Jonathan H Bowen's
Book:

2nd Mo. 18th; 1809.

THE

COMPENDIOUS MEASURER.

C. and R. Baldwin, Printers,
New Bridge-ſtreet, London.

THE

COMPENDIOUS MEASURER;

BEING A BRIEF, YET COMPREHENSIVE,

TREATISE ON MENSURATION,

AND

PRACTICAL GEOMETRY.

WITH

AN INTRODUCTION TO DECIMAL AND DUODECIMAL

ARITHMETIC.

ADAPTED TO PRACTICE, AND THE USE OF SCHOOLS.

BY CHARLES HUTTON, LL.D. AND F.R.S. &c.

THE SIXTH EDITION, CORRECTED AND ENLARGED;
ILLUSTRATED WITH THE PLAN OF A NEW FIELD-BOOK
ENGRAVEN ON COPPER PLATE.

LONDON:

PRINTED FOR J. JOHNSON; R. BALDWIN; G. WILKIE; AND
J. ROBINSON; J. WALKER; G. ROBINSON; SCATCHERD
AND LETTERMAN; AND LONGMAN, HURST,
REES, AND ORME.

M.DCCCVII.

PREFACE.

SOME years ſince I publiſhed a complete Treatiſe on Menſuration, both in Theory and Practice; in which the Elements of that Science are demonſtrated, and the Rules applied to the various practical purpoſes of life. That work has been well received by the Public, and honoured with the high approbation of the more learned Mathematicians.

It has however been often repreſented to me, by Tutors and others, that the great ſize and price of that work, as well as the very ſcientific manner in which it is formed, prevent it from being ſo generally uſeful in ſchools, and to practical meaſurers, as a more compendious and familiar little book might be, which they could put into the hands of their pupils, as a work containing all the practical rules of that art, in a form proper for them to copy from, and unmixed with ſuch geometrical and algebraical demonſtrations as occur in the large work.

In compliance therefore with ſuch repreſentations, I have drawn up this Compendium of Menſuration, Practical Geometry, and Arithmetic, expreſsly with the view of accommodating it to practical matters, and the uſe of ſchools. I have, for that end, here brought together all the moſt uſeful rules and precepts; have arranged them in an orderly manner, proper for the pupil to copy; and delivered them in plain and familiar language. An example, worked out at full length, is ſet down to each rule, together with drawings or repreſentations of the geometrical figures proper to illuſtrate each problem; and then are ſubjoined ſome more queſtions to each rule, as examples propoſed for the practice of the learners with the anſwer ſet down, by which he may know when his work is right.

The Introduction to Decimal and Duodecimal Arithmetic will be found uſeful, by going over thoſe branches before entering on the Menſuration, that the learner may be very ready and expert in numeral calculations.

The Practical Geometry contains a great number of geometrical conſtructions and operations; by the practice of which, the learner will acquire the free and eaſy uſe of his inſtruments, and ſo become prepared for making the drawings that are uſeful for illuſtrating the various branches of Menſuration following.

The Menſuration itſelf next ſucceeds, and is divided into various parts; firſt, Menſuration in general, and then as applied to the ſeveral practical uſes in life.

The

The whole being arranged in ſuch order as the learner may properly take in ſucceſſion; or diſtinguiſhed into ſeveral branches, of which he may ſelect and ſtudy any peculiar ones that may be more to his purpoſe than the reſt, when he has not either leiſure or inducement to go over the whole in a regular gradation. And notwithſtanding the compendious ſize of the book, and the great number of practical branches here treated, it will be found that each one is much more full and complete than the firſt appearance of ſo ſmall a form may promiſe to admit of. However, if further ſatisfaction be deſired by any one, either concerning the ſcience in general, or the demonſtrations of the rules, or the more curious and copious diſplay of properties, he may apply to my large treatiſe before mentioned, where he will find every part delivered in the moſt ample form.

To this edition is added the new method of ſurveying, now practiſed by the beſt ſurveyors, illuſtrated with a map or plan, and an engraved form of the Field Book.

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INTRODUCTION.

DECIMAL FRACTIONS.

A DECIMAL is a fraction whoſe denominator is an unit, or 1, with ſome number of ciphers annexed; as $\frac{1}{10}$ or $\frac{45}{10000}$.

Decimals are written down without their denominators, the numerators being ſo diſtinguiſhed as to ſhow what the denominators are; which is done, by ſeparating, by a point, ſo many of the right-hand figures from the reſt, as there are ciphers in the denominator; the figures on the left ſide of the point being integers, and thoſe on the right decimals.

Thus, 0·5 is underſtood to be $\frac{5}{10}$ or $\frac{1}{2}$
And 0·25 is - - $\frac{25}{100}$ or $\frac{1}{4}$
And 0·75 is - - $\frac{75}{100}$ or $\frac{3}{4}$
And 1·3 is - - $\frac{13}{10}$ or $1\frac{3}{10}$
And 24·6 is - - $24\frac{6}{10}$ or $24\frac{3}{5}$

But when there is not a ſufficient number of figures n the numerator, ciphers are prefixed to ſupply the defect.

So, ·02 is $\frac{2}{100}$ or $\frac{1}{50}$
And ·0015 is $\frac{15}{10000}$ or $\frac{3}{2000}$

So that the denominator of a decimal is always 1, with as many ciphers as there are figures in the decimal.

A finite decimal, is that which ends at a certain number of places. But an infinite decimal, that which no where ends, but is underſtood to be indefinitely continued.

A repeating decimal, has one figure or ſeveral figures continually repeated. As $20{\cdot}24\dot{3}3$ &c, which is a ſingle or ſimple repetend. And $20{\cdot}24\dot{2}\dot{4}$ &c, or $20{\cdot}2462\dot{4}\dot{6}$ &c. which are compound repetends; and are otherwiſe called circulates, or circulating decimals. A point is ſet over a ſingle repetend, and a point over the firſt and laſt figures of a circulating decimal.

The firſt place, next after the decimal mark, is 10th parts, the ſecond is 100th parts, the third is 1000th parts, and ſo on, decreaſing towards the right hand by 10ths, or increaſing towards the left by 10ths, the ſame as whole or integer numbers do. As in the following ſcale of Notation.

&c.	Millions	Hundreds of thouſands	Tens of thouſands	Thouſands	Hundreds	Tens	Units	Tenth parts	Hundreth parts	Thouſand parts	Ten thouſand parts	Hundred thouſand parts	Millionth parts	&c.
	8	8	8	8	8	8	8·	8	8	8	8	8	8	

Ciphers on the right hand of decimals do not alter their value.

For ·5 or $\frac{5}{10}$ is $\frac{1}{2}$
And ·50 or $\frac{50}{100}$ is $\frac{1}{2}$
And ·500 or $\frac{500}{1000}$ is $\frac{1}{2}$

&c, are all of equal value.

But ciphers before decimal figures, and after the ſeparating point, diminiſh their value in a ten-fold proportion for every cipher.

So, ·5 is $\frac{5}{10}$ or $\frac{1}{2}$
But ·05 is $\frac{5}{100}$ or $\frac{1}{20}$
And ·005 is $\frac{5}{1000}$ or $\frac{1}{200}$

And ſo on.

So that, in any mixed or fractional number, if the ſeparating point be moved

one , two , three , &c, places to the right-hand, every figure will be

10 , 100 , 1000 , &c, times greater than before.

But if the point be moved towards the left hand, then every figure will be diminiſhed in the ſame manner, or the whole quantity will be divided by

10 , 100 , 1000 , &c.

ADDITION OF DECIMALS.

Set the numbers under each other according to the value of their places, as in whole numbers, or ſo that the decimal points ſtand directly below each other. Then add as in whole numbers, placing the decimal point in the ſum ſtraight below the other points.

EXAMPLES.

(1)	(2)	(3)
276	7530	312·09
39·213	16·201	3·5711
72014·9	3·0142	4195·6
417·	957·13	71·498
5032·	6·72819	9739·215
2214·298	·03014	179·
79993·411	8513·10353	14500·9741

Ex. 4. What is the ſum of ·014, ·9816, ·32, ·15914, 72913, and ·0047 ?

Ex. 5. What is the ſum of 27·148, 918·73, 14016, 294304, ·7138, and 221·7 ?

Ex. 6. Required the ſum of 312·984, 21·3918, 2700·42, 3·153, 27·2, and 681·06.

SUBTRACTION OF DECIMALS.

Set the leſs number under the greater in the ſame manner as in addition. Then ſubtract as in whole numbers, and place the decimal point in the remainder ſtraight below the other points.

EXAMPLES.

	(1)	(2)	(3)
From	·9173	2·73	214·81
take	·2138	1·9185	4·90142
rem.	·7035	0·8115	209·90858

Ex. 4. What is the difference between 91·713 and 407?

Ex. 5. What is the difference between 2714 and ·916?

Ex. 6. What is the difference between 16·37 and 800·135.

MULTIPLICATION OF DECIMALS.

Set down the factors under each other, and multiply them as in whole numbers, then from the product, towards the right hand, point off as many figures for decimals, as there are decimal places in both factors together.

But if there be not as many figures in the product as there ought to be decimals, prefix the proper number of ciphers to ſupply the defect.

EXAMPLES.

(1)	(2)	(3)
520·3	91·78	·217
·417	·381	·0431
36421	9178	217
5203	73424	651
20812	27534	868
216·9651	34·96818	·0093527

Ex. 4. What is the product of 51·6 and 21?
Ex. 5. What is the product of 314 and ·029?
Ex. 6. What is the product of ·051 and ·0091?

Note. When decimals are to be multiplied by 10, or 100, or 1000, &c, that is by 1 with any number of ciphers, it is done by only moving the decimal point as many places farther to the right hand, as there are ciphers in the ſaid multiplier; ſubjoining ciphers if there be not ſo many figures.

EXAMPLES.

1. The product of 51·3 and 10 is 513
2. The product of 2·714 and 100 is
3. The product of ·9163 and 1000 is
4. The product of 21·81 and 10000 is

CONTRACTION.

When the product would contain ſeveral more decimals than are neceſſary for the purpoſe in hand, the work may be much contracted thus, retaining only the proper number of decimals.

Set the units figure of the multiplier ſtraight under ſuch decimal place of the multiplicand as you intend the laſt of your product ſhall be, writing the other figures of the multiplier in an inverted order: then in multiplying, reject all the figures in the multiplicand which are on the right of the figure you are multiplying by; ſetting the products down ſo, that their right-hand figures fall ſtraight below each other; and carrying to ſuch right-hand figures from the product of the two preceding figures in the multiplicand thus, viz. 1 from 5 to 14, 2 from 15 to 24, 3 from 25 to 34, &c, incluſively; and the ſum of the lines will be the product to the number of decimals required, and will commonly be to the neareſt unit in the laſt figure.

EXAMPLES.

1. Multiply 27·14986 by 92·41035, ſo as to retain only four places of decimals in the product.

Contracted.	Common way.	
27·14986	27·14986	
53014·29	92·41035	
24434874	13	574930
542997	81	44958
108599	2714	986
2715	108599	44
81	542997	2
14	24434874	
2508·9280	2508·9280	650510

2. Multiply 480·14936 by 2·72416, retaining four decimals in the product.

3. Multiply 2490·3048 by .573286, retaining five decimals in the product.

4. Multiply 325·701428 by ·7218393, retaining three decimals in the product.

DIVISION OF DECIMALS.

Divide as in whole numbers. And to know how many decimals to point off in the quotient, obſerve the following rules:

1. There muſt be as many decimals in the dividend, as in both the diviſor and quotient, together; therefore, point off for decimals in the quotient, as many figures, as the decimal places in the dividend exceed thoſe in the diviſor.

2. If the figures in the quotient be not ſo many as the rule requires, ſupply the defect by prefixing ciphers.

3. If the decimal places in the diviſor be more than thoſe in the dividend, add ciphers as decimals to the dividend, till the number of decimals in the dividend be equal to thoſe in the diviſor, and the quotient will be integers till all theſe decimals are uſed. And in caſe of a remainder after all the figures of the dividend are uſed, and more figures are wanted in the quotient, annex ciphers to the remainder, to continue the diviſion as far as neceſſary.

4. The firſt figure of the quotient will poſſeſs the ſame place, of integers or decimals, as that figure of the dividend which ſtands over the units place of the firſt product.

EXAMPLES.

1. Divide 3424·6056 by 43·6.	2. Divide 3877875 by ·675.
43·6) 3424·6056 (78·546	·675) 3877875 000 (5745000
3726	5028
2380	3037
2005	3375
2616	000

3. Divide

3. Divide ·0081892 by ·347.
4. Divide 7·13 by ·18.
5. Divide 3·15 by 375.
6. Divide 109 by ·215.

CONTRACTIONS.

1. If the deviſor be an integer with any number of ciphers at the end; cut them off, and remove the decimal point in the dividend ſo many places farther to the left as there were ciphers cut off, prefixing ciphers if need be; then proceed as before.

EXAMPLES.

1. Divide 953 by 21000.

21;000) ·953
7) 31766·
·04538 &c.

Here, firſt divide by 3, and then by 7, becauſe 3 times 7 is 21.

2. Divide 41020 by 32000.

32·000) 41·020
8) 10·255
1·281875

Here, firſt divide by 4, and then by 8, becauſe 4 times 8 is 32.

3. Divide 43·5 by 2170.
4. Divide 61 by 79000.

2. Whence, if the diviſor be 1 with ciphers, the quotient will be the ſame figures with the dividend, having the decimal point ſo many places farther to the left as there are ciphers in the diviſor.

EXAMPLES.

2173 by 100 = 2·173
5·16 by 1000 =
419 by 10 =
·21 by 1000 =

3. When the number of figures in the diviſor is great, the diviſion at large will be very troubleſome, but may be contracted thus:

Having by the fourth general rule, found what place of decimals or integers the firſt figure of the quotient will poſſeſs; conſider how many figures of the quotient will ſerve the preſent purpoſe; then take the ſame number of the left-hand figures of the diviſor, and as many of the dividend figures as will contain them (leſs than 10 times); by theſe find the firſt figure of the quotient; and for each following figure divide the laſt remainder by the diviſor, wanting one figure to the right more than before, but obſerving what muſt be carried to the firſt product for ſuch omitted figures, as in the contraction of Multiplication; and continue the operation till the diviſor is exhauſted.

When there are not ſo many figures in the diviſor as are required to be in the quotient, begin the diviſion with all the figures as uſual, and continue it till the number of figures in the diviſor, and thoſe remaining to be found in the quotient, be equal, after which uſe the contraction.

EXAMPLES.

1. Divide 2508·92806 by 92·41035, ſo as to have four decimals in the quotient.—In this caſe the quotient will contain ſix figures. Hence

```
92·4103,5 ) 2508·928,06 ( 27·1498
                 660721 ·
                  13849 · ·
                   4608 · · ·
                    912 · · · ·
                     80 · · · · :
                      6 · · · · · ·
```

2. Divide 4109·2351 by 230·409 ſo that the quotient may contain four decimals.

4. Divide 37·10438 by 5713·96 that the quotient may contain five decimals.

4. Divide 913·08 by 2137·2 that the quotient may contain three decimals.

REDUCTION OF DECIMALS.

1. To reduce a vulgar fraction to a Decimal. Divide the numerator, with as many decimal ciphers annexed, as may be neceſſary, by the denominator; and the quotient will be the decimal ſought.

EXAMPLES.

1. Reduce $\frac{1}{99}$ to a decimal.

$9\,)\,1{\cdot}000000$

$11\,)\,0.1111\dot{1}$

$0{\cdot}0101\dot{0}\dot{1}$ &c $= \frac{1}{99}$

Here divide by 9 and 11, becauſe 9 times 11 is 99. And the decimal value of $\frac{1}{99}$ is the circulate $\cdot\dot{0}\dot{1}$.

2. Reduce $\frac{1}{75}$ to a decimal.

$5\,)\,1{\cdot}0$

$5\,)\,0{\cdot}20$

$3\,)\,0{\cdot}04$

$0{\cdot}0133\dot{3}$ &c $= \frac{1}{75}$

Here divide by 5, 5, and 3, becauſe 5 × 5 × 3 = 75, And the decimal value of $\frac{1}{75}$ is the repetend $\cdot01\dot{3}$.

OTHER EXAMPLES.

$\frac{1}{2} = \cdot5$	$\frac{1}{5} = \cdot2$	$\frac{1}{8} = \cdot125.$
$\frac{1}{3} = \cdot\dot{3}$	$\frac{1}{6} = \cdot1\dot{6}$	$\frac{1}{9} = \cdot\dot{1}.$
$\frac{1}{4} = \cdot25$	$\frac{1}{7} = \cdot\dot{1}4285\dot{7}$	$\frac{1}{10} = \cdot1$
$\frac{2}{7} =$	$\frac{14}{791} =$	$\frac{1}{758} =$

So that whenever we meet with the repetend $\cdot\dot{3}$, in any operation, we may ſubſtitute $\frac{1}{3}$ for it: in like manner we may take $\frac{2}{3}$ for $\cdot\dot{6}$, and $\frac{1}{6}$ for $\cdot1\dot{6}$, and $\frac{1}{9}$ for $\dot{1}$, and $\frac{9}{9}$ or 1 for $\cdot\dot{9}$, &c.

Note, When a great many figures are required in the decimal, and the denominator of the given fraction is a prime number greater than 11, the operation will be beſt performed as follows.

Suppoſe, for inſtance, we would find the reciprocal of the prime number 29, or the value of the fraction $\frac{1}{29}$ in decimal numbers. Firſt divide 1·000 by 29, in the common way, ſo far as to find two or three of the firſt figures, or till the remainder becomes a ſingle figure, and then aſſume the ſupplement to complete the quotient. Thus we ſhall have $\frac{1}{29} = 0{\cdot}03448\frac{8}{29}$ for the complete quotient; which equation multiply by the numerator 8, and it will give $\frac{8}{29} = 0{\cdot}27584\frac{64}{29}$ or rather $\frac{8}{29} = 0{\cdot}27586\frac{6}{29}$. Subſtitute this inſtead of the fraction in the firſt equation, and we ſhall have $\frac{1}{29} = 0{\cdot}0344827586\frac{6}{29}$. Again, multiply this equation by 6, and it will give $\frac{6}{29} = 0{\cdot}2068965517\frac{7}{29}$, and then by ſubſtitution $\frac{1}{29} = 0{\cdot}03448275862068965517\frac{7}{29}$. Again, multiply this equation by 7, and it becomes $\frac{7}{29} = 0{\cdot}24137931034482758620\frac{10}{29}$, and then by ſubſtitution $\frac{1}{29} = 0{\cdot}0344827586206896551724137931034482758620\frac{10}{29}$; where every operation will at leaſt double the number of figures found by the preceding operation. And this will be an eaſy expedient for converting diviſion into multiplication in all caſes. For this reciprocal of the diviſor being thus found, it may be multiplied by the dividend to produce the quotient.

II. To reduce a Decimal to a Vulgar Fraction.

Under the figures of the given Decimal write its proper denominator; which fraction, abbreviated as much as it can be, will be the vulgar fraction ſought.

EXAMPLES.

So $\cdot 5 = \frac{5}{10} = \frac{1}{2}$

And $\cdot 25 = \frac{25}{100} = \frac{1}{4}$

And $\cdot 75 = \frac{75}{100} = \frac{3}{4}$

And $\cdot 6 = \frac{6}{10} = \frac{3}{5}$

And $\cdot 625 = \frac{625}{1000} = \frac{5}{8}$

And $\cdot 5625 = \frac{5625}{10000} = \frac{9}{16}$

III. To find the Value of a decimal, in the Lower Denominations.

Multiply the given Decimal by the number of parts in the next lower denomination; from the product cut off as many decimals as are in the given number.

Multiply theſe by the parts in the next lower denomination again, cutting off the ſame number of decimals as before.

And proceed in the ſame manner to the loweſt denomination; then the ſeveral integer parts cut off on the left hand will give the value of the decimal propoſed.

EXAMPLES.

1. For the value of ·3914l.	2. For the value of ·2139lb. avoir.
·3914	·2139
20	16
s 7·8280	12834
12	2139
d 9·3960	oz. 3·4224
4	16
q 3·7440	dr. 6·7584
Anſ. 7s. 9¼d	Anſ. 3 oz 6 dr.

OTHER

OTHER EXAMPLES.

Queſtions.	*Anſwers.*
1. — ·775 l	16s 6d 16
2. — ·625 s	0 7½
3. — ·8635 l	17 3
4. — ·0125 lb troy	3 dwts
5. — ·4694 lb troy	5 oz. 12 dwts 15 gr
6. — ·625 cwt.	2 qr. 14 lb
7. — ·009943 mile	17 yd 1 f 6 in almoſt
8. — ·6875 yd cloth	2 qu 3 nl
9. — ·3375 acr	1 rd 14 pl
10. — ·2083 hhd wine	13 gl
11. — ·40625 qr corn	3 bu 1 pk
12. — ·42857 month	1 wk 5 day nearly

IV. To bring Quantities to Decimals of Higher Denominations.

CASE I.

If a ſingle integer or decimal be propoſed, reduce it to the higher denomination, by dividing as in reduction of whole numbers.

EXAMPLES.

1. Reduce 9d to the decimal of a pound.

12	9 d
20	0·75 s
Anſ.	0·0375 l

2. Reduce 1 dwt to the decimal of a lb.

20	1 dwt
12	0·05 oz
Anſ.	$0{\cdot}0041\dot{6}$ lb

Queſtions.

Queſtions.	*Anſwers.*
3. Reduce 26 d to 1 ſterl - -	·00108$\dot{3}$ l.
4. Reduce 7 drams to lb avoird -	·02734375 lb
5. Reduce 2·15 lb to a cwt - -	·019196 cwt
6. Reduce 24 yds to a mile - -	·0136$\dot{3}\dot{6}$ mile
7. Reduce ·056 pole to an acre -	·00035 acre
8. Reduce 1·2 pint to hd wine -	·00238 hd
9. Reduce 14 min to a day - -	·00972$\dot{2}$ day
10. Reduce ·21 pint to a peck -	·013125 peck

CASE II.

A compound number may be reduced to a ſuperior name by reducing each of its parts, and taking the ſum of the decimals: the beſt way to do which is thus:

Write the given numbers under each other, proceeding orderly from the leaſt to the greateſt name, for dividends; draw a perpendicular line on the left of theſe, and on the left of it write oppoſite to each dividend ſuch a number, for a diviſor, as will reduce it to the next ſuperior name; then begin with the upper diviſion, and affix the quotient of each to the next dividend, as a decimal part of it, before it is divided, and the laſt ſum will be the anſwer.

EXAMPLES.

1. Reduce 3l 12s 6¾d to the denomination of l.

4	3
12	6·75
20	12·5625
Anſ.	·628125

2. Reduce 5 oz 12 dwt 16 gr to the denom. of lb.

24	16
20	12·6$\dot{6}$
12	5·63$\dot{3}$
Anſ.	0·469$\dot{4}$

Queſtions,

Queſtions.	*Anſwers.*
3. Reduce 19 l 17 s 4¾ d to l	19·8635416 l
4. Reduce 15 s 6 d to l	·775 l
5. Reduce 7½d to a ſhil	·625 s
6. Reduce 3 cwt 2 qr 14 lb to cwt	3·625 cwt
7. Reduce 17 yd 1 ft 6 in to a mile	·00994318 mil
8. Reduce 2 qr 3 nls to a yard	·6875 yd
9. Reduce 13 ac 1 r 14 pol to acres	13·3375 acr
10. Reduce 13 gal 1 pint to hd wine	·2083 hd
11. Reduce 3 buſh 1 pec to a qr	·40625 qr
12. Reduce 3 mo 1 we 5 da to mon	3·42857 mon

CIRCULATING DECIMALS.

It has already been obſerved, that when an infinite decimal repeats always one figure, it is a ſingle repetend; and when more than one, a compound repetend, or a circulate: alſo that a point is ſet over a ſingle repetend, and a point over the firſt and laſt figures of a circulate.

It may further be obſerved, that when other decimal figures precede a repetend, in any numbers, it is called a mixed number or quantity, as $\cdot 2\dot{3}$, or $\cdot 104\dot{1}2\dot{3}$: otherwiſe it is a pure repetend, as $\cdot\dot{3}$ and $\cdot\dot{1}2\dot{3}$.

Similar repetends begin at the ſame place, and conſiſt of the ſame number of figures: as $\cdot\dot{3}$ and $\cdot\dot{6}$, or $1\cdot\dot{3}4\dot{1}$ and $2\cdot\dot{1}5\dot{6}$.

Diſſimilar repetends begin at different places, and conſiſt of an unequal number of figures.

Similar and conterminous repetends begin and end at the ſame place, as $2\cdot\dot{9}1\dot{0}4$ and $\cdot 0\dot{6}1\dot{3}$.

REDUC-

REDUCTION OF REPEATING DECIMALS.

CASE I.

To reduce a ſingle Repetend to a Vulgar Fraction.

Make the given decimal the numerator; and for a denominator take as many nines as there are recurring places in the given repetend.

If one or more of the left-hand places, in the given decimal, be ciphers, annex as many ciphers to the right-hand of the nines in the denominator.

EXAMPLES.

1 So $\cdot\dot{3} = \frac{3}{9} = \frac{1}{3}$. 4 And $2\cdot\dot{6}\dot{3} = 2\frac{63}{99} = 2\frac{7}{11}$.

2 And $\cdot0\dot{5} = \frac{5}{99} = \frac{1}{18}$. 5 And $\cdot05\dot{9}440\dot{5} = \frac{594405}{9999999} = \frac{17}{286}$.

3 And $\cdot\dot{1}2\dot{3} = \frac{123}{999} = \frac{41}{333}$. 6 And $\cdot\dot{7}692\dot{3}0 = \frac{799230}{999999} = \frac{10}{13}$.

CASE II.

To reduce a mixed Repetend to a Vulgar Fraction.

To as many nines as there are figures in the repetend, annex as many ciphers as there are finite places, for the denominator of the vulgar fraction.

Multiply the nines in the denominator by the finite part of the decimal, and to the product add the repeating part, for the numerator.

Or find the vulgar fraction as before, anſwering to the repetend, then join it to the finite part, and reduce them to a common denominator.

EXAM-

EXAMPLES.

1\. So $\cdot 5\dot{3} = \frac{9 \times 5 + 3}{90} = \frac{48}{90} = \frac{8}{15}$

2\. And $\cdot 58\dot{3} = \frac{58 \times 9 + 3}{900} = \frac{525}{900} = \frac{7}{12}$

3\. And $\cdot 13\dot{8} = \frac{13 \times 9 + 8}{,} = \frac{125}{900} = \frac{5}{30}$

4\. And $\cdot 5\dot{9}2\dot{5} = \frac{5 \times 999 + 925}{9990} = \frac{5920}{9990} = \frac{16}{27}$

ADDITION OF REPETENDS.

MAKE every line to begin and end at the ſame place, by extending the repetends, and filling up the vacancies with the proper figures and ciphers. Then add as in common numbers; only increaſe the ſum of the right-hand row, or laſt row of the repetends, by as many units as the firſt row of repetends contains nines. And the ſum will circulate at the ſame places as the other lines.

EXAMPLES.

(1)	(2)
$39\cdot 6548 = 39\cdot 6548\dot{0}$	$91\cdot 357 = 91\cdot 357\dot{0}$
$81\cdot 04\dot{6} = 81\cdot 0466\dot{6}$	$72\cdot 3\dot{8} = 72\cdot 388\dot{8}$
$42\cdot 3\dot{5} = 42\cdot 3555\dot{5}$	$7\cdot 2\dot{1} = 7\cdot 211\dot{1}$
$9\cdot 83\dot{7} = 9\cdot 8377\dot{7}$	$4\cdot 296\dot{5} = 4\cdot 296\dot{5}$
Sum $172\cdot 8948\dot{0}$	Sum $175\cdot 253\dot{5}$

(3)

(3)

$9{\cdot}\dot{8}1\dot{4} = 9{\cdot}814\dot{8}148\dot{1}$

$1{\cdot}5 = 1{\cdot}50000000$

$87{\cdot}2\dot{6} = 87{\cdot}2666666\dot{6}$

$0{\cdot}8\dot{3} = 0{\cdot}3333333\dot{3}$

$125{\cdot}\dot{0}\dot{9} = 125{\cdot}090\dot{9}090\dot{9}$

Sum $223{\cdot}50572390$

(4)

$2{\cdot}41 = 2{\cdot}41$

$13{\cdot}\dot{2}1\dot{5} = 13{\cdot}215151\dot{5}\dot{1}$

$5{\cdot}8 = 5{\cdot}8$

$27{\cdot}0\dot{9}\dot{6} = 27{\cdot}09696\dot{9}6\dot{9}$

$0{\cdot}\dot{9}1\dot{3} = 0{\cdot}91\dot{3}91\dot{3}9\dot{1}$

Sum $49{\cdot}43\dot{6}0351\dot{2}$

SUBTRACTION OF REPETENDS.

MAKE the repetends to begin and end together, as in addition. Then ſubtract as uſual; only, if the repetend of the number to be ſubtracted exceed the repetend of the other number, make the laſt figure of the remainder 1 leſs than it otherwiſe would be.

EXAMPLES.

(1)

$76{\cdot}3\dot{2} = 76{\cdot}322\dot{2}$

$54{\cdot}761\dot{7} = 54{\cdot}761\dot{7}$

Diff. $21{\cdot}560\dot{4}$

(2)

$89{\cdot}576 = 89{\cdot}576\dot{0}$

$12{\cdot}584\dot{6} = 12{\cdot}584\dot{6}$

Diff. $76{\cdot}991\dot{3}$

(3)

$29{\cdot}\dot{2}\dot{1} = 29{\cdot}\dot{2}1212\dot{1}$

$3{\cdot}\dot{5}6\dot{1} = 3{\cdot}\dot{5}6156\dot{1}$

Diff. $25{\cdot}\dot{6}5055\dot{9}$

(4)

$87{\cdot}\dot{4}16\dot{1} = 87{\cdot}4\dot{1}61\dot{4}$

$3{\cdot}\dot{5}3\dot{2} = 0{\cdot}5\dot{3}23\dot{2}$

Diff. $86{\cdot}8\dot{8}38\dot{1}$

MULTIPLICATION OF REPETENDS.

1. When a repetend is to be multiplied by a finite number: Multiply as in common numbers; only obſerve what muſt be carried from the beginning of the repetend to the end of it. And make all the lines begin and end together when they are to be added.

2. In multiplying a finite decimal by a ſingle repetend; multiply by the repetend, and divide by ·9 or $\frac{9}{10}$.

In more complex caſes, reduce the repetends to vulgar fractions; then divide theſe, and reduce the quotient to a decimal, if neceſſary.

EXAMPLES.

$$\begin{array}{rcrcr}
(\ 1\ ) & & (\ 2\ ) & & (\ 3\ ) \\
716{\cdot}293\dot{5} & & 2{\cdot}10\dot{4} & & 3{\cdot}02\dot{8} \\
{\cdot}27 & & 1{\cdot}2 & & 17 \\
\hline
5014054\dot{8} & & 420\dot{8} & & 2120\dot{2} \\
14325871\dot{1} & & 2104\dot{4} & & 3028\dot{8} \\
\hline
1933{\cdot}9926\dot{0} & & 2{\cdot}525\dot{3} & & 51{\cdot}491 \\
\hline
\end{array}$$

$$\begin{array}{rr}
(\ 4\ ) & \\
27{\cdot}1241 & \\
3{\cdot}\dot{6} & \text{Or } 3{\cdot}\dot{6} = 3\tfrac{6}{9} = 3\tfrac{2}{3} = \tfrac{11}{3}. \\
\hline
9\ )\ 1627446 & \text{Then } 27{\cdot}1241 \\
 & 11 \\
\hline
180827\dot{3} & 3\ )\ 298{\cdot}3651 \\
813723 & 994550\dot{3} \\
\hline
99{\cdot}4550\dot{3} & \\
\hline
\end{array}$$

(5.)

(5)

Mult. $1{\cdot}2\dot{0}\dot{6}$ by $3{\cdot}\dot{5}$

$1{\cdot}2\dot{0}\dot{6} = 1{\cdot}2\frac{6}{99} = 1{\cdot}2\frac{2}{33} = 1\frac{68}{330} = \frac{398}{330}$

and $3{\cdot}\dot{5} = 3\frac{5}{9} = \frac{32}{9}$;

then $\frac{308}{330} \times \frac{32}{9} = \frac{12736}{2970} = 4{\cdot}2\dot{8}8215\dot{4}$.

DIVISION OR REPETENDS.

1. If the dividend only be a repetend, divide as in common numbers, bringing down always the recurring figures, till the quotient become as exact as requiſite.

2. And if the diviſor only be a repetend, it will be beſt to change it into its equivalent vulgar fraction, then multiply by its denominator, and divide by its numerator.

3. But if both diviſor and dividend be repetends, change them both to vulgar fractions.

EXAMPLES.

(1)

$1{\cdot}2\ )\ 2{\cdot}525\dot{3}$

$2{\cdot}10\dot{4}$

(2)

$8\ )\ 27{\cdot}91\dot{2}$

$3{\cdot}48902\dot{7}$

(3)

$17\ )\ 51{\cdot}49\dot{1}\ (\ 3{\cdot}02\dot{8}$

49

151

151

(4)

$27\ )\ 193{\cdot}39926$

$9\ )\ 64{\cdot}46642$

$7{\cdot}16293\dot{5}$

5. Divide

5. Divide $99\cdot450\dot{3}$ by $3\cdot\dot{6}$ or $3\frac{6}{9} = 3\frac{2}{3} = \frac{11}{3}$

$$\begin{array}{r} 3 \\ \hline 11\ )\ 298\cdot36510 \\ 27\cdot1241 \end{array}$$

6. Divide $4\cdot\dot{2}88215\dot{4}$ by $3\cdot\dot{5}$ or $3\frac{5}{9} = \frac{32}{9}$

$$\begin{array}{r|l} & 9 \\ \hline 32 & 38\cdot\dot{5}9\dot{3}9363 \\ 8 & 9\ \dot{6}4\dot{8} \\ & 1\cdot\dot{2}0\dot{6} \end{array}$$

7. Divide $4\cdot2\dot{8}8215\dot{4}$ by $1\cdot\dot{2}0\dot{6}$.

Here $1\cdot\dot{2}0\dot{6} = 1\cdot2\frac{6}{99} = 1\cdot2\frac{2}{33} = \frac{298}{330}$

And $4\cdot\dot{2}88215\dot{4} = 4\cdot2\frac{882154}{999999} = \frac{42882112}{99999990}$.

Then $\frac{42882112}{99999990} + \frac{330}{398} = \frac{42882112}{30303 \times 398} = 3\cdot\dot{5}$

Or rather thus:

Having found $1\cdot\dot{2}0\dot{6} = \frac{398}{330} = \frac{3980}{10 \times 3 \times 11}$, then

$$\begin{array}{r} 4\cdot\dot{2}88215\dot{4} \\ 10 \\ \hline 42\cdot\dot{8}8215\dot{4} \\ 3 \\ \hline 128\cdot\dot{6}\dot{4}6464 \\ 11 \\ \hline 398\ )\ 1415\cdot1\dot{1}\ (\ 3\cdot\dot{5} \\ 2211 \\ 221 \end{array}$$

IN-

INVOLUTION;

OR

RAISING OF POWERS.

A POWER is a number produced by multiplying any given number continually by itſelf a certain number of times.

Any number is called the firſt power of itſelf; if it be multiplied by itſelf, the product is called the ſecond power, and ſometimes the ſquare; if this be multiplied by the firſt power again, the product is called the third power, and ſometimes the cube; and if this be multiplied by the firſt power again, the product is called the fourth power, and ſo on: that is, the power is denominated from the number which exceeds the multiplications by 1.

Thus: 3 is the firſt power of 3.
3 × 3 = 9 is the ſecond power of 3.
3 × 3 × 3 = 27 is the third power of 3.
3 × 3 × 3 × 3 = 81 is the fourth power of 3.
&c. &c.

And in this manner may be calculated the following table.

TABLE

TABLE *of the Firſt Twelve Powers of Numbers.*

1ſt power	1	2	3	4	5	6	7	8	9
2d power	1	4	9	16	25	36	49	64	81
3d power	1	8	27	64	125	216	343	512	729
4th power	1	16	81	256	625	1296	2401	4096	6561
5th power	1	32	243	1024	3125	7776	16807	32768	59049
6th power	1	64	729	4096	15625	46656	117649	262144	531441
7th power	1	128	2187	16284	78125	279936	823543	2097152	4782969
8th power	1	256	6561	65536	390625	1679616	5764801	16777216	43046721
9th power	1	512	19683	262144	1953125	10077696	40353607	134217728	387420489
10th power	1	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784014
11th power	1	2048	177147	4194304	48828125	362797056	977326743	8589934592	31381059609
12th power	1	4096	531441	16777216	244140625	176782336	13841287201	68719476736	282420536481

The number which exceeds the multiplications by 1, is called the index or exponent of the power: ſo the index of the firſt power is 1, that of the ſecond power is 2, that of the third is 3, and ſo on.

Powers are commonly denoted by writing their indices above the firſt power: ſo the ſecond power of 3 is denoted thus, 3^2; the third power thus, 3^3; the fourth power thus, 3^4; and ſo on: alſo the 6th power of 503, thus, 503^6.

Involution is the finding of powers; to do which, from their definition there evidently comes this rule.

RULE.

Multiply the given number, or firſt power, continually by itſelf, till the number of multiplication be 1 leſs than the index of the power to be found, and the laſt product will be the power required.

Note 1. Becauſe fractions are multiplied by taking the products of their numerators and of their denominators, they will be involved by raiſing each of their terms to the power required. And if a mixed number be propoſed, either reduce it to an improper fraction, or reduce the vulgar fraction to a decimal, and proceed by the rule.

2. The raiſing of powers may be ſometimes ſhortened by working according to this obſervation, viz. whatever two or more powers are multiplied together, their product is the power whoſe index is the ſum of the indices of the factors; or if a power be multiplied by itſelf, the product will be the power whoſe index is double of that which is multiplied; ſo if we would find the ſixth power, we might multiply the given number twice by itſelf for the third power, then the third power into itſelf would give the ſixth power; or if we would find the ſeventh power; we might firſt find the third and fourth, and their product would be the ſeventh; or laſtly, if we would

find the eighth power, we might firſt find the ſecond, then the ſecond into itſelf would be the fourth, and this into itſelf would be the eighth.

EXAMPLE 1.

For the ſquare of 45.

45 1ſt power.
45

225
180

$2025 = 45^2$

EXAMPLE 2.

For the ſquare of ·027

·027
·027

189
54

$\cdot000729 = \cdot027^2$

EXAMPLE 3.

For the cube of 3·5

3·5
3·5

175
105

12·25
3·5

6125
3675

$42\cdot875 = 3\cdot5^3$

EXAMPLE 4.

For the fourth power of 51

5·1
5·1

51
255

$26\cdot01 = 5\cdot1^2$
26·01 ditto

2601
15606
5202

$676\cdot5201 = 5\cdot1^4$

EXAMPLE 5.

For the fifth power of ·29.

·29
·29
261
58
$\cdot0841 = \cdot29^2$
·0841 ditto
841
3364
6728
$\cdot00707281 = \cdot29^4$
·29 = 1ſt
6365529
1414562
$\cdot0020511149 = \cdot29^5$

EXAMPLE 6.

For the ſixth power of 2·6.

2·6
2·6
156
52
$6\cdot76 = 2\cdot6^2$
2·6
4056
1352
$17\cdot576 = 2\cdot6^3$
17·576 dit.
105456
123032
87880
123032
17576
$308\cdot915776 = 2\cdot6^6$

Ex. 7. The ſquare of $\frac{2}{3}$ is $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

Ex. 8. The cube of $\frac{5}{9}$ is $\frac{5}{9} \times \frac{5}{9} \times \frac{5}{9} = \frac{125}{729}$

Ex. 9. The ſquare of $3\frac{2}{5}$ or $\frac{17}{5}$ is $\frac{17}{5} \times \frac{17}{5} = \frac{289}{25}$ $= 11\,\frac{14}{25} = 11\cdot56$.

EVOLUTION;

OR

EXTRACTION OF ROOTS.

The root of any given number, or power, is ſuch a number, as being multiplied by itſelf a certain number of times, will produce the power; and it is denominated the firſt, ſecond, third, fourth, &c. root, reſpectively, as the number of multiplications made of it to produce the given power, is 0, 1, 2, 3, &c; that is, the name of the root is taken from the number which exceeds the multiplications by 1, like the name of the power in involution.

The index of the root, like that of the power in involution, is 1 more than the number of the multiplications neceſſary to produce the power or given number. So 2 is the index of the ſecond or ſquare root; and 3 the index of the 3d or cubic root; and 4 the index of the 4th root; and ſo on.

Roots are ſometimes denoted by writing $\surd$ before the power, with the index of the root againſt it: ſo the third root of 50 is $\sqrt[3]{50}$, and the ſecond root of it is $\sqrt{50}$, the index 2 being omitted; which index is always underſtood when a root is named or written without one. But if the power be expreſſed by ſeveral numbers with the ſign + or —, &c. between them, then a line is drawn from the top of the ſign of the root, or radical ſign, over all the parts of it; ſo the third root of 47 — 15, is $\sqrt[3]{47 - 15}$. And ſometimes roots are deſigned like powers, with the reciprocal of the index of the root above the given number. So the root of 3 is $3^{\frac{1}{2}}$, the root of 50 is $50^{\frac{1}{2}}$, and the third root of it is $50^{\frac{1}{3}}$; alſo the third root of 47 — 15 is $\overline{47 - 15}^{\frac{1}{3}}$ or $(47 - 15)^{\frac{1}{3}}$. And this method of notation has juſtly prevailed in the modern algebra; becauſe ſuch roots, being conſidered as fractional powers, need no other directions for any operations to be made with them, but thoſe for integral powers.

A number is called a complete power of any kind, when its root of the ſame kind can be accurately extracted; but if not, the number is called an imperfect power, and its rood a ſurd or irrational quantity. So 4 is a complete power of the ſecond kind, its root being 2; but an imperfect power of the third kind, its third root being a ſurd quantity, which cannot be accurately extracted.

Evolution is the finding of the roots of numbers, either accurately, or in decimals to any propoſed degree of accuracy.

The power is firſt to be prepared for extraction, or evolution, by dividing it, by means of points or commas, from the place of units, to the left hand in integers, and to the right in decimal fractions, into periods, containing each as many places of figures as are denoted by the index of the root, if the power contain a complete number of ſuch periods; that is, each period to have two figures for the ſquare root, three for the cube root, four for the fourth root, and ſo on. And when the laſt period in decimals is not complete, ciphers are added to complete it.

Note. The root will contain juſt as many places of figures, as there are periods or points in the given power; and they will be integers or decimals, reſpectively, as the periods are ſo, from which they are found, or to which they correſpond; that is, there will be as many integer or decimal figures in the root, as there are periods of integers or decimals in the given number.

TO EXTRACT THE SQUARE ROOT.

1. Having divided the given number into periods of two figures each, find, from the table of powers in page 24, or otherwiſe, a ſquare number either equal to, or the next leſs than the firſt period, which ſubtract from it, and place the root of the ſquare on the right of the given number, after the manner of a quotient in diviſion, for the firſt figure of the root required.

2. To the remainder annex the ſecond period for a dividend; and on the left thereof ſet the double of the root already found, after the manner of a diviſor.

3. Find how often the diviſor is contained in the dividend, wanting its laſt figure on the right hand; place that number for the next figure in the quotient, and on the right of the diviſor, as alſo below the ſame.

4. Multiply the whole increaſed diviſor by it, placing the product below the dividend, and ſubtract it from it, and to the remainder bring down the next period, for a new dividend; to which, as before, find a diviſor by doubling the figures already found in the root; and from theſe find the next figure of the root, as in the laſt article; continuing the operation ſtill in the ſame manner till all the periods be uſed, or as far as you pleaſe.

Note. Inſtead of doubling the root, to find the new diviſors, you may add the laſt diviſor to the figure below it.

To prove the work, multiply the root by itſelf, and to the product add the remainder, and the ſum will be the given number.

Ex. 1. To extract the root of 17·3056.

$$\begin{array}{r|l} \multicolumn{2}{l}{17{\cdot}30{,}56\ (\ 4{\cdot}16} \\ \multicolumn{2}{l}{16} \\ \hline 81 & 130 \\ 1 & 81 \\ \hline 826 & 4956 \\ 6 & 4956 \\ \hline \end{array}$$

Having divided the given number into three periods, namely 17, and 30, and 56, we find that 16 is the next ſquare to 17, the firſt period, which ſet below, and ſubtracting, 1 remains, to which bring down 30, the next period, and it makes 130 for a dividend. Then 4, the root of 16, is ſet on the right of the given number for the firſt figure of the root, and its double, or 8, on the left of the dividend for the firſt figure of the diviſor; which being once contained in 13, the dividend wanting its laſt figure, gives 1 for the next figure of the root, which 1 is accordingly ſet in the root, making 4·1, and in the diviſor making 81, as alſo below the ſame. Theſe multiplied, make alſo 81, which ſet below the dividend, and ſubtracting, we have 49 remaining, to which the laſt period 56 being brought down, it makes 4956 for the new dividend. Then, for a new diviſor, either double the root 4·1, or elſe, which is eaſier, to the laſt divisor add the figure 1 ſtanding below it, and either way gives 82 for the firſt part of the new diviſor. This 82 is 6 times contained in 495, and therefore 6 is the next figure, to ſet in the root, and in the diviſor, as alſo below the ſame; which being then multiplied by it, gives 4956, the ſame as the dividend; therefore nothing remains, and 4·16 is the root of 17·3056, as required.

EXAMPLE 2.

For the Root of 2025.

```
   20,25 ( 45 root
   16
  ------
85 | 425
 5 | 425
  ------
```

EXAMPLE 3.

For the root of ·000729.

```
  ·00,07,29 ( ·027 root
      4
   ------
47 | 329
 7 | 329
   ------
```

Note. When all the periods of the given number are brought down and uſed, and more figures are required to be found, the operation may be continued by adding as many periods of ciphers as we pleaſe, namely, annexing always two ciphers at once to each dividend. And when the root is to be extracted to a greater number of places, the work may be much abbreviated thus: having proceeded in the extraction after the common method till you have found one more than half the required number of figures in the root, the reſt may be found by dividing the laſt remainder by its correſponding diviſor, annexing a cipher to every dividual, as in diviſion of decimals; or rather, without annexing ciphers, by omitting continually the right hand figure of the diviſor, after the manner of the third contraction in diviſion of decimals in page 10.

So the operation for the root of 2, to 12 or 13 places, may be thus.

EXAMPLE 4.

2 (1·414213562373 root.
1

24	100
4	96
281	400
1	281
2824	11900
4	11296
28282	60400
2	56564
282841	383600
1	282841
2828423	10075900
3	8485269
2828426)	1590631 (562373
······	176418
	6712
	1055
	206
	8

Here having found the firſt ſeven figures 1·414213 by the common extraction, by adding always periods of ciphers, the laſt ſix figures 562373 are found by the method of contracted diviſion in decimals, without adding ciphers to the remainder, but only pointing off a figure at each time from the laſt diviſor.

And the ſame for the two following examples.

EXAMPLE 5.

For the root of 3.

```
    3 ( 1·732051 root
    1
 ---------
 27 | 200
  7 | 189
 ----------
 343 | 1100
   3 | 1029
 ------------
 3462 | 7100
    2 | 6924
 ------------
 3464 )  176 ( 051
 ...       3
```

EXAMPLE 6.

For the root of 5.

```
    5 ( 2·236068 root
    4
 ---------
 42 | 100
  2 |  84
 ----------
 443 | 1600
   3 | 1329
 ------------
 4466 | 27100
    6 | 26796
 ------------
 4472 )  304 ( 068
 ...      36
```

In like manner may be found the following Roots.

The root of 6 is 2·449490
The root of 7 is 2·645751
The root of 10 is 3·162278
The root of 11 is 3·316625

RULES for the Square Roots of Vulgar Fractions and Mixed Numbers.

Firſt prepare all vulgar fractions, by reducing them to their leaſt terms, both for this and all other roots. Then,

1. Take the root of the numerator and of the denominator, for the reſpective terms of the root required. And this is the beſt way if the denominator be a complete power. But if it be not,

2. Multiply the numerator and denominator together; take the root of the product; this root being made the nu-

numerator to the denominator of the given fraction, or made the denominator to the numerator of it, will form the fractional root required.

$$\text{That is, } \sqrt{\frac{a}{b}} = \frac{\sqrt{ab}}{b} = \frac{a}{\sqrt{ab}}.$$

And this rule will ſerve whether the root be finite or infinite. Or,

3. Reduce the vulgar fraction to a decimal, and extract its root.

4. Mixed numbers may be either reduced to improper fractions, and extracted by the firſt or ſecond rule: or the vulgar fraction may be reduced to a decimal, then joined to the integer, and the root of the whole extracted.

Ex. 1. $\sqrt{\frac{25}{36}}$ is $\frac{5}{6}$

Ex. 2. $\sqrt{\frac{27}{147}}$ or $\sqrt{\frac{9}{49}}$ is $\frac{3}{7}$

Ex. 3. For the root of $\frac{9}{12}$

```
Here 9/12 or 3/4 is ·75 ( ·866025 root
                        64
                     --------
                 166 | 1100
                   6 |  996
              ----------------
                1726 | 10400
                   6 | 10356
              ----------------
                1732 )    44 ( 025
                 ...       9
```

Ex. 4. For the root of $\frac{5}{12}$

Here $\frac{5}{12}$ is $= \cdot 416\dot{6}$ (·645497 root
36

124 | 566
4 | 496

1285 | 7066
5 | 6425

1290) 641 (497
••• 125
9

TO FIND A MEAN PROPORTIONAL.

There are various uſes of the ſquare root; one of which is to find a mean proportional between any two numbers, which is performed thus: Multiply the two given numbers together, then extract the ſquare root out of their product, and it will be the mean proportional ſought.

Ex. 1. To find a Mean Proportional between 3 and 12.
Here $3 \times 12 = 36$.
And $\sqrt{36}$ is 6, the mean proportional ſought.
For $3 : 6 :: 6 : 12$.

Ex. 2. To find a Mean between 2 and 5.
Here $2 \times 5 = 10$ (3·162278 the mean required.
9

61 | 100
1 | 61

626 | 3900
6 | 3756

6322 | 14400
2 | 12644

6324) 1756 (278
••• 491
49

Note. By means of the ſquare root alſo we readily find the 4th root, or the 8th root, or the 16th root, &c. that is, the root of any power whoſe index is ſome power of the number 2: namely, by extracting ſo often the ſquare root as is denoted by the index of that power of 2; that is, two extractions for the 4th root, three for the 8th root, and ſo on.

