Thomson's Arithmetical Series

IN TWO BOOKS

COMPLETE GRADED ARITHMETIC.

JAMES B. THOMSON, LL.D.

THOMSON'S ARITHMETICAL SERIES

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COMPLETE
GRADED
ARITHMETIC,

ORAL AND WRITTEN,

UPON THE

INDUCTIVE METHOD OF INSTRUCTION.

FOR SCHOOLS AND ACADEMIES.

BY

JAMES B. THOMSON, LL.D.,

AUTHOR OF MATHEMATICAL SERIES.

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GEOMETRY AND TRIGONOMETRY. (In preparation.)

SMITH & McDOUGAL, ELECTROTYPERS.
82 Beekman St., N. Y.

PREFACE.

THE book now offered to the public, unites in one volume Oral and Written Arithmetic upon the inductive method of instruction. Its aim is two-fold: to develop the intellect of the pupil, and to prepare him for the actual business of life. In securing these objects, it takes the most direct road to a practical knowledge of Arithmetic.

The pupil is led by a few simple appropriate examples to infer for himself the general principles upon which the operations and rules depend, instead of taking them upon the authority of the author without explanation. He is thus taught to put the steps of particular solutions into a concise statement, or general formula. This method of developing principles is an important feature.

It has been a cardinal point to make the explanations simple, the steps in the reasoning short and logical, and the definitions and rules brief, clear, and comprehensive.

The discussion of topics which belong exclusively to the higher departments of the science is avoided; while subjects deemed too difficult to be appreciated by beginners, but important for them when more advanced, are placed in the Appendix, to be used at the discretion of the teacher.

Arithmetical puzzles and paradoxes, and problems relating to subjects having a demoralizing tendency, as gambling, etc., are excluded. All that is obsolete in the former Tables of Weights and Measures is eliminated, and the part retained is corrected in accordance with present law and usage.

Examples for Practice, Problems for Review, and Test Questions are abundant in number and variety, and all are different from those in the Practical Arithmetic.

The arrangement of subjects is systematic; no principle is anticipated, or used in the explanation of another, until it has itself been explained. Subjects intimately connected are grouped together in the order of their dependence.

In connection with the Notation of Integers, that of Decimals is taught to three places below units, corresponding to dimes, cents, and mills. Decimal or Metric Weights and Measures are placed next after Decimal Currency. Percentage is followed by its applications in their proper order, as Profit and Loss, Commission, etc.

General Analysis, covering the several departments of Commercial Arithmetic, has received special attention. The articles devoted to Test Questions, and to the Entrance Problems of different Colleges, will be found a valuable addition and excellent practice. Thanks are due to the College Officers who have kindly furnished copies of their Examination Papers.

In the preparation of the work the author has carefully weighed the discussions in the various journals of education respecting the present wants of our schools, and has endeavored to provide for them. He has availed himself of many valuable suggestions from business men, practical teachers, and educators, all of whom he desires to thank most cordially for the aid they have rendered.

He cheerfully submits the result of his labors to his friends, the teachers, and the public, for whose favorable verdict upon his former efforts he desires to express renewed obligations.

J. B. T.

BROOKLYN, April, 1882.

CONTENTS.

ARITHMETIC.

DEFINITIONS.

Art. 1. A **Unit** is *one* or any single thing; as, one, one book, one chair.

2. A **Number** is a *unit* or a *collection* of units.

Thus, one, or one book, is a unit; five, or five books, is a collection of units.

3. The **Unit of a Number** is one of the collection forming that number.

Thus, the unit of four books is one book, of seven is one.

4. An **Abstract Number** is one that is *not* applied to any object.

Thus, four, five, thirteen, etc., are abstract numbers.

5. A **Concrete Number** is one that *is* applied to some object.

Thus, five boys, seven apples, etc., are concrete numbers.

6. **Like Numbers** are those which express units of the *same kind.*

Thus, eight pears and five pears, four and seven, are like numbers.

7. **Unlike Numbers** are those which express units of *different kinds.*

Thus, seven, six peaches, nine days, are unlike numbers.

8. **Numbers** are expressed by words, by figures, or by letters.

9. **Arithmetic** is the science which treats of *numbers* and their *applications.*

NOTATION AND NUMERATION.

10. **Notation** is expressing numbers by *figures* or *letters.*

11. **Numeration** is reading numbers expressed by figures or letters.

NOTE.—There are two methods of Notation, called the *Arabic* and the *Roman.* The former is the method in general use, and is so called because it was introduced into Europe in the 10th century by the Arabians.

12. The **Arabic Notation** expresses numbers by *ten* different characters, called **Figures**; viz.,

1,	2,	3,	4,	5,	6,	7,	8,	9,	0.
One,	**Two,**	**Three,**	**Four,**	**Five,**	**Six,**	**Seven,**	**Eight,**	**Nine,**	**Naught.**

13. The first *nine* are called **Significant figures,** because they always express *some value.* They are also called **Digits.**

14. The *last one* is called **Naught,** because when standing alone it has *no value.* It is also called **Cipher** or **Zero.**

15. The **Value** of a figure is the *number* it represents.

16. **Nine** is the largest number expressed by *one* figure.

17. The significant figures standing *alone,* express *single things* or *ones;* as, 4 apples, 5, 7.

18. To express the numbers from *nine* to *one hundred* requires *two* figures written side by side.

19. The *first* figure at the right denotes **Ones,** which are called **Units of the First Order.**

20. The figure in the *second* place denotes *ten* ones, which are called **Tens, or Units of the Second Order.**

Thus, the figures 35, denote 5 ones and 3 tens, and are read, "Thirty-five."

Write the following numbers in figures :

1. Twenty-five.	5. Thirty-six.	9. Seventy-three.
2. Thirty-eight.	6. Forty-nine.	10. Fifty-nine.
3. Fifty-six.	7. Fifty-four.	11. Eighty-eight.
4. Forty-two.	8. Sixty-eight.	12. Ninety-nine.

Read the following numbers :

13. 63.	17. 43.	21. 64.	25. 75.
14. 54.	18. 38.	22. 57.	26. 88.
15. 49.	19. 69.	23. 76.	27. 93.
16. 78.	20. 84.	24. 92.	28. 99.

21. **Ninety-nine** is the *largest* number which can be expressed by *two* figures.

22. To express the numbers from *ninety-nine* to *one thousand,* requires *three* figures written side by side.

23. The figure in the *third* place denotes *ten* tens, which are called **Hundreds,** or **Units of the Third Order.**

Thus, the figures 436, denote 4 hundred, 3 tens, and 6 units, and are read, "Four hundred thirty-six."

Write the following numbers in figures :

29. Two hundred forty-six.	33. Five hundred eight.
30. Three hundred fifty-four.	34. Six hundred seventy.
31. Five hundred thirty-two.	35. Eight hundred three.
32. Four hundred fifty.	36. Nine hundred ninety-nine.

Read the following numbers :*

37. 243.	41. 632.	45. 407.	49. 830.
38. 420.	42. 567.	46. 536.	50. 604.
39. 364.	43. 740.	47. 249.	51. 783.
40. 419.	44. 321.	48. 680.	52. 999.

24. **Nine Hundred Ninety-nine** is the *largest* number expressed by *three* figures.

---

* In reading numbers expressed by three or more figures, omit the word *and* after hundreds.

25. To express *larger* numbers, other orders of units are formed, called *thousands, ten-thousands, hundred-thousands, millions,* etc.

26. A figure in the *fourth* place denotes **Thousands,** which are called **Units of the Fourth Order.**

27. A figure in the *fifth* place denotes **Ten-thousands,** which are called **Units of the Fifth Order.**

28. A figure in the *sixth* place denotes **Hnndred-thousands,** which are called **Units of the Sixth Order.**

29. A figure in the *seventh* place denotes **Millions,** which are called **Units of the Seventh Order.**

30. If any orders are *omitted, ciphers* must be written in their places.

Thus, four thousand three hundred five, is written 4305.

The figures 5046, denote 5 thousands, 0 hundreds, 4 tens, and 6 units, and are read, "Five thousand forty-six."

Write the following in figures :

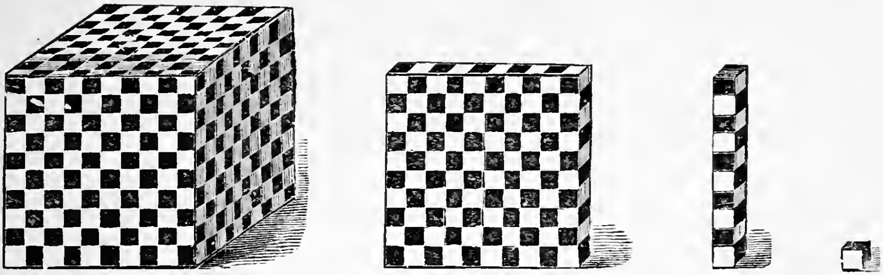
53. Three thousand two hundred sixty-eight.
54. Five thousand seventy-five.
55. Six thousand three hundred ten.
56. Seven thousand fifty-three.
57. Eight thousand seven hundred five.
58. Nine thousand nine hundred, ninety-nine.

Read the following numbers :

59.	1265.	63.	3420.	67.	5101.
60.	1503.	64.	3051.	68.	5049.
61.	2034.	65.	4036.	69.	6008.
62.	2105.	66.	4003.	70.	7059.

31. The *different values* of units expressed by the significant figures, are determined by the place they occupy, and are called *simple* and *local values.*

These values are illustrated by the following diagram:



1000. 100. 10. 1.

32. The **Simple Value** of the units represented by the significant figures is the number which they represent when standing *alone* or in *units* place.

33. The **Local Value** of these units is the number which they represent when standing on *either side* of units place.

Thus, 2 standing *alone*, or in the *first* place, denotes 2 *simple units;* in the *second* place, it denotes 2 *tens,* as in 25; in the *third* place, it denotes 2 *hundreds*, as in 246, etc.

NOTE.—These different orders of units correspond to dollars, dimes, and cents. Thus, 10 cents make 1 dime, 10 dimes 1 dollar. Now a *cent* is a unit, a *dime* is a unit, and a *dollar* is a unit; but these units have different values, corresponding to the orders of units.

34. From the above illustrations we derive the following

PRINCIPLES.

1°. Ten units of any order make a unit of the next higher order.

2°. Moving a figure one place to the left, increases its value ten times.

3°. Moving a figure one place to the right, diminishes its value ten times.

35. Hence, the *great law* of the Arabic Notation, viz.:

The **Orders of Units** *increase and decrease* by the uniform scale of **Ten.**

The Arabic Notation is therefore called the *Decimal System,* from the Latin word *decem,* which means *ten.*

36. To Express Decimal Parts of a Unit.

By the law of the decimal notation a unit of the *third* order is *ten* times a unit of the *second* order; a unit of the *second* order is *ten* times a *simple* unit or *one.*

By extending this law *below* units, a *simple unit* is *ten* times a unit of the *first decimal* order; a unit of the *first decimal* order is *ten* times a unit of the *second decimal* order, and so on. In this way a series of orders is formed below units which regularly *decrease* by the *scale* of ten.

37. The first order on the right of units is called *tenths;* the second, *hundredths;* the third, *thousandths;* etc.

38. These lower orders are separated from units by a period (.) called the **Decimal Point.**

39. The orders on the *left* of the decimal point are called **Whole Numbers** or **Integers**; those on the *right*, **Decimals.**

Thus, seven and five tenths are written 7.5; nine and fifty-three hundredths are written 9.53; sixty-five and two hundred seventy-three thousandths are written 65.273. The figures 4.7 denote four ones and seven tenths of one, and are read, "Four and seven tenths." The figures 6.35 denote six ones and thirty-five hundredths of one, etc.

Write the following in figures:

1. Five tenths.
2. Four hundredths.
3. Sixty-five hundredths.
4. Seventeen thousandths.
5. Forty-two thousandths.
6. Fifty-four thousandths.
7. 62 and 7 hundredths.
8. 3 units and 5 thousandths.
9. 245 and 25 hundredths.
10. 7 and 62 hundredths.
11. 456 and 273 thousandths.
12. 503 and 6 thousandths.

Read the following:

13.	3.7.	**17.**	62.3.	**21.**	0.25.	**25.**	42.365.
14.	5.24.	**18.**	75.21.	**22.**	0.7.	**26.**	125.034.
15.	23.9.	**19.**	36.45.	**23.**	0.253.	**27.**	245.007.
16.	31.25.	**20.**	68.4.	**24.**	0.45.	**28.**	360.248.

40. The *French Method* of writing and reading large numbers, is shown in the following

NUMERATION TABLE.

Names of Periods.	Trillions.				Billions.				Millions.				Thousands.				Units.				Thousandths.		
Orders of Units.	Hundred-trill.	Ten-trill.	Trillions.		Hundred-bill.	Ten-bill.	Billions.		Hundred-mill.	Ten-mill.	Millions.		Hundred-thou.	Ten-thou.	Thousands.		Hundreds.	Tens.	Units.	Decimal Point.	Tenths.	Hundredths.	Thousandths.
Number.	6	5	1	,	7	8	0	,	9	0	0	,	2	4	0	,	7	8	5	.	3	2	4
	5th Per.				4th Period.				3d Period.				2d Period.				1st Period.				Decimals.		

41. The *first* period on the *left* of the decimal point expresses units, tens, and hundreds, and is called **Units Period**; the *second* period denotes thousands, etc., and is called **Thousands Period**; and so on.

42. Beginning at unit's place, the orders on the *right* of the decimal point express tenths, hundredths, thousandths, etc.

The number in the table is read, "Six hundred fifty-one trillions, seven hundred eighty billions, nine hundred millions, two hundred forty thousand, seven hundred eighty-five, and three hundred twenty-four thousandths."

43. To express *larger* numbers, *other periods* are formed in like manner, called **Trillions, Quadrillions, Quintillions, Sextillions, Septillions, Octillions, Nonillions, Decillions,** etc.

44. To Express Numbers by Figures:

Begin at the left and write the figures of the given orders in succession towards the right. If any orders are omitted, supply their places by ciphers, and separate tenths from units by a decimal point.

45. To Read Numbers expressed by Figures:

Separate the number into periods of three figures each, counting each way from units place. Begin at the left, read each period separately, and add the name to each, except units period.

When there are decimals, read the figures on the right of the decimal point as if whole numbers, and add the name of the lowest order.

Thus, the figures 256,347.259 are read, "Two hundred fifty-six thousand, three hundred forty-seven, *and* two hundred fifty-nine thousandths."

NOTES.—1. In numerating decimals as well as whole numbers, the *units place* should always be made the *starting point.*

2. In reading whole numbers and decimals, the word *and* should be used between the whole number and the decimals.

Write the following numbers in figures:

1. One thousand, forty-two.
2. Thirty thousand, nine hundred seven.
3. Forty-six thousand, four hundred.
4. Ninety-two thousand, one hundred eight.
5. Sixty-eight thousand, seventy.
6. One hundred twenty-four thousand, six hundred.
7. Two hundred thousand, one hundred sixty.
8. Four hundred five thousand, forty-five.
9. Three hundred forty thousand.
10. Nine hundred thousand, seven hundred twenty.
11. One million, seven hundred thousand.
12. Thirty million, twenty thousand, fifty.
13. Three, and seven tenths.
14. Forty-five hundredths.
15. Twenty-eight, and three hundredths.
16. Two hundred fifty, and seven thousandths.
17. Thirty-five, and twenty-four thousandths.
18. Four hundred two thousand, and eight tenths.
19. Seventeen hundred, and twenty-five thousandths.
20. Nine thousand, two hundred five, and twenty-three hundredths.

Point off, numerate, and read the following numbers:

1.	520.	**15.**	207047.	**29.**	0.23.
2.	603.	**16.**	2605401.	**30.**	0.06.
3.	4506.	**17.**	4040680.	**31.**	0.235.
4.	7045.	**18.**	5604700.	**32.**	0.047.
5.	8700.	**19.**	2020105.	**33.**	4.05.
6.	25008.	**20.**	45001003.	**34.**	6.078.
7.	40625.	**21.**	30407045.	**35.**	0.265.
8.	75407.	**22.**	145560800.	**36.**	8.003.
9.	125242.	**23.**	8900401.	**37.**	9.036.
10.	240251.	**24.**	250708590.	**38.**	261.54.
11.	407203.	**25.**	803068003.	**39.**	24.06.
12.	300200.	**26.**	2175240670.	**40.**	3.807.
13.	1255673.	**27.**	7240305060.	**41.**	20.964.
14.	5704086.	**28.**	0.4.	**42.**	523.604.

[For Rom. Notation and Eng. Numeration, see Arts. **860–863**, Appendix.]

QUESTIONS.

1. What is a unit? 2. Number? 3. The unit of a number? 4. An abstract number? 5. Concrete? 6. Like numbers? 7. Unlike?

9. What is Arithmetic? 10. Notation? 11. Numeration? 12. The Arabic Notation? 15. What is the value of a figure? 19. What does the first figure at the right denote? 20. In the second place? What called?

23. In the third place? What called? 24. What is the largest number expressed by three figures?

25. How are larger numbers expressed? 26. What does a figure in the fourth place denote? 27. In the fifth? 28. In the sixth? 29. In the seventh? 30. If any orders are omitted, what is to be done?

31. What are the different values expressed by figures called? How determined? 32. What is the simple value of a figure? 33. The local value?

34. Name the three principles of Notation? 35. What is the great law of the Arabic Notation? What is it often called? Why? 36. Explain how this law is applied in expressing parts of one?

37. What is the first place at the right of units called? The second? Third? 38. How are these orders separated from units? 39. What are the orders at the left called? Those at the right?

40. Repeat the Numeration Table? 41. What is the first period on the left of the decimal point called? The second? The third? 42. The first period on the right of units? 43. How are larger numbers expressed? 44. How express numbers by figures? 45. How read them?

ADDITION.

ORAL EXERCISES.

46. 1. How many are 7 marbles, 5 marbles, and 9 marbles ?

SOLUTION.—7 and 5 are 12 and 9 are 21. *Ans.* 21 marbles.

2. How many dollars are 15 dollars, 10 dollars and 6 dollars?
3. How many are 12 books, 8 books, and 7 books ?
4. How many units are 15 units, 6 units, and 10 units?
5. How many are 6, 8, and 4 ? 7, 5, and 6 ?
6. If one tree bears 12 peaches, another 9, another 6, how many peaches will all bear?
7. A dairy-woman put 17 pounds of butter into a stone jar, and afterwards added 8 pounds more ; how many pounds of butter did the jar contain?
8. If you have 12 pens in your box, and afterwards add 7 more, how many pens will your box contain?
9. If a class of 12 pupils is united with a class of 15 pupils, how many will be in the class?
10. If 9 units and 7 units and 8 units are united in one number, how many units will the number contain?

DEFINITIONS.

47. Addition is uniting two or more numbers in *one.*

The *answer* or number found by addition is called the **Sum** or **Amount.**

NOTE.—The *Sum* or *Amount* contains as many *units* as all the numbers added.

48. The **Sign of Addition** is +. It is called **Plus,** which means *more,* and shows that the numbers between which it is placed are to be added. It is read "and," or "added to."

Thus, 5 + 3 is read, "5 plus 3," "5 and 3," or "5 added to 3."

49. The **Sign of Equality** is =. It is read, "equal" or "is equal to," and shows that the numbers between which it is placed are *equal*.

Thus, the expression 5 + 4 = 9, is read, "5 plus 4 equal 9," or "the sum of 5 and 4 is equal to 9."

How many are

11.	6+5?	**19.**	3+2+1?	**27.**	7+5+6+2?
12.	7+4?	**20.**	2+4+3?	**28.**	3+6+0+8?
13.	5+3?	**21.**	5+7+2?	**29.**	4+7+5+2?
14.	6+8?	**22.**	2+4+6?	**30.**	8+3+2+6?
15.	7+7?	**23.**	6+1+8?	**31.**	3+5+9+4?
16.	4+9?	**24.**	7+0+9?	**32.**	7+6+8+5?
17.	8+3?	**25.**	4+9+2?	**33.**	8+7+9+6?
18.	9+6?	**26.**	7+6+9?	**34.**	9+8+7+4?

35. If you pay 10 cents for an inkstand, 8 cents for paper, 4 cents for pens, how much will you pay for all?

36. If you pick 7 apples from one tree, 5 from another, and 6 from another, how many apples will you have?

37. What is the sum of 9 dollars, 7 dollars, and 4 dollars?

38. How many are 11 books, 10 books, and 6 books?

39. If you pay 10 cents for fare, 15 cents for lunch, and 8 cents for fruit, how much will you have spent?

Slate Exercises.

50. Write the following in columns and add each up and down several times, naming results only.

1. Add 4, 2, 6, 3, 5, 6, 4, 5, 3, 4, and 2.

Thus, 4, 6, 12, 15, 20, 26, etc.

2. Add 5, 3, 4, 2, 3, 5, 6, 2, 7, 4, and 3.
3. Add 4, 3, 5, 7, 6, 2, 3, 7, 4, 6, 4, and 5.
4. Add 2, 5, 4, 3, 7, 6, 4, 3, 2, 4, 5, and 6.
5. Add 8, 2, 7, 6, 5, 3, 1, 7, 6, 8, 7, 4, 3, and 7.
6. Add 9, 4, 2, 7, 3, 4, 5, 6, 8, 4, 6, 5, 4, and 8.
7. Add 7, 5, 3, 6, 4, 2, 5, 3, 8, 1, 6, 3, 4, and 9.
8. Add 6, 3, 4, 7, 2, 9, 4, 3, 7, 8, 5, 2, 6, 4, and 7.

9. Add 8, 4, 2, 7, 5, 3, 6, 2, 4, 6, 5, 8, 1, 9, 5, 6, and 8.

10. Add 6, 8, 3, 5, 2, 7, 4, 6, 3, 1, 7, 6, 5, 4, 6, 8, and 9.

11. What is the sum of 3232, 20, 4314, and 2123?

Explanation.—We write the numbers so that units of the same order stand in the same column, and naming results only, proceed thus, 3, 7, 9 units. Write the 9 under units column.

Adding the tens in the same manner, 2, 3, 5, 8 tens. Write the 8 under tens column, and add the hundreds and thousands in like manner.

We prove the work by beginning at the top and adding each column downward. The two results agree, therefore the work is right.

OPERATION.
3232
20
4314
2123
Ans. 9689

Add and prove the following:

(12.)	(13.)	(14.)	(15.)
4121	3204	5202	2320
304	4050	43	3054
1052	23	430	412
3402	612	2304	4012

16. A man paid 5423 dollars for a house, 325 dollars for repairs, and 430 dollars for painting; what did the whole cost?

17. If a steamer goes 243 miles in one day, 321 miles the next, and 402 miles the third, how far does she go in 3 days?

18. A father gave 314 acres to one son, 241 acres to another, and 432 acres to another; how many acres did he give to all?

Oral Drill.

51. 1. Add by 2's from 0 to 50, naming results only.

Thus, two, four, six, eight, ten, etc.

Add in like manner

2. By 2's from 1 to 51.
3. By 3's from 0 to 60.
4. By 3's from 1 to 61.
5. By 3's from 2 to 62.
6. By 4's from 0 to 64.
7. By 4's from 1 to 65.
8. By 4's from 2 to 66.
9. By 4's from 3 to 67.

10. Add the other digits 5, 6, 7, 8, 9 in the same manner, till the result becomes as large as may be deemed desirable.

Development of Principles.

52. 1. What is the sum of 5 peaches and 8 peaches ?

2. What is the sum of 7 apples and 5 marbles ?

Ans. Apples and marbles are *unlike* numbers and cannot be added. (Art. **7.**)

3. What is the sum of 7 units and 9 units? Of 3 tens and 5 tens ?

4. Is the sum of 4 tens and 5 units equal to 9 tens or 9 units ?

Ans. Tens and units are *unlike* orders and cannot be added to each other.

5. Which is the greater, the sum of $4+5+6$, or of $6+5+4$?
Ans. The sums are equal.

53. From the above examples we infer the following

Principles.

1°. Only like numbers and like orders of units can be added one to another.

2°. The sum is the same in whatever order numbers may be added.

Oral Exercises.

54. 1. How many are 40 pounds and 50 pounds?

Analysis.—40 is equal to 4 tens and 50 is equal to 5 tens; now 4 tens and 5 tens are 9 tens or 90. *Ans.* 90 pounds.

Note.—In adding large numbers mentally, it is more *convenient* and *expeditious* to begin with the *highest* orders.

2. How many are 30 and 40 ? 50 and 60 ? 80 and 70 ?

3. If I pay 56 dollars for a sleigh and 37 dollars for a cart, what will both cost?

Analysis.—56 equals 5 tens and 6 units, and 37 equals 3 tens and 7 units. Now 5 tens and 3 tens are 8 tens, or 80, and 6 units and 7 units are 13 units, or 1 ten and 3 units which added to 80 make 93 dollars, *Ans.*

4. A farmer had 2 wheat fields; one produced 68 bushels, the other 75 bushels; how many bushels did both produce?

5. If Charles reads 64 pages one day, and 78 pages the next day, how many pages will he read in both days?

6. A drover bought 87 sheep of one man and 98 of another; how many sheep did he buy of both?

7. A lad having spent 40 cts. finds he has 37 cts. left; how much had he at first?

8. How many are 78 and 97?

9. How many are 84 and 69?

10. A man divided his farm into 2 parts, one of which contained 77 acres and the other 88 acres; how many acres were in his farm?

11. What is the sum of $43 + 28 + 7$?

12. What is the sum of $52 + 43 + 9$?

13. The price of a horse is 94 dollars, of a buggy is 62 dollars, and a saddle is 10 dollars; what is the price of all?

Written Exercises.

55. When the Sum of a Column exceeds 9.

1. What is the sum of 4524, 276, 6745 and 5498?

OPERATION.

	4524
	276
	6745
	5498
Ans.	17043

Explanation.—Since like orders only can be added, for convenience we write them under each other, and beginning at the right, add each column separately, naming the results. Thus, adding the first column, 8, 13, 19, 23 units, or 2 tens and 3 units, we write the 3 units under the column of units, and add the 2 tens to the column of tens because they are like orders.

Adding the next column, 2, 11, 15, 22, 24 tens, or 2 hundreds and 4 tens, we set the 4, or units figure *of the sum*, under the column added, and add the tens figure to the next column.

Again, the sum of the next column is 20 hundreds, or 2 thousands and no hundreds. We place a cipher under the column, and add the 2 to the next column.

The sum of this column is 17 thousands, and being the last, we set down the whole sum.

Note.—The process of reserving the *tens* and adding them to the next column, is called "carrying the tens."

Add and explain the following in like manner :

(2.)	(3.)	(4.)	(5.)
3506 yards.	4672 rods.	7845 weeks.	8407 pounds.
4824 "	89 "	468 "	3400 "
719 "	725 "	5030 "	7902 "
3005 "	9306 "	6404 "	8434 "

56. To Add Decimals, or U. S. Money.

6. What is the sum of 235.267; 75.43; 8.624; 0.238; and 362.07?

Explanation.—We write the numbers so that the same orders stand in the same column, and beginning with the lowest add each column, and set down the result as before, placing the decimal point under those in the numbers added.

OPERATION.

```
      235.267
       75.43
        8.624
        0.238
      362.07
      -------
Ans.  681.629
```

7. Find the sum of 65.7; 248.62; 40.255; 54.07; and 6.389.

8. Find the sum of 3.036; 0.75; 23.008; 0.236; and 87.604.

57. The *denominations* of **U. S. Money** increase and decrease by *tens*, and are expressed in the same manner as *whole* numbers and *decimals*. (Art. **39.**)

58. *Dollars* are **Integers**, and occupy the place of *whole* numbers. *Cents* occupy the place of *tenths* and *hundredths*, and *mills* the place of *thousandths*. Dollars are separated from cents by the *decimal point*. (Art. **39.**)

59. The **Dollar Sign** is $. Thus, $25 is read, "25 dollars."

The **Sign for Cents** is ¢, or ct.; as, 17¢, or 17 ct.

The expression $64.735 is read, "Sixty-four dollars, seventy-three cents and five mills."

Note.—As *two* places, tenths and hundredths, are occupied by cents, if the number of cents is *less* than 10, a *cipher* must be placed before the figure representing them. Thus, seven cents are written $0.07.

9. Add \$38.273; \$80.46; \$5.073; and \$0.85.

SUGGESTION.—Write dollars under dollars, cents under cents, and add as above. *Ans.* \$124.656.

(10.)	(11.)	(12.)	(13.)
\$42.213	\$36.23	\$23.463	\$42.282
4.30	10.826	14.052	2.20
30.034	4.05	40.201	23.034
12.42	53.615	23.124	26.317

60. From the preceding Exercises we infer the following

GENERAL RULE.

I. Write the numbers so that units of the same order stand in the same column.

II. Add the right hand column, and placing the units of the sum under it, add the tens to the next order.

III. Proceed thus with the other columns, writing the whole sum of the last. If there are decimals, place the decimal point of the sum under those in the numbers added.

PROOF.—*Add the numbers from the top downward, and if the two results agree the work is right.* (Art. 3°.)

APPLICATIONS.

1. Four men formed a partnership; A furnished \$2878, B \$1784, C \$1265, and D \$894. What was the amount of their capital?

2. A man sold three house lots; for one he received \$975, for another \$763, and for the third \$586. What did the whole amount to?

3. A gentleman purchased a store for \$4500, and paid \$75 for repairs, and \$150 for having it enlarged. For how much must he sell it, in order to gain \$175?

4. A merchant paid \$375 for one package of goods, \$467 for another, \$254 for another, and \$348 for another. How much did he pay for all?

5. A certain orchard contains 256 apple trees, 119 peach trees, 83 plum trees, and 45 pear trees. How many trees are there in the orchard?

6. A man being asked his age, said he was 17 years old when he left the academy, he spent 4 years in college, 3 years in a law school, practiced law 15 years, was a member of congress 18 years, and it was 16 years since he retired from business. How old was he?

7. A shopkeeper having a note due, paid \$184 at one time, at another \$268, at another \$379, at another \$467, and there were \$350 still unpaid. What was the amount of his note?

8. A gentleman owns a house worth \$10800, a store worth \$5450, a farm worth \$3700, and has \$15000 personal property. What is the amount of his estate?

9. A man left his estate to his wife, his three sons, and two daughters; to his wife he gave \$10350, to his sons \$5450 apiece, and his daughters \$3500 apiece. How much was he worth?

Add and prove the following:

10. $261 + 31 + 256 + 17$?
11. $163 + 478 + 82 + 19$?
12. $428 + 632 + 76 + 394$?
13. $320 + 856 + 100 + 503$?
14. $641 + 108 + 138 + 710$?
15. $700 + 66 + 970 + 21$?
16. $304 + 971 + 608 + 496$?
17. $848 + 683 + 420 + 668$?
18. $868 + 45 + 17 + 25 + 27 + 38$?
19. $641 + 85 + 580 + 42 + 7 + 63$?
20. $425 + 346 + 681 + 384$?
21. $135 + 342 + 778 + 528$?
22. $460 + 845 + 576 + 723$?
23. $2345 + 4088 + 260 + 819$?
24. $8990 + 5632 + 5863 + 756$?
25. $2842 + 6361 + 523 + 836$?
26. $602 + 173 + 586 + 408 + 973$?
27. $424 + 375 + 626 + 75 + 855$?

28. A man wishing to stock his farm, paid $197 for horses, $86 for oxen, $175 for cows, and $169 for sheep. How much did he give for the whole?

29. A butcher sold to one customer 157 pounds of meat, to another 159, to another 149, to another 97, and to another 68 pounds. How much did he sell to all?

30. A carpenter received $879 for one job, for another $786, for another $693, for another $587, for another $476, and for another $368. How much did he receive in all?

31. A merchant pays $560 a year for store rent, $1386 to one clerk, $1267 to another, and $369 for various other expenses. What does his business cost him a year?

32. A man receives $568 rent for one store, $479 for another, and $276 for another. How much does he receive for them all?

(33.)	(34.)	(35.)	(36.)
$75,340	$68,901	$64,268	$346,768
6,731	50,345	405	21,380
748	75,005	1,708	4,075
68,451	29,450	43,671	126,849
396	80,063	72,049	257
7,503	91,700	492	1,305
46,075	43,621	1,760	24,350
1,290	47,834	25,357	439,871
25,738	83,276	1,434	40,306
46,803	25,327	84,162	601,734

37. What is the sum of five billions, ten millions forty-five; eight millions, eight thousand, eight; two billions, four hundred thirty millions, two hundred thousand, four hundred?

38. A man paid $2243 for a house, $825 for a barn, and for his farm as much as for his house and barn together; how much did he pay for his farm; and how much for all?

39. A man having 7 children gave a farm to each worth $2378; what was the value of all their farms?

40. A man bequeathed $6275 apiece to his 3 children, and to his wife the balance of his property, which was equal to the amount he gave all his children; what was he worth?

41. Sir Isaac Newton was born in the year 1642, and died in his eighty-fifth year; in what year did he die?

42. Four men, A, B, C, and D, built a school-house; A gave \$1500, B \$1750, C \$1975, and D gave the land, which was worth as much as A and B gave; what was the whole cost?

The Census Report of 1880 gives the population of the U. S. as follows:

43. Maine, 648,936; N. H., 346,991; Vt., 332,286; Mass., 1,783,085; R. I., 276,531; Conn., 622,700; N. Y., 5,082,871; N. J., 1,131,116; Penn., 4,282,891. What was the population of the *North Atlantic* States?

44. Delaware, 146,608; Md., 934,943; D. C., 177,624; Va., 1,512,565; West Va., 618,457; N. C., 1,399,750; S. C., 995,577; Ga., 1,542,180; Fla., 269,493. What was the population of the *South Atlantic* States?

45. Ohio, 3,198,062; Ind., 1,978,301; Ill., 3,077,871; Mich., 1,636,937; Wis., 1,315,497; Minn., 780,773; Ia., 1,624,615; Mo., 2,168,380; Dak., 135,177; Neb., 452,402; Kan., 996,096. What was the population of the *Northern Central* States?

46. Alabama, 1,262,505; Miss., 1,131,597; La., 939,946; Texas, 1,591,749; Ark., 802,525; Tenn., 1,542,359; Ky., 1,648,690. What was the population of the *Southern Central* States?

47. California, 864,694; Col., 194,327; Or., 174,768; Wash., 75,116; Id., 32,610; Mon., 39,159; Wy., 20,789; Utah, 143,963; Arizona, 40,440; Nev., 62,266; New Mex., 119,565. What was the population of the *Western* States and Territories?

48. What was the whole population of the U. S. in 1880?

NOTE.—The above is the *new grouping* of the States and Territories proposed by the Census Bureau of 1880.

QUESTIONS.

47. What is Addition? What is the answer called? 48. Make the sign of addition? What called? How read? 49. Make the sign of equality.

53. What kind of numbers can be added? What orders? 57. How do the denominations of U. S. Money increase and decrease? 58. How are they expressed? 60. Give the general rule. Proof?

SUBTRACTION.

ORAL EXERCISES.

61. 1. Edward had 12 pears in his basket and took out 5 of them; how many were left?

SOLUTION.—5 pears taken from 12 pears leave 7 pears. *Ans.* 7 pears.

2. If you take 6 cents from 14 cents, how many will be left?
3. What is the difference between 7 pounds and 15 pounds?
4. A lady bought a hat for $10 and gave in payment a $20 bill; how much change ought she to receive?
5. James is 16 years old and his sister is 7; what is the difference in their ages?
6. If 9 yards of cloth are cut from a piece containing 24 yards, how many yards will be left?
7. Charles had $25 silver dollars and gave 8 of them to the orphan asylum; how many dollars did he then have?
8. If 9 is taken from 17, how many are left?
9. How many more is 18 than 6? Than 7?
10. If a slate cost 12 cents and a reader 26 cents, how much more will one cost than the other?

DEFINITIONS.

62. **Subtraction** is taking one number from another.

63. The **Subtrahend** is the number to be subtracted.

64. The **Minuend** is the number from which the subtraction is made.

65. The *Answer*, or number found by subtraction, is called the **Difference** or **Remainder.**

NOTE.—Subtraction is the *reverse* of Addition. The one *unites* numbers, the other *separates* them.

66. The **Sign** of Subtraction is —. It is called **minus,** which means *less.* When placed between two numbers it shows that the number *after* it is to be subtracted from the one *before* it.

Thus, the expression $12 - 5 = 7$ is read, "12 minus 5 equals 7," or "is equal to 7," or "12 less 5 equals 7."

67. The **Parenthesis** ( ), and the **Vinculum** $\overline{}$, respectively show that the numbers included by them are to be considered as *one number.*

Thus, $16 - (4 + 3)$ shows that the sum of 4 and 3, or 7 is to be subtracted from 16, and the result is 9.

How many are

11. 14—6?	**17.** 16—5?	**23.** 23—7?	**29.** 52—6?
12. 16—7?	**18.** 15—7?	**24.** 27—9?	**30.** 63—7?
13. 11—6?	**19.** 18—4?	**25.** 34—6?	**31.** 74—8?
14. 13—5?	**20.** 19—8?	**26.** 42—8?	**32.** 83—5?
15. 15—8?	**21.** 18—9?	**27.** 35—6?	**33.** 97—8?
16. 17—9?	**22.** 17—8?	**28.** 44—8?	**34.** 84—9?

Development of Principles.

1. What is the difference between 15 pencils and 9 pencils?

2. What is the difference between 2 books and 5 chairs?

Ans. Books and chairs are *unlike* units, and one cannot be subtracted from the other.

3. What is the difference between 9 units and 15 units? Between 5 tens and 3 tens?

4. What is the difference between 5 tens and 3 ones?

Ans. Tens and *ones* are unlike orders of units, and one cannot be subtracted from the other.

5. If the minuend is 14 and the subtrahend 8, what is the remainder?

6. If the remainder 6 is added to the subtrahend, to what is the sum equal?

Ans. To the minuend.

7. What is the difference between 8 and 12? If you add 3 to each of the numbers 8 and 12, what is the remainder?

Ans. 4, the same as before.

68. From the examples above we infer the following

PRINCIPLES.

1°. Only like numbers and like orders of units can be subtracted one from the other.

2°. The difference and subtrahend are equal to the minuend.

3°. If two numbers are equally increased, their difference is not altered.

WRITTEN EXERCISES.

69. *When each order in the Subtrahend is less than the corresponding order of the Minuend.*

1. From 4678 subtract 1435.

OPERATION.	
4678	Min.
1435	Sub.
3243	Rem.

EXPLANATION.—We write the subtrahend under the minuend, placing units of the same order in the same column. Beginning at the right, we say, "5 units from 8 units leave 3 units," and write the 3 in *units* place, under the figure subtracted. Next, 3 tens from 7 tens leave 4 tens, which we write in *tens* place. 4 hundreds from 6 hundreds leave 2 hundreds, which we write in *hundreds* place. Finally 1 thousand from 4 thousand leave 3 thousand, which we write in *thousands* place. The remainder is 3243.

To prove the result, add the *remainder* to the *subtrahend*, and if the sum is equal to the *minuend* the work is right. (Art. **68**, *2°*.)

Subtract the following in like manner:

	(2.)	(3.)	(4.)	(5.)	(6.)
From	5374	6487	7636	8768	9689
Take	2142	3243	4212	5243	6476

7. A farmer having 876 acres of land, sold 375 acres; how many had he left?

8. A man having a note of $2365 due, had only $1231 on hand; how much more must he collect to pay the note?

9. The population of Cal. in 1880 was 864,694, that of Neb. was 452,402; what was the difference?

Oral Exercises.

1. Charles picked 10 quarts of chestnuts, and on his way home sold 4 quarts. The next day he picked 9 quarts more; how many quarts had he then?

2. John had 15 cents and his father gave him 10 more; he then spent 6 cents for candy; how many cents had he left?

3. A man having \$30, spent the sum of \$5 + \$4; how much had he left?

How many are

4.	14—6+3?	10.	18+4—6?	16.	21—(4+5)?
5.	16—9+4?	11.	24+7—3?	17.	18—(6+5)?
6.	27—8+6?	12.	17+9—8?	18.	24—(7+8)?
7.	23—7+4?	13.	28+4—6?	19.	32—(5+7)?.
8.	19—6+5?	14.	19+8—7?	20.	36—(9+7)?
9.	31—8+6?	15.	25+6—9?	21.	38—(8+10)?

22. Subtract by 2's from 40 to 0. Thus, 40, 38, 36, 34, etc.

23.	By 2's from 51 to 1.	27.	By 6's from 51 to 0.
24.	By 3's " 60 to 0.	28.	By 7's " 64 to 0.
25.	By 4's " 61 to 1.	29.	By 8's " 71 to 0.
26.	By 5's " 70 to 0.	30.	By 9's " 80 to 0.

Written Exercises.

70. *When an order in the Subtrahend is larger than the corresponding order in the Minuend.*

1. What is the difference between 5847 and 2563?

OPERATION.	
5847	Min.
2563	Sub.
3284	Rem.

Explanation.—Since like orders only can be subtracted, for convenience we write them under each other. Beginning at the right we say, 3 units from 7 units leave 4 units; write the 4 in units place. Next, since 6 tens are more than 4 tens, we take one of the hundreds (equal to 10 tens), and add it to 4 tens, making 14 tens; now 6 tens from 14 tens leave 8 tens, which we write in tens place. As we took 1 from 8 hundreds, only 7 hundreds are left, and 5 hundreds from 7 hundreds leave 2 hundreds, which we write in hundreds place. Then, 2 thousand from 5 thousand leave 3 thousand. *Ans.* 3284.

NOTES.—1. The process of taking a unit from a *higher* order in the minuend and adding it to a *lower* order, is called BORROWING TEN.

2. When we "borrow 10," it is more *logical* to take 1 from the next order of the minuend; but *practically* it is more *convenient* to add 1 to the next order of the subtrahend. (See Ex. 4.)

Subtract and explain the following in like manner:

	(2.)	(3.)	(4.)	(5.)
From	22304	30426	60000	84357
Take	12012	20343	32114	50018

71. Decimals, and dollars and cents are subtracted like integers; the decimal point in the remainder being placed under those in the given numbers.

6. What is the difference between $285.47 and $159.30?

EXPLANATION.—Subtract as in integers, placing the decimal point in the remainder under those in the given numbers.

	$285.47
	159.30
Ans.	$126.17

	(8.)	(9.)	(10.)	(11.)
From	325.2	431.58	$562.67	$6000.009
Take	108.3	249.39	320.48	2315.07

72. From the preceding examples we derive the following

GENERAL RULE.

I. Write the subtrahend under the minuend so that units of the same order stand under each other.

II. Begin at the right and subtract each order separately, placing the remainder below.

III. If any order of the subtrahend is larger than that above it, add ten to the upper order and subtract. Consider the next order of the minuend one less, and proceed as before, placing the decimal point in the remainder under those in the given numbers. (Art. **71.**)

PROOF.—*Add the remainder to the subtrahend; if the sum is equal to the minuend, the work is right.*

APPLICATIONS.

1. A man bought a piece of cloth containing 237 yds., and sold 124 yds. of it. How much had he left?

2. A merchant had on hand a quantity of flour, for which he asked $245; but for cash he sold it for $24 less. How much did he receive for his flour?

3. In a certain academy there were 357 scholars, 168 of whom were young ladies. How many gentlemen were there?

4. A farmer raised 4879 bushels of wheat, and sold 3876 bushels. How much had he left?

5. A farmer raised 1389 bu. of wheat one year, and 1763 the next. How much more did he raise the second year than the first?

6. A man bought a house and lot for $5687. The house was worth $3698; how much was the lot worth?

7. Suppose a man's income is $3268 a year, and his expenses are $2789. How much can he save in a year?

8. If a man has $3290 in real estate, and owes $1631, how much is he worth?

9. A father gave his son $8263, and his daughter $5240; how much more did he give his son than his daughter?

10. A man bought a farm for $9467, and sold it for $11230; how much did he make by his bargain?

11. If a man's income is $10000 a year, and his expenses $6253, how much will he lay up?

12.	4165 — 2340.	**19.**	45723 — 31203.
13.	5600 — 3000.	**20.**	81647 — 57025.
14.	7246 — 4161.	**21.**	265328 — 140300.
15.	8670 — 7364.	**22.**	170643 — 106340.
16.	17265 — 13167.	**23.**	465746 — 241680.
17.	21480 — 20372.	**24.**	694270 — 590395.
18.	30671 — 26140.	**25.**	920486 — 500000.

26. The captain of a ship having a cargo of goods worth $29230, threw overboard in a storm $13216 worth; what was the value of the goods left?

27. A merchant bought a quantity of goods for \$12645, and afterwards sold them for \$13960; how much did he gain by his bargain?

28. A man paid \$23645 for a ship and afterwards sold it for \$18260; how much did he lose by his bargain?

Perform the following operations in the order indicated:

29. $275+317-87+49+95-216+342-67$.

30. $436-122+63+786-678+406-309+360$.

31. $639-250+873+67-19+476-506+1000$.

32. $4678-2500+6200-4004+502-625+1600-268$.

33. $6450+476-4578+5065+250-1000+608$.

34. $87200-463+225+1800-6200-75+98+2256$.

35. $7300+163-4005-85+2640-1375-23+867$.

36. $9640+9200-7000-75+4560+125-2000+485$.

37. $1452+325+684-(631+845)=?$

38. $4850+6300-(800+3285)=?$

39. $\$256.62+(\$64.50-\$20)=?$

40. $\$5278+\$340.50-(\$480.40+\$65.75)=?$

ORAL PROBLEMS FOR REVIEW.

73. **1.** The sum of two numbers is 26 and one of them is 7; what is the other?

2. The greater of two numbers is 24 and the difference is 9; what is the less?

3. When the greater of two numbers and their difference are given, how find the *less?*

4. The less of two numbers is 28 and the difference is 12; what is the greater?

5. When the less of two numbers and their difference are given, how find the greater?

6. A boy having 75 cents, spent 32 cents for toys; how many cents did he then have?

ANALYSIS.—75 is equal to 7 tens and 5 units; and 32 is equal to 3 tens and 2 units; now 3 tens from 7 tens leave 4 tens or 40, and 2 units from 5 units, leave 3 units, which added to 40 makes 43. *Ans.* 43 cts.

NOTE.—When the numbers in subtraction are large, it is advisable, in mental operations, to begin at the highest orders, as in addition.

7. If the price of a history is 90 cts., and that of a reader is 70 cts., what is the difference in their price?

8. A farmer raised 80 bu. corn and sold 50 bu.: how many bushels had he left?

9. The united ages of two persons is 45 years, and the younger is 22; what is the age of the older?

10. William and Charles together caught 58 fish, and William caught 27: which caught the more, and how many?

11. In a school of 85 pupils, 48 are girls: how many boys are there, which department is the larger, and by how many?

12. A lady having 2 ten-dollar bills, paid \$9 for a hat, \$4 for lace, and \$2 for gloves: how much money had she left?

13. Which is the greater, 24 + 16, or 52 — 9?

14. A gentleman paid \$12 for pants, \$9 for a vest, and \$7 for boots; he paid for them with 2 ten-dollar bills and 2 fives; how much change should he have?

15. A merchant paid \$78 for a case of goods, and \$5 freight; for how much must he sell them to make \$10?

16. If you have \$47 and pay \$17 for a bicycle and \$2 for a cap, how much money will you have left?

17. A lad had 54 marbles; he gave away 28 and found 5; how many marbles had he then?

Oral Drill in Adding and Subtracting.

1. To 5 add 6, subtract 3, add 7, subtract 8, add 4, subtract 7, add 8, subtract 3; what is the result?

NOTE.—While the teacher dictates the example, "To 5 add 6, subtract 3," etc., the pupils *think* 11, 8, 15, etc. The answer may be given in concert, or by some individual designated by the teacher.

2. From 15 take 6, add 7, take 8, add 5, take 6, add 4, take 9, add 10; result?

3. To 14 add 5, take 7, add 4, take 3, add 8, take 5, add 6, take 4, add 9, take 6; result?

4. How many are 23 — 7 + 3 — 4 + 10 — 8 + 5 — 7 + 6?

5. How many are 7 + 9 — 10 + 6 — 4 + 7 — 8 + 9?

6. How many are 12 + 6 — 8 + 4 — 3 + 20 — 10 + 5?

7. How many are 27 — 8 + 9 — 10 + 7 — 6 — 8 + 9 + 7 — 5?

8. How many are $23-6+11-8+9-6+4-7+8$?
9. How many are $32-5+3-7+6-8+9-7+4-6$?
10. How many are $35+8-7+6-4+8-7+5-8+12$?
11. How many are $38+7-4+5-8+6+2-9+6$?
12. How many are $28+4-7+6-9+8-9+10$?

Written Problems for Review.

74. 1. The minuend is 3642.05, and the difference is 3202.8; what is the subtrahend?

2. Two brothers commenced business at the same time; one gained $3678 in five years, the other gained $2387. How much more did one gain than the other?

3. The subtrahend is 48206.5 and the difference is 35206.2; what is the minuend?

4. A ship having a cargo valued at $100000, was overtaken by a storm, and $27680 worth of goods were thrown overboard. How much of the cargo was saved?

5. A gentleman having $1768 on deposit, gave a check for $175 to one man, to another for $238.25, and to another for $369.50. How much remained on deposit?

6. An orchard contained 120 apple trees, 47 peach trees, and 28 pear trees. Of the apple trees 26 were cut down, 18 of the peach trees died, and 5 of the pear trees were blown down. How many trees were left?

7. A gentleman had $2700 to distribute among his three sons. To the eldest he gave $825, to the second $785, and the remainder to the youngest. How much did the youngest son receive?

8. A merchant had in his storehouse 6384 bushels of wheat, 3752 bushels of corn, 4564 bushels of oats, and 1384 bushels of rye; it was broken open and 3564 bushels of grain taken out. How many bushels remained?

9. If a man's income is $4586 a year, and he spends $384.86 for clothing, $568 for house rent, $784.75 for provisions, $568.50 for servants, and $369 for traveling, how much will he have left at the end of the year?

10. A gentleman left a fortune of \$18864 to his two sons and one daughter; to one son he gave \$6389, to the other \$6984. How much did the daughter receive?

11. A man having \$7689, invested \$689 in railroad stock, \$500 in a woolen factory, and \$1250 in bank stock. How much had he left?

12. What number added to 3645 makes 630712?

13. A man worth \$30000, lost a store by fire worth \$5000, and goods to the amount of \$3578. How much had he left?

14. From twenty-five thousand, twenty-five, take 28 hundred.

15. From 16 millions, 16 thousand, take 16 hundred.

16. The difference between 185 billions, and 185 millions?

17. What number must be added to 836.25 to make 2323?

18. How many times can 563 be subtracted from 2815 before the latter will be exhausted?

19. What number is that, from which if you take 42371, the remainder will be 19289 less 176.05?

20. What number is that, from which if you take 18268, the remainder will be 26017—17312?

21. What number is that, from which if 27239 be taken, the remainder will be 9897—3076.5?

22. A says to B, "I have 2675 sheep"; B replies, "I have 763 less than you"; C adds, "I have as many as both lacking 105." How many sheep had B and C?

23. The sum of 3 numbers is 23257; the first is 9277, the second is 1283 less than the first; what is the third number?

24. The population of the U. S. in 1840 was 17069453, in 1880 it was 50155783; what was the increase in 40 years?

QUESTIONS.

62. What is Subtraction? 63. The Subtrahend? 64. Minuend? 65. The Answer? 66. The Sign of Subtraction? What called? How read? 67. For what are the Parenthesis and Vinculum used? 68. What numbers only can be subtracted? What orders?

68. If the difference of two numbers is added to the less, to what is the sum equal? If two numbers are equally increased, how is their difference affected? 71. How subtract decimals and dollars and cents? 72. Give the general rule. How is subtraction proved?

MULTIPLICATION.

MENTAL EXERCISES.

75. 1. What will 3 pencils cost, at 4 cents apiece?

ANALYSIS.—At 4 cts. apiece, 3 pencils will cost the sum of 4 cts. + 4 cts. + 4 cts., or 4 cts. taken 3 times, which are 12 cts. Or, more briefly, 3 pencils will cost 3 times as much as 1 pencil, and 3 times 4 cts. are 12 cts.

2. At $5 each, what will 4 hats cost?
3. At 5 cts. apiece, what will 3 bananas come to?
4. In 1 gallon there are 4 qts.; how many qts. are in 5 gallons?
5. At 6 cts. a lb., what will 4 lbs. of rice cost?
6. If 1 qt. of cherries cost 6 cts., what will 3 qts. cost?
7. What will 5 vests cost, at $7 apiece?
8. If it takes 6 yds. of cloth to make 1 cloak, how many yards will it take to make 5 cloaks?
9. In 1 week there are 7 days; how many days in 4 weeks?
10. How many units in five 8's united in one number?

DEFINITIONS.

76. **Multiplication** is finding the *amount* of one number taken as many times as there are units in another.

77. The **Multiplicand** is the number to be multiplied.

78. The **Multiplier** is the number by which we multiply. It shows how many times the multiplicand is to be taken.

79. The *Answer*, or number found by multiplication, is called the **Product**.

Thus, when it is said that 4 times 6 are 24, 6 is the mutiplicand, 4 the multiplier, and 24 the product.

80. The multiplicand and multiplier which produce the product, are called its **Factors.**

81. The **Sign of Multiplication** is ×. It shows that the numbers between which it is placed are to be multiplied together, and is read "times," or "multiplied by."

Thus, 7 × 4 = 28, is read, "7 times 4," or "7 multiplied by 4 equals 28."

NOTE.—Multiplication is similar in principle to addition, and may be performed by it. Thus, the product of three times 4, is 12, which is the same as the sum of 4 + 4 + 4.

MULTIPLICATION TABLE.

2 times	3 times	4 times	5 times	6 times	7 times
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6	1 are 7
2 " 4	2 " 6	2 " 8	2 " 10	2 " 12	2 " 14
3 " 6	3 " 9	3 " 12	3 " 15	3 " 18	3 " 21
4 " 8	4 " 12	4 " 16	4 " 20	4 " 24	4 " 28
5 " 10	5 " 15	5 " 20	5 " 25	5 " 30	5 " 35
6 " 12	6 " 18	6 " 24	6 " 30	6 " 36	6 " 42
7 " 14	7 " 21	7 " 28	7 " 35	7 " 42	7 " 49
8 " 16	8 " 24	8 " 32	8 " 40	8 " 48	8 " 56
9 " 18	9 " 27	9 " 36	9 " 45	9 " 54	9 " 63
10 " 20	10 " 30	10 " 40	10 " 50	10 " 60	10 " 70
11 " 22	11 " 33	11 " 44	11 " 55	11 " 66	11 " 77
12 " 24	12 " 36	12 " 48	12 " 60	12 " 72	12 " 84

8 times	9 times	10 times	11 times	12 times
1 are 8	1 are 9	1 are 10	1 are 11	1 are 12
2 " 16	2 " 18	2 " 20	2 " 22	2 " 24
3 " 24	3 " 27	3 " 30	3 " 33	3 " 36
4 " 32	4 " 36	4 " 40	4 " 44	4 " 48
5 " 40	5 " 45	5 " 50	5 " 55	5 " 60
6 " 48	6 " 54	6 " 60	6 " 66	6 " 72
7 " 56	7 " 63	7 " 70	7 " 77	7 " 84
8 " 64	8 " 72	8 " 80	8 " 88	8 " 96
9 " 72	9 " 81	9 " 90	9 " 99	9 " 108
10 " 80	10 " 90	10 " 100	10 " 110	10 " 120
11 " 88	11 " 99	11 " 110	11 " 121	11 " 132
12 " 96	12 " 108	12 " 120	12 " 132	12 " 144

NOTE.—Promiscuous exercises upon the table should be repeated till any combinations within its limits can be answered instantly.

11. Count by 3's to 30 and back to 0.
12. Name the products by 4 to 40 and back.
13. Name the products by 5 to 50 and back.
14. Name the products by 6 to 60 and back.
15. Name the products by 7 to 70 and back.
16. Name the products by 8 to 80 and back.
17. Name the products by 9 to 90 and back.

18. How many times 7 are 28?
19. How many times 6 are 42?
20. How many times 8 are 48?
21. How many times 7 are 63?
22. Times 5 are 30?
23. Times 7 are 56?
24. Times 9 are 54?
25. Times 9 are 72?

26. $6 \times 7 - 5 \times 6 = ?$
27. $7 \times 5 - 4 \times 8 = ?$
28. $5 \times 9 - 6 \times 7 = ?$
29. $7 \times 8 - 4 \times 6$?
30. $8 \times 6 - 6 \times 8$?
31. $9 \times 7 - 6 \times 9$?
32. $8 \times 9 - 7 \times 5$?
33. $7 \times 8 - 6 \times 9$?
34. $9 \times 9 - 8 \times 8$?

DEVELOPMENT OF PRINCIPLES.

82. 1. What is the product of $7 multiplied by 4?

2. What is the product of 9 multiplied by 5?

3. What kind of numbers are these *multiplicands?*

Ans. The first is *concrete,* the second is *abstract.*

4. What kind of a number is the *multiplier?*

Ans. It is an *abstract* number in both examples.

5. What kind of a number is the product?

Ans. The *same* in each as the *multiplicand.*

6. What is the product of 7 days multiplied by 9 pounds?

Ans. Pounds are *concrete* numbers and 7 days cannot be taken 4 pounds times.

7. Which is the greater number, 3 times 4, or 4 times 3?

83. From the above examples we deduce these

PRINCIPLES.

1°. The multiplicand may be either abstract, or concrete.

2°. The multiplier must be considered an abstract number.

3°. The multiplicand and product are like numbers.

4°. The product is the same in whatever order the factors are taken.

Written Exercises.

84. When the multiplier has but one figure.

1. If a rail-car goes 538 miles a day, how far will it go in 4 days?

Analysis.—The car will go 4 times as far in 4 days as in 1 day. Write the multiplier under the multiplicand, and beginning at units say, "4 times 8 units are 32 units, or 3 tens and 2 units." We set the 2 in units place and add the 3 tens to the product of tens, as in addition. Next, 4 times 3 tens are 12 tens, and 3 tens added make 15 tens, or 1 hundred and 5 tens. We write the 5 in tens place, and add the 1 hundred to the product of hundreds. Finally, 4 times 5 hundreds are 20 hundreds, and 1 hundred added makes 21 hundreds, or 2 thousand and 1 hundred. We write the 1 in hundreds place, and the 2 in thousands place. The product is 2152 miles, *Ans.*

OPERATION.

538	Multiplicand.
4	Multiplier.
Ans. 2152 miles.	

2. If 1 bale of cotton weighs 250 pounds, what will 7 bales weigh?

3. A drover bought 6 flocks of sheep, the average number of which was 735; how many sheep did he buy in all?

(4.)	(5.)	(6.)	(7.)	(8.)
574 lbs.	725 ft.	869 yds.	4256 bu.	5178 in.
3	4	5	3	4

85. When the multiplicand or multiplier contains Decimals.

Decimals and dollars and cents are multiplied like integers, as many figures being pointed off for decimals in the product, as are found in both factors.

9. What is the product of \$64.375 multiplied by 7?

Explanation.—U. S. Money as well as Decimals is multiplied like whole numbers; from the right of the product, as many decimal figures are pointed off as there are decimal places in *both* factors.

OPERATION.

\$64.375
7
\$450.625

(10.)	(11.)	(12.)	(13.)	(14.)
384.9	67.02	54.37	8.603	87.46
5	6	8	7	9

(15.)	(16.)	(17.)	(18.)	(19.)
$4352.67	$676.238	$7283.04	$9280.23	$807.206
6	5	7	8	9

	(20.)	(21.)	(22.)	(23.)
Multiply	$34.56	$242.63	$0.96	$0.873
By	4	6	8	9

24. What cost 8 barrels of flour, at $7.50 a barrel?

25. What will 458 hats cost, at $6 apiece?

NOTE.—In this example the true multiplicand is $6. But it is more convenient to use 6 as the multiplier and 458 as the multiplicand, as follows: (Art. **83**, *4°*.)

ANALYSIS.—At $1 each, 458 hats would cost $458, and at $6 each, they will cost 6 times $458, or $2748.

```
      458
        6
Ans. $2748
```

26. What cost 375 tons of hay, at $8 per ton?

27. What cost 5265 bales of cotton, at $8 a bale?

28. At $4 a barrel, what cost 1500 barrels of apples?

29. At $5.67 a yard, what cost 8 yds. of cloth?

30. What cost 2350 clocks, at $9 each?

MENTAL EXERCISES.

86. 1. What will 4 vests cost, at $7 apiece?

1ST. ANALYSIS.—4 vests will cost 4 times as much as 1 vest, and 4 times $7 are $28, *Ans.*

2D. ANALYSIS.—Since 1 vest costs $7, 4 vests will cost 4 times $7, or $28. Therefore, 4 vests will cost $28.

NOTE.—The essentials of a good Analysis are *clearness, brevity, appropriate language*, and a *logical conclusion. Sameness* in form should be avoided.

2. If you write 8 lines a day, how many lines will you write in 6 days?

3. If there are 5 school days in 1 week, how many school days are there in 12 weeks?

4. What is the cost of 9 bananas, at 6 cents each?

5. A grocer sold a turkey weighing 8 lbs. at 11 cts. a pound; what did it come to?

6. What will 9 qts. of cherries come to, at 12 cts. a quart?

7. At 8 cts. a mile, what will it cost to ride 9 miles?

8. Carrie made 9 bouquets, each having 10 flowers; how many flowers had they all?

9. At $12 a hundred, what will 4 hundred melons cost?

10. What cost 8 dozen eggs, at 12 cts. a dozen?

11. If a man walk 47 miles a day, how many miles will he walk in 6 days?

ANALYSIS.—47 equals 4 tens and 7 units. Now 6 times 4 tens are 24 tens, or 240, and 6 times 7 units are 42 units, which added to 240 make 282. *Ans.* 282 miles in 6 days.

NOTE.—When the multiplicand is large, it is advisable in *mental* operations to begin with the higest order, as in addition. (Art. **54.**)

12. What cost 4 acres of land, at $36 an acre?

13. At $75 a share, what will 7 shares of Bank Stock come to?

14. A furniture dealer sold 10 sofas at $87 apiece; what did he get for all of them?

15. If a railroad car goes at the rate of 57 miles an hour, how far will it go in 8 hours?

16. How many are 9 times 73? 8 times 63?

17. How many are 7 times 86? 6 times 97?

18. How many are 8 times 94? 9 times 89?

19. If melodeons are $75 apiece, what will 6 cost?

WRITTEN EXERCISES.

87. When the multiplier has two or more figures.

1. What is the product of 324 multiplied by 132?

324	Multiplicand.
132	Multiplier.
648	Par. Prod.
972	" "
324	" "
42768	Entire Prod.

EXPLANATION.—We write the multiplier under the multiplicand, as in the margin, and beginning at the right, multiply by each order successively.

Thus, multiplying 324 by 2 units, the *first partial* product is 648, the right hand figure of which we set in units place. Next, multiplying by 3 tens, the *second partial* product is 972 tens, the right hand figure of which we write in tens place, under the multiplying figure. Finally, the *third partial* product is 324 hundreds, the right hand figure of which we place under the order which produced it.

Adding these *partial products*, the *sum* 42768, is the *entire product.*

NOTE.—To prove the *correctness* of the work, we multiply the multiplier 132 by 324 the multiplicand, and as the *two results agree*, the work is *correct.*

Multiply and explain the following in like manner:

(2.)	(3.)	(4.)	(5.)	(6.)
42.5	$563	$6.42	42.6 lbs.	678 yds.
34	42	27	23.4	346

88. From the preceding exercises we derive the following

GENERAL RULE.

I. Write the multiplier under the multiplicand, and beginning at the right, multiply each order of the multiplicand by each order of the multiplier, placing the right hand figure of each partial product under the order which produced it.

II. Add the partial products together, and from the right point off as many figures for decimals as there are places of decimals in the multiplicand and multiplier; the result will be the product required.

PROOF.—*Multiply the multiplier by the multiplicand; if the two results agree the work is correct.*

(For proof by casting out the 9's, see Art. **876**, Appendix.)

APPLICATIONS.

1. Allowing 365 days to a year, how many days has a man lived who is 45 years old?

2. If a garrison consume 725 pounds of beef in one day, how many pounds will they consume in 126 days?

3. What cost 243 chests of tea, at $37 per chest?

4. A man bought 268 horses, at $63 apiece; what did they come to?

5. If sound moves 1142 feet in a second, how far will it move in 60 seconds?

6. If a cattle train has 23 cars and each car contains 68 sheep, how many sheep in the train?

7. How much can a man earn in 48 months at $125 a month?

8. At $32 each, how much will it cost to furnish the outfit for 560 policemen?

9. How many bushels of corn may be raised on 485 acres which average 37 bu. to the acre?

10. There are 640 acres in a square mile; how many acres are there in 75 square miles?

What is the product of

11. 8623 by 24?
12. 2538 by 39?
13. 4752 by 43?
14. 5843 by 63?
15. $32.45 by 57?
16. $47.08 by 68?
17. 6264 by 70?
18. $29.451 by 49?
19. 420643 by 76?
20. 572062 by 84?
21. 398025 by 87?
22. 703270 by 93?
23. 2734 by 234?
24. 4803 by .325?
25. 6578 by 467?
26. 5967 by 504?
27. 43672 by 564?
28. 54865 by 647?
29. 60435 by 704?
30. 74321 by 839?
31. 543267 by 1563?
32. 684039 by 1783?
33. 709564 by 2803?
34. 894037 by 3085?

35. If a clerk has $36 a month for the first 4 months; $48 a month for the next 4; and $60 a month for the next 4; what will he receive for the year?

36. If I receive $350 a month, how much shall I have at the end of the year, after deducting $38 a month for board?

37. If it takes 385 laborers 18 months to build a railroad, how long would it take 1 man to build it?

38. A ship of war has provisions to last a crew of 645 men 90 days; how long would they last 1 man?

39. If I sell 29 bbls. of flour at $8 a barrel, and 50 bbls. of beef at $18 a barrel, and receive 5 hundred-dollar bills in payment, how much will be due me for both?

40. A farmer sold 32 sheep at $8 a head, and 9 cows for $45 apiece; how much more did he receive for the cows than the sheep?

41. What cost 169 chairs, at $3.25 apiece?

42. What cost 279 barrels of salt, at \$1.75 a barrel?

43. What cost 1565 acres of land, at \$27 per acre?

44. What cost 758 baskets of peaches, at \$2.50 a basket?

45. If a hall will seat 1250 persons, and each seat is occupied by a person weighing 135 lbs. what weight is sustained by the floor?

46. Bought 2 farms; one contained 327 acres at \$83 an acre, the other 526 acres at \$58 an acre. What did they both cost? What was the difference in their cost?

47. What is the value of 56 railway cars, at \$9550.75 each?

48. A man bought 31 colts at \$28 apiece, and paid 10 tons of hay at \$25 a ton; how much did he owe for the colts?

49. A grocer bought 21 barrels of flour at \$5 per barrel, and sold 16 barrels of it at \$7; finding the rest damaged, he put it at \$3 a barrel. How much did he make or lose, by the operation?

50. A farmer having 75 turkeys, sold 50 of them at 86 cents apiece, and the rest at 54 cts. apiece; what did they come to?

51. A man owns 7 orchards; in each orchard there are 8 rows of apple-trees, and 29 trees in each row; how many apple-trees has he?

52. In a certain school there are 3 departments, in each department there are 11 classes, and in each class there are 48 pupils; how many pupils are there in the school?

53. If you multiply 58 by 37, and this product by 29, what will be the result?

54. If 450 is multiplied by 254, and this product by 178, what will be the result?

55. A man sent 37 loads of wheat to market; every load contained 16 bags, and each bag 3 bushels; how many bushels did he send?

56. What is the product of $378 + 342$ by $(763 - 251)$?

57. What is the product of $254 + 451$ by $(836 - 434)$?

58. Required the product of $823 - 567$ by $(827 + 230)$.

59. Multiply $267 + 75 + 430$ by $(468 - 324)$.

60. Multiply $869 - 675$ by $(300 + 87 + 90)$.

61. What is the product of $(843 - 478) \times (973 + 379)$?

89. To multiply by the Factors of the Multiplier.

1. What will 15 tables cost at $7 apiece?

OPERATION.

$7	Cost of 1 Table.
5	
$35	" " 5 "
3	
$105	" " 15 "

ANALYSIS.—The factors of 15 are 5 and 3. (Art. 80.) Now as 1 table costs $7, 5 tables will cost 5 times $7 or $35. Again, since $15 = 3 \times 5$ it follows that 15 tables will cost 3 times as much as 5 tables, and 3 times $35 are $105. Hence, the

RULE.—*Multiply the multiplicand by one of the factors of the multiplier, then this product by another, and so on, till all the factors have been used.*

The last product will be the answer.

2. What will 27 sofas cost, at $85 apiece?
3. What will 24 wagons cost, at $37 apiece?
4. What will 36 cows cost, at $19 per head?
5. If a man travels at the rate of 42 miles a day, how far can he travel in 205 days?
6. What cost 45 acres of land, at 110 dollars per acre?
7. At $6 per week, how much will it cost a person to board 52 weeks?
8. At the rate of 56 bushels per acre, how much corn can be raised on 460 acres of land?
9. What cost 672 pieces of cashmere, at $24 apiece?
10. What cost 1265 yoke of oxen, at $72 per yoke?

90. When the Multiplier has ciphers on the right.

1. What is the product of 56 multiplied by 10?

SOLUTION.—When a figure is moved one place to the left, its value is increased *ten* times. (Art. 34, *2°*.) Hence, if we annex a cipher to 56 we multiply it by 10 and it becomes 560. *Ans.* 560.

2. Multiply 64 by 100.

SOLUTION.—Annexing *two* ciphers to 64 increases its value 100 times, and therefore multiplies it by 100. *Ans.* 6400.

3. Multiply 87 by 1000. *Ans.* 87000.

4. Multiply 316 by 40.

EXPLANATION.—Multiplying 316 by 4 ones, it becomes 1264. But we are required to multiply by 4 tens instead of 4 ones; therefore the true product is ten times 1264. To correct this result, we annex a cipher to it, which multiplies it by 10.

OPERATION.

$$\begin{array}{r} 316 \\ 40 \\ \hline \textit{Ans. } 12640 \end{array}$$

5. Multiply 345 by 700.

SOLUTION.—Multiplying by 7 ones only, gives 2415, which must be multiplied by 100 for the true product. This is done by annexing two ciphers. Hence,

OPERATION.

$$\begin{array}{r} 345 \\ 700 \\ \hline \textit{Ans. } 241500 \end{array}$$

91. To multiply by 10, 100, 1000, etc.

Annex as many ciphers to the multiplicand as there are ciphers in the multiplier.

When the significant figures have ciphers on the right.

Multiply by the significant figures, and to the result annex as many ciphers as are on the right of both factors. (See Art. **864**, Appendix.)

6. What will 100 bales of cotton weigh, at 468 lbs. to a bale?

7. How many pages in 2300 books, of 352 pages each?

8. $476 \times 1000 = ?$

9. $53486 \times 10000 = ?$

10. $12046708 \times 100000 = ?$

11. $26900785 \times 1000000 = ?$

12. $89063457 \times 10000000 = ?$

13. $9460305068 \times 100000 = ?$

14. $1920 \times 2000 = ?$

15. $4376 \times 2500 = ?$

16. $50634 \times 41000 = ?$

17. $630125 \times 620000 = ?$

18. $12000 \times 31 = ?$

19. $370000 \times 32 = ?$

20. $8120000 \times 46 = ?$

21. $56300000 \times 64 = ?$

22. $62300000 \times 890 = ?$

23. $54000000 \times 700 = ?$

24. $43000000 \times 600 = ?$

25. $563800 \times 7200 = ?$

26. $1230000 \times 12000 = ?$

27. $310200 \times 20000 = ?$

28. $2065000 \times 810000 = ?$

29. $2109090 \times 510000 = ?$

30. $6084201 \times 740000 = ?$

31. $7283900 \times 958300 = ?$

32. $86007400 \times 9700 = ?$

33. $90690000 \times 8600 = ?$

Oral Problems for Review.

92. 1. If 4 men can do a job of work in 6 days, how long will it take 1 man to do it?

ANALYSIS.—It will take 1 man 4 times as many days as it takes 4 men; and 4 times 6 days are 24 days, *Ans.*

2. In 1 peck are 8 quarts; how many quarts in 6 pecks?

3. How many quarts are 7 pecks and 3 quarts?

4. In 1 bushel are 4 pecks; how many pecks in 15 bushels?

5. How many pecks in 20 bushels and 3 pecks?

6. If 9 men can build a wall in 20 days, how long will it take 1 man to do it?

7. In 1 pound are 16 ounces; how many ounces in 4 pounds?

8. How many ounces in 5 lbs. 7 ounces?

9. If a barrel of flour will last 8 persons 12 days, how long will it last 1 person?

10. What will 8 lbs. of maple sugar cost, at 9 cts. a pound?

11. How many inches in 11 feet?

12. How many feet in 12 yds. and 2 feet?

13. How many quarts in 25 gallons of milk?

14. How many quarts in 30 gallons and 5 quarts?

15. If two men start from the same place and travel in opposite directions, one at the rate of 4 miles per hour, the other, 3 miles, how far apart will they be in 6 hours?

Written Problems for Review.

93. 17. George has 27 cents, and Henry has 3 times as many cents as George lacking 5; how many cents has Henry? How many have both?

18. At a military parade there were 5 regiments, in each regiment 8 companies, in each company 9 platoons, and in each platoon 10 soldiers; how many soldiers were on parade?

19. If 325 men can grade a street in 28 days, how long will it take 1 man to do it? How much will he receive for it, if he has $2 per day?

20. A and B are 20 miles apart, and travel in opposite directions, A goes 4 miles an hour and B 5 miles; how far apart will they be in 48 hours?

21. If I hire a carpenter at $28 a month, and his apprentice at $14, how much will be due them in 12 months?

22. A man bought a drove of 1560 sheep, at $4 a head; it cost him $68 to send them to market, and they brought him $5 apiece; how much did he make on them?

23. A drover bought 360 head of cattle and 96 horses; he afterwards sold the former at a profit of $19 a head, and the latter at a loss of 23 dollars a head; did he gain or lose by the operation, and how much?

24. A grocer bought 585 barrels of flour at $6 a barrel, and 117 barrels at $7; he then sold the whole at $6.50. What was the result of his speculation?

25. In music, two minims equal a semibreve; two crotchets a minim; two quavers a crotchet; two semi-quavers a quaver; and two demi-semiquavers a semi-quaver; how many demi-semiquavers are equal to 259 semi-breves?

26. Two persons start from the same place, and travel in the same direction; one at the rate of 33 miles per day, and the other at the rate of 37 miles per day; how far apart will they be at the end of a year?

27. Multiply two thousand seven, by one thousand four.

28. Multiply four thousand forty, by two thousand one hundred three.

29. Multiply forty thousand, four hundred four, by ten thousand ten.

30. Multiply one hundred five thousand seven, by sixty thousand, four hundred three.

31. Multiply five millions, two hundred six, by seventy thousand, two hundred five.

QUESTIONS.

76. What is Multiplication? 77. The Multiplicand? 78. The Multiplier? What does the Multiplier show? 79. What is the Answer called?

80. The numbers which produce the product called? 81. Make the Sign of Multiplication. What does it show? How read?

85. How proceed when the multiplicand or multiplier has decimals? 88. Give the general rule? How prove multiplication? 89. How multiply by the factors of a number?

91. How multiply by 10, 100, 1000, etc.? When there are ciphers on the right, how proceed?

DIVISION.

ORAL EXERCISES.

94. 1. How many times can 4 cents be taken from a purse containing 12 cents?

SOLUTION.—12 cents − 4 cts. = 8 cts.; 8 cts. − 4 cts. = 4 cts.; and 4 cts. − 4 cts. = 0. *Ans.*, 3 times.

2. How many times are 4 cents contained in 12 cents?

3. How many 3's in 15? How many 5's?

4. How many oranges at 4 cts. apiece, can I buy for 20 cts.?

ANALYSIS.—I can buy as many oranges as 4 cents are contained times in 20 cents. *Ans.*, 5 oranges.

5. How many times 3 in 18? How many times 6?

6. In 1 gallon are 4 quarts; how many gallons in 24 quarts?

7. In 1 week are 7 days; how many weeks in 28 days?

8. What is one of the 5 equal parts of 30?

ANALYSIS.—Since 30 is 6 times 5, one of the 5 equal parts of 30 is 6.

9. On Christmas day a father divided $25 equally among his 5 children; how many dollars did each receive?

10. How many boxes, each holding 12 lbs., will a dairyman require to pack 36 lbs. of butter?

11. A teacher having 45 pupils, formed them into classes of 9 each; how many classes did she have?

DEFINITIONS.

95. Division is finding *how many times* one number is contained in another; or finding one of the *equal parts* of a number.

96. The **Dividend** is the number to be divided.

97. The **Divisor** is the number by which we divide.

98. The *Answer*, or number found by division, is called the **Quotient**. It shows how many times the divisor is contained in the dividend.

99. The **Remainder** is the part of the dividend left when the divisor is not contained in it an *exact* number of times, and is always *less* than the divisor.

100. The **Sign of Division** is $\div$. It is read "divided by."

Thus, $8 \div 4$ is read, "8 divided by 4."

101. Division is also denoted by writing the *divisor under* the *dividend,* with a line between them.

Thus, $\frac{8}{4}$ is read, "8 divided by 4."

NOTES.—1. Division is the *reverse* of multiplication; the former *separates* numbers into *equal parts;* the latter *unites* equal parts in *one number.* The dividend corresponds to the product, and the divisor and quotient to the multiplier and multiplicand. (Art. **79.**)

2. Division is also similar in principle to *subtraction,* and may be performed by it. Thus, 5 is contained in 15, 3 times, and 5 can be subtracted from 15, 3 times.

12. How many times 3 in 36?

13.	4 in 28?	**16.**	5 in 45?	**19.**	8 in 56?
14.	4 in 32?	**17.**	6 in 54?	**20.**	9 in 54?
15.	6 in 48?	**18.**	7 in 63?	**21.**	9 in 72?

22.	$18 \div 3 = ?$	**30.**	$32 \div 4 = ?$	**38.**	$36 \div 3 = ?$
23.	$27 \div 3 = ?$	**31.**	$28 \div 4 = ?$	**39.**	$48 \div 4 = ?$
24.	$35 \div 5 = ?$	**32.**	$42 \div 7 = ?$	**40.**	$54 \div 9 = ?$
25.	$45 \div 5 = ?$	**33.**	$56 \div 8 = ?$	**41.**	$56 \div 8 = ?$
26.	$30 \div 6 = ?$	**34.**	$35 \div 7 = ?$	**42.**	$72 \div 9 = ?$
27.	$42 \div 6 = ?$	**35.**	$48 \div 8 = ?$	**43.**	$88 \div 11 = ?$
28.	$60 \div 5 = ?$	**36.**	$64 \div 8 = ?$	**44.**	$72 \div 8 = ?$
29.	$54 \div 6 = ?$	**37.**	$63 \div 7 = ?$	**45.**	$96 \div 12 = ?$

46.	$\frac{45}{9} = ?$	**49.**	$\frac{54}{6} = ?$	**52.**	$\frac{72}{8} = ?$	**55.**	$\frac{108}{12} = ?$
47.	$\frac{48}{6} = ?$	**50.**	$\frac{63}{7} = ?$	**53.**	$\frac{81}{9} = ?$	**56.**	$\frac{120}{12} = ?$
48.	$\frac{56}{7} = ?$	**51.**	$\frac{42}{6} = ?$	**54.**	$\frac{99}{11} = ?$	**57.**	$\frac{132}{12} = ?$

102. The **Name** of the equal parts into which a number or thing is divided depends upon the *number* of parts. Thus,

One of *two* equal parts is called **One-half,** written $\frac{1}{2}$.
One of *three* equal parts is called **One-third,** written $\frac{1}{3}$.
One of *four* equal parts is called **One-fourth,** written $\frac{1}{4}$.
Two of *three* equal parts are called **Two-thirds,** written $\frac{2}{3}$.
Three of *four* equal parts are called **Three-fourths,** written $\frac{3}{4}$.

103. Write the following in figures:

1. Two-ninths; three-fifths; four-fifths; five-eighths.
2. One-sixth; two-sixths; four-sixths; five-sixths.
3. Two-sevenths; three-sevenths; five-sevenths.
4. Three-tenths; five-ninths; three-elevenths; six-twelfths.

104. When a *unit* is divided into *equal* parts, the parts are called **Fractions.**

5. What part of 3 is 1? Is 2?

ANALYSIS.—If 3 is divided into 3 equal parts, one of these parts is 1-third of 3; 2 of the parts are 2-thirds of 3.

6. What part of 4 is 1? What part of 3?
7. What part of 5 is 2? Is 3? Is 4?
8. How find a half of a number? A third, a fourth, etc.

Ans., By dividing it respectively by 2, 3, 4, etc.

9. What is 1 third of 6? Of 9? Of 12? Of 18?
10. What is 1 fifth of 10? Of 15? Of 30? Of 45?
11. What is 1 eighth of 24? Of 32? Of 40? Of 48?
12. If $40 are distributed among 8 laborers, what part of the money and how many dollars will each receive?

ANALYSIS.—One is 1-eighth of 8. Therefore, 1 man will receive 1-eighth of $40, which is $5, *Ans.*

13. How many tons of coal, at $7 a ton, can be bought for $63?
14. A farmer sold 5 tons of hay, at $12 a ton, and took his pay in flour at $6 a barrel; how many barrels did he receive?
15. An express traveled 108 miles in 9 hours; at what rate was that per hour?
16. How many times is $\frac{1}{7}$ of 42 contained in 54?
17. How many times is $\frac{1}{9}$ of 63 contained in 84?

Development of Principles.

105. 1. How many times 4 cents in 20 cents?

2. What kind of a number is the quotient?

Ans. It is an *abstract* number.

3. If 30 apples are divided into 5 equal parts, how many apples will there be in one part?

4. What kind of number is the quotient?

Ans. A concrete number, the same as the dividend.

5. If 7 is the divisor and 4 the quotient, what is the dividend?

Ans. Their product, 7×4, or 28.

106. From the above examples we derive the following

Principles.

1°. When the divisor and dividend are like numbers, the quotient is an abstract number.

2°. When the divisor is an abstract number, the quotient and dividend are like numbers.

3°. The product of the divisor and quotient is equal to the dividend.

Written Exercises.

107. 1. Divide 952 by 4.

Divisor.	Divid.	Quotient.
4)	952	(238
	8	
	15	
	12	
	32	
	32	

Explanation.—Write the divisor on the left of the dividend, with a curved line between them.

First.—Beginning at the left to divide, we find 4 is contained in 9 hundreds 2 (hundreds) times, and place the 2 (in hundreds place) at the right of the dividend for the first quotient figure.

Second.—We multiply the divisor by the quotient 2, and set the remainder under the order divided.

Third.—Subtract the product from the part of the dividend used.

Fourth.—To the remainder 1 (hundred) we annex the 5 tens, making 15 tens, for a second partial dividend. Now 4 is contained in 15 tens, 3 (tens) times; write the 3 in tens place in the quotient, and multiplying the divisor by it, subtract the product from the second partial dividend.

To the remainder 3 tens, we annex the 2 units, making 32 units, a third partial dividend. 4 is contained in 32 units, 8 times. Write the 8 in units place in the quotient. Multiplying the divisor by it and subtracting the product, there is no remainder. *Ans.* 238.

PROOF.—Quotient 238 × 4 (Divisor) = 952 (Dividend). (Art. **106**, *3°*.)

2. Divide 6570 by 5, and explain the operation in like manner. *Ans.* 1314.

3. Divide 7650 by 6.
4. Divide 8211 by 7.
5. Divide 9872 by 4.
6. Divide 9219 by 7.
7. Divide 68696 by 6.
8. Divide 89240 by 8.

9. What is the quotient of 6272 divided by 4 ?

OPERATION.

$$\text{Divisor. } 4\,)\,\underline{6272}\text{ Dividend.}$$
$$1568\text{ Quotient.}$$

EXPLANATION.—We draw a line under the dividend, and begin to divide at the left as before. Dividing 6 (thousands) by 4, the quotient is 1 (thousand), which we write below the line, under the order divided. Subtracting, the remainder is 2 (thousands). To this we annex, mentally, the 2 hundreds, making 22 (hundreds) for the next partial dividend. Divide as before, and proceed in this way till all the orders are divided, carrying the multiplications and subtractions in the mind, simply setting down the quotient figures. The quotient is 1568.

PROOF.—The quotient 1568 × 4 (divisor) = 6272 (dividend).

Divide and prove the following in like manner :

(10.)	(11.)	(12.)	(13.)
3) 1134	5) 1230	6) 2562	7) 8638
(14.)	(15.)	(16.)	(17.)
6) 3276	8) 9872	7) 2345	9) 3141

108. The method of dividing in which the results of the several steps are set down, as illustrated by Ex. 1, is called **Long Division.**

109. The method in which the quotient only is set down, the results of the several steps being carried in the mind, as illustrated by Ex. 9, is called **Short Division.**

When the divisor does not exceed 12, this method is preferable.

18. Divide 13875 by 4.

NOTES.—1. To indicate the division of the final remainder, if any, it must be written over the divisor and placed at the right of the quotient as part of it.

OPERATION.

4) 13875

Ans. $3468\frac{3}{4}$

2. In proving the work, the remainder, if any, must be *added* to the product of the divisor and quotient. (Art. **99**.)

(19.)	(20.)	(21.)	(22.)
5) 4567	6) 3971	8) 6567	9) 41756

Solve the following, both by Long and Short Division :

(23.)	(24.)	(25.)	(26.)
5) 76453	4) 82354	6) 52387	7) 63874
(27.)	**(28.)**	**(29.)**	**(30.)**
7) 842952	6) 428463	8) 768345	9) 783952
(31.)	**(32.)**	**(33.)**	**(34.)**
8) 65924	9) 75327	10) 562340	12) 18396

110. Decimals are divided like integers, and from the *right* of the quotient as many figures must be pointed off for decimals, as the decimal places in the dividend *exceed* those in the divisor.

35. Divide 391.86 by 9.

EXPLANATION.—We divide as in whole numbers and point off two figures for decimals in the quotient.

OPERATION.

9) 391.86

Ans. 43.54

36. Divide \$98.752 by 8. *Ans.* \$12.344.

37. Divide 2563.48 by 4. **38.** Divide 645.328 by 8.

39. At \$4 a yard, how many yards of cloth can be bought for \$9850 ?

40. If \$48.78 are divided into 9 equal parts, what is the value of each part ?

ORAL EXERCISES.

111. 1. If the price of 7 hats is $28, what is the price of 1 hat?

ANALYSIS.—1 is 1 seventh of 7; therefore 1 hat will cost 1 seventh of $28, and 1 seventh of $28 is $4, *Ans.*

2. If 1 man can hoe a field of corn in 40 days, how long will it take 5 men to hoe it?

3. A grocer bought 8 barrels of flour for $56; what must he sell it for per barrel to gain $12?

4. A hardware merchant paid $69 for 7 kegs of nails, and $15 freight; what did each keg cost him?

5. How many times 9 in 8 times 12?

6. How many times 11 in 74 plus 13?

7. How many cords of wood at $4 a cord, will pay for 8 pairs of boots at $6 a pair?

8. A farmer gave 5 tons of hay for 2 cows, worth $30 apiece; what was the value of the hay per ton?

9. How many tons of coal, worth $6 a ton, must I give for 5 suits of clothes worth $9 a suit?

10. If I pay $12 apiece for 7 barrels of beef, and sell it so as to lose $24, what shall I get a barrel?

WRITTEN EXERCISES.

112. Ex. 1. Divide 172859 by 34.

OPERATION.

$$
\begin{array}{r}
34\ )\ 172859\ (\ 5084\tfrac{3}{34} \\
\underline{170}\phantom{859\ (\ 5084\tfrac{3}{34}} \\
285\phantom{9\ (\ 5084\tfrac{3}{34}} \\
\underline{272}\phantom{9\ (\ 5084\tfrac{3}{34}} \\
139\phantom{\ (\ 5084\tfrac{3}{34}} \\
\underline{136}\phantom{\ (\ 5084\tfrac{3}{34}} \\
3\phantom{\ (\ 5084\tfrac{3}{34}}
\end{array}
$$

EXPLANATION.—34 is contained in 172, 5 (thousands) times and 2 (thousands) rem. Setting the 5 at the right of the dividend, we annex the 8, the next figure of the dividend, to the remainder 2, making 28 for the second partial dividend.

As 32 is not contained in 28, we write a cipher in the quotient, and annex the 5 to 28, making 285 (tens). Now 34 is in 285 (tens), 8 (tens) times. Writing 8 in the quotient, we multiply, subtract, etc., and proceed as before. Finally, we write the divisor under the remainder, and place it at the right as part of the quotient.

(2.) 16) 45807 (	(3.) 28) 348072 (	(4.) 37) 516780 (
(5.) 19) 560372 (	(6.) 36) 39245 (	(7.) 37) 45567 (
(8.) 48) 9230.56 (	(9.) 29) 702345 (	(10.) 38) $45.84 (
(11.) 47) $59.85 (	(12.) 56) $75.845 (	(13.) 48) $96.354 (

113. From the preceding illustrations we derive the following

General Rule.

I. Write the divisor at the left of the dividend, and find how many times it is contained in the fewest orders that will contain it, setting the quotient at the right.

II. Multiply the divisor by this quotient, and subtract the product from the orders divided. To the remainder annex the succeeding figure of the dividend, and divide as before.

III. If there is a remainder after dividing the last order, write it over the divisor, and place the result at the right as part of the quotient.

Finally, point off as many decimal figures at the right of the quotient as the decimal places in the dividend exceed those in the divisor.

Proof.—*Multiply the divisor and quotient together, and to the product add the remainder. If the result is equal to the dividend, the work is right.*

Note.—The quotient figure both in short and long division, is always the *same order* as the *right hand order* divided.

114. To prove Multiplication by Division.

Divide the product by one of the factors, and if the quotient is equal to the other factor, the work is right

APPLICATIONS.

115. 1. A man wishes to invest $2562 in railroad stock; how many shares can he buy, at $42 per share?

2. In 1 year there are 52 weeks; how many years are there in 1640 weeks?

3. In one hogshead there are 63 gallons; how many hogsheads are there in 3065 gallons?

4. If a man can earn $75 in a month, how many months will it take him to earn $3280?

5. If it takes 18 yards of silk to make a dress, how many dresses can be made from 1350 yards?

6. If a young man's expenses are $83 a month, how long will $4265 support him?

7. A man bought a drove of 95 horses for $4750; how much did he give apiece?

8. A farmer having $1840, laid it out in land, at $25 per acre; how many acres did he buy?

9. In a cask there are 93 gallons; how many casks in 4260 gallons?

10. If a man travels 45 miles a day, how long will it take him to travel 1215 miles?

Divide and prove the following:

11.	$467.2 by 15.	19.	$84.53 by 62.
12.	$56.84 by 18.	20.	$73.56 by 48.
13.	786.3 by 21.	21.	6893 by 82.
14.	48.27 by 33.	22.	9721 by 65.
15.	6972 by 35.	23.	23456 by 28.
16.	7842 by 23.	24.	72350 by 45.
17.	8253 by 47.	25.	80854 by 84.
18.	21.08 by 32.	26.	92635 by 92.

27. A garrison had 5580 pounds of beef, which the commander wished to have last 62 days; how many pounds could be used per day?

28. A man paid $9565 for a farm, at $64 an acre; how many acres were there?

29. A grocer packed 18144 eggs in boxes holding 144 eggs each; how many boxes did he use?

30. If he had packed the same eggs in 63 equal boxes, how many eggs would he have put in a box?

NOTE.—When the divisor is large, find how many times its first figure is contained in the first or first two figures of the dividend, allowing for the addition of tens from the product of the second figure of the divisor.

31. Divide 18146 by 683. *Ans.* $26\frac{388}{683}$.
32. 62346 by 254.
33. 70893 by 532.
34. 294763 by 306.
35. 375426 by 521.
36. 2445224 by 812.
37. 3560325 by 904.
38. 4256348 by 638.
39. 5437502 by 743.
40. 89256.48 by 732.
41. 2439.2642 by 765.
42. 592348.276 by 879.
43. 569389.175 by 1247.
44. 8679538.46 by 3238.
45. 134259.8640 by 56813.
46. 396478.9523 by 75436.
47. 425367.805 by 83247.

DICTATION EXERCISES.*

116. 1. Subtract 4 from 11, add 3, multiply by 6, divide by 10, add 6, subtract 4, multiply by 5, and divide by 8; what is the result?

2. Multiply 9 by 7, subtract 3, divide by 5, add 10, divide by 11, multiply by 8, subtract 4, and add 12; result?

3. Divide 54 by 9, multiply by 3, subtract 8, add 4, divide by 7, multiply by 9, add 10, subtract 7, divide by 3; result?

4. If from 39 you take 7, divide by 8, multiply by 9, subtract 6, divide by 5, add 12, divide by 9, multiply by 11, add 3, divide by 5, add 7, and multiply by 4, what is the result?

5. $7 + 8 - 3 \times 4 \div 6 + 10 \div 3 \times 7 - 2$; result? †

6. $30 - 6 \div 8 \times 9 + 5 \div 4 + 14 \div 11 \times 12 + 6 \div 5 \times 7 + 6 \div 8 + 21$; what is the result?

7. $8 \times 7 - 8 \div 4 \times 2 \div 6 + 16 \div 4 \times 8 - 12 \div 7 + 6 \times 6$; what is the result?

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* The object of these exercises is three-fold; First, to give facility in mental combinations of numbers; Second, to cultivate the habit of fixing the attention; Third, to drill the whole class at the same time.

† Perform the successive operations indicated by the signs.

8. $48 \div 6 \times 4 + 10 - 6 \div 9 + 30 - 7 \div 3 \times 12 - 5 + 7 \div 10 \times 3$; what is the result?

9. $25 + 7 \div 8 \times 11 - 8 \div 9 + 23 \div 9 \times 7 + 12 \div 3 - 5 \times 8 + 6 \div 9$; what is the result?

10. $27 \div 9 + 15 - 10 \times 7 + 7 \div 9 \times 5 - 8 \div 3 + 12 \div 7 \times 20 - 12 \div 6 + 25 =$ how many?

117. When the Divisor has Ciphers on the right.

1. Divide 3563 by 100.

OPERATION.

1|00) 35|63

Ans. 35–63 Rem.

EXPLANATION.—Cutting off the right-hand figure of a number, removes each of its other figures one place to the right, and therefore divides it by 10. Cutting off two figures divides it by 100; cutting off three figures divides it by 1000, etc. (Art. **90**.)

2. Divide 345231 by 100.
3. Divide 672487 by 1000.
4. Divide 6423544 by 10000.
5. Divide 7364159 by 100000.

6. Divide 937643 by 4000.

OPERATION.

4|000) 937|643

Ans. 234–1643 Rem.

ANALYSIS.—By cutting off three figures at the right of the divisor and dividend, we divide each by 1000; the quotient is 937 and the remainder 643. Next, dividing by 4, the quotient is 234, and 1 remainder, which we prefix to the figures cut off, making the true remainder 1643. Hence,

118. To Divide by 10, 100, 1000.

Cut off as many figures at the right of the dividend as there are ciphers in the divisor; the remaining figures will be the quotient, and those cut off the remainder.

When the divisor is greater than 1, with ciphers on the right.

Cut off the ciphers from the divisor and as many figures from the right of the dividend.

For the quotient, divide the remaining part of the dividend by the remaining part of the divisor.

To the figures cut off, prefix the remainder, and the result will be the true remainder. (Art. **870**, Appendix.)

7. Divide 4885970 by 6000. *Ans.* 814 and 1970 rem.

8. Allowing 200 lbs. to a barrel, how many barrels will 68000 lbs. of beef make?

9. In \$1 there are 100 cents; how many dollars are in 45650 cents?

10. How many bales of cotton, each weighing 450 lbs., are in 36000 lbs.?

11. If \$96000 are divided equally among 2400 soldiers, how much will each receive?

12. A pound of cotton has been spun into a thread 76 miles long, and a pound of wool into a thread 95 miles long; how many pounds of both together will spin a thread which will reach round the world, a distance of 25000 miles?

13. If 600 steam engines can do the work of 2 million 496 thousand men, to how many men is 1 engine equivalent?

119. From the relations of the *Divisor, Dividend,* and *Quotient,* we deduce the following

General Principles of Division,

First.—Let 24 be a dividend and 6 a divisor. The quotient is 4.

$$\left.\begin{array}{ll}\text{Then} & (24 \times 2) \div 6 = 8 \\ \text{And} & 24 \div (6 \div 2) = 8\end{array}\right\} = 4 \times 2. \quad \text{Hence,}$$

1°. *Multiplying* the dividend, or *Dividing* the divisor, } *Multiplies* the quotient.

$$\left.\begin{array}{ll}\textit{Second.}\text{—} & (24 \div 2) \div 6 = 2 \\ \text{And} & 24 \div (6 \times 2) = 2\end{array}\right\} = 4 \div 2. \quad \text{Hence,}$$

2°. *Dividing* the dividend, or *Multiplying* the divisor, } *Divides* the quotient.

$$\left.\begin{array}{ll}\textit{Third.}\text{—} & (24 \times 2) \div (6 \times 2) \\ \text{Or,} & (24 \div 2) \div (6 \div 2)\end{array}\right\} = 4. \quad \text{Hence,}$$

3°. *Multiplying* or *dividing* both divisor and dividend by the *same* number, } Does *not change* the quotient.

119, *a*. 1st. When the *product* of two factors and *one* of them are given, the other is found by *dividing* the product by the given factor.

2d. When the product of *three or more* factors and *all* but one of them are given, the other factor is found by *dividing* the given product by the product of the given factors.

3d. When the *sum* and *difference* of two numbers are given, the *less* number is found by *subtracting* the difference from the sum and dividing *the remainder* by 2.

4th. The *average* of *two unequal* numbers is *half* their sum. The *average* of *three unequal* numbers is *one-third* the sum.

Oral Problems for Review.

120. 1. The dividend is 63, the quotient 9; what is the divisor?

2. When the dividend and quotient are given, how find the divisor?

3. The divisor being 11 and the quotient 10, what is the dividend?

4. When the divisor and quotient are given, how find the dividend?

5. The quotient being 9, the divisor 20, and the remainder 7, what is the dividend?

6. When the divisor, quotient, and remainder are given, how find the dividend?

7. At $7 a week, how many weeks can you board for $84?

8. How long will it take a printer to earn $132, if he gets $11 a week?

9. A farmer bought 12 yards of cloth, at $4, and paid for it in hay, at $8 a ton; how many tons did it take?

10. In 7 times 11, less 5, how many times 9?

11. In 9 times 12, less 8, how many times 5?

12. How many tons of coal, at $6 a ton, will pay for 8 barrels of flour, at $9 a barrel?

13. If 12 men can earn $100 in a week, how much can 1 man earn in the same time?

14. When wood is $4 a cord and coal is $9 a ton, how much wood is equal in value to 8 tons of coal?

15. If eggs are worth 9 cents a dozen, and butter 12 cents a pound, how many eggs are worth 6 lb. of butter?

16. A man being on a journey, finds he can reach home in 9 days by traveling 20 miles a day; but becoming lame, he traveled only 12 miles a day; in how many days did he reach home?

17. A man bought 6 hats at $4 apiece, and 5 caps at $2, and paid in apples at $6 a barrel; how many barrels and what part of a barrel did it take to pay the bill?

18. John has 12 marbles and William has 9 times as many as John, minus 11; how many marbles has William?

19. When peaches are sold at the rate of 5 for 8 cents, how many will 56 cents buy?

20. What cost 60 apples, at the rate of 10 for 7 cents?

21. George bought 12 oranges, at 4 cents apiece, and after eating 3 of them, sold the rest at 6 cents apiece; did he make or lose by his bargain, and how much?

Written Problems for Review.

121. **1.** The product of two numbers being 252, and the multiplier 18, what is the multiplicand?

2. The product of two numbers is 576, the multiplicand 48; what is the multiplier?

3. When the product of two factors and one of the factors are given, how find the other factor?

4. The sum of two numbers is 250, their difference 50; what is the smaller number? The greater?

5. At an election A and B together received 273 votes, and A had 37 more than B; how many had each?

6. A grocer mixed two kinds of tea in equal quantities, worth 63 and 75 cts. a pound respectively; what is the average price of the mixture a pound?

7. What is the average age of 3 brothers, who are respectively 76, 81, and 89 years old?

8. What is the average price of 4 horses, worth respectively $180, $273, $304, and $375?

9. The ship America of Boston, sailed 56 hours at the rate of 11 miles per hour, when she encountered a storm of 16 hours duration which drove her back at the rate of 14 miles per hour; how far from port was she at the end of 72 hours?

10. A thief fled from New York, at the rate of 85 miles a day; 5 days after an officer started in pursuit of him at the rate of 138 miles a day; how far from the thief was the officer at the end of 8 days from the time the latter started?

