FACULTY WORKING PAPER 91-0182

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*The authors are grateful to Bjorn Flesaker, Steven D'arcy and Joseph Finnerty for their helpful comments and suggestions.

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ABSTRACT

Although random variance option pricing models are theoretically more sound than the Black-Scholes model, their empirical performances have not been proved successful for equity options. We find that one of the reasons may be either because the risk premium on the price volatility has been ignored or because it might not have been captured appropriately since the equity options used in previous studies are short-term. Using scores which are deep out-of-money, long-term and European call options, we show that the volatility risk premium is important and that the random variance model taking account of the risk premium is superior to the random variance model ignoring the risk premium and the Black-Scholes model.

A COMPARISON OF A RANDOM VARIANCE MODEL AND THE BLACK-SCHOLES MODEL FOR PRICING LONG-TERM EUROPEAN OPTIONS

A number of researchers have tried to improve the Black-Scholes model by relaxing the underlying assumptions. One of the problems in the model pertains to the assumption that the volatility of stock prices is constant, which has been rejected by numerous empirical studies (e.g., Scott (1987), Clark (1973), Epps and Epps (1976), Christie (1982), and Kon (1984)). The volatility of stock prices plays a very important role in option pricing and thus has received more attention than any of the underlying factors.

In recent years, several studies have examined random variance option pricing models (e.g., Eisenberg (1985), Hull and White (1987), Scott (1987), Wiggins (1987), Chesney and Scott (1989), Johnson and Shanno (1987), Bailey and Stulz (1989), Melino and Turnbull(1990)). Hull and White (1987) derive a series solution for the price of a call option on a security with a stochastic volatility that is uncorrelated with the security price. Scott (1987), in a framework similar to Hull and White (1987), develops an option pricing model that allows the variance parameter to change randomly, and shows that the option price depends on the risk premium associated with the random variance. Wiggins (1987) examines a random variance model and shows numerical solutions. Bailey and Stulz (1989) examine the performance of a three state option pricing model for stock index options. The common approach to pricing options in these papers is to treat the volatility as a random state variable.

Although the two or three state option pricing models are theoretically more sound than the Black-Scholes model, their empirical performances have not been proved successful with an exception of Melino and Turnbull(1990) for currency options. Using Canadian dollar options, Melino and Turnbull show that allowing volatility to be stochastic results in a better fit to the distribution of the Canada-U.S. exchange rates, and a stochastic volatility option model yields significantly better prediction than non-stochastic volatility option model. However, the empirical studies of random variance models for equity options either fail to produce better predictions, or show that they are at best marginally better than the Black-Scholes model.

There are some potential reasons for the relatively poor performance of random variance models for equity options. First, random pricing models have been applied to short-term options rather than long-term options since the exchange traded equity options last up to only 9-12 months. Therefore, the effect of the stochastic price volatility might not be captured appropriately. For example, Lauterbach and Schultz (1990) show that the constant variance assumption of an adjusted Black-Scholes model causes biases in pricing warrants which have lives of a few years. Second, many authors simply ignore the risk premium on price volatility in a random variance model, assuming that the stock price volatility is uncorrelated with the aggregate consumption of investors. This also may be because it is difficult to estimate the volatility risk premium since volatility is not a traded asset. Third, in most of previous studies, the Black-Scholes model and the random variance models have been applied primarily to American options that can be exercised prior to maturity. In addition, except at-the-money options, the previous studies show that options, in particular, deep out-of-money options are sensitive to the parameter of the stochastic process describing changes in volatility.[1]

This paper develops a random variance (two-state) option pricing model based on the general equilibrium model of Cox, Ingersoll, and Ross (1985), and compares it with the Black-Scholes model, using Scores which are long-term, deep out-of-money, European call options with lives of five years. For the long-term European call options, we show that the risk-premium on the price volatility is important and that the random variance model taking account of the risk premium is superior to the Black-Scholes model. However, we find no significant difference between the two models when the volatility risk premium is ignored.

