McVICAR'S

Complete Arithmetic

PART SECOND.

Paul L. Hibbard

A

COMPLETE

ARITHMETIC,

ORAL AND WRITTEN.

PART SECOND.

BY

MALCOLM MACVICAR, PH. D., LL.D.,
PRINCIPAL STATE NORMAL SCHOOL, POTSDAM, N. Y.

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PREFACE.

THE aim of the author in the preparation of this work may be stated as follows:

1. To present each subject in arithmetic in such a manner as to lead the pupil by means of preparatory steps and propositions which he is required to examine for himself, to gain clear perceptions of the elements necessary to enable him to grasp as a reality the more complex and complete processes.

2. To present, wherever it can be done, each process objectively, so that the truth under discussion is exhibited to the eye and thus sharply defined in the mind.

3. To give such a systematic drill on oral and written exercises and review and test questions as will fix permanently in the mind the principles and processes of numbers with their applications in practical business.

4. To arrange the pupil's work in arithmetic in such a manner that he will not fail to acquire such a knowledge of principles and facts, and to receive such mental discipline, as will fit him properly for the study of the higher mathematics.

The intelligent and experienced teacher can readily determine by an examination of the work how well the author has succeeded in accomplishing his aim.

Special attention is invited to the method of presentation given in the teacher's edition. This is arranged at the beginning of each subject, just where it is required, and contains definite and full instructions regarding the order in which the subject should be presented, the points that require special attention and illustration, the kind of illustrations that should be used, a method for drill exercise, additional oral exercises where required for the teacher's use, and such other instructions as are necessary to form a complete guide to the teacher in the discussion and presentation of each subject.

The plan adopted of having a separate teacher's edition avoids entirely the injurious course usually pursued of cumbering the pupil's book with hints and suggestions which are intended strictly for the teacher.

Attention is also invited to the Properties of Numbers, Greatest Common Divisor, Fractions, Decimals, Compound Numbers, Business Arithmetic, Ratio and Proportion, Alligation, and Square and Cube Root, with the belief that the treatment will be found new and an improvement upon former methods.

The author acknowledges with pleasure his indebtedness to Prof. D. H. MacVicar, LL.D., Montreal, for valuable aid rendered in the preparation of the work, and to Charles D. McLean, A. M., Principal of the State Normal and Training School, at Brockport, N. Y., for valuable suggestions on several subjects.

M. MacVICAR.

Potsdam, *September*, 1877.

CONTENTS.

ARITHMETIC.

REVIEW OF PART FIRST.*

NOTATION AND NUMERATION.

DEFINITIONS.

11. A ***Unit*** is a single thing, or group of single things, regarded as one; as, *one* ox, *one* yard, *one* ten, *one* hundred.

12. ***Units are of two kinds*** — Mathematical and Common. A *mathematical* unit is a single thing which has a fixed value; as, one *yard,* one *quart,* one *hour,* one *ten.* A *common* unit is a single thing which has no fixed value; as, one *house,* one *tree,* one *garden,* one *farm.*

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* NOTE.—The first 78 pages of this part contains so much of the matter in Part First as is necessary for a thorough review of each subject, including all the tables of Compound Numbers. For convenience in making references, the Articles retained are numbered the same as in Part First. Hence the numbers of the Articles are not consecutive.

13. A ***Number*** is a unit, or collection of units; as, *one* man, *three* houses, *four, six* hundred.

Observe, the *number* is "*the how many*" and is represented by whatever answers the question, How many? Thus in the expression *seven* yards, *seven* represents the number.

14. The ***Unit of a Number*** is one of the things numbered.

Thus, the unit of eight bushels is *one bushel,* of five boys is *one boy,* of nine is *one.*

15. A ***Concrete Number*** is a number which is applied to *objects* that are named; as four *chairs,* ten *bells.*

16. An ***Abstract Number*** is a number which is not applied to any named objects; as *nine, five, thirteen.*

17. ***Like Numbers*** are such as have the same unit.

Thus, four *windows* and eleven *windows* are like numbers, eight and ten, three *hundred* and seven *hundred.*

18. ***Unlike Numbers*** are such as have different units.

Thus, twelve *yards* and five *days* are unlike numbers, also six *cents* and nine *minutes.*

19. ***Figures*** are characters used to express numbers.

20. The ***Value*** of a figure is the number which it represents.

21. The ***Simple*** or ***Absolute Value*** of a figure is the number it represents when standing alone, as 8.

22. The ***Local*** or ***Representative Value*** of a figure is the number it represents in consequence of the place it occupies.

Thus, in 66 the 6 in the second place from the right represents a number ten times as great as the 6 in the first place.

23. ***Notation*** is the method of writing numbers by means of figures or letters.

24. ***Numeration*** is the method of reading numbers which are expressed by figures or letters.

25. A ***Scale*** in Arithmetic is a succession of mathematical units which increase or decrease in value according to a fixed order.

26. A ***Decimal Scale*** is one in which the fixed order of increase or decrease is uniformly ten.

This is the scale used in expressing numbers by figures.

27. ***Arithmetic*** is the Science of Numbers and the Art of Computation.

REVIEW AND TEST QUESTIONS.

31. Study carefully and answer each of the following questions:

1. Define a scale. A decimal scale.

2. How many figures are required to express numbers in the decimal scale, and why?

3. Explain the use of the cipher, and illustrate by examples.

4. State reasons why a scale is necessary in expressing numbers.

5. Explain the use of each of the three elements—*figures*, *place*, and *comma*—in expressing numbers.

6. What is meant by the *simple* or *absolute* value of figures? What by the *local* or *representative* value?

7. How is the *local value* of a figure affected by changing it from the *first* to the *third* place in a number?

8. How by changing a figure from the second to the fourth? From the fourth to the ninth?

9. Explain how the names of numbers from twelve to twenty are formed. From twenty to nine hundred ninety.

10. What is meant by a *period* of figures?

11. Explain how the name for each order in any period is formed.

12. State the name of the right-hand order in each of the first six periods, commencing with units.

13. State the two things mentioned in (**9**) which must be observed when writing large numbers.

14. Give a rule for reading numbers; also for writing numbers.

ADDITION.

50. ***Addition*** is the process of uniting two or more numbers into one number.

51. ***Addends*** are the numbers added.

52. The ***Sum*** or ***Amount*** is the number found by addition.

53. The ***Process of Addition*** consists in forming units of the same order into groups of ten, so as to express their amount in terms of a higher order.

54. The ***Sign of Addition*** is $+$, and is read *plus.*

When placed between two or more numbers, thus, $8+3+6+2+9$, it means that they are to be added.

55. The ***Sign of Equality*** is $=$, and is read *equals*, or *equal to;* thus, $9+4=13$ is read, *nine plus four equals thirteen.*

56. PRINCIPLES.—*I. Only numbers of the same denomination and units of the same order can be added.*

II. The sum is of the same denomination as the addends.

III. The whole is equal to the sum of all the parts.

REVIEW AND TEST QUESTIONS.

57. 1. Define Addition, Addends, and Sum or Amount.

2. Name each step in the process of Addition.

3. Why place the numbers, preparatory to adding, units under units, tens under tens, etc.?

4. Why commence adding with the units' column?

5. What objections to adding the columns in an irregular order? Illustrate by an example.

6. Construct, and explain the use of the addition table.

7. How many combinations in the table, and how found?

8. Explain *carrying* in addition. What objection to the use of the word?

9. Define counting and illustrate by an example.

10. Write five examples illustrating the general problem of addition, "Given all the parts to find the whole."

11. State the difference between the addition of objects and the addition of numbers.

12. Show how addition is performed by using the addition table.

13. What is meant by the denomination of a number? What by units of the same order?

14. Show by analysis that in adding numbers of two or more places, the orders are treated as independent of each other.

SUBTRACTION.

70. ***Subtraction*** is the process of finding the difference between two numbers.

71. The ***Minuend*** is the greater of two numbers whose difference is to be found.

72. The ***Subtrahend*** is the smaller of two numbers whose difference is to be found.

73. The ***Difference*** or ***Remainder*** is the result obtained by subtraction.

74. The ***Process of Subtraction*** consists in comparing two numbers, and resolving the greater into two parts, one of which is equal to the less and the other to the difference of the numbers.

75. The ***Sign of Subtraction*** is $-$, and is called *minus.*

When placed between two numbers it indicates that their difference is to be found; thus, $14 - 6$ is read, 14 minus 6, and means that the difference between 14 and 6 is to be found.

76. ***Parentheses*** ( ) denote that the numbers inclosed between them are to be considered as one number.

77. A ***Vinculum*** $\overline{\quad\quad}$ affects numbers in the same manner as parentheses.

Thus, $19+(13-5)$, or $19+\overline{13-5}$ signifies that the difference between 13 and 5 is to be added to 19.

78. PRINCIPLES.—*I. Only like numbers and units of the same order can be subtracted.*

II. The minuend is the sum of the subtrahend and difference, or the minuend is the whole of which the subtrahend and difference are the parts.

III. An equal increase or decrease of the minuend and subtrahend does not change the difference.

REVIEW AND TEST QUESTIONS.

79. 1. Define the process of subtraction. Illustrate each step by an example.

2. Explain how subtraction should be performed when an order in the subtrahend is greater than the corresponding order in the minuend. Illustrate by an example.

3. Indicate the difference between the subtraction of numbers and the subtraction of objects.

4. When is the result in subtraction a remainder, and when a difference?

5. Show that so far as the process with numbers is concerned, the result is always a difference.

6. Prepare four original examples under each of the following problems and explain the method of solution:

PROB. I.—*Given the whole and one of the parts to find the other part.*

PROB. II.—*Given the sum of four numbers and three of them to find the fourth.*

7. Construct a Subtraction Table.

8. Define counting by subtraction.

9. Show that counting by addition, when we add a number larger than *one*, necessarily involves counting by subtraction.

10. What is the difference between the meaning of *denomination* and *orders of units?*

11. State Principle III and illustrate its meaning by an example.

12. Show that the difference between 63 and 9 is the same as the difference between (63 + 10) and (9 + 10).

13. Show that 28 can be subtracted from 92, without analyzing the minuend as in (**64**), by adding 10 to each number.

14. What must be added to each number, to subtract 275 from 829 without analyzing the minuend as in (**64**)?

15. What is meant by *borrowing* and *carrying* in subtraction?

MULTIPLICATION.

ILLUSTRATION OF PROCESS.

92. Step II.—*To multiply by using the parts of the multiplier.*

1. The multiplier may be made into any desired parts, and the multiplicand taken separately the number of times expressed by each part. The sum of the products thus found is the required product.

Thus, to find 9 times 12 we may take 4 times 12 which are 48, then 5 times 12 which are 60. 4 times 12 plus 5 times 12 are 9 times 12; hence, 48 plus 60, or 108, are 9 times 12.

2. When we multiply by one of the equal parts of the multiplier, we find one of the equal parts of the required product. Hence, by multiplying the part thus found by the number of such parts, we find the required product.

For example, to find 12 times 64 we may proceed thus:

(1.) ANALYSIS.

$$\left.\begin{array}{l} 64 \times 4 = 256 \\ 64 \times 4 = 256 \\ 64 \times 4 = 256 \end{array}\right\} = 3 \text{ times } 256$$

$$64 \times 12 = 768$$

(2.)

$$\begin{array}{r} 64 \\ 4 \\ \hline 256 \\ 3 \\ \hline 768 \end{array}$$

(1.) Observe, that $12 = 4 + 4 + 4$; hence, 4 is one of the 3 equal parts of 12.

(2.) That 64 is taken 12 times by taking it 4 times + 4 times + 4 times, as shown in the analysis.

(3.) That 4 times 64, or 256, is one of the 3 equal parts of 12 times 64. Hence, multiplying 256 by 3 gives 12 times 64, or 768.

3. In multiplying by 20, 30, and so on up to 90, we invariably multiply by 10, one of the equal parts of these numbers, and then by the number of such parts.

For example, to multiply 43 by 30, we take 10 times 43, or 430, and multiply this product by 3; $430 \times 3 = 1290$, which is 30 times 43. We multiply in the same manner by 200, 300, etc., 2000, 3000, etc.

93. PROB. II.—**To multiply by a number containing only one order of units.**

1. Multiply 347 by 500.

(1.) ANALYSIS.

First step, $347 \times 100 = 34700$

Second step, $34700 \times 5 = 173500$

(2.)

$$\begin{array}{r} 347 \\ 500 \\ \hline 173500 \end{array}$$

EXPLANATION.—500 is equal to 5 times 100; hence, by taking 347, as in *first step*, 100 times, 5 times this result, or 5 times 34700, as shown in *second step*, will make 500 times 347. Hence 173500 is 500 times 347.

In practice we multiply first by the significant figure, and annex to the product as many ciphers as there are ciphers in the multiplier, as shown in (2).

96. PROB. III.—**To multiply by a number containing two or more orders of units.**

1. Multiply 539 by 374.

(1.) ANALYSIS.

$$539 \times 374 = \begin{cases} 539 \times 4 = 2156 \\ 539 \times 70 = 37730 \\ 539 \times 300 = 161700 \end{cases}$$

(2.)

$$\begin{array}{rl} 539 & \text{Multiplicand.} \\ 374 & \text{Multiplier.} \\ \hline 2156 & \text{1st partial product.} \\ 37730 & \text{2d partial product.} \\ 161700 & \text{3d partial product.} \\ \hline 201586 & \text{Whole product.} \end{array}$$

EXPLANATION.—1. The multiplier, 374, is analyzed into the parts 4, 70, and 300, according to (**92**).

2. The multiplicand, 539, is taken first 4 times = 2156 (**86**); then 70 times = 37730 (**93**); then 300 times = 161700 (**93**).

3. 4 times + 70 times + 300 times are equal to 374 times; hence the sum of the partial products, 2156, 37730, and 161700, is equal to 374 times 539 = 201586.

4. Observe, that in practice we arrange the partial products as shown in (2), omitting the ciphers at the right, and placing the first significant figure of each product under the order to which it belongs.

DEFINITIONS.

100. ***Multiplication*** is the process of taking one number as many times as there are units in another.

101. The ***Multiplicand*** is the number taken, or multiplied.

102. The ***Multiplier*** is the number which denotes how many times the multiplicand is taken.

103. The ***Product*** is the result obtained by multiplication.

104. A ***Partial Product*** is the result obtained by multiplying by one order of units in the multiplier, or by any part of the multiplier.

105. The ***Total*** or ***Whole Product*** is the sum of all the partial products.

106. The ***Process of Multiplication*** consists, *first*, in finding partial products by using the memorized results of the Multiplication Table; *seccnd*, in uniting these partial products by addition into a total product.

107. A ***Factor*** is one of the *equal parts* of a number.

Thus, 12 is composed of six 2's, four 3's, three 4's, or two 6's; hence, 2, 3, 4, and 6 are factors of 12.

The multiplicand and multiplier are factors of the product. Thus, $37 \times 25 = 925$. The product 925 is composed of *twenty-five* 37's, or *thirty-seven* 25's. Hence, both 37 and 25 are equal parts or factors of 925.

108. The ***Sign of Multiplication*** is $\times$, and is read *times*, or *multiplied by*.

When placed between two numbers, it denotes that either is to be multiplied by the other. Thus, 8×6 shows that 8 is to be taken 6 times, or that 6 is to be taken 8 times; hence it may be read either 8 times 6 or 6 times 8.

109. Principles.—*I. The multiplicand may be either an abstract or concrete number.*

II. The multiplier is always an abstract number.

III. The product is of the same denomination as the multiplicand.

REVIEW AND TEST QUESTIONS.

110. 1. Define Multiplication, Multiplicand, Multiplier, and Product.

2. What is meant by Partial Product? Illustrate by an example.

3. Define Factor, and illustrate by examples.

4. What are the factors of 6? 14? 15? 9? 20? 24? 25? 27? 32? 10? 30? 50? and 70?

5. Show that the multiplicand and multiplier are factors of the product.

6. What must the denomination of the product always be, and why?

7. Explain the process in each of the following cases and illustrate by examples:

I. To multiply by numbers less than 10.
II. To multiply by 10, 100, 1000, and so on.
III. To multiply by one order of units.
IV. To multiply by two or more orders of units.
V. To multiply by the factors of a number (**92**—2).

8. Give a rule for the third, fourth, and fifth cases.

9. Give a rule for the shortest method of working examples where both the multiplicand and multiplier have one or more ciphers on the right.

10. Show how multiplication may be performed by addition.

11. Explain the construction of the Multiplication Table, and illustrate its use in multiplying.

12. Why may the ciphers be omitted at the right of partial products?

13. Why commence multiplying the units' order in the multiplicand first, then the tens', and so on? Illustrate your answer by an example.

14. Multiply 8795 by 629, multiplying first by the tens, then by the hundreds, and last by the units.

15. Multiply 3572 by 483, commencing with the thousands of the multiplicand and hundreds of the multiplier.

16. Show that *hundreds* multiplied by *hundreds* will give *ten thousands* in the product.

17. Multiplying *thousands* by *thousands*, what order will the product be?

18. Name at sight the *lowest order* which each of the following examples will give in the product:

(1.) 8000 × 3000; 2000000 × 3000; 5000000000 × 7000.

(2.) 40000 × 20000; 7000000 × 4000000.

19. What orders in 3928 can be multiplied by each order in 473, and not have any order in the product less than thousands?

DIVISION.

ILLUSTRATION OF PROCESS.

119. Prob. I.—**To divide any number by any divisor not greater than 12.**

1. Divide 986 by 4.

Explanation.—Follow the *analysis* and notice each step in the process; thus,

1. We commence by dividing the higher order of units. We know that 9, the figure expressing hundreds, contains twice the divisor 4, and 1 remaining. Hence 900 contains, according to (**117**—2), 200 times the divisor 4, and 100 remaining. We multiply the divisor 4 by 200, and subtract the product 800 from 986, leaving 186 of the dividend yet to be divided.

ANALYSIS.

	4)986(200	
4 × 200 =	800	40
	186	
4 × 40 =	160	6
	26	$246\frac{2}{4}$
4 × 6 =	24	
	2	

2. We know that 18, the number expressed by the two left-hand figures of the undivided dividend, contains 4 times 4, and 2 remaining. Hence 18 tens, or 180, contains, according to (**117**—2), 40 times 4, and 20 remaining. We multiply the divisor 4 by 40, and subtract the product 160 from 186, leaving 26 yet to be divided.

3. We know that 26 contains 6 times 4, and 2 remaining, which is less than the divisor; hence the division is completed.

4. We have now found that there are 200 fours in 800, 40 fours in 160, and 6 fours in 26, and 2 remaining; and we know that 800 + 160 + 26 = 986. Hence 986 contains (200 + 40 + 6) or 246 fours, and 2 remaining. The remainder is placed over the divisor and written after the quotient; thus, $246\frac{2}{4}$.

SHORT AND LONG DIVISION COMPARED.

121. Compare carefully the following forms of writing the work in division:

(1.)			(2.)	(3.)
FORM USED FOR EXPLANATION. Two steps in the process written.			LONG DIVISION. One step written.	SHORT DIVISION. Entirely mental.
	4) 986 (	200	4) 986 (246	4) 986
4 × 200 =	800	40	8	$246\frac{2}{4}$
	186	6	18	
4 × 40 =	160	246	16	
	26		26	
4 × 6 =	24		24	

129. Prob. II.—**To divide any number by any given divisor.**

1. Divide 21524 by 59.

```
59)21524(364
   177
   ---
    382
    354
    ---
     284
     236
     ---
      48
```

Explanation.—1. We find how many times the divisor is contained in the fewest of the left-hand figures of the dividend which will contain it.

59 is contained 3 times in 215, with a remainder 38, hence, according to (**115**—1), it is contained 300 times in 21500, with a remainder 3800.

2. We annex the figure in the next lower order of the dividend to the remainder of the previous division, and divide the number thus found by the divisor. 2 tens annexed to 380 tens make 382 tens.

59 is contained 6 times in 382, with a remainder 28; hence, according to (**115**—1), it is contained 60 times in 3820, with a remainder 280.

3. We annex the next lower figure and proceed as before.

137. Prob. III.—**To divide by using the factors of the divisor.**

Ex. 1. Divide 315 by 35.

```
      5 ) 3 1 5
7 fives ) 6 3 fives
          9
```

Explanation.—1. The divisor 35 = 7 *fives.*

2. Dividing the 315 by 5, we find that 315 = 63 *fives.* (**138.**)

3. The 63 *fives* contain 9 times 7 *fives*; hence 315 contains 9 times 7 *fives* or 9 times 35.

Ex. 2. Divide 359 by 24.

```
           2 | 3 5 9
      3 twos | 1 7 9 twos      and 1 remaining           =  1
  4 (3 twos)     | 5 9 (3 twos) and 2 twos remaining     =  4
    Quotient,    1 4         and 3 (3 twos) remaining    = 18
                                    True remainder,        23
```

Explanation.—1. The divisor $24 = 4 \times 3 \times 2 = 4$ (3 *twos*).

2. Dividing 359 by 2, we find that 359=179 *twos* and 1 unit remaining.

3. Dividing 179 *twos* by 3 *twos*, we find that 179 *twos* = 59 (3 *twos*) and 2 *twos* remaining.

4. Dividing 59 (3 *twos*) by 4 (3 *twos*), we find that 59 (3 *twos*) contain 4 (3 *twos*) 14 times and 3 (3 *twos*) remaining.

Hence 359, which is equal to 59 (3 *twos*) and 2 *twos* + 1, contains 4 (3 *twos*), or 24, 14 times, and 3 (3 *twos*) + 2 *twos* + 1, or 23, remaining.

142. Prob. IV.—**To divide when the divisor consists of only one order of units.**

1. Divide 8736 by 500.

```
5 ) 8 7 | 3 6
    1 7 and 236 remaining.
```

Explanation.—1. We divide first by the factor 100. This is done by cutting off 36, the units and tens at the right of the dividend.

2. We divide the quotient, 87 hundreds, by the factor 5, which gives a quotient of 17 and 2 hundred remaining, which added to 36 gives 236, the true remainder.

DEFINITIONS.

144. ***Division*** is the process of finding how many times one number is contained in another.

145. The ***Dividend*** is the number divided.

146. The ***Divisor*** is the number by which the dividend is divided.

147. The ***Quotient*** is the result obtained by division.

148. The ***Remainder*** is the part of the dividend left after the division is performed.

149. A ***Partial Dividend*** is any part of the dividend which is divided in one operation.

150. A ***Partial Quotient*** is any part of the quotient which expresses the number of times the divisor is contained in a partial dividend.

151. The ***Process of Division*** consists, *first,* in finding the partial quotients by means of memorized results; *second,* in multiplying the divisor by the partial quotients to find the partial dividends; *third,* in subtracting the partial dividends from the part of the dividend that remains undivided to find the part yet to be divided.

152. ***Short Division*** is that form of division in which no step of the process is written.

153. ***Long Division*** is that form of division in which the *third* step of the process is written.

154. The ***Sign of Division*** is ÷, and is read *divided by.* When placed between two numbers, it denotes that the number before it is to be divided by the number after it; thus, 28 ÷ 7 is read, 28 divided by 7.

Division is also expressed by placing the dividend above the divisor, with a short horizontal line between them; thus, $\frac{35}{5}$ is read, 35 divided by 5.

155. Principles.—*I. The dividend and divisor must be numbers of the same denomination.*

II. The denomination of the quotient is determined by the nature of the problem solved.

III. The remainder is of the same denomination as the dividend.

REVIEW AND TEST QUESTIONS.

156. 1. Define Division, and illustrate each step in the process by examples.

2. Explain and illustrate by examples Partial Dividend, Partial Quotient, and Remainder.

3. Prepare two examples illustrating each of the following problems:

I. Given all the parts, to find the whole.

II. Given the whole and one of the parts, to find the other part.

III. Given one of the equal parts and the number of parts, to find the whole.

IV. Given the whole and the size of one of the parts, to find the number of parts.

V. Given the whole and the number of equal parts, to find the size of one of the parts.

4. Show that 45 can be expressed as *nines*, as *fives*, as *threes.*

5. What is meant by true remainder, and how found?

6. Explain division by factors. Illustrate by an example.

7. Why cut off as many figures at the right of the dividend as there are ciphers at the right of the divisor? Illustrate by an example.

8. Give a rule for dividing by a number with one or more ciphers at the right. Illustrate the steps in the process by an example.

9. Explain the difference between Long and Short Division, and show that the process in both cases is performed mentally.

10. Illustrate each of the following problems by three examples:

VI. Given the final quotient of a continued division, the true remainder, and the several divisors, to find the dividend.

VII. Given the product of a continued multiplication and the several multipliers, to find the multiplicand.

VIII. Given the sum and the difference of two numbers, to find the numbers.

PROPERTIES OF NUMBERS.

163. An ***Integer*** is a number that expresses how many there are in a collection of *whole* things.

Thus, 8 yards, 12 houses, 32 dollars.

164. An ***Exact Divisor*** is a number that will divide another number without a remainder.

Thus, 3 or 5 is an exact divisor of 15.

All numbers with reference to exact divisors are either prime or composite.

165. A ***Prime Number*** is a number that has no exact divisor besides 1 and itself.

Thus, 1, 3, 5, 7, 11, 13, etc., are prime numbers.

166. A ***Composite Number*** is a number that has other exact divisors besides 1 and itself.

Thus, 6 is divisible by either 2 or 3; hence is composite.

167. A ***Prime Divisor*** is a prime number used as a divisor.

Thus, in $35 \div 7$, 7 is a prime divisor.

168. A ***Composite Divisor*** is a composite number used as a divisor.

Thus, in $18 \div 6$, 6 is a composite divisor.

EXACT DIVISION.

169. The following *tests* of exact division should be carefully studied and fixed in the memory for future use.

PROP. I.—*A divisor of any number is a divisor of any number of times that number.*

Thus, $12 = 3$ fours. Hence, $12 \times 6 = 3$ fours $\times 6 = 18$ fours. But 18 fours are divisible by 4. Hence, 12×6, or 72, is divisible by 4.

PROP. II.—*A divisor of each of two or more numbers is a divisor of their sum.*

Thus, 5 is a divisor of 10 and 30; that is, 10 = 2 fives and 30 = 6 fives. Hence, 10 + 30 = 2 fives + 6 fives = 8 fives. But 8 fives are divisible by 5. Hence, 5 is a divisor of the sum of 10 and 30.

PROP. III.—*A divisor of each of two numbers is a divisor of their difference.*

Thus, 3 is a divisor of 27 and 15; that is, 27 = 9 threes and 15 = 5 threes. Hence, 27 − 15 = 9 threes − 5 threes = 4 threes. But 4 threes are divisible by 3. Hence 3 is a divisor of the difference between 27 and 15.

PROP. IV.—*Any number ending with a cipher is divisible by the divisors of* 10, *viz.,* 2 *and* 5.

Thus, 370 = 37 times 10. Hence is divisible by 2 and 5, the divisors of 10, according to Prop. I.

PROP. V.—*Any number is divisible by either of the divisors of* 10, *when its right-hand figure is divisible by the same.*

Thus, 498 = 490 + 8. Each of these parts is divisible by 2. Hence the number 498 is divisible by 2, according to Prop. II.

In the same way it may be shown that 495 is divisible by 5.

PROP. VI.—*Any number ending with two ciphers is divisible by the divisors of* 100, *viz.,* 2, 4, 5, 10, 20, 25, *and* 50.

Thus, 8900 = 89 times 100. Hence is divisible by any of the divisors of 100, according to Prop. I.

PROP. VII.—*Any number is divisible by any one of the divisors of* 100, *when the number expressed by its two right-hand figures is divisible by the same.*

Thus, 4975 = 4900 + 75. Any divisor of 100 is a divisor of 4900 (Prop. VI). Hence, any divisor of 100 which will divide 75 is a divisor of 4975 (Prop. II).

PROP. VIII.—*Any number ending with three ciphers is divisible by the divisors of* 1000, *viz.,* 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 125, 200, 250, *and* 500.

Thus, 83000 = 83 times 1000. Hence is divisible by any of the divisors of 1000, according to Prop. I.

Prop. IX.—*Any number is divisible by any one of the divisors of* 1000, *when the number expressed by its three right-hand figures is divisible by the same.*

Thus, $92625 = 92000 + 625$. Any divisor of 1000 is a divisor of 92000 (Prop. VIII). Hence, any divisor of 1000 which will divide 625 is a divisor of 92625 (Prop. II).

Prop. X.—*Any number is divisible by* 9, *if the sum of its digits is divisible by* 9.

This proposition may be shown thus:

(1.) $486 = 400 + 80 + 6$.

(2.) $100 = 99 + 1 = 11$ nines + 1. Hence, $400 = 44$ nines + 4, and is divisible by 9 with a remainder 4.

(3.) $10 = 9 + 1 = 1$ nine + 1. Hence, $80 = 8$ nines + 8, and is divisible by 9 with a remainder 8.

(4.) From the foregoing it follows that $400 + 80 + 6$, or 486, is divisible by 9 with a remainder $4 + 8 + 6$, the sum of the digits. Hence, if the sum of the digits is divisible by 9, the number 486 is divisible by 9 (Prop. II).

Prop. XI.—*Any number is divisible by* 3, *if the sum of its digits is divisible by* 3.

This proposition is shown in the same manner as Prop. X; as 3 divides 10, 100, 1000, etc., with a remainder 1 in each case.

Prop. XII.—*Any number is divisible by* 11, *if the difference of the sums of the digits in the odd and even places is zero or is divisible by* 11.

This may be shown thus:

(1.) $4928 = 4000 + 900 + 20 + 8$.

(2.) $1000 = 91$ elevens $- 1$. Hence, $4000 = 364$ elevens $- 4$.

(3.) $100 = 9$ elevens $+ 1$. Hence, $900 = 81$ elevens $+ 9$.

(4.) $10 = 1$ eleven $- 1$. Hence, $20 = 2$ elevens $- 2$.

(5.) From the foregoing it follows that $4928 = 364$ elevens + 81 elevens + 2 elevens $- 4 + 9 - 2 + 8$.

But $- 4 + 9 - 2 + 8 = 11$. Hence, $4928 = 364$ elevens + 81 elevens + 2 elevens + 1 eleven = 448 elevens, and is therefore divisible by 11.

The same course of reasoning applies where the difference is minus or zero.

PRIME NUMBERS.

PREPARATORY PROPOSITIONS.

171. PROP. I.—*All even numbers are divisible by* 2, *and consequently all even numbers, except* 2, *are composite.*

Hence, in finding the prime numbers, we cancel as composite all even numbers except 2.

Thus, 3, ~~4~~, 5, ~~6~~, 7, ~~8~~, 9, ~~10~~, 11, ~~12~~, and so on.

PROP. II.—*Each number in the series of odd numbers is* 2 *greater than the number immediately preceding it.*

Thus, the numbers left after cancelling the even numbers are

$$3 \quad \overbrace{3+2}^{5} \quad \overbrace{5+2}^{7} \quad \overbrace{7+2}^{9} \quad \overbrace{9+2}^{11} \quad \overbrace{11+2}^{13}, \text{ and so on.}$$

PROP. III.—*In the series of odd numbers, every* THIRD *number from* 3 *is divisible by* 3, *every* FIFTH *number from* 5 *is divisible by* 5, *and so on with each number in the series.*

This proposition may be shown thus :

According to Prop. II, the series of odd numbers increase by 2's. Hence the *third* number from 3 is found by adding 2 *three* times, thus :

$$3 \qquad \overbrace{3+2}^{5} \qquad \overbrace{3+2+2}^{7} \qquad \overbrace{3+2+2+2}^{9}$$

From this it will be seen that 9, the third number from 3, is composed of 3, plus 3 *twos*, and is divisible by 3 (Prop. II); and so with the third number from 9, and so on.

By the same course of reasoning, each fifth number in the series, counting from 5, may be shown to be divisible by 5 ; and so with any other number in the series ; hence the following method of finding the prime numbers.

ILLUSTRATION OF PROCESS.

172. PROB.—**To find all the Prime Numbers from 1 to any given number.**

Find all the prime numbers from 1 to 63.

1	3	5	7	9	11	13
				3		
15	17	19	21	23	25	27
3 5			3 7		5	3 9
29	31	33	35	37	39	41
		3 11	5 7		3 13	
43	45	47	49	51	53	55
	3 5 9 15		7	3 17		5 11
	57	59	61	63		
	3 19			3 7 9 21		

EXPLANATION.—1. Arrange the series of odd numbers in lines, at convenient distances from each other, as shown in illustration.

2. Write 3 under every *third* number from 3, 5 under every *fifth* number from 5, 7 under every *seventh* number from 7, and so on with each of the other numbers.

3. The terms under which the numbers are written are composite, and the numbers written under are their factors, according to Prop. III. All the remaining numbers are prime.

Hence all the prime numbers from 1 to 63 are 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61.

FACTORING.

175. A ***Factor*** is one of the *equal parts* of a number, or one of its exact divisors.

Thus, 15 is composed of 3 *fives* or 5 *threes;* hence, 5 and 3 are factors of 15.

176. A ***Prime Factor*** is a prime number which is a factor of a given number.

Thus, 5 is a prime factor of 30.

177. A ***Composite Factor*** is a composite number which is a factor of a given number.

Thus, 6 is a composite factor of 24.

178. ***Factoring*** is the process of resolving a composite number into its factors.

179. An ***Exponent*** is a small figure placed at the right of a number and a little above, to show how many times the number is used as a factor.

Thus, $3^5 = 3 \times 3 \times 3 \times 3 \times 3$. The 5 at the right of 3 denotes that the 3 is used 5 times as a factor.

180. A ***Common Factor*** is a number that is a factor of each of two or more numbers.

Thus, 3 is a factor of 6, 9, 12, and 15; hence is a common factor.

181. The ***Greatest Common Factor*** is the greatest number that is a factor of each of two or more numbers.

Thus, 4 is the greatest number that is a factor of 8 and also of 12. Hence 4 is the greatest common factor of 8 and 12.

ILLUSTRATION OF PROCESS.

182. Find the prime factors of 462.

```
2 ) 462
3 ) 231
  7 ) 77
      11
```

EXPLANATION.—1. We observe that the number 462 is divisible by 2, the smallest prime number. Hence we divide by 2.

2. We observe that the first quotient, 231, is divisible by 3, which is a prime number. Hence we divide by 3.

3. We observe that the second quotient, 77, is divisible by 7, which is a prime number. Hence we divide by 7.

4. The third quotient, 11, is a prime number. Hence the prime factors of 462 are 2, 3, 7, and 11; that is, $462 = 2 \times 3 \times 7 \times 11$.

CANCELLATION.

PREPARATORY PROPOSITIONS.

185. PROP. I.—*Rejecting a factor from a number divides the number by that factor.*

Thus, $72 = 24 \times 3$. Hence, rejecting the factor 3 from 72, we have 24, the quotient of 72 divided by 3.

PROP. II.—*Dividing both dividend and divisor by the same number does not change the quotient.*

Thus, $60 \div 12 = 20$ threes $\div$ 4 threes $= 5$.

Observe that the unit *three*, in 20 *threes* $\div$ 4 *threes*, does not in any way affect the size of the quotient; therefore, it may be rejected and the quotient will not be changed.

Hence, dividing both the dividend 60 and the divisor 12 by 3 does not change the quotient.

ILLUSTRATION OF PROCESS.

186. Ex. 1. Divide 462 by 42.

$$\frac{6)\,462}{6)\,42} = \frac{77}{7} = 11.$$

EXPLANATION.—We divide both the divisor and dividend by 6. According to Prop. II, the quotient is not changed.

Hence, $77 \div 7 = 462 \div 42 = 11$.

Ex. 2. Divide $65 \times 24 \times 55$ by $39 \times 15 \times 35$.

$$\frac{\overset{13}{\not{6}\not{5}} \times \overset{8}{\not{2}\not{4}} \times \overset{11}{\not{5}\not{5}}}{\underset{3}{\not{3}\not{9}} \times \underset{\not{3}}{\not{1}\not{5}} \times \underset{7}{\not{3}\not{5}}} = \frac{8 \times 11}{3 \times 7} = \frac{88}{21} = 4\tfrac{4}{21}.$$

EXPLANATION.—1. We divide any factor in the dividend by any number that will divide a factor in the divisor.

Thus, 65 in the dividend and 15 in the divisor are divided each by 5. In the same manner, 55 and 35, 13 and 39, 24 and 3 are divided.

The remaining factors, 8 and 11, in the dividend are prime to each of the remaining factors in the divisor. Hence, no further division can be performed.

2. We divide the product of 8 and 11, the remaining factors in the dividend, by the product of 3 and 7, the remaining factors in the divisor, and find as a quotient $4\frac{4}{21}$, which, according to (**185**—II), is equal to the quotient of $65 \times 24 \times 55$ divided by $39 \times 15 \times 35$.

All similar cases may be treated in the same manner.

GREATEST COMMON DIVISOR.

190. A ***Common Divisor*** is a number that is an exact divisor of each of two or more numbers.

Thus, 5 is a divisor of 10, 15, and 20.

191. The ***Greatest Common Divisor*** is the greatest number that is an exact divisor of each of two or more numbers.

Thus, 3 is the greatest exact divisor of each of the numbers 6 and 15. Hence 3 is their greatest common divisor.

192. Numbers are ***prime to each other*** when they have no common divisor besides 1; thus, 8, 9, 25.

METHOD BY FACTORING.

PREPARATORY PROPOSITION.

193. Illustrate the following proposition by examples.

The greatest common divisor is the product of the prime factors that are common to all the given numbers; thus,

$$42 = 7 \times 2 \times 3 = 7 \text{ sixes};$$

$$66 = 11 \times 2 \times 3 = 11 \text{ sixes}.$$

7 and 11 being prime to each other, 6 must be the greatest common divisor of 7 sixes and 11 sixes. But 6 is the product of 2 and 3, the common prime factors; hence the greatest common divisor of 42 and 64 is the product of their common prime factors.

ILLUSTRATION OF PROCESS.

194. Prob. I.—**To find the Greatest Common Divisor of two or more numbers by factoring.**

Find the greatest common divisor of 98, 70, and 154.

(1.)

2) 98	2) 70	2) 154
7) 49	7) 35	7) 77
7	5	11

Or, (2.)

2)	98	70	154
7)	49	35	77
	7	5	11

$2 \times 7 =$ greatest common divisor.

EXPLANATION.—1. We resolve each of the numbers into their prime factors, as shown in (1) or (2).

2. We observe that 2 and 7 are the only prime factors common to all the numbers. Hence the product of 2 and 7, or 14, according to (**193**), is the greatest common divisor of 98, 70, and 154.

The greatest common divisor of any two or more numbers is found in the same manner.

METHOD BY DIVISION.

PREPARATORY PROPOSITIONS.

197. Let the two following propositions be carefully studied and illustrated by other examples, before attempting to find the greatest common divisor by this method.

PROP. I.—*The greatest common divisor of two numbers is the greatest common divisor of the smaller number and their difference.*

Thus, 3 is the greatest common divisor of 15 and 27.

Hence, $15 = 5$ threes and $27 = 9$ threes;

and 9 threes − 5 threes = 4 threes.

But 9 and 5 are prime to each other; hence, 4 and 5 must be prime to each other, for if not, their common divisor will divide their sum, according to (**169**—II), and be a common divisor of 9 and 5.

Therefore, 3 is the greatest common divisor of 5 threes and 4 threes, or of 15 and 12. Hence, the greatest common divisor of two numbers is the greatest common divisor of the smaller number and their difference.

PROP. II.—*The greatest common divisor of two numbers is the greatest common divisor of the smaller number and the remainder after the division of the greater by the less.*

This proposition may be illustrated thus:

$$22 - 6 = 16$$
$$16 - 6 = 10$$
$$10 - 6 = 4$$

1. Subtract 6 from 22, then from the difference, 16, etc., until a remainder less than 6 is obtained.

2. Observe that the number of times 6 has been subtracted is the quotient of 22 divided by 6, and hence that the remainder, 4, is the remainder after the division of 22 by 6.

3. According to Prop. 1, the greatest common divisor of 22 and 6 is

the greatest common divisor of their difference, 16, and 6. It is also, according to the same Proposition, the greatest common divisor of 10 and 6, and of 4 and 6. But 4 is the remainder after division and 6 the smaller number. Hence the greatest common divisor of 22 and 6 is the greatest common divisor of the *smaller number* and the *remainder* after division.

ILLUSTRATION OF PROCESS.

198. Prob. II.—**To find the Greatest Common Divisor of two or more numbers by continued division.**

Find the greatest common divisor of 28 and 176.

```
28)176(6
   168
   ---
     8)28(3
       24
       --
        4)8(2
          8
          -
          0
```

Explanation.—1. We divide 176 by 28 and find 8 for a remainder; then we divide 28 by 8, and find 4 for a remainder; then we divide 8 by 4, and find 0 for a remainder.

2. According to Prop. II, the greatest common divisor of 28 and 176 is the same as the greatest common divisor of 28 and 8, also of 8 and 4. But 4 is the greatest common divisor of 8 and 4. Hence 4 is the greatest common divisor of 28 and 176.

LEAST COMMON MULTIPLE.

PREPARATORY PROPOSITIONS.

211. Prop. I.—*A multiple of a number contains as a factor each prime factor of the number as many times as it enters into the number.*

Thus, 60, which is a multiple of 12, contains 5 times 12, or 5 times $2 \times 2 \times 3$, the prime factors of 12. Hence, each of the prime factors of 12 enters as a factor into 60 as many times as it enters into 12.

Prop. II.—*The least common multiple of two or more given numbers must contain, as a factor, each prime factor in those numbers the greatest number of times that it enters into any one of them.*

Thus, $12 = 2 \times 2 \times 3$, and $9 = 3 \times 3$. The prime factors in 12 and 9 are 2 and 3. A multiple of 12, according to Prop. I, must contain 2 as a factor twice and 3 once. A multiple of 9, according to the same proposition, must contain 3 as a factor twice. Hence a number which is a multiple of both 12 and 9 must contain 2 as a factor twice and 3 twice, which is equal to $2 \times 2 \times 3 \times 3 = 36$. Hence 36 is the least common multiple of 12 and 9.

DEFINITIONS.

212. A ***Multiple*** of a number is a number that is exactly divisible by the given number.

Thus, 24 is divisible by 8; hence, 24 is a multiple of 8.

213. A ***Common Multiple*** of two or more numbers is a number that is exactly divisible by each of them.

Thus, 36 is divisible by each of the numbers 4, 9, and 12; hence, 36 is a common multiple of 4, 9, and 12.

214. The ***Least Common Multiple*** of two or more numbers is the least number that is exactly divisible by each of them.

Thus, 24 is the least number that is divisible by each of the numbers 6 and 8; hence, 24 is the least common multiple of 6 and 8.

METHOD BY FACTORING.

ILLUSTRATION OF PROCESS.

215. PROB. I.—**To find, by factoring, the least common multiple of two or more numbers.**

Find the least common multiple of 18, 24, 15, and 35.

3	18	24	15	35
2	6	8	5	35
5	3	4	5	35
	3	4		7

EXPLANATION.—1. We observe that 3 is a factor of 18, 24, and 15. Dividing these numbers by 3, we write the quotients with 35, in the second line.

2. Observing that 2 is a factor of 6 and 8, we divide as before, and find the third line of numbers. Dividing

by 5, we find the fourth line of numbers, which are prime to each other ; hence cannot be further divided.

3. Observe the divisors 3, 2, and 5 are all the factors that are common to any two or more of the given numbers, and the quotients 3, 4, and 7 are the factors that belong each only to one number. Therefore the divisors and quotients together contain each of the prime factors of 18, 24, 15, and 35 as many times as it enters into any one of these numbers. Thus, the divisors 3 and 2, with the quotient 3, are the prime factors of 18; and so with the other numbers.

Hence, according to (**211**—II), the continued product of the divisors 3, 2, and 5, and the quotients 3, 4, and 7, which is equal to 2520, is the least common multiple of 18, 24, 15, and 35.

METHOD BY GREATEST COMMON DIVISOR.

ILLUSTRATION OF PROCESS.

218. Prob. II.—**To find, by using the greatest common divisor, the least common multiple of two or more numbers.**

Find the least common multiple of 195 and 255.

Explanation.—1. We find the greatest common divisor of 195 and 255, which is 15.

2. The greatest common divisor, 15, according to (**193**), contains all the prime factors that are common to 195 and 255. Dividing each of these numbers by 15, we find the factors that are not common, namely, 13 and 17.

3. The common divisor 15 and the quotient 13 contain all the prime factors of 195, and the common divisor 15 and the quotient 17 contain all the prime factors of 255.

Hence, according to (**211**—II), the continued product of the common divisor 15 and the quotients 13 and 17, which is 3315, is the least common multiple of 195 and 255.

The least common multiple of any two numbers is found in the same manner; and of three or more by finding first the least common multiple of two of them, then finding the least common multiple of the multiple thus found and the third number, and so on.

REVIEW AND TEST QUESTIONS.

223. 1. Define Prime Number, Composite Number, and Exact Divisor, and illustrate each by an example.

2. What is meant by an Odd Number? An Even Number?

3. Show that if an even number is divisible by an odd number, the quotient must be even.

4. Name the prime numbers from 1 to 40.

5. Why are all even numbers except 2 composite?

6. State how you would show, in the series of odd numbers, that every fifth number from 5 is divisible by 5.

7. What is a Factor? A Prime Factor?

8. What are the prime factors of 81? Of 64? Of 125?

9. Show that rejecting the same factor from the divisor and dividend does not change the quotient.

10. Explain Cancellation, and illustrate by an example.

11. Give reasons for calling an exact divisor a *measure*.

12. What is a Common Measure? The Greatest Common Measure? Illustrate each answer by an example.

13. Show that the greatest common divisor of 42 and 114 is the greatest common divisor of 42 and the remainder after the division of 114 by 42.

14. Explain the rule for finding the greatest common divisor by factoring; by division.

15. Why must we finally get a common divisor if the greater of two numbers be divided by the less, and the divisor by the remainder, and so on?

16. What is a Multiple? The Least Common Multiple?

17. Explain how the Least Common Multiple of two or more numbers is found by using their greatest common divisor.

18. Prove that a number is divisible by 9 when the sum of its digits is divisible by 9.

19. Prove that a number is divisible by 11 when the difference of the sums of the digits in the odd and even places is zero.

FRACTIONS.

225. A ***Fractional Unit*** is one of the *equal parts* of anything regarded as a whole.

226. A ***Fraction*** is *one* or *more* of the equal parts of anything regarded as a whole.

227. The ***Unit of a Fraction*** is the unit or whole which is considered as divided into equal parts.

228. The ***Numerator*** is the number above the dividing line in the expression of a fraction, and indicates how many *equal parts* are in the *fraction.*

229. The ***Denominator*** is the number below the dividing line in the expression of a fraction, and indicates how many *equal parts* are in the *whole.*

230. The ***Terms*** of a fraction are the numerator and denominator.

231. Taken together, the *terms* of a *fraction* are called a ***Fraction***, or ***Fractional Number.***

232. Hence, the word ***Fraction*** means one or more of the equal parts of anything, or the *expression* that denotes one or more of the equal parts of anything.

PRINCIPLES OF REDUCTION.

235. Illustrate the following principles with examples:

PRIN. I.—*The numerator and denominator of a fraction represent, each, parts of the same size;* thus,

$\frac{3}{7}$ $\frac{4}{5}$

(1.)  (2.) 

Observe in illustration (1) the denominator 7 represents the whole or 7 *sevenths,* and the numerator 3 represents 3 *sevenths;* in illustration (2), the denominator represents 5 *fifths,* and the numerator 4 *fifths.* Hence the numerator and denominator of a fraction represent parts of the same size.

PRIN. II.—*Multiplying both the terms of a fraction by the same number does not change the value of the fraction*; thus,

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Be particular to observe in the illustration that the *amount* expressed by the 2 in the numerator or the 3 in the denominator of $\frac{2}{3}$ is not changed by making each part into 4 equal parts; therefore $\frac{2}{3}$ and $\frac{8}{12}$ express, each, the same amount of the same whole.

Hence, multiplying the numerator and denominator by the same number means, so far as the real fraction is concerned, *dividing the equal parts in each into as many equal parts as there are units in the number by which they are multiplied.*

PRIN. III.—*Dividing both terms of a fraction by the same number does not change the value of the fraction*; thus,

$$\frac{9 \div 3}{12 \div 3} = \frac{3}{4}$$

The amount expressed by the 9 in the numerator or the 12 in the denominator of $\frac{9}{12}$ is not changed by putting every 3 parts into *one*, as will be seen from the illustration.

Hence $\frac{9}{12}$ and $\frac{3}{4}$ express each the same amount of the same whole, and dividing the numerator and denominator by the same number means *putting as many parts in each into one as there are units in the number by which they are divided.*

DEFINITIONS.

236. The ***Value*** of a fraction is the amount which it represents.

237. ***Reduction*** is the process of changing the terms of a fraction without altering its value.

238. A fraction is reduced to ***Higher Terms*** when its numerator and denominator are expressed by larger numbers. Thus, $\frac{4}{5} = \frac{8}{10}$.

239. A fraction is reduced to ***Lower Terms*** when its numerator and denominator are expressed by smaller numbers. Thus, $\frac{8}{12} = \frac{4}{6}$.

240. A fraction is expressed in its ***Lowest Terms*** when its numerator and denominator are *prime* to each other.

Thus, in $\frac{4}{9}$, the numerator and denominator 4 and 9 are prime to each other; hence the fraction is expressed in its *lowest terms.*

241. A ***Common Denominator*** is a denominator that belongs to two or more fractions.

242. The ***Least Common Denominator*** of two or more fractions is the least denominator to which they can all be reduced.

243. A ***Proper Fraction*** is one whose numerator is less than the denominator; as $\frac{2}{3}$, $\frac{5}{7}$.

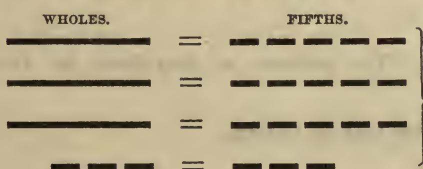
244. An ***Improper Fraction*** is one whose numerator is equal to, or greater than, the denominator; as $\frac{5}{5}$, $\frac{8}{3}$.

245. A ***Mixed Number*** is a number composed of an integer and a fraction; as $4\frac{2}{3}$, $18\frac{5}{7}$.

ILLUSTRATION OF PROCESS.

246. Prob. I.—**To reduce a whole or mixed number to an improper fraction.**

1. Reduce $3\frac{3}{5}$ equal lines to *fifths.*



$= 18$ *fifths* $= \frac{18}{5}$.

Explanation,—Each whole line is equal to 5 *fifths*, as shown in the illustration; 3 lines must therefore be equal to 15 *fifths*. 15 *fifths* + 3 *fifths* = 18 *fifths*. Hence in $3\frac{3}{5}$ lines there are $\frac{18}{5}$ of a line.

249. Prob. II.—**To reduce an improper fraction to an integer or a mixed number.**

1. Reduce 9 *fourths* of a line to whole lines.

$$\frac{9}{4} = 9 \div 4 \quad = \quad 2\frac{1}{4}$$

Explanation.—A whole line is composed of 4 *fourths.* Hence, to make the 9 *fourths* of a line into whole lines, we put every *four* parts into *one,* as shown in the illustration, or divide the 9 by 4, which gives 2 wholes and 1 of the *fourths* remaining.

252. Prob. III.—**To reduce a fraction to higher terms.**

1. Reduce $\frac{2}{3}$ of a line to twelfths.

$$\frac{2}{3} \quad = \quad \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Explanation.—1. To make a *whole,* which is already in *thirds,* into 12 equal parts, *each third* must be made into *four* equal parts.

2. The numerator of the given fraction expresses 2 *thirds,* and the denominator 3 *thirds;* making each third in both into *four* equal parts (**245**—II), as shown in the illustration, the new numerator and denominator will each contain 4 times as many parts as in the given fraction.

Hence, $\frac{2}{3}$ of a line is reduced to *twelfths* by multiplying both numerator and denominator by 4.

255. Prob. IV.—**To reduce a fraction to lower terms.**

Reduce $\frac{9}{12}$ of a given line to *fourths.*

$$\frac{9 \div 3}{12 \div 3} \quad = \quad \frac{3}{4}$$

Explanation.—1. To make into 4 equal parts or *fourths* a whole, which is already in 12 equal parts, or twelfths, every 3 of the 12 parts must be put into one.

2. The numerator of the given fraction expresses 9 *twelfths*, and the denominator 12 *twelfths;* putting every 3 *twelfths* into *one*, in both (**235**—III), as shown in the illustration, the new numerator and denominator will each contain *one-third* as many parts as in the given fraction.

Hence, $\frac{9}{12}$ of a line is reduced to *fourths* by dividing both numerator and denominator by 3.

258. Prob. V.—**To change fractions to equivalent ones having a common denominator.**

1. Reduce $\frac{2}{3}$ and $\frac{3}{4}$ of a line to fractions having a common denominator.

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

(1.) 

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

(2.) 

Explanation.—1. We find the least common multiple of the denominators 3 and 4, which is 12.

2. We reduce each of the fractions to twelfths (**252**—III), as shown in illustrations (1) and (2).

ADDITION AND SUBTRACTION.

PREPARATORY PROPOSITIONS.

261. Prop. I.—*Fractional units of the* SAME KIND, *that are fractions of the same whole, are added or subtracted in the same manner as integral units.*

Thus, $\frac{2}{5}$ of a yard can be added to or subtracted from $\frac{4}{5}$ of a yard, because they are each fifths of *one yard.* But $\frac{2}{5}$ of a *yard* cannot be added to or subtracted from $\frac{4}{5}$ of a *day.*

PROP. II.—*Fractions expressed in different fractional units must be changed to equivalent fractions having the same fractional unit, before they can be added or subtracted.*

262. PROB. I.—**To find the sum of any two or more given fractions.**

1. Find the sum of $\frac{4}{9} + \frac{5}{6} + \frac{7}{8}$.

$$\left.\begin{array}{l}\frac{4}{9}=\frac{32}{72}\\ \frac{5}{6}=\frac{60}{72}\\ \frac{7}{8}=\frac{63}{72}\end{array}\right\}=\frac{155}{72}=2\tfrac{11}{24}.$$

EXPLANATION.—1. We reduce the fractions to the same fractional unit, by reducing them to their least common denominator, which is 72 (**258**—V).

2. We find the sum of the numerators, 155, and write it over the common denominator, 72, and reduce $\frac{155}{72}$ to $2\frac{11}{24}$.

The sum of any number of fractions may be found in the same manner.

266. PROB. I.—**To find the difference of any two given fractions.**

1. Find the difference between $\frac{7}{8}$ and $\frac{5}{12}$.

$$\frac{7}{8}-\frac{5}{12}=\frac{21}{24}-\frac{10}{24}=\frac{11}{24}$$

EXPLANATION.—1. We reduce the given fractions to their least common denominator, which is 24.

2. We find the difference of the numerators, 21 and 10, and write it over the common denominator, giving $\frac{11}{24}$, the required difference.

2. Find the difference between $35\frac{2}{3}$ and $16\frac{3}{4}$.

$$\begin{array}{r}35\tfrac{2}{3}=35\tfrac{8}{12}\\ 16\tfrac{3}{4}=\underline{16\tfrac{9}{12}}\\ 18\tfrac{11}{12}\end{array}$$

EXPLANATION.—1. We reduce the $\frac{2}{3}$ and $\frac{3}{4}$ to their least common denominator.

2. $\frac{9}{12}$ cannot be taken from $\frac{8}{12}$; hence, we increase the $\frac{8}{12}$ by $\frac{12}{12}$ or 1, taken from the 35. We now subtract $\frac{9}{12}$ from $\frac{20}{12}$, leaving $\frac{11}{12}$.

3. We subtract 16 from the remaining 34, leaving 18, which united with $\frac{11}{12}$ gives $18\frac{11}{12}$, the required difference.

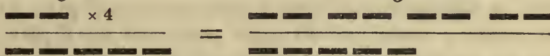
MULTIPLICATION.

PREPARATORY PROPOSITIONS.

269. The following propositions must be mastered perfectly, to understand and explain the process in multiplication and division of fractions.

PROP. I.—*Multiplying the numerator of a fraction, while the denominator remains unchanged, multiplies the fraction;* thus,

$$\frac{2 \times 4}{5} = \frac{8}{5}$$



Observe that since the denominator is not changed, the size of the parts remain the same. Hence the fraction $\frac{2}{5}$ is multiplied by 4, as shown in the illustration, by multiplying the numerator by 4.

PROP. II.—*Dividing the denominator of a fraction while the numerator remains unchanged multiplies the fraction;* thus,

$$\frac{2}{12 \div 4} = \frac{2}{3}$$

(1.) (2.)

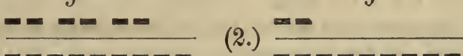
Observe that in (1) the whole is made into 12 equal parts. By putting every 4 of these parts into one, or dividing the denominator by 4, the whole, as shown in (2), is made into 3 equal parts, and each of the 2 parts in the numerator is 4 *times* 1 *twelfth.*

Hence, dividing the denominator of $\frac{2}{12}$ by 4, the number of parts in the numerator remaining the same, multiplies the fraction by 4.

PROP. III.—*Dividing the numerator of a fraction while the denominator remains unchanged divides the fraction;* thus,

$$\frac{6 \div 3}{9} = \frac{2}{9}$$

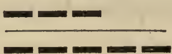
(1.) $\frac{1}{3}$ of (2.)



In (1) the numerator 6 expresses the parts taken, and *one-third* of these 6 parts, as shown by comparing (1) and (2), the denominator remaining the same, is one-third of the value of the fraction. Hence, the fraction $\frac{6}{9}$ is divided by 3 by dividing the numerator by 3.

PROP. IV.—*Multiplying the denominator of a fraction while the numerator remains unchanged divides the fraction;* thus,

$$\frac{3}{5 \times 2} = \frac{3}{10}$$

(1.)  (2.) 

In (1) the whole is made into 5 equal parts; multiplying the denominator by 2, or making each of these 5 parts into 2 equal parts, as shown in (2), the whole is made into 10 equal parts, and the 3 parts in the numerator are one-half the size they were before.

Hence, multiplying the denominator of $\frac{3}{5}$ by 2, the numerator remaining the same, divides the fraction by 2.

ILLUSTRATION OF PROCESS.

271. PROB. I.—**To multiply a fraction by an integer.**

1. Multiply $\frac{4}{9}$ by 7.

SOLUTION.—1. According to (**269**—I), multiplying the numerator, the denominator remaining the same, multiplies the fraction. Hence, 7 times $\frac{4}{9}$ is equal to $\frac{4 \times 7}{9} = \frac{28}{9} = 3\frac{1}{9}$.

2. According to (**269**—II), a fraction is also multiplied by dividing the denominator.

274. PROB. II.—**To find any given part of an integer,** or *To multiply an integer by a fraction.*

1. Find $\frac{4}{5}$ of \$395.

SOLUTION.—1. We find the $\frac{1}{5}$ of \$395 by dividing it by 5. Hence the *first step*, \$395 ÷ 5 = \$79.

2. Since \$79 is 1 fifth of \$395, four times \$79 will be 4 fifths. Hence the *second step*, \$79 × 4 = \$316.

To avoid fractions until the final result, we multiply by the numerator first, then divide by the denominator.

276. PROB. III.—**To find any given part of a fraction,** or *To multiply a fraction by a fraction.*

Find the $\frac{2}{3}$ of $\frac{3}{4}$ of a given line.

FIRST STEP.

$$\frac{1}{3} \text{ of } \frac{3}{4} = \frac{3}{4 \times 3} = \frac{3}{12}$$

SECOND STEP.

$$\frac{3}{12} \times 2 = \frac{6}{12} = \frac{1}{2}$$

EXPLANATION.—According to (**269**—IV), a fraction is divided by multiplying its denominator. Hence we find the $\frac{1}{3}$ of $\frac{3}{4}$ by multiplying the denominator 4 by 3, as shown in *First Step.*

Having found 1 *third* of $\frac{3}{4}$, we find 2 *thirds*, according to (**269**—I), by multiplying the numerator of $\frac{3}{12}$ by 2, as shown in *Second Step.*

In practice, the Second Step is usually made the First.

