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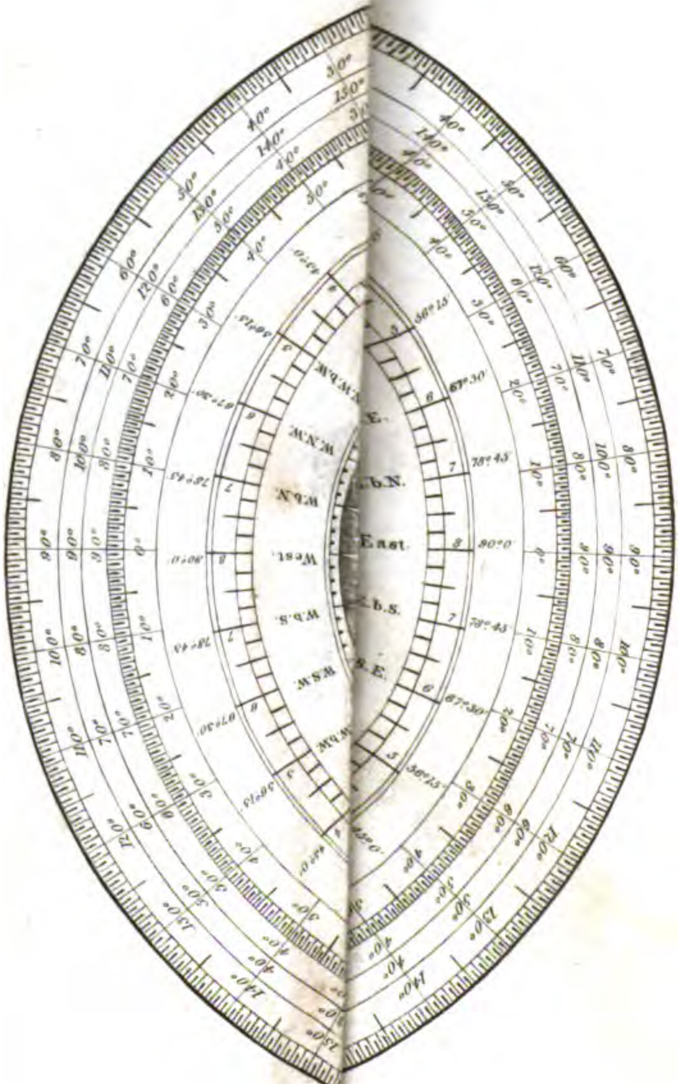

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TO THE RIGHT HONOURABLE

THE (LATE) LORDS COMMISSIONERS

FOR EXECUTING THE OFFICE OF LORD HIGH ADMIRAL

OF THE

UNITED KINGDOMS OF GREAT BRITAIN AND IRELAND,

VIZ.

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WILLIAM ROBERT KEITH DOUGLAS, ESQ.

MY LORDS,

IN bringing to a close the following Treatise, I feel that it cannot with so much propriety be inscribed to any other department in the State, as to that which has so successfully presided over, and so long and judiciously directed the Naval operations of Great Britain.

It will ever be to me, my Lords, a cause of the most sincere gratitude, that to the condescension of your Lordships, in accepting the DEDICATION of my mathematical labours, I am principally indebted for the encouragement and support which I have received, in presenting the result of those labours to the Royal Naval Service of His Majesty, and to the Merchant Service, in general, of the British Empire.

I have the honour to be,

My Lords,

With the utmost deference,

Your Lordships' most humble,

And most obedient Servant,

THOMAS KERIGAN.

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December, 1827.

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DESCRIPTION OF THE TABLES.

TO

HIS ROYAL HIGHNESS WILLIAM HENRY, DUKE OF CLARENCE AND ST. ANDREWS, K.G., &c. &c. &c.

LORD HIGH ADMIRAL

OF THE

UNITED KINGDOMS OF GREAT BRITAIN AND IRELAND.

MAY IT PLEASE YOUR ROYAL HIGHNESS;

THIS Treatise, which I am graciously permitted to lay before your Royal Highness, is the result of long study and labour; the chief aim of which has been, to contribute, in some measure, to the benefit of the Naval Service of His Majesty. To this end, I have sought to combine simplicity, perspicuity, and conciseness, in trigonometrical calculations, in a greater degree than has hitherto been attempted by the writers of nautical works; and to comprise, in one book, a compendium of all the sciences that may be useful or interesting to the practical navigator.

That my humble attempt has met with your Royal Highness's approbation and high sanction, I shall ever esteem to be the most honourable circumstance of my life; that it has been deemed worthy of the honour of your Royal Highness's patronage, I cannot but feel as the greatest mark of the condescension of your Royal Highness.

I have the honour to subscribe myself,

With the most profound respect,

Your Royal Highness's

Most obedient, most devoted,

And most grateful Servant,

THOMAS KERIGAN.

Portsmouth,
December, 1827.

PREFACE.

Although the importance and general utility of the subjects treated of in this work are sufficient to recommend it to public attention, without the aid of prefatory matter, yet, since there is an extensive variety of nautical publications now extant, I think it right to say something relative to what I have done, were it for no other purpose than that of satisfying the reader that the present work is widely different from any former treatise on nautical and mathematical subjects. The following observations will develope my motives for commencing so laborious an undertaking.

In perusing the various nautical publications which have appeared for many years past, I observed that they all fell considerably short of the objects at which they professed to aim;—some, by being too much contracted, and others by not including all the necessary tables, or by being generally defective: and that, therefore, a great deal remained to be done, particularly in the tabular parts, beyond what had yet been brought before the public.

Of the nautical works that came under my notice, some have proved, on examination, to be so inaccurately executed, as to be entirely unfit for the consultation of any person not sufficiently skilled in the mathematics to detect their numerous errors. Many of the works in question are extremely incomplete, through their want of particular tables, and their logarithms not being extended to a sufficient number of decimal places: such as those by Mendoza Rios, where the decimals are only continued to *five places* of figures, and where the logarithmic tangents are entirely wanting; for, although the addition of a logarithmic sine and a logarithmic secant will always produce a logarithmic tangent, yet there are few mariners so far acquainted with the peculiar properties of the trigonometrical canon, as to be able to find by Rios' tables the arch corresponding to a given logarithmic tangent.* Hence, when the course and the distance between two places are to be deduced from their respective latitudes and longitudes, by logarithmical computation, the mariner is invariably obliged to have recourse to some other work for the necessary table of logarithmic tangents. Besides, since none of the nautical works now in use exhibit the principles upon which the tables contained therein have been constructed, the mariner is left without the means of examining such tables, or of satisfying himself as to their accuracy; though it is to them that he is obliged to make continual reference, and on their correctness that the safety of the ship and stores, with the lives of all on board, so materially depend.

Notwithstanding that Mr. Taylor's Logarithmical Tables are the most extensive, the best arranged, and by far the most useful for astronomical purposes, of any that have ever appeared in print,—yet, since they do not contain the necessary navigation tables, they are but of little use, if of any, to the practical navigator: and, since the same objection is applicable to the very excellent system of tables published by the learned Dr. Hutton, these are, also, ill adapted to nautical purposes, and but rarely consulted by mariners.

Being thus convinced that there was something either deficient or very defective in all the works that had hitherto been published on this subject, I was ultimately led to the conclusion that a *general and complete set of Nautical Tables* was still a desideratum to mariners: with this conviction on my mind, I was at length induced to undertake the laborious task of drawing up the following work; in the prosecution of which I found it necessary to exercise the most determined perseverance and industry, in order to surmount the fatigue and anxiety attendant on such a long series of difficult calculations.

These points premised, it remains to present to the reader a familiar and comparative view of the nature of this work, and of the improvements that have been made in the tables immediately connected with the elements of navigation and nautical astronomy: confining the attention to those that possess the greatest claims to originality, or in which the most useful improvements have been made.

Table VI. contains the parallaxes of the planets in altitude; and will be found particularly useful in deducing the apparent time from the altitudes of the planets, and, also, in problems relating to the longitude. The hint respecting this was originally taken from the Copenhagen edition of "The Distances of the Planets from the Moon's Centre, for the Year 1823;" but this design has been considerably enlarged and improved upon.

* See Remark, page 98; with diagram and calculations, page 99.

Table VIII. is so arranged that the mean astronomical refraction may be taken out at first sight, without subjecting the mariner to the necessity of making proportion for the odd minutes of altitude. This improvement will have a tendency to facilitate nautical calculations.

Table X.—The arrangement of this table is an improvement of that originally given by the author, in his treatise called "The Young Navigator's Guide to the Sidereal and Planetary Parts of Nautical Astronomy." By this improved table, the correction of the polar star's altitude may be readily taken out, at sight, to the nearest second of a degree, by means of five columns of proportional parts; and, to render the table permanent for at least half a century, the annual variation of that star's correction has been carefully determined to the hundredth part of a second. By means of this table, and that which immediately follows (Table XI.), the latitude may be very correctly inferred at any hour of the night, in the northern hemisphere, to every degree of accuracy desirable for nautical purposes.

Tables XIII. and XIV. contain the equations to equal altitudes of the sun: these have been computed on a new principle, so as to adapt them to proportional logarithms, by means of which they are rendered infinitely more simple than those given under the same denomination in other treatises on nautical subjects; they will be found strictly correct, and, from their simplicity, a hope may be entertained that the truly correct and excellent method of finding the error of a watch or chronometer by equal altitudes of the sun, will be brought into more general use.

Tables XV. and XVI., which are entirely new, contain correct equations for readily reducing the longitudes, right ascensions, declinations, &c. &c., of the sun and moon, as given in the Nautical Almanac, to any given meridian, and to any given time under that meridian.

Table XVII. contains the equation corresponding to the mean second difference of the moon's place in longitude, latitude, right ascension, or declination: this table, besides being newly-arranged, will be found more extensive than those under a similar denomination, usually met with in books on navigation.

Table XVIII. is so arranged as to exhibit the true correction of the moon's apparent altitude corresponding to every second of horizontal parallax, and to every minute of altitude from the horizon to the zenith: and will prove very serviceable in all problems where the moon's altitude forms one of the arguments either given or required.

Table XIX. is fully adapted to the reduction of the true altitudes of the heavenly bodies, obtained by calculation, to their apparent central altitudes: the reductions of altitude may be very readily taken out to the decimal part of a second. This table will be found of considerable utility in deducing the longitude from the lunar observations, when the distance only has been observed.

Table XX. is new; and by its means the operation of reducing the apparent central distance between the moon and sun, a fixed star, or planet, to the true central distance, is very much abridged, as will appear evident by referring to Method I., vol. i., page 433, where the true central distance is found by the simple addition of five natural versed sines.

Table XXI., which is also new, contains the correction of the auxiliary angle when the moon's distance from a planet is observed: this will be of great use in finding the longitude by the moon's central distance from a planet.

Table XXIV.—The form of this table is entirely original; and though it is comprised in nine pages, yet it is so arranged that the logarithmic difference may be obtained, strictly correct, to the nearest minute of the moon's apparent altitude, and to every second of her horizontal parallax. This table will be found of almost general use in the problem for finding the longitude by the lunar observations.

Table XXVI., which is original, contains the correction of the logarithmic difference when the moon's distance from a planet is observed: this table will be found of great use in computing the lunar observations whenever the moon's distance from the planets appears in the Nautical Almanac; an improvement which, from the advertisement prefixed to the late Almanacs, may be shortly expected to take place.

Table XXVII., Natural Versed Sines, &c.—The numbers corresponding to the first 90 degrees of this table are expressed by the arithmetical complements of those contained in the Table of Natural Co-sines published by the author in "The Young Navigator's Guide," &c.; the arithmetical complement of the natural co-sine of an arch being the natural versed sine of the same arch. The numbers contained in the remaining 90 degrees of this table are expressed by the natural sines, from the abovementioned work, augmented by the radius.

This table is so arranged as to render it general for every arch contained in the whole semi-circle, and conversely, whether that arch or its correlative be expressed as a natural versed sine, natural versed sine supplement, natural co-versed sine, natural sine, or natural co-sine.

Table XXVIII. is an extension of that published by the author in "The Young Navigator's Guide," &c.: it is arranged in a familiar manner, and, though concise, contains all the numbers that can be usefully employed in the elements of navigation; for, by means of nine columns of proportional parts, the logarithmic value of any natural number under 1839999 may be obtained nearly at sight, and conversely.

Tables XXX., XXXI., and XXXII., have been carefully drawn up, and proportional parts adapted to them, by means of which the logarithmic half-elapsed time, middle time, and logarithmic rising may be very readily taken out at the first sight, and conversely.

Table XXXV., Logarithmic Secants.—The arrangement of this table is original, as well as its length: the numbers contained therein are expressed by the arithmetical complements of those contained in the table of logarithmic co-sines published by the author in "The Young Navigator's Guide," &c.

This table is so drawn up as to be properly adapted to every arch expressed in degrees, minutes, and seconds, in the whole semi-circle, whether that arch or its correlative be considered as a secant or a co-secant; and by means of proportional parts, the absolute value of any arch, and conversely, may be readily obtained at sight.

Table XXXVI., Logarithmic Sines.—This table is rendered general for every degree, minute, and second, in the whole semicircle. The Table of Logarithmic Tangents, which immediately follows, is also rendered general to the same extent; and by means of proportional parts, the true value of any arch, and conversely, may be instantly obtained, without the trouble of either multiplying or dividing: this improvement, to the practical navigator, must be an object of great importance, in reducing the labour attendant on computations in Nautical Astronomy.

Table XXXVIII. has been newly computed to the nearest second of time, so that the mariner may be readily enabled to reduce the time of the moon's passage over the meridian of Greenwich to that of her passage over any other meridian. This table will be found very useful in determining the apparent time of the moon's rising or setting, and also in ascertaining the time of high water at any given place by means of Table XXXIX.

Table XLII.—This general Traverse Table, so useful in practical navigation, is arranged in a very different manner from the Traverse Tables given in the generality of nautical books; and although comprised in 38 pages, is more comprehensive than the two combined tables of 61 pages usually found in those books, under the head "Difference of Latitude and Departure." In this table, every page exhibits all the angles that a ship's course can possibly make with the meridian, expressed both in points and degrees; which does away with the necessity of consulting two tables in finding the difference of latitude and departure corresponding to any given course and distance.

Table XLIV. contains the mean right ascensions and declinations of the principal fixed stars. The eighth column of this table, which is original, and is intended to facilitate the method of finding the latitude by the altitudes of two fixed stars observed at any hour of the night, contains the true spherical distance between the stars therein contained and those preceding or abreast of them on the same horizontal line. The ninth or last column of the page contains the annual variation of that distance, expressed in seconds and decimal parts of a second. Great pains have been taken, in order to find the absolute value of the annual variation of the true spherical distance between the fixed stars; and the author trusts that he has so far succeeded as to render this part of the table permanent for a *long period of years subsequent to* 1824.

Tables XLV. and XLVI., which are adapted to the reduction of sidereal time into mean solar time, and conversely, have been newly constructed: these will be found considerably more extensive and uniform, than those generally given under the same denomination.

Tables LI. and LII. are entirely new: these will be found exceedingly useful in finding the latitude by the altitude of a celestial object observed at certain intervals from the meridian; and since they are adapted to proportional logarithms, the operation of finding the latitude thereby becomes extremely simple, and yet far more accurate than that resulting from *double altitudes*, even after *repeating* a troublesome operation, and then applying *correction* to *correction*.

Table LIV.—This table will be of service to Masters in the Royal Navy, to officers employed in maritime surveys, and to all others who may be desirous of constructing charts agreeably to Mercator's principles of projection.

Table LVI. will be found essentially useful in reducing the French centesimal division of the circle into the English sexagesimal division, and conversely; and since most of the modern French works on astronomy are now adapted to the centesimal principle, this table will be found of assistance in consulting those works;—nor will it be of less advantage to the French navigator, in enabling him readily to consult the works of the English astronomers, where the degrees, &c., are expressed agreeably to the original or sexagesimal principle.

Table LVII. is new; and although it may not immediately affect the interest of the mariner, yet it cannot fail to be useful to officers in charge of His Majesty's Victualling Stores, in consequence of the late Act of Parliament for the establishment of a new general standard or imperial gallon measure throughout the United Kingdoms.—See Practical Gauging, page 596 to 606.

Table LVIII. contains the latitudes and longitudes of all the principal sea-ports, islands, capes, shoals, rocks, &c. &c., in the known world; these are so arranged as to exhibit to the navigator the whole line of coast along which he may have occasion to sail, or on which he may chance to be employed, agreeably to the manner in which it unfolds to his view on a Mercator's chart; a mode of arrangement much better adapted to nautical purposes than the alphabetical. But since the table is not intended for general geographical purposes, the positions of places inland, which do not immediately concern the mariner, have, with a few exceptions, been purposely omitted. The time of high water, at the full and change of the moon, is given at all places where it is known; which will be found considerably more convenient than referring for it to a separate table.

The series of latitudes and longitudes that have been established, astronomically and chronometrically, by Captain William Fitzwilliam Owen, of His Majesty's ship Eden, during his recent and extensive survey along the coasts of Africa, Arabia, Madagascar, Brazil, &c., follow as an Appendix to the last-mentioned table. These series are published by the express permission of Captain Owen; and from his general knowledge as a navigator, hydrographer, and practical astronomer, there is every reason to believe that the geographical positions have been determined with astronomical exactness.

A general Victualling Table forms an addition to the Appendix; and as this exhibits the full allowance of sea provisions (calculated agreeably to the new Victualling Scale), from one man to any given number of men, it will be found useful to the Pursers of the Royal Navy, to Lieutenants serving as Commanders and Pursers, and to the gentlemen who are officially employed in the auditing of the Naval Victualling Accounts.

The sun's declination is not given in this work; nor is it necessary that it should be, since it is contained, in the most ample manner, in the *Nautical Almanac*; a work which is so truly valuable to mariners that few now go to sea without it; the judicious never will.

Having thus taken a survey of the principal part of the Tables, I must briefly notice their *description and use*;—these will be found at the commencement of the first Volume. The principles and methods of their computation are here fully detailed; and the reader is furnished with the means, in the most simple formulæ, of examining any part of the Tables; which is far more satisfactory than trusting to the author's mere word for their entire accuracy; though, I flatter myself with the hope that, in this extensive mass of figures, very few errors will be found;—at all events, none of *principle*.

My original plan had been to close the work with the description and use of the Tables, but being apprehensive that a series of Tables alone, however well arranged, or clearly illustrated, would not be sufficient to ensure general acceptation, I was induced to show their direct application to the different elements connected with the sciences of navigation and nautical astronomy, as well as to other subjects of a highly interesting nature, such as the art of gunnery, &c. &c. In this part of the work, since my design did not extend beyond an ample illustration of the various mathematical purposes to which these tables may be applied, I have restricted myself to the practical parts of the sciences on which I have had occasion to touch; because those are the points which most concern the mariner, and the commercial interests of this maritime nation. Nevertheless, wherever it has appeared necessary to notice the elementary parts of the sciences, reference has been made to relative problems in "The Young Navigator's Guide," where, it is hoped, the reader will find his inquiries fully satisfied.

The various sciences touched upon commence with a concise system of decimal arithmetic, and complete courses of plane and spherical trigonometry. In the latter, the solution of the quadrantal triangles will be found much simplified.

The practical parts of Navigation begin with parallel sailing; but, with the view of preventing the work from swelling to an unnecessary size, the cases of plane sailing, usually met with in other nautical books, have been omitted in this; as these are, in effect, no more than a mere repetition of the cases of right angled plane trigonometry under a different denomination. Middle latitude sailing will be found exceedingly simplified by means of a series of familiar analogies or proportions: and in Mercator's sailing a series of rational proportions is given; which, it is hoped, may tend to induce mariners to substitute the rules of reason for the *rules of rote;* and thus do away with the mistaken system of getting *canons by heart;* a system which has too long prevailed in the Royal Navy.

The two very useful sailings, oblique and windward, which have been hitherto little noticed by mariners, are also rendered so simple, particularly the latter, that it is to be hoped they will, ere long, be brought into general use.

In current sailing (Example 3,) the true principles of steering a vessel in a current, or tideway are familiarly illustrated. This problem cannot fail of being interesting to every person who is at all curious in the art of navigation.

The solution of a problem in great circle sailing is given, which will be found essentially useful to ships bound from the Cape of Good Hope to New South Wales: comprising a table which exhibits, at sight, all the scientific particulars attendant on the true spherical track between those two places; by which it will be seen that a saving of 585 miles may be effected by sailing near the arc of a great circle as laid down in that table; which saving ought to be an object of very high consideration to all ships bound from the Cape of Good Hope to Van Diemen's Land, or to his Majesty's colony at New South Wales with either troops or convicts; because the length of the voyage on the old track, or that deduced from the common principles of navigation, generally occasions a great scarcity of fresh water, and this, eventually, adds distress to the many privations under which those on board usually labour. In the same problem, there is a table showing the true spherical route from Port Jackson, in New South Wales, to Valparaiso, on the coast of Chili: in this route there is a saving of 745 miles when compared with that resulting from Mercator's sailing; and this must be of considerable importance to the captain of a ship sailing between these places, who is desirous of making his port in the shortest space of time; particularly since few ships can carry a liberal allowance of fresh water to serve during a passage which measures very nearly one fourth of the earth's circumference.

The introductory problems to Nautical Astronomy will be found ranged in the most natural order; all of which, except those relating to the altitudes of the objects, are concisely solved by proportional logarithms: the greater part of these will appear entirely new to the navigator. The VIth problem relating to the latitude exhibits the method of finding the latitude by an altitude of the north polar star taken at any hour of the night, which will be found very useful in all parts of the northern hemisphere.—The VIIIth problem shows the method of finding the latitude by the altitudes of two stars taken at any time of the night, agreeably to the computed spherical distance between them contained in Table XLIV; this method of ascertaining the latitude is general; it will be found very correct, and far less troublesome than that by *double altitudes* which immediately precedes it.—Problems IX, X, XI, and XII, contain *new* and *accurate methods* of deducing the latitude from the altitudes of the celestial bodies observed at given intervals from the meridian: the operation consists of very little more than the common *addition* of *three proportional logarithms*, and yet the latitude resulting from it will always be as correct as that deduced from the object's meridional altitudes, provided the watch shows apparent time at the place of observation, and the altitudes be taken within the limits prescribed. These problems will be found highly advantageous to the practical navigator; because, in the event of the sun's, or other celestial object's meridional altitude being neglected to be taken, or of it's being obscured by clouds at the time of transit, he is, thus, provided with the most safe and ready means of determining his latitude with as much certainty as if the altitude of the object had been observed actually upon the meridian either above or below the pole. See remark, page 368.

A most ingenious problem in this part of the work, for determining the latitude, which for neatness and general utility stands unrivalled, has been communicated to the author by the scientific Captain W. F. W. Owen.

In the methods of computing the altitudes of the heavenly bodies, the solutions to the several problems are rendered exceedingly concise and explicit.

The IIIrd, IVth, Vth, and VIth problems relating to the longitude contain the methods of finding the longitude by a chronometer and the respective altitudes of the sun, stars, planets, and the moon; the three last of which will be found considerably elucidated.

The lunar observations commence with the VIIth problem on the longitude. In this problem *thirteen methods* are given for reducing the apparent central distance between the moon and sun, a fixed star, or planet, to the true central distance; several of which are entirely original, and all of them adapted to solve this interesting and important problem in the most simple and expeditious manner.

• In the series of problems relative to finding the variation of the compass by amplitudes, azimuths, transits of the fixed stars and planets, and by observations of the circumpolar stars, Problem II exhibits a *new method* for computing the true azimuth of a celestial object: and Problems V and VI, contain the methods of reducing or correcting the true and the magnetic courses, between two places, agreeably to any given variation of the compass.—An improved azimuth compass card is described in this part of the work, which may be applied to the determination of the longitude by the lunar observations:—See the last two paragraphs in page 499.

The series of problems for finding the apparent times of the rising or setting of the celestial bodies, and of the beginning or the end of twilight;—and that for determining the interval of time between the rising or setting of the sun's upper and lower limbs, it is hoped will prove acceptable to the lovers of the science of Nautical Astronomy;—likewise the art of Dialling, which, although it may appear foreign or irrelevant to the pursuits of the mariner, cannot fail to be interesting as a branch of science. It is here treated of in a familiar manner.

The IVth Problem in the mensuration of heights and distances, exhibits the method whereby the officers on board two ships of war can readily ascertain their absolute distance from any fort or garrison which they may be directed to cannonade;—after which follow several problems that will be found exceedingly useful on many military occasions.—See remark at page 533, and also at page 543. Problem XI. showing the method of reducing a base line, measured on any elevated horizontal plane, to its true level at the surface of the sea; and Problem XIII. exhibiting a new rule for finding the height of a mountain, or other eminence, by means of two barometers and two thermometers, may be of considerable use to engineers, or to others employed in conducting surveys. A problem is also given for finding the direct course steered by a ship seen at a distance; and being a subject highly interesting to all nautical persons, it is reduced to every desirable degree of simplicity both by geometry and trigonometry.

All the problems in Practical Gunnery are readily solved by logarithms: it contains three very concise tables which considerably facilitate the operation for finding the greatest range of a shot or shell, and the elevation of the piece to produce that range. A small table is also given, which will be found extremely useful in problems relating to shells, when it is required that they should strike an object at a given distance.—The rules and operations for computing the time of flight of a shell in Problems XXVII, XXXIV, and XXXVI, will be found very simple and concise.

Although the art of gunnery may, in some measure, be considered as not being immediately connected with that of navigation; yet it is a subject with which all naval officers ought to have some acquaintance; since it very frequently happens, in time of war, that they are called upon to go on shore with a party of men for the purpose of working the great guns of the besieging batteries in co-operation with his Majesty's Land Forces:—and since this truly interesting art is here, for the first time, unveiled of its mystic dress, and reduced to a state of simplicity, every officer may make himself thoroughly acquainted with it in a very little time, without any other assistance than that afforded in this treatise.

The problems on the mensuration of planes may be found useful on many occasions; particularly to persons employed in carrying on surveys on shore.

Practical Gauging contains a few interesting problems; the last of which will be found essentially useful to such persons as may have occasion to purchase wine, or spirits on his Majesty's account in foreign countries; because it enables them to ascertain, in a very few minutes, the absolute number of gallons contained in any given quantity of foreign liquor, agreeably to the newly established standard or Imperial gallon measure.

The compendium of Practical Navigation, given in this volume, exhibiting the direct manner of making out a *day's work* at sea, is intended for the benefit of such persons as may be unacquainted with the elements of geometry and trigonometry: and includes the true method of finding the index error of a sextant or quadrant so as to guard against the error arising from the elasticity or spring of the index bar, with the method of applying the corrections to altitudes taken on shore by means of an artificial horizon.

A new and correct method of finding the longitude of a place on shore by means of the moon's altitude (observed in an artificial horizon,) and the apparent time of observation, follows the above compendium; and will be found of considerable utility in settling the geographical positions of places inland or along the sea coast. An Appendix, which concludes the first volume, contains every thing relating to the doctrine of compound interest; and developes the extraordinary powers of logarithmical numbers in a more striking point of view than any other department of science to which they have been applied.

I have thus given a brief account of the more original parts of the subjects comprised in this work, the completion of which has cost me several years of incessant labour; during which time I had to contend with as many infirmities, vexations, and disappointments as generally fall to the lot of persons doomed to drudge through the toils of life: but stimulated by the hope of ultimately succeeding in rendering myself useful to the Naval Service of his Majesty, and to the nautical world in general, I have been so far enabled to bear up against the vicissitudes of health and fortune, as to bring my long and arduous task to a close.

How far I have succeeded in my endeavour to supply the desideratum which has been hitherto felt by navigators, it is not for me, but for a generous British public to determine: to their decision I submit my labours, under the conviction that, whatever may be the defects in its execution, they will do justice to my motives, in this attempt to lessen the existing obstructions in the way of attaining a practical knowledge of the elements of Navigation and Nautical Astronomy.

THOMAS KERIGAN.

Portsmouth, December 1st.,
1827.

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THE

DESCRIPTION AND USE

OF THE

TABLES;

WITH THE

PRINCIPLES UPON WHICH THEY HAVE BEEN COMPUTED.

TABLE I.

To convert Longitude, or Degrees, into Time, and conversely.

THIS Table consists of six compartments, each of which is divided into two columns: the left-hand column of each compartment contains the longitude, expressed either in degrees, minutes, or seconds; and the right-hand column the corresponding time, either in hours, minutes, seconds, or thirds. The proper signs, for degrees and time, are placed at the top and bottom of their respective columns in each compartment, with the view of simplifying the use of the Table:—hence it will appear evident that if the longitude be expressed in degrees, the corresponding time will be either in hours or minutes; if it be expressed in minutes, the corresponding time will be either in minutes or seconds; and if it be expressed in seconds, the corresponding time will be expressed either in seconds or thirds. The converse of this takes place in converting time into longitude.

The extreme simplicity of the Table dispenses with the formality of a rule in showing its use, as will obviously appear by attending to the following examples.

Example 1.

Required the time corresponding to $47^\circ 47' 47''$ of longitude?

47 degrees, time answering to which in the Table is	$3^h\ 8^m\ 0^s\ 0^t$
47 minutes, answering to which is	0. 3. 8. 0
47 seconds, answering to which is	0. 0. 3. 8
Lon. $47^\circ 47' 47''$, the time corresponding to which is .	$3^h\ 11^m\ 11^s\ 8^t$

Example 2.

Required the longitude corresponding to the given time	8h 52m 28s ?
8 hours, longitude answering to which in the Table is	120°0′0″
. 52 minutes, answering to which is	13.0.0
. . 28 seconds, answering to which is	0.0.7
Time 8h 52m 28s, the longitude corresponding to which is	133°0′7″

Besides the use of this Table in the reduction of longitude into time, and the contrary, it will also be found very convenient in problems relating to the Moon, where it becomes necessary to turn the right ascension of that object into time.

Example.

The right ascension of the Moon is 355°44′48″; required the corresponding time?

355 degrees, time answering to which in the Table is	23h 40m 0s 0t
. 44 minutes, answering to which is	0. 2. 56. 0
. . 48 secs., answering to which is	0. 0. 3. 12
Right ascension 355°44′48″, the time corresponding to which is	23h 42m 59s 12t

Since the Earth makes one complete revolution on its axis in the space of 24 hours, it is evident that every part of the equator will describe a great circle of 360 degrees in that time, and, consequently, pass the plane of any given meridian once in every 24 hours; whence it is manifest that any given number of degrees of the equator will bear the same proportion to the great circle of 360 degrees that the corresponding time does to 24 hours; and that any given portion of time will be in the same ratio to 24 hours that its corresponding number of degrees is to 360.

Now since 24 hours are correspondent or equal to 360 degrees, 1 hour must, therefore, be equal to 15 degrees; 1 minute of time equal to 15 minutes of a degree; 1 second of time to 15 seconds of a degree, and so on. And as 1 minute of time is thus evidently equal to 15 minutes or one fourth of a degree, it is very clear that 4 minutes of time are exactly equal to 1 degree; wherefore since degrees and time are similarly divided, we have the following general rule for converting longitude into time, and *vice versa*.

Multiply the given degrees by 4, and the product will be the corresponding time:—observing that seconds multiplied by 4 produce thirds; minutes, so multiplied, produce seconds, and degrees minutes; which, divided by 60, will give hours. The converse of this is evident:—thus, reduce the hours to minutes; then these minutes, divided by 4, will give degrees; the seconds, so divided, will give minutes, and the thirds, if any, seconds. Hence the principles upon which the Table has been computed. The following examples are given for the purpose of illustrating the above rule.

Example 1.

Required the time corresponding to 36°44′32″?

Given degrees = 36°44′32″
Multiplied by 4

Corresponding time $2^h 26^m 58^s 8^t$

Example 2.

Required the degrees corresponding to $3^h 45^m 48^s 20^t$?

Given time = $3^h 45^m 48^s 20^t$
60

Divide by 4) 225 . 48 . 20

Corresponding degs. 56°27′5″

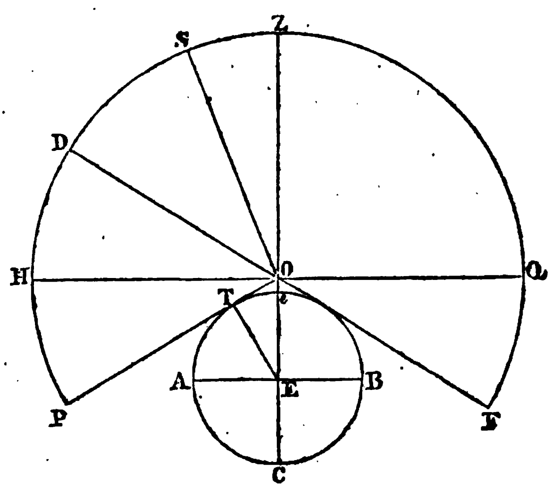
Table II.

Depression of the Horizon.

The depression or dip of the horizon is the angle contained between a horizontal line passing through the eye of an observer, and a line joining his eye and the visible horizon.

This Table contains the measure of that angle, which is a correction expressed in minutes and seconds answering to the height of the observer's eye above the horizon; and which being subtracted from the observed central altitude of a celestial object, when the fore observation is used, or added thereto in the back observation, will show its apparent central altitude. The corrections in this Table were deduced from the following considerations, and agreeably to the principles established in the annexed diagram.

Let the small circle A B C *e* represent the terrestrial globe, and *e* O the height of the observer's eye above its surface; then H O Q, drawn parallel to a tangent line to the surface at *e*, will be the true or sensible horizon of the observer at O; and O P, touching the surface at T, the apparent horizon.



Let S be an object whose altitude is to be taken by a fore observation, by bringing its image in contact with the apparent horizon at P; then the angle S O P will be the apparent altitude, which is evidently greater than the true altitude S O H by the arc P H, expressed by the angle of horizontal depression P O H. But if the altitude of the object S is to be taken by a back observation, then, the observer's back being necessarily turned to the object, his apparent horizon will be in the direction O F, and his whole horizontal plane represented by the line D O F; in which case his back horizon O D, to which he brings the object S, will be as much elevated above the plane of the true horizon H O Q as the apparent horizon O F will be depressed below it; because, when two straight lines intersect each other, the opposite angles will be equal. (Euclid, Book I., Prop. 15.) In this case it is evident that the arc or apparent altitude S D is too little; and that it must be augmented by the arc D H = the angle of horizontal depression F O Q, in order to obtain the true altitude S H. Hence it is manifest that altitudes taken by the fore observation must be diminished by the angle of horizontal depression, and that in back observations the altitudes must be increased by the value of that angle.

The absolute value of the horizontal depression may be established in the following manner:—From where the apparent horizon O P becomes a tangent to the earth's surface at T (the point of contact where the sky and water seem to meet) let a straight line be drawn to the centre E, and it will be perpendicular to O P (Euclid, Book III., Prop. 18): hence it is obvious that the triangle E T O is right-angled at T. Now, because O T is a straight line making angles from the point O upon the same side of the straight line O E, the two angles E O T and T O H are together equal to the angle E O H (Euclid, Book I., Prop. 13); but the angle E O H is a right angle; therefore the angle of depression T O H is the complement of the angle E O T, or what the latter wants of being a right angle: but the angle T E O is also the complement of the angle E O T (Euclid, Book I., Prop. 32); therefore the angle T E O is equal to the angle of horizontal depression; for magnitudes which coincide with one another, and which exactly fill up the same space, are equal to one another. Then, in the right-angled rectilineal triangle E T O, there are given the perpendicular T E, = the earth's semidiameter, and the hypothenuse E O, = the sum of the earth's semidiameter and the height of the observer's eye, to find the angle T E O = the angle of horizontal depression T O H:—hence the proportion will be, as the hypothenuse E O is to radius, so is the perpendicular T E to the cosine of the angle T E O, which angle has been demonstrated to be equal to the angle of horizontal depression H O P. But because very small arcs cannot be strictly determined by cosines, on account of the differences being so very trivial at the beginning of the quadrant as to run several seconds without producing any sensible alteration, and there being no rule for showing why one second should be preferred to another in a choice of so many, the following method is therefore given as the most eligible for computing the true value of the horizontal depression, and which is deduced from the 36th Prop. of the third Book of Euclid.

Because the apparent horizon O P touches the earth's surface at T, the square of the line O T is equal to the rectangle contained under the two lines C O and *e* O. Now as the earth's diameter is known to be 41804400 English feet, and admitting the height of the observer's eye *e* O to be 290 feet above the plane of the horizon; then, by the proposition, the square root of C O, 41804690 × *e* O, 290 = the line O T, 110105.75 feet; the distance of the visible horizon from the eye of the observer independent of terrestrial refraction.

Then, in the right-angled rectilineal triangle E T O, there are given the perpendicular E T = 20902200 feet, the earth's semidiameter, and the base O T = 110105.75, to find the angle T E O. Hence,

As the perpendicular T E =	20902200 feet,	log. arith. compt. =	2.679808
Is to the radius	90°0′0″	log. sine . . .	10.000000
So is the base O T = . .	110105.75 feet,	log.	5.041810
To the angle T E O = .	18′7″ =	log. tang. . . .	7.721618

But it has been shown that the angle T E O, thus found, is equal to the angle H O P; therefore the true value of the angle of horizontal depression H O P, is 18′7″. Now, according to Dr. Maskelyne, the horizontal depression is affected by terrestrial refraction, in the proportion of about one-tenth of the whole angle; wherefore, if from the angle of horizontal depression 18′7″ we take away the one-tenth, viz. 1′49″, the allowance for terrestrial refraction, there will remain 16′18″ for the true horizontal depression, answering to 290 feet above the level of the sea. The principles being thus clearly established, it is easy to deduce many simple formulæ therefrom, for the more ready computation of the horizontal depression; of which the following will serve as an example.

To the proportional log. of the height of the eye in feet, (estimated as seconds,) add the constant log. .4236, and half the sum will be the proportional log. of an arc; which being diminished by one-tenth, for terrestrial refraction, will leave the true angle of horizontal depression.

Example.

Let the height of the eye above the level of the sea be 290 feet, required the depression of the horizon corresponding thereto?

Height of the eye 290 feet, esteemed as secs.=4′50″, propor.log.=1.5710
Constant log. 4236

Sum = 1.9946

Arc = ... 18′7″ Proportional log. .9973
Deduct one-tenth = 1.49, for terrestrial refraction.

True horizontal depression 16′18″, the same as by the direct method.

In using the Table, it may not be unnecessary to remark that it is to be entered with the height of the eye above the level of the sea, in the column marked *Height, &c.*; opposite to which, in the following column, stands the corresponding correction; which is to be subtracted from the observed altitude of a celestial object when taken by the fore observation; but to be added thereto when the back observation is used, as before stated. Thus the dip, answering to 20 feet above the level of the sea, is 4′17″

Table III.

Dip of the Horizon at different Distances from the Observer.

If a ship be nearer to the land than to the visible horizon when unconfined, and that an observer on board brings the image of a celestial object in contact with the line of separation betwixt the sea and land, the dip of the horizon will then be considerably greater than that given in the preceding Table, and will increase as the distance of the ship from the land diminishes: in this case the ship's distance from the land is to be estimated, with which and the height of the eye above the level of the sea, the angle of depression is to be taken from the present Table. Thus, let the distance of a ship from the land be 1 mile, and the height of the eye above the sea 30 feet; with these elements enter the Table, and in the angle of meeting under the latter and opposite to the former will be found 17′ which, therefore, is the correction to be applied by subtraction to the observed altitude of a celestial object when the fore observation is used, and *vice versa.*

The corrections in this Table were computed after the following manner; viz.,—

Let the estimated distance of the ship from the land represent the base of a right-angled triangle, and the height of the eye above the level of the sea its perpendicular; then the dip of the horizon will be expressed by the measure of the angle opposite to the perpendicular: hence, since the base and perpendicular of that triangle are known, we have the following general

Rule.—As the base or ship's distance from the land, is to the radius, so is the perpendicular, or height of the eye above the level of the sea to the tangent of its opposite angle, which being diminished by one-tenth, on account of terrestrial refraction, will leave the correct horizontal dip, as in the subjoined example.

Let the distance of a ship from the land be 1 mile, and the height of the eye above the level of the sea 25 feet, required the corresponding horizontal dip

As distance 1 mile, or 5280 feet,		Logarithm Ar. Comp. =	6.277366
Is to radius . . .	90°,	Logarithmic Sine . .	10.000000
So is height of the eye	25 feet,	Logarithm	1.397940
To Angle	16′. 17″	= Log. Tang. =	7.675306
Deduct one-tenth for terrestrial refraction	1.37		
True horizontal dip = . . .	14′.40″, or 15′ nearly as in the Table.		

Remark.—Although a skilful mariner can always estimate the distance of a ship from the shore horizon to a sufficient degree of accuracy for taking out the horizontal dip from the Table, yet since some may be desirous of obtaining the value of that dip independently of the ship's distance from the land, and consequently of the Table, the following rule is given for their guidance in such cases:—

Let two observers, the one being as near the mast head as possible, and the other on deck immediately under, take the sun's altitude at the same instant. Then to the arithmetical complement of the logarithm of the difference of the heights, add the logarithm of their sum, and the logarithmic sine of the difference of the observed altitudes; the sum, rejecting 10 from the index, will be the log. sine of an arch; half the sum of which and the difference of the observed altitudes will be the horizontal dip corresponding to the greatest altitude, and half their difference will be that corresponding to the least altitude.

Example.

Admit the height of an observer's eye at the main-topmast head of a ship close in with the land, to be 96 feet, that of another (immediately under) on deck 24 feet; the altitude of the sun's lower limb found by the former to be 39°37′, and by the latter, taken at the same instant, 39°21′; required the dip of the shore horizon corresponding to each altitude?

Height of mast head observer	96 feet.		
Height of deck observer .	24 do.		
Difference of heights . .	72 do.,	Log. Ar. Comp. =	8.142667
Sum of ditto	120 do.	Logarithm .	2.079181
Difference of altitudes .	16′	Log. sine .	7.667845
Arch	= 26′40″	Log. sine	7.889693
Sum	= 42′40″, ½ = 21′20″ = dip to the greatest height		
Diff.	= 10.40, ½ = 5.20 = dip to the least height.		

Note.—When the dip answering to an obstructed horizon is thus carefully determined, the ship's distance from the land may be ascertained to the greatest degree of accuracy by the following rule: *viz.* As the Log. tangent of the horizontal dip of the shore horizon is to the logarithm of the height of the eye at which that dip was determined, so is radius to the true distance.

Thus, in the above example where the horizontal dip has been determined to the corresponding height of the eye and difference of altitudes,

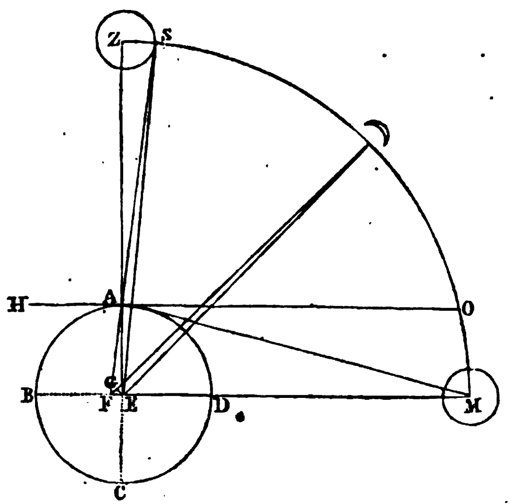
As horizontal dip =	5′20″	Log. tang. ar. compt. =	2.809275
Is to the height of the eye	24 feet,	Logarithm . . .	1.380211
So is radius	90°	Logarithmic sine .	10.000000
To true distance . .	15469.8 feet .	Logarithm =	4.[illegible]89486

The same result will be obtained by using the greatest dip and its corresponding height; and since the operation is so very simple, it cannot fail of being extremely useful in determining a ship's true distance from the shore.

TABLE IV.

Augmentation of the Moon's Semidiameter.

Since it is the property of an object to increase its apparent diameter in proportion to the rate in which its distance from the eye of an observer is diminished; and, since the moon is nearer to an observer, on the earth, when she is in the zenith than when in the horizon, by the earth's semidiameter; she must, therefore, increase her semidiameter by a certain quantity as she increases her altitude from the horizon to the zenith. This increase is called the augmentation of the moon's semidiameter, and depends upon the following principles,



Let the circle A B C D represent the earth; A E its semidiameter, and M the moon in the horizon. Let A represent the place of an observer on the earth's surface; B D M his rational horizon, and H A O, drawn parallel thereto, his sensible horizon extended to the moon's orbit; join A M, then A M E is the angle under which the earth's semidiameter A E is seen from the moon M, which is equal to the angle M A O, the moon's horizontal parallax; because the straight line A M which falls upon the two parallel straight lines E M and A O makes the alternate angles equal to one another. (Euclid, Book I. Prop. 29.) Let the moon's horizontal parallax be assumed at 57′.30″, which is about the parallax she has at her mean distance from the earth; then in the right angled triangle A E M, there are given the angle A M E=57′.30″, the moon's horizontal parallax, and the side A E=3958.75 miles, the earth's semidiameter; to find the hypothenuse A M=the moon's distance from the observer at A: hence by trigonometry,

As the angle at the moon, A M E=57′.30″	Log. sine ar. comp.	1.776626
Is to the earth's semidiameter=A E=3958.75 miles,	Log. .	3.597558
So is radius . . . . 90° . . .	Log. sine	10.000000
To moon's horizontal distance A M=236692.35 miles,	Log. .	5.374184

Now, because the moon is nearer to the observer at A, by a complete semidiameter of the earth when in the zenith Z, than she is when in the horizon M, as appears very evident by the projection; and, because the earth's semidiameter A E thus bears a sensible ratio to the moon's distance; it hence follows that the moon's semidiameter will be apparently increased when in the zenith, by a small quantity called its augmentation; and which may be very clearly illustrated as follows, viz.

Let the arc Z O M represent a quarter of the moon's orbit; Z her place in the zenith, and Z S her semidiameter: join E Z, A S, and E S; then the angles Z E S and Z A S will represent the angles under which the moon's semidiameter is seen from the centre and surface of the earth; their difference, viz., the angle A S E is, therefore, the augmentation of the moon's semidiameter, which may be easily computed; thus—

In the oblique angled triangle A S E, there are given the side A E =3958.75 miles, the earth's semidiameter; the side A S,=A M—A E= 232733.6 miles, the moon's distance when in the zenith from the observer at A; and the angle A E S=15′ 30″, the moon's mean semidiameter; to find the angle A S E=the greatest augmentation corresponding to the given horizontal parallax and horizontal semidiameter: therefore,

As moon's zenith distance =	A Z=232733.6 miles,	Log. ar. co.	4.633141
Is to moon's semidiameter	A E S = 15′.30″	Log. sine	7.654056
So is earth's semidiameter	E A = 3958.75 miles,	Log. . .	3.597558
To augment. of semidiam.	A S E = 0′.16″	Log. sine	5.884755

Now, having thus found the augmentation of the Moon's semidiameter, when in the zenith, answering to the assumed horizontal parallax and horizontal semidiameter; the increase of semidiameter at any given altitude, from the horizon to the zenith, may be computed in the following manner.

Let S A be produced to F. and draw E F parallel to Z S; then will E F represent the greatest augmentation to the radius E Z. Let the moon be in any other part of her orbit, as at ☽ with an altitude of 45 degrees; join ☽ E, and ☽ F, and make ☽ G= ☽ E; then will E G (the measure of the angle E ☽ G to the radius E ☽,) be the augmentation corresponding to the given altitude. Then, in the right angled triangle E G F, right angled at G, there are given the angle E F G=45 degrees, the moon's apparent altitude, and the side E F=16 seconds, the augmentation of semidiameter when in the zenith; to find the side E G, which expresses the augmentation of semidiameter at the given altitude. And, since the angles expressing the augmentations are so very small, the measure of each may be substituted for its sine, which will simplify the calculation; thus,

As radius	90°0′.0″	Log. sine ar. comp.	0.000000
Is to moon's greatest augment. of semidiam.=	E F 16″,	Log. =	1.204120
So is moon's given apparent alt.= ∠	E F G, 45°	Log. sine =	9.849485
To the augmentation, or side	E G = 11″.31.	Log. =	1.053605

which, therefore, is the augmentation of the moon's semidiameter corresponding to the given apparent altitude of 45 degrees; horizontal semidiameter 15′.30″ and horizontal parallax 57′.30″

Explanation of the Table.

This Table contains the augmentation of the moon's semidiameter (determined after the above manner,) to every third degree of altitude: the augmentation is expressed in seconds, and is to be taken out by entering the Table with the moon's horizontal semidiameter at the top, as given in the Nautical Almanac, and the apparent altitude in the left-hand column; in the angle of meeting will be found a correction, which being applied by addition to the moon's horizontal semidiameter will give the true semidiameter, corresponding to the given altitude. Thus the augmentation answering to moon's apparent altitude 30 degrees, and horizontal semidiameter 16′30″ is 9 seconds; and that corresponding to altitude 60° and semidiameter 16′ is 14 seconds.

TABLE V.

Contraction of the semidiameters of the Sun and Moon.

Since all parts of the horizontal semidiameter of the sun or moon are equally elevated above the horizon, all those parts must be equally affected by refraction, and thereby cause the horizontal semidiameter to remain invariable. But when the semidiameter is inclined to the plane of the horizon, the lower extremity will be so much more affected by refraction than the upper, as to suffer a sensible contraction, and thus cause the semidiameter, so inclined, to be something less than the horizontal semidiameter given in the Nautical Almanac. Hence it is manifest that the semidiameter of a celestial object, measured in any other manner than that parallel to the plane of the horizon will be always less than the true semidiameter by a certain quantity:—this quantity, called the contraction of semidiameter is contained in the present Table; the arguments of which are, the apparent altitude of the object in the left-hand column, and at the top the angle comprehended between the measured diameter and that parallel to the plane of the horizon; in the angle of meeting will be found a correction, which being subtracted from the horizontal semidiameter in the Nautical Almanac, will leave the true semi-diameter. Thus, let the sun's or moon's apparent altitude be 5 degrees, and the inclination of its semidiameter 72 degrees; now, in the angle of meeting, of these arguments, stands 23 seconds; which, therefore, is the contraction of semidiameter, and which is to be applied by subtraction to the semidiameter given in the Nautical Almanac.

To compute the contraction of Semidiameter.

Rule.—Find by Table VIII. the refraction corresponding to the object's apparent central altitude, and also the refraction answering to that altitude augmented by the semidiameter; (which, for this purpose, may be estimated at 16 minutes,) and their difference will be the contraction of the vertical semidiameter. Now, having thus found the contraction corresponding to the vertical semidiameter, that answering to a semidiameter which forms any given angle with the plane of the horizon, will be found by multiplying the vertical contraction by the square of the angle of inclination.

Example.

Let the sun's or moon's apparent central altitude be 3° and the inclination of its semidiameter to the plane of the horizon 72°; required the contraction of the semidiameter?

Apparent central altitude . 3° 0′ Refraction=14′36″
Do. augmented by semidiam. = 3°16′ Ditto . =13.46.

Contraction of the vertical semidiameter . . . 0′50″ Log.=1.698970
Inclination of semidiameter . =72° twice the log. sine . =19.956412

Required contraction of semidiameter . . . . 45″.22 Log.=1.655382

And so on of the rest.—It is to be remarked, however, that the correction arising from the contraction of the semidiameter of a celestial object is very seldom attended to in practice at sea.

Table VI.

Parallax of the Planets in Altitude.

The arguments of this Table are the apparent altitude of a planet in the left or right-hand margin, and its horizontal parallax at the top; under the latter, and opposite the former, stands the corresponding parallax in altitude; which is always to be applied by addition to the planets apparent altitude. Hence, if the apparent altitude of a planet be 30 degrees, and its horizontal parallax 27 seconds, the corresponding parallax in altitude will be 23 seconds; additive to the apparent altitude.

The parallaxes of Altitude in this Table were computed by the following

Rule.—To the proportional logarithm of the planet's horizontal parallax add the log. secant of its apparent altitude, and the sum, abating 10 in the index, will be the proportional logarithm of the parallax in altitude.

Example.

If the horizontal parallax of a planet be 23 seconds, and its apparent altitude 30 degrees; required the parallax in altitude?

Horizontal parallax of the planet=23 Seconds, proportional log.=		2. 6717
Apparent altitude of ditto . .	30 Degrees, log. secant . .	10. 0625
Parallax in altitude	20 Seconds, proportional log.	2. 7342

Table VII.

Parallax of the Sun in Altitude.

The difference between the places of the sun, as seen from the surface and centre of the earth at the same instant, is called his parallax in altitude, which may be computed in the following manner.

To the log. cosine of the sun's apparent altitude, add the constant log. 0. 945124, (the log. of the sun's mean horizontal parallax estimated at 8″. 813,) and the sum, rejecting 10 from the index, will be the log. of the parallax in altitude; as thus,

Given the sun's apparent altitude 20 degrees; required the corresponding parallax in altitude?

Sun's apparent altitude 20 degrees, log. cosine . .	9. 972986
Constant log.	0. 945124
Parall. corresponding to the given altitude 8″. 282 Log.	0. 918110

This Table, which contains the correction for parallax, is to be entered with the sun's apparent altitude in the left-hand column; opposite to which, in the adjoining column, stands the corresponding parallax in altitude;—thus, the parallax answering to 10° apparent altitude is 9 seconds; that answering to 40° apparent altitude is 7 seconds, &c. &c.—And since the parallax of a celestial object causes it to appear something lower in the heavens, than it really is; this correction for parallax, therefore, becomes always additive to the sun's apparent altitude.

Table VIII.

Mean Astronomical Refraction.

Since the density of the atmosphere increases in proportion to its proximity to the earth's surface, it therefore causes the ray of light issuing from a celestial object to describe a curve, in its passage to the horizon; the convex side of which is directed to that part of the heavens to which a tangent to that curve at the extremity of it which meets the earth, would be directed. Hence it is, that the celestial objects are apparently more elevated in the heavens than they are in reality; and this apparent increase of elevation or altitude is called the refraction of the heavenly bodies; the effects of which are greatest at the horizon, but gradually diminish as the altitude increases, so as to entirely vanish at the zenith.

In this Table the refraction is computed to every minute in the first 8 degrees of apparent altitude; consequently this part of the Table is to be entered with the degrees of apparent altitude at the top or bottom, and the minutes in the left-hand column: in the angle of meeting, stands the refraction.

In the rest of the Table the apparent altitude is given in the vertical columns, opposite to which in the adjoining columns will be found the corresponding refraction. Thus, the refraction answering to 3°27′ apparent altitude, is 13′14″; that corresponding to 9°46′ is 5′52″; that corresponding to 17°55′ is 2′54″, and so on. The refraction is always to be applied by subtraction to the apparent altitude of a celestial object, on account of its causing such object to appear under too great an angle of altitude. The refractions in this Table are adapted to a medium state of the atmosphere; that is, when the Barometer stands at 29.6 inches, and the Thermometer at 50 degrees; and were computed by the following *general rule*, the horizontal refraction being assumed at 33 minutes of a degree.

To the constant log. 9.999279 (the log. cosine of 6 times the horizontal refraction) add the log. cosine of the apparent altitude; and the sum, abating 10 in the index, will be the log. cosine of an arch. Now, one-sixth the difference between this arch and the given apparent altitude will be the mean astronomical refraction answering to that altitude.

Example.

Let the apparent altitude of a celestial object be 45°, required the corresponding refraction?

Constant log.			9.999279
Given apparent altitude	45°0′0″	Log. cosine	9.849485
Arch	45°5′42″	Log. cosine	9.848764

Difference . . . . . 0°5′42″ ÷ 6 = 0′57″; which, therefore, is the mean astronomical refraction answering to the given apparent altitude.

TABLE IX.

Correction of the Mean Astronomical Refraction.

Since the refraction of the heavenly-bodies depends on the density and temperature of the atmosphere, which are ever subject to numberless variations; and since the corrections contained in the foregoing Table are adapted to a medium state of the atmosphere, or when the barometer stands at 29.6 inches, and the thermometer at 50 degrees: it hence follows, that when the density and temperature of the atmosphere differ from those quantities, the amount of refraction will also differ, in some measure, from that contained in the said foregoing Table. To reduce, therefore, the corrections in that Table to other states of the atmosphere, the present Table has been computed; the arguments of which are, the apparent altitude in the left or right hand margin, the height of the thermometer at the top, and that of the barometer at the bottom of the Table; the corresponding corrections will be found in the angle of meeting of those arguments respectively, and are to be applied, agreeably to their signs, to the mean refraction taken from Table VIII, in the following manner:—

Let the apparent altitude of a celestial object be 5 degrees; the height of the barometer 29.15 inches, and that of the thermometer 48 degrees; required the true atmospheric refraction?

Apparent altitude 5 degrees,—mean refraction in Table VIII = . .		9′.54″
Opposite to 5 degrees, and over 29.15, in Table IX, stands . .	−	0. 9
Opposite to 5 degrees, and under 48 degrees, in ditto	+	0. 3
True atmospheric refraction, as required		9′.48″

The correction of the mean astronomical refraction, may be computed by the following rule, viz.

As the mean height of the barometer, 29.6 inches, is to its observed height, so is the mean refraction to the corrected refraction; now, the difference between this and the mean refraction will be the correction for barometer, which will be affirmative or negative, according as it is greater or less than the latter.—And,

As 350 degrees* increased by the observed height of Fahrenheit's thermometer, are to 400 degrees†, so is the mean refraction to the corrected refraction; the difference between which, and the mean refraction, will be the correction for thermometer; which will be affirmative or negative, according as it is greater or less than the latter.

* Seven times 50 degrees, the mean temperature of the atmosphere.

† Eight times 50 degrees, the mean temperature of the atmosphere.

Example 1.

Let the apparent altitude be 1 degree, the mean refraction 24′.29″, the height of the barometer 28.56 inches, and that of the thermometer 32 degrees; required the respective corrections for barometer and thermometer?

As mean height of barometer . .	29.60.	Log. ar. co. . .	8.528708
Is to observed height of ditto . .	28.56.	Log.	1.455758
So is mean refraction 24′.29″= .	1469″	Log.	3.167022
To corrected refraction	1417″	Log.	3.151488

Correction for barometer . . . . − 52″, which is negative, because the corrected refraction is the least.

And

As 350° + 32°=	382°	Log. ar. co. . .	7.417937
Is to	400°	Log.	2.602060
So is mean refraction 24′.29″= .	1469″	Log.	3.167022
To corrected refraction . . .	1538″	Log.	3.187019

Correction for thermometer . . + 69″= 1′.9″, which is affirmative, because the corrected refraction is the greatest.

Example 2.

Let the apparent altitude be 7 degrees, the mean refraction 7′.20″, the height of the barometer 29.75 inches, and that of the thermometer 72 degrees; required the respective corrections for barometer and thermometer?

As mean height of barometer . .	29.60.	Log. ar. co. . .	8.528708
Is to observed height of ditto . .	29.75.	Log.	1.473487
So is mean refraction 7′.20″ = . .	440″	Log.	2.643453
To corrected refraction	442″	Log.	2.645648

Correction for barometer . . . . . + 2″, which is affirmative.

And

As 350° + 72°=	422°	Log. ar. co. . .	7.374688
Is to	400°	Log.	2.602060
So is mean refraction 7′.20″ = . .	440″	Log.	2.643453
To corrected refraction	417″	Log.	2.620201

Correction for thermometer . . . − 23″, which is negative.

Table X.

To find the Latitude by an Altitude of the North Polar Star.

The correction of altitude, contained in the third column of this Table, expresses the difference of altitude between the north polar star, and the north celestial pole, in its apparent revolution round its orbit, as seen from the equator: the correction of altitude is particularly adapted to the beginning of the year 1824; but by means of its annual variation, which is determined for the sake of accuracy to the hundredth part of a second, it may be readily reduced to any subsequent period, (with a sufficient degree of exactness for all nautical purposes,) for upwards of half a century, as will be seen presently.

The Table consists of five compartments; the left and right hand ones of which, are each divided into two columns containing the right ascension of the meridian: the second compartment, which forms the third column in the Table, contains the correction of the polar star's altitude: the third compartment consists of five small columns, in which are contained the proportional parts corresponding to the intermediate minutes of right ascension of the meridian; by means of which the correction of altitude, at any given time, may be accurately taken out at the first sight: the fourth compartment contains the annual variation of the polar star's correction, which enables the mariner to reduce the tabular correction of altitude to any future period: for, the product of the annual variation, by the number of years and parts of a year elapsed between the beginning of 1824, and any given subsequent time, being applied to the correction of the polar star's altitude by addition or subtraction, according to the prefixed sign, will give the true correction at such subsequent given time.

Example 1.

Required the correction of the polar star's altitude in January 1834, the right ascension of the meridian being 6 hours and 22 minutes?

Correction of altitude answering to 6h 20m, is		0°16′ 9″
Proportional part to 2 minutes of right ascension		0.50
Correction of polar star's altitude in January 1824= . . .		0.15.19
Annual variation of correction . .	+ 2″.90	
Number of years after 1824	10	
Product	+ 29″.0= . . .	+ 0.29
Correction of the polar star's altitude in Jan. 1834, as required		0°15′48″

Example 2.

Required the correction of the polar star's altitude in January 1854, the right ascension of the meridian being 5 hours and 13 minutes?

Correction of altitude answering to 5h.10m. is		0°.44'.24".
Proportional part to 3 minutes of right ascension		1. 8
Correction of polar star's altitude in Jan. 1824		0. 43. 16
Annual variation of correction	−3".34	
Number of years after 1824	30	
Product	−100".20 =	— 1. 40

Correction of the polar star's altitude in Jan. 1854, as required, 0°.41'.36". which differs but 8 seconds from the true result by spherical trigonometry, as will be shown hereafter; and which evidently demonstrates that the column of annual variation may be safely employed in reducing the correction of altitude to any future period, for a long series of years, since the error in the space of thirty years only amounts to 8 seconds of a degree, which becomes insensible in determining the latitude at sea.

The corrections of altitude contained in the present Table were computed in conformity with the following principles:—

Since to an observer placed at the equator, the poles of the world will appear to be posited in the horizon, the polar star will, to such observer, apparently revolve round the north celestial pole in its diurnal motion round its orbit. In this apparent revolution round the celestial pole, the star's meridional or greatest altitude above the horizon will be always equal to its distance from that pole; which will ever take place, when the right ascension of the meridian is equal to the right ascension of the star. In six hours after this, the star will be seen in the horizon, west of the pole; in six hours more it will be depressed beneath the horizon (on the meridian below the pole), the angle of depression being equal to its polar distance; in six hours after, it will be seen in the horizon east of the pole; and in six hours more, it will be seen again on the meridian above the pole: allowance being made, in each case, for its daily acceleration.

Now, since the north celestial pole represents a fixed point in the heavens, and that the star apparently moves round it in an uniform manner, making determinable angles with the meridian; it is, therefore, easy to compute what altitude the star will have, as seen from the equator, in every part of its orbit; for, in this computation, we have a spherical triangle to work in, whose three sides are expressed by the complement of the latitude, the complement of the polar star's altitude, and the complement of its declination; in which there are given two sides and the included angle to find the third side; viz., the star's co-declination or polar distance and the complement of the latitude, with the comprehended angle, equal to the star's distance from the meridian, to find the star's co-altitude; the difference between which and 90 degrees will be the correction of altitude, or the difference of altitude between the polar star and the north celestial pole, as seen from the equator.

Example.

In January 1854, the mean right ascension of the north polar star will be $1^h 5^m 23^s$, and its polar distance $1°28'5''$; now, admitting the right ascension of the meridian to be $5^h 13^m$, the correction of the polar star's altitude, as seen from the equator, is required?

Right asc. of the merid. $5^h 13^m 0^s$
Right asc. of the pol. star 1. 5. 23

P. star's dist. from merid. $4^h 7^m 37^s = 61°54'15''$

Half do. do, in degrees	30.57 . 7½	Twice the log. sine=	19.422469
Star's polar distance	1.28 . 5	Log. sine	8.408572
Complement of the latitude . . .	90. 0 . 0	Log. sine	10.000000
		Sum	37.831041
Diff. between polar dist. and co-lat.	88°31'55''	Half sum	18.915520½
Half do.	44°15'57½''	Log. sine	9.843849
Arch = 6°43'35½''		Log. tang.	9.071671½
Log. sine of this arch . . .			9.068670
Half the polar star's co-altitude . .	44°39'16''	Log. sine	9.846850½
Polar star's co-altitude	89°18'32''		

Cor. of polar star's alt. in Jan. 1854 = $0°41'28''$. Now, by comparing this result with that shown in Example 2 (page 18), it will be seen that the correction of altitude, deduced directly from the Table, may be reduced to any period subsequent to 1824, without its being affected by any error of sufficient magnitude to endanger the interest of the mariner in any respect whatever.

Note.—For further information on this subject, the reader is referred to the author's Treatise on the Sidereal and Planetary Parts of Nautical Astronomy, page 144 to 156.

Table XI.

Correction of the Latitude deduced from the preceding Table.

Although the latitude deduced from Table X. will be always sufficiently correct for most nautical purposes, yet, since observation has shown that it will be something less than the truth in places distant from the equator, the present Table has been computed; which contains the number of minutes and seconds that the latitude, so deduced, will be less than what would result from actual observation at every tenth or fifth degree from the equator, to within five degrees of the north pole of the world.

The elements of this Table are, the approximate latitude, deduced from Table X., at top, and the right ascension of the meridian in the left or right-hand column; in the angle of meeting will be found the corresponding correction, which is always to be applied by *addition* to the approximate latitude. Hence, if the approximate latitude be 50 degrees, and the right ascension of the meridian $6^h 40^m$, the corresponding correction will be $1' 38''$ additive.

Remark.—Since the corrections of altitude in Table X. have been computed on the assumption that the motions of the polar star were witnessed from the equator, they ought, therefore, to show what altitude that star will have at any given time, in north latitude, when applied to such latitude with a contrary sign to that expressed in the Table; this, however, is not the case; because when the altitude of the polar star is computed by spherical trigonometry, or otherwise, it will always prove to be something less than that immediately deduced from Table X.: it is this difference, then, that becomes the correction of latitude in Table XI., and which is very easily determined, as may be seen in the following

Example.

Let the right ascension of the meridian in January 1824 be $6^h 40^m$, and the latitude 60 degrees north; required the true altitude of the polar star, and thence the correction of latitude?

Latitude or elevation of the pole	$60° 0' 0''$ north.
Correction in Table X., answ. to $6^h 40^m$, is +	0. 7. 41
Altitude of polar star, per Table X. = . .	$60° 7' 41''$

Now, to compute the true altitude of the polar star, on spherical principles, at the given time and place, we may either proceed as in last example, or, more readily, as follows:—

Right ascension of the merid.	6h 40m 0s		
Star's right ascension . .	0.58. 1		
Star's dist. from the meridian	5h 41m 59s	. . . Log. rising	5.964481
Star's polar distance . .	1°37′48″	. . . Log. sine	8.454006
Complement of the latitude	30. 0. 0	. . . Log. sine	9.698970
Difference	28.22.12	Nat. cos. 879897	
Natural number		013106 Log.=	4.117457
Star's true altitude . .	60°5′16″	Nat. sine 866791	
Star's alt. per Tab. as above	60.7.41		

Difference . . . . . . 0°2′25″; which, therefore, is the correction of latitude.

Note.—The correction of latitude, thus found, differs 4 seconds from that given in Table XI.: this difference is owing to the star's apparent polar distance having been taken, inadvertently, from the Nautical Almanac of 1824, instead of its mean polar distance; but since this can only lead to a trifling difference, *and not to any error*, it was not, therefore, deemed necessary to alter or recompute the Table.

Table XII.

Mean Right Ascension of the Sun.

This Table may be used for the purpose of finding the approximate time of transit of a fixed star, when a Nautical Almanac is not at hand; it may also be employed in finding the right ascension of the meridian, or mid-heaven, when the latitude is to be determined by an altitude of the north polar star: for, if to the sun's right ascension, as given in this Table, the apparent time be added, the sum (rejecting 24 hours if necessary) will be the right ascension of the meridian, sufficiently near the truth for determining the latitude.

Table XIII.

Equations to equal Altitudes.—FIRST PART.

The arguments of this Table are, the interval between the observations at top or bottom, and the latitude in either of the side columns; in the angle of meeting stands the corresponding equation, expressed in seconds and thirds: hence the equation to interval 6 hours 40 minutes and latitude 50 degrees, is 15 seconds and 33 thirds.

The equations in this Table were computed by the following rule, viz.:—

To the log. co-tangent of the latitude, add the log. sine of half the interval in degrees; the proportional log. of the whole interval in time (esteemed as minutes and seconds), and the constant log. 8.8239;* the sum of these four logs., rejecting 29 from the index, will be the proportional log. of the corresponding equation in minutes and seconds, which are to be considered as seconds and thirds.

Example.

Let the latitude be 50 degrees, and the interval between the observed equal altitudes of the sun 4 hours; required the corresponding equation?

Latitude	50°0′0″	Log. co-tang.	9.9238
Half int. = 2 hours, in degs.=	30.0.0	Log. sine .	9.6990
Whole interval 4 hours, esteemed as 4 min., propor. log.			1.6532
Constant log.			8.8239
Required equation	14″18‴	Propor. log.	1.0999

The equations in the abovementioned Table were computed by Mrs. T. Kerigan.

Table XIV.

Equations to equal Altitudes.—PART SECOND.

In this Table, the interval between the observations is marked at top or bottom, and the sun's declination in the left or right-hand margin; under or over the former, and opposite to the latter, stands the corresponding equation, expressed in seconds and thirds: thus, the equation answering to 6 hours 40 minutes, and declination 18°30′, is 2 seconds and 48 thirds.

The equations contained in this Table were computed as follows, viz.:—

To the log. co-tangent of the declination, add the log. tang. of half the interval in degrees; the proportional log. of the whole interval in time (esteemed as minutes and seconds), and the constant log. 8.8239;† the

* † The arithmetical complement of 12 hours considered as minutes.

sum of these four logs., rejecting 29 from the index, will be the proportional log. of the corresponding equation in minutes and seconds, which are to be considered as seconds and thirds.

Example.

Let the sun's declination be 18°30′, and the interval between the observed equal altitudes of the sun 4 hours; required the corresponding equation?

Sun's declination	18°30′	Log. co-tang.	10.4755
Half interval = 2 ho. in degs. =	30. 0.	Log. tang. .	9.7614
Whole interval 4 ho. esteemed as 4 min.		Prop. log.	1.6532
Constant log.			8.8239
Required equation = . . .	3″29‴	Prop. log. .	1.7140

The equations in the abovementioned Table were, also, computed by Mrs. T. Kerigan.

To find the Equation of Equal Altitudes by Tables XIII. and XIV.

Rule.

Enter Table XIII., with the latitude in the side column and the interval between the observations at top; and find the corresponding equation, to which prefix the sign + if the sun be *receding* from the elevated pole, but the sign − if it be advancing towards that pole.

Enter Table XIV., with the declination in the side column, and the interval between the observations at top, and take out the corresponding equation, to which prefix the sign + when the sun's declination is *increasing*, but the sign − when it is *decreasing*.

Now, if those two equations are of the same signs; that is, both affirmative or both negative, let their sum be taken; but if contrary signs, namely, one affirmative and the other negative, their difference is to be taken: then,

To the proportional log. of this sum or difference, considered as minutes and seconds, add the proportional log. of the daily variation of the sun's declination; and the sum, rejecting 1 from the index, will be the proportional log. of the true equation of equal altitudes in minutes and seconds, which are to be esteemed as *seconds and thirds*, and which will be always of the same name with the greater equation.

Example 1.

In latitude 49° south, the interval between equal altitudes of the sun was 7ʰ20ᵐ; the sun's declination 18° north, increasing, and the variation of declination 15′12″; required the true equation of equal altitudes?

Opposite lat. 49° under $7^h 20^m$ Tab. XIII. stands + 15″27‴
Opposite dec. 18° under 7′20″ Tab. XIV. stands + 2.30

Sum	17″57‴	Pro. log. 1.0012
Variation of declination . .	15′12″	Pro. log. 1.0734
True equation, as required	+ 15″10‴	Pro. log. 1.0746

Example 2.

In latitude 50° north, the interval between equal altitudes of the sun was $5^h 20^m$; the sun's declination 18°30′ north, increasing, and the daily variation of declination 14′34″; required the true equation of equal altitudes?

Op. lat. 50° under $5^h 20^m$ Tab. XIII. stands − 14″50‴
Op. dec. 18°30′ under 5.20 Tab. XIV. stands + 3.11

Difference	−11″39‴	Pro. log. = 1.1889
Variation of declination	14′34″	Pro. log. = 1.0919
True equation, as required	− 9″26‴	Pro. log. = 1.2808

Remark.—In north latitude the sun *recedes* from the elevated pole from the summer to the winter solstice; that is, from the 21st June to the 21st December; but *advances* towards that pole from the winter to the summer solstice; viz., from the 21st December to the 21st June. The converse of this takes place in south latitude: thus, from the 21st June to the 21st December, the sun *advances* towards the south elevated pole; but *recedes* from that pole the rest of the year, viz., from the 21st December to the 21st June.

Here it may be necessary to observe, that in taking out the equations from Tables XIII. and XIV., allowance is to be made for the excess of the given, above the next less tabular arguments, as in the following examples;

Example 1.

Required the equation from Table XIII., answering to latitude 50°48′, and interval between the observations 5 hours 10 minutes?

Equation to latitude 50°, and interval $4^h 40^m$ = . . . 14″33‴

Tabular diff. to 1° of lat. = + 31‴; now, $\frac{31''' \times 48'}{60'}$ = + 0.24⅘

Tab. diff. to 40′ of inter. = + 17‴; now, $\frac{17''' \times 30'}{40'}$ = + 0.12¾

Equation, as required . . . . 15″10‴

Example 2.

Required the equation from Table XIV., answering to sun's declination 20°47′, and interval between the observations 5 hours 10 minutes?

Equation to declination 20°30′ and interval $4^h 40^m =$ 3″44‴

Tabular diff. to 30′ declination $= + 6‴$; now, $\frac{6‴ \times 17'}{30'} = + 0.\ 3\frac{1}{2}$

Tabular diff. to 40′ interval $= - 10‴$; now, $\frac{10‴ \times 30'}{40'} = - 0.\ 7\frac{1}{2}$

Equation, as required . . . . 3″40½‴

Note.—Should the latitude exceed the limits of Table XIII., which is only extended so far as to comprehend the ordinary bounds of navigation, viz., to 60 degrees, the first part of the equation, in this case, must be determined by the rule under which that Table was computed, as in page 22.

TABLE XV.

To reduce the Sun's Longitude, Right Ascension, and Declination; and also the Equation of Time, as given in the Nautical Almanac, to any given Meridian, and to any given Time under that Meridian.

This Table is so arranged, that the proportional part corresponding to any given time, or longitude, and to any variation of the sun's right ascension, declination, &c. &c., may be taken out to the greatest degree of accuracy,—even to *the two hundred and sixteen thousandth part of a second, if necessary*.

Precepts.

In the general use of this Table it will be advisable to abide by the solar day; and hence, to estimate the time from noon to noon, or from 0 to 24 hours, after the manner of astronomers, without paying any attention to either the nautical or the civil division of time at midnight. And to guard against falling into error, in applying the tabular proportional part to the sun's right ascension, declination, &c. &c., it will be best to reduce the apparent time at ship or place, to Greenwich time; as thus:

Turn the longitude into time (by Table I.), and *add* it to the given time at ship or place, if it be *west*; but *subtract* it if *east*; and the sum, or difference, will be the corresponding time at Greenwich.

From page II. of the month in the Nautical Almanac, take out the sun's right ascension, declination, &c. &c., for the *noons immediately preceding and following the Greenwich time,* and find their difference, which will express the variation of those elements in 24 hours; then,

Enter the Table with the variation, thus found, at top, and the Greenwich time in the left-hand column; under the former and opposite the latter will be found the corresponding equation, or proportional part. And, since the Greenwich time may be estimated in hours, minutes, or seconds, and the variation of right ascension, &c. &c. &c., either in minutes or seconds; the sum of the several proportional parts making up the whole of such time and variation will, therefore, express the required proportional part. The proportional part, so obtained, is always to be applied by *addition* to the sun's longitude and right ascension at the *preceding noon;* but it is to be applied by *addition,* or *subtraction,* to the sun's declination and the equation of time at that noon, according as they are *increasing* or *decreasing.*—See the following examples:—

Example 1.

Required the sun's right ascension and declination, and also the equation of time May 6th, 1824, at 5ʰ 10ᵐ, in longitude 64°45′ west of the meridian of Greenwich?

Apparent time at ship or place	5ʰ 10ᵐ
Longitude 64°45′ west, in time = . . .	+ 4. 19
Greenwich time	9ʰ 29ᵐ

To find the Sun's Right Ascension:—

Sun's right ascension at noon, May 6th, 1824, per Nautical Almanac, ... 2ʰ 53ᵐ 31ˢ 42ᵗ

Variation in 24ʰ = 3′ 52″

Pro. part	to 9ʰ 0ᵐ and 3′ 0″ =	1′ 7″ 30‴ 0⁗	
Do.	to 0. 29 and 3. 0 =	0. 3. 37. 30	
Do.	to 9. 0 and 0. 50 =	0. 18. 45. 0	
Do.	to 0. 29 and 0. 50 =	0. 1. 0. 25	
Do.	to 9. 0 and 0. 2 =	0. 0. 45. 0	
Do.	to 0. 29 and 0. 2 =	0. 0. 2. 25	
Pro. part	to 9ʰ 29ᵐ and 3′ 52″ is	1. 31. 40. 20 =	+ 1ˢ 31ᵗ 40ᵗʰ

Sun's right ascension, as required 2ʰ 55ᵐ 3ˢ 22ᵗ

To find the Sun's Declination :—

Sun's declination at noon, May 6th, 1824, per Nautical Almanac, ... 16°36′5″
north, increasing, and var. in 24 ho.=16′38″

Pro. part	to	$9^h\ 0^m$ and	16′ 0″	=	6′ 0″ 0‴ 0⁗	
Do.	to	0.29 and	16. 0	=	0.19.20. 0	
Do.	to	9. 0 and	0.30	=	0.11.15. 0	
Do.	to	0.29 and	0.30	=	0. 0.36.15	
Do.	to	9. 0 and	0. 8	=	0. 3. 0. 0	
Do.	to	0.29 and	0. 8	=	0. 0. 9.40	
Pro. part	to	$9^h 29^m$ and	16′38″ is		6.34.20.55	= + 6′34″

Sun's declination, as required . . . . 16°42′39″

To find the Equation of Time :—

Equation of time at noon, May 6th, 1824, per Nautical Almanac, ... $3^m 36^s\ 6^t$
increasing, and variation in 24 hours = $4^s 30^t$

Pro. part	to	$9^h\ 0^m$ and	$4^s\ 0^t$	=	$1^s 30^t\ 0$	
Do.	to	0.29 and	4. 0	=	0. 4.50	
Do.	to	9. 0 and	0.30	=	0.11.15	
Do.	to	0.29 and	0.30	=	0. 0.36	
Pro. part	to	$9^h 29^m$ is	$4^s 30^t$	=	1.46.41	= + $1^s 47^t$

Equation of time, as required . . . . $3^m 37^s 53^t$

Example 2.

Required the sun's right ascension and declination, and also the equation of time, August 2d, 1824, at $19^h 22^m$, in longitude 98°45′ east of the meridian of Greenwich?

Apparent time at ship or place	$19^h 22^m$
Longitude 98°45′ east, in time = ...	− 6.35
Greenwich time	$12^h 47^m$

To find the Sun's Right Ascension :—

Sun's right ascension at noon, August 2d, 1824, per Nautical Almanac, ...				$8^h50^m0^s48^t$
Variation in 24 hours = $3'52''$				
Pro. part	to $12^h\ 0^m$	and $3'\ 0''$	= $1'30''\ 0'''\ 0''''$	
Do.	to 0.47	and 3. 0	= 0. 5.52.30	
Do.	to 12. 0	and 0.50	= 0.25. 0. 0	
Do.	to 0.47	and 0.50	= 0. 1.37.55	
Do.	to 12. 0	and 0. 2	= 0. 1. 0. 0	
Do.	to 0.47	and 0. 2	= 0. 0. 3.55	
Pro. part	to 12^h47^m	and $3'52''$ is	2. 3.34.20 =	$+2'3''34'''$
Sun's right ascension, as required				$8^h52^m4^s22^t$

To find the Sun's Declination :—

Sun's declination at noon, August 2d, 1824, per Nautical Almanac, ...				$17°44'41''$
north, decreasing, and var. in $24^h = 15'\ 36''$				
Pro. part	to $12^h\ 0^m$	and $15'\ 0''$	= $7'30''\ 0'''\ 0''''$	
Do.	to 0.47	and 15. 0	= 0.29.22.30	
Do.	to 12. 0	and 0.30	= 0.15. 0. 0	
Do.	to 0.47	and 0.30	= 0. 0.58.45	
Do.	to 12. 0	and 0. 6	= 0. 3. 0. 0	
Do.	to 0.47	and 0. 6	= 0. 0.11.45	
Pro. part	to 12^h47^m	and $15'36''$ is	8.18.33. 0 =	$-\ 8'19''$
Sun's declination, as required				$17°36'22''$

To find the Equation of Time :—

Equation of time at noon, August 2d, 1824, per Nautical Almanac, ...				$5^m54^s24^t$
decreasing, and variation in 24 hours = $4''30'''$				
Pro. part	to $12^h\ 0^m$	and $4''\ 0'''$	= $2''\ 0'''\ 0''''$	
Do.	to 0.47	and 4. 0	= 0. 7.50	
Do.	to 12. 0	and 0.30	= 0.15. 0	
Do.	to 0.47	and 0.30	= 0. 0.58	
Pro. part	to 12^h47^m	and $4''30'''$ is	2.23.48 =	$-\ 2''24'''$
Equation of time, as required				$5^m52^s\ 0^t$

Remark.—Should the proportional part corresponding to the daily variation of the sun's longitude and any given time be required, it may be taken from the first page of the Table, by esteeming the seconds of variation, in that page, as minutes, and then raising the signs of the corresponding proportional parts one grade higher than what are marked at the top of the said page: the seconds of variation will, of course, be taken out after the usual manner. Thus,

Suppose that the daily variation of the sun's longitude be 57′ 40″, and the Greenwich time 9 hours 50 minutes, to find the corresponding equation, or proportional part.

Pro. part	to 9h 0m and 50′ 0″	=	18′45″ 0‴ 0″″
Do.	to 9. 0 and 7. 0	=	2.37.30. 0
Do.	to 0.50 and 50. 0	=	1.44.10. 0
Do.	to 0.50 and 7. 0	=	0.14.35. 0
Do.	to 9. 0 and 0.40	=	0.15. 0. 0
Do.	to 0.50 and 0.40	=	0. 1.23.20
Pro. part	to 9h 50m and 57′40″	is	23.37.38.20 = 23′38″ +

Note.—It is easy to perceive that the foregoing operations might have been much contracted, by taking out two or more of the proportional parts at once; but, lest doing so should appear anywise ambiguous to such as are not well acquainted with the method of taking out tabular numbers, it was deemed prudent to arrange the said operations according to their present extended form, so as to render them perfectly intelligible to every capacity.

The present Table was computed agreeably to the established principles of the rule of proportion; viz., As one day, or 24 hours, is to the variation of the sun's right ascension, declination, &c. &c., in that time, so is any other portion of time to the corresponding proportional part of such variation.

Table XVI.

To reduce the Moon's Longitude, Latitude, Right Ascension, Declination, Semidiameter, and Horizontal Parallax, as given in the Nautical Almanac, to any given Meridian, and to any given Time under that Meridian.

This Table is arranged in a manner so nearly similar to the preceding, that any explanation of its use may be considered almost unnecessary; the only difference being, that the proportional parts are computed to variation in 12 hours, instead of 24. By means of the present Table, the proportional part corresponding to any variation of the moon's longitude, latitude, right ascension, &c. &c. &c., may be easily obtained, to the greatest degree of accuracy, as follows; viz.

Turn the longitude of the ship or place into time (by Table I.), and *add* it to the apparent time at such ship or place, if it be *west;* but subtract it if *east:* and the sum, or difference, will be the corresponding time at Greenwich.

Take from pages V., VI., and VII. of the month, in the Nautical Almanac, the moon's longitude, latitude, right ascension, declination, semidiameter, and horizontal parallax, (or any one of these elements, according to circumstances,) for the noon and midnight immediately preceding and following the Greenwich time, and find their difference; which difference will express the variation of those elements in 12 hours.

Enter the Table with the variation, thus found, at top, and the Greenwich time in the left-hand column; in the angle of meeting will be found the corresponding equation, or proportional part, which is always to be *added* to the moon's longitude and right ascension at the *preceding* noon or midnight, but to be applied by *addition*, or *subtraction*, to the moon's latitude, declination, semidiameter, and horizontal parallax, according as they are *increasing* or *decreasing*. And, since the Greenwich time and the variation in 12 hours will be very seldom found to correspond exactly; it is the sum, therefore, of the several equations making up those terms, that will, in general, express the required proportional part.

Example.

Required the moon's longitude, latitude, right ascension, declination, semidiameter, and horizontal parallax, August 2d, 1824, at 3h 10m, in longitude 60° 30′, west of the meridian of Greenwich?

Apparent time at ship or place	3h 10m
Longitude 60° 30′ west, in time = ...	4. 2
Greenwich time	7h 12m

To find the Moon's Longitude :—

Moon's longitude at noon, August 2d, 1824, per Nautical Almanac, ... 7s 17° 16′ 27″

Variation in 12h = 6° 31′ 59″

Propor. part	to 7h 0m	and 6° 0′ 0″	=	3° 30′ 0″ 0‴	
Do.	to 0.12	and 6. 0. 0	=	0. 6. 0. 0	
Do.	to 7. 0	and 0.30. 0	=	0. 17. 30. 0	
Do.	to 0.12	and 0.30. 0	=	0. 0. 30. 0	
Do.	to 7. 0	and 0. 1. 0	=	0. 0. 35. 0	
Do.	to 0.12	and 0. 1. 0	=	0. 0. 1. 0	
Do.	to 7. 0	and 0. 0. 50	=	0. 0. 29. 10	
Do.	to 0.12	and 0. 0. 50	=	0. 0. 0. 50	
Do.	to 7. 0	and 0. 0. 9	=	0. 0. 5. 15	
Do.	to 0.12	and 0. 0. 9	=	0. 0. 0. 9	
Propor. part	to 7h 12m	and 6° 31′ 59″	is	3. 55. 11. 24	= + 3° 55′ 11″

Moon's longitude, as required . . . . 7s 21° 11′ 38″

To find the Moon's Latitude :—

Moon's latitude at noon, August 2d, 1824, per Nautical Almanac, ... 4° 6′ 59″

south, decreasing, and var. in 12 hours = 23′ 35″

Proportional part	to 7h 0m	and 20′ 0″	=	11′ 40″ 0‴	
Do.	to 0.12	and 20. 0	=	0. 20. 0	
Do.	to 7. 0	and 3. 0	=	1. 45. 0	
Do.	to 0.12	and 3. 0	=	0. 3. 0	
Do.	to 7. 0	and 0. 30	=	0. 17. 30	
Do.	to 0.12	and 0. 30	=	0. 0. 30	
Do.	to 7. 0	and 0. 5	=	0. 2. 55	
Do.	to 0.12	and 0. 5	=	0. 0. 5	
Proportional part	to 7h 12m	and 23′ 35″	is	14. 9. 0	= − 14′ 9″

Moon's latitude, as required . . . . 3° 52′ 50″

Note.—In consequence of the unequal motion of the moon in 12 hours, (when her place is to be determined with astronomical precision,) the proportional part of the variation of her longitude and latitude, found as above, must be corrected by the equation of second difference contained in Table XVII.; and the same may be observed of her right ascension and declination.

To find the Moon's Right Ascension :—

Moon's right ascension at noon, August 2d, 1824, per Nautical Almanac, ... 223°33′36″

Var. in 12^{h} = 6°51′ 49″

Propor. part	to $7^{h}\ 0^{m}$	and 6° 0′ 0″	=	3°30′ 0″ 0‴
Do.	to 0. 12	and 6. 0. 0	=	0. 6. 0. 0
Do.	to 7. 0	and 0. 50. 0	=	0. 29. 10. 0
Do.	to 0. 12	and 0. 50. 0	=	0. 0. 50. 0
Do.	to 7. 0	and 0. 1. 0	=	0. 0. 35. 0
Do.	to 0. 12	and 0. 1. 0	=	0. 0. 1. 0
Do.	to 7. 0	and 0. 0. 40	=	0. 0. 23. 20
Do.	to 0. 12	and 0. 0. 40	=	0. 0. 0. 40
Do.	to 7. 0	and 0. 0. 9	=	0. 0. 5. 15
Do.	to 0. 12	and 0. 0. 9	=	0. 0. 0. 9

Propor. part to $7^{h}\,12^{m}$ and 6°51′49″ is 4. 7. 5. 24 = +4° 7′ 5″

Moon's right ascension, as required . . . . 227°40′41″*

To find the Moon's Declination :—

Moon's declination at noon, August 2d, 1824, per Nautical Almanac, ... 20°57′ 7″

south, increasing, and var. in 12 ho. = 1°23′43″

Propor. part	to $7^{h}\ 0^{m}$	and 1° 0′ 0″	=	35′ 0″ 0‴
Do.	to 0. 12	and 1. 0. 0	=	1. 0. 0
Do.	to 7. 0	and 0. 20. 0	=	11. 40. 0
Do.	to 0. 12	and 0. 20. 0	=	0. 20. 0
Do.	to 7. 0	and 0. 3. 0	=	1. 45. 0
Do.	to 0. 12	and 0. 3. 0	=	0. 3. 0
Do.	to 7. 0	and 0. 0. 40	=	0. 23. 20
Do.	to 0. 12	and 0. 0. 40	=	0. 0. 40
Do.	to 7. 0	and 0. 0. 3	=	0. 1. 45
Do.	to 0. 12	and 0. 0. 3	=	0. 0. 3

Propor. part to $7^{h}\,12^{m}$ and 1°23′43″ is 50. 13. 48 = + 50′14″

Moon's declination, as required . . . . 21°47′21″*

* When accuracy is required, the moon's right ascension and declination must be corrected by the equation of second difference, on account of the irregularities of her motion in 12 hours.

To find the Moon's Semidiameter :—

Moon's semidiameter at noon, August 2d, 1824, per Nautical Almanac, ... 15′ 33″
decreasing, and var. in 12 hours = 6″

Proportional part to $7^h\ 0^m$ and 6″ = 3″ 30‴
Do. to 0.12 and 6 = 0. 6

Proportional part to $7^h\ 12^m$ and 6″ is 3.36 = − 4″

Moon's semidiameter, as required . . . . 15′ 29″

To find the Moon's Horizontal Parallax :—

Moon's horizontal parallax at noon, August 2d, 1824, per Nautical Almanac, ... 57′ 6″
decreasing, and var. in 12 hours = 23″

Proportional part to $7^h\ 0^m$ and 20″ = 11″ 40‴
Do. to 0.12 and 20 = 0.20
Do. to 7. 0 and 3 = 1.45
Do. to 0.12 and 3 = 0. 3

Proportional part to $7^h\ 12^m$ and 23″ is 13.48 = − 14″

Moon's horizontal parallax, as required . . . . 56′ 52″

Remarks.—1. It is evident that, in the above operations, the greater part of the figures might have been dispensed with, by taking out two or more of the proportional parts at once; but since they were merely intended to simplify and render familiar the use of the Table, the whole of the proportional parts have been put down at length.

2. This Table was computed according to the rule of proportion; viz.:— As 12 hours are to the variation of the moon's longitude, latitude, right ascension, &c. &c. &c., in that interval, so is any other given portion of time to the corresponding proportional part of such variation.

TABLE XVII.

Equation of Second Difference.