11. A is worth $1265, B is worth 4 times as much as A, and $183, and C is worth three times as much as A and B lacking $2348; how much are B and C worth respectively; and how much are they all worth?

12. If a man's salary is $3176 a year, and he spends $7 a day, how much can he lay up?

13. In a single city, $2170 are spent daily for cigars; how many free schools will this support, at $1085 each per annum?

14. A man bought 467 acres of land, at $16 per acre, and sold it for $9340; how much did he get per acre; and how much did he gain or lose by his bargain?

15. A man bought 563 horses, at $65 apiece, and sold them so as to make $860; how much did he get apiece?

16. Which are worth more, 863 cows at $38 apiece, or 356 horses at $75 apiece? How much?

17. A owns 1368 acres of wild land, which is 6 times as much as B owns, and B owns twice as much as C; how much land do B and C own; and how much do all own?

18. The smaller of two numbers is contained 14 times in 252, the greater is 49 times the smaller; what are the numbers?

19. A man bought a drove of oxen for $18130, and after selling 84 of them at $51 apiece, the rest stood him in $43 apiece; how many did he buy?

20. What is the difference between 9313702853 divided by 1987, and 46481 multiplied by 936?

21. A man sold 155 acres of land at $34 per acre, and took in payment for it, 19 horses at $65 apiece, and 15 cows at $17 apiece; how much was still due him?

22. What number besides 137 will exactly divide 11371?

23. The quotient being 275, the divisor 383, and the remainder 49, what is the dividend?

24. If the dividend is 2756, the quotient 184, and the remainder 180, what is the divisor?

25. What must 5376 be multiplied by, to make 6521088?

26. How many times can 437 be subtracted from 18791?

27. If the sum of 14350 and 7845 is divided by 965, the quotient multiplied by 386, and the product diminished by 761, what will the remainder be?

28. The sum of 250 and 173, being multiplied by their difference, and the product divided by 45, what is the quotient?

29. How many men will it take to do as much work in 1 day, as 368 men can do in 134 days?

30. How many men would be required to do the same work in 16 days?

31. Four men, A, B, C, and D, bought a ship together for $16256; A paid $4756, B paid $763 more than A, and C $256 less than B; how much did D pay?

32. Bought sofas for $9212 and selling them at $67 gained $20 on each; how many were bought?

QUESTIONS.

95. What is Division? 96. The Dividend? 97. Divisor? 98. What is the answer called? What does it show? 99. Remainder?

100. Make the sign of division. How is it read? 101. How else is division denoted?

102. When a number is divided into two equal parts, what is one of the parts called? 104. When a unit is divided into equal parts, what are the parts called?

106. When the divisor and dividend are like numbers, what is the quotient? When the divisor is an abstract number, what are the dividend and quotient? To what is the product of the divisor and quotient equal?

110. How divide when the dividend has decimals? 113. What is the general rule? How prove division? 118. How divide by 10, 100, 1000, etc.? When the divisor is greater than 1, with ciphers on the right, how proceed?

119. What is the effect of multiplying the dividend or dividing the divisor? Of dividing the dividend or multiplying the divisor? Of multiplying or dividing both by the same number?

CANCELLATION.

Development of Principles.

122. 1. What is the quotient of 24 divided by 6? *Ans.* 4.

2. Separate the dividend and divisor into factors, and write them thus, $\frac{2\times3\times4}{2\times3}$; what factors are common to both?

3. If you *cancel* the factor 2, which is common to both, what is the quotient? *Ans.* 4.

Note.—To *cancel* means to cross out or reject.

4. If you cancel both the 2's and the 3's, what is the effect? *Ans.* The quotient is not altered. Hence, the following

Principles.

123. *1°. Cancelling a factor of a number divides the number by that factor.*

2°. Cancelling equal factors of the divisor and dividend does not change the quotient. (Art. **119**, *3°*.)

124. Cancellation is the method of shortening Division, by rejecting equal factors from the divisor and dividend.

The **Sign of Cancellation** is an oblique mark drawn across the face of a figure; as, $\cancel{3}$, $\cancel{5}$, $\cancel{7}$, etc.

125. To divide by Cancellation.

5. Divide the product of $14\times15\times56$ by $8\times45\times7$.

1st Form.

$$\frac{\overset{2}{\cancel{14}}\times\cancel{15}\times\overset{7}{\cancel{56}}}{\cancel{8}\times\underset{3}{\cancel{45}}\times\cancel{7}}=\frac{2\times7}{3}=4\tfrac{2}{3},\ \textit{Ans.}$$

2d Form.

$$\begin{array}{r|l} \cancel{8} & \cancel{14}^{2} \\ {}^{3}\cancel{45} & \cancel{15} \\ \cancel{7} & \cancel{56}^{7} \\ \hline 3 & 14 = 4\tfrac{2}{3},\ \textit{Ans.} \end{array}$$

Explanation.—Since 8 in the divisor is a factor of 56 in the dividend, cancel the 8 in both, retaining 7, the other factor of 56. Also cancel 15, a factor of 45, and 7 a factor of 14, retaining the prime factor 2 in the dividend, and 3 in the divisor; then $(7 \times 2) \div 3 = 4\frac{2}{3}$, *Ans.* Hence, the

RULE.—*Cancel all the factors common to the divisor and dividend, and divide the product of those remaining in the dividend by the product of those remaining in the divisor.* (Art. **123**, *2°*.)

NOTE.—When a factor cancelled is *equal* to the number itself, the unit 1 always remains. If the 1 is in the *dividend* it must be *retained;* if in the *divisor*, it may be *disregarded.*

What is the quotient of

6. $28 \times 56 \times 15 \div 14 \times 5 \times 3$?

7. $112 \times 40 \times 18 \div 56 \times 3 \times 4$?

8. $48 \times 72 \times 20 \div 48 \times 15 \times 7$?

9. $54 \times 36 \times 25 \div 45 \times 7 \times 30$?

10. $1365 \div 21 \times 5$?

11. $2850 \div 125$?

12. $3236 \div 256$?

13. $1728 \div 576$?

14. $120 \times 24 \times 35 \times 9 \div 42 \times 15 \times 54 \times 7$?

15. An agent sold 176 boxes of starch, of 15 lbs. each, at 12 cts.; how many loads of corn, having 9 sacks of 5 bu. each, worth 44 cts. a bushel, will it require to pay for the starch?

$$\left.\begin{array}{l}\text{The val. of starch} = 176 \times 15 \times 12 \\ \text{“ “ corn} = 9 \times 5 \times 44\end{array}\right\} \text{and } \frac{\overset{16}{\cancel{176}} \times \overset{3}{\cancel{15}} \times \overset{4}{\cancel{12}}}{\underset{3}{\cancel{9}} \times \cancel{5} \times \underset{11}{\cancel{44}}} = 16, \textit{Ans.}$$

NOTE.—Practical Problems, should first be *analyzed*, and the operations *indicated.* Then cancel as before.

16. A farmer bought 9 cows at \$25 apiece, and paid for them in hay at \$15 a ton; how many tons of hay did it require?

17. How many bags of coffee containing 56 lbs., at 28 cts. a pound, must be given for 8 pieces of muslin, each containing 40 yards, at 8 cts. a yard?

18. How many barrels of flour worth \$8 a barrel, must be given for 45 tons of coal at \$6 a ton?

19. A miller bought 7 loads of wheat, each containing 28 bags of 3 bushels each, worth \$1.50 a bushel, and paid for it in flour at \$7 a barrel; how much flour was required?

PROPERTIES OF NUMBERS.

DEFINITIONS.

126. Numbers are divided into *Odd, Even, Prime,* and *Composite.*

127. An **Even Number** is one that can be exactly divided by 2.

128. An **Odd Number** is one that cannot be exactly divided by 2; as 3, 5, 7, etc.

129. A **Prime Number** is one that cannot be exactly divided by any number, except a unit and itself; as 5, 7, 11, etc.

NOTE.—All prime numbers except 2 are odd.

130. Two numbers are **Prime to each other** when the only number by which both can be exactly divided is a unit or one; as 5 and 6.

131. A **Composite Number** is the product of two or more factors, each of which is greater than 1; as $21 = 3 \times 7$.

NOTE.—The least divisor of a Composite Number is a *prime* number.

132. An **Exact Divisor** of a number is one which will divide it without a *remainder.*

One number is said to be *divisible* by another when there is no *remainder.*

133. The **Factors** of a number are the numbers whose *product* equals that number. (Art. **80.**)

Thus, 7 and 9 are the factors of 63; 3, 4 and 5, of 60.

134. A **Prime Factor** is a prime number used as a factor.

NOTE.—The prime factors of a number are also *exact* divisors of it.

135. The **Reciprocal** of a *number* is 1 divided by that number. Thus, the reciprocal of 4 is $1 \div 4$, or $\frac{1}{4}$.

ORAL EXERCISES.

136. 1. Name the even numbers up to 31.

2. Name the odd numbers less than 30.

3. Name the prime numbers less than 30.

4. Name the composite numbers up to 30.

5. Name an exact divisor of 18, 27, 42.

6. Name all the exact divisors of 24; of 36.

7. What is the smallest number except 1, that will exactly divide 10? 15? 25? 35? 49?

8. What is the largest number, except itself, that will exactly divide 18? 22? 24? 30? 36?

9. What numbers multiplied together produce 21? 35? 42? 27? 45? 48?

10. What will produce 33? 54? 63? 36? 72?

DEVELOPMENT OF PRINCIPLES.

137. *First.*—Take any number, as 12, and separate it into the factors 3 and 4.

If we multiply 12 by 2 the product is 24, if we multiply it by 3 the product is 36, etc. Now each of these products is divisible by 3 and by 4. Hence,

1°. If one number is a factor of another, the former is also a factor of any **Product** *or* **Multiple** *of the latter.*

Second.—Take any number, as 2, which is a common factor of 4 and 12.

The sum of $4+12=16$; their difference $12-4=8$, and their product $12\times4=48$. By inspection we see that 2 is a factor of 16, of 8, and of 48. Hence,

2°. A factor common to two or more numbers, is also a factor of their **Sum,** *their* **Difference,** *and their* **Product.**

Third.—Take any composite number, as 30.

30 is divisible by 2, 3 and 5; also by 2×3, or 6; by 2×5, or 10; by 3×5, or 15; and by no other number. But 2, 3 and 5 are its prime factors; 6, 10 and 15 are the different products of them. Hence,

3°. Every composite number is divisible by each of its **Prime** *factors; and by the* **Product** *of any two or more of them.*

FACTORING.

DEFINITIONS.

138. **Factoring** a number is separating it into factors.

Thus, the factors of 21 are 3 and 7 ; the factors of 32 are 4 and 8.

139. A **Composite Number** is separated into *two factors* by dividing it by any *exact divisor.*

NOTE.—It is not customary to consider the unit 1 and the number itself as factors ; if they were, all numbers would be composite. (Art. **131.**)

140. A number that is a *factor* or *divisor* of two or more numbers is called a **Common Factor** or **Common Measure** of those numbers.

141. The following facts will assist the learner in separating large numbers into factors :

All numbers are divisible

1°. By 2, which end with a cipher, or a digit divisible by 2.

2°. By 3, when the sum of the digits is divisible by 3.

3°. By 4, when the number expressed by the two right hand figures is divisible by 4.

4°. By 5, which end with a cipher or 5.

5°. By 6, when divisible by 2 and 3.

6°. By 8, when the three right hand figures are ciphers, or when the number expressed by them is divisible by 8.

7°. By 9, when the sum of the digits is divisible by 9.
(See Art. **875**, Appendix.)

8°. By 10, 100 or 1000, which end with an equal number of ciphers.

NOTE.—For 7, no convenient rule can be given.

ORAL EXERCISES.

142. 1. What prime factors will exactly divide 12 ? 18 ? 26 ?

2. What prime factors will exactly divide 30 ? 36 ? 40 ?

3. What prime factors are common to 18, 24, and 36 ?

4. Name the prime factors common to 45, 27, and 60 ?

WRITTEN EXERCISES.

143. To Separate a Number into Prime Factors.

1. What are the prime factors of 2310 ?

2	2310	Given Number.
3	1155	1st Quotient.
5	385	2d Quotient.
7	77	3d Quotient.
	11	4th Quotient.

EXPLANATION.—We divide the given number by any prime factor, as 2, and the successive quotients by the prime factors 3, 5 and 7, and the last quotient 11, is a prime number. Therefore, the several divisors with the last quotient are the prime factors required.

PROOF.—$2 \times 3 \times 5 \times 7 \times 11 = 2310$. Hence, the

RULE.—*Divide the given number by any prime factor; then divide this quotient by another prime factor; and so on until the quotient obtained is a prime number. The several divisors, with the last quotient, are the prime factors required.*

Find the prime factors of

2. 225.	6. 672.	10. 3420.	14. 10376.
3. 376.	7. 796.	11. 18500.	15. 25600.
4. 344.	8. 864.	12. 46096.	16. 64384.
5. 576.	9. 945.	13. 96464.	17. 98816.

144. To find the Prime Factors common to two or more numbers.

18. Find the prime factors common to 168, 42, and 210 ?

2)	168,	42,	210
3)	84,	21,	105
7)	28,	7,	35
	4,	1,	5

EXPLANATION.—Dividing by the prime factor 2, the quotients are 84, 21, and 105. Dividing these by 3, we have 28, 7, and 35. Dividing by 7, the quotients are prime to each other (Art. **130**). The divisors 2, 3, and 7, are the prime factors required. Hence, the

RULE.—*Divide the given numbers by any common prime factor, and the quotients thence arising in like manner, till they have no common factor; the several divisors will be the prime factors required.*

19. What are the prime factors common to 24, 76, and 32?

20. 28, 54, and 48?

21. 58, 64, and 84?

22. 436, 308, and 506?

23. 252, 126, and 210?

24. 120, 96, and 384?

25. 168, 320, and 256?

26. 225, 350, and 475?

27. 144, 276, and 524?

COMMON DIVISORS.

145. A **Common Divisor** is one that will divide *two* or *more* numbers without a remainder. It is often called a **Common Measure.**

146. The **Greatest Common Divisor** or **Greatest Common Measure** of two or more numbers is the *greatest* number that will divide *each* of them without a remainder.*

Thus, the greatest common divisor of 18 and 30 is 6.

NOTE.—Numbers which are *prime* to each other have no common divisor or measure greater than 1.

ORAL EXERCISES.

147. **1.** What divisor is common to 15 and 27?

2. What divisor is common to 16 and 20?

3. Find a common factor of 15, 18, and 24.

4. What is the greatest number that will divide 21 and 35 without a remainder?

5. What is the greatest divisor common to 30 and 48? Hence,

148. *The greatest common divisor of two or more numbers is the product of all their common prime factors.*

ILLUSTRATION.—Take any two numbers, as 30 and 42, and separate them into their prime factors; thus,

$$30 = 2 \times 3 \times 5; \qquad 42 = 2 \times 3 \times 7.$$

Now 2 and 3 are the only prime factors common to both numbers, and their product, 6, is the greatest divisor common to both.

* The letters *g. c. d.* stand for Greatest Common Divisor.

WRITTEN EXERCISES.

149. To find the *g. c. d.* of two or more numbers by Prime Factors.

1. What is the ***g. c. d.*** of 45, 30, and 105?

$$\begin{array}{r|rrr} 3 & 45, & 30, & 105 \\ \hline 5 & 15, & 10, & 35 \\ \hline & 3, & 2, & 7 \end{array} \qquad \text{Or,} \quad \begin{aligned} 45 &= 5 \times 3 \times 3 \\ 30 &= 5 \times 3 \times 2 \\ 105 &= 5 \times 3 \times 7 \end{aligned}$$

$5 \times 3 = 15$, ***g. c. d.***, *Ans.*

EXPLANATION.—Separating the numbers into their prime factors, we find 5 and 3 common to each; therefore their product is the ***g. c. d.*** required. (Art. **134**.) Hence, the

RULE.—*Separate the numbers into their prime factors; the product of those that are common to each is the greatest common divisor.*

2. Find the ***g. c. d.*** of 63 and 147.
3. 91 and 117.
4. 247 and 323.
5. 285 and 465.
6. 63, 105, and 240.
7. 16, 124, and 300.
8. 492, 744; and 1044.
9. 485, 145, and 3471.
10. 6430 and 8945.

150. To find the *g. c. d.* by Continued Division.

1. What is the greatest common divisor of 30 and 42?

ANALYSIS.—Dividing the *greater* by the *less*, the quotient is 1 and 12 remainder. Next, dividing the *first divisor* by the *first remainder*, the quotient is 2 and 6 remainder. Again, dividing the *second divisor* by the *second remainder*, the quotient is 2 and no remainder. Therefore, 6 is the ***g. c. d.*** of 30 and 42. Hence, the

$$\begin{array}{l} 30\,)\,42\,(\,1 \\ \quad\;\; \underline{30} \\ \quad\;\; 12\,)\,30\,(\,2 \\ \qquad\quad\;\; \underline{24} \\ \qquad\quad\;\; 6\,)\,12\,(\,2 \\ \qquad\qquad\;\;\; \underline{12} \end{array}$$

RULE.—*Divide the greater number by the less; then divide the first divisor by the first remainder, and so on, until nothing remains; the last divisor will be the greatest common divisor.*

If there are more than two numbers, find the greatest common divisor of two of them; then of this divisor and a third number, and so on, until all the numbers have been taken.

NOTE.—The greatest common divisor of two or more *prime* numbers, or numbers *prime* to *each other* is 1. (Art. **130**.)

(For demonstration, see Art. **873**, Appendix.)

1. Find the ***g. c. d.*** of 246 and 324.

2. 285 and 465.

3. 72, 96, and 132.

4. 2145 and 3472.

5. 464320 and 18945.

6. 638296 and 33888.

7. 18996 and 29932.

8. 54428 and 262424.

9. 143168 and 2064888.

10. A farmer has 664 bushels of oats and 316 bushels of corn, which he wishes to send to market in the largest possible bags of equal size that will hold each kind of grain; how many bushels must each bag hold?

11. A man bought three pieces of land containing 28, 36, and 44 acres respectively, which he wished to fence into the largest possible fields, each having the same number of acres; how many acres can he put in a field?

12. A grocer had 42 oranges and 63 pears which he wished to put in bags each containing the largest number possible; how many could he put in each bag?

13. A man having 3 plots of land fronting a street, the width of which was 600 ft., 120 ft., and 900 ft., respectively, wished to divide each into house-lots of equal width; how wide will the lots be, and how many can be made from each plot?

14. Three men having \$1260, \$2268, and \$2772 respectively, agreed to buy horses at the highest price per head that will allow each man to invest all his money; how many horses can each man buy?

COMMON MULTIPLES.

151. A **Multiple** of a number is one which is exactly divisible by that number.

Thus, 12 is a multiple of 4; 18 of 6.

152. A **Common Multiple** of two or more numbers is a number that is exactly divisible by each of them.

Thus, 18 is a common multiple of 2, 3, 6, and 9.

153. The **Least Common Multiple** of two or more numbers, is the *least* number exactly divisible by each of them.*

Thus, 15 is the least common multiple of 3 and 5.

154. A *composite* number contains all the *prime factors* of each of the numbers which produce it.

DEVELOPMENT OF PRINCIPLES.

155. 1. Name two numbers each of which can be divided by 3 and 5 without a remainder.

2. What is the smallest number that can be exactly divided by 3 and 5?

3. Name two numbers which can be exactly divided by 6 and 8?

4. What is the smallest number that can be exactly divided by 6 and 8?

5. By what two prime numbers can 35 be divided?

6. What is the least number that is exactly divisible by 2, 3, and 5?

7. What is the least number that can be exactly divided by 3, 5, and 6? Hence, we derive the following

PRINCIPLES.

156. *1°. A multiple of a number must contain all the prime factors of that number.*

2°. A common multiple of two or more numbers must contain all the prime factors of each of the given numbers.

---

* The letters *l. c. m.* stand for "Least Common Multiple."

3°. The least common multiple of two or more numbers is the least number which contains all their prime factors, each factor being taken the greatest number of times it occurs in either of the given numbers.

Written Exercises.

157. To find the Least Common Multiple of two or more numbers.

1. What is the ***l. c. m.*** of 10, 21, 66?

OPERATION.

$$
\begin{array}{r|rrr}
2 & 10, & 21, & 66 \\ \hline
3 & 5, & 21, & 33 \\ \hline
 & 5, & 7, & 11
\end{array}
$$

$2 \times 3 \times 5 \times 7 \times 11 = 2310$, *Ans.*

Explanation.—Write the numbers in a line, and divide them by any prime number as 2, that will exactly divide two or more of them, setting the quotients and undivided numbers in a line below. Divide these by the prime number 3, and set the results below as before. Now, as all the numbers in the third line are *prime*, we can carry the division no further. (Art. **129**.) Hence, the divisors 2 and 3, with the numbers in the last line, 5, 7, and 11, are all prime factors of the given numbers, and each is taken as many times as it occurs in either of them. Therefore, the continued product of these factors, or 2310, is the ***l. c. m.*** required. (Art. **156**, *2°*.) Hence,

158. Rule.—*Write the numbers in a line, and divide by any prime number that will divide two or more of them without a remainder, placing the quotients and undivided numbers in a line below.*

Next, divide this line as before, and thus proceed till no two numbers are divisible by any number greater than 1. The continued product of the divisors and numbers in the last line will be the answer.

Notes.—1. The operation may often be shortened by *cancelling* any number which is a *factor of another number* in the same line.

2. When the given numbers are *prime* or *prime to each other*, their continued product will be the least common multiple.

2. Find the ***l. c. m.*** of 24, 16, 15, and 20.
3. 25, 60, 72, and 35.
4. 63, 12, 84, and 72.
5. 54, 81, 14, and 63.
6. 12, 72, 36, and 144.
7. 17, 29, 53, and 85.
8. 18, 55, 49, 33, and 121.
9. 720, 336, and 1736.
10. 8, 12, 16, 24, 36, 48, 72, 144.

11. Find the least common multiple of the nine digits.

12. Of 720, 336, 576, and 1820.

13. Of 642, 876, 984, and 2000.

14. Required the smallest number of pears that a farmer can exactly divide among 3 classes of children containing 18, 24, and 30 respectively.

15. A bell-hanger wishes to find the shortest piece of wire which may be cut into pieces of 16, 18, or 22 feet long.

16. What is the least sum with which a dealer can buy an exact number of hats at $3, $4, $5, or $6 each?

17. What is the smallest number of gallons that can exactly be measured by each of 4 casks holding 15, 30, 40, and 42 gal. respectively?

18. Two lads start at the same time and place to travel round a pond; one can travel the distance in 3 hours, the other in 4 hours. In what time will they first meet at the starting-place?

19. Three boats start to sail round an island at the same place and time; one of them can perform the trip in 6 hours, another in 8 hours, and the other in 12 hours; how long before they will all meet at the place of starting?

20. Three messengers start at the same time from New York to go to Philadelphia and back, one of whom can perform the journey in 8 hours, another in 10 hours, and the other in 12 hours. In what time will they all meet at New York?

QUESTIONS.

127. An even number? 128. What is an odd number? 129. A prime number? 131. A composite number?

130. When are numbers prime to each other? 132. What is an exact divisor? 133. What are factors? 134. A prime factor? 137. Name the first principle respecting factors. Second. Third.

138. What is factoring? 143. How separate a number into its prime factors? 144. How find the prime factors common to two or more numbers?

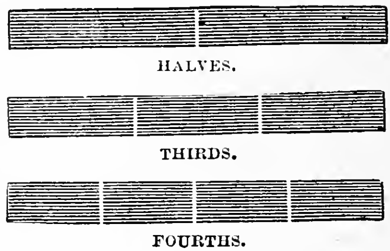
145. What is a common divisor? 146. The greatest common divisor? 149. How find the *g. c. d.*?

156. Name the first principle respecting multiples. The second. The third.

157. How find the least common multiple of two or more numbers?

FRACTIONS.

ORAL EXERCISES.



159. 1. If a unit is divided into two equal parts, what is each part called ?

2. If divided into three equal parts, what are the parts called ?

3. If divided into four equal parts, what ?

4. When divided into 5 equal parts, what are 2 of the parts called ? 3 of the parts ?

5. When divided into 7 equal parts, what is 1 of the parts called ? 2 of the parts ? 4 of the parts ? 6 of the parts ?

6. If a sheet of paper is divided into 4 equal parts, what part of the sheet is 3 of the parts ? 5 parts ?

7. How many *halves* in a unit ? Thirds ? Fifths ?

8. Which is the larger, halves or thirds ?

160. A **Fraction** is *one or more* of the *equal parts* of a unit.

161. The **Unit of a Fraction** is the number or thing of which the fraction is a part.

162. A **Fractional Unit** is *one* of the equal parts into which the number or thing is divided.

Thus, in the expression *two-thirds* of a pear, the *unit* of the fraction is *one pear ;* and the *fractional unit* is *one-third* of a pear.

163. *Fractional units* take their name from the **Number** of *equal parts* into which the unit is divided ; as, *thirds, fourths.*

164. The *number* of equal parts into which the unit is *divided* is called the **Denominator,** because it *names* the parts.

165. The *number* of *parts taken* is called the **Numerator,** because it *numbers* the parts.

Thus, in the fraction *three-fourths* ($\frac{3}{4}$), the denominator is 4 and the numerator is 3.

166. The **Terms** of a fraction are the *numerator* and *denominator*.

167. Fractions are divided into **Common and Decimal.**

168. A **Common Fraction** is one in which the unit is divided into *any number* of equal parts.

169. Common fractions are expressed by writing the denominator *under* the numerator with a line between them.

Thus, the fraction three-fifths is written $\frac{3}{5}$; four-sevenths, $\frac{4}{7}$.

Express the following fractions by figures :

1. Two-thirds.
2. Three-fourths.
3. Two-fifths.
4. Five-sevenths.
5. Five-eighths.
6. Six-sevenths.
7. Four-tenths.
8. Seven-twelfths.
9. Two-twentieths.
10. Fifteen-thirtieths.
11. Twenty-fiftieths.
12. Fifty-hundredths.

Copy and read the following:

13. $\frac{5}{11}$, $\frac{8}{13}$, $\frac{9}{17}$, $\frac{14}{93}$.
14. $\frac{17}{63}$, $\frac{43}{78}$, $\frac{61}{97}$, $\frac{58}{79}$.
15. $\frac{83}{100}$, $\frac{97}{126}$, $\frac{67}{150}$, $\frac{99}{200}$.
16. $\frac{110}{327}$, $\frac{203}{568}$, $\frac{326}{723}$, $\frac{500}{974}$.

170. An **Integer** may be expressed in the form of a fraction by writing 1 under it for a denominator.

Thus, 3 may be written $\frac{3}{1}$, and read "3 ones."

171. A **Proper Fraction** is one whose numerator is *less* than the denominator ; as, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$.

172. An **Improper Fraction** is one whose numerator *equals* or *exceeds* the denominator ; as, $\frac{4}{4}$, $\frac{5}{3}$.

173. A **Simple Fraction** is one whose terms are integers, and may be proper or improper.

174. A **Compound Fraction** is a fraction of a fraction; as, $\frac{1}{3}$ of $\frac{3}{4}$.

175. A **Mixed Number** is a *whole* number and a *fraction* expressed together; as, $5\frac{3}{7}$, $34\frac{13}{23}$.

176. Fractions arise from division, the *numerator* being the *dividend*, and the *denominator* the *divisor*. (Art. **101.**) Hence,

177. The **Value of a Fraction** is the *quotient* of the numerator divided by the denominator.

Thus, the value of 1 fourth is $1 \div 4$, or $\frac{1}{4}$; of 6 halves is $6 \div 2$, or 3; of 3 thirds is $3 \div 3$, or 1.

NOTE.—This value depends upon the *size* of the number or thing divided, and upon the *number of parts* into which it is divided.

178. The **Reciprocal of a Fraction** is 1 divided by the fraction, or the fraction inverted. (Art. **135.**)

Thus, the reciprocal of $\frac{2}{3}$ is $1 \div \frac{2}{3} = \frac{3}{2}$; of $\frac{7}{12}$ is $\frac{12}{7}$.

GENERAL PRINCIPLES OF FRACTIONS.

179. Since fractions arise from division, the *numerator* being a dividend and the *denominator* a divisor, the *general principles* of division are true of fractions. (Art. **119.**) That is,

1°. Multiplying the numerator, or *Dividing* the denominator,	*Multiplies the fraction.*
2°. Dividing the numerator, or *Multiplying* the denominator,	*Divides the fraction.*
3°. Multiplying or *Dividing* both terms of a fraction by the same number,	*Does not change its value.*

REDUCTION OF FRACTIONS.

ORAL EXERCISES.

180. 1. How many halves in a whole apple? How many fourths?

2. How many fourths in $\frac{1}{2}$ of an apple?

ANALYSIS.—The required denominator 4, is twice the given denominator 3. Multiplying both terms of $\frac{1}{2}$ by 2, it becomes $\frac{2}{4}$, *Ans.*

3. How many sixths in $\frac{2}{3}$? How many ninths?
4. How many eighths in $\frac{3}{4}$? How many twelfths?
5. Change $\frac{1}{2}$, $\frac{1}{5}$ to tenths.
6. Change $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{2}{8}$ to sixteenths.
7. Change $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{6}$, $\frac{3}{8}$ to twenty-fourths.
8. Change $\frac{2}{7}$ to twenty-eighths; $\frac{2}{9}$ to forty-fifths.
9. Change $\frac{5}{13}$ to thirty-ninths; $\frac{3}{10}$ to sixtieths.

181. Reduction of Fractions is changing their terms without altering the *value* of the fractions. (Art. **179**, *3°*.)

WRITTEN EXERCISES.

182. To Reduce a Fraction to Larger Terms.

1. Change $\frac{14}{27}$ to a fraction whose denominator is 81?

$$81 \div 27 = 3$$
$$\frac{14 \times 3}{27 \times 3} = \frac{42}{81}$$

ANALYSIS.—The given denominator 27 is contained in the required denominator 81, 3 times. Multiplying both terms of the given fraction by 3, we have $\frac{42}{81}$, the fraction required. (Art. **179**, *3°*.) Hence, the

RULE.—*Divide the required denominator by the denominator of the given fraction, and multiply both terms by the quotient.*

2. Change $\frac{10}{26}$ to 104ths.
3. Change $\frac{13}{24}$ to 120ths.
4. Change $\frac{17}{22}$ to 176ths.
5. Change $\frac{15}{48}$ to 144ths.
6. Change $\frac{15}{28}$ to 196ths.
7. Change $\frac{25}{32}$ to 288ths.
8. Change $\frac{7}{8}$ to 192ds.
9. Change $\frac{23}{72}$ to 576ths.

ORAL EXERCISES.

183. Reduce the following at sight to their *lowest* terms:

1. $\frac{2}{6}, \frac{3}{6}, \frac{2}{8}, \frac{2}{10}, \frac{2}{16}$. *Ans.* $\frac{1}{3}, \frac{1}{2}, \frac{1}{4}, \frac{1}{5}, \frac{1}{8}$.
2. $\frac{3}{12}, \frac{3}{18}, \frac{3}{24}, \frac{3}{39}, \frac{3}{60}$.
3. $\frac{4}{12}, \frac{4}{20}, \frac{4}{32}, \frac{4}{48}, \frac{4}{68}$.
4. $\frac{7}{28}, \frac{7}{35}, \frac{7}{56}, \frac{7}{63}, \frac{7}{84}$.
5. $\frac{8}{24}, \frac{16}{40}, \frac{8}{56}, \frac{16}{48}, \frac{24}{88}$.
6. $\frac{9}{27}, \frac{18}{45}, \frac{18}{63}, \frac{18}{99}, \frac{27}{108}$.
7. $\frac{6}{18}, \frac{6}{24}, \frac{6}{42}, \frac{6}{72}, \frac{6}{96}$.

184. A Fraction is expressed in its **Lowest Terms** when its numerator and denominator have no *common divisor.*

WRITTEN EXERCISES.

185. To Reduce Fractions to their Lowest Terms.

1. Reduce $\frac{48}{72}$ to an equivalent fraction in its lowest terms.

1ST OPERATION.

$$\frac{\cancel{48}}{\cancel{72}} = \frac{\cancel{8}}{\cancel{12}} = \frac{2}{3}$$

1ST ANALYSIS.—Cancelling the factor 6 from both terms of the given fraction, we have $\frac{8}{12}$. Again, cancelling the factor 4 from both terms of this fraction, we have $\frac{2}{3}$, the terms of which are *prime to each other.* (Art. **130**.)

2D OPERATION.

$$\frac{48 \div 24}{72 \div 24} = \frac{2}{3}$$

2D ANALYSIS.—Dividing both terms of $\frac{48}{72}$ by their *g. c. d.* 24, (Art. **150**) we obtain the same result. Hence, the

RULE.—*Cancel all the factors common to both terms of the fraction.*

Or, Divide both terms by their greatest common divisor.

Change the following to the lowest terms:

2.	$\frac{63}{165}$.	**6.**	$\frac{55}{605}$.	**10.**	$\frac{750}{900}$.	**14.**	$\frac{126}{162}$.
3.	$\frac{240}{312}$.	**7.**	$\frac{302}{894}$.	**11.**	$\frac{475}{525}$.	**15.**	$\frac{435}{957}$.
4.	$\frac{272}{425}$.	**8.**	$\frac{253}{782}$.	**12.**	$\frac{594}{2142}$.	**16.**	$\frac{1740}{2900}$.
5.	$\frac{384}{1152}$.	**9.**	$\frac{528}{984}$.	**13.**	$\frac{55}{1210}$.	**17.**	$\frac{6465}{7335}$.

ORAL EXERCISES.

186. 1. Reduce $\frac{65}{7}$ to a whole or mixed number.

ANALYSIS.—In 7 sevenths there is 1 unit, and in 65 sevenths there are as many units as 7's in 65, or $9\frac{2}{7}$, *Ans.*

Change the following to whole or mixed numbers :

2. $\frac{24}{5}$.	**5.** $\frac{35}{7}$.	**8.** $\frac{60}{7}$.	**11.** $\frac{756}{25}$.
3. $\frac{21}{8}$.	**6.** $\frac{45}{6}$.	**9.** $\frac{75}{8}$.	**12.** $\frac{800}{40}$.
4. $\frac{18}{18}$.	**7.** $\frac{600}{12}$.	**10.** $\frac{88}{9}$.	**13.** $\frac{960}{12}$.

14. How many dollars in 28 half dollars ? In 36 quarter dollars ? In \$$\frac{54}{6}$? In \$$\frac{63}{5}$? In \$$\frac{75}{8}$?

WRITTEN EXERCISES.

187. To reduce Improper Fractions to Whole or Mixed Numbers.

1. Reduce $\frac{112}{8}$ to a whole or mixed number.

OPERATION.

$$8\,)\,\underline{112}$$
$$14,\ Ans.$$

ANALYSIS.—Since 8 eighths make a unit, in 112 eighths there are as many units as 8's in 112, or 14 units. Hence, the

RULE.—*Divide the numerator by the denominator.*

NOTE.—If there is a fraction in the answer, it should be reduced to the *lowest* terms.

Reduce the following to whole or mixed numbers :

1. $\frac{448}{12}$.	**6.** $\frac{835}{160}$.	**11.** $\frac{6786}{314}$.	**16.** $\frac{28342}{256}$.
2. $\frac{576}{13}$.	**7.** $\frac{783}{55}$.	**12.** $\frac{8573}{450}$.	**17.** $\frac{98536}{750}$.
3. $\frac{750}{25}$.	**8.** $\frac{8437}{298}$.	**13.** $\frac{9568}{235}$.	**18.** $\frac{100000}{59000}$.
4. $\frac{845}{30}$.	**9.** $\frac{7243}{320}$.	**14.** $\frac{12000}{121}$.	**19.** $\frac{410035}{72146}$.
5. $\frac{500}{16}$.	**10.** $\frac{5805}{126}$.	**15.** $\frac{15720}{1168}$.	**20.** $\frac{879920}{72840}$.

21. In $\frac{25460}{420}$ of a rod how many rods ?

22. In $\frac{360275}{2354}$ of a dollar how many dollars ?

ORAL EXERCISES.

188. 1. Reduce $7\frac{3}{4}$ to an improper fraction.

ANALYSIS.—Since in 1 unit there are 4 fourths, in 7 units there are 7 times 4, or 28 fourths, and 3 added make 31 fourths. *Ans.* $\frac{31}{4}$.

Reduce the following to improper fractions :

2. $4\frac{1}{2}$.	**5.** $7\frac{1}{3}$.	**8.** $11\frac{3}{5}$.	**11.** $15\frac{2}{3}$.
3. $5\frac{3}{4}$.	**6.** $8\frac{5}{6}$.	**9.** $8\frac{7}{9}$.	**12.** $20\frac{1}{5}$.
4. $7\frac{8}{9}$.	**7.** $9\frac{2}{7}$.	**10.** $12\frac{5}{7}$.	**13.** $25\frac{3}{4}$.

14. Change 7 to ninths ; 8 to sevenths ; 11 to eighths.

15. Change 14 to thirds ; 12 to ninths ; 15 to fourths.

WRITTEN EXERCISES.

189. To reduce Whole or Mixed Numbers to Improper Fractions.

1. Reduce $18\frac{3}{5}$ to an improper fraction.

ANALYSIS.—Since in 1 there are 5 fifths, in 18 there are 18 times 5 fifths, or 90, and 3 fifths added make 93 fifths. Hence, the *Ans.* $\frac{93}{5}$.

$$\begin{array}{r} 18\frac{3}{5} \\ \underline{5} \\ 93 \text{ fifths.} \end{array}$$

RULE.—*Multiply the whole number by the given denominator; to the product add the numerator, and place the sum over the denominator.*

NOTE.—1. A *whole* number is reduced to an *improper* fraction by making 1 its denominator. Thus, $4 = \frac{4}{1}$. (Art. **170.**)

2. For reducing a Compound Fraction to a Simple one, See Art. **211.**

Reduce the following to improper fractions:

2. Reduce $19\frac{2}{3}$.
3. Reduce $23\frac{7}{9}$.
4. Reduce $64\frac{8}{15}$.
5. Reduce $304\frac{1}{3}$.
6. Reduce 45 to fifths.
7. Reduce 830 to sixths.
8. Reduce $26\frac{5}{8}$.
9. Reduce $45\frac{5}{12}$.
10. Reduce $56\frac{23}{45}$.
11. Reduce $725\frac{1}{5}$.
12. Reduce 72 to eighths.
13. Reduce 743 to 15ths.

COMMON DENOMINATORS.

ORAL EXERCISES.

190. 1. Change $\frac{1}{2}$ and $\frac{2}{3}$ to fractions whose denominator is 6.

ANALYSIS.—Multiplying both terms of $\frac{1}{2}$ by 3, we have $\frac{3}{6}$; and multiplying both terms of $\frac{2}{3}$ by 2, we have $\frac{4}{6}$. *Ans.* $\frac{3}{6}$ and $\frac{4}{6}$.

2. By what must $\frac{1}{3}$ and $\frac{3}{4}$ be multiplied to become twelfths?
3. Change $\frac{2}{3}$ and $\frac{5}{6}$ to the same denominator.
4. Change $\frac{3}{4}$ and $\frac{4}{5}$ to the same denominator.
5. Name two multiples of 3; of 4; of 6; of 7.
6. Name a multiple common to 3 and 5.
7. Change $\frac{1}{6}$ and $\frac{3}{8}$ to twenty-fourths.

191. Fractions which have the *same* denominator, have a **Common Denominator.**

192. The **Least Common Denominator** (***l. c. d.***) of two or more fractions, is the *smallest* number *divisible* by each of their denominators.

193. The *smallest* number divisible by any two or more numbers is their **Least Common Multiple.** (Art. **153**.) Hence,

194. The **Least Common Denominator** of two or more fractions, is the **Least Common Multiple** of their denominators.

NOTE.—When the denominators are *prime* to each other, their *continued product* is their ***l. c. d.***

WRITTEN EXERCISES.

195. To Reduce Fractions to a Common Denominator.

1. Reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{4}{5}$ to equivalent fractions having a ***c. d.***

ANALYSIS.—The product of the denominators 2, 3, and 5, is 30, the common denominator. (Art. **191.**)

Multiplying each numerator by all the denominators except its own, we have $\frac{15}{30}, \frac{20}{30}, \frac{24}{30}$. (Art. **179**, *3°*.) Hence, the

$\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$, Given Fraction.
$2 \times 3 \times 5 = 30$, ***c. d.***
$1 \times 3 \times 5 = 15$, 1st. ***n.***
$2 \times 2 \times 5 = 20$, 2d ***n.***
$4 \times 2 \times 3 = 24$, 3d ***n.***
Ans. $\frac{15}{30}$, $\frac{20}{30}$, $\frac{24}{30}$.

RULE.—*Multiply the denominators together for the common denominator, and each numerator by all the denominators, except its own, for the numerators.*

Reduce to a common denominator,

2. $\frac{4}{5}$ and $\frac{6}{7}$.
3. $\frac{7}{9}$ and $\frac{3}{5}$.
4. $\frac{3}{8}$ and $\frac{7}{10}$.
5. $\frac{3}{9}$ and $\frac{7}{16}$.
6. $\frac{35}{63}$, $\frac{18}{45}$, and $\frac{7}{100}$.
7. $\frac{31}{84}$, $\frac{23}{36}$, and $\frac{32}{108}$.

196. To Reduce Fractions to the Least Common Denominator.

8. Reduce $\frac{2}{3}$, $\frac{5}{9}$, and $\frac{11}{15}$, to their ***l. c. d.***

ANALYSIS.—Reducing the given denominators to the ***l. c. m.***, we have 45 for the ***l. c. d.*** (Art. **157.**) Multiply the terms of each fraction by the quotient of 45 divided by each given denominator, and the products will be $\frac{30}{45}$, $\frac{25}{45}$, $\frac{33}{45}$, *Ans.* Hence, the

$3 \times 3 \times 5 = 45$, ***l. c. d.***
$45 \div 3 = 15$ and $\frac{2 \times 15}{3 \times 15} = \frac{30}{45}$
$45 \div 9 = 5$ and $\frac{5 \times 5}{9 \times 5} = \frac{25}{45}$
$45 \div 15 = 3$ and $\frac{11 \times 3}{15 \times 3} = \frac{33}{45}$

RULE.—*Find the least common multiple of all the denominators for the least common denominator.*

Divide this multiple by the denominator of each fraction, and multiply its numerator by the quotient.

NOTE.—Mixed numbers must be reduced to improper fractions and all to their *lowest* terms before the rule is applied.

Reduce the following fractions to the ***l. c. d.***:

9. $\frac{2}{5}$, $\frac{5}{12}$, $\frac{4}{15}$.
10. $\frac{3}{7}$, $\frac{5}{8}$, $\frac{4}{11}$.
11. $\frac{5}{12}$, $\frac{8}{27}$, $\frac{5}{36}$.
12. $\frac{7}{8}$, $\frac{5}{7}$, $\frac{8}{10}$, $\frac{7}{12}$.
13. $\frac{9}{10}$, $\frac{6}{7}$, $\frac{12}{15}$, $\frac{2}{5}$.
14. $\frac{12}{25}$, $\frac{62}{70}$, $\frac{27}{40}$.
15. $\frac{8}{20}$, $\frac{79}{105}$, $\frac{52}{25}$.
16. $\frac{7}{21}$, $8\frac{5}{7}$, $\frac{4}{35}$.
17. $\frac{3}{70}$, $\frac{5}{8}$, $2\frac{3}{7}$.
18. $\frac{22}{35}$, $10\frac{4}{7}$, $\frac{36}{49}$.
19. $7\frac{4}{11}$, $\frac{63}{44}$, $\frac{75}{99}$.
20. $\frac{39}{63}$, $\frac{85}{189}$, $\frac{48}{144}$.

ADDITION OF FRACTIONS.

DEFINITIONS.

197. Like Fractions are those which express *like* parts of *like* units.

Thus, $\frac{3}{8}$ yard and $\frac{5}{8}$ yard; also $\frac{4}{9}$ and $\frac{7}{9}$ are like fractions.

198. Unlike Fractions are those which express *unlike* parts of *like* units, or parts of *unlike* units. (Art. **7.**)

Thus, $\frac{2}{3}$ pound and $\frac{5}{8}$ pound; also $\frac{4}{5}$ and $\frac{3}{7}$ are unlike fractions.

ORAL EXERCISES.

199. 1. What is the sum of $\frac{3}{7}$, $\frac{1}{7}$, and $\frac{2}{7}$?

SOLUTION.—$\frac{3}{7}$ and $\frac{1}{7}$ are $\frac{4}{7}$, and $\frac{2}{7}$ are $\frac{6}{7}$, *Ans.*

2. What is the sum of $\frac{3}{12}+\frac{5}{12}+\frac{7}{12}$? Of $\frac{7}{23}+\frac{8}{23}+\frac{4}{23}$?

3. A dealer sold $\frac{5}{10}$ ton of coal to one person, $\frac{7}{10}$ to another, and $\frac{4}{10}$ to another; how much coal did he sell to all?

4. A Reader costs $\$\frac{1}{2}$ and a History $\$\frac{3}{4}$; what will both cost?

ANALYSIS.—Halves and fourths are *unlike* parts of the unit dollar, and cannot be added in their present form. But $\frac{1}{2}$ is equal to $\frac{2}{4}$, and $\frac{2}{4}+\frac{3}{4}$ are $\frac{5}{4}$, which equals $\$1\frac{1}{4}$, *Ans.* (Art. **197.**)

5. What is the sum of $\frac{2}{3}$ and $\frac{1}{6}$? Of $\frac{3}{4}$ and $\frac{5}{8}$?

6. How much wood is there in $\frac{2}{3}$ cord and $\frac{5}{6}$ cord?

7. What is the sum of $\frac{3}{5}$ pear and $\frac{4}{5}$ melon?

Ans. Pears and melons are *unlike* units, and parts of unlike units cannot be added. (Art. **53**, *1°*.) Hence, the following

PRINCIPLES.

200. *1°. Only like fractions can be added.* (Art. **197.**)

2°. Like fractions are added the same as like integers.

8.	$\frac{2}{3}$ and $\frac{1}{2}=?$	11.	$\frac{4}{5}$ and $\frac{2}{15}=?$	14.	$\frac{5}{12}$ and $\frac{7}{15}=?$
9.	$\frac{1}{4}$ and $\frac{2}{3}=?$	12.	$\frac{3}{4}$ and $\frac{8}{10}=?$	15.	$\frac{4}{9}$ and $\frac{11}{18}=?$
10.	$\frac{5}{6}$ and $\frac{3}{4}=?$	13.	$\frac{7}{8}$ and $\frac{11}{12}=?$	16.	$\frac{5}{18}$ and $\frac{8}{12}=?$

17. A farmer sold $\frac{5}{7}$ tons of hay to one neighbor, and $\frac{4}{5}$ of a ton to another; how much hay did he sell to both?

18. If a newsboy makes $\$\frac{1}{4}$ in one day, $\$\frac{1}{2}$ the next, and $\$\frac{2}{3}$ the third day, how much will he make in 3 days?

19. If a pupil is absent $\frac{1}{2}$ day 1 week, $\frac{2}{3}$ day the next, and $\frac{5}{6}$ day a third week, how much time has he lost in 3 weeks.

WRITTEN EXERCISES.

201. To find the Sum of two or more Fractions.

1. What is the sum of $\frac{3}{8}$, $\frac{5}{6}$, and $\frac{7}{12}$?

ANALYSIS.—Reducing the given fractions to the *l. c. d.* 24, they become $\frac{9}{24}$, $\frac{20}{24}$, and $\frac{14}{24}$, which are like fractions, the sum of whose numerators is 43; and $\frac{43}{24}=1\frac{19}{24}$, the answer required. Hence, the

$$\frac{3}{8}=\frac{9}{24}$$
$$\frac{5}{6}=\frac{20}{24}$$
$$\frac{7}{12}=\frac{14}{24}$$
$$\frac{9}{24}+\frac{20}{24}+\frac{14}{24}=\frac{43}{24},$$
$$\frac{43}{24}=1\frac{19}{24},\ \textit{Ans.}$$

RULE.—*Reduce the given fractions to a common denominator, and over it write the sum of their numerators.*

NOTES.—1. If there are mixed numbers, add the *fractions* and *integers* separately, and unite the results.

2. The answer should be reduced to lowest terms, and if improper fractions, to whole or mixed numbers.

3. It is advisable in most cases to reduce the fractions to the *l. c. d.*

2. What is the sum of $12\frac{2}{3}$, $19\frac{3}{4}$, and 15 ?

$$12\tfrac{2}{3} = 12\tfrac{8}{12}$$
$$19\tfrac{3}{4} = 19\tfrac{9}{12}$$
$$15 = 15$$
$$\textit{Ans. } 47\tfrac{5}{12}$$

ANALYSIS.—Reducing the fractional parts to a common denominator, $\frac{2}{3}$ and $\frac{3}{4}$ are equal to $\frac{8}{12}$ and $\frac{9}{12}$. Now $\frac{8}{12} + \frac{9}{12} = \frac{17}{12}$, or $1\frac{5}{12}$. Adding the 1 to the sum of the integral parts, we have $47 + \frac{5}{12} = 47\frac{5}{12}$, *Ans.*

Add the following:

3. $\frac{3}{5}$, $\frac{2}{11}$, and $\frac{6}{18}$.
4. $\frac{4}{13}$, $\frac{7}{8}$, and $\frac{12}{7}$.
5. $\frac{1}{8}$, $\frac{3}{5}$, $\frac{6}{3}$, and $\frac{8}{7}$.
6. $\frac{1}{10}$, $\frac{2}{7}$, and $\frac{5}{6}$.
7. $\frac{1}{3}$, $\frac{1}{6}$, $\frac{2}{8}$, and $\frac{9}{2}$.
8. $\frac{6}{8}$, $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{7}{12}$.
9. $\frac{3}{5}$, $\frac{2}{3}$, $\frac{7}{8}$, $\frac{1}{4}$ and $\frac{1}{6}$.
10. $2\frac{1}{2}$, $6\frac{1}{3}$, and $\frac{2}{3}$.
11. $\frac{24}{30}$, $\frac{81}{45}$, and $\frac{19}{10}$.
12. $\frac{1}{7}$, 3, $\frac{3}{5}$, $\frac{1}{3}$, and $\frac{5}{8}$.
13. $\frac{2}{3}$, 2, $3\frac{1}{2}$, and $5\frac{2}{7}$.
14. $35\frac{1}{8}$, $\frac{53}{12}$, $\frac{2}{3}$, and $\frac{7}{8}$.
15. $\frac{25}{4}$, $6\frac{1}{2}$, $1\frac{2}{3}$, and $\frac{5}{8}$.
16. $\frac{11}{45}$, 85, $\frac{21}{35}$, and $3\frac{5}{8}$.
17. $24\frac{7}{8}$, $82\frac{3}{7}$, and $\frac{19}{20}$.
18. $25\frac{3}{8}$, $18\frac{11}{19}$, and $\frac{23}{34}$.
19. $\frac{336}{448}$, $\frac{315}{693}$, and $\frac{588}{756}$.
20. $263\frac{3}{5}$, $\frac{4}{5}$, $\frac{19}{20}$, and $385\frac{2}{3}$.

21. A grocer sold $47\frac{3}{7}$ pounds of sugar to one customer, $83\frac{3}{5}$ pounds to another, and $68\frac{5}{9}$ pounds to another; how much did he sell to all?

22. If you travel $85\frac{5}{12}$ miles in one day, $78\frac{9}{15}$ in another, and $125\frac{17}{35}$ in another, how far will you travel in all?

23. If a man buys 3 pieces of cloth, containing $127\frac{7}{8}$ yards, $168\frac{9}{16}$ yards, and $256\frac{3}{5}$ yards, how much will he then have?

24. If a hat costs $\$4\frac{3}{4}$ and a vest $\$5\frac{1}{2}$, what will both cost?

ANALYSIS.—$4 and $5 are $9; $\frac{1}{2} = \frac{2}{4}$, and $\frac{2}{4} + \frac{3}{4}$ are $\frac{5}{4} = \$1\frac{1}{4}$, which added to $9 make $\$10\frac{1}{4}$, *Ans.*

25. What is the sum of $6\frac{2}{3}$ and $5\frac{1}{6}$?
26. $9\frac{1}{4}$ and $7\frac{3}{8}$?
27. $15\frac{3}{5}$ and $18\frac{1}{4}$?
28. $7\frac{1}{6}$ and $9\frac{3}{8}$?
29. $12\frac{4}{5}$ and $20\frac{5}{7}$?
30. $15\frac{11}{12}$ and $21\frac{7}{8}$?
31. $7\frac{5}{11}$ and $15\frac{7}{33}$?
32. $20\frac{7}{12}$ and $8\frac{9}{15}$?
33. $30\frac{3}{7}$ and $20\frac{5}{8}$?

34. A man bought 3 pieces of cloth, one of which contained $45\frac{3}{4}$ yards, another $63\frac{5}{8}$ yards, and the other $56\frac{4}{7}$ yards; how many yards did he buy?

35. If you travel $85\frac{7}{8}$ miles in one day, $95\frac{11}{12}$ miles the second, and $115\frac{5}{16}$ miles the third day, how far will you travel in all?

SUBTRACTION OF FRACTIONS.

MENTAL EXERCISES.

202. 1. What is the difference between $\frac{3}{9}$ and $\frac{7}{9}$? (Art. **197**.)

2. Find the difference between $\frac{5}{11}$ and $\frac{9}{11}$. $\frac{11}{23}$ and $\frac{7}{23}$.

3. If you have $\frac{7}{8}$ pound of candy and give away $\frac{3}{8}$, how much will you have left?

4. George had $\$\frac{9}{10}$ and gave $\$\frac{4}{10}$ for a Reader, how much money did he have left?

5. What is the difference between $\$\frac{1}{3}$ and $\$\frac{5}{6}$?

ANALYSIS.—Thirds and sixths are *unlike* parts of the unit \$1, and one cannot be taken from the other in their present form. But $\frac{1}{3}$ is equal to $\frac{2}{6}$, and $\frac{2}{6}$ from $\frac{5}{6}$ leave $\frac{3}{6}$, or $\$\frac{1}{2}$, *Ans.*

6. Florence has two pieces of ribbon, one is $\frac{5}{6}$ of a yard long and the other $\frac{3}{8}$ of a yard; what is the difference in length?

7. If a cap costs $\$\frac{7}{8}$ and a pair of mittens $\$\frac{3}{4}$, what is the difference in their price?

8. What is the difference between $\frac{5}{8}$ quart and $\frac{5}{8}$ yard?

ANALYSIS.—Quarts and yards are *unlike units,* and parts of one cannot be taken from parts of the other.

203. From the above examples we infer the following

PRINCIPLE.—*Only like fractions can be subtracted.*

9. What is the difference between $\frac{2}{3}$ and $\frac{3}{4}$?

10. What is the difference between $\frac{3}{4}$ and $\frac{5}{6}$?

11. One man owned $\frac{4}{5}$ of a ship and another $\frac{2}{7}$; what was the difference in their ownership?

12. Richard's kite-line was $18\frac{4}{5}$ yards long and he cut off $6\frac{2}{10}$ yards; how long was the part left?

ANALYSIS.—$\frac{2}{10}$ are equal to $\frac{1}{5}$, and $\frac{1}{5}$ from $\frac{4}{5}$ leaves $\frac{3}{5}$. Again, 6 yd. from 18 yd. leave 12 yd., and $\frac{3}{5}$ added make $12\frac{3}{5}$ yards, *Ans.*

Required the difference between

13.	$\frac{4}{5}$ and $\frac{3}{5}$.	**17.**	$\frac{7}{10}$ and $\frac{5}{6}$.	**21.**	$8\frac{3}{4}$ and $5\frac{1}{4}$.
14.	$\frac{3}{8}$ and $\frac{5}{6}$.	**18.**	$\frac{4}{11}$ and $\frac{10}{33}$.	**22.**	$15\frac{2}{3}$ and $18\frac{3}{4}$.
15.	$\frac{4}{7}$ and $\frac{3}{5}$.	**19.**	$\frac{7}{12}$ and $\frac{75}{18}$.	**23.**	$25\frac{5}{6}$ and $20\frac{2}{5}$.
16.	$\frac{7}{8}$ and $\frac{8}{7}$.	**20.**	$\frac{9}{15}$ and $\frac{12}{20}$.	**24.**	$34\frac{7}{8}$ and $25\frac{11}{12}$.

WRITTEN EXERCISES.

204. To find the Difference between two fractions.

1. From $\frac{5}{6}$ subtract $\frac{4}{9}$.

ANALYSIS.—Reducing the given fractions to the *l. c. d.* 18, they become $\frac{15}{18}$ and $\frac{8}{18}$, and $\frac{15}{18}-\frac{8}{18}=\frac{7}{18}$, *Ans.*

OPERATION.

$$\frac{5}{6}=\frac{15}{18};\ \frac{4}{9}=\frac{8}{18};$$
$$\frac{15}{18}-\frac{8}{18}=\frac{7}{18},\ \textit{Ans.}$$

2. From $246\frac{2}{5}$ subtract $132\frac{3}{4}$.

ANALYSIS.—Reducing the fractions $\frac{2}{5}$ and $\frac{3}{4}$ to the *l. c. d.* 20, we have $\frac{2}{5}=\frac{8}{20}$, $\frac{3}{4}=\frac{15}{20}$. Now to subtract $\frac{15}{20}$ from $\frac{8}{20}$, we take $1=(\frac{20}{20})$ from 6, and add it to $\frac{8}{20}$, making $\frac{28}{20}$, then subtract, and take 2 from 5, etc. Hence, the

OPERATION.

$$246\tfrac{2}{5}=246\tfrac{8}{20}$$
$$132\tfrac{3}{4}=132\tfrac{15}{20}$$
$$\textit{Ans.}\ 113\tfrac{13}{20}$$

RULE.—*I. Reduce the given fractions to a common denominator, and over it write the difference of the numerators.* (Art. **195.**)

II. If there are mixed numbers, subtract the fractional and integral parts separately, and unite the results. (Ex. 2.)

NOTE.—In most cases it is better to reduce the fractions to the *least common denominator.*

What is the difference between

3. $\frac{37}{42}$ and $\frac{13}{125}$.	8. $\frac{16}{21}$ and $\frac{8}{15}$.	13. 2 and $\frac{3}{5}$.
4. $\frac{9}{55}$ and $\frac{1}{15}$.	9. $\frac{2}{7}$ and $\frac{2}{11}$.	14. 65 and $25\frac{2}{3}$.
5. $\frac{27}{35}$ and $\frac{23}{48}$.	10. $\frac{14}{75}$ and $\frac{13}{25}$.	15. $21\frac{2}{5}$ and $9\frac{7}{8}$.
6. $8\frac{1}{3}$ and $5\frac{2}{3}$.	11. $12\frac{2}{3}$ and $8\frac{3}{4}$.	16. $25\frac{3}{5}$ and $17\frac{4}{5}$.
7. $12\frac{5}{8}$ and $7\frac{1}{4}$.	12. $15\frac{3}{7}$ and $9\frac{1}{2}$.	17. $37\frac{4}{7}$ and $19\frac{3}{4}$.

18. From $385\frac{1}{4}$ rods take $67\frac{1}{3}$ rods.

19. From $573\frac{4}{5}$ tons take $216\frac{5}{8}$ tons.

20. From $563\frac{7}{8}$ pounds take $260\frac{1}{2}$ pounds.

21. From $1673\frac{5}{7}$ bushels take $356\frac{3}{4}$ bushels.

22. A man bought a wagon for $\$85\frac{5}{8}$, and a sleigh for $\$69\frac{3}{4}$; how much more did he pay for one than the other?

23. A man having $246\frac{7}{16}$ acres of land, sold $195\frac{2}{3}$ acres; how many acres did he have left?

24. If from a piece of cloth containing $125\frac{19}{20}$ yards, you cut $87\frac{7}{16}$ yards, how many yards will be left?

Multiplication of Fractions.

Oral Exercises.

205. 1. What is the cost of 5 books, at $\$\frac{3}{8}$ apiece?

Analysis.—Since 1 book costs $\$\frac{3}{8}$, 5 books will cost 5 times $\$\frac{3}{8}$, which are $\$\frac{15}{8} = \$1\frac{7}{8}$, *Ans.*

2. What cost 6 bushels of apples, at $\$\frac{3}{10}$ a bushel?
3. At $\$\frac{5}{12}$ a pound, what will 10 pounds of butter come to?
4. Multiply $\frac{5}{9}$ by 8.
5. Multiply $\frac{7}{8}$ by 12.
6. How many units in 7 times $\frac{4}{7}$?
7. At $\$\frac{11}{12}$ a pound, what will 4 pounds of tea come to?

Analysis.—Dividing the denominator 12 by 4, multiplies the fraction (Art. **179**), the result is $\$\frac{11}{3} = \$3\frac{2}{3}$, *Ans.*

8. Multiply $\frac{7}{9}$ by 3.
9. " $\frac{9}{15}$ by 5.
10. " $\frac{7}{20}$ by 4.
11. Multiply $\frac{8}{21}$ by 7.
12. " $\frac{7}{30}$ by 10.
13. " $\frac{12}{30}$ by 15.

14. What will 4 yds. of braid come to, at $5\frac{2}{3}$ cts. a yard?

Analysis.—Since 1 yd. is worth $5\frac{2}{3}$ cents, 4 yards are worth 4 times $5\frac{2}{3}$ cents. Now 4 times 5 are 20 cts. and 4 times 2 thirds are 8 thirds equal to $2\frac{2}{3}$ cents, which added to 20 make $22\frac{2}{3}$ cents. Therefore, etc.

15. At $6\frac{1}{4}$ cents each, what must I pay for 8 oranges?
16. At $\$5\frac{3}{4}$ a yard, what is the cost of 7 yds. of cloth?
17. What must a lady pay for 8 yds. of silk at $\$3\frac{4}{8}$ a yard?
18. How many are 7 times $8\frac{5}{6}$?
19. Multiply $10\frac{5}{7}$ by 8.
20. What is the product of $12\frac{5}{8}$ by 9?

Written Exercises.

206. **Multiplying a Fraction by an Integer.**

A Fraction is multiplied by multiplying its numerator or by dividing its denominator. (Art. **179**, *1°.*)

1. Multiply $\frac{13}{45}$ by 9.

EXPLANATION. — Multiplying the numerator 13 by 9, the result equals $2\frac{3}{5}$.

Cancelling the factor 9, which is common to both terms, the result is the same.

Or, dividing the denominator 45 by 9, the result is $2\frac{3}{5}$, as before.

1ST OPERATION.

$$\frac{13 \times 9}{45} = \frac{117}{45} = 2\frac{3}{5}, \quad \textit{Ans.}$$

$$\text{Or, } \frac{13}{\cancel{45}_{5}} \times \cancel{9} = \frac{13}{5} = 2\frac{3}{5},$$

2D OPERATION.

$$\frac{13}{45 \div 9} = \frac{13}{5} = 2\frac{2}{3}, \quad \textit{Ans.}$$

NOTE.—In the 1st operation, the *number* of parts is *increased,* while their *size* is *unchanged.* In the 2d operation, the *size* of the parts is *increased,* while their *number* is unchanged.

2. Multiply $\frac{9}{35}$ by 14. *Ans.* $3\frac{3}{5}$.

3. $\frac{5}{27} \times 9 = ?$	**6.** $\frac{9}{75} \times 45 = ?$	**9.** $\frac{11}{105} \times 40 = ?$
4. $\frac{7}{75} \times 12 = ?$	**7.** $\frac{35}{240} \times 48 = ?$	**10.** $\frac{20}{164} \times 41 = ?$
5. $\frac{12}{63} \times 21 = ?$	**8.** $\frac{43}{129} \times 86 = ?$	**11.** $\frac{25}{270} \times 54 = ?$

12. Multiply $15\frac{3}{4}$ by 7.

$$\begin{array}{r} 15\frac{3}{4} \\ \underline{7} \\ \textit{Ans. } 110\frac{1}{4} \end{array}$$

EXPLANATION. — Multiply the fractional and integral parts of $15\frac{3}{4}$ separately, and uniting the results, we have $110\frac{1}{4}$, the product required.

NOTE.—When the multiplicand is a *mixed* number, the *fractional* and *integral* part should be multiplied separately, and the *results* be united.

13. $87\frac{3}{4} \times 8 = ?$	**15.** $205\frac{7}{8} \times 24 = ?$	**17.** $256\frac{3}{8} \times 3 = ?$
14. $165\frac{2}{3} \times 12 = ?$	**16.** $196\frac{5}{10} \times 18 = ?$	**18.** $575\frac{3}{12} \times 48 = ?$

ORAL EXERCISES.

207. When the Multiplier is a Fraction.

1. If a barrel of flour is worth \$6, what is $\frac{1}{2}$ barrel worth?

ANALYSIS.—1 half barrel is worth 1 half as much as a whole barrel, and 1 half of \$6 is \$3. Therefore, etc.

2. If a stage goes 9 miles an hour, how far will it go in $\frac{1}{3}$ of an hour?

3. What is $\frac{1}{2}$ of 14 apples? $\frac{1}{3}$ of 15 pounds? $\frac{1}{4}$ of 28 days?

4. At \$5 a yard, what will $\frac{3}{4}$ of a yard of cloth cost?

ANALYSIS.—$\frac{3}{4}$ of a yard will cost $\frac{3}{4}$ times \$5, or 3 times $\frac{1}{4}$ of \$5. Now $\frac{1}{4}$ of \$5 = \$$1\frac{1}{4}$, and 3 times \$$1\frac{1}{4}$ are \$$3\frac{3}{4}$.

5. What is $\frac{3}{4}$ of $\frac{5}{9}$?

Analysis.—$\frac{3}{4}$ of $\frac{5}{9}$ are equal to 3 times $\frac{1}{4}$ of $\frac{5}{9}$. Now $\frac{1}{4}$ of $\frac{5}{9}$ is $\frac{5}{36}$, and 3 fourths are 3 times $\frac{5}{36}$ or $\frac{15}{36} = \frac{5}{12}$, *Ans.*

6. At $\$\frac{5}{6}$ a pound, what will $\frac{2}{3}$ pound of tea cost?
7. What costs $\frac{5}{8}$ of a box of lemons, at \$6 a box?
8. At \$8 a barrel, what will $\frac{7}{9}$ of a barrel of flour cost?

9. $\frac{5}{8}$ of 12 = ?	12. $\frac{3}{8}$ of 42 feet = ?	15. $\frac{5}{8}$ of 60 = ?
10. $\frac{6}{7}$ of 13 = ?	13. $\frac{5}{9}$ of 40 yds. = ?	16. $\frac{7}{12}$ of 72 = ?
11. $\frac{4}{9}$ of 16 = ?	14. $\frac{6}{11}$ of 25 lbs. = ?	17. $\frac{5}{20}$ of 200 = ?

18. At 8 cts. a yd. what will be the cost of $5\frac{3}{4}$ yds. of muslin?

Analysis.—$5\frac{3}{4}$ yds. will cost $5\frac{3}{4}$ times 8 cts. Now 5 times 8 cts. are 40 cts.; 1 fourth of 8 cts. is 2 cts. and 3 fourths are 3 times 2 cts. or 6 cts. which added to 40 cts. make 46 cents. Therefore, etc.

19. At 7 cts. a pound, what will $5\frac{1}{2}$ pounds of sal soda cost?
20. How many are $8\frac{2}{3}$ times 9?
21. How many are $7\frac{5}{6}$ times 12?
22. At 6 shillings a pound, what cost $5\frac{2}{3}$ pounds of tea.
23. What cost $7\frac{2}{5}$ acres of land, at \$10 per acre?

Written Exercises.

208. *Multiplying by a fraction is taking a certain* **part** *of the multiplicand as many times, as there are like parts of a unit in the multiplier.* Thus,

Multiplying by $\frac{1}{2}$, is taking 1 *half* of the multiplicand *once.*

Multiplying by $\frac{1}{3}$, is taking 1 *third* of the multiplicand *once.*

Multiplying by $\frac{2}{3}$, is taking 1 *third* of the multiplicand *twice.*

Note.—1. To find a *half* of a number, *divide it by* 2. To find a *third* of a number, *divide it by* 3. To find a *fourth* of a number, *divide it by* 4, etc.

1. Multiply 63 by $\frac{4}{7}$.

Analysis.—Multiplying 63 by $\frac{4}{7}$, is finding $\frac{4}{7}$ of 63. Now $\frac{4}{7}$ of 63 = 4 times $\frac{1}{7}$ of 63; $\frac{1}{7}$ of 63 is 9 and $\frac{4}{7}$ is 4 times 9 = 36.

$$\frac{63 \times 4}{7} = \frac{252}{7} = 36, \text{ Ans.}$$

$$\text{Or, } \frac{\overset{9}{\cancel{63}} \times 4}{\cancel{7}} = 36, \text{ Ans.}$$

Or, cancelling the factor 7, common to both terms, we have $9 \times 4 = 36$.

NOTE.—2. A fraction is multiplied by a number equal to its *denominator* by *cancelling its denominator.* (Arts. **123**, *1°*; **179**, *1°*.)

In like manner a fraction is multiplied by any *factor* of its denominator by *cancelling that factor.*

Find the product of

2. 60 by $\frac{2}{18}$.
3. 63 by $\frac{5}{21}$.
4. 70 by $\frac{9}{14}$.
5. 112 by $\frac{5}{32}$.
6. 168 by $\frac{15}{42}$.
7. 105 by $\frac{17}{7}$.
8. 39 by $\frac{12}{13}$.
9. 896 by $\frac{27}{56}$.
10. 572 by $\frac{10}{48}$.

11. Multiply 160 by $5\frac{3}{4}$.

NOTE. — 3. When the multiplier is a *mixed* number, multiply by the fractional and integral parts separately and *unite* the results.

OPERATION.

$$160 \times \tfrac{3}{4} = 120$$
$$160 \times 5 = 800$$

Ans. 920

Multiply the following:

12. 93 by $12\frac{2}{3}$.
13. 184 by $18\frac{3}{4}$.
14. 125 by $10\frac{6}{25}$.
15. 268 by $12\frac{19}{26}$.
16. 256 by $17\frac{9}{16}$.
17. 196 by $41\frac{11}{28}$.
18. 341 by $30\frac{7}{12}$.
19. 457 by $12\frac{21}{75}$.
20. 107 by $47\frac{21}{39}$.
21. 510 by $85\frac{13}{65}$.
22. 834 by $89\frac{1}{19}$.
23. 963 by $95\frac{31}{93}$.

ORAL EXERCISES.

209. **1.** If I cut $\frac{1}{2}$ sheet of paper into 2 equal parts, what part of 1 sheet will there be in each piece?

Ans. $\frac{1}{2}$ of a *half* sheet, which is equal to $\frac{1}{4}$ sheet.

2. If a bushel of apples costs $\$\frac{1}{2}$, what will $\frac{1}{3}$ bushel cost?

ANALYSIS.—$\frac{1}{3}$ bushel will cost $\frac{1}{3}$ as much as a whole bushel; and $\frac{1}{3}$ of $\$\frac{1}{2}$ is $\$\frac{1}{6}$. (Art. **179**, *2°*.)

3. What part of 1 is $\frac{1}{4}$ of $\frac{1}{3}$? $\frac{1}{3}$ of $\frac{1}{5}$? $\frac{1}{5}$ of $\frac{1}{8}$?

4. Which is greater $\frac{1}{4}$ or $\frac{1}{9}$? $\frac{1}{11}$ or $\frac{1}{13}$? $\frac{2}{8}$ or $\frac{2}{10}$?

5. What is $\frac{2}{3}$ of $\frac{5}{6}$? $\frac{3}{4}$ of $\frac{5}{8}$? $\frac{4}{5}$ of $\frac{7}{12}$? $\frac{5}{7}$ of $\frac{4}{20}$?

6. If a pound of tea is worth $\$\frac{4}{5}$, what is $\frac{3}{4}$ pound worth?

7. What cost $\frac{7}{8}$ yard of ribbon, at $\$\frac{4}{5}$ a yard?

8. If a yard of cashmere is worth $\$\frac{3}{4}$, what is $\frac{2}{9}$ yd. worth?

9. A man owning $\frac{4}{5}$ of a yacht, sold $\frac{3}{4}$ of it to his neighbor; what part of the yacht did each then own?

WRITTEN EXERCISES.

210. Multiplying a Fraction by a Fraction.

1. At $\$\frac{8}{9}$ a yard, what will $\frac{3}{4}$ yd. of silk cost?

EXPLANATION. — One-fourth of a yard will cost $\frac{1}{4}$ as much as 1 yard, and $\frac{1}{4}$ of $\$\frac{8}{9} = \$\frac{8}{36}$, and 3 fourths yard will cost 3 times $\$\frac{8}{36}$, or $\$\frac{24}{36} = \$\frac{2}{3}$, *Ans.*

OPERATION.

$$\$\frac{8}{9} \times \frac{3}{4} = \frac{24}{36} \text{ or } \$\frac{2}{3}.$$

$$\text{Or, } \frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{9}}} \times \frac{\cancel{3}}{\cancel{4}} = \$\frac{2}{3}, \text{ Ans.}$$

Or, indicating the operation, and cancelling the factors common to the terms of the fractions, we have $\$\frac{2}{3}$ the same as before.

NOTE.—The above solution is the same in effect as multiplying the numerators together for the *numerator*, and the denominators for the *denominator* of the required product.

2. Multiply $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{5}{6}$ by $\frac{1}{2}$ of $\frac{4}{10}$.

EXPLANATION.—The product of the numerators $\frac{3}{4}$, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{1}{2}$, $\frac{4}{10}$, is 120; the product of the denominators is 1440; and $\frac{120}{1440} = \frac{1}{12}$, *Ans.*

Or, cancelling the factors common to the numerators and denominators, the result is $\frac{1}{12}$, the *Ans.* required.

$$\frac{3}{4} \times \frac{2}{3} \times \frac{5}{6} \times \frac{1}{2} \times \frac{4}{10} =$$

$$\frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{5}}{6} \times \frac{1}{2} \times \frac{\cancel{4}}{\cancel{10}} = \frac{1}{12}$$

211. The word *of*, in Compound Fractions, has the force of the *sign* of multiplication $\times$. *Multiplying* compound fractions together reduces them to a *simple fraction.*

Thus, $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ is a compound fraction, and is equivalent to $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$, which is equal to $\frac{24}{60}$, a simple fraction.

3. Multiply $\frac{2}{3}$ of $\frac{3}{7}$ by $\frac{5}{6}$ of $\frac{7}{8}$ of 14. *Ans.* $\frac{35}{12} = 2\frac{11}{12}$.

4. At $\$6\frac{3}{4}$ a barrel, what are $5\frac{1}{3}$ barrels of flour worth?

NOTE.—If either factor is a *mixed* or *whole* number it may be reduced to an *improper fraction*, and the operation becomes the same as multiplying a fraction by a fraction.

OPERATION.

$$6\frac{3}{4} = \frac{27}{4} \text{ and } 5\frac{1}{3} = \frac{16}{3}$$

$$\frac{27}{4} \times \frac{16}{3} = \frac{432}{12} = \$36, \text{ Ans.}$$

$$\text{Or, } \frac{\overset{9}{\cancel{27}}}{\cancel{4}} \times \frac{\overset{4}{\cancel{16}}}{\cancel{3}} = \$36, \text{ Ans.}$$

5. $\frac{8}{13} \times \frac{13}{24} = ?$

6. $\frac{51}{77} \times \frac{11}{17} = ?$

7. $\frac{9}{11} \times \frac{21}{45} = ?$

8. $\frac{5}{7} \times \frac{3}{5} \times \frac{2}{3} = ?$

9. $\frac{3}{7}$ of $\frac{5}{8}$ of $\frac{2}{10} = ?$

10. $\frac{45}{50}$ of $\frac{10}{42}$ of $\frac{12}{30} = ?$

11. $\frac{3}{7}$ of $25 \times \frac{2}{3}$ of $\frac{3}{4} = ?$

12. $\frac{5}{8}$ of $30 \times \frac{10}{12}$ of $\frac{3}{4} = ?$

13. How many are $\frac{8}{10}$ of $45 \times \frac{11}{15}$ of $\frac{4}{5}$?

14. If a quart of chestnuts costs $\frac{1}{3}$ of $\frac{3}{4}$ of 40 cents what will $\frac{7}{8}$ of $\frac{14}{16}$ of a quart cost?

15. What cost $15\frac{1}{3}$ tons of coal, at $\$6\frac{3}{4}$ a ton?

212. The preceding principles may be summed up in the following

General Rule.

Reduce whole and mixed numbers to improper fractions, compound fractions to simple ones, and cancelling the common factors, write the product of the numerators over the product of the denominators.

Applications.

213. 1. At $\$\frac{5}{8}$ a cord, how much will the sawing of $20\frac{1}{4}$ cords of wood amount to?

2. What cost 16 pounds of cheese, at $8\frac{1}{2}$ cents a pound?
3. What cost 9 dozen of eggs, at $12\frac{1}{2}$ cents per dozen?
4. What cost $15\frac{2}{3}$ yards of cambric, at 15 cents per yard?
5. What cost $11\frac{1}{9}$ cords of wood, at $\$3\frac{1}{2}$ per cord?
6. At $12\frac{1}{2}$ cents a pound, what cost $2\frac{2}{3}$ pounds of pepper?
7. What cost 18 ounces of nutmegs, at $16\frac{1}{4}$ cts. an ounce?
8. At $12\frac{3}{4}$ cents a yard, what will 27 yards of cotton cost?
9. At $\$3\frac{11}{15}$ a yard, what cost $15\frac{1}{2}$ yards of broadcloth?
10. What cost $15\frac{3}{4}$ yards of ribbon, at 40 cents per yard?
11. What cost 22 penknives, at $\$\frac{12}{16}$ apiece?
12. At $\$\frac{5}{10}$ a yard, what cost $8\frac{3}{4}$ yards of silk?
13. At $\$\frac{3}{8}$ a yard, what will $9\frac{7}{9}$ yards of muslin cost?
14. At $\$\frac{3}{4}$ a bushel, what cost $7\frac{9}{10}$ bushels of wheat?
15. What will $8\frac{6}{7}$ pounds of tea cost, at $\$\frac{5}{8}$ a pound?
16. What cost 66 bushels of apples, at $18\frac{3}{4}$ cents a bushel?
17. At $32\frac{1}{2}$ cents a yard, what cost $12\frac{1}{2}$ yards of gingham?
18. What cost $18\frac{1}{2}$ yards of lace, at $16\frac{1}{4}$ cents per yard?
19. What cost 43 bushels of oats, at $18\frac{3}{4}$ cents a bushel?
20. What cost $31\frac{1}{2}$ yards of sheeting, at $\$\frac{2}{9}$ per yard?
21. At $\$\frac{7}{12}$ a quart, what cost $18\frac{1}{2}$ quarts of cherries?
22. What cost $14\frac{3}{5}$ bushels of potatoes, at $18\frac{3}{4}$ cents a bushel?
23. At $\$\frac{3}{8}$ a yard, what cost $8\frac{3}{8}$ yards of velvet?
24. At $\$\frac{7}{8}$ a bushel, what costs $47\frac{4}{5}$ bushels of pears?

25. What cost $63\frac{3}{4}$ pounds of sugar, at $9\frac{3}{4}$ cents per pound?
26. What cost $22\frac{3}{4}$ yards of velvet, at $\$3\frac{2}{3}$ a yard?
27. What cost $25\frac{1}{4}$ pounds of figs, at $15\frac{1}{2}$ cents a pound?
28. What cost $35\frac{3}{5}$ cords of wood, at $\$3\frac{3}{4}$ per cord?
29. What cost $175\frac{1}{2}$ bushels of corn, at $\$\frac{3}{8}$ a bushel?
30. What cost $38\frac{3}{4}$ tons of hay, at $\$15\frac{7}{8}$ a ton?
31. At $42\frac{1}{2}$ miles a day, how far can you travel in $17\frac{1}{2}$ days?

32. Mult. 126 by $\frac{5}{9}$ of 33.
33. Mult. $\frac{6}{7}$ of 9 by $\frac{3}{5}$ of 7.
34. Mult. $\frac{5}{9}$ of $18\frac{1}{7}$ by $\frac{4}{5}$ of $24\frac{1}{2}$.
35. Mult. $217\frac{1}{8}$ by $\frac{2}{3}$ of $\frac{3}{4}$ of 8.
36. Mult. $\frac{356}{943}$ by $\frac{5}{7}$ of $\frac{119}{428}$.
37. Mult. $\frac{786}{967}$ by $\frac{3}{10}$ of $\frac{245}{393}$.
38. Mult. $\frac{46}{75}$ by $14\frac{1}{2}$.
39. Mult. $\frac{5}{9}$ of $\frac{3}{4}$ by $\frac{2}{3}$ of $\frac{4}{5}$.
40. Mult. $16\frac{2}{3}$ by $\frac{4}{5}$ of 6.
41. Mult. $468\frac{5}{11}$ by $\frac{3}{4}$ of $\frac{21}{47}$.

42. Multiply $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{7}{11}$ of $\frac{19}{20}$ of 11 by $\frac{7}{8}$ of $\frac{5}{6}$ of 45.
43. Multiply $\frac{5}{7}$ of $\frac{6}{13}$ of $\frac{26}{35}$ of $\frac{1}{2}$ of 29 by $\frac{11}{15}$ of $\frac{6}{10}$ of $\frac{3}{11}$.
44. Multiply $\frac{7}{8}$ of $\frac{14}{42}$ of $\frac{8}{56}$ of $16\frac{1}{2}$ by $\frac{21}{33}$ of $\frac{28}{35}$ of $\frac{3}{8}$ of 49.

Division of Fractions.

Oral Exercises.

214. 1. If 2 melons cost $\$\frac{4}{5}$, what will 1 melon cost?

Analysis.—1 melon is $\frac{1}{2}$ of 2 melons; therefore, 1 melon will cost $\frac{1}{2}$ of $\$\frac{4}{5}$, and $\frac{1}{2}$ of $\$\frac{4}{5}$ is $\$\frac{2}{5}$, *Ans.*

2. If 3 knives are worth $\$\frac{9}{10}$, what is 1 knife worth?
3. If I pay $\$\frac{8}{12}$ for 4 slates, what do I pay for 1 slate?
4. If 2 pears cost $\frac{2}{3}$ of a dime, how much will 1 pear cost?
5. If $\frac{3}{4}$ of a yard of cloth is divided into 3 equal pieces, what part of a yard will 1 piece contain?
6. By what do you divide to find 1-half a number? To find 1-third? 1-fourth? 1-fifth?
7. How do you multiply by $\frac{1}{2}$? By $\frac{1}{3}$? By $\frac{2}{3}$?
8. What is the difference between multiplying by $\frac{1}{2}$ and dividing a number by 2? Between multiplying by $\frac{1}{3}$ and dividing by 3?
9. If 5 fans are worth $\$\frac{10}{12}$, what is 1 fan worth?
10. If 4 melons cost $\$\frac{8}{12}$, what will 1 melon cost?

11. If 5 apples are worth $\frac{3}{4}$ dime, what is 1 apple worth?

ANALYSIS.—A Fraction is divided by dividing its numerator or multiplying its denominator. Since the numerator 3 cannot be divided by 5 without a remainder, we multiply the denominator 4 by it, and 5 times 4 are 20. Therefore, 1 apple is worth $\frac{3}{20}$ dime, *Ans.* (Art. **179**, 2°.)

12. A grocer divided $\frac{2}{3}$ of a cocoanut among 6 boys; what part of a cocoanut did each receive?

13. Paid $\$\frac{3}{4}$ for 5 Table-books; what was the price of each?

14. How many ways can you divide a fraction?

15. Divide $\frac{6}{7}$ by 3. $\frac{8}{11}$ by 4. $\frac{12}{15}$ by 6. $\frac{18}{23}$ by 9.

16. Divide $\frac{5}{6}$ by 3. $\frac{7}{8}$ by 4. $\frac{8}{10}$ by 6. $\frac{11}{12}$ by 5. $\frac{18}{7}$ by 11.

17. What is the quotient of $\frac{27}{8} \div 9$? Of $\frac{31}{10} \div 8$?

18. What is the quotient of $\frac{36}{7}$ divided by 9? $\frac{42}{8} \div 11$?

19. A man had $\frac{45}{2}$ of a dollar, and gave all for 9 hats; how much did each hat cost him?

20. At \$4 a bushel, how many bushels of quinces can be had for $\$10\frac{2}{3}$?

ANALYSIS.—As many bushels, at \$4, may be had, as \$4 are contained times in $\$10\frac{2}{3}$. Now $\$10\frac{2}{3} = \$\frac{32}{3}$, and $\frac{32}{3} \div 4 = \frac{8}{3}$, or $2\frac{2}{3}$ bushels, *Ans.*

21. Divide $6\frac{2}{3}$ by 4. $8\frac{1}{3}$ by 5. $7\frac{3}{4}$ by 8. $11\frac{2}{5}$ by 9.

22. If $8\frac{3}{4}$ pounds of candy are divided equally among 5 children, what part and how much will each receive?

WRITTEN EXERCISES.

215. Dividing a Fraction by an Integer.

1. If 4 yds. of muslin cost $\$\frac{8}{12}$, what will 1 yd. cost?

FIRST.—Dividing the numerator by 4, we have $\$\frac{2}{12} = \$\frac{1}{6}$. (Art. **179**, 2°.)

$$\frac{8 \div 4}{12} = \$\frac{2}{12} = \$\frac{1}{6}, \textit{ Ans.}$$

SECOND.—Multiplying the denominator by 4, we have $\frac{8}{12 \times 4} = \frac{8}{48}$, or $\$\frac{1}{6}$, *Ans.*

$$\frac{8}{12 \times 4} = \frac{8}{48} = \$\frac{1}{6};$$

Or, cancelling the factors common to both terms, we have $\$\frac{1}{6}$, as before.

$$\frac{\overset{2}{\cancel{8}}}{\underset{6}{\cancel{12}} \times \cancel{4}} = \$\frac{1}{6}, \textit{ Ans.}$$

NOTES.—1. It is better to divide the numerator when it can be done without a remainder.

2. By the *first* operation the *number* of parts is *diminished,* but their *size remains* the same. By the *second* operation the *number* of parts *remains* the same, but their *size* is *diminished.*

2. Divide $\frac{17}{8}$ by 9.
3. $\frac{48}{31} \div 8 = ?$
4. $\frac{73}{63} \div 12 = ?$
5. $\frac{66}{74} \div 22 = ?$
6. $2\frac{19}{45} \div 70 = ?$
7. $3\frac{21}{50} \div 37 = ?$
8. $\frac{175}{94} \div 25 = ?$
9. $\frac{348}{100} \div 93 = ?$
10. $\frac{478}{108} \div 120 = ?$
11. $\frac{573}{63} \div 114 = ?$
12. $\frac{1024}{1200} \div 75 = ?$
13. $\frac{1728}{273} \div 144 = ?$

14. At \$7 a barrel, how many barrels of cranberries can be bought for \$$25\frac{2}{3}$?

NOTE.—When the *dividend* is a mixed number, it should be reduced to an *improper fraction;* then proceed as above. *Ans.* \$$3\frac{2}{3}$.

OPERATION.

\$$25\frac{2}{3}$ = \$$\frac{77}{3}$

\$$\frac{77}{3} \div 7$ = \$$\frac{11}{3}$ = \$$3\frac{2}{3}$.

15. If $2\frac{12}{14}$ of a ton of hay were fed to 6 horses, what part and how much would each receive?

16. Paid \$$7\frac{8}{10}$ for 12 books; what was the price of each?

ORAL EXERCISES.

216. **1.** How long will it take a lad to earn \$5, if he earns \$$\frac{2}{3}$ a day?

ANALYSIS.—At \$$\frac{2}{3}$ a day, it will take as many days as $\frac{2}{3}$ are contained times in 5. In 1 there are 3 thirds, and in 5, 5 times 3, or 15 thirds. Now 2 thirds are contained in 15 thirds, $7\frac{1}{2}$ times. *Ans.* $7\frac{1}{2}$ days.

2. How many times is $\frac{1}{3}$ contained in 4? In 5? In 9?

3. How many times $\frac{1}{4}$ in 5? $\frac{1}{5}$ in 7? $\frac{1}{6}$ in 8? $\frac{1}{8}$ in 9?

4. If you earn \$$\frac{3}{4}$ in 1 day, how long will it take you to earn \$12?

5. How many times $\frac{2}{3}$ in 7? In 8? In 10?

6. At \$$\frac{3}{4}$ apiece, how many books can you buy for \$12?

7. If a boy saws $\frac{4}{5}$ of a cord of wood in 1 day, how long will it take him to saw 8 cords?

8. How many times are $\frac{3}{8}$ contained in 5? In 6? In 10?

9. At \$$\frac{5}{6}$ a bushel, how much corn can you buy for \$10?

10. If I burn $\frac{3}{7}$ ton of coal in 1 day, how long will 6 tons last me?

11. How many times are $\frac{2}{3}$ contained in $\frac{3}{4}$ of 16?

12. How many times $\frac{4}{5}$ in $\frac{3}{8}$ of 32? In $\frac{5}{8}$ of 40?

13. If $2\frac{1}{2}$ yards of cloth will make a coat, how many coats can be made from 20 yds. of cloth?

ANALYSIS.—In $2\frac{1}{2}$ yards there are 5 half-yards, and in 20 yds. there are 40 half-yards. Now 5 is contained in 40, 8 times. *Ans.* 8 coats.

14. How much wood at $\$3\frac{1}{4}$ a cord can be had for \$26?

15. How many barrels of potatoes at $\$2\frac{3}{4}$ can you buy for \$22?

16. At $\$6\frac{2}{5}$ a week, how long can a man board for \$100?

WRITTEN EXERCISES.

217. Dividing an Integer by a Fraction.

1. How many times are $\frac{3}{4}$ contained in 21?

EXPLANATION.—Reducing 21 to fourths we have $21 = \frac{84}{4}$. Now $\frac{3}{4}$ and $\frac{84}{4}$ are like fractions, and one numerator is divided by the other like integers.

Or, Multiply the integer by the fraction *inverted*.

OPERATION.

$$21 \times 4 = 84$$
$$\frac{84}{4} \div \frac{3}{4} = 28, \text{ Ans.}$$
$$\text{Or, } 21 \times \frac{4}{3} = 28$$

2. 56 by $\frac{8}{42}$.	**5.** 240 by $\frac{8}{45}$.	**8.** 384 by $\frac{12}{42}$.
3. 72 by $\frac{9}{54}$.	**6.** 256 by $\frac{16}{3}$.	**9.** 576 by $\frac{48}{67}$.
4. 132 by $\frac{11}{48}$.	**7.** 110 by $\frac{45}{7}$.	**10.** 1880 by $\frac{8}{47}$.

11. At $\$\frac{4}{5}$ a yard, how many yards of silk can be had for \$37?

12. If you pay $\$\frac{7}{8}$ a day for board, how many days can you board for \$126?

13. How many cloaks can be made from 72 yds. of cloth, allowing $4\frac{1}{2}$ yds. for a cloak?

NOTE.—When the divisor is a *mixed* number, it should be reduced to an improper fraction before dividing; then multiply the integer by the fraction inverted. (Art. **217**, Ex. 1.)

OPERATION.

$$4\frac{1}{2} = \frac{9}{2}$$
$$72 \div \frac{9}{2} = 16$$
$$\text{Or, } 72 \times \frac{2}{9} = 16, \text{ Ans.}$$

14. $120 \div 12\frac{1}{2} = ?$	**16.** $240 \div \frac{7}{12} = ?$	**18.** $785 \div 62\frac{1}{2} = ?$
15. $192 \div 10\frac{2}{3} = ?$	**17.** $552 \div \frac{25}{37} = ?$	**19.** $2000 \div 87\frac{1}{2} = ?$

20. At $\$3\frac{1}{8}$ apiece, how many sheep can be had for \$1500?

21. How many yards of silk, at $\$3\frac{1}{4}$ can be had for \$185 ?

22. Allowing $4\frac{1}{2}$ yards of cloth for a cloak, how many cloaks can be made from 154 yards ?

23. At $\$4\frac{3}{4}$ each, how many chairs can be bought for \$250 and what remainder.

24. If a stage coach travels at the rate of $10\frac{5}{8}$ miles per hour, how long will it be in going 320 miles ?

Oral Exercises.

218. **1.** How many slates at $\$\frac{4}{15}$ can be bought for $\$\frac{12}{15}$?

ANALYSIS.—Since these fractions express *like parts* of *like units*, it is plain that as many slates can be bought as $\frac{4}{15}$ are contained times in $\frac{12}{15}$, or 3. *Ans.* 3 slates. (Art. **197.**)

2. If a vest can be made from $\frac{2}{3}$ yd. of velvet, how many vests can be made from $\frac{10}{3}$ yards?

3. How many times are $\frac{3}{4}$ contained in $\frac{18}{4}$? In $\frac{21}{4}$? In $\frac{27}{4}$?

4. Divide $\frac{8}{10}$ by $\frac{4}{10}$. $\frac{9}{20}$ by $\frac{3}{20}$. $\frac{18}{23}$ by $\frac{6}{23}$. $\frac{36}{40}$ by $\frac{9}{40}$.

5. If pen-knives are $\$\frac{3}{8}$ apiece, how many can you buy for $\$\frac{15}{8}$?

6. How many melons at $\$\frac{3}{8}$ apiece can a person buy for $\$\frac{3}{4}$?

ANALYSIS.—He can buy as many as $\frac{3}{8}$ are contained times in $\frac{3}{4}$. Now $\frac{3}{4} = \frac{6}{8}$, and $\frac{3}{8}$ are contained in $\frac{6}{8}$, 2 times. *Ans.* 2 melons.

7. How many books, at $\$\frac{3}{4}$, can be bought with $\$\frac{6}{8}$?

8. At $\$\frac{2}{3}$ a yard, how much fringe will $\$\frac{5}{6}$ buy?

9. At $\$\frac{2}{5}$ a pound, how many pounds of spice can be had for $\$\frac{4}{10}$? For $\$\frac{12}{10}$?

10. Divide $\frac{6}{9}$ by $\frac{3}{18}$. $\frac{8}{14}$ by $\frac{2}{7}$. $\frac{10}{13}$ by $\frac{2}{30}$. $\frac{12}{20}$ by $\frac{4}{5}$.

11. At $\$\frac{3}{4}$ a yard, how many yards of flannel can you buy for $\$\frac{11}{12}$?

12. How many pounds of tea at $\$\frac{4}{5}$, can be bought for $\$\frac{19}{20}$?

13. At $\$\frac{1}{6}$, how many yards of calico can be had for $\$\frac{9}{10}$?

14. If cinnamon is $\$\frac{2}{5}$ a pound, how much can be bought for $\$\frac{14}{20}$?

15. How much coffee can be bought for $\$\frac{21}{24}$ when the price is $\$\frac{3}{8}$ a pound ?

Written Exercises.

219. Dividing a Fraction by a Fraction.

1. How much tea, at $\$\frac{2}{3}$ a pound, can be had for $\$\frac{3}{4}$?

1st Method.—Reducing the given fractions to a *c. d.*, $\frac{2}{3} = \frac{8}{12}$, and $\frac{3}{4} = \frac{9}{12}$. Now if $\$\frac{8}{12}$ will buy 1 pound, $\$\frac{9}{12}$ will buy as many pounds as $\frac{8}{12}$ are contained times in $\frac{9}{12}$, and $9 \div 8 = 1\frac{1}{8}$. *Ans.* $1\frac{1}{8}$ pounds. (Art. **191.**)

1st operation.

$$\frac{2}{3} = \frac{8}{12};\quad \frac{3}{4} = \frac{9}{12}$$
$$\frac{9}{12} \div \frac{8}{12} = 9 \div 8$$
$$9 \div 8 = 1\tfrac{1}{8}\text{ lb., } \textit{Ans.}$$

2d Method.—The above process may be shortened by *inverting* the *divisor* and multiplying the two fractions together as in the margin. (Art. **210.**)

2d operation.

$$\$\tfrac{3}{4} \div \$\tfrac{2}{3} = \tfrac{3}{4} \times \tfrac{3}{2}$$
$$\tfrac{3}{4} \times \tfrac{3}{2} = \tfrac{9}{8}\text{, or } 1\tfrac{1}{8}\text{ lb.}$$

Note.—1. It will be seen by inspection that the 2d method in effect reduces the fraction to a *c. d.* and divides one numerator by the other at the same time, the numerators only being used, as in 1st method.

2. Divide $10\frac{1}{8}$ by $6\frac{3}{4}$.

Solution. — Reducing the mixed numbers, to improper fractions and dividing, the result is $1\frac{1}{2}$.

operation.

$$10\tfrac{1}{8} = \tfrac{81}{8};\quad 6\tfrac{3}{4} = \tfrac{27}{4}$$
$$\tfrac{81}{8} \div \tfrac{27}{4} = \tfrac{81}{8} \times \tfrac{4}{27} = 1\tfrac{1}{2},\ \textit{Ans.}$$

3. Divide $\frac{3}{4}$ of $\frac{4}{9}$ by $\frac{3}{7}$ of $\frac{7}{12}$.

Solution.—$\frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{\cancel{9}_{3}} \times \frac{\cancel{7}}{\cancel{3}} \times \frac{\cancel{12}^{4}}{\cancel{7}} = \frac{4}{3}$, or $1\frac{1}{3}$, *Ans.*

220. The preceding principles may be summed up in the following

General Rule.

Reduce whole and mixed numbers to improper fractions, and multiply the dividend by the divisor inverted.

Or, Reduce the fractions to a common denominator and divide the numerator of the dividend by that of the divisor.

Note.—The object of inverting the divisor is *convenience* in multiplying. After inverting the divisor, cancel the common factors.

4. Divide $\frac{7}{12}$ by $\frac{1}{8}$.
5. Divide 75 by $3\frac{3}{4}$.
6. Divide $81\frac{5}{8}$ by $45\frac{2}{3}$.
7. Divide $\frac{8}{15}$ of $\frac{5}{9}$ by 30.

APPLICATIONS.

221. 1. At $16\frac{1}{4}$ cents per pound, how many pounds of figs can you buy for $87\frac{1}{2}$ cents?

2. How many cords of wood, at $\$6\frac{1}{2}$ per cord, will it take to pay a debt of $\$67\frac{1}{2}$?

3. How many barrels of pork, at $\$11\frac{3}{5}$ per barrel, can be obtained for $\$95\frac{1}{2}$?

4. A man bought $15\frac{1}{2}$ barrels of beef for $\$124\frac{5}{8}$; how much did he give per barrel?

5. A man bought $13\frac{1}{2}$ pounds of sugar for $94\frac{1}{2}$ cents; how much did his sugar cost him a pound?

6. A lady bought $15\frac{3}{4}$ yards of silk for $145\frac{5}{12}$ shillings; how much did she pay per yard?

7. Bought $15\frac{1}{8}$ baskets of peaches for $\$24\frac{1}{4}$; how much was the cost per basket?

8. Bought $30\frac{1}{4}$ yards of broadcloth for $\$181\frac{1}{2}$; what was the price per yard?

9. Paid $375 for $125\frac{1}{5}$ pounds of indigo; what was the cost per pound?

10. How many tons of hay, at $\$16\frac{1}{2}$ per ton, can be bought for $\$196\frac{1}{8}$?

11. How many sacks of wool, at $\$17\frac{1}{8}$ per sack, can be purchased for $1500?

12. How many bales of cotton, at $\$15\frac{7}{8}$ per bale, can be bought for $2500?

13. Divide $\frac{3}{5}$ of $\frac{8}{15}$ by $6\frac{1}{2}$.
14. Divide $\frac{9}{10}$ of 30 by 19.
15. Divide $\frac{8}{11}$ of $\frac{19}{20}$ by 31.
16. Divide $4\frac{4}{5}$ by $\frac{2}{3}$ of 12.
17. Divide $\frac{12}{23}$ by $18\frac{1}{2}$.
18. Divide $42\frac{1}{4}$ by $\frac{4}{5}$ of 7.
19. Divide $\frac{4}{7}$ of 16 by $\frac{2}{3}$ of $\frac{4}{5}$.
20. Divide $\frac{3}{7}$ of $\frac{2}{9}$ by 21.
21. Divide $\frac{4}{15}$ of $\frac{2}{21}$ by $\frac{1}{2}$ of [illegible]
22. Divide $223\frac{7}{8}$ by $\frac{3}{4}$ of 51.
23. Divide $\frac{2}{3}$ of $\frac{3}{5}$ by 48.
24. Divide $42\frac{1}{4}$ by $\frac{4}{5}$ of $53\frac{1}{3}$.

25. Divide $\frac{7}{8}$ of $\frac{3}{4}$ of $\frac{8}{14}$ of $\frac{2}{5}$ of $\frac{19}{20}$ by $\frac{3}{5}$ of $\frac{13}{20}$.
26. Divide $\frac{5}{12}$ of $\frac{8}{9}$ of $\frac{12}{15}$ of $\frac{9}{10}$ by $\frac{14}{25}$ of $\frac{15}{11}$ of 18.
27. Divide $\frac{13}{25}$ of $\frac{16}{43}$ of 67 by $\frac{31}{43}$ of $\frac{67}{95}$ of 25.
28. Divide $\frac{23}{24}$ of $\frac{31}{60}$ of $41\frac{1}{2}$ by $\frac{32}{60}$ of $\frac{31}{24}$ of 31.
29. Divide $\frac{47}{65}$ of $\frac{23}{47}$ of $\frac{84}{91}$ of $82\frac{4}{5}$ by $\frac{69}{91}$ of $\frac{84}{23}$ of $42\frac{3}{5}$.