Section I briefly describes the characteristics of the scores. Section II develops a two-state option pricing model when the security price volatility is stochastic, based on the general equilibrium model of Cox, Ingersoll, and Ross (1985). In Section III, we estimate the parameters of the stochastic process. In Section IV, we present empirical results comparing the Black-Scholes model with a random variance model in the absence and presence of the volatility risk premium. Section V contains a brief conclusion.

I. Review of Scores

Americus Shareowners Service Corporation has created Americus Trusts on 26 blue chips, the purpose of which was to divide an existing share of common stock into two distinct tradeable instruments: prime and score.[2] They include trusts on American Express, American Home Product, ATT-Series 2, Amoco, ARCO, Bristol Myers, Chevron, Coca-Cola, Dow, DuPont, Kodak, Exxon, Ford, GE, GM, GTE, Hewlett Packard, IBM, Johnson & Johnson, Merck, Mobil, Philip Morris, Procter & Gamble, Sears, Union Pacific, and Xerox. The conversion of one share into a score and a prime allows investors to separate the potential capital appreciation in excess of a stipulated dollar amount from the right to receive dividends and all other attributes of share ownership. The life of each trust is set to be five years.

At the beginning of each trust, a shareholder can elect to tender each share to receive a unit. Each unit consists of one prime and one score. The owner of the prime (primeholder) receives dividends and any appreciation in price up to a predetermined termination value, while the owner of the score (scoreholder) receives the capital appreciation on the underlying stock, if any, over the predetermined termination value.[3]

The payoff on a score at the termination date can be written as $f_T = MAX[V_T - X, 0]$ where f_T is the value of the score at time T; V_T is the value of the underlying stock at time T; X is the termination value; and T is the termination date. In all of the scores, the termination claims were set to be far greater than the current stock prices. Therefore, the scores are equivalent to deep out-of-money European call options with maturities of five years. The data of the scores give us a unique opportunity to examine a random variance option pricing model relative to the Black-Scholes model in that they are deep out-of-money, European and long term (five years) call options.

II. Pricing Scores with Stochastic Variance of the Underlying Security

A. Assumptions

(1) The market is frictionless and borrowing and lending are allowed without restriction.

(2) There is a riskless asset whose rate of return per unit time, r, is known and constant over time.[4]

(3) There are no transaction costs or taxes.

(4) Securities are traded in continuous time.

(5) The stock price follows the stochastic differential equations as

$$dV = (\alpha V - c)\,dt + \sigma V dz_1$$

$$d\sigma = \beta(\underline{\sigma} - \sigma)\,dt + \theta dz_2$$

where z_1 and z_2 are standard Wiener processes with the correlation coefficient, δ; V is the stock price; α is the instantaneous expected return on the stock per unit time; σ^2 is the instantaneous variance of the rate of return; c is the instantaneous cash outflow per unit time; β is the speed of adjustment coefficient of the stock price volatility; $\underline{\sigma}$ is the mean reverting level of the volatility; θ^2 is the instantaneous variance of the volatility process.

However, for empirical works later in this paper, V and σ will be transformed into ln V and ln σ, repectively, to have nonnegativity of the volatility and tractability of computation.

(6) Dividends are continuously paid to the primeholder at a rate of π and are proportional to the stock price.

B. Partial Differential Equation of Score

The assumption (5) for the volatility process creates some complications. Scott (1987) shows that unlike the Black-Scholes framework, a dynamic portfolio with only one option and one stock is not sufficient any longer for creating a riskless strategy. The problem is that we cannot value the score by arbitrage methods since the price of the state variable such as the variance of stock prices is not observable or traded. Cox and Rubinstein (1985) point out this problem stating that "the volatility may depend on random variables other than the stock price or interest rates may fluctuate randomly over time. How can we value an option when these factors are too important to be ignored? Here we are reaching a point where option pricing theory ceases to be a separate area and becomes part of a general theory of asset valuation." (p. 420).

