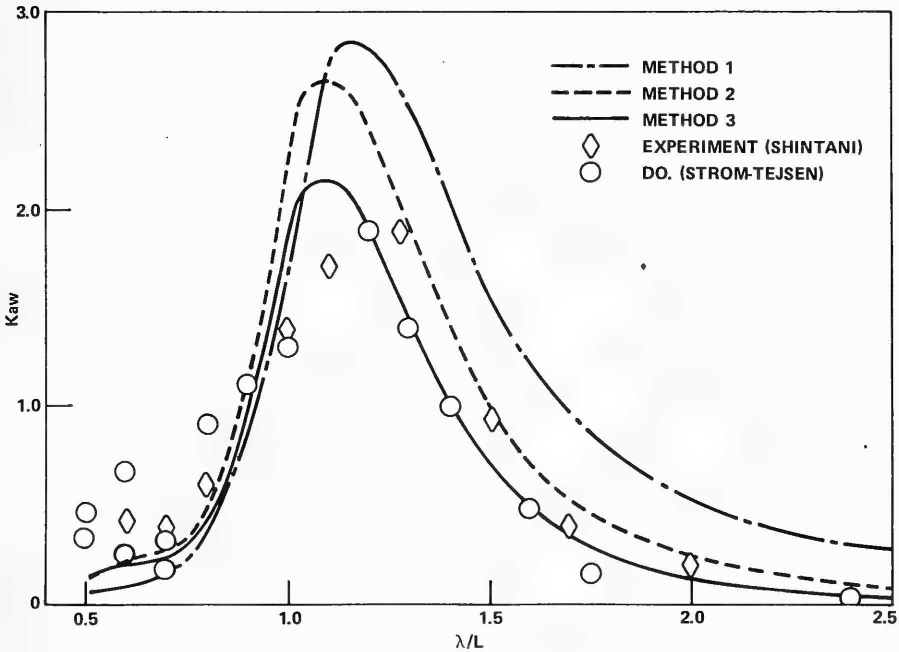

Figure 13 - Added Resistance Coefficient of Series 60 Model for $C_b = 0.7$ and $f_n = 0.2$

added resistance in waves can show much better agreement. Furthermore, what is really surprising is that the most rational formulation brings sometimes most unrealistic results, while much simpler approximations can often result in much better predictions. This is the attribute of engineering and the fun of engineering. Mathematics has nothing to do with this respect.

Concluding this lecture, I wish to express my sincere thanks to Mr. Justin McCarthy for his kind arrangements which enabled me to have this opportunity, and to Dr. Kwang June Bai and Dr. Ming Shun Chang for taking care of me every day. My thanks also to Dr. Allen Powell, Dr. William Morgan, Dr. Wen-Chin Lin, Dr. Bohyun Yim, Dr. Choung M. Lee, Dr. Thomas Huang and all friends at DTNSRDC for their warm hospitality.

I should be happy if this lecture series can contribute to everlasting friendship between Japan and the United States.

APPENDIX A - OSCILLATION OF A SHIP WITH SIX DEGREES OF FREEDOM

Consider a ship floating on a uniform stream and making oscillations of small amplitude about a fixed point. Take the x-axis in the direction of the uniform stream and the z-axis vertically upwards. The fluid motion around the ship is expressed by the velocity potential of the form

$$\phi = Ux + U\phi_0 + \phi_1 \tag{301}$$

where U is the velocity of the uniform flow and ϕ_1 gives the periodical disturbance. Since the amplitude of oscillation is small, the steady potential, $U\phi_0$, is determined from the boundary condition when the hull is fixed at an average position and identical with the velocity potential for the uniform motion of the ship. Now we consider the coordinate system x_0, y_0, z_0, which is fixed to the ship with the origin at the center of gravity. The coordinates of the center of gravity at the average position are written as x_G, y_G, z_G, and the displacement of the center of gravity as ξ_1, ξ_2, ξ_3. Designating the angle of rotation about each axis by θ_1, θ_2, θ_3, and omitting higher order terms, one can write

$$\left.\begin{aligned} x_0 &= x - x_G - \xi_1 + \theta_3(y-y_G) - \theta_2(z-z_G) \\ y_0 &= y - y_G - \xi_2 + \theta_1(z-z_G) - \theta_3(x-x_G) \\ z_0 &= z - z_G - \xi_3 + \theta_2(x-x_G) - \theta_1(y-y_G) \end{aligned}\right\} \tag{302}$$