Thus for the 4th root of 97·41.

```
 97·41,00,00  ( 9·86,96,50,50 ( 3·14159999 anſ.
 81             9
-----------    ------
188 | 1641       61 | 86
  8 | 1504        1 | 61
-----------    ------------
1966 | 13700    624 | 2596
   6 | 11796      4 | 2496
-------------  --------------
19729 | 190400  6281 | 10050
    9 | 177561     1 |  6281
---------------  ----------------
19738 ) 12839   62825 | 376950
          996       5 | 314125
            9   ------------------
                62830 ) 62825 ( 9999
                         6278
                          623
                           58
```

So that the 4th root of 97·41 is 3·14159999, which expreſſes the circumference of a circle whoſe diameter is 1 nearly.

OTHER EXAMPLES.

The 4th root of 21035·8 is 12·0431407.
The 4th root of - 2 is 1·189207.

TO EXTRACT THE CUBE ROOT.

RULE 1.

1. Point the given number into periods of three places each, beginning at units; and there will be as many integral places in the root, as there are points over the integers in the given number.

2. Seek the greateſt cube in the left-hand period; write the root in the quotient, and the cube under the period; from which ſubtract it, and to the remainder bring down the next period: Call this the reſolvend, under which draw a line.

3. Under the reſolvend, write the triple ſquare of the root, ſo that units in the latter may ſtand under the place of hundreds in the former; under the triple ſquare of the root, write the triple root, removed one place to the right; and the ſum of theſe two lines call a diviſor; under which draw a line.

4. Seek how often this diviſor may be had in the reſolvend, its right-hand place excepted, and write the reſult in the quotient.

5. Under the diviſor, write the product of the triple ſquare of the root by the laſt quotient figure, ſetting the units place of this line, under that of tens in the diviſor; under this line, write the product of the triple root by the ſquare of the laſt quotient figure, let this line be removed one place beyond the right of the former: and under this line, removed one place forward to the right, ſet the cube of the laſt quotient figure; the ſum of theſe three lines call the ſubtrahend, under which draw a line.

6. Subtract the ſubtrahend from the reſolvend; to the remainder bring down the next period for a new reſolvend; the diviſor to this, muſt be the triple ſquare of all the quotient added to the triple thereof, and ſo on as in the third article &c.

EXAMPLE 1.

What is the cube root of 48228544?

	48228544 (364		
	27		
	21228	Reſolvend	
add	27	Triple ſquare of 3	the root.
	09	Triple of 3	
	279	Diviſor	
add	162	Triple ſquare of 3 multiplied by 6	
	324	Triple of 3 multiplied by ſquare of 6	
	216	Cube of 6	
	19656	Subtrahend	
	1572544	Reſolvend	
add	3888	Triple ſquare of 36	the root
	108	Triple of 36 -	
	38988	Diviſor	
add	15552	Triple ſquare of 36 mult. by 4	
	1728	Triple of 36 mult. by ſquare of 4	
	64	Cube of 4	
	1572544	Subtrahend	

If the work of this example be well conſidered, and compared with the foregoing rule, it will be eaſy to conceive how any other example of the ſame kind may be wrought. And here obſerve, that when the cube root is extracted to more than two places, there is a neceſſity of doing ſome work upon a ſpare piece of paper, in order to come at the root's triple ſquare, and the product of the triple root by the ſquare of the quotient figure, &c.

In this example, the given number is a cubic number, and therefore at the end of the operation there remained nothing; for 364 multiplied by 364, and the product multiplied by 364 again, gives 48228544, the given number.

But if the number given be not a cubic number; then, to the laſt remainder always bring down three ciphers, and work anew for a decimal fraction if needful.

MORE EXAMPLES.

What is the cube root of

389017		73
1092727		103
27054036008	Anſwers.	3002
219365327791		6031
122615327232		4968

Theſe examples are all performed in the ſame manner as the foregoing one.

TO FIND TWO MEAN PROPORTIONALS.

There are many uſes of the cube root: one is to find two mean proportionals between two given numbers; which is performed thus:

Divide the greater extreme by the leſs, and the cube root of the quotient multiplied by the leſs extreme, gives the leſs mean. Multiply the ſaid cube root by the leſs mean, and the product is the greater mean proportional.

Note. This is only underſtood of thoſe numbers that are in continued geometric proportion.

EXAMPLE 1.

What are the two mean proportionals between 4 and 108?

108

108 Divided by 4 gives 27, whoſe cube root is 3: and the leſs extreme 4, multiplied by 3, gives 12 for the leſs mean; and 12 multiplied by the ſaid root 3, gives 36 for the greater mean.

For 4 is to 12 as 12 to 36 and as 36 to 108.

EXAMPLE II.

To find two geometrical means between 8 and 1728?

Here 8) 1728 (216, whoſe cube root is 6. Then 6 times 8 is 48, the leſs mean, and 6 times 48 is 288, the greater mean.

For 8 is to 48 as 48 to 288 and as 288 to 1728.

If the rule already given for the cube root be thought too tedious, the following one will be found much more eaſy and ready for uſe.

RULE II.

FOR THE CUBE ROOT.

1. By trials take the neareſt rational cube to the given cube or number, and call it the aſſumed cube.

2. Then ſay, as the ſum of the given number and double the aſſumed cube, is to the ſum of the aſſumed cube and double the given number, ſo is the root of the aſſumed cube, to the root required, nearly. Or as the firſt ſum is to the difference of the given and aſſumed cube, ſo is the aſſumed root, to the difference of the roots nearly.

3. Again, by uſing, in like manner, the cube of the root laſt found as a new aſſumed cube, another root will be obtained ſtill nearer. And ſo on as far as we pleaſe; uſing always the cube of the laſt-found root, for the aſſumed cube.

EXAM-

EXAMPLE.

To find the cube root of 21035·8.

Here we ſoon find, that the root lies between 20 and 30, and then between 27 and 28. Taking therefore 27, its cube is 19683 the aſſumed cube. Then

	19683	21035·8
	2	2
	39366	42071·6
	21035·8	19683
As	60401·8 :	61754·6 : : 27 : 27·6047
		27
		4322822
		1235092
	60401·8)	1667374·2 (27·6047 the root nearly.
		459338
		36525
		284
		42

Again, for a ſecond operation, the cube of this root is 21015·318645155823, and the proceſs by the latter method will be thus:

	21035·318645 &c.	
	2	
	42070·637290	21035·8
	21035·8	21035·318645 &c.
As	63106·437290 :	dif. ·481355 : : 27·6047 :
		the dif. ·000210834.

conſeq. the root req. is 27·604910834.

TO EXTRACT ANY ROOT WHATEVER.

Let G be the given power or number, n the index of the power, A the aſſumed power, r its root, R the required root of G. Then

As the ſum of $n + 1$ times A and $n - 1$ times G, is to the ſum of $n + 1$ times G and $n - 1$ times A, ſo is the aſſumed root r, to the required root R.

Or, as half the ſaid ſum of $n + 1$ times A and $n - 1$ times G, is to the difference between the given and aſſumed powers, ſo is the aſſumed root r, to the difference between the true and aſſumed roots: which difference added or ſubtracted, gives the true root nearly.

That is, $\overline{n+1}.\,A + \overline{n-1}.\,G : \overline{n+1}.\,G + \overline{n-1}.\,A :: r : R$.

Or, $\overline{n+1}.\,\frac{1}{2}A + \overline{n-1}.\,\frac{1}{2}G : A \backsim G :: r : R \backsim r$.

And the operation may be repeated as often as we pleaſe, by uſing always the laſt found root for the aſſumed root, and its nth power for the aſſumed power A.

EXAMPLE.

To extract the fifth root of 21035·8.

Here it appears that the 5th root is between 7·3 and 7·4. Taking 7·3, its 5th power is 20730·71593. Hence then we have

G =

$G = 21035{\cdot}8\,;\ r = 7{\cdot}3\,;\ n = 5\,;\ \tfrac{1}{2}.\,\overline{n + 1} = 3\,;$

$A = 20730{\cdot}716 \qquad \text{and } \tfrac{1}{2}.\,\overline{n - 1} = 2$

$G - A = 305{\cdot}084$

$A = 20730{\cdot}716 \qquad G = 21035{\cdot}8$

$\phantom{A = 20730{\cdot}71}3 \qquad \phantom{G = 21035{\cdot}}2$

$3\,A = 62192{\cdot}148 \qquad 42071{\cdot}6$

$2\,G = 42071{\cdot}6$

As $104263{\cdot}7 : 305{\cdot}084 :: 7{\cdot}3 : {\cdot}0213605$

$7{\cdot}3$

915252
2135588

$104263{\cdot}7\ )\ 2227{\cdot}1132\ (\ {\cdot}0213605$

14184 — $7{\cdot}3 = r$ add

3758 — $7{\cdot}321360 = R$ the root true to the

630 — laſt figure.

5

OTHER EXAMPLES.

1. What is the 3d root of 2? Anſ. 1·259921.
2. What is the 4th root of 2? Anſ. 1·189207.
3. What is the 4th root of 97·41? Anſ. 3·141599.
4. What is the 5th root of 2? Anſ. 1·148699.
5. What is the 6th root of 21035·8? Anſ. 5·254037.
6. What is the 6th root of 2? Anſ. 1·122462.
7. What is the 7th root of 21035·8? Anſ. 4·145392.
8. What is the 7th root of 2? Anſ. 1·104089.
9. What is the 8th root of 21035·8? Anſ. 3·470323.
10. What is the 8th root of 2? Anſ. 1·090508.
11. What is the 9th root of 21035·8? Anſ. 3·022239.
12. What is the 9th root of 2? Anſ. 1·080059.

GENERAL RULES for extracting any Root out of a Vulgar Fraction or Mixed Number.

If the given fraction have a finite root of the kind required, it is beſt to extract the root out of the numerator and denominator, for the terms of the root required.

2. But if the fraction be not a complete power; it may be thrown into a decimal, and then extracted. Or,

3. Take either of the terms of the given fraction for the correſponding term of the root; and for the other term of the root, extract the required root of the product, ariſing from the multiplication of ſuch a power of the firſt aſſigned term of the root whoſe index is leſs by 1 than that of the given power, by the other term of the given number.

This rule will do when the root is either finite or infinite.

$$\text{That is, } \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{ab^{n-1}}}{b} = \frac{a}{\sqrt[n]{ba^{n-1}}}.$$

4. Mixed numbers may be reduced either to improper fractions or decimals, and then extracted.

EXAMPLES.

1. What is the cube root of $\frac{8}{27}$? Anſ. - - - $\frac{2}{3}$.
2. What is the 4th root of $\frac{80}{405}$? Anſ. - - - $\frac{2}{3}$.
3. What is the cube root of $\frac{1}{2}$? Anſ. ·7937005.
4. What is the cube root of $2\frac{10}{27}$? Anſ. - $\frac{4}{3}$ or $1\frac{1}{3}$.
5. What is the third root of $7\frac{1}{5}$? Anſ. 1·930979.

DUODECIMALS;

OR

CROSS MULTIPLICATION.

DUODECIMALS are the calculations by feet, inches, and parts, and are ſo called, becauſe they decreaſe by twelves, from the place of feet, towards the right-hand. Inches are ſometimes called primes, and are marked thus ′; the next diviſion after inches are called parts, or ſeconds, and are marked thus″; the next are thirds, and marked thus‴; and ſo on.

This rule is otherwiſe called Croſs Multiplication, becauſe the factors are ſometimes multiplied croſs ways. And it is commonly uſed by workmen and artificers in computing the contents of their work; the dimenſions being taken in feet, inches, and parts; though a much better way would be by a decimal ſcale of diviſions.

RULE I.

1. Under the multiplicand write the ſame names or denominations of the multiplier; that is, feet under feet, inches under inches, parts under parts, &c.

2. Multiply each term in the multiplicand, beginning at the loweſt by the feet in the multiplier, and ſet each reſult under its reſpective term, obſerving to carry an unit for every 12, from each lower denomination to its next ſuperior.

3. In the ſame manner multiply every term in the multiplicand by the inches in the multiplier, and ſet the reſult of each term one place removed to the right of thoſe in the multiplicand.

4. Proceed in like manner with the ſeconds, and all the reſt of the denominations, if there be any more, ſetting the product of each line always one place more towards the right-hand than the line next before, and the ſum of all the lines will be the whole product required.

Or the denominations of the particular products will be as follow:

Feet by feet, give feet.
Feet by primes, give primes.
Feet by ſeconds, give ſeconds,
&c.

Primes by primes, give ſeconds.
Primes by ſeconds, give thirds.
Primes by thirds, give fourths,
&c.

Seconds by ſeconds, give fourths.
Seconds by thirds, give fifths.
Seconds by fourths, give ſixths,
&c.

Thirds by thirds, give ſixths.
Thirds by fourths, give ſevenths.
Thirds by fifths, give eighths,
&c.

In general thus:

When feet are concerned, the product is of the ſame denomination with the term multiplying the feet.

When feet are not concerned, the name of the product is expreſſed by the ſum of the indices of the two factors.

	f	′	″		f	′	″
Ex. 1. Multiply	10	4	5	by	7	8	6
	7	8	6				
	72	6	11				
	6	10	11	4			
		5	2	2	6		
	79	11	0	6	6	Anſwer.	

RULE II.

When the feet in the multiplicand are expreſſed by a large number.

Multiply firſt by the feet of the multiplier, as before.

Then, inſtead of multiplying by the inches and parts, &c. proceed as in the Rule of Practice, by taking ſuch aliquot parts of the multiplicand as correſpond with the inches and ſeconds, &c. of the multiplier. Then the ſum of them all will be the product required.

	f	′	″		f	′	″
Ex. 2. Multiply	240	10	8	by	9	4	6
	9	4	6				
	2168	0	0				
$4 = \frac{1}{3}$ - -	80	3	6	8			
$6 = \frac{1}{8}$ - -	10	0	5	4			
	2258	4	0	0	Anſwer.		

RULE III.

If the feet in both the multiplicand and multiplier be large numbers.

Multiply the feet only into each other: then, for the inches and ſeconds in the multiplier, take parts of the multi-

multiplicand ; and for the inches and ſeconds of the multiplicand, take aliquot parts of the feet only in the multiplier. Then the ſum of all will be the whole product.

		f	′	″		f	′	″
Ex. 3.	Multiply	368	7	5	by	137	8	4

	368	7	5		
	137	8	4		
	2576				
	1104.				
	368..				
6′ = $\frac{1}{2}$ - -	184	3	8	6	
2′ = $\frac{1}{3}$ - - -	61	5	2	10	
4″ = $\frac{1}{6}$ - -	10	2	10	5	8
6′ = $\frac{1}{2}$ - -	68	6	.	.	.
1′ = $\frac{1}{6}$ - -	11	5	.	.	.
4″ = $\frac{1}{3}$ - -	3	9	8	.	.
1″ = $\frac{1}{4}$ - -	1	1	5	.	.
	50756	7	10	9	8 Anſwer.

OTHER EXAMPLES.

Queſtions.						*Anſwers.*						
	f	′	″	‴	iv	f	′	″	‴	iv	v	vi
4. Mult.	4	7	.	.	.	29	0	4	.	.	.	.
by	6	4	.	.	.							
5. Mult.	14	9	9	.	.	66	4	6	.	.	.	.
by	4	6	.	.	.							
6. Mult.	4	7	8	.	.	44	0	10	.	.	.	.
by	9	6	.	.	.							
7. Mult.	7	8	6	.	.	79	11	0	6	6	.	.
by	10	4	5	.	.							
8. Mult.	39	10	7	.	.	745	6	10	2	4	.	.
by	18	8	4	.	.							
9. Mult.	44	2	9	2	4	126	2	10	8	10	11	.
by	2	10	3	.	.							
10. Mult.	24	10	8	7	5	233	4	5	9	6	4	.
by	9	4	6	.	.							

A
TABLE
OF
SQUARES AND CUBES, ALSO SQUARE ROOTS AND CUBE ROOTS.

Number.	Square.	Cube.	Square Root.	Cube Root.
1	1	1	1·0000000	1·000000
2	4	8	1·4142136	1·259921
3	9	27	1·7320508	1·442250
4	16	64	2·0000000	1·587401
5	25	125	2·2360680	1·709976
6	36	216	2·4494897	1·817121
7	49	343	2·6457513	1·912933
8	64	512	2·8284271	2·000000
9	81	729	3·0000000	2·080084
10	100	1000	3·1622777	2·154435
11	121	1331	3·3166248	2·223980
12	144	1728	3·4641016	2·289428
13	169	2197	3·6055513	2·351335
14	196	2744	3·7416574	2·410142
15	225	3375	3·8729833	2·466212
16	256	4096	4·0000000	2·519842
17	289	4913	4·1231056	2·571282
18	324	5832	4·2426407	2·620741
19	361	6859	4·3588989	2·668402
20	400	8000	4·4721360	2·714418
21	441	9261	4·5825757	2·758923
22	484	10648	4·6904158	2·802039
23	529	12167	4·7958315	2·843867
24	576	13824	4·8989795	2·884499
25	625	15625	5·0000000	2.924018

Number.	Square.	Cube.	Square Root.	Cube Root.
26	676	17576	5·0990195	2·962496
27	729	19683	5·1961524	3·000000
28	784	21952	5·2915026	3·036589
29	841	24389	5·3851648	3·072317
30	900	27000	5·4772256	3·107232
31	961	29791	5·5677644	3·141381
32	1024	32768	5·6568542	3·174802
33	1089	35937	5·7445626	3·207534
34	1156	39304	5·8309519	3·239612
35	1225	42875	5·9160798	3·271066
36	1296	46656	6·0000000	3·301927
37	1369	50653	6·0827625	3·332222
38	1444	54872	6·1644140	3·361975
39	1521	59319	6·2449980	3·391211
40	1600	64000	6·3245553	3·419952
41	1681	68921	6·4031242	3·448217
42	1764	74088	6·4807407	3·476027
43	1849	79507	6·5574385	3·50 398
44	1936	85184	6·6332496	3·530348
45	2025	91125	6·7082039	3·556893
46	2116	97336	6·7823300	3·583048
47	2209	103823	6·8556546	3·608826
48	2304	110592	6·9282032	3·634241
49	2401	117649	7·0000000	3·659306
50	2500	125000	7·0710678	3·684031
51	2601	132651	7·1414284	3·708430
52	2704	140608	7·2111026	3·732511
53	2809	148877	7·2801099	3·756286
54	2916	157464	7·3484692	3·779763
55	3025	166375	7·4161985	3·802953
56	3136	175616	6·4893148	3·825862
57	3249	185193	7·5498344	3·848501
58	3364	195112	7·6157731	3·870877
59	3481	205379	7·6811457	3·892996
60	3600	216000	7·7459667	3·914867

Number.	Square.	Cube.	Square Root.	Cube Root.
61	3721	226981	7·8102497	3·936497
62	3844	238328	7·8740079	3·957892
63	3969	250047	7·9372539	3·979057
64	4096	262144	8·0000000	4·000000
65	4225	274625	8·0622577	4·020726
66	4356	287496	8·1240384	4·041240
67	4489	300763	8·1853528	4·061548
68	4624	314432	8·2462113	4·081656
69	4761	328509	8 3066239	4·101566
70	4900	343000	8·3666003	4·121285
71	5041	357911	8·4261498	4·140818
72	5184	373248	8·4852814	4·160168
73	5329	389017	8·5440037	4·179339
74	5476	405224	8·6023253	4·198336
75	5625	421875	8·6602540	4·217163
76	5776	438976	8·7177979	4·235824
77	5929	456533	8·7749644	4·254321
78	6084	474552	8·8317609	4·272659
79	6241	493039	8·8881944	4·290841
80	6400	512000	8·9442719	4·308870
81	6561	531441	9·0000000	4·326749
82	6724	551368	9·0553851	4·344481
83	6889	571787	9·1104336	4·362071
84	7056	592704	9·1651514	4·379519
85	7225	614125	9·2195445	4·396830
86	7396	636056	9·2736185	4·414005
87	7569	658503	9·3273791	4·431047
88	7744	681272	9·3828315	4·447960
89	7921	704969	9·4339811	4·464745
90	8100	729000	9·4868330	4·481405
91	8281	753571	9·5393920	4·497942
92	8464	778688	9·5916630	4·514357
93	8649	804357	9·6436508	4·530655
94	8836	830584	9·6953597	4·546836
95	9025	857375	9·7467943	4·562903

Number.	Square.	Cube.	Square Root.	Cube Root.
96	9216	884736	9·7979590	4·578857
97	9409	912673	9·8488578	4·594701
98	9604	941192	9·8994949	4·610436
99	9801	970299	9·9498744	4·626065
100	10000	1000000	10·0000000	4·641589
101	10201	1030301	10·0498756	4·657010
102	10404	1061208	10·0995049	4·672330
103	10609	1092727	10·1488916	4·687548
104	10816	1124864	10·1980390	4·702669
105	11025	1157625	10·2469508	4·717694
106	11236	1191016	10·2956301	5·732624
107	11449	1225043	10·3440804	4·747459
108	11664	1259712	10·3923018	4·762203
109	11881	1295029	10·4403065	4·776856
110	12100	1331000	10·4880885	4·791420
111	12321	1367631	10·5356538	4·805896
112	12544	1404928	10·5830052	4·820284
113	12769	1442897	10·6301458	4·834588
114	12996	1481544	10·6770783	4·848808
115	13225	1520875	10·7238053	4·862944
116	13456	1560896	10·7703296	4·876999
117	13689	1601613	10·8166538	4·890973
118	13924	1643032	10·8627805	4·904868
119	14161	1685159	10·9087121	4·918685
120	14400	1728000	10·9544512	4·932424
121	14641	1771561	11·0000000	4·946088
122	14884	1815848	11·0453610	4·959675
123	15129	1860867	11·0905365	4·973190
124	15376	1906624	11·1355287	4·986631
125	15625	1953125	11·1803399	5·000000
126	15876	2000376	11·2249722	5·013298
127	16129	2048383	11·2694277	5·026526
128	16384	2097152	11·3137085	5·039684
129	16641	2146689	11·3578167	5·052774
130	16900	2197000	11·4017543	5·065797

Number.	Square.	Cube.	Square Root.	Cube Root.
131	17161	2248091	11·4455231	5·078753
132	17424	2299968	11·4891253	5·091643
133	17689	2352637	11·5325626	5·104469
134	17956	2406104	11·5758369	5·117230
135	18225	2460375	11·6189500	5·129928
136	18496	2515456	11·6619038	5·142563
137	18769	2571353	11·7046999	5·155137
138	19044	2628072	11·7473444	5·167649
139	19321	2685619	11·7898261	5 180101
140	19600	2744000	11·8321596	5·192494
141	19881	2803221	11·8743421	5·204828
142	20164	2863288	11·9163753	5·217103
143	20449	2924207	11·9582607	5·229321
144	20736	2985984	12·0000000	5·241482
145	21025	3048625	12·0415946	5·253588
146	21316	3112136	12·0830460	5·265637
147	21609	3176523	12·1243557	5·277632
148	21904	3241792	12·1655251	5·289572
149	22201	3307949	12·2065556	5·301459
150	22500	3375000	12·2474487	5·313293
151	22801	3442951	12·2882057	5·325074
152	23104	3511808	12·3288280	5·336803
153	23409	3581577	12·3693169	5·348481
154	23716	3652264	12·4096736	5·360108
155	24025	3723875	12·4498996	5·371685
156	24336	3796416	12·4899960	5·383213
157	24649	3869893	12·5299641	5·394690
158	24964	3944312	12·5698051	5·406120
159	25281	4019619	12 6095202	5·417501
160	25600	4096000	12·6491106	5·428835
161	25921	4173281	12·6885775	5·440122
162	26244	4251528	12·7279221	5·451362
163	26569	4330747	12·7671453	5·462556
164	26896	4410944	12·8062485	5·473703
165	27225	4492125	12·8452326	5·484806

Number.	Square.	Cube.	Square Root.	Cube Root.
166	27556	4574296	12·8840987	5·495865
167	27889	4657463	12·9228480	5·500879
168	28224	4741632	12·9614814	5·517848
169	28561	4826809	13·0000000	5·528775
170	28900	4913000	13·0384048	5·539658
171	29241	5000211	13·0766968	5·550499
172	29584	5088448	13·1148770	5·561298
173	29929	5177717	13·1529464	5·572054
174	30276	5268024	13·1909060	5·582770
175	30625	5359375	13·2287566	5·593445
176	30976	5451776	13·2664992	5·604079
177	31329	5545233	13·3041347	5·614673
178	31684	5639752	13·3416641	5·625226
179	32041	5735339	13·3790882	5·635741
180	32400	5832000	13·4164079	5·646216
181	32761	5929741	13·4536240	5·656652
182	33124	6028568	13·4907376	5·667051
183	33489	6128487	13·5277493	5·677411
184	33856	6229504	13·5646600	5·687734
185	34225	6331625	13·6014705	5·698019
186	34596	6434856	13·6381817	5·708267
187	34969	6539203	13·6747943	5·718479
188	35344	6644672	13·7113092	5·728654
189	35721	6751269	13·7477271	5·738794
190	35100	6859000	13·7840488	5·748897
191	36481	6967871	13·8202750	5·758965
192	36864	7077888	13·8564065	5·768998
193	37429	7189057	13·8924440	5·778996
194	37636	7301384	13·9283883	5·788660
195	38025	7414875	13·9642400	5·798890
196	38416	7529536	14·0000000	5·808786
197	38809	7645373	14·0356688	5·818648
198	39204	7762392	14·0712473	4·828476
199	39601	7880599	14·1067360	5·838272
200	40000	8000000	14·1421356	5·848035

MENSURATION.

MENSURATION is the meaſuring and eſtimating the magnitude and dimenſions of bodies and figures: and it is either angular, lineal, ſuperficial, or ſolid, according to the objects it is concerned with. It is accordingly treated in ſeveral parts: as 1ſt, Practical Geometry, which treats of the definitions and conſtruction of geometrical figures; 2d, Trigonometry, which teaches the calculation and conſtruction of triangles, or three-ſided figures, and, by application, of other figures depending on them: 3d, Superficial Menſuration, or the meaſuring the ſurfaces of bodies; 4th, Solid Menſuration, or meaſuring the capacities or ſolid contents of bodies. Beſide theſe general heads, there are ſeveral other ſubordinate diviſions, as alſo the application of them to the practical concerns of life. Of each of which in their order: excepting Trigonometry, which is fully treated of in my large book of Menſuration, as alſo in my New Courſe of Mathematics.

PRACTICAL GEOMETRY.

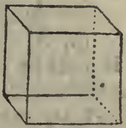
DEFINITIONS.

1. A POINT has poſition, but no parts, nor dimenſions, neither length, breadth, nor thickneſs.

2. A line is length, without breadth or thickneſs.

3. A ſurface or ſuperficies, is an extenſion, or a figure of two dimenſions, length and breadth; but without thickneſs.

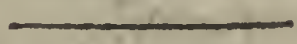
4. A body or ſolid, is a figure of three dimenſions, namely, length, breadth, and thickneſs.



Hence ſurfaces are the extremities of ſolids; lines the extremities of ſurfaces; and points the extremities of lines.

5. Lines are either right, or curved, or mixed of theſe two.

6. A right line, or ſtraight line, lies all in the ſame direction, between its extremities; and is the ſhorteſt diſtance between two points.

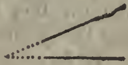


7. A curve continually changes its direction between its extreme points.

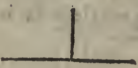
8. Lines are either parallel, oblique, perpendicular, or tangential.

9. Parallel lines are always at the ſame diſtance; and never meet though ever ſo far produced.

10. Oblique right lines change their diſtance, and would meet, if produced, on the ſide of the leaſt diſtance.



11. One line is perpendicular to another, when it inclines not more on the one ſide than on the other.

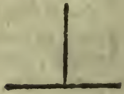


12. One line is tangential, or a tangent to another, when it touches it without cutting, when both are produced,

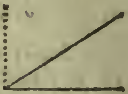
13. An angle is the inclination, or opening of two lines, having different directions, and meeting in a point.

14. Angles are right or oblique, acute or obtuſe.

15. A right angle, is that which is made by one line perpendicular to another. Or when the angles on each ſide are equal to one another, they are right angles.



16. An oblique angle, is that which is made of two oblique lines; and is either leſs or greater than a right angle.



17. An acute angle is leſs than a right angle.

18. An obtuſe angle is greater than a right angle.

19. Superficies are either plane or curved.

20. A plane, or plane ſuperficies, is that with which a right line may every way, coincide. But if not, it is curved.

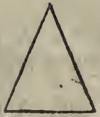
21. Plane figures are bounded either by right lines or curves.

22. Plane figures that are bounded by right lines, have names according to the number of their ſides, or of their angles; for they have as many ſides as angles; the leaſt number being three.

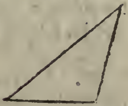
23. A figure of three ſides and angles, is called a triangle. And it receives particular denominations from the relations of its ſides and angles.

24. An equilateral triangle, is that whoſe three ſides are all equal.

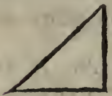
25 An



25. An iſoſceles triangle, is that which has two ſides equal.

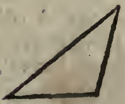


26. A ſcalene triangle, is that whoſe ſides are all unequal.



27. A right-angled triangle, is that which has one right angle.

28. Other triangles are oblique-angled, and are either obtuſe or acute.



29. An obtuſe-angled triangle has one obtuſe angle.

30. An acute-angled triangle has all its three angles acute.

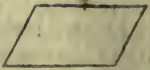
31. A figure of four ſides and angles, is called a quadrangle, or a quadrilateral.

32. A parallelogram is a quadrilateral which has both its pairs of oppoſite ſides parallel. And it takes the following particular names.

33. A rectangle is a parallelogram, having all its angles right ones

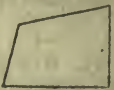
34. A ſquare is an equilateral rectangle; having all its ſides equal, and all its angles right ones.

35. A rhomboid is an oblique angled parallelogram.



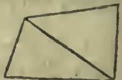
36. A rhombus is an equilateral rhomboid; having all its ſides equal; but its angles oblique.

37. A trapezium is a quadrilateral which hath not both its pairs of oppoſite ſides parallel.



38. A trapezoid hath only one pair of oppoſite ſides parallel.

39. A diagonal is a right line joining any two oppoſite angles of a quadrilateral.



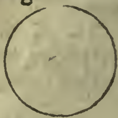
40. Plane figures that have more than four ſides are, in general, called polygons; and they receive other particular names according to the number of their ſides or angles.

41. A pentagon is a polygon of five ſides; a hexagon hath ſix ſides; a heptagon, ſeven; an octagon, eight; a nonagon, nine; a decagon, ten; an undecagon, eleven; and a dodecagon hath twelve ſides.

42. A regular polygon hath all its ſides and all its angles equal. If they are not both equal, the polygon is irregular.

43. An equilateral triangle is alſo a regular figure of three ſides, and the ſquare is one of four: the former being alſo called a trigon, and the latter a tetragon.

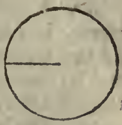
44. A circle is a plane figure bounded by a curve line, called the circumference, which is every where equi-diſtant from a certain point within, called its centre.



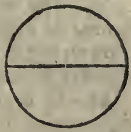
Note, The circumference itſelf is often called a circle.

45. The

45. The radius of a circle is a right line drawn from the centre to the circumference.



46. The diameter of a circle is a right line drawn through the centre, and terminating in the circumference on both ſides.

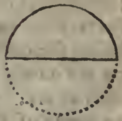


47. An arc of a circle, is any part of the circumference.

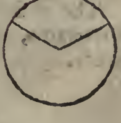
48. A chord, is a right line joining the extremities of an arc.

49. A ſegment, is any part of a circle, bounded by an arc and its chord.

50. A ſemicircle, is half the circle or a ſegment cut off by a diameter.



51. A ſector, is any part of a circle, bounded by an arc, and two radii drawn to its extremities.

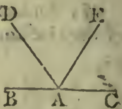


52. A quadrant, or quarter of a circle, is a ſector having a quarter of the circumference for its arc, and its two radii are perpendicular to each other.

53. The height or altitude of a figure, is a perpendicular let fall from an angle, or its vertex, to the oppoſite ſide, called the baſe.

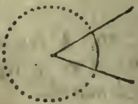
54. In a right-angled triangle, the ſide oppoſite the right angle, is called the hypothenuſe; and the other two ſides, the legs, or ſometimes the baſe and perpendicular.

55. When an angle is denoted by three letters, of which one ſtands at the angular point, and the other two on the two ſides, that which ſtands at the angular point is read in the middle.



56. The circumference of every circle is ſuppoſed to be divided into 360 equal parts, called degrees; and each degree into 60 minutes, each minute into 60 ſeconds, and ſo on. Hence a ſemicircle contains 180 degrees, and a quadrant 90 degrees.

57. The meaſure of a right-lined angle, is an arc of any circle contained between the two lines which form that angle, the angular point being the centre; and it is eſtimated by the number of degrees contained in that arc. Hence a right angle is an angle of 90 degrees.



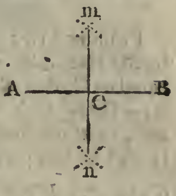
The definition of ſolids, or bodies, will be given afterwards, when we come to treat of the menſuration of ſolids.

PROBLEMS.

PROBLEM I.

To divide a Given Line AB *into Two Equal Parts.*

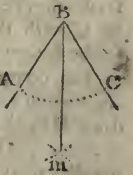
From the centres A and B, with any diſtance greater than half AB. deſcribe arcs cutting each other in m and n. Draw the line mCn, and it will cut the given line into two equal parts in the middle point C.



PROBLEM II.

To divide a Given Angle ABC *into Two Equal Parts.*

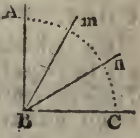
From the centre B, with any diſtance, deſcribe the arc AC. From A and C, with one and the ſame radius, deſcribe arcs interſecting in m. Draw the line Bm, and it will biſect the angle as required.



PROBLEM III.

To divide a Right Angle ABC *into Three Equal Parts.*

From the centre B, with any diſtance, deſcribe the arc AC. From the centre A, with the ſame radius, croſs the arc AC in n. And with the centre C, and the ſame radius, cut the arc AC in m. Then through the points m and n draw Bm and Bn, and they will triſect the right angle as required.

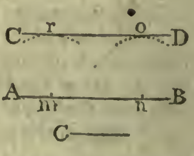


PROBLEM IV.

To draw a Line Parallel to a Given Line AB.

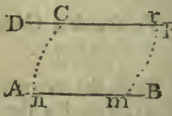
CASE 1. *When the Parallel Line is to be at a Given Diſtance* C.

From any two points m and n, in the line AB, with a diſtance equal to C, deſcribe the arcs r and o:—Draw CD to touch theſe arcs, without cutting them, and it will be the parallel required.



CASE 2. *When the Parallel Line is to paſs through a Given Point* C.

From any point m, in the line AB, with the diſtance mC, deſcribe the arc Cn.—From the centre C with the ſame radius deſcribe the arc mr. Take the arc Cn in the compaſſes, and apply it from m to r.—Through C and r draw DE, the parallel required.



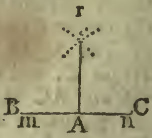
Note. This problem is more eaſily effected with a parallel ruler.

PROBLEM V.

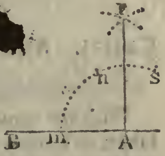
To erect a Perpendicular from a Given Point A *in a Given Line* BC.

CASE 1. *When the Point is near the Middle of the Line.*

On each ſide of the point A take any two equal diſtances Am, An. From the centres m, n, with any radius greater than Am or An; deſcribe two arcs cutting in r.—Through A and r draw the line Ar, and it will be the perpendicular as required.

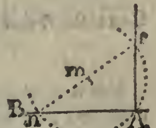


CASE 2. *When the Point is near the end of the* [illegible].



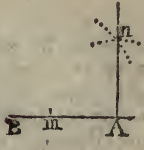
With the centre A, and any diſtance describe the arc m n s.—From the point m, with the ſame radius, turn the compaſſes twice over on the arc, as at n and s.—Again, with the centres n and s, deſcribe arcs interſecting in r.—Then draw Ar, and it will be the perpendicular as required.

Another Method.



From any point m as a centre, with the radius or diſtance mA, deſcribe an arc cutting the given line in n and A.—Through n and m draw a right line cutting the arc in r.—Laſtly, draw A r, and it will be the perpendicular as required.

Another Method.



From any plane ſcale of equal parts, ſet off Am equal to 4 parts.—With centre A, and diſtance of 3 parts, deſcribe an arc—And with centre m, and radius of 5 parts, croſs it at n.—Draw An for the perpendicular required.

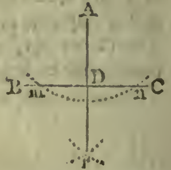
Or any other numbers in the ſame proportion, as 3, 4, 5, will do the ſame; ſuch as 6, 8, 10, &c.

PROBLEM VI.

From a Given Point A, *out of a Given Line* BC, *to let fall a Perpendicular.*

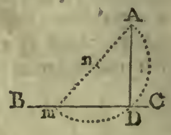
CASE 1. *When the Point is nearly oppoſite the Middle of the Line.*

With the centre A, and any diſtance, deſcribe an arc cutting BC in m and n.—With the centres m and n, and the ſame, or any other radius, deſcribe arcs interſecting in r.—Draw ADr, ſor the perpendicular required.



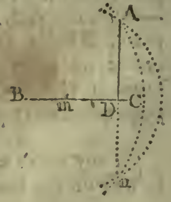
CASE 2. *When the Point is nearly oppoſite the End of the Line.*

From A draw any line Am to meet BC, in any point m.—Biſect Am at n, and with the centre n, and diſtance An or mn, deſcribe an arc, cutting BC in D.—Draw AD the perpendicular as required.



Another Method.

From B, or any point in BC, as a centre, deſcribe an arc through the point A.—From any other centre m in BC, deſcribe another arc through A, and cutting the former arc again in n.—Through A and n draw the line ADn; and AD will be the perpendicular as required.

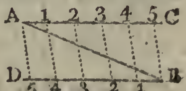


Note. Perpendiculars may be more readily raiſed and let fall, in practice, by means of a ſquare, or by the common parallelogram protractor.

PROBLEM VII.

To divide a Given Line AB *into any propoſed Number of Equal Parts.*

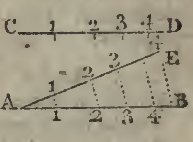
From A draw any line AC at random, and from B draw BD parallel to it.—On each of theſe lines, beginning at A and B, ſet off as many equal parts of any length, as AB is to be divided into. Join the oppoſite points of diviſion by the lines A 5, 1 4, 2 3, &c. and they will divide the given line AB as required.



PROBLEM VIII.

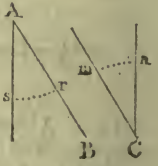
To divide a Given Line AB *in the ſame Proportion as another Line* CD *is Divided.*

From A draw any line AE equal to CD, and upon it tranſfer the diviſions of the line CD.—Join BE, and parallel to it draw the lines 1 1, 2 2, 3 3, &c. and they will divide the line AB as required.



PROBLEM IX.

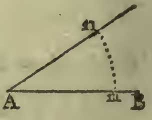
At a Given Point A, *in a Given Line* AB, *to make an Angle Equal to a Given Angle* C.



With the centre C, and any diſtance, deſcribe an arc mn.—With the centre A, and the ſame radius, deſcribe the arc rs.—Take the diſtance mn between the compaſſes, and apply it from r to s—Then a line drawn through A and s, will make the angle A equal to the angle C as required.

PROBLEM X.

At a Given Point A, *in a Given Line* AB, *to make an Angle of any propoſed Number of Degrees.*



With the centre A, and radius equal to 60 degrees, taken from a ſcale of chords, deſcribe an arc, cutting AB in m.—Then take between the compaſſes the propoſed number of degrees from the ſame ſcale of chords, and apply them from m to n. Through the point n draw An, and it will make the angle A of the number of degrees propoſed.

Note. Angles of more than 90 degrees are uſually taken off at twice.

Or the angle may be made with the protractor or other inſtrument, by laying the centre to the point A, and its radius along AB; then make a mark n at the propoſed number of degrees, through which draw the line An as before.

PROBLEM XI.

To meaſure a Given Angle A.

(*See the laſt Figure.*)

Deſcribe the arc mn with the chord of 60 degrees, as in the laſt problem.—Take the arc mn between the compaſſes, and that extent, applied to the chords, will ſhew the degrees in the given angle.

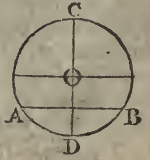
Note. When the diſtance mn exceeds 90 degrees, it muſt be taken off at twice as before.

Or the angle may be meaſured by applying the radius of a graduated arc, of any inſtrument, to AB, as in the laſt problem; and then noting the degrees cut off by the other leg An of the angle.

PROBLEM XII.

To find the Centre of a Circle.

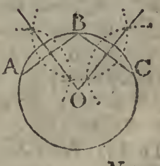
Draw any chord AB; and biſect it perpendicularly with CD, which will be a diameter. Biſect CD in the point O; and that will be the centre.



PROBLEM XIII.

To deſcribe the Circumference of a Circle through Three Given Points.

From the middle point B draw chords to the two other points.—Biſect theſe chords perpendicularly by lines meeting in O, which will be the centre.—Then from the centre O, at the diſtance OA, or OB, or OC, deſcribe the circle.



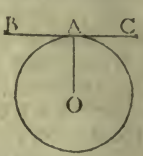
Note. In the ſame manner may the centre of an arc of a circle be found.

PROBLEM XIV.

Through a Given Point A, *to draw a Tangent to a Given Circle.*

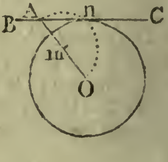
CASE 1. *When* A *is in the Circumference of the Circle.*

From the given point A, draw AO to the centre of the circle.—Then through A draw BC perpendicular to AO, and it will be the tangent as required.



CASE 2. *When* A *is out of the Circumference.*

From the given point A, draw AO to the centre, which biſect in the point m.—With the centre m, and radius mA or mO, deſcribe an arc cutting the given circle in n.—Through the points A and n, draw the tangent BC.

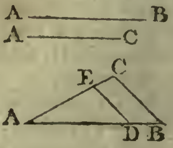


PROBLEM XV.

To find a Third Proportional to Two Given Lines. AB, AC.

Place the two given Lines, AB, AC, making any angle at A, and join BC.—In AB take AD equal to AC, and draw DE parallel to BC. So ſhall AE be the third proportional to AB and AC.

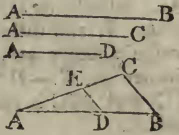
That is, AB : AC :: AC : AE.



PROBLEM XVI.

To find a Fourth Proportional to Three Given Lines, AB, AC, AD.

Place two of them AB, AC, making any angle at A, and join BC. Place AD on AB, and draw DE parallel to BC. So ſhall AE be the fourth proportional required.

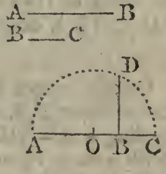


That is, AB : AC : : AD : AE.

PROBLEM XVII.

To find a Mean Proportional between Two Given Lines, AB, BC.

Join AB and BC in one ſtraight line AC, and biſect it in the point O.—With the centre O, and radius OA or OC, deſcribe a ſemicircle.—Erect the perpendicular BD, and it will be the mean proportional required.



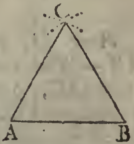
That is, AB : BD : : BD : BC.

PROBLEM XVIII.

To make an Equilateral Triangle on a Given Line AB.

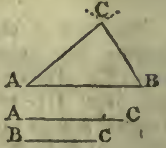
From the centres A and B, with the diſtance AB, deſcribe arcs, interſecting in C.—Draw AC and BC, and it is done.

Note. An iſoſceles triangle may be made in the ſame manner, taking for the diſtance the given length of one of the equal ſides.



PROBLEM XIX.

To make a Triangle with Three Given Lines, AB, AC, BC.



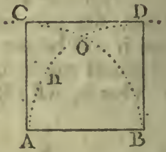
With the centre A and diſtance AC, deſcribe an arc.—With the centre B, and diſtance BC, deſcribe another arc, cutting the former in C.—Draw AC and BC, and ABC is the triangle required.

PROBLEM XX.

To make a Square on a Given Line AB.

Draw BC perpendicular and equal to AB. From A and C, with the diſtance AB, deſcribe arcs interſecting in D —Draw AD and CD, and it is done.

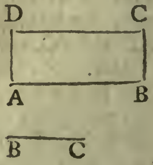
Another Way.



On the centres A and B, with the diſtance AB, deſcribe arcs croſſing at o.—Biſect Ao in n.—With centre o, and radius o n, croſs the two arcs in C and D.—Then draw AD, BC, CD.

PROBLEM XXI.

To deſcribe a Rectangle, or a Parallelogram, of a Given Length and Breadth.



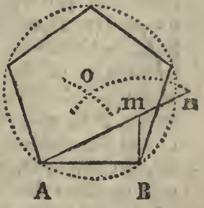
Place BC perpendicular to AB.—With centre A, and diſtance AC, deſcribe an arc.—With centre C, and radius AB, deſcribe another arc, cutting the former in D.—Draw AD and CD, and it is done.

Note.

Note. In the ſame manner is deſcribed any oblique parallelogram, only drawing BC, to make the given oblique angle with AB, inſtead of perpendicular to it.

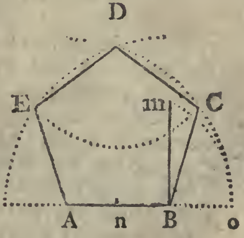
PROBLEM XXII.

To make a Regular Pentagon on a Given Line AB.



Make Bm perpendicular and equal to half AB.—Draw Am, and produce it till mn be equal to Bm.—With centres A and B, and diſtance Bn, deſcribe arcs interſecting in o, which will be the centre of the circumſcribing circle.—Then with the centre o, and the ſame radius, deſcribe the circle; and about the circumference of it apply AB the proper number of times.

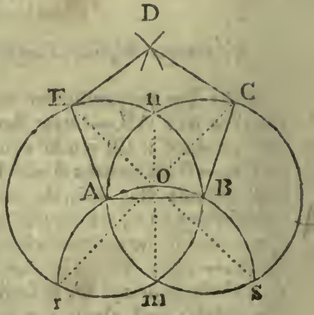
Another Method.



Make Bm perpendicular and equal to AB.—Biſect AB in n; then with the centre n, and diſtance nm, croſs AB produced in o.—With the centres A and B, and diſtance Ao, deſcribe arcs interſecting in D, which will be the oppoſite angle of the pentagon.—Laſtly with centre D, and radius AB, croſs thoſe arcs again in C and E, the other two angles of the figure.—Then draw the lines from angle to angle, to complete the figure.

A Third Method nearly true.

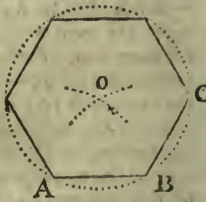
On the centres A and B, with the diſtance AB, deſcribe two circles interſecting in m and n. ——With the ſame radius, and the centre m, deſcribe rAoBS, and draw mn cutting it in o.——Draw roC and SoE, which will give two angles of the pentagon.——Laſtly, with radius AB, and centres C and E, deſcribe arcs interſecting in D, which will be the other angle of the pentagon nearly.



PROBLEM XXIII.

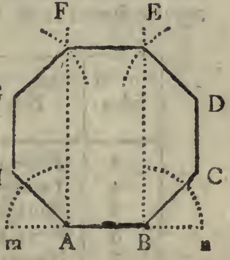
To make a Hexagon on a Given Line AB.

With the diſtance AB, and the centres A and B, deſcribe arcs interſecting in o.—With the ſame radius, and centre o, deſcribe a circle, which will circumſcribe the hexagon.—— Then apply the line AB ſix times round the circumference, marking out the angular points; and connect them with right lines.



PROBLEM XXIV.

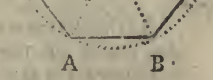
To make an Octagon on a Given Line AB.



Erect AF and BE perpendicular to AB.——Produce AB both ways, and biſect the angles m A F and nBE with the lines AH and BC, each equal to AB.——Draw CD and HG parallel to AF or BE, and each equal to AB.—With the diſtance AB, and centres G and D, croſs AF and BE in F and E.—Then join GF, FE, ED, and it is done.

PROBLEM XXV.

To make any Regular Polygon on a Given Line AB.



Draw Ao and Bo making the angles A and B each equal to half the angle of the polygon.—With the centre o and diſtance oA, deſcribe a circle.—Then apply the line AB continually round the circumference the proper number of times, and it is done.

Note. The angle of any polygon, of which the angles oAB and oBA are each one half, is found thus: Divide the whole 360 degrees by the number of ſides, and the quotient will be the angle at the centre o; then ſubtract that from 180 degrees, and the remainder will be the angle of the polygon, and is double of oAB or of oBA. And thus you will find the numbers of the following table, containing the degrees in the angle o, at the centre, and the angle of the polygon, for all the regular figures from 3 to 12 ſides.

No. of ſides	Name of the Polygon	Angle o at the centre	Angle of the polyg.	Angle OAB or OBA
3	Trigon	120°	60°	30°
4	Tetragon	90	90	45
5	Pentagon	72	108	54
6	Hexagon	60	120	60
7	Heptagon	$51\frac{3}{7}$	$128\frac{4}{7}$	$64\frac{2}{7}$
8	Octagon	45	135	67
9	Nonagon	40	140	70
10	Decagon	36	144	72
11	Undecagon	$32\frac{8}{11}$	$147\frac{3}{11}$	$73\frac{7}{11}$
12	Dodecagon	30	150	75

PROBLEM XXVI.

In a Given Circle to Inſcribe any Regular Polygon; or to divide the Circumference into any Number of Equal Parts.

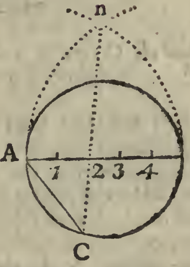
(See the laſt figure.)

At the centre o make an angle equal to the angle at the centre of the polygon, as contained in the third column of the above table of polygons.—Then the diſtance AB will be one ſide of the polygon; which being carried round the circumference the proper number of times, will complete the figure. Or, the arc AB will be one of the equal parts of the circumference.

Another

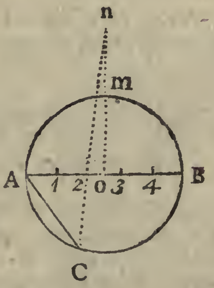
Another Method, nearly true.

