ON

LAYING OUT CURVES

AN ESSAY WRITTEN IN 1868

BY

"FIELDAT"

JOSE ANTONIO DE LAVALLE Y ROMERO, [illegible] PREMIO-REAL, M.A., C.E.,
ETC.

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EXPLANATIONS.

1. *Equivalent curves* are such as subtend equal central angles.

2. These angles at the centre form what is called the *degree of curvature*, or solely the *curvature*.

3. *Corresponding points in different curves* are any points where the tangents, and of course the radii as perpendicular to said tangents are parallel.

4. *A compound curve* is constructed by two curves of different radii, said two curves turning in the same direction and having a common tangent at the point of meeting.

5. This point of meeting is called the *point of compound curvature.*

6. A *reversed curve* is composed of two curves turning in opposite directions and having a common tangent at their point of meeting. The radii may be the same or different.

7. Such point is named the *point of reversed curvature.*

8. A *differential curve* is one whose radius is equal to the difference between the radii of any two curves to which it is applied.

9. An *integral curve* is that whose radius equals the sum of the radii of two other curves.

REMARKS.

1. As all the curves described in this work ar circular the words *curve* and *circle* will be us synonymously.

2. All measurements are referred to the engineer chain of 100 feet, and so the vocables *chain* an *chord* mean here both the same.

3. The point where a curve commences is the termination of a tangent, and the point in whic the curve ends is the origin of a next tangent Therefore the terms *origin* (or point of curve, o point of tangency) and *termination* are applie in reference to the course of location.

4. When simply as often said "*set up and adjus the transit*" it is in the supposition that the reade fully understands such instrument and is able t detect and remedy the accidents and errors whicl in practice so frequently occur.

PROPOSITIONS.

The following are demonstrated in most works on geometry, so that it is only necessary to refer to them here.

Fig. I.

PROPOSITION I.

Two tangents (J A & J B) *drawn to a circle from any given point* (I) *are equal, and a chord* (A B) *joining the points of tangency* (A & B) *forms equal angles* (I A C & I B C) *with said tangents.*

PROPOSITION II. Fig. I.

A tangential angle (J A D), or the smaller of the two angles (J A D & G A D) formed between any tangent (G K) and a chord (A D) drawn from the point of tangency (A), *is measured by half of the intercepted arc* ($\frac{1}{2}$ A D=A M or M D) *subtended by said chord; such tangential angle being thus equal to half of the central one* ($\frac{1}{2}$ A C D = A C M or M C D) *which the whole arc* (A D) *measures.*

PROPOSITION III. Fig. I.

The circumferential angle (D A E), having its vertex (A) in the circumference of a circle, subtended by a chord (D E) and measured by half the corresponding arc ($\frac{1}{2}$ D E), *is equal to half of the central angle* ($\frac{1}{2}$ D C E), *and therefore* (see preceding proposition) *to the tangential one* (I A D).

PROPOSITION IV. Fig. I.

Equal chords (A D, D E, E F & F B) *subtend equal angles at the centre* (A C D, D C E, E C F & F C B) *and also at the circumference* (A E D, D A E, E A F & F A B).

PROPOSITION V. Fig. I.

The exterior or *deflection angle* (L D E) formed between any chord (D E) and the extension (D L) of the preceding one, *is double the tangential angle*

(J A D) *and equal to that of the curvature or at th* *centre* (A C D).

Fig. I.

PROPOSITION VI.

An angle (A J B) *figured by the meeting of tw tangents* (J A & J B) *is equal to the supplement o the central angle* (sup. A C B) *subtended by th chord which joins the points of tangency* (A & B *and said supplement equals the other angle* (B J K *not very properly described as that of intersection of the tangents.*

PRELIMINARY PROBLEMS.

Fig. I.

PROBLEM I.

To find the distance between two points A and B.

When they are accessible and visible from each other, measure it with the engineer's chain ; but if they cannot be seen, take several bearings and distances, and resolve them as a traverse in land surveying.

PROBLEM II.

To find the angle A I B formed by the meeting of the two tangents I A and I B when the point I is inaccessible.

If the included figure has only three sides, add together the supplements of the angles G A B and H B A, and subtract their sum from 180°, the remainder will be the angle A I B. Fig. II.

If the included figure has four sides, subtract the sum of the three interior angles A, C & B marked by dotted sectors of circles from 360°, and the difference will be the angle A I B. Figs. III & IV

If the included figure has more than four sides, add together all the interior angles A, C, D, E, F and B, marked by dotted sectors of circles, and subtract their sum from twice as many right angles as the figure has sides less four, and the remainder will be the angle A I B. Fig. V.

PROBLEM III. Fig. I.

To unite two straight lines of road, as G A and H B, by a curve.

The angle of meeting of the tangent is measured, and then a radius for the curve may be assumed and the length of the tangents calculated ; or the tangents may be assumed of a practical length and the radius calculated.

Fig. I.

PROBLEM IV.

To find the radius C A of a curve, given the an A I B at the meeting of the tangents I A a I B (= 120°), *and the tangent I A* (= 16 *feet*)

C A : I A : : r : n. t. < A C I (= 30°)

C A : 1654 : : 1 : .57735

C A + .57735 = 1654 + 1

$$C\ A = \frac{1644}{.57735} = 2865$$ feet, the radius of the curv

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Fig. I.

PROBLEM V.

To find the tangent J A, given the ang J B to the meeting of the tangents J A and J B (= 120°), *and the radius of the curve* (= 286 *feet.*)

C A : J A : : r : n. t. < A C J (= 30°)

2865 : J A : : 1 : .57735

J A × 1 = 2865 + .57735

J A = 1654 feet

Fig. I.

PROBLEM VI.

To find the angle A J B at the meeting of th tangents J A and J B, given the radius C A o *e curve* (= 2865 *feet*), *and the tangent J A* (= 1654 *feet*).

$$C A : J A :: r : n.t. \angle A C J$$

$$2865 : 1654 :: 1 \; n.t. \angle A C J$$

$$2865 \times n.t. \angle A C J = 1654 \times 1$$

$$n.t. \angle A C J = \frac{1654}{2865} = .57835$$

$\angle$ A C J = 30° (by table of natural tangents.)