Since the moon's longitude and latitude, and also her right ascension and declination, require to be strictly determined on various astronomical occasions; particularly the *two latter* when the *apparent time* is to be *inferred* from the *true altitude of that object;* and since the reduction of these elements, to a given instant, cannot be performed by even proportion, on account of the great inequalities to which the lunar motions are subject;—a *correction*, therefore, resulting from these inequalities, must be applied to the proportional part of the moon's longitude or latitude, right ascension or declination, answering to a given period after noon or midnight, as deduced from the preceding Table or otherwise, in order to have it truly accurate. This *correction* is contained in the present Table, the arguments of which are,—the mean second difference of the moon's place at top; and the apparent or Greenwich time past noon, or midnight, in the left or right-hand column; in the angle of meeting stands the corresponding equation or correction.

The Table is divided into two parts: the upper part is adapted to the mean second difference of the moon's place in seconds of a degree, and in which the equations are expressed in seconds and decimal parts of a second; the lower part is adapted to minutes of mean second difference; the equations being expressed in minutes and seconds, and decimal parts of a second.

In using this Table, should the mean second difference of the moon's place exceed its limits, the sum of the equations corresponding to the several terms which make up the mean second difference, in both parts of the Table, is in such case to be taken. The manner of applying the equation of second difference to the proportional part of the moon's motion in latitude, longitude, right ascension, or declination, as deduced from the preceding Table, or obtained by even proportion, will be seen in the solution to the following

Problem.

To reduce the Moon's Latitude, Longitude, Right Ascension, and Declination, as given in the Nautical Almanac, to any given Time under a known Meridian.

Rule.

Turn the longitude into time, (by Table I.) and apply it to the apparent time at ship or place by *addition* in *west*, or *subtraction* in *east* longitude; and the *sum*, or *difference*, will be the corresponding time at Greenwich.

Take from the Nautical Almanac the two longitudes, latitudes, right ascensions, and declinations immediately *preceding* and *following* the Greenwich time, and find the difference between each pair successively; find also the second difference, and let its mean be taken.

Find the proportional part of the middle *first* difference, (the variation of the moon's motion in 12 hours,) by Table XVI., answering to the Greenwich time.

With the mean second difference, found as above, and the Greenwich time, enter Table XVII., and take out the corresponding equation. Now, this equation being *added* to the proportional part of the moon's motion if the first *first difference* is *greater* than the third *first difference*, but *subtracted* if it be *less*, the sum or difference will be the correct proportional part of the moon's motion in 12 hours.

The correct proportional part, thus found, is always to be *added* to the moon's longitude and right ascension at the noon or midnight preceding the Greenwich time; but to be applied by *addition* or *subtraction* to her latitude and declination, according as they may be *increasing* or *decreasing*.

Example.

Required the moon's correct longitude, latitude, right ascension, and declination, August 2d, 1824, at 3h. 10m. apparent time, in longitude 60°30′, west of the meridian of Greenwich?

Apparent time at ship or place	3h. 10m.
Longitude 60°30′ west, in time = ...	4. 2
Greenwich time	7h. 12m.

To find the Moon's correct Longitude:—

				First Diff.	Second Diff.	Mean 2d Diff.
Moon's long. Aug.	1st,	at midnt.	7s. 10°38′49″			
				6°37′38″		
Do.	2	at noon	7. 17. 16. 27		5′39″	
				6. 31. 59		5′27½″
Do.	2	at midnt.	7. 23. 48. 26		5. 16	
				6. 26. 43		
Do.	3	at noon	8. 0. 15. 9			

Propor. part from Table XVI., ans. to 7h. 12m. and 6°31′59″ is 3°55′11″24‴
Eq. from Tab. XVII., corres. to 7h. 12m. and 5′ 0″ = 36″.0
and 0. 20 = 2 .4
and 0. 7½ = 0 .9

Eq. of mean second diff. ans. to 7h. 12m. and 5′27½″ is 39 .3 = + 39″18‴

Correct proportional part of the moon's motion in longitude	3°55′50″42‴
Moon's longitude at noon, August 2d, 1824	7s. 17. 16. 27. 0
Moon's correct longitude, at the given time	7s. 21°12′17″42‴

To find the Moon's correct Latitude :—

				First Diff.	Second Diff.	Mean 2d Diff.
Moon's lat. Aug.	1st,	at midnt.	4°27′37″ S.	20′38″	2′57″	2′44″
Do.	2	at noon	4. 6.59	23.35	2.31	
Do.	2	at midnt.	3.43.24	26. 6		
Do.	3	at noon	3.17.18			

Pro. part from Table XVI., ans. to $7^h 12^m$ and 23′35″ is 0°14′ 9″
Eq. from Tab. XVII., cor. to $7^h 12^m$ and 2′ 0″ = 14″.4
and 0.40 = 4 .8
and 0. 4 = 0 .5

Eq. of mean sec. diff., ans. to $7^h 12^m$ and 2′44″ is 19 .7 = – 19″.7

Correct proportional part of the moon's motion in lat. 0°13′49″.3
Moon's latitude at noon, August 2d, 1824 . . . . . 4. 6.59 .0 S.

Moon's correct latitude at the given time . . . . . 3°53′ 9″.7 south.

To find the Moon's correct Right Ascension :—

				First Diff.	Second Diff.	Mean 2d Diff.
Moon's R. A. Aug.	1st,	at midnt.	216°44′43″	6°48′53″	2′56″	2′32″
Do.	2	at noon	223.33.36	6.51.49	2. 8	
Do.	2	at midnt.	230.25.25	6.53.57		
Do.	3	at noon	237.19.22			

Propor. part from Table XVI., ans. to $7^h 12^m$ and 6°51′49″ is 4° 7′ 5″24‴
Eq. from Table XVII., ans. to $7^h 12^m$ and 2′ 0″ = 14″.4
and 0.30 = 3 .6
and 0. 2 = 0 .2

Eq. of mean sec. diff., ans. to $7^h 12^m$ and 2′32″ is 18 .2 = – 18″12‴

Correct propor. part of the moon's motion in right ascension 4° 6′47″12‴
Moon's right ascension at noon, August 2d, 1824 . . 223.33.36. 0

Moon's correct right ascension at the given time . . . 227°40′23″12‴

To find the Moon's correct Declination :

				First Diff.	Second Diff.	Mean 2d Diff.
Moon's dec. Aug.	1st,	at midnt.	19°15′49″ S.	1°41′18″	17′35″	17′58″
Do.	2	at noon	20.57. 7	1.23.43	18.21	
Do.	2	at midnt.	22.20.50	1. 5.22		
Do.	3	at noon	23.26.12			

Pro. part fr. Tab. XVI., ans. to 7ʰ 12ᵐ and 1°23′43″ is 0°50′13″48‴
Eq. fr. Tab. XVII. cor. to 7ʰ 12ᵐ and 15′ 0″=1′48″.0
and 2. 0 =0. 14 .4
and 0. 50 =0. 6 .0
and 0. 8 =0. 1 .0

Eq. of mn sec. diff., ans. to 7ʰ 12ᵐ and 17′58″ is 2. 9 .4=+2′ 9″24‴

Correct prop. part of the moon's motion in declination 0°52′23″12‴
Moon's declination at noon, August 2d, 1824 . . . 20.57. 7. 0 S.

Moon's correct declination at the given time . . 21°49′30″12‴ south.

Note.—It frequently happens that the three *first differences* first increase and then decrease, or *vice versa*, first decrease and then increase; in this case *half the difference* of the two second differences is to be esteemed as the mean second difference of the moon's place: as thus,

			First Diff.	Second Diff.	Mean 2d Diff.
Mn's dec. Aug. 18th, 1824,	at midt.	24°23′26″N.	18′21″	14′26″	4′26″
Do. 19	at noon	24.41.47	3.55	23.18	
Do. 19	at midt.	24.37.52	27.13		
Do. 20	at noon	24.10.39			

Here the two second differences are 14′26″, and 23′18″ respectively; therefore half their difference, viz., 8′52″ ÷ 2 = 4′26″ is the mean second difference. Now, if the Greenwich time be 5ʰ40ᵐ past noon of the 19th, the corresponding equation in Table XVII. will be 33″ *subtractive*, because the first *first difference* is less than the third *first difference*; had it been greater, the equation would be *additive*.

Remark.—When the apparent time is to be inferred from the true altitude of the moon's centre, the right ascension and declination of that object ought, in general, to be corrected by the equation of second difference; because an inattention to that correction may produce an error of about 2½ minutes in the right ascension, and about 4 minutes in the declination; which, of course, will affect the accuracy of the apparent time.—See the author's Treatise on the Sidereal and Planetary Parts of Nautical Astronomy, pages 171 and 172.

The equation of second difference, contained in the present Table, was computed by the following

Rule.

To the constant log. 7.540607 add the log. of the mean second difference reduced to seconds; the log. of the time from noon, and the log. of the difference of that time to 12 hours (both expressed in hours and decimal parts of an hour): the sum, rejecting 10 from the index, will be the log. of the equation of second difference in seconds of a degree.

Example.

Let the mean second difference of the moon's place be 8 minutes, and the apparent time past noon or midnight $3^h.20^m$; required the corresponding equation?

Mean second difference, 8 minutes =	480 seconds.	Log. = 2.681241
Apparent time past noon or midnight =	$3^h.333$	Log. = 0.522835
Difference of do. to 12 hours	$8^h.666$	Log. = 0.937819
Constant log. (ar. co. of log. of 288 = 24 × 12) . .		= 7.540607
Required equation	48″.14	Log. = 1.682502

Table XVIII.

Correction of the Moon's Apparent Altitude.

By the correction of the moon's apparent altitude is meant, the difference between the parallax of that object, at any given altitude, and the refraction corresponding to that altitude.

This correction was computed by the following rule; viz.

To the log. secant of the moon's apparent altitude, add the proportional log. of her horizontal parallax; and the sum, abating 10 in the index, will be the proportional log. of the parallax in altitude; which, being diminished by the refraction, will leave the correction of the moon's apparent altitude.

Example.

Let the moon's apparent altitude be 25°40′, and her horizontal parallax 59 minutes; required the correction of the apparent altitude?

Moon's apparent altitude	25°40′	Log. secant = 10.0451
Moon's horizontal parallax	0.59	Propor. log. = 0.4844
Moon's parallax in altitude . . .	53′11″ =	Propor. log. = 0.5295
Refraction ans. to app. alt. in Tab. VIII.	1.58	
Correction of the moon's appar. altitude	51′13″	

The correction, thus computed, is arranged in the present Table, where it is given to every tenth minute of apparent altitude, and to each minute of horizontal parallax. The proportional part for the excess of the given above the next *less* tabular altitude, is contained in the right-hand column of each page; and that answering to the seconds of parallax is given in the intermediate part of the Table.

This correction is to be taken out of the Table in the following manner; viz.

Enter the Table with the moon's apparent altitude in the left-hand column, or the altitude next *less* if there be any odd minutes; opposite to which, and under the minutes of the moon's horizontal parallax, will be found the approximate correction. Enter the compartment of the "Proportional parts to seconds of parallax," abreast of the approximate correction, with the tenths of seconds of the moon's horizontal parallax in the vertical column, and the units at the top; in the angle of meeting will be found the proportional part for seconds, which *add* to the approximate correction. Then,

Enter the last or right-hand column of the page, abreast of the approximate correction or nearly so, and find the proportional part corresponding to the odd minutes of altitude. Now, this being added to or subtracted from the approximate correction, according to its sign, will leave the true correction of the moon's apparent altitude. And since the apparent altitude of a celestial object is depressed by parallax and raised by refraction, and the lunar parallax being always greater than the refraction to the same altitude, it hence follows that the correction, thus deduced, is always to be applied by *addition* to the moon's apparent altitude.

Example 1.

Let the moon's apparent altitude be 8°38′, and her horizontal parallax 57′46″; required the corresponding correction?

Correction to alt. 8°30′ and horiz. parallax 57′0″ is		50′14″
Propor. part to 46 seconds of horiz. parallax . .	+	0. 46
Do. to 8 min. of alt. (8′ × 0″.5 = 4″.0) =	+	0. 4
Correction of the moon's apparent altitude, as required		51′ 4″

Example 2.

Let the moon's apparent altitude be 33°16′, and her horizontal parallax 59′34″; required the corresponding correction?

Correction to alt. 33°10′ and horiz. parallax 59′0″ is		47′56″
Propor. part to 34 seconds of horiz. parallax . . .	+	28
Do. to 6 minutes of altitude	−	3
Correction of the moon's apparent altitude, as required		48′21″

Table XIX.

To reduce the True Altitudes of the Sun, Moon, Stars, and Planets, to their Apparent Altitudes.

This Table is particularly useful in that method of finding the longitude by lunar observations, where the distance only is given, and where, of course, the altitudes of the objects must be obtained by computation.

The Table consists of two pages, each page being divided into two parts: the left-hand part contains four columns; the first of which comprehends the true altitude of the sun or star; the second the reduction of the sun's true altitude; the third the reduction of a star's true altitude; and the fourth the common difference of those reductions to 1 minute of altitude for sun or star.

The other part of the Table is appropriated to the moon; in which the true altitude of that object is given in the column marked "Moon's true altitude," and her horizontal parallax at top or bottom; the two last or right hand columns of each page contain the difference to 1 minute of altitude, and 1 second of parallax respectively; by means of which the reduction may be easily taken out to minutes of altitude and seconds of horizontal parallax.

The first part of the Table is to be entered with the sun's or star's true altitude (or the altitude next *less* when there are any odd minutes, as there generally will be,) in the left-hand column; abreast of which, in the proper column, will be found the approximate reduction; from which let the product of the difference to 1 minute by the excess of the odd minutes above the tabular altitude, be subtracted, and the remainder will be the true reduction of altitude for sun or star.

Example 1.

Let the true altitude of the sun's centre be 8°15′; required the reduction to apparent altitude?

Correction corresponding to altitude 8 degrees . . . . 6′15″
Cor. for min. of alt.; viz. diff. to 1 min. of alt.=0″.70×15′=10″.5 = −10

Required reduction = ... 6′5″

Example 2.

Given the true altitude of a star 19°45′; the reduction to apparent altitude is required?

Correction corresponding to altitude 19 degrees . . . . . 2'44"
Cor. for min. of alt.; viz. diff. to 1 min. of alt. $= 0''.15 \times 45' = 6''.75 = -7$

Required reduction = ... 2'37"

The reduction of the moon's true altitude is to be taken from the second part of the Table, by entering that part with the true altitude in the proper column (or the altitude next *less* when there are any odd minutes) and the horizontal parallax at top or bottom; in the angle of meeting will be found a correction; to which apply the product of the difference to 1 minute by the excess of the odd minutes above the tabular altitude by *subtraction*, and the product of the difference to 1 second by the odd seconds of parallax by *addition:* and the true reduction will be obtained, as may be seen in the following

Example.

Let the true altitude of the moon's centre be 29°13', and her horizontal parallax 58'37"; required the corresponding reduction to apparent altitude?

Correc. corres. to alt. 29 degs., and horiz. parallax 58' = . . 49'22"
Cor. for min. of alt.; viz., diff. to 1 min. of alt. $= 0''.41 \times 13' = 5''.3 = -5$
Cor. for secs. of par.; viz., diff. to 1 sec. of par. $= 0''.90 \times 37'' = 33''.3 = +33$

Required reduction . . . . 49'50"

Remark.—The reduction of the sun's true altitude is obtained by increasing that altitude by the difference between the refraction and parallax corresponding thereto: then, the difference between the refraction and parallax answering to that augmented altitude, will be the reduction of the true altitude.

Example.

Let the true altitude of the sun's centre be 5 degrees; required the reduction to apparent altitude?

Sun's true altitude . . . . 5° 0' 0"
Refract. Tab. VIII. = 9'54" } diff. + 9'45"
Paral. Table VII. 0. 9 }

Augmented altitude . . 5° 9'45", refrac. ans. to which is 9'38"
and parallax . 0. 9

Required reduction = ... 9'29"

The correction for reducing a star's true altitude to its apparent, is obtained in the same manner, omitting what relates to parallax. Thus, if the true altitude of a star be 8 degrees, and the corresponding refraction 6′ 29″, their sum, viz., 8° 6′ 29″ will be the augmented altitude; the refraction answering to this is 6′ 24″, which, therefore, is the reduction of the true to the apparent altitude of the star.

The correction for reducing the true altitude of the moon to the apparent, is found by diminishing the true altitude by the difference between the parallax and refraction answering thereto; then the difference between the parallax and refraction corresponding to the altitude so diminished, will be the reduction of the true to the apparent altitude. As thus:—

Let the true altitude of the moon's centre be 10 degrees, and her horizontal parallax 57 minutes; required the reduction to apparent altitude?

Moon's true altitude	10° 0′ 0″	Log. secant	10.0066
Do. horizontal parallax	57′ 0″	Propor. log.	0.4994
Parallax in altitude	56′ 8″	Propor. log.	0.5060
Refrac. to altitude 10°, Table VIII. =	5.15		
Difference between parallax and refrac. =	50′ 53″		
Diminished altitude	9° 9′ 7″	Log. secant	10.0056
Horizontal parallax	57. 0	Propor. log.	0.4994
Parallax in altitude	56′ 16″	Propor. log.	0.5050
Refrac. to diminished alt. Table VIII.	5.42		
Difference	50′ 34″; which, therefore, is the required reduction.		

Table XX.

Auxiliary Angles.

Since the solution of the Problem for finding the longitude at sea, by celestial observation, is very considerably abridged by the introduction of an *auxiliary angle* into the operation, the true central distance being hence readily determined to the nearest second of a degree by the simple addition of five natural versed sines; this Table has, therefore, been computed; and to render it as convenient as possible, it is extended to every tenth minute of the moon's apparent altitude, and to each minute of her horizontal parallax; with proportional parts adapted to the intermediate minutes of altitude, and to the seconds of horizontal parallax.

This Table was calculated in the following manner:—

To the moon's apparent altitude apply the correction from Table XVIII., and the sum will be her true altitude; from the log. cosine of which (the index being augmented by 10) subtract the log. cosine of her apparent altitude, and the remainder will be a log., which, being diminished by the constant log. .300910,* will give the logarithmic cosine of the auxiliary angle.

Example.

Let the moon's apparent altitude be 4 degrees, and her horizontal parallax 55 minutes; required the corresponding auxiliary angle?

Moon's apparent altitude .	4° 0′ 0″	Log. cosine .	9.998941
Correction from Table XVIII.	+ 43. 2		
Moon's true altitude . .	4°43′ 2″	Log. cosine .	9.998527
		Log. . . .	9.999586
		Constant log.	0.300910
Auxiliary angle, as required	60° 1′21″ =	Log. cosine .	9.698676

The correction of the auxiliary angle for the sun's or star's apparent altitude, given at the bottom of each page of the Table, was computed by the following rule—viz.

From the log. cosine of the sun's or star's true altitude subtract the log. cosine of the apparent altitude, and find the difference between the remainder and the constant log. .000120.† Now this difference, being subtracted from the log. cosine of 60 degrees, will leave the log. cosine of an arch; the difference between which and 60 degrees will be the correction of the auxiliary angle depending on the apparent altitude of the sun or star.

Example.

Let the sun's or star's apparent altitude be 3 degrees; required the correction of the auxiliary angle?

* This is the log. secant, less radius, of 60 degrees diminished by .000120, the difference between the log. cosines of a star's true and apparent altitude betwixt 30 and 90 degrees.

† This is the difference between the log. cosines of a star's true and apparent altitude, between 30 and 90 degrees.

Sun's apparent altitude	3° 0′ 0″	Log. cosine	9.999404
Refract. Table VIII. 14′36″ / Parallax. Table VII. 9 } difference	−14′27″		
Sun's true altitude	2°45′33″	Log. cosine	9.999497
		Remainder	0.000093
		Const. log.	0.000120
		Difference	0.000027
	60° 0′ 0″	Log. cosine	9.698970
Arch	60. 0. 8	Log. cosine	9.698943
Difference	0° 0′ 8″		

Difference . . . . 0° 0′ 8″; which, therefore, is the required correction of the auxiliary angle.

In this Table the auxiliary angle is given to every tenth minute of the moon's apparent altitude (as has been before observed) from the horizon to the zenith, and to each minute of horizontal parallax. The proportional part for the excess of the given, above the next *less* tabular altitude is contained in the right-hand column of each page; and that answering to the seconds of parallax is given in the intermediate part of the Table. The correction depending on the sun's or star's apparent altitude is placed at the bottom of the Table in each page.

As the size of the paper would not admit of the complete insertion of the auxiliary angle, except in the first vertical column of each page under or over 53′; therefore, in the eight following columns, it is only the excess of the auxiliary angle above 60 degrees that is given: hence, in taking out the auxiliary angle from those columns, it is always to be prefixed with 60 degrees.

The auxiliary angle is to be taken out of the Table, as thus:—

Enter the Table with the moon's apparent altitude in the left-hand column of the page, or the altitude next *less* if there be any odd minutes, opposite to which and under the minutes of the moon's horizontal parallax at top, will be found the approximate auxiliary angle.

Enter the compartment of the "Proportional parts to seconds of parallax," abreast of the approximate auxiliary angle, with the tenths of seconds of the moon's horizontal parallax in the vertical column, and the units at the top; in the angle of meeting will be found a correction, which place under the approximate auxiliary angle; then enter the last or right-hand column of the page abreast of where the approximate auxiliary angle was found, or nearly so, and find the proportional part corresponding to the odd minutes of altitude, which place under the former. To these *three* let the correction, at the bottom of the Table, answering to the sun's or star's apparent altitude, be applied, and the *sum* will be the correct auxiliary angle.

Example.

Let the moon's apparent altitude be 25°37′, the sun's apparent altitude 58°20′, and the moon's horizontal parallax 59′47″; required the corresponding auxiliary angle?

Aux. angle ans. to moon's app. alt. 25°30′, and hor. par. 59′ is	60°13′47″
Proportional parts to 47 seconds of horizontal parallax is .	12
Proportional part to 7 minutes of altitude is	4
Correction corresponding to sun's app. alt. (58°20′) is . . .	4
Auxiliary angle, as required	60°14′ 7″

Table XXI.

Correction of the Auxiliary Angle when the Moon's Distance from a Planet is observed.

The arguments of this Table are, a planet's apparent altitude in the left or right-hand column, and its horizontal parallax at top; in the angle of meeting stands the correction, which is always to be applied by *addition* to the auxiliary angle deduced from the preceding Table: hence, if the apparent altitude of a planet be 26 degrees, and its horizontal parallax 23 seconds, the correction of the auxiliary angle will be 6 seconds, additive.

This Table was calculated by a modification of the rule (page 43) for computing the correction of the auxiliary angle, answering to the sun's or star's apparent altitude; as thus:—

To the logarithmic secant of the planet's apparent altitude, add the logarithmic cosine of its true altitude, and the constant logarithm 9.698850;* and the sum (abating 20 in the index) will be the logarithmic cosine of an arch; the difference between which and 60 degrees will be the required correction.

* This is the log. cosine of 60 degrees diminished by .000120, the difference between the log. cosines of the true and apparent altitude of a fixed star between 30 and 90 degrees.

Example.

Let the apparent altitude of a planet be 30 degrees, and its horizontal parallax 23 seconds; required the correction of the auxiliary angle?

Planet's apparent altitude . . .	30° 0′ 0″	Log. secant	10.062469
Refrac. Table VIII. 1′.38″ } difference — Parallax, Table VI. 0.20 }	1′.18″		
		Const. log.	9.698850
True altitude of the planet . . .	29°58′.42″	Log. cosine	9.937626
Arch = ...	60° 0′ 7″ =	Log. cosine	9.698945
	60. 0. 0		
Difference	0° 0′ 7″; which is the required correction.		

TABLE XXII.

Error arising from a Deviation of one Minute in the Parallelism of the Surfaces of the Central Mirror of the Circular Instrument of Reflection.

This Table contains the error of observation arising from a deviation of *one minute* in the parallelism of the surfaces of the central mirror of the reflecting circle, the axis of the telescope being supposed to make an angle of 80 degrees with the horizon mirror; it is very useful in finding the verification of the parallelism of the surfaces of the central mirror in the reflecting circle, or of the index glass in the sextant; as thus:—

Let the instrument be carefully adjusted, and then take four or five observations of the angular distance between two *well-defined* objects, whose distance is not less than 100 degrees; the sum of these, divided by their number, will be the mean observation. Then,

Take out the central mirror, and turn it so that the edge which was before uppermost may now be downwards, or next the plane of the instrument; rectify its position, and take an equal number of observations of the angular distance between the same two objects, and find their mean, as before: now, half the difference between the mean of these and that of the former, will be the error of the mirror answering to the observed angle. If the first mean exceeds the second, the error is subtractive; otherwise additive: the mirror being in its first or natural position. Hence, if the mean of the first set of observations be 115°0'.40", and that of the second 114°59'.20", half their difference, viz., 1'.20" ÷ 2 = 40", will be the error of the observed angle, and is subtractive; because the first mean angular distance, or that taken with the mirror in its natural position, is greater than the second, or that taken with the mirror inverted.

Having thus determined the error of the observed angle, that answering to any given angle may be readily computed by means of the present Table, as follows :—

Enter the left-hand column of the Table with the angular distance, by which the error of the central mirror was determined, and take out the corresponding number from the adjoining column, or that marked "Observation to the right;" in the same manner take out the number answering to the given angle; then,

To the arithmetical complement of the proportional log. of the *first* number, add the proportional log. of the second, and the proportional log. of the observed error; the sum of these three logs., rejecting 10 from the index, will be the proportional log. of the error answering to such given angle.

Example.

Having found the error arising from a defect of parallelism in the central mirror, at an angle of 115 degrees, to be 40 seconds subtractive; required the error corresponding to an angle of 85 degrees?

Obs. ang. 115 deg. opp. to which is 3'.23"	Arith. comp. prop. log.	=	8.2741
Given ang. 85 deg. opp. to which is 1'.15"	Propor. log. . .	=	2.1584
Observed error of central mirror 0.40	Propor. log. . .	=	2.4313
Required error = ... − 0'.15" =	Propor. log. .	=	2.8638

Table XXIII.

Error of Observation arising from an Inclination of the Line of Collimation to the Plane of the Sextant, or to that of the circular Instrument of Reflection.

If the line of sight is not parallel to the plane of the instrument, the angle measured by such instrument will always be greater than the true angle. This Table contains the error arising from that cause, adapted to the most probable limits of the inclination of the line of collimation, and to any angle under 120 degrees: hence the arguments of the Table are, the observed angle in the left-hand column, and the inclination of the line of collimation at top; opposite the former, and under the latter, will be found the corresponding correction.

Thus, if the observed angle be 80 degrees, and the inclination of the line of collimation 30 minutes, the corresponding error will be 13 seconds. The error or correction taken from this Table is always to be applied by *subtraction* to the observed angle.

The corrections in this Table were computed by the following

Rule.

To the log. sine of half the observed angle, add the log. cosine of the inclination of the line of collimation; and the sum, rejecting 10 in the index, will be the log. sine of an arch. Now, the difference between twice this arch and the observed angle, will be the error of the line of collimation.

Example.

Let the observed angle be 80 degrees, and the inclination of the line of collimation 1°30′; required the corresponding correction?

Obs. angle 80 degs. and 80° ÷ 2 =	40°	Log. sine	9.808068
Inclinat. of line of collim.	1°30′	Log. cosine	9.999851
Arch = 39°59′ 1″ =		Log. sine	9.807919
Twice the arch = 79°58′ 2″			

Difference 0° 1′58″, which, therefore, is the required error,

Table XXIV.

Logarithmic Difference.

This Table contains the logarithmic difference, adapted to every tenth minute of the moon's apparent altitude from the horizon to the zenith, and to each minute of horizontal parallax. The proportional part for the excess of the given above the next *less* tabular altitude, is contained in the right-hand compartment of each page, and that answering to the seconds of parallax is given in the intermediate part of the Table.

As the size of the paper would not admit of the complete insertion of the logarithmic difference, except in the first vertical column of each page, under or over 53′, therefore in the eight following columns it is only the four last figures of the logarithmic difference that are given: hence, in taking out the numbers from these columns, they are always to be prefixed by the characteristic, and the two leading figures in the first column. The logarithmic difference is to be taken out in the following manner.

Enter the Table with the moon's apparent altitude in the left-hand column of the page, or the altitude next *less* if there be any odd minutes, opposite to which, and under the minutes of the moon's horizontal parallax, at top, will be found a number, which call *the approximate logarithmic difference.*

Enter the compartment of the "Proportional parts to seconds of parallax," abreast of the approximate logarithmic difference, with the tenths of seconds of the moon's horizontal parallax in the vertical column, and the units at the top, and take out the corresponding correction. Enter the right-hand compartment of the page,* abreast of where the approximate logarithmic difference was found, or nearly so, with the odd minutes of altitude, and take out the corresponding correction, which place under the former. Enter Table XXV. or XXVI., with the sun's, star's, or planet's apparent altitude, and take out the corresponding correction, which also place under the former. Now, the sum of these three corrections being taken from the approximate logarithmic difference, will leave the correct logarithmic difference.

Example 1.

Let the moon's apparent altitude be 19°25′, her horizontal parallax 60′38″, and the sun's apparent altitude 33 degrees; required the logarithmic difference?

Log. difference to app. alt. 19°20′, and hor. par. 60′ is			9.997669
Propor. part to 38 seconds of parallax is .	28	} sum =	− 49
Propor. part to 5 minutes of altitude is. . .	11		
Cor. from Tab. XXV. ans. to sun's apparent alt. is	10		
Logarithmic difference, as required			9.997620

* In taking out the correction corresponding to the odd minutes of altitude in this compartment, attention is to be paid to the moon's horizontal parallax: thus, if the parallax be between 53′ and 56′, the correction is to be taken out of the first column, or that adjoining the minutes of altitude; if it be between 56′ and 59′, the correction is to be taken out of the second, or middle column; and if it be between 59′ and 62′, the correction is to be taken out of the third, or last column.

Example 2.

Let the moon's apparent altitude be 63°37′, her horizontal parallax 58′43″, the apparent altitude of a planet 35°10′, and its horizontal parallax 23″; required the logarithmic difference?

Log. difference to appar. alt. 63°30′, and hor. par. 58′, is 9.993622
Propor. part to 43″ of parallax is . . . . . 83 ⎫
Propor. part to 7′ of altitude is . . . . . . 7 ⎬ sum = −118
Cor. from Tab. XXVI. ans. to planet's appar. alt. 28 ⎭

Logarithmic difference, as required 9.993504

Remark.—The logarithmic difference was computed by the following

Rule.

To the logarithmic secant of the moon's apparent altitude, add the logarithmic cosine of her true altitude, and the constant log. .000120;* the sum of these three logs., abating 10 in the index, will be the logarithmic difference.

Example.

Let the moon's apparent altitude be 19°20′, and her horizontal parallax 60 minutes; required the logarithmic difference?

Moon's apparent altitude . .	19°20′ 0″	Log. secant	10.025208
Correction from Table XVIII.	53.56	Constant log.	0.000120
Moon's true altitude	20.13.56	Log. cosine	9.972341

Logarithmic difference, as required 9.997669

* The difference between the log. cosines of the true and apparent altitude of a star betwixt 30 and 90 degrees.

Table XXV.

Correction of the Logarithmic Difference.

This Table is divided into two parts: the first, or left-hand part, contains the correction of the logarithmic difference when the moon's distance from the sun is observed; and the second, or right-hand part, the correction of that log. when the moon's distance from a star is observed. Thus, if the sun's apparent altitude be 35 degrees, the corresponding correction will be 11; if a star's apparent altitude be 20 degrees, the corresponding correction will be 1; and so on. These corrections are always to be applied by *subtraction* to the logarithmic difference deduced from the preceding Table.

The corrections contained in this Table were obtained in the following manner, viz.

To the log. secant of the apparent altitude, add the log. cosine of the true altitude; and the sum, rejecting 10 from the index, will be a log.; which being subtracted from the constant log. .000120,* will leave the tabular correction.

Example 1.

Let the sun's apparent altitude be 35 degrees; required the tabular correction?

Given apparent altitude =	35° 0′ 0″	Log. secant =	10.086635
Refrac. Table VIII. 1′21″ } diff.=−1.14			
Parallax Table VII. 7 }			
Sun's true altitude	34°58′46″	Log. cosine .	9.913474
		Sum . . .	0.000109
		Constant log.	0.000120
Tabular correction, as required			0.000011

Example 2.

Let the apparent altitude of a star be 10 degrees; required the tabular correction?

* See Note, page 50.

Star's apparent altitude	10° 0′ 0″	Log. secant =	10.006649
Refraction Table VIII.	− 5.15		
Star's true altitude . . .	9.54.45	Log. cosine =	9.993467
		Sum = . .	0.000116
		Constant log.	0.000120
Tabular correction, as required			0.000004

Table XXVI.

Correction of the Logarithmic Difference when the Moon's Distance from a Planet is observed.

The arguments of this Table are, the apparent altitude of a planet in the left or right-hand marginal column, and its horizontal parallax at top; in the angle of meeting stands the corresponding correction, which is to be applied by *subtraction* to the logarithmic difference deduced from Table XXIV., when the moon's distance from a planet is observed. Hence, if the apparent altitude of a planet be 20 degrees, and its horizontal parallax 21 seconds, the corresponding correction will be 16 subtractive, and so on.

This Table was computed by the rule in page 51, under which the correction corresponding to the sun's apparent altitude in Table XXV. was obtained, as thus:—

Let the apparent altitude of a planet be 23 degrees, and its horizontal parallax 21 seconds; required the correction of the logarithmic difference?

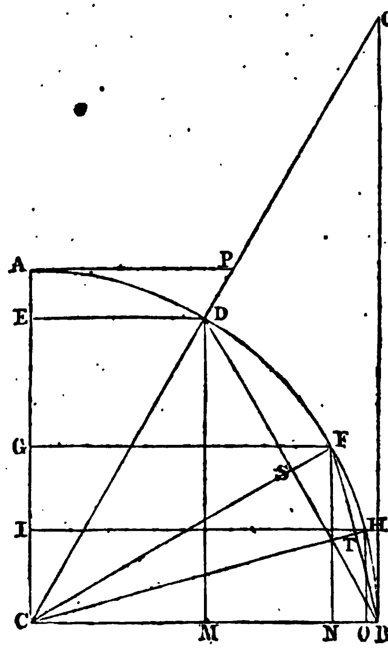
Planet's apparent altitude . . .	23° 0′ 0″	Log. secant	10.035974
Refrac. Table VIII. = 2′ 14″ / Parallax Table VI. = 20″ } diff. =	− 1.54		
Planet's true altitude	22°58′ 6″	Log. cosine	9.964128
		Sum . .	0.000102
		Constant log.	0.000120
Correction of the logarithmic difference, as required . . .			0.000018

Table XXVII.

Natural Versed Sines, and Natural Sines.

Since the methods of computing the true altitudes of the heavenly bodies, the apparent time at ship or place, and the true central distance between the moon and sun, or a fixed star, are considerably facilitated by the application of natural versed sines, or *natural sines*, this Table is given; which, with the view of rendering it generally useful and convenient, is extended to every tenth second of the semicircle, with proportional parts corresponding to the intermediate seconds; so that either the natural versed sine, natural versed sine supplement, natural co-versed sine, *natural sine* or *natural cosine* of any arch, may be readily taken out at sight.

The numbers expressed in this Table may be obtained in the following manner:—



Let A B C represent a quadrant, or the fourth part of a circle; and let the radius C B = unity or 1, be divided into an indefinite number of decimal parts: as thus, 1.0000000000, &c. Make B D = the radius C B; and since the radius of a circle is equal to the chord of 60 degrees, the arc B D is equal to 60 degrees: draw D M, the sine of the arc B D, and, at right-angles thereto, the cosine D E: bisect the arc B D in F, and draw F N and F G at right-angles to each other; then will the former represent the sine, and the latter the cosine of the arc B F = 30 degrees: bisect B F in H; then H O will express the sine, and H I the cosine of the arc B H = 15 degrees.

Proceeding in this manner, after 12 bisections, we come to an arc of $0^\circ 0' 52'' 44''' 3'''' 45'''''$, the cosine of which approximates so very closely to the radius C B, that they may be considered as being of equal value. Now, the absolute measure of this arc may be obtained by numerical calculation, as follows, viz.

Because the chord line B D is the side of a hexagon, inscribed in a circle, it is the subtense of 60 degrees, and, consequently, equal to the radius C B (corollary to Prop. 15, Book IV., of Euclid); wherefore half the radius B S = B M, will be the sine of 30 degrees = F N, which, therefore, is .5000000000. Now, having found the sine of 30 degrees, its cosine may be obtained by Euclid, Book I., Prop. 47: for in the right-angled triangle F N C, the hypothenuse F C is given = the radius, or 1.0000000000, and the perpendicular F N = half the radius, or .5000000000, to find the base C N = the cosine G F; therefore

$\sqrt{FC \times FC - FN \times FN} = CN$ .8660254037, or its equal G F; hence the sine of 30 degrees is .5000000000, and its cosine .8660254037. Again,

In the triangle F N B, the perpendicular F N is given = .5000000000, and the base C B − C N = N B = .1339745963, to find the hypothenuse B F: but half the side of a polygon, inscribed in a circle, is equal to the sine of half the circumscribing arc; therefore its half, B T = H O, will be the sine of the arc of 15 degrees: hence $\sqrt{FN \times FN + NB \times NB}$ = B F .5176380902; the half of which, viz., .2588190451, is therefore equal to B T, or to its equal H O, the sine of 15 degrees, and from which its cosine H I may be easily obtained; for, in the triangle C O H, the hypothenuse C H is given = 1.0000000000, and the perpendicular H O = .2588190451, to find the base C O = the cosine H I. Now,

$\sqrt{CH \times CH - HO \times HO} = CO$ .9659258263 = the cosine H I; hence the sine of 15 degrees is .2588190451, and its cosine .9659258263.

Thus proceeding, the sine of the 12th bisection, viz., 52″44‴3⁗45″‴, will be found = .0002556634. And because small arcs are very nearly as their corresponding sines, the measure of 1 minute may be easily deduced from the sine of the small arc, or 12th bisection determined as above; for,

As the arc of 52″44‴3⁗45″‴ is to an arc of 1 minute, so is the sine of the former to the sine of the latter: that is, as 52″44‴3⁗45″‴ : 1′ :: .0002556634 : .0002908882; which, therefore, is the sine of 1 minute, the cosine of which is .9999999577; but this approximates so very closely to the radius, that it may be esteemed as being actually equal to it in all calculations; and hence, that the cosine of 1′ is 1.0000000000.

Now, having thus found the sine and cosine of *one minute*, the sines of every minute in the quadrant may be obtained by the following rule; viz.

As radius is to twice the cosine of 1 minute, so is the sine of a mean arc to the sum of the sines of the two equidistant extremes; from which let either extreme be subtracted, and the remainder will be the sine of the other extreme: as thus, *To find the Sine of the Arc of 2 Minutes.*

As radius = 1 : 2 :: .0002908882 to .0005817764, and .0005817764 − .0000000000 = .0005817764; which, therefore, is the sine of the arc of 2 minutes.

To find the Sine of 3 Minutes.

As radius = 1 : 2 :: .0005817764 to .0011635528, and .0011635528 − .0002908882 = .0008726646; which, therefore, is the sine of an arc of 3 minutes.

To find the Sine of 4 Minutes.

As radius = 1 : 2 :: .0008726646 to .0017453292, and .0017453292 − .0005817764 = .0011635528; which, therefore, is the sine of the arc of 4 minutes.

To find the Sine of 5 Minutes.

As radius = 1 : 2 :: .0011635528 to .0023271056, and .0023271056 − .008726646 = .0014544407; which, therefore, is the sine of the arc of 5 minutes.

In this manner, the sines may be found to 60 degrees; from which, to the end of the quadrant, they may be obtained by addition only; for the sine of an arc greater than 60 degrees, is equal to the sine of an arc as much less than 60, augmented by the sine of the excess of the given arc above 60 degrees: thus,

All the sines being found to 60 degrees; required the sine of 61 degrees?

Solution.—Sine of 59° = .8571673, and sine of 1° = .0174524; their sum = .8746197; which, therefore, is the sine of 61 degrees, as required. Again,

All the sines being found to 60 degrees; required the sine of 62 degrees?

Solution.—Sine of 58° = .8480481, and sine 2° = .0348995; their sum = .8829476; which, therefore, is the sine of 62 degrees, as required.

Now, the natural sines being thus found, the natural versed sines, natural tangents, and natural secants, may be readily deduced therefrom, agreeably to the principles of similar triangles, as demonstrated in Euclid, Book VI., Prop. 4. Thus, *To find the Natural Versed Sine of* 30 *Degrees* = N B, *in the Diagram.*

Since the versed sine of an arc is represented by that part of the diameter which is contained between the sine and the arc; therefore N B is the versed sine of the arc B F, which is the arc of 30 degrees; and since the versed sines are measured upon the diameter, from the extremity B to C continued to the other extremity, the natural versed sines under 90 degrees are expressed by the difference between the radius and the cosine, and those above 90 degrees by the sum of the radius and the sine: hence, the radius C B 1.0000000 − the cosine F G, or its equal N C . 8660254 = N B . 1339746; which, therefore, is the natural versed sine of 30 degrees.

To find the Natural Tangent of 60 *degrees* = B Q, *in the Diagram.*

As the cosine C M is to the sine D M, so is the radius C B to the tangent B Q: that is,

As C M . 5000000 : D M . 8660254 :: C B 1.0000000 : B Q = 1.7320508; which is the natural tangent of 60 degrees.

To find the Natural Secant of 60 *Degrees* = C Q, *in the Diagram.*

As the cosine CM is to the radius C D, so is the radius C B to the secant C Q: that is,

As C M . 5000000 : C D 1.0000000 :: C B 1.0000000 : C Q = 2.0000000; which is the natural secant of 60 degrees. Hence, the manner of computing the natural co-tangent A P, the natural co-secant C P, and the natural co-versed sine E A, will be obvious. The versed sine supplement of an arc is represented by the difference between the versed sine of that arc and the diameter or twice the radius: thus, the versed sine supplement of the arc B F is expressed by the difference between twice the radius C B, and the versed sine N B; viz., twice C B = 2.0000000 − N B . 1339746 = 1.8660254; which, therefore, is the natural versed sine supplement of the arc B F or the arc of 30 degrees, and so of any other.

Now, the natural sines, versed sines, tangents, and secants, found as above, being principally decimal numbers, on account of the radius being assumed at unity or 1; therefore, in order to render these numbers all affirmative, they are to be multiplied by ten thousand millions respectively; and then the common logs. corresponding thereto will be the logarithmic sines, versed sines, tangents, and secants, which are generally given in the different mathematical Tables under these denominations.

Of ● *Table.*

In this Table, the natural versed sines are given to every tenth second of the semicircle; the corresponding arcs being arranged at the top, in numerical order, from 0 to 180 degrees. The natural versed sines supplement are given to the same extent; but their corresponding arcs are placed at the bottom of the Table, and numbered from the right hand towards the left, or contrary to the order of the versed sines. The natural co-versed sines begin at the end of the first quadrant, or of the 90th degree of the versed sines; the arcs corresponding to which are given at the bottom of the page and numbered, like the versed sines supplement, towards the left hand from 0 to 90 degrees, and then continued at top of the page from 90 to 180 degrees, towards the right hand, until they terminate at the 90th degree of the versed sines, where they first began. The *natural sines* begin where the co-versed sines end; viz., at the end of the first quadrant, or 90th degree of the versed sines, with which they increase by equal increments; the arcs corresponding to those are placed at the top of the page to every tenth second of the quadrant, the 90th degree of which terminates with the 180th of the versed sines. The *natural cosines* begin with the versed sines supplement; the arcs corresponding to which are given at the bottom of the page, being numbered, like the latter, contrary to the order of the versed sines and *natural sines*, to every tenth second from 0 to 90 degrees, or to the end of the first quadrant of the versed sines, thus ending where the co-versed sines begin.

Note.—In the general use of this Table, it is to be remarked, that the natural versed sine supplement, natural co-versed sine under 90 degrees, or *natural cosine*, of a given degree, is found in the same page with the *next less* degree in the column marked 0″ at top, it being the first number in that column; that answering to a given degree and minute is found on the same line with the *next less* minute in the column marked 60″ at the bottom of the page; and that corresponding to an arch expressed in degrees, minutes, and seconds, is obtained by deducting the proportional part, at bottom of the page, from the natural versed sine supplement, natural co-versed sine under 90 degrees, or *natural cosine* of the given degree, minute, and *less* tenth second.

PROBLEMS TO ILLUSTRATE THE USE OF THE TABLE.

Problem.

To find the Natural Versed Sine, Natural Versed Sine Supplement, Natural Co-versed Sine, Natural Sine, and Natural Cosine, of any given Arch, expressed in Degrees, Minutes, and Seconds.

Rule.

Enter the Table, and find the natural versed sine, versed sine supplement, co-versed sine, *natural sine*, or *natural cosine*, answering to the given degree, minute, and *next less tenth* second; to which add the proportional part answering to the odd seconds, found at the bottom of the page, if a natural versed sine, co-versed sine above 90°, or *natural sine* be wanted; but subtract the proportional part, if a versed sine supplement, co-versed sine under 90°, or *natural cosine*, be required: and the sum, or remainder, will be the natural versed sine, *natural sine*, natural versed sine supplement, co-versed sine, or *natural cosine*, of the given arch.

Example 1.

Required the natural versed sine, versed sine supplement, co-versed sine, *natural sine*, and *natural cosine*, answering to 42°12′36″?

To find the Natural Versed Sine:—

Natural versed sine to	42°12′30″ =	. . . .	259293
Proportional part to	6″ =	. . . . Add	20
Given Arch	42°12′36″	Natural versed sine =	259313

To find the Versed Sine Supplement:—

Versed sine supplement to	42°12′30″	. . . .	1.740707
Proportional part to . .	6″	. . . Subtract	20
Given arch . .	42°12′36″	Versed sine sup. =	1.740687

To find the Co-versed Sine :—

Co-versed sine to . .	42°12′30″	. . . .	328172
Proportional part to . . .	6″	. . Subtract	21
Given arch	42°12′36″	Co-vers. sine =	328151

To find the Natural Sine :—

Natural sine to . .	42°12′30″	. . . .	671828
Proportional part to . . .	6″	. . . Add	21
Given arch	42°12′36″	Nat. sine =	671849

To find the Natural Cosine :—

Natural cosine to .	42°12′30″	. . . .	740707
Proportional part to	6″	. . Subtract	20
Given arch	42°12′36″	Nat. cosine =	740687

Example 2.

Required the natural versed sine, versed sine supplement, co-versed sine, *natural sine*, and *natural cosine*, answering to 109°53′45″?

To find the Natural Versed Sine :—

Natural versed sine to	109°53′40″	=	1.340288
Proportional part to	5″	is . . . . Add	23
Given arch . .	109°53′45″	Nat. versed sine =	1.340311

To find the Versed Sine Supplement :—

Versed sine sup. to	109°53′40″	. . . .	659712
Proportional part to	5″	. . Subtract	23
Given arch	109°53′45″	Vers. sine sup. =	659689

To find the Co-versed Sine :—

Co-versed sine to	109°53′40″	. . . .	059679
Proportional part to	5″	. . . . . Add	8
Given arch	109°53′45″	Co-versed sine =	059687

To find the Natural Sine :—

Natural sine to	70°6′10″	Sup. to 109°53′50″		940305
Proportional part to	5″		Add	8
Supplement	70°6′15″	to given arch, nat. sine =		940313

To find the Natural Cosine :—

Natural cosine to	70°6′10″	Sup. to 109°53′50″		340334
Proportional part to	5″		Subtract	23
Supplement	70°6′15″	to given arch, nat. cosine =		340311

Remark.—Since the *natural sines* and *natural cosines* are not extended beyond 90 degrees, therefore, when the given arch exceeds that quantity, its supplement, or what it wants of 180 degrees, is to be taken, as in the above example. And when the given arch is expressed in degrees and minutes, the corresponding versed sine supplement, co-versed sine under 90 degrees, and *natural cosine*, are to be taken out agreeably to the note in page 57, which see.

Problem II.

To find the Arch corresponding to a given Natural Versed Sine, Versed Sine Supplement, Co-versed Sine, Natural Sine, and Natural Cosine.

Rule.

Enter the Table, and find the arch answering to the *next less* natural versed sine, or *natural sine*, but to the next greater versed sine supplement, co-versed sine, or *natural cosine;* the difference between which and that given, being found in the bottom of the page, will give a number of seconds, which, being added to the arch found as above, will give the required arch.

Example 1.

Required the arch answering to the natural versed sine 363985 ?

Solution.—The next *less* natural versed sine is 363959, corresponding to which is 50°30′10″; the difference between 363959 and the given natural versed sine, is 26; corresponding to which, at the bottom of the Table, is 7″, which, being added to the above-found arch, gives 50°30′17″, the required arch.

Note.—The arch corresponding to a given *natural sine* is obtained precisely in the same manner.

Example 2.

Required the arch corresponding to the natural versed sine supplement 1.464138?

Solution.—The next *greater* natural versed sine supplement is 1.464155; corresponding to which is 62°20′40″; the difference between 1.464155 and the given natural versed sine supplement, is 17; answering to which, at the bottom of the Table. is 4″, which, being added to the above-found arch, gives 62°20′44″, the required arch.

Note.—The arch corresponding to a given co-versed sine, or *natural cosine,* is obtained in a similar manner.

Remark 1.

The logarithmic versed sine of an arch may be found by taking out the common logarithm of the product of the natural versed sine of such arch by 10000000000; as thus:

Required the logarithmic versed sine of 78°30′45″?

The natural versed sine of 78°30′45″ is .800846, which, being multiplied by 10000000000, gives 8008460000; the common log. of this is 9.903549; which, therefore, is the logarithmic versed sine of the given arch, as required.

Remark 2.

The Table of Logarithmic Rising may be readily deduced from the natural versed sines; as thus:

Reduce the meridian distance to degrees, by Table I., and find the natural versed sine corresponding thereto; now, let this be esteemed as an integral number, and its corresponding common log. will be the logarithmic rising.

Example.

Required the logarithmic rising answering to $4^h 50^m 45^s$?

$4^h 50^m 45^s = 72^\circ 41' 15''$, the natural versed sine of which is 702417; the common log. of this is 5.846595, which, therefore, is the logarithmic rising required.

Table XXVIII.

Logarithms of Numbers.

Logarithms are a series of numbers invented, and first published in 1614, by Lord Napier, Baron of Merchiston in Scotland, for the purpose of facilitating troublesome calculations in plane and spherical trigonometry. These numbers are so contrived, and adapted to other numbers, that the sums and differences of the former shall correspond to, and show, the products and quotients of the latter.

Logarithms may be defined to be the numerical exponents of ratios, or a series of numbers in arithmetical progression, answering to another series of numbers in geometrical progression; as,

Thus:

0.	1.	2.	3.	4.	5.	6.	7.	8.	ind. or log.
1.	2.	4.	8.	16.	32.	64.	128.	256.	geo. prog.

Or,

0.	1.	2.	3.	4.	5.	6.	7.	8.	ind. or log.
1.	3.	9.	27.	81.	243.	729.	2187.	6561.	geo. pro.

Or,

0.	1.	2.	3.	4.	5.	6.	7.	8.	ind. or log.
1.	10.	100.	1000.	10000.	100000.	1000000.	10000000.	100000000	ge. pro.

Whence it is evident, that the same indices serve equally for any geometrical series; and, consequently, there may be an endless variety of systems of logarithms to the same common number, by only changing the second term 2. 3. or 10. &c. of the geometrical series of whole numbers.

In these series it is obvious, that if any two indices be added together, their sum will be the index of that number which is equal to the product of the two terms, in the geometrical progression to which those indices belong: thus, the indices 2. and 6. being added together, make 8; and the corresponding terms 4. and 64. to those indices (in the first series), being multiplied together, produce 256, which is the number corresponding to the index 8.

It is also obvious, that if any one index be subtracted from another, the difference will be the index of that number which is equal to the quotient of the two corresponding terms: thus, the index 8. minus the index 3 = 5; and the terms corresponding to these indices are 256 and 8, the quotient of which, viz., 32, is the number corresponding to the index 5, in the first series.

And, if the logarithm of any number be multiplied by the index of its power, the product will be equal to the logarithm of that power; thus, the index, or logarithm of 16, in the first series, is 4; now, if this be multiplied by 2, the product will be 8, which is the logarithm of 256, or the square of 16.

Again,—if the logarithm of any number be divided by the index of its root, the quotient will be equal to the logarithm of that root: thus, the index or logarithm of 256 is 8; now, 8 divided by 2 gives 4; which is the logarithm of 16, or the square root of 256, according to the first series.

The logarithms most convenient for practice are such as are adapted to a geometrical series increasing in a tenfold ratio, as in the last of the foregoing series; being those which are generally found in most mathematical works, and which are usually called *common logarithms*, in order to distinguish them from other species of logarithms.

In this system of logarithms, the index or logarithm of 1, is 0; that of 10, is 1; that of 100, is 2; that of 1000, is 3; that of 10000, is 4, &c. &c.; whence it is manifest, that the logarithms of the intermediate numbers between 1 and 10, must be 0, and some fractional parts; that of a number between 10 and 100, must be 1, and some fractional parts; and so on for any other number: those fractional parts may be computed by the following

Rule.—To the geometrical series 1. 10. 100. 1000. 10000. &c., apply the arithmetical series 0. 1. 2. 3. 4. &c., as logarithms. Find a geometrical mean between 1 and 10, or between 10 and 100, or any other two adjacent terms of the series between which the proposed number lies. Between the mean thus found and the nearest extreme, find another geometrical mean in the same manner, and so on till you arrive at the number whose logarithm is sought. Find as many arithmetical means, according to the order in which the geometrical ones were found, and they will be the logarithms of the said geometrical means; the last of which will be the logarithm of the proposed number.

Example.

To compute the Log. of 2 to eight Places of Decimals:—

Here the proposed number lies between 1 and 10.

First, The log. of 1 is 0, and the log. of 10 is 1;
therefore $0 + 1 \div 2 = .5$ is the arithmetical mean,
and $\sqrt{1 \times 10} = 3.1622777$ is the geometrical mean:
hence the log. of 3.1622777 is .5.

Second, The log. of 1 is 0, and the log. of 3.1622777 is .5;
therefore $0 + 5 \div 2 = .25$ is the arithmetical mean,
and $\sqrt{1 \times 3.1622777} = 1.7782794$ the geometrical mean:
hence the log. of 1.7782794 is .25.

Third, The log. of 1.7782794 is .25, and the log. of 3.1622777 is .5;
therefore $.25 + .5 \div 2 = .375$ is the arithmetical mean,
and $\sqrt{1.7782794 \times 3.1622777} = 2.3713741$ the geo. mean:
hence the log. of 2.3713741 is .375.

Fourth, The log. of 1.7782794 is .25, and the log. of 2.3713741 is .375;
therefore $.25 + .375 \div 2 = .3125$ is the arithmetical mean,
and $\sqrt{1.7782794 \times 2.3713741} = 2.0535252$ the geo. mean:
hence the log. of 2.0535252 is .3125.

Fifth, The log. of 1.7782794 is .25, and the log. of 2.0535252 is .3125;
therefore $.25 + .3125 \div 2 = .28125$ is the arith. mean,
and $\sqrt{1.7782794 \times 2.0535252} = 1.9109530$ the geo. mean:
hence the log. of 1.9109530 is .28125.

Sixth, The log. of 1.9109530 is .28125, & the log. of 2.0535252 is .3125;
therefore $.28125 + .3125 \div 2 = .296875$ is the arith. mean,
and $\sqrt{1.9109530 \times 2.0535252} = 1.9809568$ the geo. mean:
hence the log. of 1.9809568 is .296875.

Seventh, The log. of 1.9809568 is .296875, & the log. of 2.0535252 is .3125;
therefore $.296875 + .3125 \div 2 = .3046875$ is the arith. mean,
and $\sqrt{1.9809568 \times 2.0535252} = 2.0169146$ the geo. mean:
hence the log. of 2.0169146 is .3046875.

Eighth, The log. of 2.0169146 is .3046875, & log. of 1.9809568 is 296875;
therefore $.3046875 + .296875 \div 2 = .30078125$ is the ar. mean,
and $\sqrt{2.0169146 \times 1.9809568} = 1.9988548$ the geo. mean:
hence the log. of 1.9988548 is .30078125.

Proceeding in this manner, it will be found, after 25 extractions, that the log. of 1.9999999 is .30103000; and since 1.9999999 may be considered as being essentially equal to 2 in all the practical purposes to which it can be applied, therefore the log. of 2 is .30103000.

If the log. of 3 be determined, in the same manner, it will be found that the twenty-fifth arithmetical mean will be .47712125, and the geometrical mean 2.9999999; and since this may be considered as being in every respect equal to 3, therefore the log. of 3 is .47712125.

Now, from the logs. of 2 and 3, thus found, and the log. of 10, which is given=1, a great many other logarithms may be readily raised; because the sum of the logs. of any two numbers gives the log. of their product; and the difference of their logs. the log. of the quotient; the log. of any number, being multiplied by 2, will give the log. of the square of that number; or, multiplied by 3, will give the log. of its cube; as in the following examples:—

Example 1.

To find the Log. of 4:—

To the log. of 2 =	.30103000
Add the log. of 2 =	.30103000
Sum is the log. of 4 =	.60206000

Example 2.

To find the Log. of 5:—

From the log. of 10=	1.00000000
Take the log. of 2 =	.30103000
Rem. is the log. of 5 =	.69897000

Example 3.

To find the Log. of 6:—

To the log. of 3 =	.47712125
Add the log. of 2 =	.30103000
Sum is the log. of 6 =	.77815125

Example 4.

To find the Log. of 8:—

To the log. of 4 =	.60206000
Add the log. of 2 =	.30103000
Sum is the log. of 8 =	.90309000

Example 5.

To find the Log. of 9:—

To the log. of 3 =	.47712125
Add the log. of 3 =	.47712125
Sum is the log. of 9 =	.95424250

Example 6.

To find the Log. of 15:—

To the log. of 5 =	.69897000
Add the log. of 3 =	.47712125
Sum is the log. of 15=	1.17609125

Example 7.

To find the Log. of 81 = the square of 9:—

Log. of 9 = . . .	.95424250
Multiply by . . .	2
Pro. is the log. of 81=	1.90848500

Example 8.

To find the Log. of 729 = the cube of 9:

Log. of 9 = . . .	.95424250
Multiply by . . .	3
Pro. is the log. of 729=	2.86272750

Since the odd numbers 7. 11. 13. 17. 19. 23. 29. &c. cannot be exactly deduced from the multiplication or division of any two numbers, the logs. of those must be computed agreeably to the rule by which the logs. of 2 and 3 were obtained; after which, the labour attending the construction of a table of logarithms will be greatly diminished, because the principal part of the numbers may then be very readily found by addition, subtraction, and composition.

Of the Table.

This Table, which is particularly adapted to the reduction of the apparent to the true central distance, by certain concise methods of computation, to be treated of in the Lunar Observations, is divided into two parts: the *first* of which contains the decimal parts of the logs., to six places of figures, of all the natural numbers from unity, or 1, to 999999; and the *second*, the logs. to the same extent, of all the natural numbers from 1000000 to 1839999;—and although the logs. apparently commence at the natural number 100, yet the logs. of all the natural numbers under that are also given: thus, the log. of 1, or 10, is the same as that of 100; the log. of 2, or 20, is the same as that of 200; the log. of 3, or 30, is equal to that of 300; that of 11, to 110; that of 17, to 170; that of 99, to 990; and so on: using, however, a different index. And as the indices are not affixed to the logs., they must therefore be supplied by the computer: these indices are always to be considered as being *one less* than the number of *integer* figures in the corresponding natural number. Hence the index to the log. of any natural number, from 1 to 9 inclusive, is 0; the index to the log. of any number from 10 to 99 inclusive, is 1; that to the log. of any number from 100 to 999, is 2; that to the log. of any number from 1000 to 9999, is 3; &c. &c. &c. The second part of the Table will be found very useful in computing the lunar observations, by certain methods to be given hereafter, when the apparent distance exceeds 90 degrees, or when it becomes necessary to take out the log. of a natural number consisting of seven places of figures, and conversely.

In the left-hand column of the Table, and in the upper or lower horizontal row, are given the natural numbers, proceeding in regular succession; and, in the ten adjacent vertical columns, their corresponding logarithms.

As the size of the paper would not admit of the ample insertion of the logs., except in the first column, therefore only the four last figures of each log. are given in the nine following columns; the two preceding figures belonging to which will be found in the first column under 0 at top, or over 0 at bottom; and where these two preceding figures change, in the body of the Table, large dots are introduced instead of 0's, to catch the eye and to indicate that from thence, through the rest of the line, the said two preceding figures are to be taken from the next lower line in the column under or over 0: those dots are to be accounted as ciphers in taking out the logarithms.

The log. of any natural number consisting of four figures, or under, and conversely, is found directly by the Table; but because the log. of a natural number consisting of five or more places of figures, and the converse, is frequently required in the reduction of the apparent to the true central distance, and also in many other astronomical calculations; proportional parts are, therefore, adapted to the Table, and arranged in the nine small columns on the right-hand side of each page; by means of which the logarithms of all the natural numbers, not consisting of more than seven places of figures, and *vice versa*, may be found to a sufficient degree of accuracy for all nautical purposes, as may be seen in the following problems.

Problem I.

Given a Natural Number consisting of five, six, or seven Places of Figures, to find the corresponding Logarithm.

Rule.

Look for the three first figures of the given natural number in the left-hand column; opposite to which, and under the fourth figure, in the horizontal column at top, will be found the log. to the four first figures of the given natural number: on the same line with this, and under the fifth figure of the natural number at top, in the proportional parts, will be found a number, which, being added to the above, will give the log. to five places of figures of the given natural number; on the same line of proportional parts, and under the sixth figure of the natural number at top, will be found a number, which, being divided by 10, and the quotient added to the last found log., will give the log. to six places of figures of the given natural number. In the same manner, the log. may be taken out to seven places of figures; observing, that the number in the proportional parts, corresponding to the seventh figure of the natural number, is to be divided by 100.

Note.—In dividing by 10 or 100, we have only to strike off the right-hand, or two right-hand figures.

Example.

Required the log. corresponding to the given natural number 1378978?

Log. corresponding to 1378		(four first figures) is	139249
5th fig. of the nat. num.	. 9	ans. to which in the pro. parts is	284
6th fig. of do.	. . 7	ans. to which in the pro. parts is 221, which, divided by 10, gives 22.1	22
7th fig. of do.	. . . 8	ans. to which in the pro. parts is 252, which, divided by 100, gives 2.52	2
Given natural number	1378978	Corresponding log. = . . .	6.139557*

Problem II.

To find the Natural Number to five, six, or seven Places of Figures, corresponding to a given Logarithm.

Rule.

Find the next *less* log. answering to the given one in the column under 0; continue the sight along the horizontal line, and a log., either the same as that given, or somewhat near it, will be found; then, the three first figures of the corresponding natural number will be found in the left-hand column, and the fourth figure, above the log., at the top of the Table. Should the given log. be found exactly, let one, two, or three ciphers be annexed to the natural number found as above, according to the number of figures wanted, and it will be the natural number required. But, if the log. cannot be exactly found (which in general will be the case), find the difference between the given log. and the next *less* log. in the Table: with this difference, enter the proportional parts, on the same horizontal line in which the next *less* log. was found, and find the next *less* proportional part; answering to which, at the top or bottom, will be found the fifth figure of the required natural number: find the difference between the above-found difference and the aforesaid next *less* proportional part; which being multiplied by 10, and the *product* found in the same line of proportional parts, the number corresponding thereto, at top or bottom, will be the sixth figure of the required natural number. Now, the difference between the above *product* and its next *less* proportional part, being multiplied by 10, also, and the product found in the same line of proportional parts, the number answering thereto at top or bottom will be the seventh figure of the required natural number.

* The index 6 is prefixed, because the given natural number consists of seven places of figures.

Example.

Required the natural number corresponding to the given log. 6. 119558?

	Given log.	6. 119558	
1316	= four first figs. of the required nat. num. answering to next less log. . . .	. 119256	
	Difference	302	
. 9	= fifth fig. of the required nat. num. ans. to the pro. part next less . . .	297	
	Difference	5 × 10=50 product.	
. . 1	=sixth fig. of the required nat. num. ans. to pro. part next less		33
	Difference		17 × 10=170
. . . 5	=seventh fig. of the required nat. num. ans. to the nearest pro. part		165

1316915 which is the natural number corresponding to the given log. 6. 119558, as required.

Note.—From the above Problems, the manner of using the second part of the Table will appear obvious.

Remarks.

1. The whole of the operation is inserted at length, for the purpose of illustrating, more clearly, the use of the Table; but in practice, the logs. may, in most cases, be taken out at sight, and conversely; particularly from the *second part*, where the natural numbers are given to five places of figures, from 1000000 to 1839999.

2. In taking out the log. of a decimal fraction, or any number less than unity, if the first decimal place be a significant figure, the index of its log. is to be accounted as 9; but if the first significant figure of the decimal stands in the second, third, or fourth place, &c., the index of the corresponding log. is to be taken as 8, 7, or 6, &c. The converse of this,—that is, finding the significant decimals corresponding to a given log., will appear obvious.

3. The arithmetical complement of a log. is what that log. wants of the

radius of the Table; viz., of 10. 000000: this is most easily found, by beginning at the left hand, and subtracting each figure from 9, except the last significant one, which is to be taken from 10; as thus:

The arithmetical complement of the log. 4. 372853 is 5. 627147; and so on.

Problem III.

To perform Multiplication by Logarithms.

Rule.

To the log. of the multiplicand, add the log. of the multiplier, or add the logs. of the factors together, and the sum will be the log. of the product; the natural number corresponding to which will be the product required.

Example 1.

Multiply 436 by 19. 7.

436	Log. = . .	2. 639486
19. 7	Log. = . .	1. 294466
Prod.=8589. 18	Log.=	3. 933952

Example 2.

Multiply 437. 8 by 14. 07, and also by 0. 239.

437. 8	Log. = .	2. 641276
14. 07	Log. = .	1. 148294
0. 239	Log. = .	9. 378398
Pro.=1472. 204	Log.=	3. 167968

Example 3.

What is the product of 0. 049, 9. 875, and 0. 753?

0. 049	Log. = . .	8. 690196
9. 875	Log. = . .	0. 994537
0. 753	Log. = . .	9. 876795
Prod.=0. 3642	Log. =	9. 561528

Example 4.

What is the product of 0. 0567 and 0. 00339?

0. 0567	Log. = .	8. 753583
0. 00339	Log. = .	7. 530200
Pro.=0. 0001922	Log.=	6. 283783

Note.—Respecting the index of a decimal fraction, and conversely, see Remark 2, page 69.

Problem IV.

To perform Division by Logarithms.

Rule.

From the log. of the dividend, subtract the log. of the divisor, and the remainder will be the log. of the quotient; the natural number corresponding to which will be the quotient required.

Example 1.

Divide 1497 by 93.

1497	Log. =	3.175222
93	Log. =	1.968483
Quo. = 16.0968	Log. =	1.206739

Example 2.

Divide 469.76 by 0.937.

469.76	Log. =	2.671876
0.937	Log. =	9.971740
Quo. = 501.343	Log. =	2.700136

Example 3.

Divide 49.73 by 0.0632.

49.73	Log. =	1.696618
0.0632	Log. =	8.800717
Quo. = 786.869 =	Log. =	2.895901

Example 4.

Divide 0.00815 by 0.000275.

0.00815	Log. =	7.911158
0.000275	Log. =	6.439333
Quo. = 29.6368	Log. =	1.471825

Problem V.

To perform Proportion, or the Rule of Three, or Golden Rule, by Logarithms.

Rule.

To the arithmetical complement of the log. of the first term, add the logs. of the second and third terms; and the sum will be the log. of the fourth term, or answer.

Example 1.

If a ship sails $19\frac{1}{2}$ miles in $2\frac{1}{4}$ hours, how many miles will she run, at the same rate, in 24 hours?

As 2.25 hours, arith. comp. log. =	9.647817
Is to 19.5 miles, log.	1.290035
So is 24 hours, log.	1.380211
To 208 miles, log. =	2.318063

Example 2.

If the interest of 100*l.* for 365 days be 4*l.* 10*s.*, what will be the interest of 178*l.* 15*s.* for 213 days?

As { 100	Arith. comp. of log.	= . .	8.000000
{ 365	Do. do.	. .	7.437707
Is to { 178.75	Log.	. . . .	2.252246
{ 213.	Log.	. . . .	2.328380
So is 4.5	Log.	. . . .	0.653213
To 4.69403	Log.	. . . .	0.671546

Example 3.

A man of war, sailing at the rate of 9 knots an hour, descried a ship, distant 26 miles, sailing at the rate of 6½ knots, to which she gave chase: after two hours' chase, the breeze freshened, and increased the man of war's rate of sailing to 11½ knots, and that of the chase to 8¾. In what time did the man of war come up with the chase?

Solution.—Since the man of war gained, at the commencement, 2½ miles an hour on the chase, therefore, at the end of the first two hours, the distance between them was reduced to 21 miles; during the rest of the chase, the hourly gain of the man of war was 2¾ miles.

Hence, As the hourly gain	2.75	Ar. comp. log.	9.560667
Is to	1 hour,	Log. . . .	0.000000
So is distance .	21 miles,	Log. . . .	1.322219
To	7.6363	Log. =	0.882886,

or 7 hours and 38 minutes from the time the breeze freshened.

Problem VI.

To perform Involution by Logarithms.

Rule.

Multiply the log. of the given number by the index of the power to which it is to be raised, and the product will be the log. of the required power.

Example 1.

Required the square of 346?

346	Log. . . .	2.539076
	Ind. of the power=	2
Answer 119716	Log.=	5.078152

Example 2.

Required the cube of 754?

754	Log.	2.877371
	Ind. of the power=	3
Ans. 428661064	Log.=	8.632113

Problem VII.

To perform Evolution by Logarithms.

Rule.

Divide the log. of the given number by the index of the power, and the quotient will be the log. of the root.

Example 1.

Required the square root of 76176?

76176	Log. . .	2/4.881818
Ans. 276	Log. =	2.440909

Example 2.

Required the cube root of 21952000?

21952000	Log. . .	3/7.341475
Ans. 280	Log. =	2.447158⅓

Problem VIII.

To find the Tonnage of a Ship by Logarithms.

Rule.

To the log. of the length of the keel, *reduced to tonnage*, add the log. of the breadth of the beam, the log. of half the breadth of the beam, and the constant log. 8.026872*; the sum, rejecting 10 from the index, will be the log. of the required tonnage.

Example.

Let the length of a ship's keel, *reduced to tonnage*, be 120.5 feet, and the breadth of the beam 35.75 feet; required the ship's tonnage?

---

* This is the arithmetical complement of the log. of 94; the common divisor for finding the tonnage of ships.

Length of the keel for tonnage .	120.5	feet	Log. 2.080987
Breadth of the beam	35.75	feet	Log. 1.553276
Half ditto	17.875	feet	Log. 1.252246
Constant log.			8.026872
Required tonnage	819.18	. . . .	Log. 2.913381

Problem IX.

Given the Measured Length of a Knot, the Number of Seconds run by the Glass, and the Distance sailed per Log, to find the true Distance by Logarithms.

Rule.

To the arithmetical complement of the log. of the number of seconds run by the glass, add the log. of the measured length of a knot, the log. of the distance sailed, and the constant log. 9.795880*; the sum of these four logs., rejecting 20 from the index, will be the log. of the true distance.

Example 1.

The distance sailed by the log is 180 miles; the measured length of a knot is 43 feet, and the time by the glass 32 seconds; required the true distance?

32 seconds, arith. comp. log.	8.494850
43 feet, log. = ...	1.633469
180 miles, log. = ...	2.255273
Constant log. =	9.795880
True distance = 151.2 miles. Log. =	2.179472

Example 2.

The distance sailed by the log is 210 miles; the measured length of a knot is 51 feet, and the time by the glass 27 seconds; required the true distance?

27 seconds, arith. comp. log. = . . .	8.568636
51 feet, log.	1.707570
210 miles, log.	2.322219
Constant log.	9.795880
True distance = 247.9 miles. Log. =	2.394305

* This is the sum of the arithmetical complement of the log. of 48 (the general length of a knot) and the log. of 30 seconds, the true measure of the half-minute glass.

Table XXIX.

Proportional Logarithms.

This Table contains the proportional log. corresponding to all portions of time under three hours, and to every second under three degrees. It was originally computed by Dr. Maskelyne, and particularly adapted to the operation for finding the apparent time at Greenwich answering to a given distance between the moon and sun, or a fixed star; but it is now applied to many other important purposes, as will be seen hereafter.

Proportional Logarithms may be computed by the following

Rule.

From the common log. of 3 hours, reduced to seconds, subtract the common log. of the given time in seconds; and the remainder will be the proportional log. corresponding thereto.

Example.

Required the proportional log. corresponding to $0^h 40^m 26^s$?

3 hours reduced to seconds	=	10800ˢ	Log.	=	4.033424
$40^m 26^s$ given time, in secs.	=	2426ˢ	Log.	=	3.384891

Proportional log. corresponding to the given time = 0.6485.33

As hours and degrees are similarly divided, therefore, in the general use of this Table, the hours and parts of an hour, may be considered as degrees and parts of a degree, and conversely. And to render the use of it more extensive, one minute may be esteemed as being either one degree, or one second, and *vice versa*.

Since proportion is performed by adding together the arithmetical complement of the proportional logarithm of the first term, and the proportional logarithms of the second and third terms, rejecting 10 from the index, the present Table is of great use in reducing the altitudes of the moon and sun, or a fixed star, to the mean time and distance, when the observations are made by one person, as will appear evident by the following

Example.

Let the first altitude of the moon's lower limb be $27°25'20''$, and the corresponding time per watch $21^h 42^m 8^s$, and the last altitude $25°24'20''$, and its corresponding time $21^h 55^m 57^s$; it is required to reduce the first altitude to what it should be at $21^h 49^m 33^s$, the time at which the mean lunar distance was taken?

1st time	$21^h 42^m 8^s$	1st time	$21^h 42^m 8^s$	1st alt.	$27° 25' 20''$	$27° 25' 20''$
Last do.	21.55.57	Mean do.	21.49.33	Last do.	25.24.20	
Diff. .	0.13.49	Diff. .	0. 7.25	Diff .	2. 1. 0	

As	$13^m 49^s$, arithmetical comp. prop. log.	= 8.8851
Is to	$7^m 25^s$ proportional log.	= 1.3851
So is	$2° 1'$ proportional log.	= 0.1725
To prop. log. of reduction of Moon's alt. .		= 0.4427 = − $1° 4' 57''$
Moon's alt. reduced to mean time of observation . .		= $26° 20' 23''$

And in the same manner may the altitude of the sun, or a fixed star, be reduced to the time of taking the mean lunar distance.

Remark.—Although this Table is only extended to 3 hours or 3 degrees, yet by taking such terms as exceed those quantities one grade lower, that is, the hours, or degrees, to be esteemed as minutes, and the minutes as seconds, the proportion may be worked as above: hence it is evident that the Table may be very conveniently applied to the reduction of the sun's, moon's, or a planet's right ascension and declination to any given time after noon or midnight; and, also, to the equation of time;—for the illustration of which the following Problems are given.

Problem I.

To reduce the Sun's Longitude, Right Ascension and Declination; and, also, the Equation of Time, as given in the Nautical Almanac, to any given Meridian, and to any given time under that Meridian.

Rule.

To the apparent time at ship, or place, (to be always reckoned from the *preceding noon* *,) add the longitude, in time, if it be west, but subtract it if east; and the sum, or difference, will be the Greenwich time.

From page II. of the month in the Nautical Almanac, take out the sun's

* See precepts to Table XV.—page 25.

longitude, right ascension, declination, or equation of time for the noons immediately preceding and following the Greenwich time, and find their difference; then,

To the proportional log. of this difference, add the proportional log. of the Greenwich time (reckoning the hours as *minutes*, and the minutes as *seconds*,) and the constant log. 9. 1249*; the sum of these three logs., rejecting 10 from the index, will be the proportional log. of a correction which is always to be *added* to the sun's longitude and right ascension at the *noon preceding* the Greenwich time; but to be applied by addition or subtraction to the sun's declination and the equation of time, at that noon, according as they may be increasing or decreasing.

Example 1.

Required the sun's longitude, right ascension and declination, and also the equation of time, May 6th, 1824, at 5ʰ 10ᵐ, in longitude 64°45′ west of the meridian of Greenwich?

Apparent time at ship or place, = ...	5ʰ 10ᵐ
Longitude 64°45′ west, in time, = ...	+4. 19
Greenwich time, = ...	9ʰ 29ᵐ

To find the Sun's Longitude.

Diff. in 24 hours = 57′59″ prop. log.	=	.4920
Greenwich time = 9ʰ29ᵐ prop. log.	=	1.2783
Constant log.	=	9.1249
Correction of sun's long. . . . . . = . + 22′55″ p. log.	=	0.8952
Sun's long. at noon, May 6, 1824 . = 1ˢ15°51′13″		
Sun's long. as required . . . . . = 1ˢ16°14′ 8″		

To find the Sun's Right Ascension.

Diff. in 24 hours = 3′52″ prop. log.	=	1.6679
Greenwich time = 9ʰ29ᵐ prop. log.	=	1.2783
Constant log.	=	9.1249
Correction of sun's right asc. . . . . = + 1′32″ p. log.	=	2.0711
Sun's right asc. at noon, May 6, 1824, = 2ʰ53ᵐ31ˢ.7		
Sun's right asc. as required . . . . . = 2ʰ55ᵐ 3ˢ.7		

† The arithmetical complement of the proportional log. of 24 hours esteemed as minutes.

To find the Sun's Declination.

Diff. in 24 hours = 16′38″ prop. log.	. . . .	= 1.0343
Greenwich time = 9h29m prop. log.	. . . .	= 1.2783
Constant log.	. . . .	= 9.1249
Correction of sun's dec.	=+ 6′34″ p. log.	= 1.4375
Sun's dec. at noon, May 6, 1824, . .	=16°36′ 5″ north.	
Sun's dec. as required	=16°42′39″ north.	

To find the Equation of Time.

Diff. in 24 hours = 4″.5 prop. log.	. . . .	= 3.3829
Greenwich time = 9h29m prop. log.	. . . .	= 1.2783
Constant log.	. . . .	= 9.1249
Correction of the equation of time .	= + 1s.8 p. log.	= 3.7861
Equation of time, May 6, 1824 . .	= 3m36s.1	
Equation of time as required	= 3m37s.9	

Remark.—Since the daily difference of the equation of time is expressed, in the Nautical Almanac, in seconds and tenths of a second; if, therefore, these tenths be multiplied by 6, the daily difference will be reduced to seconds and thirds :—Now, if those seconds and thirds be esteemed as minutes and seconds, the operation of reducing the equation of time will become infinitely more simple; because the necessity of making proportion for the tenths, as above, will then be done away with :—remembering, however, that the minutes and seconds corresponding to the sum of the three logs. are to be considered as seconds and thirds.

Example 2.

Required the sun's longitude, right ascension, and declination, and also the equation of time, August 2d, 1824, at 19h22m, in longitude 98°45′ east of the meridian of Greenwich?

Apparent time at ship, or place, ...	19h22m
Longitude 98°45′ east, in time, = ...	− 6.35.
Greenwich time, ...	12h47m

To find the Sun's Longitude.

Diff. in 24 hours = 57′ 28″ prop. log.	. . . .	= .4959
Greenwich time = $12^h 47^m$ prop. log.	. . . .	= 1.1486
Constant log.	. . . .	= 9.1249
Correction of sun's longitude, . . .	= + 30′ 37″ p. log.	= 0.7694
Sun's long. at noon, Aug. 2, 1824,	= 4^s 10° 3′ 8″	
Sun's long. as required	= 4^s 10° 33′ 45″	

To find the Sun's Right Ascension.

Diff. in 24 hours = 3′ 52″ prop. log.	. . . .	= 1.6679
Greenwich time = $12^h 47^m$ prop. log.	. . . .	= 1.1486
Constant log.	. . . .	= 9.1249
Correction of sun's right asc. . . .	= + 2′ 4″ p. log. .	= 1.9414
Sun's right asc. at noon, Aug. 2, 1824,	= $8^h 50^m$ 0. 8	
Sun's right asc. as required . . .	= $8^h 52^m$ 4″.8	

To find the Sun's Declination.

Diff. in 24 hours = 15′ 36″ prop. log.	. . . .	= 1.0621
Greenwich time = $12^h 47^m$ prop. log.	. . . .	= 1.1486
Constant log.	. . . .	= 9.1249
Correction of sun's dec.	= − 8′ 19″ p. log.	= 1.3356
Sun's dec. at noon, Aug. 2, 1824, .	= 17° 44′ 41″ north.	
Sun's dec. as required, ...	= 17° 36′ 22″ north.	

To find the Equation of Time.

Diff. in 24 hours = 4^s 30‴ prop. log.	. . . .	= 1.6021
Greenwich time = $12^h 47^m$ prop. log.	. . . .	= 1.1486
Constant log.	. . . .	= 9.1249
Correction of the equation of time .	= − 2^s 24‴ p. log.	= 1.8756
Eqn. of time, at noon, Aug. 2, 1824,	= 5^m 54^s 24^t	
Eqn. of time as required, . . .	= 5^m 52^s 0^t	

Problem II.

To reduce the Moon's Longitude, Latitude, Right Ascension, Declination, Semi-diameter and Horizontal Parallax, as given in the Nautical Almanac, to any given Meridian, and to any given time under that Meridian.

Rule.

To the apparent time at ship, or place, (reckoned from the preceding noon or midnight,) add the longitude, in time, if it be west, but subtract it if east, and the sum, or difference, will be the Greenwich time past that noon or midnight, according as it may be.

Take from page V. VI. or VII. of the month in the Nautical Almanac, the moon's longitude, latitude, right ascension, declination, semidiameter, or horizontal parallax for the noon and midnight immediately *preceding* and *following* the Greenwich time, and find their difference; then,

To the proportional log. of this difference, add the proportional log. of the Greenwich time past the preceding noon or midnight, (reckoning the hours as *minutes*, and the minutes as *seconds*,) and the constant logarithm 8.8239 *; the sum of these three logs., abating 10 in the index, will be the proportional log. of a correction which is always to be *added* to the moon's longitude or right ascension at the noon or midnight preceding the Greenwich time; but to be applied by addition or subtraction to her latitude, declination, semidiameter or horizontal parallax, at that noon or midnight, according as it may be increasing or decreasing.

Note.—Since the difference of the moon's longitude and right ascension, in 12 hours, will always exceed the limits of the Table, and also the difference of her declination in that interval, at times; if, therefore, the one half or one third of such difference be taken, and the correction, resulting therefrom, multiplied by 2 or 3, the required correction will be obtained.

Example.

Required the moon's longitude, latitude, right ascension, declination, semidiameter and horizontal parallax, Aug. 2d, 1824, at $3^h\,10^m$ past noon, in longitude $60°30'$ west of the meridian of Greenwich?

Apparent time at ship, or place, ...	=	$3^h\,10^m$
Longitude $60°30'$ west, in time, ...	=	+ 4. 2.
Greenwich time, past noon, Aug. 2, 1824,	=	$7^h\,12^m$

* The arithmetical complement of the proportional log. of 12 hours esteemed as *minutes*.

To find the Moon's Longitude.

Diff. in 12 hours = 6°31′59″ ÷ 3 = 2°10′39⅔″, prop. log. = .1391

Greenwich time = ... $7^h 12^m$ = prop. log. = 1.3979

Constant log. . . . . = 8.8239

One third the corr. of the moon's long. = 1°18′25″ p. log. = 0.3609

Multiply by . . . . . . 3.

Corr. of moon's long. . . . . + 3°55′15″

Moon's long. at noon, Aug. 2, 1824 . = 7^s17°16′27″

Moon's long. as required . . . . 7^s21°11′42″

To find the Moon's Latitude.

Diff. in 12 hours = 23′35″ prop. log. . . . . = .8827

Greenwich time = $7^h 12^m$ prop. log. . . . . = 1.3979

Constant log. . . . . = 8.8239

Correction of moon's lat. . . . . −14′ 9″ p. log. = 1.1045

Moon's lat. at noon, Aug. 2, 1824, . = 4° 6′59″ south.

Moon's lat. as required . . . . 3°52′50″ south.

To find the Moon's Right Ascension :—

Diff. in 12 hours = 6°51′49″ ÷ 3 = 2°17′16⅓″, prop. log. = .1177

Greenwich time, = ... $7^h 12^m$ prop. log. = 1.3979

Constant log. . . . . = 8.8239

One third the corr. of the moon's rt. asc. = 1°22′22″ p. log. = 0.3395

Multiply by . . . . . . 3.

Corr. of moon's right asc. . . . . + 4° 7′ 6″

Moon's rt. asc. at noon, Aug. 2, 1824, = 223.33.36.

Moon's right asc. as required . . . 227°40′42″

To find the Moon's Declination :—

Diff. in 12 hours = 1°23′43″, prop. log.		= .3325
Greenwich time = 7h12m prop. log.		= 1.3979
Constant log.		= 8.8239
Correction of moon's dec.	+ 50′14″ p. log.	= 0.5543
Moon's dec. Aug. 2, 1824, at noon,	= 20°57′ 7″ south.	
Moon's declination as required	21°47′21″ south.	

Note.—The correction, or proportional part of the moon's motion, found as above, must be corrected by the equation of second difference contained in Table XVII., as explained in pages 33 and 34.

To find the Moon's Semidiameter :—

Diff. in 12 hours = 6″ prop. log.		= 3.2553
Greenwich time = 7h12m prop. log.		= 1.3979
Constant log.		= 8.8239
Corr. of the moon's semidiameter	— 4″ p. log.	= 3.4771
Moon's semidiameter at noon, Aug. 2, 1824,	= 15′33″	
Moon's semidiameter as required	15′29″	

To find the Moon's Horizontal Parallax :—

Diff. in 12 hours = 28″ prop. log.		= 2.6717
Greenwich time = 7h12m prop. log.		= 1.3979
Constant log.		= 8.8239
Corr. of moon's horiz. paral.	— 14″ p. log.	= 2.8935
Moon's horiz. paral. at noon, Aug. 2, 1824,	= 57′ 6″	
Moon's horiz. paral. as required	56′52″	

Note.—The moon's semidiameter, thus found, must be augmented by the correction contained in Table IV., as explained in page 10.

Problem III.

To reduce the Right Ascension and Declination of a Planet, as given in the Nautical Almanac, to any given time under a known Meridian.

Rule.

Turn the longitude into time, and add it to the apparent time at ship or place if it be west, but subtract it if east; and the sum, or difference, will be the corresponding time at Greenwich.

From page IV. of the month in the Nautical Almanac, take out the planet's right ascension and declination for the nearest days preceding and following the Greenwich time, and find the difference; find, also, the difference between the Greenwich time and the nearest preceding day; then,—

To the proportional log. of this difference, add the proportional log. of the difference of right ascension, or declination, and the constant log. 9.9031 *; the sum of these three logs., rejecting 10 from the index, will be the proportional log. of a correction, which being applied by addition, or subtraction, to the right ascension, or declination, (on the nearest day preceding the Greenwich time,) according as it may be increasing or decreasing, the sum, or difference, will be the correct right ascension or declination at the given time and place.

Example.

Required the right ascension of the planet Venus, July 3, 1824, at 10^h20^m apparent time, at a place 75°30′ west of the meridian of Greenwich?

Apparent time at given place,	=	10^h20^m
Longitude 75°30′ west, in time	=	+ 5. 2.
Greenwich time	. . . . = . .	8 days, 15^h22^m

To find the Right Ascension:—

R. A. of Venus, July 1	= 6^h 8^m	. . . . . .	1^d 0^h 0^m	
Ditto 7	= 6.40.	Gr. time = .	3. 15. 22	
Difference .	= 0^h32^m	Diff. . = .	$2^d15^h22^m$	= 63^h22^m;

which are to be esteemed as minutes and seconds:—hence,

* The arithmetical complement of the proportional log. of 144 hours (6 days,) esteemed as minutes; and, hence taken as 2 hours and 24 minutes.

Diff. bet. G. time and nearest preceding day 63ʰ22ᵐ prop. log. = .4534
Diff. of right ascension in 6 days . . . . 0ʰ32ᵐ prop. log. = .7501
Constant log. 9.9031

Correction of right ascension . . . . +14ᵐ 5ˢ p. log. = 1.1066
Planet's R. A. on July 1, 1824 = ... 6ʰ 8ᵐ 0ˢ

R. A. of Venus, as required 6ʰ22ᵐ 5ˢ

To find the Declination:—

Dec. of Venus, July 1 = 23°36′ N. 1ᵈ 0ʰ 0ᵐ
Ditto . . . . . 7 = 23.32. N. Gr. time = 3.15.22

Difference . . 0° 4′ Diff. = 2ᵈ15ʰ22ᵐ = 63ʰ22ᵐ;
which are to be esteemed as minutes and seconds; hence,
Diff. bet. G. time and nearest day preceding 63ʰ22ᵐ prop. log. = .4534
Diff. of declination in 6 days = ... 0° 4′ prop. log. 1.6532
Constant log. 9.9031

Correction of declination = . . . — 1′46″ p. log. = 2.0097
Planet's dec. on July 1, 1824, . . = 23.36. 0. north.

Dec. of Venus, as required . . . . . 23°34′14″ north.

Table XXX.

Logarithmic Half-elapsed Time.

This Table is useful in finding the latitude by two altitudes of the sun; and also in other astronomical calculations, as will be shown hereafter. The Table is extended to every fifth second of time under 6 hours, with proportional parts, adapted to the intermediate seconds, in the right hand margin of each page; by means of which, the logarithmic half-elapsed time answering to any given period, and conversely, may be readily obtained at sight.

As the size of the page would not admit of the indices being prefixed to the logs. except in the first column, under 0ˢ, therefore where the indices change in the other columns, a bar is placed over the 9, or left hand figure of the log., as thus, $\bar{9}$, to catch the eye, and to indicate that from thence, through the rest of the line, the index is to be taken from the next lower line in the first column, or that marked 0ˢ at top and bottom. It is to be observed, however, that the indices are only susceptible of change when the half-elapsed time is under 23 minutes.

The logarithmic half-elapsed time corresponding to any given period, is to be taken out by entering the Table with the hours and fifths of seconds at the top, or next less fifth if there be any odd seconds, and the minutes in the left-hand column; in the angle of meeting will be found a number, which being *diminished* by the proportional part answering to the odd seconds, in the right hand margin, will give the required logarithm.

Example.

Required the logarithmic half-elapsed time answering to $2^h 47^m 28^s$?

	$2^h 47^m 25^s$ answering to which is	0. 17572
Odd seconds . . . 3.	pro. part answering to which is . . —	11
Given time = $2^h 47^m 28^s$	corresponding log. hf.-elapsed time .	0. 17561

In the converse of this, that is, in finding the time corresponding to a given log.;—if the given log. can be exactly found, the corresponding hours, minutes, and seconds, will be the time required:—but if it cannot be exactly found (which in general will be the case), take out the hours, minutes, and seconds answering to the *next greater* log.; the difference between which, and that given, being found in the column of proportional parts, abreast of where the *next greater* log. was found, or nearly so, will give a certain number of seconds, which being added to the hours, minutes and seconds, found as above, will give the required time.

Example.

Required the time corresponding to the logarithmic half-elapsed time 0. 14964?

Solution.—The next *greater* log. is 0. 14973, corresponding to which is $3^h 0^m 25^s$; the difference between this log. and that given is 9; answering to which in the column of proportional parts is 3 seconds, which being added to the above found time gives $3^h 0^m 28^s$ for that required.

Remark.—The numbers in this Table are expressed by the logarithmic co-secants adapted to given intervals of time, the index being diminished by radius, as thus:

Let the half-elapsed time be $3^h 20^m 45^s$; to compute the corresponding logarithm.