If the hull surface is expressed by the equation

$$F(x_0,y_0,z_0) = 0 \tag{303}$$

the boundary condition on the hull surface is given by the equation

$$\left(\frac{\partial}{\partial t} + \phi_x \frac{\partial}{\partial x} + \phi_y \frac{\partial}{\partial y} + \phi_z \frac{\partial}{\partial z}\right) F(x_0,y_0,z_0)$$

$$= - \{\dot{\xi}_1-\dot{\theta}_3(y-y_G)+\dot{\theta}_2(z-z_G)\} F_{x_0} - \{\dot{\xi}_2-\dot{\theta}_1(z-z_G)+\dot{\theta}_3(x-x_G)\} F_{y_0}$$

$$- \{\dot{\xi}_3-\dot{\theta}_2(x-x_G)+\dot{\theta}_1(y-y_G)\} F_{z_0} + \phi_x(F_{x_0}-\theta_3F_{y_0}+\theta_2F_{z_0})$$

$$+ \phi_y(F_{y_0}+\theta_3F_{x_0}-\theta_1F_{z_0}) + \phi_z(F_{z_0}-\theta_2F_{x_0}+\theta_1F_{y_0}) = 0 \tag{304}$$

where the dot means the differentiation with respect to t. If we write the direction cosines of the outward normal as

$$n_1 = - F_{x_0}/\Delta, \quad n_2 = - F_{y_0}/\Delta, \quad n_3 = - F_{z_0}/\Delta \tag{305}$$

where

$$\Delta = \sqrt{F_{x_0}^{\ 2} + F_{y_0}^{\ 2} + F_{z_0}^{\ 2}}$$

the boundary condition on the hull surface becomes

$$\{\dot{\xi}_1-\dot{\theta}_3(y-y_G)+\dot{\theta}_2(z-z_G)\} n_1 + \{\dot{\xi}_2-\dot{\theta}_1(z-z_G)+\dot{\theta}_3(x-x_G)\} n_2$$

$$+ \{\dot{\xi}_3-\dot{\theta}_2(x-x_G)+\dot{\theta}_1(y-y_G)\} n_3 - \phi_x(n_1-\theta_3n_2+\theta_2n_3)$$

$$- \phi_y(n_2+\theta_3n_1-\theta_1n_3) - \phi_z(n_3-\theta_2n_1+\theta_1n_2) = 0 \tag{306}$$

Now we write the steady portion of the fluid velocity as

$$u = U + U\partial\phi_0/\partial x, \quad v = U\partial\phi_0/\partial y, \quad w = U\partial\phi_0/\partial z \tag{307}$$

and define vectors $\underline{V}_0 = (U,0,0)$ and $\underline{V} = (u,v,w)$. If we employ vector notations

$$\underline{\xi} = (\xi_1,\xi_2,\xi_3), \quad \underline{\theta} = (\theta_1,\theta_2,\theta_3)$$
$$\underline{r} = (x-x_G, y-y_G z-z_G) \tag{308}$$

the fluid velocity at the moving hull surface is expressed by

$$\underline{V} = \underline{V}_0 + [(\underline{\xi}+\underline{\theta}\times\underline{r})\nabla]\ \underline{V} \tag{309}$$

Substituting Equation (301) for ϕ_x, ϕ_y, ϕ_z in Equation (306) and making use of Equation (309) together with the boundary condition for ϕ_0 given by

$$n_1\phi_{0x} + n_2\phi_{0y} + n_3\phi_{0z} + n_1 = 0 \tag{310}$$

and the irrotationality condition

$$u_y = v_x, \quad v_z = w_y, \quad w_x = u_z \tag{311}$$

we obtain eventually

$$\begin{aligned}
&\dot{\xi}_1 n_1 + \dot{\xi}_2 n_2 + \dot{\xi}_3\ n_3 + \dot{\theta}_1\ \{(y-y_G)n_3-(z-z_G)n_2\} \\
&+ \dot{\theta}_2\ \{(z-z_G)n_1-(x-x_G)n_3\} + \dot{\theta}_3\ \{(x-x_G)n_2-(y-y_G)n_1\} \\
&- \xi_1\ \partial u/\partial n - \xi_2\ \partial v/\partial n - \xi_3\ \partial w/\partial n \\
&- \theta_1 \frac{\partial}{\partial n}\{(y-y_G)w-(z-z_G)v\} - \theta_2 \frac{\partial}{\partial n}\{(z-z_G)u-(x-x_G)w\} \\
&- \theta_3 \frac{\partial}{\partial n}\{(x-x_G)v-(y-y_G)u\} = \frac{\partial\phi_1}{\partial n}
\end{aligned} \tag{312}$$

