Faculty Working Papers

A COMPARATIVE ANALYSIS OF THE PREDICTIVE ABILITY OF ADAPTIVE FORECASTING, REESTIMATION AND REIDENTIFICATION USING BOX-JENKINS TIME SERIES ANALYSIS

James C. McKeown and Kenneth S. Lorek

#327

College of Commerce and Business Administration
University of Illinois at Urbana-Champaign

FACULTY WORKING PAPERS

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

August 19, 1976

A COMPARATIVE ANALYSIS OF THE PREDICTIVE ABILITY OF ADAPTIVE FORECASTING, REESTIMATION AND REIDENTIFICATION USING BOX-JENKINS TIME SERIES ANALYSIS

James C. McKeown and Kenneth S. Lorek

#327

A COMPARATIVE ANALYSIS OF THE PREDICTIVE ABILITY OF ADAPTIVE FORECASTING, REESTIMATION AND REIDENTIFICATION USING BOX-JENKINS TIME SERIES ANALYSIS

By

James C. McKeown

and

Kenneth S. Lorek*

* *Associate Professor, Department of Accountancy, University of Illinois at Urbana-Champaign and Assistant Professor, College of Business, The Florida State University, respectively.*

ABSTRACT

This paper assesses the predictive ability of the Box-Jenkins methodology when utilized in an ongoing setting. Three procedures are utilized to update the original forecasts generated from the Box-Jenkins models: adaptive forecasting, reestimation and re-identification. The results indicate that constant monitoring of the structure and parameters of the time series models are necessary through time. It appears that adaptive forecasting techniques are insufficient in performing this function.

A COMPARATIVE ANALYSIS OF THE PREDICTIVE ABILITY OF ADAPTIVE FORECASTING, REESTIMATION AND REIDENTIFICATION USING BOX-JENKINS TIME SERIES ANALYSIS

INTRODUCTION

A considerable amount of evidence has been reported recently concerning the applicability of Box-Jenkins (hereafter referred to as BJ) time series analysis in predictive settings.*[1][3][4][5][6][7]* These studies have assessed the predictive ability of BJ models in various manners: (1) simple presentation of forecast accuracy without significance testing *[1][5]*, (2) comparative analysis of the forecasting accuracy of BJ time series models relative to certain simplistic models *[7]*, (3) comparative analysis of the forecasting accuracy of the BJ time series models and econometric models *[6]*, and (4) comparative analysis of the accuracy of BJ models and management forecasts.*[3][4]*

The results of the above studies have consistently demonstrated the powerful nature of the autoregressive integrated moving average (hereafter referred to as ARIMA) models. These models are particularly attractive considering the possibility of describing highly variant time series behavior. Moreover, the BJ methodology utilizes an inherent structure in the determination of the functional form of the model eliminating potential arbitrariness in the model selection process.

An important issue which has not been systematically researched concerns the predictive ability of the BJ ARIMA models in an on-going setting. As more new observations of a time series become available, what procedures should be utilized in updating either the identified time series model or its resultant forecasts?[1]

In this paper, we examine specifically three possible techniques which may be employed to update BJ predictions as more data become available: adaptive forecasting, reestimation and respecification. The time series data bases which we examined are comprised of quarterly earnings data from a sample of 30 New York Stock Exchange firms.

ARIMA MODELS

The class of linear models most appropriate for quarterly earnings data is the ARIMA model adjusted for seasonality factors, commonly referred to as the general multiplicative seasonal model. Let the difference operator ∇ be defined by:

$$\nabla Z_t = (1-B)\, Z_t = Z_t - Z_{t-1} \tag{1}$$

where B is called a backshift operator in the general form: $B^n Z_t = Z_{t-n}$. The entire general multiplicative seasonal ARIMA model can be represented as follows:

---

[1] *We are not concerned with the detection of radical departures in the nature of the time series data under examination. Detection of such turning points may be undertaken utilizing the parabolic mask procedures suggested by Brown [2].*

$$\phi_p(B)\,\Phi_P(B^s)\nabla^d\nabla_s^D\,\tilde{Z}_t = \theta_q(B)\,\Theta_Q(B^s)\,a_t + \theta_0 \qquad (2)$$

where:

Z_t = the value of the original series at time t.