Given time = $3^h 20^m 45^s$ in degrees = $50° 11' 15''$; log. co-secant less radius = 0. 114557; which, therefore, is the required log.; and since it is not necessary that this number should be extended beyond five decimal places, the sixth, or right hand figure, may be struck off; observing, however, to increase the fifth figure by unity or 1, when the right hand figure, so struck off, amounts to 5 or upwards:—hence, the tabular number corresponding to $3^h 20^m 45^s$ is 0. 11456; and so of others.

Table XXXI.

Logarithmic Middle Time.

This Table is, also, useful in finding the latitude by two altitudes of the sun; for which purpose it is extended to every fifth second under 6 hours, with proportional parts for the intermediate seconds, in the right-hand margin of each page; by means of which the logarithmic middle time answering to any given period, and conversely, may be readily taken out at sight.

As the indices are only prefixed to the logs. in the first column, therefore where those change in the other columns a bar is placed over the cypher, as thus, $\bar{0}$, to catch the eye, and to indicate that from thence through the rest of the line, the index is to be taken from the next lower line, in the first column.

The logarithmic middle time answering to any given period is to be taken out by entering the Table with the hours and fifths of seconds at the top, or the *next less fifth* second (when there are any odd seconds, as there generally will be), and the minutes in the left-hand column; in the angle of meeting will be found a number, which being *augmented* by the proportional part answering to the odd seconds, in the compartment abreast of the angle of meeting, will give the log. required.

Example.

Required the logarithmic middle time answering to . $3^h 17^m 23^s$?
$3^h 17^m 20^s$ answering to which is 6.18099
Odd seconds . . 3. pro. part answering to which is . . + 8

Given time = $3^h 17^m 23^s$ corresponding log. middle time . . . 6.18107

The time corresponding to a given logarithmical number, is found by taking out the hours, minutes, and seconds, answering to the *next less* tabular number; the difference between which and that given, being found in the compartment of proportional parts, abreast of the said *next less* tabular number, will give a certain number of seconds, which being added to the hours, minutes, and seconds found as above, will be the time required.

Example.

Required the time corresponding to the log. middle time, 6.01767 ?

Solution.—The *next less* tabular log. is 6.01757, answering to which is $2^h 5^m 30^s$; the difference between this log. and that given is 10, answering to which in the column of proportional parts is 2 seconds, which being added to the time found, as above, gives $2^h 5^m 32^s$, for that required.

Remark.—The logarithmic middle time may be readily computed by the following rule; viz:—

To the logarithmic sine of the given time expressed in degrees, add the constant log. 6.301030, and the sum, abating 10 in the index, will be the required logarithm.

Example.

Let the middle time be 4^h 10^m 25^s, required the corresponding log.?

Given time = 4^h 10^m 25^s, in degs. = 62° 36′ 15″ log. sine = 9.948339
Constant log. . . . . = 6.301030

Logarithmic middle time, as required . . . . = 6.249369; and since it is not necessary that this log. should be extended beyond five places of decimals, the sixth, or right-hand figure may, therefore, be struck off; observing, however, to increase the fifth figure by unity or 1, when the right-hand figure, so struck off, amounts to 5 or upwards; hence the tabular number corresponding to 4^h 10^m 25^s, is 6.24937, and so on.

Table XXXII.

Logarithmic Rising.

This Table, with the two preceding, is particularly useful in finding the latitude by two altitudes of the sun; it is also of considerable use in many other astronomical calculations, such as in computing the apparent time from the altitude of a celestial object; determining the altitude of a celestial object from the apparent time, &c. &c.—The arrangement of the present Table is so very uniform with the preceding, that it is not deemed necessary to enter into its description any farther than by observing that the indices are only prefixed to the logs. in the first column:—that where those change in the other columns, large dots are introduced instead of 0's to catch the eye, for the purpose of indicating that from thence through the rest of the line, the index is to be taken from the next lower line in the first column; and that, in the general use of the Table, these dots are to be accounted as cyphers.

Example.

Required the logarithmic rising answering to $1^h 43^m 27^s$?

	$1^h 43^m 25^s$, answering to which is	5.00040
Odd seconds . .	2, pro. part answering to which is . .	28
Given time = $1^h 43^m 27^s$, corresponding logarithmic rising		5.00068

The converse of this, that is, finding the time corresponding to a given log. will appear obvious; thus,

Let the given logarithmic rising be 5.69088, to find the corresponding time.

The *next less* tabular log. is 5.69071, answering to which is $3^h 57^m 30^s$; the difference between this log. and that given is 17, answering to which in the column of proportional parts, abreast of the tabular log., is 3 seconds; now, this being added to the time found, as above, gives $3^h 57^m 33^s$; which, therefore, is the time corresponding to the given logarithmic rising.

Note.—The numbers in this Table were computed by the following rule, viz:—

To twice the logarithmic sine of half the meridian distance, in degrees, add the constant log. 6.301030, and the sum, rejecting 20 from the index, will be the logarithmic rising.

Example.

Required the logarithmic rising answering to $4^h 10^m 45^s$?

Given meridian distance = $4^h 10^m 45^s$, in degrees = $62° 41' 15''$.

Half the meridian distance, in degrees . . . . = 31.20.37½; twice the logarithmic sine	19.432293
Constant log.	6.301030
Logarithmic rising answering to the given meridian distance =	5.733323

The numbers in the present Table may be also computed by means of the natural versed sines contained in Table XXVII., as thus;

Reduce the meridian distance to degrees, and find the natural versed sine corresponding thereto; the common log. of which will be the logarithmic rising.

Example.

Required the logarithmic rising answering to 4ʰ 22ᵐ 30ˢ, or 65°37′30″? Meridian distance in degrees = 65°37′30″, natural versed sine = 587293, log. = 5.768855; which, therefore, is the logarithmic rising answering to the given meridian distance.

In this method of computing the logarithmic rising, the natural versed sine is to be conceived as being multiplied by 1000000, the radius of the Table, and thus reduced to a whole number.

Table XXXIII.

To reduce Points of the Compass to Degrees, and conversely.

This Table is divided into six columns; the two first and two last of which contain the names of the several points and quarter points of the compass; the third column contains the corresponding number of points and quarter points reckoned from the meridian; and the fourth column the degrees and parts of a degree answering thereto.—The manner of using this Table is obvious; and so is the method by which it was computed:—for since the whole compass card is divided into 32 points, and the whole circle into 360 degrees; it is evident that any given number of points will be to their corresponding degrees in the ratio of 32 to 360; and *vice versa*, that any given number of degrees will be to their corresponding points as 360 is to 32:—Hence, to find the degrees corresponding to one point.—As 32ᵖ : 360° :: 1ᵖ : 11°15′; so that one point contains 11 degrees and 15 minutes;—two points, 22 degrees 30 minutes, &c. &c.

Table XXXIV.

Logarithmic Sines, Tangents, and Secants, to every Point and Quarter Point of the Compass.

In this Table the points and quarter points are contained in the left and right hand marginal columns, and the log. sines, tangents, and secants, corresponding thereto, in the intermediate columns.

If the course be given in points, it will be found more convenient to take the log. sine, tangent, or secant of it from this Table, than to reduce those points to degrees, and then find the corresponding log. sine, &c. &c. in either of the following Tables.—The manner of using this Table must appear obvious at first sight.

Table XXXV.

Logarithmic Secants.

In the first 10 degrees of this Table, the logarithmic secants are given to every *tenth* second, with proportional parts, answering to the intermediate seconds, in the right hand marginal column.—Thence to 88 degrees, the log. secants are given to every *fifth* second, with proportional parts, adapted to the intermediate seconds, in the right hand column of each page:—and because the numbers increase rapidly between 80 and 88 degrees, producing very considerable differences between any two adjacent logs.; therefore betwixt those limits, there are two pages allotted to a degree; every page being divided into two parts of 15 minutes each, so that no portion whatever of the proportional parts might be lost, and that the whole might have room to be fully inserted.—In the two last degrees, viz. from 88 to 90, the log. secants are given to every second.—The Table is so arranged as to be extended to every second in the semicircle, or from 0 to 180 degrees; as thus: the arcs corresponding to the log. secants are given in regular succession at top from 0 to 90 degrees, and then continued at bottom, reckoning towards the left hand, from 90 to 180 degrees:—the arcs corresponding to the co-secants are placed at the bottom of the Table, in numerical order, from the right hand towards the left (like the secants in the second quadrant), from 0 to 90 degrees, and then continued at top, agreeably to the order of the secants in the first quadrant, from 90 degrees to the end of the semicircle.—This mode of arrangement, besides doing away with the necessity of finding the supplement of an arch when it exceeds 90 degrees, possesses the peculiar advantage of enabling the reader to take out the log. secant, or co-secant of any arch whatever, and conversely, at sight, as will appear evident by the following problems.

Note.—The log. co-secant of a given degree, or secant of a degree above 90, will be found in the same *page* with the *next less* degree in the first column under 0″ at top, it being the first number in that column; and the log. co-secant of a given degree and minute, or secant of a degree and minute above 90, will be found on the same line with the *next less* minute in the column marked 60″ at bottom of the Table.

Problem I.

To find the Logarithmic Secant, and Co-secant of any given Arch, expressed in Degrees, Minutes, and Seconds.

Rule.

If the given arch be comprised within the limits of the two last degrees of the first quadrant, that is, between 88 and 90 degrees, the Table will directly exhibit its corresponding log. secant or co-secant;—but when it falls without those limits, then find the log. secant, or co-secant, in the angle of meeting made by the given degree and next less fifth or tenth second at top, and the minutes in the left hand column; to which, add the proportional part corresponding to the odd seconds from the right hand column abreast of the angle of meeting, if a secant be wanted, or a *co-secant above 90 degrees;* but subtract that part when a co-secant is required, *or a secant above* 90 *degrees;* and the sum, or difference, will be the log. secant or co-secant answering to the given arch.

Example 1.

Required the logarithmic secant, and co-secant, corresponding to 23°14′23″?

To find the Log. Secant:—

	23°14′20″,	ans. to which is . . .	10.036747
Odd seconds	3	propor. part to which is +	3
Given arch =	23°14′23″	Corres. log. secant =	10.036750

To find the Log. Co-secant:—

	23°14′20″,	ans. to which is . . .	10.403881
Odd seconds	3	propor. part to which is −	15
Given arch =	23°14′23″	Corres. log. co-secant =	10.403866

Example 2.

Required the log. secant, and co-secant, corresponding to 113°23′47″?

To find the Log. Secant:—

	113°23′45″,	ans. to which is . .	10.401121
Odd seconds	2	propor. part to which is −	10
Given arch =	113°23′47″	Corres. log. secant =	10.401111

To find the Log. Co-secant:—

	113°23′45″,	ans. to which is . .	10.037260
Odd seconds	2	propor. part to which is +	2
Given arch =	113°23′47″	Corres. log. co-secant=	10.037262

Note.—In that part of the Table which lies between 10 and 80 degrees, the size of the page would not admit of the indices being prefixed to any other logs. than those contained in the first column of each page; nor, indeed, is it necessary that they should be, since they are uniformly the same as those contained in the said first column; viz., 10 for each log. secant or co-secant.

Problem II.

To find the Arch corresponding to a given Logarithmic Secant or Co-secant:

Rule.

If the given log. secant, or co-secant, exceeds the secant of 88 degrees, viz., 11.457181, its corresponding arch will be found at first sight in the Table; but if it be under that number, find the arch answering to the next less secant, or next greater co-secant; the difference between which and that given, being found in the column of proportional parts, abreast of the tabular log., will give a certain number of seconds, which, being added to the above-found arch, will give that required.

Example 1.

Required the arch corresponding to the given log. secant 10.235421?

Solution.—The *next less* secant, in the Table, is 10.235412, corresponding to which is 54°26′25″; the difference between this log. secant, and that given, is 9; answering to which, in the column of proportional parts abreast of the tabular log., is 3″; which, being *added* to the above-found arch, gives 54°26′48″ for that required.

Example 2.

Required the arch corresponding to the given log. co-secant 10.562114?

Solution.—The *next greater* co-secant, in the Table, is 10.562129, corresponding to which is 15°45′25″; the difference between this log. co-secant and that given, is 15; answering to which, in the column of proportional parts abreast of the tabular log., is 2″; which, being added to the above-found arch, gives 15°45′27″ for that required.

Remark.—The log. secant of any arch is expressed by the difference between twice the radius and the log. co-sine of that arch; and the co-secant of an arch, by the difference between twice the radius and the log. sine of such arch. Hence, to find the log. secant of 50°40′.—The log. co-sine of 50°40′ is 9.801974, which, being taken from twice the radius, viz., 20.000000, leaves 10.198026 for the log. secant: from this, the manner of computing the co-secant will be obvious.

Table XXXVI.

Logarithmic Sines.

Of all the Logarithmic Tables in this work, this is, by far, the most generally useful, particularly in the sciences of Navigation and Nautical Astronomy; and, therefore, much pains have been taken in reducing it to that state of simplicity which appears to be best adapted to its direct application to the many other purposes for which it is intended, besides those above-mentioned.

In this Table, the log. sines of the two first degrees of the quadrant are given to every second. The next eight degrees, viz., from 2 to 10, have their corresponding log. sines to every fifth second, with proportional parts answering to the intermediate seconds in the *adjacent* right-hand column; and because the log.sines increase rapidly in those degrees, two pages are allotted to a degree; every page being divided into two parts, and each part containing 15 minutes of a degree: so that no portion whatever of the proportional parts might be lost, and that the whole might have room to be fully inserted. In the following seventy degrees, that is, from 10 to 80, the log. sines are also given to every fifth second, with proportional parts corresponding to the intermediate seconds in the right-hand column of each page. In this part of the Table, each page contains a degree; and, for want of sufficient room, the indices are only prefixed to the logs. expressed in the first column.

From 80 to 90 degrees, the log. sines are only given to every tenth second, because of the small increments by which the sines increase towards the end of the first quadrant; the proportional parts for the intermediate seconds are given in the right-hand column of each page, as in the preceding part of the Table.

The Table is so arranged, as to be extended to every second in the semicircle, or from 0 to 180 degrees; as thus: the arcs corresponding to the log. sines are given in regular succession at top, from 0 to 90 degrees, and then continued, at bottom, reckoning towards the left hand, from 90 to 180 degrees. The arcs corresponding to the co-sines are given at bottom of the Table, and ranged in numerical order towards the left hand, from 0 to 90 degrees, (according to the order of the sines between 90 and 180 degrees,) and then continued at top, from 90 degrees to the end of the semicircle, agreeably to the order of the sines in the first quadrant. This mode of arrangement does away with the necessity of finding the supplement of an arch when it exceeds 90 degrees, and possesses the peculiar advantage of enabling the navigator to take out the log. sine or co-sine of any arch, and conversely, at sight, as will appear obvious by the following Problems.

Note.—The log. co-sine of a given degree is found in the same page with the *next less* degree in the column marked 0″ at top, it being the first number in that column; and the co-sine of a given degree and minute is found on the same line with the *next less* minute in the column marked 60′ at bottom of the page.

PROBLEM I.

To find the Logarithmic Sine, and Co-sine of any given Arch, expressed in Degrees, Minutes, and Seconds.

RULE.

If the given arch be comprised within the limits of the two first degrees of the quadrant, the Table will directly exhibit its corresponding log. sine or co-sine; but when it exceeds those limits, then find the log. sine, or co-sine, in the angle of meeting made by the given degree and next less fifth or tenth second at top, and the minutes in the left-hand column; to which add the proportional part corresponding to the odd seconds in the right-hand column abreast of the angle of meeting, if a sine be wanted, *or a co-sine above* 90 degrees; but subtract that part when a co-sine is required, *or a sine above* 90 *degrees:* and the sum, or difference, will be the log. sine, or co-sine, answering to the given arch.

Example 1.

Required the log. sine, and co-sine, corresponding to 23°14′23″?

To find the Log. Sine :—

	23°14′20″,	ans. to which is . . .	9.596119.
Odd seconds	3	propor. part to which is +	15
Given arch =	23°14′23″	Corresponding log. sine	9.596134

To find the Log. Co-sine :—

	23°14′20″,	ans. to which is . . .	9.963253
Odd seconds	3	propor. part to which is −	3
Given arch =	23°14′23″	Corresponding log. co-sine	9.963250

Example 2.

Required the log. sine, and co-sine, corresponding to 113°23′47″?

To find the Log. Sine :—

	113°23′45″,	ans. to which is . .	9.962740
Odd seconds	2	propor. part to which is −	2
Given arch =	113°23′47″	Corresponding log. sine	9.962738

To find the Log. Co-sine :—

	113°23′45″,	ans. to which is . .	9.598879
Odd seconds	2	propor. part to which is +	10
Given arch =	113°23′47″	Corresponding log. co-s.	9.598889

Problem II.

To find the Arch corresponding to a given Logarithmic Sine, or Co-sine.

Rule.

If the given log. sine, or co-sine, be less than the sine of 2 degrees, viz., 8.542819, its corresponding arch will be found at first sight in the Table; but if it exceeds that number, find the arch answering to the next less sine, or next greater co-sine; the difference between which and that given, being found in the column of proportional parts abreast of the tabular log., will give a certain number of seconds, which, being *added* to the above-found arch, will give that required.

Note.—Since the arcs corresponding to the sines between 90 and 180 degrees are found at the bottom of the Table, and those corresponding to the co-sines between the same limits at its top; if, therefore, it be required to find the arch above 90 degrees answering to a given log. sine, or co-sine, the first term is to be taken out as if it were a *co-sine under* 90 *degrees*, and the other term as if it were a *sine under* 90 *degrees*.

Example 1.

Required the arch corresponding to the given log. sine 9.437886?

Solution.—The next *less* log. sine in the Table is 9.437871, corresponding to which is 15°54′25″; the difference between this and that given, is 15; answering to which, in the column of proportional parts, abreast of the tabular log., is 2″; which, being added to the above-found arch, gives 15°54′27″ for that required.

Example 2.

Required the arch corresponding to the given log. co-sine 9.764579?

Solution.—The next *greater* co-sine in the Table is 9.764588, corresponding to which is 54°26′25″; the difference between this and that given, is 9; answering to which, in the column of proportional parts, abreast of the tabular log., is 3″; which, being added to the above-found arch, gives 54°26′28″ for that required.

Remark.—The log. sines are deduced directly from the natural sines; as thus:—

Multiply the natural sine by 10000000000; find the common log. of the product, and it will be the log. sine.

Example 1.

Required the log. sine of 39°30′?

Solution.—The natural sine of 39°30′ is .636078, which, being multiplied by 10000000000, gives 6360780000.000000, the common log. of which is 9.803511; which, therefore, is the log. sine of 39 degrees and 30 minutes, as required.

Example 2.

Required the log. co-sine of 68 degrees?

Solution.—The natural co-sine of 68 degrees is .374607, which, being multiplied by 10000000000, gives 3746070000.000000, the common log. of which is 9.573575; which, therefore, is the log. co-sine of 68 degrees, as required.

Table XXXVII.

Logarithmic Tangents.

This Table is arranged in a manner so very nearly similar to that of the log. sines, that it is not deemed necessary to enter into its description any farther than by observing, that it is computed to every second in the two first and two last degrees of the quadrant, or semicircle, and to every fifth second in the intermediate degrees. The log. tangent, or co-tangent, of a given arch, and conversely, is to be found by the rules for the log. sines in pages 94 and 95.

Example 1.

Required the log. tangent, and co-tangent, corresponding to 31°10′47″?

To find the Log. Tangent:—

	31°10′45″,	ans. to which is	9.781845
Odd seconds	2	propor. part to which is +	10
Given arch =	31°10′47″	Corresponding log. tang.=	9.781855

To find the Log. Co-tangent:—

	31°10′45″,	ans. to which is . .	10.218155
Odd seconds	2	propor. part to which is −	10
Given arch =	31°10′47″	Corres. log. co-tang. =	10.218145

Example 2.

Required the log. tangent, and co-tangent, corresponding to 139°11′53″?

To find the Log. Tangent:—

	139°11′50″,	ans. to which is . .	9.936142
Odd seconds	3	propor. part to which is −	13
Given arch =	139°11′53″	Corres. log. tang. = .	9.936129

To find the Log. Co-tangent:—

	139°11′50″,	ans. to which is . . .	10.063858
Odd seconds	3	propor. part to which is +	13
Given arch =	139°11′53″	Corres. log. co-tang. =	10.063871

Example 3.

Required the arch corresponding to the given log. tang. 10. 155436?

Solution.—The next *less* log. tangent in the Table is 10. 155423, corresponding to which is 55°2′25″; the difference between this log. tangent and that given, is 13; answering to which, in the column of proportional parts abreast of the tabular log., is 3″; which, being added to the above-found arch, gives 55°2′28″ for that required.

Example 4.

Required the arch corresponding to the given log. co-tang. 9. 792048?

Solution.—The next *greater* log. co-tangent in the Table is 9. 792057, corresponding to which is 58°13′15″; the difference between this log. co-tangent and that given, is 9; answering to which, in the column of proportional parts abreast of the tabular log., is 2″; which, being added to the above-found arch, gives 58°13′17″ for that required.

Remark.

The arch corresponding to a given log. tangent may be found by means of a Table of log. sines, in the following manner; viz.,

Find the natural number corresponding to twice the given log. tangent, rejecting the index, to which add the radius, and find the common log. of the sum; now, half this log. will be the log. secant, less radius, of the required arch; and which, being subtracted from the given log. tangent, will leave the log. sine corresponding to that arch.

Example.

Let the given log. tangent be 10. 084153; required the arch corresponding thereto by a table of log. sines?

Given log. tang. . 084153 × 2 = . 168306, Nat. num. = 1. 473349
to which add the radius = 1. 000000

Sum = ... 2. 473349, the
common log. of which is 0. 393286; the half of this is 0. 196643, the
secant, less radius of the required arch.

Given log. tangent = ... 10. 084153

Corresponding log. sine = ... 9. 887510,
answering to which is 50°31′; and which, therefore, is the required arch corresponding to the given log. tangent.

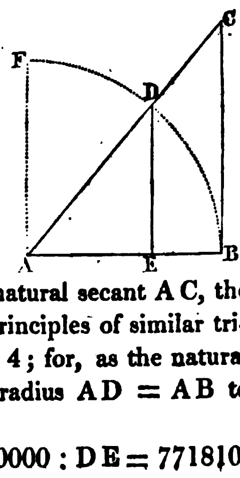
The arch corresponding to a given log. tangent may also be found in the following manner, which, it is presumed, will prove both interesting and instructive to the student in this department of science.

Find the natural tangent, that is, the natural number corresponding to the given log. tangent, to the square of which add the square of the radius; extract the square root of the sum, and it will be the natural secant corresponding to the required arch; then, say, as the natural secant, thus found, is to the natural tangent, so is the radius to the natural sine: now, the degrees, &c. answering to this in the *Table of Natural Sines*, will be the arch required, or that corresponding to the given log. tangent.

Example.

Let the given log. tangent be 10.084153; it is required to find the arch corresponding thereto by a Table of Natural Sines?

Solution.—Given log. tangent = .084153; the natural number corresponding to this is 1.213816; which, therefore, is the natural tangent answering to the given log. tangent.



In the annexed diagram, let B C represent the natural tangent = 1.213816, and A B the radius = 1.000000. Now, since the base and perpendicular of the right-angled triangle A B C are known, the hypothenuse or secant A C may be determined by Euclid, Book I., Prop. 47. Hence $\sqrt{BC^2 = 1.213816^2 + AB^2 = 1.000000^2} =$ A C = 1.572689, the natural secant corresponding to the given log. tangent. Having thus found the natural secant A C, the natural sine D E may be found agreeably to the principles of similar triangles, as demonstrated in Euclid, Book VI., Prop. 4; for, as the natural secant A C is to the natural tangent B C, so is the radius A D = A B to the natural sine D E: hence,

As A C 1.572689 : B C 1.213816 :: A D 1.000000 : D E = 771810, the corresponding natural sine; now, the arch answering to this, in the Table of Natural Sines, is 50°31′; which, therefore, is the arch corresponding to the given log. tangent, as required.

Note.—The Table of log. tangents may be very readily deduced from Tables XXXV. and XXXVI., as thus:—To the log. secant of any given arch, add its log. sine; and the sum, abating 10 in the index, will be the log. tangent of that arch; the difference between which and twice the radius, will be its co-tangent.

Example.

Required the log. tangent, and co-tangent, of 25°27′35″?

Log. secant of the given arch 25°27′35″ =	10.044366
Log. sine of ditto	9.633344
Log. tangent corres. to the given arch = .	9.677710
Log. co-tangent corres. to ditto	10.322290

The Table of log. tangents may also be computed in the following manner; viz.,

From the log. sine of the given arch, the index being increased by 10, subtract its log. co-sine, and the remainder will be the log. tangent of that arch; the difference between which and twice the radius, will be its log. co-tangent.

Example.

Required the log. tangent, and co-tangent, of 32°39′40″?

Log. sine of the given arch 32°39′40″ = . .	9.732128
Log. co-sine of ditto	9.925249
Log. tangent corres. to the given arch = . .	9.806879
Log. co-tangent corresponding to ditto . .	10.193121

Table XXXVIII.

To reduce the Time of the Moon's Passage over the Meridian of Greenwich, to the Time of her Passage over any other Meridian.

The daily retardation of the moon's passage over the meridian, given at the top of the Table, signifies the difference between two successive transits of that object over the same meridian, diminished by 24 hours; as thus: the moon's passage over the meridian of Greenwich, July 22d, 1824, is $21^h.7^m$, and that on the following day $22^h.9^m$; the interval of time between these two transits is $25^h.2^m$, in which interval it is evident that the moon is $1^h.2^m$ later in coming to the meridian; and which, therefore, is the daily retardation of her passage over the meridian.

This Table contains the proportional part corresponding to that retardation and any given interval of time or longitude; in computing which, it is easy to perceive that the proportion was,

As 24 hours, augmented by the daily retardation of the moon's transit over the meridian, are to the said daily retardation of transit, so is any given interval of time, or longitude, to the corresponding proportional part of such retardation. The operation was performed by proportional logs., as in the following

Example.

Let the daily retardation of the moon's transit over the meridian be 1h 2m; required the proportional part corresponding thereto, and 9h 40m of time, or 145 degrees of longitude?

As 24 hours + 1h 2m (daily retard.) =	25h 2m	Ar. comp. pro. log.	9.1432
Is to daily retardation of transit = .	1. 2	Propor. log. . .	0.4629
So is given interval of time = . . .	9.40	Propor. log. . .	1.2700
To corresponding proportional part =	23m 57s =	Pro. log. =	0.8761;

and in this manner were all the numbers in the Table obtained.

The corrections or proportional parts contained in this Table are expressed in minutes and seconds, and are extended to every twentieth minute of time, or fifth degree of longitude: these are to be taken out and applied to the time of the moon's transit, as given in the Nautical Almanac, in the following manner:—

Find, in page VI. of the month in the Nautical Almanac, the difference between the moon's transit on the given day (reckoned astronomically) and that on the day *following*, if the longitude be *west;* but on the day *preceding*, if it be *east*. With this difference enter the Table at the top, and the given time in the left-hand, or the longitude in the right-hand column; in the angle of meeting will be found a correction, which, being applied by *addition* to the time of transit on the given day, if the longitude be *west*, but by *subtraction*, if *east*, the sum, or difference, will be the reduced time of transit.

Example 1.

Required the apparent time of the moon's passage over a meridian 80 degrees west of Greenwich, July 22d, 1824?

Mn's pas. over mer. of Greenw. on giv. day is $21^h 7^m$ . . . $21^h\ 7^m\ 0^s$
Ditto on the day *following* = 22.9

Retardation of moon's transit = . . . $1^h 2^m$; ans. to which
and 80 degs. is + 13. 13

Apparent time of the moon's transit over the given meridian = $21^h 20^m 13^s$

Example 2.

Required the apparent time of the moon's passage over a meridian 120 degrees east of Greenwich, August 20th, 1824?

Mn's pas. over mer. of Greenw. on giv. day is $20^h 54^m$ . . . $20^h 54^m\ 0^s$
Ditto on the day *preceding* = 19. 54

Retardation of the moon's transit = . . $1^h\ 0^m$; ans. to which
and 120 degs. is − 19. 12

Apparent time of the moon's transit over the given meridian = $20^h 34^m 48^s$

Table XXXIX.

Correction to be applied to the Time of the Moon's Transit in finding the Time of High Water.

Since the moon is the principal agent in raising the tides, it might be expected that the time of high water would take place at the moment of her passage over the meridian; but observation has shown that this is not the case, and that the tide does not cease flowing for some time after: for, since the attractive influence of the moon is only diminished, and not entirely destroyed, in passing the meridian of any place, the ascending impulse previously communicated to the waters at that place must, therefore, continue to act for some time after the moon's meridional passage. The ascending impulse, thus imparted to the waters, ought to cause the time of the highest tide to be about 80 minutes after the moon's passage over the meridian; but owing to the disturbing force of the sun, the actual time of high water differs, at times, very considerably from that period.

The effect of the moon in raising the tides exceeds that of the sun in the ratio of about 2½ to 1; but this effect is far from being uniform: for, since the moon's distance from the earth bears a very sensible proportion to the diameter of this planet, and since she is constantly changing that distance, (being sometimes nearer, and at other times more remote in every lunation,) it is evident that she must attract the waters of the ocean with very unequal forces: but the sun's distance from the earth being so very immense, that, compared with it, the diameter of this planet is rendered nearly insensible, his attraction is consequently more uniform, and therefore it affects the different parts of the ocean with nearly an equal force.

By the combined effect of these two forces, the tides come on *sooner* when the moon is in her *first* and *third* quarters, and *later* when in the *second* and *fourth* quarters, than they would do if raised by the sole lunar agency: it is, therefore, the mean quantity of this acceleration and retardation that is contained in the present Table, the arguments of which are, the apparent times of the moon's reduced transit; answering to which, in the adjoining column, stands a correction, which, being applied to the apparent time of the moon's passage over the meridian of any given place by addition or subtraction, according to its title, the sum, or difference, will be the corrected time of transit. Now, to the corrected time of transit, thus found, let the time of high water on full and change days, at any given place in Table LVI., be applied by *addition*, and the *sum* will be the time of high water at that place, reckoning from the noon of the given day: should the *sum* exceed $12^h 24^m$, or $24^h 48^m$, subtract one of those quantities from it, and the remainder will be the time of high water very near the truth.

Example 1.

Required the time of high water at Cape Florida, America, March 7th, 1824; the longitude being 80°5′ west, and the time of high water on full and change days $7^h 30^m$?

Moon's transit over the meridian of Greenwich, per Nautical Almanac, March 7th, 1824, is	$5^h\ 2^m\ 0^s$
Correction from Table XXXVIII., answering to retardation of transit 58^m, and longitude 80°5′ west = ...	+ 12.23
Moon's transit reduced to the meridian of Cape Florida . . .	$5^h 14^m 23^s$
Correction answering to reduced transit ($5^h 14^m 23^s$) in Table XXXIX., is	− 1. 5. 0
Corrected time of transit	$4^h\ 9^m 23^s$
Time of high water at Cape Florida on full and change days	7.30. 0
Time of high water at Cape Florida on the given day = . . .	$11^h 39^m 23^s$

Example 2.

Required the time of high water in Queen Charlotte's Sound, New Zealand, April 13th, 1824; the longitude being 174°56′ east, and the time of high water on full and change days $9^h 0^m$?

Moon's transit over the meridian of Greenwich, per Nautical Almanac, April 13th, 1824, is	$10^{h}27^{m}0^{s}$
Correction from Table XXXVIII., answering to retardation of transit 50^{m}, and longitude $174°56'$ west = ...	− 23.29
Moon's transit reduced to the meridian of Queen Charlotte's Sound	$10^{h}3^{m}31^{s}$
Correction answering to reduced time of transit ($10^{h}3^{m}31^{s}$) in Table XXXIX., is	+ 23. 0
Corrected time of transit	$10^{h}26^{m}31^{s}$
Time of high water at given place on full and change days .	9. 0. 0
Time of high water at Queen Charlotte's Sound, past noon of the given day	$19^{h}26^{m}31^{s}$
Subtract	12. 24. 0
Time of high water at given place, as required	$7^{h}\ 2^{m}31^{s}$

TABLE XL.

Reduction of the Moon's Horizontal Parallax on account of the Spheroidal Figure of the Earth.

Since the moon's equatorial horizontal parallax, given in the Nautical Almanac, is determined on spherical principles, a *correction* becomes necessary to be applied thereto, in places distant from the equator, in order to reduce it to the spheroidal principles, on the assumption that the polar axis of the earth is to its equatorial in the ratio of 299 to 300; and, when *very great accuracy* is required, this *correction* ought to be attended to, since it may produce an error of seven or eight seconds in the computed lunar distance. The correction, thus depending on the spheroidal figure of the earth, is contained in this Table; the arguments of which are, the moon's horizontal parallax at the top, and the latitude in the left-hand column; in the angle of meeting will be found a correction, expressed in seconds, which being *subtracted* from the horizontal parallax given in the Nautical Almanac, will leave the horizontal parallax agreeably to the spheroidal hypothesis.

Thus, if the moon's horizontal parallax, in the Nautical Almanac, be $57'58''$, and the latitude $51°48'$; the corresponding correction will be 7 seconds *subtractive*. Hence the moon's horizontal parallax on the spheroidal hypothesis, in the given latitude, is $57'51''$.

Remark.—The corrections contained in this Table may be computed by the following

Rule.

To the logarithm of the moon's equatorial horizontal parallax, reduced to seconds, add twice the log. sine of the latitude, and the constant log. 7.522879;* the sum, rejecting the tens from the index, will be the logarithm of the corresponding reduction of parallax.

Example.

Let the moon's horizontal parallax be 57′58″, and the latitude 51°48′; required the reduction of parallax agreeably to the spheroidal hypothesis?

Moon's equatorial horiz. par.	57′58″ = 3478″	Log. =	3.541330
Latitude	51°48′	Twice the log. sine =	19.790688
		Constant log. . .	7.522879
Reduction of horizontal parallax =	. . 7″.159	Log. =	0.854897

TABLE XLI.

Reduction of Latitude on account of the Spheroidal Figure of the Earth.

Since the figure of the earth is that of an oblate spheroid, the latitude of a place, as deduced directly from celestial observation, agreeably to the spherical hypothesis, must be greater than the true latitude expressed by the angle, at the earth's centre, contained between the equatorial radius and a line joining the centre of the earth and the place of observation. This excess, which is extended to every second degree of latitude from the equator to the poles, is contained in the present Table; and which, being *subtracted* from the latitude of any given place, will reduce that latitude to what it would be on the spheroidal hypothesis: thus, if the latitude be 50 degrees, the corresponding reduction will be 11′42″, subtractive; which, therefore, gives 49°48′18″ for the reduced or spheroidal latitude.

Remark.—The corrections contained in this Table may be computed by the following rule; viz.,

To the constant log. .003003,† add the log. co-tangent of the latitude,

---

* The arithmetical complement of the log. of the earth's ellipticity assumed at $\frac{1}{300}$.

† The excess of the spherical above the elliptic arch in the parallel of 45 degrees from the equator, is 11′.887, or 11′53″ (Robertson's Navigation, Book VIII., Article 134): hence 45° − 11′53″ = 44°48′7″, the log. co-tangent of which, rejecting the index, is .003003.

and the sum will be the log. co-tangent of an arch; the difference between which and the given latitude will be the required reduction.

Example.

Let it be required to reduce the spherical latitude 50°48′ to what it would be if determined on the spheroidal principles; and, hence, to find the reduction of that latitude.

Latitude	50°48′ 0″	Log. co-tang.=9.911467
		Constant log.= .003003
Reduced or spheroidal latitude =	50°36′21″	Log. co-tang.=9.914470
Reduction of latitude, as required	0°11′39″	

Table XLII.

A General Traverse Table; or Difference of Latitude and Departure.

This Table, so exceedingly useful in the art of navigation, is drawn up in a manner quite different from those that are given, under the same denomination, in the generality of nautical books; and, although it occupies but 38 pages, yet it is more extensive than the two combined Tables of 61 pages, which are contained in those books. In this Traverse Table, every page exhibits all the angles that a ship's course can make with the meridian, expressed both in points and degrees; which does away with the necessity of consulting two Tables in finding the difference of latitude and the departure corresponding to any given course and distance. If the course be *under* 4 points, or 45 degrees, it will be found in the left-hand compartment of each page; but that *above* 4 points, or 45 degrees, in the right-hand compartment of the page. The distance is given, in numerical order, at the top and bottom of the page, from unity, or 1, to 304 miles; which comprehends all the probable limits of a ship's run in 24 hours; and, by this arrangement, the mariner is spared the trouble of turning over and consulting twenty-three additional pages. Although the manner of using this Table must appear obvious at first sight, yet since its mode of arrangement differs so very considerably from the Tables with which the reader may have been hitherto acquainted, the following Problems are given for its illustration.

Problem I.

Given the Course and Distance sailed, or between two Places, to find the Difference of Latitude and the Departure.

Rule.

Enter the Table with the course in the left or right-hand column, and the distance at the top or bottom; opposite to the former, and under or over the latter, will be found the corresponding difference of latitude and departure: these are to be taken out as marked at the *top* of the respective columns if the course be *under* 4 points or 45 degrees, but as marked at the *bottom* if the course be *more* than either of those quantities.

Note.—If the distance exceed the limits of the Table, an aliquot part thereof may be taken, as a half, third, fourth, &c.; then the difference of latitude and departure corresponding to this and the given course, being multiplied by 2, 3, 4, &c., (that is, the figure by which such aliquot part was found,) the product will be the difference of latitude and departure answering to the given course and distance.

Example 1.

A ship sails S.S.W. ½ W. 176 miles; required the difference of latitude and the departure?

Opposite 2½ points and under 176 miles, stand 155.2 and 83.0: hence the difference of latitude is 155.2, and the departure 83.0 miles.

Example 2.

A ship sails N. 57° E. 236 miles; required the difference of latitude and the departure?

Opposite to 57°, and under 236 miles, stand 128.5 and 197.9: hence the difference of latitude is 128.5, and the departure 197.9 miles.

Example 3.

The course between two places is E. b. S. ¾ S., and the distance 540 miles; required the difference of latitude and the departure?

Distance divided by 2, gives 270 miles; under or over which, and opposite to 6½ points, stand . . . 91.0 and 254.2

Multiply by	2	2
Products =	182.0	and 508.4

: hence the difference of latitude is 182.0, and the departure 508.4 miles.

Example 4.

The course between two places is N. 61 W. and the distance 1176 miles; required the difference of latitude and the departure?

Distance 1176 divided by 4, gives 294 miles; under or over which, and opposite to 61°, stand . . . . 142.5 and 257.1
Multiply by 4 4

Product = 570.0 and 1028.4 : hence the difference of latitude is 570.0, and the departure 1028.4 miles.

Problem II.

Given the Difference of Latitude and the Departure, to find the Course and Distance.

Rule.

With the given difference of latitude and departure, enter the Table and find, in the proper columns abreast of each other, the tabular difference of latitude and departure either corresponding or nearest to those given; then the course will be found on the same horizontal line therewith in the left or right-hand column, and the distance at the top or bottom of the compartment where the tabular numbers were so found.

Note.—If the difference of latitude be *greater* than the departure, the course will be *less* than 4 points, or 45 degrees; and, therefore, it is to be taken from the left-hand column: but when the difference of latitude is *less* than the departure, the course will be *more* than 4 points or 45 degrees, and, consequently, it must be taken from the right-hand column.

Note, also, that when the difference of latitude and the departure, or either of them, exceed the limits of the Table, aliquot parts are to be taken, as a half, third, fourth, &c., with which find the course and distance as before; then the *distance*, thus found, being multiplied by 2, 3, 4, &c., the product will be the *whole distance* corresponding to the given difference of latitude and departure. The course is *never* to be multiplied, because the angle will be the same whether determined agreeably to the whole difference of latitude and the departure, or according to their corresponding aliquot parts.

Example 1.

If the difference of latitude made by a ship in 24 hours be 177.4 miles north, and the departure 102.6 miles east, required the course and distance made good?

Solution.—The tabular difference of latitude and departure, nearest corresponding to those given, are 177.5 and 102.5 respectively: these are found in the compartment under or over 205, and opposite to 30 degrees; hence the course made good is N. 30 E., and the distance 205 miles.

Example 2.

The difference of latitude made by a ship in 24 hours, is 98.5 miles south, and the departure 140.6 miles west; required the course and distance made good?

Solution.—The tabular difference of latitude and departure, nearest to those given, are 98.7 and 140.9 respectively: these are found in the compartment under or over 172, and opposite to 55 degrees; hence the course made good is S. 55° W., and the distance 172 miles.

Example 3.

The difference of latitude is 700 miles south, and the departure 928 miles west; required the course and distance?

Solution.—Since the difference of latitude and the departure exceed the limits of the Table, take therefore any aliquot part of them, as one fourth, and they will be 175 and 232 respectively: now, the tabular numbers, answering nearest to those, are 175.1 and 232.4; these are found in the compartment under or over 291, and opposite to 53 degrees: hence the course is S. 53° W., and the distance 291 × 4 = 1164 miles, as required.

Remark.—Whenever it becomes necessary to take aliquot parts of the difference of latitude, the same must be taken of the departure, whether it falls without the limits of the Table or not; and, *vice versa*, whenever it becomes necessary to take aliquot parts of the departure, the same must be taken of the difference of latitude.

And, in all cases where the tabular numbers differ considerably from those given, proportion must be made for that difference.

Problem III.

Given the proper Difference of Latitude between two Places, the Meridional Difference of Latitude, and the Departure, to find the Course, Distance, and Difference of Longitude.

Rule.

With the proper difference of latitude and the departure, find the course and distance by Problem II.; then, with the course thus found and the meridional difference of latitude, (in a latitude column,) take out the corresponding departure, and it will be the difference of longitude required; as thus: run the eye along the horizontal line answering to the course, from where the proper difference of latitude was found, (*always to the right hand,*) and find, in a latitude column, the tabular difference of latitude answering nearest to the given meridional difference of latitude; abreast of which, in the departure column, will be found the difference of longitude.

Example.

The proper difference of latitude between two places, is 142 miles north, the departure 107 miles west, and the meridional difference of latitude 169 miles; required the course, distance, and difference of longitude?

Solution.—The tabular difference of latitude and departure answering nearest to those given, are 142.2 and 107.3 respectively; these are found in the compartment under or over 178, and opposite to 37 degrees: hence the course is N. 37° W., and the distance 178 miles. Now, with the course 37 degrees, and the meridional difference of latitude 169 miles, the difference of longitude is found, as thus: from where the proper difference of latitude was found, run the eye along the horizontal line answering to 37 degrees, (*always towards the right hand,*) and the tabular difference of latitude answering nearest to the given meridional difference of latitude will be found in the compartment under or over 212, viz. 169.3; corresponding to which, in the departure column, is 127.6; and which, therefore, is the difference of longitude, as required.

Problem IV.

Given the proper Difference of Latitude, the Meridional Difference of Latitude, and the Difference of Longitude, to find the Course and Distance.

Rule.

With the meridional difference of latitude and the difference of longitude, esteemed as difference of latitude and departure, find the course by Problem II.; then with the course, thus found, and the proper difference of latitude, the distance is to be obtained, as thus: run the eye (*always to the left hand*) along the horizontal line answering to the course, from where the meridional difference of latitude was found, and seek, in the proper column, the difference of latitude answering nearest to that given; over or under which, at the top or bottom of the column, will be found the required distance.

Note.—When the meridional difference of latitude *exceeds* the difference of longitude, the course is to be taken from the left-hand column; but otherwise from the right.

Example.

The proper difference of latitude between two places is 78 miles south, the meridional difference of latitude 107 miles south, and the difference of longitude 119 miles east; required the course and distance?

Solution.—The tabular difference of latitude and departure, answering nearest to the meridional difference of latitude and the difference of longitude, are 107.1 and 118.9 respectively; these are found in the compartment under or over 160, and opposite to 48 degrees: hence the course is S. 48° E. Now, the eye being run along the horizontal line answering to 48, (*towards the left hand,*) the nearest tabular difference of latitude, answering to the proper difference of latitude, will be found in the compartment under or over 117: hence the distance is 117 miles.

Problem V.

Given the middle Latitude, and the Meridian Distance or Departure, to find the Difference of Longitude.

Rule.

Enter the Table with the middle latitude, *taken as a course*, and the departure in a latitude column; run the eye along the horizontal line answering to that course (towards the right hand or the left, according as the first tabular difference of latitude which meets the eye therein is greater or less than the given departure), and find a difference of latitude that either agrees with, or comes nearest to, the given departure; then the distance corresponding to this, at the top or bottom of the column, will be the difference of longitude.

Example.

The middle latitude between two places is 20° north, and the meridian distance or departure 140 miles; required the difference of longitude?

Solution.—The middle latitude, 20 degrees, taken as a course, and the departure 140, as difference of latitude, will be found to correspond in the compartment under or over 149: hence the difference of longitude is 149 miles, as required.

Problem VI.

Given the middle Latitude, the Difference of Latitude, and the Difference of Longitude between two Places, to find the Course and Distance.

Rule.

Enter the Table with the difference of longitude, esteemed as distance, at the top or bottom of the page, and the middle latitude, *taken as a course,* in the left or right-hand column; answering to which, in the *difference of latitude column,* will be found the departure. Now, with this departure and the given difference of latitude, the course and distance are to be found by Problem II.

Example.

The middle latitude is 26 degrees north, the difference of latitude 200 miles north, and the difference of longitude 208 miles east; required the course and distance?

Solution.—In the compartment under or over 208 miles (the given longitude), and opposite to 26 degrees (the middle latitude taken as a course), stands 186.9 in the difference of latitude column, which, therefore, is the departure. Now, the tabular numbers answering nearest to the given difference of latitude and the departure, thus found, are 200.4 and 186.9 respectively; these are found in the compartment under or over 274, and opposite to 43 degrees: hence the course is N. 43° E., and the distance 274 miles.

Remark.—The numbers in the general Traverse Table were computed agreeably to the following rule; viz.,

As radius is to the distance, so is the co-sine of the course to the difference of latitude; and so is the sine of the course to the departure.

Example.

Given the course 35 degrees, and the distance 147 miles; to compute the difference of latitude and the departure.

To find the Difference of Latitude.

As radius	= 90° log. sine	. . =	10.000000
Is to distance	147 miles	. . log. =	2.167317
So is the course . . .	= 35° log. co-sine	. =	9.913365
To difference of lat.	= 120.4, miles	. . log. =	2.080682

To find the Departure.

As radius	= 90° log. sine	. . =	10.000000
Is to distance	147 miles	. . log. =	2.167317
So is the course	= 35°, log. sine	. . =	9.758591
To departure . . =	84.3 miles	. . log. =	1.925908

Table XLIII.

Meridional Parts.

This Table contains the meridional parts answering to each degree and minute of latitude from the equator to the poles; the arguments of which are, the degrees at the top, and the minutes in the left or right hand marginal columns; under the former, and opposite to the latter, in any given latitude, will be found the meridional parts corresponding thereto, and conversely. Thus, if the latitude be 50°48′, the corresponding meridional parts will be 3549.8 miles.

Remark.—The Table of meridional parts may be computed by the following rule; viz.,

Find the logarithmic co-tangent *less radius* of half the complement of any latitude, and let it be esteemed as an *integral number*; now, from the common logarithm of this, subtract the constant log. 2.101510 *, and the remainder will be the log. of the meridional parts answering to that latitude.

Example 1.

Required the meridional parts corresponding to latitude 50°48′ ?
Given lat. = 50°48′ ; complement = 39°12′ ÷ 2 = 19°36′, the half complement ; hence,
Half comp. = 19°36′ log. co-tangent *less radius* = .448448, the log. of which is 5.651712
Constant log. 2.101510
Meridional parts corresponding to given lat. 3549.78 = log. = 3.550202

Example 2.

Required the meridional parts corresponding to latitude 89°30′ ?
Given lat. = 89°30′ ; comp. = 0°30′ ÷ 2 = 0°15′, the half complement ; hence,
Half comp. = 0°15′ log. co-tangent *less radius* = 2.360180, the log. of which is 6.372945
Constant log. 2.101510
Meridional parts corresponding to given lat. 18682.49 = log. = 4.271435

Table XLIV.

The Mean Right Ascensions and Declinations of the principal fixed Stars.

This Table contains the mean right ascensions and declinations of the principal fixed stars adapted to the beginning of the year 1824.—The stars are arranged in the Table according to the order of right ascension in which they respectively come to the meridian ; the annual variation, in right ascension and declination, is given in seconds and decimal parts of a second ; that of the former being expressed in time, and that of the latter motion.

The stars marked †, have been taken from the Nautical Almanac for the year 1824.—The stars that have asteriks prefixed to them are those from which the moon's distance is computed in the Nautical Almanac for the purpose of finding the longitude at sea.

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* The measure of the arc of 1 minute (page 54,) is .0002908882 ; which being multiplied by 10000000000, (the radius of the Tables) produces 290.8882000000 ; and, this being multiplied by the modulus of the common logarithms, viz., .4342944819, gives 126.331140109823580 ; the common log. of which is 2.101510, as above.

The places of the stars, as given in this Table, may be reduced to any future period by multiplying the annual variation by the number of years and parts of a year elapsed between the beginning of 1824, and such future period: the product of right ascension is to be *added* to the right ascensions of all the stars, except β and δ, in Ursa Minor, from whose right ascensions it is to be subtracted: but the product of declination is to be applied, according to *the sign* prefixed to the annual variation in the Table, to the declinations of all the stars without any exception;—thus,

To find the right ascension and the declination of α Arietis, Jan. 1st, 1834.

R. A. of α Arietis, per Tab.	$1^h57^m16^s$;	and its dec. . .		$22°37'33''$ N.
Annual var. .	$+3''.35$	Ann. var.	$+17''.40$.	
Number of years after 1824 =	10	Num. of yrs. after 1824 =	10	
Product .	$+33''.5$	$+0'38''.5$	Prod. $+174''.0 =$	$+\ 2'54''$

Rt. asc. of α Arietis, as req. $1^h57^m49^s.5$, and its declination $22°40'27''$ N.

Should the places of the stars be required for any period antecedent to 1824, it is evident that the products of right ascension and declination must be applied in a contrary manner.

The eighth column of this Table contains the *true spherical distance* and the approximate bearing between the stars therein contained and those preceding, or abreast of them on the same horizontal line; and the ninth, or last column of the page, the annual variation of that distance expressed in seconds and decimal parts of a second.—By means of the last column, the tabular distance may be reduced very readily to any future period, by multiplying the years and parts of a year between any such period and the epoch of the Table, by the annual variation of distance; the product being applied by addition or subtraction to the tabular distance, according as the sign may be affirmative or negative, the sum or difference will be the distance reduced to that period.

Example.

Required the distance between α Arietis and Aldebaran, Jan. 1st, 1844?

Tabular dist. between the two given stars = . . . $35°32'7''$

Annual var. of distance . $-\ 0''.02$

Number of years after 1824 = 20

Product . . . $-\ 0''.40$ = ... $-\ 0''.40$

True spherical distance between the two given stars, as required. $35°32'6''.60$.

Remark.—The true spherical distance between any two stars, whose right ascensions and declinations are known, may be computed by the following rule; viz.,

To twice the log. sine of half the difference of right ascension, in degr ees add the log. sines of the polar distances of the objects; from half the sum of these three logs. subtract the log. sine of half the difference of the polar distances, and the remainder will be the log. tangent of an arch; the log. sine of which being subtracted from the half sum of the three logs., will leave the log. sine of half the true distance between the two given stars.

Example.

Let it be required to compute the true spherical distance between α Arietis and Aldebaran, January 1, 1844.

R. A. of α Arietis red. to 1844 = 1ʰ.58ᵐ.23ˢ., and its dec. = 22°.43′.21″ N.
R. A. of Aldebaran red. to 1844 = 4. 26. 58. 6, and its dec. = 16. 11. 28 N.

Difference of right ascension = 2ʰ.28ᵐ.35ˢ.6 =
37°.8′.54″ ÷ 2 = 18°.34′.27″

Half difference of R. A. in degrees = ...	18°.34′.27″	Twice the Log. sine	19.0063060
N. polar dist. of α Arietis =	67. 16. 39	Log. sine	9.9649129
N. polar dist. of Aldebaran =	73. 48. 32	Log. sine	9.9824236
		Sum . .	38.9536425

Diff. of Polar dists. 6°.31′.53″ Half = 19.4768212½ . . . 19.4768212.5

Half diff. of ditto 3°.15′.56½″ Log S. 8.7556177½

Arch 79°.14′.27″.5826 log. tang. . 10.7212035 Log. S. 9.9922976.3

Half the req. dist. . . . . . 17°.46′.3″.4424. Log. S. 9.4845236.2

True spher. dist. between
the two given stars . . 35°.32′.6″.8848 on Jan. 1, 1844.

Now, by comparing this computed distance with that directly deduced from the Table, as in the preceding example, it will be seen that the difference amounts to very little more than the fifth part of a second in twenty years; which evidently demonstrates that the tabular distances may be reduced to any subsequent period, for a considerable series of years, with all the accuracy that may be necessary for the common purposes of navigation.

Note.—The tabular distances will be found particularly useful in determining the latitude, at sea, by the altitudes of two stars, as will be shown hereafter.

Table XLV.

Acceleration of the Fixed Stars; or to reduce Sidereal to Mean Solar Time.

Observation has shown that the interval between any two consecutive transits of a fixed star over the same meridian is only 23ʰ.56ᵐ.4ˢ.09, whilst that of the sun is 24 hours:—the former is called a sidereal day, and the latter a solar day; the difference between those intervals is 3ᵐ.55ˢ.91, and which difference is called the acceleration of the fixed stars.

This acceleration is occasioned by the earth's annual motion round its orbit: and since that motion is from west to east at the mean rate of 59′.8″.3 of a degree each day; if, therefore, the sun and a fixed star be observed on any day to pass the meridian of a given place at the same instant, it will be found the next day when the star returns to the same meridian, that the sun will be nearly a degree short of it; that is, the star will have gained 3ᵐ.56ˢ.55 sidereal time, on the sun, or 3ᵐ.55ˢ.91 in mean solar time; and which amounts to one sidereal day in the course of a year:—for 3ᵐ.55ˢ.91 × 365ᵈ.5ʰ.48ᵐ.48ˢ = 23ʰ.56ᵐ.4ˢ:—hence in 365 days as measured by the transits of the sun over the same meridian, there are 366 days as measured by those of a fixed star.

Now, because of the earth's equable or uniform motion on its axis, any given meridian will revolve from any particular star to the same star again in every diurnal revolution of the earth, without the least perceptible difference of time shewn by a watch, or clock, that goes well:—and this presents us with an easy and infallible method of ascertaining the error and the rate of a watch or clock:—to do which we have only to observe the instant of the disappearance of any bright star, during several successive nights, behind some fixed object, as a chimney or corner of a house at a little distance, the position of the eye being fixed at some particular spot, such as at a small hole in a window-shutter nearly in the plane of the meridian; then if the observed times of disappearance correspond with the acceleration contained in the second column of the first compartment of the present Table, it will be an undoubted proof that the watch is well regulated:—hence, if the watch be exactly true, the disappearance of the same star will be $3^m.56^s$ earlier every night; that is, it will disappear $3^m.56^s$ sooner the first night; $7^m.52^s$ sooner the second night; $11^m.48^s$ sooner the third night, and so on, as in the Table.—Should the watch, or clock deviate from those times, it must be corrected accordingly; and since the disappearance of a star is instantaneous, we may thus determine the rate of a watch to at least half a second.

The first compartment of this Table consists of two columns; the first of which contains the sidereal days, or the interval between two successive transits of a fixed star over the same meridian, and the second the acceleration of the stars expressed in mean solar time; which is extended to 30 days, so as to afford ample opportunities for the due regulation of clocks or watches.—The five following compartments consist of two columns each, and are particularly adapted to the reduction of sidereal time into mean solar time:—the correction expressed in the column marked *acceleration*, &c. being subtracted from its corresponding sidereal time, will reduce it to mean solar time; as thus.

Required the mean solar time corresponding to $14^h.40^m.55^s$. sidereal time?

Given sidereal time =	. . . .		$14^h.40^m.55^s$
Corresponding to 14 hours is . .	$2^m.17^s.61$	} Sum =	
Do. 40 minutes . .	0. 6 .55	} Sum =	− $2^m.24$ .31
Do. 55 seconds . .	0. 0 .15	}	
Mean solar time as required	. . . .		$14^h.38^m.30^s.69$

Remark.—This Table was computed in the following manner; viz.,

Since the earth performs its revolution round its orbit, that is, round the sun, in a solar year; therefore as $365^d.5^h.48^m.48^s$: $360°$ :: 1^d : $59'.8''.3$; which, therefore, is the earth's daily advance in its orbit: but while the earth is going through this daily portion of its orbit, it turns once round on its axis, from west to east, and thereby describes an arc of $360°59'.8''.3$ in a mean solar day, and an arc of $360°$ in a sidereal day.

Hence, as $360°59'.8''.3$: 24^h :: $360°$: $23^h.56^m.4^s.09$, the length of a sidereal day in mean solar time; and which, therefore, evidently anticipates $3^m.55^s.91$ upon the solar day as before-mentioned. Now, As one sidereal day, is to $3^m 55^s.91$, so is any given portion of sidereal time to its corresponding portion of mean solar time :—and hence, the method by which the Table was computed.

Table XLVI.

To reduce Mean Solar Time into Sidereal Time.

Since this Table is merely the converse of the preceding, it is presumed that it does not require any explanation farther than by observing, that the correction is to be applied by addition to the corresponding mean solar time, in order to reduce it into sidereal time ; as thus.

Required the sidereal time corresponding to $20^h 15^m 33^s$ mean solar time?

Given mean solar time =			. . . .	$20^h 15^m 33^s$
Corresponding to 20 hours is	$3^m 17^s.13$	}		
Do. 15 minutes	0. 2 .46	} Sum = .	+	$3^m 19^s.68$
Do. 33 seconds	0. 0 .09	}		———
Sidereal time as required			. . . .	$20^h 18^m 52^s.68$

Table XLVII.

Time from Noon when the Sun's Centre is in the Prime Vertical; being the instant at which the Altitude of that Object should be observed in order to ascertain the apparent Time with the greatest Accuracy.

Since the change of altitude of a celestial object is quickest when that object is in the prime vertical, the most proper time for observing an altitude from which the apparent time is to be inferred, is therefore when the object is due east or west; because then the apparent time is not likely to be affected by the unavoidable errors of observation, nor by the inaccuracy of the assumed latitude.—This Table contains the apparent time when a celestial object is in the above position.—The declination is marked at top and bottom, and the latitude in the left and right hand marginal columns: hence, if the latitude be 50 degrees, and the declination 10 degrees, both being of the same name, the object will be due east or west at $5^h 26^m$ from its time of transit or meridional passage.

Remark.—This Table was computed by the following rule; viz., To the log. co-tangent of the latitude, add the log. tangent of the declination; and the sum, abating 10 in the index, will be the log. co-sine of the hour angle, or the object's distance from the meridian when its true bearing is either east or west.

Example.

Let the latitude be 50 degrees, north or south, and the sun's declination 10 degrees, north or south; required the apparent time when that object will bear due east or west?

Given latitude = ... 50° log. co-tangent = 9.923814
Declination of the sun = . . . 10° log. tangent = 9.246319

Hour angle = . . . 81°29′30″ = log. co-sine = 9.170133

In time = . . . 5h25m58s; which, therefore, is the apparent time when the sun bears due east or west.

Note.—During one half of the year, or while the sun is on the other side of the equator, with respect to the observer, that object is not due east or west while above the horizon; in this case, therefore, the observations for determining the apparent time must be made while the sun is near to the horizon; the altitude, however, should not be under 3 or 4 degrees, on account of the uncertainty of the effects of the atmospheric refraction on low altitudes.

Table XLVIII.

Altitude of a Celestial Object (when its centre is in the Prime Vertical,) most proper for determining the apparent Time with the greatest Accuracy.

This Table is nearly similar to the preceding; the only difference being that that Table shows the apparent time when a celestial object bears due east or west, and this Table the true altitude of the object when in that position; being the altitude most proper to be observed in order to ascertain the apparent time with the greatest accuracy:—thus, if the latitude be 50 degrees, and the declination 10 degrees, both being of the same name, the altitude of the object will be 13°6′, when it bears due east or west from the observer; which, therefore, is the altitude most proper to be observed, for the reasons assigned in the explanation to Table XLVII.

Note.—This Table was computed by the following rule; viz., If the declination be less than the latitude; from the log. sine of the former (the index being increased by 10), subtract the log. sine of the latter, and the remainder will be the log. sine of the altitude of the object when its centre is in the prime vertical:—But, if the latitude be less than the declination, a contrary operation is to be used; viz., from the log. sine of the latitude, the index being increased by 10, subtract the log. sine of the declination, and the remainder will be the log. sine of the altitude of the object when its centre is in the prime vertical, or when it bears due east or west.

Example 1.

Let the latitude be 50°, and the declination of a celestial object 10°, both being of the same name; required the altitude of that object when its centre is in the prime vertical.

Declination of the object =	10°	log. sine =	9.239670
Latitude	50.	log. sine =	9.884254
Altitude required . . .	13°6′6″	log. sine =	9.355416

Example 2.

Let the latitude be 3°, and the declination of a celestial object 14°, both being of the same name; required the altitude of that object when its in the prime vertical.

Latitude	3°	log. sine =	8.718800
Declination of the object =	14	log. sine =	9.383675
Altitude required .	12°29′38″	log. sine =	9.335125

Note.—Altitudes under 3 or 4 degrees should not be made use of in computing the apparent time, on account of the uncertainty of the atmospheric refraction near the horizon.

And since the Table only shows the altitude of a celestial object most favourable for observation when the latitude and declination are of the same name; therefore during that half of the year in which the sun is on the other side of the equator, with respect to the observer, and in which he does not come to the prime vertical while above the horizon, the altitude is to be taken whenever it appears to have exceeded the limits ascribed to the uncertainty of the atmospheric refraction in page 120.

TABLE XLIX.

Amplitudes of a Celestial Object, reckoned from the true East, or West Point of the Horizon.

The arguments of this Table are, the declination of a celestial object at top or bottom, and the latitude in the left, or right hand column; in the angle of meeting will be found the amplitude: proportion, however, is to be made for the excess of the minutes above the next less tabular arguments.

Example 1.

Let the latitude be 50°48′ north, and the sun's declination 10°25′ north; required the sun's true amplitude at its setting?

True amplitude corresp. to lat. 50°, and dec. 10°, =W. 15° 40′ N.

Tab. diff. to 1° of lat. =21′; now $\frac{21' \times 48'}{60'}$ = + 17, nearly;

T.diff. to 1° of dec.=1°36′, or 96′; now $\frac{96' \times 25'}{60'}$ = + 40

Sun's true amplitude as required . . . . . . = W. 16. 37. N.

Example 2.

Let the latitude be 34°24′ north, and the sun's declination 16°48′ south; required the sun's true amplitude at the time of its rising?

True amplitude corresponding to latitude 34° N. and declination 16°30′ S. = ... E. 20° 2′ S.

Tab. diff. to 1° of lat. = 15′; now $\frac{15' \times 24'}{60'}$ = . . + 6

Tab. diff. to 30′ of dec. = 37′; now $\frac{37' \times 18'}{30'}$ = . . + 22, nearly.

Sun's true amplitude as required = ... E. 20°30′ S.

Remark.

This Table was computed agreeably to the following rule; viz.,

To the log. secant of the latitude, add the log. sine of the declination, and the sum, abating 10 in the index, will be the log. sine of the true amplitude.

Example.

Let the latitude be 50°48′, and the declination of a celestial object 10°25′; required the true amplitude of that object?

Latitude	50°48′	log. secant	10.199263
Declination	10.25	log. sine	9.257211
True amplitude as required	16°37′22″	log. sine . .	9.456474

Table L.

To find the Times of the Rising and Setting of a Celestial Object.

This Table contains the semidiurnal arch, or the time of half the continuance of a celestial object above the horizon when its declination is of the same name with the latitude of the place of observation; or the time of half its continuance below the horizon when its declination and the latitude are of different denominations.—*The semi-diurnal arch expresses the time that a celestial object takes in ascending from the eastern horizon to the meridian; or of its descending from the meridian to the western horizon.*

As the Table is only extended to 23½ degrees of declination, being the greatest declination of the sun, and to no more than 60 degrees of latitude; therefore, when the declination of any other celestial object and the latitude of the place of observation exceed those limits, the semi-diurnal arch is to be computed by the following rule; viz.,

To the log. tangent of the latitude, add the log. tangent of the declination, and the sum, rejecting 10 in the index, will be the log. sine of an arch; which being converted into time, and added to 6 hours when the latitude and declination are of the same name; or subtracted from 6 hours when these elements are of contrary names; the sum, or difference, will be the semi-diurnal arch.

Example 1.

Let the latitude be 61 degrees, north, and the declination of a celestial object 25°10′, north; required the corresponding semi-diurnal arch?

Latitude	61° 0′ north,	log. tangent	10.256248
Declination	25.10 north,	log. tangent	9.671963
Arch = . . .	57°57′21″ =	log. sine . . .	9.928211

Arch conv. into time 3h51m49s + 6h = 9h51m49s, the semidiurnal arch, as required.

Example 2.

Let the latitude be 20°40′, south, and the declination of a celestial object 30°29′, north; required the corresponding semi-diurnal arch?

Latitude.	20°40′ south,	log. tangent . .	9.576576
Declination . . .	30.29 north,	log. tangent . .	9.769860
Arch = ...	12°49′45″ =	log. sine . . .	9.346436

Arch conv. into time 0h51m19s; and 6h − 0h51m19s = 5h8m41s, the semi-diurnal arch.

The present Table has been computed agreeably to the first example; but as in most nautical computations, it is not absolutely necessary that the semi-diurnal arch should be determined to a greater degree of accuracy than the nearest minute; the seconds have, therefore, been rejected, and the nearest minute retained accordingly.

Since the Table for finding the time of the rising or setting of a celestial object (commonly called a Table of semi-diurnal and semi-nocturnal arcs,) is scarcely applied to any other purpose, by the generality of nautical persons, than that of merely finding the approximate time of the rising or setting of the sun; the following problems are, therefore, given for the purpose of illustrating and simplifying the use of this Table; and of showing how it may be employed in determining the apparent times of the rising and setting of all the celestial objects whose declinations come within its limits.

Problem I.

Given the Latitude and the Sun's Declination, to find the Time of its Rising or Setting.

Rule.

Let the sun's declination, as given in the Nautical Almanac, be reduced to the meridian of the given place by Table XV., or by Problem I., page 76; then,

Enter the Table with this reduced declination at top, or bottom, and the latitude in either of the side columns; under or over the former, and opposite to the latter, will be found the approximate time of the sun's setting when the latitude and declination are of the same name; or that of its rising when they are of contrary names.—The time of setting being taken from 12 hours will leave the time of rising, and *vice versa*, the time of rising being taken from 12 hours will leave that of setting.

Note.—Proportion must be made, as usual, for the excess of the minutes of latitude and declination above the next less tabular arguments.

Example 1.

Required the approximate times of the sun's rising and setting July 13, 1824, in latitude 50°48′, north, and longitude 120 degrees west?

Sun's declination July 13th. per Nautical
  Almanac, is . . . . 21°49′51″ north.
Correction from Table XV., answering to
  var. of dec. 8′58″, and long. 120° W. − 2′59″

Sun's dec. reduced to given meridian . . 21°46′52″; or 21°47′, N. *
Time, in Table L., ans. to lat. 50°, north, and
  dec. 21°30′, north = ... 7h52m

Tabular difference to 1° of lat. = 4′; now $\frac{4' \times 48'}{60'}$ = + 3

Tab. difference to 30′ of dec. = 3′; now $\frac{3' \times 17'}{30'}$ = + 2, nearly.

Approximate time of the sun's setting 7h57m

Approximate time of the sun's rising 4h 3m

Note.—Twice the time of the sun's setting will give the length of the day; and twice the time of its rising will give the length of the night.

Example 2.

Required the approximate times of the sun's rising and setting October 1st, 1824, in latitude 40°30′ north, and longitude 105 degrees east?

Sun's declination October 1st. per Nautical
  Almanac, is . . . . 3°16′ 6″ south.
Correction from Table XV., answering to
  var. of dec. 23′20″, and long. 105° E. − 6′48″

Sun's dec. reduced to the given meridian 3° 9′18″, or 3°9′ south.
Time in Table L., ans. to lat. 40° north, and
  dec. 3° south, is 6h10m

Tab. diff. to 1° of lat. = 0′; now $\frac{0' \times 30'}{60'}$ = 0

Tab. diff. to 1° of dec. = 3′; now $\frac{3' \times 9'}{60'}$ = 0

Approximate time of the sun's rising 6h10m

Approximate time of the sun's setting 5h50m

* The nearest minute of declination is sufficiently exact for the purpose of finding the approximate times of the rising and setting of a celestial object.

Remark.

Since the times of the sun's rising and setting, found as above, will differ a few minutes from the observed, or apparent times in consequence of no notice having been taken of the combined effects of the horizontal refraction and the height of the observer's eye above the level of the sea, by which the time of rising of a celestial object is accelerated, and that of its setting retarded; nor of the horizontal parallax which affects these times in a contrary manner; a correction, therefore, must be applied to the approximate times of rising and setting, in order to reduce them to the apparent times.—This correction may be computed by the following rule; by which the apparent times of the sun's rising and setting will be always found to within a few seconds of the truth.

Rule.—To the approximate times of rising and setting, let the longitude, in time, be applied by addition or subtraction, according as it is west or east, and the corresponding times at Greenwich will be obtained: to these times, respectively, let the sun's declination be reduced by Table XV., or by Problem I., page 76; then,

Find the sum and the difference of the natural sine of the latitude, and the natural co-sine of the declination (rejecting the two right hand figures from each term), and take out the common log. answering thereto, rejecting also the two right hand figures from each:—now, to half the sum of these two logs. add the proportional log. of the sum of the horizontal refraction and the dip of the horizon diminished by the sun's horizontal parallax, and the constant log. 1.1761 *; the sum of these three logs., abating 4 in the index, will be the proportional log. of a correction; which being subtracted from the approximate time of rising, and added to that of setting, the apparent times of the sun's rising and setting will be obtained.

Thus,—Let it be required to reduce the approximate times of the sun's rising and setting, as found in the last Example, to the respective apparent times; the horizontal refraction being 33′; the dip of the horizon 5′ 15″, and the sun's horizontal parallax 9 seconds.

The sun's declination reduced to the approximate time of rising, is 3°3′37″, and to that of setting 3°14′58″ south.

* This is the proportional log. of 12 hours esteemed as minutes.

Latitude . . 40°30′ 0″ nat. sine . . = 6494				
Declination . 3° 3′37″ nat. co-sine . = 9986				
	Sum	16480 log.	=	4.2170
	Difference	3492 log.	=	3.5431
		Sum		7.7601
		Half-sum = . . .		3.8800½
	33′+5′15″−9″=38′6″, prop. log. . . .			0.6743
	Constant log.			1.1761
Correction . . . . − 3′21″ prop. log. =				1.7304½
Approximate time of rising = . . . 6ʰ10ᵐ 0ˢ				
Apparent time of sun's rising = . . 6ʰ 6ᵐ39ˢ				

Latitude . . 40°30′ 0″ nat. sine . . = 6494				
Declination . 3°14′58″ nat. co-sine . = 9984				
	Sum	16478 log.	=	4.2169
	Difference	3490 log.	=	3.5428
		Sum		7.7597
		Half sum = . . .		3.8798½
	33′+5′15″−9″=38′6″, prop. log. = .			0.6743
	Constant log.			1.1761
Correction . . . . + 3′21″ prop. log. =				1.7302½
Approximate time of setting = . . 5ʰ50ᵐ 0ˢ				
Apparent time of sun's setting = . 5ʰ53ᵐ21ˢ				

Note.—In this method of reducing the approximate to the apparent time of rising or setting, it is immaterial whether the latitude and declination be of the same, or of contrary names:—nor is it of any consequence whether the declination be reduced to the approximate times of rising and setting or not, since the declination at noon will be always sufficiently exact to determine the *correction* within two or three seconds of the truth, on account of its natural co-sine being only required to four places of figures;—this will appear evident by referring to the above example, where, although there is a difference of 11.′21.″ between the reduced declinations at the approximate times of rising and setting; yet this difference has no sensible effect on the correction corresponding to those times.

Problem II.

Given the Latitude of a Place and the Declination of a fixed Star, to find the Times of its Rising and Setting.

Rule.

Let the right ascension and declination of the star, as given in Table XLIV, be reduced to the given day; then, from the right ascension of the star, increased by 24 hours if necessary, subtract that of the sun, at noon of the given day; and the remainder will be the approximate time of the star's transit, or passage over the meridian; from which, let the correction answering thereto and the daily variation of the sun's declination (Table XV.,) be subtracted, and the apparent time of the star's transit will be obtained.

If much accuracy be required, and the place of observation be under a meridian different from that of Greenwich, a correction depending on the longitude and variation of the sun's right ascension (Table XV.,) must be applied to the time of transit:—this correction is subtractive in west, and additive in east longitude; the time being always reckoned from the *preceding* noon: now,

Enter Table L., with the declination at top or bottom, and the latitude in the side column; and in the angle of meeting will be found the semi-diurnal arch, or the time of half the star's continuance above the horizon, when the latitude and declination are of the same name; but if these elements are of different names, the time, so found, is to be subtracted from 12 hours, in order to obtain the half continuance above the horizon: then this half continuance* being applied by subtraction and addition to the apparent time of transit, will give the approximate times of the star's rising and setting.

Example 1.

At what times will the star α Arietis rise and set January 1st. 1824, in latitude 50.°48.′ north?

---

* In strictness the semi-diurnal arch, or half continuance above the horizon ought to be corrected by subtracting therefrom the proportional part (Table XV.,) corresponding to it and the variation of the sun's right ascension for the given day.

Star's dec. on given day is 22°37′33″, or 22°38′ north, and its right ascension		$1^h 57^m 16^s$
Sun's right ascension at noon of the given day is . . .		18.43.58
Approximate time of star's transit		7.13.18
Correction from Tab. XV., ans. to $7^h 13^m 18^s$, and 4′24″, the var. of the sun's right ascension		− 1.20
Apparent time of star's transit, or passage over the meridian		$7^h 11^m 58^s$
Time, in Tab. L. ans. to lat. 50° N., and dec. 22°30′ N. = ...	$7^h 58^m$	
Tabular diff. to 1° of lat. = 5′; now $\frac{5' \times 48'}{60'}$ =	+ 4	
Tab. diff. to 30′ of dec. = 4′; now $\frac{4' \times 8'}{30'}$ =	+ 1	
Semi-diurnal arch, or time of half the star's continuance above the horizon . . . =	$8^h\ 3^m$ . .	$8^h\ 3^m\ 0^s$
Approx. time of star's rising, past noon of Dec. 31st, 1823		$23^h\ 8^m 58^s$
Approx. time of star's setting, past noon of the *given day* .		$15^h 14^m 58^s$

Example 2.

At what times will the star Sirius rise and set January 1st, 1824, in lat. 40°30′ north, and long. 120 degrees, west of the meridian of Greenwich?

Star's dec. on given day is 16°28′53′ or 16°29′ south, and its right ascension		$6^h 37^m 23^s$
Sun's right ascension at noon of the given day is . . .		18.43.58
Approximate time of the star's transit		11.53.25
Corr. from Table XV., ans. to $11^h 53^m 25^s$, and 4′24″, the var. of the sun's right ascension		− 2.11
Corr. from ditto, ans. to long. 120° west, and 4′24″ the var. of the sun's right ascension		− 1.28
Appar. time of star's transit over the given meridian . . .		$11^h 49^m 46^s$
Time, in Table L., ans. to lat. 40° north, and declination 16° S. = ...	$6^h 56^m$	
Tab. diff. to 1° of lat. = 2′; now $\frac{2' \times 30'}{60'}$ =	+ 1	
Tab. diff. to 30′ of dec. = 2′; now $\frac{2' \times 29'}{30'}$ =	+ 2, nearly.	
Semi-nocturnal arch	$6^h 59^m$,	
which being subtracted from 12^h leaves		$5^h\ 1^m\ 0^s$
Approximate time of the star's rising		$6^h 48^m 46^s$
Approximate time of the star's setting		$16^h 50^m 46^s$

Remark.—The approximate times of the rising and setting of a fixed star may be readily reduced to the respective apparent times by the rule given for those of the sun, in page 126; omitting, however, the first part, or that which relates to the reduction of declination: and, since the fixed stars have no sensible parallax, the words "horizontal parallax" are, also, to be omitted; thus:—

To reduce the approximate times of rising and setting, as found in the last example, to the respective apparent times, the dip of the horizon being assumed at 6′.30″

Lat. of place of observ. 40°.30′. Nat. sine = 6494
Declin. of the star = 16.29 Nat. co-sine = 9589

Sum = ...	16083	Log. = .	4.2064
Difference = .	3095	Log. = .	3.4907
		Sum = .	7.6971
		Half sum =	3.8485½
Horiz. refrac. = 33′ + dip of horiz. = 6′.30″ = 39′.30″		Prop. log. =	0.6587
Constant log. = ...			1.1761
Correction = ...	3′.44″	Prop. log. =	1.6833½

Now, this correction being subtracted from the approximate time of rising, and added to that of setting, shows the former to be 6h.45m.2s., and the latter 16h.54m.30s.

PROBLEM III.

Given the Latitude of a Place, and the Declination of a Planet, to find the Times of its Rising and Setting.

RULE.

Take, from page IV. of the month in the Nautical Almanac, the times of the planet's transits for the days nearest preceding and following the given day, and find their difference; then say, as 6 days are to this difference, so is the interval between the given day and the nearest preceding day, to a correction; which, being applied by addition or subtraction to the time of transit on the nearest preceding day, according as it is increasing or decreasing, the sum or difference will be the approximate time of transit. Find the interval between the times of transit on the days nearest preceding and following the given day; and then say, as the interval between the times of transit is to the difference of transit in that interval, so is the longitude, in time, to a correction; which, being added to the approximate time of transit if the longitude be west and the transit increasing, or subtracted if decreasing, the sum or difference will be the apparent time of the planet's transit over the meridian of the given place; but if the longitude be east, a contrary process is to be observed: that is, the correction is to be subtracted from the approximate time of transit if the transit be increasing, but to be added thereto if it be decreasing.

To the apparent time of transit, thus found, apply the longitude, in time, by addition or subtraction, according as it is west or east; and the sum or difference will be the corresponding time at Greenwich. To this time, let the planet's declination be reduced by Problem III., page 83; or as thus:—

Take, from the Nautical Almanac, the planet's declination for the days nearest preceding and following the Greenwich time, and find the difference; find, also, the difference between the Greenwich time and the nearest preceding day: then say, as 6 days are to the difference of declination, so is the difference between the Greenwich time and the nearest preceding day, to a correction; which, being applied to the declination on the nearest preceding day, by addition or subtraction, according as it may be increasing or decreasing, the sum or difference will be the planet's correct declination at the time of its transit over the given meridian. Now,

With the planet's declination and the latitude of the given place, enter Table L., and find the corresponding semidiurnal arch* by Problem II., page 128; and, thence, the approximate times of rising and setting, in the same manner as if it were a fixed star that was under consideration.

Example 1.

At what times will the planet Jupiter rise and set, January 4th, 1824, in latitude 36° north, and longitude 135° west of the meridian of Greenwich?

* In strictness the semidiurnal arch ought to be corrected by adding thereto, or subtracting therefrom, the proportional part corresponding to it and the daily variation of transit, according as the transit may be increasing or decreasing.

Time of preced. trans. Jan. 1, is $11^h 38^m$ nearest prec. day 1st, trans. $11^h 38^m 0^s$
Time of follow. trans. Jan. 7, is 11. 8 given day 4th

As 6^d is to $0^h 30^m$, so is 3^d to — $15^m 0^s$

Approximate time of transit on the given day = ... $11^h 23^m 0^s$
Time of preceding transit = . . $1^d 11^h 38^m$
Time of following transit = . . . 7. 11. 8

Interval between the times of trans. = $5^d 23^h 30^m$

As interval between times of trans. = $5^d 23^h 30^m$: diff. of transit = 30^m :: longitude in time = 9^h to . . . — 1.53

Apparent time of transit over given merid. Jan. 4th, 1824 = $11^h 21^m 7^s$
Longitude 135 degrees west, in time = ... 9. 0. 0

Corresponding time at Greenwich = ... $20^h 21^m 7^s$

Planet's dec. Jan. 1 is = 23° 17′ N.; near. prec. $1^d 0^h 0^m 0^s$ dec. 23° 17′ 0″ N.
Ditto 7 is = 23. 20 N.; Gr. tim. = 4. 20. 21. 7

As 6^d is to 0° 3′ so is . . $3^d 20^h 21^m 7^s$ to + 1.55

Jupiter's dec. reduced to his app. time of transit over the given meridian = ... 23° 18′ 55″ N.

Time, in Table L., ans. to lat. 36° north, and dec. 23° N. = $7^h 12^m 0^s$
Tabular difference to 30′ of dec. = 2′; now, $\frac{2' \times 19'}{30'}$ = . + 1.16

Semidiur. arch, or time of half planet's contin. above the hor. = $7^h 13^m 16^s$
Apparent time of Jupiter's transit over the given meridian = 11. 21. 7

Approximate time of Jupiter's rising at the given meridian = $4^h 7^m 51^s$

Approximate time of Jupiter's setting at ditto = ... $18^h 34^m 23^s$

Example 2.

At what times will the planet Mars rise and set, January 16th, 1824, in latitude 40° north, and longitude 140° east of the meridian of Greenwich?

Time of preced. trans. 13th, is 16^h54^m; near. prec. day 13th, trans. = $16^h54^m\ 0^s$
Time of follow. trans. 19th, is 16. 34 ; given day 16th

As 6^d is to 0^h20^m, so is . . 3^d to . — $10^m\ 0^s$

Approximate time of transit on the given day = ... $16^h44^m\ 0^s$

Interval between the times of transit = $5^d23^h40^m$

As interval between times of transit = $5^d23^h40^m$: diff. of trans. = 20^m :: long. in time = 9^h20^m, to . . . . + 1.18

Apparent time of trans. over given merid. Jan. 16th, 1824 = $16^h45^m18^s$
Longitude of the given merid. = 140° east, in time = . . 9. 20. 0

Corresponding time at Greenwich = ... $7^h25^m18^s$

Dec. of Mars, Jan. 13, is 0°37′ S.; near. prec. $13^d\ 0^h\ 0^m\ 0^s$ dec. 0°37′ 0″ S.
Ditto 19, is 1. 11 S.; Gr. time = 16. 7. 25. 18

As 6^d is to 0°34′, so is . $3^d\ 7^h25^m18^s$ to + 18. 45

Dec. of Mars reduced to his apparent time of transit over the given meridian = ... 0°55′45″ S.

Semidiurnal arch in Table L., answering to lat. 40° N. and dec. 0°55′45″ S., is $6^h2^m48^s$; sub. from 12^h leaves . . . $5^h57^m12^s$
Apparent time of the planet's transit over the given meridian = 16. 45. 18

Approximate time of rising of the planet Mars = ... $10^h48^m\ 6^s$

Approximate time of setting of ditto = ... $22^h42^m30^s$

Remark.—The approximate times of a planet's rising and setting may be reduced to the respective apparent times, by the rule in page 126, for reducing those of the sun; omitting, however, the first part, or that which relates to the reduction of declination, and reading planet's instead of sun's horizontal parallax: this, it is presumed, does not require to be illustrated by an example.

Problem IV.

Given the Latitude of a Place, and the Moon's Declination, to find the Times of her Rising and Setting.

Rule.

Take, from page VI. of the month in the Nautical Almanac, the moon's transit, or passage over the meridian of Greenwich, on the given day, and also her declination. Let the time of transit be reduced to the meridian of the given place by Table XXXVIII.; to which apply the longitude, in time, by addition or subtraction, according as it is west or east; and the sum, or difference, will be the corresponding time at Greenwich: to this time, let the declination be reduced by Table XVI., or by Problem II., page 80;—then,

With this reduced declination, and the latitude of the given place, find the moon's semidiurnal arch, or the time of half her continuance above the horizon, by Problem II., page 128, and, thence, the approximate times of rising and setting, in the same manner precisely as if it were a fixed star that was under consideration: call these the *estimated* times of rising and setting.

To the *estimated* times of rising and setting, thus found, let the longitude, in time, be applied by addition or subtraction, according as it is west or east; and the sum, or difference, will be the corresponding times at Greenwich.

To these times respectively, let the moon's declination be reduced by Table XVI., or by Problem II., page 80; with which, and the latitude, find the moon's semidiurnal arch at each of the *estimated* times.

To the respective semidiurnal arches, thus found, apply the corrections corresponding thereto, and the retardation of the moon's transit (Table XXXVIII.) by addition, and the correct semidiurnal arches will be obtained.

Now, the semidiurnal arch answering to the *estimated* time of rising, being subtracted from the moon's reduced transit, will leave the approximate time of her rising at the given place; and that corresponding to the *estimated* time of setting, being added to the moon's reduced transit, will give the approximate time of her setting at the said place.

Example 1.

Required the times of the moon's rising and setting, Jan. 17th, 1824, in latitude 51°29′ north, and longitude 78°45′ west of the meridian of Greenwich?

Moon's transit over merid. of Greenwich on the given day is	$13^h34^m\ 0^s$
Corr. fr. Tab. XXXVIII., ans. to retard. 53′, and long. 75° west	+ 10.39
App. time of moon's transit reduced to the given meridian .	$13^h44^m39^s$
Longitude 78°45′ west, in time = ...	5.15. 0
Corresponding time at Greenwich	$18^h59^m39^s$

Moon's dec. red. to Gr. time, by Table XVI., is 10°25′30″N.

Semidiurnal arch, in Table L., answering to lat. 51°29′N., and declination 10°25′N., is	$6^h54^m\ 0^s$
Moon's reduced transit	13.44.39
Estimated time of the moon's rising	$6^h50^m39^s$
Estimated time of the moon's setting	$20^h38^m39^s$

To find the approximate Time of Rising:—

Estimated time of rising	$6^h50^m39^s$
Longitude 78°45′ west, in time = ...	5.15. 0
Greenwich time past noon of the given day	$12^h\ 5^m39^s$

Moon's dec. reduced to Greenwich time, is 12°10′53″N.

Time, in Table L., ans. to lat. 51°29′N. and dec. 12°11′N., is	$7^h\ 3^m\ 0^s$
Correction, Table XXXVIII., ans. to 53′ and 7^h3^m = . .	+ 15. 0
Moon's correct semidiurnal arch at rising	$7^h18^m\ 0^s$
Moon's reduced transit	13.44.39
Approximate time of moon's rising	$6^h26^m39^s$

To find the approximate Time of Setting:—

Estimated time of setting	$20^h38^m39^s$
Longitude 78°45′ west, in time = ...	5.15. 0
Greenwich time past noon of the 18th	$1^h53^m39^s$

Moon's dec. reduced to Greenwich time, is 8°41′11″N.

Time, in Table L., answ. to lat. 51°29′N. and dec. 8°41′N., is	$6^h44^m\ 0^s$
Correction, Table XXXVIII., ans. to 53′ and 6^h44^m = . .	+ 14. 0
Moon's correct semidiurnal arch at setting	$6^h58^m\ 0^s$
Moon's reduced transit	13.44.39
Approximate time of moon's setting	$20^h42^m39^s$

Example 2.

Required the approximate times of the moon's rising and setting, January 20th, 1824, in latitude 40°30′ north, and longitude 80 degrees east of the meridian of Greenwich?

Moon's transit over the merid. of Greenwich on the given day is	$16^h\ 6^m\ 0^s$
Cor. fr. Tab. XXXVIII., ans. to retard. 49′ and long. 80° east	− 10.32
Moon's transit reduced to the given meridian	$15^h 55^m 28^s$
Longitude 80 degrees east, in time = ...	5.20. 0
Greenwich time	$10^h 35^m 28^s$

Moon's dec. red. to Green. time, by Table XVI., is 5°55′40″S.

Seminocturnal arch, in Table L., answering to lat. 40°30′N. and dec. 5°56′S. = $6^h 20^m$, subtracted from 12^h, leaves	$5^h 40^m\ 0^s$
Moon's reduced transit	15.55.28
Estimated time of the moon's rising	$10^h 15^m 28^s$
Estimated time of the moon's setting	$21^h 35^m 28^s$

To find the approximate Time of Rising:—

Estimated time of rising	$10^h 15^m 28^s$
Longitude 80 degrees. east, in time = ...	5.20. 0
Greenwich time = ...	$4^h 55^m 28^s$

Moon's dec. reduced to this time, is 4°30′49″S.

Time, in Table L., answering to lat. 40°30′N., and dec. 4°31′S. is $6^h 15^m$, which, subtracted from 12^h, leaves .	$5^h 45^m\ 0^s$
Corr. Table XXXVIII., answering to 49′ and $5^h 45^m$ . .	+ 11. 0
Moon's correct semidiurnal arch at rising	$5^h 56^m\ 0^s$
Moon's reduced transit	15.55.28
Approximate time of moon's rising	$9^h 59^m 28^s$

To find the approximate Time of Setting:—

Estimated time of setting	$21^h 35^m 28^s$
Longitude 80 degs. east, in time = ...	5.20. 0
Greenwich time = ...	$16^h 15^m 28^s$

Moon's dec. reduced to this time, is 7°17′52″S,

Time, in Table L., answering to lat. 40°30′N. and dec. 7°18′S., is 6^h25^m, which, subtracted from 12^h, leaves .	$5^h35^m0^s$
Corr. Table XXXVIII., ans. to 49′ and 5^h35^m	+ 11. 0
Moon's correct semidiurnal arch at setting	$5^h46^m0^s$
Moon's reduced transit	15.55.28
Approximate time of moon's setting	$21^h41^m28^s$

Remark.—The approximate times of the moon's rising and setting may be reduced to the respective apparent times by the following rule; viz.,

Find the sum and the difference of the natural sine of the latitude and the natural co-sine of the declination at the *estimated* times of rising and setting (rejecting the two right-hand figures from each term), and find the common log. answering thereto, rejecting also the two right-hand figures from each. Now, to half the sum of these two logs. add the constant log. 1.1761,* and the proportional log. of the difference between the horizontal parallax and the sum of the horizontal refraction and dip of the horizon: the sum of these three logs., abating 4 in the index, will be the proportional log. of a correction, which, being *added* to the approximate time of rising and *subtracted* from that of setting, the respective apparent times of rising and setting will be obtained: thus,

Let it be required to reduce the approximate times of rising and setting, as found in the last example, to the respective apparent times, the dip of the horizon being 4′50″.

Note.—The moon's horizontal parallax computed to the *reduced estimated* time of rising, is 59′6″, and that at the reduced time of setting 58′40″.

Latitude 40°30′	Nat. sine	6494		
Declination 4.31	Nat. co-sine	9969		
Sum		16463	Log.	4.2165
Difference		3475	Log.	3.5410
			Sum	7.7575
			Half sum . .	3.8787½
59′6″ − 37′50″ (33′ + 4′50″) = 21′16″			Prop. log. . .	0.9276
Constant log.				1.1761
Correction		+ 1′52″	Prop. log. . .	1.9824½
Approximate time of rising =		$9^h59^m28^s$		
Apparent time of moon's rising =		$10^h1^m20^s$		

* This is the proportional log. of 12 hours esteemed as minutes.

Latitude	40°30′	Nat. sine	6494		
Declination	7 18	Nat. co-sine	9919		
Sum			16413	Log.	4.2152
Difference			3425	Log.	3.5347
				Sum	7.7499
				Half sum . .	3.8749½

58′40″ − 37′50″ (33′ + 4′50″) = 20′50″ Prop. log. . . . 0.9365
Constant log. 1.1761

Correction . . . . − 1′51″ Prop. log. . . . 1.9875½
Approximate time of setting = . 21h41m28s

Apparent time of moon's setting = 21h39m37s

Note.—The direct method of solving this and the three preceding Problems, by spherical trigonometry, is given in some of the subsequent pages of this work.