$\angle$ A C B (double of $\angle$ A C J) = 60°

$\angle$ A J B (supplement of $\angle$ A C B) = 120°

PROBLEM VII.

To find the tangential angle J A D with which to commence a curve at either of the points of tangency A or B, given the radius C A of the curve (= 2865 *feet*). (Fig. I.)

Bisect the chord A D by the line C M, and in the right angled triangle C A M we have.

$$C A : A M :: r : n.s. \angle A C M \; (= \angle J A D)$$

$$2865 : 50 :: 1 : n.s. \angle A C M$$

$$2865 \times n.s. \angle A C M = 50 \times 1$$

$$n.s. \angle A C M = \frac{50}{2865} = .01745$$

$\angle$ A C M = 1° (by table of natural sines)

$\angle$ J A D = 1°.

PROBLEM VIII.

(Fig 1.) *To find the number of chords to construct a cur given the radius C A of the curve (=2865 feet) a the angle A I B at the meeting of the tangents A and I B (=120°).*

Once known, by problem VII, the tangenti angle J A D, twice the same must be divided in the supplement of A J B (which is the angle A B) and the quotient will be the number of chorc to construct the curve.

$$180° - \angle A I B = 180° - 120° = 50° = \angle A C B$$

$$\frac{60°}{2°} = \frac{\angle A C B}{2 \times \angle I A D} = 30 \text{ chords.}$$

PROBLEM IX.

To resolve the preceding problem when the divisio of twice the tangential angle I A D into A C B does not go evenly.

(Fig. 1.) For completing or finishing the curve, in this case, a subchord will be required, and mind that the subtangential angle is to the tangential one as the subchord is to a whole chord.

Given the radius C A=2865 feet, and supposing $\angle A I B = 119°$, then $\angle I A D = 1°$ (by problem VII), and the number of chords must be, per probm VIII, 30½ ; that is, 30 chords and 1 subchord f 50 feet.

PROBLEM X.

To find the greatest radius that can be used to construct solely with it the reversed or serpentine curve A Ba, uniting G A and ga, and given the angles A I B (=120°) and aiB (=119°) and the distance Ii (=3342 feet). (Fig. VI.)

$180° - \angle A I B = 180° - 120° = 60° = \angle A C B$

$\angle B C I (= \frac{1}{2} A C B) = 30°$

$180° - \angle aiB = 180° - 119° = 61° = \angle acB$

$\angle Bci (= \frac{1}{2} acB) = 30° 30'$

n. t. $\angle$ B C I (= 30°) =.57735

n. t. $\angle$ B C I (= 30° 30') =.58905

Sum of the n. t. t. . . . 1.16640

Now

$1.1664 : .57735 :: 3342 : I B$

$$I B = \frac{.57735 + 3342}{1.1664} = 1654$$

$I i - I B = 3342 - 1654 = 1688 = i B.$

In the triangle J B C

$C B : J B :: r : \text{n. t.} \angle B C I (= 30°)$

$C B : 1654 :: 1 : .57735$

$$C B = \frac{1654 + 1}{.57735} = 2865.$$

Again in the triangle i B c.

c B : i B : : r : n. t. ∠B c i (=30° 30')

c B : 1688 : : 1 : .58905

$$c\ B = \frac{1688 \times 1}{.58901} = 2865 \text{ (same as above)}$$

PROBLEM XI.

(Fig. VII.) *To find two different radii C B and c B to be used in the construction of a reversed curve A B uniting the lines G A and a g; given the A I angles B (= 120°) and ai B (= 130°) and distance Ii (= 3342 feet.)*

180°–∠A I B=180°–120°=60°=∠A C B

B C I (=½ A C B)=30°

180°–∠a i B=180°–130°=50°=∠a c B

B c i (=½ a c B)=25°

n. t. ∠B C I (=30°)=. 57735

n. t. ∠B c i (=25°)=. 46631

Sum of the n. t. t.....1.04366

Now

1.04366 : . 57735 : : 3342 : I B

$$I\ B = \frac{57735 \times 3342}{1.04366} = 1850$$

I i – I B=3342–1850=1492=i B.

In the triangle I B C

$C B : I B :: r : n. t. \angle B C I \; (=30°)$

$C B : 1850 :: 1 : .57735$

$$C B = \frac{1850 \times 1}{.57735} = 3204$$

Again, in the triangle i B c

$c B : i B :: r : n. t. \angle B c i \; (=25°)$

$\therefore c B : 1492 :: 1 : .46631$

$$c B = \frac{1492 \times 1}{.46631} = 3221$$

PROBLEM XII.

Between Parallel Roads (same radius). *Given the* (Fig. VIII.) *perpendicular distance ad = 955 feet, between two parallel roads G A and ga, and the distance between the points of tangency A and a = 3308 feet, to find the common radius of the reversed curve.*

Rule.—Divide four times the perpendicular distance *ad* into the square of the distance between the points of tangency *Aa*, and the quotient will be the common radius of the reversed curve.

$$r = \frac{(A\,a)\,2}{4\,ad} = \frac{(3308)\,2}{4 \times 955} = \frac{10942864}{3820} \quad 2865 \text{ feet.}$$

PROBLEM XIII.

(Fig. IX.) BETWEEN PARALLEL ROADS (DIFFERENT RAI
Given the perpendicular distance ad = 615 *f*
between two parallel roads G A A and *ga, the dista*
between the points of tangency A and a =3308, *a*
the radius C A = 2865 feet. to find the other rad
ca.

In the similar triangles A a d and C A E we ha

Aa : ad : : C A : A E (= $\frac{1}{2}$ A B

3308 : 615 :: 2865 + 2 : A B,

$$A\ C = \frac{615 \times 2865 - 2}{3308} = 1066\ ;$$

A a—A B= 3308—1066= 2242=aB

$$\frac{aB}{2} = \frac{2242}{2} = 1121 = a\ B$$

In the similar triangles Aad and caF we have

Aa : ad :: ca : cF,

3308 : 615 :: c : e121.

$$ca = \frac{3308 + 1121}{615} = 6030.$$

ON ORDINATES.

