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COMMUNICATION REQUIREMENTS AND STRATEGIC MECHANISMS FOR MARKET ORGANIZATION

by

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Abstract and Headnote

A systematic analysis of the incremental communication requirements or informational costs of Nash-implementation is carried out. The increment is measured relative to the requirements of the competitive allocation mechanism (which realizes Walrasian performance in the absence of strategic behavior). We establish upper bounds on the incremental communication requirements corresponding to implementation mechanisms that satisfy alternative lists of desired properties. In general, these bounds are lower when fewer properties are insisted upon. New methods in mechanism design are explored which yield new mechanisms and also modify existing ones. A surprising implication of these results is that we reject two crucial hypotheses derived from a reading of the literature on information and incentives.

Keywords: Communication requirements, implementation, Walrasian performance

JEL Classification: O26.

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COMMUNICATION REQUIREMENTS AND STRATEGIC MECHANISMS FOR MARKET ORGANIZATION

1. Motivation

Recent advances in the theory of incentives provide numerous ways of overcoming the problem of unobservability of private information, by designing appropriate mechanisms for the implementation of desired performance. The motivation for the widespread interest in such mechanisms comes from the fact that the costs of auditing individuals and verifying their private information are too high. What the bulk of the literature on incentives ignores, however, are the costs of operating these mechanisms themselves in terms of communication requirements and computational complexity. In this paper, we present a framework within which the communication or informational costs of incentive schemes can be analyzed. The issue of computational complexity, being the more difficult one to tackle, is deferred to future research.

Consider the problem of resource allocation that confronts a social planner. An enormously complex body of information about the basic parameters of the economy needs to be transmitted for an efficient allocation of the available resources. Economists argue that the *competitive (Walrasian) allocation mechanism* (see Hurwicz (1977)) encodes the complex information into finite-dimensional signals, and determines a Pareto-efficient allocation of resources. Given that the social planner operating this giant information processing system cannot directly observe the information dispersed among the agents, the competitive mechanism (which ignores strategic play) can be manipulated. This raises the following crucial questions: can the Pareto-efficiency and the information encoding properties of the competitive mechanism be duplicated by an "acceptable" mechanism that takes account of strategic behavior? Are there mechanisms whose communication requirements are "close" to that of the competitive mechanism, and which satisfy other desirable properties? This paper presents a menu of mechanisms for the social planner. Depending on the planner's constraints, different properties will be required from an acceptable market mechanism.

Two important questions arise at this point. First, why is the availability of such a menu important? Second, what is a minimal requirement for "acceptability"? Each of these questions will be addressed in turn.

An enormous body of literature has emerged since the 70's which studies the *implementability* of economic performance standards by games. In particular, the contributions of Hurwicz (1979a), Schmeidler (1980), Hurwicz, Maskin and Postlewaite (1984) among others assure us that the Walrasian standard can be implemented, given certain interiority assumptions, i.e. there is a game such that, for every economy, the set of interior Walrasian allocations coincide with the set of Nash equilibrium allocations of the game. Thus, the Pareto-efficiency properties can be mimicked by a game or "strategic market mechanism". Do we pay a price for such mechanisms in terms of more information being required from agents when compared with the communication requirements of the competitive mechanism? Moreover, if we insist on other desirable properties from the mechanisms, do the communication requirements increase? A menu of alternative strategic market mechanisms specifies the informational costs of implementability, together with a variety of other desirable properties. It must be noted that even though we restrict our attention to the implementation of Walrasian performance, the implications are more general: from the results in Hurwicz

(1979b), we know that implementation of Walrasian performance is a sufficient condition for (partial) implementation of almost every implementable individually rational and Pareto-efficient performance standard.

This brings us to the question of minimal requirements for acceptability. Obviously, we would require that the mechanism should implement the Walrasian performance standard. If we do not impose any restrictions on the class of admissible mechanisms, the information encoding problem becomes trivial and our search would end immediately. This can be seen in the following manner. We suppose that the planner asks the agents to transmit information via a signal or a message. A natural measure of the communication requirements of a mechanism is the size of the message space. For the case where the space is Euclidean, the size is given by its dimensionality. This provides a measure of the amount of information that a planner must be prepared to process in the worst situation. It is known that the competitive mechanism requires a message space of finite dimensionality (see Hurwicz (1977)). Also, members of the Hurwicz-Schmeidler class of strategic mechanisms use finite-dimensional message spaces. Thus, by the application of the inverse of a "space-filling" device, such as the Peano mapping, any finite-dimensional space can be smuggled into a space which has at least the same dimensionality as the competitive mechanism's message space.

Clearly, the problem becomes interesting only if we were to impose some further restrictions on the type of mechanism we are looking for. The appeal of the competitive mechanism lies in the fact that it encodes information by relying on the ability of agents to perform basic economic calculations. Artificial smuggling devices are simply mathematical tricks that are beyond the reach of the average trader or social planner. Moreover, as Marschak (1986) points out, the apparent cheapness of such smuggled information is illusory. In practice, a continuum of messages would have to be approximated by a finite collection of messages and a small mistake by an agent could translate into a major error after unscrambling the signal. This has prompted information economists (Hurwicz (1977), Mount and Reiter (1974)) to impose the requirement that an acceptable mechanism must not resort to the use of such smuggling devices. Thus, our minimal requirement for an acceptable strategic mechanism will be that the mechanism cannot include a space-filling function as part of its specification. This modest condition does the job of preventing smuggling of information since a particular space-filling function must be specified *a priori* by the planner as part of the description of the mechanism. Note that if the planner were to leave it to the agents to decide on the encoding device, it would completely defeat the purpose since the agents would then have to report a function to the planner and their messages would not be finite-dimensional.

This restriction is naturally the weakest one that we could make. Typically, much stronger requirements have been imposed in the literature. These involve certain regularity conditions on the mechanism. Motivated by a condition of local threadedness of the message correspondence used by Mount and Reiter (1974), a continuity assumption on the outcome mappings of the strategic mechanisms has been suggested by Reichelstein and Reiter (1985). Quite apart from the fact that continuity is by itself a nice property to have, it is much too strong a requirement for ruling out information smuggling *per se*. As we shall show, imposing such a strong, and also indirect, restriction can prevent the planner from taking advantage of savings in terms of communication requirements. In fact, by using a restriction which is weaker than continuity, we give ourselves the freedom of checking exactly how sensitive the communication requirements are when we add continuity to our list of desired properties.