In this paper, we rely on the general equilibrium asset pricing model developed by Cox, Ingersoll and Ross (1985).[5] From the results of Theorem 3 and Lemma 4 of CIR, the expected return on the score can be written

$$E(df/f) = \{r + f_V V/f(\alpha - r) + f_\sigma / f\mu*\}\,dt \qquad (1)$$

where subscripts indicate partial derivatives and $\mu*$ represents the risk premium on the stock price volatility.[6]

The score, f(V,σ,t), can be expressed then as follows by Ito's Lemma

$$df = [\frac{1}{2}\sigma^2 V^2 f_{VV} + \delta\sigma\theta V f_{V\sigma} + \frac{1}{2}\theta^2 f_{\sigma\sigma} + (\alpha V - c) f_V$$
$$+ \beta(\underline{\sigma} - \sigma) f_\sigma - f_\tau]\,dt + \sigma V f_V dz_1 + \theta f_\sigma dz_2 .$$

The expected value of df is

$$E(df) = [\frac{1}{2}\sigma^2 V^2 f_{VV} + \delta\sigma\theta V f_{V\sigma} + \frac{1}{2}\theta^2 f_{\sigma\sigma} + (\alpha V - c) f_V$$
$$+ \beta(\underline{\sigma} - \sigma) f_\sigma - f_\tau]\,dt. \qquad (2)$$

From (1) and (2), we obtain the P.D.E. for the score as

$$\frac{1}{2}\sigma^2 V^2 f_{VV} + \delta\sigma\theta V f_{V\sigma} + \frac{1}{2}\theta^2 f_{\sigma\sigma} + \beta(\underline{\sigma}-\sigma) f_\sigma - f_\tau - rf + f_V Vr - f_\sigma \mu * - cf_V = 0 \quad (3)$$

s.t. $f(V,\sigma,0) = \text{Max}(V\text{-}X,0)$, $f(0,\sigma,\tau) = 0$, and $f(V,\sigma,\tau) \leq V$.

III. Estimating Parameters of the Stochastic Process

A. Methodology

To estimate the stochastic parameters, β, θ and $\underline{\sigma}$, of the stock price, we use the generalized method of moments (GMM) developed by Hansen (1982), assuming that the unobservable variance information is imbedded in the stock price movements. Hansen shows that the GMM estimator is consistent and asymptotically normal. Applications of the GMM to estimating stochastic processes can be found in many studies(e.g., Scott (1987) and Wiggins (1987)).

Following the previous studies, we assume that stock prices follow a lognormal distribution. If we take the log transformation of V and apply Ito's Lemma,

$$y = \ln V$$

$$dy = (\alpha - \frac{1}{2}\sigma^2)\, dt + \sigma dz$$

The discrete approximation of the stock price process can be written

$$\Delta \ln V(t) = (\alpha - \frac{1}{2}\sigma_{t-1}^2)\Delta t + \sigma_{t-1}\theta z \quad (4)$$

To get the discrete approximation of the variance of the stock price process, a standard Orstein-Uhlenbeck process is assumed for ln σ, which gives us the maximum likelihood estimators.[7] From the results of Vasicek (1977), σ_t is normally distributed with the mean, $e^{-\beta\Delta t}\sigma_{t-1} + \underline{\sigma}(1-e^{-\beta\Delta t})$, and the variance, $\gamma^2(1-e^{-2\beta\Delta t})/2\beta$, when σ follows an O-U process. Therefore, the discrete approximation of σ can be written as

$$\sigma_t = e^{-\beta}\sigma_{t-1} + \underline{\sigma}(1-e^{-\beta}) + \varepsilon_t$$

where Δt is assumed to be 1.

The discrete approximation for ln σ is

$$\ln \sigma_t = e^{-\beta}\ln \sigma_{t-1} + \underline{\sigma}(1-e^{-\beta}) + \varepsilon_t \qquad (5)$$

Equations (4) and (5) can be transformed for empirical analysis as

$$\Delta\ln V_t = \pi + \sigma_{t-1}u_t \qquad (6)$$

$$\ln \sigma_t = a + \rho \ln \sigma_{t-1} + \varepsilon_t \qquad (7)$$

From equations (4), (5), (6) and (7),

$$\underline{\sigma} = a/(1-\rho)$$

$$\beta = -\ln \rho \qquad (8)$$

$$\theta^2 = \sigma_\varepsilon^2(-2 \ln \rho)/(1-\rho^2)$$

Following Scott (1987), we use $X_t = \Delta\ln V_t - \mu = \sigma_{t-1}u_t$, where μ is a constant term, to estimate the parameters of the stochastic process of the random variance. Using ARMA(1,1) model, ρ can be estimated, and β, $\underline{\sigma}$, and θ can be derived from equation (8).