This is the boundary condition for the periodical potential ϕ_1. Then the periodical potential is constituted by 12 components such as

$$\begin{aligned}
\phi_1 = &\ \dot{\xi}_1\varphi_1 + \dot{\xi}_2\varphi_2 + \dot{\xi}_3\varphi_3 + \dot{\theta}_1\varphi_4 + \dot{\theta}_2\varphi_5 + \dot{\theta}_3\varphi_6 \\
&+ \xi_1\psi_1 + \xi_2\psi_2 + \xi_3\psi_3 + \theta_1\psi_4 + \theta_2\psi_5 + \theta_3\psi_6
\end{aligned} \tag{313}$$

Boundary conditions which are satisfied by each component on the hull surface are

$$\left.\begin{aligned}
&\partial\varphi_1/\partial n = n_1, \quad \partial\varphi_2/\partial n = n_2, \quad \partial\varphi_3/\partial n = n_3 \\
&\partial\varphi_4/\partial n = (y-y_G)\, n_3 - (z-z_G)\, n_2 \equiv n_4 \\
&\partial\varphi_5/\partial n = (z-z_G)\, n_1 - (x-x_G)\, n_3 \equiv n_5 \\
&\partial\varphi_6/\partial n = (x-x_G)\, n_2 - (y-y_G)\, n_1 \equiv n_6 \\
&\partial\psi_1/\partial n = -\, \partial u/\partial n \equiv m_1 \\
&\partial\psi_2/\partial n = -\, \partial v/\partial n \equiv m_2 \\
&\partial\psi_3/\partial n = -\, \partial w/\partial n \equiv m_3 \\
&\partial\psi_4/\partial n = -\frac{\partial}{\partial n}\{(y-y_G)w-(z-z_G)v\} \equiv m_4 \\
&\partial\psi_5/\partial n = -\frac{\partial}{\partial n}\{(z-z_G)u-(x-x_G)w\} \equiv m_5 \\
&\partial\psi_6/\partial n = -\frac{\partial}{\partial n}\{(x-x_G)v-(y-y_G)u\} \equiv m_6
\end{aligned}\right\} \tag{314}$$

Forces and moments acting on the hull are expressed by the integral

$$F_i = -\iint_S n_i\, p\, dS \qquad \begin{array}{ll} n = 1,\ 2,\ 3 & \text{for force} \\ n = 4,\ 5,\ 6 & \text{for moment} \end{array} \tag{315}$$

Omitting terms of the second order with respect to the oscillation amplitude, the fluid pressure is given by

$$\frac{p - p_s}{\rho} = U^2\,\frac{\partial\phi_0}{\partial x} + \frac{U^2}{2}\,|\nabla\phi_0|^2 + \frac{\partial\phi_1}{\partial t} + u\,\frac{\partial\phi_1}{\partial x} + v\,\frac{\partial\phi_1}{\partial y} + w\,\frac{\partial\phi_1}{\partial z} \tag{316}$$

where p_s is the hydrostatic pressure. We divide the above pressure into a part due to $U\phi_0$ and that due to ϕ_1 such as

$$\left.\begin{aligned} p_0 &= -\rho U^2\left(\frac{\partial\phi_0}{\partial x}+\frac{1}{2}\,|\nabla\phi_0|^2\right) \\ p_1 &= -\rho\left(\frac{\partial\phi_1}{\partial t}+u\,\frac{\partial\phi_1}{\partial x}+v\,\frac{\partial\phi_1}{\partial y}+w\,\frac{\partial\phi_1}{\partial z}\right) \end{aligned}\right\} \tag{317}$$