279. PROB. IV.—**To multiply by a mixed number.**

Multiply 372 by $6\frac{5}{7}$.

$$\begin{array}{r} 372 \\ 6\frac{5}{7} \\ \hline 2233 \\ 265\frac{5}{7} \\ \hline 2497\frac{5}{7} \end{array}$$

EXPLANATION. — 1. In multiplying by a mixed number, the multiplicand is taken separately (**92**) as many times as there are units in the multiplier, and such a part of a time as is indicated by the fraction in the multiplier; hence,

2. We multiply 372 by $6\frac{5}{7}$ by multiplying first by 6, which gives the product 2232, and adding to this product $\frac{5}{7}$ of 372, which is $265\frac{5}{7}$ (**274**), giving $2497\frac{5}{7}$, the product of 372 and $6\frac{5}{7}$.

282. PROB. V.—**To multiply when both multiplicand and multiplier are mixed numbers.**

Multiply $86\frac{2}{3}$ by $54\frac{5}{7}$.

(1.) $86\frac{2}{3} = \frac{260}{3}$; $54\frac{5}{7} = \frac{383}{7}$.

(2.) $\frac{260}{3} \times \frac{383}{7} = \frac{99580}{21} = 4741\frac{19}{21}$.

EXPLANATION.—1. We reduce, as shown in (1), both multiplicand and multiplier to improper fractions.

2. We multiply, as shown in (2), the numerators together for the numerator of the product, and the denominators together for the denominators of the product (**275**), then reduce the result to a mixed number.

DIVISION.

ILLUSTRATION OF PROCESS.

287. PROB. I.—**To divide a fraction by an integer.**

1. Divide $\frac{8}{9}$ by 4.

(1.) $\frac{8}{9} \div 4 = \frac{8 \div 4}{9} = \frac{2}{9}$

(2.) $\frac{8}{9} \div 4 = \frac{8}{9 \times 4} = \frac{8}{36} = \frac{2}{9}$

EXPLANATION.—1. According to (**269**—III), a fraction is divided by dividing the numerator. Hence we divide $\frac{8}{9}$ by 4, as shown in (1), by dividing the numerator 8 by 4.

2. According to (**269**—IV), a fraction is divided by multiplying the denominator. Hence we divide $\frac{8}{9}$ by 4, as shown in (2), by multiplying the denominator by 4, and reducing the result to its lowest terms.

290. PROB. II.—**To divide by a fraction.**

1. How many times is $\frac{3}{5}$ of a given line contained in twice the same line?

FIRST STEP.

2 lines $=$ $\frac{10}{5}$ of a line.

SECOND STEP.

$\frac{10}{5} \div \frac{3}{5} = \frac{10}{3} = 3\frac{1}{3}$.

EXPLANATION.—1. We can find how many times one number is contained in another, only when both are of the same denomination (**155**).

Hence we first reduce, as shown in *First Step*, the 2 lines to 10 *fifths* of a line; the same *fractional* denomination as the divisor, 3 *fifths*.

2. The 3 fifths in the divisor, as shown in *Second Step*, are contained in the 10 fifths in the dividend 3 times, and 1 part remaining, which makes $\frac{1}{3}$ of a time. Hence 2 equal lines contain $\frac{3}{5}$ of one of them $3\frac{1}{3}$ times.

Observe the following regarding this solution:

(1.) The dividend is reduced to the same fractional denomination as the divisor by multiplying it by the denominator of the divisor; and when reduced, the division is performed by dividing the numerator of the dividend by the numerator of the divisor.

(2.) By inverting the terms of the divisor, these two operations are expressed by the sign of multiplication. Thus, $2 \div \frac{3}{5} = 2 \times \frac{5}{3}$, which means that 2 is to be multiplied by 5, and the product divided by 3.

2. How many times is $\frac{1}{2}$ of a given line contained in $\frac{2}{3}$ of it?

FIRST STEP.

$$\frac{2}{3} = \frac{4}{6}$$

$$\frac{1}{2} = \frac{3}{6}$$

SECOND STEP.

$$\frac{4}{6} \div \frac{3}{6} = \frac{4}{3} = 1\frac{1}{3}$$

EXPLANATION.—1. We reduce, as shown in *First Step*, the dividend $\frac{2}{3}$ and the divisor $\frac{1}{2}$ both to *sixths* (**155**—1).

2. We divide the $\frac{4}{6}$ by $\frac{3}{6}$ by dividing the numerator of the dividend by the numerator of the divisor. The $\frac{3}{6}$ is contained in $\frac{4}{6}$, as shown in *Second Step*, $1\frac{1}{3}$ times. Hence $\frac{1}{2}$ is contained $1\frac{1}{3}$ times in $\frac{2}{3}$.

291. When dividing by a fraction, we abbreviate the work by *inverting the divisor*, as follows:

1. In reducing the dividend and divisor to the same fractional unit, the product of the denominators is taken as the common denominator,

and each numerator is multiplied by the denominator of the other fraction, thus,

$$\frac{5}{7} \div \frac{2}{3} = \frac{5 \times 3}{7 \times 3} \div \frac{2 \times 7}{3 \times 7} = \frac{15}{21} \div \frac{14}{21} = \frac{15}{14} \begin{array}{l}\text{Numerator of dividend.}\\ \text{Numerator of divisor.}\end{array}$$

2. By inverting the divisor, thus, $\frac{5}{7} \div \frac{2}{3} = \frac{5}{7} \times \frac{3}{2} = \frac{15}{14}$, the numerators 15 and 14 are found at once, without going through the operation of finding the common denominator. Hence the rule, *Invert the terms of the divisor and proceed as in multiplication.*

294. PROB. III.—**To divide when the divisor or dividend is a mixed number, or both.**

1. Divide 48 by $4\frac{2}{3}$.

(1.) $48 \div 4\frac{2}{3} = 48 \div \frac{14}{3}$

(2.) $48 \div \frac{14}{3} = 48 \times \frac{3}{14} = 10\frac{2}{7}$

EXPLANATION.—1. We reduce the divisor $4\frac{2}{3}$, as shown in (1), to the improper fraction $\frac{14}{3}$.

2. We invert the divisor, as shown in (2), according to (**291**), and multiply the 48 by $\frac{3}{14}$, giving $10\frac{2}{7}$ as the quotient of 48 divided by $4\frac{2}{3}$.

2. Divide $8\frac{6}{7}$ by $3\frac{7}{8}$.

(1.) $8\frac{6}{7} \div 3\frac{7}{8} = \frac{62}{7} \div \frac{31}{8}$

(2.) $\frac{62}{7} \div \frac{31}{8} = \frac{\overset{2}{\cancel{62}}}{7} \times \frac{8}{\cancel{31}} = \frac{16}{7} = 2\frac{2}{7}$

EXPLANATION.—1. We reduce the dividend and divisor, as shown in (1), to improper fractions, giving $\frac{62}{7} \div \frac{31}{8}$.

2. We invert the divisor, $\frac{31}{8}$, as shown in (2), according to (**291**), and cancel 31 in the numerator 62 and denominator 31 (**186**), giving $\frac{16}{7}$, or $2\frac{2}{7}$. Hence, $8\frac{6}{7} \div 3\frac{7}{8} = 2\frac{2}{7}$.

COMPLEX FRACTIONS.

297. Certain results are obtained by dividing the numerator and denominator of a fraction by a number that is not an exact divisor of each, which are fractional in form, but are not fractions according to the *definition* of a fraction. These fractional forms are called *Complex Fractions.*

298. A ***Complex Fraction*** is an expression in the form of a fraction, having a fraction in its numerator or denominator, or in both; thus, $\frac{\frac{3}{5}}{7}$, $\frac{4}{5\frac{2}{3}}$, $\frac{2\frac{1}{2}}{6\frac{1}{5}}$.

299. A ***Simple Fraction*** is a fraction having a whole number for its numerator and for its denominator.

PROBLEMS IN COMPLEX FRACTIONS.

300. PROB. I.—**To reduce a complex fraction to a simple fraction.**

Reduce $\frac{4\frac{2}{3}}{7\frac{3}{4}}$ to a simple fraction.

$$\frac{4\frac{2}{3}}{7\frac{3}{4}} = \frac{4\frac{2}{3} \times 12}{7\frac{3}{4} \times 12} = \frac{56}{93}$$

EXPLANATION.—1. We find the least common multiple of the denominators of the partial fractions $\frac{2}{3}$ and $\frac{3}{4}$, which is 12.

2. Multiplying both terms of the complex fraction by 12 (**235**—II), which is divisible by the denominators of the partial fractions, $\frac{2}{3}$ and $\frac{3}{4}$, reduces each term to a whole number. $4\frac{2}{3} \times 12 = 56$; $7\frac{3}{4} \times 12 = 93$. Therefore $\frac{4\frac{2}{3}}{7\frac{3}{4}}$ is equal to the simple fraction $\frac{56}{93}$.

302. The three classes of complex fractions are forms of expressing three cases of division; thus,

(1.) $\frac{5\frac{2}{3}}{7} = 5\frac{2}{3} \div 7$. A mixed number divided by an integer.

(2.) $\frac{32}{9\frac{5}{6}} = 32 \div 9\frac{5}{6}$. An integer divided by a mixed number.

(3.) $\frac{8\frac{3}{5}}{2\frac{2}{3}} = 8\frac{3}{5} \div 2\frac{2}{3}$. A mixed number divided by a mixed number.

Hence, when we reduce a complex fraction to a simple fraction, as directed (**301**), we in fact reduce the dividend and divisor to a common denominator, and reject the denominator by indicating the division of the numerator of the dividend by the numerator of the divisor; thus,

(1.) $\frac{5\frac{3}{4}}{2\frac{2}{3}} = \frac{5\frac{3}{4} \times 12}{2\frac{2}{3} \times 12} = \frac{69}{32}$, according to (**300**).

(2.) $5\frac{3}{4} \div 2\frac{2}{3} = \frac{23}{4} \div \frac{8}{3}$, and $\frac{23}{4} \div \frac{8}{3} = \frac{69}{12} \div \frac{32}{12} = \frac{69}{32}$,

the same result as obtained by the method of multiplying by the least common multiple of the denominators of the partial fractions.

304. PROB. II.—**To reduce a fraction to any given denominator.**

1. *Examples where the denominator of the required fraction is a factor of the denominator of the given fraction.*

Reduce $\frac{17}{24}$ to a fraction whose denominator is 8.

$$\frac{17}{24} = \frac{17 \div 3}{24 \div 3} = \frac{5\frac{2}{3}}{8}$$

EXPLANATION.—We observe that 8, the denominator of the required fraction, is a factor of 24, the denominator of the given fraction. Hence, dividing both terms of $\frac{17}{24}$ by 3, the other factor of 24, the fraction is reduced (**235**—III) to $\frac{5\frac{2}{3}}{8}$, a fraction whose denominator is 8.

2. *Examples where the denominator of the required fraction is not a factor of the denominator of the given fraction.*

Reduce $\frac{8}{13}$ to a fraction whose denominator is 10.

$$(1.)\quad \frac{8}{13} = \frac{8 \times 10}{13 \times 10} = \frac{80}{130}$$

$$(2.)\quad \frac{80}{130} = \frac{80 \div 13}{130 \div 13} = \frac{6\frac{2}{13}}{10}$$

EXPLANATION.—1. We introduce the given denominator 10 as a factor into the denominator of $\frac{8}{13}$ by multiplying, as shown in (1), both terms of the fraction by 10 (**235**—II).

2. The denominator 130 now contains the factors 13 and 10. Hence, dividing both terms of the fraction $\frac{80}{130}$ by 13 (**235**—III), as shown in (2), the result is $\frac{6\frac{2}{13}}{10}$, a fraction whose denominator is 10.

REVIEW EXAMPLES.

307. 1. How many *thirtieths* in $\frac{5}{6}$, and why? In $\frac{2}{5}$?

2. Reduce $\frac{3}{7}$, $\frac{5}{14}$, $\frac{3}{4\frac{2}{3}}$, $\frac{9}{56}$, and $\frac{27}{84}$ each to twenty-eighths.

3. Reduce to a common denominator $\frac{5}{6\frac{2}{3}}$, $\frac{3\frac{1}{2}}{8}$, and $\frac{7}{32}$.

4. State the reason why $\frac{5}{9 \div 4} = \frac{5 \times 4}{9}$ (**269**).

5. Reduce $\frac{4}{5}$ to a fraction whose numerator is 12; is 20; is 2; is 3; is 7 (**235**).

6. Reduce to a common numerator $\frac{3}{5}$ and $\frac{5}{7}$ (**252**).

7. Find the sum of $\frac{8}{9}$, $\frac{5}{12}$, $\frac{7}{8}$, $\frac{3}{4}$, and $\frac{11}{18}$.

8. Find the value of $(\frac{5}{7} \text{ of } \frac{4}{15} - \frac{1}{14}) \div \left(\frac{3}{4} + \frac{2}{3\frac{1}{2}}\right)$.

9. If $\frac{3}{8}$ of an estate is worth $3460, what is $\frac{4}{7}$ of it worth?

10. $4 is what part of $8? Of $12? Of $32? Of $48?

Write the solution of this example, with *reason* for each step.

11. If a man can do a piece of work in 150 days, what part of it can he do in 5 days? In 15 days? In 25 days? In $7\frac{1}{2}$ days? In $3\frac{3}{4}$ days? In $12\frac{1}{2}$ days?

12. A's farm contains 120 acres and B's 280; what part of B's farm is A's?

13. 42 is $\frac{6}{7}$ of what number?

Write the solution of this example, with *reason* for each step.

14. $897 is $\frac{5}{9}$ of how many dollars?

15. $\frac{3}{7}$ of 76 tons of coal is $\frac{8}{10}$ of how many tons?

16. A piece of cloth containing 73 yards is $\frac{8}{9}$ of another piece. How many yards in the latter?

17. Bought a horse for $286, and sold him for $\frac{7}{9}$ of what he cost; how much did I lose?

18. 84 is $\frac{5}{12}$ of 8 times what number?

Write the solution of this example, with *reason* for each step.

19. A has $694 in a bank, which is $\frac{4}{9}$ of 3 times the amount B has in the same bank; what is B's money?

20. Two men are $86\frac{3}{4}$ miles apart; when they meet, one has traveled $8\frac{7}{8}$ miles more than the other; how far has each traveled?

21. If $\frac{7}{12}$ of a farm is valued at $\$4732\frac{5}{8}$, what is the value of the whole farm?

22. The less of two numbers is $432\frac{5}{9}$, and their difference $123\frac{7}{12}$. Find the greater number.

23. A man owning $\frac{4}{9}$ of a saw-mill, sold $\frac{3}{8}$ of his share for $2800; what was the value of the mill?

24. What number diminished by $\frac{5}{7}$ and $\frac{2}{9}$ of itself leaves a remainder of 32?

25. I put $\frac{4}{9}$ of my money in the bank and gave $\frac{3}{8}$ of what I had left to a friend, and had still remaining $400. How much had I at first?

26. Sold 342 bushels of wheat at $\$1\frac{5}{7}$ a bushel, and expended the amount received in buying wood at $\$4\frac{3}{4}$ a cord. How many cords of wood did I purchase?

27. If $\frac{3}{8}$ of 4 pounds of tea cost $\$2\frac{1}{2}$, how many pounds of tea can be bought for $\$7\frac{1}{3}$? For $\$12\frac{3}{9}$? For $\$\frac{7}{12}$?

28. If 5 be added to both terms of the fraction $\frac{3}{7}$, how much will its value be changed, and why?

29. I exchanged $47\frac{3}{8}$ bushels of corn, at $\$\frac{5}{7}$ per bushel, for $24\frac{2}{3}$ bushels of wheat; how much did the wheat cost a bushel?

30. A can do a piece of work in 5 days, B can do the same work in 7 days; in what time can both together do it?

31. Bought $\frac{2}{7}$ of $84\frac{4}{5}$ acres of land for $\frac{4}{9}$ of $\$3584\frac{7}{8}$; what was the price per acre?

32. A boy while fishing lost $\frac{2}{3}$ of his line; he then added 8 feet, which was $\frac{4}{5}$ of what he lost; what was the length of the line at first?

33. A merchant bought a quantity of cloth for $\$2849\frac{4}{5}$, and sold it for $\frac{7}{10}$ of what it cost him, thereby losing $\$\frac{5}{7}$ a yard. How many yards did he purchase, and at what price per yard?

34. A tailor having $276\frac{2}{3}$ yards of cloth, sold $\frac{2}{9}$ of it at one time and $\frac{3}{7}$ at another; what is the value of the remainder at $3 a yard?

35. A man sold $\frac{5}{12}$ of his farm at one time, $\frac{2}{7}$ at another, and the remainder for $180 at $45 an acre; how many acres were there in the farm?

36. A merchant owning $\frac{12}{25}$ of a ship, sells $\frac{1}{3}$ of his share to B, and $\frac{3}{8}$ of the remainder to C for $\$600\frac{3}{4}$; what is the value of the ship?

REVIEW AND TEST QUESTIONS.

308. 1. Define Fractional Unit, Numerator, Denominator, Improper Fraction, Reduction, Lowest Terms, Simple Fraction, Common Denominator, and Complex Fraction.

2. What is meant by the *unit* of a fraction? Illustrate by an example.

3. When may $\frac{1}{3}$ be greater than $\frac{1}{2}$? $\frac{1}{6}$ than $\frac{1}{4}$?

4. State the three principles of Reduction of Fractions, and illustrate each by lines.

5. Illustrate with lines or objects each of the following propositions:

   I. To diminish the numerator, the denominator remaining the same, diminishes the value of the fraction.

   II. To increase the denominator, the numerator remaining the same, diminishes the value of the fraction.

   III. To increase the numerator, the denominator remaining the same, increases the value of the fraction.

   IV. To diminish the denominator, the numerator remaining the same, increases the value of the fraction.

6. What is meant by the Least Common Denominator?

7. When the denominators of the given fractions are prime to each other, how is the Least Common Denominator found, and why?

8. State the five problems in reduction of fractions, and illustrate each by the use of lines or objects.

9. Show that multiplying the denominator of a fraction by any number divides the fraction by that number (**269**).

10. Show by the use of lines or objects the truth of the following:

(1.) $\frac{1}{3}$ of 2 equals $\frac{2}{3}$ of 1.
(2.) $\frac{3}{5}$ of 1 equals $\frac{1}{5}$ of 3.
(3.) $\frac{1}{6}$ of 5 equals $\frac{5}{6}$ of 1.
(4.) $\frac{4}{7}$ of 9 equals 4 times $\frac{1}{7}$ of 9.

11. To give to another person $\frac{3}{5}$ of 14 silver dollars, how many of the dollar-pieces must you change, and what is the largest denomination of change you can use?

12. Show by the use of objects that the quotient of 1 divided by a fraction is the given fraction inverted.

13. Why is it impossible to perform the operation in $\frac{4}{5}+\frac{2}{3}$, or in $\frac{5}{9}-\frac{3}{7}$, without reducing the fractions to a common denominator?

14. Why do we invert the divisor when dividing by a fraction? Illustrate your answer by an example.

15. What objection to calling $\frac{5}{7\frac{2}{3}}$ a fraction (**226**)?

16. State, and illustrate with lines or objects, each of the three classes of *so-called* Complex Fractions.

17. Which is the greater fraction, $\frac{8}{9}$ or $\frac{37}{40}$, and how much?

18. To compare the value of two or more fractions, what must be done with them, and why?

19. Compare $\frac{3\frac{1}{2}}{7}$ and $\frac{4}{8}$; $\frac{2\frac{4}{5}}{9}$ and $\frac{3}{10}$; $\frac{5\frac{2}{3}}{7\frac{4}{5}}$ and $\frac{2\frac{3}{5}}{3\frac{1}{2}}$, and show in each case which is the greater fraction, and how much.

20. State the rule for working each of the following examples:

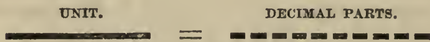
(1.) $3\frac{2}{3} + 4\frac{5}{7} + 8\frac{2}{9}$.
(2.) $(7\frac{3}{4} + 5\frac{1}{2}) - (8 - 2\frac{1}{3})$.
(3.) $5 \times \frac{4}{9}$ of $\frac{7}{8}$ of 27.
(4.) $8\frac{3}{7} \times 5\frac{2}{3}$.
(5.) $\frac{7}{9} \times \frac{5}{8}$. Explain by objects.
(6.) $\frac{5}{12} \div \frac{7}{9}$. Explain by objects.

21. Illustrate by an example the application of Cancellation in multiplication and division of fractions.

DECIMAL FRACTIONS.

DEFINITIONS.

309. A ***unit*** is separated into ***decimal parts*** when it is divided into *tenths;* thus,



310. A ***Decimal Fractional Unit*** is one of the *decimal parts* of anything.

311. By making a *whole* or *unit* into decimal parts, and one of these parts into decimal parts, and so on, we obtain a series of distinct orders of *decimal fractional units,* each $\frac{1}{10}$ of the preceding, having as denominators, respectively, 10, 100, 1000, and so on.

Thus, separating a whole into decimal parts, we have, according to (**246**), $1 = \frac{10}{10}$; making $\frac{1}{10}$ into decimal parts, we have, according to (**252**), $\frac{1}{10} = \frac{10}{100}$; in the same manner, $\frac{1}{100} = \frac{10}{1000}$, $\frac{1}{1000} = \frac{10}{10000}$ and so on. Hence, in the series of fractional units, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, and so on, each unit is *one-tenth* of the preceding unit.

312. A ***Decimal Fraction*** is a fraction whose denominator is 10, 100, 1000, etc., or 1 with any number of ciphers annexed. Thus, $\frac{3}{10}$, $\frac{7}{100}$, $\frac{43}{1000}$, are decimal fractions.

313. The ***Decimal Sign*** (**.**), called the *decimal point,* is used to express a decimal fraction without writing the denominator, and to distinguish it from an integer.

315. A ***decimal fraction*** expressed without writing the denominator is called simply a ***Decimal.***

Thus, we speak of .79 as the *decimal* seventy-nine, yet we mean the *decimal fraction* seventy-nine hundredths.

NOTATION AND NUMERATION.

314. Prop. I.—*A decimal fraction is expressed without writing the denominator by using the decimal point, and placing the numerator at the right of the period.*

Thus, $\frac{7}{10}$ is expressed .7; $\frac{35}{100}$ is expressed .35.

316. Prop. II.—*Ciphers at the left of significant figures do not increase or diminish the number expressed by these figures.*

Thus, 0034 is thirty-four, the same as if written 34 without the two ciphers.

317. Prop. III.—*When the fraction in a mixed number is expressed decimally, it is written after the integer, with the decimal point between them.*

Thus, 57 and .09 are written 57.09; 8 and .0034 are written 8.0034.

320. Prop. IV.—*Every figure in the numerator of a decimal fraction represents a distinct order of decimal units.*

Thus, $\frac{537}{1000}$ is equal to $\frac{500}{1000} + \frac{30}{1000} + \frac{7}{1000}$. But, according to (**255**), $\frac{500}{1000} = \frac{5}{10}$, and $\frac{30}{1000} = \frac{3}{100}$. Hence, 5, 3, and 7 each represent a distinct order of decimal fractional units, and $\frac{537}{1000}$, or .537 may be read 5 *tenths* 3 *hundredths* and 7 *thousandths.*

321. Prop. V.—*A decimal is read correctly by reading it as if it were an integer and giving the name of the right-hand order.*

Thus, $.975 = \frac{900}{1000} + \frac{70}{1000} + \frac{5}{1000}$. Hence is read, *nine hundred seventy-five thousandths.*

1. Observe that when there are ciphers at the left of the decimal, according to (**316**), they are not regarded in reading the number; thus, .062 is read sixty-two *thousandths.*

2. The name of the lowest order is found, according to (**314**), by prefixing 1 to as many ciphers as there are figures in the decimal. For example, in .00209 there are five figures; hence the denominator is 1 with five ciphers; thus, 100000, read hundred-thousandths.

REDUCTION.

PREPARATORY PROPOSITIONS.

The following preparatory propositions should be *very carefully* studied.

325. PROP. I.—*Annexing a cipher or multiplying a number by 10 introduces into the number the two prime factors 2 and 5.*

Thus, 10 being equal 2×5, 7×10 or $70 = 7 \times (2 \times 5)$. Hence a number must contain 2 and 5 as a factor at least as many times as there are ciphers annexed.

326. PROP. II.—*A fraction in its lowest terms, whose denominator contains no other prime factors than 2 or 5, can be reduced to a simple decimal.*

Observe that every cipher annexed to the numerator and denominator makes each divisible once by 2 and 5 (**325**). Hence, if the denominator of the given fraction contains no other factors except 2 and 5, by annexing ciphers the numerator can be made divisible by the denominator, and the fraction reduced to a *decimal.*

Thus, $\frac{7}{8} = \frac{7000}{8000}$ (**235**—II). Dividing both terms of the fraction by 8 (**235**—III), we have $\frac{7000}{8000} = \frac{875}{1000} = .875$.

327. PROP. III.—*A fraction in its lowest terms, whose denominator contains any other prime factors than 2 or 5, can be reduced only to a complex decimal.*

Observe that in this case annexing ciphers to the numerator and denominator, which (**325**) introduces only the factors 2 and 5, cannot make the numerator divisible by the given denominator, which contains other prime factors than 2 or 5.

Hence, a fraction will remain in the numerator, after dividing the numerator and denominator by the denominator of the given fraction, however far the division may be carried.

Thus, $\frac{11}{21} = \frac{11000}{21000}$ (**235**—II). Dividing both numerator and denominator by 21, we have $\frac{11000}{21000} = \frac{523\frac{17}{21}}{1000} = .523\frac{17}{21}$, a complex decimal.

328. PROP. IV.—*The same set of figures must recur indefinitely in the same order in a complex decimal which cannot be reduced to a simple decimal.*

Thus, $\frac{7}{11}=\frac{70000}{110000}=\frac{6363\frac{7}{11}}{10000}=.6363\frac{7}{11}$.

Observe carefully the following:

1. In any division, the number of different remainders that can occur is 1 less than the number of units in the divisor.

Thus, if 5 is the divisor, 4 must be the greatest remainder we can have, and 4, 3, 2, and 1 are the only possible different remainders ; hence, if the division is continued, any one of these remainders *may recur*.

2. Since in dividing the numerator by the denominator of the given fraction, each partial dividend is formed by annexing a cipher to the remainder of the previous division, when a remainder recurs the partial dividend must again be the same as was used when this remainder occurred before ; hence the same remainders and quotient figures must recur in the same order as at first.

3. If we stop the division at any point where the given numerator recurs as a remainder, we have the same fraction remaining in the numerator of the decimal as the fraction from which the decimal is derived.

Thus, $\frac{7}{11}=\frac{700}{1100}=\frac{63\frac{7}{11}}{1000}=.63\frac{7}{11}$;

or $\frac{7}{11}=\frac{70000}{110000}=\frac{6363\frac{7}{11}}{10000}=.6363\frac{7}{11}$, and so on.

329. PROP. V.—*The value of a fraction which can only be reduced to a complex decimal is expressed, nearly, as a simple decimal, by rejecting the fraction from the numerator.*

Thus, $\frac{3}{11}=\frac{27\frac{3}{11}}{100}$ (**327**). Rejecting the $\frac{3}{11}$ from the numerator, we have $\frac{27}{100}$, a simple fraction, which is only $\frac{3}{11}$ of $\frac{1}{100}$ smaller than the given fraction $\frac{3}{11}$ or $\frac{27\frac{3}{11}}{100}$.

Observe the following :

2. By taking a sufficient number of places in the decimal, the true value of a complex decimal can be expressed so nearly that what is rejected is of no consequence.

Thus, $\frac{3}{11}=\frac{27272727\frac{3}{11}}{100000000}$; rejecting the $\frac{3}{11}$ from the numerator, we have $\frac{27272727}{100000000}$, or .27272727, a simple decimal, which is only $\frac{3}{11}$ of 1 hundred-millionths smaller than the given fraction.

2. The *approximate value* of a complex decimal which is expressed by rejecting the given fraction from its numerator is called a *Circulating Decimal*, because the same figure or set of figures constantly recur.

330. PROP. VI.—*Diminishing the numerator and denominator by the same fractional part of each does not change the value of a fraction.*

Be particular to master the following, as the reduction of circulating decimals to common fractions depends upon this proposition.

1. The truth of the proposition may be shown thus:

$$\frac{9}{12}=\frac{9-\frac{1}{3}\text{ of }9}{12-\frac{1}{3}\text{ of }12}=\frac{9-3}{12-4}=\frac{6}{8}=\frac{3}{4}$$

Observe that to diminish the numerator and denominator each by $\frac{1}{3}$ of itself is the same as multiplying each by $\frac{2}{3}$. But to multiply each by $\frac{2}{3}$, we multiply each by 2 (**235**—II), and then divide each by 3 (**235**—III), which does not change the value of the fraction; hence the truth of the proposition.

2. From this proposition it follows that the value of a fraction is not changed by subtracting 1 from the denominator and the fraction itself from the numerator.

Thus, $\frac{3}{5}=\frac{3-\frac{3}{5}}{5-1}=\frac{2\frac{2}{5}}{4}$. Observe that 1 is the $\frac{1}{5}$ of the denominator 5, and $\frac{3}{5}$ is $\frac{1}{5}$ of the numerator 3; hence, the numerator and denominator being each diminished by the same fractional part, the value of the fraction is not changed.

DEFINITIONS.

331. A ***Simple Decimal*** is a decimal whose numerator is a whole number; thus, $\frac{93}{100}$ or .93.

Simple decimals are also called *Finite Decimals.*

332. A ***Complex Decimal*** is a decimal whose numerator is a mixed number; as $\frac{26\frac{2}{3}}{100}$ or $.26\frac{2}{3}$.

There are two classes of *complex decimals:*

1. Those whose value can be expressed as a simple decimal (**326**), as $.23\frac{1}{2} = .235$; $.32\frac{3}{4} = .3275$.

2. Those whose value cannot be expressed as a simple decimal (**327**), as $.53\frac{1}{3} = .53333$ and so on, leaving, however far we may carry the decimal places, $\frac{1}{3}$ of 1 of the lowest order unexpressed. See (**328**).

333. A ***Circulating Decimal*** is an *approximate value* for a complex decimal which *cannot be reduced* to a simple decimal.

Thus, $.\dot{6}6\dot{6}$ is an approximate value for $.666\frac{2}{3}$ (**329**).

334. A ***Repetend*** is the figure or set of figures that are repeated in a circulating decimal.

335. A ***Circulating Decimal is expressed*** by writing the repetend once. When the repetend consists of one figure, a point is placed over it; when of more than one figure, points are placed over the first and last figures; thus, .333 and so on, and .592592 + are written $.\dot{3}$ and $.\dot{5}9\dot{2}$.

336. A ***Pure Circulating Decimal*** is one which commences with a repetend, as $.\dot{8}$ or $.\dot{3}9\dot{4}$.

337. A ***Mixed Circulating Decimal*** is one in which the repetend is preceded by one or more decimal places, called the *finite part* of the decimal, as $.7\dot{3}$ or $.004\dot{7}2\dot{5}$, in which .7 or .004 is the finite part.

ILLUSTRATION OF PROCESS.

338. PROB. I.—**To reduce a common fraction to a decimal.**

Reduce $\frac{3}{8}$ to a decimal.

$$\frac{3}{8} = \frac{3000}{8000} = \frac{375}{1000} = .375$$

EXPLANATION.—1. We annex the same number of ciphers to both terms of the fraction (**235**—II), and divide the resulting terms by 8, the significant figure in the denominator which must give a decimal denominator. Hence, $\frac{3}{8}$ expressed decimally is .375.

2. In case annexing ciphers does not make the numerator divisible (**327**) by the significant figures in the denominator, the number of places in the decimal can be extended indefinitely.

In practice, we abbreviate the work by annexing the ciphers to the numerator only, and dividing by the denominator of the given fraction, pointing off as many decimal places in the result as there were ciphers annexed.

341. PROB. II.—**To reduce a simple decimal to a common fraction.**

Reduce .35 to a common fraction.

$$.35 = \frac{35}{100} = \frac{7}{20}$$

EXPLANATION.—We write the decimal with the denominator, and reduce the fraction (**255**) to its lowest terms

344. PROB. III.—**To find the true value of a pure circulating decimal.**

Find the true value of $.\dot{7}\dot{2}$.

$$.\dot{7}\dot{2} = \frac{\dot{7}\dot{2}}{100} = \frac{72}{100 - 1} = \frac{72}{99} = \frac{8}{11}$$

EXPLANATION.—In taking $.\dot{7}\dot{2}$ as the *approximate value* of a given fraction, we have subtracted the given fraction from its own numerator, as shown in (**329**—V). Hence, to find the true value of $\frac{\dot{7}\dot{2}}{100}$, we must, according to (**330**—VI, 2), subtract 1 from the denominator 100, which makes the denominator as many 9's as there are places in the repetend.

347. PROB. IV.—**To find the true value of a mixed circulating decimal.**

Find the true value of $.3\dot{1}\dot{8}$.

$$(1.)\quad .3\dot{1}\dot{8} = .3\tfrac{18}{99} = \frac{3\frac{18}{99}}{10} = \frac{315}{990} = \frac{7}{22}.$$

EXPLANATION.—1. We find, according to (**344**), the true value of the repetend $.0\dot{1}\dot{8}$, which is $.0\frac{18}{99}$. Annexing this to the .3, the finite part, we have $.3\frac{18}{99}$. the true value of $.3\dot{1}\dot{8}$ in the form of a *complex decimal.*

2. We reduce the complex decimal $.3\frac{18}{99}$, or $\frac{3\frac{18}{99}}{10}$, to a simple fraction by multiplying, according to (**300**), both terms of the fraction by 99, giving $\frac{3\frac{18}{99}}{10} = \frac{315}{990} = \frac{7}{22}$. Hence the true value of $.3\dot{1}\dot{8}$ is $\frac{7}{22}$.

(2.) .318 Given decimal.
3 Finite part.
315 $\frac{315}{990} = \frac{7}{22}$.

ABBREVIATED SOLUTION.—Observe that in simplifying $\frac{3\frac{18}{99}}{10}$, we multiplied both terms by 99. Instead of multiplying the 3 by 99, we may multiply by 100 and subtract 3 from the product. Hence we add the 18 to 300, and subtract 3 from the result, which gives us the true numerator.

To abbreviate the work:

From the given decimal subtract the finite part for a numerator, and for a denominator write as many 9's as there are figures in the repetend, with as many ciphers annexed as there are figures in the finite part.

ADDITION.

ILLUSTRATION OF PROCESS.

351. Find the sum of 34.8, 6.037, and 27.62.

(1.)	(2.)
34.800	34.8
6.037	6.037
27.620	27.62
68.457	68.457

EXPLANATION.—1. We arrange the numbers so that units of the same order stand in the same column.

2. We reduce the decimals to a common denominator, as shown in (1), by annexing ciphers.

3. We add as in integers, placing the decimal point before the tenths in the sum.

SUBTRACTION.

353. Find the difference between 83.7 and 45.392.

83.700
45.392
38.308

EXPLANATION.—1. We arrange the numbers so that units of the same order stand in the same column.

2. We reduce the decimals, or regard them as reduced to a common denominator, and then subtract as in whole numbers.

The reason of this course is the same as given in addition. The ciphers are also usually omitted.

MULTIPLICATION.

355. Multiply 3.27 by 8.3.

(1.) $$3.27 \times 8.3 = \frac{327}{100} \times \frac{83}{10}$$

(2.) $$\frac{327}{100} \times \frac{83}{10} = \frac{27141}{1000} = 27.141$$

EXPLANATION.—1. Observe that 3.27 and 8.3 are mixed numbers; hence, according to (**282**), they are reduced before being multiplied to improper fractions, as shown in (1).

2. According to (**276**), $\frac{327}{100} \times \frac{83}{10}$, as shown in (2), equals 27.141. Hence 27.141 is the product of 3.27 and 8.3.

The work is abbreviated thus:

(3.)
```
   3.27
    8.3
 ------
    981
  2616
 ------
 27.141
```

We observe, as shown in (2), that the product of 3.27 and 8.3 must contain as many decimal places as there are decimal places in both numbers. Hence we multiply the numbers as if integers, as shown in (3), and point off in the product as many decimal places as there are decimal places in both numbers.

DIVISION.

PREPARATORY PROPOSITIONS.

358. PROP. I.—*When the divisor is greater than the dividend, the quotient expresses the part the dividend is of the divisor.*

Thus, $4 \div 6 = \frac{4}{6} = \frac{2}{3}$. The quotient $\frac{2}{3}$ expresses the part the 4 is of 6.

1. Observe that the process in examples of this kind consists in reducing the fraction formed by placing the divisor over the dividend to its lowest terms. Thus, $32 \div 56 = \frac{32}{56}$, which reduced to its lowest terms gives $\frac{4}{7}$.

2. In case the result is to be expressed decimally, the process then consists in reducing to a decimal according to (**338**), the fraction formed by placing the dividend over the divisor. Thus, $5 \div 8 = \frac{5}{8}$, reduced to a decimal equals .625.

359. PROP. II.—*The fraction remaining after the division of one integer by another expresses the part the* REMAINDER *is of the divisor.*

Thus, $42 \div 11 = 3\frac{9}{11}$. The divisor 11 is contained 3 times in 42 and 9 left, which is 9 parts or $\frac{9}{11}$ of the divisor 11. Hence we say that the divisor 11 is contained $3\frac{9}{11}$ times in 42. We express the $\frac{9}{11}$ decimally by reducing it according to (**338**). Hence, $3\frac{9}{11} = 3.\dot{8}\dot{1}$.

360. PROP. III.—*Division is possible only when the dividend and divisor are both of the same denomination* (**155**—I).

For example, $\frac{3}{10} \div \frac{7}{100}$, or $.3 \div .07$ is impossible until the dividend and divisor are reduced to the same fractional denomination; thus, $.3 \div .07 = .30 \div .07 = 4\frac{2}{7} = 4.\dot{2}8571\dot{4}$.

ILLUSTRATION OF PROCESS.

361. Ex. 1. Divide .6 by .64.

(1.) $.8 \div .64 = .60 \div .64$

(2.) $60 \div 64 = \frac{60}{64} = .9375$

EXPLANATION.—1. We reduce, as shown in (1), the dividend and divisor to the same decimal unit or denomination (**290**).

2. We divide, according to (**290**), as shown in (2), the numerator 60 by the numerator 64, which gives $\frac{60}{64}$. Reducing $\frac{60}{64}$ to a decimal (**338**), we have $.6 \div .64 = .9375$.

Ex. 2. Divide .63 by .0022.

(1.) $.63 \div .0022 = .6300 \div .0022$

(2.) $6300 \div 22 = 286\frac{4}{11} = 386.\dot{3}\dot{6}$

EXPLANATION.—1. We reduce, as shown in (1), the dividend and divisor to the same decimal unit by annexing ciphers to the dividend (**350**).

2. We divide, according to (**290**), as shown in (2), the numerator 6300 by the numerator 22, giving as a quotient $286\frac{4}{11}$.

3. We reduce, according to (**338**), the $\frac{4}{11}$ in the quotient to a decimal, giving the repetend $.\dot{3}\dot{6}$. Hence, $.63 \div .0022 = 286.\dot{3}\dot{6}$.

Ex. 3. Divide 16.821 by 2.7.

(1.) $16.821 \div 2.7 = 16.821 \div 2.700$

(2.) $16.821 \div 2.700 = \frac{16821}{1000} \div \frac{2700}{1000}$

(3.)
```
27|00)168|21(6.23
      162
      ---
        62
        54
        --
         81
         81
         --
```

EXPLANATION.—1. We reduce, as shown in (1), the dividend and divisor to the same decimal unit by annexing ciphers to the divisor (**350**).

2. The dividend and divisor each express thousandths as shown in (2). Hence we reject the denominators and divide as in integers (**290**).

3. Since there are ciphers at the right of the divisor, they may be cut off by cutting off the same number of figures at the right of the dividend (**142**). Dividing by 27, we find that it is contained 6 times in 168, with 6 remaining.

4. The 6 remaining, with the two figures cut off, make a remainder of 621 or $\frac{621}{2700}$. This is reduced to a decimal by dividing both terms by 27. Hence, as shown in (3), we continue dividing by 27 by taking down the two figures cut off.

The work is abbreviated thus:

We reduce the dividend and divisor to the same decimal unit by cutting off from the right of the dividend the figures that express lower decimal units than the divisor. We then divide as shown in (3), prefixing the *remainder* to the figures cut off and reducing the result to a decimal.

REVIEW EXAMPLES.

364. Answers involving decimals, unless otherwise stated, are carried to four decimal places.

Reduce each of the following examples to decimals:

8. $\frac{11}{12}$.

9. $\frac{8}{9}$.

10. $\frac{2\frac{1}{2}}{5}$.

11. $\frac{3\frac{2}{3}}{7\frac{1}{2}}$.

12. $\frac{3}{7}$ of $1\frac{4}{5}$.

13. $5\frac{8}{11} - 5\frac{2}{9}$.

14. $\frac{(3\frac{2}{5} + \frac{1}{2}) \times \frac{4}{5}}{8}$.

15. $\frac{\frac{2}{7} \text{ of } .3}{8\frac{3}{5} - 4.3}$.

20. Four loads of hay weighed respectively 2583.07, $3007\frac{3}{7}$, $2567\frac{5}{8}$, and $3074\frac{11}{16}$ pounds; what was the total weight?

22. At \$1.75 per 100, what is the cost of 5384 oranges?

24. If freight from St. Louis to New York is \$.39$\frac{4}{5}$ per 100 pounds, what is the cost of transporting 3 boxes of goods, weighing respectively 783$\frac{2}{3}$, 325$\frac{3}{5}$, and 286$\frac{7}{8}$ pounds?

25. A piece of broadcloth cost \$195.38$\frac{1}{4}$, at \$3.27 per yard. How many yards does it contain?

26. A person having \$1142.49$\frac{3}{4}$, wishes to buy an equal number of bushels of wheat, corn, and oats; the wheat at \$1.37, the corn at \$.87$\frac{1}{2}$, and the oats at \$.35$\frac{3}{4}$. How many bushels of each can he buy?

29. A produce dealer exchanged 48$\frac{3}{5}$ bushels oats at 39$\frac{3}{4}$ cts. per bushel, and 13$\frac{1}{2}$ barrels of apples at \$3.85 a barrel, for butter at 37$\frac{1}{2}$ cts. a pound; how many pounds of butter did he receive?

30. A grain merchant bought 1830 bushels of wheat at \$1.25 a bushel, 570 bushels corn at 73$\frac{1}{2}$ cts. a bushel, and 468 bushels oats at 35$\frac{3}{4}$ cts. a bushel. He sold the wheat at an advance of 17$\frac{1}{2}$ cts. a bushel, the corn at an advance of 9$\frac{3}{4}$ cts. a bushel, and the oats at a loss of 3 cts. a bushel. How much did he pay for the entire quantity, and what was his gain on the transaction?

REVIEW AND TEST QUESTIONS.

365. 1. Define Decimal Unit, Decimal Fraction, Repetend, Circulating Decimal, Mixed Circulating Decimal, Finite Decimal, and Complex Decimal.

2. In how many ways may $\frac{3}{5}$ be expressed as a decimal fraction, and why?

3. What effect have ciphers written at the left of an integer? At the left of a decimal, and why in each case (**316**)?

4. Show that each figure in the numerator of a decimal represents a distinct order of decimal units (**320**).

5. How are *integral orders* and *decimal orders* each related to the *units* (**323**)? Illustrate your answer by lines or objects.

6. Why in reading a decimal is the lowest order the only one named? Illustrate by examples (**321**).

7. Give reasons for not regarding the ciphers at the left in reading the numerator of the decimal .000403.

8. Reduce $\frac{7}{8}$ to a decimal, and give a reason for each step in the process.

9. When expressed decimally, how many places must $\frac{13}{125}$ give, and why? How many must $\frac{5}{32}$ give, and why?

10. Illustrate by an example the reason why $\frac{17}{21}$ cannot be expressed as a *simple decimal* (**327**).

11. State what fractions can and what fractions cannot be expressed as simple decimals (**326** and **327**). Illustrate by examples.

12. In reducing $\frac{5}{7}$ to a complex decimal, why must the numerator 5 recur as a remainder (**328**—1 and 2)?

13. Show that, according to (**235**—II and III), the value of $\frac{15}{45}$ will not be changed if we diminish the numerator and denominator each by $\frac{2}{5}$ of itself.

14. Show that multiplying 9 by $1\frac{2}{3}$ increases the 9 by $\frac{2}{3}$ of itself.

15. Multiplying the numerator and denominator of $\frac{14}{35}$ each by $1\frac{3}{7}$ produces what change in the fraction, and why?

16. Show that in diminishing the *numerator* of $\frac{4}{5}$ by $\frac{4}{5}$ and the *denominator* by 1 we diminish each by the same part of itself.

17. In taking $.\dot{3}$ as the value of $\frac{1}{3}$, what fraction has been rejected from the numerator? What must be rejected from the denominator to make $.\dot{3} = \frac{1}{3}$, and why?

18. Show that the true value of $.\dot{8}\dot{1}$ is $\frac{81}{99}$. Give a reason for each step.

19. Explain the process of reducing a mixed circulating decimal to a fraction. Give a reason for each step.

20. How much is .33333 less than $\frac{1}{3}$, and why?

21. How much is .571428 less than $\frac{4}{7}$, and why?

22. Find the sum of .73, .0049, .089, 6.58, and 9.08703, and explain each step in the process (**261**—I and II).

23. If *tenths* are multiplied by *hundredths*, how many decimal places will there be in the product, and why (**355**)?

24. Show that a number is multiplied by 10 by moving the decimal point one place to the right; by 100 by moving it two places; by 1000 three places, and so on.

25. State a rule for *pointing off* the decimal places in the product of two decimals. Illustrate by an example, and give reasons for your rule.

26. Multiply 385.23 by .742, multiplying *first* by the 4 *hundredths*, then by the 7 *tenths*, and *last* by the 2 *thousandths*.

27. Why is the quotient of an integer divided by a *proper fraction* greater than the dividend?

28. Show that a number is divided by 10 by moving the decimal point one place to the left; by 100 by moving it two places; by 1000, three places; by 10000, four places, and so on.

29. Divide 4.9 by 1.305, and give a reason for each step in the process. Carry the decimal to three places.

30. Give a rule for division of decimals.

DENOMINATE NUMBERS.

DEFINITIONS.

366. A ***Related Unit*** is a unit which has an invariable relation to one or more other units.

Thus, 1 foot = 12 inches, or $\frac{1}{3}$ of a yard; hence, 1 foot has an invariable relation to the units *inch* and *yard*, and is therefore a *related unit.*

367. A ***Denominate Number*** is a concrete number (**15**) whose unit (**14**) is a *related unit.*

Thus, 17 yards is a denominate number, because its unit, *yard*, has an invariable relation to the units *foot* and *inch*, 1 yard making always 3 feet or 36 inches.

368. A ***Denominate Fraction*** is a fraction of a *related unit.*

Thus, $\frac{3}{4}$ of a yard is a denominate fraction.

369. The ***Orders*** of related units are called ***Denominations.***

Thus, *yards*, *feet*, and *inches* are denominations of length; *dollars*, *dimes*, and *cents* are denominations of money.

370. A ***Compound Number*** consists of several numbers expressing *related denominations*, written together in the order of the relation of their units, and read as one number.

Thus, 23 yd. 2 ft. 9 in. is a compound number.

371. A ***Standard Unit*** is a unit established by law or custom, from which other units of the same kind are derived.

Thus, the standard unit of measures of extension is the yard. By dividing the yard into three equal parts, we obtain the unit *foot;* into 36 equal parts, we obtain the unit *inch;* multiplying it by $5\frac{1}{2}$, we obtain the unit *rod*, and so on.

373. ***Reduction of Denominate Numbers*** is the process of changing their denomination without altering their value.

UNITS OF WEIGHT.

374. The ***Troy*** pound of the mint is the ***Standard Unit*** of weight.

TROY WEIGHT.

TABLE OF UNITS.

24 *gr.*	=	1 *pwt.*
20 *pwt.*	=	1 *oz.*
12 *oz.*	=	1 *lb.*
3.2 *gr.*	=	1 *carat.*

1. ***Denominations.*** — Grains (*gr.*), Pennyweights (*pwt.*), Ounces (*oz.*), Pounds (*lb.*), and Carats.

2. ***Equivalents.***—1 lb. = 12 oz. = 240 pwt. = 5760 gr.

3. ***Use.***—Used in weighing gold, silver, and precious stones, and in philosophical experiments.

AVOIRDUPOIS WEIGHT.

TABLE OF UNITS.

16 *oz.*	=	1 *lb.*
100 *lb.*	=	1 *cwt.*
20 *cwt.*	=	1 *T.*

1. ***Denominations.*** — Ounces (*oz.*), pounds (*lb.*), hundredweights (*cwt.*), tons (*T.*).

2. ***Equivalents.***—1 Ton = 20 cwt. = 2000 lb. = 32000 oz.

3. ***Use.*** — Used in weighing groceries, drugs at wholesale, and all coarse and heavy articles.

4. In the United States Custom House, and in wholesale transactions in coal and iron, 1 quarter = 28 lbs., 1 cwt. = 112 lb., 1 T. = 2240 lb. This is usually called the *Long Ton* table.

APOTHECARIES' WEIGHT.

TABLE OF UNITS.

20 *gr.*	=	1 *sc.* or ℈.
3 ℈	=	1 *dr.* or ʒ.
8 ʒ	=	1 *oz.* or ℥.
12 *oz.*	=	1 *lb.*

1. ***Denominations.*** — Grains (*gr.*), Scruples (℈), Drams (ʒ), Ounces (℥), Pounds (*lb.*).

2. ***Equivalents.*** — *lb.* 1 = ℥ 12 = ʒ 96 = ℈ 288 = gr. 5760.

3. ***Use.***—Used in medical prescriptions.

4. Medical prescriptions are usually written in Roman notation. The number is written after the symbol, and the final "i" is always written j. Thus, ℥ vij is 7 ounces.

Comparative Table of Units of Weight.

		TROY.		AVOIRDUPOIS.		APOTHECARIES.
1 pound	=	5760 grains	=	7000 grains	=	5760 grains.
1 ounce	=	480 "	=	437.5 "	=	480 "

Table of Avoirdupois Pounds in a Bushel, as Established by Law in the States named.

	Cal.	Conn.	Del.	Ill.	Ind.	Ia.	Ky.	La.	Me.	Mass.	Mich.	Minn.	Mo.	N. H.	N. J.	N. Y.	O.	Qr.	Penna.	Vt.	Wash. T.	Wis.	N. C.
Wheat	60	56	60	60	60	60	60	60		60	60	60	60		60	60	60	60	60	60	60	60	60
Indian Corn	52	56	56	52	56	50	56	56		56	56	56	52		56	58	56	56	56	56	56	56	54
Oats	32	28		32	32	35	$33\frac{1}{3}$	32	30	30	32	32	35	30	30	32	32	34	32	32	36	32	
Barley	50			48	48	48	48	32		46	48	48	48		48	48	48	46	47	46	45	48	48
Buckwheat	40	45		40	50	52	52			46	42	42	52		50	48		42	48	46	42	42	50
Rye	54	56		54	56	56	56	32		56	56	56	56		56	56	56	56	56	56	56	56	
Clover Seed				60	60	60	60				60	60	60		64	60	60	60			60	60	
Timothy Seed				45	45	45	45						45			44						46	

Peas, Beans, and Potatoes are usually weighed 60 lb. to the bushel.

The following are also in use:

100 lb. of Grain or Flour = 1 Cental. 200 lb. Pork or Beef = 1 Barrel.
100 " of Dry Fish = 1 Quintal. 196 " Flour = 1 Barrel.
100 " of Nails = 1 Keg. 240 " Lime = 1 Cask.
280 lb. salt at N. Y. Salt Works = 1 Barrel.

PROBLEMS ON RELATED UNITS.

375. PROB. I.—**To reduce a denominate or a compound number to a lower denomination.**

Reduce 23 lb. 7 oz. 9 pwt. to pennyweights.

```
 2 3 lb. 7 oz. 9 pwt.
   1 2
 ------
 2 8 3 oz.
   2 0
 -------
5 6 6 9 pwt.
```

SOLUTION.—1. Since 12 oz. make 1 lb., in any number of pounds there are 12 times as many ounces as pounds. Hence we multiply the 23 lb. by 12, and add the 7 oz., giving 283 oz.

2. Again, since 20 pwt. make 1 oz., in any number of ounces there are 20 times as many pennyweights as ounces. Hence we multiply the 283 oz. by 20, and add the 9 pwt., giving 5669 pwt.

378. PROB. II.—**To reduce a denominate number to a compound or a higher denominate number.**

Reduce 7487 sc. to a compound number.

3) 7487 sc.
8) 2495 dr. + 2 sc.
12) 311 oz. + 7 dr.
25 lb. 11 oz.

SOLUTION.—Since 3 sc. make 1 dr., 7487 sc. must make as many drams as 3 is contained times in 7487, or 2495 dr. + 2 sc.

2. Since 8 dr. make 1 oz., 2495 dr. must make as many ounces as 8 is contained times in 2495, or 311 oz. + 7 dr.

3. Since 12 oz. make 1 lb., 311 oz. must make as many pounds as 12 is contained times in 311, or 25 lb. + 11 oz. Hence, 7487 sc. are equal to the compound number 25 lb. 11 oz. 7 dr. 2 sc.

381. PROB. III.—**To reduce a denominate fraction or decimal to integers of lower denominations.**

Reduce $\frac{5}{7}$ of a ton to lower denominations.

(1.) $\frac{5}{7}$ T. $= \frac{5}{7}$ of 20 cwt. $= \frac{5}{7} \times 20 = 14$ cwt. $+ \frac{2}{7}$ cwt.
(2.) $\frac{2}{7}$ cwt. $= \frac{2}{7}$ of 100 lb. $= \frac{2}{7} \times 100 = 28$ lb. $+ \frac{4}{7}$ lb.
(3.) $\frac{4}{7}$ lb. $= \frac{4}{7}$ of 16 oz. $= \frac{4}{7} \times 16 = 9\frac{1}{7}$ oz.

SOLUTION.—Since 20 cwt. is equal 1 T., $\frac{5}{7}$ of 20 cwt., or $14\frac{2}{7}$ cwt., equals $\frac{5}{7}$ of 1 T. Hence, to reduce the $\frac{5}{7}$ of a ton to hundredweights, we take $\frac{5}{7}$ of 20 cwt., or multiply, as shown in (1), the $\frac{5}{7}$ by 20, the number of hundredweights in a ton.

In the same manner we reduce the $\frac{2}{7}$ cwt. remaining to pounds, as shown in (2), and the $\frac{4}{7}$ lb. remaining to ounces, as shown in (3).

384. PROB. IV.—**To reduce a denominate fraction or decimal of a lower to a fraction or decimal of a higher denomination.**

Reduce $\frac{3}{5}$ of a dram to a fraction of a pound.

(1.) $\frac{3}{5}$ dr. $= \frac{1}{8}$ oz. $\times \frac{3}{5} = \frac{3}{40}$ oz.
(2.) $\frac{3}{40}$ oz. $= \frac{1}{12}$ lb. $\times \frac{3}{40} = \frac{1}{160}$ lb.

SOLUTION.—1. Since 8 drams = 1 ounce, 1 dram is equal $\frac{1}{8}$ of an oz., and $\frac{3}{5}$ of a dram is equal $\frac{3}{5}$ of $\frac{1}{8}$ oz. Hence, as shown in (1), $\frac{3}{5}$ dr. $= \frac{3}{40}$ oz.

2. Since 12 ounces = 1 pound, 1 ounce is equal $\frac{1}{12}$ of a pound, and, as shown in (2), $\frac{3}{40}$ of an ounce is equal $\frac{3}{40}$ of $\frac{1}{12}$ lb., or $\frac{1}{160}$ lb. Hence, $\frac{3}{5}$ dr. = $\frac{1}{160}$ lb.

387. Prob. V.—**To reduce a compound number to a fraction of a higher denomination.**

Reduce ℥4 ʒ6 ℈2 to a fraction of 1 pound.

(1.) ℥4 ʒ6 ℈3 = ℈116; lb. 1 = ℈288.

(2.) $\frac{116}{288} = \frac{29}{72}$; hence, ℥4 ʒ6 ℈2 = lb. $\frac{29}{72}$.

Solution.—1. Two numbers can be compared only when they are the same denomination. Hence we reduce, as shown in (1), the ℥4 ʒ6 ℈2 and the lb. 1 to scruples, the lowest denomination mentioned in either number.

2. ℥4 ʒ6 ℈2 being equal ℈116, and lb. 1 being equal ℈288, ℥4 ʒ6 ℈2 is the same part of lb. 1 as ℈116 is of ℈288, which is $\frac{116}{288}$, or $\frac{29}{72}$. Hence ℥4 ʒ6 ℈2 = lb. $\frac{29}{72}$, or $.00402\dot{7}$.

15. Reduce 8 cwt. 3 qr. 16 lb. to the decimal of a ton.

25) 16.00 lb.
4) 3.64 qr.
20) 8.91 cwt.
.4455 T.

Abbreviated Solution.—Since the 16 pounds are reduced to a decimal of a quarter by reducing $\frac{16}{25}$ to a decimal, we annex two ciphers to the 16, as shown in the margin, and divide by 25, giving .64 qr.

To this result we prefix the 3 quarters, giving 3.64 qr., which is equivalent to $\frac{3.64}{4}$ hundredweights; hence we divide by 4, as shown in the margin, giving .91 cwt.

To the result we again prefix the 8 cwt., giving 8.91, which is equivalent to $\frac{8.91}{20}$ of a ton, equal .4455 T. Hence, 8 cwt. 3 qr. 16 lb. = .4455 T.

16. Reduce 8 oz. 6 dr. 2 sc. to the decimal of a pound.
17. What decimal of 24 lb. Troy is 2 lb. 8 oz. 16 pwt.?
18. 9 oz. 16 pwt. 12 gr. are what decimal of a pound?
19. Reduce 12 cwt. 2 qr. 18 lb. to the decimal of a ton.
20. What decimal of a pound are ℥9 ʒ5 ℈2 gr. 18?
21. Reduce 11 oz. 16 pwt. 20 gr. to the decimal of a pound.
22. Reduce 7 lb. 5 oz. Avoir. to a decimal of 12 lb. 5 oz. 3 pwt. Troy.
23. 1 lb. 9 oz. 8 pwt. is what part of 3 lb. Apoth. weight?

390. Prob. VI.—**To find the sum of two or more denominate or compound numbers, or of two or more denominate fractions.**

1. Find the sum of 7 cwt. 84 lb. 14 oz., 5 cwt. 97 lb. 8 oz., and 2 cwt. 9 lb. 15 oz.

cwt.	lb.	oz.
7	84	14
5	97	8
2	9	15
15	92	5

Solution.—1. We write numbers of the same denomination under each other, as shown in the margin.

2. We add as in Simple Numbers, commencing with the lowest denomination. Thus, 15, 8, and 14 ounces make 37 ounces, equal 2 lb. 5 oz. We write the 5 oz. under the ounces and add the 2 lb. to the pounds.

We proceed in the same manner with each denomination until the entire sum, 15 cwt. 92 lb. 5 oz., is found.

2. Find the sum of $\frac{4}{9}$ lb., $\frac{4}{5}$ dr., and $\frac{3}{4}$ sc.

	oz.	dr.	sc.	gr.
$\frac{4}{9}$ lb. =	5	2	2	0
$\frac{4}{5}$ dr. =			2	8
$\frac{3}{4}$ sc. =				15
	5	3	2	3

Solution.—1. According to (**261**), only fractional units of the same kind and of the same whole can be added; hence we reduce $\frac{4}{9}$ lb., $\frac{4}{5}$ dr., and $\frac{3}{4}$ sc. to integers of lower denominations (**381**), and then add the results, as shown in the margin. Or,

2. The given fractions may be reduced to fractions of the same denomination (**384**), and the results added according to (**261**), and the value of the sum expressed in integers of lower denominations according to (**381**).

392. Prob. VII.—**To find the difference between any two denominate or compound numbers, or denominate fractions.**

Find the difference between 27 lb. 7 oz. 15 pwt. and 13 lb. 9 oz. 18 pwt.

lb.	oz.	pwt.
27	7	15
13	9	18
13	9	17

Solution.—1. We write numbers of the same denomination under each other.

