Johnson Series

# SCHOOL ARITHMETIC

Advanced Book

WITHOUT ANSWERS

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# SCHOOL ARITHMETIC

## ADVANCED BOOK

BY

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# PREFACE.

To meet the needs of progressive teachers in the best public schools has been the first aim of the authors in writing the present work. In the effort to carry out this purpose great care has been taken to make the book *modern* yet *conservative*. It is believed that an examination will show the book to be sound in theory, modern in method, strong in inductive work, clear and accurate in all statements, correct in its teaching of business practice, and copious and varied in its supply of practical work and problems. Special attention is called to the following features:

**1. Inductive Method.** The inductive method is applied throughout the major part of the book. New topics are introduced by carefully prepared questions and suggestions designed to develop in the pupil's mind a correct understanding of the principles to be taught, and to give clear insight into arithmetical relations and processes.

**2. Oral and Written Work** are given with every *appropriate* subject, not only to impress the principles thereof, but also to provide ample *practice* in making solutions and abundant material for *mental discipline*. To lead the pupil to think in *general* terms, as well as to familiarize him with the use of symbols and lay a foundation for the study of algebra, numerical exercises are followed by others involving the same processes with letters.

**3. Rules and Definitions.** Few rules are given, and these usually in the form of directions. At the same time there has been no hesitancy in giving a few rules where it was thought they would assist the pupil in gaining a correct and intelligent understanding of the processes, or help him to neat and orderly methods, but in no place is dependence on rules encouraged. The definitions are brief but accurate; *unnecessary* ones are omitted.

**4. Form of Solutions.** The solutions given are intended as far as possible to suggest the reasons for the various steps and to render unnecessary the lengthy, cumbrous, and tedious explanations and analyses often given in arithmetical text-books.

**5. Solutions of Problems, with Examples for Practice.** This chapter affords the pupil not only examples of neatness and orderly procedure in making solutions, but also a basis for the recognition of similar problems and an aid in devising definite processes for their solution.

**6. Practice Work and Problems.** The book is believed to be unsurpassed in the abundance, variety, and excellent character of its practice work and problems, yet the teacher may find it advisable at times to supply additional work along special lines.

**7. Arrangement of Topics.** The topical plan of treatment has in the main been adhered to as giving the best adaptation to a wider range of schools; but where it could be advantageously done, much valuable information relating to subsequent topics has been introduced in the examples of the earlier sections. In the Primary book the order of topics is not preserved, but the subject matter is presented in the order that has been found to be most helpful, most stimulating, and most practical, the arrangement being determined by the order of the child's mental development rather than by the logic of the subject itself. In this book the usual arrangement of topics has been departed from in a few instances. We note the following:

(a) **Factoring** is clearly taught, but the G. C. D., which is now rarely met with in practical life and which is difficult for beginners to understand, is presented where it can be given in review at a time when the pupil is better able to grasp the logical reasoning it presents.

(b) **U. S. Money,** easily understood by young pupils, is used as an introduction to decimal fractions.

(c) **Decimal Fractions** are treated before common fractions, because now more generally employed, and because the *formal* treatment in connection with integers is more easily understood, the notation on the right of the decimal point being as easily understood as that on the left if the decimal fractions are compared with integers instead of with common fractions. In the Primary book, however, the common fraction is first introduced, because the first fractions with which very young children become acquainted are the half, quarter, etc., and because these are more simple and within the range of the child's visualizing power. In a *formal* treatment, however, designed for older pupils, the reasons are reversed.

(d) **Ratio and** (Simple) **Proportion.** Ratio is given in connection with Relation of Numbers on account of their intimate connection. Simple Proportion, furnishing as it does a good basis of arithmetical reasoning, is given earlier than usual as an aid in improving the reasoning ability of the pupil. The treatment is simple and not burdened with unnecessary matter. Compound proportion being little used is given in the Appendix for later review.

(e) **Practical Mensuration** is treated in connection with square and cubic measures, thus enriching the treatment, giving greater variety, and adding interest to the work by showing the practical uses to which these measures are put.

(f) The **Metric System** is given directly after Compound Numbers for purposes of comparison as well as to provide a variety of problems in the *supplementary exercises* intended for advanced classes.

(g) Subjects of minor importance and such as are suited to later reviews are placed in the Appendix, while subjects of no practical value in a modern arithmetic are omitted.

**8. Chapter on the Equation.** The importance of introducing this simple and easy chapter cannot well be overestimated. Arithmetical methods are adhered to, and by simple inductive exercises and comparative solutions the conceptions of the use of letters as the general representatives of positive numbers, and of the equation to express their relations, are developed as far as the purpose of the chapter demands. By the aid of this simple chapter, equations which have a meaning to the pupil can be substituted for the dead formulas sometimes used in percentage problems and interest problems, and a much clearer understanding can be had of such subjects as the greatest common divisor, the square and cube roots, etc. The subject is further developed in the brief but attractive chapter on "Introduction to Algebra" given in the Appendix.

**9. Division of Problems.** Experience has shown that the same problems should not be solved by the same class from term to term. It is wiser to add fresh fuel than to be constantly stirring the old coals. The pupil's interest must be kept alive. With this in view, we have divided the problems into two parts; the first to be used when the class first goes over a subject, the second when it reviews that subject. For a similar reason the *reviews* have also been so divided.

**10. Character of Problems.** In the preparation of problems the actual business practice of to-day has been kept in mind. In addition to their practical and mathematical value, many of the problems furnish much useful and scientific information that is reliable and strictly up-to-date. The aim has been to make them as *practical* and *useful* as possible.

**11. Scope of the Work.** The series meets the needs not only of the primary and the grammar school, but also of the high school. For the latter there is sufficient theory, and an abundance of practice work, owing to the "Supplementary

Exercises" for advanced classes, and matter included in the Appendix.

**12. Bank Practice** of to-day is given in a concise and accurate chapter. The information has been furnished by officials of prominent banks in the States mentioned in the exercises.

**13.** The unique treatment of various processes will appeal to the intelligent and progressive teacher. The merits of these processes can be determined only by careful examination of the text itself.

**14.** The matter and method of the Primary book are designed to serve as an introduction to and foundation for the more formal and rigorous treatment presented in these pages.

The authors desire to express their thanks for valuable criticisms and suggestions from many educators during the preparation of this series. They especially acknowledge their indebtedness to Prof. Frank W. Duke, of Hollins Institute, Hollins, Va., for help of this kind.

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## APPENDIX.

# SCHOOL ARITHMETIC.

## ILLUSTRATIONS AND DEFINITIONS.

A pail contains a *quantity* of milk which we wish to *measure.* We take a quart measure and fill it six *times,* thus finding *how much* milk we have.

(a). The **unit of measure** here is the quart.

(b). The number of *times* we filled it tells us **how many** quarts of milk there are.

(c). *Six quarts* tells us two things: (1) **how many** units of measure the pail contains; (2) **how much** milk is in the pail, i.e., the *quantity* of it.

**1.** A **Unit,** or unit of measure, is any quantity by comparison with which any other quantity of the same kind is measured.

Thus, when we buy a piece of cloth containing ten yards, *one yard* is the unit. When we measure a wall with a ten-foot pole, the unit is *ten feet.* If we use a ruler $\frac{1}{2}$ yard long, $\frac{1}{2}$ *yard* is the unit. If a box contains 6 dozen pencils, *a dozen* is the unit. When we say we have ten apples, the unit (of reference) is *one apple.*

**2. Number** denotes how many units make a measured quantity. It tells how many times the unit of measure is applied or repeated.

Thus, 5, 3 pounds, \$8, 12 feet are numbers.

A number always answers the question, "How many?"

**3.** A **Pure,** or **Abstract,** number is one that tells how many times the unit of measure is applied or repeated. It expresses the ratio of one quantity to another of the same kind, that is, the ratio of a given quantity to its measuring unit.

Thus, in the first illustration, *six* is an abstract number.

**4.** Measured *quantities,* as well as the measuring units, are called **concrete numbers.** They denote *how much* of anything.

Thus, 4 inches, 2 bushels, 5 horses, 12 hours, 6 quarts, 1 quart, are concrete numbers.

NOTE.—In the expression 6 *quarts,* 6, as denoting how many times the quantity contains the unit of measure, is strictly the *number* and is *abstract;* while the unit of measure, 1 quart, as well as the measured quantity, 6 *quarts,* is concrete. But in all cases where the unit is specified we apply the term "concrete number," disregarding the distinction for the sake of convenience.

**5. Arithmetic** treats of numbers, and the application of numbers to business and science.

## NOTATION AND NUMERATION OF INTEGERS.

**6.** An **Integer, or Integral Number,** is a whole number; that is, one or more *ones.* The unit of measure is the whole of a quantity—not one of its equal parts.

**7.** The method of expressing integral numbers by figures or letters is called **Notation of Integers.**

**8.** The method of reading integral numbers expressed by figures or letters is called **Numeration of Integers.**

**9.** The method of expressing numbers by figures is called **Arabic Notation** because the Arabs first introduced it into Europe. The symbols of this notation, except the zero, originated among the Hindus in India more than 2,000 years ago. The zero did not appear until about the fifth century A.D. The Europeans learned the new system from the Arabs during the twelfth century.

**10.** In this system of notation ten *figures,* or *digits,* are used to express numbers, viz.:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| zero | one | two | three | four | five | six | seven | eight | nine |

The first figure, *zero,* is also called *naught* and *cipher,* and signifies *none.* It is used with other figures to express numbers larger than 9.

Figures are not numbers ; they are characters or symbols used to express or represent numbers.

### ONES, TENS, AND HUNDREDS.

**11.** In counting up to nine we count by *ones.* After 9 we say *ten,* but we have no single figure to express this number. Hence to express ten *ones,* or 1 ten, we combine zero with 1, and write the number thus : 10.

**12.** PRINCIPLE.—*When a figure stands alone, it expresses ones. When two figures stand side by side, the one on the right expresses* **ones,** *the other* **tens.**

Thus, in 24 the 4 represents 4 *ones,* and the 2 represents 2 *tens.*

2 tens, or 20, is called *twenty.*
3 " " 30, " " thirty.
4 " " 40, " " forty.
5 " " 50, " " fifty.
6 " " 60, " " sixty.
7 " " 70, " " seventy.
8 " " 80, " " eighty.
9 " " 90, " " ninety.

**13.** In counting more than nine we count by tens, as above, or by tens and ones, as follows :

1 ten and 1 one, or 11, is called *eleven.*
1 ten " 2 ones, or 12, " " twelve.
1 ten " 3 ones, or 13, " " thirteen.
1 ten " 4 ones, or 14, " " fourteen.
1 ten " 5 ones, or 15, " " fifteen.

| | | | |
|---|---|---|---|
| 1 ten and | 6 ones, | or 16, is called | sixteen. |
| 1 ten " | 7 ones, | or 17, " " | seventeen. |
| 1 ten " | 8 ones, | or 18, " " | eighteen. |
| 1 ten " | 9 ones, | or 19, " " | nineteen. |
| 2 tens, | | or 20, " " | twenty. |
| 2 tens " | 1 one, | or 21, " " | twenty-one. |
| 3 tens " | 2 ones, | or 32, " " | thirty-two. |
| 4 tens " | 3 ones, | or 43, " " | forty-three. |
| 8 tens " | 7 ones, | or 87, " " | eighty-seven. |
| 9 tens " | 9 ones, | or 99, " " | ninety-nine. |

**14.** What does each figure in the following express?

| | | | | |
|---|---|---|---|---|
| **1.** 15 | **5.** 38 | **9.** 61 | **13.** 84 | **17.** 92 |
| **2.** 19 | **6.** 43 | **10.** 68 | **14.** 88 | **18.** 97 |
| **3.** 21 | **7.** 47 | **11.** 73 | **15.** 79 | **19.** 90 |
| **4.** 26 | **8.** 54 | **12.** 59 | **16.** 81 | **20.** 99 |

**15.** When we count one more than 99, we have nine tens and 10 ones, or 10 tens, which is called *one hundred.* To express this the figure 1 is written at the left of two ciphers, thus: 100.

**16.** Principle.—*When three figures are written side by side, the one at the left expresses hundreds.*

Thus, in 324 the 4 represents 4 ones, the 2 represents 2 tens, and the 3 represents 3 hundreds.

**17.** A number expressed by three figures is read without the word *and.*

Thus, 324 is read three hundred twenty-four.

**18.** Read the following:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 17 | 24 | 48 | 27 | 32 | 39 | 41 | 46 | 29 |
| 50 | 53 | 66 | 65 | 77 | 84 | 90 | 72 | 124 |
| 231 | 132 | 213 | 346 | 427 | 536 | 175 | 381 | 450 |
| 619 | 680 | 700 | 798 | 800 | 870 | 660 | 739 | 808 |
| 987 | 711 | 444 | 330 | 602 | 101 | 507 | 600 | 789 |

Write the third vertical column in words.

**19.** Express the following by figures:

**1.** Fifty.
**2.** Forty-two.
**3.** Sixty-nine.
**4.** Seventy-six.
**5.** Thirty-seven.
**6.** Ninety-five.
**7.** Eighty-eight.
**8.** Sixty-seven.
**9.** Twenty-four.
**10.** Ninety-nine.
**11.** Thirty-eight.
**12.** Seventy-three
**13.** Eighty-seven.
**14.** One hundred fifty.
**15.** Two hundred fifty-four.
**16.** Three hundred seven.
**17.** Six hundred forty-five.
**18.** Eight hundred sixteen.
**19.** Four hundred fifty-six.
**20.** Five hundred twenty.
**21.** Five hundred nine.
**22.** Seven hundred eighteen.
**23.** Nine hundred three.
**24.** Six hundred sixty-six.
**25.** Eight hundred eighty.
**26.** Three hundred three.
**27.** Nine hundreds, three tens, seven ones.
**28.** Seven hundreds, eight ones, nine tens.
**29.** Three ones, seven tens, four hundreds.
**30.** Write from dictation the numbers in Art. 18.

QUERIES.—1. When 1 stands alone, how many ones does it express? How many when it stands at the left of a cipher? At the left of two ciphers?

2. In the number 11, which 1 expresses the greater value? How many times as great? In the number 111?

**20.** PRINCIPLE.—*Any figure represents ten times the value it would represent in the next place to the right, or one tenth of the value it would represent in the next place to the left.*

**21.** The system of counting by *tens* is called the *Decimal System.* In the Arabic notation *ten* of any place or order make *one* of the next higher order, hence the system is a decimal one. This system derives its greatest importance from the use of zero, which renders possible the giving of *place value* to the figures.

**22.** The *ones* of a number are called *units of the first order*, or simply *units;* the *tens* are called *units of the second* *order;* the *hundreds* are called *units of the third order,* and so on.

**23.** For convenience in writing and reading numbers the figures are divided into groups of three figures each, beginning at the right. Each group is called a *period,* and contains *ones, tens,* and *hundreds* of *that period.* The right-hand group is called the *period of units;* the second group, the *period of thousands;* the third group, the *period of millions,* and so on.

**24.** The system of notation is shown in the following

TABLE:

| *Orders.* | Hundred-trillions | Ten-trillions | Trillions | Hundred-billions | Ten-billions | Billions | Hundred-millions | Ten-millions | Millions | Hundred-thousands | Ten-thousands | Thousands | Hundreds | Tens | Ones (Units) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 2 | 0 | 5 | 6 | 1 | 9 | 0 | 7 | 0 | 8 | 4 | 0 | 5 | 3 | 2 |
| | 5th Period. *Trillions.* | | | 4th Period. *Billions.* | | | 3d Period. *Millions.* | | | 2d Period. *Thousands.* | | | 1st Period. *Units.* | | |

The number in the table is read two hundred five trillion, six hundred nineteen billion, seventy million, eight hundred forty thousand, five hundred thirty-two.

1. The left-hand period may have only one or two figures; the others must contain three figures.

2. In writing large numbers, the periods may be separated by commas or written slightly apart, as an aid in reading them.

3. Each period is read as if it stood alone, the *name* of the period being added except in the case of the units' period.

4. The names of periods above trillions are, in order, quadrillions, quintillions, sextillions, septillions, octillions, nonillions, decillions, etc.

### EXERCISES IN NUMERATION.

**25.** Read the following numbers :

| | | | | | |
|---|---|---|---|---|---|
| **1.** | 2,375, | 52374, | 327195, | 6,382,541, | 2 906 738. |
| **2.** | 5,732, | 48106, | 245680, | 7,328,911, | 1 540 006. |
| **3.** | 8,360, | 90245, | 503276, | 5,374,802, | 8 000 325. |
| **4.** | 4,076, | 13004, | 100538, | 2,704,190, | 6 030 008. |
| **5.** | 8,007, | 86730, | 736451, | 1,398,453, | 5 111 290. |

| | | | |
|---|---|---|---|
| **6.** | 71,248,359,024, | 370,689,421, | 4,210,893,657. |
| **7.** | 42,095,384,217, | 124,986,073, | 7,563,980,124. |
| **8.** | 14,703,692,581, | 581,470,369, | 2,581,470,369. |
| **9.** | 60,482,604,826, | 726,934,851, | 8,276,354,090. |
| **10.** | 90,007,080,500, | 900,604,003, | 3,415,628,072. |

| | | |
|---|---|---|
| **11.** | 1,928,374,650,437,689, | 7,129,384,756,140,632,687. |
| **12.** | 1,234,567,891,098,765, | 3,470,153,647,005,296,304. |

**13.** Write in words the numbers in the 1st, 10th, and 12th lines.

### EXERCISES IN NOTATION.

**26.** The periods are written in regular order, beginning with the highest. If any order of units of a period is lacking, a cipher must be put in its place; and if any entire period is lacking, its three places must be filled with ciphers.

**27.** Write the following in figures :

**1.** Three thousand, eight hundred forty-five.

**2.** Eighty-three thousand, seven hundred forty.

**3.** Seventy-two thousand, three hundred five.

**4.** Thirty-seven thousand, five hundred twenty-one.

**5.** Ninety thousand, ninety. Four thousand, six.

**6.** Fifty thousand, five. Seventeen thousand, one.

**7.** Ten thousand. Six thousand, one hundred nine.

**8.** Three hundred forty-two thousand, six hundred seventy-five.

**9.** One hundred six thousand, four.

**10.** Three hundred thousand, three hundred.

**11.** Eight hundred six thousand, seventy-five.

**12.** One hundred seventeen thousand, four hundred two.
**13.** Eight million, eight thousand, eight.
**14.** Fifteen million, fifteen thousand, thirty.
**15.** Five hundred two million, three hundred four.
**16.** Two million, six thousand, ninety-eight.
**17.** Six hundred twelve thousand, four hundred sixty-two.
**18.** Five hundred sixty thousand, four hundred fifty-six.
**19.** Ten million, one thousand, one hundred.
**20.** Sixty-five million, one hundred eight thousand.
**21.** Nine hundred eighty-seven million, four.
**22.** Four hundred thousand, fifty-six.
**23.** One hundred ten thousand, ninety.
**24.** Nine hundred million, three hundred eighty-one.
**25.** Six million, six hundred thousand, sixty.
**26.** Fifteen million, nine thousand, seventeen.
**27.** Four thousand, three hundred six.
**28.** Fifty-four thousand, three hundred fifteen.
**29.** Five hundred seven million, sixty-thousand, two.
**30.** Nine billion, four hundred million, eighty.
**31.** Write from dictation all the numbers in Art. 25.

## UNITED STATES MONEY.

**28.** The money of the United States has a decimal system; *dollars* are written as integers, *cents* and *mills* as decimal parts of a dollar. Instead of writing the word dollars after a number, we write the "dollar mark" ($) before the number.

Thus, 10 dollars is written $10.

**29.** A point, called the **Decimal Point,** is placed at the left of the cents, but at the right of the dollar mark.

Thus, 25 cents is written $.25. Five cents is written $.05.

1. In writing both dollars and cents, the decimal point is placed between them.

Thus, 6 dollars, 15 cents, is written $6.15.

2. The first *two* places at the right of the decimal point are read as *cents*, and the *third* as *mills*.

Thus, $4.205 is read 4 dollars, 20 cents, 5 mills.

**30.** Read the following:

| | | | | | | |
|---|---|---|---|---|---|---|
| **1.** | \$2.75, | \$12.82, | \$375.40, | \$.87, | \$230.05, | \$100. |
| **2.** | \$16.08, | \$43.00, | \$501.09, | \$6.90, | \$.54, | \$9.86. |
| **3.** | \$107.09, | \$36.80, | \$275.005, | \$690.13, | \$7.56, | \$5.245. |
| **4.** | \$260.306, | \$150.05, | \$309.401, | \$3060.40, | \$58.039. | |

**31.** Write in figures:

**1.** Eight dollars, ten cents. Twelve dollars. Eight cents. Nine dollars, six cents.

**2.** Fourteen cents. Three dollars, sixty cents. Fifteen dollars, four cents. Five cents.

**3.** Sixty dollars, six cents. Two hundred ninety dollars, seventy-two cents. Six dollars.

**4.** Five hundred one dollars, eight cents. Nine hundred dollars, thirty cents.

**5.** Nine thousand, seven hundred four dollars, forty cents. Five dollars, nine cents.

**6.** Seven thousand four dollars, fifty-one cents. Sixty-five cents. Nine dollars.

**32.** Read and name the abstract numbers:

| | | | |
|---|---|---|---|
| **1.** \$275. | **5.** 7016. | **9.** \$.05 | **13.** 24 birds. |
| **2.** \$3.24. | **6.** \$1. | **10.** 200. | **14.** 12 feet. |
| **3.** 500. | **7.** 3 boys. | **11.** 6 cows. | **15.** 3250. |
| **4.** 5 cents. | **8.** 3. | **12.** 1 man. | **16.** 3 pounds. |

## THE ROMAN NOTATION.

**33.** This system employs seven capital letters to express numbers, viz.:

| I | V | X | L | C | D | M |
|---|---|---|---|---|---|---|
| 1 | 5 | 10 | 50 | 100 | 500 | 1000 |

Other numbers are expressed by combining these letters in accordance with the following

**34.** PRINCIPLE.—1. *Repeating a letter repeats its value.*

Thus, III represents *three;* XX, *twenty.*

2. *When a letter precedes one of greater value, the* **difference** *of their values is the number represented.*

Thus, IV represents the difference between *five* and *one*, or four.

3. *When a letter follows one of greater value, the* **sum** *of their values is the number represented.*

Thus, VI represents the sum of *five* and *one*, or six.

A dash over a letter increases its value a thousand-fold.

Thus, $\overline{\text{V}}$ represents five thousand.

These principles are illustrated in the following table :

| | | | |
|---|---|---|---|
| I......1. | XI....11. | XXI...21. | XC.....90. |
| II......2. | XII....12. | XXIV...24. | C....100. |
| III......3. | XIII....13. | XXV...25. | CC....200. |
| IV......4. | XIV....14. | XXIX...29. | CCC....300. |
| V......5. | XV....15. | XXX...30. | CD....400. |
| VI......6. | XVI....16. | XL...40. | D....500. |
| VII......7. | XVII....17. | L...50. | DC....600. |
| VIII......8. | XVIII....18. | LX...60. | M...1000. |
| IX......9. | XIX....19. | LXX...70. | MM...2000. |
| X.....10. | XX....20. | LXXX...80. | $\overline{\text{X}}$..10000. |

This method of notation is called the Roman, because the Romans invented and used it. It was in common use prior to the introduction of the Hindu system, but its use is now very limited. Let the pupil ascertain by observation for what purposes it is now used.

**35.** Express by the Arabic notation :

1. XXVII, XVIII, XLIX, XLVII, LVI, LIX.
2. LXII, LXIX, LXXV, LXXXIX, XCII, XCVII.
3. CVIII, CXIV, CCIV, CCCLXV, DVII, DCVII.
4. DXLVI, DXCI, DCXI, DCXLIX, MDCCCC.
5. MCDXCII, MDCCLXXV, MDCCCXCIII.

**36.** Express by the Roman notation :

1. 18, 27, 36, 45, 54, 63, 72, 81, 94, 97, 101.
2. 110, 203, 470, 509, 747, 931, 1009, 1865, 1789.
3. 1607, 1682, 1498, 1620, 1783, 1812, 1901, 2222.

## ADDITION.

**37.** In one box I have 4 bushels of corn, in another 2 bushels, in a third 3 bushels. If I put all into one box, how much corn will be in that box?

Here we have three **numbers to be added (addends),** and the result—9 bushels—is the *how much,* the quantity, which we wished to find. It is the **sum** of the parts or addends.

**38.** The process of finding the sum of two or more like numbers is called **Addition.**

**1.** What is the sum of 5 feet, 3 feet, and 7 feet?

**2.** What is the sum of 6 quarts and 4 pounds?

**3.** Are 6 quarts and 4 pounds *like* numbers?

**4.** Is the sum of two or more numbers the same in whatever order they are added? Find out by trial.

**39. Like Numbers** are those that have the same unit.

Thus, 5 yards and 3 yards are like numbers; but 5 yards and 3 pecks are not like numbers.

**1.** The sum of 3 feet and 6 feet is 9 what?

**2.** Is the sum always like the numbers added?

**40.** PRINCIPLE.—*Only like numbers can be added.*

Find the sum of:

| | | |
|---|---|---|
| **1.** 3 and 5. | **4.** 2, 3, and 6. | **7.** 6 boys and 2 boys. |
| **2.** 6 and 2. | **5.** 3, 5, and 4. | **8.** 3 birds and 6 birds. |
| **3.** 4 and 7. | **6.** 4, 2, and 7. | **9.** $a$ cents and $a$ cents. |

**41.** The **Sign of Addition** (+) is called *plus,* and is placed between the numbers to be added.

Thus, 5 + 3 is read *5 plus 3,* and means that 3 is to be added to 5.

**42.** The **Sign of Equality** (=) is read *equals,* or *is equal to.*

Thus, 5 + 3 = 8 is read *five plus three equals eight.*

**43.** A statement that two numbers or expressions of number are equal is called an **Equation.**

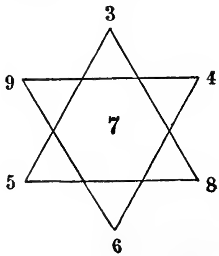
Thus, $5 + 3 = 8$ is an equation.

**44.** Find the sum of the following, and read the complete expressions :

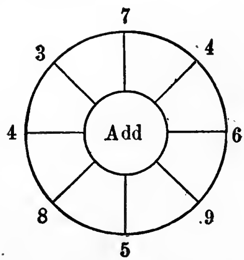
**1.** $1 + 1 = (\ \ )$.
**2.** $1 + 2 = (\ \ )$.
**3.** $1 + 3 = (\ \ )$.
**4.** $1 + 4 = (\ \ )$.
**5.** $1 + 5 = (\ \ )$.
**6.** $1 + 6 = (\ \ )$.
**7.** $1 + 7 = (\ \ )$.
**8.** $1 + 8 = (\ \ )$.
**9.** $1 + 9 = (\ \ )$.
**10.** $2 + 2 = (\ \ )$.
**11.** $2 + 3 = (\ \ )$.
**12.** $2 + 4 = (\ \ )$.
**13.** $2 + 5 = (\ \ )$.
**14.** $2 + 6 = (\ \ )$.
**15.** $2 + 7 = (\ \ )$.
**16.** $2 + 8 = (\ \ )$.
**17.** $2 + 9 = (\ \ )$.
**18.** $3 + 3 = (\ \ )$.
**19.** $3 + 4 = (\ \ )$.
**20.** $3 + 5 = (\ \ )$.
**21.** $3 + 6 = (\ \ )$.
**22.** $3 + 7 = (\ \ )$.
**23.** $3 + 8 = (\ \ )$.
**24.** $3 + 9 = (\ \ )$.
**25.** $4 + 4 = (\ \ )$.
**26.** $4 + 5 = (\ \ )$.
**27.** $4 + 6 = (\ \ )$.
**28.** $4 + 7 = (\ \ )$.
**29.** $4 + 8 = (\ \ )$.
**30.** $4 + 9 = (\ \ )$.
**31.** $5 + 5 = (\ \ )$.
**32.** $5 + 6 = (\ \ )$.
**33.** $5 + 7 = (\ \ )$.
**34.** $5 + 8 = (\ \ )$.
**35.** $5 + 9 = (\ \ )$.
**36.** $6 + 6 = (\ \ )$.
**37.** $6 + 7 = (\ \ )$.
**38.** $6 + 8 = (\ \ )$.
**39.** $6 + 9 = (\ \ )$.
**40.** $7 + 7 = (\ \ )$.
**41.** $7 + 8 = (\ \ )$.
**42.** $7 + 9 = (\ \ )$.
**43.** $8 + 8 = (\ \ )$.
**44.** $8 + 9 = (\ \ )$.
**45.** $9 + 9 = (\ \ )$.

All the combinations of numbers from 1 to 9, taken two and two, are given in these 45 equations. The pupil should practice adding the numbers until able to state the sums at a glance.

**45.** In blackboard exercises various devices may be used to advantage. Place these diagrams on the blackboard, and indicate with a pointer the numbers to be added.




| | | |
|---|---|---|
| 5 | 3 | 8 |
| 7 | **4** | 6 |
| 2 | 4 | 9 |




**1.** Combine the numbers written at the center and points of the star into a convenient number of exercises; as $4 + 7$, $5 + 8 + 6$, etc.

**2.** Add the number in the center of the square to each of the other numbers.

**3.** Using the diagram at the right, begin at a certain number and add in either direction to 50, 100, etc. These exercises should be continued until quick and correct answers can be given.

### ORAL EXERCISES.

**46. 1.** Edna has 6 books and Ida has 7. How many have they both?

**2.** Marie paid 8 cents for paper, 5 cents for ink, and 7 cents for pencils. How much did she pay for all?

**3.** James is 9 years old, and Tom is 6 years older. How old is Tom?

**4.** Six chickens are on the fence, 8 are in the barn, and 5 are picking grass. How many are there in all?

**5.** I paid \$4 for a hat, \$9 for a coat, and had \$7 left. How much had I at first?

**6.** A boy walked 3 miles to the station, then rode 10 miles in the cars, and then went 8 miles in a coach. How far did he travel?

**7.** A farmer killed 3 pigs and 2 sheep, and had 9 pigs and 4 sheep left. How many of both had he at first?

**8.** How many days are 6 days, 5 days, and a week?

**9.** How many letters in the names of all the days of the week?

**10.** What is the sum of the numbers represented by the ten digits?

**11.** Harry ran twice around a house 3 rods long and 2 rods wide. How far did he run?

**12.** Mary has 5 cents, Kate has 4 cents, and Jane has 7 cents more than Kate. How much have Jane and Mary together?

**13.** A has $3, B has $5, and C has as much as both. How much have they all?

**14.** One man has $a$ dollars and another has $2a$ dollars. How much have both?

**15.** Ella has $2b$ cents, May has $3b$ cents, and Tillie has as many as both. How many cents have they all?

**16.** What is the sum of $a$, $2a$, $4a$, and $5a$?

**47.** In the following exercise add 3 to each number in column A, then to each in column B, and so on. Thus, 10 + 3, 20 + 3, etc.; then, 31 + 3, 41 + 3, etc.

| A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|
| 10 | 31 | 52 | 83 | 24 | 95 | 46 | 67 | 78 |
| 20 | 41 | 92 | 53 | 64 | 35 | 16 | 77 | 88 |
| 60 | 81 | 12 | 73 | 54 | 45 | 36 | 27 | 98 |
| 80 | 91 | 42 | 23 | 14 | 75 | 66 | 57 | 38 |
| 30 | 21 | 62 | 13 | 74 | 55 | 86 | 97 | 48 |
| 90 | 51 | 32 | 63 | 84 | 15 | 76 | 47 | 28 |
| 50 | 71 | 82 | 43 | 34 | 25 | 96 | 17 | 68 |
| 40 | 11 | 72 | 33 | 94 | 65 | 26 | 87 | 58 |
| 70 | 61 | 22 | 93 | 44 | 85 | 56 | 37 | 18 |

When the class can announce these results at sight, the exercise may be varied by substituting one of the other digits for 3. Using all the digits gives 729 examples.

**48.**

| | |
|---|---|
| **1.** Add by 2's from 2 to 50. | From 1 to 49. |
| **2.** Add by 3's from 3 to 30. | From 1 to 31. |
| **3.** Add by 3's from 2 to 32. | From 52 to 70. |
| **4.** Add by 4's from 4 to 40. | From 1 to 45. |
| **5.** Add by 4's from 2 to 34. | From 3 to 51. |
| **6.** Add by 5's from 1 to 41. | From 2 to 52. |
| **7.** Add by 5's from 3 to 43. | From 4 to 64. |
| **8.** Add by 6's from 6 to 72. | From 1 to 49. |
| **9.** Add by 6's from 2 to 50. | From 3 to 45. |
| **10.** Add by 6's from 4 to 46. | From 5 to 53. |
| **11.** Add by 7's from 7 to 84. | From 1 to 50. |

**12.** Add by 7's from 2 to 58. From 3 to 66.
**13.** Add by 7's from 4 to 60. From 5 to 75.
**14.** Add by 7's from 6 to 83. From 0 to 49.
**15.** Add by 8's from 8 to 96. From 1 to 97.
**16.** Add by 8's from 2 to 82. From 3 to 83.
**17.** Add by 8's from 4 to 100. From 5 to 93.
**18.** Add by 8's from 6 to 94. From 7 to 167.
**19.** Add by 9's from 9 to 108. From 1 to 109.
**20.** Add by 9's from 2 to 92. From 3 to 102.
**21.** Add 9 eight times to 4. To 5. To 6. To 7.

WRITTEN EXERCISES.

**49.** Copy and add each column from bottom to top, then from top to bottom; also add each line, (as **a**), from left to right, then from right to left.

In adding, do not say 7 and 2 are 9, and 4 are 13, etc., but give results only. Thus, 7, 9, 13, 19, etc.

| | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **a** | 5 | 3 | 6 | 7 | 5 | 9 | 8 | 7 | 4 | 9 | 3 |
| **b** | 3 | 6 | 5 | 4 | 7 | 6 | 4 | 6 | 8 | 4 | 8 |
| **c** | 6 | 4 | 7 | 5 | 6 | 4 | 5 | 9 | 7 | 6 | 7 |
| **d** | 4 | 7 | 3 | 9 | 4 | 7 | 9 | 5 | 6 | 5 | 6 |
| **e** | 2 | 8 | 4 | 3 | 9 | 8 | 7 | 8 | 3 | 7 | 9 |
| **f** | 7 | 5 | 9 | 8 | 8 | 5 | 3 | 4 | 9 | 8 | 5 |

| | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **g** | 6 | 9 | 3 | 8 | 8 | 9 | 7 | 5 | 4 | 7 | 6 |
| **h** | 5 | 8 | 4 | 7 | 3 | 5 | 3 | 1 | 8 | 9 | 7 |
| **i** | 8 | 4 | 5 | 6 | 9 | 7 | 8 | 4 | 7 | 6 | 5 |
| **j** | 7 | 6 | 8 | 3 | 6 | 8 | 4 | 9 | 5 | 3 | 8 |
| **k** | 3 | 7 | 7 | 4 | 5 | 6 | 5 | 2 | 9 | 8 | 4 |
| **l** | 2 | 5 | 6 | 5 | 8 | 3 | 9 | 8 | 7 | 5 | 9 |
| **m** | 9 | 9 | 9 | 9 | 6 | 9 | 6 | 6 | 6 | 9 | 6 |
| **n** | 7 | 3 | 7 | 5 | 7 | 5 | 7 | 7 | 8 | 2 | 8 |
| **o** | 4 | 5 | 8 | 7 | 5 | 9 | 6 | 8 | 3 | 7 | 7 |

| | **23** | **24** | **25** | **26** | **27** | **28** | **29** | **30** | **31** | **32** | **33** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **p** | 2 | 6 | 4 | 5 | 6 | 7 | 8 | 9 | 8 | 7 | 6 |
| **q** | 4 | 5 | 6 | 7 | 8 | 9 | 7 | 6 | 5 | 4 | 3 |
| **r** | 6 | 7 | 8 | 9 | 7 | 6 | 5 | 4 | 3 | 2 | 9 |
| **s** | 8 | 9 | 7 | 6 | 5 | 4 | 9 | 7 | 6 | 8 | 7 |
| **t** | 3 | 4 | 5 | 7 | 6 | 5 | 7 | 5 | 4 | 5 | 8 |
| **u** | 5 | 8 | 9 | 8 | 4 | 8 | 6 | 7 | 7 | 9 | 6 |
| **v** | 9 | 6 | 5 | 4 | 3 | 7 | 9 | 6 | 9 | 6 | 9 |
| **w** | 7 | 9 | 6 | 9 | 8 | 9 | 8 | 8 | 8 | 7 | 7 |
| **x** | 4 | 5 | 7 | 6 | 9 | 6 | 5 | 9 | 6 | 8 | 6 |
| **y** | 8 | 7 | 8 | 7 | 6 | 4 | 7 | 7 | 5 | 3 | 9 |
| **z** | 6 | 8 | 9 | 8 | 7 | 3 | 4 | 6 | 7 | 9 | 8 |

ORAL EXERCISES.

**50. 1.** Nell paid 20 cents for a slate and 30 cents for a book. How much did she pay for both ?

**2.** There are 30 days in April and 31 in May. How many days in both months ?

**3.** In an orchard there are 40 apple trees, 25 peach trees, and 12 pear trees. How many trees are there in the orchard ?

**4.** William is 12 years old, and his father is 27 years older. How old is his father ?

**5.** How many days are there in May, June, and July ?

**6.** If Edgar has $25, and Allen has $20, and I have $15 more than they both have, how much have I ?

**7.** An army marched 25 miles one day and 4 miles farther the next day. How far did it march in both days ?

**8.** A drover sold 25 sheep to one man and 42 to another, and had 23 left. How many had he at first ?

**9.** A field is 40 rods long and 16 rods wide. What is the distance around the field ?

**10.** A newsboy sold 40 morning papers, 25 evening papers, and had 12 left on his hands. How many did he buy ?

**11.** A dealer sold 25 bushels of coal to one man, 35 to another, and 32 to another. How many bushels did he sell to all ?

**12.** After paying \$33 for a cow and \$51 for a wagon, I had left a ten-dollar bill, a five-dollar bill, and a silver dollar. How much money had I at first ?

**13.** Forty-one years ago Jane's grandpa was 36 years of age. How old will he be if he lives 5 years longer?

**14.** A lady bought 3 webs of cloth, the first containing 32 yards, the second 41 yards, and the third 36 yards. How many yards did she buy ?

**15.** On Monday Ada's teacher gave her 20 words to spell, on Tuesday 25, on Wednesday 25, on Thursday 30, and on Friday 18. How many words were given in the week ?

**16.** Willie has $a$ marbles, Harry has $4a$, and Rob has as many as both. How many have they all ?

**17.** A man had $a$ pigs and bought $b$ more. How many did he then have ? $(a + b)$.

**18.** Sarah has $a$ cents, Alice has $b$ cents, and Ellen has as many as Sarah. How many have they all ?

**19.** What is the sum of $a$, $3a$, $9a$, and $2b$ ?

**20.** If one stone weighs $p$ pounds and another weighs $q$ pounds, what is the weight of both together ?

### WRITTEN EXERCISES.

**51. 1.** What is the sum of 7597, 1368, and 643 ?

$$\begin{array}{r} 7597 \\ 1368 \\ 643 \\ \hline 9608 \end{array}$$

May the numbers be added without placing them in vertical columns? Which way is the more convenient?

Are the tens all in the same column? In which column are the ones? The hundreds?

What is the sum of the ones? Where is the 8 written? What is done with the 1 ten?

What is the sum of the tens? What is done with the 0? With the 2 hundreds? Explain how the 9 thousands is obtained.

**2.** Find the sum of 372, 458, and 765.

| | | | |
|---|---|---|---|
| 372 = | 300 + | 70 + | 2 |
| 458 = | 400 + | 50 + | 8 |
| 765 = | 700 + | 60 + | 5 |
| 1595 = | 1400 + | 180 + | 15 |
| | or 1500 + | 90 + | 5 |

We here see that the sum of the ones is 15; the sum of the tens 18, or 180 ones; the sum of the hundreds 14, or 1400 ones. Since 15 = 1 ten and 5 ones, we have 18 + 1, or 19 tens, or 190 ones. And since 190 = 1 *hundred* and 90 ones (9 tens), we have 15 hundreds, or 1500 ones. Hence, 1500 + 90 + 5 = 1595, the sum.

DIRECTION.—*Arrange the numbers so that units of the same order shall stand in the same column.*

*Begin at the right and add each column separately. If the sum is less than* 10, *write it under the column added. If it is* 10 *or more, place only the right-hand figure under that column, and add the remainder with the next column.*

*To insure accuracy, add each column from top downwards as well as from bottom upwards.*

Write from dictation and add:

| 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|
| 476 | 418 | 374 | 293 | 416 | 567 |
| 358 | 736 | 627 | 431 | 907 | 891 |
| 924 | 375 | 258 | 827 | 592 | 234 |
| 265 | 632 | 196 | 546 | 375 | 432 |

When long columns are to be added, the following method will be found practical. Explain the process.

$$\begin{array}{r}
327 \\
841 \\
576 \\
439 \\
765 \\
\underline{192} \\
30 \\
31\phantom{0} \\
\underline{28\phantom{00}} \\
3140
\end{array}$$

Add in two minutes:

| 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|
| 5643 | 1234 | 9876 | 9 | 568 | 6677 |
| 7328 | 5678 | 5432 | 87 | 7329 | 896 |
| 3576 | 9123 | 2347 | 654 | 8695 | 54 |
| 8794 | 4567 | 678 | 3210 | 87 | 7 |
| 4887 | 8923 | 4706 | 123 | 9764 | 88 |
| 6452 | 4567 | 934 | 45 | 9 | 945 |
| 7968 | 9876 | 87 | 6 | 846 | 8776 |
| 5679 | 5432 | 5878 | 5000 | 6375 | 9438 |
| 9785 | 7777 | 6666 | 9999 | 8888 | 7897 |

Find the value of the following:

**15.** 728 + 2076 + 7583 + 7127 + 928 + 8267.
**16.** 83267 + 48 + 2498 + 6735 + 9476 + 10783.
**17.** 9847 + 3084 + 86540 + 4306 + 89 + 9008.
**18.** 63075 + 350 + 4703 + 3715 + 937 + 15008.
**19.** 16160 + 6048 + 260 + 4826 + 50598 + 8763.
**20.** 357 + 913 + 579 + 135 + 791 + 638 + 794 + 867.
**21.** 5894 + 876 + 93 + 4672 + 8975 + 8456 + 9784.
**22.** 9768 + 87 + 69 + 764 + 893 + 5768 + 88 + 8769.
**23.** Add 8568, 3864, 6846, 5976, 7249, 4839.
**24.** Add 3925, 6868, 4857, 8394, 8426, 9397.
**25.** Add 3987, 4876, 9254, 7983, 8427, 2935.
**26.** Add 5763, 3854, 87, 609, 975, 4508, 7358.
**27.** Add 4978, 9834, 734, 5627, 8, 3764, 47, 835.
**28.** Add 6984, 8592, 5807, 74, 96, 8958, 64, 789.
**29.** Add 5926, 7859, 4768, 8729, 384, 769, 8943.
**30.** Add 8592, 5678, 6854, 4673, 7968, 6843, 4396, 78.
**31.** Add 476, 5814, 6820, 21, 657.
**32.** Add 5215, 4863, 9211, 26781.
**33.** Add 4862, 9712, 48, 63, 14.
**34.** Add 97, 86741, 2248, 6127, 487, 57120.
**35.** Add 282, 4789, 63175.

## RAPID ADDITION.

In rapid addition various methods are employed by accountants, one of the most commonly used being what may be called the "grouping method," which is here briefly illustrated.

| * (a) | (b) | (c) | (d) | (e) | (f) | (g) | (h) |
|---|---|---|---|---|---|---|---|
| 5 | 9 | 6 | 9 | 8 | 7 | 4 | 7 |
| 2 | 1 | 1 | 3 | 9 | 3 | 6 | 5 |
| 3 | 8 | 7 | 8 | 7 | 1 | 9 | 2 |
| 7 | 6 | 4 | 6 | 6 | 6 | 2 | 8 |
| 3 | 1 | 5 | 4 | 2 | 4 | 3 | 6 |
| 8 | 3 | 3 | 9 | 5 | 2 | 7 | 9 |
| 6 | 4 | 6 | 5 | 8 | 4 | 1 | 4 |
| 4 | 5 | 8 | 7 | 6 | 2 | 4 | 6 |
| 38 | 37 | 40 | 51 | 51 | 8 | 5 | 8 |

In adding column (a), we may group two or more numbers whose sum is 10, and add thus: 10, 18, 28, 38.

In (b) the sum may be 10 or less, and we say, 9, 19, 27, 37.

In (c) we may say, 8, 17, 26, 33, 40. Or, making the sum greater than 10, we may add thus: 14, 26, 40.

In (d) we may proceed thus: 12, 25, 39, 51.

In (e) our count may be, 14, 27, 43, 51; or, beginning at the top, 17, 32, 45, 51.

* For the first practice in grouping, the teacher should provide numbers that readily fall into groups whose sum is 10, as in (a). These exercises should be followed by others where the sum is 10 or less, as in (b), and these in turn by more difficult ones where the sum of the group is from 10 to 19.

As a preparation for this work, the pupil should be given abundant practice in announcing at sight the sum of any two numbers between 10 and 20; as 11 + 11, 11 + 12, 12 + 13, etc. There are 81 of these combinations.

Exercises in rapid addition should be regular, not spasmodic. Surprising results may be obtained in a single term by devoting *five minutes each day* to intelligent practice.

**36.** Try to group the numbers in (f). Which do you find the better place to begin—top or bottom?

**37.** Group the numbers in (g), and add.

**38.** Group the numbers in (h) in as many ways as you can. Which way is best?

### DOUBLE-COLUMN ADDING.

Many persons add two columns at once, some three columns. The process is as follows:

```
 5731
 3287
 8629
 4856
 6943
 7664
 ----
  310
 368
 ----
37110
```

Beginning at the bottom we add tens and ones, saying (or thinking), 6-4, 10-7, 16-3 (that is, 15-13), 19-2 (or 18-12), 27-9, 31-0 (or 30-10). The sum is 31 tens and no ones.

When we get 10 or more ones, we "carry" the 1 ten over to the tens column. Thus, when our sums are 15 tens-13 ones, we say 16-3. In this way we avoid having to remember more than one figure in the right-hand column.

Taking the second two columns we count thus: 7-6, 14-5, 19-3, 27-9, 31-1, 36-8. The sum is 36 thousands and 8 hundreds.

As before, whenever we have a sum of 10 or more in the right-hand column, the 1 ten is added with the other column. Thus, instead of 13-15 we have 14-5; instead of 18-13 we have 19-3.

Add each of the following in two minutes:

| **39.** | **40.** | **41.** |
|---|---|---|
| 1 2 3 4 5 6 7 8 | 1 2 3 4 5 6 7 8 9 0 | 7 8 5 1 7 6 8 9 |
| 7 4 8 6 3 9 2 5 | 9 8 7 6 5 4 5 6 7 8 | 5 6 3 9 5 4 6 7 |
| 5 7 6 8 7 4 3 6 | 2 3 5 7 8 9 8 7 6 5 | 3 4 1 7 3 2 4 5 |
| 8 3 7 9 6 8 9 4 | 4 2 1 3 4 5 6 8 4 3 | 1 2 9 5 1 0 2 3 |
| 4 6 8 7 8 5 1 3 | 8 7 6 5 9 8 7 6 5 4 | 8 9 6 2 8 7 9 0 |
| 8 9 4 5 4 7 8 9 | 3 9 8 9 7 3 9 4 3 7 | 5 6 3 9 5 4 6 7 |
| 6 5 9 8 7 6 6 5 | 8 7 6 6 8 7 4 9 8 9 | 2 3 0 6 2 1 3 4 |
| 5 4 6 6 8 9 3 7 | 7 8 9 8 7 6 5 3 4 6 | 9 5 7 3 9 8 7 8 |
| 7 8 7 9 9 8 7 6 | 6 3 7 4 9 4 2 1 0 5 | 7 7 8 8 6 9 9 2 |
| 3 7 8 4 5 4 9 8 | 8 3 8 7 6 5 9 8 7 2 | 8 6 9 4 7 6 8 7 |
| 9 9 5 7 6 7 4 7 | 9 8 7 6 5 4 3 2 1 8 | 6 8 5 9 8 7 6 9 |
| 6 7 8 9 8 7 6 5 | 8 7 6 5 4 6 6 7 8 9 | 9 9 6 7 9 5 7 8 |

**42.** Add 382 thousand, 4 hundred 53; 514 thousand, 6 hundred 85; 684 thousand, 3 hundred 25; 298 thousand, 5 hundred 76; 176 thousand, 7 hundred 92.

**43.** Add 24 million, 356 thousand, 8 hundred 13; 92 million, 75 thousand, 3 hundred 46; 7 million, 310 thousand, 1 hundred 6; 30 million, 30 thousand, 3 hundred 30; 8 million, 8 thousand, 8 hundred 8.

**44.** Add 376 million, 724 thousand, 9 hundred 86; 4 million, 315 thousand, 8; 591 million, 304 thousand, 81; 79 million, 58 thousand, 627; 83 million, 726; 819 million, 10 thousand, 50.

**45.** Add six million, two thousand, five hundred forty-one; eight million, seven hundred thirty-eight thousand, four hundred three; one million, seven thousand, nine; thirty million, eighty-nine thousand, fourteen; five hundred nine thousand, eight hundred thirty.

**46.** Find the sum of two million, nine thousand, four hundred seventy-six; seven million, forty thousand, sixteen; three million, twenty-four; nine hundred three thousand, ten; six million, six thousand, six hundred six.

**47.** A railroad train ran 376 miles on Monday, 298 miles on Tuesday, 437 miles on Wednesday, 326 miles on Thursday, 265 miles on Friday, 368 miles on Saturday, and 20 miles on Sunday. How many miles did it run in the week?

**48.** In one year a man pays $375 for rent, $537 more than that for other expenses, and has $513 left out of his salary. What is his salary?

**49.** A farmer sold 289 bushels of corn to one man, 397 to another, 568 to another, 197 to another, and then had 685 bushels left. How many bushels had he at first?

**50.** A man owns five lots. The first cost $325, the second $275, the third $450, the fourth $580, and the fifth $240 more than the other four. How much did they all cost?

**51.** The area of New York is 49,170 square miles; of

Pennsylvania, 45,215; of New Jersey, 7,815; of Delaware, 2,050. What is the entire area of these four States?

**52.** A man traveled 328 miles a day for three days, and 276 miles a day for the next three days. How far did he travel in the six days?

**53.** The first year a man worked in an iron mill he received $450. If his salary was increased $125 a year, for four years, how much did he earn in the five years?

---

**54.** In a stock-yard there are as many cows as calves, and as many sheep as hogs. If there are 68 calves and 97 hogs, how many animals are in the yard?

**55.** The school-yard fence has 289 pickets on the front, and an equal number on the back. On each end there are 257. How many pickets are there on the whole fence?

**56.** One side of a square farm is 2854 feet long. What is the distance around the farm?

**57.** A book-case has six shelves. The first two contain 28 books each, the second two 34 each, and the last two 39 each. How many books are there on the six shelves?

**58.** A merchant sold goods to the amount of $485 on Monday, and during the remainder of the week he increased his sales $98 each day. What was the total value of his week's sales?

**59.** A girl has $a$ dollars, her brother has $b$ dollars more than she has, and their father has $b$ dollars more than both. How much have all three?

**60.** A train makes the round trip between Philadelphia and Pittsburg every two days. If the distance between these cities is 354 miles, how many miles does the train run in 4 days?

**61.** Mr. A bought a lot for $875, and sold it so as to gain $750. At what price did he sell?

**62.** A man owning a large tract of land divided all of it except 160 acres among his 4 sons and 3 daughters. To the eldest son he gave 320 acres, and to each of the others he gave 180 acres. To each of his daughters he gave 240 acres. How many acres were in the tract?

**63.** The number of lobsters caught in 1889 was as follows: Maine, 12,552,866; Massachusetts, 2,624,218; Connecticut, 687,994; Rhode Island, 538,315; New Hampshire, 176,733; New York, 206,875; New Jersey, 74,866; Delaware, 3,750. How many were caught?

**64.** The peanut crop of Virginia in 1890 was 1,171,624 bushels; of West Virginia, 39 bushels; of Georgia, 624,528 bushels; of Pennsylvania, 22 bushels; of New York, 106 bushels; of Illinois, 481 bushels; of Tennessee, 523,088 bushels; of North Carolina, 421,138 bushels. What was their combined production?

**65.** In the year ending January 1, 1895, Kentucky produced 183,618,425 pounds of tobacco; Virginia, 35,593,984 pounds; Ohio, 32,468,938 pounds; Massachusetts, 3,449,655 pounds; Maryland, 7,010,380 pounds; and West Virginia, 2,634,585 pounds. How many pounds did all six States produce?

**66.** The bank clearings June 11, 1900, in Charleston, S. C., were as follows: People's National Bank, $32,125; Bank of Charleston, $30,219; South Carolina Loan and Trust Co., $21,041; First National Bank, $25,968. What was the total clearings of these banks?

# SUBTRACTION.

**52.** **1.** If Kate has \$5 and spends \$3, how much has she left?

**2.** How many apples are left when 4 apples are taken from 6 apples?

**3.** Seven marbles are how many more than 5 marbles?

**4.** What number of cents added to 5 cents will make 9 cents?

**5.** How many dollars are left when \$5 are taken from \$8?

**53.** The number that is left when one number is taken from another is called the **Remainder** or **Difference.**

**54.** The process of finding the remainder or the difference between two like numbers is called **Subtraction.**

**1.** Can two yards be taken from 3 pounds? Why not?

**2.** What is the remainder when \$2 is taken from \$5? How does \$5 compare with the sum of \$2 and this remainder?

**3.** Subtract one number from another and see what the remainder added to the smaller number equals.

**55.** Principles.—1. *Numbers can be subtracted from like numbers only.*

2. *The larger number is equal to the sum of the remainder and the smaller number.*

**56.** The larger number, or the one from which another is subtracted, is called the **Minuend.**

**57.** The smaller number, or the one subtracted, is called the **Subtrahend.**

**58.** The **Sign of Subtraction** (−) is called *minus*, and is placed before the number to be snbtracted.

Thus, $5 - 3$ is read *5 minus 3*, and means that 3 is to be subtracted from 5.

**59.** Practice should be continued upon the following exercises until pupils can give quick and correct answers.

**1.** $1 + (\quad) = 2.$
**2.** $2 - 1 = (\quad).$
**3.** $1 + (\quad) = 3.$
**4.** $3 - 2 = (\quad).$
**5.** $3 - 1 = (\quad).$
**6.** $1 + (\quad) = 4.$
**7.** $4 - 3 = (\quad).$
**8.** $4 - 2 = (\quad).$
**9.** $4 - 1 = (\quad).$
**10.** $1 + (\quad) = 5.$
**11.** $5 - 4 = (\quad).$
**12.** $2 + (\quad) = 5.$
**13.** $5 - 3 = (\quad).$
**14.** $5 - 2 = (\quad).$
**15.** $6 - 5 = (\quad).$
**16.** $6 - (\quad) = 2.$
**17.** $6 - 3 = (\quad).$
**18.** $6 - (\quad) = 5.$
**19.** $7 - 6 = (\quad).$
**20.** $7 - (\quad) = 2.$
**21.** $7 - (\quad) = 3.$
**22.** $7 - 3 = (\quad).$
**23.** $7 - 2 = (\quad).$
**24.** $8 - 7 = (\quad).$
**25.** $8 - (\quad) = 2.$
**26.** $8 - (\quad) = 3.$
**27.** $8 - 4 = (\quad).$
**28.** $(\quad) - 3 = 5.$
**29.** $8 - (\quad) = 6.$
**30.** $9 - 8 = (\quad).$
**31.** $9 - (\quad) = 2.$
**32.** $9 - 6 = (\quad).$
**33.** $9 - (\quad) = 4.$
**34.** $(\quad) - 4 = 5.$
**35.** $9 - 3 = (\quad).$
**36.** $9 - (\quad) = 7.$
**37.** $10 - 9 = (\quad).$
**38.** $10 - (\quad) = 2.$
**39.** $10 - 7 = (\quad).$
**40.** $10 - 6 = (\quad).$
**41.** $10 - (\quad) = 5.$
**42.** $10 - 4 = (\quad).$
**43.** $10 - 3 = (\quad).$
**44.** $11 - 9 = (\quad).$
**45.** $11 - 8 = (\quad).$
**46.** $11 - (\quad) = 4.$
**47.** $11 - (\quad) = 5.$
**48.** $12 - 9 = (\quad).$
**49.** $12 - 8 = (\quad).$
**50.** $12 - 7 = (\quad).$
**51.** $12 - (\quad) = 6.$
**52.** $13 - 9 = (\quad).$
**53.** $13 - 8 = (\quad).$
**54.** $13 - 7 = (\quad).$
**55.** $13 - (\quad) = 7.$
**56.** $13 - 5 = (\quad).$

57. 13 − ( ) = 9.
58. 14 − 9 = ( ).
59. 14 − 8 = ( ).
60. 14 − ( ) = 7.
61. 14 − 6 = ( ).
62. 14 − ( ) = 9.
63. 15 − 9 = ( ).
64. 15 − 8 = ( ).
65. 15 − ( ) = 8.
66. 15 − 6 = ( ).
67. 16 − 9 = ( ).
68. 16 − ( ) = 8.
69. 16 − 7 = ( ).
70. 17 − 9 = ( ).
71. 17 − ( ) = 9.
72. 18 − 9 = ( ).
73. Subtract by 2's from 30. From 31 to 1.
74. Subtract by 3's from 36. From 34 to 1.
75. Subtract by 4's from 40. From 35 to 1.

**60.** In the following exercises name only *sums* and *differences*. Thus, in the first example, say, 8, 12, 7, 13, 6, 1.

Find the value of:

1. 8 + 4 − 5 + 6 − 7 − 5.
2. 7 − 3 + 8 − 5 − 4 + 6.
3. 4 + 5 − 6 + 8 − 7 + 9.
4. 9 − 7 + 6 + 5 − 8 − 3.
5. 3 + 8 − 6 + 9 − 6 − 5.
6. 7 + 6 + 2 − 6 + 7 − 9.
7. 8 + 9 − 4 − 7 + 12 − 9.
8. 17 − 8 + 6 − 7 + 9 − 8.
9. 7 + 9 − 8 + 5 − 7 + 14.
10. 6 + 8 − 9 + 6 − 4 + 17.
11. 5 + 11 − 7 − 5 + 9 − 8.
12. 9 − 7 + 8 + 6 − 7 + 11.

**61.** In the following name only remainders, thus: 46, 43, 38, 36, etc.

1. 50 − 4 − 3 − 5 − 2 − 4 − 3 − 5 − 6 − 4 − 3 − 2 − 5 = ( ).
2. 50 − 5 − 4 − 2 − 6 − 4 − 7 − 5 − 3 − 5 − 2 − 3 − 4 = ( ).
3. 50 − 3 − 5 − 5 − 3 − 2 − 3 − 5 − 5 − 6 − 4 − 7 − 2 = ( ).
4. 50 − 6 − 3 − 4 − 5 − 4 − 6 − 7 − 4 − 3 − 5 − 2 − 1 = ( ).
5. 50 − 7 − 4 − 5 − 6 − 3 − 4 − 2 − 3 − 5 − 4 − 3 − 3 = ( ).
6. 50 − 3 − 2 − 8 − 5 − 4 − 6 − 1 − 9 − 3 − 4 − 3 − 2 = ( ).

**62.** When several numbers are included in a **Parenthesis** ( ), they are to be treated as a single number.

Thus, (5 + 3) − (8 − 6) means that the difference between 8 and 6 is to be taken from the sum of 5 and 3.

The parenthesis as a sign of aggregation was first used by Girard in 1629.

**63.** Find the value of:

**1.** $8 + 5 - (3 + 6)$.
**2.** $7 + 9 + (6 - 4)$.
**3.** $15 - (2 + 7) - 4$.
**4.** $(17 - 9) - (13 - 7)$.
**5.** $(16 - 7) + 6 - 8$.
**6.** $15 - 6 + 5 - 7$.
**7.** $4 + 5 + 6 - (8 - 3)$.
**8.** $3 + 7 + (6 - 4) - 5$.
**9.** $5 + 8 - (9 + 7 - 6)$.
**10.** $(16 - 9) + 8 - (12 - 5)$.
**11.** $(16 - 9) + 8 + (12 - 5)$.
**12.** $(18 - 4) - (13 - 9 + 8)$.

**13.** Willie bought 12 marbles and gave 5 of them to his brother. How many did he keep?

**14.** Mary earns $12 a month and spends $7. How much does she save a month?

**15.** Tillie has 9 cents and Lottie has 4. How many more cents has Tillie than Lottie?

**16.** Howard wants to buy a book for 17 cents, but has only 9 cents. How many cents must he get before he can buy the book?

**17.** Two pieces of cloth contain 16 yards. If there are 7 yards in one piece, how many are in the other?

**18.** In a class of 14 boys and girls there are 5 boys. How many girls are in the class?

**19.** Henry paid $19 for a cart and sold it for $9. How much did he lose?

**20.** If a farmer who had 19 sheep sold 11, how many had he left?

**21.** Mrs. E had two ten-dollar bills, and spent $4 for shoes and $8 for a dress. How much money had she left?

**22.** A farmer sold 7 pigs and 2 more died. If he had 11 left, how many had he at first?

**23.** Carl is 11 years old and his sister is 7 years younger. How old is his sister?

**24.** Sarah is 6 years old and her brother is 14. What is the difference in their ages?

**25.** Alice has 12 plants, of which 7 are geraniums and the remainder pinks. How many pinks has she?

**26.** A hunter shot 15 rabbits and squirrels. If he shot 5 rabbits, how many squirrels did he shoot?

**27.** A man has \$19 in gold, silver, and paper. If he has 4 silver dollars and 5 paper dollars, how much has he in gold?

**28.** A boy has 18 rows of potatoes to hoe. In the forenoon he hoed 5 rows, in the afternoon 7 rows, and finished the next day. How many rows did he hoe the next day?

**29.** A has 19 acres of land and B has 9 acres. How many acres must B purchase from A so that B may have 10 acres more than A?

**30.** Harry had \$7, Carrie had \$9, and Ella had \$8 less than both together. How much had they all?

**31.** A lady who had 18 chickens sold 5 to one man and 8 to another, and gave a pair to her sister. How many had she left?

**32.** A farmer having 7 horses sold them, and bought 9 from one man and 6 from another. He afterwards sold 8, and 1 died. How many had he then?

**33.** James had $3a$ cents and spent $a$ cents. How many cents had he left?

**34.** Warren having $5b$ marbles lost $2b$ of them. How many had he left?

**35.** Hattie is $a$ years old and Martha is $b$ years old. What is the difference in their ages? $(a - b)$.

**36.** Walter has $a$ dollars, George has $2a$ dollars, and Albert has $b$ dollars less than both. How many dollars has Albert?

**37.** A lady who had $7a$ hens sold $a$ hens to one man and $4a$ hens to another. How many had she left?

**38.** Nettie weighs $a$ pounds, Ida weighs $b$ pounds less, and Mabel weighs $a$ pounds more than both. What is Mabel's weight?

**39.** Find the value of $a + 3a + a - 4a + b$.

**40.** What is the value of $5a + (3a - a) - 4a - 2b$?

**41.** A man having 16 sheep sold 9, and after buying some more had 13. How many did he buy?

**42.** Tom was 5 years old 10 years ago. How old was he 5 years ago?

**43.** Jerry was $a$ years old $b$ years ago. How old was he $a$ years ago?

**44.** David will be 18 years old 7 years hence. How old was he 7 years ago?

### WRITTEN EXERCISES.

**64. 1.** What is the difference between 978 and 435?

$$\begin{array}{r} 978 \\ \underline{435} \\ 543 \end{array}$$

Is it necessary to write the smaller number under the larger? Is it convenient?

Under what is the 5 ones written? Where is the 3 tens placed? The 4 hundreds?

5 ones from 8 ones leaves —— ones; 3 tens from 7 tens leaves —— tens; 4 hundreds from 9 hundreds leaves —— hundreds. The required difference is 543.

QUERY.—Is the sum of 543 and 435 equal to 978? Does that prove the work to be correct?

Find the differences, and prove:

| 2. | 3. | 4. | 5. |
|---|---|---|---|
| 84327 | 76594 | 69748 | $86.75 |
| 53124 | 43251 | 27413 | 14.32 |

| 6. | 7. | 8. |
|---|---|---|
| 83749676 | 99977856 | 66887795 |
| 51427343 | 65424351 | 34465672 |

| 9. | 10. |
|---|---|
| 9668987957845 | 8379568978694 |
| 4357534825532 | 1234567876543 |

**11.**

8978564837695874968977896708
3526153726460530733845421602

Find the value of:

**12.** 9889 − 6345 + 234.

**13.** 7658 − 324 − 5123.

**14.** 9768 − 43 − 6402.

**15.** 6995 − 4070 − 825.

**16.** 5683 − 5461 + 1012.

**17.** 7968 − 6725 + 3009.

**18.** 2436 + 7432 − 6143.

**19.** 3254 + 435 − 634.

**20.** 8776 − 4345 − 2231.

**21.** 9000 + 678 − 7075.

**22.** Mr. C bought a lot for \$5375 and sold it for \$6798. How much did he gain?

**23.** A farmer who raised 876 bushels of corn kept 243 bushels for his own use and sold the remainder. How many bushels did he sell?

**24.** A man was born in 1873 and married in 1898. At what age did he marry?

**25.** A contractor agreed to build a house for \$5875. If his expenses were \$3725, how much did he make?

**26.** In a school of 2879 pupils there are 1324 boys. How many girls are in the school?

**27.** A man gave a wagon and \$135 for a horse worth \$189. How much was he allowed for his wagon?

**28.** A and B were 198 miles apart. They traveled toward each other one day, A going 42 miles and B 35 miles. How far apart were they then?

**29.** Twelve years ago Ella's grandpa was 59 years of age. How old will he be if he lives till 1909?

**30.** Tom ran around a barn 45 feet long and 23 feet wide, and Pat ran 287 feet. How much farther did Pat run than Tom ran?

### ORAL EXERCISES.

**65.** **1.** Subtract by 5's from 49 to 4. From 47 to 2.

**2.** Subtract by 6's from 42 to 6. From 41 to 5.

**3.** Take 7 from 58 eight times. From 60.

**4.** Take 8 from 49 six times. From 51.

**5.** Subtract by 9's from 54 to 0. From 64 to 1.

**6.** Count by 7's from 2 to 65, and back from 65 to 2.

**7.** How many are 4 tens less 3 tens? 40 — 30?

**8.** How many are 7 tens less 4 tens? 70 — 40?

**9.** How many are 8 tens less 3 tens? 80 — 30?

**10.** How many are 9 tens less 6 tens? 90 — 60?

**11.** If you buy a dozen eggs for 17 cents, how much change should you get out of a quarter?

**12.** I paid $27 for a chair and a clock. If the chair cost $18, what did the clock cost?

**13.** From a cask containing 25 gallons, 16 gallons were drawn. How many gallons remained?

**14.** An agent purchased books for $17 and sold them for $32. How much did he gain?

**15.** A farmer sold a cow for $41, which was $13 more than the cost. Find the cost.

**16.** James earns $35 a month and spends $17. How much does he save monthly?

**17.** A man having 49 sheep sold 19 and killed 11. How many had he left?

**18.** A farmer having $21a$ sheep sold $13a$ and killed $2a$. How many had he left?

**19.** When the minuend and the remainder are given, how can the subtrahend be found? Why?

**20.** If the minuend is $7a$, and the remainder $2a$, what is the subtrahend?

**21.** When the remainder and subtrahend are given, how is the minuend found? Why?

**22.** If the remainder is $b$ and the subtrahend $5b$, what is the minuend?

**23.** Mr. B had a twenty-dollar bill, a ten-dollar bill, and a five-dollar bill. He bought a hat for $6 and a coat for $13. How much money had he left?

### WRITTEN EXERCISES.

**66.** **1.** Subtract 375 from 632.

$$\begin{array}{r} 632 \\ 375 \\ \hline 257 \end{array}$$

Since we cannot take 5 ones from 2 ones, we add 1 of the 3 tens, or 10 ones, to the 2 ones, making 12 ones. Then 5 ones from 12 ones leaves 7 ones.

Having taken 1 ten from the 3 tens, but 2 tens remain, from which we cannot take 7 tens. Hence, we take 1 of the 6 hundreds, or 10 tens, and add it to the 2 tens, making 12 tens. Then 7 tens from 12 tens leaves 5 tens.

From the 6 hundreds we have already taken 1 hundred, leaving 5 hundreds, from which we now take 3 hundreds.

QUERIES.—Is the sum of the subtrahend and the remainder equal to the minuend? What does that prove? If we say 5 and **7**, 12; 7 and 1 and **5**, 13; 3 and 1 and **2**, 6, we subtract by the common method of "making change." Can you explain the process?

**2.** From 543 take 296.

$$\begin{array}{l}543 = 500 + 40 + 3 = 500 + 30 + 13 = 400 + 130 + 13 \\ \underline{296} = 200 + 90 + 6 = 200 + 90 + 6 = \underline{200 + 90 + 6} \\ 247 = \qquad\qquad\qquad\qquad\qquad\qquad 200 + 40 + 7\end{array}$$

Subtract and prove:

| 3. | 4. | 5. | 6. | 7. | 8. | 9. |
|---|---|---|---|---|---|---|
| 211 | 311 | 773 | 521 | 626 | 889 | 342 |
| 199 | 279 | 539 | 234 | 481 | 691 | 146 |

| 10. | 11. | 12. | 13. | 14. | 15. | 16. |
|---|---|---|---|---|---|---|
| 817 | 824 | 609 | 508 | 712 | 793 | 690 |
| 677 | 459 | 368 | 146 | 299 | 679 | 664 |

| 17. | 18. | 19. | 20. | 21. | 22. | 23. |
|---|---|---|---|---|---|---|
| 820 | 924 | 873 | 856 | 964 | 903 | 735 |
| 698 | 554 | 297 | 344 | 299 | 315 | 285 |

| 24. | 25. | 26. | 27. | 28. | 29. | 30. |
|---|---|---|---|---|---|---|
| 355 | 871 | 664 | 124 | 110 | 485 | 672 |
| 179 | 73 | 493 | 97 | 20 | 142 | 210 |

| 31. | 32. | 33. | 34. | 35. | 36. | 37. |
|---|---|---|---|---|---|---|
| 527 | 956 | 415 | 133 | 384 | 694 | 955 |
| 329 | 492 | 116 | 42 | 115 | 295 | 61 |

| 38. | 39. | 40. | 41. | 42. | 43. | 44. |
|---|---|---|---|---|---|---|
| 469 | 783 | 865 | 614 | 468 | 821 | 613 |
| 252 | 694 | 572 | 156 | 79 | 209 | 108 |

Find the value of :

**45.** 73251 − 23679.

**46.** 52175 − 37896.

**47.** 64037 − 45069.

**48.** 30524 − 8765.

**49.** 54321 − 12345.

**50.** 756328 − 467439.

**51.** 231562 − 87968.

**52.** 936061 − 847076.

**53.** 238013 − 199815.

**54.** 832415 − 243749.

**55.** Subtract 236 from 500.

$$\begin{array}{l} 500 = 400 + 100 + 0 = 400 + 90 + 10 \\ \underline{236} = 200 + \phantom{0}30 + 6 = \underline{200 + 30 + \phantom{0}6} \\ 264 = \qquad\qquad\qquad\quad 200 + 60 + \phantom{0}4 \end{array}$$

We cannot take 6 ones from 0 ones, neither can we add one of the tens, for there are no tens. Hence, we take one of the hundreds and change it to 10 tens, then take one of the tens and change it to 10 ones, as illustrated in the operation.

Then 6 ones from 10 ones leaves 4 ones ; 3 tens from 9 tens leaves 6 tens ; 2 hundreds from 4 hundreds leaves 2 hundreds. Hence the remainder is 264.

Subtract as indicated and prove :

**56.** 7250 − 2894.

**57.** 5460 − 3291.

**58.** 8300 − 5867.

**59.** 6500 − 5678.

**60.** 7000 − 3269.

**61.** 8000 − 4753.

**62.** 6007 − 2308.

**63.** 12003 − 7036.

**64.** 37200 − 17501.

**65.** 17500 − 2351.

**66.** 91306 − 4007.

**67.** 10000 − 6789.

**68.** 1962057 − 873698.

**69.** 2123456 − 1745798.

**70.** 7300892 − 2006975.

**71.** 5437001 − 1008024.

**72.** 8200345 − 6832456.

**73.** 1000001 − 101002.

**74.** 6347543 − 945876.

**75.** 3702674 − 2803879.

**76.** 9008007 − 900809.

**77.** 7685342 − 6776387.

**78.** 4433225 − 3344668.

**79.** 8076539 − 8067845.

Find the value of :

**80.** 6792051 − (139678 + 2005128) − 1076053.

**81.** 8076004 − 139678 + 2005128 − 1076053.

**82.** 1000000 − 54321 − (326000 + 240807).

**83.** 7236408 – (6487543 – 5328796) – 4009007.

**84.** 9001 + (7200 – 6835) – (8003 – 3009) – 4372.

**85.** 59007 – (32046 + 3423) – (4008 + 13009 – 8729).

**86.** 43700 + 56300 – (3725 + 6275) – (90000 – 1234).

**87.** Add one million to the difference between four thousand seven and seven thousand four.

**88.** Subtract the difference between five thousand eighty-one and three thousand ninety-seven from the difference between four thousand nine and nine thousand four.

**89.** A man bought a farm for $17325 and sold it for $20000. How much did he gain?

**90.** How long is it since the discovery of America by Columbus in 1492?

**91.** A man owes $2150, but has only $975. How much must he borrow to pay the debt?

**92.** Mont Blanc is 15572 feet high, and is 3572 feet higher than Pike's Peak. What is the height of the latter?

**93.** Independence was declared in 1776. How long was that after the discovery of America?

**94.** The area of England is 50922 square miles, and that of Pennsylvania is 45215 square miles. How much larger is England than Pennsylvania?

**95.** Mr. B having $32700 gave his son $10320, and his daughter $8367. How much had he left?

**96.** In 1890 the population of Pittsburg was 238617, and that of Philadelphia was 1046964. Which had the larger population, and how much?

**97.** A train left Atlanta with 273 passengers. At the first station 24 got off and 9 got on; at the next 18 got off and 12 got on; at the third 17 left and 23 got on; at the fourth 69 got off. How many yet remained on the train?

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**98.** A, B, and C bought a store for $19325. A paid $6105, and B paid $753 more than A. How much did C pay?

**99.** A man had $10000 in bank. He bought a house for

$5670, paid $1125 for repairs, $37 for insurance, and then sold the property for $8500, putting the money in bank. What sum did he then have in bank?

**100.** Mr. B bought two horses, paying $120 for one and $145 for the other. He kept them six weeks, the feed for each costing him $13, and then sold one for $170 and the other for $210. How much did he gain?

**101.** The remainder is 1786, the subtrahend 2467. What is the minuend?

**102.** The remainder is $p$, the subtrahend $q$. What is the minuend?

**103.** The difference between two numbers is 796, and the larger number is 1275. What is the smaller number?

**104.** The difference between two numbers is $a$, and the larger number is $b$. What is the smaller number?

**105.** From what must 189 be taken to leave 981?

**106.** From what must $a$ be taken to leave $b$?

**107.** If the moon is 240000 miles from the earth, and the sun 92 million miles, how much farther is it to the sun than to the moon?

**108.** At an election there were 21635 votes cast for A, B, and C. A got 9675, B got 327, and C the remainder. How many people voted for C?

**109.** The signs + and − were used by Widman in an arithmetic published at Leipzig in 1489, and the symbol = by Recorde in an algebra published in 1557. How many years elapsed from the time + and − were first used until the = was used by Recorde?

**110.** The greatest depth of water yet measured is 29400 feet, and the greatest height to which a balloon has ascended is 37000 feet. By how many feet does the greatest height reached exceed the greatest depth measured?

**111.** The area of Virginia is 42450 square miles, and that of West Virginia is 24780. How many more square miles has Virginia than West Virginia?

**112.** The number of bushels of corn raised in the U. S. in 1889 was 1,924,185,000, and the number of bushels of wheat was 675,149,000. How many more bushels of corn were raised than of wheat?

**113.** In 1889 the total production of maple sugar in the U. S. was 32,952,927 pounds. In 1879 the production was 36,576,061 pounds. What was the decrease in 10 years?

**114.** The total oyster product of the U. S. in 1898 was 25,349,668 bushels. Maryland produced 10,282,752 bushels and Virginia 6,572,493 bushels. How many bushels did all the other states produce?

**Adding and subtracting equal numbers.**—If we add 10 to each side of the equation

$$12 + 8 - 9 = 25 - 16 + 2,$$

we have $12 + 8 - 9 + 10 = 25 - 16 + 2 + 10.$

Or, subtracting 6 from each of the equals in the same equation, we have

$$12 + 8 - 9 - 6 = 25 - 16 + 2 - 6.$$

**1.** Does adding equal numbers to equals destroy equality?

**2.** Does subtracting equal numbers from equals affect equality?

**67.** Principles.—1. *If equal numbers are added to equals, the sums are equal.*

2. *If equal numbers are subtracted from equals, the remainders are equal.*

## MULTIPLICATION.

**68. 1.** Suppose you have a roll of $5 bills and wish to know how many one-dollars you have. How can you find out?

**2.** Laying the bills out one by one, you may count by 5's, thus: 5, 10, 15, 20, 25, 30. In doing this you think merely of the *sum* of the addends, and not of the *number* of $5 bills.

**3.** Or you may first count the bills—1, 2, 3, 4, 5, 6. Six bills, each $5. How many *times* is $5 repeated to make your $30. Did you *think* of this *six* in the former process?

Observe that this process introduces the idea of *times*—an idea not present in addition.

**4.** Your money is measured in two ways, by $5 and by $1. When the unit of measure is $5, how many times is it repeated? How many when the unit is $1?

**5.** It should be remarked that this higher process is founded upon addition. We learn that six 5's are 30 (ones) by first finding the *sum*, and then noticing *how many times* the 5 is repeated to make that sum.

**6.** Six times 2 quarts are how many "one-quarts"? How many "six-quarts"?

**7.** A man measuring his corn filled a 2-peck measure 8 times; how much corn had he?

**8.** In the last example what is the unit of measure? How many times was it taken? What quantity did he measure? Did he wish to find how many "two-pecks" he had, or how many *pecks?*

**9.** Ella bought 10 yards of cloth at $2 a yard. How much did it cost her?

How many times is \$2 repeated or taken to make \$20? What is here used as a unit of measure? Is \$2 itself a measured quantity? By what unit is it measured?

The cost is 10 × \$2—ten units of \$2 each—but by the process of multiplication we change this to \$20, that is, to 20 units of \$1 each.

**69.** The process of taking one number as many times as there are units in another is called **Multiplication.**

**70.** The number multiplied by another—the number taken so many times—is called the **Multiplicand.** It is regarded as a unit of measure.

**71.** The number that tells how many times the multiplicand is taken is called the **Multiplier.** It is pure number.

**72.** The result obtained by multiplying is called the **Product.** It is regarded as measured quantity.

**73.** The multiplicand and multiplier are the **Factors** of the **product.**

**74.** The product of two numbers is the same whichever is taken as the multiplier.

Of course it is absurd to say that 3 times \$5 is the same as \$5 times 3; but the meaning is that

$$3 \times \$5 = 3 \times 5 \times \$1 = 5 \times 3 \times \$1 = 5 \times \$3.$$

**75.** The **Sign of Multiplication** (×) is read **times** when it follows the multiplier, and **multiplied by** when it precedes the multiplier.

Thus, 3 × \$5 is read 3 times \$5; \$5 × 3 is read \$5 multiplied by 3. This symbol was first used by Oughtred in 1631.

**1.** When we multiply \$5 by 3, is the multiplicand concrete or abstract? Is the multiplier concrete? Is the product?

**2.** Can we multiply \$4 by \$3? 4 by \$3? \$5 by 2 feet? By what can we multiply \$5?

**76.** PRINCIPLES.—1. *The multiplier is always an abstract number.*

2. *The product is always like the multiplicand.*

MULTIPLICATION TABLE.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 |
| 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 |
| 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
| 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 |
| 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 | 77 | 84 |
| 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 | 96 |
| 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 | 99 | 108 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | 110 | 121 | 132 |
| 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 |

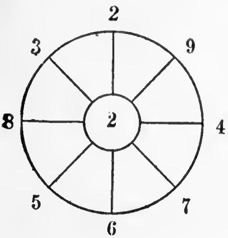
This table may be used in two ways. Find out the two ways.

**77.** The following exercises should be practiced daily until the pupil is able to announce at sight the product of any two numbers from 2 to 12.

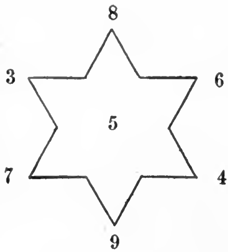
Announce products at sight :

| | | | | |
|---|---|---|---|---|
| **1.** 5 × 7 | 4 × 9 | 2 × 11 | 7 × 2 | 11 × 4 |
| **2.** 9 × 5 | 3 × 6 | 3 × 9 | 6 × 10 | 7 × 8 |
| **3.** 5 × 11 | 7 × 10 | 3 × 12 | 7 × 4 | 2 × 9 |
| **4.** 4 × 4 | 6 × 3 | 8 × 6 | 10 × 5 | 9 × 8 |
| **5.** 5 × 3 | 4 × 5 | 4 × 12 | 7 × 6 | 11 × 11 |
| **6.** 7 × 9 | 5 × 8 | 6 × 5 | 8 × 7 | 4 × 6 |
| **7.** 5 × 6 | 3 × 8 | 10 × 2 | 5 × 9 | 11 × 8 |

| | | | | | |
|---|---|---|---|---|---|
| **8.** | 6 × 4 | 3 × 7 | 6 × 12 | 8 × 10 | 9 × 7 |
| **9.** | 5 × 5 | 12 × 3 | 8 × 8 | 7 × 5 | 9 × 9 |
| **10.** | 6 × 6 | 7 × 12 | 10 × 11 | 12 × 5 | 10 × 4 |
| **11.** | 9 × 4 | 10 × 3 | 8 × 4 | 4 × 11 | 3 × 11 |
| **12.** | 4 × 10 | 12 × 11 | 10 × 7 | 7 × 3 | 11 × 6 |
| **13.** | 9 × 2 | 8 × 3 | 10 × 6 | 6 × 8 | 12 × 12 |
| **14.** | 4 × 8 | 10 × 10 | 8 × 8 | 5 × 10 | 8 × 11 |
| **15.** | 7 × 7 | 12 × 9 | 4 × 7 | 8 × 12 | 10 × 9 |
| **16.** | 9 × 12 | 11 × 3 | 12 × 8 | 9 × 10 | 12 × 10 |
| **17.** | 6 × 11 | 2 × 12 | 11 × 9 | 12 × 4 | 6 × 9 |
| **18.** | 5 × 12 | 11 × 10 | 10 × 8 | 9 × 11 | 6 × 7 |
| **19.** | 8 × 5 | 12 × 7 | 7 × 11 | 5 × 4 | 12 × 2 |
| **20.** | 5 × 11 | 11 × 2 | 12 × 6 | 10 × 12 | 11 × 12 |




For further practice in rapid work place these diagrams on the blackboard, and then, using the numbers at the centers as multipliers, indicate the multiplicands with the pointer.




ORAL EXERCISES.

**78. 1.** At $4 a day, how much can I earn in 6 days?

**2.** There are 12 things in a dozen. How many things are in 8 dozen?

**3.** How much will 9 yards of calico cost at 6 cents a yard?

**4.** How far does a man walk in 10 hours at the rate of 4 miles an hour?

**5.** At $6 each, how much must be paid for 12 sheep?

**6.** Ben picked 9 quarts of cherries, and Fred picked 8 times as many. How many quarts did Fred pick?

**7.** If my hens lay 7 eggs each day, how many will they lay in 9 days?

**8.** There are 8 quarts in a peck. How many quarts in 7 pecks? In a bushel?

**9.** Frank went to the post office 8 days, and each time got no letters. How many letters did he get in the 8 days? Then $8 \times 0 =$ what?

**10.** A man bought a dozen slates at 10 cents each. How much change did he get out of a two-dollar bill?

**11.** Find the cost of 8 yards of velvet at $5 a yard.

(a). Since one yard costs $5, 8 yards cost 8 times $5, or $40.

(b). At $1 a yard 8 yards would cost $8; hence, at $5 a yard, the cost is 5 times $8, or $40.

Note.—The pupil should compare these solutions and make himself thoroughly familiar with the process in each.

**12.** James bought 15 papers at 3 cents each and sold them for 5 cents each. How much did he make?

**13.** Dick earns $23 a week, and Charles earns $10 a week. In 6 weeks Dick earns how much more than Charles?

**14.** Two men start from the same place, one going west at the rate of 7 miles an hour, the other going east at the rate of 4 miles an hour. How far apart are they in 4 hours?

**15.** John has twice as many apples as Mark. How many have both, if Mark has 2 times 8 apples?

**16.** A and B start from Boston and travel in the same direction, A going 25 miles an hour, and B 11 miles. How far are they apart in 3 hours?

**17.** A has $4, B has 3 times as much, and C has twice as much as B. How much have they all?

**18.** How much will 5 pencils cost at $b$ cents each?

**19.** If a man walks $a$ miles an hour, how far will he walk in $b$ hours?

**20.** A man having $12a$ dollars bought 5 sheep at $2a$ dollars each. How much had he left?

**21.** If a man spends $b$ dollars a day for $c$ days, how much does he spend?

**22.** A man having $a$ dollars paid $p$ dollars each for $q$ cows. How much had he left?

**23.** Harry has $p$ marbles, Ira has $q$ marbles, and Sam has $r$ times as many as both together. How many has Sam?

**24.** When hats are worth \$5 each, how much are 3 hats worth? How do you know?

**25.** Why do 3 hats cost 3 times \$5?

**26.** Complete the following: When hats are worth *a* dollars each, 7 hats are worth —— dollars, because 7 hats are worth —— as much as ——.

**79.** Announce products at sight:

| | | |
|---|---|---|
| **1.** 3 × 4 × 6. | **9.** 2 × 5 × 12. | **17.** 3 × 3 × 11. |
| **2.** 5 × 3 × 2. | **10.** 3 × 5 × 4. | **18.** 4 × 3 × 12. |
| **3.** 6 × 3 × 2. | **11.** 4 × 4 × 3. | **19.** 2 × 2 × 20. |
| **4.** 4 × 5 × 3. | **12.** 8 × 2 × 2. | **20.** 3 × 5 × 3. |
| **5.** 2 × 7 × 2. | **13.** 9 × 2 × 3. | **21.** 8 × 3 × 2. |
| **6.** 3 × 6 × 3. | **14.** 6 × 4 × 2. | **22.** 5 × 5 × 3. |
| **7.** 2 × 3 × 11. | **15.** 5 × 6 × 3. | **23.** 2 × 5 × 11. |
| **8.** 4 × 2 × 12. | **16.** 7 × 3 × 2. | **24.** 6 × 3 × 2. |

### WRITTEN EXERCISES.

**80. 1.** Multiply 365 by 4.

(a)

$$
\begin{array}{rcrcrcr}
365 & = & 300 & + & 60 & + & 5 \\
4 & = & & & & & 4 \\
\hline
1460 & = & 1200 & + & 240 & + & 20 \\
 & \text{or} & 1200 & + & 260 & + & 0 \\
 & \text{or} & 1400 & + & 60 & + & 0
\end{array}
$$

(b)

$$
\begin{array}{r}
365 \\
365 \\
365 \\
365 \\
\hline
1460
\end{array}
$$

4 times 5 ones are 20 ones, or 2 tens and no ones. We write the 0 in ones' place in the product, and add the 2 tens to 4 times 6 tens, making 26 tens, or 2 hundreds and 6 tens. We write the 6 in tens' place in the product, and add the 2 hundreds to 4 times 3 hundreds, making 14 hundreds, or 1 thousand and 4 hundreds.

How is the same result obtained in (b)? Why do we write the multiplier under the multiplicand? Find by trial if you can multiply, beginning *at the left.*

Multiply the following:

| | | |
|---|---|---|
| **2.** 253 by 3. | **8.** 697 by 8. | **14.** 287 by 9. |
| **3.** 426 by 3. | **9.** 853 by 9. | **15.** 913 by 8. |
| **4.** 736 by 4. | **10.** 437 by 4. | **16.** 368 by 7. |
| **5.** 594 by 6. | **11.** 978 by 5. | **17.** 627 by 6. |
| **6.** 867 by 5. | **12.** 634 by 6. | **18.** 598 by 5. |
| **7.** 539 by 7. | **13.** 549 by 7. | **19.** 817 by 9. |

**20.** 62, 417, 685 by 4.
**21.** 85, 236, 417 by 6.
**22.** 47, 395, 173 by 5.
**23.** 18, 236, 456 by 7.
**24.** 54, 923, 687 by 9.
**25.** 37, 281, 459 by 8.
**26.** 98, 765, 432 by 6.
**27.** 24, 681, 357 by 9.
**28.** 36, 925, 814 by 8.
**29.** 62, 847, 418 by 7.
**30.** 56, 847, 389 by 5.
**31.** 12, 345, 679 by 9.

**81.** In expressions like $4 + 2 \times 3$, the operation indicated by the sign $\times$ must be first performed.

Thus, $4 + 2 \times 3$ means $4 + 6$, not $6 \times 3$. $18 - 8 \times 2$ means $18 - 16$, not $10 \times 2$; but $(18 - 8) \times 2$ means $10 \times 2$. $(4 + 2) \times 3 = 18$; but $4 + 2 \times 3 = 10$. Of course $4 + (2 \times 3) = 10$, but when two or more numbers are connected by the sign $\times$ the parentheses are superfluous.

Find the value of:

**1.** $6 + 4 \times 2$.
**2.** $7 + 3 \times 6$.
**3.** $2 + 8 \times 3$.
**4.** $8 - 3 \times 2$.
**5.** $16 - 5 \times 3$.
**6.** $12 + 4 \times 7$.
**7.** $20 - 9 \times 2 + 3 \times 5 - 7$.
**8.** $25 + 4 \times 3 - 6 \times 6 + 9$.
**9.** $12 \times 4 - 3 \times 7 + 10 \times 5$.
**10.** $12 \times (4 - 3) \times 7 + 10 \times 5$.
**11.** $36 + 175 \times 4 - 89 \times 6 - 200$.
**12.** $1000 - 97 \times 8 + 1 - 3 \times 75$.

**82. To multiply by 10, 100, 1000, etc.**

**1.** When a number is multiplied by 10, what is annexed to it?

Then what is the shortest method of finding 10 times any number?

**2.** When a number is multiplied by 100, what is annexed to it?

Then what is the shortest method of finding 100 times any number?

**3.** When the multiplier is 10, 100, 1000, etc., how many ciphers must be annexed to the multiplicand? Find out by trial.

**83.** PRINCIPLE.—*A number is multiplied by* 10, 100, 1000, *etc., by annexing as many ciphers to the multiplicand as there are ciphers in the multiplier.*

WRITTEN EXERCISES.

**84.** Copy, and complete the equations :

| | | |
|---|---|---|
| **1.** $25 \times 10 = (\quad)$. | $37 \times 100 = (\quad)$. | $59 \times 1000 = (\quad)$. |
| **2.** $50 \times 10 = (\quad)$. | $175 \times 100 = (\quad)$. | $324 \times 1000 = (\quad)$. |
| **3.** $100 \times 10 = (\quad)$. | $387 \times 100 = (\quad)$. | $872 \times 1000 = (\quad)$. |
| **4.** $416 \times 10 = (\quad)$. | $600 \times 100 = (\quad)$. | $903 \times 1000 = (\quad)$. |
| **5.** $708 \times 10 = (\quad)$. | $914 \times 100 = (\quad)$. | $1000 \times 1000 = (\quad)$. |

**6.** Multiply 743 by 5000.

$$\begin{array}{r} 743000 \\ 5 \\ \hline 3715000 \end{array}$$

5000 is 5 times 1000. Hence we first annex 3 ciphers, which multiplies 743 by 1000, and then multiply by 5.

Could we multiply first by 5 and then by 1000 ?

Find the products of :

| | | |
|---|---|---|
| **7.** $578 \times 30$. | **12.** $924 \times 700$. | **17.** 8125 by 6000. |
| **8.** $639 \times 20$. | **13.** $538 \times 400$. | **18.** 5346 by 8000. |
| **9.** $825 \times 40$. | **14.** $190 \times 600$. | **19.** 4325 by 9000. |
| **10.** $347 \times 50$. | **15.** $467 \times 800$. | **20.** 6913 by 7000. |
| **11.** $718 \times 60$. | **16.** $286 \times 900$. | **21.** 9000 by 6000. |

**85. To find the product when the right-hand figure of the multiplier is significant ; that is, is not *0*.**

**1.** Multiply 327 by 245.

$$\begin{array}{rcl} 327 & & \\ 245 & = & 200 + 40 + 5 \\ \hline 1635 & = & 5 \times 327 \\ 13080 & = & 40 \times 327 \\ 65400 & = & 200 \times 327 \\ \hline 80115 & = & 245 \times 327 \end{array} \qquad \begin{array}{l} \phantom{00}327 \\ \phantom{00}245 \\ \hline \phantom{0}1635 \\ 1308 \\ 654 \\ \hline 80115 \end{array}$$

The multiplier $245 = 200 + 40 + 5$. Hence we multiply first by 5, then by 40, then by 200, and then add the partial products.

Or, since 40 is 4 tens, we may multiply by 4 tens instead of 40. The product is 1308 *tens*, hence the 8 must be written under the tens, and the ones' place left vacant. Since 200 is 2 hundreds, we may multiply by 2 hundreds instead of 200. The product is 654 *hundreds*, hence the right-hand figure must be written under hundreds, leaving vacant two places to the right.

DIRECTION.—*Write the multiplier under the multiplicand, ones under ones, tens under tens, etc. Multiply the multiplicand first by the ones of the multiplier, then by the tens, and so on, placing the right-hand figure of each product directly under the figure of the multiplier used to obtain it; then add the several products thus obtained.*

To insure accuracy, review the work carefully, or multiply the multiplier by the multiplicand, and compare results.

WRITTEN EXERCISES.

Find the product of:

**2.** 375 × 24.
**3.** 628 × 35.
**4.** 296 × 16.
**5.** 418 × 29.
**6.** 537 × 32.
**7.** 704 × 58.
**8.** 650 × 43.
**9.** 486 × 74.
**10.** 825 × 86.
**11.** 943 × 235.
**12.** 627 × 364.
**13.** 548 × 621.
**14.** 703 × 513.
**15.** 864 × 207.
**16.** 597 × 648.
**17.** 486 × 753.
**18.** 398 × 462.
**19.** 987 × 789.
**20.** 8076 × 607.
**21.** 4109 × 385.
**22.** 7340 × 468.
**23.** 5897 × 903.
**24.** 8645 × 795.
**25.** 9857 × 606.
**26.** 7469 × 729.
**27.** 7208 × 905.
**28.** 8900 × 547.

Multiply as indicated:

**29.** 37 × 425 × 8000.
**30.** 72 × 618 × 6000.
**31.** 43 × 703 × 5000.
**32.** 40 × 900 × 7823.
**33.** 83 × 750 × 8600.
**34.** 69 × 583 × 4009.
**35.** 456 × 723 × 800.
**36.** 925 × 860 × 400.
**37.** 378 × 409 × 700.
**38.** 846 × 520 × 290.
**39.** 750 × 693 × 804.
**40.** 623 × 954 × 287.

**41.** 421896 × 215.
**42.** 373842 × 327.
**43.** 654083 × 456.
**44.** 937584 × 274.
**45.** 160835 × 621.
**46.** 395827 × 768.
**47.** 284936 × 492.
**48.** 516057 × 579.
**49.** 439104 × 806.
**50.** 182564 × 975.
**51.** 869375 × 658.
**52.** 82047 × 4306.
**53.** 47638 × 5219.
**54.** 68049 × 3168.

**55.** 17583 × 6574.
**56.** 38065 × 4018.
**57.** 92736 × 8539.
**58.** 72069 × 3964.
**59.** 18254 × 9651.
**60.** 75864 × 3972.
**61.** 68357 × 4098.
**62.** 37528 × 6573.

Find the value of :

**63.** (763 + 815) × (5403 − 765) − 860 × 1354.
**64.** 876 × 234 − 98 + 56 × 1009 − 12895 × 8.
**65.** (345721 − 18693) × 8700 − 63594 × 27.
**66.** (5389 − 76 × 59) × 86 − (51839 + 11 × 1987).
**67.** (5678 − 79) × 84 × 68 − 52389 − 8760 × 937.
**68.** 62007 × 503 − 8460 × 790 − 37625 × 89.
**69.** 4587 + 7854 × 79000 − 78 × (673005 + 378900).
**70.** (99 − 88) × 180 + 10000 − 77 × 66 − 179 × 38.

**71.** I paid $2678 for 426 bushels of clover seed, which I sold at $7 a bushel. How much did I gain?

**72.** John drives the cows home to be milked every morning and evening. If the pasture is 80 rods from home, how far does John travel in 4 weeks?

**73.** Two trains left Chicago at the same time, one going west at the rate of 36 miles an hour, the other east at the rate of 28 miles an hour. How far apart were they in 17 hours?

**74.** A has $95, B has three times as much, and C has twice as much as both. How much more than $1000 have they all?

**75.** There are 5280 feet in a mile. If steel rails weigh 36 pounds to the foot, what is the weight of the rails in two miles of double-track railway?

**76.** If a cow is worth 7 sheep, and a horse is worth 6 cows, how many sheep are worth as much as 19 horses and 28 cows?

**77.** A boy who lives 167 rods from the schoolhouse goes to school regularly. If he goes home for dinner every day, how far does he walk in a week?

**78.** A drover bought 286 sheep for $1430. After feeding them a week at an expense of \$175, he sold 97 of them at \$8 each, and 10 of them died. He sold the remainder for \$900. How much did he gain on all?

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**79.** A lady bought 15 yards of calico at 6 cents a yard, 3 yards of velvet at \$1.49 a yard, and 27 yards of muslin at 8 cents a yard. She gave the clerk a ten-dollar bill. How much change should she receive?

**80.** If the bill just mentioned was counterfeit, how much did the store lose?

**81.** Mr. A bought 17 five-dollar bills at 2 pigs each. How many pigs did he pay for them?

**82.** Since the product is the sum of two or more equal numbers, one of which is the multiplicand, how may the multiplier be found when the product and multiplicand are given?

**83.** When the product is 36, how often can the multiplicand 4 be subtracted from the product? Then what is the multiplier?

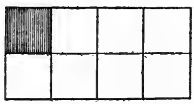
**84.** The product is 90, the multiplicand 6. What is the multiplier?

**85.** The multiplicand is $18a$, the product $54a$. What is the multiplier?

**86.** When the product is $45b$ and the multiplicand $5b$, what is the multiplier?

**87.** How many groups of $32p = 192p$?

**86. 1.** If a rectangle 4 ft. long and 2 ft. wide is divided as in the figure, what will each of the small squares represent? What is the area of one of the strips? ($4 \times 1$ sq. ft.). Then what is the area of the two strips, or of the rectangle?



**2.** How many square feet in the surface of a table 5 ft. long and 3 ft. wide?

**3.** What is the area of a rectangular field 40 rods long and 35 rods wide?

**4.** How many square feet of carpet will cover a room 16 feet square?

**5.** A rectangle is 8 feet long and 6 feet wide. What is its area?

**6.** What is the area of a rectangle $a$ feet long and $b$ feet wide?

**7.** How many square feet of oilcloth will cover a floor $b$ feet square?

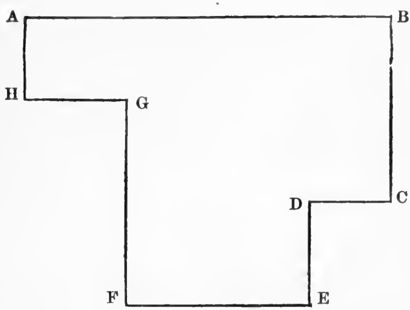
**8.** The area of a rectangle 5 feet long is 20 square feet. How wide is it?

**9.** How many square miles in a tract of land 300 miles long and half as wide?

**10.** In this figure let AB = 16 ft., BC = 8 ft., DE = 5 ft., EF = 7 ft., FG = 9 ft., GH = 5 ft.

(a) Find the length of DC and of AH.

(b) Divide the figure into rectangles and find its area.