B. Data and Results

The daily stock returns obtained from the CRSP tapes are used to estimate the parameters of the stochastic process of volatility. We use only the stocks whose scores are available without missing observations since July 1, 1987. These include American Home Products, Amoco, ARCO, ATT, GM, Exxon, Kodak, GE, DuPont, Dow, Union, Ford, Procter & Gamble, Chevron, Mobil, GTE, and Sears. The estimation period runs from July 1978 to June 1987. The samples have 2,528 observations for each company. The trading volumes for primes are, in general, low but scores have been very active since the inception of their trading.

We compute first the log of deviations from the sample means, which are used later to estimate the parameters of the stochastic process of volatility. Table 1 shows the means, variances, and kurtosis of the deviations. To apply the GMM, kurtosis should be greater than three to avoid negative variances. Sample estimates of kurtosis range from 4.259 to 6.545. The kurtosis of normal distribution is three, so the distribution of the deviations from their sample means is longer tailed than normal distribution. Table 2 presents the parameter estimates of the ARMA model. The range of ρ is from 0.87 for Dow to 0.998 for GM. The ρ estimates are all close to one, which are consistent with the estimates of Scott (1987). Table 3 presents the estimates for $\underline{\sigma}$, β, and θ for the seventeen companies. On average, the target variance is 0.217, the speed of adjustment coefficient of variance is 0.02, and the standard deviation of proportional changes in variance is 0.058. These parameter estimates are used in calculating score values in the random variance model. The correlation coefficient between the stock price and its variance, δ, is assumed to be zero in this study.[8]

IV. Pricing Scores Using Random Variance Model

A. Methodology

Based on the parameter estimates, the volatility risk premiums and the score values are estimated for the testing period, July 1987-June 1989. However, it is very difficult to estimate directly the risk premiums from the past stock prices. In this paper, following Melino and Turnbull(1990), we treat the risk premium as a free parameter and estimate it from the observed option prices. By minimizing the differences between theoretical score values and actual score prices, the (implied) risk premium can be estimated.[9] A score value can be found by solving the P.D.E. of the score for a given parameter, μ^*.

Most P.D.E. do not have an exact solution so they often must be approximately solved by numerical methods. The binomial approach, the Monte Carlo approach, and the finite difference method have been used, in general, to get approximate solutions for P.D.E.'s. The line hopscotch method proposed by Gourley (1970), which is a mixed explicit and implicit finite difference method, is used in this paper.

Let $y = \ln V$, $x = \ln \sigma$, and $w(y,x,\tau) = f(V,\sigma,\tau)$. Then, the P.D.E. for the score can be transformed into the following equation,[10]

$$\frac{1}{2}\sigma^2(w_{yy}-w_y) + (r-\pi)w_y - rw - w_\tau + \frac{1}{2}\theta^2(w_{xx}-w_x) + \delta\theta\sigma w_{yx}$$

$$+ w_x\left(\frac{1}{2}\theta^2+\beta(\underline{\alpha}-\ln\sigma)-\mu^*/\sigma\right) = 0$$

This equation can be changed into an explicit version of the finite difference method

$$_{t+1}w_{y,x} = E_1\ _tw_{y,x} + E_2\ _tw_{y+1,x} + E_3\ _tw_{y-1,x} + E_4\ _tw_{y,x+1} + E_5\ _tw_{y,x-1}$$

$$+ E_6\ _tw_{y+1,x+1} + E_7 w_{y-1,x+1}$$

where $_{t+1}w_{y,x}$ is the value of w at time t+1 given the values of x and y,

$$E_1 = 1 - r\Delta t - \Delta te^{2x}/(\Delta y)^2 - \Delta t\theta^2/(\Delta x)^2 - \Delta t\delta\theta e^x/\Delta x\Delta y$$