PROBLEM I.

(Fig. X.)

Given the radius C A = 2865 feet, and the chord B as usual = 100, to find D E the middle dinate.

C A : A E : : r : n. s. ∠ A C E,

2865 : 50 ($= \frac{1}{2}$ A B) : : 1 : n. s. ∠ A C E.

$$\text{n. s.} \angle \text{ A C E} = \frac{50 \times 1}{2865} = .01745,$$

∠ A C E, therefore, by table of n. s. s. = 1°.

ow

r : n. cos. ∠ A C E : : C A : C E,

1 : .9998477 (= n. cos. 1°) : : 2865 : C E,

$$\text{C E} = \frac{.9998477 \times 2865}{1} = 2864.564;$$

E = C D—C E = C A—C E = 2865–2864.564 = .436.

PROBLEM II.

(Fig. X.)

Given the radius C F = 2865 feet, the middle inate D E = .436 of a foot, and the distance E = 25 feet, to find the ordinate F G.

(Fig. X.) In the triangle C F H we have

$$C F : F H :: r : n. s. \angle F C H,$$

$$2865 : 25 :: 1 : n. s. \angle F C H,$$

$$n. s. \angle F C H = \frac{25 \times 1}{2865} = .00873,$$

∠ F C H, therefore, by table of n. s. s. = 0°,, 3

Now

$$r : n. cos \angle F C H :: C F : C H,$$

$$1 : .9999619 \text{ (n. cos. 30')} :: 2865 : C H,$$

$$C H = \frac{9999619 \times 2865}{1} = 2864.891 ;$$

$$F G = H E = C H - C E = 2864.891 - 2864.564 = 327.$$

ON LAYING OUT CURVES.

PROBLEM I.

Fig. XI.

To lay out a curve, with the transit and chain, tangential angles and chords.

Let N A be the tangent and A the point of tan-ency. Set up and adjust the transit at A ; place e index at zero and take a backsight to N. Then verse the telescope and turn the index until it ows the tangential angle I A B. and measure the ne A B equal to one chain. Lay off the angles A C, C A D, D A E. E A F and F A G. and easure the lines B C, C D, D E, E F and F G ith the chain as before. B, C. D, E, F and G ill be points or stations in the curve, which may e constructed all the way round by repeating the bove process most carefully, and bearing also in ind that should there be a subchord, as G H, the btangential angle G A H must be the same part f the tangential angle I A B that the subchord G I is of the chord A B

PROBLEM II.

Fig. XI.

To complete the tangent, that is, after setting out *a number of stations, as B, C, D and E in curve, to find the tangent E J at the last statio*

Produce A E to K and I E to J ; then, in triangle A I E we have the angle I A E = I E K E J = the sum of all the tangential angles the preceding tangent N A. Now remains to lay off the angle K E J, and the tangent E found. But suppose the curve is afterwards tinued to H, the tangent H L will be found laying off the angle L H M = J H E = the sum all the tangential and subtangential angles the preceding tangent E J ; and should anyt prevent our seeing from A further than the sta E, the curve may be continued from E, in the s manner as it was commenced at A, by laying the tangential angles J E F, F E G, etc.

Fig. XII.

PROBLEM III.

To lay out a curve, with the transit and ch by deflection angles and chords.

Let N A be the tangent and A the point of gency. Set up and adjust the transit at A ; the index at zero, and take a backsight to N. reverse the telescope, and turn the index un shows the tangential angle I A B, and measure line A B equal to one chain. Remove afterw e transit to the station B and lay off the deflection angle O B C equal to twice the tangential angle A B, and measure the line B C equal to one chain. Do the same at the different stations C, D and E, making each exterior angle P C D and D E equal to twice the tangential angle I A B, and measure each of the chords C D and D E equal to one chain. The points B, C, D and E will be stations in the curve; but if after setting out a number of them it becomes expedient or necessary to find at the last station, E for instance, the tangent I, lay off simply the tangential angle R E J equal to half the deflection angle O B C, and the desired tangent E I is found. This being altogether the most usual method of setting out a curve on the ground.

PROBLEM IV.

Fig. XIII.

To find the tangential distance to lay out a curve with the chain and rod, or "by the eye" as it is called, given the tangential angle and the radius of said curve.

Let N A be the tangent and A the point of tangency; the tangential angle O A B $=1^{\circ}$ and the chord A B $=$ one chain. Draw B O perpendicular to N A produced in O. In the right angled triangle A B O we have

A B : B O : : r : n. s. $\angle$ O A B (=1°),
100 : B O : : 1 : .01745,

$$B\,O = \frac{100 + .01745}{1} = 1.745,$$ the tangen

distance, which is known to be the sine of tangential angle, but it is used as the chord w laying out a curve "by the eye." The deflec distance is taken double the tangential one, and s deflection distance is also to be used as a chord the above practical rule, that, for railroad pr tice and curves of more than 300 feet radius a chords of one chain, will be found to produce, any, a very trifling error.

To construct now the desired curve, from t point of tangency A and in line with N A, measu A O equal to one chain, and stick a pin at O. Al from A measure the chord A B equal to one chai at the same time measuring with the graduated r from the pin O the tangential distance O B equal 1.745 feet and place a stake at B. The pin at O now to be removed. Next make B P equal to o chain, and, in a line with B A, stick a pin at Also from B measure the chord B C equal to o chain, at the same time measuring with the rod fro the pin P the deflection distance P C equal twice the tangential distance O B, thus 1.745 1 = 3.490. In this manner proceed to find oth stations in the curve, viz D, E, T, etc ; and order to pass from the curve at F to a tangent

M, taking care only to measure the tangential istance T H instead of the deflection distance T G, roceed as before in this method which is consiered sufficiently accurate for curves on a canal or ommon road and will answer very well, if careally performed, for railroad curves in the absence f a transit instrument.

PROBLEM V.

Fig. XIV.

To lay out a curve commencing with a subchord.

Given, for instance, the tangential angle equal 1°, and the radius of the curve 2865 feet, to begin he curve with a subchord 50 feet at A, point of angency of N A.