Given this minimal requirement, we can make certain other demands on the strategic mechanism, which is completely specified by a message space and an outcome mapping. Typically, these restrictions relate to the outcome mapping. Moreover, the extent to which a planner feels that such restrictions are necessary depends on his/her level of confidence in the ability of the agents to exactly arrive at an equilibrium of the game associated with the mechanism. If the agents are known to be highly-trained or experienced professionals, then the planner may not worry about them making mistakes while computing their Nash-optimal strategies. In such instances the planner need only worry about the equilibrium properties of the mechanism. In situations where the possibility of agents making mistakes cannot entirely be ruled out, out-of-equilibrium properties of the mechanism must be taken into account. The most crucial properties of the outcome mapping that are desirable in such situations are: *global feasibility*, *continuity* and *single-valuedness*. We shall briefly discuss the importance of each one of these properties in turn.

The global feasibility condition guarantees that every outcome of the game is such that (a) no agent goes bankrupt and is asked to give away more than his/her endowment and (b) all net trades are balanced. The continuity condition ensures that small mistakes by agents do not lead to large changes in the outcome, i.e. the outcomes do not stray far from the equilibrium outcomes in the event of small perturbations of strategies. Finally, the single-valuedness property ensures that a unique outcome is picked for every configuration of messages. The last property is, in general, satisfied by most mechanisms introduced in the literature. The significance of this requirement will be discussed later.

Our research strategy will be as follows. We shall require a mechanism to satisfy a given list of properties and ask what is the maximal level of incremental communication requirements (relative to the competitive allocation mechanism) that such a mechanism would need. The possibilities with different lists of properties will be explored. In addition, we shall formulate two hypotheses regarding the communication requirements for different types of mechanisms. These are suggested by a reading of the current state of the art on this subject and give us a benchmark by which we can evaluate the success of our findings. The first one is called the *Information vs Incentives Trade-off Hypothesis (IIT)*, which says that a social planner interested simply in implementation will always need to use a mechanism with a message space with strictly greater dimensionality than that of the message space associated with the competitive allocation mechanism. The second one is called the *Doubling of Communication Requirements Hypothesis (DCR)*, which says that a social planner interested in globally feasible implementation will always need to use a mechanism with a message space with at least twice the dimensionality of the message space of the competitive allocation mechanism. The findings of this paper are extremely optimistic: both the hypotheses are rejected. Why is such a conclusion surprising? Saijo (1986b) and Reichelstein and Reiter (1985) have respectively argued that under certain conditions the disappointing conclusions of the two hypotheses are indeed true. This raises the obvious question: can their conclusions be generalized? We shall show that the conditions imposed by these authors are too restrictive and conceal a happier result.

The arguments presented here are in a constructive vein. Some novel methods of mechanism design are demonstrated which provide savings in terms of communication requirements. These include the introduction of *price specialists* and *quasi-games*, with unambiguous equilibrium outcomes, in the design of strategic mechanisms. (Price specialists are a sub-group of agents who compute market-clearing prices and they are the only ones who do so. Quasi-games are games with outcome correspondences (see Thomson (1984)). In general, an equilibrium for a quasi-game is hard to define.) In the process, we shall also show that the communication requirements of mechanisms available in the literature can be further reduced.

The next section sets up the basic framework. The results of the paper are given in the section that follows. The final section concludes.

2. Preliminaries

We consider a class of exchange economies with m goods and n agents with $m > 1$, $n > 1$. N is the *set of agents*. Each $i \in N$ is characterized by a list $\{C_i, r_i, \omega_i\}$ where $C_i = \mathbb{R}^m_+$ is *agent i's consumption set*, whose typical element is denoted z_i; r_i is *agent i's preference relation* defined on C_i and is assumed to be binary, reflexive, complete, transitive and strictly monotonic; $\omega_i \in \mathbb{R}^m_{++}$ is *agent i's initial endowment*. We assume that for the class of economies under consideration, for all i, C_i and ω_i are fixed and known. Let $\Omega = \sum_{i \in N} \omega_i$. Let $\mathcal{R}_i$ denote the domain of admissible preference relations for agent i. An *economy* is completely characterized by a profile of preference relations, $r \equiv (r_i)_{i \in N}$. The *class of economies*, correspondingly, is given by $\mathcal{R} \equiv \times_{i \in N} \mathcal{R}_i$.

$A \equiv \{z \in \mathbb{R}^{mn}_+ : \sum_{i \in N} z_i = \Omega\}$ is the *set of feasible allocations* with $A_i \equiv \{z_i \in \mathbb{R}^m_+ : z_i \leq \Omega\}$. $L(z, r_i) \equiv \{z_i' \in C_i : z_i r_i z_i'\}$ is *agent i's r-lower contour set at z.* Unless otherwise specified, let y denote $(y_i)_{i \in N}$ and let y_{-i} denote $(y_j)_{j \in N \setminus \{i\}}$.

Our focus will be on a mapping which associates with every economy the set of perfectly competitive allocations in the interior of the feasible set. For any set X let

$\wp(X)$ denote the set of all subsets of X.

The *Walrasian performance standard*, is defined by $W : \mathcal{R} \to \wp(int(A))$ satisfying the following:

$\forall r \in \mathcal{R}, \forall z \in W(r), \exists p \in \Delta^{m-1}$ such that $\forall i \in N, z_i \in B_i(p)$ and $B_i(p) \subseteq L(z, r_i)$, where

Δ^{m-1} is the $m-1$-dimensional unit simplex and $B_i(p) \equiv \{z_i \in A_i : pz_i = p\omega_i\}$ is ***agent i's constrained budget set, given a price vector p.***

Let $W(\mathcal{R}) \equiv \{z \in A : \exists r \in \mathcal{R}$ such that $z \in W(r)\}$. We assume that for all $r \in \mathcal{R}$, $W(r) \neq \emptyset$. (See Debreu (1959) for the underlying assumptions.)

The *competitive allocation mechanism* is given by a triple $\{M^c, \mu^c, \xi^c\}$ where $M^c \equiv \{(p, z) \in \Delta^{m-1} \times A : \forall i \in N, pz_i = p\omega_i\}$ is a *message space*, $\mu^c : \mathcal{R} \to M$ is a *message correspondence* where for all $i \in N, \mu_i^c : \mathcal{R}_i \to M$ is given by $\mu_i^c(r_i) \equiv \{(p, z_i) \in \Delta^{m-1} \times C_i : z_i \in B_i(p)$ and $B_i(p) \subseteq L(z, r_i)\}$ and $\xi^c : M^c \to A$ is an *outcome function* that projects every element of M on A.