$$E_2 = \Delta te^{2x}/2(\Delta y)^2 + \Delta t\delta\theta\sigma/2(\Delta y)^2 + \Delta t/\Delta y(-e^{2x}/4+r/2-\pi/2)$$

$$E_3 = \Delta t\delta\theta e^x/\Delta x\Delta y + \Delta t/\Delta y(e^{2x}/4-r/2-\pi/2) + \Delta te^{2x}/2(\Delta y)^2$$

$$E_4 = \Delta t\theta^2/2(\Delta x)^2 + \Delta t\delta\theta e^x/2(\Delta x)(\Delta y) + \Delta t/2\Delta x(\beta(\underline{\alpha}-x)-\mu*/e^x)$$

$$E_5 = \Delta t\theta^2/2(\Delta x)^2 + \Delta t\rho\delta e^x/2(\Delta x)(\Delta y) + \Delta t/2\Delta x(-\beta(\underline{\alpha}-x)+\mu*/e^x)$$

$$E_6 = -\Delta t\delta\theta e^x/2(\Delta x)(\Delta y)$$

$$E_7 = -\Delta t\delta\theta e^x/2(\Delta x)(\Delta y)$$

subject to the boundary conditions

$$w_{0,x} = 0$$

$$w_{M+1,x} = w_{M,x} + \exp(y_{M+1}) - \exp(y_M)$$

$$x_{y,N+1} = w_{y,N}$$

$$w_{y,0} = w_{y,1}$$

where M and N are the number of steps in y and x, respectively.

B. Results

The score values are calculated using line hopscotch FORTRAN 77 routines. The score values are calculated using a space grid with 50 steps for ln V and ln σ. The time grid has 52 steps per year, roughly one step per week. Step sizes for ln V and ln σ are each set at 0.05. Truncation error can be reduced by using small time grids, but smaller time grids require longer computing time. The control variate technique is useful in reducing truncation error when large time grids are used.

For a comparison purpose, the volatility risk premium is set to be zero for initial application of the random variance option pricing model. The dividend yields, calculated using dividends and stock prices at the ex-dividend dates from June 1978 to June 1987, are used in the Black-Scholes and random variance pricing models.

Table 4 presents the results. Overall, it does not appear that there are significant differences between the random variance model in the absence of the volatility risk premium and the Black-Scholes model. To investigate further, the mean absolute deviations are compared. Table 5 presents the results. The random variance model ignoring the volatility risk premium is marginally better than the Black-Scholes model in predicting score values. The average difference is 0.01 but the difference is not statistically significant.

Next, the volatility risk premiums are estimated by minimizing the differences between model prices and actual prices. Table 6 presents the estimates of the volatility risk premiums. The risk premiums are significantly positive for twelve out of seventeen companies at the five percent level. This result gives us a clue now to why we did not see much difference between the two models in Table 5 where we assumed no risk premiums on the stock price volatility. Table 7 compares the Black-Scholes model with the random variance model taking account of the volatility risk premium. The results indicate that the random variance model incorporating the volatilty risk premium is superior to the

Black-Scholes model for valuing scores in terms of the mean absolute values of the differences between the model and actual prices. The random variance model incorporating the volatility risk premium has smaller mean absolute deviations than the Black-Scholes model for fifteen out of seventeen companies, and the differences between the two models for nine of these companies are significant at the five percent level. This result suggests that the Black-Scholes model tends to under or overestimate the scores more than the random variance model taking account of the volatility risk premium. The only exceptions are GE and Union but the differences are statistically insignificant.

Overall, the results in this paper contrast well with previous studies on pricing equity options. Some studies simply ignore the volatility risk premium, assuming that the volatility of stock prices is uncorrelated with the aggregate consumption of investors(e.g. Hull and White(1987)). Other empirical studies have not been able to find convincing evidence that a random variance model outperforms the Black-Scholes model for equity options. The results in this paper suggest that it may be due to the characteristics of the data they used, e.g., short-term options and thus the risk premium might not be appropriately captured.