**11.** The top of a desk is a rectangle, 5 feet long and 3 feet wide. What is its area in square feet?

**12.** How much available storage space is in a building 300 feet long, 150 feet wide, and six stories high?

## DIVISION.

**87.** **1.** What number multiplied by 3 gives $15 as a product

**2.** By what must 6¢ be multiplied to give 18¢ as a product?

**3.** If a man earns $15 in 3 days, how much is that a day? 3 × ($——) = $15.

**4.** Product 42, one factor 7; what is the other factor?

**5.** In multiplication two factors are given to find the product. In division the product and one factor are given. What is to be found?

**88.** The process of finding either factor when the other factor and their product are given is called **Division.**

**89.** One of the two factors is **one of the equal parts** of a quantity or number, the other being the **number** of equal parts. Hence division includes—

(A). Finding how many equal parts of a given size compose a number.

This involves the idea of **measuring, being contained in,** as seen in the following exercises:

**1.** Divide a 12-foot line into equal parts, each 3 feet in length. *How many equal parts* are there? How many 3's in 12?

**2.** There are 10 apples on a plate. How many times can you take 2 apples away? How many 2's in 10?

**3.** How many groups of 4 books each can be made from 24 books? How many 4's in 24?

**4.** How many five-dollar bills make \$20, or how many times is \$5 contained in \$20?

**5.** A man has \$40 in packages of \$5 each. How many packages has he? Of how many equal parts is his \$40 made up, or composed?

**6.** How many times must we take \$5 to make \$40?

8 × \$5 = \$40; then \$40 ÷ \$5 = ——. That is, \$5 is contained 8 times in \$40.

(B). Finding one of the equal parts of a number.

This involves the idea of **separation, partition,** as may be seen in the following exercises:

**1.** Divide 6 feet into 2 equal parts. What is the length of each part?

**2.** What is one of the 4 equal parts of \$12? \$12 ÷ 4 = \$——.

**3.** What must be multiplied by 3 to give \$18 as a product? \$18 ÷ 3 = \$——.

**4.** What is the weight of each package when 20 lb. of butter is divided into 4 equal parts? 20 lb. ÷ 4 = —— lb.

**5.** When \$40 is divided equally among 8 boys, how much does each boy get? What is one of the 8 equal parts of \$40? \$40 ÷ 8 = \$——.

**90.** When the product and given factor are like numbers, we have the process mentioned in (A); that mentioned in (B) applies when they are unlike numbers.

**91.** The given factor shows either the number of equal parts or the size of them, and is called the **Divisor.**

**92.** The factor to be found is called the **Quotient.**

**93.** The given product is called the **Dividend.** It is the sum of the equal parts.

**1.** When books are \$2 apiece, how many can you buy for \$9? How much money will you have left?

**2.** How many 3's in 9? How many in 11? How many left over?

**3.** When you take three 5's from 17, how many remain?

**4.** When you take 5 cents from 17 cents three times, how many cents remain? Is the remainder a part of the 17 cents? Is it greater or less than the 5 cents?

**94.** The number left over, or remaining, when the division is not exact is called the **Remainder.** It is a part of the dividend, and always less than the divisor.

**95.** The **Sign of Division** (÷) is read *divided by*, and when placed between two numbers shows that the one on the left is to be divided by the one on the right.

Thus, 24 ÷ 3 is read *twenty-four divided by three.* Division may also be indicated by writing the divisor under the dividend, thus, $\frac{24}{3}$.

This sign of division (÷) was first used by Rahn in an algebra published at Zurich in 1659.

**96.** *The product of divisor and quotient, plus the remainder, if any, is equal to the dividend.*

**1.** $5 \times \$4 =$ how many dollars?

**2.** How many times $\$4 = \$20$? $\$20 \div \$4 =$ what?

**3.** In example 1, what is the product? What is the dividend in example 2?

**4.** Name the multiplicand in example 1. In example 2, what is the divisor?

**97.** Division is the *converse of multiplication.* Dividend corresponds to product, divisor to one factor, and quotient to the other. Hence to find a quotient we need only recall how many of the divisors equal the dividend.

Thus, to find the quotient of 18 ÷ 3, we recall the fact that *six* 3's are 18; hence we know that 6 is the quotient.

**98.** Name quotients at sight :

| | | | |
|---|---|---|---|
| **1.** 24 ÷ 4. | **16.** 35 ÷ 5. | **31.** 28 ÷ 4. | **46.** 48 ÷ 4. |
| **2.** 45 ÷ 5. | **17.** 36 ÷ 4. | **32.** 30 ÷ 6. | **47.** 60 ÷ 5. |
| **3.** 36 ÷ 6. | **18.** 49 ÷ 7. | **33.** 77 ÷ 7. | **48.** 96 ÷ 8. |
| **4.** 35 ÷ 7. | **19.** 54 ÷ 6. | **34.** 40 ÷ 8. | **49.** 108 ÷ 9. |
| **5.** 60 ÷ 5. | **20.** 63 ÷ 9. | **35.** 36 ÷ 3. | **50.** 110 ÷ 10. |
| **6.** 45 ÷ 9. | **21.** 81 ÷ 9. | **36.** 27 ÷ 9. | **51.** 72 ÷ 12. |
| **7.** 32 ÷ 8. | **22.** 42 ÷ 6. | **37.** 56 ÷ 8. | **52.** 121 ÷ 11. |
| **8.** 42 ÷ 7. | **23.** 48 ÷ 8. | **38.** 72 ÷ 8. | **53.** 48 ÷ 12. |
| **9.** 32 ÷ 4. | **24.** 63 ÷ 7. | **39.** 88 ÷ 8. | **54.** 36 ÷ 3. |
| **10.** 40 ÷ 4. | **25.** 72 ÷ 9. | **40.** 90 ÷ 9. | **55.** 60 ÷ 12. |
| **11.** 48 ÷ 6. | **26.** 70 ÷ 7. | **41.** 84 ÷ 7. | **56.** 132 ÷ 11. |
| **12.** 36 ÷ 9. | **27.** 66 ÷ 6. | **42.** 99 ÷ 9. | **57.** 80 ÷ 10. |
| **13.** 56 ÷ 7. | **28.** 72 ÷ 6. | **43.** 90 ÷ 10. | **58.** 120 ÷ 12. |
| **14.** 60 ÷ 6. | **29.** 44 ÷ 4. | **44.** 28 ÷ 7. | **59.** 144 ÷ 12. |
| **15.** 64 ÷ 8. | **30.** 80 ÷ 8. | **45.** 27 ÷ 3. | **60.** 132 ÷ 12. |

Name quotients and remainders :

| | | | | |
|---|---|---|---|---|
| **61.** 6)44 | **71.** 8)83 | **81.** 6)51 | **91.** 4)51 | **101.** 8)95 |
| **62.** 4)35 | **72.** 4)46 | **82.** 4)22 | **92.** 7)89 | **102.** 11)105 |
| **63.** 9)56 | **73.** 7)75 | **83.** 6)39 | **93.** 9)87 | **103.** 12)107 |
| **64.** 8)26 | **74.** 9)66 | **84.** 9)38 | **94.** 8)91 | **104.** 12)117 |
| **65.** 5)38 | **75.** 8)30 | **85.** 8)47 | **95.** 9)53 | **105.** 10)126 |
| **66.** 8)49 | **76.** 9)48 | **86.** 9)78 | **96.** 7)83 | **106.** 12)149 |
| **67.** 9)29 | **77.** 8)52 | **87.** 5)63 | **97.** 8)61 | **107.** 11)138 |
| **68.** 7)65 | **78.** 3)37 | **88.** 6)77 | **98.** 9)103 | **108.** 9)86 |
| **69.** 6)63 | **79.** 4)47 | **89.** 3)29 | **99.** 8)101 | **109.** 7)69 |
| **70.** 7)72 | **80.** 5)58 | **90.** 8)99 | **100.** 6)79 | **110.** 12)141 |

### ORAL EXERCISES.

**99.** The first 13 examples following involve applications of the process mentioned in (A). The quotient in each is *abstract ;* it corresponds to the multiplier.

**1.** When apples are \$3 a barrel, how many barrels can be bought for \$21 ?

At \$3 a barrel, as many barrels can be bought for \$21 as \$3 is contained times in \$21. \$3 is contained 7 times in \$21. Hence 7 barrels can be bought.

It will be observed that in the division the quotient 7 is not 7 barrels, but is an abstract number, which is interpreted or applied in the *conclusion*.

**2.** At 5 cents each, how many tops can be bought for 40 cents ?

**3.** There are 4 quarts in one gallon. How many gallons in 32 quarts ?

**4.** If 10 yards of cloth make a dress, how many dresses can be made from 90 yards ?

**5.** A farmer put 36 bushels of wheat into three-bushel bags. How many bags did he fill ?

**6.** In an orchard are 72 apple trees. If there are 12 trees in a row, how many rows are there ?

**7.** A gardener tied 48 onions in bunches of 6 each. How many bunches did he make ?

**8.** Mr. B gave \$56 to some boys. If each boy got \$8, to how many boys did he give money ?

**9.** How many weeks in 63 days ?

**10.** Mrs. A has 54 chickens in coops. If she has 9 chickens in a coop, how many coops has she ?

**11.** I paid \$56 for coal at \$7 a ton. How many tons did I get ?

**12.** How many slates at 8 cents each can be bought for 72 cents ?

**13.** A butcher invested \$84 in fat pigs, paying \$12 for each. How many did he get ?

**14.** What number multiplied by 7 equals 56 ?

**15.** What number divided by 12 gives the quotient 6 ?

**16.** How many times can 6 be subtracted from 54 ?

**17.** How many apples at $a$ cents each can be bought for $5a$ cents ?

**18**. A man spent $18b$ dollars for books at $3b$ dollars apiece. How many did he get?

**19**. What number divided by $7a$ gives the quotient 4?

**20**. How many two-cent stamps can be bought for a cent and a quarter?

**100.** One of the two equal parts of a number is *one-half* of it, one of the three equal parts, *one-third,* one of the four equal parts, *one-fourth,* three of the four equal parts, *three-fourths,* and so on.

| | | | | | | |
|---|---|---|---|---|---|---|
| One-half is written | | $\frac{1}{2}$. | Three-tenths is written | | | $\frac{3}{10}$. |
| One-third | " | $\frac{1}{3}$. | Two-thirds | " | " | $\frac{2}{3}$. |
| One-fourth | " | $\frac{1}{4}$. | Three-fourths | " | " | $\frac{3}{4}$. |
| One-fifth | " | $\frac{1}{5}$. | Four-fifths | " | " | $\frac{4}{5}$. |
| One-tenth | " | $\frac{1}{10}$. | Five-fourteenths | | " | $\frac{5}{14}$. |

**101.** Read the following:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| (a). | $\frac{3}{5}$ | $\frac{5}{10}$ | $\frac{4}{7}$ | $\frac{5}{6}$ | $\frac{7}{8}$ | $\frac{2}{9}$ | $\frac{5}{8}$ | $\frac{6}{9}$ | $\frac{3}{7}$. |
| (b). | $\frac{9}{10}$ | $\frac{7}{11}$ | $\frac{3}{14}$ | $\frac{8}{13}$ | $\frac{5}{17}$ | $\frac{9}{20}$ | $\frac{7}{15}$ | $\frac{5}{12}$ | $\frac{8}{19}$. |
| (c). | $\frac{11}{20}$ | $\frac{12}{19}$ | $\frac{16}{23}$ | $\frac{18}{29}$ | $\frac{13}{25}$ | $\frac{19}{30}$ | $\frac{14}{27}$ | $\frac{15}{24}$ | $\frac{17}{28}$. |
| (d). | $\frac{20}{30}$ | $\frac{25}{37}$ | $\frac{36}{43}$ | $\frac{47}{64}$ | $\frac{58}{75}$ | $\frac{69}{82}$ | $\frac{72}{99}$ | $\frac{84}{100}$ | $\frac{97}{134}$. |

**1**. What is $\frac{1}{2}$ of 6? Of 10? Of 16? Of 18?

**2**. How do you find $\frac{1}{2}$ of a number?

**3**. What is $\frac{1}{3}$ of 6? Of 9? Of 18? Of 24?

**4**. How do you find $\frac{1}{3}$ of a number?

**5**. What is $\frac{1}{4}$ of 8? Of 12? Of 16? Of 20?

**6**. How is $\frac{1}{4}$ of a number found?

**7**. What is $\frac{1}{5}$ of 20? Of 25? Of 35? Of 60?

**8**. What is $\frac{1}{6}$ of 12? Of 18? Of 30? Of 42? Of 54?

**9**. What is $\frac{1}{7}$ of 21? Of 42? Of 35? Of 56? Of 63?

**10**. What is $\frac{1}{2}$ of $4a$? Of $8a$? Of $10b$? Of $16b$?

**11**. What is $\frac{1}{3}$ of $6c$? Of $18b$? Of $21a$? Of $36x$?

**12**. What is $\frac{1}{8}$ of 24? Of $32x$? Of 48? Of $56a$? Of 72?

**13**. Are there as many ones in $\frac{1}{8}$ of 24 as there are 8's in 24?

**14.** Since $\frac{1}{8}$ of 24 is 3, how many are 3 eighths of 24 ?

**15.** Find $\frac{3}{8}$ of 40. Of 32. Of 16. Of 48. Of 64. Of $24b$.

**16.** Find $\frac{5}{8}$ of 24. Of 40. $\frac{7}{8}$ of 32. Of 56. Of 80.

**17.** What is $\frac{1}{9}$ of 27 ? $\frac{5}{9}$ of 27 ? $\frac{7}{9}$ of 45 ? $\frac{8}{9}$ of 54 ?

To find one of the equal parts of a number requires *division*. Thus, to find $\frac{1}{2}$ of 8 we must divide 8 by 2. To find $\frac{1}{3}$ of 12 we must divide 12 by 3; that is, find one of the 3 equal parts of 12.

**102.** The first 13 of the following examples involve the process mentioned in (B). The quotient in each is *concrete;* it corresponds to the multiplicand.

**1.** Mr. A divided $10 equally between two boys. How much did each boy get ?

Since two boys get $10, each boy got $\frac{1}{2}$ of $10, or $5. $\$10 \div 2 = \$5$.

**2.** I paid $15 for 3 chairs. How much was that apiece ?

**3.** If 4 bags contain 28 bushels, how many bushels are in each bag ?

**4.** John earned $30 in 6 weeks. How much was that per week ?

**5.** If 45 people live in 9 houses, what is the average number in a house ?

**6.** How much was calico a yard when 8 yards cost 88 cents ?

**7.** A merchant paid $96 for 8 stoves. How much did he pay for each stove ?

**8.** A drover has 84 cattle in 7 stables. How many has he in each stable ?

**9.** There are 10 rows of trees in an orchard containing 120 trees. How many trees in a row ? In 2 rows ?

**10.** For 9 clocks a jeweler paid $108. How much did he pay for one clock ? For 2 ? For 3 ?

**11.** If 5 hats cost $15, how much will 8 hats cost ?

**12.** How much will 10 sheep cost, if 7 sheep cost $28 ?

**13.** A lady paid $25a$ dollars for 5 yards of fine cloth. How much did she pay for one yard ?

**14.** When wheat is $b$ cents a bushel, how much must be paid for $a$ bushels?

**15.** I divided $6b$ dollars equally among 3 boys, and then gave one of the boys $a$ dollars more. How much had that boy then?

**103.** In the following exercises let the pupil determine which use of division each problem illustrates; that is, whether *one* of the equal parts or the *number* of them is to be found.

**1.** At $10 each, how many carts can be bought for $80?

**2.** If 8 coats cost $80, how much does one cost?

**3.** My milk bill for one week was 96 cents. If I paid 8 cents a quart, how many quarts did I get?

**4.** A lady gave 48 apples to 12 boys. How many apples did each boy get?

**5.** If 8 apples cost 24 cents, what is the cost of a dozen apples?

**6.** Seven hunters shot 84 rabbits. What was each one's share of the game?

**7.** If 5 tons of coal cost $30, how many tons can be bought for $54?

**8.** Mr. A paid 110 cents for cloth at 11 cents a yard, and Mr. B paid 96 cents for cloth at 12 cents a yard. How many yards did both buy?

**9.** One jeweler paid $72 for watches, and another paid $132 for some of the same kind. If the first got 6 watches, how many did the second get?

**10.** How many pencils at 3 cents each can be purchased for a quarter? How many cents will be left?

**11.** At 4 cents a yard, how many yards of cloth can be bought for half a dollar?

12 yards cost 48 cents. What part of a yard will the 2 cents that remain buy?

**12.** Why is not $8\frac{1}{3}$ pencils the answer to the 10th problem?

**13.** In how many days can a man earn \$50, if he receive \$6 a day?

**14.** How many sheep, at \$7 a head, can be purchased by a man who lacks \$5 of having \$50?

**15.** What is the cost of one table, if 4 tables cost \$34?

**16.** In how many days can a man earn $54a$ dollars, if he receives $6a$ dollars a day?

**17.** In 8 weeks a boy earned $72b$ dollars. How much did he earn in a week? In 2 weeks?

### WRITTEN EXERCISES.

**104. 1.** Divide 8255 by 6.

(a)

$$6\overline{)8255}$$
$$1375\tfrac{5}{6}$$

(b)

```
6)8255(1375 quotient.
  6
  --
  22
  18
  --
   45
   42
   --
    35
    30
    --
     5
```

For convenience we write the divisor at the left of the dividend, with a line between them, and the quotient either as in (a) or in (b).

In 8 there is one 6, with a remainder of 2. Since the 8 is thousands, the quotient 1 and the remainder 2 are thousands. 2 thousands = 20 hundreds, and 20 hundreds and 2 hundreds = 22 hundreds.

In 22 there are three 6's, with a remainder of 4. Since the 22 is hundreds, the quotient 3 and the remainder 4 are hundreds. 4 hundreds = 40 tens, and 40 tens + 5 tens = 45 tens.

In 45 there are seven 6's, with a remainder of 3. Since the 45 is tens, the quotient 7 and the remainder 3 are tens. 3 tens = 30 ones, and 30 ones + 5 ones = 35 ones.

In 35 there are five 6's, with a remainder of 5, which may be written as in (a) or left to stand as in (b).

PROOF.—$1375 \times 6 + 5 = 8255$, the dividend. Hence the work is correct.

The process as shown in (a) is called *Short Division;* as shown in (b) it is called *Long Division.* Wherein do the two processes differ?

Find the quotients by short division:

**2.** 1237 ÷ 3.
**3.** 1924 ÷ 4.
**4.** 2180 ÷ 5.
**5.** 3265 ÷ 4.
**6.** 5375 ÷ 5.
**7.** 6894 ÷ 6.
**8.** 4506 ÷ 3.
**9.** 6025 ÷ 5.
**10.** 6132 ÷ 4.
**11.** 4306 ÷ 7.
**12.** 4955 ÷ 6.
**13.** 5005 ÷ 5.
**14.** 1216 ÷ 8.
**15.** 2032 ÷ 4.
**16.** 7294 ÷ 7.
**17.** 3735 ÷ 3.
**18.** 4405 ÷ 5.
**19.** 3264 ÷ 6.
**20.** 1896 ÷ 8.
**21.** 6188 ÷ 4.
**22.** 1809 ÷ 9.
**23.** 2808 ÷ 9.
**24.** 3629 ÷ 7.
**25.** 8736 ÷ 6.
**26.** 6320 ÷ 5.
**27.** 9144 ÷ 8.
**28.** 5706 ÷ 6.
**29.** 3800 ÷ 5.
**30.** 4008 ÷ 8.
**31.** 7398 ÷ 9.
**32.** 4543 ÷ 7.
**33.** 8888 ÷ 6.
**34.** 268735 ÷ 3.
**35.** 281076 ÷ 6.
**36.** 340125 ÷ 5.
**37.** 532024 ÷ 4.
**38.** 659134 ÷ 7.
**39.** 386937 ÷ 9.
**40.** 726859 ÷ 8.
**41.** 400002 ÷ 6.
**42.** 590023 ÷ 7.
**43.** 646398 ÷ 9.
**44.** 200000 ÷ 3.
**45.** 486018 ÷ 6.
**46.** $598524 ÷ $6.
**47.** $567014 ÷ $7.
**48.** $630927 ÷ $9.
**49.** $803449 ÷ $8.

**50.** 9306 ÷ 11.
**51.** 3432 ÷ 12.
**52.** 5964 ÷ 12.
**53.** 10186 ÷ 11.
**54.** 10188 ÷ 12.
**55.** $69576 ÷ $12.

## RULE FOR LONG DIVISION.

**105.** *Find how many times the divisor is contained in the number represented by the fewest left-hand figures of the dividend that will contain it.*

*Multiply the divisor by the quotient thus obtained, write the product under the left-hand figures used, and subtract.*

*To the remainder annex the next figure of the dividend, and then proceed as before, until all the figures have been annexed.*

*If any partial dividend is less than the divisor, place a cipher in the quotient, annex the next figure of the dividend, and proceed as before.*

PROOF.—Multiply the divisor by the quotient, and add the remainder, if any, to the product. If the result is equal to the dividend the work is correct.

QUERIES.—1. Why must each remainder be less than the divisor ?

2. When the product of the divisor and the quotient figure is greater than the partial dividend from which it is to be subtracted, what must be done ?

**1.** Divide 1728 by 24.

```
24)1728(72
   168
   ---
    48
    48
    --
```

As 24 is greater than 17, it is necessary to take the number represented by three figures of the dividend for the first partial dividend. 172 tens ÷ 24 = 7 tens, and a remainder. 24 × 7 tens = 168 tens. The remainder is 4 tens, and the new dividend is 4 tens and 8 ones, or 48 ones. 48 ones ÷ 24 = 2 ones. 24 × 2 ones = 48 ones. Is there any remainder ?

Find the quotients by long division :

**2.** 4536 ÷ 21.
**3.** 9175 ÷ 25.
**4.** 7998 ÷ 31.
**5.** 7052 ÷ 41.
**6.** 21879 ÷ 51.
**7.** 11792 ÷ 22.
**8.** 15136 ÷ 32.
**9.** 26250 ÷ 42.
**10.** 42224 ÷ 52.
**11.** 37015 ÷ 55.
**12.** 11914 ÷ 23.
**13.** 14058 ÷ 33.
**14.** 14964 ÷ 43.
**15.** 13409 ÷ 53.
**16.** 17400 ÷ 24.
**17.** 21556 ÷ 34.
**18.** 22836 ÷ 44.
**19.** 50058 ÷ 54.
**20.** 30660 ÷ 35.
**21.** 32715 ÷ 45.
**22.** 61620 ÷ 26.
**23.** 22212 ÷ 36.
**24.** 24472 ÷ 46.
**25.** 32088 ÷ 56.
**26.** 88020 ÷ 27.
**27.** 16206 ÷ 37.
**28.** 35344 ÷ 47.
**29.** 21888 ÷ 57.
**30.** 10388 ÷ 28.
**31.** 18696 ÷ 38.
**32.** 52542 ÷ 63.
**33.** 10725 ÷ 75.
**34.** 26487 ÷ 81.
**35.** 33943 ÷ 91.
**36.** 43368 ÷ 74.
**37.** 66864 ÷ 84.
**38.** 52776 ÷ 72.
**39.** 25568 ÷ 68.
**40.** 64108 ÷ 94.

**41.** 376859328 ÷ 48.
**42.** 384710564 ÷ 58.
**43.** 238311937 ÷ 29.
**44.** 287135862 ÷ 39.
**45.** 429436049 ÷ 49.
**46.** 728395002 ÷ 59.
**47.** 169135679 ÷ 137.
**48.** 207407256 ÷ 168.
**49.** 181481349 ÷ 147.
**50.** 219752926 ÷ 178.
**51.** 657662754 ÷ 201.
**52.** 197393584 ÷ 304.

**53.** 204963822 ÷ 406.
**54.** 304123450 ÷ 510.
**55.** 442680498 ÷ 611.
**56.** 584093472 ÷ 712.
**57.** 263090646 ÷ 819.
**58.** 383748326 ÷ 923.
**59.** 677510968 ÷ 937.
**60.** 618875970 ÷ 745.
**61.** 353628594 ÷ 1023.
**62.** 512763462 ÷ 2186.
**63.** 498933150 ÷ 3275.
**64.** 108825952 ÷ 4528.
**65.** 230884080 ÷ 5684.
**66.** 597126784 ÷ 6788.
**67.** 941108532 ÷ 7638.
**68.** 265283625 ÷ 8725.
**69.** 397046588 ÷ 9337.
**70.** 308196056 ÷ 3962.
**71.** 535673956 ÷ 8009.
**72.** 810891081 ÷ 9009.
**73.** 103031370 ÷ 8346.
**74.** 462017992 ÷ 4678.

**106.** **1.** A drover bought some cattle for $17616. If the average price of each was $48, how many did he buy?

**2.** The salary of a Congressman is $5000 a year. How much is that a day?

**3.** A owes B $5200. If he pays him $650 a year, in how many years will the debt be canceled?

**4.** The circumference of the earth is about 8 million rods, and there are 320 rods in a mile. How many miles is it around the earth?

**5.** A grocer bought 368 barrels of flour for $2208, and sold them for $2944. How much did he gain per barrel?

**6.** The W. V. R. R. is 268 miles long, and cost $5,660,728. What was the average cost per mile?

**7.** The salary of the President of the United States is $50000 a year. How much is that per day in leap year?

**8.** The product of two numbers is 1,259,375. One of the numbers is 97 less than 500. What is the other?

**9.** If a man receives $1600 a year for his labor, and spends $832, in how many years can he save enough to buy a farm of 132 acres, at $24 an acre?

**10.** By selling a farm of 240 acres for $12720, I gained $1200. How much did I pay per acre for the farm?

**11.** How often can 436 be subtracted from 34444?

**12.** The dividend is 9689, the quotient 134, and the remainder 41. What is the divisor?

**13.** A double-track street railway is 5 miles long. How many rails does it contain, if each rail is 24 feet long, there being 5280 feet in a mile?

**14.** A train of fifteen cars contained 279300 pounds of flour in barrels. How many barrels were in each car, a barrel of flour weighing 196 pounds?

**15.** Two men leave Memphis, Tenn., to travel around the earth, one going east at the rate of 154 miles a day, the other going west at the rate of 144 miles a day. In how many days will they meet, if the distance around is 16390 miles?

**16.** A railroad train makes 2 round trips daily between New York and Philadelphia. How far apart are these cities if the train runs 131400 miles in a common year?

### To divide by 10, 100, 1000, etc.

**107. 1.** $80 \div 10 =$ what? $750 \div 10 =$ what? $3200 \div 10 =$ what? How do the quotients compare with the dividends?

**2.** $700 \div 100 =$ what? $2400 \div 100 =$ what? Compare quotients with dividends, and tell how the latter have been changed.

**3.** $75 \div 10 =$ what? What is the remainder?

**4.** $3825 \div 100 =$ what? What is the remainder?

**5.** $43875 \div 1000 =$ what? What is the remainder?

Where are these remainders seen in the dividends? Can the quotients be seen in the dividends? Where?

**108.** Principle.—*A number may be divided by 10, 100, 1000, etc., by cutting off from the right of the dividend as many figures as there are ciphers in the divisor.*

The part cut off is the remainder, and the rest of the dividend is the quotient. Thus, $4100 \div 100 = 41$; $4125 \div 100 = 41$, with a remainder 25. The quotient may be written $41\frac{25}{100}$.

Divide the following :

| | | |
|---|---|---|
| **1.** 380 by 10. | **7.** 500 by 100. | **13.** 66000 by 1000. |
| **2.** 275 by 10. | **8.** 320 by 100. | **14.** 9300 by 1000. |
| **3.** 420 by 10. | **9.** 875 by 100. | **15.** 2460 by 1000. |
| **4.** 600 by 10. | **10.** 7200 by 100. | **16.** 8725 by 1000. |
| **5.** 775 by 10. | **11.** 2450 by 100. | **17.** 35000 by 1000. |
| **6.** 905 by 10. | **12.** 4315 by 100. | **18.** 40009 by 1000. |

**19.** Divide 31275 by 500.

$$5|00)\underline{312 \mid 75}$$
$$62\tfrac{275}{500}$$

Cutting off two figures from the right of the dividend divides it by 100, the quotient being 312, with the remainder 75. Since 500 is 5 times 100, the quotient is 5 times as large as it should be. Hence we divide it by 5, getting a quotient of 62 and a remainder of 2, which is hundreds. 2 hundreds + 75 (the first remainder) = 275, the entire remainder. Hence the quotient is $62\frac{275}{500}$.

Find the quotients :

| | | |
|---|---|---|
| **20.** $2765 \div 20$. | **26.** $78960 \div 80$. | **32.** $102030 \div 900$. |
| **21.** $4275 \div 30$. | **27.** $62845 \div 90$. | **33.** $510075 \div 700$. |
| **22.** $5180 \div 40$. | **28.** $59320 \div 300$. | **34.** $246783 \div 800$. |
| **23.** $3625 \div 50$. | **29.** $32856 \div 500$. | **35.** $987654 \div 600$. |
| **24.** $7338 \div 60$. | **30.** $47623 \div 600$. | **36.** $100000 \div 800$. |
| **25.** $6774 \div 70$. | **31.** $89974 \div 800$. | **37.** $808080 \div 700$. |

**38.** Divide 1728 by 12, by dividing first by 2, and then the quotient by 6.

$$1728 \div 2 = 864. \qquad 864 \div 6 = 144.$$

**39.** Divide 2625 by the factors of 15.

**40.** Divide 4536 by 21 ; also by 3 and 7, and compare results.

**109.** In expressions like $18 + 24 \div 6$, the operation indicated by the sign $\div$ must be first performed.

Thus, $18 + 24 \div 6$ means $18 + 4$, not $42 \div 6$. $120 - 80 \div 2$ means $120 - 40$.

Find the value of :

1. $43 + 18 \div 3$.
2. $19 - 36 \div 4$.
3. $27 \div 9 + 8$.
4. $35 - 21 \div 7$.
5. $22 + 9 \times 3 - 49 \div 7$.
6. $68 - 35 \div 5 + 7 \times 10$.
7. $54 - 57 \div 19 - 3 \times 17$.
8. $36 + 7 \times 9 - 63 \div 9$.

GENERAL PRINCIPLES OF DIVISION.

**110.** The value of the quotient depends upon the *relative* values of the dividend and divisor. Hence, if either dividend or divisor is changed, the quotient will be changed. If both are changed equally (as to ratio), the quotient will not be changed, as may be seen in equations (e) and (f) below.

The following equations illustrate all the changes :

GIVEN EQUATION, $24 \div 6 = 4$.

| | | |
|---|---|---|
| Changing | (a) $48 \div 6 = 8$. | (a) Multiplying the dividend by 2 multiplies the quotient by 2. |
| dividend. | (b) $12 \div 6 = 2$. | (b) Dividing the dividend by 2 divides the quotient by 2. |
| Changing | (c) $24 \div 12 = 2$. | (c) Multiplying the divisor by 2 divides the quotient by 2. |
| divisor. | (d) $24 \div 3 = 8$. | (d) Dividing the divisor by 2 multiplies the quotient by 2. |
| Changing both | (e) $48 \div 12 = 4$. | (e) Multiplying both dividend and divisor by 2 does not change the quotient. |
| equally. | (f) $12 \div 3 = 4$. | (f) Dividing both dividend and divisor by 2 does not change the quotient. |

From these examples are deduced the following general

**111.** PRINCIPLES.—1. *Multiplying the dividend multiplies the quotient, and dividing the dividend divides the quotient.*

2. *Multiplying the divisor divides the quotient, and dividing the divisor multiplies the quotient.*

3. *Multiplying or dividing both dividend and divisor by the same number does not change the quotient.*

QUERIES.—1. If a number equal to the divisor should be added to the dividend, what change would occur in the quotient?

2. Subtracting twice the divisor from the dividend would have what effect on the quotient?

3. Would adding the same number to both dividend and divisor increase or diminish the quotient?

4. If the same number were subtracted from dividend and divisor, would the quotient be increased or diminished?

5. Does subtracting any number from the divisor increase or diminish the quotient?

# REVIEW WORK.

## ORAL EXERCISES.

**112. 1.** What number is represented by 45 ?

**2.** What number is represented by 8 ones of the first period and 7 tens of the second period ?

**3.** In 42 tens how many ones ?

**4.** How many ones in 3 hundreds, 6 tens, and 5 ones ?

**5.** If 9 hats cost \$27, what will 5 hats cost ?

Since 9 hats cost \$27, 1 hat costs $\frac{1}{9}$ of \$27, or \$3; since 1 hat costs \$3, 5 hats will cost 5 times \$3, or \$15.

QUERY.—Why will 5 hats cost 5 times \$3 ?

**6.** If 6 sheep cost \$30, how much will 11 sheep cost ?

**7.** A man bought 10 books for \$40, and sold 7 of them at the same rate. How much did he receive for them ?

**8.** How much will 12 yards of cloth cost, if 5 yards cost 55 cents ?

**9.** How much will 13 pounds of meat cost, if 9 pounds cost 72 cents ?

**10.** A sold 5 pigs and B sold 3, each getting the same price per head. How much did each get if both got \$64 ?

**11.** I sold a calf for \$19, which was \$7 more than it cost me. How much did I pay for it ?

**12.** A man bought 6 barrels of flour for \$30, and gave half of them for potatoes at \$3 a barrel. How many barrels of potatoes did he get ?

**13.** Which is cheaper, and how much per dozen—eggs at 25 cents a dozen, or at 3 cents each ?

**14.** How long will it take A to earn \$99, if he earns \$18 in 2 weeks ?

**15.** How many days can three men live on the provisions that 5 men require for 9 days?

**16.** If 7 men can dig a ditch in 9 days, how long would it take 3 men?

**17.** If a load of hay lasts 8 cows a week, how long would it last 14 cows?

**18.** Twelve times 7 are how many times 4?

**19.** If 3 apples are worth 1 lemon, and 2 lemons are worth 12 pears, how many pears are worth 18 apples?

**20.** When rice is 6 cents a pound, how many pounds should I receive in exchange for 9 dozen eggs at a cent apiece?

**21.** In 85 days how many weeks?

**22.** How many days in 8 weeks and 5 days?

**23.** Jack bought a dollar's worth of apples at the rate of 2 for 5 cents. How many did he get?

**24.** May bought 2 dozen eggs at 25 cents a dozen, and sold them at the rate of 3 for a dime. How much did she gain?

**25.** Owen bought 9 oranges for 7 cents each and 11 lemons for 5 cents each; he gave in exchange 9 pounds of butter at 15 cents a pound. How much was due him?

**26.** I gave half a dozen dozen pencils worth 5 cents each for 6 knives. What was each knife worth?

**27.** How many letters are required to write $2.41 in words?

**28.** At $2.88 a dozen, what is the value of 10 hoes?

**29.** If $a$ boys earn $5a$ cents per day, how much do $b$ boys earn in one day?

**30.** If $q$ cows eat $a$ tons of hay in a month, how much will $p$ cows eat?

**31.** John was $b$ years of age $a$ years ago. How old will he be in $a + b$ years?

**32.** At $\$a$ each, what will be the cost of $c$ rabbits?

**33.** A has $\$b$, B has $\$c$ more than A, and C has as much as the difference between A's and B's money. How much have they together?

WRITTEN EXERCISES.

**113. 1.** Miss B teaches 9 months in the year at a salary of $1350. How long does it take her to earn $900 ?

**2.** A and B bought a farm of 80 acres for $7360. If A paid $3128, how many acres did he pay for ?

**3.** The President of the U. S. receives $50000 a year. If his salary were increased $5 a year, how much would he receive a day ?

**4.** How long a string will it take to reach around a barn 42 feet long and 36 feet wide ?

**5.** Mr. A bought a piano for $450, paying one-half in cash, and the remainder at the rate of $15 a month. If he made the purchase January 1, 1899, when did he make the last payment ?

**6.** Rome was founded 753 years before the birth of Christ. How long was that before Columbus discovered America ?

**7.** There are 369600 feet in 70 miles. How many feet in 5 miles and a half ?

**8.** What number besides 269 will exactly divide 36853 ?

**9.** In 100 years the population of the U. S. increased from 3,929,214 to 62,622,250. What was the average increase per year ?

**10.** If I spend a quarter a day for books, a dollar a day for rent, and $35 a month for groceries, how much do I save in a leap year if my salary is $2000 ?

**11.** Find the sum of the five largest numbers that can be expressed by the figures 9, 8, 0, 4, and 2.

**12.** The minuend is 7019, the remainder 3107. The subtrahend is how many times the sum of 3, 2, and 7 ?

**13.** The divisor is 437, the quotient 86, and the remainder 50. What is the dividend ?

**14.** Two men had $7583 divided between them. The difference between their shares was $223. How much did each man get ?

**15.** How many times can 461 be subtracted from 57820, and what is the remainder?

**16.** Marvin read from chapter LXXVII to chapter XCIX. How many chapters did he read?

**17.** How many quarts of oats will two horses eat in 30 days, if each horse eats 4 quarts 3 times a day?

**18.** The skull has 8 bones, the face 14, the ear 3, the trunk 53, the shoulders 4, an arm 3, the wrists 16, the hands 38, the legs 8, the ankles 14, and the feet 36. Allowing 32 teeth, how many bones are in the body?

**19.** If a lot of hay lasts 18 horses 27 months, how long would it last 27 horses?

**20.** The product is 5832 and the multiplier is 324. What is the multiplicand?

**21.** If a horse travels 8 miles an hour and a locomotive 40 miles an hour, how much sooner can a man go 120 miles by traveling on the cars than by going on horseback?

**22.** The product of three numbers is 13824. Two of the numbers are 18 and 32. What is the third?

**23.** Find a number to which if 369 be added the sum will be 1001 less than 9090.

**24.** A man bought 68 horses at $84 each; 11 of them died. At what price must he sell the others to gain $444?

**25.** Mr. H bought 160 acres of land at $75 an acre. After spending $1200 dollars for improvements, he sold it at a gain of $2000. At what price per acre did he sell?

**26.** A newsboy bought papers at 3 cents each, and sold them at 5 cents each, thereby gaining 90 cents. How many papers did he sell?

**27.** A mile is 5280 feet. How many steps of 2 feet each will a boy take in walking 5 miles?

**28.** How many years does it take to make the difference between saving $2 a month and $6 a month amount to a saving of $100?

**29.** I traded 120 head of cattle at $64 a head for 160 acres of land. What price per acre did I pay?

**30.** If 8 horse shoes weigh 16 pounds, how many horses can be shod with shoes that weigh 152 pounds?

**31.** Horace rode 31680 yards on his bicycle, the wheel of which was 12 feet in circumference. How many turns did the wheel make?

**32.** What number multiplied by twice 37 will produce 2664?

**33.** A wagon weighing 1000 pounds contains 6 barrels of flour and 7 of pork, and is drawn by two horses. A barrel of pork weighs 200 pounds and a barrel of flour 196 pounds. How many pounds does each horse draw?

**34.** The distance from Pittsburg to Philadelphia is 354 miles. If a railroad conductor makes a round trip every two days, how many miles does he travel in 4 weeks?

**35.** At $15 per uniform, how many companies of 85 soldiers each can be uniformed for $10200?

**36.** The smaller of two numbers is 3782, and their difference is 1218. What is the larger number?

**37.** Mr. E paid $125 an acre for 80 acres of coal land. He sold the coal for $64000, and the land at $75 an acre. How much did he gain?

**38.** How many weeks would a man take to walk 1344 miles, if he walks 4 miles an hour, 7 hours a day, and 6 days a week?

**39.** The sum of 18 equal numbers is 96346 less than a million. Find one of the numbers.

---

**40.** What is the value of $175 + 92 \times 105$?

**41.** If a train runs 28 miles an hour, in how many hours can it run to a place 420 miles distant and return?

**42.** A miller owing $500 gave in part payment 250 bushels of wheat at $1.50 a bushel, and paid the remainder with flour at $5 a barrel. How many barrels were required?

**43.** A drover bought 45 horses at $85 each, and sold them so as to gain $720. How much a head did he receive for them ?

**44.** An extension table is 12 feet long when its four boards are in, and 7 feet long when they are out. How wide is each board ?

**45.** Henry was 36 years old when Edward was born. Edward was 14 years old in 1876. How old will Henry be in 1910 ?

**46.** If one hen lays 180 eggs in a year, how many dozen eggs should 2 dozen hens lay in 2 years ?

**47.** From New York to Havana is 1260 miles, from Havana to Aspinwall is 1046 miles, from there to Panama is 60 miles, and from Panama to San Francisco is 3616 miles. What is the distance between San Francisco and Havana ?

**48.** A party of 64, eight more than half of whom were ladies, took a boat ride at an expense of $3 each. If all expenses were paid by the gentlemen, how much did each pay ?

**49.** I paid $7500 for two lots, one of them costing $1000 more than the other. What did I pay for each ?

**50.** A and B together have $500, and A has $100 more than B. How much has each ?

**51.** The area of Texas is 265780 square miles, and that of Pennsylvania is 45215 square miles. Into how many States of the size of Pennsylvania could Texas be divided, and how many square miles would be left over ?

**52.** At $4 a ton, what is the value of a carload of coal weighing 17920 pounds, counting 2240 pounds to a ton ?

**53.** There are 640 acres in a square mile. How many acres in Rhode Island, whose area is 1250 square miles ?

**54.** Each front wheel of a carriage is 10 feet in circumference, and each hind wheel 12 feet. The front wheels will make how many more turns than the hind wheels in going 5 miles, there being 5280 feet in a mile ?

**55.** If steel rails weigh 72 pounds to the yard, and 2000 pounds are a ton, how many tons of rails will be required to lay 2 miles of railroad, half of which is to have double track?

SUPPLEMENTARY EXERCISES. (FOR ADVANCED CLASSES.)

**114. 1.** Prove that if we multiply by 4, and divide the product by 100, we obtain the result of dividing by 25.

**2.** Find the value of $720 + 964 \times 8 - 154 \times 0 \times 6$.

**3.** Prove that $\$17 \times 11 = \$11 \times 17$.

**4.** By what number must we divide a given number to obtain the same result as when the given number is multiplied by 2 and divided by 70?

**5.** The remainder is 723. What is the minuend if it is twice as great as the subtrahend?

**6.** The minuend is $a + a$, and the remainder is equal to the subtrahend. Find the remainder.

**7.** What number multiplied by 100 and divided by 4 gives the same result as is obtained by multiplying 18 by 25?

**8.** The sum of two numbers is 60, and their difference is 24. What are the numbers?

**9.** The sum of two numbers is $a$, and their difference is $b$. What are the numbers?

**10.** What is the quotient when $12 is divided by $4? When $\$a$ is divided by $\$b$?

**11.** A man living at the rate of $3500 a year for 6 years finds that he is exceeding his income, and reduces his expenditures to $2500 a year. At the end of 4 years he finds that he is just out of debt. What is his income?

## FACTORS AND MULTIPLES.

**115.** **1.** What two numbers multiplied together will make 6? Then what are the factors of 6?

**2.** Is each factor of 6 an exact divisor of 6?

**116.** The integers which multiplied together will produce a number are called the **Factors** of that number.

Thus, 5 and 6, or 2, 3, and 5 are the factors of 30. The factors of a number are exact divisors of it.

**1.** What are the exact divisors of 8? Of 7? Of 10? Of 13? Of 18? Of 19? Of 23?

**117.** A number whose only exact divisors are itself and 1 is called a **Prime Number.**

A number that has other exact divisors is called a **Composite Number.**

Thus, 3, 5, 11, 17, etc., are prime numbers, and 4, 9, 12, 20, etc., are composite numbers.

**1.** Make a list of all the prime numbers between 0 and 100.

**2.** Make a list of all the numbers from 1 to 100 that are exactly divisible by 2.

**3.** Make a list of all the numbers between 0 and 144 that are not exactly divisible by 2.

**118.** A number that is exactly divisible by 2 is called an **Even Number.** All other numbers are called **Odd Numbers.**

**119.** The exact divisors, or factors, of a number must be found by inspection or by trial. The following facts are very helpful in finding factors:

Any number is exactly divisible

**1.** By 2, when the right-hand figure is 0, 2, 4, 6, or 8.

**2.** By 3, when the sum of the numbers represented by its digits is divisible by 3.

**3.** By 4, when the number represented by the two right-hand digits is divisible by 4.

**4.** By 5, when the right-hand figure is 0 or 5.

**5.** By 6, when it is divisible by 2 and 3.

**6.** By 8, when the number represented by the three right-hand digits is divisible by 8.

**7.** By 9, when the sum of the numbers represented by its digits is divisible by 9.

Find some divisors of the following by inspection :

| | | | |
|---|---|---|---|
| **1.** 324. | **6.** 8406. | **11.** 9072. | **16.** 84,306. |
| **2.** 175. | **7.** 7300. | **12.** 8100. | **17.** 52,146. |
| **3.** 260. | **8.** 2904. | **13.** 3285. | **18.** 93,528. |
| **4.** 513. | **9.** 5344. | **14.** 7824. | **19.** 60,000. |
| **5.** 400. | **10.** 4563. | **15.** 5259. | **20.** 78,327. |

## FACTORING.

**120. 1.** What prime numbers multiplied together will produce 6 ? 10 ? 14 ? 22 ? 12 ?

**2.** What prime numbers will exactly divide 18, or what are the *prime factors* of 18 ?

**121.** Prime numbers used as factors are called **Prime Factors.**

Thus, 3 and 7 are the prime factors of 21.

**1.** Can 11 and 18 both be divided by the same number ? Can 12 and 25 ?

**2.** Which of the numbers in the preceding example are prime numbers ?

**122.** Two numbers that have no common factor except unity are said to be **Prime to each other,** though one or both of them may be composite.

**1.** Since 3 is a factor of 6, must it be a factor of two 6's, or 12 ? Of three 6's, or 18 ? Of any number of 6's ?

**2.** Since 12 is a factor of 36, are all the factors of 12 also factors of 36 ? Find by trial.

**3.** Can all the numbers of which 12 is a factor be exactly divided by the factors of 12 ? Investigate.

**4.** Any exact divisor of a factor is always a factor of what ?

**123.** PRINCIPLE.—*An exact divisor of a* **factor** *of a number is a factor of the number itself.*

An exact divisor may be a fraction, but in "*factoring*" only integral divisors or factors are considered.

**124.** The process of finding the factors of a number is called **Factoring.**

WRITTEN EXERCISES.

**125. 1.** What are the prime factors of 360 ?

| | |
|---|---|
| 2 | 360 |
| 2 | 180 |
| 2 | 90 |
| 3 | 45 |
| 3 | 15 |
| | 5. |

Since the prime number 2 is a divisor of 360, it is one of the factors, and 180 is another. Since 2 is an exact divisor of 180, it is a factor of 360 (Art. 123), as is 90 also. Since 2 is an exact divisor of 90, it is a factor of 360. Likewise 3 and 5 being exact divisors of 45 and 15 are also factors of 360. Hence 2, 2, 2, 3, 3, and 5 are the prime factors of 360.

NOTE.—The number of times any factor occurs in a product may be indicated by an *exponent*. Thus, $2^3$, $3^2$, 5 are the prime factors of 360. The small figures written above and to the right of the factors 2 and 3 are exponents.

Find the prime factors of :

| | | | |
|---|---|---|---|
| **2.** 60. | **8.** 480. | **14.** 2956. | **20.** 2310. |
| **3.** 108. | **9.** 672. | **15.** 4620. | **21.** 7644. |
| **4.** 144. | **10.** 1056. | **16.** 9170. | **22.** 64,384. |
| **5.** 180. | **11.** 1872. | **17.** 5432. | **23.** 20,000. |
| **6.** 315. | **12.** 2310. | **18.** 2002. | **24.** 242,424. |
| **7.** 308. | **13.** 3204. | **19.** 6006. | **25.** 714,510. |

## LEAST COMMON MULTIPLE.

**126.** The product of two or more integers is called a **Multiple** of those numbers. It follows that any number is a multiple of another when it is exactly divisible by that number. Every number is a multiple of its factors.

**1.** Does 10 exactly contain both 2 and 5 ?

**2.** What number will contain 7 and 3 without a remainder ?

**3.** Name a multiple that is *common* to 5 and 11. To 4 and 6. To 2, 3, and 4.

**127.** A multiple that is common to two or more numbers is called a **Common Multiple.**

**1.** What is the least number that will exactly contain 3 and 5 ? 4 and 6 ? 2, 3, and 4 ?

**2.** Is 24 a common multiple of 3 and 4 ? Is it their *smallest* common multiple ? What is their least common multiple ?

**128.** The least number that is exactly divisible by each of two or more numbers is called their **Least Common Multiple,** *written* L. C. M.

**1.** What are the prime factors of 6 ? Of 10 ? What is their L. C. M.?

**2.** What are the prime factors of 30 ? How do they compare with those of 6 and 10 ?

**3.** What factor is common to 6 and 10 ? Does it occur twice in the factors of 30 ?

**4.** Since 30 contains both 6 and 10, must it contain all their prime factors ?

**5.** The factors of 6 are 2 and 3, and those of 14 are 2 and 7. Which of these factors must be multiplied together to produce the L. C. M. of 6 and 14?

**129.** PRINCIPLE.—*The least common multiple of two or more numbers contains all the prime factors of those numbers, and no others.*

If a factor is common to two or more numbers, it is contained in the L. C. M. only the greatest number of times it enters into any *one* of the numbers—not as often as it occurs in all of them.

WRITTEN EXERCISES.

**130. 1.** Find the least common multiple of 25, 30, and 42.

$25 = 5 \times 5.$
$30 = 2 \times 3 \times 5.$
$42 = 2 \times 3 \times 7.$
$2 \times 3 \times 5 \times 5 \times 7 = 1050.$

The least common multiple must contain all the prime factors of 25, 30, and 42, that is, 2, 3, 5, and 7. Each of these must be contained as often as it occurs in any one set of factors.

The only factor that occurs twice in one number is 5. Hence the factors of the L. C. M. are 2, 3, 5, 5, 7. Their product is 1050, the L. C. M. The following method, which is in common use, is based upon the same principle:

$$\begin{array}{r|rrr} 2 & 25, & 30, & 42. \\ \hline 3 & 25. & 15, & 21. \\ \hline 5 & 25, & 5, & 7. \\ \hline & 5, & 1, & 7. \end{array}$$

Since 2 is an exact divisor of some of the numbers, it is a factor of the L. C. M. Since 3 is an exact divisor of some of the quotients, it is a factor of the L. C. M. (Art. 129). We find in the same manner that 5 is also a factor of the L. C. M. The last quotients which are prime to each other are also factors of the L. C. M. Hence the L. C. M. is $2 \times 3 \times 5 \times 5 \times 7$, or 1050.

Find the L. C. M. of the following:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **2.** 12, | 24, | 30. | **7.** 4, | 5, | 9, | 8, | 12, | 6. |
| **3.** 18, | 27, | 32. | **8.** 7, | 2, | 3, | 4, | 5, | 6. |
| **4.** 22, | 33, | 55. | **9.** 6, | 7, | 8, | 10, | 14, | 16. |
| **5.** 28, | 30, | 60. | **10.** 4, | 6, | 8, | 16, | 24, | 48. |
| **6.** 36, | 50, | 70. | **11.** 3, | 5, | 7, | 11, | 13, | 17. |

1. When one of the numbers is a factor of another, it may be disregarded, as its multiple contains the same factors.

2. When several numbers have no common factor, their product is the L. C. M.

# CANCELLATION.

**131.** Cancellation is a process of shortening the work in problems that involve multiplication and division. It is based on two principles.

**1.** What is the product of $6 \times 5$? Of $3 \times 5$? How do the products compare?

**2.** What is the product of $8 \times 2$? Of $4 \times 2$? How do the products compare? How could you get the second product from the first?

**3.** Does dividing one factor by any number divide the product by the same number? Find by trial.

**132.** Principles.—1. *Dividing any factor of a series of factors by any number divides the product by the same number.*

2. *Dividing both dividend and divisor by the same number does not change the quotient.* (Art. 110.)

### WRITTEN EXERCISES.

**133. 1.** Divide the product of 3, 21, and 25 by the product of 3, 7, and 10.

$$\frac{\cancel{3} \times \overset{3}{\cancel{21}} \times \overset{5}{\cancel{25}}}{\cancel{3} \times \cancel{7} \times \underset{2}{\cancel{10}}}$$

The division is indicated by writing the dividend above and the divisor below the line. Dividing both by 3 cancels that common factor. Dividing both by 7 cancels 7 in the divisor and 21 in the dividend, leaving the quotient 3 in the latter. Dividing both by 5 cancels 10 and 25, leaving the quotient 2 in the divisor and the quotient 5 in the dividend. The product of the remaining factors of the dividend is 15. Hence the quotient is $15 \div 2$, or $7\frac{1}{2}$.

Find the quotients of the following:

2. $\frac{4 \times 5 \times 6 \times 10}{2 \times 3 \times 5 \times 8}$.

3. $\frac{8 \times 14 \times 9 \times 12}{6 \times 15 \times 7 \times 4}$.

4. $\frac{12 \times 8 \times 9 \times 30}{4 \times 80 \times 6 \times 9}$.

5. $\frac{25 \times 32 \times 18 \times 7}{16 \times 15 \times 28 \times 9}$.

6. $\frac{13 \times 14 \times 15 \times 16}{7 \times 8 \times 9 \times 10}$.

7. $\frac{45 \times 8 \times 11 \times 72}{24 \times 18 \times 15 \times 33}$.

8. $\frac{63 \times 13 \times 93 \times 23}{39 \times 31 \times 21 \times 69}$.

9. $\frac{121 \times 54 \times 28 \times 35}{44 \times 219 \times 30}$.

10. $\frac{84 \times 65 \times 55 \times 49}{56 \times 63 \times 70 \times 22}$.

11. $\frac{132 \times 52 \times 68 \times 45}{77 \times 65 \times 51 \times 20}$.

**12.** How often is $12 \times 13 \times 50$ contained in $65 \times 10 \times 84 \times 3$?

**13.** How many pounds of butter at 28 cents a pound must be paid for 25 yards of cloth at 56 cents a yard?

**14.** How many barrels of apples, each containing 3 bushels, worth 70 cents a bushel, are worth as much as 20 boxes of crackers, containing 15 pounds each, if 2 pounds are worth 28 cents?

**15.** A miller sold 20 barrels of flour, 196 pounds each, at 3 cents a pound, and received his pay in wheat at 84 cents a bushel. If there were 2 bushels in a bag, how many bags did he get?

**16.** If 36 men, working 8 hours a day, can do a piece of work in 57 days, how long would it take 27 men, working 9 hours a day?

**17.** The factors of the dividend are 10, 14, 9, 25, and 32; the factors of the divisor are 5, 16, 7, and 25. What is the quotient?

## UNITED STATES MONEY.

### (An Introduction to Decimal Fractions.)

**134.** United States money has a decimal currency. It is written as dollars and decimal parts of a dollar, called *dimes, cents,* and *mills.*

**1.** A dime is what part of a dollar? How is it written? (\$.1.) How may this be read? (One *tenth* of a dollar.) Then how would you write 2 dimes, or 2 tenths of a dollar?

**2.** How is 3 tenths of a dollar written? 5 tenths? 7 tenths? 9 tenths? 8 dimes? 15 dimes? 25 tenths?

**3.** Write 3 dollars and 5 dimes. 7 dollars and 9 tenths of a dollar. 20 dollars and 3 tenths of a dollar. 5 and one-tenth dollars.

**4.** What is always written in the first place to the right of dollars? (Tenths of a dollar.) What separates the dollars from the *tenths* of a dollar?

**5.** How is \$.01 read? A cent is what part of a dollar? Then how would you write 2 cents, or 2 *hundredths* of a dollar?

**6.** Write 3 cents, or 3 hundredths of a dollar. 5 hundredths. 7 hundredths. 8 cents. 9 hundredths.

**7.** Write 2 dollars and 6 cents. 5 dollars and 8 hundredths of a dollar. 9 dollars and 9 hundredths. Why put a cipher between dollars and hundredths?

**8.** What is always written in the second place to the right of dollars? (Hundredths of a dollar.)

**9.** Since there are ten cents in a dime, what is the difference between \$.1 and \$.10? Are they read in the same way?

**10.** How many *hundredths* of a dollar in a *tenth* of a dollar? In 2 tenths? In 2 tenths and 5 hundredths?

**11.** \$.25 may be read 25 cents; or 2 dimes and 5 cents; or 2 tenths and 5 hundredths, or 25 hundredths of a dollar.

**12.** A cent is what part of a dime? Then a hundredth of a dollar is what part of a tenth of a dollar? One tenth is equal to how many hundredths?

**13.** In \$.22, which 2 has the greater value? Its value is how many times the value of the other?

**14.** Write 24 hundredths of a dollar. 3 tenths and 5 hundredths. How many tenths of a dollar can be written with one figure? How many hundredths? With two figures?

**15.** How is \$.001 read? Since there are 1000 mills in a dollar, what part of a dollar is 1 mill? Then how is 2 mills, or 2 *thousandths* of a dollar, written?

**16.** Write 3 mills, or 3 thousandths of a dollar. 5 thousandths. 9 thousandths. 7 mills. 5 dollars and 5 thousandths of a dollar.

**17.** What is always written in the third place to the right of dollars? (Thousandths of a dollar.)

**18.** How many mills in a cent? Then how many *thousandths* of a dollar in a *hundredth* of a dollar? In 10 hundredths, or a tenth? One dollar equals how many tenths of a dollar? How many hundredths? How many thousandths?

**19.** \$.375 may be read 375 mills; or 37 cents, 5 mills; or 3 dimes, 7 cents, 5 mills; or 3 tenths, 7 hundredths, 5 thousandths of a dollar, or 375 thousandths of a dollar.

**20.** Since 5 mills equal half a cent, \$.375 may be read 37 and one-half cents, and written \$.37$\frac{1}{2}$.

**135.** Copy and complete the following:

| | | | | | |
|---|---|---|---|---|---|
| **1.** 7 dimes, or 7 | tenths | of a | dollar | = | \$.7, or \$.70. |
| **2.** 3 dimes, or —— | " | " | " | = | (           ). |
| **3.** 5 cents, or —— | " | " | " | = | (           ). |
| **4.** 9 cents, or —— | " | " | " | = | (           ). |

**136.** The processes of adding and subtracting in U. S. money are the same as in simple numbers, or integers. Dollars should be written under dollars, cents under cents, mills under mills. The decimal points should be in a vertical line.

Perform the operations indicated:

| | |
|---|---|
| **1.** \$72.65 + \$18.23. | **8.** \$6 + \$.6 + \$.08 + \$.008. |
| **2.** \$39.47 + \$26.82. | **9.** \$271.83 − \$187.93. |
| **3.** \$53.90 + \$18.25. | **10.** \$1,000 − \$100.75. |
| **4.** \$91.03 + \$4.775. | **11.** \$86.37 − \$24.80. |
| **5.** \$.325 + \$10.584. | **12.** \$9.875 − \$5.312. |
| **6.** \$.875 + \$.75 + \$.093. | **13.** \$5.003 − \$2.008. |
| **7.** \$3.40 + \$.205 + \$80. | **14.** \$100 − \$.975. |

**15.** \$73.806 + \$16.194 − \$89.98.

**16.** \$1 + \$.1 − \$.01 + \$10.

**17.** To 3 tenths of a dollar add 7 hundredths of a dollar.

**18.** From 7 tenths of a dollar subtract 35 hundredths of a dollar.

**19.** What is the difference between 4 hundredths of a dollar and 9 tenths of a dollar?

**20.** From the sum of 8 tenths and 5 thousandths of a dollar subtract 9 hundredths of a dollar.

**21.** A farmer bought two cows, giving \$29.50 for one and \$36.75 for the other. He gave in payment a wagon worth \$42.25, and the rest in cash. How much money did he give?

**22.** One month a man worked 24 days at \$2.75 a day. His expenses were \$41.70. How much did he save?

Dollars and decimal parts of a dollar are multiplied and divided just as integers are. Care must be taken to point off the proper number of places for cents and mills.

**23.** From \$12.375 × 25 subtract 12 times \$20.50.

**137.** United States money has a decimal scale; that is, 1 of any order or denomination is equal to 10 of the next lower order.

Thus, \$1 = 10 dimes ; 1 dime = 10 cents ; 1 cent = 10 mills.

**1.** How many cents in 5 dimes ? How many mills ?

**2.** How many dimes in 4 dollars ? How many cents ? How many mills ?

**138.** Carefully examine the following :

| | \$ | dimes | cents | mills |
|---|---|---|---|---|
| (a) | 8. | = 80. | = 800. | = 8000. |
| (b) | 3.25 | = 32.5 | = 325. | = 3250. |

QUERY.—1. In (a), how have we changed dollars to lower denominations ? By annexing what ?

2. In (b), how have the changes been made ? By moving what ? In which direction ?

PRINCIPLE.—*Any denomination is changed to a lower by annexing one or more ciphers, or by moving the decimal point one or more places to the right.*

Change to mills, or thousandths of a dollar :

**1.** \$5.
**2.** 3 dimes.
**3.** 9 cents.
**4.** 21 dimes.
**5.** 75 cents.
**6.** 370 dimes.
**7.** 435 cents.
**8.** \$7.65.
**9.** \$1.5.
**10.** 7 dollars.
**11.** \$2.05.
**12.** \$6.005.
**13.** \$.34.
**14.** \$2.125.
**15.** \$.875.

**139.** Carefully examine the following :

| | mills | cents | dimes | \$ |
|---|---|---|---|---|
| (a) | 2000 | = 200 | = 20 | = 2 |
| (b) | 3125. | = 312.5 | = 31.25 | = 3.125. |

QUERY.—1. In (a), how have we changed mills to higher denominations ? By cutting off what ?

2. In (b), how have the changes been made ? By moving what ? In which direction ?

PRINCIPLE.—*Any denomination is changed to a higher by cutting off one or more ciphers, or by moving the decimal point one or more places to the left.*

Change to dollars, or to dollars and decimal parts of a dollar:

**1.** 7000 mills.
**2.** 500 cents.
**3.** 80 dimes.
**4.** 600 dimes.
**5.** 9000 cents.
**6.** 35 dimes.
**7.** 865 cents.
**8.** 750 mills.
**9.** 1000 dimes.
**10.** 8625 mills.
**11.** 1250 cents.
**12.** 12375 mills.

**140.** It was learned in multiplication of integers that annexing one cipher to a number multiplies it by 10; annexing two ciphers, multiplies it by 100; and so on. The same is true in U. S. money.

Thus, $2.00 × 10 = $20.00; $2.00 × 100 = $200.00.

**141.** Moving the decimal point one place to the *right* multiplies expressions of U. S. money by 10; moving it two places, multiplies by 100; and so on.

Thus, $3.25 × 10 = $32.50; $3.25 × 100 = $325.

**142.** Moving the decimal point one place to the *left* divides expressions of U. S. money by 10; moving it two places, divides by 100; and so on.

Thus, $125.50 ÷ 10 = $12.55; $125.50 ÷ 100 = $1.255.

Find the value of the following:

(Omitting the dollar mark does not affect the operation.)

**1.** $52 × 10.
**2.** $7.25 × 10.
**3.** $1.50 × 100.
**4.** $.75 × 100.
**5.** $4.07 × 10.
**6.** $6 × 100.
**7.** 8.25 × 100.
**8.** $3.245 × 10.
**9.** $.625 × 100.
**10.** $.004 × 1000.
**11.** $330.00 ÷ $10.
**12.** 73.5 ÷ 10.
**13.** 47.3 ÷ 100.
**14.** 219 ÷ 100.
**15.** $300 × 100.
**16.** $60.50 × 100.
**17.** $1.627 × 1000.
**18.** $700.00 ÷ $100.
**19.** 300.00 ÷ 100.
**20.** 82.50 ÷ 10.
**21.** 513.7 ÷ 100.

# FRACTIONS.

**143.** **1.** In measuring milk Kate uses a can that holds half a gallon. She fills it twice. How many *half-gallons* has she? How many *gallons?*

**2.** A merchant measures a piece of silk with a ruler one third of a yard long, and finds the piece to contain *two thirds* of a yard. How many times did he apply the measuring unit?

(a). Is the piece of silk a yard in length?

(b). What unit of measure did he use?

(c). How many times did he take the unit?

**3.** A grocer selling molasses filled a jar three times. If the jar held *one fourth* of a gallon, how much molasses did he sell?

(a). What measuring unit did he use?

(b). What number tells how many times he took or repeated the unit?

(c). Is the unit of measure one of the *equal parts* of a gallon?

(d). What expresses the quantity of molasses sold?

**4.** A farmer used a *half-bushel* to measure his wheat, filling it five times.

(a). What was the measuring unit?

(b). How many such units in a bushel?

(c). How many *half-bushels* had he?

**144.** A **Fraction** is a number whose unit of measure is one of the equal *parts* of a certain whole or quantity.

Thus, three fourths of a yard (3 fourth-yards) is a fraction, its unit of measure being *one* fourth-yard—one of the four equal parts of a yard.

Five *half-bushels* (5 halves of a bushel) is a fraction ; its unit of measure is one *half-bushel—one* of the two equal parts of a bushel.

1. An integer (Art. 6) is a number whose unit of measure is an entire quantity—not one of the equal *parts* of a larger quantity.

2. The fraction $\frac{3}{4}$ yard may be regarded as 3 *fourth-yards* or as three fourths of a yard. The unit of measure is one of the four equal parts of a *yard*, and this unit is repeated 3 times in measuring the quantity, which compared with a *yard* is three fourths as great.

**145.** The general method of expressing fractions is by two numbers, written one above the other, with a line between them ; as, $\frac{1}{2}$. But a *special class* of fractions, called **Decimal Fractions,** is expressed in a notation peculiar to themselves.

## DECIMAL FRACTIONS.

**146. 1.** When anything is divided into ten equal parts, what is one part called ?

**2.** When each of these ten parts is divided into ten equal parts, how many parts are there ? What is one part called ?

**3.** When each of these 100 parts is divided into ten equal parts, how many parts are there ?

**147.** When anything is divided into *tenths, hundredths, thousandths,* etc., the parts are called **Decimal Parts** ; that is, *tenth-parts,* the word decimal being derived from *decem,* the Latin word for *ten.*

**148.** A **Decimal Fraction** is a number whose unit of measure is one of the decimal or tenth parts of a certain quantity.

Thus, 9 tenths (unit 1 tenth), 25 hundredths (unit 1 hundredth), 13 thousandths (unit 1 thousandth), etc., are decimal fractions.

Decimal fractions are often called simply *decimals.*

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**149.** The decimal fractions *one tenth, one hundredth, one thousandth,* etc., are obtained by dividing a quantity or whole into 10 equal parts (*tenths*), and each of these into 10 equal parts (*hundredths*), and each of these again into 10 equal parts (*thousandths*); hence,

1 (whole) = 10 tenths,
1 tenth = 10 hundredths,
1 hundredth = 10 thousandths.

10 thousandths = 1 hundredth,
10 hundredths = 1 tenth,
10 tenths = 1 (whole).

**150.** It is seen that tenths, hundredths, thousandths, etc., taken in order, decrease in value from left to right by the scale of *tens,* just as integers do. That is, 1 in any place or order is equal to 10 in the next place to the right; and 10 in any place is equal to 1 in the next place to the left.

**151.** Since the notation of decimals follows the same law as that of integers, an integer and a decimal fraction may be written as one expression, as in U. S. money.

The first place to the right of ones is *tenths;* the second, *hundredths;* the third, *thousandths,* etc.

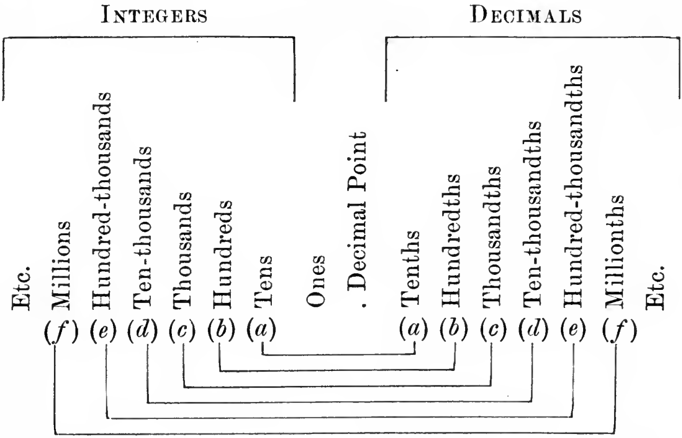
**152.** A point (.), called the **Decimal Point,** is placed before *tenths* to locate ones.

Thus, *three tenths* is written .3

**153.** An integer and a decimal written together as one number is called a **Mixed Number.**

In writing mixed numbers the decimal point is placed between the integral part and the fractional part, thus: 6.9; 2.03. When there is no integral part, what may be written in ones' place?

**154.** The relation of decimals to integers is clearly shown by the following diagram:




It will be noticed that:

**1.** Orders at equal distances to the left and right of ones' place have corresponding names; the names at the right having the fractional ending *ths*.

**2.** The increase and decrease according to the decimal scale go right along, without regard to the decimal point.

**3.** For every *part* of the unit expressed by any order on the right, there is a corresponding *multiple* of the unit expressed by the corresponding order on the left.

**4.** All orders, both higher and lower, are derived from ones. *Tens* denotes tens of ones, and *tenths* denotes tenths of one.

### READING DECIMALS.

**155.** In reading integers, we give the numbers, but omit the *name;* thus, 25 is read *twenty-five.* The name omitted is *ones*, which is the name of the right-hand order of the integer.

**156.** In reading decimals, we give both number and name. Thus, .25 is read, not *twenty-five,* but *twenty-five hundredths.* The name given is *hundredths,* which is the name of the right-hand order of the decimal.

☞ With the exception of giving the name, decimals are read precisely as integers are read.

RULE.—*Read the decimal as an integral number, and give it the name of the right-hand order.*

Read the following decimals:

| | | | |
|---|---|---|---|
| **1.** .3. | **6.** .87. | **11.** .23742. | **16.** .003761. |
| **2.** .9. | **7.** .087. | **12.** .00013. | **17.** .009042. |
| **3.** .15. | **8.** .235. | **13.** .00304. | **18.** .0007163. |
| **4.** .35. | **9.** .101. | **14.** .01238. | **19.** .00000001. |
| **5.** .07. | **10.** .3065. | **15.** .000013. | **20.** .327604385. |

1. In reading mixed numbers, read the integral part first, then the decimal part, connecting them with the word *and.* Thus, 205.03 is read, two hundred five and three hundredths.

2. It is sometimes necessary to make a pause before giving the name of the decimal. Thus, in .300, read three hundred——*thousandths;* and in .00003, read three——*hundred-thousandths.*

3. Expressions like .12½ and .33⅓ are read *twelve and one-half hundredths,* and *thirty-three and one-third hundredths,* respectively. The former may be written .125.

**157.** Since *per cent* means *hundredths,* in reading decimals we may say *per cent* instead of *hundredths.* The symbol % means either *per cent* or *hundredths.*

Thus, .25 = 25 per cent = 25%. .12½ = 12½ per cent = 12½%.
.50 = 50 per cent = 50%. .05 = 5 per cent = 5%.

Read the following mixed numbers:

| | | |
|---|---|---|
| **1.** 2.25. | **6.** 800.2035. | **11.** 136.00004. |
| **2.** 50.07. | **7.** 70.005. | **12.** 4000.004. |
| **3.** 13.033. | **8.** 30.078. | **13.** 3050.0507. |
| **4.** 310.09. | **9.** 3826.7. | **14.** 17005.017. |
| **5.** 7,394. | **10.** 4002.006. | **15.** 12345.12345. |

Read both ways:

| | | | |
|---|---|---|---|
| **16.** .15. | **19.** .01. | **22.** .37½. | **25.** 10%. |
| **17.** .27. | **20.** .09. | **23.** .80. | **26.** 35%. |
| **18.** .40. | **21.** .75. | **24.** .62½. | **27.** 33⅓%. |

REMARK.—Decimals may be read in different ways. Thus, .125 may be read 125 thousandths; or 1 tenth, 2 hundredths, 5 thousandths; or 12 hundredths, 5 thousandths. In practice, however, it is desirable to follow the method indicated in the *Rule*.

### WRITING DECIMALS.

**158. 1.** Express decimally thirty-four hundred-thousandths.

Hundred-thousandths is the *fifth* decimal order, hence 5 places are needed to express the decimal. But 34, written as an integer, occupies only 2 places, leaving 3 places on the left to be filled with ciphers. Hence the decimal is written .00034.

☞ Pupils should become thoroughly familiar with the *names* of the decimal orders at least to millionths.

Since the name or denomination of the decimal is indicated by the position of the right-hand figure with respect to ones' place, we have the following

RULE.—*The denomination of the decimal determines the number of places necessary to express it;* therefore,

*Write the decimal as an integral number, and prefix ciphers, when necessary, to supply the required number of places, placing the decimal point directly before tenths.*

Express decimally:

**2.** Three tenths. One tenth. Nine tenths. Six tenths.

**3.** Twelve hundredths. Four hundredths. Fifty-five hundredths. Ten hundredths. 15%. 9 per cent. 12½%.

**4.** Six thousandths. Fifteen thousandths. Two hundred three thousandths. Twenty-five ten-thousandths. Seven ten-thousandths. Four hundred fifty-one ten-thousandths.

**5.** Eight hundred-thousandths. 725 hundred-thousandths.

5 millionths. 75 hundred-millionths. 351 thousandths. 15 ten-millionths. 12 hundred-thousandths.

**6**. Four, and four hundredths. Three hundred-thousandths. One, and five thousandths. Six millionths. Five hundred thousandths.

**7**. Thirteen hundredths. 47005 billionths. Six hundred, and seven thousandths. Six hundred seven thousandths.

**8**. Sixty-nine, and 903 thousandths. Forty-nine, and 900 ten-thousandths. One millionth.

Write in words:

**9**. .7 ; .05 ; .01 ; .016 ; .203 ; .25 ; .324 ; 8%.

**10**. .601 ; .0015 ; .0125 ; .2405 ; .00025 ; .00123.

**11**. 2.7 ; 5.04 ; 6.008 ; 4.0019 ; 12.02301 ; 202.0202.

**12**. How many tenths can be expressed by one figure? How is 10 tenths written?

**13**. How many hundredths can be expressed by one figure? By two? How is 100 hundredths written? How is 100 *per cent* written?

**14**. Since 1 tenth equals 10 hundredths, 100 thousandths, 1000 ten-thousandths, etc., it is plain that .1 = .10 = .100 = .1000 = .10000, etc. Hence,

**159.** Principle.—*Annexing ciphers to a decimal reduces it to a lower denomination without changing its value.*

Query.—Does omitting ciphers from the right of a decimal change its value?

**1**. Change .5, .03, .027, and .4850 to thousandths.

| | |
|---|---|
| .5 = .500 | In the first two decimals we annex ciphers enough |
| .03 = .030 | to make the 3 places required to express thou- |
| .027 = .027 | sandths. The third needs no changing. Why? The |
| .4850 = .485 | last is changed by omitting the cipher at the right. |

This process is called reducing to a common name or denomination (or denominator).

**2**. Change .8, .25, .030, .4600, and .07 to thousandths.

**3**. Reduce .75, .013, .020, and .0146 to ten-thousandths.

**4.** Reduce .09, .0240, .3275, .1, .00010 to millionths.

**5.** Change .30, 5, .400, .8000, and 1.7 to tenths.

**6.** Change .0032, .2, .470, and .835000 to ten-thousandths.

**7.** Reduce 5 ones to tenths, 3 ones to hundredths, 10 ones to tenths, and 2 ones to *per cent.*

**8.** How does .1 compare in *value* with .01? How in form? Then what is the effect of *prefixing* a cipher to .1?

**9.** How does .01 compare in value with .001? Then how is .01 affected by *prefixing* a cipher? Prefix ciphers to other decimals, and compare values.

**160.** PRINCIPLE.—*Prefixing a decimal cipher to a decimal divides the value of the decimal by ten.*

QUERIES.—1. How does prefixing a cipher affect the *place* of each figure in the decimal?

2. Does a figure in that place express as much value as it did before being moved?

3. What part of its former value does it express? Then by what has the decimal been divided?

### ADDITION AND SUBTRACTION.

**161.** In addition and subtraction of decimals the operations are the same as the like operations in integral numbers.

**1.** What is the sum of .613, .0176, .2, and .601?

$$\begin{array}{r} .613\phantom{0} \\ .0176 \\ .2\phantom{000} \\ \underline{.601\phantom{0}} \\ 1.4316 \end{array}$$

By arranging the decimal points in a vertical line, we make units of the same order stand in the same vertical column. The numbers are added precisely as in integers, and the decimal point is placed before tenths. (Is 14 tenths a fraction? How is it written?)

**2.** From .3 subtract .1235.

$$\begin{array}{lcl} \text{(a)} & & \text{(b)} \\ .3 & = & .3000 \\ \underline{.1235} & = & \underline{.1235} \\ .1765 & & .1765 \end{array}$$

By arranging the decimal points in a vertical line, we cause units of the same order to stand in the same column. We subtract as in integers or U. S. money.

QUERIES.—Why may .3 be written as in (b)? (See Art. 159.) Is it necessary to annex ciphers to the minuend? When the remainder is a mixed number, where is the decimal point placed?

Find the value of the following:

3. .17 + .002 + .2509.
4. .005 + .301 + .29.
5. 19.909 + 100.01 + 199.
6. .375 + .048 + 255.0.
7. 4.372 + .4293 + 3.87.
8. 5.0008 + 124 + .016.
9. 86.45 + .001 + .05.
10. 2.3 + .004 + .2 + .88.
11. .75 − .25.
12. .5 − .005.
13. 100.01−25.001.
14. 10 − .0678.
15. .16 −.06814.
16. 1000 − .1000.
17. .6504 − .067.
18. .1 − .0053.
19. 94.61 + .00421 + .0003 + .0044 + 10.
20. 84.56 + 9.245 + .8763 + 8.009 + 7.7.
21. 1 million − one millionth.
22. 10. − 10 ten-thousandths.
23. 94 thousandths − 253 ten-millionths.
24. 25 thousandths − 25 ten-thousandths.
25. 1 − 1 thousandth + 1 tenth + 100 hundredths.

26. The minuend is the sum of .3 and .003; the subtrahend is .02875. What is the remainder?

27. From what number must .0105 be subtracted to leave the remainder 1.807?

28. The larger of two numbers is 3822.078; their difference is 1934.124. What is the less number?

29. Find the least decimal which added to 1.4142 − .0022 will make the result an integer.

30. A owes $1,000 to B, and $1,347.55 to C. He has in cash $1,955.75. If he pays C in full, how much will he lack of having enough to pay B?

31. Mr. Slaven bought an organ for $85.50 on a credit of three months. He concluded to pay cash, and was allowed a discount of $1.27. How much had he left out of a $100 bill?

32. Bishop Brothers sold goods amounting to $190.50 on Monday, $250 on Tuesday, $117.25 on Wednesday, $57 on Thursday, $135.75 on Friday, and $427.37 on Saturday. What were the total sales for the week?

**33.** Find the sum of 345 millionths, forty and 40 millionths, seven and 7 thousandths, thirty-eight and 87 ten-thousandths.

**34.** In a corncrib that will hold 572.5 bushels of corn there are 329.375 bushels. How many bushels will be required to fill it ?

**35.** One side of a square field is 42.375 rods long. If 12.5 rods of the fence around it are blown down, how many rods will remain standing ?

**36.** A tank that will hold 1050.75 gallons contains 396.7 gallons. If 135.5 gallons be added, how much will still be needed to fill the tank ?

**37.** A man bought a farm for $1,750 and a lot for $975.75. For what amount must he sell both to gain $289.50 ?

### MULTIPLICATION AND DIVISION.

**162.** The processes of multiplication and division of decimals are the same as the like processes in integers, the locating of the decimal point being the only thing that needs special attention.

| (a) | (b) | (c) | (d) |
|---|---|---|---|
| .1 | 1. | .25 | 2.5 |

**1.** In (a), the figure 1 expresses 1 tenth, in (b) it expresses 1 one. What has been the effect of moving the decimal point one place to the right ?

**2.** In (d), the 2 expresses 2 ones, in (c) 2 tenths. The 5 in (d) expresses 5 tenths, in (c) 5 hundredths. What has been the effect of moving the decimal point one place to the left ?

**163.** PRINCIPLE.—*Each removal of the decimal point one place to the right multiplies the decimal by 10 ; each removal one place to the left divides the decimal by 10.*

Thus, by moving the point one place to the right, .325 becomes 3.25 ; that is, 3 tenths have become 3 ones, the 2 hundredths have become 2 tenths, and the 5 thousandths have become 5 hundredths. Since the value of each figure has been multiplied by 10, the value of the entire decimal has been multiplied by 10.

☞ Have the pupil illustrate the second part of the principle, which is the converse of the first.

**164. To multiply or divide a decimal by 10, 100, 1000, etc.**

RULES.—1. *To multiply a decimal by 10, 100, 1000, etc., move the decimal point as many places to the right as there are ciphers in the multiplier, annexing ciphers when necessary.*

2. *To divide a decimal by 10, 100, 1000, etc., move the decimal point as many places to the left as there are ciphers in the divisor, prefixing ciphers when necessary.*

**1.** Multiply .275 by 100 ; also by 10000.

$.275 \times 100 = 27.5. \quad .275 \times 10000 = 2750.$

**2.** Divide .275 and 62.5 each by 100.

$.275 \div 100 = .00275. \quad 62.5 \div 100 = .625.$

Find the value of :

| | | |
|---|---|---|
| **3.** $3.25 \times 10.$ | **8.** $37.5 \div 10.$ | **13.** $.37685 \times 1000.$ |
| **4.** $69.3 \times 10.$ | **9.** $6.25 \div 10.$ | **14.** $52.16 \times 1000.$ |
| **5.** $.75 \times 100.$ | **10.** $.314 \div 10.$ | **15.** $7.013 \div 100.$ |
| **6.** $.486 \times 1000.$ | **11.** $.209 \div 100.$ | **16.** $3875 \div 1000.$ |
| **7.** $1.625 \times 100.$ | **12.** $632 \div 100.$ | **17.** $41.065 \div 100.$ |

**165. To multiply or divide a decimal by .1, .01, .001, etc.**

RULES.—1. *To multiply by .1, .01, .001, etc., move the decimal point as many places to the left as there are decimal places in the multiplier.*

2. *To divide by .1, .01, .001, etc., move the decimal point as many places to the right as there are decimal places in the divisor.*

**1.** Multiply 1.093 by .1 ; also by .01.

$1.093 \times .1 = 1.093 \div 10 = .1093.$

$1.093 \times .01 = 1.093 \div 100 = .01093.$

To multiply a number by .1 is to take one tenth of it ; that is, to multiply by .1 is to divide by 10 ; to multiply by .01 is to divide by 100, etc. By comparing the products with the multiplicands, we find that the decimal point has been moved to the left as many places as there are ciphers in the multiplier. (See Art. 164.)

**2.** Divide 32.5 by .1 ; also by .01.

$$32.5 \div .1 = 32.5 \times 10 = 325.$$
$$32.5 \div .01 = 32.5 \times 100 = 3250.$$

Since there are 10 tenths in 1, *one tenth* is contained in any number 10 times as often as *one* is contained in it. But dividing a number by 1 does not alter its value. Hence to divide a number by .1 is to multiply it by 10 ; to divide by .01 is to multiply by 100, etc.

| Multiply : | Divide : |
|---|---|
| **3.** .258 by 10. | **14.** 37.5 by 100. |
| **4.** 7.07 by 100. | **15.** 436 by 1000. |
| **5.** 3.916 by 1000. | **16.** .900 by 100. |
| **6.** .846 by 10000. | **17.** 24.57 by 1000. |
| **7.** 7.5 by .1. | **18.** 5 by 1000. |
| **8.** 83.7 by .01. | **19.** .99 by .1. |
| **9.** 3.25 by .01. | **20.** .0075 by .01. |
| **10.** .3004 by .001. | **21.** .0003 by .001. |
| **11.** 179.5 by .001. | **22.** 4444 by .0001. |
| **12.** 3.428 by .0001. | **23.** 18 by .01. |
| **13.** .5 by .0001. | **24.** 100 by .1000. |

**25.** Which is the greater, $.5 \times 100$, or $.5 \div .01$ ?

**26.** How much greater is $.75 \times 1000$ than $.25 \div .001$ ?

**166. To multiply or divide in decimals—universal case.**

PRINCIPLES.—1. *The product of two decimals contains as many decimal places as there are decimal places in both factors.*

2. *The quotient of two decimals contains as many decimal places as the number of decimal places in the dividend exceeds the number in the divisor.*

The number of decimal places in the dividend can be increased as you please, by principle in Art. 159.

**1.** Multiply .036 by .27.

$$\begin{array}{r} .036 \\ .27 \\ \hline 252 \\ 72\phantom{0} \\ \hline .00972 \end{array}$$

The multiplier $.27 = 27 \times .01$. We therefore multiply first by 27, then the resulting product by .01. 36 *thousandths* $\times 27 = 972$ *thousandths*, or .972. Multiplying this product by .01 moves the decimal point two places to the left (Art. 155). Hence the required product is .00972. It has as many decimal places as both factors have.

If we multiply as in integers, we get the product 972, to which we prefix two ciphers to make the required *five* places.

**2.** Divide .00972 by .27.

$$\begin{array}{l} .27).00972(.036 \\ \phantom{.27).0}81 \\ \phantom{.27).0}\overline{162} \\ \phantom{.27).0}162 \\ \phantom{.27).0}\overline{\phantom{162}} \end{array}$$

The dividend being the product of divisor and quotient must contain as many decimal places as both of them. Since the dividend contains 5 decimal places and the divisor 2, the quotient must contain 5 − 2, or 3 decimal places. Dividing as in integers, we get the quotient 36, to which we prefix a cipher to make the required *three* places.

**167.** RULES.—1. *In the multiplication of decimals multiply as in integers, and from the right of the product point off as many decimal places as there are in both factors, prefixing ciphers, if necessary, to make the required number of decimal places.*

2. *In the division of decimals, divide as in integers (annexing ciphers, if necessary, to the dividend), and point off from the right of the quotient as many decimal places as those of the dividend exceed those of the divisor.*

If the quotient does not contain a sufficient number of decimal places, ciphers must be prefixed to make the required number.

(*a*) Find the product of :

**1.** $.28 \times 4.8$.

**2.** $.6 \times .7$.

**3.** $.35 \times .16$.

**4.** $10 \times .1$.

**5.** $.134 \times 25$.

**6.** $216 \times .24$.

**7.** $.478 \times .152$.

**8.** $.0017 \times .09$.

**9.** $10000 \times .0001$.

**10.** $7.5 \times .0005$.

**11.** $1000000 \times .000001$.

**12.** $.001 \times 10000$.

**13.** $.1 \times .1$.

**14.** $.5 \times .5$.

**15.** $.5 \times .05$.

**16.** $.05 \times .005$.

(*a*) Find the product of:

17. $.01 \times .001$.
18. $150 \times .1$.
19. $\$1 \times .1$.
20. $7 \times 1.1$.
21. $2.5 \times 2.5$.
22. $\$100 \times .06$.
23. $.017 \times 3.7$.
24. $101 \times 1.01$.
25. $1.03 \times 1.09$.
26. $5.005 \times .005$.
27. $.008 \times 800$.
28. 5 tenths $\times$ 50 hundredths.
29. $.01 \times .1 \times 1$.
30. $.05 \times 5 \times .50$.
31. $72.5 \times 10$.
32. $.1225 \times .1$.
33. $25.6 \times .20$.
34. $.054 \times 100$.
35. $125 \times 1.05$.

(*b*) Find the quotient of:

1. $.00125 \div .5$.
2. $.0075 \div 1.5$.
3. $1 \div .1$.
4. $.01 \div 100$.
5. $16.84 \div .02$.
6. $.00884 \div .34$.
7. $.0355 \div .71$.
8. $16.025 \div .045$.
9. $10000 \div .0001$.
10. $.000375 \div .0005$.
11. $1000000 \div .000001$.
12. $.000001 \div 1000000$.
13. $\$150 \div \$ .06$
14. $\$1 \div \$ .05$.
15. $159.750 \div .00375$.
16. $14400 \div .32$.
17. $14400 \div 3.2$.
18. $200 \div .002$.
19. $.735 \div 500$.
20. $78.13 \div 5$.
21. $78.39 \div 3$.
22. $125 \div 25000$.
23. $12 \div .0012$.
24. $5.4768 \div 22.82$.
25. $.025 \div 250$.
26. $.0567 \div 43$.
27. $1 \div 3.1416$.
28. ten $\div .01$.
29. 1 millionth $\div .01$.
30. 300 hundredths $\div$ 15 tenths.
31. $3.1416 \div .31416$.
32. $.25 \div .0025$.
33. 9 ones $\div$ 40 tenths.
34. 25 tenths $\div$ 25 hundredths.
35. 25 hundredths $\div .025$.
36. $27.45 \div 1.5$.
37. $250 \div .025$.
38. $2750 \div .25$.
39. $3.609 \div .9$.
40. $27.63 \div .003$.
41. $4.914 \div 70$.
42. $.026 \div .000013$.

**168.** **1.** Multiply 4.65 by 700, and divide the product by 300.

$4.65 \times 100 = 465$. Then $465 \times 7 = 3255$.
$3255 \div 100 = 32.55$; and $32.55 \div 3 = 10.85$.

Find the value of:

**2.** $526.53 \times 50$.
**3.** $245.6 \times 400$.
**4.** $.864 \times 900$.
**5.** $.7854 \times 700$.
**6.** $150 \times 25\%$.
**7.** $630 \div 500$.
**8.** $.844 \div 400$.
**9.** $307.2 \div 1200$.
**10.** $2697.5 \div 8300$.
**11.** $150 \div 25\%$.

**12.** What is the value of $.05 \times .07 + .28 \div .5$?

**13.** If I give 3 pigs for $7.50, how many must I give for $37.50?

**14.** A man paid $17.25 for 300 pounds of sugar. What did it cost per pound?

**15.** How many eggs in a crate containing 24.5 dozen?

**16.** If 8 pounds of coffee cost $1.74, what will 5 pounds cost?

**17.** At $2\frac{1}{2}$¢ each, how much will 3.25 dozen lemons cost?

**18.** A man paid $15 for rice, at the rate of 4 pounds for a quarter. How many pounds did he get?

**19.** A has $1.40 and B has 2.5 times as much. How much must B give A so that each may have the same amount?

## BILLS AND ACCOUNTS.

**169.** Prof. Samuel Andrews bought of J. R. Weldin & Co. the following: 6 dozen lead pencils at $.30 a dozen, 2 gross pens at $.85, 5 reams note paper at $1.50, and 20 arithmetics at $.75.

In a few days he received the following *bill:*

COLUMBIA, S. C., May 1, 1899.

MR. SAMUEL ANDREWS,

Bought of J. R. WELDIN & Co.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | To 6 dozen Lead Pencils | @ | $.30 | 1 | 80 | | |
| | | " 2 gross Pens | " | .85 | 1 | 70 | | |
| | | " 5 reams Note Paper | " | 1.50 | 7 | 50 | | |
| | | " 20 Arithmetics | " | .75 | 15 | 00 | 26 | 00 |

When this bill was paid, the following was written on it as a *receipt:*

"Received payment,

J. R. WELDIN & Co.

G."

(The "G" is the initial of Mr. Greene, who receipted the bill.)

**1.** Mrs. R. D. White ordered the following from Davis & Russell, New Orleans, La.:

18 yd. Scotch Gingham @ 21¢.
$36\frac{1}{2}$ yd. Calico @ 6¢.
$12\frac{1}{4}$ yd. India Silk @ 45¢.
25 yd. Cashmere @ $1.25.

Make out her bill, and receipt it.

**170.** The following is a specimen of a receipted bill, with a discount, and credits:

JACKSON, MISS., Oct. 1, 1899.

Mr. T. B. DEARMIT,

To GORDON, HAY & Co., *Dr.*

| | | | | | |
|---|---|---|---|---|---|
| 1899. | | | | | |
| Jan. 13 | To 50 Grammars... $.40 | 20 | 00 | | |
| May 21 | " 24 Arithmetics.. .60 | 14 | 40 | | |
| Aug. 9 | " 42 Histories.... 1.00 | 42 | 00 | | |
| | | | | 76 | 40 |
| | Less 10%........... | ......... | .... | 7 | 64 |
| | | | | 68 | 76 |
| | *Cr.* | | | | |
| July 25 | By Cash.........$25.00 | | | | |
| Sept. 8 | " " ......... 25.00 | | | | |
| | | | | 50 | 00 |
| | | | | 18 | 76 |

Received payment,

GORDON, HAY & CO.

By WILSON.

**171.** A **Debt** is the amount which one person owes another. A **Debtor** is a person or firm that owes a debt.

**172.** A **Credit** is the amount paid on a debt. A **Creditor** is a person or firm to whom a debt is due.

In the transaction mentioned in Art. 170, who is the debtor? Who is the creditor? Name the credits.

**173.** An **Account** is a record of the debts and credits between two parties—a debtor and a creditor.

**174.** A **Bill** is a creditor's written statement of the quantity and price of each item in his account with a debtor, together with the discount and credits, if any, and the net amount due.

Bills are commonly called *invoices*.

**175.** A **Statement** is a written summary of an account between two parties, rendered at stated intervals, usually monthly.

**176.** Make out and receipt the following bills. Supply dates and names where needed.

**1.** Mr. R. P. Lougeay bought of McAllister & Co. 25 pounds of coffee at 28 cents a pound, 75 pounds of sugar at $5\frac{1}{2}$ cents a pound, and 20 pounds of prunes at 12 cents a pound.

**2.** Mrs. M. B. Kifer bought of Campbell & Smith 10 yards of silk at $1.50 a yard, 36 yards of muslin at 7 cents a yard, 15 yards of flannel at $.75 a yard, and 2 pairs of shoes at $3.25 a pair.

**3.** Mrs. A. C. McLean bought of Kauffman Bros. 3 tablecloths @ $3.50, 1 piano cover @ $4.75, 4 pairs of lace curtains @ $5.25, 2 doz. towels @ $3.60 a dozen, and 12 yards cashmere @ $1.25 a yard.

**4.** Miss Nannie Mackrell bought of W. M. Laird 2 pairs ladies' shoes @ $2.75 a pair, 6 pairs overshoes @ $.75 a pair, 1 pair Oxford ties @ $1.25, 3 pairs misses' shoes @ $2.15, and 1 pair gum boots @ $3.25.

**5.** Mr. S. M. Brinton bought of Hopper Bros. & Co. 2 doz. silver knives @ $36 a dozen, 4 doz. silver teaspoons @ $16 a dozen, 2 doz. silver tablespoons @ $10.25 a set, and 1 silver spoonholder for $9.

**6.** Mr. Wm. Hasley bought of Fred Gray 12.5 tons of coal @ $3.25, 40 bushels of apples @ $.75, 200 lb. grapes @ 3 cents a pound, and 25 bushels of potatoes @ $.85.

**7.** On May 25, J. M. Logan bought of W. H. Keech 5 bedsteads @ $14, 1 bookcase for $35, and 18 chairs @ $15 a dozen. On July 3, he bought 3 hammocks @ $2.25, and a leather couch for $45. On June 15, he paid $50 in cash, and on the 19th $37.50 more.

## REVIEW WORK.

### ORAL EXERCISES.

**177. 1.** How many tenths in 80 hundredths?

**2.** How many hundredths in 7 tenths and 15 hundredths?

**3.** I paid 3 tenths of a dollar for 3 cakes of soap. At the same rate, how much would I pay for a dozen cakes?

**4.** A lady spent .1 of her money for a hat, and .4 for a shawl, and the remainder for a dress which cost $15. How much had she at first?

**5.** At $.09 each, how many slates can be bought for $3.60?

**6.** If 80 is divided into 10 equal parts, what is one part called? Three parts? Nine parts? How many are 7 tenths (.7) of 80?

**7.** If 18 is .3 of some number, what is the number?

**8.** Thirty-five is .5 of what number?

**9.** Of what number is 9 three tenths?

**10.** The sum of .2 and .05 is .5 of what number?

**11.** A has $1.50, and .3 of his money is .1 of B's money. How much has B?

**12.** B and C together have $40. If .3 of B's money equals .9 of C's, how much has each?

**13.** How often must .3 be added to itself to make 3?

**14.** How many times must .7 be subtracted from 3.5 to leave a remainder of 1.4?

**15.** How many hundredths can be taken from 25 tenths?

### WRITTEN EXERCISES.

**178. 1.** If seven sheep are worth $31.50, how many sheep can be bought for $184.50?

**2.** A man divided his farm of 227.5 acres into 14 equal fields. How many acres in 5 of the fields?

**3.** At $2.625 a yard, how many yards of cloth can be bought for $55.125?

**4.** If a person's taxes are 5.8 mills on $1, how much will they be on $2500?

**5.** Find the cost of 237.25 bushels of oats at .42 of a dollar a bushel.

**6.** At $.08 each, how many copy books can be bought for $24?

**7.** Gold weighs 19.36 times as much as an equal bulk of water, and a cubic foot of water weighs 62.5 pounds. How many cubic feet of gold weigh a ton, or 2,000 pounds?

**8.** One pound of dry oak wood when burnt yields .023 of a pound of ashes. How many pounds must be burnt to produce 46 pounds of ashes?

**9.** Every day a newsboy buys 70 papers at 30 cents a dozen, and sells them at 5 cents each. How much money does he make in 6 days if 40 papers remain unsold?

**10.** If a boy saves 6 dimes a week, in how many days can he save enough to buy a suit worth $5.40?

**11.** How often can .013 be subtracted from 26?

**12.** Find the cost of 8 bushels 3 pecks of turnips at $.125 a peck.

**13.** The divisor is 27.125, the quotient 7.32, and the remainder 18.0825. What is the dividend?

**14.** Divide 3 ten-millionths by 10 millionths, and multiply the quotient by 30.

**15.** At $1.50 a thousand, what will 1,750 envelopes cost?

**16.** The distance around a circle is about 3.1416 times the distance across it through the center. If the distance around a circular pond is 50 feet, what is the distance across it?

**17.** Two men start from the same place at the same time and travel in the same direction, one going 3.28 miles an hour, the other 4.07 miles an hour. How far apart will they be in 9 hours?

**18.** The circumference of the wheel of a bicycle is 11.28 feet. How many times will it turn in going 2.5 miles, there being 5280 feet in a mile?

**19.** Find the cost of 8375 feet of lumber, when lumber is worth $18 a thousand feet.

**20.** In a city of 240,000 inhabitants .125 of the population are school children. If each teacher has 50 pupils, how many teachers are in that city?

**21.** Add 155 ones, 155 tenths, 155 hundredths, 155 thousandths.

**22.** The divisor 5.125 is 5 times the quotient; what is the dividend?

**23.** The product of three factors is 78.66; two of the factors are respectively 6.9 and 7.125; what is the third factor?

**24.** Find the least decimal fraction which added to the sum of 87.43 and 169.578 will make the sum an integer.

**25.** A man paid .15 of his money for rent, .02 for wood, .18 for clothing, and had $812.50 left. How much had he at first?

**26.** Find the product of the two smallest decimals that can be expressed by the figures 0, 0, 9, and 3.

**27.** Gunpowder is composed of .76 nitre, .14 charcoal, and .10 sulphur. How much of each is required to make 2000 pounds of powder?

**28.** At $0.34 a bushel, how many barrels of apples can be had for $13.60, allowing 2.5 bushels to the barrel?

**29.** How many pounds of butter could be made from 46 cows during the month of June, each cow averaging 2.5 gallons of milk daily, and each gallon making .5 of a pound of butter?

**30.** If 4 cords of wood are worth as much as 13.4 bushels of rye, how much rye can be obtained for 15 cords of wood?

**31.** If a Mexican dollar is worth \$0.85, how many Mexican dollars equal the value of \$680 in U. S. money?

**32.** If the land that produces a bale of cotton yields 30 bushels of cotton seed, what is the value @ \$.20 per bushel of the cotton seed produced by the land that yields 21 bales of cotton?

**33.** In one manufacturing establishment the average weekly wages paid to 262 operators was \$12.85; in another, to 355 operators, \$13.84; and in a third, to 128 operators, \$15.11. Find the average weekly wages in all three establishments.

**34.** If a railroad train runs 350 miles in 19.5 hours, but makes three stops of 20 minutes each, and ten stops of 6 minutes each, what is the average rate per hour while running?

**35.** A franc is 19.3 cents. Find the cost in United States money of goods bought in Paris amounting to 1,000 francs.

**36.** A cubic foot of water weighs 1000 ounces. How many pounds does a cubic foot of gold weigh, gold being 19.4 times as heavy as water?

**37.** If oysters yield 1.25 gallons to the bushel, how many bushels in the shell must I buy so that when opened they will fill a 10-gallon can?