Draw the diameter AB, which divide into as many equal parts as the figure has ſides.—With the diſtance AB, and centres A and B, deſcribe arcs croſſing at n: from thence draw nC through the ſecond diviſion on the diameter; ſo ſhall AC be a ſide of the polygon, nearly.



Another Method, ſtill nearer.

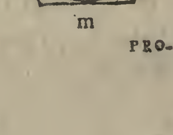
Divide the diameter AB, as before, into as many equal parts as the figure has ſides. From the centre o raiſe the perpendicular om, which produce till mn be equal to three fourths of the radius om.—From n draw nC through the ſecond diviſion of the diameter, and the line AC will be the ſide of the polygon ſtill nearer than before; or the arc AC one of the equal parts into which the circumference is to be divided.



PROBLEM XXVII.

About a Given Circle to Circumſcribe any Polygon.

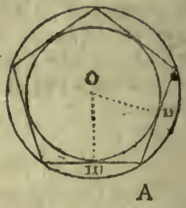
Find the points m, n, p, &c. as in the laſt problem; to which draw radii mo, no, &c. to the centre of the circle.——Then through theſe points m, n, &c. and perpendicular to theſe radii, draw the ſides of the polygon.



PROBLEM XXVIII.

To find the Centre of a Given Polygon, or the Centre of its Inſcribed or Circumſcribed Circle.

Biſect any two ſides with the perpendiculars mo, no; and their interſection will be the centre.—Then with the centre o, and the diſtance om, deſcribe the inſcribed circle; or with the diſtance to one of the angles, as A, deſcribe the circumſcribing circle.

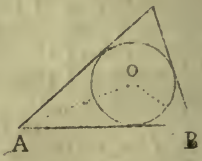


Note. This method will alſo circumſcribe a circle about any given oblique triangle.

PROBLEM XXIX.

In any Given Triangle to Inſcribe a Circle.

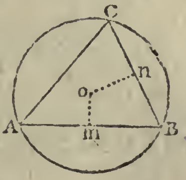
Biſect any two of the angles with the lines Ao, Bo; and o will be the centre of the circle.—Then with the centre o, and radius the neareſt diſtance to any one of the ſides, deſcribe the circle.



PROBLEM XXX.

About any Given Triangle to Circumſcribe a Circle.

Biſect any two of the ſides AB, BC, with the perpendiculars mo, no.——With the centre o, and diſtance to any one of the angles, deſcribe the circle.



PROBLEM XXXI.

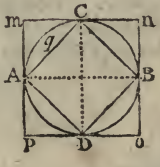
In, or About, a Given Square, to deſcribe a Circle.

Draw the two diagonals of the ſquare, and their interſection o will be the centre of both the circles.—Then with that centre, and the neareſt diſtance to one ſide, deſcribe the inner circle; and with the diſtance to one angle, deſcribe the outer circle.

PROBLEM XXXII.

In, or About, a Given Circle, to deſcribe a Square, or an Octagon.

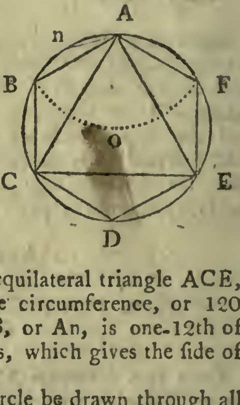
Draw two diameters AB, CD, perpendicular to each other.——Then connect their extremities, and that will give the inſcribed ſquare ACBD.——Alſo through their extremities draw tangents parallel to them, and they will form the outer ſquare mnop.



Note. If any quadrant, as AC, be biſected in q, it will give one-eighth of the circumference, or the ſide of the octagon.

PROBLEM XXXIII.

In a Given Circle, to Inſcribe a Trigon, a Hexagon, or a Dodecagon.

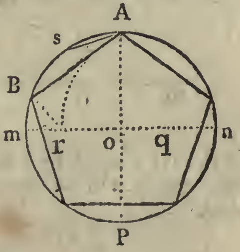


The radius of the circle is the ſide of the hexagon. Therefore from any point A in the circumference, with the diſtance of the radius, deſcribe the arc BOF. Then is AB the ſide of the hexagon; and therefore carrying it ſix times round will form the hexagon, or will divide the circumference into ſix equal parts, each containing 60 degrees—The ſecond of theſe C, will give AC the ſide of the trigon, or equilateral triangle ACE, and the arc AC one-third of the circumference, or 120 degrees.—Alſo the half of AB, or An, is one-12th of the circumference, or 30 degrees, which gives the ſide of the dodecagon.

Note. If tangents to the circle be drawn through all the angular points of any inſcribed figure, they will form the ſides of a like circumſcribing figure.

PROBLEM XXXIV.

In a Given Circle to Inſcribe a Pentagon, or a Decagon.



Draw the two diameters AP, mn perpendicular to each other, and biſect the radius on at q.—With the centre q and diſtance qA, deſcribe the arc Ar; and with the centre A, and radius Ar, deſcribe the arc rB. Then is AB one-fifth of the circumference; and AB carried five times over will form the pentagon. Alſo the arc AB biſected in s, will give A s the tenth part of the circumference, or the ſide of the decagon.

Note. Tangents being drawn through the angular points, will form the circumſcribing pentagon or decagon.

PROBLEM XXXV.

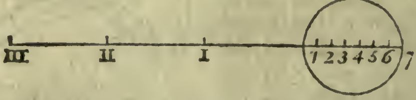
To divide the Circumference of a Given Circle into 12 *Equal Parts, each of* 30 *Degrees.*

Or to Inſcribe a Dedecagon by another Method.

Draw two diameters 1 7 and 4 10 perpendicular to each other.—Then with the radius of the circle, and the four extremities, 1, 4, 7, 10, as centres, deſcribe arcs, through the centre of the circle; and they will cut the circumference in the points required, dividing it into 12 equal parts, at the points marked with the numbers in the figure.

PROBLEM XXXVI.

To draw a Right Line equal to the Circumference of a Given Circle.

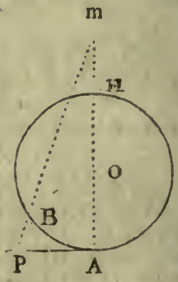


Take III 1 equal to 3 times the diameter and $\frac{1}{7}$ part more: and it will be equal to the circumference, very nearly.

PROBLEM XXXVII.

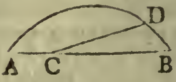
To find a Right Line equal to any Given Arc AB *of a Circle.*

Through the point A and the centre draw Am, making mn equal to $\frac{1}{4}$ of the radius n o.—Alſo draw the indefinite tangent AP perpendicular to it.—Then through m and B draw mB: ſo ſhall AP be equal to the arc AB very nearly.



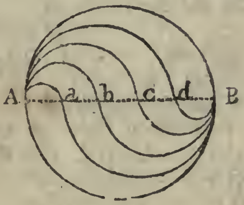
Otherwiſe.

Divide the chord AB into 4 equal parts.—Set one part AC on the arc from B to D.—Draw CD, and the double of it will be nearly equal to the arc ADB.



PROBLEM XXXVIII.

To divide a Given Circle into any propoſed Number of Parts by Equal Lines, ſo that theſe Parts ſhall be mutually Equal both in Area and Perimeter.

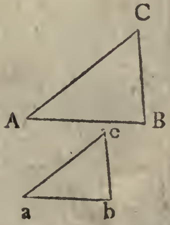


Divide the diameter AB into the propoſed number of equal parts at the points a, b, c, &c.——Then on Aa, Ab, Ac, &c. as diameters, deſcribe ſemicircles on one ſide of the diameter AB; and on Bd, Bc, Bb, &c. deſcribe ſemicircles on the other ſide of the diameter. So ſhall the correſponding joining ſemicircles divide the given circle in the manner propoſed.

And in like manner we may proceed when the ſpaces are to be in any given proportion.—As to the perimeters, they are always equal, whatever be the proportion of the ſpaces.

PROBLEM XXXIX.

To make a Triangle Similar to a Given Triangle ABC.

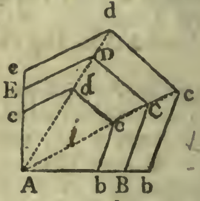


Let ab be one ſide of the required Triangle. Make the angle a equal to the angle A, and the angle b equal to the angle B; then the triangle abc will be ſimilar to ABC as propoſed.

Note. If ab be equal to AB, the triangles will alſo be equal, as well as ſimilar.

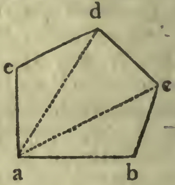
PROBLEM XL.

To make a Figure Similar to any other Given Figure ABCDE.



From any angle A draw diagonals to the other angles. —Take Ab a ſide of the figure required. Then draw bc parallel to BC, and cd to CD, and de to DE, &c.

Otherwiſe

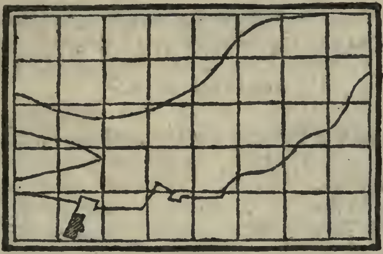


Make the angles at a, b, e, reſpectively equal to the angles at A, B, E, and the lines will interſect in the corners of the figure required.

PROBLEM XLI.

To reduce a Complex Figure from one Scale to another, alſo to copy ſuch a Figure of the ſame Size, mechanically, by means of Squares.

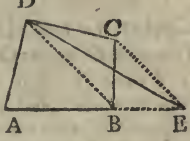
Divide



Divide the given figure, by croſs lines, into ſquares, as ſmall as may be thought neceſſary.—Then divide another paper into the ſame number of ſquares, and either greater, equal or leſs, in the given proportion.—This done, obſerve what ſquares the ſeveral parts of the given figure are in, and draw with a pencil, ſimilar parts in the correſponding ſquares of the new figure. And ſo proceed till the whole is copied.

PROBLEM XLII.

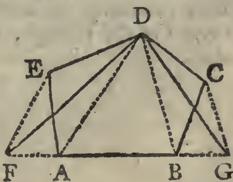
To make a Triangle Equal to a Given Trapezium ABCD.



Draw the diagonal DB, alſo CE parallel to it, meeting AB produced in E.—Join DE; ſo ſhall the triangle ADE be equal to the trapezium ABCD.

PROBLEM XLIII.

To make a Triangle equal to the Figure ABCDEA.

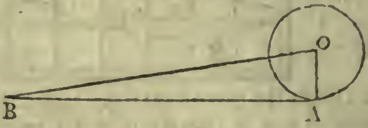


Draw the diagonals DA, DB, and the lines EF, CG parallel to them, meeting the baſe AB, both ways produced, in F and G.—Join DF, DG; and DFG will be the triangle required equal to the given figure ABCDE.

Note. Nearly in the ſame manner may a triangle be made equal to any right-lined figure whatever.

PROBLEM XLIV.

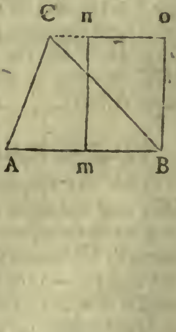
To make a Triangle Equal to a Given Circle.



Draw any radius AO, and the tangent AB perpendicular to it.—On which take AB equal to the circumference of the circle by Problem xxxvi.—Join BO; ſo ſhall ABO be the triangle required, equal to the given circle, nearly.

PROBLEM XLV.

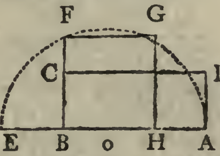
To make a Rectangle, or a Parallelogram, Equal to a Given Triangle ABC.



Biſect the baſe AB in m.—Through C draw Cno parallel to AB.—Through m and B draw mn and BO parallel to each other, and either perpendicular to A B, or making any angle with it. And the rectangle or parallelogram mnoB will be equal to the triangle, as required.

PROBLEM XLVI.

To make a Square Equal to a Given Rectangle ABCD.

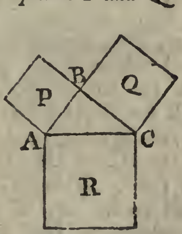


Produce one ſide, AB, till BE be equal to the other ſide BC.—Biſect AE in o; on which as a centre, with radius Ao, deſcribe a ſemicircle, and produce BC to meet it at F.—On BF make the ſquare BFGH, and it will be equal to the rectangle ABCD, as required.

*** Thus the circle, and all right-lined figures, have been reduced to equivalent ſquares.

PROBLEM XLVII.

To make a Square Equal to Two Given Squares P *and* Q.



Set two ſides AB, BC, of the given ſquares, perpendicular to each other.—Join their extremities AC; ſo ſhall the ſquare R, conſtructed on AC, be equal to the two P and Q taken together.

Note. Circles or any other ſimilar figures are added in the ſame manner. For, if AB and BC be the diameters of two circles, AC will be the diameter of a third circle equal to both the other two. And if AB and BC be the like ſides of any two ſimilar figures, then AC will be the like ſide of another ſimilar-figure equal to both the two former, and on which the third figure may be conſtructed by Problem XL.

PROBLEM XLVIII.

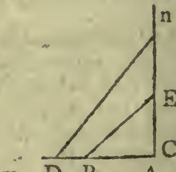
To make a Square Equal to the Difference between Two Given Squares P, R.

(*See the laſt Figure.*)

On the ſide AC of the greater ſquare, as a diameter, deſcribe a ſemicircle; in which apply AB the ſide of the leſs ſquare.—Join BC, and it will be the ſide of a ſquare equal to the difference between the two P and R, as required.

PROBLEM XLIX.

To make a Square Equal to the Sum of any Number of Squares taken together.

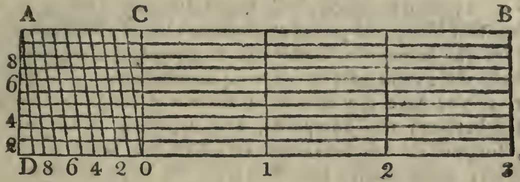


Draw two indefinite lines Am, An, perpendicular to each other at the point A. On the one of theſe ſet off AB the ſide of one of the given ſquares, and on the other AC the ſide of another of them. Join BC, and it will be the ſide of a ſquare equal to the two together. Then take AD equal to BC, and AE equal to the ſide of the third given ſquare. So ſhall DE be the ſide of a ſquare equal to the ſum of the three given ſquares.—And ſo on continually, always ſetting more ſides of the given ſquares on the line An, and the ſides of the ſucceſſive ſums on the other line Am.

Note. And thus any number of any ſort of figures may be added together.

PROBLEM L.

To make Plane Diagonal Scales.



Draw any line as AB, of any convenient length. Divide it into 11 equal parts*. Complete theſe into rectangles of a convenient height, by drawing parallel and perpendicular lines. Divide the altitude into 10 equal parts, if it be for a decimal ſcale for common numbers, or into 12 equal parts, if it be for feet and inches; and through theſe points of diviſion draw as many parallel lines, the whole length of the ſcale.—Then divide the length of the firſt diviſion AC into 10 equal parts, both above and below; and connect theſe points of diviſion by diagonal lines, and the ſcale is finiſhed, after being numbered as you pleaſe.

Note. Theſe diagonal ſcales ſerve to take off large dimenſions or numbers of three figures. If the firſt large diviſions be units; the ſecond ſet of diviſions along AC, will be 10th parts; and the diviſions in the altitude, along AD will be 100th parts. If CD be tens, AC will be units, and AD will be the 10th parts. If CB be hundreds, AC will be tens, and AD units. If CB be thouſands, AC will be hundreds, and AD will be tens. And ſo on, each ſet of diviſions being tenth parts of the former one.

For example, ſuppoſe it were required to take off 243 from the ſcale. Fix one foot of the compaſſes at 2 of the greateſt diviſions, at the bottom of the ſcale, and

---

* Only 4 parts are here drawn, for want of room.

extend

extend the other to 4 of the ſecond diviſions, along the bottom ; then, for the 3, ſlide up both points of the compaſſes by a parallel motion, till they fall upon the third longitudinal line; and in that poſition extend the ſecond point of the compaſſes to the fourth diagonal line, and you have the extent of three figures as required.

Or, if you have any line to meaſure the length of.— Take it between the compaſſes, and applying it to the ſcale, ſuppoſe it fall between 3 and 4 of the large diviſions: or, more nearly, that it is 3 of the large diviſions, or 3 hundreds, and between 5 and 6 of the ſecond diviſions, or 5 tens or 50, and a little more. Slide up the points of the compaſſes by a parallel motion, keeping one foot always on the vertical diviſion of 3 hundred, till the other point fall exactly on one of the diagonal lines, which ſuppoſe to be 8, being 8 units, which ſhows that the length of the line, propoſed to be meaſured, is 358.

PLANE SCALES FOR TWO FIGURES.

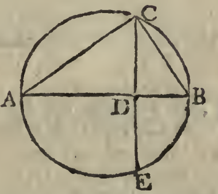
The above are three other forms of ſcales, the firſt of which is a decimal ſcale, for taking off common numbers conſiſting of two figures. The other two are duodecimal ſcales, and ſerve for feet and inches, &c.

Theſe

Theſe and other ſcales, engraved on ivory, are fitteſt for practical uſe. And the moſt convenient form of a plane ſcale of equal diviſions, is on the very edge of the ivory made thin at the edge for laying along any line, and then marking the paper oppoſite any diviſion required: which is better than taking lengths off a ſcale with compaſſes.

REMARKS.

Note 1. That in a circle, the half chord DC, is a mean proportional between the ſegments AD, DB of the diameter AB perpendicular to it. That is AD : DC : ; DC : DB.



2. The chord AC is a mean proportional between AD and the diameter AB. And the chord BC a mean proportional between DB and AB.

That is, AD : AC : : AC : AB.
and BD : BC : : BC : AB.

3. The angle ACB, in a ſemicircle, is always a right angle.

4. The ſquare of the hypothenuſe of a right-angled triangle, is equal to the ſquare of both the ſides.

That is, $AC^2 = AD^2 + CD^2$,
and $BC^2 = BD^2 + DC^2$,
and $AB^2 = AC^2 + BC^2$.

5. Triangles that have all the three angles of the one reſpectively equal to all the three of the other, are called equiangular triangles, or ſimilar triangles.

6. In ſimilar triangles, the like ſides, or ſides oppoſite the equal angles, are proportional.

7. The areas, or ſpaces, of ſimilar triangles, are to each other, as the ſquares of their like ſides.

MENSURATION

OF

SUPERFICIES.

THE area of any figure, is the meaſure of its ſurface, or the ſpace contained within the bounds of the ſurface, without any regard to thickneſs.

The area is eſtimated by the number of ſquares contained in the ſurface, the ſide of thoſe ſquares being either an inch, or a foot, or a yard, &c. And hence the area is ſaid to be ſo many ſquare inches, or ſquare feet, or ſquare yards, &c.

Our ordinary lineal meaſures, or meaſures of length, are as in the firſt table here below; and the annexed table of ſquare meaſures is taken from it, by ſquaring the ſeveral numbers.

Lineal Meaſures.		*Square Meaſures.*	
12 inches -	1 foot	144 inches -	1 foot
3 feet - -	1 yard	9 feet -	1 yard
6 feet - -	1 fathom	36 feet -	1 fathom
$16\frac{1}{2}$ feet, or $5\frac{1}{2}$ yards	1 pole or rod	$272\frac{1}{4}$ feet or $30\frac{1}{4}$ yards	1 pole or rod
40 poles -	1 furlong	1600 poles -	1 furlong
8 furlongs	1 mile	64 furlongs	1 mile

PROBLEM I.

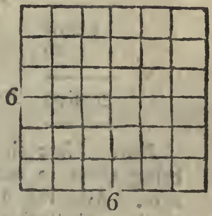
To find the Area of a Parallelogram; whether it be a Square, a Rectangle, a Rhombus, or a Rhomboid.

Multiply the length by the breadth, or perpendicular height, and the product will be the area.

EXAMPLES.

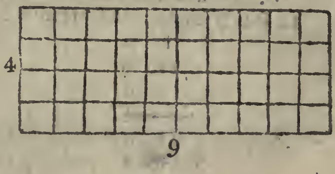
1. To find the area of a ſquare, whoſe ſide is 6 inches, or ſix feet, &c.

6
6
—
36
—
Anſwer 36



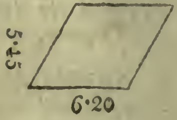
2. To find the area of a rectangle, whoſe length is 9, and breadth 4 inches, or feet, &c.

9
4
—
36
—
Anſwer 36



3. To

3. To find the area of a rhombus, whoſe length is 6·20 chains, and perpendicular height 5·45

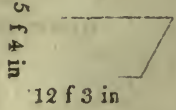


5·45
6·20
10900
3270
10) 33·7900
3·379
4
1·516
40
20·640 Anſ. 3 acres, 1 rood, 20 perches.

Note. Here the ſquare chains are divided by 10 to bring them to acres, becauſe 10 ſquare chains make an acre. Alſo the decimals of an acre are multiplied by 4 roods, and theſe by 40 perches, becauſe 4 roods make 1 acre, and 40 perches 1 rood.

4. To find the area of the rhomboid, whoſe length is 12 feet 3 inches, and breadth 5 feet 4 inches.

f i
12 3
5 4
61 3
4 1
65 4



Anſwer $65\frac{1}{3}$ ſquare feet.

5. To find the area of a ſquare, whoſe ſide is 35·25 chains.
Anſ. 124 ac 1 ro 1 perch.

6. To find the area of a parallelogram, whoſe length is 12·25 chains, and breadth 8·5 chains.
Anſ. 10 ac 1 ro 26 perch.

7. To find the area of a rectangular board, whoſe length is 12·5 feet, and breadth 9 inches. Anſ. $9\frac{3}{8}$ feet.

8. To find the ſquare yards of painting in a rhomboid, whoſe length is 37 feet, and breadth $5\frac{1}{4}$ feet.
Anſ. $21\frac{7}{12}$ ſquare yards.

PROBLEM II.

To find the Area of a Triangle.

Rule 1. Multiply the baſe by the perpendicular height, and take half the product for the area.

Rule 2. When the three ſides only are given: Add the three ſides all together, and take half the ſum; from the half ſum ſubtract each ſide ſeparately; multiply the half ſum and the three remainders continually together; and take the ſquare root of the laſt product for the area of the triangle.

EXAMPLES.

1. Required the area of the triangle, whoſe baſe is 6·25 chains, and perpendicular height 5·20 chains.

6·25

6·25
5·20

12500
3125

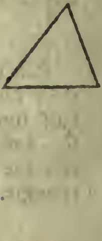
20) 32·5000
1·625
4

2·500
40

20·000

Anſ. 1 ac 2 ro 20 perches.

2. To find the number of ſquare yards in the triangle whoſe three ſides are 13, 14, 15 feet.



13
14
15

2) 42

$\frac{1}{2}$ ſum 21 21 21 21
13 14 15 6

remainders 8 7 6 126
7

882
8

9)
7056 (84 feet
64 $9\frac{1}{3}$ ſq. yds.

164 | 656
4 | 656

Anſ. $9\frac{1}{3}$ ſq. yards.

3. How

3. How many ſquare yards are in a right-angled triangle, whoſe baſe is 40, and perpendicular 30 feet?

Anſ. $66\frac{2}{3}$ ſquare yards.

4. To find the area of the triangle, whoſe three ſides are 20, 30, 40 chains. Anſ. 29 ac 0 ro 7 per.

5. How many ſquare yards contains the triangle, whoſe baſe is 49 feet, and height $25\frac{1}{4}$ feet?

Anſ. $68\frac{53}{72}$ or 68·7361.

6. How many acres, &c. in the triangle, whoſe three ſides are 380, 420, 765 yards? Anſ. 9 ac 0 ro 38 per.

7. To find the area of the triangle, whoſe baſe is 18 feet 4 inches, and height 11 feet 10 inches.

Anſ. 108 feet 5 inches 8″.

8. How many acres, &c. contains the triangle, whoſe three ſides are 49·00, 50·25, 25·69 chains?

An. 61 ac 1 ro 39·68 per.

PROBLEM III.

To find one Side of a Right-angled Triangle, having the other two Sides given.

The ſquare of the hypothenuſe is equal to both the ſquares of the two legs. Therefore,

1. To find the hypothenuſe; add the ſquares of the two legs together, and extract the ſquare root of the ſum.

2. To find one leg; ſubtract the ſquare of the other leg from the ſquare of the hypothenuſe, and extract the ſquare root of the difference.

EXAMPLES.

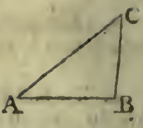
1. Required the hypothenuſe of a right angled triangle whoſe baſe is 40, and perpendicular 30.

40 30
40 30

1600 900
900

2500 (50 the hypothenuſe AC
25

00



2. What is the perpendicular of a right-angled triangle, whoſe baſe AB is 56, and the hypothenuſe AC 65?

56 65
56 65

336 325
280 390

3136 4225
3136

1089 (33 the perp. BC.
9

63 | 189
3 | 189

3. Required the length of a ſcaling ladder to reach the top of a wall whoſe height is 28 feet, the breadth of the ditch before it being 45 feet. Anſ. 53 feet.

4. To find the length of a ſhoar, which, ſtrutting 12 feet from the upright of a building, may ſupport a jaumb 20 feet from the ground. Anſ. 23·32380 feet.

5. A line of 320 feet will reach from the top of a precipice, ſtanding cloſe by the ſide of a brook, to the oppoſite bank: required the breadth of the brook; the height of the precipice being 103 feet. Anſ. 302·9703 feet.

6. A

6. A ladder of 50 feet long being placed in a ſtreet, reached a window 28 feet from the ground on one ſide; and by turning the ladder over, without removing the foot out of its place, it touched a moulding 36 feet high on the other ſide: required the breadth of the ſtreet?

Anſ. 76·1233335 feet.

PROBLEM IV.

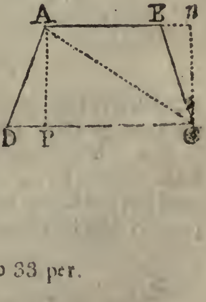
To find the Area of a Trapezoid.

Add together the two parallel ſides; multiply that ſum by the perpendicular diſtance between them, and take half the product for the area.

EXAMPLES.

1. In a trapezoid the parallel lines are AB 7·5, and DC 12·25, alſo the perpendicular diſtance AP or Cn is 15·4 chains; required the area.

12·25
7·5
19·75
15·4
7900
9875
1975
20) 304·150
15·2075
4
·8300 anſ. 15 ac 0 ro 33 per.
40
33·2000



2. How many ſquare feet contains the plank, whoſe length is 12 feet 6 inches, the breadth at the greater end 1 foot 3 inches, and at the leſs end 11 inches? Anſ. $13\frac{13}{24}$ feet.

3. Re-

3. Required the area of a trapezoid, the parallel ſides being 21 feet 3 inches and 18 feet 6 inches, and the diſtance between them 8 feet 5 inches.

Anſ. 167 feet 3 inches 4″ 6‴.

4. In meaſuring along one ſide AB of a quadrangular field, that ſide and the two perpendiculars upon it from the oppoſite corners, meaſured as below: required the content. Anſ. 4 ac 3 r 17·92 p.

chains

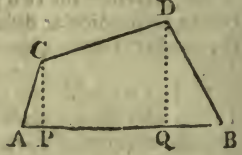
AP = 1·10

AQ = 7·45

AB = 11·10

PC = 3·52

QD = 5·95



PROBLEM V.

To find the Area of a Trapezium.

CASE 1.

For any Trapezium.

Divide it into two Triangles by a diagonal; then find the areas of theſe triangles, and add them together.

Note. If two perpendiculars be let fall on the diagonal, from the other two oppoſite angles, the ſum of theſe perpendiculars being multiplied by the diagonal, half the product will be the area of the trapezium.

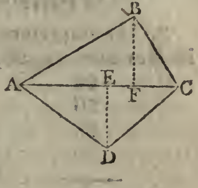
CASE 2.

When the Trapezium can be inſcribed in a Circle.

Add all the four ſides together, and take half the ſum; next ſubtract each ſide ſeparately from the half ſum: then multiply the four remainders continually together, and take the ſquare root of the laſt product for the area of the trapezium.

EXAMPLES.

1. To find the area of the trapezium ABCD, the diagonal AC being 42, the perpendicular BF 18, and the perpendicular DE 16.



18
16
—
34 Sum
42
—
68
136
—
2) 1428
714 the anſwer.

2. In the trapezium ABCD, the ſide AB is 15, BC 13, CD 14, AD 12, and the diagonal AC is 16: required the area.

AC 16				AC 16			
AB 15				CD 14			
BC 13				AD 12			
2) 44				2) 42			
22	22	22	half ſum	21	21	21	half ſum
16	15	13		16	14	12	
6	7	9		5	7	9	
7				7			
42				35			
9				9			
378				315			
22				21			
756				315			
756				630			
8316 (91·1921				6615 (81·3326			

The

The triangle ABC	- - -	91·1921
The triangle ADC	- - -	81·3326
The trapezium ABCD		172·5247 the anſwer.

3. If a trapezium can be inſcribed in a circle, and have its four ſides 24, 26, 28, 30; required its area.

24			
26			
28			
30			
2) 108			
54	54	54	54 half ſum
24	26	28	30
30	28	26	24
28		24	
840		104	
		52	
		624	
		840	
		24960	
		4992	
		524160 (723·9889488 anſwer.	

4. How many ſquare yards of paving are in the trapezium, whoſe diagonal is 65 feet, and the two perpendiculars let fall on it 28 and 33·5 feet? Anſ. 222$\frac{1}{12}$ yards.

5. What is the area of a trapezium, whoſe ſouth ſide is 27·40 chains, eaſt ſide 35·75 chains, north ſide 37·55 chains, weſt ſide 41·05 chains, and the diagonal from ſouth-weſt to north-eaſt 48·35 chains?

Anſ. 123 ac 0 ro 11·8672 per.

6. What

6. What is the area of a trapezium, whoſe diagonal is $108\frac{1}{2}$ feet, and the perpendiculars $56\frac{1}{4}$ and $60\frac{3}{4}$ feet?
Anſ. $6347\frac{1}{4}$ feet.

7. What is the area of a trapezium inſcribed in a circle, the ſour ſides being 12, 13, 14, 15?
Anſ. 180·9972372.

8. In the four-ſided field ABCD, on account of obſtructions in the two ſides AB, CD, and in the perpendiculars BF, DE, the following meaſures only could be taken: namely, the two ſides BC 265 and AD 220 yards, the diagonal AC 378 yards, and the two diſtances of the perpendiculars from the ends of the diagonal, namely AE 100, and CF 70 yards: required the area in acres, when 4840 ſquare yards make an acre. Anſ. 17 ac 2 ro 21 per.

PROBLEM VI.

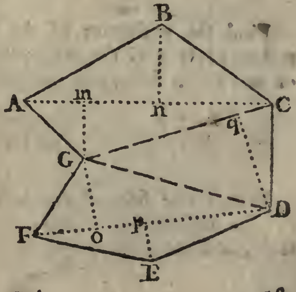
To find the Area of an Irregular Polygon.

Draw diagonals dividing the figure into trapeziums and triangles. Then find the areas of all theſe ſeparately, and add them together for the content of the whole figure.

EXAMPLE.

To find the content of the irregular figure ABCDEFG A, in which are given the following diagonals and perpendiculars: namely,

AC	5·5
FD	5·2
GC	4·4
Gm	1·3
Bn	1·8
Go	1·2
Ep	0·8
Dq	2·3



1ſt, For trapez. ABCG.	2d, For trapez. GDEF.	3d, For triangle GCD.
1·3	1·2	4·4
1·8	0·8	2·3
3·1	2·0	132
5·5	5·2	88
155	10·4	10·12
155		

17·05 double ABCG
10·40 double GDEF
10·12 double GCD

2) 37·57 double the whole
18·785 the anſwer.

PROBLEM VII.

To find the Area of a Regular Polygon.

RULE 1.

Find the perimeter of the figure, or ſum of its ſides, and multiply it by the perpendicular falling from its centre on one of its ſides, and take half the product for the area.

RULE 2.

Square one ſide of the polygon; multiply that ſquare by the multiplier ſet againſt its name in the following table, and the product will be the area.

No of ſides.	Names.	Multipliers.
3	Trigon or equ. tri.	0·4350127
4	Tetragon or ſquare	1·0000000
5	Pentagon	1·7204774
6	Hexagon	2·5980762
7	Heptagon	3·6339124
8	Octagon	4·8284271
9	Nonagon	6·1818242
10	Decagon	7·6942088
11	Undecagon	9·3656399
12	Dodecagon	11·1961524

EXAMPLES.

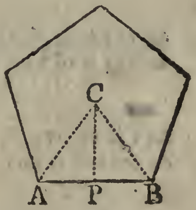
1. Required the area of the regular pentagon, whoſe ſide AB is 25 feet, and perpendicular CP 17·204774.

By the 1ſt Rule.

17·204774 perp.
125 perim.

86023870
34409548
17204774

2) 2150·596750
1075·298375 anſ.



By the 2d Rule.

First 25
25

125
50

625

Then 1·7204774
625

86023870
34409548
103228644

1075·2983750 anſwer.

2. To

2. To find the area of the hexagon, whoſe ſide is 20.
Anſ. 1039·23048.

3. To find the area of the trigon, or equilateral triangle, whoſe ſide is 20. Anſ. 173·20508.

4. Required the area of an octagon, whoſe ſide is 20.
Anſ. 1931·37084.

5. What is the area of a decagon, whoſe ſide is 20?
Anſ. 3077·68351.

PROBLEM VIII.

To find the Diameter and Circumference of a Circle, the one from the other.

RULE 1.

As 7 is to 22, ſo is the diameter to the circumference.
As 22 is to 7, ſo is the circumference to the diameter.

RULE 2.

As 113 is to 355, ſo is the diameter to the circumf.
As 355 is to 113, ſo is the circumf. to the diameter.

RULE 3.

As 1 is to 3·1416, ſo is the diameter to the circumf.
As 3·1416 is to 1, ſo is the circumf. to the diameter.

EXAMPLES.

1. To find the circumference of a circle, whoſe diameter AB is 10.

By Rule 1.

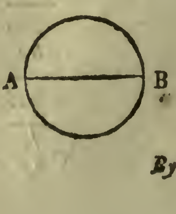
$7 : 22 :: 10 : 31\frac{3}{7}$

10

$7\,)\,220$

$31\frac{3}{7}$

or 31·42857 anſ.



By Rule 2.

113 : 355 : : 10 : $31\frac{47}{113}$

10

113) 3550 (31·41593

160 the anſ.

470

180

670

1050

330

By Rule 3.

1 : 3·1416 : : 10 : 31·416 the circumference nearly, the true circumference being 31·4159265358979 &c.

So that the 2d rule is neareſt the truth.

2. To find the diameter when the circumference is 50.

By Rule 1.

$$22 : 7 : : 50 : \frac{7 \times 25}{11} = \frac{175}{11} = 15\tfrac{10}{11} = 15{\cdot}90\dot{9}\dot{0}$$ anſ.

By Rule 2.

355 : 113 : : 50 : $15\frac{65}{71}$

50

355 | 5650

71 | 1130 (15·9155 , ...

420

650

110

390

330

By Rule 3.

3·1416 : 1 : : 50 : 15·9156

50

3·1416) 50·000 (15·9156

18484

2876

49

18

2

3. If the diameter of the earth be 7958 miles, as it is very nearly, what is the circumference, ſuppoſing it to be exactly round? Anſ. 25000·8528 miles.

4. To find the diameter of the globe of the earth, ſuppoſing its circumference to be 25000 miles.

Anſ. $7957\frac{3}{4}$ nearly.

PROBLEM IX.

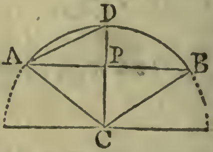
To find the Length of any Arc of a Circle.

RULE 1.

As 180 is to the number of degrees in the arc,
So is 3·1416 times the radius, to its length.
Or as 3 is to the number of degrees in the arc,
So is ·05236 times the radius, to its length.

Ex. 1. To find the length of an arc ADB of 30 degrees, the radius being 9 feet.

3·1416
9
As 180 : 30———
Or 6 : 1 : : 28·2744 : 4·7124
Cr 3 : 30 : : ·05236 × 9 : 4·7124
90
———
4·7124 the anſwer.
———



RULE 2.

From 8 times the chord of half the arc ſubtract the chord of the whole arc, and take $\frac{1}{3}$ of the remainder for the length of the arc nearly.

Ex. 2. The chord AB of the whole arc being 4·65874, and the chord AD of the half arc 2·34947; required the length of the arc.

2·34947
8
———
18·79576
4·65874
———
3) 14.13702
4·71234 anſwer.
———

Ex. 3. Required the length of an arc of 12 degrees 10 minutes, or 12$\frac{1}{6}$ degrees, the radius being 10 feet.

Anſ. 2·1234$\frac{8}{9}$.

Ex. 4.

Ex. 4. To find the length of an arc whoſe chord is 6, and the chord of its half is $3\frac{1}{2}$. Anſ. $7\frac{1}{3}$.

Ex. 5. Required the length of the arc, whoſe chord is 8, and the height PD 3. — — Anſ. $10\frac{2}{3}$.

Ex. 6. Required the length of the arc, whoſe chord is 6, the radius being 9. Anſ. 6·11706.

PROBLEM X.

To find the Area of a Circle.

The area of a circle may be found from the diameter and circumference together, or from either of them alone, by theſe rules following.

Rule 1. Multiply half the circumference by half the diameter. Or multiply the whole circumference by the whole diameter, and take $\frac{1}{4}$ of the product.

Rule 2. Multiply the ſquare of the diameter by ·7854.

Rule 3. Multiply the ſquare of the circumference by ·07958.

Rule 4. As 14 to 11, ſo is the ſquare of the diameter to the area.

Rule 5. As 88 to 7, ſo is the ſquare of the circumference to the area.

EXAMPLES.

1. To find the area of a circle whoſe diameter is 10, and circumference 31·4159265.

By Rule 1.	*By Rule* 2.	*By Rule* 4.
		100
31·4159265		11
10	·7854	
	100	14 | 1100
4) 314·159265		7 | 550
area 78·539816	78·54	78·57

By

By Rule 3.

ſq. circ. 986·96044
invert. 85970

6908723
888264
49348
7896

78·54231 area

By Rule 5.

31·4159265 circum.
562951413 invert.

94247779
3141593
1256637
31416
15708
2827
63
19
2

88 : 7 : : 986·96044
7

8 | 6908·72308
11 | 863·59038
78·50821

Ex. 2. Required the area of the circle, whoſe diameter is 7, and circumference 22. Ans. $38\frac{1}{2}$.

Ex. 3. What is the area of a circle, whoſe diameter is 1, and circumference 3·1416? Anſ. ·7854.

Ex. 4. What is the area of a circle, whoſe diameter is 7? Anſ. 38·4846.

Ex. 5. How many ſquare yards are in a circle whoſe diameter is $3\frac{1}{2}$ feet? Anſ. 1·069.

Ex. 6. How many ſquare feet does a circle contain, the circumference being 10·9956 yards? Anſ. 86·19266.

PROBLEM XI.

To find the Area of the Sector of a Circle.

RULE 1.

Multiply the radius, or half the diameter, by half the arc of the ſector, for the area. Or, multiply the diameter by the arc of the ſector, and take $\frac{1}{4}$ of the product.

Note. The arc may be found by problem IX. RULE

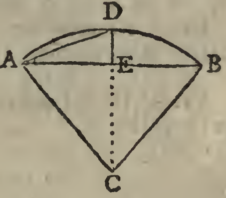
RULE 2.

As 360 is to the degrees in the arc of the ſector, ſo is the whole area of the circle, to the area of the ſector.

Note. For a ſemicircle take one half, for a quadrant one quarter, &c. of the whole circle.

EXAMPLES.

1. What is the area of the ſector CADB, the radius being 10, and the chord AB 16?



$100 = AC^2$

$64 = AE^2$

$36 \,(6 = CE$

$10 = CD$

$4 = DE$

$16 = DE^2$

$64 = AE^2$

$80 \,(8{\cdot}9442719 = AD$

8

$71{\cdot}5541752$

16

$3\,)\,55{\cdot}5541752$

$2\,)\,18{\cdot}5180584$ arc ADB

$9{\cdot}2590297 =$ half arc

$10 =$ radius

$92{\cdot}590297$ anſwer.

Ex. 2. Required the area of the ſector, whoſe arc contains 18 degrees; the diameter being 3 feet.

·7854
9

Then, as 360 : 18 : : 7·0686 the area of the whole circle,
Or as 20 : 1 : : 7·0686 : ·35343 the anſwer.

Ex 3. What is the area of the ſector, whoſe radius is 10, and arc 20? Anſ. 100.

Ex. 4. What is the area of the ſector, whoſe radius is 9, and the chord of its arc 6? Anſ. 27·52678.

Ex. 5. Required the area of the ſector, whoſe radius is 25, its arc containing 147 degrees 29 minutes. Anſ. 804·4017.

Ex. 6. To find the area of a quadrant and a ſemicircle, to the radius 13. Anſ. 132·7326 and 265·4652.

PROBLEM XII.

To find the Area of a Segment of a Circle.

RULE 1.

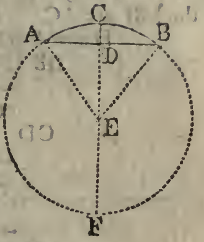
Find the area of the ſector having the ſame arc with the ſegment, by the laſt problem.

Find alſo the area of the triangle, formed by the chord of the ſegment and the two radii of the ſector.

Then add theſe two together for the anſwer when the ſegment is greater than a ſemicircle; but ſubtract them for the anſwer when it is leſs than a ſemicircle.

EXAMPLE 1.

Required the area of the ſegment ACBDA, its chord AB being 12, and the radius AE or CE 10.



100 AE^2
36 AD^2

64 DE^2

its root 8 DE
from 10 CE

leaves 2 CD

4 CD^2
36 AD^2

40 chord AC^2

its root 6·324555 chord AC
8

50·596440
12·

3) 38·59644
2) 12·86548 arc ACB
6·43274 half arc
10 radius

64·3274 area of ſect. or EACB
48·0000 area of triangle EAB

anſ. 16·3274 area of ſegm. ACBA.

9 AD
8 DE

48 area of △ EAB

RULE 2.

To the chord of the whole arc add $\frac{4}{3}$ of the chord of half the arc, or add the latter chord and $\frac{1}{3}$ of it more. Multiply the ſum by the verſed ſine or height of the ſegment, and take $\frac{4}{10}$ of the product for the area of the ſegment.

Ex.

Ex. Take the ſame example, in which the radius is 10, and the chord AB 12.

Then, as before, are found CD 2, and the chord of the half arc AC 6·324555

Hence $\frac{1}{3}$ is 2·108185

AB 12·

20·432740

CD - - 2

40·86548

·4

Anſ. 16·346192 area nearly.

RULE 3.

Divide the height of the ſegment by the diameter, and find the quotient in the column of heights or verſed ſines, at the end of the book.

Take out the correſponding area in the next column on the right hand, and multiply it by the ſquare of the diameter, for the anſwer.

Ex. The example being the ſame as before, we have CD equal to 2, and the diameter 20.

Then 20) 2 (·1

And to ·1 anſwers ·040875

Sq. of diam. - - - 400

Anſwer 16·3500

OTHER EXAMPLES.

Ex. 2. What is the area of the ſegment, whoſe height is 2, and the chord 20? Anſ. 26·878787.

Ex. 3. What is the area of the ſegment, whoſe height is 18, and diameter of the circle 50? Anſ. 636·375.

Ex. 4. Required the area of the ſegment whoſe chord is 16, the diameter being 20. Anſ. 44.7292.

PRO-

PROBLEM XIII.

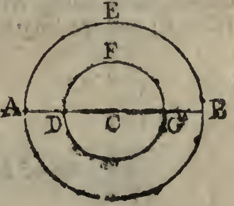
To find the Area of a Circular Ring, or Space included between two Concentric Circles.

Take the difference between the two circles, for the ring; or multiply the ſum of the diameters by their difference, and multiply the product by ·7854, for the anſwer.

EXAMPLES.

1. The diameters of the two concentric circles being AB 10 and DG 6, required the area of the ring contained between their circumferences AEBA, and DFGD.

	10	·7854
	6	64
ſum	16	31416
dif.	4	47124
	64	50·2656 anſ.



Ex. 2. The diameters of two concentric circles being 20 and 10; required the area of the ring between their circumferences. Anſ. 235·62.

Ex. 3. What is the area of a ring, the diameters of whoſe bounding circles are 6 and 4? Anſ. 15·708.

PROBLEM XIV.

To meaſure long Irregular Figures.

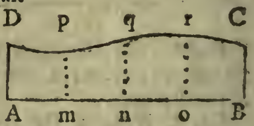
Take the breadth in ſeveral places, at equal diſtances. Add the firſt and laſt two breadths together, and divide the ſum by 2, for the half ſum, or arithmetical mean between thoſe two. Then add together this mean and all the other breadths, omitting the firſt and laſt, and divide their ſum by by the number of parts ſo added, which will give a medium breadth among the whole; then multiply it by the length, to give the true area.

If the breadths be not taken at equal diſtances; then compute all the little trapezoids ſeparately, and add them all together.—Or, add all the breadths together, and divide the ſum by the whole number of them for the mean breadth, to multiply by the length for the whole area, which will not be far from the truth.

EXAMPLES.

1. The breadths of an irregular figure, at five equidiſtant places being AD 8·2, mp 7·4, nq 9·2, or 10·2 BC 8·6; and the length AB 39; required the area.

8·2
8·6
2) 16·8
8·4 mean of firſt and laſt
7·4
9·2
10·2
4) 35·2
8·8 mean of all
39 length
343·2 anſwer.



Ex. 2. The length of an irregular figure being 84, and the breadths at 6 equi-diſtant places 17·4, 20·6, 14·2, 16·5, 20·1, 24·4; what is the area? Anſ. 1550·64.

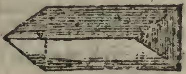
MENSURATION OF SOLIDS.

DEFINITIONS.

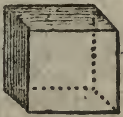
SOLIDS, or bodies, are figures having length, breadth, and thickneſs.

2. A priſm is a ſolid, or body, whoſe ends are any plane figures, which are parallel, equal, and ſimilar; and its ſides are parallelograms.

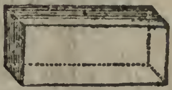
A priſm is called a triangular one, when its ends are triangles; a ſquare priſm, when its ends are ſquares; a pentagonal priſm, when its ends are pentagons; and ſo on.



3. A cube is a ſquare priſm, having ſix ſides, which are all ſquares. It is like a die, having its ſides perpendicular to one another.



4. A parallelopipedon is a ſolid having ſix rectangular ſides, every oppoſite pair of which are equal and parallel.

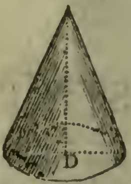


5. A cylinder is a round priſm, having circles for its ends.

6. A pyramid is a ſolid having any plane figure for a baſe, and its ſides are triangles whoſe vertices meet in a point at the top, called the vertex of the pyramid.

The pyramid takes names according to the ſigure of its baſe, like the priſm; being triangular, or ſquare, or hexagonal, &c

7. A cone is a round pyramid, having a circular baſe.



8. A ſphere is a ſolid bounded by one continued convex ſurface, every point of which is equally diſtant from a point within, called the centre.—The ſphere may be conceived to be formed by the revolution of a ſemicircle about its diameter, which remains fixed.

9. The axis of a ſolid, is a line drawn from the middle of one end, to the middle of the oppoſite end; as between the oppoſite ends of a priſm. Hence the axis of a pyramid, is the line from the vertex to the middle of the baſe, or the end on which it is ſuppoſed to ſtand. And the axis of a ſphere, is the ſame as a diameter, or a line paſſing through the centre, and terminated by the ſurface on both ſides.

10. When the axis is perpendicular to the baſe, it is a right priſm or pyramid; otherwiſe, it is oblique.

11. The height or altitude of a ſolid, is a line drawn from its vertex or top, perpendicular to its baſe.—This is equal to the axis in a right priſm or pyramid; but in an oblique one, the height is the perpendicular ſide of a right-angled triangle, whoſe hypothenuſe is the axis.

12. Alſo a priſm or pyramid is regular or irregular, as its baſe is a regular or an irregular plane figure.

13. The ſegment of a pyramid, ſphere, or any other ſolid, is a part cut off the top by a plane parallel to the baſe of that figure.

14. A fruſtrum or trunk, is the part that remains at the bottom, after the ſegment is cut off.

15. A zone of a ſphere, is a part intercepted between two parallel planes; and is the difference between two ſegments. When the ends, or planes, are equally diſtant from the centre, on both ſides, the figure is called the middle zone.

16. The ſector of a ſphere, is compoſed of a ſegment leſs than a hemiſphere or half ſphere, and of a cone having the ſame baſe with the ſegment, and its vertex in the centre of the ſphere.

17. A circular ſpindle, is a ſolid generated by the revolution of a ſegment of a circle about its chord, which remains fixed.

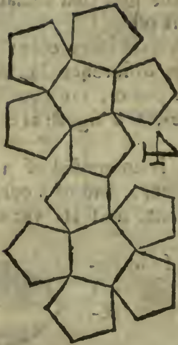
18. A regular body, is a ſolid contained under a certain number of equal and regular plane figures of the ſame ſort.

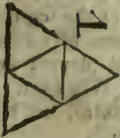
19. The faces of the ſolid are the plane figures under which it is contained. And the linear ſides, or edges of the ſolid, are the ſides of the plane faces.

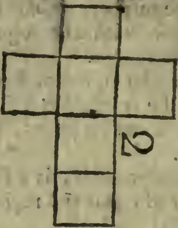
20. There are only five regular bodies: namely, 1ſt, the tetraedon, which is a regular pyramid, having four triangular faces: 2d, the hexaedron, or cube, which has 6 equal ſquare faces; 3d, the octaedron, which has 8 triangular faces; 4th, the dodecaedron, which has 12 pentagonal faces; 5th, the icoſaedron, which has 20 triangular faces.

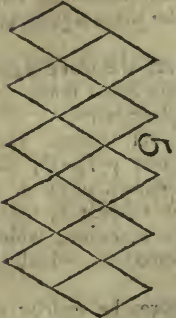
Note. If the following figures be exactly drawn on paſteboard, and the lines cut half through, ſo that the parts be turned up and their edges glued together, they will repreſent the five regular bodies: namely, figure 1 the tetraedron, figure 2 the hexaedron, figure 3 the octaedron, figure 4 the dodecaedron, and figure 5 the icoſaedron.