V. Conclusion

This paper develops a random variance (two-state) option pricing model based on the general equilibrium model of Cox, Ingersoll, and Ross (1985) and compares it with the Black-Scholes model for pricing long-term European options. We find that the results in previous studies (not much difference between the Black-Scholes model and a random variance model for equity options) may be due to the charateristics of the options they use, e.g., short-term, among others. Using scores, which are deep out-of-money long-term European call options, we show that the risk premium on the price volatility is important and that the random variance model taking account of the risk premium is superior to the random variance model ignoring the risk premium and the Black-Scholes model. Therefore, it appears that investors require an ex-ante premium for bearing the volatility risk when they trade long-term options.

Footnotes

1. For example, Stein and Stein(1991) demonstrate that stochastic volatility is more important for away-from-the-money options(particularly out-of-money options) than at-the-money options.

2.The term "prime" stands for "prescribed right to income and to maximum equity," while "score" for "special claim on residual equity."

3.See Barron's on March 4, 1988 and Jarrow and O'Hara (1989) for details.

4.Even though this assumption is standard, it may not be plausible since we are dealing with long term options. If the volatility risk is systematic in general equilibrium, volatility shocks may shift the interest rate. However, allowing interest rate to be stochastic imposes computational challenge. We leave this for future research.

5.See Hull and White (1987), Scott (1987) and Wiggins (1987) for other examples of applying the equilibrium model of CIR to develop random variance option pricing models.

6.The risk premium on the stock price can be expressed as the expected rate of return minus riskfree rate of return, but the risk premium on the volatility of stock prices cannot be expressed by the conventional form unless the volatility of stock prices is itself the market value of some asset (see Cox and Rubinstein (1985, p. 422)).

7.See Lo (1986) for details in the maximum likelihood estimation.

8.Scott (1987) and Hull and White (1987) assume that δ and μ are zero and Wiggins (1987) shows that δ is close to zero.

9.Dietrich-Campbell and Schwarz (1986) estimate the market price of short-term interest rate risk by minimizing the sum of squared errors between theoretical bond values and market bond prices.

10.Assuming a lognormal process, the stochastic differential equation for σ is $d\sigma = \sigma\{\frac{1}{2}\theta^2+\beta[\underline{\sigma}-\ln\sigma)]\}dt + \sigma\theta dZ_2$.

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Table 1

Summary statistics of $X_t = \Delta \ln V_t - \alpha \Delta t$, deviations from the sample means. X_t is calculated using daily returns over the time period July 1978-June 1987.[a]

Company	N	mean	standard deviation	kurtosis
AHP	2528	0.000353	0.0132	5.951
ARCO	2528	0.000436	0.0174	4.595
AMOCO	2528	0.000552	0.0170	5.267
ATT-2	2528	0.000524	0.0098	5.727
CHEVRON	2528	0.000654	0.0171	5.239
DOW	2528	0.000189	0.0178	5.495
DUPONT	2528	0.000408	0.0152	5.446
EXXON	2528	0.000704	0.0119	4.259
FORD	2528	0.000598	0.0182	5.339
GE	2528	0.000657	0.0134	4.629
GM	2528	0.000653	0.0149	5.246
GTE	2528	0.000669	0.0119	5.647
KODAK	2528	0.000218	0.0156	4.913
MOBIL	2528	0.000704	0.0173	6.545
PROCTER	2528	0.000287	0.0111	4.529
SEARS	2528	0.000316	0.0162	4.801
UNION	2528	0.000487	0.0173	5.324

[a]X_t: log deviation from the sample mean at t.

V_t: stock price at t.

α: sample mean of log of stock price during the sample period.