$\tilde{Z}_t = (Z_t - \mu)$ when $d = D = 0$.

a_t = the residuals or "white noise" at time t.

$\phi_p(B) = 1 - \phi_1 B - \cdots\cdots - \phi_p B^p$

$\Phi_P(B^s) = 1 - \Phi_1 B^s - \cdots\cdots - \Phi_P B^{sP}$

$\nabla^d = (1-B)^d$

$\nabla_s^D = (1-B^s)^D$

$\theta_q(B) = 1 - \theta_1 B - \cdots\cdots - \theta_q B^q$

$\Theta_Q(B^s) = 1 - \Theta_1 B^s - \cdots\cdots - \Theta_Q B^{sQ}$

θ_0 = deterministic trend parameter.

Equation (2) is referred to as the ARIMA pdq X PDQ model. The designations p and q refer to the order of the autoregressive and moving average polynomials $\phi(B)$ and $\theta(B)$. The letters d and D designate the degree of consecutive and seasonal differencing necessary to attain stationarity. The letters P and Q refer to the order of the seasonal autoregressive and moving average polynomials $\Phi(B^s)$ and $\Theta(B^s)$.

UPDATING PROCEDURES

Assuming that we have a pdq X PDQ model for a given time series $(Z_1, Z_2, \cdots Z_n)$, what updating techniques can

be implemented when observations (Z_{n+1}, Z_{n+2}, ... Z_{n+m}) become available? In other words, what adjustments to the original model or the predictions of the original model are necessary to take into consideration the information impounded in the new observations? Three possible alternatives are explored in this paper: adaptive forecasting, reestimation and reidentification.

In the utilization of adaptive forecasting, the functional form of the original time series model remains unchanged. However, the remaining predictions in the forecast horizon are modified to incorporate the informational content of the new data. For example, assume data base (Z_1, Z_2, ..., Z_t) is utilized to predict Z_{t+1} and Z_{t+2}. Then,

$\hat{Z}_t(1)$ = forecasted value of Z_{t+1} at time t

$\hat{Z}_t(2)$ = forecasted value of Z_{t+2} at time t.

As a new observation of data becomes available (Z_{t+1}), adaptive forecasting may be utilized to update the remaining forecasts, $\hat{Z}_t(2)$ in this case. The change in the original forecast of Z_{t+2} may be represented as follows:

$$\hat{Z}_{t+1}(1) - \hat{Z}_t(2) = \text{factor} \cdot (Z_{t+1} - \hat{Z}_t(1)) \qquad (3)$$

The LHS of equation (3) is the difference between the updated forecast of Z_{t+2} and the original forecast of Z_{t+2}. This difference is due to the informational content of the

additional data. The RHS of equation (3) reveals the specific nature of the updating process, a model specific factor[2] multiplied by the forecast error associated with the prediction of Z_{t+1}.

In utilizing reestimation procedures the identification stage of the BJ iterative model building process is bypassed and the parameters of the original model are simply reestimated. The new data are appended to the existing data base and rerun through the non-linear least squares algorithm. For example, if the original model were an autoregressive process of order one with a ϕ_1 parameter equal to .6, the functional form of the model would remain the same but the ϕ_1 parameter might be changed dependent upon the impact of the new data. This technique differs from adaptive forecasting in that parameter values of the original model may be changed. It is similar to adaptive forecasting in that the structure of the model remains unchanged.

Finally, the third alternative, reidentification, involves a complete reapplication of the BJ methodology whenever a new observation of data becomes available. The adjusted data base is input sequentially through the iterative stages of identification, estimation and diagnostic checking. This technique differs from the first two alternatives by allowing for structural change in the original model. For example, if the original model were an autoregressive process

---

[2]Nelson [6, pp. 157-159] provides several examples of how the factor term is derived. In actuality, the factor term is dependent upon the parameters of the original model.

of order one, the reidentification technique would ignore this information and allow for the possibility of any ARIMA model to be identified.

Thus, adaptive forecasting is clearly the most convenient mechanism available for updating BJ forecasts. Reestimation is somewhat more complex considering that parameter values must be re-examined. Reidentification is the most involved updating technique examined; it employs all three modelling stages in the BJ methodology with the addition of each new data point.