Using the following conditions: (i) $\sum_{i \in N}(z_i - \omega_i) = 0$, (ii) $\forall i \in N, pz_i = p\omega_i$ and (iii) zero degree homogeneity of demand, the dimensionality of the "competitive" message space M^c, denoted $\dim(M^c)$, is equal to $n(m-1)$.

The competitive allocation mechanism *realizes* W, i.e.

$\forall r \in \mathcal{R}, \xi^c(\mu^c(r)) = W(r)$.

The competitive allocation mechanism requires that the agents must follow the prescribed rules of behavior and transmit information consistent with the true privately observed preferences. However, an uninformed operator of the mechanism cannot prevent the agents from manipulating it by reporting false information. To take account of such strategic behavior, we model mechanisms as non-cooperative games. The strategy space of the game is a proxy for the message space of the associated mechanism. Next, we shall define and discuss a generalization of the usual concept of a game.

A *quasi-game form* or, simply, *quasi-game* or *mechanism*, Γ, is triple $\{N, S, \xi\}$ where $S \equiv \times_{i \in N} S_i$ is a *strategy space* and $\xi \equiv (\xi_i)_{i \in N}$, where $\xi_i : S \rightarrow \mathbf{R}^m$ is an *outcome mapping for agent i.*

For all $i \in N$ and $s_{-i} \in S_{-i}$, $\xi_i(S_i, s_{-i}) \equiv \{z_i \in \mathbf{R}^m : z_i \in \xi_i(s_i, s_{-i})$, for some $s_i \in S_i\}$.

Given that a game Γ is played in an economy $r \in \mathcal{R}$, a pair $(s, z) \in S \times A$ with $z \in \xi(s)$ is a *strict Nash equilibrium* if

$\forall i \in N, \xi_i(S_i, s_{-i}) \subseteq L(z, r_i)$.

Let $E(\Gamma, r) \subseteq S \times A$ denote the *set of strict Nash equilibria of* Γ *played in* r and let $E_S(\Gamma, r) \subseteq S$ and $E_A(\Gamma, r) \subseteq A$ denote, respectively, the projections of the set $E(\Gamma, r)$ on S and A.

Remark 1: A "quasi"-game is a generalization of the usual concept of a game (see Thomson (1984)) where the outcome mapping may be multi-valued. A game is a special case of a quasi-game where the outcome mapping is a function and not a correspondence. Correspondingly, a *Nash equilibrium* is a special case of a strict Nash equilibrium for games with outcome functions. Quasi-games have been studied very infrequently in the literature. The basic difficulty with such a concept is that an appropriate equilibrium notion is hard to define. The problem arises from the fact that any unilateral change in strategy by an agent requires a comparison of the status quo outcome with a list of outcomes. In the absence of any clear domination of one over the other, a best-response calculation is difficult. The concept of strict Nash equilibria was initially used by Otani and Sicilian (1982) and Thomson (1984) for the purpose of analyzing the manipulability of economic performance standards (which are set-valued mappings, in general). This concept essentially assumes that agents are optimistic, i.e. when they compare the utility payoffs from two lists of outcomes, they simply look at the best (in utility terms) outcome in each list. The literature on mechanism design and implementation, (with the exception of Thomson (1983) and Chakravorti (1985)) on the other hand, has avoided the use of such quasi-games since the equilibrium depends on whether agents are optimistic or pessimistic or, even, Bayesian. In this paper, we shall use only one quasi-game which has an outcome correspondence. However, the attractive feature of this mechanism is that the criticisms mentioned above will not apply. This mechanism will require agents to compare a list of outcomes with a status quo outcome only when one clearly dominates the other. Thus, the concept of strict Nash equilibrium will yield an unambiguous prediction.

The job of an uninformed implementor of the Walrasian performance standard is to find a strategic mechanism which exactly mimics the competitive allocation mechanism, i.e. for every economy, the equilibrium allocations of the mechanism are Walrasian. This property can be stated formally in the following manner:

P1 (Implementation): Γ *strictly Nash implements* W if $\forall r \in \mathcal{R}, E_A(\Gamma, r) = W(r)$.

We denote the class of all admissible strategic mechanisms by $\mathcal{G}$. A quasi-game Γ belongs to $\mathcal{G}$ if and only if it is completely characterized by a triple $\{N, S, \xi\}$ and does not include any space-filling functions in its description. This restricts what the implementor can specify as part of the "rules of the game". In addition, we consider other properties:

P2 (Single-valuedness): $\Gamma = \{N, S, \xi\}$ satisfies the following: $\forall i \in N, \forall s \in S, \xi_i(s)$ is a singleton.

P3 (Continuity): $\Gamma = \{N, S, \xi\}$ satisfies the following: $\forall i \in N, \xi_i$ is continuous.

P4 (Global feasibility): $\Gamma = \{N, S, \xi\}$ satisfies the following:

(i) $\forall i \in N, \forall s \in S, \xi_i(s) \in C_i$ and

(ii) $\forall i \in N, \forall s \in S, \sum_{i \in N}(\xi_i(s) - \omega_i) = 0$.

Note that P1 implies Pareto-efficiency and in the sequel we shall use P2 and P3 together, so by "continuity" we mean continuity of a function.

Given a game $\Gamma = \{N, S, \xi\}$ with a Euclidean strategy space S, let $\dim(S)$ denote its dimensionality. The incremental communication requirements, i.e. over and above that represented by $\dim(M^c)$, is the *informational cost of implementation* denoted $\rho(\Gamma, m, n) \equiv \dim(S) - \dim(M^c)$.

Given these definitions, we can set up some benchmarks by stating two hypotheses:

The *Information vs Incentives Trade-off Hypothesis (IIT):* $\exists \Gamma \in \mathcal{G}$ satisfying P1 $\Longrightarrow \rho(\Gamma, m, n) > 0$.

The *Doubling of Communication Requirements Hypothesis (DCR):* $\exists \Gamma \in \mathcal{G}$ satisfying P1 and P4 $\Longrightarrow \rho(\Gamma) \geq \dim(M^c)$.