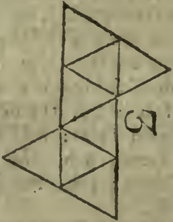
Fig. 1.











Note alſo, that, in cubic meaſure,

1728 inches	make	1 foot
27 ſeet	- -	1 yard
166$\frac{3}{8}$ yards	-	1 pole
64000 poles	- -	1 furlong
512 furlongs		1 mile.

PROBLEM I.

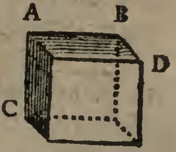
To find the Solidity of a Cube.

Cube one of its ſides for the content; that is, multiply the ſide by itſelf, and that product by the ſide again.

EXAMPLES.

1. If the ſide AB, or AC, or BD, of a cube be 24 inches, what is its ſolidity or content?

$$\begin{array}{r} 24 \\ 24 \\ \hline 96 \\ 48 \\ \hline 576 \\ 24 \\ \hline 2304 \\ 1152 \\ \hline 13824 \text{ anſwer.} \\ \hline \end{array}$$



Ex. 2. How many ſolid feet are in the cube whoſe ſide is 22 feet?
Anſ. 10648.

Ex. 3 Required how many ſolid feet are in the cube whoſe ſide is 18 inches?
Anſ. $3\frac{3}{8}$.

PROBLEM II.

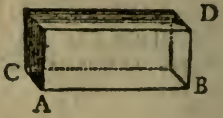
To find the Solidity of a Parallelopipedon.

Multiply the length, breadth, and depth, or altitude, all continually together, for the ſolid content; that is, multiply the length by the breadth, and that product by the depth.

EXAMPLES.

1. Required the content of the parallelopipedon, whoſe length AB is 6 feet, its breadth AC $2\frac{1}{2}$ feet, and altitude BD $1\frac{3}{4}$ feet?

$$\begin{array}{r l} 1{\cdot}75 & \text{BD} \\ 6 & \text{AB} \\ \hline 10{\cdot}50 & \\ 2{\cdot}5 & \text{AC} \\ \hline 5250 & \\ 2100 & \\ \hline 26{\cdot}250 & \text{anſwer.} \\ \hline \end{array}$$



Ex. 2. Required the content of a parallelopipedon, whoſe length is 10·5, breadth 4·2, and height 3·4? Anſ. 149·94.

Ex. 3. How many cubic feet are in a block of marble, whoſe length is 3 feet 2 inches, breadth 2 feet 8 inches, and depth 2 feet 6 inches? Anſ. $21\frac{1}{9}$.

PROBLEM III.

To find the Solidity of any Priſm.

Find the area of the baſe, or end; which multiply by the height, or length, and it will give the content.

Which rule will do, whether the priſm be triangular, or ſquare, or pentagonal, &c. or round, as a cylinder.

EXAMPLES.

1. What is the content of a triangular priſm, whoſe length AC is 12 feet, and each ſide AB of its equilateral baſe $2\frac{1}{2}$ feet?

Here $\frac{5}{2} \times \frac{5}{2} = \frac{25}{4} = 6\frac{1}{4}$.

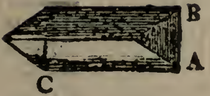
Then

Then ·433013 tabular n°
$6\frac{1}{4}$

2·598078
108253

2·706331 area of end
12 length

32·475972 anſwer.



Ex. 2. Required the ſolidity of a triangular priſm, whoſe length is 10 feet, and the three ſides of its triangular end or baſe, are 5, 4, 3 feet? Anſ. 60.

Ex. 3. What is the content of a hexagonal priſm, the length being 8 feet, and each ſide of its end 1 foot 6 inches? Anſ. 46·765368.

Ex. 4. Required the content of a cylinder, whoſe length is 20 feet, and circumference $5\frac{1}{2}$ feet? Anſ. 48·1459.

Ex. 5. What is the content of a round pillar, whoſe height is 16 feet, and diameter 2 feet 3 inches? Anſ. 63·6174.

PROBLEM IV.

To find the Convex Surface of a Cylinder.

Multiply the circumference by the height or length of the cylinder.

Note. The upright ſurface of any priſm is found in the ſame manner, viz. by multiplying the perimeter of the end by the length. And the ſolidity of a cylinder is found as the priſm in the laſt problem.

EXAMPLES.

1. What is the convex ſurface of a cylinder, whoſe length is 16 feet, and its diameter 2 feet 3 inches.

3·1416

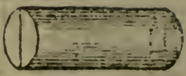
3·1416
$2\frac{1}{4}$ diameter

6·2832
·7854

7·0686 circumf.
16

424116
70686

113·0976 anſwer.



Ex. 2. Required the convex ſurface of the cylinder; whoſe length is 20 feet, and its diameter 2 feet?
Anſ. 125·664.

Ex. 3. What is the convex ſurface of a cylinder, whoſe length is 18 feet 6 inches, and circumference 5 feet 4 inches?
Anſ. $98\frac{2}{3}$.

PROBLEM V.

To find the Convex Surface of a Right Cone.

Multiply the circumference of the baſe by the ſlant height, or length of the ſide, and take half the product for the ſurface.

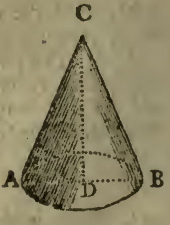
Ex. 1. If the diameter of the baſe be AB 5 feet, and the ſide of the cone AC 18, required the convex ſurface?

3·1416
5 diameter

15·7080 circumf.
18

125664
15708

2) 282·744
141·372 anſwer.



Ex. 2.

Ex. 2. What is the convex ſurface of a cone, whoſe ſide is 20, and the circumference of its baſe 9?

Anſ. 90.

Ex. 3. Required the convex ſurface of a cone, whoſe ſlant height is 50 feet, and the diameter of its baſe 8 feet 6 inches? Anſ. 667·59.

PROBLEM VI.

To find the Convex Surface of the Fruſtum of a Right Cone.

Add together the perimeters of the two ends; then multiply that ſum by the ſlant height, or ſide of the fruſtum, and take half the product for the ſurface.

EXAMPLES.

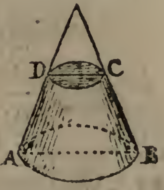
1. If the circumferences of the two ends be 12·5 and 10·3 and the ſlant height AD 14, required the convex ſurface of the fruſtum ABCD?

12·5
10·3

22·8 ſum.
14

912
228

2) 319·2
159·6 anſwer.



Ex. 2. What is the convex ſurface of the fruſtum of a cone, the ſlant height of the fruſtum being 12·5, and the circumferences of the two ends 6 and 8·4? Anſ. 90.

Ex. 3. Required the convex ſurface of the fruſtum of a cone, the ſide of the fruſtum being 10 feet 6 inches, and the circumferences of the two ends 2 feet 3 inches, and 5 feet 4 inches? Anſ. $39\frac{13}{16}$.

PROBLEM VII.

To find the Solidity of a Cone, or any Pyramid.

Compute the area of the baſe, then multiply that area by the height, and take $\frac{1}{3}$ of the product for the content.

EXAMPLES.

1. What is the ſolidity of a cone, whoſe height CD is $12\frac{1}{2}$ feet, and the diameter AB of the baſe $2\frac{1}{2}$?

Here $2\frac{1}{2} \times 2\frac{1}{2} = \frac{5}{2} \times \frac{5}{2} = \frac{25}{4} = 6\frac{1}{4}$.

Then ·7854
$6\frac{1}{4}$

4·7124
19635

4·90875 area of baſe.
$12\frac{1}{2}$ height.

5890500
2454375

3) 61·359375
20·453125 anſwer.

Ex. 2. What is the ſolid content of a pentagonal pyramid, its height being 12 feet, and each ſide of its baſe 2 feet?

1·720477 tab. area
4 ſq. ſide

6·881908 area baſe
4 $\frac{1}{3}$ of height

27·527632

Ex. 3.

Ex. 3. What is the content of a cone, its height being $10\frac{1}{2}$ feet, and the circumference of its baſe 9 feet?

Anſ. 22·56093.

Ex. 4. Required the content of a triangular pyramid, its height being 14 feet 6 inches, and the three ſides of its baſe 5, 6, 7? Anſ. 71·0352.

Ex. 5. What is the content of a hexagonal pyramid, whoſe height is 6·4, and each ſide of its baſe 6 inches?

Anſ. 1·38561.

PROBLEM VIII.

To find the Solidity of the Fruſtum of a Cone or any Pyramid.

RULES.

1. Add into one ſum, the areas of the two ends, and the mean proportional between them, or the ſquare root of their product; and take $\frac{1}{3}$ of that ſum for a mean area; which multiplied by the height of the fruſtum, will give the content.

2. When the ends are regular plane figures; the mean area will be found by multiplying $\frac{1}{3}$ of the correſponding tabular number belonging to the polygon, either by the ſum ariſing by adding together the ſquare of a ſide of each end and the product of the two ſides, or by the quotient of the difference of their cubes divided by their difference, or by the ſum ariſing from the ſquare of their half difference added to 3 times the ſquare of their half ſum.

3. And in the fruſtum of a cone, the mean area is found by multiplying ·2618, or $\frac{1}{3}$ of ·7854, either by the ſum ariſing by adding together the ſquares of the two diameters and the product of the two, or by the difference of their cubes divided by their difference, or by the ſquare of half their difference added to 3 times the ſquare of their half ſum.

Or, if the circumferences be uſed in like manner, inſtead of their diameters, the multiplier will be ·02654, inſtead of ·2618.

EXAMPLES.

1. What is the content of a fruſtum of a cone, whoſe height is 20 inches, and the diameters of its two ends 28 and 20 inches?

```
 28       28      20
 28       20      20
----     ----    ----
224      560     400
56       784     ----
----     400
784     -----
----     1744
         ·2618
        ------
         13952
         1744
        10464
        3488
       ---------
       456·5792
             20
       ---------
       131·5840 anſwer.
       ---------
```

See Fig. to Prob. vi.

Ex. 2. Required the content of a pentagonal fruſtum, whoſe height is 5 feet, each ſide of the baſe 1 foot 6 inches, and each ſide of the leſs end 6 inches?

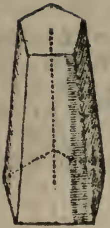
18	18	6
18	6	6
144	108	36
18	324	
324	36	

3) 468

156 ⅓ of ſum
1·720477 tab. area

10322862
8602385
1720477

268·394412 mean area
5 height.



144 { 12 12 | 1341·972060
111·831005
9·319250 anſwer in cubic feet.

Ex. 3. What is the ſolidity of the fruſtum of a cone, the altitude being 25, the circumference at the greater end 20, and at the leſs end 10? Anſ. 464·205.

Ex. 4. How many ſolid feet are in a piece of timber, whoſe baſes are ſquares, each ſide of the greater end being 15 inches, and each ſide of the leſs end 6 inches; alſo the length, or perpendicular altitude is 24 feet? Anſ. $19\frac{1}{2}$.

Ex. 5. To find the content of the fruſtum of a cone, the altitude being 18, the greateſt diameter 8, and the leaſt 4. Anſ. 527·7888.

Ex. 6. What is the ſolidity of a hexagonal fruſtum, the height being 6 feet, the ſide of the greater end 18 inches, and of the leſs 12 inches? Anſ. 24·681722.

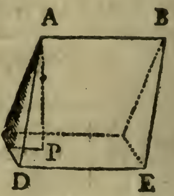
PROBLEM IX.

To find the Solidity of a Wedge.

To the length of the edge add twice the length of the back or baſe, and reſerve the ſum; multiply the height of the wedge by the breadth of the baſe; then multiply this product by the reſerved ſum, and take $\frac{1}{6}$ of the laſt product for the content.

EXAMPLES.

1. What is the content in feet of a wedge, whoſe altitude AP is 14 inches, its edge AB 21 inches, and the length of its baſe DE 32 inches, and its breadth CD $4\frac{1}{2}$ inches?



```
21      14
32      4½
32      ——
——      56
85       7
——      ——
        63
        85
        ———
        315
        504
        ———
     ⎧ 6 | 5355
1728 ⎨12 |  892·5 anſ. in cubic inches
     ⎪12 |   74·375
     ⎩12 |    6·197916
         |     ·516493 anſ. in feet, or little more
                         than half a cubic foot.
```

Ex. 2. Required the content of a wedge, the length and breadth of the baſe being 70 and 30 inches, the length of the edge 110 inches, and the height 34·29016?

Anſ. 24·8048.

PROBLEM X.

To find the Solidity of a Priſmoid.

Definition.

A priſmoid differs only from the fruſtum of a pyramid, in not having its oppoſite ends ſimilar planes.

RULE.

RULE.

Add into one ſum, the areas of the two ends and 4 times the middle ſection parallel to them, and $\frac{1}{6}$ of that ſum will be a mean area; which being multiplied by the height, will give the content.

Note. For the length of the middle ſection, take half the ſum of the lengths of the two ends; and for its breadth, take half the ſum of the breadths of the two ends.

EXAMPLES.

1. How many cubic feet are there in a ſtone, whoſe ends are rectangles, the length and breadth of the one being 14 and 12 inches; and the correſponding ſides of the other 6 and 4 inches; the perpendicular height being $30\frac{1}{2}$ feet?

14 10 6
12 8 4
168 80 24
4
320
168
24
6) 512
$85\frac{1}{3}$ mean area in inches.
$30\frac{1}{2}$ height
2560
$42\frac{2}{3}$
$144 \begin{cases} 12 \\ 12 \end{cases}$ $2602{\cdot}\dot{6}$
$216{\cdot}\dot{8}$
$18{\cdot}\dot{0}7\dot{4}$ anſwer.

Ex. 2. Required the content of a rectangular priſmoid, whoſe greater end meaſures 12 inches by 8, the leſſer end 8 inches by 6, and the perpendicular height 5 feet?

Anſ. 2·453 feet.

Ex. 3. What is the content of a cart or waggon, whoſe inſide dimenſions are as follow: at the top the length and breadth $81\frac{1}{2}$ and 55 inches, at the bottom the length and breadth 41 and $29\frac{1}{2}$ inches, and the height $47\frac{1}{4}$ inches?

Anſ. 126340·59375 cubic inches.

PROBLEM XI.

To find the Convex Surface of a Sphere or Globe.

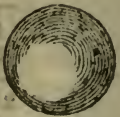
Multiply its circumference by its diameter.

Note. In like manner the convex ſurface of any zone or ſegment is found, by multiplying its height by the whole circumference of the ſphere.

EXAMPLES.

1. Required the convex ſuperficies of a globe, whoſe diameter or axis is 24 inches.

```
 3·1416
     24 diam.
 ------
 125664
 62832
 ------
 75·3984 circumf.
      24
 -------
 3015936
1507968
 -------
1809·5616 anſwer.
```



Ex. 2. What is the convex ſurface of a ſphere, whoſe diameter is 7, and circumference 22? Anſ. 154.

Ex. 3.

Ex. 3. Required the area of the ſurface of the earth, its diameter, or axis, being 7957$\frac{3}{4}$ miles, and its circumference 25000 miles? Anſ. 198943750 ſq. miles.

Ex. 4. The axis of a ſphere being 42 inches, what is the convex ſuperficies of the ſegment, whoſe height is 9 inches? Anſ. 1187·5248 inches.

Ex. 5. Required the convex ſurface of a ſpherical zone, whoſe breadth or height is 2 feet, and cut from a ſphere of 12$\frac{1}{2}$ feet diameter? Anſ. 78·54 feet.

PROBLEM XII.

To find the Solidity of a Sphere or Globe.

Find the cube of the axis, and multiply it by ·5236.

EXAMPLES.

1. What is the ſolidity of the ſphere, whoſe axis is 12?

	Or thus
12	·5236
12	12
144	6·2832
12	12
1728	75·3984
·5236	12
10368	904·7808 anſ.
5184	
3456	
8640	
904·7808 anſ.	

Ex. 2. To find the content of the ſphere, whoſe axis is 2 feet 8 inches. Anſ. 9·9288 feet.

Ex. 3. Required the ſolid content of the earth, ſuppoſing its circumference to be 25000 miles?

Anſ. 263858149120 miles.

PROBLEM XIII.

To find the Solidity of a Spherical Segment.

To three times the ſquare of the radius of its baſe, add the ſquare of its height; then multiply the ſum by the height, and the product again by ·5236.

EXAMPLES.

1. Required the content of a ſpherical ſegment, its height being 4 inches, and the radius of its baſe 8?

8	4	·5236
8	4	832
64	16	10472
3	192	15708
		41888
192	208	
	4	435·6352 anſ.
	832	

Ex. 2. What is the ſolidity of the ſegment of a ſphere, whoſe height is 9, and the diameter of its baſe 20?

Anſ. 1795·4244.

Ex. 3. Required the content of the ſpherical ſegment, whoſe height is $2\frac{1}{4}$, and the diameter of its baſe 8·61684?

Anſ. 71·5695.

PROBLEM XIV.

To find the Solidity of a Spherical Zone or Fruſtum.

Add together the ſquare of the radius of each end, and $\frac{1}{3}$ of the ſquare of their diſtance, or of the height; then multiply the ſum by the ſaid height, and the product again by 1·5708.

EX-

EXAMPLES.

1. What is the ſolid content of a zone, whoſe greater diameter is 12 inches, the leſs 8, and the height 10 inches?

$$\begin{array}{rrr}
6 & 4 & 10 \\
6 & 4 & 10 \\
\hline
36 & 16 & 3\,)\,100 \\
\cline{1-1}
 & 36 & 33\frac{1}{3} \\
\cline{3-3}
 & 33\frac{1}{3} & \\
\cline{2-2}
 & 85\frac{1}{3} & \\
 & 1\cdot5708 & \\
\cline{2-2}
 & 78540 & \\
 & 125664 & \\
 & 5236 & \\
\cline{2-2}
 & 134\cdot0416 & \\
 & 10 & \\
\cline{2-2}
 & 1340\cdot416 \text{ anſ.} & \\
\cline{2-2}
\end{array}$$

Ex. 2. Required the content of a zone, whoſe greater diameter is 12, leſs diameter 10, and height 2?

Anſ. 195·8264.

Ex. 3. What is the content of a middle zone, whoſe height is 8 feet, and the diameter of each end 6?

Anſ. 494·2784 feet.

PROBLEM XV.

To find the Surface of a Circular Spindle.

Multiply the length AB of the ſpindle by the radius OC of the revolving arc. Multiply alſo the ſaid arc ACB by the central diſtance OE, or diſtance between the

the centre of the ſpindle and centre of the revolving arc. Subtract the latter product from the former, and multiply double the remainder by 3·1416, or the ſingle remainder by 6·2832, for the ſurface.

Note. The ſame rule will ſerve for any ſegment or zone cut off perpendicular to the chord of the revolving arc, only uſing the particular length of the part, and the part of the arc which deſcribes it, inſtead of the whole length and whole arc.

EXAMPLES.

1. Required the ſurface of a circular ſpindle, whoſe length AB is 40, and its thickneſs CD 30 inches?

Here, by the notes at pa. 91.

The chord AC $= \sqrt{AE^2 + CE^2} = \sqrt{20^2 + 15^2} = 25$,

and 2CE : AC : : AC : CO $= \frac{25^2}{30} = 20\frac{5}{6}$,

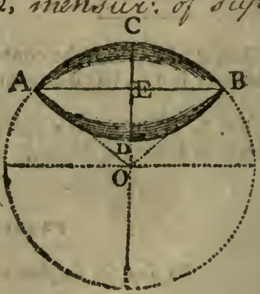
hence OE $=$ OC $-$ CE $= 20\frac{5}{6} - 15 = 5\frac{5}{6}$.

Alſo, by problem ix, rule 2, *mensur: of super:*

25 AC
8

200
40 AB

3) 160
53⅓ arc ACB



Then,

Then, by the rule,

$20\frac{5}{6}$	$53\frac{1}{3}$
40	$5\frac{5}{6}$
800	$266\frac{2}{3}$
$33\frac{1}{3}$	$44\frac{4}{9}$
$833\frac{1}{3}$	$311\frac{1}{9}$
$311\frac{1}{9}$	
$522\frac{2}{9}$ or $522{\cdot}\dot{2}$ or $\frac{4700}{9}$	
$6{\cdot}2832$	Or thus,
10444	$6{\cdot}2832$
156666	4700
4177777	439824
10444444	251328
313333333	
	$9\,)\,29531{\cdot}04$
$3281{\cdot}2266\dot{6}$	$3281{\cdot}22\dot{6}$ anſ. nearly.

Ex. 2. What is the ſurface of a circular ſpindle, whoſe length is 24, and thickneſs in the middle 18?

Anſ. 1177·4485.

PROBLEM XVI.

To find the Solidity of a Circular Spindle.

Find the area of the revolving ſegment ACBEA, which multiply by half the central diſtance OE. Subtract the product from $\frac{1}{3}$ of the cube of AE, half the length of the ſpindle. Then multiply the remainder by 12·5664, or 4 times 3·1416, for the whole content.

EXAMPLES.

1. Required the content of the circular ſpindle, whoſe length AB is 40, and middle diameter CD 30?

[*See the laſt Figure.*]

By

By the work of the laſt problem,
we have OE = $6\frac{5}{6}$
and arc AC = $26\frac{2}{3}$
and rad. OC = $20\frac{5}{6}$

$533\frac{1}{3}$
$22\frac{2}{9}$

Sector OACB $555\frac{5}{9}$
AE × OE = OAB $116\frac{2}{3}$

2) $438\frac{8}{9}$

$\frac{1}{2}$ ſeg. ACE $219\frac{4}{9}$
OE $5\frac{5}{6}$

$1097\frac{2}{9}$
183 nearly

$1280\frac{2}{9}$

20 half length
20

400
20

3) 8000

$2666\frac{2}{3}$
$1280\frac{2}{9}$

$1386\frac{4}{9}$

or 1386·44
4665·21 mult. inver.

138644
27739
6932
832
83
5

17423·5 Anſ.

Ex. 2. What is the ſolidity of a circular ſpindle, whoſe length is 24, and middle diameter 18?

Anſ. 3739·93.

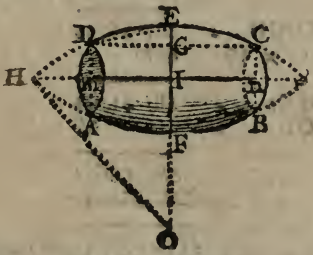
PROBLEM XVII.

To find the Solidity of the Middle Fruſtum or Zone of a Circular Spindle.

From the ſquare of half the length of the whole ſpindle, take $\frac{1}{3}$ of the ſquare of half the length of the middle fruſtum, and multiply the remainder by the ſaid half length of the fruſtum.—Multiply the central diſtance by by the revolving area, which generates the middle fruſtum.—Subtract this latter product from the former; then the remainder multiplied by 6·2832, or 2 times 3·1416, will give the content.

EXAMPLES.

1. Required the ſolidity of the fruſtum, whoſe length mn is 40 inches, alſo its greateſt diameter EF is 32, and leaſt diameter AD or BC 24?



Draw DG parallel to mn, then we
have $DG = \frac{1}{2}mn = 20$,
and $EG = \frac{1}{2}EF - \frac{1}{2}AD = 4$,
chord $DE^2 = DG^2 + GE^2 = 416$,

and $DE^2 \div EG = \frac{416}{4} = 104$ the diameter of the generating circle,
or the radius $OE = 52$,
hence $OI = 52 - 16 = 36$ the central diſtance,
and $HI^2 = OH^2 - OI^2 = 52^2 - 36^2 = 1408$,
$\frac{1}{3}DG^2 = \frac{1}{3}$ of $400 = \;.\;.\;.\;133\frac{1}{3}$

$1274\frac{2}{3}$
DG . . . 20

$25493\frac{1}{3}$ 1ſt prod.
GE

$$GE + 2\,OE = \frac{4}{104} = \frac{1}{26} = \cdot 038$$

Its tab. ſegment - -	·00994
but 104^2 is - -	10816
	43264
	97344
	97344
area of ſeg. DECGD -	107·51104
$mD \times mn = 12 \times 40$	408·
gener. area mDECn -	587·51104
OI -	36
	352506624
	176253312
	21150·39744
	25493·33333
	4342·93589 c
	2382·6 n.
	260576
	8686
	3474
	130
	9
	27287·5 anſwer.

Ex. 2. What is the content of the middle fruſtum of a circular ſpindle, whoſe length is 20, greateſt diameter 18, and leaſt diameter 8? Anſ. 3657·1613.

PRO-

PROBLEM XVIII.

To find the Superficies or Solidity of any Regular Body.

1. Multiply the proper tabular area (taken from the following table) by the ſquare of the linear edge of the ſolid, for the ſuperficies.

2. Multiply the tabular ſolidity by the cube of the linear edge, for the ſolid content.

Surfaces and Solidities of Regular Bodies.

No. of faces	Names.	Surfaces	Solidities
4	Tetraedron	1·73205	0·11785
6	Hexaedron	6·00000	1·00000
8	Octaedron	3·46410	0·47140
12	Dodecaedron	20·64573	7·66312
20	Icoſaedron	8·66025	2·18169

EXAMPLES.

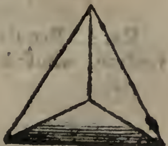
1. If the linear edge or ſide of a tetraedron by 3, required its ſurface and ſolidity?

The ſquare of 3 is 9, and the cube 27. Then,

ta . ſurf. 1·73205 0·11785 tab. ſol.
9 27

ſuperf. 15·58846 82495
23570

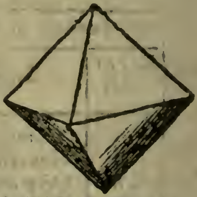
ſolidity 3·18195



Ex. 2. What is the ſuperficies and ſolidity of the hexaedron, whoſe lineal ſide is 2?

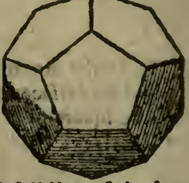
Anſ. { ſuperficies 24
ſolidity 8

Ex. 3. Required the ſuperficies and ſolidity of the octaedron, whoſe linear ſide is 2?



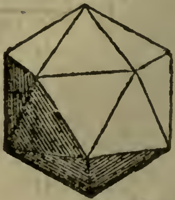
Anſ. { ſuperficies 13·85640
ſolidity 3·77120

Ex. 4. What is the ſuperficies and ſolidity of the dodecaedron, whoſe linear ſide is 2?



Anſ. { ſuperficies 82·58292
ſolidity 61·30496

Ex. 5. Required the ſuperficies and ſolidity of the icoſaedron, whoſe linear ſide is 2?



Anſ. { ſuperficies 34·64100
ſolidity 17·45352

PRO.

PROBLEM XIX.

To find the Surface of a Cylindrical Ring.

This figure being only a cylinder bent round into a ring, its ſurface and ſolidity may be found as in the cylinder, namely, by multiplying the axis, or length of the cylinder, by the circumference of the ring, or of the ſection, for the ſurface; and by the area of a ſection, for the ſolidity.

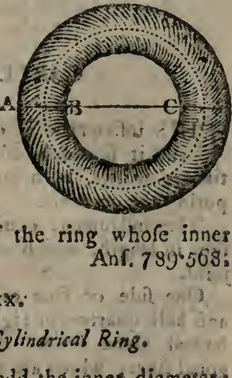
Or uſe the following rules:

For the ſurface.—To the thickneſs of the ring add the inner diameter; multiply this ſum by the thickneſs, and the product again by 9·8696, or the ſquare of 3·1416.

EXAMPLES.

1. Required the ſuperficies of a ring, whoſe thickneſs AB is 2 inches, and inner diameter BC is 12 inches?

12	9·8696
2	28
14	789568
2	197392
28	276·3488 Anſ.



Ex. 2. What is the ſurface of the ring whoſe inner diameter is 16, and thickneſs 4? Anſ. 789·568.

PROBLEM XX.

To find the Solidity of a Cylindrical Ring.

To the thickneſs of the ring, add the inner diameter; then multiply that ſum by the ſquare of the thickneſs, and the product again by 2·4674, or $\frac{1}{4}$ of the ſquare of 3·1416, for the ſolidity.

EXAMPLES.

1. Required the ſolidity of the ring, whoſe thickneſs is 2 inches, and its inner diameter 12?

12	2·4674
2	·56
14	148044
4	123370
56	138·1744 anſ.

Ex. 2. What is the ſolidity of a cylindrical ring, whoſe thickneſs is 4, and inner diameter 16?

Anſ. 789·568.

OF THE

CARPENTERS' RULE.

THIS inſtrument is otherwiſe called the ſliding rule; and it is much uſed in meaſuring of timber and artificers' works, both for taking the dimenſions, and computing the contents.

The inſtrument conſiſts of two equal pieces, each a foot in length, which are connected together by a folding joint.

One ſide or face of the rule, is divided into inches, and half quarters or eighths. On the ſame face alſo are ſeveral plane ſcales, divided into 12th parts by diagonal lines; which are uſed in planning dimenſions that are taken in feet and inches. The edge of the rule is commonly divided decimally, or into tenths; namely, each foot into 10 equal parts, and each of thoſe into 10 parts again;

again: ſo that, by means of this laſt ſcale, dimenſions are taken in feet and tenths and hundredths, and then multiplied as common decimal numbers, which is the beſt way.

On the one part of the other face are four lines, marked A, B, C, D; the two middle ones, B and C, being on a ſlider, which runs in a groove made in the ſtock. The ſame numbers ſerve for both theſe two middle lines, the one being above the numbers, and the other below.

Theſe four lines are logarithmic ones, and the three A, B, C, which are all equal to one another, are double lines, as they proceed twice over from 1 to 10. The other or loweſt line D, is a ſingle one, proceeding from 4 to 40. It is alſo called the girt line, from its uſe in computing the contents of trees and timber. Upon it are alſo marked WG at 17·15, and AG at 18·95, the wine and ale gage points, to make this inſtrument ſerve the purpoſe of a gaging rule.

On the other part of this face there is a table of the value of a load, or 50 cubic feet of timber, at all prices, from 6 pence to 2 ſhillings a-foot.

When 1 at the beginning of any line is accounted 1, or unit, then the 1 in the middle will be 10, and the 1 at the end 100; and when 1 at the beginning is accounted 10, then the 1 in the middle is 100, and the 1 at the end 1000; and ſo on. All the ſmaller diviſions being altered proportionally.

PROBLEM I.

To multiply Numbers together.

Suppoſe the two numbers 13 and 24.—Set 1 on B to 13 on A; then againſt 24 on B ſtands 312 on A, which is the required product of the two given numbers 13 and 24.

Note. In any operations, when a number runs beyond the end of the line, ſeek it on the other radius, or other part of the line; that is, take the 10th part of it, or the

100th part of it, &c. and increaſe the reſult proportionally 10 fold, or 100 fold, &c.

In like manner the product of 35 and 19 is 665.
and the product of 270 and 54 is 14580.

PROBLEM II.

To divide by the Sliding Rule.

As ſuppoſe to divide 312 by 24.—Set the diviſor 24 on B to the dividend 312 on A; then againſt 1 on B ſtands 13, the quotient, on A.

Alſo 396 divided by 27 gives 14·6.
And 741 divided by 42 gives 17·6.

PROBLEM III.

To ſquare any Number.

Suppoſe to ſquare 23.—Set 1 on B to 23 on A; then againſt 23 on B, ſtands 529 on A, which is the ſquare of 23.

Or, by the other two lines, ſet 1 or 100 on C to the 10 on D, then againſt every number on D, ſtands its ſquare in the line C. So againſt 23 ſtands 529
againſt 20 ſtands 400
againſt 30 ſtands 900
and ſo on.

If the given number be hundreds, &c. reckon the 1 on D for 100, or 1000, &c. then the correſponding 1 on C is 10000, or 1000000, &c. So the ſquare of 230 is found to be 52900.

PROBLEM IV.

To extract the Square Root.

Set 1 or 100, &c. on C to 1 or 10, &c. on D; then againſt every number found on C, ſtands its ſquare root on D.

So,

So, againſt 529 ſtands its root 23
againſt 400 ſtands its root 20
againſt 900 ſtands its root 30
againſt 300 ſtands its root 17·3
and ſo on.

PROBLEM V.

To find a Mean Proportional between two Numbers.

As ſuppoſe between 29 and 430.—Set the one number 29 on C to the ſame on D; then againſt the other number 430 on C, ſtands their mean proportional 111 on D.

Alſo the mean between 29 and 320 is 96·3.

And the mean between 71 and 274 is 139.

PROBLEM VI.

To find a Third Proportional to two Numbers.

Suppoſe to 21 and 32.—Set the firſt 21 on B to the ſecond 32 on A; then againſt the ſecond 32 on B, ſtands 48·8 on A; which is the third proportional ſought.

Alſo the 3d proportional to 17 and 29 is 49·4.

And the 3d proportional to 73 and 14 is 2·5.

PROBLEM VII.

To find a Fourth Proportional to three Numbers.

Or, to perform the Rule-of-Three.

Suppoſe to find a fourth proportional to 12, 28, and 114.—Set the firſt term 12 on B to the 2d term 28 on A; then againſt the third term 114 on B, ſtands 266 on A, which is the fourth proportional ſought.

Alſo the 4th proportional to 6, 14, 29, is 67·6.

And the 4th proportional to 27, 20, 73, is 54·0.

TIMBER MEASURING.

PROBLEM I.

To find the Area, or Superficial Content, of a Board or Plank.

MULTIPLY the length by the mean breadth.

Note. When the board is tapering, add the breadths at the two ends together, and take half the ſum for the mean breadth.

By the Sliding Rule.

Set 12 on B to the breadth in inches on A; then againſt the length in feet on B, is the content on A, in feet and fractional parts.

EXAMPLES.

1. What is the value of a plank, at $1\frac{1}{2}$d. per foot, whoſe length is 12 feet 6 inches, and mean breadth 11 inches?

By Decimals.

	12·5
	11
12	137·5
	1146
$1\frac{1}{2}$d.is $\frac{1}{8}$	1s. 5d anſ.

By Duodecimals.

	12	6	
		11	
$1\frac{1}{2}$d.is $\frac{1}{8}$	11	5	6
	is	$4\frac{1}{4}$d	
5 is	0	$\frac{1}{2}$	
	1s	5d anſ.	

By the Sliding Rule.

As 12 B : 11 A : : $12\frac{1}{2}$ B ; $11\frac{1}{2}$ A.

That is, as 12 on B is to 11 on A, ſo is $12\frac{1}{2}$ on B to $11\frac{1}{2}$ on A.

Ex. 2. Required the content of a board, whoſe length is 11 feet 2 inches, and breadth 1 foot 10 inches.

Anſ. 20^{f} 5^{i} $8''$

Ex. 3. What is the value of a plank, which is 12 feet 9 inches long, and 1 foot 3 inches broad, at $2\frac{1}{2}$d. a foot?

Anſ. 3s. $3\frac{3}{4}$d.

Ex. 4. Required the value of 5 oaken planks at 3d. per foot, each of them being $17\frac{1}{2}$ feet long; and their ſeveral breadths are as follow, namely, two of $13\frac{1}{2}$ inches in the middle, one of $14\frac{1}{4}$ inches in the middle, and the two remaining ones, each 18 inches at the broader end, and $11\frac{1}{4}$ at the narrower.

Anſ. £1 5 $8\frac{1}{4}$.

PROBLEM II.

To find the Solid Content of Squared or Four-ſided Timber.

Multiply the mean breadth by the mean thickneſs, and the product again by the length, and the laſt product will give the content.

By the Sliding Rule.

C D D C

As length : 12 or 10 : : quarter girt : content.

That is, as the length in feet on C, is to 12 on D when the quarter girt is in inches, or to 10 on D when it is in tenths of feet; ſo is the quarter girt on D, to the content on C.

Note 1. If the tree taper regularly from the one end to the other, either take the mean breadth and thickneſs in the middle, or take the dimenſions at the two ends, and half their ſum for the mean dimenſions.

2. If the piece do not taper regularly, but is unequally thick in ſome parts and ſmall in others; take ſeveral different dimenſions, add them all together, and divide their ſum by the number of them, for the mean dimenſions.

3. The quarter girt is a geometrical mean proportional between the mean breadth and thickneſs, that is the ſquare root of their product. Sometimes unſkilful

meaſurers uſe the arithmetical mean inſtead of it, that is half their ſum; but this is always attended with error, and the more ſo, as the breadth and depth differ the more from each other.

EXAMPLES.

1. The length of a piece of timber is 18 feet 6 inches, the breadths at the greater and leſs end 1 foot 6 inches and 1 foot 3 inches, and the thickneſs at the greater and leſs end 1 foot 3 inches and 1 foot: required the ſolid content.

Decimals.			*Duodecimals.*			
1·5			1	6		
1·25			1	3		
2) 2·75		2)	2	9		
1·375	mean breadth		1	4	6	
1·25			1	3		
1· 0			1	0		
2) 2·25		2	2	3		
1·125	mean depth		1	1	6	
1·375	mean breadth		1	4	6	
'5625			1	1	6	
7875				4	6	
3375					6	
1125						
1·546875			1	6	6	9
18·5	length		18	6		
7734375						
2375000			27	10	1	
11546875				9	3	4
28·6171875	content		28	7	4	10

By

By the Sliding Rule.

B A B A
As 1 : $13\frac{1}{2}$: : $16\frac{1}{2}$: 223, the mean ſquare.
C D C D
As 1 : 1 : : 223 : 14·9, quarter girt.
C D D C
As $18\frac{1}{2}$: 12 : : 14·9 : 28·6, the content.

Ex. 2. What is the content of the piece of timber, whoſe length is $24\frac{1}{2}$ feet, and the mean breadth and thickneſs each 1·04 feet? Anſ. $26\frac{1}{2}$ feet.

Ex. 3. Required the content of a piece of timber, whoſe length is 20·38 feet. and its ends unequal ſquares, the ſide of the greater being $19\frac{1}{8}$, and the ſide of the leſs $9\frac{7}{8}$ inches? Anſ. 29·756 feet.

Ex. 4. Required the content of the piece of timber, whoſe length is 27·36 feet; at the greater end the breadth is 1·78, and the thickneſs 1·23; and at the leſs end the breadth is 1·04, and thickneſs 0·91? Anſ. 41·278 feet.

PROBLEM III.

To find the Solidity of Round or unſquared Timber.

Rule 1, *or Common Rule.*

Multiply the ſquare of the quarter girt, or of $\frac{1}{4}$ of the mean circumferenee, by the length, for the content.

By the Sliding Rule.

As the length upon C : 12 or 10 upon D : : quarter girt, in 12ths or 10ths, on D : content on C.

Note 1. When the tree is tapering, take the mean dimenſions as in the former problems, either by girting it in the middle, for the mean girt, or at the two ends, and take half the ſum of the two. But when the tree is very irregular, divide it into ſeveral lengths, and find the content of each part ſeparately: or elſe, add all the girts together; and divide the ſum by the number of them, for the mean girt.

2. This rule, which is commonly uſed, gives the anſwer about $\frac{1}{4}$ leſs than the true quantity in the tree, or nearly what the quantity would be after the tree is hewed ſquare in the uſual way; ſo that it ſeems intended to make an allowance for the ſquaring of the tree. When the true quantity is deſired, uſe the 2d rule, given below.

EXAMPLES.

1. A piece of round timber being 9 feet 6 inches long, and its mean quarter girt 42 inches; what is the content?

Decimals.		*Duodecimals.*		
3·5	quarter girt	3	6	
3·5		3	6	
175		10	6	
105		1	9	
12·25		12	3	
9·5	length	9	6	
6125		110	3	
11025		6	1	6
116·375	content	116	4	6

By the Sliding Rule.

C D D C
As 9·5 : 10 : : 35 : 116$\frac{1}{3}$
Or 9·5 : 12 : : 42 : 116$\frac{1}{3}$

Ex. 2. The length of a tree is 24 feet, its girt at the thicker end 14 feet, and at the ſmaller end 2 feet; required the content? Anſ. 96 feet.

Ex. 3.

Ex. 3. What is the content of a tree, whoſe mean girt is 3·15 feet, and length 14 feet 6 inches?

Anſ. 8·9922 feet.

Ex. 4. Required the content of a tree, whoſe length is $17\frac{1}{4}$ feet, which girts in five different places as follows, namely, in the firſt place 9·43 feet, in the ſecond 7·92, in the third 6·15, in the fourth 4·74, and in the fifth 3·16?

Anſ. 42·5195.

RULE II.

Multiply the ſquare of $\frac{1}{5}$ of the mean girt by double the length, and the product will be the content, very near the truth.

By the Sliding Rule.

As the double length on C : 12 or 10 on D : : $\frac{1}{5}$ of the girt, in 12ths or 10ths, on D : content on C.

EXAMPLES.

1. What is the content of a tree, its length being 9 feet 6 inches, and its mean girt 14 feet?

Decimals.		*Duodecimals.*		
2·8	$\frac{1}{5}$ of girt	2	9	7
2·8		2	9	7
224		5	7	2
56		2	1	3
			1	8
7·84				
19				
		7	10	1
7056		19		
784				
148·96	content	148	11	7

By the Sliding Rule.

C D D C
As 19 : 10 : : 28 : 149
Or 19 : 12 : : $33\frac{6}{10}$: 149

Ex. 2. Required the content of a tree, which is 24 feet long, and mean girt 8 feet? Anſ. 122·88 feet.

Ex. 3. The length of a tree is $14\frac{1}{2}$ feet, and mean girt 3·15 feet; what is the content? Anſ. 11·51 feet.

Ex. 4. The length of a tree is $17\frac{1}{4}$ feet, and its mean girt 6·28; what is the content? Anſ. 54·4065 feet.

Other curious problems relating to the cutting of timber, ſo as to produce uncommon effects, may be found in my large Treatiſe on Menſuration.

ARTIFICERS' WORK.

Artificers compute the contents of their works by ſeveral different meaſures.

As glazing and maſonry by the foot.

Painting, plaſtering, paving, &c. by the yard, of 9 ſquare feet.

Flooring, partitioning, roofing, tiling, &c. by the ſquare, of 100 ſquare feet.

And brick-work, either by the yard of 9 ſquare feet. or by the perch, or ſquare rod or pole, containing $272\frac{1}{4}$ ſquare feet, or $30\frac{1}{4}$ ſquare yards, being the ſquare of the rod or pole, of $16\frac{1}{2}$ feet or $5\frac{1}{2}$ yards long.

As

As this number $272\frac{1}{4}$ is a troubleſome number to divide by, the $\frac{1}{4}$ is often omitted in practice, and the content in feet divided only by the 272. But when the exact diviſor $272\frac{1}{4}$ is to be uſed, it will be eaſier to multiply the feet by 4, and then divide ſucceſſively by 9, 11, and 11. Alſo to divide ſquare yards by $30\frac{1}{4}$, firſt multiply them by 4, and then divide twice by 11.

All works, whether ſuperficial or ſolid, are computed by the rules proper to the figure of them, whether it be a triangle, or rectangle, a parallelopiped, or any other figure.

BRICKLAYERS' WORK.

BRICK-WORK is eſtimated at the rate of a brick and a half thick; ſo that if a wall be more or leſs than this ſtandard thickneſs, it muſt be reduced to it, as follows: Multiply the ſuperficial content of the wall by the number of half bricks in the thickneſs, and divide the product by 3. And to find the ſuperficial content of a wall, multiply the length by the height, for the content.

Chimneys are by ſome meaſured as if they were ſolid, deducting only the vacuity from the hearth to the mantle, on account of the trouble of them.

And by others they are girt or meaſured round for their breadth, and the height of the ſtory is their height, taking the depth of the jambs for their thickneſs. And in this caſe no deduction is made for the vacuity from the floor to the mantle-tree, becauſe of the gathering of the breaſt and wings, to make room for the hearth in the next ſtory.

All windows, doors, &c. are to be deducted out of the contents of the walls in which they are placed.

EXAMPLES.

1. How many yards and rods of ſtandard brick-work are in a wall whoſe length or compaſs is 57 feet 3 inches, and height 24 feet 6 inches; the walls being $2\frac{1}{2}$ bricks, or 5 half bricks thick?

	Decimals.		*Duodecimals.*		
	57·15		57	3	
	24·5		24	6	
	28625		234	0	
	22900		114		
	11450		28	7	6
	1402·625		1402	7	6
	5 half bricks thick				5
3	7013·125	3	7013	1	6
9	2337·708$\frac{1}{3}$ ſq. feet	9	2337	8	6
	259·745$\frac{10}{27}$ yds.		2596	8	6
	4		4		
11	1038·981$\frac{13}{27}$	11	1036		
11	94·4528	11	94	2	
rods	8·5866 Anſ.		8 r 17yds 6f 8′ 6″		

By the Sliding Rule.

B A B A
As 1 : $24\frac{1}{2}$: : $57\frac{1}{4}$: 1403.

Ex. 2. Required the content of a wall 62 feet 6 inches long, and 14 feet 8 inches high, and $2\frac{1}{2}$ bricks thick?

Anſ. 169 753 yards.

Ex. 3.

Ex. 3. A triangular gable is raiſed 17$\frac{1}{2}$ feet high, on an end wall whoſe length is 24 feet 9 inches, the thickneſs being 2 bricks; required the reduced content?

Anſ. 32·08$\frac{1}{3}$ yards.

Ex. 4. The end wall of a houſe is 28 feet 10 inches long, and 55 feet 8 inches high to the eaves, 20 feet high is 2$\frac{1}{2}$ bricks thick, other 20 feet high is 2 bricks thick, and the remaining 15 feet 8 inches is 1$\frac{1}{2}$ brick thick, above which is a triangular gable of 1 brick thick, which riſes 42 courſes of bricks, of which every 4 courſes make a foot. What is the whole content in ſtandard meaſure?

Anſ. 253·62 yards.

MASONS' WORK.

TO maſonry belongs all ſorts of ſtone-work; and the meaſure made uſe of is a foot, either ſuperficial or ſolid.

Walls, columns, blocks of ſtone or marble, &c. are meaſured by the cubic foot; and pavements, ſlabs, chimney-pieces, &c. by the ſuperficial or ſquare foot.

Cubick or ſolid meaſure is uſed for the materials, and ſquare meaſure for the workmanſhip.

In the ſolid meaſure, the true length, breadth, and thickneſs, are taken, and multiplied continually together. In the ſuperficial, there muſt be taken the length and breadth of every part of the projection, which is ſeen without the general upright face of the building.

EXAMPLES.

1. Required the ſolid content of a wall, 53 feet 6 inches long, 12 feet 3 inches high, and 2 feet thick.

Decimals.	*Duodecimals.*
53·5	53 6
$12\frac{1}{4}$	12 3
642·0	642 0
13·375	13 4 6
655·375	655 4 6
2	2
1310·750	Anſ. 1310 9 0

By the Sliding Rule.

B A B A
1 : $53\frac{1}{2}$: : $12\frac{1}{4}$: 655
1 : 655 : : 2 : 1310

Ex. 2. What is the ſolid content of a wall, the length being 24 feet 3 inches, height 10 feet 9 inches, and 2 feet thick? Anſ. 521·375 feet.

Ex. 3. Required the value of a marble ſlab, at 8s. per foot; the length being 5 feet 7 inches, and breadth 1 foot 10 inches. Anſ. £4 1 $10\frac{1}{2}$.

Ex. 4. In a chimney piece, ſuppoſe the

length of the mantle and ſlab, each,	4f	6in
breadth of both together - -	3	2
length of each jamb - -	4	4
breadth of both together - -	1	9

Required the ſuperficial content? Anſ. 21f 10in.

CARPENTERS

AND

JOINERS' WORK.

TO this branch belongs all the wood-work of a houſe, ſuch as flooring, partitioning, roofing, &c.

Note. Large and plain articles are uſually meaſured by the ſquare foot or yard, &c. but enriched mouldings, and ſome other articles, are often eſtimated by running or lineal meaſure, and ſome things are rated by the piece.

In meaſuring of joiſts, multiply the depth, breadth, and length all together, for the content of one joiſt, multiply that by the number of the joiſts, note that the length of the joiſts will exceed the breadth of the room by the thickneſs of the wall, and $\frac{1}{3}$ of the ſame, becauſe each end is let into the wall about $\frac{2}{3}$ of its thickneſs.

Partitions are meaſured from wall to wall for one dimenſion, and from floor to floor, as far as they extend, for the other; then multiply the length by the height.

In meaſuring of joiners' work, the ſtring is made to ply cloſe to every part of the work over which it paſſes.

The meaſure of centering for cellars is found by making a ſtring paſs over the ſurface of the arch for the one dimenſion, and taking the length of the cellar for the other; but in groin centering, it is uſual to allow double meaſure, on account of their extraordinary trouble.

In roofing, the length of the rafters is equal to the length of a ſtring ſtretched from the ridge down the rafter,

rafter, and along the eaves-board, till it meets with the top of the wall. This length multiplied by the common depth and breadth of the rafters, gives the content, and that multiplied by the numbers of them, gives the content of all the rafters.

For ſtair-caſes, take the breadth of all the ſteps, by making a line ply cloſe over them, from the top to the bottom, and multiply the length of this line by the length of a ſtep for the whole area.—By the length of a ſtep is meant the length of the front and the returns at the two ends; and by the breadth, is to be underſtood the girt of its two outer ſurfaces, or the tread and riſe.

For the baluſtrade, take the whole length of the upper part of the hand-rail, and girt over its end till it meet the top of the newel poſt, for one dimenſion; and twice the length of the baluſter upon the landing, with the girt of the hand-rail, for the other dimenſion.

For wainſcotting, take the compaſs of the room for one dimenſion; and the height from the floor to the ceiling, making the ſtring ply cloſe into all the mouldings, for the other dimenſion.—Out of this muſt be made deductions, for windows, doors, and chimneys, &c.

For doors, it is uſual to allow for their thickneſs, by adding it into both the dimenſions of length and breadth, and then multiply them together for the area.—If the door be pannelled on both ſides, take double its meaſure for the workmanſhip: but if one ſide only be pannelled, take the area and its half for the workmanſhip.—*For the ſurrounding architrave*, gird it about the outermoſt part for one dimenſion, and meaſure over it as far as it can be ſeen when the door is open, for the other.

Window-ſhutters, baſes, &c. are meaſured in the ſame manner.

In the meaſuring of roofing, the holes for chimney ſhafts and ſky-lights are generally deducted.

EXAMPLES.

1. Required the content of a floor 48 feet 6 inches long, and 24 feet 3 inches broad.

Decimals.		*Duodecimals.*		
48·5		48	6	
24¼		24	3	
1940		204	0	
970		96		
12·125		12	1	6
1176·125	feet	1176	1	6
11·76125	ſquares Anſ.	11·76	1	6

Ex. 2. A floor being 36 feet 3 inches long, and 16 feet 6 inches broad, how many ſquares are in it?