**38.** In the year 1897, the total ordinary expenditures of the United States government were \$365,774,159, which was \$5.02 to each person. What was the population in that year, to the nearest 1000?

### SUPPLEMENTARY EXERCISES (FOR ADVANCED CLASSES).

**179. 1.** Can 75 tenths be written as a decimal fraction? Why not? Can it be written as a mixed number?

**2.** Divide one by seven, carrying the quotient to 12 decimal places, and carefully note the result.

**3.** Investigate the result of dividing one by 3, 11, 13, and 17, carrying the division as far as may be necessary.

**4.** After spending .015 of his money, and 2 tenths of the remainder, B had $15.76 left. How much did he spend ?

**5.** Find the cost, at $8 75 per thousand, of the rails for 640 panels of 14-rail fence.

**6.** The multiplicand is .005, and the product is 1. By what must the multiplier be divided to give a quotient equal to the product ?

**7.** I gave .44 of my money for a farm, and .75 of the remainder for a store. If the farm cost $250 more than the store, how much did I pay for the store ?

**8.** Cork weighs 15 pounds per cubic foot, and its weight is .24 of the weight of water. Find the weight of 10 cubic feet of oak, if the weight of oak is .934 of the weight of water.

**9.** The distance of the moon from the earth is 59.97 times the earth's radius. If this radius is 3962.824 miles, find the distance to the moon.

**10.** In 1890 the native population of the United States was 85.23% of the whole. The foreign born was what decimal part of the native ?

**11.** The population of Italy is 29,699,785. The total indebtedness of the country is $2,324,826,329. Find the rate of debt for each person.

**12.** The population of New Orleans in 1897 was 280,000. The assessed valuation of taxable property was $140,654,475. Supposing the whole population to be divided into families of five, compute the average wealth of each family.

## COMMON FRACTIONS.

**180. 1.** When anything is divided into ten equal parts, what is one part called? What name is given to *tenth-parts* of anything? What kind of fraction is one expressing decimal parts of any whole or quantity? Then what kind of fraction is one-tenth (.1)?

**2.** When anything is divided into 100 equal parts, what is one part called? 25 parts? What kind of fraction is 6 hundredths? Fifty thousandths? How are they written?

**3.** Is one *fourth* yard a decimal fraction? $\$\frac{4}{5}$? 5 *half-bushels?* Why not?

**181.** When a measuring unit is a *decimal* part of a certain whole or quantity, any number of these units (parts) expressed in decimal notation is called a *decimal fraction;* but when the unit of measure is one of *any* number of equal parts, one or more of such parts, expressed by two numbers one above the other with a line between them, is called a **Common Fraction.**

Thus, 5 tenths (.5) and 25 hundredths (.25) are decimal fractions, while 3 fourths ($\frac{3}{4}$) and 10 thirds ($\frac{10}{3}$) are common fractions.

NOTES.—1. The decimal point was first used about the beginning of the 17th century, but 100 years elapsed before decimal fractions were extensively employed.

2. During the introduction of this new class of fractions, the "old style" fractions were given the name "vulgar" or "common" fractions to distinguish them from the others, which were written in a new and special way by the aid of the decimal point.

3. Strictly speaking, a number of decimal parts, to be called a decimal fraction, must be written in the form peculiar to decimals, since it is only in that *form* that the practices exemplified in decimals are applicable.

**182.** The **Unit** in fractions is a *part* of one whole, and is called a **Fractional Unit**; that is, a fourth is the unit of measure in three fourths.

**183.** The **Denominator** (*namer*), the number written below the line, shows the number of equal parts into which a certain whole or quantity is divided in order to obtain the fractional unit.

Thus, in $\frac{5}{2}$ bushels, the denominator 2 shows that the one *bushel* is divided into two equal parts and the unit is a half-bushel. How many fractional units are used to express the value of the quantity?

**184.** The **Numerator** (*numberer*), the number written *above* the line, shows how many of the fractional units are used.

QUERIES.—In the fraction $\$\frac{3}{4}$, which is the denominator? What does it show? What does the 3 show? Of what does $\$\frac{3}{4}$ express the value?

NOTE.—A fraction whose numerator is less than the denominator is called a *proper fraction;* otherwise it is usually called an *improper fraction.*

**185.** A **Mixed Number** is the sum of an integer and a fraction, and is expressed by writing the fraction immediately after the whole number.

Thus, $\$5 + \$\frac{2}{3} = \$5\frac{2}{3}$, is a mixed number.

NOTES.—1. The integer and fraction must be *like* numbers.

2. Five silver quarters are *equivalent* to $\$1\frac{1}{4}$; but in this case the mixed number is *not* the sum of an integer and a fraction. The quantity is a *fraction* ($\$\frac{5}{4}$) made up of 5 units of measure (quarters). In practice, however, no attention is paid to this distinction.

3. The definition of a fraction in Art. 181 includes only fractions whose denominators are integral, hence excludes such expressions as $\frac{\frac{2}{3}}{\frac{3}{4}}$ and $\frac{5}{\frac{2}{3}}$, which are properly expressions of unexecuted *division*. (See Art. 95.) However, in algebra the definition of a fraction is extended so as to embrace any expression in the fractional *form.*

**186.** An *Integer* or a *Mixed Number* may be expressed in the form of a fraction, and treated as a fraction.

Thus, $\$4 = \$\frac{4}{1} = 4$ *one dollars;* $\$3\frac{1}{2} = \$\frac{7}{2} = 7$ *half-dollars;* $1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3}$, etc.

**187.** The number of equal parts into which anything is divided gives the parts their name.

Thus, any quantity divided into 2 equal parts = 2 halves, or $\frac{2}{2}$;
any quantity divided into 3 equal parts = 3 thirds, or $\frac{3}{3}$;
any quantity divided into 4 equal parts = 4 fourths, or $\frac{4}{4}$;
any quantity divided into 5 equal parts = 5 fifths, or $\frac{5}{5}$;
etc.

Therefore, to read fractions,

Direction.—*State the number of fractional units (units of measure) and give them the name indicated by the denominator.*

(*a*) Read, and write in words:

**1.** $\frac{2}{3}$, $\frac{5}{6}$, $\frac{2}{8}$.
**2.** $\frac{3}{4}$, $\frac{4}{5}$, $\frac{1}{7}$.
**3.** $\frac{8}{9}$, $\frac{6}{7}$, $\frac{9}{11}$.
**4.** $\frac{11}{17}$, $\frac{21}{23}$, $\frac{33}{44}$.
**5.** $\frac{13}{30}$, $\frac{16}{31}$, $\frac{1}{32}$.
**6.** $\frac{27}{110}$, $\frac{9}{123}$, $\frac{63}{241}$.
**7.** $\frac{50}{60}$, $\frac{17}{365}$, $\frac{37}{73}$.
**8.** $\frac{40}{200}$, $\frac{42}{100}$, .42.
**9.** $\frac{3\frac{1}{2}}{4}$, $\frac{\frac{2}{3}}{5}$, $6\frac{2}{3}$.

(*b*) Write in figures:

**1.** Three fourths.
**2.** Four fifths.
**3.** Ten elevenths.
**4.** Eight twenty-firsts.
**5.** Twelve thirty-thirds.
**6.** Two hundredths.
**7.** Fifty-two hundredths.
**8.** Sixty two-hundredths.
**9.** Nine eightieths.
**10.** Eighty ninetieths.
**11.** Forty three-thousandths.
**12.** Two and one-half thirds.

## CHANGE OF FORM.

### 188. To change fractions to larger denominators.

**1.** How many fourths in a gallon? In $\frac{1}{2}$ of a gallon? Then $\frac{1}{2}$ is equal to how many fourths?

**2.** How many eighths in an apple? In $\frac{1}{2}$ of an apple? Then $\frac{1}{2}$ is equal to how many eighths?

**3.** Since $\frac{1}{2} = \frac{2}{4}$, and $\frac{1}{2} = \frac{4}{8}$, what has been done to the numerator and denominator of the fraction $\frac{1}{2}$ without changing its value? How has it been done?

**4.** Change $\frac{1}{2}$ to sixths. To twelfths. To 16ths. To 20ths.

**5.** Change $\frac{2}{3}$ to sixths. To ninths. To 12ths. To 18ths.

**6.** In 1 how many 24ths? Then how many 24ths are there in $\frac{1}{8}$? In $\frac{3}{8}$? In $\frac{5}{8}$? In $\frac{7}{12}$?

**7.** Name three fractions each equal to $\frac{3}{4}$.

**8.** What may be done to the numerator and denominator of a fraction without changing its value?

**9.** Express $\frac{3}{4}$ with numerator and denominator 4 times as large, and explain why the *value* of the fraction is not changed.

Multiplying numerator and denominator by 4, we have $\frac{12}{16}$. Since the *fractional units* are $\frac{1}{4}$ as large, while the number of them taken is 4 times as great, $\frac{12}{16}$ is equivalent to $\frac{3}{4}$. Hence the following

**189.** Principle.—*Multiplying numerator and denominator by the same number does not change the value of the fraction.*

WRITTEN EXERCISES.

**190. 1.** Change $\frac{2}{3}$ to twelfths.

$$12 \div 3 = 4$$
$$\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Multiplying numerator and denominator by the same number does not change the value. Since the denominator is to be 12, both numerator and denominator must be multiplied by $12 \div 3$, or 4; therefore, $\frac{2}{3} = \frac{8}{12}$.

Analysis.—In $\frac{3}{3}$, or *one*, there are 12 twelfths, hence in $\frac{1}{3}$ there are $\frac{1}{3}$ of 12 twelfths, or 4 twelfths, and in $\frac{2}{3}$ there are 2 times 4 twelfths, or $\frac{8}{12}$.

Direction.—*Multiply numerator and denominator by the quotient arising from the division of the required denominator by the given denominator.*

Change

**2.** $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$ to twelfths.

**3.** $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{7}{12}$ to twenty-fourths.

**4.** 1, $\frac{3}{5}$, $\frac{7}{10}$, and $\frac{19}{20}$ to fortieths.

**5.** 3, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{9}$, and $\frac{5}{18}$ to 36ths.

**6.** $\frac{3}{4}$, $\frac{2}{5}$, $\frac{9}{20}$, $\frac{31}{50}$, and $\frac{9}{10}$ to 100ths.

**7.** $\frac{7}{12}$, $\frac{5}{24}$, $\frac{9}{16}$, $\frac{19}{36}$, and $\frac{51}{72}$ to 144ths.

**8.** $\frac{5}{7}$, $\frac{4}{5}$, $\frac{7}{11}$, $\frac{21}{35}$, and $\frac{31}{55}$ to 385ths.

**191. To change fractions to smaller denominators.**

**1.** How many *twelfths* in 1? How many *halves?* Then 12 twelfths = how many halves?

**2.** How many halves in 6 twelfths?

**3.** How many thirds in 1? Then how many thirds in 12 twelfths? In 4 twelfths? In eight 12ths?

**4.** Since $\frac{6}{12} = \frac{1}{2}$, and $\frac{4}{12} = \frac{1}{3}$, what has been done to the numerator and denominator of the fractions $\frac{6}{12}$ and $\frac{4}{12}$ without changing the values? How has it been done?

**5.** Change $\frac{6}{12}$ to *fourths.* $\frac{9}{12}$, $\frac{12}{24}$, $\frac{15}{20}$.

**6.** How many fourths in $\frac{6}{8}$? Is $\frac{6}{8} = \frac{3}{4}$? What change has been made in the denominator? What corresponding change in the numerator?

Dividing numerator and denominator by 2, we have $\frac{3}{4}$. Since the *fractional units* are twice as large, while the number of them is $\frac{1}{2}$ as great, $\frac{3}{4}$ is equivalent to $\frac{6}{8}$. Hence the following

**192.** PRINCIPLE.—*Dividing denominator and numerator of a fraction by the same number does not change the value of the fraction.*

WRITTEN EXERCISES.

**193. 1.** Change $\frac{16}{24}$ to twelfths.

$$24 \div 12 = 2$$

$$\frac{16 \div 2}{24 \div 2} = \frac{8}{12}$$

Dividing the given denominator by the required denominator we get 2, which must be used as divisor of both numerator and denominator.

QUERIES.—1. In this process, how has the *size* of the fractional unit been changed? 2. How correspondingly has the *number* of fractional units been changed? 3. Which gives more definite idea of the value of the quantity, $\frac{12}{24}$ yd. or $\frac{1}{2}$ yd.?

Change to smaller denominators:

**2.** $\frac{6}{8}$; $\frac{3}{9}$; $\frac{8}{12}$; $\frac{10}{15}$; $\frac{12}{14}$; $\frac{8}{16}$; $\frac{15}{18}$.

**3.** $\frac{16}{20}$; $\frac{11}{22}$; $\frac{16}{28}$; $\frac{20}{24}$; $\frac{25}{30}$; $\frac{22}{26}$; $\frac{21}{33}$.

**4.** $\frac{24}{32}$; $\frac{32}{40}$; $\frac{36}{54}$; $\frac{49}{63}$; $\frac{54}{72}$; $\frac{60}{90}$; $\frac{66}{99}$.

QUERIES.—(a) When should you express a fraction with smaller denominator ? (b) Can $\frac{8}{12}$ be changed to smaller denominator without a change of value ? Can $\frac{4}{6}$ ? Can $\frac{2}{3}$ ? Are 2 and 3 *prime* to each other ? Why ?

**194.** *A fraction is expressed with its smallest denominator when its numerator and denominator are prime to each other.*

### WRITTEN EXERCISES.

**195. 1.** Change $\frac{30}{45}$ to its smallest denominator.

$30 \div 3 = 10$
$45 \div 3 = 15$
Therefore,
$\frac{30}{45} = \frac{10}{15}$.
$10 \div 5 = 2$
$15 \div 5 = 3$
Therefore,
$\frac{10}{15} = \frac{2}{3}$.

Divide numerator and denominator by 3, then by 5. Since no number will exactly divide both numerator and denominator of the fraction $\frac{2}{3}$, the fraction is changed, or *reduced*, to its *smallest denominator.*

DIRECTION.—*Divide numerator and denominator by common factors successively until they are prime to each other.*

Change to smallest denominators :

| | | |
|---|---|---|
| **2.** $\frac{12}{16}$, $\frac{18}{27}$, $\frac{27}{45}$. | **8.** $\frac{64}{144}$, $\frac{105}{126}$, $\frac{41}{164}$. | **14.** $\frac{196}{444}$, $\frac{182}{196}$, $\frac{252}{396}$. |
| **3.** $\frac{60}{75}$, $\frac{63}{72}$, $\frac{80}{100}$. | **9.** $\frac{144}{198}$, $\frac{192}{240}$, $\frac{154}{196}$. | **15.** $\frac{300}{450}$, $\frac{216}{240}$, $\frac{288}{480}$. |
| **4.** $\frac{55}{66}$, $\frac{36}{54}$, $\frac{48}{72}$. | **10.** $\frac{121}{132}$, $\frac{125}{375}$, $\frac{121}{144}$. | **16.** $\frac{195}{455}$, $\frac{345}{552}$, $\frac{275}{875}$. |
| **5.** $\frac{30}{45}$, $\frac{18}{60}$, $\frac{70}{95}$. | **11.** $\frac{240}{312}$, $\frac{93}{217}$, $\frac{147}{210}$. | **17.** $\frac{105}{259}$, $\frac{400}{640}$, $\frac{333}{351}$. |
| **6.** $\frac{45}{75}$, $\frac{56}{84}$, $\frac{28}{42}$. | **12.** $\frac{176}{208}$, $\frac{75}{125}$, $\frac{119}{147}$. | **18.** $\frac{144}{432}$, $\frac{120}{372}$, $\frac{121}{363}$. |
| **7.** $\frac{63}{81}$, $\frac{64}{96}$, $\frac{80}{88}$. | **13.** $\frac{144}{156}$, $\frac{120}{192}$, $\frac{121}{143}$. | **19.** $\frac{64}{512}$, $\frac{81}{729}$, $\frac{125}{625}$. |

**196. To change integers or mixed numbers to fractional form, and the reverse.**

**1.** How many quarter-dollars in one dollar?

**2.** How many fourths in a dollar ? In 2 dollars ? In $2\frac{1}{4}$ dollars ? In $2\frac{3}{4}$ apples ? Illustrate with objects.

**3.** How many apples have you when you have eight quarters, or fourths ? Ten fourths ? 17 fourths ?

**4.** How many eighths in 1 ? In 2 ? In 3 ? In 5 ? In $2\frac{3}{8}$ ? In $3\frac{5}{8}$ ?

**5.** How many 1's in eight eighths ? In 16 eighths ? In $\frac{24}{8}$ ? In $\frac{40}{8}$ ? In $\frac{11}{8}$ ? In $\frac{21}{8}$ ?

**6.** Twenty-five half-dollars are how many dollars?

**7.** How many quarters will pay for a pig that costs $\$2\frac{1}{4}$?

**8.** How many fractional units, thirds, are there in $3\frac{1}{3}$? In $5\frac{2}{3}$?

**9.** How many ones are there in $\frac{24}{3}$? In $\frac{28}{4}$? In $\frac{30}{6}$? In $\frac{49}{7}$? In $\frac{25}{2}$? In $\frac{25}{4}$?

### WRITTEN EXERCISES.

**197. 1.** Change $15\frac{3}{4}$ to fourths.

$15 \times \frac{4}{4} = \frac{60}{4}$

$\frac{60}{4} + \frac{3}{4} = \frac{63}{4}$

In 1 there are 4 fourths, and in 15 there are 15 times 4 fourths, or 60 fourths; 60 fourths + 3 fourths = 63 fourths. Why do we add 3 fourths to 60 fourths?

Change to fractional form:

| | | | |
|---|---|---|---|
| **2.** $1\frac{2}{5}$. | **9.** $9\frac{3}{4}$. | **16.** $12\frac{11}{13}$. | **23.** $115\frac{3}{40}$. |
| **3.** $4\frac{3}{7}$. | **10.** $3\frac{2}{12}$. | **17.** $18\frac{9}{14}$. | **24.** $208\frac{7}{64}$. |
| **4.** $6\frac{2}{3}$. | **11.** $5\frac{11}{13}$. | **18.** $25\frac{9}{20}$. | **25.** $365\frac{9}{28}$. |
| **5.** $7\frac{1}{8}$. | **12.** 8. | **19.** $29\frac{12}{23}$. | **26.** $500\frac{50}{83}$. |
| **6.** $5\frac{3}{8}$. | **13.** $7\frac{6}{17}$. | **20.** $37\frac{1}{41}$. | **27.** $710\frac{91}{92}$. |
| **7.** $8\frac{2}{7}$. | **14.** $6\frac{3}{19}$. | **21.** $72\frac{37}{72}$. | **28.** 802. |
| **8.** $7\frac{9}{11}$. | **15.** 10. | **22.** $90\frac{19}{50}$. | **29.** 613.25. |

**30.** Change $\frac{29}{3}$ to a mixed number.

$29 \div 3 = 9\frac{2}{3}$

Since there are 3 thirds in *one*, in 29 thirds there are as many *ones* as there are 3's in 29, or $9\frac{2}{3}$; therefore, $\frac{29}{3} = 9\frac{2}{3}$.

Change to integers or mixed numbers:

| | | |
|---|---|---|
| **31.** $\frac{7}{3}$, $\frac{9}{7}$, $\frac{23}{5}$. | **35.** $\frac{325}{7}$, $\frac{290}{8}$. | **39.** $\frac{1327}{34}$, $\frac{2154}{27}$. |
| **32.** $\frac{25}{3}$, $\frac{17}{8}$, $\frac{19}{6}$. | **36.** $\frac{500}{111}$, $\frac{782}{12}$. | **40.** $\frac{1809}{44}$, $\frac{3006}{57}$. |
| **33.** $\frac{23}{2}$, $\frac{33}{3}$, $\frac{24}{9}$. | **37.** $\frac{400}{13}$, $\frac{927}{14}$. | **41.** $\frac{5728}{75}$, $\frac{7231}{83}$. |
| **34.** $\frac{52}{4}$, $\frac{36}{7}$, $\frac{44}{5}$. | **38.** $\frac{876}{15}$. $\frac{735}{16}$. | **42.** $\frac{7125}{60}$, $\frac{9320}{79}$. |

Complete the following equations:

| | | |
|---|---|---|
| **43.** $38\frac{2}{9} = (\quad)$. | **46.** $24\frac{5}{17} = (\quad)$. | **49.** $\frac{825}{33} = (\quad)$. |
| **44.** $\frac{143}{8} = (\quad)$. | **47.** $\frac{1000}{13} = (\quad)$. | **50.** $1001\frac{1}{11} = (\quad)$. |
| **45.** $10\frac{10}{11} = (\quad)$. | **48.** $365\frac{1}{4} = (\quad)$. | **51.** $\frac{5000}{19} = (\quad)$. |

**198. To change decimal fractions to common fractions.**

**1.** (*a*) Express .7 as a common fraction; (*b*) change .125 to a common fraction.

(*a*) $.7 = \frac{7}{10}$.

(*b*) $.125 = \frac{125}{1000} = \frac{5}{40} = \frac{1}{8}$.

We write the figures of the decimal for the numerator, and 1 with as many ciphers after it as there are figures after the decimal point for the denominator.

When desirable, we change to *smallest denominator*, as in (*b*).

Change to common fractions with smallest denominators:

| | | | |
|---|---|---|---|
| **2.** .25. | **9.** .39. | **16.** .375. | **23.** .0125. |
| **3.** .35. | **10.** .43. | **17.** .425. | **24.** .3750. |
| **4.** .85. | **11.** .38. | **18.** .500. | **25.** .0875. |
| **5.** .75. | **12.** .50. | **19.** .625. | **26.** .1872. |
| **6.** .24. | **13.** .95. | **20.** .205. | **27.** .4020. |
| **7.** .72. | **14.** .05. | **21.** .875. | **28.** .0075. |
| **8.** .60. | **15.** .03. | **22.** .945. | **29.** .15625. |

**199. To change common fractions to decimal fractions.**

**1.** How many tenths in 4 twentieths? In $\frac{8}{20}$? In $\frac{12}{20}$? $\frac{16}{20}$ = how many 10ths?

**2.** How many tenths in $\frac{1}{5}$? In $\frac{2}{5}$? In $\frac{3}{5}$? $\frac{4}{5}$ = how many 10ths?

**3.** How many hundredths in $\frac{8}{200}$? In $\frac{12}{400}$? In $\frac{50}{500}$? $\frac{45}{300}$ = how many 100ths?

**4.** How many hundredths in $\frac{3}{20}$? In $\frac{7}{20}$? In $\frac{4}{25}$? In $\frac{9}{50}$? $\frac{3}{25}$ = how many 100ths?

**5.** What have you been changing in these examples? Since 10ths and 100ths are *decimal* divisions, to what have you been changing the common fractions?

**200.** To change a common fraction to a decimal fraction is to change it to *larger* or *smaller* denominators, and to express it in the notation peculiar to decimals.

Thus, $\frac{1}{2} = \frac{5}{10} = .5$; $\frac{3}{4} = \frac{75}{100} = .75$; $\frac{4}{200} = \frac{2}{100} = .02$.

**1.** Change $\frac{3}{8}$ to a decimal fraction, that is, to a fraction whose denominator is 10 or 100 or 1000, etc.

$$1000 \div 8 = 125$$
$$\frac{3 \times 125}{8 \times 125} = \frac{375}{1000} = .375.$$

The required denominator must be a multiple of 8. The least decimal denominator that is a multiple of 8 is 1000. Hence $\frac{3}{8}$ must be changed to 1000ths. (Art. 189.)

The required denominator can be found only by inspection or by trial. Since it must be divided by the given denominator, and the given numerator multiplied by the quotient, we may, for convenience, combine the processes and obtain the required decimal at once, by dividing the numerator by the denominator, annexing ciphers, and pointing off decimal places as in division of decimals.

$$\begin{array}{r} 8)\underline{3.000} \\ .375 \end{array}$$

**2.** Change $\frac{26}{200}$ to a decimal fraction.

(a)

$$\frac{26}{200} = \frac{13}{100} = .13$$

In many cases, reducing a fraction to a smaller denominator changes it to a decimal fraction, as in (a).

It may also be solved as in (b).

(b)

$$\begin{array}{r} 200)\underline{26.00} \\ .13 \end{array}$$

RULE.—*Annex ciphers to the numerator and divide by the denominator, pointing off decimal places as in division of decimals.*

NOTE.—When the division will not terminate, the *remainder* may be expressed as a common fraction, or the sign + may be placed after the decimal figures to show that the division is not complete.

Thus, $\frac{1}{3} = .33\frac{1}{3}$, or .33 +.

Change to decimal fractions:

| | | | |
|---|---|---|---|
| **3.** $\frac{3}{4}$. | **11.** $\frac{13}{20}$. | **19.** $\frac{1}{12}$. | **27.** $\frac{36}{75}$. |
| **4.** $\frac{2}{5}$. | **12.** $\frac{7}{25}$. | **20.** $\frac{2}{3}$. | **28.** $\frac{22}{55}$. |
| **5.** $\frac{5}{8}$. | **13.** $\frac{19}{20}$. | **21.** $\frac{3}{7}$. | **29.** $\frac{17}{20}$. |
| **6.** $\frac{1}{4}$. | **14.** $\frac{21}{25}$. | **22.** $\frac{5}{9}$. | **30.** $\frac{4}{40}$. |
| **7.** $\frac{3}{8}$. | **15.** $\frac{13}{40}$. | **23.** $\frac{4}{13}$. | **31.** $\frac{15}{50}$. |
| **8.** $\frac{4}{5}$. | **16.** $\frac{3}{40}$. | **24.** $\frac{3}{30}$. | **32.** $\frac{21}{63}$. |
| **9.** $\frac{7}{8}$. | **17.** $\frac{31}{50}$. | **25.** $\frac{12}{60}$. | **33.** $\frac{3}{90}$. |
| **10.** $\frac{9}{20}$. | **18.** $\frac{9}{50}$. | **26.** $\frac{16}{68}$. | **34.** $\frac{9}{131}$. |

**201. To change dissimilar to similar fractions.**

**1.** Have $\frac{2}{3}$ and $\frac{4}{5}$ the same denominators? Have $\frac{3}{4}$ and $\frac{5}{6}$? Have $\frac{3}{7}$ and $\frac{5}{7}$? Have $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{3}{8}$?

**202.** Fractions that have the same denominators are said to have a *common denominator,* and are called **Similar Fractions.** They have a common unit of measure.

**203.** Fractions that do not have the same denominators are called **Dissimilar Fractions.** Their units of measure are *not* the same.

Tell which are similar fractions:

**1.** $\frac{2}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{1}{4}$.

**2.** $\frac{3}{7}$, $\frac{5}{8}$, $\frac{4}{5}$, $\frac{5}{7}$, $\frac{7}{8}$, $\frac{1}{5}$.

**3.** $\frac{5}{9}$, $\frac{7}{12}$, $\frac{6}{7}$, $\frac{1}{9}$, $\frac{5}{12}$, $\frac{4}{7}$.

**4.** $\frac{3}{11}$, $\frac{3}{13}$, $\frac{3}{14}$, $\frac{5}{11}$, $\frac{5}{14}$, $\frac{5}{13}$.

**5.** How many sixths in 1? In $\frac{1}{2}$? In $\frac{1}{3}$? Then what common denominator may $\frac{1}{2}$ and $\frac{1}{3}$ have?

**6.** How many eighths in 1? In $\frac{1}{2}$? In $\frac{3}{4}$? Then what common denominator may $\frac{1}{2}$ and $\frac{3}{4}$ have?

**7.** When fractions have a common denominator, what are they called?

Change to similar fractions:

**8.** $\frac{1}{2}$ and $\frac{1}{3}$.

**9.** $\frac{1}{4}$ and $\frac{1}{2}$.

**10.** $\frac{1}{3}$ and $\frac{1}{4}$.

**11.** $\frac{1}{2}$ and $\frac{1}{8}$.

**12.** $\frac{1}{4}$ and $\frac{1}{8}$.

**13.** $\frac{1}{2}$ and $\frac{2}{3}$.

**14.** $\frac{2}{3}$ and $\frac{5}{6}$.

**15.** $\frac{3}{4}$ and $\frac{2}{3}$.

**16.** $\frac{5}{6}$ and $\frac{3}{4}$.

**17.** $\frac{3}{5}$ and $\frac{2}{3}$.

**18.** $\frac{3}{5}$ and $\frac{5}{8}$.

**19.** $\frac{3}{6}$ and $\frac{3}{5}$.

**20.** $\frac{2}{3}$ and $\frac{3}{7}$.

**21.** $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

**22.** $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{2}{5}$.

**23.** $\frac{1}{6}$, $\frac{3}{8}$, and $\frac{2}{3}$.

**24.** $\frac{3}{5}$, $\frac{1}{3}$, and $\frac{7}{15}$.

**25.** $\frac{3}{4}$, $\frac{2}{5}$, and $\frac{9}{20}$.

**204.** A common denominator of two or more fractions is a **Common Multiple** of their denominators.

### WRITTEN EXERCISES.

**205.** **1.** Change $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ to similar fractions.

$$\frac{2 \times 20}{3 \times 20} = \frac{40}{60}$$
$$\frac{3 \times 15}{4 \times 15} = \frac{45}{60}$$
$$\frac{4 \times 12}{5 \times 12} = \frac{48}{60}$$

Since the product of the given denominators is a common multiple of each, $3 \times 4 \times 5$, or 60, is a common denominator. Hence the fractions must be changed to 60ths, which is done by multiplying numerator and denominator of each by the quotient that arises from dividing the required denominator by its given denominator.

RULES.—1. *Multiply numerator and denominator of each fraction by the number of times its denominator is contained in a common multiple of all the given denominators.* Or,

2. *Multiply numerator and denominator of each fraction by the product of the denominators of all the other fractions.*

Change to similar fractions:

**2.** $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.
**3.** $\frac{1}{4}$, $\frac{3}{8}$, $\frac{5}{12}$.
**4.** $\frac{3}{4}$, $\frac{7}{9}$, $\frac{2}{3}$.
**5.** $\frac{3}{5}$, $\frac{7}{8}$, $\frac{6}{7}$.
**6.** $\frac{3}{8}$, $\frac{5}{6}$, $\frac{7}{12}$.
**7.** $\frac{3}{5}$, .7, $\frac{9}{20}$.
**8.** $\frac{2}{3}$, $\frac{5}{9}$, $\frac{11}{12}$.
**9.** $\frac{7}{15}$, .8, $\frac{13}{25}$.
**10.** $\frac{1}{6}$, $\frac{7}{8}$, $\frac{5}{24}$.
**11.** $\frac{2}{5}$, $\frac{3}{8}$, $\frac{5}{9}$.
**12.** $\frac{9}{16}$, $\frac{7}{8}$, $\frac{3}{4}$.
**13.** $\frac{11}{15}$, $\frac{7}{9}$, $\frac{2}{5}$.
**14.** $\frac{4}{5}$, $\frac{11}{30}$, .9.
**15.** $\frac{5}{12}$, $\frac{5}{24}$, $\frac{5}{48}$.
**16.** $\frac{13}{20}$, .5, $\frac{3}{5}$, $\frac{17}{40}$.
**17.** $\frac{11}{28}$, $\frac{7}{60}$, $\frac{2}{15}$.
**18.** $\frac{12}{25}$, $\frac{7}{27}$, $\frac{8}{45}$, $\frac{5}{36}$, $\frac{7}{60}$.

**206.** Fractions may have more than one common denominator. The *smallest* one they can have is called their **Least Common Denominator.** The *smaller* the denominator, the *greater* the unit of measure.

**207.** The *least common multiple* of the denominators of several fractions is their *least common denominator* (L. C. D.).

WRITTEN EXERCISES.

**208. 1.** Change $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{7}{9}$ to similar fractions having their least common denominator.

$$\frac{3 \times 18}{4 \times 18} = \frac{54}{72}$$
$$\frac{5 \times 9}{8 \times 9} = \frac{45}{72}$$
$$\frac{7 \times 8}{9 \times 8} = \frac{56}{72}$$

We find 72 to be the L. C. M. of the denominators, and the least common denominator of the fractions, which must, therefore, be changed to 72nds. This is done by multiplying numerator and denominator of each fraction by the number of times its denominator is contained in the L. C. M.

☞ Explain why fractions having their *least* common denominator have their *greatest* common *fractional unit.*

Change to similar fractions with their L. C. D. :

**2.** $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$.
**3.** $\frac{3}{5}$, $\frac{5}{6}$, $\frac{7}{8}$.
**4.** $\frac{5}{6}$, $\frac{7}{18}$, $\frac{17}{36}$.
**5.** $\frac{5}{7}$, $\frac{2}{3}$, $\frac{3}{4}$.
**6.** $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{12}$.
**7.** $\frac{2}{5}$, $\frac{1}{6}$, $\frac{4}{15}$.
**8.** $\frac{5}{9}$, $\frac{5}{12}$, $\frac{5}{6}$.
**9.** $\frac{11}{15}$, .5, $\frac{4}{5}$.
**10.** $\frac{5}{6}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{6}{7}$.
**11.** $\frac{2}{5}$, $\frac{3}{20}$, $\frac{5}{12}$, $\frac{7}{30}$.
**12.** $\frac{7}{11}$, $\frac{8}{13}$, $\frac{16}{44}$, $\frac{18}{26}$.
**13.** $\frac{6}{7}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{11}$.
**14.** $\frac{5}{9}$, $\frac{10}{18}$, $\frac{15}{27}$, $\frac{20}{36}$.
**15.** $\frac{1}{7}$, $\frac{1}{2}$, $\frac{5}{28}$, $\frac{9}{16}$.
**16.** $\frac{19}{30}$, .8, $\frac{11}{25}$, $\frac{18}{60}$.
**17.** $\frac{1}{7}$, $\frac{2}{9}$, $\frac{3}{11}$, $\frac{4}{13}$.
**18.** $\frac{4}{11}$, $\frac{5}{7}$, $\frac{1}{22}$.
**19.** $\frac{5}{9}$, $\frac{2}{6}$, $\frac{5}{4}$.

**20.** Change $\frac{2}{3}$ to a number whose fractional unit is $\frac{1}{15}$.

**21.** How many fractional units are there in $\frac{3}{4}$ if changed to 16ths ?

**22.** A fraction whose fractional unit is an eighth has the numerator 6. What is the value of the fraction in fourths ?

**23.** What fraction has sixteen fractional units, each of which is $\frac{1}{17}$ ?

## ADDITION.

**209. 1.** Ella has \$$\frac{5}{8}$, and Jane has \$$\frac{3}{8}$. How much have they both ?

**2.** Mary paid \$$\frac{1}{2}$ for a knife and \$$\frac{3}{4}$ for a book. How much did she pay for both ?

**3.** George bought $\frac{2}{3}$ of an acre from one man, and $\frac{3}{4}$ of an acre from another. How much land did he buy ?

4. When fractions have different denominators, what must be done to them before they can be added? Why?

5. Can $\frac{1}{4}$ of a dollar and $\frac{1}{3}$ of a bushel be added? Why not?

**210.** Principle.—*Fractions can be added only when they express like quantities, and have a common denominator.*

WRITTEN EXERCISES.

**211.** 1. Find the sum of $\frac{3}{5}$, $\frac{7}{8}$, and $\frac{11}{20}$.

$$\begin{aligned} \tfrac{3}{5} &= \tfrac{24}{40} \\ \tfrac{7}{8} &= \tfrac{35}{40} \\ \tfrac{11}{20} &= \tfrac{22}{40} \\ \hline \tfrac{81}{40} &= 2\tfrac{1}{40} \end{aligned}$$

Changing the fractions to similar ones having their least common denominator, we have 24, 35, and 22 fortieths, and the sum of these is $\frac{81}{40}$, or $2\frac{1}{40}$.

2. What is the sum of $3\frac{1}{3}$, $7\frac{4}{9}$, and $12\frac{5}{6}$?

$$\begin{aligned} 3\tfrac{1}{3} &= 3\tfrac{6}{18} \\ 7\tfrac{4}{9} &= 7\tfrac{8}{18} \\ 12\tfrac{5}{6} &= 12\tfrac{15}{18} \\ \hline & 23\tfrac{11}{18} \end{aligned}$$

Changing the *fractions* to similar ones having their least common denominator, we find their sum to be $\frac{29}{18}$, or $1\frac{11}{18}$; the sum of the integers is 22; and the sum of both is $23\frac{11}{18}$.

Supply the words that are wanting in the following

Rule.—*Change the given fractions to ——— ———, add their ———, and write the sum over the ——— ———. When there are integers or mixed numbers, add ——— and ——— separately, and then ——— the two sums.*

Add the following:

3. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.
4. $\frac{5}{6}$, $\frac{3}{8}$, $\frac{1}{12}$, $\frac{3}{4}$.
5. $\frac{3}{5}$, .7, $\frac{8}{15}$, $\frac{9}{20}$.
6. $\frac{3}{7}$, $\frac{5}{14}$, $\frac{1}{2}$, $\frac{9}{14}$.
7. $\frac{2}{3}$, $\frac{3}{8}$, $\frac{5}{24}$, $\frac{7}{12}$.
8. $\frac{7}{9}$, $\frac{1}{3}$, $\frac{5}{11}$, $\frac{25}{33}$.
9. $\frac{5}{16}$, $\frac{11}{12}$, $\frac{17}{20}$, $\frac{7}{18}$.
10. $\frac{8}{9}$, $\frac{5}{18}$, $\frac{11}{16}$, $\frac{3}{28}$.
11. $\frac{3}{5}$, $\frac{5}{9}$, $\frac{7}{12}$, $\frac{1}{8}$.
12. $\frac{7}{50}$, $\frac{13}{25}$, $\frac{19}{40}$, .25
13. $7\frac{1}{2}$, $4\frac{3}{4}$, $2\frac{3}{5}$.
14. $6\frac{2}{3}$, $5\frac{1}{6}$, 3.
15. $8\frac{5}{7}$, $9\frac{3}{14}$, 2.5.
16. $3\frac{5}{6}$, 8.7, $12\frac{1}{5}$.
17. $13\frac{3}{4}$, $17\frac{8}{15}$, $20\frac{5}{12}$.
18. $22\frac{2}{3}$, $18\frac{5}{8}$, $7\frac{7}{16}$.
19. $30\frac{1}{2}$, $20\frac{1}{5}$, 40.3.
20. $5\frac{3}{8}$, $2\frac{7}{18}$, $8\frac{19}{24}$.

**21.** One week Jasper earned $\$3\frac{2}{5}$, Homer, \$4.7, David $\$12\frac{3}{4}$, and James, $\$9\frac{1}{20}$. How much did all earn?

**22.** Mary weighs $75\frac{1}{4}$ pounds, Edna $12\frac{3}{8}$ pounds more than Mary, and Charles as much as both. How much does Charles weigh?

**23.** The product of two numbers is the sum of $258\frac{8}{18}$ and $173\frac{5}{9}$, and one of the numbers is 24. Find the other.

**24.** Find the distance around a rectangular field whose length is $281\frac{3}{4}$ rods, and whose width is $190\frac{5}{8}$ rods.

**25.** A certain minuend would be $\frac{19}{20}$, if it were $\frac{1}{40}$ less. What would it be if increased by $\frac{1}{40}$?

**26.** A locomotive runs $354\frac{3}{8}$ miles every other day. How far does it run in a week, not counting Sunday?

**27.** What is the minuend when the remainder is $17\frac{2}{39}$, and the subtrahend $10\frac{3}{13}$?

**28.** The number of fractional units in the greater of two fractions is 15, the number in the less is 6. The denominator of the greater fraction is 27, and that of the other is 18. Find the sum of the fractions.

SUPPLEMENTARY EXERCISES (FOR ADVANCED CLASSES).

**212. 1.** Mr. A has 5 fields, the smallest of which contains $6\frac{2}{5}$ acres, and the largest 12.375 acres. Each of the others contains $9\frac{1}{3}$ acres. How much land has Mr. A?

**2.** John has $\$3\frac{7}{10}$, Harry has $\$1\frac{1}{2}$ more than John, Ben has \$3.05 more than both, and their father has \$10.25 more than the three boys together have. How much have all four?

**3.** A man earned $\$16.66\frac{2}{3}$ in January. Each month thereafter he earned \$16.125 more than he earned the preceding month. How much did he earn in a year?

**4.** A lady spent $\frac{1}{7}$ of her money for shoes, $\frac{3}{8}$ for a dress, and $\frac{2}{9}$ for a hat. If she spent \$37.30 for all, how much had she left?

**5.** Find the value of $3 \times (6\frac{4}{5} + 5.2) - 7.5 \div (.5 + 1)$.

**6.** A can do $\frac{3}{8}$ of a piece of work in a day, B $\frac{2}{5}$ of it, C $\frac{7}{8}$ of it, and D $\frac{1}{4}$ of it. In what time can they do the work, all working together?

**7.** I sold to different customers $13\frac{3}{4}$ gallons, 11.5 gallons, $15\frac{1}{4}$ gallons, $3\frac{1}{8}$ gallons, and 11.75 gallons of oil. How many gallons did I sell to all?

**8.** Find the sum of all the proper fractions that can be formed having one figure each in the numerator and denominator.

**9.** Ten is added to a certain mixed decimal. The *point* is then moved one place to the left and 10 is added. The sum is equal to 4.5 times the original number. Find the original number.

## SUBTRACTION.

**213. 1.** Ella has $\$\frac{5}{8}$, and Jane has $\$\frac{3}{8}$. How much more has Ella than Jane?

**2.** Mary paid $\$\frac{1}{2}$ for a knife and $\$\frac{3}{4}$ for a book. How much did the book cost more than the knife?

**3.** A man who had $\frac{3}{4}$ of an acre of land sold $\frac{2}{3}$ of an acre. How much had he left?

**4.** When fractions have different denominators, what must be done to them before they can be subtracted? Why?

**5.** Can $\frac{2}{3}$ of a dollar be subtracted from $\frac{3}{4}$ of a foot? Why not?

**214.** PRINCIPLE.—*Fractions can be subtracted only when they express like quantities, and have a common denominator.*

### WRITTEN EXERCISES.

**215. 1.** Subtract $\frac{3}{4}$ from $\frac{8}{9}$.

$$\begin{aligned} \tfrac{8}{9} &= \tfrac{32}{36} \\ \tfrac{3}{4} &= \tfrac{27}{36} \\ \hline & \tfrac{5}{36} \end{aligned}$$

Changing to similar fractions, we have 32 thirty-sixths, and 27 thirty-sixths, whose difference is $\frac{5}{36}$.

**2.** From $6\frac{3}{8}$ take $4\frac{2}{3}$.

| (a) | (b) |
|---|---|
| $6\frac{3}{8} = 6\frac{9}{24}$ | $= 5\frac{33}{24}$ |
| $4\frac{2}{3} = 4\frac{16}{24}$ | $= 4\frac{16}{24}$ |
| | $1\frac{17}{24}$ |

The *fractions* must be made similar, as in (a). $\frac{16}{24}$ cannot be taken from $\frac{9}{24}$, hence 1, or $\frac{24}{24}$, is taken from 6 and added to $\frac{9}{24}$, making $\frac{33}{24}$, as in (b). Then $\frac{16}{24}$ from $\frac{33}{24}$ leaves $\frac{17}{24}$, and 4 from 5 leaves 1. Hence the remainder is $1\frac{17}{24}$.

☞ In practice, the numbers under (b) should not be *written;* the work should be done mentally.

Supply the blanks in the following

Rule.—*Change the fractions to ——— ———; find the difference between the ———, and write it over the ——— ———.*

*When there are mixed numbers or integers, subtract fractions and integers separately.*

Query.—May integers or mixed numbers be changed to fractional form and subtracted according to the first part of the rule?

Find the value of:

| | | |
|---|---|---|
| **3.** $\frac{7}{8} - \frac{3}{4}$. | **10.** $\frac{1}{6} + \frac{1}{3} - \frac{1}{2}$. | **17.** $6.7 - 2\frac{3}{7}$. |
| **4.** $\frac{8}{11} - \frac{3}{22}$. | **11.** $.9 + 2 - \frac{2}{5}$. | **18.** $7\frac{5}{6} - 6\frac{8}{9}$. |
| **5.** $\frac{9}{13} - \frac{6}{26}$. | **12.** $\frac{6}{7} - \frac{5}{21} + \frac{1}{3}$. | **19.** $21.7 - 16\frac{7}{20}$. |
| **6.** $\frac{7}{24} - \frac{1}{9}$. | **13.** $\frac{8}{15} - \frac{1}{5} - .1$. | **20.** $14\frac{1}{4} - \frac{17}{30}$. |
| **7.** $\frac{11}{18} - \frac{5}{24}$. | **14.** $\frac{11}{12} + \frac{5}{8} - 1$. | **21.** $12\frac{1}{5} - 3\frac{2}{19}$. |
| **8.** $5 - \frac{11}{12}$. | **15.** $7\frac{5}{8} - 3\frac{2}{5}$. | **22.** $2\frac{1}{3} + 3\frac{4}{7} + 5\frac{11}{21}$. |
| **9.** $\frac{31}{33} - \frac{5}{21}$. | **16.** $13\frac{3}{20} - 4.25$. | **23.** $4\frac{3}{4} + 6\frac{2}{5} - 3.7$. |

**24.** Ella is $16\frac{3}{4}$ years old, and Nellie is $19\frac{1}{2}$ years old. How much older is Nellie than Ella?

**25.** A pole $14\frac{5}{8}$ feet long was broken into two pieces, one of which was $5\frac{2}{3}$ feet long. How long was the other?

**26.** A train ran from Lynchburg to Roanoke in $1\frac{1}{5}$ hours; it reached Roanoke at 9 A.M. At what time did it start?

**27.** Carl bought a book for $\$2\frac{3}{10}$, a knife for $\$\frac{3}{4}$, a hat for $\$1\frac{3}{5}$, and a coat for $\$2\frac{1}{3}$. If he had a ten-dollar bill at first, how much had he left?

**28.** The sum of two numbers is $21\frac{3}{4}$, and one of them is $8\frac{2}{3}$. What is the other?

**29.** What fraction added to $\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$ will make $2\frac{5}{24}$?

**30.** The minuend is $\frac{25}{36}$, and the remainder is $\frac{2}{9}$. What is the subtrahend?

**31.** Two fractions have the common denominator 12. One has 8 fractional units, the other 11. How much greater is one than the other?

**32.** Subtract $\frac{1}{7}$ from the greatest possible fractional unit.

**33.** If 4 is added to numerator and denominator of $\frac{3}{4}$, how much is the value of the fraction increased or diminished? How much if the fraction is $\frac{8}{5}$?

### SUPPLEMENTARY EXERCISES (FOR ADVANCED CLASSES).

**216. 1.** If numerator and denominator of $\frac{5}{6}$ are each diminished by 2, will the value of the fraction be increased or diminished, and how much?

**2.** What number is that, $\frac{2}{5}$ of which exceeds $\frac{1}{4}$ of it by $11\frac{3}{4}$?

**3.** A owns $\frac{2}{7}$ of a store, and B the remainder. If $\frac{5}{8}$ of the store is worth $575 more than .5 of it, what is the value of B's share.

**4.** What number is that, to which if its $\frac{7}{8}$ and its .25 are added, the sum will be 170?

**5.** The difference between the subtrahend and minuend is $5\frac{7}{8}$. If the subtrahend is $8\frac{9}{11}$ less than $9\frac{8}{11}$, what is the minuend?

**6.** What fraction is as much larger than $\frac{5}{6}$, as $\frac{4}{5}$ is less than $\frac{5}{6}$?

**7.** The highest score in an inning was $\frac{2}{3}$ of the total, and the next highest was $\frac{5}{18}$ of the total less. The scores in these two innings differed by 5 runs. What was the total score?

## MULTIPLICATION.

**217. 1.** James has $\$\frac{1}{2}$, and Henry has twice as much. How much has Henry? Then 2 times $\frac{1}{2} = (\quad)$.

**2.** Rose has $\frac{1}{4}$ of a pie, and Orville has three times as much. How much has he? Then $3 \times \frac{1}{4} = (\quad)$.

**3.** May worked $\frac{2}{7}$ of a week, and Arthur worked 4 times as long. How long did he work?

**4.** How much is 5 times 3 twentieths?

**5.** $5 \times \frac{3}{20} = \frac{15}{20}$. Have $\frac{3}{20}$ and $\frac{15}{20}$ the same denominators? How do their numerators compare? How do the fractions compare in value? Which has the greater number of fractional units?

**218.** PRINCIPLES.—1. *Multiplying the numerator of a fraction by any number multiplies the value of the fraction by that number.*

For, since the numerator tells how many fractional units are taken, multiplying it multiplies the number of fractional units, each of which has the same size or value as before.

Conversely,

2. *Dividing the numerator of a fraction by any number divides the value of the fraction by that number.*

For, since the numerator tells how many parts are taken, dividing it divides the number of parts, each of which is of the same size as before.

WRITTEN EXERCISES.

**219. 1.** Multiply $\frac{3}{20}$ by 9.

$9 \times \frac{3}{20} = \frac{27}{20} = 1\frac{7}{20}$ 9 times 3 twentieths = 27 twentieths, just as 9 times 3 tops = 27 tops.

**2.** Multiply $3\frac{1}{4}$ by 7.

$$\begin{array}{ll} 3\frac{1}{4} & \\ \underline{7} & \\ 21 & = 7 \times 3 \\ \underline{1\frac{3}{4}} & = 7 \times \frac{1}{4} \\ 22\frac{3}{4} & = 7 \times 3\frac{1}{4} \end{array}$$

When the multiplier is an integer, it is often more convenient to multiply the integer and fraction separately, and then add the products. Or we may proceed as in the first example, thus: $7 \times 3\frac{1}{4} = 7 \times \frac{13}{4} = \frac{91}{4} = 22\frac{3}{4}$.

Multiply the following:

| | | |
|---|---|---|
| **3.** $\frac{7}{8}$ by 12. | **8.** $\frac{5}{18}$ by 15. | **13.** $5\frac{3}{4}$ by 10. |
| **4.** $\frac{5}{9}$ by 13. | **9.** $\frac{13}{20}$ by 11. | **14.** $11\frac{3}{7}$ by 14. |
| **5.** $\frac{11}{12}$ by 8. | **10.** $\frac{8}{17}$ by 21. | **15.** $13\frac{2}{3}$ by 5. |
| **6.** $\frac{8}{13}$ by 11. | **11.** $\frac{10}{19}$ by 38. | **16.** $15\frac{3}{5}$ by 7. |
| **7.** $\frac{6}{17}$ by 16. | **12.** $2\frac{1}{3}$ by 8. | **17.** $23\frac{5}{8}$ by 12. |

**18.** Multiply each of the following by its own denominator, and note the results: $\frac{3}{4}$, $\frac{5}{8}$, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{6}{7}$, $\frac{5}{9}$, $\frac{4}{5}$, $\frac{8}{13}$.

**19.** How does it affect a fraction to multiply it by its denominator? Find by trial.

**20.** Multiply each of the following mixed numbers by the denominator of the fractional part, and compare each multiplicand with its product: $2\frac{1}{2}$, $1\frac{1}{4}$, $4\frac{1}{3}$, $3\frac{2}{5}$, $6\frac{7}{8}$.

**21.** How does it affect a mixed number to multiply it by the denominator of the fractional part?

NOTE.—As regards their form, it is usual to make a distinction between *simple* and *complex* fractions. They are called *simple* when numerator and denominator are integers; otherwise *complex*.

**22.** Multiply the complex fraction $\dfrac{\frac{3}{4}}{5}$ by 4.

$$4 \times \frac{\frac{3}{4}}{5} = \frac{\frac{12}{4}}{5} = \frac{3}{5}.$$

**23.** Change $\dfrac{\frac{2}{3}}{4}$ to a simple fraction.

$$\frac{\frac{2}{3} \times 3}{4 \times 3} = \frac{\frac{6}{3}}{12} = \frac{2}{12} = \frac{1}{6}$$

A fraction is made integral by multiplying it by its denominator. Hence we multiply both numerator and denominator by the denominator of the fraction $\frac{2}{3}$; that is, by the denominator which is in the numerator of the complex fraction.

Change to simple fractions:

| | | |
|---|---|---|
| **24.** $\dfrac{\frac{3}{4}}{5}$. | **28.** $\dfrac{\frac{2}{5}}{8}$. | **32.** $\dfrac{5\frac{3}{4}}{7}$. |
| **25.** $\dfrac{\frac{2}{3}}{7}$. | **29.** $\dfrac{\frac{5}{9}}{15}$. | **33.** $\dfrac{\frac{10}{11}}{20}$. |
| **26.** $\dfrac{\frac{3}{8}}{4}$. | **30.** $\dfrac{2\frac{1}{2}}{5}$. | **34.** $\dfrac{17\frac{3}{5}}{22}$. |
| **27.** $\dfrac{\frac{1}{2}}{6}$. | **31.** $\dfrac{3\frac{1}{3}}{4}$. | **35.** $\dfrac{20\frac{7}{11}}{21}$. |

**36.** Multiply 12 by $\frac{2}{3}$.

$12 \div 3 = 4$
$2 \times 4 = 8$
or
$12 \times \frac{2}{3} = \frac{24}{3} = 8$

To multiply 12 by $\frac{2}{3}$ is to find $\frac{2}{3}$ of 12, or 2 times $\frac{1}{3}$ of 12.

$\frac{1}{3}$ of $12 = 12 \div 3$, or 4; and $\frac{2}{3}$ of $12 = 2$ times 4, or 8.

Or, since $\frac{2}{3} \times 12 = 12 \times \frac{2}{3}$, we have 12 times $\frac{2}{3}$, or $\frac{24}{3}$, or 8.

Find the value of :

**37.** $15 \times \frac{3}{5}$.
**38.** $16 \times \frac{3}{4}$.
**39.** $12 \times \frac{3}{8}$.
**40.** $14 \times \frac{5}{7}$.
**41.** $18 \times \frac{2}{3}$.
**42.** $24 \times \frac{5}{6}$.
**43.** $27 \times \frac{4}{9}$.
**44.** $\frac{3}{11} \times 22$.
**45.** $\frac{4}{13} \times 17$.
**46.** $2\frac{1}{2} \times 16$.
**47.** $18 \times 3\frac{1}{4}$.
**48.** $21 \times 5\frac{3}{4}$.
**49.** $24 \times 6\frac{4}{5}$.
**50.** $25 \times 4\frac{2}{3}$.
**51.** $7\frac{3}{8} \times 30$.
**52.** $5\frac{3}{5} \times 42$.
**53.** $9.8 \times 14$.
**54.** $18\frac{3}{7} \times 12$.
**55.** $\dfrac{2\frac{1}{2}}{4} \times 16$.
**56.** $25 \times \dfrac{3\frac{1}{5}}{8}$.
**57.** $42 \times \frac{6}{7}$.

**58.** Multiply $\frac{3}{5}$ by $\frac{4}{7}$.

$$\frac{3}{5} \times \frac{4}{7} = \frac{\frac{12}{7}}{5} = \frac{12}{35}$$

Multiplying the numerator of a fraction by any number multiplies the fraction by that number. (See Art. 218.) Hence we multiply 3 by $\frac{4}{7}$ (as in example 36), which gives $\frac{12}{7}$. Writing this product over the denominator, this is readily changed to the simple fraction $\frac{12}{35}$. (See example 23.)

Multiply the following :

**59.** $\frac{4}{7}$ by $\frac{1}{2}$.
**60.** $\frac{3}{5}$ by $\frac{1}{3}$.
**61.** $\frac{5}{6}$ by $\frac{1}{5}$.
**62.** $\frac{8}{9}$ by $\frac{3}{4}$.
**63.** $\frac{7}{11}$ by $\frac{3}{7}$.
**64.** $\frac{3}{13}$ by $\frac{5}{6}$.
**65.** $2\frac{1}{4}$ by $1\frac{1}{2}$.
**66.** $3\frac{2}{3}$ by $2\frac{1}{2}$.
**67.** 6 by $\frac{2}{3}$.
**68.** 12 by $\frac{3}{5}$.
**69.** $15\frac{1}{3}$ by $\frac{1}{2}$.
**70.** $\frac{7}{8}$ by $2\frac{7}{9}$.

☞ In the product of $\frac{3}{5} \times \frac{4}{7}$, or in the product of any two or more fractions, it will be seen that the numerator of the product is the product of the numerators of the factors, and the denominator the product of the denominators of the factors.

Hence the following *convenient method* may be used :

RULE.—*Change all integers and mixed numbers to frac-*

*tional form; then write the product of the numerators over the product of the denominators.*

NOTES.—1. The work may often be very much shortened by *cancellation.*

2. The sign × after a fraction is sometimes read "of." Thus, $\frac{4}{5} \times \$3$ may be read "four fifths of three dollars." In expressions like $\frac{3}{8}$ of $\frac{5}{6}$ the sign × may be substituted for "of."

Find the value of:

**71.** $\frac{3}{7} \times \frac{5}{6}$.

**72.** $\frac{4}{9} \times \frac{3}{5}$.

**73.** $\frac{5}{8} \times \frac{4}{7}$.

**74.** $\frac{7}{12} \times \frac{5}{14}$.

**75.** $\frac{9}{16} \times \frac{8}{12}$.

**76.** $\frac{3}{4} \times \frac{8}{9}$.

**77.** $\frac{11}{60} \times \frac{30}{44}$.

**78.** $\frac{13}{24} \times \frac{18}{26}$.

**79.** $\frac{17}{19} \times \frac{5}{13}$.

**80.** $.9 \times \frac{9}{11}$.

**81.** $8\frac{9}{17} \times 34$.

**82.** $10\frac{8}{11} \times 33$.

**83.** $21\frac{8}{9} \times 6\frac{2}{3}$.

**84.** $13\frac{3}{8} \times 11\frac{8}{11}$.

**85.** $324 \times \frac{7}{9}$.

**86.** $\frac{15}{27} \times \frac{3}{5} \times \frac{18}{7}$.

**87.** $\frac{16}{44} \times \frac{11}{12} \times \frac{5}{8}$.

**88.** $\frac{48}{72} \times \frac{18}{16} \times \frac{4}{3}$.

**89.** $\frac{2}{3} \times \frac{4}{5} \times 3\frac{1}{2}$.

**90.** $\frac{3}{4} \times \frac{5}{6} \times 2\frac{1}{5}$.

**91.** $\frac{3}{5} \times 4\frac{1}{3} \times \frac{15}{16}$.

**92.** $3\frac{1}{4} \times 2\frac{1}{2} \times \frac{26}{5}$.

**93.** $\frac{5}{8} \times 3.1 \times 5\frac{1}{3}$.

**94.** $90 \times 3\frac{1}{5} \times \frac{10}{12}$.

**95.** $\frac{16}{15} \times \frac{5}{8} \times 4\frac{1}{2}$.

**96.** Find the weight of $2\frac{3}{4}$ bushels of oats, allowing 32 pounds to a bushel.

**97.** If a man can walk $3\frac{5}{16}$ miles in an hour, how far can he walk in $7\frac{3}{4}$ hours?

**98.** A and B paid \$160 for a horse. If A paid $\frac{9}{20}$ of the cost, how much money did B pay?

**99.** The multiplier is $\frac{7}{9}$, and the multiplicand is $\frac{3}{14}$. What is the product?

---

**100.** Mr. E owned $\frac{5}{8}$ of a mill which was valued at \$12000. He sold B $\frac{7}{15}$ of his share. What was the value of the part retained?

**101.** A farmer bought $12\frac{3}{4}$ acres of land at $\$37\frac{1}{2}$ an acre, and sold it at $\$52\frac{1}{4}$ an acre. How much did he gain?

**102.** A man who had $\frac{8}{9}$ of an acre of land sold $\frac{3}{4}$ of his share at the rate of \$300 an acre. How much did he get for it?

**103.** What part of a gallon is $\frac{2}{3}$ of $\frac{3}{8}$ of a gallon ?

**104.** If a ton of hay is worth \$$15\frac{3}{4}$, what is the value of 7.5 tons ?

**105.** When cloth is worth \$$1\frac{1}{3}$ a yard, how much must be paid for $\frac{7}{8}$ of a yard ?

**106.** A has $\frac{3}{4}$ as much money as B, who has $\frac{4}{5}$ of \$80. How much money has A ?

**107.** Find the cost of $12\frac{1}{2}$ pounds of butter at $18\frac{3}{4}$ cents a pound.

$$\begin{array}{r} \$.18\frac{3}{4} \\ 12\frac{1}{2} \\ \hline 216 \\ 9 \\ 9 \\ \frac{3}{8} \\ \hline \$2.34\frac{3}{8} \end{array}$$

In examples like this it is often better to multiply as in the solution given. Thus,

$12 \times 18$ cents $= 216$ cents.
$\frac{1}{2} \times 18$ cents $= 9$ cents.
$12 \times \frac{3}{4}$ of a cent $= 9$ cents.
$\frac{1}{2} \times \frac{3}{4}$ of a cent $= \frac{3}{8}$ of a cent.

The sum of all is $234\frac{3}{8}$ cents, or \$$2.34\frac{3}{8}$.

**108.** What is the value of 7 barrels of sugar, each containing $344\frac{1}{2}$ pounds, at \$$.04\frac{3}{4}$ a pound ?

**109.** What is the price of a 1000-mile book of tickets at $1\frac{5}{6}$ cents a mile ?

**110.** If a cubic foot of water weighs $62\frac{1}{2}$ pounds, and iron is $7\frac{9}{10}$ times as heavy as water, what is the weight of a cubic foot of iron ?

**111.** How many pounds of ship-biscuit will be required for a cruise of 30 days, if the daily allowance is $\frac{9}{16}$ of a pound to each of the 250 men in the crew ?

**112.** If the freight rate is $1\frac{7}{10}$ cents a ton for each mile, what will it cost to ship $5\frac{4}{5}$ tons of produce 100 miles ?

### SUPPLEMENTARY EXERCISES (FOR ADVANCED CLASSES).

**220. 1.** A ship sailed 3 days, each day sailing $1\frac{5}{11}$ times as far as on the preceding day. If she traveled 55 miles the first day, how far did she travel the last two days ?

**2.** How much must be paid for the rent of a store for $2\frac{2}{3}$ years at \$$62\frac{1}{2}$ a month ?

**3.** If a man plants $\frac{7}{8}$ of an acre of corn in a day, how many acres will 7 men plant in 3.5 days?

**4.** I bought 51 pounds of sugar at $6\frac{1}{2}$ cents a pound, and $\frac{2}{3}$ as much coffee at 4 times the price paid for the sugar. What did I pay for the coffee?

**5.** Divide $4.09 between two boys so that one will receive 40 cents more than twice what the other receives.

**6.** If I breathe 17 times a minute, and take in at each breath $\frac{5}{7}$ of a quart of air, how many quarts of air do I need in $\frac{1}{2}$ hour?

**7.** If each soldier walks $\frac{3}{4}$ of a mile in $\frac{1}{2}$ of an hour, what is the combined distance marched by a regiment of 800 soldiers in the same time?

**8.** The factors of the multiplicand are 2, 3, 4, 5, and $\frac{5}{7}$; those of the product are $\frac{2}{3}$, and the prime factors of 144. What is the multiplier?

**9.** A clock loses $1\frac{9}{10}$ minutes in a day. If it is correct at noon on the 4th of July, what time will the clock indicate at noon on the 14th of July?

**10.** A clock loses $\frac{1}{4}$ of a second every 5 minutes. How much will it lose in 5 days and 12 hours?

**11.** Prove that the sum of two fractions, the numerator of each of which is 1, is equal to the *sum* of the denominators divided by their product.

## DIVISION.

**221. 1.** A man having 4 fifths of an acre divided it into 2 equal lots. How much land was in each lot? Then $\frac{4}{5} \div 2 = (\quad)$.

**2.** A lady divided 3 fourths of a pie equally among 3 boys. How much did each boy get? Then $\frac{3}{4} \div 3 = (\quad)$.

**3.** Mrs. A gave $\frac{8}{9}$ of a cake to 4 girls. How much did each get? Then $\frac{8}{9} \div 4 = (\quad)$.

**4.** $\frac{8}{9} \div 4 = \frac{2}{9}$. Are the fractional units of this dividend and quotient equal in size? Are they equal in number? The number in the quotient is what part of the number in the dividend? Then, how is a fraction divided by an integer? (See Art. 218.)

WRITTEN EXERCISES.

**222.** **1.** Divide $\frac{9}{20}$ by 3.

$\frac{9}{20} \div 3 = \frac{3}{20}$ Dividing the numerator of a fraction by any number divides the value of the fraction by that number. (Art. 218.)

**2.** Divide $\frac{3}{8}$ by 6.

$\frac{3}{8} \div 6 = \frac{\frac{3}{6}}{8} = \frac{3}{48} = \frac{1}{16}$ Dividing the numerator by 6, as in the preceding example, we get a complex fraction as a quotient, which is changed to a simple fraction, as in Art. 219.

Find the quotients of:

**3.** $\frac{8}{9} \div 2$.
**4.** $\frac{10}{13} \div 5$.
**5.** $\frac{12}{17} \div 4$.
**6.** $\frac{21}{30} \div 7$.
**7.** $\frac{27}{33} \div 9$.
**8.** $\frac{28}{36} \div 14$.
**9.** $\frac{39}{40} \div 13$.
**10.** $\frac{45}{47} \div 15$.
**11.** $\frac{54}{60} \div 18$.
**12.** $\frac{70}{75} \div 14$.
**13.** $\frac{9}{11} \div 4$.
**14.** $\frac{12}{13} \div 5$.
**15.** $\frac{19}{20} \div 9$.
**16.** $\frac{1}{3} \div 11$.
**17.** $\frac{100}{124} \div 25$.
**18.** $\frac{1}{7} \div 12$.
**19.** $\frac{3}{8} \div 9$.
**20.** $\frac{120}{135} \div 24$.
**21.** $\frac{168}{180} \div 28$.
**22.** $\frac{320}{480} \div 32$.
**23.** $\frac{432}{492} \div 36$.

**24.** Find the quotient of $127\frac{2}{3}$ divided by 9.

$$9)\underline{127\tfrac{2}{3}}$$
$$14\frac{1\frac{2}{3}}{9} = 14\tfrac{5}{27}$$

The dividend may be changed to fractional form and divided as in the preceding examples. But it is often more convenient to proceed as follows:

9 is contained in 127 *fourteen* times, with a remainder of $1\frac{2}{3}$, and $1\frac{2}{3} \div 9 = \frac{5}{27}$. Hence the quotient is $14\frac{5}{27}$.

Divide the following:

**25.** $32\frac{1}{2}$ by 4.
**26.** $48\frac{2}{3}$ by 3.
**27.** $65\frac{1}{4}$ by 5.
**28.** $73\frac{3}{4}$ by 6.
**29.** $92\frac{4}{5}$ by 7.
**30.** $93\frac{3}{5}$ by 9.
**31.** $134\frac{4}{5}$ by 12.
**32.** $176\frac{3}{7}$ by 5.
**33.** $248\frac{4}{9}$ by 6.
**34.** $325\frac{1}{8}$ by 3.
**35.** $540\frac{5}{6}$ by 7.
**36.** $809\frac{5}{11}$ by 8.
**37.** $\frac{70}{74}$ by 5.
**38.** $2\frac{3}{8}$ by 19.
**39.** $7\frac{5}{7}$ by 18.
**40.** $\frac{12}{19}$ by 5.
**41.** $170\frac{3}{4}$ by 20.
**42.** $200\frac{7}{8}$ by 10.

**43.** A man earned $\$13594\frac{1}{2}$ in 9 years. What was his average yearly income?

**44.** How often is $\frac{3}{4}$ contained in 9?

$9 = \frac{36}{4}$

$\frac{36}{4} \div \frac{3}{4} = 12$

Changing 9 to *fourths*, we have 36 fourths ÷ 3 fourths. 36 fourths contains 3 fourths 12 times, just as \$36 contains \$3 twelve times.

**45.** How often is $\frac{2}{11}$ contained in $\frac{8}{9}$?

$\frac{8}{9} = \frac{88}{99}$

$\frac{2}{11} = \frac{18}{99}$

$\frac{88}{99} \div \frac{18}{99} = 4\frac{8}{9}$

Changing dividend and divisor to similar fractions, we proceed as in example 44. Any example in division of fractions may be solved in this manner.

Find the quotients of:

| | | |
|---|---|---|
| **46.** $\frac{7}{8} \div \frac{1}{3}$. | **52.** $\frac{11}{18} \div \frac{5}{9}$. | **58.** $9 \div \frac{9}{11}$. |
| **47.** $\frac{4}{5} \div \frac{1}{2}$. | **53.** $5 \div \frac{2}{7}$. | **59.** $3\frac{3}{4} \div 2\frac{1}{2}$. |
| **48.** $\frac{5}{6} \div \frac{3}{4}$. | **54.** $8 \div \frac{4}{5}$. | **60.** $4\frac{5}{7} \div 5\frac{1}{2}$. |
| **49.** $\frac{5}{12} \div \frac{3}{4}$. | **55.** $2\frac{1}{2} \div \frac{1}{2}$. | **61.** $9\frac{1}{4} \div 1\frac{1}{3}$. |
| **50.** $\frac{3}{5} \div \frac{5}{7}$. | **56.** $7\frac{1}{3} \div \frac{3}{5}$. | **62.** $7\frac{3}{5} \div \frac{19}{20}$. |
| **51.** $\frac{7}{15} \div \frac{4}{5}$. | **57.** $3\frac{2}{5} \div \frac{5}{6}$. | **63.** $\frac{7}{20} \div 2\frac{1}{3}$. |

**64.** How many times is $\frac{3}{8}$ contained in $\frac{4}{5}$?

$1 \div \frac{1}{8} = 8$

$1 \div \frac{3}{8} = \frac{8}{3}$ (divisor inverted)

$\frac{4}{5} \div \frac{3}{8} = \frac{4}{5} \times \frac{8}{3} = \frac{32}{15}$, or $2\frac{2}{15}$

Since $\frac{1}{8}$ is contained in 1 eight times, $\frac{3}{8}$ is contained in 1 one-third of 8 times, or $\frac{8}{3}$ times; and in $\frac{4}{5}$ of 1, or $\frac{4}{5}$, it is contained $\frac{4}{5}$ of $\frac{8}{3}$ times, or $\frac{32}{15}$ times.

The inverted divisor shows how many times the divisor is contained in 1; it is called the *reciprocal* of the divisor.

RULE.—*Multiply the inverted divisor by the dividend.*

Mixed numbers and integers must be expressed in fractional form. Cancellation should be used whenever possible.

Find the value of:

| | | |
|---|---|---|
| **65.** $\frac{7}{15} \div \frac{3}{11}$. | **72.** $\frac{18}{32} \div \frac{14}{30}$. | **79.** $5\frac{11}{14} \div 7\frac{3}{7}$. |
| **66.** $\frac{9}{13} \div \frac{5}{12}$. | **73.** $\frac{63}{72} \div \frac{21}{36}$. | **80.** $\frac{8}{15} \div 12$. |
| **67.** $\frac{8}{15} \div \frac{10}{18}$. | **74.** $7 \div \frac{4}{11}$. | **81.** $\frac{13}{27} \div 20$. |
| **68.** $\frac{13}{24} \div \frac{12}{39}$. | **75.** $8 \div \frac{9}{13}$. | **82.** $\frac{30}{31} \div 60$. |
| **69.** $\frac{17}{44} \div \frac{11}{12}$. | **76.** $3\frac{2}{3} \div \frac{5}{8}$. | **83.** $15\frac{1}{9} \div 7$. |
| **70.** $\frac{32}{39} \div \frac{13}{26}$. | **77.** $8\frac{5}{9} \div \frac{5}{6}$. | **84.** $18\frac{1}{2} \div 2.2$. |
| **71.** $\frac{25}{28} \div \frac{15}{42}$. | **78.** $11\frac{3}{7} \div \frac{9}{14}$. | **85.** $7.5 \div 22\frac{1}{2}$. |

**86.** Find the value of $\frac{2\frac{1}{3}}{\frac{3}{4}}$.

This expression is equivalent to $2\frac{1}{3} \div \frac{3}{4}$, and may be treated accordingly. (Note 3, Art. 185.)

**87.** What is the quotient of $\frac{8}{15} \div \frac{2}{3}$?

$\frac{8}{15} \div \frac{2}{3} = \frac{4}{5}$ This process is the converse of that employed in multiplication of fractions, second rule.

The dividend $\frac{8}{15}$ is the product of two factors, one of which is $\frac{2}{3}$. Since the numerator 8 is the product of two factors, one of which is 2, the other factor is $8 \div 2$, or 4, which is the numerator of the required quotient. Since the denominator 15 is the product of two factors (or denominators), one of which is 3, the other factor is $15 \div 3$, or 5, which is the denominator of the required quotient. Hence the quotient is $\frac{4}{5}$.

☞ This is the most convenient method when the numerator and denominator of the dividend are respectively multiples of the numerator and denominator of the divisor.

Divide, using this method:

**88.** $\frac{9}{12}$ by $\frac{3}{4}$.
**89.** $\frac{10}{15}$ by $\frac{2}{5}$.
**90.** $\frac{12}{18}$ by $\frac{4}{9}$.
**91.** $\frac{16}{27}$ by $\frac{8}{9}$.
**92.** $\frac{18}{30}$ by $\frac{9}{15}$.
**93.** $\frac{24}{39}$ by $\frac{8}{13}$.
**94.** $\frac{32}{49}$ by $\frac{8}{7}$.
**95.** $3\frac{9}{15}$ by $\frac{6}{5}$.
**96.** $5\frac{8}{16}$ by $2\frac{3}{4}$.
**97.** $7\frac{3}{11}$ by $1\frac{9}{11}$.
**98.** $\frac{45}{80}$ by $\frac{15}{20}$.
**99.** $\frac{132}{153}$ by $\frac{12}{51}$.

**100.** At $\$2\frac{3}{4}$ a day, how long will it take to earn $\$37\frac{1}{2}$?

**101.** If Thomas can walk $3\frac{1}{3}$ miles an hour, in what time can he walk 20 miles?

**102.** A man gave $56 for sheep, paying $\$5\frac{3}{5}$ a head. How many did he buy?

**103.** If a bankrupt's property is worth $3100, and his debts amount to $7000, how many cents on a dollar can he pay?

**104.** A man paid $55 for $3\frac{2}{5}$ tons of hay. What was the price a ton?

**105.** A horse traveled $24\frac{3}{4}$ miles in 4 hours. At what rate an hour was that?

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**106.** How many times must $\frac{2}{3}$ be added to itself to make $7\frac{1}{3}$?

**107.** How many times can $\frac{7}{8}$ be subtracted from 7?

**108.** A man having 10 acres of land sold each of several men $\frac{3}{4}$ of an acre, and had $2\frac{1}{2}$ acres left. To how many men did he sell?

**109.** The distance around a barn is $212\frac{1}{2}$ feet. How wide is the barn, if it is $60\frac{3}{4}$ feet long?

**110.** I bought 25 bushels of potatoes at $\$\frac{5}{8}$ a bushel, and sold them for $\$18\frac{3}{4}$. How much did I gain on one bushel?

**111.** What must $3\frac{2}{5}$ be multiplied by to make $11\frac{1}{3}$?

**112.** A man bought 396 pounds of oats, at $\$\frac{2}{5}$ a bushel. Allowing 32 pounds to a bushel, what did he pay for them?

**113.** A man paid $\$1359\frac{3}{8}$ for $21\frac{3}{4}$ acres of land. How much was that an acre?

**114.** The divisor is .8, the dividend $\frac{8}{9}$. Find the quotient.

**115.** When eggs are worth $18\frac{3}{4}$ cents a dozen, how much must be paid for 5 dozen and 6 eggs?

SUPPLEMENTARY EXERCISES (FOR ADVANCED CLASSES).

**223.** **1.** A boy bought lemons at the rate of 8 for 7 cents, and sold them at the rate of 7 for 8 cents. If he made $1.35 in 7 days, how many did he sell each day?

**2.** A merchant purchased a cargo of flour for $\$2173\frac{1}{2}$, and sold it for $\frac{2}{2}\frac{2}{3}$ of its cost, thereby losing $\$\frac{1}{4}$ on a barrel. How many barrels did he purchase?

**3.** The product of a number multiplied by 3 is how many times the product of the same number multiplied by $\frac{3}{8}$?

**4.** A owns $2\frac{1}{3}$ times as much land as B, C owns $1\frac{1}{4}$ times as much as both A and B, and D owns $3\frac{5}{8}$ times as much as B and C. If B has $36\frac{1}{6}$ acres, how much has D?

**5.** A boy bought $\frac{7}{8}$ of a bushel of nuts, and sold $\frac{5}{7}$ of them for what he paid for all, and the remainder at cost. If he gained $\$1\frac{1}{2}$ by the transaction, how much money had he invested?

**6.** Change $\frac{5}{125}$ to a decimal, and divide by 5000.

**7.** The owner of $\frac{3}{11}$ of a mine sold $\frac{9}{10}$ of his share for $40,500. What should he who owns $\frac{3}{5}$ of the mine get for $\frac{5}{9}$ of his share?

**8.** A can mow a field in 3 days, B in 4 days, C in 5 days, and D in 6 days. If A can earn \$20 a week, how much can B, C, and D together earn in the same time?

**9.** If $8\frac{6}{7}$ tons of hay are worth 36 sheep, and 11 sheep are worth 2 cows, and 9 cows are worth \$200, how many dollars is hay worth a ton?

**10.** $1 + \dfrac{1 + 1.5}{5} \div \dfrac{5}{1 + 1.5} = (\quad).$

**11.** (a) $.00001 \div 10000 = (\quad)$; (b) $1000 \div .0001 = (\quad)$; (c) $.001 \div .000001 = (\quad)$; (d) $400 \div 10000 = (\quad)$.

**12.** Find value of $2\frac{3}{4} \div \frac{5}{8} \times 24\frac{3}{16} \times 0$.

**13.** The sum of two numbers is 1000; the difference is 648. Divide twice the larger by $\frac{1}{2}$.

**14.** A owns $\frac{2}{7}$ of a farm, and B the remainder. $\frac{5}{8}$ of the difference between their shares is \$10500. What is the value of the farm?

**15.** The minuend is 8 times $\frac{1}{4} \times .5$; the remainder is $\frac{1}{8}$ of $.7 \div \frac{7}{9}$. What is the subtrahend?

**16.** A can dig $16\frac{1}{2}$ rows of potatoes in a day, and B can pick $33\frac{1}{2}$ rows in a day. If A has 93.5 rows dug when B begins, how many rows must B pick before he overtakes A?

## ILLUSTRATIVE SOLUTIONS,

### With Problems for Practice.

**224.** The solution of all problems in the applications of arithmetic requires **Analysis** of some kind. A type of *arithmetical analysis* much employed involves the process of reasoning from a given number to *one,* and then passing from *one* to the *required number.*

**225.** As many formal solutions begin with a "since" and end with a "therefore," it is convenient to represent these two words by signs. By general usage the sign (∵) is read "since," and the sign (∴) is read "therefore."

**226. 1.** In a town $\frac{3}{35}$ of the people are sick and 512 are well. How many are sick?

$\frac{35}{35}$ of the people $-$ $\frac{3}{35}$ of the people $= \frac{32}{35}$ of them.
∵ $\frac{32}{35}$ of the people $=$ 512 persons,
$\frac{1}{35}$ of the people $=$ 16 persons,
and $\frac{3}{35}$ of the people $=$ 48 persons.

**2.** If electricity passes through 7200 miles of wire in $\frac{2}{3}$ of a second, what is its rate a second?

**3.** If a horse trots $\frac{5}{6}$ of a mile in $2\frac{1}{3}$ minutes, in what time can he trot a mile?

**4.** One half of a post is in the water, $\frac{1}{8}$ in mud, and 4 ft. 6 in. in the air. Find length of the post.

**5.** After selling .32 of his sheep to one man and .38 of them to another, a drover had 312 remaining. How many had he at first?

**6.** A boy gave $\frac{1}{2}$ of his marbles to A, $\frac{2}{3}$ of the remainder to B, and then had 15 marbles left. At first he had how many?

After giving $\frac{1}{2}$ of his marbles to A he had $\frac{1}{2}$ of them left. To B he gave $\frac{2}{3}$ of this $\frac{1}{2}$, or $\frac{1}{3}$, and had $\frac{1}{2} - \frac{1}{3}$, or $\frac{1}{6}$ of them left.

$\therefore$ $\frac{1}{6}$ of his marbles $= 15$, and $\frac{6}{6}$ of them $= 90$.

**7.** A man invested $\frac{1}{2}$ of his money in land, $\frac{1}{5}$ of the remainder in cattle, and had \$1000 left. How much money had he at first?

**8.** In a certain school $\frac{3}{8}$ of the pupils belong to the third grade, $\frac{7}{9}$ of the remaining pupils belong to the second grade, and the remainder, which is 20, belong to the first grade. How many pupils are there in the school?

**9.** A man gives $\frac{1}{4}$ of his property to one son, $\frac{1}{6}$ of it to another, $\frac{1}{8}$ of it to the third son, and the remainder, \$550, to his daughter. What is the value of the whole property?

**10.** A person loses $\frac{1}{10}$ of his fortune and then $\frac{1}{15}$ of the remainder, and then $\frac{1}{7}$ of what he then had, and finds that he has \$3600 left. How much had he at first?

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**11.** Find the average weight of three men weighing, respectively, 130 lb., 145 lb., and 175 lb.

They together weigh 130 lb. + 145 lb. + 175 lb. = 450 lb. $\therefore$ the average weight of the three men is $\frac{1}{3}$ of 450 lb., or 150 lb.

**12.** If 130 pupils attend school on Monday, 126 on Tuesday, 122 on Wednesday, 125 on Thursday, and 122 on Friday, what is the average attendance for the week?

**13.** If the temperature as indicated by a thermometer at eight different times on a certain day was, respectively, 37°, 36°, 36°, 38°, 40°, 38°, 36°, and 35°, what was the average temperature for the day?

**14.** The *per capita* indebtedness of France is \$116, of Prussia, \$37, Great Britain and Ireland, \$88; Russia, \$31; Spain, \$74, and the U. S. \$15. What is the average *per capita* indebtedness of these six countries?

**15.** If the heart beats 140 times a minute during the first 3 years of life, 120 times a minute for the next 3 years, 100 times a minute for the next 6 years, 90 times a minute for the next 10 years, and 75 times a minute for the next 28 years, what is the average number of beats a minute in a life of 50 years?

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**16.** The sum of two numbers is 84, and their difference is 14. Find the numbers.

$$\begin{aligned} \text{The greater} + \text{the less} &= 84. \\ \text{The greater} - \text{the less} &= 14. \\ \therefore 2 \times \text{the less} &= 70, \\ \text{and the less} &= 35. \end{aligned}$$

**17.** The sum of two numbers is 603, and their difference is 273. Find the numbers.

**18.** The difference between A's money and B's is $16.50, and they together have $166.50. How much has each?

**19.** George has $\frac{1}{2}$ doz. eggs more than Kate, and both have 42 eggs. How many has each?

**20.** The sum of two numbers exceeds their difference by 198. What is the smaller number?

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**21.** A crew can row 60 miles down stream in 3 hours, but requires 4 hours to row back. What is the rate of the current?

$$\begin{aligned} \text{Rate in still water} + \text{rate of current} &= 20 \text{ miles an hour.} \\ \text{Rate in still water} - \text{rate of current} &= 15 \text{ miles an hour.} \\ \therefore 2 \times \text{rate of current} &= 5 \text{ miles an hour,} \\ \text{and the current's rate} &= 2\tfrac{1}{2} \text{ miles an hour.} \end{aligned}$$

**22.** A steamboat goes 72 miles down stream in 6 hours, but is 8 hours returning. What is the rate of the stream?

**23.** Going down stream A can row 11 miles in 2 hours. Going up stream he can row $1\frac{1}{2}$ miles in a half hour. Find the rate of the current.

**24.** If a man can row $6\frac{1}{2}$ miles an hour down stream and $4\frac{1}{2}$ miles an hour up stream, how far can he row in an hour in still water?

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**25.** A garrison of 1200 men had provisions to last 90 days, but 30 days later 300 more men arrived. How long did the provisions last after the increase in the number of men?

They would have lasted 1200 men 60 days,
or 1 man $1200 \times 60$ days.

$\therefore$ they lasted 1500 men $\frac{1200 \times 60}{1500}$, or 48 days.

**26.** A garrison at Manila, consisting of 2500 men, had provisions for 30 days, but 1500 men were withdrawn. How long did the provisions last the remainder?

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**27.** A can do a piece of work in 3 days, and B can do it in 5 days. How long will it take both together?

In 1 day A does $\frac{1}{3}$ of the work.
In 1 day B does $\frac{1}{5}$ of the work.
In 1 day A and B do $\frac{1}{3} + \frac{1}{5}$, or $\frac{8}{15}$ of it.
To do $\frac{15}{15}$, or the whole work, will require $\frac{15}{15} \div \frac{8}{15}$ or $1\frac{7}{8}$ days.

**28.** A can do a piece of work in 5, B in 6, and C in 8 days. How long would it take them to do it together?

**29.** A and B can build a house in 3 months. B alone can complete it in 8 months. In what time can A build the house?

**30.** If A can lay a certain wall in $4\frac{1}{2}$ days, and B in $5\frac{1}{2}$ days, how long will it take both together?

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**31.** Two pipes lead into a tank. One can fill it in 50 minutes, the other in 40 minutes. There is a discharging pipe which can empty the tank in 25 min. In what time will the tank be filled if all three pipes are in operation?

The L. C. M. of 50, 40, and 25 is 200. Hence one pipe fills the tank 4 times in 200 min., the other 5 times, and both 9 times. The third pipe empties the tank 8 times in 200 min.

$\therefore$ the tank is filled $9 - 8$, or 1 time, in 200 min., or 3 hr. 20 min.

**32.** One pipe will fill a cistern in 3 hours; a waste-pipe will empty it in 2 hours. If the cistern is full and both pipes are opened, in what time will the cistern be emptied?

**33.** An empty cistern has two pipes. One fills it in 40 minutes, the other empties it in 60 minutes. If both are opened, in what time will the cistern be filled?

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**34.** If 8 horses eat 48 bushels of corn in 24 days, in how many days will 4 horses eat 38 bushels?

8 horses eat 48 bu. in 24 days.
1 horse eats 48 bu. in $8 \times 24$ days.
1 horse eats 1 bu. in $\frac{8 \times 24}{48}$ days.
$\therefore$ 4 horses eat 1 bu. in $\frac{8 \times 24}{48 \times 4}$ days,
and 4 horses eat 38 bu. in $\frac{8 \times 24 \times 38}{48 \times 4}$ days $= 38$ days.

**35.** If 8 men in 7 days can reap a field of 40 acres, how many acres will be cut by 24 men in 28 days?

**36.** If 3 men earn \$150 in 20 days, how many men will earn \$157.50 in 9 days, at the same rate?

**37.** If 5 yards of cloth $\frac{3}{4}$ yd. wide cost \$3.12$\frac{1}{2}$, how much will 15 yards of that cloth 1 yd. wide cost at the same rate?

**38.** If a 5-cent loaf weighs 1.5 pounds when wheat is 50 cents a bushel, what should it weigh when wheat is \$.75 a bushel?

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**39.** When a certain number is multiplied by 9, the product divided by 12, the quotient increased by 96, and the sum divided by 3, the quotient is 36. What is the number?

$$36 \times 3 = 108;\ 108 - 96 = 12;$$
$$12 \times 12 = 144;\ 144 \div 9 = 16.$$

**40.** If a certain number is diminished by 76, the remainder multiplied by 2, the product increased by 148, and the sum divided by 12, the quotient is 21$\frac{1}{3}$. Find the number.

**41.** At what time between 1 and 2 o'clock are the hour and minute hands of a clock together?

FIRST SOLUTION.

In 1 hour the minute hand moves over 60 minute-spaces, and the hour hand over 5. Hence the former gains 55 minute-spaces in that time. To gain 1 space requires $\frac{1}{55}$ of an hour. At 1 o'clock the hands are 5 spaces apart. Hence to gain this 5 spaces will require $5 \times \frac{1}{55}$ hr. $= \frac{1}{11}$ hr., or $5\frac{5}{11}$ min. Hence the time is $5\frac{5}{11}$ min. past 1 o'clock.

SECOND SOLUTION.

The minute hand gains 11 hour-spaces in going 12, that is, in 1 hour. Hence to gain 1 space requires $\frac{1}{11}$ of an hour. At 1 o'clock the hands are one hour-space apart, which the minute hand will gain in $\frac{1}{11}$ hr., or $5\frac{5}{11}$ min. Hence it will overtake the hour hand at $5\frac{5}{11}$ min. past 1 o'clock.

**42.** At what time between 4 and 5 o'clock do the hour and minute hands of a clock coincide?

**43.** At what time between 10 and 11 o'clock do the hour and minute hands of a watch coincide?

**44.** At what time between 1 and 2 o'clock are the hands of a clock exactly opposite each other?

**45.** A and B start from the same point and travel in the same direction. A goes 7 miles an hour and B 3 miles an hour. If B has a start of 5 hours, when will he be overtaken by A?

## RELATION OF NUMBERS.

**227.** The **Relation of Numbers** is their comparative value.

Thus, comparing 2 with 4, we say 2 is $\frac{1}{2}$ of 4.

**228. To find a number when part of it is given.**

**1.** \$2 is $\frac{1}{2}$ of my money. How much have I?

**2.** Nettie spent \$5, which was $\frac{1}{3}$ of her money. How much money had she?

**3.** Harry lost 3 marbles, which was $\frac{1}{4}$ of all he had. How many had he?

**4.** Two is $\frac{1}{2}$ of what number? $\frac{1}{3}$? $\frac{1}{4}$?

**5.** Five is $\frac{1}{2}$ of what number? $\frac{1}{3}$?

**6.** Eight is $\frac{2}{3}$ of what number?

$\frac{2}{3}$ of the number $= 8$.
$\therefore \frac{1}{3}$ " " " $= \frac{1}{2}$ of 8, or 4.
$\therefore \frac{3}{3}$ " " " $= 3 \times 4$, or 12.

Since 8 is $\frac{2}{3}$ of some number, 1 third of that number is $\frac{1}{2}$ of 8, or 4; and since 4 is 1 third of the number, 3 thirds, or the number, equals 3 times 4, or 12. Hence 8 is $\frac{2}{3}$ of 12.

**7.** Nine is $\frac{3}{4}$ of what number?

**8.** Ten is $\frac{2}{5}$ of what number?

Find the number of which

**9.** 12 is $\frac{4}{7}$.
**10.** 15 is $\frac{3}{8}$.
**11.** 24 is $\frac{4}{9}$.
**12.** 29 is $\frac{1}{3}$.
**13.** 32 is $\frac{8}{11}$.
**14.** 39 is $\frac{3}{10}$.
**15.** 40 is $\frac{10}{11}$.
**16.** 52 is $\frac{4}{5}$.
**17.** 72 is $\frac{12}{13}$.
**18.** 65 is $\frac{13}{15}$.
**19.** 120 is $\frac{5}{8}$.
**20.** 217 is $\frac{7}{9}$.
**21.** 210 is $\frac{6}{7}$.
**22.** 225 is $\frac{15}{17}$.
**23.** 414 is $\frac{23}{25}$.
**24.** 1000 is .5.

**25.** 2 is $\frac{3}{4}$ of what number?

$\frac{3}{4}$ of the number $= 2$.
$\therefore \frac{1}{4}$ " " " $= \frac{1}{3}$ of 2, or $\frac{2}{3}$.
$\therefore \frac{4}{4}$ " " " $= 4 \times \frac{2}{3}$, or $\frac{8}{3}$.

Since 2 is $\frac{3}{4}$ of some number, 1 fourth of that number is $\frac{1}{3}$ of 2, or $\frac{2}{3}$; and since $\frac{2}{3}$ is $\frac{1}{4}$ of the number, 4 fourths, or the number, equals 4 times $\frac{2}{3}$, or $\frac{8}{3}$. Hence 2 is $\frac{3}{4}$ of $2\frac{2}{3}$.

**26.** 9 is $\frac{4}{5}$ of what number?

**27.** 16 is $\frac{5}{8}$ of what number?

**28.** A lady spent \$20, which was $\frac{3}{4}$ of her money. How much had she at first?

**29.** $\frac{8}{9}$ is $\frac{2}{7}$ of what number?

$\frac{2}{7}$ of the number $= \frac{8}{9}$.
$\therefore \frac{1}{7}$ " " " $= \frac{1}{2}$ of $\frac{8}{9}$, or $\frac{4}{9}$.
$\therefore \frac{7}{7}$ " " " $= 7 \times \frac{4}{9}$, or $3\frac{1}{9}$.

Since $\frac{8}{9}$ is $\frac{2}{7}$ of some number, 1 seventh of that number is $\frac{1}{2}$ of $\frac{8}{9}$, or $\frac{4}{9}$; and since $\frac{4}{9}$ is $\frac{1}{7}$ of the number, $\frac{7}{7}$, or the number, equals 7 times $\frac{4}{9}$, or $\frac{28}{9}$, or $3\frac{1}{9}$. Hence $\frac{8}{9}$ is $\frac{2}{7}$ of $3\frac{1}{9}$.

**30.** $\frac{10}{11}$ is $\frac{5}{9}$ of what number?

**31.** A hat cost \$$1\frac{3}{5}$, which was $\frac{2}{3}$ of the cost of a vest. How much did the vest cost?

**32.** William walked 27 miles in one day, which was $\frac{3}{4}$ of the distance Joseph walked. How far did Joseph walk?

**33.** A lot cost \$450, which was $\frac{3}{13}$ of the cost of a house. Find the cost of the house.

**34.** Three fifths of the distance from Pittsburg to Philadelphia is 213 miles. How far apart are the two cities?

**35.** A bought a horse for \$126, which was $\frac{9}{11}$ of what he sold him for. How much did he gain?

**36.** In $\frac{27}{40}$ of a mile are 216 rods. How many rods in a mile?

**37.** \$18 is $\frac{3}{5}$ of what B paid for a cow. His horse cost twice as much as the cow. How much did he pay for the horse?

## ALIQUOT PARTS.

**229.** The **Aliquot Parts** of a number are the parts that will exactly divide it. Thus, 2 and 5 are aliquot parts of 10.

The relation of

| | | | |
|---|---|---|---|
| 50 to 100 is $\frac{1}{2}$. | Hence | $50 = \frac{1}{2}$ of 100. |
| $33\frac{1}{3}$ to 100 is $\frac{1}{3}$. | " | $33\frac{1}{3} = \frac{1}{3}$ of 100. |
| 25 to 100 is $\frac{1}{4}$. | " | $25 = \frac{1}{4}$ of 100. |
| 20 to 100 is $\frac{1}{5}$. | " | $20 = \frac{1}{5}$ of 100. |
| $16\frac{2}{3}$ to 100 is $\frac{1}{6}$. | " | $16\frac{2}{3} = \frac{1}{6}$ of 100. |
| $14\frac{2}{7}$ to 100 is $\frac{1}{7}$. | " | $14\frac{2}{7} = \frac{1}{7}$ of 100. |
| $12\frac{1}{2}$ to 100 is $\frac{1}{8}$. | " | $12\frac{1}{2} = \frac{1}{8}$ of 100. |
| $11\frac{1}{9}$ to 100 is $\frac{1}{9}$. | " | $11\frac{1}{9} = \frac{1}{9}$ of 100. |
| 10 to 100 is $\frac{1}{10}$. | " | $10 = \frac{1}{10}$ of 100. |
| $8\frac{1}{3}$ to 100 is $\frac{1}{12}$. | " | $8\frac{1}{3} = \frac{1}{12}$ of 100. |
| $6\frac{1}{4}$ to 100 is $\frac{1}{16}$. | " | $6\frac{1}{4} = \frac{1}{16}$ of 100. |

The numbers in the first column are *aliquot parts* of 100. From them may be found the following other parts of 100:

| | | |
|---|---|---|
| $40 = \frac{2}{5}$ of 100. | $37\frac{1}{2} = \frac{3}{8}$ of 100. | $83\frac{1}{3} = \frac{5}{6}$ of 100. |
| $60 = \frac{3}{5}$ of 100. | $62\frac{1}{2} = \frac{5}{8}$ of 100. | $41\frac{2}{3} = \frac{5}{12}$ of 100. |
| $80 = \frac{4}{5}$ of 100. | $87\frac{1}{2} = \frac{7}{8}$ of 100. | $58\frac{1}{3} = \frac{7}{12}$ of 100. |
| $75 = \frac{3}{4}$ of 100. | $66\frac{2}{3} = \frac{2}{3}$ of 100. | $31\frac{1}{4} = \frac{5}{16}$ of 100. |

**230. To multiply by the aliquot parts of 100.**

**1.** Multiply 2856 by 25.

$$4\overline{)285600}$$
$$71400$$

Since 25 is $\frac{1}{4}$ of 100, we multiply by 100—which is done by annexing two ciphers—and take $\frac{1}{4}$ of the product.

Multiply in a similar manner:

**2.** 3576 by $33\frac{1}{3}$.
**3.** 2748 by 50.
**4.** 1728 by $12\frac{1}{2}$.
**5.** 3270 by $16\frac{2}{3}$.
**6.** 4368 by $8\frac{1}{3}$.
**7.** 5138 by $14\frac{2}{7}$.
**8.** 2946 by $66\frac{2}{3}$.
**9.** 35768 by $37\frac{1}{2}$.

**10.** At $\$.62\frac{1}{2}$ a bushel, what must be paid for 1648 bushels of wheat?

**11.** Find the cost of 75 dozen shovels at \$.87½ each?

**12.** When potatoes are worth 66⅔¢ a bushel, how much must be paid for 240 barrels, each containing 3¼ bushels?

**231. To divide by the aliquot parts of 100.**

**1.** Divide 1257 by 33⅓.

$$\begin{array}{r} 12.57 \\ 3 \\ \hline 37.71 \end{array}$$

Since 33⅓ is ⅓ of 100, we divide by 100—which is done by pointing off two decimal places—and multiply the quotient by 3.

Divide in a similar manner:

**2.** 2576 by 16⅔.
**3.** 2718 by 25.
**4.** 3592 by 12½.
**5.** 5340 by 14²⁄₇.
**6.** 4825 by 8⅓.
**7.** 3344 by 11⅑.
**8.** 76512 by 6¼.
**9.** 37584 by 37.5.
**10.** 21360 by 66⅔.
**11.** 576900 by 62½.

**12.** If pears are worth \$.33⅓ a peck, how many can be bought for \$12.50?

**13.** When butter is selling at \$.16⅔ a pound, how much can be purchased for \$1.50?

**14.** At \$⅛ a yard, how many yards of cloth can be bought for \$¾?

**15.** James can walk 33⅓ miles in a day. How far can he walk in six weeks?

**16.** A farmer has 2400 bushels of corn to husk. In a day he can husk 75 bushels. In how many days can he husk the entire crop?

**17.** In an orchard ½ of the trees bear apples, ¼ peaches, ⅙ pears, and the rest, 5 of them, cherries. How many trees in the orchard?

**18.** A father willed ³⁄₇ of his estate to one son, ³⁄₇ of the remainder to another, and the rest to his wife. If one son received \$900 more than the other, how much did the widow receive?

## RATIO.

**232. To find the relation of one number to another.**

**1.** 2 feet is what part of 4 feet? Of 6 feet?

**2.** How does \$5 compare with \$10? With \$20?

**3.** What is the relation of 3 to 6? 4 to 8? 5 to 15?

**4.** How does \$10 compare with \$2? 12 with 3? 16 with 8? 12 with 4?

**5.** What is the relation of 3 to 8, or what part of 8 is 3?

Since 1 is $\frac{1}{8}$ of 8, 3 is 3 times $\frac{1}{8}$ of 8, or $\frac{3}{8}$ of 8. Hence $\frac{3}{8}$ is the relation of 3 to 8.

**6.** What is the relation of 4 to 9? Of 8 to 12? Of 6 to 14?

**233.** The relation of one number to another of the same kind is **Ratio.**

NOTE.—There is no ratio between \$3 and 6 *hats*, nor can the ratio between 4 *feet* and 8 *yards* be determined in this form; but if we reduce the 8 yards to feet, the ratio is found to be $\frac{4}{24}$ or $\frac{1}{6}$.

**234.** The **Sign** of ratio is ( : ).

Thus, the ratio of 3 to 8 is written 3 : 8. This was first used as a sign of division by Leibnitz. The ratio of 3 to 8 may be expressed in three ways—3 : 8, 3 ÷ 8, and $\frac{3}{8}$.

**235.** The two numbers compared are together called a **Couplet.**

**236.** The first is called the **Antecedent,** the second the **Consequent.**

Thus, in the ratio 3 : 8, 3 is the antecedent, and 8 the consequent.

NOTES.—1. The ratio of one number to another is found by dividing the antecedent by the consequent.

2. The ratio being the quotient of one number divided by another of the same kind is *always* an *abstract* number. (Art. 99.)