Table 2

Estimation of ρ in X_t Processes

$$(1-\rho L)\ln|X_t| = \mu + (1-\rho L)\ln|u_t| + \epsilon_t$$

Company	ρ	μ
AHP	0.963 (0.017)[a]	-5.219 (0.049)[a]
ARCO	0.992 (0.004)	-4.847 (0.098)
AMOCO	0.996 (0.002)	-4.885 (0.150)
AT&T-2	0.996 (0.002)	-5.431 (0.137)
CHEVRON	0.997 (0.002)	-4.839 (0.141)
DOW	0.870 (0.149)	-4.912 (0.032)
DUPONT	0.984 (0.009)	-4.975 (0.051)
EXXON	0.977 (0.010)	-5.134 (0.048)
FORD	0.997 (0.010)	-4.811 (0.159)
GE	0.984 (0.006)	-5.016 (0.068)
GM	0.998 (0.001)	-4.897 (0.191)
GTE	0.984 (0.005)	-5.257 (0.087)
KODAK	0.988 (0.006)	-4.926 (0.069)
MOBIL	0.992 (0.003)	-4.875 (0.111)
PROCTER	0.971 (0.012)	-5.293 (0.052)
SEARS	0.995 (0.003)	-5.014 (0.106)
UNION	0.996 (0.002)	-4.905 (0.139)

[a]Parentheses represent standard errors.

X_t: deviation from the sample mean.

ρ: first order correlation coefficient for X_t process.

μ: constant term in the ARMA(1,1) process of X_t process.

Table 3

Parameter Estimates for the Stochastic Process of Variance

$$\ln \sigma_t = \beta(\underline{\sigma} - \ln \sigma) + \theta dz$$

Company	$\underline{\sigma}$	β	θ
AHP	-4.19	0.036	0.110
ARCO	-4.15	0.007	0.040
AMOCO	-4.21	0.003	0.030
AT&T-2	-4.16	0.032	0.030
CHEVRON	-4.20	0.002	0.032
DOW	-4.17	0.139	0.205
DUPONT	-4.32	0.015	0.067
EXXON	-4.51	0.023	0.064
FORD	-4.15	0.003	0.028
GE	-4.11	0.015	0.057
GM	-4.34	0.001	0.018
GTE	-4.58	0.015	0.070
KODAK	-4.27	0.011	0.052
MOBIL	-4.24	0.007	0.053
PROCTER	-4.59	0.029	0.077
SEARS	-4.23	0.005	0.034
UNION	-4.19	0.003	0.032

$\underline{\sigma}$: mean reverting level of $\ln \sigma$.

β: speed of adjustment coefficient of $\ln \sigma$.

θ: standard deviation of proportional changes in $\ln \sigma$.

Table 4

Actual and Model Prices: The Black-Scholes Model and the Random Variance Model with No Risk Premium

	Actual	R-V Price[a] With No Risk Premium	B-S Price[b]
		American Home Product	
Mean	12.49	13.44	13.45
Max.[c]	22.50	19.85	19.75
Min.[d]	8.50	9.61	9.45
		AMOCO	
Mean	7.09	8.29	8.27
Max.	10.75	13.10	13.09
Min.	4.75	4.07	4.05
		ARCO	
Mean	8.41	6.98	6.92
Max.	12.38	14.42	14.41
Min.	5.63	3.63	3.56
		ATT2	
Mean	6.98	7.31	7.29
Max.	12.75	11.64	11.62
Min.	4.25	3.81	3.76
		CHEVRON	
Mean	4.48	2.24	2.23
Max.	7.13	4.49	4.48
Min.	3.00	0.82	0.81
		DOW	
Mean	15.86	15.99	15.32
Max.	21.50	21.67	21.86
Min.	12.38	8.21	7.49

Table 4 (continued)

	Actual	R-V Price[a] With No Risk Premium	B-S Price[b]
		DUPONT	
Mean	15.32	17.34	17.26
Max.	27.00	28.27	28.21
Min.	7.38	7.95	7.84
		EXXON	
Mean	6.37	7.25	7.22
Max.	9.24	10.66	10.71
Min.	3.18	4.23	4.38
		FORD	
Mean	20.59	23.96	23.96
Max.	25.25	31.38	31.78
Min.	15.75	14.84	14.81
		GE	
Mean	9.09	7.17	7.29
Max.	19.38	13.14	13.03
Min.	7.00	3.68	3.54
		GTE	
Mean	8.01	9.53	9.54
Max.	17.63	17.14	17.13
Min.	3.63	4.48	4.47
		GM	
Mean	8.79	11.42	11.44
Max.	14.25	17.04	17.04
Min.	6.00	5.89	5.89

Table 4 (continued)