Anſ. 5 ſq. 98⅛ feet.

Ex. 3. How many ſquares are there in 173 feet 10 inches in length, and 10 feet 7 inches height, of partitioning?

Anſ. 18·3972 ſquares.

Ex. 4. What coſt the roofing of a houſe at 10s. 6d. a ſquare; the length, within the walls, being 52 feet 8 inches, and the breadth 30 feet 6 inches; reckoning the roof $\frac{3}{2}$ of the flat?

Anſ. £12 12 11¾.

Ex. 5. To how much, at 6s. per ſquare yard, amounts the wainſcotting of a room; the height, taking in the cornice and mouldings, being 12 feet 6 inches, and the whole compaſs 83 feet 8 inches; alſo the three window ſhutters are each 7 feet 8 inches by 3 feet 6 inches, and the door 7 feet by 3 feet 6 inches; the door and ſhutters, being worked on both ſides, are reckoned work and half work?

Anſ. £36 12 2½.

SLATERS

AND

TILERS' WORK.

IN theſe articles, the content of a roof is found by multiplying the length of the ridge by the girt over from eaves to eaves; making allowance in this girt for the double row of ſlates at the bottom, or for how much one row of ſlates or tiles is laid over another.

When the roof is of a true pitch, that is, forming a right angle at top; then the breadth of the building with its half added, is the girt over both ſides.

In angles formed in a roof, running from the ridge to the eaves, when the angle bends inwards, it is called a valley; but when outwards, it is called a hip.

Deductions are made for chimney ſhafts or window holes.

EXAMPLES.

1. Required the content of a ſlated roof, the length being 45 feet 9 inches, and the whole girt 34 feet 3 inches?

Decimals.	*Duodecimals.*
45·75	45 9
34¼	34 3
18300	205 6
13725	135
114375	11 5 3
9) 1566·9375 feet	9) 1566 11 3
yds 174·104	174^{y} 11^{i} $3''$

Ex. 2. To how much amounts the tiling of a houſe, at 25s. 6d. per ſquare; the length being 43 feet 10 inches, and the breadth on the flat 27 feet 5 inches, alſo the eaves projecting 16 inches on each ſide, and the roof of a true pitch? Anſ. £24 9 5½.

PLASTERERS' WORK.

PLASTERERS' work is of two kinds, namely, ceiling, which is plaſtering upon laths; and rendering which is plaſtering upon walls: which are meaſured ſeparately.

The contents are eſtimated either by the foot or yard, or ſquare of 100 feet. Enriched mouldings, &c. are rated by running or lineal meaſure.

Deductions are to be made for chimneys, doors, windows, &c.

EXAMPLES.

1. How many yards contains the ceiling, which is 43 feet 3 inches long, and 25 feet 6 inches broad?

Decimals.		*Duodecimals.*
43·25		43 3
25½		25 6
21625		221 3
8650		86
21625		21 7 6
9) 1102·875		9) 1102· 10 6
yds 122·541	Anſwer	122^y 4^f 10^i 6″

Ex. 2. To how much amounts the ceiling of a room, at 10d. per yard; the length being 21 feet 8 inches, and the breadth 14 feet 10 inches? Anſ. £1 9 8$\frac{3}{4}$.

Ex. 3. The length of a room is 18 feet 6 inches, the breadth 12 feet 3 inches, and height 10 feet 6 inches; to how much amounts the ceiling and rendering, the former at 8d. and the latter at 3d. per yard; allowing for the door of 7 feet by 3 feet 8, and a fire place of 5 feet ſquare?

Anſ. £1 13 3.

Ex. 4. Required the quantity of plaſtering in a room, the length being 14 feet 5 inches, breadth 13 feet 2 inches, and height 9 feet 3 inches to the under ſide of the cornice, which girts 8$\frac{1}{2}$ inches, and projects 5 inches from the wall on the upper part next the ceiling: deducting only for a door 7 feet by 4.

Anſ. 53^{yd} 5^{f} 3^{i} of rendering
18 5 6 of ceiling
39 0$\frac{1}{1}\frac{1}{2}$ of cornice.

PAINTERS' WORK.

PAINTERS' work is computed in ſquare yards. Every part is meaſured where the colour lies; and the meaſuring line is forced into all the mouldings, and corners.

Windows are done at ſo much a piece. And it is uſual to allow double meaſure for carved mouldings, &c.

EXAMPLES.

1. How many yards of painting contains the room which is 65 feet 6 inches in compaſs, and 12 feet 4 inches high?

De-

Decimals.		*Duodecimals.*
65·5		65 6
$12\frac{1}{3}$		12 4
786·0		786 0
21·83		21 10
9) $807{\cdot}8\dot{3}$		9) 807 10
89·7888	Anſwer	89 6 10

Ex. 2. The length of a room being 20 feet, its breadth 14 feet 6 inches, and height 10 feet 4 inches; how many yards of painting are in it, deducting a fire-place of 4 feet by 4 feet 4 inches, and two windows each 6 feet by 3 feet 2 inches? Anſ. $73\frac{2}{27}$ yards.

Ex. 3. What coſt the painting of a room, at 6d. per yard; its length being 24 feet 6 inches, its breadth 16 feet 3 inches, and height 12 feet 9 inches; alſo the door is 7 feet by 3 feet 6, and the window ſhutters to two windows each 7 feet 9 by three feet 6, but the breaks of the windows themſelves are 8 feet 6 inches high, and 1 foot 3 inches deep: deducting the fire-place of 5 feet by 5 feet 6? Anſ. £3 3 $10\frac{1}{2}$.

GLAZIERS' WORK.

Glaziers take their dimenſions either in feet, inches and parts, or feet, tenths and hundreths. And they compute their work in ſquare feet.

In taking the length and breadth of a window, the croſs bars between the ſquares are included. Alſo windows of round or oval forms are meaſured as ſquare, meaſuring them to their greateſt length and breadth, on account of the waſte in cutting the glaſs.

EXAMPLES.

1. How many ſquare feet contains the window which is 4·25 feet long, and 2·75 feet broad?

Decimals.		*Duodecimals.*
2·75		2 9
$4\frac{1}{4}$		4 3
11·00		11 0
·6875		8 3
11·6875	Anſwer	11 8 3

2. What will the glazing a triangular ſky-light come to at 10d. per foot; the baſe being 12 feet 6 inches, and the perpendicular height 6 feet 9 inches?

Anſ. £1 15 $1\frac{3}{4}$.

3. There is a houſe with three tier of windows, three windows in each tier, their common breadth 3 feet 11 inches;

now the height of the firſt tier is 7^{f} 10^{in}
of the ſecond 6 8
of the third 5 4

Required the expence of glazing at 14d. per foot?

Anſ. £13 11 $10\frac{1}{2}$.

4. Required the expence of glazing the windows of a houſe at 13d. a foot; there being three ſtories, and three windows in each ſtory:

the height of the lower tier is 7^{f} 9^{in}
of the middle 6 6
of the upper 5 $3\frac{1}{4}$
and of an oval window over the door 1 $10\frac{1}{2}$

The common breadth of all the windows being 3 feet 9 inches.

Anſ. £12 5 6.

PA-

PAVERS' WORK.

PAVERS' work is done by the ſquare yard. And the content is found by multiplying the length by the breadth.

EXAMPLES.

1. What coſt the paving a foot-path at 3s. 4d. a-yard; the length being 35 feet 4 inches, and breadth 8 feet 3 inches?

Decimals.		*Duodecimals.*
$35{\cdot}\dot{3}$		35 4
$8\frac{1}{4}$		8 3
$282{\cdot}6\dot{6}$		282 8
$8{\cdot}8\dot{3}$		8 10
9) 291·5		9) 291 6
$32{\cdot}3\dot{8}$	Content	32 3 6
2s is $\frac{1}{10}$ $3{\cdot}238\dot{8}$		£0 3 4
1s is $\frac{1}{2}$ $1{\cdot}619\dot{4}$		4
4d is $\frac{1}{3}$ 5398		
1 5·3981		0 13 4
20		8
s 7·9620		5 6 8
12	3yd is $\frac{1}{3}$	1 $1\frac{1}{4}$
	6f is $\frac{1}{6}$	$2\frac{1}{4}$
d 11·5440	Anſwer	£5 7 $11\frac{1}{2}$

Ex. 2. What coſt the paving a court, at 3s. 2d. per yard; the length being 27 feet 10 inches, and the breadth 14 feet 9 inches? Anſ. 7 4 $5\frac{1}{4}$.

Ex.

Ex. 3. What will be the expence of paving a rectangular court yard, whoſe length is 63 feet, and breadth 45 feet; in which there is laid a foot path of 5 feet 3 inches broad, running the whole length with broad ſtones, at 3s. a-yard; the reſt being paved with pebbles at 2s. 6d. a yard? Anſ. 40 5 10½.

PLUMBERS' WORK.

PLUMBERS' work is rated at ſo much a pound, or elſe by the hundred weight, of 112 pounds.

Sheet lead uſed in roofing, guttering, &c. is from 7 to 12lb to the ſquare foot. And a pipe of an inch bore is commonly 13 or 14lb to the yard in length.

EXAMPLES.

1. How much weighs the lead which is 39 feet 6 inches long, and 3 feet 3 inches broad, at 8½lb to the ſquare foot?

Decimals.		*Duodecimals.*
39·5		39 6
3¼		3 3
118·5		118 6
9·875		9 10 6
128·375		128 4 6
8½		8½
		1024
1027·000		64
64·1875		$2\frac{5}{6}$
		$0\frac{17}{48}$
1091·1875	Anſwer	$1091\frac{9}{84}$lb.

Ex.

Ex. 2. What coſt the covering and guttering a roof with lead, at 18s. the cwt.; the length of the roof being 43 feet, and breadth or girt over it 32 feet; the guttering 57 feet long, and 2 feet wide; the former 9·831lb, and the latter 7·373lb to the ſquare foot?

Anſ. £115 9 1½.

VAULTED
AND
ARCHED ROOFS.

ARCHED roofs are either vaults, domes, ſaloons, or groins.

Vaulted roofs are formed by arches ſpringing from the oppoſite walls, and meeting in a line at the top.

Domes are made by arches ſpringing from a circular or polygonal baſe, and meeting in a point at the top.

Saloons are formed by arches connecting the ſide walls to a flat roof, or ceiling, in the middle.

Groins are formed by the interſection of vaults with each other.

Vaulted roofs are commonly of the three following ſorts:

1. *Circular roofs*, or thoſe whoſe arch is ſome part of the circumference of a circle.

2. *Elliptical* or *oval roofs*, or thoſe whoſe arch is an oval, or ſome part of the circumference of an ellipſis.

3. *Gothic roofs*, or thoſe which are formed by two circular arcs, ſtruck from different centres, and meeting in a point over the middle of the breadth, or ſpan of the arch.

PROBLEM I.

To find the Surface of a Vaulted Roof.

Multiply the length of the arch by the length of the vault, and the product will be the ſuperficies.

Note. To find the length of the arch, make a line or ſtring ply cloſe to it, quite acroſs from ſide to ſide.

EXAMPLES.

1. Required the ſurface of a vaulted roof, the length of the arch being 31·2 feet, and the length of the vault 120 feet?

31·2
120

Anſ. 3744·0 ſquare feet.

Ex. 2. How many ſquare yards are in the vaulted roof, whoſe arch is 42·4 feet, and the length of the vault 106 feet? Anſ. 499·37 yds.

PROBLEM II.

To find the Content of the Concavity of a Vaulted Roof.

Multiply the length of the vault by the area of one end, that is, by the area of a vertical tranſverſe ſection, for the content.

Note. When the arch is an oval, multiply the ſpan by the height, and the product by ·7854, for the area.

EXAMPLES.

1. Required the content of the concavity of a ſemicircular vaulted roof, the ſpan or diameter being 30 feet, and the length of the vault 150 feet?

·7854
900 the ſquare of 30.

2) 706·86
353·43 area of the end
150 the length.

1767150
35343

5301450 the content.

Ex.

Ex. 2. What is the content of the vacuity of an oval vault, whoſe ſpan is 30 feet, and height 12 feet; the length of the vault being 60 feet? Anſ. 1694·64.

Ex. 3. Required the content of the vacuity of a Gothic vault, whoſe ſpan is 50 feet, the chord of each arch 50 feet, and the diſtance of each arch from the middle of theſe chords 10 feet; alſo the length of the vault 20.

Anſ. 35401·7.

PROBLEM III.

To find the Superficies of a Dome.

Find the area of the baſe, and double it; then ſay, as the radius of the baſe, is to the height of the dome, ſo is the double area of the baſe, to the ſuperficies.

Note. For the ſuperficies of a hemiſpherical dome, take the double area of the baſe only.

EXAMPLES.

1. To how much comes the painting of an octagonal ſpherical dome, at 8d. per yard; each ſide of the baſe being 20 feet?

4·828427 tabular area
400 ſquare of 20

1931·3708 area of the baſe
2

9) 3862·7416 ſuperficies in feet
429·1934 yards
8

12 | 3433·5472
2,0 | 28,6 1½
| £14 6 1½ anſwer.

Ex. 2. Required the ſuperficies of a hexagonal ſpherical dome, each ſide of the baſe being 10 feet.

Anſ. 519·6152.

Ex. 3. What is the ſuperficies of a dome with a circular baſe, whoſe circumference is 100 feet, and height 20 feet? Anſ. 2000 feet.

PROBLEM IV.

To find the Solid Content of a Dome.

Multiply the area of the baſe by the height, and take $\frac{2}{3}$ of the product.

EXAMPLES.

1. Required the ſolid content of an octagonal dome, each ſide of the baſe being 20 feet, and the height 21 feet?

4·828427
400

1931·3708 area of the baſe
14 $\frac{2}{3}$ of height

77254832
19313708

27039·1912 anſwer.

Ex. 2. What is the ſolid content of a ſpherical dome, the diameter of whoſe circular baſe is 30 feet?

Anſ. 7068·6 feet.

PROBLEM V.

To find the Superficies of a Saloon.

Find its breadth by applying a ſtring cloſe to it acroſs the ſurface. Find alſo its length by meaſuring along the middle of it, quite round the room.

Then multiply theſe two together for the ſurface.

EXAMPLE.

The girt acroſs the face of a ſaloon being 5 feet, and its mean compaſs 100 feet, required the area or ſuperficies?

100
5

500 anſwer.

PRO-

PROBLEM VI.

To find the Solid Content of a Saloon.

Multiply the area of a tranſverſe ſection by the compaſs taken round the middle part. Subtract this product from the whole vacuity of the room, ſuppoſing the walls to go upright all the height to the flat ceiling. And the difference will be the anſwer.

EXAMPLE.

If the height AB of the ſaloon be 3·2 feet, the chord ADC of its front 4·5, and the diſtance DE of its middle part from the arch be 9 inches; required the ſolidity, ſuppoſing the mean compaſs to be 50 feet?

2) 4·5

2·25 AD
2·25

1125
450
450

5·0625 AD^2
·5625 DE^2

5·6250 (2·37 AE
4

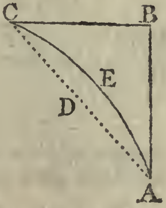
43 | 162
3 | 129

46 | 33

0·75 DE
0·75

375
525

·5625



2·37
4

3) 9·48

3·16 = $\frac{4}{3}$ AE
4·50 AC

7·66
·3 = $\frac{4}{10}$ DE

2·298 area ſeg. ADCEA.

Again, 3·2 4·5
3·2 4·5
64 225
96 180
10·24 20·25 AC^2
10·24 AB^2
10·01 (3·16 = BC
9 1·6 = $\frac{1}{2}$ AB
61 | 1·01 1896
1 | ·61 316
40 5·056 area of triangle ABC
2·298 area ſeg.
2·738 area of ſection AECBA
50 compaſs
137·900 content of the ſolid part:

Then this taken from the whole upright ſpace, will leave the content of the vacuity contained within the room.

PROBLEM VII.

To find the Concave Superficies of a Groin.

To the area of the baſe add $\frac{1}{7}$ part of itſelf, for the ſuperficial content.

EXAMPLES.

1. What is the ſuperficial content of the groin arch, raiſed on a ſquare baſe of 15 feet on each ſide?

15
15

75
15

7) 225 area of the baſe
$32\frac{1}{7}$ its 7th part

$257\frac{1}{7}$ anſwer.

Ex. 2. Required the ſuperficies of a groin arch, raiſed on a rectangular baſe, whoſe dimenſions are 20 feet by 16.
Anſ. $365\frac{5}{7}$.

PROBLEM VIII.

To find the Solid Content of a Groin Arch.

Multiply the area of the baſe by the height: from the product ſubtract $\frac{1}{10}$ of itſelf; and the remainder will be the content of the vacuity.

EXAMPLES.

1. Required the content of the vacuity within a groin arch, ſpringing from the ſides of a ſquare baſe, each ſide of which is 16 feet.

16
16

96
16

256 area of baſe
8 height or radius

2048
$204\frac{4}{5}$ $\frac{1}{10}$ ſubtract

$1843\frac{1}{5}$ anſwer.

2. What

2. What is the content of a vacuity below an oval groin, the ſide of its ſquare baſe being 24 feet, and its height 8 feet? Anſ. $4147\frac{1}{5}$.

NOTES.

1. To find the ſolid content of the brick or ſtone-work, which forms any arch or vault: Multiply the area of the baſe by the height, including the work over the top of the arch; and from the product ſubtract the content of the vacuity, found by the foregoing problems; then the remainder will be the content of the ſolid materials.

2. In groin arches, however, it is uſual to take the whole as ſolid, without deducting the vacuity, on account of the trouble and waſte of materials, attending the cutting and fitting them to the arch.

LAND SURVEYING.

CHAPTER I.

Deſcription and Uſe of the Inſtruments.

I. OF THE CHAIN.

LAND is meaſured with a chain, called Gunter's chain, of 4 poles or 22 yards in length, which conſiſts of 100 equal links, the length of each link being $\frac{22}{100}$ of a yard, or $\frac{66}{100}$ of a foot, or 7·92 inches, that is nearly 8 inches or $\frac{2}{3}$ of a foot.

An acre of land is equal to 10 ſquare chains, that is, 10 chains in length and 1 chain in breadth. Or it is 220 × 22 or 4840 ſquare yards. Or it is 40 × 4 or 160 ſquare poles. Or it is 1000 × 100 or 100000 ſquare links. Theſe being all the ſame quantity.

Alſo,

Alſo, an acre is divided into 4 parts called roods, and a rood into 40 parts called perches, which are ſquare poles, or the ſquare of a pole of $5\frac{1}{2}$ yards long, or the ſquare of $\frac{1}{4}$ of a chain, or of 25 links, which is 625 ſquare links. So that the diviſions of land meaſure will be thus:

625 ſq. links = 1 pole or perch
40 perches = 1 rood
4 roods = 1 acre.

The length of lines, meaſured with a chain, are beſt ſet down in links as integers, every chain, in length being 100 links; and not in chains and decimals. Therefore after the content is found, it will be in ſquare links; then cut off 5 of the figures on the right-hand for decimals, and the reſt will be acres. Thoſe decimals are then multiplied by 4 for roods, and the decimals of theſe again by 40 for perches.

EXAMPLE.

Suppoſe the length of a rectangular piece of ground be 792 links, and its breadth 385; to find the area in acres, roods, and perches.

```
    792
   ·385
   ----
   3960
  6336
 2376
 -------        ac  ro  p
 3·04920  Anſ. 3   0   7
       4
 -------
  ·19680
     .40
 -------
 7·87200
 -------
```

2. OF THE PLAIN TABLE.

This inſtrument conſiſts of a plane rectangular board of any convenient ſize, the centre of which, when uſed, is fixed by means of ſcrews to a three-legged ſtand, having a ball and ſocket, or joint, at the top, by means of which, when the legs are fixed on the ground, the table is inclined in any direction.

To the table belong ſeveral parts: viz.

1. A frame of wood, made to fit round its edges, and to be taken off, for the convenience of putting a ſheet of paper on the table. The one ſide of this frame is uſually divided into equal parts, for drawing lines acroſs the table, parallel or perpendicular to the ſides; and the other ſide of the frame is divided into 360 degrees, from a centre, which is in the middle of the table; by means of which the table is to be uſed as a theodolite, &c.

2. A needle and compaſs ſcrewed into the ſide of the table, or elſe in the middle of the ſupport, to point out the directions; and to be a check upon the ſights.

3. An index, which is a braſs two-foot ſcale, with either a ſmall teleſcope, or open ſights erected perpendicularly on the ends. Theſe ſights and one edge of the index are in the ſame plane, and that edge is called the fiducial edge of the index.

Before uſing this inſtrument, take a ſheet of paper which will cover it, and wet it to make it expand; then ſpread it flat on the table, preſſing down the frame on the edges, to ſtretch it and keep it fixed there; and when the paper is become dry, it will, by contracting again, ſtretch itſelf ſmooth and flat from any cramps and unevenneſs. On this paper is to be drawn the plan or form of the thing meaſured.

In uſing this inſtrument, begin at any part of the ground you think the moſt proper, and make a point on a convenient part of the paper or table, to repreſent that point of the ground; then fix in that point one leg of the compaſſes, or a fine ſteel pin, and apply to it the fiducial edge of the index, moving it round, till through the ſights you perceive ſome remarkable object, as the corner of a field, &c. and from the ſtation point draw a line with the point of the compaſſes along the fiducial edge of the index; then ſet another object or corner, and draw its line; do the ſame by another, and ſo on, till as many objects are ſet as may be thought neceſſary. Then meaſure from the ſtation towards as many of the objects as may be neceſſary, and no more, taking the requiſite offsets to corners or crooks in the hedges, &c. and lay the meaſures down on their reſpective lines on the table. Then, at any convenient place, meaſured to, fix the table in the ſame poſition, and ſet the objects which appear from thence, &c. as before; and thus continue till the work is finiſhed, meaſuring ſuch lines as are neceſſary, and determining as many as you can by interſecting lines of direction drawn from different ſtations.

And in theſe operations, obſerve the following particular cautions and directions: 1. Let the lines on which you make ſtations be directed towards objects as far diſtant as poſſible; and when you have ſet any ſuch object, go round the table and look through the ſights from the other end of the index, to ſee if any other remarkable object lie directly oppoſite: if there be not ſuch an one, endeavour to find another forward object, ſuch as ſhall have a remarkable backward oppoſite one, and make uſe of it, rather than the other; becauſe the back object will be of uſe in fixing the table in the original poſition, either when you have meaſured too near to the forward object, or when it may be hid from your ſight at any neceſſary ſtation by intervening hedges, &c.

2. Let the ſaid lines, on which the ſtations are taken, be purſued as far as you conveniently can; for that will be the means of preſerving more accuracy in the work.

3. At each ſtation, it will be neceſſary to prove the truth of it; that is, whether the table be ſtraight in the line towards the object, and alſo whether the diſtance

be rightly meaſured and laid down on the paper.—To know if the table be ſet down ſtraight in the line; lay the index on the table in any manner, and move the table about, till through the ſights you perceive either the fore or back object; then, without moving the table, go round it, and look through the ſights by the other end of the index, to ſee if the other object can be perceived; if it be, the table is in the line; if not, it muſt be ſhifted to one ſide, according to your judgment, till through the ſights both objects can be ſeen.—The aforeſaid operation only informs you if the ſtation be ſtraight in the line: but to know if it be in the right part of the line, that is, if the diſtance has been rightly laid down; fix the table in the original poſition, by laying the index along the ſtation line, and turning the table about till the fore and back objects appear through the ſights, and then alſo will the needle point at the ſame degree as at firſt; then lay the index over the ſtation point and any other point on the paper repreſenting an object which can be ſeen from the ſtation; and if the ſaid object appear ſtraight through the ſights, the ſtation may be depended on as right; if not, the diſtance ſhould be examined and corrected till the object can be ſo ſeen. And for this very uſeful purpoſe, it is adviſable to have ſome high object or two, which can be ſeen from the greateſt part of the ground, accurately laid down on the paper from the beginning of the ſurvey, to ſerve continually as proof objects.

When from any ſtation, the fore and back objects cannot both be ſeen, the agreement of the needle with one of them may be depended on for placing the table ſtraight on the line, and for fixing it in the original poſition.

Of ſhifting the Paper on the Plain Table.

When one paper is full, and there is occaſion for more; draw a line in any manner through the fartheſt point of the laſt ſtation line, to which the work can be conveniently ly laid down; then take the ſheet off the table, and fix another on, drawing a line on it, in a part the moſt convenient for the reſt of the work; then fold or cut the old ſheet by the line drawn on it, apply the edge to the line on the new ſheet, and, as they lie in that poſition, continue the laſt ſtation line on the new paper, placing on it the reſt of the meaſure, beginning at where the old ſheet left off. And ſo on from ſheet to ſheet.

When the work is done, and you would faſten all the ſheets together into one piece, or rough plan, the aforeſaid lines are to be accurately joined together, as when the lines were transferred from the old ſheets to the new ones.

But it is to be noted, that if the ſaid joining lines, on the old and new ſheet, have not the ſame inclination to the ſide of the table, the needle will not point to the original degree when the table is rectified; and if the needle be required to reſpect ſtill the ſame degree of the compaſs, the eaſieſt way of drawing the lines in the ſame poſition, is to draw them both parallel to the ſame ſides of the table, by means of the equal diviſions marked on the other two ſides.

3. OF THE THEODOLITE.

The theodolite is a brazen circular ring, divided into 360 degrees, and having an index with ſights, or a teleſcope, placed on the centre, about which the index is moveable; alſo a compaſs fixed to the centre, to point out courſes and check the ſights; the whole being fixed by the centre on a ſtand of a convenient height for uſe.

In uſing this inſtrument, an exact account, or field-book, of all meaſures and things neceſſary to be remarked in the plan, muſt be kept, from which to make out the plan on returning home from the ground.

Begin at ſuch part of the ground, and meaſure in ſuch directions, as you judge moſt convenient; taking angles or directions to objects, and meaſuring ſuch diſtances as appear neceſſary, under the ſame reſtrictions as in the

the uſe of the plain table. And it is ſafeſt to fix the theodolite in the original poſition at every ſtation by means of fore and back objects, and the compaſs, exactly as in uſing the plain table; regiſtering the number of degrees cut off by the index when directed to each object; and at any ſtation, placing the index at the ſame degree as when the direction towards that ſtation was taken from the laſt preceding one, to fix the theodolite there in the original poſition, after the ſame manner as the plain table is fixed in the original poſition, by laying its index along the line of the laſt direction.

The beſt method of laying down the aforeſaid lines of direction, is to deſcribe a pretty large circle, quarter it, and lay on the circumference, the ſeveral numbers of degrees cut off by the index in each direction, marking the points they reach to; then draw lines from the centre to all theſe points in the circumference; laſtly, parallel to the ſaid lines, draw other lines from ſtation to ſtation.

4. OF THE CROSS.

The croſs conſiſts of two pair of ſights ſet at right angles to each other, on a ſtaff having a ſharp point at the bottom to ſtick in the ground.

The croſs is very uſeful to meaſure ſmall and crooked pieces of ground. The method is to meaſure a baſe or chief line, uſually in the longeſt direction of the piece from corner to corner; and while meaſuring it, finding the places where perpendiculars would fall on this line, from the ſeveral corners and bends in the boundary of the piece, with the croſs, by fixing it, by trials, on ſuch parts of the line as that through one pair of the ſights both ends of the line may appear, and through the other pair you can perceive the correſponding bends or corners; and then meaſuring the lengths of the ſaid perpendiculars.

REMARKS.

Of all the inſtruments for meaſuring, the plain table is

on

on many occaſions the beſt; not only becauſe it may be uſed as a theodolite or ſemi-circle, by turning uppermoſt that ſide of the frame which has the 360 degrees on it; but becauſe it is, in its own proper uſe, by much the eaſieſt, ſafeſt, and moſt accurate for the purpoſe; for, by planning every part immediately on the ſpot, as ſoon as meaſured, there is not only ſaved a great deal of writing in the field-book, but every thing can alſo be planned more eaſily and accurately while it is in view, than it can afterwards from a field-book, in which many little things may be either neglected or miſtaken; and beſides, the opportunities which the plain table affords of correcting the work, or proving if it be right, at every ſtation, are ſuch advantages as can never be balanced by any other inſtrument. But though the plain table be the moſt generally uſeful inſtrument, it is not *always* ſo; there being many caſes in which ſometimes one inſtrument is the propereſt, and ſometimes another; nor is that ſurveyor maſter of his buſineſs, who cannot in any caſe diſtinguiſh which is the fitteſt inſtrument or method, and uſe it accordingly: nay often no inſtrument at all, but barely the chain itſelf is the beſt method, particularly in regular open fields lying together; and even when you are uſing the plain table, it is often of advantage to meaſure ſuch large open parts with the chain only, and from thoſe meaſures lay them down on the table.

The perambulator is uſed for meaſuring roads, and other great diſtances on level ground, and by the ſides of rivers. It has a wheel of $8\frac{1}{4}$ feet, or half a pole in circumference, on which the machine turns; and the diſtance meaſured is pointed out by an index, which is moved round by clock work.

Levels, with teleſcopic or other ſights, are uſed to find the level between place and place, or how much one place is higher or lower than another. And in meaſuring any ſloping or oblique line, either aſcending or deſcending, a ſmall pocket level is uſeful for ſhowing how many links

for

for each chain are to be deducted, to reduce the line to the true horizontal length.

An offset ſtaff is a very uſeful and neceſſary inſtrument, for meaſuring the offsets and other ſhort diſtances. It is 10 links in length, being divided and marked at each of the 10 links.

Ten ſmall arrows, or rods of iron or wood, are uſed to mark the end of every chain length, in meaſuring lines. And ſometimes pickets, or ſtaves with flags, are ſet up as marks or objects of direction.

Various ſcales are alſo uſed, in protracting and meaſuring on the plan or paper; ſuch as plane ſcales, line of chords, protractor, compaſſes, reducing ſcale, parallel and perpendicular rules, &c. Of plane ſcales, there ſhould be ſeveral ſizes, as a chain in 1 inch, a chain in $\frac{3}{4}$ of an inch, a chain in $\frac{1}{2}$ an inch, &c. And of theſe, the beſt for uſe are thoſe that are laid on the very edges of the ivory ſcale, to prick off diſtances by, without compaſſes.

THE FIELD-BOOK.

In ſurveying with the plain table, a field-book is not uſed, as every thing is drawn on the table immediately when it is meaſured. But in ſurveying with the theodolite, or any other inſtrument, ſome ſort of a field-book muſt be uſed, to write down in it a regiſter or account of all that is done and occurs relative to the ſurvey in hand.

This book every one contrives and rules as he thinks fitteſt for himſelf. The following is a ſpecimen of a form very generally uſed. It is ruled into 3 columns: the middle, or principal column, is for the ſtations, angles, bearings, diſtances meaſured, &c.; and thoſe on the right and left are for the offsets on the right and left, which are ſet againſt their correſponding diſtances in the middle column; as alſo for ſuch remarks as may occur, and be proper to note in drawing the plan, &c.

Here ⊙ 1 is the firſt ſtation, where the angle or bearing is 105° 25′. On the left, at 73 links in the diſtance or principal line, is an offset of 92; and at 610 an offset of 24 to a croſs hedge. On the right, at o, or the beginning, an offset 25 to the corner of the field; at 248 Brown's boundary hedge commences; at 610 an offset 35; and at 954, the end of the firſt line, the o denotes its terminating in the hedge. And ſo on for the other ſtations.

A line is drawn under the work, at the end of every ſtation line, to prevent confuſion.

Form of the Field-Book.

Offsets and Remarks on the left.	Stations, Bearings, and Diſtances.	Offsets and Remarks on the right.
	⊙ 1	
	105°25′	
	00	25 corner
92	73	
	248	Brown's hedge
croſs a hedge 24	610	35
	954	00
	⊙ 2	
	53°10′	
	00	00
houſe corner 51	25	21
	120	29 a tree
34	734	40 a ſtyle
	⊙ 3	
	67°20′	
	61	35
a brook 30	248	
	639	16 a ſpring
foot-path 16	810	
croſs hedge 18	973	20 a pond

The learner will here draw a plan to this field-book. But

But ſome ſkilful ſurveyors now make uſe of a different method for the field-book, namely, beginning at the bottom of the page, and writing upward; by which they ſketch a neat boundary on either hand, as they paſs along: an example of which will be given further on, in the method of ſurveying a large eſtate.

In ſmaller ſurveys and meaſurements, a good way of ſetting down the work, is, to draw, by the eye, on a piece of paper, a figure reſembling that which is to be meaſured; and ſo writing the dimenſions, as they are found, againſt the correſponding parts of the figure. And this method may be practiſed to a conſiderable extent, even in the larger ſurveys.

CHAPTER II.

THE PRACTICE OF SURVEYING.

THIS part contains the ſeveral works proper to be done in the field, or the ways of meaſuring by all the inſtruments, and in all ſituations.

PROBLEM I.

To meaſure a Line or Diſtance.

To meaſure a line on the ground with a chain: Having provided a chain, with 10 ſmall arrows, or rods, to ſtick one into the ground, as a mark, at the end of every chain; two perſons take hold of the chain, one at each end of it, and all the 10 arrows are taken by one of them, who is to go foremoſt, and is called the leader; the other being called the follower, for diſtinction ſake.

A picket or ſtation ſtaff, being ſet up in the direction of the line to be meaſured, if there do not appear ſome marks

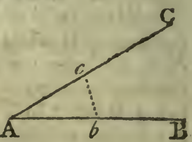
marks naturally in that direction; the follower ſtands at the beginning of the line, holding the ring at the end of the chain in his hand, while the leader drags forward the chain by the other end of it, till it is ſtretched ſtraight, and laid or held level, and the leader directed, by the follower waving his hand, to the right or left, till the follower ſee him exactly in a line with the mark or direction to be meaſured to; there both of them ſtretching the chain ſtraight, and ſtooping and holding it level, the leader having the head of one of his arrows in the ſame hand by which he holds the end of the chain, he there ſticks one of them down with it while he holds the chain ſtretched. This done, he leaves the arrow in the ground, as a mark for the follower to come to, and advances another chain forward, being directed in his poſition by the follower, ſtanding at the arrow, as before; as alſo by himſelf now, and at every ſucceeding chain's length, by moving himſelf from ſide to ſide, till he brings the follower and the back mark into a line. Having then ſtretched the chain, and ſtuck down an arrow, as before, the follower takes up his arrow, and they advance again in the ſame manner another chain length. And thus they proceed, till all the 10 arrows are employed, and are in the hands of the follower; and the leader, without an arrow, is arrived at the end of the 11th chain-length. The follower then ſends or brings the 10 arrows to the leader, who puts one of them down at the end of his chain, and advances with the chain as before; and thus the arrows are changed from the one to the other at every 10 chains' length, till the whole line is finiſhed; then the number of changes of the arrows ſhows the number of tens, to which the follower adds the arrows he holds in his hand, and the number of links of another chain over to the mark or end of the line. So if there have been 3 changes of the arrows, and the follower hold 6 arrows, and the end of the line cut off 45 links more, the whole length of the line is ſet down in links thus, 3645.

When the ground is ſloping, aſcending or deſcending; at at every chain length, lay the offset ſtaff, or link ſtaff down in the ſlope of the chain, on which lay the ſmall pocket level, to ſhow how many links or parts the ſlope line is longer than the true level one; then draw the chain forward ſo many links or parts, which reduces the line horizontal. Or, holding the chain level every time, will perhaps be the better way to have the true length of the line.

PROBLEM II.

To take Angles and Bearings.

Let B and C be two objects, or two pickets ſet up perpendicular, and let it be required to take their bearings, or the angle formed between them at any ſtation A.



1. *With the Plain Table.*

The table being covered with a paper, and fixed on its ſtand; plant it at the ſtation A, and fix a fine pin, or a point of the compaſſes in a proper point of the paper, to repreſent the point A: Cloſe by the ſide of this pin lay the fiducial edge of the index, and turn it about, ſtill touching the pin till one object B can be ſeen through the ſights: then by the fiducial edge of the index draw a line. In the very ſame manner draw another line in the direction of the other object C. And it is done.

2. *With the Theodolite, &c.*

Direct the fixed ſights along one of the lines, as AB, by turning the inſtrument about till you ſee the mark B through theſe ſights; and there ſcrew the inſtrument faſt. Then turn the moveable index about till, through its ſights, you ſee the other mark C. Then the degrees cut by the index, on the graduated limb or ring of the inſtrument, ſhew the quantity of the angle.

3. *With the Magnetic Needle and Compaſs.*

Turn the inſtrument, or compaſs, ſo that the north end of the needle point to the flower-de-luce. Then direct the ſights to one mark as B, and note the degrees cut by the needle. Next direct the ſights to the other mark C, and note again the degrees cut by the needle. Then their ſum or difference, as the caſe is, will give the quantity of the angle BAC.

4. *By Meaſurement with the Chain, &c.*

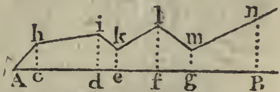
Meaſure one chain length, or any other length, along both directions, as to b and c. Then meaſure the diſtance b c, and it is done.—This is eaſily transferred to paper, by making a triangle A b c with theſe three lengths, and then meaſuring the angle A as in Practical Geometry, prob. XI.

PROBLEM III.

To meaſure the Offſets.

A h i k l m n being a crooked hedge, or river, &c. From A meaſure in a ſtraight direction along the ſide of it to B. And in meaſuring along this line AB, obſerve when you are directly oppoſite any bends or corners of the fence, as at c, d, e, &c. and thence meaſure the perpendicular offſets c h, d i, &c. with the offſet-ſtaff, if they are not very large, otherwiſe with the chain itſelf. And the work is done. The regiſter or field-book of which may be as follows:

Offs. left.		Baſe line A B.	
	0	⊙	A
c h	62	45	A c
d i	84	220	A d
e k	70	340	A e
f l	88	510	A f
g m	57	634	A g
B n	91	785	A B



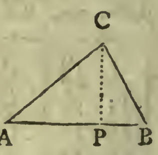
Note. When the offſets are not very large, their places c, d, e, &c. on the baſe line, can be very well determined by by the eye, eſpecially when aſſiſted by laying down the offſet-ſtaff in a croſs or perpendicular direction. But when theſe perpendiculars are very large, find their poſitions by the croſs, or by the inſtrument which you happen to be uſing, in this manner: In meaſuring along AB, when you come nearly oppoſite C, where you judge a perpendicular will ſtand, plant the inſtrument in the line, and turn the index till the marks A and B can be ſeen through both the ſights, looking both backward and forward; then look along the croſs ſights, or the croſs line on the index; and if it point directly to the corner or bend h, the place of c is right. Otherwiſe move the inſtrument backward or forward on the line A B, till the croſs line points ſtraight to h. This being found, ſet down the diſtance meaſured from A to c: then meaſure the offſet c h, and ſet it down oppoſite the former, and on the left hand ſide. Then proceed forward in the line A B, till you arrive oppoſite another corner, and determine the place of the perpendicular as before. And ſo on throughout the whole length.

PROBLEM IV.

To ſurvey a Triangular Field ABC.

1. *By the Chain.*

AP	794
AB	1321
PC	826



Having ſet up marks at the corners, which is to be done in all caſes where there are not marks naturally; meaſure with the chain from A to P, where a perpendicular would fall from the angle C, and ſet up a mark at P, noting down the diſtance AP. Then complete the diſtance AB by by meaſuring from P to B. Having ſet down this meaſure, return to P, and meaſure the perpendicular PC. And thus, having the baſe and perpendicular, the area from them is eaſily found. Or having the place P of the perpendicular, the triangle is eaſily conſtructed.

Or, meaſure all the three ſides with the chain, and note them down. From which the content is eaſily found, or the figure conſtructed.

2. *By taking one or more of the Angles.*

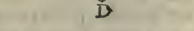
Meaſure two ſides AB, AC, and the angle A between them. Or meaſure one ſide AB, and the two adjacent angles A and B. From either of theſe ways the figure is eaſily planned; then by meaſuring the perpendicular CP on the plan, and multiplying it by half AB, you have the content.

PROBLEM V.

To meaſure a Four-ſided Field.

1. *By the Chain.*

AE	214	210	DE
AF	362	260	BF
AC	592		

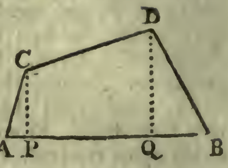


Meaſure along either of the diagonals, as AC; and either of the two perpendiculars DE, BF, as in the laſt problem; or elſe the ſides AB, BC, CD, DA. From either of theſe ways may the figure be planned and computed, as before directed.

Other-

Otherwiſe by the Chain.

AP	110	352	PC
AQ	745	595	QD
AB	1110		



Meaſure on the longeſt ſide, the diſtances AP, AQ, AB; and the perpendiculars PC, QD.

2. *By taking one or more of the Angles.*

Meaſure the diagonal AC (ſee the laſt fig. but one,) and the angles CAB, CAD, ACB, ACD.—Or meaſure the four ſides, and any one of the angles at BAD.

Thus		Or thus	
AC	591	AB	486
CAB	37°20′	BC	394
CAD	41 15	CD	410
ACB	72 25	DA	462
ACD	54 40	BAD	78°35′

PROBLEM VI.

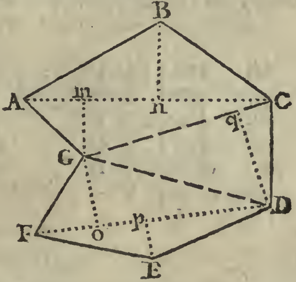
To ſurvey any Field by the Chain only.

Having ſet up marks at the corners, where neceſſary, of the propoſed field ABCDEFG. Walk over the ground, and conſider how it can beſt be divided into triangles and trapeziums; and meaſure them ſeparately as in the laſt two problems. And in this way it will be proper to divide it into triangles and trapeziums, by drawing diagonals from corner to corner; and ſo as that all the perpendiculars may fall within the figures. Thus, the following figure is divided into the two trapeziums ABCG, GDEF, and the triangle GCD. Then at the firſt trapezium, beginning at A, meaſure the diagonal AC, and

and the two perpendiculars G m, B n. Then the baſe G C, and the perpendicular D q. Laſtly, the Diagonal D F, and the two perpendiculars p E, o G. All which meaſures write againſt the correſponding parts of a rough figure, drawn to reſemble the figure to be ſurveyed, or ſet them down in any other form you chooſe.

Thus

A m 135	130 m G
A n 410	180 n B
A C 550	
C q 152	230 q D
C G 440	
F o 206	120 o G
F p 288	80 p E
F D 520	



Or Thus,

Meaſure all the ſides AB, BC, CD, DE, EF, FG, GA; and the diagonals AC, CG, GD, DF.

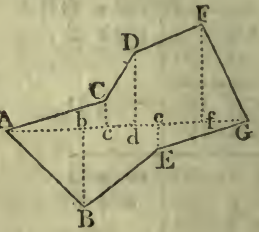
Otherwiſe.

Many pieces of land may be very well ſurveyed, by meaſuring any baſe line, either within or without them, together with the perpendiculars let fall on it from every corner of them. For they are by thoſe means divided into ſeveral triangles, and trapezoids, all whoſe parallel ſides are perpendicular to the baſe line; and the ſum of theſe triangles and trapeziums will be equal to the figure propoſed if the baſe line fall within it; if not, the ſum of the parts which are without being taken from the ſum of the whole which are both within and without, will leave the area of the figure propoſed.

In pieces that are not very large, it will be ſufficiently exact to find the points, in the baſe line, where the ſeveral perpendiculars will fall, by means of the *Croſs*, and thence meaſuring to the corners for the lengths of the perpendiculars.—And it will be moſt convenient to draw the line ſo, as that all the perpendiculars may fall within the figure.

Thus, in the following figure, beginning at A, and meaſuring along the line AG, the diſtances and perpendiculars, on the right and left, are as below.

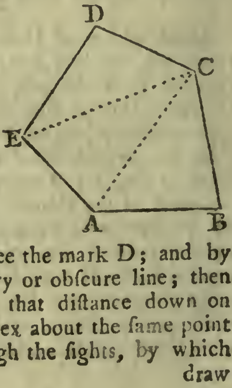
Ab	315	350 bB
Ac	440	70 cC
Ad	585	320 dD
Ae	610	50 eE
A f	990	470 fF
AG	1020	0



PROBLEM VII.

To ſurvey any Field with the Plain Table.

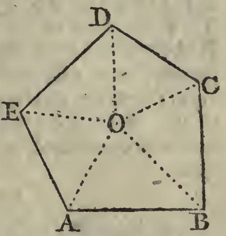
1. *From one Station.*



Plant the table at any angle, as C, from which all the other angles, or marks ſet up, can be ſeen. Then turn the table about till the needle point to the flower-de-luce; and there ſcrew it faſt. Make a point for C on the paper on the table, and lay the edge of the index to C, turning it about that point till through the ſights you ſee the mark D; and by the edge of the index draw a dry or obſcure line; then meaſure the diſtance CD, and lay that diſtance down on the line CD. Then turn the index about the ſame point C, till the mark E be ſeen through the ſights, by which draw

draw a line, and meaſure the diſtance to E, laying it on the line from C to E. In like manner determine the poſitions of CA and CB, by turning the ſights ſucceſſively to A and B; and lay the lengths of thoſe lines down. Then connect the points with the boundaries of the field, by drawing the black lines CD, DE, EA, AB, BC.

2. *From a Station Within or Without the Field.*



When all the other parts cannot be ſeen from one angle, chooſe ſome place O within; or even without, if more convenient: from which the other parts can be ſeen. Plant the table at O, then fix it with the needle north, and mark the point O on it. Apply the index ſucceſſively to O, turning it round with the ſights to each angle A, B, C, D, E, drawing dry lines to them along the edge of the index; then meaſuring the diſtances OA, OB, &c. and laying them down on thoſe lines. Laſtly, draw the boundaries AB, BC, CD, DE, EA.

3. *By going Round the Figure.*

When the figure is a wood, or water, or when from ſome other obſtruction you cannot meaſure lines acroſs it; begin at any point A, and meaſure round it, either within or without the figure, and draw the directions of all the ſides thus: Plant the table at A, turn it with the needle to the north of flower-de-luce, fix it, and mark the point A. Apply the index to A, turning it till you can ſee the point E, there draw a line; and then the point B, and there draw a line: then meaſure theſe lines, and lay them down from A to E and B. Next move the table to B, lay the index along the line AB, and turn the table about till

till you can ſee the mark A, and ſcrew faſt the table; in which poſition alſo the needle will again point to the flower-de-luce, as it will do indeed at every ſtation when the table is in the right poſition. Here turn the index about B till through the ſights you ſee the mark C; there draw a line, meaſure BC, and lay the diſtance on that line after you have ſet down the table at C. Turn it then again into its proper poſition, and in like manner find the next line CD. And ſo on, quite round by E, to A again. Then the proof of the work will be the joining at A: for if the work be all right, the laſt direction EA on the ground, will paſs exactly through the point A on the paper; and the meaſured diſtance will alſo reach exactly to A. If theſe do not coincide, or nearly ſo, ſome error has been committed, and the work muſt be examined over again.

PROBLEM VIII.

To ſurvey a Field with the Theodolite, &c.

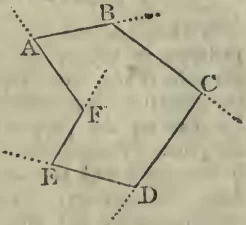
1. *From One Point or Station.*

When all the angles can be ſeen from one point, as the angle C, (firſt fig. to laſt prob.); place the inſtrument at C, and turn it about till, through the fixed ſights, you ſee the mark B, and there fix it. Then turn the moveable index about, till the mark A is ſeen through the ſights, and note the degrees on the inſtrument. Next turn the index ſucceſſively to E and D, noting the degrees cut off at each; which gives all the angles BCA, BCE, BCD. Laſtly, meaſure the lines CB, CA, CE, CD; and enter the meaſures in a field-book, or rather againſt the correſponding parts of a rough figure, drawn by gueſs, to reſemble the field.

2. *From a Point Within or Without.*

Plant the inſtrument at O, (laſt fig.) and turn it about till the fixed ſights point to any object as A; and there ſcrew it faſt. Then turn the moveable index round, till the ſights point ſucceſſively to the other points E, D, C, B, noting the degrees cut off at each of them; which gives all the angles round the point O. Laſtly, meaſure the diſtances OA, OB, OC, OD, OE, noting them down as before, and the work is done.

3. *By going Round the Field.*



By meaſuring round, either within or without the field, proceed thus. Having ſet up marks at B, C, &c. near the corners as uſual, plant the inſtrument at any point A, and turn it till the fixed index be in the direction AB, and there ſcrew it faſt: then turn the moveable index to the direction AF; and the degrees cut off will be the angle A. Meaſure the line AB, and plant the inſtrument at B, and there in the ſame manner obſerve the angle A. Then meaſure BC, and obſerve the angle C. Then meaſure the diſtance CD, and take the angle D. Then meaſure DE, and take the angle E. Then meaſure EF, and take the angle F. And laſtly meaſure the diſtance FA.

To prove the work; add all the inward angles A, B, C, &c. together, and when the work is right, their ſum will be equal to twice as many right angles, as the figure has ſides, wanting 4 right angles. But when there is an angle, as F, that bends inwards, and you meaſure the external angle, which is leſs than 2 right angles, ſubtract it from 4 right angles, or 360 degrees, to give the internal angle greater than a ſemicircle or 180 degrees.

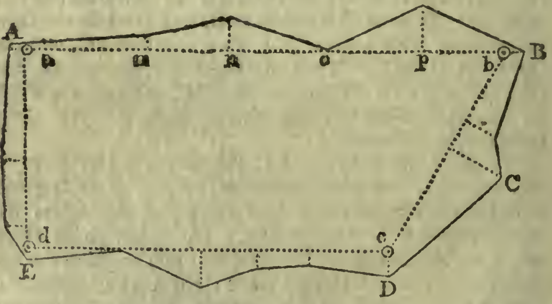
Otherwiſe.

Inſtead of obſerving the internal angles, you may take the external angles, formed without the figure by producing the ſides farther out. And in this caſe, when the work is right, their ſum altogether will be equal to 360 degrees. But when one of them, as F, runs inwards, ſubtract it from the ſum of the reſt, to leave 360 degrees.

PROBLEM IX.

To ſurvey a Field with Crooked Hedges.