Remark 2: These hypotheses provide benchmarks for our study. Reichelstein and Reiter (1985) have shown that the IIT hypothesis can be supported if we require that Γ have a differentiable outcome function. Intuitively, also it seems that we must pay a price (in terms of communication costs) for settling the manipulability issue. Thus, IIT seems to be a very natural hypothesis to postulate. On the other hand, the literature on globally feasible implementation of arbitrary performance standards (Maskin (1977), Saijo (1986a), Mckelvey (1986)) has demonstrated the possibility of designing games where each agent reports the preferences of at least two agents. This immediately doubles the communication requirements, since a mechanism where every agent reports his/her own preferences would realize any given performance standard in the absence of strategic behavior. In the more specific context of globally feasible implementation of Walrasian performance, the Hurwicz-Maskin-Postlewaite (1984) game also uses a strategy space which is twice that of the competitive allocation mechanism. Moreover, Saijo (1986b) argues that if we are interested in constructing globally feasible implementation mechanisms from realization mechanisms as Williams (1986) does, then the dimensionality of the message space must be doubled in general. Given the regularity with which this "doubling" of the message space has occurred, it is natural to ask if such a doubling is a necessary condition.

3. Results

In this section, we consider mechanisms satisfying different combinations of the desirable properties listed earlier, and check for the maximum value that $\rho(\Gamma, m, n)$ must take. The first result shows that there does exist a strategic mechanism which strictly Nash implements W and uses a message (strategy) space of dimensionality $n(m-1)$. This mechanism employs two devices by which it economizes on communication requirements – an outcome correspondence and a sub-group of *price specialists.* The other results give an indication of the sensitivity of $\rho(\Gamma, m, n)$ to the imposition of additional properties.

Theorem 1: *If there exists a mechanism* $\Gamma \in \mathcal{G}$ *satisfying P1, then* $\rho(\Gamma, m, n) \leq 0$.

Consider the following quasi-game, denoted Γ_1:

(I) N contains a sub-group, T who are *price specialists.* $|T| = 2$.

(II) $\forall i \in T, S_i \equiv \{s_i : \text{either } s_i = p_i \in \Delta^{m-1} \text{ or } s_i = z_i \in A\}$.

$\forall j \in N \backslash T, S_j \equiv \{s_j : s_j = z_j \in A\}$.

Given $s \in S$, we consider three cases:

Case 1: (i) Given $\{i,k\} = T, s_i = s_k = p$ and (ii) $\forall j \in N \backslash T, s_j = z_j$ with $z_j \in B_j(p)$.

Case 2: (i) Given $\{i,k\} = T, s_i = p_i, s_k = z_k$ with $z_k \in B_k(p_i)$ and (ii) $\forall j \in N \backslash T, s_j = z_j$ with $z_j \in B_j(p_i)$.

Case 3: Otherwise.

(III) $\xi : S \to \bigcup_{i \in N} C_i$ is given by the following rules:

Rule 1: Case 1 $\Longrightarrow \forall t \in T, \xi_t(s) = B_t(p)$ and $\forall j \in N \backslash T, \xi_j(s) = z_j$.

Rule 2: Case 2 $\Longrightarrow \xi_k(s) = z_k$ and $\forall l \in N \backslash \{k\}$, $\xi_l(s) = 0$.

Rule 3: Case 3 $\Longrightarrow \forall j \in N, \xi_j(s) = 0$.

Remark 3: The strategy space of Γ_1 is of dimensionality $n(m-1)$. By the fact that prices are elements of the unit simplex, and given that whenever an announced consumption vector appears as an outcome of the game it must always be in the budget set for a unique price vector, it is adequate for the purposes of playing the game if the announced price and consumption vectors are always $(m-1)$ dimensional. In other words, agents need transmit the prices or their consumption demands for only $m-1$ goods.

Remark 4: Given that for every i, ω_i is in R^m_{++}, observe that an equilibrium of the game occurs only when Case 1 is true. For all other cases, the outcomes recommended by the quasi-game are not elements of the set A. Next, we shall check that the concept of strict Nash equilibrium yields an unambiguous prediction, in the sense that there are no other allocations which could be equilibria. Let $s \in E_S(\Gamma_1, r)$ for some $r \in \mathcal{R}$. By definition of Case 1, given $T = \{i,k\}$, we have $s_i = s_k = p$. By Rule 1, each of the two agents in T gets a set of consumptions as an outcome of the game. These sets coincide with the respective budget sets defined by the price p. We need to locate the points on this budget set from which no agent can hope to do better by changing strategy. Also, we would need to make sure that such an alteration in strategy will not yield a list of outcomes some of which make the agent better off and some which make him/her worse off because in the latter case a pessimistic or a Bayesian agent may not change his/her strategy. Clearly, for all agents not in T, unilateral deviation yields a unique outcome. We shall argue that even for agents in T, unilateral deviation yields a unique outcome which can be strictly compared with the outcomes in $\xi(s)$. Consider unilateral deviation by agent $i \in T$. Given the rules 2 and 3, unless agent i's alternative strategy s_i' is such that $s_i' = z_i' \in B_i(p)$, agent i will get nothing. Thus, $\xi_i(S_i, s_{-i}) = B_i(p) \cup \{0\}$. Given strict monotonicity of preferences, agent i will not change strategy if and only if $z_i \in \xi_i(s)$ is such that $B_i(p) \subseteq L(z, r_i)$, which is exactly what the definition of strict Nash equilibria predicts. No other point on $B_i(p)$ can be supported as an equilibrium since there always exists an alternative strategy for agent i which makes him/her better off.

The theorem can now be proved with the help of the following lemmata.

Lemma 1: $\forall r \in \mathcal{R}, W(r) \subseteq E_A(\Gamma_1, r)$.

Proof of Lemma 1: Choose $r \in \mathcal{R}$ and let $z \in W(r)$. We need to show that $z \in E_A(\Gamma_1, r)$. Consider $s = ((s_k = p)_{k \in T}, (s_j = z_j)_{j \in N \setminus T})$ where $p \in \Delta^{m-1}$ is such that for all $i \in N, z_i \in B_i(p) \subseteq L(z, r_i)$. In other words, p is a Walrasian price associated with (z, r). Since s satisfies Case 1, by Rule 1, for all $k \in T, \xi_k(s) = B_k(p)$ and for all $j \in N \setminus T, \xi_j(s) = z_j$. Next, consider unilateral deviation by some $i \in N$ to some arbitrary $s_i' \in S_i, s_i' \neq s_i$. There are two possibilities:

(i) Suppose $i \in T$: There are two further possibilities depending on whether Case

2 or Case 3 occurs. Thus, $\xi_i(s_i', s_{-i}) \in \{0, z_i'\}$ for some $z_i' \in C_i$. If the outcome is z_i', then Case 2 must have occurred and it must be true that $z_i' \in B_i(p)$. By definition of W and p, we have $B_i(p) \subseteq L(z, r_i)$. Thus, $z_i' \in L(z, r_i)$. By strict monotonicity of prefernces, $0 \in L(z, r_i)$.