	Actual	R-V Price[a] With No Risk Premium	B-S Price[b]
		KODAK	
Mean	8.87	8.75	8.72
Max.	17.38	15.33	15.73
Min.	6.38	6.48	6.44
		MOBIL	
Mean	5.43	7.15	7.11
Max.	8.13	9.59	9.56
Min.	3.75	4.82	4.78
		PROCTER	
Mean	13.32	14.99	14.98
Max.	31.00	30.57	30.54
Min.	7.50	8.96	8.92
		SEARS	
Mean	4.37	4.10	4.10
Max.	7.13	8.87	8.88
Min.	3.00	1.58	1.56
		UNION	
Mean	9.07	9.92	9.91
Max.	14.88	15.72	15.74
Min.	5.75	5.39	5.37

[a]R-V price is based on the random variance option pricing model without risk premium.

[b]B-S price is based on the Black-Scholes option pricing model.

[c]Max. represents the maximum value during the testing period.

[d]Min. represents the minimum value during the testing period.

Table 5

Pricing Errors as Measured by Mean Absolute Deviation Between Model Prices and Actual Prices: Black-Scholes and Random Variance Model With No Risk Premium[a]

	R-V Model With No Risk Premium	B-S Model	t-value[b]
AHP	1.445	1.491	-0.235
AMOCO	1.372	1.359	0.077
ARCO	2.265	2.291	-0.099
AT&T-2 SERIES	0.636	0.639	-0.048
CHEVRON	2.250	2.266	-0.081
DOW	2.610	2.777	-0.380
DUPONT	2.093	2.026	0.193
EXXON	1.781	1.810	-0.013
FORD	3.748	3.751	-0.005
GE	2.351	2.398	-0.143
GM	2.895	2.913	-0.048
GTE	1.616	1.618	-0.072
KODAK	1.719	1.727	-0.029
MOBIL	1.830	1.802	0.156
PROCTER	1.950	1.967	-0.071
SEARS	1.789	1.802	-0.068
UNION	1.994	1.996	-0.005

[a]Mean absolute deviation = |Actual price - Model price| / n where n is the sample size.

[b]t-value is calculated to test whether the difference of mean absolute deviations between the two models is significant.

Table 6

Estimation of the Risk Premium on the Volatility of Stock Prices

Company	Risk premium[a]
American Home Product	0.0367 (0.002)
Amoco	0.0241 (0.001)
Arco	-0.0127 (0.002)
AT&T-2series	0.0182 (0.003)
Chevron	-0.0685 (0.021)
Dow	0.0095 (0.002)
DuPont	0.0051 (0.002)
Exxon	0.0173 (0.004)
Ford	0.0628 (0.003)
General Electric	-0.0335 (0.025)
GM	0.0210 (0.002)
GTE	0.0365 (0.001)
Kodak	0.0055 (0.002)
Mobil	0.0240 (0.002)
Procter & Gamble	0.0114 (0.003)
Sears	-0.0064 (0.001)
Union	-0.0047 (0.001)

[a]Parentheses represent standard errors.

Table 7

Pricing Errors as Measured by Mean Absolute Deviation Between Model Prices and Actual Prices: Black-Scholes Model and Random Variance Model With Risk Premium[a]

	R-V Model With Risk Premium	B-S Model	t-value[b]
AHP	1.297	1.491	-0.827
AMOCO	1.120	1.359	-1.659*
ARCO	1.988	2.291	-1.083
AT&T-2 SERIES	0.511	0.639	-1.981*
CHEVRON	1.528	2.266	-3.323*
DOW	2.511	2.777	-0.237
DUPONT	1.758	2.026	-0.992
EXXON	0.899	1.507	-3.824*
FORD	1.972	3.748	-4.560*
GE	2.603	2.398	0.752
GM	1.984	2.913	-3.049*
GTE	0.633	1.618	-10.770*
KODAK	1.695	1.727	-0.089
MOBIL	0.919	1.802	-5.954*
PROCTER	1.266	1.967	-3.149*
SEARS	1.782	1.802	-0.033
UNION	2.131	1.996	0.462

[a]Mean absolute deviation = |Score price - Model price| / n where n is the sample size.

* represents that the statistics are significant at the five percent level.