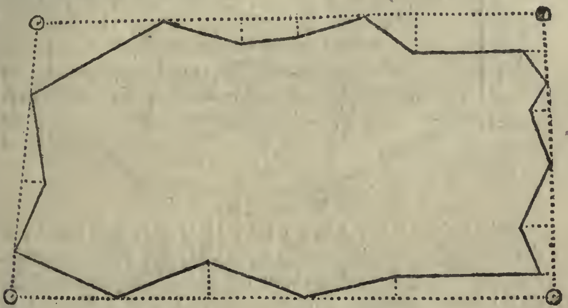
With any of the inſtruments meaſure the lengths and poſitions of imaginary lines running as near the ſides of the field as you can; and, in going along them, meaſure the offsets in the manner before taught; and you will have the plan on the paper in uſing the plain table, drawing the crooked hedges through the ends of the offsets; but in ſurveying with the theodolite, or other inſtrument, ſet down the meaſures properly in a field-book, or memorandum book, and plan them after returning from the field, by laying down all the lines and angles.



So, in ſurveying the piece ABCDE, ſet up marks a, b, c, d, dividing it into as few ſides as may be, commonly 4. Then begin at any ſtation a, and meaſure the lines ab, bc, cd, da, and take their poſitions, or the angles a, b, c, d; and, in going along the lines, meaſure all the offsets, as at m, n, o, p, &c. along every ſtation line.

And

And this is done either within the field, or without, as may be moſt convenient. When there are obſtructions within, as wood, water, hills, &c. then meaſure without, as in the figure here below.



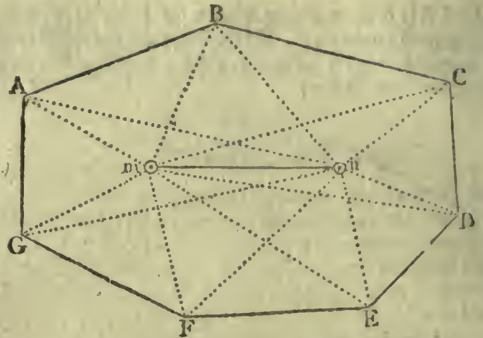
PROBLEM X.

To ſurvey a Field or any other thing, by Two Stations.

This is performed by chooſing two ſtations, from which all the marks and objects can be ſeen; then meaſuring the diſtance between the ſtations, and at each ſtation taking the angles formed by every object, from the ſtation line or diſtance.

The two ſtations may be taken either within the bounds, or in one of the ſides, or in the direction of two of the objects, or quite at a diſtance and without the bounds of the objects, or part to be ſurveyed.

In this manner, not only grounds may be ſurveyed, without even entering them, but a map may be taken of the principal parts of a county, or the chief places of a town, or any part of a river or coaſt ſurveyed, or any other inacceſſible objects; by taking two ſtations, on two towers, or two hills, or ſuch like.



When the plain table is uſed; plant it at one ſtation m, draw a line mn on it, along which lay the edge of the index, and turn the table about till the ſights point directly to the other ſtation; and there ſcrew it faſt. Then turn the ſights round m ſucceſſively to all the objects A, B, C, &c. drawing a dry line by the edge of the index at each, as m A, m B, m C, &c. Then meaſure the diſtance to the other ſtation, there plant the table, and lay that diſtance down on the ſtation line from m to n. Next lay the index by the line nm, and turn the table about till the ſights point to the other ſtation m, and there ſcrew it faſt. Then direct the ſights ſucceſſively to all the objects A, B, C, &c. as before, drawing lines each time, as n A, n B, n C, &c. and their interſection with the former lines, will give the places of all the objects, or corners, A, B, C, &c.

When the theodolite, or any other inſtrument for taking angles, is uſed; proceed in the ſame way, meaſuring the ſtation diſtance m n, planting the inſtrument firſt at one ſtation and then at the other; then placing the fixed ſights in the direction m n, and directing the moveable ſights to every object, noting the degrees cut off at each time. Then, theſe obſervations being planned, the interſections of the lines will give the objects as before.

When

When all the objects to be ſurveyed cannot be ſeen from two ſtations; then three ſtations may be uſed, or four, or as many as neceſſary; meaſuring always the diſtance from one ſtation to another; placing the inſtrument in the ſame poſition at every ſtation, by means deſcribed before; and from each ſtation obſerving or ſetting every object that can be ſeen from it, by taking its direction or angular poſition, till every object be determined by the interſection of two or more lines of direction, the more the better. And thus may very extenſive ſurveys be taken, as of large commons, rivers, coaſts, countries, hilly grounds, and ſuch like.

PROBLEM XI.

To ſurvey a Large Eſtate.

If the eſtate be very large, and contain a great number of fields, it cannot well be done by ſurveying all the fields ſingly, and then putting them together; nor can it be done by taking all the angles and boundaries that incloſe it. For in theſe caſes, any ſmall errors will be ſo multiplied, as to render it very much diſtorted.

1. Walk over the eſtate two or three times, in order to get a perfect idea of it, and till you can carry the map of it tolerably in your head. And to help your memory, draw an eye draught of it on paper, or at leaſt of the principal parts of it, to guide you; ſetting the names within the fields in that draught.

2. Chooſe two or more eminent places in the eſtate, for your ſtations, from which you can ſee all the principal parts of it: and let theſe ſtations be as far diſtant from one another as poſſible, as the fewer ſtations you have to command the whole, the more exact your work will be; and they will be fitter for your purpoſe, if theſe ſtation lines be in or near the boundaries of the ground, eſpecially if two lines or more proceed from one ſtation.

3. Take what angles, between the ſtations, you think neceſſary, and meaſure the diſtances from ſtation to ſtation,

always in a right line: theſe things muſt be done, till you get as many lines and angles as are ſufficient for determining all the ſtation points. And in meaſuring any of theſe ſtation diſtances, mark accurately where theſe lines meet with any hedges, ditches, roads, lanes, paths, rivulets, &c. and where any remarkable object is placed, by meaſuring its diſtance from the ſtation line; and where a perpendicular from it cuts that line; and always mind, in any of theſe obſervations, that you be in a right line, which you will know by taking a backſight and foreſight, along the ſtation line. And thus in going along any main ſtation line, take offsets to the ends of all hedges, and to any pond, houſe, mill, bridge, &c. omitting nothing that is remarkable. And all theſe things muſt be noted down: for theſe are the data, by which the places of ſuch objects are to be determined on the plan. And be ſure to ſet up marks at the interſections of all hedges with the ſtation line, that you may know where to meaſure from, when you come to ſurvey theſe particular fields, which muſt immediately be done, as ſoon as you have meaſured that ſtation line, while they are freſh in memory. In this way all the ſtation lines are to be meaſured, and the ſituation of all places adjoining to them determined, which is the firſt grand point to be obtained. It will be proper to lay down the work on paper every night, when you go home, that you may ſee how you go on.

4. As to the inner parts of the eſtate, they muſt be determined in like manner, by new ſtation lines: for, after the main ſtations are determined, and every thing adjoining to them, then the eſtate muſt be ſubdivided into two or three parts by new ſtation lines; taking inner ſtations at proper places, where you can have the beſt view. Meaſure theſe ſtation lines as you did the firſt, and all their interſections with hedges, and all offsets to ſuch objects as appear. Then proceed to ſurvey the adjoining fields, by taking the angles that the ſides make with the ſtation line, at the interſections, and meaſuring the diſ-

diſtances to each corner, from the interſections. For the ſtation lines will be the baſes to all the future operations; the ſituations of all parts being entirely dependent on them; and therefore they ſhould be taken of as great length as poſſible; and it is beſt for them to run along ſome of the hedges or boundaries of one or more fields, or to paſs through ſome of their angles. All things being determined for theſe ſtations, you muſt take more inner ſtations, and continue to divide and ſubdivide till at laſt you come to ſingle fields; repeating the ſame work for the inner ſtations, as for the outer ones, till all be done; and cloſe the work as often as you can, and in as few lines as poſſible. And that you may chooſe ſtations the moſt conveniently, ſo as to cauſe the leaſt labour, let the ſtation lines run as far as may be along ſome hedges, and through as many corners of the fields, and other remarkable points, as you can. And take notice how one field lies by another; that you may not miſplace them in the draught.

5. An eſtate may be ſo ſituated, that the whole cannot be ſurveyed together; becauſe one part of the eſtate may not be ſeen from another. In this caſe you may divide it into three or four parts, and ſurvey theſe parts ſeparately, as if they were lands belonging to different perſons; and at laſt join them together.

6. As it is neceſſary to protract or lay down the work as you proceed in it, you muſt have a ſcale of a due length to do it by. To get ſuch a ſcale, meaſure the whole length of the eſtate in chains; then conſider how many inches long the map is to be; and from theſe you will know how many chains you muſt have in an inch; then make your ſcale accordingly, or chooſe one already made.

7. The trees in every hedge row may be placed in their proper ſituation, which is ſoon done by the plain table; but may be done by the eye without an inſtrument; and being thus taken by gueſs in a rough draught, they will be exact enough, being only to look at; except it be ſuch as are at any remarkable places, as at the ends of hedges,

at ſtiles, gates, &c. and theſe muſt be meaſured, or taken with the plain table. But all this need not be done till the draught is finiſhed. And obſerve in all the hedges, what ſide the gutter or ditch is on, and to whom the fences belong.

8. When you have long ſtations, you ought to have a good inſtrument to take angles with, and the plain table may very properly be made uſe of, to take the ſeveral ſmall internal parts, and ſuch as cannot be taken from the main ſtations: as it is a very quick and ready inſtrument.

The New Method of Surveying.

Inſtead of the foregoing method, an ingenious friend (Mr. Abraham Crocker), after mentioning the new and improved method of keeping the field-book, by writing from bottom to top of the pages, obſerves that "In the former method of meaſuring a large eſtate, the accuracy of it depends on the correctneſs of the inſtruments uſed in taking the angles. To avoid the errors incident to ſuch a multitude of angles, other methods have of late years been uſed by ſome few ſkilful ſurveyors: the moſt practical, expeditious, and correct, ſeems to be the following:

"As was adviſed in the foregoing method, ſo in this, chooſe two or more eminences, as grand ſtations, and meaſure a principal baſe line from one ſtation to the other, noting every hedge, brook or other remarkable object as you paſs by it; meaſuring alſo ſuch ſhort perpendicular lines to ſuch bends of hedges as may be near at hand. From the extremities of this baſe line, or from any convenient parts of the ſame, go off with other lines to ſome remarkable object ſituated towards the ſides of the eſtate, without regarding the angles they make with the baſe line or with one another; ſtill remembering to note every hedge, brook, or other object that you paſs by. Theſe lines, when laid down by interſections, will with the baſe line form a grand triangle on the eſtate; ſeveral of which, if if need be, being thus laid down, you may proceed to form other ſmaller triangles and trapezoids, on the ſides of the former: and ſo on, until you finiſh with the encloſures individually.

"This grand triangle being completed, and laid down on the rough plan paper, the parts, exterior as well as interior, are to be completed by ſmaller triangles and trapezoids.

"When the whole plan is laid down on paper, the contents of each field might be calculated by the methods laid down below, at prob. 2. chap. 3.

"In countries where the lands are encloſed with high hedges, and where many lanes paſs through an eſtate, a theodolite may be uſed to advantage, in meaſuring the angles of ſuch lands; by which means, a kind of ſkeleton of the eſtate may be obtained, and the lane lines ſerve as the baſes of ſuch triangles and trapezoids as are neceſſary to fill up the interior parts."

The method of meaſuring the other croſs lines, offsets, and interior parts and encloſures, appears in the plan fig. 1, pl. 28. Dictionary.

Another ingenious correſpondent (Mr. John Rodham, of Richmond, Yorkſhire), has alſo communicated the following example of the new method of ſurveying, accompanied by the field-book, and its correſponding plan. His account of the method is as follows.

"The field-book is ruled into three columns. In the middle one are ſet down the diſtances on the chain line at which any mark, offset, or other obſervation is made; and in the right and left hand columns are entered the offsets and obſervations made on the right and left hand reſpectively of the chain line.

"It is of great advantage, both for brevity and perſpicuity, to begin at the bottom of the leaf and write upwards, denoting the croſſing of fences, by lines drawn acroſs the middle column, or only part of ſuch a line on the right and left oppoſite the figures, to avoid confuſion:

fuſion: and the corners of fields, and other remarkable turns in the fences where offsets are taken to, by lines joining in the manner the fences do, as will be beſt ſeen by comparing the book with the plan annexed to the field-book, in p. 208.

" The marks called, *a*, *b*, *c*, &c. are beſt made in the fields, by making a ſmall hole with a ſpade, and a chip or ſmall bit of wood, with the particular letter upon it, may be put in, to prevent one mark being taken for another, on any return to it. But in general, the name of a mark is very eaſily had by referring in the book to the line it was made in. After the ſmall alphabet is gone through, the capitals may be next, the print letters afterwards, and ſo on, which anſwer the purpoſe of ſo many different letters; or the marks may be numbered.

" The letter in the left hand corner at beginning of every line, is the mark or place meaſured *from;* and, that at the right hand corner at the end, is the mark meaſured *to:* But when it is not convenient to go exactly from a mark, the place meaſured from, is deſcribed *ſuch a diſtance* from *one mark* towards *another;* and where a mark is not meaſured to, the exact place is aſcertained by ſaying, turn to the right or left hand, *ſuch a diſtance* to *ſuch a mark;* it being always underſtood that thoſe diſtances are taken in the chain line.

" The characters uſed, are ⌈ for *turn to the right hand,* ⌉ for *turn to the left hand,* and ∧ placed over an offset, to ſhow that is not taken at right angles with the chain line, but in the line with ſome ſtraight fence; being chiefly uſed when croſſing their directions, and it is a better way of obtaining their true places than by offsets at right angles.

" When a line is meaſured whoſe poſition is determined, either by former work (as in the caſe of producing a given line, or meaſuring from one known place or mark to another) or by itſelf, (as in the third ſide of a triangle) it is called a *faſt line*, and a double line acroſs the book is drawn at the concluſion of it: but if its poſition is not determined,

termined, as in the ſecond ſide of a triangle, it is called a *looſe line*; and a ſingle line is drawn acroſs the book. When a line becomes determined in poſition, and is afterwards continued, a double line half through the book is drawn.

"When a looſe line is meaſured, it becomes abſolutely neceſſary to meaſure ſome line that will determine its poſition. Thus, the firſt line *ab*, being the baſe of a triangle, is always determined; but the poſition of the ſecond ſide *bj*, does not become determined, till the third ſide *jb* is meaſured; then the triangle may be conſtructed, and the poſition of both is determined.

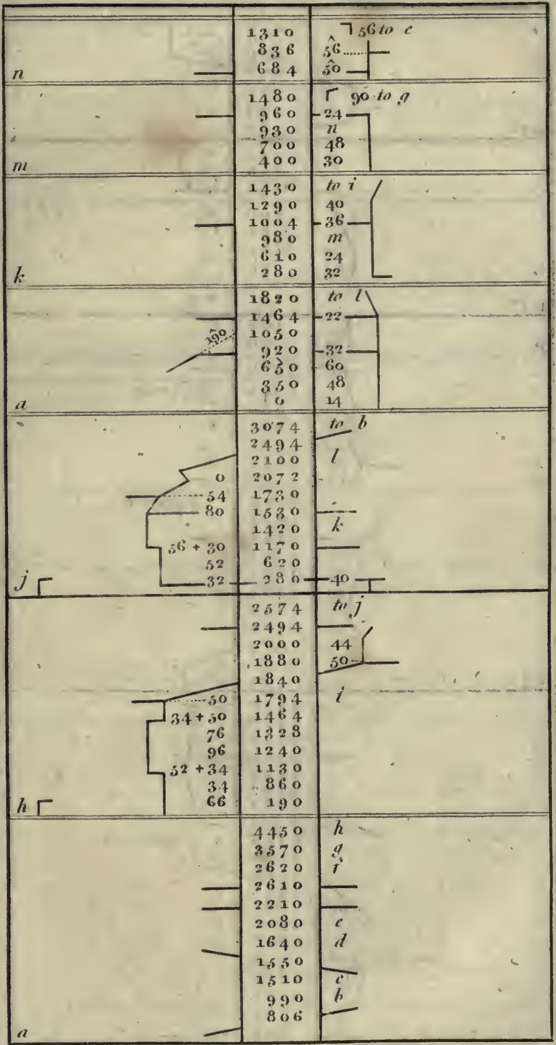
"At the beginning of a line, to fix a looſe line to the mark or place meaſured from, the ſign of turning to the right or left hand muſt be added (as at *j* in the third line;) otherwiſe a ſtranger, when laying down the work, may as eaſily conſtruct the triangle *bjb* on the wrong ſide of the line *ab*, as on the right one; but this error cannot be fallen into, if the ſign above named be carefully obſerved.

"In chooſing a line to fix a looſe one, care muſt be taken that it does not make a very acute or obtuſe angle; as in the triangle *p*B*r*, by the angle at B being very obtuſe, a ſmall deviation from truth, even the breadth of a point, at *p* or *r*, would make the error at B, when conſtructed, very conſiderable; but by conſtructing the triangle *p*B*q*, ſuch a deviation is of no conſequence.

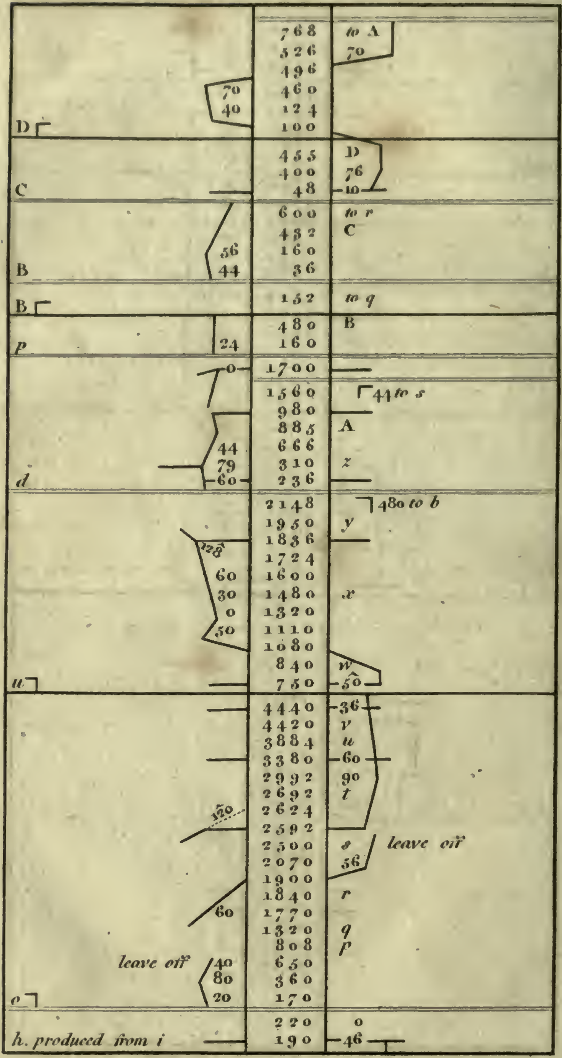
"Where the words *leave off* are written in the field-book, it is to ſignify that the taking of offsets is from thence diſcontinued; and of courſe ſomething is wanting between that and the next offset."

The field book for this method, and the plan drawn from it, are contained in the four following pages, engraven on copper-plates. After the manner of which, the pupil muſt lay down a plan, to a larger ſcale, from the field-book entirely; and then computing the contents of every ſeparate field, and adding all the contents together, the ſum will amount to between 103 and 104 acres, when the work is all right.

Field Book.

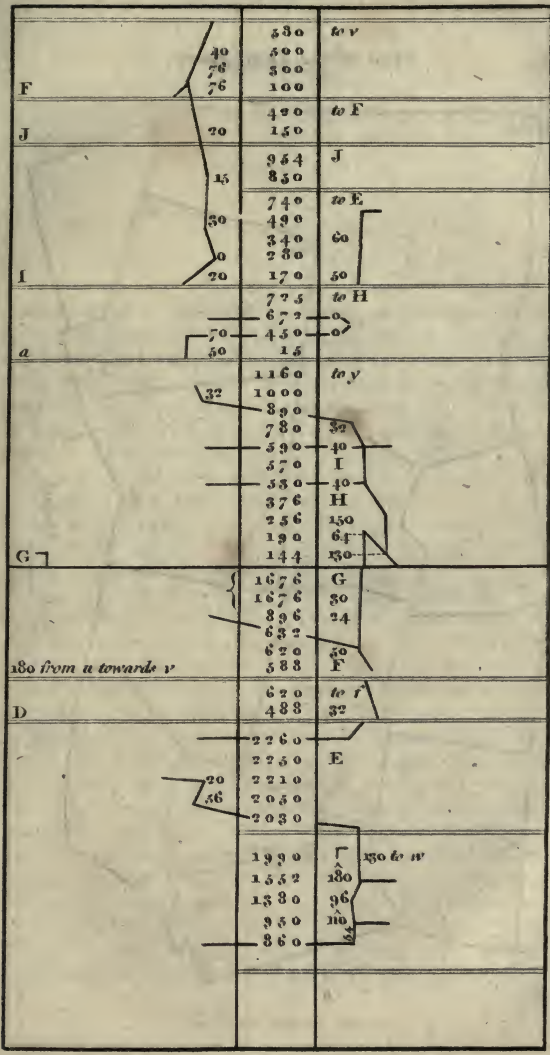


1310
56 to e
836
56
684
50
n
1480
90 to g
960
24
930
n
700
48
400
30
m
1430
to i
1290
40
1004
36
980
m
610
24
280
32
k
1820
to l
1464
22
190
1050
920
32
650
60
350
48
0
14
a
3074
to b
2494
2100
l
0
2072
54
1730
80
1530
1420
k
56 + 30
1170
52
620
32
280
40
j
2574
to j
2494
2000
44
1880
50
1840
50
1794
i
34 + 50
1464
76
1328
96
1240
52 + 34
1130
34
860
66
190
h
4450
h
3570
g
2620
f
2610
2210
2080
e
1640
d
1550
1510
c
990
b
806
a



		768	to A
		526	70
		496	
	70	460	
	40	124	
D ┌		100	
		455	D
		400	76
C		48	10
		600	to r
		432	C
	56	160	
B	44	36	
B ┌		152	to q
		480	B
p	24	160	
	0	1700	
		1560	┌ 44 to s
		980	
		885	A
	44	666	
	79	310	z
d	60	236	
		2148	┐ 480 to b
		1950	y
	128	1836	
		1724	
	60	1600	
	30	1480	x
	0	1320	
	50	1110	
		1080	
		840	w
u ┐		750	50
		4440	36
		4420	v
		3884	u
		3380	60
		2992	90
		2692	t
	120	2624	
		2592	
		2500	s leave off
		2070	56
		1900	
		1840	r
	60	1770	
		1320	q
		808	p
leave off	40	650	
	80	360	
o ┐	20	170	
		220	o
h. produced from i		190	46

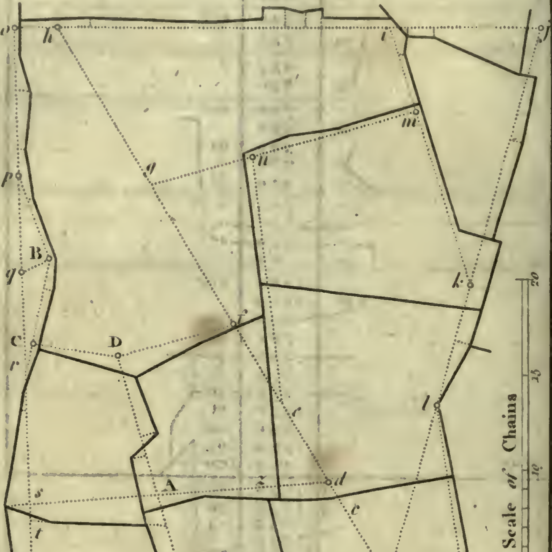
Field Book.



		580	*to v*
	40	500	
	76	300	
F	76	100	
		420	*to* F
J	20	150	
		954	J
	15	850	
		740	*to* E
	30	490	
		340	60
	0	280	
I	20	170	50
		725	*to* H
		672	0
	70	450	0
a	50	15	
		1160	*to y*
	32	1000	
		890	
		780	32
		590	40
		570	I
		530	40
		376	H
		256	150
		190	64
G		144	130
		1676	G
		1676	30
		896	24
		632	
		620	50
180 *from u towards v*		588	F
		620	*to f*
D		488	32
		2260	
		2250	E
	20	2210	
	56	2050	
		2030	
		1990	130 *to w*
		1552	1̂80
		1380	96
		950	1̂10
		860	54

Plan from the foregoing Field Book

Plan of the Field Book.



Whole content $\overset{A}{103}.\overset{R}{2}.\overset{P}{10}$.

PROBLEM XII.

To ſurvey a County, or Large Tract of Land.

1. Chooſe two, three, or four eminent places for ſtations; ſuch as the tops of high hills or mountains, towers, or church ſteeples, which may be ſeen from one another; and from which moſt of the towns, and other places of note, may alſo be ſeen. And let them be as far diſtant from another as poſſible. On theſe places raiſe beacons, or long poles, with flags of different colours flying at them: ſo as to be viſible from all the other ſtations.

2. At all the places, which you would ſet down in the map, plant long poles with flags at them of ſeveral colours, to diſtinguiſh the places from one another; fixing them on the tops of church ſteeples, or the tops of houſes, or in the centres of leſſer towns.

But you need not have theſe marks at many places at once, as ſuppoſe half a ſcore at a time. For when the angles have been taken, at the two ſtations, to all theſe places, the marks may be removed to new ones; and ſo ſucceſſively to all the places wanted. Theſe marks then being ſet up at a convenient number of places, and ſuch as may be ſeen from both ſtations; go to one of theſe ſtations, and, with an inſtrument to take angles, ſtanding at that ſtation, take all the angles between the other ſtation, and each of theſe marks, obſerving which is blue, which is red, &c. and which hand they lie on; and ſet all down with their colours. Then go to the other ſtation, and take all the angles between the firſt ſtation, and each of the former marks, and ſet them down with the others, each againſt its fellow with the ſame colour. You may, if you can, alſo take the angles at ſome third ſtation, which may ſerve to prove the work, if the three lines interſect

terſect in that point where any mark ſtands. The marks muſt ſtand till the obſervations are finiſhed at both ſtations; and then they muſt be taken down, and ſet up at freſh places. The ſame operations muſt be performed, at both ſtations, for theſe freſh places; and the like for others. The inſtrument for taking angles muſt be an exceeding good one, made on purpoſe with teleſcopic ſights; and of a good length of radius.

3. And though it be not abſolutely neceſſary to meaſure any diſtance, becauſe a ſtationary line being once laid down from any ſcale, all the other lines will be proportional to it; yet it is better to meaſure ſome of the lines, to aſcertain the diſtances of places in miles; and to know how many geometrical miles there are in any length; and from that to make a ſcale to meaſure any diſtance in miles. In meaſuring any diſtance, it will not be exact enough to go along the high roads; by reaſon of their turnings and windings, and hardly ever lying in a right line between the ſtations, which muſt cauſe infinite reductions, and create endleſs trouble to make it a right line; for which reaſon it can never be exact. But a better way is to meaſure in a right line with a chain, between ſtation and ſtation, over hills and dales or level fields, and all obſtacles. Only in caſe of water, woods, towns, rocks, banks, &c. where one cannot paſs; ſuch parts of the line muſt be meaſured by the methods of inacceſſible diſtances; and beſides, allowing for aſcents and deſcents, when we meet with them. And a good compaſs, that ſhows the bearing of the two ſtations, will always direct you to go ſtraight, when you do not ſee the two ſtations; and in the progreſs, if you can go ſtraight, you may take offsets to any remarkable places, likewiſe note the interſection of your ſtationary line with all roads, rivers, &c.

4. And from all the ſtations, and in the whole progreſs, be very particular in obſerving ſea-coaſts, river-mouths, towns, caſtles, houſes, churches, windmills, watermills, trees, rocks, ſands, roads, bridges, fords, ferries,

ferries, woods, hills, mountains, rills, brooks, parks, beacons, ſluices, floodgates, locks, &c. and in general all things that are remarkable.

5. After you have done with the firſt and main ſtation lines, which command the whole county; you muſt then take inner ſtations, at ſome places already determined; which will divide the whole into ſeveral partitions; and from theſe ſtations you muſt determine the places of as many of the remaining towns as you can. And if any remain in that part, you muſt take more ſtations, at ſome places already determined; from which you may determine the reſt. And thus we muſt go through all the parts of the country, taking ſtation after ſtation, till we have determined all we want. And in general the ſtation diſtances muſt always paſs through ſuch remarkable points as have been determined before, by the former ſtations.

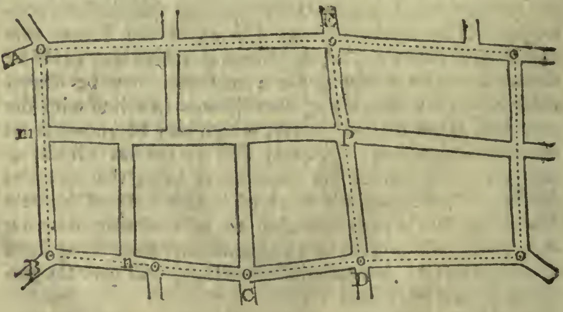
6. Laſtly, the poſition of the ſtation line meaſured, or the point of the compaſs it lies on, muſt be determined by aſtronomical obſervation. Hang up a thread and plummet in the ſun, over ſome part of the ſtation line, and obſerve when the ſhadow runs along that line, and at that moment take the ſun's altitude; then having his declination, and the latitude, the azimuth will be found by ſpherical trigonometry. The azimuth being the angle the ſtation line makes with the meridian, therefore a meridian may eaſily be drawn through the map. Or a meridian may be drawn through it, by hanging up two threads in a line with the pole ſtar, when it is juſt north, which may be known from aſtronomical tables. Or thus; obſerve the ſtar Alioth, or that in the rump of the great bear, being that next the ſquare, by a line and plummet when that ſtar and the pole ſtar come into a perpendicular; for at that time they are due north. Therefore two perpendicular lines being fixed at that moment, towards theſe two ſtars, will give the poſition of the meridian.

PROBLEM XIII.

To ſurvey a Town or City.

This is beſt performed with the plain table, where every minute part is drawn while in ſight. It is beſt alſo to have a chain of 50 feet long, divided into 50 links, each 1 foot in length, and an offset-ſtaff of 10 feet long.

Begin at the meeting of two or more of the principal ſtreets, through which you can have the longeſt proſpects, to get the longeſt ſtation lines. There having fixed the inſtrument, draw lines of direction along thoſe ſtreets, uſing two men as marks, or poles ſet in wooden pedeſtals, or perhaps ſome remarkable places in the houſes at the farther ends, as windows, doors, corners, &c. Meaſure theſe lines with the chain, taking offsets with the ſtaff, at all corners of ſtreets, bendings, or windings, and to all remarkable things, as churches, markets, halls, colleges, eminent houſes, &c. Then remove the inſtrument to another ſtation, along one of theſe lines, and there repeat the ſame proceſs as before. And ſo on till the whole is finiſhed.



Thus, fix the inſtrument at A, and draw lines in the direction of all the ſtreets meeting there; then meaſure AB, AB, noting the ſtreet on the left at m. At the ſecond ſtation B, draw the directions of the ſtreets meeting there; meaſure from B to C, noting the places of the ſtreets at n and o as you paſs by them. At the 3d ſtation C, take the direction of all the ſtreets meeting there, and meaſure CD. At D do the ſame, and meaſure DE, noting the place of the croſs ſtreets at p. And in this manner go through all the principal ſtreets. This done, proceed to the ſmaller and intermediate ſtreets; and laſtly to the lanes, alleys, courts, yards, and every part which it may be thought proper to repreſent in the plan.

CHAPTER III.

OF PLANNING, COMPUTING, AND DIVIDING.

PROBLEM I.

To lay down the Plan of any Survey.

IF the ſurvey was taken with a plain table, you have a rough plan of it already on the paper which covered the table But if the ſurvey was with any other inſtrument, a plan of it is to be drawn from the meaſures that were taken in the ſurvey, and firſt of all a rough plan on paper.

To do this, you muſt have a ſet of proper inſtruments, for laying down both lines and angles, &c. as ſcales of various ſizes, the more of them, and the more accurate, the better; ſcales of chords, protractors, perpendicular and parallel rulers, &c. Diagonal ſcales are beſt for the lines, becauſe they extend to three figures, or chains and links, which are hundredth parts of chains. But in uſing the diagonal ſcale, a pair of compaſſes muſt be employed to take off the lengths of the principal lines very accurately.

accurately. But a ſcale with a thin edge divided, is much readier for laying down the perpendicular offſets to crooked hedges, and for marking the places of thoſe offſets on the ſtation line; which is done at only one application of the edge of the ſcale to that line, and then pricking off all at once the diſtances along it. Angles are to be laid down, either with a good ſcale of chords, which is perhaps the moſt accurate way; or with a large protractor, which is much readier when many angles are to be laid down at one point, as they are pricked off all at once round the edge of the protractor.

Very particular directions for laying down all ſorts of figures cannot be neceſſary in this place, to any perſon who has learned practical geometry, and the conſtruction of figures, with the uſe of his inſtruments. It may therefore be ſufficient to obſerve, that all lines and angles muſt be laid down on the plan in the ſame order in which they were meaſured in the field, and in which they are written in the field-book; laying down firſt the angles for the poſition of lines, then the lengths of the lines, with the places of the offſets, and then the lengths of the offſets themſelves, all with dry or obſcure lines; then a black line drawn through the extremities of all the offſets, will be the hedge or bounding line of the field, &c. After the principal bounds and lines are laid down, and made to fit or cloſe properly, proceed next to the ſmaller objects, till you have entered every thing that ought to appear in the plan, as houſes, brooks, trees, hills, gates, ſtiles, roads, lanes, mills, bridges, woodlands, &c. &c.

The north ſide of a map or plan is commonly placed uppermoſt, and a meridian ſomewhere drawn, with the compaſs or flower-de-luce pointing north. Alſo, in a vacant place, a ſcale of equal parts or chains muſt be drawn, with the title of the map in conſpicuous characters, and embelliſhed with a compartment. All hills muſt be ſhadowed, to diſtinguiſh them in the map. Colour the hedges with different colours; repreſent hilly grounds by broken

broken hills and valleys; draw ſingle dotted lines for footpaths, and double ones for horſe or carriage roads. Write the name of each field and remarkable place within it, and, if you chooſe, its content in acres, roods, and perches.

In a very large eſtate, or a county, draw vertical and horizontal lines through the map, denoting the ſpaces between them by letters placed at the top, and bottom, and ſides, for readily finding any field or other object, mentioned in a table.

In mapping counties, and eſtates that have uneven grounds of hills and valleys, reduce all oblique lines, meaſured up hill and down hill, to horizontal ſtraight lines, if that was not done during the ſurvey, before they were entered in the field-book, by making a proper allowance to ſhorten them. For which purpoſe there is commonly a ſmall table engraven on ſome of the inſtruments for ſurveying. Or it may be done by holding the chain, in meaſuring, quite level, and then dropping the arrow from the hand.

PROBLEM II.

To Compute the Contents of Fields.

1. Compute the contents of all the figures, whether triangles, or trapeziums, &c. by the proper rules for the ſeveral figures laid down in meaſuring; multiply the lengths by the breadths, both in links, and divide by 2; the quotient is acres, after you have cut off five figures on the right for decimals. Then bring theſe decimals to roods and perches, by multiplying firſt by 4, and then by 40. An example of which is before given, in the deſcription of the chain.

2. In ſmall and ſeparate pieces, it is uſual to compute their contents from the meaſures of the lines taken in ſurveying them, without making a correct plan of them.

Thus, in the triangle in prob. iv. page 190, where we had AP = 794, and AB = 1321

PC = 826

7926

2642

10568

2) 10·91146

5·45573   ac r p

4 Anſ. 5 1 33

1·82292

40

32·91680

Or the firſt example to prob. v. page 191, thus:

AE 214	210 ED
AF 362	306 FB
AC 592	
	516 ſum of perps.
	592 AC

1032

4644

2580

2) 3·05472

1·52736   ac r p

4   Anſ. 1 2 4

2·10944

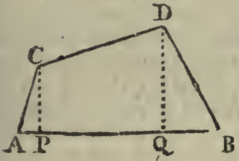
40

4·37760

Or

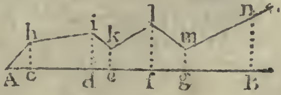
Or the 2d example to the ſame prob. v. thus:

AP	110	352	PC	QD 595	
AQ	745	595	QD	QB 365	
AB	1110				
PC	352	PC	352	2975	
AP	110	QD	595	3570	
				1785	
2APC	38720	ſum	947		
		PQ	635	217175 = 2 QDB	
				601345 = 2 PCDQ	
			4735	38720 = 2 APC	
			2841		
			5682	2) 8·57240 = dou. the whole	
				4·2862	
2PCDQ			601345	4	ac r p
				1·1448	Anſ. 4 1 5
				40	
				5·7920	



3. In pieces bounded by very crooked and winding hedges, meaſured by offsets, all the parts between the offsets are moſt accurately meaſured ſeparately as ſmall trapezoids. Thus, for the example to prob. III. p. 189, where

Ac	45	62	ch
Ad	220	84	di
Ae	340	70	ek
Af	510	88	fl
Ag	634	57	gm
AB	785	91	Bn



Then

Ac 45	ch 62	di 84	dk 70	f l 88	gm 57
ch 62	di 84	k 70	f l 98	gm 57	Bn 91
90	140	154	168	145	148
270	cd 175	de 120	ef 170	fg 124	gB 151
2790	730	18480	11760	580	148
	1022		168	290	740
	146			105	148
			28560		
	25550			17980	22348

2790
25550
18480
28560
17980
22348

2) 1.15708 — ac r p
·57854 Content 0 2 12
4

2·31416
40

12·56640

4. Sometimes ſuch pieces as that above, are computed by finding a mean breadth, by dividing the ſum of the offsets by the number of them, accounting that for one of them where the boundary meets the ſtation line, as at A; then multiply the length AB by that mean breadth.

Thus:

Thus:

00	785	A B
62	66	mean breadth
84	——	
70	4710	ac r p
98	4710	Content 0 2 2 by this method,
57	——	which is 10 perches too little.
91	·51810	
——	4	
7) 462	——	
66	2·07240	For this method is always erroneous,
——	40	except when the offsets ſtand at equal
	——	diſtances from one another.
	2·89600	
	——	

5. But in larger pieces, and whole eſtates, conſiſting of many fields, it is the common practice to make a rough plan of the whole, and from it compute the contents quite independent of the meaſures of the lines and angles that were taken in ſurveying. For then new lines are drawn in the fields in the plan, ſo as to divide them into trapeziums and triangles, the baſes and perpendiculars of which are meaſured on the plan by means of the ſcale from which it was drawn, and ſo multiplied together for the contents. In this way, the work is very expeditiouſly done, and ſufficiently correct; for ſuch dimenſions are taken, as afford the moſt eaſy method of calculation; and, among a number of parts, thus taken, and applied to a ſcale, it is likely that ſome of the parts will be taken a ſmall matter too little, and others too great; ſo that they will, upon the whole, in all probability, very nearly balance one another. After all the fields, and particular parts, are thus computed ſeparately, and added all together into one ſum; calculate the whole eſtate independent of the fields, by dividing it into large and arbitrary triangles and trapeziums, and add theſe all together. Then if this ſum be equal to the former, or nearly ſo, the work is right; but

the ſums have any conſiderable difference, it is wrong, and hey muſt be examined and recomputed till they nearly agree.

A ſpecimen of dividing into one triangle, or one trapezium, which will do for moſt ſingle fields, may be ſeen in the examples to the laſt problem; and a ſpecimen of dividing a large tract into ſeveral ſuch trapeziums and triangles, in prob. vi of chapter 11 of Surveying, page 193, where a piece is ſo divided, and its dimenſions taken and ſet down; and again at prob. vi of Menſuration of Surfaces, where the contents of the ſame piece are computed.

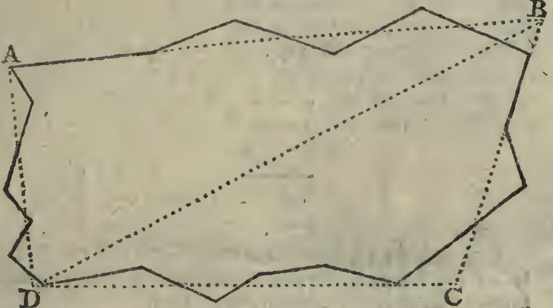
6. But the chief ſecret in computing, conſiſts in finding the contents of pieces bounded by curved or very irregular ines, or in reducing ſuch crooked ſides of fields or boundaries to ſtraight lines, that ſhall incloſe the ſame or equal area with thoſe crooked ſides, and ſo obtain the area of the curved figure by means of the right lined one, which will commonly be a trapezium. Now this reducing the crooked ſides to ſtraight ones, is very eaſily and accurately performed, thus: Apply the ſtraight edge of a thin clear piece of lanthorn horn to the crooked line, which is to be reduced, in ſuch a manner, that the ſmall parts cut off from the crooked figure by it, may be equal to thoſe which are taken in: which equality of the parts included and excluded you will preſently be able to judge of very nicely by a little practice: then with a pencil, or point of a tracer, draw a line by the ſtraight edge of the horn. Do the ſame by the other ſides of the field or figure. So ſhall you have a ſtraight ſided figure equal to the curved one; the content of which, being computed as before directed, will be the content of the curved figure propoſed.

Or, inſtead of the ſtraight edge of the horn, a horſehair line thread may be applied acroſs the crooked ſides in the ſame manner; and the eaſieſt way of uſing the hair is to ſtring a ſmall ſlender bow with it, either of wire, or cane, or whale-bone, or ſuchlike ſlender elaſtic matter; for, the bow keeping it always ſtretched, it can be eaſily and neatly applied with one hand, while the other is at liberty

liberty to make two marks by the ſide of it, to draw the ſtraight line by.

EXAMPLE.

Thus, let it be required to find the contents of the ſame figure as in prob. ix of the laſt chapter, page 198, to a ſcale of 4 chains to an inch.



Draw the four dotted ſtraight lines AB, BC, CD, DA, cutting off equal quantities on both ſides of them, which they do as near as the eye can judge: ſo is the crooked figure reduced to an equivalent right lined one of four ſides ABCD. Then draw the diagonal BD, which, by applying a proper ſcale to it, meaſures 1256 Alſo the perpendicular or neareſt diſtance, from A to this diagonal, meaſures 456; and the diſtance of C from it, is 428.

L Then

$$\begin{array}{lrl}
\text{Then} & 456 & \\
 & 428 & \\
\hline
 & 884 & \\
 & 1256 & \\
\hline
 & 5024 & \\
 & 10048 & \\
 & 10048 & \\
\hline
2\,) & 11{\cdot}10304 & \\
 & 5{\cdot}55153 & \\
 & 4 & \\
\hline
 & 2{\cdot}20608 & \\
 & 40 & \\
\hline
 & & \text{ac r p} \\
 & 8{\cdot}24320 & \text{Content } 5\ 2\ 8 \\
\hline
\end{array}$$

And thus the content of the trapezium, and conſequently of the irregular figure, to which it is equal, is eaſily found to be 5 acres, 2 roods, 8 perches.

PROBLEM III.

To transfer a Plan to another Paper, &c.

After the rough plan is completed, and a fair one is wanted; this may be done, either on paper or vellum, by any of the following methods.

FIRST METHOD.

Lay the rough plan over the clean paper, and keep them always preſſed flat and cloſe together, by weights laid on them. Then, with the point of a fine pin or pricker, prick through all the corners of the plan to be copied. Take them aſunder, and connect the pricked points on the clean paper, with lines; and it is done. This

This method is only to be practiſed in plans of ſuch figures as are ſmall and tolerably regular, or bounded by right lines.

SECOND METHOD.

Rub the back of the rough plan over with black lead powder; and lay the ſaid black part on the clean paper on which the plan is to be copied, and in the proper poſition. Then with the blunt point of ſome hard ſubſtance, as braſs or ſuch like, trace over the lines of the whole plan; preſſing the tracer ſo much as that the black lead under the lines may be transferred to the clean paper; after which take off the rough plan, and trace over the leaden marks with common ink, or with Indian ink, &c.—Or, inſtead of blacking the rough plan, you may keep conſtantly a blacked paper to lay between the plans.

THIRD METHOD.

Another method of copying plans, is by means of ſquares. This is performed by dividing both ends and ſides of the plan which is to be copied, into any convenient number of equal parts, and connecting the correſponding points of diviſion with lines; which will divide the plan into a number of ſmall ſquares. Then divide the paper, on which the plan is to be copied, into the ſame number of ſquares, each equal to the former when the plan is to be copied of the ſame ſize, but greater or leſs than the others, in the proportion in which the plan is to be increaſed or diminiſhed, when of a different ſize. Laſtly, copy into the clean ſquares the parts contained in the correſponding ſquares of the old plan; and you will have the copy, either of the ſame ſize, or greater or leſs in any proportion.

FOURTH METHOD.

A fourth method is by the inſtrument called a pentagraph, which alſo copies the plan in any ſize required.

FIFTH METHOD.

But the neateſt method of any is this. Procure a copying frame or glaſs, made in this manner; namely, a large ſquare of the beſt window glaſs, ſet in a broad frame of wood, which can be raiſed up to any angle, when the lower ſide of it reſts on a table. Set this frame up to any angle before you, facing a ſtrong light; fix the old plan and clean paper together with ſeveral pins quite around, to keep them together, the clean paper being laid uppermoſt, and over the face of the plan to be copied. Lay them with the back of the old plan, over the glaſs, namely, that part which you intend to begin at to copy firſt; and, by means of the light ſhining through the papers, you will very diſtinctly perceive every line of the plan through the clean paper. In this ſtate then trace all the lines on the paper with a pencil. Having drawn that part which firſt covers the glaſs, ſlide another part over the glaſs, and copy it in the ſame manner: then another part. And ſo on, till the whole be copied.

Then, take them aſunder, and trace all the pencil-lines over with a fine pen and Indian ink, or with common ink.

And thus you may copy the fineſt plan, without injuring it in the leaſt.

When the lines, &c. are copied on the clean paper or vellum, the next buſineſs is to write in ſuch names, remarks, or explanations as may be judged neceſſary: laying down the ſcale for taking the lengths of any parts, a flower-de-luce to point out the direction, and the proper title, ornamented with a compartment: and illuſtrating or colouring every part, in ſuch manner as ſhall ſeem moſt natural, ſuch as ſhading rivers or brooks with crooked lines, drawing the repreſentation of trees, buſhes, hills, woods, hedges, houſes, gates, roads, &c. in their proper places; running a ſingle dotted line for a foot path, and a double one for a carriage road; and either repreſenting the baſes or the elevation of buildings, &c.

CONIC SECTIONS

AND THEIR

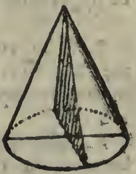
SOLIDS.

DEFINITIONS.

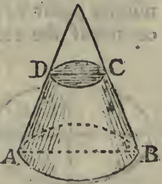
1. CONIC Sections are the plane figures formed by cutting a cone.

According to the different poſitions of the cutting plane there will ariſe five different figures or ſections.

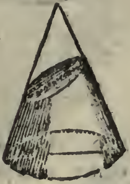
2. If the cutting plane paſs through the vertex, and any part of the baſe, the ſection will be a triangle.



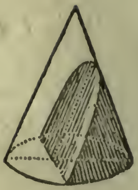
3. If the cone be cut parallel to the baſe, the ſection will be a circle.



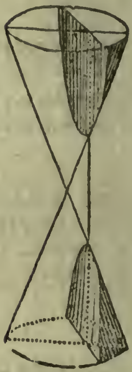
4. The ſection is called an ellipſis, when the cone is cut obliquely through both ſides.



5. The ſection is a parabola, when the cone is cut parallel to one of its ſides.

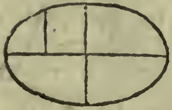


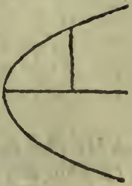
6. The ſection is an hyperbola, when the cutting plane meets the oppoſite cone continued above the vertex, where it will make another ſection or hyperbola.

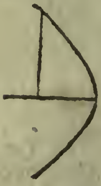


7. The vertices of any ſection, are the points where the cutting plane meets the oppoſite ſides of the cone.

8. The tranſverſe axis, is the line between the two vertices. And the middle point of the tranſverſe is the centre of the conic ſection.



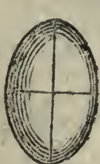




9. The conjugate axis, is a line drawn through the centre, and perpendicular to the tranſverſe.

10. An ordinate, is a line perpendicular to the axis.

11. An abſciſs, is a part of the axis between the ordinate and the vertex.



12. A ſpheroid, or ellipſoid, is a ſolid generated by the revolution of an ellipſe about one of its axis. It is a prolate one, when the revolution is made about the tranſverſe axis; and oblate, when about the conjugate.

13. A conoid is a ſolid formed by the revolution of a parabola, or hyperbola, about the axis. And is accordingly called parabolic, or hyperbolic.—The parabolic conoid is alſo called a paraboloid; and the hyperbolic conoid, an hyperboloid.

14. A ſpindle is formed by any of the three ſections revolving about a double ordinate, like the circular ſpindle.

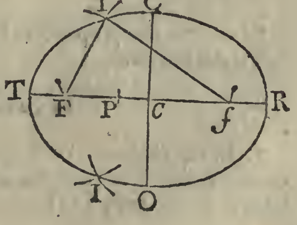
15. A ſegment of any of theſe figures, is a part cut off at the top, by a plane parallel to the baſe.

16. And a fruſtum is the part left next the baſe, after the ſegment is cut off.

PROBLEM I.

To deſcribe an Ellipſe.

Let TR be the tranſverſe, CO the conjugate, and *c* the centre. With the radius T*c* and centre C, deſcribe an arc cutting TR in the points F, *f*; which are called the two foci of the ellipſe.



Aſſume any point P in the tranſverſe; then with the radii PT, PR, and centres F, *f*, deſcribe two arcs interſecting in I; which will be a point in the curve of the ellipſe.

And thus, by aſſuming a number of points P in the tranſverſe, there will be found as many points in the curve as you pleaſe. Then, with a ſteady hand, draw the curve through all theſe points.

Otherwiſe with a Thread.

Take a thread of the length of the tranſverſe TR, and faſten its ends with two pins in the foci F, *f*. Then ſtretch the thread, and it will reach to I in the curve: and by moving a pencil round, within the thread, keeping it always ſtretched, it will trace out the ellipſe.

PROBLEM II.

In an Ellipſe, to find the Tranſverſe, or Conjugate, or Ordinate, or Abſciſs: having the other three given.