On integrating the pressure over the moving hull surface, a periodical term comes from the integration of p_0 because of the periodical variation of the hull surface. We regard this as a correction to the restoring forces due to the forward motion. Forces and moments due to the periodical potential are given by

$$F_i = \rho \iint_{S_0} n_i(\dot{\phi}_1 + \underline{V}\cdot\nabla\phi_1)\ dS \tag{318}$$

where S_0 is the hull surface at the average position and

$$\dot{\phi}_1 = \partial\phi_1/\partial t$$

APPENDIX B - THEOREM OF TUCK AND RELATIONS DERIVED THEREFROM

Tuck has proved a theorem with respect to the integral in Equation (318) as follows

Tuck's Theorem

If ϕ is a harmonic function defined in the lower half space outside the surface S_0 which has a vertical tangent at $z = 0$, and $\underline{V} = (u,v,w)$ is the velocity of an irrotational motion, which satisfies the boundary condition

$$un_1 + vn_2 + wn_3 = 0 \tag{319}$$

on S_0, the following relation is valid.

$$\iint_{S_0} [m_i\phi + n_i(\underline{V}\cdot\nabla\phi)]\ dS = -\int_{L_0} n_i\phi w ds \tag{320}$$

where m_i is defined in Equation (314) and L_0 is the intersection of S_0 with the plane z = 0. This theorem can be proved by means of Stokes' theorem as Tuck did, but here let us derive the same result by integration by parts.

In the first place, we put i = 1 and consider an integral

$$I_1 = \iint_{S_0} (u\phi_x + v\phi_y + w\phi_z)\ n_1 dS$$

$$= \iint \mathrm{sgn}x (u\phi_x + v\phi_y + w\phi_z)\ dydz$$

If we write the equation of S_0 as

$$x = f(y,z)$$

we have

$$\frac{\partial}{\partial y} (\phi)_{x=f(y,z)} = \phi_y + \phi_x \frac{\partial x}{\partial y}$$

Integrating by parts

$$\int dz \int v\phi_y dy = \int dz \int \left[v \frac{\partial (\phi)_{x=f}}{\partial y} - v\phi_x \frac{\partial x}{\partial y} \right] dy$$

$$= - \int dz \int \left(v_y + v_x \frac{\partial x}{\partial y} \right) \phi dy - \int dz \int v\phi_x \frac{\partial x}{\partial y} dy$$

$$= \iint (v_y \phi n_1 - v_x \phi n_2 - v\phi_x n_2)\ dS$$

By a similar way

$$\int dy \int w\phi_z dz = \int dy \int \left[w \frac{\partial (\phi)_{x=f}}{\partial z} - w\phi_x \frac{\partial x}{\partial z} \right] dz$$

$$= \int dy \cdot w\phi_{z=0} - \int dy \int \left(w_z + w_x \frac{\partial x}{\partial z} \right) \phi dz - \int dy \int w\phi_x \frac{\partial x}{\partial z} dz$$

$$= - \int_{L_0} n_1 \phi w ds - \iint (w_z \phi n_1 - w_x \phi n_3 - w\phi_x n_3)\ dS$$

Adding the above equations and making use of relations of continuity and irrotationality,

$$v_y + w_z = - u_x, \quad v_x = u_y, \quad w_x = u_z$$

together with Equation (319), we obtain

$$I_1 = \iint (u_x n_1 + u_y n_2 + u_z n_3) \ \phi dS - \int_{L_0} n_1 \phi w ds$$

$$= \iint \frac{\partial u}{\partial n} \ \phi dS - \int_{L_0} n_1 \phi w ds$$

$$= - \iint m_1 \phi dS - \int_{L_0} n_1 \phi w ds$$

A similar relation is obtained when i = 2,

$$I_2 = - \iint m_2 \phi dS - \int_{L_0} n_2 \phi w ds$$

In the case of i = 3, we write

$$I_3 = - \iint (u\phi_x + v\phi_y + w\phi_z) dxdy$$

We get similarly, as before

$$- \iint u\phi_x dxdy = - \int dy \int \left(u \ \frac{\partial \phi}{\partial x} - u\phi_z \ \frac{\partial z}{\partial x} \right) dx$$

$$= \iint \left(u_x + u_z \ \frac{\partial z}{\partial x} \right) \phi dxdy + \iint u\phi_z dydz$$

$$= - \iint (u_x n_3 - u_z n_1) \ \phi dS + \iint u\phi_z n_1 dS$$

$$- \iint v\phi_y dxdy = - \int dx \int \left(v \ \frac{\partial \phi}{\partial y} - v\phi_z \ \frac{\partial z}{\partial y} \right) dy$$