222. To Reduce Complex Fractions to Simple Ones.

Expressions which have a Fraction in the numerator or denominator or in both, are called **Complex Fractions.**

Thus, $\frac{1\frac{1}{2}}{3}$; $\frac{4}{3\frac{1}{2}}$; $\frac{2\frac{1}{2}}{5\frac{3}{8}}$; $\frac{\frac{2}{3}}{\frac{6}{7}}$, are complex fractions, and are a form of indicating Division of Fractions.

1. What is the value of $\frac{3\frac{1}{3}}{9\frac{3}{5}}$.

OPERATION.

$3\frac{1}{3} = \frac{10}{3}$
$9\frac{3}{5} = \frac{48}{5}$
$\frac{10}{3} \div \frac{48}{5} = \frac{50}{144}$
$\frac{50}{144} = \frac{25}{72}$, *Ans.*

ANALYSIS.—Reducing the mixed numbers to improper fractions, we divide the numerator by the denominator according to the rule. (Art. **220**.) The result is $\frac{50}{144} = \frac{25}{72}$, *Ans.* Hence, the

RULE.—*Treat the numerator as a dividend and the denominator as a divisor, and divide one by the other according to the rule for division of fractions.*

2. Reduce $\frac{2\frac{1}{3}}{5\frac{3}{4}}$ to a simple fraction. *Ans.* $\frac{28}{69}$.

Reduce the following to their simplest form:

3. $\frac{6}{3\frac{1}{2}}$.	6. $\frac{12\frac{1}{4}}{6\frac{1}{9}}$.	9. $\frac{9\frac{1}{2}}{7\frac{1}{8}}$.	12. $\frac{251}{\frac{4}{5}}$.
4. $\frac{\frac{3}{5}}{\frac{2}{3}}$.	7. $\frac{5\frac{1}{2}}{6}$.	10. $\frac{18\frac{1}{3}}{12\frac{1}{2}}$.	13. $\frac{\frac{23}{27}}{\frac{31}{35}}$.
5. $\frac{8}{5\frac{1}{2}}$.	8. $\frac{4\frac{5}{9}}{6}$.	11. $\frac{20\frac{3}{8}}{25\frac{3}{4}}$.	14. $\frac{\frac{25}{35}}{476}$.

NOTE.—Complex Fractions, when reduced to Simple Fractions, are added, subtracted, multiplied, and divided like other fractions.

15. Find the sum of the 2d and 3d.
16. Find the difference of the 4th and 5th.
17. What is the product of the 6th by the 7th?
18. What is the quotient of the 10th divided by the 9th?
19. What is the product of the 7th and 8th?
20. What is the quotient of the 12th divided by the 13th?

223. **Finding what Part one Number is of Another.**

1. What part of 9 inches is 2 inches? 4 in.? 7 in.?

2. What part of a yard is 1 foot?

ANALYSIS.—In 1 yard there are 3 feet, and 1 foot is $\frac{1}{3}$ of 3 feet, *Ans.*

3. What part of a week is 1 day? 2 days? 5 days?

4. What part of 3 days is 1 foot?

Ans. Days and feet are *unlike* numbers, and therefore one cannot be compared with the other.

5. What part of $\frac{5}{7}$ is $\frac{3}{7}$?

ANALYSIS.—Since these fractions have a *c. d.* they are like fractions, and their numerators are compared like integers. *Ans.* $\frac{3}{5}$.

224. From the examples above are derived the following

PRINCIPLES.

1°. Only like numbers, or those which are so far of the same kind that one may be said to be a part of the other, can be compared.

2°. When fractions have a common denominator, their numerators are compared like integers.

ORAL EXERCISES.

225. 1. What part of 30 cents are 5 cents? *Ans.* $\frac{5}{30}$ or $\frac{1}{6}$.

2. What part of 21 yards are 7 yards? Of 45 days are 9 days?

3. \$7 are what part of \$15? Of \$45? Of \$63?

Find what part one of the following numbers is of the other, expressed in lowest terms:

4. Of 30 is 12?
5. Of 63 is 14?
6. Of 96 is 48?
7. Of 120 is 30?
8. Of 65 is 100?
9. Of 108 is 144?

10. If an acre of land is worth \$63, what part of an acre will \$9 buy?

11. If a piece of carpeting can be bought for \$120, what part of a piece can be bought for \$12?

12. 23 is what part of 69? 48 of 72? 84 of 99?

13. $\frac{5}{9}$ is what part of $\frac{7}{9}$? $\frac{8}{12}$ of $\frac{15}{12}$? $\frac{19}{20}$ of $\frac{37}{20}$?

14. What part of $\frac{4}{5}$ is $\frac{3}{10}$?

SUGGESTION.—$\frac{4}{5} = \frac{8}{10}$. *Ans.* $\frac{3}{8}$.

15. What part of $\frac{11}{12}$ is $\frac{2}{6}$? Of $\frac{7}{9}$ is $\frac{10}{27}$?

16. What part of $\frac{19}{20}$ is $\frac{7}{10}$?

WRITTEN EXERCISES.

226. To find what part one number is of another.

1. What part of 49 is 28?

ANALYSIS.—28 is $\frac{28}{49}$ of 49, or $\frac{4}{7}$ of 49, *Ans.*

2. What part of $\frac{7}{12}$ is $\frac{11}{30}$?

ANALYSIS.—Reduced to a *c. d.* the given fractions become $\frac{35}{60}$ and $\frac{22}{60}$, which are *like fractions.* Now 22 is $\frac{22}{35}$ of 35, *Ans.* (Art. **224.**) Hence, the

OPERATION.

$$\frac{7}{12} = \frac{35}{60}$$
$$\frac{11}{30} = \frac{22}{60}$$
$$22 \div 35 = \frac{22}{35}$$

RULE. — *Make the number denoting the part the numerator, and that with which it is compared the denominator.*

NOTE.—If either or both the given numbers are fractional, they should be reduced to a *c. d.;* their numerators are then compared like integers.

3. What part of 36 is $\frac{4}{5}$?

4. What part of 62 is $\frac{7}{8}$?

5. What part of 86 is $\frac{11}{12}$?

6. What part of 58 is $7\frac{3}{5}$?

7. What part of 112 is $\frac{4}{5}$?

8. What part of 325 is $\frac{7}{8}$?

9. $100\frac{2}{3}$ is what part of $175\frac{4}{9}$?

10. $6\frac{1}{4}$ is what part of 45?

11. 40 is what part of 954?

12. $\frac{14}{48}$ is what part of $\frac{7}{16}$?

13. $\frac{20}{56}$ is what part of $\frac{11}{28}$?

14. $18\frac{5}{6}$ is what part of $46\frac{2}{3}$?

15. At \$23 per acre, how much land will \$17 buy?

16. A man paid \$185 for a horse, and sold it for \$150; what part of the cost did he get?

17. A man 76 years old has a son whose age is 54 years; what part of the father's age is that of his son?

18. If from a piece of silk containing $27\frac{3}{4}$ yds., you cut $11\frac{1}{2}$ yds., what part of the piece will be left?

19. If a man can perform a journey in 24 days, what part of it can he go in 9 days?

20. What part of $\$268\frac{3}{5}$ is $\$175\frac{3}{4}$?

21. If A can do a job of work in 20 days, and B in 10 days, what part will each do in 1 day? What part will both do?

ORAL EXERCISES.

227. **1.** 4 is $\frac{1}{7}$ of what number?

ANALYSIS.—4 is $\frac{1}{7}$ of 4 times 7, or 28. Therefore, 4 is $\frac{1}{7}$ of 28, *Ans.*

2. 36 is $\frac{3}{4}$ of what number?

ANALYSIS.—Since 36 is $\frac{3}{4}$ of a number, $\frac{1}{4}$ of that number is $\frac{1}{3}$ of 36, which is 12, and 4 fourths are 4 times 12, or 48. Therefore, etc.

NOTE.—If the learner is at a loss which term of the fraction to take for the *divisor,* let him substitute the word *parts* for the denominator, and his difficulty will vanish.

3. 15 is $\frac{3}{9}$ of what?

4. 16 is $\frac{4}{5}$ of what?

5. 45 is $\frac{5}{7}$ of what?

6. 210 is $\frac{7}{9}$ of what?

7. $15\frac{3}{4}$ is $\frac{3}{7}$ of what?

8. $10\frac{2}{3}$ is $\frac{4}{7}$ of what?

9. $45 = \frac{15}{28}$ of what?

10. $48 = \frac{12}{45}$ of what?

WRITTEN EXERCISES.

228. To find a Number when a Part of it is given.

1. 56 is $\frac{7}{9}$ of what number?

ANALYSIS.—Since $\frac{7}{9}$ of a number is 56, 1 ninth is $\frac{1}{7}$ of 56, which is 8, and 9 ninths are 9 times 8, or 72, *Ans,* Hence, the

OPERATION.

$56 \div 7 = 8$

$8 \times 9 = 72$, *Ans.*

RULE.—*Divide the number denoting the part by the numerator, and multiply the quotient by the denominator.* (Art. **208**.)

2. 48 is $\frac{3}{4}$ of what?

3. 56 is $\frac{4}{5}$ of what?

4. 75 is $\frac{5}{9}$ of what?

5. 96 is $\frac{6}{10}$ of what?

6. 132 is $\frac{12}{15}$ of what?

7. 257 is $\frac{8}{9}$ of what?

8. 394 is $\frac{7}{15}$ of what?

9. 859 is $\frac{11}{16}$ of what?

10. A merchant lost $4368, which was $\frac{3}{10}$ of his capital; what was his capital?

11. If $\frac{3}{9}$ of a farm is worth \$2360, what is the whole worth?

12. A drover being asked how many sheep he had replied that 147 was equal to $\frac{7}{15}$ of them; how many sheep had he?

13. A man lost $\frac{3}{7}$ of his money and had \$260 left; how much had he at first?

ORAL PROBLEMS FOR REVIEW.

229. **1.** A lad having \$5, paid $\$2\frac{1}{2}$ for a pair of skates and $\$1\frac{1}{4}$ for a sled; how much did he have left?

2. A lady went shopping with \$15 in her purse; she paid $\$\frac{3}{4}$ for a handkerchief, $\$2\frac{1}{4}$ for a pair of gloves, and the rest for a shawl; what did the shawl cost her?

3. A laborer earned $\$1\frac{1}{4}$ one day, $\$1\frac{1}{2}$ the next, and paid $\$1\frac{3}{4}$ for board; how much had he left?

4. A grocer bought a load of apples at $\$\frac{5}{8}$ a bushel, and sold them at $\$\frac{3}{4}$; how much did he make on a bushel?

5. A man owning $\frac{15}{16}$ of a ship, sold $\frac{3}{8}$ of her; what part had he left?

6. The sum of two fractions is $\frac{9}{15}$, and one of them is $\frac{2}{5}$; what is the other? What is their difference?

7. The greater of two numbers is $6\frac{3}{4}$, and their difference is $2\frac{1}{2}$; what is the less number?

8. The less of two numbers is $5\frac{2}{3}$, and their difference $2\frac{1}{6}$; what is the greater number?

9. The product of two fractions is $\frac{1}{12}$, and one of the fractions is $\frac{3}{4}$; what is the other fraction?

10. If the dividend is $\frac{18}{20}$, and the quotient is $\frac{2}{5}$, what is the divisor?

11. What number divided by $\frac{3}{5}$ will give a quotient of $7\frac{1}{5}$?

12. A teacher spends $\frac{2}{5}$ of his salary for board and $\frac{1}{10}$ for clothing; what part of his salary is left?

13. If a man earns \$60 a month and spends $\frac{3}{5}$ of it, how much can he lay up?

14. I sold $\frac{5}{8}$ of my farm and had 48 acres left; how many acres did my farm contain?

15. What is the difference between $4\frac{1}{8}$ and $5\frac{3}{4}$?

16. What number subtracted from $15\frac{4}{5}$ will leave $10\frac{3}{10}$?

17. The sum of two fractions is $17\frac{3}{4}$, and one of them is $12\frac{2}{5}$; what is the other?

18. At $\$12\frac{7}{8}$ a sack, what are 5 sacks of coffee worth?

19. At $\$5\frac{3}{5}$ a barrel, what will 10 barrels of flour cost?

20. At $6 a ton, what will $15\frac{2}{3}$ tons of coal come to?

21. What will 9 cords of wood cost, at $\$3\frac{1}{4}$ a cord?

22. Bought a horse and sleigh for $175, and the sleigh was worth $\frac{2}{5}$ as much as the horse; what was the value of each?

23. A lady bought 6 neck-ties at $\$\frac{3}{5}$ each, and has $15 left; how much money had she at first?

Written Problems for Review.

230. **1.** Reduce $\frac{13}{27}$ to the denominator 243.

2. Reduce $\frac{1458}{2482}$ to lowest terms.

3. Find the prime factors of 486, 576, and 972.

4. What is the ***l. c. d.*** of $\frac{34}{100}$, $\frac{6}{8}$, and $\frac{19}{40}$?

5. What is the sum of $\frac{2}{3}$ of $\frac{3}{4}$, $\frac{1}{6}$, $\frac{5}{8}$, and $5\frac{1}{2}$?

6. What is the difference between $14\frac{1}{2}+25\frac{2}{3}$, and $25\frac{7}{8}+19\frac{1}{4}$?

7. The greater of two numbers is $375\frac{3}{8}$, and their difference $273\frac{5}{7}$; what is the less?

8. If I buy $\frac{11}{16}$ of a ship, and sell $\frac{2}{3}$ of what I buy, how much shall I then own?

9. Required the sum and difference of $\frac{3}{4}$ and $\frac{3}{10}$?

10. A railroad car goes $225\frac{3}{4}$ miles in a day and a steamer $185\frac{1}{2}$ miles; how far do both go in a day, and what is the difference in the distance they go?

11. A grocer bought a quantity of apples for $\$162\frac{1}{2}$ and sold them for $\$210\frac{6}{8}$; what was his profit?

Find the sum of the following:

12. $8\frac{4}{5}+6\frac{2}{10}-3\frac{1}{2}$.

13. $14\frac{7}{8}+6\frac{1}{4}-7\frac{3}{4}$.

14. $20\frac{8}{9}-8\frac{1}{3}+4\frac{5}{6}$.

15. $26\frac{3}{4}+(4\frac{1}{4}-2\frac{1}{2})+3\frac{1}{2}$.

16. $(24\frac{3}{8}+12\frac{1}{2})-(11\frac{3}{4}+2\frac{1}{4})$.

17. $(28-2\frac{3}{7})+(16-2\frac{3}{14})$.

18. $(140+1\frac{5}{6})-(8\frac{7}{12}-1\frac{1}{6})$.

19. $145+\frac{3}{20}+(112\frac{2}{3}-8\frac{4}{6})$.

20. A farmer sold a cow for $\$26\frac{2}{3}$, 15 sheep for $\$52\frac{1}{2}$, and the buyer handed him a $100 bill; how much change should he return?

21. Paid $\$275\frac{4}{5}$ for a quantity of wheat, $\$320\frac{7}{8}$ for a quantity of corn, and sold the former for $\$316\frac{3}{4}$, and the latter for $\$410\frac{1}{2}$; what was my profit?

22. If $\frac{4}{5}$ of a factory cost $23245, what is the whole worth?

23. What number multiplied by $7\frac{3}{4}$ will produce $872\frac{5}{8}$?

24. If the divisor is $\frac{5}{37}$, and the quotient $\frac{7}{9}$, what must be the dividend?

25. The dividend is $42\frac{3}{4}$, the quotient $8\frac{1}{2}$, what is the divisor?

26. A father bequeathed $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ of his property to his 3 children, and had $4800 left for his wife; what was the amount of his property?

27. A merchant lost $\frac{4}{9}$ of his capital by one creditor, and $\frac{2}{7}$ by another, and had $500 left; what was his capital?

28. My neighbor having $356\frac{1}{2}$ acres of land, sold $\frac{1}{4}$ of it to one man, and $\frac{2}{5}$ of it to another; what was the value of the remainder at $\$25\frac{3}{4}$ per acre?

29. A man gave his check for $\$1675\frac{3}{4}$, which was $\frac{5}{8}$ of what he had on deposit; how much had he in bank?

30. A man had $6\frac{5}{7}$ acres of land, which he divided into building lots each containing $\frac{6}{14}$ acres; how many lots did he have?

31. Bought a horse for $\$160\frac{1}{2}$, and sold it for $\frac{4}{5}$ of the cost; how much did I lose?

32. How many books, at $\$\frac{7}{8}$ apiece, can be bought for $\$10\frac{1}{8}$?

33. At $\$3\frac{1}{4}$ a day, how much can a man earn in $25\frac{1}{2}$ days?

34. What cost $34\frac{1}{2}$ bushels of flaxseed, at $\$2\frac{1}{5}$ a bushel?

35. A market-woman being asked how many eggs she had, replied, "244 equals $\frac{4}{5}$ of them;" how many had she?

36. A man paid $5250 for a house, which was $\frac{3}{7}$ of all his property; how much was he worth?

37. If $\frac{5}{16}$ of a ship cost $8360, what is the whole ship worth?

38. A lady teacher paid $750 for a piano, which was $\frac{3}{4}$ of her salary for a year; what was her salary?

39. A tree casts a shadow of 48 feet, which is $\frac{3}{4}$ of its height; how high is the tree?

40. Nine feet of a flag-staff stands in the ground, which is $\frac{3}{25}$ of its whole length; what is its length?

41. A lad being asked how many marbles he had, said he had $\frac{2}{3}$ as many as his friend, and that both together had 255; how many had he?

42. A goldsmith paid $75 for a watch, which was $\frac{2}{3}$ of what he got for it; how much did he get for the watch?

43. A can build a school-house in 90 days, which is $\frac{5}{6}$ of the time it would take C; how long would it take C to build it?

44. An army lost $\frac{1}{8}$ of its men in battle and $\frac{1}{4}$ by sickness, and had 9600 left; what was its whole number?

45.	$16\frac{2}{3}$ is $\frac{1}{6}$ of what?	**48.**	$\frac{8}{9}$ of $6\frac{2}{3}$ is $\frac{4}{9}$ of what?
46.	$18\frac{3}{4}$ is $\frac{3}{16}$ of what?	**49.**	$\frac{5}{8}$ of $\frac{16}{29}$ is $\frac{4}{10}$ of what?
47.	$25\frac{3}{9}$ is $\frac{2}{3}$ of what?	**50.**	$\frac{4}{5}$ of 48 is how many times 10?

51. A man bought a buggy for $185, which was $\frac{4}{5}$ the price of his horses; what did his horses cost?

52. A whale-ship lost $\frac{1}{10}$ of the bread, and the men were allowed 12 ounces per day apiece; what had each at first?

53. A man sold his farm for $4760, and thereby gained $\frac{1}{4}$ of the cost; what did he pay for it?

54. A man bequeathed to his son $7600, which was $1\frac{3}{5}$ of what he gave his daughter; what was his daughter's portion?

QUESTIONS.

160. What is a fraction? 161. The unit of a fraction? 162. A fractional unit? 164. What is the denominator? 165. The numerator?

166. What are the terms of a fraction? 171. What is a proper fraction? 172. Improper? 173. Simple? 174. A Compound? 175. A mixed number?

176. From what do fractions arise? 177. What is the value of a fraction? 179. Name the three general principles of fractions.

181. What is reduction of fractions? 182. How is a fraction reduced to higher terms? 185. How to the lowest terms? 187. Improper fractions to mixed numbers? 189. Mixed numbers to improper fractions?

191. What is a common denominator? 192. The least common denominator? 196. How found? 197. What are like fractions? 198. Unlike? 200. What fractions can be added? 201. Rule for adding fractions? 204. Rule for subtracting fractions?

211. The force of the word "of" in compound fractions? 212. General rule for multiplying fractions? 220. General rule for dividing fractions?

222. What are complex fractions? How reduce complex fractions to simple ones? 226. How find what part one number is of another? 228. How find a number when a part of it is given?

DECIMAL FRACTIONS.

ORAL EXERCISES.

231. 1. If a *unit* is divided into 10 equal parts, what is each part called?

2. If one of these *tenths* is subdivided into 10 equal parts, what part of the unit is *one* of them?

Ans. $\frac{1}{10}$ of $\frac{1}{10} = \frac{1}{100}$, or *one hundredth.*

3. What part of the unit is 2 of these parts? 3 of them? 6 of them? 11 of them?

4. If 1 hundredth of a dollar is divided into 10 equal parts, what part of a dollar is one of these parts?

Ans. $\frac{1}{10}$ of $\frac{1}{100} = \frac{1}{1000}$, or *one-thousandth.*

5. What part of a dollar is 2 of these parts? 4 parts?
6. What is meant by a *tenth?* 3 tenths? 7 tenths?
7. What is meant by a *hundredth?* 4 hundredths?
8. What is meant by a *thousandth?* 5 thousandths?

DEFINITIONS.

232. A **Decimal Fraction** is one or more of the equal parts of a unit divided into *tenths, hundredths, thousandths,* etc.

NOTE.—They are called Decimals from the Latin *decem, ten,* which indicates their origin and scale of decrease.

233. A **Mixed Decimal** is an integer and decimal expressed together.

Thus, 34.153, and 42.65 are mixed decimals.

234. Decimals are expressed by writing the numerator only, with a decimal point (.) on the left.

235. The **Denominator** of a decimal is always 10, 100, 1000, etc.; or 1 with as *many ciphers annexed* to it as there are *decimal places* in the given numerator.

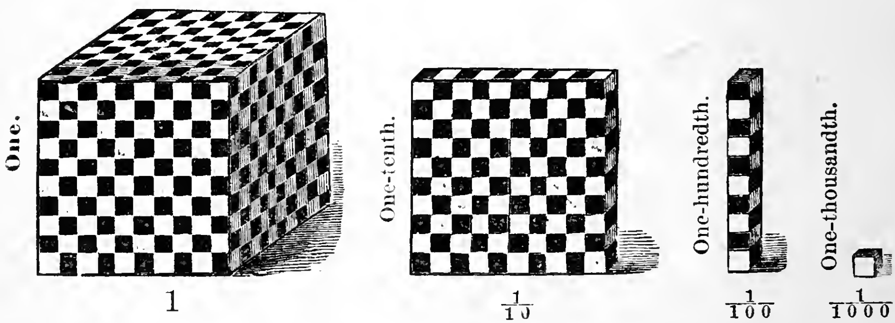
236. The **Notation of Decimals** is an extension of the Notation of *Integers.* (Art. **36.**)

TABLE.

Names of Orders.	Hundreds of millions.	Tens of millions.	**Millions.**		Hundreds of thousands.	Tens of thousands.	**Thousands.**		Hundreds.	Tens.	**Units.**		Tenths.	Hundredths.		**Thousandths.**	Ten-thousandths.	Hundred-thousandths.		**Millionths.**	Ten-millionths.	Hundred-millionths.
Number.	6	3	8	,	4	2	5	,	6	7	2	.	3	2	,	6	7	2	,	5	4	5
Orders.	9th.	8th.	7th.		6th.	5th.	4th.		3d.	2d.	1st.		2d.	3d.		4th.	5th.	6th.		7th.	8th.	9th.
	Integers.												**Decimals.**									

The number is read, Six hundred thirty-eight millions, four hundred twenty-five thousands, six hundred seventy-two, and thirty-two million six hundred seventy-two thousand, five hundred forty-five hundred-millionths.

The scale of decrease of decimal orders may be illustrated by the following diagram:



237. The *value* of each figure in decimals, as well as in integers, is determined by the place it occupies, counting from *units.*

Thus, a figure in the first place on the *right* of the decimal point, has *ten times* the value of the same figure in the next lower order, or *hundredths* place, and only *one-tenth* the value it would have in *units* place.

238. Orders *equally* distant on the *right* and *left* from units place, have *corresponding* names. Thus, *tenths* correspond to *tens, hundredths* to *hundreds,* etc. Hence,

239. The **Numerator** of a decimal fraction, when written alone, must contain as *many figures* as there are *ciphers* in its denominator. If it has not significant figures enough, the deficiency must be supplied by prefixing ciphers.

Thus, $\frac{5}{100}$ expressed decimally is .05 ; $\frac{5}{1000}$ is .005, etc. Hence,

240. *To write decimals,* we have the following

RULE.—*Write tenths in the first decimal place, hundredths in the second, thousandths in the third, etc.*

Write the following fractions decimally: (Art. **239.**)

1.	$\frac{7}{10}$.	**4.**	$\frac{5}{1000}$.	**7.**	$\frac{87}{10000}$.	**10.**	$\frac{306}{100000}$.
2.	$\frac{7}{100}$.	**5.**	$\frac{82}{1000}$.	**8.**	$\frac{204}{10000}$.	**11.**	$\frac{4075}{100000}$.
3.	$\frac{16}{100}$.	**6.**	$\frac{105}{1000}$.	**9.**	$\frac{506}{10000}$.	**12.**	$\frac{6803}{100000}$.

13. Forty-two hundredths.
14. Twenty-one thousandths.
15. Six ten-thousandths.
16. 43 ten-thousandths.
17. 65 hundred-thousandths.
18. 426 millionths.

19.	$18\frac{7}{100}$.	**22.**	$60\frac{38}{1000}$.	**25.**	$28\frac{43}{10000}$.
20.	$23\frac{5}{1000}$.	**23.**	$100\frac{7}{1000}$.	**26.**	$64\frac{291}{10000}$.
21.	$28\frac{16}{1000}$.	**24.**	$243\frac{125}{1000}$.	**27.**	$93\frac{406}{10000}$.

241. To Read Decimals expressed by Figures.

1. Read the decimal 0.000427.

EXPLANATION.—Beginning at units, we say, "units, tenths, hundredths, thousandths," etc., to the lowest order, which is millionths. We now read the significant figures as if integers, and pronounce the name *millionths. Ans.* Four hundred twenty-seven millionths. Hence, the

RULE.—*Read the significant figures of the decimal as integers, and give it the name of the lowest order.*

NOTES.—1. In *mixed decimals,* read the *integral* part as if it stood alone, then read the *decimal.* Or, having read the integral part, pronounce the word "*decimal,*" then read the decimal figures as if integers.

2. In reading *mixed decimals*, the word "and" should not be used except *between* integers and decimals.

242. Copy and read the following:

2.	.07.	**10.**	7.042.	**18.**	.0072.
3.	.005.	**11.**	16.0039.	**19.**	.00201.
4.	.102.	**12.**	23.0142.	**20.**	.400025.
5.	.0624.	**13.**	62.00301.	**21.**	.000367.
6.	.00206.	**14.**	73.04605.	**22.**	6.043216.
7.	.024542.	**15.**	8.20304.	**23.**	45.002064.
8.	.000821.	**16.**	68.207308.	**24.**	.00004607.
9.	.0000265.	**17.**	95.000206.	**25.**	.020605027.

Reduction of Decimals.

Oral Exercises.

243. **1.** How many tenths in 1? How many hundredths? How many thousandths?

2. How many tenths in 2? In 5? In 6?

3. How many hundredths in 3? In 4? In 7?

4. How many thousandths in 4? In 6? In 8?

5. How many thousandths in 5? In 7? In 9?

Illustration of Principles.

244. Since the orders of decimals decrease from *left* to *right* by **Tens**, it follows:

1°. Prefixing a cipher to a decimal, diminishes its value 10 times, and reduces it to the next lower order.

Thus, $.5 = \frac{5}{10}$; but $.05 = \frac{5}{100}$; $.005 = \frac{5}{1000}$, etc.

2°. Removing a cipher from the left of a decimal, increases its value 10 times, and reduces it to the next higher order.

Thus, $.005 = \frac{5}{1000}$; but $.05 = \frac{5}{100}$; $.5 = \frac{5}{10}$, etc.

3°. Annexing a cipher to a decimal, or removing one from its right, does not change its value.

Thus, $.5 = \frac{5}{10}$; also $.50 = \frac{50}{100}$; $.500 = \frac{500}{1000}$, all of which are equal.

WRITTEN EXERCISES.

245. To Reduce Decimals to a Common Denominator.

1. Reduce .5, .42, and .006, to a common denominator.

ANALYSIS.—The lowest order of the given decimals is thousandths. Annexing ciphers to decimals does not change their value. The fractions are .500, .420, and .006, *Ans.* (Art. **244**, *3°*.) Hence, the

OPERATION.

$.5 = 0.500$
$.42 = 0.420$
$.006 = 0.006$

RULE.—*Annex to each as many ciphers as may be required to make their decimal places equal.*

2. Reduce .20, 2.0004, and 7.008, to a ***c. d.***
3. Reduce 2 tenths, 6 hundredths, and 8 thousandths, to a common denominator.
4. Reduce .03, .125, .7, and .2362, to a ***c. d.***
5. Reduce .26, .275, .0236, and .206, to a ***c. d.***
6. Reduce .045, .61, .0035, and .108, to a ***c. d.***

ORAL EXERCISES.

246. 1. Reduce .5 to a common fraction.

ANALYSIS.—$0.5 = \frac{5}{10}$, and $\frac{5}{10}$ reduced to its lowest terms, equals $\frac{1}{2}$, *Ans*

2. How many halves in .50? In .500?
3. How many fifths in .4? In .6?
4. How many fourths in .25? Fifths in .20?
5. How many tenths in .40? In .60?
6. How many twentieths in .60? In .80?

WRITTEN EXERCISES.

247. To Reduce Decimals to Common Fractions.

1. Reduce .68 to a common fraction.

SOLUTION.—The denominator of .68 is 100. Therefore, $.68 = \frac{68}{100}$ or $\frac{17}{25}$. (Art. **235**.) Hence, the

RULE.—*Erase the decimal point, write the numerator over its denominator, and reduce the fraction to its lowest terms.* (Art. **185**.)

2. Reduce $.33\frac{1}{3}$ to a common fraction in the lowest terms.

Ans. $.33\frac{1}{3} = \dfrac{33\frac{1}{3}}{100} = \frac{100}{300}$, or $\frac{1}{3}$.

Reduce the following:

3. 0.28.	7. 0.05.	11. 0.005.	15. 0.410007.
4. 0.56.	8. 0.008.	12. 0.0006.	16. 0.0000002.
5. $0.12\frac{1}{2}$.	9. $0.6\frac{1}{4}$.	13. $0.16\frac{2}{3}$.	17. $0.008\frac{1}{4}$.
6. $0.37\frac{1}{2}$.	10. $0.31\frac{1}{4}$.	14. $0.24\frac{4}{5}$.	18. $0.944\frac{4}{9}$.

ORAL EXERCISES.

248. 1. How many tenths in $\frac{1}{2}$?

ANALYSIS.—Since there are 10 tenths in 1, in 1 half there is 1 half of 10 tenths, or 5 tenths, *Ans.*

2. How many hundredths in $\frac{1}{2}$? How many thousandths?

3. How many decimal places are required to express tenths? To express hundredths? Thousandths? (Art. **239**.)

4. In $\frac{1}{5}$ how many tenths? In $\frac{3}{5}$? In $\frac{4}{5}$?

5. In $\frac{3}{5}$ how many hundredths? In $\frac{4}{5}$?

6. How many hundredths in $\frac{3}{20}$? In $\frac{5}{20}$? In $\frac{7}{20}$?

7. How many thousandths in $\frac{3}{25}$? In $\frac{7}{25}$? In $\frac{9}{25}$?

WRITTEN EXERCISES.

249. To Reduce Common Fractions to Decimals.

1. Reduce $\frac{3}{8}$ to a decimal fraction.

OPERATION.

$8\ )\ 3.000$

$.375$, *Ans.*

ANALYSIS.—$\frac{3}{8}$ of 1 equals $\frac{1}{8}$ of 3. Since we cannot divide 3 by 8, we reduce it to tenths by annexing a cipher. (Art. **244**, *3°*.) Now $\frac{1}{8}$ of 30 tenths is 3 tenths and 6 tenths over. 6 tenths = 60 hundredths, and $\frac{1}{8}$ of 60 hundredths = 7 hundredths and 4 hundredths over. But 4 hundredths = 40 thousandths, and $\frac{1}{8}$ of 40 thousandths = 5 thousandths. Therefore $\frac{3}{8} = .375$. Hence, the

RULE.—*Annex ciphers to the numerator and divide by the denominator.*

From the right of the quotient point off as many decimal figures as there are ciphers annexed.

NOTES.—1. If the number of figures in the quotient is *less* than the number of ciphers annexed to the numerator, supply the *deficiency* by *prefixing ciphers*.

2. When the division has been carried as far as desirable, the remainder may be written over the divisor and annexed to the quotient.

3. If the remainder is $\frac{1}{2}$ or more, the last decimal figure may be increased by 1. If the remainder is less than $\frac{1}{2}$ the divisor, it may be omitted and the sign + annexed to the result.

Reduce the following fractions to decimals:

2. $\frac{1}{4}$.	**6.** $\frac{1}{8}$.	**10.** $\frac{9}{20}$.	**14.** $\frac{3}{5}$ of $\frac{7}{10}$.
3. $\frac{3}{4}$.	**7.** $\frac{3}{8}$.	**11.** $\frac{7}{20}$.	**15.** $\frac{1}{10}$ of $\frac{8}{25}$.
4. $\frac{5}{8}$.	**8.** $\frac{6}{8}$.	**12.** $\frac{8}{20}$.	**16.** $\frac{3}{4}$ of $\frac{1}{8}$.
5. $\frac{4}{5}$.	**9.** $\frac{7}{8}$.	**13.** $\frac{13}{20}$.	**17.** $\frac{4}{5}$ of $\frac{60}{145}$.

Reduce the following mixed numbers to decimals:

18. $75\frac{3}{5}$.	**20.** $39\frac{5}{8}$.	**22.** $8.07\frac{1}{2}$.	**24.** $27.81\frac{1}{4}$.
19. $89\frac{3}{4}$.	**21.** $65\frac{1}{8}$.	**23.** $0.8\frac{5}{25}$.	**25.** $93.18\frac{3}{4}$.

Reduce the following to five decimal places:

26. $\frac{2}{3}$. **27.** $\frac{3}{9}$. **28.** $\frac{4}{9}$. **29.** $\frac{7}{43}$. **30.** $\frac{41}{300}$.

31. Reduce $\frac{1}{3}$ to the form of a decimal.

ANALYSIS.—Annexing ciphers to the numerator and dividing by the denominator, the quotient is 3 continually repeated and the remainder is always 1. Therefore $\frac{1}{3}$ cannot be exactly expressed by decimals.

OPERATION.

$$3\,)\,\underline{1.0000}$$
$$.3333 \text{ etc.}$$

32. Reduce $\frac{5}{37}$ to the form of a decimal.

ANALYSIS.—The first three quotient figures are 135, and the remainder is 5, the same as the numerator; the second three are 135, the same set of figures as before, and so on.

OPERATION.

$$37\,)\,\underline{5.000000}$$
$$.135135 \text{ etc.}$$

250. When the *numerator*, with ciphers annexed, is *exactly divisible* by the denominator, the decimal is called a **Terminate** decimal.

251. When it is *not exactly divisible*, and the *same figure* or *set of figures* continually recurs in the quotient, the decimal is called an **Interminate** or **Circulating** decimal.

(For Circulating Decimals, see Art. **877**, Appendix.)

Addition of Decimals.

252. Since decimals *increase* and *decrease* regularly by *the scale of ten,* it is plain they may be treated like *integers.* (Arts. **60, 72.**)

Oral Exercises.

253. 1. What is the sum of .4 and .5?

Analysis.—.4 = $\frac{4}{10}$, and .5 = $\frac{5}{10}$. Now 4 tenths and 5 tenths are $\frac{9}{10}$, or .9, *Ans.*

2. What is the sum of .04 and .12? Of .09 and .15?
3. Find the sum of .006 and .007. Of .008 and.012.
4. Find the sum of $.07 and $.05. How many cents in $.12 and $.18?
5. Find the sum of $.60 and $.40. How many dollars?
6. Find the sum of $.004 and $.006. How many cents?
7. How many dollars in 80 cts. and 90 cts.?

Written Exercises.

254. For **Adding Decimals,** see Art. **60.**

1. What is the sum of 236.0503, .63, 25.432, and 345.6414? *Ans.* 607.7537.

Note.—Placing tenths under tenths, hundredths under hundredths, etc., reduces the decimals to a *common denominator;* hence the ciphers on the right may be omitted. (Arts. **244,** *3°.*)

2. What is the sum of $53.07 + $7.923 + $61.033 + $60.705?
3. Find the sum of 15.063 + 8.0023 + 2.05 + 213.306.
4. Find the sum of 40.103 + 217.054 + 385.0063 + 430.00057.
5. What is the sum of 48.05 + 125.006 + 7.0364 + 206.42?
6. What is the sum of 2.0707 + 100.04 + 24.084 + 7.034?
7. A man bought a horse for $375.50, which was $35.625 less than what he sold it for; what did he get for it?
8. A lady paid $65.375 for a dress, $375.50 for a shawl, and $287.125 for a set of furs; what was the price of all?

SUBTRACTION OF DECIMALS.

ORAL EXERCISES.

255. 1. From .9 subtract .4.

ANALYSIS.— .9 = $\frac{9}{10}$, and .4 = $\frac{4}{10}$. Now 4 tenths taken from 9 tenth leaves $\frac{5}{10}$, or .5, *Ans.*

2. From .23 take .12. From .32 subtract .24.
3. From .25 take .08. From .42 take .12.
4. What is the difference between $\frac{3}{5}$ and $\frac{8}{10}$?
5. What is the difference between $\frac{9}{10}$ and .2?
6. What is the difference between \$.50 and \$.25?
7. What is the difference between \$.50 and \$.75?

WRITTEN EXERCISES.

256. For **Subtracting Decimals,** see Arts. **71, 72.**

1. From 24.35 subtract 6.2875. *Ans.* 18.0625.

NOTE.—Writing the same orders in the same column, in effect reduces the given numbers to a common denominator.

2. From \$372.06 take \$168.234. *Ans.* \$203.826.
3. A lad bought a bicycle for \$15.37½ and sold it for \$12.75; how much did he lose?
4. A real estate dealer bought a house for \$8256.75, and sold it for \$10000; how much did he make?
5. A farmer gave 20 sheep worth \$65.50, 3 cows worth \$100, and 2 tons of hay worth \$28.75, for a horse, and afterwards sold the horse for \$200; how much did he make or lose by his trades?
6. What is the value of \$5263.5 — \$4236.40 + \$278.80?
7. What is the value of \$375.40 + \$478.375 — \$683.10?
8. What is the value of \$756.25 — (\$175 + \$30 + \$28.60)?
9. Two ships start from the same island; one sails due north 461.25 miles, the other due south 345.16 miles; how far apart are they, and how much farther has one sailed than the other?

MULTIPLICATION OF DECIMALS.

ORAL EXERCISES.

257. 1. What is the product of 3 times .2?

ANALYSIS.— $.2 = \frac{2}{10}$, and 3 times $\frac{2}{10}$ are $\frac{6}{10} = .6$, *Ans.*

2. What is 4 times .2? 3 times .3? 6 times .4?
3. How many decimal figures in the product of tenths by units?
4. If a pound of tea costs \$.5 what will 3 pounds cost?
5. What is 4 times .03?

ANALYSIS.—$.03 = \frac{3}{100}$, and 4 times $\frac{3}{100}$ are $\frac{12}{100} = .12$, *Ans.*

6. What is 5 times .07? 7 times .06? 6 times .08?
7. How many decimal figures in the product of hundredths by units?
8. What will 5 inkstands cost at \$.08 apiece?
9. What is .2 times .3?

ANALYSIS.—$.2 = \frac{2}{10}$ and $.3 = \frac{3}{10}$. Now $\frac{2}{10} \times \frac{3}{10} = \frac{6}{100}$, and $\frac{6}{100}$ expressed decimally is .06, *Ans.*

10. What is .6 multiplied by .4? .5 by .7? .8 by .6?
11. How many decimal figures in the product of tenths by tenths?
12. Multiply .07 by .4, using the slate if necessary.

ANALYSIS.— $.07 = \frac{7}{100}$ and $.4 = \frac{4}{10}$. Now $\frac{7}{100} \times \frac{4}{10} = \frac{28}{1000}$, which expressed decimally is .028, *Ans.*

13. How many decimal figures in the product of hundredths by tenths?
14. At \$.06 a pound, what will .5 pound of sal soda cost?
15. Multiply 8 thousandths by 5 tenths? 6 thousandths by 7 hundredths?

258. From these examples we derive the following

PRINCIPLE.—*The product of any two decimals has as many decimal figures, as both factors.*

WRITTEN EXERCISES.

259. To Multiply Decimals.

1. Multiply .29 by .7.

ANALYSIS.—Multiplying the decimals as integers the product is 203; but as the multiplicand has two decimal figures and the multiplier one, the product must have three. (Art. **258**.) Hence, the

$$\begin{array}{r} .29 \\ .7 \\ \hline .203, \end{array} \quad \textit{Ans.}$$

RULE.—*Multiply the numbers as integers, and from the right of the product point off as many figures for decimals as there are decimal places in both factors.*

NOTE.—If the product has not as many figures as there are decimals in both factors, supply the deficiency by *prefixing ciphers*.

2. Multiply 23.006 by .004. *Ans.* .092024.
3. 5.034 by .027.
4. 4.0304 by 4.005.
5. 6.4203 by 4.28.
6. 63.0048 by 7.003.
7. $235\frac{1}{2}$ by 3.45.
8. $75\frac{3}{4}$ by 68.
9. $37\frac{1}{2}$ by $.7 \times 6\frac{1}{4}$.
10. 28.3 by $12\frac{1}{2} \times 4\frac{1}{4}$.
11. $23.504\frac{1}{2}$ by 2.0006.
12. $2456\frac{1}{4}$ by .00007.
13. $62\frac{1}{2}$ by $5\frac{3}{5} \times 10.5$.
14. $87\frac{1}{4}$ by $2\frac{3}{4} \times .075$.
15. .000781 by 2.40002.
16. 278.5 by $3.87\frac{1}{2} \times 2\frac{1}{5}$.
17. What cost 465 pounds of coffee, at $31\frac{1}{2}$ cts. a pound?
18. Find the cost of 608 pounds of tea, at $87\frac{1}{2}$ cts. a pound.
19. Find the cost of 563.5 tons of hay, at $6.75 a ton.

260. When the Multiplier is 10, 100, 1000, etc.

20. Multiply 4.506 by 100.

ANALYSIS.—Moving a figure one place to the left multiplies its value by 10 (Art. **34**, 2°), hence, moving the decimal point one place to the right multiplies the number by 10, moving it two places, multiplies the number by 100, etc. Hence, the

OPERATION.

$$\begin{array}{r} 4.506 \\ 100 \\ \hline \textit{Ans.}\ 450.600 \end{array}$$

RULE.—*Move the decimal point in the multiplicand as many places to the right as there are ciphers in the multiplier.* (Art. **91**.)

21. Mult. 46.3842 by 1000.
22. Mult. 6.42302 by 1000.
23. Mult. 25.46 by 1000.
24. Mult. 3.004 by 1000.

DIVISION OF DECIMALS.

DEVELOPMENT OF PRINCIPLES.

261. 1. What is the quotient of .8 divided by .2 ?

ANALYSIS.—$.8 = \frac{8}{10}$ and $.2 = \frac{2}{10}$. Now $\frac{8}{10} \div \frac{2}{10} = 4$, *Ans.*

2. What is the quotient of .06 divided by .03 ?

3. When *tenths* are divided by *tenths*, and *hundredths* by *hundredths*, what is the quotient ? Why ?

Ans. Because they have common denominators. (Art. **224**, 2°.)

4. The product of two numbers is 3.2 and one of the factors is 4 ; how do you find the other factor ? (Art. **114**.)

5. Of what is division the reverse ? (Art. **101**.)

6. What corresponds to the product ? To the factors ?

7. What is the product of .06 by .4 ? How many decimal figures has it ?

8. If .024 is divided by .4, what is the quotient ? How many decimal figures has it ? Why ?

Ans. Since division is the reverse of multiplication, the *dividend* must have as many decimal figures as the divisor and quotient.

9. How many decimal figures are there in the product of any two factors ?

10. When *tenths* are divided by *units*, how many decimal figures in the quotient ? *Hundredths* by *units?* By *tenths?* *Thousandths* by *hundredths?*

11. If the dividend has five decimal places, and the divisor three, how many decimal places will there be in the quotient ?

262. From these examples we derive the following

PRINCIPLES.

1°. When the decimal places in the divisor and dividend are equal, the quotient is a whole number.

2°. The number of decimal places in the divisor and quotient must equal those in the dividend.

WRITTEN EXERCISES.

263. To Divide Decimals.

1. What is the quotient of .98 divided by .7 ?

SOLUTION.—We divide as in whole numbers, and point off as many figures for decimals in the quotient as the decimal places in the dividend exceed those in the divisor, which is one. Hence, the

OPERATION.

$$.7\,)\,\underline{.98}$$

Ans. 1.4

RULE.—*Divide as in whole numbers, and from the right of the quotient point off as many figures for decimals, as the decimal places in the dividend exceed those in the divisor.*

If the quotient does not contain figures enough, supply the deficiency by prefixing ciphers.

NOTES.—1. When there are *more* decimals in the divisor than in the dividend, make them equal by annexing ciphers to the latter before dividing. (Ex. 3.)

2. After all the figures of the dividend are divided, if there is a remainder, ciphers may be annexed to it as decimals, and the division continued at pleasure.

3. For ordinary purposes, it will be sufficiently exact to carry the quotient to four or five places of decimals; but when great accuracy is required, it must be carried farther.

When there is a remainder at the close of the operation, the sign + should be annexed to the quotient to show that it is not complete.

2. Divide 177.6 by 2.4. *Ans.* 74.

3. Divide 428.1 by .346. *Ans.* 1237.28+.

Perform the following divisions .

4. $3.560 \div 3.9$.	**9.** $.00634 \div 62$.	**14.** $4356.2 \div .436$.
5. $4.234 \div 4.5$.	**10.** $283.25 \div 82$.	**15.** $643.003 \div .072$.
6. $.04634 \div 5.2$.	**11.** $6432.42 \div 7.6$.	**16.** $4873.02 \div .0064$.
7. $.072 \div .8$.	**12.** $.280 \div 2.4$.	**17.** $756.4 \div 10\frac{1}{4}$.
8. $1.25 \div .12\frac{1}{2}$.	**13.** $0.063 \div .09$.	**18.** $1268.2 \div 10\frac{1}{5}$.

19. If 1.7 yd. of cloth will make a coat, how many coats will 81.6 yds. make?

20. How much broadcloth, at $5.675 a yard., can be bought for $45.40 ?

21. If a stage goes 8.25 miles an hour, how long will it take to go 125 miles?

22. If a barrel of beef is worth $14.25, how many barrels can be bought for $798?

23. If a steamer goes 215.6 miles per day, how long will it take to go 1000 miles?

264. When the Divisor is 10, 100, 1000, etc.

24. Divide 324.56 by 100.

ANALYSIS.—As each removal of a figure one place to the right diminishes its value ten times, moving a decimal point one place to the left divides the number by ten, two places to the left divides it by 100, etc. Hence, the

OPERATION.

100) 324.56

Ans. 3.2456

RULE.—*Remove the decimal point in the dividend as many places to the left, as there are ciphers in the divisor.* (Art. **34**, *3°*.)

Perform the following divisions as indicated:

25. 24.25 ÷ 100.

26. 456.31 ÷ 1000.

27. 32.463 ÷ 1000.

28. 0.08534 ÷ 1000.

29. 64.2564 ÷ 10000.

30. 56345.27 ÷ 100000.

QUESTIONS.

232. What is a Decimal Fraction? Why called Decimals? 234. How are they expressed? 235. What is the denominator of a decimal? 237. How determine the value of a decimal figure?

236. In writing and reading decimals, what should be made the starting point? 240. How write decimals? How read them?

244. What is the effect of removing the decimal point one place to the left? One place to the right? 244. What is the effect of annexing a cipher to a decimal or removing one from its right?

245. How reduce decimals to a common denominator? 247. How reduce decimals to common fractions? 249. Common fractions to decimals? 254. How are decimals added? 256. How subtracted?

259. How are decimals multiplied? How point off the product? 263. How divided? How point off the quotient.

DECIMAL CURRENCY.

UNITED STATES COINS.

265. **Coins** are pieces of metal stamped at the Mint, authorized by Government to be used as *money* at fixed values.

266. **Money** is the measure of value.

267. **Currency** is the money employed in trade. It consists of coins, bank bills, bonds, bills of exchange, etc.

268. A **Decimal Currency** is one whose orders or denominations increase and decrease by the scale of tens.

UNITED STATES MONEY.

269. U. S. Money is the legal currency of the United States. Its denominations are Eagles, Dollars, Dimes, Cents, and Mills, which increase and decrease by tens.*

TABLE.

10 mills	are	1 cent, - -	*ct.*
10 cents	"	1 dime,- -	*d.*
10 dimes, or 100 cts.	"	1 dollar, -	*dol.*, or $.
10 dollars	"	1 eagle,- -	*E.*

270. The U. S. coins are gold, silver, nickel, and bronze.

271. The **Gold** coins are the *double eagle, eagle, half eagle, quarter eagle, three-dollar* piece, and *dollar*.

NOTES—1. The gold dollar is the *unit* of *value*. Its standard weight is 25.8 gr. Troy.

272. The **Silver** coins are the *dollar, half dollar, quarter dollar*, and *dime*.

2. The weight of the silver dollar is 412½ grains. The standard purity of gold and silver coins is *nine-tenths* pure metal and *one-tenth* alloy.

273. The **Nickel** coins are the *5-cent* and *3-cent* pieces.

274. The **Bronze** coin is the *1-cent* piece.

275. The **Dollar** is the **Unit**; hence, dollars are written as *integers* with the *sign* ($) prefixed to them, and the *decimal point* placed after them.

Cents occupy hundredths place on the right, and mills the place of thousandths.

NOTES.—1. Eagles and dimes are seldom used in business calculations; the former are read as *dollars*, the latter as *cents*. Thus, 15 eagles are read as $150, and 6 dimes as 60 cents.

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* The United States adopted the decimal system of currency in 1789. Since then it has been adopted by France, Belgium, Brazil, Bolivia, Canada, Chili, Denmark, Ecuador, Greece, Germany, Italy, Japan, Mexico, Norway, Peru, Portugal, Spain, Sweden, Switzerland, Turkey, U. S. of Colombia, and Venezuela.

2. *Cents* occupy *two* places, hence if the number to be expressed is *less* than 10, a *cipher* must be prefixed to the figure denoting them.

3. In business calculations, if the mills in the *result* are 5 or more, they are considered a *cent;* if less than 5, they are omitted.

276. To reduce dollars to cents, multiply them by 100.
To reduce dollars to mills, multiply them by 1000.
To reduce cents to mills, multiply them by 10.

277. To reduce cents to dollars divide them by 100.
To reduce mills to dollars divide them by 1000.
To reduce mills to cents, divide them by 10.

278. Dollars, cents, and mills correspond to the orders of integers and decimals, and are expressed in the same manner.

Thus, 78 dollars 47 cents 5 mills are written, $78.475.

Write the following in like manner:

1. 50 dols. 10 cts. 5 mills.
2. 75 dols. 5 cts. 8 mills.
3. 627 cents 5 mills.
4. $372\frac{25}{100}$ dollars.
5. 407 dols. $12\frac{1}{2}$ cts.
6. $5260\frac{37}{100}$ dols.

Oral Exercises.

1. Change 5 cents to mills.
2. Change 8 cents to mills.
3. Change 40 mills to cents.
4. Change 65 mills to cents.
5. Change $3 to cents.
6. Change $5.20 to cents.
7. Reduce $4 to mills.
8. Reduce $6.10 to mills.
9. Reduce 600 cents to dollars.
10. Reduce 7000 mills to dollars.
11. Reduce 460 cents to dollars.
12. Reduce 5420 mills to dollars.

13. A lad bought a History for $1.10, and gave a two-dollar bill in payment; what change did he receive?

14. A dealer paid $4.30 for a pair of boots, and sold them for $5.25; how much did he make?

15. If a laborer earns $1.25 in one day, how much can he earn in 4 days?

16. What cost 5 barrels of flour, at $6.50 a barrel?

17. If 5 caps cost $3.25, what will 1 cap cost?

18. At 20 cents apiece, how many citrons can be bought for $2.40?

WRITTEN EXERCISES.

279. United States Money is added, subtracted, multiplied, and divided in all respects like Decimal Fractions. (Arts. **252-261.**)

1. A man bought a cow for \$15.75, a calf for \$2.375, a sheep for \$3.875, and a load of hay for \$8.68; how much did he pay for all?

2. A farmer sold a firkin of butter for \$9.28, a cheese for \$1.17, a quarter of veal for \$.56, and a bushel of wheat for \$1.12; how much did he receive for the whole?

3. A man bought a hat for \$5.375, a cloak for \$35.68, and a pair of boots for \$4.75; how much did he pay for all?

4. What is the sum of 63 dols. and 4 cts., 86 dols. and 10 cts., and 47 dols. and 37 cts.?

5. If I pay \$217 for a horse and \$145.50 for a buggy; what is the cost of both? What is the difference in cost?

6. What is the difference between \$137.25 + \$65.07 and \$126.12½ + \$93.06?

7. A man paid \$63.87½ for a sleigh and \$27.50 for a robe, and sold them both for \$185; how much did he make?

8. What will 145 loads of wood cost, at \$3.25 a load?

9. Bought 115 barrels of apples, at \$3 a barrel, and sold 20 barrels at \$2.50 and the remainder at \$4.25 a barrel; did I gain or lose by the operation? How much?

10. A paid \$15 per acre for his farm of 365 acres, and B paid \$23 per acre for his farm of 285 acres; required the difference in the cost of their farms?

11. A farmer bought 165 sheep at \$6 a head, 16 cows at \$34, and 27 tons of hay at \$21 a ton, and paid \$500 down; how much did he then owe for them?

12. If a man has a salary of \$1800 a year, and pays \$225 for his board, and spends \$175 for clothes and \$220 for incidentals, how much will he lay up in a year?

13. A grocer bought 1365 sacks of coffee at \$20 per sack; he sold 563 sacks at \$25 and the balance at \$27 a sack; how much did he gain or lose?

14. A butcher bought 116 head of cattle at $47 a head, and 3 times as many sheep at $6 a head; how much did he pay for his cattle and sheep?

15. How many hats at $3.75 apiece can you buy for $18.75?

16. If a man pays $7.25 a week for board, how long can he board for $258.50?

17. A mason received $194.375 for doing a job, which took him 75½ days; how much did he receive per day?

18. At $1.12½ per bushel, how many bushels of wheat can be bought for $523.75?

19. If $1285.25 were divided equally among 125 men, what would each receive?

20. The salary of the President of the United States is $50000 a year; how much does he receive per day?

21. A man paid $66.51 for broadcloth, which was $7.39 per yard; how many yards did he buy?

22. If flour is $8.12½ per barrel, how many barrels can be bought for $2047.50?

23. If 556.25 lbs of tobacco cost $69.532, how much is that a pound?

24. At $47.184 per ton, how many tons of railroad iron can be bought for $28310.40?

SHORT METHODS.

280. An **Aliquot Part** of a number is an *exact divisor* of that number.

Thus, 2, 2½, 3⅓, and 5, are aliquot parts of 10.

ALIQUOT PARTS OF A DOLLAR.

50 cents = $\$\frac{1}{2}$.	12½ cents = $\$\frac{1}{8}$.
33⅓ cents = $\$\frac{1}{3}$.	10 cents = $\$\frac{1}{10}$.
25 cents = $\$\frac{1}{4}$.	8⅓ cents = $\$\frac{1}{12}$.
20 cents = $\$\frac{1}{5}$.	6¼ cents = $\$\frac{1}{16}$.
16⅔ cents = $\$\frac{1}{6}$.	5 cents = $\$\frac{1}{20}$.

Oral Exercises.

281. 1. What part of $1 is 50 cts.? 25 cts.? 20 cts.?

2. What part of $1 is $12\frac{1}{2}$ cts.? 10 cts.? $8\frac{1}{3}$ cts.? $6\frac{1}{4}$ cts.?

3. What will 27 yds. of delaine cost at 50 cts. a yard?

Analysis.—50 cents are $\$\frac{1}{2}$; therefore 27 yds. will cost 27 times $\$\frac{1}{2}$, or $\$\frac{27}{2}$, which are equal to $\$13\frac{1}{2}$, or $13.50, *Ans.*

4. At 25 cts. a pair, what cost 75 pairs of mittens?

5. At $12\frac{1}{2}$ cts. each, what will be the cost of 72 slates?

6. If you pay 20 cts. a day for car-fare, what will be your fare for 60 days?

7. At $33\frac{1}{3}$ cts. a bushel, what will be the cost of 31 bushels of apples? Of 36 bushels? Of 45 bushels? Of 63 bushels?

8. At $16\frac{2}{3}$ cts. a pound, what cost 30 pounds of butter?

9. What cost 64 qts. of milk at $6\frac{1}{4}$ cts. a quart? 80 quarts?

10. How many melons, at $12\frac{1}{2}$ cts. each, can be had for $6?

Analysis.—Since $12\frac{1}{2}$ cts. are $\$\frac{1}{8}$, $6 will buy as many melons as $\$\frac{1}{8}$ is contained times in $6, or 48 melons, *Ans.*

11. At $.50 a pound, how many pounds of tea can be bought for $11? For $\$18\frac{1}{2}$? For $25? For $50?

12. A farmer sold 36 bushels of oats at $\$.33\frac{1}{3}$ a bushel, and took his pay in raisins at $12\frac{1}{2}$ cts. a pound; how many pounds of raisins did he receive?

Written Exercises.

282. Price is the money value of a unit of like things.

283. Cost is the sum paid for a given number of like things.

284. To find the *Cost* of a number of like things, when the Price of one is an Aliquot Part of $1.

1. What is the cost of 675 Histories, at $33\frac{1}{3}$ cts. each?

Analysis.—At $1 apiece, they would cost $675. But the price is only $\frac{1}{3}$ of $1 each; therefore, the cost is $\frac{1}{3}$ of $675, which is $225, *Ans.* Hence, the

$$\begin{array}{r} 3\,\underline{)\,675} \\ \textit{Ans. } 225 \end{array}$$

Rule.—*Multiply the given number of things by the fractional part of $1 which expresses the price of One: the result is the cost.* (Art. **208.**)

2. At \$0.50 a bushel, what cost 876 bu. of potatoes?
3. At 25 cts. a yard, what will 1200 yards of ribbon cost?
4. If I pay 20 cts. a bu. for apples, what must I pay for 688 bu.?
5. What cost 898 Spellers, at $12\frac{1}{2}$ cts. each?
6. At $33\frac{1}{3}$ cents a pound, what cost 750 pounds of butter?

7. What cost 450 boxes of lemons, at \$1.25 a box?

Analysis.—At \$1 a box, they would cost \$450. But the price is $\$1 + \$\frac{1}{4}$; therefore, the cost is \$450 $+ \frac{1}{4}$ of \$450, which is \$562.50, *Ans.*

$$\begin{array}{r|l} 4\,) & \$450 \text{ at } \$1. \\ & \underline{\$112.50} \text{ at } \$\tfrac{1}{4}. \\ & \$562.50,\ \textit{Ans.} \end{array}$$

8. At $\$1.33\frac{1}{3}$, what cost 796 Geographies?
9. If a man saves $\$1.16\frac{2}{3}$ each week, how much will he save in 312 weeks?
10. A shoe dealer sold at wholesale 250 pairs of slippers for \$1.20 a pair; what was the amount of his bill?

285. To find the *Number of Things* when their Cost is given, and the Price of *One* is an Aliquot Part of \$1.

11. How many gallons of milk, at $\$.33\frac{1}{3}$ a gallon, can be bought for \$175?

Analysis.—Since \$1 will pay for 3 gallons, \$175 will pay for 175 times 3 gallons, or 525 gallons. Or, at $\$\frac{1}{3}$ a gallon, \$175 will pay for as many gallons as $\$\frac{1}{3}$ is contained times in \$175, or 525 gallons, *Ans.* Hence, the

OPERATION.

$$\$.33\tfrac{1}{3} = \$\tfrac{1}{3}$$
$$\$175 \times 3 = 525$$
$$\text{Or, } \$175 \div \$\tfrac{1}{3} = 525$$

Rule.—*Divide the cost by the aliquot part of \$1 which is the price of One.*

12. How many yards of flannel, at 50 cts. a yard, can you buy for \$850?
13. A farmer sold his cheese at $16\frac{2}{3}$ cts. a pound, and received \$75 for it; how many pounds did he sell?
14. How many bushels of oats, at 50 cts. a bushel, can be had for \$975?
15. How many cans of baking powder, at 25 cts. each, can be had for \$240.50?
16. How many yards of silk, at $\$1\frac{1}{4}$ a yard, will \$160 buy?
17. How many hoes, at $\$1.33\frac{1}{3}$, can be bought for \$176?

286. To find the *Cost*, when the price per 100 or 1000 is given.

18. What is the cost of 475 oysters, at $1.65 per 100?

ANALYSIS.—At $1.65 apiece, 475 oysters would cost $1.65 × 475 = $783.75. But the price is $1.65 per *hundred;* therefore, $783.75 is 100 times the true cost. To correct this result, we divide it by 100, or remove the decimal point two places to the left.

$$\begin{array}{r} \$1.65 \\ \underline{475} \\ 100\ )\ \underline{\$783.75} \\ \textit{Ans.}\ 7.8375 \end{array}$$

19. At $12.60 a thousand, what will 2845 bricks cost?

SOLUTION.—Multiplying the price of 1000 by the number of bricks, and dividing the product by 1000, the result is $35.847, the answer required. Hence, the

$$\begin{array}{r} \$12.60 \\ \underline{2845} \\ 1|000\ )\ 35.847 \end{array}$$

RULE.—*Multiply the price per hundred or thousand by the given number of things, and divide the product by 100 or 1000, as the case may require.* (Art. **118.**)

NOTE.—The letter C is sometimes put for *hundred,* and M for *thousand.*

20. What will 2842 lb. of sugar cost, at $12.50 per hundred?
21. At $3¼ per C., what will 21264 pounds of flour come to?
22. At $12.50 a thousand, what will 25260 oranges cost?
23. At $25.50 per hundred, what cost 18564 feet of boards?
24. What cost 1276 cedar posts, at $8.75 per C.?
25. What cost 12250 envelopes, at $3.60 per M.?
26. At $6.50 per thousand, what cost 15460 shingles?
27. At $12.25 per hundred, what cost 15240 pineapples?
28. At $8.50 per M., what cost 22580 bricks?

287. To find the *Cost*, when the price of 2000 pounds is given.

29. What cost 2460 pounds of coal at $6.50 per ton?

ANALYSIS.—At $6.50 a pound, 2460 pounds will cost $6.50 × 2460 = $15990.00. But the price is $6.50 per ton of 2000 pounds; therefore, $15990.00 is 2000 times the true cost. To correct this result we divide it by 2000; or divide by 2 and remove the decimal point three places to the left. Hence, the

OPERATION.

$$\begin{array}{r} \$6.50 \\ \underline{2460} \\ 2000\ )\ \underline{15990.00} \\ \textit{Ans.}\ \$7.99500 \end{array}$$

RULE.—*Multiply the price of 1 ton by the given number of pounds and divide the product by 2000.*

30. At \$12.50 per ton, what is the value of 8 loads of hay, each weighing 1525 pounds? *Ans.* \$76.25.

31. What is the cost of 12 sacks of wool, each weighing 450 pounds, at \$25.30 per ton?

32. What is the freight from London to New York on a quantity of goods weighing 8540 pounds, at \$4.60 per ton?

33. What cost 16250 pounds of guano, at \$80¼ per ton?

Accounts and Bills.

288. An **Account** is a record of business transactions.

289. Every *business* transaction has *two* parties, a *buyer* and a *seller*, called a **Debtor** and a **Creditor**.

290. A **Debtor** is a party who *owes* another.

291. A **Creditor** is a party to whom a *debt* is *due*.

292. A **Ledger** is the principal Book of Accounts kept by business men. To the Ledger is transferred for preservation and reference, a brief statement of all the items of the *Day Book or Journal*, where they are fully recorded.

293. The **Debits** or **Debts** are placed on the left, marked *Dr.*, the **Credits** or **Payments** on the right, marked *Cr.*

294. The **Balance** of an account is the *difference* between the Debit and Credit sides.

295. A **Bill** is a written statement of goods sold, or services rendered, with their prices, etc.

296. An **Invoice** is a written statement of items sent with merchandise.

Note.—Accounts and bills should always state the names of both parties, the place and time of each transaction, the name and price of each item, and the entire cost.

297. A **Bill is Receipted** when the words "*Received Payment*" are written at the bottom, and it is *signed* by the creditor, or by some person duly authorized.

298. The following abbreviations are often used:

Acct. or %,	Account.	Inst.,	This month.
Amt.,	Amount.	Mdse.,	Merchandise.
@,	At.	Net.,	Without Discount.
Bal.,	Balance.	Prox.,	Next month.
Do.,	The same.	Ult.,	Last month.

299. Copy, extend the items, and balance the following:

(1.)

BOSTON, May 25th, 1881.

JAMES BROWNELL, ESQ.,

Bought of FAIRMAN & LINCOLN.

5 yds. broadcloth,	@ \$3.25	- - - - - -		
3 yds. cambric,	@ .12½	- - - - - -		
3 doz. buttons,	@ .15	- - - - - -		
6 skeins sewing silk,	@ .06¼	- - - - - -		
4 yds. wadding,	@ .08	- - - - - -		
	Amount,	- -		

Received Payment,

FAIRMAN & LINCOLN.

(2.)

NEW YORK, Feb. 13th, 1881.

HORACE FOOTE & CO.,

To GEO. SPENCER & CO., *Dr.*

1881.					
Feb.	10	For 85 lbs. Coffee,	@ 25 cts.		
"	12	" 36 lbs. Tea,	@ 94 cts.		
"	"	" 63 gal. Molasses,	@ 37½ cts.		
"	13	" 125 lbs. Rice,	@ 8⅓ cts.		
"	"	" 75 boxes Oswego Stch.,	@ 87½ cts.		
"	"	" 56 lbs. Bar-soap,	@ 6¼ cts.		
		Amount, - -			

Received Payment,

GEO. SPENCER & CO.

(3.)

CHICAGO, May 15th, 1881.

MESSRS. J. C. GRIGGS & CO.,

Bought of CLARK & MAYNARD.

1881.					
May	1	150 Spellers,	@ $6\frac{1}{4}$ cts.		
"	"	110 Geographies,	@ $1.20		
"	2	72 Roman Histories,	@ $1.15		
"	"	96 Grammars,	@ 65 cts.		
"	4	48 Philosophies,	@ 56 cts.		
"	8	75 Astronomies,	@ 63 cts.		
		Amount, - -			

Received Payment by draft on Boston,

For CLARK & MAYNARD,

J. S. MANN.

(4.)

SAN FRANCISCO, Oct. 3, 1881.

HENRY STANDART & BROTHER,

In Acct. with G. ATWATER & CO., *Dr.*

1881.					
June	4	65 tons R.R. iron,	@ $45.25		
"	15	15 cwt. Bessemer steel,	@ $20.50		
July	8	18 doz. Axes,	@ $10.40		
Aug.	10	25 Saws,	@ $3.75		
Sept.	20	42 cwt. Lead,	@ $7.40		
		Cr.			
July	1	500 bbls. Flour,	@ $5.40		
"	20	356 bu. Wheat,	@ $1.17		
Aug.	10	Dft. on New York, - - - - - - -		300	
Sept.	25	12 shares Mining Stock,	@ $70.00		
		Bal. due, - -			

Received Payment,

G. ATWATER & CO.,

Per CHARLES KING.

Put the following items into the form of bills and find the amount of each:

5. Bought 35 doz. gloves, at \$4.50 per doz.; 95 yds. black silk, at \$.87½ per yard; 115 yds. colored ditto, at \$.78; 36 crape shawls, at \$32.50 apiece; 65 Broché ditto, at \$17.83; what was the amount of the bill?

6. Bought 85 ploughs, at \$9.63; 125 hoes, at 63 cents; 94 shovels, at 84 cents; 56 rakes, at 28 cents; 67 axes, at \$1.13; what was the amount of the bill?

7. Bought 96 pair black silk hose, at 83 cents; 85 ditto white, at 87½ cents; 135 ditto worsted, at 56¼ cents; 87 pair men's gloves, at 67 cents; 120 pair ladies' ditto, at 58 cents; 75 cravats, at 96 cents; what was the amount?

8. Bought 67 Latin Readers, at 63 cents; 60 Greek Readers, at \$1.09; 84 Greek Grammars, at 68 cents; 95 Latin ditto, at 62½ cents; 35 Virgil, at \$2.13; 45 Sallust, at 78 cents; 52 Cicero's Orations, at 75 cents; what was the amount of the bill?

9. Bought 36 pair of boots, at \$5.17; 216 pair thick shoes, at \$1.37½; 135 pair gaiters, at \$1.38; 240 pair buskins, at 83 cents; 134 pair slippers, at 68 cents; 87 pair rubbers, at \$1.13; what was the amount of the bill?

QUESTIONS.

266. What is money? 267. What is currency? 268. Decimal currency? 269. U. S. Money? Repeat the table. 265. What are coins? 271. Name the gold coins of U. S. 272. The silver. 273. The nickel. 274. The bronze.

276. How reduce dollars to cents? To mills? Cents to mills? 277. Cents to dollars? Mills to dollars? Mills to cents? 279. Rules for calculating U. S. Money?

280. What is an aliquot part of a number? Name the aliquot parts of a dollar. 284. How find the cost, when the price of one is an aliquot part of \$1? 285. How find the number of things, when the cost is given and the price of one is an aliquot part of \$1? 286. How find the cost, when the price per 100 or 1000 is given?

288. What is an account? 290. A debtor? 291. A creditor? 294. The balance of an account? 295. What is a bill? 297. How receipted? 296. An invoice?

METRIC SYSTEM.

DEFINITIONS.

300. **Metric Weights and Measures** are those whose units increase and decrease regularly by the **Decimal Scale.**

301. The **Meter** is the **Base,** and from it the **Metric System** derives its name.*

302. The **Meter** is *one ten-millionth* part of the distance from the *Equator* to the *Pole,* and is equal to 39.37 inches, nearly.

NOTE.—The term *Meter* is from the Greek *metron,* a *measure.*

303. The Metric System has three principal units, the *Méter* (meeter), *Líter* (leeter), and *Gram.* To these are added the *Ar* and *Ster,*† for square and cubic measure. Each of these units has its *multiples* and *subdivisions.*

304. The names of the *higher* metric denominations are formed by prefixing to the name of the *unit,* the *Greek* numerals, *Dek'a, Hek'to, Kil'o,* and *Myr'ia.*

Thus, from	**Dek'a,**	10,	we have	Dek'améter,	10	meters.
"	**Hek'to,**	100,	"	Hek'tométer,	100	"
"	**Kil'o,**	1000,	"	Kil'ométer,	1000	"
"	**Myr'ia,**	10000,	"	Myr'iaméter,	10000	"

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* This system had its origin in France near the close of the last century. Its simplicity and comprehensiveness have secured its adoption in nearly all the countries of Europe and South America.

Its use was legalized in Great Britain in 1864, and in the United States in 1866.

It is adopted by the U. S. Coast Survey, and is extensively used in the Arts and Sciences, and partially in the Mint and Post Office.

† The spelling, pronunciation, and abbreviation of metric terms in this work, are the same as adopted by the American Metric Bureau, Boston, and the Metrological Soc., N. Y.

305. The *lower denominations* are formed by prefixing to the name of the unit the *Latin* numerals, *Dec'i, Cen'ti,* and *Mil'li.*

Thus, from	**Dec'i,**	$\frac{1}{10}$,	we have	Dec'iméter,	$\frac{1}{10}$ meter.
"	**Cen'ti,**	$\frac{1}{100}$,	"	Cen'timéter,	$\frac{1}{100}$ "
"	**Mil'li,**	$\frac{1}{1000}$,	"	Mil'liméter,	$\frac{1}{1000}$ "

NOTE.—The numeral prefixes are the Key to the whole system, and should be thoroughly committed to memory.

MEASURES OF LENGTH.

306. The principal unit of each table is printed in capital letters; those in common use in full-faced Roman.

TABLE.

10 *mil'li-me'ters (mm.)*	=	1 **cen'ti-me'ter,** - -	*cm.*
10 cen'ti-me'ters	=	1 dec'i-me'ter, - -	*dm.*
10 dec'i-me'ters	=	1 **METER,** - -	*m.*
10 me'ters	=	1 dek'a-me'ter, - -	*Dm.*
10 dek'a-me'ters	=	1 hek'to-me'ter, - -	*Hm.*
10 hek'to-me'ters	=	1 **kil'o-me'ter,** - -	*Km.*
10 kil'o-me'ters	=	1 myr'ia-me'ter, - -	*Mm.*

NOTES.—1. The *Accent* of each *unit* and *prefix* is on the *first* syllable, and remains so in the compound words.

2. Abbreviations of the higher denominations begin with a *capital,* those *of the lower* begin with a *small* letter.

COMMON EQUIVALENTS.

1 cen'timeter	=	0.3937 inches.
1 dec'imeter	=	3.937 "
1 me'ter	=	39.37* "
1 kil'ometer	=	0.6214 mile.

* Established by Act of Congress in 1866.

ONE DECIMETER.

100 *Millimeters.*

307. The **Meter** is the **Standard Unit** of length, and, like the *yard,* is used in measuring cloths, laces, short distances, etc.*

308. The **Kilometer,** like the *mile,* is used in measuring long distances.

309. The **Centimeter** and **Millimeter** are used for minute measurements, as the thickness of glass, paper, etc.

NOTE.—The compound words may be abbreviated by using only the prefix and the first syllable or letter of the unit; thus, centimeter, millimeter, centiliter, milliliter, centigram, decigram, may be called centim, millim, centil, decig, etc.

310. The *approximate length* of 1 meter is 40 inches; of 1 decimeter, 4 inches; of 5 meters, 1 rod; of 1 kilometer, $\frac{5}{8}$ mile.

NOTE.—*Decimeters, dekameters, hektometers,* like dimes and eagles, are seldom used.

311. Since meters, centimeters, and millimeters, correspond to dollars, cents, and mills, it follows that metric numbers may be read like U. S. Money. Thus, $28.375 is read, "28 and 375 thousandths dollars," or "28 dollars, 37 cents, 5 mills."

In like manner, 28.375 meters are read, "28 and 375 thousandths meters," or "28 meters, 37 centimeters, 5 millimeters.

312. Read the following:

1.	14.5 m.	5.	47.3 Dm.	9.	89.63 Hm.
2.	236.4 m.	6.	83.25 Dm.	10.	434.5 Km.
3.	78.35 m.	7.	568 Hm.	11.	65.48 Km.
4.	23.7 Dm.	8.	648.8 Hm.	12.	9.237 Km.

* It is important for the teacher to show the class a meter stick, with its subdivisions marked on one side, and halves, quarters, etc., on the other.

313. To write Metric Numbers decimally in terms of a given Unit.

1. Write 7 Hm. 9 m. 3 dm. 5 cm. in terms of a meter.

OPERATION.

709.35 m., *Ans.*

EXPLANATION. — We write meters in *units* place, on the *left* of the decimal point, the Dm. in *tens* place, the Hm. in *hundreds* place, etc., and the decims. in *tenths* place, centims. in *hundredths*, etc., as we write the orders of integers and decimals in simple numbers. Hence, the

RULE.—*Write the given unit and the higher denominations in their order, on the left of a decimal point, as integers, and those below the unit, on the right, as decimals.*

NOTE.—If any intervening denominations are omitted in the given number, their places must be supplied by *ciphers.*

Write the following as meters and decimals:

2. 256 millimeters. *Ans.* 0.256 m.
3. 8 decimeters 4 centimeters.
4. 25 meters 3 centimeters.
5. 348 dekameters 43 centimeters.
6. 465 hektometers 48 millimeters.
7. 4725 meters 25 centimeters.
8. 4 Km. 8 Hm. 6 Dm. 4 dm. 5 cm. 3 mm.
9. 23 Km. 6 Hm. 8 dm. 6 cm.

314. To reduce Metric Numbers from higher denominations to lower, and from lower to higher.

1. Reduce 45 meters to millimeters.

SOLUTION.—Since 1 m. = 1000 mm., 45 meters must equal 45×1000, or 45000 mm., *Ans.*

$$\begin{array}{r} 45 \text{ m.} \\ \underline{1000} \\ \textit{Ans. } 45000 \text{ mm.} \end{array}$$

2. Reduce 64000 millimeters to meters.

SOLUTION.—In 1000 mm. there is 1 m., and in 64000 mm. there are as many meters as 1000 is contained times in 64000, or 64 meters. Hence, the

$$\begin{array}{r} 1000 \,\underline{)\ 64000} \\ \textit{Ans. } 64 \text{ m.} \end{array}$$

RULE.—*Move the decimal point one place to the right or left, as the case may require, for each denomination to which the given number is to be reduced.*

3. Reduce 25.7 Km. to meters. *Ans.* 25700 m.
4. Reduce 43.4 m. to millimeters.
5. Reduce 65.3 Dm. to decimeters.
6. Reduce 84.25 Km. to centimeters.
7. Reduce 4823.6 meters to Hektometers. *Ans.* 48.236 Hm.
8. Reduce 36482.9 m. to kilometers. *Ans.* 36.4829 Km.
9. Reduce 28526 mm. to meters and decimals.
10. Reduce 48639 cm. to meters and decimals.
11. Reduce 438.6 m. to millimeters.
12. Reduce 738.4 Dm. to centimeters.

MEASURES OF SURFACE.

315. A **Surface** is that which has length and breadth only.

316. The **Measuring Unit** of Surfaces is a *Square,* each side of which is a *Linear Unit.*

317. A **Square** is a figure which has *four equal* sides and *four equal* angles, called right angles.

TABLE.

100 sq. mil′li-me′ters (*sq. mm.*)	=	1 sq. **cen′ti-me′ter,**	*sq. cm.*
100 sq. cen′ti-me′ters	=	1 sq. dec′i-me′ter,	*sq. dm.*
100 sq. dec′i-me′ters	=	1 **SQ. METER,** or cent′ar,	*sq. m.* *ca.*
100 sq. me′ters	=	1 sq. dek′a-me′ter, or Ar,	*sq. Dm.* *A.*
100 sq. dek′a-me′ters	=	1 sq. hek′to-me′ter, or hek′tar,	*sq. Hm.* *Ha.*
100 sq. hek′to-me′ters	=	1 sq. **kil′o-me′ter,**	*sq. Km.*

COMMON EQUIVALENTS.

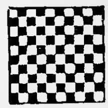
1 sq. centim.	=	0.1550 sq. in.
1 sq. decim.	=	0.1076 sq. ft.
1 sq. meter	=	1.196 sq. yd.
1 ar	=	3.954 sq. rods.
1 hektar	=	2.471 acres.
1 sq. kilo	=	0.3861 sq. mile.

318. The sq. meter is used in measuring ordinary surfaces, as floors, ceilings, etc.; the ar and hektar in measuring land; and the sq. kilometer in measuring States and Territories.

NOTE.—The term *ar* is from the Latin *area*, a *surface.*

319. The approximate area of a sq. meter is $10\frac{3}{4}$ sq. ft., or $1\frac{1}{5}$ sq. yd., and of the hektar about $2\frac{1}{2}$ acres.

320. The scale of surface measure is 100 (10×10). That is, 100 units of a lower denomination make a unit of the next higher; hence, each denomination must have *two* places of figures.



Sq. Centim.

Thus, 23 Ha. 19 a. 25 ca., written as ars, is 2319.25 a., and may be read "2319 ars and 25 centars." If written as hektars, it is 23.1925 Ha., and may be read "23 hektars and 1925 centars."

1. Write 78.29 a. as centars, also as hektars.
2. Write 9 sq. m. as sq. dm. Write 7 sq. cm. as sq. mm.
3. In 3246 ca., how many ars? In 63.42 ars, how many Ha.?

MEASURES OF SOLIDS.

321. A **Solid** is that which has length, breadth, and thickness.

TABLE.

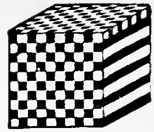
1000 cu. mil′li-me′ters (*cu. mm.*)	=	1 cu. cen′ti-me′ter,	*cu. cm.*
1000 cu. cen′ti-me′ters	=	1 cu. dec′i-me′ter,	*cu. dm.*
1000 cu. dec′i-me′ters	=	1 **CU. METER,**	*cu. m.*
10 dec′i-sters	=	1 **STER,**	*st.*
10 sters	=	1 dek′a-ster.	*Dst.*

COMMON EQUIVALENTS.

1 cu. centimeter	=	0.061 cu. in.
1 cu. decimeter	=	61.022 cu. in.
1 cu. meter	=	1.308 cu. yds.

NOTE.—The *ster* = .2759 cord is seldom used.

322. The **Measuring Unit** of solids is a *Cube,* the edge of which is a *Linear Unit.*



Cu. Cm.

323. A **Cube** is a regular solid bounded by *six equal squares* called its faces. Hence, its *length, breadth,* and *thickness* are equal.

A **Cubic Centimeter** is a cube, each side of which is a *square centimeter.*

324. The *cubic meter* is used in measuring ordinary solids, as timber, excavations, embankments, etc.

When applied to fire-wood, it is sometimes called a **Ster,** and is equal to about $35\frac{1}{2}$ cubic feet.

NOTE.—The *cubic decimeter* when used as a unit of dry or liquid measure is called a **Liter.**

325. The units of cubic measure increase by the *scale* of 1000 $(10 \times 10 \times 10)$; hence, each denomination must have *three* places of figures.

1. Express 6000 cu. mm. as cu. centimeters.
2. Express 8000 cu. dm. as cubic meters.
3. Express 86.005 cu. dm. as cu. meters; as cu. cm.
4. Write 0.6235 cu. m. as cu. dm.; as cu. cm.
5. In 862 cu. dm., how many cu. meters? In 250 cu. m. how many cubic decimeters?

MEASURES OF CAPACITY.

326. The **Liter** is the principal unit of *Dry* and *Liquid* Measure, and is equal in volume to a cubic decimeter.

TABLE.

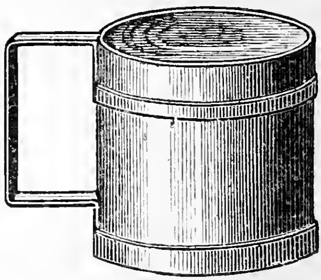
10 mil'li-li'ters (*ml.*)	=	1 **cen'ti-li'ter** - - -	*cl.*
10 cen'ti-li'ters	=	1 dec'i-li'ter - - -	*dl.*
10 dec'i-li'ters	=	1 **LITER** - - - -	*l.*
10 li'ters	=	1 dek'a-li'ter - - -	*Dl.*
10 dek'a-li'ters	=	1 hek'to-li'ter - -	*Hl.*
10 hek'to-li'ters	=	1 kil'o-li'ter - - -	*Kl.*
10 kil'o-li'ters	=	1 myr'ia-li'ter - -	*Ml.*

1 cubic centimeter = 1 milliliter of water.

Common Equivalents.

1 liter	=	61.022 cu. inches.
1 liter	=	1.0567 liquid quarts.
1 liter	=	0.908 dry quarts.
1 hektoliter	=	3.531 cu. feet.
1 hektoliter	=	26.417 gallons.
1 hektoliter	=	2.837 bushels.

327. The **Centiliter** is a little less than $\frac{1}{8}$ gill, and is used for measuring liquids in small quantities.



The **Liter** is used in measuring milk, wine, and small fruits, and is about equal to a quart.

The **Hektoliter** is used in measuring grain and liquids in casks, and is equal to about $26\frac{1}{2}$ gal., or $2\frac{5}{6}$ bushels.

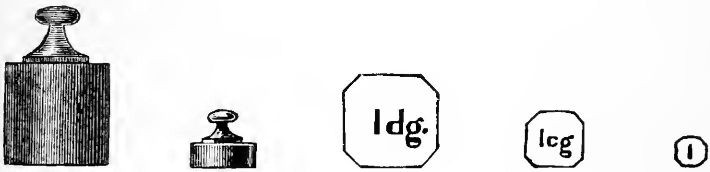
1. Express 8.53 l. as centiliters. As deciliters.
2. Express 4.640 kiloliters as liters. As hektoliters.
3. How many deciliters in 8 liters? In 9.35 liters?
4. How many liters in 6.358 centiliters? In 800 cl.?
5. In 8500 liters how many kiloliters? How many Hl.?

Weight.

328. The **Gram** is the *principal unit* of weight, and is equal to a *cubic centimeter* of distilled water at its greatest density, viz., at 4° Centigrade, or 39.2° Fahrenheit.

TABLE.

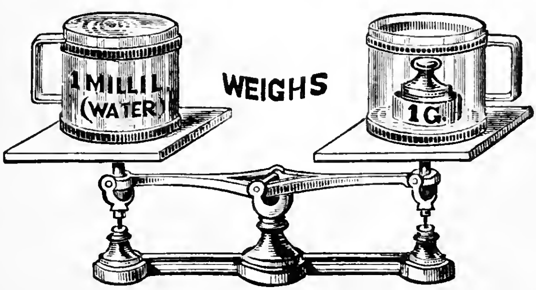
10	mil′li-grams (*mg.*)	=	1 **cen′ti-gram**	- -	*cg.*
10	cen′ti-grams	=	1 dec′i-gram	- -	*dg.*
10	dec′i-grams	=	1 **GRAM**	- - -	*g.*
10	grams	=	1 dek′a-gram	- -	*Dg.*
10	dek′a-grams	=	1 hek′to-gram	- -	*Hg.*
10	hek′to-grams	=	1 **kil′o-gram**	- -	*Kg.*
10	kil′o-grams	=	1 myr′ia-gram	- -	*Mg.*
100	myr′ia-grams	=	1 tonneau or Ton	-	*T.*



1 Dg. 1 gram. 1 dg. 1 cg. 1 mg.

COMMON EQUIVALENTS.

1 gram	=	1 cu. centim., or 1 millil. of water.
1 kilogram	=	1 cu. decim., or 1 liter of water.
1 metric ton	=	1 cu. meter, or 1 kiloliter of water.
1 gram	=	15.432 grs. Troy.
1 gram	=	0.03527 oz. Av.
1 kilogram	=	2.2046 lbs. Av.
1 metric ton	=	1.1023 tons.



329. The *Gram* is used in weighing gold, silver, jewels, and letters, and in mixing medicines.

330. The *Kilogram*, (often called *kilo*) is used in weighing common articles; as sugar, tea, butter, etc.

The *Metric ton* is used in weighing heavy articles; as hay, coal, etc.

NOTES.—1. The kilo is equal to $2\frac{1}{5}$ lbs., nearly; the metric ton about 2200 pounds.

2. The nickel 5-cent piece weighs 5 grams. The silver $\frac{1}{2}$ dollar $12\frac{1}{2}$ grams. The silver dime weighs $2\frac{1}{2}$ grams. The silver $\frac{1}{4}$ dollar $6\frac{1}{4}$ grams.

3. The weight of a letter for single postage must not exceed 15 grams, or 3 nickels.

1. Express 6.354 g. as decigrams. As centigrams.
2. Write 5834 mg. as dg. As cg. As grams.
3. How many grams in 78.45 Dg.? How many Kg.?
4. How many kilos in 3.54 T.? How many Dg.?
5. Express 1 g. in the decimal part of a kilo.
6. Express a kilo in the decimal part of a ton.
7. Express 2.0005 T. as grams.

331. To Add, Subtract, Multiply, and Divide Metric numbers.

Apply the corresponding rules of decimals or U. S. money. (Art. **279.**)

1. What is the sum of 45.68 Dm., 63.4 Hm., and 6845 cm.?

SOLUTION.—Writing the numbers as meters and decimals of a meter, the principal unit of the table, and adding, we have 6865.25 meters.

$$\begin{array}{r} 456.8 \\ 6340.0 \\ 68.45 \\ \hline \end{array}$$

Ans. 6865.25 m.

2. Find the sum of 24.35 m., 6.425 m., 32.7 m., and 42.26 m.

3. What is the difference between 8.5 kilograms and 976 grams?

SOLUTION.—8.5 kilos − .976 kilos = 7.524 kilos, *Ans.*

4. From 1 hektoliter of oil, 36 liters were drawn out; how many liters remained?

5. How much silk is there in $12\frac{1}{2}$ pieces, each containing 48.75 meters?

SOLUTION.—48.75 m. × 12.5 = 609.375 m., *Ans.*

6. It is 285 meters around my garden; how many Km. shall I walk in a week by going twice around it every day?

7. At $6.50 a meter, what will 37 meters of silk cost?

8. What cost 24 meters of fringe, at $2.25 a meter?

9. How many cloaks, each containing 5.68 meters, can be made from 426 meters of cloth?

SOLUTION.—426 m. ÷ 5.68 m. = 75 cloaks, *Ans.*

10. If a car goes 160 Km. in 6 hours, how far does it go in 1 hour?

11. How many Km. in 85.72 m. multiplied by 2036?

12. If the price of 1 liter of milk is 6 cents, what cost 75 liters?

13. At 12 cents a liter, what cost 4.5 liters blackberries?

14. If 1 hektoliter of wheat costs $3.50, what will 234 hektoliters cost?

15. A man paid $281.75 for 245 hektoliters of oats; what was the price of 1 hektoliter?

16. What cost 46.25 kilos of butter, at $0.50 per kilo?

17. At $1.28 per kilo, what will 82.5 kilos of tea come to?

18. At $16 a ton, what will the coal cost to supply a factory a week, if 25 kilos are burned each day?

19. If 735 kilos of flour are distributed among 35 persons, how many kilos will each person receive?*

332. The contents of *Rectangular Surfaces* are found by multiplying the *length* by the *breadth*.

20. A garden is 18 meters long and 12.5 meters wide; how many square meters does it contain?

SOLUTION.—The product of 18 × 12.5 = 225 sq. m., *Ans.*

21. How many sq. meters in a blackboard 2.5 meters long and 1.2 meters wide?

22. If a room is 8.4 meters long and 4.5 meters wide, how many square meters of carpeting will it take to cover the floor?

23. How many sq. meters of flagging in a side-walk 35.5 meters long, and 2.4 meters wide?

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* For reducing Metric to common Weights and Measures, etc., see Art. 405.

24. How many centars in a piece of land 45 meters long, and 23.2 meters wide?

333. The contents of *Rectangular Solids* are found by multiplying the *length*, *breadth*, and *thickness* together.

25. How many cu. meters of earth in a mound whose length, breadth, and height are each 6.4 meters.

SOLUTION.—$6.4 \times 6.4 \times 6.4 = 262.144$ cu. meters, *Ans.*

26. How many cu. meters of earth must be removed in digging a cellar 23.4 meters long, 15.2 m. wide, and 2.4 m. deep?

27. How many loads of earth each equal to a cu. meter, will it take to fill an excavation 4 dekameters long, 8 meters wide, and 2.4 meters deep?

28. At $1.45 a cu. meter, what will be the cost of digging a trench 2 dekameters long, 2 meters wide, and 1.5 meters deep?

29. At $2.50 a ster, what is the cost of a pile of wood 3 meters long, 1.5 m. wide, and 1.1 m. high?

30. What is the value of a nugget of gold 2.6 cm. long, 2.3 cm. wide, and 0.65 cm. thick, at $15.40 a cu. centimeter?

QUESTIONS.

300. What are Metric weights and measures? 301. What is the Base? 304. How are the names of the higher denominations formed? 305. The lower? 306. Repeat the table of measures of length.

307. What is the standard unit of length? For what used? 308. The kilometer? 311. How read metric numbers? 313. How write them?

314. How reduce metric numbers from higher to lower denominations? From lower to higher? 317. Repeat the table of measures of surface. 318. For what is the square meter used? The sq. kilometer? The ar and hektar?

321. Repeat the measures of solids. 324. For what is the cu. meter used? When called a ster? 326. Repeat the table of measures of capacity. 327. For what is the liter used? The hektoliter?

328. Repeat the table of weight. 329. For what is the gram used? 330. The kilogram? The metric ton? 331. How are metric numbers added, subtracted, etc.?

COMPOUND NUMBERS.

DEFINITIONS.

334. A **Simple Number** is one which expresses one or more units of the same *name* or *denomination;* as five, 4 feet, etc.

335. A **Compound Number** expresses units of *two* or *more* denominations of the *same kind,* which increase and decrease by varying scales; as, 3 yards 2 feet 4 inches. But 2 feet and 4 pounds is not a compound number, for the units are *unlike.*

NOTE.—*Compound Numbers* are often called *Denominate Numbers.* The term *denomination* is a name given to the different units of weights and measures.

LINEAR MEASURE.

336. A **Measure** is a *standard unit* established by law or custom, by which the length, surface, capacity, and weight of things are estimated.

337. **Linear Measure** is used in measuring lines and distances.

338. A **Line** is that which has *length* only.

TABLE.

12	inches (*in.*)	=	1 foot, - - -	*ft.*
3	feet	=	1 yard, - - -	*yd.*
$5\frac{1}{2}$	yds., or $16\frac{1}{2}$ ft.	=	1 rod, - - -	*rd.*
40	rods	=	1 furlong, - -	*fur.*
320	rods, or 5280 ft.	=	1 mile, - - -	*mi.*
3	miles	=	1 league, - -	*l.*

339. The **Standard Unit** of length is the **Yard,** which is used in measuring cloths, laces, ribbons, etc. (Art. **900**, App.)

ORAL EXERCISES.

340. 1. Draw a line 4 inches long. A foot. A yard.
2. How long is this book? Your slate? How wide?
3. How long is this table? How wide? How high?
4. In 6 feet how many inches? In 8 ft.? In 9 ft.?
5. How many feet in 7 yards? In 15 yds.? In 20 yds.?
6. In 120 inches how many feet? How many yards?
7. How many feet in 4 rods? In 5 rods?

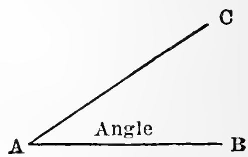
SQUARE MEASURE.

341. **Square Measure** is used in measuring surfaces; as, flooring, land, etc.

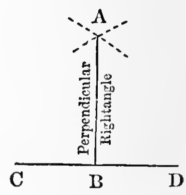
342. A **Surface** is that which has *length* and *breadth* only.

343. An **Angle** is the opening between two lines which meet at a point, as BAC.

The *Lines* AB and AC are called the sides; and the *Point* A, at which they meet, the **Vertex** of the angle.



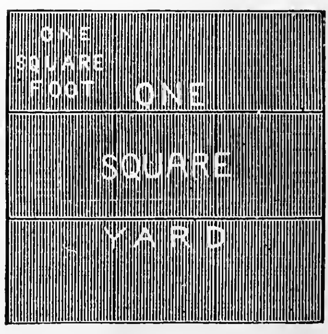
344. When two *straight* lines meet so as to make the *two adjacent angles equal*, the lines are **Perpendicular** to each other, and the two angles thus formed are called **Right Angles**; as, ABC, ABD.



345. A **Square** is a *rectilinear* figure which has *four equal sides*, and *four right angles*.

346. The *measuring unit* of surfaces is a **Square**, each side of which is a *linear unit*.

347. The **Area** of a *figure* is the *quantity of surface* it contains.



3 ft. × 3 ft. = 1 sq. yd.

TABLE.

144	square inches (*sq. in.*)	=	1 square foot,	- -	*sq. ft.*
9	square feet	=	1 square yard,	- -	*sq. yd.*
$30\frac{1}{4}$	sq. yds., or $272\frac{1}{4}$ sq. ft.	=	1 square rod,	- -	*sq. rd.*
160	square rods	=	1 acre,	- - - - -	*A.*
640	acres	=	1 square mile,	- -	*sq. mi.*

(For Surveyor's Measure, see Art. **889**, Appendix.)

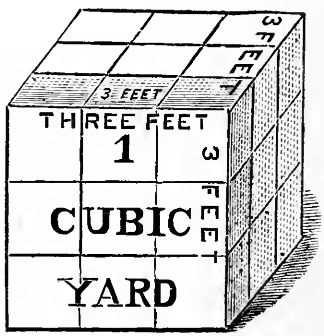
ORAL EXERCISES.

348. 1. How many sq. inches in 2 sq. feet?

2. In 8 sq. yds. how many sq. ft.? In 15 sq. yds.?
3. In 2 acres how many sq. rods? In 3 acres and 5 sq. rd.?
4. How many sq. ft. in 288 sq. inches?
5. In 320 sq. rods how many acres?

CUBIC MEASURE.

349. **Cubic Measure** is used in measuring *solids* or *volume*.



350. A **Solid** is that which has *length, breadth,* and *thickness;* as, timber, boxes of goods, etc.

351. A **Cube** is a regular solid bounded by *six equal squares* called its faces. Hence, its *length, breadth,* and *thickness* are equal to each other.

352. The *measuring unit* of solids is a **Cube** the edge of which is a *linear* unit.

TABLE.

1728	cubic inches (*cu. in.*)	=	1 cubic foot,	- -	*cu. ft.*
27	cubic feet	=	1 cubic yard,	- -	*cu. yd.*
128	cubic feet	=	1 cord of wood,	-	*C.*

353. A **Cord** of wood is a pile 8 ft. long, 4 ft. wide, and 4 ft. high; for $8 \times 4 \times 4 = 128$.

354. A **Cord Foot** is *one* foot in length of such a pile; hence, 1 cord foot = 16 cu. feet; 8 cord ft. = 1 cord.

Oral Exercises.

355. 1. How many cubic inches in 2 cubic feet?

2. How many cu. feet in 2 cubic yards? In 3 cu. yards?
3. How many cubic feet in 2 cords? In 3 cords?
4. In 54 cu. feet, how many cu. yards? In 126 cubic feet?
5. How many cord feet in 3 cords of wood? In 5 cords?
6. In 32 cord feet how many cords? In 72 cord feet?

Liquid Measure.

356. **Liquid Measure** is used in measuring *milk, oil, wine,* etc.

Table.

4	gills (*gi.*)	=	1 pint, - - -	*pt.*
2	pints	=	1 quart, - - -	*qt.*
4	quarts	=	1 gallon, - - -	*gal.*
$31\frac{1}{2}$	gallons	=	1 barrel, - - -	*bar.* or *bbl.*
63	gallons	=	1 hogshead, - -	*hhd.*

357. The **Standard Unit** of Liquid Measure is the *gallon,* which contains 231 cubic inches.

Note.—The *barrel* and *hogshead,* as units of measure, are chiefly used in estimating the contents of cisterns, reservoirs, etc.

ORAL EXERCISES.

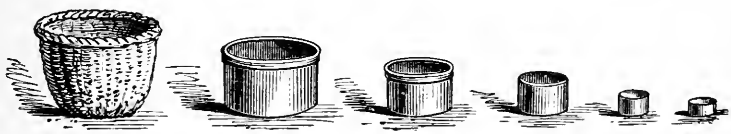
358. 1. How many quarts in 20 pints? In 36 pts.?
2. In 24 pints how many quarts? In 40 pts.?
3. How many gallons in 2 hogsheads? In 5 hhd.?
4. How many qts. in 12 gal. of milk? In 15 gallons?
5. What is the cost of 5 gal. of syrup at 60 cts. a gal.?

DRY MEASURE.

359. Dry Measure is used in measuring *grain, fruit, salt,* etc.

TABLE.

2 pints (*pt.*)	=	1 quart, - - -	*qt.*
8 quarts	=	1 peck, - - -	*pk.*
4 pecks, or 32 qts.	=	1 bushel, - -	*bu.*



bu. ½ bu. pk. ½ pk. qt. pt.

360. The **Standard Unit** of Dry Measure is the *bushel*, which contains 2150.4 cubic inches.*

NOTE.—The *dry quart* is equal $1\frac{1}{6}$ liquid quart nearly.

ORAL EXERCISES.

361. 1. How many pints in 12 quarts? In 25 quarts?
2. How many pecks in 40 qts. of chestnuts?
3. How many bushels in 72 pecks?
4. If you pay 40 cts. for ½ bushel of apples, what must you pay for 5 bushels?
5. If I buy a bushel of walnuts for $3, and sell them at 5 cts. a pt., how much shall I make?
6. How many bu. in 36 pecks? In 96 quarts?

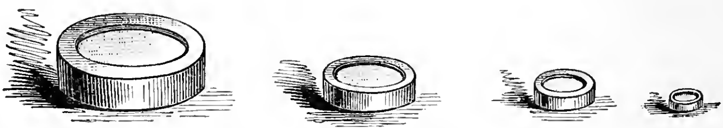
* For the standard weight of a bushel of different grains, see Art. 896, Appendix.