2. We subtract as in simple numbers. When the number of any denomination of the subtrahend cannot be taken from the number of the

same denomination in the minuend, we add as in simple numbers (**65**—III) one from the next higher denomination. Thus, 18 pwt. cannot be taken from 15 pwt.; we add 1 of the 7 oz. to the 15 pwt., making 35 pwt. 18 pwt. from 35 pwt. leaves 17 pwt., which we write under the pennyweights.

We proceed in the same manner with each denomination until the entire difference, 13 lb. 9 oz. 17 pwt., is found.

To subtract denominate fractions, we reduce as directed in addition, and then subtract.

394. PROB. VIII.—**To multiply a denominate or compound number by an abstract number.**

Multiply 18 cwt. 74 lb. 9 oz. by 6.

	18 cwt.	74 lb.	9 oz.
			6
5 T.	12 cwt.	47 lb.	6 oz.

SOLUTION.—We multiply as in simple numbers, commencing with the lowest denomination. Thus, 6 times 9 oz. equals 54 oz. We reduce the 54 oz. to pounds (**378**), equal 3 lb. 6 oz. We write the 6 oz. under the ounces, and add the 3 lb. to the product of the pounds.

We proceed in this manner with each denomination until the entire product, 5 T. 12 cwt. 47 lb. 6 oz., is found.

396. PROB. IX.—**To divide a denominate or compound number by any abstract number.**

Divide 29 lb. 7 oz. 2 dr. by 7.

	lb.	oz.	dr.
7)	29	7	2
	4	2	6

SOLUTION.—1. The object of the division is to find $\frac{1}{7}$ of the compound number. This is done by finding the $\frac{1}{7}$ of each denomination separately. Hence the process is the same as in finding one of the equal parts of a concrete number.

Thus, the $\frac{1}{7}$ of 29 lb. is 4 lb. and 1 lb. remaining. We write the 4 lb. in the quotient, and reduce the 1 lb. to ounces, which added to 7 oz. make 19 oz. We now find the $\frac{1}{7}$ of the 19 oz., and proceed as before.

1. Divide 9 T. 15 cwt. 3 qr. 18 lb. by 2; by 5; by 8; by 12.
2. If 29 lb. 7 oz. 16 pwt. are made into 7 equal parts, how much will there be in each part?

UNITS OF LENGTH.

397. A *yard* is the ***Standard Unit*** in *linear, surface,* and *solid* measure.

LINEAR MEASURE.

TABLE OF UNITS.

12	*in.*	=	1 *ft.*
3	*ft.*	=	1 *yd.*
$5\frac{1}{2}$	*yd.*	=	1 *rd.*
320	*rd.*	=	1 *mi.*

1. ***Denominations.***—Inches (in.), Feet (ft.), Yards (yd.), Rods (rd.), Miles (mi.).

2. ***Equivalents.***—1 mi. = 320 rd. = 5280 ft. = 63360 in.

3. ***Use.***—Used in measuring lines and distances.

4. In measuring cloth the yard is divided into *halves, fourths, eighths,* and *sixteenths.* In estimating duties in the Custom House, it is divided into *tenths* and *hundredths.*

Table of Special Denominations.

60	Geographic *or*	= 1 Degree	of Latitude on a Meridian or of Longitude on the Equator.
$69.1\dot{6}$	Statute Miles		
360	Degrees	= the Circumference of the Earth.	
$1.1\dot{6}$	Statute Miles	= 1 Geog. Mi.	*Used to measure distances at sea.*
3	Geographical Miles	= 1 League	
6	Feet	= 1 Fathom.	*Used to measure depths at sea.*
4	Inches	= 1 Hand.	*Used to measure the height of horses at the shoulder*

SURVEYORS' LINEAR MEASURE.

TABLE OF UNITS.

7.92	*in.*	=	1 *l.*
25	*l.*	=	1 *rd.*
4	*rd.*	=	1 *ch.*
80	*ch.*	=	1 *mi.*

1. ***Denominations.***—Link (l.), Rod (rd.), Chain (ch.), Mile (mi.).

2. ***Equivalents.***—1 mi. = 80 ch. = 320 rd. = 8000 l.

3. ***Use.*** — Used in measuring roads and boundaries of land.

4. The ***Unit*** of measure is the *Gunter's Chain,* which contains 100 links, equal 4 rods or 66 feet.

UNITS OF SURFACE.

399. A *square yard* is the ***Standard Unit*** of *surface measure.*

400. A ***Surface*** has two dimensions—*length* and *breadth.*

401. A ***Square*** is a *plane surface* bounded by four equal lines, and having four right angles.

402. A ***Rectangle*** is any *plane surface* having four sides and four right angles.

403. The ***Unit of Measure*** for surfaces is usually a square, each side of which is one unit of a known length.

Thus, in 14 sq. ft., the *unit* of *measure* is a *square foot.*

404. The ***Area*** of a rectangle is the *surface* included within its boundaries, and is expressed by the number of times it contains a given *unit* of *measure.*

Thus, since a square yard is a surface, each side of which is 3 feet long, it can be divided into 3 rows of square feet, as shown in the diagram, each row containing 3 square feet. Hence, if 1 square foot is taken as the ***Unit*** of ***Measure***, the area of a square yard is

3 sq. ft. × 3 = 9 sq. ft.

The area of any rectangle is found in the same manner.

3 feet long.

3 feet long.

3 sq. ft. × 3 = 9 sq. ft.

SQUARE MEASURE.

TABLE OF UNITS.

144 *sq. in.*	=	1 *sq. ft.*
9 *sq. ft.*	=	1 *sq. yd.*
$30\frac{1}{4}$ *sq. yd.*	=	1 *sq. rd.*, or *P.*
160 *sq. rd.*	=	1 *A.*
640 *A.*	=	1 *sq. mi.*

1. ***Denominations.***—Square Inch (sq. in.), Square Yard (sq. yd.), Square Rod (sq. rd.), Acre (A.), Square Mile (sq. mi.).

2. ***Equivalents.***—1 sq. mi. = 640 A. = 102400 sq. rd. = 3097600 sq. yd. = 27878400 sq. ft. = 4014489600 sq. in.

3. ***Use.***—Used in computing areas or surfaces.

4. Glazing and stone-cutting are estimated by the *square foot;* plastering, paving, painting, etc., by the *square foot* or *square yard;* roofing, flooring, etc., generally by the *square* of 100 *square feet.*

5. In laying shingles, *one thousand,* averaging 4 inches wide, and laid 5 inches to the weather, are estimated to cover a square.

SURVEYORS' SQUARE MEASURE.

TABLE OF UNITS.

625 *sq. l.*	= 1 *P.*
16 *P.*	= 1 *sq. ch.*
10 *sq. ch.*	= 1 *A.*
640 *A.*	= 1 *sq. mi.*
36 *sq. mi.*	= 1 *Tp.*

1. ***Denominations.***—Square Link (sq. l.), Poles (P.), Square Chain (sq. ch.), Acres (A.), Square Mile (sq. mi.), Township (Tp.).

2. ***Equivalents.***—1 Tp.=36 sq. mi. =23040 A.=230400 sq. ch. = 3686400 P. = 2304000000 sq. l.

3. ***Use.***—Used in computing the area of land.

4. The ***Unit*** of land measure is the *acre.* The measurement of a tract of land is usually recorded in *square miles, acres,* and *hundredths* of an acre.

UNITS OF VOLUME.

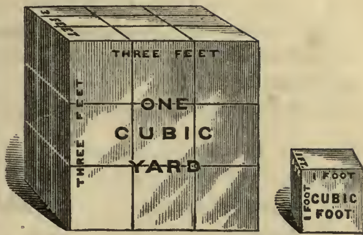
408. A ***Solid*** or ***Volume*** has three dimensions—*length, breadth,* and *thickness.*

409. A ***Rectangular Solid*** is a body bounded by six *rectangles* called *faces.*

410. A ***Cube*** is a *rectangular solid,* bounded by six equal squares.

411. The ***Unit of Measure*** is a cube whose edge is a unit of some known length.

412. The ***Volume,*** or ***Solid Contents*** of a body is expressed by the number of times it contains a given *unit of measure.* For example, the contents of a *cubic yard* is expressed as 27 *cubic feet.*



Thus, since each face of a cubic yard contains 9 square feet, if a section 1 ft. thick is taken it must contain 3 times 3 cu. ft., or 9 cu. ft., as shown in the diagram.

And since the cubic yard is 3 feet thick, it must contain 3 sections, each containing 9 cu. ft., which is 27 cu. ft.

Hence, the *volume* or *contents* of a cubic yard expressed in cubic feet, is found by taking the product of the numbers denoting its 3 dimensions in feet.

The *contents* of any rectangular solid is found in the same manner.

CUBIC MEASURE.

TABLE OF UNITS.

1728 *cu. in.* = 1 *cu. ft.*
27 *cu. ft.* = 1 *cu. yd.*

1. ***Denominations.***—Cubic Inch (cu. in.), Cubic Foot (cu. ft.), Cubic Yard (cu. yd.).

2. ***Equivalents.***—1 cu. yd. = 27 cu. ft. = 46656 cu. in.

3. ***Use.***—Used in computing the volume or contents of solids.

Table of Units for Measuring Wood and Stone.

16 *cu. ft.*	= 1 Cord Foot (*cd. ft.*)	Used for measuring both wood and stone.
8 *cd. ft.* or 128 *cu. ft.*	= 1 Cord (*cd.*)	
$24\frac{3}{4}$ *cu. ft.*	= 1 perch (*pch.*) of stone or masonry.	

1 *cu. yd.* of earth is called a load.

1. The materials for masonry are usually estimated by the *cord* or *perch*, the work by the *perch* and *cubic foot*, also by the *square foot* and square yard.

2. In estimating the mason work in a building, each wall is measured on the outside, and no allowance is ordinarily made for doors, windows, and cornices, unless specified in contract. In estimating the material, the doors, windows, and cornices are deducted.

3. Brickwork is usually estimated by the *thousand bricks.* The size of a brick varies thus: North River bricks are 8 in. $\times 3\frac{1}{2} \times 2\frac{1}{4}$, Philadelphia and Baltimore bricks are $8\frac{1}{4}$ in. $\times 4\frac{1}{8} \times 2\frac{3}{8}$, Milwaukee bricks $8\frac{1}{2}$ in. $\times 4\frac{1}{8} \times 2\frac{3}{8}$, and Maine bricks $7\frac{1}{2}$ in. $\times 3\frac{5}{8} \times 2\frac{3}{8}$.

4. *Excavations* and *embankments* are estimated by the cubic yard.

BOARD MEASURE.

TABLE OF UNITS.

12 B. in. = 1 B. ft.
12 B. ft. = 1 cu. ft.

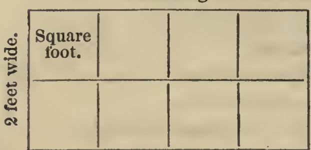
418. A ***Board Foot*** is 1 ft. long, 1 ft. wide, and 1 in. thick. Hence, 12 *board feet* equals 1 cu. ft.

419. A ***Board Inch*** is 1 ft. long, 1 in. wide, and 1 in. thick, or $\frac{1}{12}$ of a *board foot.* Hence, 12 *board inches* equals 1 *board foot.*

Observe carefully the following:

(1.)

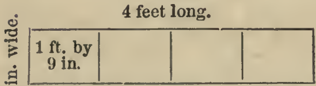
4 feet long.



4 × 2 = 8 sq. ft. or 8 B. ft.

(2.)

4 feet long.



4 × 9 = 36 B. in.; 36 B. in. ÷ 12 = 3 B. ft.

1. Diagram 1 represents a board where both dimensions are feet. Hence the product of the two dimensions gives the square feet in surface (**405**), or the number of board feet when the lumber is not more than 1 inch thick.

2. Diagram (2) represents a board where one dimension is feet and the other inches. It is evident (**418**) that a board 1 foot long, 1 inch thick, and any number of inches wide, contains as many *board inches* as there are inches in the width. Hence the number of square feet or *board feet* in a board 1 inch thick is equal to the length in feet multiplied by the width in inches divided by 12, the number of *board inches* in a *board foot.*

3. In case the lumber is more than 1 inch thick, the number of board feet is equal to the number of square feet in the surface multiplied by the thickness.

16. Find the length of a stick of timber 8 in. by 10 in., which will contain 20 cu. ft.

OPERATION.—$(1728 \times 20) \div (8 \times 10) = 432$; $432 \div 12 = 36$ ft., the length.

UNITS OF TIME.

424. The *mean solar day* is the ***Standard Unit*** of time.

TABLE OF UNITS.

60 *sec.*	=	1 *min.*
60 *min.*	=	1 *hr.*
24 *hr.*	=	1 *da.*
7 *da.*	=	1 *wk.*
365 *da.*	=	1 *common yr.*
366 *da.*	=	1 *leap yr.*
100 *yr.*	=	1 *cen.*

1. ***Denominations.***—Seconds (sec.), Minutes (min.), Hours (hr.), Days (da.), Weeks (wk.), Months (mo.), Years (yr.), Centuries (cen.).

2. There are 12 Calendar Months in a year; of these, April, June, September, and November, have 30 da. each. All the other months except February have 31 da. each. February, in *common years*, has 28 da., in *leap years* it has 29 da.

3. In computing interest, 30 days are usually considered *one month.* For business purposes the day begins and ends at 12 o'clock midnight.

425. The reason for *common* and *leap years* will be seen from the following:

The *true year* is the time the earth takes to go *once around the sun*, which is 365 days, 5 hours, 48 minutes and 49.7 seconds. Taking 365 days as a *common year*, the time lost in the calendar in 4 years will lack only 44 minutes and 41.2 seconds of 1 day. Hence we add 1 day to February every fourth year, making the year 366 days, or ***Leap Year.*** This correction is 44 min. 41.2 sec. more than should be added, amounting in 100 years to 18 hr. 37 min. 10 sec.; hence at the end of 100 years we omit adding a day, thus losing again 5 hr. 22 min. 50 sec., which we again correct by adding a day at the end of 400 years.

How many yr., mo., da. and hr. from 6 o'clock P. M., July 19, 1862, to 6 o'clock A. M., April 9, 1876.

yr.	*mo.*	*da.*	*hr.*
1876	4	9	7
1862	7	19	18
13	8	19	13

SOLUTION.—1. Since the latter date denotes the greater period of time, it is the minuend, and the earlier date, the subtrahend.

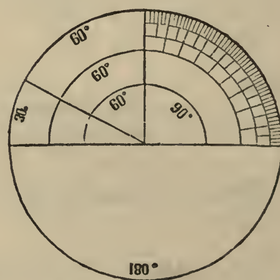
2. Since each year commences with

January, and each day with 12 o'clock midnight, 7 o'clock A. M., April 9, 1876, is the 7th hour of the 9th day of the fourth month of 1876; and 6 o'clock P. M., July 19, 1862, is the 18th hour of the 19th day of the 7th month of 1862. Hence the minuend and subtrahend are written as shown in the margin.

3. Considering 24 hours 1 day, 30 days 1 month, and 12 months 1 year, the subtraction is performed as in compound numbers (**392**), and 13 yr. 8 mo. 19 da. 13 hr. is the interval of time between the given dates.

CIRCULAR MEASURE.

428. A ***Circle*** is a plane figure bounded by a curved line, all points of which are equally distant from a point within called the centre.



429. A ***Circumference*** is the line that bounds a circle.

430. A ***Degree*** is one of the 360 equal parts into which the circumference of a circle is supposed to be divided.

431. The degree is the ***Standard Unit*** of circular measure.

TABLE OF UNITS.

60″	=	1′
60′	=	1°
30°	=	1 *S.*
12 *S.*	=	1 *Cir.*
360°	=	1 *Cir.*

1. ***Denominations.***—Seconds (″), Minutes (′), Degrees (°), Signs (S.), Circle (Cir.).

2. *One-half* of a *circumference*, or 180°, as shown by the figure in the margin, is called a *Semi-circumference; One-fourth*, or 90°, a *Quadrant; One-sixth*, or 60°, a *Sextant;* and *One-twelfth*, or 30°, a *Sign*.

3. The length of a degree varies with the size of the circle, as will be seen by examining the foregoing diagram.

4. A degree of latitude or a degree of longitude on the Equator is 69.16 statute miles. A *minute* on the earth's circumference is a *geographical* or *nautical* mile.

SPECIAL UNITS.

Table for Paper.	*Table for Counting.*
24 Sheets = 1 Quire.	12 Things = 1 Dozen (doz.)
20 Quires = 1 Ream.	12 Dozen = 1 Gross (gro.)
2 Reams = 1 Bundle.	12 Gross = 1 Great Gross (G. Gro.)
5 Bundles = 1 Bale.	20 Things = 1 Score (Sc.)

UNITS OF MONEY.

UNITED STATES MONEY.

433. The *dollar* is the ***Standard Unit*** of United States money.

TABLE OF UNITS.
10 *m.* = 1 *ct.*
10 *ct.* = 1 *d.*
10 *d.* = $1.
$10 = 1 *E.*

1. ***Denominations.***—Mills (m.), Cents (ct.), Dimes (d.), Dollars ($), Eagles (E.).

2. The United States *coin*, as fixed by the Coinage Acts of 1873 and 1878, is as follows: ***Gold,*** the double-eagle, eagle, half-eagle, quarter-eagle, three-dollar, and one-dollar; ***Silver,*** the trade-dollar, dollar, half-dollar, quarter-dollar, and dime; ***Nickel,*** the five-cent and three-cent; ***Bronze,*** one-cent.

3. ***Composition of Coins.***—*Gold coin* contains .9 pure gold and .1 silver and copper. *Silver coin* contains .9 pure silver and .1 pure copper. *Nickel coin* contains .25 nickel and .75 copper. *Bronze coin* contains .95 copper and .05 zinc and tin.

4. The *Trade-dollar* weighs 420 grains and is designed for commercial purposes solely. The silver *Dollar* weighs 412½ grains.

CANADA MONEY.

434. 1. ***Denominations.***—*Mills, Cents,* and *Dollars.* These have the same nominal value as in United States Money.

2. The *Coin* of the Dominion of Canada is as follows: ***Gold,*** the coins in use are the sovereign and half-sovereign; ***Silver,*** the fifty-cent, twenty-five cent, ten-cent, and five-cent pieces; ***Bronze,*** the one-cent piece.

ENGLISH MONEY.

435. The *pound sterling* is the ***Standard Unit*** of English money. It is equal to $4.8665 United States money.

TABLE OF UNITS.

4 *far.*	=	1 *d.*
12 *d.*	=	1 *s.*
20 *s.*	=	1 *Sov.* or £1.
2 *s.*	=	1 *fl.*
5 *s.*	=	1 *cr.*

1. ***Denominations.***—Farthings (far.), Pennies (d.), Shillings (s.), Sovereign (sov.), Pound (£), Florin (fl.), Crown (cr.).

2. The Coins in general use in Great Britain are as follows: ***Gold,*** sovereign and half-sovereign; ***Silver,*** crown, half-crown, florin, shilling, six-penny, and three-penny; ***Copper,*** penny, half-penny, and farthing.

FRENCH MONEY.

436. The *silver franc* is the ***Standard Unit*** of French money. It is equal to $.193 United States money.

TABLE OF UNITS.

10 *m.*	=	1 *ct.*
10 *ct.*	=	1 *dc.*
10 *dc.*	=	1 *fr.*

1. ***Denominations.***—Millimes (m.), Centimes (ct.), Decimes (dc.), Francs (fr.).

2. ***Equivalents.***—1 fr. = 10 dc. = 100 ct. = 1000 m.

3. The Coin of France is as follows: ***Gold,*** 100, 40, 20, 10, and 5 francs; ***Silver,*** 5, 2, and 1 franc, and 50 and 25 centimes; ***Bronze,*** 10, 5, 2, and 1 centime pieces.

GERMAN MONEY.

437. The *mark* is the ***Standard Unit*** of the *New German Empire.* It is equal to 23.85 cents United States money, and is divided into 100 equal parts, one of which is called a ***Pfennig.***

1. The *Coins* of the *New Empire* are as follows: ***Gold,*** 20, 10, and 5 marks; ***Silver,*** 2 and 1 mark; ***Nickel,*** 10 and 5 pfennig.

2. The coins most frequently referred to in the United States are the silver Thaler, equal 74.6 cents, and the silver Groschen, equal $2\frac{1}{2}$ cents.

THE METRIC SYSTEM.

439. The ***Metric System*** of *Related Units* is formed according to the *decimal scale.*

440. The ***Meter,*** which is 39.37079 inches long, or nearly *one ten-millionth* of the distance on the earth's surface from the equator to the pole, is the *base* of the system.

441. The ***Primary*** or ***Principal Units*** of the system are the *Meter*, the *Are* (air), the *Stere* (stair), the *Liter* (leeter), and the *Gram.* All other units are multiples and sub-multiples of these.

442. The *names* of ***Multiple Units*** or higher denominations are formed by prefixing to the names of the *primary units* the Greek numerals *Deka* (10), *Hecto* (100), *Kilo* (1000), and *Myria* (10000).

443. The names of ***Sub-multiple Units,*** or lower denominations, are formed by prefixing to the names of the *primary units* the Latin numerals, *Deci* ($\frac{1}{10}$), *Centi* ($\frac{1}{100}$), and *Milli* ($\frac{1}{1000}$).

UNITS OF LENGTH.

444. The ***Meter*** is the *principal unit* of *length.*

TABLE OF UNITS.

10 Millimeters,	*mm.*	=	1 Centimeter	=	.3937079 in.
10 Centimeters,	*cm.*	=	1 Decimeter	=	3.937079 in.
10 Decimeters,	*dm.*	=	1 ***Meter***	=	39.37079 in.
10 METERS,	*M.*	=	1 Dekameter	=	32.808992 ft.
10 Dekameters,	*Dm.*	=	1 Hectometer	=	19.884237 rd.
10 Hectometers,	*Hm.*	=	1 Kilometer	=	.6213824 mi.
10 Kilometers,	*Km.*	=	1 Myriameter (*Mm.*)	=	6.213824 mi.

The *meter* is used in place of one yard in measuring cloth and short distances. Long distances are usually measured by the *kilometer.*

UNITS OF SURFACE.

445. The ***Square Meter*** is the *principal unit* of surfaces.

TABLE OF UNITS.

100 Sq. Millimeters,	*sq. mm.*	=	1 Sq. Centimeter	=	.155+ sq. in.
100 Sq. Centimeters,	*sq. cm.*	=	1 Sq. Decimeter	=	15.5+ sq. in.
100 Sq. Decimeters,	*sq. dm.*	=	1 ***Sq. Meter*** (*Sq. M.*)	=	1.196+ sq. yd

446. The ***Are,*** a square whose side is 10 meters, is the *principal unit* for measuring land.

TABLE OF UNITS.

100 Centiares,	*ca.*	= 1 ***Are***	=	119.6034 sq. yd.
100 Ares,	*A.*	= 1 Hectare (*Ha.*)	=	2.47114 acres.

UNITS OF VOLUME.

447. The ***Cubic Meter*** is the *principal unit* for measuring ordinary solids, as embankments, etc.

TABLE OF UNITS.

1000 Cu. Millimeters,	*cu. mm.*	= 1 Cu. Centimeter	=	.061 cu. in.
1000 Cu Centimeters,	*cu. cm.*	= 1 Cu. Decimeter	=	61.026 cu. in.
1000 Cu. Decimeters,	*cu. dm.*	= 1 ***Cu. Meter***	=	35.316 cu. ft.

448. The ***Stere,*** or *Cu. Meter,* is the *principal unit* for measuring wood.

TABLE OF UNITS.

10 Decisteres,	*dst.*	= 1 ***Stere***	=	35.316+ cu. ft.
10 STERES,	*St.*	= 1 Dekastere (*Dst.*)	=	13.079+ cu. yd.

UNITS OF CAPACITY.

449. The ***Liter*** is the *principal unit* both of Liquid and Dry Measure. It is equal to a vessel whose volume is equal to a cube whose edge is *one-tenth* of a *meter.*

TABLE OF UNITS.

10 Milliliters,	*ml.*	= 1 Centiliter	=	.6102 cu. in.	=	.338 fl. oz.
10 Centiliters,	*cl.*	= 1 Deciliter	=	6.1022 " "	=	.845 gill.
10 Deciliters,	*dl.*	= 1 ***Liter***	=	.908 qt.	=	1.0567 qt.
10 LITERS,	*L.*	= 1 Dekaliter	=	9.08 "	=	2.6417 gal.
10 Dekaliters,	*Dl.*	= 1 Hectoliter	=	2.8372+ bu.	=	26.417 "
10 Hectoliters,	*Hl.*	= 1 Kiloliter	=	28.372+ "	=	264.17 "
10 Kiloliters,	*Kl.*	= 1 Myrialiter	=	283.72+ "	=	2641.7 "

The Hectoliter is used in measuring large quantities in both liquid and dry measure.

UNITS OF WEIGHT.

450. The ***Gram*** is the *principal unit* of *weight,* and is equal to the weight of a cube of distilled water whose edge is one centimeter.

TABLE OF UNITS.

10 Milligrams,	*mg.*	=	1 Centigram	=	.15432+ gr. Troy.
10 Centigrams,	*cg.*	=	1 Decigram	=	1.54324+ " "
10 Decigrams,	*dg.*	=	1 ***Gram***	=	15.43248+ " "
10 GRAMS,	*G.*	=	1 Dekagram	=	.3527 + oz. Avoir.
10 Dekagrams,	*Dg.*	=	1 Hectogram	=	3.52739+ " "
10 Hectograms,	*Hg.*	=	1 { Kilogram or *Kilo.* }	=	2.20462+ lb.
10 Kilograms,	*Kg.*	=	1 Myriagram	=	22.04621+ "
10 Myriagrams,	*Mg.*	=	1 Quintal	=	220.46212+ "
10 Quintals,		=	1 { Tonneau or *Ton.* }	=	2204.6212 + "

The ***Kilogram*** or Kilo., which is little more than $2\frac{1}{5}$ lb. Avoir., is the *common* weight in trade. Heavy articles are weighed by the ***Tonneau,*** which is 204 lb. more than a *common ton.*

Comparative Table of Units.

1 Inch	=	.0254 meter.	1 Cu. foot	=	.2832 Hectoliter.
1 Foot	=	.3048 "	1 Cu. yard	=	.7646 Steres.
1 Yard	=	.9144 "	1 Cord	=	3.625 Steres.
1 Mile	=	1.6093 Kilometers.	1 Fl. ounce	=	.02958 Liter.
1 Sq. inch	=	.0006452 sq. meter.	1 Gallon	=	3.786 Liters.
1 Sq. foot	=	.0929 "	1 Bushel	=	.3524 Hectoliter.
1 Sq. yard	=	.8361 "	1 Troy grain	=	.0648 Gram.
1 Acre	=	40.47 Ares.	1 Troy lb.	=	.373 Kilogram.
1 Sq. mile	=	.259 Hectares.	1 Avoir. lb.	=	.4536 Kilogram.
1 Cu. inch	=	.01639 Liter.	1 Ton	=	.9071 Tonneau.

EXAMPLES FOR PRACTICE.

451. Reduce

1. 84 lb. Avoir. to kilograms.
2. 37 T. to tonneau.
3. 96 bu. to hectoliters.
4. 75 fl. oz. to liters.
5. 89 cu. yd. to steres.
6. 328 acres to ares.
7. 4.0975 liters to cu. in.
8. 31.7718 sq. meters to sq. yd.
9. 272.592 liters to bushels.
10. 35.808 kilograms to Troy gr.
11. 133.75 steres to cords.
12. 33.307 steres to cu. ft.
13. If the price per gram is $.38, what is it per grain ?

14. If the price per liter is $1.50, what is it per quart?

15. At 26.33 cents per hectoliter, what will be the cost of 157 bushels of peas?

16. When sugar is selling at 2.168 cents per kilogram, what will be the cost of 138 lb. at the same rate?

17. Reduce 834 grams to decigrams; to dekagrams.

18. In 84 hectoliters how many liters? how many centiliters?

19. A man travels at the rate of 28.279 kilometers a day. How many miles at the same rate will he travel in 45 days?

20. If hay is sold at $18.142 per ton, what is the cost of 48 tonneau at the same rate?

21. When a kilogram of coffee costs $1.1023, what is the cost of 148 lb. at the same rate?

REVIEW AND TEST QUESTIONS.

465. 1. Define Related Unit, Denominate Number, Denominate Fraction, Denomination, and Compound Number.

2. Repeat Troy Weight and Avoirdupois Weight.

3. Reduce 9 bu. 3 pk. 5 qt. to quarts, and give a reason for each step in the process.

4. In 9 rd. 5 yd. 2 ft. how many inches, and why?

5. Repeat Square Measure and Surveyors' Linear Measure.

6. Reduce 23456 sq. in. to a compound number, and give a reason for each step in the process.

7. Define a cube, a rectangular volume, and a cord foot.

8. Show by a diagram that the contents of a rectangle is found by multiplying together its two dimensions.

9. Define a Board Foot, a Board Inch; and show by diagrams that there are 12 *board feet* in 1 cubic foot and 12 *board inches* in 1 board foot.

10. Reduce $\frac{3}{8}$ of an inch to a decimal of a foot, and give a reason for each step in the process.

11. How can a pound Troy and a pound Avoirdupois be compared?

12. Reduce .84 of an oz. Troy to a decimal of an ounce Avoirdupois, and give reason for each step in the process.

13. Explain how a compound number is reduced to a fraction or decimal of a higher denomination. Illustrate the abbreviated method, and give a reason for each step in the process.

PART SECOND.

BUSINESS ARITHMETIC

SHORT METHODS.

466. Practical devices for reaching results rapidly are of *first importance* in all business calculations. Hence the following summary of short methods should be thoroughly mastered and applied in all future work. The exercises under each problem are designed simply to illustrate the application of the contraction.

When the directions given to perform the work are not clearly understood, the references to former explanations should be carefully examined.

467. Prob. I.—**To multiply by 10, 100, 1000, etc.**

Move the decimal point in the multiplicand as many places to the right as there are ciphers in the multiplier, annexing ciphers when necessary **(91)**.

Multiply the following:

1. 84×100.	4. 3.8097×10000.	7. 3426×1000.
2. 76×1000.	5. $.89752 \times 1000$.	8. 7200×100000.
3. 5.73×100.	6. 3.0084×10000.	9. 463×1000000.

468. Prob. II.—**To multiply where there are ciphers at the right of the multiplier.**

Move the decimal point in the multiplicand as many places to the right as there are ciphers at the right of the multiplier, annexing ciphers when necessary, and multiply the result by the significant figures in the multiplier **(93)**.

Multiply the following:

1. 376×800.	4. 836.9×2000.	7. 3800×7200.
2. 42.9×420.	5. 7.648×3200.	8. 460×900.
3. 500×700.	6. 2300×5000.	9. $.8725 \times 3600$.

469. Prob. III.—**To multiply by 9, 99, 999, etc.**

Move the decimal point in the multiplicand as many places to the right as there are nines in the multiplier, annexing ciphers when necessary, and subtract the given multiplicand from the result.

Observe that by moving the decimal point as directed, we multiply by a number 1 greater than the given multiplier; hence the multiplicand is subtracted from the result. To multiply by 8, 98, 998, and so on, we move the decimal point in the same manner, and subtract from the result twice the multiplicand.

Perform the following multiplication:

1. 736458×9.	4. 53648×990.	7. 7364×998.
2. 3895×99.	5. 83960×9999.	8. 6283×9990.
3. 87634×999.	6. 26384×98.	9. 4397×998.

470. Prob. IV.—**To divide by 10, 100, 1000, etc.**

Move the decimal point in the dividend as many places to the left as there are ciphers in the divisor, prefixing ciphers when necessary.

Perform the division in the following:

1. $8736 \div 100$.	4. $23.97 \div 1000$.	7. $.54 \div 100$.
2. $437.2 \div 10$.	5. $5.236 \div 100$.	8. $.07 \div 1000$.
3. $790.3 \div 100$.	6. $.6934 \div 1000$.	9. $7.2 \div 1000$.

471. Prob. V.—**To divide where there are ciphers at the right of the divisor.**

Move the decimal point in the dividend as many places to the left as there are ciphers at the right of the divisor, prefixing ciphers when necessary (**140**), *and divide the result by the significant figures in the divisor* (**142**).

Perform the division in the following:

1. $7352 \div 40$.
2. $523.7 \div 80$.
3. $329.5 \div 3000$.
4. $5.2 \div 400$.
5. $.96 \div 120$.
6. $.08 \div 200$.
7. $364.2 \div 540$.
8. $973.5 \div 360$.
9. $8.357 \div 600$.

472. PROB. VI.—**To multiply one fraction by another.**

Cancel all factors common to a numerator and a denominator before multiplying (**185**—II).

Perform the following multiplications by canceling common factors:

1. $\frac{12}{25} \times \frac{15}{16}$.
2. $\frac{40}{99} \times \frac{33}{250}$.
3. $\frac{160}{625} \times \frac{25}{64}$.
4. $\frac{84}{125} \times \frac{5}{12}$.
5. $\frac{32}{45} \times \frac{90}{128}$.
6. $\frac{4}{5} \times \frac{15}{28} \times \frac{35}{99}$.
7. $\frac{9}{17} \times \frac{68}{81} \times \frac{3}{16}$.
8. $\frac{5}{11} \times \frac{14}{55} \times \frac{33}{70}$.
9. $\frac{8}{9} \times \frac{63}{56} \times \frac{7}{16}$.
10. $\frac{120}{162} \times \frac{27}{96} \times \frac{3}{100}$.
11. $\frac{80}{93} \times \frac{31}{100} \times \frac{5}{8}$.
12. $\frac{1000}{8462} \times \frac{16}{125} \times \frac{5}{32}$.
13. $\frac{10}{21} \times \frac{100}{625} \times \frac{7}{64}$.
14. $\frac{100}{385} \times \frac{11}{25} \times \frac{7}{8}$.
15. $\frac{100}{376} \times \frac{47}{1000} \times \frac{8}{9}$.

473. PROB. VII.—**To divide one fraction by another.**

Cancel all factors common to both numerators or common to both denominators before dividing (**291**). Or,

Invert the divisor and cancel as directed in Prob. VI.

Perform the division in the following, canceling as directed:

1. $\frac{15}{28} \div \frac{5}{7}$.
2. $\frac{12}{25} \div \frac{8}{15}$.
3. $\frac{32}{63} \div \frac{16}{54}$.
4. $.9 \div .03$.
5. $\frac{42}{125} \div \frac{14}{25}$.
6. $\frac{39}{56} \div \frac{13}{28}$.
7. $\frac{84}{275} \div \frac{36}{125}$.
8. $.28 \div .04$.
9. $\frac{100}{189} \div \frac{10}{21}$.
10. $\frac{51}{100} \div \frac{17}{1000}$.
11. $.39 \div .003$.
12. $.63 \div .0027$.

474. PROB. VIII.—**To divide one number by another.**

Cancel the factors that are common to the dividend and divisor before dividing (**185**—II).

Perform the following divisions, canceling as directed:

1. $8400 \div 300$.
2. $3900 \div 130$.
3. $4635 \div 45$.
4. $62500 \div 2500$.
5. $3420 \div 5400$.
6. $89600 \div 800$.
7. $9999 \div 63$.
8. $32000 \div 400$.
9. $75000 \div 1500$.

ALIQUOT PARTS.

475. An ***Aliquot Part*** of a number is any number, integral or mixed, which will exactly divide it.

Thus, 2, $2\frac{1}{2}$, $3\frac{1}{3}$, are aliquot parts of 10.

476. The aliquot parts of any number are found by dividing by 2, 3, 4, 5, and so on, up to 1 less than the given number.

Thus, $100 \div 2 = 50$; $100 \div 3 = 33\frac{1}{3}$; $100 \div 4 = 25$. Each of the quotients 50, $33\frac{1}{3}$, and 25, is an aliquot part of 100.

477. The character @ is followed by the price of a unit or one article. Thus, 7 cords of wood @ $4.50 means 7 cords of wood at $4.50 a cord.

478. Memorize the following aliquot parts of 100, 1000, and $1.

Table of Aliquot Parts.

of 100		of 1000		of $1	
50	$= \frac{1}{2}$	500	$= \frac{1}{2}$	50 ct.	$= \frac{1}{2}$
$33\frac{1}{3}$	$= \frac{1}{3}$	$333\frac{1}{3}$	$= \frac{1}{3}$	$33\frac{1}{3}$ ct.	$= \frac{1}{3}$
25	$= \frac{1}{4}$	250	$= \frac{1}{4}$	25 ct.	$= \frac{1}{4}$
20	$= \frac{1}{5}$	200	$= \frac{1}{5}$	20 ct.	$= \frac{1}{5}$
$16\frac{2}{3}$	$= \frac{1}{6}$	$166\frac{2}{3}$	$= \frac{1}{6}$	$16\frac{2}{3}$ ct.	$= \frac{1}{6}$
$14\frac{2}{7}$	$= \frac{1}{7}$	$142\frac{6}{7}$	$= \frac{1}{7}$	$14\frac{2}{7}$ ct.	$= \frac{1}{7}$
$12\frac{1}{2}$	$= \frac{1}{8}$	125	$= \frac{1}{8}$	$12\frac{1}{2}$ ct.	$= \frac{1}{8}$
$11\frac{1}{9}$	$= \frac{1}{9}$	$111\frac{1}{9}$	$= \frac{1}{9}$	$11\frac{1}{9}$ ct.	$= \frac{1}{9}$
10	$= \frac{1}{10}$	100	$= \frac{1}{10}$	10 ct.	$= \frac{1}{10}$

479. PROB. IX.—**To multiply by using aliquot parts.**

1. Multiply 459 by $33\frac{1}{3}$.

$$3\,)\underline{45900}$$
$$15300$$

EXPLANATION.—We multiply by 100 by annexing two ciphers to the multiplicand, or by moving the decimal point two places to the right. But 100 being equal to 3 times the multiplier $33\frac{1}{3}$, the product 45900 is 3 times as large as the required product; hence we divide by 3.

Perform the following multiplications by aliquot parts.

2. 974×50.	5. $234 \times 333\frac{1}{3}$.	8. $4.38 \times 3\frac{1}{3}$.
3. $35.8 \times 16\frac{2}{3}$.	6. $869 \times 11\frac{1}{9}$.	9. $7.63 \times 142\frac{6}{7}$.
4. 895×125.	7. $72 \times 111\frac{1}{9}$.	10. 58.9×250.

Solve the following examples orally, by aliquot parts.

11. What cost 48 lb. butter @ 25 ct.? @ 50 ct.? @ $33\frac{1}{3}$ ct.?

SOLUTION.—At \$1 a pound, 48 would cost \$48. Hence at $33\frac{1}{3}$ cts. a pound, which is $\frac{1}{3}$ of \$1, 48 pounds would cost $\frac{1}{3}$ of \$48, which is \$16.

12. What cost 96 lb. sugar @ $12\frac{1}{2}$ ct.? @ $14\frac{2}{7}$ ct.? @ $16\frac{2}{3}$ ct.?
13. What is the cost of 24 bushels wheat @ \$$1.33\frac{1}{3}$?

SOLUTION.—At \$1 a bushel, 24 bushels cost \$24; at $33\frac{1}{3}$ ct., which is $\frac{1}{3}$ of \$1 a bushel, 24 bushels cost \$8. Hence at \$$1.33\frac{1}{3}$ a bushel, 24 bushels cost the sum of \$24 and \$8, which is \$32.

14. What cost 42 yards cloth @ \$$1.16\frac{2}{3}$? @ \$$2.14\frac{2}{7}$?
15. What cost 72 cords of wood @ \$$4.12\frac{1}{2}$? @ \$3.25?

Find the cost of the following, using aliquot parts for the cents in the price.

16. 834 bu. wheat @ \$$1.33\frac{1}{3}$; @ \$1.50; @ \$1.25; at \$$1.16\frac{2}{3}$.
17. 100 tons coal @ \$4.25; at \$5.50; @ \$$6.12\frac{1}{2}$; @ \$$5.33\frac{1}{3}$.
18. 280 yd. cloth @ \$$2.14\frac{2}{7}$; @ \$$1.12\frac{1}{2}$; @ \$3.25; @ \$2.50.
19. 150 bbl. apples @ \$4.20; @ \$4.50; @ \$$4.33\frac{1}{3}$.
20. 2940 bu. oats @ 33 ct.; @ 50 ct.; @ 25 ct.
21. 896 lb. sugar @ $12\frac{1}{2}$; @ $14\frac{2}{7}$; @ $16\frac{2}{3}$.
22. What is the cost of 2960 yd. cloth at $37\frac{1}{2}$ ct. a yard?

$25 = \frac{1}{4}$ of 100, hence	4)	2960
$12\frac{1}{2} = \frac{1}{2}$ of 25, hence	2)	740
$37\frac{1}{2}$		370
		\$1110

EXPLANATION.—At \$1 a yard, 2960 yd. will cost \$2960. But 25 ct. is $\frac{1}{4}$ of \$1, hence $\frac{1}{4}$ of \$2960 which is \$740, is the cost at 25 ct. a yd.

2. Again, $12\frac{1}{2}$ ct. is the $\frac{1}{2}$ of 25 ct., hence \$740, the cost at 25 cts., divided by 2, gives the cost at $12\frac{1}{2}$ ct., which is \$370. But 25 ct. + $12\frac{1}{2}$ ct. = $37\frac{1}{2}$, hence \$740 + \$370 or \$1110 is the cost at $37\frac{1}{2}$ ct.

23. 495 bu. barley @ 75 ct.; @ $62\frac{1}{2}$ ct.; @ $87\frac{1}{2}$ ct.
24. 680 lb. coffee @ $37\frac{1}{2}$ ct.; @ 75 ct.; @ 60 ct.
25. 4384 yd. cloth @ $12\frac{1}{2}$ ct.; @ 15 ct.; @ 30 ct.; @ 35 ct.

Observe, that 10 ct. = $\frac{1}{10}$ of 100 ct., and 5 ct. = $\frac{1}{2}$ of 10 ct.

26. 870 lb. tea @ 60 ct.; @ $62\frac{1}{2}$ ct.; @ 80 ct.; @ $87\frac{1}{2}$ ct.

480. Prob. X.—To divide by using aliquot parts.

1. Divide 7258 by $33\frac{1}{3}$.

$$\begin{array}{r} 72.58 \\ 3 \\ \hline 217.74 \end{array}$$

Explanation.—1. We divide by 100 by moving the decimal point two places to the left.

2. Since 100 is 3 times $33\frac{1}{3}$, the given divisor, the quotient 72.58 is only $\frac{1}{3}$ of the required quotient; hence we multiply the 72.58 by 3, giving 217.74, the required quotient.

Perform by aliquot parts the division in the following:

2. $8730 \div 3\frac{1}{3}$.
3. $9764 \div 5$.
4. $8.375 \div 16\frac{2}{3}$.
5. $379.6 \div 33\frac{1}{3}$.
6. $98.54 \div 50$.
7. $394.8 \div 125$.
8. $460.85 \div 250$.
9. $90.638 \div 25$.
10. $73096 \div 333\frac{1}{3}$.

Solve the following examples orally, using aliquot parts.

11. At $33\frac{1}{3}$ ct., how many yards of cloth can be bought for $4?

Solution.—Since $1, or 100 ct., is 3 times $33\frac{1}{3}$ ct., we can buy 3 yards for $1. Hence for $4 dollars we can buy 4 times 3 yd., which is 12 yd.

Observe, that in this solution we divide by 100 and multiply by 3, the number of times $33\frac{1}{3}$, the given price, is contained in 100. Thus, $4=400 ct., 400÷100=4, and 4×3=12. In the solution, the reduction of the $4 to cents is omitted, as we recognize at sight that 100 ct., or $1, is contained 4 times in $4.

12. How many yards of cloth can be bought for $8 @ $12\frac{1}{2}$ ct.? @ $14\frac{2}{7}$ ct.? @ $33\frac{1}{3}$ ct.? @ $16\frac{2}{3}$ ct.? @ 25 ct.? @ 10 ct.? @ 50 ct.? @ 8 ct.? @ 5 ct.? @ 4 ct.?

13. How many pounds of butter @ $33\frac{1}{3}$ ct. can be bought for $7? For $10? For $40?

14. How much sugar can be bought at $12\frac{1}{2}$ ct. per pound for $3? For $8? For $12? For $30? For $120?

Solve the following, performing the division by aliquot parts:

15. How many acres of land can be bought for \$8954 at \$25 per acre? At \$50? At \$$33\frac{1}{3}$? At \$125? At \$$16\frac{2}{3}$? At \$250?

16. How many bushels of wheat can be bought for \$6354 at \$1.25 per bushel? At \$2.50?

Observe, \$1.25 = $\frac{1}{8}$ of \$10 and \$2.50 = $\frac{1}{4}$ of \$10. Hence by moving the decimal point one place to the left, which will give the number of bu. at \$10, and multiplying by 8, will give the number of bu. at \$1.25. Multiplying by 4 will give the number at \$2.50.

17. How many yards of cloth can be bought for \$2642 at $33\frac{1}{3}$ ct. per yard? At $14\frac{2}{7}$ ct.? At 25 ct.? At \$$3.33\frac{1}{3}$? At \$2.50? At \$$1.11\frac{1}{9}$? At \$$1.42\frac{6}{7}$?

18. What is the cost of 138 tons of hay at \$$12\frac{1}{2}$? At $14\frac{2}{7}$? At $16\frac{2}{3}$? At \$25? At \$13.50? At \$$15.33\frac{1}{3}$? At \$17.25?

BUSINESS PROBLEMS.

DEFINITIONS.

481. ***Quantity*** is the amount of any thing considered in a business transaction.

482. ***Price,*** or ***Rate,*** is the value in money allowed for a *given unit,* a *given number of units,* or a *given part* of a quantity.

Thus, in 74 bu. of wheat at \$2 per bushel, the *price* is the value of a *unit* of the quantity; in 8735 feet of boards at 45 ct. per 100 feet, the price is the value of *100 units.*

483. When the rate is the value of a given number of units, it may be expressed as a fraction or decimal.

Thus, cloth at \$3 for 4 yards may be expressed as \$$\frac{3}{4}$ per yard; 7 for every 100 in a given number may be expressed $\frac{7}{100}$ or .07. Hence, $\frac{5}{8}$ of 64 means *5 for every 8* in 64 or *5 per 8* of 64, and .08 means *8 per 100.*

484. *Cost* is the value in money allowed for an entire quantity.

Thus, in 5 barrels of apples at $4 per barrel, $4 is the *price*, and $4 × 5 or $20, the entire value of the 5 barrels, is the *cost*.

485. *Per Cent* means ***Per Hundred.***

Thus, 8 per cent of $600 means $8 out of every $100, which is $48. Hence a given per cent is the *price* or *rate* per 100.

486. The ***Sign*** of ***Per Cent*** is %. Thus, 8% is read, *8 per cent.*

Since *per cent* means *per hundred*, any given *per cent* may be expressed with the sign % or in the form of a decimal or common fraction; thus,

1 per cent is written	1%	or	.01	or	$\frac{1}{100}$.
7 per cent " "	7%	"	.07	"	$\frac{7}{100}$.
100 per cent " "	100%	"	1.00	"	$\frac{100}{100}$.
135 per cent " "	135%	"	1.35	"	$\frac{135}{100}$.
½ per cent " "	½%	"	.00½	"	$\frac{\frac{1}{2}}{100} = .005$.

487. *Percentage* is a certain number of hundredths of a given quantity.

488. *Profit and Loss* are commercial terms used to express the gain or loss in business transactions.

489. The ***Profit*** or ***Gain*** is the amount realized on business transactions in addition to the amount invested.

Thus, a man bought a farm for $8500 and sold it for $9200. The $8500 paid for the farm is the amount invested, and the $9200 is the whole sum realized on the transaction, which is $700 more than what was invested; hence the $700 is the *profit* or *gain* on the transaction.

490. The ***Loss*** is the amount which the whole sum realized on business transactions is less than the amount invested.

Thus, if a horse is bought for $270 and sold again for $170, there is a loss of $100 on the transaction.

491. The *Gain* and the *Loss* are usually expressed as a *per cent* of the amount invested.

ORAL EXERCISES.

492. Express the following decimally:

1. 5%.	5. $9\frac{1}{2}$%.	9. 207%.	13. $\frac{2}{3}$%.
2. 7%.	6. $2\frac{3}{4}$%.	10. $125\frac{1}{2}$%.	14. $1\frac{1}{6}$%.
3. 13%.	7. 112%.	11. $312\frac{3}{4}$%.	15. $3\frac{4}{5}$%.
4. 25%.	8. $\frac{1}{5}$%.	12. $\frac{7}{8}$%.	16. $\frac{9}{10}$%.

17. What is meant by 8%? By 135%? By $\frac{2}{3}$%?

18. What is the difference in the meaning of 5 *per cent* and 5 *per seven*?

19. How is 3 *per eight* expressed with figures? 7 *per five?* 13 *per twenty?* 9 *per four?*

20. What does $\frac{7}{100}$ mean, according to (**483**)? What does $\frac{4}{9}$ mean, according to the same Art.?

21. What is the difference in the meaning of $\frac{3}{4}$% and $\frac{3}{4}$ of 100?

22. What is the meaning of $.00\frac{2}{9}$? Of $.07\frac{2}{3}$? Of $.32\frac{1}{8}$?

23. Express $.00\frac{3}{7}$ with the sign % and fractionally.

24. Write in figures *three per cent*, and *nine per cent.*

Express the following as a *per cent:*

25. $\frac{4}{5}$.	28. 143.	31. $1\frac{1}{4}$.	34. 100.
26. $9\frac{1}{2}$.	29. 236.	32. 1.	35. 700.
27. $3\frac{2}{7}$.	30. $107\frac{1}{3}$.	33. 3.	36. 205.

493. In the following problems, some already given are repeated. This is done *first*, for review, and *second*, to give in a connected form the general problems that are of *constant recurrence* in actual business. Each problem should be fixed firmly in the memory, and the solution clearly understood.

It will be observed that Problems VIII, IX, X, and XI, are the same as are usually given under the head of ***Percentage.*** They are presented in a general form, as the solution is the same whether *hundredths*, or some other *fractional parts* are used.

PROBLEMS.

494. Prob. I.—**To find the cost when the number of units and the price of one unit are given.**

1. What is the cost of 35 lb. tea @ $\$\frac{5}{7}$?

Solution.—Since 1 lb. cost $\$\frac{5}{7}$, 35 lb. will cost 35 times $\$\frac{5}{7}$, which is (**271**) $25.

Find the cost and explain the following orally.

2. 64 bu. apples @ $\$\frac{7}{8}$.
3. 24 yd. cloth @ $\$2\frac{2}{3}$.
4. $6\frac{2}{5}$ yd. cloth @ $\$\frac{5}{8}$.
5. $24\frac{1}{2}$ lb. butter @ $\$\frac{2}{7}$.
6. 9 boxes oranges @ $\$4\frac{2}{3}$.
7. 18 tons coal @ $\$6\frac{5}{9}$.
8. 96 cords wood @ $\$4\frac{7}{12}$.
9. $8\frac{4}{7}$ yd. cloth @ $\$\frac{7}{10}$

Find the cost of the following, and express the answer in dollars and cents and fractions of a cent.

10. 84 bu. oats @ $\$\frac{2}{3}$.
11. 18 bbls. apples @ $\$4\frac{5}{6}$.
12. 52 yd. cloth @ $\$2\frac{7}{9}$.
13. 83 lb. coffee @ $\$\frac{3}{7}$.
14. $25\frac{3}{4}$ cords wood @ $\$5\frac{4}{7}$.
15. $63\frac{9}{16}$ lb. butter @ $\$\frac{7}{25}$.
16. 169 acr. land @ $\$27\frac{5}{9}$.
17. $32\frac{11}{16}$ lb. sugar @ $\$\frac{4}{23}$.

18. How much will a man earn in $19\frac{3}{4}$ days at $\$2\frac{2}{3}$ per day?

19. Sold Wm. Henry $25\frac{4}{5}$ lb. butter @ $28\frac{3}{4}$ ct., $17\frac{9}{16}$ lb. coffee @ $\$.33\frac{1}{3}$, and $39\frac{13}{16}$ lb. sugar @ $\$.14\frac{2}{3}$. How much was his bill?

20. A builder has 17 carpenters employed @ $2.25 per day. How much does their wages amount to for $24\frac{3}{5}$ days?

495. Prob. II.—**To find the price per unit, when the cost and number of units are given.**

1. If 9 yards cost $10.80, what is the price per yard?

Solution.—Since 9 yards cost $10.80, 1 yard will cost $\frac{1}{9}$ of it, or $\$10.80 \div 9 = \$1.20$. Hence, 1 yard cost $1.20.

Solve and explain the following orally.

2. If 9 lb. sugar cost $1.08, what is the price per pound?
3. At $4.80 for 8 yards of cloth, what is the price per yard?

4. If 12 lb. of butter cost $3.84, how much is it a pound?

5. Paid $3.42 for 9 lb. of coffee. How much did I pay per pound?

Solve and explain the following.

6. Bought 236 bu. oats for $90.80. What did I pay a bu.?

7. A piece of cloth containing 348 yd. was bought for $515.91. What did it cost per yard? *Ans.* $1.4825.

8. A farm containing 282 acres of land was sold for $22184. What was the rate per acre? *Ans.* $78.66+.

9. If 85 cords of stone cost $371.875, what is the price per cord? *Ans.* $4.375.

10. There were 25 mechanics employed on a building, each receiving the same wages; at the end of 28 days they were paid in the aggregate $1925. What was their daily wages?

11. A merchant bought 42 firkins of butter, each containing $63\frac{1}{2}$ lb., for $735.67. What did he pay per pound?

12. A farmer sold 70000 lb. of hay for $542.50. How much did he receive per ton? *Ans.* $15.50.

496. Prob. III.—**To find the cost when the number of units and the price of any multiple or part of one unit is given.**

1. What is the cost of 21 lb. sugar at 15 ct. for $\frac{7}{9}$ lb.?

Solution.—Since $\frac{7}{9}$ lb. cost 15 ct., 21 lb. must cost as many times 15 ct. as $\frac{7}{9}$ lb. is contained times in it. Hence, *First step,* $21 \div \frac{7}{9} = 27$; *Second step,* $\$.15 \times 27 = \$4.05$.

Find the cost of the following:

2. 124 acres of land at $144 for $2\frac{2}{7}$ acres; for $1\frac{4}{9}$ A.

3. 486 bu. wheat at $11 for 8 bushels; at $4.74 for 3 bushels; at $.72 for $\frac{3}{7}$ of a bushel.

4. 265 cords of wood at $21.95 for 5 cords.

5. 135 yd. broadcloth at $8.97 for $2\frac{3}{5}$ yd.; at $12.65 for $3\frac{2}{3}$ yd.

6. What is the cost of 987 lb. coal, at 35 ct. per 100 lb.?

SOLUTION.—As the price is per 100 lb., we find the number of *hundreds* is 987 by moving the decimal point two places to the left. The price multiplied by this result will give the required cost. Hence $\$.35 \times 9.87 = \$3.4545$, the cost of 987 lb. at 35 ct. per 100 lb.

Find the cost of the following bill of lumber:

7. 2345 ft. at \$1.35 per 100 (C) feet; 3628 ft. at \$.98 per C.; 1843 ft. at \$1.90 per C. ft.; 8364 ft. at \$2.84 per C.; 4384 ft. at \$27.50 per 1000 (M) ft.; 19364 ft. at \$45.75 per M.

8. What is the cost of 84690 lb. of coal at \$6.45 per ton (2000 lb.)?

Observe, that pounds are changed to tons by moving the decimal point 3 places to the left and dividing by 2

9. What is the cost of 96847 lb. coal at \$7.84 per ton?

497. PROB. IV.—**To find the number of units when the cost and price of one unit are given.**

1. How many yards of cloth can be bought for \$28 @ $\$\frac{4}{7}$?

SOLUTION.—Since 1 yard can be bought for $\$\frac{4}{7}$, as many yards can be bought for \$28 as $\$\frac{4}{7}$ is contained times in it. Hence, $\$28 \div \$\frac{4}{7} = 49$ yd.

Find the price and explain the following orally:

2. How many tons of coal can be bought for \$56 at \$4 a ton? At \$7? At \$8? At \$14? At \$6? At \$9? At \$5?

3. For \$40 how many bushels of corn can be bought at $\$\frac{5}{9}$ per bu.? At $\$\frac{4}{7}$? At $\$\frac{8}{11}$? At $\$\frac{10}{13}$? At \$.8? At $\$\frac{5}{6}$?

4. How many pounds of coffee can be bought for \$60 at $\$\frac{1}{3}$ per pound? At $\$\frac{2}{7}$? At $\$\frac{6}{13}$? At $\$\frac{10}{29}$? At $\$.33\frac{1}{2}$? At \$.4?

Solve the following:

5. The cost of a piece of cloth is \$480, and the price per yard $\$1\frac{3}{4}$; how many yards does it contain?

6. How many bushels of wheat at $\$1\frac{3}{5}$ can be purchased for \$840? At $\$1\frac{5}{8}$? At $\$1\frac{1}{3}$? At $\$1\frac{4}{7}$? At $\$1\frac{5}{6}$?

7. The cost of digging a drain at $\$3\frac{2}{3}$ per rod is \$187; what is the length of the drain? *Ans.* 51 rd.

8. A farmer paid \$14198 for his farm, at \$$65\frac{3}{7}$ per acre; how many acres does the farm contain? *Ans.* 217 A.

9. A grocer purchased \$101.65 worth of butter, at $35\frac{2}{3}$ cents a pound; how many pounds did he purchase? *Ans.* 285 lb.

10. How many yards of cloth can be bought at \$2.75 a yard for \$1086.25? *Ans.* 395.

11. A grain dealer purchased a quantity of wheat at \$1.20 per bushel, and sold it at an advance of $9\frac{6}{10}$ cents per bushel, receiving for the whole \$616.896; how many bushels did he purchase?

498. PROB. V.—**To find the number of units that can be purchased for a given sum when the cost of a multiple or part of one unit is given.**

1. At 19 ct. for $\frac{2}{3}$ of a yard, how many yards can be bought for \$8.55?

SOLUTION.—1. Since $\frac{2}{3}$ yd. cost 19 ct., $\frac{1}{3}$ must cost $\frac{1}{2}$ of 19 ct., or $9\frac{1}{2}$ ct., and $\frac{3}{3}$, or 1 yard, must cost 3 times $9\frac{1}{2}$ ct., or $28\frac{1}{2}$ ct.

2. Since 1 yard cost $28\frac{1}{2}$ ct., as many yards can be bought for \$8.55 as $28\frac{1}{2}$ ct. are contained times in it. Hence, $\$8.55 \div \$.285 = 30$, the number of yards that can be bought for \$8.55, at 19 ct. for $\frac{2}{3}$ yd.

2. How many tons of coal can be bought for \$277.50, at \$6 for $\frac{4}{5}$ of a ton? At \$8 for $\frac{5}{7}$ of a ton? *Ans.* 37 T.

3. How many bushels of corn can be bought for \$28, at 32 ct. for $\frac{4}{9}$ of a bu.? At 28 ct. for $\frac{2}{5}$ bu.?

4. A town lot was sold for \$1728, at \$3 per 8 sq. ft. The front of the lot is 48 ft. What is its depth? *Ans.* 96 ft.

5. A piece of cloth was sold for \$34.50, at 14 yards per \$1. How many yards did the piece contain? *Ans.* 483 yd.

6. A drove of cattle was sold for \$3738, at \$294 for every 7 head. How many head of cattle in the drove? *Ans.* 89.

7. A pile of wood was bought for \$275.60, at \$1.95 for 3 cord feet. How many cords in the pile? *Ans.* 53 cd.

8. A cellar was excavated for \$408.24, at \$4.41 for every 7 cu. yd. The cellar was 54 ft. by 36 ft. How deep was it?

499. Prob. VI.—**To find the cost when the quantity is a compound number and the price of a unit of one denomination is given.**

1. What is the cost of 8 bu. 3 pk. 2 qt. of wheat, at $1.44 per bushel?

2)	$1.44	
	8	
	11.52	Cost of 8 bu.
2)	72	" " 2 pk.
4)	36	" " 1 pk.
	9	" " 2 qt.
	$12.69, *Ans.*	

Solution.—1. Since $1.44 is the price per bushel, $1.44 × 8, or $11.52, is the cost of 8 bushels.