Since the antecedent may be regarded as a dividend, and the consequent as a divisor, we have the following

**237.** Principles.—1. *Multiplying the antecedent or dividing the consequent by any number multiplies the ratio by that number.*

2. *Dividing the antecedent or multiplying the consequent by any number divides the ratio by that number.*

3. *Multiplying or dividing antecedent and consequent by the same number does not change the value of the ratio.*

**238.** Since a ratio is the quotient of an antecedent by its consequent, it follows that

(a). The antecedent = the consequent × the ratio.
(b). The consequent = the antecedent ÷ the ratio.

Find the ratio of:

**1.** 12 to 16.
**2.** 7 to 21.
**3.** 8 to 18.
**4.** $10 to $25.
**5.** 30 to 15.
**6.** 20 yd. to 30 yd.
**7.** 27 to 9.
**8.** 9 to 27.
**9.** 5 to .5.
**10.** .5 to 5.
**11.** 50¢ to 100¢.
**12.** $5 to $2.
**13.** 52 to 13.
**14.** 18 to 15.
**15.** 12 to 17.
**16.** 95 to 19.

**17.** What is the ratio of 1 foot to a yard?
**18.** What is the relation of $.12$\frac{1}{2}$ to $1?
**19.** What is the ratio of 2.5 to .25?
**20.** What is the ratio of $\frac{4}{7}$ to $\frac{2}{7}$?

Query.—When the denominators are the same, why may they be disregarded?

**21.** What is the ratio of $\frac{2}{3}$ to $\frac{4}{5}$?

Suggestion. $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{10}{12}$, or $\frac{5}{6}$. Or,
$\frac{2}{3} \div \frac{4}{5} = \frac{10}{15} \div \frac{12}{15} = \frac{10}{12}$, or $\frac{5}{6}$.

What is the ratio of:

**22.** $\frac{2}{3}$ to $\frac{3}{4}$? $\frac{3}{5}$ to $\frac{1}{4}$? $\frac{8}{9}$ to $\frac{5}{6}$?
**23.** $\frac{7}{8}$ to $\frac{1}{4}$? $\frac{1}{4}$ to $\frac{7}{8}$? $\frac{6}{7}$ to $\frac{1}{2}$?

What is the ratio of:

**24.** 1 to $\frac{1}{6}$? 2 to $\frac{1}{6}$? 3 to $\frac{3}{4}$?

**25.** $\frac{1}{3}$ to 1? $\frac{1}{3}$ to 2? $\frac{2}{3}$ to 1?

**26.** $\frac{3}{5}$ to .7? 6 to 1.2? 10 to $2\frac{1}{2}$?

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**27.** What number has to 6 the ratio 3? To 7 the ratio 5?

**28.** Mention two numbers whose ratio is $\frac{5}{7}$.

**29.** How much is 6 : 2 more than 7 : 3?

**30.** Which is greater, $\frac{3}{4}$, $2 \div 3$, or 1 : 2?

**31.** Find the value of $(2 : 6) \times (5 : 2)$.

**32.** Find the value of $(5 : 3) \div (12 : \frac{3}{4})$.

**33.** If the antecedent is 12 and the ratio 3, what is the consequent?

**34.** If the consequent is 5 and the ratio 3, what is the antecedent?

**35.** What is the effect produced on the ratio 4 : 5 by multiplying the antecedent by 3? The consequent by 3? Both by 3?

**36.** What is the ratio of $4a$ to $2a$? Of $6b$ to $3b$?

**37.** What is the ratio of $5b$ to $10b$? Of $7a$ to $21a$?

**38.** What number has to 5 the ratio 3? What number has to $a$ the ratio $b$?

**39.** What is the ratio of $b$ to $a$? Of $p$ to $q$?

**40.** If the antecedent is $6a$ and the ratio 3, what is the consequent?

**41.** If $a$ is the antecedent and 2 the ratio, what is the consequent?

# PROPORTION.

**239.** **1.** What is the ratio of 2 to 4? Of 3 to 6? Are these ratios equal to each other?

**2.** Since they are equal, they may be written thus:

$$2 : 4 = 3 : 6.$$

This may be read in two ways: thus, the ratio of 2 to 4 *equals* the ratio of 3 to 6; or, 2 is to 4 as 3 is to 6.

**240.** When two ratios are equal, and written as above, they form an **Equality of Ratios.**

**241.** An equality of ratios is a **Proportion.**

Thus, $8 : 4 = 12 : 6$ is a proportion. It may also be written $8 \div 4 = 12 \div 6$, or $\frac{8}{4} = \frac{12}{6}$.

**242.** A proportion being composed of two ratios must use four numbers—two antecedents and two consequents. When any three of these are given, the fourth may be found.

Find the wanting number in the following:

**1.** $2 : 4 = 5 : (\quad)$.
**2.** $1 : 2 = 4 : (\quad)$.
**3.** $2 : (\quad) = 3 : 6$.
**4.** $1 : 3 = (\quad) : 8$.
**5.** $(\quad) : 5 = 8 : 4$.
**6.** $3 : 6 = 2 : (\quad)$.
**7.** $8 : 4 = 6 : (\quad)$.
**8.** $10 : (\quad) = 8 : 4$.
**9.** $4 : 2 = (\quad) : 5$.
**10.** $(\quad) : 6 = 3 : 9$.

QUERIES.—1. Which numbers are antecedents? 2. Which are consequents? 3. Is every proportion an equation? Why?

**243.** The sign ( : : ) is sometimes written between the equal ratios instead of the sign of equality (=).

Thus, $3 : 6 = 4 : 8$ may be written $3 : 6 :: 4 : 8$, and is read 3 is to 6 *as* 4 is to 8.

**244.** The *first* and *fourth* numbers of the proportion are the **Extremes,** and the *second* and *third* the **Means.**

Thus, in the proportion 5 : 10 = 2 : 4, 5 and 4 are the *extremes,* 10 and 2 the *means.*

**245.** The ratio of one number to another is a **Simple Ratio.**

**246.** A **Simple Proportion** is an equality of two simple ratios.

**247.** PRINCIPLES.—1. *The product of the extremes is equal to the product of the means.*

In any proportion, as 2 : 4 = 3 : 6, the ratios may be expressed in fractional form. Thus, 2 : 4 = 3 : 6 may be written $\frac{2}{4} = \frac{3}{6}$. Changing to similar fractions, we have $\frac{2 \times 6}{24} = \frac{3 \times 4}{24}$.

Hence, $2 \times 6 = 3 \times 4$. But 2 and 6 are the extremes, and 3 and 4 the means of the given proportion; therefore, the product of the extremes is equal to the product of the means.

2. *The product of the means divided by either extreme is equal to the other extreme.*

3. *The product of the extremes divided by either mean is equal to the other mean.*

☞ Have the pupils prove principles 2 and 3.

Find the number omitted in each of the following:

**1.** 6 : 8 = 9 : ( ).

**2.** 12 : 10 = ( ) : 5.

**3.** 9 : 15 = 12 : ( ).

**4.** 16 : ( ) = 18 : 27.

**5.** 50 : 20 = ( ) : 12.

**6.** ( ) : 15 = $4\frac{1}{2}$ : 9.

**7.** 6.5 : 19.5 = $\frac{1}{3}$ : ( ).

**8.** 7.5 : 2.5 = ( ) : 1.3.

**9.** 10 feet : 15 feet : : $8 : ( ).

**10.** 1 : $6\frac{1}{4}$ : : 8 : ( ).

**11.** $14 : $7 : : ( ) : 1.

**12.** $\frac{2}{5}$ : 2 : : 5 : ( ).

**13.** ( ) : $\frac{3}{8}$ : : $13\frac{1}{2}$ : $1\frac{2}{3}$.

**14.** 3.5 : $\frac{1}{2}$ : : ( ) : $2\frac{1}{7}$.

**15.** $\frac{1}{9}$ : ( ) : : $\frac{2}{3}$ : $\frac{1}{4}$.

**16.** 24 : ( ) : : 10 : $\frac{1}{10}$.

### APPLICATIONS OF SIMPLE PROPORTION.

**248.** Many problems that are usually solved by analysis can be readily solved by proportion.

**1.** If 8 hats cost \$16, how much will 12 hats cost?

FIRST STATEMENT.

$$\begin{array}{cccc} \text{hats} & \text{hats} & \$ & \$ \\ 8 & : 12 = & 16 : & (\quad) \end{array}$$

$$\frac{12 \times 16}{8} = 24$$

Since only *like numbers* can be compared in a ratio, we have for the first couplet 8 hats and 12 hats, either of which may be made the antecedent. For the second couplet we have the cost of 8 hats and the cost of 12 hats. In the first statement we made 8 hats the antecedent, and it is less than the consequent. Therefore, the antecedent in the second couplet must be less than the consequent. It is evident that 8 hats cost less than 12 hats; hence \$16 must be made the antecedent in the second couplet. Solving by Principle 2, we find the cost of 12 hats to be \$24.

SECOND STATEMENT.

$$\begin{array}{cccc} \text{hats} & \text{hats} & \$ & \$ \\ 12 & : 8 = & (\quad) : & 16 \end{array}$$

$$\frac{12 \times 6}{8} = 24$$

In the second statement we make 12 hats the first antecedent, which is greater than the consequent. Hence, the required cost, which is greater than \$16, must be made the second antecedent. Solving by Principle 3, we get the same answer as before.

☞ The pupil will note that 8 hats bears the same relation to 12 hats as the cost of 8 hats bears to the cost of twelve hats.

NOTE.—In solving problems of this kind in proportion, we apparently multiply *hats* by *dollars*. This is due, however, to the *form of work*. Since the ratio of 8 hats to 12 hats is equal to the ratio of 8 to 12, we may write $8 : 12 = \$16 : (\$\quad)$. In practice this substitution is not necessary.

**2.** If 5 men can build a house in 40 days, in how many days can 8 men build it?

$$\begin{array}{lcccc} & \text{men} & \text{men} & \text{days} & \text{days} \\ \text{(a)} & 5 & : 8 = & (\quad) : & 40 \\ \text{(b)} & 8 & : 5 = & 40 : & (\quad) \end{array}$$

It is evident that 8 men can do the work in less time than 5 men; hence the required number is less than 40, and the proportions are easily expressed as in the statements.

NOTE.—In example (a) we have a *direct proportion*. Thus, 8 hats : 12 hats = cost of 8 hats : cost of 12 hats. In example (b) we have an *inverse proportion*. Thus, 5 men : 8 men = time of 8 men : time of 5 men.

QUERY.—How does an inverse proportion differ from a direct proportion?

RULE.—*For the first couplet compare the two like numbers. For the second couplet compare the remaining number with the required number. Determine from the conditions of the problem which is the greater.*

*Arrange these two numbers as a couplet, making the greater or less the antecedent according to the arrangement of the first couplet.*

*Divide the product of the means or extremes by the single mean or extreme.*

*The quotient will be the required number.*

The work may frequently be shortened by cancellation. This rule is often called "The Rule of Three," because three numbers are given to find a fourth.

**3.** If 12 pounds of tea cost \$5, how much must be paid for 24 pounds?

**4.** At 8 cents a yard how much will 10 yards of calico cost?

**5.** A tree 60 feet high casts a shadow 80 feet long. How long a shadow is cast by a tree 48 feet high?

**6.** If 18 horses eat 12 bushels of oats in a day, how many horses would eat 20 bushels in the same time?

**7.** If a train runs 90 miles in 3 hours, how far will it run in 24 hours?

**8.** By working 9 hours a day, B can dig a ditch in 16 days. In how many days can he dig it by working 10 hours a day?

**9.** If a family uses 6 barrels of flour in 10 months, how many barrels would last a year?

**10.** A man earns \$1000 in 6 months. In how many months can he earn \$2500?

**11.** If $\frac{3}{8}$ of an acre cost \$30, how much must be paid for 6 acres?

**12.** A farmer raised 36 bushels of wheat on $1\frac{1}{2}$ acres. At this rate, how many bushels can he raise on 10 acres?

**13.** In how many days can 12 men build a wall that 5 men can build in $18\frac{1}{2}$ days?

**14.** Mr. C paid $140 for 7 months' rent. How much did he pay in a year?

**15.** How long would it take 2 men to mow a field that 7 men can mow in $3\frac{1}{7}$ days?

**16.** Find the cost of 75 sheep at $6.40 each.

**17.** A paid $15.75 for $3\frac{1}{2}$ weeks' board. At the same rate, how much did he pay in a year?

**18.** If $\frac{3}{4}$ of a yard of cloth costs $\$\frac{7}{8}$, how much must be paid for 16 yards?

**19.** If 10 tons of hay last 5 horses 8 months, how long would it last 12 horses?

**20.** If 20 bushels of wheat produce $6\frac{1}{4}$ barrels of flour, how many bushels will produce 100 barrels?

**21.** If 24 yards of carpet cover $\frac{3}{4}$ of a floor, how many yards will cover the entire floor?

**22.** If 9 compositors can set up a 6-page paper in 8 hours, in how many hours can they set up a 20-page paper?

**23.** If $a$ cows cost $b$ dollars, how much will $3a$ cows cost?

**24.** When $b$ hats cost $c$ dollars, how much must be paid for $d$ hats?

**25.** Complete the equation, $a : b = 7a : (\quad)$.

SUPPLEMENTARY EXERCISES (FOR ADVANCED CLASSES).

**249. 1.** The Washington monument casts a shadow 223 feet $6\frac{1}{2}$ inches long when a post 3 feet high casts a shadow 14.5 inches. Find the height of the monument.

**2.** Milk is worth 20¢ a gallon, but by watering it the value is reduced to 15¢ a gallon. Find the ratio of water to milk in the mixture.

**3.** If 4 is one third of a certain number, what is one half of it?

**4.** Sixty men can grade a street in 40 days. After 24 days, one third of the men are discharged. In how many days can the others finish the work?

**5.** A piece of work can be done in 5 weeks by 12 men. At the end of 2 weeks it is decided to complete the work in 6 days. How many more men must be employed?

**6.** If a 4-cent loaf weighs 9 ounces when flour is $6 a barrel, how much should a 5-cent loaf weigh when flour is $8 a barrel?

**7.** If 300 cats kill 300 rats in 300 minutes, how many cats can kill 100 rats in 300 minutes?

**8.** Two men or three boys can plow an acre in $\frac{1}{6}$ of a day. How long will it take 3 men and 2 boys to plow it?

**9.** If 4 horses or 6 cows can be kept 10 days on a ton of hay, how long will it last 2 horses and 12 cows?

**10.** If 8 men or 15 boys plow a field in 15 days of $9\frac{1}{3}$ hours, how many boys must assist 16 men to do the work in 5 days of 10 hours each?

**11.** Divide $1000 between A and B so that A shall have $3 out of every $5.

There are $1000 \div 5 = 200$ fives.
$\therefore$ A gets $200 \times \$3 = \$600$.

☞ A gets $3 for every $2 B gets. The money is divided in the ratio of 3 to 2.

**12.** Tom and Peter found a watch worth $45, and agreed to divide the value of it in the ratio of $\frac{2}{3}$ to $\frac{5}{6}$. How much was each one's share?

**13.** Gunpowder is composed of nitre, charcoal, and sulphur in the proportion of 38, 7, and 5. How many pounds of sulphur in 180 pounds of powder?

**14.** If 20 men can perform a piece of work in 12 days, how many men can do a piece of work 3 times as large in $\frac{1}{3}$ of the time?

**15.** A man rides a certain distance at the rate of 6 miles an hour, and walks back at the rate of $3\frac{1}{2}$ miles an hour. If the time of the round trip is $4\frac{3}{4}$ hours, what is the distance?

**16.** If $3a$ sheep cost $24, how much will $5b$ sheep cost, if $a : b :: 2 : 1$?

# THE EQUATION.

**250.** An **Equation** is a statement that two numbers or expressions are equal.

Thus, $1 + 1 = 2$, $7 + 3 = 2 \times 5$, and $3x - 2 = 11$ are equations.

**251.** An equation is like a balanced scale-beam—one side representing the thing weighed, and the other side the weights.

**252.** The part of an equation that is written *before* the sign = is called the *first member,* and that written *after* the sign the *second member.*

Thus, in the equation $7 + 3 = 2 \times 5$, $7 + 3$ is the first member, and $2 \times 5$ is the second member.

1. The two members may differ widely in *form,* but in *value* they must be the same.

2. In the expression $4 \div 3 - 2 = (\quad)$, the second member is to be supplied.

**253.** The numbers that compose the members are called the **Terms.** The terms of each member are separated from each other by the signs + or −.

Thus, in the equation $3 + 6 \times 7 - 8 \div 4 = 43$, the terms of the first member are 3, $6 \times 7$, and $8 \div 4$. In the expression $5 + 2 \times 4 - (\quad) = 10$, one term is to be supplied.

INDUCTIVE EXERCISES.

**254. 1.** When will a scale-beam balance?

**2.** If two one-pound packages of coffee are placed in one scale, what must be done to restore the balance?

**3.** If one of the packages is removed, what must be done to restore the balance?

**4.** Would the balance be destroyed if the coffee and weight should change scales? Why not?

**5.** Could anything else of equal weight be substituted for the coffee without destroying the balance?

**6.** If 5 pounds are added to one side, what must be done to the other side to keep the balance true?

**7.** If one pound is taken from one scale, how can the balance be restored?

**8.** What does the expression $7 + 5 + 3 = 15$ mean?

**9.** If we add one to the first member, how much must be added to the second to make the sides equal?

**10.** If we take 5 from the first member, what must be done to the second to preserve the equality?

**11.** Is there any difference in value between $7 + 3$ and $15 - 5$? Then $7 + 3 = 15 - 5$.

**12.** How much is 6 times $(7 + 3)$? How much is 6 times $(15 - 5)$? Are the products equal?

**13.** Then what may be done to both sides of an equation without destroying the equality?

**14.** What is the quotient of $(7 + 3) \div 2$? Of $(15 - 5) \div 2$? Are the quotients equal?

**15.** Then what else may be done to both sides without destroying the equality?

**16.** A 10-acre field is worth \$100 an acre, and a 20-acre field is worth \$50 an acre. In what respects are the two fields equal?

**255.** Since one member of an equation is equal in value to the other, whatever is done to one side must be done to the other in order to preserve the equality.

**256.** PRINCIPLES.—1. *Expressions which are equal to the same thing or to equal things are equal to each other.*

2. *If equals are added to or subtracted from equals, the results are equal.*

3. *If equals are multiplied or divided by the same number, the results are equal.*

4. *In general, if the same operations are performed upon both members of an equation, the results are equal.*

5. *If two expressions are equal, either can be substituted for the other wherever it occurs.*

☞ These principles are self-evident, and are called *axioms.*

### CHANGE OF FORM.

**257.** It is frequently necessary to change the form of an equation in order to simplify it. The principal change is *transposing terms.*

**258. Transposing** is the process of changing a term from one member of an equation to the other without destroying the equality.

**1.** In the equation $16 − $5 = $8 + $3, transpose $5 to the second member.

Adding $5 to each member (Prin. 2) we have

$16 − $5 + $5 = $8 + $3 + $5.

But since − $5 + $5 = 0, we may write the equation

$16 = $8 + $3 + $5.

How does this equation compare with the one given ?

It will be noticed that (− $5) has disappeared from the first member, while (+ $5) appears in the second.

**2.** In the equation $2 + 5 = 4 + 3$, transpose 3 to the first member.

Subtracting 3 from each member (Prin. 2), we have

$$2 + 5 - 3 = 4 + 3 - 3.$$

But since $+ 3 - 3 = 0$, we may write the equation

$$2 + 5 - 3 = 4.$$

How does this equation compare with the one given ?

It will be noticed that 3 has disappeared from the second member, and appears in the first, with a different sign before it.

**259.** Any term may be transposed from one member of an equation to the other by dropping it from the member in which it stands, and writing it in the other with a different sign.

Transpose so that only odd numbers will be in the first member:

1. $7 - 4 = 8 - 5$.
2. $2 + 9 = 1 + 10$.
3. $19 + 6 = 30 - 5$.
4. $5 \times 3 - 2 \times 4 = 1 + 6$.
5. $39 \div 3 - 4 \div 2 = 22 \div 2$.
6. $7 + 4 - 8 = 26 - 25 + 2$.

Transpose so that only like terms will be in each member:

7. 3 times a number $-$ 5 $=$ 2 times the number.
8. 5 times A's money $-$ \$10 $=$ 3 times A's money $+$ \$20.
9. $\frac{1}{4}$ of my money $=$ \$9 $-$ $\frac{1}{5}$ of my money.
10. $\frac{3}{5}$ of A's age $+$ 4 years $=$ $\frac{1}{4}$ of A's age $+$ 11 years.
11. \$2 $+$ 10 cents $=$ \$1 $+$ \$$\frac{1}{2}$ $+$ 60 cents.
12. 3 rods $+$ 2 yards $=$ 2 rods $+$ $7\frac{1}{2}$ yards.

Transpose all terms containing $x$ to the first member, and all others to the second member:

13. $7x - 5 = 3x + 15$.
14. $5 + 6x - 9 = 2x + 8$.
15. $5x + 6 - 8 = 4 + 3x$.
16. $9x - 36 = 24 - 6x$.
17. $8x - 2y = 14y - 16x$.
18. $3y + 10x = 10 + 11x$.

**260.** After like terms have been collected into one member, the equation may often be made still more simple by performing the operations indicated.

Thus, the equation,

5 times A's money $-$ 3 times A's money $=$ \$240 $+$ \$60,

may be written

2 times A's money $=$ \$300.

Perform indicated operations:

1. 4 times a number $-$ 3 times the number $=$ $10 + 5$.
2. $2 \times$ B's money $+$ $3 \times$ B's money $=$ \$1000 $-$ \$500.

**3.** $7 \times 5 - 64 \div 16 - 11 = 5 \times 4$.

**4.** $6 + 5 \times 6 + 18 - 2 = 2 + 2 \times 25$.

☞ This may be called *uniting like terms.*

NOTES.—1. In uniting terms we subtract the *sum* of the *minus terms* from the *sum* of the *plus terms.*

2. The sign × may be omitted between factors if one (or more) of the factors is a letter.

Supply the wanting term:

**5.** $32 - 5 \times 3 + 45 \div 9 = (\quad)$.

**6.** $46 - 18 + 16 + (\quad) = 30 + 7 \times 10$.

**7.** $75 + 13 \times (\quad) - 51 = 64 - 2 \times 7$.

**8.** $\frac{3}{4} + \frac{1}{2} \times \frac{2}{3} - (\quad) = 2 - 2 \times \frac{1}{2}$.

**261.** Expressing the conditions of a problem in the form of an equation is called *stating the problem.* The several steps of the *analysis* may be expressed in a series of equations, each derived from the one preceding, by a *change of form* under Principles 1—5.

**1.** If 3 melons are worth 60 cents, how much are 5 melons worth?

1.) The cost of 3 melons = 60 cents.
2.) ∴ the cost of 1 melon = 20 cents.
3.) ∴ the cost of 5 melons = 100 cents.

QUERIES.—How is equation 2 derived from 1? How is 3 derived from 2?

**2.** 24 is $\frac{2}{3}$ of what number?

1.) $\frac{2}{3}$ of some number = 24.
2.) ∴ $\frac{1}{3}$ of the number = 12.
3.) ∴ $\frac{3}{3}$ of the number = 36.

QUERY.—How are equations 2 and 3 derived?

**3.** Find $\frac{3}{8}$ of 40?

1.) $\frac{8}{8}$ of forty = 40.
2.) ∴ $\frac{1}{8}$ of forty = 5.
3.) ∴ $\frac{3}{8}$ of forty = 15.

QUERY.—How is equation 1 derived?

**4.** 35 is how many eighths of 40 ?

$$40 = 8 \text{ eighths of } 40.$$
$$\therefore 5 = 1 \text{ eighth of } 40.$$
$$\therefore 35 = 7 \text{ eighths of } 40.$$

**262.** The following solutions show a few of the many advantages of using the equation in arithmetic. Its utility in solving many of the problems of percentage, etc., appears in later pages of this book.

**1.** Eight times a number diminished by 46 equals 14 more than 3 times the number. Find the number.

FIRST SOLUTION.

8 times the number − 46 = 3 times the number + 14.

Transposing 46,

$$8 \times \text{the number} - 3 \times \text{the number} = 14 + 46.$$

Uniting terms,

$$5 \times \text{the number} = 60.$$
$$\therefore \text{the number} = 12.$$

SECOND SOLUTION.

Let $x =$ the number.

$$\therefore 8x - 46 = 3x + 14.$$

Transposing,

$$8x - 3x = 14 + 46.$$

Uniting terms,

$$5x = 60.$$
$$\therefore x = 12.$$

☞ In the solution of any problem, the *number to be found* is the one that must be represented by a letter. This letter may be treated just as the number itself would be if known.

**2.** A coat and a hat cost \$36. The coat cost 5 times as much as the hat. What was the cost of the hat ?

Let $x =$ cost of hat.

$5x =$ cost of coat.

$\therefore x + 5x =$ cost of both.

$\therefore 6x = \$36.$

$\therefore x = \$6$, cost of hat,

and $5x = \$30$, cost of coat.

**3.** A farmer has 100 hens and chicks. Every hen has 9 chicks. How many of each has he?

SUGGESTION.—Let $x =$ the number of hens.

**4.** If three times A's age plus 12 years equals five times his age less 8 years, what is A's age?

$$\begin{aligned} \text{Let } x &= \text{A's age.} \\ 3x + 12 \text{ yr.} &= 5x - 8 \text{ yr.} \\ 3x - 5x &= -8 \text{ yr.} - 12 \text{ yr.} \\ -2x &= -20 \text{ yr.,} \\ \text{or } 2x &= 20 \text{ yr.} \\ x &= 10 \text{ yr.} \end{aligned}$$

☞ The signs of all the terms can be changed without destroying the equality; for, by transposition, the members can be interchanged and therefore their signs changed.

QUERIES.—1. In solving problems, what is the first thing to do? (State the problem.)

2. What is the second step? The third?

3. How do you explain the work after terms have been united? Let the pupil write a *rule.*

☞ In solving the following problems great care must be exercised in making the *statement*. Solve all by using $x$.

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**5.** A and B have \$80, and for each dollar B has, A has \$3. How much has each?

**6.** \$40 is \$4 more than $\frac{4}{9}$ of my money. How much have I?

**7.** 18 is $\frac{3}{4}$ of what number?

**8.** What number added to $\frac{1}{2}$ of itself equals $2 \times 9$?

**9.** $\frac{5}{6}$ of a number is 5 more than $\frac{2}{3}$ of the number. What is the number?

**10.** A lady bought a dress for \$24, and found that she had $\frac{2}{5}$ of her money left. How much money had she at first?

**11.** Mr. E has gold dollars and silver dollars to the amount of \$30. He has one half as many silver dollars as gold dollars. How many of each has he?

**12.** A got $\frac{1}{4}$ of his father's fortune, B got $\frac{1}{5}$ of it, and C got the remainder. If A got \$2000 more than B, how much did C get?

**13.** If 18 is added to 6 times a certain number, the sum will be 22 less than $\frac{2}{3}$ of 150. What is the number?

**14.** A pole 36 feet long was broken into two unequal pieces, $\frac{3}{5}$ of the longer piece being equal to $\frac{3}{4}$ of the shorter. How long was each piece?

**15.** Three boys, A, B, and C, have 77 marbles; B has 10 more than A, and A has 8 more than C. How many has each boy?

**16.** Find a number such that if it be added to $\frac{1}{3}$ of itself the sum will be 60.

**17.** By selling a watch for \$36, I gained $\frac{4}{5}$ of its cost. What was its cost?

**18.** $\frac{3}{4}$ of A's money diminished by \$20 is equal to $\frac{1}{3}$ of his money increased by \$5. Find A's money.

**19.** A and B together sold 728 bushels of wheat, and B sold 3 times as much as A. How much did each sell?

**20.** Divide \$1000 between C and B, so that C may have $\frac{2}{3}$ as much as B.

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**21.** What number increased by one half of itself, by one third of itself, and by 18 more, will be doubled?

**22.** If with the money I now have, I had 3 times as much, and \$25 more, I should have \$125. How much money have I?

**23.** Find a number such that the sum of its half and its third may exceed the sum of its fourth and its fifth by 23.

**24.** A man left $\frac{1}{2}$ of his estate to his wife, $\frac{1}{10}$ for charity, $\frac{3}{8}$ to his children, and \$1400 to his servants. What was the amount of his estate?

**25.** A man gave \$100 to his 3 sons; to the second he gave twice as much as to the first, lacking \$8, and to the third he gave 3 times as much as to the first, lacking \$15. How much did he give to the first?

SUPPLEMENTARY EXERCISES (FOR ADVANCED CLASSES).

**263. 1.** A drover, being asked how many sheep he had, said, "If to $\frac{1}{3}$ of the number of my flock you add the number $9\frac{1}{2}$, the sum will be $99\frac{1}{2}$." How many sheep had he?

**2.** A's money added to $\frac{1}{2}$ of B's money equals $2000. How much money has each, provided A's is to B's as 3 to 4?

**3.** A boy cut off one half of the length of his kite-string. He then added $45\frac{3}{4}$ ft., and found that the new string was $\frac{4}{5}$ of the original length. What was this original length?

**4.** A lady being asked the time of day replied that $\frac{2}{3}$ of the time past noon equaled $\frac{4}{5}$ of the time to midnight, minus $\frac{4}{5}$ of an hour. What was the time?

**5.** After losing $\frac{3}{4}$ of my money I earned $12, and then spent $\frac{2}{3}$ of what I had. What I then had left was $36 less than I had at first. How much had I at first?

**6.** A thief stole $\frac{5}{9}$ of A's money and spent $\frac{2}{5}$ of the amount stolen before he was caught. The remainder, $324.75, was secured. How much money had A at first?

**7.** A, B, and C together have $434. $\frac{2}{3}$ of A's money equals $\frac{3}{4}$ of B's, and $\frac{2}{3}$ of B's equals $\frac{3}{4}$ of C's. How much has each?

**8.** If $\frac{2}{3}$ of A's money is to $\frac{4}{5}$ of B's as 3 to 4, and together they have $1520, how much has A?

**9.** A, B, and C together have $21950. $\frac{3}{4}$ of A's money, $\frac{5}{6}$ of B's, and $\frac{8}{9}$ of C's are all equal to each other. How much has each?

# REVIEW WORK.

## ORAL EXERCISES.

**264.** **1.** A wagon cost \$50, which was $\frac{5}{11}$ of the cost of a horse. What was the cost of the horse?

**2.** If three barrels of flour cost \$14.40, what will $2\frac{1}{2}$ barrels cost?

**3.** Mr. A bought $\frac{3}{4}$ of a mill, and sold $\frac{2}{3}$ of it to B. What part of the mill did he retain?

**4.** Twenty-eight is 7 tenths of how many times 4?

**5.** A had 60 acres of land; he sold $\frac{3}{4}$ of it to B, and $\frac{2}{5}$ of the remainder to C. How many acres had he left?

**6.** Eight ninths of 27 are how many times 6?

**7.** A man gave 7 beggars \$1.60 apiece, and had \$8.90 left. How much had he at first?

**8.** If $\frac{3}{8}$ of a lemon cost $\frac{3}{4}$ of a cent, how much will 3 lemons cost?

**9.** A boy sold a watch for \$16, which was \$4 less than $\frac{2}{3}$ of its cost. What did it cost?

**10.** A horse cost \$180, which was 3 times the cost of a yoke of oxen. What was the cost of an ox?

**11.** A has $\frac{2}{3}$ as much money as B, and both have \$50. How much has each?

**12.** John and Jane each has $\frac{1}{5}$ as much money as Kate, and all have \$35. How much has each?

**13.** Two thirds of A's age is $\frac{4}{5}$ of B's age, and the sum of their ages is 77 years. What is the age of each?

**14.** Ada gave $\frac{3}{7}$ of her flowers to Eli, and had 12 remaining. How many did she give to Eli?

**15.** Mr. D gave \$45 for shovels at \$1.25 each. How many dozen did he get?

**16.** A lady bought 2 bushels and 3 pecks of pears at \$1.50 a bushel. What did they cost her?

**17.** Mrs. A bought a dress for \$6.75. If she paid \$$\frac{3}{4}$ a yard, how many yards did she buy?

**18.** A farmer has a field containing 16 acres. If he can mow $\frac{1}{5}$ of it in two days, how many acres can he mow in a day?

**19.** A can dig a ditch in 2 days, and B can dig it in 3 days. If they work together, how long will it take them?

How much of it can A dig in a day? How much can B dig in a day? Then how much can both dig? How many sixths are to be dug? Then how many days will it take them?

**20.** A can build a wall in 2 days, B in 3 days, and C in 4 days. How long would it take all three working together?

**21.** A and B can cut a field of wheat in 6 days, and B alone can cut it in 10 days. In what time can A alone cut it?

**22.** A farmer bought 9 sheep for \$45, and sold them for \$14 more than $\frac{4}{5}$ of what they cost. Did he gain or lose, and how much?

**23.** A boy bought apples at the rate of 6 for 5 cents, and sold them at the rate of 5 for 6 cents. How much did he gain on each apple? How much on 10 apples?

**24.** Tom is $\frac{3}{7}$ as old as his mother, who was married 24 years ago at the age of 25. How old is Tom?

**25.** When hay is \$13.50 a ton, and coal is \$8 a ton, what part of a ton of coal can be bought for $\frac{1}{3}$ of a ton of hay?

**26.** Jane bought a dozen oranges, of which she ate two, and sold the remainder at 2 cents apiece, thereby gaining $1\frac{1}{4}$ cents on each orange bought. How much did they cost each?

**27.** After spending $\frac{1}{3}$ of his money for a cake, and $\frac{1}{5}$ of it for a ball and bat, Henry had \$1.40 left. How much had he at first?

**28.** A can build a boat in 4 days, B in 5 days, and C in 10 days. If all work together 1 day, how long will it take C alone to finish?

**29.** A and B hired a rig for \$11. A used it one day, and B used it two days. How much should each pay?

**30.** A can do a piece of work in half a day, and B can do it in $\frac{2}{3}$ of a day. How long will it take them if they work together?

**31.** Jack can eat a loaf in $1\frac{1}{2}$ days, and Jill can eat it in $2\frac{1}{2}$ days. How long will it last both?

**32.** What number is that which, being increased by its $\frac{1}{2}$, its $\frac{1}{3}$, and its $\frac{1}{6}$, will be doubled?

**33.** What number will be doubled by adding to it its $\frac{1}{3}$, its $\frac{1}{4}$, and 5 more?

**34.** Mr. Kay is 45 years of age, and $\frac{4}{9}$ of his age is $\frac{5}{11}$ of his wife's age. How old was his wife when she was married 20 years ago?

**35.** A and B earned \$110. If A earned \$10 more than B, how much did each earn?

**36.** A man spends $\frac{1}{3}$ of his money, and then loses $\frac{1}{5}$ of the remainder; he then has \$400. How much had he at first?

**37.** In a school $\frac{2}{3}$ of the pupils study arithmetic; $\frac{1}{4}$ of the remainder, algebra; and the rest, or 12, geometry. How many pupils are there?

**38.** A can do a piece of work in 3 days; B can do the same work in 4 days; if A earns \$2 a day, what does B earn a day?

**39.** $\frac{3}{4}$ of my money is 4 times my week's wages; I have \$100. What are my weekly wages?

**40.** In traveling 72 miles a man went $\frac{2}{3}$ of the distance the first day, $\frac{1}{8}$ of the distance the second day, and the remainder the third day. How far did he travel the third day?

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**41.** Owen is $\frac{3}{9}$ as old as his father, and $\frac{3}{8}$ as old as his mother. If he is 18 years old, how old are his father and mother?

**42.** $\frac{5}{6}$ of C's money is $\frac{3}{10}$ of D's, and $\frac{1}{5}$ of D's is \$20. How much money has C ?

**43.** If Mr. Fox were twice as old as he is, $\frac{1}{4}$ of his age would be 20 years. What is his age ?

**44.** It took $\frac{5}{7}$ of my money to pay a debt ; I then paid \$12 for a coat, which was $\frac{3}{4}$ of the money I had left. How much had I at first ?

**45.** $\frac{2}{3}$ of the sum of two equal numbers is 20. What are the numbers ?

**46.** What number must be added to the difference between $\frac{1}{2}$ and $\frac{1}{3}$ to make $\frac{1}{5}$ ?

**47.** The numerator of a fraction, whose value is $\frac{4}{5}$, is 20. What is the denominator ?

**48.** A has \$3.50, B has \$1.25 more than A, and C has as much as A and B. How much money have they together ?

**49.** If a locomotive moves $\frac{5}{8}$ of a mile in $\frac{1}{12}$ of an hour, what is its speed per hour ?

**50.** $\frac{5}{16}$ of a farm is in crops, and $\frac{5}{12}$ of the remainder is woodland. What part of the farm is woodland ?

**51.** A man plows $\frac{3}{8}$ of a field the first day, and $\frac{1}{4}$ of it the second day. What part of it does he plow the third day, if he finishes on that day ?

**52.** A miller keeps as toll $\frac{1}{7}$ of the corn to be ground. What is the ratio of the toll to the meal returned ?

**53.** How long will $\frac{1}{2}$ of a bushel of oats last a horse, if $2\frac{1}{2}$ bushels last him one week ?

**54.** By selling a horse for \$96 I gain $\frac{1}{5}$ of the cost. What did the horse cost ?

**55.** James had 36 cents. He lost $\frac{2}{3}$ of it and spent $\frac{1}{4}$ of it. How much has he left ?

**56.** If $\frac{3}{4}$ of a cord of wood costs \$3, how much will $\frac{1}{2}$ of a cord cost ?

**57.** A man bought 5 bushels of wheat for \$4, which was $\frac{4}{5}$ of the cost. What was the cost a bushel ?

**58.** A can do twice as much work as B. How many times B's work can both do?

**59.** Daisy traveled $\frac{3}{8}$ of her journey by rail, $\frac{2}{5}$ by water, and the remainder, which was 18 miles, by stage. How many miles did she travel?

**60.** Preston worked 16 days, and after paying for a suit of clothes with $\frac{2}{5}$ of his money had \$24 left. How much did he receive a day?

**61.** A watch and chain cost \$90; the chain cost $\frac{1}{3}$ as much as the watch. What was the cost of each?

**62.** A, B, and C can paper a room in 6 hours, B and C can paper it in 10 hours. In what time can A alone paper it?

**63.** If $\frac{3}{4}$ of a farm is worth \$150 more than $\frac{3}{5}$ of it, what is the whole farm worth?

**64.** James being asked how many marbles he had, said he had $\frac{2}{3}$ as many as Phil, and that both together had 155. How many had he?

WRITTEN EXERCISES.

**265. 1.** A man bought 20 acres of land at \$50.25 an acre. He sold $\frac{1}{3}$ of an acre to B, $8\frac{2}{5}$ acres to C, and the remainder to D. If he received \$65 an acre from B and C, and \$60 an acre from D, how much did he gain?

**2.** How far can a man walk in $3\frac{2}{3}$ hours, at the rate of $3\frac{3}{4}$ miles an hour?

**3.** If $\frac{3}{7}$ of a pole is in the ground and 24 feet are above the ground, how long is the pole?

**4.** If there are 69.16 miles in one degree of latitude, how many miles are there in $34\frac{3}{4}$ degrees?

**5.** If $2\frac{3}{4}$ yards of cloth are required for a pair of pants, $\frac{7}{8}$ of a yard for a vest, and 4 yards for a coat, how many yards will be left from a piece of 41 yards, after 5 suits have been cut off?

**6.** At the rate of 3 for 10 cents, what will 75 dozen oranges cost?

**7.** A man paid \$5 for sugar and \$5 for coffee. If sugar was $6\frac{1}{4}$ cents a pound and coffee was 25 cents a pound, how many pounds of both did he get?

**8.** A man can build a fence in 16 days by working $9\frac{1}{2}$ hours a day. How much longer would it take him working only 8 hours a day?

**9.** Find the cost of 6 spoons when 5 dozen cost \$32.

**10.** The quotient is 16.73, and the divisor is $8\frac{5}{7}$. How much must be added to the dividend to make 150?

**11.** In a square rod there are $272\frac{1}{4}$ square feet. How many square rods are there in 9000 square feet?

**12.** A jeweler cut a wire $\frac{11}{16}$ of an inch long into 11 equal parts. How many of the parts were equal to half an inch?

**13.** Walter spent $\frac{1}{2}$ of his money and \$$\frac{1}{2}$ more, then $\frac{1}{2}$ of the remainder and \$$\frac{1}{2}$ more, then $\frac{1}{2}$ of the remainder and \$$\frac{1}{2}$ more, and then had \$$1\frac{5}{8}$ left. How much had he at first?

**14.** A man owning $\frac{3}{4}$ of a store sold $\frac{1}{4}$ of his share for \$2250. What was the value of the store?

**15.** What number is that whose $\frac{5}{7}$ exceeds its $\frac{5}{9}$ by 10?

**16.** I bought a horse and a buggy, paying $\frac{8}{9}$ as much for the buggy as for the horse. If both cost \$340, what did the buggy cost?

**17.** Bought potatoes at \$$2.62\frac{1}{2}$ and sold them for \$$3\frac{1}{2}$ a barrel. If my gain was \$87.50, how many barrels did I handle?

**18.** Seven men dug a cellar in $12\frac{3}{8}$ days. How long would it have taken 3 men?

**19.** If 8 be added to numerator and denominator of $\frac{6}{7}$, will the value of the fraction be increased or diminished, and how much?

**20.** If 5 be subtracted from numerator and denominator of $\frac{6}{7}$, how will the value of the fraction be affected? Why?

**21.** Mr. S has 60 hens. He sold $\frac{2}{5}$ of them to A, and $\frac{2}{3}$ of the remainder to B. C bought what remained at \$.75 a pair. How much did C pay?

**22.** Mr. Willis sold 3 loads of hay weighing, with the wagons, $1\frac{3}{4}$, $1\frac{7}{8}$, and $2\frac{1}{2}$ tons respectively. The empty wagons weighed $\frac{2}{5}$, $\frac{3}{8}$, and .9 of a ton. What was the value of the hay at \$16.50 a ton?

**23.** A man bought 20 sheep at \$4.75 each, and 3 horses. If he paid \$470 for the sheep and horses, what was the average price of the horses?

**24.** What number multiplied by $\frac{5}{7}$ of $13\frac{3}{5}$ will produce 1?

**25.** A can walk 60 miles in $12\frac{1}{2}$ hours, and B can walk it in 15 hours. If they are 60 miles apart, and start at the same time to walk toward each other, how far apart will they be in an hour and a half?

**26.** Divide $(18\frac{3}{4} \times 18\frac{3}{4})$ by $(\frac{3}{8} \div .01)$, and add $(\frac{3}{4}$ of $.001)$ to the quotient.

**27.** What is the difference between $3\frac{1}{2} + 12\frac{1}{4} \times 2\frac{1}{3}$ and $(3\frac{1}{2} + 12\frac{1}{4}) \times 2\frac{1}{3}$?

**28.** A can fence one side of a square in 8 days, and B can fence 2 sides of it in $12\frac{1}{2}$ days. In what time can both together fence the field?

**29.** A man left $\frac{2}{5}$ of his estate to his first son, $\frac{2}{7}$ to his second son, and the remainder to his daughter, whose share was \$600 less than that of the first son. Find each one's share.

**30.** The width of a stream was measured at several points, the measurements being as follows: $42\frac{1}{3}$ feet, $37\frac{1}{2}$ feet, 35 feet, $41\frac{5}{8}$ feet, $52\frac{1}{6}$ feet, and 48.875 feet. What is the average width?

**31.** What number increased by $\frac{2}{9}$ of itself is equal to 1?

**32.** Two fifths of my money is gold, $\frac{1}{6}$ of it is silver, and the remainder is paper. I have \$8 more paper money than silver. How much gold have I?

**33.** A watch and chain cost \$125. If the chain cost $\frac{1}{4}$ as much as the watch, what was the cost of each?

**34.** Twelve years ago, A was $\frac{1}{2}$ as old as B, but now he is $\frac{3}{4}$ as old. How old is A?

**35.** A mother and 3 children use a pound of coffee in a week. When the mother is absent, two pounds last the children 6 weeks. How long would a pound last the mother alone?

**36.** If 3 be added to $\frac{3}{4}$ of a certain number, and $\frac{3}{4}$ of the sum be multiplied by 3 tenths of 3, the product will be 3 times 10.8. What is the number?

**37.** A man gives .1 of his income to charity, .24 for educating his children, .375 for other expenses, and lays by the remainder, which is \$570. What is his income?

**38.** A man sold a horse for $1\frac{3}{16}$ times the cost, gaining \$15. Find the cost.

**39.** $\frac{1}{5}$ of $\frac{3}{8}$ of what number equals $\frac{7}{18}$ of 360?

**40.** How much will 49 men earn in $17\frac{1}{2}$ days @ \$2.10 a day?

**41.** If $\frac{5}{18}$ of a ream of paper costs \$.60, how much can be purchased for \$237.60?

**42.** For what length of time will \$495 pay rent at the rate of \$24.75 a month?

**43.** The greater of two fractions is $\frac{1}{2}\frac{2}{5}$; their difference is $\frac{3}{35}$. What is the other fraction?

**44.** How many oranges @ \$.25 per dozen will pay for 36 bu. of coal worth $8\frac{1}{3}$ cents a bushel?

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**45.** A lady sold $\frac{1}{3}$, $\frac{2}{9}$, and $\frac{2}{7}$ of her fowls; she had 30 remaining. How many had she at first?

**46.** A farmer sold 65 bu. more than $\frac{1}{5}$ of his wheat, and found that the remainder was 65 bu. more than $\frac{2}{3}$ of his wheat. How many bushels had he at first?

**47.** A drover spent \$50.60, which was $\frac{11}{25}$ of what he got for 23 sheep. What did he get apiece for them?

**48.** Bought eggs at the rate of $1\frac{2}{3}$¢ each, and sold them at 27¢ a dozen. I gained \$3.65; how many dozens did I sell?

**49.** A merchant had $163\frac{3}{4}$ yd. of gingham, from which he sold $95\frac{1}{8}$ yd. The remainder was made into 25 aprons of the same size. How many yards did each apron contain?

**50.** A woman bought a shawl for \$9.75, which was $\frac{7}{9}$ of $\frac{6}{7}$ of the price asked. At how much less than the price asked did she buy the shawl?

**51.** John earns $\$24.66\frac{2}{3}$ per week; James earns $\frac{5}{8}$ as much, and Tom $\frac{3}{5}$ as much as John and James together. If Tom gives $\frac{1}{12}$ of his money to charity, how much has he left each week?

**52.** $\frac{2}{13}$ of the troops engaged in a battle were killed; $\frac{2}{7}$ of $\frac{5}{8}$ of the killed numbered 45. What was the original number of the troops?

**53.** If one horse consumes $3\frac{3}{8}$ bu. of oats per week, how many bushels will 18 horses consume in 6 weeks?

**54.** A can do a piece of work in $7\frac{1}{2}$ days, A and B can both do the same work in 5 days. In what time can B do the work?

**55.** A lady bought a watch for \$81.75, paying $\frac{1}{3}$ down. How much must she pay per month to pay the remainder in 8 months?

**56.** Mr. Drift sold $\frac{2}{7}$ of his property, $\frac{1}{6}$ of the remainder, and $\frac{3}{11}$ of that remainder. His property was then worth \$2760. What was the value of the whole property?

**57.** A and B raised 320 bushels of wheat. After paying $\frac{1}{4}$ of it for rent, they divided the remainder so that A received $\frac{3}{5}$ as much as B. How many bushels did each receive?

**58.** William bought 330 lemons at 30 cents a dozen, and $6\frac{1}{2}$ dozen oranges at 2 cents apiece; he sold $\frac{2}{3}$ of the lemons at 4 for 25 cents, the remainder at 3 cents apiece; he sold $\frac{1}{3}$ of the oranges at cost, the remainder at 30 cents a dozen. How much did he gain?

**59.** A jeweler sold a watch for \$62.50, which was $\frac{1}{4}$ more than he paid for it. How much did it cost?

**60.** Mr. D purchasing a pair of horses paid $\frac{1}{4}$ of the price in cash; $\frac{1}{2}$ the remainder he paid a month later. He still owes $210 of the debt. What was the purchase price?

**61.** Two men do a piece of work in 18 days. If the first man works three times as fast as the second, in how many days can each do the work alone?

**62.** A father left a fortune to his three children; the oldest was to have $\frac{3}{8}$ of it, the second was to have $\frac{2}{11}$ of it, and the youngest was to have the rest. The oldest received $12000 less than the youngest. What was the value of the estate?

**63.** If $\frac{2}{3}$ of a bushel of corn be worth $\frac{3}{7}$ of a bushel of wheat, and wheat be worth $1.40 a bushel, how many bushels of corn can be bought for $27?

**64.** Bought 304 pounds of rice for $31.16. One fourth of it is destroyed. I sell the remainder at a loss of 1 cent on the pound. What is my whole loss by the transaction?

**65.** $\frac{5}{16}$ of the cost of my house and barn is $\frac{15}{32}$ of the difference between their costs; and $1\frac{1}{8}$ times the difference between their costs is $3600. Required the cost of each.

**66.** A agreed to hoe $\frac{7}{9}$ of a field of corn while B hoed the remainder. After finishing, it was found that A had hoed $69\frac{4}{9}$ rows more than one-half. How many rows in the field?

**67.** The total Indian population on the reservations in 1893 was 249,366, while the area of the reservations was 134,176 square miles. What was the average quantity of land occupied by 100 Indians?

**68.** The moon's diameter is $\frac{3}{11}$ that of the earth, and the sun's diameter is 110 times that of the earth. What fraction of the sun's diameter is that of the moon?

**69.** I bought 21 pigs and 15 cows for $693. Each cow cost $27 more than each pig cost. Find cost of each pig.

**70.** Three Indians counted their trophies. Red Cloud had twice as many as Bigfoot, and the latter had $\frac{2}{3}$ as many as Horsehead, who had 4 less than Red Cloud. How many had each?

SUPPLEMENTARY EXERCISES (FOR ADVANCED CLASSES).

**266. 1.** One day a man spent $\frac{5}{8}$ of his money, and the next day $\frac{4}{9}$ of the remainder. If he had $\frac{3}{20}$ of a dollar left, how much did he spend the second day?

**2.** A can roof a house in $6\frac{1}{4}$ days, but can work only .75 of each day. If B helps him, the time required to roof the house is 2 days and $\frac{3}{4}$ of an hour. Counting 9 hours a day, in how many days can B alone do the work?

**3.** A farmer has 5 horses to each of which he gives $\frac{1}{4}$ bushel of oats three times a day. When oats are worth $\$\frac{2}{5}$ a bushel, what will be the cost of the oats required in the month of September?

**4.** Two men, starting from the same point, walk along a railroad track. When one has gone $\frac{5}{12}$ of a mile and the other $\frac{3}{8}$ of a mile, how many feet are they apart, there being $16\frac{1}{2}$ feet in a rod, and 320 rods in a mile?

**5.** If telegraph poles are $\frac{1}{44}$ of a mile apart, how many poles in a line 40.5 miles long?

**6.** When 0 is a factor, what is the product? Why?

**7.** When the multiplicand is one greater than the product, which is $\frac{5}{16}$, what is the multiplier?

**8.** A man divided \$10 equally among 5 boys. Each boy gave $\frac{1}{3}$ of his share to the poor. How much was given to charity?

**9.** A man divided $\$a$ equally among $b$ boys. Each boy gave $\frac{1}{3}$ of his share to the poor. How much was given to the poor?

**10.** At one store a lady spent $\frac{1}{2}$ her money and $\$\frac{1}{2}$ more; at another, $\frac{1}{2}$ the remainder and $\$\frac{1}{2}$ more; at another, $\frac{1}{2}$ the remainder and $\$\frac{1}{2}$ more, and then had a dollar left. How much had she at first?

**11.** In an orchard there are $a$ rows of trees, and between each two rows of trees are 5 rows of cabbage. What is the value of the cabbage at $\$b$ a row? ($\$5ab - 5b$.)

## COMPOUND NUMBERS.

**267.** A concrete number expressed in two or more denominations (units of measure) is called a **Compound Number.**

Thus, 2 feet 3 inches, 3 pounds 8 ounces, are compound numbers. But 3.5 pounds and $1.55 are not compound numbers.

**268.** A **Measure** is a unit by which *quantity,* such as value, length, weight, etc., is estimated.

Thus, the *yard* is a measure, because it is a unit by which length is estimated or measured.

**269.** A **Prime** (or *principal*) **Unit** is a unit of measure from which other units of the same kind may be derived.

Thus, a *dollar* is a prime (or principal) unit.

### MEASURES OF VALUE.

**270.** The ordinary measure of value is **Money,** which is sometimes called *currency.* **Coin** is metal money; all other currency is called **Paper Money.**

**271. United States Money** is the legal currency of the United States. The prime (or principal) unit is the *dollar.*

TABLE.

10 mills = 1 cent (ct. or ¢).
10 cents = 1 dime (d.).
10 dimes = 1 dollar ($).

| $ | | d. | | ct. | | m. |
|---|---|---|---|---|---|---|
| 1 | = | 10 | = | 100 | = | 1000 |

1. The U. S. now issues the following *coins:*

Gold.—The 20-dollar, 10-dollar, 5-dollar, and $2\frac{1}{2}$-dollar pieces.

Silver.—The dollar, half-dollar, quarter-dollar, and the dime.

Nickel.—The 5-cent piece.

Bronze.—The one-cent piece.

2. The standard weight of the *gold dollar* is 25.8 gr.; it contains 23.22 grains of pure gold, but is not now coined.

3. The silver dollar weighs $412\frac{1}{2}$ gr.

4. The *standard purity* of the gold and silver coins is 9 parts pure metal and 1 part alloy (by weight). The alloy of silver coin is pure copper. The alloy of gold coins is copper, or silver and copper. If both are used, the silver is not to exceed $\frac{1}{10}$ of the alloy.

5. The *mill* has never been coined; it is merely a convenient name for the tenth part of a cent.

**272. English Money** is the currency of Great Britain. The principal unit is the *pound sterling,* which is called the *sovereign* when coined.

TABLE.

| | | |
|---|---|---|
| 4 farthings | = | 1 penny (d.). |
| 12 pence | = | 1 shilling (s.). |
| 20 shillings | = | 1 pound (£). |
| | = | \$4.8665 in U. S. money. |

| £ | | s. | | d. | | far. |
|---|---|---|---|---|---|---|
| 1 | = | 20 | = | 240 | = | 960. |

**273.** Canada has a decimal currency, and the table and denominations are the same as those of U. S. money.

**274.** France also has a decimal currency. The unit is the *franc.*

The value of a franc in U. S. money is \$.193. The franc is divided into 100 centimes.

**275. German Money** is the legal currency of the German Empire. The unit is the *mark.*

The value of a mark in U. S. money is \$.2385. The mark is divided into 100 *pfennigs.*

**1.** How many dimes in \$2? In \$$\frac{1}{2}$? In \$$3\frac{1}{2}$?

**2.** How many cents in \$10? How many dimes? How many half-dollars?

**3.** How many pence in 3 shillings? In 8 shillings? In 4 shillings 2 pence?

**4.** How many pence in a pound? In £$\frac{1}{2}$? In £2 5s.?

**5.** How many shillings in £5? In £2 10s.? In £$\frac{1}{2}$?

**6.** Have you changed the *value* of these numbers, or their *form?*

**276. Reduction** is the process of changing the denomination of a number without changing its value; that is, changing the unit of measure and the number of the units.

**277.** Reducing a number to a lower denomination is called **Reduction Descending.**

WRITTEN EXERCISES.

**1.** Reduce £5 8s. 7d. to pence.

5 × 20s. = 100s.
100s. + 8s. = 108s.
108 × 12d. = 1296d.
1296d. + 7d. = 1303d.

Since there are 20s. in 1 pound, in £5 there are 5 times 20s., or 100s.; 100s. + 8s. = 108s. ∵ there are 12d. in 1 shilling, in 108s. there are 108 times 12d., or 1296d.; 1296d. + 7d. = 1303d.

Reduce to farthings:

**2.** 6s. 4d. 2 far.

**3.** 10s. 10d. 3 far.

**4.** 13s. 11d.

**5.** 9s. 6d. 1 far.

**6.** £2 8s.

**7.** £3 4s. 9d. 3 far.

**8.** Reduce £$\frac{1}{2}$ to pence.

**9.** Reduce £$\frac{5}{8}$ to shillings and pence.

$$£\tfrac{5}{8} = \tfrac{5}{8} \text{ of } 20\text{s.} = \tfrac{100}{8}\text{ s.} = 12\tfrac{1}{2}\text{s.}$$
$$\tfrac{1}{2}\text{s.} = \tfrac{1}{2} \text{ of } 12\text{d.} = 6\text{d.}$$
$$\therefore £\tfrac{5}{8} = 12\text{s. } 6\text{d.}$$

This may be called reducing to *lower denominations.*

Reduce to lower denominations:

**10.** £$\frac{3}{4}$.

**11.** £$\frac{9}{10}$.

**12.** $\frac{2}{3}$s.

**13.** $\frac{2}{7}$s.

**14.** £$\frac{3}{7}$.

**15.** £$1\frac{2}{3}$.

**16.** $2\frac{1}{2}$s.

**17.** $1\frac{1}{5}$s.

**18.** Reduce .36 of a shilling to lower denominations.

.36 of a shilling = .36 of 12d. = 4.32d.
.32 of a penny = .32 of 4 far. = 1.28 far.
∴ .36s. = 4d. 1.28 far.

Reduce to lower denominations:

| | | | |
|---|---|---|---|
| **19.** .37s. | **21.** £.45 | **23.** £.1$\frac{1}{3}$. | **25.** .56s. |
| **20.** .75s. | **22.** £.66 | **24.** £.$\frac{2}{3}$. | **26.** .87s. |

**278.** **1.** In 70 cents how many dimes?

**2.** How many dollars in 200 cents? In 500 ct.?

**3.** How many shillings in 24 pence? In 36d.?

**4.** In 60s. how many pounds? In £100? In £70?

**5.** Have you changed the value of these numbers? What have you changed?

**279.** Reducing a number to a higher denomination is called **Reduction Ascending.**

WRITTEN EXERCISES.

**1.** Reduce 6845 farthings to pounds, shillings, etc.

4 far.)6845 far.
12d. )1711d. + 1 far.
20s. ) 142s. + 7d.
£7 + 2s.

Since in 1 penny there are 4 far., in 6845 far. there are as many pence as 4 far. is contained times in 6845 far., or 1711, with a remainder of 1 far.

Since there are 12d. in 1 shilling, in 1711d. there are as many shillings as 12d. is contained times in 1711d., or 142, with a remainder of 7d.

Since there are 20s. in 1 pound, in 142s. there are as many pounds as 20s. is contained times in 142s., or 7, with 2s. remaining.

Therefore, 6845 far. = £7 2s. 7d. 1 far.

ANOTHER METHOD.

1 far. = $\frac{1}{4}$d.
∴ 6845 far. = 6845 × $\frac{1}{4}$d. = 1711$\frac{1}{4}$d. = 1711d. + 1 far.
1d. = $\frac{1}{12}$s.
∴ 1711d. = 1711 × $\frac{1}{12}$s. = 142$\frac{7}{12}$s. = 142s. + 7d.
1s. = £ $\frac{1}{20}$.
∴ 142s. = 142 × £$\frac{1}{20}$ = £7$\frac{2}{20}$ = £7 + 2s.
∴ 6845 far. = £7 2s. 7d. 1 far.

Reduce to higher denominations:

**2.** 7500 far. **4.** 8927d. **6.** 13785s.
**3.** 6738 far. **5.** 5360d. **7.** 23456 far.

**8.** What part of a pound is $\frac{1}{2}$d.?

(a)

1d. $= \frac{1}{12}$s.
$\therefore \frac{1}{2}$d. $= \frac{1}{2}$ of $\frac{1}{12}$s. $= \frac{1}{24}$s.
1s. $= £\frac{1}{20}$.
$\therefore \frac{1}{24}$s. $= \frac{1}{24}$ of $£\frac{1}{20} = £\frac{1}{480}$.

(b)

$\frac{1}{2} \div 12 = \frac{1}{24}$.
$\frac{1}{24} \div 20 = \frac{1}{480}$.
$\therefore \frac{1}{2}$d. $= £\frac{1}{480}$.

NOTE.—This is really reducing $\frac{1}{2}$d. to a higher denomination, but, as *fractions* are involved, it is customary to say it is reducing $\frac{1}{2}$d. to the fraction of a £.

Reduce to the fraction of a £:

**9.** $\frac{1}{4}$d. **11.** $\frac{3}{4}$ far. **13.** .35d. **15.** $\frac{5}{12}$ far.
**10.** $\frac{1}{3}$ far. **12.** $\frac{2}{5}$d. **14.** .65 far. **16.** $\frac{7}{16}$d.

**17.** What decimal part of a pound is 12s. 8d. 3 far.?

$3 \div 4 = .75$
$8.75 \div 12 = .72916 +$
$12.72916 \div 20 = £.636458 +$
or
£1 = 960 far.
12s. 8d. 3 far. = 611 far.
611 far. $= £\frac{611}{960} = £.636458 +$

The explanation of this solution is similar to that given in example 8.

The second method is simple and convenient. Reduce both numbers to farthings, and find the ratio.

**18.** Reduce 9s. 6d. 1 far. to the decimal part of a £.
**19.** Reduce £5 to U. S. money.
**20.** Reduce 18s. 9d. 3 far. to shillings.
**21.** How many francs in $19.30?
**22.** What is the value in English money of $243.325?
**23.** Reduce 1000 francs to U. S. money.
**24.** How many marks in $2385?
**25.** Reduce $\frac{5}{7}$ of a pound to lower denominations.
**26.** Reduce 1s. 4d. to the fraction of a pound.
**27.** Express 8 dimes, 7 cents, 5 mills as the decimal part of a dollar.

## MEASURES OF CAPACITY.

**280. Liquid Measure** is used in measuring liquids of all kinds. The *principal unit* is the *gallon.*

TABLE.

| | | |
|---|---|---|
| 4 gills (gi.) | = | 1 pint (pt.). |
| 2 pints | = | 1 quart (qt.). |
| 4 quarts | = | 1 gallon (gal.). |

| gal. | | qt. | | pt. | | gi. |
|---|---|---|---|---|---|---|
| 1 | = | 4 | = | 8 | = | 32. |

1. In estimating capacity, $31\frac{1}{2}$ gal. are counted a barrel, and 63 gal. a hogshead ; but in commerce they are not fixed measures.

2. The gallon contains 231 cubic inches.

**281. Dry Measure** is used in measuring grain, fruit, vegetables, etc. The *principal unit* is the *bushel.*

TABLE.

| | | |
|---|---|---|
| 2 pints (pt.) | = | 1 quart (qt.). |
| 8 quarts | = | 1 peck (pk.). |
| 4 pecks | = | 1 bushel (bu.). |

| bu. | | pk. | | qt. | | pt. |
|---|---|---|---|---|---|---|
| 1 | = | 4 | = | 32 | = | 64. |

1. The standard bushel of the United States contains 2150.42 cubic inches.

2. Grain, seeds, small fruits, etc., are sold by *even* or *stricken* measure. Coal, corn in the ear, coarse vegetables, etc., are sold by *heaped* measure.

3. In dry measure a quart contains ($2150.42 \div 32$), or 67.2 cubic inches. In liquid measure a quart contains ($231 \div 4$), or 57.75 cu. in.

### ORAL EXERCISES.

**282. 1.** How many quarts in 16 pt.? In 28 pt.? In 19 pt.?

**2.** How many gallons in 20 qt.? In 32 qt.? In 18 qt.?

**3.** What is a gallon of milk worth at 2 cents a gill?

**4.** When vinegar is 40 cents a gallon, how much must be paid for 2 qt.? 1 pt.?

**5.** At 5 cents a pint, how many gallons of molasses can be bought for $2 ?

---

**6.** How many pecks in 24 qt.? In 35 qt.?

**7.** In 2 pecks how many pints ? In $3\frac{1}{2}$ pk.?

**8.** How many pecks in 5 bushels ? In $12\frac{1}{2}$ bu.?

**9.** A man bought 12 pk. of potatoes at 10 cents a half-peck. How much did he pay for them ?

**10.** Harry sold half a bushel of chestnuts at 5 cents a pint. How much did he get for them ?

WRITTEN EXERCISES.

**283.** Reduce to lower denominations :

**1.** 5 gal. 3 qt. 1 pt.

**2.** $\frac{7}{8}$ gal.

**3.** 5 bu. 2 pk. 3 qt.

**4.** $\frac{3}{8}$ bu.

**5.** 10 gal. 2 qt. 1 pt.

**6.** .875 gal.

**7.** 3 bu. 6 qt.

**8.** .675 pk.

**284.** Reduce to higher denominations :

**1.** 1235 qt. (liquid measure).

**2.** 1323 qt. (dry measure).

**3.** 321 pk.

**4.** 2457 qt. (dry measure).

**5.** 501 pt. (dry measure).

**6.** 1620 pt. (dry measure).

---

**7.** What part of a gallon is $\frac{2}{3}$ of a pint ?

**8.** Reduce 2 qt. 1 pt. to the fraction of a gallon.

**9.** What decimal part of a gallon is 1 pt. ?

**10.** Mrs. E bought $2\frac{1}{2}$ gal. of milk at 4 cents a pint. How much did she pay for it ?

**11.** What part of a bushel is a pint and a half ?

**12.** Hiram feeds his horse 12 qt. of oats in a day. How long will 5 bu. last ?

**13.** If a bushel of salt is worth $.40, how much must be paid for 4 qt. ?

**14.** When apples are worth $1.60 a bushel, how many pecks can be purchased for $2.40 ?

## MEASURES OF WEIGHT.

**285. Weight** is the measure of quantity estimated by the scale or balance with reference to some fixed unit.

**286. Avoirdupois Weight** is used in weighing all coarse and heavy articles, such as cattle, horses, coal, grain, groceries, and all metals except gold and silver. It is the common *commercial weight*.

TABLE.

16 ounces (oz.) = 1 pound (lb.).
100 pounds = 1 hundred-weight (cwt.).
20 hundred-weight = 1 ton (T.).

T. cwt. lb. oz.
1 = 20 = 2000 = 32000.

1. The avoirdupois pound contains 7000 grains.
2. The long ton of 2240 lb. has almost gone out of use in the United States.
3. A barrel of flour weighs 196 lb.

LEGAL WEIGHT OF A BUSHEL.

| | Cal. | Conn. | Ill. | Ind. | Iowa. | Ky. | Me. | Mass. | Mich. | Mo. | Md. | N. J. | N. Y. | Ohio. | Or. | Penn. | Va. | W. Va. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Barley | 50 | 48 | 48 | 48 | 48 | 47 | 48 | 48 | 48 | 48 | | 48 | 48 | 48 | 46 | 48 | 48 | 48 |
| Buckwheat | 40 | 48 | 52 | 50 | 52 | 56 | 48 | 48 | 48 | 52 | | 50 | 48 | 50 | 42 | 48 | 52 | 52 |
| Corn | 52 | 56 | 56 | 56 | 56 | 56 | 56 | 56 | 56 | 56 | | 56 | 58 | 56 | 56 | 56 | 56 | 56 |
| Oats | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 26 | 30 | 32 | 32 | 36 | 32 | 32 | 32 |
| Potatoes | | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 56 | 60 | | 60 | 60 | 56 | 60 | 60 |
| Rye | 54 | 56 | 56 | 56 | 56 | 56 | | 56 | 56 | 56 | | 56 | 56 | 56 | 56 | 56 | 56 | 56 |
| Clover-seed | | | 60 | 60 | 60 | 60 | | | 60 | 60 | | 64 | 60 | 60 | 60 | 62 | 60 | 60 |
| Timothy-seed | | | 45 | 45 | 45 | 45 | 45 | 45 | 45 | 45 | | 45 | 44 | 45 | | 45 | 45 | 45 |
| Wheat | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | | 60 | 60 | 60 | 60 | 60 | 60 | 60 |

ORAL EXERCISES.

**287.** 1. In 4000 lb. how many tons? In 7500 lb.? In 35 cwt.?

2. Find the cost of 2 T. 15 cwt. of hay at $16 a ton.

3. What will $2\frac{1}{4}$ lb. of cinnamon cost at 2 cents an ounce?

4. At $.32 a pound, what will 7.5 oz. of butter cost?

5. What part of a pound is $\frac{1}{2}$ oz.?

6. What decimal part of a pound is 12 ounces?

WRITTEN EXERCISES.

**288.** Reduce:

1. 3 T. 4 cwt. to oz.
2. 5 cwt. 45 lb. to oz.
3. 6 cwt. 90 lb. to lb.
4. $5\frac{3}{4}$ T. to lb.
5. $7\frac{3}{8}$ cwt. to lb.
6. 625 lb. to oz.

Reduce to higher denominations:

7. 475 oz.
8. 1220 oz.
9. 2075 lb.
10. 1000 oz.
11. 150 cwt.
12. 75386 oz.

13. Reduce $\frac{2}{3}$ of an ounce to the fraction of a pound.

14. Reduce 12 cwt. to the decimal part of a ton.

15. How many grains in 3 lb.? In 5 lb.? 4 oz.?

16. How many barrels of flour in 686 lb.?

17. When hay is worth $15 a ton, how much must be paid for 250 lb.?

18. How much will 3 lb. 6 oz. of butter cost, at $.32 a pound?

19. If cloves are worth 40 cents a pound, how much must be paid for 40 ounces?

20. A man bought 1500 lb. of wheat, at $.80 a bushel. What did it cost him?

21. Mr. B bought a bag of oats weighing 112 lb. If oats were worth 40 ct. a bushel, how much did he pay for them?

**289. Troy Weight** is used in weighing gold, silver, and jewels, and in laboratory tests.

### Table.

| | | |
|---|---|---|
| 24 grains (gr.) | = | 1 pennyweight (pwt.). |
| 20 pennyweights | = | 1 ounce (oz.). |
| 12 ounces | = | 1 pound (lb.). |

| lb. | | oz. | | pwt. | | gr. |
|---|---|---|---|---|---|---|
| 1 | = | 12 | = | 240 | = | 5760. |

1. The carat is a weight used in weighing diamonds and other jewels. It is equal to 3.168 troy grains.

2. The term carat is also used to indicate the fineness of gold, and means $\frac{1}{24}$ part of the mass. Thus, gold that is **16** carats fine is $\frac{16}{24}$ gold, and $\frac{8}{24}$ alloy.

WRITTEN EXERCISES.

**290. 1.** Reduce .625 of an ounce to the fraction of a pound.

**2.** Reduce $\frac{3}{5}$ gr. to the fraction of an ounce.

**3.** What decimal part of a pound is 5 oz. 4 pwt.?

**4.** How many grains in 3 lb. troy? In 3 lb. avoirdupois?

**5.** How many rings can be made from 6 oz. of gold, if each ring contains 80 gr.?

**6.** When silver is worth \$.60 an ounce, how much must be paid for 1000 gr.?

**7.** When a ring is marked "18 carats," what part of it is pure gold?

**8.** How many knives, each weighing 3 oz., can be made from 5 lb. 9 oz. of silver?

**9.** At \$.80 a pwt., how much must be paid for a chain weighing 1 oz. 8 pwt. 12 gr.?

**10.** What is the present market value of an ounce of gold? What is the market value of an ounce of silver? What is the ratio of the market value of silver to that of gold?

**291. Apothecaries' Weight** is used in mixing medicines and in selling them at retail. Drugs in bulk are bought and sold by avoirdupois weight.

TABLE.

| | | |
|---|---|---|
| 20 grains (gr.) | = | 1 scruple (℈). |
| 3 scruples | = | 1 dram (ʒ). |
| 8 drams | = | 1 ounce (℥). |
| 12 ounces | = | 1 pound (℔). |

| ℔. | ℥ | ʒ | ℈ | gr. |
|---|---|---|---|---|
| 1 = | 12 = | 96 = | 288 = | 5760. |

COMPARISON OF WEIGHTS.

| AVOIRDUPOIS. | TROY. | APOTHECARIES'. |
|---|---|---|
| 1 lb. = 7000 gr. | 1 lb. = 5760 gr. | 1 ℔. = 5760 gr. |
| 1 oz. = $437\frac{1}{2}$ gr. | 1 oz. = 480 gr. | 1 oz. = 480 gr. |

It will be observed that the troy pound and ounce are identical with the apothecaries' pound and ounce. It will also be observed that, while the avoirdupois *pound* is heavier than the troy pound, the troy *ounce* is heavier than the avoirdupois ounce.

QUERIES.—1. What is the ratio of the troy pound to the avoirdupois pound? 2. What is the ratio of the troy ounce to the avoirdupois ounce?

NOTE.—In compounding liquid medicines, druggists make use of the following: 60 minims (♏) = 1 dram, 8 drams = 1 ounce, 16 ounces = 1 pint.

## MEASURES OF EXTENSION.

**292. Extension** has three dimensions, called length, breadth, and thickness, or height.

A line has but one dimension—*length.*

A surface has two dimensions—*length* and *breadth.*

A solid has three dimensions—*length, breadth,* and *thickness.*

**293. Measures of Extension** are used in measuring lengths, surfaces, and solids.

**294. Linear Measure** is used in measuring length. The principal unit is the *yard.*

### Table.

| | |
|---|---|
| 12 inches (in.) | = 1 foot (ft.). |
| 3 feet | = 1 yard (yd.). |
| $5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet | = 1 rod (rd.). |
| 320 rods | = 1 mile (mi.). |

| mi. | | rd. | | yd. | | ft. | | in. |
|---|---|---|---|---|---|---|---|---|
| 1 | = | 320 | = | 1760 | = | 5280 | = | 63360. |

### Supplementary Table.

| | |
|---|---|
| 3 feet | = 1 pace. |
| 4 inches | = 1 hand (used to measure height of horses) |
| 18 inches | = a cubit. |
| 21.888 inches | = 1 sacred cubit. |
| 6 feet | = 1 fathom. |
| 3 sizes | = 1 inch (used by shoemakers). |
| 8 furlongs | = 1 mile. |
| 9 inches | = 1 span. |
| 1 cable length | = 120 fathoms. |
| 1.15 statute miles | = 1 nautical, or geographic, mile. |
| 1 nautical mile | = 1 knot = 6086 feet. |
| 3 nautical miles | = 1 league. |

#### WRITTEN EXERCISES.

**295.** Reduce the following:

**1.** 3 yd. 2 ft. to in.
**2.** 1 mi. 40 rd. to rd.
**3.** $\frac{1}{2}$ mi. to feet.
**4.** 10 yd. 1 ft. 9 in. to in.
**5.** $\frac{1}{2}$ rd. to inches.
**6.** 2 rd. 4 yd. 6 in. to in.
**7.** 3520 yd. to mi.
**8.** 7920 ft. to mi.
**9.** 3000 yd. to mi., etc.
**10.** 720 rd. to miles.

Reduce to lower denominations:

**11.** $\frac{5}{9}$ rd.
**12.** $\frac{1}{6}$ mi.
**13.** .8 yd.
**14.** .375 mi.
**15.** $3\frac{1}{4}$ yd.
**16.** 7 ℔. ℥ 3.

**17.** Reduce $\frac{3}{4}$ of a yard to the fraction of a rod.

**18.** What part of a rod is 9 inches?

**19.** What part of a yard is 2 ft. 8 in.?

**20.** At \$.55 a rod, how much must be paid for digging a ditch 10 rd. 7 ft. 6 in. long?

**21.** When cloth is worth 90 cents a yard, how much must be paid for a piece 16 inches long?

**22.** How many pounds (troy) in 11520 grains?

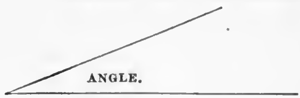
**23.** How many powders of gr. V each can be made from ℥ 6 of calomel?

**24.** A ring which weighs 120 grains is 16 carats fine. What is the value of the gold in it at \$20 an ounce?

**296.** Boundaries of solids are called **Surfaces.** A surface has *length* and *breadth,* but no *thickness.*

Note.—We usually think of a solid as a material body—*e.g.*, a block of wood—but the portion of space occupied by the block of wood is regarded as the *solid.*

**297.** Two lines proceeding from the same point form an **Angle.** The size of the angle depends upon the degree of opening between the lines.

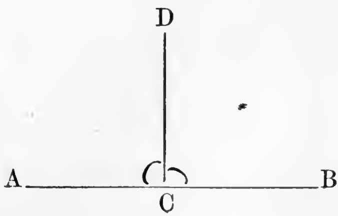



**298.** A *flat surface* bounded by straight lines or by a curved line is called a **Plane Figure.**

The distance around a plane figure is called the *perimeter.*

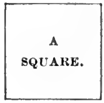
**299.** A plane figure that has four straight sides is called a **Quadrilateral.**

**300.** When the line CD meets AB, as in the figure, making the angles ACD and DCB equal, each of these angles is called a **Right Angle.**

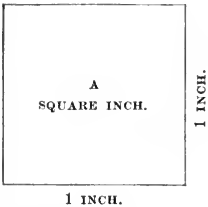


**301.** A quadrilateral all of whose angles are right angles is called a **Rectangle.**


**302.** An equilateral (equal-sided) rectangle is called a **Square.**




1. Each angle is a *right angle.*
2. How does the length of a square compare with its breadth?

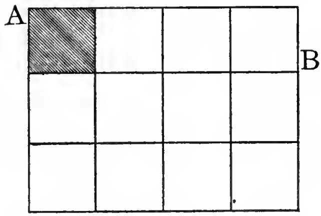



**303.** When each side of a square is *one inch* long, the figure is called a *square inch.*

1. What is a *square foot?* A *square yard?*
2. What is a *square unit?*
3. Is a square also a rectangle?

**304.** The **Area** of a figure or surface is the number of square units of measure it contains. The area is often called the *superficial contents.*

A rectangle 4 feet long and 3 feet wide may be divided into 3 strips, each 1 foot wide, and each strip into 4 equal parts, each part being 1 square foot (square unit of measure).




Then the area of the *strip* AB is $4 \times 1$ sq. ft., and the total area is $3 \times 4 \times 1$ sq. ft., or 12 sq. ft.

**305.** Principle.—*The area of a rectangle is expressed by the product of the numbers that represent its length and breadth.*

Both dimensions must be expressed in like units.

1. A board is 8 feet long and 1 foot wide. How many square feet in its surface? How many would there be if it were 2 feet wide?

**2.** A box lid is 6 inches long and 4 inches wide. How many square inches in its surface? What unit of measure is used here?

**3.** How many square feet in the surface of a square table whose sides are 2 feet? 3 feet?

**4.** How many square inches in a square whose sides are 12 inches? What may this square be called? Why?

**5.** How many square feet in a square whose sides are 3 feet? What may this square be called? Why?

**6.** The sides of a square are $16\frac{1}{2}$ feet. How many square feet in its surface? What may this square be called? Why?

## SQUARE MEASURE.

**306. Square Measure** is used in measuring surfaces.

### Table.

| | |
|---|---|
| 144 square inches (sq. in.) | = 1 square foot (sq. ft.). |
| 9 square feet | = 1 square yard (sq. yd.). |
| $30\frac{1}{4}$ square yards | = 1 square rod (sq. rd.). |
| 160 square rods | = 1 acre (A.). |
| 640 acres | = 1 square mile (sq. mi.). |

| sq. mi. | A. | sq. rd. | sq. yd. | sq. ft. | sq. in. |
|---|---|---|---|---|---|
| 1 = | 640 = | 102400 = | 3097600 = | 27878400 = | 4014489600. |

1. In square measure the term *perch* is sometimes used instead of *square rod.*

2. In roofing, slating, etc., 100 sq. ft. is called a *square.*

3. When we say a surface is a "foot square," we mean that each dimension is a foot. Hence the terms "foot square," "rod square," etc., mean *dimensions*, while "square foot," "square rod," etc., mean *area.*

#### ORAL EXERCISES.

**307. 1.** A board is 2 feet long and $\frac{1}{2}$ ft. wide. What is its area? Is it *square?*

**2.** What is the area of a board 4 yards long and $\frac{1}{4}$ yd. wide?

**3.** A lot is 20 rods long and 8 rods wide. How many sq. rd. in the lot? What may this rectangle be called? What other dimensions might it have, and yet have the same *area?*

**4.** What is a square mile? How many rods long is one side of it? Then how can we find how many sq. rd. in a square mile?

**5.** How can we find the number of acres in a square mile? Name the unit of measure.

**6.** How do you know that there are 9 sq. ft. in a square yard? Explain by a diagram.

### WRITTEN EXERCISES.

**308.** Reduce to lower denominations:

**1.** 2 sq. ft. 12 sq. in.
**2.** 3 sq. yd. 8 sq. ft.
**3.** 5 sq. yd. 7 sq. in.
**4.** 10 sq. rd. 5 sq. yd.
**5.** 2 A. 80 sq. rd.
**6.** 3 yd. 2 ft. 8 in.
**7.** 1 A. 25 sq. yd.
**8.** 1 sq. mi. 280 A.

Reduce to higher denominations:

**9.** 1000 sq. ft.
**10.** 2000 sq. in.
**11.** 3000 sq. yd.
**12.** 4000 sq. rd.
**13.** 10000 sq. rd.
**14.** 33333 sq. in.

**15.** Reduce $\frac{3}{8}$ sq. rd. to square yards, etc.

**16.** What part of an acre is $\frac{2}{3}$ of a square rod?

**17.** One square yard is what part of a square rod?

**18.** What part of a square rod is 5 sq. yd. 8 sq. ft.?

**19.** A floor is 16 ft. long and 12 ft. wide. How many square feet does it contain?

**20.** A town lot is 40 ft. wide and 120 ft. long. What is its area in square yards?

**21.** How many sq. yd. in a piece of carpet 18 ft. long and a yard wide?

**22.** A field 40 rods long and 25 rods wide contains how many acres?

**23.** At $50 an acre, how much must be paid for 12 A. 32 sq. rd. of land?

**24.** When a product and one factor are given, how may the other factor be found?

**25.** A two-acre lot is 20 rods long. How wide is it?

**26.** A field containing 16 acres is 32 rods wide. What is its length?

**27.** What will it cost to paint a ceiling 16 ft. by 12 ft., at $.25 a square yard?

**28.** A gable barn-roof, 65 ft. long and 30 ft. wide, was covered with sheet-iron. How many sq. yd. were required?

**29.** The area of a rectangular glass is 30 sq. ft., and its length is 144 inches. What is its width?

**30.** A tract of prairie land is 6 miles long and $4\frac{1}{2}$ miles wide. How many farms of 160 acres each could be sold from it?

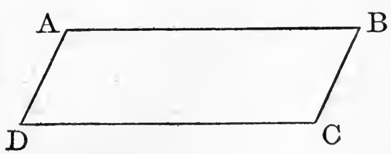
**31.** A lot 60 feet square has in its central part a reservoir 18 feet square. What is the distance from the reservoir to the fence surrounding the lot?

**32.** If steel rails weigh 24 lb. a foot, and can be bought for $30 a ton, what will be the cost of the rails for a mile of single-track railway?

**33.** A man who owns a farm 120 rods square put 10 acres in corn, 15 in rye, 20 in oats, 15 in barley, and the remainder in wheat. What part of the whole farm did he put in wheat?

### PARALLELOGRAMS.

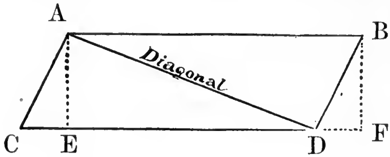
How does the figure ABCD differ from a rectangle? Are its sides equal? Are its *opposite* sides equal? Are they parallel? Are its angles equal?



**309.** A quadrilateral whose opposite sides are parallel is called a **Parallelogram.**

QUERY.—Is a rectangle a parallelogram? Why?

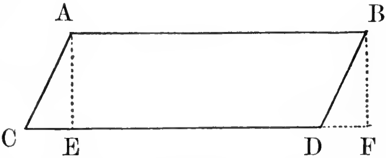
**310.** The side of a figure on which it is supposed to stand is called the **Base**; as CD. The **Altitude** is the perpendicular distance between the base (or the base extended) and the side or angle opposite.




**1.** Any side may be regarded as the base.

**2.** What is the *diagonal* of a parallelogram?

(*a*). If we cut off the end CAE, and place it at the other end, on DBF, will the *size* of the figure be changed?




(*b*). Then is the *area* of ABFE equal to that of ABDC? Is EF equal to CD?

(*c*). How is the area of the rectangle ABFE found? Then how may the area of the parallelogram ABDC be found?

Since a parallelogram is equivalent to a rectangle having the same base and altitude, it follows that

**311.** PRINCIPLE.—*The area of a parallelogram is expressed by the product of the numbers that represent its base and altitude.*

The base and altitude of all figures must be expressed in like units.

Find the area of the following parallelograms:

**1.** Base 24 ft., alt. 8 ft.

**2.** Base 35 ft., alt. 15 ft.

**3.** Base 72 ft., alt. 39 ft.

**4.** Base 16 ft., alt. 9 in.

**5.** Base 42 rd., alt. $5\frac{1}{2}$yd.

**6.** Base 132 in., alt. 30 ft.

**7.** A board in the form of a parallelogram is 6 ft. 6 in.

long on each side, and 1 ft. 4 in. wide. Draw the figure, and find the area.

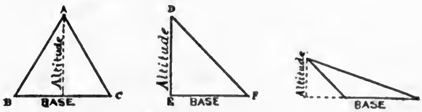
**8.** The area of a parallelogram is 518 sq. in. If its length is 37 inches, what is its altitude, or width?

**9.** A ten-acre field is in the form of a parallelogram. The shortest distance from one side to the opposite side is 25 rods. What is the length of the field?

**10.** Mr. A has a rectangular field 40 rd. long and 20 rd. wide. Mr. B has a field of equal size in the form of a parallelogram. If its length is 32 rd., what is the shortest distance across the field?

TRIANGLES.

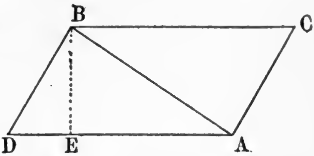
How many sides has the figure ABC? Are they all straight?




**312.** A plane figure that has three straight sides is called a **Triangle.**

The point where two sides meet is called a **Vertex;** as A.

(*a*). Into how many equal triangles does the diagonal AB divide the parallelogram BCAD?




(*b*). Then the area of each triangle is what part of the area of the parallelogram?

(*c*). Since the area of the parallelogram is expressed by AD × BE, how is one half its area, or the area of the triangle ABD, expressed?

**313.** Principle.—*The area of a triangle is expressed by one half the product of the numbers that represent its base and altitude.*

Note.—When one angle of a triangle is a right angle, the triangle is said to be *right-angled.* In such a triangle one *side* is the altitude, or height.

Find the area of the following triangles :

1. Base 12 ft., alt. $7\frac{1}{2}$ ft.
2. Base 29 ft., alt. 16 ft.
3. Base 47 rd., alt. 165 ft.
4. Base 18 in., alt. 12 ft. 6 in.

5. The base of a triangular piece of slate is 27 inches, and the altitude 33 inches. Draw the figure, and find the area in square feet.

6. How many triangles, base 5 ft. and alt. 4 ft., are equal to a parallelogram whose base is 50 ft., and whose altitude is 40 ft.?

7. A triangle whose base is 7 yards has an area of 819 sq. ft. What is its altitude ?

8. A house is 24 ft. wide, and the ridge of the roof is 12 ft. above the upper floor. Find the cost of painting the gables, at $.40 a square yard.

9. What is the area of a table $a$ feet square ?

10. A rectangle is $a$ ft. long and $b$ ft. wide. What is its area ?

11. The altitude of a parallelogram is $a$ ft., and the base is $b$ ft. What is the area ?

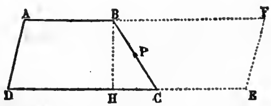
12. The base of a triangle is $2b$ inches, and the altitude is $a$ inches. Find its area.

13. What is the area of a square whose perimeter is $4p$ inches ?

14. One side of a rectangle is $2b$ ft., and the perimeter is $6b$ ft. What is the area ?

### TRAPEZOIDS.

In the figure ABCD which sides are parallel ? How many sides has the figure ?




**314.** A quadrilateral having two sides parallel is called a **Trapezoid** ; as ABCD.

($a$). Cut two equal trapezoids out of paper, and place them end to end so as to form a parallelogram.

(*b*). How does the area of one trapezoid compare with that of the parallelogram?

**315.** If the trapezoid ABCD in the figure swings about the point P, to the position ECBF, then the whole figure AFED is a parallelogram. Now the area of ABCD is one-half of that of AFED; but the area of AFED is equal to DE × BH. But DE = DC + CE = DC + AB. ∴ the area of ABCD = $\frac{1}{2}$ of (DE × BH) = $\frac{1}{2}$ of (AB + CD) × BH. Hence the area of a trapezoid equals half the area of a parallelogram having the same altitude and a base equal to the sum of the two parallel sides.

**316.** Principle.—*The area of a trapezoid is expressed by one half the product of the numbers that represent its altitude and the sum of its parallel sides.*

Find the area of the following trapezoids:

**1.** Altitude 8 in., parallel sides 12 in. and 10 in.

**2.** Altitude 13 ft., parallel sides 19 ft. and 11 ft.

**3.** Altitude 25 rd., parallel sides 45 rd. and 35 rd.

**4.** Altitude 76 ft., parallel sides 6 rd. and 23 yd.

**5.** How many sq. ft. in a board 12 ft. long, 15 in. wide at one end and 9 in. at the other?

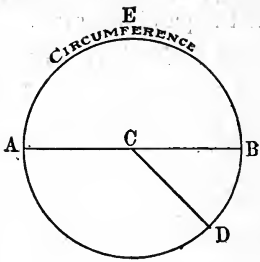
**6.** The parallel sides of a trapezoidal lot are 75 ft. and 55 ft. respectively, and the shortest distance between them is 40 ft. Draw the figure and find the area of the lot.

**7.** One side of a farm in the form of a trapezoid is 135 rd. long, the side parallel to it is 121 rd. long, and the perpendicular distance between them is 100 rd. What is the value of the farm at $75 an acre?

**8.** A twenty-acre field is in the form of a trapezoid whose parallel sides are 72 rd. and 88 rd. respectively. What is the altitude?

**9.** The distance around a triangular farm whose sides are equal is 480 rd., and the altitude is 80 rd. How many acres in the farm?

### CIRCLES.




Is the portion of the page enclosed by ADBE a plane figure?

What kind of line bounds the figure?

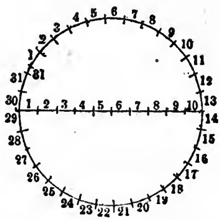
Is any part of the line nearer the center than another part?

**317.** A plane figure whose bounding line is everywhere equally distant from a point within, called the center, is a **Circle.**

**1.** The curved line that bounds a circle is the *Circumference.*

**2.** A straight line passing from one side of a circle to the other, through the center, is a *Diameter;* as AB.

**3.** A line from the center of a circle to the circumference is called a *Radius.* It is *half* a diameter; as CD or CA.



QUERIES.—1. Can there be a center without a circumference?

2. What is the surface between the center and circumference called?

☞ Divide the diameter of a circle into 10 equal parts, and step dividers or compasses around the circumference, making each step equal to one of the divisions of the diameter. There will be a little more than 31 steps. Hence the circumference is a little more than 3.1 times the diameter. In geometry it is shown to be 3.1416 times the diameter.

**318.** PRINCIPLE.—*The circumference of any circle is 3.1416 times its diameter.*

What is the circumference of

**1.** A circle whose diameter is 1 ft.? 60 ft.? 27 in.?

**2.** A circle whose radius is 6 ft.? $12\frac{1}{2}$ ft.? 2 ft. 6 in.?

What is the diameter of a circle whose

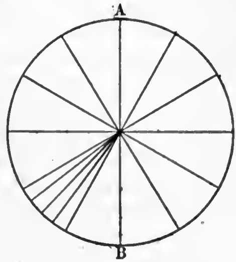
**3.** Radius is $7\frac{1}{2}$ in.? $13\frac{3}{4}$ in.? 19.08 ft.?

**4.** Circumference is 6.2832 ft.? 12.5664 yd.? 1 ft.?

**5.** What is the radius of a circular field whose circumference is 320 rods?

**6.** What is the circumference of a 6-inch stove-pipe?

**319. To find the area of a circle.**



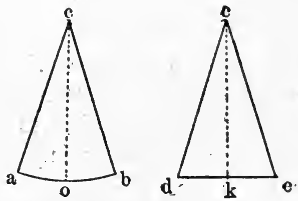

(*a*). May the circle AB be regarded as made up of a vast number of very small triangles? To what is the sum of their *bases* equal? Is the *altitude* of each triangle equal to the radius of the circle?

(*b*). Is the area of all the triangles equal to the area of the circle?

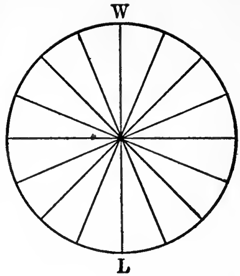
(*c*). Since the area of a triangle is found by multiplying its base by half its altitude, how may the area of a circle be found?

**320.** Principle.—*The area of a circle is expressed by the product of the numbers that represent its circumference and half its radius.*

This principle may be illustrated as follows in (*a*) and (*b*):

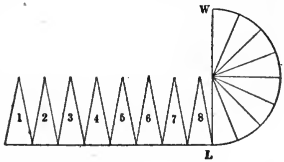



(*a*). In the figure *abc*, the base *ab* is a curved line—an arc of a circle whose center is *c*. If this arc be pressed up or stretched until it becomes a straight line, the sides *ac* and *bc* will be forced farther apart, and the figure will appear as *dec*, the line *de* being equal to the arc *ab*, and the other sides remaining unchanged. But *co*, the radius, is a little longer than the altitude *ck*. However, by increasing the number of radii the arc may be made as small as we please; and, as the arc is thus made smaller, the difference between the radius and the altitude becomes continually less. When the arc is *extremely* small, the radius and the altitude are regarded as equal.




(*b*). Take a rubber-tired wheel with spokes, as WL. Cut it through and straighten out as in the figure. The resulting figures 1, 2, 3, etc., are *nearly* triangles. The sum of all the bases is easily seen to be the circumference of the wheel, or circle. The area of all the triangles is equal to that of the circle, and their common altitude is the radius. (See Art. 319.)

(*c*). Regarding the circle, then, as made up of many triangles, we find its area by multiplying the *sum of all the bases* (which is the circumference) by *one-half the common altitude* (*i. e.*, $\frac{1}{2}$ the radius). It is proved in geometry that this is the *exact* area.




Find the area of the following circles :

**1.** Radius 10, circumference 62.832.

$$62.832 \times 5 = 314.16. \quad \text{Or,}$$
$$\tfrac{1}{2} \text{ of } 62.832 \times 10 = 314.16.$$

**2.** Diameter 10 in., circumference 31.416 in.

**3.** Diameter 15 ft., circumference 47.124 ft.

**4.** Diameter 7.9578 ft., circumference 25 ft.

**5.** Diameter 15 yd.

**6.** Radius 13.5 rd.

**7.** Circumference 100 ft.

**8.** Circumference 1.5708 ft.

**9.** At $50 an acre, what is the value of a circular field whose diameter is 100 rods ?

**10.** A horse is tied to an iron weight by a rope 20 feet long. Upon how many sq. ft. can he graze ? Draw the figure.

**11.** The fence around a circular field is 500 rods long. How many acres in the field ?

For convenience, 3.1416 is usually represented by the Greek letter $\pi$ (*pi*), the diameter by $d$, and the radius by $r$. Then the circumference is expressed by $\pi d$. But since $d = 2r$, the circumference equals $2\pi r$;

and the area equals $2\pi r \times \frac{1}{2}r$, or $\pi \times r \times r$, or $\pi r^2$. Thus, if the radius of a circle is 3 ft., the area is $3.1416 \times 3^2 \times 1$ sq. ft. = 28.2744 sq. ft.

**12.** Solve examples 2, 5, and 6 by this formula, and compare results with those already obtained.

### SUPPLEMENTARY EXERCISES (FOR ADVANCED CLASSES).

**321. 1.** The perimeter of a rectangular field is 200 rods, and three times the width is twice the length. Find the area in acres.

**2.** Each of the three sides of a field is $255\frac{3}{4}$ feet long. The field is surrounded by a fence 5 boards high, the posts being 8 feet apart. If the posts cost 25 cents each, and the boards 2 cents a linear foot, what did the posts and boards cost?

**3.** The distance around a circular lot is 1000 feet. If I plow around the lot until $\frac{1}{2}$ of it is plowed, how many square feet will remain to be plowed?

**4.** Find the area of a circular ring 4 feet wide, the radius of the outer circle being 32 feet.

**5.** Find the length of the minute-hand of a clock whose point moves 5 inches in 15 minutes.

**6.** A takes 924 steps in walking around a field 40 rods long. If there are 5 feet in two of his steps, how many acres in the field?

**7.** A carpenter had a plank 20 in. wide, from which he wished to saw off 10 sq. ft. What will be the length of the piece sawed off?

**8.** I have a rectangular farm whose perimeter is 240 rods. It is twice as long as it is wide. How many acres does it contain?

**9.** A map is 4 sq. ft. 4 sq. in. in area, and is drawn on a scale of 1 inch to a mile. How many acres are represented on the map?

**10.** A square lawn is bordered by a gravel drive 10 yards wide. The drive covers 4000 sq. yd. How many sq. yd. in the enclosed lawn?

**11.** If $b$ feet of fence cost $24.50, what will be the cost of fencing a square field, one side of which is $3b$ feet long?

**12.** What is the area of a circle whose radius is $r$ feet?

**13.** How many sq. ft. in a table which is $a$ inches long and $b$ inches wide?

**14.** The distance around a rectangular field is $p$ rods. If the field is $q$ rods long, how wide is it?

**15.** My storeroom is 75 ft. long, 35 ft. wide, and 20 ft. high, the vertex of the roof being 15 ft. above the upper floor. Making no allowance for doors and windows, what will be the cost of painting at $.18 a sq. yd.?

## MEASURES OF VOLUME.

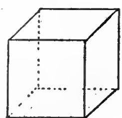
**322.** Any limited portion of space is called a **Solid.** A solid has three dimensions—*length, breadth*, and *thickness.*

**323.** A solid that has six rectangular sides, or faces, is called a **Rectangular Solid.**

**1.** A rectangular side or surface is one that has the form of a rectangle.

**2.** Is a brick a rectangular solid? Why? Name three bodies that are rectangular solids.

**324.** A regular solid with six square faces is called a **Cube.**



CUBE

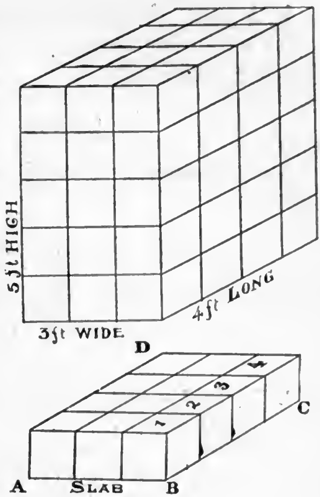
**1.** A cube has twelve *edges.* Where are they?

**325.** A **Cubic Inch** is a cube whose faces are each an inch long and an inch wide.

**1.** What is a *cubic foot?* A *cubic yard?* A cubic unit? A solid unit?

**326.** The **Volume** of any solid or body is the number of solid or cubic units it contains.

The term *solid contents* is often used instead of *volume.*




A solid 4 ft. long, 3 ft. wide, and 5 ft. high (or thick), may be divided into 5 slabs, each containing 3 rows of blocks, as ABCD.

Each row contains 4 blocks, or *cubic feet,* hence 3 rows contain 12 cubic feet.

Since there are 12 cubic feet in 1 slab, in 5 slabs there are 5 times 12 cubic feet, or 60 cubic feet, which is the *volume.*

$(4 \times 3 \times 5) \times 1$ cu. ft. $= 60$ cu. ft. in the solid. What is the unit of measure here?

**327.** PRINCIPLE.—*The volume of a rectangular solid is expressed by the product of the numbers that represent its length, breadth, and thickness.*

Since the area of one face or side of a rectangular solid 4 ft. long and 3 ft. wide is 12 sq. ft., it will be observed that the number of *cubic* feet in 1 foot of thickness is the same as the area of one face.

Hence, to find the volume of a rectangular solid,

RULE.—*Multiply the number of cubic units in one unit of thickness or height by the number that represents the height.*

All dimensions must be expressed in like units.

**1.** An iron bar is 12 inches long, 1 inch wide, and 1 inch thick. How many cubic inches does it contain?

**2.** A marble slab is 12 inches square and 1 inch thick. How many cubic inches does it contain?

**3.** A block of coal is 12 inches long, 12 inches wide, and 12 inches thick. How many cubic inches in it? What may this solid be called? Why?