$$= \iint \left(v_y + v_z \ \frac{\partial z}{\partial y} \right) \phi dxdy + \iint v\phi_z dxdz$$

$$= - \iint (v_y n_3 - v_z n_2) \ \phi dS + \iint v\phi_z n_2 dS$$

Summing up the above equations, we obtain

$$I_3 = \iint (w_z n_3 + u_z n_1 + v_z n_2)\ \phi dS$$
$$= \iint \frac{\partial w}{\partial n}\ \phi dS$$
$$= -\iint m_3 \phi dS$$

Then the relation of Equation (320) follows when i = 1, 2, 3. It can be shown that a similar derivation is applied to the cases of i = 4, 5, 6, if we introduce the definition of n_4, n_5, n_6 in Equation (314).

Reverse Flow Theorem and Haskind's Relation

Applying the above theorem to Equation (318) and making use of the relations

$$\frac{\partial \Phi_i}{\partial n} = n_i, \quad \frac{\partial \psi_i}{\partial n} = m_i$$

one can express the forces and moments in the form like

$$F_{ij} = e^{i\omega t}\ \rho\ \xi_i \iint_{S_0} (i\omega\Phi_i + \psi_i)\ \frac{\partial}{\partial n}\ (i\omega\Phi_j - \psi_j)\ dS \tag{321}$$

where the simple harmonic displacement is expressed by $e^{i\omega t}\ \xi_i$. The line integral term vanishes because we have assumed that w = 0 on the plane z = 0. Now we assume that Φ_i and ψ_i satisfy the linearized free surface condition such as

$$\left.\begin{aligned} -\omega^2\Phi_i + 2i\omega U \frac{\partial \Phi_i}{\partial x} + U^2 \frac{\partial^2 \Phi_i}{\partial x^2} + g\frac{\partial \Phi_i}{\partial z} = 0 \\ -\omega^2\psi_i + 2i\omega U \frac{\partial \psi_i}{\partial x} + U^2 \frac{\partial^2 \psi_i}{\partial x^2} + g\frac{\partial \Phi_i}{\partial z} = 0 \end{aligned}\right\} \tag{322}$$

and the radiation condition. Next we define velocity potentials Φ_i^* and ψ_i^* which satisfy the same boundary condition on the hull surface as those of Φ_i and ψ_i when the uniform flow has an opposite direction. Then there are relations

$$\frac{\partial \Phi_i}{\partial n} = \frac{\partial \Phi_i^*}{\partial n}, \quad \frac{\partial \psi_i}{\partial n} = -\frac{\partial \psi_i^*}{\partial n} \tag{323}$$

while the free surface conditions are

$$\left.\begin{aligned} -\omega^2 \Phi_i^* - 2i\omega U \frac{\partial \Phi_i^*}{\partial x} + U^2 \frac{\partial^2 \Phi_i^*}{\partial x^2} + g \frac{\partial \Phi_i^*}{\partial z} = 0 \\ -\omega^2 \psi_i^* - 2i\omega U \frac{\partial \psi_i^*}{\partial x} + U^2 \frac{\partial^2 \psi_i^*}{\partial x^2} + g \frac{\partial \psi_i^*}{\partial z} = 0 \end{aligned}\right\} \tag{324}$$

If we put, for simplicity,

$$i\omega\Phi_i + \psi_i = \Phi_i, \quad i\omega\Phi_i^* + \psi_i^* = \Phi_i^* \tag{325}$$

the expression for forces and moments takes the form

$$F_{ij} = e^{i\omega t} \rho \, \xi_i \iint_{S_0} \Phi_i \frac{\partial \Phi_j^*}{\partial n} \, dS \tag{326}$$

We apply Green's theorem in the space bounded by S, Σ_0, Σ, and Σ_B such as illustrated in Figure 3.

$$\iint_{S_0+\Sigma_0+\Sigma+\Sigma_B} \left(\Phi_i \frac{\partial \Phi_j^*}{\partial n} - \frac{\partial \Phi_i}{\partial n} \Phi_j^* \right) dS = 0$$