Set up and adjust the transit at the point A, and place the index at zero, reversing then the telescope and laying off the subtangential angle O A B (= 0° 30,) which must be the same part of the whole tangential angle that the subchord is of a whole chord. Next lay off the whole tangential angles B A C, C A D, D A E, etc, and the chords B C, C D, D E, etc, each equal to one chain. But should the view be obstructed at A, the transit must be removed to B the end of the subchord A B, and sight back to A, and lay off the deflection angle P B C equal to the subtangential and a whole tangential angle (=1°, 30'), and make the chord B C equal one chain. The rest of the curve can be laid off by whole tangential angles from B, viz C B D, B E, etc., and whole chords viz C D, D E, etc. each equal one chain. It is true that mathematically the subchord does not bear the same proportion to the whole chord that the subtangential angle does to the whole tangential angle: the error though, arising from this supposition, is really so very trifling in large railroad curves with chords of one chain that it is neglected in curves of more than 300 feet radius.

Fig. XV.

PROBLEM VI.

To lay off the curves using long chords.

This is frequently convenient in preliminary locations, and goes thus: instead of finding the stations B, C, D, E, F, G, etc., with tangential angles and single chords of one chain each, the points C, E and G may be ascertained with a great deal less trouble by using twice the tangential angle, taking then the chords A C, A E and A G that are nearly twice as long as the single chord of one chain; but the table of long chords contains the exact length of those required to subtend respectively 2, 3, 4 or 5 stations, which latter limit it is not desirable to exceed.

PROBLEM VII.

Fig. XVI.

To lay out the curve when any obstruction prevents the use of the ordinary methods.

In such case the radius of said curve (O A) and the tangential angle (J C D) must be already found or given. Suppose O A = 2865 feet J C D = 1°, and let G A be the tangent and A the point of tangency where we wish to begin a curve. Set up and adjust the transit at A ; place the index at zero, and take a back sight to G. Then reverse the telescope and turn the index until it shows such a multiple of the tangential angle J C D as will enable us to clear the house which obstructs us at B. In the figure we have taken five times the tangential angle, viz 5°, therefore the sight will pass through C (the 5th station from A) which will be found by measuring the long chord A C taken from table for long chords ; and now from the station C we can lay off the tangential angles J C D, D C E, E C F, each equal 1°, and making the chords C D, D E, E F equal one chain, find the stations D, E, F in the curve.

PROBLEM VIII.

Fig. XVII

To lay out a curve by an auxiliary one when some obstruction, as in the preceding problem, exists.

For the same case mentioned we have this other method and it is performed by running a cu: parallel to the true curve, either inside or outs: of it, in the following way: 1st. upon the rad A O measure A H of any convenient length aliquot part of said radius is always the best) a from the point H proceed to lay off the seve stations I, J, K, L, M, with the tangential angle and the auxiliary chord H I which is to be c culated as follows: multiply the distance A H one chain (100 feet) and divide the product the radius of the curve, and the quotient subtract from one chain (100 feet) will give the length the auxiliary chord if the auxiliary arc is on t inside of the true curve, but to be added to o chain (100 feet)if the auxiliary arc is on the o side. The stations of the true curve, viz C, D, F, are easily found by laying off the distances C, L D, K E, J F, each equal to A H.

Quebec; Printed by G. T. Cary, 1877.

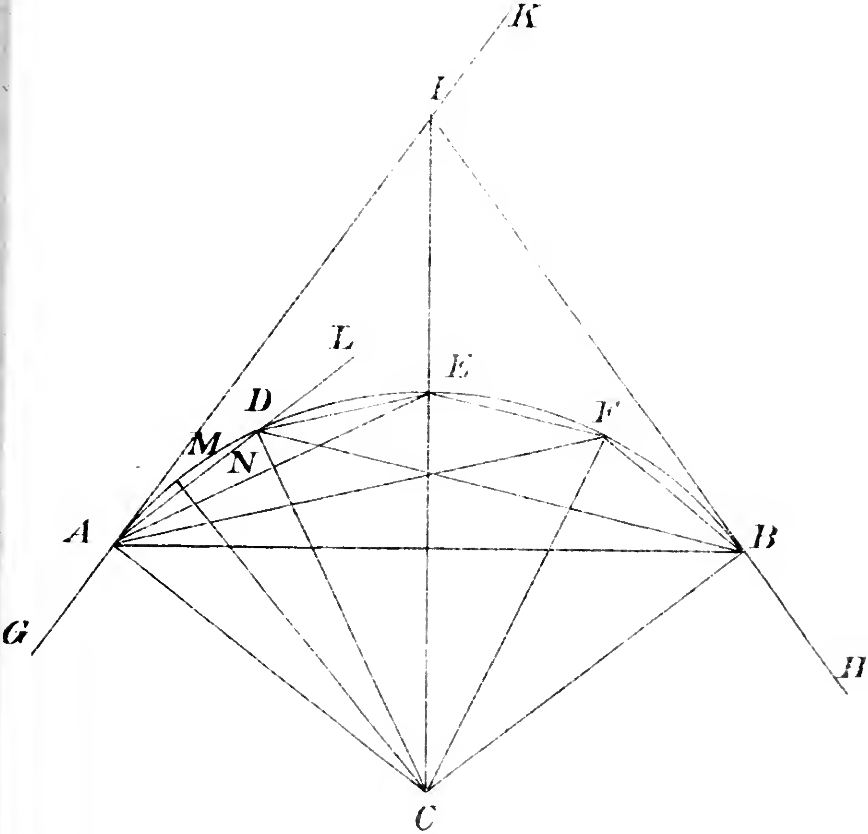

Fig. I.