**4.** A cube of stone measures 3 feet on each side. How many cubic feet does it contain? What may this cube be called? Why?

## CUBIC MEASURE.

**328. Cubic Measure** is used in measuring solids.

### Table.

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.).
27 cubic feet = 1 cubic yard (cu. yd.).

| cu. yd. | | cu. ft. | | cu. in. |
|---|---|---|---|---|
| 1 | = | 27 | = | 46656 |

1. A pile of wood 8 ft. long, 4 ft. wide, and 4 ft. high is called a *cord.* It contains 128 cu. ft.

2. In measuring masonry or stone, $24\frac{3}{4}$ cu. ft. = 1 *perch.*

3. In finding the length of walls, masons measure the outside. The corners are thus counted twice.

4. A cubic foot of distilled water at its greatest density weighs 1000 oz. avoirdupois.

WRITTEN EXERCISES.

**329.** Reduce:

**1.** 3 cu. ft. to cu. in.
**2.** 5 cu. yd. to cu. in.
**3.** 6 cu. yd. to cu. ft.
**4.** 2 A. to sq. yd.
**5.** 1 cu. yd. 1 cu. ft. to cu. in.
**6.** 108 cu. ft. to cu. yd.

**7.** Reduce 594 cu. ft. to cubic yards.

**8.** Reduce 50000 cu. in. to higher denominations.

**9.** What part of a cubic yard is $\frac{9}{10}$ of a cubic foot?

**10.** Reduce $\frac{1}{2}$ cu. yd. to lower denominations.

**11.** How many cu. in. in .75 of a cubic foot?

**12.** A log 2 feet square is 12 ft. 5 in. long. How many cubic inches does it contain?

**13.** A cellar is 30 ft. long, 24 ft. wide, and 8 ft. deep. How many cu. yd. of earth were removed?

**14.** How many cu. ft. of air in a room 16 ft. square and 12 ft. high?

**15.** My granary is 8 ft. long, 4 ft. 3 in. wide, and 5 ft. deep. How many bushels of wheat will it hold?

**16.** A cubical cistern 8 ft. deep contains 2 ft. of water. How many gallons in it?

**17.** Some gold miners sunk a shaft 390 feet. If it was 4 ft. by 8 ft., how many cu. yd. of material were removed?

**18.** A trough 2 ft. wide and 1 ft. 3 in. deep is 8 ft. long. If a cu. ft. of water weighs 1000 oz., how many pounds of water will fill the trough?

**19.** If a box car is 40 ft. long, 8 ft. wide, and 9 ft. high, how many blocks of marble, each 4 ft. by 2 ft. by 1 ft. 6 in., will it hold?

**20.** A two-quart pail was half full of water. Several frogs jumped into the pail, and then it was full. What was the volume of the frogs?

### SUPPLEMENTARY EXERCISES (FOR ADVANCED CLASSES).

**330. 1.** At 30 cents a bushel, find the cost of a box of oats, the box being 8 feet long, 4 feet wide, and 4 feet deep.

**2.** A rectangular trough is $\frac{2}{3}$ full of water. After 35 gallons are taken out, it is $\frac{3}{8}$ full. What is the depth of the trough, the length being 10 feet and the width $2\frac{3}{4}$ ft.?

**3.** A two-acre pond is covered with ice 6 in. thick. If a cu. ft. of ice weighs 896 ounces, how many tons of ice are on the pond?

**4.** A tank 8 ft. long, $1\frac{1}{2}$ ft. wide, and 8 in. deep will contain how many pounds of water, a cu. ft. of water weighing $62\frac{1}{2}$ lb.?

**5.** What is the difference between a 3-foot cube and 3 cubic feet?

**6.** A cistern 6 ft. long and $4\frac{1}{2}$ ft. wide holds 108 cu. ft. of water. How many cu. in. of zinc will be required to line the sides and bottom, the zinc being $\frac{1}{8}$ in. thick?

**7.** What are the dimensions of a rectangular box whose capacity is 50274 cu. ft., the length, breadth, and depth being to each other as 3, 2, and 1?

**8.** A contractor received $45.50 for digging a cellar 18 ft. long and 15 ft. wide, at $.65 a cu. yd. To what depth did he dig it?

**9.** A man wishes to make a bin to contain 725 bushels, the width and depth to be equal, and the length to be double the width. What must be its dimensions?

**10.** How thin is a cu. in. of gold beaten so as to cover a space 46 ft. 10 in. by 41 ft. 8 in.?

**11.** How many gallons of water can be poured into a bushel measure?

**12.** A vessel 3 inches square contains some water. A gold chain dropped into the water raises the fluid $\frac{1}{2}$ in. What is the volume of the chain? What is its weight if the gold weighs 19.2 times as much as water and a cu. ft. of the water weighs 1000 oz.?

## SURVEYORS' MEASURE.

**331.** In measuring boundaries of land, in locating railroads, etc., and in computing the area of land, surveyors and engineers make use of measures not given in the ordinary tables under "Measures of Extension." They are as follows:

### SURVEYORS' LINEAR MEASURE.

TABLE.

| | | |
|---|---|---|
| 7.92 inches | = | 1 link (l.). |
| 25 links | = | 1 rod (rd.). |
| 4 rods | = | 1 chain (ch.). |
| 80 chains | = | 1 mile (mi.). |

The unit of surveyors' measure is *Gunter's Chain*, which is 100 links long.

SURVEYORS' SQUARE MEASURE.

TABLE.

| | |
|---|---|
| 16 square rods | = 1 square chain (sq. ch.). |
| 10 square chains | = 1 acre (A.). |
| 640 acres | = 1 square mile (sq. mi.). |

| | |
|---|---|
| 1 mile square | = 1 section. |
| 36 sections | = 1 township. |

## MEASURES OF TIME.

**332.** The time required for the earth to rotate once on its axis is called a *day,* which is the *unit* of time. Other divisions of time are shown in the following

TABLE.

| | |
|---|---|
| 60 seconds (sec.) | = 1 minute (min.). |
| 60 minutes | = 1 hour (hr.). |
| 24 hours | = 1 day (da.). |
| 7 days | = 1 week (wk). |
| 365 days | = 1 year (yr.). |
| 366 days | = 1 leap year (yr.). |

| da. | | hr. | | min. | | sec. |
|---|---|---|---|---|---|---|
| 1 | = | 24 | = | 1440 | = | 86400. |

**1.** A period of 100 years is called a *century,* 10 years a *decade.*

**2.** In business transactions 30 days are usually considered a month.

**3.** The time from midnight to noon is called *forenoon*—A.M.; that from noon to midnight is called *afternoon*—P.M.

THE CALENDAR.

**333.** The system of reckoning time by years and months, as found in our almanacs, is called a **Calendar.**

1. The time of one revolution of the earth around the sun—the solar year—is 365 da. 5 hr. 48 min. 46 sec., *nearly*. For ordinary purposes it is important that the year contain an exact number of days. The present calendar year secures this result. Its length is sometimes 365 and sometimes 366 days, but its *average* length is almost exactly equal to that of the solar year. The year of 366 days is called a leap-year, *and those years are leap-years whose date numbers are exactly divisible by 4, except centennial years, whose dates must be divisible by 400.*

2. On the supposition that the length of the year was $365\frac{1}{4}$ days, Julius Cæsar introduced a calendar in which *every year* whose date number is exactly divisible by 4 was to consist of 366 days, and all other years of 365. But as the year does not contain exactly $365\frac{1}{4}$ days, Cæsar's year was thus 11 minutes and about 14 seconds too long, and this would accumulate in 400 years to a little over 3 days. In 1582, Pope Gregory XIII., aiming to correct this error, arranged that only *each fourth centennial* year (*those exactly divisible by 400*) should be a leap-year. The Gregorian Calendar still leaves a very slight error, which will not amount to a day until about A.D. 5200.

3. The calendar of Cæsar is known as the *Julian Calendar*, or Old Style (O. S.) ; that which Gregory substituted is called the *Gregorian Calendar*, or New Style (N. S.). In changing from one calendar to the other, Gregory dropped out 10 days, so that the day after Oct. 4, 1582, was called Oct. 15. England adopted the New Style in 1750, by which time it had become necessary to drop out 11 days. The difference in 1899 was 12 days. From 1900 to 2100 it will be 13 days.

EXAMPLE.—Columbus discovered America, October 12, 1492, O. S. What is the date N. S.?

WRITTEN EXERCISES.

**334.** Reduce to lower denominations :

**1.** 3 hr. 10 min. 15 sec.
**2.** 2 da. 2 hr. 30 min.
**3.** 1 wk. 7 hr. 25 min.
**4.** 1 wk. 1 da. 1 hr.
**5.** 3 sq. yd. 5 sq. ft.
**6.** 5 cwt. 87 lb.

Reduce to higher denominations :

**7.** 1000 sec.
**8.** 2345 min.
**9.** 5050 hr.
**10.** 13258 wk.
**11.** 3675 da.
**12.** 3636 cu. in.

**13.** What part of a day is $\frac{1}{2}$ of an hour ?

**14.** Reduce $\frac{3}{5}$ of a day to hours and minutes.

**15.** What decimal part of a day is 8 hr. 40 min. ?

**16.** In 1894 how many days from January 1st to March 15 ? In 1896 ? In 1900 ?

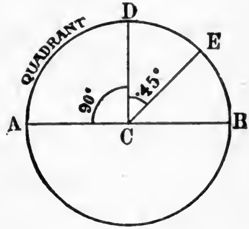
**17.** How many decades in one century and 15 years ?

**18.** John was paid $2.10 for working from 7 A.M. till 15 minutes past 12 M. How much was that an hour ?

**19.** George worked 5 da. 6 hr. 30 min. at the rate of $2 a day. How much did he earn ?

## CIRCULAR OR ANGULAR MEASURE.

**335.** An **Arc** of a circle is any part of the circumference ; as EB or BD.




**336.** When two lines are drawn from the circumference to the center, the arc between them is the *measure of the angle* formed.

Thus, the arc DE is the measure of the angle DCE.

**337.** In measuring angles, the circumference is divided into 360 equal parts, called *degrees ;* each degree into 60 parts, called *minutes ;* and each minute into 60 parts, called *seconds.*

1. The length of a degree varies. A degree is $\frac{1}{360}$ of a circumference, and the greater the circumference the greater the degree, or arc.

2. The angle measured by a degree does not vary.

**338.** **Circular** or **Angular Measure** is used in measuring angles, arcs of circles, in determining latitude and longitude, the location of vessels at sea, etc.

TABLE.

| | |
|---|---|
| 60 seconds (″) | = 1 minute (′). |
| 60 minutes | = 1 degree (°). |
| 360 degrees | = 1 circumference (cir.). |

One fourth of a circumference, or 90°, is called a *quadrant*. A *minute* of the circumference of the earth is called a *geographical* mile.

### MISCELLANEOUS TABLES.

| | |
|---|---|
| 12 things = 1 dozen. | **PAPER.** |
| 12 dozen = 1 gross. | 24 sheets = 1 quire. |
| 12 gross = 1 great gross. | 20 quires = 1 ream. |
| 2 things = 1 pair. | 2 reams = 1 bundle. |
| 6 things = 1 set. | 5 bundles = 1 bale. |
| 20 things = 1 score. | |

#### ORAL EXERCISES.

**339.** **1.** Mrs. A has 30 knives. How many sets has she?

**2.** Mary sold 5 doz. eggs at 2 cents apiece. How much did she get for them?

**3.** Harry bought 36 lemons at 20¢ a dozen. What did they cost him?

**4.** Sarah bought 5 quires of paper at 15 cents a quire, and sold it at a cent a sheet. How much did she gain?

**5.** Ten years ago Mr. Smith was 3 score and 10 years of age. How old is he now?

**6.** Half a dozen is what part of a gross?

**7.** What is the ratio of a dozen to a score?

**8.** How often is $\$\frac{1}{2}$ contained in \$.75?

**9.** A lady sold 200 eggs at 20 cents a dozen, and took her pay in sugar at 6 cents a pound. How many pounds did she get?

**10.** What has the same ratio to a foot that a yard has to an inch?

## ADDITION.

**340.** In simple numbers the scale is uniform, 10 units of any order being equal to 1 of the next higher order. In compound numbers the scale is varying.

In U. S. money the scale is uniform, and it is to be observed that while \$2.55 is a *simple* number, yet when written in the separate units \$2 5d. 5¢, it may be regarded as a *compound* number.

The addition, subtraction, multiplication, and division of compound numbers differ little in theory from the like operations in simple numbers. Their varying scale, however, causes the numbers to be written somewhat differently, and makes a slight difference in the processes.

### WRITTEN EXERCISES.

**1.** What is the sum of \$7 2d. 4¢ 5m., \$3 8d. 6¢ 7m., and \$2 7d. 5¢ 1m.?

(*a*)

| \$ | d. | ¢ | m. |
|---|---|---|---|
| 7 | 2 | 4 | 5 |
| 3 | 8 | 6 | 7 |
| 2 | 7 | 5 | 1 |
| 13 | 8 | 6 | 3 |

(*b*)

| |
|---|
| \$7.245 |
| 3.867 |
| 2.751 |
| \$13.863 |

The given numbers may be written either as in (*a*) or (*b*), since they have a decimal scale. Units of the same order should stand in the same column. The numbers in (*a*) are added as follows:

The sum of the mills is 13, which is equal to 1 cent and 3 mills. We write the 3 in the column of mills, and add the 1 to the cents, making 16¢, or 1 dime and 6¢. Writing the 6 in the column of cents, we add the 1d. to the dimes, making 18d., or \$1 and 8d. We write the 8 in the column of dimes, and add the \$1 to the dollars, making \$13.

☞ Have the pupil add the numbers in (*b*), and compare the process with that just explained.

**2.** What is the sum of 16s. 9d. 1 far., 18s. 8d. 3 far., and 9s. 10d. 3 far.?

| £ | s. | d. | far. |
|---|---|---|---|
| | 16 | 9 | 1 |
| | 18 | 8 | 3 |
| | 9 | 10 | 3 |
| 2 | 5 | 4 | 3 |

We write units of the same order in the same vertical column, regardless of the number of places they may occupy. Thus, 10d., being less than 1 shilling, must be placed in the pence column.

The sum of the farthings is 7, or 1d. 3 far. Writing the 3 in the column of farthings, we add the 1d. to the pence, making 28d., or 2s. 4d. Writing the 4 in the column of pence, we add the 2s. to the shillings, making 45s., or £2 5s., which we write in the proper columns.

Query.—Why do we not "carry" one for every *ten*?

**3.** Find the sum of 3 gal. 2 qt. 1 pt., 7 gal. 3 qt., and 8 gal. 1 qt. 1 pt.

**4.** Find the sum of 3 bu. 1 pk. 7 qt., 9 bu. 3 pk. 5 qt. 1 pt., 12 bu. 2 pk., and 7 bu. 1 pk. 6 qt.

**5.** Add 8 lb. 5 oz. 12 pwt. 16 gr., 12 lb. 9 oz. 4 pwt. 9 gr., 15 lb. 11 oz. 19 pwt. 22 gr., and 10 oz. 17 pwt.

**6.** What is the sum of 2 T. 15 cwt. 40 lb., 5 T. 8 cwt. 75 lb., 4 T. 9 cwt. 85 lb., and 13 T. 3 cwt.?

**7.** Add 1 mi. 40 rd. 5 yd. 2 ft. 6 in., 12 mi. 185 rd. 2 yd. 1 ft. 8 in., 5 mi. 316 rd. 4 yd. 7 in., 8 mi. 1 yd. 2 ft. 3 in., and 18 rd. 4 yd. 10 in.

When a fraction occurs in the sum, it should be reduced to lower denominations, and added to the proper columns.

**8.** What is the sum of 80 sq. rd. 25 sq. yd. 5 sq. ft. 75 sq. in., 136 sq. rd. 12 sq. yd. 8 sq. ft. 120 sq. in., 48 sq. rd. 9 sq. yd. 3 sq. ft. 56 sq. in., and 108 sq. rd. 136 sq. in.?

**9.** Find the sum of 18 cu. yd. 13 cu. ft. 87 cu. in., 9 cu. yd. 21 cu. ft. 1236 cu. in., 4 cu. yd. 25 cu. ft. 1600 cu. in., and 18 cu. ft. 760 cu. in.

**10.** Add 12 hr. 40 min. 34 sec., 7 hr. 8 min. 50 sec., 18 hr. 25 min. 26 sec., 19 hr. 15 min. 45 sec., and 6 hr. 50 min. 12 sec.

**11.** What is the sum of £16 12s. 7d. 3 far., £4 18s. 11d. 1 far., £2 16s. 9d. 2 far., and £3 8s. 7d. 1 far.

**12.** Find the value of $\frac{5}{8}$ bu. + $\frac{3}{4}$ pk. + $5\frac{1}{2}$ qt.

| | | | |
|---|---|---|---|
| $\frac{5}{8}$ bu. = | 2 pk. | 4 qt. | |
| $\frac{3}{4}$ pk. = | | 6 | |
| $5\frac{1}{2}$ qt. = | | 5 | 1 pt. |
| $\frac{5}{8}$ bu. + $\frac{3}{4}$ pk. + $5\frac{1}{2}$ qt. = | 3 pk. | 7 qt. | 1 pt. |

**13.** What is the value of $\frac{9}{14}$ wk. + $\frac{3}{5}$ da. + $\frac{3}{4}$ hr.?

**14.** Add $3\frac{1}{2}$ lb. 8 oz. $13\frac{1}{2}$ pwt., $1\frac{1}{4}$ lb. $3\frac{1}{3}$ oz. 10 pwt. 18 gr., and 7 lb. 6 oz. 17 pwt. 15 gr.

**15.** Add 45 gal., 3.7 qt., and 1.5 pt.

Solution same as in example 12.

**16.** Find the sum of .25 mi., 1.15 mi., and 120 rd. 18 ft.

**17.** The latitude of Pittsburg is 40° 27′ 36″ north, and that of the Cape of Good Hope is 33° 56′ 3″ south. How many degrees between the two places?

Why do we *add* to find the difference in the latitude of these places? Draw figure to illustrate.

**18.** A farmer sold corn as follows: To A, 75 bu. 1 pk.; to B, 37 bu. 3 pk.; to C, $110\frac{1}{2}$ bu.; to D, 18 bu. $3\frac{1}{4}$ pk.; to E, $42\frac{1}{5}$ bu. How much did he sell to all?

## SUBTRACTION.

### WRITTEN EXERCISES.

**341. 1.** Mr. A had \$20 8d. 5¢, and spent \$12 9d. 7¢. How much had he left?

(*a*)

| \$ | d. | ¢ |
|---|---|---|
| 20 | 8 | 5 |
| 12 | 9 | 7 |
| 7 | 8 | 8 |

(*b*)

| |
|---|
| \$20.85 |
| 12.97 |
| \$7.88 |

The given numbers may be written either as in (*a*) or (*b*), units of the same order standing in the same column. The numbers in (*a*) are subtracted as follows:

We can not take 7¢ from 5¢, hence we take 1d. (10¢) from the 8d., and add it to the 5¢, making 15¢. Then 7¢ from 15¢ leaves 8¢, which we write in the column of cents.

Having taken 1d. from the 8d., only 7d. remains, from which 9d. cannot be taken. Hence we take \$1 (10d.) from the \$20, and add it to the 7d., making 17d. Then 9d. from 17d. leaves 8d., which we write under dimes.

Having taken \$1 from the \$20, only \$19 remain. Then \$12 from \$19 leaves \$7, which we write as the dollars of the remainder.

☞ Have the pupil subtract the numbers in (*b*), and compare the process with that just explained.

**2.** B has £10 8s. 6d. and D has £6 5s. 10d. less than B. How much has D?

**3.** From 5 bu. 1 pk. 6 qt. 1 pt., take 2 bu. 3 pk. 5 qt.

**4.** From 13 gal. 3 qt., take 8 gal. 2 qt. 1 pt.

**5.** Take 2 T. 18 cwt. 90 lb. from 4 T. 15 cwt. 80 lb.

**6.** From 3 cu. yd. take 21 cu. ft. 1628 cu. in.

**7.** Mr. B had a two-acre lot from which he sold 1 A. 75 sq. rd. How much had he left?

**8.** From a barrel containing 41 gal. 2 qt. 1 pt. of molasses, 10 gal. 3 qt. were sold at one time and 23 gal. 1 qt. 1 pt. at another time. How much remained in the barrel?

**9.** From 10 lb. 10 oz. 10 pwt. 10 gr., take 5 lb. 11 oz. 12 pwt. 13 gr.

**10.** A man ate lunch at 12 o'clock 15 min. P.M., and dinner at 6 o'clock 45 min. P.M. How long was it between those meals?

**11.** The latitude of Pittsburg is 40° 27′ 36″ north, and that of Washington is 38° 53′ 39″ north. Find their difference of latitude.

**12.** From $\frac{5}{8}$ A. take 42.5 sq. rd.

| | sq. rd. | sq. yd. | sq. ft. | sq. in. |
|---|---|---|---|---|
| $\frac{5}{8}$ A. | = 100 | | | |
| 42.5 sq. rd. | = 42 | 15 | 1 | 18 |
| $\frac{5}{8}$ A.—442.5 sq. rd. | = 57 | $14\frac{1}{4}$ | 7 | 126 |
| | | $\frac{1}{4}$ | = 2 | 36 |
| " " | = 57 | 15 | 1 | 18 |

QUERY.—Why do we change $\frac{1}{4}$ sq. yd. to 2 sq. ft. 36 sq. in.?

**13.** From $\frac{5}{9}$ mi. take $234\frac{3}{4}$ rd.

**14.** From $\frac{4}{5}$ wk. take 2.6 da.

**15.** Take 1 pk. 1 qt. 1 pt. from .7 bu.

**16.** From $\frac{7}{8}$ gross take $6\frac{2}{3}$ doz.

**17.** How long was it from May 12, 1892, to July 4, 1899?

| yr. | mo. | day. |
|---|---|---|
| 1899 | 7 | 4 |
| 1892 | 5 | 12 |
| 7 | 1 | 22 |

The later date expresses the greater period of time, hence it is the minuend. The later date is the 4th day of the 7th month of 1899. The other date is the 12th day of the 5th month of 1892.

We subtract as in other compound numbers, considering 30 days as a month, as given in the table.

QUERY.—Does the remainder express the *exact time* between the given dates? Why not?

☞ To find the exact time between two dates (as, for example, Aug. 24 and Dec. 3, same year) we must proceed as follows:

$$7 + 30 + 31 + 30 + 3 = 101.$$

That is, there are 7 more days in Aug., 30 in Sept., 31 in Oct., 30 in Nov., and 3 in Dec. The sum is the difference in time expressed in *days*.

**18.** The War between the States began April 11, 1861, and ended April 9, 1865. How long did it continue?

**19.** Find the time from Oct. 15, 1812, to June 3, 1912.

**20.** A note dated June 19, 1897, was paid Oct. 12, 1898. How long did it run?

**21.** What is your age to-day?

**22.** How long is it from to-day to Feb. 29, 1920?

**23.** Independence was declared July 4, 1776. How long since that important event occurred?

## MULTIPLICATION.

### WRITTEN EXERCISES.

**342. 1.** Multiply 4 gal. 3 qt. 1 pt. by 9.

(*a*)

| gal. | qt. | pt. |
|---|---|---|
| 4 | 3 | 1 |
| | | 9 |
| 43 | 3 | 1 |

(*b*)

9 times 1 pt. = 9 pt., or 4 qt. 1 pt.
9 times 3 qt. + 4 qt. = 31 qt., or 7 gal. 3 qt.
9 times 4 gal. + 7 gal. = 43 gal.
∴ the product is 43 gal. 3 qt. 1 pt.

The calculation is conveniently made as in (*a*).

**2.** Multiply £6 12s. 8d. by 5.

**3.** Multiply 4 lb. 9 oz. 6 pwt. 13 gr. by 11.

**4.** Multiply 6 wk. 4 da. 8 hr. 20 min. by 12.

**5.** A grocer bought 8 bags of chestnuts, each containing 3 bu. 1 pk. 5 qt. How many did he buy?

**6.** If he sold the chestnuts at 6 cents a qt., how much did he get for them ?

**7.** What is the difference in time between 366 common years and 366 leap years ?

**8.** How far is it around a square that measures 4 yd. 2 ft. 8 in. on a side ?

**9.** How much calomel can be put into 20 bottles, if each bottle holds ℥ 6 ℈ 2 gr. xv ?

**10.** If $20 will buy 1 T. 6 cwt. 80 lb. of hay, how much hay will $240 buy ?

**11.** When silver is worth $.60 an ounce, how much must be paid for 5 oz. 12 pwt. 16 gr.?

**12.** When vinegar is 6 cents a quart, how much must be paid for $\frac{7}{8}$ gal. ?

## DIVISION.

### WRITTEN EXERCISES.

**343. 1.** A bbl. of syrup containing 43 gal. 3 qt. 1 pt. was shared equally by 9 persons. How much did each one get ?

(*a*)

| | gal. | qt. | pt. |
|---|---|---|---|
| 9) | 43 | 3 | 1 |
| | 4 | 3 | 1 |

(*b*)

43 gal. ÷ 9 = 4 gal., and 7 gal. (28 qt.) remaining.
28 qt. + 3 qt. = 31 qt.
31 qt. ÷ 9 = 3 qt., and 4 qt. (8 pt.) remaining.
8 pt. + 1 pt. = 9 pt.
9 pt. ÷ 9 = 1 pt.
∴ the quotient is 4 gal. 3 qt. 1 pt.

QUERY.—Is the dividend here the product in example 1 in Art. 342 ? How do we find one of the two factors when the product and the other factor are given ?

**2.** If 5 horses eat 11 T. 16 cwt. 80 lb. of hay in a year, how much does one horse eat ?

**3.** The distance around a square field is 155 rd. 5 yd. 2 ft. What is the length of one side ?

4. If 25 bu. 3 pk. of oats fill 9 bags of the same capacity, how much do 2 bags contain?

5. The weight of 8 silver chains is 18 oz. 12 pwt. 16 gr. What is the average weight?

6. A druggist made ʒ2 ℈1 gr. x of calomel into 15 powders. How much was in each powder?

7. The distance around a rectangular field is 128 rd. 12 ft., and one side is 42 rd. 4 ft. What is the length of one end?

8. How many barrels will contain 36 bu. 1 pk. 4 qt. of chestnuts, if each barrel holds 3 bu. 1 qt.?

Reduce both numbers to the same denomination, and divide.

9. If one man can earn £4 12s. 10d. in a week, how many men can earn £46 8s. 4d. in the same time?

10. How often can 1 ft. 8 in. be sawed from a board 13 ft. 4 in. long?

11. How many dress patterns of $14\frac{3}{4}$ yd. each can be cut from a piece of cloth containing $44\frac{1}{4}$ yd.?

12. How many blocks 1 ft. square can be cut from a board 12 ft. long and 2 ft. wide?

13. A two-acre lot is 8 rd. wide. How many yards is it around the lot?

14. Six men and 2 boys weigh 1152 lb. 12 oz. If each boy weighs 70 lb. 10 oz., what is the average weight of a man?

15. How many steel rails, each 24 ft. long, are required for a mile of single-track railroad?

16. How often can a bucket that holds 2 gal. 3 qt. 1 pt. be filled from a hhd. containing 60 gal. 1 qt. 1 pt.?

17. If each step measures 2 ft. 8 in., how many steps will a man take in walking $\frac{2}{3}$ of a mile?

18. A field is 165 ft. long and 66 ft. wide. What will it cost to fence it at $3 a rod?

19. If I exchange $1023.10 for English money, how many pounds, shillings, and pence do I receive?

# REVIEW WORK.

## ORAL EXERCISES.

**344. 1.** What is the difference between a square yard and 3 square feet?

**2.** What is the difference between a cubic yard and 3 cubic feet?

**3.** What will a gallon of cream cost at 12 cents a pint?

**4.** At $4 a bushel, what will be the cost of 3 pk. 4 qt. of clover seed?

**5.** How many inches in 5 feet 9 in.?

**6.** In 3.5 pwt. how many grains?

**7.** What will 2 gallons of vinegar cost, if 2 pints cost 8 cents?

**8.** A field is 40 rods long and half as wide. How many acres in it?

**9.** If 9 pints of chestnuts cost 45 cents, how much will half a bushel cost?

**10.** How many ounces in $6\frac{1}{4}$ lb. avoirdupois?

**11.** At a cent a pound, how much will a ton of iron cost?

**12.** How many square feet in 5 sides of a 4-inch cube?

**13.** The distance around a square surface is 48 inches. How many square feet does it contain?

**14.** From $\frac{3}{4}$ of a score take $\frac{2}{3}$ of a dozen.

**15.** If you sleep 8 hours each night, how many days do you sleep in 2 weeks?

**16.** Mr. B left home on Friday, and was gone 25 days. On what day of the week did he return?

**17.** My horse eats 9 qt. of oats a day. How long will 2 bu. $\frac{1}{4}$ pk. last him?

**18.** A rope is 10 yards long. Into how many pieces each $\frac{1}{4}$ ft. long can it be cut?

**19.** If there are 6 ties to every rod of track, how many ties will it take for $\frac{1}{8}$ of a mile of double-track railroad?

**20.** How many pens in $\frac{1}{4}$ gross and $1\frac{1}{6}$ dozen?

**21.** How many square inches in the surface of a rectangle 12 feet long and $\frac{1}{2}$ inch wide?

**22.** A lady put a half gallon of perfume into gill bottles. How many bottles were required?

**23.** Mr. A's horse is 15 hands high, and Mr. B's is $16\frac{1}{2}$ hands high. How many inches higher is Mr. B's horse?

**24.** How many cubic yards of air in a room 4 yards wide, 6 yards long, and 9 feet high?

**25.** How many days between Feb. 10 and March 10, 1900?

**26.** A man traveled $\frac{3}{8}$ of the distance around a circular park. Through how many degrees did he travel?

**27.** If I receive $\$\frac{1}{2}$ for working 45 minutes, how much should I receive for 9 hours' labor?

**28.** What is the entire surface of a cube whose edge is $a$ feet? What is the volume?

**29.** What is the value of $b$ gallons of milk at $a$ cents a pint?

**30.** If $p$ bales of hay weigh $q$ pounds, how much will $a$ bales weigh?

### WRITTEN EXERCISES.

**345. 1.** A lot is 120 feet deep and 60 feet front. How many square yards does it contain?

**2.** A gardener put 3 bu. 1 pk. 7 qt. of berries into quart boxes. How many boxes were required?

**3.** How many quarts of milk will fill a peck measure?

**4.** How many ounces of quinine in 5 lb. avoirdupois?

**5.** What will be the cost, at \$.35 a square yard, of painting 5 sides of a cube whose edge is 4 feet?

**6.** How many five-grain pills can a druggist make from ℥5 ℈1 of calomel?

**7.** How many gold dollars weigh a pound avoirdupois?

**8.** How many gold dollars can be coined from a pound of gold?

**9.** How many pounds of silver are required to coin 100 silver dollars?

**10.** If a boy idles away $\frac{1}{4}$ hr. a day, how much time will he lose in a year?

**11.** A 20-acre field is 80 rods long. Find the cost of fencing it at $.75 a rod.

**12.** The wheel of a bicycle is 7 ft. 4 in. in circumference. How often will it revolve in going a mile?

**13.** At $2\frac{3}{4}$ cents a pound, how many barrels of flour can be bought for $53.90?

**14.** The diameter of a circular field is 40 rods, and the side of a square field is the same. Which has the greater area, and how much?

**15.** Find the cost of fencing the fields just mentioned, at $1.50 a rod.

**16.** When land is worth $80 an acre, what is the value of a field in the form of a trapezoid, whose altitude is 30 rods, and whose parallel sides are 48 and 32 rods respectively?

**17.** Bought 2 lb. silver by avoirdupois weight, paying $8 a pound, and sold it by troy weight at 80 cents an ounce. How much did I gain?

**18.** Bought 3 lb. quinine at $4.50 a pound avoirdupois, and sold it at $6 a pound apothecaries' weight. What did I gain?

**19.** A traveler returning from Europe had £10, 10 shillings, 10 francs, and 10 marks, which he exchanged for U. S. money. How much did he get?

**20.** A man in Boston bought 2880 lb. of wheat, at $.85 a bushel, and sold it at $.91 a bushel. How much did he gain?

**21.** The altitude of a triangular field is 32 rods, and its area is 8 acres. Find the length of the base.

**22.** What is the radius of a wheel if an arc of 15° of its circumference is 1 ft. 2 in. in length?

**23.** Find the cost of 8 yd. 1 ft. 5 in. of pipe, 4 lb. to the foot, at 24¢ a pound.

**24.** What is the value of $\frac{1}{4}$ of a square mile of land at $6 an acre?

**25.** How many steel rails 32 feet long are needed to build 2 miles of double-track railway?

**26.** Bought 6 gross of lead pencils at $3.50, and sold them at 5 cents apiece. Find the gain.

**27.** My little girl is "worth her weight in gold" dollars. If she weighs 30 pounds, what is she worth?

**28.** I sold 138 eggs at 30 cents a dozen, and enough butter at 35 cents a pound to make my total receipts $4.50. How many pounds of butter did I sell?

**29.** The water in my cistern, which is 6 ft. long, 5 ft. wide, and 8 ft. deep, weighs 15000 pounds. Is the cistern full? If so, what is the weight of a cubic foot of water?

**30.** Mr. H has a feed-box 8 ft. long, 3 ft. wide, and 4 ft. high. When it is half full of oats, how many bushels does it contain?

**31.** How many square feet of zinc will be required to line the sides and bottom of a bin 12 ft. square and 5 ft. high?

**32.** If tobies cost $9.50 a thousand, and a man smokes 4 a day, what is the amount of his annual toby bill?

**33.** How many cubes measuring 6 inches each way can be cut from a cubic yard of marble?

**34.** Measure your schoolroom, and find how many cubic feet of air there are to each pupil.

**35.** What will be the cost of painting a box 2 feet square and 10 feet high, at $.25 a sq. yd.?

**36.** How many square yards of canvas will be needed to cover two boxes, each of whose bases is 6 feet square, the height of one being 4 feet and that of the other 5 feet?

**37.** What part of the annual revolution does the earth make in 24 da. 6 hr. 47 min.?

**38.** If 1 bu. 3 pk. of seed is required to plant an acre, how much must be sowed upon 2 A. 16 sq. rd.?

**39.** 3960 cu. yd. of earth were removed in digging a ditch 360 rd. long and 6 ft. deep. How wide is the ditch?

**40.** A druggist pays $53\frac{1}{3}$¢ a pound avoirdupois for 9 lb. of borax. If he sells it at the rate of $6\frac{1}{4}$¢ an ounce apothecaries', how much does he gain?

**41.** How many silver forks, each weighing 2.3 oz., can be made from 9 lb. 18 pwt. of silver?

**42.** How many sq. yd. in the walls and ceiling of a room 24 ft. long, 18 ft. wide, and 12 ft high?

**43.** How many cords of wood in a pile 336 ft. by 9 ft. by 6 ft.?

**44.** If it takes a man $\frac{5}{6}$ of a day to mow an acre of grass, how long will it take him to mow 3 A. 45 sq. rd. 16 sq. yd.?

**45.** If $\frac{15}{16}$ of a bushel of salt can be made from 54 gal. of salt water, how much salt can be made from 72 gal.?

**46.** A plank 37 ft. 4 in. long and 4 in. thick contains $4\frac{2}{3}$ cu. ft. What is the width?

**47.** A man who had 2 miles to travel, walked 5 rd. 7 ft. How far had he yet to travel?

**48.** If 5.3 T. of porcelain clay cost $106, what is the cost of 438 pounds?

**49.** How many cubic yards of air in a room 18 ft. by 15 ft. by 9 ft.?

**50.** What must be the depth of a measure $18\frac{1}{2}$ in. square to contain a bushel?

**51.** If 4 bu. 3 pk. 4 qt. 1.6 pt. of wheat makes one barrel of flour, and the toll is 4 qt. a bushel, how many bushels of wheat must I take to the mill in order to get five barrels ot flour?

**52.** When corn meal is selling at 80¢ per cwt., how many pounds will 10¢ buy?

**53.** A building lot 100 ft. front contains 1 acre. How deep is the lot ?

**54.** How many cakes of maple sugar 8 inches by 6 inches by 3 inches can be packed in a box 24 inches by 18 inches square in the clear ?

**55.** If a load of wood is 8 ft. long and 3 ft. wide, how high must it be to contain a cord ?

**56.** When each end and the middle of boards $16\frac{1}{2}$ feet long are nailed to posts, how many posts are in a fence around a field 30 rods long and 20 rods wide ?

SUPPLEMENTARY EXERCISES (FOR ADVANCED CLASSES).

**346. 1.** How many seconds in the month of February, 1899 ?

**2.** How long will it take a body moving at the rate of a mile a minute to travel from the earth to the moon, the distance being 239,000 miles ?

**3.** How many seconds in the circumference of a silver dollar?

**4.** Which is heavier, and how much—a pound and an ounce of gold, or a pound and an ounce of lead ?

**5.** How many pounds of gold are equal in weight to 6 lb. of feathers ?

**6.** How many bushels of corn will a receptacle contain that holds 5,000 gallons of water ?

**7.** How long will it take to count a million, counting 80 a minute, and 12 hours a day ?

**8.** A printer used 3 reams, 5 quires, 19 sheets of paper for printing half-sheet sale bills. How many did he print, allowing 1 quire to a ream for waste ?

**9.** There are 9 oz. of iron in the blood of one man. The blood of how many men would be required to furnish sufficient iron to make a kettle weighing $22\frac{1}{2}$ lb. ?

**10.** A merchant bought a barrel (42 gal.) of vinegar for \$6.30. How much water must be added to reduce the first cost to 10¢ a gallon ?

**11.** In $18\frac{3}{4}$ carat gold, what part is alloy ?

**12.** How many silver dollars can be made from a bar of silver weighing 11 lb. 9 oz. avoirdupois ?

**13.** How many tucks $\frac{1}{4}$ inch wide can be made in a strip of muslin a yard long, leaving $\frac{1}{8}$ of an inch between the edge of one tuck and the stitching of the next ?

## LONGITUDE AND TIME.

**347. 1.** Does the earth rotate from west to east, or from east to west ? In how many hours does it rotate *once?*

**2.** In what time does any place on the earth's surface pass through 360° ? Through how many degrees does it pass in 1 hour ? In 1 minute ? $\frac{1}{60}$ of 15° = ( )? $\frac{1}{4}$° = how many ′?

**3.** Since the earth moves 15′ in 1 minute of time, how far does it move in 1 second of time ? $\frac{1}{4}$′ = how many ″?

**4.** How long does it take the earth to turn through 15° ? 30° ? 60° ? 90° ? 180° ?

**5.** How does the number of hours compare with the number of degrees traveled ?

**6.** Which city has sunrise first—Baltimore or Chicago ? Why ?

**7.** If my watch shows the correct time when I leave Boston, will it be too slow or too fast when I reach Denver ?

**8.** A man from Cleveland arrived at another city and found his watch 20 minutes too slow. Had he traveled east or west ? How many degrees ?

**348.** A **Meridian** (mid-day line) is an imaginary line running north and south from pole to pole. All places on the same meridian have their mid-day, or noon, at the same moment ; that is, when the sun's rays are vertical on that meridian.

**349.** The **Longitude** of a place is its distance (in degrees, etc.) east or west from a given meridian.

**350.** The given meridian from which longitude is generally reckoned is called the *Prime Meridian.* It passes through Greenwich, a part of London.

NOTES.—1. From the prime meridian, longitude is reckoned east and west to 180°. West longitude is designated by the letter W; east longitude by the letter E.

2. Clocks show *later* (faster) time at places *east* of a given place, and *earlier* (slower) time at places *west* of a given place.

**351.** Since the earth turns upon its axis once in 24 hours, any place on the earth's surface passes through 360° in that time. Hence we deduce the following

TABLE OF RELATIONS.

| LONGITUDE. | | | TIME. |
|---|---|---|---|
| 360° | corresponds | to | 24 hours. |
| 15° | " | " | 1 hour. |
| 15′ | " | " | 1 minute. |
| 15″ | " | " | 1 second. |
| 1° | " | " | $\frac{1}{15}$ hr., or 4 min. |
| 1′ | " | " | $\frac{1}{15}$ min., or 4 sec. |

**352. To find the difference in time between two places, the difference in longitude being given.**

WRITTEN EXERCISES.

**1.** Boston is 71° 3′ 30″ west of London. What is the difference in time?

$$\begin{array}{r|rrr} 15) & 71° & 3' & 30'' \\ \hline & 4 & 44 & 14 \end{array}$$

It will be seen by the table that the numbers denoting the difference in time between two places are $\frac{1}{15}$ of those denoting longitude. The process is the same as in division of compound numbers, and the difference in time is 4 hours, 44 minutes, and 14 seconds.

**2.** The difference in longitude between two places is 50° 25′. What is the difference in time?

**3.** Cincinnati is 84° 29′ 45″ west from Greenwich. Find the difference in time.

**4.** When it is noon at Cincinnati, what is the time at Greenwich?

SHORT LIST OF CITIES, WITH LONGITUDE FROM GREENWICH.

| CITIES. | LONGITUDE. | | CITIES. | LONGITUDE. | |
|---|---|---|---|---|---|
| Greenwich | 0° 0′ 0″ | | St. Louis | 90° 15′ 15″ | W. |
| New York | 73° 58′ 25.5″ | W. | San Francisco | 122° 25′ 40.8″ | W. |
| Paris | 2° 20′ 15″ | E. | St. Petersburg | 30° 19′ 0″ | E. |
| Boston | 71° 3′ 30″ | W. | Richmond | 77° 26′ 4″ | W. |
| Chicago | 87° 36′ 42″ | W. | Denver | 104° 59′ 33″ | W. |
| Rome | 12° 27′ 14″ | E. | Pittsburg | 80° 2′ 0″ | W. |
| Cleveland | 81° 40′ 30″ | W. | Honolulu | 157° 51′ 48″ | W. |
| Washington | 77° 3′ 0″ | W. | Jerusalem | 35° 13′ 25″ | E. |
| Calcutta | 88° 20′ 8″ | E. | Athens | 23° 43′ 55.5″ | E. |

Find the difference in time between:

**5.** Boston and Pittsburg.

**6.** Chicago and Washington.

**7.** Rome and Calcutta.

**8.** San Francisco and Denver.

**9.** Philadelphia and Cleveland.

**10.** New York and Paris.

**11.** Richmond and St. Petersburg.

**12.** Honolulu and Jerusalem.

**13.** When it is noon at New York, what is the time at Denver?

**14.** Would a traveler's watch be too fast or too slow, and how much, when he goes from St. Louis to Athens?

**353. To find the difference in longitude between two places, the difference in time being given.**

**1.** When it is noon at San Francisco, it is 8 minutes 45 seconds past 2 P.M. at St. Louis. What is their difference in longitude?

| hr. | min. | sec. |
|---|---|---|
| 2 | 8 | 45 |
| | | 15 |
| 32 | 11 | 15 |

It will be seen by the table that there are 15 times as many °, ′, and ″ of longitude between two places as there are hours, minutes, and seconds of time. The process is the same as in multiplication of compound numbers, and the difference in longitude is 32° 11′ 15″.

**2.** What is the difference in longitude between two places whose difference in time is 4 hours, 12 minutes, and 22 seconds?

Find the difference in longitude between:

**3.** New York and Chicago.

**4.** Philadelphia and Cleveland.

**5.** Richmond and Denver.

**6.** Cleveland and Greenwich.

**7.** Boston and San Francisco.

**8.** Washington and Paris.

**9.** Pittsburg and Calcutta.

**10.** Rome and St. Petersburg.

**11.** When a traveler reached Boston he found that his watch was 4 hr. 25 min. too fast. What was the longitude of the place from which he started?

**12.** When it is 12 o'clock M. at Pittsburg, it is 9 hr. 10 min. 18 sec. A.M. at Portland, Oregon. What is the longitude of Portland?

**13.** Mr. R left Philadelphia and traveled eastward at the rate of $\frac{1}{3}°$ an hour. How much too slow was his watch at the end of two weeks?

**14.** The difference in time between two places on the equator is 2 hr. 45 min. 30 sec. In how many hours could a railroad train run from one place to the other at the rate of 30 geographic miles an hour?

**15.** Midnight comes 1 hr. 5 min. 42 sec. sooner at Quebec than at Chicago. What is the longitude of Quebec?

**16.** At Richmond the sun rises 1 hr. 2 min. 52 sec. earlier than at St. Paul, and 2 hr. 59 min. 49 sec. earlier than at San Francisco. What is the difference in longitude between St. Paul and San Francisco?

**354.** To find the difference in the longitude of two places:

1. *If both longitudes are* **east,** *or if both are* **west, subtract**; *if one is east and the other west,* **add.**

2. *If the sum of two longitudes is greater than* 180°, *the sum must be subtracted from* 360° *to obtain the correct difference of longitude.*

## STANDARD TIME.

**355.** The time we have been considering is called *Local Time,* which is determined by the rotation of the earth on its axis. To avoid the confusion and mistakes incident to such time, the railroad companies have adopted the time of the meridians of 75°, 90°, 105°, and 120° as the *standards* by which to run their trains. This railroad time is called **Standard Time.**

**356.** This new plan divides the United States and Canada into 4 belts, extending north and south, each about 15° wide. *All places in the same belt have the same time, regardless of their longitude.*

NOTE.—The division lines of the time belts are not exactly $7\frac{1}{2}°$ on either side of the hour meridians, but are somewhat irregular, passing through leading railway termini.

☞ Have the pupil turn to map in his geography showing the standard time belts. Which is the widest?

**357.** The first belt lies on both sides of the meridian of 75°, which passes 1° west of New York city, and all places therein have the time of that meridian, which is called **Eastern time.**

Thus, when a man goes from Richmond to New York, Boston, or Quebec, he finds that his watch is neither too fast nor too slow.

**358.** The second belt lies on both sides of the meridian of 90°, which passes near New Orleans and St. Louis. The time in this belt is called **Central time,** and is 1 hour slower than Eastern time.

QUERY.—Why is Eastern time just 1 hour faster than Central time?

**359.** The third belt lies on both sides of the meridian of

105°, which passes near Pike's Peak. The time in this belt is called **Mountain time,** and is 1 hour slower than Central time, and 2 hours slower than Eastern time.

QUERY.—A man who goes from Baltimore to Denver will find his watch how much too fast?

**360.** The fourth belt lies on both sides of the meridian of 120°, which passes 9 degrees east of San Francisco. The time there is 3 hours slower than Eastern time, and 1 hour slower than Mountain time. It is called **Pacific time.**

QUERY.—When it is 9 A.M. at San Francisco, what is the time at New York? At Leadville? At St. Paul?

COMPARISON OF TIMES.

| | BOSTON | CHICAGO | DENVER | PORTLAND (OR.) |
|---|---|---|---|---|
| Standard Time, | 12 M. | 11 A.M. | 10 A.M. | 9 A.M. |
| Local Time, | 12 M. | 10:53$\frac{3}{4}$ A.M. | 9:44$\frac{1}{4}$ A.M. | 8:34$\frac{2}{5}$ A.M. |

☞ On the 4 standard meridians, *local* time and *standard* time are the same. In other places standard time, being fixed arbitrarily, is not the correct or local time.

WRITTEN EXERCISES.

**361. 1.** At Pittsburg, what is the difference between local time and standard time?

| | |
|---|---|
| Longitude of Pittsburg, | 80° 2′ 0″ W. |
| Longitude of the meridian of 75°, | 75° 0′ 0″ W. |
| | 15) 5° 2′ 0″ |
| | 20 8 |

Pittsburg has the time of the meridian of 75° instead of that of her own meridian. The difference in longitude between the two meridians is readily found, and the difference in time is $\frac{1}{15}$ as great. Hence, standard time at Pittsburg is 20 min. 8 sec. faster than local time.

**2.** What is the difference at New York between standard and local time?

**3.** When it is 3 P.M., standard time, at San Francisco, what is the local time there?

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**4.** The longitude of Staunton, Va., is 79° 4′ 15″. When it is 5 A.M. there, standard time, what is the local time?

**5.** When it is 12 M. at Paris, what is the standard time at New York? The local time?

**6.** What is the local time at Denver when the standard time at Cleveland is 9 A.M.?

## DATE LINE.

**362.** The boundary line between adjoining regions in which the calendar day is different is called a **Date Line.** This line as now agreed upon nearly coincides with the meridian of 180°.

NOTE.—Suppose a man to leave London at noon on Friday and travel westward just as fast as the earth rotates. He keeps the sun directly overhead all the time, and it seems to him that it is still Friday noon when he reaches London again 24 hours after starting, when in reality it is Saturday noon. He has *lost* a day in his reckoning. Had he traveled eastward he would have *gained* a day. There is the same loss and gain when traveling is done at ordinary speed.

Hence is seen the necessity for a *fixed* place at which each new day shall begin. This place is the *date line;* there a new day begins every midnight. Thus, July 4th begins on the meridian of 180° at midnight following the 3d of July. *At that time* it is midday, July 3, at London (Greenwich); 12 hours later it is *midnight* at London, and that city enters upon the new date—July 4; it is then noon of the 4th on the date line, while at Chicago it is about 6 P.M., July 3.

Each *day* has the same *date* all around the earth. Thus, Sunday, May 1, 1899, began at midnight on the date line, and thereafter as each place on the earth had midnight it began to record the date as "Sunday, May 1."

It is a day later on one side of the date line than it is on the other. Thus, when it is Monday in San Francisco it is Tuesday in Hong Kong. Hence navigators change their calendar one day in crossing this date line. Going *east,* they count the same day twice; going *west,* they skip a day.

QUERIES.—1. When it is noon, Jan. 1, 1900, at New York, what is the *date* and *time* on the island of Luzon, longitude 120° E.?

2. When it is Saturday noon at Athens, what is the *day* and local *time* at Chicago?

3. When it is 2 A.M. on Friday at Calcutta, what is the day and hour at Boston?

SUPPLEMENTARY EXERCISES (FOR ADVANCED CLASSES).

**363. 1.** If A leaves home at 12 M. on Monday, and on Saturday finds his watch 1 hr. 15 min. slow, in what direction and how far has he traveled?

**2.** A man starts from Philadelphia and travels westward. When he stops, he ascertains that his watch is 6 hr. slow. In what longitude did he stop? Through how many degrees did he travel?

**3.** A man traveling eastward from a point on the equator stops when his watch has become 2 hr. 40 min. slow. How many miles has he traveled?

**4.** If a man starts westward from Paris and travels 181°, will his watch be too fast or too slow, and how much?

**5.** A and B start from different points and travel towards each other. When they meet, A's watch is 40 minutes slow, and B's is 1 hour fast. How far apart are the starting-points? In what direction did each travel?

**6.** When it is noon, local time, at New York, on what meridian is it midnight?

**7.** When it is Sunday noon, local time, at Chicago, what is the day and time at Jerusalem?

**8.** A vessel had sailed a certain distance on a parallel of latitude when the captain found that, although the sun was on the meridian, his Greenwich chronometer indicated 1:47 P.M. What was the ship's longitude?

**9.** The battle of Manila began at 6 A.M., May 1. Had Dewey cabled at once to Washington, at what hour and on what date would the message have reached the President, allowing 1 hour for transmission, and considering the longitude of Manila to be 120° E.?

# PRACTICAL MEASUREMENTS.

## PAINTING AND PLASTERING.

**364.** Painting and plastering are usually estimated by the *square yard.*

Allowances for openings are sometimes made, but there is no fixed rule as to how much should be deducted.

**1.** Find the cost of painting the outside of a house 36 ft. long, 28 ft. wide, and 24 ft. high, at $.25 a sq. yd.

**2.** A room 18 ft. long, 15 ft. wide, and 12 ft. high has 2 doors, each 8 ft. by 4 ft. 6 in. Find the cost of plastering the walls and ceiling at $.35 a sq. yd., making allowance for the doors.

**3.** A parlor is 15 ft. 9 in. square, and a reception-room is 12 ft. by 16 ft. The height of each is 11 ft. 6 in. What will be the cost of plastering the walls and ceilings at $.33 a sq. yd.?

**4.** How much would be saved by having the ceilings kalsomined at a cost of $.18 a sq. yd.?

**5.** Measure your schoolhouse, and prepare a problem in painting for your class.

**6.** Find the cost of wainscoting a hall 30 ft. long, 8 ft. wide, and 12 ft. 8 in. high, at $.40 a sq. yd.

**7.** Measure your schoolroom, and prepare a problem in plastering for your class.

**8.** Find the cost of painting a gable roof 45 ft. long and 24 ft. wide, at 28 ct. a sq. yd.

## MEASUREMENT OF LUMBER.

**365.** When boards are 1 inch thick or less, they are estimated by the square foot of surface, the thickness not being considered.

Thus, a board 8 ft. long, 1 ft. wide, and 1 inch (or less) thick contains 8 square feet. Its surface is a rectangle.

**366.** A board 1 foot square and 1 inch thick is called a *board foot,* or a foot *board measure.*

When lumber is more than 1 inch thick, the number of board feet depends upon the thickness.

Thus, a board 8 feet long, 1 foot wide, and 2 inches thick contains 16 board feet, or *twice* as many as if only 1 inch thick.

**1.** How many square feet in a board 12 ft. long and 12 in. wide? How many board feet in it if it is 1 inch thick?

**2.** How many feet board measure in a board 16 ft. long and 15 in. wide if it is 1 inch thick? How many if it is $\frac{1}{2}$ inch thick?

**3.** Find the cost of a board 14 ft. long, 6 in. wide, and an inch thick, at 3¢ a board foot.

**4.** An inch board 18 ft. long is 16 in. wide at one end and 8 in. wide at the other. How many board feet does it contain?

The average width of a board that tapers uniformly is one-half the sum of the end widths. A board like this has the form of a trapezoid.

**5.** A half-inch board 16 ft. long is 16 in. wide at one end and tapers to a point at the other. What is it worth at $3\frac{1}{2}$¢ a foot board measure?

What is the form of this board?

**6.** How many feet board measure in a beam 24 ft. long, 15 in. wide, and 3 in. thick?

$24 \times 1\frac{1}{4} = 30$, the number of sq. ft. in the surface, or the number of board feet in 1 inch of the thickness. Hence, in 3 inches of thickness there are $3 \times 30$ board feet, or 90 board feet.

*In any board or timber the number of board feet in 1 inch* *of thickness is equal to the number of square feet in the surface of one side.*

Hence, to find the number of feet board measure, when lumber is more than 1 inch thick,

RULE.—*Multiply the number of square feet in the surface of one side by the number representing the thickness in inches.*

**7.** How many feet board measure in a plank 14 ft. long, 16 in. wide, and 2 in. thick?

**8.** How many board feet in a log 18 ft. long, 15 in. wide, and 12 in. thick?

**9.** Mr. H bought 30 joists, each 20 ft. long, 8 in. wide, and 3 in. thick, at $18.50 per M. Find the cost.

☞ In lumber measure "per M" means "by the thousand" (board feet).

**10.** What will it cost to floor a two-story warehouse, 24 ft. by 36 ft., with two-inch planks, at $25 per M?

**11.** How many feet board measure in a piece of timber 32 ft. long, 18 in. wide at one end and 6 in. wide at the other, the thickness being 14 inches?

**12.** Two men bought a piece of timber 16 ft. long and 15 in. square, paying at the rate of $24 per M. How much should each man pay?

**13.** A farmer used inch boards to make a feed-box which measured on the outside 8 ft. in length, 3 ft. 2 in. in width, and 4 ft. 9 in. in height. What was the cost of the boards, at $18 per M?

**14.** James McKnight bought from Jas. Laird, Charleston, S. C., as follows:

| | | | | |
|---|---|---|---|---|
| 40 joists, | 2 × 6, | 18 ft. long, | @ | $25, |
| 16 beams, | 6 × 9, | 20 " " | " | 30, |
| 72 scantling, | 2 × 4, | 12 " " | " | 24, |
| 240 boards, | 1 × 10, | 12 " " | " | 18, |
| 24 planks, | 2 × 14, | 16 " " | " | 17.50. |

Make out a complete bill, and find the amount due Laird.

## BRICK WORK AND STONE WORK.

**367.** Brick work is commonly estimated by the *thousand bricks;* stone work by the *perch,* which contains $24\frac{3}{4}$ cu. ft.

The number of bricks in a cubic foot of wall depends upon the size of the bricks. Common bricks are 8 in. × 4 in. × 2 in., and 22 of these are assumed to build 1 cubic foot of wall.

In measuring walls of buildings, masons and bricklayers take the entire outside length, thus measuring the corners twice. This must not be done in estimating *material.*

In estimating the *work,* allowance for openings in the walls is sometimes made, but the amount to be deducted, if any, should be specified in a written contract.

In estimating the *material,* all openings should be deducted. In stone work $\frac{1}{9}$ is allowed for mortar and filling. In a perch of masonry, without openings, there are only 22 cu. ft. of stone.

**1.** If 22 common bricks build a cubic foot of wall, how many bricks will be required to build a wall containing 1000 cu. ft.?

**2.** How many common bricks in a wall 36 ft. long, 7 ft. high, and 16 in. thick?

**3.** How many perches of masonry in the walls of a cellar that is 40 ft. long and 24 ft. wide, the walls being 9 ft. high and 18 in. thick?

**4.** How many perches of stone in the walls of the above cellar?

**5.** Find the cost of building said walls at $1.50 a perch.

**6.** Had the walls been built at 10¢ a cu. ft., how much more or less would the mason have received?

**7.** The inside length of a storeroom is 56 ft., the width 44 ft. The outside length is 60 ft., the width 48 ft. If the walls are 22 ft. high, how many cubic feet do they contain, allowing nothing for openings, and counting corners once?

**8.** Mr. S built a house 38 ft. long, 32 ft. wide, and 26 ft. high. The front wall is built of stone, and is 2 ft. thick. The others are built of common bricks, and are 16 in. thick. There are 16 windows, each 6 by 3 ft., and 4 doors, each 7 by $3\frac{1}{2}$ ft. In the front wall are 4 windows and one door. How many bricks and perches of stone were used?

## CARPETING.

**368.** The amount of carpet that must be bought for a room depends upon the length and number of strips, and the waste in matching the patterns.

The number of strips often depends upon whether they are laid lengthwise or crosswise. Thus, in a room 15 by 17, 5 strips, each a yard wide, will be sufficient, if laid lengthwise. If laid the other way, 6 strips will be required, and one foot of width may be turned under or cut off.

$5 \times 17$ ft. $= 85$ ft., or $28\frac{1}{3}$ yd. when laid lengthwise.
$6 \times 15$ ft. $= 90$ ft., or 30 yd. " " crosswise.

The waste in matching the patterns cannot be estimated except by actual measurement of the carpet. In the following problems waste in matching is not considered.

**1.** A schoolroom is 30 ft. long and 26 ft. wide. How much matting a yard wide should be bought to cover the floor if the strips are to be laid lengthwise?

Suggestions.—1. How wide is the room? Then how many strips will be needed? 2. Can we buy $\frac{2}{3}$ of a strip (in width)? Then how many strips must be bought? What may we do with the surplus width? 3. How long must each strip be? Then how many yards must be bought?

**2.** A dining-room is 18 ft. 6 in. long and 15 ft. wide. How much ingrain carpet a yard wide must be bought to cover it if the strips are to be laid lengthwise?

**3.** What will be the cost of carpeting a room 14 ft. by 18 ft. with Brussels carpet 27 in. wide, laid lengthwise, if 3 yards cost $4.50?

**4.** A barber shop 15 ft. by 17 ft. 8 in. is covered with oilcloth 2 yd. wide, laid across the room. What was the cost of the oilcloth if $2\frac{1}{2}$ yd. cost $2.25?

**5.** In a parlor 14 ft. by 20 ft. the carpet is $\frac{3}{4}$ yd. wide, and is laid crosswise. Find its cost at $.85 a yard.

**6.** My library, which is 12 ft. square, is covered with China matting 36 in. wide, for which I paid $.65 a yard. How much did the matting cost me?

**7.** Find the cost of a rug for a room 3 yd. 2 ft. 8 in. by 4 yd. @ $1.25 a sq. yd.

**8.** How many tiles 8 in. square will be required to lay a floor 12 ft. by 32 ft.?

**9.** Brussels carpet 27 in. wide is laid lengthwise on the floor of Mr. B's reception-room, which is 13 ft. wide. One yard cost $1.75, and the entire cost was $52.50. What was the length of the room?

## PAPERING.

**369.** The amount of wall paper required to paper a room depends upon the area of the walls and ceiling and the waste in matching.

(a). Wall paper is put up in double rolls, but the prices quoted are for single rolls.

(b). A single roll contains 8 yards, 18 inches wide, its area being 36 square feet. Allowing for all waste, this will cover 30 square feet of wall.

(c). In estimating the number of rolls required for a room, some dealers deduct the *exact* area of the doors and windows, while others deduct an *approximate* area, allowing 20 square feet for each.

(d). Borders vary in width, and are sold by the *yard*.

(e). The number of single rolls required for the walls and that for the ceiling must be estimated separately, and can be found only approximately. Two methods employed by dealers are shown in (*a*) and (*b*) below.

EXAMPLE.—Room 12 × 14, 10 feet high, one window 6 × 4, one door 7 × 4.

(*a*)

| | |
|---|---|
| Area of walls.......... | 520 sq. ft. |
| Area of openings...... | 52 " " |
| Difference............. | 468 " " |

468 ÷ 36 = 13, number of single rolls required. No allowance is here made for matching and waste.

(*b*)

| | |
|---|---|
| Area of walls.......... | 520 sq. ft. |
| Two openings, 20 sq. ft. each................ | 40 " " |
| Difference............. | 480 " " |

480 ÷ 30 = 16, number of single rolls needed. This method allows for waste and matching.

The area of the ceiling divided by 36 (or by 30 to allow for waste) gives the number of rolls required for that ceiling. The number of yards of border required is the same as the perimeter of the room in *yards*.

**1.** How many single rolls of wall paper will be required to paper the walls and ceiling of a room 17 ft. long and 15 ft. wide, not allowing for waste, the height from baseboard to ceiling being 9 ft.?

**2.** Making no allowance for waste, what will be the cost of paper and border for the walls of a room 22 ft. long, 16 ft. 6 in. wide, and 14 ft. high, if paper costs $.75 a roll and border $.60 a yard?

**3.** Measure your sitting-room at home, and prepare a problem in papering for your class.

**4.** Estimate the cost of paper for your parlor at $.75 a roll.

**5.** My dining-room is 16 ft. long, 13 ft. 9 in. wide, and 10 ft. high. It has two windows, each 7 ft. by 4 ft., and a door 8 ft. by $4\frac{1}{2}$ ft. Estimate the number of single rolls required to paper walls and ceiling, allowing for waste.

## BINS, CISTERNS, ETC.

**370.** The method of computing the contents of a rectangular solid was learned in Art. 327. The number of cubic units in a box or cistern is found in the same manner.

NOTE.—Cubic inches can readily be reduced to bushels or gallons if we bear in mind that

2150.42 cu. in. = 1 bushel.
231 cu. in. = 1 gallon.

**1.** How many bushels of oats will be required to fill a feed-box 6 ft. long, 3 ft. wide, and 4 ft. deep?

Find the contents in bushels:

**2.** A box 4 ft. long, 3 ft. wide, and 2 ft. deep.

**3.** A bin 7 ft. long, 4 ft. wide, and 5 ft. high.

**4.** A granary 18 ft. long, 6 ft. 9 in. wide, and 5 ft. 6 in. high.

Find contents in gallons:

**5.** A cistern 5 ft. long, 4 ft. wide, and 6 ft. deep.

**6.** A trough 8 ft. long, 12 in. wide, and 8 in. deep.

**7.** A tank 6 yd. long, 6 ft. wide, and 6 in. deep.

**8.** A bin 8 ft. long, $4\frac{1}{2}$ ft. wide, and 5 ft. high is half full of wheat. How much is the wheat worth at $.80 a bushel?

**9.** How many gallons will fill a tank 4 ft. square and 6 ft. deep?

**10.** My cistern, which is 6 ft. long, 5 ft. wide, and 8 ft. deep, is $\frac{1}{3}$ full of water. If we use 2 bbl. of the water, how many gallons will remain in the cistern?

**11.** Fifty bushels of rye are in a bin 4 ft. 6 in. square. If the bin is 6 ft. high, how much more rye will it hold?

**12.** A cistern 6 ft. by 4 ft. will contain 1000 gallons. How deep is it?

**13.** A wagon-box is $11\frac{1}{2}$ ft. long and $3\frac{1}{2}$ feet wide. When even full of oats it contains 64.686 bushels. What is its depth?

**14.** A shed is 8 ft. long, 6 ft. wide, and 6 ft. 4 in. high. How many bushels of coal will it hold? How many of oats?

**15.** A box 3 ft. long, 2 ft. 8 in. wide, and 2 ft. high is $\frac{3}{4}$ full of wheat. What is the weight of the wheat?

**16.** A tin box is 1 foot square and 1 foot deep. Find the number of gallons it will contain, correct to 3 decimal places, and reduce the decimal to qt. and pt.

**17.** A tank 12 ft. long by 8 ft. 8 in. wide is full of oil. How many gallons must be drawn off to lower the surface 3 ft.? How many barrels?

## THE METRIC SYSTEM.

**371.** The uniform system of measures expressed in the decimal scale, which was first adopted in France in 1795, is called the **Metric System.** It is used in nearly all countries of continental Europe and South America, especially in scientific work. An act of Congress authorizes its use in the United States.

**372.** The system is based on the principal unit of length, called the **Meter** (meaning *measure*). The original standard meter is a rod of platinum which is preserved at Paris by the French government. It was intended to be .0000001 of the distance from the equator to the pole, but more careful measurements show this distance to be 10,001,887 meters.

**373.** The names of the higher and lower units are formed by attaching certain prefixes to the names of the principal units. These prefixes are, in part, as follows:

| (Greek) | | | (Latin) | | |
|---|---|---|---|---|---|
| deka, | meaning | 10 | deci, | meaning | .1 |
| hekto, | " | 100 | centi, | " | .01 |
| kilo, | " | 1000 | milli, | " | .001 |
| myria, | " | 10000 | | | |

### Table of Length.

| | | |
|---|---|---|
| A myriameter | = 10,000 | meters. |
| A *kilometer* (Km.) | = 1000 | " |
| A hektometer (Hm.) | = 100 | " |
| A dekameter (Dm.) | = 10 | " |
| *Meter* (m.) | | |
| A decimeter (dm.) | = .1 | of a meter. |
| A *centimeter* (cm.) | = .01 | " |
| A *millimeter* (mm.) | = .001 | " |

This table may be arranged thus :

| | |
|---|---|
| 10 millimeters | = 1 centimeter. |
| 10 centimeters | = 1 decimeter. |
| 10 decimeters | = 1 meter. |
| 10 meters | = 1 dekameter. |
| 10 dekameters | = 1 hektometer. |
| 10 hektometers | = 1 kilometer. |
| 10 kilometers | = 1 myriameter. |

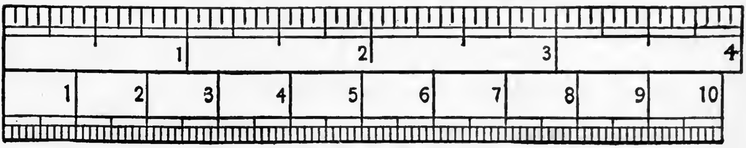
NOTES.—1. As in U. S. money we seldom speak of anything except dollars and cents, so in the metric system only the units printed in *italics* are commonly used.

2. In practice, length values are read in three denominations. Thus, 1 dm. 5 cm. is read *fifteen centimeters.* Values inconveniently large to be expressed in meters are read as kilometers.

3. A length given in one unit may be changed to another by simply moving the decimal point the requisite number of places. Thus, 75 dm. = 7.5 m., and 75 cm. = .75 m.

**374.** The annexed scale shows the decimeter divided into centimeters, and the latter into millimeters. It also compares the decimeter with *four inches.* The teacher should by all means have a metric stick for reference.

FOUR INCHES IN SIXTEENTHS OF AN INCH




ONE DECIMETER IN MILLIMETERS

## EQUIVALENTS.

| | |
|---|---|
| 1 meter | = 39.37 inches. |
| 1 decimeter | = 3.937 inches. |
| 1 centimeter | = .3937 inch. |
| 1 inch | = 2.54 centimeters. |
| 1 mile | = 1.6093 kilometers. |
| 1 kilometer | = .6214 of a mile. |

**375.** In surface measures the principal unit is the *Square Meter*.

TABLE OF SQUARE MEASURE.

| | |
|---|---|
| 100 square millimeters (qmm.) | = 1 square centimeter (qcm.). |
| 100 *square centimeters* | = 1 square decimeter (qdm.). |
| 100 square decimeters | = 1 square meter (qm.). |
| 100 *square meters* | = 1 square dekameter (qDm.). |
| 100 square dekameters | = 1 *square hektometer* (qHm.). |
| 100 *square hektometers* | = 1 *square kilometer* (qKm.). |

NOTE.—The square dekameter is also called an *are* (a.), pronounced like the word *air*, and the square hektometer is called a *hektare* (ha.). The square meter is sometimes called a *centare* (ca.). These are used in measuring land. The area of a farm is expressed in *hektares;* that of a country in *square kilometers.*

EQUIVALENTS.

| | |
|---|---|
| 1 square inch | = 6.452 sq. centimeters. |
| 1 square foot | = .0929 sq. meter. |
| 1 square yard | = .8361 sq. meter. |
| 1 square mile | = 2.59 sq. kilometers. |
| 1 acre | = .4047 hektare. |
| 1 square meter | = 1.196 sq. yards. |
| 1 hektare | = 2.471 acres. |

**376.** In measures of volume the principal unit is the *Cubic Meter*.

TABLE OF CUBIC MEASURE.

| | |
|---|---|
| 1000 cubic millimeters (cmm). | = 1 *cubic centimeter* (ccm.). |
| 1000 cubic centimeters | = 1 cubic decimeter (cdm.). |
| 1000 cubic decimeters | = 1 *cubic meter* (cu m.). |

NOTE.—When used in measuring wood, the cubic meter is called a *stere* (st.), pronounced *steer.*

EQUIVALENTS.

| | |
|---|---|
| 1 cubic inch | = 16.387 cubic centimeters. |
| 1 cubic foot | = .02832 cubic meter. |
| 1 cubic yard | = .7645 cubic meter. |
| 1 cubic meter | = 1.308 cubic yards. |

**377.** The principal unit of weight is the **Gram.** It is the weight of a cubic centimeter of distilled water at its maximum density.

TABLE OF WEIGHT.

| | |
|---|---|
| 10 *milligrams* (mg.) | = 1 centigram (cg.). |
| 10 centigrams | = 1 decigram (dg.). |
| 10 decigrams | = 1 gram (g.). |
| 10 *grams* | = 1 dekagram (Dg.). |
| 10 dekagrams | = 1 hektogram (Hg.). |
| 10 hektograms | = 1 kilogram (Kg.). |
| 1000 *kilograms* | = 1 metric ton (T.). |

| | |
|---|---|
| A quintal (Q.) | = 100,000 grams. |
| A myriagram | = 10,000 grams. |

NOTE.—The *metric ton* is the weight of a cubic meter of water ; the *kilogram* of a *cubic decimeter* or a *liter* of water, which is about 2.2 lb. The kilogram is sometimes called a *kilo,* and is the unit used in weighing ordinary articles.

EQUIVALENTS.

| | |
|---|---|
| 1 pound avoir. | = .4536 kilo. |
| 1 pound troy | = .3732 kilo. |
| 1 ton avoir. | = .9072 metric ton. |
| A gram | = 15.432 grains. |

**378.** The principal unit of capacity is the **Liter** (*lee'ter*). It is the capacity of a cube whose edge is .1 of a meter.

## Table of Capacity.

| | |
|---|---|
| 10 milliliters (ml.) | = 1 centiliter (cl.). |
| 10 *centiliters* | = 1 deciliter (dl.). |
| 10 deciliters | = 1 liter (l.). |
| 10 *liters* | = 1 dekaliter (Dl.). |
| 10 dekaliters | = 1 hektoliter (Hl.). |
| 10 *hektoliters* | = 1 kiloliter (Kl.). |

NOTE.—The *hektoliter* is used in measuring grain, vegetables, etc.; the *liter* in measuring liquids and small fruits.

## Equivalents.

| | |
|---|---|
| 1 gallon | = 3.786 liters. |
| 1 bushel | = .3524 hektoliter. |
| 1 liter | = 1.0567 liquid qt. |
| 1 hektoliter | = $2\frac{5}{6}$ bushels, nearly. |

### ORAL EXERCISES.

**379.** **1.** How are the decimal values of the principal units named?

**2.** Multiples of the principal units use what prefixes before the name of the unit?

**3.** What is the prefix which means 10? 100? 1000? .1? .01? .001?

**4.** Can you mention any advantages in the use of a metric system of weights and measures?

**5.** Since metric measures have a decimal scale, how may units of one denomination be reduced to another? How should they be written before adding or subtracting?

**6.** How many cm. long is this book? Your desk?

**7.** Cut a qdm. out of paper, and draw a sq. ft. How many of the former does the latter equal?

**8.** How many mm. in 5 Km.? In 1 Hm.? How many cm. in 25 m.?

**9.** How many qcm. in a qm.? How many cmm. in a ccm.? In a liter?

**10.** Explain why in measures of surface each unit is 100 times as large as the next smaller unit. Why 1000 times as large in measures of volume.

**11.** What part of a square meter is a square decimeter?

**12.** For what do we use the inch? The yard? The mile? For what are the mm., the cm., the m., and the Km. respectively used?

**13.** For what purposes is the liter used?

**14.** What part of a liter is 100 g. of water?

**15.** What is the weight of a cubic meter of water?

**16.** If a train travels at the rate of 20 m. a second, what is the rate in Km. an hour?

WRITTEN EXERCISES.

**380. 1.** What part of a cubic meter is a cubic foot?

**2.** A girl's hoop is 6 m. in circumference. How many times will it turn in rolling a distance of 1.08 Km.?

**3.** How many kilograms does a barrel of flour weigh?

**4.** How many square meters in a circle whose diameter is 15 meters?

**5.** If sulphuric acid is 1.84 times as heavy as water, what is the weight in dekagrams of 26 l. of the acid?

**6.** What is the area of Virginia in square kilometers?

**7.** At 16 cents a liter, what is the cost of 52.4 Hl. of olive oil?

**8.** How many kilos will a hektoliter of water weigh?

**9.** Mt. Blanc is 4800 m. high. How many feet high is it?

**10.** Find the weight in kilograms of a cubic foot of gold, if gold is 19.5 times as heavy as water.

**11.** A liter of mercury weighs 13.596 Kg. How many cu m. of mercury weigh 1 g.?

**12.** If marble is 2.7 times as heavy as water, what is the weight of a block 2 m. by 1 m. by 1 m.?

**13.** What will be the duty on 150 liters of wine at 50¢ a gallon?

**14.** A lot of land containing 62.5 ares is sold for 25 cents a square meter. For how much does the lot sell?

**15.** The capacity of a tank is 60 cu m. How long will it take a pipe to fill it at the rate of 3.8 Dl. a minute?

**16.** How many steps 2.5 ft. long will a man take in walking a kilometer?

**17.** A silver five-franc piece weighs 25 g., and is composed of 9 parts of pure silver and 1 part of pure copper. What is the total weight of silver in 200 five-franc pieces?

**18.** My cistern is 2 m. 5 dm. long, $2\frac{1}{4}$ m. wide, and 2 m. deep. How many gallons of water will it hold?

SUPPLEMENTARY EXERCISES (FOR ADVANCED CLASSES).

**381. 1.** If a rainfall is 700 liters per hectare, it is how many inches an acre?

**2.** If a ream of paper is .7872 Dm. in thickness, what is the thickness in millimeters of a single sheet?

**3.** At $7 a metric ton for coal, what will the coal for a week cost if 30 kilos are burned each day?

**4.** Air being .001276 as heavy as an equal volume of water, what is the weight of the air in a room 6 m. long, 4 m. wide, and 3 m. high?

**5.** A certain vessel when empty weighs 2.7 Kg., and when full of water weighs 4235 Dg. What does it weigh when full of petroleum, which is .7 as heavy as an equal volume of water?

**6.** If it costs $5 to travel 384 Km. by rail, what is the rate of fare in cents per mile?

**7.** A hektoliter of potatoes to the are is equivalent to how many bushels to the acre?

**8.** What is the distance in miles around the earth through the poles, if the distance from the equator to the pole is 10,001,887 meters?

**9.** The distance between two places on a map is 12.5 centimeters. What is the actual distance in miles, if the scale of the map is 1 to 6000?

# GENERAL REVIEW WORK.

## ORAL EXERCISES.

**382. 1.** A man built 11 rods of fence in 3 days. How much did he build in 2 days?

**2.** How much will a dozen lemons cost, at the rate of 2 for 5 cents?

**3.** If 4 apples cost 5 cents, how many apples can be bought for a quarter?

**4.** Bought eggs at the rate of 2 for 3 cents, and sold them at the rate of 2 for 5 cents, gaining 66 cents. How many dozen did I buy?

**5.** A lady gave $\frac{1}{2}$ of her money for a shawl, and $\frac{1}{3}$ of it for a dress, and had \$5 left. How much had she at first?

**6.** How many pigs can be bought for \$30 when 6 pigs cost \$20?

**7.** If 7 bats cost \$3.50, what will a dozen cost at the same rate?

**8.** I gave \$36 for shoes, at the rate of \$9 for 3 pairs. How many pairs did I get?

**9.** How many books can be bought for \$12.50, at the rate of 3 books for \$3.75?

**10.** If $\frac{4}{7}$ of a barrel of sugar costs \$8, what will 5 barrels cost?

**11.** If $\frac{3}{5}$ of the pupils in a school are girls, and there are 20 boys, how many pupils in the school?

**12.** A is 30 years old, and $\frac{2}{3}$ of his age is $\frac{4}{5}$ of his wife's age. How old is his wife?

**13.** B has $8\frac{3}{4}$ acres of land, and A has $\frac{5}{7}$ as many. How many acres have both?

**14.** If 12 cows are worth 3 horses, and 5 horses are worth 10 yoke of oxen, how many oxen are worth 4 cows?

**15.** If 4 bbl. of flour cost $14.40, what will $\frac{3}{4}$ of a barrel cost?

**16.** Tom spent $\frac{5}{8}$ of his money for a cap, and with the remainder bought a dozen apples at the rate of 2 for five cents. How much money had he at first?

**17.** If a 10-foot pole casts a shadow 18 feet long, what is the length of a pole that casts a shadow 72 feet long?

**18.** D has $50 more than E, and $30 is $\frac{3}{5}$ of what they both have. How much has E?

**19.** One half of a certain number is 12 more than $\frac{1}{3}$ of it. What is the number?

**20.** Two thirds of A's money equals $\frac{4}{5}$ of B's, and both have $8.80. How much has B?

**21.** How many square inches in one side of a two-foot cube?

**22.** If 5 men can build a wall in $6\frac{3}{5}$ days, how long would it take 11 men?

**23.** Fifteen men can build a house in 10 days. How many men can build it in $3\frac{1}{3}$ days?

**24.** A can cut a field of grass in 6 days, B in 8 days, and C in 12 days. In what time can they together cut it?

**25.** A man bought 2 hats for $5, and one cost $\frac{2}{3}$ as much as the other. What was the cost of each?

**26.** A suit of clothes cost $35. The pants cost $\frac{1}{2}$ as much as the coat, and the vest $\frac{1}{4}$ as much as the coat. What was the cost of each?

**27.** Two fifths of Robert's money equals $\frac{3}{7}$ of Andrew's, and Andrew has $2 more than Robert. How much has each?

**28.** Two men can husk a field of corn in 6 days. If one of them alone can husk it in 10 days, how long would it take the other?

**29.** What will be the cost of painting a sign-board 20 feet long and 9 feet wide, at the rate of 3 square yards for $1?

**30.** By selling a cow for \$45 I gained as much as she cost me. Find the cost.

**31.** Two pipes can fill a cistern in 8 hours. If one carries twice as much water as the other, in how many hours can each alone fill it?

**32.** Harry has 3 times as many marbles as Warren, and Earl has 5 times as many as Warren. If all have 36, how many has each?

**33.** A man having a lot 8 rods square, divided it into 4 equal lots. After selling one of the lots, what part of an acre did he have left?

**34.** A is 20 yards behind B, and runs 9 yards while B runs 8. How far will B run before he is overtaken?

**35.** By selling a stove for \$34, a merchant gained $\frac{5}{12}$ of what it cost. How much did it cost?

**36.** Mr. S has 20 lots, each 8 rods long and 2 rods wide. How many acres has he?

**37.** If 36 men can earn \$$a$ in 6 days, how many men can earn \$$8a$ in 24 days?

**38.** The sum of two numbers is 25, and their difference is 13. What are the numbers?

WRITTEN EXERCISES.

**383. 1.** Sold 42 horses for \$5600, thereby gaining \$13.33$\frac{1}{3}$ on each. What was the cost a head?

**2.** If a horse can trot 64 rods in half a minute, in what time can he trot 2$\frac{1}{2}$ miles?

**3.** A telegraph line is 200 miles long. If the poles are 150 feet apart, what is their value at \$1.33$\frac{1}{3}$ each?

**4.** How many boards 14 ft. long and 15 in. wide would be required to cover the sides of a shed 28 ft. long, 21 ft. wide, and 10 ft. high?

**5.** A street one fourth of a mile long and 48 feet wide is to be graded down half a yard. What will be the cost of excavating at \$.12$\frac{1}{2}$ a cubic yard?

**6.** How many boards 12 ft. long, 1 ft. wide, and one inch thick, can be made from a square log whose length is 24 feet, and whose ends are 2 ft. square?

**7.** A cistern is 22 ft. long, 14 ft. wide, and 8 ft. deep. How many gallons in it when it is $\frac{3}{4}$ full?

**8.** A log 2 ft. square is 24 ft. long. How many planks 12 ft. long, 12 in. wide, and 2 in. thick can be sawed from it?

**9.** An encyclopedia averages 764 pages to the volume, and 126 lines to the page. If the entire work contains 1443360 lines, how many volumes are there?

**10.** How many gold dollars weigh as much as a silver dollar?

**11.** A silversmith paid \$.60 an ounce for 5 lb. of silver, and made it into chains weighing 1 oz. 4 pwt. each, which he sold at \$1.50 apiece. How much did he receive for his labor?

**12.** When it is 9 A.M. on the meridian of Greenwich, what is the time on the 180th meridian?

**13.** Divide the product of $3\frac{5}{11}$ and $7\frac{1}{3}$ by their sum.

**14.** A druggist bought a pound of calomel for \$6, and sold it at the rate of 5 grains for a cent. What was his profit?

**15.** 440 lb. of copper was made into wire, a yard of which weighed 4 oz. What was the length of the wire?

**16.** If a cubic foot of granite weighs 250 lb., what is the weight, in tons, of a block 6 ft. long, 4 ft. wide, and 3 ft. thick?

**17.** If steel rails weigh 180 lb. per yard, how many tons will be required to lay 2 miles of railway, one of which has a double track?

**18.** If a wheel is 4 ft. in diameter, how many revolutions will it make in going a mile?

**19.** How many horses can be supplied with shoes from 10 lb. of iron, if 8 oz. make one shoe?

**20.** The distance over a hill is 60 rods, and the distance through on a level is 40 rods. If 81 posts are required for a fence from one side to the other on the level, how many would be needed to build a fence over the hill?

**21.** Explain the effect of removing the cipher in each of the following: 750, 025, .250, .025.

**22.** How often can a 3-bushel bag be filled from a bin containing 181 bu. 16 qt.?

**23.** A man bought 500 fence-boards, each 16 ft. long and 6 in. wide, at $14.50 per M, and 50 posts at a quarter apiece. Find the total cost.

**24.** How many perches in a pile of stone 45 ft. long, 30 ft. wide, and 4 ft. high?

**25.** A rectangular solid standing on a base 6 in. square is $5\frac{1}{2}$ ft. high. How many cubic feet does it contain?

**26.** A trough 4 ft. in length and 2 ft. square is full of water. What is the weight of the water, if a cubic foot of it weighs 1000 ounces?

**27.** A wheel of a bicycle travels 235.62 yd. in making 50 revolutions. What is the radius?

**28.** The area of a triangular field is 9 A. 65 sq. rd., and the length of its base is 70 rods. What is the altitude?

**29.** A floor is 24 feet wide at one end, 16 ft. at the other, and its area is 40 sq. rd. What is the length?

**30.** A roof is 50 ft. long and 20 ft. wide on each side. What will be the cost of roofing it at $8.75 per hundred sq. ft.?

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**31.** How many yards of lining $\frac{5}{6}$ of a yard wide will be required to line 5 coats, each containing $4\frac{2}{3}$ yards of material $1\frac{1}{2}$ yd. wide?

**32.** Willie lost $\frac{3}{16}$ of his marbles less 15; he then gave $\frac{5}{9}$ of the remainder and 8 more to John; he had 32 remaining. How many had he at first?

**33.** The point of a minute hand moves 4 inches in two hours. What is its length?

**34.** Mrs. Brown sold $\frac{5}{18}$ of her turkeys less 17; $\frac{2}{3}$ of the remainder less 3, and then had 39 remaining. How many had she at first?

**35.** How many quart, pint, half pint, and gill bottles, of each an equal number, can be filled from a vessel containing 5 gal. 2 qt. 1 pt.?

**36.** I have a lot the length of whose sides is 56, 84, 98, and 112 ft. respectively. I desire to enclose it with a fence four boards high, using boards 14 ft. long. How many will it take?

**37.** A passenger train 65 yd. long, and running at the rate of $\frac{3}{4}$ of a mile a minute, met a freight train moving $\frac{1}{2}$ as fast. They passed each other in 5 seconds. How long was the freight train?

**38.** If 9 men can do as much work as 15 women, and 70 women do as much as 25 boys, and 12 boys do as much as 36 girls, how many men would it take to do the work of 50 girls?

**39.** Find the difference between the largest fractional unit in decimals and the largest fractional unit in common fractions.

**40.** If the strips are laid lengthwise, how many yards of carpet 27 inches wide must be bought for a room 18 feet long and 16 feet wide?

**41.** Five minutes after two ships pass each other the distance between them is 2160 rods. One of them sails at the rate of 35 miles an hour. What is the hourly speed of the other?

**42.** A bin is 25 ft. long and 20 ft. wide. The oats in the bin is an inch deep, and a bushel of it weighs 32 pounds. How long will it last 5 horses, if one horse eats 18 pounds a day?

**43.** From 11 A.M. to 1.30 P.M. my watch gained 10 seconds. In how many days did it gain 10 minutes?

SUPPLEMENTARY EXERCISES (FOR ADVANCED CLASSES).

**384.** **1.** When a certain number is divided by 7, the quotient is equal to the ratio of 5 to 2. What is the number?

**2.** If $\frac{1}{3}$ of an acre is worth $\frac{2}{3}$ of \$100, how many thirds of an acre can be bought for $\frac{2}{3}$ of \$75?

**3.** How many fractional units in .375? Give three answers.

**4.** If 8 yards of cloth $1\frac{1}{6}$ yd. wide will make Lucy a dress, how many yards of 48-inch goods will make her a dress?

**5.** How many fractional units equal to .25 are there in $2\frac{3}{4}$?

**6.** In an orchard there are 18 rows of trees. Between every two rows of trees there are 8 rows of potatoes. If the average yield of a row is 12 bushels, what is the value of the potato crop at \$.75 a bushel?

**7.** Is $\frac{1}{5}$ a fraction, or a number, or a unit? May it be all three? Why?

**8.** If marble weighs 2.8 times as much as water, bulk for bulk, what is the weight of a block of marble 12 ft. 9 in. long, 4.5 ft. wide, and 3.2 ft. thick?

**9.** The average depth of a certain rainfall was .25 of an inch. What weight of water fell on a lot 40 ft. by 60 ft., if 1000 oz. of water measures a cubic foot?

**10.** If a train loses $\frac{2}{3}$ of an hour in running 80 miles at 18 miles an hour, in how many hours does it run 360 miles when running at the regular speed?

**11.** If 4 horses can draw 80 bushels of wheat, 60 lb. to the bushel, on a wagon whose weight is 900 lb., how many bushels can 2 horses draw on a wagon that weighs 600 lb.?

**12.** Does the expression $.\frac{1}{2}$ mean anything? If so, what?

**13.** A cubical box is 4.8 m. on an edge. How many hektoliters of oats will it hold? How many bushels?

**14.** How many bricks 20 cm. long and 10 cm. wide will it take to pave a sidewalk 2.4 m. wide and 1.4 Km. long?

**15.** At 43¢ a cubic meter, what will it cost to macadamize a road 1 Km. long and 7 m. wide, to the depth of 46 cm.?

## PERCENTAGE.

**385. 1.** Warren had 100 cents and spent 1 cent. What part of his money did he spend?

**2.** Boyd had $1 and spent 2 cents. What part of his dollar did he spend?

**3.** Emma misses 5 words *in a hundred.* What part does she miss?

**4.** A man had 400 sheep, and 10 of every hundred were killed by dogs. How many were killed?

**5.** A farmer having 50 hogs lost 10 *hundredths,* or 10 *per cent* of them. How many did he lose?

**6.** If $\frac{7}{100}$, or 7 per cent, of the pupils in a school of 100 are absent, how many are absent? How many are present? How many per cent are present?

**7.** What is 6 per cent, or .06, of $100? Of $300? Of $500?

**386. Per cent** means *hundredths.*

Thus, 5 per cent of a number means 5 hundredths of it.

NOTE.—The phrase "per cent" is from the Latin *per centum,* by the hundred.

**387.** The symbol % stands for the words *per cent,* and means either *per cent* or *hundredths.*

Thus, 6% is .06, and is read 6 per cent or 6 hundredths.

**1.** How do we express cents? Hundredths?

**2.** Since per cent is so many hundredths, how may we express *per cent?*

**3.** Is there any difference in value between 20%, .20, and $\frac{1}{5}$?

**4.** Express $12\frac{1}{2}$ per cent in three ways. $\frac{1}{2}$ per cent in 3 ways.

**5.** Explain how 5 per cent, or 5% = .05, or $\frac{5}{100}$, or $\frac{1}{20}$.

**6.** Express 225 per cent in two ways; 90 per cent; 100 per cent.

**388.** Change to per cent:

| | | | |
|---|---|---|---|
| **1.** .06 = 6%. | **7.** $\frac{7}{100}$ = 7%. | **13.** $\frac{1}{2}$ = 50%. | **19.** $.00\frac{1}{2}$ = $\frac{1}{2}$%. |
| **2.** .15 = | **8.** $\frac{15}{100}$ = | **14.** $\frac{3}{4}$ = | **20.** $.00\frac{3}{4}$ = |
| **3.** .25 = | **9.** $\frac{29}{100}$ = | **15.** $\frac{7}{8}$ = | **21.** .005 = |
| **4.** .39 = | **10.** $\frac{12\frac{1}{2}}{100}$ = | **16.** $\frac{4}{5}$ = | **22.** .0005 = |
| **5.** 1.35 = | **11.** 1.375 = | **17.** $\frac{2}{3}$ = | **23.** .2775 = |
| **6.** 3. = | **12.** .01 = | **18.** $\frac{3}{11}$ = | **24.** .0325 = |

**25.** 226 hundredths = how many per cent?

**389.** Change to decimal fractions:

| | | | |
|---|---|---|---|
| **1.** 6% = .06 | **6.** $\frac{1}{2}$% = .005 | **11.** 100% = 1.00 | **16.** $162\frac{1}{2}$% = 1.625 |
| **2.** 10% = | **7.** $\frac{3}{4}$% = | **12.** 227% = | **17.** $237\frac{1}{2}$% = |
| **3.** 25% = | **8.** $\frac{1}{8}$% = | **13.** 3000% = | **18.** $266\frac{2}{3}$% = |
| **4.** 99% = | **9.** $\frac{5}{8}$% = | **14.** $\frac{7}{20}$% = | **19.** $267\frac{1}{2}$% = |
| **5.** $99\frac{1}{2}$% = | **10.** $\frac{13}{25}$% = | **15.** $6\frac{1}{4}$% = | **20.** 8% = |

**390.** Change to common fractions:

| | | | |
|---|---|---|---|
| **1.** 25% = $\frac{25}{100}$ = $\frac{1}{4}$ | **5.** $16\frac{2}{3}$% = $\frac{16\frac{2}{3}}{100}$ = $\frac{1}{6}$ | **9.** $62\frac{1}{2}$% = $\frac{5}{8}$ | **13.** $87\frac{1}{2}$% = $\frac{7}{8}$ |
| **2.** 35% = | **6.** $37\frac{1}{2}$% = | **10.** 150% = | **14.** .283 = |
| **3.** 44% = | **7.** $83\frac{1}{3}$% = | **11.** 180% = | **15.** .375 = |
| **4.** 75% = | **8.** $116\frac{2}{3}$% = | **12.** 225% = | **16.** $233\frac{1}{3}$% = |

**391.** The following per cents and their equivalents are so often used that pupils should be able to give their values in the different forms at sight.

### Drill Table.

| | | | |
|---|---|---|---|
| 1 = 100%. | $\frac{3}{4}$ = 75%. | $\frac{1}{8}$ = $12\frac{1}{2}$%. | $\frac{1}{15}$ = $6\frac{2}{3}$%. |
| $\frac{1}{2}$ = 50%. | $\frac{1}{5}$ = 20%. | $\frac{3}{8}$ = $37\frac{1}{2}$%. | $\frac{1}{20}$ = 5%. |
| $\frac{1}{3}$ = $33\frac{1}{3}$%. | $\frac{3}{5}$ = 60%. | $\frac{5}{8}$ = $62\frac{1}{2}$%. | $\frac{1}{25}$ = 4%. |
| $\frac{2}{3}$ = $66\frac{2}{3}$%. | $\frac{1}{6}$ = $16\frac{2}{3}$%. | $\frac{7}{8}$ = $87\frac{1}{2}$%. | $\frac{1}{50}$ = 2%. |
| $\frac{1}{4}$ = 25%. | $\frac{5}{6}$ = $83\frac{1}{3}$%. | $\frac{1}{10}$ = 10%. | $\frac{1}{100}$ = 1%. |

**392.** The result obtained by taking a certain per cent (meaning a stated number of hundredths) of a number is called the **Percentage.**

The name *Percentage* is also applied to that portion of arithmetic which involves the taking of per cents.

**393.** The number of which the per cent is taken is called the **Base.**

**394.** The per cent taken is called the **Rate.**

Thus, in 6% of $50 = 3$, the 6% is the *rate.* The 6 alone is usually called the *rate per cent.*

**395.** Using the *first letters* of the words *percentage, base,* and *rate* to represent the *numbers* called by these names, we readily express in the form of equations the relations that these numbers bear to each other.

From the definition of percentage,

$$p = br. \qquad (1)$$

Dividing both members of (1) by $b$,

$$\frac{p}{b} = r, \text{ or } r = \frac{p}{b}. \qquad (2)$$

Dividing both members of (1) by $r$,

$$b = \frac{p}{r}. \qquad (3)$$

In (2) we have a *product* (*percentage*) and one of the *factors* (*base*) to find the other *factor* (*rate*). What have we in (1)? In (3)? What relation, then, do the base, rate, and percentage bear to each other?

**396.** One hundred per cent of a number is the number itself.

Thus, 100% of 50 is 50.

**397. To find a given per cent of any number.**

1. What part of a number is 10 hundredths of it? 10% of it?

**2.** How many hundredths in 1? How many cents in \$1? How many per cent in 1?

**3.** What per cent of a number is $\frac{1}{10}$ of it? $\frac{1}{2}$ of it? All of it?

**4.** Since 1% of a number $= \frac{1}{100}$ of it, 10% $= \frac{10}{100}$ or $\frac{1}{10}$ of it, and 50% $= \frac{50}{100}$ or $\frac{1}{2}$ of it, what does 100% equal?

**5.** Since any number is 100% of itself, and the base is the number of which the per cent is taken, the base equals what per cent?

**6.** If you lose 4% of your money, what per cent do you have left? 100% − 4% = (  ).

What is:

| | |
|---|---|
| **7.** 1% of 50? | **13.** 5% of 50? |
| **8.** 10% of 50%? | **14.** 100% of 50? |
| **9.** 12% of 40? | **15.** 50% of 100 sheep? |
| **10.** $16\frac{1}{3}$% of 66? | **16.** 75% of 120 horses? |
| **11.** $33\frac{1}{3}$% of 60? | **17.** $83\frac{1}{3}$% of 24 quarts? |
| **12.** $66\frac{2}{3}$% of 72? | **18.** 100% of 100 bushels? |

WRITTEN EXERCISES.

**398. 1.** Find 20% of \$960.

(a)

.20% = .20 or $\frac{1}{5}$.
.20 of \$960 = \$192.
Or
$\frac{1}{5}$ of \$960 = \$192.

(b)

100% of \$960 = $\frac{100}{100}$ of \$960.
∴ 1% of \$960 = $\frac{1}{100}$ of \$960, or \$9.60,
and 20% of \$960 = 20 × \$9.60, or \$192.

Have the pupil solve the above problem by substituting in the equation

$$p = br.$$

**2.** Find $\frac{3}{4}$% of \$800.

(a)

100% of \$800 = \$800.
∴ 1% of \$800 = $\frac{1}{100}$ of \$800, or \$8,
and $\frac{3}{4}$% of \$800 = $\frac{3}{4}$ × \$8, or \$6.

(b)

1% of \$800 = \$8.00.
$\frac{3}{4}$% of \$800 = $\frac{3}{4}$ × \$8.00, or \$6.

SUGGESTION.—The pupil should be taught to select and apply the method that is *most convenient* in each *particular* problem.

**3.** 12% of $240.50.

**4.** 75% of $1286.45.

**5.** $16\frac{2}{3}$% of 120 sheep.

**6.** $\frac{3}{5}$% of 1200.

**7.** 325% of 55.2 rods.

**8.** 133% of $7824.

**9.** 90% of .0577.

**10.** $1\frac{1}{4}$% of $\frac{9}{15}$.

**11.** What is the difference between 5% of $120, and 120% of $5?

QUERY.—Is the percentage a factor or a product?

**12.** How much had I left after paying out 15% of my $3000?

**13.** The owner of a threshing machine charges $2\frac{1}{2}$% for threshing a crop of 275 bushels. How much does he get?

**14.** A man who owed $1750 was able to pay only 39 per cent. How many dollars could he pay?

**15.** If a foot of rope shrinks $4\frac{1}{2}$% when wet, how much would 500 feet of rope shrink?

**16.** If A's income is $1000 a year and he saves 25% of it, how much will he save in 25 years?

**17.** I bought 40 head of cattle for $$1666\frac{2}{3}$, and sold them at a profit of $3\frac{1}{3}$%. What did I make?

**18.** Dickson and Tribby engaged in business, each with $1250. Tribby gained $33\frac{1}{2}$% of his capital, and Dickson $37\frac{1}{2}$% of his capital. How much did Dickson gain more than Tribby?

**19.** A farmer raises 500 bushels of grain, of which 29% was wheat, 47% rye, and 22% oats. How many bushels of each did he raise?

**20.** I own $\frac{3}{4}$ of a mill and sell $33\frac{1}{3}$% of my share. What part of the mill do I sell?

**21.** A barrel that will hold 42 gallons is $66\frac{2}{3}$% full. How many gallons does it contain?

**22.** A man has $1500. He spends $66\frac{2}{3}$% of it, and gives away 6% as much as he spends. How much has he left?

**23.** If pure air consists of 20.0265% of oxygen gas and 79.9735% of nitrogen, how much oxygen in 1500 cu. ft. of air? How much nitrogen?

**24.** If 25% of a certain ore is melted, and $1\frac{3}{5}$% of the metal is silver, how much silver in a ton of the ore?

**25.** A certain lot of cane has 89% juice, and the juice contains 11.4% sucrose. How much sucrose in 5 tons of cane?

**26.** A man has a library of 1600 volumes. 14% are biography, 62% are history, and $83\frac{1}{3}$% of the remainder are fiction. How many volumes of fiction in his library?

**27.** A maltman malts 1500 bushels of barley, which in the process increases $12\frac{1}{2}$%. How many bushels of malt has he?

**28.** An agent sells 25 bicycles at $60 each, and is allowed 15% of the receipts. How much does he make?

**29.** Water is composed of 88.9% of oxygen and 11.1% of hydrogen. How many pounds are there of each in 1 cu. ft. of water?