DATA

A sample of New York Stock Exchange firms was obtained by utilizing the following sampling criteria:

1. Only calendar year reporting firms were selected.
2. Quarterly earnings were reported in the Wall Street Journal Index from 1959 to 1974.
3. Random sampling techniques were employed subject to the above criteria until a sample size of 30 was obtained. (See Table 1 for a listing of the sample firms.)

The first sampling criterion was motivated by pragmatic considerations. By focusing attention on calendar year firms, data acquisition was concentrated upon specific time intervals surrounding the interim reporting dates. Since data procurement was performed manually, only specific sections of each yearly volume of the Wall Street Journal Index needed to be referenced.

Table 1

LIST OF SAMPLE FIRMS

1. American Home Products Corporation
2. Anchor Hocking Glass Corporation
3. Armco Steel Corporation
4. Bausch and Lomb Optical Corporation
5. Baxter Laboratories Incorporated
6. Belden Corporation
7. Beneficial Finance Corporation
8. Campbell Red Lake Mines Ltd.
9. Coca-Cola Corporation
10. Crane Company
11. Diebold Incorporated
12. Duquesne Light Company
13. Federal-Mogul Bower Bearings
14. Gardner-Denver Company
15. Goodyear Tire & Rubber Company
16. Hercules Incorporated
17. Interstate Power Company
18. Koppers Incorporated
19. Liggett & Myers Tobacco
20. Long Island Lighting Company
21. National Starch & Chemical Corporation
22. Niagra Mohawk Power Company
23. Pfizer Incorporated
24. Pullman Incorporated
25. Rohm & Haas Company
26. St. Regis Paper Company
27. Texas Utilities Company
28. Timken Company
29. Union Carbide Corporation
30. Wm. Wrigley Jr. Company

The second criterion was necessary to fulfill the suggested minimum data requirements of the BJ methodology. *[1, p. 18]* Although the results of this study may not be generalized to all New York Stock Exchange firms or other types of data, we feel that the quarterly earnings data are representative of the types of data previously examined in conjunction with the BJ methodology.

METHODOLOGY

Data bases were compiled for each of the 30 sample firms consisting of 64 quarterly earnings numbers. They encompassed the time period from 1959 to 1974. However, the last three years (12 quarters) were deleted initially from the model identification process. These actual quarterly earnings numbers were held back to assess the accuracy of the BJ predictions. Thus, for each sample firm, a 52 observation data base (quarterly earnings from 1959 to 1971) was utilized to identify the most appropriate BJ model. After identification of the models and the estimation of their parameters, they were then utilized to predict the next 12 quarterly earnings numbers for each firm (quarterly earnings from 1972-1974).[3]

Since our primary concern was the predictive ability of BJ models as new observations of data became available, forecasts of the remaining quarterly earnings numbers in the forecast horizon were updated each time we appended a new observation to the data base. Specifically, when we appended the actual first quarter earnings numbers for 1972 to the existing 52 observation data bases, we were interested in the impact these had upon the predictions of the 11 remaining quarterly earnings numbers for each sample firm. This process was repeated an additional 10 times until the only quarterly

---

[3] *We selected 12 quarters or three years as the forecast horizon to minimize short run aberrations in a particular year which could potentially bias the results of the study.*

earnings number remaining in the forecast horizon was the fourth quarter earnings of 1974.

Although an examination of the time series properties of quarterly earnings data is beyond the scope of this paper, the impact that reestimation and reidentification had upon the structure of the originally identified BJ model is of interest. We observe that the first updating technique examined in this paper, adaptive forecasting, does not alter the structure of the originally identified BJ model. Hence, no model information is reported for this technique.

Table 2

THE EFFECT OF REESTIMATION ON THE TIME SERIES MODELS OF GARDNER-DENVER CO.

NOB	pdq X PDQ	θ_0	θ_1	θ_2	θ_3	θ_7
52	004 X 010	244.80	-.56	-.58	-.45	.90
53	"	251.37	-.55	-.53	-.42	.90
54	"	251.99	-.56	-.54	-.41	.90
55	"	243.68	-.57	-.54	-.40	.88
56	"	246.64	-.56	-.53	-.40	.88
57	"	239.12	-.56	-.54	-.41	.88
58	"	230.15	-.57	-.55	-.41	.88
59	"	264.50	-.55	-.59	-.50	.92
60	"	186.91	-.62	-.65	-.51	.85
61	"	299.49	-.50	-.56	-.50	.97
62	"	152.45	-.80	-.96	-.59	.47
63	"	181.75	-.34	-.82	-.10	.44

Table 3

THE EFFECT OF REIDENTIFICATION ON THE TIME SERIES MODELS OF GARDNER-DENVER CO.