(ii) Suppose $i \in N \backslash T$: Again there are two possibilities depending on whether Case 2 or Case 3 occurs. Thus, $\xi_i(s_i', s_{-i}) \in \{0, z_i'\}$ for some $z_i' \in C_i$. If the outcome is z_i', then Case 1 must havė occurred and it must be true that $z_i' \in B_i(p)$. By the argument given in (i) above $\{0, z_i'\} \subseteq L(z, r_i)$.

Thus, for all $i \in N, \xi_i(S_i, s_{-i}) \subseteq L(z, r_i)$. $z \in W(r)$ implies $z \in A$. Therefore, $z \in E_A(\Gamma_1, r)$. Q.E.D.

Lemma 2: $\forall r \in \mathcal{R}, E_A(\Gamma_1, r) \subseteq W(r)$.

Proof of Lemma 2: Choose $r \in \mathcal{R}$ and let $(s, z) \in E(\Gamma_1, r)$. Given that $z \in A$ and for all $i \in N, \omega_i \in \mathbf{R}^m_{++}$, Case 2 and Case 3 could not have occurred. Thus, s must have the following form: $((s_k = p)_{k \in T}, (s_j = z_j)_{j \in N \backslash T})$, where for all $j \in N \backslash T, z_j \in B_j(p)$. We shall show that for all $i \in N, B_i(p) \subseteq L(z, r_i)$. Choose $i \in N$. There are two possibilities:

(i) Suppose $i \in T$: We shall show that $B_i(p) \subseteq \xi_i(S_i, s_{-i})$. Choose $z_i' \in B_i(p)$. To show that $z_i' \in \xi_i(S_i, s_{-i})$, consider the strategy $s_i' = z_i'$. Since $z_i' \in B_i(p)$, Case 2 applies and, by Rule 2, $\xi_i(s_i', s_{-i}) = z_i'$.

(ii) Suppose $i \in N \backslash T$: Again, we shall establish that $B_i(p) \subseteq \xi_i(S_i, s_{-i})$. Choose $z_i' \in B_i(p)$. To show that $z_i' \in \xi_i(S_i, s_{-i})$, consider $s_i' = z_i'$. Since $z_i' \in B_i(p)$, Case 1 applies. Thus, by Rule 1, $\xi_i(s_i', s_{-i}) = z_i'$.

Thus, we have

$$(s, z) \in E(\Gamma_1, r) \Longrightarrow \forall i \in N, \xi_i(S_i, s_{-i}) \subseteq L(z, r_i).$$

From (i) and (ii), we conclude that for all $i \in N, B_i(p) \subseteq L(z, r_i)$. Given that for all $i \in N, z_i \in B_i(p)$, by the definition of $W, z \in W(r)$. Q.E.D.

Remark 5: This mechanism has some additional desirable properties. The implementation result works even for two-agent economies. Most available methods for implementation need at least three agents. Besides, a weak form of feasibility is met – individual feasibility, i.e. no agent is ever required to give away more of a good than he/she was initially endowed with.

Proof of Theorem 1: Γ_1 uses a strategy space with dimensionality n(m - 1) and by Lemma 1 and Lemma 2, it strictly Nash implements W. Q.E.D.

Theorem 2: *If there exists a mechanism $\Gamma \in \mathcal{G}$ satisfying P1, P2, P3, then* $\rho(\Gamma, m, n) \leq min_\nu\{\nu \in \mathrm{N} : (n-1)\nu \geq (m-1)\}$, *where* $\mathrm{N} \equiv \{1, 2, 3, ...\}$.

Proof of Theorem 2: See Reichelstein and Reiter (1985).

Theorem 3: *Suppose* $| N | > 2$. *If there exists a mechanism $\Gamma \in \mathcal{G}$ satisfying P1, P2, P4, then* $\rho(\Gamma, m, n) \leq 3m + 2n - 9$.

Consider the following mechanism, denoted Γ_2:

(I) N contains a sub-group T with $| T | = 3$.

(II) $\forall i \in T, S_i = \{s_i = (z_i, q_i) : z_i \in A_i$ and either $q_i \in \Delta^{m-1}$ and $z_i \in B_i(q_i)$ or $q_i \in (5, 10]\}$.

$\forall i \in N \backslash T, S_i = \{s_i = (z_i, q_i) : z_i \in A_i, q_i \in (5, 10]\}$.

The following definitions shall simplify notation:

(D1) Given that there exists $j, k \in T$ such that $q_j, q_k \in \Delta^{m-1}$ and $q_j = q_k$, define $q^*(s)$ such that $q^*(s) = q_j = q_k$.

(D2) $s_{-i} = (z_k, q_k)_{k \in N \backslash \{i\}}$ satisfies *Property* $\alpha \mid i$ if $i \in T$ and the following conditions hold:

(i) $q^*(s)$ is well-defined and $\forall j \in T\backslash\{i\}, q_j = q^*(s)$

(ii) $\forall j \in N\backslash T, q_j = 10$ and

(iii) $(\Omega - \sum_{j\in N\backslash\{i\}} z_j, z_{-i}) \in W(\mathcal{R})$ with $(\Omega - \sum_{j\in N\backslash\{i\}} z_j) \in B_i(q^*(s))$ and $\forall j \in N\backslash\{i\}, z_j \in B_j(q^*(s))$.

(D3) $s_{-i} = (z_k, q_k)_{k\in N\backslash\{i\}}$ satisfies *Property* $\beta \mid i$ if $i \in N\backslash T$ and the following conditions hold:

(i) $q^*(s)$ is well-defined and $\forall j \in T, q_j = q^*(s)$

(ii) $\forall j \in N\backslash\{i \cup T\}, q_j = 10$ and

(iii) $(\Omega - \sum_{j\in N\backslash\{i\}} z_j, z_{-i}) \in W(\mathcal{R})$ with $(\Omega - \sum_{j\in N\backslash\{i\}} z_j) \in B_i(q^*(s))$ and $\forall j \in N\backslash\{i\}, z_j \in B_j(q^*(s))$.

(D4) $K(s) \equiv \{k \in N : q_k \in (5, 10)\}$.