**399. To find what per cent one number is of another.**

**1.** 12 is what part of 24? How many hundredths of 24? What per cent of 24?

**2.** 15 is what part of 45? How many hundredths or *per cent* of 45?

**3.** $5 is what part of $25? What per cent of $25? What is the ratio of $5 to $25?

What per cent of:

| | | |
|---|---|---|
| **4.** 24 is 12? | **8.** $25 is $10? | **12.** 1 is $\frac{1}{6}$? |
| **5.** 7 is 21? | **9.** 30 yd. is 20 yd.? | **13.** 9 is $\frac{3}{5}$? |
| **6.** 8 is 18? | **10.** 100¢ is 50¢? | **14.** $\frac{5}{8}$ is $\frac{1}{4}$? |
| **7.** 30 is 5? | **11.** 5 is 5? | **15.** $\frac{8}{9}$ is $\frac{2}{3}$? |

**16.** A boy having 10 cents gave his sister 5 cents? What per cent of his money did he give away?

**17.** A teacher whose salary is $1250 spends $1000. What per cent of his salary does he spend?

**18.** A's money is twice B's. What per cent of A's money is B's? What per cent of B's is A's?

WRITTEN EXERCISES.

**400. 1.** 9 is what per cent of 30?

(a)

$30 = 100\%$ of 30.

$\therefore 1 = \frac{1}{30}$ of $100\%$, or $\frac{10}{3}\%$ of 30,

and $9 = 9 \times \frac{10}{3}\%$, or $30\%$ of 30.

(b)

9 is $\frac{9}{30}$, or $\frac{3}{10}$ of 30,

and $\frac{3}{10} = .30$, or $30\%$.

(c)

$$r = \frac{p}{b} = \frac{9}{30} = .30, \text{ or } 30\%.$$

**2.** If a miller takes 4 qt. for toll from every bushel, what per cent does he take for toll?

**3.** $3\frac{1}{3}$ is what per cent of 20?

**4.** Edward bought 2 lb. of candy. He ate 4 oz. and gave away $\frac{1}{2}$ lb. What per cent of his candy did he have left?

**5.** From a cask containing $66\frac{1}{2}$ gal., 26.6 gallons were drawn. What per cent of the whole remained in the cask?

**6.** If gold coin is 9 parts pure and 1 part alloy, what per cent is pure?

**7.** An attorney charges \$68.75 for collecting \$550. What per cent does he charge?

**8.** Mr. S paid \$45 for the use of \$750. What per cent did he pay?

**9.** What per cent is a pound avoirdupois of a pound troy?

**10.** $25\%$ of $\frac{3}{5}$ of a number is what per cent of $\frac{2}{3}$ of it?

**11.** A merchant buys 5 gross of pens, and sells 5 dozen. What per cent of them does he sell?

**12.** Frank has \$10 and Ray \$4. What per cent of Ray's money is equal to Frank's, and what per cent of Frank's money equals Ray's?

**13.** What per cent of his time does a man sleep who sleeps $7\frac{1}{2}$ hr. out of 24?

**14.** If 7 lb. of a certain article lost 4 oz. by drying, what per cent of its original weight was water?

**15.** In a mixture of copper and zinc, the copper is to the zinc as $3\frac{1}{2}$ to $2\frac{1}{3}$. Express the percentage of each ingredient in the mixture.

**16.** If carpeting, which should be one yard wide, is only $34\frac{1}{2}$ inches wide, what per cent should be deducted from the price?

**17.** If I sell $\frac{1}{3}$ of my interest in a business to one man and $\frac{1}{5}$ of it to another, what per cent have I remaining?

**18.** If to 23 gallons of alcohol 2 gallons of water are added, what per cent of the mixture is water?

**19.** In an examination 50 questions were asked, of which A answered 45, B 35, and C 18. What per cent did each make?

**20.** If B's age is $33\frac{1}{3}\%$ more than A's, A's age is what per cent less than B's?

**21.** If a gold ring is 18 carats fine, what per cent of it is gold?

**22.** If a piece of bronze weighing $7\frac{2}{3}$ pounds contains 6.5 pounds of copper, what per cent of the bronze is copper?

**23.** Of 25320 votes cast in a certain city, A received 11394 and B the remainder. What per cent did B receive?

**24.** An army paymaster receives $125000, but embezzles $5000 of it. What per cent of the money does the government lose?

**25.** In a certain year (1898) 501,066,681 passengers were carried on railways in the United States, and 221 were killed. What per cent were not killed?

**26.** The area of North America is 9,350,000 sq. mi., and the area of the Missouri-Mississippi basin is 1,250,000 sq. mi. What per cent of the area of the continent is drained by these rivers?

**401. To find a number when a certain per cent of it is given.**

**1.** 15 is 3 times what? $\frac{3}{5}$ of what? .03 of what? 3 per cent of what?

2. 24 is $\frac{6}{7}$ of what? .06 of what? 6 per cent of what?

Find the number of which:

3. 10 is 10%.
4. 60 is 25%.
5. 36 is $33\frac{1}{3}$%.
6. 30 is $12\frac{1}{2}$%.
7. $150 is 50%.
8. $\frac{5}{6}$ is 100%.
9. $\frac{2}{3}$ is 50%.
10. 50 is $\frac{1}{2}$%.
11. $\frac{5}{8}$ is 5%.

12. Of passengers aboard a ship $16\frac{2}{3}$% of the number, or 800 persons, were lost. What was the number of persons aboard?

13. $\frac{1}{2}$ of $\frac{2}{5}$ of a yard is 20% of what?

14. A bin holds 60 bushels of wheat, which is 3% of a farmer's crop. How many bushels did he have?

WRITTEN EXERCISES.

**402.** **1.** 30 is 5% of what number?

(a)

5% of a number = 30.
∴ 1% of the number = $\frac{1}{5}$ of 30, or 6,
and 100% of the number = 100 × 6, or 600.

(b)

5%, or $\frac{1}{20}$ of the number = 30.
$\frac{20}{20}$ of the number =

2. 24 is $\frac{3}{8}$% of what number?

$\frac{3}{8}$% of a number = 24.
∴ $\frac{1}{8}$% of the number = $\frac{1}{3}$ of 24, or 8.
∴ $\frac{8}{8}$%, or 1%, of the number = 8 × 8, or 64,
and 100% of the number = 100 × 64, or 6400.

3. A paid B $20, which was $6\frac{1}{4}$% of what he owed him. What was the debt?

4. A farmer sold 560 bushels of wheat, which was 85% of his crop. How many bushels did he raise?

5. A boy after selling 60% of his berries received 80¢ for the remainder at 10¢ a quart. How many quarts had he at first?

6. A charity concert realized $200.85 for the benefit of the poor. The expenses were 35% of the receipts. What were the expenses?

**7.** A milkman sold milk at 7 cents a quart, which was $233\frac{1}{3}\%$ of the cost. What did it cost a gallon?

**8.** If 80% of the pages in Webster's International Dictionary is 1688, how many pages does it contain?

**9.** When gold was worth 20% more than paper money, what was the value in gold of a dollar bill?

**10.** A father gave the contents of a purse to his 3 sons. To Charles he gave $33\frac{1}{3}\%$ of the money; to John, $37\frac{1}{2}\%$ of it; and to James, 20% of the money, or \$2. How many dollars were in the purse at first?

**11.** A man sold 50 cords of wood, losing thereby \$25, or 10%. What did the wood cost him a cord?

**12.** The land surface of the earth is about 52,000,000 square miles, which is $33\frac{1}{3}\%$ of the water surface. What is the extent of the water surface?

**13.** If $2\frac{1}{2}$ bbl. of apples are $\frac{3}{4}\%$ of the yield of my orchard this year, how many bushels did the orchard produce?

**14.** By adding \$695.75 to my money in bank, I increased it 20%. How much did I have in bank before the increase?

**15.** A rug 12 feet long and 10 feet wide covers 24% of the floor of a room 25 feet long. How wide is the room?

**403. To find a number, having given another number which is a certain per cent more or less than the required number.**

**1.** What number increased by 5 times itself becomes 30?

**2.** What number increased by .06 of itself becomes 212? By 6% of itself becomes 212?

**3.** What number diminished by $\frac{2}{5}$ of itself becomes 30? By .06 of itself becomes 188? By 6% of itself becomes 188?

**4.** My salary is \$75 a month, which is 25% more than it was last year. What was my salary last year?

**5.** I pay \$12 a week for board this year, which is 20% less than I paid last year. What did I pay last year?

### WRITTEN EXERCISES.

**404.** **1.** What number increased by 15% of itself equals 1150?

100% of the number = the number.
100% + 15%, or 115% of the number = 1150.
∴ 1% of the number = 1150 ÷ 115, or 10,
and 100% of the number = 100 × 10, or 1000.

**2.** What number diminished by 90% of itself equals 126?

100% of the number = the number.
100% − 90%, or 10% of the number = 126.
∴ 1% of the number = $\frac{1}{10}$ of 126, or 12.6,
and 100% of the number = 100 × 12.6, or 1260.

**3.** A expends in a week $24, which exceeds his earnings by $33\frac{1}{3}$%. What are his earnings?

**4.** A mechanic has had his wages twice reduced 10%. What did he receive before the reductions if he now gets $2.025 a day?

**5.** If $\frac{3}{4}$ of a number is 240 more than $66\frac{2}{3}$% of it, what is the number?

**6.** A school enrolls 87 boys, which is 16% more than the number of girls. How many pupils in the school?

**7.** A flock of sheep has been increased by 250% of its number, and now numbers 1050. What was the original number?

**8.** I gave away 40% of my money, and had $2 left. How much had I at first?

**9.** If I double my capital, and in addition make 25% of it, and then have $22500, what was my capital?

**10.** The fraction $\frac{6}{11}$ is 20% more, and 20% less, than what fractions?

**11.** The Mississippi is 4200 miles long, and is 5% longer than the Nile, which is $6\frac{2}{3}$% longer than the Amazon. Find the length of each river.

**12.** There are 48 gallons in two casks, one of which contains 40% less than the other. How much is in each cask?

**13.** John spent 80% of his money, and had $400 left. How much had he at first?

**14.** The time past noon is $66\frac{2}{3}\%$ of the time to midnight. What is the time?

**15.** A man worth $990,000 is unhappy because his fortune is 1% less than his neighbor's. What is his neighbor worth?

**16.** The difference between two numbers is 425, and one of them is $25\frac{1}{2}\%$ greater than the other. What are the numbers?

**17.** After spending 20% of her money for a coat, and 80% of the remainder for a watch, a lady had $12 left. How much had she at first?

**18.** A farmer gave 40% of his land to one son, $33\frac{1}{3}\%$ of 60% to another, $87\frac{1}{2}\%$ of the remainder to another, and the remainder, 8 acres, to his daughter. How many acres had he at first?

**19.** Seventy-five per cent of a number exceeds $\frac{2}{5}$ of it by 50% of 7. What is the number?

Complete the following equations:

**20.** 5% of 75 = (  %) of 15.
**21.** 20% of 35 = 50% of (  ).
**22.** 16% of (  ) = 4% of 200.
**23.** (  %) of 27 = 27% of $33\frac{1}{3}$.
**24.** (  %) of 1 = $\frac{1}{4}\%$ of 2.
**25.** 75% of 1 = (  %) of $\frac{1}{2}$.
**26.** 50% of 2 = (  %) of 200.
**27.** $33\frac{1}{3}\%$ of $\frac{3}{4}$ = $12\frac{1}{2}\%$ of (  ).
**28.** 40% of 40% = 24% of (  %).
**29.** (  %) of .85 = 500% of .034.
**30.** $\frac{1}{2}\%$ of (  ) = 75% of $\frac{1}{2}$.
**31.** $2\frac{1}{4}\%$ of 3.6 = (  %) of .9.

**32.** 6% of $100 = 8% of ($  ).
**33.** (  %) of twice a number = 6% of thrice the number.
**34.** 5% of (  times) a number = 40% of half the number.
**35.** $62\frac{1}{2}\%$ of .24 : .6% of 50 = (  %) of 25 : 25% of 10.

### SUPPLEMENTARY EXERCISES (FOR ADVANCED CLASSES).

**405. 1.** How many pounds of tallow must be mixed with 8.5 pounds of rosin that the mixture may contain 15% of tallow?

**2.** On Jan. 1 a man weighed 150 lb. In that month he lost $2\frac{1}{2}\%$ in weight, and in February gained $2\frac{1}{2}\%$. What per cent of his weight Jan. 1 was his weight on the first day of March?

**3.** Sugar is composed of 49.856% of oxygen, 43.265% of carbon, and the remainder hydrogen. How many pounds of each in 1 ton of sugar?

**4.** The capacity of a bin 9 feet long, 5 feet wide, and 4 feet deep equals 36% of how many bushels of grain?

**5.** A tank, whose capacity is 168 gallons, discharges 72 gallons an hour, which is 25% less than it receives. In what time will it be filled?

**6.** A man has his money invested as follows: $3000 at 4%, $500 at 6%, and $1200 at $6\frac{1}{2}\%$. If he should invest the whole amount at 5%, would he gain or lose, and how much?

**7.** The earth's radius is 6370 Km., and the sun's is 10856% of the earth's. Find the radius of the sun.

**8.** In 1898 the number of pupils enrolled in the schools of the United States was 14,652,492, which was 20.53% of the population. What was the population? Of those enrolled, 10,089,620 were in daily attendance. What was the per cent of attendance?

**9.** In a certain year (1898) the gold and silver money in this country was $1,354,283,142, and the paper money was $1,143,946,669, or a total of $2,498,229,811. The *per capita* circulation was $25.19. If the population was 74,925,000, what per cent of the total was in circulation?

**10.** The wealth of the United States by the census of 1890 was 65,037 millions of dollars. The increase of wealth from 1880 to 1890 was 49%. At the same rate of increase, what wealth should the census of 1900 show?

**11.** In the Spanish-American War the deaths in the army from all causes, May 1 to Sept. 30, 1898, were 2910, and the total force was 274,837 soldiers. What was the per cent of loss by death?

**12.** Of the \$1,210,292,097 worth of exports sent abroad from the United States in 1898, 70.61% of the total were farm products. What was the value of the farm products exported that year?

**13.** In 1890 the manufacturing industries of the United States employed 3,599,292 males, which was 80.4% of the total number of employés. Of this total 2.68% were children, and 16.92% were females. How many women and children were employed in manufactures?

**14.** In the manufacturing industries, the office forces (including firm members) form 9.78% of the aggregate employés, and they receive 17.17% of the total wages. Therefore an average office employé receives how many times as much salary as an operative?

## COMMERCIAL DISCOUNT.

**406. 1.** The list price of a book is \$1, but copies are offered for introduction at 25% off. What is the introduction price?

**2.** A sewing machine sells for \$40 on 3 months time, or 10% off for cash. What is the cash price?

**3.** A bill of goods was sold at 10% less than list price, which was \$20. If 5% was deducted for cash, what was the net cash price?

**4.** Does it make any difference in the net price whether the discount is 10% off and 5% for cash, or 5% off and 10% for cash?

**407.** An allowance or reduction from a *list price*, or from the amount of a bill, is called a **Commercial Discount.**

Manufacturers, publishers, and wholesale dealers usually bill goods to the trade at fixed *list* or *catalogue prices*, subject to discounts.

1. As the market varies they revise the discounts instead of changing the prices.

2. Many houses print on their bill-heads the discounts allowed; as, "Terms: 30 days net, or 2% if paid in 10 days."

**408.** When several discounts are allowed, one is first deducted, then another is computed on the remainder and deducted, and so on for each successive discount.

Thus, 25% and 5% means a discount of 25% from the list price and then 5% from the remainder.

**409.** The list price of an article less the discounts is the *net price;* and the amount of a bill less the discounts is the *net amount.*

WRITTEN EXERCISES.

**410. 1.** A merchant buys goods listed at $1200, the discounts being 25%, 20%, and 10%. Find the net price.

(a)

| | |
|---|---|
| 4 | $1200 |
| | 300 |
| 5 | $900 |
| | 180 |
| 10 | $720 |
| | 72 |
| | $648 |

(b)

$\$1200 \times .25 = \$300$, first discount.
$\$1200 - \$300 = \$900$, first remainder.
$\$900 \times .20 = \$180$, second discount.
$\$900 - \$180 = \$720$, second remainder.
$\$720 \times .10 = \$72$, third discount.
$\$720 - \$72 = \$648$, the *net price.*

The calculation is often conveniently made as in (a).

Notes.—1. When several discounts are given, the list price is the *base* in computing the *first* discount only.

2. It is immaterial in what order the discounts are taken.

3. In stating commercial discounts, the sign % is very often omitted.

**2.** A bill of goods amounted to $2400. If 20% off was allowed, what was paid for the goods?

**3.** A bill of $850 was discounted 8% and 5%. What were the discount and the amount paid?

**4.** What is the cash price of a carriage listed at $230, 15% off, and 5% for cash?

**5.** Find the net amount of a bill of $280.50, the discounts being 15%, 10%, and 5%.

**6.** How much greater is a single discount of 40% on a bill of \$210, than two successive discounts of 30% and 10% ?

**7.** Which is better, a single discount of 19% or three successive discounts of 12%, 8%, and 1% off a bill of \$647 ?

**8.** A bill of hardware amounting to \$375 was bought March 1, "Terms: 3 mo., or 10% off, 30 da." How much money paid the bill April 1 ?

**9.** A school district bought 7000 grammars listed at \$.60, discount 20%. If 5% was allowed for cash payment, what sum did the district remit ?

**10.** Goods listed at \$830 were sold me at 30, 10, and 5 off. For what must I sell them to gain 20 per cent ?

**11.** A bill of goods at list prices amounted to \$2700. The discounts were $\frac{1}{3}$ and 5%. What was due on the bill ?

($\frac{1}{3}$ is often used for $33\frac{1}{3}$%, $\frac{1}{2}$ for 50%, and $\frac{1}{4}$ for 25%.)

**12.** What is the cost of a bill of cutlery amounting to \$272, if the discounts are $\frac{1}{2}$, 10, and 5, freight being \$2.65, package \$.60 ?

**13.** Find the net amount of a bill for \$19.20, subject to discounts of $16\frac{2}{3}$%, $7\frac{1}{2}$%, $6\frac{1}{4}$%, and 4%.

**14.** Find the net amount of a bill of \$136, discounts being 50%, 10%, and 5%. Find a single discount equivalent to these successive discounts.

The net amount $= \$58.14$.
$\therefore$ the single equivalent discount $= \$136 - \$58.14 = \$77.86$.
$\therefore$ the *rate* of a single discount $= \$77.86 \div \$136 = .5725 = .57\frac{1}{4}\%$.

**15.** What single discount is equivalent to discounts of $33\frac{1}{3}$%, 20%, and 1% ?

**16.** If the list price is \$37.50, and the rates of discount are 20%, $12\frac{1}{2}$%, and 6%, find the cost. What one rate of discount is equivalent to the several discounts given ?

**17.** If the price of paper is \$1.08 a ream after discounts of 20% and 10%, what is the list price ?

**18.** The cost of certain goods is \$49.63, and the discounts are 30%, 12%, and 6%. Find the list price.

**19.** If a book costs 6¢ less when the marked price is discounted 50% than when discounted 40% and 10%, what is the marked price?

**20.** A retail dealer buys certain goods at discounts of 30%, 10%, and 8%, and sells them at list prices. What per cent does he gain?

**21.** Show that the discounts 15%, 10%, and 3% are equivalent to the discounts 3%, 10%, and 15%, but not to the single discount 15% + 10% + 3%.

**22.** Two rival houses used the same list prices. The Union Company offered 50 and 50 off, while the National Company offered 50, 40, and 10 off. The house receiving the higher prices for its goods got what per cent more than the other?

## COMMISSION.

**411. 1.** An agent sold $100 worth of books, and received 40% of the sales. How much did he get for his services?

**2.** If a commission merchant sells $200 worth of eggs and charges 5% of the sales, how much does he receive for his services, or how much is his *commission?*

**3.** How much should be received by an agent for selling $500 worth of goods, if 3% is allowed for his commission? How much will be realized after paying the commission, or how much will be the *net proceeds?*

**4.** If an agent charges 5 cents for each $1 expended for butter, what will a dollar's worth of butter cost a grocer who buys through the agent? If each dollar's worth costs the grocer $1.05, how many dollars' worth can be bought for $105?

**412.** A person or firm employed to transact business for another is called an **Agent.**

**413.** An agent who sells or buys produce or other merchandise is called a **Commission Merchant.**

**414.** The charge made by an agent, or the sum paid him for his services, is called his **Commission.**

Commission is reckoned as a certain per cent of the amount *paid in buying* or *realized in selling*. Sometimes, however, it is a specified sum for a given transaction.

**415.** The name *Consignment* is given to goods sent to a commission merchant to be sold. The sender is called the *Consignor*, and the agent to whom they are sent, the *Consignee.*

**416.** The sum realized less the commission and other charges is the **Net Proceeds.**

In selling goods consigned to him, a commission merchant deducts his commission from the sum realized, and remits the proceeds to the consignor. In buying, he charges the sum paid *plus* his commission, and both are included in the funds remitted to him by his customer.

WRITTEN EXERCISES.

**417. 1.** A commission merchant sold a consignment of wheat for \$7240, charging $2\frac{1}{2}\%$ commission. What was his commission? How much did he remit to the consignor?

**2.** An agent receives \$510 with which to purchase goods, after deducting his commission of 2%. What was the cost of the goods? What the commission?

$$\$510 = 100\% + 2\%, \text{ or } 102\% \text{ of cost.}$$

On what is the 2% commission reckoned?

**3.** A commission merchant sold 140 barrels of flour at \$4.80 a barrel. What was his commission at $3\frac{1}{2}$ per cent?

**4.** If I receive 25% for selling 500 bushels of wheat at $87\frac{1}{2}$ cents a bushel, what sum do I remit to my employer?

**5.** An architect charges $\frac{5}{8}\%$ for plans and specifications, and $1\frac{1}{2}\%$ for superintending the construction of a building which cost \$16000. What is his commission?

**6.** A lawyer charged \$14 for collecting \$200. What was his rate of commission?

**7.** My commission for selling \$792 worth of goods was \$23.76. What rate did I charge?

**8.** A publisher received \$160 as the proceeds of a sale of books. For how much were the books sold, the agent's commission being 20% ?

**9.** An agent buys an invoice of 47,500 lb. of coffee at 13 cents, 8% off for cash. What is his commission at 2% ?

**10.** A commission merchant remits \$427.50 as the net proceeds of a sale of 100 bags of chestnuts, containing 2 bushels each, his commission being 5%. At what price a bushel did he sell them ?

**11.** A commission merchant sold produce for \$2600, and after deducting his commission and \$70 for other charges, he remitted to the consignor \$2400. What was his rate of commission ?

**12.** A commercial traveler selling goods at a commission of $2\frac{1}{2}$% had an income in a certain year of \$2500. What was the amount of his sales ?

**13.** My principal instructed me to invest \$2440 in wool, and sent me a draft for \$2440 plus 2% commission. For what amount was the draft drawn ?

**14.** A collector's commission for collecting taxes, at $1\frac{1}{2}$%, was \$413.10. What sum did he collect ?

**15.** A grain dealer received \$70 as his commission on the sale of a consignment of wheat, the rate of commission being $2\frac{1}{2}$%. If he sold the wheat at 80¢ a bushel, how many bushels did he sell ?

**16.** A real estate agent bought 10 town lots at \$850 each, at a commission of $1\frac{1}{2}$%, and paid \$4.25 a lot for the examination of title. How much did the lots cost the principal ?

**17.** How many tons of hay at \$14 a ton must a commission merchant sell so that he may remit to the shipper \$175.63 after deducting a commission of $3\frac{1}{2}$ per cent ?

**18.** A merchant sent goods to an agent to be sold on commission. The latter sold them and with the money bought 1805 bbl. of flour at \$5 a barrel after deducting 5% for selling and 5% for buying. What was his total commission ?

## PROFIT AND LOSS.

**418. 1.** A man sold a horse that cost $100 at a profit of 25%. How much did he gain? What was the selling price?

**2.** How many dollars are lost by selling a lot that cost me $1000 at a loss of 5%? How much do I get for it?

**3.** A merchant buys calico for 5 cents a yard, and sells it for 8 cents a yard. What part of the cost is gained? What per cent does he make?

**4.** Flour that cost $5 a barrel was sold for $4 a barrel. What was the loss per cent?

**5.** A man sold a saddle for $12 and gained 20%, or $\frac{1}{5}$ of the cost. What was the cost?

**6.** A sleigh sold for $24 at a loss of 20%. What was its cost?

**7.** A dealer sells corn at a profit of 12¢ a bushel and gains 20%. What did it cost?

**8.** Find the per cent gained on oil bought at 12¢ and sold at 14¢.

**9.** What must be the selling price of coffee that cost 20¢ in order to gain 25%?

**10.** By selling tea at 88¢ a pound, 10% of the cost was gained. What did the tea cost?

**11.** What per cent is gained on goods sold at double the cost?

**12.** How shall I mark goods that cost $6, so that I may deduct 10% from the marked price, and yet make 50% of the cost?

**419. Profit** is the excess of the selling price over the cost.

**420. Loss** is the amount by which the selling price is less than the cost.

**421.** The base upon which the *rate* of gain or *rate* of loss is estimated is *always the cost price.*

**422.** The simple problems of profit and loss correspond to the three general problems of percentage:

1. Given, *cost* and *rate* of gain or loss; required the *gain* or *loss*.

2. Given, *cost* and *gain* or *loss;* required the *rate* of gain or loss.

3. Given, *gain* or *loss* and *rate* of gain or loss; required the *cost*.

To these we add the special problem:

4. Given, *selling price* and *rate* of gain or loss; required the *cost*.

WRITTEN EXERCISES.

**423. 1.** What is gained by selling a horse which cost \$120 at a profit of $33\frac{1}{3}$ per cent?

**2.** A gentleman bought a house for \$1750, and sold it for \$2050. What was his rate of gain?

**3.** If a merchant sells goods at a profit of 10%, and gains \$300, what was the cost of the goods?

**4.** If the selling price of a horse is \$50 and the loss is 50%, what is the cost?

**5.** At what price must goods costing \$200 be marked in order that there may be a profit of 20% after a reduction of 20% has been made?

20% of \$200 = \$40, gain.
\$200 + \$40 = \$240, selling price.
100% − 20%, or 80% of marked price = selling price.
∴ 80% " " = \$240.
1% " " = \$3.
100% " " = \$300.

**6.** At what per cent above cost must goods be marked in order that a deduction of 20% may leave 20% profit?

cost + 20% of cost = 120% of cost, the *selling price*.
100% − 20%, or 80% of marked price = *selling price*.
∴ 80% of marked price = 120% of cost.
1% " " = 1.5% of cost.
100% " " = 150% of cost.
Hence, 150% − 100% = 50% *above cost*.

**7.** A carriage that cost \$128 was sold at a loss of $18\frac{3}{4}\%$. What was the amount received for it?

**8.** Paid \$4.80 for a barrel of flour and sold it for \$6.00. What per cent was gained?

**9.** A grocer sells tea at 30 cents a pound less than cost and loses $33\frac{1}{3}\%$. What was its cost?

**10.** Paper that cost \$2.40 a ream is sold at 18 cents a quire. What is the gain per cent?

**11.** At what price must I sell goods that cost $\$\frac{5}{8}$ to gain 20%?

**12.** What per cent was lost on a wagon which cost \$90 and sold for \$75?

**13.** A boy had 2 goats which he sold for \$6.00 each. What did they cost him if he gained 20% on one and lost 20% on the other?

**14.** What must be paid for 27 bushels of apples so that when sold at 20 cents a half-peck there may be a gain of 20%?

**15.** A drover sold 40 head of cattle for \$1820, which was $16\frac{2}{3}\%$ more than they cost. What was the average cost of each?

**16.** A grocer sold potatoes for \$2.80 a barrel and made $16\frac{2}{3}\%$. If he had sold them at \$3.20 a barrel, how much per cent would he have made on the cost price?

**17.** What per cent is gained by selling 15 ounces of tea for a pound?

**18.** If 20% was gained on flour when sold at \$6 a barrel, what per cent was gained when sold at \$7 a barrel?

**19.** If $\frac{6}{7}$ of an acre of land was sold for what the whole cost, what was the gain per cent?

**20.** When goods are sold at $\frac{2}{3}$ of their cost, what per cent is lost?

**21.** If a grocer pays \$2 for 6 pounds of tea and sells 4 pounds for \$3, what is his per cent of profit?

**22.** If I sell $\frac{3}{5}$ of an acre of land for what $\frac{1}{2}$ an acre costs me, what per cent do I lose?

**23.** A farmer sold a cow for $50, which was 80% of the cost price. What was his loss?

**24.** A druggist gained 300% by retailing quinine at $3.00 an ounce. What did it cost him?

**25.** If $\frac{4}{9}$ of the selling price is gained, what is the per cent profit?

**26.** A merchant marks an article $2.80, but takes off 5% for cash. If his profit is 33%, what was the cost of the article?

**27.** If resin is melted with $33\frac{1}{3}\%$ of its weight of tallow, what per cent of tallow does the mixture contain?

**28.** The sum of 10% of a number and 5% of half the remainder is what per cent of $\frac{1}{4}$ of the number?

**29.** If I sell $\frac{3}{4}$ of an article for the cost of the whole of it, what per cent gain do I make on the part sold?

**30.** If there is a gain of $12\frac{1}{2}\%$ on tea at 90 cents a pound, what would be the gain per cent at 84 cents a pound?

$$84 : 90 = 1.12\tfrac{1}{2} : (\quad).$$

**31.** If I lose 10% by selling goods at 28 cents a yard, for what should they be sold to gain 20%?

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**32.** If from the retail price of a book 20% is deducted, and a discount of 60% is made on the balance, and then the book sells for $1.44, what is the retail price?

**33.** Sold two lots at $2400 each, gaining 25% on one and losing 25% on the other. What was the entire gain or loss?

**34.** On two pianos sold for $550 a dealer gained 25%. What was the cost of each if one cost 20% more than the other?

**35.** Mr. H bought 50 sheep at $5 each. After 10 of them died, he sold the remainder so as to gain 20% on his investment. What price a head did he receive?

**36.** How shall a merchant mark shoes that cost $2.50 so that he may fall 20% from the marked price and still make 12%?

**37.** A speculator sold 5000 bushels of July wheat at $75\frac{7}{8}$¢, at a profit of $6\frac{1}{4}\%$. What was the cost of the wheat?

**38.** I bought a watch for \$45, which was 25% less than its real value, and sold it for 25% more than its real value. What was my gain?

**39.** A tradesman marks his goods to sell at retail at 40% above cost, but sells to wholesale customers at 12% discount from the marked price. What per cent profit does he make on the goods sold at wholesale?

**40.** A beef packer bought a lot of cattle for 15% less than they cost the shipper, and made a profit of 30% on the transaction, gaining \$1500. What did the cattle cost the shipper?

**41.** A wholesale clothier marked a lot of clothing so that the price a suit was \$18. He discounted this price 20% to a retailer, who marked them to gain 25%. Find the price of a suit at retail.

**42.** A grocer mixes in equal quantities teas costing 68 cents, 86 cents, and 96 cents a pound, and sells the mixture at 90 cents a pound. Find the gain per cent.

**43.** What per cent is gained by buying coal by the long ton, and selling it at the same price by the short ton?

**44.** A dealer sells 72 lb. of sugar for as much as 87 lb. cost him. What per cent does he gain?

**45.** What per cent above cost must a merchant mark goods so that after taking 15% from the marked price he will lose 5%?

**46.** A merchant sells goods to a customer at a profit of 60%, but the buyer becomes bankrupt and pays only 70 cents on the dollar. What per cent does the merchant gain or lose on the sale?

## TAXES.

**424.** Charges imposed by law upon persons or property for the support of government are called **Taxes.**

The annual *yield* of these charges is called *revenue.*

## State and Local Taxes.

**425.** The charges levied by a state for the support of its government are called **State Taxes**; those levied by the various minor civil divisions are called **Local Taxes.** The latter include county, city, and township taxes.

**426.** The tax on property is reckoned at *a certain per cent* of its *assessed* value. The rate of taxation is usually expressed as a certain number of *mills* on the dollar.

**427.** In addition to the tax on property, some states levy an equal tax upon each adult citizen. This is called a **Poll Tax,** and is usually $1 a year.

1. If taxes are not paid when due, the law usually requires the delinquent to pay a certain per cent of his tax additional.

2. The collector of taxes usually receives a per cent of the sum collected, *i.e.*, a commission.

### WRITTEN EXERCISES.

**428. 1.** A tax of $3600 is to be raised in a town, the taxable property of which is valued at $712500, and there are 750 persons subject to a poll tax of $1 each. What is the rate of taxation? What is A's tax, if he owns property assessed at $5900.

$\$3600 - \$750 = \$2850$, to be levied on property.
The tax on $\$712500 = \$2850$.
$\therefore$ the rate of taxation $= \$2850 \div \$712500 = .004$, or $\frac{2}{5}\%$.
$\therefore$ the property tax on $\$5900 = \frac{2}{5}\%$ of $\$5900 = \$23.60$
$\$23.60 + \$1.00 = \$24.60$, A's entire tax.

**2.** The assessed value of a property is $7500, and the rate of taxation $\frac{1}{4}\%$. What is the tax?

**3.** The assessed valuation of taxable property in a town is $3,500,000. The tax to be raised is $4900. What is the rate of taxation?

**4.** What sum must be assessed to build a schoolhouse at a cost of $5460, and pay $2\frac{1}{2}\%$ for collecting?

Compare "buying" problems in Commission.

**5.** What tax will be paid by a man whose house is valued at \$5160, personal property at \$7815, and who pays a poll tax of \$1, the rate of taxation being $\frac{1}{4}\%$, or $2\frac{1}{2}$ mills on the dollar?

**6.** At 5 mills on a dollar, how much is the tax of a man who owns a farm of 150 acres, worth \$50 an acre, but assessed for only $\frac{3}{5}$ of its value? If he does not pay the tax when due and a penalty of 5% additional is added, what is his tax?

**7.** Find the tax levy, total valuation, and poll tax in the town, city, or county in which you live, and compute the tax rate.

**8.** When the tax rate is reduced from 7 mills to $5\frac{1}{2}$ mills on the dollar, my taxes are lowered \$75. For how much am I assessed?

**9.** The taxes assessed in a town are \$24000. If $1\frac{3}{4}\%$ commission is paid for all taxes actually collected and 5% of the taxes can not be collected, what are the net proceeds?

**10.** The expenses of a city are \$7,500,000 per annum, and the assessed valuation of its property \$200,000,000. What must the tax rate be, allowing 1% of the taxes to be uncollectible?

**11.** I buy a lot for \$500 and build a house on it for \$3000. I pay a tax on the whole of 7 mills on a dollar, the property valuation being $\frac{2}{3}$ of the cost. For how much must I rent the house and lot to realize 10% a year on my money?

## National Taxes.

**429.** Taxes levied by the general government upon goods imported from other countries are called **Duties** or **Customs.** These are *indirect* taxes.

The *customs revenue* is collected at *custom houses* situated at ports of entry established by law.

**430.** When the duty is a certain per cent of the cost of the imported goods it is called an **Ad Valorem Duty.**

**431.** When the duty is a fixed charge on the quantity of goods, without regard to their cost, it is called a **Specific Duty.**

Thus, \$15 a ton is a specific duty, while 30% (of cost where bought) is ad valorem.

NOTES.—1. Some goods are by law subject to both ad valorem and specific duty.

2. A schedule of the legal rates of duties on imports is called a *tariff*.

**432.** The expenses of the general government are met partly by customs revenue, partly by **Internal Revenue.** The latter is derived chiefly from the sale of licenses to manufacture or sell certain domestic articles, as tobacco, whisky, etc.

NOTE.—In order to meet war expenditures the general government sometimes levies a special **War Revenue Tax**, as in the recent Spanish-American War (1898). The revenue is derived from the sale of *revenue stamps,* which are required to be used on certain articles, as checks, mortgages, stock certificates, patent medicines, etc.

WRITTEN EXERCISES.

**433. 1.** What is the duty, at $2\frac{1}{2}$ cents a pound, on 42 boxes of raisins, each containing 40 pounds, and costing 7 cents a pound?

NOTE.—The rates of duty given here are from the Dingley Tariff Law, which went into effect July 24, 1897.

**2.** What will be the duty, at 25% *ad valorem,* on a shipment of books from London to New York, invoiced at \$4500?

**3.** What is the duty on 4368 pounds of wool invoiced at \$1826, when the rate is 11 cents a pound?

**4.** What is the duty on a ton of tin plate at $1\frac{1}{2}$ ct. a pound?

**5.** What is the duty on 300 lb. of perfumery that cost \$2 a pound, when the duty is \$.60 a pound and 45% ad valorem ?

**6.** If 200 yards of silk are purchased at \$1.25 a yard, what is the duty at 60% ad valorem? What specific duty does it equal ?

**7.** Imported 100 half-pint bottles of champagne. If the duty is \$2 a dozen, breakage 5%, what is the duty ?

**8.** I paid 40% duty on a watch which cost me, including the duty, \$60. If it had been on the free list how much less would it have cost me ?

**9.** A manufacturer produces oleomargarine at a cost of 3 cents a pound. What per cent does he gain if he sells it at 20 ct. a pound after paying the internal revenue tax of 15 cents a pound ?

**10.** A dealer imported 20000 oriental rugs, each 2 ft. 6 in. by 6 ft., and invoiced at \$1.80. The duty was 10 cents a square foot and 40% ad valorem. How much did the government get as a result of this importation ?

## REVIEW WORK.

### ORAL EXERCISES.

**434. 1.** Mr. B bought a cow for \$30, which was 50% of his money. How much had he left ?

**2.** A man gave 25% of his money for an organ valued at \$125. How much money had he left ?

**3.** A laborer who earns \$1.50 a day saves 20% of it. How much does he spend ?

**4.** A merchant bought a stove for \$15 and sold it for \$20. What was the gain per cent ?

**5.** If a grocer gains 10% on eggs that cost 15 cents a dozen, what price is paid by the purchaser ?

**6.** When a shop-worn article that cost \$2.80 is sold for 25% less than cost, what is received for it ?

**7.** What per cent do I gain by selling for \$15 a cart that cost \$12 ?

**8.** Thirty dollars is $37\frac{1}{2}\%$ of what I paid for a wagon. How much less than $100 did I pay?

**9.** What per cent of a yard is 2 feet?

**10.** In a certain district there are 20 school days in a month. When a boy is absent 5 days each month, what per cent of the time does he lose?

**11.** When pencils that cost 1 cent each are sold for 3 cents each, what is the gain per cent?

**12.** I bought a lot for $250, and through an agent sold it for $300. If the agent's commission was 5%, how much did I gain?

**13.** If 4 lemons are sold for what 5 cost, what per cent is gained?

**14.** Mr. H bought wheat at $.80 a bushel. If he sells it at 5% gain, how many bushels must he sell to gain $5?

**15.** What is the gain per cent when an article that cost half a cent is sold for a cent and a half?

**16.** I sold two sheep for $12 each. On one I gained 20%, and on the other I lost 20%. How much did my loss exceed my gain?

WRITTEN EXERCISES.

**435. 1.** A man buys a quantity of wine at $5 a gallon; 20% of it is wasted. At what price a gallon must he sell the remainder so as to gain 20% on his outlay?

**2.** Sold 72 yards of carpet at $\$1.37\frac{1}{2}$ a yard, and thereby gained $18. What per cent did I gain?

**3.** The difference between 28% and 59% of a number is 325.5. What is the number?

**4.** Fifteen per cent of 484 is 66% of what number?

**5.** How much land at $35 an acre can an agent buy with $3965.50, after deducting his commission of 3 per cent?

**6.** What must I ask for a house that cost $4000 in order that I may gain $12\frac{1}{2}\%$ on my purchase after decreasing the amount originally asked by 10 per cent?

**7.** The taxes assessed in a town are \$24000. If 1.75% commission is paid on all taxes actually collected, and 5% of the taxes can not be collected, what are the net proceeds?

**8.** A boy spent 50% of his money at one time, and 60% of the remainder at another, and had 6 cents left. How much had he at first?

**9.** A watch that cost \$62.50 was sold for \$37.50. What per cent was lost?

**10.** A farm was bought for \$3690, which was $12\frac{1}{2}$% more than its value. What was its value?

**11.** Out of a circle 38 inches in diameter there is cut a circle 27 inches in diameter. What per cent of the larger circle is left?

**12.** A man invested $37\frac{1}{2}$% of his money in land at \$40 an acre. If he had \$4500 left, how many acres did he buy?

**13.** A bookseller for a number of books receives \$33.15, after giving a discount of 15% from the list prices. What is his gain, if he himself gets a discount of 25%?

**14.** What is the difference on a bill of \$3500 between a discount of 40% and discounts of 30% and 10%?

**15.** A store was sold at auction for \$3375, which was only $\frac{9}{13}$ of its value. What was the loss per cent?

**16.** The revenue of the post-office department for a certain year (1899) was \$95,021,384, and the expenditures were \$101,632,160. The excess of expenditures was what per cent of the revenue?

**17.** A's money is 60% of B's. What per cent of A's money is B's?

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**18.** A section of land was bought at \$24 an acre. What was received for 240 acres when sold at a gain of 30%?

**19.** A hardware merchant bought three dozen agate basins at the rate of 3 for \$5, and sold them at a gain of \$10 on the whole. What was the gain per cent?

**20.** If I sell $\frac{4}{5}$ of an article for what $\frac{7}{8}$ of it cost, what per cent do I gain?

**21.** How many pounds of flour are required to make 100 2-pound loaves of bread, if the bread weighs 30% more than the flour used?

**22.** What per cent do I lose by selling $\frac{4}{5}$ of my sheep for what $\frac{3}{4}$ of them cost?

**23.** A coin is 22 parts copper and 3 parts nickel. What per cent of the coin is copper? What per cent of the nickel is the copper?

**24.** A man sold flour for $10, thereby losing 20%. How much would he have lost by selling it for $8?

**25.** Eight pounds of coffee and 1 pound of tea are sold at an average price of 30 ct. a pound, being an advance of 35% on the cost. If the pound of tea cost $1, what did the coffee cost a pound?

**26.** Bought land at $60 an acre. How much must I ask an acre that I may deduct 25% from my asking price, and yet make 20% of the purchase price?

**27.** A man bought goods at 25% below the list price, and sold them at 20% above the list price. How much did he invest if he gained $270?

**28.** An agent received $5250 to invest at 5% commission. If he reserves his commission, and invests the remainder in wheat at $.75 a bushel, how many bushels will be purchased?

SUPPLEMENTARY EXERCISES (FOR ADVANCED CLASSES).

**436. 1.** What per cent of a bill does a merchant receive if he gives a discount of 20%, 10%, and 5%?

**2.** A man bought a carriage for $225. One fifth of the cost of the carriage was 9% of what he paid for a span of horses. Find the cost of both.

**3.** A merchant pays $1800 a year rent for a storeroom; 45% of this sum is 18% of one half of his profit. What is his profit?

**4.** What is the rate of gain when 25% of the selling price is gain?

**5.** Mr. A bought a watch for $30, which was 40% less than its value, and sold it for 50% more than its value. How much did he gain?

**6.** Josie paid $160 for her pony, which was 15% more than double the amount she paid for her cart. What did she pay for both?

**7.** An article that cost nothing was sold for $5.20. What was the gain per cent?

**8.** Mr. G said he bought a book for $2 and sold it at a loss of 100%. Did he state two facts or but one? Why?

**9.** A farmer sold a horse at such a price that $\frac{3}{4}$ of the gain equaled $\frac{1}{2}$ of the cost. What was his gain per cent?

**10.** A merchant's private key for marking goods is "p r e - c a u t i o n." How must he mark cloth that cost 10 cents a yard so as to gain 70%?

**11.** How much water must be mixed with 31 gallons 1 quart of alcohol, which cost $45.25, so that the mixture may be sold at $1.25 a gallon, and 25% gained?

**12.** A's money is 12% of B's and 16% of C's. B has $100 more than C. How much has A?

**13.** The United States produced 91,070 metric tons of zinc in a certain year (1897), and that produced by other countries constituted 79.56% of the total production. What was the total production?

**14.** If 15% can be saved by employing women when men are paid $1.60 a day, and if a man does $\frac{1}{3}$ more than a woman in the same time, what are the daily wages paid to women?

**15.** If water expands $7\frac{1}{2}$% in freezing, what will a cubical block of ice weigh that measures 3 dm. on an edge?

**16.** The operatives in a certain factory have their working hours reduced from 10 hours to 8 hours a day without any reduction in their daily wages. By what per cent are their daily wages increased by this change in time?

## INTEREST.

**437.** **1.** I borrow $100 and agree to pay $6 for the use of it for one year. What should I pay if I use the money 2 years?

**2.** I can borrow money by paying 5 cents for the use of $1 for a year. What must I pay for the use of $100 at this rate?

**3.** If I pay 6 cents for the use of $1 for 1 year, what should I pay for its use for 3 years? For $5 for 1 year? For $5 for two years? For $100 for a year?

**4.** I borrow $100 and agree to pay back the money at the end of a year, and 6% additional for its use. How much do I pay for the use of the money, or how much *interest* do I pay? How much do I pay in all at the end of the year?

**5.** What should I receive for a loan of $500 for 2 years at 6%? What interest should I receive for a loan of $300 for 5 years at the same rate?

**438.** **Interest** is a sum paid for the *use* of money.

**439.** The money for the use of which interest is paid is called the **Principal.**

**440.** The sum of the principal and interest is the **Amount.**

**441.** The **Rate of Interest** is always expressed as a certain per cent of the principal. The unit of time is the year.

Thus, interest at 6% means that the charge for a year is 6% of the principal. Hence for 6 months the charge is 3%; for 2 months, 1%; for 2 years, 12%.

NOTES.—1. In ordinary interest it is customary to regard a year as 12 months, and a month as 30 days.

2. The mills in the interest or amount are usually disregarded unless they are 5 or more, in which case they are regarded as a cent.

**442.** If $p$ stands for the principal, $r$ for the rate of interest, $t$ for the *number* of years, and $i$ for the interest, we have the equation—

$$i = p \times r \times t.$$

QUERIES.—Which is the *product?* Which are *factors?*

WRITTEN EXERCISES.

**443. 1.** What is the interest on \$250 for 3 years at 5% ?

Interest for 1 year = 5% of \$250 = \$12.50.
Interest for 3 years = 3 × \$12.50 = \$37.50.

Have the pupil solve by substituting in the equation given above.

**2.** What is the interest on \$350 for 3 years 8 months at 6% ?

(3 yr. 8 mo. = $3\frac{2}{3}$ yr.)

Find the interest on :

**3.** \$480 for 3 years at 6%.
**4.** \$531.21 for 5 years at 8%.
**5.** \$375.75 for 8 years at $5\frac{1}{2}$%.
**6.** \$870.30 for 3 years at 6%.
**7.** \$427.30 for 1 year, 3 months at 5%.
**8.** \$150.25 for 6 years, 1 month at 10%.
**9.** \$325.75 for 3 years, 11 months at $4\frac{1}{2}$%.
**10.** \$650.00 for 7 years, 7 months at 6%.
**11.** \$860.00 for 6 years, 3 months at 4%.
**12.** \$476.38 for 4 years, 8 months at 6%.
**13.** \$410.30 for 7 years, 3 months at 6%.
**14.** \$367.50 for 8 years, 7 months at 5%.
**15.** \$1.50 for 5 years, 4 months at 8%.
**16.** Find the interest on \$150 for 3 months at 6%.

| | | | |
|---|---|---|---|
| Interest for one year | = 6% of the principal. | | |
| " " two months | = 1% of principal | = | \$1.50. |
| " " one month | = $\frac{1}{2}$ of \$1.50 | = | .75. |
| Interest for 3 months | = | | \$2.25. |

**17.** What is the interest on $180 for 98 days at 6% ?

| | | | |
|---|---|---|---|
| Interest for 60 days | = 1% of principal | = | $1.80. |
| " " 30 " | = $\frac{1}{2}$ of $1.80 | = | .90. |
| " " 6 " | = $\frac{1}{5}$ of 90¢ | = | .18. |
| " " 2 " | = $\frac{1}{3}$ of 18¢ | = | .06. |
| Interest for 98 days = | | | $2.94. |

**18.** Find the interest on $860 for 7 months, 24 days at 5%.

| | | | |
|---|---|---|---|
| Interest for | 60 da. (2 mo. ) | = 1% of $860 | = $8.60. |
| " " | 120 da. (4 mo. ) | = 2 × $8.60 | = 17.20. |
| " " | 30 da. (1 mo. ) | = $\frac{1}{2}$ of $8.60 | = 4.30. |
| " " | 24 da. (.8 mo.) | = .8 of $4.30 | = 3.44. |
| Interest for 7 mo. 24 da. at 6% = | | | $33.54. |
| The interest at 1% = | | | $5.59. |
| The interest at 5% = | | | $27.95. |

It may often be more convenient to proceed as follows :

| | | |
|---|---|---|
| Interest for 2 mo. | = 1% of $860 | = $8.60. |
| " " 1 mo. | = $\frac{1}{2}$ of $8.60 | = $4.30. |
| " " 7.8 mo. | = 7.8 × $4.30 | = $33.54. |

Interest at 5% found as above.

Let it be observed that the interest at *any* rate may be found from the interest at 6%.

Find the interest at 6% on :

**19.** $250 for 6 months.
**20.** $720 for 10 months.
**21.** $150 for 15 months.
**22.** $75 for 23 months.
**23.** $240 for 69 days.
**24.** $225 for 76 days.
**25.** $412 for 93 days.
**26.** $1060 for 119 days.
**27.** $87.50 for 126 days.
**28.** $1 for 93 days.
**29.** $272 for 29 days.
**30.** $27.60 for 3 yr. 10 mo.
**31.** $840 for 5 months, 12 days.
**32.** $760 for 9 months, 24 days.
**33.** $500 for 7 months, 18 days.
**34.** $950 for 3 months, 25 days.
**35.** $375 for 3 years, 7 months.
**36.** $624 for 2 years, 11 months.
**37.** $480 for 1 yr. 4 mo. 20 da.
**38.** $320 for 2 yr. 7 mo. 13 da.
**39.** $1 for 1 yr. 8 mo. 27 da.
**40.** $5 for 5 yr. 5 mo. 5 da.
**41.** $275 for 8 yr. 1 mo. 27 da.
**42.** $360.25 for July, 1901.

**43.** Find the interest on $1400 from June 6, 1896, to July 6, 1900, at 6%.

**44.** Find the amount of \$950 from Nov. 5, 1896, to June 15, 1898, at 7%.

**45.** Find the amount of \$343.50 from Jan. 24, 1895, to Dec. 24, 1899, at 5%.

Find the interest and amount of:

**46.** \$750 for 48 days at 6%.

**47.** \$1000 for 63 days at 6%.

**48.** \$600 for 225 days at 5%.

**49.** \$120 for 93 days at 7%.

**50.** \$360 for 123 days at $3\frac{1}{2}$%.

**51.** \$630 for 63 days at $4\frac{1}{2}$%.

**52.** \$860 for 33 days at 8%.

**53.** \$9430 for 2 yr. 5 mo. 7 da. at $3\frac{1}{2}$%.

**54.** \$3875 for 100 days at 4%.

**55.** \$720 for 6 mo. 3 days at 3%.

**56.** \$842 from Jan. 1 to April 10 at 6%.

**57.** Mr. Stoney borrowed \$1250 for 94 days at 6%. How much interest did he have to pay?

**58.** Mr. Thomas bought a piano for \$350, agreeing to pay interest at 5% if he did not pay for the instrument in 30 days. He paid the bill at the end of 66 days. What amount did he pay?

**59.** A man borrowed \$6500 with which he engaged in a business that paid him a profit of 25% on his capital. If the loan cost him 6%, what was his annual net profit?

**60.** A man bought a farm for \$4200, agreeing to pay \$1000 at the end of each year, and also to pay 6% interest on all unpaid money. At the end of four years, how much did he still owe?

OLD 6 PER CENT METHOD.

**61.** Find the interest on \$360 for 1 yr. 1 mo. 1 da. at 6%.

| | | |
|---|---|---|
| Interest at 6% for 1 yr. | $= .06$ | of the principal. |
| Interest at 6% for 1 mo. | $= .005$ | of the principal. |
| Interest at 6% for 1 da. | $= .000\frac{1}{6}$ | of the principal. |
| Interest for 1 yr. 1 mo. 1 da. | $= .065\frac{1}{6}$ | of the principal. |

$\therefore$ the interest $= .065\frac{1}{6} \times \$360 = \$23.46$.

**62.** Find the interest on $400 for 8 mo. 21 da. at 6%.

| | | | |
|---|---|---|---|
| Interest for 8 mo. | $= 8 \times .005$ | $= .040$ |
| Interest for 21 da. | $= 21 \times .000\frac{1}{6}$ | $= .0035$ |
| Interest for 8 mo. 21 da. | $=$ | $.0435$ |

$\therefore$ the interest $= .0435 \times \$400 = \$17.40$.

☞ Should this method be preferred by any one, it may be employed in computing the interest in all or part of the preceding examples.

## Exact Interest.

**444.** In the preceding exercises 360 days have been regarded as a year. To find *exact interest,* however, we must reckon 365 days to a year.

Exact interest is used by the national and some state governments in their computations of interest, and also by some trust companies.

Find the exact interest on :

**1.** $1000 for 129 days at 4%.

| | | |
|---|---|---|
| Interest for a year | $= 4\%$ of $\$1000$ | $= \$40.$ |
| Exact interest for 1 day | $= \$40.00 \div 365$ | $= \$.10958.$ |
| " " " 129 days | $= 129 \times \$.10958$ | $= \$14.14.$ |

**2.** $450 for 228 days at 6%.

**3.** $2568 for 93 days at 3%.

**4.** $84.75 for 37 days at 4%.

**5.** $200 for 63 days at 5%.

**6.** $875 for 151 days at $4\frac{1}{2}\%$.

**7.** $375 for 33 days at 6%.

**8.** $725 from April 1 to July 19 at 6%.

## PROMISSORY NOTES.

**445.** A promise in writing to pay a stated sum of money on demand or at a specified time is called a **Promissory Note.**

1. The person who signs the note is called the *maker.*
2. The person to whom the note is made payable is called the *payee.*
3. The person who owns the note is called the *holder.*

## Forms of Notes.

**446.**

(a)

$350.50. Richmond, Va., May 1, 1900.

On demand I promise to pay Chas. S. McNulty three hundred fifty and $\frac{50}{100}$ dollars, with interest at 6%, for value received.

I. L. Beverage.

(b)

$500. New York, Dec. 5, 1901.

Thirty days after date, for value received, I promise to pay to Henry Wilson, or order, five hundred dollars, with interest at 6%.

John R. Logan.

(c)

$280. Chester, Pa., July 31, 1899.

Three months after date, I promise to pay G. B. M. Zerr, or bearer, two hundred eighty dollars, value received.

M. G. Brumbaugh.

1. A note payable on demand is called a *demand note.*

2. A note payable at a specified time is called a *time note,* and, if the words "with interest" are omitted, does not bear interest until it is due.

3. The sum of money named in a note is called the *face.* However, in the case of notes with interest the practice is not uniform, many leading bankers calling the *sum due at maturity* the face.

**447.** A note made payable to the order of the payee or to the bearer is called a **Negotiable Note**; that is, one that can be bought and sold. A note made payable to the payee only is called a **Non-negotiable Note**; that is, one that can not be bought or sold.

1. Which of the notes given above are negotiable?

2. What words must a note contain in order to be negotiable?

**448.** A person who places his *signature* on the back of a negotiable note is called an **Indorser.**

1. A note that is payable to the person named therein "or bearer" may be sold by the payee and transferred by *delivery.*

2. A note payable to a specified person "or order" may also be sold by the payee, but the transfer is indicated by his *indorsement.*

3. A *special* indorsement specifies the person to whose order the note is to be payable, and the indorsement of such person is in turn necessary to the further sale and transfer of the note. An indorsement *in blank* specifies no person to whom it is payable—the payee merely writing his name across the back—and a note so indorsed is payable to *bearer.*

4. By indorsing a note the payee becomes liable for its payment, unless he is permitted by the buyer to qualify his liability by adding to his signature the words "*without recourse.*"

**449.** Where *days of grace* are not allowed, a note is payable at the time specified therein; but about one half of the states allow three "days of grace" for the payment of negotiable notes before they are said to *mature,* or be *legally* due.

Thus, a note *nominally* due Sept. 2 is *legally* due Sept. 5, and maturity is indicated thus: "Due Sept. 2/5."

1. In ascertaining the date of maturity when the *time* is given in *days,* we count forward the exact number of days from the date of the note; when the time is given in *months,* we count forward by calendar months.

2. When the time is given in months, a note dated on the last day of a "long" month falls due on the *last day* of the proper month. Thus, a note dated Jan. 31, 1900, is due Feb. 28 if for one month, and April 30 if for three months.

3. For further information as to the date of maturity, see Bank Discount.

4. Grace has been abolished in the following:

| | | |
|---|---|---|
| California, | Maine,* | Wisconsin, |
| Colorado, | Maryland, | Ohio, |
| Connecticut, | Massachusetts,* | Oregon, |
| Dist. of Columbia, | Montana, | Pennsylvania, |
| Florida, | New York, | Utah, |
| Idaho, | New Jersey, | Vermont, |
| Illinois, | New Hampshire,* | Virginia. |
| | North Dakota, | |

* Except on Sight Drafts.

WRITTEN EXERCISES.

**450. 1.** Write a negotiable note for $350, with interest at 6%, payable to George H. Hugus, making J. H. Lafferty the maker.

**2.** Write a non-negotiable note for $50, making W. P. Campbell the payee, and O. A. Stephenson the maker, payable on demand.

**3.** Write a negotiable note for $275, payable on demand, to Geo. C. Rodgers, or order, with interest at 6%; Walter Newman, maker.

**4.** Write a negotiable note for $1724, payable one day after date, to J. C. Matheny, or bearer, with interest at 6%; E. A. Dudley, maker. Indorse in blank.

**5.** Write a negotiable note for $1000, due three months after date, payable to William Gibson, or order, interest at 6%; F. G. Mauzy, maker.

**6.** Write a note that will be negotiable without indorsement, binding yourself to pay to C. L. Magee $125 in two years, with interest at 6%.

**7.** Write a note that is negotiable when indorsed, binding J. C. Kendall to pay to your order $275.87 on demand, with interest at 5%.

**8.** Indorse the last note so that H. H. Dinsmore may sell it to James M. Laird. Also put Mr. Dinsmore's indorsement on it.

**9.** From the following data write a note bearing interest from date: Date, May 24, 1900; payee, Nannie Mackrell; amount named, $150; rate, 6%; maker, C. Lamb; maturity, August 24, 1900.

**10.** Find the amount due on the last note at maturity.

**11.** Find the date of maturity and the interest of the following note:

$250. BOSTON, March 16, 1900.

Sixty days after date I promise to pay W. M. McCullough, or order, two hundred fifty dollars, with interest at 6%, at the Diamond National Bank. JOSEPH TURNER.

**12.** The following note was paid Dec. 14, 1899. Find the amount paid and the date it became legally due.

$325. MONTEREY, VA., June 1, 1899.

Ninety days after date I promise to pay E. M. Arbogast, or order, three hundred twenty-five dollars, with interest at 6%, for value received.

J. R. SPIEGEL.

**13.** Write a negotiable note for $350, payable to Chas. M. Loomis, due in 30 days from date, with interest, and signed by O. A. Bird. Indorse properly for transferring to W. B. Smith, or order.

## PARTIAL PAYMENTS.

**451.** A **Partial Payment** is a payment of a part of a note or other obligation bearing simple interest.

The amount and date of a payment are usually written, or *indorsed*, upon the back of the note.

**452.** The method usually employed in computing interest on notes when partial payments have been made is expressed in what is known as the *United States Rule.* It is the legal method in most states.

**453.** UNITED STATES RULE.—1. *From the amount of the principal, computed to the time when the payment or the sum of the payments equals or exceeds the interest due, subtract such payment or the sum of the payments.*

2. *Treat the remainder as a new principal, and proceed as before.*

### WRITTEN EXERCISES.

**454.** **1.** A note for $600, bearing interest at 6%, and dated July 10, 1897, has the following payments indorsed upon it:

Jan. 25, 1900, $5; May 20, 1900, $100; Oct. 2, 1901, $200. What is due March 20, 1902?

| | |
|---|---|
| Principal | $600.00 |
| Interest to May 20, 1900 ($34\frac{1}{3}$ mo.) | 103.00 |
| Amount | $703.00 |
| First payment + second payment | 105.00 |
| Balance due | $598.00 |
| Interest to October 2, 1901 | 49.04 |
| Amount | $647.04 |
| Third payment | 200.00 |
| Balance due | $447.04 |
| Interest to March 20, 1902 | 12.52 |
| Balance due March 20, 1902 | $459.56 |

The interest from July 10, 1897 to Jan. 25, 1900 is $91.50, which is more than the payment; hence, by the rule, it cannot be added to the principal. The reason for this is that if the interest were now added and the $5 deducted, the new principal would be $686.50, and the borrower would be paying interest on $86.50 too much. We therefore compute interest to the time of the *second* payment, which together with the *first* payment is more than sufficient to pay accrued interest.

**2.** On a note for $2000, interest at 6%, dated Dec. 10, 1898, and payable in 12 mo., are found the following indorsements:

Jan. 27, 1898, $49; Feb. 5, 1899, $104; May 16, 1899, $60; July 21, 1899, $700.

The note was paid six months after it became due. What was then paid?

**3.** A note for $698, dated Jan. 24, 1899, is indorsed as follows:

Feb. 17, 1899, $115; Aug. 5, 1899, $82; Aug. 18, 1899, $129; Oct. 11, 1899, $213.

At 6%, what is due Nov. 5, 1899?

**4.** A note for $600, dated Aug. 10, 1897, and drawing interest at 5%, has indorsements as follows:

Feb. 4, 1898, $50; July 27, 1898, $10; Oct. 9, 1898, $75.

How much was due Dec. 15, 1898?

**5.**

$3150. BUFFALO, N. Y., NOV. 1, 1898.

Thirty days after date, for value received, I promise to pay T. B. DeArmit three thousand one hundred fifty dollars, with interest at 4%. C. R. McDANIEL.

Indorsements:

Dec. 27, 1898, $1080; May 15, 1899, $540; June 21, 1899, $310; Sept. 22, 1899, $770.

How much was due Oct. 21, 1899?

**455.** When partial payments are made on notes running *one year* or *less*, the balance due at settlement is sometimes computed by what is known as the *Mercantile Rule.*

**456.** MERCANTILE RULE.—*From the amount of the principal at the date of settlement subtract the amount of each payment at the same time, and the remainder will be the balance due.*

**1.** A note for $250.60, bearing interest at 6%, and dated July 17, 1899, has the following indorsements:

Sept. 20, 1899, $80; Jan. 1, 1900, $50; March 13, 1900, $50.

Find the balance due at settlement, May 5, 1900.

| | | |
|---|---|---|
| Amount of principal to May 5, 1900 .............. | | $262.63 |
| Amount of 1st payment to May 5, 1900... | $83.00 | |
| Amount of 2nd payment to " " ... | 51.03 | |
| Amount of 3d payment to " " ... | 50.43 | $184.46 |
| Balance due May 5, 1900......................... | | $78.17 |

**2.** A note for $1200, dated April 22, 1900, and due 5 months after date, bears interest at 8%. The following payments have been made:

May 3, 1900, $125; August 7, 1900, $25.

Find the balance due at maturity.

**3.** A note for $850 was made Jan. 18, 1898, with interest at 6%. On the note were the following indorsements:

April 18, 1898, $200; July 18, 1898, $250; Sept. 18, 1898, $200.

Find amount due on this note Jan. 18, 1899.

## PROBLEMS IN SIMPLE INTEREST.

**457. 1.** At 6% what principal will yield 6¢ in a year? 60¢ in a year? \$3 in a year? \$6 in a year? \$12 in two years?

**2.** What is the rate when the interest of \$100 for a year is \$6? When it is \$4? When it is \$15 for 3 years?

**3.** In what time will \$100 loaned at 6% yield \$6 interest? \$18 interest?

**4.** In what time will \$200 loaned at 6% yield \$6 interest? \$12 interest?

**458.** In Art. 442 we found that the interest is the product of three factors, expressed in the equation

$$i = prt \qquad (1)$$

In this equation, if three of the elements are given, the other can be found. Dividing both members of (1) by $rt$, we have

$$p = \frac{i}{rt} \qquad (2)$$

Dividing both members of (1) by $pt$, we have

$$r = \frac{i}{pt} \qquad (3)$$

Dividing both members of (1) by $pr$, we have

$$t = \frac{i}{pr} \qquad (4)$$

**459.** Since the *amount* equals the sum of the principal and interest, we have, if $a$ represents the *amount*,

$a = p + i$, or, replacing $i$ by its value $prt$,
$a = p + prt$, or $a = p\,(1 + rt)$.

Dividing both members by $1 + rt$, we have

$$p = \frac{a}{1 + rt} \qquad (5)$$

WRITTEN EXERCISES.

**460. 1.** What principal will in 3 yr. 6 mo. at 6% produce \$49.14 interest?

(a)

Interest of $\$x$ for $3\frac{1}{2}$ yr. = \$49.14.
Interest of \$1 for $3\frac{1}{2}$ yr. = \$.21.
$\therefore \$x = \$49.14 \div .21$, or \$234.

(b)

Using equation (2), $t = 3\frac{1}{2}$; $r = 6\% = .06$; $i = \$49.14$.

Hence, $p = \dfrac{\$49.14}{.06 \times 3\frac{1}{2}} = \$234.$

Find the principal that will produce:

**2.** \$60 interest in 2 yr. at 6%.

**3.** \$125 interest in 2 yr. 6 mo. at 8%.

**4.** \$216 interest in 8 months at 6%.

**5.** \$127.50 interest in 3 yr. 6 mo. 15 da. at 6%.

**6.** If money loaned for seven months at $4\frac{1}{2}\%$ produces \$210, how much is loaned?

**7.** How much must be invested at 5% to yield \$1500 interest quarterly?

**8.** If I receive \$2200 semi-annually from an investment yielding 5%, what is the sum invested?

**9.** At what rate will \$234 produce \$49.14 interest in 3 yr. 6 mo.?

(a)

Interest for $3\frac{1}{2}$ yr. = \$49.14.
Interest for 1 yr. = $\$49.14 \div 3\frac{1}{2} = \$14.04$.
$\therefore$ the rate = $\$14.04 \div \$234 = .06$, or 6%.

(b)

Using equation (3), $i = \$49.14$; $p = \$234$; $t = 3\frac{1}{2}$.

Hence, $r = \dfrac{\$49.14}{\$234 \times 3\frac{1}{2}} = .06$, or 6%.

Find the rate when the interest:

**10.** On \$325 for 1 yr. 6 mo. is \$19.50.

**11.** On \$1400 for 3 yr. 9 mo. is \$315.

**12.** On \$2500 for 2 yr. 10 mo. is $\$283.33\frac{1}{3}$.

**13.** On \$1576 for 1 yr. 5 mo. 18 da. is \$92.45.

**14.** In what time will \$234 yield \$49.14 interest at 6% ?

(a)

Interest for $x$ years = \$49.14.
Interest for 1 year = \$14.04.
$\therefore$ the time = \$49.14 ÷ \$14.04 = 3.5, or $3\frac{1}{2}$ yr.

(b)

Using equation (4), $i = \$49.14$; $p = \$234$; $r = .06$.

$$\text{Hence, } t = \frac{\$49.14}{\$234 \times .06} = 3.5, \text{ or } 3\tfrac{1}{2} \text{ years.}$$

In what time will :

**15.** \$300 produce \$37.50 at 5% ?

**16.** \$685 produce \$123.30 at 4%.

**17.** \$4000 produce \$1000 at $4\frac{1}{2}$% ?

**18.** What principal will in 3 yr. 6 mo. amount to \$283.14, at 6% ?

(a)

Amount of \$$x$ for $3\frac{1}{2}$ yr. = \$283.14.
Amount of \$1 for $3\frac{1}{2}$ yr. = \$1.21.
$\therefore$ \$$x$ = \$283.14 ÷ 1.21 = \$234.

(b)

Using equation (5), $a = \$283.14$; $r = .06$; $t = 3\frac{1}{2}$;
$1 + rt = 1 + .06 \times 3\frac{1}{2} = 1 + .21 = 1.21.$

$$\text{Hence, } p = \frac{\$283.14}{1.21} = \$234.$$

Find the principal that will amount to :

**19.** \$936 in 5 years at 6%.

**20.** \$843.60 in 8 years at 6%.

**21.** \$1844.40 in 222 days at 4%.

**22.** \$681.40 in 7 mo. 24 da. at 5%.

**23.** At what rate will \$240 gain \$8.96 in 6 mo. 12 da. ?

**24.** If \$1200 amounts to \$1391 in 2 yr. 7 mo. 25 da., what is the rate per annum ?

**25.** A borrowed \$700 at 6%, and paid in full \$724.50. How long did he have the money ?

**26.** In what time will \$100 amount to \$200, or *double* itself, at 6% ?

**27.** In what time will \$100, at 6%, gain \$100 ?

**28.** How much must be invested at 7% to give a semi-annual income of \$875 ?

**29.** What principal at interest at 6% will amount to \$580.72 in 7 mo. 12 da.?

**30.** What sum at 6% will amount to \$795 in a year ?

**31.** I owe \$1302 which is to be paid in a year. What cash payment to-day would pay the debt if money is worth 5% ?

**32.** A house is offered to me for \$2400 cash, or for \$2800 if not paid for 15 months. If money is worth $4\frac{1}{2}$%, how much better for me is the cash offer ?

## ANNUAL INTEREST.

**461.** Simple interest on the principal and on each year's unpaid interest is called **Annual Interest.**

Notes bearing annual interest contain the words "interest payable annually." The laws of some of the states do not allow annual interest.

### WRITTEN EXERCISES.

**462. 1.** Find the amount due at maturity on a note for \$400, due 5 years from its date, with interest at 6%, payable annually.

Face of note = \$400.
Interest on \$400 for 5 yr. at 6% = \$120.
Interest on \$24 for 4 yr. + 3 yr. + 2 yr. + 1 yr. at 6% = \$14.40.
\$400 + \$120 + \$14.40 = \$534.40, the amount.

The first year's interest (\$24) draws interest 4 yr. ; the second, 3 yr. ; the third, 2 yr. ; the fourth, 1 yr. This is the same as one year's interest drawing interest 10 years.

Find the amount at annual interest of :

**2.** \$1200 due in 3 yr. at 6%.

**3.** \$500 due in 4 yr. 6 mo. at 6%.

(Periods = $3\frac{1}{2} + 2\frac{1}{2} + 1\frac{1}{2} + \frac{1}{2}$.)

4. $350 due in 3 yr. 8 mo. 15 da. at 8%.

5. $840 due in 3 yr. at 5%.

6. $7000 due in 4 yr. at 6%.

7. What amount is due June 20, 1898, on a note for $350, dated Jan. 5, 1895, with 6% interest, payable annually, on which no payments have been made?

8. Find the amount of $1200 at annual interest for 4 yr. at 6%. Compare the amount of annual interest with the amount at simple interest.

9. Mr. H makes a note for $1000 for 3 yr. 3 mo. with interest at 6% per annum, payable semi-annually, but pays no interest. Find the amount due at maturity.

## COMPOUND INTEREST.

**463.** Interest found by adding unpaid interest to the principal at stated intervals and by using the sum as a new principal is called **Compound Interest.**

1. When the interest is added to the principal at the end of each year, the interest is said to be compounded *annually;* when it is added every three months, it is said to be compounded *quarterly;* and so on.

2. Savings banks usually allow compound interest to depositors, and in some states it is legal on funds due by or to guardians. It is also used in constructing the tables of bond values used by brokers; otherwise it is not in general use.

### WRITTEN EXERCISES.

**464. 1.** What is the compound interest on $500 for 3 years at 5%?

$500 + interest for a year = $525, a new principal.
$525 + " " " = $551.25, " "
$551.25 + interest for a year = $578.81, the amount due.
$578.81 − $500 = $78.81, the compound interest.

2. Find the compound interest on $250 for 2 years 6 months at 6%, interest compounded semi-annually.

(The interest is to be added every six months.)

**3.** Find the compound interest on $800 for 3 yr. 6 mo. 24 da. at 6%.

| | | |
|---|---|---|
| The amount at compound interest for 3 yr. | = | $952.81. |
| Interest on $952.81 for 6 mo. 24 da. | = | 32.40. |
| Amount for 3 yr. 6 mo. 24 da. | = | $985.21. |

$985.21 – $800 = $185.21, the compound interest required.

Find the compound interest on:

**4.** $360 for 3 yr. 6 mo. at 5%.

**5.** $243.12 for 3 yr. at 4%.

**6.** $300 for 2 yr. 4 mo. at 6%.

**7.** $500 for 2 yr. 4 mo. at 6%, compounded semi-annually.

**8.** $735.60 for 2 yr. 5 mo. 24 da. at 4%, compounded quarterly.

**9.** A boy deposits $200 in a savings bank which allows 4% compound interest, and adds the interest to the principal every 6 months. At the end of 5 years how much will the bank owe the boy?

**10.** On the first day of June each year a man deposits $300 in a savings bank which allows 4% interest, compounded semi-annually. How much will the bank owe the man at the expiration of 5 yr. 3 mo.?

**465.** The compound-interest table on the opposite page is such as is used by savings banks in computing compound interest, and by investors and others who wish to compute the amount resulting from the *reinvestment of interest* as it becomes due.

Find the compound interest of the following, making use of the table:

**1.** $420 for 8 years at $4\frac{1}{2}$%.

Amount of $1 for 8 yr. at $4\frac{1}{2}$% = $1.42210.
Amount of $420 = 420 × $1.42210 = $597.28.

**2.** $510 for 5 yr. 6 mo. at 5%.

**3.** $2500 for 4 yr. 3 mo. at 7%.

**4.** $1050 for 2 yr. 8 mo. at 6%.

**5.** $800 for 3 yr. 7 mo. 15 da. at 3%.

**6.** $260.75 for 4 yr. 8 mo. 10 da. at $3\frac{1}{2}$%.

## Table.

*Showing the amount of $1 at compound interest for :*

| YR. | 2 PER CENT. | 2½ PER CENT. | 3 PER CENT. | 3½ PER CENT. | 4 PER CENT. |
|---|---|---|---|---|---|
| 1 | 1.02000 | 1.02500 | 1.03000 | 1.03500 | 1.04000 |
| 2 | 1.04040 | 1.05063 | 1.06090 | 1.07123 | 1.08160 |
| 3 | 1.06121 | 1.07689 | 1.09273 | 1.10872 | 1.12486 |
| 4 | 1.08243 | 1.10381 | 1.12551 | 1.14752 | 1.16986 |
| 5 | 1.10408 | 1.13141 | 1.15927 | 1.18769 | 1.21665 |
| 6 | 1.12616 | 1.15969 | 1.19405 | 1.22926 | 1.26532 |
| 7 | 1.14869 | 1.18869 | 1.22987 | 1.27228 | 1.31593 |
| 8 | 1.17166 | 1.21840 | 1.26677 | 1.31681 | 1.36857 |
| 9 | 1.19509 | 1.24886 | 1.30477 | 1.36290 | 1.42331 |
| 10 | 1.21899 | 1.28009 | 1.34392 | 1.41060 | 1.48024 |
| 11 | 1.24337 | 1.31209 | 1.38423 | 1.45997 | 1.53945 |
| 12 | 1.26824 | 1.34489 | 1.42576 | 1.51107 | 1.60103 |
| 13 | 1.29361 | 1.37851 | 1.46853 | 1.56396 | 1.66507 |
| 14 | 1.31948 | 1.41297 | 1.51259 | 1.61870 | 1.73168 |
| 15 | 1.34587 | 1.44830 | 1.55797 | 1.67535 | 1.80094 |
| 16 | 1.37279 | 1.48451 | 1.60471 | 1.73399 | 1.87298 |
| 17 | 1.40024 | 1.52162 | 1.65285 | 1.79468 | 1.94790 |
| 18 | 1.42825 | 1.55966 | 1.70243 | 1.85749 | 2.02582 |
| 19 | 1.45681 | 1.59865 | 1.75351 | 1.92250 | 2.10685 |
| 20 | 1.48595 | 1.63862 | 1.80611 | 1.98979 | 2.19112 |

| YR. | 4½ PER CENT. | 5 PER CENT. | 5½ PER CENT. | 6 PER CENT. | 7 PER CENT. |
|---|---|---|---|---|---|
| 1 | 1.04500 | 1.05000 | 1.05500 | 1.06000 | 1.07000 |
| 2 | 1.09203 | 1.10250 | 1.11303 | 1.12360 | 1.14490 |
| 3 | 1.14117 | 1.15763 | 1.17424 | 1.19102 | 1.22504 |
| 4 | 1.19252 | 1.21551 | 1.23882 | 1.26248 | 1.31080 |
| 5 | 1.24618 | 1.27628 | 1.30696 | 1.33823 | 1.40255 |
| 6 | 1.30226 | 1.34010 | 1.37884 | 1.41852 | 1.50073 |
| 7 | 1.36086 | 1.40710 | 1.45468 | 1.50363 | 1.60578 |
| 8 | 1.42210 | 1.47746 | 1.53469 | 1.59385 | 1.71819 |
| 9 | 1.48610 | 1.55133 | 1.61909 | 1.68948 | 1.83846 |
| 10 | 1.55297 | 1.62889 | 1.70814 | 1.79085 | 1.96715 |
| 11 | 1.62285 | 1.71034 | 1.80209 | 1.89830 | 2.10485 |
| 12 | 1.69588 | 1.79586 | 1.90121 | 2.01220 | 2 25219 |
| 13 | 1.77220 | 1.88565 | 2.00577 | 2.13293 | 2.40985 |
| 14 | 1.85194 | 1.97993 | 2.11609 | 2.26090 | 2.57853 |
| 15 | 1.93528 | 2.07893 | 2.23248 | 2.39656 | 2.75903 |
| 16 | 2.02237 | 2.18287 | 2.35526 | 2.54035 | 2.95216 |
| 17 | 2.11338 | 2.29202 | 2.48480 | 2.69277 | 3.15882 |
| 18 | 2.20848 | 2.40662 | 2.62147 | 2.85434 | 3.37993 |
| 19 | 2.30786 | 2.52695 | 2.76565 | 3.02560 | 3.61653 |
| 20 | 2.41171 | 2.65330 | 2.91776 | 3.20714 | 3.86968 |

## BANKS AND BANK DISCOUNT.

**466.** A **Bank** is an institution whose ordinary business is to receive deposits of money, to make loans, to discount notes, and to sell and collect drafts.

**467.** A **Check** is the written order of a depositor, directing a bank to pay a specified sum of money to a certain person, or to his order.

The following is a common form:

| | |
|---|---|
| *No. 185.* | |
| | *Pensacola, Fla., March 15, 1900.* |
| **First National Bank of Pensacola.** | |
| *Pay to the order of J. A. Jones* | |
| *Seventy-five and* ~~~~~~~~ $\frac{50}{100}$ | *Dollars.* |
| *\$75.50* | *John Doe.* |

NOTE.—If a check is made payable to the order of the payee, as above, the payee must indorse it.

**468.** A *certified check* is a check upon the face of which the cashier of the bank has stamped the word "Certified," with date and name of bank, and has written his signature as cashier. The bank is then responsible for its payment.

**469.** **Bank Discount** is the *sum retained* by a bank in cashing a note before it is due, or on which it loans money. It is simple interest on the *maturity value* of the note.

**470.** The **Proceeds** of a note is its *maturity value* less the *discount*. It is the sum *paid out* by a bank for or on the note.

**471.** The **Day of Maturity** is the day on which the note becomes legally due and payable.

Banks do no business on Sundays or legal holidays, hence notes falling due then are payable on the succeeding or the preceding business day, usually the former. Find out whether in your locality notes falling due on Saturday are payable on that day or on the next succeeding business day.

**472.** The **Term of Discount** is the time for which the bank computes discount—the period for which it charges interest. It begins on the day of discount and ends on the day of maturity.

Some bankers include the day of discount in the time for which they take interest.

**473.** In states where *days of grace* have not been abolished, *three* days are allowed in addition to the time stated in the note before the note is legally due. (See Art. 449.)

**474.** A **Protest** is a notice in writing by a *notary public* to the indorsers that a note has not been paid on the day of maturity. If a note is not protested before the day of maturity ends, the indorsers are released from their obligation.

**475.** In finding *maturity*, banks generally count forward by months or by days, as the note specifies. The *term of discount* is usually found in *days*. In the examples given in this book, this practice is observed.

## MAKING LOANS.

**476.** A person wishing to borrow money from a bank usually makes a note payable to the order of some person who is willing to indorse it.

The following is a common form of such note :

| | |
|---|---|
| *$100.* | *Richmond, Va., March 15, 1902.* |
| *Two months after date I promise to pay to the order of Richard Roe* | |
| *One Hundred ~~~~~~~~ Dollars* | |
| *at The First National Bank, Richmond, Va.* | |
| *Value received.* | |
| *Due, ____________* | *John Doe.* |

When Richard Roe indorses this note, John Doe takes it to the bank, which loans him $100 less the interest (discount) for 61 days *at the legal rate.* This discount is $1.02, hence Mr. Doe gets $98.98 ; but in two months he must pay the bank $100.

1. By indorsing the note Richard Roe binds himself to pay it if John Doe does not ; and since he indorses the note to enable the latter to secure the loan, that is, to accommodate the maker, he is called an *accommodation indorser.*

2. If this note is not paid May 15, it goes to *protest.*

3. No interest is *specified* in notes of this kind, but the *legal rate* is charged.

### WRITTEN EXERCISES.

**477. 1.**

$1000. NEW YORK, Dec. 31, 1900.

Two months after date I promise to pay J. A. Greene, or order, one thousand dollars, value received.

J. B. MURDOCK.

This note was discounted Dec. 31 at 6%, the legal rate. Find the proceeds.

The date of maturity = Feb. 28, 1901.
The term of discount = 59 days.
The discount = interest on \$1000 for 59 da. = \$9.83.
The proceeds = \$1000 − \$9.83 = \$990.17.

NOTE.—In Pennsylvania, and in other states where the day of discount is included, the term of discount in the above note would be 60 days; where grace is allowed, it would be 3 days more. In Boston, and in some other places, when the time a note has to run is expressed in *months*, the term of discount is computed for this number of months, and not for the exact number of days.

**2.**

\$600. PHILADELPHIA, PA., Feb. 28, 1898.

Three months after date I promise to pay to William Post, or order, six hundred dollars, value received.

W. E. SANKEY.

Discounted Feb. 28. Find proceeds.

Day of maturity = May 31 (the 28th being Saturday, and the 30th a legal holiday). Term of discount = 93 days.

**3.**

\$4000. RICHMOND, VA., June 30, 1899.

Sixty days after date I promise to pay to J. F. Guffey, or order, four thousand dollars, value received.

W. R. FORD.

Discounted June 30, at 6%. Find proceeds.

**4.**

\$2500. COLUMBIA, S. C., Aug. 31, 1901.

Three months after date I promise to pay to the order of J. F. Bunn, two thousand five hundred dollars, value received.

W. H. MCKELVEY.

Discounted Aug. 31, at 6%. Find the proceeds. (Grace allowed.)

**5.**

$3000. HARTFORD, CONN., May 4, 1902.

Sixty days after date I promise to pay J. M. Clark, or order, three thousand dollars, value received.

R. J. STONEY, JR.

Discounted May 4.

**6.**

$750. FRANKFORT, KY., April 7, 1900.

Three months after date I promise to pay to the order of W. M. Gill, seven hundred fifty dollars, value rec'd.

GEO. H. WELSHONS.

Discounted April 7. (Grace allowed, and day of discount included.)

**7.**

$1500. BALTIMORE, MD., March 30, 1902.

Two months after date I promise to pay to Howard Welsh, or order, one thousand five hundred dollars, value rec'd.

W. S. FINNEY.

Discounted Mar. 30. (Day of discount included.)

**8.**

$1000. WILMINGTON, DEL., May 18, 1901.

Sixty days after date I promise to pay William Pollock, or order, one thousand dollars, value received.

H. W. WALKER.

Discounted May 18.

**9.**

$500. CHARLOTTE, N. C., April 15, 1900.

Three months after date I promise to pay to the order of Anna Bamford, five hundred dollars, value received.

GEORGE DEWEY.

Discounted April 15. (Grace allowed.)

**10.**

$10,000. SAVANNAH, GA., May 25, 1901.

Two months after date I promise to pay to the order of W. H. McCleary, ten thousand dollars, value received.

JOHN D. MILLER.

Discounted May 25. (Grace allowed.)

**11.**

$400. NASHVILLE, TENN., May 4, 1900.

Sixty days after date I promise to pay to Wm. H. McGary, or order, four hundred dollars, value received.

SAMUEL HARPER.

Discounted May 4. (Grace allowed.)

**12.**

$2000. AUSTIN, TEX., Jan. 15, 1901.

Sixty days after date I promise to pay to the order of H. M. Jones, two thousand dollars, value received.

W. W. ULERICH.

Discounted Jan. 15, 1901. (Grace allowed.)

**13.**

$6000. LITTLE ROCK, ARK., May 9, 1901.

Ninety days after date I promise to pay to the order of R. S. Latham, six thousand dollars, value received.

J. D. ANDERSON.

Discounted May 9, 1901. (Grace allowed.)

## DISCOUNTING NOTES.

**478.** John Mason bought a lot from Richard Adams for $2000, but not having the ready money agreed to pay the $2000 in sixty days, together with interest at 6%.

He gave the following note:

$2000. NEW YORK, March 16, 1900.

Sixty days after date, for value received, I promise to pay to the order of Richard Adams, two thousand dollars, with interest at 6%. JOHN MASON.

Needing money, Richard Adams took the note to a bank and had it discounted the same day, March 16, transferring the note to the bank by indorsement. The bank paid to Mr. Adams the maturity value, less 6% interest thereon for 60 days.

1. The note matures May 15, at which time John Mason must pay the bank $2020, the maturity value. The bank retains the interest on this at 6% for 60 days, or $20.20; hence the owner of the note, Mr. Adams, gets $2020 − $20.20 = $1999.80.

2. So far as the bank is concerned, this process of discounting notes purchased by way of discount—being in effect a mode of lending money—is essentially the same as that of lending money on an indorsed promissory note; but there is the important distinction that the *indorser* is not now an *accommodation* indorser; he is the *owner of the note* and is the *party who receives the money from the bank.*

3. Notice, also, that the note in the latter case usually draws interest, and is frequently not discounted on the day of its making. When it is discounted at a subsequent date, the discount is reckoned on the maturity value for the time from the day of discount to the day of maturity.

WRITTEN EXERCISES.

**479. 1.**

$800. RICHMOND, VA, June 17, 1900.

Three months after date I promise to pay to the order of L. F. Graham, eight hundred dollars, value received, at the First National Bank, with interest at 6%. JNO. B. HEAD.

Discounted July 12, at 6%. Find the proceeds.

Date of maturity = Sept. 17.
Maturity value = $800 + interest for 3 mo. = $812.
Term of discount = 67 days (*i.e.*, 19 in July, 31 in Aug., 17 in Sept.).
The discount = the interest on $812 for 67 days at 6% = $9.07.
The proceeds = $812 − $9.07 = $802.93.

(Many banks in Virginia charge for day of discount.)

**2.**

$300. PITTSBURG, PA., July 10, 1900.

Sixty days after date I promise to pay to J. F. Miller, or order, three hundred dollars, with interest at 6%, value received. LEWIS MORAN.

Discounted Aug. 6, at 6%. Find the proceeds. (Day of discount included.)

**3.**

$500. DAYTON, O., Sept. 29, 1901.

Two months after date I promise to pay to James H. Platt, or order, five hundred dollars, value received.

C. HORNUNG.

Discounted Sept. 29, at 6%. Find proceeds.

**4.**

$1000. JACKSON, MISS., Dec. 31, 1900.

Two months after date I promise to pay to Thomas E. Boyd, or order, one thousand dollars, with interest at 6%, value received. S. C. HEPLER.

Discounted Dec. 31, at 6%. Find proceeds. (Grace allowed.)

**5.**

James H. Lafferty wishing to borrow some money for two months from the Atlas National Bank of Boston gives a promissory note for $1800, George Gosser being the accommodation indorser. Draw the note, dating it May 4, 1900, and find the proceeds.

**6.**

Prepare a 90-day note for $250 on which you can obtain a loan from a bank in your locality. Compute the bank's charge for discounting it in accordance with local practice.

**7.**

I have a note for $5000 dated June 5, and payable three months after date, with 6% interest. How much would a bank in your locality give me for the note on July 3?

**8.**

For what sum is a 60-day note given when a bank discounting it at 8% gives the maker \$725, allowing days of grace?

The discount on \$1 for 63 days at 8% = \$.014.
The proceeds of a \$1 note = \$1 − \$.014 = \$.986.
The face required = \$725 ÷ .986 = \$735.29.

**9.**

If you wish to procure \$1000 from a bank for ninety days at 6%, for what sum must you write the note?

**10.**

The National Union Bank of New York (which counts neither grace nor day of discount) loans J. O. Brown \$7500 on a 3-month note dated April 4, J. W. Lee being the indorser. Write the note.

## PRESENT WORTH AND TRUE DISCOUNT.

**480.** The **Present Worth** of a debt is the sum which, if put at simple interest, would amount to the debt when due.

**481.** The difference between the amount of the debt and its present worth is called the **True Discount.**

WRITTEN EXERCISES.

**482. 1.** Find the present worth and true discount of \$621 due in 2 yr. 6 mo., if money is worth 6%.

(a)

Amount of $\$x$ for $2\frac{1}{2}$ yr. = \$621.
Amount of \$1 for $2\frac{1}{2}$ yr. = \$1.15.
$\$x = \$621 \div 1.15 = \$540$, the present worth.
The true discount = \$621 − \$540 = \$81.

(b)

In equation (5), Art. 459, $a = \$621$; $r = .06$; $t = 2\frac{1}{2}$; $1 + rt = 1.15$.
Hence $p = \$621 \div 1.15 = \$540$.

**2.** Find the present worth of \$590 due in 1 yr. 6 mo., the current rate of interest being 6%.

**3.** Find the true discount on a debt due in 4 mo. 10 da., the debt being \$450, and money being worth 5%.

**4.** Find the present worth and true discount of \$1235 due in 1 yr. 7 mo. 12 da., the current rate being 6%.

**5.** A man buys flour for \$2840 on six months' time. If payment is made at the time of purchase, how much should be deducted from the bill, money being worth 6%?

**6.** What is the difference between the true discount of \$640, due in 1 yr. 3 mo. 15 da., and the interest on the same amount for the same time, money being worth 6% ?

**7.** What sum must I put at interest at 8% to liquidate a debt of \$1250 due 2 yr. 6 mo. hence ?

**8.** I am offered \$8250 cash for my farm by one man, and another offers me \$8580 in 4 months, without interest. Which offer is the better, if money is worth 6% ?

**9.** What is the difference between the bank discount and the true discount, each at 5%, on a note for \$654 due in 90 days ? Show that the bank discount equals the true discount plus the interest on the true discount.

## STOCKS AND BONDS.

**483.** When a number of persons wish to engage in any extensive business, they usually form themselves into an association called a **Stock Company.**

**484.** The sum of money subscribed by the members of the company to inaugurate the business is called the **Capital Stock.**

**485.** The capital stock is usually divided into a definite number of *shares* of a specified value, and is issued in the form of *certificates of stock,* each stating that the person named therein owns so many shares.

**486.** The value of a share named in the certificate of stock is called the **Par Value.** The stock is usually divided into shares of the face or par value of $100 each.

**1.** If the value of each share is $100, what is the par value of 10 shares? 100 shares?

**487.** The price at which stocks are selling in the market is called their **Market Value.**

**1.** When stock sells at 5% above par value, what is the market value of a $100 share?

**488.** A stock is said to be *at a premium*, or *above par*, when it sells for *more than its face value;* it is said to be *at a discount*, or *below par*, when it sells for *less than its face value.*

Thus, if a stock is quoted at 107, $100 stock sells for $107, and the stock is at 7% premium; if it is quoted at 93, $100 stock sells for $93, and the stock is at 7% discount.

**1.** If stock is quoted at 106, what is the rate of premium? Why do stocks vary in price?

**489.** If the company makes more than its expenses, part or all of the surplus is divided among the stockholders as **Dividends.** The dividends are usually expressed as a certain per cent of the *par value,* but sometimes as a certain number of dollars a share.

**1.** If a company declares a dividend of 10%, how much does the owner of a $100 share get?

**490.** A written obligation under seal securing the payment of a sum of money at a specified time, and bearing a certain rate of interest, is called a **Bond.**

**1.** Which would be preferable to own, $1000 of stock in a company, or one of its $1000 bonds, if each pays 5%?

**491.** Bonds are issued by governments (local, state, or national), or by stock companies, for the purpose of effecting loans. They are of two kinds—*registered* bonds and *coupon* bonds.

**492.** *Registered* bonds are recorded by their numbers and the names of the persons owning them, and cannot be transferred without a change in the record kept by the party issuing them. *Coupon* bonds have attached to them coupons, or certificates of interest, which are detached as interest becomes due, and presented for payment.

**493.** Bonds may be bought and sold in the market in the same manner as stocks, and are *designated* in quotations by the name of the company or government issuing them, and the rate of interest they bear, with the date of maturity, and whether *registered* or *coupon.*

Thus, "U. S. 4's, coup., 1925" means United States coupon bonds bearing 4 per cent interest, the principal payable in 1925.

Note.—Bonds pay interest on their *face* value at a fixed rate, hence their *market price* does not affect the interest they yield. The income from stocks is variable, as it depends upon the prosperity of the business.

**494.** Persons who buy and sell stocks and bonds are called **Stock Brokers,** and their commission is called **Brokerage.** The brokerage is usually $\frac{1}{8}$% *of the par value.* This is charged for *buying* and also for *selling.*

**1.** If a broker charges $\frac{1}{8}$% for selling 10 shares of stock, how much will be the brokerage?

**495.** The market values of stocks and bonds as given daily in the newspapers are called *Stock Quotations.*

A quotation of 127 means that \$100 of stock is selling for \$127. In this case the seller receives \$127 − \$$\frac{1}{8}$, or \$126$\frac{7}{8}$ for each share, and the buyer pays \$127 + \$$\frac{1}{8}$, or \$127$\frac{1}{8}$, for each share, provided the deal is made through a broker.

**1.** What will a seller receive from his broker for one share of stock sold at 71$\frac{3}{8}$, brokerage $\frac{1}{8}$%? What for 10 shares?

**2.** What will a buyer have to pay for one share of stock purchased at 97$\frac{1}{4}$, brokerage $\frac{1}{8}$%? What for 10 shares?

**496.** The following are quotations of U. S. bonds in the market of June 7, 1900:

| | BID. | ASKED. |
|---|---|---|
| U. S. 3's, reg. | $109\frac{1}{4}$ | $109\frac{3}{4}$ |
| U. S. 3's, coup. | $109\frac{1}{4}$ | $109\frac{3}{4}$ |
| 4's, reg., 1907 | $114\frac{1}{2}$ | 115 |
| 4's, coup., 1907 | $115\frac{1}{2}$ | 116 |
| 4's, reg., 1925 | $134\frac{1}{2}$ | 135 |
| 4's, coup., 1925 | $134\frac{1}{2}$ | 135 |
| 5's, reg., 1904 | $113\frac{1}{2}$ | 114 |
| 5's, coup., 1904 | $113\frac{1}{2}$ | 114 |

**497.** The following are from the stock quotations of the same day:

| STOCKS. | HIGHEST. | LOWEST. | CLOSING. |
|---|---|---|---|
| Am. Sugar Ref. | $116\frac{1}{8}$ | $114\frac{3}{8}$ | $114\frac{3}{4}$ |
| Am. Tobacco Co. | 128 | 128 | 128 |
| Brooklyn Rap. Tr. | $69\frac{3}{8}$ | $68\frac{5}{8}$ | $68\frac{3}{4}$ |
| C. B. & Q. | $129\frac{1}{4}$ | $128\frac{1}{4}$ | $128\frac{1}{4}$ |
| Del. & Hud. | $113\frac{1}{2}$ | 113 | 113 |
| Ches. & Ohio | $27\frac{3}{4}$ | $27\frac{3}{4}$ | $27\frac{3}{4}$ |
| N. Y. Central | $131\frac{1}{8}$ | $130\frac{1}{2}$ | $130\frac{7}{8}$ |
| Pennsylvania R.R. | $130\frac{3}{4}$ | $129\frac{3}{4}$ | $128\frac{7}{8}$ |
| Northern Pacific | 60 | $59\frac{3}{8}$ | $59\frac{3}{8}$ |
| Southern Railway | 12 | 12 | 12 |

**1.** What would I receive for one share of N. Y. Central at the highest quotation, brokerage $\frac{1}{8}\%$? What for 10 shares?

**2.** What would one share of Pennsylvania R.R. stock cost at the closing quotation, brokerage $\frac{1}{8}\%$? 10 shares? How many shares could be bought for $2580?

**3.** What is the difference between the highest and the lowest quotations of Brooklyn Rapid Transit stock on the given date?

WRITTEN EXERCISES.

**498. 1.** What is the cost of 84 shares of bank stock at $95\frac{1}{4}$, brokerage $\frac{1}{8}\%$?

$$1 \text{ share costs } \$95\tfrac{1}{4} + \$\tfrac{1}{8} = \$95\tfrac{3}{8}.$$
$$84 \text{ shares cost } 84 \times \$95\tfrac{3}{8} = \$8011.50.$$

**2.** What will 40 shares of Northern Pacific stock cost at $59\frac{3}{8}$, brokerage $\frac{1}{8}$ per cent ?

**3.** How much must be paid for 125 shares of Delaware and Hudson stock at 113, brokerage $\frac{1}{8}$ per cent ?

**4.** My broker bought for me 75 shares of C. B. & Q. stock at $129\frac{1}{4}$, charging $\frac{1}{8}\%$ brokerage. Find the cost.

**5.** How many shares of railroad stock at $104\frac{7}{8}$ can be bought for \$9450, brokerage $\frac{1}{8}$ per cent ?

1 share costs $\$104\frac{7}{8} + \$\frac{1}{8} = \$105$.
$\therefore$ the number of shares $= \$9450 \div \$105 = 90$.

**6.** How many shares of Brooklyn Rapid Transit stock can be bought for \$3475 if the quotation is $69\frac{3}{8}$, and brokerage $\frac{1}{8}\%$ ?

**7.** When Federal Steel is quoted at $33\frac{3}{4}$, how many shares can be bought for \$1355, brokerage $\frac{1}{8}$ per cent ?

**8.** I sent my broker \$7938 with which to buy Canadian Pacific stock at $94\frac{3}{8}$, brokerage $\frac{1}{8}\%$. How many shares did I get ?

**9.** What annual income will be realized from \$4982 invested in 4% stock at $105\frac{7}{8}$, brokerage $\frac{1}{8}$ per cent ?

1 share costs $\$105\frac{7}{8} + \$\frac{1}{8} = \$106$.
The number of shares $= \$4982 \div \$106 = 47$.
$\therefore$ the income $= 47 \times \$4 = \$188$.

**10.** What income will be derived from \$4565 invested in railroad stock at 114, brokerage $\frac{1}{8}\%$, if the stock pays 6% dividends annually ?

**11.** What annual income will a man receive if he invests \$12830 in bank stock paying quarterly dividends of 3%, provided the stock is bought at $160\frac{3}{8}$, and no brokerage paid ?

**12.** I invested \$4535 in U. S. 4's at $113\frac{1}{4}$, brokerage $\frac{1}{8}\%$. What does the government pay me annually ?

**13.** What sum of money must be invested in 3% stock at 90, brokerage $\frac{1}{8}\%$, to realize an annual income of \$1800 ?

1 share yields an income of \$3.
The number of shares $= \$1800 \div \$3 = 600$.
1 share costs $\$90 + \$\frac{1}{8} = \$90\frac{1}{8}$.
600 shares cost $600 \times \$90\frac{1}{8} = \$54075$, the investment.

**14.** A man has 4% U. S. bonds on which the quarterly interest amounts to \$480. He bought the bonds at 105. How much did he pay for them?

**15.** A certain bank pays 4% semi-annual dividends, my share of which amounts to \$360. If the stock cost me $152\frac{1}{2}$ and $\frac{1}{8}$% brokerage, how much have I invested in that security?

**16.** A man sold 144 shares of railroad stock at $2\frac{1}{2}$% below par, paying $\frac{1}{8}$% brokerage. With the proceeds he bought Cotton Oil stock at $36\frac{3}{8}$, brokerage $\frac{1}{8}$%. How many shares did he get, and how many dollars were left over?

**17.** If 250 shares of Southern Railway are bought at $12\frac{1}{4}$ and sold at $13\frac{1}{8}$, brokerage $\frac{1}{8}$% in each case, what is the profit?