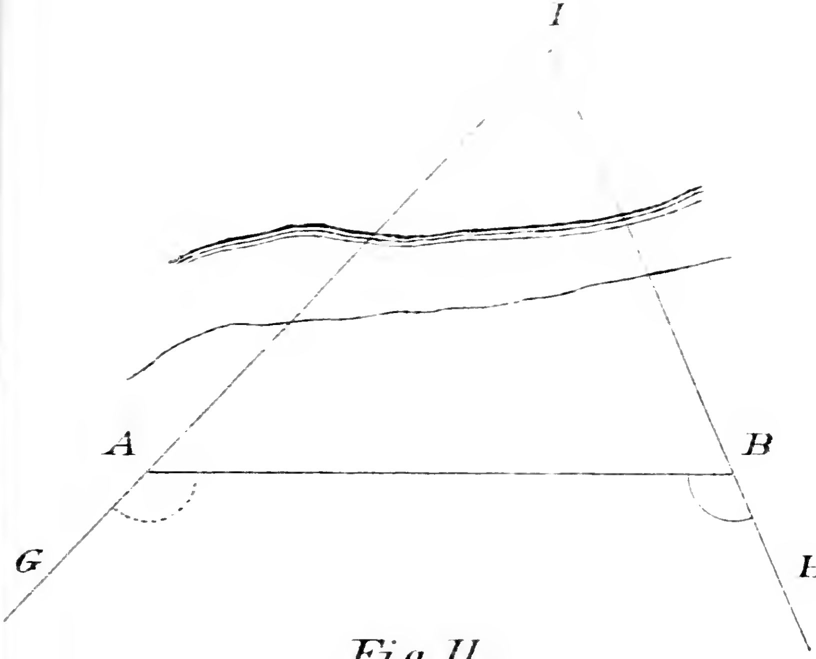

Fig. II.

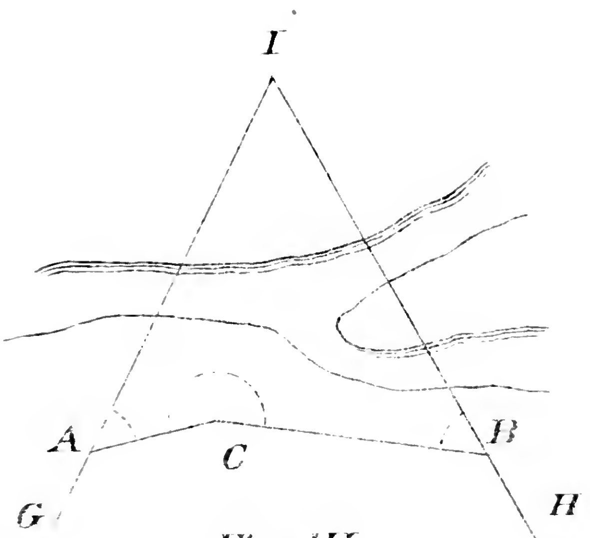

Fig. III.

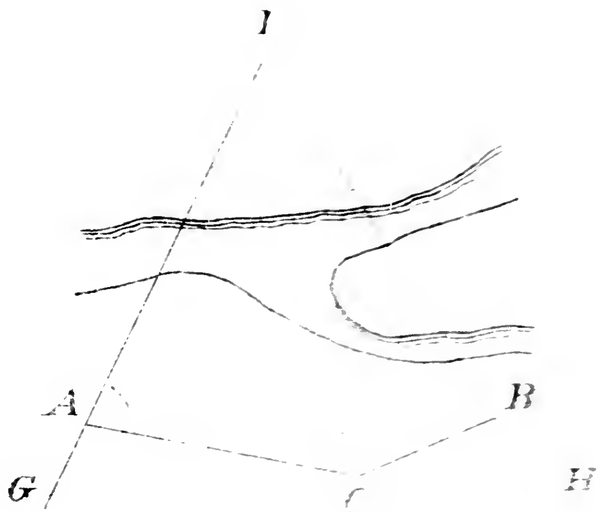

Fig. IV.

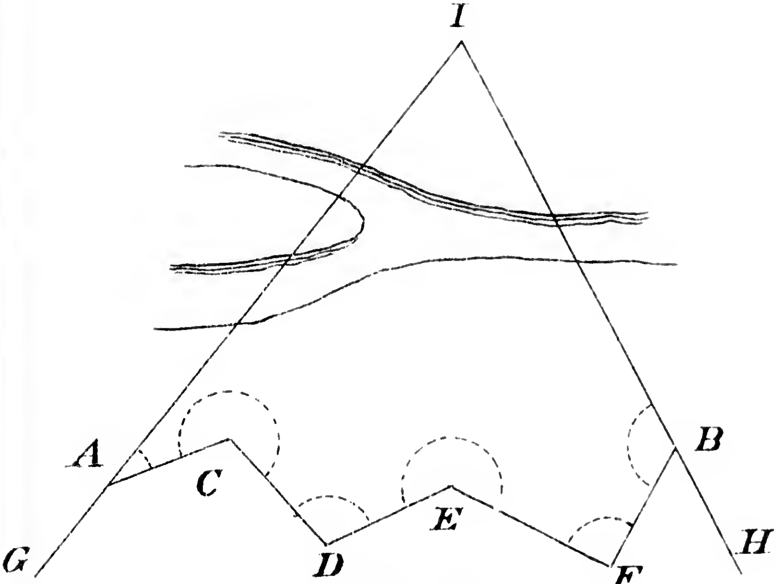

Fig. V.

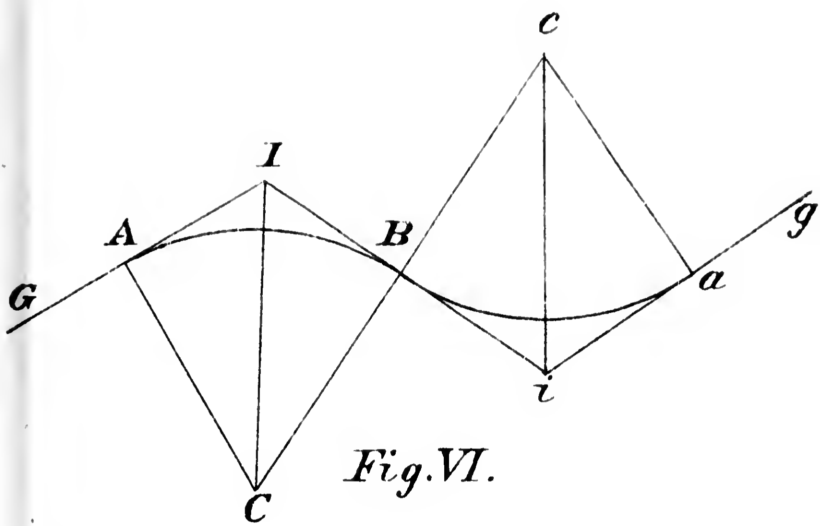

Fig. VI.