If we consider the case of infinite depth, the integral on Σ_B vanishes, while the integral on Σ vanishes as the radius of the cylinder tends to infinity. The integral on the horizontal plane is

$$-\iint_{\Sigma_0} \left(\Phi_i \frac{\partial \Phi_j^*}{\partial z} - \frac{\partial \Phi_i}{\partial z} \Phi_j^* \right) dxdy$$

Making use of relations from Equations (322) and (324) such as

$$\frac{\partial \Phi_i}{\partial z} = \frac{1}{g}\left(\omega^2 \Phi_i - 2i\omega U \frac{\partial \Phi_i}{\partial x} - U^2 \frac{\partial^2 \Phi_i}{\partial x^2}\right)$$

$$\frac{\partial \Phi_j^*}{\partial z} = \frac{1}{g}\left(\omega^2 \Phi_j^* + 2i\omega U \frac{\partial \Phi_j^*}{\partial x} - U^2 \frac{\partial^2 \Phi_j^*}{\partial x^2}\right)$$

and integrating by parts with respect to x, we obtain

$$-\iint_{\Sigma_0} \left(\Phi_i \frac{\partial \Phi_j^*}{\partial z} - \frac{\partial \Phi_i}{\partial z} \Phi_j^*\right) dxdy$$

$$= \int_{L_0} \left\{ \frac{U^2}{g}\left(\frac{\partial \Phi_i}{\partial x} \Phi_j^* - \Phi_i \frac{\partial \Phi_j^*}{\partial x}\right) + \frac{2i\omega U}{g} \Phi_i \Phi_j^* \right\} dy$$

Therefore, we have

$$F_{ij} = e^{i\omega t} \rho\, \xi_i \iint_{S_0} \Phi_i \frac{\partial \Phi_j^*}{\partial n} dS$$

$$= e^{i\omega t} \rho\, \xi_i \left[\iint_{S_0} \Phi_j^* \frac{\partial \Phi_i}{\partial n} dS - \int_{L_0} \left\{\frac{U^2}{g}\left(\frac{\partial \Phi_i}{\partial x} \Phi_j^* - \Phi_i \frac{\partial \Phi_j^*}{\partial z}\right) + \frac{2i\omega U}{g} \Phi_i \Phi_j^* \right\} dy\right] \tag{327}$$

Now we put

$$F_{ij} = e^{i\omega t} \rho\, \xi_i T_{ij} \tag{328}$$

and write the force or moment when the uniform flow has an opposite direction as

$$F_{ij}^* = e^{i\omega t} \rho\, \xi_i\, T_{ij} \tag{329}$$

Then the following relation is valid.

$$T_{ij} = T_{ji}^* + L_{ij} \tag{330}$$

where L_{ij} is the line integral in Equation (325). If the waterline of the ship is slender, L_{ij} becomes small. By neglecting L_{ij} we have the reverse flow theorem

$$T_{ij} = T_{ji}^{*} \tag{331}$$

This reciprocal relation was first shown by Hanaoka in the case of a thin ship. Timman and Newman gave a proof to the general validity for non-slender ships, with a somewhat intuitive hypothesis.

When a train of regular waves is superimposed on the uniform flow, the periodical potential is composed of the incident wave potential ϕ_w and the diffraction potential ϕ_D if the ship is fixed in the stream. The wave excitation forces and moments are given by Equation (318) if ϕ_i is substituted by $\phi_w + \phi_D$, or

$$F_{7j} = \rho \iint (\phi_w + \phi_D) \frac{\partial \Phi_j^{*}}{\partial n} \, dS \tag{332}$$

If we apply Green's theorem as before and make use of the boundary condition

$$\frac{\partial \phi_w}{\partial n} + \frac{\partial \phi_D}{\partial n} = 0 \tag{333}$$

on S_0, we can write

$$F_{7j} = - \rho \iint_{\Sigma} \left(\phi_w \frac{\partial \Phi_j^{*}}{\partial n} - \frac{\partial \phi_w}{\partial n} \Phi_j^{*} \right) dS + L_{7j} \tag{334}$$

If the line integral is omitted, the result is similar to the relation which was given by Haskind in the case of zero forward speed. The integral on the large cylindrical surface Σ is related to the Kochin function for the radiation problem. Therefore, the exciting forces and moments are derived from the solution of the radiation problem in still water. It should be noted that the above formula does not give a consistent expression for the wave excitation if the finite forward speed is present as mentioned in the section on wave pressure on slender ships.

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