**18.** A speculator sold through his broker 240 shares of Am. Sugar stock at $152\frac{3}{8}$. It cost him \$27720. What was his gain?

**19.** Mr. Jones bought 75 shares of Am. Tobacco at $142\frac{1}{4}$, held it a year, and sold it at 210. If he received a 15% dividend and paid $\frac{1}{8}$% brokerage each way, what was his total gain?

**20.** If I buy 100 shares of Delaware and Hudson stock at 115 and sell them 6 months later at $112\frac{1}{4}$, after receiving a $3\frac{1}{2}$% dividend, what is my loss, money being worth 4% to me?

**21.** When Missouri Pacific stock is quoted at $47\frac{1}{2}$ and is paying $2\frac{1}{2}$% dividends, what sum invested in it will give me an income of \$2000 a year?

**22.** How many shares of Chesapeake and Ohio stock at $28\frac{1}{2}$ are worth as much as 50 shares of Air Brake at 228?

**23.** One morning a man bought 1000 shares of railroad stock at $133\frac{1}{4}$; in the afternoon he sold it at 135. What was his profit after paying brokerage of $\frac{1}{8}$% each way?

**24.** In 1893 I bought thirty 5% city bonds, par value \$100, at $108\frac{7}{8}$, brokerage $\frac{1}{8}$%. They fell due 6 years later, when the city redeemed them (paid me their par value). What was my net profit?

**25.** Jan. 3, 1890, Henry Howard bought 220 shares of railroad stock at $92\frac{3}{8}$, which paid him dividends as follows: in 1890, 4%; in '91, $4\frac{1}{2}$%; in '92, 3%; in '93, none; in '94, 1%; in '95, $2\frac{1}{2}$%; in '96, 4%; in '97, 5%. Jan. 3, 1898, he sold the stock at $118\frac{1}{2}$. How much would he have lost or gained by lending his money at 6% interest?

**26.** Mr. C. C. Davis bought 80 shares of bank stock at $149\frac{7}{8}$, brokerage $\frac{1}{8}$%. If his investment yielded him 10%, what quarterly dividends did the bank declare?

**27.** What per cent on my investment will I receive if I buy 5% bonds, interest payable annually, at $79\frac{7}{8}$, brokerage $\frac{1}{8}$%, not considering the length of time the bonds run?

Each \$80 invested yields \$5.
$\therefore$ the rate of income $= \frac{5}{80}$, or $6\frac{1}{4}$%.

**28.** Including $\frac{1}{8}$% brokerage, how much must I pay for 4% bonds, interest payable annually, so that I may realize an income of 5% on the investment, not considering the length of time the bonds run?

5% of the cost of each bond (\$100) = \$4.
100% " " " " " " = \$80
(The quotation = $80 - \frac{1}{8} = 79\frac{7}{8}$.)

## REVIEW WORK.

### ORAL EXERCISES.

**499. 1.** What is the interest on \$200 for 5 yr. 6 mo. at 6 per cent?

**2.** If I lend my money at 5%, and receive \$100 a year interest, how much have I loaned?

**3.** Five per cent of William's money is \$1 more than 4% of it. Should he loan it at 5%, in how many years would the interest amount to \$50?

**4.** In how many years will one dollar amount to three dollars, if money is worth 10% ?

**5.** If the interest I pay at 6% is a dollar a month, how much have I borrowed ?

**6.** A man whose salary is $2000 a year spends 70% of it. How much does he save a month ?

**7.** If an organ costs $60, at what price must it be sold to gain 60 per cent ?

**8.** Hart spent 40% of his money for a cart, 50% for a watch, and the remainder, which was $5, for a hat. How much had he at first ?

**9.** After paying out 60% of his money for rent, and 60% of the remainder for coal, Mr. A had $32 left. How much did he pay for rent ?

**10.** When the interest at 8% is $\frac{2}{5}$ of the principal, how many years' interest is due ?

**11.** In how many years will $5 at 5% amount to $15 ?

**12.** The width of a board is 10% of its length, and the perimeter of the board is 22 feet. What is its length ?

**13.** Mr. A received $80 for selling $200 worth of books. What rate of commission did he receive ?

**14.** When goods that cost 30 cents a yard are sold at $37\frac{1}{2}$ cents a yard, what per cent is gained ?

**15.** When bicycles are marked down in price from $75 to $50, by what per cent is the price reduced ?

**16.** One eighth of an acre is what per cent of half an acre ?

**17.** A real estate agent sold a lot for me, and, retaining his commission of 3%, remitted $291. For what sum did he sell the lot ?

WRITTEN EXERCISES.

**500.** **1.** What is the amount of $1000 for 7 yr. 10 mo. 18 da. at 6% simple interest ?

**2.** Find the difference between the simple interest and the compound interest of $1200 for 3 years at 6 per cent.

**3.** How much does the annual interest exceed the simple interest of \$460 for $2\frac{1}{2}$ years at 5 per cent?

**4.** I paid \$165.375 for the use of \$1350 at 7%. How long did I have it?

**5.** What principal at 6% will amount to \$139.86 in 7 months and 6 days?

**6.** A note for \$700 due in 3 months, without interest, is purchased by a broker for \$650. What rate of interest does he receive?

**7.** What sum at simple interest for 5 years at 5% will amount to \$1075.50?

**8.** A man loaned a neighbor a sum of money at $4\frac{1}{2}$% interest. At the end of 18 months the debt was paid in full by a check for \$1814.75. How much more than the sum borrowed did he return?

**9.** Mr. A bought a horse for \$124.80, to be paid in 8 months without interest, and sold him at once for \$180. What was his gain per cent, money being worth 6 per cent?

**10.** What is the interest on a sum of money that amounts to \$653.48 in 243 days at 8 per cent?

**11.** In 1 year 4 months \$311.50 amounted to \$348.88, at simple interest. What was the rate?

**12.** A man bought a horse in New York and paid \$20 for his transportation into Canada, where he sold him at a loss of \$13. Had he paid no transportation charges, he would have gained $3\frac{1}{2}$%. How much was paid for the horse?

**13.** After spending 25% of his money, and 25% of the remainder, Harry had \$731.25 left. How much had he at first?

**14.** If $\frac{4}{9}$ of the selling price is gain, what is the per cent of profit?

**15.** What sum of money compounded semi-annually at 6% will amount to \$2500 in 10 years?

**16.** What is the present worth and true discount of a debt of \$1000 due in 1 year 6 months, the rate being 6%?

**17.** What would be the annual income from an investment of \$8250 in 6% stocks at $82\frac{3}{8}$, brokerage $\frac{1}{8}$ per cent ?

**18.** If 5% stock is purchased at 125, what per cent of the investment is the income ?

**19.** For how much must I give my note at bank to obtain \$1000 for 90 days, the rate of discount being 6 per cent ?

**20.** A boy sold two knives at the same price, gaining 20% on one, and losing 20% on the other. He lost 2 cents by the transaction. What did each cost ?

**21.** An agent sells a typewriter for \$100, taking a note to be paid in 10 equal monthly payments without interest. After half the payments have been made he takes the note to bank and has it discounted at 6%. Find the proceeds.

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**22.** Mr. Henry bought land at \$30 an acre. How much must he ask an acre in order that he may reduce his asking price 25% and yet make 20% on the purchase money ?

**23.** A man invested a certain amount and sold at a loss of 30%. He invested the proceeds and sold at a gain of 30%. What per cent did he lose on the two speculations ?

**24.** By selling a hat at an advance of $16\frac{2}{3}$%, a merchant gained 50 cents. What did a dozen such hats cost the merchant ?

**25.** A dealer sold two books for \$1 each. On one he made 100%, on the other he lost 50%. What did each book cost ?

**26.** Allowing 10% for delinquent taxes, and 5% for collection, what must be the assessment to pay an indebtedness of \$72675 ?

**27.** A glazier bought of Murphy & Diebold, July 1, 1901, glass to the amount of \$324, list price, from which he received the regular discount of 40%. He got an additional discount of 5% for cash. Make out and receipt the bill.

**28.** The amount at 6% for 2 years, 5 months, 18 days is \$746.20. What is the interest ?

**29.** Bought 8000 bushels of wheat in Chicago at \$.75 a bushel, and shipped it to New York, where my agent sold it at \$.87½ a bushel. His commission was 2%, and other expenses were \$325. What was my gain?

**30.** A druggist marked goods to sell at 40% gain. He lost 10% of sales in bad debts, and paid 10% for collecting. What was his net gain per cent?

**31.** The net earnings of a stock company are \$22425, and the capital stock is \$215000. What per cent dividend can be declared, no surplus being reserved?

**32.** In selling hay for \$15 a ton I lose 10%. At what price must I sell it to gain 10%?

**33.** When green hams, bought at 8 cents a pound, waste 10% in curing, at what price must they be sold to gain 30% on the cost?

**34.** A rectangular garden, 40 feet long and 20 feet wide, is enlarged 10% in each dimension. Find the per cent of increase in area.

**35.** The proceeds of a 30-day note, discounted at 6%, are \$143.28. Find the face.

**36.** In October a contractor counted the cost of a proposed building, and adding 10% for his profit, put in his bid, which was accepted. Before he began work in April, labor and material had advanced 5%. What per cent profit did he realize?

**37.** A note for \$486, dated Sept. 7, 1895, was endorsed as follows: Rec'd March 22, 1896, \$125; Nov. 29, 1896, \$150; May 13, 1897, \$120. What was due April 19, 1898, the rate of interest being 6%?

**38.** A merchant buys goods on 6 months' credit. At the end of one month he borrows money at 6% per annum and pays the bill, getting a discount of 5%. How much does he save on a bill of \$3650?

**39.** What is the difference between the annual and the compound interest on \$300 for 2 years at 6 per cent?

**40.** A and B each had \$52900. A invested his money in 7% railroad bonds at $132\frac{1}{8}$, brokerage $\frac{1}{8}$. B loaned his at 5% simple interest. Which received the larger income, and how much?

SUPPLEMENTARY EXERCISES (FOR ADVANCED CLASSES).

**501. 1.** The difference between the interest on \$600 and that on \$750 at 5% for a certain time is \$18.75. What is the time?

**2.** Find the simple interest at 8% on \$5000 belonging to a boy 12 yr. 6 mo. 15 da. old, and remaining on interest until he is of age.

**3.** If the use of \$3750 for 8 months is worth \$68.75, what sum is that whose use for two years 4 months is worth \$250?

**4.** A owes B \$1500 due in 1 yr. 18 mo.; he pays him \$300 cash, and gives a six-month note for the balance. What is the face of the note, money being worth 6 per cent?

**5.** Bought goods at 25, 20, 15, and 10% off. If the sum of my discounts was \$162.30, what was the list price?

**6.** I sent my agent \$1364.76 to be invested in pork at \$6 a barrel, after deducting his commission of 2%. How many barrels of pork did he buy?

**7.** I sold a pig at a loss of 25%. Had it cost me \$1 more, my loss would have been 40%. Find its cost.

**8.** A merchant bought goods for \$150, and sold $\frac{1}{3}$ of them at a loss of 4%. What per cent must that selling price be increased so that by selling the rest at the increased rate he may gain 4% on the whole transaction?

**9.** A man has an opportunity of investing \$10000 in mining stock at 125, paying 8% dividend; in railroad stock at 85, paying 5%; or he can loan his money at 6%. Which would yield the largest income, and how much?

**10.** If I sell oats at $42\frac{1}{2}$¢ a bushel, my gain is only $\frac{2}{3}$ of what it would be if I should sell at $56\frac{1}{4}$¢ a bushel. Find the price paid for them.

**11.** A farmer paid \$76 for calves and sheep, paying \$3 for calves and \$2 for sheep. He sold $\frac{1}{4}$ of his calves and $\frac{2}{5}$ of his sheep for \$23, thereby losing 8% on their cost. How many of each did he buy?

**12.** The cost of a bridge was \$1260.52, which was raised by a tax upon the property of the town. The tax levy was $3\frac{1}{4}$ mills, and the collector's commission was $3\frac{1}{2}$%. What was the valuation of the property?

**13.** I wish to borrow \$350 from a bank for 90 days. For what sum must I give my note, if the bank discounts it at 6%? Write the note.

## PROPORTIONAL PARTS.

**502. 1.** A and B bought 15 horses. For every one A bought, B bought 2. How many did each buy?

QUERIES.—How many did both buy at one purchase? How many such purchases were made? Then how many did A buy?

**2.** Divide \$25 between May and Ned, giving Ned \$2 as often as you give May \$3.

**3.** A, B, and C ate 27 peaches. While A ate two, B ate three, and C ate four. How many did each eat?

QUERIES.—How many did all eat while A ate two? Then how often did A eat two?

### WRITTEN EXERCISES.

**503. 1.** Divide \$132 between two persons so that their share shall be in the ratio of 5 to 7.

$\$5 + \$7 = \$12.$
$\$132 \div \$12 = 11.$
$11 \times \$5 = \$55.$
$11 \times \$7 = \$77.$

One gets \$5 as often as the other gets \$7. These sums can be distributed 11 times.

**2.** Divide \$100 into parts proportional to $\frac{1}{2}$ and $\frac{1}{3}$.

SUGGESTION. $\frac{1}{2} = \frac{3}{6}$, and $\frac{1}{3} = \frac{2}{6}$; hence, $\frac{1}{2} : \frac{1}{3} = 3 : 2$.

**3.** The sum of two numbers is 187. What are the numbers if they are to each other as 6 to 11 ?

**4.** Divide 533 into three parts which shall be to one another as 2, 3, and 5.

**5.** Three numbers are to one another as 3, 5, and 9, and one half their sum is 153. What are the numbers ?

**6.** Divide 693 into five parts which shall be proportional to 12, 7, 6, 5, and 3.

**7.** For every \$15 earned by a man, his three sons each earned \$9, and his two daughters each \$6. If all earned \$5616 in a year, how much did each earn in a week ?

## PARTNERSHIP.

**504. 1.** A and B engaged in business, A investing \$500 and B \$1000. If the gain was \$750, what was each one's share ?

QUERIES.—Should A and B share the gain in proportion to their investments ? Then \$750 must be divided into parts proportional to what ? Since A furnishes $\frac{1}{3}$ of the capital, what part of the gain should he receive ? Is the ratio $\frac{1}{3} : \frac{2}{3}$ equal to the ratio \$500 : \$1000 ?

**2.** D, A, and E formed a partnership, D putting in \$1000, A \$2000, and E \$3000. At the end of the year they had gained \$1800. What was each one's share ?

**505.** An association of two or more persons for the purpose of carrying on business is called a **Partnership.**

The persons thus associated are called *partners,* and together they form a *company* or *firm.*

**506.** The money (or its equivalent) invested by all the partners is called the *capital* of a firm. Its debts are called *liabilities.*

**507.** PRINCIPLE.—*The profits and losses of a firm are shared in proportion to the parts of the capital invested by each partner.*

WRITTEN EXERCISES.

**508. 1.** A, B, and C formed a partnership, A furnishing \$2000 of the capital, B \$3000, and C \$7000. What was each partner's share of the \$4800 gained?

(a)

\$2000 + \$3000 + \$7000 = \$12000.
\$4800 ÷ \$12000 = .4, or $\frac{2}{5}$.
$\frac{2}{5}$ of \$2000 = \$800, A's share.
$\frac{2}{5}$ of \$3000 = \$1200, B's "
$\frac{2}{5}$ of \$7000 = \$2800, C's "

(b)

\$2000 + \$3000 + \$7000 = \$12000.
Gain on \$1 of capital = \$4800 ÷ 12000, or \$$\frac{2}{5}$.
A's gain on \$2000 = 2000 × \$$\frac{2}{5}$, or \$800.

**2.** R, O, and H are partners. R put into the business \$4500, O \$6000, and H \$7500. Their first year's gain was \$3750. What was each one's share?

**3.** G, H, and K formed a partnership, G furnishing \$3750, H \$6250, and K \$8750. When they dissolved, H received \$1350 as his share of the gain. How much did G and K receive?

**4.** Smith invested \$4200 in a business, and his first year's profit was \$840. His partner, Jones, received \$1120 profit in the same time. How much did Jones invest?

**5.** B and C formed a partnership with a capital of \$7500, and realized a profit of \$12000. What was each one's share of the gain if B's investment was $\frac{2}{5}$ of C's?

**6.** R, Q, P, and Y, partners, have a capital of \$300,000. At the end of a year's business R received \$12000 as his share of the profit, Q received \$10000, P \$15000, and Y \$13000. How much did each invest?

**7.** D, L, and G, who were engaged in business three years, made an annual profit of \$7200. During the first year D owned $\frac{1}{2}$, L $\frac{1}{3}$, and G $\frac{1}{6}$ of the stock; during the second year each owned $\frac{1}{3}$ of it; and during the last year G owned $\frac{1}{2}$ of the stock, while the other half was equally divided between L and D. What was each partner's share of the total profits?

**8.** A and B bought a lot for \$1500, agreeing to pay \$500 cash and \$1000 in 3 months. A pays the \$500 cash, and B pays the \$1000 three months later. Nine months after the purchase they sold the lot for \$1750. What was each one's share of the gain ?

The use of \$500 for 9 mo. = the use of 9 × \$500, or \$4500, for 1 mo.
The use of \$1000 for 6 mo. = the use of 6 × \$1000, or \$6000, for 1 mo.

Hence the respective gains are proportional to 4500 and 6000.

**9.** D, E, and F gain in trade \$8000. D furnishes \$12000 for 6 mo.; E, \$10000 for 8 mo.; and F, \$8000 for 11 mo. What is each man's share of the gain ?

**10.** X, Y, and Z hired a pasture for \$420. X put in 6 horses for 9 weeks, Y 9 horses for 8 weeks, and Z 12 horses for 7 weeks. How much should each pay ?

**11.** A, B, and C contribute capital to a business as follows: A \$3000 for 12 months, B \$4000 for 10 months, and C \$5000 for 8 months. Their profits are \$900. What is the gain of each ?

**12.** A, B, and C hired a pasture for \$452. A put in 12 horses for 15 weeks, B 80 sheep for 8 weeks, and C 18 cows for 20 weeks. How much should each pay if a cow eats as much as 3 sheep, and 5 sheep eat as much as a horse and 2 sheep ?

**13.** K rented a house for \$720 a year. After 3 months B moved in with him, agreeing to pay his share of the rent. Five months later C also moved in on the same conditions. How much of the \$720 did each pay ?

**14.** In a certain company B has 3 times as much capital as A, and C has $\frac{1}{2}$ as much as the other two. What is each one's share of a profit of \$393 ?

**15.** M hired a rig for \$10 to drive from Salem to Manor, a distance of 10 miles, and back again. At Derby, midway between the two places, he took in L, who agreed to pay his proportional share of the expense if allowed to ride to Manor and back to Derby. How much should L pay ?

# GENERAL REVIEW WORK.

## ORAL EXERCISES.

**509.** **1.** If $\frac{2}{3}$ of an acre of land costs $\frac{4}{5}$ of \$120, what will 5 acres cost?

**2.** What will 3 ounces of silver cost if half a pound costs \$4.20?

**3.** How many 3-inch squares are there in a piece of paper 2 yards long and 3 feet wide?

**4.** At two cents a foot, how much will 8 rods of wire cost?

**5.** If .3 of John's money equals $\frac{6}{7}$ of Kate's, and both have 27 cents, how much has each?

**6.** How many cubic inches are there in a piece of scantling $\frac{5}{9}$ of yard long and a foot square at the ends?

**7.** After spending $\frac{5}{9}$ of his money, Henry had \$2 less than half his money left. How much had he at first?

**8.** What is the average cost of 25 cows, if 13 of them are bought at \$50 a head, and the others at \$75 a head?

**9.** If $\frac{5}{7}$ of one number is 135, and $\frac{2}{5}$ of another is $7\frac{1}{2}$, what is the sum of the two numbers?

**10.** What is the number whose half exceeds its third by 126?

**11.** Forty per cent of George's marbles equals $\frac{1}{2}$ of Tom's, and both have 54. How many has each?

**12.** A man sold 50 acres of his land, and had $37\frac{1}{2}\%$ of it left. How many acres had he at first?

**13.** In a mixture of grain there are 50 bushels of oats, 40 of corn, and 15 of wheat. What part of the mixture is each?

**14.** A is 5 miles ahead of B, and walks $3\frac{1}{2}$ miles while B walks 4. How many miles will B walk before overtaking A?

**15.** Divide 42 apples between Edna and May so that Edna will have $\frac{1}{2}$ of $\frac{4}{5}$ as many as May.

**16.** When money is worth 5%, what must I pay for the use of $30 for 3 years 8 months?

**17.** If 7 be added to both numerator and denominator of the fraction $\frac{2}{3}$, how much will the value of the fraction be increased or diminished?

**18.** When money was worth 6% a year, I paid $32 for the use of $200. How long did I have it?

**19.** At $1.50 a cord, what is it worth to saw a cubical pile of wood 16 feet long?

**20.** Ten years ago A was half as old as B. Ten years hence B will be three score years of age. How old is each now?

**21.** Ten years ago Mr. H was $\frac{1}{3}$ as old as he will be 30 years hence. What is his age?

**22.** Bought shirts at $15 a dozen, and marked each to be sold at a profit of 20%. What was the marked price?

**23.** What rate of income do I receive when I buy 6% stocks at 50% premium?

**24.** A fruit dealer paid 15 cents a dozen for oranges, and sold them at the rate of five cents for two. What per cent did he gain?

**25.** What is the difference between .1 and 1%?

**26.** Sixty per cent of Nell's money is 75% of Ada's, and both have $18. How much has each?

**27.** By selling eggs at 4 cents a dozen more than cost, a grocer made 25%. At what price did he sell them?

**28.** A man gained 80 cents on a bushel of berries sold at the rate of 25 cents for two quarts. What was the cost a quart?

**29.** Divide $112 among 2 men and 3 women, giving each man twice as much as each woman.

**30.** What per cent of a yard is a foot and three inches?

**31.** What is the least sum of money with which a trader can buy sheep at $6 apiece or cows at $26?

**32.** How many tiles each 6 inches square will be required to cover a space 6 feet square?

**33.** A boy sold papers at the rate of $2\frac{1}{2}$ for 5 cents, 50% of which is profit. How many papers could he buy for a quarter?

**34.** How many square yards are there in the surface of two cubes whose edges are each 2 feet 6 inches?

**35.** A room is $\frac{1}{3}$ as long as it is wide, and its perimeter is 84 feet. What are its dimensions?

**36.** Sugar worth $1.55 is weighed in a false balance which gives only $15\frac{1}{2}$oz. to the pound. What is the selling price of the sugar?

**37.** What per cent of a score is a dozen? What per cent of a dozen is a score?

**38.** If I charge $1.50 a cord for sawing wood into three pieces, how much should I charge for sawing it into five pieces?

**39.** One square rug is $1\frac{1}{4}$ yd. on a side, and another is $1\frac{1}{2}$ yd. on a side. If the larger rug costs $1.44, what will the smaller cost at the same rate a square yard?

**40.** A, B, and C bought a horse for $100, A paying $20, B $30, and C $50. They sold him for $175. How much did each gain?

**41.** F, G, and H have $510. If G has $\frac{3}{4}$ as much as F, and H has $\frac{4}{5}$ as much as F, how much has each?

**42.** If $2\frac{1}{2}$ yards of cloth make a pair of pants, how many pairs can be made from a piece of cloth containing 40 yards?

**43.** What number increased by 6, the sum multiplied by 5, and the product divided by 10, gives 3 as a product?

**44.** A man started westward from London and traveled through 360°. Did his watch then indicate the correct time? Why?

**45.** How many sheep are worth as much as a cow, if 4 cows are worth one horse, and 2 horses are worth 48 sheep?

**46.** James, who lives $1\frac{3}{4}$ miles from the schoolhouse, goes to school 5 days each week. If he goes home for lunch every other day, in how many days does he walk 21 miles?

**47.** A can do as much work in $\frac{3}{5}$ of a day as B can do in $\frac{3}{4}$ of a day. How long will it take B to paint a house that A can paint in 18 days?

**48.** A and B ran a mile, A beating B by 40 rods. In what time can B run a mile, if A's time in the race was 7 minutes?

**49.** W can build 30 rods of fence in 4 days, and R can build as much in 6 days as W can in 8 days. In what time can R build 75 rods of fence?

**50.** A man who had $3b$ sheep bought three times as many as he had, and then sold $\frac{1}{4}$ of all. How many had he left?

**51.** Harry has $6a$ cents, which is $\frac{3}{7}$ as many as Marie has. How many cents have they both?

**52.** How many square feet are there in a board $a$ feet long and $b$ inches wide?

**53.** The dimensions of a cube are $a$ inches. How many square inches are there in 5 of its sides?

**54.** In a school there are $p$ pupils, and $q$ of them are girls. How many boys are there in the school?

**55.** If $a$ yards of cloth cost $b$ dollars, what will 3 yards cost?

**56.** What will $b$ yards of cloth cost if $c$ yards cost $d$ cents?

**57.** The area of a field is $ab$ square rods, and the length is $a$ rods. What is the distance around the field?

**58.** What is the volume of a cube whose edge is $b$ feet?

**59.** A living-room 12 feet square and 10 feet high is occupied by 5 persons. How many cubic feet of air are there to each person?

**60.** A lot two rods wide is planted in corn, the rows being a yard apart. How many rows are there, no row being nearer the fence than 1 foot 6 inches?

**61.** I have work for either 8 men or 12 boys. If I employ 6 men, to how many boys can I give employment?

**62.** B sold a buggy to C, gaining 20%, and C sold it to D at a loss of 20%. If D paid $150 for it, what did B gain?

**63.** After a rain it was found that there was half an inch of water in a box 3 feet square. What was the volume of the water in the box?

**64.** Said A to B, "I have as many quarters as you have half-dollars, and we together have $9." How much had each?

**65.** How many ounces in $p$ pounds and $q$ ounces?

**66.** What is the quotient of $a$ fourths $\div$ $a$ eighths?

**67.** There are 946 pupils in a school. If $\frac{2}{3}$ of the number of girls is equal to $\frac{4}{5}$ of the number of boys, how many of each are there in the school?

**68.** What per cent does a merchant gain on his investment if 20% of his sales is profit?

**69.** I sold a piece of land so that $\frac{4}{5}$ of the profit equalled $\frac{3}{8}$ of the cost. Find the gain per cent.

**70.** If 8 men can do $\frac{2}{3}$ of a piece of work in 9 days, how many men can do the whole of it in 4 days?

**71.** How many hours a day must 4 men work to do half as much work in 10 days as 15 men can do in 4 days, working 10 hours a day?

**72.** A can hoe $\frac{1}{20}$ of a row of corn in 1 minute, and B can hoe $2\frac{2}{5}$ rows in an hour. In what time can they together hoe a row?

**73.** Had a bin contained twice as much oats, and the oats been used one-fourth as fast, the oats would have lasted 48 weeks. How many days did they last?

**74.** My pony is 13 hands high. How many feet is that?

**75.** A lady gave $\frac{3}{4}$ of her money to the poor, and then found $\frac{2}{3}$ as much as she had given away, and then had $30. How much had she at first?

WRITTEN EXERCISES.

**510. 1.** A wagon box is 8 ft. long, 3 ft. 6 in. wide, and 2 ft. deep. How many bushels of coal will it hold?

**2.** A and B can do a piece of work in 5 days, B and C in 6 days, and A and C in 10 days. How long would it take each alone?

**3.** A saddle cost $\frac{1}{8}$ as much as a horse, and the horse cost $\frac{1}{2}$ as much as a buggy. If all cost $500, what was the cost of each?

**4.** What number is that from which if $7\frac{1}{2}$ is subtracted, $\frac{2}{3}$ of the remainder is 25?

**5.** When the gold dollar was worth 7% more than the greenback dollar, how much gold was $371.29 in greenbacks worth?

**6.** Sold 3 acres of land for $100 more than 5 acres cost, and thus gained 100% on the amount sold. What was the cost an acre?

**7.** After losing $\frac{3}{7}$ of his money, A found $15, and then lacked $\frac{4}{21}$ of having his original amount. How much did he lose?

**8.** Divide and multiply 1 by .001, and to the sum of the quotient and product add the quotient of $.01 \div 50$.

**9.** A man bought 5 shares of 10% bank stock ($100), which yielded him 8%. What did it cost him a share?

**10.** A's money is to B's as 7 : 11, but if each had $9 more, A's would be to B's as 5 : 7. How much has each?

**11.** A ship consumes $\frac{1}{30}$ of its coal supply each day. It starts with its bunkers $\frac{4}{5}$ full, and when it reaches port has only $\frac{2}{15}$ of its supply left. How many days were occupied on the voyage?

**12.** Two men hired a pasture for $56. A puts in 10 cows and B puts in 36 horses. If a cow eats twice as much as a horse, how much should each pay?

**13.** Four pipes, each 2 inches in diameter, empty a tank in 9 hours. What must be the diameter of a single pipe that will empty the tank in the same time?

**14.** A, B, and C start together to walk around a race-track. A goes once around in 2 hours, B in 3 hours, and C in 4 hours. In how many hours will all be together again at the starting point?

**15.** A's money added to $\frac{1}{2}$ of B's equals \$2000. How much has each, if A's money is to B's as 3 to 4?

**16.** Bought 5% stock at $124\frac{3}{4}$, brokerage $\frac{1}{4}$%. What rate of interest do I receive on my investment?

**17.** If 24 sheep are worth 6 cows, 8 cows worth 2 horses, and 3 horses worth 96 pigs, how many pigs are worth a dozen sheep?

**18.** Sold a lot for \$600, payable in 90 days without interest. Bought it back the same day for \$500, payable in 60 days without interest. How much did I gain, money being worth 8%?

**19.** The assessed valuation of a town is \$760,000. What rate must be levied to raise \$3610, exclusive of the collector's commission of 5%?

**20.** A offers \$300 cash for a lot, and B offers \$325, payable in 9 months without interest. Which is the better offer, and how much, money being worth 6%?

**21.** Bought a horse for \$150 on a year's credit, without interest, and sold him at once for \$150 cash. How much did I make, money being worth 5%?

**22.** The shadow of a man 6 feet tall is 8 ft. 6 in. long. Another man's shadow is 7 ft. 9 in. long. How tall is the latter?

**23.** What is the least quantity of milk from which if 1 quart be taken the remainder can be exactly measured by either a 2-quart, a 4-quart, a 6-quart, or an 8-quart measure?

**24.** B walked twice as far as C; but if he had walked 4 miles less, and C 6 miles more, he would have walked $\frac{1}{3}$ farther than C. How far did each walk?

**25.** If Tom gives May a penny, each will have the same sum; but if May gives Tom a dollar, he will have twice as much as she has left. How much has each?

**26.** A and B run a race, their rates of running being as 17 : 18. A runs $2\frac{1}{3}$ miles in 16 minutes, 48 seconds. B the whole distance in 34 minutes. What is the distance run?

**27.** A owned $\frac{5}{8}$ and B $\frac{3}{8}$ of a store, but they took C into the firm and reorganized as equal partners. If C paid them $4000, what was A's share of it?

**28.** If 16 yd. of cloth cost $56 when wool is $.75 a pound and labor $.20 an hour, what would it cost when wool is $.60 a pound and labor $.25 an hour, if 24 lb. of wool and 60 hr. of labor are required to make it?

**29.** Two houses cost $8100, and $\frac{3}{5}$ of the cost of one is equal to $\frac{9}{10}$ of the cost of the other. What is the cost of each?

**30.** A sold B a horse for $\frac{1}{5}$ more than it cost, and B sold it for $80, losing $\frac{1}{6}$ of its cost. How much did A pay for the horse?

**31.** I paid $214.20 for a piano after discounts of 20%, 15%, and 10% had been allowed. What was the list price?

**32.** Schley and Sampson were partners for two years, making an annual profit of $5460. During the first year Sampson owned $\frac{2}{3}$ of the stock, but during the second year Schley owned $\frac{3}{4}$ of the stock. What was each one's share of the profit?

**33.** A steamer sails a mile down stream in five minutes, and a mile up stream in 7 minutes. How far down stream can she go and return in one hour?

**34.** A stable 30 ft. long, 20 ft. wide, and 18 ft. high has two gables each 12 ft. high. Find cost of painting the outside at 50¢ a sq. yd.

**35.** How many yards of carpeting 27 inches wide will cover a hall 45 ft. long and 32 ft. wide, the strips running lengthwise, and there being a waste of $\frac{1}{4}$ yard in matching the pattern?

**36.** What will it cost to plaster the walls of a room $18\frac{1}{2}$ ft. long, $16\frac{1}{2}$ ft. wide, 12 ft. high, at 11¢ a square yard, allowing nothing for openings?

**37.** How many board feet of siding 5 in. wide will be required to cover the sides of a house 40 ft. long, 28 ft. wide, 20 ft. high, if they are laid 4 inches to the weather, and 150 sq. ft. are deducted for doors and windows?

**38.** If as many silver dollars as possible are laid on the bottom of a box 18 inches long by 12 inches wide, how much space will be left uncovered?

**39.** The fence around a circular field is 1.19 miles in length. How many acres inside the fence?

**40.** Rome is 20° 27′ 14″ E., and Washington 77° 3′ W. When it is 9 A.M. at Washington, what is the time at Rome?

**41.** How many ounces of gold in a 16-carat chain that weighs $3\frac{1}{2}$ ounces?

**42.** At one point an eclipse of the moon was seen at 9 A.M., at another point at 11:30 A.M. What is the difference in the longitude of the two places?

**43.** A town has a water supply of 104 gal. a day for every house. If the number of houses increases $\frac{1}{12}$, and the total supply diminishes $\frac{1}{12}$, what will be the daily supply to a house?

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**44.** Either 30 pears and 20 apples or 14 apples and 42 pears will just fill a basket. How many of either will fill it?

**45.** A and B receive $1000 for grading a street. A furnishes 3 teams 20 days, B 5 teams 30 days. If A receives $100 for overseeing the work, what does each receive of the $1000?

**46.** A ten-acre field was divided into lots, each containing $\frac{3}{8}$ of an acre. The partial lot was sold at the rate of $300 an acre, and the others at $150 each. What was received for the field?

**47.** In a spelling contest there were 75 words given; 6 contestants spelled 74 words each, and 13 spelled 70 words each. Find the average per cent made by these contestants.

**48.** City lots, 200 feet deep, sell for \$80 a front foot. What is the value of an acre at that rate?

**49.** A train runs 25 miles an hour. How far can I ride on it and walk back at the rate of $3\frac{1}{2}$ miles an hour, and be gone just 5 hours?

**50.** A room 16 ft. by 18 ft. is covered with carpet 27 inches wide, and the smallest possible number of yards of the carpet is in use. How many yards?

**51.** Find the cost of a bushel of ground feed, the ingredients of which are 60 bushels of corn at 45¢, 90 bushels of oats at 32¢, and 26 bushels of rye at 64¢, the cost of grinding being \$6.20.

**52.** Two men together received \$97.75, but one received \$18.25 more than the other. How much did each receive?

**53.** An agent, having in his hands \$3150 of his principal's funds, is instructed to invest it in barley at \$.48 a bushel, after retaining his commission of 5%. How many bushels should he buy?

**54.** Two districts buy a road machine for \$285, and pay the freight from the factory, one district paying $\frac{3}{7}$ and the other $\frac{4}{7}$ of the entire cost. The cost of the first district being \$127.50, how much was charged for freight?

**55.** A commission merchant sold 1014 bushels of oats at 41 cents a bushel, paid \$33.74 freight charges, and retained $3\frac{1}{2}$% commission. How much should he remit to the consignor?

**56.** The Columbian souvenir half-dollar weighs 192.9 grains. How many of them weigh as much as 50 ordinary silver dollars?

**57.** An upright pole 16 ft. long casts a shadow 5 ft. 4 in. long, and at the same time the shadow of a tree is found to be 26 ft. 9 in. How high is the tree?

**58.** If one fifth be allowed for matching and waste, how many board feet of inch lumber will be required for flooring and ceiling a porch 17 ft. 4 in. by 7 ft. 6 in.?

**59.** By the introduction of improved machinery in a certain factory it was found that 7 men could do the work formerly done by 11 men. What per cent of the labor required to turn out the same product was saved by using the improved machinery?

**60.** If Tennessee 3% bonds are selling at 95, how much money must be invested in them to secure an annual income of \$750?

**61.** When the grade on a road is 1320 feet to the mile, what is the per cent of grade?

**62.** If the interest is \$19.07, the time 8 mo. 2 da., and the rate $5\frac{1}{2}$%, what is the principal?

**63.** If it costs \$110 to dig a cellar 40 ft. long, 27 ft. wide, and 4 ft. deep, how much will it cost to dig a cellar 36 ft. long, 30 ft. wide, and 5 ft. deep?

**64.** The running time of a train from New York to Buffalo is $8\frac{1}{2}$ hours, and the distance is 440 miles. If stops of 5 minutes each are made at Albany, Utica, Syracuse, and Rochester, what is the average speed an hour?

**65.** At what price is $4\frac{1}{2}$% stock equal as an investment to $3\frac{1}{2}$% stock at \$87.50 a share?

**66.** A man sells $22\frac{1}{2}$ shares of 5 per cent stock at 105, and buys 4 per cent stock at $94\frac{1}{2}$. How much is his income diminished?

**67.** A cooper paid \$78.32 for 16488 barrel staves. Required the price per M.

**68.** Bought stock at 8% below par and sold it $12\frac{1}{2}$% below par, thereby losing \$99. How much did I invest?

**69.** Bought 4 loads of hay, 2750 lb. each, at \$20 a ton, and paid for it with a 60-day note without interest. What will be the proceeds of the note if discounted at bank immediately at 6%?

**70.** Railroad stock that cost \$121.75 a share pays a semi-annual dividend of 4%. Required the rate per annum of income on the investment.

**71.** Divide \$744 among A, B, and C so that $\frac{3}{5}$ of A's money will equal $\frac{2}{3}$ of B's or $\frac{1}{2}$ of C's.

**72.** The sum of three numbers is 940. The first equals $\frac{5}{9}$ of the second, and the second equals $\frac{7}{10}$ of the third. Find the numbers.

**73.** Write a 30-day note the proceeds of which, when discounted at a New York bank on the day of making, shall be \$514.

**74.** Dewit purchased a house and lot for \$3300 ; paid \$975 for repairs, and now rents the premises for \$30 a month. If he expends annually for taxes \$48.70, and for incidental repairs \$35, what is his per cent of annual income on his investment ?

**75.** What is the difference between the true and the bank discount of \$200 for 60 days, at 6%? (No grace.)

**76.** A farm is worth 10% less than a store, and the store 20% more than a lot. The owner of the lot exchanges it for 80% of the farm, thereby losing \$850. What is the farm worth ?

**77.** In a proportion whose ratio is $12\frac{1}{2}$, the first number is 25 and the last number is 8. What is the third number ?

**78.** B owns a square mile of land, and D owns a farm of equal area whose width is 128 rods. At \$1.25 a rod, how much more will it cost to fence D's land than B's ?

**79.** A farmer agreed to give his hired man \$100 and two cows for a year's labor. The man quit work at the end of 10 months, receiving the cows and \$70 as a fair settlement. At how much were the cows valued ?

**80.** When a railroad company is declaring quarterly dividends of $1\frac{1}{2}$%, and its stock is quoted at 125, what annual rate of income does an owner of that stock receive ?

**81.** A young man puts \$10 in a savings bank each month, making his first deposit Jan. 1, 1901. How much will there be to his credit Jan. 1, 1902, if the bank pays 4% per annum, and adds the interest at the end of each quarter ?

SUPPLEMENTARY EXERCISES (FOR ADVANCED CLASSES).

**511. 1.** If postage stamps are $\frac{7}{8}$ of an inch long and $\frac{3}{4}$ of an inch wide, how many will be required to cover a ceiling 12 ft. 3 in. by 13 ft. 6 in.?

**2.** How must I invest in 3% stock at 90 so as to get the same income as if I invested $4950 in the same stock when it is quoted at 99?

**3.** Divide $14600 among 3 boys, aged 9, 13, and 17 years respectively, so that if invested at 5% simple interest each will receive the same amount at the age of 21.

**4.** A owns a mine worth $11000, which pays 6% on his investment. Paying $100 brokerage, he exchanges the mine for bank stock at 109, thus increasing his annual income $340. What dividend does the bank stock pay?

**5.** A pole was $\frac{2}{7}$ under water. The water rose 8 feet, and then there was as much of the pole above the water as was previously under it. Find the length of the pole.

**6.** Two equal annual payments have been made on an 8% note for $200, dated two years ago to-day. The balance due is $44. What was the annual payment?

**7.** A cubic foot of water weighs 62.5 lb., and lead is 11.44 times as heavy as water. How many cubic inches are there in a piece of lead weighing 35 lb. 6 oz.?

**8.** M and N, equal partners, found on settlement that M owed the firm $240, and that the firm owed N $260. How much should M have given N to square the account?

**9.** Snow has fallen to the depth of 25 cm. If 12 cu m. of snow produces 1 cu m. of water, find the volume of water produced by this snow on one acre of land.

**10.** A rented a farm from B, agreeing to give B $\frac{2}{5}$ of all the produce. During the year A used 45 bu. of wheat, and at settlement first gave B 18 bu. to balance the 45 bu., and then divided the remainder as if neither had received any. How much did B lose?

# POWERS AND ROOTS.

## INVOLUTION.

**512. 1.** What is the product of 2 multiplied by itself, or used twice as a factor ?

**2.** How often must 3 be used as a factor to produce 9 ? To produce 27 ?

**3.** What is the product of $a$ used twice as a factor ? Used three times as a factor ?

**513.** The product of two or more equal factors is called a **Power.** The product of two equal factors is called the *second power ;* of three equal factors the *third power,* and so on.

The second power of a number is also called the *square* of the number, because the area of a square is expressed by the product of two equal factors. Why is the third power of a number also called its *cube ?*

**1.** What is the square of 5 ? Of 7 ? Of 8 ? Of 10 ? Of $\frac{1}{2}$ ? Of $\frac{2}{3}$ ?

**2.** Find the third power of 2. Of 4. Of 7. Of $\frac{1}{3}$. Of $\frac{3}{4}$. Of .3.

**514.** The number of times a number is to be used as a factor is indicated by a small figure placed at the right of the number. This figure is called an **Exponent.**

Thus, $a^2$ is read "$a$ square," or "$a$ to the second power," and means $a \times a$; $a^3$ is read "$a$ cube," or "$a$ to the third power," and means $a \times a \times a$; $a^4$ is read "$a$ to the fourth power," and so on.

Write the following products as powers :

| | | |
|---|---|---|
| **1.** $3 \times 3$. | **4.** $5 \times 5$. | **7.** $9 \times 9$. |
| **2.** $2 \times 2 \times 2$. | **5.** $23 \times 23$. | **8.** $7 \times 7 \times 7$. |
| **3.** $a \times a$. | **6.** $a \times a \times a$. | **9.** $b \times b \times b \times b \times b$. |

Find the following indicated powers:

**10.** $8^2 = (\quad)$. $11^2 = (\quad)$. $(\frac{1}{2})^2 = (\quad)$ $(1\frac{1}{3})^2 = (\quad)$.

**11.** $5^3 = (\quad)$. $8^3 = (\quad)$. $(\frac{2}{3})^3 = (\quad)$ $.3^3 = (\quad)$.

**12.** $2^4 = (\quad)$. $3^4 = (\quad)$. $(\frac{1}{2})^4 = (\quad)$ $.3^4 = (\quad)$.

**515. Involution** is the process of finding a power of a number.

WRITTEN EXERCISES.

**516. 1.** Find the third power of 12.

**2.** Find the square of 10, 13, 15, 25, 36.

**3.** What is the cube of 8? 12? 20? 44?

**4.** Find the fourth power of 6, 10, .5, $\frac{3}{4}$.

The second power of a number multiplied by the second power equals the fourth power. $a^2 \times a^2 = a^4$. What is the product of $a^2 \times a^3$? $a^4 \times a^4 = (\quad)$.

**5.** Since $2^2 \times 2^2 = 2^4$, and $2^4 \times 2^4 = 2^8$, what is the shortest method of finding the 16th power of 2?

**6.** Find the third power of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{7}{8}$, .7.

**7.** What is the square of 1? .1? 100? .01? 2.5?

**8.** Can a number ending in 2, 3, 7, or 8 be a perfect square? Why not?

**9.** How many figures are used to express the square of a number of two figures? The square of a number of one figure?

**10.** What is the difference between the square of 48 and the cube of 24?

**11.** How much does the cube of 15 exceed twice its square?

**12.** Which is greater and how much—the cube of $\frac{1}{2}$ or its square?

Find the value of:

| | | | |
|---|---|---|---|
| **13.** $43^2$. | **17.** $6.25^2$. | **21.** $(\frac{3}{5})^4$. | **25.** $14^3 - 14^2$. |
| **14.** $46^3$. | **18.** $.005^3$. | **22.** $(2\frac{3}{4})^2$. | **26.** $.500^2$. |
| **15.** $14^4$. | **19.** $(\frac{13}{14})^2$. | **23.** $5^4 - 2^5$. | **27.** $99^2$. |
| **16.** $3.75^2$. | **20.** $(\frac{12}{13})^3$. | **24.** $(\frac{3}{7})^2 - (\frac{2}{7})^3$. | **28.** $3^3 + 2 \times 5^4$. |

**517.** Since $53 = 50 + 3$, the square of 53 may be obtained as follows:

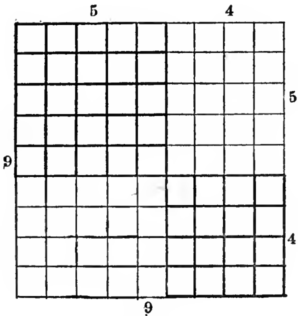
(a)

$$\begin{array}{rl}
(50 + 3) \times \ \ 3 = & \qquad 50 \times 3 + 3 \times 3 \\
(50 + 3) \times 50 = & 50^2 + 50 \times 3 \\
\therefore 53^2 = & 50^2 + 2\,(50 \times 3) + 3^2
\end{array}$$

That is, $53^2$ = square of tens + twice (tens × ones) + square of ones.

(b)

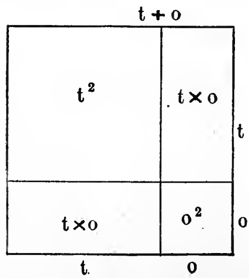
$$\begin{array}{rrl}
 & 53 & \\
 & 53 & \\
53 & 9 = & 3^2 \\
53 & 150 = & 50 \times 3 \\
159 & 150 = & 50 \times 3 \\
265 & 2500 = & 50^2 \\
2809 = & 2809 = & 50^2 + 2\,(50 \times 3) + 3^2.
\end{array}$$




**518.** Since any integral number expressed by two or more figures may be regarded as composed of tens and ones, if we represent the number of tens by $t$ and the number of ones by $o$, we have

$(t + o)^2 = t^2 + 2to + o^2$. Hence,

*The square of a number is equal to the square of the tens, plus twice the tens multiplied by the ones, plus the square of the ones.*




We may write the formula thus: $(t + o)^2 = t^2 + (2t + o) \times o$. Show why.

Show by the diagrams that

(a). $9^2 = (5 + 4)^2 = 5^2 + 4^2 + 2 \times (5 \times 4)$.

(b). $(t + o)^2 = t^2 + o^2 + 2to$.

Square the following by the above method :

| | | | |
|---|---|---|---|
| **1.** 25. | **4.** 56. | **7.** 87. | **10.** $(4 + 3)$. |
| **2.** 32. | **5.** 64. | **8.** 33. | **11.** $(a + b)$. |
| **3.** 43. | **6.** 71. | **9.** 98. | **12.** $(a + 1)$. |

**519.** The cube of a number may also be found by the above method.

**1.** Raise 25 to the third power.

By Art. 517, $25^2 = 20^2 + 2\,(20 \times 5) + 5^2$, which must be multiplied by 25, or $20 + 5$.

$$\begin{aligned} 25^2 \times 5 &= \qquad\quad\; 20^2 \times 5 \;+ 2\,(20 \times 5^2) + 5^3 \\ 25^2 \times 20 &= 20^3 + 2\,(20^2 \times 5) + 20 \times 5^2 \\ \hline 25^3 &= 20^3 + 3\,(20^2 \times 5) + 3\,(20 \times 5^2) + 5^3. \end{aligned}$$

If we represent the number of tens by $t$ and of ones by $o$, we have

$$(t + o)^3 = t^3 + 3t^2o + 3to^2 + o^3. \quad \text{Hence,}$$

*The cube of a number is equal to the cube of the tens, plus three times the product of the square of the tens by the ones, plus three times the product of the tens by the square of the ones, plus the cube of the ones.*

We may write the above formula thus:

$$(t + o)^3 = t^3 + (3t^2 + 3to + o^2) \times o. \quad \text{Show why.}$$

Cube by the above method :

| | | | |
|---|---|---|---|
| **2.** 12. | **5.** 45. | **8.** 76. | **11.** $(a + b)$. |
| **3.** 21. | **6.** 54. | **9.** 89. | **12.** $(2 + 3)$. |
| **4.** 33. | **7.** 67. | **10.** 98. | **13.** $(a + 1)$. |

## EVOLUTION.

**520. 1.** What is one of the two equal factors of 16 ? Of 36 ? Of 64 ?

**2.** What is one of the three equal factors of 8 ? Of 27 ? Of 125 ?

**521.** One of the equal factors of a number is called a **Root** of the number.

The *square root* is one of the two equal factors; the *cube root* one of the three equal factors, and so on.

**522.** The process of finding a root of a number is called **Evolution.**

The symbol $\sqrt{\ }$ is called the **Radical Sign,** and calls for the square root. The radical sign with *index* 3 calls for the cube root; with index 4 it calls for one of the four equal factors, and so on.

NOTES.—1. The symbol $\sqrt{\ }$ was first used in this form by Rudolff in 1525.

2. The square root is also indicated by the fractional exponent $\frac{1}{2}$; the cube root by the exponent $\frac{1}{3}$, etc.

Find the root called for:

| | | | | |
|---|---|---|---|---|
| 1. $\sqrt{25}$. | 4. $\sqrt[3]{27}$. | 7. $\sqrt{\frac{4}{9}}$. | 10. $\sqrt[4]{16}$. | 13. $9^{\frac{1}{2}}$. |
| 2. $\sqrt{400}$. | 5. $\sqrt[3]{1}$. | 8. $\sqrt{a^2}$. | 11. $\sqrt[4]{81}$. | 14. $64^{\frac{1}{3}}$. |
| 3. $\sqrt{64}$. | 6. $\sqrt[3]{125}$. | 9. $\sqrt[3]{b^6}$. | 12. $\sqrt[3]{c^3}$. | 15. $\sqrt{(a+x)^6}$. |

## SQUARE ROOT.

**523.** 1. Since $1 = 1^2$, $100 = 10^2$, $10{,}000 = 100^2$, and so on, the square root of any integral number between 1 and 100 lies between what two numbers? By how many figures is it expressed?

2. The square root of any integral number between 100 and 10,000 lies between what two numbers? By how many figures is the root expressed?

**524.** 1. The square root of any integral number expressed by *one* or *two* figures is a number of *one* figure; expressed by *three* or *four* figures is a number of *two* figures, and so on.

2. If an integral number is divided into groups of two

figures each, from right to left, the number of groups will be equal to the number of figures in the root.

QUERY.—May the left-hand group have but one figure? Why?

**525.** The method of finding the square root of numbers is derived from the identity,

$$(t + o)^2 = t^2 + 2to + o^2 = t^2 + (2t + o) \times o. \quad \text{(Art. 518.)}$$

WRITTEN EXERCISES.

**526.** Find the square root of 1369.

(a)

$$\begin{array}{lr|l} & & t + o \\ & 13\ 69 & 30 + 7 = 37 \\ \hline & 9\ 00 = & t^2 \\ 2t = 2 \times 30 = & 60 \quad 4\ 69 = & (2t + o) \times o. \\ o = & 7 \quad\quad\quad & \\ \hline 2t + o = & 67 \quad 4\ 69 & \end{array}$$

(b)

$$\begin{array}{r|l} & 13\ 69 \,\big|\, 37 \\ & 9 \\ 67 & 4\ 69 \\ & 4\ 69 \\ \hline \end{array}$$

The four figures of the number show that the root is expressed by *two* figures. In 1300 the greatest tens-square is 900, and its square root is 30, which is the tens of the root.

The square of the tens, $t^2$, is subtracted, and the remainder, 469, contains twice the tens × the ones + the square of the ones. This remainder is largely the product of two factors—twice the tens × the ones, of which twice the tens, or 60, is so much the larger that it may be used as a *trial* divisor; using it thus, we find 7 to be the ones' figure of the root. Since $2to + o^2$ is equal to $(2t + o) \times o$, the seven units are added to twice the tens, and the sum, 67, is multiplied by 7.

In the above, $2t$, or 60, is called the *trial* divisor, while $2t + o$, or 67, is the complete divisor. In using the trial divisor, if the quotient is found to be too great, it must be diminished.

In practice the operation is conveniently performed as in (b), omitting unnecessary ciphers. The first group, 13, contains the square of the tens' number of the root. The greatest square in 13 is 9, and its square root is 3. The square of the tens is subtracted. Twice the 3 tens is 6 tens, and 6 tens is contained in the 46 tens of the remainder 7 times, giving the ones' figure of the root.

**527.** We may illustrate square root by the problem of finding the side of the square whose area is 1369 square inches.

The square root of 1369 is 37. The square of $37 = (30 + 7)^2 = 30^2 + 2(30 \times 7) + 7^2$. The 30 may be represented by a square $t = 30$ in. on a side. The $2(30 \times 7)$ may be represented by two strips each $t = 30$ in. long and $o = 7$ in. wide, while the 7 may be represented by the small square $o = 7$ in. on a side (Fig. 1).

$$\begin{aligned} t^2 + 2to + o^2 &= 1369 \\ t^2 \phantom{+ 2to + o^2} &= \phantom{1}900 \\ \hline 2to + o &= \phantom{1}469 \end{aligned}$$

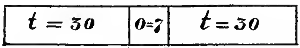


Fig. 2

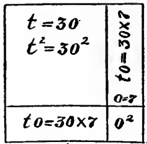


Fig. 1

That is, in extracting the square root of 1369, the large square, which is $t = 30$ in. on a side, is first removed, and a surface of 469 sq. in. remains. What is this remainder the area of?

The two rectangles and the small square have one dimension, $o = 7$, in common. If placed as in Fig. 2, they form one rectangle whose width is $o = 7$, and whose length is $2t + o = 60 + 7$. The area is therefore expressed by $(2t + o) \times o$, or $(60 + 7) \times 7$.

In finding the width $o$, we are obliged to use $2t$, or 60, as a trial divisor, since the whole length is as yet unknown.

$$469 \div 2t = 469 \div 60 = 7.$$

This gives $(2t + o) \times o = (60 + 7) \times 7 = 469$.

$\therefore \sqrt{1369} = 30 + 7 = 37$, the number of inches in the side.

**528.** We may apply the method given above to numbers of more than two groups of figures, by always regarding *the part of the root already found as so many tens* with respect to the next figure of the root.

**1.** What is the square root of 54756?

(a)

| | | | |
|---|---|---|---|
| | | 5 47 56 | 234 |
| | | 4 | |
| $2 \times 20 =$ | 40 | 1 47 | |
| $40 + 3 =$ | 43 | 1 29 | |
| $2 \times 230 =$ | 460 | 18 56 | |
| $460 + 4 =$ | 464 | 18 56 | |

(b)

| | | |
|---|---|---|
| | 5 47 56 | 234 |
| | 4 | |
| 43 | 1 47 | |
| | 1 29 | |
| 464 | 18 56 | |
| | 18 56 | |

The first trial and complete divisors are obtained as they would be if the given number were 547; that is $t = 20$ and $o = 3$. For the second divisors, $t = 230$ and $o = 4$.

**529.** RULE.—*Beginning at ones, separate the number into groups of two figures each.*

*Find the greatest square in the left-hand group, and write its root for the first part of the required root.*

*Subtract the square of this root from the left-hand group, and to the remainder annex the next group for a dividend.*

*Divide this dividend by twice the root already found, considered as tens. The quotient (or the quotient diminished) will be the next figure of the root.*

*To the last trial divisor add the part of the root last found for a complete divisor. Multiply this complete divisor by the part of the root last found, subtract the product from the dividend, to the remainder annex the next group for a new dividend, and proceed as before until all of the groups have been thus annexed.*

1. A decimal is separated into groups of two figures each by beginning at the decimal point, and its root is found precisely as the root of an integer is found.

2. The square root of a common fraction is found by extracting the square root of numerator and denominator separately, or by reducing it to a decimal and then finding the root.

3. If a number is not a perfect square, ciphers may be annexed, and the value of the root found to any required degree of approximation.

Find one of the two equal factors of:

| | | | |
|---|---|---|---|
| **1.** 256. | **4.** 1024. | **7.** 4356. | **10.** 2025. |
| **2.** 441. | **5.** 2401. | **8.** 6080. | **11.** .3249. |
| **3.** 625. | **6.** 2809. | **9.** 7225. | **12.** .000144. |

Find the square root of the following:

| | | | |
|---|---|---|---|
| **13.** 1225. | **17.** 13225. | **21.** 143641. | **25.** 1234321. |
| **14.** 1849. | **18.** 15625. | **22.** 173056. | **26.** 5416.96. |
| **15.** 4489. | **19.** 26001. | **23.** 499849. | **27.** 97.8121. |
| **16.** 9216. | **20.** 60516. | **24.** 801025. | **28.** 31.4721. |

Extract the square root of :

**29.** $\frac{169}{196}$. **32.** $\frac{484}{961}$. **35.** $6\frac{1}{4}$.
**30.** $\frac{324}{2916}$. **33.** $\frac{3025}{11881}$. **36.** $3\frac{6}{25}$.
**31.** $\frac{3136}{5329}$. **34.** $\frac{46656}{81225}$. **37.** $169\frac{81}{144}$.

Find the value to four decimal places :

**38.** $\sqrt{14.}$ **40.** $\sqrt{3.}$ **42.** $\sqrt{.5.}$ **44.** $\sqrt{91.}$
**39.** $\sqrt{2.}$ **41.** $\sqrt{20.}$ **43.** $\sqrt{3.4.}$ **45.** $\sqrt{52.321.}$

### APPLICATIONS OF SQUARE ROOT.

**530. 1.** A square field contains 40 acres. Find the length of a side.

**2.** A square court is paved with 3844 marble slabs 8 inches square. What is the distance around the court ?

**3.** A 40-acre field is three times as long as wide. Find its length.

**4.** A rectangular field is 60 rods long and 40 rods wide. What is the side of a square field of equal area ?

**5.** How many rods of fence will be required to enclose a square farm of 160 acres ?

**6.** A rectangular farm 80 rods wide contains 160 acres. At $1.45 a rod, how much will it cost to fence it ?

**7.** A farmer has a square ten-acre field of grass. How many times will he have to mow around it to cut the grass, each swath being 5 feet wide ?

**8.** A rectangular field, the sides of which are in the ratio of 4 to 7, contains 4032 sq. rd. Find the cost of fencing it at $2 a rod.

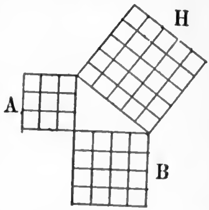
**9.** What is the difference between the perimeters of two fields, one of which is 20.25 rd. square, and the other 20.25 sq. rd. ?

**10.** An army of 7056 men is arranged in a solid square. How many men in rank and file ? How many soldiers would be required to make another row around the square ?

**531.** In a right-angled triangle, the side opposite the right angle is called the **Hypotenuse.**

1. Cut from a cardboard a right triangle with a base 3 inches in length and an altitude of 4 inches. Find by actual measurement the length of the hypotenuse.

2. Square the numbers representing the lengths of the three sides, and find whether one of the squares is equal to the sum of the other two. Which one?




3. In the figure, H is the square on the hypotenuse, B the square on the base, and A the square on the altitude. How many small squares in H? How many in A and B together?

4. Explain by the figure that $5^2 = 3^2 + 4^2$. Also, $H^2 = A^2 + B^2$.

**532.** *The square on the hypotenuse of a right-angled triangle is in area equal to the sum of the squares on the other two sides.*

NOTE.—This is known in geometry as the Pythagorean theorem, because it is supposed to have been first proved by Pythagoras (about 500 B.C.).

**533.** The above relation is expressed in the equation $h^2 = b^2 + a^2$, where $b$ and $a$ represent the base and altitude respectively, and $h$ the hypotenuse.

1. Cut a cardboard as in figure 1. If the triangles 1, 2, 3, 4 are taken away, the square on the hypotenuse of a right-angled triangle remains; and if the two rectangles AP, PB, are taken away from the whole figure, the sum of the squares on the two sides of the triangle remains. Do the four triangles together equal the two rectangles? Does this prove the relation $h^2 = b^2 + a^2$?

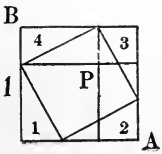
Fig. 1


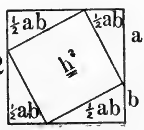
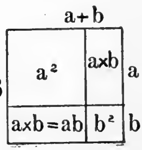

Fig 2



Fig.3


2. Again, cut two equal squares as in the figures 2 and 3.

From Fig. 2, $(a + b)^2 = h^2 + 4(\frac{1}{2}ab) = h^2 + 2ab$...(1)
From Fig. 3, $(a + b)^2 = a^2 + b^2 + 2ab$............(2)

Show from (1) and (2) that $h^2 = a^2 + b^2$.

3. If the sides of a right-angled triangle are 6 ft. and 8 ft., what is the length of the hypotenuse?

$6 \times 6 \times 1$ sq. ft. $= 36$ sq. ft., the square on one side.
$8 \times 8 \times 1$ sq. ft. $= 64$ sq. ft., the square on the other side.
36 sq. ft. + 64 sq. ft. = 100 sq. ft., the square on the hypotenuse.

$\therefore$ the number of units in the hypotenuse $= \sqrt{100} = 10$; hence the hypotenuse is 10 ft. long. $h = \sqrt{(b^2 + a^2)} = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$.

NOTE.—Since $h^2 = b^2 + a^2$, we get, by subtracting $a^2$ from both sides, $h^2 - a^2 = b^2$, or $b^2 = h^2 - a^2$. Hence $b = \sqrt{h^2 - a^2}$. How find the value of $a$?

Find the wanting side:

| | BASE. | ALTITUDE. | HYPOTENUSE. |
|---|---|---|---|
| 1. | 12 | 9 | ( ) |
| 2. | 12 | 16 | ( ) |
| 3. | 15 | ( ) | 25 |
| 4. | ( ) | 28 | 35 |

5. A pole 100 feet long rests against the top of a wall 60 feet high. How wide a stream could flow between the foot of the pole and the wall?

6. Find the diagonal of a room 18 feet square.

7. How far will a man walk in going half way round a square farm of 640 acres? In going diagonally across it?

8. Find, to the nearest inch, the altitude of an equilateral (equal-sided) triangle whose side is 6 inches.

9. While a ship is sailing at the rate of 12 miles an hour, a sailor walks across the deck at the rate of 5 miles an hour. Find his rate of motion.

10. What is the grade to the mile on a road on which the horizontal distance to each mile is 5026 feet?

11. If the ridge of a roof is 9 feet above the upper floor of a house 24 feet wide, how long must the rafters be, allowing 12 inches for cornice?

**12.** A room is 16 feet long, 12 feet wide, and 15 feet high. Find the distance from any lower corner to the opposite upper corner.

**13.** Two trees, 80 and 120 feet high respectively, are 30 yards apart. What is the distance between their tops?

**14.** Two trains leave Richmond at the same time, one running north at the rate of 30 miles an hour, the other east at the rate of 40 miles an hour. How far apart will they be in 30 minutes?

**15.** Around a garden 80 feet square is a walk containing one sixth as much area as the garden contains. Find the width of the walk.

**16.** A ladder 52 feet long stands close against a building. How far must the foot be drawn out that the top may be lowered 4 feet?

**17.** How long a rope will wind once around a cylinder 10 feet long and 6 feet in diameter, commencing at one end and going spirally around to the other?

## CUBE ROOT.

**534.** 1. Since $1 = 1^3$, $1000 = 10^3$, $1,000,000 = 100^3$, and so on, the cube of any integral number between 1 and 1000 lies between what two numbers? By how many figures is it expressed?

2. The cube root of any integral number between 1000 and 1,000,000 lies between what two numbers? By how many figures is the root expressed?

**535.** 1. The cube root of any integral number expressed by *one, two,* or *three* figures is a number of *one* figure; expressed by *four, five,* or *six* figures is a number of *two* figures, and so on.

2. If an integral number is divided into groups of three figures each, from right to left, the number of groups will be equal to the number of figures in the root.

QUERY.—May the left-hand group have but one or two figures? Why?

**536.** The method of finding the cube root of numbers is derived from the identity (Art. 519):

$$(t + o)^3 = t^3 + 3t^2o + 3to^2 + o^3 = t^3 + (3t^2 + 3to + o^2) \times o.$$

WRITTEN EXERCISES.

**537. 1.** Find the cube root of 46656.

(a)

$$
\begin{array}{r|l}
 & \begin{array}{l|l} & t + o. \\ 46\ 656 & 30 + 6 = 36. \end{array} \\
 & 27\ 000 = t^3 \\
\hline
3t^2 = 3 \times 30^2 = 2700 & 19\ 656 = (3t^2 + 3to + o^2) \times o. \\
3to = 3 \times 30 \times 6 = \phantom{0}540 & \\
o^2 = 6^2 = \phantom{00}36 & \\
\hline
3t^2 + 3to + o^2 = 3276 & 19\ 656 \\
\hline
\end{array}
$$

(b)

$$
\begin{array}{r|l}
 & 46\ 656 \mid \underline{36} \\
 & 27 \\
\hline
3 \times 30^2 = 2700 & 19\ 656 \\
3 \times 30 \times 6 = \phantom{0}540 & \\
6^2 = \phantom{00}36 & \\
\hline
3272 & 19\ 656 \\
\hline
\end{array}
$$

The five figures of the number show that the root is expressed by *two* figures. In 46000 the greatest tens-cube is 27000, and its cube root is 30, which is the *tens* of the root.

The cube of the tens, $t^3$, is subtracted, and the remainder, 19656, contains three times the product of the square of the tens by the ones + three times the product of the tens by the square of the ones + the cube of the ones. Each of these parts contains the ones' number as a factor.

Hence the 19656 consists of two factors, one of which is the ones' number of the root; the other is three times the square of the tens + three times the product of the tens by the ones + the square of the ones. Of this last factor, three times the square of the tens, or 2700, is so much the larger part that it may be used as a *trial* divisor; using it thus, we find 7 to be probably the ones' figure of the root. But by trial we find this value too large, and we must take one less, or 6, for the ones' figure of the root.

Since $3t^2o + 3to^2 + o^3$ is equal to $(3t^2 + 3to + o^2) \times o$, the trial divisor is completed by adding to the 2700 the $3 \times (30 \times 6) = 540$, and $6^2 = 36$; the sum, 3276, is multiplied by 6.

NOTES.—1. In finding the ones' figure, we have given a product and the greater portion of one factor to find the other factor.

2. In practice, ciphers are omitted for convenience, as in (b). The first group, 46, contains the cube of the tens' number of the root. The greatest cube in 46 is 27, and the cube root of 27 is 3. Hence 3 is the tens' figure of the root. We then divide the 196 hundreds of the remainder by the $3 \times 30^2 = 27$ hundreds to get the ones' number of the root.

**538.** We may illustrate cube root by the problem of finding the edge of a cube whose volume is 46656 cubic inches.

Here $t^3 + 3t^2o + 3to^2 + o^3 = 46656$, whose cube root is 36.

The cube of $36 = (30 + 6)^3 = 30^3 + 3\,(30^2 \times 6) + 3\,(30 \times 6^2) + 6^3$.

The $30^3$ may be represented by a cube whose edge is 30 inches.

The $3\,(30^2 \times 6)$ may be represented by three rectangular solids, each 30 in. long, 30 in. wide, and 6 in. thick, to be added to three adjacent faces of Fig. 1.

The $3\,(30 \times 6^2)$ may be represented by three equal rectangular solids 30 in. long, 6 in. wide, and 6 in. thick, to be added to Fig. 2.

The $6^3$ may be represented by the small cube required to complete the cube of Fig. 3.

$$
\begin{array}{r}
t^3 + 3t^2o + 3to^2 + o^3 = 46656 \\
t^3 \qquad\qquad\qquad\quad = 27000 \\
\hline
3t^2o + 3to^2 + o^3 = 19656
\end{array}
$$

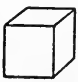


Fig. 1.

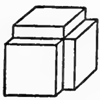


Fig. 2.

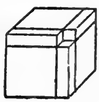


Fig. 3.

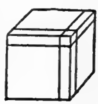


Fig. 4.

That is, in extracting the cube root of 46656, the large cube, whose edge is $t = 30$ in., is first removed. There remain 19656 cu. in. Of what is this the volume?

The seven additions to the cube of the tens have one dimension, $o = 6$, in common. If these seven solids were laid side by side so as to form one solid, the area of its base would be $3t^2 + 3to + o^2$, or $3 \times 30^2 + 3 (30 \times 6) + 6^2$; its height would be $o = 6$; and its volume would be the product of these factors.

In finding the height $o$, we are obliged to use $3t^2$, or 2700 (trial divisor), as the area of the base, since the whole area is as yet unknown. If this gives too large a value for $o$, we must take one less.

$$19656 \div 3t^2 = 19656 \div (3 \times 30^2) = 7 + .$$

By trial we find this too large; hence we must take $o = 6$. This gives

$$(3t^2 + 3to + o^2) \times o = (3 \times 30^2 + 3 \times 30 \times 6 + 6^2) \times 6 = 19656.$$

$\therefore \sqrt[3]{46656} = 30 + 6 = 36$, the number of inches in the edge of the cube.

**539.** The methods given above will apply to numbers of more than two groups of figures if we regard *the part of the root already found as so many tens* with respect to the next figure of the root.

**1.** What is the cube root of 1906624 ?

| | | |
|---|---|---|
| | | 1 906 624 \| 124 |
| | | 1 |
| $3 \times 10^2 =$ | 300 | 906 |
| $3 \times (10 \times 2) =$ | 60 | |
| $2^2 =$ | 4 | |
| | 364 | 728 |
| $3 \times 120^2 =$ | 43200 | 178 624 |
| $3 \times (120 \times 4) =$ | 1440 | |
| $4^2 =$ | 16 | |
| | 44656 | 178 624 |

**540.** RULE.—*Beginning at ones, separate the number into groups of three figures each.*

*Find the greatest cube in the left-hand group, and write its root for the first part of the required root.*

*Subtract the cube of this root from the left-hand group, and to the remainder annex the next group for a dividend.*

*Divide this dividend by three times the square of the root* *already found, considered as tens. The quotient (or the quotient diminished) will be the next figure of the root.*

*To the last (trial) divisor add three times the product of the first part of the root, considered as tens, by the part last found, and also the square of the last part, for a complete divisor.*

*Multiply the complete divisor by the part of the root last found, subtract the product from the dividend, to the remainder annex the next group for a new dividend, and proceed as before, until all of the groups have been thus annexed.*

NOTES.—1. A decimal is separated into groups of three figures each by beginning at the decimal point.

2. To find the cube root of a common fraction, extract the cube root of numerator and denominator separately ; or change to a decimal and then extract the root.

3. If a number is not a perfect square, ciphers may be annexed, and the value of the root found to any required degree of approximation.

Find the cube root of the following :

| | | |
|---|---|---|
| **1.** 2744. | **7.** 74088. | **13.** $\frac{512}{1331}$. |
| **2.** 4096. | **8.** 140608. | **14.** $\frac{4913}{6859}$. |
| **3.** 8000. | **9.** 226981. | **15.** $\frac{5832}{9261}$. |
| **4.** 24389. | **10.** 1860867. | **16.** 140.608. |
| **5.** 10648. | **11.** 12167000. | **17.** 250.047. |
| **6.** 42875. | **12.** 926859375. | **18.** .970299. |

### APPLICATIONS OF CUBE ROOT.

**541. 1.** A cubical block of marble contains 13824 cubic inches. What is the length of a side ?

**2.** A cubical cistern holds 1000 gallons of water. How deep is it ?

**3.** How many square inches in one face of a cube of granite whose contents are 5832 cubic inches ?

**4.** Find the length of the diagonal of a cube whose volume is 8000 cubic inches.

**5.** A cubical bin contains 500 bushels of wheat. How deep is it if it is half full of wheat?

**6.** A square box 16 in. deep will hold 9.64276 bushels of grain. What is the length of its side (inside measure)?

**7.** A miller wishes to make a cubical bin that will hold 200 bushels of grain. What must be its depth?

**8.** What is the length of the edge of a cubical box that will hold one half as much as one whose edge is 4 feet?

**9.** Two thousand gallons just fill a vat whose length is twice its width, and whose height is $\frac{1}{2}$ of its length. Find the length.

**10.** Assuming that cast iron weighs 7.15 times as much as water, find the edge of a cube of such iron that would weigh a ton.

**11.** The width and depth of a cistern are equal, its length is twice its width, and it will hold $2489\frac{1}{7}$ gallons of water. What are its dimensions?

**12.** By cooling a red-hot cube of iron, the length of each of its edges was diminished by 6%. Find correct to three decimals the ratio of decrease in the volume of the cube.

# MISCELLANEOUS PROBLEMS.

## ORAL EXERCISES.

**542. 1.** The sum of two numbers is 60, and their difference is 12. What are the numbers ?

**2.** If a man can build .3 of a fence in a day, in what time can he build $\frac{3}{5}$ of it ?

**3.** How many pies will be needed to give each of 48 boys $\frac{3}{8}$ of a pie ?

**4.** If a man can do $\frac{5}{8}$ of a piece of work in a day, how long will it take 2 men to do .5 of it ?

**5.** If a bushel and 3 pecks of potatoes last a family 5 weeks, how many days will a peck and a half last them ?

**6.** A man lost $12\frac{1}{2}\%$ by selling a cow for \$35. What per cent would he have gained by selling her for \$50 ?

**7.** The sum of two numbers is 45, and their difference is 25% of the smaller number. What are the numbers ?

**8.** I bought eggs at 20 cents a dozen. Had I paid a quarter a dozen they would have cost a dollar more. How many did I buy ?

**9.** A man spent $\frac{2}{9}$ of his money for a sleigh and $\frac{1}{3}$ of it for a book-case. What per cent of his money had he left ?

**10.** By selling books at a profit of 40% an agent gained \$250. What did the books cost him ?

**11.** What number increased by its half, third, and fourth equals 50 ?

**12.** If a man can earn a dollar in $\frac{3}{4}$ of a day, how much can he earn in $\frac{2}{3}$ of a day ?

**13.** What number diminished by 12 is equal to 4 times $\frac{2}{11}$ of the number ?

**14.** A man divided his farm of 160 acres between his two sons in the ratio of $\frac{1}{2}$ to $\frac{1}{3}$. How many acres did each receive?

**15.** One half of the money in my purse is quarters, $\frac{1}{4}$ is nickels, $\frac{1}{5}$ is dimes, and the remainder is 5 pennies. What sum is in my purse?

**16.** B and C together have \$50, and B has a dollar more than C. How much has each?

**17.** What is the diagonal of a table whose length is 4 feet and whose width is 75% of the length?

**18.** I borrowed a sum of money for 3 years, at the end of which time I repaid the loan by a check for \$22.50 more than I borrowed. If the rate was 5%, what was the sum borrowed?

**19.** A, B, and C shared a tract of land in the ratio of 1, $\frac{1}{2}$, and .2. C received $4\frac{1}{2}$ acres less than B. What was the share of each?

**20.** What is the gain per cent when half a yard of cloth is sold for what $\frac{5}{8}$ of a yard cost?

**21.** A man engaged to work 10 days for \$30, agreeing to forfeit \$2 for every day he failed to work. If he received \$22.50, how many days was he idle?

**22.** A lady, being asked how many children she had, replied, "If I had twice as many and 6 more I would have a dozen." How many had she?

**23.** A fox is 80 rods ahead of a hound, and runs 20 rods while the hound runs 25. How far will the hound run before he catches the fox?

**24.** A man paid \$45 for some sheep. Three of them died and $\frac{4}{5}$ of the remainder were sold for cost, which was \$30. How many were sold?

**25.** Either 8 turkeys or 12 ducks are needed for a dinner. If only 3 ducks can be had, how many turkeys must be taken?

**26.** A starts on a journey and travels 27 miles a day; 7 days later B starts, and travels 36 miles a day. In how many days will B overtake A?

**27.** A farmer raised 60 bushels of potatoes, and the crop was 1500% of the seed. How many bushels were planted?

**28.** A man sold $\frac{2}{5}$ of his sheep to A, $\frac{1}{12}$ of the remainder to B, and then had 33 left. How many did he sell to B?

**29.** If $\frac{3}{4}$ of A's age is $\frac{2}{3}$ of B's, and the sum of their ages is 51 years, what is the age of each?

**30.** Three men hired a pasture for $114. A put in 3 horses 16 weeks; B, 5 horses 12 weeks; C, 8 horses 15 weeks. How much should A pay?

**31.** If an orange and a half cost a cent and a half, how many oranges may be bought for a dime?

**32.** I sold a watch to A for $\frac{1}{4}$ more than it cost me; he sold it for $18, thereby losing $\frac{2}{5}$ of what it cost him. How much did I pay for it?

**33.** A is $\frac{3}{4}$ as old as B, but if he were 6 years older, he would be .9 as old as B. How old is each?

**34.** D's money is $3 more than $\frac{3}{5}$ of B's, and $5 less than $\frac{2}{3}$ of B's. How much has each?

**35.** If 3 boys do a piece of work in 9 hours, how long will it take a man who works $4\frac{1}{2}$ times as fast as a boy?

**36.** If 6 men can dig a ditch in $3\frac{1}{2}$ days, how much time will be saved by employing 2 more men?

**37.** A boat goes 10 miles an hour up stream, and 15 miles an hour down stream. How far can she go and return in 10 hours?

**38.** A party of 6 hired a coach. If there had been 2 more, the expense would have been $1 less for each person. How much was paid for the coach?

**39.** A can do a piece of work in 12 days, or in 8 days with B's assistance. After they work together 6 days, B finishes the work, for which he receives $10. How much should A receive?

**40.** What time after midnight are the hour and minute hands of a clock first together?

WRITTEN EXERCISES.

**543. 1.** What is the value of a lot .625 of which is worth $1250 ?

**2.** Of the people in a building $\frac{3}{7}$ are boys, .375 are girls, and the remainder, which is 22, are men. How many girls in the building ?

**3.** The quotient is 3, the remainder $\frac{2}{11}$, and the divisor $\frac{3}{11}$. What is the dividend ?

**4.** How often can .125 be subtracted from the sum of 125 tenths and 125 hundredths ?

**5.** If $\frac{7}{9}$ of an acre of land costs $$41\frac{2}{3}$, what will $4\frac{3}{5}$ acres cost ?

**6.** Find $\sqrt{2} \times \sqrt{3}$ correct to two decimal places.

**7.** The minuend is .875, and the remainder is one less than 1500 thousandths. What is the subtrahend ?

**8.** Explain the short process of dividing by $33\frac{1}{3}$; by 125.

**9.** What will be the cost of 6160 lb. of coal, at $5.50 a ton ?

**10.** Divide, by using factors, 2875 by 48, and explain the process.

**11.** Between the lightning and the thunder I noted $12\frac{3}{8}$ seconds. How far away was the thunder, if sound traveled 1140 feet a second ?

**12.** What number divided by either 3, 4, 8, 9, 12, 18, 24, or 36 leaves a remainder of 3 ?

**13.** Making no allowance for mortar, how many bricks 8 in. long and 4 in. wide will be required to pave a walk 40 yards in length and 5 feet in width ?

**14.** If the velocity of electricity is 288,000 mi. a second, how long would it take electricity to travel around the earth, considering the circumference to be 24,900 mi. ?

**15.** A and B hire a pasture. A puts in 21 cows and B puts in 35. If B's part of the rent is $185, how much is A's ?

**16.** If 40 pupils use 6 boxes of crayons, 200 in a box, in

3 mo., how many boxes, 150 in a box, will be required, at the same rate, to supply 75 pupils for 2 mo.?

**17.** How many books the size of this would be required to cover the floor of your schoolroom?

**18.** How many cubic feet of ice will an ice-house hold whose dimensions are 50 feet by 30 feet, and 18 ft. high, allowing 2 ft. above and below and on each side for sawdust?

**19.** What is the value of $7\frac{1}{2} + .3 + 18 \div 4.5 - 2\frac{1}{3} \times 1\frac{3}{4}$?

**20.** A lot 40 ft. by 120 ft. is enclosed by a wire fence 3 wires high. If 25 feet of wire weighs a pound, and a pound costs 5 cents, what did the wire for the fence cost?

**21.** A party of 60 hired a boat. Had there been 20 more, the expense of each would have been reduced $1. How much was paid for the boat?

**22.** What is the area of a square field whose perimeter is 160 rods?

**23.** The perimeter of a rectangular field is 240 rods, and the width is $\frac{1}{2}$ the length. How many acres in the field?

**24.** One half the diagonal of a rectangular field is 25 rods, and the width is 30 rods. What is the area of the field?

**25.** What is the volume of a rectangular solid $a$ feet square and $b$ feet long?

**26.** One evening a tree 45 feet high cast a shadow 75 feet long. At the same time a shadow of another tree was 160 feet. How high was the other tree?

**27.** If 10 ounces of cotton make $6\frac{2}{3}$ yards of cloth a yard wide, how much will be required to make 12 yards 48 inches wide?

**28.** The rafters of a barn are 25 feet long, and their ends are 40 feet apart. What is the height of the gable?

**29.** If 29 cows average 9 quarts of milk each per day throughout the year, and the milk is sold at an average of 7 cents a quart, what is the total annual profit if the expenses are $78 a head?

**30.** How many feet of lumber in 12 planks, each 18 feet long, 10 in. wide, and 3 in. thick?

**31.** The extreme end of the minute hand of a town clock moves 19 inches in 12 minutes. What is the length of the minute hand?

**32.** Each board in a floor 56 feet long and 28 feet wide is 14 feet long and 6 inches wide, and it is held in place by 8 nails. If 68 nails weigh a pound, what is the weight of the nails in the floor?

**33.** If 36 yards of cloth cost \$54 when wool is 25¢ a pound, what will 25 yards cost when wool is 20¢ a pound?

**34.** A carpenter, a mason, and a painter built a house by contract for \$3000. The carpenter worked 108 days, the mason 72 days, and the painter 45 days. The material used cost \$1775. How much did each man earn if carpenter's wages were \$3 a day, mason's \$4, and painter's \$2.40?

**35.** A load of 120 bushels consists of corn and rye in the ratio of 7 bu. of corn to 3 bu. of rye. How much rye must be taken away that the corn may be to the rye as 10 is to 4?

**36.** A garden 40 yards square is surrounded by a walk 24 feet wide. What part of an acre does the walk contain?

**37.** What is the diameter of a circular field containing 10 acres?

**38.** An orchard contains 7200 square rods, and its length is to its breadth as $\frac{3}{4}$ to $\frac{2}{3}$. What is its length?

**39.** A, B, and C can build a fence in 10 days. A can build twice as much as B, and C $\frac{2}{3}$ as much as A. In what time can each alone build the fence?

**40.** The property of an insolvent debtor amounts to \$3560, and his liabilities to \$8900. How many cents on the dollar will his creditors receive?

**41.** What is the distance from a lower corner to the opposite upper corner of a room 16 feet long, 12 feet wide, and 10 feet high?

**42.** The entire surface of a cube is 1014 square inches. How many cubic inches does it contain?

**43.** Required the cost of fencing a square 40-acre field at $1.50 a rod.

**44.** Measure your schoolroom and calculate the distance from the center of the ceiling to a lower corner.

**45.** If steel rails weigh 180 lb. to the yard and cost $12 a ton, what will be the cost of rails for 2 miles of railroad, one of which has double track?

**46.** A cow is tied by a rope 20 feet long. Upon how many square yards can she graze?

**47.** An iron slab is 4 feet long, 3 feet wide, and an inch thick. If drawn out until only $\frac{1}{8}$ of an inch thick, how many square feet will its sides contain?

**48.** A square park is surrounded by a walk 2 rods wide. The area of the walk is 2 acres. What is the area of the square?

**49.** Find the value of the following lumber at $24 per M:

Six $6 \times 9$ sills, 18 ft. long.
Thirty-two $2 \times 8$ joists, 18 ft. long.
Thirty-four $2 \times 6$ rafters, 22 ft. long.

**50.** Bought apples at $3 a barrel. Half of them rotted. At what price must I sell the remainder in order to gain $33\frac{1}{3}\%$ on the amount bought?

**51.** If a man, starting at noon from Pittsburg, could travel at the rate of 15 degrees an hour, where would he be in 24 hours? Would his watch be too fast?

**52.** How many revolutions will a wheel whose radius is 2 ft. 3 in. make in rolling a mile and a half?

**53.** The troy pound is what per cent of the avoirdupois pound?

**54.** The base of an isosceles (two sides equal) triangle is 32 feet, and the altitude is 12 feet. What is the perimeter? Draw the figure.

**55.** A vessel can sail 25 miles an hour with the current, and 15 miles against it. What is her rate of sailing in still water ?

**56.** A ship, whose rate of sailing in still water is 10 miles an hour, sails 40 miles up a stream in 5 hours. What is the rate of the current ?

**57.** Last year my rent was $350, which is $12\frac{1}{2}\%$ less than it is this year. How much has my rent been increased this year ?

**58.** If the water is 25% of the milk, how many gallons of each in a mixture containing $62\frac{1}{2}$ gallons ?

**59.** In a company of 120 persons, the children are $33\frac{1}{3}\%$ of the men, who are 75% of the women. How many children are there ?

**60.** Each of two dealers, A and B, wishes to sell an organ to C. A asked a certain price, and B $33\frac{1}{3}\%$ more. A then reduced his price 25%, and B his price 30%, and C took both organs, paying $202. What was B's price ?

**61.** A druggist pays $3.50 a pound avoirdupois for 3 lb. of a certain article. If he sells it at the rate of 60 cents a troy ounce, how much does he gain ?

**62.** Find the cost, at $24 per M, of 225 2-inch planks 18 ft. long and 8 inches wide.

**63.** The rainfall one day was .08 in. How many cubic feet of water fell on an acre ?

**64.** The cost of polishing 5 sides of a cubical block of marble, at 20 cents a square foot, was $144. What are its solid contents ?

**65.** A man sold two lots for $3600 each. On one he gained $33\frac{1}{3}\%$ and on the other he lost $33\frac{1}{3}\%$. Did he gain or lose, and how much ?

**66.** In a mixture of silver and copper weighing 55 ounces, there are 15 ounces of silver. How much copper must there be added that there may be $1\frac{2}{3}$ ounces of silver in 10 ounces of the mixture ?

**67.** Forrest and John receive the same salary. Forrest saves $\frac{1}{8}$ of his, but John spends \$200 a year more than Forrest, and at the end of three years finds himself \$150 in debt. Required the salary of each.

**68.** A 2-inch plank is 16 feet long, 18 inches wide at one end, and tapers regularly to a point. How many feet board measure does it contain?

**69.** How long will it take $a$ men to do a piece of work that $b$ men can do in $c$ days?

**70.** A man receives a salary of \$1250 a year, and $3\frac{1}{3}\%$ of his salary equals $16\frac{2}{3}\%$ of his savings. What sum does he save per annum?

**71.** How shall I mark goods which cost me \$240 that I may fall $14\frac{2}{7}\%$ and still gain 20 per cent?

**72.** From an excavation whose length, width, and height are equal, 2197 cu. ft. of earth were taken. How many feet of boards will be required to cover the sides and bottom of the cavity?

**73.** A family consumes $47\frac{1}{4}$ lb. of meat a week. If each member of the family consumes $\frac{3}{4}$ lb. daily, how many in the family?

**74.** A grocer sold 18 dozen and 9 boxes of matches from 2 gross. What part of the whole did he sell?

**75.** How many sheets of tin, each $18 \times 30$ in., will it take to cover a roof, each side being 24 ft. long and 16 ft. 3 in. wide?

**76.** A commission merchant after paying \$1605 for various expenses found that he had cleared \$2494. What amount did he collect if his rate of commission was 5 per cent?

**77.** A, B, and C invested \$10,800 in business, B paying $\frac{2}{3}$ as much as A, and C $\frac{1}{2}$ as much as A and B together. The profit the first year was $33\frac{1}{3}\%$ of the capital. How much should each receive?

**78.** Three men rent a room for one year, four months, at the rate of \$210 a year. The first man paid \$122.50 for the time he occupied the room; the second man occupied the room for 4 mo., and the third for the remaining time. If each paid according to the time he occupied the room, how much should the last two pay?

**79.** A man drew out of the bank $\frac{1}{9}$ of his money and \$16 more; then $\frac{1}{4}$ of the remainder less \$20. He then had \$182 remaining in the bank. How much had he at first?

**80.** How many times will a wagon wheel 12 ft. 6 in. in circumference turn round in going 10 mi. 24 rd. 4 ft.?

**81.** In a pile of wood 16 feet long and 16 ft. wide, there are 12 cords. How high is it?

**82.** How many times can .013 be subtracted from 125.78, and by what must the remainder be divided to give 350,000 as a quotient?

**83.** What length of a road 32 feet wide will have an area of half an acre?

**84.** Find the weight in tons per acre of a rainfall of an inch, one cubic foot of water weighing 62.5 pounds.

**85.** If the gas for 5 burners, 5 hours each evening for 10 days, costs \$1, what will be the cost of gas for 75 burners which are lighted 4 hours every evening for 15 evenings?

**86.** The average age of 200 boys is 14.75 years; what will be the average if 10 boys are added whose average age is 11.6 years?

**87.** The combined weight of 2 bars of silver is 27 lb. 3 pwt. 5 gr. The larger one weighs 12 lb. 19 cwt. 21 gr. more than the smaller one. Required the weight of each.

**88.** A and B each have \$10 so invested that A receives 4 per cent and B 5 per cent interest. What per cent of A's interest is B's?

**89.** At the rate of a ton of coal every 21 days, what will be the cost of the coal used by a family from Oct. 17, 1904, to April 25, 1905, excluding both of the days named, at \$4.50 a ton?

**90.** A publisher sells to the wholesale trade 40 copies of a book at the retail price of 24 copies. What does he receive wholesale for a book which retails at $1.50?

**91.** A's weight is $\frac{3}{4}$ of B's, and C's is as much as A's and B's together; the sum of their weights is 490 lb. How much does each weigh?

**92.** The duty on a shipment of blankets being 33 cents a pound and 40% *ad valorem*, what is the invoice price, if they cost the importer $3874, including the duty, the total weight of the blankets being 5800 lb.?

**93.** If a man buys 100 shares of railroad stock at $107\frac{1}{4}$, and sells it a month later when quoted at $115\frac{3}{8}$, what is the gain, if in the meanwhile the stock paid a 2% dividend?

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**94.** The first of four cog wheels which work together has 21 cogs, the second has 18, the third 15; and the number of cogs that the fourth has is to the number the third has as 2 to 3. After how many revolutions of the smallest wheel will they all be in their position at starting?

**95.** What is the area of a circle 20 ft. in diameter?

**96.** Divide $34.50 among 6 men and 11 boys, giving each boy .5 of a man's share.

**97.** The great pyramid, whose base is square, measures 763 ft. on each side. How many acres does it cover?

**98.** To cover the floor of a gentleman's carriage-house, it took 170 planks 16 ft. long, 9 inches wide, and 2 inches thick. How much did the planks cost at $12 a thousand feet?

**99.** How many yards of carpet, 28 inches wide, laid lengthwise, will be required to cover a floor 9 ft. 4 in. wide and $18\frac{1}{2}$ ft. in length?

**100.** A house which cost $4800 rents for $300. If the taxes are 2 per cent, insurance $\frac{3}{4}$ per cent, and repairs $5.50 per annum, what per cent is the net income?

**101.** There are in the library of a certain school 683 books, which number will give 23 books to each pupil, and 16 books over. What is the number of pupils?

**102.** If one pound of zinc covers a square yard, and it is worth 45¢ a pound, what will it cost to line a tank 10 ft. square and 5 ft. deep?

**103.** When it is 12 o'clock M. at St. Louis, 90° 15′ 15″ W., what is the time at Richmond, 77° 20′ 4″ W.?

**104.** How many shingles will be required to cover the roof of a building 54 feet long, the two sides each $16\frac{1}{2}$ feet wide, if one shingle covers a space 6 in. square?

**105.** What is the amount of the following bill?

$32\frac{1}{2}$ yd. of muslin at $6\frac{2}{3}$ ct.
$15\frac{1}{2}$ lb. of lard at 12 lb. for $1.00.
23 lb. 3 oz. of butter, at 25¢ a lb.
$13\frac{1}{2}$ lb. of sugar at 16 lb. for $1.
2 doz. bananas at 4 for a dime.

**106.** Find the cost of 25 joists 10 in. wide, 18 ft. long, and 4 in. thick at $35 per M.

**107.** Edward and Peter hire a pasture for $14. Edward puts in 8 horses; Peter puts in 50 sheep. If 21 sheep eat as much as 2 horses, what must Edward pay?

**108.** If a miller takes $\frac{1}{16}$ for toll, and a bushel of wheat produces 40 lb. of flour, how many bushels of wheat must be taken to the mill to obtain a barrel of flour?

**109.** The valuation of the real property of a village is $125000. How many mills on the dollar must be levied to net $475 after paying the collector 5 per cent?

**110.** How much money must I put in a bank which allows 4% interest on deposits in order to receive $100 at the end of 9 months?

**111.** Ten horses and 12 cows cost $1160; 4 horses and 7 cows cost $530. What is the value of a horse?

**112.** My shoemaker sends me a bill of $18 for a pair of boots and two pairs of shoes. Some months afterwards he sends me a bill of $32 for two pairs of boots and three pairs of shoes. What do the boots cost a pair?

**113.** In a lot of eggs 7 of the largest, or 10 of the smallest, weigh a pound. When the largest are worth 15¢ a dozen, what are the smallest worth ?

**114.** When I was married I was 3 times as old as my wife, but 15 years after our marriage I was only twice her age. Find my age at marriage.

**115.** The sum of $\frac{2}{3}$ of A's money and $\frac{1}{2}$ of B's being on interest for 8 years at 6 per cent, gives $960 interest. How much money has each if $\frac{1}{2}$ of B's is 3 times $\frac{2}{3}$ of A's ?

**116.** Take the proportion $8 : 3 = 5 : 1\frac{7}{8}$. If the second and third numbers each be increased by 7, what multiplier will be needed by the first to make a proportion ?

**117.** Find the diagonal of a room 40 ft. long, 30 ft. wide, and 12 ft. high.

**118.** The end of the minute hand of a town clock passes over 30 inches in 12 minutes. What is the length of the hand ?

**119.** What length of rope will enable a horse to graze on 2 acres of grass ?

**120.** Out of a piece of paper 5 ft. 10 in. square is cut the greatest possible circle. How many square inches of paper are cut away ?

**121.** How many pounds of flour should I give for $42.30, at the rate of $4.90 a barrel ?

**122.** How many acres are in a square the diagonal of which is 20 rods more than either side ?

**123.** A bin 6 ft. long, 2 ft. wide, and $1\frac{1}{2}$ ft. high is filled with oats worth 40¢ a bushel. What is the value of the oats ?

**124.** Find the cost of paving and curbing one mile of street, the paving being 30 feet wide and costing $2.75 a square yard, and each line of curbing costing 30¢ a linear foot.

**125.** Two men enter into a partnership, one putting in $5000, the other $2000. The partner that puts in the less sum is to receive $300 extra from the proceeds for his experience in the business. They gain \$4725. What is the share of each?

**126.** Required the area in acres of a piece of land .5 of a mile long and .3 of a mile broad.

**127.** A wine merchant mixes 12 gallons of wine worth \$1 a gallon with 5 gallons of brandy worth \$1.50 a gallon, and 3 gallons of water of no value. What is the value of one gallon of the mixture?

**128.** How much clover seed at \$5.50 a bushel could a man buy for \$1017, after deducting his commission of $2\frac{1}{2}\%$ on amount paid for seed, and drayage at the rate of one and a fourth cents a bushel?

**129.** A man has \$4400. How much must he borrow at 4% and put with it so that the two sums invested in a business that pays 12 per cent per annum may net him a gain of \$600 a year?

**130.** John can write a page in $a$ minutes, and Sam can do the same in $r$ minutes. How much can they both write in 5 minutes?

**131.** In the A class there are twice as many girls as boys. Each girl makes a bow to every other girl, to every boy, and to the teacher. Each boy makes a bow to every other boy, to every girl, and to the teacher. In all there are 900 bows made. How many boys in the class?

**132.** Sold wheat on commission at 6%, and invested the net proceeds in flour at 4% commission, my whole commission being \$625. What was the value of the wheat and flour?

**133.** A man invested a certain sum of money in 5% stock at 80, and twice as much in 4% stock. If his income from the former is \$300, and from the latter $1\frac{1}{3}$ times as much, what was the price of a share in the latter investment?

**134.** If water expands $\frac{1}{24}$ in volume in being heated from the freezing point to the boiling point, find the weight of a cubic foot of boiling water, the weight at freezing point being 62.5 pounds.

**135.** If the pressure of the air on the surface of water is 15 lb. per square inch, and if 1 cu. ft. of water weighs 1000 oz., find the pressure per square foot of a column of water 100 ft. deep.

**136.** If each person in breathing spoils the air of a closed room at the rate of 8 cu. ft. a minute, how long can the windows and doors of a schoolroom be safely kept closed when occupied by 50 pupils, if the room is 25 ft. by 20 ft., and 10 ft. high?

**137.** By raising the temperature of a cube of iron, the length of the edge was increased 5%. Find the ratio of increase in the volume of the cube.

**138.** Each side of a hexagonal (six-sided) field is 20 rd., and the distance from the center of the field to the middle point of each side is also 20 rd. What is the area of the field?

**139.** During a heavy rainstorm, a circular pond is formed in a circular field. If the diameter of the field is 250 rd. and that of the pond 125 rd., what is the ratio of the land area to the water area of the field?

**140.** If the average velocity of a bullet is 1342 feet a second and that of sound 1122 feet a second, how much time elapses, on a range of 3000 feet, between the time the bullet strikes and the time the sound reaches the target?

### SUPPLEMENTARY EXERCISES (FOR ADVANCED CLASSES).

**544.** **1.** Is a billion a million million? Explain.

**2.** Multiply 789627 by 834, beginning at the left to multiply.

**3.** Reduce $\frac{1}{5}$ to a fraction whose denominator is 11.

**4.** Change $\frac{3}{5}$, $\frac{7}{8}$, $\frac{6}{11}$, and .7 to fractions having a common numerator.

**5.** Find the value of $\dfrac{\frac{3}{4} \text{ of } \dfrac{\frac{1}{2}}{\frac{1}{3}}}{\dfrac{\frac{1}{4}}{\frac{1}{2}} \times \dfrac{4\frac{1}{2}}{.5}} \div .125.$

**6.** If I receive a discount of 20%, 10%, and 5%, and sell at a discount of 10%, 5%, $2\frac{1}{2}$%, what is my per cent of gain?

**7.** At noon the three hands—hour, minute, and second—of a clock are together. At what time will they first be together again?

**8.** A merchant bought cloth at $3.25 a yard, and after keeping it 6 months sold it at $3.75 a yard. What was his gain per cent, counting 6% per annum for the use of money?

**9.** Bought 10 bushels of corn and 20 bushels of turnips for $11; at another time 20 bushels of corn and 10 bushels of turnips for $13. What did the corn cost a bushel?

**10.** A man wishing to sell a horse and a cow asked 3 times as much for the horse as for the cow; but finding no purchaser, he reduced the price of the horse 20%, and the price of the cow 10%, and sold both for $165. How much did he get for the cow?

**11.** If $\frac{1}{2}$ of an article is sold for the cost of $\frac{1}{3}$ of it, what is the rate of loss?

**12.** A man sold a pig and a sheep for $18, gaining 25% on the cost of the sheep, and 20% on that of the pig. If $\frac{2}{3}$ of the cost of the pig equaled $\frac{3}{4}$ of the cost of the sheep, what was the cost of each?

**13.** Mr. G spent $260 for apples at $1.30 a bushel. Retaining a part for his own use, he sold the rest at a profit of 40%, clearing $13 on the entire cost. How many bushels did he keep?

**14.** If 32 men have food for 5 days, how many men must leave, so that the food may last the remaining men 70 days?

**15.** What is the difference in area between a square whose diagonal is one foot and a circle whose diameter is one foot?