NOB	pdq X PDQ	ϕ_1	θ_0	θ_1	θ_2	θ_3	θ_7	Θ_1	Θ_2
52	004 X 010		244.80	-.56	-.58	-.45	.90		
53	102 X 011	.62	122.60			-.18		.51	
54	004 X 010		250.59	-.56	-.54	-.41	.90		
55	"		242.86	-.57	-.54	-.40	.88		
56	"		246.57	-.56	-.53	-.40	.88		
57	"		239.19	-.56	-.54	-.41	.88		
58	"		229.15	-.57	-.55	-.41	.87		
59	101 X 011	.64	136.10					.55	
60	006 X 012		340.73	-.65	-.46	-.34	.79	.36	.39
61	005 X 011		328.22	-.57	-.35	-.27	.83	.52	
62	101 X 011	.63	108.13					.63	
63	002 X 010		309.54	-.56			.62		

where: NOB = the number of observations in the data base.

The Gardner-Denver Co. time series models were selected as indicative of the types of changes reestimation and reidentification had upon the original time series models. The intention of this example is to provide an intuitive feeling as to the differential effects of the two methods. Tables 2 and 3 depict the changes in the original model of Gardner-Denver Co. when reestimation and reidentification procedures were undertaken. The original BJ model identified for Gardner-Denver Co. using a 52 observation data base can be represented in pdq X PDQ notation as 004 X 010. There are four moving average parameters at lags one, two, three and seven. Seasonal differencing was utilized to attain stationarity and a deterministic trend parameter was detected.

When the 53rd observation was appended to the data base, reestimation procedures required that the identification stage of the BJ methodology be bypassed. The 004 X 010 model structure was assumed to be still appropriate and the existing parameters were simply updated by inputing the expanded data base into the estimation stage of the BJ methodology. The parameters of the 004 X 010 model appear very stable when the number of observations ranges from 52-58. However, the parameters oscillate markedly between 59-63 observations; perhaps casting doubt on the propriety of the 004 X 010 model for these expanded data bases.

Since reidentification procedures avoid any a-priori assumptions regarding the propriety of the original BJ model, the functional form of the most appropriate model (see pdq X PDQ column in Table 3) may change. This procedure resulted in a different BJ model for 53 observations: 102 X 011 with a return to the original structure between observations 54-58. This pattern is disrupted between observations 59-63. Thus, in addition to parameter changes the functional form of the model changes as well.

HYPOTHESES

Although Box and Jenkins *[1, pp. 134-35]* and Nelson *[6, pp. 157-59]* discuss the use of adaptive forecasting to update predictions, no comparative assessment is available concerning the use of more involved updating procedures, i.e., reestimation and reidentification. In order to provide

information regarding the comparative predictive ability of these three updating techniques, the following hypotheses were tested:

H_o:1 As more data become available, the updating of BJ predictions using reestimation results in more accurate predictions than adaptive forecasting.

H_o:2 As more data become available, the updating of BJ predictions using reidentification results in more accurate predictions than reestimation.

RESULTS

The accuracy of the predictions generated from adaptive forecasting, reestimation and reidentification was assessed by utilizing Theil's U Coefficient. *[8]* The U Coefficient is confined to the interval between zero and one. That is, the closer the value of the U Coefficient is to zero the more accurate is the forecast, and the farther removed from zero the less accurate. The U Coefficient is computed in the following manner:

$$U = \frac{\sqrt{\frac{1}{n}\Sigma\,(P_i - A_i)^2}}{\sqrt{\frac{1}{n}\Sigma\,P_i^2} + \sqrt{\frac{1}{n}\Sigma\,A_i^2}} \tag{4}$$

where:

P_i = predictions of quarterly earnings

A_i = actual quarterly earnings

n = the number of predictions

The evaluations made in this paper were based on paired comparisons between the sample means of the U Coefficients. The U Coefficients were calculated by reference to the predictions generated from the BJ models using the three alternative techniques. Sample means were compiled by computing the average value of the U Coefficients across all 30 firms. These values are reported in Table 4.