Tables LI. and LII.

For computing the Meridional Altitude of a Celestial Object.

Since it frequently happens, at sea, that the meridional altitude of the sun, or other celestial object, cannot be taken, in consequence of the interposition of clouds at the time of its coming to the meridian; and since it is of the utmost importance to the mariner to be provided at all times with the means of determining the meridional altitude of the heavenly bodies, for the purpose of ascertaining the exact position of his ship with respect to latitude, these Tables have therefore been carefully computed; by means of which the meridional altitude of the sun, or any other celestial object whose declination does not exceed 28 degrees, may be very readily obtained to a sufficient degree of accuracy for all nautical purposes, provided the altitude be observed within certain intervals of noon, or time of transit, to be governed by the meridional zenith distance of the object: thus, *for the sun*, the number of minutes and parts of a minute contained in the interval between the time of observation and noon, must not exceed the number of degrees and parts of a degree contained in the object's meridional zenith distance at the place of observation. And since the meridional zenith distance of a celestial object is expressed by the difference between the latitude and the declination when they are of the same name, or by their sum when of contrary names; therefore the extent of the interval from noon (within which the altitude should be observed) may be determined by means of the difference between the latitude and the declination when they are both north or both south, or by their sum when one is north and the other south. Thus, if the latitude be 40 degrees, and the declination 8 degrees, both of the same name, the interval between the time of taking the altitude and noon must not exceed 32 minutes; but if they be of different names, the altitude may be taken at any time within 48 minutes before or after noon: if the latitude be 60 degrees, and the declination 10 degrees, both of the same name, the interval between the time of observation and noon ought not to exceed 50 minutes; but if one be north and the other south, the interval may be extended, if necessary, to 70 minutes before or after noon, and so on.

The limits within which the altitudes of the other celestial objects should be observed, may be determined in the same manner; taking care, however, to estimate the interval from the time of transit or passage over the meridian, instead of from noon.

Now, if the altitude of the sun or other celestial object be observed *at any time* within the limits thus prescribed, and the time of observation be carefully noted by a well-regulated watch, the meridional altitude of such object may then be readily determined, to every desirable degree of accuracy, by the following rule; viz.,

Enter Table LI. or LII., according as the latitude and the declination are of the same or of contrary names, and with the latitude in the side column, and the declination (reduced to the meridian of the place of observation) at the top or bottom; take out the corresponding correction in seconds and thirds, which are to be esteemed as *minutes and seconds*;—then,

To the proportional log. of this correction,* add twice the proportional log. of the interval between the time of observation and noon, or time of transit, and the constant log. 7.2730; and the sum will be the proportional log. of a correction, which, being *added* to the true altitude deduced from observation, will give the correct meridional altitude of the object.

Note 1.—In taking out the numbers from Tables LI. and LII., proportion must be made for the excess of the given latitude and declination above the next less tabular arguments.

* When the object either comes to, or within, one degree of the zenith, the angle of meeting made by the latitude and declination will fall within the zigzag double lines which run through the body of Table LI., and through the upper left-hand corner of Table LII: in this case, since the interval between the time of observation and noon, or meridional passage, must not exceed one minute, the corresponding number will be the correction of altitude direct, independently of any calculation whatever.

2.—The interval between the time of observation and noon may be always known by means of a chronometer, or any well-regulated watch; making proper allowance, however, for the time comprehended under the change of longitude since the last observation for determining the error of such watch or chronometer.

Example 1.

In latitude 45° north, at 34^m40^s before noon, the sun's true altitude was found to be 54°12′49″, when his declination was 10° north; required the meridional altitude?

Corr. in Table LI., ans. to lat. 45° and dec. 10°, is 2″23‴.1; the propor. log. of which is		. . . .	1.8778
Interval between time of obs. and noon, 34^m40^s, twice prop. log.		=	1.4308
Constant log.		. . . .	7.2730
Correction of altitude . .	0°47′10″	Prop. log. = . .	0.5816
True alt. at time of observ.	54.12.49		
Sun's meridional altitude	54°59′59″		

which is but one second less than the truth.

Example 2.

In latitude 48° north, at $1^h5^m48^s$ past noon, the sun's true altitude was found to be 20°25′5″, when his declination was 20 degs. south; required the meridional altitude?

Corr. in Table LII., answering to lat. 48°N. and dec. 20°S., is 1″19‴.9, the propor log. of which is		. . . .	2.1308
Interval between time of obs. and noon $1^h5^m48^s$, twice prop. log.		=	0.8740
Constant log.		. . . .	7.2730
Correction of altitude . .	1°34′57″	Prop. log. = .	0.2778
True alt. at time of observ.	20.25. 5		
Sun's meridional altitude .	22° 0′ 2″		

which is but two seconds more than the truth.

Example 3.

At sea, March 22d, 1824, in latitude 51°16′ north, at 50^m32^s past noon, the sun's true altitude was found to be 38°20′56″; required the meridional altitude, the declination being 0°43′51″ north?

Corr. in Table LI., answering to lat. 51°16′ and dec. 0°43′51″ is 1″35‴.6,* the propor. log. of which is			2.0530
Interval between time of obs. and noon 50^m32^s, twice prop. log. =			1.1034
Constant log.			7.2730
Correction of altitude	1° 6′58″	Prop. log. =	0.4294
True alt. at time of observ.	38.20.56		
Sun's meridional altitude	39°27′54″; which differs but three seconds from the truth.		

Example 4.

At sea, December 21st, 1824, in latitude 60°22′ north, at $10^h36^m10^s$ A.M., or $1^h23^m50^s$ before noon, the sun's true altitude was found to be 4°26′38″; required his meridional altitude, the declination being 23°27′45″ south?

Corr. in Table LII., ans. to lat. 60°22′ and dec. 23°27′45″, is 0″53‴.8,† the propor. log. of which is			2.3026
Interval between time of obs. and noon $1^h23^m50^s$, twice prop.log. =			0.6638
Constant log.			7.2730
Correction of altitude	1°43′43″	Prop. log. =	0.2394
True alt. at time of observ.	4.26.38		
Sun's meridional altitude	6°10′21″; which differs but six seconds from the truth.		

After this manner may the meridional altitude of the moon, a planet, or a fixed star be obtained, when the declination does not exceed the limits of the Table.

Remarks, &c.

From the above examples it is manifest, that by means of the present Tables the meridional altitude of a celestial object may be readily inferred

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* Corr. to lat. 50° and dec. 0° =	1″38‴.8
Diff. to 2° lat. = 6‴.8; now, 6‴.8 × 76′ ÷ 120′ =	− 4 .3
Diff. to 1° dec. = 1‴.5; now, 1‴.5 × 44′ ÷ 60′ =	+ 1 .1
Corr. to lat. 50°32′ and dec, 0°43′51″ =	1″35‴.6

† Corr. to lat. 60° and dec. 23° =	0″54‴.6
Diff. to 2° lat. = 3‴.5; now 3‴.5 × 22′ ÷ 120′ =	− 0 .6
Diff. to 1° dec. = 0‴.5; now 0‴.5 × 28′ ÷ 60′ =	− 0 .2
Corr. to lat. 60°22′ and dec. 23°27′45″ =	0″53‴.8

from its true altitude observed at a known interval from noon (within the limits before prescribed), with all the accuracy to be desired in nautical operations; and that it is immaterial whether the observation is made before or after noon, or time of transit, provided the time be but correctly known; and, since most sea-going ships are furnished with chronometers, there can be but very little difficulty in ascertaining the apparent time to within a few seconds of the truth.

It is to be observed, however, that the nearer to noon or time of transit the observation is made, the less susceptible will it be of being affected by any error in the time indicated by the watch: thus, in *example* 4, where the interval or time from noon is $1^h.23^m.50^s.$, an error of one minute in that interval would produce an error of 2½ minutes in the sun's meridional altitude; but if the observation had been made within a quarter of an hour of noon, an error of *five minutes* in the time would scarcely affect the meridional altitude to the value of 2 minutes: hence it is evident, that although the observation may be safely made at any time from noon to the full extent of the interval, when dependance can be placed on the time shown by the watch, yet when there is any reason to doubt the truth of that time, it will be advisable to take the altitude as near to noon, or the time of transit, as circumstances may render convenient.

In all narrow seas trending in an easterly or westerly direction, where the meridional altitude of a celestial object is of the greatest consideration, such as in the British Channel, the mariner will do well to avail himself of this certain method for its actual determination; particularly during the winter months, when the sun is so very frequently obscured by clouds at the time of its coming to the meridian.

These Tables were computed by the following rule; viz.,

To the constant log. 0. 978604,* add the log. co-sines of the latitude and the declination; the sum, rejecting 20 from the index, will be the log. of a natural number, which, being subtracted from the natural co-sine of the difference between the latitude and the declination, when they are of the same name, or from that of their sum if of contrary names, will leave the natural co-sine of an arch; now, the difference between this arch, and the difference or sum of the latitude and the declination, according as they are of the same or of contrary names, will be the change of altitude in one minute from noon.

Example 1.

Let the latitude be 13 degrees, and the declination of a celestial object 2 degrees, both of the same name; required the variation or change of altitude in one minute from noon?

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* This is the log. versed sine, or log. rising, of one minute of time.

Constant log. =	. . . .		0.978604
Latitude = .	13 degrees.	Log. co-sine .	9.988724
Declination =	2 degrees.	Log. co-sine .	9.999735
Difference = .	11 degrees. Nat. co-sine=981627		
	Nat. number=	9.269=Log.	0.967063
Arch = . .	11° 0′ 10″ = Nat. co-s.=981617.731		
Difference = .	0° 0′ 10″		

Difference = . 0° 0′ 10″; which, therefore, is the change of altitude in one minute from noon.

Example 2.

Let the latitude be 40 degrees, and the declination of a celestial object 8 degrees, of a contrary name to that of the latitude; required the variation or change of altitude in one minute from noon?

Constant log. =	. . . .		0.978604
Latitude =	40 degrees.	Log. co-sine .	9.884254
Declination =	8 degrees.	Log. co-sine .	9.995753
Sum = . .	48 degrees. Nat. co-sine=669131		
	Nat. num. =	7.221 Log.=	0.858611
Arch = . .	48° 0′ 2″=Nat.co-sine=669123.779		

Difference = 0° 0′ 2″; which, therefore, is the change of altitude in one minute from noon.

It is to be observed, however, that, with the view of introducing every possible degree of accuracy into the present Tables, the natural and log. co-sines, &c., employed in their construction, have had their respective numbers extended to seven places of decimals.

Note.—The difference between the meridional altitude of a celestial object and its altitude at a given interval from noon, is found, by actual observation, to be very nearly proportional to the square of that interval, under certain limitations, as pointed out in page 138; and hence the rule, in page 139, for computing the meridional altitude of a celestial object.

Table LIII.

The Miles and Parts of a Mile in a Degree of Longitude at every Degree of Latitude.

This Table consists of seven compartments: the first column in each compartment contains the degrees of latitude, and the second column the miles and parts of a mile in a degree of longitude corresponding thereto. In taking out the numbers from this Table, proportion is to be made, as usual, for the minutes of latitude; this proportion is subtractive from the miles, &c., answering to the given degree of latitude.

Example.

Required the number of miles contained in a degree of longitude in latitude 37°48′?

Miles in a degree of longitude, in latitude 37 degrees = . . . 47.92

Difference to 1 degree of latitude = .64; now $\frac{64 \times 48'}{60'}$ = — .51

Miles in a degree of long. in latitude 37 degs. 48 min., as required = 47.41

Remarks.—Since the difference of longitude between two places on the earth is measured by an arch of the equator intercepted between the meridians of those places; and since the meridians gradually approach each other from the equator to the poles, where they meet, it hence follows that the number of miles contained in a degree of longitude will decrease in proportion to the increase of the latitude; the ratio of decrease being as radius to the co-sine of the latitude. Now, since a degree of longitude at the equator contains 60 miles, we have the following rule for computing the present Table; viz.,

As radius is to the co-sine of the latitude of any given parallel, so is the measure of a degree of longitude at the equator to the measure of a degree in the given parallel of latitude.

Example.

Required the number of miles contained in a degree of longitude in the parallel of latitude 37 degrees?

As radius . .	90 degrees	Log. sine = . .	10.000000
Is to latitude =	37 degrees	Log. co-sine . .	9.902349
So is	60 miles	Log. = . . .	1.778151
To	47.92 miles	Log. = . . .	1.680500;

Hence the measure of a degree of longitude in the given parallel of latitude, is 47.92 miles.

Table LIV.

Proportional Miles for constructing Marine or Sea Charts.

In this Table the parallels of latitude are ranged in the upper horizontal column, beginning at 0°, and numbered 10°, 20°, 30°, &c., to 89°; the horizontal column immediately under the parallels of latitude contains the number of miles of longitude corresponding to each parallel's distance from the equator; under which, in the horizontal column marked "Difference of the Parallels, &c.," stands the number of miles of longitude contained between the parallel under which it is placed and that immediately preceding it.

The left-hand vertical column contains the intermediate or odd degrees of latitude, from 0° to 10°; opposite to which, and under the respective parallels of latitude, will be found the number of miles of longitude corresponding to each degree of latitude in those parallels: these are intended to facilitate, and render more accurate, the subdivision of the different parallels of latitude into degrees and minutes.

To make a Chart of the World, in which the Parallels of Latitude and Longitude are to consist of 10 *Degrees each.*

Draw a straight, or meridian, line along the right hand, or east margin of the paper intended to receive the projection; bisect that line, and from the point of bisection draw a straight line perpendicular to the former, which continue to the left-hand or west margin of the paper, and it will represent the equator.

From any diagonal scale of convenient size take 600 miles in the compasses (the number of miles of the equator contained in 10 degrees of longitude), and lay it off from the point of bisection along the equator, and it will graduate it into 36 equal parts of 10 degrees each; through which let straight lines be drawn at right angles to the equator, and parallel to that drawn along the right-hand margin, and they will represent the meridians or parallels of longitude. Take, from the same scale, 60 miles in the compasses, and it will subdivide each of those 36 divisions, or parallels of longitude, into ten equal parts consisting of one degree each; and then will the equator be divided into 360 degrees of 60 miles each.

On the meridian lines drawn along the right and left-hand margins of the paper, let the parallels of latitude be laid down, as thus:—For the first parallel, or 10 degrees from the equator, take 603.1 miles in the compasses (found in the horizontal column immediately under the parallels of latitude, and marked "Ditto in miles of the Equator, &c."); place one foot on the equator, and where the other falls upon the right and left-hand marginal lines, when turned northward and southward, there make points; through which let straight lines be drawn parallel to the equator, and they will represent the parallels of latitude at 10 degrees north and south of the equator: in the same manner, for 20 degrees, lay off 1225. 1 miles; for 30 degrees, 1888. 4 miles; for 40 degrees, 2622. 6 miles, and so on.

But since the common compasses are generally too small for taking off such high numbers, it will be found more convenient to lay down the parallels of latitude by the numbers contained in the third horizontal column, or that marked "Difference of the Parallels, &c." Thus, for 10 degrees, take 603. 1 miles in the compasses; place one foot on the equator, and with the other make points north and south thereof on the east and west marginal lines, through which let straight lines be drawn, and they will represent the parallels of latitude at 10 degrees north and south of the equator. From these parallels respectively, lay off 622. 0 miles, by placing one foot of the compasses on the respective parallels and the other on the east and west marginal lines; through the points thus made by the compasses draw straight lines, and they will represent the parallels of latitude at 20 degrees north and south of the equator. From the parallels, thus obtained, lay off 663. 3 miles, and the parallel of 30 degrees will be determined: thence lay off 734. 2 miles, and it will show the parallel of 40 degrees; and so on for the succeeding parallels.

The numbers for subdividing those parallels will be found in the vertical columns under each respectively, and are to be applied as follows; thus, to graduate the parallel between 50 and 60 degrees: take 94. 3 miles in the compasses, and lay it off from 50 degrees towards 60 degrees, and it will give the parallel of 51 degrees; from which lay off 96. 4 miles, and it will show the parallel of 52 degrees; from this lay off 98. 6 miles, and the parallel of 53 degrees will be obtained; and so on of the rest. In the same manner let the other parallels of latitude be subdivided; then let the parallels of latitude be numbered along the east and west marginal columns, from the equator towards the poles, according to the number of degrees contained in that arc of the meridian which is intercepted between them and the equator, as 10°, 20°, 30°, 40°, &c. &c.; and let the parallels of longitude be numbered at the top and bottom, and also along the equator; these are to be reckoned east and west of the first meridian, as 10°, 20°, 30°, 40°, &c., to 180°, both ways; and since the first meridian is entirely arbitrary, it may be assumed as passing through any particular place on the earth, such as Greenwich Observatory: then will the chart be ready for receiving the latitudes and longitudes of all the principal places on the earth, and which are to be placed thereon by the following rule; viz.,

Lay a ruler over the given longitude found at the top and bottom of the chart, and with a pair of compasses take the latitude from the east or west marginal columns; which being applied to the edge of the ruler, placing one foot on the equator or on the parallel that the latitude was counted from, the other foot turned north or south according to the name of the latitude, will point out or fall upon the true position of the given latitude and longitude.

From what has been thus laid down, the manner of constructing a chart for any particular place or coast must appear obvious.

Note.—Since this Table is merely an extract from the Table of Meridional parts, the reader is referred to page 113 for the method of computing the different numbers contained therein.

Table LV.

To find the Distance of Terrestrial Objects at Sea.

If an observer be elevated to any height above the level of the earth or sea, he can not only discern the distant surrounding objects much plainer than he could when standing on its surface, but also discover objects which are still more remote by increasing his elevation. Now, although the great irregularity of the surface of the land cannot be subjected to any definite rule for determining the distance at which objects may be seen from different elevations; yet, at sea, where there is generally an uniform curvature of the water, on account of the spherical figure of the earth, the distance at which objects may be seen on its surface may be readily obtained by means of the present Table; in which the distance answering to the height of the eye, or to that of a given remote object, is expressed in nautical miles and hundredth parts of a mile; allowance having been made for terrestrial refraction, in the ratio of the one-twelfth of the intercepted arch.

Note.—The distance between two objects whose heights are given, is found by adding together the tabular distances corresponding to those heights. And, when the given height exceeds the limits of the Table, an aliquot part thereof is to be taken; as one fourth, one ninth, or one sixteenth, &c.; then, the distance corresponding thereto in the Table, being multiplied by the *square root* of such aliquot part, viz., by 2, 3, or 4, &c., according as it may be, will give the required distance.

Example 1.

The look-out man at the mast-head of a man-of-war, at an elevation of 160 feet above the level of the sea, saw the top of a light-house in the horizon whose height was known to be 290 feet; required the ship's distance therefrom?

The distance answering to 160 feet is . .	14.57 miles.
Ditto to 290 feet is . .	19.62 do.
Required distance = ...	34.19 miles;

which, therefore, is the ship's distance from the light-house.

Example 2.

The Peak of Teneriffe is about 15300 feet above the level of the sea; at what distance can it be seen by an observer at the mast-head of a ship, supposing his eye to be 170 feet above the level of the water?

One ninth of 15300 is 1700, answering to which is 47.50 miles; this being multiplied by 3 (the square root of one ninth) gives 142.50 miles.

Distance ans. to 170 feet (height of the eye) is .	15.03 do.
Required distance = ...	157.53 miles.

Remark 1.—Since the distances given in this Table are expressed in nautical miles, whereof 60 are contained in one degree, and there being 69.1 English miles in the same portion of the sphere; if, therefore, the distance be required in English miles, it is to be found as follows; viz.,

As 60, is to 69.1; so is the tabular distance to the corresponding distance in English miles; which may be reduced to a logarithmic expression, as thus:—

To the log. of the given tabular distance, add the constant logarithm 0.061327,* and the sum will be the log. of the given distance in English miles.

Example.

Let it be required to reduce 157.53 nautical miles into English miles?

Given distance in nautical miles = 157.53, log. =	2.197364
Constant log.	0.061327
Distance reduced to English miles 181.42 = Log. =	2.258691

* The log. of 69.1 = 1.839478, less the log. of 60 = 1.778151 is 0.061327; which, therefore, is the constant logarithm.

The converse of this (that is, to reduce English miles into nautical miles,) must appear obvious.

Remark 2.—This Table was computed by the following rule; viz.,

To the earth's diameter in feet, add the height of the eye above the level of the sea, and multiply the sum by that height; then, the square root of the product being divided by 6080 (the number of feet in a nautical mile), will give the distance at which an object may be seen in the visible horizon, independent of terrestrial refraction. This rule may be adapted to logarithms, as thus:—

Let the earth's diameter in feet be augmented by the height of the eye; then, to the log. thereof add the log. of the height of the eye; from half the sum of these two logs. subtract the constant log. 3.783904,* and the remainder will be the log. of the distance in nautical miles, which is to be increased by a twelfth part, of itself, on account of the terrestrial refraction.

Example.

At what distance can an object be seen, in the visible horizon, by an observer whose eye is elevated 290 feet above the level of the sea?

Diameter of the earth in feet =	41804400		
Height of the eye	290	Log. =	2.462398
Sum = ...	41804690	Log. =	7.621225
		Sum .	10.083623
		Half sum	5.041811½
Constant log. = ...			3.783904
Distance uncorrected by refraction	18.11 =	Log. =	1.257907½
Add one-12th part on acc. of refrac.	1.51		
Distance, as required = . . .	19.62 nautical miles.		

Note.—For the principles of this rule, see how the distance of the visible horizon, expressed by the line O T, is determined in page 5.

* This is the log. of 6080, the number of feet in a nautical mile.

Table LVI.

To reduce the French Centesimal Division of the Circle into the English Sexagesimal Division; or, to reduce French Degrees, &c., into English Degrees, &c., and conversely.

This Table is intended to facilitate the reduction of French degrees of the circle into English degrees, and conversely. The Table is divided into two parts: the first or upper part exhibits the number of English degrees and parts of a degree contained in any given number of French degrees and parts of a degree; and the second or lower part exhibits the number of French degrees, &c., contained in any given number of English degrees, &c.

Note.—In the general use of this Table, when any given number of French degrees exceeds the limits of the first part, take out for 100 degrees first, and then for as many more as will make up the given number; and, when any given number of English degrees exceeds the limits of the second part, take out for 90 degrees first, and then for as many more as will make up the given number.

Example 1.

If the distance between the moon and a fixed star, according to the French division of the circle, be 128°93′96″, required the distance agreeably to the English division of the circle?

100 French degrees	are equal to . .	90° 0′ 0″	English.
28 Ditto	are equal to . .	25. 12. 0	do.
93 French minutes	are equal to . .	0. 50. 13 .20	do.
96 French seconds	are equal to . .	0. 0. 31 .10	do.

Distance reduced to English degs., as required 116° 2′44″.30

Example 2.

If the distance between the moon and sun, according to the English division of the circle, be 116°53′47″, required the distance agreeably to the French division of the circle?

90 English degrees	are equal to . .	100° 0′ 0″	French.
26 Ditto	are equal to . .	28. 88. 88 .89	do.
53 English minutes	are equal to . .	0. 98. 14 .81	do.
47 English seconds	are equal to . .	0. 1. 45 .06	do.

Distance reduced to French degs. as required =129°88′48″.76

Remark 1.—This Table was computed in conformity with the following considerations and principles; viz.,

The French writers on trigonometry have recently adopted the centesimal division of the circle, as originally proposed by our excellent countryman Mr. Henry Briggs, about the year 1600. In this division, the circle is divided into 400 equal parts or degrees, and the quadrant into 100 equal parts or degrees; each degree being divided into 100 equal parts or minutes, and each minute into 100 equal parts or seconds: these degrees, &c. &c., are written in the usual manner and with the customary signs, as thus; $128^\circ 93' 96''$.

Hence, the French degree is evidently less than the English, in the ratio of 100 to 90; a French minute is less than an English minute, in the ratio of $100^\circ \times 100'$ to $90^\circ \times 60'$; and a French second is less than an English second, in the ratio of $100^\circ \times 100' \times 100''$ to $90^\circ \times 60' \times 60''$: now, the converse of this being obvious, we have the following general rule for converting French degrees into English, and the contrary.

As 100, the number of degrees in the French quadrant, is to 90, the number of degrees in the English quadrant; so is any given number of French degrees to the corresponding number of English degrees.

As 10000, the number of minutes in the French quadrant, is to 5400, the number of minutes in the English quadrant; so is any given number of French minutes to the corresponding number of English minutes. And,

As 1000000, the number of seconds in the French quadrant, is to 324000, the number of seconds in the English quadrant; so is any given number of French seconds to the corresponding number of English seconds.

English degrees, minutes, and seconds, are reduced into French by a converse proportion; viz.,

As 90, is to 100; so is any given number of English degrees to the corresponding number of French degrees.

As 5400, is to 10000; so is any given number of English minutes to the corresponding number of French minutes. And,

As 324000, is to 1000000; so is any given number of English seconds to the corresponding number of French seconds.

Remark 2.—French degrees and parts of a degree may be turned into English, independently of the Table, by the following rule; viz.,

Let the French degrees be esteemed as a whole number, to which annex the minutes and seconds as decimals; then one-tenth of this mixed number, deducted from itself, will give the corresponding English degrees, &c.

Example.

The latitude of Paris, according to the French division of the quadrant, is 54°26′36″ north; required the latitude agreeably to the English division of the quadrant?

Given latitude = 54°26′36″ =	54°.2636
Deduct one-tenth . . .	5 .42636
English degrees, &c. . . .	48°.83724
	60
	50′.23440
	60
	14″.06400

Hence, the latitude of Paris, reduced to the English division of the quadrant, is 48°50′14″ north.

Remark 3.—English degrees and parts of a degree may be turned into French, independently of the Table, as thus:—

Reduce the English minutes and seconds to the decimal of a degree, and annex it to the given degrees; then one-ninth of this mixed number, being added to itself, will give the corresponding French degrees, &c.

Example.

The latitude of the Royal Observatory at Greenwich is 51°28′40″ north, agreeably to the English division of the quadrant; required the latitude according to the French division of the quadrant?

Given latitude = 51°28′40″ =	51°.4777777, &c.
Add one-ninth	5 .7197530, &c.
French degrees, &c.	57°.1975307 = 57°19′75″.307

Hence, the latitude of Greenwich Observatory, according to the French division of the quadrant, is 57°19′75″.307 N.

TABLE LVII.

A general Table for Gauging, or finding the Content of all Circular-headed Casks.

Although this Table may not directly affect the interest of the mariner; yet, since it cannot fail of being exceedingly useful to officers in charge of

His Majesty's victualling stores (such as Pursers of the Royal Navy, Lieutenants commanding gun-brigs, &c. &c.), it has therefore been deemed advisable to give it a place in this work, particularly since it may be found interesting to those whom it immediately concerns.

This Table is divided into two parts: the first part consists of five compartments, and each compartment of three columns; the first of which contains the quotient of the head diameter of a cask divided by the bung diameter; the second the corresponding log. adapted to ale gallons; and the third the log. for wine gallons. The second part of the Table contains the bung diameter and its corresponding logarithm.

The use of this Table will be exemplified in the following

Problem.

Given the Dimensions of a Cask, to find its Contents in Ale and Wine Gallons.

Rule.

Divide the head diameter by the bung diameter to two places of decimals in the quotient; then add together the log. for ale or wine gallons, corresponding to this quotient, in the first part of the Table; the log. corresponding to the bung diameter, in the second part of the Table, and the common log. of the length of the cask; the sum of these three logs., rejecting 10 in the index, will be the log. of the true content of the cask, in ale or wine gallons, according as the content may be required.

Example.

Let the bung diameter of a cask be 25 inches, the head diameter 19.5 inches, and its length 31 inches; required the contents in ale and wine gallons?

```
25)19.50(.78, quotient of the head diameter divided by the bung
   175              diameter.
   ---
    200
    200
    ---
    ...
```

.78 = quotient, log. for ale gallons = . . .	7.362671
25 inches, bung diameter, corresponding log. =	2.795880
31 inches, length of the cask, common log. =	1.491362
Content in ale gallons = 44.66 common log. =	**1.649913**

.78 = quotient, log. for wine gallons = . .	7.449340
25 inches, bung diameter, corresponding log. =	2.795880
31 inches, length of the cask, common log. =	1.491362
Content in wine gallons = 54.52 common log.=	1.736582

Remark.—Should the bung diameter not come within the limits of the second part of the Table; that is, should it be under 10 or above 50 inches, then twice the common log. corresponding thereto will express the log. of the said bung diameter, with which proceed as before: hence, the rule becomes universal for all circular-headed casks, be the size ever so great or ever so trivial.

This subject will be revived in a subsequent page of the present work.

Table LVIII.

Latitudes and Longitudes of the principal Sea-Ports, Islands, Capes, &c. &c., with the Time of High Water at the Full and Change of the Moon at all Places where it is known.

In drawing up this Table, the greatest pains have been taken to render it not only the most accurate, but also the most extensive of any now extant. Perfect accuracy, however, is not to be expected in a Table which principally depends on the observations made, at different periods, by the navigators of most civilized nations; because, in those periods, or at the time when a very considerable portion of the latitudes and longitudes were established, the nautical instruments and tables employed in their determination were far from being in that highly-improved state in which they are found at present: besides, it is a fact well known to the generality of nautical persons, that if two or more navigators be directed to ascertain the position of any particular place, they will, in most cases, differ four or five miles in the latitude, and perhaps thrice as many in the longitude.

In constructing all the other Tables in this work, there were fixed data to work upon, with certain means of detecting and exterminating errors; but, in this, there were no determinate means of ensuring the desired degree of accuracy, except in those positions where chance or professional duties happened, from time to time, to conduct the author. Hence, although every possible degree of attention has been paid in consulting the most approved works of the present day, and in collating *this* with the best modern Tables; yet the mariner must not expect to find it perfectly free from blemishes; though, doubtless, he will find it considerably less so than any with which he may have been hitherto acquainted.

Since this Table is not intended for general geographical purposes, the positions of places *inland*, which do not concern the navigator, have, with one or two exceptions, been purposely omitted: hence, the latitudes and longitudes are limited to maritime places. These are so arranged as to exhibit to the mariner the whole line of coast along which he may chance to sail, or on which he may be employed, agreeably to the manner in which it unfolds to his view on a Mercator's chart. This mode of arrangement is evidently much better adapted to nautical purposes than the alphabetical mode.

With the view of keeping up the identity of the Table with the line of coast laid down on particular charts, a *few* positions have been inserted a second time. This, it is presumed, if not conducive to good, will not, at least, be productive of any evil, since the repetition is so very trivial as not to embrace, in the whole, more than ten or twelve positions.

The time of high water, at the full and change of the moon, is given at all places where it is known. This, it is hoped, will be found not a little convenient, since it does away with the necessity of consulting a separate Table for that particular purpose.

In order to render this Table still more complete, an alphabetical reference has been annexed, which will very essentially contribute towards assisting the mariner in readily finding out most of the principal coasts and islands contained in that Table.

The page which immediately follows the alphabetical reference to Table LVIII. contains the form of a Transit Table, and the next page a variety of numbers with their corresponding logarithms, &c., which may, perhaps, be found useful on many occasions. At the foot of these numbers there is a small Table, showing the absolute time at which the hour and minute hands of a well-regulated watch or clock should exactly be in conjunction, and also in opposition, in every revolution.

Having thus completed the Description and Use of the Tables contained in this work, it now remains to show their application to the different elements connected with the sciences of navigation and nautical astronomy. In doing this, since the author's design carries him no farther than that of giving an ample illustration of the various purposes to which they may be applied; the reader must not, therefore, expect to find the elementary part of the sciences treated of. Hence, in this part of the work, the author will endeavour to confine himself to such Problems and subject matters as may appear to be most interesting and useful to nautical persons, without entering into particulars or the minutiæ of the sciences, and thus swelling the work to an unnecessary size;—a thing which he most anxiously wishes to avoid.

A CONCISE SYSTEM

OF

DECIMAL ARITHMETIC.

Although, from what has been said in the last paragraph, it may appear somewhat irregular, and even contrary to the general tenor of this work, to introduce any subject therein that does not come immediately under the cognizance of logarithms ; yet, since the reader may be desirous of having some little acquaintance with the nature of decimal fractions previously to his entering on the logarithmical computations, the following concise system is given for that purpose.—It has been deemed advisable to touch upon this subject for two cogent reasons ;—first, because a short account of decimals may be acceptable to the mariner whose early entrance on a sea life prevents him from going through a regular course of scholastic education on shore ; and, second, that he may have directly under his view all that is essentially necessary to be known in the practically useful branches of science, without being under the necessity of consulting any other author for the purpose of assisting him in the comprehension of the different subjects contained in this work.

DECIMAL FRACTIONS.

A decimal fraction signifies the artificial manner of setting down and expressing natural vulgar fractions as if they were whole numbers.—A decimal fraction has always for its denominator an unit (1,) with as many ciphers annexed to it as there are places in the numerator ; and it is generally expressed by setting down the numerator only, with a point before it, on the left hand ;—thus, $\frac{5}{10}$ is .5 ; $\frac{75}{100}$ is .75 ; $\frac{25}{1000}$ is .025 ; $\frac{114}{100000}$ is .00114, &c. &c. :—hence the numerator must always consist of as many figures as there are ciphers in the denominator.

A mixed number is made up of a whole number and a decimal fraction, the one being separated from the other by a point; thus 5.75 is the same as 5 $\frac{75}{100}$, or $\frac{575}{1000}$.

Ciphers on the right hand of decimals do not increase their value; for .5 . 50 . 500 . 5000, &c., are decimal fractions of the same value, each being equal to $\frac{5}{10}$, or $\frac{1}{2}$.—But when ciphers are placed on the left hand of a decimal they decrease its value in a tenfold proportion;—thus, .5 is $\frac{5}{10}$ or 5 tenths; but .05 is only $\frac{5}{100}$ or 5 hundredths; .005 is only $\frac{5}{1000}$ or 5 thousandths, and so on:—hence it is evident that in decimals as well as in whole numbers, the value of the place of the figure increases towards the left hand, and decreases towards the right, each being in the same tenfold proportion.

ADDITION OF DECIMALS.

Addition of decimals is performed in the same way as addition of whole numbers, observing to place the numbers right; that is, all the decimal points under each other, units under units, tenths under tenths, hundredths under hundredths, &c.; taking care to point off from the total or sum as many places for decimals as there are in the line containing the greatest number of decimal places.

Example 1.

Add together 41.37; 3.762; 137.03; 409, and .3976.

```
 41.37
  3.762
137.03
409.
   .3976
--------
591.5596, the sum.
```

Example 2.

Add together 3.268; 208.1; 276; 4.7845, and 1.07.

```
  3.268
208.1
276.
  4.7845
  1.07
--------
493.2225, the sum.
```

SUBTRACTION OF DECIMALS.

Subtraction of decimals is likewise performed the same way as in whole numbers; observing to place the numbers right; that is, the decimal points under each other, units under units, tenths under tenths, hundredths under hundredths, &c. &c.

Example 1.		*Example* 2.	
From	439.7265	From	179.037
Take	98.283	Take	54.932468
Remains . . .	341.4435	Remains . . .	124.104532

MULTIPLICATION OF DECIMALS.

Multiplication of decimals is also performed the same way as in whole numbers; observing to cut off as many decimal places in the product as there are decimal places in both factors; that is, in the multiplicand and multiplier.

Example 1.		*Example* 2.	
Multiply	2.4362	Multiply . . .	376.09
By	.275	By	13.43
	121810		112827
	170534		150436
	48724		112827
			37609
Product = .	0.6699550	Product = .	5050.8887

Note.—If a decimal fraction be multiplied by a decimal fraction the product will be less than either the multiplicand or the multiplier.—And if any number either whole or mixed, be multiplied by a decimal fraction, the product will be always less than the multiplicand, as in example 1;—hence if a decimal fraction be multiplied by itself, its value will *decrease* in the proportion of its multiple:—thus,

Multiply . . .	.25	Multiply . . .	.75
By	.25	By	.75
	125		375
	50		525
Product = .	.0625	Product = .	.5625

DIVISION OF DECIMALS.

Division of decimals is performed in the same manner as in whole numbers; observing to point off as many decimal places in the quotient as the decimal places in the dividend exceed those in the divisor:— But if there be not as many figures in the quotient as there are in that excess, the deficiency must be supplied by prefixing ciphers, with a point before them;—for the decimal places in the divisor and quotient taken together, must be always equal to those in the dividend.—When there happens to be a remainder after the division; or when the decimal places in the divisor are more than those in the dividend, then ciphers may be annexed to the latter, and the quotient carried on as far as may be necessary.

Example 1.

Divide .6699550 by .275

```
                Dividend.   Quotient.
Divisor.275 ) .6699550   ( 2.4362
                550
                ----
                1199
                1100
                ----
                ..995
                  825
                  ---
                  1705
                  1650
                  ----
                  ..550
                    550
                    ---
                    ...
```

Example 2.

Divide 5050.8887 by 13.43

```
              Dividend.    Quotient.
Div. 13.43 ) 5050.8887  ( 376.09
             4029
             -----
             10218
              9401
              ----
              .8178
               8058
               ----
              .12087
               12087
               -----
               .....
```

Note.—If a decimal fraction be divided by a decimal fraction, the quotient will be greater than either the divisor or dividend, as in Example 1. And, if any whole, or mixed number be divided by a decimal fraction, the quotient will be greater than the dividend; but if a decimal fraction be divided by a whole, or mixed number, the quotient will be less than the dividend.—If a decimal fraction be divided by itself, its value will increase in the proportion of its division, or of the *decrease of the parts* into which the decimal is divided; because, in this case, the quotient will be a natural number:—thus, .25 divided by .25, quotes 1.—And, .5625, divided by .5625, quotes 1 also. Hence it is manifest that the dividing of a decimal fraction by itself increases its value.

REDUCTION OF DECIMALS.

CASE I.

To reduce a Vulgar Fraction to a Decimal Fraction of equal value.

RULE.

Annex a cipher or ciphers to the numerator; then divide by the denominator, as in whole numbers, and the quotient will be the required decimal.

Examples.

Reduce $\frac{1}{4}$ to a decimal fraction.

4)100

Required dec. = .25

Reduce $\frac{3}{4}$ to a decimal fraction.

4)300

Required dec. = .75

Examples.

Reduce $\frac{1}{2}$ to a decimal fraction.

2)10

Required dec. = .5

Reduce $\frac{5}{8}$ to a decimal fraction.

8)5000

Req. dec. = .625

CASE II.

To reduce Numbers of different Denominations, such as Degrees, Time, Coin, Measure, &c. into Decimals.

RULE.

Reduce the given degrees, time, coin, measure, &c. into the lowest denomination mentioned, for a dividend, annex ciphers thereto, and then divide by the integer, reduced also into the lowest denomination mentioned; the quotient will be the required decimal fraction.

Examples.

Reduce 30 minutes to the decimal of a degree.

The given number being in the lowest denomination required, annex a cipher and divide by 60, the number of minutes in a degree; the quotient will be the required decimal;—thus,

```
60)300(.5, the Answer.
   300
   ...
```

Examples.

Reduce 49′ 30″ to the decimal of a degree.

The given number being reduced to the lowest denomination mentioned, gives 2970″; to this annex ciphers, and divide by 3600, the seconds in a degree; the quotient will be the required decimal:—thus,

```
3600)2970.000(.825, Answer.
     28800
     ..9000
       7200
      18000
      18000
      .....
```

Reduce $15^m 50^s$ to the decimal of an hour.

The given terms being reduced to the lowest denomination give 950 seconds; annex ciphers and divide by 3600, the seconds in an hour; as thus,

```
3600)950.0000(.2639 nearly
     7200              Ans.
     23000
     21600
     .14000
      10800
      .32000
       32400
       .....
```

Reduce $4^h 10^m 50^s$ to the decimal of a day.

The given time being reduced to the lowest denomination mentioned is 15050 seconds; annex ciphers and divide by 86400, the seconds in a day, or 24 hours;—thus,

```
86400)15050.00000(.17419
       86400       nearly Ans.
      641000
      604800
      .362000
       345600
       .164000
         86400
        776000
```

Reduce $3^s 4^d$ to the decimal of a pound sterling.

The given sum being reduced to the lowest denomination mentioned gives 40 pence, annex ciphers and divide by 240, the pence in a pound sterling; as thus,

```
240)40.0000(.1666 Answer.
    240
    1600
    1440
    .1600
     1440
     .1600
      1440
      .160
```

Reduce 45 minutes to the decimal of an hour.

The given number being in the lowest denomination mentioned, annex ciphers and divide by 60, the minutes in an hour; as thus,

```
60)45.00(.75 which is the
   420          Ans.
   ----
   .300
    300
   ----
    ...
```

Reduce 100 fathoms and 2 feet to the decimal of a nautical mile.

The given measure being reduced to the lowest denomination mentioned is 602 feet; annex ciphers and divide by 6080, the number of feet in a sea mile; as thus,

```
6080)602.00000(.09901 Ans.
     54720
     -----
     .54800
      54720
     -----
     ...8000
        6080
        1920
```

Reduce 3 qrs. 21 lb. to the decimal of a hundred weight.

The given weight being reduced to the lowest denomination mentioned is 105 lbs. annex ciphers, and divide by 112, the number of pounds in a hundred weight; as thus,

```
112)105.0000(.9375 Ans.
    1008
    ..420
      336
     .840
      784
     .560
      560
      ...
```

Case III.

To find the value of any Decimal Fraction in the known parts of an Integer; such as Degrees, Time, Coin, Weight, Measure, &c.

Rule.

Multiply the given decimal by the number of parts contained in the next inferior denomination; and, from the right hand of the product, point off so many figures as the given decimal consists of.—Multiply those figures so pointed off by the number of parts contained in the next inferior denomination, and from the result cut off the decimal places as before:—proceed in this manner till the least known, or required parts of the integer are brought out;—then, the several denominations on the left hand of the decimal points, will express the value of the given decimal fraction.

Example 1.

Required the value of .825 of a degree.

Given decimal .825
Multiply by 60 minutes.

$49'.500$
Multiply by 60 seconds.

$30''.000$

Hence, the required value is $49'30''$

Example 2.

Required the value of .2639 of an hour.

Given decimal = .2639
Multiply by 60 min.

$15^{m}.8340$
Multiply by 60 seconds.

$50^{s}.040$

Hence, the required value is 15^{m} $50^{s}.040$.

Example 3.

Required the value of .166666 of a pound sterling.

Given decimal = .166666
Multiply by 20 shill.

$3^{s}.333320$
Multiply by 12pence

$3^{d}.999840$

Hence, the required value is $3^{s}4^{d}$ very nearly.

Example 4.

Required the value of .09901 of a nautical or sea mile.

Given decimal = .09901
Multiply by 6080, the ft. in a sea mile.

792080
594060

601.98080

Hence, the required value is 602 feet very nearly.

THE RULE OF PROPORTION IN DECIMALS.

Prepare the terms by reducing the fractional parts to the highest denomination mentioned; then state the question and proceed as in the common Rule of Three Direct;—thus, place the numbers in such order that the first and third may be of the same kind, and the second the same as the number required:—bring the first and third terms into the same name, and the second into the highest denomination mentioned.—Then,

Multiply the second and third terms together; divide the product by the first term, and the quotient will be the answer in the same denomination as the second number;—observing, however, to point off the decimal places; the value of which is to be found by the Rule to Case III., page 162.

Note.—In the rule of proportion there are always three numbers given to find a fourth proportional; two of these are of supposition and one of demand; the latter must ever be the third term in the statement of the question; and, as this is interrogatory, it may, therefore, be known by the words—What will? What cost? How many? How far? How much?, &c.—The *first* term must always be of the same name as the third; the fourth, or term sought, will be of the same kind and denomination as the second term in the proportion.

Example 1.

If a degree of longitude, measured on the surface of the earth under the equator, be 69.092 English miles; how many miles are contained in the earth's circumference under the same parallel, it being divided into 360 degrees?

As . . . 1° : 69^{mi}.092 :: 360°
360

4145520
207276

Answer . . . 24873.120 English miles.

Example 2.

The earth turns round upon its axis in 23^{h}56^m; at what rate per hour are the inhabitants carried from west to east by this rotation under the equator where the earth's circumference measures 24873. 12 English miles; and at what rate per hour are the inhabitants of London carried in the same direction, where a degree of longitude measures 42. 99 miles.

FIRST.—*For the Inhabitants at the Equator.*

23 hours 56 minutes are equal to 23. 9333 hours.—Now,

As $23^{h}.9333 : 24873^{mi}.12 :: 1^{h} : 1039$ miles.

```
24873.1200
239333
------
..939820
  717999
  ------
  2218210
  2153997
  -------
  ..64213
```

SECOND.—*For the Inhabitants of London.*

360 degrees multiplied by 42. 99 miles, give 15476. 4 miles;—And,

As $23^{h}.9333 : 15476^{mi}.4 :: 1^{h} : 646$ miles.

```
15476.4000
1435998
-------
.1116420
  957332
  ------
 .1590880
  1435998
  -------
  .154882
```

Hence, the inhabitants under the equator are carried at the rate of 1039 miles every hour, and those of London 646 miles per hour, by the earth's motion round its axis.

Example 3.

If a ship sails at the rate of 11¼ knots per hour; in what time would she circumnavigate the globe, the circumference of which is 24873. 12 miles?

11¼ knots are equal to 11.25 miles.—Now,

As 11^{m}.25 : 1^{h} :: 24873^{m}.12 : 2210.9 hours.

```
2250
────
.2373
 2250
 ────
 .1231
  1125
  ────
  .10620
   10125
   ─────
   ..495
```

Hence, the required time is 2210.9 hours; or 92 days, 2 hours, and 54 minutes.

PROPORTION, AND PROPERTIES OF NUMBERS.

If three quantities be proportional, the product or rectangle of the two extremes will be equal to the square of the mean.

If four quantities be proportional, the product of the two extremes will be equal to the rectangle or product of the two means.—Thus,

Let 2.4.8.16 be the four quantities; then, the rectangle of the extremes, viz. 16 × 2, is equal to the rectangle of the means, viz. 4 × 8, or 32.

If the product of any two quantities be equal to the product of two others, the four quantities may be turned into a proportion by making the terms of one product the *means*, and the terms of the other product the extremes.—Thus,

Let the terms of two products be 10 and 6, and 15 and 4, each of which is equal to 60; then, As 10 : 4 :: 15 : 6. As 4 : 6 :: 10 : 15. As 6 : 15 :: 4 : 10, &c. &c.

If four quantities be proportional, they shall also be proportional when taken inversely and alternately.

If four quantities be proportional, the sum, or difference, of the first and second will be to the second, as the sum, or difference of the third and fourth is to the fourth.—Thus, let 2.4.8.16 be the four proportional quantities; then

As 2 + 4 : 4 :: 8+16 : 16; or, as 4 − 2 : 4 :: 16 − 8 : 16.

If from the sum of any two quantities either quantity be taken, the remainder will be the other quantity.

If the difference of any two quantities be added to the less, the sum will be the greater quantity; or if subtracted from the greater, the remainder will be the less quantity.

If half the difference of any two quantities be added to half their sum, the total will give the greater quantity; or if subtracted, the remainder will be the less quantity.

If the product of any two quantities be divided by either quantity, the quotient will be the other quantity.

If the quotient of any two quantities be multiplied by the less, the product will be the greater quantity.

The rectangle or product of the sum and difference of any two quantities, is equal to the difference of their squares.—Thus,

Let 4 and 10 be the two quantities; then $4+10=14$; $10-4=6$, and $14\times6=84$.—Now, $10\times10=100$; $4\times4=16$, and $100-16=84$.

The difference of the squares of the sum and difference of any two quantities, is equal to four times the rectangle of those quantities.—Thus,

Let 10 and 6 be the two quantities; then $10+6=16\times16=256$;— $10-6=4\times4=16$.—Now, $256-16=240$; and $10\times6\times4=240$.

The sum of the squares of the sum and difference of any two quantities, is equal to twice the sum of their squares.—Thus,

$10+6=16\times16=256$; and $10-6=4\times4=16$; then $256+16=272$. Again, $10\times10=100$; $6\times6=36$, and $100+36=136\times2=272$.

If the sum and difference of any two numbers be added together, the total will be twice the greater number.—Thus,

$10+6=16$; and $10-6=4$; then $16+4=20$; and $10\times2=20$.

If the difference of any two numbers be subtracted from their sum, the remainder will be twice the less number.—Thus,

$10-6=4$; and $10+6=16$; then $16-4=12$;—and $6\times2=12$.

The square of the sum of any two numbers is equal to the sum of their squares, together with twice their rectangle.—Thus,

$10+6=16$; and $16\times16=256$. Again, $10\times10=100$; $6\times6=36$, and $100+36=136$; then, $10\times6\times2=120$; and $120+136=256$.

The sum, or difference, of any two numbers will measure the sum, or difference, of the cubes of the same numbers; that is, the sum will measure the sum, and the difference the difference.

The difference of any two numbers will measure the difference of the squares of those numbers.

The sum of any two numbers differing by an unit (1,) is equal to the difference of the squares of those numbers.—Thus,

$9+8=17$; and $9\times9=81$; $8\times8=64$; now, $81-64=17$.

If the sum of any two numbers be multiplied by each number respectively, the sum of the two rectangles will be equal to the square of the sum of those numbers.

Thus, $10+6=16$; now, $16\times10=160$; $16\times6=96$; and $160+96=256$.

Again, $10+6=16$; and $16\times16=256$.

The square of the sum of any two numbers is equal to four times the square of half their sum.—Thus,

$10+6=16$; and $16\times16=256$; then $10+6=16\div2=8$, and $8\times8\times4=256$.

The sum of the squares of any two numbers is equal to the square of their difference, together with twice the rectangle of those numbers.—Thus,

$10\times10=100$; $6\times6=36$; and $100+36=136$.—Again,

$10-6=4$; and $4\times4=16$; $10\times6\times2=120$; and $120+16=136$.

The numbers 3, 4 and 5, or their multiples 6, 8 and 10, &c. &c., will express the three sides of a right angled plane triangle.

The sum of any two square numbers whatever, their difference, and twice the product of their roots, will also express the three sides of a right angled plane triangle.—Thus,

Let 9 and 49 be the two square numbers:—then $9+49=58$; $49-9=40$.—Now, the root of 9 is 3, and that of 49 is 7;—then $7\times3\times2=42$: hence the three sides of the right angled plane triangle will be 58, 40, and 42.

The sum of the squares of the base and perpendicular of a right angled plane triangle, is equal to the square of the hypothenuse.

The difference of the squares of the hypothenuse and one leg of a right angled plane triangle, is equal to the square of the other leg.

The rectangle or product of the sum and difference of the hypothenuse and one leg of a right angled plane triangle, is equal to the square of the other leg.

The cube of any number divided by 6 will leave the same remainder as the number itself when divided by 6.—The difference between any number and its cube will divide by 6, and leave no remainder.

Any even square number will divide by 4, and leave no remainder; but an uneven square number divided by 4 will leave 1 for a remainder.

PLANE TRIGONOMETRY.

The Resolution of the different Problems, or Cases, in Plane Trigonometry, by Logarithms.

ALTHOUGH it is not the author's intention (as has been already observed,) to enter into the elementary parts of the sciences on which he may have occasion to touch in elucidating a few of the many important purposes to which these Tables may be applied; yet, since this work may, probably, fall into the hands of persons not very conversant with trigonometrical subjects, he therefore thinks it right briefly to set forth such definitions, &c. as appear to be indispensably necessary towards giving such persons some little insight into this particular department of science.

PLANE TRIGONOMETRY is that branch of the mathematics which teaches how to find the measures of the unknown sides and angles of plane triangles from some that are already known.—It is divided into two parts; right angled and oblique angled :—in the former case one of the angles is a right angle, or 90°; in the latter they are all oblique.

Every plane triangle consists of six parts; viz., three sides and three angles; any three of which being given (except the three angles), the other three may be readily found by logarithmical calculation.

In every triangle the greatest side is opposite to the greatest angle; and, *vice versa*, the greatest angle opposite to the greatest side.—But, equal sides are subtended by equal angles, and conversely.

The three angles of every plane triangle are, together, equal to two right angles, or 180 degrees.

If one angle of a plane triangle be obtuse, or more than 90°, the other two are acute, or each less than that quantity: and if one angle be right, or 90°, the other two taken together, make 90° :—hence, if one of the angles of a right angled triangle be known, the other is found by subtracting the known one from 90°.—If one angle of any plane triangle be known, the sum of the other two is found by subtracting that which is given from 180°; and if two of the angles be known, the third is found by subtracting their sum from 180°

The complement of an angle is what it wants of 90°; and the supplement of an angle is what it wants of 180°

In every right angled triangle, the side subtending the right angle is called the *hypothenuse;* the lower or horizontal side is called the *base*, and that which stands upright, the *perpendicular*.

If the hypothenuse be assumed equal to the radius, the sides, that is, the base and the perpendicular, will be the sines of their opposite angles. And, if either of the sides be considered as the radius, the other side will be the tangent of its opposite angle, and the hypothenuse the secant of the same angle.

Thus.—Let A B C be a right angled plane triangle; if the hypothenuse A C be made radius, the side B C will be the sine of the angle A, and A B the sine of the angle C.—If the side A B be made radius, B C will be the tangent, and A C the secant, of the angle A :—And, if B C be the radius, A B will be the tangent, and A C the secant of the angle C.

For, if we make the hypothenuse A C radius (Fig. 1.), and upon A, as a centre, describe the arch C D to meet A B produced to D; then it is evident that B C is the sine of the arch D C, which is the measure of the angle B A C; and that A B is the co-sine of the same arch :—and if the arch A E be described about the centre C, to meet C B produced to E, then will A B be the sine of the arch A E, or the sine of the angle A C B, and B C its co-sine.

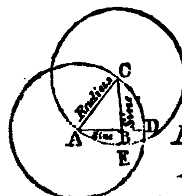


Fig 1.

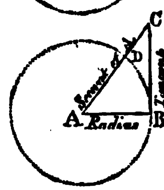


Fig 2.

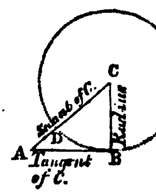


Fig 3.

Again, with the extent A B as a radius (Fig. 2.), describe the circle B D; then B C is the tangent of the arch B D, which is evidently the measure of the angle B A C; and A C is the secant of the same arch, or angle.

Lastly, with C B as a radius (Fig. 3.), describe the arch B D; then A B is the tangent of the arch B D, the measure of the angle A C B, and A C the secant of the same arch or angle.

In the computation of right angled triangles, any side, whether given or required, may be made radius to find a *side*; but a given side must be made radius to find an angle : thus,

To find a Side :—

Call any one of the sides of the triangle radius, and write upon it the word *radius* :—observe whether the other sides become sines, tangents, or secants, and write these words on them accordingly, as in the three preceding figures : then say, as the name of the given side, is to the given side; so is the name of the side required, to the side required.

And, to find an Angle :—

Call one of the *given sides* the radius, and write upon it the word radius: observe whether the other sides become sines, tangents, or secants, and write these words on them accordingly, as in the three foregoing figures; then say, as the side made radius, is to radius; so is the other *given side* to its name: that is, to the sine, tangent, or secant by it represented.

Now, since in plane trigonometry the sides of a triangle may be considered, without much impropriety, as being in a direct ratio to the sines of their opposite angles, and conversely; the proportion may, therefore, be stated agreeably to the established principles of the *Rule of Three Direct*, by saying

As the name of a given angle, is to its opposite given side; so is the name of any other given angle to its opposite side.—And, as a given side, is to the name of its opposite given angle; so is any other given side to the name of its opposite angle.

The proportion, thus stated, is to be worked by logarithms, in the following manner; viz.,

To the arithmetical complement of the first term, add the logs. of the second and third terms, and the sum (rejecting 20, or 10 from the index, according as the required term may be a side or an angle,) will be the logarithm of the required, or fourth term.

Remarks.—1. The arithmetical complement of a logarithm is what that logarithm wants of the radius of the Table; viz., what it is short of 10.000000; and the arithmetical complement of a log. sine, tangent, or secant, is what such logarithmic sine, &c. &c. wants of twice the radius of the Tables, viz., 20.000000.

2. The arithmetical complement of a log. is most readily found by beginning at the left hand and subtracting each figure from 9 except the last significant one, which is to be taken from 10, as thus;—if the given log. be 2.376843, its arithmetical complement will be 7.623157:—if a given log. sine be 9.476284, its arithmetical complement will be 10.523716, and so on.

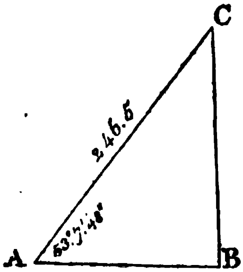
3. The arithmetical complement of the log. sine of an arch, is the log. co-secant of that arch;—the arithmetical complement of the log. tangent of an arch, is the log. co-tangent of that arch; and conversely, in both cases.

Solution of Right-angled Plane Triangles, by Logarithms.

PROBLEM I.

Given the Angles and the Hypothenuse, to find the Base and the Perpendicular.

Example.



Let the hypothenuse A C, of the annexed triangle A B C, be 246.5, and the angle A 53°7′48″; required the base A B, and the perpendicular B C?

Note.—*Since there is no more intended, in this place, than merely to show the use of the Tables; the geometrical construction of the diagrams is, therefore, purposely omitted.*

By making the hypothenuse A C radius; B C becomes the sine of the angle A, and A B the co-sine of the same angle.—Hence,

To find the Perpendicular B C:—

As radius = ... 90° =	Log. sine = . .	10.000000
Is to hypothenuse A C = 246.5	Log. = ...	2.391817
So is the angle A = 53°7′48″	Log. sine = . .	9.903090
To the perpendicular B C = 197.2 =	Log. = . . .	2.294907

To find the Base A B:—

As radius = ... 90° =	Log. sine = . .	10.000000
Is to hypothenuse A C = 246.5	Log. = . . .	2.391817
So is the angle A = 53°7′48″	Log. co-sine = .	9.778153
To the base A B = 147.9 =	Log. = . . .	2.169970

Making the base A B radius; B C becomes the tangent of the angle A, and A C the secant of the same angle.—Hence,

To find the Perpendicular B C:—

As the angle A = . 53°7′48″	Log. secant Ar. comp. =	9.778153
Is to hypothenuse A C = 246.5	Log. = ...	2.391817
So is the angle A = 53°7′48″	Log. tangent = . .	10.124937
To the perpendicular B C = 179.2 =	Log. = . . .	2.294907

To find the Base A B :—

As the angle A = 53°7′48″	Log. secant Ar. compt. =	9.778153
Is to hypothenuse A C = 246.5	Log. = ...	2.391817
So is radius = 90°	Log. sine = ...	10.000000
To the base A B 147.9 =	Log. = ...	2.169970

The perpendicular B C being made radius; the base A B becomes the tangent of the angle C, or co-tangent of the angle A, and the hypothenuse A C the secant of the angle C, or co-secant of the angle A.—Hence,

To find the Perpendicular B C:

As the angle A = 53°7′48″	Log. co-secant. Ar. compt. =	9.903090
Is to hypothenuse A C = 246.5	Log.	2.391817
So is radius = 90° . . .	Log. sine	10.000000
To the perpendicular B C = 197.2 =	Log. = ...	2.294907

To find the Base A B :—

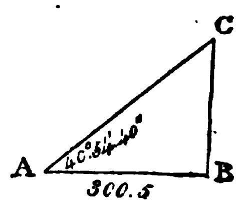
As the angle A = 53°7′48″	Log. co-secant Ar. compt. =	9.903090
Is to hypothenuse A C = 246.5	Log. = ...	2.391817
So is the angle A = 53°7′48″	Log. co-tangent . .	9.875063
To the base A B = 147.9 =	Log. = ...	2.169970

Problem II.

Given the Angles and One Side, to find the Hypothenuse and the other Side.

Example.

Let the base A B of the annexed triangle A B C, be 300.5, and the angle A 40°54′40″; required the hypothenuse A C, and the perpendicular B C?



The hypothenuse A C being made radius; the perpendicular B C will be the sine of the angle A, and the base A B the co-sine of the same angle.

To find the Hypothenuse A C :—

As the angle A = 40°54′40″	Log. co-sine Ar. compt. =	10. 121635
Is to the base A B = 300. 5	Log. = ...	2. 477845
So is radius = 90°	Log. sine = ...	10. 000000
To the hypothenuse A C = 397. 6 =	Log. = . . .	2. 599480

To find the Perpendicular B C :—

As the angle A = 40°54′40″	Log. co-sine Ar. compt. =	10. 121635
Is to the base A B = 300. 5	Log. = ...	2. 477845
So is the angle A = 40°54′40″	Log. sine = ...	9. 816167
To the perpendicular B C = 260. 4 =	Log. = ...	2. 415647

The base A B being made radius; the perpendicular B C will be the tangent of the angle A, and the hypothenuse A C the secant thereof.—Hence,

To find the Hypothenuse A C :—

As radius = 90°	Log. sine = ...	10. 000000
Is to the base A B = 300. 5	Log. = ...	2. 477845
So is the angle A = 40°54′40″	Log. secant	10. 121635
To the hypothenuse A C = 397. 6 =	Log. = ...	2. 599480

To find the Perpendicular B C :—

As radius = 90° =	Log. sine = ...	10. 000000
Is to the base A B = 300. 5	Log.	2. 477845
So is the angle A = 40°54′40″	Log. tangent	9. 937802
To the perpendicular B C = 260. 4 =	Log. = . . .	2. 415647

The perpendicular B C being made radius; the base A B will be the tangent of the angle C, or co-tangent of the angle A, and the hypothenuse the secant of the angle C, or co-secant of A.—Hence,

To find the Hypothenuse A C :—

As the angle A = 40°54′40″	Log. co-tang. Ar. compt. =	9. 937802
Is to the base A B = 300. 5	Log. = ...	2. 477845
So is the angle A = 40°54′40″	Log. co-secant = . .	10. 183833
To the hypothenuse A C = 397. 6 =	Log. = ...	2. 599480

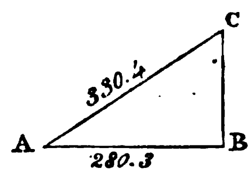
To find the Perpendicular B C :—

As the angle A = 40°54′40″ Log. co-tang. Ar. compt. =	9.937802
Is to the base A B = 300.5 Log. = ...	2.477845
So is radius = 90° Log. sine = ...	10.000000
To the perpendicular B C = 260.4 = Log. = . . .	2.415647

Problem III.

Given the Hypothenuse and One Side, to find the Angles and the Other Side.

Example.



Let the hypothenuse A C, of the annexed triangle A B C, be 330.4, and the base A B 280.3; required the angles A and C, and the perpendicular B C?

By making the hypothenuse A C radius; the perpendicular B C becomes the sine of the angle A, and the base A B the co-sine of the same angle.—Hence,

To find the Angle A :—

As the hypothenuse A C = 330.4 Log. Ar. compt. =	7.480960
Is to radius = 90° Log. sine = ...	10.000000
So is the base A B = 280.3 Log. = ...	2.447623
To the angle A = 31°57′56″ Log. co-sine = . . .	9.928583

To find the Perpendicular B C.

As radius = 90° Log. sine = ...	10.000000
Is to hypothenuse A C = 330.4 Log. = ...	2.519040
So is the angle A = 31°57′56″ Log. sine = . . .	9.723791
To the perpendicular B C = 174.9 = Log. = . . .	2.242831

The base A B being made radius; the perpendicular B C becomes the tangent of the angle A, and the hypothenuse A C the secant of that angle. —Hence, To find the Angle A :—

As the base AB = 280.3 Log. Ar. compt. = . . .	7.552377
Is to the radius = 90° Log. sine = ...	10.000000
So is the hypothenuse AC = 330.4 = Log. = . . .	2.519040
To the angle A = 31°57′56″ Log. secant = . . .	10.071417

To find the Perpendicular BC :—

As radius = 90°. Log. sine = ...	10.000000
Is to the base AB = 280.3 Log. = ...	2.447623
So is the angle A = 31°57′56″ Log. tangent = . .	9.795208
To the perpendicular BC = 174.9 = Log. = ...	2.242831

Remark.—The perpendicular BC may be found independently of the angles by the following rule (deduced from Euclid, Book I. Prop. 47, and Book II. Prop. 5), viz.,

To the log. of the sum of the hypothenuse and given side, add the log. of their difference; then, half the sum of these two logs. will be the log. of the required side :—as thus;

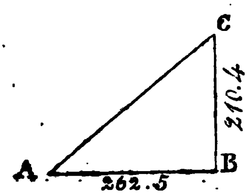
Hypothenuse AC	= 330.4		
Base . . . AB	= 280.3		
Sum . . .	= 610.7	Log. . . .	= 2.785828
Difference . . .	50.1	Log. . . .	= 1.699838
		Sum	= 4.485666
Perpendicular BC	= 174.9	= Log. . .	= 2.242833

Problem IV.

Given the Base and the Perpendicular, to find the Angles and the Hypothenuse.

Example.

Let the base AB, of the annexed triangle ABC, be 262.5, and the perpendicular BC 210.4; required the angles, and the hypothenuse AC?



By making the base A B radius; the perpendicular B C becomes the tangent of the angle A, and the hypothenuse A C the secant thereof. —Hence,

To find the Angle A :—

As the base A B = 262.5 Log. Ar. compt. = ...	7.580871
Is to radius = 90° Log. sine = ...	10.000000
So is the perpendicular B C = 210.4 Log. = ...	2.323046
To the angle A = 38°42′47″ Log. tangent = . . .	9.903917

To find the Hypothenuse A C :—

As radius = 90° Log. sine = ...	10.000000
Is to the base A B = 262.5 Log. = ...	2.419129
So is the angle A = 38°42′47″ Log. secant = . . .	10.107745
To the hypothenuse A C = 336.4 = Log. = . . .	2.526874

The perpendicular B C being made radius; the base A B will be the tangent of the angle C, or co-tangent of the angle A, and the hypothenuse A C will be the secant of C, or the co-secant of the angle A.—Hence,

To find the Angle A :—

As the perpendicular B C = 210.4 Log. Ar. compt. = .	7.676954
Is to radius = 90° Log. sine = ...	10.000000
So is the base A B = 262.5 Log. = ...	2.419129
To the angle A = 38°42′47″ = Log. co-tangent = .	10.096083

To find the Hypothenuse A C :—

As radius = 90° Log. sine = ...	10.000000
Is to the perpendicular B C = 210.4 Log. = . . .	2.323046
So is the angle A = 38°42′47″ Log. co-secant = . .	10.203828
To the hypothenuse A C = 336.4 = Log. = . . .	2.526874

The angle A subtracted from 90° leaves the angle C; thus 90° − 38° 42′47″ = 51°17′13″ the measure of the angle C.

Remark.—The hypothenuse A C may be found independently of the angles by the following rule, deduced principally from Euclid, Book I. Prop. 47; Book II. Prop. 5; and Book VI. Prop. 8, viz., From twice the log. of the base subtract the log. of the perpendicular, and add the corresponding natural number to the perpendicular; then, to the log. of this sum add the log. of the perpendicular, and half the sum of these two logs. will be the log. of the hypothenuse. As thus:—

Base A B = . . .	262.5	twice the log. =	4.838258	
Perpendicular B C =	210.4	Log. . . . =	2.323046	. . 2.323046
Natural number =	327.5	Log. . . . =	2.515212	
Sum	537.9	Log. = ...		2.730702
			Sum =	5.053748
Hypothenuse A C =	336.4	Log. = ...		2.526874

Solution of Oblique-angled Plane Triangles by Logarithms.

Problem I.

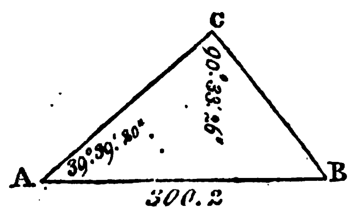
Given the Angles and One Side of an Oblique-angled Plane Triangle, to find the other Sides.

Rule.

As the Log. sine of any given angle, is to its opposite given side; so is the log. sine of any other given angle to its opposite side.

Example.

Let the side A B, of the triangle A B C, be 300.2, the angle A 39°39′20″ the angle C 90°33′26″ and, hence, the angle B 49°47′14″; to find the sides A C and B C.



To find the Side A C:—

As the angle C = 90°33′26″ Log. sine ar. compt. =	10.000021
Is to the side B C = 300.2 Log.	2.477411
So is the angle B = 49°47′14″ Log. sine	9.882895
To the side A C = 229.3 = Log. = ...	2.360327

To find the Side B C:—

As the angle C = 90°33′26″ Log. sine ar. compt. =		10.000021
Is to the side B C = 300.2 Log. =	. . . .	2.477411
So is the angle A = 39°39′20″ Log. sine =	. . . . .	9.804937
To the side B C = 191.6 = Log. =	. . . .	2.282369

Note.—When a log. sine, or log. co-sine, is the first term in the proportion, the arithmetical complement thereof may be taken directly from the Table of secants by using a log. co-secant in the former case, and a log. secant in the latter.

Problem II.

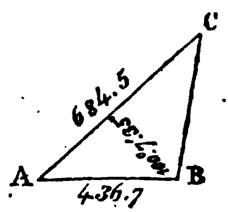
Given two Sides and an Angle opposite to one of them, to find the other Angles and the third Side.

Rule.

As any given side of a triangle is to the log. sine of its opposite given angle, so is any other given side to the log. sine of the angle opposite thereto.

The angles being thus found, the third side is to be computed by the preceding Problem.

Example.



Let the side A B, of the triangle A B C, be 436.7, the side A C 684.5, and the angle B 100°7′35″; required the angles A and C, and the side B C?

To find the angle C:—

As the side A C = . 684.5	Log. ar. comp.	7.164626
Is to the angle A = 100°7′35″	Log. sine =	9.993181
So is the side A B = 436.7	Log. = . .	2.640183
To the angle C = 38°54′22″	Log. = . .	9.797990

To find the side B C :—

As the angle B = . 100°7′35″	Log. sine ar. comp. =	10.006819
Is to the side A C = 684.5	Log. = ...	2.835374
So is the angle A = 40°58′3″	Log. sine = . . .	9.816659
To the side B C = 455.9 =	Log. = ...	2.658852

Note.—The angle A = 100°7′35″ + the angle C = 38°54′22″ = 139°1′57″; and 180° − 139°1′57″ = the angle A = 40°58′3″

Remark.—An angle found by this rule is ambiguous when the given side opposite to the given angle is *less* than the other given side; that is, the angle opposite to the greater side may be either acute or obtuse: for trigonometry only gives the sine of an angle, which sine may either represent the measure of the angle itself, or of its supplement to 180 degrees. But when the given side opposite to the given angle is greater than the other given side, then the angle opposite to that (other given) side is always acute, as in the above example.

Problem III.

Given two Sides and the included Angle, to find the other Angles and the third Side.

Rule.

Find the sum and difference of the two given sides; subtract the given angle from 180°; take half the remainder, and it will be half the sum of the unknown angles; then say,

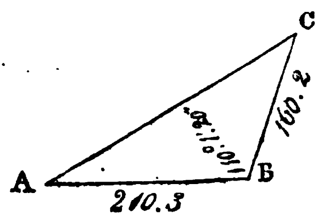
As the sum of the sides is to their difference; so is the log. tangent of half the sum of the unknown angles, to the log. tangent of half their difference.

Now, half the difference of the angles, thus found, added to half their sum, gives the greater angle, or that which is opposite to the greater side; and being subtracted, leaves the angle opposite to the less side.

The angles being thus determined, the third side is to be computed by Problem I., page 177.

Example.

Let the side A B, of the triangle A B C, be 210. 3, the side B C 160. 2, and the angle B 110°1′20″; required the angles A and C, and the side A C?



180° − the angle B 110°1′20″ = 69°58′40″ ÷ 2 = 34°59′20″ = half the sum of the angles A and C.

Side AB = ...	210.3		
Side BC = ...	160.2		
As sum = ...	370.5	Log ar. comp. =	7.431212
Is to difference = . . .	50.1	Log. = . .	1.699838
So is ½ sum of angles =	34°59′20″	Log. tang. =	9.845048
To ½ differ. of angles =	5°24′24″	Log. tang. =	8.976098
Angle C = ...	40°23′44″		
Angle A = ...	29°34′56″		

To find the side AC:

As the angle A = .	29°34′56″	Log. sine ar. comp. =	10.306561
Is to the side BC = .	160.2	Log. = ...	2.204663
So is the angle B = .	110°1′20″	Log. sine = ...	9.972925
To the side AC = .	304.9 =	Log. = ...	2.484149

Problem IV.

Given the three Sides of a Plane Triangle, to find the Angles.

Rule.

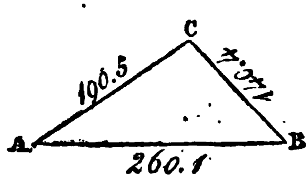
Add the three sides together, and take half their sum; the difference between which and the side opposite to the required angle call the *remainder;* then,

To the arithmetical complements of the logs. of the other two sides, add the logs. of the half sum and of the *remainder:* half the sum of these four logs. will be the log. co-sine of an arch; which, being doubled, will give the required angle.

Now, one angle being thus found, either of the other two angles may be computed by Problem II., page 178.

Example.

Let the side AB, of the triangle ABC, be 260.1, the side AC 190.5, and the side BC 140.4; required the angles A, B, and C?



The side AB =	260.1			
BC =	140.4	Log. ar. comp.	. .	7.852633
AC =	190.5	Log. ar. comp.	. .	7.720105
Sum = . . .	591.0			
Half sum = . .	295.5	Log. =	. . . .	2.470558
Remainder = .	35.4	Log. =	. . . .	1.549003
			Sum =	19.592299
Arch = . .	51°17′22″	Log. co-sine =	. .	9.796149½
Angle C = .	102°34′44″			

To find the angle B:—

As the side AB =	260.1	Log. ar. comp. =		7.584860
Is to the angle C =	102°34′44″	Log. sine =	. . .	9.989448
So is the side AC =	190.5	Log. =	. . .	2.279895
To the angle B =	45°37′45″	Log. =	. . .	9.854203

Now, angle C 102°34′44″ + angle B 45°37′45″ = 148°12′29″; and 180° − 148°12′29″ = 31°47′31″ = the angle A.

THE RESOLUTION OF THE DIFFERENT PROBLEMS, OR CASES, IN SPHERICAL TRIGONOMETRY, BY LOGARITHMS.

Spherical Trigonometry is that branch of the mathematics which shows how to find the measures of the unknown sides and angles of spherical triangles from some that are already known. It is divided into three parts; viz., right-angled, quadrantal, and oblique-angled.

A right-angled spherical triangle has one right angle; the sides including the right angle are called legs, and that opposite thereto the hypothenuse.

A quadrantal spherical triangle has one side equal to 90°, or the fourth part of a circle.

An oblique-angled spherical triangle has neither a side nor an angle equal to 90°

A spherical triangle is formed by the intersection of three great circles on the surface of the sphere.

The three angles of a spherical triangle are always more than two, but less than six, right angles.

The three sides of a spherical triangle are always less than two semicircles, or 360°.

Any two sides of a spherical triangle, taken together, are greater than the third.

The greater side subtends the greater angle; the lesser side the lesser angle, and conversely.

Equal sides subtend equal angles, and, *vice versa*, equal angles are subtended by equal sides.

The two sides or two angles of a spherical triangle, when compared together, are said to be alike, or of the same affection, when both are less or both greater than 90°; but when one is greater and the other less than 90°, they are said to be unlike, or of different affections.

Every side of a right-angled spherical triangle exceeding 90°, is greater than the hypothenuse; but every side less than that quantity, is less than the hypothenuse.

The hypothenuse is less than a quadrant, if the legs be of the same affection; but greater than a quadrant, if they be of different affections.

The hypothenuse is, also, less or greater than a quadrant, according as the adjacent angles are of the same or of different affections.

When the hypothenuse and one leg, or its opposite angle, are of the same or of different affections, the other side, or its opposite angle, will be, accordingly, less or greater than a quadrant.

The legs and their opposite angles are always of the same affection.

The sides of a spherical triangle may be changed into angles, and conversely.

Every spherical triangle consists of six parts: viz., three sides and three angles; of which, if any three be given, the remaining three may be readily computed; but in right-angled spherical triangles, it is sufficient that two only be given, because the right angle is always known.

SOLUTION OF RIGHT-ANGLED SPHERICAL TRIANGLES, BY LOGARITHMS, AGREEABLY TO LORD NAPIER'S RULES.

In every right-angled spherical triangle there are five circular parts, exclusive of the right angle, which is not taken into consideration. These five parts consist of the *two legs*, or sides; the *complement of the hypothenuse;* and the *complements of the two angles.* They are called circular parts, because each of them is measured by the arc of a great circle.

Three of these circular parts, besides the radius, enter into every proportion; two of which are given, and the third required. One is called the *middle part*, and the other two the *extremes conjunct* or *disjunct*.

The *middle part*, and also the *extremes conjunct* or *disjunct*, may be determined by the following rules.

Rule 1.—When the three circular parts under consideration are joined together, or follow each other in successive order, the middle one is termed the *middle part*, and the other two the *extremes conjunct*, because they are directly conjoined thereto.

Rule 2.—When the three circular parts do not join, or follow each other in successive order, that which stands alone, or disjoined from the other two, is termed the *middle part*, and the other two the *extremes disjunct*, because they are separated or disjoined therefrom by the intervention of a side, or an angle not concerned in the proportion.

Note.—In determining the *middle part*, it is to be observed, that the right angle does not separate or disjoin the legs: therefore, when these are under consideration, they are always to follow each other in succession.

These things being premised, the required parts are to be computed by the two following equations; viz.,

1st.—*The product of radius and the sine of the middle part, is equal to the product of the tangents of the extremes conjunct.*

2d.—*The product of radius and the sine of the middle part, is equal to the product of the co-sines of the extremes disjunct.*

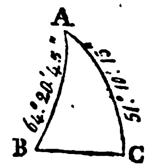
Since these equations are adapted to the complements of the hypothenuse and angles, and since the sine or the tangent of the complement of an arch is represented directly by the co-sine or co-tangent of that arch,—therefore, to save the trouble of finding the complements, let a co-sine or co-tangent be used instead of a sine or tangent, and a sine instead of a co-sine, &c. &c., when the angles or the hypothenuse are in question.

Now, the *middle part* being determined by the rules 1 or 2, as above, according as the extremes are *conjunct* or *disjunct*, the terms under consideration are then to be reduced to a proportion, as thus:—Put the unknown or required term *last*, that with which it is connected *first*, and the remaining two in the middle, in any order; this being done, the equation will then be ready for a direct solution by logarithmical numbers.

Problem I.

Given the Hypothenuse and one Leg, to find the Angles and the other Leg.

Example.



Let the hypothenuse A B, of the spherical triangle A B C, be 64°20′45″, and the leg A C 51°10′15″; required the angles A and B, and the leg B C?

To find the angle A:—

Here the hypothenuse A B, the given leg A C, and the required angle A, are the three circular parts which enter the proportion; and since the angle A evidently connects the hypothenuse and the given leg, it is therefore the *middle part*, and the other two the *extremes conjunct*, according to rule 1, page 183; therefore, by equation 1, page 183,

Radius × co-sine of angle A = tangent of A C × co-tangent of A B.

Now, since radius is *connected* with the required term, it is to be the first term in the proportion. Hence,

As radius = ...	90° 0′ 0″	Log. sine ar. comp. =	10.000000
Is to the leg A C = . .	51. 10. 15	Log. tangent = . .	10.094280
So is the hypothenuse A B =	64. 20. 45	Log. co-tangent =	9.681497
To the angle A = . . .	53°21′50″	Log. co-sine = .	9.775777

Note.—The angle A is acute, because the hypothenuse and the given leg are both of the same affection.

To find the angle B:—

The three circular parts which enter the proportion, in this case, are the hypothenuse A B, the given leg A C, and the required angle B; and since the leg A C is disjoined from the other two parts by the angle A, it is therefore the *middle part*, and the other two the *extremes disjunct*, according to rule 2, page 183; therefore, by equation 2, page 183,

Radius × sine leg A C = sine hyp. A B × sine of angle B.

Now, since the hypothenuse is connected with the required term, it is to stand first in the proportion. Hence,

As the hypothenuse A B =	64°20′45″	Log. sine ar. comp. =	10.045071
Is to radius = . . .	90. 0. 0	Log. sine = . .	10.000000
So is the leg A C = . .	51.10.15	Log. sine = . .	9.891548
To the angle B = . .	59°47.34″	Log. sine = . .	9.936619

Note.—The angle B is acute, because the hypothenuse and the given leg are of the same affection.

To find the leg B C :—

In this case the three circular parts which enter the proportion, are the hypothenuse and the two legs; and since the hypothenuse is disjoined from the legs by the angles A and B, it is the *middle part*, and the other two are the *extremes disjunct*; therefore,

Radius × co-sine hyp. A B = co-sine leg A C × co-sine leg B C.

Now, the leg A C, being connected with the required term, is, therefore, to stand first in the proportion. Hence,

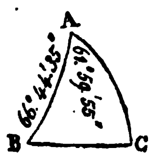
As the leg A C = . .	51°10′15″	Log. co-sine ar. comp. =	10.202732
Is to radius = . . .	90. 0. 0	Log. sine = . . .	10.000000
So is hypothenuse A B =	64.20.45	Log. co-sine = . .	9.636426
To the leg B C = . .	46°19′52″	Log. co-sine = . .	9.839158

Note.—The leg B C is acute, because the hypothenuse and the given leg are of the same affection.

Problem II.

Given the Hypothenuse and one Angle, to find the other Angle and the two Legs.

Example.



Let the hypothenuse A B, of the spherical triangle A B C, be 66°44′35″, and the angle A 61°59′55″; required the angle B and the legs A C and B C?

To find the angle B :—

Here the three circular parts are connected or joined together; therefore the hypothenuse A B is the *middle part*, and the angles A and B *extremes conjunct* (rule 1, page 183); therefore, by equation 1, page 183, Radius × co-sine hyp. A B = co-tangent angle A × co-tangent angle B.

Now, the angle A, being connected with the required part, is therefore to stand first in the proportion. Hence,

As the angle A = .	61°59′55″	Log. co-tang. ar. comp. =	10.274300
Is to radius = . .	90. 0. 0	Log. sine = ...	10.000000
So is the hyp. A B =	66.44.35	Log. co-sine	9.596438
To the angle B = .	53°24′12″	Log. co-tangent = . .	9.870738

Note.—The angle B is acute, because the hypothenuse and the given angle are of the same affection.

To find the leg A C :—

In this case, the three circular parts are joined together; therefore the angle A is the *middle part*, and the hypothenuse A B and required leg A C are the *extremes conjunct*; therefore,

Radius × co-sine of angle A = co-tangent A B × tangent A C.

And since the hypothenuse is connected with the required part, it is therefore to be the first term in the proportion. Hence,

As the hyp. A B =	66°44′35″	Log. co-tang. ar. comp. =	10.366756
Is to radius = . .	90. 0. 0	Log. sine = ...	10.000000
So is the angle A =	61.59.55	Log. co-sine	9.671629
To the leg A C = .	47°31′42″	Log. tangent = . . .	10.038385

Note.—The leg A C is acute, because the hypothenuse and the given angle are of the same affection.

To find the leg B C :—

In this case the leg B C is the *middle part*, because it stands alone, or is disjoined from the other two circular parts concerned, by the angle B : hence the hypothenuse A B and the given angle A are *extremes disjunct*, according to rule 2, page 183; therefore, by equation 2, page 183,

Radius × sine of leg B C = sine of hyp. A B × sine of angle A.

And since radius is connected with the required part, it is to be the first term in the proportion. Hence,

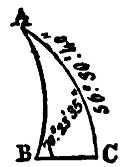
As radius = . . .	90° 0′ 0″	Log. sine ar. comp. =	10.000000
Is to hypothenuse A B =	66.44.35	Log. sine = . . .	9.963194
So is the angle A = .	61.59.55	Log. sine = . . .	9.945929
To the leg B C = .	54°12′45″	Log. sine = . . .	9.909123

Note.—The leg B C is acute, because the hypothenuse and the given angle are of the same affection.

Problem III.

Given a Leg and its opposite Angle, to find the other Angle, the other Leg, and the Hypothenuse.

Example.



Let the leg A C, of the spherical triangle A B C, be 56°30′40″, and the angle B 70°23′35″; required the angle A, the leg B C, and the hypothenuse A B?

To find the angle A :—

Here the three circular parts which enter the proportion, are the given angle B, the given leg A C, and the required angle A; and since the angle B is disjoined from the other two parts by the intervention of the hypothenuse A B, it is the *middle part*, and the other two are the *extremes disjunct*, according to rule 2, page 183; therefore, by equation 2, page 183,

Radius × co-sine of the angle B = sine of the angle A × co-sine of the leg A C.

And since A C is connected with the required part, it is to be the first term in the proportion. Hence,

As the leg A C =	56°30′40″	Log. co-sine ar. comp. =	10.258238
Is to radius = .	90. 0. 0	Log. sine = ...	10.000000
So is the angle B =	70.23.35	Log. co-sine	9.525778
To the angle A =	{ 37°27′23″ / 142.32.37 }	Log. sine = ...	9.784016

Note.—The angle A is *ambiguous*, since it cannot be determined, from the parts given, whether it is acute or obtuse.

To find the leg B C :—

The three circular parts concerned in this case, are the legs A C and B C, and the given angle A; and since the right angle *never separates the legs*, B C is the *middle part*, and A C and the angle B are the *extremes conjunct*, by rule 1, page 183; therefore, by equation 1, page 183,

Radius × sine of the leg B = tangent leg A C × co-tangent angle B.

Now, since radius is connected with the required term, it is to stand first in the proportion. Hence,

As radius = . .	90° 0′ 0″	Log. sine ar. comp. =	10.000000
Is to the leg A C =	56. 30. 40	Log. tangent = . .	10.179400
So is the angle B =	70. 23. 35	Log. co-tangent = .	9.551719
To the leg B C =	{ 32°34′33″ / 147. 25. 27 }	Log. sine = . . .	9.731119

Note.—The leg B C is *ambiguous*, since it cannot be determined, from the parts given, whether it is acute or obtuse.

To find the hypothenuse A B:—

Here the given leg A C is the *middle part*, because it is disjoined from the other two circular parts concerned, by the intervention of the angle A: hence the angle B and the hypothenuse A B are *extremes disjunct*; therefore,

Radius × sine of leg A C = sine of hyp. A B × sine of angle B.

And since the angle B is connected with the required term, it is to stand first in the proportion. Hence,

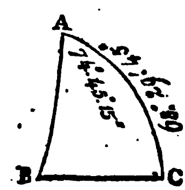
As the angle B = . .	70°23′35″	Log. sine ar. comp. =	10.025941
Is to the leg A C = . .	56. 30. 40	Log. sine = . . .	9.921162
So is radius = ...	90. 0. 0	Log. sine = . . .	10.000000
To the hyp. A B =	{ 62°17′30″ / 117. 42. 30 }	Log. sine = . . .	9.947103

Note.—The hypothenuse A B is *ambiguous*; that is, it may be either acute or obtuse, from the parts given.

Problem IV.

Given a Leg and its adjacent Angle, to find the other Angle, the other Leg, and the Hypothenuse.

Example.



Let the leg A C, of the spherical triangle A B C, be 68°29′45″, and the angle A 74°45′15″; required the angle B, the leg B C, and the hypothenuse A B?

To find the Angle B :—

Here the circular parts concerned are, the leg A C, the given angle A, and the required angle B; and since the angle B is disjoined from the other two parts by the hypothenuse A B, it is the *middle part*, and the other two are the *extremes disjunct*, by rule 2, page 183; therefore, by equation 2, page 183,

Radius × co-sine angle B = sine of angle A × co-sine leg A C.

Now, since radius is connected with the required term, it is to stand first in the proportion. Hence,

As radius = . .	90° 0′ 0″	Log. sine ar. comp. =	10.000000
Is to the angle A =	74.45.15	Log. sine = . . .	9.984440
So is the leg A C =	68.29.45	Log. co-sine = . .	9.564156
To the angle B =	69°17′17″	Log. co-sine = . .	9.548596

Note.—The angle B is acute, or of the same affection with its opposite given leg A C.

To find the Leg B C :—

In this case, since the *right angle never separates the legs*, the three circular parts are joined together: hence the leg A C is the *middle part*, and the leg B C and the angle A are the *extremes conjunct*, according to rule 1, page 183; therefore, by equation 1, page 183,

Radius × sine of leg A C = co-tangent angle A × tangent of leg B C.

And since the angle A is connected with the required part, it is to be the first term in the proportion. Hence,

As the angle A =	74°45′15″	Log. co-tang. ar. comp. =	10.564549
Is to radius = . .	90. 0. 0	Log. sine = ...	10.000000
So is the leg A C =	68.29.45	Log. sine = ...	9.968666
To the leg B C =	73°40′20½″	Log. tangent = ...	10.533215

Note.—The leg B C is acute, or of the same affection with its opposite given angle A.

To find the Hypothenuse A B :—

In this case, since the three circular parts which enter the proportion are joined together, the given angle A is the *middle part*, and the leg A C and the hypothenuse A B are the *extremes conjunct*: therefore,

Radius × co-sine of angle A = tangent of leg A C × co-tangent hypothenuse A B.

Now, the leg A C, being connected with the required part, is therefore to be the first term in the proportion. Hence,

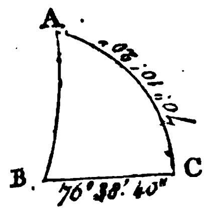
As the leg A C = . . .	68°29′45″	Log. tang. ar. comp. =	9.595490
Is to radius = ...	90. 0. 0	Log. sine = . . .	10.000000
So is the angle A = .	74.45.15	Log. co-sine = . . .	9.419891
To the hypothenuse A C =	84° 5′ 6″	Log. co-tangent =	9.015381

Note.—The hypothenuse is acute, because the given leg and angle are of the same affection.

Problem V.

Given the two Legs, to find the Angles and the Hypothenuse.

Example.



Let the leg A C, of the spherical triangle A B C, be 70°10′20″, and the leg B C 76°38′40″; required the angles A and B, and the hypothénuse A B?

To find the Angle A :—

Here, since the right angle *never separates the legs,* the leg A C is the *middle part,* and the leg B C and the required angle A are the *extremes conjunct,* agreeably to rule 1, page 183; therefore, by equation 1, page 183,

Radius × sine leg A C = tangent leg B C × co-tangent angle A.

Now, since the leg B C is connected with the required part, it is to be the first term in the proportion. Hence,

As the leg B C =	76°38′40″	Log. tangent ar. comp. =	9.375506
Is to radius = .	90. 0. 0	Log. sine = ...	10.000000
So is the leg A C =	70. 10. 20	Log. sine = ...	9.973459
To the angle A =	77°24′37″	Log. co-tangent = . .	9.348965

Note.—The angle A is acute, or of the same affection with its opposite given leg B C.

To find the Angle B :—

In this case the leg B C is *the middle part,* and the leg A C and the required angle B are the *extremes conjunct*, according to rule 1, page 183; therefore, by equation 1, page 183,

Radius × sine of the leg B C = tangent of leg A C × co-tangent angle B.

And since the leg A C is connected with the required part, it is to be the first term in the proportion. Hence,

As the leg A C =	70°10′20″	Log. tangent ar. comp. =	9.556990
Is to radius = . .	90. 0. 0	Log. sine = ...	10.000000
So is the leg B C =	76.38.40	Log. sine = ...	9.988093
To the angle B =	70°40′ 5½″	Log. co-tangent = . .	9.545083

Note.—The angle B is acute, or of the same affection with its opposite given leg A C.

To find the Hypothenuse A B:—

Here the hypothenuse A B is the *middle part*, because it is disjoined from the legs by the angles A and B: hence A C and B C are *extremes disjunct*, agreeably to rule 2, page 183; therefore, by equation 2, page 183,

Radius × co-sine hypothenuse A B = co-sine leg A C × co-sine leg B C.