CASE I.

To find the Ordinate.

As the tranſverſe :
Is to the conjugate : :
So is the mean proportional between the two abſciſſes :
To the ordinate.

EXAMPLES.

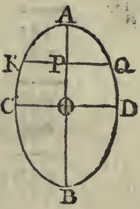
1. In the ellipſe ADBC, the tranſverſe AB is 70, the conjugate CD is 50, and the abſciſſes AP 14, and BP 56; what is the ordinate PQ?

Firſt 56 = PB
14 = AP

224
56

784 (28 mean
4 between 56
and 14

48 | 384
| 384



Then 70 : 50 : : 28 : 20 = PQ the ordinate.

Ex. 2. If the tranſverſe be 80, the conjugate 60, and an abſciſs 16 ; required the ordinate ? Anſ. 24.

CASE II.

To find the Abſciſs.

From the ſquare of half the conjugate, take the ſquare of the ordinate; and extract the ſquare root of the remainder. Then ſay,

As the conjugate :
Is to the tranſverſe : :
So is that ſquare root :
To half the difference of the abſciſſes.

Then add this half difference to half the tranſverſe, for the greater abſciſs ; and ſubtract it, for the leſs abſciſs.

EXAMPLES.

1. The tranſverſe AB is 70, the conjugate CD is 50 and the ordinate PQ is 20 ; required the abſciſſes AP and PB ?

First

Firſt 25
25

125
50

$625 = CO^2$
$400 = PQ^2$

225 (15
1

25 | 125
125

Then
35 half AB
As 50 : 70 : : 15 : 21 half dif.

56 = PB
14 = AP

Ex. 2. What are the two abſciſſes to the ordinate 24, the axes being 80 and 60? Anſ. 16 and 64.

CASE III.

To find the Conjugate.

As the mean proportional between the abſciſſes :
Is to the ordinate : :
So is the tranſverſe :
To the conjugate.

Note. In the ſame manner, the tranſverſe may be found from the conjugate; uſing here the abſciſſes of the conjugate, and their ordinate perpendicular to the conjugate.

EXAMPLES.

1. The tranſverſe being 180, the ordinate 16; and the greater abſciſs 144; required the conjugate?

180 tranſ-

180 tranſverſe
144 greater abſ.
36 leſs abſ.

864
432

5184 (72 : 16 : : 180 : 40 the conjugate.
49 16

142 | 284 1080
| 284 18

72) 2830 (40
288

Ex. 2. The tranſverſe being 70, the ordinate 20, and abſciſs 14; what is the conjugate? Anſ. 50.

CASE IV.

To find the Tranſverſe.

From the ſquare of half the conjugate, ſubtract the ſquare of the ordinate; and extract the root of the remainder. Next add this root to the half conjugate, if the leſs abſciſs be given, but ſubtract it when the greater abſciſs is given, reſerving the ſum or difference.

Then ſay,
As the ſquare of the ordinate :
Is to the rectangle of the abſciſs and conjugate : :
So is the reſerved ſum or difference :
To the tranſverſe.

EXAMPLES.

1. If the conjugate be 50, the ordinate 20, and the leſs abſciſs 14; what is the tranſverſe?

Firſt

Firſt Then
25 20 14
25 20 50
125 400 : 700 : : 40 : 70 the tranſ.
50
625
400
225 (15
1 25
25 | 125 40
125

Ex. 2. The conjugate being 40, the ordinate 16, and the leſs abſciſs 36; required the tranſverſe? Anſ. 180.

PROBLEM III.

To find the Circumference of an Ellipſe.

Add the two axes together, and multiply the ſum by 1·5708, for the circumference nearly.

EXAMPLES.

1. Required the circumference of the ellipſe whoſe two axes are 70 and 50?

70
50
120 ſum.
1·5708
188·4960 circumf. nearly.

Ex. 2.

Ex. 2. What is the periphery of an ellipſe whoſe two axes are 24 and 20? Anſ. 69·1152.

PROBLEM IV.

To find the Area of an Ellipſe.

Multiply the tranſverſe by the conjugate; then that product multiplied by ·7854, will be the area.

Or multiply ·7854 firſt by the one axe, and the product again by the other.

EXAMPLES.

1. To find the area of the ellipſe whoſe two axes are 70 and 50.

$$\begin{array}{r} \cdot 7854 \\ 50 \\ \hline 39{\cdot}2700 \\ 70 \\ \hline 2748{\cdot}9000 \text{ anſ.} \\ \hline \end{array}$$

Ex. 2. What is the area of the ellipſe whoſe two axes are 24 and 18? Anſ. 339·2928.

PROBLEM V.

To find the Area of an Elliptic Segment.

Divide the height of the ſegment by that axis of the ellipſe of which it is a part; and find, in the table of circular ſegments at the end of the book, a circular ſegment having the ſame verſed ſine as this quotient. Then multiply continually together, this ſegment, and the two axes, for the area required.

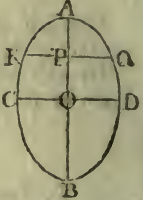
EXAMPLES.

1. What is the area of an elliptic ſegment RAQ, whoſe height AP is 20; the tranſverſe AB being 70, and the conjugate CD 50?

70) 20 (·285$\frac{5}{7}$ the tab. verſ.
The correſpond. ſeg.
is ·185166
70

12·961620
50

648·081000



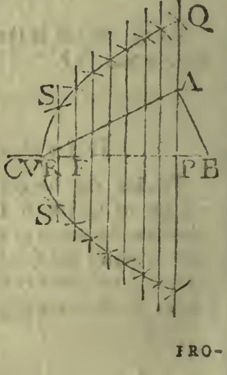
Ex. 2. What is the area of an elliptic ſegment, cut off parallel to the ſhorter axis, the height being 10, and the axes 25 and 35? Anſ. 162·0210.

Ex. 3. What is the area of the elliptic ſegment, cut off parallel to the longer axis, the height being 5, and the axes 25 and 35? Anſ. 97·8458.

PROBLEM VI.

To Deſcribe or Conſtruct a Parabola.

VP being an abſciſs, and PQ its given ordinate; biſect PQ in A, join AV, and draw AP perpendicular to it; then transfer PB to VF and VC in the axis produced. So ſhall F be wh is called the focus.



Draw ſeveral double ordinates SRS, &c. perpendicular to VP. Then with the radii CR, &c. and the centre F, deſcribe arcs cutting the correſponding ordinates in the points S, &c. Then draw the curve through all the points S, &c.

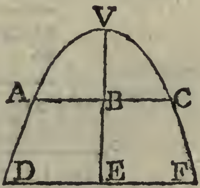
PROBLEM VII.

To find any Parabolic Abſciſs or Ordinate.

The abſciſſes are to each other as the ſquares of their ordinates; that is,

As any abſciſs is to the ſquare of its ordinate,
So is any other abſciſs, to the ſquare of its ordinate.
Or as the ſquare root of any abſciſs, is to its ordinate,
So is the ſquare root of another abſciſs, to its ordinate.

EXAMPLES.



1. The abſciſs VB is 9, and its ordinate AB is 6; required the ordinate DE whoſe abſciſs VE is 16.

Here $\surd 9$ is 3, and $\surd 16$ is 4.
Then $3 : 6 : : 4 : 8 = DE$ required.

Or if the ordinate DE were given $= 8$, to find its abſciſs VE. Then $6^2 = 36$, and $8^2 = 64$. Hence $36 : 64 : : 9 : 16 = VE$ required.

Ex. 2. If an abſciſs be 8, and its ordinate 10; required the ordinate whoſe abſciſs is 18? Anſ. 15.

Ex. 3. If an abſciſs be 18, and its ordinate 18; what is the abſciſs whoſe ordinate is 10? Anſ. 8.

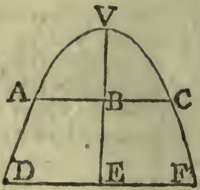
PROBLEM VIII.

To find the Length of a Parabolic Curve.

To the ſquare of the ordinate, add $\frac{4}{3}$ of the ſquare of the abſciſs; extract the ſquare root of the ſum, and double it for the length of the curve, cut off by the double ordinate, nearly.

EXAMPLES.

1. The abſciſs VB being 2, and the ordinate AB 6; required the length of the curve AVC?



```
          2 = VB
          2
          —
          4 = VB²
          4
          —
    3 ) 16
        ———
        5·3333
        36·    = AB²
        ———————
        41·3333 ( 64·291 root
        36               2
        ———————   ———————
  124 | 533       12·8582 = arc AVC nearly.
    4 | 496       ———————
  ——————————
  1282 | 3733
     2 | 2564
  ——————————
  1284 ) 1169 ( 91
         1156
         ————
           13
           —
```

Ex. 2. What is the length of the parabolic curve, whoſe abſciſs is 3, and ordinate 8? Anſ. 17·4356.

PROBLEM IX.

To find the Area of a Parabola.

Multiply the baſe by the height, and take $\frac{2}{3}$ of the product for the area.

EX.

EXAMPLES.

1. Required the area of the parabola AVCA, the abſciſs VB being 2, and the ordinate AB 6?

12
2
24
2
3) 48
16 anſ.

Ex. 2. What is the area of a parabola whoſe abſciſs is 10, and ordinate 8? Anſ. $106\frac{2}{3}$.

PROBLEM X.

To find the Area of a Parabolic Fruſtum.

Cube each end of the fruſtum, and ſubtract the one cube from the other; then multiply that difference by double the altitude, and divide the product by triple the difference of their ſquares, for the area.

EXAMPLES.

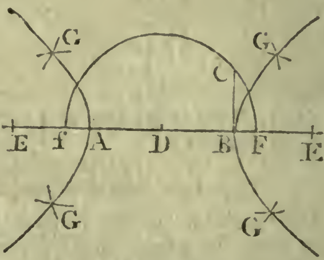
1. Required the area of the parabolic fruſtum ACFD, AC being 6, DF 10, and the altitude BE 4.

Ends	Sqrs.		Cubes
10	100		1000
6	36		216
	64	dif.	784
	3		8
	192	)	6272 ($32\frac{128}{192} = 32\frac{2}{3}$ anſ.
			576
			512
			384
			128

Ex. 2. What is the area of the parabolic fruſtum, whoſe two ends are 6 and 10, and its altitude 3? Anſ. 24$\frac{1}{2}$.

PROBLEM XI.

To Conſtruct or Deſcribe an Hyperbola.



Let D be the centre of the hyperbola, or the middle of the tranſverſe AB; and BC perpendicular to AB, and equal to half the conjugate.

With centre D, and radius DC, deſcribe an arc, meeting AB produced in F and f, which are the two focus points of the hyperbola.

Then aſſuming ſeveral points E in the tranſverſe produced, with the radii AE, BE, and centres f, F, deſcribe arcs interſecting in the ſeveral points G; through all which points draw the hyperbolic curve.

PROBLEM XII.

In an Hyperbola to find the Tranſverſe, or Conjugate, or Ordinate, or Abſciſs.

CASE I

To find the Ordinate.

As the tranſverſe :
Is to the conjugate : :
So is the mean propor. between the two abſciſſes :
To the ordinate. *Note.*

Note. In the hyperbola, the leſs abſciſs added to the axis, gives the greater abſciſs.

EXAMPLES.

1. If the tranſverſe be 24, the conjugate 21, and the leſs abſciſs VB 8; what is the ordinate AB?

24 tranſ.
8 leſs abſ.

---

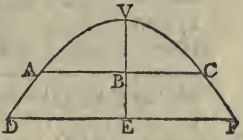
32 greater abſ.
8

---

256 (16 mean propor.
1

---

26 | 156
| 156



Then 24 : 21 : : 16 : 14 = AB required.

Ex. 2. The tranſverſe being 60, the conjugate 36, and the leſs abſciſs 20, required the ordinate? Anſ. 24.

CASE II.

To find the Abſciſs.

To the ſquare of half the conjugate, add the ſquare of the ordinate; and extract the ſquare root of the ſum. Then ſay,

As the conjugate :
Is to the tranſverſe : :
So is that ſquare root :
To half the ſum of the abſciſſes.

Then, to this ſum, add half the tranſverſe, for the greater abſciſs; and ſubtract it, for the leſs abſciſs.

EX-

EXAMPLES.

1. The tranſverſe being 24, and the conjugate 21; required the two abſciſſes to the ordinate AB 14?

Firſt 10·5 = $\frac{1}{2}$ conj. 14 ord.

10·5	14
525	56
1050	14
110·25	
196·	196

306·25 (17·5 the ſquare root. Then
1

27	206	As 21 : 24 : : 17·5 :	20 half ſum
7	189	8	12 half tranſ.
345	1725	7) 140·0	32 greater abſ.
5	1725	20	8 leſs abſcifs.

Ex. 2. The tranſverſe being 60, the conjugate 36; required the two abciſſes to the ordinate 24?

Anſ. 80 and 20.

CASE III.

To find the Conjugate.

As the mean proportion between the abſciſſes :
Is to the ordinate : :
So is the tranſverſe :
To the conjugate.

EX-

EXAMPLES.

1. The tranſverſe being 24, the leſs abſciſs VB 8, and its ordinate AB 14, what is the conjugate?

Firſt 24
8
———
32
8
———
256 (16 the mean.
1
———
26 | 156
6 | 156
———

Then
As 16 : 14 : : 24 : 21 Anſ.
7
———
8) 168
21
———

Ex. 2. What is the conjugate to the hyperbola, whoſe tranſverſe is 60, and ordinate 24, and the leſs abſciſs 20?

Anſ. 36.

CASE IV.

To find the Tranſverſe.

To the ſquare of half the conjugate add the ſquare of the ordinate, and extract the ſquare root of the ſum.

Next, to this root add the half conjugate when the leſs abſciſs is uſed, but ſubtract it when the greater abſciſs is uſed; reſerving the ſum or difference. Then ſay,

As the ſquare of the ordinate :
Is to the product of the abſciſs and conjugate : :
So is the reſerved ſum or difference :
To the tranſverſe.

EXAMPLES.

1. The leſs abſciſs VB being 8, and its ordinate AB 14; required the tranſverſe to the conjugate 21?

```
Firſt  10·5      14
       10·5      14
       ————      ——
        525      56
       105       14
       ————     ————
       110·25    196
       196·     ————
       ——————
       306·25 ( 17·5
       1        10·5
       ——————   ————
 27 | 206       28·0
  7 | 189       ————          Then
 ——————————               As 196 : 168 : : 28 :
345 | 1725                Or   7 :   6 : : 28 : 24 Anſ.
  5 | 1725
 ——————————
```

Ex. 2. What is the tranſverſe of the hyperbola, whoſe conjugate is 36; the leſs abſciſs being 20, and its ordinate 24? Anſ. 60.

PROBLEM XIII.

To find the Length of an Hyperbolic curve.

1. To 21 times the ſquare of the conjugate, add 9 times the ſquare of the tranſverſe; and to the ſame 21 times the ſquare of the conjugate, add 19 times the ſquare of the tranſverſe; and multiply each ſum by the abſciſs.

2. To each of theſe two products add 15 times the product of the tranſverſe and ſquare of the conjugate.

3. Then as the leſs ſum is to the greater, ſo is the double ordinate, to the length of the curve nearly.

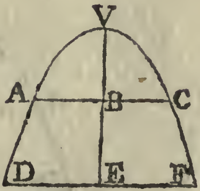
EXAMPLES.

1. Required the length of the curve AVC to the abſciſs VB 20 and ordinate AB 24; the two axes being 60 and 36

	36	27216	27216	1296
	36	32400	68400	60
	216	59616	95616	77760
	108	20	20	15
ſq. conju.	1296	1192320	1912320	388800
	21	1166400	1166400	77760
	1296	2358720	3078720	1166400
	2592			
	27216			

Then 2358720 : 3078720 : : 48 : 62·6520 the whole curve

```
            48
         -------
         2462976
        1231488
        --------
2358772 ) 14777856 ( 62·6520 Anſ.
.......   1415232
          -------
            62553
            47174
            -----
            15379
            14152
            -----
             1227
             1179
             ----
               48
               47
               --
                1
```



Ex. 2.

Ex. 2. What is the length of the whole curve to the ordinate 10, the tranſverſe and conjugate axes being 80 and 60? Anſ. 20·601.

PROBLEM XIV.

To find the Area of an Hyperbola.

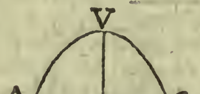
1. To $\frac{5}{7}$ of the abſciſs add the tranſverſe: multiply the ſum by the abſciſs; and extract the ſquare root of the product.

2. Multiply the tranſverſe by the abſciſs, and extract the root of that product alſo.

3. To 21 times the firſt root, add 4 times the ſecond root; multiply the ſum by double the product of the conjugate and abſciſs; then divide by 75 times the tranſverſe, for the area nearly.

EXAMPLES.

1. Required the area of the hyperbola AVCA, whoſe abſciſs VB is 10, the tranſverſe and conjugate being 30 and 18?



10

```
      10
       5                                    30
      —                                     10
  7 ) 50                                   ————
      7·1428571                             300 ( 17·3205081
      30·                                   1              4
      ——————————                          ——————————————————
      37·1428571                      27 | 200   69·2820324
             10                        7 | 189
                                      ——————————
      ——————————                     343 | 1100
      371·428571 ( 19·2724822          3 | 1029
      1                     21       ——————————
      ————            ——————————    3462 | 7100
  29 | 271         192724822           2 | 6924
   9 | 261         385449644        ————————————
  ——————————       ——————————     34640 | 17600 ( 5081
  382 | 1042       404·7221262     ···· | 17320
    2 |  764        69·2820324            ——————
  ——————————       ——————————                280
  3847 | 27885     474·0041586               277
     7 | 26929     ———————————               ———
  ————————————                                 3
  38542 | 95671                                —
      2 | 77084
  ————————————
 38544 ) 18587 ( 4822
  ····   15418
         —————
          3169
          3083
          ————
            86
            77
            ——
             9
             —
```

		474·0041586
		720
18	75	94800831720
40	30	33180291102
720	2250	3412829941920
	4	4
	9·000 (	1365·131·9767680
		151·68133 area required.

Ex. 2. What is the area of the hyperbola to the abſciſs 25, the two axes being 50 and 30?

Anſ. 805·090868.

PROBLEM XV.

To find the Solidity of a Spheroid.

Square the revolving axis, multiply that ſquare by the fixed axis, and multiply the product by ·5236 for the content.

EXAMPLES.

1. Required the ſolidity of the prolate ſpheroid ACBD, whoſe axes are AB 50 and CD 30?

30	·5236
30	45000
900	26180000
50	20944
45000	23562·0000 Anſ.

Ex. 2.

Ex. 2. What is the content of an oblate ſpheroid, whoſe axes are 50 and 30? Anſ. 39270.

Ex. 3. What is the ſolidity of a prolate ſpheroid, whoſe axes are 9 and 7? Anſ. 230·9076.

PROBLEM XVI.

To find the Solidity of a Segment of a Spheroid.

CASE I.

When the Baſe is Circular, or Parallel to the Revolving Axis.

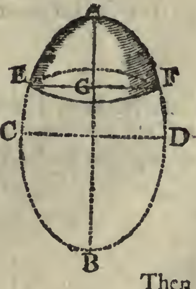
From triple the fixed axe, take double the height of the ſegment; multiply that difference by the ſquare of the height, and the product again by ·5236.

Then as the ſquare of the fixed axe is to the ſquare of the revolving axe, ſo is the laſt product to the content of the ſegment.

EXAMPLES.

1. Required the content of the ſegment of a prolate ſpheroid, the height AG being 5, the fixed axe AB 50, and the revolving axe CD 30?

150	·5236
10	3500
140	2618000
25	15708
700	1832·6000
28	
3500	



Then as 25 : 9 : : 1832·6 :
Or as 100 : 36 : : 1832·6 : 659·736

36

109956
54978

100) 65973·6 (659·736 Anſwer.

Ex. 2. If the axes of a prolate ſpheroid be 10 and 6, required the content of the ſegment whoſe height is 1, and its baſe parallel to the revolving axe? Anſ. 5·277888.

Ex. 3. The axes of an oblate ſpheroid being 50 and 30, what is the content of the ſegment, the height being 6, and its baſe parallel to the revolving axe?

Anſ. 4084·07.

CASE II.

When the Baſe is Elliptical, or Perpendicular to the Revolving Axe.

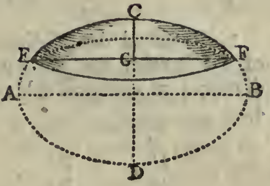
From triple the revolving axe, take double the height of the ſegment; multiply that difference by the ſquare of the height, and the product again by ·5236.

Then as the revolving axe, is to the fixed axe,
So is the laſt product, to the content.

EXAMPLES.

1. In the prolate ſpheroid ACBD, the fixed axe AB is 50, the revolving axe CD 30; required the ſolidity of the ſegment CEF, its height CG being 6?

90	·5236
12	2808
78	41888
36	41888
	10472
468	
234	1470·2688
2808	



Then as 30 : 50 : : 1470·2688 : 2450·448

5

3) 7351·3440

2450·4180 Anſwer

Ex. 2. In an oblate ſpheroid, whoſe axes are 50 and 30, required the content of the ſegment whoſe height is 5, its baſe being perpendicular to the revolving axe ?

Anſ. 1099·56.

PROBLEM XVII.

To find the Content of the Middle Fruſtum of a Spheroid.

CASE I.

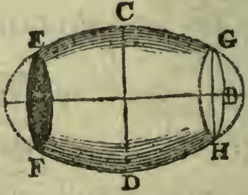
When the Ends are Circular, or Parallel to the Revolving Axe.

To double the ſquare of the middle diameter, add the ſquare of the diameter of one end ; multiply this ſum by the length of the fruſtum, and the product again by ·2618 for the content.

EXAMPLES.

1. Required the ſolidity of the middle fruſtum EGHF of a ſpheroid, the greateſt diameter CD being 30, the diameter of each end EF or GH 18, and the length AB 40.

18	30
18	30
144	900
18	2
324	1800
	324
	2124
	40
	84960
	·2618
	679680
	8496
	50976
	16992
	22242·5280 Anſwer.



Ex. 2. What is the ſolidity of the middle fruſtum of an oblate ſpheroid, having the diameter of each circular end 40, the middle 50, and the length 18?

Anſ. 31101·84.

CASE II.

When the Ends are Elliptical, or Perpendicular to the Revolving Axe.

To double the product of the tranſverſe and conjugate diameters of the middle ſection, add the product of the tranſverſe and conjugate of one end; multiply the ſum by the length of the fruſtum, and the product again by ·2618 for the content.

EXAMPLES.

1. In the middle fruſtum EFHG of an oblate ſpheroid the diameters of the middle or greateſt elliptic ſection AB are 50 and 30, and of one end EF or GH 40 and 24; required the content, the height IK being 9?

24	50
40	30
960	1500
	2
	3000
	960
	3960
	9
	35640
	·2618
	285120
	3564
	21384
	7128
	9330·5520 Anſ.

Ex. 2. In the middle fruſtum of an oblate ſpheroid, the axes of the middle ellipſe are 50 and 30, and thoſe of each end are 30 and 18; required the content, the height being 40? Anſ. 37070·88.

PROBLEM XVIII.

To find the Solidity of an Elliptic Spindle.

RULE I.

1. Take the difference between 3 times the ſquare of the middle or greateſt diameter, and 4 times the ſquare of the diameter at $\frac{1}{4}$ of the length, or equally diſtant between the middle and one end; alſo take the difference between 3 times the greateſt diameter, and 4 times the ſaid middle diameter. Then the former difference divided by the latter, will be quadruple the central diſtance, or diſtance between the centre of the ſpindle and centre of the generating ellipſe.

2. Then find the axes of the ellipſe by Problem 11, and the area of the ſegment which generated the ſpindle by problem v.

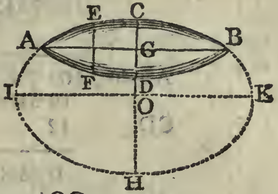
3. Divide 3 times that area by the length of the ſpindle; from the quotient ſubtract the greateſt diameter; and multiply the remainder by 4 times the central diſtance, before found.

4. Subtract this product from the ſquare of the greateſt diameter; and multiply the remainder by the length of the ſpindle, and again by ·5236, for the ſolidity.

EXAMPLES.

1. Required the ſolidity of the elliptic ſpindle ACBDA, the length AB being 40, the greateſt diameter CD 12, and the diameter EF, at $\frac{1}{4}$ of the length, 9·49546?

1. *For the Central Diſtance, and Axes of the Ellipſe.*



4EF	37·98184
3CD	36·00000
dif.	1·98184
$3CD^2$	432·0000
$4EF^2$	360·0546

1·98184) 71·3454 (36 = 4OG
59·4582 9 = OG
6 = CG
118872
118916 15 = OC
30 = CH the conj.
24 = GH
6 = CG
144
its root 12 = mean between CG & GH.

Then as 12 : 20 (or AG) : : 30 (or CH) : 50 = IK the tranſverſe.

2. *For the Generating Elliptic Segment.*

CH 30) 6 CG
·2 tab. verſ.
·111823 tab. area correſp.
50 IK
5·591150
30 CH
167·734500 area generating ſeg. ACBA.

3. For the Solidity of the Spindle.

	167·7345
	3
AB 40)	503·2035
	12·5800875
CD	12
	0·5800875
4 OG	36
	34805250
	17402625
prod.	20·8831500
ſrom	144·
rem.	123·11685
AB	40
	4924·67400
	·5236
	29548044
	14774022
	9849348
	24623370
ſolidity	2578·5593064 Anſwer.

Ex. 2. Required the ſolidity of the elliptic ſpindle, whoſe length is 20, the greateſt diameter 6, and the diameter at $\frac{1}{4}$ of the length 4·74773? Anſ. 322·32.

RULE II.

To the ſquare of the greateſt diameter, add the ſquare of double the diameter at $\frac{1}{4}$ of the length; multiply the ſum by the length, and the product again by ·1309 for the ſolidity, very nearly. *Note.*

Note. This rule will alſo ſerve for any other ſolid formed by the revolution of any conic ſection.

EXAMPLE.

What is the ſolid content of the elliptic ſpindle, whoſe length is 20, the greateſt diameter 6, and the diameter at $\frac{1}{4}$ of the length 4·74773?

4·74773
2

9·49546 double the diam.
645949 ditto inverted.

8545914
379818
85459
4748
380
56

90·16375 ſq. of double diam.
36·00000 ſq. of other diam.

126·16375 ſum
20 length

2523·27500
9031 or 1309 inverted

2523
757
22

Anſ. 330·2 the ſolidity nearly.

PROBLEM XIX.

To find the Solidity of a Fruſtum or Segment of an Elliptic Spindle.

Proceed as in the laſt rule, for this, or any other ſolid formed

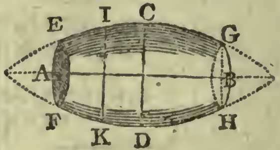
formed by the revolution of a conic ſection about an axis namely.

Add together the ſquares of the greateſt and leaſt diameters, and the ſquare of double the diameter in the middle between the two; multiply the ſum by the length, and the product again by ·1309 for the ſolidity.

Note. For all ſuch ſolids, this rule is exact when the body is formed from the conic ſection, or a part of it, revolved about the axis of the ſection. And will always be very near the truth when the figure revolves about another line.

EXAMPLES.

1. Required the content of the middle fruſtum EGHF of any ſpindle, the length AB being 40, the greateſt or middle diameter CD 32, the leaſt or diameter at either end EF or GH 24, and the diameter IK, in the middle between EF and CD, 30·157568?



32	30·157568
32	2
64	60·315136 double
96	51306 invert
1024	361890
	1809
24	60
24	30
96	3637·89 ſq. of 2 IK
48	1024·00 ſq. of CD
	576·00 ſq. of EF
576	
	5237·89 ſum
	40 length
	209515·60
	9031 inverted
	20951
	6285
	188
	27424 Anſwer.

Ex. 2.

Ex. 2. What is the content of the ſegment of any ſpindle, the length being 10, the greateſt diameter 8, and the middle diameter 6? Anſ. 272·272.

Ex. 3. Required the ſolidity of the fruſtum of an hyperbolic conoid, the height being 12, the greateſt diameter 10, the leaſt diameter 6, and the middle diameter $8\frac{1}{2}$? Anſ. 667·59.

Ex. 4. What is the content of the middle fruſtum of an hyperbolic ſpindle, the length being 20, the middle or greateſt diameter 16, the diameter at each end 12, and the diameter at $\frac{1}{4}$ of the length $14\frac{1}{2}$? Anſ. 3248·938.

PROBLEM XX.

To find the Solidity of a Parabolic Conoid.

Square the diameter of the baſe, multiply that by the altitude, and the product again by ·3927, for the content.

EXAMPLES.

1. Required the ſolidity of the paraboloid whoſe height BD is 30, and the diameter of its baſe AC is 40?

$$\begin{array}{r}
40\\
40\\
\hline
1600\\
30\\
\hline
48000\\
\cdot3927\\
\hline
31416\\
15708\\
\hline
1884\cdot96000
\end{array}\text{ Anſwer.}$$

Ex. 2.

Ex. 2. What is the content of the parabolic conoid whoſe altitude is 42, and the diameter of its baſe 24?

Anſ. 9500·1984,

PROBLEM XXI.

To find the Solidity of the Fruſtum of a Paraboloid.

Square the diameter of the two ends, add thoſe two ſquares together, multiply that ſum by the height, and the product again by ·3927, for the content.

EXAMPLES.

1. Required the content of the paraboloidal fruſtum ABCD, the diameter AB being 20, the diameter DC 40, and the height EF 22$\frac{1}{2}$?

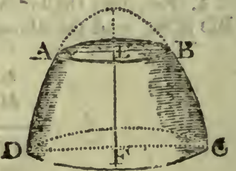
1600 DC^2
400 AB^2

2000 ſum
22$\frac{1}{2}$ EF

45000
·3927

19635000
15708

17671·5000



Ex. 2. What is the content of the fruſtum of a paraboloid, the greateſt diameter being 30, the leaſt 24, and the altitude 9?

Anſ. 5216·6266.

PROBLEM XXII.

To find the Solidity of a Parabolic Spindle.

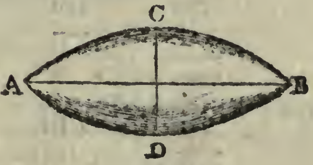
Take the ſquare of the middle or greateſt diameter, multiply it by the length, and the product again by ·41888, for the content.

EX-

EXAMPLES.

1. Required the content of the parabolic ſpindle ACBD, whoſe length AB is 40, and the greateſt diameter CD 16?

16 CD
16
—
96
16
—
256 CD^2
40 AB
—
10240
·41888
—
1675520
83776
41888
—
4289·33120 Anſwer.



Ex. 2. What is the ſolidity of a parabolic ſpindle whoſe length is 18, and its middle diameter 6 feet?
Anſ. 271·4336.

PROBLEM XXIII.

To find the Solidity of the Middle Fruſtum of a Parabolic Spindle.

Add altogether, 8 times the ſquare of the greateſt diameter, 3 times the ſquare of the leaſt diameter, and 4 times the product of the two diameters; multiply the ſum by the length, and the product again by ·05236 for the ſolidity.

EXAMPLES.

1. Required the content of the fruſtum of a parabolic ſpindle EGHF, the length AB being 20, the greateſt diameter CD 16, and the leaſt diameter EF 12?

16	12	16
16	12	12
96	144	192
16	3	4
256	432	768
8		

2048 8 CD^2
432 3 EF^2
768 4 CD × EF

3248 ſum
20 AB

64960 *See Fig. in p.* 260.
·05246

389760
19488
12992
32480

3401·30560 Anſwer.

Ex. 2. What is the content of the fruſtum of a parabolic ſpindle, whoſe length is 18, greateſt diameter 18, and leaſt diameter 10? Anſ. 3404·23776.

Note. The ſolidities of the hyperboloid and hyperbolic ſpindle, are to be found by rule 2 to prob. XVIII. And thoſe of their fruſtums by prob. XIX; where ſome examples of them are given.

OF

OF

GAUGING.

THE buſineſs of caſk gauging is commonly performed by two inſtruments, namely, the gauging or ſliding-rule, and the gauging or diagonal rod.

1. OF THE GAUGING RULE.

This inſtrument ſerves to compute the contents of caſks, &c. after the dimenſions have been taken. It is a ſquare rule, having various logarithmic lines on its four ſides or faces; and three ſliding pieces, running in grooves in three of them.

On the firſt face are three lines, namely, two marked A, B, for multiplying and dividing; and the third, MD, for malt depth, becauſe it ſerves to gauge malt. The middle one B is on the ſlider, and is a kind of double line, being marked at both the edges of the ſlider, for applying it to both the lines A and MD. Theſe three lines are all of the ſame radius, or diſtance from 1 to 10, each containing twice the length of the radius. A and B are placed and numbered exactly alike, each beginning at 1, which may be either 1, or 10, or 100, &c. or ·1, or ·01, or ·001, &c. but whatever it is, the middle diviſion, 10, will be ten times as much, and the laſt diviſion

100

100 times as much. But 1 on the line MD is oppoſite 215, or more exactly 2150·4 on the other lines, which number 2150·4 denotes the cubic inches in a malt buſhel; and its diviſions numbered retrograde to thoſe of A and B. On theſe two lines are alſo ſeveral other marks and letters: thus, on the line A are MB, for malt buſhel, at the number 2150·4; and A for ale, at 282, the cubic inches in an ale gallon; and on the line B, is W, for wine, at 231, the cubic inches in a wine gallon; alſo *s i*, for ſquare inſcribed, at ·707, the ſide of a ſquare inſcribed in a circle whoſe diameter is 1; *s e*, for ſquare equal at ·886, the ſide of a ſquare which is equal to the ſame circle; and *c*, for circumference, at 3·1416, the circumference of the ſame circle.

On the ſecond face, or that oppoſite the firſt, are a ſlider and four lines, marked D, C, D, E, at one end, and root, ſquare, root, cube, at the other; the lines C and D containing reſpectively the ſquares and cubes of the oppoſite numbers on the lines D, D; the radius of D being double to that of A, B, C, and triple to that of E; ſo that whatever the firſt 1 on D denotes, the firſt on C is the ſquare of it, and the firſt on E the cube of it; ſo if D begin with 1, C and E will begin with 1; but if D begin with 10, C will begin with 100, and E with 1000; and ſo on. On the line C are marked *o c* at ·0796, for the area of the circle whoſe circumference is 1; and *o d* at ·7854, for the area of the circle whoſe diameter is 1. Alſo on the line D, are WG, for wine gauge, at 17·15; and AG for ale gauge, at 18·95; and MR, for malt round, at 52·32; theſe three being the gauge points for round and circular meaſure, and are found by dividing the ſquare roots of 231, 282, and 2150·4 by the ſquare root of ·7854: alſo MS, for malt ſquare, are marked at 46·37, the malt gauge point for ſquare meaſure, being the ſquare root of 2150·4.

On the third face are three lines, one on a ſlider marked N; and two on the ſtock, marked S S and SL, for ſegment ſtanding and ſegment lying, which ſerve for ullaging ſtanding and lying caſks.

And

And on the fourth, or oppoſite face, are a ſcale of inches, and three other ſcales, marked ſpheroid, or 1ſt variety, 2d variety, 3d variety; the ſcale for the 4th, or conic variety, being on the inſide of the ſlider in the third face. The uſe of theſe lines is to find the mean diameters of caſks.

Beſides all thoſe lines, there are two others on the inſides of the two firſt ſliders, being continued from the one ſlider to the other. The one of theſe is a ſcale of inches, from $12\frac{1}{2}$ to 36; and the other is a ſcale of ale gallons, between the correſponding numbers 435 and 3·61; which form a table to ſhow, in ale gallons, the contents of all cylinders whoſe diameters are from $12\frac{1}{2}$ to 36 inches, their common altitude being 1 inch.

The Uſe of the Gauging Rule.

PROBLEM I.

To Multiply two Numbers, as 12 *and* 25.

Set 1 on B, to either of the given numbers, as 12, on A; then againſt 25 on B, ſtands 300 on A; which is the product.

PROBLEM II.

To Divide one Number by another, as 300 *by* 25.

Set 1 on B, to 25 on A; then againſt 300 on A, ſtands 12 on B, for the quotient.

PROBLEM III.

To find a Fourth Proportional, as to 8, 24, *and* 96.

Set 8 on B, to 24 on A; then againſt 96 on B, is 288 on A, the 4th proportional to 8, 24, 96, required.

PROBLEM IV.

To Extract the Square Root, as of 225.

The firſt 1 on C ſtanding oppoſite the one on D, on the ſtock; then againſt 225 on C, ſtands its ſquare root 15 on D.

PROBLEM V.

To Extract the Cube Root, as of 3375.

The line D on the ſlide being ſet ſtraight with E; then oppoſite 3375 on E, ſtands its cube root 15 on D.

PROBLEM VI.

To find a Mean Proportional, as between 4 *and* 9.

Set 4 on C, to the ſame 4 on D; then againſt 9 on C, ſtands the mean proportional 6 on D.

PROBLEM VII.

To find Numbers in Duplicate Proportion.

As to find a Number which ſhall be to 120, *as the Square of* 3 *to the Square of* 2.

Set 2 on D, to 120 on C; then againſt 3 on D, ſtands 270 on C, for the anſwer.

PROBLEM VIII.

To find Numbers in Subduplicate Proportion.

As to find a Number which ſhall be to 2 *as the Root of* 270 *to the Root of* 120.

Set 2 on D, to 120 on C; then againſt 270 on C, ſtands 3 on D, for the anſwer.

PRO-

PROBLEM IX.

To find Numbers in Triplicate Proportion.

As, to find a Number which ſhall be to 100, *as the Cube of* 36 *is to the Cube of* 40.

Set 40 on D, to 100 on E; then againſt 36 on D, ſtands 72·9 on E, for the anſwer.

PROBLEM X.

To find Numbers in Subtriplicate Proportion.

As, to find a Number which ſhall be to 40, *as the Cube Root of* 72·9 *is to the Cube Root of* 100.

Set 40 on D, to 100 on E; then againſt 72·9 on E, ſtands 36 on D, for the anſwer.

PROBLEM XI.

To Compute Malt Buſhels by the Line MD.

As, to find the Malt Buſhels in the Couch, Floor, or Ciſtern, whoſe Length is 230, *Breadth* 58·2, *and Depth* 5·4 *Inches.*

Set 230 on B, to 5·4 on MD; then againſt 58·2 on A ſtands 33·6 buſhels on B, for the anſwer.

Note. The uſes of the other marks on the rule, will appear in the examples farther on.

OF THE GAUGING OR DIAGONAL ROD.

The diagonal rod is a ſquare rule, having four faces; being commonly 4 feet long, and folding together by joints. This inſtrument is uſed both for gauging or meaſuring caſks, and computing their contents, and that from one dimenſion only, namely the diagonal of the caſk, or the length from the middle of the bung hole to the meeting of the head of the caſk with the ſtave oppoſite to the bung; being the longeſt line that can be drawn within the caſk from the middle of the bung. And, accordingly, on one face of the rule is a ſcale of inches for meaſuring this diagonal; to which are placed the areas, in ale gallons, of circles to the correſponding diameters, in like manner as the lines on the under ſides of the three ſlides in the ſliding rule.

On the oppoſite face, are two ſcales of ale and wine gallons, expreſſing the contents of caſks having the correſponding diagonals. And theſe are the lines which chiefly form the difference between this inſtrument and the ſliding rule; for all their other lines are the ſame, and are to be uſed in the ſame manner.

EXAMPLE.

The rod being applied within the caſk at the bung-hole, the diagonal was found to be 34·4 inches; required the content in gallons?

Now to 34·4 inches correſpond, on the rod, 90¾ ale gallons, or 111 wine gallons, the content required.

Note. The contents exhibited by the rod, anſwer to the moſt common form of caſks, and fall in between the 2d and 3d varieties following.

OF CASKS AS DIVIDED INTO VARIETIES.

It is uſual to divide caſks into four caſes or varieties, which are judged of from the greater or leſs apparent curvature of their ſides; namely,

1. The middle fruſtum of a ſpheroid,
2. The middle fruſtum of a parabolic ſpindle,
3. The two equal fruſtums of a paraboloid,
4. The two equal fruſtums of a cone. And

And if the content of any of theſe be computed in inches, by their proper rules, and this be divided by 282, or 231, or 2150·4, the quotient will be the content in ale gallons, or wine gallons, or malt buſhels, reſpectively. Becauſe

282 cubic inches make 1 ale gallon
231 .... 1 wine gallon
2150·4 .... 1 malt buſhel.

And the particular rule will be for each as in the following problems:

PROBLEM XII.

To find the Content of a Caſk of the Firſt Form.

To the ſquare of the head diameter, add double the ſquare of the bung diameter; and multiply the ſum by the length of the caſk. Then let the product

be multiplied by $\cdot 0009\frac{1}{4}$, or divided by 1077, for ale gallons;
and multiplied by $\cdot 0011\frac{1}{3}$, or divided by 882, for wine gallons.

EXAMPLES.

1. Required the content of a ſpheroidal caſk, whoſe length is 40, and bung and head diameters 32 and 24 inches.

24	32		
24	32		
96	64		
48	96		
576	1024		
	2		
	2048	104960	104960
	576	·0009$\frac{1}{4}$	·0011$\frac{1}{3}$
	2624	944640	1154560
	40	26240	34987
	104960	ale 97·0880 gallons	118·9547 wine

By the Gauging Rule.

Having ſet 40 on C, to the ale gauge 32·82 on D,
againſt
24 on D, ſtands 21·3 on C
32 on D, ſtands 38·0 on C
the ſame 38·0

ſum 97·3 ale gallons.

And having ſet 40 on C, to the wine gauge 29·7 on D,
againſt
24 on D, ſtands 26·1 on C
32 on D, ſtands 46·5 on C
the ſame 46·5

ſum 119·1 wine gallons.

Ex. 2. Required the content of the ſpheroidal caſk, whoſe length is 20, and diameters 12 and 16 inches.

Anſwer { 12·136 ale gallons,
14·869 wine gallons.

PRO-

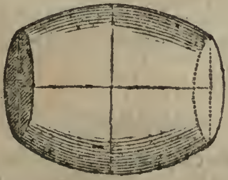
PROBLEM XIII.

To find the Content of a Caſk of the Second Form.

To the ſquare of the head diameter, add double the ſquare of the bung diameter, and from the ſum take $\frac{2}{5}$ or $\frac{4}{10}$ of the ſquare of the difference of the diameters; then multiply the remainder by the length, and the product again by ·0009$\frac{1}{4}$ for ale gallons, or by ·0011$\frac{1}{3}$ for wine gallons.

EXAMPLES.

1. The length being 40, and diameters 24 and 32, required the content.



32			
24			
8	2624·0	103936	103936
8	25·6	·0009$\frac{1}{4}$	·0011$\frac{1}{3}$
64	2598·4	935424	1143296
4	40	25984	34645
25·6	103936	ale 96·1408 gall.	117·7941 wine

By the Gauging Rule.

Having ſet 40 on C, to 32·82 on D, againſt 8 on D, ſtands 2·4 on C; the $\frac{4}{10}$ of which is 0·96. This taken from the 97·3 in the laſt form, leaves 96·3 ale gallons.

And having ſet 40 on C, to 29·7 on D, againſt 8 on D, ſtands 2·9 on C: the $\frac{4}{10}$ of which is 1·16. This taken from the 119·1 in the laſt form, leaves 117·9 wine gallons.

Ex. 2. Required the content when the length is 20, and the diameters 12 and 16.

Anſw { 12·018 ale gallons,
14·724 wine gallons.

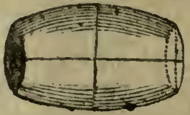
PROBLEM XIV.

To find the Content of a Caſk of the Third Form.

To the ſquare of the bung diameter, add the ſquare of the head diameter; multiply the ſum by the length, and the product again by ·0014 for ale gallons, or by ·0017 for wine gallons.

EXAMPLES.

1. Required the content of a caſk of the third form, when the length is 40, and the diameters 24 and 32.



1024		64000		64000	
576		·0024		·0017	
1600		256		449	
40		64		64	
64000	ale	89·6	gallons	108·8	wine

By

By the Gauging Rule.

Set 40 on C, to 26·8 on D, then againſt
24 on D, ſtands 32·0 on C
32 on D, ſtands 57·3 on C

ſum 89·3 ale gallons.

And having ſet 40 on C. to 24·25 on D; then againſt
24 on D ſtands 39·1 on C
32 on D, ſtands 69·8 on C

ſum 108·9 wine gallons.

Ex. 2. Required the content when the length is 20, and the diameters 12 and 16.

Anſwer $\begin{cases} 11\cdot 2 \text{ ale gallons.} \\ 13\cdot 6 \text{ wine gallons.} \end{cases}$

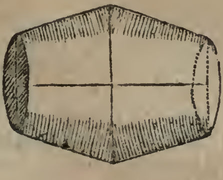
PROBLEM XV.

To find the Content of a Caſk of the Fourth Form.

Add the ſquare of the difference of the diameters, to 3 times the ſquare of their ſum; then multiply the ſum by the length, and the product again by $\cdot 00023\frac{1}{5}$ for ale gallons, or by $\cdot 00028\frac{1}{3}$ for wine gallons.

EXAMPLES.

1. Required the content, when the length is 40, and the diameters 24 and 32 inches.



56	8		
56	8		
		378880	378880
336	64	·00023$\frac{1}{5}$	·00028$\frac{1}{3}$
280	9408		
		1136640	3031040
3136	9472	757760	757760
3	40	75776	126293
9408	378880 ale	87·90016 gall.	107·34933 wine.

By the Sliding Rule.

Set 40 on C, to 65·64 on D; then againſt
8 on D, ſtands 0·6 on C
56 on D, ſtands 29·1 on C
29·1
29·1
ſum 87·9 ale gallons.

And ſet 40 on C, to 59·41 on D; then againſt
8 on D, ſtands 0·7
56 on D, ſtands 35·6
35·6
35·6
ſum 107·5 wine gal.

Ex. 2. What is the content of a conical caſk, the length being 20, and the bung and head diameters 16 and 12 inches?

Anſwer { 10·985 ale gallons,
13·416 wine gallons.

PRO-

PROBLEM XVI.

To find the Content of a Caſk by Four Dimenſions.

Add together, the ſquares of the bung and head diameters, and the ſquare of double the diameter taken in the middle between the bung and head; then multiply the ſum by the length of the caſk, and the product again by ·0004$\frac{2}{3}$ for ale gallons, or by ·0005$\frac{2}{3}$ for wine gallons.

EXAMPLES.

1. Required the content of any caſk whoſe length is 40, the bung diameter being 32, the head diameter 24, and the middle diameter between the bung and head 28$\frac{3}{4}$ inches.

57·5	24	32
57·5	24	32
2875	96	64
4025	48	96
2875	576	1024
3306·25		
1024		
576		
4906·25		
40		
196250		196250
·0004$\frac{2}{3}$		·0005$\frac{2}{3}$
785000		981250
130833		130833
ale 91·5833	gallons	111·2083 wine.

By the Sliding Rule.

Set 40 on C, to 46·4 on D; then againſt
24 on D, ſtands 10·5
32 on D, ſtands 19·0
$57\frac{1}{2}$ on D, ſtands 62·0

ſum 91·5 ale gallons.

Set 40 on C, to 42·0 on D; then againſt
24 on D, ſtands 13·0
32 on D, ſtands 23·2
$57\frac{1}{2}$ on D, ſtands 75·0

ſum 111·2 wine gallons.

Ex. 2. What is the content of a caſk, whoſe length is 20, the bung diameter being 16, the head diameter 12, and the diameter in the middle between them $14\frac{3}{8}$?

Anſwer $\begin{cases} 11{\cdot}4479 \text{ ale gallons,} \\ 13{\cdot}9010 \text{ wine gallons.} \end{cases}$

PROBLEM XVII.

To find the Content of any Caſk from Three Dimenſions only.

Add into one ſum, 39 times the ſquare of the bung diameter, 25 times the ſquare of the head diameter, and 26 times the product of the two diameters: then multiply the ſum by the length, and the product again by $\frac{\cdot 00034}{9}$ for wine gallons, or by $\frac{\cdot 00034}{11}$ or $\cdot 00003\frac{1}{11}$ for ale gallons.

EXAMPLES.

1. Required the content of a caſk, whoſe length is 40, and the bung and head diameters 32 and 24?

32	24	32
32	24	24
64	96	128
96	48	64
1024	576	768
39	25	26
9216	2880	4608
3072	1152	1536
39936	14400	19968
	39936	
	19968	
	74304	
	40	
	2972160	
	·00034	2972160
		·00003$\frac{1}{11}$
	11888640	
	8916480	8916480
		270196
	9) 1010·53440	
	112·2816 wine gal.	91·86676 ale gallons.

Ex. 2. What is the content of a caſk, whoſe length is 20, and the bung and head diameters 16 and 12?

Anſwer { 11·4833 ale gallons,
14·0352 wine gallons.

Note. This is the moſt exact rule of any, for three dimenſions only; and agrees nearly with the diagonal rod.

OF THE ULLAGE OF CASKS.

The ullage of a caſk, is what it contains when only partly filled. And it is conſidered in two poſitions, namely, as ſtanding on its end with the axis perpendicular to the horizon, or as lying on its ſide with the axis parallel to the horizon.

PROBLEM XVIII.

To find the Ullage by the Sliding Rule.

By one of the preceding problems find the whole content of the caſk. Then ſet the length on N, to 100 on SS, for a ſegment ſtanding, or ſet the bung diameter on N, to 100 on SL, for a ſegment lying; then againſt the wet inches on N, is a number on SS or SL, to be reſerved.

Next, Set 100 on B, to the reſerved number on A; then againſt the whole content on B, will be found the ullage on A.

EXAMPLES.

1. Required the ullage anſwering to 10 wet inches of a ſtanding caſk, the whole content of which is 92 gallons, and length 40 inches.

Having ſet 40 on N, to 100 on SS; then
againſt 10 on N, is 23 on SS, the reſerved numb.

Then ſet 100 on B to 23 on A; and
againſt 92 on B, is 21·2 on A, the ullage required.

Ex. 2. What is the ullage of a ſtanding caſk whoſe whole length is 20 inches, and content 11$\frac{1}{2}$ gallons; the wet inches being 5? Anſ. 2·65 gallons.

Ex. 3.

Ex. 3. The content of a caſk being 92 gallons, and the bung diameter 32, required the ullage of the ſegment lying when the wet inches are 8? Anſ. 16·4 gallons.

PROBLEM XIX.

To Ullage a Standing caſk by the Pen.