Table 4

SAMPLE MEANS OF U COEFFICIENTS

NOB	P	Adaptive	Reestimation	Reidentification
53	11	.238	.228	.230
54	10	.244	.224	.215
55	9	.247	.215	.214
56	8	.258	.226	.219
57	7	.260	.216	.201
58	6	.265	.198	.180
59	5	.279	.209	.196
60	4	.284	.205	.197
61	3	.319	.237	.209
62	2	.323	.250	.202
63	1	.421	.352	.334

where: NOB = number of observations in data base
P = number of quarterly earnings predictions generated

For purposes of testing the statistical significance of H_0:1 and H_0:2, the one-tailed "student's matched-pair" t-test was utilized. Table 5 summarizes the results of the statistical analysis.

Table 5

SIGNIFICANCE TESTING

NOB	P	H_o:1	H_o:2
53	11	.10	NS
54	10	.05	NS
55	9	.05	NS
56	8	.05	.10
57	7	.05	.05
58	6	.01	.05
59	5	.01	.10
60	4	.01	.10
61	3	.05	.05
62	2	.05	.05
63	1	.05	NS

where: NOB = number of observations in data base
P = number of quarterly earnings predictions generated
NS = non-significant difference

Table 5 is interpreted in the following manner: If there is a significant difference in the updating techniques an α level between .01-.10 is indicated. If there is a non-significant difference in the predictions, NS is indicated. Specifically, when NOB = 53 predictions are generated for 11 quarterly earnings numbers using three different updating techniques. The eleven forecasts generated by reestimation were significantly superior (α=.10) to the eleven forecasts generated by adaptive forecasting (H_o:1). However, there was no significant difference (NS) between the forecasts generated by reestimation and reidentification (H_o:2).

INTERPRETATION OF RESULTS AND CONCLUSIONS

The results reported herein indicate that adaptive forecasting provided significantly larger forecast errors when compared to reestimation. This finding is consistently demonstrated regardless of the number of predictions in the forecast horizon. The comparisons between reestimation and reidentification are not as straightforward. When the number of predictions in the forecast horizon varied from 2-8, reestimation provided significantly larger forecast errors. When the number of predictions ranged between 9-11 and 1 there was no significant difference.

Of course, these results are subject to the sampling procedures employed, the specific data examined and the error measure selected. However, given the recent proliferation of BJ applications in the literature, more information is necessary regarding the adaptive ability of the BJ models. The results suggest that constant monitoring of the adequacy of the structure of the BJ model as well as its parameters is necessary to maintain the predictive ability of the model. If appears that adaptive forecasting may be insufficient to provide this function.

REFERENCES

[1] Box, G.E.P. and G.J. Jenkins. Time Series Analysis: Forecasting and Control. San Francisco: Holden-Day, 1970.

[2] Brown, Robert G. "Detection of Turning Points in a Time Series," Decision Sciences, Vol. 2, No. 4 (October, 1971), pp. 383-404.

[3] Lorek, Kenneth S., McDonald, Charles L. and Dennis H. Patz. "A Comparative Examination of Management Forecasts and Box-Jenkins Forecasts of Earnings," The Accounting Review, Vol. LI, No. 2 (April, 1972), pp. 321-330.

[4] Mabert, Vincent A. "Statistical Versus Sales Force-Executive Opinion Short Range Forecasts: A Time Series Analysis Case Study," Decision Sciences, Vol. 7, No. 1 (January, 1976), pp. 310-318.

[5] Mabert, Vincent A. and R.C. Radcliffe. "A Forecasting Methodology as Applied to Financial Time Series," The Accounting Review, Vol. XLIX, No. 1 (January, 1974), pp. 61-75.

[6] Nelson, Charles R. Applied Time Series Analysis for Managerial Forecasting. San Francisco: Holden-Day, 1973.

[7] Rogalski, Richard J. "Bond Yields: Trends or Random Walks?" Decision Sciences, Vol. 6, No. 4 (October, 1975), pp. 688-99.

[8] Theil, Henri. Economic Forecasts and Policy. Amsterdam: North Holland Publishing Company, 1961.