2. Since 2 pk. = $\frac{1}{2}$ bu., $1.44 ÷ 2, or 72 cts., is the cost of 2 pk., and the $\frac{1}{2}$ of 72 ct., or 36 ct., is the cost of 1 pk.

3. Since there are 8 qt. in 1 pk., 2 qt. = $\frac{1}{4}$ pk. Hence, the cost of 1 pk., 36 ct. ÷ 4, or 9 ct., is the cost of 2 qt.

4. The sum of the cost of the parts must equal the cost of the whole quantity. Hence, $12.69 is the cost of 8 bu. 3 pk. 2 qt., at $1.44 per bu.

Find the cost of the following orally:

2. 9 lb. 8 oz. sugar, at 12 ct. per pound; @ 14 ct.; @ 20 ct.
3. 7$\frac{3}{4}$ yd. ribbon @ 16 ct.; @ 40 ct.; @ 30 ct.
4. 15 bu. 3 pk. 6 qt. of apples, @ $1 per bushel.
5. 3 lb. 12 oz. butter, at 34 ct. per pound; at 40 ct.

Solve the following:

6. What will 5 T. 15 cwt. 50 lb. sugar cost, at $240 per ton?

7. Find the cost of 48 lb. 9 oz. 10 pwt. of block silver, at $12 per pound. *Ans.* $585.50.

8. Find the cost of excavating 240 cu. yd. 13$\frac{1}{2}$ cu. ft. of earth, at 50 cts. per cubic yard.

9. How much will a man receive for 2 yr. 9 mo. 25 da. service, at $1800 per year? *Ans.* $5075.

10. Sold 48 T. 15 cwt. 75 lb. of hay at $15 per ton, and 32 bu. 3 pk. 6 qt. timothy seed at $3.50 per bushel. How much did I receive for the whole?

11. How much will it cost to grade 8 mi. 230 rd. of a road, at $4640 per mile? *Ans.* $40455.

500. PROB. VII.—**To find what part one number is of another.**

1. What part of 12 is 4?

SOLUTION.—1 is $\frac{1}{12}$ of 12 and 4 being 4 times 1, is 4 times $\frac{1}{12}$ of 12, which is $\frac{4}{12} = \frac{1}{3}$; hence 4 is $\frac{1}{3}$ of 12.

Observe, that to ascertain what part one number is of another, we may at once write the former as the numerator and the latter as the denominator of a fraction, and reduce the fraction to its lowest terms (**256**).

2. What part is 15 of 18? Of 25? Of 24? Of 45?
3. What part is 36 of 48? Of 38? Of 42? Of 72?
4. $\frac{2}{3}$ is what part of $\frac{6}{7}$?

SOLUTION.—1. Only units of the *same* integral and fractional denomination can be compared (**155**); hence we reduce $\frac{2}{3}$ and $\frac{6}{7}$ to $\frac{14}{21}$ and $\frac{18}{21}$, and place the numerator 14 over the numerator 18, giving $\frac{14}{18} = \frac{7}{9}$; hence $\frac{2}{3}$ is $\frac{7}{9}$ of $\frac{6}{7}$.

2. We may express the relation of the fractions in the form of a complex fraction, and reduce the result to a simple fraction (**300**). Thus $\frac{\frac{2}{3}}{\frac{6}{7}} = \frac{14}{18} = \frac{7}{9}$. Hence $\frac{2}{3}$ is $\frac{7}{9}$ of $\frac{6}{7}$.

5. $\frac{3}{4}$ is what part of 11? $\frac{4}{5}$ is what part of $2\frac{1}{2}$?
6. $5\frac{3}{7}$ inches is what part of $2\frac{1}{3}$ yards? (See **387.**)
7. $29\frac{7}{12}$ rods is what part of 1 mile?
8. $7\frac{1}{2}$ is how many times $\frac{2}{5}$?
9. $11\frac{4}{9}$ is how many times $2\frac{3}{5}$?
10. What part of a year is 24 weeks? 8 weeks 10 days?
11. A man's yearly wages is \$950, and his whole yearly expenses \$590.80. What part of his wages does he save each year?
12. Out of \$750 I paid \$240. What part of my money have I still left? *Ans.* $\frac{17}{25}$, or .68.
13. A man owning a farm of $240\frac{4}{5}$ acres, sold $117\frac{5}{7}$ acres. What part of his whole farm has he still left?
14. 4% is what part of 12%? 8% is what part of 14%?
15. $3\frac{2}{5}$% is what part of 9%? $7\frac{1}{4}$% is what part of $8\frac{5}{6}$%?

16. Illustrate in full the process in the 14th and 15th examples.

501. Prob. VIII.—**To find a given fractional part of a given number.**

1. Find $\frac{3}{7}$ of 238.

Solution.—We find $\frac{1}{7}$ of 238 by dividing it by 7; hence $238 \div 7 = 34$, the $\frac{1}{7}$ of 238. But $\frac{3}{7}$ is 3 times $\frac{1}{7}$; hence $34 \times 3 = 102$, the $\frac{3}{7}$ of 238.

2. Find $\frac{7}{8}$ of 48; of 96; of 376; of 1035.
3. Find $\frac{6}{100}$ of 340; $\frac{8}{100}$ of 972; $\frac{9}{10}$ of 560.
4. Find $\frac{4}{5}$ of \$75; $\frac{7}{9}$ of \$824.60; $\frac{5}{12}$ of \$3.25.

Observe, that $\frac{4}{5}$ of \$75 means such a number of dollars as will contain \$4 for every \$5 in \$75; hence, to find the $\frac{4}{5}$ of \$75, we divide by 5 and multiply the quotient by 4.

5. Find 7% of 328.

Solution.—1. 7% means $\frac{7}{100}$. We find $\frac{1}{100}$ by moving the decimal point two places to the left (**470**). Hence 7% or $\frac{7}{100}$ of 328 is equal to $3.28 \times 7 = 22.96$.

2. We usually multiply by the rate first, then point off two decimal places in the product, which divides it by 100.

6. What is 8% of \$736? 4% of 395 lb. butter?
7. How much is $\frac{2\frac{1}{2}}{5}$ of 157 acres? $\frac{2\frac{2}{3}}{7}$ of 84 bu. wheat?

Find
8. 7% of 28 yd.
9. 5% of 300 men.
10. 9% of 278 lb.

Find
11. $4\frac{2}{3}$% of 284 mi.
12. $12\frac{1}{2}$% of 732.
13. $\frac{3}{4}$% of \$860.

14. Find the amount of \$832 + $\frac{5}{8}$% of itself.
15. Find the amount of \$325 + 7% of itself.
16. A firkin of butter contained $72\frac{3}{5}$ lb.; $\frac{3}{8}$ of it was sold: how many pounds are there left?
17. A piece of cloth contained 142 yd.; 15% was sold: how many yards yet remained unsold?

18. James Smith's farm contained 284 acres, and H. A. Watkins' $8\frac{1}{2}\%$ less. How many acres in H. A. Watkins' farm?

19. A merchant bought 276 yards cloth at \$3.40 per yard. He sold it at 25% profit. How much did he realize, and what was his selling price?

20. If tea cost 96 ct. per pound and is sold at a loss of $12\frac{1}{2}\%$, what is the selling price?

502. Prob. IX.—**To find a number when a fractional part is given.**

1. Find the number of which 84 is $\frac{7}{9}$.

Solution.—Since 84 is $\frac{7}{9}$ of the number, $\frac{1}{7}$ of 84 must be $\frac{1}{9}$; hence $84 \div 7 = 12$ is the $\frac{1}{9}$ of the required number. But 9 times $\frac{1}{9}$ is equal to the whole; hence, $12 \times 9 = 108$, the required number.

2. \$36 is $\frac{4}{5}$ of how many dollars? \$49 is $\frac{7}{9}$ of how many dollars?

3. Find the number of yards of cloth of which 135 yd. is $\frac{5}{12}$.

4. James has \$756, which is $\frac{4}{7}$ of George's money; how many dollars has George? *Ans.* \$1323.

5. The profits of a grocery for one year are \$3537, which is $\frac{3}{20}$ of the capital invested. How much is the capital?

6. Find the number of dollars of which \$296 are 8%, or .08.

First Solution.—Since \$296 are $\frac{8}{100}$ of the number, $\frac{1}{8}$ of \$296 or \$37, are $\frac{1}{100}$; hence $\frac{100}{100}$, or the whole, is 100 times \$37, or $37 \times 100 = \$3700$.

Second Solution.—Since \$296 are $\frac{8}{100}$ of the number, $\frac{1}{8}$ of \$296 is $\frac{1}{100}$, and $\frac{1}{8}$ of 100 times \$296 is $\frac{100}{100}$, or the required number. Hence, $\$296 \times 100 = \$29600$, and $\frac{1}{8}$ of $\$29600 = \$3700$, the required number.

From these solutions we obtain the following rule for finding a number when a decimal part of it is given:

503. Rule.—*Move the decimal point as many places to the right as there are places in the given decimal, annexing ciphers if necessary, and divide the result by the number expressed by the significant figures in the given decimal.*

Find what number	Find what number	Find what number
7. 16 is 8% of.	13. $\frac{4}{5}$ is 4% of.	19. $3\frac{4}{5}$ is 9% of.
8. 24 is 6% of.	14. \$$\frac{2}{3}$ is 7% of.	20. \$2.16 is 6% of.
9. 84 is 7% of.	15. $\frac{5}{9}$ pk. is 8% of.	21. $7\frac{7}{8}$ is 9% of.
10. \$72 are 9% of.	16. .7 ft. is 5% of.	22. $27\frac{1}{2}$ is 5% of.
11. 120 yd. are 5% of.	17. .09 is 4% of.	23. $\frac{4}{5}$ yd. is 8% of.
12. 56 bu. are 8% of.	18. .48 is 12% of.	24. .96 is 12% of.

25. A man's profits for one year amount to \$2840, which is 8% of the amount he has invested in business. What is his investment?

26. A merchant sells a piece of cloth at a profit of 30 ct. a yard, which is 20% of what it cost him. What was the buying price per yard?

27. A grocer purchased 186 lb. butter on Saturday, which is 6% of the entire quantity purchased during the week. What was the week's purchase?

28. A mechanic pays \$12 a month for house rent, which is 16% of his wages. What does he receive per month?

29. 12% of $\frac{3}{4}$ is 9% of what number?

30. How many acres in a farm 14% of which contains 42 acres?

31. An attorney receives \$1.75 for collecting a bill, which is $2\frac{1}{3}$ per cent. of the bill. What is the amount of the bill?

32. A man having failed in business is allowed to cancel his debts by paying 20%. What does he owe a man who receives \$270? *Ans.* \$1350.

33. A man sold his house for \$1000, which was 12% of the sum he received for his farm. What was the price of the farm? *Ans.* $\$8333.33\frac{1}{3}$.

34. If in a certain town \$3093.75 was raised from a $\frac{3}{4}$% tax, what was the value of property in the town?

35. S. T. Esty has 25% of his property invested in a house, 10% in a farm, 5% in a barn, and the rest in a grove worth \$4800. What is the amount of his property?

504. PROB. X.—**To express the part one number is of another in any given fractional unit.**

1. How many *fifths* of 3 is 8.

SOLUTION.—Since $\frac{3}{5}$ is $\frac{1}{5}$ of 3, there must be as many *fifths* of 3 in 8 as $\frac{3}{5}$ is contained times in it. $8 \div \frac{3}{5} = 13\frac{1}{3}$. Hence, 8 is $\frac{13\frac{1}{3}}{5}$ of three.

Solve the following orally:

2. How many *fourths* of 9 is 7? Is 5? Is 12? Is 20?

3. How many *hundredths* of 36 is 9? Is 4? Is 18? Is 12?

4. \$12 are how many *tenths* of \$5? Of \$8? Of \$15?

5. 42 yards are how many sixths of 2 yd.? Of 7 yd.? Of 3 yd.?

6. What *per cent* of \$11 are \$3, or \$3 are how many hundredths of \$11?

FIRST SOLUTION.—Since $\frac{11}{100}$ is $\frac{1}{100}$ of \$11, there must be as many hundredths of \$11 in \$3 as $\frac{11}{100}$ is contained times in \$3. $\$3 \div \frac{11}{100} = 3 \times \frac{100}{11} = \frac{300}{11} = 27\frac{3}{11}$. Hence, \$3 are $\frac{27\frac{3}{11}}{100}$, or $27\frac{3}{11}\%$ of \$11.

SECOND SOLUTION.—Since (**500**) \$3 are $\frac{3}{11}$ of \$11, we have only to reduce $\frac{3}{11}$ to hundredths to find what per cent \$3 are of \$11. $\frac{3}{11} = \frac{300}{1100} = \frac{27\frac{3}{11}}{100} = 27\frac{3}{11}\%$. Hence, \$3 are $27\frac{3}{11}\%$ of \$11.

From these solutions we obtain the following rule for finding what per cent or what decimal part one number is of another:

505. RULE.—*Express the former number as a fraction of the latter* (**500**), *and reduce this fraction to hundredths or to the required decimal* (**338**).

Find what per cent

7. 16 is of 64.

8. 12 is of 72.

9. \$36 are of \$180.

10. \$46 are of \$414.

11. 7 feet are of 8 yards.

12. 13 oz. are of 5 lb.

13. 284 acres are of 1 sq. mi.

14. 2 bu. 3 pk. are of 28 bu.

15. \$$\frac{2}{7}$ is of \$$\frac{4}{5}$; of \$5; of \$$2\frac{3}{5}$.

16. 3 lb. 13 oz. are of 9 lb.

17. $\frac{4}{9}$ of a cu. ft. is of 1 cu. yd.

18. 48 min. are of 3 hr.

19. $\frac{2}{3}$ of a sq. yd. is of $\frac{7}{8}$ of a sq. yd.

20. 3 bu. 2 pk. are of 8 bu. 3 pk. 5 qt.

21. A man paid $24 for the use of $300 for one year. What rate per cent did he pay?

22. A merchant invested $3485 in goods which he had to sell for $2973. What per cent of his investment did he lose?

23. A druggist paid 84 ct. an ounce for a certain medicine, and sold it at $1.36 an ounce. What per cent. profit did he make?

SOLUTION.—$\$1.36 - \$.84 = \$.52$; $\frac{52}{84} = \frac{5200}{8400} = \frac{61\frac{19}{21}}{100} = 61\frac{19}{21}\%$.

24. A farmer owning 386 acres sold 148 acres. What per cent of his original farm does he still own?

25. When a yard of silk is bought for $1.20 and sold for $1.60, what per cent is the profit of the buying price?

26. A man owed me $350, but fearing he would not pay it I agreed to take $306.25; what per cent. did I allow him?

27. Hawkins deposited $2500 in a bank, and again deposited enough to make the whole amount to $2750. What per cent of the first deposit was the last? *Ans.* 10.

28. Gave away $77\frac{1}{2}$ bushels of potatoes, and my whole crop was 500 bushels; what % of the crop did I give away?

29. A man pays $215.34 per acre for $4\frac{1}{2}$ acres of land, and lets it a year for $33.916; what % of the cost is the rent?

506. PROB. XI.—**To find a number which is a given fraction of itself greater or less than a given number.**

1. Find a number which is $\frac{2}{5}$ of itself less than 28.

SOLUTION.—1. Since the required number is $\frac{5}{5}$ of itself, and is $\frac{2}{5}$ of itself less than 28, hence 28 is $\frac{5}{5}+\frac{2}{5}$ or $\frac{7}{5}$ of it.

2. Since 28 is $\frac{7}{5}$ of the number, $\frac{1}{7}$ of 28, or 4, is $\frac{1}{5}$. Hence $\frac{5}{5}$, or the whole of the required number, is 5 times 4 or 20.

Solve the following orally:

2. What number is $\frac{2}{3}$ of itself less than 15? Less than 40? Less than 75? Less than 26? Less than 32?

3. What number increased $\frac{3}{7}$ of itself is equal 100? Is equal 80? Is equal 120? Is equal 12? Is equal 7?

4. Find a number which diminished by $\frac{3}{9}$ of itself is equal 56. Is equal 70. Is equal 15. Is equal 5.

Solve and explain the following:

5. What number increased by 7% or $\frac{7}{100}$ of itself is equal 642?

SOLUTION.—1. Since a number increased by 7% or $\frac{7}{100}$ of itself is $\frac{100}{100}+\frac{7}{100}=\frac{107}{100}$ of itself, 642 is $\frac{107}{100}$ or 107% of the required number.

2. Since 642 is $\frac{107}{100}$ of the required number, for every 107 in 642 there must be 100 in the required number. Hence, $642 \div 107 = 6$, and $6 \times 100 = 600$, the required number.

Observe, that $642 \div 1.07$ is the same as dividing by 107 and multiplying by 100 (**360**); hence the following rule, when a number has been increased or diminished by a given per cent or any decimal of itself:

507. RULE.—*Divide the given number, according as it is more or less than the required number, by 1 increased or diminished by the given decimal.*

6. What number increased by 15% of itself is equal 248.40?

7. A certain number increased by 80% of itself is 331.2; what is that number? *Ans.* 184.

8. By running 15% faster than usual, a locomotive runs 644 miles a day; what was the usual distance per day?

9. What number diminished by 25% of itself is 654?

10. A regiment after losing 8% of its number contained 736 men; what was its original number? *Ans.* 800.

11. A man who has had his salary increased 5% now receives $1050 a year; what was his former salary? *Ans.* $1000.

12. A merchant sells a coat for $8, thereby gaining 25%; what did the coat cost him? *Ans.* $6.40.

13. A clergyman lays up 12½% of his salary, which leaves him $1750 to spend; what is his salary? *Ans.* $2000.

14. J. Fayette sold his farm for $3960, which was 10% less than he gave for it, and he gave 10% more than it was worth; what was its actual value? *Ans.* $4000.

APPLICATIONS.

508. Profit and Loss, Commission, Insurance, Stocks, Taxes, and Duties, are applications of Business Problems VIII, IX, X, XI. The *rate* in these subjects is usually a *per cent*. Hence, for convenience in expressing rules, we denote the quantities by letters as follows:

1. ***B*** represents the ***Base,*** or number on which the percentage is reckoned.

2. ***R*** represents the ***Rate per cent*** expressed decimally.

3. ***P*** represents the ***Percentage,*** or the part of the *Base* which is denoted by the *Rate.*

4. ***A*** represents the ***Amount,*** or sum of the *Base* and *Percentage.*

5. ***D*** represents the ***Difference,*** or *Base* less the *Percentage.*

Formulæ, or Rules for Percentage.

509. Prob. VIII. $P = B \times R$. Read, { *The percentage is equal to the base multiplied by the rate.* }

510. Prob. IX. $B = \frac{P}{R}$. Read, { *The base is equal to the percentage divided by the rate.* }

511. Prob. X. $R = \frac{P}{B}$. Read, { *The rate is equal to the percentage divided by the base.* }

512. Prob. XI.
{ $B = \frac{A}{1+R}$. Read, { *The base is equal to the amount divided by* 1 *plus the rate.* }
{ $B = \frac{D}{1-R}$. Read, { *The base is equal to the diff'nce divided by* 1 *minus the rate.* }

513. Refer to the problems on pages 222 to 226 inclusive, and answer the following questions regarding these formulæ:

1. What is meant by $B \times R$, and why is $P = B \times R$? Illustrate your answer by an example, giving a reason for each step.

2. Why is $P \div R$ equal B? Give reasons in full for your answer.

3. If R is 135%, which is the greater, P or B, and why?

4. If R is 248%, how would you express R without the sign %?

5. Why is R equal to $P \div B$, and how must the quotient of $P \div B$ be expressed to represent R correctly?

6. What is meant by A? How many times R in P (**502**)? How many times 1 in B? How many times $1 + R$ must there be in A and why?

7. How many times R in P (**502**)? D is equal to B minus how many times R (**501**)?

8. Why is B equal to $D \div (1 - R)$? Give reasons in full for your answer.

PROFIT AND LOSS.

514. The quantities considered in Profit and Loss correspond with those in Percentage thus:

1. The ***Cost,*** or Capital invested, is the ***Base.***
2. The ***Per cent*** of ***Profit*** or ***Loss*** is the ***Rate.***
3. The ***Profit*** or ***Loss*** is the ***Percentage.***
4. The ***Selling Price*** when equal the *Cost* plus the *Profit* is the ***Amount,*** when equal the *Cost* minus the *Loss* is the ***Difference.***

EXAMPLES FOR PRACTICE.

515. 1. A firkin of butter was bought for $19 and sold at a profit of 16%. What was the gain?

Formula P = B × R. Read, *Profit* or *Loss* = *Cost* × *Rate %*.

Find the profit on the sale

2. Of 320 yd. cloth bought @ $1.50, sold at a gain of 17%.
3. Of 84 cd. wood bought @ $4.43⅓, sold at a gain of 20%.
4. Of 873 bu. wheat bought @ $1.25, sold at a gain of 14½%.

Find the loss on the sale

5. Of 180 T. coal bought @ $7.85, sold at a loss of 8¼%.

6. Of 124 A. land bought @ \$84.50, sold at a loss of $21\frac{3}{4}\%$.

7. If a farm was bought for \$4860 and sold for \$729 more than the cost, what was the gain per cent

Formula R = P ÷ B. Read, *Rate % Gain = Profit ÷ Cost.*

8. A piece of cloth is bought at \$2.85 per yard and sold at \$2.10 per yard. What is the loss per cent? *Ans.*

9. If $\frac{5}{8}$ of a cord of wood is sold for $\frac{3}{4}$ of the cost of 1 cord, what is the gain per cent? *Ans.*

10. Find the *selling* price of a house bought at \$5385.90, and sold at a gain of 18%.

Formula A=B × (1 + R). Read, *Selling* Price=*Cost* × (1 + Rate % *Gain*).

11. Corn that cost 65 ct. a bushel was sold at 20% gain. What was the selling price? *Ans.* 78 ct. a bu.

12. A grocer bought 43 bu. clover seed @ \$4.50, and sold it in small quantities at a gain of 40%. What was the selling price per bu. and total gain?

13. Bought 184 barrels of flour for \$1650, and sold the whole at a loss of 8%. What was the selling price per barrel?

Formula D=B (1−R). Read, *Selling* Price = *Cost* × (1−Rate % *Loss*).

14. Flour was bought at \$8.40 a barrel, and sold so as to lose 15%. What was the selling price?

15. C. Baldwin bought coal at \$6.25 per ton, and sold it at a loss of 18%. What was the selling price?

16. Sold a house at a loss of \$879, which was 15% of the cost. What was the cost?

Formula B = P ÷ R. Read, *Cost = Profit* or *Loss* ÷ Rate %.

17. A grain merchant sold 284 barrels of flour at a loss of \$674.50, which was 25% of the cost. What was the buying and selling price per barrel?

18. A drover wished to realize on the sale of a flock of 236 sheep \$531, which is 30% of the cost. At what price per head must he sell the flock?

19. Two men engaged in business, each having \$4380. A

gained $33\frac{1}{3}\%$ and B 75%. How much was B's gain more than A's?

20. If I buy 72 head of cattle at \$36 a head, and sell $33\frac{1}{3}\%$ of them at a gain of 18%, and the remainder at a gain of 24%, what is my gain?

21. A grocer sells coffee that costs $13\frac{1}{2}$ cents per pound, for $10\frac{4}{5}$ cents a pound. What is the loss per cent?

22. Fisk and Gould sold stock for \$3300 at a profit of $33\frac{1}{3}\%$. What was the cost of it?

23. A man bought 24 acres of land at \$75 an acre, and sold it at a profit of $8\frac{1}{2}\%$. What was his total gain?

24. A merchant sold cloth for \$3.84 a yard, and thus made 20%. What was the cost price?

25. Bought wood at \$3.25 a cord, and sold it at an average gain of 30%. What did it bring per cord?

26. If land when sold at a loss of $12\frac{1}{2}\%$ brings \$11.20 per acre, what would be the gain per cent if sold for \$15.36?

27. Bought a barrel of syrup for \$20; what must I charge a gallon in order to gain 20% on the whole?

COMMISSION.

516. A ***Commission Merchant*** or ***Agent*** is a person who transacts business for another for a percentage.

517. A ***Broker*** is a person who buys or sells stocks, bills of exchange, etc., for a percentage.

518. ***Commission*** is the amount paid a commission merchant or agent for the transaction of business.

519. ***Brokerage*** is the amount paid a broker for the transaction of business.

520. The ***Net Proceeds*** of any transaction is the sum of money that is left after all expenses of commission, etc., are paid.

521. The quantities considered in commission correspond with those in percentage thus:

1. The *amount* of money *invested* or *collected* is the ***Base.***
2. The *per cent* allowed for services is the ***Rate.***
3. The *Commission* or *Brokerage* is the ***Percentage.***
4. The sum invested or collected, plus the commission, is the ***Amount,*** minus the commission is the ***Difference.***

EXAMPLES FOR PRACTICE.

522. Let the pupil write out the formulæ for each kind of examples in commission in the same manner as they are given in Profit and Loss.

What is the commission or brokerage on the following:

1. The sale of 85 cords of wood @ \$4.75, commission $3\frac{1}{2}\%$?
2. The sale of 484 yds. cloth @ \$2.15, commission $1\frac{3}{4}\%$?
3. The sale of 176 shares stocks at \$87.50 a share, brokerage $\frac{3}{8}\%$?
4. The collection of \$3462.84, commission $2\frac{1}{8}\%$?

What is the rate of commission on the following:

5. Selling a farm for \$4800, commission \$120?
6. Collecting a debt of \$7500, commission \$350?
7. Selling wheat worth \$1.80 a bu., commission 4 ct. a bushel?

What is the amount of the sale in the following:

8. The commission is \$360, rate of commission $2\frac{1}{2}\%$?
9. The brokerage is \$754.85, rate of brokerage $1\frac{3}{8}\%$?
10. The commission is \$26.86, rate of commission $1\frac{5}{8}\%$?

Find the amount of the sales in the following:

Observe, that the commission is on the amount of the sales. Hence the formula for finding the amount of the sales when the net proceeds are given is (**506**)

Amount of sales = *Net proceeds* ÷ (1 − *Rate* %).

11. Net proceeds, \$8360; rate of commission, $3\frac{1}{2}\%$.

12. Net proceeds, \$3640; rate of commission, $\frac{3}{8}\%$.

13. Net proceeds, \$1850; rate of commission, $\frac{3}{4}\%$.

Find the amount to be invested in the following:

Observe, that when an agent is to deduct his commission from the amount of money in his hand the formula is (**506**)

Sum invested = *Amount in hand* ÷ (1 + *Rate* %).

14. Amount in hand, \$3401.01; rate of commission, $3\frac{1}{2}\%$.

15. Remittance was \$393.17; rate of commission, $2\frac{3}{8}\%$.

16. Amount in hand, \$606.43; rate of commission, $1\frac{3}{4}\%$.

17. A lawyer collects bills amounting to \$492; what is his commission at 5%? *Ans.* \$24.60.

18. An agent sold 824 barrels of beef, averaging $202\frac{1}{2}$ lb. each at 9 cents a pound; what was his commission at $2\frac{1}{2}\%$?

19. A merchant has sent me \$582.40 to invest in apples, at \$5 a barrel; how many can I buy, commission being 4%?

20. I have remitted \$1120 to my correspondent in Lynn to invest in shares, after deducting his commission of $1\frac{1}{4}\%$; what is his commission?

21. An auctioneer sold goods at auction for \$13825, and others at a private sale for \$12050; what was his commission at $\frac{1}{2}\%$? *Ans.* \$129.3750.

22. A man sends \$6897.12 to his agent in New Orleans, requesting him to invest in cotton after deducting his commission of 2%; what was the amount invested?

INSURANCE.

523. ***Insurance*** is a contract which binds one party to indemnify another against possible loss or damage. It is of two kinds: insurance on *property* and insurance on *life*.

524. The ***Policy*** is the written contract made between the parties.

525. The ***Premium*** is the percentage paid for insurance.

526. The quantities considered in insurance correspond with those in percentage; thus,

1. The *amount insured* is the ***Base.***
2. The *per cent* of premium is the ***Rate.***
3. The premium is the ***Percentage.***

EXAMPLES FOR PRACTICE.

527. Let the pupil write out the formulæ as in Profit and Loss.

1. What is the premium on a policy for \$3500, at 3%?

2. My house is insured for \$7250; what is the yearly premium, at $2\frac{3}{4}$%?

3. Justus Weston's house is insured for \$3250 at $3\frac{1}{2}$ per cent, his furniture for \$945 at $1\frac{3}{4}$ per cent, and his barn for \$1220 at $1\frac{1}{2}$ per cent; what is the amount of premium on the whole property?

4. A factory is insured for \$27430, and the premium is \$685.75; what is the rate of insurance?

5. The Pacific Mills of Lawrence, worth \$28000, being insured for $\frac{5}{6}$ their value, were destroyed by fire; at $2\frac{2}{3}$ per cent, what is the actual loss of the insurance company?

6. The premium on a house, at $\frac{2}{3}$ per cent, is \$40; what is the sum insured?

7. It costs me \$72 annually to keep my house insured for \$18000; what is the rate?

8. What must be paid to insure from Boston to New Orleans a ship valued at \$37600, at $\frac{2}{3}$ of 1%?

9. A cargo of 800 bundles of hay, worth \$4.80 a bundle, is insured at $1\frac{1}{2}$% on $\frac{1}{2}$ of its full value. If the cargo be destroyed, how much will the owner lose?

10. My dwelling-house is insured for \$4800 at $\frac{5}{8}$%; my furniture, library, etc., for \$2500 at $\frac{7}{8}$%; my horses, cattle, etc., for \$3900 at $\frac{3}{8}$%; and a carriage manufactory, including machinery, for \$4700 at $1\frac{3}{4}$%. What is my annual premium?

STOCKS.

528. A ***Corporation*** is a body of individuals or company authorized by law to transact business as one person.

529. The ***Capital Stock*** is the money contributed and employed by the company or corporation to carry on its business.

The term *stock* is also used to denote Government and State bonds, etc.

530. A ***Share*** is one of the equal parts into which the *capital stock* is divided.

531. A ***Certificate of Stock,*** or ***Scrip,*** is a paper issued by a corporation, securing to the holder a given number of shares of the *capital stock.*

532. The ***Par Value*** of stock is the sum for which the scrip or certificate is issued.

533. The ***Market Value*** of stock is the price per share for which it can be sold.

534. The ***Premium, Discount,*** and ***Brokerage*** are *always computed* on the *par value* of the stock.

535. The ***Net Earnings*** are the moneys left after deducting all expenses, losses, and interest upon borrowed capital.

536. A ***Bond*** is a written instrument, securing the payment of a sum of money at or before a specified time.

537. A ***Coupon*** is a certificate of interest attached to a bond, which is cut off and delivered to the payor when the interest is discharged.

538. ***U. S. Bonds*** may be regarded as of two classes: those payable at a fixed date, and those payable at any time between two fixed dates, at the option of the government.

539. In commercial language, the two classes of U. S. bonds are distinguished from each other thus:

(1.) ***U. S. 6's***, bonds payable at a fixed time.

(2.) ***U. S. 6's 5-20***, bonds payable, at the option of the Government, at any time from 5 to 20 years from their date.

EXAMPLES FOR PRACTICE.

540. Let the pupil write out the formula for each class of examples, as shown in Profit and Loss:

1. Find the cost of 120 shares N. Y. Central stock, the market value of which is 108, brokerage $\frac{1}{2}$%.

SOLUTION.—Since 1 share cost 108% + $\frac{1}{2}$%, or 108$\frac{1}{2}$% of \$100 = 108$\frac{1}{2}$, the cost of 120 shares will be \$108$\frac{1}{2}$ × 120 = \$13020.

2. What is the market value of 86 shares in the Salem and Lowell Railroad, at 3$\frac{1}{2}$% premium, brokerage $\frac{3}{4}$%?

3. Find the cost of 95 shares bank stock, at 6% premium, brokerage $\frac{3}{8}$%.

4. How many shares of Erie Railroad stock at 8% discount can be bought for \$7030, brokerage $\frac{1}{2}$%?

SOLUTION.—Since 1 share cost 100% − 8% + $\frac{1}{2}$%, or 92$\frac{1}{2}$% of \$100 = \$92.50, as many shares can be bought as \$92.50 are contained times in \$7020, which is 76.

How many shares of stock can be bought

5. For \$10092, at a premium of 5%, brokerage $\frac{1}{8}$%?

6. For \$13428, at a discount of 7%, brokerage $\frac{1}{4}$%?

7. For \$16830, at a premium of 9$\frac{3}{4}$%, brokerage $\frac{1}{4}$%?

8. What sum must be invested in stocks at 112, paying 9%, to obtain a yearly income of \$1260?

SOLUTION.—Since \$9 is the annual income on 1 share, the number of shares must be equal \$1260 ÷ \$9, or 140 shares, and 140 shares at \$112 a share amount to \$15680, the required investment.

Find the investment for the following:

9. Income \$2660, stock purchased at 105$\frac{1}{2}$, yielding 7%.

10. Income \$1800, stock purchased at 109$\frac{3}{4}$, yielding 12%.

11. Income \$3900, stock purchased at 92, yielding 6%.

12. What must be paid for stocks yielding 7% dividends, that 10% may be realized annually from the investment?

SOLUTION.—Since \$7, the annual income on 1 share, must be 10% of the cost of 1 share, $\frac{1}{10}$ of \$7, or 70 ct., is 1%. Hence 100%, or 70 ct. $\times$ 100 = \$70, is the amount that must be paid for the stock.

What must be paid for stocks yielding

13. 5% dividends to obtain an annual income of 8%?

14. 7% dividends to obtain an annual income of 12%?

15. 9% dividends to obtain an annual income of 7%?

16. How much currency can be bought for \$350 in gold, when the latter is at 12% premium?

SOLUTION.—Since \$1 in gold is worth \$1.12 in currency, \$350 in gold are equal \$1.12 $\times$ 350 = \$392.

How much currency can be bought

17. For \$780 in gold, when it is at a premium of 9%?

18. For \$396 in gold, when it is at a premium of 13$\frac{1}{4}$%?

19. For \$520 in gold, when it is at a premium of 12$\frac{1}{8}$%?

20. How much is \$507.50 in currency worth in gold, the latter being at a premium of 12$\frac{1}{2}$%?

SOLUTION.—Since \$1 of gold is equal to \$1.12$\frac{1}{2}$ in currency, \$507.50 in currency must be worth as many dollars in gold as \$1.12$\frac{1}{2}$ is contained times in \$507.50, which is \$451.11$\frac{1}{9}$.

How much gold can be bought

21. For \$1053.17 currency, when gold is at a premium of 9$\frac{1}{4}$%?

22. For \$317.47 currency, when gold is at a premium of 11$\frac{2}{5}$%?

23. For \$418.14 currency, when gold is at a premium of 13$\frac{5}{8}$%?

24. Bought 80 shares in Boston and Maine Railroad, at a discount of 2$\frac{1}{2}$%, and sold the same at an advance of 12%; what did I gain? *Ans.* \$1160.

25. An agent sells 415 barrels of flour, at $6 a barrel, commission 5%, and invests the proceeds in stocks of the Suffolk Bank, Boston, at $17\frac{1}{4}$% discount, brokerage $\frac{1}{4}$%; how many shares did he buy?

26. Bought 84 shares in Michigan Southern Railroad, at 7% discount, and sold them at $6\frac{1}{4}$% advance; what was my profit, the brokerage in buying and selling being $\frac{1}{2}$ per cent?

27. Bought bonds at 70%, bearing $4\frac{1}{5}$% interest; what is the rate of income? *Ans.* 6%.

28. I invest $2397.50 in Empire Iron Foundry stock, whose shares, worth $50 each, are sold at $43.50, brokerage $\frac{1}{2}$%; what annual income shall I derive, the stock yielding 7%?

29. O. E. Bonney sold $6000 Pacific Railroad 6's at 107, and with a part of the proceeds bought St. Lawrence County bonds at 90, yielding 6% dividends sufficient to give an annual income of $180; how much has he left?

30. What rate of income can be derived from money invested in the stock of a company paying a semi-annual dividend of 5%, purchased at $84\frac{1}{2}$%, brokerage $\frac{1}{2}$%?

31. What must I pay for bonds yielding $4\frac{1}{2}$% annually, that my investment may pay 6%?

32. What must be paid for stocks paying 5 per cent, that the investment may return 8%?

33. How much more is $1400 gold worth than $1515 currency, when gold is 112%?

34. A man bought a farm, giving a note for $3400, payable in gold in 5 years; at the expiration of the time gold was 175%: what did his farm cost in currency?

35. I invested $785.40 of currency in gold when it was worth $115\frac{1}{2}$%; what amount of gold did I purchase?

36. How much gold at a premium of $9\frac{1}{2}$% can be purchased for $876.90 currency? For $85.50? For $136.80?

37. What is the difference in the value of $800 in gold and $900 in currency, when gold is at a premium of $13\frac{1}{2}$%?

TAXES.

541. A ***Tax*** is a sum of money assessed upon a person or property, for any public purpose.

542. ***Property*** is of two kinds: *Real Estate,* such as houses and lands; *Personal Property,* such as merchandise, cash, furniture, ships, notes, bonds, mortgages, etc.

543. ***Taxes*** are of two kinds: *Property Tax,* which is assessed upon taxable property according to its estimated value; *Poll Tax,* which is a sum assessed without regard to property upon each male citizen liable to taxation.

544. An Assessment Roll is a schedule or list which contains the names and the taxable value of the property of all persons subject to a given tax.

545. The ***Rate of Property Tax*** is the *rate per cent* on the valuation of the property.

546. An ***Assessor*** is an officer appointed to prepare the *Assessment Roll* and apportion to each person his tax.

Method of Apportioning a Tax.

547. 1. The Assessor determines by a personal examination the *taxable* value of the real estate and personal property of each person subject to the tax, and fills an Assessment Roll, thus:

ASSESSMENT ROLL.

NAMES.	REAL ESTATE.	PERSONAL PROPERTY.	TOTAL PROPERTY.	POLLS.	AMOUNT OF TAX.
L. Henry,	$6984	$1862	$8846	1	$45.48
W. Mann,	8095	1983	10078	1	51.64
P. Duncan,	9709	2300	12009	1	61.295
R. Storey,	6092	1975	8067	1	41.585
Totals,	30880	8120	39000	4	$200.

2. If the amount to be raised on this Assessment Roll is $195, the collector's fees $2\frac{1}{2}\%$, and the poll tax $1.25 per poll, the Assessor, or party authorized to do so, would proceed to apportion the tax thus:

(1.) Since the collector is paid $2\frac{1}{2}\%$ of the whole tax for collecting, the $195 is $97\frac{1}{2}\%$ of the assessment which must be made. Hence, $\$195 \div .97\frac{1}{2} = \$200$, the whole tax.

(2.) Since each poll pays $1.25, the 4 polls will pay $5, and the amount which must be assessed on the property is $\$200 - \$5 = \$195$.

(3.) Since $195 are to be assessed on $39000, the whole amount of property, the rate per dollar is $\$195 \div \$39000 = .005$, or 5 mills.

(4.) Multiplying each man's taxable property by .005 will give his property tax, to which we add the poll tax; hence,

J. Henry's tax $= \$8846 \times .005 + \$1.25 = \$45.48$.
W. Mann's tax $= \$10078 \times .005 + \$1.25 = \$51.64$.
P. Duncan's tax $= \$12009 \times .005 + \$1.25 = \$61.295$.
R. Storey's tax $= \$8067 \times .005 + \$1.25 = \$41.585$.

(5.) These results are now inserted in the blank under "Amount of Tax," and the Assessment Roll is thus completed ready for the collector.

EXAMPLES FOR PRACTICE.

548. Prepare an Assessment Roll ready for the collector for each of the following:

1. Net tax to be raised $1930, collector's fee $3\frac{1}{2}\%$, poll tax $2.50 per poll.

Property taxed.—A, real estate $10800, personal property $3200; B, real estate $9600, personal property $5200; C, personal property $4200; D, real estate $12800, personal property $4000; E, real estate $20000, personal property $6200; *Polls* without property 35.

2. Net tax to be assessed $2387.50, collector's fee $4\frac{1}{2}\%$, poll tax $1.25 per poll.

Property taxed.—A, real estate $9700, personal property $5000; B, real estate $14600, personal property $5400; C, real estate $8900, personal property $3100; D, real estate $40000, personal property $12000; E, real estate $21600, personal property $3700; *Polls* without property 11.

DUTIES OR CUSTOMS.

549. ***Duties*** or ***Customs*** are taxes levied by the government upon imported goods.

550. A ***Specific Duty*** is a certain sum imposed upon an article without regard to its value.

551. An ***Ad Valorem Duty*** is a *per cent* assessed upon the value of an article in the country from which it is brought.

552. A ***Tariff*** is a schedule giving the rates of duties fixed by law.

553. The following deductions or allowances are made before computing specific duties:

1. ***Tare.***—An allowance for the box, cask, bag, etc., containing the merchandise.

2. ***Leakage.***—An allowance for waste of liquors imported in casks or barrels.

3. ***Breakage.***—An allowance for loss of liquors imported in bottles

EXAMPLES FOR PRACTICE.

554. 1. What is the duty on 420 boxes of raisins, each containing 40 pounds, bought for 8 cents a pound, at 20 per cent ad valorem?

2. Imported 21 barrels of wine, each containing 31 gallons; 2% being allowed for leakage, what is the duty at 40 cents per gallon?

3. A merchant imported from Havana 100 boxes oranges @ $2.25 per box; 75 hogsheads of molasses, each containing 63 gal., @ 23 cents per gal.; 50 hogsheads of sugar, each containing 340 lb., @ 6 cents per lb. The duty on the molasses was 25%, on the sugar 30%, and on the oranges 20%. What was the duty on the whole?

4. What is the duty on 320 yards of cloth, invoiced at $1.15 per yard, at 20% ad valorem?

5. At 12% ad valorem, what is the duty on 100 barrels of kerosene, invoiced at $.18 a gallon, 2% leakage?

REVIEW AND TEST QUESTIONS.

555. 1. When a fraction is to be divided by a fraction, why can the factors that are common to the denominators of the dividend and divisor be cancelled?

2. How does moving the decimal point one or more places to the left or right affect a number, and why?

3. Show that multiplying by 1000 and subtracting three times the multiplicand from the product is the same as multiplying by 997?

4. Define Base, Percentage, Amount and Difference.

5. When the amount and rate per cent is given to find the base, why add the rate expressed decimally to 1 and divide by the result?

6. Represent the quantities by letters and write a formula for solving each of the following problems (**508**).

I. Given, the *Cost* and the *Profit*, to find the rate per cent profit.

II. Given, the *rate per cent profit* and the *selling price*, to find the *buying price*.

III. Given, the *amount of money* sent to an agent to purchase goods and the rate per cent commission, to find the amount of the purchase.

IV. Given, the rate at which stocks can be purchased, to find how much can be secured for a given sum.

V. Given, the rate at which stocks can be purchased and the rate per cent of dividend, to find the rate per cent of income on the investment.

VI. Given, the premium on gold, to find how much can be purchased for a given sum in currency.

INTEREST

DEFINITIONS.

556. ***Interest*** is a sum paid for the *use* of money.

Thus, I owe Wm. Henry $200, which he allows me to use for one year after it is due. At the end of the year I pay him the $200 and $14 for its use. The $14 is called the *Interest* and the $200 the *Principal.*

557. ***Principal*** is a sum of money for the use of which *interest* is paid.

558. ***Rate of Interest*** is the number of units of any denomination of money paid for the use of 100 units of the same denomination for one year or some given interval of time.

559. The ***Amount*** is the sum of the principal and interest.

560. ***Simple Interest*** is interest which falls due when the principal is paid, or when a partial payment is made.

561. ***Legal Interest*** is interest reckoned at the rate per cent *fixed by law.*

562. ***Usury*** is interest reckoned at a higher rate than is allowed by law.

563. The following table gives the *legal rates* of interest in the different States.

Where two rates are given, any rate between these limits is allowed, *if specified in writing.* When no rate is named in a paper involving interest, the *legal* or lowest *rate* is always understood.

STATES.	RATE %.		STATES.	RATE %.		STATES.	RATE %.		STATES.	RATE %.	
Ala......	8		Ill........	6	10	Mo.......	6	10	S. C......	7	Any
Ark......	6	Any	Ind......	6	10	Montana.	10		Tenn.....	6	10
Arizona..	10	Any	Iowa.....	6	10	N. H.....	6		Texas....	8	12
Cal.......	10	Any	Kan......	7	12	N. J......	7		Utah.....	10	Any
Conn.....	7		Ken......	6	10	N. Y.....	7		Vt........	6	
Colo......	10	Any	La..	5	8	N. C.	6	8	Va........	6	12
Dakota...	7	Any	Maine....	6	Any	Neb......	10	15	W. Va....	6	
Del.......	6		Md.......	6		Nevada .	10	Any	W. T.....	10	Any
D. C......	6	10	Mass.....	6	Any	Ohio.....	6	8	Wis......	7	10
Flor......	8	Any	Mich.....	7	10	Oregon,..	10	12	Wy......	12	
Geo.. ...	7	10	Minn....	7	12	Penn.....	6	7			
Idaho	10		Miss.....	6	10	R. I......	6	Any			

The legal rate for England and France is 5%; for Canada and Ireland, 6%.

564. PROB. I.—**To find the simple interest of any given sum for one or more years.**

1. Find the interest on $384 for 5 years, at 7%.

SOLUTION.—1. Since the interest of $100 for one year is $7, the interest of $1 for one year is $.07. Hence the interest of $1 for 5 years is $.07 × 5 = $.35.

2. Since the interest of $1 for 5 yr. is $.35, the interest of $384 for the same time must be 384 times $.35, or $134.40. Hence the following

565. RULE.—*I. Find the interest of $1 at the given rate for the given time, and multiply this result by the number of dollars in the given principal.*

II. To find the amount add the interest and principal.

EXAMPLES FOR PRACTICE.

566. Find the interest on the following orally:

1. $800 for 2 years at 4%.
2. $1200 for 3 years at 3%.
3. $200 for 5 years at 6%.
4. $600 for 4 years at 5%.
5. $90 for 2 years at 7%.
6. $70 for 4 years at 8%.
7. $400 for 8 years at 5%.
8. $100 for 12 years at 9%.
9. $600 for 7 years at 10%.
10. $1000 for 5 years at 8%.
11. $20 for 3 years at 9%.
12. $500 for 5 years at 5%.

Find the interest on the following:

13. $245.36 for 3 years at 7%.
14. $784.25 for 9 years at 4%.
15. $836.95 for 2 years at $\frac{1}{2}$%.
16. $795.86 for 7 years at $\frac{3}{4}$%.
17. $896.84 for $3\frac{1}{2}$ years at $2\frac{3}{4}$%.
18. $28.95 for $1\frac{3}{10}$ years at $4\frac{2}{5}$%.
19. $414.14 for 4 years at $\frac{4}{5}$%.
20. $375.84 for 3 years at $9\frac{1}{2}$%.
21. $293.50 for 6 years at $4\frac{5}{6}$%.
22. $899.00 for 12 years at $7\frac{2}{3}$%.
23. $600.80 for 9 years at $8\frac{5}{8}$%.
24. $50.84 for 5 years at $1\frac{3}{8}$%.
25. $95.60 for $\frac{4}{5}$ of a yr. at $7\frac{1}{2}$%.
26. $262.62 for 6 years at $6\frac{1}{6}$%.

METHOD BY ALIQUOT PARTS.

567. Prob. II.—**To find the interest on any sum at any rate for years, months, and days by aliquot parts.**

1. In business transactions involving interest, 30 days are usually considered *one month*, and 12 months 1 year. Hence the interest for days and months may be found according to (**499**), by regarding the time as a compound number; thus,

Find the interest and amount of $840 for 2 yr. 7 mo. 20 da., at 7%.

	$840	Principal.
	.07	Rate of Interest.
6 mo. = $\frac{1}{2}$ of 1 yr., hence 2)	58.80	Interest for 1 yr.
	2	
	117.60	Interest for 2 yr.
1 mo. = $\frac{1}{6}$ of 6 mo., hence 6)	29.40	" " 6 mo.
15 da. = $\frac{1}{2}$ of 1 mo., hence 2)	4.90	" " 1 mo.
5 da. = $\frac{1}{3}$ of 15 da. hence 3)	2.45	" " 15 da.
	$81\frac{2}{3}$	" " 5 da.
	$$155.16\frac{2}{3}$	" " 2 yr. 7 mo. 20 da.
	840.00	Principal.
	$$995.16\frac{2}{3}$	Amount for 2 yr. 7 mo. 20 da

568. The interest, by the method of aliquot parts, is usually found by finding first the interest of $1 for the given time, and multiplying the given principal by the decimal expressing the interest of $1; thus,

Find the interest of $680 for 4 yr. 9 mo. 15 da. at 8%.

1. We first find the interest of $1 for the given time thus:

8 ct.	= Int. of $1 for 1 yr.,	8 ct. × 4 =	Int. for 4 yr.	=	32 ct.
6 mo.	= $\frac{1}{2}$ of 1 yr., hence,	$\frac{1}{2}$ of 8 ct. =	" " 6 mo.	=	4 ct.
3 mo.	= $\frac{1}{2}$ of 6 mo., "	$\frac{1}{2}$ of 4 ct. =	" " 3 mo.	=	2 ct.
15 da.	= $\frac{1}{6}$ of 3 mo., "	$\frac{1}{6}$ of 2 ct. =	" " 15 da.	=	.03$\frac{1}{3}$ m.

Hence the interest on $1 for 4 yr. 9 mo. 15 da. = $.383$\frac{1}{3}$.

2. The decimal .383$\frac{1}{3}$ expresses the part of $1 which is the interest of $1 for the given time at the given rate. Hence, $680 × .383$\frac{1}{3}$ = 260.66\frac{2}{3}$, is the interest of $680 for 4 yr. 9 mo. 15 da., at 8%; hence the following

569. Rule.—*I. Find by aliquot parts the interest of $1 for the given rate and time.*

II. Multiply the principal by the decimal expressing the interest for $1, and the product will be the required interest.

III. To find the Amount, add the interest to the principal.

EXAMPLES FOR PRACTICE.

570. Find the interest

1. Of $284 for 3 yr. 8 mo. 12 da. at 6%; at 8$\frac{1}{2}$%.
2. Of $560.40 for 2 yr. 10 mo. 18 da. at 7%; at 9%.
3. Of $296.85 for 4 yr. 11 mo. 24 da. at 8%; at 5%.
4. Of $860 for 1 yr. 7 mo. 27 da. at 4$\frac{1}{2}$%; at 7$\frac{1}{2}$%.
5. Of $2940.75 for 3 yr. 11 mo. 17 da. at 7%; at 9$\frac{1}{4}$%.
6. Find the amount of $250.70 for 2 yr. 28 da. at 8%.
7. Find the amount of $38.90 for 3 yr. 13 da. at 9%.
8. A man invested $795 at 8% for 4 yr. 8 mo. 13 da. How much was the amount of principal and interest?
9. Paid a debt of $384.60, which was upon interest for 11 mo. 16 da. at 7%. What was the amount of the payment?
10. Find the amount of $1000 for 9 yr. 11 mo. 29 da. at 7%.

METHOD BY SIX PER CENT.

PREPARATORY STEPS.

571. STEP I.—*To find the interest for any number of months at 6%.*

1. Since the interest of $1 for 12 months, or 1 yr., at 6%, is 6 cents, the interest for two months, which is $\frac{1}{6}$ of 12 months, must be 1 cent, or $\frac{1}{100}$ part of the principal.

2. Since the interest for 2 months is $\frac{1}{100}$ of the principal, the interest for any number of months will be as many times $\frac{1}{100}$ of the principal as 2 is contained times in the given number of months. Hence the following

572. RULE.—*I. Move the decimal point in the principal* TWO PLACES *to the left* (**470**), *prefixing ciphers, if necessary.*

II. Multiply this result by one-half the number of months.

Or, *Multiply $\frac{1}{100}$ of the principal by the number of months and divide the result by 2.*

EXAMPLES FOR PRACTICE.

573. Find the interest at 6%

1. Of $896 for 8 mo.
2. Of $973.50 for 10 mo.
3. Of $486.80 for 18 mo.
4. Of $364.40 for 7 mo.
5. Of $432.90 for 13 mo.
6. Of $398 for 1 yr. 6 mo. = 18 mo.
7. Of $750 for 2 yr. 8 mo.
8. Of $186 for 4 yr. 2 mo.
9. Of $268 for 2 yr. 6 mo.
10. Of $873 for 1 yr. 11 mo.

574. STEP II.—*To find the interest for any number of days at 6%.*

1. Since the interest of $1 for 2 months at 6% is 1 cent, the interest for 1 month, or 30 days, must be $\frac{1}{2}$ cent or 5 mills. And since 6 days are $\frac{1}{5}$ of 30 days, the interest for 6 days must be $\frac{1}{5}$ of 5 mills, or 1 mill, which is $\frac{1}{1000}$ of the principal.

2. Since the interest for 6 days is $\frac{1}{1000}$ of the principal, the interest for any number of days will be as many times $\frac{1}{1000}$ of the principal as 6 is contained times in the given number of days. Hence the following

575. RULE.—*I. Move the decimal point in the principal* THREE PLACES *to the left* (**470**), *prefixing ciphers, if necessary.*

II. Multiply this result by one-sixth the number of days.

Or, *Multiply $\frac{1}{1000}$ of the principal by the number of days and divide the result by 6.*

EXAMPLES FOR PRACTICE.

576. Find the interest at 6%

1. Of $790 for 12 da.	6. Of $584 for 19 da.
2. Of $384 for 24 da.	7. Of $730 for 22 da.
3. Of $850 for 15 da.	8. Of $809 for 28 da.
4. Of $935 for 27 da.	9. Of $396 for 17 da.
5. Of $580 for 16 da.	10. Of $840 for 14 da.

577. PROB. III.—**To find the interest on any sum at any rate for years, months, and days, by the six per cent method.**

Find the interest of $542 for 4 years 9 months 17 days at 8 per cent.

SOLUTION.—1. The interest of $542 for 4 years at 6%, according to (**564**), is $\$542 \times .06 \times 4 = \$130.08$.

2. The interest for 9 months, according to (**571**), is $\frac{1}{100}$ of $542 or $5.42 multiplied by 9, and this product divided by 2 = $24.39.

3. The interest for 17 days, according to (**574**), is $\frac{1}{1000}$ of $542 or $.542 multiplied by 17, and this product divided by 6 = $1.535+.

Hence $130.08 + $24.39 + $1.54 = $156.01, the interest of $542 for 4 years 9 months and 17 days.

4. Having found the interest of $542 at 6%, to find the interest at 8% we have 8% = 6% + 2%, and 2% is $\frac{1}{3}$ of 6%. Hence, $156.01 + $\frac{1}{3}$ of $156.01 = $208.013, the interest of $542 at 8% for 4 yr. 9 mo. 17 da.

EXAMPLES FOR PRACTICE.

578. Find the interest by the 6% method

1. Of \$384.96 for 2 yr. 8 mo. 12 da. at 6%; at 9%; at 8%.
2. Of \$890.70 for 4 yr. 10 mo. 15 da. at 7%; at 10%; at 4%.
3. Of \$280.60 for 11 mo. 27 da. at 8%; at 4%; at 7%.
4. Of \$480 for 2 yr. 7 mo. 15 da. at 9%; at 12%; at 4½%.
5. Of \$890 for 9 mo. 13 da. at 6½%; at 8¼%; at 9½%.

METHOD BY DECIMALS.

579. In this method the time is regarded as a compound number, and the months and days expressed as a decimal of a year.

When the principal is a small sum, sufficient accuracy will be secured by carrying the decimal to three places; but when a large sum, a greater number of decimal places should be taken.

580. Prob. IV.—**To find the interest on any sum at any rate for years, months, and days, by decimals.**

What is the amount of \$450 for 5 yr. 7 mo. 16 da. at 6%?

(1.)

```
30 ) 16       da.
12 )  7.533   mo.
      5.627   yr.
         27
     39 389
    112 54
   $151.929   Interest.
    450
   $601.93    Amount.
```

(2.)

```
 $450
  .06
$27.00
```

Explanation.—1. We express, according to (**389**—15), the days and months as a decimal of a year, as shown in (1).

2. We find the interest on \$450, the given principal, for 1 year, which is \$27, as shown in (2).

3. Since \$27 is the interest on \$450 for 1 year, the interest for 5.627 years is 5.627 times \$27, which is \$151.929, as shown in (1).

4. The amount is equal to the principal plus the interest (**559**); hence, \$151.93 + \$450 = \$601.93 is the amount. Hence the following

(249)

581. RULE.—*I. To find the interest, multiply the principal by the rate, and this product by the time, expressed in years and decimals of a year.*

II. To find the amount, add the interest to the principal.

EXAMPLES FOR PRACTICE.

582. Find the interest by the decimal method

1. Of \$290 for 1 yr. 8 mo. 12 da. at 5%; at 8%; at 7%.
2. Of \$374.05 for 2 yr. 9 mo. 15 da. at 6%; at 9%; at 4%.
3. Of \$790.80 for 5 yr. 3 mo. 7 da. at 7%; at 11%; at 3%.
4. Of \$460.90 for 3 yr. 5 mo. 13 da. at $6\frac{1}{2}$%; at $8\frac{1}{4}$%; at $3\frac{5}{8}$%.
5. Of \$700 for 11 mo. 27 da. at $9\frac{1}{2}$%; at $7\frac{1}{4}$%; at $2\frac{3}{4}$%.
6. Of \$580.40 for 17 da. at $6\frac{3}{4}$%; at $9\frac{1}{2}$%; at $5\frac{1}{2}$%.
7. Of \$890 for 7 yr. 19 da. at 6%; at 8%; at 5%.

EXACT INTEREST.

583. In the foregoing methods of reckoning interest the year is regarded as 360 days, which is 5 days *less* than a *common year*, and 6 days *less* than a *leap year;* hence, the interest when found for a part of a year is incorrect.

Thus, if the interest of \$100 is \$7 for a *common year* or 365 days, the interest for 75 days at the same rate must be $\frac{75}{365}$ of \$7; but by the foregoing method $\frac{75}{360}$ of \$7 is taken as the interest, which is too great.

Observe, that in using $\frac{75}{360}$ instead of $\frac{75}{365}$, the denominator is diminished $\frac{5}{365} = \frac{1}{73}$ part of itself, and consequently (**330**) the result is $\frac{1}{73}$ part of itself too great.

Hence, when interest is calculated by the foregoing methods, it must be diminished by $\frac{1}{73}$ of itself for a *common year*, and for like reasons $\frac{1}{61}$ of itself for a *leap year*.

To find the exact interest we have the following:

584. RULE.—*I. Find the interest for the given number of years* (**563**).

II. Find the exact number of days in the given months and days, and take such a part of the interest *of the principal for one year, as the whole number of days is of 365 days.*

Or, *Find the interest for the given months and days by either of the foregoing methods, then subtract $\frac{1}{73}$ part of itself for a common year, and $\frac{1}{61}$ for a leap year.*

III. Add the result to the interest for the given number of years.

EXAMPLES FOR PRACTICE.

585. Find the exact interest by both rules

1. Of \$836 for 84 da. at 6%.
2. Of \$260 for 55 da. at 8%.
3. Of \$690 for 25 da. at 7%.
4. Of \$985 for 13 da. at 9%.
5. Of \$2360 for 7 da. at $7\frac{1}{2}$%.
6. Of \$120 for 133 da. at $8\frac{1}{4}$%.
7. Of \$380.50 for 93 da. at $6\frac{3}{4}$%.
8. Of \$260.80 for 17 da. at 12%.

9. Required the exact interest of \$385.75 at 7%, from January 15, 1875, to Aug 23 following.

10. What is the difference between the exact interest of \$896 at 7% from January 11, 1872, to November 19, 1876, and the interest reckoned by the six per cent method?

11. A note for \$360.80, bearing interest at 8%, was given March 1st, 1873, and is due August 23, 1876. How much will be required to pay the note when due?