(III) $\xi : S \longrightarrow A$ is given by Figure 1.

[Insert Figure 1 here]

Remark 6: First, consider the case, $m > 2$. Given the fact that prices are elements of the unit simplex and given the constraint $q_i(z_i - \omega_i) = 0$ for all $q_i \in \Delta^{m-1}$, a strategy pair (z_i, q_i) for agent $i \in T$ requires at most a price and quantity announcement for $m - 1$ goods. If agent $i \in T$ announces $q_i \in (5, 10]$, then the quantity announcement by i must be for all m goods. Thus, the strategy space for each agent in T is contained in max $\{2(m-1), (m+1)\} = 2(m-1)$ dimensions for $m > 2$. Every member of the set $N\backslash T$ has an $m+1$-dimensional strategy space. Therefore, for $m > 2, \Gamma_2$ has a strategy space of dimensionality $(n-3)(m+1) + 6(m-1)$ which can be re-written as $n(m-1) + 3m + 2n - 9$. Also, note that Γ_2 satisfies the conditions for global feasibility and has a single-valued outcome mapping.

The case where $m = 2$ will be dealt with later. First we shall establish the following results:

Lemma 3: *Let $r \in \mathcal{R}$ and $(s, z) \in E(\Gamma_2, r)$ be given. $\forall i \in N, s_{-i}$ does not satisfy either Property $\alpha \mid i$ or $\beta \mid i \Longrightarrow L(z, r_i) = A_i$.*

Proof of Lemma 3: Choose $i \in N$. We shall first show that if s_{-i} does not satisfy either Property $\alpha \mid i$ or Property $\beta \mid i$, then $A_i \subseteq \xi_i(S_i, s_{-i})$. Consider $z'_i \in A_i$. To show that $z'_i \in \xi_i(S_i, s_{-i})$, consider $s'_i = (z'_i, q'_i) \in S_i$ where $q'_i \in (5, 10)$ is such that, given $s' \equiv (s'_i, s_{-i})$, either $K(s') = \{i\}$ or $q'_i < q_j$, for all $j \in K(s')\backslash\{i\}$. Observe that this strategy preserves the characteristic of the vector s_{-i} that it does not satisfy either Property $\alpha \mid i$ or Property $\beta \mid i$. Furthermore, since $q'_i \neq 10$ and $q'_i \notin \Delta^{m-1}$, for no $j \in N\backslash\{i\}$ is either Property $\alpha \mid i$ or Property $\beta \mid i$ satisfied. Since Case 2 applies, there are two possibilities:

(a) Case 2A is applicable. Since Case 2AA applies, $\xi_i(s'_i, s_{-i}) = z'_i$.

(b) Case 2B is applicable. Again $\xi_i(s'_i, s_{-i}) = z'_i$.

$(s, z) \in E(\Gamma_2, r)$ implies $\xi_i(S_i, s_{-i}) \subseteq L(z, r_i)$. Thus, for all $i \in N, A_i \subseteq L(z, r_i)$. By definition, for all $i \in N, L(z, r_i) \subseteq A_i$. Q.E.D.

Lemma 4: $\forall r \in \mathcal{R}, W(r) \subseteq E_A(\Gamma_2, r)$.

Proof of Lemma 4: Choose $r \in \mathcal{R}$. The assumptions on $\mathcal{R}$ are sufficient to guarantee that $W(r) \neq \emptyset$. Choose $z \in W(r)$. To show that $z \in E_A(\Gamma_2, r)$, consider $s = ((z_i, q^*(s))_{i \in T}, (z_j, 10)_{j \in N\backslash T})$ where $q^*(s)$ is a Walrasian price associated with (z, r). Since, $z \in W(\mathcal{R})$, Case 2B applies. Therefore, $\xi(s) = z$. Consider unilateral deviation from s by some $i \in N$ to an arbitrary $s'_i = (z'_i, q'_i) \in S_i$. Observe that s_{-i} satisfies either Property $\alpha \mid i$ or Property $\beta \mid i$. There are two possibilities:

(i) $z'_i = z_i$. Since Case 2B applies, $\xi_i(s'_i, s_{-i}) = z_i \in L(z, r_i)$.

(ii) $z_i' \neq z_i$. Since Case 1 is applicable, $\xi_i(s_i', s_{-i}) \in \{z_i', \omega_i\}$. If the outcome is z_i', then it must be true that Case 1A is applicable and $z_i' \in B_i(q^*(s))$. By definition of W, given that $q^*(s)$ is a Walrasian price associated with (z, r), it must be the case that $B_i(q^*(s)) \subseteq L(z, r_i)$. Thus, $z_i' \in L(z, r_i)$. If the outcome is ω_i, by the definition of W, and given $z \in W(r)$, we have $\omega \in L(z, r_i)$ by individual rationality of W.

Thus, $\xi_i(S_i, s_{-i}) \subseteq L(z, r_i)$ for all $i \in N$. Therefore, $z \in E_A(\Gamma_2, r)$. Q.E.D.

Lemma 5: $\forall r \in \mathcal{R}, E_A(\Gamma_2, r) \subseteq W(r)$.

Proof of Lemma 5: Choose $r \in R$ and let $(s, z^*) \in E(\Gamma_2, r)$. We examine the possibilities arising from the fact that any one of the two cases could have occurred and the associated outcome rules were used to obtain z^*.

(i) Suppose Case 1 had occurred: We can distinguish between two possibilities:

Possibility 1: Suppose Case 1A had occurred. In this case, it must be true that there exists $i \in N$ such that $K(s) = \{i\}$. This implies that there does not exist $j \in N \backslash \{i\}$ such that s_{-j} satisfies either Property $\alpha \mid j$ or Property $\beta \mid j$. By Lemma 3, for all $j \in N \backslash \{i\}, L(z^*, r_j) = A_j$. Given strict monotonicity of r and $\mid N \backslash \{i\} \mid \geq 2$, there cannot exist $z^* \in A$ satisfying this condition. Thus, Case 1A could not have occurred.

Possibility 2: Suppose Case 1B had occurred. In this case, $\xi(s) = \omega$. We shall show that $\omega \in W(r)$. Let $s = (z_j, q_j)_{j \in N}$. Observe that, by definition of Case 1, $q^*(s)$ is well-defined. Choose $j \in N$. There are two possibilities:

(i-a) s_{-j} does not satisfy either Property $\alpha \mid j$ or Property $\beta \mid j$, in which case, by Lemma 3, $L(\omega, r_j) = A_j$. Thus, trivially, $B_j(q^*(s)) \subseteq L(\omega, r_j)$.