And radius, being connected with the middle part, is therefore to be the first term in the proportion. Hence,

As radius = . . .	90° 0′ 0″	Log. sine ar. comp. = . .	10.000000
Is to the leg A C =	70. 10. 20	Log. co-sine = . . .	9.530448
So is the leg B C =	76. 38. 40	Log. co-sine = . . .	9.363599
To the hyp. A B =	85°30′22″	Log. co-sine = . . .	8.894047

Note.—The hypothenuse A B is acute, because the given legs A C and B C are of the same affection.

Problem VI.

Given the two Angles, to find the Hypothenuse and the two Legs.

Example.

Let the angle A, of the spherical triangle A B C, be 50°10′20″, and the angle B 64°20′25″; required the legs A C and B C, and the hypothenuse A B?

To find the Hypothenuse AB :—

Here, because the three circular parts are joined together, the hypothenuse AB is the *middle part*, and the angles A and B are the *extremes conjunct*, agreeably to rule 1, page 183; therefore, by equation 1, page 183,

Radius × co-sine hypothenuse AB = co-tangent angle A × co-tangent angle B.

Now, since radius is connected with the required part, it is to be the first term in the proportion. Hence,

As radius = . . .	90° 0′ 0″	Log. sine ar. comp. =	10.000000
Is to the angle A =	50. 10. 20	Log. co-tangent = .	9.921161
So is the angle B =	64. 20. 25	Log. co-tangent = .	9.681605
To the hyp. AB =	66°22′52″	Log. co-sine = . .	9.602766

Note.—The hypothenuse AB is acute, because the given angles A and C are of the same affection.

To find the leg AC :—

Here, since the angle B is disjoined by the hypothenuse AB from the other two circular parts concerned, it is the *middle part*, and the angle A and the required leg AC are the *extremes disjunct*, agreeably to rule 2, page 183; therefore, by equation 2, page 183,

Radius × co-sine angle B = sine of angle A × co-sine of leg AC.

And because the angle A is connected with the required part, it is to stand first in the proportion. Hence,

As the angle A =	50°10′20″	Log. sine ar. comp. = .	10.114654
Is to radius = .	90. 0. 0	Log. sine = ...	10.000000
So is the angle B =	64. 20. 25	Log. co-sine	9.636514
To the leg AC =	55°40′38″	Log. co-sine = . . .	9.751168

Note.—The leg AC is acute, or of the same affection with its opposite given angle B.

To find the Leg BC :—

In this case the angle A is the *middle part*, because it is disjoined from the other two circular parts by the hypothenuse AB: hence the angle B and the required leg BC are *extremes disjunct*; therefore,

Radius × co-sine of angle A = sine of angle B × co-sine of leg BC.

And as the angle B is connected with the required part, it is to be the first term in the proportion. Hence,

As the angle B =	64°20′45″	Log. sine ar. comp. = .	10.045091
Is to radius = .	90. 0. 0	Log. sine = ...	10.000000
So is the angle A =	50. 10. 20	Log. co-sine	9.806507
To the leg B C =	44°43′11½″	Log. co-sine = ...	9.851598

Note.—The leg B C is acute, or of the same affection with its opposite given angle A.

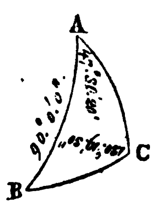
SOLUTION OF QUADRANTAL SPHERICAL TRIANGLES, BY LOGARITHMS.

Problem I.

Given a Quadrantal Side, its opposite Angle, and an adjacent Angle, to find the remaining Angle and the other two Sides.

Remark.—Since the sides of a spherical triangle may be turned into angles, and, *vice versa*, the angles into sides, all the cases of quadrantal spherical triangles may be resolved agreeably to the principles of right-angled spherical triangles; as thus: let the quadrantal side be esteemed the radius; the *supplement of the angle* subtending that side, the hypothenuse; and the other angles legs, or the legs angles, as the case may be. Then the middle part, and the extremes conjunct or disjunct, being established, the required parts are to be computed, and the affections of the angles and sides determined, in the same manner precisely as if it were a right-angled spherical triangle that was under consideration.

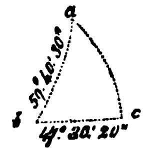
Example.



Let A B, in the spherical triangle A B C, be the quadrantal side = 90°, the angle C 120°19′30″, and the angle A 47°30′20″; required the sides A C and B C, and the angle B?

Solution.—Let the supplement of the angle C (59°40′30″), subtending the quadrantal side A B, represent the hypothenuse *a b* of the *dotted* spherical triangle *a b c*. Let the given angle A 47°30′20″ represent the leg *b c* of the said dotted triangle, and the required angle B the leg *a c*.

Then, in the right-angled spherical triangle *a b c*, given the hypothenuse *a b* 59°40′30″, and the leg *b c* 47°30′20″, to find the leg *a c* = the angle B in the quadrantal triangle; the angle *a* = the leg B C, and the angle *b* = the leg A C, of the said quadrantal triangle.



To find the Leg *a c* = the Angle B in the Quadrantal Triangle :—

Here the hypothenuse *a b* is the *middle part*, and the legs *b c* and *a c* are the *extremes disjunct*; therefore,

Radius × co-sine hyp. *a b* = co-sine leg *b c* × co-sine leg *a c*.

Now, since the leg *b c* is connected with the required part, it is to be the first term in the proportion. Hence,

As the leg *b c* =	47°30′20″	Log. co-sine ar. comp. = .	10.170363
Is to radius = .	90. 0. 0	Log. sine = ...	10.000000
So is the hyp. *a b* =	59.40.30	Log. co-sine = ...	9.703209
To the leg *a c* =	41°37′54″	Log. co-sine = ...	9.873572

Note.—The leg *a c* is acute, because the hypothenuse and the given leg are of the same affection: hence the angle B (in the quadrantal triangle), represented by the leg *a c*, is also acute = 41°37′54″.

To find the Angle *a* = the Leg B C in the Quadrantal Triangle :—

Here the leg *b c* is the *middle part*, and the hypothenuse *a b* and angle *a* are the *extremes disjunct*; therefore,

Radius × sine of leg *b c* = sine of hypothenuse *a b* × sine of the angle *a*.

And since the hypothenuse is connected with the required part, it is to be the first term in the proportion. Hence,

As the hyp. *a c* = .	59°40′30″	Log. sine ar. comp. =	10.063901
Is to radius = . .	90. 0. 0	Log. sine = . . .	10.000000
So is the leg *b c* = .	47.30.20	Log. sine = . . .	9.867670
To the angle *a* = .	58°40′26″	Log. sine = . . .	9.931571

Note.—The angle *a* is acute, because the hypothenuse and the given leg are of the same affection: hence the leg B C (of the quadrantal triangle), represented by the angle *a*, is also acute = 58°40′26″.

To find the angle b = the Leg A C in the Quadrantal Triangle:—

In this case the angle b is the *middle part*, and the hypothenuse $a\,b$ and the leg $b\,c$ are the *extremes conjunct*; therefore,

Radius × co-sine of the angle b = co-tangent hypothenuse $a\,b$ × tangent of leg $b\,c$.

And radius, being connected with the required part, is, therefore, to stand first in the proportion. Hence,

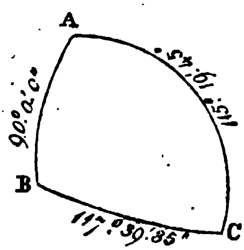
As radius = ...	90° 0′ 0″	Log. sine ar. comp. =	10.000000
Is to the hyp. $a\,b$ = .	59.40.30	Log. co-tangent =	9.767110
So is the leg $b\,c$ = . .	47.30.20	Log. tangent = .	10.038032
To the angle b = . .	50°19′19″	Log. co-sine = . .	9.805142

Note.—The angle b is acute, because the hypothenuse and the given leg are of the same affection. Hence, the leg A C (of the quadrantal triangle), represented by the angle b, is also acute = 50°19′19″.

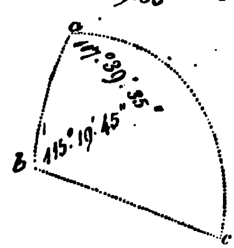
Problem II.

Given the Quadrantal Side and the other two Sides, to find the three Angles.

Example.



Let AB, in the spherical triangle ABC, be the quadrantal side = 90°; the side AC, 115°19′45″; and the side BC, 117°39′35″: required the angles A, B, and C?



Solution.—Let the angle c, in the *dotted* spherical triangle $a\,b\,c$, be radius, and represent the side AB = 90° of the quadrantal triangle ABC. Let the angle a, of the dotted triangle, represent the side BC of the quadrantal triangle = 117°39′45″, and let the angle b represent the side AC of the said quadrantal triangle = 115°19′45″. Then, in the right-angled spherical triangle $a\,b\,c$, right-angled at c, given the angle a = 117°39′35″, and the angle b = 115°19′45″, to find the hypothenuse $a\,b$, the leg $a\,c$, and the leg $b\,c$; the first of which represents the supplement of the angle

C opposite to the quadrantal side AB, in the triangle ABC; the second represents the angle B; and the third the angle A, in the said quadrantal triangle.

To find the Hypothenuse $a\,b$ = the Supplement of the Angle C, subtending the Quadrantal Side AB:—

Here the hypothenuse $a\,b$ is the *middle part*, and the given angles a and b are the *extremes conjunct*; therefore,

Radius × co-sine hypothenuse $a\,b$ = co-tangent of angle a × co-tangent of angle b.—Now, since radius is connected with the required part, it is to be the first term in the proportion.—Hence,

As radius = . . .	90° 0′ 0″	Log. sine ar. compt. =	10.000000
Is to the angle a =	117.39.35	Log. co-tangent =	9.719427
So is the angle b =	115.19.45	Log. co-tangent =	9.675156
To the hypo. $a\,b$ =	75°38′11″	Log. co-sine = . . .	9.394583

Note.—The hypothenuse $a\,b$ is acute because the given angles are of the same affection:—but since it only represents the supplement of the angle C; therefore the angle C is obtuse, or 104°21′49″.

To find the Leg $a\,c$ = the Angle B in the Quadrantal Triangle.

The angle b, in this case, is the *middle part*, and the angle a and leg $a\,c$ *extremes disjunct*.—Therefore, radius × co-sine of angle b = sine of angle a × co-sine of leg $a\,c$.

And the angle a being connected with the required part, is, therefore, to be the first term in the proportion.—Hence,

As the angle a =	117°19′35″	Log. sine ar. compt. =	10.052703
Is to radius = . .	90. 0. 0	Log. sine = . . .	10.000000
So is the angle b =	115.19.45	Log. co-sine = . .	9.631259
To the side $a\,c$ =	118°52′57″	Log. co-sine = . .	9.683962

Note.—The side $a\,c$ is obtuse, or of the same affection with its opposite angle b:—and since $a\,c$ represents the angle B; therefore the angle B, in the quadrantal triangle, is obtuse, or 118°52′57″.

To find the Leg $b\,c$ = the Angle A in the Quadrantal Triangle.

In this case the angle a is the *middle part*; and the angle b and leg $b\,c$ *extremes disjunct*.—Therefore, radius × co-sine of the angle a = sine of the angle b × co-sine of the leg $b\,c$.

And since the angle b is connected with the required part, it is to be the first term in the proportion.—Hence,

As the angle b =	115°19′45″	Log. sine ar. compt. =	10.043896
Is to radius = . .	90. 0. 0	Log. sine = ...	10.000000
So is the angle a =	117.39.35	Log. co-sine = . . .	9.666723
To the leg $b\,c$ =	120°54′12″	Log. co-sine = . . .	9.710619

Note.—The leg $b\,c$ is obtuse, or of the same affection with its opposite angle a:—and since the leg $b\,c$ represents the angle A, in the quadrantal triangle; therefore the angle A is obtuse, or 120°54′12″.

Remark.—From the ample solutions of the two preceding Problems, it must appear obvious, that all the cases of quadrantal spherical triangles may be easily resolved by the equations for right-angled spherical triangles. And if the analogies of those two Problems be well understood, all the *apparent* difficulty attending the trigonometrical solution of quadrantal triangles will entirely vanish.

SOLUTION OF OBLIQUE-ANGLED SPHERICAL TRIANGLES BY LOGARITHMS.

The most natural, and, perhaps, the easiest method of solving the *four first Problems*, or cases of oblique-angled spherical triangles, is by means of a perpendicular let fall from an angle to its opposite side, continued if necessary; and thus reducing the oblique into two right-angled spherical triangles.—The perpendicular, however, should be let fall in such a manner that *two of the given parts* in the oblique triangle may remain known in one of the right-angled triangles:—Then, the other parts may be readily computed by means of Lord Napier's analogies, as given in the equations 1 and 2, page 183.—But, since the solution of oblique-angled spherical triangles without a perpendicular is possessed of many advantages in astronomical calculations; and, besides, since the author's object is to establish the use of the Tables contained in this work by a variety of rules and formulæ which, it is hoped, may not be found quite uninstructive to persons but slightly informed on trigonometrical subjects; the different cases of oblique triangles will, therefore, be resolved independently of a perpendicular, agreeably to the propositions generally used in such cases.

Problem I.

Given Two Sides of an Oblique-angled Spherical Triangle, and an Angle opposite to one of them; to find the remaining Angles and the Third Side.

Rule.

1.—*To find an angle opposite to one of the given sides.*

As the log. sine of the side opposite to the given angle, is to the log. sine of the given angle; so is the log. sine of the other given side, to the log. sine of its opposite angle.

Now, to know whether the angle thus found is determinate; that is, whether it is ambiguous, acute, or obtuse, proceed in the following manner, viz.—To the angle so found, and its supplement, add the given angle, or that used in the proportion.—Then, if each of these sums be of the *same affection* with respect to 180° as the *sum of the two given sides*, or those used in the proportion, the angle is *ambiguous;* that is, it may be either acute or obtuse; and, therefore, indeterminate.—But, if those sums are of *different affections with respect to the sum of the sides*, the angle is determinate, and, therefore, *not* ambiguous:—In this case that value of the angle is to be taken, whether acute or obtuse, which, when added to the given angle, produces a quantity of the same affection with the sum of the two sides.

2.—*To find the angle contained between the two given sides.*

Find half the difference, and half the sum of the two given sides:—find, also, half the difference of their opposite angles. Then say,

As the log. sine of half the difference of the sides, is to the log. sine of half their sum; so is the log. tangent of half the difference of their opposite angles, to the log. co-tangent of half the angle contained between the two given sides; the double of which will be the angle sought.

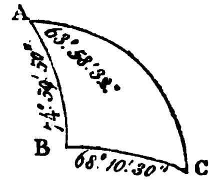
3.—*To find the third side.*

Since the sides are proportional to the sines of their opposite angles; therefore the third side may be found by the converse of the first part of the rule; as thus:

As the log. sine of a given angle opposite to a given side, is to the log. sine of that side; so is the log. sine of the given angle opposite to the required side, to the log. sine of the required side.

Note.—When the angle comes out ambiguous, or indeterminate, in the first proportion; the contained angle and the third side, found by the other proportions, will also be ambiguous.

Example.



In the oblique-angled spherical triangle A B C, let the side A B be 74°59'50", the side B C 68°10'30" and the angle A 63°58'32"; required the angles B and C, and the side A C?

To find the Angle C:—

As the side B C =	68°10'30"	Log. sine ar. compt. =	10.032301
Is to the angle A =	63. 58. 32	Log. sine . . . =	9.953570
So is the side AB =	74. 59. 50	Log. sine . . . =	9.984938
To the angle C =	69°13'37"	Log. sine . . . =	9.970809

To determine whether the Angle C is Ambiguous, Acute, or Obtuse:—

Angle C =	69°13'37"	Sup. =	110°46'23"	Side B C =	68°10'30"
Angle A =	63. 58. 32	Angle A =	63. 58. 32	Side A B =	74. 59. 50
Sum =	133°12' 9"	Sum =	174°44'55"	Sum =	143°10'20"

Here, since the three sums are of the same affection with respect to 180° the angle C is ambiguous; therefore it may be either 69°13'37" or the supplement thereof; viz., 110°46'23".

To find the Angle B:—

As the side AB—the side BC÷2=3°24'40"	Log. S. ar. compt.	11.225483
Is to the S. AB+the S. BC÷2=71. 35. 10	Log. sine = . .	9.977174
So is the ang. C—the ang. A÷2=2. 37. 32½	Log. tangent =	8.661426
To half the angle B = . . . 53°49'22"	Log. co-tangent	9.864083

Angle B = ... 107. 38. 44; which is ambiguous because the angle C came out indeterminate.

To find the Side A C:—

As the angle A = . .	63°58'32"	Log. sine ar. compt.	10.046430
Is to the side B C = .	68. 10. 30.	Log. sine = ...	9.967699
So is the angle B =	107. 38. 44.	Log. sine = ...	9.979070
To the side A C =	100° 6'47"	Log. sine = ...	9.993199

The side A C is also ambiguous because the angle C came out indeterminate.

Problem II.

Given Two Angles of an Oblique Angled Spherical Triangle, and a Side opposite to one of them; to find the remaining Angle and the other Two Sides.

Rule.

1.—*To find a side opposite to one of the given angles.*

As the log. sine of the angle opposite to the given side, is to the log. sine of the given side: so is the log. sine of the other given angle, to the log. sine of its opposite side.

Now, to know whether the side thus found is ambiguous, acute, or obtuse, proceed as follows; viz.,

To the side so found, and its supplement, add the given side, or that used in the proportion.—Then, if each of these sums be of the *same affection* with respect to 180° as the *sum of the two given angles*, or those used in the proportion, the side is *ambiguous*; that is, it may be either acute, or obtuse; and, therefore, indeterminate.

But, if those sums are of *different affections with respect to the sum of the angles*, the side is *not* ambiguous: in this case that value of the side is to be taken, whether acute or obtuse, which, when added to the given side, produces a quantity of the same affection with the sum of the angles.

2.—*To find the side contained between the two given angles.*

Find half the difference, and half the sum of the two given angles:—find, also, half the difference of their opposite sides.—Then say,

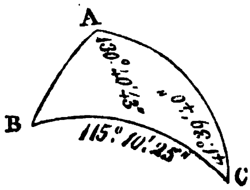
As the log. sine of half the difference of the angles, is to the log. sine of half their sum; so is the log. tangent of half the difference of their opposite sides, to the log. tangent of half the side contained between the two given angles; the double of which will be the side sought.

3.—*To find the third, or remaining angle.*

As the log. sine of a given side opposite to a given angle, is to the log. sine of that angle; so is the log. sine of the side opposite to the required angle, to the log. sine of the required angle.

Note.—When the side comes out ambiguous, or indeterminate, in the first proportion; the contained side and the third angle, found by the other proportions, will also be ambiguous.

Example.



Let the angle A, of the spherical triangle A B C, be 130°40'43", the angle C 41°39'40", and the side B C 115°10'25"; required the angle B, and the sides A B and A C?

To find the Side A B :—

As the angle A =	130°40'43"	Log. sine ar. compt. =	10.120114
Is to the side B C =	115.10.25	Log. sine = ...	9.956660
So is the angle C =	41.39.40	Log. sine = ...	9.822641
To the side A B =	52°29'28"	Log. sine = ...	9.899415

To determine whether the Side A B is Ambiguous, Acute, or Obtuse :—

Side A B	52°29'28"	Supplement =	127°30'32"	Angle A	130°40'43"
Side B C	115.10.25	Side B C =	115.10.25	Angle C	41.39.40
Sum =	167°39'53"	Sum = . .	242°50'57"	Sum =	172°20'23"

Here, since the two first sums, viz. A B and B C, and the supplement of A B and B C, are of different affections with respect to 180°, the side A B is not ambiguous;—and since the sum of the acute value of A B added to B C is of the same affection with the sum of the angles; therefore the side A B is acute = 52°29'28".

To find the Side A C :—

As the ang. A—the ang. C÷2=	44°30'31½"	Log. S. ar. compt.	10.154271
Is to angle A + angle C ÷ 2=	86.10.11½	Log. sine = . .	9.999029
So is the S. B C—S. A B÷2=	31.10.28½	Log. tangent = .	9.784614
To half the side A C = . .	40°55'6"	Log. tangent . .	9.937914

Side A C = ... 81°50'12"; which is acute, because the side A B came out determinate, and that its acute value applied to B C is of the same affection with the sum of the angles.

To find the Angle B :—

As the side B C = . . .	115°10'25"	Log. S. ar. compt. . .	10.043340
Is to the angle A = .	130.40.43	Log. sine = ...	9.879886
So is the side A C = .	81.50.12	Log. sine = ...	9.995577
To the angle B = . .	56°2'41½"	Log. sine = ...	9.918803

Note.—The angle B is acute like its opposite side A C, because the side A B is not ambiguous; and that its acute value applied to the side B C is of the same affection with the sum of the angles.

Problem III.

Given Two Sides of an Oblique-angled Spherical Triangle, and the Angle contained between them; to find the other Two Angles and the Third Side.

Rule.

1.—*To find the other two angles.*

As the log. co-sine of half the sum of the two given sides, is to the log. co-sine of half their difference; so is the log. co-tangent of half the contained angle, to the log. tangent of half the sum of the other two angles.

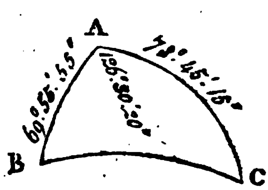
Half the sum of the angles thus found, will be of the same affection with half the sum of the sides.—Again: As the log. sine of half the sum of the two given sides, is to the log. sine of half their difference; so is the log. co-tangent of half the contained angle, to the log. tangent of half the difference of the other two angles.—Half the difference of the angles, thus found, will always be acute.

Now, half the sum of the two angles, added to half their difference, will give the greater angle; and half the difference of the angles subtracted from half their sum will leave the lesser angle.

2.—*To find the third side.*

The angles being known, the third or remaining side is to be computed by Rule 3, Problem I., page 198.

Example.



Let the side A C, of the spherical triangle A B C, be 78°45′15″, the side A B 69°55′55″, and the contained angle 126°30′20″; required the angles B and C, and the side B C?

To find the Angle B :—

As the side AC + AB ÷ 2 =	74° 20′ 55″	Log. co-sine ar. comp. =	10. 568984
Is to the side AC − AB ÷ 2 =	4. 24. 40	Log. co-sine = . . .	9. 998712
So is the angle A ÷ 2 =	63. 15. 10	Log. co-tang. = . . .	9. 702414
To ½ the sum of the an. =	61° 46′ 8″	Log. tangent = . . .	10. 270110
Half diff. of the angles =	2. 18. 19, as below		
Sum = . . .	64° 4′ 27″ = Angle B.		

To find the Angle C :—

As the side AC + AB ÷ 2 =	74° 20′ 55″	Log. sine ar. compt. =	10. 016409
Is to the side AC − AB ÷ 2 =	4. 24. 40	Log. sine = . . .	8. 885996
So is the angle A ÷ 2 =	63. 15. 10	Log. co-tangent = .	9. 702414
To half the diff. of the ang. =	2° 18′ 19″	Log. tangent = . . .	8. 604819
Half sum of the angles =	61. 46. 8, as above		
Difference = . . .	59° 27′ 49″ = Angle C.		

Note.—The half sum of the angles came out acute, because the half sum of the sides is acute ; the half difference of the angles *is always acute.*

To find the Side B C :—

As the angle B =	64° 4′ 27″	Log. sine ar. compt. =	10. 046066
Is to the side A C =	78. 45. 15	Log. sine = ...	9. 991580
So is the angle A =	126. 30. 20	Log. sine = ...	9. 905148
To the side B C =	118° 46′ 1″	Log. sine = ...	9. 942794

Remark 1.—The side B C may be found directly, independently of the angles B and C, by the following general Rule.

To twice the log. sine of half the contained angle, add the log. sines of the two containing sides ; from half the sum of these three logs. subtract the log. sine of half the difference of the sides, and the remainder will be the log. tangent of an arch : the log. sine of which being subtracted from the half sum of the three logs. will leave the log. sine of half the required side.

Example.

Let the side A C, of a spherical triangle, be 62° 10′ 25″, the side A B 50° 14′ 45″, and the included angle A 123° 11′ 40″ ; required the side B C?

Half ang. A =	61°35′50″	{Twice the Log. sine} =	19.888596
Side A C =	62.10.25	Log. sine =	9.946632
Side A B =	50.14.45	Log. sine =	9.885811
		Sum =	39.721039
Diff. of Sides	11°55′40″	Half =	19.860519½ . . . 19.860519½
Half ditto =	5°57′50″	Log. sine =	9.016622
Arch = . .	81°50′52″	Log. tang. =	10.843897½ Log. S. = 9.995588½
½ Side B C =	47° 6′50″	= Log. sine =	. . . . 9.864931
Side B C =	94°13′40″, as required.		

Remark 2.—The side B C may be also computed by the following general rule, viz.

To twice the log. sine of half the contained angle, add the log. sines of the two containing sides, and the sum (rejecting 30 from the index,) will be the log. of a natural number.—Now, the sum of twice this natural number and the natural versed sine of the difference of the containing sides, will be the natural versed sine of the third side.

Thus, to find the side B C in the above example.

Half included ang. A =	61°35′50″	twice the log. sine = . .	19.888596
Side A C = . . .	62.10.25	Log. sine = ...	9.946632
Side A B = . . .	50.14.45	Log. sine = ...	9.885811
Natural number = ...	526065	= Log.	9.721039
Twice the nat. numb. = ...	1052130		
Diff. of the given sides 11°55′40″ N.V.S. =	021591		
Side B C = . . . 94°13′40″ N. V. S.	1073721	; the same as above.	

Note.—In taking out the natural number corresponding to the sum of the three logs.: if the index be 9, the natural number is to be taken out to six places of figures; if 8, to five places of figures; if 7, to four places of figures, &c.

PROBLEM IV.

Given Two Angles of a Spherical Triangle, and the Side comprehended between them; to find the remaining Angle and the other Two Sides.

RULE.

1.—*To find the other two sides.*

As the log. co-sine of half the sum of the two given angles, is to the log. co-sine of half their difference; so is the log. tangent of half the comprehended side, to the log. tangent of half the sum of the other two sides.

Half the sum of the sides, thus found, will be of the same affection with the half sum of the angles.

Again.—As the log. sine of half the sum of the two given angles, is to the log. sine of half their difference; so is the log. tangent of half the comprehended side, to the log. tangent of half the difference of the other two sides.

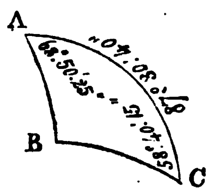
Half the difference of the angles, thus found, will always be acute.

Now, half the sum of the two sides, added to half their difference, will give the greater side; and half the difference of the two sides, subtracted from half their sum, will leave the lesser side.

2.—*To find the remaining angle.*

The sides and two angles being known, the remaining or third angle is to be computed by Rule 3, Problem II., page 200.

Example.



Let the angle A, of the spherical triangle A B C, be 63°50′25″; the angle C 58°40′15″, and the comprehended side A C 87°30′40″; required the sides A B and B C, and the remaining angle B?

To find the Side B C:—

As the angle A+angleC÷2=	61°15′20″	L. co-sine ar. com. =	10.317942
Is to the ang. A−ang.C÷2=	2.35. 5	Log co-sine = . .	9.999557
So is the side A C ÷ 2 =	43.45.20	Log. tangent = . .	9.981129
To half the sum of the sides=	63°18′28″	Log. tangent = .	10.298628
Half difference of the sides =	2.49. 10,	as in the next operation.	
Sum =	66° 7′38″	= the side B C.	

To find the Side A B :—

As the angle A+angleC÷2=61°15′20″		L. sine ar. compt. =	10.057113
Is to angle A − angle C÷2=	2.35. 5	Log. sine = . . .	8.654144
So is the side A C ÷ 2 =	43.45.20	Log tangent . .	9.981129
To half the diff. of the sides =2°49′10″		Log tangent = .	8.692386

Half sum of the sides = . 63.18.28, as in the last operation.

Difference = 60°29′18″ = the side A B.

Note.—The half sum of the sides came out acute because the half sum of the angles is acute; the half difference of the sides must be *always* acute.

To find the Angle B :—

As the Side B C =	66° 7′38″	Log. sine ar. compt. =	10.038842
Is to the angle A =	63.50.25	Log. sine = ...	9.953068
So is the side A C =	87.30.40	Log. sine = ...	9.999590
To the angle B = .	78°42′ 3″	Log. sine = ...	9.991500

Remark 1.—The angle B may be found directly by the following general rule.

To twice the log. co-sine of half the given side, comprehended between the two given angles, add the log. sines of those angles : from half the sum of these three logs. subtract the log. sine of half the difference of the angles, and the remainder will be the log. tangent of an arch.—Now, the log. sine of this arch being subtracted from the half sum of the three logs. will leave the log. sine of half the required angle.

Thus, to find the angle B in the above example.

Half side A C =	43°45′20″	{Twice the Log. co-sine . .} =	19.717432	
Angle A =	63.50.25	Log sine =	9.953068	
Angle C =	58.40.15	Log. sine =	9.931557	
		Sum =	39.602057	
Diff. of the ang.=5°10′10″		Half =	19.801028½ . .	19.801028½
Half diff. of do.=2.35. 5		Log. sine =	8.654144	
Arch = 85°55′17″ .		Log. tangent=	11.146884½	Lg.S.=9.998899
Half the required angle =	39°21′1″	Log. sine = . . .		9.802129½

Hence, the angle B is = 78°42′2″; which differs 1″ from the angle found as above.

Remark 2.—The angle B may be also very readily computed by the following general Rule; viz.,

To twice the log. co-sine of half the given side, comprehended between the two given angles, add the log. sines of those angles, and the sum (rejecting 30 from the index), will be the log. of a natural number.—Now, the sum of twice this natural number and the natural versed sine of the difference of the angles, will be the natural versed sine of the required angle.

Thus, to find the angle B in the last example.

Half the given side A C =	43°45′20″	twice the log. co-sine =	19.717432
Angle A = ...	63.50.25	Log. sine = ...	9.953068
Angle C = ...	58.40.15	Log. sine = ...	9.931557
	Natural number = ...	399998 = Log.	9.602057
	Twice the natural number = .	799996	
Diff. of the ang. = 5°10′10″	nat. versed sine =	004067	

Angle B = 78°42′ 2″ nat. versed sine = 804063; the same as by the former Rule.

Problem V.

Given the Three Sides of a Spherical Triangle, to find the Angles.

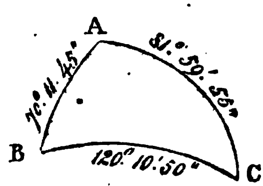
Rule.

Add the three sides together and take half their sum; find the difference between this half sum and the side opposite to the required angle, which call the remainder; then,

To the log. co-secants, less radius, of the other two sides, add the log. sines of the half sum and the remainder:—half the sum of these four logs. will be the log. co-sine of an arch, which being doubled will be the required angle.

One angle being thus found, the remaining angles may be computed by Rule 3, Problem II., page 200.

Example.



In the spherical triangle A B C, let the side A B be 70°11′45″, the side A C 81°59′55″, and the side B C 120°10′50″; required the angles A, B, and C?

To find the Angle A:—

Side B C = . . .	120°10′50″		
Side A C = . . .	81.59.55	Log. co-secant, less radius=	0.004248
Side A B = . . .	70.11.45	Log. co-secant, less radius=	0.026477
Sum . .	272.22.30		
Half sum =	136.11.15	Log. sine = ...	9.840295
Remainder=	16. 0.25	Log. sine = ...	9.440522
		Sum = ...	19.311542
Arch = . .	63° 5′ 8″	= Log. co-sine = . .	9.655771
Angle A =	126°10′16″		

To find the Angle B:—

As the side B C = ...	120°10′50″	Log. co-secant =	10.063262
Is to the angle A = . .	126.10.16	Log. sine = . .	9.907012
So is the side A C = . .	81.59.55	Log. sine = . .	9.995752
To the angle B = ...	67°37′52″	Log. sine = . . .	9.966026

To find the Angle C.—

As the side B C = ...	120°10′50″	Log. co-secant =	10.063262
Is to the angle A = . .	126.10.16	Log. sine = . .	9.907012
So is the side A B = . .	70.11.45	Log. sine = . . .	9.973523
To the angle C = ...	61°28′31″	Log. sine = . .	9.943797

Remark.—The required angle of a spherical triangle (when the three sides are given), may be also found by the following general Rule; viz.,

Add the three sides together and take half their sum: find the difference between this half sum and each of the sides containing the required angle, and note the remainders.—Then,

To the log. co-secants, less radius, of those sides, add the log. sines of the two remainders:—half the sum of these four logs. will be the log. sine of half the required angle.

Thus, to find the angle A in the last example.

Side B C = . . .	120°10′50″		
Side A C = . .	81.59.55	Log. co-secant, less radius=	0.004248.
Side A B = . .	70.11.45	Log. co-secant, less radius=	0.026477
Sum = ...	272.22.30		
Half sum . .	136°11′15″		
Remainder, by A C =	54.11.20	Log. sine = ...	9.908994
Remainder, by A B =	65.59.30	Log. sine = ...	9.960702
		Sum = . .	19.900421
Half the angle A =	63° 5′ 8″	Log. sine = ...	9.950210½

Which being doubled, shows the angle A to be 126°10′16″; the same as by the former rule.

Problem VI.

Given the Three Angles of a Spherical Triangle, to find the Sides.

Rule.

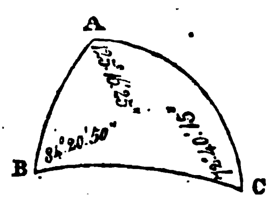
Add the three angles together and take half their sum; find the difference between the half sum and the angle opposite to the required side, which call the remainder.—Then,

To the log. co-secants, less radius, of the other two angles, add the log. co-sines of the half sum, and the remainder; half the sum of these four logs. will be the log. sine of half the required side.

One side being thus found, the remaining sides may be computed by Rule 3. Problem I., page 198.

Example.

In the spherical triangle A B C, let the angle A be 125°16′25″; the angle B 84°20′50″, and the angle C 72°40′15″; required the sides B C, A B, and A C?



To find the side B C :—

Angle A = . . .	125°16′25″		
Angle B = . . .	84. 20. 50	Log. co-secant, less radius =	0.002117
Angle C = . . .	72. 40. 15	Log. co-secant, less radius =	0.020174
Sum = ...	282. 17. 30		
Half sum = . .	141° 8′45″	Log. co-sine = ...	9.891395
Remainder = .	15. 52. 20	Log. co-sine = ...	9.983118
		Sum = . .	19.896804
Half the side B C = . .	62°37′13″	Log. sine = ...	9.948402

The double of which gives 125°14′26″, for the whole side B C.

To find the Side A B :—

As the angle A = . . .	125°16′25″	Log. co-secant = . .	10.088095
Is to the side B C = .	125. 14. 26	Log. sine = ...	9.912083
So is the angle C = .	72. 40. 15	Log. sine = ...	9.979826
To the side A B = . .	72°44′46″	Log. sine = ...	9.980004

To find the Side A C :—

As the angle A =	125°16′25″	Log. co-secant = . . .	10.088095
Is to the side B C =	125. 14. 26	Log. sine = ...	9.912083
So is the angle B =	84. 20. 50	Log. sine = ...	9.997883
To the side A C =	84°35′25″	Log. sine = ...	9.998061

Remark.—The required side of a spherical triangle (when the three angles are given,) may be also found by the following general rule; viz.,

Add the three angles together and take half their sum; find the difference between the half sum and each of the angles comprehending the required side, and note the remainders.—Then to the log. co-secants less radius, of those angles, add the log. co-sines of the two remainders: half the sum of these four logs. will be the log. co-sine of half the required side.

Thus, to find the side B C in the last example.

Angle A = . .	125°16′25″		
Angle B = .	84.20.50	Log. co-secant, less radius =	0.002117
Angle C = .	72.40.15	Log. co-secant, less radius =	0.020174
Sum =	282.17.30		
Half sum = . .	141° 8′45″		
Remainder by B =	56.47.55	Log. co-sine = ...	9.738450
Remainder by C =	68.28.30	Log. co-sine = ...	9.564556
		Sum =	19.325297
Half Side B C = ...	62°37′13″	Log. co-sine = .	9.662648½

Which being doubled gives = 125°14′26″, for the side B C; the same as by the former rule.

THE RESOLUTION OF PROBLEMS IN NAVIGATION BY LOGARITHMS; AND, ALSO, BY THE GENERAL TRAVERSE TABLE.

Lest the mariner should feel some degree of disappointment in not finding a regular course of navigation in this work: the author thinks it right to *remind him*, that his present intention carries him no farther than merely to show the proper application of the Tables to some of the most useful parts of the sciences on which he may touch:—it being completely at variance with the plan of this work, to enter into such parts of the sciences as could reasonably be dispensed with, without entirely losing sight of their principles.—Hence it is, that the cases of plane sailing, usually met with in books on navigation, will not be noticed in this.—However, since it is not improbable that this volume may fall into the hands of persons not very deeply versed in nautical matters; it therefore may not be deemed unnecessary to give a few introductory definitions, &c. for their immediate guidance, previously to entering upon the essentially useful parts of the sailings.

Navigation is the art of conducting a ship, through the wide and pathless ocean, from one part of the world to another.—Or, it is the method of finding the latitude and longitude of a ship's place at sea; and of thence determining her course and distance from that place, to any other given place.

The *Equator* is a great circle circumscribing the earth, every point of which is equally distant from the poles; thus dividing the globe into two equal parts, called hemispheres: that towards the North Pole is called the northern hemisphere, and the other, the southern hemisphere.—The equator, like all other great circles, is divided into 360 equal parts, called degrees; each degree into 60 equal parts, called minutes; each minute into 60 equal parts, called seconds, and so on.

The *Meridian* of any place on the earth is a great circle passing through that place and the poles, and cutting the equator at right angles.—Every point on the surface of the sphere may be conceived to have a meridian line passing through it;—hence there may be as many meridians as there are points in the equator.—Since the *First Meridian* is merely an imaginary circle passing through any remarkable place and the poles of the world; therefore it is entirely arbitrary.—Hence it is that the British reckon their *first meridian* to be that which passes through the Royal Observatory at Greenwich: the French esteem their *first meridian* to be that which passes through the Royal Observatory at Paris; the Spaniards that which passes through Cadiz, &c. &c. &c.

Every meridian line may be said, with respect to the place through which it passes, to divide the surface of the sphere into two equal parts, called the eastern and western hemispheres.

The *Latitude* of any place on the earth is that portion of its meridian which is intercepted between the equator and the given place; and is named north or south, according as the given place is in the northern or southern hemisphere.—As the latitude begins at the equator, where it is nothing, and is reckoned thence to the poles, where it terminates; therefore the greatest latitude any place can have, is 90 degrees.

The *Difference of Latitude* between two places on the earth is an arc of the meridian intercepted between their corresponding parallels of latitude; showing how far one of them is to the northward or southward of the other:—The difference of latitude between two places can never exceed 180 degrees.

The *Longitude* of any place on the earth is that arc or portion of the equator which is contained between the *first meridian* and the meridian of the given place; and is denominated east, or west, according as it may be situated with respect to the *first meridian*.—As the longitude is reckoned both ways from the *first meridian* (east and west) till it meets at the same meridian on the opposite part of the equator; therefore the longitude of any place can never exceed 180 degrees.

The *difference of Longitude* between two places on the earth is an arc of the equator intercepted between the meridians of those places; showing how far one of them is to the eastward or westward of the other:—The difference of longitude between two places can never exceed 180 degrees.

When the latitudes of two places on the earth are both north or both south; or their longitudes both east or both west, they are said to be of the same name.—But, when one latitude is north and the other south; or one longitude east and the other west; then they are said to be of different names.

The *Horizon* is that great circle which is equally distant from the zenith and nadir, and divides the visible from the invisible hemisphere; this is called the rational horizon.—The sensible horizon is that which terminates the view of a spectator in any part of the world.

The *Mariner's Compass* is an artificial representation of the horizon:—it is divided into 32 equal parts, called points; each point consisting of $11°15'$.—Hence the whole compass card contains 360 degrees; for $11°15'$ multiplied by 32 points = 360 degrees.

A *Rhumb Line* is a right line, or rather curve, drawn from the centre of the compass to the horizon, and obtains its name from the point of the horizon it falls in with.—Hence there may be as many rhumb-lines as there are points in the horizon.

The *Course* steered by a ship is the angle contained between the meridian of the place sailed from, and the rhumb-line on which she sails; and is either estimated in points or degrees.

The *Distance* is the number of miles intercepted between any two places, reckoned on the rhumb line of the course; or it is the absolute length that a ship has sailed in a given time.

The *Departure* is the distance of the ship from the meridian of the place sailed from, reckoned on the parallel of latitude at which she arrives; and is named east or west, according as the course is in the eastern or western hemisphere.

If a ship's course be due north or south, she sails on a meridian, and therefore makes no departure:—hence the distance sailed will be equal to the difference of latitude.

If a ship's course be due east or west, she sails either on the equator, or on some parallel of latitude; in this case since she makes no difference of latitude, the distance sailed will, therefore, be equal to the departure.

When the course is 4 points, or 45 degrees, the difference of latitude and departure are equal.

When the course is *less than* 4 points, or 45 degrees, the difference of latitude exceeds the departure; but when it is *more* than 4 points, or 45 degrees, the departure exceeds the difference of latitude.

Note.—Since the distance sailed, the difference of latitude, and the departure form the sides of a right angled plane triangle; in which the hypothenuse is represented by the distance; the perpendicular, by the difference of latitude; the base, by the departure; the angle opposite to the base, by the course; and the angle opposite to the perpendicular, by the complement of the course; therefore any two of these five parts being given, the remaining three may be readily found by the analogies for right angled plane trigonometry.

These being premised, we will now proceed to the following *Introductory Problems.*

Problem I.

Given the Latitudes of Two Places on the Earth, to find the difference of Latitude.

Rule.

When the latitudes are of the same name; that is, both north, or both south, their difference will be the difference of latitude; but when one is north and the other south, their sum will express the difference of latitude.

Note.—The same Rule is to be observed in finding the meridional difference of latitude between two places.

Example 1.

Required the difference of latitude between Portsmouth and Cape Trafalgar?

Lat. of Portsmouth =	50°47′	N.
Lat. of C. Trafalgar =	36. 10	N.
Diff. of Lat. = . .	14°37′	
Ditto in Miles = . .	877	

Example 2.

Required the difference of latitude between Portsmouth and James Town, St. Helena?

Lat. of Portsmouth =	50°47′	N.
Lat. of James Town =	15. 55	S.
Diff. of Lat. = . .	46°42′	
Ditto in Miles = . .	2802	

Note.—In finding the difference of latitude, or the difference of longitude between two places (when any of the sailings are under consideration), it will be sufficiently exact to take out the latitudes and longitudes from Table LVIII. to the nearest minute of a degree, as above.

Problem II.

Given the Latitude left and the difference of Latitude, to find the Latitude in.

Rule.

When the latitude left and the difference of latitude are of the same name their sum will be the latitude; but when they are of contrary denominations, their difference will be the latitude required :—This latitude will always be of the same name with the greater quantity.

Example 1.

A ship, from a place in latitude 30°45′ north sailed 497 miles in a northerly direction; required the latitude at which she arrived?

Latitude left = . . 30°45′ N.
Diff. of Lat. = 497 ms. or 8. 17 N.

Lat. arrived at = . 39° 2′ N.

Example 2.

A ship from a place in latitude 2°50′ north, sails 530 miles in a southerly direction; required the latitude come to?

Latitude left = . . . 2°50′ N.
Diff. of lat. = 530 ms, or 8. 50 S.

Lat. come to = . . 6° 0′ S.

Problem III.

Given the Longitudes of Two Places on the Earth, to find the difference of Longitude.

Rule.

When the longitudes are of the same name: that is, both east, or both west, their difference will express the difference of longitude; but when one is east and the other west, their sum will be the difference of longitude. If the sum of the longitudes exceed 180°, subtract it from 360°, and the remainder will be the difference of longitude.

Example 1.

Required the difference of longitude between Portsmouth and Fayal, one of the western islands?

Long. of Portsmouth= 1° 6′ W.
Long. of Fayal, Horta, 28. 43 W.

Diff. of long. = . . 27°37′

Ditto in miles = . . 1657

Example 2.

Required the difference of longitude between Canton and Point Venus, in the island of Otaheite?

Long. of Canton = 113° 3′ E.
Long. of Point Venus = 149. 36 W.

Sum = . . . 262°39′

Diff. of Long. = . . 97°21′

Ditto in miles = . . 5841

Problem IV.

Given the Longitude left and the difference of Longitude, to find the Longitude in.

Rule.

When the longitude left and the difference of longitude are of the same name, their sum will be the longitude in; should that sum exceed 180°, subtract it from 360°; and the remainder will be the longitude in, of a contrary name to the longitude left.—But, when the longitude left and the difference of longitude are of contrary names, their difference will be the longitude in, of the same name with the greater quantity.

Example 1.

A ship from a place in longitude 50°40′ west, sails westward till her difference of longitude is 410 miles; required the longitude in?

Long. left = . . . 50°40′ W.
Diff. of long. = 410 ms. or 6. 50 W.

Longitude in = . . 57°30′ W.

Example 2.

Let the long. left be 174°45′ west, and the difference of longitude 13°17′ west; required the longitude in?

Longitude left = . 174°45′ W.
Diff. of Long. = . 13. 17 W.

Sum = . . . 188. 2

Longitude in = . . 171°58′ E.

Example 3.

Let the longitude left be 41°37′ east, and the difference of longitude 11°20′ west; required the longitude come to?

Longitude left = . .	41°37′ E.
Diff. of long. = . .	11. 20 W.
Longitude in = . . .	30°17′ E.

Example 4.

Let the longitude left be 5°40′ east, and the difference of longitude 10°17′ west; required the longitude in?

Longitude left = .	5°40′ E.
Diff. of long. = . .	10. 17 W.
Long. in = . . .	4°37′ W.

Remarks.—If a ship be in north latitude sailing northerly, or in south latitude sailing southerly, she *increases* her latitude, and therefore the difference of latitude must be *added* to the latitude left, in order to find the latitude in :—but, in north latitude sailing southerly, or in south latitude, northerly, she *decreases* her latitude; therefore the difference of latitude *subtracted* from the latitude left will give the latitude in :—should the difference of latitude be the greatest, the latitude left is to be taken from it; in this case the ship will be on the opposite side of the equator with respect to the latitude sailed from.—Again,

If a ship be in east longitude sailing easterly, or in west longitude sailing westerly, she *increases* her longitude; therefore the difference of longitude *added* to the longitude left will give the longitude in; should the sum exceed 180°, the ship will be on the opposite side of the *first meridian* with respect to the longitude sailed from.—But, in east longitude sailing westerly, or in west longitude sailing easterly, she *decreases* her longitude, and therefore the difference of longitude is to be *subtracted* from the longitude left, in order to find the longitude in;—should the difference of longitude be the greatest, the longitude left is to be taken from it; in this case the ship will, also, be on the opposite side of the *first meridian* with respect to the longitude sailed from. These remarks will appear evident on a comparison with the above Examples.

SOLUTION OF PROBLEMS IN PARALLEL SAILING.

Parallel Sailing is the method of finding the distance between two places situate under the same parallel of latitude; or of finding the difference of longitude corresponding to the meridional distance, when a ship sails due east or west.

Problem I.

Given the Difference of Longitude between two Places, both in the same Parallel of Latitude, to find their Distance.

Rule.

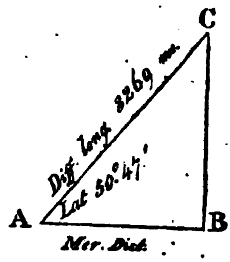
As radius, is to the co-sine of the latitude; so is the difference of longitude, to the distance.

Example.

Required the distance between Portsmouth, in longitude 1°6′ W., and Green Island, Newfoundland, in longitude 55°35′ W.; their common latitude being 50°47′ N.?

Long. of Portsmouth = 1° 6′ W.
Long. of Green Island = 55. 35 W.

Diff. of long. = . 54°29′ = 3269 miles.



Solution.

In the right-angled triangle A B C, where the hypothenuse A C represents the difference of longitude between the two given places, the angle A the latitude of the parallel of those places, and the base A B their meridional distance: given the side A C = 3269 miles, and the angle A = 50°47′, to find the side A B. Hence, by right-angled plane trigonometry, problem I., page 171,

As radius = ...	90° 0′ 0″	Log. co-secant = .	10.000000
Is to the diff. of long. A C =	3269 miles	Log. = . .	3.514415
So is the lat. = the angle A =	50°47′ 0″	Log. co-sine = . .	9.800892
To the merid. dist. A B =	2066.8 miles	Log. = . .	3.315307

To find the Meridional Distance by *Inspection* in the general Traverse Table:—

Note.—This case may be solved by Problem I., page 107, as thus:

To latitude 50° as a course, and one-eleventh of the difference of longitude (viz. 297.2) as a distance, the corresponding difference of latitude is 190.9; and to latitude 51°, and distance 297.2, the difference of latitude is 186.9 : hence the change of meridional distance (represented by difference of latitude,) to 1° or 60′ of latitude, is 4′. Now, 4′ × 47′ ÷ 60′ = 3′.1; this being subtracted from the first difference of latitude, because it is decreasing, gives 187.8; and 187.8 multiplied by 11, the aliquot part, gives 2065.8 for the meridional distance; which comes within one mile of the result by calculation.

Problem II.

Given the Distance between two Places, both in the same Parallel of Latitude, to find the Difference of Longitude between those Places.

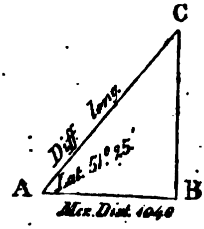
Rule.

As the co-sine of the latitude, is to radius; so is the distance, to the difference of longitude.

Example.

A ship from Cape Clear, in latitude 51°25′ N. and longitude 9°29′ W., sailed due west 1040 miles; required the longitude at which she then arrived?

Solution.—In the right angled triangle A B C, let the hypothenuse A C represent the difference of longitude; the angle A, the latitude of the parallel on which the ship sailed; and the base A B, the meridional distance: then, in this triangle, there are given, the angle A = 51°25′, and the base A B = 1040 miles, to find the side A C. Hence, by right angled plane trigonometry, Problem II., page 172,



As radius = ...	90°. 0′. 0″.	Log. co-secant =	10.000000
Is to the merid. dist. A B =	1040 miles.	Log. = .	3.017033
So is the lat. = the angle A =	51°25′. 0″.	Log. secant = .	10.205057
To the difference of long. A C =	1667.6 miles.	Log. = .	3.222090

Longitude of Cape Clear = ...	9°29′ west.
Difference of longitude 1667.6 miles, or . .	27.48 west.
Longitude at which the ship arrived = . .	37°17′ west.

To find the Difference of Longitude by *Inspection* in the general Traverse Table :—

Note.—This case falls under Problem V., page 111 : hence,

To latitude 51° as a course, and one-eighth of the meridional distance $\doteq$ 130, in a difference of latitude column, the corresponding distance is 207 ; and to latitude 52°, and difference of latitude 130, the distance is 211 : hence, the difference of distance to 1° of latitude, is 4 miles. Now, $4' \times 25' \div 60' = 1'.6$, which being added to the first distance, because it is increasing, gives 208.6 ; this being multiplied by 8 (the aliquot part), gives 1668.8 for the difference of longitude.

PROBLEM III.

Given the Difference of Longitude and the Distance between two Places, in the same Parallel of Latitude, to find the Latitude of that Parallel.

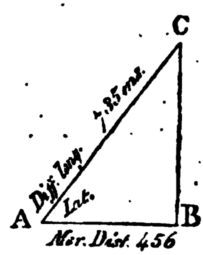
RULE.

As the difference of longitude, is to the distance ; so is radius, to the co-sine of the latitude.

Example.

A ship, from a place in longitude 16°30′ W., sailed due east 456 miles, and then by observation was found to be in the longitude of 4°15′ W. ; required the latitude of the parallel on which she sailed ?

Long. sailed from =	16°30′ W.	
Long. come to =	4.15 W.	
Diff. of long. =	12°15′	= 735 miles.



Solution.—In the right angled triangle A B C, let the hypothenuse A C represent the difference of longitude ; the angle A, the latitude of the parallel ; and the base A B, the meridional distance : then, there are given, the side A C = 735 miles, and the leg A B = 456 miles, to find the angle A. Hence, by right angled plane trigonometry, Problem III., page 174,

As the diff. of longitude AC = 735 miles.	Log. ar. comp. =	7.133713
Is to radius = ... 90° 0′ 0″	Log. sine = .	10.000000
So is the merid. distance AB = 456 miles.	Log. = . . .	2.658965
To lat. of parall. = ang. A=51°39′14″	Log. co-sine =	9.792678

To find the Latitude of the Parallel by *Inspection* in the general Traverse Table:—

Enter the Table with one-third the difference of longitude = 245 as a distance, and one-third the meridional distance = 152, in a difference of latitude column; and the latitude corresponding to them will be found to lie between 51° and 52°. Now, to latitude 51°, and distance 245, the corresponding difference of latitude is 154.2, which exceeds half the meridional distance by 2′.2; and, to latitude 52°, and distance 245, the difference of latitude is 150.8, which is 1′.8 less than half the meridional distance. Hence, 1′.8 + 2′.2 = 4′ is the change of meridional distance to 1° of latitude. And, as 4′ : 2′.2 :: 60′ : 38′; this, being added to 51°, gives 51°38′ for the required latitude.

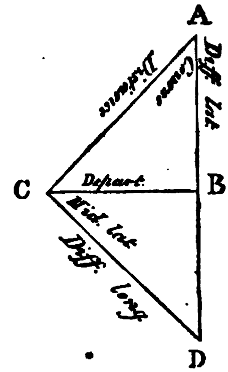
SOLUTION OF PROBLEMS IN MIDDLE LATITUDE SAILING.

Middle Latitude Sailing is the method of solving the several cases, or problems, in Mercator's sailing, by principles compounded of plane and parallel sailing. This method is founded on the supposition that the meridional distance, at that point which is a middle parallel between the latitude left and the latitude bound to, is equal to the departure which the ship makes in sailing from one parallel of latitude to the other.

This method of sailing, though not quite accurate, is, nevertheless, sufficiently so for a *single day's run*, particularly in low latitudes, or when the ship's course is not more than two or three points from a parallel. But, in high latitudes, or places considerably distant from the equator, it fails of the desired accuracy: in such places, therefore, the mariner should never employ it in the determination of a ship's place, when he wishes to draw correct nautical conclusions from his operations.

With the intention of avoiding prolixity and unnecessary repetition, in resolving the different problems in this method of sailing, we will here briefly give a general view of the principles on which the solutions of those problems are founded; as thus:—

In the annexed diagram, let the triangle A B C be a figure in plane sailing, in which A C represents the distance, A B the difference of latitude, B C the departure, and the angle A the course. Again, let D B C be a figure in parallel sailing, in which D C represents the difference of longitude, B C the meridional distance, and the angle C the middle latitude. Hence, the parts concerned form two connected right angled triangles, in which the departure or meridional distance B C is a side common to both.



Now, in one of these triangles, there will be always two terms given, and in the other one term, at least, to find the required terms. The required parts in that triangle which has two terms given, may be readily found by the analogies for right angled plane trigonometry, page 171 to 177; and, hence, the unknown terms in the other triangle.

When the departure B C is not under consideration, the two connected triangles may be considered as one oblique angled triangle, and resolved as such. In this case, if the course, distance, middle latitude, and difference of longitude, are the terms in question, any three of them being given, the fourth may be found by one direct proportion. Thus, in the oblique angled triangle A C D, the side A C is the distance; the angle A, the course; the angle B C D, the middle latitude; and, consequently, the angle D its complement, and the side D C the difference of longitude. Now, if any three of these be known, the fourth may be found by one of the following analogies; viz.,

1. As co-sine middle latitude = C : sine of course = A :: distance = A C : difference of longitude = D C.

2. As sine of course = A : co-sine middle latitude = C :: difference of longitude = D C : distance = A C.

3. As distance = A C : difference of longitude = D C :: co-sine of middle latitude = C : sine of course = A.

4. As difference of longitude = D C : distance = A C :: sine of course = A : co-sine of middle latitude = C.

Again, if the course, middle latitude, difference of latitude, and difference of longitude, be the terms under consideration, the resulting analogies will be,

5. As difference of latitude = A B : difference of longitude = D C :: co-sine of middle latitude = C : tangent of course = A.

6. As difference of longitude = D C : difference of latitude = A B :: tangent of course = A : co-sine of middle latitude = C.

7. As co-sine of middle latitude = C : tangent of course = A :: difference of latitude = A B : difference of longitude = A C.

8. As tangent of course = A : co-sine of middle latitude = C :: difference of longitude = D C : difference of latitude = A B.

In these four analogies, it is evident that the course must be a tangent, because the difference of latitude A B is concerned.

Note.—Since the sine complement of the middle latitude = the angle D, is expressed directly by the co-sine of the angle B C D, therefore, with the view of abridging the preceding analogies, the co-sine of the middle latitude has been used instead of its sine complement; and, in the operations which follow, the same term will be invariably employed.

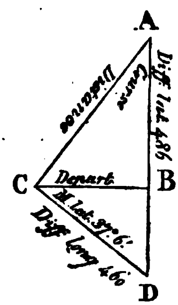
Remark.—The middle latitude between two places is found by taking half the sum of the two latitudes, when they are both of the same name, or half their difference if of contrary names.

Problem I.

Given the Latitudes and Longitudes of two Places, to find the Course and Distance between them.

Example.

Required the course and distance from Oporto, in latitude 41°9′ N. and longitude 8°37′ W. to Porto Santo, in latitude 33°3′ N. and longitude 16°17′ W.?



Latitude of Oporto	41° 9′ N.	Longitude = ...	8°37′ W.
Lat. of Porto Santo	33. 3 N.	Longitude = ...	16. 17 W.

Diff. of latitude = 8° 6′ = 486 miles. Diff. of long. = 7°40′ = 460 ms.
Sum of latitudes = 74°12′ ÷ 2 = 37°6′ = the middle latitude.

To find the Course = Angle A:—

Here, since the departure is not in question, the parts concerned come under the 5th analogy in page 222: hence,

As the diff. of latitude =	486 miles,	Log. ar. comp. =	7.313364
Is to the diff. of long. =	460 miles,	Log. = . . .	2.662758
So is the mid. latitude =	37°6′.	Log. co-sine = .	9.901776
To the course = . .	37°3′.	Log. tangent =	9.877898

To find the Distance = A C :—

The course being thus found, the distance may be determined by trigonometry, Problem II., page 172 : hence,

As radius = . . .	90°0′.	Log. co-secant =	10.000000
Is to the diff. of lat. =	486 miles,	Log. = . . .	2.686636
So is the course = .	37°3′.	Log. secant = .	10.097937
To the distance = .	608.9 miles,	Log. = . . .	2.784573

Hence, the true course from Oporto to Porto Santo is S. 37°3′. W., or S.W. ¾ S. nearly, and the distance 609 miles.

To find the Course and Distance by Inspection in the general Traverse Table :—

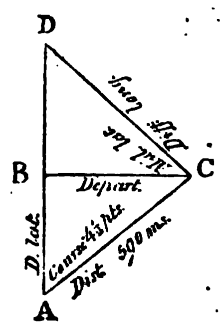
To middle latitude = 37° as a course, and one-fourth the difference of longitude = 115, as a distance, the corresponding difference of latitude is 91.8 = the meridional distance. Now, one-fourth the difference of latitude = 121.5, and the meridional distance 91.8 in a departure column, are found to agree nearest at 37°, under distance 152. Hence, the course is S. 37° W., and the distance 152 × 4 = 608 miles.

Problem II.

Given the Latitude and Longitude of the Place sailed from, the Course, and Distance; to find the Latitude and Longitude of the Place come to.

Example.

A ship from Corvo, in latitude 39°41′. N., and longitude 31°3′. W., sailed N.E. ½ E., 590 miles; required the latitude and longitude come to?



To find the Difference of Latitude = A B :—

Here the course = A, and the distance = A C, being given, the difference of latitude = A B may be found by trigonometry, Problem I., page 171; as thus:

As radius = . . .	90°0′	Log. co-secant =	10.000000
Is to the distance =	590 miles,	Log. = . . .	2.770852
So is the course = .	4½ points,	Log. co-sine = .	9.802359
To the diff. of lat. =	374.3 miles,	Log. = . . .	2.573211

Latitude left = ...	39°41′ N.		39°41′ N.
Diff. of lat. = 374.3 miles, or =	6.14 N.	Half =	3. 7 N.
Latitude come to = ...	45°55′ N.	Mid. lat. =	42°48′

To find the Difference of Longitude = C D :—

Here, since the departure is not concerned, the parts in question come under the 1st analogy in page 222: hence,

As the mid. lat. = . .	42°48′	Log. secant =	10.134464
Is to the course = . .	4½ points,	Log. sine = .	9.888185
So is the distance = .	590 miles,	Log. = . . .	2.770852
To the diff. of longitude =	621.6 miles,	Log. = . . .	2.793501

Longitude left = ...	31° 3′ W.
Diff. of longitude = 621.6 miles, or =	10.22 E.
Longitude come to = ...	20°41′ W.

To find the Difference of Latitude and Difference of Longitude by Inspection :—

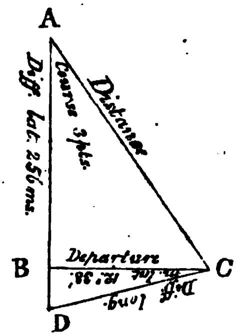
Under or over one-fifth of the given distance = 118, and opposite to the course = 4½ points, is difference of latitude 74.9, and departure 91.2. Tabular difference of latitude 74.9 × 5 = 374.5, the whole difference of latitude; whence the latitude in, is 45°55′ N., and the middle latitude 42°48′. Now, to middle latitude 42°, and departure 91.2, in a latitude column, the corresponding distance is 123 miles; and to middle latitude 43°, and departure 91.2, the distance is 125 miles: hence, the difference of distance to 1° of latitude, is 2 miles; and 2′ × 48′ ÷ 60′ = 1′.6, which, added to 123, gives 124.6; this, being multiplied by 5 (the aliquot part), gives 623 miles = the difference of longitude, or 10°23′ E.

Problem III.

Given both Latitudes and the Course; to find the Distance and the Longitude in.

Example.

A ship from Brava, in latitude 14°46′ N., and longitude 24°46′ W., sailed S.E. b. S., until, by observation, she was found to be in latitude 10°30′ N.; required the distance sailed and her present longitude?



Lat. of Brava = 14°46′ N.	. . .	14°46′ N.
Lat. by obs. = 10.30 N.	. . . .	10.30 N.
Diff. of lat. = 4°16′ = 256 m.	Sum =	25°16′

Middle latitude = 12°38′

To find the Distance = A C :—

With the course = A, and the difference of latitude = A B, the distance is found by trigonometry, Problem II., page 172; as thus:

As radius = ...	90°0′	Log. co-secant = .	10.000000
Is to the diff. of latitude =	256 miles .	Log. = . . .	2.408240
So is the course = . . .	3 points,	Log. secant = ,	10.080154
To the distance = . . .	307.9 miles,	Log. = . . .	2.488394

To find the Difference of Longitude = C D :—

Here, since the departure is not in question, the parts concerned fall under the 7th analogy, page 222 : hence,

As the middle latitude =	12°38′	Log. secant =	10.010644
Is to the course = . .	3 points,	Log. tangent =	9.824893
So is the diff. of lat. =	256 miles,	Log. = . .	2.408240
To the diff. of long. =	175.2 miles,	Log. = . .	2.243777

Longitude of Brava, the place sailed from =	24°46′ W.
Difference of longitude = 175.2 miles, or =	2.55 E.
Longitude of the ship = ...	21°51′ W.

To find the Distance sailed, and the Difference of Longitude, by Inspection:—

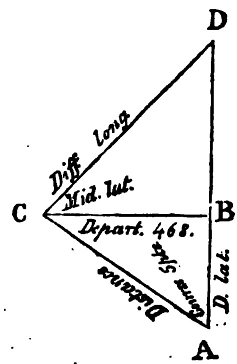
To the course 3 points, and half the difference of latitude = 128, the distance is 154, and the departure 85.5. Now, 154 × 2 = 308 miles, is the required distance. Again, to middle latitude 12°, and departure 85.5, in a latitude column, the corresponding distance is 87; and to latitude 13°, and departure 85.5, the distance is 88: hence, to middle latitude 12°38′, and departure 85.5, the distance is 87½; the double of which = 175 miles, is the difference of longitude, as required.

Problem IV.

Given the Latitude and Longitude of the Place sailed from, the Course, and the Departure; to find the Distance sailed, and the Latitude and Longitude of the Place come to.

Example.

A ship from Cape Finisterre, in latitude 42°54′ N., and longitude 9°16′ W., sailed N.W. b. W., till her departure was 468 miles; required the distance sailed, and the latitude and longitude come to?



To find the Distance = A C:—

With the course = A. and the departure B C, the distance may be found by trigonometry, Problem II., page 172; as thus:

As radius = ...	90°0′	Log. co-secant =	10.000000
Is to the departure = . .	468 miles,	Log. = . . .	2.670246
So is the course = . . .	5 points,	Log. co-secant =	10.080154
To the distance = . . .	562.9 miles,	Log. = . . .	2.750400

To find the Difference of Latitude = A B:—

With the course = A, and the departure B C, the distance is found by trigonometry, Problem II., page 172; as thus:

As the course = ...	5 points,	Log. co-tangent =	9.824893
Is to the departure = . .	468 miles,	Log. = . . .	2.670246
So is radius = ...	90°0′	Log. sine = . .	10.000000
To the diff. of latitude = .	312.7	Log. = . . .	2.495139

Latitude of Cape Finisterre =	42°54′ N.		42°54′ N.
Diff. of lat. = 313 miles, or =	5.13 N.	Half diff. of lat. =	2.36 N.
Latitude of the ship = . .	48° 7′ N.	Middle lat. =	45°30′

To find the Difference of Longitude = C D:—

With the middle latitude = B C D, and the departure B C, the difference of longitude is found by trigonometry, Problem II., page 172.—

As radius = ...	90° 0′	Log. co-secant =	10.000000
Is to the departure = . .	468 miles,	Log. = . .	2.670246
So is the mid. lat. = . .	45°30′	Log. secant =	10.154338
To the diff. of long. = .	667.7 miles,	Log. = . . .	2.824584

Longitude of Cape Finisterre = ...	9°16′	W.
Difference of long. = 667.7 miles, or =	11. 8	W.
Longitude of the ship = ...	20.24	W.

To find the Distance, Difference of Latitude, and Difference of Longitude, by Inspection:—

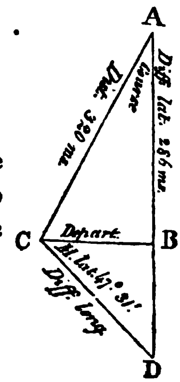
To course 5 points, and one-fourth of the departure = 117, the distance is 141, and the difference of latitude 78.3. Now, 141 × 4 = 564 miles, the distance, and 78.3 × 4 = 313.2, or 5°13′, the difference of latitude; whence the latitude in, is 48°7′N., and the middle latitude 45°30′. Again, to middle latitude 45°, and one-fourth the departure = 117, in a latitude column, the distance is 166; and to middle latitude 46°, and departure 117, the distance is 168: hence, to middle latitude 45°30′, and departure 117, the difference of longitude is 167 × 4 = 668 miles; nearly the same as by calculation.

Problem V.

Given both Latitudes and the Distance; to find the Course and Difference of Longitude.

Example.

A ship from St. Agnes, Scilly, in latitude 49°54′ N., and longitude 6°19′ W., sailed 320 miles between the south and west, and then, by observation, was found to be in latitude 45°8′ N.; required the course, and the longitude come to?



Latitude of St. Agnes =	49°54′ N.	. . . . . .	49°54′ N.
Latitude of the ship =	45. 8 N.	. . . . . .	45. 8 N.
Difference of latitude =	4°46′ = 286 miles.	Sum =	95° 2′
		Middle latitude =	47°31′

To find the Course = A:—

With the distance A C, and the difference of latitude = A B, the course may be found by trigonometry, Problem III., page 174; as thus:

As the distance = . . .	320 miles,	Log. ar. comp. =	7.494850
Is to radius = ...	90° 0′ 0″	Log. sine = . .	10.000000
So is the diff. of lat. = . .	286 miles,	Log. = . . .	2.456366
To the course = ...	26°39′ 6″	Log. co-sine = .	9.951216

To find the Difference of Longitude = C D:—

With the course, middle latitude, and distance, the difference of longitude is found by the 1st analogy, page 222; as thus:

As middle latitude = . . .	47°31′ 0″	Log. secant =	10.170455
Is to the course = ...	26. 39. 6	Log. sine = .	9.651825
So is the distance = . . .	320 miles,	Log. = . .	2.505150
To the diff. of long. = . . .	212.5	Log. = . .	2.327430

Longitude of St. Agnes =	. . .	6° 19′ W.
Diff. of long. = 212.5 miles, or =		3. 33 W.
Longitude of the ship =	. . . . .	9° 52′ W.

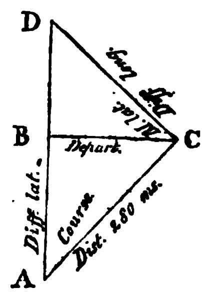
The course is S. 26° 39′ W., or S.S.W. ¾ W., nearly.

To find the Course and Difference of Longitude by Inspection :—

To half the distance = 160, and half the difference of latitude = 143, the course nearest agreeing is 27, and the departure 72.6. Now, to middle latitude 47° as a course, and departure 72.6, in a latitude column, the distance is 105; and to middle latitude 48°, and departure 72.6, the distance is 108: hence, the difference of distance to 1° of latitude, is 3 miles; therefore, 3′ × 31′ ÷ 60 = 1′.5, which, added to 105, makes 106.5: this, being multiplied by 2, gives 213 miles = the difference of longitude.

Problem VI.

Given one Latitude, Distance, and Departure; to find the other Latitude, the Course, and the Difference of Longitude.



Example.

A ship from Cape Bajoli, Minorca, in latitude 40° 3′ N., and longitude 3° 52′ E., sailed 280 miles between the north and east, upon a direct course, and made 186 miles of departure; required the course, and the latitude and longitude come to?

To find the Course = A :—

The distance = A C, and the departure B C, being given, the course may be found by trigonometry, Problem III., page 174; as thus:

As the distance =	. . .	280 miles,	Log. ar. comp. =	7.552842
Is to radius =	. . . . .	90° 0′ 0″	Log. sine = .	10.000000
So is the departure =	. .	186 miles,	Log. = . . .	2.269513
To the course =	. . . . .	41° 37′ 39″	Log. sine = .	9.822355

To find the Difference of Latitude = A B:—

The course = A, and the distance, being thus known, the difference of latitude may be computed by trigonometry, Problem III., page 174.—

As radius = . . .	90° 0′ 0″	Log. co-secant =	10.000000
Is to the distance = .	280 miles,	Log. = . . .	2.447158
So is the course = . .	41°37′39″	Log. co-sine = .	9.873599
To the diff. of lat. = .	209.3 miles,	Log. = . . .	2.320757

Latitude of Cape Bajoli = . .	40° 3′ N.	. . . . . .	40° 3′ N.
Diff. of lat. = 209.3 miles, or =	3.29 N.	Half = .	1.44½ N.
Latitude come to = ...	43°32′ N.	Middle latitude =	41°47½′ N.

To find the Difference of Longitude = C D:—

The middle latitude = angle B C D, and the departure B C, being given, the difference of longitude may be found by trigonometry, Problem II., page 172; as thus:

As radius = ...	90° 0′	Log. co-secant =	10.000000
Is to the departure = .	186 miles,	Log. = . . .	1.269513
So is the mid. lat. = .	41°47½′	Log. secant = .	10.127510
To the diff. of long. = .	249.5 miles,	Log. = . . .	1.397023

Longitude of Cape Bajoli = ...	3°52′ E.
Diff. of long. = 249.5 miles, or = . . .	4. 9 E.
Longitude come to = ...	8. 1 E.

The course is N. 41°38′ E., or N.E. ¾ N., nearly.

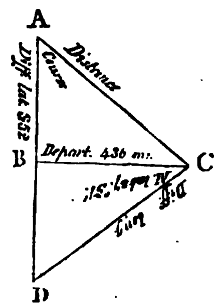
To find the Course, Difference of Latitude, and Difference of Longitude, by Inspection:—

The distance 280, and departure 186, are found to agree between 41° and 42°, and the corresponding difference of latitude 208.1: whence the middle latitude is 41°46′. Now, to middle latitude 41°, and departure 186, in a latitude column, the corresponding distance is 247; and to latitude 42°, and departure 186, the distance is 250: hence, the difference of distance to 1° of latitude, is 3 miles; and 3′ × 46 ÷ 60′ = 2′.3, which, added to 247, gives 249.3 = the difference of longitude, as required; which nearly agrees with the result by calculation.

Problem VII.

Given both Latitudes and Departure; to find the Course, Distance, and Difference of Longitude.

Example.



A ship from Cape Agulhas, in latitude 34°55′ S., and longitude 20°18′ E., sailed upon a direct course between the south and east, till she was found, by observation, to be in latitude 40°47′ S., and to have made 436 miles of easting; required the course, distance, and longitude at which the ship arrived?

Latitude of Cape Agulhas =	34°55′ S.	. . . .	34°55′ S.
Latitude of the ship =	40.47 S.	. . . .	40.47 S.
Diff. of latitude =	5°52′=352 miles.	Sum =	75.42
		Middle latitude =	37°51′

To find the Course = Angle A:—

Here, the difference of latitude = AB, and the departure BC, being given, the course is found by trigonometry, Problem IV., page 175; as thus:

As the diff. of lat. =	352 miles,	Log. ar. comp. =	7.453457
Is to radius =	90°0′0″	Log. sine =	10.000000
So is the departure =	436 miles,	Log. =	2.639486
To the course =	51°5′5″	Log. tangent =	10.092943

To find the Distance = AC:—

With the course, thus found, and the difference of latitude AB, the distance may be computed by trigonometry, Problem IV., page 175: hence,

As radius =	90°0′0″	Log. co-secant =	10.000000
Is to the diff. of lat. =	352 miles,	Log. =	2.546543
So is the course =	51°5′5″	Log. secant =	10.201922
To the distance =	560.4 miles,	Log. =	2.748465

Hence, the course is S. 51°5′ E., or S.E. ½ E., nearly, and the distance 560.4 miles.

To find the Difference of Longitude = C D :—

With the middle latitude = B C D, and the departure B C, the difference of longitude is found by trigonometry, Problem IV., page 175; as thus:

As radius = ...	90° 0′	Log. co-secant =	10.000000
Is to the departure =	436 miles,	Log. = . . .	2.639486
So is the middle lat. =	37°51′	Log. secant = .	10.102582
To the diff. of long. =	552.2 miles,	Log. = . . .	2.742068

Longitude of Cape Agulhas = ...	20°18′ E.
Diff. of longitude = 552.2 miles, or = .	9.12 E.
Longitude at which the ship arrived = .	29.30 E.

To find the Course, Distance, and Difference of Longitude, by Inspection:—

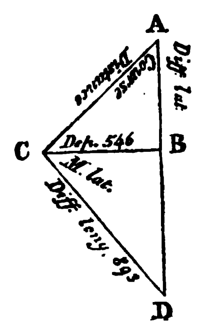
Half the difference of latitude = 176, and half the departure = 218, are found to agree nearest at 51° under or over distance 280: hence, 280 × 2 = 560 miles, is the distance. Again, to middle latitude 37° as a course, and departure 218, in a latitude column, the corresponding distance is 273; and to latitude 38° and departure 218, the distance is 277: hence, the change of distance to 1° of latitude, is 4 miles. Now, 4′ × 51′ ÷ 60 = 3′.4, which, added to 273, gives 276.4; and this, being multiplied by 2, gives 552.8 miles; which very nearly corresponds with the result by calculation.

PROBLEM VIII.

Given one Latitude, Departure, and Difference of Longitude; to find the other Latitude, Course, and Distance.

Example.

A ship from the Snares, New Zealand, in latitude 48°3′ S., and longitude 166°20′ E., sailed upon a direct course between the south and west, till she was found by observation to be in longitude 151°27′ E., and to have made 546 miles of departure; required the latitude come to, the course steered, and the distance sailed?



Longitude of the Snares = . .	166°20′ E.	
Long. of the ship by observation =	151.27 E.	
Difference of longitude = . . .	14°53′=893 miles.	

To find the Middle Latitude = the Angle B C D :—

With the departure = B C, and the difference of longitude = C D, the angle of the middle latitude may be found by trigonometry, Problem III., page 174; as thus:

As the diff. of long. =	893 miles,	Log. ar. comp. =	7.049148
Is to radius = .	90° 0′ 0″	Log. sine = .	10.000000
So is the departure =	546 miles,	Log. = . .	2.737193
To the mid. lat. =	52°18′28″	Log. co-sine =	9.786341

Twice mid. lat. = 104°37′ 0″ nearly.
Lat. of the Snares = 48. 3. 0 S.

Latitude come to = 56°34′ 0″S.

Diff. of latitude = 8°31′ 0″ = 511 miles.

To find the Course = the Angle A :—

With the difference of latitude A B, and the departure B C, the course may be found by trigonometry, Problem IV., page 175; as thus:

As the diff. of lat. =	511 miles,	Log. ar. comp. =	7.291579
Is to radius = . .	90° 0′ 0″	Log. sine = .	10.000000
So is the departure =	546 miles,	Log. = . .	2.737193
To the course = . .	46°53′48″	Log. tangent =	10.028772

To find the Distance = A C :—

With the angle of the course, thus found, and the difference of latitude A B, the distance may be computed by trigonometry, Problem IV., page 175: hence,

As radius = . .	90° 0′ 0″	Log. co-secant =	10.000000
Is to diff. of lat. =	511 miles,	Log. = . . .	2.708421
So is the course =	46°53′48″	Log. secant = .	10.165378
To the distance = .	747.8 miles,	Log. = . . .	2.873799

Hence, the course is S. 46°54′ W., or S.W. ¼ W. nearly, and the distance 747.8 miles.

To find the Latitude come to, Course, and Distance, by Inspection in the general Traverse Table:—

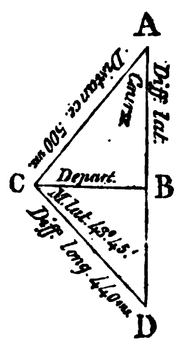
One-fourth of the difference of longitude = 223¼, taken as distance, and one-fourth of the departure = 136.5, in a latitude column, will be found to agree between 52° and 53°. Now, to latitude 52°, and distance 223, the difference of latitude is 137.3, which is 0′.8 more than 136.5; and to latitude 53°, and distance 223, the difference of latitude is 134.2, being 2′.3 less than 136.5: hence, the difference of meridional distance to 1° of latitude is 0′.8 + 2′.3 = 3′.1: therefore, as 3′.1 : 0′.8 :: 60′ : 16′, which, added to 52° (proportion being made for the quarter of a mile in the distance), gives the middle latitude = 52°18½′: hence, the latitude come to is 56°34′ S., and the difference of latitude 511 miles. Again, to one-fourth of the difference of latitude = 127.75, and one-fourth of the departure = 136.5, the course is 47°, and the distance 187; which, multiplied by 4, gives 748 miles = the whole distance.

Problem IX.

Given the Distance, Difference of Longitude, and Middle Latitude; to find the Course and both Latitudes.

Example.

A ship, in north latitude, sailed 500 miles upon a direct course between the south and west, until her difference of longitude was 440 miles; required the course steered, the latitude sailed from, and the latitude come to; allowing the middle latitude to be 43°45′ north?



To find the Angle of the Course = A:—

The course may be found by the 3d analogy, page 222, as thus:

As the distance = . . .	500 miles,	Log. ar. comp. =	7.301030
Is to the diff. of longitude =	440 miles,	Log. = . . .	2.643453
So is the middle latitude =	43°45′ 0″	Log. co-sine =	9.858756
To the course = . .	S. 39°28′14″ W.	Log. sine =	9.803239

To find the Difference of Latitude = A B :—

The difference of latitude may be found by the 8th analogy, page 222, as thus :

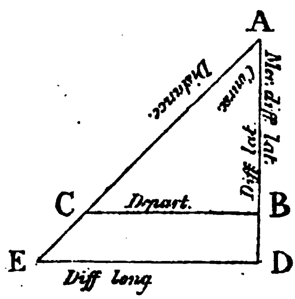
As the course = . . .	39°28′14″	Log. co-tangent =	10.084350
Is to the middle latitude =	43.45. 0	Log. co-sine = . .	9.858756
So is the diff. of long. = .	440 miles,	Log. = ...	2.643453
To the diff. of latitude =	386 miles,	Log. = ...	2.586559

Middle latitude = ...	43°45′	N.
Half the diff. of lat. = 193 miles, or = . .	3. 13	S.
Latitude of the place sailed from = . .	46°58′	N.
Latitude of the place come to = . . .	40. 32	N.

SOLUTION OF PROBLEMS IN MERCATOR'S SAILING.

Mercator's Sailing is the method of finding, on a plane surface, the motion of a ship upon any assigned point of the compass, which shall be true in latitude, longitude, and distance sailed.

Mariners, generally speaking, solve all the practical cases in Mercator's Sailing by stated rules, called *canons*, which they early commit to memory, and, ever after, employ in the determination of a ship's place at sea. Those *canons*, certainly, hold good in most cases; but since they are destructive of the best principles of science, inasmuch as that they have a direct tendency to remove from the mind every trace of the elements of trigonometry, the very doctrine from which they were originally deduced, and on which the whole art of navigation is founded, the following observations and consequent analogies are, therefore, submitted to the attention of naval people, under the hope that they will serve as an inducement to the substitution of the rules of reason for the *rules of rote*; and thus do away with the necessity of getting *canons by heart*.



In the annexed diagram, let the triangle A B C be a figure in plane sailing, in which the angle A represents the course, A C the distance, A B the difference of latitude, and B C the departure. If A B be produced to D, until it is made equal to the meridional difference of latitude, and D E be drawn at right angles thereto, and parallel to B C; then the triangle A D E will be a figure in Mercator's sailing, in which the angle A represents the course, the side A D the meridional difference of latitude, and the side D E the difference of longitude. Now, since the two triangles A B C and A D E are right angled, and that the angle A is common to both; therefore they are equi-angular: and because they are equi-angular, they are also similar; therefore the sides containing the equal angles of the one are proportional to the sides containing the equal angles of the other.—Euclid, Book VI., Prop. 4.

Now, from the relative properties of those two triangles, all the analogies for the solution of the different cases in Mercator's sailing may be readily deduced agreeably to the established principles of right angled trigonometry, as given in page 171, and thence to 177; as thus:—

First, in the triangle A B C, if the distance A C be made radius, the analogies will be,

1. As radius : distance A C :: sine of the course A : departure B C; and :: co-sine of the course A : difference of latitude A B.

2. As sine of the course A : departure B C :: radius : distance A C; and :: co-sine of the course A : difference of latitude A B.

3. As co-sine of the course A : difference of latitude A B :: radius : distance A C; and :: sine of the course A : departure B C.

4. As the distance A C : radius :: departure B C : sine of the course A; and :: difference of latitude A B : co-sine of the course A.

Again, by making the difference of latitude A B radius, the analogies will be,

5. As the difference of latitude A B : radius :: departure B C : tangent of the course A; and :: distance A C : secant of the course A.

6. As radius : difference of latitude A B :: tangent of the course A : departure B C; and :: secant of the course A : distance A C.

And by making the departure B C radius, it will be,

7. As the departure B C : radius :: difference of latitude A B : co-tangent of the course A; and :: distance A C : co-secant of the course A.

8. As radius : departure B C :: co-tangent of the course A : difference of latitude A B; and :: co-secant of the course A : distance A C.

Now, in the triangle A D E, if the meridional difference of latitude A D be made radius, the analogies will be,

9. As the meridional difference of latitude A D : radius :: difference of longitude D E : tangent of the course A.

10. As radius : meridional difference of latitude A D :: tangent of the course A : difference of longitude D E.

And by making the difference of longitude D E radius, it will be,

11. As the difference of longitude D E : radius :: meridional difference of latitude D E : co-tangent of the course A.

12. As radius : difference of longitude D E :: co-tangent of the course A : meridional difference of latitude A D.

Finally, since the triangles A B C and A D E are equi-angular and similar, we have,

13. As the difference of latitude A B : departure B C :: meridional difference of latitude A D : difference of longitude D E.

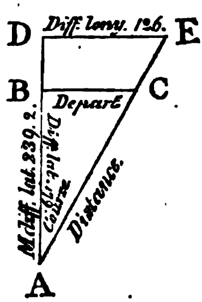
The meridional difference of latitude is found by means of Table XLIII., by the same rules as those for the difference of latitude given at page 214; as thus :—If the two given latitudes be of the same name, the difference of their corresponding meridional parts will be the meridional difference of latitude; but if the latitudes be of contrary names, the sum of these parts will be the meridional difference of latitude.

Problem I.

Given the Latitudes and Longitudes of two Places; to find the Course and Distance between them.

Example.

Required the course and distance between Cape Bajoli, in latitude 40°3′ N., and longitude 3°52′ E., and Cape Sicie, in latitude 43°2′ N., and longitude 5°58′ E.?



Lat. of C. Bajoli	40° 3′N.	Merid. pts. 2626.6.	Longitude 3°52′E.
Lat. of C. Sicie	43. 2 N.	Merid. pts. 2865.8.	Longitude 5.58 E.
Diff. of latitude	2°59′	Merid. diff. lat. 239.2.	Diff. long. 2° 6′
	= 179 miles.		= 126 miles.

To find the Course = Angle A :—

This comes under the 9th analogy, in page 237 : hence,

As the merid. diff. of lat. =	239.2 miles,	Log. ar. comp. =	7.621239
Is to radius = ...	90° 0′ 0″	Log. sine = .	10.000000
So is the diff. of long. = .	126 miles,	Log. = . .	2.100371
To the course = ...	27°46′42″	Log. tangent =	9.721610

To find the Distance = A C :—

This comes under the 6th analogy, in page 237 : hence,

As radius = . . .	90° 0′ 0″	Log. co-secant = . .	10.000000
Is to the diff. of lat. =	179 miles,	Log. = ...	2.252853
So is the course = .	27°46′42″	Log. secant = . . .	10.053176
To the distance = . .	202.3 miles,	Log. = ...	2.306029

Hence, the true course from Cape Bajoli to Cape Sicie is N. 27°47′ E., or N.N.E. ¼ E. nearly, and the distance 202.3 miles.

To find the Course and Distance, by Inspection in the general Traverse Table :—

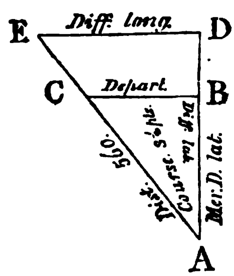
The meridional difference of latitude 239.2, and the difference of longitude 126, as departure, are found to agree nearest at 28°, which, therefore, is the course. Now, to course 28°, and difference of latitude 179, the corresponding distance is 203 miles; which nearly agrees with the result by calculation.

PROBLEM II.

Given the Latitude and Longitude of the Place sailed from, the Course and Distance; to find the Latitude and Longitude of the Place come to.

Example.

A ship from Cape Ortegal, in latitude 43°47′ N., and longitude 7°49′ W., sailed N.W. ¼ N. 560 miles; required the latitude and longitude come to?



To find the Difference of Latitude = A B :—

This comes under the 1st analogy, page 237 : hence,

As radius = ...	90°0′	Log. co-secant = .	10.000000
Is to the distance = .	560 miles,	Log. = ...	2.748188
So is the course = . .	3½ points,	Log. co-sine = . .	9.888185
To the diff. of latitude =	432.9 miles,	Log. = ...	2.636373

To find the Difference of Longitude = D E :—

This comes under the 10th analogy, in page 237 : hence,

As radius = . . .	90°0′	Log. co-secant = . . .	10.000000
Is to merid. diff. of lat.=641 miles,		Log. = ...	2.806856
So is the course = .	3½ points,	Log. tangent = . . .	9.914173
To the diff. of long.=526.1 miles,		Log. = ...	2.721029

Lat. of C. Ortegal 43°47′N.	Mer. pts 2927.8.	Long. of C. Ortegal 7°49′W.
Diff. lat.=433 m. or 7.13 N.		Diff. long.=526 m. or 8.46 W.
Latitude come to=51° 0′N.	Mer. pts 3568.8	Long. come to = 16°35′W.
	Merid. diff. of lat. = 641.0	

To find the Difference of Latitude and Difference of Longitude, by Inspection :—

To course 3¼ points, and half the distance = 280, the difference of latitude is 216.4; the double of which, or 432.8, is the difference of latitude : hence, the latitude come to is 51°0′ N., and the meridional difference of latitude 641.

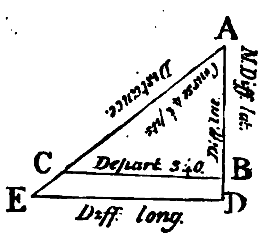
Now, to course 3½ points, and one-third of the meridional difference of latitude = 213.7, the corresponding departure is 175.4, proportion being made for the excess of the given, above the tabular difference of latitude; then 175.4 × 3 = 526 miles; which, therefore, is the difference of longitude.

Problem III.

Given the Latitude and Longitude of the Place sailed from, the Course, and the Departure, to find the Distance sailed, and the Latitude and Longitude of the Place come to.

Example.

A ship from Wreck Hill, Bermudas, in latitude 32°15′ N., and longitude 64°47′ W., sailed S.W. ½ W., and made 340 miles of departure; required the distance sailed, and the latitude and longitude come to?



To find the Distance = A C :—

This comes under the 8th analogy, page 237 : hence,

As radius =	. . 90°0′	. Log. co-secant =	10.000000
Is to the departure =	340 miles,	Log. = . . .	2.531479
So is the course =	4½ points,	Log. co-secant =	10.111815
To the distance =	439.8 miles,	Log. = . . .	2.643294

To find the Difference of Latitude = A B :—

This comes under the 8th analogy, page 237 : hence,

As radius =	. 90°	. Log. co-secant = .	10.000000
Is to the departure	340 miles,	Log. = ...	2.531479
So is the course =	4½ points,	Log. co-tangent = .	9.914173
To the diff. of lat. =	279 miles,	Log. = ...	2.445652

Lat. of Wreck Hill, Bermudas,	32°15′ N.	Merid. parts =	2046.1
Diff. of latitude = 279 miles, or	4.39 S.		
Latitude come to = . . .	27°36′ N.	Merid. parts =	1724.0
Meridional difference of latitude = ...			322.1

To find the Difference of Longitude D E :—

This comes under the 10th analogy, page 237 : hence,

As radius =	. . 90°0′	Log. co-secant =	10.000000
Is to merid. diff. of lat. =	322.1 miles,	Log. = .	2.507991
So is the course =	4½ points,	Log. tangent = .	10.085827
To the diff. of long. =	392.5 miles,	Log. = . .	2.593818

Longitude of Wreck Hill, Bermudas, = . . .	64°47′ W.
Difference of longitude = 392.5 miles, or =	6.32 W.
Longitude come to = ...	71°19′ W.

The distance sailed is 440 miles, very nearly.

To find the Distance sailed, and the Latitude and Longitude come to, by Inspection :—

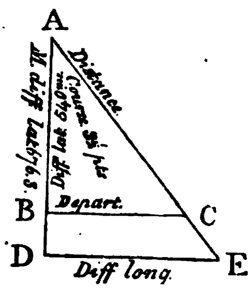
To the course 4½ points, and half the departure = 170, the corresponding difference of latitude is 139.6, under distance 220; twice the latter, or 440 miles, is, therefore, the distance sailed; and twice 139.6 = 279.2 miles, or 4°39′, is the difference of latitude: whence the latitude in, is 27°36′ N., and the meridional difference of latitude 322.1. Now, to course 4½ points, and half the meridional difference of latitude = 161 miles, in a latitude column, the corresponding departure is 196.3; the double of which, or 392.6 miles, is the difference of longitude: hence, the longitude come to is 71°19½′ W.

PROBLEM IV.