TROY WEIGHT.

362. Troy Weight is used in weighing *gold, silver,* etc.

TABLE.

24 grains (*gr.*)	=	1 pennyweight,	- -	*pwt.*
20 pennyweights	=	1 ounce,	- - - -	*oz.*
12 ounces	=	1 pound,	- - - -	*lb.*



lb. oz. pwt. gr.

363. The **Standard Unit** of weight in the United States, is the *Troy Pound.*

1. How many grains in 6 pennyweights?
2. How many pwt. in 6 lb. 7 oz.? In 8 lb. 5 oz.?
3. How many ounces in 8 pounds of silver?
4. Change 4 pwt. to grains. 25 oz. to pwt.

AVOIRDUPOIS WEIGHT.

364. Avoirdupois Weight is used in weighing coarse articles; as *hay, cotton, groceries,* etc., and all metals except *gold* and *silver.*

TABLE.

16 ounces (*oz.*)	=	1	pound, - - -	*lb.*
100 pounds	=	1	{ cental, or - - hundredweight,	*ctl.* *cwt.*
2000 lb., or 20 cwt.	=	1	ton, - - - -	*T.*

NOTE.—In calculating duties, etc., 112 lb. are called a *cwt.,* and 2240 lb. a *long ton.*

365. Gross Weight is the weight of goods including the boxes, etc., which contain them.

Net Weight is the weight of goods after deducting all allowances.

366. *Comparison of Avoirdupois and Troy Weight.*

7000	grains Troy	=	1 lb. Avoirdupois.
5760	grains "	=	1 lb. Troy.
$437\frac{1}{2}$	grains "	=	1 oz. Avoirdupois.
480	grains "	=	1 oz. Troy.

Apothecaries Weight.

367. Apothecaries Weight is used by Apothecaries in mixing medicines. (Art. **898**, Appendix.)

Table.

20 grains (*gr.*)	=	1 scruple, - -	*sc.*,	or ℈.
3 scruples	=	1 dram, - - -	*dr.*,	or ʒ.
8 drams	=	1 ounce, - - -	*oz.*,	or ℥.
12 ounces	=	1 pound, - -	*lb.*,	or ℔.

Note.—The *pound, ounce*, and *grain* are the same as Troy weight.

Oral Exercises.

368. 1. How many ounces in 5 pounds? In 100 pounds?

2. How many tons in 4000 lbs.? In 6200 lbs.?

3. How many pounds in $\frac{1}{2}$ ton? In $\frac{1}{4}$ ton?

4. At 90 cts., what will $\frac{1}{2}$ lb. of tea cost?

5. At $20 a ton, what will $\frac{3}{4}$ ton of hay cost?

Time.

369. Time is a measured portion of duration. Its divisions are shown in the following

Table.

60 seconds (*sec.*)	=	1 minute, - -	*min.*
60 minutes	=	1 hour, - - -	*hr.*
24 hours	=	1 day, - - -	*d.*
7 days	=	1 week - - -	*wk.*
365 days	=	1 common year,	*c. yr.*
366 days	=	1 leap year, -	*l. yr.*
12 calendar months (*mo.*)	=	1 civil year, -	*yr.*
100 years	=	1 century, - -	*C.*

370. A **Civil Day** is the day adopted by government for business purposes. It begins and ends at midnight, and is divided into two parts of 12 hours each; the former being designated A. M., the latter P. M.

371. The **Solar Year** is equal to 365 d. 5 hr. 48 min. 49.7 sec., or $365\frac{1}{4}$ d. nearly. In 4 years this fraction amounts nearly to 1 day. To provide for this excess, 1 day is added to the mo. of Feb. every 4th year, which is called **Leap Year.***

372. The **Civil** year includes both *common* and *leap* years, and is divided into 12 Calendar months, viz:

January	(Jan.)	31 days.	July	(July)	31 days.
February	(Feb.)	28 "	August	(Aug.)	31 "
March	(Mar.)	31 "	September	(Sept.)	30 "
April	(Apr.)	30 "	October	(Oct.)	31 "
May	(May)	31 "	November	(Nov.)	30 "
June	(June)	30 "	December	(Dec.)	31 "

NOTE.—The following couplet will aid the learner in remembering the months that have 30 days each:

"Thirty days hath September,
April, June, and November."

All the rest have 31 days, except *February*, which in *common* years has 28 days; in *leap* years, 29.

ORAL EXERCISES.

373. 1. How many days in 7 weeks? In 9 weeks?

2. How many weeks in 42 days? In 63 days? In 90 days?

3. How many months in 6 years? In 8 years? In 11 years?

4. In 48 months how many years? In 72 months?

5. How many centuries in 500 years? In 1800 years?

6. At $9 a week, how much will a man earn in 6 weeks?

7. If you pay $3 a week, how long can you board for $60?

8. How many days has a person lived who is 12 years old?

9. If you count 60 a minute, how long will it take to count 1800?

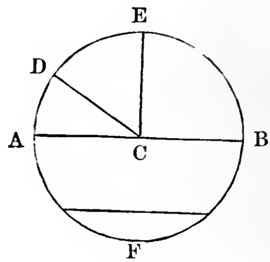
* For an explanation of the mean Solar days, leap years, etc., see Art. 901, Appendix.

CIRCULAR MEASURE.

374. **Circular Measure** is used in *measuring angles, latitude* and *longitude, heavenly bodies,* etc.

375. A **Circle** is a plane figure bounded by a curve line every part of which is *equally distant* from a point within, called the *center.*

376. The **Circumference** of a circle is the curve line by which it is bounded; as ADEBF.



377. The **Diameter** is a *straight line* drawn through the *center*, terminating at each end in the *circumference;* as AB.

378. The **Radius** is a straight line drawn from the *center* to the *circumference,* and is equal to *half* the diameter; as AC, DC.

379. An **Arc** is any part of the circumference; as AD

TABLE.

60 seconds (″)	=	1 minute, - -	′.
60 minutes	=	1 degree, - -	°, or *deg.*
30 degrees	=	1 sign, - - -	*S.*
12 signs, or 360°	=	1 circumference,	*Cir.*

380. The **Measure** of an angle is the *arc* of a circle included between its two sides, as the arc DE.

The **Standard Unit** for measuring angles is the **Degree.**

381. A **Degree** is the angle measured by the arc of $\frac{1}{360}$ part of the *circumference* of a circle.

The length of the *arc* which measures an angle of 1°, varies according to the size of different circles, while the angle remains the same.

A degree at the equator, also the *average* degree of latitude, adopted by the U. S. Coast Survey, is equal 69.16 miles, or $69\frac{1}{6}$ miles, nearly.

382. A **Semi-circumference** (*half* a circumference) is an arc of 180°, as AFB.

383. A **Quadrant**, or *one-fourth* of a circumference, is an arc of 90°, as EB.

A *right angle* contains 90°; for the quadrant, which measures it, is an arc of 90°.

ORAL EXERCISES.

384. 1. How many degrees in $\frac{1}{2}$ a cir.? In $\frac{1}{4}$ cir.?

2. How many degrees in a quadrant? In a right angle?

3. How many miles in 2°? 3°? 5°?

4. Through how many deg. does the hour-hand of a clock move in 12 hours? In 3 hrs.? In 6 hrs.? In 1 hr.?

5. Through how many degrees does the minute-hand of a clock pass in 1 hour? In $\frac{1}{2}$ hr.? In $\frac{1}{4}$ hr.? In 1 minute?

6. In making a voyage around the world, through how many degrees would you sail?

FOREIGN MONEYS.

385. **English** or **Sterling Money** is the *currency* of Great Britain.

TABLE.

4 farthings (*qr.* or *far.*)	=	1 penny, - - - -	*d.*
12 pence	=	1 shilling, - - - -	*s.*
20 shillings	=	1 pound or sovereign,	£.
10 florins (*fl.*)	=	1 pound, - - - -	£.

386. The **Unit** of English Money is the **Pound Sterling**, which is represented by a gold *Sovereign* equal in value to $4.8665.

387. **Canada Money** is expressed in *dollars, cents,* and *mills,* which have the same nominal value as the corresponding denominations of U. S. money.

388. French Money is the *currency* of France.

TABLE.

10	centimes	=	1	decime.
10	decimes	=	1	franc.

389. The **Unit** of French Money is the **Franc**, the value of which in U. S. money is 19.3 cts., or about $\frac{1}{5}$ of a dollar.

NOTE.—The system is founded upon the *decimal* notation; hence, all operations in it are the same as those in U. S. money.

390. The Money Unit of the German Empire is the **Mark**, which is divided into 100 pennies.

The value of a Mark is $0.238, or $$\frac{1}{4}$ nearly.*

MISCELLANEOUS TABLES.

12	things	=	1	dozen.
12	dozen	=	1	gross.
12	gross	=	1	great gross.
20	things	=	1	score.
24	sheets	=	1	quire of paper.
20	quires	=	1	ream.
2	reams	=	1	bundle.
5	bundles	=	1	bale.
2	leaves	=	1	folio.
4	leaves	=	1	quarto, or 4to.
8	leaves	=	1	octavo, or 8vo.
12	leaves	=	1	duodecimo, or 12mo.

NOTE.—The terms *folio, quarto, octavo,* etc., denote the *number* of leaves into which a sheet of paper is folded in making books.

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* For Table of Foreign Coins, see Art. 631.

ORAL EXERCISES.

391. 1. How many farthings in 5 shillings?

2. How many pence in £3?

3. What cost 8 meters of lace, at 12 francs a meter?

4. How many Sovereigns will 12 yards of silk cost, at 10s. a yard?

5. What will 8 doz. eggs cost, at a cent apiece?

6. What will 18 quires of paper cost, at 20 cts. a quire?

7. What will a gross of buttons cost, at 15 cts. a dozen?

QUESTIONS.

334. What is a simple number? 335. Compound? 336. What is a measure? 337. For what is linear measure used? 338. What is a line? Recite the table. 339. What is the standard unit of length?

341. For what is square measure used? 342. What is a surface? 343. An angle? The vertex? 344. A right angle? 345. What is a square? 347. What is the area of a figure? Repeat the table.

349. For what is cubic measure used? 350. What is a solid? 351. What is a cube? Recite the table.

356. For what is liquid measure used? Recite the table.

359. For what is dry measure used? Repeat the table?

362. For what is Troy weight used? Recite the table. 363. The standard unit of weight? 364. For what is Avoirdupois weight used? Recite the table. What is a long ton? 367. For what is Apothecaries weight used?

369. What is Time? Recite the table. 370. What is a civil day? The meaning of A.M.? Of P.M.? 371. Length of a Solar year? 372. How many calendar months in a civil year? Name them.

374. For what is circular measure used? 375. What is a circle? 376. The circumference? 377. Diameter? 378. Radius? 379. An arc? Table? 380. The measure of an angle? 381. What is a degree? 382. A semi-circumference? 383. A quadrant? How many degrees in a right angle?

385. What is English or Sterling money? Repeat the table. 386. The unit of English money? Its value? 387. How is Canada money expressed? 388. What is French money? Recite the table. 389. The unit? Its value? 390. What is the money unit of the German Empire? Its value? Recite the miscellaneous tables.

REDUCTION.

ORAL EXERCISES.

392. 1. How many pints in 3 gallons?

ANALYSIS.—In 1 gal. there are 4 qt., and in 3 gal., 3 times 4, or 12 qt. In 1 qt. there are 2 pints, and in 12 qts., 12 times 2, or 24 pints, *Ans.*

2. How many feet in 4 yd.? In 8 yd.?
3. In 6 sq. yd. how many square feet?
4. How many gills in 9 quarts? In 12 quarts?
5. In 10 bushels how many pints?
6. In 5 days how many minutes?

393. Reduction is changing Compound Numbers from one denomination to another without *altering* their *values.* It is of two kinds, *Descending* and *Ascending.*

394. Reduction Descending is changing *higher* denominations to *lower;* as, yards to feet, etc.

395. To reduce *Higher* Denominations to *Lower.*

1. Reduce 34 rods 4 yds. 2 ft. to feet.

ANALYSIS.—As $5\frac{1}{2}$ yds. make 1 rod, there must be $5\frac{1}{2}$ times as many yards as rods; and $(34 \times 5\frac{1}{2}) + 4$ (the given yds.) = 191 yds. (Art. **208.**) Again, as 3 ft. make 1 yd. there must be 3 times as many feet as yards; and $(191 \times 3) + 2$ (the given ft.) = 575 feet. Hence, the

$$\begin{array}{l} 34 \text{ r. } 4 \text{ yd. } 2 \text{ ft.} \\ \underline{\quad 5\frac{1}{2}} \\ 191 \text{ yds.} \\ \underline{\quad 3} \\ 575 \text{ ft., } \textit{Ans.} \end{array}$$

RULE.—*Multiply the highest denomination by the number required of the next lower to make a unit of the higher, and to the product add the lower denomination.*

Proceed in this manner with the successive denominations, till the one required is reached.

2. In 5 mi. 12 rd. 4 yd. 2 ft. how many feet?
3. Reduce 143 lb. 3 oz. 6 pwt. to grains.
4. Reduce 217 tons 35 lb. to pounds.
5. Reduce 106 tons 68 lb. to ounces.

396. Reduce the following:

6. 23 mi. 5 rd. 6 ft. to feet.
7. 24 lb. 4 oz. 6 pwt. to gr.
8. 48 T. 2 cwt. 36 lb. to oz.
9. 328 gal. 3 qt. 1 pt. to gills.
10. 85 hhd. 15 gal. to pints.
11. 45 bu. 3 pk. 4 qt. to pints.
12. 124 sq. yd. 8 sq. ft. to sq. in.
13. 32 A. 6 sq. rd. to sq. feet.
14. 26 C. 7 cu. ft. to cu. ft.
15. 36 wk. 1 d. 5 hr. to min.
16. 21 yr. 26 d. to hours.
17. 145° 28″ to seconds.
18. £68 3s. 6d. to pence.
19. £205 7s. 3½d. to far.

20. How many sec. in 3 yr. 42 wk. 5 d. 9 hr. 17 min.?

21. What will 7 bu. 3 pk. of cranberries cost at 8 cts. a quart?

22. Bought 84 gal. syrup at 75 cts. a gal., and sold it at 22 cts. a quart; what was the gain?

23. What is the value of 12 lb. 5 oz. 6 pwt. of gold, at 87 cts. a pwt.?

ORAL EXERCISES.

397. 1. In 64 pints how many quarts? How many gallons?

ANALYSIS.—Since in 2 pints there is 1 qt., in 64 pints there are 32 quarts. In 4 qts. there is 1 gallon, and in 32 qts. there are 8 gallons, *Ans.*

2. How many feet in 120 inches? How many yards?
3. In 60 ounces Troy, how many pounds? In 168 ounces?
4. In 72 hr. how many days? In 96 hours?
5. How many cords of wood in 72 cord feet?
6. Change 120s. to pounds, and 240 pence to shillings.

398. **Reduction Ascending** is changing *lower* denominations to *higher*; as, feet to yards, etc.

399. To reduce *Lower* denominations to *Higher*.

1. Reduce 6900 inches to rods, etc.

ANALYSIS.—Since 12 in. make 1 ft., 6900 in. = as many feet as 12 is contained times in 6900, or 575 ft. As 3 ft. make 1 yd., 575 ft. = as many yd. as 3 is contained times in 575, or 191 yd. and 2 ft. over. Finally, as $5\frac{1}{2}$ yd. make 1 rod, 191 yd. = as many rods as $5\frac{1}{2}$ is contained times in 191, or 34 rd. and 8 half yards, or 4 yd. over. (Art. **217.**) *Ans.* 34 rd. 4 yd. 2 ft. Hence, the

```
12 ) 6900  in.
  3 ) 575  ft.
5½ ) 191  yd. 2 ft.
       2
11 ) 382
      34  rd. 4 yd.
```

RULE.—*Divide the given denomination by the number required to make one of the next higher.*

Proceed in this manner with the successive denominations, till the one required is reached. The last quotient, with the several remainders annexed, will be the answer.

NOTE.—The *remainders* are the same denomination as the respective *dividends* from which they arise.

400. PROOF.—Reduction *Ascending* and *Descending prove* each other; for, one is the *reverse* of the other.

2. In 245640 ft. how many miles, rods, etc.?

Ans. 46 mi. 4 fur. 7 rd. 1 yd. 1 ft. 6 in.

Reduce the following to the denominations indicated:

3. 34248 gills to bbl.

4. 46840 pt. to hhd.

5. 653674 pwt. to lb.

6. 426508 gr. to lb.

7. 35624 oz. to cwt.

8. 8420724 oz. to tons.

9. 29728 in. to rods.

10. 48400 ft. to miles.

11. 85264 sq. ft. to sq. rods.

12. $2118165\frac{1}{2}$ sq. yd. to acres.

13. 16568 cu. ft. to cords.

14. 43228 qt. to bushels.

15. 28956 pt. to bushels.

16. 5685720 hr. to com. yr.

17. 856700 d. to weeks.

18. 4683248 far. to pounds.

19. What will a can of milk containing 28 gal. 3 qt. cost, at 6 cts. a quart?

20. At $0.75 a yd., what will it cost to build a wall 182 r. long?

21. If a grocer buys 3 bu. of cranberries at $2.25 a bu. and sells them at 9 cts. a quart, how much does he make?

DENOMINATE FRACTIONS.

401. Denominate Fractions are fractions of denominate Integers, and may be common or decimal.

402. To reduce Denominate Fractions, Common or Decimal, of higher denominations, to Integers of lower denominations.

1. Reduce $\frac{7}{8}$ yard to integers of lower denominations.

SOLUTION. — 1 yd. = 3 ft., and $\frac{7}{8}$ yd $\times 3 = \frac{21}{8}$ ft., or $2\frac{5}{8}$ ft. Again, $\frac{5}{8}$ ft. $\times 12 = \frac{60}{8}$ in., or $7\frac{1}{2}$ in.

$$\frac{7}{8}\text{ yd.} \times 3 = \frac{21}{8},\text{ or } 2\frac{5}{8}\text{ ft.}$$
$$\frac{5}{8}\text{ ft.} \times 12 = \frac{60}{8},\text{ or } 7\frac{4}{8}\text{ in.}$$
Ans. 2 ft. $7\frac{1}{2}$ in.

2. Reduce .875 yard to integers of lower denominations.

SOLUTION.—1 yd. = 3 ft., and .875 yd. $\times$ 3 = 2.625 ft. Again, .625 $\times$ 12 = 7.500 inches.
The answer is 2 ft. 7.5 inches, the same as above.

$$\begin{array}{r} .875\text{ yd.} \\ 3 \\ \hline 2.625\text{ ft.} \\ 12 \\ \hline 7.500\text{ in.} \end{array}$$

NOTE.—Pointing off 3 figures in the several products is equivalent to dividing them by 1000, the denominator of the given decimal. Hence, the

RULE.—*Multiply the given numerator, whether common or decimal, and the remainder, if any, by the successive numbers which will reduce a* ***unit*** *of the given fraction to the denomination required, and divide the several products by the given denominator.*

3. In $\frac{7}{18}$ day, how many hours and minutes?
4. In $\frac{12}{15}$ week, how many days, hours, etc.
5. Reduce $\frac{20}{45}$ mile to furlongs, etc.
6. Reduce $\frac{36}{84}$ bu. to pecks, quarts, etc.
7. Reduce $\frac{3}{7}$ sq. mile to acres, rods, and yards.
8. Reduce $\frac{1}{256}$ gal. to the fraction of a gill.
9. What part of a pint is $\frac{4}{2565}$ of a bushel?
10. Reduce £.4625 to shillings and pence.
11. Reduce .756 gallons to quarts and pints.
12. Reduce .6254 days to hours, minutes, and seconds.
13. Reduce .856 cwt. to ounces.
14. Reduce .7582 of a bushel to pecks, etc.
15. Reduce 0.98 rod to yards, feet, and inches.

403. To Reduce Denominate Integers or Fractions of lower, to Fractions, either Common or Decimal, of higher denominations.

16. Reduce 7s. 6d. to the *common* fraction of a pound.

SOLUTION.—7s. 6d. = 90d., and £1 = 240d. Now, $£\frac{90}{240} = £\frac{3}{8}$, *Ans.*

17. Reduce 3 quarts 1 pint 2 gills to the *decimal* of a gallon.

SOLUTION.—Writing the numbers under each other, the lowest denomination at the top, we divide the 2 gi. by 4, and place the quotient .5 below, at the right of the next higher denomination. Thus, 1.5 pt. ÷ 2 = .75 qt., and so on. Hence, the

$$\begin{array}{r|l} 4 & 2.\ \text{gi.} \\ \hline 2 & 1.5\ \text{pt.} \\ \hline 4 & 3.75\ \text{qt.} \\ \hline \end{array}$$

Ans. .9375 gal.

RULE.—*Reduce the given compound number to the lowest denomination mentioned for the numerator, and a* ***unit*** *of the required fraction to the same denomination for the denominator.*

For decimals, divide the given numbers as in reducing integers to higher denominations. (Art. **399.**)

NOTE.—If the lowest denomination of the given number contains a fraction, the number must be reduced to the parts indicated by the denominator of the fraction.

18. Reduce $\frac{4}{5}$ pint to the fraction of a bu. (Art. **179,** *2°*.)

SOLUTION.—$\frac{4}{5 \times 2 \times 8 \times 4} = \frac{\cancel{4}}{5 \times 2 \times 8 \times \cancel{4}} = \frac{1}{80}$ bu., *Ans.*

19. What part of a bushel is 3 pk. 5 qt. 1 pt. ?
20. What part of a gallon is 3 qt. 1 pt. 3 gills?
21. Reduce 9 hr. 15 min. 12 sec. to the fraction of a week.
22. Reduce $15\frac{3}{4}$ gr. to the fraction of a pound Troy.
23. What part of an acre is $18\frac{4}{5}$ square feet?
24. Reduce 3 pk. 2 qt. 1 pt. to the decimal of a bushel.
25. Change 18 hr. 9 min. to the decimal of a day.
26. Change 2 ft. 8 in. to the decimal of a yard.
27. Change 8 oz. 7 pwt. 12 gr. to the decimal of a lb. Troy.
28. Change .4 of a pt. to the decimal of a gallon.
29. Change .25 lb. to the decimal of a ton.
30. Reduce 2 yr. 3 mo. 18 d. to the decimal of a year.

404. **To find what part one Compound Number is of another:**

Reduce the numbers to the same denomination, and make the number denoting the part the numerator, and that with which it is compared the denominator. (Arts. **226**, **249**.)

31. What part of 2 gal. 3 qt. 1 pt. is 1 gal. 2 qt.?
32. What part of 4 wk. 2 d. 6 hr. is 3 d. 12 hours?
33. What part of 15 miles 40 rd. is 6 mi. 30 rods?
34. What decimal of 4 lb. 2 oz. 12 pwt. is 6 oz. 8 pwt.?
35. What decimal of 10 bu. 3 pk. 4 qt. is 4 bu. 1 pk. 5 qt.

405. **To Reduce Metric to Common Weights and Measures.**

1. Reduce 84 decimeters to feet.

OPERATION.

```
      39.37 in.
        8.4 m.
      -------
      15748
     31496
     -------
12 ) 330.708 in.
Ans.  27.559 ft.
```

ANALYSIS.—Taking 39.37 in., the value of the principal metric unit, as the standard, we multiply it by the given metric number expressed in the same metric unit; and 84 dm. = 8.4 m.

Since 1 m. is equal to 39.37 in., 8.4 m. are equal to 8.4 times 39.37 in., or 330.708 in., and 330.708 in. = 27.559 ft., *Ans.* Hence, the

RULE.—*Multiply the value of the principal metric unit of the Table by the given metric number expressed in the same unit, and reduce the product to the denomination required.* (Art. **399**.)

2. In 45 kilos, how many pounds? *Ans.* 99.207 lb.
3. In 63 kilometers, how many miles?
4. Reduce 75 liters to gallons.
5. Reduce 56 dekaliters to bushels.
6. Reduce 120 grams to ounces.
7. Reduce 137.75 kilos to pounds.
8. In 36 ars, how many square rods?

ANALYSIS.—In 1 ar there are 119.6 sq. yd.; hence in 36 ars there are 36 times as many. Now $119.6 \times 36 = 4305.6$ sq. yd., and 4305.6 sq. yd. $\div 30\frac{1}{4} = 142.33$ sq. rods, *Ans.*

9. In 60.25 hektars, how many acres?
10. In 120 cu. meters, how many cu. feet?

406. To reduce Common to Metric Weights and Measures.

11. Reduce 2190 yds. 2 ft. 11 in. to kilometers.

OPERATION.

$$\begin{array}{r} 2190 \text{ yd. } 2 \text{ ft. } 11 \text{ in.} \\ 3 \\ \hline 6572 \text{ ft.} \\ 12 \\ \hline 39.37\ )\ 78875 \text{ in.} \\ \hline 2003.429+ \text{ m.} \\ \textit{Ans. } 2.003429+ \text{ Km.} \end{array}$$

EXPLANATION.—Reducing the given number to inches we have 78875 in. Dividing this number by 39.37, the number of inches in a meter, we have 2003.429+ m. To reduce meters to Km. we remove the decimal point 3 places to the left. *Ans.* 2.003429+ Km. Hence, the

RULE.—*Divide the given number by the value of the principal metric unit of the Table, and reduce the quotient to the denomination required.*

NOTE.—Before dividing, the given number should be reduced to the denomination in which the *value* of the *principal unit* is expressed.

12. In $63\frac{3}{4}$ yards, how many meters?
13. Reduce 13750 pounds to kilograms.
14. Reduce 250 liquid quarts to liters.
15. Reduce 2056 bu. 3 pecks to kiloliters.
16. In 3 cwt. 15 lb. 12 oz., how many kilos?
17. In 7176 sq. yards, how many sq. meters?
18. In 40.471 acres, how many hektars?
19. In 14506 cu. feet, how many cu. meters?
20. In 36570 cu. yards, how many cu. meters?

ADDITION.

407. The method of Adding, Subtracting, Multiplying, and Dividing Compound Numbers is the same as the corresponding operations in simple numbers and special rules are unnecessary.

NOTE.—1. The apparent difference arises from their scales of increase, one being *variable* and *irregular*, the other *decimal* and *uniform*.

1. What is the sum of 18 bu. 3 pk. 5 qt. 1 pt., 24 bu. 2 pk. 6 qt., 6 bu. 2 pk. 7 qt. 1 pt., 8 bu. 3 pk. 4 qt. 1 pt.?

EXPLANATION.—The sum of the right-hand col. is 3 pt. = 1 qt. 1 pt. Set the 1 pt. under the col. of pints, and adding the 1 qt. to the col. of qt., the sum is 23 qt. = 2 pk. 7 qt. Write the 7 qt. in the col. of quarts, and adding the 2 pk. to the col. of pk., proceed as before. *Ans.* 59 bu. 0 pk. 7 qt. 1 pt.

OPERATION.

bu.	pk.	qt.	pt.
18	3	5	1
24	2	6	
6	2	7	1
8	3	4	1
59	0	7	1

(2.)

£	s.	d.	far.
5	4	2	3
6	7	8	2
5	6	7	1
12	8	6	2

(3.)

gal.	qt.	pt.	gi.
4	2	1	2
6	3	0	1
7	0	1	2
4	3	1	0

(4.)

wk.	da.	hr.	min.
2	3	8	40
4	6	5	10
2	5	20	35
6	4	18	23

5. What is the sum of 5 rd. 4 yd. 2 ft. 7 in., 6 rd. 5 yd. 2 ft. 6 in., 4 rd. 4 yd. 0 ft. 4 in., 3 rd. 3 yd. 2 ft. 8 in.?

NOTE.—2. When a *fraction* occurs in the amount in any denomination except the *lowest*, it should be reduced to integers of lower denominations, and united with like integers. Thus, in Ex. 5 the $\frac{1}{2}$ yd. = 1 ft. 6 in., which added to 2 ft. 1 in. make 3 ft. 7 in.; and 1 yd. plus 3 ft. plus 7 in. equals 2 yd. 0 ft. 7 in.

	rd.	yd.	ft.	in.
	5	4	2	7
	6	5	2	6
	4	4	0	4
	3	3	2	8
	21	$1\frac{1}{2}$	2	1
		$\frac{1}{2} =$	1	6
Ans.	21	2	0	7

6. What is the capacity of 3 bins holding respectively 35 bu. 3 pk. 4 qt., 42 bu. 1 pk. 6 qt., and 56 bu. 2 pk. 5 qt.?

7. How much land in 3 farms containing 87 A. 48 sq. rd., 97 A. 67 sq. rd., and 65 A. 42 sq. rd.?

8. Bought 3 casks of oil; holding 2 hhd. 30 gal. 2 qt.; 3 hhd. 10 gal.; 1 hhd. 13 gal. 1 qt.; how much did all hold?

9. Add together 23 yr. 2 mo. 3 wk. 5 d., 68 yr. 3 mo. 2 wk. 3 da., 60 yr. 4 mo. 1 wk. 6 d., 49 yr. and 4 d.

10. Required the number of miles, etc. in 3 roads, measuring 23 mi. 67 rd.; 32 mi. 65 rd.; and 46 mi. 28 rods.

11. A mason plastered one room containing 45 square yards 7 ft. 6 in., another 25 sq. yd. 6 ft. 95 in., another 38 sq. yd. 4 ft. 41 in.; what was the amount of his plastering?

12. One pile of wood contains 10 C. 38 ft. 39 in., another 15 C. 56 ft. 73 in., another 30 C. 19 ft. 44 in., another 17 C. 84 ft. 21 in.; how much do they all contain?

13. Find the sum of 45 mi. 17 rd. 5 yd. 2 ft. 9 in., 43 mi. 44 yd. 1 ft. 8 in., 89 mi. 216 rd. 3 yd. 2 ft. 5 in.

14. What is the sum of £$\frac{1}{6}$, $\frac{1}{8}$s., and $\frac{1}{3}$d.?

NOTE.—3. *Denominate Fractions* should be reduced to integers of lower denominations, then added as above. (Art. **402**.)

OPERATION.

£$\frac{1}{6}$	=	3s.	4d.	0	far.
$\frac{1}{8}$s.	=	0s.	1d.	2	far.
$\frac{1}{3}$d.	=	0s.	0d.	$1\frac{1}{3}$	far.
	Ans.	3s.	5d.	$3\frac{1}{3}$	far.

15. Add $\frac{5}{8}$ bu. $\frac{11}{16}$ pk. $\frac{3}{8}$ qt. $\frac{1}{4}$ pt., $\frac{7}{12}$ bu. $\frac{1}{2}$ pk. $\frac{2}{3}$ qt. $\frac{1}{4}$ pt.

16. Add $\frac{2}{3}$ of $\frac{7}{12}$ day, $\frac{2}{5}$ of $\frac{5}{7}$ hr., $\frac{4}{15}$ of $\frac{19}{20}$ min., and $\frac{3}{4}$ of $2\frac{4}{5}$ sec.

17. Add $\frac{5}{8}$ lb. to $\frac{2}{3}$ oz. $\frac{3}{5}$ pwt.

18. Add $\frac{5}{7}$ wk. to $\frac{3}{5}$ d. $1\frac{5}{7}$ hr.

19. $\frac{3}{4}$ gal. to $\frac{1}{4}$ qt. $1\frac{1}{2}$ pt.

20. £$\frac{7}{8}$ to $\frac{3}{4}$s. $2\frac{2}{3}$d.

SUBTRACTION.

408. **1.** From 35 rd. 2 yd. 1 ft. 8 in., take 22 rd. 2 yd. 2 ft. 6 in.

OPERATION.

	35 rd.	2 yd.	1 ft.	8 in.
	22	2	2	6
Ans.	12	$4\frac{1}{2}$	2	2
		$\frac{1}{2}$ =	1	6
Or,	12	5	0	8

EXPLANATION. — Write the numbers and proceed as in simple subtraction. Taking 6 in. from 8 in. leaves 2 inches. As 2 ft. cannot be taken from 1 ft., we take 1 yd. = 3 ft. from 2 yd., and adding it to 1 ft., we have 4 ft., and 2 ft. from 4 ft. leave 2 ft. Again, 2 yd. cannot be taken from the 1 yd. remaining. But 1 rd. = $5\frac{1}{2}$ yd., added to 1 yd. make $6\frac{1}{2}$ yd., from which subtract 2 yd., and $4\frac{1}{2}$ yd. remain. Finally, 22 rd. from 34 rd. leave 12 rd. The $\frac{1}{2}$ yd. = 1 ft. 6 in., which added to the above make 12 rd. 5 yd. 0 ft. 8 in., *Ans.*

2. From 121 hhd. 28 gal. 1 qt., take 63 hhd. 21 gal. 3 qt.

3. Bought 2 silver pitchers, one weighing 2 lb. 10 oz. 10 pwt. 7 gr., the other 2 lb. 3 oz. 12 pwt. 5 gr.; what is the difference in their weight?

4. A merchant had $228\frac{3}{4}$ yards of cloth, and sold $115\frac{5}{8}$ yards; how much had he left?

5. From 25 mi. 7 fur. 8 rd. 12 ft. 6 in., take 16 mi. 6 fur. 30 rd. 4 ft. 8 in.

6. A man owning 95 A. 75 rd. 67 sq. ft. of land, sold 40 A. 86 rd. 29 ft.; how much had he left?

7. A tanner built two cubical vats, one containing 116 ft. 149 in., the other 245 ft. 73 in.; what is the difference between them?

8. A man having 65 C. 95 ft. 123 in. of wood in his shed, sold 16 C. 117 ft. 65 in.; how much had he left?

409. To find the Exact Number of Years, Months, and Days, between two dates.

9. What is the difference of time between July 4th, 1879, and Nov. 15th, 1882?

ANALYSIS.—	The time from July 4th, 1879 to July 4th, 1882	= 3 yr.
	The time from July 4th to Nov. 4th	= 4 mo
	The time from Nov. 4th to Nov. 15th.	= 11 d.
	Ans. 3 yr. 4 mo. 11 d. Hence, the	

RULE.—*First find the number of entire years, next the number of entire months, then the days in the parts of a month.*

NOTE.—1. The day on which a note or draft is *dated*, and that on which it becomes *due*, must *not both* be reckoned. It is customary to omit the *former* and count the *latter.*

10. A ship started on a trading voyage round the world Mar. 3d, 1875, and arrived back Aug. 24th, 1878; how long was she gone?

11. What is the time from Oct. 15th, 1875, to March 10th, 1882?

12. A note dated Oct. 2d, 1870, was paid Dec. 25th, 1882; how long was it from its date to its payment?

13. A ship sailed on a whaling voyage, Aug. 25th, 1880, and returned April 15th, 1882; how long was she gone?

14. A mortgage was dated April 10th, 1875, and was paid Aug. 25, 1880; how long did it run?

15. How many days did a note run which was dated Sept. 18th, 1879, and paid Jan. 15th, 1880?

ANALYSIS.—In Sept. it had 30−18=12 days; in Oct., 31 d.; in Nov., 30 d.; in Dec., 31 d.; in Jan., 15 d. Hence,

OPERATION.

Sept.	30 − 18	= 12 d.
Oct.		= 31 d.
Nov.		= 30 d.
Dec.		= 31 d.
Jan.		= 15 d.
	Ans.	119 d.

NOTE.—2. To find the number of days between two dates, write in a col. the number of days remaining in the first mo., and the number in each succeeding month, including those in the last; the sum will be the number of days required.

16. A note dated May 21st, 1879, was paid Nov. 28th, 1879; how many days did it run?

17. What is the number of days between Oct. 5th, 1879, and March 3d, 1880?

18. A person started on a journey Aug. 19th, 1869, and returned Nov. 1st, 1869; how long was he absent?

19. A note dated Jan. 31st, 1870, was paid June 30th, 1870; how many days did it run?

20. How many days from May 23d, 1868, to Dec. 31st, following?

21. The latitude of New York is 40° 42′ 43″ N., that of St. Augustine, Fla., is 29° 48′ 30″; what is the difference of their latitude?

NOTE.—3. When two places are on opposite sides of the *Equator*, the *difference* of latitude is found by *adding* their latitudes.

22. The latitude of Cape Horn is 55° 59′ S., that of Cape Cod is 42° 1′ 57″ N.; what is the difference of their latitude?

23. The longitude of Cambridge, Mass., is 71° 7′ 22″, that of St. Louis is 90° 15′ 16″; what is the difference of their longitude?

24. The lon. of Paris is 2° 20′ E., that of Washington D. C. is 77° 0′ 15″ W.; what is the difference of their longitude?

MULTIPLICATION.

410. **1.** If a man can build a fence 12 rd. 1 yd. 2 ft. 5 in. long in one day, how long a fence can he build in 6 days?

ANALYSIS.—In 6 d. he can build 6 times as much as in 1 d. 6 times 5 in. are 30 in. = 2 ft. 6 in. Write the 6 under the in., and add the 2 ft. to the next product. Proceed in this way till all the denominations are multiplied.

	12 rd.	1 yd.	2 ft.	5 in.
				6
	73 rd.	$4\frac{1}{2}$ yd.	2 ft.	6 in.
		$(\frac{1}{2})$ =	1	6
Ans.	73 rd.	5 yd.	1 ft.	0 in.

NOTE.—If a fraction occurs in the product of any denomination except the lowest, it should be reduced to *lower denominations*, and be united to those of the same name as in Compound Addition.

2. Multiply 8 lb. 6 oz. 3 pwt. by 8.

3. Multiply 27 gal. 3 qt. 1 pt. 3 gi. by 7.

4. Multiply 26 mi. 87 rd. 4 yd. 2 ft. by 9.

5. What is the weight of 12 silver cups, each weighing 8 oz. 17 pwt. 6 gr.?

6. How much water in 28 casks, each containing 54 gal. 3 qt. 1 pt. 2 gi.?

7. If a railroad car goes 21 mi. 2 fur. 10 rd. per hour, how far will it go in 25 hours?

DIVISION.

411. **1.** A grocer paid £5 2s. 9d. for 4 boxes of sugar; how much was that a box?

OPERATION.

4)	£5	2s.	9d.
Ans.	£1	5s.	$8\frac{1}{4}$d.

ANALYSIS.—Since 4 boxes cost £5 2s. 9d., 1 box will cost $\frac{1}{4}$ as much, and £5÷4 = £1 and £1 over. Reducing the remainder to the next lower denomination, and adding the 2s., we have 22s., which divided by 4 = 5s. and 2s. over. Reducing 2s. as before, continue the division till each denomination is divided.

2. A silversmith melted up 2 lb. 8 oz. 10 pwt. of silver, which he made into 6 spoons; what was the weight of each?

3. If 8 persons consume 85 lb. 12 oz. of meat in a month, how much is that apiece?

4. A man traveled 50 mi. and 32 rd. in 11 hours; at what rate did he travel per hour?

5. A man had 285 bu. 3 pk. 6 qt. of grain, which he wished to carry to market in 15 equal loads; how much must he carry at a load?

LONGITUDE AND TIME.

412. The Earth *turns* on its axis once in 24 hours; hence, $\frac{1}{24}$ part of 360°, or 15° of longitude, passes under the sun in 1 hour.

Again, $\frac{1}{60}$ of 15° lon., or 15′, passes under the sun in 1 min. of time. And $\frac{1}{60}$ of 15′, or 15″ lon., passes under the sun in 1 sec. of time, as seen in the following

413. *Comparison of Longitude and Time.*

360°	lon.	make a	difference of	24 hrs.	of time.
15°	"	"	"	1 hr.	"
1°	"	"	"	4 min.	"
1′	"	"	"	4 sec.	"
1″	"	"	"	$\frac{1}{15}$ sec.	"

414. The **Longitude** of a place is the number of deg., min., and sec., reckoned on the equator, between a *standard meridian* (marked 0°) and the *meridian* of the given place.

All places are in *East* or *West* longitude, according as they are East or West of the *Standard* Meridian, until 180°, or half the circumference of the Earth is reached.

NOTES.—1. The English reckon lon. from the meridian of Greenwich; the French from that of Paris. Americans generally reckon it from the meridian of Greenwich; sometimes from that of Washington.

2. When two places are on *opposite* sides of the *Standard Meridian*, the *difference* of lon. is found by *adding* their longitudes. (Art. **409**, N 3.)

415. To find the Difference of Longitude between two places, the Difference of Time being known.

1. The difference of time between New York and Chicago is 54 min. 19 sec. What is the difference of longitude?

ANALYSIS. — Since 15′ of lon. make a difference of 1 min. of time, there must be 15 times as many min. of lon. as there are min. and sec. of time, and (54 min. 19 sec.) $\times 15 = 13° \; 34' \; 45''$, *Ans.* Hence, the

OPERATION.

	54 m.	19 sec.
		15
13°	34′	45″, *Ans.*

RULE.—*Multiply the difference of time, expressed in hours, minutes, and seconds, by 15; the product will be the difference of longitude in degrees, minutes, and seconds.* (Art. **412.**)

2. The difference of time between Boston and Albany is 9 min. 2 sec.; what is the difference of longitude?

3. The difference of time between Savannah, Ga., and Portland, Me., is 43 min. 32.13 sec.; what is the dif. of longitude?

4. The difference of time between Boston and Detroit is 47 min. 56 sec.; what is the difference of longitude?

5. The difference of time between Philadelphia and Cincinnati is 37 min. 8.4 sec.; what is the difference of longitude?

6. The difference of time between Louisville, Ky., and Burlington, Vt., is 49 min. 20 sec.; what is the dif. of longitude?

416. To find the Difference of Time between two places, the Difference of Longitude being known.

1. The difference of longitude between Chicago and Boston is 16° 34′ 15″; what is the difference of time?

ANALYSIS.—Since 15° lon. make a difference of 1 hour of time, there must be $\frac{1}{15}$ as many hours, minutes, and seconds, as there are deg., min., and sec. of lon., and (16° 34′ 15″) ÷ 15 = 1 hr. 6 min. 17 sec. Hence, the

OPERATION.

15) 16° 34′ 15″

Ans. 1 hr. 6 min. 17 sec.

RULE.—*Divide the difference of longitude, in degrees, minutes, and seconds, by 15; the quotient will be the difference of time in hours, minutes, and seconds.*

2. The difference of longitude between Cambridge, Mass., and Charlottesville, Va., is 7° 23′ 49″; what is the difference of time?

3. The lon. of St. Louis is 90° 15′ 15″, that of Charleston, S. C., is 79° 55′ 38″; what is the difference of time?

4. The lon. of Berlin is 13° 23′ 45″ E., that of New Haven, Ct., is 72° 55′ 24″ W.; what is the difference of time?

5. The lon. of Montreal is 73° 25′ W., that of New Orleans is 90° 2′ 30″ W.; what is the difference of time?

6. The lon. of Paris is 2° 20′ E., Rome is 12° 27′ E.; what is the difference of time?

7. The lon. of West Point is 73° 57′ W., that of Washington, D. C., 77° 0′ 15″ W.; what is the difference of time?

8. How much earlier does the sun rise in Albany, lon. 73° 44′ 50″, than in St. Paul, Min., lon. 93° 4′ 55″? Than in Astoria, Oregon, lon. 124°?

9. When it is 9 A.M. in New York, lon. 74° 3′, what is the time in Richmond, Va., lon. 77° 25′ 45″? In San Francisco, lon. 122° 26′ 45″?

QUESTIONS.

392. What is reduction? 393. Descending? 396. Rule? 397. Reduction Ascending? 399. Rule? How proved?

401. What is a denominate fraction? 402. How reduce them from higher denominations to integers of lower? 403. How reduce denominate integers to fractions of higher denominations?

404. How find what part one number is of another? 405. How reduce metric to common weights and measures? 406. How reduce common to metric weights and measures?

407. How are compound numbers added, subtracted, multiplied, and divided? From what does the apparent difference arise? 409. How find the difference between two dates in years, months, and days? How find the difference of latitude between two places on opposite sides of the equator?

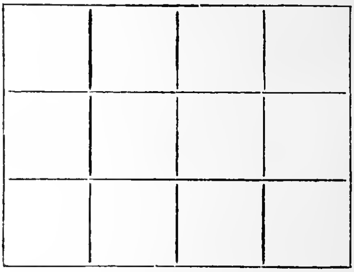
414. What is the longitude of a place? When is a place in East longitude? When in West? From what meridian do the English reckon longitude? The French? Americans?

415. How find the difference of longitude when the difference of time is given? 416. How find the difference of time when the difference of longitude is given?

MEASUREMENT OF SURFACES.

ORAL EXERCISES.

417. 1. How many sq. feet in the surface of a blackboard 4 ft. long and 3 ft. wide?



ANALYSIS.—Let the sides of the blackboard be divided into 4 equal parts, and the ends into 3 equal parts, each denoting a linear foot. The blackboard contains as many sq. feet as there are squares in the figure. Since there are 4 squares in 1 row, in 3 rows there are 3 times 4, or 12 squares. *Ans.* 12 sq. feet.

2. How many sq. feet in a flagging stone 8 ft. long and 4 feet wide?

3. How many sq. feet in a strawberry bed 20 ft. long and 5 ft. wide?

4. How many sq. yards in a lawn whose length is 9 yards and its breadth 7 yards?

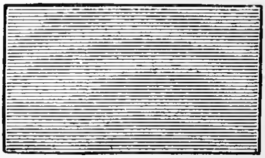
5. If a meadow is 12 rods long and 8 rods wide, how many sq. rods does it contain?

6. A house lot containing 84 sq. rods is 7 rods wide; what is its length?

7. I wish to lay out an orchard 12 rods in width; what must be its length to contain 240 sq. rods?

8. What is the difference between 4 square feet and 4 feet square?

WRITTEN EXERCISES.



418. A **Plane Figure** is one which represents a plane or flat surface.

419. The **Perimeter** of a plane figure is the line which *bounds* it.

420. The **Area** of a plane figure is the quantity of surface it contains.

421. A **Rectangle** is a plane figure having four sides and four right-angles. (Art. **418.**)

422. When all the sides of a rectangle are equal it is called a **Square.**

423. The **Dimensions** of a rectangular figure are its *length* and *breadth.*

424. To find the Area of Rectangular Surfaces.

1. How many square rods in a garden 18 rods long and 12 rods wide?

SOLUTION.—A rectangle 18 rods long and 1 rod wide will contain 18 sq. rods. And a garden 18 rods long and 12 rods wide will contain 12 times 18, or 216 sq. rods, *Ans.* Hence, the

OPERATION.

$$\begin{array}{r} 18 \text{ rods.} \\ \underline{12} \\ Ans.\ 216 \text{ sq. rods.} \end{array}$$

RULE.—*Multiply the length by the breadth.*

NOTES.—1. Both *dimensions* should be reduced to the *same denomination* before they are multiplied.

2. One *line* is said to be multiplied by another, when the *number* of units in the former are taken as many times as there are *like units* in the latter. (Art. 83, *1°.*)

3. The *area* and *one side* of a rectangular surface being given, the *other* side is found by *dividing the area by the given side.* (Art. **119*a*.**)

2. How many yards of carpeting 1 yd. wide will it take to cover a floor 22 ft. long and 15 ft. wide?

3. How many yards of carpeting 27 in. wide will it take to cover the same floor.

4. In a meadow 68 rd. long and 43 rd. wide, how many acres?

5. A building lot is 50 ft. front, and contains half an acre; how far back does it extend?

6. At 25 cts. per sq. foot, what is the cost of an acre of land?

7. Bought a rectangular farm 240 rods long and 88 rods wide, at $15 per acre; what was the cost?

8. The length of a pasture is 234 meters, and its breadth is 87 meters: what is its area in sq. meters?

9. The area of a meadow is 210.6 sq. meters, and its length is 64.8 meters; what is its width?

10. If I pay \$276 for 92 meters of broadcloth 1.5 meters wide, what is that per square meter?

11. How many acres in a field 800 rods long, and 128 rods wide?

12. Find the area of a square field whose sides are 65 rods in length.

13. A man fenced off a rectangular field containing 3750 sq. rods, the length of which was 75 rods; what was its breadth?

14. How many hektars in a rectangular field 475.5 meters long and 246 meters wide?

15. The length of the Capitol at Washington is 751 ft., its width 348 ft.; how many sq. rods, and how many acres does it cover?

16. What is the difference between two asparagus beds one of which is 2 rods square, and the other contains 2 sq. rods?

17. The length of the main Centennial building in Philadelphia was 1880 ft., and the width 464 ft.; how many acres did it cover?

18. A speculator bought 50 acres of land at \$50 per acre, and sold it in villa lots of 5 rods by 4 rods, at \$150 a lot; what did he make by the operation?

19. A garden 27 yd. long and 15 yd. wide has a gravel walk round it 6 feet wide; what did the walk cost, at 50 cts. per square yard?

20. What will it cost to carpet a floor 18 by 16 ft., the carpet being 27 in. wide, and its cost \$1.12 a yard?

21. What is the cost of paving a street 628 ft. long and $60\frac{1}{2}$ ft. wide, at \$2.25 a sq. yard?

22. How many tiles 10 in. square are required to lay a side walk 168 ft. long and $5\frac{1}{2}$ ft. wide?

23. What will it cost to concrete a court 168 ft. square, at \$3.75 per sq. yard?

24. A farm containing 150 acres, is 200 rods long; what is its width? What will it cost to build a wall around it, at \$4 a rod?

25. How many planks 15 ft. long and 6 in. wide will it take to floor a room 20 ft. long and $15\frac{1}{2}$ ft. wide?

MEASUREMENT OF SOLIDS.

425. A **Rectangular Body** is one bounded by *six rectangular* sides, each *opposite* pair being *equal* and *parallel ;* as, boxes of goods, blocks of hewn stone, etc.

426. When *all* the sides are equal, it is a **Cube**; when the *opposite* sides only are equal, it is a **Parallelopiped.**

427. The **Contents** or **Volume** of a body is the *quantity* of *matter* or *space* it contains.

428. The **Dimensions** of a *rectangular* body are its length, breadth, and thickness.

429. To find the contents or volume of Rectangular Bodies.

1. How many cubic feet in a block of granite 4 ft. long, 3 ft. wide, and 2 ft. thick?

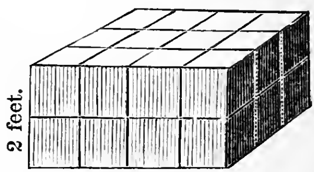


ILLUSTRATION.—Let the block be represented by the adjoining figure, the length of which is divided into 4 equal parts, the width into 3, and the thickness into 2 parts, each of which is a *linear* foot. Since the block is 4 ft. long and 3 ft. wide, in the upper face there are 3 times 4, or 12 sq. feet. Now, if the block were 1 foot thick it must have as many cu. feet as there are sq. feet in the upper face. But the given block is 2 ft. thick; therefore, it contains 2 times (4×3), or 24 cu. feet, *Ans.* Hence, the

RULE.—*Multiply the length, breadth, and thickness together.* (Art. **424.**)

NOTES.—1. When the contents and two dimensions are given, the other dimension may be found by dividing the contents by the product of the two given dimensions. (Art. **119*a*.**)

2. *Excavations* and *embankments* are estimated by the cubic yard. In removing earth, a cu. yard is called a *load.*

2. How many cu. feet of air in a school-room 20 ft. square and $10\frac{1}{2}$ ft. high?

3. How many cu. feet in a mound 54 ft. long, 36 ft. wide, and 12 ft. high?

4. How many loads of earth must be removed in digging a cellar 48 ft. long, 25 ft. wide, and $8\frac{1}{2}$ ft. deep?

5. What will it cost to dig such a cellar, at $33\frac{1}{3}$ cts. a cu. yard?

6. What will it cost to fill in a street 55 feet wide, 600 ft. long, and $5\frac{1}{2}$ ft. below grade, at 42 cents a cu. yard?

7. What is the volume of a cube whose edge is 5 yd. 2 ft. 6 in.?

8. Find the volume of a cube whose edge is $15\frac{1}{2}$ ft.?

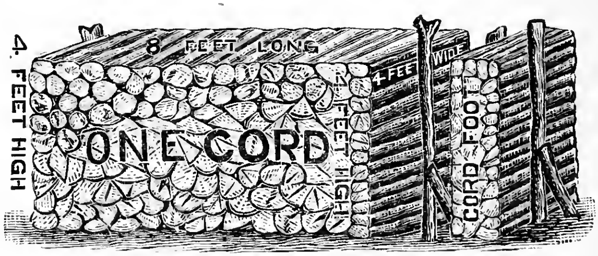
9. The width of a reservoir is 24 ft., its depth 8 ft., and its volume 5760 cu. ft.; what is its length?

Wood Measure.

430. A **Cord of Wood** is a pile 8 feet long, 4 feet wide, and 4 feet high. (Art. **353.**)

431. A **Cord Foot** is 1 foot in length of such a pile. Hence,

1 cord foot = 16 cubic feet
8 cord feet = 1 cord.



1. How many cords of wood in a pile 35 ft. long, 6 ft. high, and 4 ft. wide?

2. Find the number of cords in a pile of wood 42 ft. long, $5\frac{1}{2}$ ft. high, and 8 ft. wide.

3. At $4.25 a cord, what will a pile of wood 26 ft. long, 4 ft. wide, and 4 ft. high cost?

4. In 3 cord feet, how many cu. feet? In 5 cord feet?

5. How many cubic feet in 12 cords? In 24 cords?

6. If a pile of wood is 28 ft. long, 4 ft. wide, how high must it be to contain 112 cords?

7. How many cord feet in a load of wood 8 ft. long, 4 ft. high, and 3 ft. wide?

8. What is the worth of a pile of wood 4 ft. in height, 6 ft. in length, and $3\frac{1}{2}$ in width, at $4.50 per cord?

9. What must be the height of a load of wood that is 6 ft. long and 4 ft. wide, to contain a cord?

Masonry.

432. Stone Masonry is sometimes estimated by the *perch*. **Brickwork** is estimated by the thousand bricks.

NOTES.—1. A perch of stone masonry is $16\frac{1}{2}$ ft. long, $1\frac{1}{2}$ ft. wide, and 1 ft. high, which is equal to $24\frac{3}{4}$ cu. ft. It is customary, however, to call 25 cu. ft. a perch.

2. The *average size* of bricks is 8 in. long, 4 in. wide, and 2 in. thick.

In estimating the labor of brickwork by cu. feet, it is customary to measure the length of each wall on the outside; no allowance being made for windows, doors, or corners. But a deduction of $\frac{1}{10}$ the solid contents is made for the mortar.

1. How many perch (25 cu. ft.) in the walls of a cellar, the thickness of which is 1 ft. 6 in., the height 8 ft., each side wall being 42 ft.. and each end wall 24 feet?

2. At $4.75 a perch, what will it cost to build the walls of the above cellar?

3. How many bricks will it take to build the walls of a house 50 ft. long, 25 ft. wide, 21 ft. high, and 1 ft. thick, deducting $\frac{1}{10}$ of the contents for the mortar, but making no allowance for windows and doors?

4. How many bricks will be required to build a house, the walls of which are 48 ft. long, 24 ft. wide, 42 ft. high, and 1 ft. thick, making no allowance for windows, doors, or corners?

5. At $3.50 per M. for bricks, deducting $\frac{1}{10}$ for mortar, and $4.25 per M. for laying them, what will the walls of such a house cost?

BOARD MEASURE.

433. A **Board Foot** is 1 ft. long, 1 ft. wide, and 1 in. thick; that is, a *square foot* 1 inch thick.

434. A **Board Inch** is $\frac{1}{12}$ of a board foot; that is, 1 inch long by 12 inches wide and 1 inch thick. Hence, **Twelve** board feet are equal to 1 cubic foot.

435. *Sawed timber,* as plank, joists, etc., is estimated by cu. feet; *hewn timber,* as beams, etc., either by board feet or cu. feet; *round timber,* as masts, etc., by cu. feet.

WRITTEN EXERCISES.

436. To find the Contents of Boards, Planks, etc.

1. How many board feet in a board 11 ft. long, 18 in. wide, and 1 inch thick?

EXPLANATION.—Multiplying the length in feet by the width and thickness expressed in inches, we have 198 board inches. Dividing this product by 12, the result is $16\frac{1}{2}$ board feet, *Ans.*

OPERATION.

$$11 \times 18 \times 1 = 198 \text{ in.}$$
$$198 \div 12 = 16\tfrac{1}{2} \text{ ft.}$$
Ans. $16\frac{1}{2}$ ft.

2. How many board feet in a scantling 14 ft. long, 4 in. wide, and $2\frac{1}{2}$ in. thick?

SOLUTION.—Multiplying the length in feet by the width and thickness expressed in inches, we have 14 ft. $\times\ 4 \times 2\frac{1}{2} = 140$, and $140 \div 12 = 11\frac{2}{3}$ board ft., *Ans.* Hence, the

RULE.—*Multiply the length in feet by the width and thickness expressed in inches, and divide the product by 12; the quotient will be in board feet.*

NOTES.—1. The *standard thickness* of a board is 1 inch. If *less* than 1 inch, it is *disregarded;* if *more* than 1 inch. it becomes a *factor* in finding the contents of plank, scantling, etc.

2. If a board is *tapering,* multiply the length by *half* the *sum* of the two ends.

3. The *approximate* contents of *round timber* or *logs* may be found by *multiplying* $\frac{1}{4}$ of the mean circumference by *itself*, and this product by the *length.*

3. What is the number of feet in a board 14 ft. long and 17 in. wide?

4. Find the contents of a tapering board 15 ft. long, 17 in. wide at one end and 11 in. at the other?

5. Required the contents of 8 boards, 11 ft. long and 15 in. wide?

6. What is the worth of 120 boards of the above size at 4 cents a board foot?

7. Find the contents of a board 16 ft. long, 15 in. wide, and $\frac{1}{2}$ in. thick?

8. What are the contents of 8 scantlings 15 ft. long, 4 in. wide, and 3 in. thick, board measure?

9. How many feet in a beam 16 ft. long, 8 in. wide, and 4 in. thick, board measure? Cubic measure?

10. What cost 24 joists whose dimensions are 4 in. by 3 in. and 11 ft. long, at 25 cts. a cu. foot?

11. What must be the length of a piece of timber 16 in. by 15 in., to contain 20 cu. feet?

12. How many cu. feet in a log 65 ft. long, whose mean circumference is 8 ft.?

13. How many cords of wood in such a log?

14. How many feet of inch boards will it take to build a fence 4 ft. high and 125 ft. long?

15. At $2.25 per 100 ft., what will the boards cost for such a fence?

16. What amount of inch boards would be required to make a box 4 ft. long, $3\frac{1}{2}$ ft. wide, and $2\frac{1}{2}$ ft. deep?

17. What is the cost of a stock of 9 boards 14 ft. long 15 in. wide, at $23.50 per 1000 ft.

18. How many cu. feet in a mast 54 ft. long, the circumference of which is 9 ft.; and what will it cost at $1.09 a cu. foot?

19. What cost 12 planks 14 ft. long, $12\frac{1}{2}$ in. wide, and $2\frac{1}{2}$ in. thick; at $18 per M.?

20. How many cu. feet in a log 62 ft. long and 28 ft. in circumference?

Rectangular Cisterns, Bins, Etc.

437. The *Capacity* of rectangular cisterns, bins, etc., is measured by *cubic* measure, but the results are commonly expressed in *units* of *Liquid* and *Dry* Measure.

438. To find the Number of Gallons in Rectangular Cisterns, etc.

1. How many gal. of water will a rectangular cistern 6 ft. long, 4 ft. wide, and 3 ft. deep contain?

Analysis.—The product of 6 ft. × 4 × 3 = 72 cu. feet in the cistern; and 72 × 1728 = 124416 cu. inches. Again, in 1 gallon there are 231 cu. inches, and 124416 ÷ 231 = $538\frac{46}{77}$ gal., *Ans.* (Art. **357.**)

2. How many bushels in a bin 11 ft. long, 4 ft. wide, and 3 ft. high?

Analysis.—11 ft. × 4 × 3 = 132 cu. feet, and 132 cu. ft. × 1728 = 228096 cu. inches. Now 1 bu. contains 2150.4 cu. in. and 228096 cu. in. ÷ 2150.4 = $106\frac{1}{14}$ bu., *Ans.* (Art. **360.**) Hence, the

Rule.—*Find the number of cubic inches in the thing measured, and reduce them to liquid or dry measure, as may be required.* (Arts. **356, 359.**)

3. Find the number of gallons in a cistern 8 ft. long by 6 ft. wide and 5 ft. deep.

4. How many hogsheads in a tank 12 ft. square and 8 feet deep?

5. In a reservoir 40 ft. long, 30 ft. wide, and 15 ft. high, how many hogsheads?

6. I wish to build a cistern containing 5000 gal., whose base is 12 ft. by 8; what must be its height?

7. If a reservoir 45 ft. long, 28 ft. wide, contains 40000 hhd., how high must it be?

8. At $1.12½ a bushel, what is the value of a bin of wheat 9 ft. long, 5 ft. wide, and 4 ft. deep?

9. How many cu. feet in a bin which will contain 300 bushels of grain?

439. *Shorter Methods.*—Since 2150.4 cu. inches ÷ 1728 cu. in. = $1\frac{1}{4}$, it follows that a bushel must contain $1\frac{1}{4}$ cu. ft. nearly. (Art. **360.**) Hence, we have the following methods:

1st. *Divide* the number of cu. feet in a bin by $1\frac{1}{4}$ and the quotient will be the approximate number of bu. in the bin.

2d. *Multiply* the number of bu. in a bin by $1\frac{1}{4}$, and the product will be the approximate number of cu. feet in the bin.

3rd. A *ton* (2000 lbs.) of Lehigh white ash, egg size, *coal* in bins measures $34\frac{1}{2}$ cu. ft.

A *ton* of white ash Schuylkill, egg size, measures 35 cu. ft.

A *ton* of pink, gray, and red ash, egg size, measures 36 cu. ft.

4th. A *ton* of *hay* upon a scaffold measures about 500 cu. ft.; when in a mow, 400 cu. feet; and in well settled stacks, 10 cubic yards.

10. How many bushels of corn can be put into a bin 6 ft. long, 5 ft. wide, and 4 ft. deep? *Ans.* 96 bushels.

11. A farmer has a bin 10 ft. long, $6\frac{1}{2}$ ft. wide, and 4 ft. deep; how many bushels does it hold?

12. A bin holding 150 bu. is 6 ft. wide and 4 ft. deep; what is its length?

13. A bin containing 280 bushels is 10 ft. long and 7 ft. wide; what is its depth?

14. What must be the length of a bin 8 ft. wide, 5 ft. deep, to contain 320 bushels?

15. At $$1\frac{1}{2}$ a bushel, what is the value of a bin of wheat 12.5 ft. long, 6 ft. wide, and 4 ft. deep?

16. A farmer filled a bin 8 ft. long, 7 ft. wide, and 5 ft. deep, with the corn raised on 5 acres; how many bushels was that per acre?

17. How many tons of Lehigh white ash, egg size coal, will fill a bin 12 ft. long, 8 ft. wide, 6 ft. high?

18. How many tons of hay in a mow 20 ft. long, 18 ft. wide, and 14 ft. high?

Oral Problems for Review.

440. **1.** At 3 cts. a yd., what will 5 mi. of telegraph wire cost?

2. My neighbor's farm is $\frac{3}{4}$ mile square; how many acres did it contain?

3. Bought 40 acres of land at 75 cts. per sq. rod, and sold it so as to double my money; required my gain?

4. At 25 cts. a gallon, what is a family's milk bill for 60 days, taking 2 qts. daily?

5. If a man lives $2\frac{1}{2}$ miles from the City Hall, how many miles will he travel in 6 days, making 1 trip a day?

6. The length of a blackboard is 6 ft., its width 4 ft.; how many sq. yards does it contain?

7. If a garden is 5 rods long and 4 rods wide, how many rods in its perimeter?

8. If $\frac{1}{2}$ oz. of spice costs 8 cents, what will $2\frac{1}{2}$ pounds cost?

9. At $1 a sq. yard, what will it cost to carpet a room 18 feet long and 15 ft. wide?

10. A stationer paid $1.25 a gross for pencils, and sold them for a cent apiece; how much did he gain on 5 gross?

11. In a certain school are 72 girls, and $\frac{5}{9}$ of the pupils are boys; how many pupils in the school?

12. The surface of a cube is 150 sq. inches; what is the surface of one side?

13. What fraction of a semi-circumference is 45 degrees?

14. How many writing books of 36 pages each can be made from a half ream of paper?

15. How many days in 7 of the longest months?

16. A can do a job in 2 days, B in 3 days; what part will each do in one day?

17. How long will it take both to do the same job working together?

18. How many yards of carpeting $\frac{3}{4}$ yd. wide, will carpet a room 18 ft. square?

19. How many sq. yards in the pavement of a street 60 ft. wide and 800 ft. long?

20. How many suits of clothes can be made from 648 yards, allowing 4 yds. to a suit?

Written Problems for Review.

441. 1. How many acres in a piece of land 184 rods long and 96 rods wide?

2. What will 16568 cu. feet of wood cost, at $\$3\frac{1}{2}$ a cord?

3. How many dollars can be made out of 50 lb. 9 oz. of silver, allowing $412\frac{1}{2}$ grains to a dollar?

4. How many cubic inches in a box whose length is 30 inches, its breadth 18, and its depth 15 inches?

5. How many cubic inches in a block of marble 43 inches long, 18 inches broad, and 12 inches thick?

6. How many cubic feet of air in a school-room 16 feet long, 15 feet wide, and 9 feet high?

7. How many cubic feet in a pile of wood 16 feet long, 6 feet wide, and 5 feet high? How many cords?

X 8. How many cords of wood in a pile 140 feet long, $4\frac{1}{2}$ feet wide, and $6\frac{1}{2}$ feet high?

9. At 50 cts. per decister, what will a ster of wood cost?

10. What will a metric ton of hemp cost, at 25 cts. per kilo?

11. At 6 cts. per liter, what cost a hektoliter of milk?

X 12. How many square yards in the four sides of a room 18 feet long, $17\frac{1}{2}$ feet wide, and $14\frac{1}{2}$ feet high?

X 13. How many square yards of plastering will it take to cover the four sides and the ceiling of a room 18 feet square, and 15 feet high?

14. How many yards of muslin 3 qrs. wide, are equal to 36 yds. brocatelle, which is $1\frac{1}{2}$ yard wide?

15. How many yards of silk 3 qrs. wide, will 51 yds. of cambric line, which is $1\frac{1}{4}$ yd. wide?

16. What will it cost to pave a street 3 mi. 115 rods long, and 2 rods wide, at $\$15\frac{1}{2}$ a square rod?

X 17. A man having 15 acres and 60 rods of land, laid it out in lots each containing 12 sq. rods, and sold the lots at \$150 apiece; how much did he realize for his land?

18. What is the worth of a pile of wood 18 ft. long, $10\frac{1}{2}$ ft. high, and $9\frac{1}{2}$ wide, at $\$3\frac{1}{2}$ a cord?

X 19. How many times will a wheel of a railroad car, 9 ft. in circumference, turn round in going 1500 miles?

20. How long would it take a cannon ball, flying at the rate of 8 miles per minute, to reach the moon, a distance of 240000 miles?

21. The velocity of light is 11875000 miles per minute, and it takes 8 minutes for it to pass from the sun to the earth; how far from the sun is the earth; and how many weeks would it take to travel this distance, 30 miles an hour?

22. How many bricks will it take to pave a sidewalk 75 feet long and 8 feet wide, each brick being 8 inches long and 4 inches wide?

23. Required to reduce 5 mi. 6 fur. 23 rods 5 yd. and 8 in. to inches, and prove the operation.

24. Allowing 1 shingle to cover 24 sq. inches, how many shingles will be required to cover the roof of a house 50 feet long, the rafters on each side being 29 feet long?

25. How many farms of 160 A. in a township 6 miles square?

26. How many bricks will it take to build a prison 60 feet long, 25 feet wide, and 48 feet high, whose walls are 1 foot thick, the bricks 8 in. long, 4 in. wide, and 2 in. thick?

27. If the pendulum of a clock vibrates 65 times per minute, how much time will it gain in a common year?

28. How many years would it take to count a billion, counting 60 a minute, working 10 hours a day, and allowing 365 days to a year?

QUESTIONS.

418. What is a plane figure? 419. The perimeter of a plane figure? 420. The area?

421. What is a rectangle? 423. The dimensions of a rectangular figure? 424. How find the area of rectangular surfaces? When the area and one side are given, how find the other?

425. What is a rectangular solid? 427. The contents? 428. The dimensions?

430. What are the dimensions of a cord of wood? 431. How many cubic feet does it contain? Cord feet?

432. How is stone masonry estimated? Brickwork?

433. What is a board foot? 434. A board inch? How many board feet in a cu. foot? 435. How are sawed and hewn timber estimated? Round timber? 436. How find the contents of boards, plank, etc.?

438. How find the contents of cubical bins, cisterns, etc.?

PERCENTAGE.

ORAL EXERCISES.

442. 1. When a number is divided into a *hundred* equal parts, what is one of the parts called? Two of the parts? Five? Ten?

2. A man paid $100 for a horse and sold it for $105; how many dollars did he gain? How many *hundredths* of the *cost* did he gain?

3. What part of $100 is $5? (Art. **226**.)

4. If I pay $100 for a sofa and sell it for $94, how many dollars shall I lose? How many *hundredths* of the *cost?*

443. The number of hundredths gained or lost is called the *Rate per cent.*

444. **Per Cent,** means by the *hundred,* or simply *hundredths.*

Thus, 3 per cent is 3 hundredths of a number; 5 per cent is 5 hundredths, etc.

445. The **Sign of Per Cent** is %. Thus, 4% means 4 per cent.

446. The *process* of calculating by *hundredths* is called **Percentage.**

5. A farmer lost 8 sheep out of every 100 of his flock; what per cent of them did he lose?

6. A man gave away $10 out of every $100 of his income; what per cent of his income did he give away?

7. A teacher having a class of 150 pupils, promoted 10% of them; how many were promoted?

447. Per cent is expressed by decimals, by %, or by fractions.

TABLE.

Sign.	Decimal.		Fraction.	Sign.	Decimal.		Fraction.
1%	.01	=	$\frac{1}{100}$	$\frac{1}{2}$%	.005	=	$\frac{1}{200}$
5%	.05	=	$\frac{1}{20}$	$3\frac{1}{2}$%	.035	=	$\frac{7}{200}$
10%	.10	=	$\frac{1}{10}$	$\frac{1}{4}$%	.0025	=	$\frac{1}{400}$
25%	.25	=	$\frac{1}{4}$	$6\frac{1}{4}$%	.0625	=	$\frac{1}{16}$
50%	.50	=	$\frac{1}{2}$	$18\frac{3}{4}$%	.1875	=	$\frac{3}{16}$
75%	.75	=	$\frac{3}{4}$	$33\frac{1}{3}$%	$.33\frac{1}{3}$	=	$\frac{1}{3}$
100%	1.00	=	$\frac{1}{1}$	$112\frac{1}{2}$%	1.125	=	$1\frac{1}{8}$

448. Since *hundredths* occupy two decimal places, every *per cent* requires, at least, two decimal figures. Hence, if the given per cent is less than 10, a *cipher* must be prefixed to the figure denoting it. Thus, 2% is written .02; 6%, .06, etc.

NOTES.—1. A *hundred* per cent of a number is equal to the *number itself;* for $\frac{100}{100}$ is equal to 1.

2. In expressing per cent, when the *decimal point* is used, the words *per cent* and the *sign* (%) must be omitted, and *vice versa.* Thus, .05 denotes 5 per cent, and is equal to $\frac{5}{100}$ or $\frac{1}{20}$; but .05 per cent or .05% denotes $\frac{5}{100}$ of $\frac{1}{100}$, and is equal to $\frac{5}{10000}$ or $\frac{1}{2000}$.

449. To read any given Per Cent, expressed Decimally.

Call the first two decimal figures per cent; and those on the right, decimal parts of 1 per cent.

NOTE.—Parts of 1 per cent, when easily reduced to a common fraction, are often read as such. Thus, .105 is read 10 and a half per cent; .0125 is read one and a quarter per cent.

Read the following as rates per cent:

8. .06; .052; .085; .094.

9. .012; .174; .0836; .154.

10. .1857; .2352; .1685; .7225.

11. $.12\frac{1}{2}$; $.08\frac{1}{3}$; $.16\frac{1}{2}$; .5775.

12. 1.07; 2.53; 4.65; 2.338.

13. $5.33\frac{1}{3}$; 4.125; 8.0623; $6.73\frac{1}{3}$.

450. Express the following by Com. Frac. in lowest terms:

14.	4%.	**17.**	20%.	**20.**	75%.	**23.**	150%.
15.	6%.	**18.**	25%.	**21.**	100%.	**24.**	200%.
16.	10%.	**19.**	50%.	**22.**	125%.	**25.**	500%.

26. To what common fraction is $8\frac{1}{3}\%$ equal?

ANALYSIS.—$8\frac{1}{3}\% = \frac{8\frac{1}{3}}{100}$, or $8\frac{1}{3} \div 100$; and $8\frac{1}{3} \div 100 = \frac{25}{3} \times \frac{1}{100} = \frac{25}{300}$, or $\frac{1}{12}$, *Ans.* (Art. **220**.)

27. To what common fraction is $\frac{1}{2}\%$ equal?

ANALYSIS.—$\frac{1}{2}\% = .005$; and $.005 = \frac{5}{1000}$, or $\frac{1}{200}$, *Ans.* (Art. **185**.)

28. $6\frac{1}{4}\%$ = what fraction? $37\frac{1}{2}\%$? $18\frac{3}{4}\%$? $2\frac{2}{5}\%$?

MENTAL EXERCISES.

451. 1. What per cent of a number is $\frac{1}{3}$ of it?

ANALYSIS.—Since any number equals 100% of itself, $\frac{1}{3}$ of a number must equal $\frac{1}{3}$ of 100%, or $33\frac{1}{3}\%$, *Ans.*

2. What per cent of a number is $\frac{1}{2}$? Is $\frac{2}{3}$? $\frac{3}{4}$? $\frac{1}{5}$?
3. What per cent of a number is $\frac{3}{5}$? $\frac{1}{4}$? $\frac{4}{5}$? $\frac{3}{8}$?
4. What per cent of a number is $\frac{3}{10}$? $\frac{7}{10}$? $\frac{1}{8}$? $\frac{5}{8}$?

WRITTEN EXERCISES.

452. To change a Common Fraction to an equivalent per cent.

1. What per cent of a number is $\frac{24}{60}$ of it?

ANALYSIS.—Every number is equal to 100% of itself; hence, $\frac{24}{60}$ of a number $= \frac{24}{60}$ of 100%, or $\frac{1}{60}$ of 2400%. Therefore, annexing ciphers to the numerator and dividing by the denominator, we have .40 or 40%. Hence, the

OPERATION.

$\frac{24}{60} = 24 \div 60$

60) 24.00 (.40, *Ans.*
240

RULE.—*Annex ciphers to the numerator, and divide it by the denominator.* (Art. **249**.)

2. What per cent of a number is $\frac{25}{40}$ of it?
Ans. .625, or $62\frac{1}{2}\%$.
3. What per cent of a number is $\frac{24}{30}$ of it? Is $\frac{30}{75}$?

4. What per cent of a number is $\frac{75}{120}$ of it? Is $\frac{8}{45}$?
5. What per cent of a number is $\frac{35}{95}$? Is $\frac{72}{240}$?
6. What per cent of a number is $\frac{90}{270}$ of it? Is $\frac{240}{500}$?

453. The **Parts** or **Elements** employed in calculating percentage are the *Base*, the *Rate per cent*, the *Percentage*, and the *Amount* or *Difference*.

454. The **Base** is the number on which the percentage is calculated.

455. The **Rate** is the number of *hundredths* of the *base* taken.

456. The **Percentage** is the part of the base indicated by the *rate per cent.*

Thus, when it is said that 4% of $50 is $2, the rate is .04, the base $50, and the percentage $2.

457. The **Amount** is the *sum* of the base and percentage.

458. The **Difference** is the *base less* the *percentage.*

Thus, if the base is $75 and the percentage $4, the amount is $75 + 4 = $79; the difference is $75 − $4 = $71.

The *relation* between these parts is such, that if any *two* of them are *given*, the *other three* may be found.

PROBLEM I.

Oral Exercises.

459. 1. What is 5% of $60?

Analysis.—5% of a number equals $\frac{5}{100}$, or $\frac{1}{20}$ of the number, and $\frac{1}{20}$ of $60 is $3. Therefore, 5% of $60 is $3.

How much is

2. 4% of $80?
3. 6% of $100?
4. 7% of $200?
5. 8% of $400?
6. 10% of $250?

How much is

7. $12\frac{1}{2}$% of 320 rods?
8. 20% of 275 gallons?
9. 25% of 260 acres?
10. 50% of 700 men?
11. 100% of $2000?

12. A teacher who received $30 a month, had her salary increased 10%; what was the increase per month?

13. From a cistern of water holding a hogshead $33\frac{1}{3}\%$ leaked out; how many gallons remained?

Written Exercises.

460. To find the *Percentage* when the Base and Rate are given.

1. What is 9% of $3465?

OPERATION.

$3465	B.
.09	R.
$311.85	P.

Analysis.—9% of a number equals $\frac{9}{100}$, or .09 of it; therefore, the percentage must be .09 times $3465, which is equal to $311.85, *Ans.* Hence, the

Rule.—*Multiply the base by the rate, expressed in decimals.*

Formula.—*Percentage = Base × Rate.*

Notes.—1. When the *rate* is an *aliquot* part of 100, the *percentage* may be found by taking a *like part* of the base. (Art. **447.**) Thus, for 20%, take $\frac{1}{5}$; for 25%, take $\frac{1}{4}$, etc.

2. When the *base* is a *compound number*, the *lower* denominations should be reduced to a *decimal* of the *highest;* or the whole number to the *lowest* denomination mentioned; then apply the rule. (Ex. 5.)

3. Finding a *per cent* of a number is the same as finding a *fractional part* of it. (Art. **226.**)

2. What is 37% of 1546 pounds 8 ounces?
Ans. 572.205 pounds.

Find the percentage of the following:

3. 25% of $5068.
4. 42% of £6248.
5. 75% of 8675 bu. 3 pk.
6. 100% of 2240 pounds.
7. $6\frac{1}{4}\%$ of $1000.
8. $37\frac{1}{2}\%$ of $1568.
9. 50% of £2436.
10. $12\frac{1}{2}\%$ of $2874.
11. $22\frac{1}{2}\%$ of 865 acres.
12. $42\frac{1}{2}\%$ of $4820.
13. $62\frac{1}{2}\%$ of 4360 feet 6 in.
14. $33\frac{1}{3}\%$ of $564175.

15. Which is greater, 7 per cent of $6300, or 6 per cent of $7200?

16. Which is less, 9 per cent of \$82000, or 6 per cent of \$93000?

17. A man had \$8750 in bank and drew out 8% of it at one time, and then 10% of the remainder; how much had he left on deposit?

18. A man who owed \$9584 failed in business and paid 40% of his debts; how much did he pay?

19. A land speculator paid \$6075 for a farm, and sold it at 15% less than cost; how much did he lose?

461. The **Amount** is found by *adding* the percentage to the base.

462. The **Difference** by *subtracting* the percentage from the base.

FORMULAS.— $\begin{cases} \textit{Amount} = \textit{Base} + \textit{Percentage}. \\ \textit{Difference} = \textit{Base} - \textit{Percentage}. \end{cases}$

20. A began business with \$4200 capital, and increased it 7 per cent the first year; what amount of capital did he then have?

SOLUTION.—\$4200 × .07 = \$294.00, and \$4200 + \$294 = \$4494, *Ans.*

21. B commenced business with \$6500 capital, and lost 6 per cent of it the first year; how much capital had he then?

SOLUTION.—\$6500 × .06 = \$390.00, and \$6500 − \$390 = \$6110, *Ans.*

22. A man sold his house, which cost him \$5760, at $12\frac{1}{2}$% above cost; what amount did he receive for his house?

23. A farmer raised 4256 Hl. of grain, and sold $12\frac{1}{2}$% of it; how many hektoliters did he have left?

24. What is the amount of \$252500 increased by 20% of itself?

25. A commander having an army of 16293 men, lost $33\frac{1}{3}$% of them by sickness and desertion; how many soldiers remained?

26. A farmer owning 3560 sheep, lost 50 per cent of them by disease; how many had he left?

27. If my annual income is \$3560, and I spend 25% of it each year, how much shall I save in 4 years?

PROBLEM II.

ORAL EXERCISES.

463. 1. A farmer had 100 sheep and lost 50 of them; what part of them did he lose? How many hundredths? How many per cent?

ANALYSIS.—50 is equal to $\frac{50}{100}$, or 1 half; and since per cent means hundredths, $\frac{50}{100}$ equals 50 per cent.

2. A man spent \$25 for a suit of clothes, which was $\frac{1}{4}$ of his money; what per cent of his money did he spend?

3. A pupil missed $\frac{1}{5}$ of his questions; how many hundredths did he miss? How many per cent?

4. What part of \$12 is \$3? What per cent?

5. What per cent of 20 is 7?

ANALYSIS.—7 is $\frac{7}{20}$ of 20; and 20 is 100% of itself. Now $\frac{1}{20}$ of 100% is 5%, and $\frac{7}{20}$ of 100% is 7 times 5, or 35%.

What per cent

6. Of \$25 are \$8?
7. Of \$10 are \$9?
8. Of 5 is 3?
9. Of 16 is 4?
10. Of 36 is 9?
11. Of 63 is $31\frac{1}{2}$?
12. Of $16\frac{2}{3}$ is $8\frac{1}{3}$?

What per cent

13. Of \$24 are \$18?
14. Of 20 pears are 12 pears?
15. Of 25 gal. are 16 gal.?
16. Of 50 lb. are 45 lb.?
17. Of \$50 are $\$12\frac{1}{2}$?
18. Of \$1 are $6\frac{1}{4}$ cents?
19. Of \$1 are $33\frac{1}{3}$ cents?

20. If you pay \$5 for the use of \$50 for a year, what per cent do you pay?

21. What per cent of 30 kilograms are 6 kilograms?

22. If a pint of water is added to a gallon of milk, what per cent of it is water?

23. If a man earns \$80 a month and spends \$30, what per cent does he spend?

WRITTEN EXERCISES.

464. To find the *Rate* when the Base and Percentage are given.

1. What per cent of \$63 is \$42 ?

OPERATION.

63) 42.00

Ans. $.66\frac{2}{3}$

ANALYSIS.—Percentage is the product of the *base* and the *rate;* therefore, the percentage \$42, divided by the base \$63, gives $.66\frac{2}{3}$, or $66\frac{2}{3}\%$, the rate. Hence, the

RULE.—*Divide the percentage by the base.*

FORMULA.—*Rate* = *Percentage* ÷ *Base.*

2. What % of £18 is 15s. *Ans.* $4\frac{1}{6}\%$.
3. What % of 96 meters is 28 meters ?
4. What % of \$18 is 12 cts. ?
5. What % of 168 is 15 ?
6. What % of 275 is 18 ?
7. What % of \$5 is $5\frac{1}{2}$ dimes ?
8. What % of $\frac{4}{5}$ ton is $\frac{1}{2}$ ton and 16 pounds ?
9. Henry spelled 225 words out of 250, and his sister 235; what per cent of the words did each spell correctly ?
10. From a cask of kerosene containing 52 gal., 6 gal. 2 qts. leaked out; what per cent of it was lost ?
11. A farmer having 250 bu. of wheat, sold $\frac{5}{8}$ of it; how many bushels and what per cent did he sell ?
12. A man worth \$12500, bequeathed \$3125 to his wife and the rest to his 3 children; what per cent of it did his wife have, and how much had each child ?
13. What per cent of 365 days are 30 days ?
14. Of 1880 years are 4000 years ?
15. Of 27 lb. Avoir. are 12 oz. ?
16. Of 125 miles are 250 rods ?
17. Of 88 kilograms are 75 grams ?
18. If a man owns $\frac{3}{4}$ of a ship and sells $\frac{1}{8}$ of her, what per cent of his part does he sell ?
19. What per cent of 75 bu. 3 pk. are 50 bu. 2 pk. ?
20. A man gave \$9863 to 3 charities; to the first \$2500, to the second \$4500; how much was left for the third and what per cent did each receive ?

PROBLEM III.

ORAL EXERCISES.

465. **1.** \$48 are 6% of what number?

ANALYSIS.—Since \$48 are 6% of the number, 1% is $\frac{1}{6}$ of \$48, which is 8, and 100% is 100 times 8, or \$800, *Ans.*

2. 24 is 4% of what?
3. 32 is 5% of what?
4. 48 is 20% of what?
5. $12\frac{1}{2}$ is 10% of what?
6. $6\frac{1}{4}$ is 25% of what?
7. 40 gal. are 20% of what?
8. \$68 are 12% of what?
9. 25 yd. are 40% of what?
10. 12d. are 30% of what?
11. 25 doz. are $12\frac{1}{2}$% of what?

WRITTEN EXERCISES.

466. To find the *Base* when the Rate and Percentage are given.

1. 192 is 25% of what number?

ANALYSIS.—Percentage is the product of the base by the rate. The base $192 \div .25 = 768$, the base required. Hence, the

$$\begin{array}{r|l} .25\ )\ 192.00 & \text{P.} \\ \hline \textit{Ans.}\ 768 & \text{B.} \end{array}$$

RULE.—*Divide the percentage by the rate.*

FORMULA.—*Base* = *Percentage* ÷ *Rate.*

2. 84 is $12\frac{1}{2}$% of what number? *Ans.* 672. (Art. **447.**)

3. $96 = 33\frac{1}{3}$% of what?
4. 234 = 10% of what?
5. £240 = 7% of what?
6. $62.5 = 6\frac{1}{4}$% of what?
7. 60 yd. = $\frac{3}{4}$% of what?
8. 78 = 25% of what?
9. $31.25 = 12\frac{1}{2}$% of what?
10. 60 cts. = $\frac{1}{2}$% of what?
11. \$100 = $\frac{1}{4}$% of what?
12. \$42.30 = $\frac{1}{5}$% of what?
13. 94 = 150% of what?
14. $58\frac{1}{2}$ = 125% of what?

15. The number of children of age to attend school is 862, which is 20% of the population; what is the whole population?

16. A man sold a house, making \$360, which was 5% more than it cost him; what did he pay for the house?

17. 4% of \$230 is 5% of what? $12\frac{1}{2}$% of \$530 is $6\frac{1}{4}$% of what?

18. A man paid a war tax of \$73.50, which was 2% on the value of his property; what was he worth?

PROBLEM IV.

ORAL EXERCISES.

467. 1. A man sold a cow for \$40, which was 25% more than she cost him; what did he pay for her?

ANALYSIS.—\$40 is the cost increased by 25% of itself; and since the cost is $\frac{100}{100}$ of itself, \$40 must be $\frac{125}{100}$, or $\frac{5}{4}$ of the cost. Now, as \$40 = $\frac{5}{4}$ of the cost, $\frac{1}{4}$ is $\frac{1}{5}$ of \$40, which is \$8, and $\frac{4}{4}$ are 4 times 8, or \$32, *Ans.*

2. What number increased by 25% of itself, is 100?

3. A furniture dealer sold a bureau for \$20, which was 10% more than it cost him; how much did it cost him?

4. What number plus $12\frac{1}{2}$% of itself amounts to 96?

. A grocer sold a barrel of apples for \$5.50, and gained 20% on the sum it cost him; what did he pay for it?

6. A jeweller sold a watch for \$150, which was 50% more than it cost him; what did he pay for it?

7. What number diminished by 25% of itself is 60?

ANALYSIS.—As 60 is the number after it is diminished, 60 must be 100% − 25% = $\frac{75}{100}$, or $\frac{3}{4}$ of the number. Now if 60 is $\frac{3}{4}$ of the number, $\frac{1}{4}$ of it is 60 ÷ 3 = 20, and 4 fourths are 4 times 20, or 80, *Ans.*

8. What number diminished by 20% of itself is 48?

9. A pupil answered on examination 45 questions correctly, which was 10% less than the number asked him; how many were asked him, and how many did he miss?

WRITTEN EXERCISES.

468. To find the *Base* when the Amount or Difference, and the Rate are given.

1. What number increased by 25% of itself is 3500?

ANALYSIS.—Since 3500 is the number after it is increased by 25% of itself, 3500 must be 125% of the number, or 1.25 times the number, and 3500 ÷ 125 = 2800, *Ans.*

$$1 + .25 = 1.25$$
$$1.25\,)\,3500.00$$
$$\textit{Ans.}\ 2800$$

2. What number diminished by 20% of itself is 2560?

ANALYSIS.—Since 2560 is the number after it is diminished by 20% of itself, 2560 must be 80%, or .80 times the number, and 2560 ÷ .80 = 3200, *Ans.*

$$1 - .20 = .80$$
$$.80\,)\,2560.00$$
$$\textit{Ans.}\ 3200$$

469. From the operations above, we derive the following

RULE.—*Divide the amount by 1 increased by the rate.* Or, *Divide the difference by 1 diminished by the rate.*

FORMULAS.—$Base = \begin{cases} Amount \div (1 + Rate). \\ Difference \div (1 - Rate). \end{cases}$

What number increased	What number diminished
3. By 10% of itself is 5342?	9. By 25% of itself is 3900?
4. By 6% of itself = 2418?	10. By 6% of itself = 2100?
5. By 10% of itself = 28600?	11. By 12% of itself = 1200?
6. By 16% of itself = 2552?	12. By 15% of itself = 2300 bu.?
7. By 20% of itself = \$3720?	13. By $7\frac{1}{2}$% of itself = \$6475?
8. By $28\frac{1}{2}$% of itself = \$8995?	14. By $12\frac{1}{2}$% of itself = 13125?

15. At the end of the year, a merchant's stock was \$8400, which was 17% more than his capital; what was his capital?

16. A man sold his house for \$2700 and lost $12\frac{1}{2}$%; what did the house cost him?

17. A grocer sold 950 barrels of flour for \$5760, which was 20% advance on the cost; what was the entire cost, and the cost per barrel?

18. A provision dealer sold 800 barrels of beef for \$12000, which was a loss of 25%; what was the whole cost, and how much per barrel?

470. Percentage is applied to two classes of problems.

First.—Those which are *independent* of Time; as, Profit and Loss, Commission and Brokerage, Insurance, Taxes, Duties.

Second.—Those in which Time is an element; as, Interest, Discount, Equation of Payments, Averaging Accounts, Stocks and Exchange.

NOTE.—In applying the PRINCIPLES of Percentage to these subjects, the pupil should carefully observe what elements or parts are given and what required in each example, and then apply the corresponding rule or formula.

Profit and Loss.

Oral Exercises.

471. 1. A man paid $60 for a watch, and sold it at 10% above the cost; how much did he gain?

Analysis.—He gained 10% of $60. Now 10% of a number is $\frac{10}{100}$, or $\frac{1}{10}$; and $\frac{1}{10}$ of $60 is $6. Therefore, etc.

2. If a man pays $40 for a cow, and sells her at 20% advance, what will be his profit?

3. A jockey bought a horse for $80, and sold it at a loss of 5%; how much did he lose?

4. A man having 120 acres of land, bought 25% more; how many acres did he buy?

5. What part of a number is $12\frac{1}{2}$% of it?

6. What is $12\frac{1}{2}$% of 32? Of 48? Of 96?

7. What is $6\frac{1}{4}$% of 32? Of 64? Of 80?

8. What is $33\frac{1}{3}$% of $15? Of $50? Of $60?

9. A merchant sells flannel at a profit of 10 cts. on a yard, and gains $12\frac{1}{2}$%; what is the cost?

Analysis.—$12\frac{1}{2}\% = \frac{1}{8}$; hence, 10 cts. = $\frac{1}{8}$ the cost; and $\frac{8}{8}$ are 8 times 10 cts., or 80 cts., *Ans.*

10. A farmer lost $32.40 on a reaping machine, which was $33\frac{1}{3}$% of the cost; what was the cost?

11. A goldsmith sold a watch at 25% profit, and made $26; what was the cost?

12. A tradesman sold out his stock of goods for $2760, which was 8% less than he paid; what did they cost him?

13. A grocer sold strawberries at 15 cts. a liter and made 20%; what did he pay for them?

14. A fruit dealer sold a barrel of apples for $1.50, which was a loss of 50%; what did he pay for them?

15. A newsboy sells papers at 5 cts. apiece, and makes 100%; what does he pay for them?

16. A man sold his house for $7500, which was $33\frac{1}{3}$% more than he paid for it; required the cost?

Written Exercises.

472. Profit and Loss denote the *gain* or *loss* in business transactions. They are calculated by *percentage.*

The *cost* is the *base;* the *per cent* of gain or loss, the *rate;* the *gain* or *loss,* the *percentage;* the *selling price,* the *amount* or *difference.*

473. To find the Profit or Loss. (Art. **460.**)

FORMULA.—*Profit or Loss = Cost × Rate.*

1. A house bought for $5860 was sold for 23% above cost; what was the gain?

2. A grocer bought a cask of oil for $96.50, and retailed it at a profit of 6 per cent; how much did he make on his oil?

3. A pedlar bought a lot of goods for $2150, and retailed them at 25 per cent advance; how much was his profit?

4. A merchant bought a cargo of coal for $450, which he sold for 12½ per cent less than cost; what was his loss?

5. What is the loss on a piano that cost $1260, and sold at 20% loss?

6. What was the gain on a farm that cost $3585, and sold at a profit of 12½%?

7. What is the profit on wool which cost $2538 and sold at an advance of 15%?

8. A dealer bought a quantity of grain for $1375, and sold it for 8% profit; what amount did he receive? (Art. **461.**)

9. A young man having $2750, lost 35% of it in speculation; how much had he left?

10. Bought a quantity of produce for $989.33, which I sold at 20% loss; how much did I receive for it?

11. A drover bought a flock of sheep for $2275, and sold them at 25% advance; for how much did he sell them?

12. A merchant had a quantity of groceries on hand, which cost him $367.13; to close up his business he sold them at 15% less than cost; how much did he get for them?

13. A man bought a farm for $875, and was offered 33% advance for his bargain; how much was he offered?

14. A merchant bought a cargo of cotton for \$30000; the price declining, he sold it at $2\frac{1}{2}\%$ less than cost; for how much did he sell it?

474. To find the Rate of Profit or Loss. (Art. 464.)

FORMULA.—*Rate* = *Profit or Loss* ÷ *Cost.*

15. A dealer bought a span of horses for \$450, and sold them for \$600; what per cent was his profit?

16. A mowing machine was sold for \$175, which cost \$225; what per cent was the loss?

What is the rate per cent profit

17. On coffee bought at 25 cts. and sold at 30 cts.?

18. On tea bought at 55 cts. and sold at 67 cts.?

19. On starch bought at 10 cts. and sold at 13 cts.?

20. On goods sold at *double* the cost?

21. On goods sold at $1\frac{1}{4}$ the cost?

22. A merchant bought a quantity of goods for \$155.63, and sold them for \$148.28; what per cent did he lose?

23. A gentleman bought a house for \$3500, and sold it for \$150 more than he gave; what per cent was his profit?

24. A speculator laid out \$7500 in land, and afterwards sold for \$10000; what per cent did he make?

25. A merchant bought \$10000 worth of wool, and sold it for \$12362; what per cent, and how much was his profit?

475. To find the Cost. (Art. 466.)

FORMULA.—*Cost* = *Gain or Loss* ÷ *Rate.*

26. The loss on a cargo of lumber was \$1260, which was 23% of the cost; what was the cost?

27. A speculator gained \$3748 in land, which was 22% of the cost; required the cost?

28. An importer made \$3900 on a cargo of goods, which was 16% of the cost; required the cost?

29. If a grocer pays \$3584 for a cargo of flour, for how much must he sell it to gain $16\frac{1}{2}\%$? (Arts. **460**, **461**.)

30. A merchant paid $8500 for a case of silks; at what price must he sell it to lose 18%?

31. A merchant bought butter for $322.75; for how much must he sell it to gain 15% by his bargain?

32. Bought tea for $437.50; for how much must I sell it, to make 18% by the operation?

33. What is the selling price of hay bought for $845 and sold at 16% gain?

34. What is the selling price of land costing $1868.25 and sold at $12\frac{1}{2}$% loss?

35. What is the selling price of goods costing $2576.40 and sold at $33\frac{1}{3}$% profit?

36. What is the selling price of furniture costing $1848.75 and sold at a loss of $8\frac{1}{4}$%?

476. To Find the Cost from the Selling Price and the Rate per cent of Profit or Loss.

37. A manufacturer sold a carriage for $432, which was 20% above cost; what was the cost?

ANALYSIS.—$432 is the cost, plus 20% of itself; hence, the cost was $432 ÷ (1 + .20) = $360, *Ans.* (Art. **461.**)

38. Another carriage sold for $432, which was 20% less than cost; what was the cost?

ANALYSIS.—$432 is the cost, minus 20% of itself; hence, the cost was $432 ÷ (1 − .20) = $540, *Ans.* (Art. **462.**) Hence, the

$$\text{FORMULAS.—}\textit{Cost} = \begin{cases} \textit{Selling Price} \div (1 + \textit{Rate of Gain}). \\ \textit{Selling Price} \div (1 - \textit{Rate of Loss}). \end{cases}$$

39. A farmer sold land for $86.50 a hektar, and made 12%; what was the cost?

40. A merchant sold a bill of goods for $$675\frac{1}{4}$, and made $10\frac{1}{2}$% profit; what did he pay for the goods?

41. A drover sold cattle for $1750, which was a profit of $12\frac{1}{2}$%; what did they cost him?

42. A dealer sold 525 hektoliters of grain for $2750, which was a loss of 15%; what was the cost?

Commission and Brokerage.

477. A **Commission Merchant, Agent,** or **Factor** is a person who buys or sells goods or transacts business for another.

478. **Commission** is the *Percentage* allowed the agent on the money *invested* or *collected.*

479. A **Broker** is one who buys and sells Stocks, Bills of Exchange, etc., and his commission is called **Brokerage.**

480. A **Consignment** is Goods sent to an agent to sell.

The **Consignor** is the person sending them.

The **Consignee** is the person to whom they are sent.

481. The **Net Proceeds** are the *gross amount* of a business transaction, *minus* the commission and other charges.

482. The computation of commission and brokerage is the same as Percentage; the *money employed* being the *base;* the *per cent* for services, the *rate;* the *commission,* the *percentage.*

483. 1. Find $4\frac{1}{2}\%$ com. on sales for \$3468. (Art. **460.**)

2. An agent sold a house for \$7265; what was his commission at $1\frac{1}{2}\%$?

3. Find $5\frac{1}{2}\%$ com. on 375 bbl. apples, sold at \$2.25 a barrel.

4. Find $6\frac{1}{4}\%$ com. on a ton of wool, at $87\frac{1}{2}$ cts. a pound.

5. A commission merchant sold goods amounting to \$7468, at 5% for commission and guaranty. How much did he receive, and how much did he pay the owner?

6. An auctioneer sold a farm for \$12482, and charged $3\frac{1}{2}\%$ com., and \$50 for advertising it. What was his whole bill, and what the net proceeds?

7. When a commission of \$150, at $6\frac{1}{4}\%$, is received for goods sold, what is the amount of sales? (Art. **466.**)

8. When the commission, at $3\frac{1}{2}\%$ is \$294?

9. When the commission, at 6% is \$105?

10. When the commission, at $1\frac{1}{2}\%$ is \$270?

11. An auctioneer charged \$405 for selling a saw-mill, which was $1\frac{1}{2}\%$; for what did he sell it and what did the owner receive?

12. A commission merchant charged $1\frac{1}{2}\%$ com., and $3\frac{1}{2}\%$ for guaranty; he received $105.30. What were the net proceeds?

484. To Find the Amt. of Sales from the Net Proceeds and Rate.

FORMULA.—*Amount of Sales* = *Net Proceeds* ÷ (1 — *Rate*).

13. The net proceeds of goods sold were $4845, and the agent charged $2\frac{1}{2}\%$ commission and $2\frac{1}{2}\%$ for guaranty. What was the amount of sales? *Ans.* $5100.

14. When the net proceeds are $229.80 and the rate 3%, what is the amount of sales?

15. My agent charged $1\frac{1}{4}\%$ commission and $62.40 expenses for selling my house, and sent me $15250. For how much did the house sell?

485. To find the sum to be invested, after deducting the per cent commission from the amount remitted.

16. If $7098 are remitted to an agent to buy cotton, after deducting 4% com., how much will be left to be invested?

ANALYSIS.—The money remitted includes both the commission and the investment. The money invested is 100% of itself, and 100% + 4% = 104%. Therefore, $7098 ÷ 1.04 = $6825, the money to be invested. Hence, the

OPERATION.

1.04) $7098

Ans. $6825

FORMULA.—*Sum Invested* = *Remittance* ÷ (1 + *Rate*).

17. When the remittance is $1623.10, and the commission $2\frac{1}{2}\%$, how much remains to be invested?

18. When the remittance is $4454, and the commission $2\frac{1}{2}\%$?

19. When $4908 are sent, and the commission is $4\frac{1}{2}\%$?

20. How many apples at $2 a barrel can be bought for $6720.80, after deducting 5% commission?

21. Sent an agent $50000 to buy a ship. How much did the owner receive after deducting $1\frac{1}{2}\%$ commission?

22. How many buffalo robes at $5 each can be bought with a remittance of $2575, after deducting 3% commission?

23. A college sent an agent $10250 to be invested in a library; how much remained after deducting $2\frac{1}{2}\%$ commission?