(i-b) s_{-j} satisfies either Property $\alpha \mid j$ or Property $\beta \mid j$. We shall show that $B_j(q^*(s)) \subseteq L(\omega, r_j)$. First, we establish that $B_j(q^*(s)) \subseteq \xi_j(S_j, s_{-j})$. Choose $z_j' \in$

$B_j(q^*(s))$. To show that $z'_j \in \xi_j(S_j, s_{-j})$, consider the strategy $s'_j = (z'_j, q'_j) \in S_j$, where $q'_j \in (5, 10)$. Observe that $K(s'_j, s_{-j}) = j$. There are two further possibilities:

(i-b-a) $(z'_j, z_{-j}) \in W(\mathcal{R})$. Thus, Case 2B applies and $\xi_j(s'_j, s_{-j}) = z'_j$.

(i-b-b) $(z'_j, z_{-j}) \notin W(\mathcal{R})$. Given that s_{-j} satisfies either Property $\alpha \mid j$ or Property $\beta \mid j$, $z'_j \in B_j(q^*(s))$ and $K(s'_j, s_{-j}) = j$, Case 1A applies and $\xi_j(s'_j, s_{-j}) = z'_j$.

$(s, \omega) \in E(\Gamma_2, r)$ implies that $\xi_j(S_j, s_{-j}) \subseteq L(\omega, r_j)$. Therefore, $B_j(q^*(s)) \subseteq L(\omega, r_j)$.

Since $q^*(s)$ is well-defined, we conclude from cases (i-a) and (i-b) that for all $k \in N, B_k(q^*(s)) \subseteq L(\omega, r_k)$. By definition of $W, \omega \in W(r)$.

(ii) Suppose Case 2 had occurred: There are two possibilities:

Possibility 1: There exists no $i \in N$ such that either Property $\alpha \mid i$ or Property $\beta \mid i$ is satisfied. By Lemma 3, for all $i \in N, L(z^*, r_i) = A_i$. By strict monotonicity of r, since $\mid N \mid > 2$, there exists no $z^* \in A$ such that this is possible. Thus, s cannot be such that neither Property $\alpha \mid i$ nor Property $\beta \mid i$ is met for any i.

Possibility 2: There exists $i \in N$ such that either Property $\alpha \mid i$ or Property $\beta \mid i$ is satisfied. Observe that $q^*(s)$ is well-defined. By the definition of Cases 1 and 2, $s = (z_j, q_j)_{j \in N}$ must be such that $z \in W(\mathcal{R})$. Thus, Case 2A could not have occurred. Since Case 2B has occurred, $\xi(s) = z$. We shall show that $z \in W(r)$. Choose $j \in N$. There are two possibilities:

(ii-a) s_{-j} does not satisfy either Property $\alpha \mid j$ or Property $\beta \mid j$, in which case, by Lemma 3, $L(z, r_j) = A$. Thus, trivially, $B_j(q^*(s)) \subseteq L(z, r_i)$.

(ii-b) s_{-j} satisfies either Property $\alpha \mid j$ or Property $\beta \mid j$. We shall show that $B_j(q^*(s)) \subseteq \xi_j(S_j, s_{-j})$. Choose $z'_j \in B_j(q^*(s))$ such that $z'_j \neq z_j$. To show that $z'_j \in \xi_j(S_j, s_{-j})$, consider $s'_j = (z'_j, q'_j) \in S_j$ with $q'_j \in (5, 10)$. Observe that $K(s'_j, s_{-j}) =$

$\{j\}$. Since $(z'_j, z_{-j}) \neq z$, given that s_{-j} satisfies either Property $\alpha \mid j$ or Property $\beta \mid j, z'_j \in B_j(q^*(s))$ and $K(s'_j, s_{-j}) = \{j\}$, Case 1A applies and $\xi_j(s'_j, s_{-j}) = z'_j$.

$(s, z) \in E(\Gamma_2, r)$ implies $\xi_j(S_j, s_{-j}) \subseteq L(z, r_j)$. Therefore, $B_j(q^*(s)) \subseteq L(z, r_j)$.

Since $q^*(s)$ is well-defined, we conclude from the cases (ii-a) and (ii-b) above that for all $k \in N, B_k(q^*(s)) \subseteq L(z, r_k)$. By definition of the properties α and β, for all $k \in N, z \in B_k(q^*(s))$. By definition of $W, z \in W(r)$. Q.E.D.

Next, assuming that $m = 2$ with the two goods being denoted x and y, consider the following game, denoted Γ_3:

(I) $\mid N \mid > 2$.

$\forall i \in N$, let $\zeta_i : \mathrm{R}_+ \times \Delta^{m-1} \to A_i$ be defined by $p\zeta_i(x_i, p) = p\omega_i$.

Remark 7: Note that by the fact that $m = 2, \zeta_i$ is a well-defined function.

(II) $\forall i \in N, S_i = \{(x_i, p_i) \in \mathrm{R}_+ \times \Delta^{m-1} : p_i\zeta_i(x_i, p_i) = p_i\omega_i\}$.

For all $i \in N$, given $s_i = (x_i, p_i)$ and $z_i \equiv \zeta_i(x_i, p_i)$, consider the following cases:

Case 1: $\exists i, j, k \in N$ such that p_i, p_j, p_k are distinct.

Case 2: $\exists$ only two distinct announced prices p', p'' and at least two agents announce each p' and p''.

Case 3: (i) $\exists p \in \Delta^{m-1}$ such that $\forall i \in N, p_i = p$ and (ii) $\sum_{i \in N} z_i \neq \Omega$.

Case 4: (i) $\exists p \in \Delta^{m-1}$ such that $\forall i \in N, p_i = p$ and (ii) $\sum_{i \in N} z_i = \Omega$.

Case 5: (i) $\exists p \in \Delta^{m-1}$ and $i \in N$ such that $\forall j \in N \backslash \{i\}, p_j = p \neq p_i$ and (ii) $\frac{p\omega_i}{pz_i} z_i \leq \Omega$.

Case 6: (i) $\exists p \in \Delta^{m-1}$ and $i \in N$ such that $\forall j \in N \backslash \{i\}, p_j = p \neq p_i$ and (ii) $\frac{p\omega_i}{pz_i} z_i > \Omega$.