Given both Latitudes and the Course; to find the Distance and the Longitude in.

Example.

A ship from the east end of Martha's Vineyard, in latitude 41°21′ N., and longitude 70°24′ W., sailed S.E. ½ S., and, by observation, was found to be in latitude 32°21′ N.; required the distance sailed, and the longitude at which she arrived?



Lat. of the east end of Martha's Vineyard =	41°21′ N.	Merid. parts =	2729.5
Lat. in, by observation =	32.21 N.	Merid. parts =	2053.2
Difference of latitude =	9° 0′ = 540 miles.	Merid. diff. of lat. =	676.3

To find the Distance = A C:—

This comes under the 6th analogy, page 237; therefore,

As radius = . . 90°0′ Log. co-secant = . .	10.000000
Is to the diff. of lat. 540 miles, Log. = . .	2.732394
So is the course = 3½ pts. Log. secant = . . .	10.111815
To the distance = 698.6 miles, Log. = ...	2.844209

To find the Difference of Longitude = D E:—

This comes under the 10th analogy, page 237; therefore, To find the Course and Distance made good :—

To the whole difference of latitude and departure, so found, find the corresponding course and distance by Problem II, page 108, and thus the course and distance made good will be obtained.

To find the Latitude in, by Account, or Dead Reckoning :—

If the difference of latitude and the latitude of the place from which the ship's departure was taken, or the yesterday's latitude, be of the same name, their sum will be the latitude in, by account; but if they are of contrary names, their difference will be the latitude in, of the same name with the greater quantity.

To find the Difference of Longitude :—

With the course made good, and the meridional difference of latitude, in a latitude column, find the corresponding departure, by Problem III. page 110, and it will be the difference of longitude.

Or.—With the middle latitude as a course, and the departure, in a latitude column, find the corresponding distance, by Problem V., page 111, and it will be the difference of longitude.

To find the Longitude in, by Account, or Dead Reckoning :—

If the difference of longitude and the longitude of the place from which the ship's departure was taken, or the yesterday's longitude, be of the same name, their sum will be the longitude in, by account, when it does not exceed 180°; otherwise, it is to be taken from 360°, and the remainder will be the longitude in, of a contrary name to that left :—but, if the difference of longitude and the longitude left are of contrary names, their difference will be the longitude in, of the same name with the greater quantity.

To find the Bearing and Distance from the Ship to the Port to which she is bound :—

By Mercator's Sailing.

With the meridional difference of latitude, in a latitude column, and the difference of longitude, as departure, find the course, by Problem IV. page 111; then, with the course, thus found, and the difference of latitude, the distance is to be obtained by the same Problem.—Or,

By Middle Latitude Sailing.

With the middle latitude between the ship and the proposed place, as a course, and the difference of longitude, as distance, find the corresponding meridional distance, or departure, by Problem VI. page 112; then, with this departure, and the difference of latitude, the course and distance are to be obtained by the same Problem.

Note.—The true bearing or course, thus found, may be reduced to the magnetic, or compass bearing, if necessary, by allowing the value of the variation to the right hand if westerly; and to the left hand if easterly; being the converse of reducing the course steered by compass, to the true course.

And, this rule comprises the substance of that nautical operation which is generally termed *a day's work* at sea.

Example 1.

A ship from Cape Espichell, in latitude 38°25′ north, and longitude 9°13′ west, bound for Porto Santo, in latitude 33°3′ north, and longitude 16°17′ west, by reason of contrary winds was obliged to sail upon the following compass courses; viz.—W. by S. 56 miles; N.W. by W. 110 miles; W. N.W. 95 miles; S. by E. ½ E. 50 miles; S. by W. ¾ W. 103 miles, and S. S. W. 116 miles; the variation was 2 points westerly on the three first courses, and 1¾ point on the three last: required the course, and distance made good, the latitude and longitude at which the ship arrived; with the direct course, and distance from thence to her intended port?

TRAVERSE TABLE.

Corrected Courses.	Distances.	Difference of Latitude.		Departure.	
		N.	S.	E.	W.
S. W. by W.	56	”	31.1	”	46.6
W. by N.	110	21.5	”	”	107.9
West.	95	”	”	”	95.0
S. E. ¾ S.	50	”	40.2	29.8	”
South.	103	”	103.0	”	”
S. ¼ W.	116	”	115.9	”	5.7
		21.5	290.2	29.8	255.2
			21.5		29.8
		Diff. Lat.=	268.7	Departure=	225.4

To find the Course and Distance made good :—

Half the difference of latitude = 134.35, and half the departure = 112.7, are found to agree nearest abreast of 40° under distance 175;—now, 175 × 2 = 350 miles.—Hence, the course made good is S. 40° W. or, S. W. ½ S. nearly, and the distance 350 miles.

To find the Latitude and Longitude come to, by Account :—

Lat. of C. Espichell =	38°25′ N.	Mer. pts. 2500.1.	Long. = 9°13′ W.
Diff. of lat. = 269 ms., or	4.29 S.	. . . . .	Diff. long. = 4.40 W.
Latitude come to =	33°56′ N.	Mer. pts. 2166.7.	Long. = 13°53′ W.
	Merid. diff. of lat. . . =	333.4	

To find the Difference of Longitude made good :—

To the course made good = 40° and half the meridional difference of latitude = 166.7 the corresponding departure is 140.1, which, multiplied by 2, gives the difference of longitude 280.2 miles = 4°40′ west.—Or,

With the middle latitude = 36°10′, and half the departure = 112.7, in a latitude column, the corresponding distance is 139.3 (proportion being made for the 10 minutes of latitude); hence, 139.3 × 2 = 278.6 miles, the difference of longitude; being about a mile and a half less than the result by Mercator's sailing.

To find the Course and Distance from the Ship to her intended Port :—

Lat. of the ship	33°56′ N.	M. pts. 2166.7.	Longitude 13°53′ W.
Lat. Porto Santo	33. 3 N.	M. pts. 2103.1.	Longitude 16.17 W.
Diff. of Lat. =	0°53′ = 53 ms.	M. diff. L. 63.6.	Diff. Long. 2°24′ = 144 miles.

By Mercator's Sailing.

The meridional difference of latitude = 63.6 in a latitude column, and the difference of longitude = 144, in a departure column, are found to agree nearest abreast of 66° the course.—Now, to course 66° and difference of latitude 53, the corresponding distance is 130 miles.—Or, With the middle latitude = 33°30′ as a course, and the difference of longitude = 144 as distance, the corresponding difference of latitude is 120.1.—Now, with 120.1 in a departure column, and the difference of latitude = 53, in its proper column, the corresponding course is 66° and the distance 131 miles.

Hence,—The course made good is S. 40° W. or S. W. ¼ S. nearly.
The distance made good is 350 miles.
The latitude by account is 33°56′ north.
The long. by account is 13.53 west.

And,

Porto Santo bears from the ship S. 66° W. or W.S.W. nearly.

Distance 130 miles as required.

Note.—If the latitude and longitude of the ship, or either of them, have been deduced from celestial observations, they are to be made use of, instead of those by account, in determining the course and distance between the ship and the place to which she is bound.—See the compendium of Practical Navigation near the end of this Volume.

Example 2.

A ship from Port Royal, Jamaica, in latitude 17°58′ north, and longitude 76°53′ west, got under weigh for Hayti, St. Domingo, in latitude 18°30′ north, and longitude 69°49′ west, and sailed upon the following courses, viz.; S. 40 miles; S. E. by S. 97 miles; N. by E. 72 miles; S. E. ½ S. 108 miles; N. by E. ½ E. 114 miles; S. E. 126 miles; N. N. E. 86 miles; and then by observation was found to be in latitude 16°55′ N., and longitude 72°30′ W.;—the lee-way on each of those courses was a quarter of a point (the wind being between E. S. E. ½ S. and E. by N. ½ N.), and the variation of the compass half a point easterly:—required the true course and distance made good; the latitude and longitude at which the ship arrived by account, with the direct course and distance between her true place by observation and the port to which she is bound?

Traverse Table.

Corrected Courses.	Distances.	Difference of Latitude.		Departure.	
		N.	S.	E.	W.
S. ¾ W.	40	"	39.6	"	5.9
S. S. E. ¼ E.	97	"	87.7	41.5	"
N. by E. ¼ E.	72	69.8	"	17.5	"
S. S. E. ¾ E.	108	"	92.6	55.5	"
N. by E. ¾ E.	114	107.3	"	38.4	"
S. E. by S. ¼ E.	126	"	101.2	75.1	"
N. N. E. ¼ E.	86	77.7	"	36.8	"
		254.8	321.1	264.8	5.9
			254.8	5.9	
		Diff. Lat. =	66.3	258.9 =	Departure

To find the Course and Distance made good :—

Half the difference of Latitude = 33.15, and half the departure = 129.45, are found to agree nearest between 75° and 76°, under distance 134; and by making proportion for the difference between the given and the tabular numbers, the true course will be found = 75°38′; and the distance 134, × 2= 168 miles.—Hence the course made good is S. 75°88′ E. or E. by S. ¼ S. nearly; and the distance 268 miles.

To find the Latitude and Longitude come to, by Account :—

Lat. of Port Royal = 17°58′ N.	Mer. pts. 1096.1	Long. =	76°53′ W.
Diff. Lat. 66.3, or 1. 6 S.		Diff. Long. =	4.30 E.
Lat. come to by ac. = 16°52′ N.	Mer. pts. 1026.9	Long. by Acc.	72°23′ W.
Merid. diff. of Lat. =	69.2		

To find the Difference of Longitude made good :—

To the course made good = 75°38′ and the meridional difference of latitude = 69.2, the corresponding departure is 270.3, proportion being made for the 38′ in the course beyond 75°.—Hence, the difference of longitude is 270.3, or 4°30′ east.—Or, with the middle latitude = 17°25′ as a course, and half the departure made good = 129.45 in a latitude column, the corresponding distance, at top or bottom, is 135; which, multiplied by 2, gives the difference of longitude = 270 miles.

To find the Course and Distance from the Ship to her intended Port:—

Lat. of ship by ob.	16°55′ N.	Mer. pts.	1030.1	Long. by ob.	72°30′ W.
Lat. of Hayti =	18.30 N.	Mer. pts.	1129.8	Lg. of Hayti	69.49 W.
Diff. of Lat. =	1°35′	M.D.L. =	99.7	Diff. of Long.	2°41′
	= 95 miles.				= 161 miles.

The meridional difference of latitude = 99.7, and difference of longitude = 161, in a departure column, are found to agree nearest between 58° and 59° under distances 188 and 194; and by making proportion for the difference between the given and the tabular numbers, the true course will be found = 58°14′.—Now, to course 58°14′ and difference of latitude 95, the corresponding distance is 180 miles.—Or, with the middle latitude = 17°42½′ as a course, and the difference of longitude = 161 as a distance, the corresponding difference of latitude is 153.4:—now, with 153.4, in a departure column, and the difference of latitude = 95, in its proper column, the course, nearest agreeing, is 58 degrees, and the distance 181 miles.—Hence,

The Course made good is S. 75°38′ E. or E. by S. ¾ S. nearly.
Distance made good = 268 miles.
Latitude come to by account = 16°52′ N.
Latitude by observation = . . 16°55′ N.
Long. come to by account = . 72°23′ W.
Long. by observation = . . 72°30′ W.
Hayti bears from the ship N. 58°14′ E. or N. E. by E. ¼ E. nearly.
Distance 180 miles, as required.

Note.—This example and the preceding exhibit all the particulars attendant on making out a *day's work at sea.*—See more of this in the compendium of Practical Navigation near the end of this Volume.

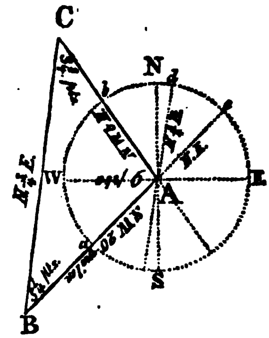
SOLUTION OF CASES IN OBLIQUE SAILING.

Oblique sailing is the application of oblique angled plane trigonometry to the solution of certain cases at sea: such as in coasting along shore; approaching, or leaving the land; surveying coasts and harbours, &c., where it becomes necessary to determine the distance of particular places from the ship, and from each other.—And, also, when it is required to settle the position of any place, cape, or head-land from a ship, by observations taken on board.

Example 1.

A ship being about to take her departure from Madeira, set the Lizard Point, which bore, by azimuth compass, N. W. by N.; and after sailing S. W. 20 miles, it was again set and found to bear N. ¾ E.; required the ship's distance from the Lizard at both stations.

Solution.—In the annexed diagram let the point C represent the Lizard, and the points A and B the stations or places of the ship, whence the bearings of the point C were taken.—Now, the difference between the bearing A C = N. W. by N. and the ship's course A B = S. W. is 9 points, which is the value of the angle B A C, measured by the arc *a b*:—The difference between N. W. by N. and N. ¾ E. is 3¾ points = the angle A C B, measured by the arc *b d*; and the difference between N. ¾ E. and N. E. (the opposite point to S. W.) is 3¼ points = the angle A B C, measured by the arc *d e*.— Then, in the oblique angled triangle A B C, given the angles and the side A B, to find the sides A C and B C = the distance of the ship from the Lizard at the respective stations.—Hence, by oblique angled trigonometry, Problem I., page 177.



To find the Distance A C:—

As the angle C = . . . 3¾ pts.	Log. co-secant =	10.172916
Is to the distance A B = 20 ms.,	Log. = ...	1.301030
So is the angle B = 3¼ pts.	Log. sine = . .	9.775027
To the dist. A C = 17.74 ms.,	Log. = ...	1.248973

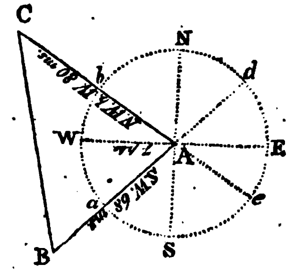
To find the Distance B C:—

As the angle C = . . 3¾ pts.	Log. co-secant =	10.172916
Is to the distance A B = 20 ms.,	Log. = ...	1.301030
So is the angle A = 9 pts.	Log. sine = . .	9.991574
To the dist. B C = 29.21 miles,	Log. = . . .	2.465520

Hence, the distance of the ship from the Lizard at the first station is 17¾ miles; and at the second station 29¼ miles nearly.

Example 2.

Two ships sail from the same port, one N. W. by W. 80 miles, and the other S. W. 68 miles; required the bearing and distance of those ships from each other?



Solution.—In the annexed diagram let the side A C represent the course steered by one of the ships, and the side AB the course steered by the other ship; and let the side BC represent the relative bearing and distance of the ships from each other.—Now, the difference between the bearing A = N. W. by W. and the bearing A B = S. W. is 7 points = the angle BAC, measured by the arc *a b*.—Hence, in the oblique angled triangle A B C, given the side AC 80 miles; the side AB 68 miles, and the included angle A = 7 points; to find the other angles, and the side BC.—Therefore, by oblique angled trigonometry, Problem III., page 179,

To find the Angles B and C:—

As the sum of A B and A C =	148 miles,	Log. ar. compt. = 7.829738
Is to difference of A B and A C =	12 miles,	Log. = . . . 1.079181
So is ½ sum of angles B and C =	50°37′30″	Log. tangent = 10.085827
To ½ diff. of angles B and C =	5°38′32″	Log. tangent = 8.994746

Angle B = . . . 56°16′ 2″
Angle C = . . . 44°58′58″

To find the Side B C = the Distance between the Ships:—

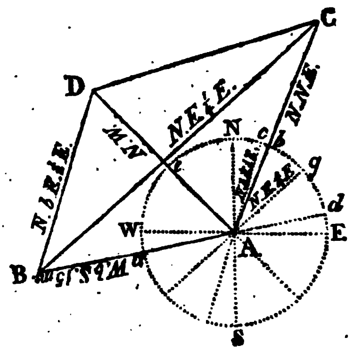
As the angle B =	56°16′2″	Log. co-secant = . .	10.080066
Is to the side A C =	80 miles,	Log. = ...	1.903090
So is the angle A =	7 points	Log. sine = ...	9.991574
To distance B C =	94.34 miles,	Log. = ...	1.974730

To find the relative Bearings of the Ships :—

From the angle B = 56°16′2″ subtract the course from A to B = 45°, and the remainder = 11°16′2″ is the bearing of C from B = N. 11°16′. W. or N. by W. nearly.—And from the course A C = 56°15′ subtract the angle C = 44°58′58″ and the remainder = 11°16′2″ is the course from C to B = S. 11°16′ E. or S. by E. nearly.

Example 3.

Coasting along shore two head-lands were observed ; the first bore, by azimuth compass, N.N.E., the second N.W. :—after sailing W. by S. 15 miles, the first bore N. E. ¼ E. and the second N. by E. ½ E. ; required the relative bearing and distance of those head-lands from each other ?



Solution.—In the diagram A B D C, let the side A B represent the course steered by the ship ; A C the bearing of the first head-land, and A D the bearing of the second head-land from the place of the ship at A ; and, let B C represent the bearing of the first head-land, and B D the bearing of the second head-land from the ship's place at B.—Now, in the triangle A B D, the angles and the side A B are given, to find the side A D.—Thus, the difference between N. W. and W. by S. is 5 points = the angle B A D, measured by the arc *a e* ;—the difference between N. by E. ½ E. and E. by N. (the opposite point to W. by S. the ship's course,) is 5½ points = the angle D B A, measured by the arc *c d*, and the difference between N. by E. ½ E. and N. W. is 5½ points = the angle A D B, measured by the arc *c e ;* and the side A B = 15 miles ; to find the side A D.—Hence, by oblique angled trigonometry, Problem I., page 177,

As the angle A D B =	5½ pts.	Log. co-secant =	10.054570
Is to the side A B =	15 miles,	Log. = . . .	1.176091
So is the angle A B D =	5½ pts.	Log. sine = . .	9.945430
To the side A D. =	15 miles,	Log. = . .	1.176091

Note.—The side A D might be determined independently of calculation, as thus ; the angles B and D are equal, for each is measured by an arc of

5½ points; and since equal angles are subtended by equal sides, therefore the side A D is equal to the side A B = 15 miles.

Again.—In the triangle A B C, the angles and the side A B are given, to find the side A C; thus, the difference between N. N. E. and W. by S. is 11 points = the angle B A C, measured by the arc *a b*; the difference between N. N. E. and N. E. ¾ E. is 2¾ points = the angle A C B, measured by the arc *b g*, and the difference between N. E. ¾ E. and E. by N. (the opposite point to W. by S. the ship's course,) is 2¼ points = the angle A B C, measured by the arc *g d*:—hence, the side A C may be found by the above-mentioned Problem; as thus:

As the angle A C B = 2¾ points	Log. co-secant =	10.369008
Is to the side A B = 15 miles,	Log. = . . .	1.176091
So is the angle A B C = 2¼ points	Log. sine = .	9.711050
To the side A C = 18.03 miles,	Log. = . .	1.256149

Now, in the triangle A D C there are given, the side A D = 15 miles; the side A C = 18.03 miles, and the included angle D A C, 6 points = the difference between N. N. E. and N. W. measured by the arc *e b*, to find the angles A D C and A C D, and the side D C.—Hence, by trigonometry, Problem III., page 179,

As the sum of A C and A B = 33.03 miles,	Log. ar. compt. =	8.481091
Is to difference of A C and A B = 3.03.	Log. = . .	0.481443
So is ½ sum of angs. A D C and A C D = 56°15′ 0″	Log. tang. =	10.175107
To ½ difference of those angles = . . 7°49′ 2″	Log. tang. =	9.137641

Angle A D C = . . 64° 4′ 2″
Angle A C D = . . 48°25′58″

To find the Side D C:—

As the angle A C D = 48°25′58″	Log. co-secant = . .	10.125995
Is to the side A D = 15 miles,	Log. = ...	1.176091
So is the angle D A C = 6 points	Log. sine = ...	9.965615
To the side D C = 18.52 miles,	Log. = ...	1.267701

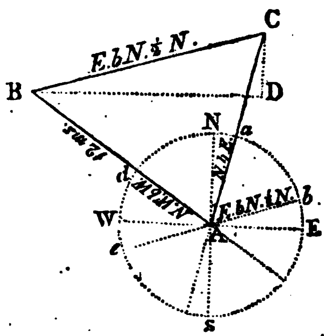
Hence, the distance between the two head-lands is 15½ miles.

To find the relative Bearings of the two given Head-lands :—

To the angle A C D = 48°25′58″ add the course or bearing from A to C = 2 points, or 22°30′ and the sum = 70°55′58″ is the bearing of D from C = S. 70°56′ W., or W. by S. ¾ S. nearly.—And, to the angle A D C = 64°4′2″ add the bearing from A to D = 4 points, or 45 degrees, and the sum = 109°4′2″ being taken from 180° gives 70°55′58″ = the bearing of C from D = N. 70°56′ E. or E. by N. ¾ N. nearly.

Example 4.

Being desirous of ascertaining the exact position of a head-land, with respect to latitude and longitude, it was carefully set, by an azimuth compass, and found to bear N. b. E., and after sailing N.W. b. W. 12 miles, it was again set, and observed to bear E. b. N. ½ N., due allowance being made for the variation of the compass. Now, the correct latitude of the ship at the last place of observation was 21°50′21″ N., and the longitude 85°9′6″ W.; required the latitude and longitude of the said head-land?



Solution.—In the oblique angled triangle ABC, where the side AC represents the first bearing of the head-land, the side BC the second bearing, and the side AB the distance sailed; given the three angles and the side AB = 12 miles, to find the side BC = the ship's distance from the headland at the second station. Thus, the difference between N. b. E., and N.W. b. W., is 6 points = the angle CAB, measured by the arc *ad*; the difference between N. W. b. W., and E. b. S. ½ S., the opposite point to E. b. N. ½ N., is 4½ points = the angle ABC, measured by the arc *de*, and the difference between E. b. N. ½ N., and N. b. E., is 5½ points = the angle ACB, measured by the arc *ab*. Hence, by oblique angled trigonometry, Problem I., page 107, to find the side BC = the ship's distance from the head-land at the second station.

As the angle ACB =	5½ points,	Log. co-secant =	10.054570
Is to the side AB =	12 miles,	Log. = . . .	1.079181
So is the angle CAB =	6 points,	Log. sine = .	9.965615
To the side BC =	12.57 miles,	Log. = . . .	1.099366

Hence, the distance of the ship from the head-land at the second station is 12½ miles, nearly.

To find the Difference of Latitude and Difference of Longitude between the Ship's Place at B, and the Head-Land C:—

In the right angled triangle B C D, given the angle C; 6½ points = the bearing of B from C, and the distance B C = 12.57 miles, to find the difference of latitude C D, and the difference of longitude B D; therefore, by Mercator's Sailing, Problem II., page 239,

As radius = 90°0′ Log. secant = . . 10.000000
Is to distance B C = 12.57 miles, Log. = 1.099366
So is the course C = 6½ points, Log. co-sine = 9.462824

To the diff. of lat. CD = 3.65 miles, Log. = 0.562190

As radius = 90°0′. Log. co-secant = . . 10.000000
Is to mer. diff. of lat. = 3.9 miles, Log. = 0.591065
So is the course C = 6½ points, Log. tangent = 10.518061

To the diff. of long. = 12.85 miles, Log. = 1.109126

Lat. of ship = 21°50′21″N.	M. pts = 1343.3	Lon. of ship = 85° 9′ 6″W.
Diff. lat. 3.65, or 3′39″N.		Diff. lon. 12.85, or 12.51 E.
Lat. of hd. ld. 21°54′ 0″N.	M. pts = 1347.2	Lon. of hd. ld. 84°56′15″W.

Meridional difference of latitude = 3.9 miles.

Hence, the latitude of the head-land is 21°54′0″ N., and its longitude 84°56′15″ W.

Note.—The foregoing examples contain all the cases in oblique sailing that are of any immediate import to the mariner. Other examples, indeed, might be given; but since they would rather tend to the exercise of the mind on trigonometrical subjects, than to any useful nautical purpose, they have therefore been intentionally omitted.

The two last examples will be found particularly useful in maritime surveying, when the operations are conducted on board of a ship or vessel.

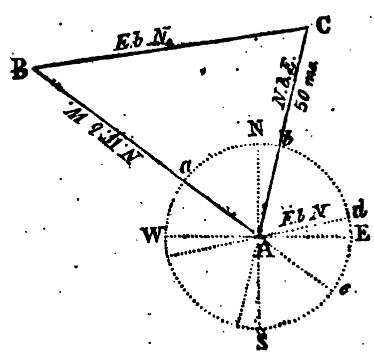
SOLUTION OF CASES IN WINDWARD SAILING.

Windward Sailing is the method of reaching the port or place bound to by the shortest route, when the wind is in a direction contrary to the direct course between the ship and the place to which she is bound.

When the wind is opposed to the course which a ship should steer from any one port to another, she is obliged to sail upon different tacks, close-hauled to the wind, in order to reach the port bound to. The object, therefore, of this method of sailing, is to find the proper course to be *conned* on each tack, so that the ship may arrive at the place to which she is bound, in the shortest time possible.

Example 1.

A ship that can lie within 6 points of the wind is bound to a port 50 miles directly to windward, which it is intended she shall reach on two tacks; the first being on the starboard tack, and the wind steady at N. b. E.; required the course and distance to be run upon each tack?



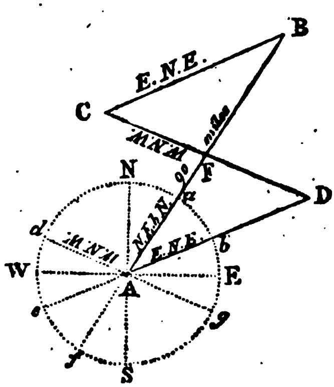
Solution.—Since the ship can lie within six points of the wind, which is at N. b. E., the course on the starboard tack will be N.W. b. W., and that on the larboard tack E. b. N. Now, in the annexed diagram, let the side A C represent the course and distance between the ship and her intended port; A B the course and distance to be made good on the starboard tack; and B C the course and distance to be made good on the larboard tack. Then, in the triangle A B C, the three angles are given to find the side A B or B C, which sides are mutually equal to each other, because the triangle is isosceles, and its vertex at B = the angle comprehended between those sides. Thus, the difference between N. b. E., and N.W. b. W., is 6 points = the angle B A C, measured by the arc *a b*; the difference between E. b. N., and S.E. b. E. (the opposite point to N.W. b. W.), is 4 points, measured by the arc *d e*; and the difference between N. b. E., and E. b. N., is 6 points, measured by the are $b d$; and since the distance A C is given = 50 miles, the side A B, or its equal B C,* may be readily determined by oblique angled trigonometry, Problem I., page 177; as thus:—

As the angle B = 4 points, Log. co-secant =	10.150515
Is to the distance A C = 50 miles, Log. =	1.698970
So is the angle C = 6 points, Log. sine =	9.965615
To the distance A B = 65.33 miles, Log. =	1.815100

Hence, it is evident that the ship must run 65.33 miles on the starboard tack, and 65.33 miles on the larboard tack, before she can reach her intended port.

Example 2.

A ship that can lie within 6 points of the wind is bound to a port bearing N.E. b. N., distance 90 miles, which it is intended she shall reach on three tacks, with the wind steady at north; required the course and distance to be run upon each tack, the first course being on the larboard tack?



Solution.—Since the wind is at north, and that the ship can lie within 6 points thereof, the course on the larboard tack will be E.N.E., and that on the starboard tack W.N.W.

In the annexed diagram, let the N.E. b. N. line A B = 90 miles, represent the bearing and distance between the ship and her intended port; let the E.N.E. line A D represent the first board on the larboard tack, and, parallel thereto, the line B C = the second board on that tack. And, since the ship is to make her port in three tacks, it is evident that the board on the starboard tack, represented by the W.N.W. line C D (parallel to $d g$), must bisect the line A B in the point F; and that, therefore, A F and F B are equal to one another, each being equal to 45 miles = half the line, or distance A B.

Now, since the straight line A B falls upon the two parallel straight lines C B and A D, it makes the alternate angles equal to one another; there-

* Since the angles A and C are equal to one another, the sides which subtend, or are opposite to those angles (viz., B C and A B), are also equal to one another.—Euclid, Book I., Prop. 6.

fore the angle A B C is equal to the angle B A D.—Euclid, Book I., Prop. 29. And because the straight line C D falls upon the two parallel straight lines C B and A D, it makes the angle A D B equal to the angle B C D, by the aforesaid proposition. And because the two triangles A D F and B C F have, thus, two angles of the one equal to two angles of the other, viz., the angle F A D to the angle F B C, and the angle A D F to the angle B C F; and the side A F of the one equal to the side B F of the other: therefore the remaining sides A D and D F of the one are equal to the remaining sides B C and C F of the other, each to each; and the third angle A F D of the one equal to the third angle B F C of the other.—Euclid, Book I., Prop. 26. Now, since the two triangles A F D and B F C are, thus, evidently equal to one another, we have only to compute the unknown sides of one, viz., of the triangle A F D, where the three angles are given, and the side A F, to find the sides A D and F D; thus, the difference between N.E. b. N. and E.N.E., is 3 points = the angle F A D, measured by the arc *a b;* the difference between E.N.E. and E.S.E. (the opposite point to W.N.W.), is 4 points = the angle A D F, measured by the arc *b g;* and the difference between W.N.W. and N.E. b. N., is 9 points = the angle A F D, measured by the arc *a d:* hence, by oblique angled trigonometry, Problem I., page 177,

To find the Side A D:—

As the angle D = 4 points,	Log. co-secant =	10. 150515
Is to the side A F = 45 miles,	Log. = .	1. 653213
So is the angle F = 9 points,	Log. sine =	9. 991574
To the side A D = 62. 42 miles,	Log. =	1. 795302

To find the Side F D:—

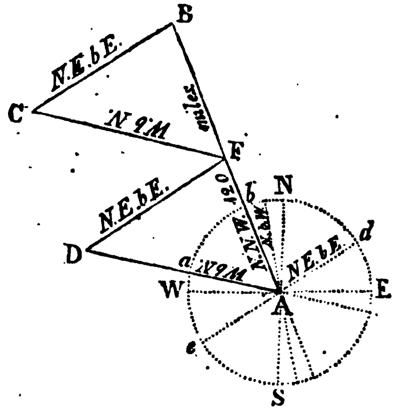
As the angle D = 4 points,	Log. co-secant =	10. 150515
Is to the side A F = 45 miles,	Log. = ...	1. 653213
So is the angle A = 3 points,	Log. sine = .	9. 744739
To the side F D = 35. 35 miles,	Log. = . .	1. 548467
Side D C = . . . 70. 70 miles.		

Hence it is evident that the ship must first run 62. 42 miles on the larboard tack; then 70. 70 miles on the starboard tack; and, again, 62. 42 miles on the larboard tack, before she can reach her intended port.

Example 3.

A ship that can lie within 6 points of the wind is bound to a port bearing N.N.W., distance 120 miles, which it is intended she shall make on four tacks, with the wind at N. b. W. The coast, which is to the eastward, trends in a direction nearly parallel to the bearing of the port, so that the ship must go about as soon as she reaches the straight line joining the two ports; required the course and distance to be run upon each tack, on the supposition that the ship's progress is not affected by either leeway or currents?

Solution.—Since the wind is N. b. W., and the land trends in a N.N.W. direction, the first board, therefore, must be on the starboard tack; and, as the ship can lie within 6 points of the wind, the course on the starboard tacks will be W. b. N., and that on the larboard tacks N.E. b. E.



In the annexed diagram, let the N.N.W. line AB, 120 miles, represent the bearing and distance between the ship and the port to which she is bound; let the W. b. N. line A D represent the first board on the starboard tack, and F C, parallel to A D, the second board on that tack; let the N.E. b. E. line D F represent the first board on the larboard tack, and, parallel thereto, the line C B = the second board on this tack. And, since the ship is to make her port in four tacks, without going to the eastward of the line A B, therefore, at the end of the second tack, she must reach the point F, which bisects or divides the distance A B into two equal parts, of 60 miles each; thus making A F = to A B.

Now, because the straight line A B falls upon the two parallel straight lines A D and F C, it makes the angle B F C equal to the interior and opposite angle F A D: and, because the straight line A B falls upon the two parallel straight lines F D and C B, it makes the angle A F D equal to the interior and opposite angle C B F,—Euclid, Book I., Prop. 29. And, since the two triangles A F D and F B C have, thus, two angles of the one equal to two angles of the other, viz., the angle A F D to the angle F B C, and the angle F A D to the angle B F C, and the side A F of the one equal to the side F B of the other,—therefore the remaining sides A D and D F of the one, are equal to the remaining sides F C and C B of the other, each to each; and the third angle A D F of the one equal to the third angle F C B of the other.—Euclid, Book I., Prop. 26.

The two triangles A D F and F C B, being, thus, clearly equal to one another in every respect, we have only to compute the unknown sides of one, viz., of the triangle A F D, where the three angles are given, and the side A F = 60 miles, to find the sides A D and D F; thus the difference between N.N.W. and W. b. N., is 5 points = the angle F A D, measured by the arc *a b*; the difference between W. b. N. and S.W. b. W. (the opposite point to N.E. b. E.), is 4 points = the angle A D F, measured by the arc *a e*; and the difference between N.N.W. and N.E. b. E., is 7 points = the angle A F D, measured by the arc *b d*.

Hence, by oblique angled trigonometry, Problem I., page 177,

To find the Side A D = F C:—

As the angle D = 4 points, Log. co-secant =	10.150515
Is to the side A F = 60 miles, Log. = . . .	1.778151
So is the angle F = 7 points, Log. sine = .	9.991574
To the side A D = 83.22 miles, Log. = . .	1.920240

To find the Side D F = C B:—

As the angle D = 4 points, Log. co-secant =	10.150515
Is to the side A F = 60 miles, Log. = ...	1.778151
So is the angle A = 5 points, Log. sine = .	9.919846
To the side D F = 70.55 miles, Log. = .	1.848512

From this it is manifest, that the ship must first run 83.22 miles upon the starboard tack; then 70.55 miles upon the larboard tack; then 83.22 miles again upon the starboard tack; and 70.55 miles upon the larboard tack, before she can reach the port to which she is bound.

SOLUTION OF CASES IN CURRENT SAILING.

Current Sailing is the method of determining the true course and distance made good by a ship, when her own motion is affected or combined with that of the current in which she sails.

A *current* is a progressive motion of the water, causing all floating bodies thereon to move in the direction to which its stream is impelled. The *setting* of a current is that point of the compass towards which the water runs; and the *drift* of a current is the rate at which it runs per hour.

When a ship sails in the direction of a current, her velocity will be equal to the sum of her own proper motion and the current's drift; but when she sails directly against a current, her velocity will be expressed by the difference between her own proper motion and the drift of the current: in this case, the absolute motion of a ship will be a-head, if her proper velocity exceeds the drift of the current; but if it be less, she will make sternway. When a ship's course is oblique to the direction of a current, her true course and distance will be compounded of the course and distance given by the log, and of the observed setting and drift of the current.

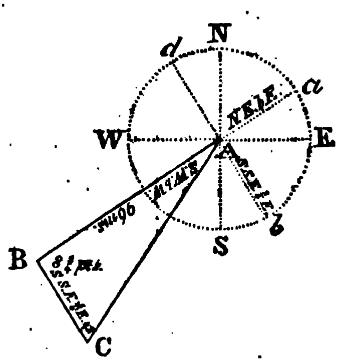
When a ship's course and distance by the log, and the setting and drift of the current in which she sails are given, the true course and distance made good may be found by a trigonometrical solution of the triangles forming the figure; but the easiest and most expeditious method of finding the course and distance made good, particularly when a ship sails upon different courses, is by resolving a traverse, in which the setting and drift of the current are to be esteemed as an additional course and distance to those exhibited by the log.

Example 1.

If a ship sails S.W. b. W., at the rate of 4 knots an hour, in a current setting S.S.E. ¾ E., at the rate of 1¾ miles an hour; required the course and distance made good in 24 hours?

Solution.—$4^{m} \times 24^{h} = 96$ miles, the distance sailed, by log, in 24 hours;

And $1\frac{3}{4}^{m} \times 24 = 42$ miles, the observed drift of the current in 24 hours.



In the annexed diagram, let the side AB of the triangle ABC represent the course and distance sailed by the log, and the side BC parallel to db the setting and drift of the current; then, the side AC will represent the course and distance made good in the given time. Now, in the triangle ABC, given the side AB = 96 miles, the side BC = 42 miles, and the included angle B = 8¼ points, being the difference between S.S.E. ¾ E. and N.E. b. E. (the opposite point to S.W. b. W.), measured by the arc ab, to find the angles A and C, and the true distance AC. Hence, by oblique angled trigonometry, Problem III., page 179.

To find the Angles A and C :—

As AB + BC = 138 miles,	Log. ar. comp. =	7.860121
Is to AB − BC = 54 miles,	Log. =	1.732394
So is ½ sum of the angles = 43°35′37½″	Log. tangent =	9.978673
To ½ diff. of the angles = 20.25.59	Log. tangent =	9.571188

Angle C = ... 64° 1′36½″
Angle A = ... 23° 9′38½″

To find the true Distance = AC :—

As the angle C = 64°1′36½″	Log. co-secant =	10.046241
Is to the side AB = 96 miles,	Log. = ...	1.982271
So is the angle B = 8¼ points,	Log. sine = . .	9.999477
To the true distance = AC = 106.7 miles,	Log. =	2.027989

To find the Course made good :—

From the angle SAB = S.W. b. W., or 56°15′, subtract the angle CAB = 23°9′38½″, and the remainder, 33°5′21½″ = the angle SAC, is the course made good.

Hence the course made good is S. 33°5′ W., or S.W. b. S. nearly, and the distance 106¾ miles nearly.

To find the Course and Distance made good by the Traverse Table :—

Traverse Table.

Corrected Courses.	Distance.	Difference of Latitude.		Departure.	
		N.	S.	E.	W.
S.W. b. W.	96	—	53.3	—	79.8
Current S.S.E ¾ E.	42	—	36.0	21.6	—
		Diff. lat. =	89.3	21.6	79.8
				—	21.6
				Depart. =	58.2

Now, by Problem II., page 108,

The difference of latitude 89.3, and the departure 58.2, are found to agree nearest abreast of 33°, under or over distance 107.

Hence the course made good is S. 33° W., or S.W. b. S., and the distance 107 miles; which nearly agrees with the above result.

Example 2.

Suppose a ship sails N.W. 65 miles, W.N.W. 70 miles, and N. b. E. 71 miles, in a current that sets S.E. b. S. 36 miles in the same time; required the true course and distance made good?

TRAVERSE TABLE.

Corrected Courses.	Distance.	Difference of Latitude.		Departure.	
		N.	S.	E.	W.
N.W.	65	46.0	—	—	46.0
W.N.W.	70	26.8	—	—	64.7
N. b. E.	71	69.6	—	13.9	—
Current S.E. b. S.	36	—	29.9	20.0	—
		142.4	29.9	33.9	110.7
		29.9			33.9
		112.5			76.8

Solution.—With the difference of latitude and departure, thus found, the course and distance made good may be determined by Problem II., page 108; as thus:

The difference of latitude 112.5, and the departure 76.8, are found to agree nearest abreast of 34° under or over 136.

Hence the direct course made good is N. 34° W., or N.W. b. N. nearly, and the distance 136 miles.

To find the Course and Distance made good by Calculation:—

This may be done by means of the 5th analogy, page 237; as thus:

To find the true Course:—

As the diff. of lat. =	112.5	Log. ar. comp. =	7.948848
Is to radius = . . .	90°0′	Log. sine = . .	10.000000
So is the departure =	76.8	Log. = ...	1.885361
To the true course =	34°19′12″	Log. tangent =	9.834209

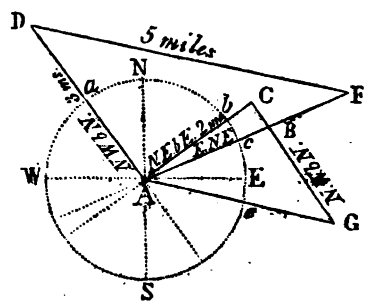
To find the true Distance :—

As radius = . . .	90°0′	Log. co-secant =	10.000000
Is to diff. of lat. =	112.5	Log. = . . .	2.051152
So is the true course =	34°19′12″	Log. secant = .	10.083072
To the distance =	136.2 miles,	Log. = . . .	2.134224

Hence the course made good is N. 34°19′ W., or N.W. b. N. nearly, and the distance 136 miles.

Example 3.

There is a harbour 2 miles broad, in which the tide is running N.W. b. N. at the rate of 3 miles an hour. Now, a waterman who can pull his boat at the rate of 5 miles an hour, wishes to cross the harbour to a point on the opposite side bearing E.N.E.; required the direction in which he should pull, so as to meet with the least possible resistance from the force of the tide in gaining the intended point, and the time that it will take him to reach that point?



Solution.—Since the principles of this Problem are but little understood by the generality of young navigators, a brief account of the geometrical construction will be given, with the view of elucidating and rendering familiar the nature of the corresponding calculations. Thus,

With the chord of 60° describe the arch N E S W; draw the north and south line N S, and, at right angles thereto, the east and west line W E; make the arc N a = 3 points, and draw the N.W. b. N. line A a D, which make equal to 3 miles (taken from any scale of equal parts), to represent the direction of the harbour; perpendicular thereto draw the N.E. b. E. line A b C, which make equal to 2 miles, to represent the breadth of the harbour; and, from the point C, draw the line C G parallel to A D, which lines will represent the eastern and western shores of the harbour respectively.

Make N c equal to 6 points, and draw the E.N.E. line A c F, cutting C G in B; then will B represent the point to which the waterman intends to cross. Take 5 miles in the compasses; place one foot on the point D; and where the other falls upon the E.N.E. line A F, there make a point, as at F, and draw the line D F; parallel to which, draw the line A G, and it will represent the distance and direction in which the waterman must pull to gain the point B: for in the time that he would reach the point G, by pulling at the rate of 5 miles an hour, the tide, running at the rate of 3 miles an hour, would carry him to the point B; because B G bears the same proportion to 3 miles an hour that A G does to 5. Now, A G, being applied to the same scale of equal parts from which the other sides were taken, will measure 2.95 miles, and the angle G A E, or e A E, being applied to the line of chords, will measure 13°33′; hence the direction in which he should pull, is E. 13°33′ S, or E. b. S. ¼ S. nearly.

Now, in the triangle A D F, given the side A D = 3 miles, the side D F = 5 miles, and the angle D A F = 9 points (being the difference between E.N.E. and N.W. b. N., measured by the arc a c), to find the angle A F D. Hence, by oblique angled trigonometry, Problem I., page 177,

As the side D F = 5 miles, Log. ar. comp. =	9.301030
Is to the angle A = 9 points, Log. sine = .	9.991574
So is the side A D = 3 miles, Log. = . . .	0.477121
To the angle A F D = 36°2′55″ Log. = .	9.769725

Now, because the straight line A F falls upon the two parallel straight lines D F and A G, it makes the alternate angles equal to one another; therefore the angle D F A is equal to the angle F A G,—Euclid, Book I., Prop. 29; but the angle D F A is known to be 36°2′55″; therefore the angle F A G, measured by the arc c e, is also equal to 36°2′55″; and if to the angle F A G we add the angle B A C = 11°15′ (being the difference between N.E. b. E. and E.N.E., measured by the arc b c), the sum = 47°17′55″ is the angle C A G, measured by the arc b e. Then,

In the right angled triangle A C G, given the angle C A G = 47°17′55″ and the side A C = 2 miles, the breadth of the harbour, to find the side A G equal to the distance which the waterman must pull before he can reach the point B. Hence, by right angled trigonometry, Problem II., page 172, making A C radius,

As radius = . . 90°0′ Log. co-secant =	10.000000
Is to the side A C = 2 miles, Log. = . . .	0.301030
So is the angle C A G = 47°17′55″ Log. secant =	10.168657
To the distance = A G = 2.949 miles, Log. =	0.469687

To find the Time requisite to reach the Point B:—

As distance 5 miles, Log. ar. comp. =	9.301030
Is to 1 hour, or 60 minutes, Log. =	1.778151
So is A G = 2.949 miles, Log. =	0.469687
To the time = 35ᵐ 23ˢ.34 = 35ᵐ.389 Log. =	1.548868

To find the Direction in which he should pull or steer:—

From the angle $bAe = 47°17'55''$, take away the angle $bAE = 33°45'$, and the remaining angle $EAe = 13°32'55''$ is the direct course which he should steer; viz., E. 13°33′ S., or E. b. S. ¾ S. nearly.

Hence it is evident, that if the waterman pulls in the direction of E. 13°33′ S. or E. b. S. ¾ S. nearly, he will reach the intended point in the space of about 35 minutes and 23 seconds.

SOLUTION OF PROBLEMS RELATIVE TO THE ERRORS OF THE LOG-LINE AND THE HALF-MINUTE GLASS, BY LOGARITHMS.

The instruments generally employed at sea, for finding the distance run by a ship in a given time, are the log-line and the half-minute glass. Now, since a ship's reckoning is kept in nautical miles, of which 60 make a degree, the distance between any two adjacent knots on the log-line should bear the same proportion to a nautical mile that half a minute does to an hour; viz., the *one hundred and twentieth part.* And, since a nautical mile contains 6080 feet, the true length of a knot is equal to 6080 divided by 120; that is, 50 feet and 8 inches: but, because it is advisable at all times to have the reckoning a-head of the ship, so that the mariner may be looking out for the land in sufficient time, instead of his making it unexpectedly, or in an unprepared moment, 48 feet, therefore, is the customary measure allowed to a knot. And, to make up for any time that may be unavoidably lost, in turning the half-minute glass, its absolute measure should not exceed *twenty-nine seconds and a half.*

The method of finding the hourly rate of sailing, or distance run in a given time by the log-line and the half-minute glass, is subject to many errors: thus, a new log-line, though divided with the utmost care and attention, is generally found to contract after being first used;

and, after some wear, it stretches so very considerably as to be out of due proportion to the measure of the half-minute glass. Nor is the half-minute glass itself free from error: for this instrument is so very liable to be affected by various changes of weather, from moist to dry, and conversely, that notwithstanding its being perfectly correct when first taken on board, yet it alters so sensibly at sea, that at one time it will run out in the short space of 26 or 27 seconds, and at another not till it has passed the half-minute by several seconds. Hence it becomes indispensably necessary to examine those instruments frequently; and, if found erroneous, to correct the ship's run accordingly. This may be done by means of the following *rules*, which are adapted to a log-line of 48 feet to a knot, and to a glass measuring 30 seconds.

Problem I.

Given the Distance sailed by the Log, and the Number of Seconds run by the Glass; to find the true Distance, the Line being truly divided.

Rule.

To the arithmetical complement of the logarithm of the number of seconds run by the glass, add the logarithm of the distance given by the log, and the constant logarithm 1.477121*; the sum of these three logarithms, abating 10 in the index, will be the logarithm of the true distance sailed.

Example 1.

Let the hourly rate of sailing be 11 knots, and the time measured by the glass 33 seconds; required the true rate of sailing?

Seconds run by the glass = 33, Log. ar. comp. =	8.481486
Rate of sailing, by log = 11 knots, Log. = . .	1.041393
Constant log. = ...	1.477121
True rate of sailing = 10 knots, Log. = . . .	1.000000

Example 2.

If a ship sails 198 miles by the log, and the glass is found, on examination, to run out in 26 seconds, required the true distance sailed?

Seconds run by the glass = 26, Log. ar. comp. =	8.585027
Distance sailed by log = 198 miles, Log. = .	2.296665
Constant log. = ...	1.477121
True distance sailed = 228.46 miles, Log. = .	2.358813

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* This is the logarithm of 30 seconds, the true measure of the half-minute glass.

Problem II.

Given the Distance sailed by the Log, and the measured Length of a Knot; to find the true Distance, the Glass being correct.

Rule.

To the logarithm of the distance given by the log, add the logarithm of the measured length of a knot, and the constant logarithm 8.318759*; the sum of these three logarithms, rejecting 10 in the index, will be the logarithm of the true distance sailed.

Example 1.

Let the hourly rate of sailing be 9 knots, by a log-line which measures 53 feet to a knot; required the true rate of sailing?

Hourly rate of sailing =	9 knots, Log. = . .	0.954243
Measured length of a knot =	53 feet, Log. = . .	1.724276
Constant log. =	. . . .	8.318759
True rate of sailing =	9.937 knots, Log. = . .	0.997278

Example 2.

Let the distance sailed be 240 miles, by a log-line which measures 43 feet to a knot; required the true distance sailed?

Distance sailed by log =	240 miles, Log. =	2.380211
Measured length of a knot =	43 feet, Log. =	1.633469
Constant log. =	. . . .	8.318759
True distance sailed =	215 miles, Log. = .	2.332439

Problem III.

Given the measured Length of a Knot, the Number of Seconds run by the Glass, and the Distance sailed by the Log; to find the true Distance sailed.

Rule.

To the arithmetical complement of the logarithm of the number of seconds run by the glass, add the logarithm of the measured length of a knot, the logarithm of the distanee sailed by the log, and the constant

---

* This is the arithmetical complement of the logarithm of 48, the generally-approved length of a knot.

logarithm 9.795880*; the sum of these four logarithms, rejecting 20 from the index, will be the logarithm of the true distance sailed.

Example 1.

Let the hourly rate of sailing be 12 knots, the measured length of a knot 44 feet, and the time noted by the glass 25 seconds; required the true rate of sailing?

Seconds run by the glass = 25, Log. ar. comp.=	8.602060
Measured length of a knot=44 feet, Log. =	1.643453
Rate of sailing by log = 12 knots, Log. =	1.079181
Constant log. = ...	9.795880
True rate of sailing = 13.2 knots, Log. = .	1.120574

Example 2.

Let the distance sailed by the log be 354 miles, the measured length of a knot 52 feet, and the interval run by the glass 34 seconds; required the true distance sailed?

Seconds run by the glass = 34, Log. ar. comp.=	8.468521
Measured length of a knot = 52 feet, Log. = .	1.716003
Distance sailed by log = 354 miles, Log. =	2.549003
Constant log. = ...	9.795880
True distance = 338.38 miles, Log. = . . .	2.529407

Problem IV.

Given the Number of Seconds run by any Glass whatever, to find the corresponding Length of a Knot, which shall be truly proportional to the Measure of that Glass.

Rule.

To the logarithm of 10 times the number of seconds run by the glass, add the constant logarithm 9.204120, and the sum, abating 10 in the index, will be the logarithm of the proportional length of a knot, in feet, corresponding to the given glass.

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* This is the sum of the two preceding constant logarithms; thus 1.477121 + 8.318759 = 9.795880.

Example 1.

Required the length of a knot corresponding to a glass that runs 27 seconds?

Number of seconds 27 × 10 = 270	Log. =	2.431364
Constant log. = ...		9.204120
True length of a knot, in feet, = 43.2	Log. =	1.635484

Example 2.

Required the length of a knot corresponding to a glass that runs 34 seconds?

Number of seconds 34 × 10 = 340	Log. =	2.531479
Constant log. = ...		9.204120
True length of a knot, in feet, = 54.4	Log. =	1.735599

SOLUTION OF A PROBLEM IN GREAT CIRCLE SAILING,

Very useful to Ships going to Van Diemen's Land, or to New South Wales, by the way of the Cape of Good Hope.

Great Circle Sailing is the method of finding the successive latitudes and longitudes which a ship must make; with the courses that she must steer, and the distances to be run upon such courses, so that her track may be nearly in the arc of a great circle, passing through the place sailed from and that to which she is bound.

The angle of position is an angle which a great circle, passing through two places on the sphere, makes with the meridian of one of them; and shows the true position of each place, in relation to the intercepted arc of the great circle and the respective meridians of those places.

The *polar angle* is an arc of the equator intercepted between the meridians, or circles of longitude, of two given places on the sphere.

On the sphere, the shortest distance between two places is expressed by the arc of a great circle intercepted between those places: consequently the spiral, or rhumb line, passing through two places on the sphere, can never represent the shortest distance between those places, unless such rhumb line coincides with the arc of a great circle; and this can never happen but when the places are situate under the equator, or under a meridian. Hence, although Mercator's Sailing resolves correctly all the cases incident to a ship's course along the rhumb line passing through two places,—yet, since there is no case in which the course along the direct rhumb line indicates the shortest distance between those places, except when they both lie under the same meridian, or under the equator, the distance, therefore, obtained by that method of sailing, must always exceed the truth (the above-mentioned positions excepted); and the nearer the places are to a parallel of latitude, and the farther they are removed from the equator, the greater will be the error in distance.

Now, since it is frequently an object of the greatest importance, to the commander of a ship, to reach the port to which he is bound by the shortest route, and in the least time possible,—particularly to the commander of a ship bound from the Cape of Good Hope to Van Diemen's Land, or to His Majesty's Colony at New South Wales, where the length of the voyage generally occasions a great scarcity of fresh water,—the following Problem is, therefore, given, by which all the particulars connected with the shortest possible route between those places will be fully and clearly exhibited.

Were a ship to sail exactly in the arc of a great circle (not under the equator or upon a meridian), the navigator would be obliged to keep continually altering her course; but, as this would be attended with more trouble and inconvenienee than could be reasonably admitted into the general practice of navigation, it has been deemed sufficiently exact to determine a certain number of latitudes and longitudes through which a ship should pass, with the relative courses and distances between them; so that the track, thus indicated, though not exactly in the arc of a great circle may, notwithstanding, approximate so very near thereto, as not to produce any sensible difference between it and the true spherical track.

Problem.

Given the Latitudes and Longitudes of two Places on the Globe, to determine the true spherical Distance between them; together with the angular Position of those Places with respect to each other, and the successive Positions at which a Ship should arrive when sailing on or near to the Arc of a great Circle, agreeably to any proposed Change of Longitude.

Rule.

1. Find the true spherical distance between the two given places, by oblique angled spherical trigonometry, Problem III., page 202.

2. Find the highest latitude which the great circle touches that passes through the two given places; that is, find the perpendicular from the pole

to that circle by right angled spherical trigonometry, Problem II., page 185; and find, also, the several polar angles (made by the proposed alterations of longitude,) contained between the perpendicular, thus found, and the several meridians corresponding to the successive changes of longitude.

3. With the co-latitude or perpendicular, so found, and the several polar angles, compute as many corresponding co-latitudes by right angled spherical trigonometry, Problem IV., page 188.

4. With the several latitudes and longitudes through which the ship is to pass, compute the corresponding courses and distances by Mercator's Sailing, Problem I., page 238; and they will indicate the path along which a ship must sail, so as to keep nearly in the arc of a great circle.

Note.—**The smaller the alterations are in the longitude, the nearer will the track, thus determined, approximate to the truth; because, in very small arcs, the difference between the arc and its corresponding chord, sine, or tangent, is so very trifling, that the one may be substituted for the other, in most nautical calculations, without producing any sensible difference in the result.**

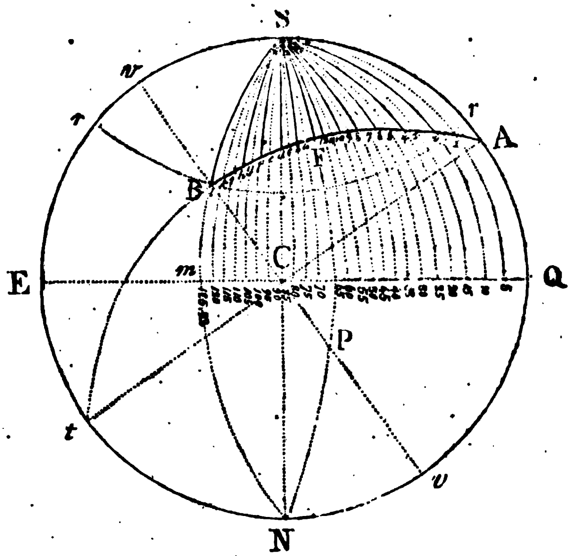
Example 1.

A captain of a ship bound from the Cape of Good Hope (in latitude 34°24′ S., and longitude 18°32′ E.) to New South Wales, being desirous of making the north point of King's Island, at the western entrance to Bass' Strait (in latitude 39°37′ S., and longitude 143°54′ E.), by the shortest possible route, proposes, therefore, to sail as near to the arc of a great circle as he can, by altering the ship's course at every 5 degrees of longitude; required the latitude at each time of altering the course, and, also, the respective courses and distances between those several latitudes and longitudes made by the proposed changes?

Cape of Good Hope, Latitude =	34°24′S.	Longitude =	18°32′E.
King's Island, N. point, Latitude =	39.37 S.	Longitude =	143.54 E.
		Difference of longitude =	125°22′

Stereographic Projection.

With the chord of 60 degrees, describe the primitive circle S E N Q on the plane of the meridian, or circle of longitude passing through the Cape of Good Hope; draw the line E Q to represent the equator, and, at right angles thereto, the line S N for the earth's axis; then S represents the south, or elevated pole, and N the north, or depressed pole. Take the latitude of the Cape of Good Hope in the compasses from the line of chords = 34°24′, and lay it off from Q to A; draw the diameter A C *t*, and, at right angles thereto, the diameter *v c v*. Take the latitude of King's Island = 39°37′ in the compasses from the line of chords, and lay it off from Q to *r*, and also from E to *r*; and, with the tangent of its complement = 50°23′ draw the parallel circle *r r*. Take the difference of longitude 125°22′ from the scale of semi-tangents, and lay it off on the equator from Q to *m*: thus 90° will reach from Q to C; then the



excess above 90°, viz., 35°22′, will reach from C to *m*. With the secant of the complement of the excess of the difference of longitude above 90° = 54°38′ (being the supplement of the difference of longitude to 180°), describe the great circle S *m* N; the intersection of which with the parallel circle *r r* at B shows the position of King's Island. Then, the great circle S B *m* N represents the meridian of King's Island. Through the three points A B *t* describe a great circle, and then will the arc A B represent the true spherical distance between the Cape of Good Hope and King's Island; in which A represents the place of the former, and B that of the latter. Through P, the pole of the great circle A B *t*, draw the great circle S F P N; then the arc S F will be perpendicular to the arc A B. Hence, S F represents the least co-latitude at which the ship should arrive in her spherical passage from the Cape of Good Hope to King's Island; which, being reduced to the primitive circle, and measured on the scale of chords, gives about 31¼ degrees. The arc A B, reduced to the primitive circle, and measured on the line of chords, shows the true spherical distance to be about 90½ degrees. The angle S A B is the angle of position which the meridian of the Cape of Good Hope makes with King's Island; and the angle S B A is the angle of position which the meridian of King's Island makes with the Cape of Good Hope. These angles, being reduced to the primitive circle, and measured on the line of chords, give about 39° for the former, and 42¼° for the latter.

Note.—The remaining parts of the projection will be explained hereafter.

Calculation.

In the oblique angled spherical triangle A S B, there are given two sides and the included angle, to find the remaining angles and the third side; viz., the side A S = 55°36′, the co-latitude of the Cape of Good Hope; the side B S = 50°23′, the co-latitude of King's Island; and the angle A S B = 125°22′, the difference of longitude between those places, to find the true spherical distance A B, and the respective angles of position S A B and S B A. The distance may be readily found by Remark 1 or 2, to Problem III., page 203 or 204; as thus:

Diff. of long. A S B = 125°22′ ÷ 2 =	62°41′	Twice log. sine	19.897300	
Co-lat. of Cape of Good Hope = AS	55.36	Log. sine =	9.916514	
Co-lat. of King's Island = B S	50.23	Log. sine =	9.886676	
		Sum =	39.700490	
Diff. of co-lats. =	5°13′	Half =	19.850245	. . 19.850245
Half diff. of do. =	2°36′30″	Log. sine =	8.658090	
Arch = ...	86°19′27″	Log. T. =	11.192155	Log. sine 9.999106
Half the side AB =	45.13. 8½			Log. sine = 9.851139

Side AB = . . . 90°26′17″; which is the true spherical distance between the two given places.

To find the Angle of Position at Cape of Good Hope = Angle S A B:—

This is found by Problem I., page 198; as thus:

As the distance	AB	90°26′17″	Log. co-secant =	10.000013
Is to diff. of long.	ASB	125.22. 0	Log. sine =	9.911405
So is the co-lat.	BS	50.23. 0	Log. sine =	9.886676
To the ang. of posit.	SAB	38°55′ 1″	Log. sine =	9.798094

To find the Angle of Position at King's Island = Angle S B A:—

This is found by Problem I., page 198; as thus:

As the distance	AB	90°26′17″	Log. co-secant =	10.000013
Is to diff. of long.	ASB	125.22. 0	Log. sine =	9.911405
So is the co-lat.	AS	55.36. 0	Log. sine =	9.916514
To the ang. of pos.	SBA	42°17′20½″	Log. sine =	9.827932

To find the Perpendicular F S = the Complement of the highest southern Latitude at which the Ship should arrive in the proposed Route :—

Here we have a choice of two right angled spherical triangles, viz., A S F and B S F; in each of which the hypothenuse and the angle at the base are given, to find the perpendicular. Thus, in the triangle A S F, given the hypothenuse A S, 55°36′ = the co-latitude of the Cape of Good Hope, and the angle at the base, S A F 38°55′1″ = the angle of position at that place, to find the perpendicular F S = the complement of the highest latitude at which the ship should arrive. Hence, by right angled spherical trigonometry, Problem II., page 185,

As radius = ...	90° 0′ 0″	Log. co-sec.=	10.000000
Is to co-lat. C. Good Hope A S =	55. 36. 0	Log. sine =	9.916514
So is the ang. of position S A F =	38. 55. 1	Log. sine =	9.798094
To the perpendicular F S = .	31. 13. 13½	Log. sine =	9.714608
Highest lat. at which the ship should arrive = ...	58°46′46½″ south.		

Hence the true spherical distance between the Cape of Good Hope and the north point of King's Island, is 90°26′17″, or 5426. 3 miles; the angle of position at the Cape of Good Hope, is 38°55′1″; and that at King's Island, 42°17′20″; and the highest southern latitude at which the ship should arrive, 58°46′46″. Now, by Mercator's Sailing, the course from the Cape of Good Hope to King's Island is S. 87°1′ E., or E. ¼ S. nearly, and the distance 6011. 2 miles; whence it is evident, that if a ship sails on the direct rhumb line indicated by Mercator's Sailing, she will have to run a distance of no less than 585 miles more than if her course had been shaped along the arc of a great circle passing through the two given places.

Now, since it is extremely difficult for persons unacquainted with the doctrine of spherics to reconcile a route to their senses, as the shortest distance between two places, which carries them nearly 22 degrees to the southward of the middle latitude between the two given places; and since, in sailing on the arc of a great circle, the course ought to be changing constantly, with the view of keeping the side of the polygon on which the ship sails as near to the arc of its circumscribing circle as possible, or that the difference between the arc and its chord may be so small that the one may be substituted for the other without sensibly affecting the result in nautical operations,—I shall, therefore, show the successive latitudes at which the ship should arrive at every 5 degrees of longitude, as proposed (which is sufficiently near to preserve the desired ratio between the arc and its chord); together with the respective courses and distances, by Mercator's Sailing, between those several successive latitudes and longitudes: then, if the sum of the several distances coincide, or nearly so, with the true spherical distance found as above, the senses must become reconciled to the propriety of adopting that high southern route at which they originally seemed to recoil.

In order to determine the several successive latitudes at which the ship must arrive, we must previously compute the vertical or polar angles A S F and B S: then, if the sum of these angles makes up the whole difference of longitude, or polar angle between the two given places, it will be a convincing and satisfactory proof that, for so far, the operations will have been properly conducted. Now, in the right angled spherical triangle A S F, given the hypothenuse A S, 55°36′ = the co-latitude of the Cape of Good Hope, and the perpendicular F S, 31°13′13½″ = the complement of the highest latitude at which the ship should arrive, to find the vertical or polar angle F S A. And, in the right angled spherical triangle B S F, given the hypothenuse B S, 50°23′ = the co-latitude of King's Island, and the perpendicular F S, 31°13′13½″, to find the vertical or polar angle B S F. Hence, by right angled spherical trigonometry, Problem I., page 184,

To find the Polar Angle A S F.—

As radius = ...	90° 0′ 0″	Log. co-secant =	10.000000
Is to the co-latitude A S =	55.36. 0	Log. co-tangent =	9.835509
So is the co-latitude F S =	31.13.13½	Log. tangent =	9.782550
To the polar angle A S F =	65°28′48″	Log. co-sine =	9.618059

To find the Polar Angle B S F:—

As radius = ...	90° 0′ 0″	Log. co-secant =	10.000000
Is to the co-latitude B S =	50.23. 0	Log. co-tangent =	9.917906
So is the co-latitude F S =	31.13.13½	Log. tangent =	9.782550
To the polar angle B S F =	59°53′12″	Log. co-sine =	9.700456

And, since the sum of the polar angles, thus obtained, viz., A S F 65°28′48″ + B S F 59°53′12″ = 125°22′0″, makes up the whole difference of longitude between the two given places expressed by the whole angle A S B, it shows that thus far the work is right.

Now, on the equator, from Q to *m*, lay off the proposed changes of longitude, viz., 5°, 10°, 15°, 20°, 25°, &c. These are to be taken respectively, in the compasses, from the scale of semi-tangents, reckoning *backwards* from 90° towards 0°, till the proposed changes of longitude reach the centre C; and then *forwards* on that scale, or from 0° towards 90°, till those changes of longitude meet the point *m*: thus, the extent from 90°

to 85° will reach from Q to 5°; the extent from 90° to 80°, will reach from Q to 10°, and so on to the centre C; then, the extent from 0° to 5°, will reach from C to 95°; the extent from 0° to 10°, will reach from C to 100°, and so on to the point *m*. Through the points S and N, and the several points made by the proposed changes of longitude on the equator, draw arcs of great circles, viz., S 1, 5°; S 2, 10°; S 3, 15°; S 4, 20°, &c. &c.; and then the arcs S 1, S 2, S 3, &c. &c., will represent the respective complements of the several latitudes at which the ship should arrive at the given changes of longitude; the true values of which may be found in the following manner, viz.,

From the polar angle A S F, subtract the proposed changes of longitude continually; and the several polar angles made by those changes, and contained between the perpendicular F S and the co-latitude of the Cape of Good Hope = S A, will be obtained. Thus, from the polar angle A S F = 65°28′48″, let 5° be continually *subtracted*, and the results will be F S 1 = 60°28′48″; F S 2 = 55°28′48″; F S 3 = 50°28′48″, &c. &c. And, since the last subtraction in this triangle leaves the remainder, or polar angle, F S 12 = 5°28′48″, which is 28′48″ greater than the proposed alteration of longitude, therefore, in the triangle B S F, where the polar angle S is 59°53′12″ (and where the several polar angles contained between the perpendicular F S and the co-latitude of King's Island are to be determined by a *contrary process* to that which was observed in the preceding triangle), the first polar angle is expressed by 5° − 28′48″ = 4°31′12″ = the angle F S *a*; to which let the proposed alterations of longitude be continually *added*, and the sums will be F S *b* = 9°31′12″; F S *c* = 14°31′12″, &c. &c. Those various results are to be arranged agreeably to the form exhibited in the first column of the following Table; and, since they respectively express the true measures of the several polar angles contained between the meridians of the given places and those of the several co-latitudes to which they correspond, it is, therefore, manifest that those results reduce the two right angled spherical triangles (A S F and B S F) into a series of right angled spherical triangles; to each of which the perpendicular F S is common. Then, in each of these triangles, we have the perpendicular and the angle adjacent, to find the hypothenuse or co-latitude. Thus, in the right angled spherical triangle F S 1, right angled at F, given the perpendicular F S = 31°13′13½″, and the polar angle F S 1 = 60°28′48″, to find the hypothenuse or co-latitude S 1; in the right angled spherical triangle F S 2, given the perpendicular F S = 31°13′13½″, and the polar angle F S 2 = 55°28′48″, to find the hypothenuse or co-latitude S 2, &c. &c. Hence, by right angled spherical trigonometry, Problem IV., page 188, To find the Hypothenuse, or Co-Latitude = S 1 :—

As the perpendicular F S =	31°13′13½″	Log. co-tangent =	10.217450*
Is to the radius = . .	90. 0. 0	Log. sine = .	10.000000
So is the angle F S 1 = .	60.28.48	Log. co-sine =	9.692607
To the co-latitude S 1 =	50.53.28	Log. co-tangent =	9.910057
First latitude = ...	39° 6′32″S., at which the ship should arrive.		

To find the Hypothenuse, or Co-Latitude = S 2 :—

As the perpendicular F S =	31°13′13½″	Log. co-tangent =	10.217450*
Is to the radius = . .	90. 0. 0	Log. sine = . .	10.000000
So is the angle F S 2 =	55.28.48	Log. co-sine = .	9.753349
To the co-latitude S 2 =	46.55.29	Log. co-tangent =	9.970799
Second latitude = . .	43° 4′31″S., at which the ship should arrive.		

Hence, the first latitude at which the ship should arrive, is 39°6′32″S.; and the second latitude 43°4′31″ S.: and, since it is the latitude itself, and not its complement, that is required, if the log. tangent of the sum of the three logarithms be taken, it will give the latitude direct; and, by rejecting the radius, the work will be considerably facilitated. Proceeding in this manner, the several successive latitudes corresponding to the proposed alterations of longitude will be found, as in the third column of the following Table.

Now, let the several successive longitudes be arranged (agreeably to the proposed change, and to the measure of the corresponding polar angles,) as given in the second column of the following Table; and find the difference between every two adjacent longitudes, as shown in the fourth column of that Table. Find the difference between every two successive latitudes, and place them in the fifth column of the Table. Take out from Table XLIII. the meridional parts corresponding to the several successive latitudes, as given in column 6, and find the difference between every two adjacent numbers, as given in the seventh column. Then find, by Mercator's Sailing, Problem I., page 238, the respective courses and distances between the several successive latitudes and longitudes; and let those courses and distances, so found, be arranged as in the two last columns of the following Table: viz.,

* The log. co-tangent is used, so as to avoid the trouble of finding the arithmetical complement of the log. tangent.

A TABLE,

Exhibiting, at Sight, all the Principal Elements attendant on the Computation of the Approximate Spherical Route from the Cape of Good Hope to the North Point of King's Island, at the Western Entrance to Bass' Strait.

Polar Angles.	Successive Longitudes.	Successive Latitudes.	Differences of Longitude.	Differences of Latitude.	Meridional Parts.	Meridional Difference of Latitude.	By Mercator's Sailing. Courses.	By Mercator's Sailing. Distances.
F S A = 65° 28′ 48″	18° 32′ 0″ E.	34° 24′ 0″ S.	Miles.	Miles.	2200.50	Miles.	S. 40° 22′ E.	370.85
F S 1 = 60.28.48	23.32. 0	39. 6.32	300.0	282.53	2553.34	352.84	43.31	328.17
F S 2 = 55.28.48	28.32. 0	43. 4.31	300.0	237.98	2869.28	315.94	46.56	291.60
F S 3 = 50.28.48	33.32. 0	46.23.39	300.0	199.13	3149.71	280.43	50.33	261.01
F S 4 = 45.28.48	38.32. 0	49. 9.30	300.0	165.85	3396.58	246.87	54.19	235.81
F S 5 = 40.28.48	43.32. 0	51.27. 2	300.0	137.53	3611.97	215.39	58.14	215.27
F S 6 = 35.28.48	48.32. 0	53.20.21	300.0	113.32	3797.71	185.74	62.15	198.70
F S 7 = 30.28.48	53.32. 0	54.52.53	300.0	92.53	3955.57	157.86	66.21	185.54
F S 8 = 25.28.48	58.32. 0	56. 7.19	300.0	74.44	4086.97	131.40	70.29	175.10
F S 9 = 20.28.48	63.32. 0	57. 5.49	300.0	58.50	4193.31	106.34	74.41	167.26
F S 10 = 15.28.48	68.32. 0	57.50. 0	300.0	44.18	4275.47	82.16	78.56	161.57
F S 11 = 10.28.48	73.32. 0	58.21. 2	300.0	31.03	4334.18	58.71	83.41	157.80
F S 12 = 5.28.48	78.32. 0	58.39.47	300.0	18.75	4370.08	35.90	87.39	170.53
F S = 0. 0. 0	84.00.48	58.46.46½	329.8	6.98	4383.55	13.47	N. 88.53	140.20
F S *a* = 4.31.12	88.32. 0	58.44. 1	271.2	2.75	4378.23	5.32	83.17	157.40
F S *b* = 9.31.12	93.32. 0	58.25.35	300.0	18.43	4342.86	35.37	79.44	160.75
F S *c* = 14.31.12	98.32. 0	57.56.57	300.0	28.64	4288.54	54.32	75.30	166.06
F S *d* = 19.31.12	103.32. 0	57.15.22	300.0	41.59	4210.93	77.61	71.18	173.47
F S *e* = 24.31.12	108.32. 0	56.19.44	300.0	55.64	4109.34	101.59	67. 8	183.29
F S *f* = 29.31.12	113.32. 0	55. 8.30	300.0	71.23	3982.81	126.53	63. 2	195.93
F S *g* = 34.31.12	118.32. 0	53.39.38	300.0	88.87	3830.13	152.68	59. 0	211.79
F S *h* = 39.31.12	123.32. 0	51.50.33	300.0	109.08	3649.87	180.26	55. 4	231.53
F S *i* = 44.31.12	128.32. 0	49.37.59	300.0	132.57	3440.35	209.52	51.16	255.79
F S *k* = 49.31.12	133.32. 0	46.57.55	300.0	160.07	3199.66	240.69	47.37	285.28
F S *l* = 54.31.12	138.32. 0	43.45.35	300.0	192.33	2925.81	273.85	44. 2	345.76
F S B = 59.53.12	143.54. 0	39.37. 0	322.0	248.58	2592.75	333.06		
			7522.0	2612.53		3973.85		5426.46

Now, the sum of the several successive differences of longitude = 7522 miles, makes up the whole difference of longitude between the two given places; the sum of the successive differences of latitude = 2612.53 miles, is equal to the whole difference of latitude comprehended under the highest latitude at which the ship should arrive, and the latitudes of the two given places, viz. 34°24′0″ S., 58°46′46½″ S., and 39°37′0″ S.—And, the sum of the several meridional differences of latitude = 3973.85 miles, coincides exactly with the whole meridional difference of latitude corresponding to the highest latitude, and the latitudes of the two given places; which several agreements, form an incontestable proof that the work has been carefully conducted.

The sum of the several distances measured on the consecutive rhumb lines intercepted between the successive latitudes and longitudes, as exhibited in the last column of the Table, is 5426.46 miles;—but the true spherical distance on the arc of a great circle is 5426.30 miles; the difference, therefore, is only 0′.16; or, about $\frac{1}{6}$ of a mile; which is very trifling, considering the extent of the arc.—The distance by Mercator's sailing is 6011.2 miles; which is 585 miles more than by great circle sailing.

Hence, it is evident that the shortest and most direct route from the Cape of Good Hope to King's Island is by the latitude of 58°46′46½″ S.; and that the ship must make, successively, the several longitudes and latitudes contained in the 2nd and 3rd columns of the Table, in the same manner, precisely, as if they were so many headlands, or places of rendezvous, at which she was required to touch.—The first course, therefore, from the Cape of Good Hope is S. 40°22′ E. distance 371 miles, which will bring the ship to longitude 23°32′ E. and latitude 39°6′32″ S.;—the second course is S. 43°31′ E. distance 328 miles, which brings the ship to longitude 28°32′ E. and latitude 43°4′31″ S.; the third course is S. 46°56′ E. distance 292 miles, which brings the ship to longitude 33°32′ E. and latitude 46°23′39″ S;—and so on of the rest.—Whence, it is evident that if the ship sails upon the several courses, and runs the corresponding distances respectively set forth in the two last columns of the Table, she will, most assuredly, arrive at the several successive longitudes and latitudes pointed out in the 2nd and 3rd columns of that Table; and thus will she reach King's Island, the place which it is intended she shall make, by a track 585 miles shorter than if such track had been determined agreeably to the principles of Mercator's sailing.

And, in a long voyage, like the present, in which ships generally experience a great scarcity of fresh water, particularly those bound to His Majesty's colony at New South Wales with troops, or convicts, the saving of 585 miles run at sea becomes a consideration of no inconsiderable importance.

Nor is there any more difficulty in sailing on the arc of a great circle, thus determined, than there is in sailing on a parallel of latitude; for, if the ship's compass be but tolerably good, the variation thereof carefully attended to, and proper attention paid to the steerage, the courses and distances expressed in the two last columns of the Table will, undoubtedly, carry the ship direct from the Cape of Good Hope to the north point of King's Island, without ever referring to celestial observation for either latitude or longitude; provided, indeed, that the ship's way is not affected by currents:—but, since the courses contained in the 8th column of the Table, express the true bearings between the several successive latitudes and longitudes through which the ship must pass; these must, therefore, be reduced to the magnetic, or compass course, by allowing the observed variation to the right hand thereof if it be westerly, but to the left hand if easterly; this being the converse process of reducing the magnetic, or course steered by compass, to the true course.—And, if the spherical track, so determined, be delineated on a Mercator's chart, it will, perhaps, not only simplify the navigation, but also point out to the mariner any *known land* that may be adjacent thereto *; and thus enable him to alter his course as occasion may require.—The spherical track may be readily delineated on a chart by means of the angles of meeting made by the several latitudes and longitudes, which show the places or points where the ship is to alter her course:—Now, those points being joined by right lines will indicate the true courses and distances, or the absolute route on which the ship must sail from the Cape of Good Hope to King's Island; then, if each day's run be carefully measured on the track, so delineated, the navigator can always know his distance from the place to which he is bound, without resorting to the trouble of calculation.

I have dwelt at considerable length upon this Problem for the express purpose of simplifying a subject which is but very little understood by the generality of maritime people:—and, with the view of rendering it still more familiar, another example will be given by which the approximate spherical route, as performed by His Majesty's ship Dauntless, under the command of George Cornish Gambier, esq. on her voyage from Port Jackson to Valparaiso, in the year 1822, will be clearly illustrated.

Example 2.

His Majesty's ship Dauntless being bound from Port Jackson, in latitude 33°52′ S. and longitude 151°16′ E. to Valparaiso, in latitude 33°1′ S. and longitude 71°52′ W., the captain, G. C. Gambier, Esq., proposed to navigate her as near to the arc of a great circle as he could, by altering her course at every 5 degrees of longitude; required the latitude at each

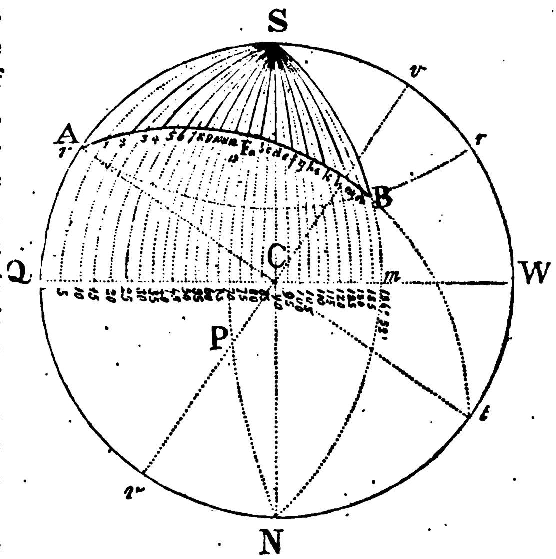
* It is presumed that there is not any land to intercept a ship's progress in this track.

time of altering the course, together with the respective courses and distances between those several latitudes and longitudes, occasioned by the proposed changes?

Port Jackson, . .	Latitude 33°52′ S.	Longitude =	151°16′ E.
Valparaiso, . .	Latitude 33. 1 S.	Longitude =	71.52 W.
		Sum = . . .	223°.8′

Difference of longitude between the two given places = 136°52′

Calculation.



Since the elements of this Example are analogous to those of the last; it is not, therefore, deemed necessary to repeat the mode of projection; the only difference in the construction being that, in the preceding diagram, because the ship is bound to a place to the eastward of that from which she is to sail; the latter is, therefore, for the sake of uniformity, placed on the primitive circle in the western hemisphere:—and, in the present diagram, because the ship is bound to a port to the westward of that from which she is to sail, the latter (for the sake of uniformity also) is placed on the primitive circle in the eastern hemisphere:—the letter Q representing the western hemisphere in the former case, and the eastern hemisphere in the latter.

Now, the figure being thus constructed on the plane of the meridian passing through Port Jackson; let the point A represent that place; the point B, the place of Valparaiso, and the arc A B, the true spherical distance between those places;—then, A S represents the co-latitude of Port Jackson; B S, that of Valparaiso; S A B, the angle of position at the former place, and S B A, the angle of position at the latter place.—The arc F S, which is drawn perpendicular to A B, represents the complement of the highest latitude at which the ship should arrive; and the several arcs S 1;

S 2; S 3; S 4; &c. &c. &c., represent the complements of the successive latitudes through which the ship must pass.—Hence, in the oblique angled spherical triangle A B S, two sides and the included angle are given to find the third side and the remaining angles; viz., the side A S = 56°8′ the co-latitude of Port Jackson; the side B S = 56°59′ the co-latitude of Valparaiso, and the angle A S B = 136°52′ the difference of longitude between those places; to find the spherical distance AB, and the respective angles of position = S A B and S B A :—the distance may be readily found by Remark 1, or 2, to Problem III., page 203 or 204; as thus:

Diff. long. A S B 136°52′ ÷ 2 =	68°26′	Twice log. S. =	19.936958	
Co-latitude of Port Jackson = A S	56° 8′	Log. sine =	9.919254	
Co-lat. of Valparaiso = B S	56°59′	Log. sine =	9.923509	
		Sum = . .	39.779721	
Diff. of co-lat. =	0°51′	Half = . .	19.889860½	19.889860½
Half diff. of ditto =	0°25′30″	Log. S. =	7.870262	
Arch = ...	89°25′47″	Log. tang. =	12.019598½	Log S. = 9.999979
Half the arc A B =	50°53′56″	Log. sine = ...		9.889881½

Side A B = . . 101°47′52″ = the true spherical distance between the two given places.

To find the Angle of Position at Port Jackson = Angle S A B :—

This is found by Problem I., page 198; as thus:

As the distance A B =	101°47′52″	Log. co-secant =	10.009273
Is to diff. long. A S B =	136. 52. 0	Log. sine = . .	9.834865
So is the co-latitude B S =	56. 59. 0	Log. sine = . .	9.923509
To angle of pos. = S A B =	35°50′59″	Log. sine = . .	9.767647

To find the Angle of Position at Valparaiso = Angle S B A :—

This is found by Problem I., page 198; as thus:

As the distance A B =	101°47′52″	Log. co-secant =	10.009273
Is to diff. long. A S B =	136. 52. 0	Log. sine = . .	9.834865
So is the co-lat. A S =	56. 8. 0	Log. sine = . .	9.919254
To angle of pos. S B A =	35°26′50″	Log. sine = . .	9.763392

To find the Perpendicular F S = the Complement of the highest Southern Latitude at which the Ship should arrive :—

Here we have a choice of two right angled spherical triangles, viz. A S F and B S F; in each of which the hypothenuse and the angle at the base are given to find the perpendicular;—thus, in the triangle A S F, given the hypothenuse AS = 56°8′ the co-latitude of Port Jackson; and the angle at the base S A B = 35°50′59″ the angle of position at that place, to find the perpendicular F S = the complement of the highest southern latitude at which the ship should arrive :—

Hence, by right angled spherical trigonometry, Problem II., page 185,

As radius = ...	90° 0′ 0″	Log. co-secant=	10.000000
Is to co-lat. Port Jackson = A S	56. 8. 0	Log. sine = . .	9.919254
So is ang. of position = S A F	35. 50. 59	Log. sine = . .	9.767647
To the perpendicular F S =	29° 5′51″	Log. sine = . .	9.686901
Highest lat. at which the ship should arrive =	60°54′ 9″ south.		

From the above calculations it appears evident that the true spherical distance between Port Jackson and Valparaiso is 101°47′52″, or 6107.87 miles; the angle of position at Port Jackson = 35°50′59″, and that at Valparaiso = 35°26′50″, and the highest southern latitude at which the ship should arrive = 60°54′9″.—Now, by Mercator's sailing, the course from Port Jackson to Valparaiso is N. 89°34′ E. and the distance 6853.16 miles;—whence it is manifest, that if a ship sails on the direct rhumb line between those places, as indicated by that mode of sailing, she will have to run 745¼ miles more than by shaping her course along the arc of a great circle.

To compute the vertical, or Polar Angles A S F, and B S F :—

In the right angled spherical triangle A S F, given the hypothenuse A S 56°8′ = the co-latitude of Port Jackson, and the perpendicular F S 29°5′51″ = the complement of the highest latitude at which the ship should arrive; to find the vertical, or polar angle A S F.—And, in the right angled triangle B S F, given the hypothenuse B S, 56°59′ = the co-latitude of Valparaiso, and the perpendicular F S, 29°5′51″; to find the polar angle B S F.—Hence, by right angled spherical trigonometry, Prob. I., page 184,

To find the Polar Angle A S F:—

As radius = ...	90°0′ 0″	Log. co-secant =	10.000000
Is to the co-latitude A S = . .	56. 8. 0	Log. co-tangent =	9.826805
So is the co-latitude F S = .	29. 5. 51	Log. tangent = .	9.745493
To the polar angle A S F =	68°4′5″	Log. co-sine = . .	9.572298

To find the Polar Angle B S F:—

As radius = ...	90° 0′ 0″	Log. co-secant =	10.000000
Is to the co-latitude B S =	56. 59. 0	Log. co-tangent =	9.812794
So is the co-latitude F S =	29. 5. 51	Log. tangent =	9.745493
To the polar angle B S F =	68°47′55″	Log. co-sine = .	9.558287

Now, since the sum of the polar angles, thus obtained, viz. A S F, 68°4′5″ + B S F, 68°47′55″ = 136°52′, makes up the whole difference of longitude between the two given places, expressed by the whole angle A S B, it shows that, thus far, the work is right.

To find the several successive Polar Angles made by the proposed changes of Longitude.

From the polar angle A S F, subtract the proposed alteration of longitude continually, as far as subtraction can be made; and the several polar angles occasioned by those alterations, and contained between the perpendicular F S, and the co-latitude of Port Jackson = A S, will be obtained.—Thus, from the polar angle A S F = 68°4′5″, let 5° be continually *subtracted*, and the results will be F S 1 = 63°4′5″ F S 2 = 58°4′5″; F S 3 = 53°4′5″, &c. &c., the last remainder being 3°4′5″ = the polar angle F S 13.—Now, the polar angles contained between the perpendicular F S, and the co-latitude of Valparaiso = B S, are to be determined by a contrary process; and, since the last subtraction in the triangle F S A, left the remainder, or polar angle F S 13 = 3°4′5″, which is 1°55′55″, less than the proposed alteration of longitude; therefore, the first polar angle in the triangle F B S, must be 1°55′55″ = the polar angle F S *a*; to which, let 5° be continually added, as far as the measure of the angle F S B will allow, and we shall have F S *b* = 6°55′55″; F S *c* = 11°55′55″; F S *d* = 16°55′55″, and so on; as expressed in the first column of the following Table.

To compute the successive Latitudes at which the Ship should arrive :—

Since the several successive polar angles, obtained as above, evidently reduce the two right angled spherical triangles A F S and B F S, into a series of right angled spherical triangles, to each of which the perpendicular F S is common ; therefore, in each triangle of this series we have the perpendicular and the angle adjacent, to find the hypothenuse, or co-latitude.—Thus, in the right angled spherical triangle F S 1, right angled at F, given the perpendicular F S = 29°5′51″, and the polar angle F S 1 = 63°4′5″ ; to find the hypothenuse, or co-latitude S 1 ;—In the right angled spherical triangle F S 2, given the perpendicular F S = 29°5′51″, and the polar angle F S 2 = 58°4′5″ ; to find the hypothenuse, or co-latitude S 2, &c. &c. &c.

Hence, by right angled spherical trigonometry, Problem IV., page 188,

To find the Hypothenuse, or Co-latitude S 1 :—

As the perpendicular F S =	29° 5′ 51″	Log. co-tang. =	10.254507*
Is to the radius = ...	90. 0. 0	Log. sine = .	10.000000
So is the polar angle F S 1 =	63. 4. 5	Log. co-sine .	9.656033
To the co-latitude S 1 =	50. 51. 36	Log. co-tangent =	9.910540
First latitude = . .	39° 8′ 24″ S. at which ship should arrive.		

To find the Hypothenuse, or Co-latitude S 2 :—

As the perpendicular F S =	29° 5′ 51″	Log. co-tang. =	10.254507*
Is to the radius = ...	90. 0. 0	Log. sine = . .	10.000000
So is the polar angle F S 2 =	58. 4. 5	Log. co-sine = .	9.723383
To the co-latitude S 2 =	46. 27. 28	Log. co-tang. =	9.977890
Second latitude =	43°32′32″ S. at which ship should arrive.		

Hence, the first latitude at which the ship should arrive is 39°8′24″ S. ; and the second latitude 43°32′32″ S.—And since it is the latitude, and not its complement that is required ; therefore, if the log. tangent of the sum of the three logs. be taken, it will give the latitude direct ; and, by rejecting the radius from the calculation, the work will be considerably facilitated.—Proceeding in this manner, the several successive latitudes cor-

* The log. co-tangent is used, so as to save the trouble of finding the arithmetical complement of the log. tangent.

responding to the proposed alterations of longitude will be found as shown in the 3d. column of the following Table.

Now, let the several successive longitudes be arranged (agreeably to the proposed change, and to the measure of the corresponding polar angles,) as exhibited in the 2d column of the following Table; and find the difference between every two adjacent longitudes, as shown in the 4th column of that Table.—Find the difference between every two adjacent latitudes, and place those differences in the 5th column.—Find the meridional parts corresponding to the several successive latitudes, which place in the 6th column; and find the difference between every two adjacent meridional altitudes, as shown in the 7th column.—Then, find, by Mercator's sailing, Problem I., page 238, the respective courses and distances between the several successive latitudes and longitudes; and, let the courses and distances, so found, be arranged in regular succession, as exhibited in the two last columns of the Table.—Then, will this Table be duly prepared for navigating a ship on the arc of a great circle, agreeably to the proposed alterations of longitude.—And, should the sum of the several successive differences of longitude, contained in the Table, coincide with the whole difference of longitude between the two given places; —the sum of the several successive differences of latitude be found to agree with the whole difference of latitude comprehended under the mean, or highest latitude, and its corresponding extremes;—the sum of the several meridional differences of latitude to be equal to the whole meridional difference of latitude corresponding to the mean, or highest latitude, and its respective extremes,—and the sum of the several successive distances to make up the whole spherical distance (or nearly so,) between the two given places; then, those several concurring equalities will be so many satisfactory proofs that the work is right.

Note.—In the spherical track laid down in the following Table, it is presumed that there is not any land to intercept a ship's progress: but since this track will take the navigator into high southern latitudes, it will be indispensibly necessary to keep a sharp look-out at all times, particularly during the night, so as to guard against any of the ice-bergs that may be floating to the northward of the Antarctic circle;—though if the track be made in the months of November, December, January, or February, there will be no real night or darkness to experience; for during these months there will be a strong twilight between the latitudes of 53, and 61 degrees south; and thus the navigating at night will be attended with very little more danger than that by day.

A TABLE,

Exhibiting, at Sight, all the Particulars attendant on the Computation of the Approximate Spherical Route from Port Jackson, in New South Wales, to Valparaiso on the Coast of Chili.