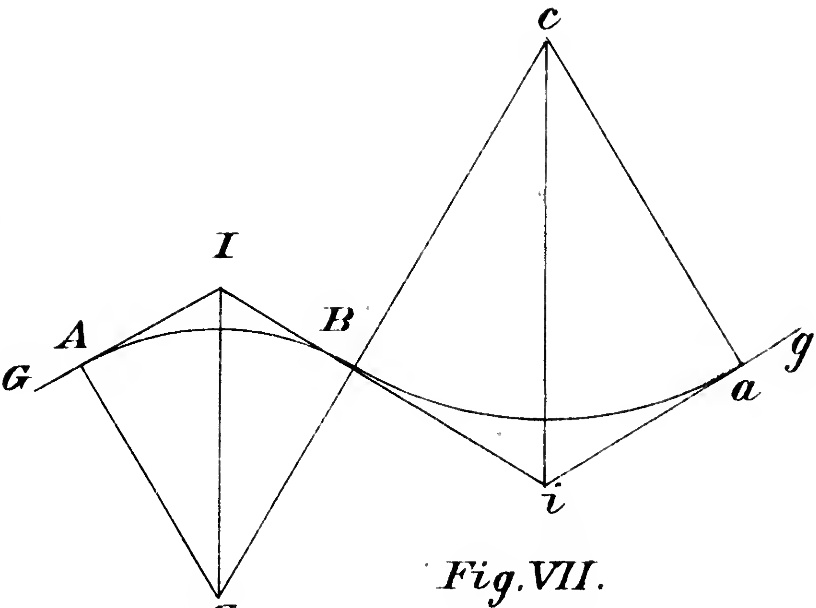

Fig. VII.

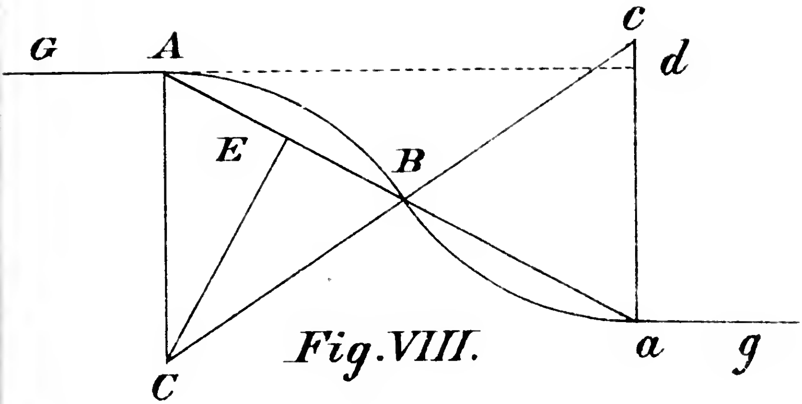

Fig. VIII.

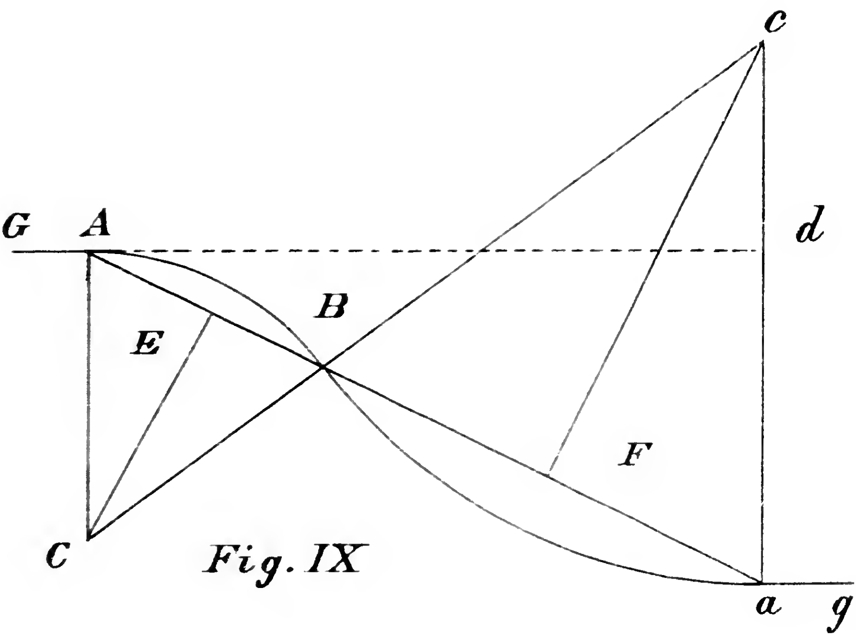

Fig. IX

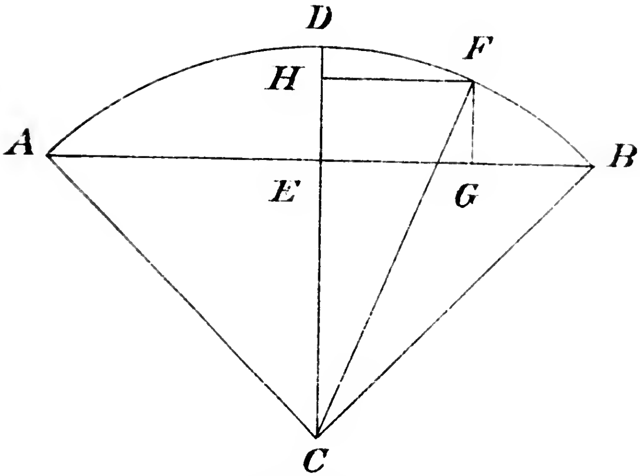

Fig. X.