12. What is the exact interest of \$586.90 from March 13 to October 23 of the same year, at 7%?

586. Prob. V.—**To find the principal when the interest, time, and rate are given.**

Observe, that the *interest* of any principal for a given time at a given rate, is the interest of \$1 taken (**563**) as many times as there are dollars in the principal; hence, the following

587. Rule.—*Divide the given interest by the interest of \$1 for the given time at the given rate.*

EXAMPLES FOR PRACTICE.

588. 1. What sum of money will gain $110.25 in 3 yr. 9 mo. at 7%?

SOLUTION.—The interest of $1 for 3 yr. 9 mo. at 7%, is $.2625. Now since $.2625 is the interest of $1 for the given time at the given rate, $110.25 is the interest of as many dollars for the same time and rate as $.2625 is contained times in $110.25. Hence $110.25 ÷ .2625 = $420, the required principal.

What principal or sum of money

2. Will gain $95.456 in 3 yr. 8 mo. 25 da. at 7%?
3. Will gain $63.488 in 2 yr. 9 mo. 16 da. at 8%?
4. Will gain $106.611 in 3 yr. 6 mo. 18 da. at 6½%?
5. Will gain $235.609 in 4 yr. 7 mo. 24 da. at 9%?
6. Will gain $30.636 in 1 yr. 9 mo. 18 da. at 5¾%?
7. Will gain $74.221 in 2 yr. 3 mo. 9 da. at 7½%?

589. PROB. VI.—**To find the principal when the amount, time, and rate are given.**

Observe, that the amount is the principal plus the interest, and that the interest contains the interest (**564**) of $1 as many times as there are dollars in the principal; consequently the amount must contain (**506**) $1 plus the interest of $1 for the given time at the given rate as many times as there are dollars in the principal; hence, the following

590. RULE.—*Divide the amount by the amount of $1 for the given time at the given rate.*

EXAMPLES FOR PRACTICE.

591. 1. What sum of money will amount to $290.50 in 2 yr. 8 mo. 12 da. at 6%?

SOLUTION.—The amount of $1 for 2 yr. 8 mo. 12 da. at 6% is $1.162. Now since $1.162 is the amount of $1 for the given time at the given rate, $290.50 is the amount of as many dollars as $1.162 is contained times in it. Hence, $290.50 ÷ $1.162 = $250, is the required principal.

2. What principal will amount to $310.60 in 3 yr. 5 mo. 9 da. at 5% ?

3. What is the interest for 1 yr. 7 mo. 13 da. on a sum of money which in this time amounts to $487.65, at 7% ?

4. What sum of money at 10% will amount to $436.02 in 4 yr. 8 mo. 23 da.

5. At 8% a certain principal in 2 yr. 9 mo. 6 da. amounted to $699.82. Find the principal and the interest.

592. PROB. VII.—**To find the rate when the principal, interest and time are given.**

Observe, that the given interest must be as many times 1% of the given principal for the given time as there are units in the rate ; hence the following

593. RULE.—*Divide the given interest by the interest of the given principal for the given time at 1 per cent.*

EXAMPLES FOR PRACTICE.

594. 1. At what rate will $260 gain $45.50 in 2 yr. 6 mo.?

SOLUTION.—The interest of $260 for 2 yr. 6 mo. at 1% is $6.50. Now since $6.50 is 1% of $260 for the given time, $45.50 is as many per cent as $6.50 is contained times in $45.50 ; hence, $45.50 ÷ $6.50 = 7, is the required rate.

At what rate *per cent*

2. Will $732 gain $99.674 in 2 yr. 3 mo. 7 da.?

3. Will $524 gain $206.63 in 5 yr. 7 mo. 18 da.?

4. Will $873 gain $132.89 in 1 yr. 10 mo. 25 da.?

5. Will $395.80 gain $53.873 in 2 yr. 8 mo. 20 da.?

6. Will $908.50 gain $325.422 in 4 yr. 2 mo. 17 da.?

7. A man purchased a house for $3486, which rents for $418.32. What rate per cent does he make on the investment?

8. Which is the better investment and what rate per cent

per annum, \$4360 which yields in 5 years \$1635, or \$3860 which yields in 9 years \$2692.45 ?

9. At what rate per cent per annum will a sum of money double itself in 7 years ?

SOLUTION.—Since in 100 years at 1% any sum doubles itself, to double itself in 7 years the rate per cent must be as many times 1% as 7 is contained times in 100, which is $14\frac{2}{7}$. Hence, etc.

10. At what rate per cent per annum will any sum double itself in 4, 8, 9, 12, and 25 years respectively ?

11. At what rate per cent per annum will any sum triple or quadruple itself in 6, 9, 14, and 18 years respectively ?

12. Invested \$3648 in a business that yields \$1659.84 in 5 years. What per cent annual interest did I receive on my investment ?

595. PROB. VIII.—**To find the time when the principal, interest, and rate are given.**

Observe, that the interest is found (**580**) by multiplying the interest of the given principal for 1 year at the given rate by the time expressed in years ; hence the following

596. RULE.—*I. Divide the given interest by the interest of the given principal for 1 year at the given rate.*

II. Reduce (**579**), *when called for, fractions of a year to months and days.*

EXAMPLES FOR PRACTICE.

597. 1. In what time will \$350 gain \$63 at 8% ?

SOLUTION.—The interest of \$350 for 1 yr. at 8% is \$28. Now since \$28 is the interest of \$350 at 8% for 1 year, it will take as many years to gain \$63 as \$28 is contained times in \$63 ; hence $\$63 \div \$28 = 2\frac{1}{4}$ yr., or 2 yr. 3 mo., the required time.

In what time will

2. \$80 gain \$36 at $7\frac{1}{2}$%?
3. \$460 gain \$80.50 at 5% ?
4. \$260 gain \$98.80 at 8% ?
5. \$477 gain \$152.64 at 12%?
6. \$690 gain \$301.392 at $7\frac{4}{5}$%?
7. \$385 gain \$214.72 at $8\frac{4}{7}$% ?

8. My total gain on an investment of \$860 at 7% per annum, is \$455.70. How long has the investment been made?

9. How long will it take any sum of money to double itself at 7% per annum?

SOLUTION.—At 100% any sum will double itself in 1 year; hence to double itself at 7% it will require as many years as 7% is contained times in 100%, which is $14\frac{2}{7}$. Hence, etc.

Observe, that to find how long it will take to triple, quadruple, etc., any sum, we must take 200%, 300%, etc.

10. How long will it take any sum of money at 5%, 8%, $6\frac{1}{2}$%, or 9% per annum to double itself? To triple itself, etc.?

11. At 7% the interest of \$480 is equal to 5 times the principal. How long has the money been on interest?

COMPOUND INTEREST.

598. ***Compound Interest*** is interest upon principal and interest united, at given intervals of time.

Observe, that the interest may be made a part of the principal, or compounded at any interval of time *agreed upon;* as, annually, semi-annually, quarterly, etc.

599. PROB. IX.—**To find the compound interest on any sum for any given time.**

Find the compound interest of \$850 for 2 yr. 6 mo. at 6%.

\$850	Prin. for 1st yr.
1.06	
\$901	Prin. for 2d yr.
1.06	
\$955.06	Prin. for 6 mo.
1.03	
\$983.71	Total amount.
\$850	Given Prin.
\$133.71	Compound Int.

EXPLANATION.—Since at 6% the *amount* is 1.06 of the principal, we multiply \$850, the principal for the first year, by 1.06, giving \$901, the *amount* at the end of the first year, which forms the *principal* for the *second* year. In the same manner we find \$955.06, the amount at the end of the second year which forms the principal for the 6 months.

2. Since 6% for one year is 3% for 6 months, we multiply \$955.06, the principal for the 6 months, by 1.03, which gives the *total* amount at the end of the 2 years 6 months.

3. From the total amount we subtract $850, the given principal, which gives $133.71, the compound interest of $850 for 2 years 6 months at 6%. Hence the following

600. RULE.—*I. Find the amount of the principal for the first interval of time at the end of which interest is due, and make it the principal for the second interval.*

II. Find the amount of this principal for the second interval of time, and so continue for each successive interval and fraction of an interval, if any.

III. Subtract the given principal from the last amount and the remainder will be the compound interest.

EXAMPLES FOR PRACTICE.

601. 1. What is the compound interest of $650 for 3 years, at 7%, payable annually?

2. Find the amount of $870 for 2 years at 6% compound interest.

3. Find the compound interest of $380.80 for 1 year at 8%, interest payable quarterly.

4. What is the amount of $1500 for 2 years 9 months at 8% compound interest, payable annually?

5. What is the amount of $600 for 1 year 9 months at 5% compound interest, payable quarterly?

6. What is the difference in the simple interest and compound interest of $480 for 4 yr. and 6 mo. at 7%?

7. What is the annual income from an investment of $2860 at 7% compound interest, payable quarterly?

8. What will be the compound interest at the end of 2 yr. 5 mo. on a note for $600 at 7%, payable semi-annually?

9. A man invests $3750 for 3 years at 7% compound interest, payable semi-annually, and the same amount for the same time at 7¼% simple interest. Which will yield the greater amount of interest at the end of the time, and how much?

INTEREST TABLES.

602. Interest, both *simple* and *compound,* is now almost invariably reckoned by means of tables, which give the interest or amount of $1 at different rates for years, months, and days. The following illustrate the nature and use of such tables.

TABLE *showing the simple interest of $1 at 6, 7, and 8%, for years, months, and days.*

Years.	6%.	7%.	8%.	*Years.*	6%.	7%.	8%.
1	.06	.07	.08	4	.24	.28	.32
2	.12	.14	.16	5	.30	.35	.40
3	.18	.21	.24	6	.36	.42	.48
Months.				*Months.*			
1	.005	.00583	.00666	7	.035	.04083	.04666
2	.01	.01166	.01333	8	.04	.04666	.05333
3	.015	.01750	.02000	9	.045	.05250	.06000
4	.02	.02333	.02666	10	.05	.05833	.06666
5	.025	.02916	.03333	11	.055	.06416	.07333
6	.03	.03500	.04000				
Days.				*Days.*			
1	.00016	00019	.00022	16	.00266	.00311	.00355
2	.00033	.00038	.00044	17	.00283	.00330	.00377
3	.00050	.00058	.00066	18	.00300	.00350	.00400
4	.00066	.00077	.00088	19	.00316	.00369	.00422
5	.00083	.00097	.00111	20	.00333	.00388	.00444
6	.00100	.00116	.00133	21	.00350	.00408	.00466
7	.00116	.00136	.00155	22	.00366	.00427	.00488
8	.00133	.00155	.00177	23	.00383	.00447	.00511
9	.00150	.00175	.00200	24	.00400	.00466	.00533
10	.00166	.00194	.00222	25	.00416	.00486	.00555
11	.00183	.00213	.00244	26	.00433	.00505	.00577
12	.00200	.00233	.00266	27	.00450	.00525	.00600
13	.00216	.00252	.00288	28	.00466	.00544	.00622
14	.00233	.00272	.00311	29	.00483	.00563	.00644
15	.00250	.00291	.00333				

Method of using the Simple Interest Table.

603. Find the interest of $250 for 5 yr. 9 mo. 18 da. at 7%.

1. We *find the interest of* $1 *for the given time* = { .35 interest in table for 5 yr.; .0525 " " " " 9 mo.; .0035 " " " " 18 da. }

Interest of $1 for 5 yr. 9 mo. 18 da is .406 of $1.

2. Since the interest of $1 for 5 yr. 9 mo. 18 da. is .406 of $1, the interest of $250 for the same time is .406 of $250.

Hence, $250 × .406 = $101.50, the required interest.

EXAMPLES FOR PRACTICE.

604. Find by using the table the interest, at 7%, of

1. $860 for 3 yr. 7 mo. 23 da.
2. $438 for 5 yr. 11 mo. 19 da.
3. $283 for 6 yr. 8 mo. 27 da.
4. $325.86 for 5 yr. 13 da.
5. $796.50 for 11 mo. 28 da.
6. $395.75 for 3 yr. 7 mo.

TABLE *showing the amount of $1 at 6, 7, and 8% compound interest from 1 to 12 years.*

YRS.	6%.	7%.	8%.	YRS.	6%.	7%.	8%.
1	1.060000	1.070000	1.080000	7	1.503630	1.605781	1.713824
2	1.123600	1.144900	1.166400	8	1.593848	1.718186	1.850930
3	1.191016	1.225043	1.269712	9	1.689479	1.838459	1.999005
4	1.262477	1.310796	1.360489	10	1.790848	1.967151	2.158925
5	1.338226	1.402552	1.469328	11	1.898299	2.104852	2.331639
6	1.418519	1.500730	1.586874	12	2.012197	2.252192	2.518170

Method of using the Compound Interest Table.

605. Find the compound interest of $2800 for 7 years at 6%.

1. The amount of $1 for 7 years at 6% in the table is 1.50363.

2. Since the amount of $1 for 7 years is 1.50363, the amount of $2800 for the same time must be 2800 times $1.50363 = $1.50363 × 2800 = $4210.164. Hence, $4210.164 − $2800 = $1410.164, the required interest.

EXAMPLES FOR PRACTICE.

606. Find by using the table the compound interest of

1. \$560 for 9 yr. at 7%.
2. \$2600 for 5 yr. at 8%.
3. \$870 for 11 yr. at 6%.
4. \$3860 for 7 yr. at 8%.
5. \$2500 for $3\frac{1}{2}$ yr. at 6%.
6. \$640 for $4\frac{1}{4}$ yr. at 8%.
7. \$285 for $9\frac{1}{2}$ yr. at 7%.
8. \$384.50 for 8 yr. at 6%.
9. \$400 for 4 yr. 7 mo. at 7%.
10. \$900 for 6 yr. 3 mo. at 8%.
11. \$690 for 12 yr. 8 mo. at 6%.
12. \$4000 for 9 yr. 2 mo. at 7%.
13. \$3900 for 4 yr. 3 mo. at 6%.
14. \$600 for 11 yr. 6 mo. at 8%.

ANNUAL INTEREST.

607. ***Annual Interest*** is *simple interest* on the principal, and each year's interest remaining unpaid.

Annual interest is allowed on promissory notes and other contracts which contain the words, "interest payable annually if the interest remains unpaid."

608. Prob. X.—**To find the annual interest on a promissory note or contract.**

What is the interest on a note for \$600 at 7% at the end of 3 yr. 6 mo., interest payable annually, but remaining unpaid.

Solution.—1. At 7% the payment of interest on \$600 due at the end of each year is \$42, and the simple interest for 3 yr. 6 mo. is \$147.

2. The first payment of \$42 of interest is due at the end of the first year and must bear simple interest for 2 yr. 6 mo. The second payment is due at the end of the second year and must bear simple interest for 1 yr. 6 mo., and the third payment being due at the end of the third year must bear interest for 6 mo.

Hence, there is simple interest on \$42 for 2 yr. 6 mo. + 1 yr. 6 mo. + 6 mo. = 4 yr. 6 mo., and the interest of the \$42 for this time at 7% is \$13.23.

3. The simple interest on \$600 being \$147, and the simple interest on the interest remaining unpaid being \$13.23, the total interest on the note at the end of the given time is \$160.23.

EXAMPLES FOR PRACTICE.

609. 1. How much interest is due at the end of 4 yr. 9 mo. on a note for $460 at 6%, interest payable annually, but remaining unpaid?

2. Wilbur H. Reynolds has J. G. MacVicar's note dated July 29, 1876, for $800, interest payable annually; what will be due November 29, 1880, at 7%?

3. Find the amount of $780 at 7% annual interest for 5½ yr.

4. What is the difference between the annual interest and the compound interest of $1800 for 7 yr. at 7%?

5. What is the annual interest of $830 for 4 yr. 9 mo. at 8%?

6. What is the difference in the simple, annual, and compound interest of $790 for 5 years at 8%?

PARTIAL PAYMENTS.

610. A ***Promissory Note*** is a *written promise* to pay a sum of money at a specified time or on demand.

The *Face* of a note is the sum of money made payable by it.

The *Maker* or *Drawer* of a note is the person who signs the note.

The *Payee* is the person to whom or to whose order the money is paid.

An *Indorser* is a person who signs his name on the back of the note, and thus makes himself responsible for its payment.

611. A ***Negotiable Note*** is a note made payable to the bearer, or to some person's order.

When a note is so written it can be bought and sold in the same manner as any other property.

612. A ***Partial Payment*** is a payment in part of a note, bond, or other obligation.

613. An ***Indorsement*** is a written acknowledgment of a partial payment, placed on the back of a note, bond, etc., stating the time and amount of the same.

MERCANTILE RULE.

614. The method of reckoning partial payments known as the *Mercantile Rule* is very commonly used in computing interest on notes and amounts running for a year or less. The rule is as follows:

615. RULE.—*I. Find the amount of the note or debt from the time it begins to bear interest, and of each payment until the date of settlement.*

II. Subtract the sum of the amounts of payments from the amount of the note or debt; the remainder will be the balance due.

Observe, that an accurate application of the rule requires that the *exact interest* should be found according to (**583**).

EXAMPLES FOR PRACTICE.

616. 1. $900. POTSDAM, N. Y., *Sept. 1st,* 1876.

On demand I promise to pay Henry Watkins, or order, nine hundred dollars with interest, value received.

WARREN MANN.

Indorsed as follows: Oct. 18th, 1876, $150; Dec. 22d, 1876, $200; March 15th, 1877, $300. What is due on the note July 19th, 1877?

2. A note for $600 bearing interest at 8% from July 1st, 1874, was paid May 16th, 1875. The indorsements were: July 12th, 1874, $185; Sept. 15, 1874, $76; Jan. 13, 1875, $230; and March 2, 1875, $115. What was due on the note at the time of payment?

3. An account amounting to $485 was due Sept. 3, 1875, and was not settled until Aug. 15, 1876. The payments made upon it were: $125, Dec. 4, 1875; $84, Jan. 17, 1876; $95, June 23, 1876. What was due at the time of settlement, allowing interest at 7%?

4. $250. OGDENSBURG, N. Y., *March* 25, 1876.

Ninety-eight days after date I promise to pay E. D. Brooks, or order, two hundred fifty dollars with interest, value received.

SILAS JONES.

Indorsements: $87, April 12, 1876; $48, May 9. What is to pay when the note is due?

UNITED STATES RULE.

617. The United States courts have adopted the following for reckoning the interest on partial payments:

618. RULE.—*I. Find the amount of the given principal to the time of the first payment; if the payment equals or exceeds the interest then due, subtract it from the amount obtained and regard the remainder as the new principal.*

II. If the payment is less than the interest due, find the amount of the given principal to a time when the sum of the payments equals or exceeds the interest then due, and subtract the sum of the payments from this amount, and regard the remainder as the new principal.

III. Proceed with this new principal and with each succeeding principal in the same manner.

619. The method of applying the above rule will be seen from the following example:

1. A note for $900, dated Syracuse, Jan. 5th, 1876, and paid Dec. 20th, 1876, had endorsed upon it the following payments: Feb. 23d, 1876, $40; April 26th, $6; July 19th, 1876, $70. How much was the payment Dec. 20th, 1876, interest at 7%?

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SOLUTION.

First Step.

1. The first principal is the face of the note		$900
2. We find the interest from the date of the note to the first payment, Feb. 23, 1876 (49 da.), at 7%		8.43
Amount		$908.43
3. The first payment, $40, being greater than the interest then due, is subtracted from the amount		40.00
Second principal		$868.43

Second Step.

1. The second principal is the remainder after subtracting the first payment from the *amount* at that date		$868.43
2. The interest on $868.43, from Feb. 23 to Apr. 26, 1876 (63 da.), is	$10.463	
3. This interest being greater than the second payment ($6), we find the interest on $868.55 from April 26 to July 19, 1876, (84 da.), which is	13.951	
Interest from first to third payment .	$24.414	24.414
Amount		$892.844
4. The sum of the second and third payments being greater than the interest due, we subtract it from the amount .		76
Third principal		$816.844

Third Step.

We find the interest on $816.844, from July 19 to Dec. 20, 1876 (154 da.), which is	24.057
Payment due Dec. 20, 1876	$840.90

In the above example, the interest has been reckoned according to (**583**); in the following, 360 days have been regarded as a year.

EXAMPLES FOR PRACTICE.

2. A note for $1630 at 8% interest was dated March 18, 1872, and was paid Aug. 13, 1875. The following sums were endorsed upon it: $160, Feb. 12, 1873; $48, March 7, 1874; and $350, Aug. 25, 1874. How much was paid Aug. 13?

(263)

3. A mortgage for \$3500 was dated Aug. 24, 1873. It had endorsed upon it the following payments: May 17, 1874, \$89; Sept. 12, 1874, \$635; March 4, 1875, \$420. How much was due upon it Feb. 9, 1876, interest at 7%?

4. What was the last payment on a note for \$1000 at 7%, which was dated Jan. 7, 1876, and paid Dec. 26, 1876, endorsed as follows: April 12, 1876, \$16; July 10, 1876, \$250; and Oct. 26, 1876, \$370?

5. A mortgage for \$4600, dated Leavenworth, Kansas, Sept. 25, 1871, had endorsed upon it: \$400, June 23, 1872; \$125, Aug. 3, 1873; \$580, May 7, 1874; and \$86, Mar. 5, 1875. How much was due upon it Sept. 25, 1875, interest at 11%?

DISCOUNT.

620. ***Discount*** is a deduction made for any reason from an account, debt, price of goods, and the like, or for the interest of money advanced upon a bill or note due at a future date.

621. The ***Present Worth*** of a note, debt, or other obligation, payable at a future time without interest, is such a sum as, being placed at interest at a legal rate, will amount to the given sum when it becomes due.

622. ***True Discount*** is a difference between any sum of money payable at a future time and its *present worth.*

623. PROB. XI.—**To find the present worth of any sum.**

Find the present worth of a debt of \$890, due in 2 yr. 6 mo. without interest, allowing 8% discount.

SOLUTION.—Since \$1 placed at interest for 2 yr. 6 mo. at 8% amounts to \$1.20, the *present worth* (**621**) of \$1.20, due in 2 yr. 6 mo., is \$1. Hence the present worth of \$890, which is due without interest in 2 yr. 6 mo., must contain as many dollars as \$1.20 is contained times in \$890 = \$741.66.

Observe, that this problem is an application of (**506**, Prob. XI).

EXAMPLES FOR PRACTICE.

624. What is the *present worth*

1. Of \$800 at 6%, due in 6 mo.? At 8%, due in 9 mo.?
2. Of \$360 at 7%, due in 2 yr.? At 5%, due in 8 mo.?
3. Of \$490 at 8%, due in 42 da.? At 7%, due in 128 da.?

What is the *true discount*

4. Of \$580 at 7%, due in 90 da.? At 8½%, due in 4 yr. 17 da.?
5. Of \$260 at 6½%, due in 120 da.? At 9%, due in 2 yr. 25 da.?
6. Of \$860 at 7%, due in 93 da.? At 12%, due in 3 yr. 19 da.?
7. What is the true discount at 8% on a debt of \$3200, due in 2 yr. 5 mo. and 24 da.?
8. Sold my farm for \$3800 cash and a mortgage for \$6500 running for 3 years without interest. The use of money being worth 7% per annum, what is the cash value of the farm?
9. What is the difference between the interest and *true discount* at 7% of \$460, due 8 months hence?
10. A man is offered a house for \$4800 cash, or for \$5250 payable in 2 yr. 6 mo. without interest. If he accepts the former, how much will he lose when money is worth 8%?
11. Which is more profitable, and how much, to buy wood at \$4.50 a cord cash, or at \$4.66 payable in 9 months without interest, money being worth 8%?
12. A merchant buys \$2645.50 worth of goods on 3 mo. credit, but is offered 3% discount for cash. Which is the better bargain, and how much, when money is at 7% per annum?
13. A grain merchant sold 2400 bu. of wheat for \$3600, for which he took a note at 4 mo. without interest. What was the cash price per bushel, when money is at 6%?

BANK DISCOUNT.

625. ***Bank Discount*** is the interest on the amount of a note at maturity, computed from the date the note is discounted to the date of maturity.

1. *Observe* that when a note bears no interest, its amount at maturity is its face.

2. When the time of a note is given in months, calendar months are understood; if the month in which the note falls due has no day corresponding to the date of the note, then the note is due on the last day of that month.

3. In computing bank discount, it is customary to reckon the time in days.

4. When a note becomes due on Sunday or a legal holiday, it must be paid on the day previous.

626. ***Days of Grace*** are three days usually allowed by law for the payment of a note after the expiration of the time specified in it.

627. The ***Maturity*** of a note is the expiration of the time for which it is made, including days of grace.

628. The ***Proceeds*** or ***Avails*** of a note is the sum left after deducting the discount.

629. A ***Protest*** is a declaration in writing by a Notary Public, giving legal notice to the maker and endorsers of a note of its non-payment.

630. PROB. XII.—**To find the bank discount and proceeds of a note for any given rate and time.**

Observe, that the *bank discount* is the interest on the amount of the note at maturity, computed from the time the note is discounted to the date of maturity; and the proceeds is the amount of the note at maturity, minus the bank discount. Hence the following

631. RULE.—*I. Find the amount of the note at maturity, compute the interest upon this sum from the date of discounting the note to the date of maturity; the result is the bank discount.*

II. Subtract the bank discount from the amount of the note at maturity; the remainder is the proceeds.

EXAMPLES FOR PRACTICE.

632. What are the bank discount and proceeds of a note

1. Of \$280 for 3 mo. 15 da. at 7%? For 6 mo. 9 da. at 8%?

2. Of \$790 for 154 da. at 6%? For 2 mo. 12 da. at 7%?

3. Of \$1600 for 80 da. at 7%? For 140 da. at $8\frac{1}{2}$%?

4. What is the difference between the *bank* and *true* discount on a note of \$1000 at 7%, payable in 90 days?

5. Valuing my horse at \$212, I sold him and took a note for \$235 payable in 60 days, which I discounted at the bank. How much did I gain on the transaction?

6. A man bought 130 acres of land at \$16 per acre. He paid for the land by discounting a note at the bank for \$2140.37 for 90 da. at 6%. How much cash has he left?

Find the *date of maturity*, the *time*, and the *proceeds* of the following notes:

7. \$480.90. ROCHESTER, N. Y., *Mar.* 15, 1876.

Seventy days after date I promise to pay to the order of N. L. Sage, four hundred eighty $\frac{90}{100}$ dollars for value received.

Discounted Mar. 29. DUNCAN MACVICAR.

8. \$590. POTSDAM, N. Y., *May* 13, 1876.

Three months after date I promise to pay to the order of Wm. Flint, five hundred ninety dollars, for value received.

Discounted June 2. PETER HENDERSON.

9. \$1600. ROME, N. Y., *Jan.* 19, 1876.

Seven months after date we jointly and severally agree to pay James Richards, or order, one thousand six hundred dollars at the National Bank, Potsdam, N. Y., value received.

Discounted May 23. ROBERT BUTTON,
JAMES JACKSON.

633. Prob. XIII.—**To find the face of a note when the *proceeds,* time, and rate are given.**

Observe, that the proceeds is the *face of the note* minus the *interest* on it for the given time and rate, and consequently that the proceeds must contain $1 minus the interest of $1 for the given time and rate as many times as there are dollars in the face of the note. Hence the following

634. Rule.—*Divide the given proceeds by the proceeds of $1 for the given time and rate; the quotient is the face of the note.*

EXAMPLES FOR PRACTICE.

635. What must be the face of a note which will give

1. For 3 mo. 17 da., at 6%, $860 proceeds? $290? $530.80?

2. For 90 da., at 7%, $450 proceeds? $186.25? $97.32?

3. For 73 da., at $8%, $234.60 proceeds? $1800? $506.94?

4. What must be the face of a note for 80 days, at 7%, on which I can raise at a bank $472.86?

5. The avails of a note for 50 days when discounted at a bank were $350.80; what was the face of the note?

6. How much must I make my note at a bank for 40 da., at 7%, to pay a debt of $296.40?

7. A merchant paid a bill of goods amounting to $2850 by discounting three notes at a bank at 7%, the proceeds of each paying one-third of the bill; the time of the first note was 60 days, of the second 90 days, and of the third 154 days. What was the *face* of each note?

8. For what sum must I draw my note March 23, 1876, for 90 days, so that when discounted at 7% on May 1 the proceeds may be $490?

9. Settled a bill of $2380 by giving my note for $890 at 30 days, bearing interest, and another note at 90 days, which when discounted at 7% will settle the balance. What is the face of the latter note?

EXCHANGE

636. ***Exchange*** is a method of paying debts or other obligations at a distance without transmitting the money.

Thus, a merchant in Chicago desiring to pay a debt of $1800 in New York, pays a bank in Chicago $1800, plus a small per cent for their trouble, and obtains an order for this amount on a bank in New York, which he remits to his creditor, who receives the money from the New York bank.

Exchange between places in the same country is called *Inland* or *Domestic Exchange*, and between different countries *Foreign Exchange*.

637. A ***Draft*** or ***Bill of Exchange*** is a written order for the payment of money at a specified time, drawn in one place and payable in another.

1. The *Drawer* of a bill or draft is the person who signs it; the *Drawee*, the person directed to pay it; the *Payee*, the person to whom the money is directed to be paid; the *Indorser*, the person who transfers his right to a bill or draft by indorsing it; and the *Holder*, the person who has legal possession of it.

2. A *Sight Draft* or *Bill* is one which requires payment to be made when *presented* to the *payor*.

3. A *Time Draft* or *Bill* is one which requires payment to be made at a specified time after date, or after *sight* or being *presented* to the *payor*.

Three days of grace are usually allowed on bills of exchange.

4. The *Acceptance* of a bill or draft is the agreement of the party on whom it is drawn to pay it at maturity. This is indicated by writing the word "Accepted" across the face of the bill and signing it.

When a bill is protested for non-acceptance, the drawer is bound to pay it immediately.

5. Foreign bills of exchange are usually drawn in duplicate or triplicate, and sent by different conveyances, to provide against miscarriage, each copy being valid until the bill is paid.

638. The ***Par of Exchange*** is the relative value of the coins of two countries.

Thus, the par of exchange between the United States and England is the number of gold dollars, the standard unit of United States money, which is equal to a pound sterling, the standard unit of English money. Hence $4.8665 = £1 is the par of exchange.

DOMESTIC EXCHANGE.

639. ***Domestic Exchange*** is a method of paying debts or other obligations at distant places in the same country, without transmitting the money.

Forms of Sight and Time Drafts.

THIRD NATIONAL BANK OF ROCHESTER,
$890. ROCHESTER, N. Y., *May* 4, 1876.

At sight, pay to the order of Chas. D. McLean, eight hundred ninety dollars.

WILLIAM ROBERTS, *Cashier.*

To the SEVENTH NATIONAL BANK,
NEW YORK, N. Y.

This is the usual form of a draft drawn by one bank upon another.

$2700. SYRACUSE, N. Y., *July* 25, 1876.

At fifteen days sight, pay to the order of Taintor Brothers & Co., two thousand seven hundred dollars, value received, and charge the same to the account of

A. B. STEWART & CO.

To the TENTH NATIONAL BANK,
NEW YORK, N. Y.

1. This is the usual form of a draft drawn by a firm or individual upon a bank. It may also be made payable at a given time after date.

2. All time drafts should be presented for acceptance as soon as received. When the cashier writes the word "Accepted," with the date of acceptance across the face, and signs his name, the bank is responsible for the payment of the draft when due.

METHODS OF DOMESTIC EXCHANGE.

640. First Method.—*The party desiring to transmit money, purchases a draft for the amount at a bank, and sends it by mail to its destination.*

Observe carefully the following:

1. Banks can sell drafts only upon others in which they have deposits in money or equivalent security. Hence banks throughout the country, in order to give them this facility, have such deposits at centres of trade, such as New York, Boston, Chicago, etc.

2. A *Bank Draft* will usually be purchased by banks in any part of the country, in case the person offering it is fully identified as the party to whom the draft is payable. Hence, a debt or other liability may be discharged at any place by a draft on a New York bank.

3. A draft may be made payable to the person to whom it is sent, or to the person buying it. In the latter case the person buying it must write on the back "Pay to the order of" (name of party to whom it is sent), and sign his own name.

Second Method.—*I. The party desiring to transmit money, deposits the amount in a bank and takes a certificate of deposit, which he sends as by first method.* Or,

II. If he has a deposit already in a bank, subject to his check or order, it is customary to send his check, certified to be good by the cashier of the bank.

This method, in either of these forms, is ordinarily followed in making payments at a distance by persons in New York and other large centres of trade. Banks in such places have no deposits in cities and villages throughout the country, and hence do not sell drafts.

Certificates of deposits and certified checks are purchased by banks in the same manner as bank drafts.

Third Method.—*The party desiring to transmit money, obtains a Post Office order for the amount and remits it as before.*

As the amount that can be included in one Post Office order is limited, this method is restricted in its application. It is usually employed in remitting small sums of money.

FOURTH METHOD.—*The party desiring to transmit money, makes a draft or order for the amount upon a party owing him, at the place where the money is to be sent, and remits this as previously directed.*

1. By this method one person is said to *draw* upon another. Such drafts should be presented for payment as soon as received, and if not paid or accepted should be protested for non-payment immediately.

2. Large business firms have deposits in banks at business centres, and credit with other business firms; hence, their drafts are used by themselves and others the same as bank drafts.

641. The *Premium* or *Discount* on a draft depends chiefly on the *condition of trade* between the place where it is purchased and the place on which it is made.

Thus, for example, merchants and other business men at Buffalo contract more obligations in New York, for which they pay by draft, than New York business men contract in Buffalo; consequently, banks at Buffalo must actually send money to New York by Express or other conveyance. Hence, for the expense thus incurred and other trouble in handling the money, a small premium is charged at Buffalo on New York drafts.

EXAMPLES FOR PRACTICE.

642. 1. What is the cost of a sight draft for \$2400, at $\frac{2}{3}$% premium?

SOLUTION.—Cost = \$2400 + $\frac{2}{3}$% of \$2400 = \$2416.

2. What is the cost of a draft for \$3200, at $\frac{1}{8}$% premium?

SOLUTION.—Cost = \$3200 + $\frac{1}{8}$% of \$3200 = \$3204.

Find the cost of sight draft

3. For \$834, premium 2%.
4. For \$6300, premium $\frac{1}{2}$%.
5. For \$132.80, premium $\frac{3}{4}$%.
6. For \$1500, discount $\frac{3}{5}$%.
7. For \$384.50, discount $\frac{4}{7}$%.
8. For \$295.20, discount $1\frac{3}{8}$%.

9. The cost of a sight draft purchased at $1\frac{1}{2}$% premium is \$493.29; what is the face of the draft?

SOLUTION.—At $1\frac{1}{2}$% premium, \$1 of the face of the draft cost \$1.015. Hence the face of the draft is as many dollars as \$1.015 is contained times in \$493.29, which is \$486.

Find the face of a draft which cost

10. $575.41, premium $2\frac{3}{4}\%$.
11. $731.70, premium $1\frac{5}{8}\%$.
12. $483.20, premium, $\frac{2}{3}\%$.
13. $819.88, discount $\frac{1}{2}\%$.
14. $273.847, discount $\frac{3}{5}\%$.
15. $315.65, discount $1\frac{2}{3}\%$.

16. What is the cost of a draft for $400, payable in 3 mo., premium $1\frac{1}{2}\%$, the bank allowing interest at 4% until the draft is paid?

Solution.—A sight draft for $400, at $1\frac{1}{2}\%$ premium, costs $406, but the bank allows interest at 4% on the face, $400, for 3 mo., which is $4. Hence the draft will cost $406 − $4 = $402.

Find the cost of drafts

17. For $700, premium $\frac{1}{4}\%$, time 60 da., interest at 3%.
18. For $1600, premium $1\frac{1}{2}\%$, time 50 da., interest at 4%.
19. For $2460, discount $\frac{2}{3}\%$, time 90 da., interest at $4\frac{1}{2}\%$.
20. For $1800, discount $\frac{3}{4}\%$, time 30 da., interest at 5%.

21. A merchant in Albany wishing to pay a debt of $498.48 in Chicago, sends a draft on New York, exchange on New York being at $\frac{1}{2}\%$ premium in Chicago; what did he pay for the draft?

Solution.—The draft cashed in Chicago commands a premium of $\frac{1}{2}\%$ on its face. The man requires, therefore, to purchase a draft whose face plus $\frac{1}{2}\%$ of it equals $498.48. Hence, according to (**506**—5), the amount paid, or face of the draft, is $498.48 ÷ 1.005 = $496.

22. Exchange being at $98\frac{3}{4}$ ($1\frac{1}{4}\%$ discount), what is the cost of a draft, time 4 mo., interest at 5%?

23. The face of a draft which was purchased at $1\frac{1}{2}\%$ premium is $2500, the time 40 da., rate of interest allowed 4%; what was its cost?

24. My agent in Detroit sold a consignment of goods for $8260, commission on the sale $2\frac{1}{2}\%$. He remitted the proceeds by draft on New York, at a premium of $\frac{1}{2}\%$. What is the amount remitted?

FOREIGN EXCHANGE.

643. ***Foreign Exchange*** is a method of paying debts or other obligations in foreign countries without transmitting the money.

Observe, that *foreign exchange* is based upon the fact that different countries *exchange products*, securities, etc., with each other.

Thus, the United States sells wheat, etc., to England, and England in return sells manufactnred goods, etc., to the United States. Hence, parties in each country *become indebted* to parties in the other. For this reason, a merchant in the United States can pay for goods purchased in England by buying an order upon a firm in England which is *indebted* to a firm in the United States.

Form of a Bill or Set of Exchange.

£400. NEW YORK, *July* 13, 1876.

At sight of this FIRST *of* EXCHANGE *(second and third of the same date and tenor unpaid), pay to the order of* E. D. BLAKESLEE FOUR HUNDRED POUNDS STERLING, *for value received, and charge the same to the account of*

WILLIAMS, BROWN & CO.

To MARTIN, WILLIAMS & CO., *London.*

The person purchasing the exchange receives three bills, which he sends by different mails to avoid miscarriage. When one has been received and paid, the others are void.

The above is the form of the first bill. In the Second Bill the word "FIRST" is used instead of "SECOND," and the parenthesis reads, "*First* and *Third* of the same date and tenor unpaid." A similar change is made in the Third Bill.

644. ***Exchange*** with ***Europe*** is conducted chiefly through prominent financial centres, as London, Paris, Berlin, Antwerp, Amsterdam, etc.

645. ***Quotations*** are the published rates at which bills of exchange, stocks, bonds, etc., are bought and sold in the money market from day to day.

These quotations give the market gold value in United States money of one or more units of the foreign coin.

Thus, quotations on London give the value of £1 *sterling* in *dollars;* on Paris, Antwerp, and Geneva, the value of $1 in *francs;* on Hamburg, Berlin, Bremen, and Frankfort, the value of 4 *marks* in *cents;* on Amsterdam, the value of a *guilder* in *cents.*

646. The following table gives the *par of exchange,* or gold value of foreign monetary units, as published by the Secretary of the Treasury, January 1, 1876:

TABLE OF PAR OF EXCHANGE.

COUNTRIES.	MONETARY UNIT.	STANDARD.	VALUE IN U. S. MONEY.
Austria	Florin	Silver	.45, 3
Belgium	Franc	Gold and silver.	.19, 3
Bolivia	Dollar	Gold and silver.	.96, 5
Brazil	Milreis of 1000 reis	Gold	.54, 5
Bogota	Peso	Gold	.96, 5
Canada	Dollar	Gold	$1.00
Central America.	Dollar	Silver	.91, 8
Chili	Peso	Gold	.91, 2
Denmark	Crown	Gold	.26, 8
Ecuador	Dollar	Silver	.91, 8
Egypt	Pound of 100 piasters	Gold	4.97, 4
France	Franc	Gold and silver.	.19, 3
Great Britain	Pound sterling	Gold	4.86, 6½
Greece	Drachma	Gold and silver.	.19, 3
German Empire	Mark	Gold	.23, 8
Japan	Yen	Gold	.99, 7
India	Rupee of 16 annas	Silver	.43, 6
Italy	Lira	Gold and silver.	.19, 3
Liberia	Dollar	Gold	1.00
Mexico	Dollar	Silver	.99, 8
Netherlands	Florin	Gold and silver.	.38, 5
Norway	Crown	Gold	.26, 8
Peru	Dollar	Silver	.91, 8
Portugal	Milreis of 1000 reis	Gold	1.08
Russia	Rouble of 100 copecks	Silver	.73, 4
Sandwich Islands	Dollar	Gold	1.00
Spain	Peseta of 100 centimes	Gold and silver.	.19, 3
Sweden	Crown	Gold	.26, 8
Switzerland	Franc	Gold and silver.	.19, 3
Tripoli	Mahbub of 20 piasters	Silver	.82, 9
Tunis	Piaster of 16 caroubs	Silver	.11, 8
Turkey	Piaster	Gold	.04, 3
U. S. of Colombia	Peso	Silver	.91, 8

METHODS OF DIRECT EXCHANGE.

647. *Direct Exchange* is a method of making payments in a foreign country at the quoted rate of exchange with that country.

FIRST METHOD.—*The person desiring to transmit the money purchases a SET of EXCHANGE for the amount on the country to which the money is to be sent, and forwards the three bills by different mails or routes to their destination.*

SECOND METHOD.—*The person desiring to transmit the money instructs his creditor in the foreign country to DRAW upon him, that is, to SELL a SET of EXCHANGE upon him, which he pays in his own country when presented.*

EXAMPLES FOR PRACTICE.

648. 1. What is the cost in currency of a bill of exchange on Liverpool for £285 9s. 6d., exchange being quoted at \$4.88, and gold at 1.12, brokerage $\frac{1}{4}\%$?

£285 9s. 6d. = £285.475
\$4.88 × 285.475 = \$1393.118
\$1.1225 × 1393.118 = 1563.77+

SOLUTION.—1. We reduce the 9s. 6d. to a decimal of £1. Hence £285 9s. 6d. = £285.475.

2. Since £1 = \$4.88, £285.475 must be equal \$4.88 × 285.475 = \$1393.118, the gold value of the bill without brokerage.

3. Since \$1 gold is equal \$1.12 currency, and the brokerage is $\frac{1}{4}\%$, the cost of \$1 gold in currency is \$1.1225. Hence the bill cost in currency \$1.1225 × 1393.118 = \$1563.77+.

What is the cost of a bill on

2. *London* for £436 8s. 3d., sterling at $4.84\frac{1}{2}$, brokerage $\frac{1}{2}\%$?
3. *Paris* for 4500 francs at .198, brokerage $\frac{1}{4}\%$?
4. *Geneva*, Switzerland, for 8690 francs at .189?
5. *Antwerp* for 4000 francs at .175, in currency, gold at 1.09?
6. *Amsterdam* for 8400 guilders at $41\frac{1}{4}$, brokerage $\frac{1}{8}\%$?
7. *Frankfort* for 2500 marks, quoted at $.97\frac{1}{2}$?

8. A merchant in Boston instructed his agent at Berlin to draw on him for a bill of goods of 43000 marks, exchange at $24\frac{1}{4}$, gold being at $1.08\frac{1}{2}$, brokerage $\frac{1}{4}\%$; what did the merchant pay in currency for the goods?

METHODS OF INDIRECT EXCHANGE.

649. *Indirect Exchange* is a method of making payments in a foreign country by taking advantage of the rate of exchange between that country and one or more other countries.

Observe carefully the following:

1. The advantage of *indirect* over *direct* exchange under certain financial conditions which sometimes, owing to various causes, exist between different countries, may be shown as follows:

Suppose exchange in New York to be at par on London, but on Paris at 17 cents for 1 franc, and at Paris on London at 24 francs for £1. With these conditions, a bill on London for £100 will cost in New York \$486.65; but a bill on London for £100 will cost in Paris 24 francs × 100 = 2400 francs, and a bill on Paris for 2400 francs will cost in New York 17 cents × 2400 = \$408.

Hence £100 can be sent from New York to London by direct exchange for \$486.65, and by indirect exchange or through Paris for \$408, giving an advantage of \$486.65 − \$408 = \$78.65 in favor of the latter method.

2. The process of computing indirect exchange is called *Arbitration of Exchange.* When there is only one intermediate place, it is called *Simple Arbitration;* when there are two or more intermediate places, it is called *Compound Arbitration.*

Either of the following methods may be pursued:

First Method.—*The person desiring to transmit the money may buy a bill of exchange for the amount on an intermediate place, which he sends to his agent at that place with instructions to buy a bill with the proceeds on the place to which the money is to be sent, and to forward it to the proper party.*

This is called the *method by remittance.*

SECOND METHOD.—*The person desiring to send the money instructs his creditor to draw for the amount on his agent at an intermediate place, and his agent to draw upon him for the same amount.*

This is called the *method by drawing.*

THIRD METHOD.—*The person desiring to send the money instructs his agent at an intermediate place to draw upon him for the amount, and buy a bill on the place to which the money is to be sent, and forward it to the proper party.*

This is called the method by *drawing and remitting.*

These methods are equally applicable when the exchange is made through two or more intermediate places, and the solution of examples under each is only an application of compound numbers and business. Probs. VIII, IX, X, and XI.

EXAMPLES FOR PRACTICE.

650. 1. Exchange in New York on London is 4.83, and on Paris in London is $24\frac{1}{2}$; what is the cost of transmitting 63994 francs to Paris through London?

SOLUTION.—1. We find the cost of a bill of exchange in London for 63994 francs. Since $24\frac{1}{2}$ francs = £1, $63994 \div 24\frac{1}{2}$ is equal the number of £ in 63994 francs, which is £2612.

2. We find the cost of a bill of exchange in New York for £2612. Since £1 = \$4.83, the bill must cost $\$4.83 \times 2612 = \$12615.96$.

2. A merchant in New York wishes to pay a debt in Berlin of 7000 marks. He finds he can buy exchange on Berlin at .25, and on Paris at .18, and in Paris on Berlin at 1 mark for 1.15 francs. Will he gain or lose by remitting by *indirect exchange,* and how much?

3. What will be the cost to remit 4800 guilders from New York to Amsterdam through Paris and London, exchange being quoted as follows: at New York on Paris, $.18\frac{1}{2}$; at Paris on London, $24\frac{1}{3}$ francs to a £; and at London on

Amsterdam, $12\frac{1}{8}$ guilders to the £. How much more would it cost by direct exchange at $39\frac{1}{2}$ cents for 1 guilder?

4. An American residing in Berlin wishing to obtain $6000 from the United States, directs his agent in Paris to draw on Boston and remit the proceeds by draft to Berlin. Exchange on Boston at Paris being .18, and on Berlin at Paris 1 mark for 1.2 francs, the agent's commission being $\frac{1}{2}\%$ both for drawing and remitting, how much would he gain by drawing directly on the United States at $24\frac{1}{2}$ cents per mark?

EQUATION OF PAYMENTS.

651. An ***Account*** is a written statement of the *debit* and *credit* transactions between two persons with their dates.

The *debit* or left-hand side of an account (marked *Dr.*) shows the sums due to the *Creditor*, or person keeping the account; the *credit* or right-hand side (marked *Cr.*) shows the sums paid by the *Debtor*, or person against whom the account is made.

652. The ***Balance*** of an account is the difference between the sum of the items on the debit and credit sides.

653. ***Equation of Payments*** is the process of finding a date at which a debtor may pay a creditor in one payment several sums of money due at different times, without loss of interest to either party.

654. The ***Equated Time*** is the date at which several debts may be equitably discharged by one payment.

655. The ***Maturity*** of any *obligation* is the date at which it becomes due or draws interest.

656. The ***Term of Credit*** is the interval of time from the date a debt is contracted until its *maturity*.

657. The ***Average Term of Credit*** is the interval of time from the *maturity* of the first item in an account to the *Equated Time*.

PREPARATORY PROPOSITIONS.

658. The method of settling accounts by *equation of payments* depends upon the following propositions; hence they should be carefully studied:

PROP. I.—*When, by agreement, no interest is to be paid on a debt from a specified time, if any part of the amount is paid by the debtor, he is entitled to interest until the expiration of the specified time.*

Thus, A owes B $100, payable in 12 months without interest, which means that A is entitled by agreement to the use of $100 of B's money for 12 months. Hence, if he pays any part of it before the expiration of the 12 months, he is entitled to interest.

Observe, that when credit is given without charging interest, the profits or advantage of the transaction are such as to give the creditor an equivalent for the loss of the interest of his money.

PROP. II.—*After a debt is due, or the time expires for which by agreement no interest is charged, the creditor is entitled to interest on the amount until it is paid.*

Thus, A owes B $300, due in 10 days. When the 10 days expire, the $300 should be paid by A to B. If not paid, B loses the use of the money, and is hence entitled to interest until it is paid.

PROP. III.—*When a* TERM *of* CREDIT *is allowed upon any of the items of an account, the date at which such items are due or commence to draw interest is found by adding its term of credit to the date of each item.*

Thus, goods purchased March 10 on 40 days' *credit* would be due or draw interest March 10 + 40 da., or April 19.

659. PROB. I.—**To settle equitably an account containing only debit items.**

R. Bates bought merchandise of H. P. Emerson as follows: May 17, 1875, on 3 months' *credit*, $265; July 11, on 25 days, $460; Sept. 15, on 65 days, $650.

Find the equated time and the amount that will equitably settle the account at the date when the last item is due, 7% interest being allowed on each item from maturity.

SOLUTION BY INTEREST METHOD.

1. We find the date of maturity of each item thus:

$265 on 3 mo. is due May 17 + 3 mo. = Aug. 17
$460 on 25 da. is due July 11 + 25 da. = Aug. 5.
$650 on 65 da. is due Sept. 15 + 65 da. = Nov. 19.

2. As the items of the debt are due at these dates, it is evident that when they all remain unpaid until the latest maturity, H. P. Emerson is entitled to legal interest

On $265 from Aug. 17 to Nov. 19 = 94 da.
On $460 from Aug. 5 to Nov. 19 = 106 da.

The $650 being due Nov. 19 bears no interest before this date.

3. On Nov. 19, H. P. Emerson is entitled to receive $1375, the sum of the items of the debt and the interest on $265 for 94 da. plus the interest on $460 for 106 da. at 7%, which is $14.12.

Hence the account may be equitably settled on Nov. 19 by R. Bates paying H. P. Emerson $1375 + $14.12 = $1389.12.

4. Since H. P. Emerson is entitled to receive Nov. 19, $1375 + $14.12 interest, it is evident that if he is paid $1375 a sufficient time before Nov. 19 to yield $14.12 interest at this date, the debt will be equitably settled. But $1375, according to (**596**), will yield $14.12 in 53 + a fraction of a day.

Hence the equated time of settlement is Sept. 26, which is 53 days previous to Nov. 19, the assumed date of settlement.

SOLUTION BY PRODUCT METHOD.

1. We find in the same manner as in the *interest method* the dates of maturity and the number of days each item bears interest.

2. Assuming Nov. 19, the latest maturity, as the date of settlement, it is evident that H. P. Emerson should be paid at this date $1375, the sum of the items of the account and the interest on $265 for 94 days plus the interest on $460 for 106 days.

3. Since the interest on $265 for 94 days at any given rate is equal to the interest on $265 × 94, or $24910, for 1 day at the same rate, and the interest on $460 for 106 days is equal to the interest on $460 × 106, or

$48760, for 1 day, the interest due H. P. Emerson Nov. 19 is equal the interest of $24910 + $48760, or $73670, for 1 day.

4. Since the interest on $73670 for 1 day is equal to the interest on $1375 for as many days as $1375 is contained times in $73670, which is $53\frac{159}{275}$, it is evident that if H. P. Emerson receive the use of $1375 for 53 days previous to Nov. 19, it will be equal to the interest on $73670 for 1 day paid at that date. Consequently, R. Bates by paying $1375 Sept. 26, which is 53 days before Nov. 19, discharges equitably the indebtedness.

Hence, Sept. 26 is the *equated time*, and from Aug. 5 to Sept. 26, or 52 days, is the *average term of credit*.

Observe, that R. Bates may discharge equitably the indebtedness in one of three ways:

(1.) *By paying Nov.* 19, *the latest maturity*, $1375, *the sum of the items of the account, and the interest of* $73670 *for* 1 *day*.

In this case the payment is $1375 + $14.12 interest = $1389.12.

(2.) *By paying* $1375, *the sum of the items in cash, on Sept.* 26, *the* EQUATED TIME.

(3.) *By giving his note for* $1375, *the sum of the items of the account, bearing interest from Sept.* 26, *the equated time*.

Observe this is equivalent to paying the $1375 in cash Sept. 26.

From these illustrations we obtain the following

660. RULE.—*I. Find the date of maturity of each item.*

II. Assume as the date of settlement the latest maturity, and find the number of days from this date to the maturity of each item.

In case the indebtedness is discharged at the assumed date of settlement:

III. Find the interest on each item from its maturity to the date of settlement. The sum of the items plus this interest is the amount that must be paid the creditor.

In case the *equated time* or *term of credit* is to be found and the indebtedness discharged in one payment, either by cash or note:

IV. Multiply each item by the number of days from its maturity to the latest maturity in the account, and divide the sum of these products by the sum of the items; the quotient is the number of days which must be counted back from the latest maturity to give the equated time.

V. The first maturity subtracted from the equated time gives the average term of credit.

EXAMPLES FOR PRACTICE.

661. 1. Henry Ross purchases Jan. 1, 1876, \$1600 worth of goods from James Mann, payable as follows: April 1, 1876, \$700; June 1, 1876, \$400; and Dec. 1, 1876, \$500. At what date can he equitably settle the bill in one payment?

When the interval between the maturity of each item and the date of settlement is months, as in this example, the months should not be reduced to days; thus,

SOLUTION.—1. Assuming that no payment is made until Dec. 1, James Mann is entitled to interest

On \$700 for 8 mo. = \$700 × 8 or \$5600 for 1 month.
On \$400 for 6 mo. = \$400 × 6 or \$2400 for 1 month.
Hence he is entitled to the use of \$8000 for 1 month.

2. \$8000 ÷ \$1600 = 5, the number of months (**659**—4) which must be counted back from Dec. 1 to find the equated time, which is July 1. Hence the bill can be equitably settled in one payment July 1, 1876.

2. A man purchased a farm May 23, 1876, for \$8600, on which he paid \$2600, and was to pay the balance, without interest, as follows: Aug. 10, 1876, \$2500; Jan. 4, 1877, \$1500; and June 14, 1877, \$2000. Afterwards it was agreed that the whole should be settled in one payment. At what date must the payment be made?

3. Bought merchandise as follows: Feb. 3, 1875, \$380; April 13, \$520; May 18, \$260; and Aug. 12, \$350, each item on interest from date. What must be the date of a note for the sum of the items bearing interest which will equitably settle the bill?

Find the date at which a note bearing interest can be given as an equitable settlement for the amount of each of the following bills, each item being on interest from the date of purchase:

4. Purchased as follows:
July 9, 1876, $380;
Sept. 13, " $270;
Nov. 24, " $840;
Dec. 29, " $260.

5. Purchased as follows:
May 5, 1876, $186;
Aug. 10, " $230;
Oct. 15, " $170;
Dec. 20, " $195.

6. Purchased as follows:
April 17, 1877, $185;
June 24, " $250;
Sept. 13, " $462.

7. Purchased as follows:
Aug. 25, 1877, $280;
Oct. 10, " $193;
Dec. 18, " $290.

8. Find the average term of credit on goods purchased as follows: Mar. 23, $700, on 95 da. credit; May 17, $480, on 45 da.; Aug. 25, $690, on 60 da.; and Oct. 2, $380 on 35 da.

9. Sold A. Williams the following bills of goods: July 10, $2300, on 6 mo. credit; Aug. 15, $900, on 5 mo.; and Oct. 13, $830, on 7 mo. What must be the date of note for the three amounts, bearing interest which will equitably settle the account?

662. Prob. II.—**To settle equitably an account containing both debit and credit items.**

Find the amount equitably due at the latest maturity of either the debit or credit side of the following account, and the equated time of paying the balance:

Dr. B. Whitney. *Cr.*

1876.			1876.		
Mar. 17	To mdse. . . .	$400	Apr. 13	By cash	$300
May 10	" " at 4 mo.	380	June 15	" draft at 30 da.	450
Aug. 7	" " at 2 mo.	540			

Before examining the following solution, *study carefully* the three propositions under (**658**).

SOLUTION BY PRODUCT METHOD.

Due.	*Amt.*	*Days.*	*Products.*	*Paid.*	*Amt.*	*Days.*	*Products.*
Mar. 17.	400 ×	204 =	81600	Apr. 13.	200 ×	177 =	35400
Sept. 10.	380 ×	27 =	10260	July 15.	250 ×	84 =	21000
Oct. 7.	540				450		56400
Total debt,	$1320		$91860	Amt. whose Int. for 1 da. is due to Creditor.			
Total paid,	450		56400	Amt: whose Int. for 1 da. is due to Debtor.			
Balance,	$870		$35460	Bal. whose Int. for 1 day is due to Creditor.			

EXPLANATION.—Assuming Oct. 7, the latest maturity on either side of the account, as the date of settlement, the creditor is entitled to interest on each item of the debit side, and the debtor on each item of the credit side to this date (**658**). Hence, we find, according to (**659**—3), the amount whose interest for 1 day both creditor and debtor are entitled to Oct. 7.

2. The creditor being entitled to the most interest, we subtract the amount whose interest for 1 day the debtor is entitled to from the creditor's amount, leaving $3540, the amount whose interest for 1 day the creditor is still entitled to receive.

3. We find the sum of the debit and credit items, and subtract the latter from the former, leaving $870 yet unpaid. This, with 68 cents, the interest on $3540, is the amount equitably due Oct. 7, equal $870.68.

4. According to (**658**—4), $\$3540 \div \$870 = 40\frac{60}{87}$, the number of days previous to Oct. 7 when the debt can be discharged by paying the balance, $870, in cash, or by a note bearing interest. Hence the equated time of paying the balance is Aug. 27.

The following points regarding the foregoing solution should be carefully studied:

1. In the given example, the sum of the debit is greater than the sum of the credit items; consequently the balance on the account is due to the creditor. But the balance of interest being also due him, it is evident that to settle the account equitably he should be paid the $870 before the assumed date of settlement. Hence the equated time of paying the balance must be before Oct. 7.

2. Had the balance of interest been on the credit side, it is evident the debtor would be entitled to keep the balance on the account until the

interest upon it would be equal the interest due him. Hence the equated time of paying the balance would be after Oct. 7.

3. Had the balance of the account been on the credit side, the creditor would be overpaid, and hence the balance would be due to the debtor.

Now in case the balance of interest is also on the credit side and due to the debtor, it is evident that to settle the account equitably the debtor should be paid the amount of the balance before the assumed date of settlement. Hence the equated time would be before Oct. 7.

In case the balance of interest is on the debtor side, it is evident that while the creditor has been overpaid on the account, he is entitled to a balance of interest, and consequently should keep the amount he has been overpaid until the interest upon it would be equal to the interest due him. Hence the equated time would be after Oct. 7.

4. The *interest method* given (**659**) can be used to advantage in finding the *equated time* when the time is long between the *maturity* of the items and the *assumed date of settlement*. In case this method of solution is adopted, the foregoing conditions are equally applicable.

From these illustrations we obtain the following

663. RULE.—*I. Find the maturity of each item on the debit and credit side of the account.*

II. Assume as the date of settlement the latest maturity on either side of the account, and find the number of days from this date to the maturity of each, on both sides of the account.

III. Multiply each debit and credit item by the number of days from its maturity to the date of settlement, and divide the balance of the debit and credit products by the balance of the debit and credit items; the quotient is the number of days the equated time is from the assumed date of settlement.

IV. In case the balance of items and balance of interest are both on the same side of the account, subtract this number of days from the assumed date of settlement, but add it in case they are on opposite sides; the result is the equated time.

EXAMPLES FOR PRACTICE.