Polar Angles.	Successive Longitudes.	Successive Latitudes.	Differences of Longitude.	Differences of Latitude.	Meridional Parts.	Meridional Difference of Latitude.	By Mercator's Sailing. Courses.	By Mercator's Sailing. Distances.
F S A = 68° 4′ 5″	151° 16′ 0″ E.	33° 52′ 0″ S.	Miles.	Miles.	2161.84	Miles.	S. 37° 18′ E.	397.71
F S 1 = 63. 4. 5	156. 16. 0	39. 8. 24	300.00	316.40	2556.75	393.91	40. 26	347.04
F S 2 = 58. 4. 5	161. 16. 0	43. 32. 32	300.00	264.14	2907.79	352.04	43. 53	303.92
F S 3 = 53. 4. 5	166. 16. 0	47. 11. 36	300.00	219.06	3219.75	311.96	47. 33	268.26
F S 4 = 48. 4. 5	171. 16. 0	50. 12. 41	300.00	181.09	3494.24	274.49	51. 23	239.17
F S 5 = 43. 4. 5	176. 16. 0	52. 41. 58	300.00	149.29	3733.92	239.68	55. 22	215.70
F S 6 = 38. 4. 5	178. 44. 0 W.	54. 44. 35	300.00	122.60	3941.17	207.25	59. 26	196.91
F S 7 = 33. 4. 5	173. 44. 0	56. 24. 42	300.00	100.12	4118.31	177.14	63. 36	181.90
F S 8 = 28. 4. 5	168. 44. 0	57. 45. 35	300.00	80.89	4267.26	148.95	67. 51	170.31
F S 9 = 23. 4. 5	163. 44. 0	58. 49. 48	300.00	64.21	4389.38	122.12	72. 7	161.20
F S 10 = 18. 4. 5	158. 44. 0	59. 39. 19	300.00	49.52	4486.22	96.84	76. 26	154.50
F S 11 = 13. 4. 5	153. 44. 0	60. 15. 33	300.00	36.23	4558.59	72.37	80. 46	149.85
F S 12 = 8. 4. 5	148. 44. 0	60. 39. 35	300.00	24.04	4607.35	48.76	85. 8	147.06
F S 13 = 3. 4. 5	143. 44. 0	60. 52. 3	300.00	12.46	4632.86	25.51	88. 39	89.50
F S = 0. 0. 0	140. 39. 55	60. 54. 9	184.08	2.10	4637.18	4.32	N. 89. 9	56.26
F S *a* = 1. 55. 55	138. 44. 0	60. 53. 19	115.92	0.83	4635.47	1.71	86. 8	146.71
F S *b* = 6. 55. 55	133. 44. 0	60. 43. 25	300.00	9.91	4615.16	20.31	81. 46	149.01
F S *c* = 11. 55. 55	128. 44. 0	60. 22. 4	300.00	21.34	4571.75	43.41	77. 24	153.06
F S *d* = 16. 55. 55	123. 44. 0	59. 48. 40	300.00	33.40	4504.67	67.08	73. 7	159.65
F S *e* = 21. 55. 55	118. 44. 0	59. 2. 17	300.00	46.37	4413.61	91.06	68. 48	167.95
F S *f* = 26. 55. 55	113. 44. 0	58. 1. 34	300.00	60.73	4297.26	116.35	64. 34	179.04
F S *g* = 31. 55. 55	108. 44. 0	56. 44. 40	300.00	76.90	4154.57	142.69	60. 17	193.38
F S *h* = 36. 55. 55	103. 44. 0	55. 8. 50	300.00	95.84	3983.30	171.18	56. 21	210.88
F S *i* = 41. 55. 55	98. 44. 0	53. 12. 0	300.00	116.83	3783.75	199.64	52. 18	233.51
F S *k* = 46. 55. 55	93. 44. 0	50. 49. 12	300.00	142.80	3551.88	231.87	48. 23	260.99
F S *l* = 51. 55. 55	88. 44. 0	47. 55. 51	300.00	173.35	3285.33	266.55	44. 42	295.19
F S *m* = 56. 55. 55	83. 44. 0	44. 26. 0	300.00	209.85	2982.09	303.24	41. 12	336.55
F S *n* = 61. 55. 55	78. 44. 0	40. 12. 46	300.00	253.23	2639.39	342.70	37. 59	385.38
F S *o* = 66. 55. 55	73. 44. 0	35. 8. 50	300.00	303.94	2255.07	384.32	35. 58	157.94
F S B = 68. 47. 55	71. 52. 0	33. 1. 0	112.00	127.63	2100.72	154.35		
			8212.00	3295.30		5011.80		6108.73

Now, the sum of the several successive differences of longitude, viz. 8212 miles, coincides exactly with the whole difference of longitude between the two given places; the sum of the successive differences of latitude = 3295.30 miles, agrees with the whole difference of latitude comprehended under the highest latitude at which the ship should arrive, and the latitudes of the two given places; viz. 33°52′0″ S; 60°54′9″ S, and 33°1′0″ S:—and, the sum of the several meridional differences of latitude = 5011.80 miles, makes up the whole difference of latitude corresponding to the highest latitude and the latitudes of its respective extremes:—these several concurrences or agreements, form, therefore, the most satisfactory and indisputable proofs that the work has been properly conducted.

The sum of the several distances, measured on the respective rhumb-lines intercepted between the successive longitudes and latitudes, as given in the last column of the Table, is 6108.73 miles;—but the true spherical distance on the arc of a great circle is 6107.87 miles; the difference, therefore, is only 0.86, or a little more than three fourths of a mile; which is a very close approximation in the measure of so great an arc.

The distance by Mercator's sailing is 6853.16 miles; which is 745.29, or about 745¼ miles more than by great circle sailing.—Hence, it is evident that the shortest and most direct route from Port Jackson to Valparaiso is by the latitude of 60°54′9″ S; and that the ship must make, successively, the several longitudes and latitudes contained in the 2nd and 3rd columns of the Table, in the same manner precisely, as if they were so many ports or places of rendezvous, at which she was directed to touch.

The first course, therefore, from Port Jackson to Valparaiso, is S. 37°18′ E. distance 398 miles; which will bring the ship to longitude 156°16′0″ E. and latitude 39°8′24″ S;—the second course is S. 40°26′ E. distance 347 miles; which brings the ship to longitude 161°16′0″ E. and latitude 43°32′32″ S;—the third course is S. 43°53′ E. distance 304 miles, which brings the ship to 166°16′0″ E. and latitude 47°11′36″ S.

Whence it is evident that Captain Gambier saved a distance of 745¼ miles in that judicious and well-planned route: And this saving of distance should be an object of the highest consideration to every captain who wishes to recruit the strength and spirits of his ship's company by a generous supply of fresh provisions after a fatiguing and tedious voyage; the measure of which falls very little short of being equal to one fourth of the earth's circumference as taken under the equator, or to the one third of that circumference if taken under the given parallel of latitude.

SOLUTION OF PROBLEMS IN NAUTICAL ASTRONOMY.

NAUTICAL ASTRONOMY is the method of finding, by celestial observation, the latitude and longitude of a ship at sea; the variation of the compass; the apparent time at ship; the altitudes of the heavenly bodies, &c. &c. &c. —Or, it is that branch of mathematical astronomy which shows how to solve all the important Problems in navigation by means of spherical operations, when the altitudes, or distances of the celestial objects are under consideration.

Introductory Problems to the Science of Nautical Astronomy.

PROBLEM I.

To convert Longitude or Parts of the Equator into Time.

RULE.

Multiply the given degrees by 4, and the product will be the corresponding time:—observing that seconds multiplied by 4 produce thirds; minutes multiplied by 4 produce seconds, and degrees multiplied by 4 produce minutes, which, divided by 60, give hours, &c.

Example 1.

Required the time corresponding to 12°40′45″?

Given degrees = 12°40′45″
Multiply by 4

Corresp. time = 0ʰ50ᵐ43ˢ0ᵗ

Example 2.

Required the time corresponding to 76°20′30″?

Given degrees = 76°20′30″
Multiply by 4

Corresp. time = 5ʰ5ᵐ22ˢ0ᵗ

PROBLEM II.

To convert Time into Longitude, or Parts of the Equator.

RULE.

Reduce the hours to minutes, to which add the odd minutes, if any; then, the minutes divided by 4 give degrees; the seconds divided by 4 give minutes, and the thirds divided by 4 give seconds.

Example 1.

Required the degrees corresponding to 0ʰ47ᵐ36ˢ?

Given time = . 0ʰ47ᵐ36ˢ

Divide by . . . 4) 47ᵐ36ˢ

Corresp. deg. = . 11°54′ 0″

Example 2.

Required the degrees corresponding to 9ʰ25ᵐ37ˢ?

Given time = . . 9ʰ25ᵐ37ˢ

Divide by . 4) 565ᵐ37ˢ

Corresp. deg. = 141°24′15″

Note.—The two preceding Problems are readily solved by means of Table I;—see explanation, pages 1 and 2.

Problem III.

Given the Time under any known Meridian, to find the corresponding Time at Greenwich.

Rule.

Let the given time be reckoned from the *preceding noon*, to which apply the longitude of the place in time (reduced by Problem I., as above,) by addition if it be west, or subtraction if east; and the sum, or difference will be the corresponding time at Greenwich.

Example 1.

Required the time at Greenwich, when it is 4h 40m 13s at a ship in longitude 80°53′15″ W.?

Time at ship = . . . 4h 40m 13s
Long. 80°53′15″ W.
in time = . . . + 5. 23. 33

Corresp. time at Greenwich = . . . 10h 3m 46s

Example 2.

Required the time at Greenwich, when it is 20h 11m 41s at a ship in longitude 98°14′45″ E?

Time at ship = . . . 20h 11m 41s
Long. 98°14′45″ E.
in time = . . . − 6. 32. 59

Corresp. time at Greenwich = ... 13h 38m 42s

Problem IV.

Given the Time at Greenwich, to find the corresponding Time under a known Meridian.

Rule.

Let the given time be reckoned from the *preceding noon*, to which apply the longitude of the place in time (reduced by Problem I., as above,) by addition if it be east, or subtraction if west; and the sum, or difference will be the corresponding time under the given meridian.

Example 1.

When may the emersion of the first satellite of Jupiter be observed at Trincomalee, in longitude 81°22′ E., which, by the Nautical Almanac, happens at Greenwich, March 4th, 1825, at 9h 9m 28s?

Apparent time of emersion at Greenwich = . . . 9h 9m 28s
Longitude of Trincomalee 81°22′ E., in time = . 5. 25. 28

Apparent time of emersion at Trincomalee = . . 14h 34m 56s

Example 2.

When may the immersion of the first satellite of Jupiter be observed at Port Royal, Jamaica, in longitude 76°52′30″ W., which, by the Nautical Almanac, happens at Greenwich Nov. 1st. 1825, at 18ʰ17ᵐ45ˢ ?

Apparent time of immersion at Greenwich = . .	18ʰ17ᵐ45ˢ
Longitude of Port Royal 76°52′30″ W., in time =	5. 7.30
Apparent time of immersion at Port Royal = . .	13ʰ10ᵐ15ˢ

Problem V.

To reduce the Sun's Longitude, Right Ascension, and Declination; and, also, the Equation of Time, as given in the Nautical Almanac, to any other Meridian, and to any time under that Meridian.

Rule.

Let the given apparent time at ship, or place, be *always* reckoned from the *preceding noon*; to which apply the longitude in time (reduced by Problem I., page 296,) by addition if it be west, or subtraction if east, and the sum or difference will be the corresponding time at Greenwich.

Take, from page II. of the month in the Nautical Almanac, the sun's longitude, right ascension and declination, or the equation of time, as the case may be, for the noons immediately preceding and following the Greenwich time, and find their difference; then,

To the proportional log. of this difference, add the proportional log. of the Greenwich time (reckoning the hours as minutes, and the minutes as seconds), and the constant log. 9.1249; * the sum of these three logs. rejecting 10 from the index, will be the proportional log. of a correction which is always to be *added* to the sun's longitude, or right ascension, at the noon preceding the Greenwich time; but to be applied by addition or subtraction to the sun's declination, or the equation of time at that noon, according as these elements may be increasing or decreasing.

Remark.—Since the daily difference of the equation of time is expressed, in the Nautical Almanac, in seconds and tenths of a second; if, therefore, these tenths be multiplied by 6 they will be reduced to thirds: hence, the daily difference will be obtained in seconds and thirds.—Now, if those seconds and thirds be esteemed as minutes and seconds, the operation of reducing the equation of time will become as simple as that of the sun's declination;—observing, however, that the minutes and seconds, corresponding to the sum of the three logs., are to be considered as seconds and thirds.

* This is the arithmetical complement of the proportional log. of 24 hours esteemed as *minutes*.

To find the Moon's Declination :—

Difference in 12 hours = ...	2°47′57″	Prop. log. =	.0301
Greenwich time = ...	7h20m53s	Prop. log. =	1.3891
Constant log. = ...			8.8239
Correction of moon's declination =	+ 1°42′51″	Prop. log. =	0.2431
Moon's declin. at noon March 6th =	7.58. 6	south.	
Moon's declination, as required = . .	9°40′57″	south.	

To find the Moon's Semi-diameter :—

Diff. in 12 hours = ...	4″	Prop. log. =	3.4314
Greenwich time = . . .	7h20m53s	Prop. log. =	1.3891
Constant log. = ...			8.8239
Correction of the moon's semi-diameter =	− 2″	Prop. log. =	3.6444
Moon's semi-diam. at noon March 6th =	16′38″		
Moon's semi-diameter, as required = .	16′36″		

To find the Moon's Horizontal Parallax :—

Difference in 12 hours = ...	16″	Prop. log. =	2.8293
Greenwich time = ...	7h20m53s	Prop. log. =	1.3891
Constant log. = ...			8.8239
Corr. of the moon's horizontal parallax =	10″	Prop. log. =	3.0423
Moon's horiz. par. at noon, March 6th	61′ 2″		
Moon's horizontal parallax, as required =	60′52″		

Remark.—When much accuracy is required, the proportional part of the moon's motion in 12 hours, found as above, must be corrected by the equation of second difference contained in Table XVII., as explained in pages 33, 34, and 35. And, in all cases, the moon's semi-diameter, so found, must be increased by the augmentation given in Table IV., as explained between pages 8 and 11.

Example 2.

Required the moon's longitude, latitude, right ascension, declination, semi-diameter, and horizontal parallax, March 26th, 1825, at $1^h 30^m 47^s$, in longitude 94°15′30″ east; the apparent altitude of that object being 24°?

Apparent time at ship or place = ...	$1^h 30^m 47^s$
Longitude of the ship or place = 94°15′30″ E., in time = .	6. 17. 2
Greenwich time past *midnight*, March 25th, 1825 = . . .	$7^h 13^m 45^s$

To find the Moon's Longitude :—

Diff. in 12 hours = 6°22′57″ ÷ 3 =	2°7′39″	Prop. log. =	.1493
Greenwich time = $7^h 13^m 45^s$		Prop. log. =	1.3962
Constant log. = ...			8.8239
One-third of the proportional part =	1°16′53″	Prop. log. =	0.3694
Multiply by	3		
Prop. part of ☽'s motion in long. =	3°50′39″		
Equation from Table XVII. = . . .	− 33		
Proportional part corrected = . . .	3°50′ 6″		
☽'s long. at midnt., March 25th =	2ˢ15°31′56″		
Moon's true longitude = ...	2ˢ19°22′ 2″		

To find the Moon's Latitude :—

Difference in 12 hours = ...	33′37″	Prop. log. =	.7287
Greenwich time = $7^h 13^m 45^s$		Prop. log. =	1.3962
Constant log. = ...			8.8239
Proportional part of ☽'s latitude =	− 20′15″	Prop. log. =	0.9488
Equation from Table XVII. = . . .	− 6		
Proportional part corrected = . . .	− 20′ 9″		
☽'s lat. at midnight, March 25th =	0°46′36″ north.		
Moon's true latitude = ...	0°26′27″ north.		

To find the Moon's Right Ascension:—

Diff. in 12 hours = 6°59′1″ ÷ 3 =	2°19′40⅓″	Prop. log. =	.1102
Greenwich time = 7ʰ13ᵐ45ˢ		Prop. log. =	1.3962
Constant log. =			8.8239
One-third of the proportional part =	1°24′ 8″	Prop. log. =	0.3303
Multiply by	3		
Prop. part of ☽'s motion in right asc. =	4°12′24″		
Equation from Table XVII. =	− 36		
Proportional part corrected =	4°11′48″		
☽'s right asc. at midnt., March 25th =	74.11.56		
Moon's true right ascension =	78°23′44″		

To find the Moon's Declination:—

Difference in 12 hours =	1′0″	Prop. log. =	2.2553
Greenwich time = 7ʰ13ᵐ45ˢ		Prop. log. =	1.3962
Constant log. =			8.8239
Proportional part of ☽'s declination =	− 0′36″	Prop. log. =	2.4754
Equation from Table XVII. =	− 10		
Proportional part corrected =	− 0′26″		
☽'s dec. at midnt., March 25th =	23°26′53″ north.		
Moon's true declination =	23°26′27″ north.		

To find the Moon's Semi-diameter:—

Difference in 12 hours =	6″	Prop. log. =	3.2553
Greenwich time = 7ʰ13ᵐ45ˢ		Prop. log. =	1.3962
Constant log. =			8.8239
Proportional part of ☽'s semi-diameter	+ 4″	Prop. log. =	3.4754
☽'s semi-diam. at midnt., March 25th =	15′19″		
Moon's apparent semi-diameter =	15′23″		
Augment. from Tab. IV., for alt. 24° =	6		
Moon's true semi-diameter =	15′29″		

To find the Moon's Horizontal Parallax:—

Difference in 12 hours = 22″		Prop. log. =	2.6910
Greenwich time = 7h 13m 45s		Prop. log. =	1,3962
Constant log. =			8.8239
Proportional part of ☽'s hor. parallax =	+ 13″	Prop. log. =	2.9111
☽'s hor. par. at midnight, March 25th =	56′ 13″		
Moon's true horizontal parallax =	56′ 26″		

Note.—The examples to the foregoing Problem may be very correctly solved by means of Table XVI.—See explanation, page 80.

Problem VII.

To reduce the Right Ascension and Declination of a Planet, as given in the Nautical Almanac, to any given Time under a known Meridian.

Rule.

Let the apparent time at the ship or place be reckoned from the preceding noon, to which apply the longitude in time, (reduced by Problem I., page 296,) by addition if it be west, or subtraction if east; and the sum or difference will be the corresponding time at Greenwich.

From page IV. of the month in the Nautical Almanac, take out the planet's right ascension and declination for the nearest days preceding and following the Greenwich time, and find the difference; find, also, the difference between the Greenwich time and the nearest preceding day; then,

To the proportional logarithm of this difference, esteemed as *minutes* and *seconds*, add the proportional logarithm of the difference of right ascension, or declination, and the constant logarithm 9.9031*; the sum of these three logarithms, rejecting 10 from the index, will be the proportional logarithm of a correction; which being applied, by addition or subtraction, to the right ascension or declination (on the nearest day preceding the Greenwich time), according as it may be increasing or decreasing, the sum or difference will be the correct right ascension or declination.

Example 1.

Required the right ascension and declination of the planet Mars, March 16th, 1825, at 4h 40m apparent time, in longitude 68° 12′ west of the meridian of Greenwich?

---

* This is the arithmetical complement of the proportional logarithm of 144 hours = 6 days, esteemed as *minutes*; and, hence, taken as 2 hours and 24 minutes.

Apparent time at ship or place = . . .	March, 16 days,	$4^h 40^m 0^s$
Longitude of ship or place = 68° 12′ W., in time = . . .		4. 32. 48
Greenwich time = ...	16 days,	$9^h 12^m 48^s$

To find the Right Ascension:—

R. A. of Mars, March 13th = $0^h 41^m$		$13^d\ 0^h\ 0^m\ 0^s$
Ditto 19th = 0. 58	Gr. time =	16. 9. 12. 48
Difference = ... $0^h 17^m$	Diff. =	$3^d\ 9^h 12^m 48^s = 81^h 12^m 48^s$

Diff. of time = $81^h 12^m 48^s$, or $1^h 21^m 12^s 48^t$	Prop. log. =	.3456
Difference of right ascension = $0^h 17^m$ . . .	Prop. log. =	1.0248
Constant log. = ...		9.9031
Correction of right ascension = . . + $9^m 35^s$	Prop. log. =	1.2735
Planet's right ascension, March 13th = $0^h 41^m\ 0^s$		
Planet's right ascension, as required = $0^h 50^m 35^s$.		

To find the Declination:—

Dec. of Mars, March 13th = 3° 53′ N.		$13^d\ 0^h\ 0^m\ 0^s$
Ditto 19th = 5. 43 N.	Gr. time =	16. 9. 12. 48
Difference = ... 1° 50′		$3^d\ 9^h 12^m 48^s = 81^h 12^m 48^s$

Difference of time = $81^h 12^m 48^s$, or $1^h 21^m 12^s 48^t$	Prop. log. =	.3456
Difference of declination = 1° 50′	Prop. log. =	.2139
Constant log. = ...		9.9031
Correction of declination = . . + 1° 2′ 2″	Prop. log. =	0.4626
Planet's declination, March 13th = 3. 53. 0 north.		
Planet's declination, as required = . 4° 55′ 2″ north.		

Example 2.

Required the right ascension and declination of the planet Mars, Sept. 23d, 1825, at $1^h 23^m 19^s$, apparent time, in longitude 100° 40′ 30″ east of the meridian of Greenwich?

Apparent time at ship or place = . . .	Sept. 23 days,	$1^h 23^m 19^s$
Longitude of ship or place = 100° 40′ 30″ E., in time = . .		6. 42. 42
Greenwich time (past noon of the 22d Sept.) =	22 days,	$18^h 40^m 37^s$

To find the Right Ascension :—

R. A. of Mars, Sept. 19th = 9h 37m			19d 0h 0m 0s
Ditto 25th = 9. 52		Gr. time	22. 18. 40. 37
Difference = . . . 0h 15m		Diff. =	3d 18h 40m 37s = 90h 40m 37s

Difference of time = 90h 40m 37s, or 1h 30m 40s 37t Prop. log. = .2977
Difference of right ascension = 0h 15m . . . Prop. log. = 1.0792
Constant log. = ... 9.9031

Correction of right ascension = . + 9m 27s Prop. log. = 1.2800
Planet's right ascension, Sept. 19th = 9h 37m 0s

Planet's right ascension, as required = 9h 46m 27s

To find the Declination :—

Dec. of Mars, Sept. 19th = 15° 30′ N.			19d 0h 0m 0s
Ditto 25th = 14. 18 N.		Gr. time	22. 18. 40. 37
Difference = . . . 1° 12′		Diff. =	3d 18h 40m 37s = 90h 40m 37s

Difference of time = 90h 40m 37s, or 1h 30m 40s 37t Prop. log. = .2977
Difference of declination = 1° 12′ . . . . Prop. log. = .3979
Constant log. = ... 9.9031

Correction of declination = − 45′ 21″ Prop. log. = 0.5987
Planet's declination, Sept. 19th = 15. 30. 0 north.

Planet's declination, as required = 14° 44′ 39″ north.

Problem VIII.

To compute the Apparent Time of the Moon's Transit over the Meridian of Greenwich.

Since the moon's transit over the meridian of Greenwich is only given to the nearest minute in the Nautical Almanac; and, since it is absolutely necessary, on many astronomical occasions, to have it more strictly determined: the following rule is, therefore, given, by which the apparent time of the moon's transit over the meridian of Greenwich may be obtained true to the decimal part of a second.

Rule.

From the moon's right ascension at noon of the given day (converted into time, and increased by 24 hours if necessary,) subtract the sun's right ascension at that noon, and the remainder will be the approximate time of the moon's transit over the meridian of Greenwich.

Find the excess of the moon's motion in right ascension over the sun's in 12 hours; then say, as 12 hours, diminished by this excess, is to 12 hours, so is the approximate time of transit to the apparent time of transit.

Note.—If the three terms be reduced to seconds, the operation may be readily performed by logarithms.

Example 1.

Required the apparent time of the moon's transit over the meridian of Greenwich, March 26th, 1825?

Moon's R. A. at noon of given day = $81°10'57''$, in time = $5^h 24^m 43^s.8$
Sun's right ascension at that noon = ... $0.20.24.0$

Approx. time of the moon's tr. over the merid. of Greenw. = $5^h\ 4^m 19^s.8$

Sun's right ascension at noon, March 26th, = $0^h 20^m 24^s$
Sun's ditto 27th = $0.24.2$

Sun's motion in 24 hours = ... $0^h\ 3^m 38^s$

Sun's motion in 12 hours = ... $0^h\ 1^m 49^s$

Moon's R. A. at noon, March 26th, = $81°10'57''$, in time = $5^h 24^m 43^s.8$
Moon's ditto, at midnt., March 26th, = $88°14'15''$, in time = $5.52.57.0$

Moon's motion in 12 hours = ... $0^h 28^m 13^s.2$
Sun's motion in 12 hours = ... $0.\ 1.49.0$

Excess of the moon's motion over the sun's in 12 hours = $26^m 24^s.2$

As $12^h - 26^m 24^s.2 = 11^h 33^m 35^s.8$,
in seconds = ... 41615.8 Log. ar. comp. = 5.380742
Is to 12 hours, in seconds = . . . 43200. Log. = . . . 4.635484
So is the approximate time of transit
= $5^h 4^m 19^s.8$, in seconds = . 18259.8 Log. = . . . 4.261496

To the apparent time of the moon's
transit = $5^h 15^m 54^s.9$, in secs. = 18954.9 Log. = . . . 4.277722

Example 2.

Required the apparent time of the moon's transit over the meridian of Greenwich, April 10th, 1825?

Moon's R. A. at noon of given day = 294°59′11″, in time = 19h39m56s.7
Sun's right ascension at that noon = ... 1. 15. 1 .6

Approx. time of the moon's tr. over the merid. of Greenw. = 18. 24. 55 .1

Sun's right ascension at noon, April 10th, = 1h15m 1s.6
Sun's ditto 11th, = 1. 18. 41 .6

Sun's motion in 24 hours = ... 0h 3m40s.0

Sun's motion in 12 hours = ... 0h 1m50s.0

Moon's R. A. at midnt., April 10th = 301°15′6″, in time = 20h 5m 0s.4
Moon's ditto at noon, April 11th = 307. 20. 12, in time = 20. 29. 20 .8

Moon's motion in 12 hours = ... 0h24m20s.4
Sun's motion in 12 hours = ... 0. 1. 50 .0

Excess of the moon's motion over the sun's in 12 hours = 0. 22. 30 .4

As 12h − 22m30s.4 = 11h37m29s.6,
in seconds = ... 41849.6 Log. ar. comp. = 5.378309
Is to 12 hours, in seconds = . . . 43200. Log. = . . 4.635484
So is the approximate time of transit
= 18h24m55s.1, in seconds = 66295.1 Log. = . . . 4.821481

To the apparent time of the moon's
transit = 19h0m34s.3, in secs. = 68434.3 Log. = . . . 4.835274

Note.—In strictness the apparent time of transit, thus found, should be corrected by the equation of second difference answering thereto, and the mean second difference of the moon's place in right ascension; but, at sea, this correction may safely be dispensed with.

Problem IX.

Given the Apparent Time of the Moon's Transit over the Meridian of Greenwich, to find the Apparent Time of Transit over any other Meridian.

Rule.

Take, from page VI. of the month in the Nautical Almanac, the moon's transit over the meridian of Greenwich on the given day, and also on the day *following* if the longitude be *west*, but on the day *preceding* it if it be *east*, and find the difference; which difference will be the daily retardation of transit: then say,

As the sum of 24 hours and the daily retardation of the moon's transit, thus found, is to the daily retardation of transit; so is the longitude of the given meridian, in time, to a correction, which, being applied by addition to the apparent time of transit over the meridian of Greenwich on the given day, if the longitude be west, but by subtraction if east; the sum, or difference, will be the apparent time of transit over the given meridian.

Note.—This proportion may be readily performed by proportional logarithms, esteeming the hours and minutes in the *first* and *third* terms as minutes and seconds.

Example 1.

Required the apparent time of the moon's transit over a meridian 94°30′30″ west of Greenwich, March 26th, 1825, the computed apparent time of transit at Greenwich being 5ʰ15ᵐ54ˢ.9?

Moon's transit over the merid. of Greenwich on the given day = 5ʰ16ᵐ
Moon's ditto on the day *following* = 6.11

Daily retardation of moon's transit = ... 0ʰ55ᵐ

As 24 hours + 0ʰ55ᵐ (daily retard.) = 24ʰ55ᵐ Prop. log. ar. comp. = 9.1412
Is to the daily retard. of transit = 0.55 Prop. log. = . . . 0.5149
So is the lon. 94°30′30″W., in time = 6ʰ18ᵐ2ˢ Prop. log. = 1.4559

To the correction of retardation = . + 13ᵐ54ˢ.5 Prop. log. = 1.1120
Computed apparent time of moon's
transit over the merid. of Greenwich = 5ʰ15ᵐ54ˢ.9

App. time of ☽'s tr. over the given mer. = 5ʰ29ᵐ49ˢ.4

Example 2.

Required the apparent time of the moon's transit over a meridian 105°10′45″ east of Greenwich, April 10th, 1825, the computed apparent time of transit at Greenwich being $19^h 0^m 34^s.3$?

Moon's transit over the merid. of Greenwich on the given day =	$19^h\ 1^m$
Moon's ditto on the day *preceding* =	18. 13
Daily retardation of moon's transit = ...	$0^h 48^m$

As 24 hours + $0^h 48^m$ (daily retard.) =	$24^h 48^m$	Prop. log. ar. comp. =	9. 1392
Is to the daily variation of transit =	0. 48	Prop. log. = . .	0. 5740
So is the long. 105°10′45″E., in time =	$7^h\ 0^m 43^s$	Prop. log. =	1. 4094
To the correction of retardation =	$13^m 34^s.4$	Prop. log. =	1. 1226
Computed apparent time of moon's tr. over the merid. of Greenwich =	$19^h\ 0^m 34^s.3$		
App. time of ☽'s tr. over given mer. =	$18^h 46^m 59^s.9$		

Note.—The above problem may be readily solved by means of Table XXXVIII.—See explanation, page 100.

Problem X.

To compute the Apparent Time of a Planet's Transit over the Meridian of Greenwich.

Rule.

Find the planet's right ascension at noon of the given day, by Problem VII., page 307; from which (increased by 24 hours if necessary), subtract the sun's right ascension at that noon, and the remainder will be the approximate time of the planet's transit over the meridian of Greenwich.

Take the difference of the sun's and the planet's daily variations, or motions, in right ascension, if the planet's motion be *progressive*, but the sum if it be *retrograde**: then say,

As 24 hours, diminished or augmented by this difference or sum (according as the planet's diurnal motion in right ascension is greater or less than

* When the daily variation of the planet's right ascension is greater than that of the sun's, its motion is *progressive*; but when less, its motion is *retrograde*.

the sun's), is to 24 hours, so is the approximate time of transit to the apparent time of the planet's transit over the meridian of Greenwich.

Note.—If the terms be reduced to seconds, the operation may be easily performed by logarithms.

Example 1.

Required the apparent time that the planet Mars will pass the meridian of Greenwich, March 16th, 1825?

Planet's right ascension at noon of the given day = . . $0^h 49^m 30^s.0$
Sun's right ascension at that noon = ... 23. 44. 0 .3

Approx. time of the planet's transit over the mer. of Greenw. = $1^h\ 5^m 29^s.7$

Planet's right ascension at noon, March 16th, = $0^h 49^m 30^s$
Planet's ditto 17th, = 0. 52. 20

Planet's motion in 24 hours = ... $0^h\ 2^m 50^s$ $0^h\ 2^m 50^s$

Sun's right ascension at noon, March 16th, = $23^h 44^m\ 0^s.3$
Sun's ditto 17th, = 23. 47. 39 .3

Sun's motion in 24 hours = ... $0^h\ 3^m 39^s.0$ $0^h\ 3^m 39^s$

Sum of the motions = $0^h\ 6^m 29^s$

Note.—The sum is taken because the planet's motion is *retrograde.*

As 24 ho. + $6^m 29^s$ = $24^h 6^m 29^s$, in secs. = 86789 Log. ar. comp. = 5. 061535
Is to 24 hours, in seconds = . . . 86400 Log. = . . 4. 936514
So is the approximate time of transit
= $1^h 5^m 29^s.7$, in seconds = . . . 3929. 7 Log. = 3. 594359

To the apparent time of the planet's
transit = $1^h 5^m 12^s.1$, in seconds = 3912. 1 Log. = 3. 592408

Example 2.

Required the apparent time that the planet Venus will pass the meridian of Greenwich, Sept. 23d, 1825?

Planet's right ascension at noon of the given day = . . $9^h 34^m 40^s.0$
Sun's right ascension at that noon = ... 12. 0. 29 .7

Approx. time of the planet's tr. over the mer. of Greenw. = $21^h 34^m 10^s.3$

Planet's right ascension at noon, Sept. 23d,	$= 9^h34^m40^s$		
Planet's ditto	24th, $= 9.39.20$		
Planet's motion in 24 hours $=$	$0^h\ 4^m40^s$	$0^h\ 4^m40^s$	
Sun's right ascension at noon, Sept. 23d,	$= 12^h\ 0^m29^s.7$		
Sun's ditto	24th, $= 12.\ 4.\ 5\ .5$		
Sun's motion in 24 hours $=$	$0^h\ 3^m35^s.8$	$0.\ 3.35\ .8$	
	Difference of motion $=$	$0^h\ 1^m\ 4^s.2$	

Note.—The difference is taken because the planet's motion is *progressive.*

As $24^h - 1^m4^s.2 = 23^h58^m55^s.8$, in secs. $=$	86335.8	Log.ar.co. $=$	5.063809
Is to 24 hours, in seconds $=$ . . .	86400.	Log. $=$.	4.936514
So is the approx. time of transit $= 21^h34^m10^s.3$, in secs. $=$.	77650.3	Log. $=$.	4.890143
To the apparent time of the planet's transit $= 21^h35^m8^s.1$, in secs. $=$.	77708.1	Log. $=$.	4.890466

Problem XI.

Given the Apparent Time of a Planet's Transit over the Meridian of Greenwich, to find the Apparent Time of Transit over any other Meridian.

Rule.

Take, from page IV. of the month in the Nautical Almanac, the apparent times of the planet's transits over the meridian of Greenwich on the days nearest preceding and following the given day, and find the interval between those times; find, also, the difference of transit in that interval: then say,

As the interval between the times of transit is to the difference of transit, so is the longitude, in time, to a correction; which, being added to the *computed apparent time of transit,* if the longitude be west and the planet's transit increasing, or subtracted if decreasing, the sum or difference will be the apparent time of transit over the meridian of the given place; but, if the longitude be east, a contrary process is to be observed; that is, the correction is to be subtracted from the approximate time of transit, if the transit be increasing, but to be added thereto if decreasing.

Note.—If the *first* and *third* terms of the proportion be esteemed as *minutes* and *seconds,* the operation may be performed by proportional logarithms.

Example 1.

Required the apparent time that the planet Mars will pass the meridian of a place $145°30'$ west of Greenwich, March 16th, 1825, the computed apparent time of transit at Greenwich being $1^h 5^m 12^s.1$?

Time of preceding transit = ...	$13^d\ 1^h\ 8^m$
Time of following transit = ...	$19.\ 1.\ 3$
Interval between the times of transit =	$5^d 23^h 55^m$
Difference of transit in that interval = .	5 minutes.

As the interval $=5^d 23^h 55^m = 143^h 55^m = 2^h 23^m 55^s$		P. log. ar. co. =	9.9028
Is to the difference of transit = . .	5^m	Prop. log. =	1.5563
So is the long. $145°30'$ W. in time =	$9^h 42^m\ 0^s$	Prop. log. =	1.2685
To the correction of transit = ...	$-\ 20^s$	Prop. log. =	2.7276
Computed time of planet's transit over the meridian of Greenwich = .	$1^h\ 5^m 12^s.1$		
Apparent time of planet's transit over the given meridian = ...	$1^h\ 4^m 52^s.1$		

Example 2.

Required the apparent time that the planet Venus will pass the meridian of a place $175°40'$ east of Greenwich, Sept. 23d, 1825, the computed apparent time of transit at Greenwich being $21^h 35^m 8^s.1$?

Time of preceding transit = ...	$19^d 21^h 31^m$
Time of following transit = ...	$25.\ 21.\ 37$
Interval between the times of transit =	$6^d\ 0^h\ 6^m$
Difference of transit in that interval = .	6 minutes.

As the interval $=6^d 0^h 6^m = 144^h 6^m = 2^h 24^m\ 6^s$		P. log. ar. comp. =	9.9034
Is to the difference of transit =	6^m	Prop. log. = . .	1.4771
So is the long. $175°40'$E., in time =	$11^h 42^m 40^s$	Prop. log. = .	1.1867
To the correction of transit = ...	$-\ 29^s$	Prop. log. =	2.5672
Computed time of planet's transit over the meridian of Greenwich = ...	$21^h 35^m\ 8^s.1$		
Apparent time of planet's transit over the given meridian = ...	$21^h 34^m 39^s.1$		

Problem XII.

To find the Apparent Time of a Star's Transit, or Passage over the Meridian of any known Place.

Since the plane of the meridian of any given place may be conceived to be extended to the sphere of the fixed stars,—therefore, when the diurnal motion of the earth round its axis brings the plane of that meridian to any particular star, such star is then said to transit, or pass over the meridian of that place. This observation is applicable to all other celestial objects.

The apparent time of transit of a known fixed star is to be computed by the following

Rule.

Reduce the right ascension of the star, as given in Table XLIV., to the given day; from which (increased by 24 hours if necessary,) subtract the sun's right ascension at noon of that day, as given in the Nautical Almanac, and the remainder will be the approximate time of transit.

Turn the longitude of the given meridian or place into time, by Problem I., page 296, and add it to the approximate time of transit if the longitude be west, but subtract it if east; and the sum, or difference, will be the corresponding time at Greenwich; and let it be noted whether that time *precedes* or *follows* the noon of the given day.

Find, in the Nautical Almanac, the variation of the sun's right ascension between the noons preceding and following the Greenwich time; then,

To the proportional logarithm of this variation, add the proportional logarithm of the *difference* between the Greenwich time and the noon of the given day (esteeming the hours as *minutes*, and the minutes as seconds), and the constant logarithm 9.1249*; the sum of these three logarithms, abating 10 in the index, will be the proportional logarithm of a correction, which, being added to the approximate time of transit if the Greenwich time *precedes* the noon of the given day, or subtracted therefrom if it *follows* that noon, the sum or difference will be the apparent time of the star's transit over the given meridian.

Example 1.

At what time on the 2d of January, 1825, will the star Rigel transit, or come to the meridian of a place 165°30′ east of Greenwich?

* This is the arithmetical complement of the proportional logarithm of 24 hours, esteemed as *minutes*.

Right ascension of Rigel, reduced to the given day, = . . .	$5^h\ 6^m\ 8^s$
Sun's right ascension at noon of the given day = ...	18.51.44
Approximate time of transit = ...	$10^h 14^m 24^s$
Longitude 165°30′ east, in time = ...	11. 2. 0
Greenwich time past noon of January 1st = ...	$23^h 12^m 24^s$

which is $47^m 36^s$ *before* noon of the given day.

Sun's right ascension at noon, January 1st = ...	$18^h 47^m 19^s$
Sun's ditto 2d = ...	18.51.44
Variation of right ascension in 24 hours = ...	$0^h\ 4^m 25^s$

Variation of right ascension =	$4^m 25^s$	Prop. log. =	1.6102
Diff. of Gr. time from noon =	47.36	Prop. log. =	2.3558
Constant log. = ...			9.1249
Correction of star's transit =	$+\ 0^m\ 9^s$	Prop. log. =	3.0909
Approximate time of transit =	$10^h 14^m 24^s$		
Apparent time of transit =	$10^h 14^m 33^s$, as required.*		

Example 2.

At what time on the 2d of January, 1825, will the star Markab transit, or come to the meridian of a place 140°40′ west of Greenwich?

Right ascension of Markab, reduced to the given day, = .	$22^h 56^m\ 3^s$
Sun's right ascension at noon of the given day = ...	18.51.44
Approximate time of transit = ...	$4^h\ 4^m 19^s$
Longitude 140°40′ west, in time = ...	9.22.40
Greenwich time = ...	$13^h 26^m 59^s$

which, of course, is *past* the noon of the given day.

Sun's right ascension at noon, January 2d, = ...	$18^h 41^m 44^s$
Sun's ditto 3d, = ...	18.56. 8
Variation of right ascension in 24 hours = ...	$0^h\ 4^m 24^s$

* If 12 hours, diminished by half the variation of the sun's right ascension, be added to the apparent time of transit, thus found, the sum, abating 24 hours if necessary, will give the apparent time of transit below the pole.

Variation of right ascension =	4^{m}24^s	Prop. log. =	1.6118
Diff. of Gr. time from noon =	13^{h}26^{m}59^s	Prop. log. =	1.1266
Constant log. =	. . . .		9.1249
Correction of star's transit =	− 2^{m}28^s	Prop. log. =	1.8633
Approximate time of transit =	4^{h}4^{m}19^s		
Apparent time of transit =	4^{h}1^{m}51^s, as required.*		

Note.—The correction of a star's approximate time of transit may be readily found by means of Table XV., in the same manner, precisely, as if it were the proportional part of the sun's right ascension that was under consideration.—See explanation, page 25, and examples, pages 26 and 28.

Problem XIII.

To find what Stars will be on, or nearest to, the Meridian at any given Time.

Rule.

To the sun's right ascension, at noon of the given day, add the apparent time at ship, and the sum will be the right ascension of the meridian or mid-heaven; with which enter Table XLIV., and find what stars' right ascensions correspond with, or come nearest thereto, and they will be the stars required.

If much accuracy be required, the sun's right ascension at noon of the given day must be previously reduced to the given time and place, by Problem V., page 298; at sea, however, this reduction may be dispensed with.

Example 1.

What star will be nearest to the meridian, April 6th, 1825, at 9^{h}40^{m}20^s apparent time?

Sun's right ascension at noon of the given day =	1^h 0^{m}24^s
Given apparent time at ship or place = . .	9.40.20
Right ascension of the meridian or mid-heaven =	10^{h}40^{m}44^s

Now, this being looked for among the right ascensions of the stars, in

* See Note, page 318.

Table XLIV., it will be found that the star's right ascension corresponding nearest thereto, is that of η Argûs Navis; which, therefore, is the star required, or the one nearest to the meridian at the given time.

Example 2.

What star will be nearest to the meridian, December 31st, 1825, at $10^h 12^m 41^s$ apparent time?

Sun's right ascension at noon of the given day =	$18^h 41^m 49^s$
Given apparent time at ship or place = . . .	10. 12. 41
Right ascension of the meridian or mid-heaven =	$4^h 54^m 30^s$

Now, this being looked for among the right ascensions of the stars, in Table XLIV., it will be found that the star's right ascension corresponding nearest thereto, is that of β Eridani; which, therefore, is the star required, or the one nearest to the meridian at the given time.

Note.—When the sum of the sun's right ascension and the apparent time exceeds 24 hours, let 24 hours be subtracted therefrom; and the remainder will be the right ascension of the meridian, as in the last example.

Problem XIV.

Given the observed Altitude of the lower or upper Limb of the Sun, to find the true Altitude of its Centre.

Rule.

For the Fore Observation.

To the observed altitude of the sun's lower limb (corrected for index error, if any,) add the difference between its semi-diameter * and the dip of the horizon †; and the sum will be the apparent altitude of the sun's centre: or, from the corrected observed altitude of the sun's upper limb subtract the sum of the semi-diameter * and the dip of the horizon †; and the remainder will be the apparent central altitude.

For the Back Observation.

From the observed altitude of the sun's lower limb subtract the differ-

* Page III. of the month in the Nautical Almanac. † Table II.

ence between its semidiameter and the dip of the horizon: or, to the observed altitude of its upper limb add the sum of the semi-diameter and the dip of the horizon, and the sun's apparent central altitude will be obtained.

Now, from the apparent altitude of the sun's centre, thus found, subtract the difference between the refraction* corresponding thereto, and the parallax in altitude†, and the remainder will be the true altitude of the sun's centre.

Example 1.

Let the observed altitude of the sun's lower limb, by a *fore observation*, be 16°29′, the height of the eye above the level of the sea 24 feet, and the sun's semi-diameter 16′18″; required the sun's true central altitude?

Observed altitude of the sun's lower limb =		16°29′ 0″
Sun's semidiameter = . . 16′18″	Diff. =	+ 11.36
Dip of the horiz. for 24 feet = 4.42		
Apparent altitude of the sun's centre = . .		16°40′36″
Refraction = 3′8″	Difference = . . .	− 3. 0
Parallax = 0.8		
True altitude of the sun's centre = . . .		16°37′36″

Example 2.

Let the observed altitude of the sun's upper limb, by a *fore observation*, be 18°37′, the height of the eye above the surface of the water 30 feet, and the sun's semi-diameter 15′46″; required the true central altitude?

Observed altitude of the sun's upper limb =		18°37′ 0″
Sun's semi-diameter = . . 15′46″	Sum =	−21. 1
Dip of the horizon for 30 feet = 5.15		
Apparent altitude of the sun's centre = . .		18°15′59″
Refraction = 2′51″	Difference = . . .	− 2.43
Parallax = 0. 8		
True altitude of the sun's centre = . . .		18°13′16″

Example 3.

Let the observed altitude of the sun's lower limb, by a *back observation*, be 20°10′, the height of the eye above the level of the sea 25 feet, and the sun's semi-diameter 15′55″; required the true central altitude?

* Table VIII. † Table VII.

Observed altitude of the sun's lower limb =			20°10′ 0″
Sun's semi-diameter = . . .	15′55″	Diff. =	11. 8
Dip of the horizon for 25 feet =	4.47		
Apparent altitude of the sun's centre = . .			19°58′52″
Refraction =	2′35″	Difference = . . .	− 2.27
Parallax = .	0. 8		
True altitude of the sun's centre = . . .			19°56′25″

Example 4.

Let the observed altitude of the sun's upper limb, by a ***back observation***, be 25°31′, the height of the eye above the surface of the water 27 feet, and the sun's semi-diameter 15′49″; required the true central altitude?

Observed altitude of the sun's upper limb =			25°31′ 0″
Sun's semi-diameter = .	15′49″	Sum =	+ 20.47
Dip of the horizon for 27 feet =	4.58		
Apparent altitude of the sun's centre = . .			25°51′47″
Refraction =	1′57″	Difference = . .	− 1.49
Parallax =	0. 8		
True altitude of the sun's centre = . . .			25°49′58″

Remark.—I think it my duty, in this place, to caution the mariner against the mistaken rule for the ***back observation***, given in some treatises on Navigation;—because, if that rule be adopted, the ship's place will, most assuredly, be affected by an error in latitude equal to the full measure of the sun's diameter, or about 32 miles: and this, to a ship approaching or drawing in with the land, becomes an object of the most serious consideration, since it so very materially affects the lives and interests of those concerned. To set the mariner right in this matter, I will here work an

Example.

December 25th, 1825, in longitude 35° W., the meridian altitude of the sun's lower limb, by a *back observation*, was 16°28′ south, the height of the eye being 20 feet; required the latitude?

Observed altitude of the sun's lower limb =		16°28′ 0″
Sun's semi-diameter = . 16′18″ Dip of the horizon for 20 feet=4. 17	Diff. =	− 12. 1
Apparent altitude of the sun's centre = . .		16°15′59″
Refraction = 3′13″ Parallax = . . 0. 8	Difference = . . .	− 3. 5
True altitude of the sun's centre = . . .		16°12′54″
Sun's meridional zenith distance = . . .		73°47′ 6″ north.
Sun's corrected declination = ...		23. 24. 46 south.
Required latitude = ...		50°22′20″ north.

By the old rule, the latitude is only 49°50′ north, which is evidently erroneous, it being 32 miles and 20 seconds less than the truth.

Problem XV.

Given the observed Altitude of the upper or lower Limb of the Moon, to find the true central Altitude.

Rule.

Turn the longitude into time, and add it to the apparent time of observation if it be west, or subtract it therefrom if east, and it will give the corresponding time at Greenwich.

To this time let the moon's semi-diameter and horizontal parallax be reduced, by Problem VI., page 302, (or by Table XVI., as explained in pages 30 and 33,) and let the reduced semi-diameter be increased by the correction contained in Table IV., answering to it and the observed altitude; then,

To the observed altitude of the moon's lower limb (correeted for index error, if any), add the difference between the true semi-diameter and the dip of the horizon; or, from the observed altitude of the upper limb subtract the sum of the semi-diameter and dip, and the apparent central altitude of the moon will be obtained; to which let the correction (Table XVIII.) answering to the moon's reduced horizontal parallax and apparent central altitude be added, and the sum will be the altitude of the moon's centre.

Example 1.

In a certain latitude, March 10th, 1825, at 3h40m20s apparent time, the observed altitude of the moon's lower limb was 20°10′40″, and the height of the eye above the level of the sea 24 feet; required the true altitude of the moon's centre, the longitude of the place of observation being 35°40′ west?

Apparent time of observation = ...	3h40m20s
Longitude 35°40′ W., in time = . . .	2.22.40
Greenwich time = ...	6h 3m 0s
Moon's reduced semi-diameter = . . .	15′40″
Augmentation, Table IV. = ...	0. 6
Moon's true semi-diameter = ...	15′46″
Moon's reduced horizontal parallax = . .	57′32″
Observed altitude of moon's lower limb =	20°10′40″
Moon's true semi-diam. = 15′46″; Dip of the horiz. for 24 feet = 4. 42; Diff. =	+11. 4
Apparent altitude of the moon's centre =	20°21′44″
Correction to altitude 20°21′44″, and horiz. parallax 57′32″, Table XVIII. =	+ 51.24
True altitude of the moon's centre = .	21°13′ 8″

Example 2.

In a certain latitude, March 26th, 1825, at 1h30m47s apparent time, the observed altitude of the moon's upper limb was 30°17′30″, and the height of the eye above the level of the sea 30 feet; required the true altitude of the moon's centre, the longitude of the place of observation being 94°15′30″ east?

Apparent time of observation = ...	1h30m47s
Longitude 94°15′30″ E., in time = .	6. 17. 2
Greenwich time past midnight, March 25th =	7h13m45s
Moon's reduced semi-diameter = ...	15′23″
Augmentation, Table IV. = ...	0. 8
Moon's true semi-diameter = ...	15′31″
Moon's reduced horizontal parallax = .	56′26″

Observed altitude of moon's upper limb =	30°17′30″
Moon's true semi-diam. = 15′31″ } Dip of the horiz. for 30 feet = 5. 15 } Sum =	−20. 46
Apparent altitude of moon's centre = .	29°56′44″
Correction to altitude 29°56′44″, and horiz. parallax 56′26″, Table XVIII. =	+ 47. 16
True altitude of the moon's centre = .	30°44′ 0″

Note.—In the above examples, the altitudes are supposed to be taken by the *fore observation;* and since this mode of observing is not only the most natural, but, also, the most simple, it will, therefore, be constantly made use of throughout the subsequent parts of this work. Hence the necessity of making constant reference to the particular mode of observation may, in future, be dispensed with.

Problem XVI.

Given the observed Altitude of a Planet's Centre, to find its true Altitude.

Rule.

From the planet's observed central altitude (corrected for index error, if any,) subtract the dip of the horizon, and the remainder will be the apparent central altitude.

Find the difference between the planet's parallax in altitude (Table VI.) and its refraction in altitude (Table VIII.); now, this difference being applied by addition to the apparent central altitude when the parallax is greater than the refraction, but by subtraction when it is less, the sum or remainder will be the true central altitude of the planet.

Example 1.

Let the observed central altitude of Venus be 16°40′, the index error 2′30″ subtractive, and the height of the eye above the level of the sea 28 feet; required the true altitude of that planet, allowing her horizontal parallax to be 31 seconds?

Observed central altitude of Venus = .	16°40′ 0″
Index error = ...	− 2.30
Dip of the horizon for 28 feet = . . .	− 5. 5
Apparent central altitude of Venus = .	16°32′25″
Refraction, Table VIII.,=3′10″ } Diff.=	
Parallax, Table VI., = 0.29 }	− 2.41
True ~~Apparent~~ central altitude of Venus = .	16°29′24″

Example 2.

Let the observed central altitude of Mars be 17°29′40″, the index error 3′45″ additive, and the height of the eye above the surface of the water 26 feet; required the true central altitude of that planet, allowing his horizontal parallax to be 17 seconds?

Observed central altitude of Mars = .	17°29′40″
Index error = ...	+ 3.45
Dip of the horizon for 26 feet = . .	− 4.52
Apparent central altitude of Mars = .	17°28′33″
Refraction, Table VIII.,=2′59″ } Diff. =	
Parallax, Table VI., = 0.16 }	− 2.43
True ~~Apparent~~ central altitude of Mars = .	17°25′50″

Remark.—In taking the altitude of a planet, its centre should be brought down to the horizon. Neither the semi-diameters nor the horizontal parallaxes of the planets are given in the Nautical Almanac, but it is to be hoped that they soon will be. If the parallaxes of the planets be determined by means of a comparison of their respective distances (from the earth's centre) with the earth's semi-diameter, they will be found to be as follows, very nearly; viz.,

Venus' greatest horizontal parallax, about 32 seconds; and her least parallax about 5 seconds.

Mars' greatest horizontal parallax, about 17 seconds; and his least parallax, about 3 seconds.

Jupiter's mean horizontal parallax, about 2 seconds; and that of Saturn about 1 second.

The parallaxes of the two last planets are subject to very little alteration, because the distances at which those objects are placed from the earth's centre are so exceedingly great as to render any variations in their parallaxes almost insensible.

Problem XVII.

Given the observed Altitude of a fixed Star, to find the true Altitude.

Rule.

To the observed altitude of the star apply the index error, if any; from which subtract the dip of the horizon, and the remainder will be the star's apparent altitude.

From the apparent altitude, thus found, let the refraction corresponding thereto be subtracted, and the remainder will be the true altitude of the star.

Example 1.

Let the observed altitude of Spica Virginis be 18°30′, the index error 3′.20″ subtractive, and the height of the eye above the level of the water 18 feet; required the true altitude of that star?

Observed altitude of Spica Virginis =	18°30′ 0″
Index error = ...	− 3.20
Dip of the horizon for 18 feet = .	− 4. 4
Apparent altitude of Spica Virginis =	18°22′.36″
Refraction = ...	− 2.50
True altitude of Spica Virginis = .	18°19′.46″

Example 2.

Let the observed altitude of Regulus be 20°43′, the index error 1′.47″ additive, and the height of the eye above the level of the sea 20 feet; required the true altitude of that star?

Observed altitude of Regulus =	20°43′ 0″
Index error = ...	+ 1.47
Dip of the horizon for 20 feet =	− 4.17
Apparent altitude of Regulus =	20°40′.30″
Refraction = ...	− 2.29
True altitude of Regulus = .	20°38′ 1″

Note.—The fixed stars do not exhibit any apparent semi-diameter, nor any sensible parallax; because the immense and inconceivable distance at which they are placed from the earth's surface causes them to appear, at all times, as so many mere luminous indivisible points in the heavens.

SOLUTION OF PROBLEMS RELATIVE TO THE LATITUDE.

The *Latitude* of any place on the earth is expressed by the distance of such place from the equator, either north or south, and is measured by an arc of the meridian intercepted between the said place and the equator.—Or,

The *Latitude* of any place on the earth is equal to the elevation of the pole of the equator above the horizon of such place; or (which amounts to the same), it is equal to the distance of the zenith of the place from the equinoctial in the heavens. The *complement* of the latitude is the distance of the zenith of any place from the pole of the equator, and is expressed by what the latitude wants of 90 degrees. The latitude is named north or south, according as the place is situate with respect to the equator.

Problem I.

Given the Sun's Meridian Altitude, to find the Latitude of the Place of Observation.

Rule.

Find the true altitude of the sun's centre, by Problem XIV., page 320, and call it north or south, according as that object may be situate with respect to the observer at the time of observation; which, subtracted from 90°, will give the sun's meridional zenith distance of a contrary denomination to that of its altitude.

Reduce the sun's declination to the meridian of the place of observation, by Problem V., page 298, or, more readily, by Table XV. Then, if the meridional zenith distance and the declination are both north or both south, their sum will be the latitude of the place of observation; but if one be north and the other south, their difference will be the latitude, and always of the same name with the greater term.*

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* The principles upon which this rule is founded may be seen by referring to "The Young Navigator's Guide to the Sidereal and Planetary Parts of Nautical Astronomy," page 98; reading, however, the word *sun* instead of *star*.

Example 1.

April 10th, 1825, in longitude 75° W., the meridian altitude of the sun's lower limb was 57°40′30″ S., and the height of the eye above the level of the sea 22 feet; required the latitude?

Observed altitude of the sun's lower limb =		57°40′30″ S.	
Sun's semidiameter = . 15′59″ Dip of the horiz. for 22 feet = 4. 30	Diff. =	+ 11. 29	
Apparent altitude of the sun's centre =	.	57°51′59″ S.	
Refraction = 0′35″ Parallax = 0. 5	Difference = .	− 0. 30	
True altitude of the sun's centre = . . .	.	57°51′29″ S.	
Sun's meridional zenith distance = . . .	.	32° 8′31″ N.	32°8′31″ N.
Sun's declination at noon, April 10th =	7°56′42″ N.		
Correction for longitude 75° W. = .	+ 4. 36		
Sun's reduced declination = ...	8° 1′18″ N.		8°1′18″ N.
Latitude, as required = ...			40°9′49″ N.

Note.—The meridional zenith distance and the declination are added together, because they are both of the same name: hence, the latitude is 40°9′49″ N.

Example 2.

October 24th, 1825, in longitude 90° east, the meridian altitude of the sun's lower limb was 27°31′20″ S., and the height of the eye above the surface of the sea 23 feet; required the latitude?

Observed altitude of the sun's lower limb =		27°31′20″ S.
Sun's semi-diameter = . 16′ 8″ Dip of the horiz. for 23 feet = 4. 36	Diff. =	+ 11. 32
Apparent altitude of the sun's centre = . .		27°42′52″ S.
Refraction = 1′48″ Parallax = 0. 8	Difference = . . .	− 1. 40
True altitude of the sun's centre = . . .		27°41′12″ S.
Sun's meridional zenith distance = . . .		62°18′48″ N.

Sun's declination at noon, Oct. 24th =	11°45′42″ S.
Correction for longitude 90° east = .	− 5.15
Sun's reduced declination = . . .	11°40′27″ S.
Sun's meridional zenith distance = .	62.18.48 N.
Latitude, as required = ...	50°38′21″ N.

Note.—The difference between the meridional zenith distance and the declination is taken, because they are of contrary names: hence, the latitude is 50°38′21″ N.

Problem II.

Given the Moon's Meridional Altitude, to find the Latitude of the Place of Observation.

Rule.

Reduce the moon's passage over the meridian of Greenwich, on the given day, to the meridian of the place of observation, by applying thereto the correction in Table XXXVIII., by addition or subtraction, according as the longitude is west or east; as explained in examples 1 and 2, pages 101 and 102.

To the time of the moon's passage over the meridian of the place of observation, thus found, let the longitude of that meridian, in time, be added if it be west, or subtracted if east; and the sum, or difference, will be the corresponding time at Greenwich: to which let the moon's declination, horizontal parallax, and semi-diameter, be reduced by Problem VI., page 302, (or by means of Table XVI., as explained in page 30,) and let the moon's reduced semi-diameter be corrected by the augmentation contained in Table IV.

Find the true altitude of the moon's centre, by Problem XV., page 323, and call it north or south, according as it may be situate with respect to the observer at the time of observation; which, subtracted from 90°, will give the moon's meridional zenith distance of a contrary denomination to that of its altitude.

Then, if the meridional zenith distance and the declination are of the same name, their sum will be the latitude of the place of observation; but if they are of contrary names, their difference will be the latitude, of the same name with the greater term.

Note.—In strictness, the moon's declination should be corrected by the equation of second difference contained in Table XVII, as explained between pages 33 and 37.

Example 1.

January 27th, 1825, in longitude 55° W., the meridian altitude of the moon's lower limb was 58°40′ S., and the height of the eye above the level of the sea 26 feet; required the latitude?

Time of ☽'s passage over the meridian of Greenwich = . . .	5h54m 0s
Correction, Table XXXVIII., for longitude 55° W. = . . .	+ 7.23
Time of ☽'s pass. over the merid. of the place of observation =	6h 1m23s
Longitude 55° W., in time = ...	+ 3.40. 0
Greenwich time = ...	9h41m23s

Moon's horizontal parallax at noon, Jan. 27th =	55′20″
Correction of parallax for 9h41m23s = . .	+ 0.17
Moon's reduced horizontal parallax = ...	55′37″
Moon's semi-diameter at noon, Jan. 27th = . .	15′ 5″
Correction of semi-diameter for 9h41m23s = .	+ 4
Augmentation of semi-diameter, Table IV. = .	+12
Moon's true semi-diameter = ...	15′21″

Moon's declination at noon, Jan. 27th =	18°19′18″N.
Correction of declination for 9h41m23s =	+1. 16. 7
Moon's reduced declination = ...	19°35′25″N.

Observed altitude of the moon's lower limb = . . .		58°40′ 0″ S.
Moon's true semi-diameter = 15′21″	} Diff. =	+ 10. 29
Dip of the horiz. for 26 feet = . 4. 52		
Apparent altitude of the moon's centre = ...		58°50′29″ S.
Correction of altitude, Table XVIII. = ...		+ 28. 12
True altitude of the moon's centre = ...		59°18′41″ S.
Moon's meridional zenith distance = ...		30°41′19″ N.
Moon's reduced declination = ...		19. 35. 25 N.
Latitude of the place of observation = ...		50°16′44″ N.

Example 2.

February 3d, 1825, in longitude 65° E., the meridian altitude of the moon's upper limb was 62°45′ north, and the height of the eye above the level of the sea 29 feet; required the latitude?

Time of ☽'s passage over the meridian of Greenwich = .	$12^h25^m0^s$
Correction, Table XXXVIII., for longitude 65° east = . . .	− 9.44
Time of ☽'s pass. over the merid. of the place of observation =	$12^h15^m16^s$
Longitude 65° E., in time = ...	− 4.20. 0
Greenwich time = ...	$7^h55^m16^s$

Moon's semi-diameter at noon, February 3d =	16′34″
Correction of semi-diameter for $7^h55^m16^s$ =	+ 1
Augmentation of semi-diameter, Table IV. =	+ 16
Moon's true semi-diameter = ...	16′51″

Moon's horizontal parallax at noon, February 3d =	60′49″
Correction of parallax for $7^h55^m16^s$ = . . .	+ 5
Moon's reduced horizontal parallax = ...	60′54″

Moon's declination at noon, February 3d =	12°52′50″ N.
Correction of declination for $7^h55^m16^s$ =	− 1.47.29
Moon's reduced declination = ...	11° 5′21″ N.

Observed altitude of the moon's upper limb =		62°45′ 0″ N.
Moon's true semi-diameter = 16′51″	Sum =	− 22. 1
Dip of the horiz. for 29 feet = 5.10		
Apparent altitude of the moon's centre = . . .		62°22′59″ N.
Correction of altitude, Table XVIII. = ...		+ 29.33
True altitude of the moon's centre = . . .		62°52′32″ N.

Moon's meridional zenith distance = . .	27° 7′28″ S.
Moon's reduced declination = ...	11. 5.21 N.
Latitude of the place of observation = . .	16° 2′ 7″ S.

Remark.—Although this method of finding the latitude at sea is strictly correct when the longitude of the place of observation is well determined; yet, in some cases, it is subject to such peculiarities as to render it inconvenient to the practical navigator: this happens in high latitudes, and when the variation in the moon's declination is very considerable; because, under such circumstances, the moon's altitude sometimes continues to increase after she has actually passed the meridian. To provide against this, the observer should be furnished with a chronometer, or other well-regulated watch, to show the instant of the moon's coming to the meridian of the ship or place; at which time her altitude should be taken, without waiting for its ceasing to rise or *beginning to dip*, as it is generally termed at sea: then this altitude is to be considered as the observed meridional altitude of that object, and to be acted upon accordingly.

Problem III.

Given the Meridional Altitude of a Planet, to find the Latitude of the Place of Observation.

Rule.

To the apparent time of observation (always reckoning from the preceding noon,) apply the longitude, in time, by addition or subtraction, according as it is west or east; and the sum, or difference, will be the corresponding time at Greenwich, to which let the planet's declination be reduced, by Problem VII., page 307.

Find the true altitude of the planet's centre, by Problem XVI., page 325; and hence its meridional zenith distance, noting whether it be north or south: then, if the meridional zenith distance and the declination are of the same name, their sum will be the latitude of the place of observation; but if they are of contrary names, their difference will be the latitude, of the same name with the greater term.

Example 1.

February 3d, 1825, in longitude 80° W., at 11h28m30s apparent time, the meridional central altitude of the planet Jupiter was 58°22′ S., the height of the eye above the level of the sea 24 feet, and the planet's horizontal parallax 2 seconds; required the latitude?

Apparent time of observation, February =	3d11h28m30s
Longitude 80° W., in time = ...	+ 5.20. 0
Greenwich time = ...	3d16h48m30s

Jupiter's declination, February 1st = .	19° 3′ 0″ N.
Correction of ditto for $2^d16^h48^m30^s$.	+ 5.51
Jupiter's reduced declination = ...	19° 8′51″ N.

Jupiter's observed central altitude =	58°22′ 0″ S.
Dip of the horizon for 24 feet =	− 4.42
Jupiter's apparent central altitude =	58°17′18″ S.
Refraction, Tab. VIII. = 0′34″ } Parallax, Table VI. = 0. 1 } Diff. =	− 0.33
Jupiter's true central altitude = .	58°16′45″ S.
Jupiter's meridional zenith distance =	31°43′15″ N.
Jupiter's reduced declination = .	19. 8.51 N.
Latitude of the place of observation =	50°52′ 6″ N.

Example 2.

March 16th, 1825, in longitude 75° E., at 2^h49^m apparent time, the meridional central altitude of the planet Venus was 31°10′ N., the height of the eye above the level of the horizon 18 feet, and the planet's horizontal parallax 23 seconds; required the latitude?

Apparent time of observation, March =	$16^d\ 2^h49^m$
Longitude, 75° E., in time = ...	− 5. 0
Greenwich time = ...	$15^d21^h49^m$

Venus' declination, March 13th =	17°15′ 0″ N.
Correction of ditto for $2^d21^h49^m$ =	+ 1. 5.57
Venus' reduced declination = . .	18°20′57″ N.

Venus' observed central altitude = .	31°10′ 0″ N.
Dip of the horizon for 18 feet = . .	− 4. 4
Venus' apparent central altitude = .	31° 5′56″ N.
Refraction, Table VIII. = 1′35″ } Parallax, Table VI. = 0.20 } Diff. =	− 1.15
Venus' true central altitude = . . .	31° 4′41″ N.
Venus' meridional zenith distance = .	58°55′19″ S.
Venus' reduced declination = . . .	18.20.57 N.
Latitude of the place of observation =	40°34′22″ S.

Note.—The principles of finding the latitude by the meridional altitude of a celestial object may be seen by referring to "the Young Navigator's Guide to the Sidereal and Planetary Parts of Nautical Astronomy," between pages 98 and 105.

Problem IV.

Given the Meridional Altitude of a fixed Star, to find the Latitude of the Place of Observation.

Rule.

Find the true altitude of the star, by Problem XVII., page 327; and hence its meridional zenith distance, noting whether it be north or south. Take the declination of the star from Table XLIV., and reduce it to the time of observation. Now, if the star's meridional zenith distance and its declination be of the same name, their sum will be the latitude of the place of observation; but if they are of contrary names, their difference will be the latitude, of the same name with the greater term.

Example 1.

January 1st, 1825, in longitude 85°3′ W., at $12^h 39^m 26^s$ apparent time, the meridian altitude of Procyon was 44°49′ S., and the height of the eye above the level of the horizon 16 feet; required the true latitude?

Observed altitude of Procyon = . .	44°49′ 0″ S.
Dip of the horizon for 16 feet = . .	− 3.50
Procyon's apparent altitude = . . .	44°45′10″ S.
Refraction = ...	− 0.57
Procyon's true altitude = ...	44°44′13″ S.
Procyon's meridional zenith distance =	45°15′47″N.
Procyon's reduced declination = . .	5.40.16 N.
Latitude of the place of observation =	50°56′ 3″N.

Example 2.

January 2d, 1825, in longitude 165°30′ E., at $10^h 14^m 33^s$ apparent time, the meridian altitude of Rigel was 30°39′ S., and the height of the eye above the level of the sea 21 feet; required the true latitude?

Observed altitude of Rigel = . . .	30°39′ 0″ S.
Dip of the horizon for 21 feet = . .	— 4.24
Rigel's apparent altitude = . . .	30°34′36″ S.
Refraction = ...	— 1.37
Rigel's true altitude = ...	30°32′59″ S.
Rigel's meridional zenith distance =	59°27′ 1″N.
Rigel's reduced declination = . . .	8.24.35 S.
Latitude of the place of observation =	51° 2′26″N.

Note.—The principles upon which the above rule is founded, are given in "the Young Navigator's Guide to the Sidereal and Planetary Parts of Nautical Astronomy," between pages 98 and 105.

Problem V.

Given the Meridional Altitude of a Celestial Object observed below the Pole, to find the Latitude of the Place of Observation.

Rule.

Find the true altitude of the object, as before; to which let the polar distance of that object, or the complement of its corrected declination, be added, and the sum will be the latitude of the place of observation, of the same name with the declination.

Example 1.

June 20th, 1825, in longitude 65° W., the meridian altitude of the sun's lower limb, observed below the pole, was 9°12′, and the height of the eye 20 feet; required the latitude?

Observed altitude of the sun's lower limb = .		9°12′ 0″
Sun's semi-diameter = . 15′46″	Diff. =	+ 11.29
Dip of the horizon for 20 feet=4. 17		
Apparent altitude of the sun's centre = . .		9°23′29″
Refraction = 5′34″	Difference = ...	— 5.25
Parallax = 0. 9		
True meridian altitude below the pole = . .		9°18′ 4″
Sun's corrected polar distance, or co-declination=		66. 32. 17 N.
Latitude of the place of observation = . . .		75°50′21″N.

Example 2.

June 1st, 1825, in longitude 90° E., at 11h 26m 40s apparent time, the observed altitude of Capella, when on the meridian below the pole, was 11°48′, and the height of the eye above the level of the sea 25 feet; required the latitude?

Observed altitude of Capella = ...	11°48′ 0″
Dip of the horizon for 25 feet = ...	— 4.47
Capella's apparent altitude = ...	11°43′13″
Refraction = ...	— 4.29
Capella's true meridian altitude below the pole =	11°38′44″
Capella's corrected polar distance, or co-declination=	44.11.28 N.
Latitude of the place of observation = ...	55°50′12″N.

Remarks.—1. When the polar distance or co-declination of a celestial object is less than the latitude of the place of observation (both being of the same name), such celestial object will not set, or go below the horizon of that place: in this case, the celestial object is said to be circumpolar, because it revolves round the pole of the equator, or equinoctial, without disappearing in the horizon.

2. If 12 hours, diminished by half the daily variation of the sun's right ascension, be added to the apparent time of the superior transit of a *fixed star*, it will give the apparent time of its inferior transit over the opposite meridian; that is, the apparent time of its coming to the meridian below the pole.

3. The least altitude of a circumpolar celestial object indicates its being on the meridian below the pole.

PROBLEM VI.

Given the Altitude of the North Polar Star, taken at any Hour of the Night, to find the Latitude of the Place of Observation.

Although the proposed method of finding the latitude at sea is only applicable to places situate to the northward of the equator, yet, since it can be resorted to at any time of the night, it deserves the particular attention of the mariner.

Of all the heavenly bodies, the polar star seems best calculated for finding the latitude in the northern hemisphere by nocturnal observation; because a single altitude, taken at any hour of the night by a careful observer, will give the latitude to a sufficient degree of accuracy, provided the apparent time of observation be but known within a *few* minutes of the truth: however, an error in the apparent time, even as considerable as 20 minutes, will not affect the latitude to the value of half a minute, when the polar star is on the meridian, either above or below the pole; nor will it ever affect the latitude more than about 8½ minutes, even at the star's greatest distance from the meridian. But, as it is highly improbable, in the present improved state of watches, that the apparent time at the ship can ever be so far out as five minutes, the latitude resulting from this method will, in general, be as near to the truth as the common purposes of navigation require.

Rule.

To the sun's right ascension, as given in the Nautical Almanac, or in Table XII. (reduced to the meridian of the place of observation, by Problem V., page 298,) add the apparent time of observation; and the sum (rejecting 24 hours, if necessary,) will be the right ascension of the meridian, or mid-heaven; with which enter Table X., and take out the corresponding correction. Find the true altitude of the star, by Problem XVII., page 327; to which let the correction, so found, be applied by addition or subtraction, according to the directions contained in the Table, and the sum or difference will be the approximate latitude.

Enter Table XI., with the approximate latitude, thus found, at top of the page, and the right ascension of the meridian in one of the side columns; in the angle of meeting will be found a correction, which, being applied by *addition* to the approximate latitude, will give the true latitude of the place of observation.

Remark.—Since the corrections of the polar star's altitude, in Table X., have been computed for the beginning of the year 1824, a reduction therefore becomes necessary, in order to adapt them to subsequent years and parts of a year. The method of finding this reduction is illustrated in examples 1 and 2, pages 17 and 18.

Example 1.

January 2d, 1825, in longitude 60° west, at 8h 10m 40s apparent time, the observed altitude of the polar star was 52° 15′ 20″, and the height of the eye above the level of the sea 16 feet; required the latitude?

Sun's reduced right ascension = ...	$18^h 53^m 58^s$
Apparent time of observation = ...	8. 10. 40
Right ascension of the meridian = ...	$3^h\ 4^m 38^s$

Correction of altitude, Table X., answering to 3^h = . . .	1°24′16″
Proportional part to $4^m 38^s$ of right ascension = ...	− 1. 1
Annual var. of correction = 13″. 67; which × by 1 year, gives	− 0. 14
Correction of altitude, reduced to time of observation = .	1°23′ 1″;

which is *subtractive*, because the right ascension of the meridian falls in one of the left-hand columns.

Observed altitude of the polar star = ...	52°15′20″
Dip of the horizon for 16 feet = ...	− 3. 50
Apparent altitude of the polar star = ...	52°11′30″
Refraction = ...	− 0. 44
True altitude of the polar star = ...	52°10′46″
Correction from Table X., ans. to $3^h 4^m 38^s$ =	− 1. 23. 1
Approximate latitude = ...	50°47′45″N.
Correction of ditto from Table XI. = ...	+ 0′28″
Latitude of the place of observation = . . .	50°48′13″N.

Example 2.

January 1st, 1830, in longitude 75° W., at $9^h 3^m$ apparent time, let the observed altitude of the north polar star be 19°15′, and the height of the eye above the level of the horizon 23 feet; required the latitude?

Sun's R.A., Table XII., reduced to given time=	$18^h 49^m$
Apparent time of observation = ...	9. 3
Right ascension of the meridian = ...	$3^h 52^m$

Correction of altitude, Table X., answering to $3^h 50^m$ = . .	1°11′32″
Proportional part to 2^m of right ascension = ...	− 0. 34
Annual var. of correction=10″. 06, which × by 6 years, gives	− 1. 1
Correction of altitude, reduced to time of observation, = .	1° 9′57″;

which is subtractive, because the right ascension of the meridian falls in one of the left-hand columns.

Observed altitude of the polar star = . . .	19°15′ 0″
Dip of the horizon for 23 feet = ...	− 4.36
Apparent altitude of the polar star = . . .	19°10′24″
Refraction = ...	− 2.43
True altitude of the polar star = ...	19° 7′41″
Correction of altitude from Table X. = .	− 1. 9.57
Approximate latitude = ...	17°57′44″N.
Correction of ditto, Table XI. = ...	+ 0.15
Latitude of the place of observation = . .	17°57′59″N.

Example 3.

Let the true altitude of the north polar star, January 1st, 1854, be 50°5′41″, and the right ascension of the meridian 17ʰ13ᵐ; required the latitude?

True altitude of the north polar star = ...		50° 5′41″
Cor. from Tab. X., answ. to 17ʰ10ᵐ=0°44′24″	Addit.=	+0.41.36
Proportional part to = . . 3 = −1. 8		
Annual var.= −3″.34×30 years = − 1.40		
Approximate latitude = ...		50°47′17″N.
Correction of ditto from Table XI., answ. to 17ʰ13ᵐ = .		+ 1.17
True latitude, as required = ...		50°48′34″N.

Note.—The true latitude, computed with the most rigorous degree of accuracy, by spherical trigonometry, is 50°48′13″ N.; the difference, therefore, between the true spherical latitude, thus deduced, and that resulting from Tables X. and XI., as above, is only 21 seconds in the long period of 30 years: hence it is evident, that the latitude may be always determined by means of those Tables, to every degree of exactness desirable in most nautical operations.

The elementary principles of computing the latitude by an altitude of the north polar star, are given in "the Young Navigator's Guide to the Sidereal and Planetary Parts of Nautical Astronomy," between pages 144 and 156, where a diagram may be seen, illustrative of the star's *apparent motion* round its orbit.

Problem VII.

Given the Latitude by Account, the Sun's Declination, and two observed Altitudes of its lower or upper Limb, the elapsed Time, and the Course and Distance run between the Observations; to find the Latitude of the Ship at the Time of Observation of the greatest Altitude.

Rule.

To reduce the least Altitude to what it would be, if taken at the Place where the greatest Altitude was observed:—

Find the angle contained between the ship's course, (corrected for leeway, if any,) and the sun's bearing at the time of taking the least altitude; with which, if less than 8, or with what it wants of 16 points if it be more than 8, enter the general Traverse Table, and find the difference of latitude corresponding thereto and the distance made good between the observations, which call the *reduction of altitude.*

Now, if the least altitude be observed in the forenoon, the reduction of altitude is to be applied thereto by *addition* when the above angle is less than 8 points, but by *subtraction* when it is more than 8 points; the sum, or difference, will show what the less altitude would be if observed at the same place with the greater altitude. Again, if the less altitude be observed in the afternoon, a contrary process is to be observed; viz., the reduction of altitude is to be *subtracted* therefrom, when the above angle is less than 8 points, but to be *added* thereto when it is greater.

To compute the Latitude:—

Reduce the sun's declination to the time and place where the greatest altitude was observed; then, to the log. secant of the latitude by account, add the log. secant of the corrected declination; the sum, rejecting 20 from the index, will be the *logarithmic ratio.*

To the log. ratio, thus found, add the logarithm of the difference of the natural co-versed sines of the two corrected altitudes, and the logarithm of the half-elapsed time (Table XXX.); the sum of these three logarithms will be the logarithmic middle time. Find the time corresponding to this in Table XXXI.; the difference between which and the half-elapsed time will be the time from noon when the greatest altitude was observed.* From the log. rising (Table XXXII.), answering to this time, subtract the log. ratio; and the remainder will be the logarithm of a natural number, which, being subtracted from the natural co-versed sine of the greatest altitude, will leave the natural versed sine of the sun's meridional zenith distance; to which let the corrected declination be applied by addition or subtraction, according as it is of the same or of a contrary name: and the sum, or difference, will be the latitude of the ship at the time that the greatest altitude was taken; which may be reduced to noon, by means of the log, if necessary.

If the latitude, thus found, differ considerably from that by account, the *operation must be repeated*, using the computed latitude in place of that by account, until the latitude last found agrees nearly with the latitude used in the computation.

Remarks.—1. Since this method is only an approximation to the truth, it requires to be used under certain restrictions; viz., the observations must be taken between nine o'clock in the forenoon, and three in the afternoon. If both observations be in the forenoon, or both in the afternoon, the elapsed time must not be less than the distance of the observation of the greatest altitude from noon. If one observation be in the forenoon, and the other in the afternoon, the elapsed time must not exceed four hours and a half; and, in all cases, the nearer the greater altitude is to noon, the better.

2. If the sun's meridional zenith distance be less than the latitude, the limitations are still more contracted. If the latitude be double the meridian zenith distance, the observations must be taken between half-past nine in the forenoon and half-past two in the afternoon; and the elapsed time must not exceed three hours and a half. The observations must be taken still nearer to noon, if the latitude exceeds the meridian zenith distance in a greater proportion.

Example 1.

At sea, January 9th, 1825, in latitude 50° 12′ N., by account, and longitude 30° 10′ W., at $21^h 30^m 0^s$ apparent time, the observed altitude of the sun's lower limb was 10° 27′ 30″, and the bearing of its centre, by azimuth compass, S.E. ¾ S.; and at $23^h 10^m 10^s$ the observed altitude was 17° 6′ 40″, and the height of the eye above the level of the sea 20 feet; the ship's course during the elapsed time was S.S.E., at the rate of 10 knots an hour; required the latitude of the ship at the time of observing the greater altitude?

* When the middle time is greater than the half-elapsed time, both observations will be on the same side of the meridian; otherwise, on different sides.

Sun's bearing at 1st observation = S.E. ¾ S., or 3¼ points.
Ship's course = ... S.S.E., or 2 points.

Contained angle = ... 1¼ point.

Time elapsed between the observations = 1ʰ40ᵐ10ˢ.—And,
As 1ʰ : 10ᵐ :: 1ʰ40ᵐ10ˢ : 16′42″, or 17 miles nearly = the distance run between the two observations.

Now, to course 1¼ point, and distance 17 miles, the difference of latitude is 16.5 miles, the *reduction of altitude;* which is *additive* to the least altitude,* because the contained angle is less than 8 points, and the observation made in the forenoon.

Time of observing the greatest altitude = . 23ʰ10ᵐ10ˢ
Longitude 30°10′ W., in time = . . . + 2. 0.40

Greenwich time past noon of the 10th Jan. = 1ʰ10ᵐ50ˢ

Sun's declination at noon, January 10th = . . 21°57′50″ S.
Correction of declination, Table XV., for 1ʰ10ᵐ50ˢ = − 0.28

Sun's corrected declination = ... 21°57′22″ S.

First observed altitude of the sun's lower limb = 10°27′30″
Sun's semi-diameter = . 16′18″ } Diff. = + 12. 1
Dip of the horiz. for 20 feet = 4.17 }

Apparent altitude of the sun's centre = . . 10°39′31″
Refraction = 4′56″ } Difference = . . . − 4.47
Parallax = 0. 9 }
Reduction of altitude = ... +16.30

Reduced altitude = ... 10°51′14″

Second observed altitude of the sun's lower limb = 17° 6′40″
Sun's semi-diameter = . . 16′18″ } Diff. = . + 12. 1
Dip of the horiz. for 20 feet = 4.17 }

Apparent altitude of the sun's centre = ... 17°18′41″
Refraction = 3′1″ } Difference = ... − 2.53
Parallax = . 0.8 }

True altitude of the sun's centre = ... 17°15′48″ S.

Latitude by account = 50°12′ 0″N. Log. sec. = 10.193746

Time of obs.	Altitude.	Nat. co-v. sine.	Red. dec.	
21ʰ30ᵐ 0ˢ	10°51′14″	811695.	21°57′22″S. Log. sec. =	10.032700
23.10.10	17.15.48	703190.	Log. ratio = . .	0.226446
1ʰ40ᵐ10ˢ elaps. time.		Diff. = 108505.	Log. = ...	5.035450
0ʰ50ᵐ 5ˢ half-elapsed time.			Log. half-elapsed time =	0.663950
1.39.43 middle time.			Log. middle time = . .	5.925846

0ʰ49ᵐ38ˢ time from noon when the greatest altitude was taken. Log. rising = ... 4.368450
Nat. co-versed sine of the greatest alt. = 703190. Log. ratio = 0.226446

Natural number = ... 13868. Log. = 4.142004

Sun's mer. zen. dis. = 71°54′ 0″N. Nat. V.S. = 689322
Do. reduced dec. = 21.57.22 S.

Latitude of ship = 49°56′38″ north. And, since this latitude differs so much from that by account, it will be necessary to *repeat the operation.*

Computed latitude = . . .	49°56′38″	Log. secant =	10.191427
Reduced declination = . .	21.57.22	Log. secant =	10.032700
		Log. ratio =	0.224127
Diff. of nat. co-versed sines =	108505.	Log. = ...	5.035450
Half-elapsed time = . .	0ʰ50ᵐ5ˢ	Log. half-elaps. time =	0.663950
Middle time = ...	1.39.9	Log. middle time =	5.923527

Time from noon when greatest altitude was taken = 0ʰ49ᵐ4ˢ Log. rising = . . . 4.358520
Nat. co-versed sine of the greatest alt. = 703190. Log. ratio = 0.224127

Natural number = 13627. Log. = . 4.134393

Sun's mer. z. dis. = 71°54′52″N. Nat. V.S. = 689563.
Sun's red. dec. = 21.57.22 S.

Lat. of the ship = 49°57′30″ north. And, since this latitude differs only 52 seconds from the last, it may, therefore, be esteemed as the true latitude.

Note.—The correct latitude, by spherical trigonometry, is 49°56′0″ north.

Example 2.

At sea, April 14th, 1825, in latitude 43°47′ S., by account, and longitude 60°25′ E., at 23h20m40s apparent time, the observed altitude of the sun's lower limb was 35°54′, and at 2h10m10s apparent time, April 15th, the observed altitude of that limb was 28°42′15″, and the bearing of the sun's centre, by azimuth compass, N.W. ¾ N.; the height of the eye above the level of the horizon was 24 feet, and the ship's course during the elapsed time S.W., at the rate of 9 knots an hour; required the latitude of the ship at the time of observation of the greater altitude?

Sun's bearing at 2d observation = N.W. ¾ N.,	or =	3¾ points.
Ship's course = ... S.W..	or =	4 points.
Contained angle = ...		8¾ points.

Time elapsed between the observations = 2h49m30s And,
As 1h : 9m. :: 2h49m30s : 25′26″ = the distance made good between the observations.

Now, to course 7¼ points, and distance 25 miles, the difference of latitude is 3.7 miles, the *reduction of altitude;* which is *additive* to the least altitude, because the contained angle is less than 8 points, and the observation was made in the afternoon.

Time of observing the greatest altitude = . . .	23h20m40s
Longitude 60°25′ E., in time = ...	4. 1.40
Greenwich time past noon, April 14th, = . . .	19h19m 0s

Sun's declination at noon, April 14th, = . .	9°24′15″N.
Correction from Table XV., for 19h19m0s = .	+ 17.19
Sun's reduced declination = ...	9°41′34″N.

First observed altitude of the sun's lower limb =		35°54′ 0″N.
Sun's semi-diameter = . . 15′58″ Dip of the horiz. for 24 feet = 4.42	Diff. =	+ 11.16
Apparent altitude of the sun's centre = . . .		36° 5′16″N.
Refraction = 1′18″ Parallax = 0. 7	Difference = ...	− 1.11
True altitude of the sun's centre = ...		36° 4′ 5″N.

Second observed altitude of sun's lower limb = 28°42′15″
Sun's semi-diameter = . 15′58″ } Diff. = + 11.16
Dip of the horiz. for 24 feet = 4.42 }

Apparent altitude of the sun's centre = . . 28°53′31″
Refraction = 1′43″ } Difference = . . . − 1.35
Parallax = 0. 8 }
Reduction of altitude = ... + 3.42

Reduced altitude = ... 28°55′38″

Latitude by account = 43°47′ 0″ Log. sec. = 10.141486

Time of obs.	Altitude.	Nat. co-v. sine.	Red. dec.	
23h20m40s	36° 4′ 5″	411255	9°41′34″ Log. sec. =	10.006244
2.10.10	28.55.38	516302	Log. ratio = . . .	0.147730
2.49.30 elaps. time.	Diff. =	105047	Log. = ...	5.021384
1h24m45s half-elapsed time		. .	Log. half-elaps. time =	0.441990
0.47. 8 middle time		. . . .	Log. middle time =	5.611104

0h37m37s time from noon when the
greatest altitude was taken. Log. rising = . . 4.128390
Natural co-versed sine of the greatest alt. = 411255 Log. ratio = 0.147730

Natural number = 9564 Log. = 3.980660

Sun's mer. z. dist. = 53°15′ 4″ S. Nat. ver. S. = 401691
Sun's red. dec. = 9.41.34 N.

Lat. of the ship = 43°33′30″ S. But, since this latitude differs so much from that by account, it becomes necessary to repeat the operation.

Computed latitude = . . 43°33′30″ S. Log. secant = 10.139858
Reduced declination = . 9.41.34 N. Log. secant = 10.006244

Log. ratio = . 0.146102
Diff. of nat. co-versed sines = 105047 Log. = . . 5.021384
Half-elapsed time = . . 1h24m45s Log. ½-elaps. time = 0.441990

Middle time = ... 0, 46, 57 Log. middle time = 5.609476

Time from noon when the
greatest alt. was taken = 0ʰ.37ᵐ.48ˢ. Log. rising = . 4.132610
Nat. co-vers. sine of the greatest altitude = 411255 Log. ratio = 0.146102

Natural number = 9694 Log. = . 3.986508

Sun's mer. z. dis. = 53°.14'.30".S. = Nat. V. S. = 401561
Sun's red. dec. = 9.41.34 N.

Latitude = . 43°.32'.56" south. And, since this latitude only differs 34 seconds from the last, it may be considered as being the latitude of the ship at the time of observation of the greater altitude. The correct latitude, however, by spherical trigonometry, is 43°.29'.30" south: hence the method by double altitudes, even after *repeating the operation*, differs from the truth by 3 minutes and 26 seconds.

Note.—The method of finding the latitude by double altitudes, being a very tedious and indirect operation, and generally a very inaccurate one, unless the limitations pointed out in the remarks (page 342) are strictly attended to, no notice, therefore, would have been taken of it in this work, were it not for the purpose of giving the most ample illustration of the general use of the Tables. And, notwithstanding what has been said in favour of double altitudes by *theoretical writers*, this method of finding the latitude at sea is evidently far from being one of the most advantageous in practical navigation: for the operation, besides being rather circuitous, requires a considerable portion of time to go through with it correctly; and, after all, it frequently happens, that although every seeming precaution has been taken, the mariner's hopes are disappointed in the result. We will now proceed to a more direct and universal method of finding the latitude, either at sea or on shore.

Problem VIII.

Given the Altitudes of two known fixed Stars observed at the same instant, at any Time of the Night, to find the Latitude of the Place of Observation, independent of the Latitude by Account, the Longitude, or the Apparent Time.

In the preceding problems for finding the latitude (the two last excepted), the meridional altitudes of the celestial objects were the principal elements under consideration: however, since it frequently happens that, in consequence of the interposition of clouds, or other causes, the altitudes of the heavenly bodies cannot always be taken at their respective times of transit, the present problem is, therefore, proposed, which possesses the peculiar advantage of enabling the mariner to determine the position of his ship, with respect to latitude, by the altitudes of two known fixed stars, observed at the same instant and at any hour of the night, either before or after their passing the meridian, and independent of the latitude by account, the longitude, or the apparent time of observation. Nor will the mariner, in this method, be subjected to the necessity of *repeating the operation*, or of puzzling himself with a variety of cases and corrections, in finding an approximate latitude.

Rule.

Let the altitudes of two stars be observed, at the same moment, whose computed spherical distance asunder is given in Table XLIV.; and let those *observed* altitudes be reduced to the *true* by Problem XVII., page 327. Take the right ascensions and declinations of the two stars, and also their computed spherical distance, from Table XLIV., and let these be reduced, respectively, to the *night* of observation. Let the star which is adjacent, or nearest to the elevated pole, be distinguished by the letter A, and that which is remote, or farthest, by the letter R.—Now,

To the log. sine of the tabular distance between the two stars, add the log. secant of the declination of the star A, and the log. half-elapsed time of the difference of right ascension; the sum, rejecting 20 from the index, will be the log. half-elapsed time of *arch the first*.

From the natural co-versed sine of the altitude of the star A, subtract the natural co-versed sine of the sum of the tabular distance between the stars and the altitude of the star R, and find the logarithm of the remainder; to which add the log. co-secant of the tabular distance, and the log. secant of the altitude of the star R;—the sum of these three logarithms, abating 20 in the index, will be the log. rising of *arch the second;* the difference between which and *arch the first*, will be *arch the third.*

To the log. rising of *arch the third*, add the log. co-sines of the declination and altitude of the star R, and the sum, abating 20 in the index, will be the logarithm of a natural number; which, being added to the natural versed sine of the difference between the altitude and declination of the star R, when its polar distance is less than 90°, or to that of their sum when it is more than 90°, the sum will be the natural co-versed sine of the latitude.