INSURANCE.

486. **Insurance** is security against *loss.*

487. **Fire Insurance** is security against the loss of property by fire.

488. **Marine Insurance** is security against the loss of property at sea.

NOTE.—Risks of transportation partly by land and partly by water, are called *Transit Insurance.* The same policy often covers both Marine and Transit Insurance.

489. **Accident Insurance** is security against loss by accidents.

490. **Health Insurance** secures a certain sum during sickness.

491. **Life Insurance** secures a stated sum to the heirs and assigns of the insured in case of death.

492. The parties who agree to make good the loss, are called **Insurance Companies** or **Underwriters.**

NOTE.—When only a part of the property insured is destroyed, the underwriters are required to make good only the estimated loss.

493. The **Premium** is the sum paid for insurance.

494. The **Policy** is the *written contract* between the insurers and the insured.

495. Insurance Companies are of two kinds: *Stock Companies* and *Mutual Companies.*

496. A **Stock Company** is one which has a paid up capital, and divides the profit and loss among its stockholders.

497. A **Mutual Company** is one in which the losses are shared by the parties insured.

498. Insurance is calculated by Percentage; the *sum insured* being the *base;* the *per cent* premium, the *rate;* the *premium* itself, the *percentage.*

Mental Exercises.

499. 1. How much must be paid for insuring a house for \$5000, at $\frac{1}{2}\%$ premium?

Analysis.—Since the premium is $\frac{1}{2}\%$, the sum paid must be $\frac{1}{2}\%$ of \$5000. Now 1% of \$5000 is \$50, and $\frac{1}{2}\%$ is $\frac{1}{2}$ of \$50, or \$25, *Ans.*

2. Find the annual premium of insurance, at $1\frac{1}{4}\%$ on a store and goods valued at \$800.

3. I paid \$8 for insuring \$400; what was the rate?

Analysis.—As the premium on \$400 is \$8, the premium on \$1 is $\frac{1}{400}$ of \$8 which is \$.02, or 2%, *Ans.* (Art. **464.**)

4. Paid \$15 for insuring \$1000; required the rate?

5. What amount of insurance, at 2% can be obtained for \$40?

Analysis.—Since 2% is $\frac{2}{100}$ or $\frac{1}{50}$ of the amount insured, the premium \$40 is $\frac{1}{50}$ of this amount; and \$40 is $\frac{1}{50}$ of 50 times \$40 or \$2000. (Art. **466.**)

6. What amount of insurance, at 4% can be obtained on a vessel for \$100?

Written Exercises.

500. 1. What is the premium at $2\frac{1}{4}\%$ for insuring \$16000?

2. What is the premium for insuring a store and goods valued at \$7500, at $1\frac{1}{2}\%$?

3. What is the premium for insuring a house and furniture valued at \$65000, at $\frac{3}{4}\%$?

4. If \$72 are paid for insuring \$4800, what is the rate?

5. If \$420 are paid for insurance on \$18000, what is the rate?

6. If \$860 are paid for insurance of \$1720, what is the rate?

7. A merchant paid \$157.80 to insure his store, at $1\frac{1}{2}\%$; what amount did he insure?

8. Paid \$187 to insure half the value of a ship at $2\frac{3}{4}\%$; what was the total value of the ship?

501. To find the sum to be insured to cover the value of the goods and premium.

9. Bought goods in London for \$7194. What sum insured, at $3\frac{1}{4}\%$ will cover the value of the goods and the premium?

ANALYSIS.—The bill is 100% of itself, and the premium is $3\frac{1}{4}\%$ of that sum; therefore, $\$7194 = 100\% - 3\frac{1}{4}\% = 96\frac{3}{4}\%$, or .9675 times the sum; now $\$7194 \div .9675 = \$7435.658$, the sum required. (Art. **468.**)

FORMULA.—*Sum insured* = *Value* ÷ (1 − *Rate*).

10. If a store and goods are worth \$16625, what sum must be insured, at 2% to cover the property and premium?

11. What sum must be insured, at $2\frac{1}{2}\%$ on a consignment of tea which cost \$352.50 to cover property and premium?

12. A dealer shipped 1000 bbls. flour worth $\$6\frac{1}{2}$ a bbl.; for what sum must he take out a policy, at $2\frac{1}{2}\%$ to cover the value of the flour and the premium?

LIFE INSURANCE.

502. Life Insurance Policies are of *different kinds*, and the premium *varies* according to the *expectation* of life.

503. Life Policies, are payable at the death of the party named in the policy, the annual premium continuing through life.

504. Term Policies are payable at the death of the insured, if he dies during a given term of years, the annual premium continuing till the policy expires.

505. Endowment Policies are payable to the insured at a given age, or to his heirs if he dies before that age, the annual premium continuing till the policy expires.

NOTE.—The *expectation of life* is the average duration of the life of individuals after any specified age.

13. What premium must a man, at the age of 27, pay annually for a life policy of \$4500, at $4\frac{1}{2}\%$?

14. What is the annual premium on \$5000, at $5\frac{1}{4}\%$, and what will it amount to in 20 years?

15. A man took an endowment policy of \$25000 for 20 yrs., at 5%. Which was the greater, the sum paid or the sum insured?

TAXES.

506. A **Tax** is a sum assessed upon the person, property, or income of citizens, for public purposes.

507. A **Property Tax** is a tax upon *property.*

508. A **Personal Tax** is a tax upon the *person,* and is called a *poll or capitation tax.*

NOTE.—The term *poll* is from the German *polle,* the head; *capitation,* from the Latin *caput,* the *head.*

509. **Property** is of two kinds, personal and real estate.

510. **Personal Property** is that which is *movable;* as, money, stocks, etc.

511. **Real Estate** is that which is *fixed;* as, houses and lands.

512. **Assessors** are persons appointed to make a list of taxable property and estimate its value for the purpose of taxation.

513. Property taxes are computed by Percentage.
The *valuation* of the property is the *Base.*
The *tax* on \$1 is the *Rate.*
The *net sum* to be raised, the *Percentage.*

514. To assess a Property Tax, when the sum to be raised and the valuation of the property are given.

1. A tax of \$12500 is to be raised in a town the property of which is valued at \$1500000, and there are 250 polls, each taxed at \$2; what is the rate of the tax, and what is A's tax whose real estate is valued at \$6000, and personal at \$3000?

OPERATION.

Town tax	\$12500
Poll "	500
1500000)	\$12000
Rate	.008

ANALYSIS.—The sum to be raised is \$12500 less \$500 on the polls, which is equal to \$12000, and \$12000 ÷ \$1500000 = \$.008, $\frac{8}{10}\%$, or 8 mills.

A's property is \$6000 + \$3000 = \$9000. As he pays 8 mills on \$1, on \$9000 he pays 9000 × .008 = \$72, and \$72 + \$2 (his poll tax) = \$74. *Ans.* The rate is 8 mills or $\frac{8}{10}\%$, and his tax \$74. Hence, the

RULE.—*I. From the sum to be raised subtract the poll tax and divide the remainder by the amount of taxable property; the quotient will be the rate.*

II. Multiply the valuation of each man's property by the rate, and the product plus his poll tax will be his entire tax.

NOTES.—1. If a poll tax is included, the *sum* arising from the polls must be *subtracted* from the *sum* to be raised, before it is divided by the value of taxable property.

2. The computation of taxes may be shortened by finding the rate, and giving the tax on $1 to $10, etc., as in the following

TAX TABLE.

515. Showing the tax on various sums at the rate of 8 mills on $1.

Prop.	Tax.	Prop.	Tax.	Prop.	Tax.	Prop.	Tax.
$1	$0.008	$7	$0.056	$40	$0.32	$100	$0.80
2	0.016	8	0.064	50	0.40	200	1.60
3	0.024	9	0.072	60	0.48	300	2.40
4	0.032	10	0.08	70	0.56	400	3.20
5	0.040	20	0.16	80	0.64	500	4.00
6	0.048	30	0.24	90	0.72	1000	8.00

2. Find by the table B's tax whose property is valued at $7256, and who pays for 3 polls at $1.50.

3. Find C's tax on property valued at $9480, who pays for 3 polls at $1.25.

4. What is D's tax on a valuation of $15676, and pays for 2 polls at $1.50?

5. A tax of $250000 is levied on a County whose real estate is valued at $3000000, and has 500 polls taxed at $2 each. Required the rate of tax, a tax table for that rate, and a person's tax whose property is valued at $5250, and who pays for 3 polls at $2 each.

Duties or Customs.

516. **Duties or Customs** are taxes levied upon *imported goods* for revenue, or the encouragement of home industry.

517. An **Invoice** or **Manifest** containing a description of the goods and their cost in the country from which they are imported, is required by law to be exhibited to the *Collector of the Port* on the arrival of the ship.

518. Duties are either **Ad valorem** or **Specific.**

519. An **Ad valorem Duty** is a certain per cent laid on the cost of goods in the country from which they are imported.

520. A **Specific Duty** is a fixed sum laid on a given article or quantity, without regard to its value.

521. Before calculating specific duties, certain allowances are made called *Tare, Leakage,* and *Breakage.*

Tare is an allowance for weight of box, bag, cask, etc.

Leakage is an allowance for loss of liquids in casks.

Breakage is an allowance for loss of liquids in bottles.

522. 1. What is the *Specific duty* on 75 hogsheads of alcohol, at 1s. per gallon, 4% leakage ?

SOLUTION.

The number of gal.	$= 63 \times 75 = 4725$ gal.
The leakage at 4%	$= 4725 \times .04 = 189$ gal.
The net gallons	$= 4725 - 189 = 4536$ gal.
The duty at 1s.	$= 4536$s. $= \$1103.7222$, *Ans.*

2. What is the specific duty, at $0.75 a meter, on 150 pieces of broadcloth, each containing 45 meters ?

3. What is the specific duty on 182 cases of shawls, containing 75 each, at $1.50 per shawl ?

4. What is the duty, at 5 cts. a pound, on 400 sacks of coffee, each containing 63 lb., the tare being 2% ?

5. What is the *Ad Valorem* duty on $2\frac{1}{2}$ doz. clocks, invoiced at \$32.75, and 5 doz. watches invoiced at $\$45\frac{1}{4}$, at 25% ?

ANALYSIS.—The cost of 30 clocks	= \$32.75 × 30 =	\$982.50
The cost of 60 watches	= \$45.25 × 60 =	\$2715.00
The cost of both	=	\$3697.50
The ad valorem duty at 25%	= \$3697.50 × .25 =	\$924.375

6. What is the ad valorem duty, at $33\frac{1}{3}\%$, on 150 chests of tea, each weighing 60 lb., and invoiced at 48 cts. a pound, the tare being 5 lbs. a chest ?

7. Find the ad valorem duty, at $2\frac{1}{2}\%$, on 110 boxes of raisins, 25 lb. in a box, invoiced at \$0.12 a pound, the tare being $3\frac{1}{2}$ lb. a box ?

8. At $12\frac{1}{2}\%$ what is the ad valorem duty on 5250 kilograms of Russia iron, invoiced at 75 cts. a kilogram.

QUESTIONS.

444. What does per cent mean ? 447. How expressed ? 452. How change a common fraction to a per cent ? 454. What is the base ? 455. The rate per cent ? 456. The percentage ? 457. The amount ? 458. The difference ?

460. How find the percentage when base and rate are given ? 461. How find amount ? 462. Difference ? 464. How find rate from base and percentage ? 468. How find base from the rate and amount or difference ?

472. What are profit and loss ? What are the corresponding parts ? 473. How find the profit or loss ? 474. The rate ? 475. The cost ? 476. How find cost from selling price and rate ?

478. What is commission ? 479. Brokerage ? 482. How calculated ? Corresponding parts ? 485. How find the sum to be invested after deducting commission ?

486. What is insurance ? 493. The premium ? 494. Policy ? 498. How calculated ? 501. How find sum to be insured to cover loss and premium ?

506. What are Taxes ? 513. How computed ? Corresponding parts ?

516. What are duties or customs ? 519. Ad valorem ? 520. Specific ? 521. What deductions are made in specific duties ? What is tare ? Leakage ? Breakage ?

INTEREST.

523. 1. How much must I pay you for the use of $100 for 1 year at 6%, and what shall I owe you at the end of the year?

ANALYSIS.—6% is $\frac{6}{100}$; hence, I must pay you $\frac{6}{100}$ of $100, or $6, for its use.

Again, I shall owe you at the end of 1 year the sum borrowed together with $6 for its use, and $100 + $6 = $106, the amount due.

NOTE.—In this solution four elements or parts are considered, called the *Interest*, the *Principal*, the *Per cent*, and the *Amount*.

DEFINITIONS.

524. **Interest** is the money paid for the use of money.

525. The **Principal** is the money for which interest is paid.

526. The **Rate** is the per cent of the principal, paid for its use 1 year, or a specified time.

527. The **Amount** is the *sum* of the principal and interest.

528. **Simple Interest** is the interest on the principal only.

529. **Legal Interest** is the rate established by law.

530. **Usury** is a *higher* than the legal rate.

531. Interest differs from the preceding applications of Percentage only by introducing *time* as an *element* in connection with the *rate per cent.*

532. The *Principal* is the **Base**; the *Per cent per annum* is the **Rate**; the *Interest* is the **Percentage**; the *Sum* of principal and interest, the **Amount**.

TABLE.

533. Showing the legal rates of interest in the several States, compiled from the latest official sources.

States.	*Rate %.*		*States.*	*Rate %.*		*States.*	*Rate %.*		*States.*	*Rate %.*	
Ala.....	8		Ind. ..	6	8	Montana	10	Any.	S. C.....	7	Any.
Ark....	6		Iowa. .	6	10	N. H....	6		Tenn...	6	
Arizona	10	Any*	Kan....	7	12	N. J.....	6		Texas ..	8	12
Cal.....	7	Any.	Ky.	6	8	N. Mex..	6	12	Utah....	10	Any.
Conn...	6		La......	5	8	N. Y....	6		Vt......	6	
Colo....	10	Any.	Maine..	6	Any.	N. C....	6	8	Va......	6	8
Dakota.	7	12	Md.. ..	6		Neb.....	7	10	W. Va..	6	
Del.....	6		Mass...	6	Any.	Nev.....	10	Any.	W. T....	10	Any.
Fla.....	8	Any.	Mich....	7	10	Ohio....	6	8	Wis.....	7	10
Ga.	7	Any.	Minn...	7	12	Oregon .	10	12	Wy.....	12	Any.
Idaho...	10	24	Miss....	6	10	Penn....	6		D. C....	6	10
Ill......	6	8	Mo.....	6	10	R. I.....	6	Any.			

534. In computing interest, a legal year is 12 calendar months.

ORAL EXERCISES.

535. 1. What is the interest of $40 for 1 year at 5%?

ANALYSIS.—At 5%, the interest for 1 yr. is $\frac{5}{100}$ of the principal, and $\frac{5}{100}$ of $40 = $2, *Ans.*

2. What is the int. of $50 for 1 yr. at 5%? 2 yr.? 5 yr.?
3. Of $100 for 1 yr. at 6%? At 8%? At 7%?
4. Of $200 for 2 yr. at 7%? At 4%? At 8%?
5. Of $500 for $2\frac{1}{2}$ yr. at 5%? At 6%? At 10%?
6. Of $400 for 3 yr. at 5%? At 6%? At 10%?
7. What part of a year is 6 months? 4 mo.? 3 mo.? 2 mo.? 1 mo.? 8 mo.? 7 mo.? 9 mo.? 10 mo.? 11 mo.? 12 mo.?
8. What part of 1 year's interest is the interest on the same sum for 6 mo.? For 3 mo.? For 4 mo.? For 2 mo.?
9. At 4%, what is the interest of $600 for 1 yr. and 6 mo.?
10. Calling a month 30 days, what part of 1 mo. is 15 days? Is 10 days? 6 days? 5 days? 3 days? 2 days? 1 day?
11. If the interest on a sum for 1 year is $48, what is it for 1 month? For 3 months? 5 months? 7 months?

* By special agreement.

PROBLEM I.

GENERAL METHOD.

536. To find the Interest and Amount, when the Principal, Rate, and Time are given.

I. *By the time expressed decimally in years.* (Art. **403.**)

1. What is the interest of \$250 for 3 yr. 1 mo. 6 d., at 7%? What is the amount?

EXPLANATION.

The given principal	=	\$250	Prin.
The int. of \$250, at 7% for 1 yr. is \$250 × .07	=	17.50	Int. 1 yr.
1 mo. 6 d. = .1 yr. (Art. **403**); hence, the time	=	3.1	Yr.
Int. for 1 yr. \$17.50 × 3.1 gives int. for 3.1 yr.	=	\$54.25	Int.
The amount = prin. \$250 + \$54.25 int.	=	\$304.25	Amt.

Hence, the

RULE.—I. *Multiply the principal by the given rate, and this product by the time expressed in years.*

II. *Add the interest to the principal for the amount.*

2. Find the interest of \$75.36 for 1 yr. 7 mo. 18 d. at 5%. What is the amount?

SOLUTION.—\$75.36 × .05 × $1.63\frac{1}{3}$ (time) = \$6.154, Int. And \$6.154 + \$75.36 = \$81.514, Amt.

3. What is the int. of \$340.20, at 6%, for 2 yr. 8 mo. 12 d.? What is the amount?

II. *By Aliquot Parts.* (Art. **280.**)

EXPLANATION.

Taking example first, the given Principal is		\$250	Prin.
For 1 yr. the int. at 7% is \$250 × .07	=	17.50	Int. 1 yr.
For 3 yr. the int. is \$17.50 × 3	=	\$52.50	Int. 3 yr.
For 1 month the int. is \$17.50 ÷ 12	=	$1.4583\frac{1}{3}$	Int. 1 mo.
For 6 d. ($\frac{1}{5}$ of 30 d.) the int. is $\$1.4583\frac{1}{3} \div 5$	=	$.2916\frac{2}{3}$	Int. 6 d.
The entire int. = $\$52.50 + 1.4583\frac{1}{3} + .2916\frac{2}{3}$	=	\$54.2500	Int.
The prin. \$250 + \$54.25 interest	=	\$304.25	Amt.

537. From the above illustrations we derive the following

RULE.—FOR ONE YEAR.—*Multiply the principal by the rate.*

FOR TWO OR MORE YEARS.—*Multiply the interest for 1 year by the number of years.*

FOR MONTHS.—*Take the aliquot part of 1 year's interest.*

FOR DAYS.—*Take the aliquot part of 1 month's interest.*

The entire interest is the sum of the partial interests.

FOR THE AMOUNT.—*Add the interest to the principal.*

NOTES.—1. For 1 month take $\frac{1}{12}$ of the interest for 1 year; for 2 months, $\frac{1}{6}$; for 3 months, $\frac{1}{4}$, etc.

2. For 1 day take $\frac{1}{30}$ of the interest for 1 month; for 2 days, $\frac{1}{15}$; for 6 days, $\frac{1}{5}$; for 10 days, $\frac{1}{3}$, etc.

3. In computing interest 30 days are commonly considered a month.

Solve the following by either or both methods:

4. What is the interest of \$684 for 1 yr. 9 mo. 10 d. at 6%?
5. At 4%, what is the amt. of \$1125 for 1 yr. 2 mo. 3 d.?
6. At 5%, what is the amt. of \$1056 for 10 mo. 24 d.?
7. At 6%, what is the int. of \$1340 for 1 mo. 15 d.?
8. At 7%, what is the int. of \$815 for 3 yr. 2 mo. 21 d.?
9. At 8%, what is the amt. of \$961 for 2 yr. 4 mo. 10 d.?
10. What is the int. of \$3500 for 11 mo. 20 d., at 10%?
11. What is the amt. of \$39.275 for 2 yr. 6 mo., at $12\frac{1}{2}$%?
12. What is the int. of \$113.61 for 5 yr. 5 mo., at 5%?
13. What is the int. of \$1000 for 2 yr. 3 mo. 10 d., at $4\frac{1}{2}$%?
14. What is the int. of \$1260.34 for 10 yr., at 3%?
15. What is the int. of \$234.56 for 2 yr. 4 mo. 5 d., at 6%?
16. What is the amt. of \$600 for 1 yr. 6 mo. 10 d., at 5%?
17. Find the amount of \$60 for 7 mo., at 8%.
18. What is the interest of \$96 for 10 months, at 6%?
19. At 6%, what is the amt. of \$700 for 1 yr. 2 mo. 12 d.?
20. At 4%, what is the amt. of \$470 for 10 days?
21. Find the int. of \$1000 for 1 yr. 1 mo. 1 d., at 6%.
22. At 6%, what is the amt. of \$4565.61 for 4 mo. 7 days?

23. What is the interest of \$5625.43 for 4 mo. 18 d., at $6\frac{1}{4}\%$?
24. At $5\frac{1}{2}\%$, what is the int. of \$624.625 for 7 mo. 3 days?
25. At 8%, what is the int. of $\$11261.18\frac{3}{4}$ for 3 mo. 3 days?
26. At 7%, what is the amt. of \$9208.95 for 11 mo. 5 days?
27. What is the amt. of \$15206.843, at $7\frac{1}{2}\%$, for 1 year 8 months 25 days?
28. The amt. of \$10050.69, at $5\frac{1}{2}\%$, for 2 yr. 9 mo. 5 d.?
29. What is the amt. of \$11607.858, at 7%, for 3 years 6 months 9 days?
30. The amt. of \$41361.18, at 6%, for 5 yr. 7 mo. 3 d.?
31. What is the interest on \$1145 from July 20th, 1881, to Dec. 7th, 1881, at 7%? (Art. **409**, N. 2.)

NOTE.—The time is 4 mo. and 11 d. (July) + 7 d. (Dec.) = 4 mo. 18 d.

32. What is the interest on a note of \$568.45 from May 21st, 1881, to March 25th, 1882, at 5%?
33. Required the amount of \$2576.81 from Jan. 21st, 1881, to Dec. 18th, 1881, at 7%.

SIX PER CENT METHOD.

DEVELOPMENT OF PRINCIPLES.

538. The interest of \$1 at 6%

For 1 yr., or 12 mo., is 6 cts., = .06 of the principal.
For $\frac{1}{6}$ yr., or 2 mo., is 1 cent, = .01 of the principal.
For $\frac{1}{12}$ yr., or 1 mo., is 5 m., = .005 of the principal.
For $\frac{1}{5}$ mo., or 6 d., is 1 m., = .001 of the principal.
For $\frac{1}{30}$ mo., or 1 d., is $\frac{1}{6}$ m., = $.000\frac{1}{6}$ of the principal.

Hence, we derive the following

PRINCIPLES.

1°. The interest of \$1 at 6%, is half as many cents as there are months in the given time.

2°. The interest of \$1 at 6%, is one-sixth as many mills as there are days in the given time.

10

539. To find the Interest, when the Principal, Rate, and Time are given.

1. What is the interest of \$250.26 for 1 yr. 3 mo. 21 d., at 6%? What is the amount?

EXPLA.—The interest of \$1 for 15 mo. = .075
By 2°, int. of \$1 for 21 d. = .0035
Int. of \$1 for 1 yr. 3 mo. 21 d. = .0785

As the interest of \$1 for the given time and rate is \$.0785, the interest of \$250.26 must be \$250.26 × .0785 = \$19.64541 interest.

The prin. \$250.26 + \$19.64541 = \$269.90541, Amount. Hence, the

OPERATION.

\$250.26	Prin.
.0785	Int. \$1.
125130	
2.00208	
17.5182	
\$19.645410,	*Ans.*

RULE.—*Multiply the principal by the interest of \$1 for the given time, and rate.*

NOTES.—1. When the rate is *greater* or *less* than 6%, find the interest of the principal at 6% for the given time; then *add* to or *subtract* from it *such a part* of itself, as the given rate *exceeds* or *falls* short of 6 per cent.

2. If the *mills* are 5 or more, it is customary to add 1 to the cents; if less than 5, they are disregarded.

3. Only *three decimals* are retained in the following Answers, and each answer is found by the rule under which the Example is placed.

4. In finding the interest of \$1 for days, it is sufficient for ordinary purposes to carry the decimals to four places.

2. What is the amt. of \$350.60 for 1 yr. 5 mo. 15 d., at 6%?
3. What is the int. of \$56.19 for 4 mo. 3 d., at 7%?
4. What is the int. of \$242.83 for 7 mos. 18 d., at 5%?
5. Find the int. of \$781.13 for 11 mo. 21 d., at 6%.
6. Find the int of \$968.84 for 2 yr. 10 mo. 26 d., at 6%.
7. What is the int. of \$639 for 18 mo. 29 d., at 7%?
8. What is the int. of \$745.13 for 17 d., at 5%?
9. What is the int. of \$1237.63 for 8 mo. 3 d., at 8%?
10. What is the int. of \$2046$\frac{1}{4}$ for 25 d., at 4%?
11. Find the amount of \$640.37$\frac{1}{2}$ for 9 mo. 15 d., at 10%.
12. Find the amount of \$2835.20 for 2 mo. 3 d., at 9%.
13. Find the amount of \$4356.81 for 3 mo. 10 d., at 5$\frac{1}{2}$%.
14. What is the int. of \$12240 for 63 d., at 4$\frac{1}{2}$%?
15. What is the int. of \$350000 for 10 d., at 3$\frac{1}{2}$%?

METHOD BY DAYS.

540. 1. What is the interest of $248.60 for 93 days, at 6%?

ANALYSIS.—Since the interest for 30 days is 5 mills, or $\frac{5}{1000}$ of the principal, for 1 day it is $\frac{1}{30}$ of $\frac{5}{1000}$, or $\frac{1}{6000}$; hence, the interest for 93 days is $\frac{93}{6000}$ of the principal. And $\frac{93}{6000}$ of $248.60=($248.60 × 93)÷6000 = $3.85. Hence, the

OPERATION.

$248.60	Prin.
93	No. d.
74580	
223740	
6|000) 23|119.80	
$3.853, *Ans.*	

RULE.—*Multiply the principal by the number of days, and divide the product by 6000.*

2. Find the interest of $360 for 95 d., at 7%. *Ans.* $6.65.

What is the interest of

3. $450 for 63 d. at 6%?
4. $245.50 for 33 d. at 6%?
5. $278.68 for 75 days at 6%?
6. $500.75 for 130 days at 6%?
7. $600 for 63 days at 5%?
8. $735 for 45 days at 7%?
9. $1200 for 60 d. at 5%?
10. $1500 for 93 d. at 8%?

EXACT INTEREST.

541. The methods based upon the supposition that 360 days make a year and 30 days a month, though common, are not strictly *accurate*. As a year contains 365 days, the int. found by these methods is $\frac{5}{365}$, or $\frac{1}{73}$ part of itself too large. Hence,

542. *To compute exact interest for months and days, find the interest by the 6% method and subtract from it $\frac{1}{73}$ part of itself.* (An. Int., Art. **905**, App.)

1. What is the exact interest, at 6%, of $248.60 for 3 mo. 3 d.?

Ans. The interest at 6% is $3.853, $\frac{1}{73}$ part of which is $.053, and $3.853—$.053 = $3.80.

2. What is the exact interest of $2568 for 93 d., at 6%?
3. What is the exact interest of $5000 for 12 d., at 7%?

Partial Payments.

543. **Partial Payments** are parts of a note paid at different times.

544. A **Promissory Note** is a written promise to pay a specified sum at a given time.

545. The **Maker** is the person who signs the note.

546. The **Payee** is the person to whom it is to be paid.

547. The **Holder** is the person who has the note in his possession.

548. **Indorsements** are partial payments, the amount and date of which are written upon the back of notes and bonds.

549. The **Face** of a note is the sum named in it.

550. A **Negotiable Note** is one payable to the bearer, or to the order of the person named in it.

Notes.—1. A note payable to A. B., or "order," is transferable by *indorsement;* if to A. B., or "bearer," it is transferable by *delivery.* Treasury notes and bank bills belong to this class.

2. If the words "order" and "bearer" are both omitted, the note can be collected only by the *party* named in it.

551. An **Indorser** is a person who writes his name on the back of a note as security for its payment.

552. The **Maturity** of a note is the day it becomes legally due. In most States a note does not mature until 3 days after the time named for its payment.

These three days are called *Days of Grace.*

553. **To compute Interest on notes and bonds, when *Partial Payments* have been made.**

United States Rule.

Find the amount of the principal to the time of the first payment, and subtracting the payment from it, find the amount of the remainder as a new principal, to the time of the next payment.

If the payment is less than the interest, find the amount of the principal to the time when the sum of the payments equals or exceeds the interest due; and subtract the sum of the payments from this amount.

Proceed in this manner to the time of settlement.

NOTES.—1. The principles upon which the preceding rule is founded are, 1st. That payments must be applied first to discharge accrued interest, and then the remainder, if any, toward the discharge of the principal.

2d. That only unpaid principal can draw interest.

2. The following examples show the common forms of promissory notes. The first is *negotiable* by *indorsement*; the second by *delivery*; the third is a *joint* note, but *not negotiable.*

$850. WASHINGTON, Jan. 1st, 1880.

1. *On demand I promise to pay to the order of* ALEXANDER HUNTER, *eight hundred fifty dollars, with interest at 6 per cent, value received.* JOHN FRANKLIN.

The following payments were endorsed on this note:

July 1st, 1880, received $100.62.
Dec. 1st, 1880, received $15.28.
Aug. 13th, 1881, received $175.75.

What was due on taking up the note Jan 1st, 1882?

SOLUTION.

Principal, dated Jan. 1st, 1880,			$850.00
Int. to 1st payt. July 1st, 1880 (6 mo.) (Art. **539**),			25.50
Amount,		=	875.50
1st payment, July 1st, 1880,			100.62
Remainder, or new principal,		=	774.88
Int. from 1st payt. to Dec. 1st (5 mo.)			19.37
2d payt. less than int. due,	$15.28		
Int. on same prin. to 3d payt., Aug. 13 (8 mo. 12 d.)			32.54
Amount,		=	826.79
3d payt., to be added to 2d,	$175.75	=	191.03
Remainder, or new principal,		=	635.76
Int. to Jan. 1st, 1882 (4 mo. 18 d.)			14.62
Balance due Jan. 1st, 1882,		=	$650.38

692\frac{35}{100}$. BOSTON, Aug. 15th, 1879.

2. *Three months after date, I promise to pay* JOHN WARNER, *or bearer, six hundred and ninety-two dollars and thirty-five cents, with interest at 6 per cent, value received.*

SAMUEL JOHNSON.

Endorsed Nov. 15th, 1879, $250.375.
Endorsed March 1st, 1880, $65.625.

How much was due on the note, July 4th, 1881?

$500. NEW YORK, May 10th, 1878.

3. *For value received, we jointly and severally promise to pay* JAMES MONROE & SONS, *five hundred dollars on demand, with interest at 7 per cent.*

GEORGE JOHNSON.
HENRY SMITH.

The following sums were endorsed upon it:

Received, Nov. 10th, 1878, $75.
Received, March 22d, 1879, $100.

What was due on taking up the note, Sept. 28th, 1879?

$1000. PHILADELPHIA, June 20th, 1878.

4. *Six months after date, I promise to pay Messrs.* CAREY, HART & CO., *or order, one thousand dollars, with interest at 5 per cent, value received.*

HORACE PRESTON.

Endorsed Jan. 10th, 1879, $125.
Endorsed June 16th, 1879, $93.
Endorsed Feb. 20th, 1880, $200.

What was the balance due on the note, Aug. 1st, 1880?

NOTE.— Massachusetts, New York, Pennsylvania, Ohio, Illinois, and most of the other States have adopted this rule. (For Connecticut, Vermont, and New Hampshire methods, see Art 906-908, Appendix.)

Mercantile Method.

554. When Partial Payments are made on *short notes* or *interest accounts*, business men commonly employ the following method:

Find the amount of the whole debt to the time of settlement; also find the amount of each payment from the time it was made to the time of settlement.

Subtract the amount of the payments from the amount of the debt; the remainder will be the balance due.

$416. ALBANY, March 21st, 1880.

5. *On demand, I promise to pay to the order of* HENRY PATTON, *four hundred and sixteen dollars, with interest at 7 per cent, value received.*

JOHN MARSHALL.

Received on the above note the following sums:

June 15th, 1880, $35.00.
Oct. 9th, 1880, $23.00.
Jan. 12th, 1881, $68.00.

What was due on the note, Sept. 21st, 1881?

SOLUTION.

Principal, dated March 21st, 1880,			$416.000
Int. to settlement (1 yr. 6 mo.), at 7%,			43.680
Amount, Sept. 21st, 1881,		=	459.680
1st payt., $35.00, Time (1 yr. 3 mo. 6 d.), Amount	= $38.103		
2d payt., $23.00, Time (11 mo. 12 d.), Amount	= 24.530		
3d payt., $68.00, Time (8 mo. 9 d.), Amount	= 71.292		
Amount of the payments,		=	133.925
Balance due Sept. 21st, 1881,			$325.755

6. A bill of goods amounting to $750, was to be paid Jan. 1st, 1880. Received June 10th, $145; Sept. 23d, $465; Oct. 3d, $23; what was due on the bill Dec. 31st, 1880, int. 6%?

7. An account of $1200 due March 3d, received the following payments: June 1st, $310; Aug. 7th, $219; Oct. 17th, $200; what was due on the 27th of the following Dec., allowing 7% interest?

PROBLEM II.

555. To find the *Rate,* when the Principal, Interest, and Time are given.

1. At what rate of interest must \$236 be loaned, to gain \$17.70 in 1 year and 3 months?

ANALYSIS.—The int. of \$236 for 1 yr. at 1% = \$236 × .01 = \$2.36
The int. for 3 mo. ($\frac{1}{4}$ yr.) = \$2.36 × $\frac{1}{4}$ = .59
The int. for 1 yr. 3 mo. at 1% = \$2.95
Now as \$2.95 gain requires 1%, \$17.70 gain requires as many per cent as \$2.95 are contained times in \$17.70, or 6%, *Ans.* Hence, the

RULE.—*Divide the given interest by the interest of the principal, at 1 per cent for the time.*

FORMULA.—*Rate* = *Interest* ÷ (*Prin.* × *1%* × *Time*).

NOTE.—When the *amount* is given the *principal* and *interest* may be said to be *given.* For, the amt. = the prin. + int.; hence, amt. − int. = the prin.; and amt. − prin. = the int.

2. At what rate per cent. must \$450 be loaned, to gain \$56.50 interest in 1 year and 6 months?

3. At what per cent must \$750 be loaned, to gain \$225 in 4 years?

4. A man has \$8000 which he wishes to loan for \$500 per annum; at what per cent must he loan it?

5. A gentleman deposited \$1250 in a savings bank, for which he received \$31.25 every 6 months; what per cent interest did he receive on his money?

6. A capitalist invested \$9260 in railroad stock, and drew a semi-annual dividend of \$416.70; what rate per cent interest did he receive on his money?

7. A man built a hotel costing \$175000, and rented it for \$8750 per year; what per cent int. did his money yield him?

8. A man gave his note payable in 1 year and 3 months for \$640, and at its maturity paid \$688; what was the rate of interest?

9. At what rate must \$865 be loaned for 2 years to yield \$129.75 interest?

PROBLEM III.

556. To find the *Time* when the Principal, Interest, and Rate are given.

1. In what time will \$500 gain \$45 at 6%?

ANALYSIS.—The interest of \$500 for 1 yr. at 6% is \$30. Therefore, to gain \$45 will require the same principal as many years as \$30 are contained times in \$45; and \$45 ÷ \$30 = 1.5 or $1\frac{1}{2}$ years, *Ans.* Hence, the

OPERATION.

30) \$45.00

Ans. 1.5

RULE.—*Divide the given interest by the interest of the principal for 1 year, at the given rate.*

FORMULA.—*Time* = *Int.* ÷ (*Prin.* × *Rate*).

NOTES.—1. If the quotient contains *decimals*, reduce them to months and days. (Art. **402.**)

2. If the *amount* is given instead of the principal or the interest, find the part omitted, and proceed as above.

3. At 100%, any sum will *double* itself in 1 year; therefore, any per cent will require as *many years* to double the principal, as the given per cent is contained times in 100%.

2. In what time will \$4500 gain \$430 at 5%?
3. How long will it take \$5000 to earn \$5000 at 6%?
4. How long will it take any sum to double itself at 4%? 5%? 6%? 7%? 10%?

PROBLEM IV.

557. To find the *Principal*, when the Interest, Rate, and Time are given.

1. What principal at 6% will yield \$225 interest in 2 yr. 6 mo.?

ANALYSIS.—At 6%, the interest of \$1 for 2 yr. 6 mo. is \$.15, therefore, \$225 must be the int. of as many dollars as \$.15 are contained times in \$225, and \$225 ÷ \$.15 = \$1500, *Ans.* Hence, the

OPERATION.

.15) 225.00

Ans. \$1500

RULE.—*Divide the given interest by the interest of \$1 for the given time and rate, expressed decimally.*

FORMULA.—*Principal* = *Interest* ÷ (*Rate* × *Time*).

2. What principal at 7% will yield \$500 in 1 year?

3. At 6% what principal will yield \$350 in 6 months?

4. What principal at 5% will yield \$400 in 7 mo. 15 d.?

5. What sum must a father invest at 6%, that his son, now 18 yr. old may have \$5000 when he is 21? (Art. **556**, N. 2.)

6. What sum loaned at 1% a mo. will amount to \$500 in 1 yr.?

7. What sum must be loaned at 4% a year to amount to \$1200 in 8 months?

Compound Interest.

558. Compound Interest is the interest of the *principal* and of the unpaid *interest* after it becomes due.

559. To compute *Compound Interest*, when the Principal, Rate and time of compounding it are given.

1. What is the compound interest of \$500 for 3 years at 6%?

Principal,	=	\$500
Int. for 1st year, \$500 × .06,		30
Amt. for 1 yr., or 2d prin.,	=	530
Int. for 2d year, \$530 × .06,		31.80
Amt. for 2 yr., or 3d prin.,	=	561.80
Int. for 3d year, \$561.80 × .06,		33.71
Amt. for 3 years,	=	595.51
Original principal to be subtracted,		500
Compound int. for 3 years,	=	95.51

Hence, the

RULE.—I. *Find the amount of the principal for the first period. Treat this amount as a new principal, and find the amount due on it for the next period, and so on through the whole time*

II. *Subtract the given principal from the last amount, and the remainder will be the compound interest.*

NOTE.—If there are *months* or *days* after the last regular period at which the interest is compounded, find the interest on the amount last obtained for them, and add it to the same, before subtracting the principal.

2. What is the compound int. of \$450 for 3 yr. 6 mo., at 6%?
3. What is the compound int. of \$550 for 3 yr. 4 mo., at 7%?
4. What is the compound int. of \$850 for 4 yr. 6 mo., at 5%?
5. What is the com. int. of \$865 for 5 yr., at 7%?
6. What is the amt. of \$950 for 6 yr. 3 mo., at 5%, com. int.?

560. TABLE *showing the amount* of \$1, at 3, 3½, 4, 5, and 6% *compound interest,* for any number of years from 1 to 20.

Yrs.	3%.	3½%.	4%.	5%.	6%.
1.	1.030 000	1.035 000	1.040 000	1.050 000	1.060 000
2.	1.060 900	1.071 225	1.081 600	1.102 500	1.123 600
3.	1.092 727	1.108 718	1.124 864	1.157 625	1.191 016
4.	1.125 509	1.147 523	1.169 859	1.215 506	1.262 477
5.	1.159 274	1.187 686	1.216 653	1.276 282	1.338 226
6.	1.194 052	1.229 255	1.265 319	1.340 096	1.418 519
7.	1.229 874	1.272 279	1.315 932	1.407 100	1.503 630
8.	1.266 770	1.316 809	1.368 569	1.477 455	1.593 848
9.	1.304 773	1.362 897	1.423 312	1.551 328	1.689 479
10.	1.343 916	1.410 599	1.480 244	1.628 895	1.790 848
11.	1.384 234	1.459 970	1.539 451	1.710 339	1.898 299
12.	1.425 761	1.511 069	1.601 032	1.795 856	2.012 196
13.	1.468 534	1.563 956	1.665 074	1.885 649	2.132 928
14.	1.512 590	1.618 695	1.731 676	1.979 932	2.260 904
15.	1.557 967	1.675 349	1.800 944	2.078 928	2.396 558
16.	1.604 706	1.733 986	1.872 981	2.182 875	2.540 352
17.	1.652 848	1.794 676	1.947 900	2.292 018	2.692 773
18.	1.702 433	1.857 489	2.025 817	2.406 619	2.854 339
19.	1.753 506	1.922 501	2.106 849	2.526 950	3.025 600
20.	1.806 111	1.989 789	2.191 123	2.653 298	3.207 135

NOTE.—Compound interest cannot be collected by *law;* but a creditor may receive it, without incurring the penalty of *usury.* Savings Banks pay it to all depositors who do not draw their interest when due.

561. 1. What is the int. and amt. of \$2000 for 10 yr. at 3%?

SOLUTION.—Tabular amt. of \$1 for 10 yr. at 3%, \$1.343916 × 2000 = \$2687.832, amt. for 10 yr. And \$2687.832 − \$2000 prin. = \$687.832, *Com. Int.* for 10 years. Hence, the

RULE.—I. *Multiply the tabular amount of \$1 for the given time and rate by the principal; the product will be the amount.*

II. *From the amount subtract the principal, and the remainder will be the compound interest.*

NOTES.—1. If the given number of years exceed that in the Table, find the *amount* for any *convenient period*, as half the given years; then on this amount for the remaining period.

2. If interest is compounded semi-annually take $\frac{1}{2}$ the given rate and twice the number of years; if compounded quarterly, take $\frac{1}{4}$ the given rate and 4 times the number of years.

2. What is the amt. of \$3500 for 6 yr., at 5% com. interest?
3. What is the amount of \$350 for 12 years, at 4%?
4. What is the com. int. of \$469 for 15 years, at 3%?
5. What is the com. int. of \$500 for 24 years, at 6%?
6. What is the com. int. of \$650 for 30 years, at $3\frac{1}{2}$%?
7. What is the amount of \$1000 for 3 yr., at 6% compound interest, payable semi-annually?
8. What is the amount of \$1200 for 2 years, at 12% compound interest, payable quarterly?
9. What is the amt. of \$1500 for 5 yr. 3 mo., at 5% com. int.?

QUESTIONS.

524. What is interest? 525. Principal? 526. Rate? 527. Amount? 528. What is simple int.? 529. Legal interest? 530. Usury?

536. The general method of computing interest? 539. The 6% method? 540. The method by days? 542. Exact interest?

543. What are Partial payments? 544. Promissory note? 549. The face of a note? 550. A negotiable note? 552. The maturity of a note? 548. Indorsements? 553. U. S. Rule for partial payments?

555. How find the rate? 556. The time? 557. The principal? 558. Compound Interest? 559. How computed?

DISCOUNT.

ORAL EXERCISES.

562. 1. The price of a watch was $50, but for cash it was sold at 10% off; how much was the deduction?

ANALYSIS.—1% of $50 is 50 cents, and 10% is 10 times .50, or $5; hence the deduction was $5, *Ans.*

2. A man asked $200 for a horse, but for cash would take 5% off; how much was deducted?

3. A merchant sold a bill of goods amounting to $500, and for cash deducted 6%; how much was deducted?

4. A man owed $800 on Acct., and settled it for cash at 4% off; what was the deduction?

5. If you borrow $300 and pay 6% in advance for its use, how much is deducted from the loan?

TRUE DISCOUNT.

563. Discount is a deduction from a stated price, or from a debt paid before it is due.

564. True Discount is the difference between the face of a debt and its present worth.

The **Present Worth** of a debt, due at some future time without interest, is the *sum* which put at interest at the legal rate will *amount* to the debt when it becomes due.

565. To find the Present Worth and True Discount.

1. What is the present worth and true discount of $378, due in 1 year and 8 months, at 6%?

ANALYSIS.—The amount of \$1, at 6%, for 1 yr. 8 mo. = \$1.10. Since \$1.10 is the amt. of \$1, at 6% for the given time, \$378 is the amt. of as many dollars for the same time and rate, as \$1.10 is contained times in \$378, and \$378 ÷ \$1.10 = \$343.64, present worth. Then \$378 − \$343.64 = \$34.36, the true discount. Hence, the

RULE.—I. *Divide the debt by the amount of \$1 for the given time and rate; the quotient will be the present worth.*

II. *Subtract the present worth from the debt, and the remainder will be the true discount.*

Find the present worth and true discount of

2. \$850.25, due in $1\frac{1}{2}$ years, at 6%.
3. \$1272.50, due in 1 yr. 3 mo., at 7%.
4. \$2895, payable in 2 years, at 5%.
5. \$5650.75, payable in $3\frac{1}{2}$ years, at $4\frac{1}{2}$%.
6. \$10000, due in 1 yr. 5 mo., at $3\frac{1}{2}$%.

7. What is the difference between the interest and true discount of \$12250, for 1 year, at 6%?

8. Bought a farm for \$4822, payable in $2\frac{1}{2}$ years without interest, but for cash 20% discount; what was the true discount?

9. When money is worth 5%, which is preferable, \$12000 cash, or \$13000 payable in 1 year?

BANK DISCOUNT.

566. **Bank Discount** is simple interest, paid in advance.

567. The **Proceeds** of a note are the part paid to the owner; the **Discount** is the part deducted.

568. The **Maturity** of a note is on its last day of grace.

NOTE.—If the last day of grace occurs on *Sunday* or a *legal holiday*, the note matures on the preceding day.

569. The **Term of Discount** is the time from the date of discount to the maturity of the note.

570. To find the *Bank Discount* and *Proceeds*, when the Face of a note, Rate, and Time are given.

1. What is the bank discount of $368 for 3 mo. at 6%? What are the proceeds?

SOLUTION.—The face of the note	=	$368
Int. of $1 for 3 mo. and grace at 6%	=	.0155
Discount	=	$5.704
Proceeds, $368−$5.704 = $362.296.		Hence, the

RULE.—*Find the interest of the note at the given rate for three days more than the specified time; the result is the discount.*

Subtract the discount from the face of the note; the remainder will be the proceeds.

NOTE.—If a note is on interest, find its *amount* at maturity, and taking this as the *face* of the note, cast the interest on it as above.

2. Find the proceeds of a note of $650, due in 3 mo., at 6%.
3. Find the proceeds of a draft of $825, on 60 days, at 6%.
4. Find the maturity and term of discount of a note of $1250, at 5% int., on 60 days, dated July 1st, 1880, and discounted Aug. 21st, 1880, at 5%. What were the proceeds?
5. Find the difference between the true and bank discount on $4000 for 1 year, allowing each 3 days grace, at 7%?
6. A merchant bought $6500 worth of goods for cash, sold them on 4 months, at 15% advance, and got the note discounted at 6% to pay the bill. How much did he make?

571. To find the *Face* of a note, when the Proceeds, Rate, and Time are given.

1. For what sum must a note be made on 4 months, that the proceeds may be $640, discounted at 6%?

SOLUTION.—	The bank discount of $1 for 4 mo. 3 d.	=	$.0205
	The proceeds of $1 = $1−$.0205	=	$.9795
Therefore,	The face of the note is $640÷$.9795	=	$653.394
Hence, the			

RULE.—*Divide the given proceeds by the proceeds of $1 for the given time and rate.*

2. What must be the face of a note on 6 months, discounted at 7%, that the proceeds may be $500 ?

3. The avails of a note were $4350.90, the term 4 months, and the rate of discount 8% ; what was the face of the note?

4. How large a note on 3 months, must I have discounted at 6%, to realize $5260 ready money?

COMMERCIAL DISCOUNT.

572. **Commercial Discount** is a per cent deducted from the face of bills, the list price of goods, etc.

573. The **Net Price** of goods is the sum *received* for them.

574. To find *Commercial Discount,* when the rate is given.

1. What is the commercial discount on goods, the list price of which is $235, sold at 5% off ?

SOLUTION.—5% is .05, and $235 × .05 = $11.75, *Ans.*

2. What was the net price received for a parlor organ, whose list price was $450 on 3 mo., at 7% off for cash ?

3. What is the net value of a bill of books, amounting to $568.50, on 60 days, at 10% off for cash ?

4. After 5% had been deducted from the list price, a bill of goods was sold for $625 ; what was the list price ?

5. Sold a bill of goods amounting to $850, on 4 mo., at 8% discount, and deducted 5% for cash ; what was the net price ?

SOLUTION.—$850 × .08 = $68. And $850 − $68 = $782. Again, $782 × .05 = $39.10, and $782 − $39.10 = $742.90. Hence, the

RULE.—*Deduct the discount from the marked price, and from the remainder take the discount for cash.*

6. What is the net value of a bill of goods amounting to $2560, sold at 10% discount and 4% off for cash ?

7. What is the net value of a cargo of flour invoiced at $3765, at 12% discount and 5% off for cash ?

8. Find the net value of a bill amounting to $4372, at 15% discount and 2½% off for cash ?

9. Find the sum received for a sale of goods marked at \$6500, at 8% discount and $6\frac{1}{4}$% off for cash?

10. What is the cash value of a bill of \$10000, at 7% discount and $4\frac{1}{2}$% off for cash?

11. Find the net value of the following: 63 lb. tea, at 88 cts., sold on 3 mo., 8% off; 95 boxes of starch, at 68 cts., 4% off; 54 drums of figs, at 75 cts., 4% off; 85 bbl. flour, at \$7.50, 10% off; allowing 4% discount for cash.

575. To Mark goods so that a given per cent may be deducted and leave a given per cent profit.

1. Bought ladies' hats at \$5.10; what price must they be marked, that 15% may be deducted and leave 20% profit?

Analysis.—The selling price is 120% of \$5.10, and \$5.10 × 1.20 = \$6.12. But the marked price is to be diminished by 15% of itself, and 100% − 15% = 85%; hence, \$6.12 is 85% of the marked price. Now \$6.12 ÷ .85 = \$7.20, the marked price. (Art. **466**.) Hence, the

Rule.—*Find the selling price and divide it by 1 minus the given per cent to be deducted; the quotient will be the marked price.*

2. Paid \$56 for a sewing machine; what must I ask for it that I may abate 5% and sell it at a gain of 25%?

3. A shoe dealer paid \$3.60 a pair for boots; what must he ask for them that he may deduct $12\frac{1}{2}$% and make $16\frac{2}{3}$%?

4. A jeweller bought diamond rings at \$120; what must he ask for them that he may abate 4% and still make 20%?

5. Bought a piano for \$250; what must I ask for it that I may deduct 20% and leave a profit of 20%?

Questions.

563. What is discount? 564. True discount? The present worth of a debt? 565. How found?

566. What is bank discount? 567. The proceeds of a note? 568. When does a note mature? 569. What is the term of discount?

570. How find the discount of a note? The proceeds? 571. How find the face of a note that the proceeds may be a given sum? 572. What is commercial discount? 573. What is the net price of goods?

EQUATION OF PAYMENTS.

DEVELOPMENT OF PRINCIPLES.

576. 1. How long must \$1 be kept on interest to equal the interest of \$2 for 3 mo. at the same rate per cent?

ANALYSIS.—As \$2 are twice \$1, at the same rate \$1 must be kept on interest twice as long as \$2, and 2 times 3 mo. are 6 months, *Ans.*

2. How long must \$2 be kept on interest to equal the interest of \$8 for 3 months?

3. How long must \$3 be kept on interest to balance the interest of \$9 for 4 months?

4. How long must \$10 be kept to balance the interest of \$5 for 4 months?

ANALYSIS.—\$10 is twice \$5; therefore, \$10 must be kept half as long as \$5, and $\frac{1}{2}$ of 4 mo. is 2 months, *Ans.*

5. How long will it take \$30 to balance the interest of \$10 for 6 months?

6. In what time will the interest of \$200 balance the interest of \$50 for 8 months?

577. From the examples above we derive the following

PRINCIPLES.

1°. The rate and time remaining the same,
Double *the principal produces* ***twice*** *the interest.*
Half *the principal produces* ***half*** *the interest, etc.*

2°. The rate and principal remaining the same,
Double *the time produces* ***twice*** *the interest.*
Half *the time produces* ***half*** *the interest, etc.* Hence,

578. *The interest of any given principal for 1 year, 1 month, or 1 day, is the same as the interest of 1 dollar for as many years, months, or days, as there are dollars in the given principal.*

579. **Equation of Payments** is the method of finding the *average time* for the payment of several debts, due at different times, without loss of interest to either party.

580. The **Average** or **Equated** time is the *date* when the several payments may be made at one time.

581. The **Term of Credit** is the time before a debt becomes due.

582. The **Average Term of Credit** is the time at which debts due at *different* times may be equitably paid.

Written Exercises.

583. To find the *Average Time*, when the items have the same date, but different terms of credit.

1. Bought Oct. 10th, 1880, the following bills of goods, for which I was to pay \$485 cash, \$200 in 2 mo.; \$275 in 4 mo.; and \$360 in 5 mo.; what is the average time and the date, when these bills may be paid without loss to either party?

OPERATION.

\$485	×	0	=	0
200	×	2	=	400
275	×	4	=	1100
360	×	5	=	1800
\$1320				3300

Explanation.—The first bill is cash and has no interest. The int. of \$200 for 2 mo. is the same as the int. of \$1 for 400 mo. (Prin. *1*°.) The int. of \$275 for 4 mo. is the same as that of \$1 for 1100 mo. The int. of \$360 for 5 mo. is the same as that of \$1 for 1800 months. Therefore, the amount of interest due on the whole debt, is equal to the interest on \$1 for 3300 mo. Now as \$1 is entitled to int. for 3300 months, the whole debt \$1320 is entitled to interest for $\frac{1}{1320}$ of 3300 mo., and $3300 \div 1320 = 2\frac{1}{2}$ months, the average term of credit.

And $2\frac{1}{2}$ mo. added to Oct. 10th, 1880 = Dec. 25th, 1880, the date of payment. Hence, the

Rule.—*Multiply each item by its term of credit, and divide the sum of the products by the sum of the items. The quotient will be the average term of credit.*

Adding the average term of credit to the date of the Bill, will give the date of payment.

Notes.—1. When an item contains cents, if less than 50, they are rejected; if 50 or more, \$1 is added

2. In the quotient, a fraction less than ½ d., is rejected ; if ½ d. or more, 1 day is added.

2. A merchant buys goods, and agrees to pay $400 down, $400 in 4 months, and $400 in 8 months ; what is the average time of the whole?

3. A man borrows $600, and agrees to pay $100 in 2 months, $200 in 5 months, and the balance in 8 months; when can he justly pay the whole at once?

4. A man buys a house for $1600, and agrees to pay $400 down, and the rest in 3 equal annual instalments ; what is the average term of credit ?

5. I have $1200 owing to me, ½ of which is now due ; ¼ of it will be due in 4 months, and the remainder in 8 months ; what is the average term of credit ?

6. A grocer bought goods amounting to $1500, for which he was to pay $250 down, $300 in 4 months, and $950 in 9 months ; when may he pay the whole at once ?

7. A young man bought a farm for $2000, and agrees to pay $500 down, and the balance in 5 equal annual instalments ; what is the average term of credit ?

584. To find the *Average Time*, when the terms of credit are different, and begin at different dates.

8. Bought goods as follows : March 1st, 1880, $200 on 2 mo.; April 6th, $800 on 4 mo.; June 17th, $1000 on 3 mo.; what is the average time and date of payment ?

OPERATION.

$200	due	May	1,	00 d.	×	200	=	00
800	"	Aug.	6,	97 d.	×	800	=	77600
1000	"	Sept.	17,	139 d.	×	1000	=	139000
						2000		) 216600
								108

The average time is 108 d. (Art. **583**, N.)
Date of payment 108 d. from May 1st, or Aug. 17th, 1880.

EXPLANATION.—Taking as the standard the earliest date at which either of the items becomes due (May 1st), the term of credit to Aug. 6, is 97 d., to Sept. 17th, 139 days. The average term of credit is therefore 108 days, and the date of payment is Aug. 17th, 1880. Hence, the

RULE.—I. *Find the date when each item matures. Take the first day of the month in which the earliest item becomes due as a standard, and find the number of days from this to the maturity of each of the other items.*

II. *Multiply each item by its number of days, and divide the sum of the products by the sum of the items The quotient will be the average term of credit.*

III. *Add the average time to the standard date, and the result will be the equitable date of payment.*

NOTE.—Any date may be assumed as the *standard*, but it is most convenient to take the *first* day of the month in which the *earliest* item falls due.

9. Bought the following amount of goods on 4 months' credit: March 10th, 1879, $200; April 15th, $160; May 1st, $440; at what time is the amount payable?

10. Bought the following bills on 8 months: July 5th, 1879, $620.25; Aug. 11th, $240.56; Sept. 20th, $321.64; Oct. 12th, $510.38; Nov. 1st, $308.17; when ought a note for the whole amount to be dated?

11. A merchant bought the following bills of goods: March 19th, $350 on 4 mo.; April 1st, $430 on 130 days; May 16th, $540 on 95 days; June 10th, $730 on 3 mo.; what is the average time for payment of the whole?

12. Bought the following bills of goods on 90 days' credit: May 10th, $375.63; May 18th, $738.45; June 3d, $860.40; June 17th, $692.38; July 3d, $379.68; July 12th, $417.13; at what time will the whole be due at once?

13. A grocer sold the following amount of goods: June 3d, $380 on 90 days' credit; June 10th, $485 on 30 d.; July 21st, $834 on 60 d.; July 27th, $573 on 110 d.; Aug. 2d, $485 on 80 d.; when will the whole be due?

14. Sold the following bills of goods on 3 months; Sept. 5th, 1880, $1163.25; Sept. 20th, $2368.41; Oct. 7th, $3561.34; Oct. 23d, $840.90; Nov. 13th, $1307.63; at what time must a note for the whole amount be dated to give the buyer the specified credit?

AVERAGING ACCOUNTS.

585. Averaging an Account is finding the equated time at which the balance may be paid.

586. To find the *Average Time* for settling an account.

1. Find the equated time and date of paying the balance of the following account:

Dr. JOHN HAMILTON in acct. with HENRY MORGAN. *Cr.*

1881.			1881.		
Jan. 5	For Mdse. 2 mo.	$300	Jan. 25	By Draft 90 d.	$200
Feb. 26	" " 3 mo.	200	March 28	" Cash.	300
March 28	" " 1 mo.	500	May 25	" Cash.	100

OPERATION.

Due.	Amt.	Time.	Prod.	Due.	Amt.	Time.	Prod.
March 5	$300	4 d.	1200	Apr. 28	$200	58 d.	11600
May 26	200	86	17200	March 28	300	27	8100
April 28	500	58	29000	May 25	100	85	8500
	$1000		47400		600		28200
	600		28200				
Bal.	$400		) 19200 (48 days.				

Ans. Bal. $400, due in 48 days from March 1st, or April 18th.

EXPLANATION.—Having found when each item of debt and credit becomes due, by adding its term of credit to its date, we assume as the standard date the *first* day of the month in which the *earliest* item on either side of the account matures, viz.: March 1st.

Multiply each item on both sides by the number of days between the standard date and the maturity of each item, and divide the *difference* between the *sums* of the products (19200), by the *difference* between the *sums* of the items (400). The quotient is the average time of payment.

Since the equated time requires the interest of $1 for 19200 days, it will require $400, $\frac{1}{400}$ part as long, and $19200 \div 400 = 48$; and 48 days added to March 1st gives April 18th. Hence, the

RULE.—I. *Write the date at which each item on both sides matures, and assume the first day of the month in which the earliest item on either side becomes due, as the standard date. Find the number of days from this standard to the maturity of the respective items.* (Art. **583**, N.)

II. *Multiply each item by its number of days, and divide the difference between the sums of products by the difference between the sums of items; the quotient will be the average time.*

III. *If the greater sum of items and the greater sum of products are both on the same side, add the average time to the assumed date; if on opposite sides, subtract it; and the result will be the date when the balance of the account is equitably due.*

NOTES.—1. In finding the maturity of notes and drafts, 3 days grace should be added to the specified time of payment.

2. When no time of credit is mentioned, the transaction is understood to be for cash, and its payment due at once.

2. Find the average time of paying the following account:

Dr. GEORGE HADLEY. *Cr.*

1880.			1880.		
March 1	To Mdse.	$500	Apr. 12	By Draft, 20 d.	$300
Apr. 5	" " 2 mo.	700	May 10	" Cash.	540
May 20	" " 4 mo.	650	June 4	" "	500

3. At what date can the balance of the following account be equitably paid?

Dr. W. H. HENDERSON. *Cr.*

1881.			1881.		
Apr. 7	To Mdse., 2 mo.	$300	May 1	To Mdse., 60 d.	$350
July 5	" " 1 mo.	500	June 10	" " 30 d.	500
Aug. 10	" " 1 mo.	400	Aug. 30	" Cash.	200

4. Average the following account:

Dr. JAMES BROWN & CO. *Cr.*

1882.					1882.			
Jan.	10	To Mdse., 3 mo.	$400		Jan.	1	By Bal. of Acct.	$485
"	25	" " 30 d.	265		Feb.	10	" Note, 3 mo.	2500
Apr.	20	" " 2 mo.	850		March	1	" Draft, 30 d.	260

5. Balance the following account:

Dr. C. J. HAMMOND. *Cr.*

1880.					1880.			
Jan.	20	To Sundries, 30 d.	$500		Jan.	20	By real estate 60 d.	$400
Feb.	12	" " 60 d.	340		March	1	" Draft 60 d.	200
March	1	" " 30 d.	300		"	20	" Cash.	400

6. Average the following account:

Dr. HENRY RAYMOND & CO. *Cr.*

1881.					1881.			
Aug.	10	To Mdse., 60 d.	$150		Aug.	25	By Mdse., 30 d.	$500
Oct.	1	" Cash.	350		Sept.	20	" " 20 d.	300
"	18	" Dft. 30 d.	200					

7. Find when the balance of the following account becomes due:

A. B. bought of C. D., July 16th, 1882, merchandise $350; Aug. 11th, $460; Sept. 9th, $570; Sept. 14th, $840; Oct. 18th, $780. The former paid August 1st, $260; Sept. 30th, in grain $340; Oct. 5th, cash $500; Oct. 21st, $625.

QUESTIONS.

577. When the time and rate of interest remain the same, what is the effect of doubling the principal? The principal and rate remaining the same, what is the effect of doubling the time?

579. What is equation of payments? 580. What is average or equated time? 581. The term of credit? 586. Describe the process of averaging accounts.

STOCKS.

DEFINITIONS.

587. A **Corporation** is a company authorized by law to transact business as a single individual, having the *same rights* and *obligations*.

588. **Stock** is the **Capital** or money used by a corporation in carrying on its business.

589. A **Share** is one of the equal parts into which the stock is divided.

NOTE.—The *value* of a share varies in different companies. It is usually $100, and will be so regarded in this work, unless otherwise stated.

590. A **Certificate of Stock** is a written instrument issued by a corporation, stating the number of shares to which the holder is entitled, and the original value of each share.

591. The **Par Value** of stock is the sum named in the certificate.

592. The **Market Value** is the sum for which it sells.

NOTES.—1. When shares sell for their *nominal* value, they are at *par;* when they sell for *more,* they are *above par,* or at a *premium;* when they sell for *less,* they are *below par,* or at a *discount.*

2. When stocks sell at par they are often quoted at 100; when at 7% above par, they are quoted at 107, or at 7% premium; when at 15% below par, they are quoted at 85, or at 15% discount

593. An **Assessment** is a percentage required of stockholders to replace losses, etc.

594. The **Gross Earnings** of a company are its entire receipts.

595. The **Net Earnings** are the remainder after all expenses are deducted.

596. A **Dividend** is a percentage divided among the stock-holders.

597. A **Bond** is a written agreement to pay a sum of money at or before a specified time.

NOTES.—1. U. S. Bonds are generally designated according to the rates of interest they bear. Thus, U. S. 5's denote bonds issued by the United States bearing 5% interest; U. S. 4's, those bearing 4%, etc.

2. Bonds of States, cities, corporations, etc., are named by combining the rate of interest they bear with the name of the State, corporation, etc., by which they are issued; as, Ohio 6's, N. Y. Central 5's, etc.

598. A **Coupon** is a certificate of interest due on a bond, to be cut off when paid, as a receipt.

599. The term **Stocks** is applied to government, state, city, and railroad bonds, to the capital of banks, etc.

600. Premiums, discounts, dividends, and assessments are calculated by Percentage.

The *par value* of the stock is the *base;* the *per cent of premium* or *discount* is the *rate;* the *premium* or *discount* is the *percentage;* the *par value plus* the premium is the *amount;* and the *par value minus* the discount is the *difference.*

WRITTEN EXERCISES.

601. To find the Premium, Discount, Dividend, etc., from the Par Value and Rate.

FORMULA.—*Premium, etc. = Par Value × Rate.*

1. What is the premium, at 7%, on 40 shares of bank stock?

2. What is the discount, at 15%, on 50 shares of railroad stock?

3. What is the dividend, at 6%, on 85 shares of telegraph stock?

4. Find the assessment, at 10%, on 42 shares of oil stock?

602. To find the *Market Value* of Stock from the Par Value and the Premium or Discount.

FORMULA.—*Market Value* $= \begin{cases} \textit{Par Value} + \textit{Premium.} \\ \textit{Par Value} - \textit{Discount.} \end{cases}$

5. Required the market value of 23 shares of bank stock, at 7% premium?

6. Find the market value of 28 shares of telegraph stock, at 7% discount?

7. What cost 87 shares of iron mountain stock, at 15% premium and brokerage $\frac{1}{2}$%?

8. Find the cost of 150 shares of insurance stock, at $8\frac{1}{4}$% discount, brokerage $\frac{1}{4}$%?

9. What is the cost of 100 shares of N. Y. and New Haven R. R. stock, at 125, brokerage $\frac{1}{4}$%?

ORAL EXERCISES.

603. 1. A premium of $20 was paid on 4 shares of bank stock; what was the rate per cent?

ANALYSIS.—Since 4 shares pay $20, one share ($100) pays $\frac{1}{4}$ of $20, or $5. Therefore, the rate was $\frac{5}{100}$, or 5%.

2. Bought 10 shares of stock for which a premium of $40 was paid; what was the rate of premium?

3. Paid a premium of $50 on 20 shares of oil stock; what was the rate per cent?

4. Sold 15 shares of mining stock for $75; what was the rate of discount?

WRITTEN EXERCISES.

604. To find the Rate from the Par Value, the Premium, Discount, Dividend, etc.

1. The gross receipts of a manufacturing company are $17250, the expenses are $6250, and its capital $50000; what per cent dividend can it make?

ANALYSIS.—The receipts less expenses are \$17250 − \$6250 = \$11000. Now as \$50000 are entitled to \$11000, \$1 is entitled to \$11000 ÷ \$50000 = .22, or 22%. Hence, the

FORMULA.—*Rate* = {*Premium, Discount, Dividend, etc.*} ÷ *Par Value.*

2. A premium of \$375 was paid for 25 shares of R. R. stock; at what rate was the premium?

3. The discount on 50 shares of the Pacific Railroad was \$625; what was the rate of the discount?

4. If the income on \$2356 is \$268.50, what is the rate %?

5. What per cent of 3648 acres is 456 acres?

605. To find the Cost of a given number of shares, the market value of one share and the rate of brokerage being given.

6. What cost 15 shares of R. R. stock, at 120, brokerage $\frac{1}{4}$%?

ANALYSIS.—The cost of 1 share is 120% + $\frac{1}{4}$% brokerage, or 120$\frac{1}{4}$% of \$100 = \$120.25, and 15 shares will cost \$1803.75, *Ans.* Hence, the

FORMULA.—*Cost* = {*Market Value of 1 share + Brokerage × Number of shares.*}

7. What is the cost of 78 shares of R. R. stock at 124$\frac{3}{4}$, and brokerage at $\frac{1}{4}$%?

8. Find the cost of 121 shares, at 89$\frac{1}{2}$, and brokerage $\frac{1}{2}$%?

9. Sold 250 shares of bank stock at 87$\frac{3}{4}$, and paid $\frac{1}{4}$% brokerage; how much did I receive for it?

10. What is the cost of 375 shares of National Express stock, at 25% premium and brokerage $\frac{1}{2}$%?

606. To find the *Number* of Shares, when the investment and the cost of 1 share are given.

11. How many shares of bank stock at 5% discount and brokerage $\frac{1}{4}$%, can be bought for \$7620?

ANALYSIS.—Since the discount is 5% and brokerage $\frac{1}{4}$%, the cost of 1 share is 95% + $\frac{1}{4}$%, or 95$\frac{1}{4}$% of \$100 = \$95.25. As \$95.25 will buy 1 share, \$7620 will buy as many shares as \$95.25 are contained times in \$7620, and \$7620 ÷ \$95.25 = 80 shares, *Ans.* Hence, the

FORMULA.—*Number of Shares = Investment ÷ Cost of 1 Share.*

12. How many shares of telegraph stock, at $7\frac{3}{4}\%$ premium and brokerage $\frac{1}{4}\%$, can you buy for \$13500?

13. Find the number of shares of mining stock at $102\frac{5}{8}$, that can be bought for \$5150, and brokerage $\frac{3}{8}\%$.

14. What number of railroad shares at 125, brokerage $\frac{1}{4}\%$, will \$7515 pay for?

15. How many shares of express stock, at 10% premium, can be bought for \$8030?

16. Find the number of shares, at 20% discount, that can be bought for \$3200?

607. To find how stock must be bought which pays a given per cent dividend, to realize a given per cent on the investment.

17. At what price must I buy Western R. R. stock which pays 6% dividend, so as to realize 8% on the investment?

Analysis.—Dividend $.06 \div .08$ income $= .75$, or 75%, price of stock.

Formula.—*Price = Dividend ÷ Rate of Income.*

18. What must be paid for 4% bonds that the investment may yield 6%?

19. What must be paid for U. S. 5's that 8% may be received on the investment?

20. What must be paid for stock that yields 10% dividends, so as to realize $7\frac{1}{2}\%$ on the investment?

608. To find what sum to invest to yield a given income, the cost of 1 share, rate of interest, or dividend being given.

21. What sum must be invested in N. Y. 5's, at $108\frac{1}{2}$, to produce an annual income of \$1500?

Analysis.—The income $\$1500 \div \$5$ (int. on 1 share) $= 300$ shares, and $108\frac{1}{2}$ (price of 1 share) $\times 300 = \$32550$. Hence, the

Formula.—*Investment = Cost 1 Share × Number of Shares.*

22. What sum must be invested in U. S. 4's, at 105, to yield \$3000 annually?

23. What sum must be invested in Nebraska 8's, at 75, to yield an income of \$1540 annually?

24. What sum must be invested in stock at 112, which pays 10% annually, to obtain an income of $2200?

25. What sum must be invested in Alabama 6's, at 85, to realize $2000 a year?

26. How much must be invested in stock at 106, to yield an income of $600, the stock paying 10% dividend annually?

609. To find the rate per cent of income from bonds paying a given rate of interest, and bought at a given premium or discount without regard to their maturity.

27. What is the rate of income on bonds paying 8% interest, bought at 112?

SOLUTION.—Interest on 1 share $8÷112, cost per share = $7\frac{1}{7}$%, *Ans.*

28. Bought bonds paying 6% interest, at 75; what was the rate per cent of income?

SOLUTION.—Interest of 1 share $6÷75 cost per share = 8%, *Ans.*

FORMULA.—*Rate % Income* = $\left\{\begin{array}{l}\textit{Interest per Share}\\ \div\ \textit{Cost per Share.}\end{array}\right.$

29. Find the per cent of income on U. S. 5's, bought at 110.

30. What is the per cent of income on Iowa 6's, bought at 108, brokerage $\frac{1}{2}$%?

31. Which is the more profitable, $10000 invested in $3\frac{1}{2}$ per cents at 75, or in 7 per cents at 105?

610. To find the rate per cent of income from bonds paying a given rate of interest, bought at a given premium or discount and payable at par in a given time.

32. What rate per cent income will be realized from N. Y. 5's, bought at a premium of 8%, and paid at par in 10 years?

ANALYSIS.—Since the bond matures in 10 years, the premium on 1 share ($8) decreases $\frac{8}{10}$, or $$\frac{4}{5}$ each year. Now the interest $5—$$\frac{4}{5}$=$$4\frac{1}{5}$, annual income on 1 share. And $$4\frac{1}{5}$ ÷ 108, cost of 1 share = .03$\frac{8}{9}$, or $3\frac{8}{9}$%, the rate required.

33. What rate per cent income will be realized from North Carolina 8's, bought at 90, if paid at par in 20 years?

ANALYSIS.—Since the bond matures in 20 years, the average decrease of the discount on 1 share is $\$10 \div \$20 = \$\frac{1}{2}$ each year. Now the interest $\$8 + \$\frac{1}{2} = \$8\frac{1}{2}$, the annual income on 1 share. And $\$8.50 \div \$90$ (cost of 1 share) $= \$.09\frac{4}{9}$, or $9\frac{4}{9}\%$, the rate required. Hence, the

RULE.—*First find the average annual decrease of the premium or discount.*

If the bonds are at a premium, subtract it from the given rate of interest; if at a discount, add it to the interest; the result will be the average income of one share.

Divide the average income of one share by the cost of one share, and the quotient will be the rate per cent of income.

NOTES.—1. When bonds are at a *premium*, the *longer* the time before maturity, the *greater* will be the rate per cent of income.

2. When bonds are at a *discount*, the *longer* the time before maturity, the *less* will be the rate per cent of income.

34. What rate per cent of income will be received on U. S. 4's at 106, and payable at par in 15 years?

35. Bought Milwaukee and St. Paul bonds at 90, due at par in 30 years, drawing 10% interest; what is the rate per cent of income?

QUESTIONS.

587. What is a corporation? 588. Stock? 589. A share? 590. A certificate of stock? 591. The par value? 592. Market value?

593. An assessment? 594. Gross earnings? 595. Net earnings? 596. A dividend? 597. A bond? 598. A coupon?

599. To what is the term stocks applied? 600. How are premiums, etc., computed?

601. What does the premium equal? 602. The market value? 604. The rate? 605. The cost? 606. The number of shares? 607. How find the price? 609. How find the rate of income without regard to maturity? 610. On bonds payable at par at maturity?

EXCHANGE.

611. Exchange is a method of making payments between distant places without sending the money.

612. A **Draft** or **Bill of Exchange** is a written order directing one person to pay another a certain sum, at a specified time.

613. The **Drawer** is the person who signs the draft.

614. The **Drawee** is the person to whom it is addressed.

615. The **Payee** is the person to whom the money is to be paid.

616. A **Sight Draft** is one payable on its *presentation.*

617 A **Time Draft** is one payable at a specified time *after* date or presentation.

NOTE.—Drafts or Bills of Exchange are negotiable like promissory notes, and the laws respecting them are essentially the same.

618. An **Acceptance** of a draft is an *engagement* to pay it. As evidence, the drawee writes the word *accepted* across the face of the draft, with the date and his name.

619. The **Par of Exchange** is the *standard* by which the value of the currency of different countries is *compared,* and is either *intrinsic* or *commercial.*

620. Intrinsic Par is a standard having a *real* and *fixed value* represented by *gold* or *silver coin.*

621. Commercial Par is a *conventional* standard, having any assumed value which convenience may suggest.

NOTE.—The *fluctuation* in the *price* of bills from their par value, is called the *Course of Exchange.*

DOMESTIC EXCHANGE.

622. Domestic Exchange is a method of making payments between distant places in the *same* country.

623. To find the *Cost* of a Draft, when the Face and Rate of Exchange are given.

1. What cost the following *sight draft,* at $2\frac{1}{2}\%$ premium?

$2500. NEW YORK, June 30th, 1881.

At sight, pay to the order of JAMES CLARK, *twenty-five hundred dollars, value received, and charge the same to the account of*

SMITH BROS., & CO.

To S. BARRETT & CO.,
New Orleans, La.

ANALYSIS. — Since exchange on N. O. is $2\frac{1}{2}\%$ prem., the cost of \$1 draft is \$1.025, and \$2500 will cost $\$1.025 \times 2500 = \$2562.50$, *Ans.*

OPERATION.

$$1 + .025 = \$1.025$$
$$\$1.025 \times 2500 = \$2562.50$$

2. What is the cost of a sight draft on San Francisco for \$3000, at $2\frac{1}{2}\%$ discount?

SOLUTION.—A draft of \$1 at $\$2\frac{1}{2}\%$ discount will cost \$0.975, and $\$3000 \times .975 = \$2925.00$, *Ans.* Hence, the

RULE.—*Multiply the face of the draft by the cost of \$1.*

624. On *time* drafts, both the rate of exchange and bank discount are commonly included in the *rate,* which in quotations for *time* drafts is enough *less* than for *sight* drafts, to allow for bank discount.

Required the cost of a sight draft

3. On St. Louis, at $1\frac{1}{2}\%$ premium, for \$850?
4. On Buffalo, at $\frac{1}{2}\%$ discount, for \$975?
5. On Savannah, Ga., for \$2000, at $1\frac{3}{4}\%$ premium?

6. What is the cost of the following *time draft*, at $1\frac{1}{2}\%$ premium, and interest at 6%?

$4000. PHILADELPHIA, July 5th, 1881.

Sixty days after sight, pay to the order of GEORGE WILCOX, *four thousand dollars, value received, and charge the same to the account of*

H. ADAMS & Co.

To S. PARKHURST,
Trenton, N. J.

ANALYSIS.—At $1\frac{1}{2}\%$ premium, the cost of \$1 draft at sight is \$1.015. But the draft is subject to interest for 60 d. + 3 d. grace. The int. of \$1 for 63 d., at 6% is \$0.0105, and \$1.015−0.0105 = \$1.0045 the cost of \$1 draft, and a draft of \$4000 will cost 4000 times \$1.0045, or \$4018, *Ans.*

OPERATION.

$$\$1 + \$0.015 = \$1.015$$
$$\$1 \times \$.0105 = \$0.0105$$
$$\text{Cost } \$1 \text{ draft} = \$1.0045$$
$$\$1.0045 \times 4000 = \$4018$$

7. Find the cost in Omaha of a draft on New York at 90 days sight, for \$5265, at 2% premium, interest being $6\frac{1}{4}\%$.

8. Required the worth in Memphis of a draft on Boston for \$3500, at 30 days sight, at 1% discount and interest 6%.

9. What is the worth of a draft of \$5000 on St. Louis, at 30 days sight, premium $1\frac{1}{2}\%$, including interest?

625. To find the *Face* of a Draft, when the Cost and Rate of Exchange are given.

10. How large a draft on Philadelphia can be bought in Charleston, at 60 days sight, for \$3000, the premium being $1\frac{1}{2}\%$, and interest 6%?

ANALYSIS —Since the premium is $1\frac{1}{2}\%$, the cost of \$1 sight draft would be \$1.015. But the bank discount on \$1 for 63 d. is .0105, hence, the cost of \$1 draft is \$1.0045. Now if \$1 draft cost \$1.0045, \$3000 will buy a draft of as many dollars as \$1.0045 is contained times in \$3000, or \$2986.56, *Ans.* Hence, the

OPERATION.

$$\$1 + .015 = \$1.015$$
$$\text{Bank dis. 63 d.} = .0105$$
$$\text{Cost of } \$1 \text{ draft.} = \$1.0045$$
$$\$3000 \div 1.0045 = \$2986.56$$

RULE.—*Divide the cost of the draft by the cost of $1 exchange.*

11. What was the face of a sight draft for which $2500 was paid, exchange being at $2\frac{1}{2}\%$ premium?

12. What is the face of a sight draft bought for $3300 in Memphis on Boston, exchange being $3\frac{1}{2}\%$ discount?

13. What is the face of a draft on 4 months for which $450 is paid, exchange 1% premium, and int. 6%?

14. Find the face of a draft on New York at 90 days sight, bought in Cincinnati for $2250, exchange at $1\frac{1}{4}\%$ discount, interest 5%?

15. A merchant of Galveston paid $4265 for a draft on St. Louis at 30 d. sight, exchange at $3\frac{1}{4}\%$ premium, interest 8%; what was the face of the draft?

16. What is the face of a draft on Cincinnati at 90 days sight, bought for $3000, exchange $2\frac{1}{2}\%$ premium, interest 6%?

FOREIGN EXCHANGE.

626. Foreign Exchange is the method of making payments between *different* countries.

627. A **Set of Exchange** consists of three bills of the *same date* and *tenor, First, Second,* and *Third* of exchange. They are sent by different mails in order to save time in case of miscarriage. When one is *paid,* the others are *void.*

628. Exchange with Europe is chiefly done through large commercial centers, as London, Paris, Geneva, Amsterdam, Antwerp, Hamburg, Frankfort, and Berlin.

629. Bills drawn on England, Scotland, or Ireland, are called **Sterling Bills**, and the value of a Pound Sterling is quoted in U. S. money.

630. The present **Par of Exchange** on Great Britain is $4.8665 gold to the *pound sterling,* which is the intrinsic value of a Sovereign, as estimated at the U. S. Mint.

631. In quoting exchange on a foreign country, it is customary to quote the *value* of the *money unit* of that country in U. S. money.

NOTE.—These values are published annually by the Secretary of the Treasury. Those given on the 1st day of Jan., 1882, are as follows:

Country.	Monetary Unit.	Standard.	Value in U. S. Money.
Austria	Florin	Silver	.40,7
Belgium	Franc	Gold and silver	.19,3
Bolivia	Boliviano	Silver	.82,3
Brazil	Milreis of 1000 reis	Gold	.54,6
British N. A.	Dollar	Gold	$1.00
Chili	Peso	Gold and silver	.91,2
Cuba	Peso	Gold and silver	.93,2
Denmark	Crown	Gold	.26,8
Ecuador	Peso	Silver	.82,3
Egypt	Piaster	Gold	.04,9
France	Franc	Gold and silver	.19,3
Great Britain	Pound sterling	Gold	4.86.6½
Greece	Drachma	Gold and silver	.19,3
German Empire	Mark	Gold	.23,8
Hayti	Gourde	Gold and silver	.96,5
India	Rupee of 16 annas	Silver	.39
Italy	Lira	Gold and silver	.19,3
Japan	Yen	Silver	.88,8
Liberia	Dollar	Gold	1.00
Mexico	Dollar	Silver	.89,4
Netherlands	Florin	Gold and silver	.40,2
Norway	Crown	Gold	.26,8
Peru	Sol	Silver	.82,3
Portugal	Milreis of 1000 reis	Gold	1.08
Russia	Rouble of 100 copecks	Silver	.65,8
Sandwich Islands	Dollar	Gold	1.00
Spain	Peseta of 100 centimes	Gold and silver	.19,3
Sweden	Crown	Gold	.26,8
Switzerland	Franc	Gold and silver	.19,3
Tripoli	Mahbub of 20 piasters	Silver	.74,3
Turkey	Piaster	Gold	.04,4
U. S. of Colombia	Peso	Silver	.82,3
Venezuela	Bolivar	Gold and silver	.19,3

632. The method of finding the cost of foreign bills is essentially the same as that of domestic bills. (Art. **624.**)

1. What is the cost of the following bill on London, at \$4.8665 to the £ sterling?

£354 12s. NEW YORK, July 4th, 1880.

At sight of this first of exchange (the second and third of the same date and tenor unpaid), pay to the order of HENRY CROSBY, *three hundred fifty-four pounds, twelve shillings sterling, value received, and charge the same to the account of*

J. KING & Co.

TO GEORGE PEABODY, Esq., London.

ANALYSIS.—£354 12s. = £354.6. (Art. **403.**) As £1 is worth \$4.8665, £354.6 are worth 354.6 times as much; and \$4.8665 × 354.6 = \$1725.661, the cost.

$$\begin{array}{r} 4.8665 \\ 354.6 \\ \hline \textit{Ans.}\ \$1725.661 \end{array}$$

2. What is the cost of a bill on Liverpool for £345 5s. 6d., at \$4.875 to the pound sterling?

3. What is the cost in currency of a bill on Edinburgh for £360.5, exchange being at par and gold 5% premium?

633. Bills on *Paris*, *Antwerp*, and *Geneva*, are quoted by the number of francs and centimes to a dollar in gold.

NOTE.—*Centimes* are commonly written as decimals of a *Franc.*

4. What is the cost of a bill on Paris for 575 francs, at 5.16 francs to a dollar in gold?

ANALYSIS.—As 5.16 francs cost \$1, 575 francs will cost as many dollars as 5.16 is contained times in 575, and 575 ÷ 5.16 = \$111.43, *Ans.*

5. Find the cost of a bill on Geneva for 750.25 francs, at $5.15\frac{1}{2}$ fr. to the dollar in gold.

6. Find the cost of a bill for 1000 francs on Antwerp, at $5.17\frac{1}{4}$ fr. to a dollar, gold at 1% premium?

634. Bills on *Bremen, Frankfort, Hamburg,* and *Berlin,* are quoted by the value in U. S. Money of four marks (reichs-marks) in gold.

7. What cost a bill on Frankfort for 540 marks, at \$.94½?

ANALYSIS.—Since 4 marks are worth \$.945, the worth of 540 marks is 540 times ¼ of \$.945, or \$127.58, *Ans.* (Art. **632.**)

8. What cost a bill on Berlin for 2800 marks at \$.96½ in gold?

635. The method of finding the *face* of a foreign bill of exchange is essentially the same as that of domestic bills.

9. What is the face of a bill of exchange on London, bought for \$4500 at \$4.87½ in gold?

ANALYSIS.—Since \$4.875 will buy a bill of £1, \$4500 will buy as many pounds as \$4.875 are contained times in \$4500, and \$4500 ÷ 4.875 = £923.076, or £923 1s. 6¼d., *Ans.*

10. What is the face of a bill on Dublin for which \$6500 was paid in gold, at \$4.86?

11. What is the face of a bill on Paris for \$2400, exchange being 5.15 fr. to a dollar?

12. Find the face of a bill on Geneva, which cost \$1500 gold, exchange 5.16.

13. Find the face of a bill on Frankfort costing \$762 in gold, exchange at 95¼.

14. Paid \$2000 for a bill on Berlin, exchange 93¾; what was the amount of the bill?

QUESTIONS.

611. What is Exchange? 612. A draft or bill of exchange? 613. The drawer of a bill? 614. The drawee? 615. The payee? 616. A sight draft or bill? 617. A time draft or bill? 618. What is the acceptance of a draft? 619. The par of exchange?

622. What is domestic exchange? 623. How find the cost of a draft? 625. How find the face?

626. What is foreign exchange? 627. A set of exchange? 629. What are sterling bills? 631. How is foreign exchange quoted? 632. How find the cost of foreign bills? 635. How find the face?

GENERAL ANALYSIS.

636. Business men have a method of solving practical questions, which is frequently *shorter* and *more expeditious,* than that of arithmeticians fresh from the schools.

637. Their method consists in *Analysis,* and may, with propriety, be called the **Common Sense Method.**

638. No specific directions can be given for the *analysis* of problems. The learner must be taught to depend on his *judgment* as a guide.

639. He may, however, be aided by the following:

GENERAL PRINCIPLES.

1°. We reason from that which is self-evident, or known, to that which is unknown, or required.

2°. We reason from a part to the whole; as, when the value of one is given, and the value of two or more of the same kind is required.

3°. We reason from the whole to a part; as, when the value of two or more is given, and that of a part is required.

4°. We reason from a given cause to its effect, or from a given effect to its cause; as, when different combinations of numbers are given, to find the result.

Thus, If 3 men can mow 6 acres in 1 day, how many acres can they mow in 5 days?

Or, If to draw 4 tons requires 6 horses, how many horses will be required to draw 8 tons?

ORAL EXERCISES.

640. 1. If 8 tons of coal cost \$40, how much will 6 tons cost?

ANALYSIS.—1 is $\frac{1}{8}$ of 8; therefore, 1 ton will cost 1 eighth of \$40, which is \$5. As 1 ton costs \$5, 6 tons will cost 6 times \$5, or \$30, *Ans.*

Or, thus: 6 tons are $\frac{6}{8}$ of 8 tons; therefore, 6 tons will cost $\frac{6}{8}$ of \$40. Now 1 eighth of \$40 is \$5, and 6 eighths are 6 times \$5, or \$30, the same as before.

2. If 7 lb. of tea cost 42 shillings, what will 10 lb. cost

3. If 9 sheep are worth \$27, how much are 15 sheep worth?

4. If 10 barrels of flour cost \$60, what will 12 barrels cost?

5. If a man walks 54 kilometers in 6 days, how far does he walk in 15 days?

6. If 12 men can build 48 rods of wall in a day, how many rods can 20 men build in the same time?

7. Suppose 75 kilos of butter last a family 25 days, how many kilos will supply them 12 days?

8. If 7 meters of cloth cost \$30, how much will 9 meters cost?

9. If 10 bbl. of beef cost \$72, how much will 8 bbl. cost?

10. If 7 acres of land cost \$50, what will 12 acres cost?

11. If $\frac{3}{4}$ ton coal cost \$6, what will 5 tons cost?

ANALYSIS.—Since 3 fourths ton cost \$6, 1 fourth will cost $\frac{1}{3}$ of \$6, or \$2, and 4 fourths, or 1 ton will cost 4 times \$2, or \$8. Now at \$8 a ton, 5 tons will cost 5 times \$8, or \$40, *Ans.*

12. If $\frac{5}{7}$ lb. tea cost 40 cts., what will 12 lb. cost?

13. If $5\frac{1}{2}$ bbl. apples cost \$22, what will 9 bbl. cost?

14. If $\frac{7}{8}$ acre land cost \$28, what will 10 acres cost?

15. If 16 cords of wood are worth \$48, how much is $\frac{3}{4}$ cord worth?

16. If $\frac{4}{5}$ of a citron cost 28 cents, what must you pay for 12 citrons?

17. If $\frac{5}{8}$ yd. cloth cost \$2, what is that a yard?

18. If 4 lb. ginger cost \$$\frac{8}{9}$, what will 11 lb. cost?

19. If 3 melons cost \$$\frac{9}{10}$, what will 20 melons cost?

20. Paid \$$\frac{8}{10}$ for 4 slates; what must I pay for 18 slates?

WRITTEN EXERCISES.

641. The pupil should be required to Analyze each of the following examples, giving results as he proceeds, and be encouraged to invent different solutions.

1. If 60 bbl. flour cost $300, what will 42 bbl. cost?

ANALYSIS.—Since 60 barrels cost $300, 1 barrel costs $\frac{1}{60}$ of $300, and $\$300 \div 60 = \$5.00$.

Again, 42 barrels will cost 42 times as much as 1 barrel. Therefore, $\$5 \times 42 = \$210$, *Ans.*

NOTE.—Much labor may often be saved by indicating the operations required, and cancelling the common factors before the multiplications and divisions are performed.

Thus, $\$\frac{300}{60} \times 42 = \$\frac{\overset{5}{\cancel{300}}}{\cancel{60}} \times 42 = \$210$, *Ans.*

2. A man bought 30 cords of wood for $76.80; how much must he pay for 195 cords?

3. A gentleman bought 85 meters of carpeting for $106.25; how much would 38 meters cost?

4. A drover bought 350 sheep for $525; how much would 65 cost, at the same rate?

5. If $12\frac{1}{2}$ pounds of coffee cost $1.25, how much will 245 pounds cost?

6. If 126 bushels of corn are worth $52.92, how much are 84 bushels worth?

7. Paid $20 for 60 pounds of tea; how much would $112\frac{1}{2}$ pounds cost, at the same rate?

8. Bought 41 meters of flannel for $16.40; how much would $18\frac{3}{5}$ meters cost?

9. Bought 18 pounds of ginger for $1.50; how much will $20\frac{3}{5}$ pounds cost?

10. If a stage goes 84 miles in 12 hours, how far will it go in 108 hours?

11. If 16 horses eat 72 bushels of oats in a week, how many bushels will 125 horses eat in the same time?

12. If a railroad car goes 120 miles in 5 hours, how far will it go in $212\frac{3}{4}$ hours?

13. If a steamboat goes 189 kilometers in 12 hours, how far will it go in $5\frac{3}{8}$ hours?

14. If $\frac{5}{12}$ of a cord of wood costs $\frac{5}{8}$ of a dollar, how much will $\frac{3}{4}$ of a cord cost?

ANALYSIS.—Since $\frac{5}{12}$ cord costs $\$\frac{5}{8}$, $\frac{1}{12}$ cord will cost $\frac{1}{5}$ of $\$\frac{5}{8}$, or $\$\frac{1}{8}$, and $\frac{12}{12}$ or 1 cord will cost, $\$\frac{1}{8}\times 12 = \$\frac{12}{8}$, or $1\frac{1}{2}$. Again, since 1 cord costs $\$\frac{12}{8}$, $\frac{3}{4}$ cord will cost $\frac{3}{4}$ of $\$\frac{12}{8} = \frac{36}{32}$, or $\$1\frac{1}{8}$.

Or, $\$\frac{5}{8}\times\frac{12}{5}\times\frac{3}{4} = \$\frac{\cancel{5}}{8}\times\frac{\overset{3}{\cancel{12}}}{\cancel{5}}\times\frac{3}{\cancel{4}} = \$\frac{9}{8} = \$1\frac{1}{8}$, *Ans.*

15. If $\frac{3}{4}$ of a yard of cloth cost £$\frac{6}{7}$, how much will $\frac{7}{8}$ of a yard cost?

16. If $\frac{3}{16}$ of a ship cost \$16000, how much is $\frac{5}{8}$ of her worth?

17. If a man pays \$47 for building $23\frac{1}{2}$ rods of fence, how much would it cost him to build $213\frac{3}{4}$ rods?

18. A farmer paid \$45.42 for building $36\frac{2}{5}$ rods of stone wall; how much will it cost him to build $60\frac{7}{10}$ rods?

19. If $7\frac{1}{2}$ meters of satinet cost $\$9\frac{3}{8}$, how much will $18\frac{1}{2}$ meters cost?

20. A ship's company of 30 men have 4500 pounds of flour; how long will it last them, allowing each man $2\frac{1}{2}$ lb. per day?

21. How long will 56700 pounds of meat last a garrison of 756 men, allowing $1\frac{1}{2}$ lb. apiece per day?

22. A can chop a cord of wood in 4 hours, and B in 6 hours; how long will it take both to chop a cord?

ANALYSIS.—Since A can chop a cord in 4 hours, in 1 hour he can chop $\frac{1}{4}$ of a cord; and since B can chop a cord in 6 hr., he can chop $\frac{1}{6}$ of a cord; hence both can chop $\frac{1}{4}+\frac{1}{6}$ cord $= \frac{5}{12}$ cord in 1 hr.

Again, if to chop $\frac{5}{12}$ cord takes both 1 hr., to chop $\frac{1}{12}$ cord will take $\frac{1}{5}$ hour, and $\frac{12}{12}$ or a whole cord will take them 12 times $\frac{1}{5}$ hr., or $2\frac{2}{5}$ hours.

23. If a man can plant a field in 8 days, and a boy in 12 days; how long will it take both to plant it?

24. A can do a piece of work in 20, B in 40, and C in 60 minutes; how long would it take all together to do it?

25. A cistern has 3 faucets, one of which will empty it in 5 hr., another in 10 hr., and the other in 15 hr.; how long will it take all 3 to empty it?

26. If 10 men require $8\frac{4}{5}$ days to finish a piece of work, how long will it take 11 men to finish it?

27. A water-tank has 3 pipes; the first will empty it in 12 hr., the second will fill it in 6 hr., the third in 8 hours; in how many hours will the tank be filled if all run together?

28. A deer starting 150 rods before a dog, runs 30 rods a minute; the dog follows at the rate of 42 rods a minute. In what time will the dog overtake the deer?

ORAL EXERCISES.

642. 1. A grocer sold 8 lb. of sugar at 12 cents a pound, and took his pay in lard, at 10 cents a pound; how much lard did it take to pay for the sugar?

ANALYSIS.—Since 1 lb. of sugar is worth 12 cents, 8 lb. are worth 8 times 12, or 96 cents.

Again, since 10 cents will pay for 1 lb. of lard, 96 cents will pay for as many pounds as 10 cents are contained times in 96 cents, or $9\frac{3}{5}$ pounds.

Or, thus: 12 cents are $\frac{12}{10}$ of 10 cents; therefore, 1 lb. of sugar is worth $\frac{12}{10}$ lb. of lard; and 8 lb. of sugar are worth 8 times $\frac{12}{10}$ lb. of lard, which is $\frac{96}{10}$ lb. $= 9\frac{6}{10}$ or $9\frac{3}{5}$ lb., *Ans.*

2. How many dozen eggs, at 15 cents a dozen, will pay for $12\frac{1}{2}$ yards of muslin, worth 8 cents a yard?

3. A farmer exchanged 8 tons of hay, worth \$20 per ton, for flour worth \$6 a barrel; how many barrels of flour did he receive for his hay?

4. A man exchanged 50 lb. of wool, valued at $37\frac{1}{2}$ cents a pound, for flannel worth $87\frac{1}{2}$ cents a yard; how many yards did he obtain?

5. A lad bought 75 apples, at the rate of 3 for a cent, and exchanged them for oranges worth 5 cents apiece; how many oranges did he receive?

6. How many slate pencils worth $\frac{1}{2}$ cent apiece, can you buy for 150 steel pens worth 4 cents per dozen?

7. How many acres of farming land worth $\$12\frac{1}{2}$ per acre, must be given in exchange for 4 building lots in the city, valued at \$75 per lot?

8. How much lard at 27 cents a kilo, will pay for 153 Kg. of rice, worth 9 cents a kilo?

9. How many oranges at $7\frac{1}{2}$ cents apiece, can you buy for $\frac{1}{7}$ of 35 quarts of strawberries, at $12\frac{1}{2}$ cents a quart?

10. A lad bought 12 peaches, at the rate of 3 for 4 cts., and afterwards exchanged them for oranges which were 3 for 8 cents; how many oranges did he obtain?

11. Frank sold 10 apples, which was $\frac{2}{9}$ of all he had; he then divided the remainder equally among 5 companions; how many did each receive?

12. Lincoln spent 60 cents for a book, which was $\frac{10}{12}$ of his money; with the remainder he bought oranges, at 4 cents apiece; how many oranges did he buy?

13. A man paid away $35, which was $\frac{5}{7}$ of all he had; he then spent the rest in cloth at $2 per yard; how many yards did he obtain?

14. A farmer bought a quantity of goods, and paid $20 down, which was $\frac{2}{3}$ of the bill; how many cords of wood, at $3 per cord, will it take to pay the balance?

WRITTEN EXERCISES.

643. 1. A merchant sold 75 yd. silk, at $0.84 a yd., and took his pay in corn, at $0.60 a bu.; how many bu. did he receive?

ANALYSIS.—At $.84 a yd. 75 yards are worth $\$.84 \times 75 = 63$.
Again, to pay $63.00 will require as many bushels of corn as $.60 are contained times in $63.00, and $\$63.00 \div .60 = 105$ bu., *Ans.*

2. How many pounds of butter, at 35 cents, must be given in exchange for 186 yards of calico, at $18\frac{3}{4}$ cents per yard?

3. How many pounds of tobacco, at $16\frac{1}{4}$ cents, must be given for 256 pounds of sugar, at $6\frac{1}{4}$ cents a pound?

4. A farmer bought 325 sheep at $\$2\frac{1}{4}$ apiece, and paid for them in hay, at $\$10\frac{1}{2}$ per ton; how many tons did it take?

5. A man bought a hogshead of vinegar, worth $37\frac{1}{2}$ cents per gallon, and gave $331\frac{1}{2}$ pounds of cheese in exchange; how much was the cheese a pound?

6. Bought 274 bushels of salt, at $42\frac{1}{2}$ cents per bushel, and paid in wheat at $1.25 per bushel; how many bushels of wheat did it require?

7. A bookseller exchanges 400 dictionaries worth $$1.87\frac{1}{2}$ cents apiece, for 900 grammars; how much did the grammars cost apiece?

8. How many meters of silk, worth $$1\frac{3}{4}$ per meter, will pay for $249\frac{1}{2}$ meters of cloth worth $$5\frac{1}{4}$ per yard?

9. Bought $263\frac{3}{4}$ yds. of satinet, at $$1\frac{3}{4}$ per yard, and paid for it in cheese, at $$9\frac{1}{2}$ per hundred; how much cheese did it take?

10. Bought 25 hhds. 22 gals. 3 qts. of molasses at $37\frac{1}{2}$ cents per gallon, and paid for it in wool, at $62\frac{1}{2}$ cents a pound; how much wool did it take?

11. Bought 432 sheep at $$2\frac{1}{4}$ apiece, for which I paid 144 barrels of flour; what was the flour per barrel?

12. If 15 yards of flannel are worth 25 yards of muslin, how many yards of flannel are worth 315 yards of muslin?

13. A market-woman bought 10 dozen oranges, at the rate of 3 for 4 cents, and then exchanged them for eggs, at the rate of 4 for 5 cents; how many eggs did she receive?

14. If 15 lbs. of pepper are worth 25 lbs. of ginger, how many pounds of ginger must be given for 195 lbs. of pepper?

ORAL EXERCISES.