(III) $\xi : S \to A$ is given by the following rules:

Rule 1: Case 1 $\Longrightarrow \forall i \in N, \xi_i(s) = \frac{\|z_i\|}{\sum_{j \in N} \|z_j\|} \Omega$.

Rule 2: Case 2 or Case 3 or Case 6 $\Longrightarrow \xi(s) = \omega$.

Rule 3: Case 4 $\Longrightarrow \xi(s) = z$.

Rule 4: Case 5 $\Longrightarrow \xi_i(s) = \frac{p\omega_i}{pz_i} z_i$ and $\forall j \in N\backslash\{i\}, \xi_j(s) = \frac{\Omega - \xi_i(s)}{n-1}$.

Remark 8: This mechanism is a modified version of the mechanism devised in Hurwicz, Maskin and Postlewaite (1984). The outcome rules are exactly the same. The strategy space has been reduced by using the fact that $m = 2$. Thus, for the case $m = 2$, we have the dimensionality of Γ_3's strategy space equal to $n(m-1)+n$. Given $n > 2$, this does not exceed $n(m-1) + 3m + 2n - 9$.

Lemma 6: $\forall r \in \mathcal{R}, E_A(\Gamma_3, r) = W(r)$.

Proof of Lemma 6: See Hurwicz, Maskin and Postlewaite (1984).

Proof of Theorem 3: This follows from Lemmata 3-6 and Remarks 6 and 8. Q.E.D.

Theorem 4: *Suppose $\mid N \mid > 2$. If there exists a mechanism $\Gamma \in \mathcal{G}$ satisfying P1, P2, P3 and P4, then $\rho(\Gamma, m, n) \leq 3m + 2n - 3$.*

Consider the following game, denoted Γ_4:

(I) N contains a sub-group, T, with $\mid T \mid = 3$.

(II) $\forall i \in T, S_i = \{s_i = (z_i, p_i, t_i) \in A_i \times \Delta^{m-1} \times \mathrm{R}_+\}$.

$\forall i \in N\backslash T, S_i = \{s_i = (z_i, t_i) \in A_i \times \mathrm{R}_+\}$

Let $a_i \equiv \sum_{j,k \in T\backslash\{i\}} (p_j - p_k)^2$, and $\forall i \in T$,

Let

$$b_i = \begin{cases} \frac{a_i}{\sum_{j \in T} a_j}, & \text{if } \sum_{j \in T} a_j \neq 0; \\ \frac{1}{3} & \text{if } \sum_{j \in T} a_j = 0. \end{cases}$$

Let $p^* \equiv \sum_{i \in T} b_i p_i$.

Given p^* as defined above, $s_i = (z_i, p_i, t_i)$ for $i \in T$, and $s_i = (z_i, t_i)$ for $i \in N\backslash T$, define $\theta_i(s_i)$ to be the closest point to z_i such that $\theta_i(s_i) \in B_i(p^*)$.

Let $\mathcal{T} \equiv \{t \in \mathrm{R}_{++} : t \times t_i \leq 1, \forall i \in N$, and $t \times \sum_{i \in N} t_i z_i \leq \Omega\}$.

Let $t^* \equiv \max_{t \in \mathcal{T}} t$.

(III) $\forall i \in N, \forall s \in S, \xi_i(s) = t^* t_i \theta_i(s_i)$.

Remark 9: This mechanism is a modified version of a mechanism devised by Postlewaite and Wettstein (1983). The outcome function is a modified version of the outcome function used in that paper. The strategy space has been reduced by introducing price-specialists who are the only agents who announce prices. The dimensionality of the strategy space of this mechanism is $(n-3)(m+1)+6m$, which can be re-written as $n(m-1)+2n+3m-3$.

Lemma 7: $\forall r \in \mathcal{R}, E_A(\Gamma_4, r) = W(r)$.

Proof of Lemma 7: This lemma follows from a slight modification of the proof of this result by Postlewaite and Wettstein (1983).

Proof of Theorem 4: This follows from Lemma 7 and Remark 9. Q.E.D.

4. Concluding Remarks

The results of this paper can be summarized by the table in Figure 2.

[Insert Figure 2 here]

Apart from the fact that we have established upper bounds on the degree of communication requirements for implementation schemes with differing desirable properties, certain broad implications emerge from these numbers. For given (m, n), the level of incremental communication requirements, i.e. $\rho(\Gamma, m, n)$, increases as the number of properties desired of the mechanism Γ increases. Moreover, as for the two benchmarks we had set up in terms of the IIT and DCR hypotheses, from the table it is clear that both can be rejected. In other words, the pessimistic findings of Reichelstein and Reiter (1985) and Saijo (1986b) do *not* generalize. Finally, as far as further extensions of this study are concerned, more concrete conclusions will be given once lower bounds or minimal dimensionality requirements are established for the different cases considered here. As mentioned earlier, measures of computational complexity should also be incorporated to give a complete idea of the true "costs" of achieving incentive compatibility.

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FIGURE 1

Let $s = (z_k, q_k)_{k \in N} \in S$.

Case 1: (i) $z \notin W(R)$ and (ii) $\exists i \in N$ such that s_{-i} satisfies either Property $\alpha|i$ or Property $\beta|i$.

Case 1A

(i) $K(s) = \{i\}$ and (ii) $z_i \in B_i(q^*(s))$

$\xi_i(s) = z_i$,

$\forall j \in N \backslash \{i\}$, $\xi_j(s) = (\Omega - z_i)/(n - 1)$

Case 1B

Otherwise

$\xi(s) = \omega$

Case 2:

Otherwise

Case 2A

$z \notin W(R)$

Case 2AA

$\exists i \in N$ such that $K(s) = \{i\}$ or

$\exists i \in N$ such that $\forall j \in K(s)\setminus\{i\}$, $q_i < q_j$

$\xi_i(s) = z_i$,

$\forall j \in N\setminus\{i\}$, $\xi_j(s) = (\Omega - z_i)/(n - 1)$

Case 2AB

Otherwise

$\forall k \in N$, $\xi_k(s) = \omega$

Case 2B

$z \in W(R)$

$\xi(s) = z$

FIGURE 2

Upper bound on $\rho(\Gamma, m, n)$.	P1	P2	P3	P4
0	+	—	—	—
$\min_{\nu}\{\nu \in \mathbb{N}: (n-1)\nu \geq (m-1)\}$ $\mathbb{N} = \{1, 2, \ldots\}$	+	+	+	—
$3m + 2n - 9$	+	+	—	+
$3m + 2n - 3$	+	+	+	+

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