664. 1. Find the face of a note and the date from which it must bear interest to settle equitably the following account:

Dr. JAMES HAND *in acct. with* P. ANSTEAD. *Cr.*

1876.			1876.		
Jan. 7	To mdse. on 3 mo.	$430	Mar. 15	By draft at 90 da.	$500
May 11	" " " 2 mo.	390	May 17	" cash	280
June 6	" " " 5 mo.	570	Aug. 9	" mdse. on 30 da.	400

2. Equate the following account, and find the cash payment Dec. 7, 1876:

Dr. WILLIAM HENDERSON. *Cr.*

1876.			1876.		
Mar. 23	To mdse. on 45 da.	$470	Apr. 16	By cash	$490
May 16	" " " 25 da.	380	June 25	" mdse. on 30 da.	650
Aug. 7	" " " 35 da.	590	July 13	" draft at 60 da.	200

3. Find the equated time of paying the balance on the following account:

Dr. HUGH MACVICAR. *Cr.*

1876.			1876.		
Jan. 13	To mdse. on 60 da.	$840	Feb. 15	By note at 60 da.	$700
Mar. 24	" " " 40 da.	580	Apr. 17	" cash	460
June 7	" " " 4 mo.	360	June 9	" draft at 30 da.	1150
July 14	" " " 80 da.	730			

4. I purchased of Wm. Rodgers, March 10, 1876, $930 worth of goods; June 23, $680; and paid, April 3, $870 cash, and gave a note May 24 on 30 days for $500. What must be the date of a note bearing interest that will equitably settle the balance?

REVIEW AND TEST QUESTIONS.

665. 1. Define Simple, Compound, and Annual Interest.

2. Illustrate by an example every step in the six per cent. method.

3. Show that 12% may be used as conveniently as 6%, and write a rule for finding the interest for months by this method.

4. Explain the method of finding the *exact interest* of any sum for any given time. Give reasons for each step in the process.

5. Show by an example the difference between *true* and *bank* discount. Give reasons for your answer.

6. Explain the method of finding the *present worth.*

7. Explain how the face of a note is found when the proceeds are given. Illustrate each step in the process.

8. Define Exchange, and state the difference between Domestic and Foreign Exchange.

9. State the difference in the three bills in a Set of Exchange.

10. What is meant by Par of Exchange?

11. State the various methods of Domestic Exchange, and illustrate each by an example.

12. Illustrate the method of finding the cost of a draft when exchange is at a discount and brokerage allowed. Give reasons for each step.

13. State the methods of Foreign Exchange.

14. Illustrate by an example the difference between *Direct* and *Indirect* exchange.

15. Define Equation of Payments, an Account, Equated Time, and Term of Credit.

16. Illustrate the Interest Method of finding the Equated Time when there are but debit items.

17. State when and why you count forward from the assumed date of settlement to find the equated time.

RATIO

PREPARATORY PROPOSITIONS.

666. *Two numbers are compared and their relation determined by dividing the first by the second.*

For example, the relation of \$8 to \$4 is determined thus, $\$8 \div \$4 = 2$. *Observe,* the quotient 2 indicates that for every *one dollar* in the \$4, there are *two dollars* in the \$8.

Be particular to observe the following:

1. When the greater of two numbers is compared with the less, the relation of the numbers is expressed either by the relation of an *integer* or of a *mixed number* to the *unit* 1, that is, by an improper fraction whose denominator is 1.

Thus, 20 compared with 4 gives $20 \div 4 = 5$; that is, for every 1 in the 4 there are 5 in the 20. Hence the relation of 20 to 4 is that of the integer 5 to the unit 1, expressed fractionally thus, $\frac{5}{1}$.

Again, 29 compared with 4 gives $29 \div 4 = 7\frac{1}{4}$; that is, for every 1 in the 4 there are $7\frac{1}{4}$ in 29. Hence, the relation of 29 to 4 is that of the mixed number $7\frac{1}{4}$ to the *unit* 1.

2. When the less of two numbers is compared with the greater, the relation is expressed by a proper fraction.

Thus, 6 compared with 14 gives $6 \div 14 = \frac{6}{14} = \frac{3}{7}$ (**255**); that is, for every 3 in the 6 there is a 7 in the 14. Hence, the relation of 6 to 14 is that of 3 to 7, expressed fractionally thus, $\frac{3}{7}$.

Observe, that the relation in this case may be expressed, if desired, as that of the *unit* 1 to a mixed number. Thus, $6 \div 14 = \frac{6}{14} = \frac{1}{2\frac{1}{3}}$ (**255**); that is, the relation of 6 to 14 is that of the unit 1 to $2\frac{1}{3}$.

EXAMPLES FOR PRACTICE.

667. Find orally the relation

1. Of 24 to 3.
2. Of 56 to 8.
3. Of 76 to 4.
4. Of 38 to 5.
5. Of 113 to 9.
6. Of 25 to 100.
7. Of 16 to 48.
8. Of 13 to 90.
9. Of 42 to 77.
10. Of 85 to 9.
11. Of 10 to 1000.
12. Of 75 to 300.

668. Prop. II.—*No numbers can be compared but those which are of the same denomination.*

Thus we can compare \$8 with \$2, and 7 inches with 2 inches, but we cannot compare \$8 with 2 inches (**155**—I).

Observe carefully the following:

1. Denominate numbers must be reduced to the lowest denomination named, before they can be compared.

For example, to compare 1 yd. 2 ft. with 1 ft. 3 in., both numbers must be reduced to inches. Thus, 1 yd. 2 ft. = 60 in., 1 ft. 3 in. = 15 in., and 60 in. ÷ 15 in. = 4; hence, 1 yd. 2 ft. are 4 times 1 ft. 3 in.

2. Fractions must be reduced to the same *fractional* denomination before they can be compared.

For example, to compare $3\frac{1}{2}$ lb. with $\frac{4}{5}$ oz. we must first reduce the $3\frac{1}{2}$ lb. to oz., then reduce both members to the same fractional unit. Thus, (1) $3\frac{1}{2}$ lb. = 56 oz.; (2) 56 oz. = $\frac{180}{5}$ oz.; (3) $\frac{180}{5}$ oz. ÷ $\frac{4}{5}$ oz. = $\frac{180}{4}$ = 45 (**290**); hence, the relation $3\frac{1}{2}$ lb. to $\frac{4}{5}$ oz. is that of 45 to 1.

EXAMPLES FOR PRACTICE.

669. Find orally the relation

1. Of 4 yd. to 2 ft.
2. Of \$2 to 25 ct.
3. Of $2\frac{1}{4}$ gal. to $\frac{3}{4}$ qt.
4. Of $\frac{3}{5}$ to $\frac{2}{3}$.
5. Of $\frac{5}{6}$ to $1\frac{1}{9}$.
6. Of $\frac{2}{3}$ oz. to 2 lb.
7. Of $1\frac{1}{2}$ pk. to 3 bu.
8. Of 3 cd. to 6 cd. ft.
9. Of $1\frac{2}{3}$ to $2\frac{1}{4}$.

Find the relation

10. Of 105 to 28.
11. Of $6\frac{3}{8}$ to $\frac{5}{6}$.
12. Of $9\frac{4}{5}$ bu. to $3\frac{1}{2}$ pk.
13. Of \$$36\frac{4}{7}$ to \$$4\frac{2}{5}$.
14. Of 2 yd. $1\frac{2}{3}$ ft. to $\frac{3}{4}$ in.
15. Of $1\frac{1}{2}$ pt. to $2\frac{2}{3}$ gal.

(290)

DEFINITIONS.

670. A ***Ratio*** is a fraction which expresses the relation which the first of two numbers of the same denomination has to the second.

Thus the *relation* of \$6 to \$15 is expressed by $\frac{6}{15}$; that is, \$6 is $\frac{6}{15}$ of \$15, or for every \$2 in \$6 there are \$5 in \$15. In like manner the relation of \$12 to \$10 is expressed by $\frac{12}{10}$.

671. The ***Special Sign*** of ***Ratio*** is a colon (:).

Thus 4:7 denotes that 4 and 7 express the ratio $\frac{4}{7}$; hence, 4:7 and $\frac{4}{7}$ are two ways of expressing the same thing. The fractional form being the more convenient, should be used in preference to the form with the colon.

672. The ***Terms*** of a ***Ratio*** are the numerator and denominator of the fraction that expresses the relation between the quantities compared.

The *first term* or numerator is called the ***Antecedent***, the *second term* or denominator is called the ***Consequent***.

673. A ***Simple Ratio*** is a ratio in which each term is a single integer. Thus 9:3, or $\frac{9}{3}$, is a simple ratio.

674. A ***Compound Ratio*** is a ratio whose terms are formed by multiplying together the corresponding terms of two or more simple ratios.

Thus, multiplying together the corresponding terms of the simple ratios 7 : 3 and 5 : 2, we have the compound ratio $5 \times 7 : 3 \times 2 = 35 : 6$, or expressed fractionally $\frac{7}{3} \times \frac{5}{2} = \frac{7 \times 5}{3 \times 2} = \frac{35}{6}$.

Observe, that when the multiplication of the corresponding terms is performed, the *compound ratio* is reduced to a *simple ratio.*

675. The ***Reciprocal*** of a number is 1 divided by that number. Thus, the reciprocal of 8 is $1 \div 8 = \frac{1}{8}$.

676. The ***Reciprocal*** of a ***Ratio*** is 1 divided by the ratio.

Thus, the ratio of 7 to 4 is 7 : 4 or $\frac{7}{4}$, and its reciprocal is $1 \div \frac{7}{4} = \frac{4}{7}$, according to (**291**). Hence the reciprocal of a ratio is the ratio inverted, or the *consequent* divided by the *antecedent*.

677. A ***Ratio*** is in its ***Simplest Terms*** when the antecedent and consequent are prime to each other.

678. The ***Reduction*** of a ***Ratio*** is the process of changing its terms without changing the relation they express.

Thus $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, each express the same relation.

PROBLEMS ON RATIO.

679. Since every ratio is either a proper or improper fraction, the principles of reduction discussed in (**235**) apply to the reduction of ratios. The wording of the principles must be slightly modified thus:

PRIN. I.—*The terms of a ratio must each represent units of the same kind.*

PRIN. II.—*Multiplying both terms of a ratio by the same number does not change the value of the ratio.*

PRIN. III.—*Dividing both terms of a ratio by the same number does not change the value of the ratio.*

For the illustration of these principles refer to (**235**).

680. PROB. I.—**To find the ratio between two given numbers.**

Ex. 1. Find the ratio of \$56 to \$84.

SOLUTION.—Since, according to (**666**), two numbers are compared by dividing the first by the second, we divide \$56 by \$84, giving $\$56 \div \$84 = \frac{56}{84}$; that is, \$56 is $\frac{56}{84}$ of \$84. Hence the ratio of \$56 to \$84 is $\frac{56}{84}$.

Ex. 2. Find the ratio of 1 yd. 2 ft. to 1 ft. 3 in.

SOLUTION.—1. Since, according to (**668**), only numbers of the same denomination can be compared, we reduce both terms to inches, giving 60 in. and 15 in.

2. Dividing 60 in. by 15 in. we have 60 in. ÷ 15 in. = 4; that is, 60 in. is 4 times 15 in. Hence the ratio of 1 yd. 2 ft. to 1 ft. 3 in. is $\frac{4}{1}$.

EXAMPLES FOR PRACTICE.

681. Find the ratio

1. Of $512 to $256.
2. Of 143 yd. to 365 yd.
3. Of 982 da. to 2946 da.
4. Of 73 A. to 365 A.
5. Of £41 5s. 6d. to £2 3s. 6d.
6. Of 20 T. 6 cwt. 93 lb. to 25 cwt. 43 lb. 5 oz.

682. PROB. II.—**To reduce a ratio to its simplest terms.**

Reduce the ratio $\frac{15}{9}$ to its simplest terms.

SOLUTION.—Since, according to (**679**—III), the value of the ratio $\frac{15}{9}$ is not changed by dividing both terms by the same number, we divide the antecedent 15 and the consequent 9 by 3, their greatest common divisor, giving $\frac{15 \div 3}{9 \div 3} = \frac{5}{3}$. But having divided 15 and 9 by their greatest common divisor, the quotients 5 and 3 must be prime to each other. Hence (**677**) $\frac{5}{3}$ are the simplest terms of the ratio $\frac{15}{9}$.

EXAMPLES FOR PRACTICE.

683. Reduce to its simplest terms

1. The ratio 6 : 9.
2. The ratio 21 : 56.
3. The ratio $\frac{45}{63}$.
4. The ratio $\frac{165}{195}$.
5. The ratio 65 : 85.
6. The ratio 195 : 39.

Express in its simplest terms the ratio (see **668**)

7. Of 96 T. to 56 T.
8. Of $\frac{2}{3}$ ft. to 2 yd.
9. Of 8s. 9d. to £1.
10. Of 3 pk. 5 qt. to 1 bu. 2 pk.

684. PROB. III.—**To find a number that has a given ratio to a given number.**

How many dollars are $\frac{5}{8}$ of $72 ?

SOLUTION.—The fraction $\frac{5}{8}$ denotes the ratio of the required number to $72; namely, for every $8 in $72 there are $5 in the required number. Consequently we divide the $72 by $8, and multiply $5 by the quotient. Hence, *first step*, $72 ÷ $8 = 9; *second step* $5 × 9 = $45, the required number.

Observe, that this problem is the same as PROB. VIII, **501**, and PROB. II, **274**. Compare this solution with the solution in each of these problems.

EXAMPLES FOR PRACTICE.

685. Solve and explain each of the following examples, regarding the fraction in every case as a ratio.

1. How many days are $\frac{5}{12}$ of 360 days?

2. A man owning a farm of 243 acres, sold $\frac{5}{27}$ of it; how many acres did he sell?

3. James has $796 and John has $\frac{3}{4}$ as much; how much has John?

4. A man's capital is $4500, and he gains $\frac{7}{15}$ of his capital; how much does he gain?

5. Mr. Jones has a quantity of flour worth $3140; part of it being damaged he sells the whole for $\frac{18}{25}$ of its value; how much does he receive for it?

686. PROB. IV.—**To find a number to which a given number has a given ratio.**

$42 are $\frac{7}{4}$ of how many dollars?

SOLUTION.—The fraction $\frac{7}{4}$ denotes the ratio of $42 to the required number; namely, for every $7 in $42 there are $4 in the required number. Consequently we divide the $42 by $7 and multiply $4 by the quotient. Hence, *first step*, $42 ÷ $7 = 6; *second step*, $4 × 6 = $24, the required number.

Observe, that this problem is the same as PROB. IX, **502**. Compare the solutions and notice the points of difference.

EXAMPLES FOR PRACTICE.

687. Solve and explain each of the following examples, regarding the fraction in every case as a ratio.

1. 96 acres are $\frac{12}{13}$ of how many acres?
2. I received $75, which is $\frac{3}{5}$ of my wages; how much is still due?
3. James attended school 117 days, or $\frac{9}{10}$ of the term; how many days in the term?
4. Sold my house for $2150, which was $\frac{19}{25}$ of what I paid for it; how much did I lose?
5. Henry reviewed 249 lines of Latin, or $\frac{3}{5}$ of the term's work; how many lines did he read during the term?
6. 48 cd. 3 cd. ft. of wood is $\frac{3}{11}$ of what I bought; how much did I buy?
7. Mr. Smith's expenses are $\frac{3}{4}$ of his income. He spends $1500 per year; what is his income?
8. 4 gal. 3 qt. 1 pt. are $\frac{3}{11}$ of how many gallons?
9. A merchant sells a piece of cloth at a profit of $2.50, which is $\frac{5}{27}$ of what it cost him; how much did he pay for it?

688. PROB. V.—**To find a number to which a given number has the same ratio that two other given numbers have to each other.**

To how many dollars have $18 the same ratio that 6 yd. have to 15 yd.?

SOLUTION.—1. We find by (**680**—I) the ratio of 6 yd. to 15 yd., which is $\frac{6}{15} = \frac{2}{5}$, according to (**677**).

2. Since $\frac{2}{5}$ denotes the ratio of the $18 to the required number, the $18 must be the antecedent; hence we have, according to (**686**), *first step*, $18 ÷ $2 = 9; *second step*, $5 × 9 = $45, the required number.

Observe, that in this problem we have the antecedent of a ratio given to find the consequent. In the following we have the consequent given to find the antecedent.

689. PROB. VI.—**To find a number that has the same ratio to a given number that two other given numbers have to each other.**

How many acres have the same ratio to 12 acres that $56 have to $84 ?

SOLUTION.—1. We find by (**680**—I) the ratio of $56 to $84, which is $\frac{56}{84} = \frac{2}{3}$, according to (**677**).

2. Since $\frac{2}{3}$ denotes the ratio of the required number to 12 acres, the 12 acres must be the consequent ; hence we have, according to (**684**), *first step,* 12 acr. ÷ 3 acr. = 4 ; *second step,* 2 acr. × 4 = 8 acres, the required number.

EXAMPLES FOR PRACTICE.

690. The following are applications of PROB. V and VI.

1. If 12 bu. of wheat cost $15, what will 42 bu. cost?

Regarding the solution of examples of this kind, observe that *the price or rate per unit is assumed to be the same for each of the quantities given.*

Thus, since the 12 bu. cost $15. the price per bushel or unit is $1.25, and the example asks for the cost of 42 bu. at this price per bushel. Consequently whatever part the 12 bu. are of 42 bu., the $15. the cost of 12 bu., must be the same part of the cost of 42 bu. Hence we find the ratio of 12 bu. to 42 bu. and solve the example by PROB. V.

2. What will 16 cords of wood cost, if 2 cords cost $9 ?

3. If a man earn $18 in 2 weeks, how much will he earn in 52 weeks ?

4. If 24 bu. of wheat cost $18, what will 36 bu. cost ?

5. If 24 cords of wood cost $60, what will 40 cords cost ?

6. Bought 170 pounds of butter for $51; what would 680 pounds cost, at the same price ?

7. Two numbers are to each other as 10 to 15, and the less number is 329; what is the greater?

8. At the rate of 16 yards for $7, how many yards of cloth can be bought for $100 ?

PROPORTION

DEFINITIONS.

691. A ***Proportion*** is an equality of ratios, the terms of the ratios being expressed.

Thus the ratio $\frac{3}{5}$ is equal to the ratio $\frac{12}{20}$; hence $\frac{3}{5} = \frac{12}{20}$ is a proportion, and is read, The ratio of 3 to 5 is equal to the ratio of 12 to 20, or 3 is to 5 as 12 is to 20.

692. The equality of two ratios constituting a proportion is indicated either by a double colon (::) or by the sign (=).

Thus, $\frac{3}{4} = \frac{9}{12}$, or $3 : 4 = 9 : 12$, or $3 : 4 :: 9 : 12$.

693. A ***Simple Proportion*** is an expression of the equality of two simple ratios.

Thus, $\frac{8}{12} = \frac{32}{48}$, or $8 : 12 :: 3 : 48$, or $8 : 12 = 3 : 8$ is a simple proportion. Hence a simple proportion contains four terms.

694. A ***Compound Proportion*** is an expression of the equality of a *compound* (**674**) and a *simple* ratio (**673**).

Thus, $\left.\begin{matrix} 2 : 3 \\ 6 : 5 \end{matrix}\right\} :: 48 : 60$, or $\frac{2}{3} \times \frac{6}{5} = \frac{48}{60}$, is a compound proportion. It is read, The ratio of 2 into 6 is to 3 into 5 as 48 is to 60.

695. A ***Proportional*** is a number used as a term in a proportion.

Thus in the simple proportion $2 : 5 :: 6 : 15$ the numbers 2, 5, 6, and 15 are its terms; hence, each one of these numbers is called a proportional, and the four numbers together are called proportionals.

696. A ***Mean Proportional*** is a number that is the *Consequent* of one and the *Antecedent* of the other of the two ratios forming a proportion.

Thus in the proportion 4 : 8 : : 8 : 16, the number 8 is the consequent of the first ratio and the antecedent of the second; hence is a *mean proportional.*

697. The ***Antecedents*** of a proportion are the *first* and *third* terms, and the ***Consequents*** are the *second* and *fourth* terms.

698. The ***Extremes*** of a proportion are its *first* and *fourth* terms, and the ***Means*** are its *second* and *third* terms.

SIMPLE PROPORTION.

PREPARATORY STEPS.

699. The following preparatory steps should be perfectly mastered before applying proportion in the solution of problems. The solution of each example under STEP I should be given in full, as shown in (**688** and **689**), and STEP II and III should be illustrated by the pupil, in the manner shown, by a number of examples.

700. STEP I.—*Find by* PROB. V *and* VI, *in ratio, the missing term in the following proportions:*

The required term is represented by the letter x.

1. 6 : 42 :: 5 : x.
2. 24 : 60 :: x : 15.
3. 84 : x :: 21 : 68.
4. 5 bu. 2 pk. : 3 pk. :: x : 4 bu.
5. 2 yd. : 8 in. :: x : 3 ft. 4 in.
6. x : £3 2s. :: 49 T. : 18 cwt.

STEP II.—*Show that the product of the extremes of a proportion is equal to the product of the means.*

Thus the proportion 2 : 3 : : 6 : 9 expressed fractionally gives $\frac{2}{3} = \frac{6}{9}$.

Now if both terms of this equality be multiplied by 3 and by 9, the consequents of the given ratios, the equality is not changed; hence, $\frac{2 \times 9 \times 3}{3} = \frac{6 \times 3 \times 9}{9}$. Cancelling (**186**) the factor 3 in the left-hand term and 9 in the right-hand term we have $2 \times 9 = 6 \times 3$. But 2 and 9 are the extremes of the proportion and 6 and 3 are the means; hence the truth of the proposition.

STEP III.—*Show that, since the product of the extremes is equal to the product of the means, any term of a proportion can be found when the other three are known.*

Thus in the proportion $3 : x :: 9 : 15$ we have known the two extremes 3 and 15 and the mean 9. But by STEP II, 3×15, or 45, is equal to 9 times the required mean; hence $45 \div 9 = 5$, the required mean. In the same manner any one of the terms may be found; hence the truth of the proposition.

Find by this method the missing term in the following:

1. $14 : 3 :: x : 12$.
2. $x : 24 :: 7 : 8$.
3. $27 : x :: 9 : 5$.
4. \$13 : x :: 5 yd. : 3 yd.
5. 128 bu. : 3 pk. :: x : \$1.25.
6. 64 cwt. : x :: \$120 : \$15.

Solution by Simple Proportion.

701. The quantities considered in problems that occur in practical business are so related that when certain conditions are assumed as invariable, they form ratios that must be equal to each other, and hence can be stated as a proportion thus,

If 4 yd. of cloth cost \$10, what will 18 yd. cost?

Observe, that in this example the price per yard is assumed to be *invariable,* that is, the price is the same in both cases; consequently whatever part the 4 yd. are of the 18 yd., the \$10 are the same part of the cost of the 18 yd., hence the ratio of the 4 yd. to the 18 yd. is equal the ratio of the \$10 to the required cost, giving the proportion 4 yd. : 18 yd. :: \$10 : \$$x$.

EXAMPLES FOR PRACTICE.

702. Examine carefully the following proportions and state what must be considered in each case as *invariable,* and why, in order that the proportion may be correct.

1.	The number of units bought in one case	*is to*	The number of units bought in another case	*as*	The cost in the first case	*is to*	The cost in the second case.
2.	The Principal in one case	:	The Principal in another case	: :	The interest in the first case	:	The interest in the second case.
3.	The number of men that can do a piece of work in one case	:	The number of men that can do the same work in another case	: :	The number of days the second work	:	The number of days the first work.

Why is the second ratio of this proportion made the ratio of the number of days the second work to the number of days the first work? Illustrate this arrangement of the terms of the ratio by other examples.

In solving examples by simple proportion, the following course should be pursued:

I. Represent the required term by x, and make it the last extreme or consequent of the second ratio in the proportion.

II. Find the term in the example that is of the same denomination as the required term, and make it the second mean or the antecedent of the second ratio of the proportion.

III. Determine, by inspecting carefully the conditions given in the example, whether x, the required term of the ratio now expressed, must be greater or less than the given term.

IV. If x, the required term of the ratio expressed, must be greater than the given term, make the greater of the remaining terms in the example the consequent of the first ratio of the proportion; if less, make it the antecedent.

V. When the proportion is stated, find the required term either as shown in **(688)** *or in* **(689)**.

Observe, that in either way of finding the required term, any factor that is common to the given extreme and either of the given means should be cancelled, as shown in **(186)**.

4. How many bushels of wheat would be required to make 39 barrels of flour, if 15 bushels will make 3 barrels?

5. If 77 pounds of sugar cost \$8.25, what will 84 pounds cost?

6. I raised 245 bushels of corn on 7 acres of land; how many bushels grow on 2 acres?

7. If 6 men put up 73 feet of fence in 3 days, how many feet will they put up in 33 days?

8. What will 168 pounds of salt cost, if 3½ pounds cost 37½ cents?

9. If 25 cwt. of iron cost \$84.50, what will 24¾ cwt. cost?

10. Paid \$2225 for 18 cows, and sold them for \$2675; what should I gain on 120 cows at the same rate?

11. If 5 lb. 10 oz. of tea cost \$5.25; what will 7 lb. 8 oz. cost?

12. If a piece of cloth containing 18 yards is worth \$10.80, what are 4 yards of it worth?

13. My horse can travel 2 mi. 107 rd. in 20 minutes; how far can he travel in 2 hr. 20 min.

14. If 18 gal. 3 qt. 1 pt. of water leaks out of a cistern in 4 hours, how much will leak out in 36 hours?

15. Bought 28 yards of cloth for \$20; what price per yard would give me a gain of \$7.50 on the whole?

16. If I lend a man \$69.60 for 8½ months, how long should he lend me \$17.40 to counterbalance it?

17. My annual income on U. S. 6%'s is \$337.50 when gold is at 112½; what would it be if gold were at 125?

COMPOUND PROPORTION.

PREPARATORY STEPS.

703. Step I.—*A compound ratio is reduced to a simple one by multiplying the antecedents together for an antecedent and the consequents for a consequent* (**674**).

Thus the compound ratio $\left\{\begin{array}{c} 6 : 7 \\ 4 : 3 \end{array}\right\}$ is reduced to a simple ratio by multiplying the antecedents 6 and 4 together, and the consequents 7 and 3. Expressing the ratios fractionally we have $\frac{6}{7} \times \frac{4}{3} = \frac{24}{21} = \frac{8}{7}$ (**682**).

Observe, that any factor that is common to any antecedent and consequent may be cancelled before the terms are multiplied.

Reduce the following compound ratios to simple ratios in their *simplest terms.*

1. $\left\{\begin{array}{r} 9 : 25 \\ 15 : 18 \\ 28 : 50 \\ 3 : 7 \end{array}\right\}$ 2. $\left\{\begin{array}{c} \frac{8}{35} \\ \frac{5}{16} \end{array}\right\}$ 3. $\left\{\begin{array}{c} 16 : 9 \\ 27 : 15 \\ \frac{28}{8} \end{array}\right\}$

Step II.—*A compound proportion is reduced to a simple proportion by reducing the compound ratio to a simple ratio.*

Thus, in the compound proportion $\left\{\begin{array}{c} 8 : 9 \\ 6 : 4 \end{array}\right\}$ :: 24 : 18, the compound ratio $\frac{8}{9} \times \frac{6}{4}$ is equal the simple ratio $\frac{4}{3}$; substituting this in the proportion for the compound ratio we have the simple proportion 4 : 3 :: 24 : 18.

Observe, that when a compound proportion is reduced to a simple proportion, the missing term is found according to (**688**), or (**689**).

Find the missing term in the following:

1. $\left\{\begin{array}{r} 24 : 15 \\ 7 : 16 \\ 25 : 21 \end{array}\right\} :: 40 : x.$ 2. $\left\{\begin{array}{c} 48 : 20 \\ \frac{3}{6} \end{array}\right\} :: 28 : x.$

Solution by Compound Proportion.

704. The following preparatory propositions should be carefully studied and the course indicated observed in solving problems involving compound proportion.

PROP. I.—*There are one or more conditions in every example involving proportion, which must be regarded as* INVARIABLE *in order that a solution may be given,* thus

If 9 horses can subsist on 50 bu. of oats for 20 days, how long can 6 horses subsist on 70 bu.

In this example there are two *conditions* that must be considered as *invariable* in order to give a solution :

1. The fact that each horse subsists on the same quantity of oats each day.

2. The fact that each bushel of oats contains the same amount of food.

PROP. II.—*To solve a problem involving a compound proportion, the effect of each ratio, which forms the compound ratio, on the required term must be considered separately, thus:*

If 5 men can build 40 yards of a fence in 12 days, how many yards can 8 men build in 9 days.

1. We observe that the *invariable conditions* in this example are

(1.) That each man in both cases does the *same amount of work* in the *same time*.

(2.) That the *same amount of work* is required in each case *to build one yard* of the fence.

2. We determine by examining the problem how the required term is affected by the relations of the given term, thus:

(1.) We observe that the 5 men in 12 days can build 40 yards. Now since each man can build the same extent of the fence in one day, it is evident that if the 8 men work 12 days the same as the 5 men, the 40 yards built by the 5 men in 12 days must have the same ratio to the number of yards that can be built by the 8 men in 12 days as 5 men have to 8 men; hence the proportion

$$5 \text{ men} : 8 \text{ men} :: 40 \text{ yards} : x \text{ yards}.$$

This proportion will give the number of yards the 8 men can build in 12 days.

(2.) We now observe that the 8 men work only 9 days; and since they can do the same amount of work each day, the work done in 12 days must have the same ratio to the work they can do in 9 days that 12 days have to 9 days. Hence we have the compound proportion

$$\left.\begin{matrix} 5 \text{ men} : 8 \text{ men} \\ 12 \text{ days} : 9 \text{ days} \end{matrix}\right\} :: 40 \text{ yards} : x \text{ yards}.$$

We find from this proportion, according to (**703**—II), that the 8 men can build 48 yards of fence in 9 days.

EXAMPLES FOR PRACTICE.

705. 1. If it cost $88 to hire 12 horses for 5 days, what will it cost to hire 10 horses for 18 days?

2. If 12 men can saw 45 cords of wood in 3 days, working 9 hours a day, how much can 4 men saw in 18 days, working 12 hours a day?

3. If 28 horses consume 240 bushels of corn in 112 days, how many bushels will 12 horses consume in 196 days?

4. When the charge for carrying 20 centals of grain 50 miles is $4.50, what is the charge for carrying 40 centals 100 miles?

5. The average cost of keeping 25 soldiers 1 year is $3000; what would it cost to keep 139 soldiers 7 years?

6. If 1 pound of thread makes 3 yards of linen, $1\frac{1}{4}$ yard wide, how many pounds would make 45 yards of linen, 1 yard wide?

7. 64 men dig a ditch 72 feet long, 4 feet wide, and 2 feet deep, in 8 days; how long a ditch, $2\frac{1}{2}$ feet wide and $1\frac{1}{2}$ feet deep, can 96 men dig in 60 days?

8. If it requires 8400 yd. of cloth $1\frac{1}{4}$ yd. wide to clothe 3500 soldiers, how many yards $\frac{7}{8}$ wide will clothe 6720?

PARTNERSHIP

DEFINITIONS.

706. A ***Partnership*** is an *association* of two or more persons for the transaction of business.

The persons associated are called *partners,* and the *Association* is called a *Company, Firm,* or *House.*

707. The ***Capital*** is the money or other property invested in the business.

The *Capital* is also called the *Investment* or *Joint-stock* of the Company.

708. The ***Assets*** or ***Effects*** of a Company are the property of all kinds belonging to it, together with all the amounts due to it.

709. The ***Liabilities*** of a company are its debts.

PREPARATORY PROPOSITIONS.

710. PROP. I.—*The* PROFITS *and the* LOSSES *of a company are divided among the partners, according to the value of each man's investment at the time the division is made.*

Observe carefully the following regarding this proposition:

Since the ***use*** of money or property is itself ***value***, it is evident that the value of an investment at any time after it is made, depends *first* upon the amount invested, *second* on the length of the time the investment has been made, and *third* the rate of interest.

Thus the *value* of an investment of $500 *at the time it is made* is just $500; but *at the end of* 9 *years*, reckoning its *use* to be worth 7% per annum, its *value* will be $500 + $315 = $815.

PROP. II.—*The value of any investment made for a given number of intervals of time, can be represented by another investment made for one interval of time.*

Thus, for example, the value of an investment of $40 for 5 months at any given rate of interest is the same as the value of 5 times $40, or $200, for one month.

EXAMPLES FOR PRACTICE.

711. Find the *value* at simple interest

1. Of $800 invested 4 years at 6% per annum.
2. Of $350 invested 2 yr. 3 mo. at 7% per annum.
3. Of $2860 invested 19 months at 8% per annum.

Solve the following by applying (**710**—II).

4. An investment of $200 for 6 months is equal in value to what investment for 4 months?

5. A man invests $600 for 9 months, $700 for 3 months, and $300 for 7 months, each at the same rate of interest. What sum can he invest for 4 months at the given rate of interest, to be equal in value to the three investments?

ILLUSTRATION OF PROCESS.

712. PROB. I.—**To apportion gains or losses when each partner's capital is invested the same length of time.**

Observe, that when each partner's capital is used for the *same length of time*, it is evident that his share of the gain or loss must be the same fraction of the whole gain or loss that his capital is of the whole capital. Hence, examples under this problem may be solved—

I. By Proportion thus:

The whole capital invested : *Each man's capital invested* :: *Whole gain or loss* : *Each man's gain or loss.*

II. By Percentage thus:

Find what per cent (**504**) *the whole gain or loss is of the whole capital invested, and take the same per cent of each man's investment as his share of the gain or loss.*

III. By Fractions thus:

Find what fractional part each man's investment is of the whole capital invested, and take the same fractional part of the gain or loss as each man's share of the gain or loss.

EXAMPLES FOR PRACTICE.

713. 1. Three men, A, B, and C, form a company; A puts in \$6000; B \$4000; and C \$5600; they gain \$4320; what is each man's share?

2. A man failing in business owes A \$9600, B \$7000, and C \$5400, and his available property amounts to \$5460; what is each man's share of the property?

3. Three men agree to liquidate a church debt of \$7890, each paying in proportion to his property; A's property is valued at \$6470, B's at \$3780, and C's at \$7890; what portion of the debt does each man pay?

4. A building worth \$28500 is insured in the Ætna for \$3200, in the Home for \$4200, and in the Mutual for \$6500; it having been partially destroyed, the damage is set at \$10500; what should each company pay?

5. The sum of \$2600 is to be divided among four school districts in proportion to the number of scholars in each; in the first there are 108, in the second 84, in the third 72, in the fourth 48; what part should each receive?

714. PROB. II.—**To apportion gains or losses when each partner's capital is invested different lengths of time.**

Observe carefully the following:

1. According to (**710**—II) we can find for each partner an amount whose value invested one interval of time is equal to the value of his capital for the given intervals of time.

2. Having found this we can, by adding these amounts, find an amount whose value invested one interval of time is equal to the total value of the whole capital invested.

When this is done it is evident that each man's share of the gain or loss must be the same fraction of the whole gain or loss that the *value* of his investment is of the total value of the whole capital invested. Hence the problem from this point can be solved by either of the three methods given under PROB. I (**712**).

EXAMPLES FOR PRACTICE.

715. 1. A and B engage in business; A puts in $1120 for 5 months and B $480 for 8 months; they gain $354; what is each man's share of the gain?

2. Three men hire a pasture for $136.50; A puts in 16 cows for 8 weeks, B puts in 6 cows for 12 weeks, and C the same number for 8 weeks; what should each man pay?

3. The joint capital of a company was $7800, which was doubled at the end of the year. A put in $\frac{1}{3}$ for 9 mo., B $\frac{1}{4}$ for 8 mo., and C the remainder for 1 year. What is each one's stock at the end of the year?

4. Jan. 1, 1875, three persons began business. A put in $1200, B put in $500 and May 1 $800 more, C put in $700 and July 1 $400 more; at the end of the year the profits were $875; how shall it be divided?

5. A and B formed a partnership Jan. 1, 1876. A put in $6000 and at the end of 3 mo. $900 more, and at the end of 10 mo. drew out $300; B put in $9000 and 8 mo. after $1500 more, and drew out $500 Dec. 1; at the end of the year the net profits were $8900. Find the share of each.

ALLIGATION

ALLIGATION MEDIAL.

716. ***Alligation*** Medial is the process of finding the *mean* or *average* price or quality of a mixture composed of several ingredients of different prices or qualities.

EXAMPLES FOR PRACTICE.

717. 1. A grocer mixed 7 lb. of coffee worth 30 ct. a pound with 4 lb. @ 25 ct. and 10 lb. @ 32; in order that he may neither gain or lose, at what price must he sell the mixture?

7 lb.	@ 30 ct.	=	$2.10
4 lb.	@ 25 ct.	=	1.00
10 lb.	@ 32 ct.	=	3.20
21 lb.		=	$6.30

$6.30 ÷ 21 = 30 ct.

SOLUTION.—1. Since the value of each kind of coffee is not changed by mixing, we find the value of the entire mixture by finding the value of each kind at the given price, and taking the sum of these values as shown in illustration.

2. Having found that the 21 lb. of coffee are worth at the given prices $6.30, it is evident that to realize this amount from the sale of the 21 lb. at a uniform price per pound, he must get for each pound $\frac{1}{21}$ of $6.30; hence, $6.30 ÷ 21 = 30 cents, the selling price of the mixture.

2. A wine merchant mixes 2 gallons of wine worth $1.20 a gallon with 4 gallons worth $1.40 a gallon, 4 gallons worth $.90 and 8 gallons worth $.80 a gallon; what is the mixture worth per gallon?

3. A grocer mixes 48 lb. of sugar at 17 ct. a pound with 58 lb. at 13 ct. and 94 lb. at 11 ct.; what is a pound of the mixture worth?

4. A goldsmith melts together 6 ounces of gold 22 carats fine, 30 ounces 20 carats fine, and 12 ounces 14 carats fine; how many carats fine is the mixture?

5. A merchant purchased 60 gallons of molasses at 30 ct. per gallon and 40 gallons at 25 cents, which he mixed with 8 gallons of water. He sold the entire mixture so as to gain 20 per cent on the original cost; what was his selling price per gallon?

ALLIGATION ALTERNATE.

718. ***Alligation Alternate*** is the process of finding the proportional quantities of ingredients of different prices or qualities that must be used to form any required mixture, when the price or quality of the mixture is given.

PREPARATORY PROPOSITIONS.

719. Prop. I.—*In forming any mixture, it is assumed that the value of the entire mixture must be equal to the aggregate value of its ingredients at their given prices.*

Thus, if 10 pounds of tea at 45 ct. and 5 pounds at 60 ct. be mixed, the value of the mixture must be the value of the 10 pounds plus the value of the 5 pounds at the given prices, which is equal $4.50 + $3.00 = $7.50. Hence there is neither gain or loss in forming a mixture.

Prop. II.—*The price of a mixture must be less than the highest and greater than the lowest price of any ingredient used in forming the mixture.*

Thus, if sugar at 10 ct. and at 15 ct. per pound be mixed, it is evident the price of the mixture must be less than 15 cents and greater than 10 cents; that is, it must be some price between 10 and 15 cents.

ILLUSTRATION OF PROCESS.

720. If tea at 56 ct., 60 ct., 75 ct., and 90 ct. per pound be mixed and sold at 66 ct. per pound, how much of each kind of tea can be put in the mixture?

First Step in Solution.

We find the gain or loss on one unit of each ingredient thus:

(1.) $\begin{cases} 66 \text{ ct.} - 56 \text{ ct.} = 10 \text{ ct. gain.} \\ 66 \text{ ct.} - 60 \text{ ct.} = 6 \text{ ct. gain.} \end{cases}$

(2.) $\begin{cases} 75 \text{ ct.} - 66 \text{ ct.} = 9 \text{ ct. loss.} \\ 90 \text{ ct.} - 66 \text{ ct.} = 24 \text{ ct. loss.} \end{cases}$

Second Step in Solution.

We now take an ingredient on which there is a gain, and one on which there is a loss, and ascertain how much of each must be put in the mixture to make the gain and loss equal; thus:

	Producing Gain.		Gained and Lost.		Producing Loss.
(1.)	9 lb. at 10 ct. per lb. gain.	=	90 ct.	=	10 lb. at 9 ct. per lb. loss.
(2.)	4 lb. at 24 ct. per lb. gain.	=	24 ct.	=	1 lb. at 24 ct. per lb. loss.

Hence the mixture must contain 9 lb. at 56 cts. per pound, 10 lb. at 75 ct. per pound, 4 lb. at 60 ct. per pound, and 1 lb. at 9 ct. per pound.

721. *Observe* carefully the following:

1. The gain and loss on any two ingredients may be balanced by assuming any amount as the sum gained and lost.

Thus, instead of taking 90 cents, as in (1) in the above solution, as the amount gained and lost, we might take 360 cents; and dividing 360 cents by 10 cents would give 36, the number of pounds of 56 ct. tea that would gain this sum. Again, dividing 360 cents by 9 cents would give 40, the number of pounds of 75 ct. tea that would lose this sum.

2. To obtain *integral* proportional parts the amount assumed must be a *multiple* of the gain and loss on one unit of the ingredients balanced, and to obtain the *least integral* proportional parts it must be the least common multiple.

3. When a number of ingredients are given on which there is a gain and also on which there is a loss, they may be balanced with each other in several ways; hence a series of different mixtures may be formed as follows:

Taking the foregoing example we have

A Second Mixture thus:

	Producing Gain.		Gained and Lost.		Producing Loss.
(1.)	24 lb. at 10 ct. per lb. gain.	=	240 ct.	=	10 lb. at 24 ct. per lb. loss.
(2.)	9 lb. at 6 ct. per lb. gain.	=	54 ct.	=	6 lb. at 9 ct. per lb. loss.

Hence the mixture is composed of 24 lb. @ 56 ct., 9 lb. @ 60 ct., 10 lb. @ 90 ct., and 6 lb. @ 75 ct.

A Third Mixture thus:

	Producing Gain.		Gained and Lost.		Producing Loss.
(1.)	9 lb. at 10 ct. per lb. gain.	=	90 ct.	=	10 lb. at 9 ct. per lb. loss.
(2.)	24 lb. at 10 ct. per lb. gain.	=	240 ct.	=	10 lb. at 24 ct. per lb. loss.
(3.)	9 lb. at 6 ct. per lb. gain.	=	54 ct.	=	6 lb. at 9 ct. per lb. loss.

Observe, that in (1) and (2) we have balanced the loss on the 75 ct. and 90 ct. tea by the gain on the 56 ct. tea; hence we have 9 lb. + 24 lb., or 33 lb. of the 56 ct. tea in the mixture.

Observe, also, that in (3) we have balanced the gain on the 60 ct. tea by a loss on the 75 ct. tea; hence we have 10 lb. + 6 lb., or 16 lb. of the 75 ct. tea in the mixture.

Hence the mixture is composed of 33 lb. @ 56 ct., 9 lb. @ 60 ct., 16 lb. @ 75 ct., and 10 lb. @ 90 ct.

4. Mixtures may be formed as follows:

I. Take any pair of ingredients, one giving a gain and the other a loss, and find the gain and loss on one unit of each.

II. Assume the least common multiple of the gain and loss on one unit as the amount gained and lost, by putting the two ingredients in the mixture.

III. Divide the amount thus assumed by the gain and then by the loss on one unit; the results will be respectively the *number of units of each ingredient that must be in the mixture that the gain and loss may balance each other.*

IV. Proceed in the same manner with other ingredients; the results will be the proportional parts.

EXAMPLES FOR PRACTICE.

722. 1. How much sugar at 10, 9, 7, and 5 ct. will produce a mixture worth 8 cents a pound?

2. A man wishes to mix sufficient water with molasses worth 40 cents a gallon to make the mixture worth 24 cents a gallon; what amount must he take of each?

3. A jeweller has gold 16, 18, 22, and 24 carats fine; how much of each must he use to form gold 20 carats fine?

4. A merchant desires to mix flour worth \$6, \$7½, and \$10 a barrel so as to sell the mixture at \$9; what proportion of each kind can he use?

5. A farmer has wheat worth 40, 55, 80, and 90 cents a bushel; how many bushels of each must be mixed with 290 @ 40 ct. to form a mixture worth 70 cents a bushel?

Examples like this where the quantity of one or more ingredients is limited may be solved thus:

First, we find the gain or loss on one unit as in (**720**).

Second, we balance the whole gain or loss on an ingredient where the quantity is limited, by using any ingredient giving an opposite result thus:

	Producing Gain.	Gained and Lost.	Producing Loss.
(1.)	270 bu. at 30 ct. per bu. gain. =	\$81.00 =	405 bu. at 20 ct. per bu. loss.
(2.)	2 bu. at 15 ct. per bu. gain. =	.30 =	3 bu. at 10 ct. per bu. loss.
	272 bu.	+	408 bu. = 680 bu. in mixture.

Observe, the gain on the 270 bu. may be balanced with the other ingredient that produces a loss, or with both ingredients that produce a loss, and these may be put in the mixture in different proportions; hence a series of different mixtures may thus be formed.

6. A merchant having good flour worth \$7, \$9, and \$12 a barrel, and 240 barrels of a poorer quality worth \$5 a barrel, wishes to sell enough of each kind to realize an average price of \$10 a barrel on the entire quantity sold. How many barrels of each kind can he sell?

7. I wish to mix vinegar worth 18, 21, and 27 cents a gallon with 8 gallons of water, making a mixture worth 25 cents a gallon; how much of each kind of vinegar can I use?

8. A man bought a lot of sheep at an average price of \$2 apiece. He paid for 50 of them \$2.50 per head, and for the rest \$1.50, \$1.75, and \$3.25 per head; how many sheep could there be in the lot at each price?

9. A milkman mixes milk worth 8 cents a quart with water, making 24 quarts worth 6 cents a quart; how much water did he use?

Examples like this, where the quantity of the mixture is limited, may be solved thus:

SOLUTION.—1. We find, according to (**720**), the smallest proportional parts that can be used, namely, 3 quarts of milk and 1 quart of water, making a mixture of 4 quarts.

2. Now, since in 4 qt. of the mixture there are 3 qt. of milk and 1 qt. of water, in 24 qt. there must be as many times 3 qt. of milk and 1 qt. of water as 4 qt. are contained times in 24 qt. Consequently we have as the *first step* 24 qt. ÷ 4 qt. = 6, *second step* 3 qt. × 6 = 18 qt. and 1 qt. × 6 = 6 qt. Hence in 24 qt. of the mixture there are 18 qt. of milk and 6 qt. of water.

10. A grocer has four kinds of coffee worth 20, 25, 35, and 40 cents a pound, from which he fills an order for 135 pounds worth 32 cents a pound; how may he form the mixture?

11. A jeweler melts together gold 14, 18, and 24 carats fine, so as to make 240 oz. 22 carats fine; how much of each kind did it require?

12. I wish to fill an order for 224 lb. of sugar at 12 cents, by forming a mixture from 8, 10, and 16 cent sugar; how much of each must I take?

(314)

INVOLUTION

DEFINITIONS.

723. A ***Power*** of a number is either the number itself or the product obtained by taking the number two or more times as a factor.

Thus 25 is the product of 5×5 or of 5 taken twice as a factor; hence 25 is a power of 5.

724. An ***Exponent*** is a number written at the right and a little above a number to indicate:

(1.) The number of times the given number is taken as a factor. Thus in 7^3 the 3 indicates that the 7 is taken 3 times as a factor; hence $7^3 = 7 \times 7 \times 7 = 343$.

(2.) The *degree* of the power or the order of the power with reference to the other powers of the given number. Thus, in 5^4 the 4 indicates that the given power is the *fourth power* of 5, and hence there are three powers of 5 below 5^4; namely, 5, 5^2, and 5^3.

725. The ***Square*** of a number is its *second power*, so called because in finding the superficial contents of a given *square* we take the *second power* of the number of linear units in one of its sides (**404**).

726. The ***Cube*** of a number is its *third power*, so called because in finding the cubic contents of a given *cube* we take the *third power* of the number of linear units in one of its *edges* (**412**).

727. ***Involution*** is the process of finding any required power of a given number.

PROBLEMS IN INVOLUTION.

728. PROB. I. — **To find any power of any given number.**

1. Find the fourth power of 17.

SOLUTION.—Since according to (**721**) the fourth power of 17 is the product of 17 taken as a factor 4 times, we have $17 \times 17 \times 17 \times 17 = 83521$, the required power.

2. Find the second power of 48. Of 65. Of 432.
3. Find the square of 294. Of 386. Of 497. Of 253.
4. Find the cube of 63. Of 25. Of 76. Of 392.
5. Find the third power of $\frac{3}{7}$. Of $\frac{4}{9}$. Of $\frac{7}{10}$. Of .8.

Observe, any power of a fraction is found by involving each of its terms separately to the required power (**267**).

Find the required power of the following:

6. 237^2. 8. $(\frac{13}{42})^3$. 10. $(.25)^4$. 12. $(.7\frac{2}{3})^2$. 14. $(.005\frac{1}{2})^3$.
7. 45^4. 9. $(\frac{17}{100})^2$. 11. $(.3\frac{2}{5})^3$. 13. $(.1\frac{3}{10})^4$. 15. $.0302^2$.

729. PROB. II.—**To find the exponent of the product of two or more powers of a given number.**

1. Find the exponent of product of 7^3 and 7^2.

SOLUTION.—Since $7^3 = 7 \times 7 \times 7$ and $7^2 = 7 \times 7$, the product of 7^3 and 7^2 must be $(7 \times 7 \times 7) \times (7 \times 7)$, or 7 taken as a factor as many times as the sum of the exponents 3 and 2. Hence to find the exponent of the product of two or more powers of a given number, we take the sum of the given exponents.

Find the exponent of the product

2. Of $35^4 \times 35^3$. 4. Of $18^2 \times 18^7$. 6. Of $23^7 \times 23^5$.
3. Of $(\frac{2}{3})^5 \times (\frac{2}{3})^2$. 5. Of $(\frac{4}{9})^7 \times (\frac{4}{9})^8$. 7. Of $(\frac{8}{11})^4 \times (\frac{8}{11})^9$.
8. Of $(7^4)^2$. *Observe,* $(7^4)^2 = 7^4 \times 7^4 = 7^{4 \times 2} = 7^8$.

Hence the required exponent is the product of the given exponents.

9. Of $(12^3)^4$. 10. Of $(9^6)^5$. 11. Of $(16^3)^8$. 12. Of $[(\frac{4}{5})^3]^4$.

EVOLUTION

DEFINITIONS.

730. A ***Root*** of a number is either the number itself or *one* of the *equal factors* into which it can be resolved.

Thus, since $7 \times 7 = 49$, the factor 7 is a root of 49.

731. The ***Second*** or ***Square Root*** is one of the *two equal factors* of a number. Thus, 5 is the square root of 25.

732. The ***Third*** or ***Cube Root*** is one of the *three equal factors* of a number. Thus, 2 is the cube root of 8.

733. The ***Radical*** or ***Root Sign*** is $\sqrt{}$, or a *fractional exponent.*

When the sign, $\sqrt{}$, is used, the *degree* or name of the root is indicated by a small figure written over the sign; when the *fractional exponent* is used, the denominator indicates the name of the root; thus,

$\sqrt[2]{9}$ or $9^{\frac{1}{2}}$ indicates that the second or square root is to be found.

$\sqrt[3]{27}$ or $27^{\frac{1}{3}}$ indicates that the third or cube root is to be found.

Any required root is expressed in the same manner. The *index* is usually omitted when the square root is required.

734. A ***Perfect Power*** is a number whose exact root can be found.

735. An ***Imperfect Power*** is a number whose exact root cannot be found.

The indicated root of an *imperfect power* is called a *surd*; thus $\sqrt{5}$.

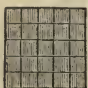
736. ***Evolution*** is the process of finding the roots of numbers.

SQUARE ROOT.

PREPARATORY PROPOSITIONS.

737. Prop. I.—*Any* PERFECT *second power may be represented to the eye by a square, and the number of units in the side of such square will represent the* SECOND *or* SQUARE ROOT *of the given power.*

For example, if 25 is the given power, we can suppose the number represents 25 small squares and arrange them thus:



1. Since $25 = 5 \times 5$, we can arrange the 25 squares 5 in a row, making 5 rows, and hence forming a square as shown in the illustration.

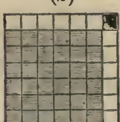
2. Since the side of the square is 5 units, it represents the square root of 25, the given power; hence the truth of the proposition.

738. Prop. II.—*Any number being given, by supposing it to represent small squares, we can find by arranging these squares in a large square the largest perfect second power the given number contains, and hence its square root.*

For example, if we take 83 as the given number and suppose it to represent 83 small squares, we can proceed, thus:

(1)

(2)



1. We can take any number of the 83 squares, as 36, that we know will form a perfect square (Prop. I), and arrange them in a square, as shown in (1), leaving 47 of the 83 squares yet to be disposed of.

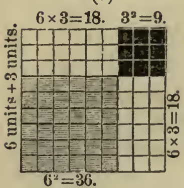
2. We can now place a row of squares on two adjacent sides of the square in (1) and a square in the corner, and still have a perfect square as shown in (2).

3. *Observe,* that in putting one row of small squares on each of two *adjacent* sides of the square *first formed,* we must use twice as many squares as there are units in the side of the square.

4. Now since it takes twice 6 or 12 squares to put one row on each of two adjacent sides, we can put on as many rows as

12 is contained times in 47, the number of squares remaining. Hence we can put on 3 rows as shown in (3) and have 11 squares still remaining.

(3)



5. Again, having put 3 rows of squares on each of two adjacent sides, it takes 3×3 or 9 squares to fill the corner thus formed, as shown in (3), leaving only 2 of the 11 squares.

Hence, the square in (3) represents the greatest perfect power in 83, namely 81; and 9, the number of units in its side, represents the *square root* of 81.

6. Now *observe* that the length of the side of the square in (3) is $6+3$ units, and that the number of small squares may be represented in terms of $6+3$; thus,

(1.) $(6+3)^2 = 6^2+3^2+$ *twice* $6 \times 3 = 36+9+36 = 81.$

Again, suppose 5 units had been taken as the side of the first square, the number of small squares would be represented thus:

(2.) $(5+4)^2 = 5^2+4^2+$ *twice* $5 \times 4 = 25+16+40 = 81.$

In the same manner it may be shown that the square of the sum of any two numbers expressed in terms of the numbers, is the *square* of *each* of the numbers plus *twice* their *product*.

Hence the square of any number may be expressed in terms of its tens and units; thus $57 = 50+7$; hence

(3.) $57^2 = (50+7)^2 = 50^2+7^2+$ *twice* $50 \times 7 = 3249.$

This may also be shown by actual multiplication. Thus, in multiplying 57 by 57 we have, *first*, $57 \times 7 = 7 \times 7 + 50 \times 7 = 7^2 + 50 \times 7$; we have, *second*, $57 \times 50 = 50 \times 7 + 50 \times 50 = 50 \times 7 + 50^2$; hence, $57^2 = 50^2 + 7^2 +$ *twice* 50×7.

Find, by constructing a diagram as above, the square root of each of the following:

Observe, that when the number is large enough to give tens in the root, we can take as the side of the first square we construct the greatest number of tens whose square can be taken out of the given number.

1. Of 144.	4. Of 529.	7. Of 1125.	10. Of 1054.
2. Of 196.	5. Of 729.	8. Of 584.	11. Of 2760.
3. Of 289.	6. Of 1089.	9. Of 793.	12. Of 3832.

739. PROP. III.—*The square of any number must contain twice as many figures as the number, or twice as many less one.*

This proposition may be shown thus:

1. *Observe*, the square of either of the digits 1, 2, 3, is expressed by one figure, and the square of either of the digits 4, 5, 6, 7, 8, 9, is expressed by two figures; thus, $2 \times 2 = 4$, $3 \times 3 = 9$, and $4 \times 4 = 16$, $5 \times 5 = 25$, and so on.

2. Since $10 \times 10 = 100$, it is evident the square of any number of tens must have two ciphers at the right; thus, $20^2 = 20 \times 20 = 400$.

Now since the square of either of the digits 1, 2, 3, is expressed by one figure, if we have 1, 2, or 3 *tens*, the square of the number must be expressed by 3 figures; that is, one figure less than twice as many as are required to express the number.

Again, since the square of either of the digits 4, 5, 6, 7, 8, 9, is expressed by two figures, if we have 4, 5, 6, 7, 8, or 9 *tens*, the square of the number must contain four figures; that is, twice as many figures as are required to express the number. Hence it is evident that, in the square of a number, the square of the tens must occupy the *third* or the *third* and *fourth* place.

By the same method it may be shown that the square of *hundreds* must occupy the *fifth* or the *fifth* and *sixth* places, the square of *thousands* the *seventh* or the *seventh* and *eighth* places, and so on; hence the truth of the proposition.

From this proposition we have the following conclusions:

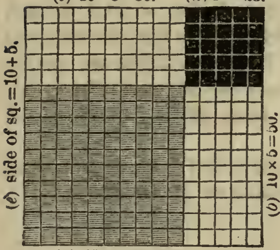
740. *I. If any number be separated into periods of two figures each, beginning with the units place, the number of periods will be equal to the number of places in the square root of the greatest perfect power which the given number contains.*

II. In the square of any number the square of the units are found in the UNITS *and* TENS *place, the square of the tens in the* HUNDREDS *and* THOUSANDS *place, the square of the hundreds in the* TENS *and* HUNDREDS *of* THOUSANDS *place,* and so on.

ILLUSTRATION OF PROCESS.

741. 1. Find the square root of 225.

(c) $10 \times 5 = 50$. (d) $5^2 = 25$.



(a) $10^2 = 100$.

$$
\begin{array}{llr}
 & & 225\,(\,10 \\
\textit{1st Step.} & 10^2 = 10 \times 10 = & \underline{100} \\
\textit{2d Step.} & (1)\ \textit{T. divisor}\ 10 \times 2 = 20)125\,(\ \ 5 & \textit{root}\ \overline{15} \\
 & (2) \left\{ \begin{array}{l} 20 \times 5 = 100 \\ 5 \times 5 = \ \ 25 \end{array} \right\} = & \underline{125}
\end{array}
$$

EXPLANATION. — 1. We observe, as shown in (*a*), that 1 ten is the largest number of tens whose square is contained in 225. Hence in *1st step* we subtract $10^2 = 100$ from 225, leaving 125.

2. Having formed a square whose side is 10 units, we observe, as shown in (*b*) and (*c*), that it will take *twice* ten to put one row on two adjacent sides. Hence the *Trial Divisor* is $10 \times 2 = 20$.

3. We observe that 20 is contained 6 times in 125, but if we add 6 units to the side of the square (*a*) we will not have enough left for the corner (*d*), hence we add 5 units.

4. Having added 5 units to the side of the square (*a*), we observe, as shown in (*b*) and (*c*), that it requires *twice* 10 or 20 multiplied by 5 plus 5×5, as shown in (*d*), to complete the square; hence (2) in *2d step.*

Solution with every Operation Indicated.

742. 2. Find the square root of 466489.

$$
\begin{array}{llrl}
 & & \dot{4}6\dot{6}4\dot{8}9\ (& 600 \\
\text{FIRST STEP.} & 600 \times 600 & \underline{360000} & \ \ 80 \\
\text{SECOND STEP.} & (1)\ \textit{Trial divisor}\ 600 \times 2 = 1200)106489 & & \ \ \underline{\ 3} \\
 & (2) \left\{ \begin{array}{l} 1200 \times 80 = 96000 \\ \ \ 80 \times 80 = \ \ 6400 \end{array} \right\} = & \underline{102400} & 683 \text{ required root.} \\
\text{THIRD STEP.} & (1)\ \textit{Trial divisor}\ 680 \times 2 = 1360) & 4089 & \\
 & (2) \left\{ \begin{array}{l} 1360 \times 3 = 4080 \\ \ \ 3 \times 3 = \ \ \ \ \ 9 \end{array} \right\} = & \underline{4089} &
\end{array}
$$

EXPLANATION.—1. We place a point over every second figure beginning with the units, and thus find, according to (**740**), that the root must have three places. Hence the first figure of the root expresses hundreds.

2. We observe that the square of 600 is the greatest second power of hundreds contained in 466489. Hence in the first step we subtract $600 \times 600 = 360000$ from 466489, leaving 106489.

3. We now double the 600, the root found, for a trial divisor, according to (**741**—2). Dividing 106489 by 1200 we find, according to (**741**—2), that we can add 80 to the root. For this addition we use, as shown in (2), *second step*, $1200 \times 80 = 96000$ and $80 \times 80 = 6400$ (**741**—3), making in all 102400. Subtracting 102400 from 106489, we have still remaining 4089.