**16.** A dealer bought 3000 bushels of oats at $.30 a bushel, and sold them out by the bushel at 10% above cost. When all had been sold, it was found that the quantity of oats had shrunk 2%. What per cent did the dealer make on the investment?

**17.** After measuring $\frac{3}{8}$ of a mile with a chain 100 feet long, the surveyor discovers a kink in the chain which shortens its length $\frac{1}{2}$ inch. How much less than $\frac{3}{8}$ of a mile was the distance measured?

**18.** A man sold two horses for \$210; on one he gained 25%, on the other he lost 25%. How much did he gain, supposing the second horse cost $\frac{2}{3}$ as much as the first?

**19.** A merchant sold goods at 20% gain, but had their cost been \$49 more, he would have lost 15% by selling at the same price. How much did the goods cost him?

**20.** Had an article cost 20% more, the gain would have been 25% less. What was the gain per cent?

**21.** A, B, and C, having 4 loaves for which A paid 5¢, B 8¢, and C 11¢, eat 3 loaves, and sell the fourth to D for 24¢. How many cents should each receive?

**22.** The distance from the center of a circle to the middle of a chord 10 inches long is one foot. What is the area of the circle?

**23.** A man bequeathed \$9000 to his three sons, aged 13 yr., 15 yr., and 17 yr., in such a manner that the share of each, placed at compound interest at 6 per cent until he arrived at the age of 21 years, should amount to the same sum. Find the share of each.

**24.** What is the distance from the lower corner to the opposite upper corner of a room 15 ft. by 12 ft., and 10 ft. high?

**25.** If $\frac{2}{3}$ of the time past noon equals $\frac{3}{8}$ of the time to midnight, what time is it?

**26.** Between 2 and 3 o'clock I mistook the minute hand for the hour hand, and consequently thought the time 55 minutes earlier than it was. What was the correct time?

**27.** If sound travels in still air 1090 ft. a second when the temperature is 32° Fahrenheit, and if the velocity increases 1.1 ft. for every degree of increase in temperature, how far off is an explosion when the report follows in 8 seconds, the temperature being 70°?

**28.** If it takes 75 Kg. of saltpeter, 12.5 Kg. of charcoal, and 12.5 Kg. of sulphur to make 100 Kg. of powder, how many kilograms of each will be required to make 10,000,000 cartridges, each containing 5 g. of powder?

**29.** In digging a cellar, 15625 cubic feet of earth were removed. The length is twice the width, and the width is twice the depth. What are the dimensions?

**30.** What is the width of a doorway 8 feet high that is just wide enough to allow a circular mirror 31.416 feet in circumference to pass through?

**31.** B has a circular garden containing 75 sq. rd. What is the area of the largest square garden he can make in it?

**32.** What is the area of a square whose diagonal is 100 feet?

**33.** A boy weighing 96 lb. is seated on one end of a seesaw 16 ft. long, and a boy weighing 120 lb. is seated on the other end. Find the distance of each boy from the point of support, the lengths of the two arms of the plank being inversely proportional to the weights at their ends.

**34.** How many apple trees can be planted in an orchard 15 rods square, allowing no two to be nearer each other than $1\frac{1}{4}$ rods?

**35.** A cube of water 1.8 dm. on an edge weighs how many kilograms?

**36.** B lends A $150 for 6 months, and a year later A lends B $100. How long may B keep the $100 to balance the use of his loan to A?

**37.** Starting from Dayton, I go 200 miles due east, then 150 miles due south, then 100 miles due west, then 350 miles due north. How far and in what direction from Dayton am I?

**38.** How many rods is it from the center to one corner of a square field containing 20 acres?

**39.** What is the difference between half a cubic foot and a cubic half foot?

**40.** If my horse had cost me 20% less, my rate of gain in selling him would have been 30% greater. What was my gain per cent?

> 100% of the cost = the cost.
> 30% of 80% = 24% of *cost*, yielded by 20% of cost.
> 100% of cost yields 5 × 24% = 120% of cost.
> ∴ the gain = 20%.

**41.** A squirrel goes spirally up a cylindrical post, making a circuit in each 5 feet. How many feet does it travel if the post is 20 feet high and 6 feet in circumference?

**42.** A man agrees to pay $6000 for a lot in three equal payments, including 6% interest on unpaid money. What is the yearly payment?

> Amount of $6000 at 6% compound interest for 3 yr. = $7146.096.
> $1.00 + $1.06 + $1.1236 = $3.1836, or 3.1836 × $1.
> $7146.096 ÷ 3.1836 = $2244.66, the payment.

**43.** Two-fifths of a mixture of wine and water is wine; but if 10 gallons of water be added to it, then only $\frac{7}{20}$ of the mixture will be wine. How many gallons of each liquid in the mixture?

**44.** In the center of a circular island 100 feet in diameter stands a tree 140 feet high. A line 500 feet long will reach from the top of the tree to the farther shore. What is the width of the river, the island being in the middle?

**45.** What is the length of the shortest possible route by which a fly can crawl from a lower corner to the opposite upper corner of a room 16 feet long, 12 feet wide, and 8 feet high?

**46.** In a certain game A can make 20 points while B makes 30; B can make 20 points while C makes 18. How many points can C make while A makes 100?

**47.** Two trains start at the same time, one from Jacksonville to Savannah, the other from Savannah to Jacksonville. If they arrive at destinations 1 hour and 4 hours after passing, what are their relative rates of running?

# APPENDIX.

## SUPPLEMENTARY WORK.

## MENSURATION.

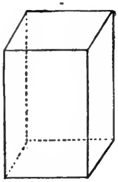
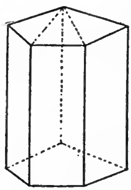
**545.** The process of measuring lines, surfaces, and solids is called **Mensuration.**

The principles of mensuration that apply to rectangles, parallelograms, triangles, trapezoids, circles, and rectangular solids have already been given.

### PRISMS.

**546.** A **Polygon** is a plane figure bounded by straight lines.

A

B

C


Thus, the ends of the solids A, B, and C are polygons.

1. Are the two ends of each solid equal? Are they parallel? What is the form of the *sides?*

2. What is the form of the *ends* of the solid A? Of the solid B?

**547.** A solid whose ends are equal and parallel polygons and whose sides are parallelograms is called a **Prism.**

Thus, A, B, and C are prisms. The solid B is also a *rectangular solid.*

1. The polygons are called *bases,* and the prisms are named from the form of the bases.

Thus, A is called a *triangular* prism; B, a *quadrangular* prism.

2. What is the prism whose bases are square called ?

3. When the bases are parallelograms, the prism is called a *Parallelopiped;* as B.

4. Is any triangular prism equal to *half* of a parallelopiped of the same altitude and of *double* the base ? Is it therefore equal to a parallelopiped of the *same* altitude and *equal* base ?

(Any prism can be cut into triangular prisms, as in C.)

**548. To find the volume of a prism.**

How to find the volume of a rectangular solid or prism was shown in Art. 327. The volume of *any* prism is found in the same way, viz.:

*Multiply the number of cubic units in 1 unit of length by the number of units of length.* Or, *multiply the area of the base by the altitude.*

It should be kept constantly in mind that the number of cubic units in 1 unit of length is the same as the number of square units in the base.

**1.** What is the volume of a prism whose length is 4 feet, and whose base is a rectangle 4 inches by 9 inches ?

**2.** The base of a prism 9 feet long is a triangle, each of whose sides is 3 feet, and whose altitude is 2.598 feet. How many cubic feet does it contain ?

**3.** A cord of Virginia pine weighs 2700 lb. What is the weight of a single piece of timber 18 inches square and 16 feet long ?

**4.** If a cubic foot of rolled steel weighs 489.6 lb., what is the weight of a piece 4 inches square and 30 feet long ?

**5.** Find the lateral surface of a prism whose length is 12 feet, and whose base is a triangle, each of whose sides is 2 feet.

The prism has 3 equal sides, each 12 feet long and 2 feet wide. The area of one side is 24 sq. ft.; hence the surface of the 3 sides is 3 times 24 sq. ft., or 72 sq. ft., the lateral surface.

The 3 sides together are equal to one rectangle 12 ft. long and 6 ft. wide. Make a rule for finding the lateral surface of prisms.

NOTE.—The *lateral* surface is the entire surface except that of the ends.

**6.** Find the lateral surface of a prism 3 feet square and 8 feet long.

**7.** The sides of a triangular prism are each $2\frac{1}{3}$ feet, and its height is 6 feet. What is the lateral surface?

**8.** What is the lateral surface of a pentagonal (five-sided) prism whose sides are each 18 inches, and whose altitude is 14 feet?

## THE CYLINDER.

What is the form of the solid C? Of its ends? Are the ends equal? Are they parallel?

C

**549.** A solid whose ends (bases) are two equal and parallel circles, and whose lateral surface is a uniformly curved surface, is called a **Cylinder.** It is described by revolving a rectangle about one of its sides as an axis.

1. The circles are called *bases*.
2. Name four objects that are cylinders.

How does the number of square units in the base of the cylinder C compare with the number of cubic units in 1 unit of length?

Then may the volume of a cylinder be found in the same manner as the volume of a prism?

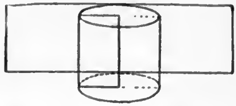
**1.** Find the volume of a cylinder whose diameter is 2 feet, and whose length is 10 feet.

**2.** In form the Winchester bushel is a cylinder, $18\frac{1}{2}$ inches in diameter and 8 inches deep. How many cubic inches does it contain?

**3.** A well is 26 feet deep and 4 feet in diameter. How many gallons of water are in it when it is half full?

**4.** A cylindrical pail 8 inches in diameter will hold 2 gallons. What is its depth?

**5.** Take an oblong paper 4 inches by 6 inches and roll it to form a cylinder. What is the length of the cylinder? The circumference?



**6.** If a hollow cylinder is cut and spread into a flat surface, what form has it?

**7.** What dimension of the cylinder is equal to the *length* of the rectangle? What to the width?

**8.** Then how may the lateral surface of the cylinder be found?

**9.** What is the lateral surface of a cylinder whose diameter is 2 feet and whose length is 6 feet?

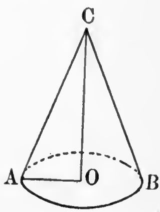
**10.** How many square feet of material in a piece of stove-pipe 6 inches in diameter and 2 feet 8 inches in length?

**11.** How many square feet of tin will be required to make 100 feet of spouting $2\frac{1}{2}$ inches in diameter?

## THE CONE AND THE PYRAMID.

What is the form of the base of the solid ABC? Notice how the solid tapers to a point.

**550.** A solid that tapers uniformly from a circular base to a point is called a **Cone.** It is described by revolving a right-angled triangle about one of its sides as an axis.




1. The point is called the vertex; as C.
2. The distance from the vertex to the center of the base is the *altitude;* as CO.
3. The shortest distance from the vertex to the circumference of the base is called the *slant height;* as CA.
4. Name several objects that have the form of a cone. Make a paper cylinder and a paper cone of equal base and

altitude. Fill the cone with salt, and empty it into the cylinder. How many conefuls will fill the cylinder?

It is shown in geometry that

**551.** *A cone has one third the volume of a cylinder of the same base and altitude.*

Then how may the volume of a cone be found?

**1.** What is the volume of a cone whose base is 12 feet in diameter, and whose altitude is 18 feet?

**2.** Find the solid contents of a cone, the diameter of whose base is 10 feet, and whose height is 15 feet.

**3.** A conical pile of grain is 3 feet high, and the diameter of its base is 6 feet. How many bushels in the pile?

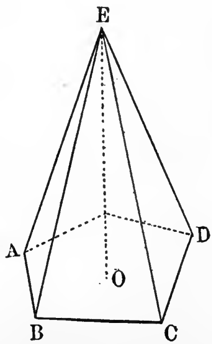
**4.** If a cubic yard of granite weighs 4700 lb., what is the weight of a granite cone 6 feet high, the diameter of the base being 4 feet?

The slant height is the hypotenuse of the right-angled triangle revolved to describe the cone, and the radius of the cone's base is the base of the triangle. (See figure.) If the slant height and radius of base are known, how can the altitude be found?

**5.** If the slant height of a cone is 10 feet, and the diameter of the base is 12 feet, what is the altitude? The volume?

What form has the base of the solid ABCDE?

What form has each side? Where do the sides meet?




**552.** A solid whose base is a polygon and whose sides are triangles meeting in a point is called a **Pyramid.**

1. The point in which the sides meet is the vertex; as E.

2. The distance from the vertex to the centre of the base is the altitude; as EO.

3. The distance from the vertex to the middle of a side of the base is the slant height.

4. Make a paper prism and a paper pyramid of the same base and altitude from pasteboard. If the latter is filled three times with salt, and the contents poured into the prism, will the latter be exactly full?

It is shown in geometry that

**553.** *A pyramid has one third the volume of a prism of the same base and altitude.*

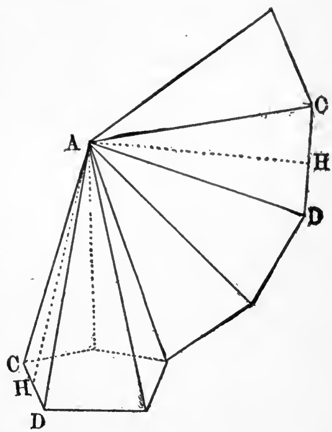
Then how may the volume of a pyramid be found?

**1.** What is the volume of a pyramid whose height is 15 feet, and whose base is 8 feet square?

**2.** Find the solid contents of a pyramid whose altitude is 20 feet, and whose base is a rectangle 12 feet by 8 feet.

**3.** Find the *lateral surface* of a pentagonal pyramid whose slant height is 20 feet, each side of the base being 8 feet.

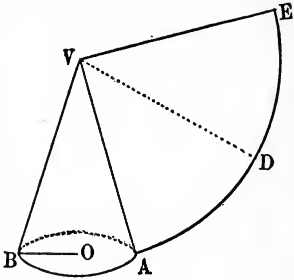
(a). The lateral surface is composed of five equal sides, each a triangle whose dimensions are given. Thus, in the triangle ACD, the base CD is 8 feet, and the slant height AH is 20 feet.




(b). Since the triangles have the same altitude, they are together equal to one triangle whose altitude is 20 feet, and whose base is 40 feet, the perimeter of the base of the pyramid. Hence the lateral surface = perimeter of base × $\frac{1}{2}$ slant height.

(c). If the number of the sides of the pyramid be increased indefinitely, the bases of the triangles will become extremely small, and the *perimeter* may be regarded as the *circumference* of the base of a cone whose lateral surface is equal to that of the pyramid. Hence the lateral surface of a cone = circumference of base × ½ slant height.

(d). If the lateral surface of a cone be imagined as unrolled from the solid itself, it will appear as a portion (sector) of a circle, as shown in the figure. Its area is equal to that of a triangle whose base equals the arc of the sector and whose altitude is the radius of the circle. The altitude VD is the slant height of the cone. What is the base ADE?




**554.** *The lateral surface of a cone = circumference of base × ½ slant height.*

**1.** Find the lateral surface of a pyramid whose base is 12 feet square, and whose slant height is 20 feet.

**2.** The slant height of a pentagonal pyramid is 9 feet, and each side of the base is 2 feet. What is the lateral surface?

**3.** Find the lateral surface of a cone whose diameter at the base is 16 feet, and whose slant height is 24 feet.

**4.** The circumference of the base of a cone is 40 feet, and the slant height is 20 feet. What is the lateral surface?

**5.** The cupola of a building is 16 feet in diameter at the base, and measures 22 feet from the vertex to the circumference of the base. What will be the cost of painting it at $.35 a square yard?

**6.** How many yards of canvas, 54 inches wide, must be bought to make a conical tent having a slant height of 11 feet, and a circumference at base of 27 feet?

**7.** The base of the pyramid Cheops in Egypt is 763.4 feet square, and the slant height is 612 feet. How many acres of surface in its sides?

**8.** Find the weight of each of the following, a cubic foot of marble weighing 170 pounds:

(a). A marble cylinder—length 3 feet, diameter 1 foot.

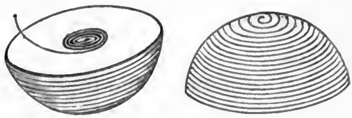
(b). A marble prism—length 3 feet, base 1 square foot.

(c). A marble cone—height 3 feet, radius of base 1 foot.
(d). A marble pyramid—height 3 feet, base 1 foot square.

## THE SPHERE.

**555.** A solid bounded by a surface whose every point is equidistant from a point within, called the center, is a **Sphere.**

(a). Bisect a sphere and observe the two surfaces exposed. These are called *great circles.* Is the diameter of these circles also the diameter of the sphere ?



(b). The curved surface of a hemisphere is larger than its flat surface. Is it twice as large? Take a wooden hemisphere and investigate by winding the surface of the hemisphere with a waxed cord, and then winding a great circle of the sphere with the cord.

(c). It is proved in geometry that the curved surface of a hemisphere is twice the flat surface. If the radius of the great circle is $r$, the area is $\pi r^2$. Then what is the area of the curved surface ?

(d). How many great circles in the curved surface of two hemispheres, or one sphere ? Then the surface of a sphere = how many times $\pi r^2$ ?

**The area of a circle = $\pi r^2$.**
**The surface of a sphere = $4\pi r^2$.**

**1.** How many square inches in the surface of an 8-inch globe ?

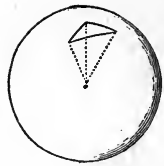
**2.** If the diameter of the moon is reckoned at 2000 miles, how many square miles in its surface ?

**3.** If the diameter of the earth is reckoned at 8000 miles, its area is how many times that of the moon ?

**4.** The surface of a sphere is 64 square feet. What is its diameter ?

**5.** The area of a great circle of a sphere is 100 square inches. Find the cost of gilding the sphere at 75¢ a square foot.

If we join three points on a sphere with the center, we mark out a solid which is *nearly* a pyramid. A sphere may be regarded as made up of a very great number of such pyramids whose common vertex is the center of the sphere, and whose bases are small portions of the surface. The altitude of each pyramid is the radius of the sphere, and the area of all the bases is equal to the surface of the sphere. Investigate this by cutting a sphere (an apple will do) into pyramids.



**Volume of pyramid = area of base × $\frac{1}{3}$ altitude.**
**Volume of sphere = surface × $\frac{1}{3}$ radius.**
" " " $= 4\pi r^2 \times \frac{1}{3}r = \frac{4}{3}\pi r^3$.

**6.** What is the volume of a sphere whose diameter is 12 inches?

**7.** How many cubic inches in a rubber ball if its diameter is 2 inches?

**8.** The diameter of a cannon ball is 6 inches. If the specific gravity of iron is 7.48, what is the ball's weight?

**9.** What is the diameter of the largest sphere that can be cut out of a cube whose edge is 10 inches? What is the volume? What per cent of the cube is cut away?

**10.** Find the number of cubic miles in the earth, considering the distance from the surface to the center as 4000 miles.

**11.** A 12-inch shell has an inside diameter of 10 inches. How many cubic inches of iron were used in casting it?

(From the entire volume subtract the volume of the hollow portion.)

## SIMILAR FIGURES.

**556.** Figures that have the same shape are called *similar figures.*

Thus, lines, squares, triangles whose angles are respectively equal, circles, cubes, or spheres are similar figures. May similar figures be regarded as enlarged or reduced copies of one another?

**557.** It is proved in geometry that—

1. *The corresponding lines of similar figures are proportional.*

2. *The surfaces (areas) of similar figures are to each other as the squares of their corresponding dimensions.* Conversely, *their corresponding dimensions are to each other as the square roots of their surfaces.*

3. *The volumes of similar figures are to each other as the cubes of their corresponding dimensions.* Conversely, *their corresponding dimensions are to each other as the cube roots of their volumes.*

**1.** If the side of one square is *twice* that of another, is its area *four times* as great? If the edge of one cube is *twice* that of another cube, is its volume *eight times* as great? Illustrate by drawing figures.

**2.** Prove that the surfaces of two spheres are to each other as the squares of their radii.

Let $S$ and $s$ represent the surfaces of two spheres, and $R$ and $r$ the radii. Then,

$$S = 4\pi R^2, \text{ and } s = 4\pi r^2.$$

$$\therefore \frac{S}{s} = 4\pi R^2 \div 4\pi r^2 = \frac{R^2}{r^2},$$

$$\text{or } S : s = R^2 : r^2.$$

**3.** Show that the circumferences of two spheres are to each other as the radii, and the volumes as the cubes of the radii.

**4.** One rectangular field containing 12.15 acres is 36 rods long and 18 rods wide; another field of the same shape is 27 rods wide. Find the length and area of the larger field.

(a) $18 : 27 = 36 \text{ rd.} : x \text{ rd.}$
(b) $18^2 : 27^2 = 12.15 \text{ A.} : x \text{ A.}$

**5.** The weight of a ball whose diameter is 5 inches is 27 lb., and the weight of a similar ball is 64 lb. What is the diameter of the larger ball?

$$5 \text{ in.} : x \text{ in.} = \sqrt[3]{27} : \sqrt[3]{64} = 3 : 4.$$

$\therefore$ the required diameter $= 6\frac{2}{3}$ in.

**6.** How many circles 2 inches in diameter are equal to a circle whose diameter is 6 inches?

**7.** An 8-inch square is equal to how many 2-inch squares?

**8.** In a park are two circular flower beds, one three times as large as the other. Find the circumference of the larger, if the smaller is 25 feet?

**9.** A rectangular lot is 20 rods long and 4 rods wide. A similar lot contains $2\frac{1}{2}$ acres, and is surrounded by a fence which cost \$1.75 a rod. Find the cost of fencing it.

**10.** A cow is tied to a stake by a rope 9 yards long, and a horse is tied to another stake by a rope 6 yards in length. Upon how much more area can the cow graze than the horse?

**11.** If 25 gallons of water flow through a pipe 2 inches in diameter in a minute, how many gallons an hour will flow through a pipe 6 inches in diameter?

**12.** If a pipe 1 ft. 6 in. in diameter fills a cistern in 6 hours, what is the diameter of a pipe that will fill it in 1 hr. 30 min.?

**13.** How many square feet of zinc will be required to line the sides and bottom of a cubical box whose capacity is equal to that of a rectangular box 4 ft. 6 in. long, 3 ft. 3 in. wide, and 2 ft. $1\frac{1}{2}$ in. deep?

**14.** A cubical box is 2 ft. deep. What is the depth of another cubical box that holds three times as much?

**15.** A pail 9 inches deep will hold 2 gallons. What is the depth of a similar pail that holds 2 quarts?

**16.** The diameter of one cannon ball is $2\frac{1}{2}$ times that of another, which weighs 27 lb. What is the larger ball worth at 1¢ a pound?

**17.** What is the edge of a cube whose contents are equal to the contents of two cubes whose edges are respectively 3 feet and 5 feet?

**18.** The Winchester bushel is $18\frac{1}{2}$ inches in diameter and 8 inches deep. What are the dimensions of a similar measure that holds $\frac{1}{2}$ peck?

SUPPLEMENTARY PROBLEMS.

**558.** **1.** The altitude of a pyramid is 12 feet, and its base is 18 feet square. What will be the cost of painting the lateral surface at \$.45 a square yard?

**2.** If a 3-inch pipe fills a cistern in $9\frac{1}{2}$ hours, how large a pipe will fill it in 12 hours?

**3.** How many 2-inch balls can be made from a ball 6 inches in diameter?

**4.** A box has a bottom 2 ft. 6 in. square, the top is 3 ft. 6 in. square, the height is 2 ft. 6 in. What will it cost to line with zinc at 20¢ a square foot?

**5.** What is the side of the largest cube that can be cut from a sphere 17 inches in diameter?

**6.** If a pipe 1.5 inches in diameter fills a cistern in 5 hours, in what time will another whose diameter is 15 inches fill it?

**7.** A conical candle is one inch thick at the bottom, and burns away the first inch in 15 minutes; it continues to burn at the same rate, and is consumed in 54 hours. Find its length.

**8.** If a $4\frac{1}{2}$-inch pipe fills a cistern in $5\frac{1}{3}$ hours, how long will it take a 3-inch pipe to fill it?

**9.** How many half-inch bullets can be made from a lead ball 5 inches in diameter?

**10.** A cubic foot of brass is to be drawn into a wire $\frac{1}{4}$ of an inch in diameter. What will be the length of the wire?

**11.** How many inch-pipes will be required to empty a reservoir as fast as a foot-pipe fills it?

**12.** The sides of 3 regular octagons are 3 ft., 4 ft., and 12 ft., respectively. Find the side of a fourth octagon whose area is equal to that of the first three.

**13.** How many square yards of cloth will be required to make a conical tent 10 ft. in diameter and $12\frac{1}{2}$ ft. high?

**14.** The diameter of the earth is about 4 times that of the moon. How many moons should weigh as much as the earth, assuming them to be composed of like material?

**15.** A conical wine glass 2 inches in diameter and 3 inches deep is $\frac{8}{27}$ full of water. What is the depth of the water?

**16.** Three men bought a grindstone 3 feet in diameter. How much of the diameter must each grind off to use up his share of the stone, making no allowance for the eye, or aperture?

**17.** Four women bought a ball of yarn 6 inches in diameter, and agreed that each should take her share in turn by winding from the outer part of the ball. How much of the diameter did each wind off?

**18.** The number of oscillations that pendulums make in a given time is inversely as the square root of the numbers representing their lengths. The length of a 1-second pendulum being .994 m., what is the length of a pendulum that beats half-seconds?

**19.** A cylinder 12 feet in diameter is equivalent to a cone 18 feet in diameter and 8 feet high. What is the height of the cylinder?

**20.** Two circular plates of the same thickness and material have diameters, the one 7 inches, the other $x$ inches. If the weight of the latter is 40% of the former, find the value of $x$.

**21.** If a ball of yarn 4 inches in diameter makes one pair of gloves, how many similar pairs will a ball 8 inches in diameter make?

**22.** Find the amount of tin necessary to make a tin pail 6 inches in diameter and 8 inches deep.

**23.** A hollow sphere 8 inches in diameter is filled with water. How many hollow cones, each 8 inches in altitude, and 8 inches in diameter at the base, can be filled with the water in the sphere?

**24.** A cylindrical tank is 1.2 m. in diameter and 3 m. long. If it is full of petroleum, which is .7 as heavy as water, what is the weight of the petroleum?

**25.** Find the dimensions of a cylinder, having its diameter equal to its height, that will hold 1 liter.

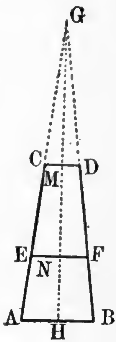
**26.** The volume of a cone is 1 cu m. What are its dimensions if its height is equal to the radius of its base?

**27.** If the air around the earth is 40 miles deep, and the diameter of the earth is taken as 7920 miles, how many cubic miles of air are there?

**28.** Find the radius of that sphere the number of square centimeters of whose surface is three times the number of cubic centimeters of its volume.

**29.** A hollow sphere is 32 cm. in diameter, and the shell 38 mm. thick. If the weight of the metal is 7.2 as heavy as water, what is the weight of the sphere? How much will it hold?

**30.** A trapezoidal board 12 feet long is 16 inches wide at one end, and 8 inches at the other. How far from either end must it be cut so that each part may contain one half of it?




Since the board, represented in the figure by ABCD, decreases 8 in. in 12 ft., its non-parallel sides will meet in a point in 24 ft., if imagined to be produced as indicated by the dotted lines.

$\therefore$ area of ABG $= \frac{1}{2} \times 24 \times 1\frac{1}{3} \times 1$ sq. ft. $= 16$ sq. ft.

Area of board ABCD $= 12$ sq. ft. Why?

$\therefore$ area of EFG $= 16$ sq. ft. $- 6$ sq. ft. $= 10$ sq. ft.

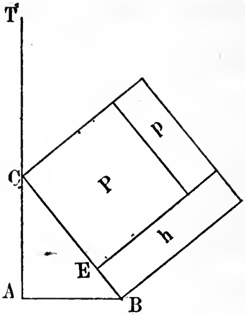
From similar triangles, $16 : 10 = 24^2 : GN^2$.

Hence $GN = \sqrt{360} = 18.97+$.

$\therefore$ 18.97 ft. $-$ 12 ft. $=$ 6.97 ft., MN, the distance from the narrow end.

**31.** A pole 120 ft. long breaks so that the top touches the ground 40 ft. from the foot. What is the height of the stump?

Let AT represent the pole, C the place it breaks, and B the point where T touches the ground.




AT = 120 ft., AB = 40 ft.

Construct the squares on BC and CE, CE being taken equal to AC.

Since P is equal to the square on AC, it is evident that the square on the hypotenuse exceeds that on the perpendicular by the two rectangles *p* and *h*, whose combined length is 120 ft., and whose area is equal to the area of the square on AB, or 1600 sq. ft.

$1600 \div 120 = 13\frac{1}{3}$. ∴ the common width of the rectangles *p* and *h* is $13\frac{1}{3}$ ft. But this width is the *difference* in the lengths of AC and BC. Their sum = 120 ft. Hence, $\frac{1}{2}$ (120 ft. + $13\frac{1}{3}$ ft.), or $66\frac{2}{3}$ ft., = BC ; and $\frac{1}{2}$ (120 ft. − $13\frac{1}{3}$ ft.), or $53\frac{1}{3}$ ft., = AC.

**32.** A limekiln measured at the bottom 50 ft. long and 20 ft. wide ; at the top 40 ft. long and 16 ft. wide. The height was 6 ft. How many cubic feet of lime in the kiln ?

The limekiln may be regarded as the frustum of a pyramid. The volume of the frustum of a pyramid (or cone) of bases $B$, $b$, and altitude $h$, as shown in geometry, is expressed by the formula

$$V = \frac{h}{3}(B + b + \sqrt{Bb}).$$

The area of the lower base = 20 × 50 sq. ft. = 1000 sq. ft.

The area of the upper base = 16 × 40 sq. ft. = 640 sq. ft.

Hence the volume = $\frac{6}{3}(1000 + 640 + \sqrt{1000 \times 640})$ cu. ft. = 4880 cu. ft.

## GREATEST COMMON DIVISOR.

**559.** The **Greatest Common Divisor** of two or more numbers is the *greatest number* that exactly divides each of them.

Thus, 7 is the greatest common divisor of 21, 35, and 42. Why ?

**1.** Find the G. C. D. of 56, 84, and 140 by the *method of factoring*.

$56 = 2^2 \times 2 \times 7.$
$84 = 2^2 \times 3 \times 7.$
$140 = 2^2 \times 5 \times 7.$

Since the greatest common divisor is the greatest factor common to the three numbers, it is $2^2 \times 7$, or 28.

**2.** Find, by factoring, the G. C. D. of :
(*a*). 32, 48, 128. (*b*). 84, 126, 128. (*c*). 187, 253, 341.

**560.** To find the greatest common divisor when the numbers cannot be easily factored, the *long division* process (also called the "Euclidean method," from Euclid, who first used it) is usually employed. This method depends upon two principles :

1. *A factor of a number is a factor of any of its multiples.*

2. *Every common factor of two numbers is also a factor of their sum and of their difference.*

Thus, 7, which is a factor of 14, is also a factor of 28, 35, etc. ; and, being a common factor of 35 and 126, it is also a factor of 161, their *sum*, and of 91, their *difference.*

1. Find the G. C. D. of 63 and 231 by the method of *continued division.*

```
63 )231( 3
    189
    ---
     42 )63( 1
         42
         --
         21 )42( 2
             42
             --
```

Since 21 is a factor of itself and of 42, it is a factor of 63, their *sum.*

Since 21 is a factor of 63, it is a factor of 3 × 63, or 189, a *multiple* of 63 ; and being a factor of 42 and of 189, it is a factor of 42 + 189, or 231, their *sum.*

∴ 21 is a common factor of 63 and 231.

Again, every common factor of 63 and 231 is a factor of 3 × 63, or 189, a *multiple* of 63 ; and also a factor of 231 − 189, or 42, their *difference.*

Since every such factor is now a common factor of 63 and 42, it is a factor of 63 − 42, or 21, their *difference.*

Since the greatest common factor of 63 and 231 is contained in 21, it cannot be greater than 21.

∴ 21 is the G. C. D. of 63 and 231.

Find by the last method the G. C. D. of :

(*a*). 9801 and 33759. (*b*). 3864, 3404, and 3657. (*c*). 4656 and 5926.

To find the G. C. D. of several numbers by this method, find the G. C. D. of two of them ; then of that result and a third number, and so on.

## COMPOUND PROPORTION.

**561.** The product of two or more ratios is called a **Compound Ratio.**

Thus, $3 \times 5 : 4 \times 7$, or $15 : 28$, is the compound ratio of $3 : 4$ and $5 : 7$.

**562.** Any equation each member of which is composed of two factors may be written as a proportion.

Take the simple proportion $5 : 8 = 15 : 24$. Solving, we have, $5 \times 24 = 8 \times 15$. It will be observed that one member contains the *extremes* of the proportion, the other the *means*. The equation $ab = cd$ may be written,

$$a : c = d : b,$$
$$\text{or, } a : d = c : b.$$

The compound ratio of $a : c$ and $a : d$ is $a \times a : c \times d$; that of $d : b$ and $c : b$ is $d \times c : b \times b$. And since the simple ratios of each proportion are equal, their products are equal. Hence

$$a \times a : c \times d = d \times c : b \times b.$$

That is, one compound ratio is equal to the other.

**563.** An expression of the equality of two compound ratios, or of a compound ratio and a simple one, is called a **Compound Proportion.**

Thus, the equation $\frac{5}{6} \times \frac{3}{7} = \frac{10}{28}$ may be written in the form of the compound proportion

$$\begin{matrix} 5 : 6 \\ 3 : 7 \end{matrix} = 10 : 28$$

By *taking the product* the proportion is reduced to the simple one $15 : 42 = 10 : 28$.

NOTE.—In a compound proportion the product of the extremes is equal to the product of the means, as in the case of a simple proportion; hence the required number is found in the same way. In problems of this class it is convenient to make the number which is of the same kind as the answer the third one, and then to consider each of the remaining *pairs* of numbers *separately*, forming a first couplet from each, as in simple proportion.

WRITTEN EXERCISES.

**564. 1.** If 8 horses eat 48 bushels of oats in 24 days, in how many days will 4 horses eat 38 bushels?

$$\begin{matrix} 4 : 8 \\ 48 : 38 \end{matrix} = 24 \text{ days} : (\quad).$$

$$\therefore \text{ the required time is } \frac{8 \times 38 \times 24 \text{ da.}}{4 \times 48} = 38 \text{ days.}$$

The problem is to determine the ratio resulting from each comparison, and how they affect the number of days which we are required to find. For convenience we make the 24 days the third number, as the answer is to be in days.

It will require *more days* for 4 horses to eat a given quantity than for 8 horses to eat the same amount. Therefore we make 4 the first number and 8 the second.

It will require *less time* for the same number of horses to eat 38 bu. than to eat 48 bu. Therefore, we make 48 the first number and 38 the second. In finding the fourth number, how may the work be simplified?

**2.** If 24 men in 5 days can build a wall 72 rd. long, how many rods of wall can 15 men build in 6 days?

**3.** If 10 men can cut 46 cords of wood in 18 days, working 10 hours a day, how many cords can 40 men cut in 24 days, working 9 hours a day?

**4.** If a railroad charges $15 for carrying 3 tons of goods 180 miles, what will it cost at the same rate to transport 15000 lb. of goods 140 miles?

**5.** If 36 men earn $1296 in 18 days, how much will 42 men earn in 87 days?

**6.** What is the weight of a block of granite 8 feet long, 4 feet wide, and 10 inches thick, if another block 10 feet long, 5 feet wide, and 16 inches thick weighs 5200 lb.?

**7.** A miller has a bin 9 ft. long, 4 ft. wide, and 2 ft. deep, holding 72 bushels of wheat. How long must he make another bin which is to be 5 ft. wide and 4 ft. deep in order that it may hold 192 bu. of wheat?

**8.** If the cost of digging a cellar 36 ft. long, 25 ft. wide, and 6 ft. deep is $90, what is the cost of digging a cellar 45 ft. long, 28 ft. wide, and 8 ft. deep?

**9.** If 120 men in 15 days can do $\frac{3}{4}$ of a certain piece of work, how many men in 30 days can do $\frac{9}{16}$ of the same work?

**10.** If 58 men working 9 hours a day require 6 days to dig a trench 100 yd. long, 2 yd. wide, and 3 yd. deep, how many men working 10 hours a day for 9 days will be required to dig a trench 50 yd. long, 6 yd. wide, and 5 yd. deep, in ground twice as hard to dig?

## INSURANCE.

**565. 1.** A house valued at $1000 was insured against fire for one year at 1%. What was the *premium*, or cost of insurance?

**2.** A building worth $10000 is insured for $\frac{1}{2}$ of its value at 1%. What is the premium?

**3.** If I pay $10 for having my property insured at 1%, for what amount do I get it insured?

**566. Insurance** is a contract by which one party agrees to indemnify another for loss sustained in the event of certain misfortunes.

The three most common forms are *Fire* insurance, *Accident* insurance, and *Life* insurance.

**567.** The written contract of insurance is called the **Policy.** It contains a promise to pay a specified sum in the event of certain contingencies. This sum is the *face* of the policy.

Fire insurance companies do not usually insure more than $\frac{3}{4}$ of the value of a property.

**568.** The sum paid for insurance is called the **Premium.** It is usually computed at a given sum for each $100 or $1000 of insurance; but sometimes at a certain *per cent* of the face of the policy.

WRITTEN EXERCISES.

**569. 1.** The owner of a store insures for \$16750, at 75¢ per \$100. How much is the annual premium?

**2.** What will it cost to insure a house for \$4800, at $1\frac{3}{4}\%$?

**3.** What will it cost to insure a mill worth \$18000 for $\frac{1}{2}$ of its value, at \$1.50 per \$100?

**4.** What is the premium for insuring property against loss by fire for 1 yr. for \$3500 at \$1.20 per \$100, and the contents for \$7500 at \$1.30 per \$100?

**5.** A house worth \$8000 was insured by 3 companies for $\frac{9}{10}$ of its value. The first took $\frac{1}{3}$ of the risk at $2\frac{1}{2}\%$, the second $\frac{1}{4}$ of the risk at 2%, and the third the remainder at $2\frac{1}{4}\%$. What was the total premium?

**6.** For how much must property worth \$21825 be insured at \$3 per \$100, to cover both property and premium?

**7.** I paid \$72 for insuring my house at 2%. What was the face of the policy?

**8.** I paid \$60.75 for insuring property worth \$2700. What was the rate of insurance?

**9.** A grain shipper paid \$525 for the insurance of a cargo of wheat at \$1.50 per \$100. For how much was the wheat insured?

**10.** If 88% of the value of a ship is insured at a cost of $\$271.33\frac{1}{3}$, at $\frac{5}{8}\%$ premium, what is the value of the ship?

**11.** A man 30 years old insures his life for \$2500, at the rate of \$22.50 for every \$1000 of insurance. What is the annual premium? How much does he pay in 20 years?

**12.** A man had his life insured for \$10000 at \$32.40 per \$1000. Should he die after having paid 18 premiums, how much more would his heirs receive than he had paid in premiums?

**13.** A man paid an insurance company for 30 years an annual premium on a life policy for \$5000 at the rate of \$22.85 per \$1000. If 15% of this premium was returned in dividends, how much did he pay for his insurance?

## EXCHANGE.

**570.** The system of making payment of debts at distant places without the transmission of money is called **Exchange.**

**571.** If John Doe, of Baltimore, owes Richard Roe, of Rochester, $500, he can cancel the debt in several ways without actually sending the money. He can send a *check*, a *draft*, a *postal money order*, or an *express money order*.

If Doe draws a check for the $500 payable to the order of Richard Roe, he sends it to the latter, who indorses it and has it cashed. The bank receiving it collects the sum named from the bank upon which it is drawn.

**572.** A **Bank Draft** is a written order from one bank directing another bank to pay a specified sum of money to the order of the person named in the draft.

The following is a common form of draft:

> **Merchants' National Bank.**
>
> *Baltimore, June 15, 1900.*
>
> *Pay to the order of Richard Roe* ~~~~ *$500.00.*
>
> ~~~~~~~~ *Five Hundred* ~~~~~~~~ *Dollars.*
>
> *B. H. Walker, Cashier.*
>
> *To the Astor Bank,*
> *New York City.*

John Doe may discharge his indebtedness to Roe by paying the money to his Baltimore bank, which in that case delivers to him a draft for the $500 on a New York bank, payable to his own order, or to the order of Richard Roe. This New York draft Doe sends to Roe, who indorses it and has it cashed as he did the check.

For its service the Baltimore bank may charge Doe a small sum (cost of exchange), probably 25¢ or 50¢, or it may be $\frac{1}{10}$ of 1% on the face of the draft. Many banks charge their patrons nothing for New York exchange.

**573.** The **cost of the exchange** is either a merely nominal sum to cover expenses, or a certain per cent of the face of the draft, usually $\frac{1}{10}$%.

**574.** If the cost of the draft is greater than the face, exchange is said to be at a **premium**; if less than the face, it is said to be at a **discount.**

If the banks of any city, say New Orleans, have not sufficient funds on deposit in New York to meet the drafts they are making on that city, they must incur the expense of sending the money to meet these drafts. This raises the cost of drafts in New Orleans, and exchange on New York is at a premium. But if the New Orleans bankers have an abundance of funds standing to their credit in New York, they sell drafts on that city at a discount in order to get money for use at home without incurring the expense and risk of having it forwarded by express.

**575.** A **Commercial Draft** is a written order from one person to another, directing him to pay a stated sum of money to the order of the bank named in the draft. Commercial drafts are extensively used by creditors to demand payment and collect debts through banks.

The following is a common form:

*$500.* *Rochester, N. Y., July 15, 1900.*

*At sight* *Pay to the*

*order of* *The First National Bank of Rochester*

*Five Hundred* $\frac{00}{100}$ *Dollars.*

*Richard Roe.*

*To John Doe,*
*98 Wood St., Baltimore, Md.*

**576.** If a draft is made payable on its presentation, it is called a *sight draft;* if payable at a specified time after sight or after date, it is called a *time draft.*

In many states 3 *days of grace* are allowed on time drafts, and in some states grace is also allowed on sight drafts.

1. If John Doe owes Richard Roe the $500 and is slow in paying it, the latter may make out a draft as above and deposit it for collection. The Rochester bank will then send it to some Baltimore bank, with a request to collect and remit. This is called "drawing on" a debtor.

2. The Baltimore bank will present it to John Doe and demand payment. If a sight draft, he may pay it on presentation. If a time draft (or a sight draft where grace is allowed on same), he may write the word "accepted," the date, and his name across the face. This is called "accepting the draft." He is then responsible for its payment, but is not liable unless and until he "accepts." At the proper time it will again be presented by the bank and payment demanded.

3. If he declines to accept it, or to pay it on presentation, it is returned to the Rochester bank and Roe is notified. If Doe pays the draft, the Baltimore bank remits to the Rochester bank, deducting a small sum (cost of exchange) for making the collection.

4. An "accepted" draft is in effect a note whose date is the date of acceptance if payable so many days after date; otherwise the date of the draft is the date of the note.

**577.** The **Postal Money Order** is an order drawn by one postmaster on another, directing him to pay a specified sum of money to the person named therein, or to his order. The fees charged (cost of exchange) vary from 3¢ to 30¢, according to the amount.

**578.** The **Express Money Order** is substantially like the postal money order. The fees are the same, except that on all orders not over $5 the fee is 5¢.

**579.** **Foreign Exchange** is subject to the same general laws as exchange between different cities of this country—**domestic exchange**—differing chiefly as to currency and the manner of making quotations.

**580.** Foreign drafts are usually called **Bills of Exchange,**

and are now generally drawn in *duplicate*, formerly in *sets of three*. These are sent by different mails to avoid loss or delay. When one is accepted or paid, the others are void.

The following is the usual form :

> £500. New York, May 1, 1901.
>
> *At sight of this First of Exchange (Second of same tenor and date unpaid) pay to the order of* Richard Roe, Five Hundred Pounds, *value received, and charge same to account of*
>
> J. E. Morgan & Co.
>
> To Baring Brothers,
> London, England.

**581.** Exchange for sight drafts was quoted in New York, on June 30, 1900, as follows :

On London, at \$4.865 for 1 pound sterling, meaning that £1 in gold was worth \$4.865 in gold, the exchange being quoted as so many dollars to the pound.

On Paris, at 5.18 francs for \$1, meaning that \$1 would buy a draft for 5.18 francs. It is sometimes quoted as so many cents to the franc.

On German cities, at 4 reichsmarks for \$.945, meaning that \$.945 will buy a draft for 4 marks. It is sometimes quoted as so many cents to the mark.

WRITTEN EXERCISES.

**582. 1.** If exchange is at a premium of $\frac{1}{2}\%$, and the bank's charge is $\frac{1}{10}\%$, find the cost of a New York draft for \$300.

The exchange $= \frac{1}{2}\%$ of \$300 = \$1.50.
The charge $= \frac{1}{10}\%$ of \$300 = .30.
Total cost = \$300 + \$1.50 + \$.30 = \$301.80.

**2.** When exchange was at a discount of $\frac{3}{8}\%$ I bought a draft for \$40, the bank's charge being 10¢. How much did the draft cost me?

**3.** Find the cost in New York of a sight draft on London for £25 8s.

$$£25\ 8s. = £25.4.$$
$$25.4 \times \$4.865 = \$123.57.$$

**4.** When New York exchange is at $\frac{1}{4}\%$ premium, what is the cost of a draft for \$600?

**5.** Find the cost of a \$200 draft at 30¢ a \$1000 discount, the charge for issuing it being 15¢.

**6.** At 50¢ a \$1000 premium, what is the cost in St. Louis of a draft on Boston for \$540, if the western bank charges $\frac{1}{10}\%$ for issuing?

**7.** When exchange was $\frac{1}{8}\%$ discount I bought a New York draft for \$1200, paying a local charge of $\frac{1}{10}\%$. Find the cost of the draft.

**8.** When a New York draft for \$10000 can be bought in Chicago for \$9800, is exchange at a premium or at a discount? What is the rate of exchange? The bank of which city, then, has a large balance to its credit in the other city?

**9.** What is the cost of a sight draft, or bill of exchange, on London for £300, exchange \$4.90? Is exchange selling at a premium or at a discount in this case?

**10.** What is the cost of a sight draft on Paris for 1000 francs, exchange $5.18\frac{1}{8}$?

**11.** What will be the cost of a bill of exchange on Berlin for 1200 marks, the rate of exchange being \$.945 for 4 marks?

**12.** If a New York merchant owes \$2500 to a dealer in London, and remits by draft, what is the face of the draft, if the rate of exchange is \$4.87?

**13.** Harry B. Naylor, of Pittsburg, draws on W. A. Saunders, of Toledo, at 30 days after sight for \$320. The latter accepts July 3. Write the draft and acceptance.

## AVERAGE OF PAYMENTS.

**583. 1.** For what time is the use of \$1 worth as much as the use of \$2 for 1 month ?

**2.** For what time is the use of \$10 worth as much as the use of \$5 for 4 months ?

**3.** If one half of a debt is paid 1 month before maturity, when may the other half be paid without loss to either party ?

**4.** If A uses \$100 of B's money for 2 months, how long may B use \$200 of A's money to balance the favor ?

**5.** A owes B \$100 due in 2 months, and \$200 due in 3 months. If the first debt were paid in 1 month, who would gain ? How much ? If the payment of the second debt were deferred 1 month after maturity, who would lose ? How much ? Then at what time may both debts be paid by a single payment without gain or loss to either party ?

### WRITTEN EXERCISES.

**584. 1.** A owes B \$300 due in 3 months, \$400 due in 4 months, and \$500 due in 7 months. In how many months can he pay the whole indebtedness at one time so that neither party shall lose ?

The use of \$300 for 3 mo. = the use for 1 mo. of \$900.
The use of \$400 for 4 mo. = the use for 1 mo. of \$1600.
The use of \$500 for 7 mo. = the use for 1 mo. of \$3500.

The use of \$1200 for $x$ mo. = the use for 1 mo. of \$6000.

$\therefore x = \$6000 \div \$1200$, or 5, the number of months.

**2.** \$800 of a debt is due in 6 months, and \$500 of it is due in 8 months. What is the average term of credit ?

**3.** Find the average (or equated) time for the payment of \$2000 due in 3 mo., \$1500 due in 4 mo., and \$2500 due in 8 mo.

**4.** Find the average time for the payment of \$300 due in 30 days, \$500 due in 60 days, and \$200 due in 90 days.

**5.** On Dec. 1, 1900, a merchant bought goods as follows: \$350 on 2 mo., \$500 on 3 mo., \$700 on 6 mo. He gave one note in payment. At what date should the note be made payable?

**6.** Find the average time for the payment of \$1000 due May 31, \$1500 due June 18, and \$2000 due July 9, reckoning the time from May 31.

**7.** \$3000 is due in 8 months. If \$1200 is paid in 5 months, and \$900 in 6 months, how long after maturity should the balance be paid?

**8.** Find the average time of payment on the following debts: Mar. 12, 1901, \$1500 due in 3 months; Apr. 16, 1901, \$1000 due in 2 months; May 19, 1901, \$1250 due in 4 months.

The earliest date at which any debt is due is June 12. The \$1000 is due 4 days after, and the \$1250 is due 99 days later.

The use of \$1500 for 0 da. =
The use of \$1000 for 4 da. = the use for 1 da. of \$4000.
The use of \$1250 for 99 da. = the use for 1 da. of \$123750.

$\therefore$ the use of \$3750 for $x$ da. = the use for 1 da. of \$127750.

$\therefore x = \$127750 \div \$3750$, or 34+.

June 12, 1901, + 34 days = July 16, 1901.

**9.** Find the average time of payment of the following debts: May 5, 1902, \$1250 due in 30 days; May 15, 1902, \$900 due in 90 days; May 25, 1902, \$1150 due in 60 days.

**10.** A man, Feb. 11, 1900, gave a note for \$850 payable in 4 mo.; but he paid Mar. 22, \$200; Apr. 20, \$110; May 10, \$150. When was the balance due?

## CASTING OUT NINES.

**585.** Every power of 10 is one more than some *multiple* of 9.

Thus, $10 = 9 + 1$; $10^2 = 11 \times 9 + 1$; $10^3 = 111 \times 9 + 1$, etc.

**586.** Every product of a power of 10 by a number of one digit is therefore some multiple of 9, plus the number represented by that digit.

Thus, $40 = 4 \times 9 + 4$; $500 = 55 \times 9 + 5$; $7000 = 777 \times 9 + 7$, etc.

**587.** As every number greater than 9 consists of the sum of such products, it follows that every such number is a multiple of 9, plus the sum of the numbers represented by its digits.

$$\begin{aligned} 7000 &= 777 \times 9 + 7 \\ 600 &= 66 \times 9 + 6 \\ 50 &= 5 \times 9 + 5 \\ 4 &= 4 \\ \hline 7654 &= 848 \times 9 + (7 + 6 + 5 + 4) \end{aligned}$$

$7654 = 850$ nines $+ 4$. $(7 + 6 + 5 + 4) = 2$ nines $+ 4$.

It is thus seen that *the excess of nines in any number equals the excess of nines in the sum of the numbers represented by its digits.* This principle may be applied to test the accuracy of the work in the simple processes of arithmetic.

**1.** Multiply 857 by 68.

| | | | |
|---|---|---|---|
| 857 | ....$8 + 5 + 7 = 2$ nines $+ 2$ | | |
| 68 | ....$6 + 8 = 1$ nine $+ 5$ | | |
| 6856 | | $10 = 1$ nine $+ 1$ | equal excesses. |
| 5142 | | | |
| 58276 | ....$5 + 8 + 2 + 7 + 6$ | $= 3$ nines $+ 1$ | |

The excess of nines in the product of the *numbers* equals the excess in the product of the *excesses in the factors.* Therefore, the work is correct, unless it contains errors that balance, which is quite improbable.

**2.** Divide 46718 by 263.

$46718 \div 263 = 177$, with remainder 167.

The excess of nines

In the dividend (46718) is. ....................8
In the divisor (263) is ...... 2
In the quotient (177) is..... 6
In the product............ (12) is.... 3
In the remainder (167) is..... ....... 5
In the sum..........................(8) is ... 8

Since the dividend equals the product of the quotient and divisor, plus the remainder, the excess of nines in the dividend = the excess in the sum of the excess in the *product* of the excesses of divisor and quotient, and the excess in the *remainder*. Therefore, the work may be assumed to be correct.

**3.** Find, by casting out the nines, which of the following products are *incorrect* :

(a). $7777 \times 864 = 6,712,328$.
(b). $67853 \times 2976 = 201,930,028$.
(c). $3769 \times 235 = 885,715$.

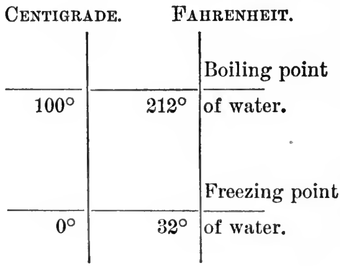
**4.** Find, by casting out the nines, which of the following quotients are *correct* :

(a). $1,348,708 \div 498 = 2708$, with remainder 129.
(b). $87614 \div 563 = 155$, with remainder 349.
(c). $4000 \div 23 = 173$, with remainder 18.

## MEASURES OF TEMPERATURE.

**588.** Temperature is measured by an instrument called a *Thermometer*.

There are three scales for measuring temperature by means of the thermometer.




The *Fahrenheit*, used in this country in ordinary business, has the freezing point of water marked 32°, and the boiling point 212°.

The *Centigrade*, generally used in science, has the freezing point 0°, and the boiling point 100°.

The *Réaumur*, which is also frequently used, has the freezing point 0°, and the boiling point 80°.

Degrees below 0° are indicated by the sign —. Thus,—20 means 20° below zero.

WRITTEN EXERCISES.

**589.** **1.** 80° Fahrenheit corresponds to what temperature Centigrade?

$212°$ F. $-$ $32°$ F., or $180°$ F. $= 100°$ C.

$1°$ F. $= \frac{100}{180}°$ C. $= \frac{5}{9}°$ C.

$80°$ F. $= 80°$ F. $- 32°$ F., or $48°$ F. above freezing.

$48 \times \frac{5}{9}° = 26.67°$.

That is, $80°$ F. $= 26.67°$ C.

**2.** 60° C. corresponds to what temperature F.?

$100°$ C. $= 180°$ F.

$60°$ C. $= \frac{60}{100}$ of $180°$ F., or $108°$ F. above freezing.

That is, $60°$ C. $= 108°$ F. $+ 32°$ F., or $140°$ F.

**3.** 80° C. corresponds to what temperature R.?

$100°$ C. $= 80°$ R.

$80°$ C. $= \frac{80}{100}$ of $80°$ R., or $64°$ R.

**4.** Express 30° F. in Centigrade scale; in Réaumur's scale.

**5.** Express −35° F. in Centigrade scale; in Réaumur's scale.

**6.** Express − 40° C. in Fahrenheit's scale; in Réaumur's scale.

**7.** Express − 33° C. in Fahrenheit's scale; in Réaumur's scale.

**8.** The temperature of a room is 63° F. Find the temperature in C. In R.

**9.** Express in Centigrade scale the following melting points: (*a*) lead, 630° F.; (*b*) ice, 32° F.; (*c*) silver, 873° F.; (*d*) tin, 455° F.

**10.** Express in Fahrenheit scale the following boiling points: (*a*) alcohol, 78° C.; (*b*) ether, 35° C.; (*c*) mercury, 357° C.

## SPECIFIC GRAVITY.

**590.** The **Specific Gravity** of any substance is the *ratio* of its weight to the weight of an equal bulk of water.

Thus, a cubic foot of zinc weighs 7000 oz., and a cubic foot of water 1000 oz. The ratio of 7000 oz. to 1000 oz. is 7; hence the specific gravity of zinc is 7. That is, zinc is 7 times as heavy as water.

TABLE OF SPECIFIC GRAVITY.

| | | | | | |
|---|---|---|---|---|---|
| Copper | 8.9 | Nickel | 8.9 | Cork | .24 |
| Gold | 19.3 | Silver | 10.5 | Granite | 2.7 |
| Lead | 11.3 | Sulphur | 2.0 | Steel | 7.8 |
| Alcohol | .79 | Petroleum | .7 | Mercury | 13.596 |

**591.** If a substance is in water, the water buoys it up by just the weight of the water displaced by it. That is, it loses a portion of its weight just equal to the weight of the water displaced.

WRITTEN EXERCISES.

**592. 1.** If a cubic foot of iron weighs 487.5 lb., and an equal volume of water weighs 62.5 lb., what is the specific gravity of the iron?

487.5 lb. ÷ 62.5 lb. = 7.8, the specific gravity.

**2.** What is the weight of a cubic inch of silver?

1 cubic foot of water weighs..............1000 oz.
1 cubic inch of water weighs ..............$\frac{1000}{1728}$ oz.
∴ 1 cubic inch of silver weighs.......10.5 × $\frac{1000}{1728}$ oz. = 6+ oz.

**3.** If a body weighs 3.71 Kg. in air and 2.38 Kg. in water, what is its specific gravity?

3.71 Kg. − 2.38 Kg. = 1.33 Kg. Since the body weighs 1.33 Kg. less in water than in air, 1.33 Kg. is the weight of the water displaced by it.
3.71 Kg. ÷ 1.33 Kg. = 2.8 *nearly*, the specific gravity of the body.

**4.** What does a bar of aluminum 113 mm. long, 17 mm. wide, and 13 mm. thick weigh if its specific gravity is 2.57?

**5.** If a bar of iron 18 in. long, $2\frac{1}{3}$ in. wide, $1\frac{3}{4}$ in. thick, weighs 18 lb. 9 oz., what is the specific gravity of the iron?

**6.** How many pounds does a man lift in raising a cubic foot of stone under water if its specific gravity is 2.5?

**7.** If the specific gravity of gold is 19.3, find the number of cubic inches of gold to the pound.

**8.** How many cubic feet of sea water weigh a ton, if its specific gravity is 1.026?

**9.** The specific gravity of ice is .92, of sea water 1.025. To what depth will a cubic foot of ice sink in sea water?

# INTRODUCTION TO ALGEBRA.

**593.** In passing from arithmetic to algebra the meaning of number and the method of representing it are extended, but there is nothing contradictory to what has been already learned in arithmetic. Algebra may be regarded as but a more comprehensive arithmetic.

The symbols, 1, 2, 3, etc., are retained in algebra with their arithmetical meanings, and the same symbols, $+$, $-$, $\times$, $\div$, ( ), $=$, are used in each. Fractions, powers, and roots have the same meaning and are written in the same form.

## Literal or General Number.

**594.** An important difference between arithmetic and algebra comes from the *frequent* and *extended* use in the latter of *letters* to represent *numbers*. Just as, in interest problems, $p$ may stand for principal, $r$ for rate per cent, $t$ for the time, $i$ for interest, and $a$ for amount, so in any case such symbols as $a$, $b$, $x$, $y$ may be used to represent *any numbers whatever*.

In arithmetic we speak of 5 books, meaning a *certain number* of books, or of \$10, meaning a *certain number* of dollars; in algebra we speak of $n$ books, meaning *any number* or an *unknown number* of books, of $x$ dollars, meaning *any number* or an *unknown number* of dollars.

**595.** Numbers represented by letters are called *Literal* or *General Numbers*. The reasoning is the same whether numbers are represented by letters or by figures.

Thus, if $a$ stands for a certain number, say the number of pupils in a room, then $2a$ stands for twice this number, $3a$ for three times this number, etc.

**1.** If $n$ stands for the number of books in my library, what is the meaning of $3n$? Of $5n$? Of $6n$? Of $\frac{1}{2}n$?

**2.** If $x$, $y$, and $z$ stand for the cost of a horse, a cow, and a sheep respectively, for what does $x + y$ stand? $x + y + z$? $2x + 3y + z$?

**3.** If $x = 5$, $y = 9$, and $z = 7$, what is the value of $x + y$? Of $x + y - z$?

**4.** If in a number of two digits, the digit in the ones' place is 5, and the digit in the tens' place is 2, the number is $10 \times 2 + 5$. Write a number containing $a$ ones and $b$ tens.

## Positive and Negative Numbers.

**596.** Sometimes quantities that are measured by the same unit are of *opposite qualities*. Thus, assets and liabilities are both measured by the unit *dollar*, the readings of a thermometer above and below zero are given in *degrees*, and dates A.D. and dates B.C. are both given in *years*.

In the case of assets and liabilities, the unit *dollar* may be taken either as a dollar of assets or as a dollar of liabilities; if as a dollar of assets, then assets are regarded as *positive*; and liabilities, for the sake of distinction, as *negative*. In order to represent quantities that have *opposite qualities* we need to extend the idea of number as given in arithmetic so as to include numbers that count *negative units*.

**597.** The numbers arithmetic deals with are *greater than zero*, and are called *positive* numbers; but algebra treats also of numbers that in relation to positive numbers are regarded as *less than zero*, and these are known as *negative numbers*.

**598.** The primary notion of a negative number is that of one which, when taken with a positive number of the same kind, goes to diminish it, cancel it, or reverse it.

Thus, liabilities neutralize so much assets, thereby diminishing the net assets, canceling them, or leaving a net liability.

**599.** Negative numbers may be regarded as arising through the extension of the operation of subtraction to the case in which the minuend is less than the subtrahend, which, from an arithmetical point of view, is impossible. Note the following:

$6 - 4 = 2.$
$5 - 4 = 1.$
$4 - 4 = 0.$
$3 - 4 = {}^{-}1$; that is, a number one unit less than 0.
$2 - 4 = {}^{-}2$; that is, a number two units less than 0.

Observe that, as the minuend decreases by 1, 2, or more units, the subtrahend remaining the same, the remainder decreases by an equal number of units, becoming 0 when the minuend is equal to the subtrahend. If, then, the minuend becomes less than the subtrahend by 1, 2, or more units, the remainder must decrease by an equal number of units, and therefore become less than 0 by 1, 2, or more units. But the operation of subtracting a greater number from a less is *possible only when numbers less than zero are introduced.*

**600.** The negative remainder, ${}^{-}1$, does not mean that more units were taken from the minuend 3 than it contained; it merely shows that the subtrahend is 1 unit *greater than the minuend.*

**601.** The *absolute value* of a number is the number of units contained in it without regard to their *quality.* The numbers of arithmetic are the absolute values of the positive and negative numbers of algebra. Since letters are used to represent numbers which may have any values whatever, they can represent either positive or negative numbers.

**602.** In the expression $6 - 4$, the *minus sign* indicates that 4 is to be taken from 6. It is a symbol of *operation,* and does *not* show 4 to be a negative number. Both 6 and 4 are positive numbers.

But the same sign has another use, namely, to denote negative numbers; and the sign + is used to denote positive numbers. When so used these signs are symbols of *quality,* and do *not* indicate any operation whatever.

Thus, ${}^{-}4$ means a number four units less than 0, and ${}^{+}4$ a number four units greater than 0.

**603.** Positive and negative numbers are called *opposite* numbers, and may represent any quantities that are opposite in their relation to each other.

Thus, degrees *above* zero on a thermometer may be called positive; degrees *below* zero, negative; distance east, positive; distance west, negative; assets, positive; liabilities, negative.

**604.** Zero is neither a positive nor a negative number; it is the starting point from which positive and negative numbers are counted.

Thus, *opposite temperatures* are counted from zero on the thermometer, $-5°$ meaning 5 degrees *below* zero, and $+5°$ meaning 5 degrees *above* zero.

**605.** *Cash received* and *cash spent* are opposite quantities, and may be represented by positive and negative numbers.

Thus, \$50 received may be represented by +\$50.
\$50 spent " " " by −\$50.

| | | | |
|---|---|---|---|
| Monday, | received | \$25, or | +\$25 |
| " | spent | \$ 8, or | −\$ 8 |
| Tuesday, | spent | \$13, or | −\$13 |
| Wednesday, | received | \$ 9, or | +\$ 9 |
| " | spent | \$16, or | −\$16 |
| Thursday | " | \$ 7, or | −\$ 7 |
| " | received | \$10, or | +\$10 |
| Friday, | Cash on hand, | 0 | 0 |

A man's cash account might be kept as in the left-hand column, or as in the right. Friday he looks over his figures to see how much cash he ought to have on hand. He adds the sums received, which amount to \$44, and the sums expended, which amount to \$44. These cancel each other. That is, \$44 *received* united with \$44 *spent* is equal to neither cash on hand nor debt; or

\$44 received + \$44 spent = 0.

Or, he may add the positive numbers in the column to the right, getting +\$44, and the negative numbers, getting −\$44. This may be expressed algebraically thus:

+\$44 + −\$44 = 0.

**606.** We have already seen that zero is the *difference* between two equal numbers. From the preceding article we learn that it is also the *sum* of two *equal* and *opposite* numbers.

Thus, *gain* \$5 + *loss* \$5 = 0.
Or, in algebraic language, $^{+}\$5 + {}^{-}\$5 = 0$.

Carefully examine the following statements:

20 dollars *gain* + 20 dollars *loss* = 0.
$^{+}20 + {}^{-}20 = 0.$

This result means neither *gain* nor *loss*.

20 dollars *gain* + 15 dollars *loss* = 5 dollars *gain*.
$^{+}20 + {}^{-}15 = {}^{+}5.$

This result means a net *gain* of 5 dollars.

20 dollars *gain* + 30 dollars *loss* = 10 dollars *loss*.
$^{+}20 + {}^{-}30 = {}^{-}10.$

This result means a net *loss* of 10 dollars.

5 miles *east* + 5 miles *west* = the starting point.
$^{+}5 + {}^{-}5 = 0.$

This result means that the traveler has returned to the point from which he started.

10 miles *north* + 15 miles *south* = 5 miles south.
$^{+}10 + {}^{-}15 = {}^{-}5.$

This result means that the traveler stopped 5 miles south of his starting point.

Make similar statements for each of the following:

**1.** \$80 gain, \$50 loss.
**2.** \$75 gain, \$100 loss.
**3.** 40 miles east, 30 miles west.
**4.** A rise of 20° in temperature, then a fall of 18°.
**5.** An army balloon ascended 3000 feet, then fell 1800 feet.

## ADDITION.

**607.** In algebra the process of adding two or more positive or negative numbers is the same as that of adding in arithmetic, except that the *sign of quality* is to be prefixed to the sum.

Thus, $^{+}7$ added to $^{+}3 = {}^{+}10$; $^{-}7$ added to $^{-}3 = {}^{-}10$.

Add the following :

| | | |
|---|---|---|
| **1.** $^+9$ to $^+16$. | **4.** $^+8.4$ to $^+9.9$. | **7.** $^+8(a+b)$ to $^+3(a+b)$. |
| **2.** $^+84$ to $^+48$. | **5.** $^+8y$ to $^+6y$. | **8.** $^-7(m-n)$ to $^-6(m-n)$. |
| **3.** $^-72$ to $^-28$. | **6.** $^-12a$ to $^-5a$. | **9.** $^+5x$ and $^+x$ to $^+9x$. |

**608.** Terms containing the same letters with the same exponents are called **Similar Terms.**

Thus, $3y^2$ and $^-5y^2$ are similar terms, as are $7(x+y)$ and $2(x+y)$.

**609.** PRINCIPLE.—*As in arithmetic only like numbers can be added, so in algebra only similar algebraic numbers can be united by addition into one term.*

Although unlike numbers can not be united by addition into one term, an *indicated* operation is regarded as their algebraic sum. Thus, $m+n$ is called the sum of $m$ and $n$.

Add the following:

| **1.** | **2.** | **3.** | **4.** | **5.** |
|---|---|---|---|---|
| $^+4x$ | $^-x$ | $5ab$ | $^-4my$ | $(a+b)$ |
| $^+x$ | $^-3x$ | $6ab$ | $^-2my$ | $5(a+b)$ |
| $^+9x$ | $^-9x$ | $ab$ | $^-7my$ | $4(a+b)$ |

When numbers are positive, the symbol of quality (+) is usually omitted, as in examples 3 and 5 above. When no symbol is written, + is understood. The sign − is never omitted.

**610.** The following equations were considered in article 606 :

$$^+20 + {}^-20 = 0 \qquad (1).$$
$$^+20 + {}^-15 = {}^+5 \qquad (2).$$
$$^+20 + {}^-30 = {}^-10 \qquad (3).$$

In (1) we see that $^+20$ and $^-20$ cancel each other, that is, that their *sum* is 0.

In (2) we see that $^-15$ cancels $^+15$, leaving 5 positive units ($^+5$), which is the *sum.*

In (3) we observe that $^+20$ cancels $^-20$, leaving 10 negative units ($^-10$), which is the *sum.*

QUERIES.—How is the 5 in (2) obtained ? Why has it the + sign ? In

(3) how is the 10 obtained? Why has it the $-$ sign? Is $-2x$ the sum of $5x$ and $-7x$? Why?

Add the following:

| | | |
|---|---|---|
| **1.** $^{+}8$ to $^{-}20$. | **4.** $^{+}3x$ to $^{-}12x$. | **7.** $3x^2y$ to $^{-}11x^2y$. |
| **2.** $-7$ to $^{+}15$. | **5.** $18ab$ to $^{-}5ab$. | **8.** $8(m+n)$ to $^{-}7(m+n)$. |
| **3.** $25$ to $^{-}16$. | **6.** $-24y$ to $^{+}30y$. | **9.** $^{-}(x-y)$ to $10(x-y)$. |

**10.** Find the algebraic sum of $5x$, $^{-}7x$, $^{+}9x$, $^{-}4x$, and $x$.

1. The sum of the positive numbers is $15x$; the sum of the negative numbers is $-11x$. These two sums united $= 4x$.

2. In this example the 5, 7, 9, and 4 are *coefficients*. When no coefficient is expressed, 1 is understood.

Add the following:

| **11.** | **12.** | **13.** | **14.** |
|---|---|---|---|
| $3ax$ | $-9mn$ | $34bcd$ | $72\,(b-a)$ |
| $-5ax$ | $-mn$ | $-25bcd$ | $-48\,(b-a)$ |
| $^{+}7ax$ | $^{+}4mn$ | $-9bcd$ | $-50\,(b-a)$ |

Express in the simplest form:

**15.** $8x + 3x - 5x + x - 4x + 12x - 7x$.

**16.** $15bx - 6bx + bx - 9bx - bx + 18bx - 10bx$.

**17.** $5(m-n) + 13(m-n) - 11(m-n) + 6(m-n) - 20(m-n)$.

**18.** Add $3x + a - 2y$, $5x - 4a + 6y$, $7a - 8y$, and $y - 4x + 6a$.

$$\begin{array}{rrr} 3x\ + & a\ - & 2y \\ 5x\ - & 4a\ + & 6y \\ & 7a\ - & 8y \\ -4x\ + & 6a\ + & y \\ \hline 4x\ + & 10a\ - & 3y \end{array}$$

For convenience we write similar terms in the same column. The sum of the first column is $+4x$, of the second $+10a$, and of the third $-3y$.

**19.** Add $5y - 4c$, $3y + 8c$, $5c - 4y$, $y - c$, $y - 10c$.

**20.** Add $6m + 2n - 5b$, $7n - 3b - 4m$, $b + 8m - 9n$, $m + 5b$.

**21.** If a man has $a$ sons, $b$ daughters, and 1 wife, how many persons are in the family?

**22.** A boy who had 15 cents found $m$ cents and earned $4m$ cents. How much had he then?

**23.** My house is $d$ feet long and $c$ feet wide. What is the distance around it?

**24.** Tom walked due east $m$ hours, then due west $n$ hours. If his rate was 3 miles an hour, at what distance from his starting point did he stop?

**25.** If $m$ in the above problem is equal to $n$, where did Tom stop?

**26.** If $m = 5$ and $n = 3$, how far, and in what direction, from his starting point did he stop?

**27.** Locate his stopping place if $m = 4$ and $n = 6$.

**28.** D earns $a$ dollars each week and spends $b$ dollars. How much will he have at the end of 8 weeks?

**29.** What will be his financial condition if $a = \$15$ and $b = \$10$? What will it be if $a = \$12$ and $b = \$16$?

## SUBTRACTION.

**611.** In algebra, as in arithmetic, the *minuend* is the *sum* of the *subtrahend* and the *remainder*.

Thus, $10 + {}^-4 = 6$, the sum, which we may regard as a minuend. Taking 10 as the subtrahend, we have

$$6 - 10 = {}^-4, \text{ the remainder.}$$

Taking $^-4$ as the subtrahend, we have

$$6 - {}^-4 = 10, \text{ the remainder.}$$

In either case 6 is the sum of 10 and $^-4$.

Carefully examine the following:

| | | |
|---|---|---|
| $8 - 2 = 6.$ | $^-11 - 2 = {}^-13.$ | $1 - 2 = {}^-1.$ |
| $8 + {}^-2 = 6.$ | $^-11 + {}^-2 = {}^-13.$ | $1 + {}^-2 = {}^-1.$ |
| $8 = 6 + 2.$ | $^-11 = {}^-13 + {}^+2.$ | $1 = {}^-1 + 2.$ |

Observe that *subtracting* $+2$ *is equivalent to adding* $^-2$; also, that the minuend is the *sum* of the remainder and the subtrahend.

Examine these equations:

| | | |
|---|---|---|
| $7 - {}^{-}4 = 11.$ | $^{-}12 - {}^{-}4 = {}^{-}8.$ | $2 - {}^{-}4 = 6.$ |
| $7 + 4 = 11.$ | $^{-}12 + 4 = {}^{-}8.$ | $2 + 4 = 6.$ |
| $7 = 11 + {}^{-}4.$ | $^{-}12 = {}^{-}8 + {}^{-}4.$ | $2 = 6 + {}^{-}4.$ |

Observe that *subtracting* $^{-}4$ *is equivalent to adding* $^{+}4$; also, that the minuend = remainder + subtrahend.

**612.** In general, to *subtract* a *positive* number is equivalent to *adding* an *equal negative* number; to *subtract* a *negative* number is equivalent to *adding* an *equal positive* number.

ILLUSTRATION I.

A man whose income is \$100 a month spends \$60, and saves \$40. If his income is reduced \$10 a month, he will save \$30. \$90 − \$60 = \$30.

Or, if his expenses are increased \$10 a month, he will save \$30.

$$\$100 - \$70 = \$30.$$

Hence, to *take away* \$10 *income* is equivalent to *adding* \$10 *expenses*. Either reduces his savings to \$30.

Calling income and savings *positive*, and expenses *negative*, we have the following algebraic expression of this relation:

$$\$40 - {}^{+}\$10 = \$30.$$
$$\$40 + {}^{-}\$10 = \$30.$$

ILLUSTRATION II.

If his income is increased \$10 a month, he will save \$50.

$$\$110 - \$60 = \$50.$$

Or, if his expenses are reduced \$10 a month, he will save \$50.

$$\$100 - \$50 = \$50.$$

Hence, to *take away* \$10 *expenses* is equivalent to *adding* \$10 *income*. Either increases his savings to \$50. The relation is algebraically expressed thus:

$$\$40 - {}^{-}\$10 = \$50.$$
$$\$40 + {}^{+}\$10 = \$50.$$

**1.** A has \$10; B has no money and is \$5 in debt. How much more is A worth than B?

**2.** Tom has \$50 in bank. Harry has \$20 in cash, but owes John \$40. Tom is worth how much more than Harry?

**3.** Albert has 24 marbles and Ed has none, but owes Albert 16. How many more marbles than Ed has Albert?

**4.** Mary has 8 jacks and Alice has $^-5$ (i.e., owes 5). Alice has how many fewer than Mary?

**5.** In the schoolroom the temperature is 70° above zero, while outside it is 5° below zero. How many degrees warmer is it inside than outside? $75° - {}^-5° = (\quad)$.

**6.** A is \$35 in debt and B is \$50 in debt. How much better off is A than B? $^-\$35 - {}^-\$50 = (\quad)$.

**613.** In algebra when the subtrahend is greater than the minuend, the remainder is *negative*, and shows *how much* the subtrahend exceeds the minuend.

Thus, $8 - 11 = {}^-3$; $5x - 9x = {}^-4x$; ${}^-12y - {}^-5y = {}^-7y$.

Subtract the following:

| | | |
|---|---|---|
| **1.** 8 from 5. | **5.** $^-1$ from $^-11$. | **9.** $15a - 22a$. |
| **2.** 9 from 1. | **6.** $^-8$ from $^-5$. | **10.** $^-4y$ from $^-11y$. |
| **3.** 25 from 7. | **7.** $^-27$ from $^-2$. | **11.** $^-9x$ from $^-20x$. |
| **4.** $^-6$ from $^-10$. | **8.** $7x - 15x$. | **12.** $^-21b$ from $^-18b$. |

**614.** In arithmetic the remainder is never greater than the minuend; but in algebra it is often greater. Note the following:

| | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| Minuend..... | +31 | −19 | +12 | −25 |
| Subtrahend .. | +16 | +13 | −8 | −24 |
| Remainder | +15 | −32 | +20 | −1 |

In (a) and (b) the remainders are *less* than the minuends; in (c) and (d) they are *greater*. Which subtrahends are positive?

These examples illustrate the following:

1. When the subtrahend is *positive* the remainder is *less*

than the minuend. Subtracting a positive number is equivalent to adding an equal negative number.

2. When the subtrahend is *negative* the remainder is *greater* than the minuend. Subtracting a negative number is equivalent to adding an equal positive number.

**1.** From $12x$ subtract $5x$.

$$\begin{array}{r} 12x \\ \underline{5x} \\ 7x \end{array} \qquad \begin{array}{l} 12x - {}^{+}5x = 7x \text{ (subtracting a positive number);} \\ \text{or } 12x + {}^{-}5x = 7x \text{ (adding a negative number).} \end{array}$$

**2.** Subtract $18y$ from $15y$.

$$\begin{array}{r} 15y \\ \underline{18y} \\ {}^{-}3y \end{array} \qquad \begin{array}{l} 15y - {}^{+}18y = {}^{-}3y. \\ 15y + {}^{-}18y = {}^{-}3y. \end{array}$$

**3.** From $7a$ take ${}^{-}5a$.

$$\begin{array}{r} 7a \\ \underline{{}^{-}5a} \\ 12a \end{array} \qquad \begin{array}{l} 7a - {}^{-}5a = 12a. \\ 7a + \phantom{-}5a = 12a. \end{array}$$

**4.** From $4x - 3y$ take $x + 2y$.

$$\begin{array}{r} 4x - 3y \\ \underline{x + 2y} \\ 3x - 5y \end{array} \qquad \begin{array}{l} 4x - x = 3x, \text{ and } {}^{-}3y - {}^{+}2y = {}^{-}5y; \\ \text{or } {}^{-}3y + {}^{-}2y = {}^{-}5y. \end{array}$$

Notice that in each of these examples we have changed the *sign* of the *subtrahend* and then *added*.

Subtract:

**5.** $8x$ from $3x$.

**6.** $13y$ from ${}^{-}17y$.

**7.** ${}^{-}5a$ from ${}^{-}12a$.

**8.** ${}^{-}6b$ from $4b$.

**9.** $7x - 2y$ from $8x - 5y$.

**10.** $9ab + 3d$ from $12ab - d$.

**11.** $a - b$ from $a + b$.

**12.** $x - y + 0$ from $x + y + z$.

**13.** $3ax - 7y + 5b$ from $6ax - 4y - 3b$.

**14.** $4c(b - d) + 6am$ from $10c(b - d) + 6am$.

**15.** Find the value of $5a + 7b - (3a + 2b)$.

Both $3a$ and $2b$ are to be subtracted from the minuend. Hence, when the parenthesis is removed, the signs of both must be changed to –, and we have $5a + 7b - 3a - 2b$, or $2a + 5b$.

**16.** $12x + 8y - (9x - y) =$ what?

$$12x + 8y - 9x + y = 3x + 9y.$$

Complete the following equations:

**17.** $3y + 17x - (y + 16x) =$

**18.** $5(m + n) - 7xy - (m + n) + 4xy =$

**19.** $12(a + b + c) - 8(a + b + c) - 4b + 20c =$

**20.** By selling a cow for $a$ dollars I gained $b$ dollars. What did the cow cost?

**21.** A and B together have 10 children, of whom A has $n$. How many has B?

**22.** Smith has $b$ dollars and Jones is $c$ dollars in debt. How much more is Smith worth than Jones?

**23.** If I have 8 children and $m$ of them are girls, how many boys have I?

**24.** A man who earns $d$ dollars a month spends $p$ dollars for rent and $q$ dollars for other purposes. How much does he save in a month?

**25.** A boy wishing to ride $n$ miles on his wheel rode $a$ miles the first day and $b$ miles on each of the next two days. How many miles had he yet to ride?

**26.** A man engaged to fence a field $c$ rods long and $d$ rods wide. If he built $(c - d)$ rods a day for 3 days, how many rods had he yet to build?

**27.** A drover paid \$$m$ for $b$ pigs, and \$$n$ for $c$ sheep. He sold the former at \$8 and the latter at \$5 a head. Find his gain.

## MULTIPLICATION.

**615.** When the multiplier is *positive*, the process of multiplication in algebra is the same as that in arithmetic, except that the sign of quality of the multiplicand is to be written before the product. Thus,

$$(a).\quad {}^{+}3 \times {}^{+}5 = {}^{+}15.$$
$$(b).\quad {}^{+}3 \times {}^{-}5 = {}^{-}15.$$

Observe that three times 5 positive units gives 15 positive units as a product, and three times 5 negative units gives 15 negative units as a product.

**616.** When the multiplier is *negative,* it gives to the product a sign opposite to that given by a positive multiplier; that is, the quality of the multiplicand is reversed in the product. Note the following :·

$$(c).\quad {}^{-}3 \times {}^{+}5 = {}^{-}15.$$
$$(d).\quad {}^{-}3 \times {}^{-}5 = {}^{+}15.$$

Since $+3 + -3 = 0$, $-3 = 0 - +3$, or $-(+3)$ ; hence in (c), $-3 \times +5$ may be regarded as signifying that $+5$ is to be taken three times, and then the result *reversed* (subtracted), giving the product the sign opposite to that of the multiplicand.

Similarly in (d), $-3 \times -5$ may be regarded as signifying that $-5$ is to be taken three times, and the result reversed, that is, $-15$ is to be subtracted. Now, subtracting $-15$ is equivalent to adding $+15$. $\therefore -3 \times -5 = +15$.

Queries.—1. In (a) and (b) the product has the *sign* of the multiplicand. Is this true in (c) and (d) where the multiplier is negative?

2. When both factors are positive, as in (a), what is the sign of the product ? When both are negative, as in (d) ? When they have unlike signs, as in (b) and (c) ?

**617.** If $a$ and $b$ stand for any two numbers, we have

$$\begin{aligned} {}^{+}a \times {}^{+}b &= {}^{+}ab, \\ {}^{+}a \times {}^{-}b &= {}^{-}ab, \\ {}^{-}a \times {}^{+}b &= {}^{-}ab, \\ {}^{-}a \times {}^{-}b &= {}^{+}ab. \end{aligned}$$

That is, when two factors have like signs the product is *positive ;* when they have unlike signs the product is *negative.*

Illustrations.

A train whose speed is 20 miles an hour runs north and south past a point P, passing at 12 m. Locate the train at 5 P.M. and at 7 A.M.

Consider the following as *positive*: (1) distances north, (2) the train's rate northward, (3) time after 12 M. Consider the opposites *negative*. Then, if the train is running northward,

(a) in 5 hours after 12 M. it will be 100 miles north of P, which is expressed algebraically by

$$^{+}5 \times {}^{+}20 = {}^{+}100.$$

(b) 5 hours before 12 M. it will be 100 miles south of P, expressed algebraically by

$$^{-}5 \times {}^{+}20 = {}^{-}100.$$

If the train is running southward,

(c) in 5 hours after 12 M. it will be 100 miles south of P, expressed algebraically by

$$^{+}5 \times {}^{-}20 = {}^{-}100.$$

(d) 5 hours before 12 M. it will be 100 miles north of P, expressed algebraically by

$$^{-}5 \times {}^{-}20 = {}^{+}100.$$

Multiply the following:

**1.** $4a$ by 3.
**2.** $^{-}b$ by 7.
**3.** $^{-}5x$ by 2.
**4.** $^{-}6y$ by $^{-}d$.
**5.** $^{-}8m$ by $^{-}1$.
**6.** $9xy$ by $^{-}a$.
**7.** $^{-}ab$ by 10.
**8.** $^{-}c^2d$ by $^{-}5$.
**9.** $^{-}12$ by $ax$.
**10.** $^{-}1$ by $15p$.

**11.** What is the product of $a - b + 2x$ multiplied by $3a$?

$$\begin{array}{l} a - b + 2x \\ 3a \\ \hline 3a^2 - 3ab + 6ax \end{array}$$

Each term of the multiplicand is multiplied by $3a$. The algebraic sum of these products is the required product.

Multiplication is usually indicated by writing letters, or a figure and one or more letters, side by side.

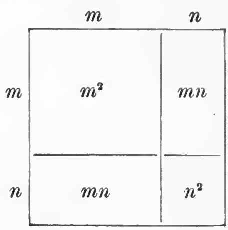
**12.** Find the product of $x^3 + 3x^2y - 3xy^2 - y^3$ multiplied by $x - y$.

$$\begin{array}{l}
x^3 + 3x^2y - 3xy^2 - y^3 \\
\underline{x \;-\; y \qquad\qquad\qquad\quad} \\
x^4 + 3x^3y - 3x^2y^2 - xy^3 = \text{product of multiplicand by } x. \\
\underline{\quad - x^3y - 3x^2y^2 + 3xy^3 + y^4} = \text{product of \quad `` \quad by } -y. \\
x^4 + 2x^3y - 6x^2y^2 + 2xy^3 + y^4 = \text{complete product.}
\end{array}$$

It is a convenient arrangement to write the multiplier under the multiplicand, and place like terms of the partial products in columns. Observe that in multiplying we take the product of the coefficients and the *sum of the exponents of the same letters.*

Multiply the following:

**13.** $a + b$ by $a + b$.
**14.** $a - b$ by $a - b$.
**15.** $a + b$ by $a - b$.
**16.** $x + 9$ by $x - 9$.
**17.** $m + 5$ by $m - 2$.
**18.** $3y + 7z$ by $4z + 5y$.
**19.** $8am + 6bn$ by $5ab - b$.
**20.** $x^2 + 2xy + y^2$ by $x + y$.
**21.** $(m - n)^2$ by $(m - n)$.
**22.** $a^2 + ab + bc - b^2$ by $a^2 + b^2$.
**23.** $r^2 - r + 1$ by $r^2 + r + 1$.
**24.** $m^3 + 3m^2n + mn^2$ by $m - n$.




**25.** How many square rods in a field $m + n$ rods square?

**26.** How many acres in a field whose length is $20m$ rods, and whose width is $16m$ rods?

**27.** A has $5b$ acres, B has $7c$ acres, and C has $12a$ acres. They offer to sell out to D at $3m$ dollars an acre. Find what D would have to pay.

**28.** A man having $2a$ horses sold $b$ of them at \$$r$ each, and the remainder at \$$2r$ each. If they cost him \$$3ar$, how much did he gain?

Find the value of the following:

**29.** $(a + b)^2$.
**30.** $(a - b)^2$.
**31.** $(c + 3)^2$.
**32.** $(y + 2)(y + 2)$.

**33.** $(x^2 + 7)(x^2 - 7)$.

**34.** $(c + 4d)(4d + c)$.

**35.** $(m - n)(m - n)$.

**36.** $(x + 4)(x + 5)$.

**37.** Show that *the square of the sum of two numbers* is equal to the square of the first number, plus twice the product of the two numbers, plus the square of the second number.

**38.** Square $a + b$ and $a - b$, and compare the results. *The square of the difference of two numbers* is equal to what?

**39.** Multiply $x + y$ by $x - y$, and note the product. *The product of the sum and difference of two numbers* is equal to the difference of what?

Write the products of the following:

**40.** $(c + d)(c + d)$.

**41.** $(c + d)(c - d)$.

**42.** $(m - n)(m - n)$.

**43.** $(3y - 5)(3y - 5)$.

**44.** $(2x + y)(2x - y)$.

**45.** $(2a + b)(2a + b)$.

**46.** What two equal factors produce $a^2 + 2ax + x^2$?

**47.** What two equal factors give the product $b^2 - 2by + y^2$?

**48.** What two factors produce $m^2 - n^2$? $x^2 - y^2$?

**49.** How many square feet in a room $a + b$ feet long and $a - b$ feet wide? How many square yards in the room, if $a = 15$ and $b = 12$?

**50.** Find the volume of a cube whose edge is $m - n$ inches.

## DIVISION.

**618.** Division is the inverse of multiplication. In the latter two factors are given, the product required. In the former the product (dividend) and one factor (divisor) are given, the other factor (quotient) required. Hence the law of signs may be derived from that in multiplication, as follows:

Since $^+3 \times {}^+5 = {}^+15$, $\therefore\ {}^+15 \div {}^+3 = {}^+5$.
Since $^+3 \times {}^-5 = {}^-15$, $\therefore\ {}^-15 \div {}^+3 = {}^-5$.
Since $^-3 \times {}^+5 = {}^-15$, $\therefore\ {}^-15 \div {}^-3 = {}^+5$.
Since $^-3 \times {}^-5 = {}^+15$, $\therefore\ {}^+15 \div {}^-3 = {}^-5$.

Or, using $a$ and $b$ to denote any two numbers, we have

$$
\begin{aligned}
{}^{+}a \times {}^{+}b &= {}^{+}ab, \quad \therefore\ {}^{+}ab \div {}^{+}a = {}^{+}b.\\
{}^{+}a \times {}^{-}b &= {}^{-}ab, \quad \therefore\ {}^{-}ab \div {}^{+}a = {}^{-}b.\\
{}^{-}a \times {}^{+}b &= {}^{-}ab, \quad \therefore\ {}^{-}ab \div {}^{-}a = {}^{+}b.\\
{}^{-}a \times {}^{-}b &= {}^{+}ab, \quad \therefore\ {}^{+}ab \div {}^{-}a = {}^{-}b.
\end{aligned}
$$

That is, *like* signs of dividend and divisor give a *positive* quotient; *unlike* signs, a *negative* quotient.

Divide the following:

**1.** $6ab$ by 2.

**2.** $9bx$ by $3b$.

**3.** $16c$ by $-4c$.

**4.** $-14y$ by 2.

**5.** $-12am$ by $-3m$.

**6.** $a^2b$ by $-ab$.

**7.** $-xy^2$ by $-xy$.

**8.** $-a^2b^2c$ by $abc$.

**9.** $20m^2$ by $-4m$.

**10.** $16a^2x^2y$ by $8axy$.

**11.** Divide $4a^2x - 6aby^2 + 2ac^2$ by $2a$.

$$
\begin{array}{r|l}
2a & \underline{4a^2x \;-\; 6aby^2 \;+\; 2ac^2} \\
 & 2ax \;-\; 3by^2 \;+\; c^2
\end{array}
$$

**12.** Divide $x^3 - 3x^2y + 3xy^2 - y^3$ by $x - y$.

$$
\begin{array}{rl|l}
 & x^3 - 3x^2y + 3xy^2 - y^3 & \underline{x - y} \\
x^2(x-y) = & \underline{x^3 - \;\; x^2y \qquad\qquad} & \underline{x^2 - 2xy + y^2}\text{, the quotient.} \\
 & \quad - 2x^2y + 3xy^2 & \\
-2xy(x-y) = & \underline{\quad - 2x^2y + 2xy^2 \qquad} & \\
 & \qquad\qquad xy^2 - y^3 & \\
y^2(x-y) = & \qquad\qquad \underline{xy^2 - y^3} &
\end{array}
$$

Divide the following:

**13.** $a^2 + 2ab + b^2$ by $a + b$.

**14.** $m^2 - 2mn + n^2$ by $m - n$.

**15.** $15b^2 - 8bc - 12c^2$ by $3b + 2c$.

**16.** $x^3 + 3x^2y + 3xy^2 + y^3$ by $x + y$.

**17.** $y^2 + 3y + 2$ by $y + 1$.

**18.** $a^2 - a - 90$ by $a + 9$.

**19.** $x^2 - y^2$ by $x - y$.

**20.** $9b^2 - 4a^2$ by $3b + 2a$.

**21.** $y^2 - 10yz - 24z^2$ by $y + 2z$.

**22.** $16x^2 - 24xy + 9y^2$ by $4x - 3y$.

**23.** There are $16bc + 24c$ square feet in a hall $8c$ feet wide. What is the length?

**24.** The area of a field $a + c$ rods wide is $a^2 + ab + 3ac + bc + 2c^2$ square rods. Find the length.

Divide the following by the highest factor common to all the terms:

**25.** $9a^2b - 18aby$.

**26.** $8bx^2 - 6ab^2x$.

**27.** $2cy^2 - 6c^2x + 4bc$.

**28.** $18m^2 - 9mn + 12mn^2$.

**29.** Divide $a^2 - b^2$ first by $a + b$ and then by $a - b$, and compare results. The difference of the squares of two numbers is divisible by what? Then what are the two factors of $a^2 - b^2$?

Write from inspection the factors of:

**30.** $x^2 - y^2$.

**31.** $x^2 - 9$.

**32.** $a^2 - 1$.

**33.** $4 - b^2$.

**34.** $x^2 - a^2y^2$.

**35.** $b^2c^2 - d^2$.

## EQUATIONS.

**619.** An equation has been defined as a statement that two numbers or expressions are equal.

The principles that apply to the transformation and solution of equations, as given in Arts. 256–262, should here be reviewed.

**620.** In solving simple integral equations the following direction will be found useful:

*Transpose all the terms containing the unknown number to the first member, and all other terms to the second member. Unite like terms, and divide both members by the coefficient of the unknown number.*

**621.** If the value found for the unknown number is substituted in the original equation, and the equation reduces to an *identity*, the *value* of the *unknown number* (called the *root of the equation*) is said to be *verified*.

Find the value of $x$, and verify the answer:

**1.** $5x = 28 - 2x$.
**2.** $-3x - 7 = -4x + 7$.
**3.** $4x - 2(2 - x) = 6$.
**4.** $4x - 14 = x - 2$.
**5.** $2x - (5x + 5) = 7$.
**6.** $ax = mx - n$.

**7.** Solve the equation $4 + \frac{x}{4} = \frac{x}{2} - 3 + \frac{x}{3}$.

$$4 + \frac{x}{4} = \frac{x}{2} - 3 + \frac{x}{3}.$$

Clearing of fractions, $48 + 3x = 6x - 36 + 4x$.
Uniting terms, $-7x = -84$.
Dividing by $-7$, $x = 12$.

An equation may be cleared of fractions by multiplying both members by the least common denominator of the fractions, which in this example is 12.

Solve the following:

**8.** $x + \frac{x}{4} = 15$.
**9.** $x + \frac{x}{5} = 18$.
**10.** $2a + \frac{a}{3} = 14$.
**11.** $2b + \frac{b}{4} = b + \frac{5b}{4}$.
**12.** $y + \frac{y}{2} + \frac{y}{3} = 11$.
**13.** $\frac{a}{3} + \frac{a}{4} + \frac{a}{6} = 18$.
**14.** $\frac{2x}{3} - \frac{x}{5} + \frac{x}{4} = 8 - \frac{x}{12}$.
**15.** $\frac{y+3}{4} - \frac{y-8}{5} = \frac{y-5}{2} = 1$.

**16.** Find the value of $x$ in the equation $x^2 - 4 = 5$.

$x^2 - 4 = 5$.
$x^2 = 5 + 4 = 9$.
$x = \pm 3$, by extracting the square root. (Art. 521.)

The sign $\pm$ before the 3, read *plus* or *minus*, shows that the root is either + or −. For $+3 \times +3 = 9$, and $-3 \times -3 = 9$. The negative value does not always have a meaning in particular problems.

Solve the following:

**17.** $5x^2 = 80$.
**18.** $1 - \frac{2}{5x^2} = x^2 - 4\frac{3}{5}$.
**19.** $(3 - x)^2 = 3(1 - x)^2$.
**20.** $3x^2 + 1 = 2x^2 + 10$.
**21.** $(x + 6)(x - 6) = 28$.
**22.** $ax^2 + b = bx^2 + a$.

## PROBLEMS.

**622.** In stating problems, it is important to remember that the letter $x$ should not be put for distance, time, weight, etc., but for the *number* of miles, of hours, of pounds, etc.

In connection with the stating and solving of problems, Arts. 261 and 262 should be re-read.

**1.** The sum of the two digits of a number is 4. If the digits are interchanged, the resulting number is equal to the original one. What is the number?

Let $x$ stand for the digit in the ones' place.

Then $4 - x$ is the digit in the tens' place.

$\therefore 10(4 - x) + x =$ the original number. Why?

$10x + (4 - x) =$ the second number.

Hence, $10x + (4 - x) = 10(4 - x) + x$, the equation of the problem.

Solving this equation, we obtain

$x = 2$, the digit in the ones' place;

whence $4 - x = 2$, the digit in the tens' place.

$\therefore$ the original number is $10(4 - x) + x$, or 22.

**2.** A son is one fourth as old as his father. Four years ago he was only one fifth as old as his father. What is the age of each?

Let $x =$ the *number* of years in the father's age.

Then $\frac{x}{4} =$ the number of years in the son's age.

$x - 4 =$ the number of years in the father's age 4 years ago.

$\frac{x}{4} - 4 =$ the number of years in the son's age 4 years ago.

$\therefore \frac{x}{4} - 4 = \frac{1}{5}(x - 4)$, whence

$x = 64$, and $\frac{x}{4} = 16$.

Therefore, the father is 54 years old and the son is 16 years old.

**3.** A certain street contains 144 square rods, and the length is 16 times the width. Find the width.

Let $x =$ the number of rods in the width of the street.
Then $16x =$ the number of rods in the length of the street.
$x \times 16x =$ the area of the street in square rods.

$$16x^2 = 144$$
$$x^2 = 9$$
$$x = \pm 3$$

Therefore, the width of the street is 3 rods. The negative root is not applicable to this particular problem.

**4.** A fulcrum is to be placed under a 3-foot lever so as to divide it into two parts such that 1.2 times the first shall equal 4.8 times the second. How far is it from either end?

**5.** A man bought 10 yards of calico and 20 yards of silk for \$30.60. The silk cost as many quarters a yard as the calico cost cents a yard. Find the price of each.

**6.** I bought a number of apples at the rate of 3 for a cent; sold one third of them at 2 for a cent, and the remainder at 5 for 3 cents, gaining 7 cents. How many did I buy?

**7.** Atmospheric air is a mixture of four parts of nitrogen with one of oxygen. How many cubic feet of oxygen are there in a room 10 yd. long, 5 yd. wide, and 12 ft. high?

**8.** In a certain family each son has as many brothers as sisters, but each daughter has twice as many brothers as sisters. How many children are in the family?

**9.** A number is composed of two digits whose sum is 8. If the digits are interchanged, the resulting number is greater by 18 than the original number. What is the number?

**10.** One third of my sheep equals one ninth of them plus 8. How many have I?

**11.** A man spends $\frac{1}{6}$ of his income for rent, $\frac{1}{5}$ for groceries, and has \$1140 left. What is his income?

**12.** Divide 15 apples between A and B so that $\frac{1}{2}$ of A's number shall equal $\frac{1}{3}$ of B's.

**13.** My wife's age plus mine equals 76 years, and $\frac{2}{3}$ of her age minus two years equals $\frac{1}{2}$ of my age plus 2 years. Find the age of each.

**14.** A has $150 more than B, and C has $\frac{1}{3}$ as much as A and B. They all have $1000. How much has each?

**15.** Sixty dollars was divided equally among a number of men. Had their number been 4 less, each would have received three times as much. How many men were there?

**16.** Find two consecutive numbers such that $\frac{1}{5}$ of the greater is 3 more than $\frac{1}{7}$ of the less.

**17.** What number added to the numerator and denominator of $\frac{2}{7}$ will give a fraction equal to $\frac{3}{4}$?

**18.** Eleven sixteenths of a certain principal was at interest at 5 per cent, and the remainder at 4 per cent. The entire income was $1500. Find the principal.

**19.** Two numbers are to each other as 3 to 4. If 10 is subtracted from each, the smaller one will be $\frac{2}{3}$ of the larger. What are the numbers?

**20.** Two numbers are to each other as 2 to 3, and their product is 150. What are the numbers?

**21.** A rectangular lot contains an acre, and its width is $\frac{2}{5}$ of its length. What is its width?

**22.** A triangular field contains 5 acres, and its altitude is $\frac{4}{9}$ of its base. What is the base?

**23.** A circular pond contains 314.16 square yards. What is its diameter?

**24.** The area of one square field is twice that of another, and they together contain 867 square rods. Find the length of a side of the smaller.

**25.** A rectangle has its length 6 feet longer and its width 5 feet shorter than the side of an equivalent square. Find its area.

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