Add all together, the ſquare of the diameter at the ſurface of the liquor, the ſquare of the diameter of the neareſt end, and the ſquare of double the diameter taken in the middle between the other two; then multiply the ſum by the length between the ſurface and neareſt end, and the product again by ·0004$\frac{2}{3}$ for ale gallons, or by ·0005$\frac{2}{3}$ for wine gallons, in the leſs part of the caſk, whether empty or filled.

EXAMPLE.

The three diameters being 24, 27, and 29 inches, required the ullage for 10 wet inches.

24	29	54		
24	29	54		2916
96	261	216		841
48	58	270		576
				4333
576	841	2916		10
		43330		43330
		·0004$\frac{2}{3}$		·0005$\frac{2}{3}$
		173320		216650
		28885		28885
	Ale	20·2205	gallons	24·5535 wine

PRO-

PROBLEM XX.

To Ullage a Lying Caſk by the Pen.

Divide the wet inches by the bung diameter; find the quotient in the column of verſed ſines, in the table of circular ſegments at the end of the book, taking out its correſponding ſegment. Then multiply this ſegment by the whole content of the caſk, and the product again by $1\frac{1}{4}$ for the ullage required, nearly.

EXAMPLE.

Suppoſing the bung diameter 32, and content 92 ale gallons; to find the ullage for 8 wet inches.

32) 8 (·25, whoſe tab. ſeg. is ·153546
92

307092
1381914

14·126232
$\frac{1}{4}$ is 3·531558

17·657790 Anſwer.

OF

SPECIFIC GRAVITY.

THE ſpecific gravities of bodies, are their relative weights, contained under the ſame given magnitude, as a cubic foot, or a cubic inch, &c.

The ſpecific gravities of ſeveral ſorts of matter are expreſſed by the numbers annexed to their names in the following table:

A Table of the Specific Gravities of Bodies.

Fine gold	19640	Brick	2000
Standard gold	18888	Light earth	1984
Quick-ſilver	13600	Solid gun-powder	1745
Lead	11325	Sand	1520
Fine ſilver	11091	Pitch	1150
Standard ſilver	10535	Box-wood	1030
Copper	9000	Sea-water	1030
Gun metal	8784	Common water	1000
Caſt braſs	8000	Oak	925
Steel	7850	Gun-powder, ſhaken	922
Iron	7645	Aſh	755
Caſt iron	7425	Maple	800
Tin	7320	Elm	600
Marble	2700	Fir	550
Common ſtone	2520	Cork	240
Loom	2160	Air	1$\frac{1}{4}$

Note. The ſeveral ſorts of wood are ſuppoſed to be dry. Alſo as a cubic foot of water weighs juſt 1000 ounces avoirdupois, the numbers in this table expreſs, not only the ſpecific gravities of the ſeveral bodies, but alſo the weight of a cubic foot of each, in avoirdupois ounces; and hence, by proportion, the weight of any other quantity, or the quantity of any other weight, may be known, as in the following problems.

PRO-

PROBLEM I.

To find the Magnitude of any Body from its Weight.

As the tabular ſpecific gravity of the body,
Is to its weight in avoirdupois ounces,
So is one cubic foot, or 1728 cubic inches,
To its content in feet, or inches, reſpectively.

EXAMPLES.

1. Required the content of an irregular block of common ſtone which weighs 1 cwt. or 112lb.

112lb.
16

672
112

2520 : 1792 : : 1728 : $1228\frac{2016}{2520}$
1728

14336
3584
12544
1792

cubic inch.
2520) 3096576 ($1228\frac{2016}{2520}$ Anſwer.
252

576
504

725
504

2217
2016

2016

Ex. 2.

Ex. 2. How many cubic inches of gun-powder are there in 1lb. weight? Anſ. 30 cubic inches nearly.

Ex. 3. How many cubic feet are there in a ton weight of dry oak? Anſ. $38\frac{138}{185}$ cubic feet.

PROBLEM II.

To find the Weight of a Body from its Magnitude.

As one cubic foot, or 1728 cubic inches,
Is to the content of the body,
So is its tabular ſpecific gravity,
To the weight of the body.

EXAMPLES.

1. Required the weight of a block of marble, whoſe length is 63 feet, and breadth and thickneſs each 12 feet; being the dimenſions of one of the ſtones in the walls of Balbeck.

$$
\begin{array}{r r l}
 & 63 & \\
 & 12 & \\
\hline
 & 756 & \\
 & 12 & \qquad \text{oz.} \qquad \text{tons} \\
\hline
 & 1 : 9072 & : : 2700 : 683\frac{4}{10} \\
 & 2700 & \\
\hline
 & 6350400 & \\
 & 18144 & \\
\hline
16\left\{\begin{array}{l}4\\4\end{array}\right. & 24494400 & \text{oz.} \\
 & 6123600 & \\
112 & 1530900 & \text{lb.} \\
20 & 13668 & \text{cwt.}
\end{array}
$$

Anſ. $683\frac{4}{10}$ ton, almoſt equal to the burthen of an East India ſhip.

Ex. 2.

Ex. 2. What is the weight of 1 pint, ale meaſure, of gun-powder? Anſ. 19 oz. nearly.

Ex. 3. What is the weight of a block of dry oak, which meaſures 10 feet long, 3 feet broad, and $2\frac{1}{2}$ feet deep? Anſ. $4335\frac{15}{16}$ lb.

PROBLEM III.

To find the Specific Gravity of a Body.

Case 1. When the body is heavier than water, weigh it both in water and out of water, and take the difference, which will be the weight loſt in water. Then ſay,

As the weight loſt in water,
Is to the whole weight,
So is the ſpecific gravity of water,
To the ſpecific gravity of the body.

EXAMPLE.

A piece of ſtone weighed 10lb. but in water only $6\frac{3}{4}$ lb. required its ſpecific gravity?

10
$6\frac{3}{4}$

$3\frac{1}{4}$: 10 : : 1000 :
or 13 : 40 : : 1000 : 3077 anſwer.
40

13) 40000 (3077
39

100
91

90

Case 2. When the body is lighter than water, ſo that it will not quite ſink; affix to it a piece of another body heavier

heavier than water, ſo that the maſs compounded of the two may ſink together. Weigh the heavier body, and the compound maſs, ſeparately, both in water and out of it; then find how much each loſes in water by ſubtracting its weight in water from its weight in air; and ſubtract the leſs of theſe remainders from the greater. Then ſay,

As this laſt remainder,
Is to the weight of the light body in air,
So is the ſpecific gravity of water,
To the ſpecific gravity of the body.

EXAMPLE.

Suppoſe a piece of elm weighs 15lb in air, and that a piece of copper, which weighs 18lb in air, and 16lb in water, is affixed to it, and that the compound weighs 8lb in water; required the ſpecific gravity of the elm?

Copper		Compound
18	in air	33
16	in water	6
2	loſs	27
		2

Then As 25 : 15 : : 1000 : 600 anſ.

PROBLEM IV.

To find the Quantities of Two Ingredients in a Given Compound.

Take the three differences of every pair of the three ſpecific gravities, namely, the ſpecific gravities of the compound and each ingredient; and multiply the difference of every two ſpecific gravities by the third Then, as the

the greateſt product is to the whole weight of the compound, ſo is each of the other products, to the two weights of the ingredients.

EXAMPLE.

A compoſition of 112lb being made of tin and copper, whoſe ſpecific gravity is found to be 8784; required the quantity of each ingredient, the ſpecific gravity of tin being 7320, and of copper 9000?

```
   9000          9000          8784
   7320          8784          7320
   ————          ————          ————
   1680           216          1464 diff.
   8784          7320          9000
                 ————      ————————
 ——————          4320      13176000
                648        ————————
 702720        1512
52704          ————
8784           1581120
———————        ———————           112
14757120 : 112 : : 13176000 : 100 copper
                       112        ——
                  ————————       12 tin
                  26352000
                 13176
                13176
                ————————
      14757120 ) 1475712000 ( 100
```

Anſwer, there is 100lb of copper, and conſequently 12lb of tin } in the compoſition.

OF

OF THE WEIGHT AND DIMENSIONS OF BALLS AND SHELLS.

THE weight and dimenſions of balls and ſhells might be found from the problems laſt given, concerning ſpecific gravity. But they may be found ſtill eaſier by means of the experimented weight of a ball of a given ſize, from the known proportion of ſimilar figures, namely, as the cubes of their diameters.

PROBLEM I.

To find the Weight of an Iron Ball, from its Diameter.

An iron ball of 4 inches diameter weighs 9lb, and the weights being as the cubes of the diameters, it will be, as 64 (which is the cube of 4) is to 9, ſo is the cube of the diameter of any other ball, to its weight. Or take $\frac{9}{64}$ of the cube of the diameter, for the weight. Or take $\frac{1}{8}$ of the cube of the diameter, and $\frac{1}{8}$ of that again, and add the two together, for the weight.

EXAMPLES.

1. The diameter of an iron ſhot being 6·7, required its weight?

6·7
6·7

469
402

44·88
6·7

31423
26934

8) 300·763

8) 37·595
4·699

Anſ. 42·294 lbs.

Ex. 2. What is the weight of an iron ball whoſe diameter is 5·54 inches? Anſ. 24lb.

PROBLEM II.

To find the Weight of a Leaden Ball.

A leaden ball of $4\frac{1}{4}$ inches diameter weighs 17lb; therefore as the cube of $4\frac{1}{4}$ is to 17, or nearly as 9 to 2, ſo is the cube of the diameter of a leaden ball, to its weight.

Or take $\frac{2}{9}$ of the cube of the diameter, for the weight, nearly.

EXAMPLES.

1. Required the weight of a leaden ball of 6·6 inches diameter?

$$\begin{array}{r}
6{\cdot}6 \\
6{\cdot}6 \\
\hline
396 \\
396 \\
\hline
43{\cdot}56 \\
6\ 6 \\
\hline
26136 \\
26136 \\
\hline
287{\cdot}496 \\
{\cdot}2 \\
\hline
9\ )\ 574{\cdot}992 \\
\text{Anſ. } 63{\cdot}888 \text{ lb.} \\
\hline
\end{array}$$

Ex. 2. What is the weight of a leaden ball of 5·24 inches diameter? Anſ. 32lb nearly.

PROBLEM III.

To find the Diameter of an Iron Ball from its Weight.

Multiply the weight by $7\frac{1}{9}$ then take the cube root of the product for the diameter.

EXAMPLES.

1. Required the diameter of a 42lb iron ball?

$$\begin{array}{r} 42 \\ 7\frac{1}{9} \\ \hline 294 \\ 4{\cdot}666 \\ \hline 298{\cdot}666 \text{ or } \frac{2}{3} \\ \hline \end{array}$$

The cube root of this is almoſt 7. Suppoſe 7, whoſe cube is 343. Then, by the 2d rule for the cube root at page 41, proceed thus:

$$\begin{array}{lrcrl}
 & 343 & & 298\frac{2}{3} & \\
 & 2 & & 2 & \\
\cline{2-2}\cline{4-4}
 & 668 & & 597\frac{1}{3} & \\
 & 298\frac{2}{5} & & 343 & \\
\cline{2-2}\cline{4-4}
\text{As} & 984\frac{2}{3} & : & 940\frac{1}{3} & ::7 \\
 & 3 & & 3 & \\
\cline{2-2}\cline{4-4}
\text{Or as} & 2954 & : & 2821 & ::7:6{\cdot}685 \text{ Anſ.} \\
 & & & 7 & \\
\cline{4-4}
 & 2954 &) & 19747 & (\,6{\cdot}685 \text{ inches} \\
 & & & 17724 & \\
\cline{4-4}
 & & & 2023 & \\
 & & & 1772 & \\
\cline{4-4}
 & & & 251 & \\
 & & & 236 & \\
\cline{4-4}
 & & & 15 & \\
 & & & 15 & \\
\cline{4-4}
\end{array}$$

Ex. 2. What is the diameter of a 24lb iron ball?

Anſ. 5·54 inches.

PRO-

PROBLEM IV.

To find the Diameter of a Leaden Ball from its Weight.

Multiply the Weight by 9, and divide the product by 2; then take the cube root of the quotient for the diameter.

EXAMPLES.

1. Required the diameter of a 64lb leaden ball?

```
      64
       9
    ----
 2 ) 576
     288
    ----
```

the cube root of which is almoſt 7, whoſe cube is 343.

```
Then 343          288
       2            2
     ---          ---
     686          576
     288          343
     ---          ---
  As 974    :     919 : : 7 : 6·605 Anſ.
                    7
                  ---
          974 ) 6433 ( 6·605 inches
                5844
                ----
                 589
                 584
                 ---
                   5
                 ---
```

Ex. 2. What is the diameter of an 8lb leaden ball?

Anſ. 3·303 inches.

PROBLEM V.

To find the Weight of an Iron Shell.

Take $\frac{9}{64}$ of the difference of the cubes of the external and internal diameter, for the weight of the ſhell.

That is, from the cube of the external diameter take the cube of the internal diameter, multiply the remainder by 9, and divide the product by 64.

EXAMPLES.

1. The outſide diameter of an iron ſhell being 12·8, and the inſide diameter 9·1 inches; required its weight?

9·1	12·8
9·1	12·8
91	1024
819	1536
82 81	163·84
9·1	12·8
8281	131072
74529	196608
753·571	2097·152
	753·571
	1343·581
	9
64 { 8)	12092·229
8)	1511·528
Anſ.	188·941 lb.

Ex. 2. What is the weight of an iron ſhell, whoſe external and internal diameters are 9·8 and 7 inches?

Anſ. 84¼ lb.

PRO-

PROBLEM VI.

To find how much Powder will fill a Shell.

Take the cube of the internal diameter, in inches, and divide it by 57·3, for the lbs. of powder.

EXAMPLES.

1. How much powder will fill the ſhell whoſe internal diameter is 9·1 inches?

9·1
9·1

91
819

82·81
9·1

8281
74529

lb
57·3) 753·571 ($13\frac{2}{13}$ nearly
573

1805
1719

86

Ex. 2. How much powder will fill the ſhell whoſe internal diameter is 7 inches?
Anſ. 6lb.

PROBLEM VII.

To find how much Powder will fill a Rectangular Box.

Find the content of the box in inches, by multiplying the length, breadth, and depth all together. Then divide by 30, for the pounds of powder.

EXAMPLES.

1. Required the quantity of powder that will fill a box; the length being 15 inches, and the breadth 12, and the depth 10 inches?

$$\begin{array}{r} 15 \\ 12 \\ \hline 180 \\ 10 \\ \hline 30\,)\,1800 \\ \hline \text{Anſ. } 60 \text{ lb.} \\ \hline \end{array}$$

Ex. 2. How much powder will fill a cubical box, whoſe ſide is 12 inches? Anſ. 57$\frac{3}{5}$lb.

PROBLEM VIII.

To find how much Powder will fill a Cylinder.

Multiply the ſquare of the diameter by the length; then divide by 38·2, for the pounds of powder.

EXAMPLES.

1. How much powder will the cylinder hold, whoſe diameter is 10 inches, and length 20 inches? 19

10
10

100
20

lb.
38·2) 2000 ($52\frac{1}{3}$ nearly
1910

900
764

136

Ex. 2. How much powder can be contained in the cylinder, whoſe diameter is 4 inches, and length 12 inches?

Anſ. $5\frac{5}{191}$ lb.

PROBLEM IX.

To find the Size of a Shell to contain a Given Weight of Powder.

Multiply the pounds of powder by 57·3; then take the cube root of the product, for the diameter in inches.

EXAMPLES.

1. What is the diameter of a ſhell that will hold $13\frac{1}{6}$lb. of powder?

57·3
$13\frac{1}{6}$

1719
573
955

754·45

The cube root of this is nearly 9, whoſe cube is 729.

```
   Then
   729        754·45
     2             2
   ————       ——————
  1458        1508·90
   754·45     729·
   ——————     ——————
As 2212·45 : 2237·90 : : 9 : 9·1 Anſwer.
                   9
              ——————
  221,245 ) 2014,10 ( 9·1 inches
            1991
            ————
              23
              22
              ——
               1
              ——
```

Ex. 2. What is the diameter of a ſhell, to contain 6lb. of powder? Anſ. 7 inches.

PROBLEM X.

To find the Size of a Cubical Box, to contain a Given Weight of Powder.

Multiply the weight in pounds by 30, then the cube root of the product will be the ſide of the box in inches.

EXAMPLES.

1. Required the ſize of a cubical box, to hold 50lb of gun-powder?

50
30

The cube root of 1500 is 11 nearly, whoſe cube is 1331.

Then 1331 1500
2 2

2662 3000
1500 1331

As 4162 : 4331 : : 11 : 11·44 Anſ.
11

4162) 47641 (11·44 inches.
45782

1859
1665

194

Ex. 2. Required the ſize of a cubical box, to hold 400lb. of gun-powder? Anſ. 22·89 inches.

PROBLEM XI.

To find what Length of a Cylinder will be filled by a Given Weight of Gun-powder.

Multiply the weight in pounds by 38·2; then divide the product by the ſquare of the diameter in inches, for the length.

EXAMPLES.

1. What length of a 36 pounder gun, of $6\frac{2}{3}$ inches diameter, will be filled with 12lb. of powder?

$6\frac{2}{3} = \frac{20}{3}$

its ſq. $= \frac{400}{9}$

$$\begin{array}{r} 38{\cdot}2 \\ 12 \\ \hline 458{\cdot}4 \\ 9 \\ \hline 400\ )\ 4125{\cdot}6 \\ \hline \end{array}$$

Anſ. 10·314 inches.

Ex. 2. What length of a cylinder of 8 inches diameter may be filled with 20lb of powder? Anſ. $11\frac{15}{16}$.

OF THE

PILING

OF

BALLS AND SHELLS.

IRON balls and ſhells are commonly piled, by horizontal courſes, either in a pyramidical or in a wedge-like form; the baſe being either an equilateral triangle, or a ſquare, or a rectangle. In the triangle and ſquare, the pile finiſhes in a ſingle ball; but in the rectangle it finiſhes in a ſingle row of balls, like an edge.

In triangular and ſquare piles, the number of horizontal rows, or courſes, is always equal to the number of balls in one ſide of the bottom row. And in rectangular piles, the

the number of rows is equal to the number of balls in the breadth of the bottom row. Alſo the number in the top row, or edge, is one or more than the difference between the length and breadth of the bottom row.

PROBLEM I.

To find the Number of Balls in a Triangular Pile.

Multiply continually together, the number in one ſide of the bottom row, that number increaſed by 1, and the ſame number increaſed by 2; then take $\frac{1}{6}$ of the laſt product for the anſwer.

EXAMPLES.

1. Required the number of balls in a triangular pile, each ſide of the baſe containing 30 balls?

$$\begin{array}{rr}
 & 32 \\
 & 31 \\
\hline
 & 32 \\
 & 96 \\
\hline
 & 992 \\
 & 30 \\
\hline
6\ ) & 29760 \\
\text{Anſ.} & 4960 \\
\hline
\end{array}$$

Ex. 2. How many balls are in the triangular pile, each ſide of the baſe containing 20? Anſ. 1540.

PROBLEM II.

To find the Number of Balls in a Square Pile.

Multiply continually together, the number in one ſide of the bottom courſe, that number increaſed by 1, and double the ſame number increaſed by 1; then take $\frac{1}{6}$ of the laſt product for the anſwer.

EXAMPLES.

1. How many balls are in a ſquare pile of 30 rows?

$$\begin{array}{r}
61 \\
31 \\
\hline
61 \\
183 \\
\hline
1891 \\
30 \\
\hline
6\ )\ 56730 \\
\text{Anſ. } 9455 \\
\hline
\end{array}$$

Ex. 2. How many balls are there in a ſquare pile of 20 rows? Anſ. 2870.

PROBLEM III.

To find the Number of Balls in a Rectangular Pile.

From 3 times the number in the length of the baſe row, ſubtract one leſs than the breadth of the ſame, multiply the remainder by the ſaid breadth, and the product by 1 more than the ſame; and divide by 6 for the anſwer.

EXAMPLES.

1. Required the number of balls in a rectangular pile, the length and breadth of the baſe row being 46 and 15;

11 46

46
3

138
14

124
15

620
124

1860
16

11160
1860

6) 29760
Anſ. 4960

Ex. 2. How many ſhot are in a rectangular complete pile, the length of the bottom courſe being 59, and its breadth 20? Anſ. 11060.

PROBLEM IV.

To find the Number of Balls in an Incomplete Pile.

From the number in the whole pile, conſidered as complete, ſubtract the number in the upper pile which is wanting at the top, both computed by the rule for their proper form; and the remainder will be the number in the fruſtum, or incomplete pile.

EXAMPLES.

1. To find the number of ſhot in the incomplete trian-

gular pile, one ſide of the bottom courſe being 40, and the top courſe 20.

19	40
20	41
380	1640
21	42
380	3280
760	6560
7980	68880
	7980
	6) 609000
	10150 Anſwer.

Ex. 2. How many ſhot are in the incomplete triangular pile, the ſide of the baſe being 24, and of the top 8? Anſ. 2516.

Ex. 3. How many balls are in the incomplete ſquare pile, the ſide of the baſe being 24, and of the top 8? Anſ. 4760.

Ex. 4. How many ſhot are in the incomplete rectangular pile of 12 courſes, the length and breadth of the baſe being 40 and 20? Anſ. 6146.

4 P

OF

DISTANCES

BY THE

VELOCITY OF SOUND.

BY various experiments it has been found that ſound flies through the air, uniformly at the rate of about 1142 feet in one ſecond of time, or a mile in $4\frac{2}{3}$ ſeconds. And therefore by proportion any diſtance may be found correſponding to any given time; namely, multiply the given time in ſeconds, by 1142, for the correſponding diſtance in feet; or take $\frac{3}{14}$ of the given time, for the diſtance in miles.

Note. The time for the paſſage of ſound, in the interval between ſeeing the flaſh of a gun, or lightning, and hearing the report, may be obſerved by a watch or a ſmall pendulum. Or, it may be obſerved by the beats of the pulſe in the wriſt, counting on an average, about 70 to a minute in perſons in moderate health, or $5\frac{1}{2}$ pulſations to a mile, and more or leſs according to circumſtances.

EXAMPLES.

1. After obſerving a flaſh of lightning, it was 12 ſeconds before I heard the thunder; required the diſtance of the cloud from whence it came?

12
3
—
14) 36 ($2\frac{4}{7}$ miles, the anſwer.

Ex. 2. How long, after firing the Tower guns, may the report be heard at Shooter's Hill, ſuppoſing the diſtance to be 8 miles in a ſtraight line?

14
8
—
3) 112
—
Anſ. $37\frac{2}{3}$ ſeconds.

Ex. 3. After obſerving the firing of a large cannon at a diſtance, it was 7 ſeconds before I heard the report; what was its diſtance? Anſ. $1\frac{1}{2}$ mile.

Ex. 4. Perceiving a man at a diſtance hewing down a tree with an axe, I remarked that 6 of my pulſations paſſed between ſeeing him ſtrike and hearing the report of the blow; what was the diſtance between us, allowing 70 pulſes to a minute? Anſ. 1 mile and 198 yards.

Ex. 5. How far off was the cloud, from which thunder iſſued, whoſe report was 5 pulſations after the flaſh of lightning; counting 75 to a minute? Anſ. 1523 yards.

MISCELLANEOUS QUESTIONS.

Qu. 1. WHAT difference is there between a floor 28 feet long by 20 broad, and two others each of half the dimenſions; and what do all three come to at 45s. per 100 ſquare feet?

Anſ. dif. 280 ſq. feet. Amount 18 guineas.

2. An

2. An elm plank is 14 feet 3 inches long, and I would have juſt a ſquare yard ſlit off it; at what diſtance from the edge muſt the line be ſtruck? Anſ. $7\frac{99}{171}$ inches.

3. A ceiling contains 114 yards 6 feet of plaſtering, and the room 28 feet broad; what is the length of it?

Anſ. $36\frac{6}{7}$ feet.

4. A common joiſt is 7 inches deep, and $2\frac{1}{2}$ thick; but I want a ſcantling juſt as big again, that ſhall be 3 inches thick; what will the other dimenſion be?

Anſ. $11\frac{2}{3}$ inches.

5. A wooden trough coſt me 3s. 2d. painting within, at 6d. per yard; the length of it was 102 inches, and the depth 21 inches; what was the width?

Anſ. $27\frac{1}{4}$ inches.

6. If my court-yard be 47 feet 9 inches ſquare, and I have laid a foot-path with Purbeck ſtone, of 4 feet wide, along one ſide of it; what will paving the reſt with flints come to, at 6d. per ſquare yard? Anſ. £5 16 $0\frac{1}{2}$.

7. A ladder, 40 feet long, may be ſo planted, that it ſhall reach a window 33 feet from the ground on one ſide of the ſtreet; and, by only turning it over, without moving the foot out of its place, it will do the ſame by a window 21 feet high on the other ſide; what is the breadth of the ſtreet? Anſ. 56 feet $7\frac{3}{4}$ inches.

8. The paving of a triangular court, at 18d. per foot, came to 100l.; the longeſt of the three ſides was 88 feet; required the ſum of the other two equal ſides.

Anſ. 106·85 feet.

9. There are two columns in the ruins of Perſepolis left ſtanding upright: the one is 64 feet above the plain, and the other 50: in a ſtraight line between theſe, ſtands an ancient ſmall ſtatue, the head of which is 97 feet from the ſummit of the higher, and 86 feet from the top of the lower column, the baſe of which meaſures juſt 76 feet to the centre of the figure's baſe. Required the diſtance between the tops of the two columns?

Anſ. 157 feet nearly.

10. The perambulator, or ſurveying wheel, is ſo contrived, as to turn juſt twice in the length of a pole, or $16\frac{1}{2}$ feet; required the diameter? Anſ. 2·626 feet.

11. In turning a one-horſe chaiſe within a ring of a certain diameter, it was obſerved that the outer-wheel made two turns while the inner made but one: the wheels were both 4 feet high; and, ſuppoſing them fixed at the ſtatutable diſtance of 5 feet aſunder on the axle-tree, what was the circumference of the track deſcribed by the outer wheel? Anſ. 63 feet nearly.

12. What is the ſide of that equilateral triangle whoſe area coſt as much paving at 8d. a-foot, as the palliſading the three ſides did, at a guinea a yard? Anſ. 72·746 feet.

13. In the trapezium ABCD are given, AB = 13, BC = $31\frac{1}{5}$, CD = 24, and DA = 18, alſo B a right angle; required the area? Anſ. 410·122.

14. A roof, which is 24 feet 8 inches by 14 feet 6 inches, is to be covered with lead at 8lb. to the ſquare foot: what will it come to at 18s. per cwt?

Anſ. £22 19 $10\frac{1}{4}$.

15. Having a rectangular marble ſlab, 58 inches by 27, I would have a ſquare foot cut off parallel to the ſhorter edge; I would then have the like quantity divided from the remainder, parallel to the longer ſide; and this alternately repeated, till there ſhall not be the quantity of a foot left: what will be the dimenſions of the remaining piece?

Anſ. 20·7 inches by 6·086.

16. Given two ſides of an obtuſe-angled triangle, which are 20 and 40 poles; required the third ſide, that the triangle may contain juſt an acre of land?

Anſ. 58·876 or 23·099.

17. The end wall of a houſe is 24 feet 6 inches in breadth, and 40 feet to the eaves; $\frac{1}{3}$ of which is two bricks thick, $\frac{1}{3}$ more is $1\frac{1}{2}$ brick thick, and the reſt 1 brick thick. Now the triangular gable riſes 38 courſes

of

of bricks, 4 of which uſually make a foot in depth, and this is but $4\frac{1}{2}$ inches, or half a brick thick: what will this piece of work come to at 5l. 10s. per ſtatute rod?

Anſ. £20 11 $7\frac{1}{2}$.

18. If from a right-angle triangle, whoſe baſe is 12, and perpendicular 16 feet, a line be drawn parallel to the perpendicular, cutting off a triangle whoſe area is 24 ſquare feet; required the ſides of this triangle?

Anſ. 6, 8, and 10.

19. The ellipſe in Groſvenor-ſquare meaſures 840 links acroſs the longeſt way, and 612 the ſhorteſt, within the rails: now the walls being 14 inches thick, what ground do they incloſe, and what do they ſtand upon?

$$\text{Anſ.}\begin{cases}\text{incloſe 4 ac 0 r 6 p}\\ \text{ſtand on } 1760\frac{1}{2}\text{ ſq. feet.}\end{cases}$$

20. If a round pillar, 7 inches over, has 4 feet of ſtone in it; of what diameter is the column, of equal length, that contains 10 times as much? Anſ. 22·136 inches.

21. A circular fiſh-pond is to be made in a garden, that ſhall take up juſt half an acre; what muſt be the length of the cord that ſtrikes the circle? Anſwer $27\frac{3}{4}$ yards.

22. When a roof is of a true pitch, the rafters are $\frac{3}{4}$ of the breadth of the building: now ſuppoſing the eaves-boards to project 10 inches on one ſide, what will the new ripping a houſe coſt, that meaſures 32 feet 9 inches long, by 22 feet 9 inches broad on the flat, at 15s. per ſquare?

Anſ. £8 15 $9\frac{1}{2}$.

23. A cable which is 3 feet long, and 9 inches in compaſs, weighs 22lb: what will a fathom of that cable weigh, which meaſures a foot about? Anſ. $78\frac{1}{2}$lb.

24. My plumber has put 28lb per ſquare foot into a ciſtern 74 inches, and twice the thickneſs of the lead long, 26 inches broad, and 46 deep; he has alſo put three ſtays acroſs it within, 16 inches deep, of the ſame ſtrength, and reckons 22s. per cwt, for work and materials. I, being a maſon, have paved him a workſhop, 22 feet 10 inches broad, with Purbeck ſtone, at 7d. per foot;

foot; and upon the balance I find there is 3s. 6d. due to him. What was the length of the workſhop?

Anſ. 32 f. 0$\frac{3}{4}$ inches.

25. The diſtance of the centres of two circles, whoſe diameters are each 50, being given equal to 30; what is the area of the ſpace incloſed by their circumferences?

Anſ. 559·119.

26. If 20 feet of iron railing weigh half a ton, when the bars are an inch and a quarter ſquare, what will 50 feet come to, at 3$\frac{1}{2}$d. per lb. the bars being put $\frac{7}{8}$ of an inch ſquare? Anſ. £20 0 2.

27. The area of an equilateral triangle, whoſe baſe falls on the diameter, and its vertex in the middle of the arc of a ſemicircle, is equal to 100: what is the diameter of the ſemicircle? Anſ. 26·32148.

28. It is required to find the thickneſs of the lead in a pipe, of an inch and quarter bore, which weighs 14lb per yard in length; the cubic foot of lead weighing 11·325 ounces. Anſ. ·20737 inches.

29. Suppoſing the expence of paving a ſemicircular plot, at 2s. 4d. per foot, come to 10l. what is the diameter of it? Anſ. 14·7737.

30. What is the length of a cord, which cuts off $\frac{1}{3}$ of the area, from a circle whoſe diameter is 289?

Anſ. 278·6716.

31. My plumber has ſet me up a ciſtern, and his ſhop-book being burnt, he has no means of bringing in the charge, and I do not chooſe to take it down, to have it weighed; but my meaſure he finds it contains 64 $\frac{3}{10}$ ſquare feet, and that it is preciſely $\frac{1}{8}$ of an inch in thickneſs. Lead was then wrought at 21l. per fother of 19$\frac{1}{2}$ cwt. It is required from theſe items to make out the bill, allowing 6$\frac{5}{9}$ oz. for the weight of a cubic inch of lead?

Anſ. £4 11 2.

32. What will the diameter of a globe be, when the ſolidity and ſuperficial content are expreſſed by the ſame number? Anſ. 6.

33. A ſack that would hold 3 buſhels of corn, is $22\frac{1}{2}$ inches broad when empty; what will that ſack contain which, being of the ſame length, has twice its breadth or circumference? Anſ. 12 buſhels.

34. A carpenter is to put an oaken curb to a round well at 8d. per foot ſquare; the breadth of the curb is to be $7\frac{1}{4}$ inches, and the diameter within $3\frac{1}{2}$ feet: what will be the expence? Anſ. 5s. $2\frac{1}{4}$d.

35. A gentleman has a garden 100 feet long, and 80 feet broad; now a gravel walk is to be made of an equal width all round it: what muſt the breadth of the walk be, to take up juſt half the ground? Anſ. 25·968 feet.

36. A may-pole whoſe top, being broken off by a blaſt of wind, ſtruck the ground at 15 feet diſtance from the foot of the pole; what was the height of the whole may-pole, ſuppoſing the length of the broken piece to be 39 feet? Anſ. 75 feet.

37. Seven men bought a grinding ſtone, of 60 inches diameter, each paying $\frac{1}{7}$ part of the expence; what part of the diameter muſt each grind down for his ſhare?

Anſ. the 1ſt 4·4508, 2d 4·8400, 3d 5·3535,
4th 6·0765, 5th 7·2079, 6th 9·3935, 7th 22·6778.

38. A maltſter has a kiln, which is 16 feet 6 inches ſquare: but he wants to pull it down, and build a new one, that may dry three times as much at once as the old one; what muſt be the length of its ſide?

Anſ. 28 feet 7 inches.

39. How many 3 inch cubes may be cut out of a 12 inch cube? Anſ. 64.

40. How long muſt be the tether of a horſe, that will allow him to graze, quite around, juſt an acre of ground?

Anſ. $39\frac{1}{4}$ yards.

41. What will the painting of a conical ſpire come to at 8d. per yard; ſuppoſing the height to be 118 feet, and the circumference of the baſe 64 feet?

Anſ. £14 0 $8\frac{3}{4}$.

42. The diameter of a ſtandard corn buſhel is $18\frac{1}{2}$ inches,

inches, and its depth 8 inches; what muſt be the diameter of that buſhel be, whoſe depth is $7\frac{1}{2}$ inches?

Anſ. 19·1067.

43. Suppoſe the ball on the top of St. Paul's church is 6 feet in diameter; what did the gilding of it coſt, at $3\frac{1}{2}$d. per ſquare inch? Anſ. £237 10 1.

44. What will a fruſtum of a marble cone come to, at 12s. per ſolid foot: the diameter of the greater end being 4 feet, that of the leſs end $1\frac{1}{2}$, and the length of the ſlant ſide 8 feet? Anſ. £30 1 $10\frac{1}{4}$.

45. To divide a cone into three equal parts by ſections parallel to the baſe, and to find the altitudes of the three parts, the height of the whole cone being 20 inches?

Anſ. the upper part 13·867,
the middle part 3·604,
the lower part 2·528.

46. A gentleman has a bowling-green 300 feet long, and 200 feet broad, which he would raiſe 1 foot higher, by means of the earth to be dug out of a ditch that goes round it; to what depth muſt the ditch be dug, ſuppoſing its breadth to be every where 8 feet? Anſ. $7\frac{23}{86}$ feet.

47. How high above the earth muſt a perſon be raiſed, that he may ſee $\frac{1}{3}$ of its ſurface?

Anſ. to the height of the earth's diameter.

48. A cubic foot of braſs is to be drawn into a wire of $\frac{1}{40}$ of an inch in diameter; what will the length of the wire be, allowing no loſs in the metal?

Anſ. 97784·797 yards, or 55 miles 984·797 yards.

49. Of what diameter muſt the bore of a cannon be, which is caſt for a ball of 24lb weight, ſo that the diameter of the bore may be $\frac{1}{10}$ of an inch more than that of the ball, and ſuppoſing a 9lb ball to meaſure 4 inches in diameter? Anſ. 5.757 inches.

50. Suppoſing the diameter of an iron 9lb ball to be 4 inches, as it is very nearly; it is required to find the diameters of the ſeveral balls weighing 1, 2, 3, 4, 6, 9, 12, 18, 24, 36, and 42 lb. and the caliber of their guns; allowing

allowing $\frac{1}{50}$ of the caliber, or $\frac{1}{40}$ of the ball's diameter for windage.

Anſwer.

Wt ball	Diameter ball	Caliber gun
1	1·9230	1·9622
2	2·4228	2·4723
3	2·7734	2·8301
4	3·0526	3·1149
6	3·4943	3·5656
9	4·0000	4·0816
12	4·4026	4·4924
18	5·0397	5·1425
24	5·5469	5·6601
36	6·3496	6·4792
42	6·6844	6·8208

51. Suppoſing the windage of all mortars be allowed to be $\frac{1}{60}$ of the caliber, and the diameter of the hollow part of the ſhell to be $\frac{7}{10}$ of the caliber of the mortar; it is required to determine the diameter and weight of the ſhell, and the quantity or weight of powder requiſite to fill it, for each of the ſeveral ſorts of mortars, namely, the 13, 10, 8, 5·8, and 4·6 inch mortar?

Anſwer.

Calib. mort.	Diameter ſhell	Wt. ſhell empty	Wt. of powder	Wt. ſhell filled
4·6	4·523	8·320	0·583	8·903
5·8	5·703	16·677	1·168	17·845
8	7·867	43·764	3·065	46·829
10	9·833	85·476	5·986	91·462
13	12·783	187·791	13·151	200·942

52. How many ſhot are in a complete ſquare pile, each ſide of the baſe containing 29? Anſ. 8555.

53. How

53. How many ſhot are in a complete oblong pile, the length of the baſe containing 49, and the breadth 19?

Anſ. 8170.

54. How many ſhot are in a triangular pile, each ſide of the baſe being 50? Anſ. 22100.

55. How many ſhot are in an unfiniſhed triangular pile, the ſide of the bottom being 50, and top 20?

Anſ. 20770.

56. How many ſhot are in an unfiniſhed oblong pile, having the corner row 12, and the ſides of the top 40 and 10? Anſ. 8606.

57. If a heavy ſphere, whoſe diameter is 4 inches, be let fall into a conical glaſs, full of water, whoſe diameter is 5, and altitude 6 inches; it is required to determine how much water will run over?

Anſ. 26·272 cubic inches, or near $\frac{35}{47}$ parts of a pint.

58. The dimenſions of the ſphere and cone being the ſame as in the laſt queſtion, and the cone only $\frac{1}{5}$ full of water; required what part of the axis of the ſphere is immerſed in the water! Anſ. ·546 parts of an inch.

59. The cone being ſtill the ſame, and $\frac{1}{5}$ full of water; required the diameter of a ſphere that may be juſt all covered by the water? Anſ. 2·445996.

60. If I ſee the flaſh of a cannon, fired by a ſhip in diſtreſs at ſea, and hear the report 33 ſeconds after, how far is ſhe off? Anſ. $7\frac{1}{14}$ miles.

61. Being one day ordered to obſerve how far a battery of cannon was from me, I counted by my watch 17 ſeconds between the time of ſeeing the flaſh, and hearing the report; how far was the battery from me?

Anſ. $3\frac{1}{2}$ miles.

62. An irregular piece of lead ore weighs in air 12 ounces, but in water only 7; and another fragment weighs in air $14\frac{1}{2}$ ounces, but in water only 9; required their comparative denſities? Anſ. as 145 to 132.

63. Suppoſing the cubic inch of common glaſs weigh 1·36 ounces troy, the ſame of ſalt water ·5427, and of brandy

brandy ·48926; then a ſeaman having a gallon of that liquor in a glaſs bottle, which weighs $3\frac{1}{2}$lb. troy out of water, and to conceal it from the officers of the cuſtoms, throws it overboard. It is required to determine, if it will ſink, how much force will juſt buoy it up?

Anſ. 12·8968 ounces.

64. Supppoſe by meaſurement it be found that a ſhip of war, with its ordnance, rigging and appointments, draws ſo much water as to diſplace 50000 cubic feet of water; required the weight of the veſſel?

Anſ. 1395 $\frac{1}{16}$ tons.

A

TABLE

OF THE

AREAS of the SEGMENTS of a CIRCLE,

Whoſe diameter is Unity, and ſuppoſed to be divided into 1000 equal parts.

Height	Area Seg	Height	Area Seg.	Height	Area Seg.
·001	·000042	·027	·005867	·053	·016007
·002	·000119	·028	·006194	·054	·016457
·003	·000219	·029	·006527	·055	·016911
·004	·000337	·030	·006865	·056	·017369
·005	·000470	·031	·007209	·057	·017831
·006	·000618	·032	·007558	·058	·018296
·007	000779	·033	·007913	·059	·018766
·008	·000951	·034	·008273	·060	·019239
·009	·001135	·035	·008698	·061	·019716
·010	·001329	·036	·009008	·062	·020196
·011	·001533	·037	·009383	·063	·020680
·012	·001746	·038	·009763	·064	·021168
·013	·001968	·039	·010148	·065	·021659
·014	·002199	·040	·010537	·066	·022154
·015	·002438	·041	·010931	·067	·022652
·016	·002685	·042	·011330	·068	·023154
·017	·002940	·043	·011734	·069	·023659
·018	·003202	·044	·012142	·070	·024168
·019	·003471	·045	·012554	·071	·024680
·020	·003748	·046	·012971	·072	·025195
·021	·004031	·047	·013392	·073	·025714
·022	·004322	·048	·013818	·074	·026236
·023	·004618	·049	·014247	·075	·020761
·024	·004921	·050	·014681	·076	·027289
·025	·005230	·051	·015119	·077	·027821
·026	·005546	·052	·015561	·078	·028356

Height	Area Seg.	Height	Area Seg.	Height	Area Seg.
·079	·028894	·114	·049528	·149	·073161
·080	·029435	·115	·050165	·150	·073874
·081	·029979	·116	·050804	·151	·074589
·082	·030526	·117	·051446	·152	·075306
·083	·031076	·118	·052090	·153	·076026
·084	·031629	·119	·052736	·154	·076747
·085	·032186	·120	·053385	·155	·077469
·086	·032745	·121	·054036	·156	·078194
·087	·033307	·122	·054689	·157	·078921
·088	·033872	·123	·055345	·158	·079649
·089	·034441	·124	·056003	·159	·080380
·090	·035011	·125	·056663	·160	·081112
·091	·035585	·126	·057326	·161	·081846
·092	·036162	·127	·057991	·162	·082582
·093	·036741	·128	·058658	·163	·083320
·094	·037323	·129	·059327	·164	·084059
·095	·037909	·130	·059999	·165	·084801
·096	·038496	·131	·060672	·166	·085514
·097	·039087	·132	·051348	·167	·086289
·098	·039680	·133	·062026	·168	·087036
·099	·040276	·134	·062707	·169	·087785
·100	·040875	·135	·063389	·170	·088535
·101	·041476	·136	·064074	·171	·089287
·102	·042080	·137	·064760	·172	·090041
·103	·042687	·138	·065449	·173	·090797
·104	·043296	·139	·066140	·174	·091554
·105	·043903	·140	·066833	·175	·092313
·106	·044522	·141	·067528	·176	·093074
·107	·045139	·142	·068225	·177	·093836
·108	·045759	·143	·068924	·178	·094601
·109	·046381	·144	·069625	·179	·095326
·110	·047005	·145	·070328	·180	·096134
·111	·047632	·146	·071033	·181	·096003
·112	·048262	·147	·071741	·182	·097674
·113	·048894	·148	·072450	·183	·098447

Height	Area Seg.	Height	Area Seg.	Height	Area Seg.
·184	·099221	·219	·127285	·254	·157019
·185	·099997	·220	·128113	·255	·157890
·186	·100774	·221	·128942	·256	·158762
·187	·101553	·222	·129773	·257	·159636
·188	·102334	·223	·130605	·258	·160510
·189	·103116	·224	·131438	·259	·161386
·190	·103900	·225	·132272	·260	·162263
·191	·104685	·226	·133108	·261	·163140
·192	·105472	·227	·133945	·262	·164019
·193	·106261	·228	·134784	·263	·164899
·194	·107051	·229	·135624	·264	·165780
·195	·107842	·230	·136465	·265	·166663
·196	·108636	·231	·137307	·266	·167546
·197	·109436	·232	·138150	·267	·168430
·198	·110226	·233	·138995	·268	·169315
·199	·111024	·234	·139841	·269	·170202
·200	·111823	·235	·140688	·270	·171089
·201	·112624	·236	·141537	·271	·171978
·202	·113426	·237	·142387	·272	·172867
·203	·114230	·238	·143238	·273	·173758
·204	·115035	·239	·144091	·274	·174649
·205	·115842	·240	·144944	·275	·175542
·206	·116650	·241	·145799	·276	·176435
·207	·117460	·242	·146655	·277	·177330
·208	·118271	·243	·147512	·278	·178225
·209	·110083	·244	·148371	·279	·179122
·210	·119897	·245	·149230	·280	·180019
·211	·120712	·246	·150091	·281	·180918
·212	·121529	·247	·150953	·282	·181817
·213	·122347	·248	·151816	·283	·182718
·214	·123167	·249	·152680	·284	·183619
·215	·123988	·250	·153546	·285	·184521
·216	·124810	·251	·154412	·286	·185425
·217	·125634	·252	·155280	·287	·186329
·218	·126459	·253	·156149	·288	·187234

Height	Area Seg.	Height	Area Seg.	Height	Area Seg.
·289	·188140	·324	·220404	·359	·253590
·290	·189047	·325	·221340	·360	·254550
·291	·189955	·326	·222277	·361	·255510
·292	·190864	·327	·223215	·362	·256471
·293	·191775	·328	·224154	·363	·257433
·294	·192684	·329	·225093	·364	·258395
·295	·193596	·330	·226033	·365	·259357
·296	·194509	·331	·226974	·366	·260320
·297	·195422	·332	·227915	·367	·261284
·298	·196337	·333	·228858	·368	·262248
·299	·197252	·334	·229801	·369	·263213
·300	·198168	·335	·230745	·370	·264178
·301	·199085	·336	·231689	·371	·265144
·302	·200003	·337	·232634	·372	·266111
·303	·200922	·338	·233580	·373	·267078
·304	·201841	·339	·234526	·374	·268045
·305	·202761	·340	·235473	·375	·269013
·306	·203683	·341	·236421	·376	·269982
·307	·204605	·342	·237369	·377	·270951
·308	·205527	·343	·238318	·378	·271920
·309	·206451	·344	·239268	·379	·272890
·310	·207376	·345	·240218	·380	·273861
·311	·208301	·346	·241169	·381	·274832
·312	·209227	·347	·242121	·382	·275803
·313	·210154	·348	·243074	·383	·276775
·314	·211082	·349	·244026	·384	·277748
·315	·212011	·350	·244980	·385	·278721
·316	·212940	·351	·245934	·386	·279694
·317	·213871	·352	·246889	·387	·280668
·318	·214802	·353	·247845	·388	·281642
·319	·215733	·354	·248801	·389	·282617
·320	·216666	·355	·249757	·390	·283592
·321	·217599	·356	·250715	·391	·284568
·322	·218533	·357	·251673	·392	·285544
·323	·219468	·358	·252631	·393	·286521

Height	Area Seg.	Height	Area Seg.	Height	Area Seg.
·394	·287498	·430	·322928	·466	·358725
·395	·288476	·431	·323918	·467	·359723
·396	·289452	·432	·324909	·468	·360721
·397	·290431	·433	·325900	·469	·361719
·398	·291411	·434	·326892	·470	·362717
·399	·292309	·435	·327882	·471	·363715
·400	·293369	·436	·328874	·472	·364713
·401	·294349	·437	·329866	·473	·365712
·402	·295330	·438	·330858	·474	·366710
·403	·296311	·439	·331850	·475	·367709
·404	·297292	·440	·332843	·476	·368708
·405	·298273	·441	·333836	·477	·369707
·406	·299255	·442	·334819	·478	·370706
·407	·300238	·443	·335822	·479	·371705
·408	·301220	·444	·336816	·480	·372704
·409	·302203	·445	·337810	·481	·373703
·410	·303187	·446	·338804	·482	·374702
·411	·304171	·447	·339798	·483	·375702
·412	·305155	·448	·340793	·484	·376702
·413	·306140	·449	·341787	·485	·377701
·414	·307125	·450	·342782	·486	·378701
·415	·308110	·451	·343777	·487	·379700
·416	·309095	·452	·344772	·488	·380700
·417	·310081	·453	·345768	·489	·381699
·418	·311068	·454	·346764	·490	·382699
·419	·312054	·455	·347759	·491	·383699
·420	·313041	·456	·348755	·492	·384699
·421	·311029	·457	·349752	·493	·385699
·422	·315016	·458	·350748	·494	·386699
·423	·316004	·459	·351745	·495	·387699
·424	·316992	·460	·352742	·496	·388699
·425	·317981	·461	·353739	·497	·389699
·426	·318970	·462	·354736	·498	·390699
·427	·319959	·463	·355732	·499	·391699
·428	·320948	·464	·356730	5·00	·392699
·429	·321938	·465	·357727		

THE USE OF THE TABLE.

IN the foregoing table, each number in the column of *area ſeg.* is the area of the circular ſegment whoſe height, or the verſe ſine of its half arc, is the number immediately on the left of it, in the column of *heights*; the diameter of the circle being 1, and its whole area ·785398.

The uſe of this table is to find, by it, the area of the ſegment of any other circle, whatever be the diameter. And this is done by firſt dividing the height of any propoſed ſegment by its own diameter, and the quotient is a decimal to be ſought in the column of heights, and againſt it is the tabular area to be taken out, which is ſimilar to the propoſed ſegment. Then this tabular area, being multiplied by the ſquare of the given diameter, will be the area of the ſegment required; becauſe ſimilar areas are to each other as the ſquares of their diameters.

EXAMPLE.

So if it be required to find the area of a ſegment of a circle, whoſe height is $3\frac{1}{4}$, the diameter being 50.

Here 50) 3·25 (·065 quo. or tabular height,
and the tab. ſeg. is ·021659
which multiply by 2500 the ſquare of the diam.

ives 54·147500 the area required.

But in dividing the given height by the diameter, if the quotient do not terminate in three places of decimals without a fractional remainder, then the area for that fractional part ought to be proportioned for, thus: Having found the tabular area anſwering to the firſt three decimals of the quotient, take the difference between it and the next following tabular area, which difference multiply by the fractional remaining part of the quotient, and the product will be the correſponding proportional part, to be added to the firſt tabular area.

So if the height of a propoſed ſegment were $3\frac{1}{3}$, to the diameter 50.

Here 50) $3\frac{1}{3}$ ($\cdot066\frac{2}{3}$ Then
to ·066 anſwers ·022154
the next area is ·022652

their difference is 498
$\frac{2}{3}$ of which is 232
which added to ·022154

gives the whole tab. area ·022386
and this multiplied by 2500

gives the area 55·965000 ſought.

FINIS.

[*C. and R. Baldwin, Printers, New Bridge-ſtreet, London.*]

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