4. We again double 680, the root found, for a trial divisor, according to (**741**—2), and proceed in the same manner as before, as shown in *third step.*

743. *Contracted Solution of the foregoing Example.*

			$4\dot{6}6\dot{4}8\dot{9}$ (683
FIRST STEP.		$6 \times 6 =$	36
SECOND STEP.	(1)	$6 \times 2 = 12$)	1064
	(2)	$128 \times 8 =$	1024
THIRD STEP.	(1)	$68 \times 2 = 136$)	4089
	(2)	$1363 \times 3 =$	4089

EXPLANATION.—1. *Observe,* in the *first step* we know that the square 600 must occupy the fifth and sixth place (**738**). Hence the ciphers are omitted.

2. *Observe,* that in (1), *second step,* we use 6 instead of 600, thus dividing the divisor by 100; hence we reject the *tens* and *units* from the right of the dividend (**142**).

3. *Observe,* also, in (2), *second step,* we unite in *one* three operations. Instead of multiplying 12 by 80, the part of the root found by dividing 1064 by 12, we multiply the 12 first by 10 by annexing the 8 to it (**91**), and having annexed the 8 we multiply the result by 8, which gives us the product of 12 by 80, plus the square of 8. But the square of 8, written, as it is, in the third and fourth place, is the square of 80.

Hence by annexing the 8 and writing the result as we do, we have united in *one* three operations; thus, $128 \times 8 = 12 \times 80 + 80 \times 80$.

(322)

From these illustrations we have the following

744. Rule.—*I. Separate the number into periods of two figures each, by placing a point over every second figure, beginning with the units figure.*

II. Find the greatest square in the left-hand period and place its root on the right. Subtract this square from the period and annex to the remainder the next period for a dividend.

III. Double the part of the root found for a trial divisor, and find how many times this divisor is contained in the dividend, omitting the right-hand figure. Annex the quotient thus found both to the root and to the divisor. Multiply the divisor thus completed by the figure of the root last obtained, and subtract the product from the dividend.

IV. If there are more periods, continue the operation in the same manner as before.

In applying this rule be particular to observe

1. When there is a remainder after the last period has been used, annex periods of ciphers and continue the root to as many decimal places as may be required.

2. We separate a number into periods of two figures by beginning at the units place and proceeding to the left if the number is an integer, and to the right if a decimal, and to the right and left if both.

3. Mixed numbers and fractions are reduced to decimals before extracting the root. But in case the *numerator* and the *denominator* are perfect powers, or the *denominator* alone, the root may be more readily formed by extracting the root of each term separately,

Thus $\sqrt{\frac{49}{81}} = \frac{\sqrt{49}}{\sqrt{81}} = \frac{7}{9}$, and $\sqrt{\frac{35}{64}} = \frac{\sqrt{35}}{\sqrt{64}} = \frac{\sqrt{35}}{8}$, and so on.

Extract the square root

1. Of $\frac{64}{169}$.
2. Of $\frac{81}{225}$.
3. Of $\frac{196}{289}$.
4. Of $\frac{256}{361}$.
5. Of $\frac{169}{625}$.
6. Of $\frac{324}{1,444}$.
7. Of $\frac{729}{2809}$.
8. Of $\frac{361}{4096}$.

EXAMPLES FOR PRACTICE.

745. Extract the square root

1. Of 4096.	7. Of $\frac{625}{7056}$.	13. Of 137641.
2. Of 3481.	8. Of $\frac{3481}{7921}$.	14. Of 4160.25.
3. Of 2809.	9. Of $\frac{1764}{6561}$.	15. Of 768427.56.
4. Of 7569.	10. Of .0225.	16. Of 28022.76.
5. Of 8649.	11. Of .2304.	17. Of 57.1536.
6. Of 9216.	12. Of .5776.	18. Of 474.8041.

Find the square root to three decimal places

19. Of 32.	22. Of .93.	25. Of 14.7.	28. Of $\frac{7}{9}$.
20. Of 59.	23. Of .8.	26. Of 86.2.	29. Of $\frac{3}{25}$.
21. Of 7.	24. Of .375.	27. Of 5.973.	30. Of $\frac{8}{49}$.

Perform the operations indicated in the following:

31. $\sqrt{6889} - \sqrt{1024}$.

32. $\sqrt{2209} + \sqrt{225}$.

33. $\sqrt{\frac{2025}{4096}} \times \sqrt{2209}$.

34. $76796^{\frac{1}{2}} \div \sqrt{2136}$.

35. $\sqrt{558009} \div (\frac{81}{1024})^{\frac{1}{2}}$.

36. $(\frac{8836}{9216})^{\frac{1}{2}} \times 131376^{\frac{1}{2}}$.

37. What is the length of a square floor containing 9025 square feet of lumber?

38. A square garden contains 237169 square feet; how many feet in one of its sides?

39. How mony yards in one of the equal sides of a square acre?

40. An orchard containing 9216 trees is planted in the form of a square, each tree an equal distance from another; how many trees in each row?

41. A triangular field contains 1966.24 P. What is the length of one side of a square field of equal area?

42. Find the square root of 2, of 5, and of 11, to 4 decimal places.

43. Find the square root of $\frac{3}{7}$, $\frac{8}{11}$, and of $\frac{13}{24}$, to 3 decimal places.

CUBE ROOT.

PREPARATORY PROPOSITIONS.

46. Prop. I.—*Any PERFECT third power may be represented to the eye by a cube, and the number of units in the side of such cube will represent the THIRD or CUBE ROOT of the given power.*

Represent to the eye by a cube 343.

(1)

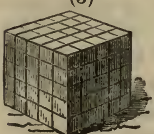
1. We can suppose the number 343 to represent small cubes, and we can take 2 or more of these cubes and arrange them in a row, as shown in (1).

2. Having formed a row of 5 cubes, as shown in (1), we can arrange 5 of these rows side by side, as shown in (2), forming a square slab containing 5×5 small cubes, or as many small cubes as the *square* of the *number* of *units* in the *side* of the slab.

(2)

3. Placing 5 such slabs together, as shown in (3), we form a cube. Now, since each slab contains 5×5 small cubes, and since 5 slabs are placed together, the cube in (3) contains $5 \times 5 \times 5$, or 125 small cubes, and hence represents the *third* power 125, and each edge of the cube represents to the eye 5, the cube root of 125.

(3)

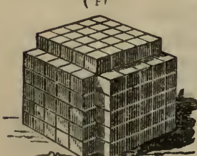


We have now remaining yet to be disposed of 343−125, or 218 small cubes.

4. Now, *observe*, that to enlarge the cube in (3) so that it may contain the 343 small cubes, we must build the same number of tiers of small cubes upon *each* of *three adjacent* sides, as shown in (4).

Observe, also, that a slab of small cubes to cover one side of the cube in (3) must contain 5×5 or 25 small cubes, as shown in (4), or as many small cubes as the square of the number of units in one edge of the cube in (3).

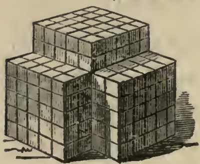
(4)



Hence, to find the number of cubes necessary to put one slab on each of three sides of the cube in (3), we multiply the square of its edge by 3 giving $5^2 \times 3 = 5 \times 5 \times 3 = 75$ small cubes.

5. Having found that 75 small cubes will put one tier on each of three adjacent sides of the cube in (3), we divide 218, the number of small cubes yet remaining, by 75, and find how many such tiers we can form. Thus, $218 \div 75 =$ 2 and 68 remaining. Hence we can put 2 tiers on each of *three* adjacent sides, as shown in (5), and have 68 small cubes remaining.

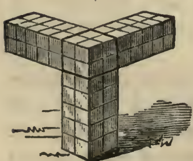
(5)



6. Now, *observe*, that to complete this cube we must fill each of the *three* corners formed by building on *three* adjacent sides.

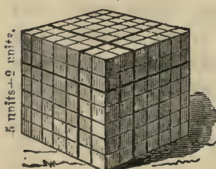
Examine carefully (6) and *observe* that to fill *one* of these *three corners* we require as many small cubes as is expressed by the *square* of the *number* of tiers added, multiplied by the *number* of units in the *side* of the *cube* to which the addition is made. Hence we require $2^2 \times 5$ or 20 small cubes. And to fill the *three* corners we require 3 times $2^2 \times 5$ or 60, leaving $68-60$ or 8 of the small cubes.

(6)



7. *Examine again* (5) and (6) and *observe* that when the *three corners* are filled we require to complete the cube as shown in (7), another *cube* whose side contains as many units as there are units added to the side of the cube on which we have built. Consequently we require 2^3 or $2 \times 2 \times 2 = 8$ small cubes.

(7)



Hence we have formed a cube containing 343 small cubes, and any one of its edges represents to the eye $5+2$ or 7 units, the cube root of 343.

From these illustrations it will be seen that the steps in finding the cube root of 343 may be stated thus:

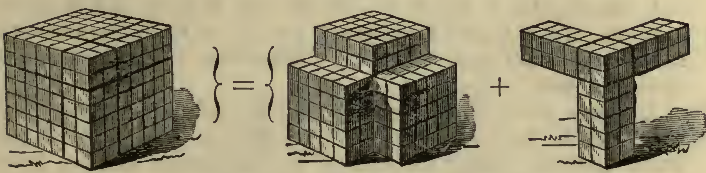
FIRST STEP.	We assume that 343 represents small cubes, and take 5 as the length of the side of a large cube formed from these. Hence we subtract the cube of 5 =	343 125
SECOND STEP.	1. We observe it takes $5^2 \times 3 = 75$ to put *one* tier on three adjacent sides. Hence we can put on	75)218(2
	2. We have now found that we can add 2 units to the side of the cube. Hence to add this we require	
	(1) For the 3 sides of the cube $5^2 \times 2 \times 3 = 150$	
	(2) For the 3 corners thus formed $2^2 \times 5 \times 3 = 60$	= 218
	(3) For the cube in the corner last formed $2^3 = 8$	

Hence the cube root of 343 is $5+2=7$.

(326)

747. *Observe,* that the number of small cubes in the cube (7) in the foregoing illustrations, are expressed in terms of $5+2$; namely, the number of units in the side of the first cube formed, plus the number of tiers added in enlarging this cube; thus:

(7) (5) (6)



$$(5+2)^3 = 5^3+5^2\times 2\times 3 + 2^2\times 5\times 3+2^2.$$

In this manner it may be shown that the cube of the sum of any two numbers is equal to the *cube* of *each* number, plus 3 *times* the *square* of the *first* multiplied by the *second* number, plus 3 *times* the *square* of the *second* multiplied by the *first* number.

Hence the cube of any number may be expressed in terms of its tens and units; thus, $74 = 70+4$; hence,

$$(70+4)^3 = 70^3+3 \textit{ times } 70^2\times 4+3 \textit{ times } 4^2\times 70+4^3 = 405224.$$

Solve each of the following examples, by applying the foregoing illustrations:

1. Find the side of a cube which contains 729 small cubes, taking 6 units as the side of the first cube formed.

2. Take 20 units as the side of the first cube formed, and find the side of the cube that contains 15625 cubic units.

3. How many must be added to 9 that the sum may be the cube root of 4096? Of 2197? Of 2744?

4. Find the cube root of 1368. Of 3405. Of 2231. Of 5832.

5. Express the cube of 83 in terms of $80+3$.

6. Express the cube of 54, of 72, of 95, of 123, of 274, in terms of the tens and units of each number.

748. Prop. II.—*The cube of any number must contain three times as many places as the number, or three times as many less one or two places.*

This proposition may be shown thus:

1. *Observe*, $1^3 = 1$, $2^3 = 9$, $3^3 = 27$, $4^3 = 64$, $5^3 = 125$, and $9^3 = 729$; hence the cube of 1 and 2 is expressed each by *one* figure, the cube of 3 and 4 each by *two* figures, and any number from 5 to 9 inclusive each by three figures.

2. *Observe*, also, that for every cipher at the right of a number there must (**91**) be three ciphers at the right of its cube; thus, $10^3 = 1,000$, $100^3 = 1,000,000$. Hence the cube of *tens* can occupy no place lower than *thousands*, the cube of *hundreds* no place lower than *millions*, and so on with higher orders.

3. From the foregoing we have the following:

(1.) Since the cube of 1 or 2 contains *one* figure, the cube of 1 or 2 *tens* must contain *four* places; of 1 or 2 *hundreds*, *seven* places, and so on with higher orders.

(2.) Since the cube of 3 or 4 contains *two* figures, the cube of 3 or 4 *tens* must contain *five* places; of 3 or 4 *hundreds*, *eight* places, and so on with higher orders.

(3.) Since the cube of any number from 5 to 9 inclusive contains *three* places, the cube of any number of *tens* from 5 to 9 *tens* inclusive must contain *six* places; of hundreds, from 5 to 9 *hundred* inclusive, *nine* places, and so on with higher orders; hence the truth of the proposition.

Hence also the following:

749. *I. If any number be separated into periods of three figures each, beginning with the units place, the number of periods will be equal to the number of places in the cube root of the greatest perfect third power which the given number contains.*

II. The cube of units contains no order higher than hundreds.

III. The cube of tens contains no order lower than thousands nor higher than hundred thousands, the cube of hundreds no order lower than millions nor higher than hundred millions, and so on with higher orders.

ILLUSTRATION OF PROCESS.

750. ***Solution with every Operation Indicated.***

Find the cube root of 92345408.

$$
\begin{array}{lllr}
 & & & \dot{9}23\dot{4}5\dot{4}08\ (\ 400 \\
\textsc{First Step.} & & 400^3 = 400 \times 400 \times 400 = & \underline{64000000} \\
\textsc{Second Step.} & (1) & \textit{Trial divisor}\ 400^2 \times 3 = 480000\ ) & 28345408\ (\quad 50 \\
 & (2) & \left\{\begin{array}{rr} 400^2 \times 50 \times 3 = & 24000000 \\ 50^2 \times 400 \times 3 = & 3000000 \\ 50^3 = & 125000 \end{array}\right\} = & \underline{27125000} \\
\textsc{Third Step.} & (1) & \textit{Trial divisor}\ 450^2 \times 3 = 607500\ ) & 1220408\ (\quad \underline{2} \\
 & & & \textit{Root}\ 452 \\
 & (2) & \left\{\begin{array}{rr} 450^2 \times 3 \times 2 = & 1215000 \\ 2^2 \times 450 \times 3 = & 5400 \\ 2^3 = & 8 \end{array}\right\} & 1220408
\end{array}
$$

Explanation.—1. We place a period over every third figure beginning with the units, and thus find, according to (**749**), that the root must have three places. Hence the first figure of the root expresses hundreds.

2. We observe that 400 is the greatest number whose cube is contained in the given number. Subtracting $400^3 = 64000000$ from 92345408, we have 28345408 remaining.

3. We find a *trial* divisor, according to (**746**—4), by taking 3 times the square of 400, as shown in (1), *second step*. Dividing by this divisor, according to (**746**—5), we find we can add 50 to the root already found.

Observe, the root now found is $400 + 50$, and that according to (**747**),

$$(400+50)^3 = 400^3 + 400^2 \times 50 \times 3 + 50^2 \times 400 \times 3 + 50^3.$$

We have already subtracted $400^3 = 640000$ from the given number. Hence we have only now to subtract,

$$400^2 \times 50 \times 3 + 50^2 \times 400 \times 3 + 50^3 = 27125000,$$

as shown in (2), *second step*, leaving 1220408.

5. We find another trial divisor and proceed in the same manner to find the next figure of the root, as shown in the *third step*.

751. ***Contracted Solution of the foregoing Example.***

			92345403 (452
FIRST STEP.	$4^3 = 4 \times 4 \times 4 =$		64
SECOND STEP.	(1) *Trial divisor* $40^2 \times 3 =$	4800)	28345
	(2) $\left\{\begin{array}{r} 40^2 \times 5 \times 3 = 24000 \\ 5^2 \times 40 \times 3 = 3000 \\ 5^3 = 125 \end{array}\right\} =$		27125
THIRD STEP.	(1) *Trial divisor* $450^2 \times 3 = 607500$	)	1220408
	(2) $\left\{\begin{array}{r} 450^2 \times 2 \times 3 = 1215000 \\ 2^2 \times 450 \times 3 = 5400 \\ 2^3 = 8 \end{array}\right\} =$		1220408

EXPLANATION.—1. *Observe*, in the *first step*, we know that the cube of 400 must occupy the *seventh* and *eighth* places (**749**—III). Hence the ciphers are omitted.

2. Observe, also, that no part of the cube of hundreds and tens is found below thousands (**749**—III). We therefore, in finding the *number* of *tens* in the root, disregard, as shown in *second step*, the right-hand period in the given number, and consider the *hundreds* and *tens* in the root as *tens* and *units* respectively.

Hence, in general, whatever number of places there are in the root, we disregard, in finding any figure, as many periods at the right of the given number as there are places in the root at the right of the figure we are finding, and consider the part of the root found as *tens*, and the figure we are finding as *units*, and proceed accordingly.

From these illustrations we have the following:

752. RULE.—*I. Separate the number into periods of three figures each, by placing a point over every third figure, beginning with the units figure.*

II. Find the greatest cube in the left-hand period, and place its root on the right. Subtract this cube from the period and annex to the remainder the next period for a dividend.

III. Divide this dividend by the trial divisor, which is 3 times the square of the root already found, con-

sidered as tens; the quotient is the next figure of the root.

IV. Subtract from the dividend 3 times the square of the root before found, considered as tens, multiplied by the figure last found, plus 3 times the square of the figure last found, multiplied by the root before found, plus the cube of the figure last found, and to the remainder annex the next period, if any, for a new dividend.

V. If there are more figures in the root, find in the same manner trial divisors and proceed as before.

In applying this rule be particular to observe:

1. In dividing by the *Trial Divisor* the quotient may be larger than the required figure in the root, on account of the addition to be made, as shown in (**746**—6) *second step.* In such case try a figure 1 less than the quotient found.

2. When there is a remainder after the last period has been used, annex periods of ciphers and continue the root to as many decimal places as may be required.

3. We separate a number into periods of three figures by beginning at the units place and proceeding to the left if the number is an integer, and to the right if a decimal, and to the right and left if both.

4. Mixed numbers and fractions are reduced to decimals before extracting the root. But in case the *numerator* and the *denominator* are perfect *third powers,* or the *denominator* alone, the root may be more readily found by extracting the root of each term separately.

EXAMPLES FOR PRACTICE.

753. Find the cube root of

1. 216.	5. 4096.	9. $\frac{2744}{9261}$.	13. 24137569.
2. 729.	6. 10648.	10. $\frac{5832}{13824}$.	14. 47245881.
3. 1331.	7. 6859.	11. 250047.	15. $\frac{4913}{42515228}$.
4. 2197.	8. $\frac{512}{4913}$.	12. 438976.	16. 113.379904.

17. Find, to two decimal places, the cube root of 11. Of 36. Of 84. Of 235. Of $\frac{5}{27}$. Of $\frac{53}{125}$. Of 75.4. Of 6.7.

18. Find to three decimal places the cube root of 3. Of 7. Of .5. Of .04. Of .009. Of 2.06.

19. Find the sixth root of 4096.

Observe, the sixth root may be found by extracting *first* the square root, *then* the cube root of the result.

For example, $\sqrt{4096} = 64$; hence, $4096 = 64 \times 64$. Now, if we extract the cube root of 64 we will have one of the *three* equal factors of 64, and hence one of the *six* equal factors or sixth root of 4096.

Thus, $\sqrt[3]{64} = 4$; hence, $64 = 4 \times 4 \times 4$. But we found by extracting its square root that $4096 = 64 \times 64$, and now by extracting the cube root that $64 = 4 \times 4 \times 4$; consequently we know that $4096 = (4 \times 4 \times 4) \times (4 \times 4 \times 4)$. Hence 4 is the required sixth root of 4096.

In this manner, it is evident, we can find any root whose index contains no other factor than 2 or 3.

20. Find the *sixth* root of 2565726409.

21. Find the *eighth* root of 43046721.

22. What is the *fourth* root of 34012224?

23. What is the *ninth* root of 134217728?

24. A pond contains 84604519 cubic feet of water; what must be the length of the side of a cubical reservoir which will exactly contain the same quantity?

25. What is the length of the inner edge of a cubical cistern that contains 2079 gal. of water?

26. How many square feet in the surface of a cube whose volume is 16777216 cubic inches?

27. A pile of cord wood is 256 ft. long, 8 ft. high, and 16 ft. wide; what would be the length of the side of a cubical pile containing the same quantity of wood?

28. What is the length of the inner edge of a cubical bin that contains 3550 bushels?

29. What are the dimensions of a cube whose volume is equal to 82881856 cubic feet?

30. What is the length in feet of the side of a cubical reservoir which contains 1221187.5 pounds avoirdupois, pure water?

PROGRESSIONS

DEFINITIONS.

754. A ***Progression*** is a series of numbers so related, that each number in the series may be found in the same manner, from the number immediately preceding it.

755. An ***Arithmetical Progression*** is a series of numbers, which increases or decreases in such a manner that the *difference* between any *two consecutive* numbers is *constant*. Thus, 3, 7, 11, 15, 19, 23.

756. A ***Geometrical Progression*** is a series of numbers, which increase or decrease in such a manner that the *ratio* between any *two consecutive* numbers is *constant*.

Thus, 5, 10, 20, 40, 80, is a geometrical progression.

757. The ***Terms*** of a progression are the numbers of which it consists. The *First* and *Last Terms* are called the *Extremes* and the intervening terms the *Means*.

758. The ***Common*** or ***Constant Difference*** of an arithmetical progression is the difference between any two consecutive terms.

759. The ***Common*** or ***Constant Ratio*** or ***Multiplier*** of a geometrical progression is the quotient obtained by dividing any term by the preceding one.

760. An ***Ascending*** or ***Increasing Progression*** is one in which each term is greater than the preceding one.

761. A ***Descending*** or ***Decreasing Progression*** is one in which each term is less than the preceding one.

ARITHMETICAL PROGRESSION.

762. There are *five* quantities considered in Arithmetical Progression, which, for convenience in expressing rules, we denote by letters, thus:

1. A represents the *First Term* of a progression.
2. L represents the *Last Term.*
3. D represents the *Constant* or *Common Difference.*
4. N represents the *Number of Terms.*
5. S represents the *Sum of all the Terms.*

763. Any *three* of these quantities being given, the *other two* may be found. This may be shown thus:

Taking 7 as the first term of an increasing series, and 5 the constant difference, the series may be written in two forms; thus:

	1st Term.	*2d Term.*	*3d Term.*	*4th Term.*
(1)	7	12	17	22, and so on.
(2)	7	$7+(5)$	$7+(5+5)$	$7+(5+5+5)$

Observe, in (2), each term is composed of the *first term* 7 plus as many times the constant difference 5 as the number of the term less 1. Thus, for example, the *ninth* term in this series would be $7+5\times(9-1)=47$.

Hence, from the manner in which each term is composed, we have the following formulæ or rules:

1. $A=L-D\times(N-1)$. Read, *The first term is equal to the last term, minus the common difference multiplied by the number of terms less 1.*

2. $L=A+D\times(N-1)$. Read, *The last term is equal to the first term, plus the common difference multiplied by the number of terms less 1.*

3. $D=\dfrac{L-A}{N-1}$. Read, *The common difference is equal to the last term, minus the first term divided by the number of terms less 1.*

4. $N = \frac{L - A}{D} + 1.$ Read, *The number of terms is equal to the last term, minus the first term divided by the common difference, plus 1.*

Observe, that in a decreasing series, the first term is the *largest* and the last term the *smallest* in the series. Hence, to make the above formulæ apply to a decreasing series, we must place L where A is, and A where L is, and read the formulæ acccordingly.

764. To show how to find the sum of a series let

(1.)	4	7	10	13	16	19	be an arithmetical series.
(2.)	19	16	13	10	7	4	be the same series reversed.
(3.)	23 +	23 +	23 +	23 +	23 +	23 =	twice the sum of the terms.

Now, *observe*, that in (3), which is equal to twice the sum of the series, each term is equal to the first term plus the last term; hence,

$S = \frac{1}{2}$ of $(A + L) \times N$. Read, *The sum of the terms of an arithmetical series is equal to one-half of the sum of the first and last term, multiplied by the number of terms.*

EXAMPLES FOR PRACTICE.

765. 1. The first term of an arithmetical progression is 4, the common difference 2; what is the 12th term?

2. The first round of an upright ladder is 12 inches from the ground, and the nineteenth 246 inches; how far apart are the rounds?

3. The tenth term of an arithmetical progression is 190, the common difference 20; what is the first term?

4. Weston traveled 14 miles the first day, increasing 4 miles each day; how far did he travel the 15th day, and how many miles did he travel in all the first 12 days?

5. The amount of $360 for 7 years at simple interest was $486; what was the yearly interest?

6. The first term of an arithmetical series of 100 terms is 150, and the last term 1338; what is the common difference?

7. What is the sum of the first 1000 numbers in their natural order?

8. A merchant bought 16 pieces of cloth, giving 10 cents for the first and $12.10 for the last, the several prices form an arithmetical series; find the cost of the cloth?

9. A man set out on a journey, going 6 miles the first day, increasing the distance 4 miles each day. The last day he went 50 miles; how long and how far did he travel?

10. How many less strokes are made daily by a clock which strikes the hours from 1 to 12, than by one which strikes from 1 to 24.

GEOMETRICAL PROGRESSION.

766. There are *five* quantities considered in geometrical progression, which we denote by letters in the same manner as in arithmetical progression; thus:

1. A = First Term.
2. L = Last Term.
3. R = Constant Ratio.
4. N = Number of Terms.
5. S = the Sum of all the terms.

767. Any *three* of these quantities being given, the *other two* may be found. This may be shown thus:

Taking 3 as the first term and 2 as the constant ratio or multiplier, the series may be written in three forms; thus:

	1st Term.	*2d Term.*	*3d Term.*	*4th Term.*	*5th Term.*
(1.)	3	6	12	24	48
(2.)	3	3×2	$3 \times (2 \times 2)$	$3 \times (2 \times 2 \times 2)$	$3 \times (2 \times 2 \times 2 \times 2)$
(3.)	3	3×2	3×2^2	3×2^3	3×2^4

Observe, in (3), each term is composed of the first term, 3, multiplied by the constant multiplier 2, raised to the power indicated by the number of the term less 1. Thus, for example, the *seventh* term would be $3 \times 2^{7-1} = 3 \times 2^6 = 192$.

Hence, from the manner in which each term is composed, we have the following formulæ or rules :

1. $A = \frac{L}{R^{n-1}}$.	Read,	*The first term is equal to the last term, divided by the constant multiplier raised to the power indicated by the number of terms less 1.*
2. $L = A \times R^{n-1}$.	Read,	*The last term is equal to the first term, multiplied by the constant multiplier raised to the power indicated by the number of terms less 1.*
3. $R = \sqrt[n-1]{\frac{L}{A}}$.	Read,	*The constant multiplier is equal to the root, whose index is indicated by the number of terms less one, of the quotient of the last term divided by the first.*
4. $R^{n-1} = \frac{L}{A}$.	Read,	*The number of terms less one is equal to the exponent of the power to which the common multiplier must be raised to be equal to the quotient of the last term divided by the first.*

768. To show how to find the sum of a geometrical series, we take a series whose common multiplier is known ; thus:

$$S = 5 + 15 + 45 + 135 + 405.$$

Multiplying each term in this series by 3, the common multiplier, we will have 3 times the sum.

(1.) $S \times 3 = 5 \times 3 + 15 \times 3 + 45 \times 3 + 135 \times 3 + 405 \times 3$, or
(2.) $S \times 3 = 15 \quad + 45 \quad + 135 \quad + 405 \quad + 405 \times 3$.

Subtracting the sum of the series from this result as expressed in (2), we have,

$$\begin{aligned} S \times 3 &= \quad\; 15 + 45 + 135 + 405 + 405 \times 3 \\ S \quad &= 5 + 15 + 45 + 135 + 405 \\ \hline S \times 2 &= 405 \times 3 - 5 \end{aligned}$$

Now, *observe,* in this remainder $S \times 2$ is $S \times (R - 1)$, and 405×3 is $L \times R$, and 5 is A. Hence, $S \times (R - 1) = L \times R - A$. And since $R - 1$ times the *Sum* is equal to $L \times R - A$, we have,

$S = \frac{L \times R - A}{R - 1}$.	Read,	*The sum of a geometrical series is equal to the difference, between the last term multiplied by the ratio and the first term, divided by the ratio minus 1.*

(337)

EXAMPLES FOR PRACTICE.

769. 1. The first term of a geometrical progression is 3, the ratio 4; what is the 8th term?

2. The first term of a geometrical progression is 1, and the ratio 2; what is the 12th term?

3. The extremes are 4 and 2916, and the ratio 3; what is the number of terms?

4. The extremes of a geometrical progression are 2 and 1458, and the ratio 3; what is the sum of all the terms?

5. The first term is 3, the seventeenth 196608; what is the sum of all the terms?

6. A man traveled 6 days; the first day he went 5 miles and doubled the distance each day; his last day's ride was 160 miles; how far did he travel?

7. Supposing an engine should start at a speed of 3 miles an hour, and the speed could be doubled each hour until it equalled 96 miles, how far would it have moved in all, and how many hours would it be in motion?

8. The first term of a geometrical progression is 4, the 7th term is 2916; what is the ratio and the sum of the series?

ANNUITIES.

770. An ***Annuity*** is a fixed sum of money, payable annually, or at the end of any equal periods of time.

771. The ***Amount*** or ***Final Value*** of annuity is the sum of all the payments, each payment being increased by its interest from the time it is due until the annuity ceases.

772. The ***Present Worth*** of an annuity is such a sum of money as will amount, at the given rate per cent, in the given time, to the *Amount* or *Final Value* of the annuity.

(338)

773. An ***Annuity*** at ***Simple Interest*** forms an *arithmetical progression* whose common difference is the interest on the given annuity for one interval of time.

Thus an annuity of \$400 for 4 years, at 7% simple interest, gives the following progression :

	1st Term.	*2d Term.*	*3d Term.*	*4th Term.*
(1.)	\$400	\$400 + (\$28)	\$400 + (\$28 + \$28)	\$400 + (\$28 + \$28 + \$28), or
(2.)	\$400	\$428	\$456	\$484.

Observe, there is no interest on the last payment; hence it forms the *1st Term.* The payment before the last bears one year's interest, hence forms the *2d Term ;* and so on with the other terms.

Hence all problems in annuities at *simple interest* are solved by *arithmetical progression.*

774. An ***Annuity*** at ***Compound Interest*** forms a geometrical progression whose *common multiplier* is represented by the *amount* of *\$1* for *one interval of time.*

Thus an annuity of \$300 for 4 years, at 6% compound interest, gives the following progression :

1st Term.	*2d Term.*	*3d Term.*	*4th Term.*
\$300	\$300 × 1.06	\$300 × 1.06 × 1.06	\$300 × 1.06 × 1.06 × 1.06.

Observe carefully the following:

(1.) The last payment bears no interest, and hence forms the *1st Term* of the progression.

(2.) The payment before the last, when not paid until the annuity ceases, bears interest for *one year ;* hence its amount is \$300 × 1.06 and forms the *2d Term.*

(3.) The second payment before the last, bears interest when the annuity ceases, for two years; hence its amount at compound interest is \$300 × 1.06, the amount for one year, multiplied by 1.06, equal \$300 × 1.06 × 1.06, and forms the *3d Term,* and so on with other terms.

Hence all problems in annuities at *compound interest* are solved by *geometrical progression.*

EXAMPLES FOR PRACTICE.

775. 1. What is the amount of an annuity of $200 for 6 years at 7% simple interest?

2. A father deposits $150 annually for the benefit of his son, beginning with his 12th birthday; what will be the amount of the annuity on his 21st birthday, allowing simple interest at 6%?

3. What is the present worth of an annuity of $600 for 5 years at 8%, simple interest?

4. What is the amount of an annuity of $400 for 4 years at 7%, compound interest?

5. What is the present worth of an annuity of $100 for 6 years at 6%, compound interest?

6. What is the present worth of an annuity of $700 at 8%, simple interest, for 10 years?

7. What is the amount of an annuity of $500 at 7%, compound interest, for 12 years?

8. What is the present worth of an annuity of $350 for 9 years at 6%, compound interest?

This example and the four following should be solved by applying the formulæ for geometrical progression on page 337.

10. At what rate % will $1000 amount to $1500.73 in 6 years, compound interest?

11. The amount of a certain sum of money for 12 years, at 7% compound interest, was $1126.096; what was the original sum?

12. What sum at compound interest 8 years, at 7%, will amount to $4295.465?

13. In how many years will $20 amount to $23.82032, at 6% compound interest?

MENSURATION

GENERAL DEFINITIONS.

776. A ***Line*** is that which has only length.

777. A ***Straight Line*** is a line which has the same direction at every point.

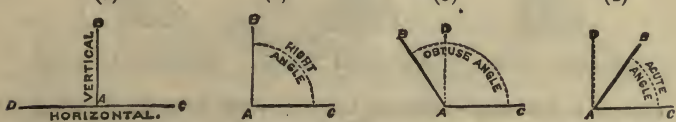
778. A ***Curved Line*** is a line which changes its direction at every point.

779. ***Parallel Lines*** are lines which have the same direction.

780. An ***Angle*** is the opening between two lines which meet in a common point, called the *vertex*.

Angles are of three kinds, thus:

(1) (2) (3) (4)



Two Right Angles. *One Right Angle.* *Obtuse Angle.* *Acute Angle.*

781. When a line *meets* another line, making, as shown in (1), two *equal angles*, each angle is a ***Right Angle***, and the lines are said to be perpendicular to each other.

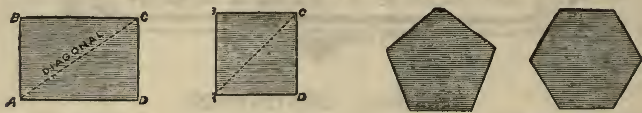
782. An ***Obtuse Angle***, as shown in (3), is greater than a right angle, and an ***Acute Angle***, as shown in (4), is less than a right angle.

Angles are read by using letters, the letter at the *vertex* being always read in the middle. Thus, in (2), we read, *the angle BAC or CAB*.

783. A ***Plane*** is a surface such that if any two points in it be joined by a straight line, every point of that line will be in the surface.

784. A ***Plane Figure*** is a plane bounded either by straight or curved lines, or by one curved line.

785. A ***Polygon*** is a plane figure bounded by straight lines. It is named by the number of sides in its boundary; thus:



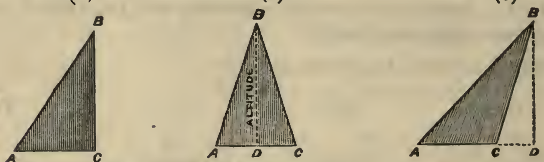
Trigon. *Tetragon.* *Pentagon.* *Hexagon*, and so on.

Observe, that a regular polygon is one that has all its sides and all its angles equal, and that the *Base* of a polygon is the side on which it stands.

786. A ***Trigon*** is a *three*-sided polygon. It is usually called a *Triangle* on account of having three angles.

Triangles are of three kinds, thus:

(1) (2) (3)



Right-angled Triangle. *Acute-angled Triangle.* *Obtuse-angled Triangle.*

Observe, a right-angled triangle has ONE *right angle*, an acute-angled triangle has THREE *acute angles*, and an obtuse-angled triangle has ONE *obtuse angle*.

Observe, also, as shown in (2) and (3), that the *Altitude* of a triangle is the perpendicular distance from one of its angles to the side opposite.

787. An ***Equilateral Triangle*** is a triangle whose three sides are equal.

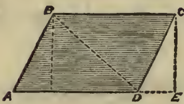
788. An ***Isosceles Triangle*** has two of its sides equal.

789. A ***Scalene Triangle*** has all of its sides unequal.

790. A ***Tetragon*** is a four-sided polygon. It is usually called a Quadrilateral.

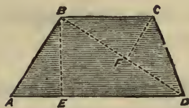
Quadrilaterals are of three kinds, thus:

(1)



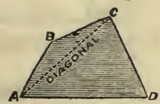
Parallelogram.

(2)



Trapezoid.

(3)



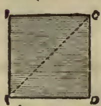
Trapezium.

Observe, that a Parallelogram has its *opposite sides* parallel, that a Trapezoid has only *two sides* parallel, and that a Trapezium has *no sides* parallel.

Observe, also, that the *Diagonal* of a quadrilateral, as shown in (1), (2) and (3), is a line joining any two opposite angles.

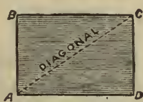
791. A ***Parallelogram*** is a quadrilateral which has its opposite sides parallel. Parallelograms are of four kinds, thus:

(1)



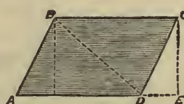
Square.

(2)



Rectangle.

(3)



Rhomboid.

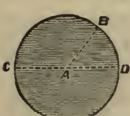
(4)

Rhombus.

Observe, that a Square has *all* its *sides* equal and *all* its *angles* right angles, that a Rectangle has its *opposite sides* equal and *all* its *angles* right angles, that a Rhomboid has its *opposite sides* equal and its angles *acute* and *obtuse,* and that a Rhombus has *all* its *sides* equal and its angles *acute* and *obtuse.*

Observe, also, that the *Altitude* of a parallelogram, as shown in (3) and (4), is the perpendicular distance between two opposite sides.

792. A ***Circle*** is a *plane* bounded by a curved line, called the *circumference,* every point of which is equally distant from a point within, called the *centre;* thus:



793. The ***Diameter*** of a circle is any straight line, as CD, passing through its centre and terminating at both ends in the circumference.

794. The ***Radius*** of a circle is any straight line, as AB, extending from the centre to the circumference.

795. The ***Perimeter*** of a polygon is the sum of all the lines which form its boundary, and of a circle the *circumference.*

796. The ***Area*** of any plane figure is the surface contained within its boundaries or boundary.

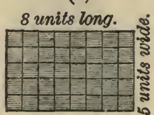
797. ***Mensuration*** treats of the method of finding the lengths of lines, the area of surfaces, and volumes of solids.

SOLUTION OF PROBLEMS.

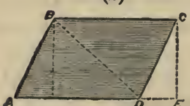
798. The solutions of problems in mensuration cannot be demonstrated except by geometry, but the general principle which underlies these solutions may be stated; thus,

The contents of any given surface or solid that can be measured can be shown to be equal to the contents of a RECTANGULAR *surface or solid, whose dimensions are equal to certain* KNOWN *dimensions of the given surface or solid, thus:*

(1)



(2)



1. *Observe,* that the number of small squares in (1) is equal to the product of the numbers denoting the length and breadth. Thus, $8 \times 5 =$ 40 small squares.

2. *Observe,* in (2), that the plane bounded by the lines FB, BC, CE, and EF, is rectangular and equal to the given parallelogram ABCD, because we have added to the right as much surface as we have taken off at the left.

Hence the contents of the parallelogram ABCD is found by taking the product of the number of units in the altitude CE or BF, and in the side BC.

3. *Observe,* again, the diagonal BD divides the parallelogram into two equal triangles, and hence the area of the triangle ABD is one-half the area of the parallelogram, and is therefore found by taking one-half of the product of the number of units in the base AD and in the altitude BF or CE.

In view of the fact that the solutions in mensuration depend upon geometry, no explanations are given. The rule, in each case, must be strictly followed.

PROBLEMS ON TRIANGLES.

799. PROB. I.—*When the base and altitude of a triangle are given, to find the area: Divide the* PRODUCT *of the* BASE *and* ALTITUDE *by 2.*

Find the area of a triangle

1. Whose base is 14 ft. and altitude 7 ft. 8 in.
2. Whose base is 3 rd. and altitude 2 rd. 7 ft.
3. Whose base is 21 chains and altitude 16 chains.
4. What is the area of a triangular park whose base is 16.76 chains and altitude 13.4 chains?
5. How many square feet of lumber will be required to board up the gable-ends of a house 30 feet wide, having the ridge of the roof 17 feet higher than the foot of the rafters?
6. How many stones, each 2 ft. 6 in. by 1 ft. 9 in. will be used in paving a triangular court whose base is 150 feet and altitude 126 feet, and what will be the expense at $.35 a square yard?

800. PROB. II.—*When the area and one dimension are given, to find the other dimension: Double the area and divide by the given dimension.*

Find the altitude of a triangle

1. Whose area is 75 square feet and base 15 feet.
2. Whose area is 264 square rods and base 24 rods.
3. Whose base is 6 ft. 1 in. and area 50 sq. ft. 100 sq. in.

Find the base of a triangle

4. Whose area is 3 A. 108 P. and altitude 28 rd.
5. Whose altitude is 2 yd. 2 ft. and area 8 sq. yd.
6. Whose area is 2 sq. rd. 19 sq. yd. 2 sq. ft. 36 sq. in. and altitude 1 rd. 1 ft. 6 in.
7. For the gable of a church 75 feet wide it required 250 stones, each 2 ft. long and 1 ft. 6 in. wide; what is the perpendicular distance from the ridge of the roof to the foot of the rafters?

801. PROB. III.—*When the three sides of a triangle are given, to find the area: From half the sum of the three sides subtract each side separately. Multiply the half sum and the three remainders together; the square root of the product is the area.*

1. Find the area of a triangle whose sides are 15, 20, 25 feet.
2. What is the area of an isosceles triangle whose base is 50 in. and each of its equal sides 35 inches?

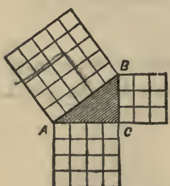
3. How many acres in a triangular field whose sides measure 16, 20, 30 rods?

4. What is the area of an equilateral triangle whose sides each measure 40 feet?

5. A piece of land in the form of an equilateral triangle requires 156 rods of fence to enclose it; how many acres are there, and what is the cost at $40 per acre?

802. PROB. IV.—*When the base and perpendicular are given in a right-angled triangle, to find the other side: Extract the square root of the sum of the squares of the base and perpendicular.*

The reason of this rule and the one in PROB. V will be seen by examining the diagram in the margin.



Observe, that the square, on the side AB opposite the right angle, contains as many small squares as the sum of the small squares in the squares on the base AC and the perpendicular BC. This is shown by geometry to be true of all right-angled triangles.

Hence, by extracting the square root of the sum of the squares of the base and perpendicular of a right-angled triangle, we have the length of the side opposite the right angle.

The side opposite the right angle is called the ***Hypothenuse.***

Find the hypothenuse of a right-angled triangle

1. Whose base is 40 ft. and perpendicular 16 ft.

2. Whose base is 15 ft. and perpendicular 36 ft.

3. A tree 104 ft. high stands upon the bank of a stream 76 feet wide; what is the distance of a man upon the opposite bank from a raven upon the top of the tree?

4. A and B start from one corner of a field a mile square, traveling at the same rate; A follows the fence around the field, and B proceeds directly across to the opposite corner; when B reaches the corner, how far will he be from A?

5. What is the length of the shortest rope by which a horse may be tied to a post in the middle of a field 20 rods square, and yet be allowed to graze upon every part of it?

803. Prob. V.—*When the base or perpendicular is to be found: Extract the square root of the difference between the square of the hypothenuse and the square of the given side.*

Find the base of a right-angled triangle

1. Whose hypothenuse is 40 ft. and perpendicular 15 feet.
2. Whose perpendicular is 20 feet and hypothenuse 45 feet.
3. Bunker Hill monument is 220 feet high; a man 360 feet from the base shot a bird hovering above the top; the man was 423 feet from the bird; how far was the bird from the top of the monument?
4. A ladder 35 feet long reaches from the middle of the street to a window 28 feet high; how wide is the street?
5. The lower ends of two opposite rafters are 48 feet apart and the length of each rafter is 30 feet; what is the elevation of the ridge above the eaves?

PROBLEMS ON QUADRILATERALS.

804. Prob. VI.—*To find the area of a parallelogram: Multiply the base by the altitude.*

1. Find the area of a parallelogram whose base is 3 ft. 9 in. and altitude 7 ft. 8 in.; whose altitude is 2 yd. 5 in. and base 3 yd. 6 in.
2. How many acres in a piece of land in the form of a parallelogram whose base is 9.86 ch. and altitude 7.5 ch?
3. How many square feet in the roof of a building 85 ft. long, and whose rafters are each 16 ft. 6 in. long?
4. The base of a rhombus is 9 ft. 8 in. and its altitude 3 ft.; how many square feet in its surface?

805. Prob. VII.—*To find the area of a trapezoid: Multiply one-half of the sum of the parallel sides by the altitude.*

Find the area of a trapezoid

1. Whose parallel sides are 15 and 25 feet and altitude 11 feet.
2. Whose parallel sides are 8 and 11 inches and altitude 6 inches.
3. How many square feet in a board 1 ft. 4 in. wide, one side of which is 32 ft. long and the other side 34 feet long?
4. One side of a field measures 47 rods, the side opposite and parallel to it measures 39 rods, and the distance between the two sides is 15 rods; how much is it worth at $40 per acre?

806. PROB. VIII.—*To find the area of a trapezium: Multiply the diagonal by half the sum of the perpendiculars to it from the opposite angles.*

Refer to diagram (3) in (**790**) and find the area of a trapezium

1. Whose diagonal is 45 in. and perpendiculars to this diagonal 11 inches and 9 inches.

2. Whose diagonal is 16 feet and perpendiculars to this diagonal 7 feet and 6 feet.

3. Whose diagonal is 37 ft. 6 in. and perpendiculars to this diagonal 7 ft. 4 in. and 8 ft. 8 in.

4. How many acres in a field in the form of a trapezium whose diagonal is $\frac{1}{4}$ mi. and the perpendiculars to this diagonal 5 ch. and 6 ch.?

807. PROB. IX.—*To find the diameter of a circle: Divide the circumference by 3.1416.*

To find the circumference: Multiply the diameter by 3.1416.

1. Find the diameter of a circle whose circumference is 94.248 inches; whose circumference is 78.54 feet.

2. Find the circumference of a circle whose diameter is 14 inches; whose radius is 9 inches.

3. What will it cost to fence a circular park 3 rods in diameter, at $4.80 per rod.

4. How many miles does the earth pass over in its revolution around the sun, its distance from the sun being 95,000,000 miles?

808. PROB. X.—*To find the area of a circle: Multiply $\frac{1}{4}$ of its diameter by the circumference; or, Multiply the square of its diameter by .7854.*

1. What is the area of a circle whose diameter is 20 feet? Whose diameter is 42 inches? Whose circumference is 157.08 feet?

2. What is the area of the largest circular plot that can be cut from a field 135 feet square? How much must be cut off at the corners in making this plot? How much less will it cost to fence this than the square, at $2.50 a rod?

3. The distance around a circular park is $1\frac{1}{4}$ miles. How many acres does it contain?

809. Prob. XI.—*To find the diameter when the area of a circle is given: Extract the square root of the quotient of the area divided by .7854.*

Observe, that when the diameter is found, the circumference can be found by multiplying the diameter by 3.1416 (**807**).

1. What is the diameter of a circle whose area is 50.2656 sq. ft.?
2. What is the circumference of a circle whose area is 153.9384 square feet?
3. The area of a circular lot is 19.635 square rods; what is its diameter?
4. The area of a circle is 113.0976 sq. in.; what is its circumference?
5. How many rods of fence will be required to inclose a circle whose area is $314\frac{4}{25}$ square rods?
6. What is the radius of a circle whose area is 804.2496 sq. in.?

PROBLEMS ON SOLIDS OR VOLUMES.

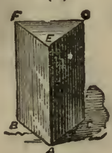
810. A ***Solid*** or ***Volume*** has three dimensions: length, breadth, and thickness.

The boundaries of a solid are planes. They are called *faces,* and their intersections *edges.*

811. A ***Prism*** is a solid or volume having two of its faces equal and parallel polygons, and its other faces parallelograms,

Observe, a prism is named by the number of sides in its equal and parallel faces or *bases;* thus:

(1)

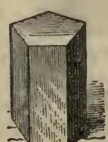


Triangular Prism.

(2)

Quadrangular Prism.

(3)



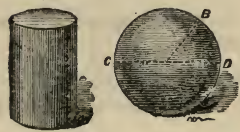
Pentagonal Prism.

Observe, a *Prism* whose parallel faces or *bases* are parallelograms, as shown in (2), is called a ***Parallelopipedon.***

Observe, also, that the *Altitude* of a prism is the perpendicular distance between its *bases.*

812. A ***Cylinder***, as shown in (1), is a round solid or volume having *two equal* and parallel circles as its *bases*.

(1) (2)



1. *Observe*, that the altitude of a cylinder is the perpendicular distance between the two circles forming its *bases*.

2. *Observe*, also, that a cylinder is conceived to be generated by revolving a rectangle about one of its sides.

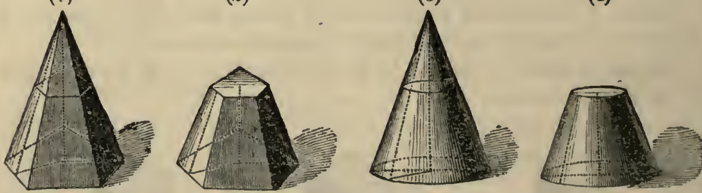
813. A ***Sphere***, as shown in (2), is a solid or volume bounded by a curved surface, such that all points in it are equally distant from a point within, called the *centre*.

814. The ***Diameter*** of a sphere is a line, as CD in (2), passing through its centre and terminating at both ends in the surface.

815. The ***Radius*** of a sphere is a line drawn from the centre to any point in the surface.

816. A ***Pyramid***, as shown in (1), is a solid or volume having as its base any polygon, and as its *other faces* triangles, which meet in a common point called the *vertex*.

(1) (2) (3) (4)



Pyramid. *Frustum.* *Cone.* *Frustum.*

817. A ***Cone***, as shown in (3), is a solid or volume whose base is a circle and whose convex surface tapers uniformly to a point, called the *vertex*.

1. *Observe*, that the *Altitude* of a pyramid or cone is the perpendicular distance between the vertex and the base.

2. *Observe*, also, that the *Slant Height* of a pyramid is the perpendicular distance between the vertex and one of the sides of the base; and of a cone the distance between the vertex and the circumference of the base.

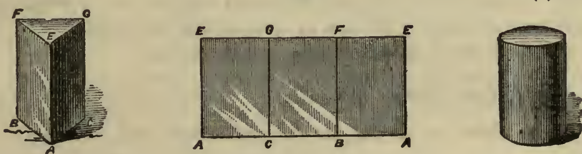
818. A ***Frustum*** of a pyramid or cone, as shown in (2) and (4), is the part which remains after cutting off the top by a plane parallel to the base.

819. Prob. XII.—*To find the convex surface of a prism or cylinder: Multiply the* PERIMETER *of the base by the* ALTITUDE.

To find the ENTIRE SURFACE *add the area of the bases.*

The reason of this rule may be shown thus:

(1) (2) (3)



Observe, that if the three faces of the prism in (1) are marked out side by side, as shown in (2), we have a rectangle which is equal to the convex surface in the prism.

Observe, also, that the surface of the cylinder in (3) may be conceived as spread out, as shown in (2); hence the reason of the rule.

Find the area of the convex surface

1. Of a prism whose altitude is 8 feet, and its base a triangle, the sides of whose base measures 4 ft., 3 ft., 3 ft. 6 in.

2. Of a cylinder whose altitude is 4 ft. 9 in. and the circumference o its base 7 ft. 8 in.

3. Of a prism whose altitude is 9 inches, and its base a hexagon, each side of which is $2\frac{1}{2}$ inches.

4. Find the entire surface of a parallelopipedon 9 ft. long, 5 ft. 6 in. wide, and 3 ft. high.

5. Find the entire surface of a cylinder 9 ft. high, the diameter of whose base is 8 ft.

820. Prob. XIII.—*To find the volume of any prism or cylinder: Multiply the area of the base by the altitude.*

1. Find the volume of a triangular prism whose altitude is 15 ft. and the sides of the base each 4 ft.

2. What is the volume of a triangular prism whose altitude is 28 ft., and the sides of its base 6 ft., 7 ft., 5 ft. respectively.

3. What is the volume of a parallelopipedon 15 ft. long, 12 ft. high, 10 ft. wide?

4. Find the contents of a cylinder whose altitude is 19 ft. and the diameter of its base 4 ft.

5. What is the value of a piece of timber 15 in. square and 50 feet long, at 40 cents a cubic foot?

6. A log is 30 ft. long and its diameter is 16 in.; how many cubic feet does it contain?

821. Prob. XIV.—*To find the convex surface of a pyramid or cone: Multiply the perimeter of the base by one-half the slant height.*

To find the entire surface, add the area of the base.

Find the convex surface of a cone

1. Whose base is 19 in. in circumference, and the slant height 12 inches.

2. Whose slant height is 15 feet, and the diameter of the base 10 feet.

Find the convex surface of a pyramid

3. Whose base is 3 ft. 6 in. square, and the slant height 5 ft.

4. Whose slant height is 19 ft., and the base a triangle whose sides are 12, 14, 8 ft.

Find the entire surface of a pyramid

5. Whose slant height is 45 feet, and the base a rectangle 7 ft. long and 8 ft. wide.

6. Whose slant height is 56 in., and its base a triangle each of whose sides is 6 in.

Find the entire surface of a cone

7. Whose slant height is 42 feet, and the circumference of the base 31.416 ft.

8. Whose slant height is 75 in., and the diameter of the base 5 inches.

822. Prob. XV.—*To find the volume of a pyramid or cone: Multiply the area of the base by one-third the altitude.*

Find the volume of a cone

1. Whose altitude is 24 ft., and the circumference of the base 6.2832 feet.

2. Whose altitude is 12 ft., and the diameter of the base 4 ft.

Find the volume of a pyramid

3. Whose altitude is 15 feet, and its base 4 feet square.

4. Whose altitude is 18 in., and the base a triangle 8 in. on each side.

5. Whose altitude is 45 ft., and its base a rectangle 15 feet by 16 feet.

823. PROB. XVI.—*To find the convex surface of a frustum of a pyramid or cone: Multiply the sum of the perimeters or circumferences by one-half the slant height.*

To find the entire surface, add the area of both the bases.

1. What is the convex surface of a frustum of a triangular pyramid whose slant height is 6 feet, each side of the greater base 3 feet, and of the less base 2 feet?

2. What is the convex surface of a frustum of a cone whose slant height is 9 inches, and the circumference of the lower base 17 inches, and of the upper base 13 inches?

3. Find the entire surface of a frustum of a pyramid whose slant height is 14 feet, and its bases triangles, each side of the larger base being 8 feet, and of the smaller base 6 feet.

4. Find the entire surface of a frustum of a cone whose slant height is 27 feet, the circumference of the greater base being 37.6992 feet, and of the less base 31.416 feet.

824. PROB. XVII.—*To find the volume of a frustum of a pyramid or cone: To the sum of the areas of both bases add the square root of their product and multiply the result by one-third of the altitude.*

1. Find the volume of a frustum of a square pyramid whose altitude is 6 feet, and each side of the lower base 16 feet, and of the upper base 12 feet.

2. How many cubic feet in a frustum of a cone whose altitude is 9 feet, the diameters of its bases 8 feet and 6 feet.

3. How many cubic feet in a section of a tree-trunk 20 feet long, the diameter of the lower base being 18 inches, and of the upper base 12 inches?

4. One of the big trees of California is 32 feet in diameter at the foot of the tree; how many cubic feet in a section of this tree 90 feet high, the upper base being 20 feet in diameter?

5. A granite rock, whose form is a frustum of a triangular pyramid, is 40 feet high, the sides of the lower base being 30 feet each, and of the upper base 16 feet each. How many cubic feet in the rock.

825. PROB. XVIII.—*To find the surface of a sphere: Multiply the diameter by the circumference of a great circle of the given sphere.*

1. Find the surface of a sphere whose diameter is 8 feet.
2. What is the surface of a globe 9 in. in diameter?
3. How many square feet in the surface of a sphere 45 feet in diameter?
4. What is the surface of a globe whose radius is 1 ft. 6 in.?
5. How many square inches in the surface of a globe 5 inches in diameter?

826. PROB. XIX.—*To find the volume of a sphere: Multiply the surface by one-sixth of the diameter.*

1. Find the volume of a sphere whose diameter is 20 inches.
2. How many cubic yards in a sphere whose diameter is 3 yards?
3. Find the solid contents of a globe 2 ft. 6 in. in diameter.
4. How many cubic feet in a globe 9 inches in diameter?
5. Find the volume of a globe whose radius is 4 inches.

827. PROB. XX.—*To find the capacity of casks in gallons: Multiply the number of inches in the length by the square of the number of inches in the mean diameter, and this product by .0034.*

Observe, that the mean diameter is found (nearly) by adding to the head diameter $\frac{2}{3}$, or if the staves are but slightly curved, $\frac{3}{5}$ of the difference between the head and bung diameters.

. The process of finding the capacity of casks is called *Gauging.*

1. How many gallons in a cask whose head diameter is 20, bung diameter 26 inches, and its length 30 inches?
2. How many gallons will a cask hold whose head diameter is 21 inches, bung diameter 30 inches, and length 42 inches?
3. What is the volume of a cask whose diameters are 18 and 24 inches respectively, and the length 32 inches?
4. A cask slightly curved is 40 inches long, its head diameter being 22 inches, and its bung diameter 27 inches; how many gallons will it hold?

REVIEW AND TEST EXAMPLES

828. In using this set of review and test examples, the following suggestions should be carefully regarded:

1. The examples cover all the important subjects in arithmetic, and are designed as a test of the pupil's strength in solving difficult problems and of his knowledge of principles and processes.

2. The teacher should require the pupil to master the thought expressed in each example before attempting a solution.

To do this he must notice carefully the meaning of each sentence, and especially the technical terms peculiar to arithmetic; he must also locate definitely the business relations involved.

3. When the solutions are given in class, the teacher should require the pupils to state clearly:

(*a*). *What is given and what is required in each example.*

(*b*). *The relations of the given quantities from which what is required can be found.*

(*c*). *The steps that must be taken in their order, and the processes that must be used to obtain the required result.*

In making these three statements, no set form should be used; each pupil should be left free to pursue his own course and give his own solution. Clearness, accuracy, and brevity should be the only conditions imposed.

When the work is written on a slate, paper, or blackboard, neatness and a logical order in arranging the steps in the solution should be invariably required.

1. A gentleman held a note for \$1643.20, payable in 8 mo., without interest. He discounted the note at 8% for ready cash, and invested the proceeds in stock at \$104 per share. How many shares did he purchase? *Ans.* 15 shares.

2. Three daughters, Mary, Jane, and Ellen, are to share an estate of \$80000, in the proportion of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$, respectively; but Ellen dies, and the whole amount is to be divided in a proper proportion between the other two. What share does each receive? *Ans.* Mary, \$48,000; Jane, \$32,000.

3. What must be the dimensions of a rectangular bin that will hold 350 bushels of grain, if its length is twice its width, and its width twice its depth?

Ans. Length, 15.16 + ft.; width, 7.58 + ft.; depth, 3.79 + ft.

4. A Chicago merchant bought 800 barrels of flour at \$7 per barrel, and sent it to New York, paying 9% of the cost for freight and other charges; his agent sold it at an advance of 25% on the original cost and charged 3% commission. What was the net gain? *Ans.* \$686.

5. What sum invested in railroad stock paying 7% annually will yield a quarterly dividend of \$325.50? *Ans.* \$18,600.

6. A, B, and C together can dig a ditch in 4 days. A can dig it alone in 10 days; B can dig it alone in 12 days. How long will it take C to do the work alone? *Ans.* 15 days.

7. A person owning $7\frac{1}{2}$ acres in the form of a rectangle 3 times as long as it is wide, wishes to tether his horse to a stake by the shortest rope that will allow him to graze upon any part of the field. What is the length of rope required?

Ans. 31.62 + rd.

8. A cubical block contains 64 cubic feet; what is the distance from one corner to the opposite diagonal corner?

Ans. 6.92 + feet.

9. A farmer bought a horse, wagon, and plow for \$134; the horse cost $\frac{7}{5}$ as much as the wagon, and the plow $\frac{1}{5}$ as much as the horse. What was the cost of each?