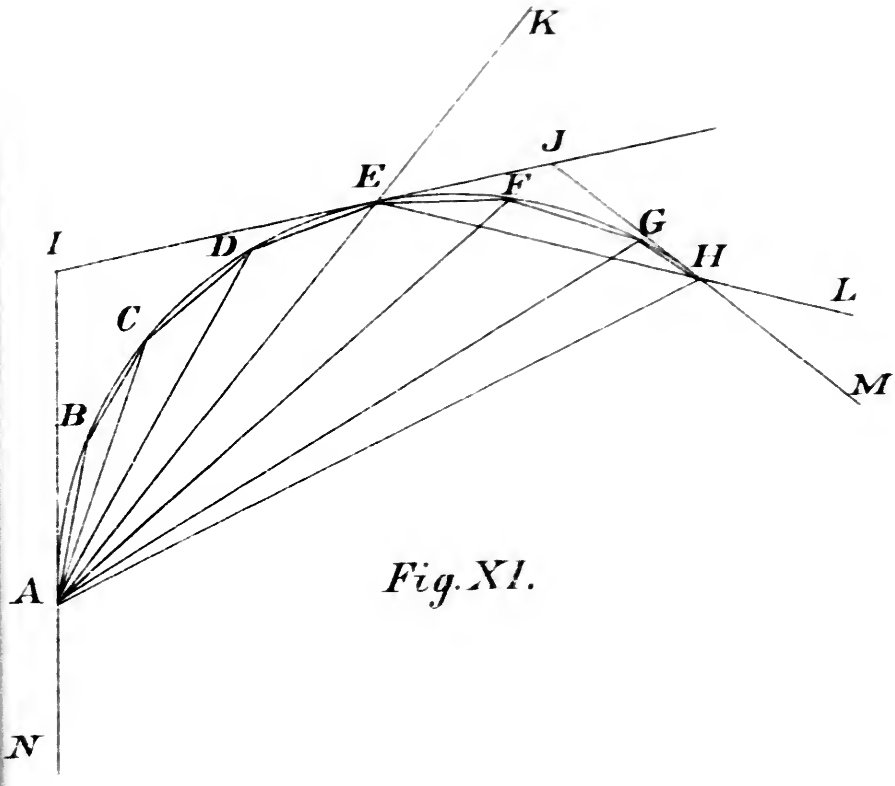

Fig. XI.

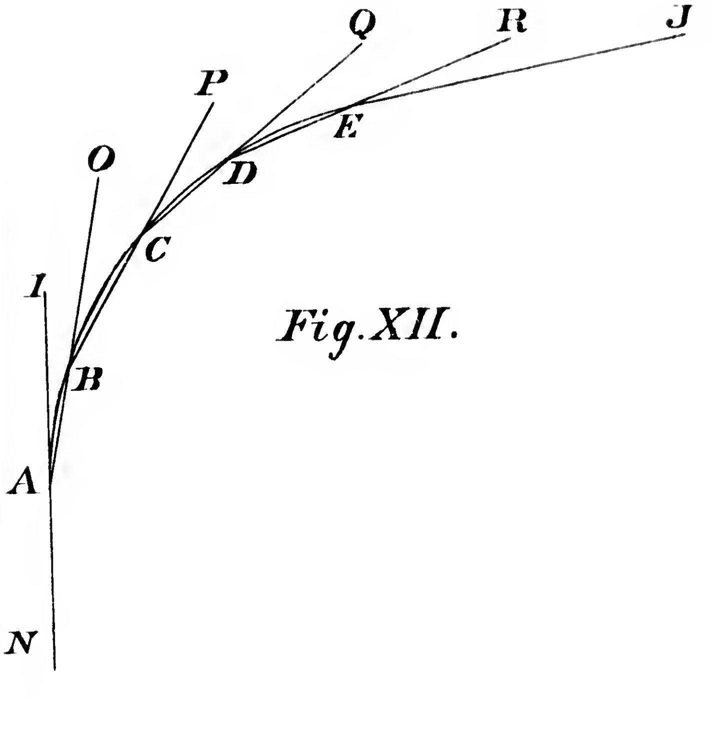

Fig. XII.

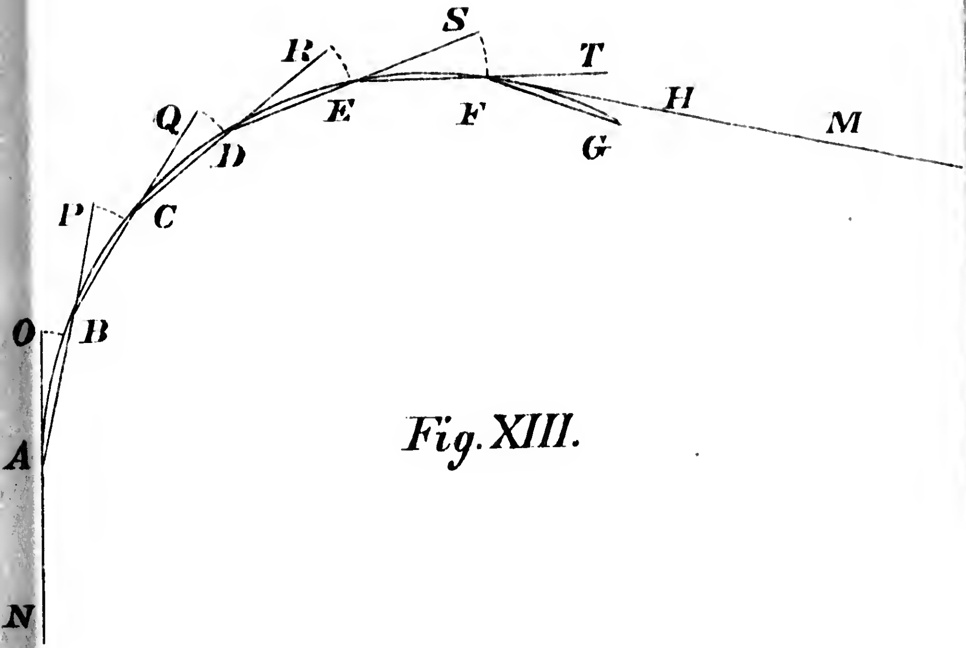

Fig. XIII.