644. 1. What cost 36 bushels of oats, at $33\frac{1}{3}$ cts. a bushel?

ANALYSIS.—At $1 a bushel 36 bu. would cost $36; but the price is $33\frac{1}{3}$ cents, or $$\frac{1}{3}$; hence, at $$\frac{1}{3}$, 36 bu. will cost $\frac{1}{3}$ of $36, which is $12, *Ans.* (Art. 280.)

2. At $12\frac{1}{2}$ cts. a pound, what cost 72 lbs. of maple sugar?
3. At 20 cts. apiece, what will 150 melons come to?
4. What cost 60 rolls of tape, at $6\frac{1}{4}$ cts. a roll?
5. What cost 72 yd. delaine, at $16\frac{2}{3}$ cts. a yard?
6. At 25 cts. a yard, what must I pay for 64 yards of ribbon?
7. At 50 cts. a bushel, what will 250 bushels of corn cost?
8. At $33\frac{1}{3}$ cts. apiece, what cost 12 doz. Grammars?

WRITTEN EXERCISES.

645. 1. What will 1268 bushels of apples come to, at 25 cts. a bushel?

ANALYSIS.—25 cts. = $\$\frac{1}{4}$; therefore the apples will cost $\frac{1}{4}$ as many dollars as there are bushels, and $1268 \div 4 = 317$. *Ans.* \$317.

2. At $8\frac{1}{3}$ cts. apiece, what cost a gross of slates?

3. What cost 480 yards of ribbon, at $16\frac{2}{3}$ cts. a yard?

4. At $33\frac{1}{3}$ cts. a hektoliter, what must I pay for 750 hektoliters of potatoes?

5. What cost 1250 melons, at 20 cents each?

6. At 50 cents apiece, how much will 1745 Readers cost?

ORAL EXERCISES.

646. 1. Two boys formed a partnership in selling newspapers; A put in 30 cts. and B 50 cts. They gained 40 cts. the first day; what was the share of each?

ANALYSIS.—Their capital was 30 cts. + 50 cts. = 80 cts.
A's part of it was $\frac{30}{80}$, or $\frac{3}{8}$; and B's part was $\frac{50}{80}$, or $\frac{5}{8}$.
Now $\frac{1}{8}$ of 40 is 5 cts., and $\frac{3}{8}$ are 3 times 5, or 15 cts., A's share.
And $\frac{5}{8}$ are 5 times 5, or 25 cts., B's share.

2. A and B bought a pony together for \$100; A put in \$60 and B \$40; they sold it so as to gain \$30; what was each one's share of the gain?

3. Two men buy a carriage together for \$500; A put in \$300 and B \$200; they sold it at a loss of \$150. What was the share of each in the loss?

4. C and D joined in a speculation and cleared \$90; C put in \$400 and D \$800; what share of the gain had each?

5. A man failed in business, owing A \$700 and B \$400; his property was valued at \$880; how much would each creditor receive?

6. B and C engaged in business; B furnished \$900 and C \$600; they made \$300; what was the share of each?

WRITTEN EXERCISES.

647. 1. A, B, and C, formed a partnership; A put in \$2000, B \$3000, and C \$4000; they gained \$2700; what was each man's share of the gain?

ANALYSIS.—The capital was \$2000 + \$3000 + \$4000 = \$9000.

Since A's part of the capital was $\frac{2000}{9000}$, or $\frac{2}{9}$, his share of the gain was $\frac{2}{9}$ of \$2700, and $\frac{2}{9}$ of \$2700 = \$600 A's gain.

B's part was $\frac{3000}{9000}$, or $\frac{3}{9}$ of \$2700, and $\frac{3}{9}$ of \$2700 = \$900 B's gain.

C's part was $\frac{4000}{9000}$, or $\frac{4}{9}$ of \$2700, and $\frac{4}{9}$ of \$2700 = \$1200 C's gain.

Proof, \$2700 Whole gain.

2. A, B, and C hired a farm together, for which they paid \$175 rent; A advanced \$75, B \$60, and C \$40. They raised 250 bushels of wheat; what was each man's share?

3. A, B, and C together spent \$1000 in mining stocks. A put in \$400, B \$250, and C \$350. They gained \$1500; how much was each man's share?

4. A, B, C, and D fitted out a whale ship; A advanced \$10000, B \$12000, C \$15000, and D \$8000. The ship brought home 3000 bbls. of oil; what was each man's share?

5. A, B, and C formed a partnership; A furnished \$900, B \$1500, and C \$1200. They lost \$1260; what was each man's share of the loss?

6. X, Y, and Z entered into a joint speculation, on a capital of \$20000, of which X furnished \$5000, Y \$7000, and Z the balance. Their net profits were \$5000 per annum; what was the share of each?

7. A bankrupt owes one of his creditors \$300, another \$400, and a third \$500. His property amounts to \$800; how much can he pay on a dollar, and how much will each of his creditors receive?

8. A bankrupt owes \$2000, and his property is appraised at \$1600; how much can he pay on a dollar?

9. A man failing in business, owes A \$156.45, B \$256.40, and C \$360.40; and his effects are valued at \$317; how much will each man receive?

10. The assets of a man failing in business amounted to \$3560; he owed \$35600; how much can he pay on a dollar, and how much will B receive, who has a claim of \$5000?

11. A man died insolvent, owing \$55645, and his property was sold at auction for \$2350; how much will his estate pay on a dollar?

12. A, B, and C sent flour by sloop from New York to Boston. A had 600 bbl., B 400 bbl., and C 200 bbl. In a gale 200 bbl. were thrown overboard; what was the loss of each?

13. A and B formed a partnership; A put in \$300 for 2 months and B \$200 for 6 months. They gained \$150; what was each man's just share of the gain?

SUGGESTION.—The gain of each depends both upon the *capital* he furnished, and the *time* it was employed. (Art. **583**.)

A's capital	$\$300 \times 2 = \$600$,	the same as	\$600	for 1 mo.
B's "	$200 \times 6 = 1200$,	" "	1200	"
	Whole capital,		\$1800	"

A's share must therefore be $\frac{600}{1800} = \frac{1}{3}$ of \$150, or \$50.
B's " " " " $\frac{1200}{1800} = \frac{2}{3}$ of \$150, or \$100.

PROOF.—$\$50 + \$100 = \$150$, the gain.

14. A, B, and C enter into partnership; A puts in \$500 for 4 mo., B \$400 for 6 mo., and C \$800 for 3 mo.; they gain \$340; what is each man's share of the gain?

15. A and B hire a pasture together for \$60; A put in 120 sheep for 6 months, and B put in 180 sheep for 4 months; what should each pay?

16. The firm A, B, and C lost \$246; A had put in \$85 for 8 mo., B \$250 for 6 mo., and C \$500 for 4 mo.; what is each man's share of the loss?

17. Smith and Jones graded a street for \$857.50. S. furnished 5 men for 20 days and 6 men for 15 days; J. furnished 10 men for 12 days and 9 men for 20 days; what was the share of each contractor?

18. Three men hire a farm of 250 A., at \$8½ an acre; A put in 244 sheep, B 325, and C 450; what rent ought each to pay?

Oral Exercises.

648. 1. If $\frac{3}{4}$ ton of hay costs \$15, what will a ton cost?

Analysis.—In this example we have a *part* of a number given, to find the *whole*. Since 15 is $\frac{3}{4}$ of the number, $\frac{1}{4}$ of it is $\frac{1}{3}$ of 15, which is 5, and $\frac{4}{4}$ are 4 times 5, or 20. *Ans.* \$20.

Note.—In solving examples of this kind a difficulty often arises from supposing that if $\frac{3}{4}$ of a certain number is 15, $\frac{1}{4}$ of it must be $\frac{1}{4}$ of 15. This mistake will be easily avoided by substituting the word *parts* for the given *denominator*.

Thus, if 3 parts cost \$15, 1 part will cost $\frac{1}{3}$ of \$15, which is \$5. But this part is a fourth. Now if 1 fourth cost \$5, 4 fourths will cost 4 times as much.

2. A builder paid \$20 for $\frac{5}{8}$ of an acre of land; what was that per acre?

3. A boy being asked how many pears he had, replied that he had 50 apples, which was $\frac{2}{3}$ the number of his pears; how many pears had he?

4. Henry lost 42 yards of his kite line, which was $\frac{6}{7}$ of his whole line; what was its length?

5. 50 is $\frac{2}{3}$ of what number? $\frac{4}{5}$ of what? $\frac{6}{7}$? $\frac{8}{11}$?

6. 75 is $\frac{3}{4}$ of what number? $\frac{8}{9}$ of what? $\frac{6}{10}$? $\frac{9}{12}$?

7. 100 is $\frac{2}{3}$ of what number? $\frac{5}{6}$ of what? $\frac{5}{7}$? $\frac{10}{12}$?

8. A man bought a yoke of oxen, and paid \$56 in cash, which was $\frac{7}{8}$ of the price of them; what did they cost?

9. A merchant bought a quantity of wood and paid \$45 in goods, which was $\frac{5}{9}$ of the whole cost; how much did he pay for the wood?

10. A man bought a buggy and paid \$45 down, which was $\frac{3}{10}$ of the price of it; what was the price, and how much did he owe?

11. The crew of a whale ship having been out 24 months, found they had consumed $\frac{4}{7}$ of their provisions; how many months' provisions had they when they embarked, and how much longer would their provisions last?

12. A man bought a meadow and paid \$75 cash which was $\frac{3}{5}$ of the price, and gave his note for the balance; how large was the note?

Written Exercises.

649. 1. A man being asked how far he had traveled, replied that 140 miles equaled $\frac{7}{25}$ of the distance; how far had he traveled?

Analysis.—Since 140 mi. is $\frac{7}{25}$, $\frac{1}{25}$ is $\frac{1}{7}$ of 140 mi., or 20 miles. As 20 mi. is $\frac{1}{25}$ of the distance, $\frac{25}{25}$ is 25 times 20 mi., or 500 miles, *Ans.*

2. 560 is $\frac{45}{69}$ of what number?

3. 1500 is $\frac{37}{63}$ of what number?

4. 2000 is $\frac{63}{84}$ of what number?

5. A man paid $150 for a carriage, which was $\frac{25}{48}$ of what he sold it for; what did he get for his carriage?

6. A builder paid $145 for $\frac{5}{8}$ A. of land; what was that an acre?

7. A man pays $0.96 for $\frac{4}{5}$ bu. of wheat, what is that a bu.?

8. A lad being asked how many pears he had, replied that he had 150 apples, which was $\frac{6}{35}$ the number of his pears; how many pears had he?

9. 680 is $\frac{8}{5}$ of what number?

10. 1260 is $\frac{63}{49}$ of what number?

11. A man traveled 240 miles by railroad, which was $\frac{8}{9}$ the distance he traveled by steamboat; how far did he go by boat?

12. If 4 times $\frac{3}{8}$ of $32 is $\frac{6}{7}$ the price paid for a cow, what did she cost?

13. 7 times $\frac{5}{7}$ of 28, is $\frac{4}{11}$ of what number?

14. 7 times $\frac{3}{4}$ of 36 cts. is $\frac{18}{20}$ of the price of a Dictionary; what is the price?

15. A man spent $560 for books which was $\frac{20}{33}$ of his money, and bought hay with the remainder, at $16 a ton; how much hay did he buy?

16. A tailor bought a horse and paid $120 in cash, which was $\frac{15}{21}$ of the price; how many coats at $24 apiece will it take to pay the balance?

17. A grocer sold 2205 lbs. butter, which was $\frac{35}{49}$ of all he had; how many tubs would hold the remainder, allowing 42 lb. to a tub?

18. A lad being asked how many peaches he had in his basket, replied that $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of them made 104; how many had he?

ANALYSIS.—The sum of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4} = \frac{13}{12}$. (Art. **195.**) Now if 104 is $\frac{13}{12}$, $\frac{1}{12}$ is $\frac{1}{13}$ of 104, which is 8; and $\frac{12}{12}$ is $8 \times 12 = 96$. *Ans.* 96 peaches.

19. A farmer lost $\frac{1}{2}$ his sheep by sickness, $\frac{1}{5}$ by wolves, and he had 72 sheep left; how many had he at first?

20. A person having spent $\frac{1}{2}$ and $\frac{1}{3}$ of his money, finds he has $48 left; what had he at first?

21. After a battle a general found that $\frac{1}{6}$ of his army had been taken prisoners, $\frac{1}{8}$ were killed, $\frac{1}{12}$ had deserted, and he had 900 left; how many had he before the battle.

22. A certain post stands $\frac{1}{3}$ in the mud, $\frac{1}{4}$ in the water, and 20 feet above the water; how long is the post?

23. Suppose I pay $185 for $\frac{5}{8}$ of an acre of land; what is that per acre?

24. A man paid $2700 for $\frac{3}{16}$ of a vessel; what is the whole vessel worth?

25. A gentleman spent $\frac{1}{3}$ of his life in Boston, $\frac{1}{4}$ of it in New York, and the rest of it, which was 30 years, in Philadelphia; how old was he?

26. What number is that, $\frac{7}{8}$ of which exceeds $\frac{5}{12}$ of it by 10?

27. In a school $\frac{1}{3}$ were studying arithmetic, $\frac{1}{4}$ algebra, $\frac{1}{6}$ geometry, and the remaining 18 were studying grammar; how many pupils were in the school?

28. A owns $\frac{1}{8}$ and B $\frac{1}{12}$ of a ship; A's part is worth $650 more than B's; what is the value of the ship?

29. In a certain orchard $\frac{1}{3}$ are apple trees, $\frac{1}{4}$ peach trees, $\frac{1}{6}$ plum trees, and the remaining 15 were cherry trees; how many trees did the orchard contain?

650. **1.** A merchant paid $1165.25 for a case of goods and sold them at 15% advance; what was the profit?

ANALYSIS.—The profit was $\frac{15}{100}$ of $1165.25. Now $\frac{1}{100} = \$11.6525$, and $\frac{15}{100} = \$11.6525 \times 15 = \$174.7875$, *Ans.*

2. A man sold a house for $2969.50, which was 25% more than it cost him; what did it cost him?

ANALYSIS.—Since he gained 25%, he received $1.25 for each dollar of cost. Now $\$2969.50 \div \$1.25 = \$2375.60$, *Ans.*

3. Received \$4100 to buy stock, after deducting $2\frac{1}{2}\%$ commission; how many shares at par can I buy?

4. What is the premium at $1\frac{3}{4}\%$ for insuring \$3560 on a house and furniture?

5. What sum must be insured on goods worth \$4760, at $3\frac{1}{2}\%$, to cover both the goods and premium?

6. What is the specific duty on 175 pieces of silk, each containing 50 yd., at 25 cents a yard?

7. What is the int. of \$765.50 for 3 yr. 8 mo., at 6%?

8. If \$850 at simple interest amounts to \$986 in 2 years, what is the rate per cent?

9. When money is at 6%, what is the present worth of \$4218, due in 1 yr. 6 months?

10. What is the bank discount on a note of \$1640.50 for 90 days, at 5%?

11. What must be the face of a note on 60 d., at 6%, to yield \$1000, if discounted at a bank?

12. What is the equated time for the payment of \$400, due in 3 mo., and \$600, due in 5 months?

13. In a mercantile house, A's capital is \$4500, B's \$5200, and C's \$5300; they make \$3000; what is the profit of each?

14. What cost a sight draft on New Orleans for \$750, at $2\frac{3}{4}\%$ premium?

15. What cost a bill of exchange on Liverpool for £800 10s., at $\$4.86\frac{1}{2}$?

651. 1. A father divided \$2700 among his 3 children in the proportion of 2, 3, and 4; how much did each receive?

ANALYSIS.—The sum of $2 + 3 + 4 = 9$. Hence, the first received \$2, the second \$3, and the third \$4, as often as 9 is contained in \$2700; and 9 is contained in \$2700, 300 times. Therefore, the first received $\$300 \times 2 = \$600$; the second $\$300 \times 3 = \$900$; the third $\$300 \times 4 = \$1200$.

PROOF.—$\$600 + \$900 + \$1200 = \$2700$.

2. A man had 756 sheep which he divided into 2 flocks in the proportion of 3 to 4; how many were in each flock?

3. Divide 1248 into 2 such parts that one shall be to the other as 2 to 6.

4. Divide 435 into 2 such parts that one shall be 3 times the other.

5. Two farmers have 755 acres of land, one having 4 times as many acres as the other; how many had each?

6. Two families, one containing 4 persons the other 5, hired board together for \$2954 a year; what proportion ought each family to pay?

7. Divide the number 720 into 3 parts in proportion to 3, 4, and 5.

8. Divide \$650 among 4 persons so that their shares shall be to each other in the proportion of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{7}{12}$.

ANALYSIS.—Since one part is $\frac{1}{2}$ or $\frac{6}{12}$ share, another $\frac{1}{3}$ or $\frac{4}{12}$ share, another $\frac{3}{4}$ or $\frac{9}{12}$, and the other $\frac{7}{12}$ of a share, the whole is $\frac{26}{12}$ share, and 1 share equals $\$650 \div \frac{26}{12} = \$300$. Hence, $\frac{1}{2}$ share is \$150, $\frac{1}{3}$ share is \$100, $\frac{3}{4}$, \$225, $\frac{7}{12}$, \$175, *Ans.*

PROOF.—$\$150 + \$100 + \$225 + \$175 = \$650$.

9. Divide 945 into 3 parts which shall be to each other in the proportion of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{5}{12}$.

10. What number added to 5 times itself will make 576?

ANALYSIS.—A number added to 5 times itself will make 6 times that number. Since 576 is the product of two factors, one of which is 6, the other factor must be $576 \div 6 = 96$, *Ans*

11. What number added to $\frac{4}{5}$ of itself will make 369?

12. What number added to $4\frac{1}{2}$ times itself will make $60\frac{1}{2}$?

13. A man being asked how far he had walked, replied that the number of kilometers he had traveled was 364, and he had ridden twice as far as he had walked; how many kilometers had he walked?

14. A lad bought apples, pears, and peaches, in all 280; the number of his apples was twice that of his pears, and the number of his pears was twice that of his peaches; how many of each did he buy?

15. Divide 192 into three such parts that the first shall be twice the second, and the third three times the second.

652. 16. If 4 men can mow 48 acres of grass in 5 days, how long will it take 9 men to mow 60 acres?

ANALYSIS.—Since 4 men can mow 48 acres in 5 d., 1 man can mow $\frac{1}{4}$ of 48 A., or 12 A. Now if 1 man can mow 12 A. in 5 d., in 1 d. he can mow $\frac{1}{5}$ of 12, or $2\frac{2}{5}$ A. Again, since to mow $2\frac{2}{5}$ A. requires 1 man 1 d., 60 A. will require him as many days as $2\frac{2}{5}$ are contained times in $60 = 25$ d.; and since it takes 1 man 25 d., it will take 9 men $\frac{1}{9}$ of $25 = 2\frac{7}{9}$ d., *Ans.*

17. If 14 men can build 84 rods of wall in 3 days, how long will it take 20 men to build 300 rods?

18. If 1000 hektoliters of provisions will support a garrison of 75 men for 3 months, how long will 3000 hektoliters support a garrison of 300?

19. If 7 men can reap 42 acres in 6 days, how many men will it take to reap 100 acres in 5 days?

20. If a man travels 320 miles in 10 days, traveling 8 hours per day, how far can he go in 15 days, traveling 12 hours per day?

21. If 24 horses eat 126 bushels of oats in 36 days, how many bushels will 32 horses eat in 48 days?

653. 22. A farmer wishes to mix a quantity of corn worth 75 cts. a bushel, with oats worth $37\frac{1}{2}$ cts. a bushel, so that the mixture may be worth 50 cts. a bushel; what part of each must he take?

ANALYSIS.—Since the mixture is worth 50 cts. a bushel, on every bushel of corn he puts in, the loss is 25 cts., and on every bushel of oats, the gain is $12\frac{1}{2}$ cts. Since it requires 1 bu. oats to gain $12\frac{1}{2}$ cts., to gain 25 cts. will require as many bu. of oats as $12\frac{1}{2}$ cts. are contained times in 25 cts., and $25 \div 12\frac{1}{2} = 2$. Hence, he must take 2 bu. oats to 1 bu. corn.

PROOF.—A mixture of 3 bushels is worth $37\frac{1}{2}$ cts. + $37\frac{1}{2}$ cts. + 75 cts. = \$1.50; hence, 1 bu. mixture is worth 50 cents.

NOTE.—The principle by which this and similar examples are solved, is that the *excess* of one article *above* the mean price of the mixture, counterbalances the *deficiency* of another article which is *below* it.

23. A tea merchant has two kinds of tea worth 40 cts. and 90 cts. a pound, and wishes to make a mixture worth 60 cts. a pound; what part of each must he take?

24. How much ginger at 24 cts. and 30 cts. a pound, will form a mixture worth 25 cts. a pound?

RATIO.

DEFINITIONS.

654. **Ratio** is the relation of one number to another. It is found by dividing one by the other.

Thus, the ratio of 6 to 3 is 6 ÷ 3, and is equal to 2.

655. The **Terms of a Ratio** are the numbers compared.

656. The **Antecedent** of a ratio is the *first* term.

657. The **Consequent** is the *second* term.

658. The two terms together are called a **Couplet.**

Thus, in the ratio 9 : 3, 9 is the *antecedent,* 3 the *consequent,* and 9 and 3 together form a *couplet.*

659. Ratio is commonly denoted by a *colon* (**:**), which is a contraction of the sign of division.

Thus, the ratio of 6 to 3 is written "6 : 3," and is equivalent to 6÷3.

660. Ratio is also denoted by writing the *consequent under* the *antecendent* in the form of a fraction.

Thus, the ratio of 8 to 4 is written $\frac{8}{4}$, and is equivalent to 8 : 4.

ORAL EXERCISES.

661. 1. What is the ratio of 48 : 6? Of 63 : 7? Of 72 : 8?

2. What is the ratio of 21 : 42? Of 15 : 45? Of 25 to 100?

3. What is the ratio of 8 lb. to 40 lb.? Of 54 yd. to 6 yd.?

4. What is the ratio of $50 : $10? Of $25 : $100?

5. Find the ratio of 6 ft. : 3 hr.

Ans. The ratio cannot be found, because one of these numbers is neither equal to nor a part of the other. Hence, the

PRINCIPLE.

662. *Only like numbers can be compared with each other.*

663. A **Simple Ratio** is the ratio of two numbers, as $8:4$.

664. A **Compound Ratio** is the product of *two or more simple* ratios. They are commonly denoted by placing the simple ratios under each other.

Thus, $\left.\begin{matrix}4:2\\9:3\end{matrix}\right\}$ or, $4 \times 9 : 2 \times 3$, is a compound ratio.

665. A Compound Ratio is reduced to a simple one by making the product of the antecedents a new antecedent, and the product of the consequents a new consequent.

666. A **Reciprocal of a Ratio** is a simple ratio *inverted,* and is the same as the ratio of the *reciprocals* of the two numbers compared.

Thus, the reciprocal of 8 to 4 is $\frac{1}{8}$ to $\frac{1}{4} = 4:8$, or $\frac{4}{8}$.

NOTE.—1. Reciprocal Ratio is sometimes called Inverse Ratio.
2. The *reciprocal* of a ratio, when a fraction is used, is expressed by inverting the terms of the *fraction* which expresses the *simple* ratio. When the *colon* is used, the *order* of the terms is inverted.

667. The ratio between *two fractions* which have a common denominator, is the same as the *ratio* of their *numerators.*

Thus, the ratio of $\frac{6}{9} : \frac{3}{9}$ is the same as $6:3$.

NOTE.—When the fractions have *different denominators,* reduce them to a *common* denominator; then compare their numerators. *Compound numbers* must be reduced to the *same denomination.*

WRITTEN EXERCISES.

668. Find the following ratios in the lowest terms:

1. 95 to 25.
2. 110 to 48.
3. 135 to 51.
4. 186 to 84.
5. 65 to 180.
6. 84 to 132.
7. 108 to 256.
8. 220 to 500.
9. $\frac{36}{17}$ to $\frac{84}{17}$.
10. $\frac{44}{27}$ to $\frac{99}{27}$.
11. $\frac{24}{5}$ to $\frac{36}{4}$.
12. $\frac{49}{8}$ to $\frac{63}{7}$.

13. 96 gal. to 24 qt.
14. 75 bu. to 160 pk.
15. 140 rd. to 20 ft.
16. 175s. to 130d.
17. 15s. to 4s. 6d.
18. 10 ounces to 95 pounds.
19. 8 yards to 9 inches.
20. 3 pints to 4 gallons.

669. Since the *antecedent* corresponds to the *numerator,* and the *consequent* to the *denominator, changes* on the terms of a ratio, have the same effect upon its value as like changes have upon the terms of a fraction. Hence, the

PRINCIPLES.

1°. Multiplying the antecedent, or Dividing the consequent.	*Multiplies the ratio.*
2°. Dividing the antecedent, or Multiplying the consequent,	*Divides the ratio.*
3°. Multiplying or dividing both terms by the same quantity,	*Does not alter the value of the ratio.*

670. The ratio, antecedent, and consequent are so related to each other, that if any two of them are given the other may be found. Hence, the

FORMULAS.
1. *The Ratio = Antecedent ÷ Consequent.*
2. *The Consequent = Antecedent ÷ Ratio.*
3. *The Antecedent = Consequent × Ratio.*

21. What is the ratio, when the antecedent is 63 and the consequent 9?

22. When the antecedent is 25 and the consequent 60, what is the ratio?

23. If the antecedent is 8 and the ratio 14, what is the consequent?

24. When the consequent is 16 and the ratio 7, what is the antecedent?

25. When the antecedent is $6\frac{3}{4}$ and the consequent 8, what is the ratio?

26. If the antecedent is $5\frac{1}{3}$ and the ratio $9\frac{3}{4}$, what is the consequent?

27. When the consequent is 24 and ratio 8, what is the antecedent?

28. If the consequent is 36 and ratio 12, what is the antecedent?

29. When the antecedent is $9\frac{2}{3}$ and ratio $8\frac{1}{5}$, what is the consequent?

PROPORTION.

ORAL EXERCISES.

671. 1. What is the ratio of 12 to 60? Of 9 to 36?

2. Which of the above ratios is the larger?

3. How does the ratio of 30 to 6 compare with 15 to 3?

4. How does the ratio of 5 to 25 compare with the ratio of 12 to 60?

5. Are all ratios equal?

6. How does the ratio of 12 to 4 compare with the ratio of 10 to 5?

7. Name two equal ratios. Name two others.

8. Name two unequal ratios. Name two others?

9. What two numbers have the same quotient as 16 divided by 4? As 28 divided by 4? As 60 divided by 12?

10. Express in both forms the ratio of two numbers which have the same ratio as 6 to 12.

11. How does the ratio of 7 to 14 compare with the ratio of 5 to 20?

12. How does the ratio of \$15 to \$5 compare with the ratio of 18 ft. to 6 ft.?

Ans. They are *equal* to each other.

672. Proportion is an equality of ratios.

Thus, the ratio $8 : 4 = 6 : 3$, is a proportion. That is,
Four quantities are in *proportion*, when the *first* is the *same multiple* or *part* of the *second*, that the *third* is of the *fourth*.

673. The **Sign of Proportion** is a double colon (**::**), or the sign (=).

Thus, the proportion above is expressed $8 : 4 :: 6 : 3$. Or, $8 : 4 = 6 : 3$

The *first* form is read "8 is to 4 as 6 to 3."

The *second* is read "the ratio of 8 to 4 equals the ratio of 6 to 3."

674. The **Terms** of a proportion are the numbers compared.

675. The **Antecedents** of a proportion are the *first* and *third* terms.

676. The **Consequents** are the *second* and *fourth* terms.

Thus, in the proportion $4:8::3:6$, the 4 and 3 are the antecedents, and 8 and 6 the consequents.

677. In every proportion there must be at least *four terms;* for the equality is between *two* or *more* ratios, and each ratio has *two* terms.

678. A proportion may, however, be formed from *three* numbers, for one of the numbers may be *repeated,* so as to form *two* terms; as, $2:4::4:8$.

NOTE.—When a proportion is formed of *three numbers,* the middle number is called a *mean proportional.*

679. The **Extremes** of a proportion are the *first* and *last* terms.

680. The **Means** are the *two middle* terms.

Thus, in the proportion $9:12::18:24$, 9 and 24 are the extremes, 12 and 18 the means.

Read the following: (Art. **673.**)

1. $35:7=60:12$.
2. $42:14=75:25$.
3. $72:24=168:56$.
4. $144:1::1728:12$.
5. $20:143\frac{3}{19}::2\frac{3}{8}:17$.
6. $4\frac{1}{2}:54::6:72$.
7. $18:54::21:63$.
8. $23:92::34:136$.
9. $37:148::41:164$.
10. $\frac{3}{8}:\frac{9}{14}::\frac{2}{8}:\frac{3}{7}$.
11. \$16.05 : \$5.35 :: \$27.03 : \$9.01.
12. $60:15::80:20$.

681. The relation of the four terms of a proportion to each other is such, that if *any three* of them are given, the *other* or *missing* term may be found.

DEVELOPMENT OF PRINCIPLES.

682. 1. If the first three terms of a proportion are 2, 4, and 5, what is the fourth or missing term ?

ANALYSIS.—Representing the missing term by x, then the proportion is $2:4::6:x$, and the ratio $\frac{2}{4}=\frac{6}{x}$. These fractions reduced to a common denom. become $\frac{2\times x}{4\times x}=\frac{4\times 6}{4\times x}$; hence the numerators are equal. (Art. **667.**) But $2\times x$ is the product of the extremes and 4×6 the product of the means. Cancelling the factor 2, which is common to both, $x=2\times 6$, or 12, is the missing term required.

683. From the preceding example we derive the following

PRINCIPLES.

1°. In every proportion the product of the extremes is equal to the product of the means.

2°. The product of the extremes divided by either of the means, gives the other mean.

3°. The product of the means divided by either extreme, gives the other extreme.

684. Find the missing term in the following:

2. $12:42=20:x$.
3. $9:153=150:x$.
4. $\$75:\$900=x:85$.
5. $x:40=120:100$.
6. $24:x=12:144$.
7. $187.5:7\frac{1}{2}=x:15$.
8. 400 rd. : 56 rd. $=195:x$.
9. x : 400 vests $=\$87.50:\$1000$.
10. 130 lb. : $x=\$150:\$850$.
11. 40 gal. : $x=180:60$.
12. 16 yd. : 10 ft. $=72:x$.
13. $x:75=\frac{6}{8}:\frac{3}{4}$.

SIMPLE PROPORTION.

685. Simple Proportion is an equality of two simple ratios.

NOTE.—Of the three given numbers, *two* must always be of the same kind, and the third the same as the answer required.

WRITTEN EXERCISES.

686. To solve problems by Simple Proportion or by Analysis.

1. If 15 books cost $45, what will be the cost of 80 books?

BY ANALYSIS.

OPERATION.

Since 15 books cost $45, 1 book costs $\frac{1}{15}$ of $45, and 80 books will cost 80 times as much, or $240, *Ans.*

$$\frac{\overset{3}{\cancel{45}} \times 80}{\cancel{15}} = 240.$$

Ans. $240.

BY PROPORTION.

OPERATION.

$$15 : 80 :: \$45 : Ans.$$

$$\text{And } \frac{\overset{3}{\cancel{45}} \times 80}{\cancel{15}} = \$240, \ Ans.$$

As the answer is to be money, we make $45 the third term; and as the cost of 80 books will be *more* than that of 15 books, we make 80 the second term and 15 the first. We now have the two means and one extreme, to find the other extreme. Hence, we divide the product of the means by the given extreme, and the quotient is the other extreme or answer. (Art. 683.)

PROOF.—$80 \times 45 = 240 \times 15$. (Art. 683, *1°*.)

687. From the preceding principles we have the following

RULE.—I. *Make that number the third term, which is the same kind as the answer.*

II. *When the answer is to be larger than the third term, make the larger of the other two numbers the second term; but when less, place the smaller for the second term, and the other for the first.*

III. *Multiply the second and third terms together, and divide the product by the first; the quotient will be the fourth term or answer.*

PROOF.—*If the product of the first and fourth terms equals that of the second and third, the answer is right.*

NOTES.—1. The arrangement of the terms in the form of a proportion is called "The Statement of the question."

2. *The factors* common to the *first* and *second*, or to the *first* and *third* terms, should be cancelled.

3. The *first* and *second* terms must be reduced to the *same denomination*. The *third* term, if a compound number, must be reduced to the *lowest denomination* it contains.

4. It is advisable for the pupil to solve the following examples both by Proportion and by Analysis.

2. If 14 vests are worth \$84, what are 23 vests worth?

BY ANALYSIS.—If 14 vests are worth \$84, 1 vest is worth 1 fourteenth of \$84, or \$6, and 23 vests are worth 23 times \$6, or \$138, *Ans.*

BY PROPORTION.—14 v. : 23 v. : : \$84 : *Ans.* That is, 14 is the same part of 23 as \$84 are of the cost of 23 vests. Cancelling, etc.,

$$\cancel{14} : 23 :: \$\overset{6}{\cancel{84}} : \$138, \ \textit{Ans.}$$

3. If 5 men can mow a meadow in 6 days, how long will it take 8 men to mow it?

4. If 6 acres and 40 rods of land cost \$125, how much will 25 acres and 120 rods cost?

5. If 15 meters of silk cost £4 10s., what will 75 meters cost?

6. If a railroad car goes 35 mi. in 1 hr. 45 min., how far will it go in 3 days?

7. If 84 lbs. of cheese cost \$5$\frac{2}{8}$, what will 60 lbs. cost?

8. If $\frac{3}{8}$ of a ship is worth \$6000, how much is $\frac{5}{16}$ of her worth?

9. If a ship has sufficient water to last a crew of 25 men for 8 months, how long will it last 15 men?

10. If the interest of \$1500 for 12 mo. is \$90, what will be the interest of the same sum for 8 mo.?

11. If a tree 20 ft. high casts a shadow 30 ft. long, how long will be the shadow of a tree 50 ft. high?

12. How long will it take a steamship to sail round the globe, allowing it to be 25000 miles in circumference, if she sails at the rate of 3000 miles in 12 days?

13. How many hektars of land can a man buy for \$840, if he pays at the rate of \$56 for every 7 hektars?

Cause and Effect.

688. The principles of proportion may also be explained by the relations of the terms to each other, as *causes* and *effects.*

689. A **Cause** is that which produces something.

An **Effect** is something which is produced.

Thus, men at work, goods bought or sold, time, money lent, etc., are *causes.* Work done, provisions consumed, cost of goods, etc., are *effects.*

690. In arithmetical operations it is assumed that *like causes* produce *like effects,* and the ratio between the *effects* is *equal* to the ratio between the *causes* which produce them.

If 2 horses as a cause can move 3 tons as an effect, 6 horses as a cause will remove 9 tons as an effect; that is

2 horses (1st C.) : 6 horses (2d C.) :: 3 T. (1st E.) : 9 T. (2d E.)

Written Exercises.

691. 1. If 4 acres produce 60 bushels of wheat, how much will 9 acres produce?

1st C.	2d C.		1st E.	
4 A. :	9 A.	::	60 bu. :	*Ans.*

$(60 \times 9) \div 4 = 135$

Ans. 135 bu.

Analysis.—In this example, the two causes are 4 acres and 9 acres; the first effect is 60 bu., the second effect is required.

We make 60 bu. the given effect, the third term, and since the second effect must be greater than the first, we make 9 A., the greater cause, the second term, and 4 A. the first. Multiplying and dividing as before, the result is 135 bu., *Ans.*

2. If it requires 4 acres to produce 60 bushels of wheat, how many acres are required to produce 135 bushels?

OPERATION.

1st. E.	2d E.		1st C.	
60 bu. :	135 bu.	::	4 A. :	*Ans.*

$(135 \times 4) \div 60 = 9$

Ans. 9 acres.

Analysis.—In this example two effects, 60 bu. and 135 bu., and one cause, 4 A., are given, the second cause is required. Since the 2d effect is greater than the first, the 2d cause must also be greater than the given cause; we therefore make 135 bu. the 2d term and 60 bu. the 1st term. The result is 9 acres.

692. When the terms of a proportion are considered in the relation of *cause* and *effect,* the operations are the same as when considered in the relation of *magnitude.* (Arts. **687, 689.**)

Solve the following by either or both the preceding methods:

3. Bought 41 yd. of flannel for \$16.40; how much would $8\frac{3}{5}$ yd. cost?

4. Bought 18 kilos of ginger for \$8.50; how much will $10\frac{3}{5}$ kilos cost?

5. If a stage goes 84 kilometers in 12 hours, how far will it go in $15\frac{1}{2}$ hours?

6. If 26 horses eat 72 hektoliters of oats in a week, how many hektoliters will 25 horses eat in the same time?

7. If a railroad car runs 125 kilometers in 5 hours, how far will it run in $12\frac{3}{4}$ hours?

8. If 9 ounces of silver will make 4 tea spoons, how many spoons will 25 pounds of silver make?

9. If $5\frac{1}{2}$ yd. of cloth are worth \$$27\frac{1}{2}$, what are $50\frac{1}{4}$ yd. worth?

10. If 60 men can build a house in $90\frac{1}{2}$ days, how long will it take 15 men to build it?

11. What will $49\frac{3}{11}$ yd. velvet cost, if $7\frac{5}{8}$ yd. cost £7 18s. 4d.?

12. At 7s. 6d. per ounce, what is the value of a silver pitcher weighing 9 oz. 13 pwt. 8 gr.?

13. If 405 yd. linen cost £69 7s. 6d., what cost 243 yd?

14. If A can saw a cord of wood in 6 hours, and B in 10 hours, how long will it take both together to saw a cord?

15. A cistern has 3 stop-cocks, the first of which will empty it in 10 min.; the second, in 15 min.; and the third, in 30 min.; how long will it take all of them together to empty it?

16. A man and a boy together can mow an acre of grass in 4 hours; the man can mow it alone in 6 hours; how long will it take the boy to mow it?

17. If the interest of \$675.25 is \$55.625 for 1 year, how much will be the interest of \$2368.85?

18. What must be the length of a board which is $9\frac{3}{4}$ in. wide, to make a square foot?

19. If $98\frac{1}{2}$ yds. carpeting $1\frac{1}{4}$ yard wide will cover a floor, how many yards $\frac{3}{4}$ yd. wide will it take to cover it?

COMPOUND PROPORTION.

693. Compound Proportion is an equality between a compound ratio and a simple one. Thus,

$$\left.\begin{matrix} 8:4 \\ 6:3 \end{matrix}\right\} :: 12:3,$$ is a compound proportion. That is,

$$8 \times 6 : 4 \times 3 :: 12 : 3; \text{ for, } 8 \times 6 \times 3 = 4 \times 3 \times 12.$$

It is read, "The ratio of 8 into 6 is to 4 into 3, as 12 to 3."

WRITTEN EXERCISES.

694. 1. If 4 men earn \$60 in 10 days, how much can 6 men earn in 8 days?

OPERATION.

$$\left.\begin{matrix} 4\text{ m.} : 6\text{ m.} \\ 10\text{ d.} : 8\text{ d.} \end{matrix}\right\} :: \$60 : Ans.$$

EXPLANATION.—Since the answer is to be money, we make \$60 the third term. We then arrange the other numbers in pairs, two of a kind, placing them according as the answer would be greater or less than the third term, if it depended on each pair alone. Now, as 6 m. can earn more than 4 m., we place the larger for the second term and the smaller for the first. Again, as they will earn less in 8 d. than in 10 d., we place the smaller for the second term, and the larger for the first.

Reducing the compound ratio to a simple one, we have,

$$4 \times 10 : 6 \times 8 :: 60 : Ans.$$

Dividing the prod. of the means by the extreme, cancelling, etc.

$$\frac{6 \times \overset{2}{\cancel{8}} \times \overset{6}{\cancel{60}}}{\cancel{4} \times \cancel{10}} = \$72, \; Ans.$$

695. From the preceding example we have the following

RULE.—I. *Make that number which is of the same kind as the answer, the third term.*

II. *Then take the other numbers in pairs, or two of a kind, and arrange them as in simple proportion.* (Art. **687**.)

III. *Multiply the second and third terms together, and divide the product by the product of the first terms. The quotient will be the answer.*

PROOF.—*If the product of the first and fourth terms equals that of the second and third terms, the work is right.*

NOTES.—1. The terms of each couplet in the compound ratio must be reduced to the same denomination, and the third term to the lowest denomination contained in it, as in Simple Proportion.

2. In Compound Proportion, all the terms are given in *couplets* or *pairs* of the *same* kind, except *one*. This is called the *odd term*, or *demand*, and is always the same kind as the *answer*.

3. Problems in Compound Proportion may also be solved by Analysis and by Simple Proportion. Take the preceding example.

BY ANALYSIS.—If 4 men can earn \$60 in 10 d., 1 man can earn in the same time, $\frac{1}{4}$ of \$60, which is \$15, and 6 men can earn 6 times 15 or \$90.

Again, if 6 men earn \$90 in 10 d., in 1 d. they can earn $\frac{1}{10}$ of \$90, which is \$9; and in 8 d. they can earn 8 times 9, or \$72, *Ans.*

BY SIMPLE PROPORTION.—4 m. : 6 m. :: \$60 : x, or \$90.
Again, 10 d. : 8 d. :: \$90 : *Ans.*, or \$72.

2. If 8 men can clear 30 acres of land in 63 days, working 10 hours a day, how many acres can 10 men clear in 72 days, working 12 hours a day?

STATEMENT.

$$\left.\begin{array}{l} 8 \text{ m.} : 10 \text{ m.} \\ 63 \text{ d.} : 72 \text{ d.} \\ 10 \text{ hr.} : 12 \text{ hr.} \end{array}\right\} :: \overset{\text{Acres.}}{30} : Ans.$$

OPERATION.

$$\frac{10 \times 72 \times 12 \times 30}{8 \times 63 \times 10} = 51\tfrac{3}{7} \text{ A., } Ans.$$

$$\begin{array}{r|l} \cancel{8} & \cancel{10} \\ {}^{7}\,\cancel{63} & \cancel{72}\;{}^{9} \\ \cancel{10} & 12 \\ & 30 \\ \hline 7 & 360 = 51\tfrac{3}{7} \text{ A., } Ans. \end{array}$$

NOTE.—When the vertical form of cancellation is used, the antecedents must be placed on the left of the line, and the consequents with the odd term on the right.

3. If a man can walk 192 miles in 4 days, traveling 12 hours a day, how far can he go in 24 days, traveling 8 hours a day?

4. If 8 men can make 9 rods of wall in 12 days, how many men will it require to make 36 rods in 4 days?

5. If 5 men make 240 pair of shoes in 24 days, how many men will it require to make 300 pair in 15 days?

6. If 60 lbs. of meat will supply 8 men 15 days, how long will 72 lbs. last 24 men?

7. If 12 men can reap 80 acres of wheat in 6 days, how long will it take 25 men to reap 200 acres?

8. If 18 horses eat 128 bushels of oats in 32 days, how many bushels will 12 horses eat in 64 days?

9. If 8 men can build a wall 20 ft. long, 6 ft. high, and 4 ft. thick, in 12 days, how long will it take 24 men to build one 200 ft. long, 8 ft. high, and 6 ft. thick?

10. If 8 men reap 36 acres in 9 days, working 9 hours per day, how many men will it take to reap 48 acres in 12 days, working 12 hours per day?

11. If \$100 gain \$6 in 12 months, how long will it take \$400 to gain \$18. *Ans.* 9 mo.

12. If \$200 gain \$12 in 12 mo., what will \$400 gain in 9 mo.?

13. If 6 men can dig a drain 20 rods long, 6 feet deep, and 4 feet wide, in 16 days, working 9 hours each day, how many days will it take 24 men to dig a drain 200 rods long, 8 feet deep, and 6 feet wide, working 8 hours per day?

14. If 3 lbs. of yarn will make 10 yards of cloth $1\frac{1}{2}$ yard wide, how many pounds will be required to make a piece 100 yards long, and $1\frac{1}{4}$ yd. wide?

15. A general wished to remove 80000 lbs. of provision from a fortress in 9 days, and it was found that in 6 days 18 men had carried away but 15 tons; how many men would be required to carry the remainder in 3 days?

16. If a man travels 130 miles in 3 days, when the days are 14 hours long, how long will it take him to travel 390 miles when the days are 7 hours long?

17. If the price of 10 oz. of bread is 5d., when corn is 4s. 2d. per bushel, what must be paid for 3 lbs. 10 oz. when corn is 5s. 5d. per bushel?

18. If 6 journeymen make 132 pair of boots in $4\frac{1}{2}$ weeks, working $5\frac{1}{2}$ days a week, and $12\frac{3}{4}$ hours per day, how many pair will 18 men make in $13\frac{1}{2}$ weeks, working $4\frac{1}{4}$ days per week, and 11 hours per day?

Partitive Proportion.

696. Partitive Proportion is dividing a number into two or more parts having a *given ratio* to each other.

Oral Exercises.

697. 1. Charles and Robert divided 28 pears between them in the ratio of 3 to 4; how many had each?

Analysis.—Since Charles had 3 parts as often as Robert had 4, both had 3+4, or 7 equal parts. Hence, Charles had $\frac{3}{7}$ and Robert $\frac{4}{7}$ of 28. Now $\frac{3}{7}$ of 28 are 12, and $\frac{4}{7}$ are 16. Therefore, Charles had 12, and Robert 16 pears.

Proof.—12 pears + 16 pears = 28 pears.

2. Divide 35 into 2 such parts that one shall be to the other as 3 to 2.

3. Divide 42 cts. into two such parts that one shall be to the other as 2 to 4.

4. A farmer had 56 acres of which he made 2 pastures in the ratio of 3 to 5; how many acres were in each?

5. A man bought a cow and a calf for $50; the cow was worth 4 times as much as the calf; what was the value of each?

6. Divide $72 into two such parts that one shall be to the other as 4 to 8.

Written Exercises.

698. To divide a number into two or more parts which shall have a given ratio to each other.

1. A and B divided $145 in the ratio of 2 to 3; how much had each?

Solution.—The sum of the proportional parts is to each separate part as the number to be divided is to each man's share. That is, 5 (2+3) is to 2 as $145 to A's share. Again, 5 is to 3 as $145 to B's share. Hence, the

OPERATION.

5 : 2 :: $145 : A's s.

5 : 3 :: $145 : B's s.

($145 × 2) ÷ 5 = $58, A's s.

($145 × 3) ÷ 5 = $87, B's s.

RULE.—I. *Make the number to be divided the third term; each proportional part successively the second term; and their sum the first.*

II. *The product of the second and third terms of each proportion, divided by the first, will be the corresponding part required.*

2. Divide 312 into three parts which shall be to each other as 3, 4, and 6.

3. A man having 198 sheep, wished to divide them into three flocks which should be to each other as 1, 3, and 5; how many will each flock contain?

4. A farmer raised 500 bushels of grain, composed of oats, wheat, and corn in the proportion of 3, 4, and $5\frac{1}{2}$; how many bushels were there of each kind?

5. A man paid $5.28 for pears, oranges, and melons, the prices of which were as 2, 4, and 6; how much did he pay for each kind?

6. A father divided $3479 among his four sons in proportion to their ages, which were as 4, 6, 8, and 10; how many dollars did each receive?

QUESTIONS.

654. What is ratio? How found? 655. What are the terms of a ratio? 656. The first term called? 657. The second? 658. The two terms? 659. Ratio denoted? 662. What numbers can be compared? 663. A simple ratio? 664. Compound? 666. Reciprocal? 669. Name the principles of ratio?

672. What is proportion? 673. The sign? 674. The terms? 677. How many terms must there be in a proportion? 678. What is a mean proportional? 679. The extremes? 680. The means? 683. Principles of proportion?

685. What is a simple proportion? 687. Which number is the third term? How arrange the other terms? How find the fourth term?

688. How else may proportion be explained? 689. What is a cause? An effect?

693. What is compound proportion? 695. Which number is made the third term? How arrange the remaining numbers? How find the required term? 696. What is partitive proportion? 698. How arrange the terms? How find the answer?

PARTNERSHIP.

699. Partnership is the association of two or more persons for the transaction of business.

700. The association is called a **Firm, Company,** or **House.**

701. The persons associated are called **Partners.**

702. The **Capital** is the *money* or *property* furnished by the Partners.

703. The **Assets** of a firm are the various kinds of property in its possession.

704. The **Liabilities** are its debts.

705. The **Net Capital** is the excess of its property above its liabilities.

WRITTEN EXERCISES.

706. To find each Partner's Share of the Profit or Loss, when their capital is employed for the same time.

1. A and B formed a partnership; A put in \$400 and B \$300; they make \$364. What was each man's share of the profit?

BY ANALYSIS.—\$400 + \$300 = \$700, the whole capital. Hence,

A had $\frac{400}{700}$, or $\frac{4}{7}$ of the gain; and $\frac{4}{7}$ of \$364 = \$208, A's.
B had $\frac{300}{700}$, or $\frac{3}{7}$ " " " $\frac{3}{7}$ of \$364 = \$156, B's. Hence,

RULE.—I. *Take such a part of the gain or loss, as each partner's stock is of the whole capital.*

BY PERCENTAGE.—The gain \$364 is $\frac{364}{700} = \frac{52}{100}$, or 52% of the whole capital. Therefore,

\$400 × .52 = \$208, A's share; and \$300 × .52 = \$156, B's share. Hence,

II. *Find the % which the profit or loss is of the whole capital, and multiply each man's capital by it.* (Art. **464.**)

NOTE.—The *per cent method* is preferable, when the partners or shareholders are numerous.

2. A and B formed a partnership; A put in \$648 and B \$1080, agreeing to divide the profit in proportion to their capital; what was each one's share of the gain?

3. A, B, and C are partners; A furnishes \$600, B \$800, and C \$1000; they lose \$480; what is each man's share of the loss?

4. A, B, C, and D make up a purse for a stock speculation; A puts in \$300, B \$400, C \$600, and D \$800; they make \$2400; what is each man's share?

707. To find each partner's share of the Profit or Loss, when their capital is employed for unequal times.

5. A and B enter into partnership; A furnishes \$400 for 8 months, and B \$600 for 4 months; they gain \$280; what is each one's share of the profit?

ANALYSIS.—In this case the profit of each partner depends on two elements, viz.: the *amount* of his capital and the *time* it is employed.

But the use or interest of \$400 for 8 months equals that of 8 times \$400, or \$3200, for 1 mo.; and of \$600 for 4 mo. equals 4 times that of \$600, or \$2400, for 1 month.

The respective capitals, then, are equivalent to \$2400 and \$3200, each employed for 1 mo. Since A furnished \$3200 and B \$2400, the whole capital = \$5600. Now \$280 ÷ \$5600 = .05 or 5%. Therefore,

\$3200 × .05 = \$160, A's share.
\$2400 × .05 = \$120, B's share. Hence, the

RULE.—*Multiply each partner's capital by the time it is employed. Consider these products as their respective capitals, and proceed as in the last article.*

708. The Rule for Partnership is also applicable to problems in **Bankruptcy**, the **General Average** of losses at sea, and other distributions; the *sum* of the debts or property in question corresponds to the *whole amount* of capital, etc.

6. A and B formed a partnership; A put in \$300 for 2 months, and B \$200 for 6 months; they gained \$150; what was each man's just share of the gain?

7. A, B, and C enter into partnership; A puts in \$500 for 4 mo., B \$400 for 6 mo., and C \$800 for 3 mo.; they gain \$340; what is each man's share of the gain?

8. A and B hire a pasture together for \$60; A put in 120 sheep for 6 months, and B put in 180 sheep for 4 months; what should each pay?

9. The firm A, B, and C lost \$246; A had put in \$85 for 8 mo., B \$250 for 6 mo., and C \$500 for 4 mo.; what is each man's share of the loss?

10. A man failing in business owed A \$1200, B \$1800, and C \$2400; his assets were \$2700; how much did each receive?

Ans. A received \$600, B \$900, and C \$1200.

PROOF.—\$600 + \$900 + \$1200 = \$2700, the assets.

11. A bankrupt owes A \$1200, B \$2300, C \$3400, and D \$4500; his whole effects are worth \$5600; how much will each creditor receive?

12. A railroad company went into bankruptcy whose liabilities were \$36300, and assets \$12100; how much did the company pay on a dollar, and how much did a creditor receive who had a claim of \$15270?

13. A, B, and C freighted a vessel with flour from New York to New Orleans; A had on board 1200 barrels, B 800, and C 400. On her passage 400 barrels were thrown overboard in a gale; what was the average loss?

14. A Liverpool packet being in distress, the master threw goods overboard to the amount of \$10000. The whole cargo was valued at \$72000, and the ship at \$28000; what per cent loss was the general average; and how much was A's loss, who had goods aboard to the amount of \$15000?

QUESTIONS.

699. What is partnership? 700. What is the association called? 702. What is the capital? 703. Assets? 704. Liabilities? 705. Net capital?
706. How find each partner's share of profit or loss, when their capital is employed for the same time? 707. When for unequal times?

INVOLUTION.

ORAL EXERCISES.

709. 1. What is the product of 3 multiplied by itself?

2. What is the product of 3 taken 3 times as a factor?

3. What is the product of 4 taken 3 times as a factor?

4. What is the product of 5 taken twice as a factor?

5. What is the product of 3 taken 4 times as a factor?

6. What is the product of $\frac{2}{3}$ multiplied by itself?

7. What is the product of $\frac{3}{4}$ taken twice as a factor?

8. What is the product of .4 taken twice as a factor? Of .3 taken three times? Of .04 twice?

DEFINITIONS.

710. Involution is finding a power of a number

711. A **Power** of a number is the product of two or more equal factors.

Thus, $2 \times 2 \times 2 = 8$, and $3 \times 3 = 9$; 8 and 9 are powers of 2 and 3.

712. Powers are *named* according to the *number of times* the factor is taken to produce the given power.

713. The **First Power** is the number itself.

714. The **Second Power** is the product of *two* equal factors, and is called a **Square.**

715. The **Third Power** is the product of *three* equal factors, and is called a **Cube.**

NOTE.—The *second* power is called a *square* because the *area* of a square is found by multiplying one of its sides by itself. The *third* power is called a *cube* because the *contents* of a cube are found by taking one of its sides three times as a factor. (Art. **429.**)

715. An **Exponent** is a small figure placed above a number on the right to denote the power.

It shows that the number above which it is placed is to be raised to the power indicated by this figure. Thus,

$2^1 = 2$, the first power, or number itself.
$2^2 = 2 \times 2$, the *second* power, or *square*.
$2^3 = 2 \times 2 \times 2$, the *third* power, or *cube*.
$2^4 = 2 \times 2 \times 2 \times 2$, the *fourth* power, etc.

NOTES.—1. The term *exponent* is from the Latin *exponere*, to represent.

2. The exponent of the first power being 1, is commonly omitted.

717. The expression 2^4 is read, "2 raised to the fourth power, or the fourth power of 2."

9. Read the following: 9^5, 12^7, 25^8, 245^6, 381^{10}, 465^{15}, 1000^{24}.

10. Read $6^3 \times 7^4$, $25^6 \times 48^5$, $140^8 - 75^3$, $256^{10} \div 97^5$.

11. Express the 4th power of 85.

12. Express the 5th power of 348.

13. The 7th power of 340.

14. The 8th power of 561.

718. To find any required Power of a Number.

1. What is the 4th power of 8 ?

SOLUTION.—$8^4 = 8 \times 8 \times 8 \times 8 = 4096$, *Ans.*

RULE.—*Take the number as many times as a factor as there are units in the exponent of the required power.*

NOTES.—1. A *common* fraction is raised to a power by involving each term. Thus, $(\frac{3}{4})^2 = \frac{9}{16}$.

2. A *mixed* number should be reduced to an *improper* fraction, or the fractional part to a *decimal* ; then proceed as above.
Thus, $(2\frac{1}{2})^2 = (\frac{5}{2})^2 = \frac{25}{4}$; or $2\frac{1}{2} = 2.5$ and $(2.5)^2 = 6.25$.

3. All powers of 1 are 1 ; for $1 \times 1 \times 1$, etc. $= 1$.

Raise the following numbers to the powers indicated:

2. 6^3.	**5.** 5^5.	**8.** 4.03^3.	**11.** $(\frac{3}{5})^4$.
3. 3^6.	**6.** 7^4.	**9.** 2.0003^3.	**12.** $(\frac{7}{8})^3$.
4. 232^2.	**7.** 35^3.	**10.** 300.05^3.	**13.** $(3\frac{1}{4})^4$.

Formation of Squares.

719. To find the *Square* of a Number in the Terms of its Parts.

1. Find the square of 5 in the terms of the parts 3 and 2.

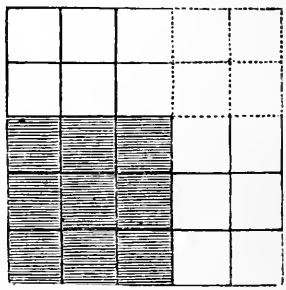


Illustration.—Let the shaded part of the diagram represent the square of 3; its contents are equal to 3×3, or 9 sq. ft.

1st. To preserve the form of the square, equal additions must be made to *two adjacent* sides; for, if made on *one side*, or on *opposite* sides, the figure will no longer be a *square*.

2d. Since 5 is 2 more than 3, it follows that *two rows* of 3 squares each, must be added at the top, and 2 rows on one of the adjacent sides, to make its *length* and *breadth* each equal to 5. Now 2×3 plus 2×3 are 12 squares, or *twice* the product of the two parts 2 and 3.

But the diagram wants 2 times 2 small squares, as represented by the dotted lines, to fill the upper corner on the right, and 2 times 2 or 4 is the square of the second part. We now have 9 (the sq. of the 1st part), 12 (twice the prod. of the two parts 3 and 2), and 4 (the square of the 2d part.) But $9 + 12 + 4 = 25$, the square required.

2. Find the square of 7 in the terms of 5 and 2.

Ans. $25 + 20 + 4$. Proof.—$7 \times 7 = 49$.

3. Find the square of 25 in the terms of its tens and units.

Analysis.—The product of 2 tens or 20 by 20 is 400 (the square of the tens); 20×5 plus 20×5 is 200 (twice the prod. of the tens by the units); and 5 by 5 is 25 (the square of the units). Now $400 + 200 + 25 = 625$, or 25^2. Hence, the

$$
\begin{array}{rcr}
25 & = & 20 + 5 \\
\underline{25} & & \underline{20 + 5} \\
125 & & 400 + 100 \\
\underline{50} & & \underline{+ 100 + 25} \\
625 & = & 400 + 200 + 25
\end{array}
$$

Rule.—*The square of any number consisting of tens and units is equal to the square of the tens, plus twice the product of the tens by the units, plus the square of the units.*

4. What is the square of 34 in the terms of its tens and units?

EVOLUTION.

ORAL EXERCISES.

720. 1. What are the two equal factors of 9? 16? 25?
2. Name the two equal factors of 36? 49? 64?
3. What are the three equal factors of 8? 27? 125?
4. Name the four equal factors of 16? Of 81?
5. Of what is 49 the square?
6. Of what is 27 the third power?
7. Of what is 125 the cube?

DEFINITIONS.

721. **Evolution** is finding a *root* of a number.

722. A **Root** is one of the *equal factors* of a number.

Roots are named according to the *number* of *equal* factors they contain.

723. The **Square Root** is one of the *two equal factors* of a number.

Thus, $5 \times 5 = 25$; therefore, 5 is the square root of 25.

724. The **Cube Root** is one of the *three equal factors* of a number.

Thus, $3 \times 3 \times 3 = 27$; therefore, 3 is the cube root of 27, etc.

725. The character ($\sqrt{\ }$) is called the **Radical Sign.** It is a corruption of the letter R, the initial of the Latin *radix*, a root.

726. *Roots* are denoted in *two ways:*

1st. By prefixing to the number the **Radical Sign,** with a figure placed over it called the **Index** of the root; as $\sqrt[2]{4}$, $\sqrt[3]{8}$.

2d. By a *fractional exponent* placed above the number on the right. Thus, $9^{\frac{1}{2}}$, $27^{\frac{1}{3}}$, denote the square root of 9, and the cube root of 27.

NOTES.—1. The figure over the radical sign and the denominator of the exponent, denote the *name* of the root.

2. In expressing the *square* root, it is customary to use simply the radical sign ($\sqrt{}$), the 2 being understood. Thus, the expression $\sqrt{25} = 5$, is read, "the square root of $25 = 5$."

727. A **Perfect Power** is a number whose exact root can be found; as, 9, 16, 25, etc.

728. An **Imperfect Power** is a number whose *exact* root can not be found. This root is called a **Surd.**

Thus, 5 is an imperfect power, and its square root 2.23+ is a surd.

NOTE.—All *roots* as well as *powers* of 1, are 1.

Read the following expressions:

8. $\sqrt{40}$.	**10.** $119^{\frac{1}{2}}$.	**12.** $1.5^{\frac{1}{4}}$.	**14.** $\sqrt[8]{256}$.	**16.** $\sqrt[4]{\frac{45}{58}}$.
9. $\sqrt[4]{15}$.	**11.** $243^{\frac{1}{3}}$.	**13.** $\sqrt[4]{29}$.	**15.** $\sqrt[15]{45.7}$.	**17.** $\sqrt[5]{\frac{125}{278}}$.

18. Express the cube root of 64 both ways; the 4th root of 25; the 7th root of 81; the 10th root of 100.

SQUARE ROOT.

729. Extracting the **Square Root** is finding one of *two* equal factors of a number.

730. To find how many figures the *Square* of a Number contains.

ILLUSTRATION.—1. Take 1 and 9, the *least* and *greatest integer* that can be expressed by *one* figure; also 10 and 99, the *least* and *greatest* that can be expressed by *two integral* figures, etc. Squaring these numbers,

$$1^2 = 1; \qquad 10^2 = 100; \qquad 100^2 = 10000.$$
$$9^2 = 81; \qquad 99^2 = 9801; \qquad 999^2 = 998001.$$

2. Take .1 and .9, the *least* and *greatest* decimals that can be expressed by *one* figure; also .01 and .99, the least and greatest that can be expressed by *two* decimal figures, etc. Squaring these,

$$.1^2 = .01; \qquad .01^2 = .0001; \qquad 001^2 = .000001.$$
$$.9^2 = .81; \qquad .99^2 = .9801; \qquad .999^2 = .998001, \text{ etc.}$$

731. From these illustrations we discover the following

PRINCIPLES.

1°. The square of a number contains twice as many figures as the root, or twice as many less one.

2°. If any number is separated into periods of two figures each beginning with units place, the number of figures in the square root will be equal to the number of periods.

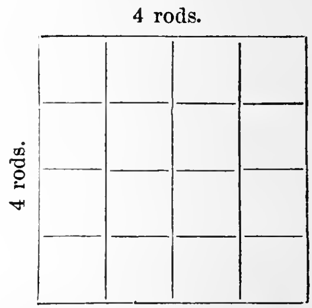
NOTE.—If the number of figures in the given number *is odd*, the *left hand* period will have but *one* figure.

732. **1.** Required the length of one side of a square garden which contains 16 sq. rods.

ILLUSTRATION.—Let the garden be represented by the adjoining diagram.

Now as the garden is square, its sides are equal, and the length of one side is one of the two equal factors, or the square root of 16. But $16 = 4 \times 4$. Hence, the length of a side is 4 rods.

PROOF.—4 rd. × 4 rd. = 16 sq. rods, the given area.



4 × 4 = 16 sq. rods.

2. What is the length of one side of a square which contains 625 square feet?

ANALYSIS.—Since 625 contains three figures, it must have two periods; its square root two figures, and first period on the left one figure.

The greatest square of 6 (hundreds) the left hand period is 4 (hundreds) and its root is 2 (tens) which we place on the right for the first figure of the root. Subtracting the square of 2 from the period used, we annex to the remainder the next period for a dividend.

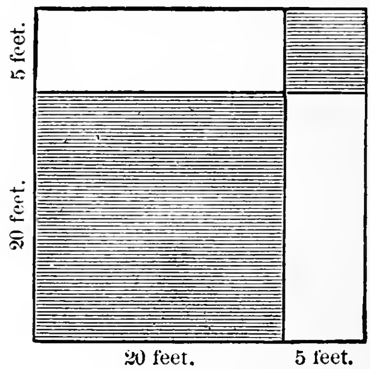
OPERATION.

$$\begin{array}{rl} & \dot{6}2\dot{5}\ )\ 25 \\ & \underline{4} \\ 45\ ) & 225 \\ & \underline{225} \end{array}$$

Since the additions are to be made on two sides of the square, we place 4, the double of the root, on the left of the dividend for a trial divisor, and find it is contained in 22, 5 times, the right hand figure being omitted. Placing the 5 on the right of the root and of the trial divisor, we multiply the divisor thus increased by this figure, and subtracting the product there is no remainder. The square root or answer, is 25 feet.

GEOMETRICAL ILLUSTRATION.

733. 1. Take any number as 625 sq. ft., the square root of which is to be found.



Let the shaded part of the diagram represent the square of 2 tens, the first figure of the root; then 20×20, or 400 sq. ft., will be its contents. Subtracting the contents from the given area, we have $625 - 400 = 225$ sq. ft. to be added to this square. To preserve its form, the addition must be made equally to two adjacent sides. The question is, what is the width of the addition.

Since the length of the square is 20 ft., adding a strip 1 foot wide to two sides will take $20 + 20$ or 40 sq. ft. Now if 40 sq. ft. will add a strip 1 foot wide to the square, 225 sq. ft. will add a strip as many ft. wide as 40 is contained times in 225; and 40 is contained in 225, 5 times and 25 over.

That is, since the addition is to be made on two sides, we double the root or length of the side found for a trial divisor, and find it is contained in 225, 5 times, which shows the width of the addition to be 5 feet.

Now the length of each side addition being 20 ft., and the width 5 ft., the area of both equals $20 \times 5 + 20 \times 5$, or $40 \times 5 = 200$ sq. feet. But there is a vacancy at the upper corner on the right, whose length and breadth are 5 ft. each; hence its area $= 5 \times 5$, or 25 sq. feet; and 200 sq. ft. + 25 sq. ft. = 225 sq. ft.

For the sake of finding the area of the two side additions and that of the corner at the same time, we place the quotient 5 on the right of the root already found, and also on the right of the trial divisor to complete it. Multiplying the divisor thus completed by 5, the figure last placed in the root, we have $45 \times 5 = 225$ sq. ft. Subtracting this product from the dividend, nothing remains.

2. What is the square root of .576?

SOLUTION.—$\sqrt{.576} = \sqrt{.5\dot{7}6\dot{0}} = .75+$, *Ans.*

3. Find the sq. root of 234.09.

SOLUTION.—$\sqrt{\dot{2}3\dot{4}.0\dot{9}} = 15.3$, *Ans.* Hence,

734. To extract the square root we have the following

GENERAL RULE.

I. *Separate the number into periods of two figures each, beginning at units, and count both ways.*

II. *Find the greatest square in the first period on the left, and place its root on the right. Subtract this square from the period, and on the right of the remainder place the next period for a dividend.*

III. *Double the part of the root thus found for a trial divisor; and finding how many times it is contained in the dividend, omitting the right hand figure, annex the quotient both to the root and to the divisor.*

IV. *Multiply the divisor thus increased by the last figure placed in the root, subtract the product from the dividend, and place the next period on the right of the remainder.*

V. *Proceed as before, till the root of all the periods is found.*

PROOF.—*Multiply the root by itself.* (Art. **722.**)

NOTES.—1. If there is a *remainder* after the root of the last period is found, annex *periods of ciphers*, and proceed as before. The figures of the root thus obtained will be *decimals.*

2. If the trial divisor is not contained in the dividend, *annex a cipher* both to the root and to the divisor, and bring down the next period.

3. It sometimes happens that the remainder is *larger* than the divisor; but it does not necessarily follow that the *figure* in the root is too *small.*

4. The *left hand* period in *whole numbers* may have but *one* figure; but in *decimals*, each period must have *two* figures. Hence, if the number of decimals is odd, a *cipher* must be annexed to complete the period.

Find the square root of the following numbers :

4.	576.	**9.**	538.245.	**14.**	287.65.
5.	1600.	**10.**	61.7646.	**15.**	.776961.
6.	1225.	**11.**	8476.124.	**16.**	1073.741824.
7.	291.64.	**12.**	1232136.	**17.**	.00053361.
8.	864.91.	**13.**	5314491.	**18.**	617230.2096.

735. To find the Square Root of Fractions.

1. What is the square root of $\frac{27}{48}$?

SOLUTION.—$\sqrt{\frac{27}{48}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$, *Ans.* Hence, the

RULE.—*Reduce the fraction to its simplest form and find the square root of each term separately.*

NOTES.—1. If either term of the given fraction, when reduced, is an imperfect square, reduce the fraction to a *decimal*, and proceed as above. (Art. **249**.)

2. Mixed numbers should be reduced to improper fractions, or the fractional part to a decimal.

2. What is the square root of $\frac{63}{448}$? *Ans.* $\frac{3}{8}$.

Find the square root of the following fractions:

3. $\frac{144}{576}$.
5. $\frac{4096}{46656}$.
7. $\frac{16384}{65536}$.
4. $\frac{256}{1296}$.
6. $\frac{1024}{7776}$.
8. $\frac{2816}{11979}$.

9. What is the square root of $20\frac{1}{4}$?

SOLUTION.—$\sqrt{20\frac{1}{4}} = \sqrt{\frac{81}{4}} = \frac{9}{2}$, or $4\frac{1}{2}$, *Ans.*

Find the square root of the following:

10. $18\frac{7}{9}$.
12. $52\frac{9}{16}$.
14. $\frac{16}{25}$ of 144.
11. $40\frac{24}{25}$.
13. $113\frac{28}{36}$.
15. $\dfrac{3\frac{1}{5}}{5\frac{2}{3}}$.

16. What is the square root of $\frac{9}{64}$ of $\frac{25}{49}$ of $\frac{81}{121}$ of 4096?
17. Required the square root of 3 to 7 decimals.
18. Required the square root of 12 to eight decimals.

APPLICATIONS.

735, *a*. 1. What is the side of a square whose area contains 2025 sq. yards?

2. A general has 906304 soldiers; how many must he place in rank and file to form them into a square?

3. A man bought a square tract of land containing 3840 acres; how many rods square is the tract?

4. What is the side of a square, whose area is equal to that of a triangle containing 5184 sq. ft.?

5. What is the side of a square equal in area to a rectangular field 32 rods long and 18 rods wide?

6. A landholder divided a tract of $3802\frac{1}{2}$ A. into four equal and square farms; what is the length of one of their sides?

7. A man having a garden 465 yards square, wished to extend it so as to make it 9 times as large; how many yards square will it then be?

736. *A mean proportional between two numbers is found by extracting the square root of their product.* (Art. **678.**)

8. What is the mean proportional between 9 and 16?

Solution.—$16 \times 9 = 144$; and $\sqrt{144} = 12$, *Ans.* Hence,

Note.—The product of any square number by another square number is always itself a square.

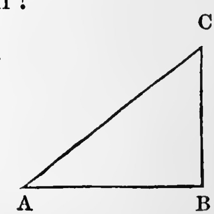
Find the mean proportional between the following numbers:

9.	4 and 16.	14.	28 and 54.	19.	$\frac{1}{4}$ and $\frac{1}{9}$.
10.	9 and 25.	15.	45 and 96.	20.	$\frac{4}{9}$ and $\frac{16}{25}$.
11.	25 and 36.	16.	.04 and .16.	21.	$\frac{25}{36}$ and $\frac{49}{81}$.
12.	49 and 64.	17.	.64 and 6.25.	22.	$\frac{36}{64}$ and $\frac{16}{100}$.
13.	81 and 64.	18.	.09 and .36.	23.	$\frac{49}{121}$ and $\frac{81}{144}$.

737. When the *length* of a rectangular field *equal* to a given *area*, is *double, triple,* etc., its *width,* its *dimensions* are found by extracting the square root of $\frac{1}{2}$, $\frac{1}{3}$, etc., of the area, as the case may be. This root will be the *width,* and being doubled, tripled, etc., will be the length.

24. The length of a rectangular field containing 80 acres, is twice its breadth; what are its length and breadth?

25. The breadth of a rectangular farm containing 160 acres, is $\frac{1}{4}$ its length; what are its length and breadth?

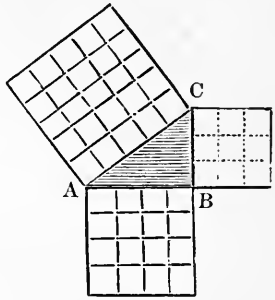


738. A **Triangle** is a figure having *three sides* and *three angles.*

739. A **Right-angled Triangle** is one which has a right angle; as ABC.

740. The **Hypothenuse** of a right-angled triangle is the side AC, opposite the right angle B; the base is AB, the perpendicular is BC.

741. The *relation* of the sides of a triangle to each other may be illustrated as follows:



Take any right-angled triangle as ABC, the base of which is 4 ft., the perpendicular 3 ft., and the hypothenuse 5 ft.

It will be seen that the square of the base contains 16 sq. ft., that of the perpendicular 9 sq. ft., and that of the hypothenuse 25 sq. ft. Now $25 = 16 + 9$. Hence we derive the following

PRINCIPLES.

1°. The sum of the squares of the Base and Perpendicular is equal to the square of the Hypothenuse.

2°. The square of the Hypothenuse diminished by the square of the Perpendicular, is equal to the square of the Base.

3°. The square of the Hypothenuse diminished by the square of the Base, is equal to the square of the Perpendicular. That is,

742. *The square described on the hypothenuse of a right-angled triangle is equal to the sum of the squares of the base and perpendicular.*

743. To find the Hypothenuse, when the Base and Perpendicular are given.

26. What is the length of a ladder which will just reach to the top of a house 32 feet high, when its foot is placed 24 feet from the house?

SOLUTION.—Perpendicular $(32)^2 = 32 \times 32 = 1024$
Base $(24)^2 = 24 \times 24 = \underline{\ 576}$
The square root of their sum, $\sqrt{1600} = 40$ ft., *Ans.*

Hence we have the following

RULE.—*Add the square of the base to the square of the perpendicular, and the square root of the sum will be the hypothenuse.*

FORMULA.—*Hypothenuse* $= \sqrt{Base^2 + Perpendicular^2}$.

27. The side of a certain school-room having square corners, is 8 yards, and its width 6 yards; what is the distance between two of its diagonal corners?

28. Two men start from the same place and at the same time; one goes exactly south 40 miles a day, the other goes exactly west 30 miles a day; how far apart will they be at the close of the first day?

29. How far apart will the same travelers be in 4 days?

744. To find the Perpendicular, when the Base and Hypothenuse are given.

30. A line 10 yd. long fastened to the top of a tree, reaches the ground 6 yd. from the base; what is the height of the tree?

SOLUTION.—Hypothenuse (10 yd.)$^2 = 10 \times 10 = 100$
Base (6 yd.)$^2 = 6 \times 6 = 36$
The square root of their difference, $\sqrt{64} = 8$ yd., *Ans.*
Hence, the

RULE.—*From the square of the hypothenuse subtract the square of the base, and the square root of the remainder will be the perpendicular.*

FORMULA.—*Perpendicular* $= \sqrt{Hypothenuse^2 - Base^2}$.

31. A line 75 feet long fastened to the top of a flag-staff reaches the ground 45 feet from its base; what is the height of the flag-staff?

32. A house is 40 ft. wide and the length of the rafters is 32 ft.; what is the perpendicular distance from the beam to the ridgepole?

33. The distance between the diagonal corners of a croquet ground is 17 yards, and its length is 15 yards; what is its width?

745. To find the Base, when the Hypothenuse and Perpendicular are given.

34. A ladder 50 ft. long was placed against the top of a house 40 ft. high; what distance was the foot of the ladder from the house?

SOLUTION.—Hypothenuse $(50 \text{ ft.})^2 = 50 \times 50 = 2500$
Perpendicular $(40 \text{ ft.})^2 = 40 \times 40 = \underline{1600}$
The square root of their difference, $900 = 30$ ft., *Ans.*
Hence, the

RULE.—*From the square of the hypothenuse subtract the square of the perpendicular, and the square root of the remainder will be the base.*

FORMULA.—*Base* $= \sqrt{\textit{Hypothenuse}^2 - \textit{Perpendicular}^2}$.

35. The slant height of a square pyramid is 40 ft., and its perpendicular height 32 ft., what is the distance from the center of the base to its side?

36. The height of a tree on the bank of a river is 100 ft., and a line stretching from its top to the opposite side is 144 ft.; what is the width of the river?

37. The side of a square field is 30 rods; how far is it between its diagonal corners?

38. If a square field contains 10 acres, what is the length of its side, and how far apart are its diagonal corners?

39. If a school room is 40 feet long, 30 feet wide, and 14 feet high, what is the length of a diagonal drawn upon the floor; and what is the length of a diagonal drawn from the floor to the ceiling?

40. A park 53 rods long and 39 rods wide has a straight walk running from its diagonal corners; what is the length of the walk?

41. The side of a square room is 40 feet; what is the distance between its diagonal corners on the floor?

42. A tree was broken 35 feet from its root, and struck the ground 21 ft. from its base; what was the height of the tree?

SIMILAR PLANE FIGURES.

746. Similar Plane Figures are those which have the *same form,* and their *like dimensions proportional.*

NOTES.—1. All *circles* and all *rectilinear figures* are *similar,* when their several angles are *equal* each to each, and their *like dimensions proportional.*

2. The like dimensions of circles are their *diameters, radii,* and *circumferences.*

747. The **Areas** of *similar surfaces* are to each other as the *squares* of their like dimensions. Conversely,

The **Like Dimensions** of *similar surfaces* are to each other as the *square roots* of their areas.

1. If one side of a triangle is 12 yards, and its area 36 square yards, what is the area of a similar triangle, the corresponding side of which is 8 yards?

SOLUTION.—$(12)^2 : (8)^2 :: 36 :$ *Ans.*, or 16 sq. yards.

2. If one side of a triangle containing 36 sq. yards is 8 yards, what is the length of a corresponding side of a similar triangle which contains 81 sq. yards?

SOLUTION.—$\sqrt{36} : \sqrt{81} :: 8 :$ *Ans.*, or 12 yards.

3. If a pipe 1 inch in diameter will fill a cistern in 60 min., in what time will a pipe 2 in. in diameter fill it?

4. If a gate 9 inches in diameter will empty a mill-pond in 16 hours, how large must a gate be to empty it in 4 hours?

5. If one side of the base of a triangular pyramid measuring 16 square feet, is 20 inches in length, what is the length of a side of a similar pyramid, which measures 36 square feet?

6. A man owns a building lot containing 20 square rods in the shape of a right-angled triangle, the perpendicular of which is 20 yards in length; what is the perpendicular of a similar lot, which contains 30 square rods?

CUBE ROOT.

ORAL EXERCISES.

748. 1. What number taken three times as a factor produces 8? 27?

2. What is one of the three equal factors of 64?

3. Name one of the three equal factors of 125?

4. Name one of the three equal factors of 1000. Of 1728.

WRITTEN EXERCISES.

749. The **Cube Root** of a number is *one* of its *three equal factors.*

750. To find the number of figures in the *Cube* of a Number, also in the *Cube Root* of a Number.

1st. Take 1 and 9, also 10 and 99, 100 and 999, etc., the least and greatest integers that can be expressed by *one, two, three,* etc., figures.

2d. In like manner take .1 and .9, also .01 and .99, etc., the least and greatest decimals that can be expressed by *one, two,* etc., *decimal figures.* Cubing these, we have

Roots.	Powers.	Roots.	Powers.
1	$1^3 = 1$,	.1	$.1^3 = .001$
9	$9^3 = 729$,	.9	$.9^3 = .729$
10	$10^3 = 1000$,	.01	$.01^3 = .000001$
99	$99^3 = 970299$,	.99	$.99^3 = .970299$
100	$100^3 = 1000000$,	.001	$.001^3 = .000000001$
999	$999^3 = 997002999$,	.999	$.999^3 = .997002999$.

By comparing these roots and their cubes, we discover the following

PRINCIPLES.

1°. The cube of a number cannot have more than three times as many figures as its root, nor but two less.

2°. If a number is separated into periods of three figures each beginning at units place, the number of figures in the cube root will be the same as the number of periods.

NOTES.—The left hand period in *whole* numbers may be *incomplete*, having only *one* or *two* figures; but each period of *decimals must* always have *three* figures. Hence, if the decimal figures in a given number are *less* than three, *annex ciphers* to complete the period.

How many figures in the cube root of the following:

1. 340566. **3.** 576.453. **5.** 32.7561.
2. 1467. **4.** 5.7321. **6.** .456785.

751. To find the Cube of a number consisting of two figures in the terms of its parts.

1. Find the cube of 35 in the terms of its tens and units.

OPERATION.

$$\begin{array}{rcr}
35 &=& 30+5 \\
35 &=& 30+5 \\
\hline
175 &=& (30\times 5)+5^2 \\
105 &=& 30^2+(30\times 5) \qquad \\
\hline
1225 &=& 30^2+2(30\times 5)+5^2 \\
35 &=& 30+5 \\
\hline
6125 &=& (30^2\times 5)+2(30\times 5^2)+5^3 \\
3675 &=& 30^3+2(30^2\times 5)+(30\times 5^2) \qquad \\
\hline
42875 &=& 30^3+3(30^2\times 5)+3(30\times 5^2)+5^3.
\end{array}$$

EXPLANATION.—The cube of the tens, $(30^3) = 27000$
3 times the square of tens by units, $3(30^2\times 5) = 13500$
3 times the tens by square of units, $3(30\times 5^2) = 2250$
and the cube of the units $5^3 = 125$
Now $27000 + 13500 + 2250 + 125 = 42875$. Hence,

752. *The cube of any number consisting of tens and units is equal to the cube of the tens, plus 3 times the square of the tens by the units, plus 3 times the tens by the square of the units, plus the cube of the units.*

NOTE.—Since the *cube* of a number consisting of *tens* and *units* is equal to the *cube of the tens*, plus 3 times the *square* of the tens by the units, etc., when a number has *two periods*, it follows that the left hand period must contain the *cube* of the *tens*, or *first figure* of the root.

2. Find the cube of 32 in the terms of its tens and units.

753. To Extract the Cube Root of a number.

1. What is the side of a cube which contains 27 solid feet?

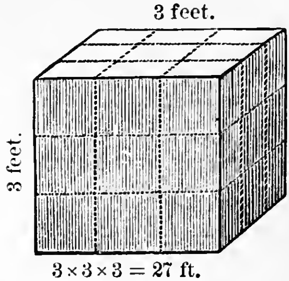


ILLUSTRATION.—Let the cube be represented by the adjoining diagram, each side of which is divided into 9 square feet. Since the length of a side is 3 feet, if we multiply 3 into 3 into 3, the product 27, will be the solid contents of the cube. (Art. **429**.) Now, if we reverse the process, dividing 27 into three equal factors, one of these factors will be the side of the cube. *Ans.* 3 ft.

2. What is the length of one side of a cubical mound containing 15625 solid feet of earth?

OPERATION.

$$
\begin{array}{r|l}
 & \dot{1}562\dot{5}\ (\ 25 \\
 & \ \ 8 \\
\hline
1200 & 7625 \\
300 & \\
25 & \\
\hline
1525 & \underline{7625}
\end{array}
$$

EXPLANATION.—1. We separate the given number into periods of three figures each, placing a point over units, then over thousands. This shows that the root must have two figures.

2. Beginning with the first period on the left, we find the greatest cube in 15 is 8, the root of which is 2. Placing the 2 on the right, we subtract its cube from the period, and to the remainder bring down the next period for a dividend. This shows that we have 7625 solid feet to be added.

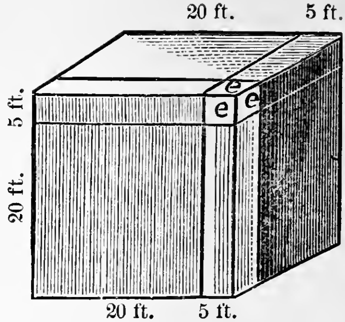
3. We square the root already found, which in reality, as there is to be another figure in the root, is 20; then multiplying its square 400 by 3, we write the product on the left of the dividend for a trial divisor; and finding it is contained in the dividend 5 times, place the 5 in the root.

4. We next multiply 20, the root already found, by 5, the last root figure; then multiply this product by 3 and write it under the divisor. We also write the square of 5, the last figure placed in the root, under the divisor. Adding these three results together, multiply their sum 1525 by 5, and subtract the product from the dividend. The answer is 25.

*ILLUSTRATION BY CUBICAL BLOCKS.**

Let the adjoining diagram represent a set of cubical blocks. Let the cube of 20, the tens of the root, be represented by the large cube. The remainder 7625 is to be added equally to three adjacent sides of this cube.

* Every school in which cube root is taught, should be furnished with a set of Cubical Blocks.



To ascertain the thickness of these side additions, we form a trial divisor by squaring 2, the first figure of the root, with a cipher annexed, for the area of one side of this cube, and multiply this square by 3 for the three side additions. Now $20^2 = 20 \times 20 = 400$; and $400 \times 3 = 1200$, the trial divisor. Dividing 7625 by 1200, the quotient 5, shows that the side additions are to be 5 ft. thick, and is placed on the right for the units' figure of the root.

To represent these additions, place the corresponding layers on the top, front, and right of the large cube. But we discover three vacancies along the edges of the large cube, each of which is 20 ft. long, 5 ft. wide, and 5 ft. thick. Filling these vacancies with the corresponding rectangular blocks, we discover another vacancy at the junction of the corners just filled, whose length, breadth, and thickness are each 5 ft. This is filled by the small cube.

To complete the trial divisor, we add the area of one side of each of the corner additions, viz., $20 \times 5 \times 3$, or 300 sq. ft., also the area of one side of the small cube $= 5 \times 5$, or 25 sq. ft. Now $1200 + 300 + 25 = 1525$. The divisor is now composed of the area of 3 sides of the large cube, plus the area of one side of each of the corner additions, plus the area of one side of the small cube, and is complete.

To ascertain the contents of the several additions, we multiply the divisor thus completed by 5, the last figure of the root; and $1525 \times 5 = 7625$. Subtracting the product from the dividend, nothing remains. Hence,

754. To extract the cube root we have the following

General Rule.

I. *Separate the given number into periods of three figures each; begin with units and count both ways.*

II. *Find the greatest cube in the first period on the left, and place its root on the right. Subtract this cube from the period, and to the right of the remainder bring down the next period for a dividend.*

III. *Multiply the square of the root thus found, considered as tens, by three, for a trial divisor; and finding how many times it is contained in the dividend, write the quotient for the second figure of the root.*

IV. *To complete the trial divisor, add to it three times the product of the root previously found with a cipher annexed, by the second root figure, also add the square of this second figure.*

V. *Multiply the divisor thus completed by the last figure placed in the root. Subtract the product from the dividend; and to the right of the remainder bring down the next period for a new dividend. Find a new trial divisor as before, and thus proceed till the root of the last period is found.*

NOTES.—1. If there is a *remainder* after the root of the last period is found, annex periods of ciphers, and proceed as before. The root figures thus obtained will be *decimals.*

2. If a trial divisor is *not contained* in the dividend, put a *cipher* in the root, *two ciphers* on the right of the divisor, and bring down the next period.

3. If the product of the divisor completed into the figure last placed in the root *exceeds* the dividend, the root figure is too large. Sometimes the remainder is *larger* than the divisor completed; but it does not necessarily follow that the root figure is *too small.*

3. What is the cube root of 130241.7?

EXPLANATION. — Having completed the period of decimals by annexing two ciphers, we find the first figure of the root as above. We place the next period on the right of the remainder, and the dividend is 5241. The trial divisor 7500 is not contained in the dividend; therefore, placing a cipher in the root and two ciphers on the right of the divisor, we bring down the next period, and proceed as before.

OPERATION.

	130241.700 (50.6 +
	125
750000	5241.700
9000	
36	
759036	4554216
	687484 Rem.

Extract the cube root of the following numbers:

4. 13824.	8. 1092727.	12. 91.125.
5. 571787.	9. 2357947691.	13. .253395799.
6. 373248.	10. 27054036008.	14. 164.566592.
7. 1953125.	11. 12.167.	15. 122615.327232.

755. *To find the cube root of a common fraction, reduce the fraction to its lowest terms, then extract the root of its numerator and denominator.*

NOTES.—1. When either the numerator or denominator is not a *perfect cube,* the fraction should be reduced to a decimal, and the root of the decimal be found as above.

2. A *mixed* number should be reduced to an improper fraction.

16. What is the cube root of $\frac{81}{192}$?

SOLUTION.—$\sqrt[3]{\frac{81}{192}} = \sqrt[3]{\frac{27}{64}} = \frac{3}{4}$, *Ans.*

Find the cube root of the following:

17. $\frac{376}{1730}$. **19.** $\frac{1520}{5130}$. **21.** $13\frac{2}{3}$.

18. $\frac{729}{4096}$. **20.** $\frac{15625}{46656}$. **22.** $37\frac{1}{27}$.

23. Find the cube root of 2 to 4 places of decimals.

24. Find the cube root of 3 to 5 places of decimals.

APPLICATIONS.

756. 1. What is the length of a side of a cubical box, which contains 389017 solid inches?

2. Find the side of a cu. vat, which contains 48228544 cu. feet?

3. What is the side of a cubical mound, which contains 1259712 solid yards?

4. What is the side of a cube equal to a stick of timber 2 feet square and 128 feet long?

5. What is the side of a cubical bin, which contains 500 bushels, allowing 2150.4 cu. in. to a bushel?

6. What is the side of a cubical cistern, which holds 100 wine hogsheads?

7. What is the side of a cube equal to a pile of wood 2421 ft long, 12 ft. wide, and 7 feet high?

SIMILAR SOLIDS.

757. Similar Solids are those which have the *same form,* and *their like dimensions proportional.*

NOTES.—1. The like dimensions of *spheres* are their *diameters, radii,* and *circumferences;* those of cubes are *their* sides.

2. The like dimensions of *cylinders* and *cones* are their *altitudes*, and the *diameters* or the *circumferences* of their bases.

3. *Pyramids* are similar, when their *bases* are similar polygons, and their *altitudes* proportional.

4. *Polyhedrons* (*i. e.*, solids included by any number of plane faces) are similar, when they are contained by the *same number* of similar *polygons*, and all their *solid angles* are *equal* each to each.

758. The **Contents** of *similar solids* are to each other as the *cubes* of their *like dimensions*. Conversely,

The **Like Dimensions** of *similar solids* are as the *cube roots* of their contents.

1. If a globe 4 inches in diameter weighs 32 lbs., what is the weight of a globe whose diameter is 5 inches?

SOLUTION.—$4^3 : 5^3 :: 32$ lbs. : *Ans.*
125×32 lbs. $= 4000$ lbs., and 4000 lbs. $\div 64 = 62.5$ lbs., *Ans.*

2. If a sphere 3 inches in diameter weighs 4 lbs., what is the diameter of a sphere which weighs 32 lbs.?

SOLUTION.—4 lbs. : 32 lbs. :: 3^3 : cube of diameter required.
Now $32 \times 27 = 864$; then $864 \div 4 = 216$, and $\sqrt[3]{216} = 6$ in., *Ans.*

3. If a cannon ball 6 inches in diameter weighs 58 lbs., what is the weight of a similar ball 8 inches in diameter?

4. If a cube of gold whose side is 3 inches is worth $6400, what is the worth of a cube of gold whose side is 8 inches?

5. If a pyramid 60 feet high contains 12500 cu. ft., how many cu. ft. are there in a similar pyramid 30 ft. high?

6. If a conical stack of hay whose height is 12 feet contains 5 tons, what is the weight of a similar stack whose height is 20 feet?

7. If a cubical block of marble whose side is 4 inches weighs 12 pounds, what will a cubic foot of marble weigh?

8. If a cylindrical cistern 6 feet in diameter will contain 30 hogsheads of water, how much will a similar cistern contain, whose diameter is 20 feet?

759. The side of a cube whose solidity is *double, triple,* etc., that of a cube whose side is given, is found by

Cubing the given side, multiplying it by the given proportion, and extracting the cube root of the product.

9. What is the side of a cubical mound, which contains 8 times as many solid feet as one whose side is 3 ft. *Ans.* 6 ft.

10. Required the side of a cubical vat, which contains 3 times as many solid feet as one whose side is 5 ft.

11. If a cube of silver whose side is 4 inches is worth $200, what is the side of a cube of silver, worth $1600?

12. I have a cubical box whose side is 6 ft.; I want another which will contain $\frac{1}{8}$ part as much. What will be the length of its side?

13. Required the side of a cubical vat which shall contain $\frac{1}{27}$ part as much as one whose side is 12 feet?

Questions.

710. What is involution? 711. What is a power? 713. The first power? 714. The second? 715. The third? 716. What is an exponent? 718. How find a power of a number?

721. What is evolution? 722. What is a root? 723. Square root? 724. Cube root? 727. A perfect power? 728. Imperfect? 729. What is extracting the square root? 731. Name the principles respecting squares and root? 734. How extract the square root. 735. How find the square root of fractions?

736. How find a mean proportional between two numbers? 739. What is a right-angled triangle? 740. Which side is the hypothenuse? What are the other two sides called? 741. Name the principles respecting right angled triangles. 742. To what is the square of the hypothenuse equal? 746. What are similar figures? 747. How do similar surfaces compare with each other?

749. What is the cube root of a number? 750. Name the principles respecting the number of periods and figures in the root? 752. To what is the cube of a number consisting of tens and units equal? 754. How extract the cube root? 755. How find the cube root of a fraction?

757. What are similar solids? What are the like dimensions of spheres? Of cubes? Of cylinders and cones? Of pyramids? 758. How do the contents of similar solids compare with each other?

PROGRESSION.

DEFINITIONS.

760. A **Progression** is a series of numbers which regularly *increase* or *decrease*.

761. The **Terms of a Progression** are the numbers which form the series. The *first* and *last* terms are the *extremes*; the others, the *means*.

762. Progressions are of two kinds, *arithmetical* and *geometrical*.

ARITHMETICAL PROGRESSION.

763. An **Arithmetical Progression** is a series which *increases* or *decreases* by a *common difference*.

764. The **Common Difference** of a progression is the difference between any two of its consecutive terms.

765. In an *ascending* series, each term is found *by adding the common difference* to the preceding term. Thus,

If the first term is 1 and the common difference 3, the series is

1, 4, 7, 10, 13, 16, 19, etc.

766. In a *descending* series, each term is found by *subtracting* the common difference from the preceding term. Thus,

If 15 is the first term and 2 the common difference, the series is

15, 13, 11, 9, 7, 5, 3, 1.

NOTES.—1. An Arithmetical Progression is sometimes called an *Equidifferent Series*. In every progression there may be an infinite number of terms.

2. An *Arithmetical Mean* between two numbers is found by taking *half their sum*.

767. In **Arithmetical Progression** there are *five elements* or *parts* to be considered: the *first* term, the *common difference,* the *last* term, the *number* of terms, and the *sum* of the terms.

Let a = the first term
l = the last term.
d = the common difference.
n = the number of terms.
s = the sum of the terms.

The relation of these five quantities to each other is such that if any *three* of them are given, the *other two* can be found.

768. To find the *Last Term,* when the First Term, the Common Difference, and Number of Terms are given.

1. Find the last term of an increasing series having 7 terms, its first term being 3, and its common difference 2.

ANALYSIS.—From the definition, each succeeding term is found by *adding the common difference* to the preceding. The series is:

$3,\quad 3+2,\quad 3+(2+2),\quad 3+(2+2+2),\quad 3+(2+2+2+2)$, etc. Or,
$3,\quad 3+2,\quad 3+(2\times 2),\quad 3+(2\times 3),\quad 3+(2\times 4)$, etc.

2. Find the last term of a decreasing series having 5 terms, the first term being 24, the common difference 2.

ANALYSIS.—In a descending series, each succeeding term is found by subtracting the common difference from the preceding. Hence, the series is

$24,\quad 24-2,\quad 24-(2+2),\quad 24-(2+2+2),\quad 24-(2+2+2\times 2)$, etc. Or,
$24,\quad 24-2,\quad 24-(2\times 2),\quad 24-(2\times 3),\quad 24-(2\times 4)$, etc. That is,

769. The *last term* is equal to the *first* term, *increased* or *diminished* by the *product* of the common difference into the number of terms *less* 1. Hence, the

RULE.—I. *Multiply the number of terms less one by the common difference.*

II. *When the series is ascending, add this product to the first term; when descending, subtract it from the first term.*

FORMULAS.—$l = \begin{cases} a + (n-1)\times d. & \text{Or,} \\ a - (n-1)\times d. \end{cases}$

770. To find the *First Term*, when the Last Term, the Common Difference, and Number of Terms are given.

1. Find the first term of a decreasing series the last term of which is 2, the common difference 3, the number of terms 6.

ANALYSIS.—The first term of a decreasing series will be the last term increased by the product of the common difference by the number of terms *less* one. The series is

$$2+3\times 5,\quad 2+3\times 4.\quad 2+3\times 3,\quad 2+3\times 2,\quad 2+3,\quad 2.$$

2. Find the first term of an increasing series, the last term of which is 45, the common difference 5, and the number of terms 7.

ANALYSIS.—The first term of an increasing series will be the last term *diminished* by the product of the common difference by the number of terms *less* one. The series is

$$45-5\times 6,\quad 45-5\times 5,\quad 45-5\times 4,\quad 45-5\times 3,\quad 45--5\times 2,\quad 45-5\times 1,\quad 45.$$

Hence, the

RULE.—I. *Multiply the number of terms less one by the common difference.*

II. *When the series is ascending, subtract this product from the last term; when descending, add it to the last term.*

FORMULAS.—$a = \begin{cases} l-(n-1)\times d. & \text{Or,} \\ l+(n-1)\times d. \end{cases}$

NOTE.—Any term in the series may be found by the preceding rules. For, the series may be supposed to stop at any term, and that may be considered the last.

3. Find the last term of an ascending series, the first term of which is 5, the common difference 3, and the number of terms 12?

4. The first term of a descending series is 40, the common difference 3, and the number of terms 11; what is the last?

5. The last term of an ascending series is 87, the number of terms 16, and the common difference 4; what is the first term?

6. What is the amount of $250, at 6% simple interest, for 21 years?

771. To find the *Number of Terms*, when the Extremes and the Common Difference are given.

1. The extremes of an arithmetical series are 4 and 37, and the common difference 3; what is the number of terms?

ANALYSIS.—The last term of a series is equal to the first term *increased* or *diminished* by the product of the common difference by the number of terms *less* 1. (Art. **769**.)

Now 37—4, or 33, is the product of the common difference 3, by the number of terms less 1. Consequently 33÷3, or 11, must be the number of terms less 1; and 11 + 1, or 12, is the answer required. Hence, the

RULE.—*Divide the difference of the extremes by the common difference, and add 1 to the quotient.*

FORMULA.—$n = \left\{ \frac{l - a}{d} + 1. \right.$

2. The age of the youngest child of a family is 1 year, the oldest 22, and the common difference of their ages 3 yr.; how many children in the family?

3. The extremes of an arithmetical series are 8 and 96, the common difference 4; what is the number of terms?

4. A laborer worked for 50 cts. the first day, 54 cts. the second, 58 cts. the third, and so on till his wages were $2 a day; how many days did he work?

772. To find the *Common Difference*, when the Extremes and the Number of Terms are given.

1. The extremes of a series are 3 and 21, and the number of terms is 10; what is the common difference?

ANALYSIS.—The difference of the extremes 21—3 = 18, is the product of the number of terms *less* 1 by the common difference, and 10—1, or 9, is the number of terms less 1; therefore 18÷9, or 2, is the common difference required. (Art. **764**.) Hence, the

RULE.—*Divide the difference of the extremes by the number of terms less 1.*

FORMULA.—$d = \left\{ \frac{l - a}{n - 1}. \right.$

2. The ages of 10 children form an arithmetical series; the youngest is 3 yr. and the eldest 30 years; what is the difference of their ages?

3. A military company appropriated $108 for 8 target prizes, the highest of which was $24, and the lowest $3; what was the common difference in the prizes?

4. The amount of $600 for 45 yr. at simple interest is $3120; what is the rate per cent?

5. The amount of $1500 for 27 years is $1620; what is the rate per cent?

773. To find the *Sum* of all the terms, when the Extremes and the Number of Terms are given.

1. Required the sum of the series having 7 terms, the extremes being 3 and 15.

ANALYSIS.—(1.) The series is 3, 5, 7, 9, 11, 13, 15.
(2.) Inverting the same, 15, 13, 11, 9, 7, 5, 3.
(3.) Adding (1.) and (2.), 18+18+18+18+18+18+18=twice the sum.
(4.) Dividing (3.) by 2, 9+ 9+ 9+ 9+ 9+ 9+ 9=63, the sum.