Ans. Horse, \$70; wagon, \$50; plow, \$14.

(356)

10. A grocer mixed 15 pounds of Hyson tea with 9 pounds of Gunpowder tea, and sold it at \$.95 per pound, thus gaining 25% on the original cost. If a pound of the Gunpowder cost 16 cents more than a pound of the Hyson, what was the cost of each per pound? *Ans.* Gunpowder, \$.86; Hyson, \$.70.

11. A farmer has a cornfield whose width is to its length as 3 to 4, and contains $4\frac{4}{5}$ acres. The hills of corn, supposing them to occupy only a mathematical point, are 2 feet apart, and no hill is nearer the fence than 3 feet. What must he pay a man to hoe his corn, at the rate of \$.50 per day, if he hoes 750 hills in a day? *Ans.* $\$34.23\frac{7}{15}$.

12. The duty at 20% *ad valorem* on a quantity of tea in chests, each weighing 75 pounds gross, and invoiced at \$.70 per pound, was \$6,552, tare being 4%. How many chests were imported? *Ans.* 650.

13. A room is 22 feet long, 18 feet wide, and 14 feet high. What is the distance from one of the lower corners to the opposite upper corner? *Ans.* 31.68 + ft.

14. A farmer sold 85 sheep at \$2, \$2.20 and \$2.80 per head, and thus realized an average price of \$2.40 per head. What number of each did the lot contain?

Ans. 17 at \$2; 34 at \$2.20; 34 at \$2.80.

15. If the ratio of increase of a certain crop is 3, and a man begins by planting 5 bushels, using all the crop for seed the next year, and so on; what will be his crop the seventh year?

Ans. 10,935 bushels.

16. A can do a piece of work in $4\frac{1}{2}$ days that requires B 6 days and C 9 days to do the same amount of work. In how many days can they do it working together? *Ans.* 2 days.

17. A father divided his property among his wife and four sons, directing that his wife should have \$8 as often as the oldest son \$6, the second son \$3 as often as the wife \$5, the youngest son \$12 as often as the third \$14, the third son \$5 as often as the oldest \$7. The youngest son received \$4,500; what was the value of the father's property? *Ans.* \$32,780.

18. If 72 men dig a trench 20 yd. long, 1 ft. 6 in. broad, and 4 ft. deep in 3 days of 10 hours each, how many men would be required to dig a trench 30 yd. long, 2 ft. 3 in. broad, and 5 ft. deep in 15 days of 9 hours each? *Ans.* 45 men.

19. Samuel Wells paid $3\frac{1}{2}$ times as much for a house as for a barn; had the barn cost him 6% more, and the house 8% more, the whole cost would have been $7260. What was the actual cost? *Ans.* $6,750.

20. Change $\frac{1}{13}$ of $\dfrac{1}{1+\dfrac{\frac{1}{3}}{3+\frac{1}{4}}}$ to a simple fraction, and reduce to lowest terms. *Ans.* $\frac{3}{43}$.

21. A person sells out $4,500 of 4% stock at 95, and invests the proceeds in bank stock at 80, which pays an annual dividend of $2\frac{3}{5}$%. How much is the gain or loss per annum? *Ans.* $37.50 loss.

22. James Griswold bought $\frac{6}{7}$ of a ship; but the property having fallen in value 8%, he sells 14% of his share for $2760. What was the value of the ship at first? *Ans.* $25,000.

23. A gentleman willed to the youngest of his five sons $2000, to the next a sum greater by one half, and so on, the oldest receiving $10,125, thus disposing of his entire estate. What was the gentleman worth? *Ans.* $26,375.

24. I sent $7847 to my agent in New Orleans, who purchased sugar at an average price of $16 per barrel; he charged $3\frac{1}{4}$% commission. How many barrels did he buy? *Ans.* 475.

25. Bought 3,000 bushels of wheat at $1.50 per bushel. What must I ask per bushel that I may fall 20% on the asking price and still make 16%, allowing 10% of the sales for bad debts? *Ans.* 2.41\frac{2}{3}$.

26. Henry Swift has $6,000 worth of 5% stock; but not being satisfied with his income, he sells at 96 and invests in stock paying $4\frac{1}{2}$%, which gives him an income greater by $45.60. At what price did he purchase the latter stock? *Ans.* At 75.

27. A drover bought a number of horses, cows, and sheep for \$3,900. For every horse he paid \$75, for each cow he paid $\frac{3}{5}$ as much as for a horse, and for each sheep $\frac{1}{5}$ as much as for a cow. He bought 3 times as many sheep as cows, and twice as many cows as horses; how many did he buy of each?

Ans. 20 horses; 40 cows; 120 sheep.

28. I shipped to my agent in Buffalo a quantity of flour, which he immediately sold at \$7.50 per barrel. I then instructed him to purchase goods for me at a commission of $3\frac{1}{2}$%; he charged me 4% commission for selling, and received as his whole commission \$800. How many barrels of flour did I send him? *Ans.* 1,472.

29. Adam Gesner gave his note for \$1,250, and at the end of 3 years 4 months, and 21 days, paid off the note, which then amounted to \$1504.375; reckoning only simple interest, what was the rate %? *Ans.* 6%.

30. A hound in pursuit of a fox runs 5 rods while the fox runs 3 rods, but the fox had 60 rods the start. How far must the hound run before he overtakes the fox? *Ans.* 150 rods.

31. A man divided his property, amounting to \$15,000, among his three sons, in such a manner that their shares put at 6% simple interest should all amount to the same sum when they were 21 years old; the ages of the children were respectively 6 yr., 9 yr., and 13 yr. What was the share of each?

Ans. Oldest, \$5683.082 +; second, \$4890.094 +; youngest, \$4426.822 +.

32. A certain garden is $12\frac{3}{4}$ rods long, and $9\frac{1}{4}$ rods wide. At $2\frac{1}{2}$ cents per cubic foot, what will it cost to dig a ditch around outside it that shall be $3\frac{1}{2}$ feet wide and 4 feet deep?

Ans. \$258.03$\frac{3}{4}$.

33. A farmer sells a merchant 40 bushels of oats at \$.60 per bushel and makes 20%; the merchant sells the farmer 4 yards of broadcloth at \$3.75 per yard, 15 yards of calico at 8 cents per yard, and 40 yards of cotton cloth at 12 cents per yard,

and makes a profit of 25%. Which gains the more by the trade and how much? *Ans.* Merchant gains \$.20.

34. A triangular cornfield consisting of 146 rows, has 437 hills in the longest row, and 2 in the shortest; how many corn hills in the field? *Ans.* 32047 hills.

35. What is the value of

$$\left[\frac{44\frac{3}{8} \text{ of } .056 - 3.04 \text{ of } \frac{4}{19}}{(8 - 2.4) + \frac{2}{9} \text{ of } 3\frac{3}{5}}\right] \times \left[\frac{\left(3\frac{1}{13} \text{ of } \frac{1\frac{6}{7}}{.4}\right) - 1}{\frac{.38}{.76} + 5\frac{2}{7}} + 2\right] \times \frac{285}{551}?$$

Ans. $\frac{41}{64}$.

36. Bought 60 barrels of flour at \$8.50 per barrel, but on account of its having been damaged, one-half of it was sold at a loss of 10%, and the remainder at \$9 per barrel. What % was lost by the operation? *Ans.* $2\frac{1}{17}$%.

37. A room 22 feet long, 16 feet wide, and 9 feet high, contains 4 windows, each of which is $5\frac{1}{2}$ feet high and 3 feet wide; also two doors, 7 feet in height and $3\frac{1}{2}$ feet in width. The base-boards are $\frac{3}{4}$ of a foot wide. What will it cost to plaster and paper the room, if the plastering cost 16 cents per square yard and the papering 10 cents? *Ans.* \$21.20$\frac{1}{18}$.

38. Two persons, A and B, each receive the same salary. A spends $76\frac{1}{2}$% of his money, and B spends as much as would equal $45\frac{1}{2}$% of what both received. At the end of the year they both together have left \$276.25; what part of it belongs to A, and what to B? *Ans.* A, \$199.75; B, \$76.50.

39. Two persons 280 miles apart travel toward each other until they meet, one at the rate of 6 miles per hour, the other at the rate of 8 miles per hour. How far does each travel?

Ans. First, 120 miles; second, 160 miles.

40. James Welch has a debt in Chicago amounting to \$4489.32. For what sum must a note be drawn at 90 days, that when discounted at 6% at a Chicago bank, will just pay the debt? *Ans.* \$4560.

41. I went to the store to buy carpeting, and found that any one of three pieces, width respectively $1\frac{1}{4}$, $1\frac{1}{2}$, and $2\frac{1}{2}$ yards, would exactly fit my room without cutting anything from the width of the carpet. What is the width of my room?

Ans. $22\frac{1}{2}$ feet.

42. If 10 horses in 25 days consume $3\frac{1}{3}$ tons of hay, how long will $6\frac{2}{3}$ tons last 6 horses, 12 cows, and 8 sheep, if each cow consumes $\frac{3}{4}$ as much as a horse, and each sheep $\frac{2}{3}$ as much as a cow? *Ans.* 25 days.

43. The distance between the opposite corners of a square field is 60 rods; how many acres in the field?

Ans. 11 A. 40 sq. rd.

44. At $225 per ton, what is the cost of 17 cwt. 2 qr. 21 lb. of sugar? *Ans.* 199.237\frac{1}{2}$.

45. A drover bought 12 sheep at $6 per head; how many must he buy at $9 and $15 per head, that he may sell them all at $12 per head and lose nothing?

Ans. 1 at $9, 25 at $15.

46. Three men bought a field of grain in circular form containing 9 A., for which they paid $192, of which the first man paid $48, the second $64, the third $80. They agreed to take their shares in the form of rings; the first man mowing around the field until he got his share, then the second, and so on. What depth of ring must each man mow to get his share of the grain? *Ans.* 1st man, 2.86 + rd.;
2d man, 4.73 + rd.;
3d man, 13.81 + rd.

47. A young man inherited an estate and spent 15% of it during the first year, and 30% of the remainder during the second year, when he had only $9401 left. How much money did he inherit? *Ans.* $15800.

48. Mr. Webster bought a house for $6750, on a credit of 10 months; after keeping it for 4 months, he sold it for $7000 on a credit of 8 months. Money being worth 6%, what was his net cash gain at the time of the sale? *Ans.* $177.37 +.

49. A and B can do a piece of work in 18 days; A can do $\frac{4}{5}$ as much as B. In how many days can each do it alone?
Ans. A, $40\frac{1}{2}$ days; B, $32\frac{2}{5}$ days.

50. If $\frac{6}{7}$ of a farm is worth \$7524 at \$45 per acre, how many acres in the whole farm? *Ans.* $195\frac{1}{15}$ A.

51. A person paid \$1450 for two building lots, the price of one being 45% that of the other; he sold the cheaper lot at a gain of 60%, and the dearer one at a loss of 25%. What % did he gain or lose on the whole transaction? *Ans.* $1\frac{11}{29}$% gain.

52. A certain sum of money, at 8% compound interest for 10 years, amounted to \$2072.568. What was the amount at interest? *Ans.* \$960.

53. There are two church towers, one 120 feet high, and the other 150 feet. A certain object upon the ground between them is 125 feet from the top of the first and 160 feet from the top of the second; how far apart are their tops?
Ans. 95.50 + feet.

54. A farmer sold to a merchant 80 bushels of wheat at \$1.90 per bushel, 70 bushels of barley at \$1.10, and 175 bushels of oats at \$.75. He took in payment a note for 5 months, and immediately got it discounted at bank at 6%; how much money did he receive? *Ans.* \$351.06 +.

55. There is a pile of 100 railroad ties, which a man is required to carry, one by one, and place in their proper places, 3 feet apart; supposing the first to be laid 3 feet from the pile, how far will the man travel in placing them all?
Ans. 30300 feet.

56. Sound travels at the rate of 1142 feet a second. If a gun be discharged at a distance of $4\frac{1}{2}$ miles, how much time will elapse, after seeing the flash, before the report is heard?
Ans. $20\frac{458}{571}$ sec.

57. If a company of 480 men have provisions for 8 months, how many men must be sent away at the end of 6 months, that the remaining provisions may last 6 months longer?
Ans. 320 men.

58. The first year a man was in business he cleared $300, and each year his profit increased by a common difference; the fourteenth year he made $950. How much did he make the third year? *Ans.* $400.

59. What number is that, which being increased by $\frac{1}{3}$, $\frac{1}{5}$, and $\frac{2}{9}$ of itself, and diminished by 25, equals 291?

Ans. 180.

60. At what time between 5 and 6 will the hour and minute hands of a clock be together?

Ans. $27\frac{3}{11}$ min. past five.

61. A field whose length is to its width as 4 to 3 contains 2 A. 2 R. 32 rd.; what are its dimensions?

Ans. Length, 24 rd.; width, 18 rd.

62. Three persons formed a partnership with a capital of $4600. The first man's stock was in trade 8 months and gained $752; the second man's stock was in trade 12 months, and gained $600; and the third man had his stock in 16 months, and gained $640. What was each man's stock?

Ans. First, $2350; second, $1250; third, $1000.

63. How many thousand shingles, 18 inches long and 4 in. wide, lying $\frac{1}{3}$ to the weather, are required to shingle the roof of a building 54 feet long, with rafters 22 feet long, the first row of shingles being double? *Ans.* $14\frac{29}{50}$.

64. Employed an agent who charges 4% commission to collect a bill of $550. He succeeded in obtaining only 85%; how much did I receive? *Ans.* $448.80.

65. A and B entered into partnership and gained $4450.50. A put in enough capital to make his gain 15% more than B's; what was each man's share of the gain?

Ans. A, $2380.50; B, $2070.

66. A building is 75 feet long and 44 feet wide, and the elevation of the roof is 14 feet. How many feet of boards will be required to cover the roof, if the rafters extend 2 feet beyond the plates, and the boarding projects $1\frac{1}{2}$ feet at each end, and $\frac{1}{5}$ allowed for waste? *Ans.* 5474.97 + feet.

67. A circular court is laid with 19 rows of flat stones, each row forming a complete circle; the outside row is 39 inches wide, and the width of each row diminishes 2 inches as it nears the centre. What is the width of the innermost row?

Ans. 3 inches.

68. Reduce $\dfrac{\frac{2}{3} \text{ of } \frac{5}{7} \text{ of } \frac{35}{9}}{\frac{1}{7} \text{ of } \frac{2}{3} \text{ of } \frac{84}{81}}$ to a simple fraction, and take the result from the sum of $10\frac{3}{4}$, $3\frac{9}{10}$, and $7\frac{24}{25}$. *Ans.* $3\frac{43}{50}$.

69. Bought 75 yards of cloth at 10% less than the first cost, and sold it at 10% more than the first cost and gained $25. What was the first cost per yard? *Ans.* $\$1.66\frac{2}{3}$.

70. A grain merchant bought 7500 bushels of corn at $1.35 per bushel, 5450 bushels of oats at $.80, 3250 bushels of barley at $.95, paid $225 for freight and $170 for storage; he immediately sold it an advance of 20% on the entire cost, on a credit of 6 months. What % did he gain at the time of the sale, money being worth 8%? *Ans.* 15 + %.

71. A farmer employs a number of men and 8 boys; he pays the boys $.65 and the men $1.10 per day. The amount that he paid to all was as much as if each had received $.92 per day; how many men were employed? *Ans.* 12 men.

72. S. Howard can mow 6 acres in 4 days, and his son can mow 7 acres in 5 days. How long will it take them both to mow $49\frac{5}{7}$ acres? *Ans.* $17\frac{1}{7}$ days.

73. I lent a friend $875, which he kept 1 year and 4 months. Some time afterward I borrowed of him $350; how long must I keep it to balance the favor? *Ans.* 3 yr. 4 mo.

74. Find the difference between the surface of a floor 80 ft. 9 in. long and 65 ft. 6 in. broad, and the sum of the surfaces of three others, the dimensions of each of which are exactly one-third of those of the other.

Ans. 391 sq. yd. 7 sq. ft. 12 sq. in.

75. A tree broken off 24 feet from the ground rests on the stump, the top touching the ground 30 feet from the foot of the tree. What was the height of the tree? *Ans.* 62.41 + ft.

76. Two persons entered into partnership for trading. A put in \$245 for 375 days and received $\frac{5}{7}$ of the gain; the number of dollars that B put in was equal to the number of days it was employed in trade. What was B's capital?

Ans. \$350.

77. How many square feet of boards $1\frac{1}{2}$ inches thick will be required to make a box, open at the top, whose inner dimensions are 6 feet long, 4 feet wide, and 3 feet deep?

Ans. $88\frac{1}{16}$ sq. ft.

78. A farmer having 80 acres of land, worth \$55 an acre, wishes to buy enough more at \$50 and \$65, respectively, so that the value of his land shall average \$60 an acre. How much of each must he buy? *Ans.* 1 A. at \$50; 82 A. at \$65.

79. What is the amount of an annuity of \$700 for 8 years, at 6% compound interest? *Ans.* $\$6928.22\frac{2}{5}$.

80. What must be the price of stock yielding $5\frac{1}{4}$%, that will yield the same profit as $4\frac{1}{2}$% stock at 96? *Ans.* 112.

81. A person after spending $\frac{1}{3}$ and $\frac{1}{4}$ of his money and \$20, had \$80 left. What had he at first? *Ans.* \$240.

82. James Harper has a large jewelry store, which with its contents he insures in the Continental Insurance Company for $\frac{3}{5}$ of its estimated value, at $3\frac{1}{4}$%. This Company immediately insures $\frac{1}{2}$ of its risk in the Astor Company, at $2\frac{1}{2}$%. After two years and a half, the store and its contents were destroyed by fire, when it was found that the Astor Company lost \$2925 more than the Continental Company. Reckoning 6% simple interest on the premiums that the owner paid, what would be his entire loss? *Ans.* \$78815.75.

83. A drover sold 42 cows and 34 oxen for \$3374, receiving \$21 per head more for the oxen than for the cows. What did he receive for each per head? *Ans.* \$35 for cows;
\$56 for oxen.

84. A certain room is 27 ft. 5 in. long, 14 ft. 7 in. wide, and 12 ft. 10 in. high. How much paper $\frac{7}{8}$ of a yard wide will be required to cover the walls? *Ans.* 136 yd. 2 ft. 8 in.

85. The area of a triangular field is 6 A. 36 rd.; the base is 64 rods. What is the perpendicular distance from the base to the angle opposite? *Ans.* $31\frac{1}{8}$ rods.

86. If the width of a building is 50 feet, and the length of the rafters 30 feet, what will it cost to board the gable ends, at \$.18 per square yard? *Ans.* \$16.58+.

87. What is the solidity of the largest ball that can be cut out of a cubical block whose sides are 6 inches square?

Ans. 113.0976 cu. in.

88. A privateer took a prize worth £348 15s., which was to be divided among 1 captain, 3 mates, and 27 privates, so that a private should have one share, a mate twice as much as a private, and the captain 6 times as much as a mate. What was the share of each?

Ans. Private, £7 15s.; mate, £15 10s.; captain, £93.

89. The width of a certain building is 38 feet, and the elevation of the roof is 16 feet; how many square feet of boards will be required to cover the gable ends? *Ans.* 608 sq. ft.

90. The length of one side of a field in the form of an equilateral triangle is 40 rods. How many acres does the field contain, and what would it cost to fence it, at \$.65 per rod?

Ans. 4 A. 52.8+ sq. rd.; \$78.

91. Change $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\frac{1}{2\frac{1}{2}}+\frac{1}{3\frac{1}{2}}+\frac{1}{4\frac{1}{2}}}$ to a simple fraction, and reduce to lowest terms. *Ans.* $1\frac{1}{88}$.

92. Two merchants, Sanford and Otis, invested equal sums in trade. Sanford gained a sum equal to $\frac{1}{5}$ of his stock and \$24 more, and Otis lost \$444; then Otis had just $\frac{1}{4}$ as much money as Sanford. What did each invest? *Ans.* \$675.

93. Henry Norton sold his farm for \$13270, \$5000 of which was to be paid in 6 months, \$4000 in one year and 6 months, and the rest in 2 years. What was the net cash value of his farm, money being worth 6%? *Ans.* \$12336.59+.

94. At what time between 10 and 11 o'clock will the hands be directly opposite? *Ans.* $21\frac{9}{11}$ min. past ten.

95. How much better is it to invest \$15000 in 6% stock, at a discount of 25%, than to let the same sum at 7% simple interest? *Ans.* \$150.

96. What is the present worth of an annuity of \$550 for 6 years, at 8% simple interest? *Ans.* \$2675.675+.

97. What is the value of $\left(\frac{\frac{4}{5} \text{ of } 9\frac{1}{3} + 4\frac{4}{7} \text{ of } \frac{1}{5}}{5 - 4\frac{3}{7}}\right) \div \frac{1.36}{.005}$?

Ans. $\frac{11}{204}$.

98. If 36 men working 8 hours a day for 16 days can dig a trench 72 yd. long, 18 ft. wide, and 12 ft. deep, in how many days will 32 men, working 12 hours a day, dig a trench 64 yd. long, 27 ft. wide, and 18 ft. deep? *Ans.* 24 days.

99. Bought a piece of broadcloth at \$2.75 per yard. At what price shall it be marked that I may sell it at 5% less than the marked price and still make 20% profit?

Ans. $\$3.47\frac{7}{19}$.

100. A man hired a mechanic for 35 days, on condition that for every day he worked he should receive \$1.75, and for every day he was absent he should forfeit \$2.50. At the end of the time he received \$40; how many days did he work?

Ans. 30 days.

101. If stock bought at 25% premium pay $7\frac{1}{8}$% on the investment, what % will it pay if bought at 4% discount?

Ans. $9\frac{3}{8}$%.

102. The interest on a note for 2 yr. 3 mo. 18 da., at 8%, was \$155.02; what was the face of the note? *Ans.* \$842.50.

103. The distance on the road around a certain park is 17 miles. If three persons start from the same point on the road at the same time and travel in the same direction around the park, how far will each have to travel before they all come together, if the first travels 5 miles an hour, the second 6, and the third 7 miles an hour?

Ans. First, 85 mi.; second, 102 mi.; third, 119 mi.

104. Two persons commence trade with the same amount of money; the first man spends 48% of his yearly, and the second spends a sum equal to 25% of what both had at first; at the end of the year they both together had $3468. How much had each at the end of the year?

Ans. $1768, first; $1700, second.

105. A roller used for leveling a lawn being 6 ft. 6 in. in circumference by 2 ft. 3 in. in width, is observed to make 12 revolutions as it rolls from one extremity of the lawn to the other. Find the area rolled when the roller has passed ten times the whole length of it. *Ans.* 195 sq. yd.

106. A and B form a copartnership: A's stock is to B's as 5 to 7. At the end of 4 months A withdraws $\frac{2}{5}$ of his stock, and B $\frac{3}{4}$ of his; their year's gain is $5650. How much does each receive? *Ans.* A, $2500; B, $3150.

107. A drover bought a number of sheep, oxen, and cows. He paid half as much more for oxen as for sheep, and half as much more for cows as for oxen; he sold the sheep at a profit of 10%, the oxen at a profit of 8%, and the cows at a loss of 4%; he received for the whole $3416. What did he pay for each lot?

Ans. $700, sheep; $1050, oxen; $1575, cows.

108. A commission merchant, who charges $1\frac{3}{4}$%, purchases for me 145 barrels of sugar, pays for freight $12.50, making the whole bill $2255.07. If there were 190 pounds of sugar in each barrel, what was the price of the sugar per pound, and what was the amount of commission?

Ans. $.08 per pound; $38.57, com.

109. A merchant in New York imported from England a quantity of goods, for which he had to pay a duty of 12%. On account of the depression in trade, he is obliged to sell at a loss of $7\frac{3}{5}$%; had he sold them two months sooner, he would have received $896 more than he did, and then would have cleared $3\frac{1}{5}$% on the transaction. What price did he pay for the goods? *Ans.* $7500.

110. How many bricks 8 inches long, 4 inches wide, and 2 inches thick will be required to build a cubical cistern, open at the top, that shall contain 2000 gallons, if the wall is made a foot thick and $\frac{1}{6}$ of the entire wall is mortar?

Ans. 5918 bricks.

111. What is the value of $2\frac{1}{2} \times \dfrac{1}{3\frac{1}{3} + \dfrac{1}{4\frac{1}{4}}}$? *Ans.* $\frac{255}{364}$.

112. If 18 men, working 10 hours per day, can dig a ditch in 20 days, how long will it take 3 men and 40 boys, working 8 hours per day, to dig a ditch twice as long, 6 men being equal to 10 boys? *Ans.* $33\frac{1}{3}$ days.

113. John Turner owes \$350 due in 7 months, \$500 in 3 months, and \$650 due in 5 months, and pays $\frac{3}{5}$ of the whole in 6 months; when ought the remainder to be paid?

Ans. 3 months.

114. A and B can do a piece of work in $4\frac{4}{9}$ days; B and C in $5\frac{5}{11}$ days; and A and C in $4\frac{4}{5}$ days. In what time can each do the work alone? *Ans.* A, 8 days; B, 10 days; C, 12 da.

115. A and B alone can do a piece of work in 15 and 18 days respectively. They work together on it for 3 days, when B leaves; but A continues, and after 3 days is joined by C; together they finish it in 4 days. In what time could C do the piece of work by himself? *Ans.* 24 days.

116. Mr. Smith paid $3\frac{1}{4}$ times as much for a horse as for a harness. If he had paid 10% less for the harness and $7\frac{1}{7}$% more for the horse, they would together have cost \$245.40. How much did he give for each?

Ans. Horse, \$182; harness, \$56.

117. James and Herbert are running around a block 25 rods square; James runs around it every $7\frac{1}{2}$ minutes, and Herbert every $8\frac{1}{3}$ minutes. If they started together from the same point, how many times must each run around the block before they will be together?

Ans. James, 10 times; Herbert, 9 times.

118. James Walker contracted to build a stone wall 180 rd. long in 21 days. He employed 45 men 12 days, who built $412\frac{1}{2}$ yards. How many more men must be employed to finish the work in the required time? *Ans.* 39.

119. I invested $6345 in Government bonds at $104\frac{1}{2}$, brokerage $1\frac{1}{4}$%. How much would I gain by selling the same at $113\frac{1}{3}$, brokerage $1\frac{1}{3}$%? *Ans.* $375.

120. A grain merchant bought 3250 bushels of wheat, at $1.25 per bushel, and sold it immediately at $1.45 per bushel, receiving in payment a note due 4 months hence, which he had discounted at bank at 6%. What was his gain?

Ans. $553.39+.

121. Two men form a partnership for trading; A's capital is $3500, B's $4800. At the end of 7 months, how much must A put in that he may receive $\frac{1}{2}$ of the year's gain?

Ans. $3120.

122. A man having lost 25% of his capital, is worth exactly as much as another who has just gained 15% on his capital; the second man's capital was originally $9000. What was the first man's capital? *Ans.* $13800.

123. A merchant imported 18 barrels of syrup, each containing 42 gallons, invoiced at $.95 per gallon; paid $85 for freight and a duty of 30%. What % will he gain by selling the whole for $1171.459? *Ans.* 15%.

ANSWERS.

Art. 307.

1. $\frac{25}{30}$; $\frac{12}{30}$.
2. $\frac{12}{28}$; $\frac{10}{28}$; $\frac{18}{28}$; $\frac{4\frac{1}{2}}{28}$; $\frac{9}{28}$.
3. $\frac{24}{32}$; $\frac{14}{32}$; $\frac{7}{32}$.
5. $\frac{12}{15}$; $\frac{20}{25}$; $\frac{2}{2\frac{1}{2}}$; $\frac{3}{3\frac{3}{4}}$; $\frac{7}{8\frac{3}{4}}$.
6. $\frac{15}{25}$; $\frac{15}{21}$.
7. $3\frac{13}{24}$.
8. $\frac{10}{111}$.
9. 5272\frac{8}{21}$.
10. $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{8}$; $\frac{1}{12}$.
11. $\frac{1}{30}$; $\frac{1}{6}$; $\frac{1}{40}$; $\frac{1}{12}$.
12. $\frac{3}{7}$.
13. 49.
14. 1614\frac{3}{5}$.
15. $40\frac{5}{7}$ tons.
16. $121\frac{2}{3}$ yards.
17. 63\frac{5}{9}$.
18. $25\frac{1}{5}$.
19. 289\frac{1}{6}$.
20. $47\frac{13}{16}$ mi.; $38\frac{15}{16}$ mi.
21. 8113\frac{1}{14}$.
22. $556\frac{5}{36}$.
23. $10500.
24. 504.
25. $1800.
26. $123\frac{3}{7}$ cords.
27. $4\frac{124}{125}$ pounds; $8\frac{56}{225}$ pounds; $\frac{28}{75}$ pounds.
28. Increased by $\frac{5}{21}$.
29. 1\frac{1541}{4144}$.
30. $2\frac{11}{12}$ days.
31. 65\frac{11605}{15264}$.
32. 15 feet.
33. $1196\frac{229}{250}$ yds.; 2\frac{8}{21}$.
34. 289\frac{53}{63}$.
35. $13\frac{11}{25}$ acres.
36. 5006\frac{1}{4}$.

Art. 451.

1. 38.1024 Kg.
2. 33.5627 Ton.
3. 33.8304 Hl.
4. 2.2185 L.
5. 68.0194 St.
6. 13274.16 A.
7. 250 cu. in.
8. 38 sq. yd.
9. 7.7353+ bu.
10. 552960 gr.
11. 36.8965+ cd.
12. 1176.15+ cu. ft.
13. $.024624.
14. $1.41975.
15. $14.5675+.
16. $1.357+.
17. 8340 dg.; 83.4 Dg.
18. 8400 L.; 840000 cl.
19. 790.75+ mi.
20. $960.
21. $74.00048+.

Art. 467.

1. 8400.
2. 76000.
3. 573.
4. 38097.
5. 897.52.
6. 30084.
7. 3426000.
8. 720000000.
9. 463000000.

Art. 468.

1. 300800.
2. 18018.
3. 350000.
4. 1673800.
5. 24473.6.
6. 11500000.
7. 27360000.
8. 414000.
9. 3141.

Art. 469.

1. 6628122.
2. 385605.
3. 87546366.
4. 53111520.
5. 839516040.
6. 2585632.
7. 7349272.
8. 62767170.
9. 4388206.

Art. 470.

1. 87.36.
2. 43.72.
3. 7.903.
4. .02397.
5. .05236.
6. .0006934
7. .0054.
8. .00007.
9. .0072.

Art. 471.

1. 183.8.
2. 6.54625.
3. $.1098\frac{1}{3}$.
4. .013.
5. .008.
6. .0004.
7. $.674\frac{4}{9}$.
8. $2.7041\frac{2}{3}$.
9. $.01392\frac{5}{6}$.

Art. 472.

1. $\frac{9}{20}$.
2. $\frac{4}{75}$.

3. $\frac{1}{10}$.
4. $\frac{7}{25}$.
5. $\frac{1}{2}$.
6. $\frac{5}{33}$.
7. $\frac{1}{12}$.
8. $\frac{3}{55}$.
9. $\frac{7}{16}$.
10. $\frac{1}{160}$.
11. $\frac{1}{6}$.
12. $\frac{10}{4281}$.
13. $\frac{1}{120}$.
14. $\frac{1}{10}$.
15. $\frac{1}{90}$.

Art. 473.

1. $\frac{3}{4}$.
2. $\frac{9}{10}$.
3. $1\frac{5}{7}$.
4. 30.
5. $\frac{3}{5}$.
6. $1\frac{1}{2}$.
7. $1\frac{2}{35}$.
8. 7.
9. $1\frac{1}{9}$.
10. 30.
11. 130.
12. $233\frac{1}{3}$.

Art. 474.

1. 28.
2. 30.
3. 103.
4. 25.
5. $\frac{19}{30}$.
6. 112.
7. $158\frac{5}{7}$.
8. 80.
9. 50.

Art. 479.

1. 15300.
2. 48700.
3. $596\frac{2}{3}$.
4. 111875.
5. 78000.
6. $9655\frac{5}{9}$.
7. 8000.
8. 14.6.
9. 1090.
10. 14725.
16. \$1112; \$1251; \$1042.50; \$973.
17. \$425; \$550; \$612.50; \$533$\frac{1}{3}$.
18. \$600; \$315; \$910; \$700.
19. \$630; \$675; \$650.
20. \$980; \$1470; \$735.
21. \$112; \$128. \$149.33$\frac{1}{3}$.
22. \$1110.
23. \$371.25; \$309.37$\frac{1}{2}$; \$433.12$\frac{1}{2}$.
24. \$255; \$510; \$408.
25. \$548; \$657.60, \$1315.20; \$1534.40.
26. \$522; \$543.75; \$696; \$761.25.

Art. 480.

1. 217.74.
2. 2619.
3. 1952.8.
4. 5025.
5. 11.388.
6. 1.9708.
7. 3.1584.
8. 1.8434.
9. 3.62552.
10. 219.288.
15. 358.16 A.; 179.08 A.; 268.62 A.; 8.954 A.; 71.632 A.; 537.24 A.; 35.816 A.
16. 5083.2 bu.; 2541.6 bu.
17. 7926 yd.; 18494 yd.; 10568 yd.; 792.6 yd.; 1056.8 yd.; 2377.8 yd.; 1849.4 yd.
18. \$1725; \$1971$\frac{3}{7}$; \$2300; \$3450; \$1863; \$2116; \$2380.50.

Art. 494.

10. \$56.
11. \$87.
12. \$144.44$\frac{4}{9}$.
13. \$35.57$\frac{1}{7}$.
14. \$143.46$\frac{3}{7}$.
15. \$17.79$\frac{3}{4}$.
16. \$4656.88$\frac{8}{9}$.
17. \$5.68$\frac{11}{23}$.
18. \$52.66$\frac{2}{3}$.
19. \$19.11$\frac{1}{12}$.
20. \$940.95.

Art. 495.

6. \$.38$\frac{28}{59}$.
7. \$1.4825.
8. \$78.66+.
9. \$4.375.
10. \$2.75.
11. \$.2758+.
12. \$15.50.

Art. 496.

1. \$4.05.
2. \$7812; \$12361$\frac{11}{13}$.
3. \$668.25; \$767.88; \$816.48.
4. \$1163.35.
5. \$465.75; \$465.75.
6. \$3.4545.
7. \$1346.2295.
8. \$273.12525.
9. \$379.64024.

Art. 497.

5. $274\frac{2}{7}$ yd.
6. 525 bu.; $516\frac{12}{13}$ bu.; 630 bu.; $534\frac{6}{11}$ bu.; $458\frac{2}{11}$ bu.
7. 51 rd.
8. 217 A.
9. 285 lb.
10. 395 yd.
11. 476 bu.

Art. 498.

1. 30 yd.
2. 37 T.; $24\frac{87}{112}$ T.
3. $38\frac{8}{9}$ bu.; 40 bu.
4. 96 ft.
5. 483 yd.
6. 89.
7. 53 cd.
8. 9 ft.

Art. 499.

6. \$1386.
7. \$585.50.
8. \$120.25.
9. \$5075.
10. \$847.09$\frac{3}{8}$.
11. \$40455.

Art. 500.

2. $\frac{5}{6}$; $\frac{3}{5}$; $\frac{5}{8}$; $\frac{1}{3}$.
3. $\frac{3}{4}$; $\frac{18}{19}$; $\frac{6}{7}$; $\frac{1}{2}$.
4. $\frac{7}{9}$.
5. $\frac{8}{44}$; $\frac{8}{25}$.
6. $\frac{19}{294}$.
7. $\frac{71}{768}$.
8. $18\frac{3}{4}$.
9. $4\frac{47}{117}$.
10. $\frac{6}{13}$ yr.; $\frac{66}{365}$ yr.
11. $\frac{898}{2375}$.
12. $\frac{17}{25}$, or .68.
13. $\frac{1077}{2107}$.
14. $\frac{1}{3}$; $\frac{4}{7}$.
15. $\frac{17}{45}$; $\frac{87}{106}$.

Art. 501.

2. 42; 84. 329; $905\frac{5}{8}$.
3. 20.4; 77.76; 504.
4. \$60; \$$641.35\frac{5}{9}$; \$$1.35\frac{5}{12}$.
5. 22.96.
6. \$58.88; 15.80 lb.
7. $78\frac{1}{2}$ A.; 32 bu.
8. 1.96 yd.
9. 15 men.
10. 25.02 lb.
11. $13.25\frac{1}{3}$ mi.
12. 91.50.
13. \$6.45.
14. \$837.2.
15. \$347.75.
16. $24\frac{1}{5}$ lb.
17. 120.7 yd.
18. 259.86 A.
19. \$1173; \$4.25 per yd.
20. 84 ct.

Art. 502.

1. 108.
2. \$45; \$63.
3. 324 yd.
4. \$1323.
5. \$23580.

Art. 503.

7. 200.
8. 400.
9. 1200.
10. 800.
11. 2400 yd.
12. 700 bu.
13. 20.
14. \$$9\frac{11}{21}$.
15. $6\frac{17}{18}$ pk.
16. 14 ft.
17. $2\frac{1}{4}$.
18. 4.
19. $42\frac{2}{9}$.
20. \$36.
21. $87\frac{1}{2}$.
22. 550.
23. 10 yd.
24. 8.
25. \$35500.
26. \$1.50.
27. 3100 lb.
28. \$75.
29. 1.
30. 300 A.
31. \$75.
32. \$1350.
33. \$$8333\frac{1}{3}$.
34. \$412500.
35. \$8000.

Art. 505.

7. 25%.
8. $16\frac{2}{3}$%.
9. 20%.
10. $11\frac{1}{9}$%.
11. $29\frac{1}{6}$%.
12. $16\frac{1}{4}$%.
13. $44\frac{3}{8}$%.
14. $9\frac{23}{28}$%.
15. $35\frac{5}{7}$%; $5\frac{5}{7}$%; $10\frac{5}{7}$%.
16. $42\frac{13}{36}$%.
17. $1\frac{157}{243}$%.
18. $26\frac{2}{3}$%.
19. $76\frac{4}{21}$%.
20. $39\frac{17}{57}$%.
21. 8%.
22. $14\frac{482}{697}$%.
23. $61\frac{19}{21}$%.
24. $61\frac{127}{193}$%.
25. $33\frac{1}{3}$%.
26. $12\frac{1}{2}$%.
27. 10%.
28. $15\frac{1}{2}$%.
29. $3\frac{48451}{96903}$%.

Art. 507.

6. 216.
7. 184.
8. 560 mi.
9. 872.
10. 800 men.
11. \$1000.
12. \$6.40.
13. \$2000.
14. \$4000.

Art. 515.

1. \$3.04.
2. \$81.60.
3. \$74.48.
4. \$$158.2312\frac{1}{2}$.
5. \$$116.57\frac{1}{4}$.
6. \$$2278.96\frac{1}{2}$.
7. 15%.
8. $26\frac{6}{19}$%.
9. 20%.
10. \$6355.362.
11. \$.78.
12. \$77.40 gain; \$6.30 selling price.
13. \$8.25.
14. \$7.14.
15. \$$5.12\frac{1}{2}$.
16. \$5860.
17. \$9.50 buying price; \$$7.12\frac{1}{2}$ selling price.
18. \$9.75.
19. \$1825.
20. \$570.24.
21. 20%.
22. \$2475.
23. \$153.
24. \$3.20 per yd.
25. \$$4.22\frac{1}{2}$ per cd.
26. 20%.
27. \$$.76\frac{4}{21}$.

Art. 522.

1. \$$14.13\frac{1}{8}$.
2. \$18.2105.
3. \$57.75.
4. \$96.95952.
5. $2\frac{1}{2}$%.
6. $4\frac{2}{3}$%.
7. $2\frac{2}{9}$%.
8. \$14400.
9. \$$47178.12\frac{1}{2}$.
10. \$$1652.92\frac{4}{13}$.
11. \$$8663.21\frac{47}{193}$.
12. \$3653.70+.
13. \$1863.97+.
14. \$3286.
15. \$384.048+.
16. \$596.
17. \$24.60.
18. \$375.435.
19. 112 bbl.
20. \$13.82.
21. \$129.375.
22. \$6761.882+.

Art. 527.

1. \$105.
2. \$$199.37\frac{1}{2}$.
3. \$$148.58\frac{3}{4}$.
4. $2\frac{1}{2}$%.
5. \$$22711\frac{1}{9}$.
6. \$6000.
7. $\frac{2}{5}$%.
8. \$$250.66\frac{2}{3}$.
9. \$1948.80.
10. \$148.75.

Art. 540.

2. \$8965.50.
3. \$$10105.62\frac{1}{2}$.
4. 76 shares.
5. 96 shares.
6. 144 shares.
7. 153 shares.
8. \$15680.
9. \$40090.
10. \$16462.50.
11. \$59800.
12. \$70.
13. \$62.50.
14. \$$58.33\frac{1}{3}$.
15. \$$128.57\frac{1}{7}$.
16. \$392.
17. \$850.20.
18. \$448.47.
19. \$583.05.
20. \$$451.11\frac{1}{9}$.
21. \$964.
22. \$284.982+.
23. \$368.
24. \$1160.
25. $28\frac{1}{2}$ shares.
26. \$1029.
27. 6%.
28. \$191.80.
29. \$3720.
30. $11\frac{13}{17}$%.

31. $75.
32. $$62\frac{1}{2}$.
33. $53.
34. $5950.
35. $680.
36. $800.821+; $78.‹82+; $124.931+.
37. $8.

Art. 554.

1. $268.80.
2. $255.192.
3. $$622.68\frac{3}{4}$.
4. $73.60.
5. $66.6792.

Art. 566.

13. $51.5256.
14. $282.33.
15. $8.3695.
16. $41.78265.
17. $86.3208+.
18. $1.6559+.
19. $13.25248.
20. $107.1144.
21. $85.115.
22. $827.08.
23. $462.616.
24. $$4.236\frac{2}{3}$.
25. $5.736.
26. $97.1694.

Art. 570.

1. $63.048; $89.318.
2. $113.1074; $145.4238.
3. $118.3442; $73.965+.
4. $64.1775; $106.9625.
5. $815.976+; $1078.254+.
6. $292.3719.
7. $49.529+.
8. $1094.096.
9. $410.475+.
10. $1699.80+.

Art. 573.

1. $35.84.
2. $48.675.
3. $43.812.
4. $12.754.
5. $28.1385.
6. $35.82.
7. $120.
8. $46.50.
9. $40.20.
10. $100.395.

Art. 576.

1. $1.58.
2. $1.536.
3. $2.125.
4. $4.2075.
5. $$1.54\frac{2}{3}$.
6. $$1.849\frac{1}{3}$.
7. $$2.67\frac{2}{3}$.
8. $$3.775\frac{1}{3}$.
9. $1.122.
10. $1.96.

Art. 578.

1. $62.36352; $93.54528; $83.15136.
2. $303.9513; $434.216; $173.6875.
3. $22.2609; $11.1304; $19.4783.
4. $113.40; $151.20; $56.70.
5. $45.4765; $57.7202; $66.4656.

Art. 582.

1. $24.65; $39.44; $34.51.
2. $62.6533 $93.9800; $41.7689.
3. $291.695; $458.378; $125.0123.
4. $103.44004; $131.2892; $57.6877.
5. $65.9458; $50.3270; $19.0895.
6. $1.85; $2.6037; $1.5074.
7. $376.6183; $502.1577; $313.8486.

Art. 585.

1. $11.5436.
2. $3.1342.
3. $3.3082.
4. $3.1574.
5. $3.3945.
6. $3.6073.
7. $6.54407.
8. $1.4576.
9. $16.2754.
10. $.8939.
11. $461.193.
12. $25.2125.

Art. 588.

2. $364.9937+.
3. $283.992+.
4. $462.019.
5. $562.984.
6. $296.
7. $434.994.

Art. 591.

2. $264.998+.
3. $49.652+.
4. $295.996+.
5. $572.996+.

Art. 594.

2. 6%.
3. 7%.
4. 8%.
5. 5%.
6. $8\frac{1}{2}$% nearly.
7. 12%.
8. $\frac{1}{4}$% better 2d.
9. $14\frac{2}{7}$%.
10. 25%; $12\frac{1}{2}$%; $11\frac{1}{9}$%; $8\frac{1}{3}$%; 4%.
11. $33\frac{1}{3}$%; 50%; $22\frac{2}{9}$%; $33\frac{1}{3}$%; $14\frac{2}{7}$%; $21\frac{3}{7}$%; $11\frac{1}{9}$%; $16\frac{2}{3}$%.
12. $9\frac{1}{10}$%.

Art. 597.

1. 2 yr. 3 mo.
2. 6 yr.
3. 3 yr. 6 mo.
4. 4 yr. 9 mo.
5. 2 yr. 8 mo.
6. 5 yr. 7 mo. 6 da.
7. 6 yr. 4 mo. 24 da.
8. 7 yr. 6 mo. 25+ da.
9. $14\frac{2}{7}$ yr.
10. 20 yr.; $12\frac{1}{2}$ yr.; $15\frac{5}{13}$ yr.; $11\frac{1}{9}$ yr.; 40 yr.; 25 yr.; $30\frac{10}{13}$ yr.; $22\frac{2}{9}$ yr.
11. $71\frac{3}{7}$ yr.

Art. 601.

1. $146.27795.
2. $977.532.
3. $31 390166+.
4. $1854.576.
5. $654.5102+.
6. $20.0034.
7. $205.516+.
8. $108.595+.
9. $44.0824+ gr. at comp. int.

Art. 604.

1. $219.558.
2. $183.0183.
3. $133.55053.
4. $114.8721672.
5. $55.4364.
6. $99.2659725.

Art. 606.

1. $469.53704.
2. $1220.2528.
3. $781.52013.
4. $2755.3606.
5. $566.8662.
6. $248.1272192.
7. $257.299443.
8. $228.3345.
9. $145.728068.
10. $556.75033.
11. $753.952567.
12. $3439.63075.
13. $1097.5152.
14. $854.942736.

Art. 609.

1. $146.004.
2. $\$1071.41\frac{1}{3}$.
3. $1128.075.
4. $23.1858.
5. $363.208.
6. $50.56 dif. between Sim. and Annual Int.; $4.209 dif. between An. and Com. Int.; $54.769+ dif. between Sim. and Com. Int.

Art. 616.

1. $\$282.46\frac{1}{4}$.
2. $11.254.
3. $202.793.
4. $117.942.

Art. 619.

2. $1491.49+.
3. $2891.527.
4. $420.292.
5. $5424.651+.

Art. 624.

1. $776.699+; $754.717+.
2. $315.789+; $348.387+.
3. $485.468+; $478.10+.
4. $9 975+.

$148.456.

5. $5.513; $40.82+.
6. $15.275; $230.578.
7. $530.367.
8. $9171.90+.
9. $.957.
10. $425.
11. $.103 more profitable at $4.66.
12. 2d $33.865 better.
13. $1.47+.

Art. 632.

1. $5.88 Bk. Dis.; $274.12 Proceeds; $\$11.94\frac{2}{3}$ Bk. Dis.; $\$268.05\frac{1}{3}$ Proceeds.
2. $\$20.671\frac{2}{3}$ Bk. Dis.; $\$769.328\frac{1}{3}$ Proc'ds; $\$11.520\frac{5}{6}$ Bk. Dis.; $\$778.479\frac{1}{6}$ Proc'ds.
3. $\$25.82\frac{2}{9}$ Bk. Dis.; $\$1574.17\frac{7}{9}$ Proc'ds; $\$54.022\frac{2}{9}$ Bk. Dis.; $\$1545.977\frac{7}{9}$ Proc'ds.
4. $.884.
5. $\$20.121\frac{1}{4}$.
6. $27.194.
7. Due May 27; 59 da. Time; $475.383+ Proc'ds.
8. Due Aug. 16; 75 da. Time; $\$581.395\frac{5}{6}$ Proc'ds.
9. Due Aug. 22; 91 da. Time; $\$1571.68\frac{8}{9}$ Proc'ds.

Art. 635.

1. $876.061+; $295.415+; $540.713.
2. $458.287; $189.68; $99.112.
3. $238.63; $1830.922; $515.648.

4. $480.616.
5. $354.452.
6. $298.899+.
7. $961.781, 1st; $967.495, 2d; $979.914, 3d.
8. $495.262.
9. $1517.440.

Art. 642.

1. $2416.
2. $3204.
3. $850.68.
4. $6331.50.
5. $133.796.
6. $1491.
7. $\$382.3028\frac{4}{7}$.
8. $291.141.
9. $486.
10. $560.
11. $720.
12. $480.
13. $824.
14. $275.50.
15. $321.
16. $402.
17. $698.25.
18. $\$1615.11\frac{1}{9}$.
19. $2415.925.
20. $1779.
21. $496.
22. $97\frac{1}{12}\%$, or $2\frac{11}{12}\%$ dis.
23. $\$2526.38\frac{8}{9}$.
24. $8013.43+.

Art. 648.

2. $2124.99065+.
3. $\$893.22\frac{3}{4}$.
4. $1642.41.
5. $763.
6. $\$3469.33\frac{1}{3}$.
7. $609.375.
8. $11456.8125.

Art. 650.

1. $12615.96.
2. $301 gain by Ind.
3. $124.852+ less by Ind.
4. 3011.58+ marks.

Art. 661.

1. July 1, 1876.
2. Dec. 27, 1876.
3. April 30, 1876.
4. Oct. 19, 1876.
5. Sept. 3, 1876.
6. July 21, 1877.
7. Oct. 19, 1877.
8. 60 da.
9. Feb. 5.

Art. 664.

1. $210, Face of note; Due Dec. 12, 1876.
2. $100 due; Dec. 7, 1876, equated time.
3. Apr. 6, 1877.
4. March 1, 1876.

Art. 681.

1. $\frac{2}{1}$.
2. $\frac{143}{365}$.
3. $\frac{982}{2941}$.
4. $\frac{73}{365}$.
5. $\frac{9906}{522}$.
6. $\frac{16}{1}$.

Art. 683.

1. $\frac{2}{3}$.
2. $\frac{3}{8}$.
3. $\frac{5}{7}$.
4. $\frac{11}{13}$.
5. $\frac{13}{17}$.
6. $\frac{5}{1}$.
7. $\frac{12}{7}$.
8. $\frac{1}{9}$.
9. $\frac{7}{16}$.
10. $\frac{29}{48}$.

Art. 685.

1. 150 da.
2. 45 A.
3. $597.
4. $2100.
5. $2386.40.

Art. 687.

1. 104 A.
2. $50.
3. 130 da.
4. $$678\frac{18}{19}$.
5. 415 lines.
6. 177 cd. 3 cd. ft.
7. $2000.
8. 17 gal. 3 qt. 1 pt.
9. $13.50.

Art. 690.

2. $72.
3. $468.
4. $27.
5. $100.
6. $204.
7. $493\frac{1}{2}$.
8. $228\frac{4}{7}$ yd.

Art. 700.

1. 35.
2. 6.
3. 272.
4. $29\frac{1}{3}$ bu.
5. 10 yd.
6. £168 15s. $6\frac{2}{3}$d.
1. 56.
2. 21.
3. 15.
4. $$7\frac{1}{5}$.
5. $213\frac{1}{3}$.
6. 8 cwt.

Art. 702.

4. 195 bu.
5. $9.
6. 70 bu.
7. 803 ft.
8. $18.
9. $83.655.
10. $3000.
11. $7.
12. $2.40.
13. 16 mi. 109 rd.
14. 169 gal. 3 qt. 1 pt.
15. $$.98\frac{3}{14}$.
16. 2 yr. 10 mo.
17. $375.

Art. 703.

1. $\frac{9}{125}$.
2. $\frac{1}{14}$.
3. $\frac{56}{5}$.
1. 48.
2. $23\frac{1}{3}$.

Art. 705.

1. 264.
2. 120 cd.
3. 180 bu.
4. $18.
5. $116760.
6. 12 lb.
7. 1728 ft.
8. 23040 yd.

Art. 711.

1. $992.
2. $405.125.
3. $$3222.26\frac{2}{3}$.
4. $300 for 4 mo.
5. $2400 for 4 mo.

Art. 713.

1. $1661.538, A's share; $1107.692, B's share; $1550.769, C's share.
2. $2382.545, A's share; $1737.272, B's share; $1340.181, C's share.
3. $2814.128, A's share; $1644.112, B's share; $3431.758, C's share;
4. $1178.947, Ætna; $1547.368, Home; $2394.736, Mutual.
5. $900, 1st district; $700, 2d district; $600, 3d district; $400, 4th district.

Art. 715.

1. $210, A's share; $144, B's share.
2. $70.451, A's share;

$39.629, B's share; $26.419, C's share.
3. $4940, A's stock; $3510, B's stock; $7150, C's stock.
4. $335.10+, A's profit; $288.56+, B's profit; $251.32+, C's profit.
5. $3666.06+, A's share; $5233.93+, B's share.

Art. 717.

2. $1.00.
3. $.13$\frac{1}{50}$.
4. 18$\frac{3}{4}$ carats.
5. $.31$\frac{1}{9}$.

Art. 722.

1. 1, 3, 2, 1 lb.
2. 3 gal. of mo. to 2 gal. of water.
3. 1 part of each.
4. 1, 2, and 6 bbl.
6. 2, 2, 604, and 240 bbl.
7. 2, 1, and 109 gal.
8. 50, 50, 5, and 1 sheep.
10. 18, 27, 63, and 27 lb.
11. 30, 30, and 180 oz.
12. 44$\frac{4}{5}$, 89$\frac{3}{5}$, and 89$\frac{3}{5}$ lb.

Art. 728.

2. 2304; 4225; 186624.
3. 86436; 148996; 247009; 64009.
4. 250047; 15625; 438976; 60236288.
5. $\frac{27}{343}$; $\frac{64}{729}$; $\frac{343}{1000}$; .512.
6. 56169.
7. 4100625.
8. $\frac{2197}{74088}$.
9. $\frac{289}{10000}$.
10. .00390625.
11. .039304.
12. $\frac{529}{900}$.
13. .00028561.
14. .000000166375.
15. .00091204.

Art. 729.

2. 7.
3. 7.
4. 9.
5. 15.
6. 12.
7. 13.
9. 12.
10. 30.
11. 24.
12. 12.

Art. 745.

1. 64.
2. 59.
3. 53.
4. 87.
5. 93.
6. 96.
7. $\frac{25}{84}$.
8. $\frac{59}{89}$.
9. $\frac{42}{81}$.
10. .15.
11. .48.
12. .76.
13. 371.
14. 64.5.
15. 876.6.
16. 167.4.
17. 7.56.
18. 21.79.
19. 5.656+.
20. 7.681+.
21. 2.645+.
22. .964+.
23. .894+.
24. .612+.
25. 3.834+.
26. 9.284+.
27. 2.443+.
28. .881+.
29. .346+.
30. .404+.
31. 51.
32. 62.
33. 33$\frac{3}{64}$.
34. 6.
35. 2656.
36. 354.906+.
37. 95 ft.
38. 487 ft.
39. 69.57+ yd.
40. 96 trees.
41. 44.342+ rd.
42. 1.4142; 2.2360; 3.3166.
43. .654; .852; .735.

Art. 753.

1. 6.
2. 9.
3. 11.
4. 13.
5. 16.
6. 22.
7. 19.
8. $\frac{8}{17}$.
9. $\frac{14}{21}$.
10. $\frac{18}{24}$.
11. 63.
12. 76.
13. 289.
14. 361.
15. $\frac{17}{162}$.
16. 4.84.
17. 2.22+; 3.30+; 4.37+; 6.17+; .56+; .75; 4.22+; 1.88+.
18. 1.442+; 1.912+; .793+; .341+; .208+; 1.272+.
19. 4.
20. 37.
21. 9.
22. 76.36+.
23. 8.
24. 439 ft.
25. 78.3+ inches.
26. 2730$\frac{2}{3}$ sq. ft.
27. 32 feet.
28. 196.9+ inches.
29. 436 feet.
30. 26.9+ feet.

Art. 765.

1. 26.
2. 13 in.
3. 10.
4. 70; 432.
5. $18.
6. 12.
7. 500500.
8. $97.60.
9. 12 days; 336 mi.
10. 144.

Art. 769.

1. 49152.
2. 2048.
3. 7.
4. 2186.
5. 393213.
6. 315 mi.
7. 189 mi.; 6.
8. 3; 4372.

Art. 775.

1. $1410.
2. $1905.
3. $2485.714+.
4. $1775.9772.
5. $491.73+.
6. $5288.88+.
7. $8944.226.
8. $2380.59+.
9. 6%.
10. 7%.
11. $500.
12. $2500.
13. 3 years.

Art. 799.

1. $53\frac{2}{3}$ sq. ft.
2. $3\frac{7}{11}$ sq. rd.
3. 168 sq. ch.
4. 112.292 sq. ch.
5. 510 sq. ft.
6. 2160 stones; $367.50.

Art. 800.

1. 10 ft.
2. 22 rd.
3. 16 ft. 8 in.
4. 42 rd.
5. 6 yd.
6. 4 rd. 4 yd. 1 ft. 9 in.
7. 20 ft.

Art. 801.

1. 150 sq. ft.
2. 612.37 sq. in.
3. .924+ A.
4. 692.82+ sq. ft.
5. $292.68+.

Art. 802.

1. 43.08+ ft.
2. 39 ft.
3. 128.80+ ft.
4. 187.45+ rd.
5. 14.14+ rd.

Art. 803.

1. 37.08+ ft.
2. 40.31+ ft.
3. 2.10+ ft.
4. 42 ft.
5. 18 ft.

Art. 804.

1. 28 sq. ft. 108 sq. in.; 6 sq. yd. 6 sq. ft. 138 sq. in.
2. 7 A. 3 sq. ch. 15 P. 125 sq. l.
3. 2805 sq. ft.
4. 29 sq. ft.

Art. 805.

1. 220 sq. ft.
2. 57 sq. in.
3. 44 sq. ft.
4. $161.25.

Art. 806.

1. 450 sq. in.
2. 104 sq. ft.
3. 300 sq. ft.
4. 11 A.

Art. 807.

1. 30 in.; 25 ft.
2. 43.9824 in.; 56.5488 in.
3. $45.239+.
4. 596904000 mi.

Art. 808.

1. 314.16 sq. ft.; 1385.4456 sq. in.; 1963.50 sq. ft.
2. 14313.915 sq. ft.; 3911.085 sq. ft.; $17.55+.
3. 79.5727+ A.

Art. 809.

1. 8 ft.
2. 43.9824 ft.
3. 5 rd.
4. 37.6992 in.
5. 62.832 rd.
6. 16 in.

Art. 819.

1. 84 sq. ft.
2. $36\frac{5}{12}$ sq. ft.
3. 135 sq. in.
4. 186 sq. ft.
5. 326.7264 sq. ft.

Art. 820.

1. 103.8 cu. ft.
2. 411.5132+ cu. ft.
3. 1800 cu. ft.
4. 238.7616 cu. ft.
5. $31.25.
6. 41.888 cu. ft.

Art. 821.

1. 114 sq. in.
2. 235.62 sq. ft.
3. 35 sq. ft.
4. 323 sq. ft.
5. 731 sq. ft.
6. 519.58+ sq. in.
7. 738.276 sq. ft.
8. 608.685 sq. in.

Art. 822.

1. 25.1328 cu. ft.
2. 50.2656 cu. ft.
3. 80 cu. ft.
4. 166.27+ cu. in.
5. 3600 cu. ft.

Art. 823.

1. 45 sq. ft.
2. 135 sq. in.
3. 337.29+ sq. ft.
4. 1124.6928 sq. ft.

Art. 824.

1. 1184 cu. ft.
2. 348.7176 cu. ft.
3. 24.871 cu. ft.
4. 48631.968 cu. ft.
5. 9445.448+ cu. ft

Art. 825.

1. 201.0624 sq. ft.
2. 254.4696 sq. in.
3. 6361.74 sq. ft.
4. 4071.5136 sq. in.
5. 78.54 sq. in.

Art. 826.

1. 4188.8 cu. in.
2. 14.1372 cu. yd.
3. 14137.2 cu. in.
4. .22089 cu ft.
5. 268.0832 cu. in.

Art. 827.

1. 58.752 gal.
2. 104.1012 gal.
3. 52.6592 gal.
4. 85 gal.