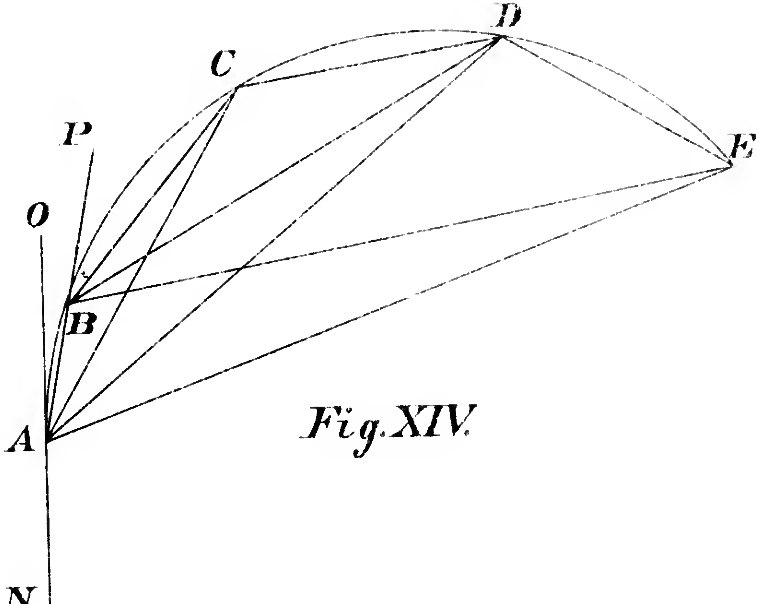

Fig. XIV.

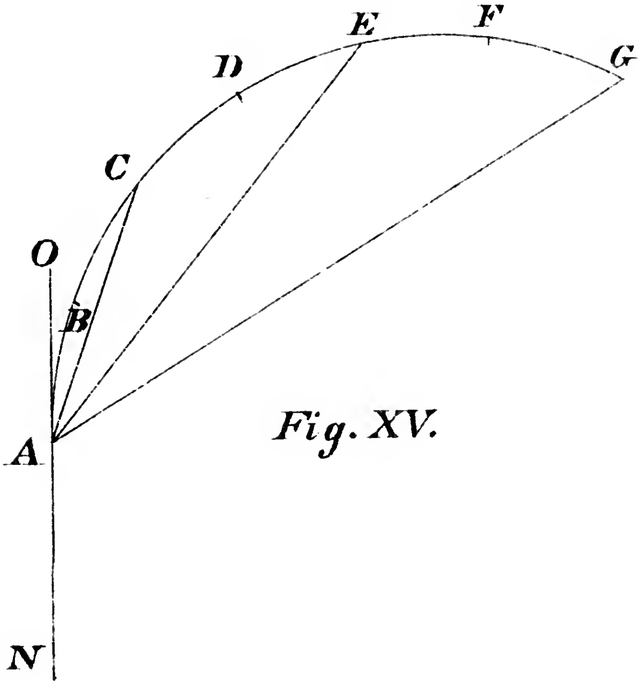

Fig. XV.

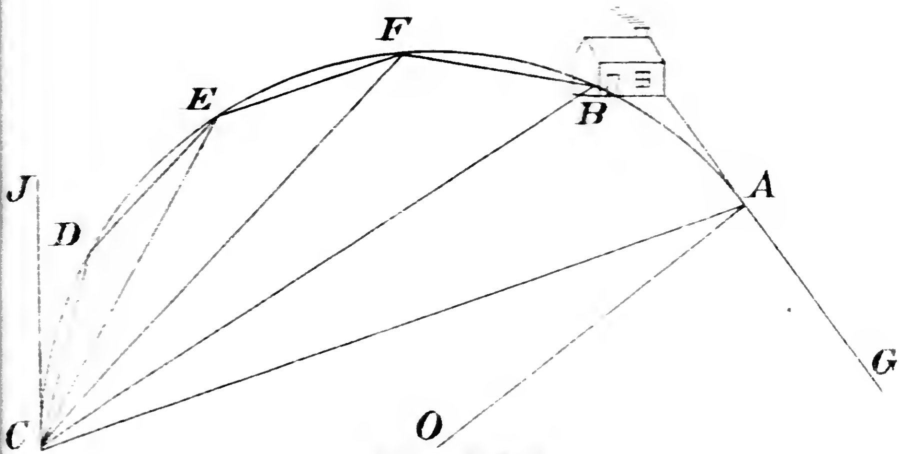

Fig. XVI.

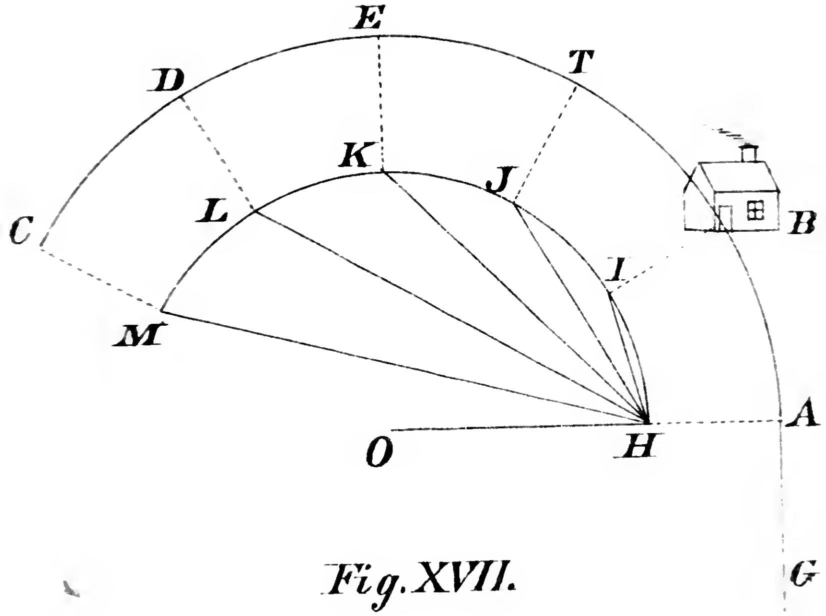

Fig. XVII.