By inspecting these series, we discover that half the sum of the extremes is equal to the average value of the terms. Hence, the

RULE.—*Multiply half the sum of the extremes by the number of terms.*

$$\text{FORMULA.}—s = \left\{\frac{a+l}{2}\right. \times n.$$

NOTE.—From the preceding illustration we see that,

The *sum* of the *extremes* is equal to the sum of any *two terms equidistant* from them; or, to *twice* the *sum* of the *middle* term, if the number of terms *be odd.*

2. How many strokes does a common clock strike in 12 hours?

3. Find the sum of all the terms, the extremes being 0 and 300, and the number of terms 1200.

4. A father deposited $1 in the bank for his daughter on her first birthday, $4 the next, $7 the next, and so on; how much did she have when she was 21 years old?

GEOMETRICAL PROGRESSION.

DEFINITIONS.

774. A **Geometrical Progression** is a series of numbers which increase or decrease by a *common ratio.*

775. The **Terms** of a geometrical progression are the numbers which form the series.

NOTE.—The series is called *Ascending* or *Descending*, according as the terms *increase* or *decrease.* (Arts. **763, 764.**)

776. In an ascending series the ratio is *greater* than one.

Thus, 2, 4, 8, 16, 32, 64, etc., is an *ascending* progression.

777. In a descending series the ratio is *less* than one.

Thus, 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, etc., is a *descending* progression.

778. In Geometrical Progression there are also *five elements* or *parts* to be considered, viz.: the *first* term, the *last* term, the *number* of terms, the *ratio*, and the *sum* of all the terms.

Let a = the first term.
l = the last term.
r = the ratio.
n = the number of terms.
s = the sum of the terms.

779. To find the *Last Term*, when the First Term, the Ratio, and the Number of Terms are given.

1. Required the last term of an ascending series having 6 terms, the first term being 3, and the ratio 2.

ANALYSIS.—From the definition, the series is

$3,\quad 3\times 2,\quad 3\times(2\times 2),\quad 3\times(2\times 2\times 2),\quad 3\times(2\times 2\times 2\times 2)$, etc. Or,
$3,\quad 3\times 2,\quad 3\times 2^2,\quad 3\times 2^3,\quad 3\times 2^4$, etc.

Now, $3\times 2^5 = 3\times 32 = 96$, *Ans.* That is,

Each successive term = 1st term × ratio raised to a power whose exponent is *one* less than the *number* of the term. Hence, the

RULE.—*Multiply the first term by that power of the ratio whose exponent is 1 less than the number of terms.*

FORMULA.—$l = a \times r^{n-1}$.

NOTES.—1. Any term in a series may be found by the preceding rule. For, the series may be supposed to stop at that term.

2. The preceding rule is applicable to *Compound Interest;* the *principal* being the first term of the series; the *amount* of \$1 for 1 year the ratio; and the *number of years* plus 1, the number of terms.

2. A father promised his son 1 ct. for the first example he solved, 2 cts. for the second, 4 cts. for the third, etc.; what would the son receive for the tenth example?

3. What is the amt. of \$375 for 4 yr., at 5% compound int.?

4. What is the amount of \$1200 for 5 years, at 6% compound interest? Of \$2500 for 4 years, at 7%?

780. To find the First Term, when the Last Term, the Ratio, and the Number of Terms are given.

1. The last term of a progression is 96, the number of terms 6, and the ratio 2; what is the first term?

ANALYSIS.—Reversing the steps of the preceding rule, we have $96 \div 2^5 = 96 \div 32 = 3$, *Ans.* Hence, the

RULE.—*Divide the last term by that power of the ratio whose exponent is 1 less than the number of terms.*

FORMULA.—$a = l \div r^{n-1}$.

2. The last term of a series is 192, the ratio 3, and the number of terms 7; what is the first term?

781. To find the *Sum of all the Terms,* when the Extremes and Ratio are given.

1. Required the sum of the series whose first and last terms are 2 and 162, and the ratio 3.

ANALYSIS.—Since each succeeding term is found by multiplying the preceding term by the ratio, the series is 2, 6, 18, 54, 162.

(1.) The sum of the series, $= 2+6+18+54+162$.

(2.) 3 times the sum $= 6+18+54+162+486$.

Subt. (1.) from (2.), we have $486-2 = 484$, or twice the sum.

Therefore, $484 \div 2 = 242$, the sum required. Hence, the

RULE.—*Multiply the last term by the ratio, and subtracting the first term from the product, divide the remainder by the ratio less 1.*

$$\text{FORMULA.}—s = \frac{(l \times r) - a}{r - 1}.$$

2. The first term is 4, the ratio 3, and the last term is 972; what is the sum of the terms?

3. What sum can be paid by 8 instalments; the first being $1, the second $2, etc., in a geometrical series?

4. A man bought a dozen sheep, agreeing to pay 1 ct. for the first, 2 cts. for the second, 4 cts. for the third, etc.; what did he pay for the 12 sheep?

5. A housekeeper bought 12 chairs, paying 2 cts. for the first, 6 cts. for the second, and so on; what did they cost?

782. To find the *Sum* of a Descending Infinite Series, when the First Term and Ratio are given.

NOTE.—In a *descending* infinite series the last term being *infinitely small,* is regarded as 0. Hence, the

RULE.—*Divide the first term by the difference between the ratio and 1, and the quotient will be the sum required.*

1. What is the sum of the series $\frac{2}{3}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{12}$, continued to infinity, the ratio being $\frac{1}{2}$? *Ans.* $1\frac{1}{3}$.

NOTE.—The preceding problems in the Progressions embrace their ordinary applications. Others might be given, but they involve principles with which the pupil is not supposed to be acquainted.

QUESTIONS.

760. What is progression? 761. The terms? 763. An arithmetical progression? 765. How is each term found in an ascending series? 766. In a descending series?

767. Name the parts. 768. How find the last term? 770. The first term? 771. Number of terms. 772. The common difference? 773. The sum of all the terms?

774. What is geometrical progression? 778. Name the parts. 779. How find the last term? 780. The first? 781. The sum of all the terms? 782. The sum of a descending infinite series?

MENSURATION.

DEFINITIONS.

783. Mensuration is the process of measuring lines, surfaces, and solids.

784. A **Line** is length without breadth or thickness.

785. A **Straight Line** is one that does not change its direction, and is the shortest distance between two points in the same plane.

786. Parallel Lines are those which are equally distant from each other at every point.

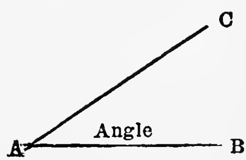
787. Curved Lines are those which change their direction at every point.

788. A **Horizontal Line** is one that is parallel to the horizon or water level.

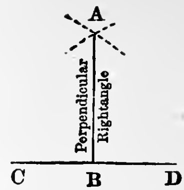
789. A **Perpendicular Line** is a straight line meeting another straight line, so as to make the two adjacent openings equal. As AB and CD. (Art. **792.**)

790. A **Perpendicular** to a *horizontal* line is called a **Vertical** line.

791. A **Plane Angle** is the opening between two straight lines drawn from the same point.

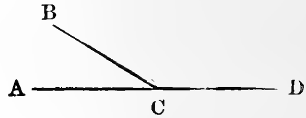


Thus, the opening between AB and AC is an *angle*, the lines AB and AC are called the *sides*, and the point A the *vertex* of the angle.



792. A **Right Angle** is one of the two equal angles formed by the meeting of two straight lines perpendicular to each other.

Thus, the adjacent angles ABC and ABD are right angles, and the lines AB and CD are perpendicular to each other.



793. An **Acute Angle** is one that is less than a right angle; as ACB.

794. An **Obtuse Angle** is one that is greater than a right angle; as BCD.

NOTE.—All angles except right angles are called *oblique angles.*

795. A **Surface** is that which has length and breadth, without thickness.

Surfaces are either *plane* or *curved.* The surface of a table is plane, that of an orange is curved.

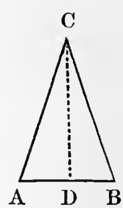
796. A **Plane Figure** is one which represents a surface all the parts of which are in the same plane.

797. A **Polygon** is a plane figure bounded by three or more *straight lines.*

798. The **Perimeter** of a polygon is the line by which it is bounded.

799. A **Regular Polygon** has all its sides and all its angles equal.

800. A polygon having three sides is called a *triangle;* four sides, a *quadrilateral;* five sides, a *pentagon;* six sides, a *hexagon;* seven sides, a *heptagon;* eight sides, an *octagon;* etc.



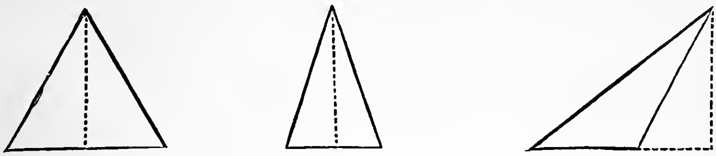
801. A **Triangle** is a polygon having three sides and three angles.

802. The **Base** of a triangle is the side AB on which it is supposed to stand.

803. A **Vertical Angle** is the angle opposite the base; as C.

804. The **Altitude** of a triangle is the perpendicular CD drawn from the vertical angle to the base.

805. An **Equilateral Triangle** is one having three *equal* sides.



Equilateral. Isosceles. Scalene.

806. An **Isosceles Triangle** is one having only *two equal* sides.

807. A **Scalene Triangle** is one having all its sides *unequal.*

Area of Triangles.

808. It is proved by Geometry that

The area of a triangle is equal to half the area of a parallelogram of equal base and altitude.

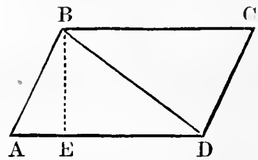


Illustration.—Let ABCD be a parallelogram whose altitude is the perpendicular EB.

Connect the diagonal corners by the straight line BD, and the parallelogram will be divided into two equal triangles, the altitude of each being EB.

The area of a parallelogram or rectangle is equal to the length multiplied by the breadth.

809. To find the Area of a Triangle when the Base and Altitude are given.

1. What is the area of a triangle whose base is 30 ft. and its altitude 12 feet?

Let the base AD of the triangle ABD be 30 ft., and EB, its altitude, be 12 ft.

Then $30 \times 12 = 360$ sq. ft., the area of the parallelogram.

And 30×6 ($\frac{1}{2}$ the altitude) $= 180$ sq. ft., area of triangle. Hence, the

Rule.—*Multiply the base by half the altitude.*

2. What is the area of a triangle whose base is 45 feet, and its altitude 20 feet?

3. What is the area of a triangle whose base is 156 feet, and its altitude 63 feet?

4. Find the number of acres in a triangular field whose base is 227 rods and altitude 65 rods.

5. What is the area of a triangle whose base is 135 yds., and its altitude is half its base?

6. Find the number of sq. feet in the gable end of a building 40 ft. wide, and $12\frac{1}{2}$ ft. from the beam to the ridgepole.

810. To find the Area of a Triangle, when the Three Sides are given.

From half the sum of the three sides subtract each side respectively; then multiply half the sum and the three remainders together, and extract the square root of the product.

1. What is the area of a triangle whose sides are respectively 10 feet, 12 feet, and 16 feet?

SOLUTION.—$(10+12+16)\div 2 = 19$ feet.
$19-10=9$; $19-12=7$; $19-16=3$.
Now $19 \times 9 \times 7 \times 3 = 3591$, and $\sqrt{3591} = 59.92+$ sq. ft.

2. What is the area of an equilateral triangle whose side is 12 yds.?

3. What is the area of an isosceles triangle whose base is 30 feet and sides 20 feet?

4. How many acres in a triangular field whose sides are 45, 53, and 64 rods?

811. To find the Altitude, when the Area and Base are given.

RULE.—*Divide the area by half the base.*

1. What is the altitude of a triangle whose area is $27\frac{1}{2}$ square yards and base 5 yards? *Ans.* 11 yards.

2. What is the altitude of a triangle whose area is 210 sq. yds. and its base 140 yards?

3. What is the altitude of a triangle whose base is 150 rods and its area 11250 square rods?

812. To find the Base, when the Area and Altitude are given.

RULE.—*Divide the area by half the altitude.*

1. What is the base of a triangle whose area is 154 sq. ft. and its altitude 14 feet? *Ans.* 22 feet.

2. What is the base of a triangle whose area is 40 acres and its altitude 160 rods?

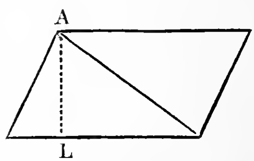
3. Find the base of a triangle whose area is 5260 sq. yd., and altitude 200 yards.

QUADRILATERALS.

813. A **Quadrilateral** is a polygon bounded by four straight lines.

A quadrilateral is either a *parallelogram*, a *trapezoid*, or a *trapezium*.

814. A **Parallelogram** is a quadrilateral having its opposite sides equal and parallel.

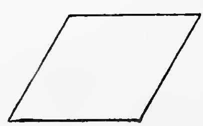


815. The **Altitude** of a quadrilateral having two parallel sides is the perpendicular distance between these sides; as, AL.

816. A **Rectangle** is a right-angled parallelogram.

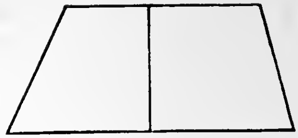
NOTE.—When the four sides of a rectangle are equal it is called a *square*. (Art. 345.)

817. A **Rhomboid** is an oblique-angled parallelogram.

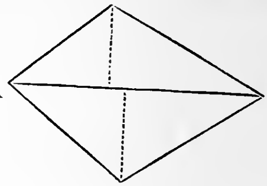


818. A **Rhombus** is an equilateral rhomboid.

819. A **Trapezoid** is a quadrilateral which has two of its sides parallel.



820. A **Trapezium** is a quadrilateral having four unequal sides, no two of which are parallel.*



NOTE.—The *Diagonal* of a plane figure is a straight line connecting two of its angles not adjacent; as AB.

821. To find the Area of a Parallelogram, when the Base and Altitude are given.

1. What is the area of a rectangle whose base is 88 feet and altitude 30 feet?

SOLUTION.—$58 \times 30 = 1740$ sq. ft., *Ans.*

2. What is the area of a rhomboid whose base is 63 feet and its altitude 40 feet?

SOLUTION.—$63 \times 40 = 2520$ sq. ft., *Ans.* Hence, the

RULE.—*Multiply the base by the altitude.*

NOTE.—The area of a square, a rectangle, a rhomboid and rhombus is found in the same manner.

3. How many acres in a field 120 rods long, and 90 rods wide?

4. How many acres in a field 800 rods long, and 128 rods wide?

5. Find the area of a square field whose sides are 65 rods in length.

6. A man fenced off a rectangular field containing 3750 sq. rods, the length of which was 75 rods; what was its breadth?

7. One side of a rectangular field is 1 mile in length, and it contains 160 acres; what is the length of the other side?

* The majority of Authors define these terms as in the text. Others, among whom are Legendre, Dr. Brewster, Young, and De Morgan, apply the definition here given of a Trapezium to the Trapezoid, and *vice versa.*

8. The length of a rhombus is 17 ft., and its perpendicular height 16 ft.; what is its area? *Ans.* 272 sq. ft.

9. What is the area of a rhomboid whose altitude is 25 rods, and its length 28.6 rods?

822. To find the Area of a *Trapezoid*, when its Parallel Sides and Altitude are given.

1. Find the area of a trapezoid whose parallel sides are 28 and 36 feet and its altitude 12 feet.

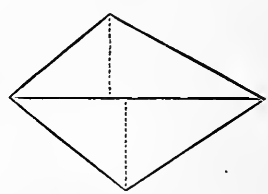
SOLUTION.—The sum of the parallel sides $28 + 36 = 64$ ft., $\frac{1}{2}$ of $64 = 32$ ft., and 32 ft. $\times$ 12 (the altitude) $=$ 384 sq. ft., *Ans.* Hence, the

RULE.—*Multiply half the sum of the parallel sides by the altitude.*

2. The parallel sides of a trapezoid are 25 yd. and 21 yd., and its altitude 16 yd.; what is its area?

3. Find the area of a trapezoid whose parallel sides are 25 rods and 37 rods, and its altitude 18 rods.

823. To find the Area of a *Trapezium*, when the Diagonal and Perpendiculars are given.



1. A man bought a city lot in the form of a trapezium, the diagonal of which was 84 ft. and perpendiculars from the opposite angles 12 ft. and 16 ft.; what was its area?

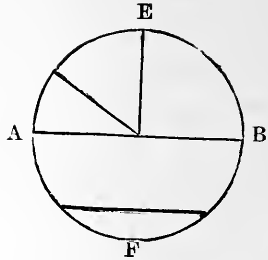
SOLUTION.—The sum of the perpendiculars is 28 ft.; $\frac{1}{2}$ of $28 = 14$ and 84 ft. $\times$ $14 = 1176$ sq. ft., *Ans.* Hence, the

RULE.—*Multiply the diagonal by half the sum of the perpendiculars to it from the opposite angles.*

2. A man bought a meadow in the form of a trapezium, the diagonal of which was 250 rods, and the perpendiculars 30 and 35 rods; how many acres did it contain?

3. Find the area of an irregular piece of land, the diagonal of which is 320 yards, and the perpendiculars 35.5 yards and $42\frac{1}{2}$ yards.

CIRCLES.



824. A **Circle** is a plane figure bounded by a curve line, every part of which is *equally distant* from a point within called the *center*.

825. The **Circumference** of a circle is the curve line by which it is bounded.

826. The **Diameter** is a *straight line* drawn through the *center*, terminating at each end in the *circumference*.

827. The **Radius** is a straight line drawn from the center to the *circumference*, and is equal to *half* the diameter.

NOTE.—From the definition of a circle, it follows that all the *radii* are *equal*; also, that all the *diameters* are *equal*.

828. From the relation of the circumference and diameter to each other, we derive from Geometry the following

PRINCIPLES.

1°. The Circumference = the Diameter × 3.1416 nearly.

2°. The Diameter of a Circle = the Circumference ÷ 3.1416 nearly.

829. To find the Circumference of a Circle, when the Diameter is given.

1. What is the circumference of a circle whose diameter is 15 feet?

SOLUTION.—15 ft. × 3.1416 = 47.125 ft., *Ans.* Hence, the

RULE. — *Multiply the given diameter by 3.1416.* (Art. **828**, *1°*.)

2. What is the circumference of a circle whose diameter is 45 yards?

3. What is the circumference of a circle whose diameter is 100 rods?

830. To find the Diameter of a Circle, when the Circumference is given.

1. What is the diameter of a circle whose circumference is $65\frac{1}{2}$ feet?

SOLUTION.—$65.5 \div 3.1416 = 20.849+$ ft., *Ans.* Hence, the

RULE.—*Divide the circumference by 3.1416.* (Prin. 2°.)

2. What is the diameter of a circle whose circumference is 94.2477 rods?

3. What is the diameter of a circle whose circumference is 628.318 yards?

NOTE.—The *diameter* of a circle may also be found by dividing the *area* by .7854, and extracting the *square root* of the quotient.

4. Required the diameter of a circle containing 50.2656 square rods.

5. Required the diameter of a circle containing 201.0624 square feet.

831. To find the Area of a Circle, when the Diameter and Circumference are given.

1. What is the area of a circle whose diameter is 10 ft. and circumference 31.416 ft.?

SOLUTION.—$\frac{31.416}{2} \times \frac{10}{2} = 78.54$ sq. ft., *Ans.*
Or, $31.416 \times (10 \div 4) = 78.54$ sq. ft., *Ans.* Hence, the

RULE.—*Multiply half the circumference by half the diameter; or,*

Multiply the circumference by a fourth of the diameter.

NOTES.—1. If only one of these dimensions are given, the other must be found before the rule can be applied. (Ex. 3, 4.)

2. The area of a circle may also be found by multiplying the square of its diameter by the decimal .7854.

2. Find the area of a circle whose diameter is 20 ft.?

SOLUTION.—$20^2 \times .7854 = 314.16$ sq. ft., *Ans.*

3. What is the area of a circle whose diameter is 100 ft.?

4. What is the area of a circle whose diameter is 120 rods?

5. What is the area of a circle whose circumference is 160 yards?

6. What is the diameter of a wheel whose circumference is 50 ft.?

7. Find the circumference of a tree whose diameter is 3 ft. 4 in.

8. What is the area of a circle whose radius is 15 ft.?

9. How many acres in a circular park whose circumference is 2 miles?

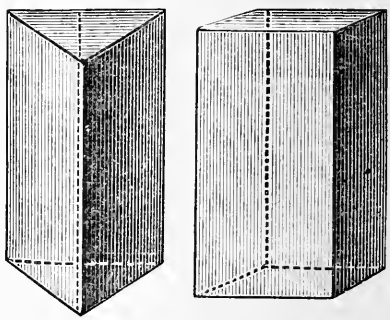
10. What is the radius of a circle which contains $1\frac{1}{4}$ acre?

SOLIDS.

832. A **Solid** is that which has length, breadth, and thickness.

833. A **Prism** is a solid whose bases are similar, equal, and parallel, and whose sides are parallelograms.

NOTE.—Prisms are named from the form of their bases, as *triangular, quadrangular, pentagonal, hexagonal,* etc.



834. A **Right Prism** is one whose sides are perpendicular to its bases.

835. A **Triangular Prism** is one whose bases are triangles.

836. A **Rectangular Prism** is one whose bases are rectangles, and its sides perpendicular to its bases.

837. The **Lateral Surface** of a prism is the sum of all its faces.

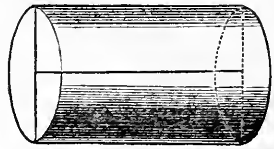
838. The **Altitude** of a prism is the perpendicular distance between its bases.

839. All rectangular solids are prisms.

NOTES.—1. When their sides are all equal to each other they are called *cubes.*

2. When their bases are parallelograms they are called *parallelopipeds,* or *parallelopipedons.*

840. A **Cylinder** is a circular body of uniform diameter, whose ends are *equal parallel circles.*



841. To find the Lateral Surface of a Prism or Cylinder.

1. What is the lateral surface of a prism whose altitude is 12 ft., and its base a pentagon each side of which is 6 feet?

SOLUTION.—6 ft. × 5 = 30 ft. the perimeter.
30 ft. × 12 = 360 sq. ft. surface, *Ans.*

2. What is the convex surface of a cylinder 32 inches in circumference and 40 inches long?

SOLUTION.—32 × 40 = 1280 sq. in., *Ans.* Hence, the

RULE.—*Multiply the perimeter of the base by the altitude.*

NOTE.—To find the *entire* surface, the area of the *bases* must be added to the lateral surface.

3. What is the surface of a triangular prism whose altitude is 9½ feet, and the sides of its base are 3, 4, and 5 ft. respectively?

4. Required the lateral surface of a triangular prism whose perimeter is 4½ inches, and its length 12 inches.

5. Required the lateral surface of a quadrangular prism whose sides are each 2 feet, and its length 19 feet.

6. Required the convex surface of a log whose circumference is 18 ft., and length 32 ft.?

7. What is the convex surface of a cylinder 16 feet in circumference and 40 feet long?

8. What is the convex surface of a cylinder whose diameter is 20 feet and its height 65 feet?

842. To find the Contents of a Prism or Cylinder, when the Perimeter of the Base and the Altitude are given.

1. What are the contents of a triangular prism whose altitude is 10 ft. and perimeter of its equilateral base 36 feet?

SOLUTION.—$12^2 - 6^2 = 108$.
$\sqrt{108} = 10.4$ nearly, altitude of base. (Art. **744.**)
Again, $12 \times 5.2 = 62.4$ sq. ft., area of base. (Art. **809.**)
62.4 sq. ft. $\times 10 = 624$ cu. ft., contents.

2. What are the contents of a cylinder whose altitude is 6 ft. 6 in. and the diameter of its base 3 ft.?

SOLUTION.—$3^2 \times .7854 = 7.0686$ sq. ft., area of base. (Art. **831**, *n.*)
7.0686 sq. ft. $\times 6.5 = 45.9459$ cu. ft., contents. Hence, the

RULE.—*Multiply the area of the base by the altitude.*

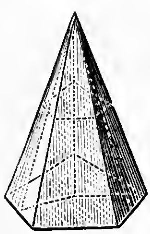
NOTE.—This rule is applicable to *all prisms*, triangular, quadrangular, etc.; also to *all parallelopipedons.*

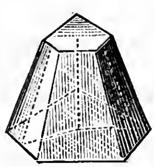
3. What is the solidity of a prism whose base is 5 feet square, and its height 15 feet?

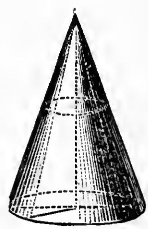
4. What is the solidity of a triangular prism whose height is 20 feet, and the area of whose base is 460 square feet?

5. Required the solidity of a cylinder 6 feet in diameter, and 20 feet high.

6. Required the solidity of a cylinder 30 feet in diameter, and 65 feet long.


Pyramid.


Frustum.


Cone.


Frustum.

843. A **Pyramid** is a solid whose base is a *triangle, square,* or *polygon,* and whose sides terminate in a point, called the vertex.

NOTE.—The sides which meet in the vertex are *triangles.*

844. A **Cone** is a solid which has a *circle* for its base, and terminates in a point called the *vertex.*

845. A **Frustum** of a *pyramid* or *cone* is the part which is left after the *top* is cut off by a plane parallel to the base.

846. To find the Contents of a Pyramid or a Cone, when the Base and Altitude are given.

1. What are the contents of a pyramid whose base is 144 sq. feet, and its altitude 30 feet?

SOLUTION.—144 sq. ft. × 10 ($\frac{1}{3}$ of altitude) = 1440 cu. ft., *Ans.*

2. What are the contents of a cone the area of whose base is 1864 sq. feet, and its altitude 36 feet?

SOLUTION.—1864 × 12 ($\frac{1}{3}$ of altitude) = 22368 cu. ft. Hence, the

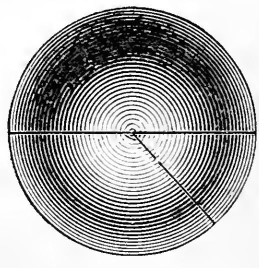
RULE.—*Multiply the area of the base by $\frac{1}{3}$ of the altitude.*

NOTE.—The contents of a frustum of a pyramid or cone are found *by adding the areas of the two ends to the square root of the product of those areas, and multiplying the sum by $\frac{1}{3}$ of the altitude.*

3. What are the contents of a pyramid whose base is 22 ft. square, and its altitude 48 ft.?

4. Of a cone 45 ft. high, whose base is 18 ft. diameter?

5. The altitude of a frustum of a pyramid is 27 ft., the ends are 5 ft. and 3 ft. square; what is its solidity?



847. A **Sphere** or **Globe** is a solid terminated by a *curve surface,* every part of which is *equally distant* from a point within, called the *center.*

848. The **Diameter** of a sphere is a straight line drawn through its center and terminated at both ends by the surface.

849. The **Radius** of a sphere is a straight line drawn from its center to any point in its surface.

850. To find the *Surface of a Sphere,* the Circumference and Diameter being given.

1. Required the surface of a globe 8 inches in diameter.

SOLUTION.—$8 \times 3.1416 = 25.1328$ in.
$25.1328 \times 8 = 201.0624$ sq. in. Hence, the

RULE.—*Multiply the circumference by the diameter.*

2. Required the surface of a 15 inch globe. *Ans.* 4.91 sq. ft.
3. Required the surface of the earth, its diameter being 8000 miles.

851. To find the *Solidity of a Sphere,* the Surface and Diameter being given.

1. Find the solidity of a sphere whose diameter is 15 inches and its surface 4.91 sq. feet?

SOLUTION.—$4.91 \times 144 = 707.04$ sq. in.
707.04 sq. in. $\times 2.5 = 1767.6$ cu. in., *Ans.* Hence, the

RULE.—*Multiply the surface by $\frac{1}{6}$ of the diameter.*

2. What is the solidity of a 10-inch globe?
Ans. 523.6 cu. in.
3. What is the solidity of the earth, its surface being 196900278 sq. miles, and its mean diameter 7916 miles?
4. Find the solidity of a cannon ball 9 inches in diameter?

852. To measure the height of an object standing in a plane.

1. What is the height of a tree standing in a plane which casts a shadow 50 feet, measured with a pole 6 ft. long, casting a shadow 10 ft.?

SOLUTION.—Take a pole of any convenient length, and placing it in a perpendicular position, measure the length of its shadow, which we will suppose to be 10 feet, then say
10 ft. (shadow of p.) : 50 ft. (shadow of t.) : : 6 ft. (l. of p.) : height of tree.
$50 \times 6 = 300$, and $300 \div 10 = 30$ feet.

2. What is the height of a pyramid standing in a plane which casts a shadow of 100 feet, measured with a pole 8 ft. long which casts a shadow of 12 feet?

Gauging of Casks.

853. **Gauging** is finding the capacity or contents of casks and other vessels.

854. The mean diameter of a cask is equal to half the sum of the head diameter and bung diameter. (Art. **766**, N. 2.)

Note.—The contents of a cask are equal to those of a cylinder having the same length and a diameter equal to the mean diameter of the cask.

855. To find the *Contents of a Cask*, when its Length, its Head, and Bung Diameters are given.

1. How many gallons in a cask whose length is 35 inches, its bung diameter 30 inches, and head diameter 26 inches?

Solution.—$(30+26)\div 2 = 28$ in., the mean diameter. (Art. **854.**)
$28^2 \times .7854 =$ area of base.
Area of base $\times$ length $=$ contents in cubic inches, which is reduced to gallons by dividing by 231.
Instead of using the factor .7854, if we divide it by 231, the number of cubic inches in a gallon, and multiply by the quotient .0034, the operation is shortened, and the result is in gallons. Thus,
$28^2 \times 35 \times .0034 = 93.296$ gal., *Ans.* Hence, the

Rule.—*Multiply the square of the mean diameter by the length in inches, and this product by .0034 for gallons, or by .0129 for liters.*

Note.—In finding the contents of cisterns, it is sufficiently accurate for ordinary purposes to call a cubic foot $= 7\frac{1}{2}$ gallons.

2. What is the capacity of a cask whose length is 30 inches, its head diameter 18, and bung diameter 24 inches?

3. Find the contents in liters of a cask whose length is 54 inches, its bung diameter 42, and head diameter 36 inches?

4. Find the contents in gallons of a rectangular cistern $4\frac{1}{2}$ ft. long, $3\frac{1}{4}$ ft. wide, and 5 ft. deep.

TONNAGE OF VESSELS.

856. Tonnage is the weight in tons which a vessel will carry. It is estimated by the following

CARPENTER'S RULE.

Multiply together the length of the keel, the breadth at the main beam, and the depth of the hold in feet, and divide the product by 95 (the cu. ft. allowed for a ton); the result will be the tonnage.

For a double decker, instead of the depth of the hold, take half the breadth of the beam.

NOTE.—A *Register Ton* = 100 cu. ft. is the legal standard.

A *Shipping Ton* = $\left\{ \begin{array}{l} \text{40 cu. ft., U. S., or} \\ \text{42 cu. ft., Eng.,} \end{array} \right\}$ used in estimating cargoes.

1. What is the tonnage of a double decker with 300 ft. keel and 40 ft. beam? *Ans.* $2526\frac{6}{19}$ tons.

2. What is the tonnage of a single decked vessel whose length is 100 ft., the breadth 30 ft., and the depth 12 ft.?

QUESTIONS.

783. What is mensuration? 784. A line? 785. Straight line? 786. Parallel lines? 789. Perpendicular line? 791. What is a plane angle? The vertex? 792. A right angle? 793. Acute? 794. Obtuse?

795. What is a surface? 796. A plane figure? 797. A polygon? 801. A triangle? 803. Vertical angle? 804. The altitude of a triangle? 805. An equilateral triangle? 806. Isosceles? 807. Scalene?

809. How find the area of a triangle from the base and altitude? 810. How when the three sides are given? 813. What is a quadrilateral? Name the three kinds of quadrilaterals. 814. A parallelogram? 815. Altitude of a quadrilateral? 816. A rectangle? 817. A rhomboid? 818. Rhombus? 821. How find the area of a parallelogram? 822. Of a trapezoid? 823. Of a trapezium?

824. What is a circle? 825. The circumference? 826. Diameter? 827. Radius? 829. How find the circumference when diameter is given? 830. How find diameter when circumference is given?

832. What is a solid? 833. A prism? 834. A right prism? 835. Triangular? 836. Rectangular? 840. What is a cylinder? 841. How find the lateral surface of a prism or cylinder? 842. When the perimeter of the base and the altitude are given, how find the contents?

847. What is a sphere? 850. How find the surface?

TEST QUESTIONS.

I. ORAL EXERCISES.

857. **1.** A lad earned $\$\frac{3}{4}$ the first day, $\$\frac{5}{8}$ the second day, and in both days he spent $\$\frac{1}{2}$; how much had he left?

2. A frog at the bottom of a well jumped up $3\frac{1}{2}$ yards the first day, and 2 yd. the second, but afterwards fell back $1\frac{1}{4}$ yd.; how far was he then from the bottom of the well?

3. From a bin containing $10\frac{1}{2}$ bushels of wheat, a miller took $3\frac{5}{10}$ bushels at one time, and $2\frac{1}{2}$ bushels at another; how much wheat remained in the bin?

4. If you buy a melon for $18\frac{3}{4}$ cents, and a quart of blackberries for $12\frac{1}{2}$ cents, and pay the market-man 50 cents, how much change ought he to give you?

5. What number must be taken from 12, that the remainder may be $5\frac{1}{4}$?

6. What number added to itself three times will make 48?

7. What number added to $\frac{1}{3}$ of itself will make 36?

8. A boy counting his money said, if he had $18\frac{3}{4}$ cents more, he would then have $62\frac{1}{2}$ cents; how much had he?

9. What is the sum of $37\frac{1}{2}$ and $6\frac{1}{4}$? What the difference?

10. A man having \$65, paid $\$24\frac{1}{2}$ for a cow, and $\$15\frac{3}{4}$ for a ton of hay; how much money had he left?

11. A man having $3\frac{1}{2}$ acres of land, bought $4\frac{1}{3}$ acres more; afterwards he sold $2\frac{1}{6}$ acres. How much land had he then?

12. A farmer paid $\$3\frac{1}{3}$ apiece for sheep; how many did he buy for \$150?

13. At $\$12\frac{1}{2}$ an acre, how many acres can be bought for \$500?

14. If a man earns $\$33\frac{1}{3}$ a month, how long will it take him to earn \$200?

15. 75 is $\frac{2}{3}$ of what number? $\frac{3}{4}$ of what number?

16. Henry lost 35 yards of his kite line, which was $\frac{5}{8}$ of its whole length; how long was it?

17. A man sold a watch for \$60, which was $\frac{5}{8}$ the cost; what did it cost?

18. If 7 barrels of peanuts cost \$42, what will 11 barrels cost?

19. Bought a bag of coffee, weighing 60 lbs., for \$20; what must I sell it for to make 12% profit?

20. Bought a horse and buggy for \$350; the price of the buggy was $\frac{3}{5}$ as much as the horse. What was the price of the horse?

21. Divide 48 into three such parts, that the first shall be twice the second, and the third 3 times the second.

22. In 48256 metres, how many kilometers?

23. If the sum of two numbers is 34, and their difference 8, what are the numbers?

24. The quantity of land in two pastures is 45 acres, and the difference in their size is 9 acres; how many acres does each contain?

25. If the difference between two numbers is 9, and their sum 32, what are the numbers?

26. If the difference of two numbers is $7\frac{1}{4}$, and their sum $22\frac{1}{2}$, what are the numbers?

27. The sum of the ages of two persons is $87\frac{1}{2}$ years, and the difference $12\frac{1}{2}$ years; what are their ages?

28. What number is that, $\frac{1}{3}$ and $\frac{1}{4}$ of which is equal to $\frac{4}{5}$ of 35?

29. The number of scholars in a certain school is 75, and the boys exceed the girls by 13; how many of each sex does the school contain?

30. A third of the trees in a certain orchard are pear trees, $\frac{1}{5}$ are peach trees, and the rest, being 21, are plum trees; how many trees are there in the orchard?

31. A man paid \$150 for a horse, and sold it at 20 per cent advance; how much did he make by the operation?

32. What is $33\frac{1}{3}$ per cent of 600? Of 660? 1500? 2100? 2700? 3600?

33. If you have \$400, and lose 40 per cent of it, how many dollars will you lose?

34. A lad having 500 marbles, lost 50 per cent of them; how many did he lose? How many did he have left?

35. If you have 200 acres of land, and sell 75 per cent of it, how many acres will you sell?

36. A man having $1000, invested it in a speculation by which he lost 100% of it; how much had he left?

37. A lad having 20 oranges, gave away 12 of them; what per cent did he give?

38. There are 6 working days to 1 sabbath; what is the percentage of sabbaths to days for labor?

39. In a certain state prison, 2 out of 3 of the inmates are intemperate; what is the percentage of intemperate?

40. What per cent of a number is $\frac{1}{4}$ of that number?

41. A farmer bought a cart and a plough for $81; the price of the cart was 8 times that of the plough. What was the price of each?

42. An agent sold a horse for $200, and received $12\frac{1}{2}$ per cent commission; how much did he receive?

43. What part of one year is 6 months? 4 mo.? 3 mo.? 2 mo.? $1\frac{1}{2}$ mo.? $1\frac{1}{3}$ mo.? 1 mo.?

44. What part of a year is 8 months? 9 mo.? 10 mo.?

45. What is the int. of $120 for 2 mo., at 4 per cent?

46. What is the int. of $250 for 6 mo., at 3 per cent?

47. What is the expense of collecting a tax of $500, at 6 per cent commission?

48. What is the commission on $600, at $12\frac{1}{2}$ per cent?

49. What is the interest of $200 for 1 year and 3 months, at 6 per cent?

50. What is the interest of $50 for 4 years, at 6 per cent?

51. What is the amount of $100 for 3 years, at 7 per cent?

52. What is the amount of $200 for 5 years, at 6 per cent?

53. The product of A, B, and C's ages is 240 years; A is 6 years, and B is 5 years old. How old is C?

54. Thomas bought 36 apples for 25 cents, and sold them at the rate of 4 for 3 cents; how much did he make or lose?

55. If he had sold them at the rate of 3 for 2 cents, what would have been the result of his bargain?

56. If a market-man buys oranges at the rate of 3 cents apiece, and sells 2 for 7 cents, what per cent is his profit?

57. A man sold a mirror for \$32, and thereby made $33\frac{1}{3}$ per cent; what per cent would he have made had he sold it for \$48?

58. A and B hired a pasture for \$36; A put in 4 horses for 12 weeks and B 6 horses for 10 weeks. How much ought each to pay?

59. If $\frac{1}{6}$ of a pole stands in the mud, $\frac{2}{3}$ of it in water, and 12 feet are above water, what is the length of the pole, and how many feet in each part?

60. If a herring and a half cost a penny and a half, how many can you buy for 11 pence?

61. A can dig a cellar in 3 weeks, B in 4 weeks, and C in 6 weeks; how long will it take all three to dig it?

62. A farmer having rye and wheat worth 6s. and 10s. a bushel, wished to make a mixture worth 9s.; what proportion of each must he put in?

63. What sum at 7% will gain \$84 int. in 1 year?

64. A grocer having two kinds of tea, worth 5s. and 7s. a pound, mixed 5 pounds of each, and sold the mixture at 6s. 6d. a pound; how much did he make by the operation?

65. If a cistern has one pipe which will fill it in 8 hours, and another which will empty it in 12 hours, how long will it take to fill it, if both run together?

66. What sum at 6% will gain \$54 interest in $1\frac{1}{2}$ year?

67. A vat holding 56 barrels has two faucets, one of which supplies 17 barrels an hour, and the other discharges 22 barrels an hour; when full, how long will it take to empty it, when both are running?

68. What is the difference between a dozen rods square and a dozen square rods?

69. The surface of a cube is 150 square inches; what is the length of its edge?

70. Four boats start at the same time from Castle Garden to sail round Governor's Island; one of them can perform the trip in 2 hours, another in 3 hours, another in 4 hours, and the other in 6 hours; how long, if they continue to sail, before all will meet at the starting place?

II. Written Exercises.

858. **1.** A teacher being asked how many pupils he had, replied that he had 140 boys, and if the number of girls were multiplied by the number of boys, the product would be 22960; how many girls had he? How many pupils?

2. The length of a rectangular meadow is 842 rods, and the product of the length and breadth is 52920 rods; what is the breadth? How many acres does it contain?

3. What is the difference between five hundred sixty-nine thousandths, and five hundred sixty-nine millionths?

4. A man having nine-tenths of an acre of land, sold nineteen thousandths of an acre; how much did he have left?

5. A pile of wood containing 150 cords is 120 feet long and $3\frac{1}{2}$ feet wide; what is its height?

6. Sold 96 yards of carpeting at \$1.87$\frac{1}{2}$ per yard, and thereby gained \$39; how much did it cost me a yard?

7. Change 22 years 122 days to days, allowing for five leap years.

8. What is the cost of 5 bu. 3 pk. and 7 qt. of clover seed, at \$4.85 per bushel?

9. What will be the cost of fencing a lot of land, 120 rods by 260 rods, at $37\frac{1}{2}$ cts. per foot?

10. The product of four numbers is 6048, and three of the numbers are 9, 12, and 8; what is the other factor?

11. A man sold 12 bu. 3 pk. 6 qt. of cranberries at \$$3\frac{1}{2}$ a bushel, and took his pay in flour at 4 cents a pound; allowing 196 lb. to a bbl., how many barrels should he receive?

12. How many steps of 30 inches must a person take in walking 42 miles?

13. A person returning from the mines had 25 lb. 10 oz. of pure gold; he sold it at \$1.04$\frac{1}{5}$ per pwt. What did he receive?

14. The product of the length, breadth, and height of a rectangular hay-mow is 3840 cubic feet; its length is 20 feet, and its breadth is 16 feet. What is its height?

15. Bought 15 cwt. 22 lb. of rice at \$4.25 a cwt., and 6 cwt. 36 lb. of pearl barley at \$5.60 a cwt.; what would be gained by selling the whole at $6\frac{1}{4}$ cts. a pound?

16. From a piece of cloth containing seventy-five and seventeen hundredths yards, thirty-six and seven thousandths yards were used; how many yards were left?

17. At an election for mayor, 8654 votes were cast for 2 persons; the successful candidate had 756 majority. How many votes had each?

18. A lady paid $1500 for an India shawl and a set of furs; the difference in their price was $575. What did she pay for each?

19. The sum of two numbers is 1876; the greater is 3 times the less. What are the numbers?

20. A and B engaged in an adventure and made $1575; they divided the gain in such a manner that A had 4 times as much as B. How much did each have?

21. The sum of two numbers is $121\frac{1}{7}$; their difference is $17\frac{1}{3}$. What are the numbers?

22. Find the greatest common divisor and least common multiple of 36, 79, 48, and 69.

23. A merchant bought 360 yd. of silk and 468 yd. of poplin, and wished them cut into equal dress patterns containing the greatest possible number of yards; how many patterns could he cut from both?

24. A man bought 3 tracts of land containing 112, 144, and 176 acres, respectively, which he fenced into equal fields of the greatest possible number of acres; how many acres did each contain?

25. The Atlantic Cable cost as follows: 2500 miles at $485 per mile; 10 miles deep sea cable, at $1450 per mile; 25 miles shore ends, at $1250 per mile. What was the total cost?

26. What is the number which divided by 453 gives the quotient 307 and the remainder 109?

27. What is a prime factor? The prime factors of 2366?

28. A man working for $2 a day, paying $4 a week for board, saved $72 in 10 weeks; how many week days was he idle?

29. Find the greatest common divisor and the least common multiple of 195, 285, and 315.

30. Divide 360 into 4 such parts, that the second shall be 2 times the first, third 3 times the first, fourth 4 times the first?

31. Reduce $\frac{4}{15}$, $\frac{5}{75}$, $\frac{32}{56}$, and $4\frac{1}{3}$ to the least common denominator.

32. What is the smallest sum with which I can stock a farm with sheep, cows, and oxen, investing the same amount in each, and paying for the first \$4, for the second \$30, and for the third \$48 each?

33. From sixteen ten thousandths take 27 millionths, and multiply the difference by 20.5.

34. Three express messengers make continuous trips between New York and Washington, one of whom can perform the round trip in 16 hr., the second in 18 hr., the third in 20 hr.; if all leave New York at the same time, how soon will they all meet there again?

35. A merchant sold 3 pieces of broadcloth, one containing $35\frac{3}{4}$ yd., another $38\frac{1}{2}$ yd., another $42\frac{1}{4}$, at $\$5\frac{1}{4}$ a yard; what was the amount of the bill?

36. What is the total of the following bill: 3 dozen eggs at 15 cents a dozen, 7 pounds of butter at 23 cents a pound, 47 yards of cotton at 12 cents a yard, and 8 pounds of coffee at 32 cents a pound?

37. A farmer sold $48\frac{3}{4}$ bu. potatoes, at $62\frac{1}{2}$ cts. a bushel, and took his pay in coffee at $33\frac{1}{3}$ cts. a pound; how much coffee did it require to pay for the potatoes?

38. A clerk in a banking house spent \$475 for house rent, \$350 for clothing, and for family expenses \$850, the sum of which was $\frac{7}{8}$ of his salary; what was his salary?

39. If $\frac{5}{16}$ of a steamboat cost \$9760, what is the whole worth?

40. A monument standing in a plane cast a shadow of 65 feet, which was $\frac{7}{8}$ of its height; what was the height of the monument?

41. What is the ***l. c. m.*** of $\frac{3}{4}$, $\frac{1}{2}$, $\frac{2}{5}$?

42. If A can do a piece of work in 7 days which A and B can do in 5 days, in how many days will B do the same work?

43. A farmer sells 7643 lb. of hay at \$9.50 per ton, and a pile of wood 6 feet high, 11 feet long, and 4 feet wide, at \$3.50 per cord. How much does he receive for both?

44. A merchant sells cloth at \$3.60 a yard, and gains 20 per cent; for what price must he sell it to lose 15 per cent?

45. Divide 429 hundredths by 5 millionths, and from the quotient subtract 425 thousandths; express the result in the lowest terms of a common fraction.

46. What will be the cost of laying a pavement 30 feet long and 8 feet 6 inches wide, at 60 cents per square yard?

47. What sum of money will yield as much interest in 2 yr., at 10 per cent, as $800 yields in 5 yr. 3 mo., at 6 per cent?

48. What is the difference between the true and the bank discount of a note of $600, payable in 40 days, at 7 per cent, without grace?

49. A sold two lots for $260 apiece, gaining 20 per cent on one and losing 20 per cent on the other; did he gain or lose, and how much?

50. A commission merchant sold 500 pieces of cloth for $130 a piece, and paid the owner $54,800; what was the rate of his commission?

51. How much will it cost to carpet a parlor 18 feet square with carpeting $\frac{5}{8}$ of a yard wide, at $1.50 per yard?

52. What is the difference between the market value and the par value of stock? Between a dividend and an assessment?

53. Two men start from the same point, one traveling 52 miles north, the other 39 west; how far apart are they?

54. If eight men cut 84 cords of wood in 12 days, working 7 hours a day, how many men will it take to cut 150 cords in 10 days, working 5 hours a day?

55. Find the cube root of 42875000.

56. A lady bought 65 yards of dress goods at 25 cts., $1\frac{3}{4}$ yd. of drilling at 15 cts., 13 yd. of cambric muslin at 12 cts., 2 spools of silk at 20 cts., 1 spool of cotton at 5 cts., 1 doz. buttons at 25 cts.; write the bill in proper form to show that it is paid, and calculate the amount.

57. If to a certain number you add its half, its third, and 28, the sum will be 3 times the number; what is the number?

58. Wichita is 40 miles on a straight line directly northwest of Winfield; how many miles will a person travel in making the journey, going on the section lines?

59. How many yards of carpeting $2\frac{1}{4}$ yards wide would be required to carpet a room $12\frac{1}{2}$ yards long and $8\frac{2}{3}$ yards wide?

60. Multiply 3 yr. 123 da. 5 hr. 17 min. 45 sec. by 63.

61. A grain merchant buys at different times, 315 bu. 15 qt., 843 bu. 19 qt., 1243 bu. 27 qt., and 734 bu. 7 qt. of oats, at 30 cents per bushel; how much money did he pay out?

62. What will 9784 pounds of hay cost, at $10 per ton?

63. In making purchases, I find I spend $\frac{1}{3}$ of my money at the first store, $\frac{1}{4}$ at the second, and $\frac{1}{5}$ at the third, and then have $13 left; how many dollars had I at first?

64. What is the total of the following bill: $3\frac{1}{2}$ yd. at $1.50, $\frac{7}{8}$ at 1.62\frac{1}{2}$, $9\frac{3}{4}$ at 2.37\frac{1}{2}$, $17\frac{1}{3}$ at 85 cents per yard?

65. Demonstrate that when we multiply the numerator of any fraction by a number, we increase the value of the fraction as many times as there are units in the multiplier.

66. Why do we invert the divisor in the division of one fraction by another?

67. Simplify $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7} \div \left(5\frac{1}{2} \times 3\frac{1}{7} \times 4\frac{1}{3} \times \frac{\frac{8}{9}}{\frac{7}{12}}\right)$.

68. If $\frac{3}{4}$ of my share of a farm is worth $510, and I own $\frac{2}{5}$ of the farm, what is the value of the farm?

69. What number must be divided by one-half of 90 to produce three-fourths of 228?

70. What does the product of all the common prime factors of two or more numbers produce?

71. If two men are 50 miles apart and travel toward each other, one going $3\frac{1}{3}$ miles per hour, and the other $3\frac{1}{2}$ miles per hour, in what time will they meet? What part of the distance will the first one travel?

72. Multiply seventy-eight ten-thousandths by five hundredths; divide the product by thirteen thousandths, and reduce the quotient to a common fraction in its lowest terms.

73. What will be the cost, at $6.50 per cord, of a load of wood consisting of two lengths of four feet each, the load being 2 ft. 9 in. wide and 3 ft. high?

74. If a man uses a pound of fertilizers on a piece of ground two yards square, how much will he use on $\frac{3}{4}$ of an acre?

75. Five cents per day is the interest on what sum at 7 per cent per annum?

76. Write a negotiable promissory note, observing the following directions: date, to-day; face, \$150; maker, John Jones; payee, George Green; drawing 6% int., payable in 4 months.

77. If a man's property is assessed at \$5125, and his State tax is five cents on a thousand dollars, his county tax one-half cent on a dollar, his school tax three mills on a dollar, and his poll tax three dollars, what is his whole tax?

78. A pole 63 feet long was broken into two pieces, the shorter being $\frac{2}{5}$ of the longer. Required the length of each.

79. A can hoe a row of corn in a certain field in 30 minutes, B can hoe a row in 20 minutes, and C can hoe a row in 35 minutes. What is the least number of rows that each can hoe in order that all may finish together?

80. What number is that from which if you take $\frac{1}{7}$ of itself, $3\frac{2}{5}$ times the remainder minus 1 will be 50?

81. What is the amount due for the following:

$700\frac{1}{2}$ feet of boards @ \$22.50 per M.;
912 pounds of hay @ \$14.50 per ton?

82. Philip Davis is debtor to William Richmond, Albion, as follows: For $16\frac{1}{4}$ yards sheeting at 22 cents per yard, $7\frac{1}{2}$ yards flannel at $62\frac{1}{2}$ cents per yard, $\frac{1}{2}$ dozen handkerchiefs at $37\frac{1}{2}$ cents each, and $2\frac{3}{4}$ yards drilling at $15\frac{1}{2}$ cents per yard. The above bill was paid November 23, 1881. Make out a receipted bill in proper form.

83. A buys a horse for \$60, and sells it to B for \$120, who sells it for \$200; what was the difference in their per cent of gain?

84. Had the cost price of an article been twenty per cent less, the rate of loss had been fifteen per cent less; what was the rate of loss?

85. A hired of B \$1000 for one year, at 12 per cent in advance, and gave his note for the \$1000, paying \$120 down. At the end of the year A said to B, "I want the \$1000 another year on the same terms." "Well," says B, "give me the \$120." A gives him a check for \$300, saying, "Take out the interest, and indorse the rest on the new note." How much should B indorse on the note?

86. A Syracuse coal dealer bought (at wholesale) at a mine in Penn. 1540 tons anthracite coal at \$3.50 per ton. The freight to Syracuse was \$2040, and the loss in transportation was estimated at \$510. The coal was retailed at \$5.50 per ton. What was the gain?

87. Sold 5000 pounds of sugar at 9 cents per pound, and lost 10 per cent; what per cent should I gain by selling it at 12 cents per pound?

88. Three persons purchased a horse together. A gave \$20, B gave 40 per cent more than A, and C gave $15\frac{1}{2}$ per cent less than both the others. What fractional part of the horse does each own?

89. A man bought 1000 bushels of wheat for \$1250. He finds 15% of it worthless. For how much must he sell the remainder per bushel to gain 20% on the cost?

90. At what rate per cent must I invest \$600, that in 2 yr. 6 mo. it may amount to \$705?

91. For what sum must a note be written in order to receive from a bank \$540 at 6% for 60 days?

92. If 6 men dig a trench 15 yards long, 4 yards broad, and 5 feet deep, in 3 days of 12 hours, in how many days of 8 hours will 8 men dig a trench 20 yd. long, 8 yd. broad, and 8 ft. deep?

93. What sum, put at simple interest at $7\frac{1}{2}\%$ per annum, will amount in 3 yr. 4 mo. to \$2500?

94. The interest on a certain sum, for $2\frac{1}{2}$ years at 7%, is \$$5.87\frac{1}{2}$. What is the true discount on the same sum for the same time, at the same rate?

95. A merchant bought a certain number of yards of cloth at \$2.50 per yard. He sold two-fifths of the cloth at a profit of 25%, and on the sale of the remainder he lost \$15. If his loss on the whole transaction amounted to 5%, how many yards of cloth did he buy?

96. If it cost \$95.60 to carpet a room 24×18 ft., how much will the same kind of carpet cost for a room 38×22 ft.?

97. What sum of money is that of which, if 80% be deposited in bank, and 20% of this deposit be withdrawn, there will remain \$5760 in bank?

98. A lawyer collecting a note at a commission of 8%, received \$6.80; what was the face of the note?

99. Bought stock at par, and sold it at 3% premium, thereby gaining \$750; how many shares of \$100 each did I buy?

100. What is the amount of \$16941.20 for 1 yr. 7 mo. 28 da., at $4\frac{3}{4}$%, simple interest.

101. An investment of \$7266.28 yields \$744.7937 annually; what is the rate of interest?

102. In what time will \$273.51 amount to \$312864, at 7%, simple interest?

103. What is the difference between the interest and the true discount of \$576, due 1 yr. 4 mo. hence, at 6%?

104. Three men gain \$2640, of which B is to have \$6 as often as C \$4 and A \$2; what is each one's share?

105. Find the square root of 10795.21 to three decimals.

106. What is the length of one side of a square piece of land containing 40 acres?

107. A room, the height of which is 11 feet and the length twice the breadth, takes 143 yards of paper 2 feet wide to cover its walls, door and windows included; how many yards of carpet 27 inches wide will be required for the floor?

108. In a rectangular cistern the length is 12 feet, the width is 5 ft., and depth 3 ft.; find the diagonal through the centre of the rectangular space. Find the weight of water in pounds it will contain, if a cubic foot of water weighs 1000 ounces.

109. A man sawed a pile of wood 40 ft. long, 4 ft. wide, $5\frac{1}{2}$ ft. high, for \$1.50 per cord. How much did he earn?

110. What will be the cost of 35 three-inch planks 22 ft. long, 16 inches wide, at \$17.50 per M.?

111. How many bushels will a bin hold that is 9 ft. long, 6 ft. wide, 6 ft. high?

112. A note was given Jan. 1, 1880, for \$700. The following payments were indorsed upon it: May 6, 1880, \$85; July 1, 1881, \$40; Aug. 20, 1881, \$100. How much was due Jan. 10, 1882, interest at 6 per cent?

113. At what price must 5 per cent bonds be bought so as to realize 7 per cent on the investment?

114. What will be the cost in Buffalo, N. Y., of a draft for \$1500 on Cleveland, O., payable 90 days after date, exchange $\frac{1}{8}$ per cent discount?

115. If I place \$1500 at interest for 18 months, and receive \$135 interest, what sum must I place at interest at the same rate, that I may receive \$275 interest in 8 months?

116. The length of a rectangular field containing 20 acres is twice its width; what is the distance around it?

117. Find the amount of \$387.20, from Jan. 1 to Oct. 20, 1881, at 7 per cent.

118. A man was offered \$3675 in cash for his house, or \$4235 in three years without interest. He accepted the latter offer; did he gain or lose, and how much, money being worth 7 per cent?

119. What are the proceeds of a note for \$368, at 90 days, discounted at bank at 6 per cent?

120. The height of the Obelisk, known as Cleopatra's needle, at N. Y. Central Park, is 70 feet, nearly; the diameter of the base is about 8 feet. What is the length of a line drawn from the apex to a point 36 ft. from the middle of one side of the base?

121. A ship sails east from Boston, long. 71° 10′, at the rate of 2° 30′ 20″ in a day; what is her long. at the end of 6 days?

122. If a man wastes 5 minutes a day, how much time will he waste in a common year?

123. How many cu. ft. of water must be drawn from a reservoir 30 ft. 6 in. long and 20 ft. 6 in. wide, to lower the surface 8 inches?

124. The Signal Service reports that $3\frac{1}{2}$ in. of rain fell in 24 hours; how many cu. yd. fell on an acre of ground?

125. What is the difference between 35 square rods and 35 rods square?

126. If a bird can fly $12\frac{3}{4}$ miles in $\frac{1}{3}$ hour, how far can it fly in $5\frac{1}{2}$ hours?

127. If 3 cheeses weigh $35\frac{5}{8}$, $44\frac{7}{16}$, and $27\frac{1}{4}$ lb., what is their entire weight? What is their average weight?

128. If 4 is added to both terms of the fraction $\frac{3}{5}$, will the value be increased or diminished, and how much?

129. A lad having 3 quarts of berries, ate $\frac{1}{4}$ of them, sold $\frac{1}{8}$ of the remainder, and divided the rest between his two companions; how many did each receive?

130. My gas bill was $12 when I burned 4800 feet of gas; what will it be when gas costs $\frac{1}{4}$ more, and I burn 1600 ft. less?

131. The difference in time between Greenwich and St. Louis is 5 hr. and 55 min.; what is the difference in longitude?

132. How many cubic feet in 10 boxes, each $7\frac{3}{4}$ ft. long, $1\frac{3}{4}$ ft. wide, and $1\frac{1}{4}$ ft. high?

133. If $\frac{9}{16}$ of a saw-mill are worth $631.89, what are $\frac{5}{14}$ of it worth?

134. Find the difference in time between two places whose difference of longitude is $5° 40'$.

135. The Hoosac Tunnel is 25000 feet long; Mount Cenis Tunnel, which connects France and Italy, is 12 kilometers. What fraction of the latter is the former?

136. A broker received $25250.50 to invest in stocks, after deducting his commission of $2\frac{1}{2}\%$; what was his commission, and how much did he invest?

137. A grain dealer sent a boat-load of corn to market, valued at $2000, and insured $62\frac{1}{2}\%$ of its value at $1\frac{1}{2}\%$; what premium did he pay?

138. If I sell wood at $7.20 per cord, and gain 20 per cent, what did it cost me per cord?

139. If 5 men can harvest a field in 12 hours, how many hours would it require if 4 more men were employed?

140. If 15 oxen or 20 horses eat 6 tons of hay in 8 weeks, how much will 12 oxen and 28 horses require in 21 weeks?

141. What must be the depth of a cubical cistern that will hold 3048.625 cubic feet of water?

142. How many tiles 8 in. square will cover a floor 18 ft. long and 12 ft. wide?

143. What per cent of a mile is 150 rd. 2 yd. $2\frac{1}{2}$ ft.?

144. The assets of a bankrupt are $65000, and his liabilities $85500; what per cent can he pay?

145. A merchant reduced the price of a piece of cloth 15 cts. per yd.; and thereby reduced his profit on the cloth from 12 to 8%; what was the cost of the cloth?

146. How many feet are there in 65% of a mile?

147. A man whose income was $2700 spent $1500; what per cent of his income remained?

148. Paid $8000 for stock 12% below par, and sold it at 115; what per cent did I gain?

149. If A loans B $500 for 6 mo., how long ought B to lend A $800 to requite the favor?

150. The ratio is $3\frac{1}{2}$, the antecedent $\frac{1}{2}$ of $\frac{5}{8}$; what is the consequent?

151. How long will it take 12 men to do a job which 7 men can do in $15\frac{1}{2}$ days?

152. What is the difference between the cube and the cube root of .027?

153. What are the dimensions of a cube equal to a bin 12 ft. 6 in. long, 10 ft. wide, and 5 ft. high?

154. The longitude of Boston is 71° 10′ W., and that of New Orleans is 90° 2′ W.; what is the time at New Orleans when it is 7 o'clock 12 min. A. M. at Boston?

155. What will be the wages of 9 men for 11 days, if the wages of 6 men for 14 days be $84?

156. For how much must I make my note at bank for 3 mo. at 6%, in order to get from the bank just $300?

157. Bought a horse for $125, and sold it for 20% advance; sold a carriage for $125, gaining 25%; sold a yoke of oxen for $125, losing 20%; bought ten sheep for $125, and sold them at a loss of 25%. What did I gain or lose on the whole?

158. Of two pieces of land, the one a circle 18 rods in diameter, the other a triangle whose hypothenuse is 30 rods, and whose base is 24 rods, which is the larger, and how much?

159. The length of a block of marble containing 105 cu. in. is 7 inches; find the length of a similar block containing 22680 cubic inches.

160. The sum of two fractions is $\frac{113}{126}$, and their difference $\frac{7}{18}$; what are the fractions?

161. A person expended 16% of all he was worth in buying 20% of the stock of a mining company. If the entire stock of the company sold for $100000, what was the person worth?

162. A trader sold 75 cords of wood for \$487.50, thereby losing 10% of the cost; what did the wood cost per cord?

163. Simplify $\frac{1}{2\frac{1}{2}} + \frac{1}{3\frac{1}{2}} + \frac{1}{4\frac{1}{2}}$.

164. Gunpowder being $\frac{3}{4}$ nitre and equal parts of sulphur and charcoal, how many pounds of each of the three are there in a ton?

165. Extract the square root of 1.225784 to four decimals.

166. Divide \$4600 into parts which are to each other as $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$.

167. What capital must be invested in 5 per cents at 95, to secure an income of \$10000?

168. Find the present worth of a note for \$175, payable in 8 mo., interest being computed at 7%?

169. $\frac{3}{4}$ is what per cent of $\frac{4}{5}$?

170. If a merchant sells goods which cost him \$1620 for \$1800 on a 9 mo. credit without interest, money being worth 6 per cent, how much does he gain?

171. A furrier asked 40% more for a set of mink furs than they cost him; but he afterwards sold them at a reduction of 10% from the price asked, thus realizing from the sale \$12.22 profit. What did the furs cost him?

172. In what time will \$560, at 8% per annum, produce \$106.40 interest?

173. I owe a debt of \$325.50, due in 1 yr. 5 mo., without interest; what will pay the debt now, money being worth 6% per annum?

174. If 15 men, working 6 hours a day, can dig a cellar 80 ft. long, 60 ft. wide, and 10 ft. deep, in 25 da., how many days will it take 25 men, working 8 hr. a day, to dig a cellar 120 ft. long, 70 ft. wide, and 8 ft. deep?

175. If by selling a house for \$12500 a builder gains 25%, what per cent would he gain or lose by selling it for \$9000?

176. What is the area of a right-angled triangle, whose perpendicular is 32 ft., and its hypothenuse 40 ft.?

177. Find the value of $\sqrt[3]{.000238328}$.

178. How much must be paid for making 52 rd. 14 ft. 8 in. of fence, at \$.75 per foot?

179. A traveler, on reaching a certain place, found that his watch, which gave the correct time for the place he left, was 2 hr. 22 min. slower than the local time. Had he traveled eastward or westward, and how many degrees?

180. Find the sum of $\frac{.06 + .03\frac{1}{3}}{3\frac{1}{5} - 2\frac{1}{2}}$ and $\frac{\frac{1}{5} \text{ of } 2\frac{1}{7}}{.5 + \frac{3}{8}}$.

181. What number diminished by 36% of itself = 336?

182. What is the value of a meadow 70 rd. long and 20 rd. wide, at $47.25 per acre?

183. What is the area of a triangle whose sides are respectively 10 ft., 12 ft., and 16 ft.?

184. What is the area of a triangle whose sides are each 24 yards?

185. A man bought a garden 3 rods wide and 4 rods long, and agreed to pay 1 cent for the first square rod, 4 cents for the second, 16 cents for the third, and so on, quadrupling each sq. rod; how much did his garden cost him?

186. Find the balance due March 4, 1882, on a note dated Jan. 1, 1879, for $580 at 5%, on which a payment of $85 had been made every 6 months; using the U. S. rule.

187. A and B enter into partnership; A furnished $240 for 8 mo., and B $559 for 5 mo. They lost $118; how much did each man lose?

188. In 25 kilogrammes how many pounds, Troy weight?

189. Reduce $\frac{18 \div \frac{1}{5}}{9 \times \frac{1}{4}}$ to its simplest form.

190. What is the area in acres of a triangle whose base is 156 rods and its altitude 63 rods?

191. Suppose a certain township is 6 miles long and $4\frac{1}{2}$ miles wide, how many lots of land of 90 acres each does it contain?

192. How many strokes would a clock which goes to 24 o'clock, strike in a day?

193. The extremes are 3 and 19, the number of terms 9; what is the com. dif., and the sum of the series?

194. A man spent $3 the first holiday, $45 the last, and each day $3 more than on the preceding; how many holidays did he have, and how much did he spend?

195. What is the area of a circle whose diameter is 120 rd.?

196. How much should be discounted on a bill of $3725.87, due in 8 mo. 10 da., if paid immediately, money being worth 5 per cent?

197. Bought bonds at 115 and sold at 110, losing $300. How many bonds of $1000 each did I buy?

198. What is the amount of $225, at 6 per cent compound interest for 4 years?

199. A steamer goes due north at the rate of 15 miles an hour, and another due west 18 miles an hour; how far apart will they be in 24 hours?

200. Find the cost, at 30 cts. per sq. yd., of plastering the bottom and sides of a cubical cistern that will hold 300 bbls.

201. Find the surface and the diagonal of a cube of granite containing 162144 cu. inches.

202. What is the area of a circle whose circumference is 160 yards?

203. What is the solidity of a prism whose height is 25 ft., and its base an equilateral triangle whose side is 12 feet?

204. What is the solidity of a prism whose base is 6 ft. square and its height 15 feet?

205. What is the solidity of a triangular prism whose height is 20 feet, and the area of whose base is 460 square feet?

206. Required the solidity of a square pyramid, the side of whose base is 25 feet, and whose height is 66 feet.

207. Required the solidity of a cone, the diameter of whose base is 30 feet, and whose height is 96 feet.

208. Required the solidity of a cylinder 20 feet in diameter and 65 feet long.

209. How many acres in a triangular field whose base is 325 yd. and its altitude 160 yd.?

210. If a scholar receive 1 credit mark for the first example he solves, 2 for the second, 4 for the third, and so on, the number being doubled for each example, how many marks will he receive for the twelfth?

211. What rate of income will U. S. $3\frac{1}{2}\%$ bonds yield, if bought at 102, and payable at par in 25 years?

212. What per cent income will Alabama 9's yield, bought at 85 and paid at par in 15 years?

III. Problems

from Entrance Examination Papers of Various Colleges.

Harvard University, 1880, '81.

859. 1. Find the greatest common divisor of 315, 504, 441.

2. Find the square root of 2 to the nearest ten-thousandth.

3. A wall which was to be 36 feet high was raised 9 feet in 6 days by 16 men; how many men will be needed to finish the work in 4 days?

4. A tradesman marks his goods at 25 per cent above cost, and deducts 12 per cent of the amount of any customer's bill, for cash. What per cent does he make?

5. A tunnel is 2 miles 21 chains 13.2 yards long. Find its length in meters. [1 mile = 1.61 kilometres.]

6. Simplify $\frac{17\frac{5}{12} - 9\frac{3}{4} + 4\frac{5}{7}}{\frac{5}{9} \times 9\frac{2}{7}}$.

7. Find the value in cubic decimeters of $\frac{11}{13}$ of 87 cu. meters 62 cu. decimeters 300 cu. centimeters.

8. If 27 men, working 10 hours a day, do a piece of work in 14 days, how many hours a day must 12 men work, to do the same amount in 45 days?

9. What sum of money, at 6 per cent annually compounded interest, will amount to $2703 in 1 yr. 4 months?

10. Arrange in order of magnitude, $\frac{22}{25}$, $\frac{49}{55}$, 0.89.

Yale College, 1880.

11. Add $(\frac{3}{8} \times \frac{4}{5} \times \frac{5}{9})$, $\frac{7}{15}$, $\frac{3}{4}$, and $\frac{9}{10}$.

12. Divide $\left(\frac{4}{6\frac{4}{7}} - \frac{1}{7}\right)$ by $\frac{8}{11}$.

13. Find the fourth term of a proportion of which the first, second, and third terms are, respectively, 3.81, 0.056, 1.67.

14. Reduce 133 sq. rd. 8 sq. ft. to a decimal of an acre.

15. In a board 4 meters long and 0.4 meters wide, how many square decimeters?

16. Divide ($\frac{3}{4}$ of $\frac{5}{16}$ of $\frac{8}{2}$) by $\frac{3\frac{4}{8}}{5\frac{7}{10}}$, and add the quotient to $\frac{3}{4} - \frac{7}{15}$.

17. Find $\sqrt{\frac{1}{21}}$, to three decimal places.

18. Find, to three decimal places, the number which has to 0.649 the same ratio which 58 has to 634.

19. A man bought a piece of ground containing 0.316 A. at 53 cents a square foot; what did he pay for the piece?

20. A grocer buys sugar at 18 cents a kilo, and sells it at 1 cent per 50 grams; how much per cent does he gain?

Columbia College, 1881.

21. Define a fraction. Give the rule for the addition of fractions, and the reason for each step of the operation.

22. Reduce 126 grams to ounces. $63\frac{3}{4}$ yd. to meters.

23. From $\frac{3}{4}$ of a gallon take $1\frac{4}{5}$ of a pint. What difference, if any, between the subtraction of compound numbers and that of simple ones? Between the subtraction of fractions and that of integers?

24. If 30 lb. of cotton will make 3 pieces of muslin 42 yd. long and $\frac{5}{8}$ yd. wide, how many pounds will it take to make 50 pieces, each containing 35 yd., $1\frac{1}{8}$ yd. wide?

25. A, B, and C formed a partnership, and cleared \$12000. A put in \$8000 for 4 mo., and then added \$2000 for 6 mo.; B put in \$16000 for 3 mo., and then withdrawing half his capital, continued the remainder 5 mo. longer; C put in \$13500 for 7 mo. How divide the profit?

26. Find the sum of $3\frac{1}{4}$, $6\frac{5}{7}$, $8\frac{9}{15}$, $65\frac{8}{9}$, reduce the fractional part to a decimal, and extract the cube root of the result.

Dartmouth College, 1879, '80.

27. Find the *l. c. m.* and the *g. c. d.* of 6, 8, 20, and 36.

28. How many metres in 25 feet?

29. Find the square root of 3530641.

30. Gold was quoted at $1.12\frac{1}{2}$; what was a \$1 greenback worth?

31. \$1200 includes a sum to be invested and a commission of 5% of the sum invested; what is the sum invested?

32. Find the sum and product of $\frac{7}{5}$, $\frac{1}{3}$, $\frac{5}{8}$.

33. Find the cube root of 3845672000.

34. Find the square root of 3534400.5.

35. A platform bears a weight of 100 lb. per square foot; what is the weight in kilograms per square meter?

36. A horse that cost $6\frac{1}{3}$ per cent of \$25000, was sold for \$1000; what was the loss per cent?

College of City of New York, 1880, '81.

37. Reduce $\frac{3}{8}$ of $\frac{4}{7}$ of $\frac{11}{2}$ to a decimal, carrying out the operation to four places.

38. If two men, working 8 hours, can carry 12000 bricks to the height of 50 feet, how many bricks can one man, working 10 hours, carry to the height of 30 feet?

39. I buy goods to the amount of \$4,978.70, payable in 4 mo., with interest at 5%, and give my note without interest. What must be the face of the note?

40. A man lost $\frac{1}{3}$, $\frac{1}{7}$, and $\frac{2}{5}$ of his money, and then had \$2600 left; what sum had he originally, and how much per cent had he lost?

41. Sold a fire engine for \$7050, and lost 6% on its cost; for how much ought I to have sold it to gain 12%?

42. What sum of money put at interest 6 yr. 5 mo. 11 da., at 7%, will gain \$3159.14?

43. For what sum must a note be drawn at 60 days to net \$1200 when discounted at 5%?

44. Extract the square root of 3286.9835 to the fourth decimal place.

45. Extract the cube root of 30.625.

Amherst College, 1881.

46. Find the greatest common divisor of 1263 and 1623.

47. Find the least common multiple of 18, 24, 36, and 126.

48. A cable that weighs one ton per mile weighs how much per foot?

49. When it is 10 o'clock in Boston what time is it in Amherst, the longitude of Boston being 71° 7′ 45″ W. from Greenwich, that of Amherst being 72° 31′ 50″?

50. Reduce 1 hr. 25 min. 30 sec. to the decimal of a day.

51. Of what number is $\frac{3}{5}$ the $\frac{7}{9}$ part?

52. What must be the face of a note which discounted at a bank at 6% for 30 days and grace, would yield \$200?

53. Sold a house for \$5000 and thereby gained 20%. Should I have gained or lost, and how much per cent, if I had sold it for \$4000?

54. Find the square root of 5.6169.

55. The meter is 39.37 inches. Find how many kilometers there are in a mile.

Vassar College, 1880.

56. Add $\dfrac{\frac{1}{5} \text{ of } 2\frac{1}{7}}{.5 + \frac{3}{8}}$ to $\dfrac{.06 + .3\frac{1}{3}}{3\frac{1}{5} - 2\frac{1}{2}}$.

57. Multiply 48 ten thousandths by two and one thousandth, and divide the result by one million.

58. Express 462 mm. in higher denominations.

59. What is $\frac{3}{4}$% of 140 books? What per cent of 30 ft. is 25 inches?

60. If I lose 10% by selling goods at 28 cents per yard, for what should they be sold to gain 20%?

61. What principal will yield an interest of \$339.20 in 5 yr. 4 mo. at 6%?

62. What must be the length of a box, 1 meter wide and 1 meter deep, to contain 4500 liters?

63. Cube .01. Square 1.001.

64. Extract the square root of 4.932841.

65. A can do a piece of work in 10 days; A and B can do the same work together in 7 days; in how many days can B working alone do the work?

New York Normal College, 1880.

66. What will it cost to floor a room $17\frac{1}{2}$ ft. long and 16 ft. wide, at the rate of \$1.10 per sq. yd.?

67. A man has a capital of \$12500; he puts 15% of it in stocks, $33\frac{1}{2}$% in land, and 25% in mortgages; how many dollars has he left?

68. A grocer bought 500 bags of coffee, each bag containing $49\frac{1}{4}$ pounds, at 12 cents a pound, and sold at a profit of $16\frac{2}{3}$%; for what did he sell it?

69. If I buy a house for \$5620 and receive \$1803 for rent in 2 yr. 3 mo. 15 da., what rate of int. do I get for my money?

70. Find the face of a note payable in 90 da. at 7%, so that the proceeds shall be \$2050 ?

71. A merchant owes \$2400, of which \$400 is payable in 6 mo., \$800 in 10 mo., and \$1200 in 16 mo.; what is the equated time?

72. If it costs \$7.20 to transport $18\frac{1}{2}$ cwt. $5\frac{1}{2}$ miles, what will it cost to transport $112\frac{3}{4}$ tons $62\frac{1}{2}$ miles ?

73. Extract the square root of 1051 to three places of decimals.

74. What is the cube root of 403583419 ?

Cornell University, 1880.

75. What is the value of 50 lb. 8 oz. of gold, at $\$20.59\frac{1}{4}$ per ounce?

76. Given the metre equal to 39.37 inches, reduce one mile to kilometers. Give the metric table of weights.

77. Divide $\frac{2}{3}$ of $7\frac{3}{4}$ by $\frac{4}{5}$ of $12\frac{11}{12}$. Prove the result by reducing the fractions to decimals and working the example anew.

78. How long must \$125 be on interest at $7\frac{1}{2}$ per cent to gain \$15 ?

79. Received 6 per cent dividend on stock bought at 25 per cent below par; what rate of interest did the investment pay?

80. How many liters in 20 bu. 3 pk. 4 qt., the bushel being 2150.42 cubic inches, and the metre 39.37 inches?

81. Simplify $\left(1 + \frac{1 + \frac{1}{5}}{5}\right) \div \left(1 + \frac{5}{1 + \frac{1}{5}}\right)$.

82. If one kilometer equals five-eighths of a mile, how many turns will a wheel make in 20 miles, the circumference of the wheel being 4 meters 5 millimeters?

83. What is the difference between the true and bank discount of \$250, due 10 mo. hence, at 7% ?

84. If 8 men spend \$32 in 13 weeks, what will 24 men spend in 52 weeks ?

Trinity College, 1880.

85. Subtract thirty million twenty-six thousand three from 45007021. Find what number must be added to the difference to make one hundred million, and write the answer in words.

86. The sum of $\frac{2}{3}$ and $\frac{4}{15}$ is diminished by $\frac{1}{10}$. How many times does the difference contain $\frac{3}{11}$ of the sum of $\frac{1}{8}$, $\frac{1}{9}$, and $\frac{1}{10}$?

87. Divide 375 by .75, and .75 by 375, and find the sum and the difference of the quotients.

88. A loaded wagon weighs 2 T. 3 cwt. 48 lb.; the wagon itself weighs 18 cwt. 75 lb. The load consists of 215 packages, each of the same weight. Find the weight of each, and reduce it to grams and kilograms.

89. Define interest, and give and explain the rule for computing the interest on any sum of money, for any time, and at any rate per cent.

90. Extract the square root of 184.2 to 3 decimals.

91. How many hektoliters of oats can be put into a bin that is 2 meters long, 1.3 meters wide, and 1.5 meters deep?

Wesleyan University, 1881.

92. Add $\dfrac{\frac{3}{4} \text{ of } 2\frac{1}{3}}{\frac{7}{8} + 4\frac{1}{5}}$ and $\dfrac{2\frac{1}{2} + 3\frac{1}{3}}{7\frac{1}{3} - 4\frac{2}{5}}$.

93. If money is worth 3 per cent, what is the premium on government $3\frac{1}{2}$ per cent bonds?

94. How many liters in 6 gallons of water?

95. How many cords of stone will it take to build a wall 2 ft. thick and 6 ft. high about a rectangular cellar whose interior dimensions, when the wall is completed, shall be 20 ft. long and 16 ft. wide?

96. How long must a note of \$243, at $3\frac{1}{2}\%$, run that its interest may equal the int. on a note of \$125, for 7 mo., at 5%?

97. Multiply $\sqrt{2}$ by $\sqrt{.123}$, and carry the result to 3 decimals.

98. Reduce 5 mi. 3 fur. 10 rd. to kilometers.

99. If 5 horses will consume 8 bu. 1 pk. 6 qt. of oats in 6 da., what quantity of oats will 7 horses consume in 11 da.?

APPENDIX.

ROMAN NOTATION.

860. The **Roman Notation** is the method of expressing numbers by *seven capital letters*, viz.:

I,	V,	X,	L,	C,	D,	M.
1,	5,	10,	50,	100,	500,	1000.

861. To express other numbers, these letters are combined as in the following

TABLE.

I	=	1	XI	=	11	XXI	=	21	CI	=	101
II	=	2	XII	=	12	XXV	=	25	CX	=	110
III	=	3	XIII	=	13	XXX	=	30	CL	=	150
IV	=	4	XIV	=	14	XXXI	=	31	CC	=	200
V	=	5	XV	=	15	XL	=	40	D	=	500
VI	=	6	XVI	=	16	L	=	50	DC	=	600
VII	=	7	XVII	=	17	LX	=	60	M	=	1000
VIII	=	8	XVIII	=	18	LXX	=	70	MC	=	1100
IX	=	9	XIX	=	19	XC	=	90	MD	=	1500
X	=	10	XX	=	20	C	=	100	MM	=	2000

MDCCLXXVI = 1776. MDCCCLXXXII = 1882.

862. The Roman Notation is based upon the following general principles:

1st. Repeating a letter *repeats* its *value*. Thus, I denotes one; II, two; III, three; X, ten; XX, twenty, etc.

2d. Placing a letter of *less* value *before* one of *greater* value, *diminishes* the value of the greater by that of the less; placing the less *after* the greater, *increases* the value of the greater by that of the less. Thus, V denotes five, but IV denotes only four, and VI six.

3d. Placing a *horizontal line* over a letter increases its value a *thousand* times. Thus, $\overline{\text{I}}$ denotes a thousand; $\overline{\text{X}}$, ten thousand; $\overline{\text{C}}$, a hundred thousand; $\overline{\text{M}}$, a million.

NOTES.—1. The letters C and M are the initials of the Latin *centum,* a *hundred,* and *mille,* a *thousand.*

2. The *radix* of the system appears to be doubtful. Some have supposed that at first it was five (V), and was subsequently changed to ten (X), forming a combination of the *quinary* and *decimal* systems.

3. Others maintain, more plausibly, that it proceeds according to the alternate scale of 5 and 2, thus uniting the *binary* with the *quinary* scale.

That is, Five times one (I) are five (V).
Two times five (V) are ten (X).
Five times ten (X) are fifty (L).
Two times fifty (L) are one hundred (C).
Five times one hundred (C) are five hundred (D).
Two times five hundred (D) are one thousand (M).

ENGLISH NUMERATION.

863. By the **English Numeration,** numbers are divided into periods of *six figures each,* and then each period is subdivided into *units, tens, hundreds, thousands, tens of thousands,* and *hundreds of thousands,* as in the following

TABLE.

Hund. of Thou. of Bill.	Tens of Thou. of Bill.	Thousands of Billions.	Hundreds of Billions.	Tens of Billions.	***Billions.***	Hund. of Thou. of Mill.	Tens of Thou. of Mill.	Thousands of Millions.	Hundreds of Millions.	Tens of Millions.	***Millions.***	Hund. of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	***Units.***
4	0	7	6	9	2	9	5	8	6	0	4	4	1	3	0	5	6
3d period.						2d period.						1st period.					

The figures in the Table are thus read: 407692 billions, 958604 millions, 413 thousand fifty-six.

CONTRACTIONS.

864. To Multiply any Number containing Two Figures by 11.

The product of any two numbers multiplied by 11 consists of the *first figure* of the number multiplied, the *sum* of the two figures, and the *last* figure.

Thus, $34 \times 11 = 374$, is composed of 3 the first figure, 7 the sum of $3 + 4$, and 4 the last figure.

NOTE.—If the sum of the two figures *exceeds* 9, the first or left-hand figure must be *increased* by 1.

1. What will 11 tons of iron cost, at $45 per ton?

ANALYSIS.—Since 1 ton costs $45, 11 tons will cost 11 times $45; and 11 times $45 are $495.

2. Alexander has 84 marbles, and Richard has 11 times as many; how many has Richard?

3. How many are 11 times 27? 11 times 33? 11 times 26? 11 times 34? 11 times 62? 11 times 54? 11 times 72?

865. To Multiply by 13, 14, 15, or 1 with a Significant Figure annexed.

If one city lot costs $3245, what will 17 lots cost?

ANALYSIS.—17 lots will cost 17 times as much as 1 lot. Placing the multiplier on the right, we multiply the multiplicand by the 7 units, set each figure one place to the right of the figure multiplied, and add the partial product to the multiplicand. The result is $55165.

OPERATION.

```
3245 × 17
 22715
$55165, Ans.
```

866. To Multiply by 21, 31, 41, etc., or 1 with either of the other Significant Figures prefixed to it.

If 21 men can do a job of work in 365 days, how long will it take 1 man to do it?

EXPLANATION.—We first multiply by the 2 tens, and set the first product figure in tens' place; then adding this partial product to the multiplicand, we have 7665 for the answer.

OPERATION.

```
 365 × 21
730
7665 days, Ans.
```

867. To Multiply Two or More Numbers by the Same Multiplier.

1. A grocer sold 4 pounds of tea to one customer, 3 lb. to another, and 5 lb. to another; how much did it all come to, at 7 dimes a pound?

SOLUTION. $(4+3+5) \times 7 = 84$ dimes, *Ans.* Hence, the

RULE.—*Multiply the sum of the numbers by the multiplier.*

868. To Multiply a Mixed Number, whose Fractional Part is $\frac{1}{2}$, by itself.

1. What is the product of $3\frac{1}{2}$ into $3\frac{1}{2}$?

SOLUTION.—The integral part of the given number is 3, and $3+1=4$. Now 3 into $4 = 12$, and $12+\frac{1}{4} = 12\frac{1}{4}$, *Ans* Hence, the

RULE.—*Multiply the integral part by one more than itself, and to this result annex* $\frac{1}{4}$.

869. To Multiply by 9, 99, 999, or any number of 9's.

1. How much will 99 carriages cost, at 235 dollars apiece?

EXPLANATION. — Since 1 carriage costs \$235, 100 carriages will cost 100 times as much, or \$23500. But 99 is 1 less than 100; therefore, subtracting the price of 1 carriage from the price of 100 gives the price of 99 carriages.

OPERATION.

\$23500	Price of 100 C.
235	" " 1 C.
\$23265	" " 99 C.

870. To Divide by 5.

1. A merchant laid out \$873 in flour, at \$5 a barrel; how many barrels did he get?

EXPLANATION.—We first multiply the dividend by 2, and then divide the product by 10, by cutting off the right-hand figure. The figure cut off is written over the divisor, and the fraction, reduced to its lowest terms, is annexed to the quotient.

$$\begin{array}{r} 873 \\ 2 \\ \hline 1|0\ )\ 174|6 \\ \hline 174\frac{3}{5},\ \textit{Ans.} \end{array}$$

NOTE.—This contraction depends upon the principle that any given divisor is contained in any given dividend just as many times as *twice* that divisor is contained in *twice* that dividend, *three times* that divisor in *three times* that dividend, etc.

871. **To Divide by 25.**

1. A farmer paid \$150 for cows, at \$25 apiece; how many cows did he buy?

OPERATION.

$$\begin{array}{r} 150 \\ \underline{4} \\ 1|00\ )\ 6|00 \end{array}$$

Ans. 6 cows.

EXPLANATION.—We first multiply the dividend by 4, and then divide the product by 100. (Art. **118.**)

872. **To Divide by 125.**

1. A man bought land for \$12150, at \$125 an acre; how many acres did he buy?

EXPLANATION.—We multiply the dividend by 8, and divide the product by 1000. (Art. **118.**)
Placing the remainder over the divisor, we reduce the fraction to lowest terms, and annex it to the quotient. The answer is $97\frac{1}{5}$ acres.

NOTE.—This contraction multiplies both the dividend and divisor by 8. (Art. **119**, 3°.)

OPERATION.

$$\begin{array}{r} 12150 \\ \underline{8} \\ 1|000\ )\ 97|200 \end{array}$$

Ans. $97\frac{1}{5}$ A.

873. **Demonstration of Finding the Greatest Common Divisor by Continued Divisions.**

1. Find the ***g. c. d.*** of 30 and 42. *Ans.* 6.

Two points are to be proved:

1st. That 6 is a *common divisor* of the given numbers.

2d. That 6 is their *greatest common divisor.*

That 6 is a common divisor of 30 and 42 is easily proved by trial.

Next, we are to prove that 6 is the *greatest* common divisor of 30 and 42. If the greatest common divisor of these numbers is not 6, it must be either *greater* or *less* than 6. But we have shown that 6 is a *common* divisor of the given numbers; therefore, no number *less* than 6 can be the *greatest* common divisor of them. The assumed number must therefore be *greater* than 6.

By the supposition, this assumed number is a divisor of 30 and 42; hence, it must be a divisor of their *difference,* 42 — 30, or 12. And as it is a divisor of 12, it must also divide the *product* of 12 into 2, or 24. Again, since the assumed number is a divisor of 30 and 24, it must also be a divisor of their *difference,* which is 6; that is, a *greater* number will divide a *less* without a *remainder,* which is impossible. Therefore, 6 must be the *greatest common divisor* of 30 and 42.

874. To find the Excess of 9's in a Number.

1. Let it be required to find the excess of 9's in 7548467.

Beginning at the left hand, add the figures together, and as soon as the sum is 9 or more, reject 9 and add the remainder to the next figure, and so on.

Adding 7 to 5, the sum is 12. Rejecting 9 from 12, leaves 3; and 3 added to 4 are 7, and 8 are 15. Rejecting 9 from 15, leaves 6; and 6 added to 4 are 10. Rejecting 9 from 10, leaves 1; and 1 added to 6 are 7, and 7 are 14. Finally, rejecting 9 from 14 leaves 5, the excess required.

875. Hence we derive this **property of the number 9:**

Any number divided by 9 will leave the same remainder as the sum of its digits divided by 9.

NOTES.—1. It will be observed that the excess of 9's in any *two* digits is always equal to the sum, or the excess in the sum, of those digits. Thus, in 15 the excess is 6, and $1+5=6$; so in 51 it is 6, and $5+1=6$.

2. The operation of finding the excess of 9's in a number is called *casting out the 9's.*

2. What is the product of 746 multiplied by 475?

OPERATION.	*Proof by Excess of 9's.*	*Proof by Mult.*
746	Excess of 9's in multiplic'd is 8	475
475	" " multiplier is 7	746
3730	Now............ 8 × 7 = 56	2850
5222	The excess of 9's in 56 is 2	1900
2984	The excess of 9's in product is 2	3325
Ans. 354350	And.... ....2 = 2	*Ans.* 354350

876. To prove Multiplication by Excess of 9's.

Find the excess of 9's in each factor separately; then multiply these excesses together, and reject the 9's from the result; if this excess agrees with the excess of 9's in the answer, the work is right.

NOTE.—The preceding is not a *necessary* but an *incidental* property of the number 9. It arises from the *law* of *increase* in the decimal notation. If the *radix* of the system were 8, it would belong to 7; if the radix were 12, it would belong to 11; and universally, it belongs to the number that is *one less* than the *radix* of the system of notation.

CIRCULATING DECIMALS.

877. A **Circulating Decimal** is one in which the *same figure* or *set of figures* is continually repeated in the same order.

878. In reducing common fractions to decimals, we find that in one class of examples the *division* is *complete*, and the quotient is an *exact decimal*.

In another class, the same figure or set of figures is repeated again and again, and the division will *not terminate*, though continued indefinitely.

The former are **Terminate Decimals**, the latter are **Circulating Decimals**, and the figure or figures repeated the **Repetend**.

Thus, $\frac{1}{2} = .5$, $\frac{2}{5} = .4$, $\frac{3}{4} = .75$, $\frac{5}{8} = .625$, etc., are exact decimals.

But $\frac{1}{3} = .333333+$, $\frac{14}{33} = .424242+$, $\frac{11}{37} = .297297+$, are interminate.

879. By inspection we see that the *denominators* of the first class are the *prime numbers* 2 or 5, or are composed of the factors 2 or 5.

In the second class, the *denominators* contain *other prime factors* than 2 and 5.

880. To find whether a common fraction will produce a *terminate* or a *circulating* decimal,

Resolve the denominator into its prime factors. If it contains no other factors than 2 and 5, the quotient will be a terminate decimal.

If it contains any other prime factors than 2 and 5, the quotient will be a circulating decimal.

881. Circulating decimals are expressed by writing the repetend *once*, and placing a *dot* over the *first* and *last* figure.

Thus, the repetends $.33333+$ and $.297297+$ are written $.\dot{3}$ and $.\dot{2}9\dot{7}$.

882. When the repetend begins with *tenths*, the decimal is called a **Pure Repetend**; as, $.\dot{4}$; $.\dot{2}9\dot{7}$.

883. When the repetend is *preceded* by one or more decimal figures, it is called a **Mixed Repetend**; as $.2\dot{7}$; $.42\dot{6}3\dot{1}$.

NOTE.—The decimal figures *before* the repetend are called the *finite part* of the decimal; as 2 and 42 in the mixed repetends above.

884. Change the following fractions to *terminate* or *circulating* decimals, and mark the repetends in each. (Art. **249.**)

1. $\frac{4}{5}$.	5. $\frac{11}{125}$.	9. $\frac{12}{36}$.	13. $\frac{10}{24}$.	17. $\frac{47}{150}$.
2. $\frac{7}{8}$.	6. $\frac{13}{16}$.	10. $\frac{16}{21}$.	14. $\frac{16}{30}$.	18. $\frac{38}{180}$.
3. $\frac{9}{12}$.	7. $\frac{7}{16}$.	11. $\frac{9}{25}$.	15. $\frac{26}{44}$.	19. $\frac{31}{64}$.
4. $\frac{23}{40}$.	8. $\frac{6}{7}$.	12. $\frac{7}{41}$.	16. $\frac{22}{60}$.	20. $\frac{3}{25}$.

885. To Reduce a Pure Repetend to a Common Fraction.

1. Reduce $.\dot{2}\dot{4}$ to a common fraction.

OPERATION.

$$0.\dot{2}\dot{4} \times 100 = 24.2424$$
$$0.\dot{2}\dot{4} \times 1 = 0.2424$$
$$0.\dot{2}\dot{4} \times 99 = 24$$
$$.\dot{2}\dot{4} = \frac{24}{99} = \frac{8}{33}, \textit{ Ans.}$$

ANALYSIS.—Since the repetend consists of two figures, if we multiply it by 100, the decimal part of the product will be the same as the repetend. Now, if we subtract the repetend from this product, the remainder will have no *decimal* and will be 99 times the given repetend. Therefore, *once* the given repetend is $\frac{24}{99}$ or $\frac{8}{33}$, *Ans.* Hence, the

RULE.—*Take the repetend for the numerator, and make the denominator as many 9's as there are figures in the repetend. Reduce the fraction thus produced to its lowest terms.*

Reduce the following to common fractions:

2. $.\dot{1}\dot{8}$.	5. $.\dot{1}2\dot{3}$.	8. $.\dot{1}06\dot{7}$.	11. $.\dot{2}512\dot{1}$.
3. $.\dot{7}\dot{2}$.	6. $.\dot{2}9\dot{7}$.	9. $.\dot{6}43\dot{5}$.	12. $.\dot{1}4285\dot{7}$.
4. $.\dot{6}\dot{9}$.	7. $.\dot{0}4\dot{5}$.	10. $.\dot{4}15\dot{8}$.	13. $.\dot{0}7692\dot{3}$.

886. To Reduce a Mixed Repetend to a Common Fraction.

14. Reduce $0.2\dot{2}\dot{7}$ to a common fraction.

OPERATION.

$.2\dot{2}\dot{7}$	Given decimal.
2	Finite part.
225	Numerator.
990	Denominator.

$\frac{225}{990} = \frac{45}{198} = \frac{5}{22}$, *Ans.*

SOLUTION.—Subtracting the finite part from the given mixed repetend, both regarded as integers, we have for the *numerator* 225, and for the *denominator* 990, that is, as many 9's as there are figures in the repetend with as many *ciphers* annexed as there are figures in the *finite* part, and the result is $\frac{225}{990} = \frac{45}{198} = \frac{5}{22}$, *Ans.* Hence, the

RULE.—*For the numerator, subtract the finite part from the given mixed repetend, both regarded as integers; and for the denominator, take as many 9's as there are figures in the repetend, with as many ciphers annexed as there are figures in the finite part.*

OPERATION.

$$0.2\dot{2}\dot{7} \times 100 = 22.7272$$
$$0.2\dot{2}\dot{7} \times 1 = .2272$$
$$0.2\dot{2}\dot{7} \times 99 = 22.5$$
$$0.2\dot{2}\dot{7} = \frac{22.5}{99} = \frac{225}{990}$$
$$\frac{225}{990} = \frac{45}{198} = \frac{5}{22}, \text{ Ans.}$$

EXPLANATION.—Since the repetend has two figures, if we multiply the given mixed repetend by 100, and from the product subtract *once* the given mixed repetend, the remainder (22.5) will be 99 times the given mixed repetend; and once the mixed repetend = $\frac{22.5}{99} = \frac{225}{990}$. But 225 is the difference between 227 the given mixed repetend, regarded as an integer, and 2 the finite part of it, regarded as an integer, and $\frac{225}{990} = \frac{5}{22}$, the same as before.

Change the following to common fractions:

15. $.64\dot{7}\dot{2}$. **17.** $.59\dot{2}\dot{5}$. **19.** $.5\dot{9}2\dot{5}$. **21.** $.9\dot{2}8571\dot{4}$.

16. $.100\dot{4}$. **18.** $.02\dot{2}\dot{7}$. **20.** $.47\dot{4}\dot{5}$. **22.** $.00849713\dot{3}$.

887. Circulating decimals, when reduced to common fractions, may be added, subtracted, multiplied, and divided, like other common fractions.

23. What is the sum of $.59\dot{2}\dot{5} + .47\dot{4}\dot{5} + .02\dot{2}\dot{7}$?

24. What is the difference between $.\dot{6}43\dot{5}$ and $.\dot{4}15\dot{8}$?

Table of Prime Numbers from 1 to 1009.

1	59	139	233	337	439	557	653	769	883
2	61	149	239	347	443	563	659	773	887
3	67	151	241	349	449	569	661	787	907
5	71	157	251	353	457	571	673	797	911
7	73	163	257	359	461	577	677	809	919
11	79	167	263	367	463	587	683	811	929
13	83	173	269	373	467	593	691	821	937
17	89	179	271	379	479	599	701	823	941
19	97	181	277	383	487	601	709	827	947
23	101	191	281	389	491	607	719	829	953
29	103	193	283	397	499	613	727	839	967
31	107	197	293	401	503	617	733	853	971
37	109	199	307	409	509	619	739	857	977
41	113	211	311	419	521	631	743	859	983
43	127	223	313	421	523	641	751	863	991
47	131	227	317	431	541	643	757	877	997
53	137	229	331	433	547	647	761	881	1009

SURVEYOR'S MEASURE.

888. **Surveyor's Measure** is used in measuring land, in laying out roads, establishing boundaries, etc.

889. The **Linear Unit** usually employed by *surveyors* is *Gunter's Chain*, which is 4 rods or 66 feet long, and contains 100 links. It is subdivided as in the following

TABLE.

7.92 inches (*in.*)	=	1 link, ...	*l.*
25 links	=	1 rod or pole, . .	*r.*
4 rods	=	1 ***chain***, . . .	*ch.*
80 chains	=	1 mile, ...	*m.*

NOTES.—1. *Gunter's* chain is so called from the name of its inventor. Measurements by it are usually given in *chains* and hundredths of a chain.

2. In measuring roads, etc., engineers use a *chain*, or *measuring tape*, 100 feet long, each foot being divided into *tenths* and *hundredths*.

3. The mile of the table is the common land mile, which contains 5280 feet.

890. The **Measuring Unit of Land** is the *Acre.*

TABLE.

625 sq. links	= 1 sq. rod or pole, .	*sq. rd.*
16 sq. rods	= 1 sq. chain, . . .	*sq. c.*
10 sq. chains, or 160 sq. rods	= 1 acre, ...	*A.*
640 acres	= 1 sq. mile, ...	*sq. mi.*

NOTES.—1. The *Rood* of 40 square rods is no longer a unit of measure.
2. A *Square,* in Architecture, is 100 square feet.

GOVERNMENT LANDS.

891. The public lands of the United States are divided into **Townships,** which are subdivided into **Sections, Half-Sections, Quarter-Sections,** etc.

A *Township* is 6 miles square, and contains 36 sq. miles.
A *Section* is 1 mile square, and contains 640 acres.
A *Half-Section* is 1 mile long by ½ mile wide, and contains 320 acres.
A *Quarter-Section* is 160 rods square, and contains 160 acres.

892. The method adopted by the Government in surveying a new territory is the following:

First.—A line is run North and South called the **Principal Meridian.**

Second.—A line is run on a parallel of latitude E. and W. called the **Base Line.**

Third.—Lines are run 6 miles apart parallel to the principal meridian.

Fourth.—Other lines are run 6 miles apart parallel to the base line, forming townships, or squares each containing 36 sq. miles.

893. **Townships** are designated by their number N. or S. of the base line.

894. A line of townships running N. and S. is called a **Range,** and is designated by its number E. or W. of the principal meridian. Thus,

T. 39 N., R. 14 E. 3d P. M., describes the township that is in the 39th tier North of the base line, and in the 14th range E. of the 3d principal meridian.

895. A Township is divided into **Sections** each 1 mile square and contains 640 acres. Thus,

A Section	= 1 mi.	×	1 mi.	=	640 acres.
A Half Section	= 1 "	×	$\frac{1}{2}$ "	=	320 "
A Quarter Section	= 1 "	×	$\frac{1}{4}$ "	=	160 "
A Half-quarter Section	= 1 "	×	$\frac{1}{8}$ "	=	80 "
A Quarter-quarter Section	= 1 "	×	$\frac{1}{16}$ "	=	40 "

The adjoining diagram represents a township divided into sections, which are numbered commencing at the N.E. corner, and running W. in the North tier, E. in the second, etc.

Each section is divided into 4 quarter sections, called N.E., S.E., N.W., and S.W. quarters, each containing 160 acres.

Thus, S.E. $\frac{1}{4}$, sec. 16, T. 39 N., R. 14 E. 3d., P. M., is read, "Southeast quarter of section 16, tier 39 north, range 14 east of third principal meridian."

A Township.

N

W — E

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

S

1. A colony of 224 persons took up a township of land and divided it equally among them; how many acres did each receive?

2. What part of a section did each colonist receive, and what did it cost him, at $1.25 an acre?

3. What will it cost to enclose a quarter section of land with a fence 5 rails high, at $2 for every 3 rods?

4. If you pay $1.75 an acre for a half section of land, and sell a quarter section for $2.50, how much will your remaining quarter cost you?

5. A company of speculators bought a township at $1.50 an acre; they sold 10 sections at $2.25 an acre, 15 sections at $3.50, 8 sections at $4, and the balance at $5 an acre; how much did they sell at $5, and what was the gain on the whole? Explain by diagram.

896. Table of Pounds Avoirdupois in a Bushel, as fixed by Law in the several States named.

It is becoming common in some parts of this country and in England to sell grain and other produce by *weight* and not by *measure,* a much more equitable system than that which has long prevailed.

COMMODITIES.	*California.*	*Connecticut.*	*Delaware.*	*Illinois.*	*Indiana.*	*Iowa.*	*Kentucky.*	*Louisiana.*	*Massachusetts.*	*Michigan.*	*Minnesota.*	*Missouri.*	*New Jersey.*	*New York.*	*North Carolina.*	*Ohio.*	*Oregon.*	*Pennsylvania.*	*Vermont.*	*Washington T.*	*Wisconsin.*
Wheat	60	56	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60
Indian Corn in ear	52	56	56	52	56	50	56	56	56	56	56	52	56	58	54	56	56	56	56	56	56
Oats	32	28		32	32	35	$33\frac{1}{3}$	32	30	32	32	35	30	32		32	34	32	32	36	32
Barley	50			48	48	48	48	32	46	48	48	48	48	48	48	48	46	47	46	45	48
Buckwheat	40	45		40	50	52	52		46	42	42	52	50	48	50		42	48	46	42	42
Rye	54	56		54	56	56	56	32	56	56	56	56	56	56		56	56	56	56	56	56
Clover Seed				60	60	60	60			60	60	60	64	60		60	60			60	60
Timothy Seed				45	45	45	45					45		45							46
Blue Grass Seed				14	14	14	14					14									56
Flax Seed				56	56	56	56					56	55	55		56					
Hemp Seed				44	44	44	44					44									

NOTES.—1. Beans, peas, and potatoes are usually estimated at 60 lb. to the bu., but the laws of N. Y. make 62 lb. of beans to a bushel.

In Illinois, 50 lb. of common salt, or 55 lb. fine, are 1 bu. In N. J., 56 lb. of salt are 1 bu. In Ind., Ky., and Iowa, 50 lb. are 1 bu. In Penn., 80 lb. coarse, 70 lb. ground, or 62 lb. fine salt are 1 bu.

In Maine, 30 lb. oats, and 64 lb. of beets or of ruta-baga turnips are 1 bu.

In New Hampshire, 30 lb. of oats are 1 bu.

2. Grain, seeds, and small fruit are sold by the bushel, *stricken* or *level measure.*

Large fruit, potatoes, and all coarse vegetables by *heaped measure.*

897. Capacity Measures, estimated by Avoirdupois Weight.

$62\frac{1}{2}$ pounds, or 1000 oz.	=	1 cubic foot of water.
100 pounds	=	1 keg of nails.
100 pounds	=	1 quintal of dry fish.
196 pounds	=	1 barrel of flour.
200 pounds	=	1 barrel of fish, beef, or pork.
280 pounds	=	1 barrel of salt.
240 pounds	=	1 cask of lime.

Apothecaries' Fluid Measure.

898. Apothecaries' Fluid Measure is used in mixing liquid medicines.

60 minims, or drops (♏ or *gtt.*)	= 1 fluid drachm, . .	*f*ʒ.
8 fluid drachms	= 1 fluid ounce, . . .	*f*℥.
16 fluid ounces	= 1 pint.	*O.*
8 pints	= 1 gallon, ...	*Cong.*

NOTE.—*Gtt.* for *guttæ*, Latin, signifying drops; *O*, for *octarius*, Latin for one-eighth; and *Cong.*, *congiarium*, Latin for gallon.

899. The following approximate measures, though not strictly accurate, are often useful in practical life:

45 drops of water, or a *common teaspoonful*	= 1 fluid drachm.
A common tablespoonful	= $\frac{1}{2}$ fluid ounce.
A small teacupful, or 1 gill	= 4 fluid ounces.
A pint of pure water	= 1 pound.
4 tablespoonfuls, or a wine-glass	= $\frac{1}{2}$ gill.
A common-sized tumbler	= $\frac{1}{2}$ pint.
4 teaspoonfuls	= 1 tablespoonful.

900. The following linear units are often used:

$1\frac{1}{6}$ statute miles	= 1 geographic or nautical mile.
60 geographic, or $69\frac{1}{2}$ statute mi., nearly,	= 1 degree on the equator.
360 degrees	= 1 circumference of the earth.

A *knot*, used for measuring distances at sea, is equivalent to a nautica mile.

4 inches	= 1 hand, for measuring the height of horses.
9 inches	= 1 span.
18 inches	= 1 cubit.
6 feet	= 1 fathom, for measuring depths at sea.
120 fathoms	= 1 cable's length.
3.3 feet	= 1 pace, for measuring approximate distances.
5 paces	= 1 rod, " " "

LEAP YEARS.

901. A **Solar Day** is the *time* between the departure of the sun from a given meridian and his return to it.

902. A **Mean Solar Day** is the average length of all the solar days in the year, and is divided into 24 *hours*, the first 12 being designated by A. M., the last by P. M.

NOTE.—A. M. is an abbreviation of *ante meridies, before midday;* P. M., of *post meridies, after midday.*

903. A **Solar Year** is the *time* in which the earth, starting from one of the *tropics* or *equinoctial* points, revolves around the sun, and returns to the same point. It is thence called the *tropical* year, and is equal to 365 da. 5 hr. 48 min. 49.7 sec.

NOTES.—1. The excess of the *solar* above the *common* year is 6 hours or $\frac{1}{4}$ of a day, nearly; hence, in 4 years it amounts to about 1 day. To provide for this excess, 1 day is added to the month of February every 4th year, which is called *Leap* year, because it *leaps over* the limit, or runs on 1 day more than a common year.

2. Every year that is exactly divisible by 4, except centennial years, is a *leap year;* the others are *common* years. Thus, 1876, '80, etc., were leap years; 1879, '81, were common. Every centennial year exactly divisible by 400 is a *leap* year; the other centennial years are *common.* Thus, 1600 and 2000 are *leap* years; 1700, 1800, and 1900 are *common.*

ANNUAL INTEREST.

904. Annual Interest is interest that is payable every year.

905. To Compute *Annual Interest*, when the Principal, Rate, and Time are given.

1. What is the amount due on a note of $500, at 6%, in 3 yr. with interest payable annually?

SOLUTION.

Principal	$500.00
Interest for 1 year is $30; for 3 years it is $30 × 3, or	90.00
Interest on 1st annual interest for 2 yr. is	3.60
" 2d " " " 1 " is	1.80
The amount is	$595.40

RULE.—*Find the interest on the principal for the given time and rate; also find the simple legal int. on each annual int. for the time it has remained unpaid.*

The sum of the principal and its int., with the int. on the unpaid annual interests, will be the amount.

NOTE.—When notes are made payable "with interest annually," simple interest can be collected, in most of the States, on the annual interest after it becomes due. This is according to the contract, and is an act of justice to the creditor, to compensate him for the damage he suffers by not receiving his money when due.

2. What is the amount of a note of $1500, payable in 4 yr. 3 mo. 10 da., with int. annually at 5%?

906. Connecticut Rule for Partial Payments.

I. When the first payment is a year or more from the time the interest commenced:

Find the amount of the principal to that time. If the payment equals or exceeds the interest due, subtract it from the amount thus found, and considering the remainder a new principal, proceed as before.

II. When a payment is made before a year's interest has accrued:

Find the amount of the principal for 1 year; also, if the payment equals or exceeds the interest due, find its amount from the time it was made to the end of the year; then subtract this amount from the amount of the principal, and treat the remainder as a new principal.

III. If the payment be less than the interest:

Subtract the payment only from the amount of the principal thus found, and proceed as before.

$650. NEW HAVEN, *April* 12, 1878.

1. On demand, I promise to pay to the order of George Selden, six hundred fifty dollars, with interest, value received.

THOMAS SAWYER.

Indorsements:—May 1, 1879, rec'd $116.20. Feb. 10, 1880, rec'd $61.50. Dec. 12, 1880, rec'd $12.10. June 20, 1881, rec'd $110. What was due Oct. 21, 1881?

SOLUTION.

Principal, dated April 12, 1878	$650.00
Interest to first payment, May 1, 1879 (1 yr. 19 da.)	41.06
Amount, May 1, '79	691.06
First payment, May 1, '79	116.20
Remainder, or New Principal, May 1, '79	574.86
Interest to May 1, '80, or 1 yr. (2d payment being short of 1 yr.)	34.49
Amount, May 1, '80	609.35
Amount of second payment to May 1, '80 (2 mo. 20 da.)	62.32
Remainder, or New Principal, May 1, '80	547.03
Amount, May 1, '81 (1 yr.)	579.86
Third payment (being less than interest due) draws no interest	12.10
Remainder, or New Principal, May 1, '81	567.76
Amount, Oct. 21, '81 (5 mo. 20 da.)	583.85
Amount of last payment to settlement (4 mo. 1 da.)	112.22
Balance due Oct. 21, '81	$471.63

NOTE.—For additional exercises in the Connecticut Rule, the student is referred to Art. **554**.

907. Vermont Rule for Partial Payments on Notes bearing Annual Interest.

I. When payments are made on notes bearing interest, such payments shall be applied,

" First, to liquidate the interest that has accrued at the time of such payments; and secondly, to the extinguishment of the principal."

II. When notes are made " with interest annually,"

The annual interests which remain unpaid shall be subject to simple interest from the time they become due to the time of settlement.

III. If payments have been made in any year, reckoning from the time such annual interest began to accrue, the amount of such payments at the end of such year, with interest thereon from the time of payment, shall be applied:

"First, to liquidate the simple interest that has accrued from the unpaid annual interests.

"Secondly, To liquidate the annual interests that have become due.

"Thirdly, To the extinguishment of the principal.

$1500. BURLINGTON, *Feb.* 1, 1877.

1. On demand, I promise to pay to the order of Jared Sparks, fifteen hundred dollars, with interest annually at 6%, value received. AUGUSTUS WARREN.

Indorsements:—Aug. 1, 1877, received $160; Nov. 1, 1880, $250. Required the amount due Feb. 1, 1881.

SOLUTION.

Principal		$1500.00
Annual interest to Feb. 1, '78 (1 yr. at 6%)		90.00
Amount		1590.00
First payment, Aug. 1, '77	$160.00	
Interest on same to Feb. 1, '78 (6 mos.)	4.80	164.80
Remainder, or New Principal		1425.20
Annual interest on same from Feb. 1, '78, to Feb. 1, '81 (3 yr.)		256.53
Interest on first annual interest from Feb. 1, '79 (2 yr.)	$10.26	
Interest on second annual int. from Feb. 1, '80 (1 yr.)	5.13	15.39
Amount		1697.12
Second payment, Nov. 1, '80	$250.00	
Interest on same to Feb. 1, '81 (3 mo.)	3.75	253.75
Balance due Feb. 1, '81		$1443.37

908. New Hampshire Rule for Partial Payments.

I. When on notes drawing annual interest,

Find the interest upon the principal from date of note to the end of the year next after the first payment, also upon each annual interest to the same date.

II. If the first payt. be larger than the sum of interests due,

Find the int. on such payt. from the time it was made to end of the year, and deduct the sum of payt. and int. from the amount of principal and interests.

III. If less than the *annual* interests accruing,

Deduct the payment without interest from the sum of annual and simple interest, and upon the balance of such interest cast the simple interest to the time of the next payment.

IV. If less than the *simple* interest due,

Deduct it from the simple interest, and add the balance without interest to the other interests due when the next payment is made.

*Proceed thus to the end of the year after the last payment, being careful to carry forward all interest unpaid at the end of each year.**

1. A agrees to pay B $2000 in 6 yr. from Jan. 1, 1870, with interest annually. On July 1, 1872, a payment of $500 was made; and Oct. 1, 1873, $50. What was due Jan. 1, 1876?

SOLUTION.

Principal		$2000.00
First year's interest	$120.00	
2 yr. simple int. thereon	14.40	134.40
Second year's interest	120.00	
1 yr. simple int. thereon	7.20	127.20
Third year's interest		120.00
		$2381.60
First payment, July 1, 1872	$500.00	
Int. thereon from July 1, '72, to Jan. 1, '73	15.00	515.00
Balance of principal		$1866.60
Interest on same for fourth year		111.99 +
Second payt. (less than the int. accruing during the year)		50.00
Balance of fourth year's interest unpaid		$61.99 +
Annual interest on balance of principal for fifth year		111.99 +
" " " " " sixth "		111.99 +
Simple int. on unpaid bal. of fourth year's int. for 2 yr.		7.43 +
Simple interest on fifth year's interest for one year		6.71 +
Balance of principal		1866.60
Amount due January 1, 1876		$2166.71

* Abstract of N. H. Court Rule, Report of Hon. C. A. Downs, State Superintendent.

909. The Twelve Per Cent Method of Computing Interest.

1. Find the int. of \$275.20, for 3 yr. 4 mo. 10 da., at 12%.

SOLUTION.

Int. of \$275.20,	1 yr. at 1% = \$275.20 × .01 = \$2.752.	
" "	1 yr. at 12% = \$2.752 × 12 = \$33.024.	
" "	1 mo. at 12% = 12 mo. at 1% = \$2.752. (Art. **578.**)	
" "	3 yr. at 12% = \$33.024 × 3.	\$99.072
" "	4 mo. ($\frac{1}{3}$ of 1 yr.) = \$33.024 ÷ 3.	11.008
" "	10 da. ($\frac{1}{3}$ of mo.) = 2.752 ÷ 3.	.917
Hence, the		*Ans.* \$110.997

RULE.—For 1 year: *Find the interest on the principal at 1%, by moving the decimal point two places to the left, and multiplying the result by 12.*

For 2 or more years: *Multiply the interest for 1 year by the number of years.*

For months and days: *Proceed as in Art.* **537.**

AVERAGE OF MIXTURES.

910. To find the *Average Value* of a Mixture, when the Quantity and Price of each Article are given.

1. A man mixed 45 bu. oats worth 25 cts. a bushel with 38 bu. corn at 50 cts., and 56 bu. rye at 60 cts.; what was the mixture worth a bushel?

SOLUTION.—The whole number of bushels mixed is 45 + 38 + 56 = 139. The whole cost of the mixture is \$11.25 + \$19.00 + \$33.60 = \$63.85.

Now, \$63.85 ÷ 139 = \$0.46 nearly, the price of 1 bushel of the mixture. Hence, the

OPERATION.

\$0.25 × 45	=	\$11.25
0.50 × 38	=	19.00
0.60 × 56	=	33.60
139)		\$63.85
	Ans.	\$0.46.

RULE.—*Divide the value of the whole mixture by the sum of the articles mixed.*

NOTES.—1. If an article costs nothing, as water, its value is 0; but the *quantity* used must be added to the other articles.

2. The process of finding the average value of mixtures is often called Alligation.

2. A grocer had three kinds of sugar, worth 6, 8, and 12 cents per pound; he mixed 112 lb. of the first, 150 lb. of the second, and 175 of the third together. What was the mixture worth per pound?

911. To find the *Proportional Parts* of a Mixture, the Mean Price and the Price of each Article being given.

3. A grocer desired to mix 4 kinds of tea, worth 5s., 8s., 11s., and 12s. a pound, so that the mixture should be worth 9s. a pound; in what proportion must they be taken?

OPERATION.

	Col.	1.	2.	3.
9s.	5s.	$\frac{1}{4}$		3
	8s.		1	2
	11s.		$\frac{1}{2}$	1
	12s.	$\frac{1}{3}$		4

ANALYSIS.—First find how much it takes of each article to gain or lose a *unit* of the *mean price*. Since the mean price is 9s. a pound, 1 lb. at 5s. gains 4s.; hence, to gain 1s. takes $\frac{1}{4}$ lb., which we place in Col. 1. Again, 1 lb. at 12s. loses 3s.; hence, to lose 1s. takes $\frac{1}{3}$ lb., which we place also in Col. 1, opposite the price compared. In like manner, 1 lb. at 8s. is required to gain 1s., while 1 lb. at 11s. loses 2s.; hence, to lose 1s. takes $\frac{1}{2}$ lb. We place these results in Col. 2, opposite their prices. Reducing the fractions in Col. 1 and 2 to a common denominator separately, the numerators are the proportional parts required. Hence, the

RULE.—I. *Write the prices of the articles in a column in their order, with the mean price on the left.*

II. *Take them in pairs, one less and the other greater than the mean price, find how much is required of each article to* GAIN *or* LOSE *a unit of the mean price, and set the results in Col. 1, opposite to its price. Compare the other couplet in like manner, setting the results in Col. 2.*

III. *Finally, reduce the numbers in each column separately to a common denominator; the numerators will be the proportional parts required.*

NOTES.—1. If there are *three* articles, compare the *price* of the one which is *greater* or *less* than the *mean* price with each of the others, and take the sum of the two numbers opposite this price.

2. The *reason* for considering the articles in *pairs*, one *above*, and the other *below* the mean price, is that the *loss* on one may be counterbalanced by the *gain* on another.

3. When the given prices are integers, the same results are readily found by taking the difference between the price of each article and the mean price, and placing it opposite the price with which it is compared, as

4. How much coffee at 9, 11, and 14 cents a pound, will form a mixture worth 12 cents a pound?

5. How much ginger at 15, 18, 21, and 22 cents a pound, will form a mixture worth 19 cents a pound?

912. To find the *Other Quantities* when the Mean Price of the Mixture and the Quantity of one of the Articles are given.

6. How many pounds of starch worth 11 and 15 cents a pound, must be mixed with 16 lb. at 10 cents, so that the mixture may be worth 13 cents a pound.

ANALYSIS.—If neither article were limited, the proportional parts would be 2, 2, and 5. (Art. **911**, N. 1.) But the quantity at 10 cts. is limited to 16 lb., and its proportional part is 2 lb. Now, $16 \div 2 = 8$. Therefore, multiplying each of the proportional parts by 8 gives 16; 16 and 40 lb. the mixture required. Hence, the

OPERATION.

		Col. 1.	2.
	10	2	2
13	11	2	2
	15	3 + 2	5

RULE.—*Find the proportional parts as if the quantity of neither article were limited.* (Art. **911.**)

Divide the limited quantity by its proportional part, and multiply each part found by this quotient; the product will be the quantity required.

NOTE.—When the quantities of *two* or *more* articles are given, find the *average* value of them, and considering their *sum* as one quantity, proceed as above.

7. How much corn at 45, 56, and 65 cents per bushel, must be mixed with 25 bu. of oats at 40 cents, so that the mixture may be worth 50 cents a bushel?

ANSWERS.

Article 50.

12. 8879.
13. 7889.
14. 7979.
15. 9798.
16. $6178.
17. 966 mi.
18. 987 Acres.

Arts. 55, 59.

2. 12054 yd.
3. 14792 rods.
4. 19747 wk.
5. 28143 lb.
7. 415.034.
8. 114.634.
10. $88.967.
11. $104.721.
12. $100.84.
13. $93.833.

Art. 60.

1. $6821.
2. $2324.
3. $4900.
4. $1444.
5. 503 trees.
6. 73 yr.
7. $1648.
8. $34950.
9. $33700.
10. 565.
11. 742.
12. 1530.
13. 1779.
14. 1597.
15. 1757.
16. 2379.
17. 2619.
18. 1020.
19. 1418.
20. 1836.
21. 1783.
22. 2604.
23. 7512.
24. 21241.
25. 10562.
26. 2742.
27. 2355.
28. $627.
29. 630 lb.
30. $3789.
31. $3582.
32. $1323.
33. $279075.
34. $595522.
35. $295306.
36. $1606895.
37. 7448208453.
38. $3068, farm.
    $6136, all.
39. $16646.
40. $37650.
41. 1727.
42. $8475.
43. 14,507,407.
44. 7,597,197.
45. 17,364,111.
46. 8,919,371.
47. 1,767,697.
48. 50,155,783.

Art. 69.

2. 3232.
3. 3244.
4. 3424.
5. 3525.
6. 3213.
7. 501 Acres.
8. $1134.
9. 412292.

Arts. 70, 71.

2. 10292.
3. 10083.
4. 27886.
5. 34339.
8. 216.9.
9. 182.19.
10. $242.19.
11. $3684.939.

Art. 72.

1. 113 yd
2. $221.
3. 189 gents.
4. 1003 bu.
5. 374 bu.
6. $1989.
7. $479.
8. $1659.
9. $3023.
10. $1763.
11. $3747.
12. 1825.
13. 2600.
14. 3085.
15. 1306.
16. 4098.
17. 1108.
18. 4531.
19. 14520.
20. 24622.
21. 125028.
22. 64303.
23. 224066.
24. 103875.
25. 420486.
26. $16014.
27. $1315.
28. $5385.
29. 708.
30. 942.
31. 2280.
32. 5583.
33. 7271.
34. 84841.
35. 5482.
36. 14935.
37. 985.
38. 7065.
39. $301.12.
40. $5072.35.

Art. 74.

1. 439.25.
2. $1291.
3. 83412.7.
4. $72320.
5. $985.25.
6. 146 trees.
7. $1090.
8. 12520 bu.
9. $1910.89.
10. $5491.
11. $5250.
12. 627067.
13. $21422.
14. 22225.
15. 16,014,400.
16. 184,815,000,
    000.
17. 1486.75.
18. 5 times.
19. 61483.95.
20. 26973.
21. 34059.5.
22. 1912, B's.
    4482, C's.
23. 5986.
24. 33086330.

Arts. 84, 85.

2. 1750 lb.

3. 4410 sheep.
4. 2022 lb.
5. 3000 ft.
6. 4345 yd.
7. 12768 bu.
8. 20712 in.
10. 1924.5.-
11. 402.12.
12. 434.96.
13. 60.221.
14. 787.14.
15. $26116.02.
16. $3381.19.
17. $50981.28.
18. $74241.84.
19. $7264,854.
20. $138.24.
21. $1455.78.
22. $7.68.
23. $7,857.
24. $60.
26. $3000.
27. $42120.
28. $6000.
29. $45.36.
30. $21150.

Art. 87.

2. 1445.
3. $23646.
4. $173.34.
5. 996.84 lb.
6. 234588 yd.

Art. 88.

1. 16425d.
2. 91350 lb.
3. $8991.
4. $16884.
5. 68520 ft.
6. 1564 sheep.
7. $6000.
8. $17920.
9. 17945 bu.
10. 48000 A.
11. 206952.
12. 98982.
13. 204336.
14. 368109.
15. $1849.65.
16. $3201.44.
17. 438480.
18. $1443.099.
19. 31,968,868.
20. 48,053,208.
21. 34,628,175.
22. 65,404,110.
23. 639,756.
24. 1560.975.
25. 3,071,926.
26. 3,007,368.
27. 24,631,008.
28. 35,497,655.
29. 42,546,240.
30. 62,355,319.
31. 849,126,321.
32. 1,219,641,537.
33. 1,988,907,892.
34. 2,758,104,145.
35. $576.
36. $3744.
37. 6930 mo.
38. 58050 ds.
39. $632.
40. $149.
41. $549.25.
42. $488.25.
43. $42255.
44. $1895.
45. 168750 lb.
46. $57649, cost.
    $3367, dif.
47. $534842.
48. $618.
49. $22.
50. $56.50.
51. 1624 trees.
52. 1584 pupils.
53. 62234.
54. 20,345,400.
55. 1776 bu.
56. 368,640.
57. 283,410.
58. 270,592.
59. 111,168.
60. 92,538.
61. 493,480.

Art. 89.

2. $2295.
3. $888.
4. $684.
5. 8610 miles.
6. $4950.
7. $312.
8. 25760 bu.
9. $16128.
10. $91080.

Art. 91.

6. 46800 lb.
7. 809600 pp.
8. 476,000.
9. 534,860,000.
10. 1,204,670,800,000.
11. 26,900,785,000,000.
12. 890,634,570,000,000.
13. 946,030,506,800,000.
14. 3,840,000.
15. 10,940,000.
16. 2,075,994,000.
17. 390,677,500,000.
18. 372,000.
19. 11,840,000.
20. 373,520,000.
21. 3,603,200,000.
22. 55,447,000,000.
23. 37,800,000,000.
24. 25,800,000,000.
25. 4,059,360,000.
26. 14,760,000,000.
27. 6,204,000,000.
28. 1,672,650,000,000.
29. 1,075,635,900,000.
30. 450,230,874,000.
31. 6,980,161,370,000.
32. 834,271,780,000.
33. 779,934,000,000.

Art. 93.

17. 103 cts.
18. 3600 soldiers.
19. $18200.
20. 452 miles.
21. $504.
22. $1492.
23. $4632.
24. $234.
25. 8288d. s. q.
26. 1460 mi.
27. 2,015,028.
28. 8,496,120.
29. 404,444,040.
30. 6,342,737,821.
31. 351,039,462,230.

Arts. 107, 110.

3. 1275.
4. 1173.
5. 2468.
6. 1317.
7. 11449.

8. 11155.
10. 378.
11. 246.
12. 427.
13. 1234.
14. 546.
15. 1234.
16. 335.
17. 349.
19. $913\frac{2}{5}$.
20. $661\frac{5}{6}$.
21. $820\frac{7}{8}$.
22. $4639\frac{5}{9}$.
23. $15290\frac{3}{5}$.
24. $20588\frac{2}{4}$.
25. $8731\frac{1}{6}$.
26. $9124\frac{6}{7}$.
27. $120421\frac{5}{7}$.
28. $71410\frac{3}{6}$.
29. $96043\frac{1}{8}$.
30. $87105\frac{7}{9}$.
31. $8240\frac{4}{8}$.
32. $8369\frac{6}{9}$.
33. 56234.
34. 1533.
37. 640.87.
38. 80.666.
39. $2462\frac{1}{2}$ yd.
40. $5.42.

Art. 112.

2. $2862\frac{15}{16}$.
3. $12431\frac{4}{28}$.
4. $13967\frac{1}{37}$.
5. $29493\frac{5}{19}$.
6. $1090\frac{5}{36}$.
7. $1231\frac{20}{37}$.
8. $192.30\frac{16}{48}$.
9. $24218\frac{23}{29}$.
10. $\$1.20\frac{24}{38}$.
11. $\$1.27\frac{16}{47}$.
12. $\$1.354\frac{21}{56}$.
13. $\$2.007\frac{18}{48}$.

Art. 115.

1. 61 shares.
2. $31\frac{28}{52}$ yr.
3. $48\frac{41}{63}$ hhd.
4. $43\frac{55}{75}$ mi.
5. 75 dresses.
6. $51\frac{32}{83}$ m.
7. $50.
8. 73.6 A.
9. $45\frac{75}{93}$ casks.
10. 27 days.
11. $\$31.14\frac{10}{15}$.
12. $\$3.15\frac{14}{18}$.
13. $37.4\frac{9}{21}$.
14. $1.46\frac{9}{33}$.
15. 199.2.
16. $340\frac{22}{23}$.
17. $175\frac{28}{47}$.
18. $.65\frac{28}{32}$.
19. $\$1.36\frac{21}{62}$.
20. $\$1.53\frac{12}{48}$.
21. $84\frac{5}{82}$.
22. $149\frac{36}{65}$.
23. $837\frac{20}{28}$.
24. $1607\frac{35}{45}$.
25. $962\frac{46}{84}$.
26. $1006\frac{83}{92}$.
27. 90 lb.
28. $149\frac{29}{64}$ A.
29. 126 boxes.
30. 288 eggs.
32. $245\frac{116}{254}$.
33. $133\frac{137}{532}$.
34. $963\frac{85}{306}$.
35. $720\frac{306}{521}$.
36. $3011\frac{292}{812}$.
37. $3938\frac{373}{904}$.
38. $6671\frac{250}{638}$.
39. $7318\frac{228}{743}$.
40. $121.93\frac{372}{732}$.
41. $3.1885\frac{617}{765}$.
42. $673.888\frac{724}{879}$.
43. $456.607\frac{246}{1247}$.
44. $2680.52\frac{1470}{3238}$.
45. $2.3631\frac{50637}{56813}$.
46. $5.2558\frac{24235}{75436}$.
47. $5.109\frac{58882}{83247}$.

Arts. 117, 118.

2. 3452 and 31 rem.
3. 672 and 487 rem.
4. 642 and 3544 rem.
5. 73 and 64159 rem.
8. 340 bar.
9. $456.50.
10. 80 bales.
11. $40.
12. $292\frac{68}{171}$ lb.
13. 4160 men.

Art. 121.

1. 14.
2. 12.
4. 100 s. 150 g.
5. 118 B., 155 A.
6. 69 cts.
7. 82 yr.
8. $283.
9. 392 mi.
10. 1 mi.
11. $5243, B.
    $17176, C.
    $23684, all.
12. $621.
13. 730 sch.
14. $20 per A.
    $1868, gain.
15. $\$66.52\frac{424}{563}$.
16. Cows; $6094.
17. 228 A., B.'s.
    114 A., C.'s.
    1710 A., all.
18. 18, sm. No.
    882, gr. No.
19. 406 oxen.
20. 38,818,897, dif.
21. $3780.
22. 83.
23. 105374.
24. 14.
25. 1213.
26. 43.
27. 8117.
28. 723.8.
29. 49312 men.
30. 3082 men.
31. $718.
32. 196 sofas.

Art. 125.

6. 112.
7. 120.
8. $13\frac{5}{7}$.

9. $5\frac{1}{7}$.
10. 13.
11. $22\frac{4}{5}$.
12. $12\frac{41}{64}$.
13. 3.
14. $3\frac{17}{21}$.
15. 15 tons.
17. $1\frac{31}{49}$ bags.
18. $33\frac{3}{4}$ bar.
19. 126 bar.

Art. 143.

2. $5 \times 5 \times 3 \times 3$.
3. $47 \times 2 \times 2 \times 2$.
4. $43 \times 2 \times 2 \times 2$.
5. $3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$.
6. $7 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2$.
7. $199 \times 2 \times 2$.
8. $3 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2$.
9. $7 \times 5 \times 3 \times 3 \times 3$.
10. $19 \times 5 \times 3 \times 3 \times 2 \times 2$.
11. $37 \times 5 \times 5 \times 5 \times 2 \times 2$.
12. $67 \times 43 \times 2 \times 2 \times 2 \times 2$.
13. $6029 \times 2 \times 2 \times 2 \times 2$.
14. $1297 \times 2 \times 2 \times 2$.
15. $5 \times 5 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$.
16. $503 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$.
17. $193 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$.

Art. 144.

19. 2 and 2.
20. 2.
21. 2.
22. 2.
23. 2, 3, and 7.
24. 2, 2, 2, and 3.
25. 2, 2, and 2.
26. 5 and 5.
27. 2 and 2.

Art. 149.

2. 21.
3. 13.
4. 19.
5. 15.
6. 3.
7. 4.
8. 12.
9. 0.
10. 5.

Art. 150.

1. 6.
2. 15.
3. 12.
4. 1.
5. 5.
6. 4.
7. 4.
8. 4.
9. 8.
10. 4 bu.
11. 4 A.
12. 21.
13. 60 ft. wide; 10 l., 2 l., and 15 l.
14. $252, price; 5 h., 9 h., and 11 h.

Art. 157.

2. 240.
3. 12600.
4. 504.
5. 1134.
6. 144.
7. 130645.
8. 533610.
9. 156240.
10. 144.
11. 2520.
12. 262080.
13. 1921506000.
14. 360.
15. 1584 ft.
16. $60.
17. 840 gal.
18. 12 hr.
19. 24 hr.
20. 120 hr.

Art. 182.

2. $\frac{40}{104}$.
3. $\frac{65}{120}$.
4. $\frac{136}{176}$.
5. $\frac{45}{144}$.
6. $\frac{105}{196}$.
7. $\frac{225}{288}$.
8. $\frac{168}{192}$.
9. $\frac{184}{576}$.

Art. 185.

2. $\frac{21}{55}$.
3. $\frac{10}{13}$.
4. $\frac{16}{25}$.
5. $\frac{1}{3}$.
6. $\frac{1}{11}$.
7. $\frac{151}{447}$.
8. $\frac{11}{34}$.
9. $\frac{22}{41}$.
10. $\frac{5}{6}$.
11. $\frac{19}{21}$.
12. $\frac{33}{119}$.
13. $\frac{1}{22}$.
14. $\frac{7}{9}$.
15. $\frac{5}{11}$.
16. $\frac{3}{5}$.
17. $\frac{431}{489}$.

Art. 187.

1. $37\frac{1}{3}$.
2. $44\frac{4}{13}$.
3. 30.
4. $28\frac{1}{6}$.
5. $31\frac{1}{4}$.
6. $5\frac{7}{32}$.
7. $14\frac{13}{55}$.
8. $28\frac{93}{298}$.
9. $22\frac{203}{320}$.
10. $46\frac{1}{14}$.
11. $21\frac{96}{157}$.
12. $19\frac{23}{450}$.
13. $40\frac{168}{235}$.
14. $99\frac{21}{121}$.
15. $13\frac{67}{146}$.
16. $110\frac{91}{128}$.
17. $131\frac{143}{375}$.
18. $1\frac{41}{59}$.
19. $5\frac{49305}{72146}$.
20. $12\frac{146}{1821}$.
21. $60\frac{13}{21}$ rd.
22. $$153\frac{113}{2354}$.

Art. 189.

2. $\frac{59}{3}$.
3. $\frac{214}{9}$.
4. $\frac{968}{15}$.
5. $\frac{913}{3}$.
6. $\frac{225}{5}$.
7. $\frac{4980}{6}$.
8. $\frac{213}{8}$.
9. $\frac{545}{12}$.
10. $\frac{2543}{45}$.
11. $\frac{3626}{5}$.
12. $\frac{576}{8}$.
13. $\frac{11145}{15}$.

Art. 195.

2. $\frac{28}{35}$, $\frac{30}{35}$.
3. $\frac{35}{45}$, $\frac{27}{45}$.
4. $\frac{30}{80}$, $\frac{56}{80}$.
5. $\frac{48}{144}$, $\frac{63}{144}$.
6. $\frac{157500}{283500}$, $\frac{113400}{283500}$, $\frac{19845}{283500}$.
7. $\frac{120528}{326592}$, $\frac{208656}{326592}$, $\frac{96768}{326592}$.

Art. 196.

9. $\frac{24}{60}$, $\frac{25}{60}$, $\frac{16}{60}$.
10. $\frac{264}{616}$, $\frac{385}{616}$, $\frac{234}{616}$.
11. $\frac{45}{108}$, $\frac{32}{108}$, $\frac{15}{108}$.
12. $\frac{735}{840}$, $\frac{600}{840}$, $\frac{672}{840}$, $\frac{490}{840}$.
13. $\frac{189}{210}$, $\frac{180}{210}$, $\frac{168}{210}$, $\frac{84}{210}$.
14. $\frac{672}{1400}$, $\frac{1240}{1400}$, $\frac{945}{1400}$.
15. $\frac{210}{525}$, $\frac{395}{525}$, $\frac{1092}{525}$.
16. $\frac{35}{105}$, $\frac{915}{105}$, $\frac{12}{105}$.
17. $\frac{12}{280}$, $\frac{175}{280}$, $\frac{680}{280}$.
18. $\frac{154}{245}$, $\frac{2590}{245}$, $\frac{180}{245}$.
19. $\frac{972}{132}$, $\frac{189}{132}$, $\frac{100}{132}$.
20. $\frac{117}{189}$, $\frac{85}{189}$, $\frac{63}{189}$.

Art. 201.

3. $1\frac{19}{165}$.
4. $2\frac{653}{728}$.
5. $3\frac{243}{280}$.
6. $1\frac{23}{105}$.
7. $5\frac{1}{4}$.
8. $2\frac{1}{2}$.
9. $2\frac{67}{120}$.
10. $9\frac{1}{2}$.
11. $4\frac{1}{2}$.
12. $4\frac{589}{840}$.
13. $11\frac{19}{42}$.
14. $41\frac{1}{12}$.
15. $15\frac{1}{24}$.
16. $89\frac{169}{360}$.
17. $108\frac{71}{280}$.
18. $44\frac{1629}{2584}$.
19. $1\frac{389}{396}$.
20. $651\frac{1}{60}$.
21. $199\frac{184}{315}$ lb.
22. $289\frac{211}{420}$ m.
23. $553\frac{3}{80}$ yd.
25. $11\frac{5}{6}$.
26. $16\frac{5}{8}$.
27. $33\frac{17}{20}$.
28. $16\frac{13}{24}$.
29 $33\frac{18}{35}$.
30. $37\frac{19}{24}$.
31. $22\frac{2}{3}$.
32. $29\frac{11}{60}$.
33. $51\frac{3}{56}$.
34. $165\frac{53}{56}$ yd.
35. $297\frac{5}{48}$ m.

Art. 204.

3. $\frac{4079}{5250}$.
4. $\frac{16}{165}$.
5. $\frac{491}{1680}$.
6. $2\frac{2}{3}$.
7. $5\frac{3}{8}$.
8. $\frac{8}{35}$.
9. $\frac{8}{77}$.
10. $\frac{1}{3}$.
11. $3\frac{11}{12}$.
12. $5\frac{13}{14}$.
13. $1\frac{2}{5}$.
14. $39\frac{1}{3}$.
15. $11\frac{21}{40}$.
16. $7\frac{4}{5}$.
17. $17\frac{23}{28}$.
18. $317\frac{11}{12}$ rd.
19. $357\frac{7}{40}$ T.
20. $303\frac{3}{8}$ lb.
21. $1316\frac{27}{28}$ bu.
22. $\$15\frac{7}{8}$.
23. $50\frac{37}{48}$ A.
24. $38\frac{41}{80}$ yd.

Art. 206.

3. $1\frac{2}{3}$.
4. $1\frac{3}{25}$.
5. 4.
6. $5\frac{2}{5}$.
7. 7.
8. $28\frac{2}{3}$.
9. $4\frac{4}{21}$.
10. 5.
11. 5.
13. 702.
14. 1988.
15. 4941.
16. 3537.
17. $769\frac{1}{8}$.
18. 27612.

Art. 208.

2. $6\frac{2}{3}$.
3. 15.
4. 45.
5. $17\frac{1}{2}$.
6. 60.
7. 255.
8. 36.
9. 432.
10. $119\frac{1}{6}$.
12. 1178.
13. 3450.
14. 1280.
15. $3411\frac{11}{13}$.
16. 4496.
17. 8113.
18. $10428\frac{11}{12}$.
19. $5611\frac{24}{25}$.
20. $5086\frac{8}{13}$.
21. 43452.
22. $74269\frac{17}{19}$.
23. 91806.

Art. 211.

5. $\frac{1}{3}$.
6. $\frac{3}{7}$.
7. $\frac{21}{55}$.
8. $\frac{2}{7}$.

9. $\frac{3}{56}$.
10. $\frac{3}{35}$.
11. $5\frac{5}{14}$.
12. $11\frac{23}{32}$.
13. $21\frac{3}{25}$.
14. $7\frac{21}{32}$.
15. $\$103\frac{1}{2}$.

Art. 213.

1. $\$12\frac{21}{32}$.
2. 136 cts.
3. $112\frac{1}{2}$ cts.
4. 235 cts.
5. $\$38\frac{8}{9}$.
6. $33\frac{1}{3}$ cts.
7. $292\frac{1}{2}$ cts.
8. $344\frac{1}{4}$ cts.
9. $\$57\frac{13}{15}$.
10. 630 cts.
11. $\$16\frac{1}{2}$.
12. $\$4\frac{3}{8}$.
13. $\$3\frac{2}{3}$.
14. $\$5\frac{37}{40}$.
15. $\$5\frac{15}{28}$.
16. $1237\frac{1}{2}$ cts.
17. $406\frac{1}{4}$ cts.
18. $300\frac{5}{8}$ cts.
19. $806\frac{1}{4}$ cts.
20. \$7.
21. $\$10\frac{19}{24}$.
22. $273\frac{3}{4}$.
23. $\$3\frac{9}{64}$.
24. $\$41\frac{33}{40}$.
25. $621\frac{9}{16}$ cts.
26. $\$83\frac{5}{12}$.
27. $391\frac{3}{8}$ cts.
28. $\$133\frac{1}{2}$.
29. $\$65\frac{13}{16}$.
30. $\$615\frac{5}{32}$.
31. $743\frac{3}{4}$ m.
32. 2310.
33. $32\frac{2}{5}$.
34. $197\frac{5}{9}$.
35. $868\frac{1}{2}$.
36. $\frac{7565}{100901}$.
37. $\frac{147}{967}$.
38. $8\frac{67}{75}$.
39. $\frac{2}{9}$.
40. 80.
41. $156\frac{2031}{2068}$.
42. $109\frac{13}{128}$.
43. $\frac{522}{1225}$.
44. $6\frac{69}{160}$.

Art. 215.

2. $\frac{17}{72}$.
3. $\frac{6}{31}$.
4. $\frac{73}{756}$.
5. $\frac{3}{74}$.
6. $\frac{109}{3150}$.
7. $\frac{171}{1850}$.
8. $\frac{7}{94}$.
9. $\frac{29}{775}$.
10. $\frac{239}{6480}$.
11. $\frac{191}{2394}$.
12. $\frac{64}{5625}$.
13. $\frac{4}{91}$.
15. $\frac{10}{21}$ T.; $\frac{1}{6}$ part.
16. $\$\frac{13}{20}$.

Art. 217.

2. 294.
3. 432.
4. 576.
5. 1350.
6. 48.
7. $17\frac{1}{9}$.
8. 1344.
9. 804.
10. 11045.
11. $46\frac{1}{4}$ yd.
12. 144 d.
14. $9\frac{3}{5}$.
15. 18.
16. $411\frac{3}{7}$.
17. $816\frac{24}{25}$.
18. $12\frac{14}{25}$.
19. $22\frac{6}{7}$.
20. 480 sh.
21. $56\frac{12}{13}$ yd.
22. $34\frac{2}{9}$ cloaks.
23. 52 c. 3 rem.
24. $30\frac{2}{17}$ hr.

Art. 220.

4. $4\frac{2}{3}$.
5. 20.
6. $1\frac{863}{1096}$.
7. $\frac{4}{405}$.

Art. 221.

1. $5\frac{5}{13}$ lb.
2. $10\frac{5}{13}$ C.
3. $8\frac{27}{116}$ bar.
4. $\$8\frac{5}{124}$.
5. 7 cts.
6. $9\frac{44}{189}$ s.
7. $\$1\frac{73}{121}$.
8. \$6.
9. $\$2\frac{623}{626}$.
10. $11\frac{39}{44}$ T.
11. $87\frac{81}{137}$ sacks.
12. $157\frac{61}{127}$ bales.
13. $\frac{16}{325}$.
14. $1\frac{8}{19}$.
15. $\frac{38}{1705}$.
16. $\frac{3}{5}$.
17. $\frac{24}{851}$.
18. $7\frac{61}{112}$.
19. $17\frac{1}{7}$.
20. $\frac{2}{441}$.
21. $\frac{128}{1575}$.
22. $5\frac{29}{34}$.
23. $\frac{1}{120}$.
24. $\frac{507}{512}$.
25. $\frac{19}{52}$.
26. $\frac{11}{567}$.
27. $1\frac{77}{3875}$.
28. $\frac{1909}{1984}$.
29. $\frac{1058}{4615}$.

Art. 222.

3. $1\frac{5}{7}$.
4. $\frac{9}{10}$.
5. $1\frac{5}{11}$.
6. $2\frac{1}{220}$.
7. $\frac{11}{12}$.
8. $\frac{41}{54}$.
9. $1\frac{1}{3}$.
10. $1\frac{7}{15}$.
11. $\frac{163}{206}$.
12. $313\frac{3}{4}$.
13. $\frac{805}{837}$.
14. $\frac{5}{3332}$.
15. $2\frac{58}{483}$.
16. $\frac{61}{110}$.
17. $1\frac{67}{80}$.
18. $1\frac{1}{10}$.
19. $\frac{451}{648}$.
20. $326\frac{143}{644}$.

Art. 226.

3. $\frac{1}{45}$.
4. $\frac{7}{496}$.
5. $\frac{11}{1032}$.
6. $\frac{19}{145}$.
7. $\frac{1}{140}$.
8. $\frac{7}{2600}$.
9. $\frac{906}{1579}$.
10. $\frac{5}{36}$.
11. $\frac{20}{477}$.
12. $\frac{2}{3}$.
13. $\frac{10}{11}$.
14. $\frac{113}{280}$.
15. $\frac{17}{23}$ A.
16. $\frac{30}{37}$.
17. $\frac{27}{38}$.
18. $\frac{65}{111}$.
19. $\frac{3}{8}$.
20. $\frac{3515}{5372}$.
21. $\frac{3}{20}$.

Art. 228.

2. 64.
3. 70.
4. 135.
5. 160.
6. 165.
7. $289\frac{1}{8}$.
8. $844\frac{2}{7}$.
9. $1249\frac{5}{11}$.
10. \$14560.
11. \$7080.
12. 315 s.
13. \$455.

Art. 230.

1. $\frac{117}{243}$.
2. $\frac{729}{1241}$.
3. $2\times3\times3\times3\times3\times3$.
   $2\times2\times2\times2\times2\times2\times3\times3$.
   $2\times2\times3\times3\times3\times3\times3$.
4. 200, **l. c. d.**
5. $6\frac{19}{24}$.
6. $4\frac{23}{24}$.
7. $101\frac{37}{56}$.
8. $\frac{11}{48}$ left.
9. $\frac{9}{20}$.
10. $411\frac{1}{4}$ m., both go.
    $40\frac{1}{4}$ m., dif.
11. $\$48\frac{1}{4}$.
12. $11\frac{1}{2}$.
13. $13\frac{3}{8}$.
14. $17\frac{7}{18}$.
15. 32.
16. $22\frac{7}{8}$.
17. $39\frac{5}{14}$.
18. $134\frac{5}{12}$.
19. $249\frac{3}{20}$.
20. $\$20\frac{5}{6}$.
21. $\$130\frac{23}{40}$.
22. $\$29056\frac{1}{4}$.
23. $112\frac{37}{62}$.
24. $\frac{35}{333}$.
25. $5\frac{1}{34}$.
26. $\$12521\frac{17}{23}$.
27. $\$1852\frac{16}{17}$.
28. $\$3212\frac{153}{160}$.
29. $\$2681\frac{1}{5}$.
30. $15\frac{2}{3}$ lots.
31. $\$32\frac{1}{10}$ lost.
32. $11\frac{4}{7}$ books.
33. $\$82\frac{7}{8}$.
34. $\$75\frac{9}{10}$.
35. 305 eggs.
36. $12250.
37. $26752.
38. $1000.
39. 64 ft.
40. 75 ft.
41. 102 marbles.
42. $\$112\frac{1}{2}$.
43. 108 ds. C.
44. 15360 men.
45. 100.
46. 100.
47. 38.
48. $13\frac{1}{3}$.
49. $\frac{25}{29}$.
50. $3\frac{21}{25}$.
51. $\$231\frac{1}{4}$.
52. $13\frac{1}{3}$ oz.
53. $3808.
54. $4750.

Art. 245.

2. .2000, 2.0004, 7.0080.
3. .200, .060, .008.
4. .0300, .1250, .7000, .2362.
5. .2600, .2750, .0236, .2060.
6. .0450, .6100, .0035, .1080.

Art. 247.

3. $\frac{7}{25}$.
4. $\frac{14}{25}$.
5. $\frac{1}{8}$.
6. $\frac{3}{8}$.
7. $\frac{1}{20}$.
8. $\frac{1}{125}$.
9. $\frac{5}{8}$.
10. $\frac{5}{16}$.
11. $\frac{1}{200}$.
12. $\frac{3}{5000}$.
13. $\frac{1}{6}$.
14. $\frac{31}{125}$.
15. $\frac{410007}{10000000}$.
16. $\frac{1}{5000000}$.
17. $\frac{33}{4000}$.
18. $\frac{17}{18}$.

Art. 249.

2. .25.
3. .75.
4. .625.
5. .8.
6. .125.
7. .375.
8. .75.
9. .875.
10. .45.
11. .35.
12. .4.
13. .65.
14. .42.
15. .032.
16. .09375.
17. $.331\frac{1}{29}$.
18. 75.6.
19. 89.75.
20. 39.625.
21. 65.125.
22. 8.075.
23. .82.
24. 27.8125.
25. 93.1875.
26. $.66666\frac{2}{3}$.
27. $.33333\frac{1}{3}$.
28. $.44444\frac{4}{9}$.
29. $.16279\frac{3}{43}$.
30. $.13666\frac{2}{3}$.

Art. 254.

2. $182.731.
3. 238.4213.
4. 1072.16387.
5. 386.5124.
6. 133.2287.
7. $411.125.
8. $728.

Art. 256.

3. $2.625.
4. $1743.25.
5. $5.75.
6. $1305.90.
7. $170.675.
8. $522.65.
9. 806.41 m.
   116.09 m.

Arts. 259, 260.

3. .135918.
4. 16.141752.
5. 27.478884.
6. 441.2226144.
7. 812.475.
8. 5151.
9. 164.0625.
10. 1503.4375.
11. 47.0231027.

12. .1719375.
13. 3675.
14. 17.9953125.
15. .00187441562.
16. 2374.2125.
17. 14647.5 cts.
18. 53200 cts.
19. $3803.625.
21. 46384.2.
22. 6423.02.
23. 25460.
24. 3004.

Art. 263.

4. .9128+.
5. .9408+.
6. .008911+.
7. .09.
8. 10.
9. .000102+.
10. 3.4542+.
11. 846.37105+.
12. .116$\frac{2}{3}$.
13. .7.
14. 9991.2844+.
15. 8930.5972+.
16. 761409.375.
17. 73.79512+.
18. 124.33$\frac{1}{3}$.
19. 48 coats.
20. 8 yd.
21. 15.15+ hr.
22. 56 bar.
23. 4.637+ d.

Art. 264.

25. .2425.
26. .45631.
27. .032463.
28. .00008534.
29. .00642564.
30. .5634527.

Art. 279.

1. $30.68.
2. $12.13.
3. $45.805.
4. $196.51.
5. $362.50, cost. $71.50, dif.
6. $16.865, dif.
7. $93.625.
8. $471.25.
9. $108.75, gain.
10. $1080.
11. $1601.
12. $1180.
13. $8429, gain.
14. $7540.
15. 5 hats.
16. 35.65$\frac{15}{29}$ w.
17. $2.574+.
18. 465.55$\frac{5}{9}$ bu.
19. $10.282.
20. $136.986+.
21. 9 yd.
22. 252 bar.
23. $.125+.
24. 600 T.

Art. 284.

2. $438.
3. $300.
4. $137.60.
5. $112.25.
6. $250.
8. 1061.33\frac{1}{3}$.
9. $364.
10. $300.

Art. 285.

12. 1700 yd.
13. 450 lb.
14. 1950 bu.
15. 962 cans.
16. 128 yd.
17. 132 hoes.

Art. 286.

20. $355.25.
21. $691.08.
22. $315.75
23. $4733 82.
24. $111.65.
25. $44.10.
26. $100.49.
27. $1866.90.
28. $191.93.

Art. 287.

31. $68.31.
32. $19.642.
33. $652 03125.

Art. 299.

1. $17.77.
2. 158.256\frac{2}{3}$.
3. $360.705.
4. $416.02.
5. $2659.275.
6. $1067.65.
7. $429.8825.
8. $372.755.
9. $1058.05.

Art. 331.

2. 105.735 m.
4. 64 liters.
6. 3.990 Km.
7. $240.50.
8. $54.
10. 26$\frac{2}{3}$ Km.
11. 174.52592 Km
12. $4.50.
13. 54 cts.
14. $819.
15. $1.15.
16. $23.125.
17. $105.60.
18. $2.40.
19. 21 kilos.

Art. 332.

21. 3 sq. m.
22. 37.8 sq. m.
23. 85.2 sq. m.
24. 1044 centars.

Art. 333.

26. 853.632 cu. m.
27. 768 cu. m.
28. $87.
29. $12.375.
30. $59.8598.

Art. 396.

2. 26612 ft.
3. 825264 gr.
4. 434035 lb.
5. 3393088 oz.
6. 121528.5 ft.
7. 140304 gr.
8. 1539776 oz.
9. 10524 gi.
10. 42960 pt.

11. 2936 pt.
12. 161856 sq. in.
13. 1395553.5 sq. ft.
14. 3335 cu. ft.
15. 364620 min.
16. 184584 hr.
17. 522028 sec.
18. 16362 d.
19. 197150 far.
20. 120475020 sec.
21. $19.84.
22. $10.92, gain.
23. $2597.82.

Art. 400.

3. 33 bbl. 30 g. 3 qt.
4. 92 hhd. 1 bbl. 27 gal. 2 qt.
5. 2723 lb. 7 oz. 14 pwt.
6. 74 lb. 11 pwt. 4 gr.
7. 22 cwt. 26 lb. 8 oz.
8. 263 T. 2 cwt. 95 lb. 4 oz.
9. 150 rd. 2 ft. 4 in.
10. 9 m. 880 ft.
11. 313sq.rd. 49.75sq.ft.
12. 437 A. 102 sq. rd.
13. 129 C. 56 cu. ft.
14. 1350 bu. 28 qt.
15. 452 bu. 14 qt.
16. 649 com. yr. 20 da.
17. 122385 wk. 5 d.
18. £4878, 7s. 8d.
19. $6.90.
20. $750.75.
21. $1.89.

Art. 402.

3. 9 hr. 20 min.
4. 5 d. 14 hr. 24 min.
5. 3 fur. 22 rd. 3 ft. 8 in.
6. 1 pk. 5 qt. $1\frac{3}{7}$ pt.
7. 274 A. 45 sq. rd. $21\frac{17}{28}$ sq. yd.
8. $\frac{1}{8}$ of a gill.
9. $\frac{256}{25665}$ pt.
10. 9s. 3d.
11. 3 qt. .048 pt.
12. 15 hr. 34.56 sec.
13. 85 lb. 9.6 oz.
14. 3 pk. .5248 pt.
15. 5 yd. 1 ft. 2.04 in.

Art. 403.

19. $\frac{59}{64}$ bu.
20. $\frac{31}{32}$ gal.
21. $\frac{347}{6300}$ wk.
22. $\frac{7}{2560}$ lb.
23. $\frac{47}{108900}$ A.
24. .828125 bu.
25. .75625 d.
26. $.88\frac{8}{9}$ yd.
27. $.69791\frac{2}{3}$ lb.
28. .05 gal.
29. .000125 T.
30. 2.3 yr.

Art. 404.

31. $\frac{12}{23}$.
32. $\frac{14}{121}$.
33. $\frac{195}{484}$.
34. .12648 +.
35. .405 +.

Art. 405.

3. 39.1482 mi.
4. 19.8131 gal.
5. 15.89 bu.
6. 4.2324 oz.
7. 303.68365 lb.
9. 148.87775 A.
10. 4237.92 cu. ft.

Art. 406.

12. 58.293 + m.
13. 6236.959 + Kg.
14. 236.58559 + li.
15. 72.497 + kl.
16. 143.223 + Kg.
17. 6000 sq. m.
18. 16.378 + hektars.
19. 410.748 + cu. m.
20. 27985.715 + cu. m

Art. 407.

2. £29, 7s. 1d.
3. 23 gal. 2 qt. 1 gi.
4. 16 wk. 6 da. 4 hr. 48 min.
6. 134 bu. 3 pk. 7 qt.
7. 249 A. 157 sq. rd.
8. 6 hhd. 53 gal. 3 qt.
9. 200 yr. 11 mo. 0 wk. 4 da.
10. 101 mi. 160 rd.
11. 109 sq. yd. 8 ft. 142 in.
12. 73 C. 69 ft. 177 in.
13. 177 mi. 242 rd. 4 yd. 2 ft. 4 in.
15. 1 bu. 2 pk. 1 qt. $\frac{11}{12}$ pt.
16. 9 hr. 37 min. $25\frac{61}{70}$ sec.
17. 8 oz. 3 pwt. 22.4 gr.
18. 5 d. 16 hr. 6 min. $51\frac{3}{7}$ sec.
19. 1 gal.
20. 18s. 5d. $2\frac{2}{3}$ far.

Art. 408.

2. 58 hhd. 6 gal. 2 qt.
3. 6 oz. 18 pwt. 2 gr.
4. $113\frac{1}{3}$ yd.
5. 9 mi. 0 fur. 18 rd. 7 ft. 10 in.
6. 54 A. 149 rd. 38 ft.
7. 128 cu. ft. 1652 cu. in.
8. 48 C. 106 ft. 58 in.
10. 3 yr. 5 mo. 21 d.
11. 6 yr. 4 mo. 26 d.
12. 12 yr. 2 mo. 23 d.
13. 1 yr. 7 mo. 21 d.
14. 5 yr. 4 mo. 15 d.
16. 191 d.
17. 150 d.
18. 74 d.
19. 150 d.
20. 222 d.
21. 10° 54′ 13″.
22. 98° 0′ 57″.
23. 19° 7′ 54″.
24. 79° 20′ 15″.

Art. 410.

2. 68 lb. 1 oz. 4 pwt.
3. 195 gal. 3 qt. 0 pt. 1 gi.
4. 236 mi. 150 rd. 3 yd. 1 ft. 6 in.
5. 8 lb. 10 oz. 7 pwt.
6. 1538 gal. 1 qt.
7. 532 mi. 0 fur. 10 rd.

Art. 411.

2. 5 oz. 8 pwt. 8 gr.
3. 10 lb. $11\frac{1}{2}$ oz.
4. 4 m. 177 r. 7 ft. 6 in.
5. 19 bu. 0 pk. 2 qt.

Art. 415.

2. 2° 15′ 30″.
3. 10° 53′ 1.95″.
4. 11° 59′.
5. 9° 17′ 6″.
6. 12° 20′.

Art. 416.

2. 29 min. $35.2\frac{2}{3}$ sec.
3. 41 min. $18\frac{7}{15}$ sec.
4. 5 h. 45 m. 16.6 s.
5. 1 hr. 6 min. 30 sec.
6. 40 min. 28 sec.
7. 12 min. 13 sec.
8. 1 hr. 17 min. $20\frac{1}{3}$ sec.
   3 hr. 21 min. $\frac{2}{3}$ sec.
9. N. Y., 9 A.M.
   Rich., Va., 8 h. 46 min. 29 sec., A.M.
   San Fr., 5 h. 46 min. 25 sec., A.M.

Art. 424.

2. $36\frac{2}{3}$ yd.
3. $48\frac{8}{9}$ yd.
4. 18 A. 44 sq. rd.
5. 435.6 ft.
6. $10890.
7. $1980.
8. 20358 sq. meters.
9. 3.25 meters.
10. $2.
11. 640 A.
12. 26 A. 65 sq. rd.
13. 50 rd.
14. 11.6973 Ha.
15. 5 A. 159 r. $260\frac{1}{4}$ sq. ft.
16. 2 sq. rods.
17. 20 A. 1120 sq. ft.
18. $57500 gain.
19. $76.
20. $\$47.78\frac{2}{3}$.
21. $9498 50.
22. 1330.56 tiles.
23. $11760.
24. 120 rd. wide.
    $2560. cost.
25. $41\frac{1}{3}$ planks.

Art. 429.

2. 4200 cu. ft.
3. 23328 cu. ft.
4. $377\frac{7}{9}$ loads.
5. $\$125.92\frac{16}{27}$.
6. $\$2823.33\frac{1}{3}$.
7. 198 cu. yd. 13 cu. ft. 648 cu. in.
8. 137 cu. yd. 24 cu. ft. 1512 cu. in.
9. 30 ft.

Art. 431.

1. $6\frac{9}{16}$ cords.
2. $14\frac{7}{16}$ cords.
3. $13.8125.
4. 48 cu. ft.; 80 cu. ft.
5. 1536; 3072 cu. ft.
6. 128 ft. high.
7. 6 cord ft.
8. $\$2.95\frac{5}{16}$.
9. $5\frac{1}{3}$ ft.

Arts. 432, 436.

1. 63 perch 9 cu. ft.
2. $300.96.
3. 76545 bricks.
4. 146966.4 bricks.
5. $1138.9896.
3. $19\frac{5}{6}$ board ft.
4. $17\frac{1}{2}$ board ft.
5. 110 board ft.
6. $66.
7. 20 board ft.
8. 120 board ft.
9. $42\frac{2}{3}$ b. ft.; $3\frac{5}{9}$ cu. ft.
10. $5 50.
11. 12 ft.
12. 260 cu. ft.
13. 2 C. 4 ft.
14. 500 ft.
15. $11.25.
16. $65\frac{1}{2}$ ft.
17. $3.70125.
18. $273\frac{3}{8}$ cu. ft.
    $\$297.97\frac{7}{8}$, cost.
19. $7.875.
20. 3038 cu. ft.

Arts. 438, 439.

3. $1795\frac{25}{77}$.
4. $136\frac{424}{539}$ hhd.
5. $2137\frac{157}{539}$ hhd.
6. 6 ft. $11\frac{317}{576}$ in.
7. $267\frac{13}{36}$ ft.
8. $162.72+.
9. $373\frac{169}{432}$ cu. ft.
11. 208 bu.
12. 7 ft. $9\frac{3}{4}$ in.
13. 5 ft.
14. 10 ft.
15. $360.
16. 44.8 bu.
17. $16\frac{16}{23}$ T.
18. 12.6 T.

Art. 441.

1. 110.4 A.
2. $453.03125.
3. $708 and 270, rem
4. 8100 cu. in.
5. 9288 cu. in.
6. 2160 cu. ft.
7. $3\frac{3}{4}$ cords.
8. $31\frac{127}{128}$ cords.
9. $5.00.
10. $250.
11. $6.
12. $114\frac{7}{18}$ sq. yd.
13. 156 sq. yd.
14. 72 yd.
15. 85 yd.
16. $33325.
17. $30750.
18. $\$49.095\frac{45}{64}$.
19. 880000 times.
20. 20 da. 20 hr.
21. $18849\frac{13}{63}$ wk.
22. 2700 bricks.
23. 369062 in.
24. 17400 shingles.
25. 144 farms.
26. 220320 bricks.
27. 30 da. 10 hr.
28. 76 yr. 37 da. 7 hr. 46 min. 40 sec.

Art. 452.

3. .40 or 40%.
4. $.17\frac{7}{9}$ or $17\frac{7}{9}$%.
5. .30 or 30%.
6. .48 or 48%.

Art. 460.

3. $1267.
4. £2624.16.
5. 6506 bu. 3 pk. 2 qt.
6. 2240 lb.
7. $62.50.
8. $588.
9. £1218.
10. $359.25.
11. 194.625 A.
12. $2048.50.
13. 2725 ft. $3\frac{3}{4}$ in.
14. $$188058.33\frac{1}{3}$.
15. $432, the first.
16. $5580, the second.
17. $7245.
18. $3833.60.
19. $911.25.

Art. 462.

22. $6480.
23. 3724 Hl.
24. $303000.
25. 10862 men.
26. 1780 sheep.
27. $10680.

Art. 464.

3. $29\frac{1}{6}$%.
4. $\frac{2}{3}$%.
5. $8\frac{13}{14}$%.
6. $6\frac{6}{11}$%.
7. 11%.
8. $63\frac{1}{2}$%.
9. 90%, Henry.
   94%, sister.
10. $12\frac{1}{2}$%.
11. $156\frac{1}{4}$ bu. sold.
    $62\frac{1}{2}$%.
12. 25%, wife.
    $3125, each child.
13. $8\frac{16}{73}$%.
14. $212\frac{36}{47}$%.
15. $2\frac{7}{9}$%.
16. $\frac{5}{8}$%.
17. $\frac{15}{176}$%.
18. $16\frac{2}{3}$%.
19. $66\frac{2}{3}$%.
20. $2863 for 3d.
    $25\frac{3425}{9863}$%, 1st.
    $45\frac{6165}{9863}$%, 2d.
    $29\frac{273}{9863}$%, 3d.

Art. 466.

3. 288.
4. 2340.
5. £$3428\frac{4}{7}$.
6. 1000.
7. 8000 yd.
8. 312.
9. 250.
10. $120.
11. $40000.
12. $21150.
13. $62\frac{2}{3}$.
14. 46.8.
15. 43100.
16. $7200.
17. $184.
    $1060.
18. $3675.

Art. 469.

3. $4856\frac{4}{11}$.
4. $2281\frac{7}{53}$.
5. 26000.
6. 2200.
7. $3100.
8. $7000.
9. 5200.
10. $2234\frac{2}{47}$.
11. $1363\frac{7}{11}$.
12. 2705 bu. 3 pk. 4 qt.
    $\frac{8}{17}$ pt.
13. $7000.
14. 15000.
15. $$7179.48\frac{3}{5}$.
16. $$3085.71\frac{3}{7}$.
17. $4800, entire cost.
    $$5.05\frac{5}{19}$, cost per bar.
18. $16000, whole cost
    $20 per bar.

Art. 473.

1. $1347.80.
2. $5.79.
3. $537.50.
4. $56.25.
5. $252.
6. $$448.12\frac{1}{2}$.
7. $380.70.
8. $1485.
9. $1787.50.
10. $791.464.
11. $2843.75.
12. $312.0605.
13. $1163.75.
14. $29250.

Art. 474.

15. $33\frac{1}{3}$%.
16. $22\frac{2}{9}$%.
17. 20%.
18. $21\frac{9}{11}$%.
19. 30%.
20. 100%.
21. 25%.
22. $4\frac{11248}{15563}$%.
23. $4\frac{2}{7}$%.
24. $33\frac{1}{3}$%.
25. $23\frac{31}{50}$%.

Art. 475.

26. $$5478.26\frac{2}{23}$.
27. $$17036.36\frac{4}{11}$.
28. $24375.
29. $4175.36.
30. $6970.
31. $$371.16\frac{1}{4}$.
32. $516.25.
33. $980.20.
34. $$1634.71\frac{7}{8}$.
35. $3435 20.
36. $$1696.2281\frac{1}{4}$.

Art. 476.

39. 77.23\frac{3}{14}$.
40. 611.08\frac{132}{221}$.
41. 1555.55\frac{5}{9}$.
42. 3235.29\frac{7}{17}$.

Art. 483.

1. $156.06.
2. $108.97½.
3. $46.41.
4. $109.37½.
5. $373.40, Com. $7094.60, p'd own'r.
6. $486.87, bill. $11995.13, net pro.
7. $2400.
8. $8400, amt. of sales.
9. $1750, amt. of sales.
10. $18000, amt. of sales.
11. $27000, selling pr. $26595, net pro.
12. $2000.70.

Art. 484.

14. $236.91.
15. $15506.23.

Art. 485.

17. 1583.512\frac{8}{41}$.
18. 4345.36\frac{24}{41}$.
19. 4696.65\frac{15}{209}$.
20. 3200.38 bbl.
21. 49261.08\frac{76}{203}$.
22. 500 robes.
23. $10024.45.

Art. 500.

1. $360.
2. $112.50.
3. $487.50.
4. 1½%.
5. 2⅓%.
6. 50%.
7. $10520.
8. $13600.

Art. 501.

10. 16964.28\frac{1}{7}$.
11. 361.538\frac{6}{13}$.
12. $6666.66⅔.

Art. 505.

13. $202.50.
14. $262.50, an. prem. $5250, amt. in 20 yr.
15. Equal.

Art. 515.

2. $62.548.
3. $79.59.
4. $128.408.
5. .083, rate. $441.75.

Art. 522.

2. $5062.50.
3. $20475.
4. $1234.80.
6. $1320.
7. $7.095.
8. $492.1875.

Art. 536.

3. $55 1124, int. $395.31, amt.

Art. 537.

4. $72.96, int.
5. $1177.875, amt.
6. $1103.52, amt.
7. $10.05, int.
8. $183.98625.
9. $1142.52, amt.
10. 340.27\frac{7}{9}$.
11. $51.548+.
12. $30.77.
13. $102.50.
14. $378.102.
15. $33.0338, int.
16. $645.83, amt.
17. $62.80, amt.
18. $4.80.
19. $750.40, amt.
20. $470.52, amt.
21. $65.16⅔, int.
22. $4662.25, amt.
23. $134.78.
24. $20.326.
25. $232.73+.
26. $9808.81, amt.
27. $17186.90, amt.
28. $11578 53+, amt.
29. $14472.096, amt.
30. $55237.86, amt.
31. $30.724.
32. $24.08.
33. $2741.15, amt.

Art. 539.

2. $281.277.
3. $1.343, int.
4. $7.689, int.
5. $45.696.
6. $168.901.
7. $70.698, int.
8. $1.759, int.
9. $66.832, int.
10. $5.684, int.
11. $691.071, amt.
12. $2879.854, amt.
13. $4423.372, amt.
14. $96.39, int.
15. $340.277, int.

Art. 540.

3. $4.725.
4. $1.35.
5. $3.483.
6. $10.85.
7. $5.25, int.
8. $6.431, int.
9. $10, int.
10. $31, int.

Art. 542.

2. $39.259.
3. $11.507.

Art. 553.

2. $426.42.
3. $366.654.
4. $672.051.

Art. 554.

6. $149.211.
7. $518.501.

Art. 555.

2. $8\frac{10}{27}\%$.
3. $7\frac{1}{2}\%$.
4. $6\frac{1}{4}\%$.
5. 5%.
6. 9%.
7. 5%.
8. 6%.
9. $7\frac{1}{2}\%$.

Arts. 556, 557.

2. 1 yr. 10 mo. 28 d.
3. 16 yr. 8 mo.
4. 10% 10 yr.
2. $7142.857.
3. $$11666.66\frac{2}{3}$.
4. $12800.
5. $4237.288.
6. $446.428.
7. $1168.831.

Art. 559

2. $102.04.
3. $139.50.
4. $209.02.
5. $348.21.
6. $1289.01.

Art. 561.

2. $4690.34.
3. $560.36.
4. $261.69.
5. $1524.46.
6. $1174.42.
7. $1194.05
8. $1520.12.
9. $1938.35.

Art. 565.

2. $780.045, pr. worth.
   $70.205, true dis.
3. $1170.11, pr. worth.
   $102.39, true dis.
4. $2631.82, pr. worth.
   $263.18, true dis.
5. $4881.86, pr. worth.
   $768.89, true dis.
6. $9527.44, pr. worth.
   $472.56, true dis.
7. $41.60.
8. $$3214\frac{2}{3}$, pr. worth.
   $1607.33, true dis.
9. $12380.95.

Art. 570.

2. $639.925.
3. $816.34.
4. $1258.84.
5. $18.61.
6. $821.76.

Art. 571.

2. $518.45.
3. $4473.01.
4. $5342.81.

Art. 574.

2. $418.50.
3. $511.65.
4. $657.90.
6. $2211.84.
7. $3147.54.
8. $3623.29.
9. $5606.25.
10. $8881.50.
11. $696.62.

Art. 575.

2. $73.68.
3. $4.80.
4. $150.
5. $375.

Art. 583.

2. 4 mo.
3. 6 mo.
4. $1\frac{1}{2}$ yr.
5. 3 mo.
6. $6\frac{1}{2}$ mo.
7. 2 yr. 3 mo.

Art. 584.

9. Aug. 15th, 1879.
10. May 4th, 1880.
11. Aug. 19th.
12. Sept. 6th.
13. Sept. 21st.
14. Jan. 5th, 1881.

Art. 586.

2. Sept. 2d.
3. Nov. 28th.
   Bal. due, $150.
4. Apr. 1st.
   Bal. due, $1730.
5. Dec. 20th, 1879.
   Bal. due, $140.
6. July 28th.
   Bal. due, $100.
7. Aug. 13th.
   Bal. due, $1275.

Art. 601.

1. $280.
2. $750.
3. $510.
4. $420.

Art. 602.

5. $2461.
6. $2604.
7. $10048.50.
8. $13725.
9. $12525.

Art. 604.

2. 15%.
3. $12\frac{1}{2}\%$.
4. $11\frac{467}{1178}\%$.
5. $12\frac{1}{2}\%$.

Art. 605.

7. $9750.
8. $10890.
9. $21875.
10. $47062.50.

Art. 606.

12. 125 shares.
13. 50 shares.
14. 60 shares.
15. 73 shares.
16. 40 shares.

Arts. 607, 10.

18. $66\frac{2}{3}\%$.
19. $62\frac{1}{2}\%$.
20. $133\frac{1}{3}\%$.
22. \$78750.
23. \$14437.50.
24. \$24640.
25. \$28333.33$\frac{1}{3}$.
26. \$6360.
29. $4\frac{6}{11}\%$.
30. $5\frac{115}{217}\%$.
31. \$666.66$\frac{2}{3}$.
34. $3\frac{21}{53}\%$.
35. $11\frac{13}{27}\%$.

Arts. 624, 25.

3. \$862.75.
4. \$970.125.
5. \$2035.
7. \$5285.29.
8. \$3445.75.
9. \$5075.
11. \$2439.02.
12. \$3419.69.
13. \$454.78.
14. \$2308.64.
15. \$4160.16.
16. \$2971.77.

Arts. 632, 34.

2. \$1683.22.
3. \$1842.092.
5. \$145.54.
6. \$195.26.
8. \$675.50.

Art. 635.

10. £1337, 8s. 11$\frac{1}{2}$d.
11. 12360 fr.
12. 7740 fr.
13. 3200 marks.
14. 8533$\frac{1}{3}$ marks.

Art. 641.

2. \$499.20.
3. \$47.50.
4. \$97.50.
5. \$24.50.
6. \$35.28.
7. \$37.50.
8. \$7.44.
9. \$5.15.
10. 756 m.
11. 562$\frac{1}{2}$ bu.
12. 5106 m.
13. 84$\frac{21}{32}$ Km.
15. £1.
16. \$53333.33.
17. \$427.50.
18. \$75.74.
19. \$23.125.
20. 60 d.
21. 50 d.
23. 4$\frac{4}{5}$ d.
24. 1$\frac{1}{11}$ min.
25. 2$\frac{8}{11}$ hr.
26. 8 ds.
27. 4$\frac{4}{5}$ hr.
28. 12$\frac{1}{2}$ min.

Art. 643.

2. 99 lb. 10$\frac{2}{7}$ oz.
3. 98 lb. 7$\frac{5}{13}$ oz.
4. 69 T. 1285$\frac{5}{7}$ lb.
5. .07$\frac{28}{221}$.
6. 93 bu. 5.12 qt.
7. \$0.83$\frac{1}{3}$.
8. 818.568 m.
9. 4858.55 + lb.
10. 958 lb. 10.4 oz.
11. \$6.75.
12. 189 yd.
13. 10$\frac{2}{3}$ doz.
14. 325 lb.

Art. 645.

2. \$12.
3. \$80.
4. \$250.
5. \$250.
6. \$872.50.

Art. 647.

2. 107 b. 4$\frac{4}{7}$ q., A.
85 b. 22$\frac{6}{7}$ q., B.
87 b. 4$\frac{4}{7}$ q., C.
3. \$600, A's.
\$375, B's.
\$525, C's.
4. 666$\frac{2}{3}$ bar., A's.
800 bar., B's.
1000 bar., C's.
533$\frac{1}{3}$ bar., D's.
5. \$315, A's.
\$525, B's.
\$420, C's.
6. \$1250, X's.
\$1750, Y's.
\$2000, Z's.
7. \$0.66$\frac{2}{3}$.
\$200, 1st.
\$266.66$\frac{2}{3}$, 2d.
\$333.33$\frac{1}{3}$, 3d.
8. \$0.80.
9. \$64.14, A's.
\$105.12, B's.
\$147.76, C's.
10. \$0.10 on \$1.
\$500, B rec'd.
11. \$0.4223 +.
12. 100 bar., A's.
66$\frac{2}{3}$ bar., B's.
33$\frac{1}{3}$ bar., C's.
14. \$100, A.
\$120, B and C.
15. \$30.
16. \$40.02, A's.
\$88.28, B's.
\$117.70, C's.
17. \$332.50, S's
\$525, Jones'.
18. \$508.83, A's.
\$677.75, B's.
\$938.42, C's.

Art. 649.

2. 858$\frac{2}{3}$.
3. 2554$\frac{2}{37}$.
4. 2666$\frac{2}{3}$.
5. \$288.
6. \$232.
7. \$1.20.
8. 875 pears.
9. 425.
10. 980.
11. 270 mi.
12. \$56.
13. 385.
14. \$2.10.
15. 22 T. 1500 lb.
16. 2 coats.
17. 21 tubs.
19. 240 sheep.
20. \$288.
21. 1440 men.
22. 48 ft.
23. \$296.
24. \$14400.
25. 72 yr.
26. 21$\frac{9}{11}$.
27. 72 pupils.
28. \$15600.
29. 60 trees.

Art. 650.

3. 40 shares.
4. \$62.30.
5. \$4932.64.
6. \$2187.50.
7. \$168.41.
8. 8%.
9. \$3869.72 +.
10. \$1619.31.
11. \$1010.50.
12. 4$\frac{1}{5}$ m.
13. \$900, A's.
\$1040, B's.
\$1060, C's.
14. \$770.625.
15. \$3894.4325.

Art. 651.

2. 324 in one.
432 in other.
3. 312 in one.
936 in other.
4. 108$\frac{3}{4}$, one.
326$\frac{1}{4}$, other.
5. 151 A., one.
604 A., other.
6. \$1312.89, one.
\$1641.11, ot'r.
7. 180, first,
240, second.
300, third.
9. 378, 1st.
252, 2d.
315, 3d.
11. 205

12. 11.
13. $121\frac{1}{3}$ Kl.
14. 40 peaches.
80 pears.
160 apples.
15. 64, 1st.
32, 2d.
96, 3d.

Art. 652.

17. $7\frac{1}{2}$ ds.
18. $2\frac{1}{4}$ mo.
19. 20 men.
20. 720 mi.
21. 224 bu.

Art. 653.

23. 90 cts., 1 pt.
40 cts., $1\frac{1}{2}$ pt.
24. 30 cts., 1 pt.
24 cts., 5 pt.

Art. 668.

1. $3\frac{4}{5}$.
2. $2\frac{7}{24}$.
3. $2\frac{11}{17}$.
4. $2\frac{3}{14}$.
5. $\frac{13}{36}$.
6. $\frac{7}{11}$.
7. $\frac{27}{64}$.
8. $\frac{11}{25}$.
9. $\frac{3}{7}$.
10. $\frac{4}{9}$.
11. $\frac{8}{15}$.
12. $\frac{49}{72}$.
13. 16.
14. $1\frac{7}{8}$.
15. $115\frac{1}{2}$.
16. $16\frac{2}{3}$.
17. $3\frac{1}{3}$.
18. $\frac{1}{152}$.
19. 32.
20. $\frac{3}{32}$.

Art. 670.

21. 7.
22. $\frac{5}{12}$.
23. $\frac{4}{7}$.
24. 112.
25. $\frac{27}{32}$.
26. $\frac{64}{117}$.
27. 192.
28. 432.
29. $1\frac{22}{123}$.

Art. 684.

2. 70.
3. 2550.
4. $7\frac{1}{12}$.
5. 48.
6. 288.
7. 375.
8. 27.3.
9. 35 vests.
10. $736\frac{2}{3}$ lb.
11. $13\frac{1}{3}$ gal.
12. 45.
13. 75.

Art. 687.

3. $3\frac{3}{4}$d.
4. $515.
5. £22, 10s.
6. 1440 min.
7. $3.75.
8. $5000.
9. $13\frac{1}{3}$ mo.
10. $60.
11. 75 ft.
12. 100d.
13. 105 hektars.

Art. 692.

3. $3.44.
4. $5.01.
5. 108.5 Kil.
6. $69\frac{3}{13}$ Hl.
7. $318\frac{3}{4}$ Km.
8. $133\frac{1}{3}$ spoons.
9. $\$251\frac{1}{4}$.
10. 362 d.
11. £51, 3s. 2d.
12. £3, 12s. 6d.
13. £41, 12s. 6d.
14. $3\frac{3}{4}$ hr.
15. 5 min.
16. 12 hr.
17. $195.14.
18. $14\frac{10}{13}$ in.
19. $164\frac{1}{6}$ yd.

Art. 695.

3. 768 m.
4. 96 men.
5. 10 men.
6. 6 d.
7. 7.2 d.
8. $170\frac{2}{3}$ bu.
9. 80 d.
10. 6 men.
12. $18.
13. 90d.
14. 25 lb.
15. 60 men.
16. 18 d.
17. 3s. 1.7d.
18. 792 pr.

Art. 698.

2. 72, 1st.
96, 2d.
144, 3d.
3. 22 sh., 1st.
66 sh., 2d.
110 sh., 3d.
4. 120 bu. oats.
160 bu. whe't.
220 bu. corn.
5. $0.88, pears.
$1.76, or'ng's.
$2.64, mel'ns.
6. $497, 1st.
$745.50, 2d.
$994, 3d.
$1242.50, 4th.

Art. 706.

2. $\frac{3}{8}$, A's.
$\frac{5}{8}$, B's.
3. $120, A's.
$160, B's.
$200, C's.
4. $342.86, A's.
$457.14, B's.
$685.71, C's.
$914.29, D's.

Art. 708.

6. $50, A's.
$100, B's.
7. $100, A's.
$120, B's.
$120, C's.
8. $30.
9. $40.02, A's.
$88.28, B's.
$177.70, C's.
10. $600, A's.
11. $589.47, A's.
$1129.82, B's.
$1670.18, C's.
$2210.53, D's.
12. $5090.
13. $16\frac{2}{3}\%$.
14. 10%, or
$1500, A's
loss.

Art. 718.

2. 216.
3. 729.
4. 53824.
5. 3125.
6. 2401.
7. 42875.
8. 65.450827.
9. 8.003600540-
027.
10. 27013502.25-
0125.
11. $\frac{81}{625}$.
12. $\frac{343}{512}$.
13. 111.56640625.

Art. 719.

4. 900 + 240
+ 16.

Art. 734.

4. 24.
5. 40.
6. 35.

7. 17.07+.
8. 29.4+.
9. 23.20+.
10. 7.859+.
11. 92.06+.
12. 1110.016+.
13. 2305.317+.
14. 16.96+.
15. .881+.
16. 32.768.
17. .0231.
18. 785.64.

Art. 735.

3. $\frac{1}{2}$.
4. $\frac{4}{9}$.
5. $\frac{8}{27}$.
6. .36288+.
7. $\frac{1}{2}$.
8. .4848+.
10. $4\frac{1}{3}$.
11. $6\frac{2}{5}$.
12. $7\frac{1}{4}$.
13. $10\frac{2}{3}$.
14. $9\frac{3}{5}$.
15. .751+.
16. $14\frac{2}{77}$.
17. 1.7320508.
18. 3.46410161+.
1. 45 yd.
2. 952.
3. 783.836 rd.
4. 72 ft.
5. 24 rds.
6. 390 rds.
7. 1395 yd. sq.

Art. 736.

9. 8.
10. 15.
11. 30.
12. 56.
13. 72.
14. 38.8844442.
15. 65.72+.
16. .08.
17. 2.
18. .18.
19. $\frac{1}{6}$.
20. $\frac{8}{15}$.
21. $\frac{35}{54}$.
22. .3.
23. $\frac{21}{44}$.

Art. 737.

24. 80 rd., width.
 160 rd., length.
25. 80 rd., breadth.
 320 rd., length.

Art. 743.

27. 10 yd.
28. 50 m.
29. 200 m.

Art. 744.

31. 60 ft.
32. 24.98 ft.
33. 8 yd.

Art. 745.

35. 24 ft.
36. 103.614+ ft.
37. 42.4264 rd.
38. 40 rd., length, side.
 56.5685 rd., diagonal.
39. 50 ft., floor diagonal.
 51.92+ ft., other diagonal.
40. 65.802+ rd.
41. 56.5685 ft.
42. 75.816+ ft.

Art. 747.

3. 15 min.
4. 18 in.
5. 30 in.
6. 24.49+ yd.

Art. 750.

1. 2 fig.
2. 2 fig.
3. 2 fig.
4. 3 fig.
5. 3 fig.
6. 2 fig.

Art. 752.

2. $30^3+3(30^2\times 2)+3(30\times 2^2)+2^3$.

Art. 754.

4. 24.
5. 83.
6. 72.
7. 125.
8. 103.
9. 1331.
10. 3002.
11. 2.3.
12. 4.5.
13. .632+.
14. 5.48.
15. 49.68.

Art. 755.

17. .601+
18. $\frac{9}{16}$.
19. $\frac{2}{3}$.
20. $\frac{25}{36}$.
21. 2.39+.
22. $3\frac{1}{3}$.
23. 1.2599+.
24. 1.442249+.

Art. 756.

1. 73 in.
2. 364 ft.
3. 108 yd.
4. 8 ft.
5. 8 ft. 6.44 in.
6. 9 ft. 5.3 in.
7. 58.8+ ft.

Art. 758.

3. 137.48+ lb.
4. $121362.96.
5. 1562.5 cu. ft.
6. $23\frac{4}{27}$ T.
7. 163840000 lb.
8. $1111\frac{1}{9}$ hhd.

Art. 759.

10. 7.2+ ft.
11. 8 in.
12. 3 ft.
13. 4 ft.

Art. 768.

1. 15.

Art. 769.

2. 16.

Art. 770.

3. 38.
4. 10.
5. 27.
6. $550.

Art. 771.

2. 8 children.
3. 23.
4. $38\frac{1}{2}$ d.

Art. 772.

2. 3 yr.
3. $3.
4. $9\frac{1}{3}\%$.
5. $\frac{8}{27}\%$.

Art. 773.

2. 78 strokes.
3. 180000.
4. $651.

Art. 779.

2. $5.12.
3. $455.81.
4. $1605.87, amt.
   $3276.975, amt.

Art. 780.

2. $\frac{64}{243}$.

Art. 781.

2. 1456.
3. $255.
4. $40.95.
5. $5314.40.

Art. 809.

2. 450 sq. ft.
3. 4914 sq. ft.
4. 46 A. $17\frac{1}{2}$ sq. rds.
5. $4556\frac{1}{4}$ sq. yd.
6. 250 sq. ft.

Art. 810.

2. 50.91 + sq. yd.
3. 198.43 + sq. ft.
4. 7 A. 58.14 sq. rd.

Art. 811.

2. 3 yd.
3. 150 rd.

Art. 812.

2. 80 rd.
3. 52.6 yd.

Art. 821.

3. $67\frac{1}{2}$ A.
4. 640 A.
5. 26 A. 65 sq. rd.
6. 50 rd.
7. 80 rd.
9. 4 A. 75 sq. rd.

Art. 822.

2. 368 sq. yd.
3. 558 sq. rd.

Art. 823.

2. 50 A. 125 sq. rd.
3. 12480 sq. yd.

Art. 829.

2. 141.372 yd.
3. 314.16 rd.

Art. 830.

2. 30 rd.
3. 200 yd.
4. 8 rd.
5. 16 ft.

Art. 831.

3. 7854 sq. ft.
4. 11309.76 sq. rd.
5. 2037.178 + sq. yd.
6. 15.91 ft.
7. 10.472 ft.
8. 706.86 sq. ft.
9. 203.7178 A.
10. 7.97 rd.

Art. 841.

3. 126 sq. ft.
4. 54 sq. in.
5. 152 sq. ft.
6. 576 sq. ft.
7. 640 sq. ft.
8. 4084.08 sq. ft.

Art. 842.

3. 375 cu. ft.
4. 9200 cu. ft.
5. 565.488 cu. ft.
6. 45945.9 cu. ft.

Art. 846.

3. 7744 cu. ft.
4. 3817.044 cu. ft.
5. 441 cu. ft.

Art. 850.

2. 4.91 sq. ft.
3. 201062400 sq. m

Art. 851.

2. 523.6 cu. in.
3. 259,777,100,108 cu miles.
4. 381.7 + cu. in.

Art. 852.

2. $66\frac{2}{3}$ ft.

Art. 855.

2. 44.982 gal.
3. 1059.5286 liters.
4. 548.4375 gal.

Art. 856.

1. $2526\frac{2}{19}$ T.
2. $378\frac{18}{19}$ T.

Art. 858.

1. 164 girls.
   304 pupils.
2. $62\frac{358}{421}$ rd., breadth.
   $330\frac{3}{4}$ A.
3. .568431.
4. .881 A.

5. $45\frac{5}{7}$ ft., height
6. $\$1.46\frac{7}{8}$.
7. 8157 da.
8. $28.95.
9. $4702.50.
10. 7.
11. 5 bbl. 152 lb.
12. 88704 steps.
13. $6460.40.
14. 12 ft.
15. $34.57.
16. 39.163 yd.
17. 3949, one.
    4705, other.
18. $462.50, one.
    $1037 50, other.
19. 469, less.
    1407, greater.
20. $315, B's share.
    $1260, A's share.
21. $51\frac{19}{21}$, one.
    $69\frac{5}{21}$, other.
22. 1 = **g. c. d.**
    261648 = **l. c. m.**
23. 23 p. of 36 yd.
24. 16 A.
25. $1258250.
26. 139180.
27. $13^2 \times 7 \times 2$.
28. 4 days.
29. 15, **g. c. d.**
    77805, **l. c. m.**
30. 72, 2d.
    144, 4th.
31. $\frac{28,\ 7,\ 60 \text{ and } 455}{105}$.
32. $720.
33. .0322465.
34. 30 da.
35. $611.625.
36. $10.26.
37 91 lb. 6.5 oz.
38. $\$1914.28\frac{4}{7}$.
39. $31232.
40. $74\frac{2}{7}$ ft.
41. 6, **l. c. m.**
42. $17\frac{1}{2}$ da.
43. $43.52.
44. $2.55.
45. $857999\frac{23}{40}$.
46. $17.
47. $1260.
48. $.04.
49. $21.67 lost.
50. $.15\frac{9}{13}$.
51. $86.40, or $90.
53. 65 mi.
54. 24 men.
55. 350.
56. $18.78.
57. 24.
58. 56.568 mi.
59. $48\frac{4}{27}$ yd., or 50 yd.
60. 210 yr. 97 da. 21 hr.
    38 min. 15 sec.
61. $941.14.
62. $48.92.
63. $60.
64. $44.56.
67. $\frac{189}{50336}$.
68. $1700.
69. 7695.
70. A. **c. m.** of the Nos.
71. 7 hr. $19\frac{1}{41}$ min.
    $24\frac{16}{41}$ min.
72. $\frac{3}{100}$.
73. $3.35.
74. $907\frac{1}{2}$ lb.
75. $18.25, int.
    $260 71, prin.
77. $44.26.
78. 45, longer.
    18, shorter.
79. 14 ; 21 ; 12 rows.
80. $17\frac{1}{2}$.
81. $22.37.
82. $10.94.
83. $33\frac{1}{3}\%$.
84. 75%.
85. $201.60.
86. $1546.40.
87. 20%.
88. $\frac{250}{1107}$, A's share.
    $\frac{350}{1107}$, B's share.
    $\frac{507}{1107}$, C's share.
89. $\$1.76\frac{8}{17}$.
90. 7%.
91. $545.73.
92. $14\frac{2}{5}$ da.
93. $2000.
94. $5.00
95. 40 yd.
96. $185.
97. $9000.
98. $85.
99. 250 shares.
100. $18277.91.
101. $10\frac{1}{4}\%$.
102. 16326 yr. 11 mo
     4.44 da.
103. $3.41.
104. $1320, A's share.
     $880, B's share.
     $440, C's share.
105. 103.9.
106. 80 rd.
107. $50\frac{2}{27}$ yd., or 52 yd.
108. 13.34+, diagonal.
     11250 lb.
109. $10.3125.
110. $53.90.
111. 260 bu. 11.424 qt.
112. $550.92.
113. $71\frac{3}{7}\%$.
114. $1525.55.
115. $6875.
116. 240 rd.
117. $408.96.
118. $175.
119. $362.30.
120. 80.6 ft.
121. 56° 8′.
122. 1 da. 6 hr. 25 min.
123. $416\frac{5}{6}$ cu. ft.
124. $470\frac{5}{9}$ cu. yd.
125. 1190 sq. rd.
126. $210\frac{3}{8}$ mi.
127. $107\frac{5}{16}$ lb., entire weight.
     $35\frac{37}{48}$ lb., average weight
128. $\frac{8}{45}$.
129. $\frac{63}{64}$ qt.
130. $10.
131. 88° 45′.
132. $169\frac{17}{32}$ cu. ft.
133. $401.20.
134. 22 min. 40 sec.
135. .635.
136. $24634.63, inves't.
     $615 87, com.
137. $18.75.

138. $6.
139. $6\frac{2}{3}$ hr.
140. $34\frac{13}{20}$ T.
141. 14.5 ft.
142. 486 tiles.
143. $47\frac{19}{528}\%$.
144. $76\frac{4}{171}\%$.
145. $3.75.
146. 3432 ft.
147. $.44\frac{4}{9}$.
148. $.30\frac{682}{1000}$.
149. $3\frac{3}{4}$ mo.
150. $\frac{5}{56}$.
151. $9\frac{1}{24}$ da.
152. .299980317.
153. 8.55 ft., nearly.
154. 5 h. 56 m. 32 s.
155. $99.
156. $304.72.
157. $12.50.
158. 38.47 sq. rd.
159. 42 in.
160. $\frac{32}{126}$, one.
$\frac{81}{126}$, other.
161. $125000.
162. $7.22.
163. $\frac{286}{315}$.
164. 1500 lb., Nit.
250 lb., S. and C.
165. 1.1071.
166. $1200, 1st.
$1600, 2d.
$1800, 3d.
167. $190000.
168. $167.20.
169. $93\frac{3}{4}\%$.
170. $107.10.
171. $9.70.
172. 2 yr. $4\frac{1}{2}$ mo.
173. $300.
174. $15\frac{3}{4}$ da.
175. $1000, or 10%.
176. 384 sq. ft.
177. .062.
178. $654.50.
179. 35° 30′ eastward.
180. $\frac{458}{735}$.
181. 525.
182. $413.44.
183. 59.92 sq. ft.
184. 249.41 sq. yd.
185. $55924.05.
186. $130.80.
187. $48.05, A's.
$69.95, B's.
188. 66.98 lb. Troy.
189. 40.
190. 30.7125 A.
191. 192 lots.
192. 300 strokes.
193. 2, com. difference.
99, sum.
194. 15 holidays.
$360.
195. 70 A. 109.76 sq. rd.
196. $3600.84.
197. 6 bonds.
198. $284.06.
199. 562.34 mi., nearly.
200. $19.40.
201. 123.888 sq. ft.
7.87 ft.
202. 2037.178 sq. yd.
203. 1558.75 cu. ft.
204. 540 cu. ft.
205. 9200 cu. ft.
206. 13750 cu. ft.
207. 22619.52 cu. ft.
208. 20420.40 cu. ft.
209. $5.37\frac{23}{121}$ A.
210. 2048.
211. $3\frac{6}{17}\%$.
212. $.11\frac{13}{17}$.

Art. 859.

1. 63, **g. c. d.**
2. 1.4142.
3. 72 men.
4. 10%.
5. 3654.7 meters.
6. $2\frac{2}{5}$.
7. 73677.6846 cu. dm.
8. 7 hr.
9. $2500.
10. $\frac{22}{25}$, .89, $\frac{49}{55}$.
11. $2\frac{17}{60}$.
12. $\frac{825}{1288}$.
13. $.0245\frac{175}{381}$.
14. $.83\frac{1249}{8712}$ A., or
.831433 + A.
15. 160 sq. dm.
16. $1\frac{1361}{1680}$.
17. .218.
18. .059.
19. $7295.43.
20. $11\frac{1}{9}\%$.
22. $4\frac{4}{9}$ oz., nearly.
58.293 + meters
23. $4\frac{1}{5}$ pt.
24. 750 lb.
25. $\$2331.12\frac{88}{151}$, A's
$\$4662.24\frac{176}{151}$, B's
$\$5006.61\frac{189}{151}$, C's.
26. 4.38 +.
27. 360, **l. c. m.**
2, **g. c. d.**
28. 7.6199 meters.
29. 1879.
30. $\$.88\frac{8}{9}$.
31. $1142.86.
32. $2.358\frac{1}{3}$, sum.
$.291\frac{2}{3}$, product.
33. 1566.712 +.
34. 1880.0001 +.
35. 488.2468 Kg.
36. $36\frac{16}{19}\%$.
37. $1.178\frac{4}{7}$.
38. 12500 bricks.
39. $5061.68.
40. $21000 at first.
$87\frac{13}{21}\%$ loss.
41. $8400.
42. $7000.
43. $1210.59.
44. 57.3322.
45. 3.128.
46. 3, **g. c. d.**
47. 504, **l. c. m.**
48. $6\frac{2}{33}$ oz.
49. 9 o'clock 54 m. $23\frac{3}{5}$ s.
50. .059375 day.
51. $\frac{27}{35}$.
52. $201.11.
53. 4% loss.
54. 2.37.
55. 1.60933 Km.
56. $1\frac{38}{735}$.

57. .0096048.
58. 4 dm. 6 cm. 2 m.
59. 1.05 books.
$6\frac{17}{18}\%$.
60. $37\frac{1}{3}$ cts.
61. \$1060.
62. 4.5 meters.
63. .000001.
1.002001.
64. 2.221.
65. $23\frac{1}{3}$ da.
66. \$34.22.
67. \$3312.50.
68. \$3447.50.
69. 14%.
70. \$2087.75.
71. $12\frac{1}{3}$ mo.
72. \$9972.97.
73. 32.419.
74. 739.
75. \$12520 24.
76. 1.60933 Km.
77. .5.
78. 1 yr. 7 mo. 6 da.
79. 8%.
80. 735.6 liters.
81. $\frac{6}{25}$.
82. 8000 turns, nearly.
83. \$.95, difference.
84. \$384.
85. 85018982.
86. $9\frac{1}{11}$.
87. 500.002, sum.
499 998, difference.
88. 5 Kg. 217.4 g.
90. 13.572+.
91. 3.9 hektoliters.
92. $2\frac{851}{2552}$.
93. $\frac{1}{2}\%$ premium.
94. 22.712+ liters.
95. $7\frac{1}{2}$ cords.
96. 5 mo. 4 da.
97. .495+.
98. 8.65 Km.
99. 21 bu. 2 pk. 5 qt.

Art. 884.

1. .8.
2. .875.
3. .75.
4. .575.
5. .088.
6. .8125.
7. .4375.
8. $.\dot{8}5714\dot{2}$.
9. $.\dot{3}$.
10. $.\dot{7}6190\dot{4}$.
11. .36.
12. $.\dot{1}707\dot{3}$.
13. $.41\dot{6}$.
14. .53.
15. $.5\dot{9}\dot{0}$.
16. $.3\dot{6}$.
17. $.31\dot{3}$.
18. $.2\dot{1}$.
19. .484375.
20. .12.

Art. 885.

2. $\frac{2}{11}$.
3. $\frac{8}{11}$.
4. $\frac{23}{33}$.
5. $\frac{41}{333}$.
6. $\frac{11}{37}$.
7. $\frac{5}{111}$.
8. $\frac{97}{909}$.
9. $\frac{65}{101}$.
10. $\frac{42}{101}$.
11. $\frac{25121}{99999}$.
12. $\frac{1}{7}$.
13. $\frac{1}{13}$.

Art. 886.

15. $\frac{178}{275}$.
16. $\frac{113}{1125}$.
17. $\frac{2933}{4950}$.
18. $\frac{1}{44}$,
19. $\frac{16}{27}$.
20. $\frac{261}{550}$.
21. $\frac{13}{14}$.
22. $\frac{127457}{15000000}$.

Art. 887.

23. $1.0\dot{8}9\dot{7}$.
24. $.\dot{2}27\dot{7}$.

Art. 895.

1. $102\frac{6}{7}$ A.
2. \$128.57.
3. \$533.33.
4. \$160.
5. 1920 A.
\$43520 gain.

Art. 905.

2. \$1826.66.

Art. 910.

2. $\$0.09\frac{39}{437}$.

Art. 911.

4. 2 lb. at 9 cts.
2 lb. at 11 cts.
4 lb. at 14 cts.
5. 3 lb. at 15 cts.
2 lb. at 18 cts.
1 lb. at 21 cts.
4 lb. at 22 cts.

Art. 912.

7. 25 bu. at 40 cts
10 bu. at 45 cts.
$8\frac{1}{3}$ bu. at 56 cts.
$16\frac{2}{3}$ bu. at 65 cts.

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