Example 1.

January 1st, 1825, in north latitude, the true altitude of the star Alphard was 16°0′12″, and, at the same instant, that of Regulus was 27°14′8″; required the latitude of the place of observation?

A, or Regulus' red. R.A. =	$9^h59^m\ 3^s$ *	and reduced dec. =	12°49′10″ * N.
R, or Alphard's ditto =	9.18.59	and reduced dec. =	7.54.13 S.

Tabular distance between the two stars = 22°59′22″ *

Diff. of right asc. =	$0^h40^m\ 4^s$	Log. half-elapsed time =	0.759620
Dist. bet. the two stars =	22°59′22″	Log. sine =	9.591690
Dec. of star A =	12.49.10	Log. secant =	10.010962
Arch the first =	$1^h42^m57^s$	Log. half-elapsed time =	0.362272
Dist. bet. the two stars =	22°59′22″	Log. co-secant =	10.408310
Altitude of the star R =	16. 0.12	Log. secant =	10.017165
Sum =	38°59′34″	Nat. co-V.S. = 370778	
Altitude of the star A =	27.14. 8	Nat. co-V.S. = 542351	
		Diff. = 171573	Log. = 5.234449
Arch the second =	$3^h48^m27^s$	Log. rising =	5.659924
Arch the first =	1.42.57		
Arch the third =	$2^h\ 5^m30^s$	Log. rising =	5.165010
Dec. of the star R =	7°54′13″S.	Log. co-sine =	9.995855
Altitude of ditto =	16. 0.12	Log. co-sine =	9.982835
Sum =	23°54′25″	Nat. vers. S. = 085795	
Natural number =		139220	Log. = 5.143700
True latitude =	50°48′13″N.	Nat. co-V.S. = 225015	

Example 2.

January 1st 1825, in north latitude, the true altitude of α Arietis was 27°12′9″, and, at the same instant, that of Aldebaran was 51°45′28″; required the latitude of the place of observation?

* The method of reducing the right ascensions, declinations, and computed spherical distances of the stars, to a given period, is shown in the explanation to Table XLIV., page 114.

True spherical distance between the two stars, reduced to night of observation = 35°32′7″*

A, or α Arietis' red. R.A. = $1^h57^m19^s$* reduced dec. = 22°37′50″N.*
R, or Aldebaran's ditto = 4.25.53 reduced dec. = 16. 8.57 N.

Diff. of right asc. =	$2^h28^m34^s$	Log. half-elapsed time =	0.219110
Dist. bet. the two stars =	35°32′ 7″	Log. sine = ...	9.764329
Dec. of star A = .	22.37.50	Log. secant = . . .	10.034796
Arch the first = .	$4^h54^m3^s$	Log. half-elapsed time =	0.018235
Dist. bet. the two stars =	35°32′ 7″	Log. co-secant = . .	10.235671
Altitude of the star R =	51.45.28	Log. secant = . . .	10.208318
Sum = ...	87°17′35″	Nat. co-V.S. = 001116	
Altitude of the star A =	27.12. 9	Nat. co-V.S. = 542863	
		Difference = 541747	Log. = 5.733796
Arch the second =	$8^h1^m33^s$	Log. rising = ...	6.177785
Arch the first = .	4.54. 3		
Arch the third = .	$3^h7^m30^s$	Log. rising = ...	5.500250
Dec. of the star R =	16° 8′57″	Log. co-sine = . . .	9.982516
Altitude of ditto =	51.45.28	Log. co-sine = . . .	9.791682
Difference = . .	35°36′31″	Nat. vers. S. = 186987	
		Natural number = 188126	Log. = 5.274448
True latitude =	38°40′26″N.	Nat. co.-V.S. = 375113	

Example 3.

March 1st, 1825, in north latitude, the true altitude of Rigel was 27°9′7″, and, at the same instant, that of Sirius 28°55′39″; required the latitude of the place of observation?

* See Note, page 349.

True spherical distance between the two given stars, reduced to night of observation = 23°40′43″.*

A, or Rigel's red. R.A. = $5^h\ 6^m\ 8^s$ reduced dec. = 8°24′35″ S.
R, or Sirius' red. R.A. = 6. 37. 26 reduced dec. = 16. 28. 58

Diff. of right asc. =	$1^h 31^m 18^s$	Log. half-elapsed time =	0.411262
Dist. bet. the two stars =	23. 40. 40	Log. sine = ...	9.603786
Dec. of the star A =	8°24′35″	Log. secant = ...	10.004695
Arch the first = .	$4^h 51^m 25^s$	Log. half-elapsed time =	0.019743
Dist. bet. the two stars =	23°40′43″	Log. co-secant = . . .	10.396200
Alt. of the star R =	28. 55. 39	Log. secant = ...	10.057877
Sum = . . .	52°36′22″	Nat. co-V.S. = 205520	
Alt. of the star A =	27. 9. 7	Nat. co-V.S. = 543648	
		Difference = 338128 Log. =	5.529081
Arch the second =	$5^h 51^m 17^s$	Log. rising = ...	5.983158
Arch the first = .	4. 51. 25		
Arch the third = .	$0^h 59^m 52^s$	Log. rising = ...	4.530500
Dec. of the star R =	16°28′58″	Log. co-sine = ...	9.981775
Altitude of ditto =	28. 55. 39	Log. co-sine = ...	9.942123
Sum = ...	45°24′37″	Nat. vers. sine = 297975	
		Natural number = 28471 Log. =	4.454398
True latitude =	42°20′31″ N.	Nat. co-V.S. 326446	

* The distance between Rigel and Sirius, as given for the year 1822, at the end of the Nautical Almanac for 1825, Table II., is 23°40′35″, and the change in 10 years + 0′5″. This is, evidently, a mistake; for the distance between those two stars, at the beginning of 1822, was 23°40′42″: and, since the annual variation of distance is − 0″.56, the change, therefore, in 10 years, is − 0′ 5″.6; being *subtractive* instead of *additive*. A similar remark is applicable to the stars Fomalhaut and Achernar; for, by the above-mentioned Table, it appears that the distance between those stars, at the beginning of 1822, was 39°7′20″, and the change in 10 years − 0′ 1″: whereas the true distance, at that period, was 39°7′13″; and, since the annual variation of distance is − 0″.17, the change, therefore, in 10 years, is − 0′ 1″.7, being very nearly two seconds of a degree. The distances and annual variations of the remaining stars in the said Table will be found equally incorrect, as may be seen by comparing them with those contained in this work, Table XLIV.

Example 4.

September 1st, 1825, in south latitude, the true altitude of Fomalhaut was 63°6′18″, and, at the same instant, that of Achernar 37°44′18″; required the latitude of the place of observation?

True spherical distance between the two given stars, reduced to the night of observation, = 39°7′13″

A, or Achernar's red. R.A. =	$1^h31^m13^s$	reduced dec. =	58° 7′27″ S.
R, or Fomalhaut's ditto =	22. 48. 0	reduced dec. =	30. 32. 38
Diff. of right ascension =	$2^h43^m13^s$	Log. half-elapsed time =	0. 184770
Dist. bet. the two stars =	39° 10′37″	Log. sine = ...	9. 800523
Dec. of the star A =	58. 7. 27	Log. secant = . . .	10. 277300
Arch the first = . .	$2^h12^m27^s$	Log. half-elapsed time =	0. 262593
Dist. bet. the two stars =	39° 7′13″	Log. co-secant = . .	10. 200004
Alt. of the star R =	63. 6. 18	Log. secant = . . .	10. 344518
Sum = ...	102°13′31″	Nat. co-V.S. =	022677
Alt. of the star A =	37. 44. 18	Nat. co-V.S. =	387944
		Difference = 365267 Log. =	5. 562610
Arch the second = .	$7^h\ 4^m59^s$	Log. rising = . . .	6. 107132
Arch the first = . .	2. 12. 27		
Arch the third . . .	$4^h52^m32^s$	Log. rising = . . .	5. 851160
Dec. of the star R =	30°32′38″	Log. co-sine = . . .	9. 935124
Altitude of ditto = .	63. 6. 18	Log. co-sine = . . .	9. 655481
Difference = . . .	32°33′40″	Nat. v. sine =	157182
		Natural number = 276544 Log. =	5. 441765
True latitude =	34°29′27″ S.	Nat. co-V.S. =	433726

Note.—The principles from which the above method is deduced, will be found in "The Young Navigator's Guide to the Sidereal and Planetary Parts of Nautical Astronomy," between pages 136 and 144.

Thus, then, is the mariner provided with a direct and *most accurate method* of finding the latitude at sea; and, since it prevents the uncertainty and confusion arising from an error in the assumed latitude, or that by account, and, besides, being free from all ambiguity, restriction, and variety of cases whatever,—it may, therefore, be employed with a certainty of success, at any hour of the night, whenever two known fixed stars are visible. Indeed, if the altitudes of the objects be determined with but common attention, the latitude resulting therefrom will be always true to the nearest second of a degree, without the necessity of *repeating the operation*, or of applying any correction whatever to the result.

Remarks.—Although it is at all times advisable for two observers to take the altitudes of the stars at the same moment of time, yet, should one person be desirous of going through the whole operation himself, he is to proceed as follows; viz.,—Let the altitude of one star be taken, and the time of observation noted by a watch that shows seconds; then let the altitude of the other star be observed, and the time noted also; and let the altitude of the *first observed star* be again taken, and the time of observation noted.

Now, find the difference between the first and last times of observation, and the altitudes of the first observed star; and find, also, the difference between the first time of observation of the first star, and the time of observing the second star. Then say, as the interval or difference of time between the two observations of the first star, is to the difference of altitude in that interval; so is the interval, or difference of time between the observations of the first and second star, to a correction; which, being applied by addition or subtraction, to the *first observed altitude* of the first star, according as it may be increasing or decreasing, the sum or difference will be the altitude of that star reduced to the time that the altitude of the second star was taken. This part of the operation may be readily performed by proportional logarithms;—see example, page 75. The interval between the observations ought, however, to be as much contracted as possible, on account of guarding against any irregularities in the change of altitude.

Caution.—In order to guard against falling into any error, by working in an impossible triangle, it will be advisable to make choice of two stars whose computed spherical distance, in Table XLIV., *is not less than* 20°, and difference of right ascension not less than a quarter of an hour; and, since the Table contains an extensive variety of distances and differences of right ascension greater than those values, the mariner can never be at a loss in finding out two eligible stars for observation. The distances in that Table are all computed to the greatest degree of accuracy; and, notwithstanding that some of those which are but of *small measure* ought not to be employed in the determination of the latitude by the above method, yet they will be found extremely useful on many occasions; particularly in assisting to distinguish the stars to which they are annexed, when the latitude is to be inferred from their meridional altitudes, agreeably to Problem IV., page 335.

Problem IX.

Given the Latitude by Account, the Altitude of the Sun's lower or upper Limb observed near the Meridian, the apparent Time of Observation, and the Longitude; to find the true Latitude.

Since it frequently happens at sea, particularly during the winter months of the year, that the sun's meridional altitude cannot be taken, in consequence of the interposition of clouds, fogs, rains, or other causes; and since the true determination of the latitude becomes an object of the greatest importance to the mariner when his ship is sailing in any narrow sea trending in an easterly or a westerly direction, such as the British Channel; the present problem is, therefore, given, by means of which the latitude may be very readily and correctly inferred from the sun's altitude taken at a given interval from noon, within the following limits; viz.,—The *number* of minutes and parts of a minute, contained in the interval between the time of observation and noon, must not exceed the *number* of degrees and parts of a degree contained in the object's meridional zenith distance at the place of observation. And, since the meridional zenith distance of a celestial object is expressed by the difference between its declination and the latitude of the place of observation, when they are of the same name, or by their sum, when of contrary names, the extent of the interval from noon, *within which the altitude should be observed*, may, therefore, be readily ascertained, by means of the difference between the latitude and the declination, when they are both north or both south, or by their sum when one is north and the other south: thus, if the latitude be 60 degrees, and the declination 23 degrees, both of the same name, the interval between the time of observation and noon ought not to exceed 37 minutes; but if one be north and the other south, the interval may be extended, if necessary, to 83 minutes before or after noon. The altitude, however, may be taken as near to noon as the mariner may think proper; the only restriction being, that the observation must not be made without the above-mentioned limits.

The interval between the apparent time of observation and noon must be accurately determined: this may be always done, by means of a chronometer or any well-regulated watch showing seconds; proper allowance being made for the difference of time answering to the change of longitude, if any, since the last observation for determining the error of such watch or chronometer.

Now, if the sun's altitude be observed *at any time within the above-mentioned limits*, the latitude of the place of observation may then be determined, to every degree of accuracy desirable in nautical operations, by the following rule; which, being performed by proportional logarithms, renders the operation nearly as simple as that of finding the latitude by the meridional altitude of a celestial object.

See explanation to Tables LI. and LII., between pages 138 and 143.

Rule.

Reduce the sun's declination to the time and place of observation, by Problem V., page 298; and let the observed altitude of the sun's lower or upper limb be reduced to the true central altitude, by Problem XIV., page 320. Then, with the sun's reduced declination, and the latitude by account, enter Table LI. or LII., (according as the latitude and the declination are of the same or of a contrary denomination,) and take out the corresponding correction in seconds and thirds, which are to be esteemed as *minutes and seconds*, agreeably to the rule in page 139. Now,

To the proportional logarithm of this correction, add *twice* the proportional logarithm of the interval between the time of observation and noon, and the constant logarithm 7.2730; the sum of these three logarithms, abating 10 in the index, will be the proportional logarithm of a correction, which being *added* to the true altitude of the sun's centre, the sum will be the meridional altitude of that object: hence the sun's meridional zenith distance will be known; to which let its declination be applied by addition or subtraction, according as it is of the same or of a contrary name, and the sum or difference will be the latitude of the place of observation.

Example 1.

At sea, January 1st, 1825, at 22ʰ45ᵐ24ˢ apparent time, in latitude 51°36′ N., by account, and longitude 10°45′30″ W., the observed altitude of the sun's lower limb was 13°33′34″*, and the height of the eye above the level of the horizon 25 feet; required the latitude of the place of observation?

Apparent time of observation = . . .	22ʰ45ᵐ24ˢ
Longitude 10°45′30″, in time = . .	+ 0. 43. 2
Greenwich time = ...	23ʰ28ᵐ26ˢ

* This is the mean of several altitudes of the sun's lower limb.

Sun's declination at noon, January 1st = 23° 0′ 59″ S.
Correction of ditto for 23ʰ 28ᵐ 26ˢ = . − 5. 6

Sun's reduced declination = ... 22° 55′ 53″ S.

The observed altitude of the sun's lower limb, reduced to the true central altitude, is 13° 41′ 24″ S.

Cor. in Table LII., answering to lat. 50° N., and dec. 22° S. = 1″ 13‴.8
Difference to 2° of lat. = −3‴.9; now, 3‴.9 × 96′ ÷ 120′ = − 3 .1
Diff. to 1° of dec. = −0‴.9; now, 0‴.9 × 55′ 53″ ÷ 60′ = − 0 .8

Cor. to lat. 51° 36′ N. and declination 22° 55′ 53″ S. = . . 1″ 9‴.9

Computed correction = 1″ 9‴.9, Proportional log. = . . . 2.1889
Time of obs. from noon 1ʰ 14ᵐ 36ˢ, twice the prop. log. = . . 0.7650
Constant log. = ... 7.2730

Correction of the sun's altitude = . . 1° 46′ 45″ Prop. log. = 0.2269
True altitude of the sun's centre = . . 13. 41. 24 S.

Sun's meridional altitude = ... 15° 28′ 9″ S.

Sun's meridional zenith distance = . . 74° 31′ 51″ N.
Sun's corrected declination = . . . 22. 55. 53 S.

Latitude of the place of observation = 51° 35′ 58″ N.; which differs but 2″ from the truth.

Example 2.

At sea, March 21st, 1825, at 0ʰ 50ᵐ 25ˢ apparent time, in latitude 51° 5′ N., by account, and longitude 35° 45′ W., the observed altitude of the sun's lower limb was 37° 55′ 27″*, and the height of the eye above the level of the sea 21 feet; required the true latitude of the place of observation?

Apparent time of observation = . . . 0ʰ 50ᵐ 25ˢ
Longitude 35° 45′ W., in time = . . . 2. 23. 0

Greenwich time = ... 3ʰ 13ᵐ 25ˢ

Sun's declination at noon, March 21st = 0° 14′ 30″ N.
Correction of ditto for 3ʰ 13ᵐ 25ˢ = . . + 3. 11

Sun's reduced declination = ... 0° 17′ 41″ N.

* See Note, page 355.

The observed altitude of the sun's lower limb, reduced to the true central altitude, is 38°6′2″ S.

Cor. in Table LI., answering to lat. 50° N. and declin. 0° = . 1″38‴.8
Difference to 2° of lat. = − 6‴.8; now, 6‴.8 × 65′ ÷ 120′ = − 3 .6
Diff. to 1° of declin. = + 1‴.5; now 1‴.5 × 17′41″ ÷ 60′ = + 0 .4

Correction to lat. 51°5′N. and declination 0°17′41″ N. = . 1″35‴.6

Computed correction = . 1″35‴.6, Prop. log. = ... 2.0530
Time of observ. from noon = 0ʰ50ᵐ25ˢ, twice the prop. log. = 1.1054
Constant log. = ... 7.2730

Correction of the sun's altitude = . . 1° 6′40″ Prop. log. = 0.4314
True altitude of the sun's centre = . . 38. 6. 2 S.

Sun's meridional altitude = ... 39°12′42″ S.

Sun's meridional zenith distance = . . 50°47′18″N.
Sun's reduced declination = ... 0.17.41 N.

Latitude of the place of observation = . 51° 4′59″N.; which differs but 1″ from the truth.

Hence it is evident, that the latitude may be determined by this method to all the accuracy desirable in nautical purposes. It possesses a decided advantage over that by *double altitudes;* and, since the operation is so extremely simple, the mariner will do well to avail himself thereof on every occasion; because the latitude, thus deduced, will be equally as correct as that resulting from the observed meridional altitude, provided the observation be made within the prescribed limits. When, however, the latitude and the declination are of different names, it will not produce any sensible error in the result, if the altitude be observed *a few seconds without* those limits, as may be seen in Example 1, above.

But it is to be remembered, that the apparent time of observation must be well determined.

PROBLEM X.

Given the Latitude by Account, the Altitude of the Moon's lower or upper Limb observed near the Meridian, the apparent Time of Observation, and the Longitude; to find the true Latitude.

RULE.

To the apparent time of observation apply the longitude, in time, by addition or subtraction, according as it is west or east, and the corresponding time at Greenwich will be obtained; to which let the sun's right ascension be reduced, by Problem V., page 298; and let the moon's right ascension, declination, semi-diameter, and horizontal parallax, be also reduced to that time, by Problem VI., page 302. Let the observed altitude of the moon's limb be reduced to the true central altitude, by Problem XV., page 323.

To the apparent time of observation add the sun's reduced right ascension, and the sum (abating 24 hours, if necessary,) will be the right ascension of the meridian; the difference between which and the moon's reduced right ascension, converted into time, will be the moon's distance from the meridian at the time of observation. Now, with the moon's reduced declination, and the latitude by account, enter Table LI. or LII., according as they are of the same or of a contrary denomination, and take out the corresponding correction, agreeably to the rule in page 139; with which, and the moon's distance from the meridian, compute the correction of altitude; and, hence, the latitude of the place of observation, by Problem IX., page 354.

Note.—The limits within which the altitude of the moon should be observed, are to be determined in the same manner, precisely, as if it were the sun that was under consideration; observing, however, to estimate the interval from the time of transit over the meridian of the place of observation, instead of from noon.

See the explanation to Tables LI. and LII., between pages 138 and 143.

Example 1.

January 23d, 1825, at 3h 55m 17s apparent time, in latitude 51° 15′ N., by account, and longitude 45° W., the observed altitude of the moon's lower limb was 39° 27′ 30″*, and the height of the eye above the level of the horizon 24 feet; required the true latitude of the place of observation?

* This is the mean of several altitudes.

Apparent time of observation = ...	$3^h 55^m 17^s$
Longitude 45° W., in time = ...	3. 0. 0
Greenwich time = ...	$6^h 55^m 17^s$
Sun's right ascension at noon, January 23d, =	$20^h 22^m 23^s$
Correction of ditto for $6^h 55^m 17^s$ = . . .	+ 1.12
Sun's reduced right ascension = ...	$20^h 23^m 35^s$
Apparent time of observation = ...	3. 55. 17
Right ascension of the meridian = . . .	$0^h 18^m 52^s$

Moon's R. A. at noon, January 23d, = 349°47′55″
Correction of ditto for $6^h 55^m 17^s$ = + 3. 6. 56

Moon's reduced right ascension = 352°54′51″ =	$23^h 31^m 39^s$
Right ascension of the meridian = ...	0. 18. 52
Moon's distance from the meridian = ...	$0^h 47^m 13^s$

Moon's declination at noon, January 23d, =	1°10′39″N.
Correction of ditto for $6^h 55^m 17^s$ = . .	+ 1. 22. 36
Moon's reduced declination = ...	2°33′15″N.
Observed altitude of moon's lower limb = .	39°27′30″ S.
Semi-diameter 14′52″ − dip 4′42″ = . .	+ 10. 10
Apparent altitude of the moon's centre = .	39°37′40″ S.
Cor., Table XVIII., ans. to hor. parallax 54′4″ =	40. 36
True altitude of the moon's centre = . . .	40°18′16″ S.

Cor. in Table LI., answering to lat. 50°N. and declin. 2°N. =	1″41‴.8
Diff. to 2° of lat. = − 7‴.2; now, 7‴.2 × 75′ ÷ 120′ = .	− 4 .5
Diff. to 1° of declin. = + 1‴.6; now, 1‴.6 × 33′15″ ÷ 60′ =	+ 0 .9
Cor. answering to lat. 51°15′N. and declin. 2°33′15″N. = .	1″38‴.2

Computed correction = 1″38‴.2, Prop. log. = ...	2.0413
Moon's mer. distance = $0^h 47^m 13^s$, twice the prop. log. = .	1.1624
Constant log. = ...	7.2730
Correction of the moon's altitude = 1° 0′ 3″ Prop. log. =	0.4767

True altitude of the moon's centre = 40. 18. 16 S.

Moon's meridional altitude = . . 41°18′19″ S.

Moon's meridional altitude = . .	41°18′19″ S.
Moon's meridional zenith distance =	48°41′41″N.
Moon's reduced declination = .	2.33.15 N.

Latitude of the place of observation = 51°14′56″N.; which differs but 4″ from the truth.

Example 2.

January 30th, 1825, at 9h45m12s apparent time, in latitude 57°40′ S., by account, and longitude 60° east, the observed altitude of the moon's lower limb was 5°37′12″*, and the height of the eye above the level of the horizon 26 feet; required the true latitude of the place of observation?

Apparent time of observation = ...	9h45m12s
Longitude 60° east, in time = ...	4. 0. 0
Greenwich time = ...	5h45m12s

Sun's right ascension at noon, January 30th, =	20h51m25s
Correction of ditto for 5h45m12s = ...	+ 0.59
Sun's reduced right ascension = ...	20h52m24s
Apparent time of observation = ...	9.45.12
Right ascension of the meridian = . . .	6h37m36s

Moon's R. A. at noon, January 30th, =	76°21′55″	
Correction of ditto for 5h45m12s =	+ 2.22.52	
Moon's reduced right ascension = .	78°44′47″	= 5h14m59s
Right ascension of the meridian = ...		6.37.36
Moon's distance from the meridian = ...		1h22m37s

Moon's declination at noon, January 30th, = . .	23°57′46″N.
Correction of ditto for 5h45m12s = ...	− 4. 3
Moon's reduced declination = ...	23°53′43″N.

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* See Note, page 358.

Observed altitude of the moon's lower limb = . .	5°37′12″N.
Semi-diameter 15′52″ − dip 4′52″ = ...	+ 11. 0
Apparent altitude of the moon's centre = . . .	5°48′12″N.
Correc., Table XVIII., ans. to hor. parallax, 58′5″=	+ 49. 6
True altitude of the moon's centre = ...	6°37′18″N.

Cor. in Tab. LII., answering to lat. 56° S. and declin. 23° N. =	1″ 1‴.8
Difference to 2° of lat. = − 3‴.6; now, 3‴.6 × 100′ ÷ 120′ =	− 3 .0
Diff. to 1° of declin. = − 0‴.7; now, 0‴.7 × 53′43″ ÷ 60′ =	− 0 .6
Cor. to lat. 57°40′ S. and declination 23°53′43″ N. = .	0″58‴.2

Computed correction = 0″58‴.2 Prop. log. = ...	2.2685
Moon's merid. distance = 1ʰ22ᵐ37ˢ Twice the prop. log. = . .	0.6764
Constant log. = ...	7.2730

Correction of the moon's altitude = .	1°48′59″	Prop. log. = 0.2179
True altitude of the moon's centre = .	6.37.18 N.	
Moon's meridional altitude = ...	8°26′17″N.	
Moon's meridional zenith distance = .	81°33′43″ S.	
Moon's reduced declination = ...	23.53.43 N.	
Latitude of the place of observation = .	57°40′ 0″ S.; which is exactly right.	

Hence it is evident, that, by this method, the latitude may be inferred from the true altitude of the moon's centre, to every degree of accuracy desirable in nautical operations, provided the altitude be observed within the proper limits; which, for the sake of assisting the memory, will be here repeated,—viz., The *number* of minutes and seconds, in the moon's distance from the meridian at the time of observation, must not exceed the *number* of degrees and minutes contained in the meridional zenith distance of that object at the place of observation. Thus, in the above example, where the moon's meridional zenith distance is 81°34′ nearly, the interval between the time of observation and the time of the moon's transit, or passage over the meridian of the place of observation, must not exceed 81ᵐ34ˢ; though the moon's altitude may be taken at any time *within that interval*, or as near to the time of transit as the observer may think proper.

Problem XI.

Given the Latitude by Account, the observed central Altitude of a Planet near the Meridian, the apparent Time of Observation, and the Longitude: to find the true Latitude.

Rule.

To the apparent time of observation apply the longitude, in time, by addition or subtraction, according as it is west or east; and the sum, or difference, will be the corresponding time at Greenwich. To this time let the planet's right ascension and declination be reduced, by Problem VII., page 307; and let the sun's right ascension at noon of the given day be also reduced to that time by Problem V., page 298.

Let the observed central altitude of the planet be reduced to its true central altitude, by Problem XVI., page 325. Then, to the apparent time of observation add the sun's reduced right ascension, and the sum (abating 24 hours, if necessary,) will be the right ascension of the meridian; the difference between which and the planet's reduced right ascension, will be that object's distance from the meridian at the time of observation. Now, with the latitude by account, and the planet's reduced declination, enter Table LI. or LII., according as they are of the same or of contrary denominations, and take out the corresponding correction, agreeably to the rule in page 139; with which, and the planet's distance from the meridian, compute the correction of altitude, and, hence, the latitude of the place of observation, by Problem IX., page 354.

Note.—The measure of the interval between the time of observation and the time of transit,—that is, the *number* of minutes and seconds contained in the planet's distance from the meridian, must not exceed the number of degrees and minutes contained in that object's meridian zenith distance at the place of observation.

See explanation to Tables LI. and LII., page 138, and thence to 143.

Example 1.

January 4th, 1825, at 12ʰ31ᵐ30ˢ apparent time, in 65°28′ S., by account, and longitude 60° east, the observed central altitude of the planet Jupiter was 5°14′35″*, and the height of the eye above the level of the horizon 25 feet; required the true latitude of the place of observation?

* This is the mean of several altitudes.

Apparent time of observation = . . .	$12^h31^m30^s$
Longitude 60° E., in time = . . .	$-\ 4.\ 0.\ 0$
Greenwich time = ...	$8^h31^m30^s$
Sun's right ascension at noon, Jan. 4th, =	$19^h\ 0^m32^s$
Correction of ditto for $8^h31^m30^s$ = . .	$+\ 1^m34^s$
Sun's reduced right ascension = . .	$19^h\ 2^m\ 6^s$
Apparent time of observation = . . .	12.31.30
Right ascension of the meridian = . .	$7^h33^m36^s$
Jupiter's right ascension at noon, Jan. 1st =	$8^h58^m\ 0^s$
Correction of ditto for $3^d8^h31^m30^s$ = .	$-\ 1^m\ 6^s$
Planet's reduced right ascension = . .	$8^h56^m54^s$
Right ascension of the meridian = . .	7.33.36
Planet's distance from the meridian = .	$1^h23^m18^s$

Jupiter's declination at noon, January 1st =	17°56′ 0″N.
Correction of ditto for $3^d8^h31^m30^s$ = . .	+ 6.43
Planet's reduced declination = ...	18° 2′43″N.

Correction in Table LII., answ. to lat. 64°S. and dec. 18°N. =	0″49‴.6
Difference to 2° of lat. = − 3‴.8; now, 3‴.8 × 88′ ÷ 120′ =	− 2 .8
Difference to 1° of dec. = − 0‴.4; now, 0‴.4 × 2′43″ ÷ 60′ =	0 .1
Correction answ. to lat. 65°28′ S. and dec. 18°2′43″ N. = .	0″46‴.7

Computed correction = . . .	0″46‴.7	Prop. log. = .	2.3642
Jupiter's dist. fr. mer. at time of obs. =	$1^h23^m18^s$	Tw. the prop. log. =	0.6692
Constant log. = ...			7.2730
Correction of Jupiter's altitude = . .	1°28′54″	Prop. log. =	0.3064
Jupiter's obs. alt., red. to true centr. alt., is =	5. 0.10		
Jupiter's meridional altitude = ...	6°29′ 4″N.		
Jupiter's meridional zenith distance = .	83°30′56″ S.		
Jupiter's reduced declination = ...	18. 2.43 N.		

Latitude of the place of observation = . 65°28′13″ S.; which differs 13″ from the truth.

Example 2.

February 4th, 1825, at $3^h36^m20^s$ apparent time, the observed central altitude of the planet Venus was $36°24'25''$*, in latitude $52°12'$ N., by account, and longitude $45°40'$ W., and the height of the eye above the level of the horizon was 26 feet; allowing the horizontal parallax of the planet, at that time, to be $17''$, the true latitude of the place of observation is required?

Apparent time of observation = . . .	$3^h36^m20^s$
Longitude $45°40'$ W., in time = .	+ 3. 2.40
Greenwich time = ...	6.39. 0
Sun's right ascension at noon, Feb. 4th =	$21^h11^m45^s$
Correction of ditto for $6^h39^m0^s$ = . .	+ 1^m 7^s
Sun's reduced right ascension = . . .	$21^h12^m52^s$
Apparent time of observation = . . .	$3^h36^m20^s$
Sun's reduced right ascension = . . .	21.12.52
Right ascension of the meridian = . .	$0^h49^m12^s$
Venus' right ascension at noon, Feb. 1st, =	23^h45^m 0^s
Correction of ditto for $3^d6^h39^m0^s$ = .	+ 13. 6
Venus' reduced right ascension = . .	23^h58^m 6^s
Right ascension of the meridian = . .	0.49.12
Planet's distance from the meridian = .	0^h51^m 6^s
Venus' declination at noon, Feb. 1st, =	$2°$ $7'$ $0''$ S.
Correction of ditto for $3^d6^h39^m0^s$ = .	− 1.42.41
Venus' reduced declination = . . .	$0°24'19''$ S.

Observed central altitude of Venus = ...	$36°24'25''$ S.
Dip of the horizon for 26 feet = ...	− 4.52
Apparent central altitude of Venus = ...	$36°19'33''$ S.
Refrac. (Tab. VIII.) $1'17''$ − Parall. (Tab. VI.) $0'14''$ =	− 1. 3
True central altitude of Venus = ...	$36°18'30''$ S.

* This is the mean of several altitudes. The altitude of Venus may be taken very correctly when the sun is above the horizon, provided the atmosphere be fine and clear.

Cor. in Tab. LII., ans. to lat. 52°N. and dec. 0° = . . .	1″32‴.0
Diff. to 2° of lat. = −6‴.4; now, 6‴.4 × 12′ ÷ 120′ = . . .	− 0 .6
Diff. to 1° of dec. = −1‴.2; now, 1‴.2 × 24′19″ ÷ 60′ = . . .	− 0 .5
Correction answering to lat. 52°12′ N. and dec. 0°24′19″S. =	1″30‴.9

Computed correction = 1″30‴.9	Prop. log. = ...	2.0749
Venus' merid. distance = 0h51m6s	Twice the prop. log. = .	1.0938
Constant log. = ...		7.2730
Correction of Venus' altitude = . .	1° 5′ 6″	Prop. log. = 0.4417
True central altitude of Venus = . .	36.18.30 S.	
Venus' meridional altitude = ...	37°23′36″S.	
Venus' meridional zenith distance = .	52°36′24″N.	
Venus' reduced declination = . . .	0.24.19 S.	
Latitude of the place of observation =	52°12′ 5″N.; which differs but 5″ from the truth.	

Problem XII.

Given the Latitude by Account, the Altitude of a fixed Star observed near the Meridian, the apparent Time of Observation; and the Longitude, to find the true Latitude.

Rule.

Turn the longitude into time, and apply it to the apparent time of observation, by addition or subtraction, according as it is west or east; and the sum, or difference, will be the corresponding time at Greenwich.

To this time let the sun's right ascension at noon of the given day be reduced by Problem V., page 298.

Let the star's right ascension and declination (Table XLIV.) be reduced to the night of observation, by the method shown in page 115; and let the star's observed altitude be reduced to the true altitude, by Problem XVII., page 327.

To the apparent time of observation add the sun's reduced right ascension, and the sum (abating 24 hours, if necessary,) will be the right ascension of the meridian; the difference between which and the star's reduced right ascension will be that object's distance from the meridian at the time of observation.

Now, with the latitude by account, and the star's reduced declination, enter Table LI. or LII., according as they are of the same or of a contrary denomination; and take out the corresponding correction, agreeably to the rule in page 139; with which, and the star's distance from the meridian, compute the correction of altitude; and, hence, the latitude, by Problem IX., page 354.

Note.—The interval between the time of observation and the time of transit must not exceed the limits pointed out in the three preceding Problems; viz., the *number* of minutes and parts of a minute contained in the star's distance from the meridian, is not to exceed the number of degrees and parts of a degree contained in that object's meridional zenith distance at the place of observation.

See explanation to Tables LI. and LII., from page 138 to 143.

Example 1.

January 1st, 1825, at 8ʰ52ᵐ17ˢ apparent time, in latitude 52°46′ N., by account, and longitude 56°15′ W., the observed altitude of the star Menkar was 39°42′40″, and the height of the eye above the level of the sea 26 feet; required the true latitude of the place of observation?

Apparent time of observation =	8ʰ52ᵐ17ˢ
Longitude 56°15′ W., in time =	+ 3.45. 0
Greenwich time =	12ʰ37ᵐ17ˢ

Sun's right ascension at noon, January 1st =	18ʰ47ᵐ19ˢ
Correction of ditto for 12ʰ37ᵐ17ˢ =	+ 2.19
Sun's reduced right ascension =	18ʰ49ᵐ38ˢ
Apparent time of observation =	8.52.17
Right ascension of the meridian =	3ʰ41ᵐ55ˢ

Menkar's right ascension, January 1st, 1824 =	2ʰ53ᵐ 5ˢ
Correction of ditto for 1 year =	+ 0′ 3″
Menkar's reduced right ascension =	2ʰ53ᵐ 8ˢ
Right ascension of the meridian =	3.41.55
Star's distance from the meridian =	0ʰ48ᵐ47ˢ

Menkar's declination, January 1st, 1824 = .	3°23′41″N.
Correction of ditto for 1 year = ...	0. 15
Menkar's reduced declination = ...	3°23′58″N.
Star's observ. alt., reduced to its true alt., is =	39°36′39″S.

Correction in Table LI. answering to lat. 52°N. and dec. 3°N. =	1″36‴.0
Difference to 2° of lat. = −7‴.0; now, 7‴.0 × 46′ ÷ 120′ =	− 2 .7
Difference to 1° of dec. = +1‴.2; now, 1‴.2 × 23′56″ ÷ 60′ =	+ 0 .5
Correction to lat. 52°46′N. and declination 3°23′56″N. =	1″33‴.8

Computed correction =	1″33‴.8	Prop. log. = ...	2.0612
Star's merid. distance =	0ʰ48ᵐ47ˢ	Twice the prop. log. =	1.1340
Constant log. = ...			7.2730
Correction of Menkar's altitude = .	1° 1′15″	Prop. log. =	0.4682
True altitude of Menkar = . . .	39.36.39 S.		
Menkar's meridional altitude = . .	40°37′54″S.		
Menkar's meridional zenith distance =	49°22′ 6″N.		
Menkar's reduced declination = . .	3.23.56 N.		

Latitude of the place of observation = 52°46′ 2″N.; which differs but 2″ from the truth.

Example 2.

September 1st, 1825, at 13ʰ28ᵐ42ˢ apparent time, in latitude 49°30′S. by account, and longitude 22°10′30″ E., the observed altitude of the star β Pegasi, or *Scheat*, was 11°37′59″, and the height of the eye above the level of the sea 19 feet; required the true latitude of the place of observation?

Apparent time of observation = ...	13ʰ28ᵐ42ˢ
Longitude 22°10′30″ E., in time = . . .	− 1.28.42
Greenwich time = ...	12ʰ 0ᵐ 0ˢ
Sun's right ascension at noon, Sept. 1st, = .	10ʰ41ᵐ16ˢ
Correction of ditto for 12ʰ0ᵐ0ˢ = ...	+ 1′49″
Sun's reduced right ascension = ...	10ʰ43ᵐ 5ˢ
Apparent time of observation = ...	13.28.42
Right ascension of the meridian = . . .	0ʰ11ᵐ47ˢ

Scheat's right ascension, January 1st, 1824, =	$22^h 55^m 5^s$
Correction of ditto for 1 year and 8 months =	$+ 0^m 5^s$
Scheat's reduced right ascension = . . .	$22^h 55^m 10^s$
Right ascension of the meridian = . . .	0. 11. 47
Star's distance from the meridian = . . .	$1^h 16^m 37^s$
Scheat's declination, January 1st, 1824, = .	$27° 7' 35''$N.
Correction of ditto for 1 year and 8 months =	+ 0. 32
Scheat's reduced declination = ...	$27° 8' 7''$N.
Scheat's observed altitude = ...	$11° 37' 59''$N.
Dip of the horizon for 19 feet = ...	− 4. 11
Scheat's apparent altitude = ...	$11° 33' 48''$N.
Refraction = ...	− 4. 33
Scheat's true altitude = ...	$11° 29' 15''$N.

Correction in Table LII., answ. to lat. 49°S. and dec. 27°N. =	$1'' 11'''.0$
Difference to 1° of latitude = $-1'''.8$; now, $1'''.8 \times 30' \div 60'$ =	− 0 .9
Difference to 1° of dec.= $-0'''.8$; now, $0'''.8 \times 8' 7'' \div 60'$ =	− 0 .1
Correction to latitude 49°30′S. and declination 27°8′7″N. =	$1'' 10'''.0$

Computed correction =	$1'' 10'''.0$	Prop. log. = ...	2. 1883
Star's merid. distance =	$1^h 16^m 37^s$	Twice the prop. log. = .	0. 7418
Constant log. = ...			7. 2730
Correction of the star's altitude = .	$1° 52' 46''$	Prop. log. =	0. 2031
Scheat's true altitude = ...	11. 29. 15 N.		
Scheat's meridional altitude = . .	$13° 22' 1''$N.		
Scheat's meridional zenith distance =	$76° 37' 59''$S.		
Scheat's reduced declination = . .	27. 8. 7 N.		

Latitude of the place of observation = $49° 29' 52''$ S.; which differs $8''$ from the truth.

Remark.—The latitude may be also very correctly inferred from the altitude of a celestial object observed near the meridian below the pole. In this case, the meridian distance of the object is to be reckoned from the apparent time of its transit below the pole; the correction answering to the latitude and the declination is *always* to be taken out of *Table LII.*, in the same manner as if those elements were of different denominations; and the correction of altitude is to be applied by *subtraction* to the true altitude of the object, deduced from observation, in order to find its meridional altitude below the pole. Then, with the meridional altitude below the pole, thus found, and the declination, the latitude is to be determined, by Problem V., page 336.

The interval, or limits within which the altitude should be observed, is to be determined in the same manner as if the celestial object were near the meridian above the pole.

Example 1.

June 20th, 1825, at 11ʰ18ᵐ30ˢ apparent time, in latitude 71°50′ N., by account, and longitude 65° W., the observed altitude of the sun's lower limb was 5°30′50″, and the height of the eye above the level of the sea 20 feet; required the true latitude of the place of observation?

Interval between the time of observation and midnight = . 0ʰ41ᵐ30ˢ

Sun's *observed* reduced to its *true* central altitude = 5°33′37″

Sun's corrected declination = 23°27′36″ N.

Sun's north polar distance = 66°32′24″

Cor. in Table LII., answering to lat. 70° and declin. 23° = .	0″37‴.1
Diff. to 2° of lat. = − 3‴.5; now, 3‴.5 × 110′ ÷ 120′ = .	− 3 .2
Diff. to 1° of dec. = − 0‴.2; now, 0‴.2 × 27′36″ ÷ 60 = .	− 0 .1
Correction to lat. 71°50′ and dec. 23°27′36″ = ...	0″33‴.8

Computed correction = 0″33‴.8, Proportional log. = . . .		2.5045
Sun's dist. from midnight = 0ʰ41ᵐ30ˢ, twice the prop. log. =		1.2744
Constant log. = ...		7.2730
Correction of altitude = ...	− 0°15′58″ Prop. log. =	1.0519
True central altitude of the sun = .	5.33.37	
Sun's meridian altitude below the pole =	5°17′39″	
Sun's north polar distance = . . .	66.32.24	

Latitude of the place of observation = 71°50′ 3″ N.; which differs but 3″ from the truth.

In case of a Fixed Star :—

Find the apparent time of the star's superior transit above the pole, at the given meridian, by Problem XII., page 317 ; to this time let 12 hours, diminished by half the variation of the sun's right ascension on the given day, be added, and the sum will be the apparent time of the star's inferior transit below the pole. Then, the rest of the operation is to be performed exactly the same as that for the sun in example 1, as above.

Example 2.

January 1st, 1825, at 11ʰ50ᵐ0ˢ apparent time, in latitude 71°30′ N., by account, and longitude 84°9′30″ W., the observed altitude of the star Albireo was 9°33′, and the height of the eye above the level of the horizon 19 feet; required the true latitude of the place of observation?

Apparent time of star's transit above the pole = .	0ʰ35ᵐ32ˢ
To which add 12ʰ − 2ᵐ12ˢ (half var. of S. R. A.) =	11.57.48
Apparent time of the star's transit below the pole =	12ʰ33ᵐ20ˢ
Apparent time of observation = ...	11.50. 0
Star's distance from the meridian = ...	0ʰ43ᵐ20ˢ

Observed altitude of the star Albireo = .	9°33′ 0″
Dip of the horizon for 19 feet = ...	− 4.11
Star's apparent altitude = ...	9°28′49″
Refraction = ...	− 5.32
Star's true altitude = ...	9°23′17″

Cor. in Table LII., answering to lat. 70° and declin. 27° = .	0″36‴.2
Diff. to 2° of lat.= − 3‴.4; now, 3‴.4×90′÷120′ = .	− 2 .5
Diff. to 1° of dec.= − 0‴.3; now, 0‴.3×35′.54″÷60 = .	− 0 .1
Correction to latitude 71°30′ and declin. 27°35′54″ = . .	0″33‴.6

Computed correction = . 0″33‴.6, Prop. log. = . .	2.5071
Star's merid. distance = . 0ʰ43ᵐ20ˢ, twice the prop. log. =	1.2370
Constant log. = ...	7.2730
Correction of the star's altitude = . − 0°17′18″ Prop. log.=	1.0171

Correction of the star's altitude = .	− 0°17′18″
True altitude of the star = ...	9.23.17
Star's meridian altitude below the pole =	9° 5′59″
Star's north polar distance = ...	62.24. 6

Latitude of the place of observation = . 71°30′ 5″N.; which differs but 5″ from the truth.

Note.—From the above examples, the method of finding the latitude by an altitude of the moon, or of a planet, observed near the meridian below the pole, will appear obvious.

Remark.

The following ingenious problem for determining the latitude, either at sea or on shore, has been communicated to the author by that scientific and enterprising officer, Captain William Fitzwilliam Owen, of His Majesty's ship Eden, who is so highly renowned for his extensive knowledge in every department of science connected with nautical subjects.

Problem.

Given the Latitude by Account, the true Altitude of the Sun's Centre, and the apparent Time; to find the true Latitude of the Place of Observation.

Rule.

Find the mean between the estimated meridian altitude, and the altitude deduced from observation, which call the middle altitude; then,

To the log. rising of the apparent time from noon, add the log. co-sine of the latitude, the log. co-sine of the corrected declination, the log. secant less radius of the middle altitude, and the constant logarithm 7.536274;* the sum of these five logarithms, abating 30 in the index, will be the logarithm of a natural number, which is to be esteemed as minutes, and which, being added to the sun's true central altitude, will give his correct meridional altitude; and, hence, the true latitude of the place of observation?

Example 1.

December 22d, 1825, in latitude 8°0′ south, by account, at 23ʰ41ᵐ15ˢ apparent time, the true altitude of the sun's centre was 74°16′; required the true latitude?

* This is the log. secant of one minute, with a modified index.

Apparent time from noon = .	$0^h 18^m 45^s$	Log. rising =	3.524365
Latitude by account = . . .	8° 0′ 0″S.	Log. co-sine =	9.995753
Sun's corrected declination = .	23.27. 0 S.	Log. co-sine =	9.962562
Estimated meridian altitude =	74°33′ 0″	Constant log.=	7.536274
True central altitude = . .	74.16. 0	74°16′ 0″	
Middle altitude = ...	74°24′30″	Log. secant =	0.570604
Correction of altitude = ...		+ 39′ 0″Log.=	1.589558

Sun's correct meridional altitude = ... 74°55′ 0″
Sun's correct declination = ... 23.27. 0 south.

True latitude of the place of observation = . 8°22′ 0″south; which exactly agrees with the result by spherical trigonometry.

Note.—By this method of computation, an error of one degree in the latitude by account, in places within the tropics, will produce little or no effect on the latitude resulting from calculation: thus, if the latitude by account be assumed at 7°0′, or at 9°0′, the resulting latitude, or that deduced from computation, will not differ more than one minute from the truth; and the same result would be obtained, if the altitude were observed at the distance of an hour from noon: provided, always, that the measure of the interval from noon be very correctly known.

Example 2.

December 23d, 1825, in latitude 50°0′ N., by account, at $1^h 14^m 15^s$ apparent time, the altitude of the sun's centre was 13°58′; required the true latitude?

Time from noon = .	$1^h 14^m 15^s$	Log. rising = . . .	4.716200
Latitude by account =	50° 0′N.	Log. co-sine = . . .	9.808068
Sun's corrected dec. =	23.27 S.	Log. co-sine = . . .	9.962562
Estimated merid. alt.=	16°33′	Constant log. = . .	7.536274
True central altitude =	13.58 . .	13°58′	
Middle altitude = .	15°15′30″	Log. secant =	0.015586
Correction of altitude = . . .	+ 1°49′=109′=	Log.=	2.038690

Sun's correct meridional altitude = . 15°47′
Sun's correct declination = . . . 23.27 south.

Co-latitude of the place of observation=39°14′north; hence the true latitude is 50°46′ north, which is 2′ less than the result by spherical trigonometry: the correct latitude being 50°48′ north.

If the latitude by account be assumed at 51°48′, the latitude by computation will be 50°50′; being, in this instance, only two minutes more than the truth.

Note.—The above method of finding the latitude is, as far as I am aware, perfectly original; it is exceedingly well arranged, and it affords a direct and general solution to the problem given, for the same purpose, in page 354: the apparent time, or the measure of the interval from noon, must, however, be *very correctly known;* although, in places distant from the equator, or where the sun does not come very near to the zenith of the place of observation, an error of a few minutes in the time will not very materially affect the latitude: thus, in the last example, an error of two minutes in the interval from noon would only produce an error of six minutes in the latitude; and in the first example, where the sun passes nearer to the zenith, it would produce an error of eight minutes in the latitude. As this method does not labour under any restraint, or since it does not require that the interval from noon should be governed by the object's meridional zenith distance, the observation may therefore be taken at any hour before or after the sun's transit; and this is a peculiarity that gives it a most decided advantage over the method contained in the above-mentioned page.

Problem XIII.

Given the Longitude of a Place, the Sun's Declination and Semi-diameter, and the Interval of Time between the Instants of his Limbs being in the Horizon; to find the Latitude of that Place.

Rule.

Reduce the apparent time, per watch, of the rising or setting of the sun's centre to the corresponding time at Greenwich, by Problem III., page 297; to which time let the sun's declination be reduced, by Problem V., page 298.

To the logarithm of the sun's semi-diameter, reduced to seconds, add the arithmetical complement of the logarithm of the interval of time, expressed in seconds, between the instants of the sun's limbs being in the horizon, and the constant logarithm 9.124939; the sum of these three logarithms, rejecting 10 in the index, will be the logarithmic co-sine of an arch. Now, To the logarithmic sine of the sum of this arch and the sun's reduced declination, add the logarithmic sine of their difference; half the sum will be the logarithmic sine of the latitude of the place of observation.

Example 1.

July 13th, 1824, in north latitude, and longitude 120° west, the sun's lower limb, at the time of its setting, was observed to touch the horizon at $7^h 59^m 58^s$ apparent time, and the upper limb at $8^h 4^m 4^s$; required the latitude of the place of observation?

Apparent time of sun's setting = $7^h59^m58^s + 8^h4^m4^s \div 2$ =	$8^h\ 2^m\ 1^s$
Longitude 120° west, in time = ...	8. 0. 0
Greenwich time of sun's setting = ...	16. 2. 1

Sun's declination at noon, July 13th, 1824, =	21°49′51″ N.
Correction of ditto for $16^h2^m1^s$ = ...	− 6. 0
Sun's reduced declination = ...	21°43′51″ N.

Sun's semi-diameter = 15′45″.8, in seconds = 945″.8		Log. =	2.975799
Interval of time between the setting of the sun's lower and upper limbs = 4^m6^s, or 246^s		Log. ar. comp. =	7.609065
Constant log. (the ar. comp. of the prop. log. of 24 hours esteemed as minutes) = ...			9.124939
Arch = ...	59° 9′39″	Log. co-sine =	9.709803
Sun's reduced declination =	21.43.51 N.		
Sum = ...	80°53′30″	Log. sine = . .	9.994489
Difference = ...	37.25.48	Log. sine = . .	9.783755
		Sum = . . .	19.778244
Latitude of the place of obs. =	50°46′34″N.	Log. sine = . .	9.889122

Example 2.

October 1st, 1824, in north latitude, and longitude 105° east, the sun's upper limb, at the time of its rising, was observed to emerge from the horizon at $6^h3^m43^s$, and the lower limb at $6^h6^m32^s$; required the latitude of the place of observation?

Apparent time of sun's rising = $6^h3^m43^s + 6^h6^m32^s \div 2$ =	$6^h\ 5^m\ 7\frac{1}{2}^s$
Longitude 105° east, in time = ...	7. 0. 0
Greenwich time past noon, September 30th = ...	$11^h\ 5^m\ 7\frac{1}{2}^s$

Sun's declination at noon, September 30th, 1824, =		2°52'46" S.
Correction of ditto for 11h.5m.7½s. = ...		+ 10.48
Sun's reduced declination = ...		3° 3'34" S.

Sun's semi-diameter = 16'1".2, in seconds = 961".2		Log. =	2.982814
Interval of time between the rising of the sun's upper and lower limbs = 2m.49s., or 169s.		Log. ar. comp. =	7.772113
Constant log. = ...			9.124939
Arch = ...	40°40'54"	Log. co-sine =	9.879866
Sun's reduced declination = .	3. 3.34 S.		
Sum = ...	43°44'28"	Log. sine = .	9.839730
Difference = ...	37.37.20	Log. sine = .	9.785652
		Sum =	19.625382
Latitude of the place of observation = 40°31' N.		Log. sine = .	9.812691

Remark.—In this method of finding the latitude, it is indispensably necessary that the interval of time (per watch) between the instants of the sun's lower and upper limbs touching the horizon be determined to the *nearest second;* otherwise the latitude resulting therefrom may be subject to a considerable error, particularly in places where the limbs of that object rise or set in a vertical position; which is frequently the case in parts within the tropics.

SOLUTION OF PROBLEMS RELATIVE TO APPARENT TIME.

Time, as inferred directly from observations of the sun, is denominated either *apparent* or *mean solar time. Apparent time* is that which is deduced from altitudes of the sun, moon, stars, or planets. *Mean time* arises from *a supposed uniform* motion of the sun: hence, a mean solar day is always of the same determinate length; but the measure of an apparent day is ever variable,—being longer at one time of the year, and shorter at another, than a mean day; the instant of apparent noon will, therefore, sometimes precede, and at other times follow, that of mean noon. The difference of those instants is called the *equation of time;* which *equation* is expressed by the difference between the sun's true right ascension and his mean longitude, corrected by the equation of the Equinoxes in right ascension, and converted into time at the rate of 1 minute to every 15 minutes of motion, &c. &c. The equation of time is always equal to the difference between the times shown by an uniform or equable going clock, and a true sun-dial.

The sun's motion in the Ecliptic is constantly varying, and so is his motion in right ascension; but since the latter is rendered further unequal, on account of the obliquity of the Ecliptic to the Equator, it hence follows that the intervals of the sun's return to the same meridian become unequal, and that he will gradually come to the meridian of the same place too late, or too early, every day, for an uniform motion, such as that shown by an equable going watch or clock.

It is this retardation, or acceleration of the sun's coming to the meridian of the same place, that is called the *equation of time;* which implies a correction additive to, or subtractive from, the *apparent time,* in order to reduce it to equable or mean time.

The equation of time vanishes at four periods in the year,—which happen, at present, about the 15th of April, the 15th of June, the 31st of August, and the 24th of December; because, at these periods, there is no difference between the sun's true right ascension and his mean longitude: hence the apparent noon, at those times, is equal to the mean noon. When the sun's true right ascension differs most from his mean longitude, the equation of time is greatest: this happens, at present, about the 11th of February, the 15th of May, the 27th of July, and the 3d of November. But, since at those times the diurnal motion of the sun in right ascension is equal to his mean motion in longitude, or 59′.8″, the length of the apparent day, at these four periods, is, therefore, equal to that of a mean day: at all other times of the year, the lengths of the apparent and mean days differ; and it is the accumulation of those differences that produces the absolute equation of time.

The equation of time is *additive* from about the 25th of December to the 15th of April, and, again, from the 16th of June to the 31st of August; because, during the interval between those periods, the sun comes to the meridian later than the times indicated by a well-regulated clock: but it is *subtractive* from about the 16th of April to the 15th of June, and, again, from the 1st of September to the 24th of December; because, during the interval between these periods, the sun comes to the meridian *earlier* than the times indicated by an equable going clock.

The equation of time is contained in page II. of the month in the Nautical Almanac; but, since it is calculated for the meridian of the Royal Observatory at Greenwich, and for noon, a correction, therefore, becomes necessary, in order to reduce it to any other meridian, and to any given time under that meridian. This correction is to be found by Problem V., page 298; or by means of Table XV., as explained in page 25.

Problem I.

To find the Error of a Watch or Chronometer, by equal Altitudes of the Sun.

Rule.

In the morning, when the sun is nearly in the prime vertical, or at least when he is not less than two hours distant from the meridian, let several altitudes of his upper or lower limb be taken, and the corresponding times (per watch) increased by 12 hours, noted down in regular succession. In the afternoon, observe the instants when the same limb of the sun, taken in the morning, comes to the same altitudes, and write down each, augmented by 24 hours, opposite to its respective altitude. Take the means of the morning and of the afternoon times of observation; add them together, and half their sum will be the time of noon, per watch, incorrect. The difference between the means of the morning and afternoon times will be the interval between the observations: with this interval, and the latitude, enter Table XIII., and with the interval and the declination, corrected for longitude, enter Table XIV.; take out the corresponding equations, noting whether they be affirmative or negative, agreeably to the rule in page 23: then, with the sum or difference of those two equations, according as they are of the same or of contrary signs, and the variation of the sun's declination for the given day, compute the equation of equal altitudes, by the said rule in page 23. Now, to the time of noon, per watch, incorrect, apply the equation of equal altitudes, by addition or subtraction, according as its sign is affirmative or negative, and the sum or difference will be the time, per watch, of apparent noon, or the instant when the sun's centre was on the meridian of the place of observation; the difference between which and noon, or 24 hours, will be the *error* of the watch for apparent time.

If the watch be regulated to mean solar time, such as a chronometer, let the equation of time (as given in the Nautical Almanac, and reduced to the meridian of the place of observation by Problem V., page 298,) be applied to noon, or 24 hours, by addition or subtraction, according to its title, and the mean time of noon will be obtained; the difference between which and the time, per watch, of apparent noon, will be the *error* of the watch for mean solar time.

Example 1.

March 1st, 1825, (*civil time*) in latitude 50°48′N., and longitude 30°W., the following equal altitudes of the sun were observed; required the error of the watch?

Alt. of Sun's Lower Limb.	Forenoon Times, p. Watch.	Afternoon Times, p. Watch.
11°56′	$19^h 59^m 47^s$	$28^h 0^m 58^s$
12. 1	20. 0.23	28. 0.22
12. 6	20. 0.59	27.59.46
12.11	20. 1.35	27.59.10
12.16	20. 2.11	27.58.34

Mean = . . . $20^h 0^m 59^s$ Mean = . . $27^h 59^m 46^s$
Afternoon mean = 27.59.46 Forenoon mean = 20. 0.59

Interval = . . $7^h 58^m 47^s$ Sum = $48^h 0^m 45^s$

Time of noon, per watch, uncorrected = . . $24^h 0^m 22\frac{1}{2}^s$

Equation, Table XIII., ans. to lat.
50°48′ and interval $7^h 58^m 47^s$ = −16″59‴; negative, because the sun is advancing towards the elevated pole.

Equation, Table XIV., ans. to dec.
7°32′25″S. and int. $7^h 58^m 47^s$ = − 0.55; negative, because the sun's dec. is decreasing.

Sum of the equations = ... −17″54‴ Prop. log. = . 1.0024
Variation of sun's declination = 22′46½″ * Prop. log. = . 0.8979

Equation of equal altitudes = . −22″39‴ Prop. log. = . 0.9003
Time of noon, p. watch, uncor. = $24^h 0^m 22\frac{1}{2}^s$

Time, per watch, of app. noon = $24^h 0^m 0^s$ . . . . $24^h 0^m 0^s$
Apparent noon = . . . 24. 0. 0 + Eq. of time $12^m 38^s$
= mean noon = 24.12.38

Watch true for apparent time = $0^h 0^m 0^s$ Watch *slow* for mean time = $12^m 38^s$

Example 2.

August 2d, 1825, (*civil or nautical time*) in latitude 50°48′ N., and longitude 30° W., the following equal altitudes of the sun were observed; required the error of the watch?

* Since the morning observations belong, astronomically, to February 28th, therefore, half the sum of the variation of the sun's declination, for the days preceding and following the given one, is to be taken for the true variation of declination.

Alt. of Sun's Lower Limb.	Forenoon Times, p. Watch.	Afternoon Times, p. Watch.
32°18′	$20^h\ 3^m 52^s$	$28^h\ 3^m 43^s$
32. 23	20. 4. 25	28. 3. 11
32. 28	20. 4. 57	28. 2. 39
32. 33	20. 5. 30	28. 2. 6
32. 38	20. 6. 2	28. 1. 34

Mean = . . . $20^h\ 4^m 57^s 12^t$ Mean = $28^h\ 2^m 38^s 36$.
Afternoon mean = 28. 2. 38. 36 Foren. mean = 20. 4. 57. 12

Interval = . . $7^h 57^m 41^s 24^t$ Sum = $48^h\ 7^m 35^s 48^t$

Time of noon, per watch, uncorrected = . . $24^h\ 3^m 47^s 54^t$

Equation, Table XIII., ans. to lat.
50°48′ and int. $7^h 57^m 41^s 24^t$ = $+16''58'''$; affirmative, because the sun is receding from the elevated pole.

Equa., Tab. XIV., ans. to dec. 17°47′9″
and interval $7^h 57^m 41^s 24^t$ = $-$ 2. 14; negative, because the sun's dec. is decreasing.

Difference of the equations = . $+14''44'''$ Prop. log. = . 1.0870
Variation of sun's declination = . 15′22½″ Prop. log. = . 1.0685

Equation of equal altitudes = . $+12''35'''$ Prop. log. = . 1.1555
Time of noon, p. watch, uncor. = $24^h\ 3^m 47^s 54^t$

Time, p. watch, of app. noon = $24^h\ 4^m\ 0^s 29^t$ . . . . $24^h\ 4^m\ 0^s 29^t$
Apparent noon = ... 24. 0. 0. 0 + Eq. of time = $5^m 54^s$ = mean noon = . 24. 5. 54. 0

Watch fast for apparent time = $4^m\ 0^s 29^t$ Watch *slow* for mean time = $1^m 53^s 31^t$

Now, since the equal altitudes in the two preceding examples have been observed at the same place, and the times of observation specified by the same watch, the daily rate of that machine may therefore be readily established, upon the assumption of an uniform motion; as follows, viz.,

March 1st, 1825, watch slow for mean time at noon = $12^m 38^s$
August 2d, 1825, watch slow for mean time at noon = 1. 53½

Interval = 154 days. Difference = $10^m 44\frac{1}{2}^s$

Now, $10^m.44\frac{1}{2}^s$., divided by 154 days, gives $4^s.185$; which, therefore, is the daily rate *gaining*.

Remarks.

In finding the rate of a watch or chronometer, if it be too fast at the time of the first observation, and the error increasing, the machine will evidently be gaining on mean time; but if decreasing, it will be losing for mean time. Again, if the watch or chronometer be too slow at the first observation, and the error increasing, the machine will be losing for mean time; but if decreasing, it will be gaining on mean time, as in the case or example above.

Since the method of finding the apparent or mean time, by equal altitudes of the sun, does not indispensably require that the latitude of the place of observation and the value of the sun's declination be strictly determined, as these elements are only employed in taking out the equations from Tables XIII. and XIV.; and since any trifling error therein will not sensibly affect the resulting equation,—this method, therefore, is the best adapted for practice on shore, where the altitudes may be taken with a sextant, by means of an artificial horizon, and the corresponding times determined with the greatest exactness. Nor is it absolutely necessary that the instrument be very rigidly adjusted, provided only, that it shows the same altitude at both observations.

In taking equal altitudes, it will be advisable for the observer to fix the index of his sextant or quadrant to some particular division on the arch, and then wait till the contact of the images takes place.

Problem II.

To find the Error of a Watch or Chronometer, by equal Altitudes of a fixed Star.

Rule.

Let several altitudes of a known fixed star be observed when in the eastern hemisphere, and the corresponding times, per watch, noted down in regular succession. When the star is in the western horizon, observe the instants when it comes to each of the former altitudes, and write down each opposite to its respective altitude. Take the means of the eastern and of the western times of observation; add them together, and half their sum will be the time, per watch, of the star's transit over the meridian of the place of observation.

Compute the apparent time of the star's transit over the given meridian, by Problem XII., page 317; the difference between which and the observed time of transit will be the error of the watch, which will be fast or slow according as the observed time of transit is greater or less than the computed time of transit.

Example 1.

April 24th, 1825, in latitude 50°15′ N., and longitude 60°45′ W., the following equal altitudes of Arcturus were observed; required the error of the watch for apparent time?

Altitudes of Arcturus.	Eastern Times, per Watch.	Western Times, per Watch.
26° 4′	$7^h\ 6^m 41^s$	$16^h 49^m 49^s$
26. 19	7. 8. 14	16. 48. 16
26. 34	7. 9. 47	16. 46. 43
26. 49	7. 11. 21	16. 45. 9
27. 4	7. 12. 55	16. 43. 35

Mean = $7^h\ 9^m 47^s 36^t$ Mean = $16^h 46^m 42^s 24^t$
Mean of eastern times = ... 7. 9. 47. 36

Sum = $23^h 56^m 30^s\ 0^t$

Time of star's transit over the given mer., per watch, = $11^h 58^m 15^s\ 0^t$

Star's R. A., reduced to night of obs. = $14^h\ 7^m 41^s.3$
Sun's R. A. at noon of the given day = 2. 6. 55 .3

Approximate time of the star's transit = $12^h\ 0^m 46^s.0$ $12^h\ 0^m 46^s$
Longitude 60°45′ W., in time = . + 4. 3. 0

Corresponding time at Greenwich = $16^h\ 3^m 46^s$
Correction of transit answering to Greenwich time $16^h 3^m 46^s$, and variation of sun's right ascension $3^m 45^s.5$ = $-\ 2^m 31^s$

App. time of star's transit over the merid. of the place of obs. = $11^h 58^m 15^s$
Apparent time of transit, per watch, = ... 11. 58. 15

Watch *true* for apparent time = ... $0^h\ 0^m\ 0^s$

Example 2.

January 1st, 1825, in latitude 30°45′ S., and longitude 75°30′ E., at $7^h 47^m 23^s$ apparent time, per watch, the observed altitude of Sirius, in the eastern hemisphere, was 33°43′40″, and at 15ʰ47ᵐ19ˢ, when the star was in the western hemisphere, it was observed again to have the same altitude; required the error of the watch for apparent time?

Apparent time, per watch, of the obs. equal alt. in east. hemis. = 7ʰ47ᵐ23ˢ
Apparent time, per watch, of the obs. equal alt. in west. hemis. = 15. 47. 19

Sum = 23ʰ34ᵐ42ˢ

Apparent time, per watch, of star's transit over the given mer. = 11ʰ47ᵐ21ˢ

Star's R. A., reduced to night of observ. = 6ʰ37ᵐ25ˢ.6
Sun's right ascension at noon, Jan. 1st, = 18. 47. 19 .1

Approximate time of star's transit = 11ʰ50ᵐ 6ˢ.5 . 11ʰ50ᵐ 6ˢ.5
Longitude 75°30′ E., in time = . . − 5. 2. 0

Corresponding time at Greenwich = . 6ʰ48ᵐ 6ˢ.5
Correction of transit answering to Greenwich time 6ʰ48ᵐ6½ˢ and variation of sun's right ascension 4ᵐ24ˢ.8 = − 1ᵐ15ˢ

Apparent time of star's transit over merid. of place of obs. = 11ʰ48ᵐ51ˢ.5
Apparent time of transit, per watch, = ... 11. 47. 21

Watch *slow* for apparent time = ... 1ᵐ31ˢ.5

Remark.—In ascertaining the error of a watch by equal altitudes of a fixed star, it will be advisable to select one whose declination is of the same name with the latitude, and which exceeds it in value. In high latitudes, the altitude most advantageous for observation may be computed by the second part of the rule in pages 120 and 121, as exemplified in the second example of those pages.

In this case, if the latitude of the place of observation be considerably distant from the Equator, the interval between the times of taking the equal altitudes will be sensibly contracted; and, therefore, any probable irregularity in the going of the watch, during that interval, will be proportionably diminished.

Example.

May 1st, 1825, in latitude 70°30′ N., and longitude 35°45′ W., at 9ʰ36ᵐ18ˢ apparent time, per watch, the observed altitude of the star Kochab, in the eastern hemisphere, was 77°33′20″, and, at 14ʰ57ᵐ11ˢ that star, in the western hemisphere, was again observed to have the same altitude; required the error of the watch for apparent time?

Apparent time, per watch, of obs. equal alt. in the east. hemis. $= 9^h 36^m 18^s$
Apparent time, per watch, of obs. equal alt. in the west. hemis. $= 14.57.11$

Sum $= 24^h 33^m 29^s$

Apparent time, per watch, of star's transit over given merid. $= 12^h 16^m 44\frac{1}{2}^s$

Star's R. A., reduced to night of observ. $= 14^h 51^m 18^s.6$
Sun's right ascension at noon, May 1st $= 2.33.23.9$

Approximate time of transit $= \ldots 12^h 17^m 54^s.7 \ldots 12^h 17^m 54^s.7$
Longitude $35°45'$ W., in time $= + 2.23.0.0$

Corresponding time at Greenwich $= 14^h 40^m 54^s.7$
Correction of transit answering to Greenwich time
$14^h 40^m 54^s.7$, and var. of sun's right ascension $3^m 49^s = - 2^m 20^s.1$

App. time of star's transit over the merid. of place of obs. $= 12^h 15^m 34^s.6$
Apparent time of transit, per watch, $= \ldots\ldots 12.16.44.5$

Watch *fast* for apparent time $= \ldots\ldots 1^m 9^s.9$

Note.—In this example, since the interval between the observations is only $5^h 20^m 53^s$ (the star being in the prime vertical; that is, bearing due east and due west at the equal altitude,) it is, therefore, evident that any probable irregularity in the going of the watch, during that interval, is less liable to affect the resulting error for apparent time, in any sensible manner, than if such error had been determined from observations comprehending an interval of $9^h 36^m 55^s$, as in the case of Example 1, page 381.

Problem III.

Given the Latitude of a Place, and the Altitude and Declination of the Sun; to find the apparent Time of Observation, and, thence, the Error of a Watch or Chronometer.

Method I.

Rule.

Reduce the sun's declination to the time and place of observation, by Problem V., page 298; which being applied to 90°, by addition or subtraction, according as it is of a different or of the same denomination with the latitude, the sum or remainder will be the sun's *polar distance.*

Reduce the observed altitude of the sun's limb to the true central altitude, by Problem XIV., page 320.

Now, add together the sun's true altitude, its polar distance, and the latitude of the place of observation; take half the sum, and call the difference between it and the sun's true altitude the *remainder*.

Then, to the log. co-secant of the polar distance, add the log. secant of the latitude, the log. co-sine of the half sum, the log. sine of the remainder, and the constant logarithm 6.301030: the sum of these five logarithms, abating 20 in the index, will be the log. rising answering to the sun's distance from the meridian; which will be the apparent time at ship or place, if the observation be made in the afternoon; but if in the forenoon, its complement to 24 hours will be the apparent time; the difference between which and the time of observation, per watch, will be the *error* of the watch, and which will be *fast* or *slow* according as the time shown thereby is later or earlier than the apparent time.

Remark.—In practice, it becomes absolutely necessary to take several altitudes of the sun's limb, and to note the corresponding times per watch; then, the sum of the altitudes, divided by their number, gives the mean altitude,—and the sum of the times, so divided, gives the mean time.

Example 1.

January 1st, 1825, in latitude 40°27′ N., and longitude 54°40′ W., the following altitudes of the sun's lower limb were observed, the height of the eye above the level of the sea being 20 feet; required the apparent time of observation and the error of the watch?

Mean time of observation, per watch, = . . .	3ʰ 2ᵐ 0ˢ
Longitude 54°40′ W., in time = ...	+ 3.38.40
Greenwich time = ...	6ʰ40ᵐ40ˢ
Sun's declination at noon, January 1st, = . . .	23° 0′59″ S.
Correction of ditto for 6ʰ40ᵐ40ˢ = ...	− 1.27
Sun's reduced declination = ...	22°59′32″ S.
Sun's north polar distance = ...	112°59′32″

Time, per Watch.	Altitude of Sun's Lower Limb.
3ʰ 0ᵐ 30ˢ	13°49′40″
3. 1. 15	13. 44. 0
3. 2. 0	13. 38. 10
3. 2. 45	13. 32. 30
3. 3. 30	13. 26. 40
10ᵐ 0ˢ	191′ 0″

Mean = 3ʰ 2ᵐ 0ˢ Mean = 13°38′12″
Sun's semi-diam. 16′18″ − dip 4′17″ = +12. 1

Sun's apparent altitude = ... 13°50′13″
Refraction 3′48″ − Parallax 0′9″ = − 3. 39

Sun's true central altitude = . . . 13°46′34″ Constant log. = 6. 301030
Sun's north polar distance = . . . 112. 59. 32 Log. co-sec.* = 0. 035949
Lat. of the place of observation = 40. 27. 0 Log. secant* = 0. 118631

Sum = . . . 167°13′ 6″

Half sum = . . 83°36′33″ Log. co-sine = 9. 046534
Remainder = . 69. 49. 59 Log. sine = 9. 972523

Apparent time of observation = . 3ʰ 1ᵐ 45ˢ Log. rising = 5. 47466. 7
Time of observation, per watch, = 3. 2. 0

Watch *fast* for apparent time = . 15 seconds.

Example 2.

June 9th, 1825, in latitude 50°40′ N., and longitude 47°56′15″ E., the following altitudes of the sun's lower limb were observed, the height of the eye above the level of the horizon being 23 feet; required the apparent time of observation, and the error of the watch?

Time of observation, per watch, = ... 19ʰ 22ᵐ 25ˢ
Longitude 47°56′15″ E., in time = . . . − 3. 11. 45

Greenwich time = ... 16ʰ 10ᵐ 40ˢ

* The 10ˢ are rejected from the indices of the logarithmic secant and co-secant; and, with the view of facilitating the future operations in this work, the same plan will be pursued in all the computations.

Sun's declination at noon, June 9th, = . .	22°56′37″N.
Correction of ditto for 16ʰ10ᵐ40ˢ = ...	+ 3.16
Sun's reduced declination = ...	22°59′53″N.
Sun's north polar distance = ...	67° 0′ 7″

Time, per Watch.	Altitude of Sun's Lower Limb.
19ʰ20ᵐ45ˢ	29°33′10″
19.21.35	29.25.10
19.22.25	29.17.30
19.23.15	29. 9.40
19.24. 5	29. 1.50
112ᵐ 5ˢ	87′20″

Mean = 19ʰ22ᵐ25ˢ Mean = 29°17′28″ : hence the true central altitude is 29°27′7″.

Sun's true central altitude = .	29°27′ 7″		
Sun's north polar distance = .	67. 0. 7	Log. co-secant =	0.035967
Latitude of the place of observ. =	50.40. 0	Log. secant =	0.198026
Sum = . . .	147° 7′14″		
Half sum = . .	73°33′37″	Log. co-sine =	9.451797
Remainder = .	44. 6.30	Log. sine = .	9.842620
Constant log. = ...			6.301030
Sun's distance from the meridian =	4ʰ44ᵐ11ˢ	Log. rising =	5.82944.0

Apparent time of observation =	19ʰ15ᵐ49ˢ
Time of observation, per watch, =	19.22.25
Watch *fast* for apparent time =	6ᵐ36ˢ

Note.—Since the log. rising, in Table XXXII., is only computed to five places of decimals, therefore, in taking out the meridian distance of a celestial object from that Table, answering to a given log. rising, the sixth or right-hand figure of such given log. rising, is to be rejected; observing, however, to increase the fifth or preceding figure by unity or 1, when the figure so rejected amounts to 5 or upwards: thus, in the preceding example, where the log. rising is 5.474667, the meridian distance is taken out for 5.47467; and so on of others.

For the principles on which the meridian distance of a celestial object is computed, and hence the apparent time, the reader is referred to "The Young Navigator's Guide to the Sidereal and Planetary Parts of Nautical Astronomy," page 156.

Remarks.

Altitudes for ascertaining the *error* of a watch ought to be taken by means of an artificial horizon: one produced by *pure* quicksilver should be preferred, because it shows, at all times, when placed in a proper position, a truly horizontal plane; and, therefore, the angles of altitude taken therein are always as correct as the divisions on the sextant with which those angles are observed; whereas, altitudes taken by means of the *sea horizon* are generally subject to some degree of uncertainty, owing to its being frequently broken or ill-defined, by atmospherical haze, at the time of observation; though such altitudes are, nevertheless, sufficiently correct for finding the longitude at sea.

In taking altitudes by means of an artificial horizon, it is to be observed, that the angle shown by the sextant will be *double the altitude* of the observed limb of the object; which is to be corrected for index error, if any: then, *half the corrected angle* will be the observed altitude of the object's limb above the *true horizontal plane;* to which, if its semi-diameter, refraction, and parallax be applied, the true central altitude of the observed object will be obtained. There is no correction necessary for *dip*, because the quicksilver shows a truly horizontal plane, as has been before remarked.*

The position of a celestial object most favourable for determining the apparent time with the greatest accuracy, is, when it is in the prime vertical; that is, when it bears either due east or due west at the place of observation, or, if it be circumpolar, when it is in that part of its diurnal path which is in contact with an azimuth circle; viz., when the log. sine of its altitude = log. sine of the latitude + radius — log. sine of its declination; because, then, the change of altitude is quickest, and the extreme accuracy of the latitude not very essentially requisite. The nearer a celestial object is to either of these positions, the nearer will the apparent time, deduced from its altitude, be to the truth; as, then, the unavoidable small errors which generally creep into the observations, or a few miles difference in the latitude, will have little or no effect on the apparent time so deduced.

Table XLVII. contains the time or distance of a celestial object from the meridian at which its altitude should be observed, in order to determine the apparent time with the greatest accuracy; and Table XLVIII. contains

* The direct rules for applying the necessary corrections to altitudes taken on shore by means of an artificial horizon, will be found at the end of the Compendium of Practical Navigation, towards the latter part of this volume.

the corresponding altitude most advantageous for observation. But, since those Tables are adapted to the declination of a celestial object when it is of the same name with the latitude of the place of observation, they will not, therefore, indicate either the proper time or the altitude when those elements are of contrary denominations: in this case, since the sun or other celestial object comes to the prime vertical before it rises, and therefore does not bear due east or west while above the horizon, the observation for determining the apparent time from its altitude must be made while the object is near to the horizon; taking care, however, not to take an altitude for that purpose under 3 or 4 degrees, on account of the uncertain manner in which the atmospheric refraction acts upon very small angles of altitude observed adjacent to the horizon.—See explanation to the above-mentioned Tables, pages 119 and 120.

Method II.

Of computing the horary Distance of a celestial Object from the Meridian.

Rule.

If the latitude of the place of observation and the declination of the celestial object be of different names, let their *sum* be taken,—otherwise, their *difference*,—and the meridional zenith distance of the object will be obtained; to which apply its observed zenith distance, by addition and subtraction, and let half the sum and half the difference be taken; then,

To the log. secant of the latitude add the log. secant of the declination, the log. sine of the half sum, the log. sine of the half difference, and the constant logarithm 6.301030; the sum of these five logarithms, abating 20 in the index, will be the log. rising of the object's horary distance from the meridian; and if this object be the sun, the apparent time will be known, as in the last method; and, hence, the error of the watch, if necessary.

Example 1.

January 10th, 1825, in latitude 40°30′ N. and longitude 59°2′30″ W., the mean of several observed altitudes of the sun's lower limb was 14°31′47″, that of the corresponding times, per watch, 3h 1m 45s, and the height of the eye above the level of the horizon 18 feet; required the apparent time of observation, and the error of the watch?

Time of observation, per watch, = . . .	3h 1m 45s
Longitude 59°2′30″ W., in time = .	+ 3.56.10
Greenwich time = ...	6h 57m 55s

Sun's declination at noon, January 10th, =	21°57′50″ S.
Correction of ditto for 6ʰ57ᵐ55ˢ = . .	− 2.40
Sun's reduced declination = ...	21°55′10″ S.

Obs. alt. of sun's l. limb=14°31′47″; hence, its true cent. alt. is 14°41′36″

Sun's true zenith distance at time of observation = . . . 75°18′24″

Latitude = . .	40°30′ 0″N.	. . . . . .	Log. secant=0.118954
Declination = .	21.55.10 S.	. . . . . .	Log. secant=0.032588
Sun's mer. z. dist.=	62°25′10″		
Obs. zenith dist. =	75.18.24	. . . .	Const. log.= 6.301030
Sum = . . .	137°43′34″	Half=68°51′47″	Log. sine = 9.969752
Difference = .	12°53′14″	Half= 6.26.37	Log. sine = 9.050091

Sun's dist. from the mer.=the app. time=3ʰ1ᵐ15ˢ Log. rising=5.47241.5
Time of observation, per watch, = . . 3.1.45

Watch *fast* for apparent time = . . 0ᵐ30ˢ

Example 2.

January 20th, 1825, in latitude 37°20′ S. and longitude 49°45′ E., the mean of several altitudes of the sun's lower limb was 26°39′15″, that of the corresponding times, per watch, 19ʰ11ᵐ45ˢ, and the height of the eye above the level of the horizon 16 feet; required the apparent time of observation, and the error of the watch?

Time of observation, per watch, = . .	19ʰ11ᵐ45ˢ
Longitude 49°45′ E., in time = . .	− 3.19. 0
Greenwich time = ...	15ʰ52ᵐ45ˢ

Sun's declination at noon, January 20th,=	20° 7′11″ S.
Correction of ditto for 15ʰ52ᵐ45ˢ = . .	− 8.45
Sun's reduced declination = ...	19°58′26″ S.

Obs. alt. of sun's l. limb = 26°39′15″; hence, its true cent. alt. is 26°49′58″

Sun's true zenith distance at the time of observation = . . 63°10′ 2″

Latitude = . .	37°20′ 0″ S.	. . . .	Log. secant=0.099567
Declination = .	19.58.26 S.	. . . . .	Log. secant=0.026942
Sun's mer. z. dist.=	17°21′34″		
Obs. zenith dist. =	63. 10. 2	. . . .	Const. log.= 6.301030
Sum = ...	80°31′36″	Half=40°15′48″	Log. sine = 9.810435
Difference = . .	45.48.28	Half=22.54.14	Log. sine = 9.590158

Sun's horary distance from the merid.=$4^h43^m42^s$ Log. rising=5.82813.2

Apparent time of observation = . . .	$19^h16^m18^s$
Time of observation, per watch, = .	19.11.45
Watch *slow* for apparent time = .	4^m33^s

Method III.

Of computing the horary Distance of a celestial Object from the Meridian.

Rule.

If the latitude of the place of observation and the declination of the celestial object are of different names, let their *sum* be taken,—otherwise, their *difference*,—and the meridional zenith distance of the object will be obtained; the natural versed sine of which, being subtracted from the natural co-versed sine of the object's true altitude, will leave a *remainder*. Now, to the logarithm of this remainder add the log. secants of the latitude and the declination, and the sum will be the log. rising of the object's horary distance from the meridian; and if this object be the sun, the apparent time will be known, and, hence, the error of the watch, if required, as shown in the first method, page 384.

Example 1.

May 1st, 1825, in latitude 40°35′ S., and longitude 63°15′ E., the mean of several altitudes of the sun's lower limb was 19°43′58″; that of the corresponding times, per watch, $20^h57^m45^s$, and the height of the eye above the level of the sea 14 feet; required the apparent time of observation, and the error of the watch?

Time of observation, per watch, = . . .	$20^h57^m45^s$
Longitude 63°15′ E., in time = . . .	− 4.13. 0
Greenwich time = ...	$16^h44^m45^s$

Sun's declination at noon, May 1st, =		15° 4′19″N.
Correction of ditto for 16ʰ44ᵐ45ˢ	=	+ 12.34
Sun's reduced declination = ...		15°16′53″N.

Obs. alt. of sun's l. limb = 19°43′58″; hence, the true cent. alt. is 19°53′47″

Latitude = . .	40°35′ 0″S.	. . . .	Log. secant = 0.119495
Reduced dec. =	15.16.53 N.	. . . .	Log. secant = 0.015634
Sun's mer. z. dis. =	55°51′53″	Nat. V.S. = 438851	
Sun's true alt. =	19.53.47	Nat. co-V.S. = 659680	
		Remainder = 220829 Log. =	5.344056
Sun's horary distance from the merid. =	3ʰ 2ᵐ45ˢ	Log. rising =	5.47918.5

Apparent time of observation = . . .	20ʰ57ᵐ15ˢ
Time of observation, per watch, = .	20.57.45
Watch *fast* for apparent time = . .	0ᵐ30ˢ

Example 2.

November 10th, 1825, in latitude 49°13′ S., and longitude 36°50′ W., the mean of several altitudes of the sun's lower limb was 22°28′30″, the mean of the corresponding times, per watch, 5ʰ4ᵐ25ˢ, and the height of the eye above the level of the horizon 20 feet; required the apparent time of observation, and the error of the watch?

Time of observation, per watch, = . . .	5ʰ 4ᵐ25ˢ
Longitude 36°50′W., in time = .	+ 2.27.20
Greenwich time = ...	7ʰ31ᵐ45ˢ

Sun's declination at noon, Nov. 10th, = .	17° 9′50″ S.
Correction for 7ʰ31ᵐ45ˢ = ...	+ 5.15
Sun's reduced declination = ...	17°15′ 5″ S.

Obs. alt. of sun's l. limb = 22°28′30″; hence, its true cent. alt. is 22°38′17″

Latitude = . .	49°13′ 0″S.	Log. secant=0.184954
Reduced dec. =	17.15. 5 S.	Log. secant=0.019991
Sun's mer. z. dist.=31°57′55″	Nat.V.S. = 151631	
Sun's true alt. = 22.38.17	Nat.co-V.S.=615091	
	Remainder = 463460	Log. = 5.666012
Sun's dist. from the mer.=the appar. time=5h 0m25s		Log.ris.=5.87095.7
Time of observation, per watch, = . . .	5. 4.25	
Watch *fast* for apparent time = . . .	4m 0s	

Method IV.

Of computing the horary Distance of a celestial Object from the Meridian.

Rule.

If the latitude of the place of observation and the declination of the celestial object be of different names, let their *sum* be taken,—otherwise, their *difference*,—and the meridional zenith distance of the object will be obtained; from the natural co-sine of which, subtract the natural sine of the object's true altitude, and to the logarithm of the remainder add the log. secants of the latitude and the declination; and the sum will be the log. rising of the object's horary distance from the meridian. Now, if this object be the sun, the apparent time is known, and, hence, the error of the watch, if required, as shown in the first method, page 384.

Example 1.

July 4th, 1825, in latitude 39°47′S., and longitude 60°50′ E., the mean of several altitudes of the sun's lower limb was 13°2′30″, that of the corresponding times, per watch, 3h10m45s, and the height of the eye above the level of the horizon 22 feet; required the apparent time, and the error of the watch?

Time of observation, per watch, = . .	3h10m45s
Longitude 60°50′ E., in time = . .	− 4. 3.20
Greenwich time past noon of July 3d =	23h 7m25s

Sun's declination at noon, July 3d, = 22°59′30″N.
Correction of ditto for 23ʰ7ᵐ25ˢ = . — 4.48

Sun's reduced declination = ... 22°54′42″N.

Obs. alt. of the sun's l. limb = 13°2′30″; hence, its true cent. alt. is 13°9′53″

Latitude = . . 39°47′ 0″S. . . . . . . Log. secant=0.114373
Reduced dec. = 22.54.42 N. . . . . . . Log. secant=0.035690

Sun's mer. z. dist. = 62°41′42″ Nat. co-sine=458727
Sun's true alt. = 13. 9.53 Nat. sine = 227751

Remainder = 230976 Log. = 5.363567

Sun's dist. from the mer. = the appar. time = 3ʰ10ᵐ35ˢ Log. ris. = 5.51363.0
Time of observation, per watch, = . . 3.10.45

Watch *fast* for apparent time = ... 0ᵐ10ˢ

Example 2.

July 19th, 1825, in latitude 40°10′50″ N., and longitude 53°20′ W., the mean of several altitudes of the sun's lower limb was 33°23′15″, that of the corresponding times, per watch, 19ʰ47ᵐ30ˢ, and the height of the eye above the level of the horizon 15 feet; required the apparent time, and the error of the watch?

Time of observation, per watch, = . . 19ʰ47ᵐ30ˢ
Longitude 53°20′ W., in time = . . + 3.33.20

Greenwich time = ... 23ʰ20ᵐ50ˢ

Sun's declination at noon, July 19th, = 20°53′11″N.
Correction of ditto for 23ʰ20ᵐ50ˢ = — 10.44

Sun's reduced declination = ... 20°42′27″N.

Obs. alt. of sun's l. limb = 33°23′15″; hence, the true cent. alt. is 33°34′1″

Latitude = . . 40°10′50″N. . . . . . . Log. secant=0.116898
Reduced dec. = 20.42.27 N. . . . . . . Log. secant=0.029004

Sun's mer. z. dist. = 19°28′23″ Nat. co-sine = 942798
Sun's true alt. = 33.34. 1 Nat. sine = 552911

Remainder = 389887 Log. = 5.590939

Sun's horary distance from the merid. = 4ʰ11ᵐ53ˢ Log. rising = 5.73684.1

Sun's horary distance from the merid. =	4h 11m 53s
Apparent time of observation = .	19h 48m 7s
Time of observation, per watch, =	19. 47. 30
Watch *slow* for apparent time = .	0m 37s

Problem IV.

Given the Latitude of a Place, the Altitude, Right Ascension, and Declination of a known fixed Star, and the Sun's Right Ascension; to find the apparent Time, and, hence, the Error of the Watch.

Rule.

Find the true altitude of the star, by Problem XVII., page 327; and let its right ascension and declination, as given in Table XLIV., be reduced to the night of observation; then,

With the latitude of the place, the star's true altitude, and its reduced declination, compute its horary distance from the meridian, by any of the methods given in the last problem.

Now, if the star be observed in the western hemisphere, let its meridian distance, thus found, be *added* to its reduced right ascension, but, if in the eastern hemisphere, subtracted from it, and the sum or remainder will be the right ascension of the meridian; from which, (increased by 24 hours, if necessary,) subtract the sun's right ascension at noon of the given day, and the remainder will be the approximate time of observation. Reduce this to Greenwich time, by Problem III., page 297, and find the proportional part of the variation of the sun's right ascension, for the given day, answering thereto and 24 hours, by Problem XII., page 317; which, being subtracted from the approximate, will give the apparent time of observation: hence the error of the watch may be known.

Note.—For the principles of this rule, see "The Young Navigator's Guide," page 156.

Example 1.

January 1st, 1825, in latitude 40°29′ N., and longitude 59°45′ W., the mean of several altitudes of α Arietis, west of the meridian, was 36°29′48″; that of the corresponding times, per watch, 11h 9m 29s, and the height of the eye above the level of the sea 19 feet; required the apparent time, and the error of the watch?

Reduced R. A. of α Arietis $= 1^h 57^m 19^s$, and its reduced dec. $= 22^\circ 37' 50''$ N.

Observed alt. of α Arietis $= 36^\circ 29' 48''$; hence, its true alt. is $36^\circ 24' 20''$

True zenith distance of α Arietis $=$ . . . . $53^\circ 35' 40''$

Latitude $=$ . . $40^\circ 29' 0''$ N. . . . . . . Log. secant $= 0.118847$
Star's red. dec. $=$ 22. 37. 50 N. . . . . . . Log. secant $= 0.034796$

Star's mer. z. dis. $= 17^\circ 51' 10''$
Star's obs. z. dist. $=$ 53. 35. 40 . . . . Const. log. $= 6.301030$

Sum $=$ . . . . . $71^\circ 26' 50''$ Half $= 35^\circ 43' 25''$ Log. sine $= 9.766321$
Difference $=$. 35. 44. 30 Half $=$ 17. 52. 15 Log. sine $= 9.486958$

Star's horary dist., west of the merid. $= 4^h\ 2^m 45^s$ Log. rising $= 5.70795.2$
Star's reduced right ascension $=$ . . . 1. 57. 19

Right ascension of the meridian $=$. $6^h\ 0^m\ 4^s$
Sun's R. A. at noon, January 1st $=$. 18. 47. 19

Approximate time $=$ . . . . . . $11^h 12^m 45^s$ . . . . . $11^h 12^m 45^s$
Longitude $59^\circ 45'$ W., in time $=$ + 3. 59. 0

Greenwich time $=$ . . . . $15^h 11^m 45^s$
Correction of approximate time, ans. to Greenwich time
$15^h 11^m 45^s$, and variation of sun's R. A. $4^m 24^s.8$, $=$ . . . $-\ 2^m 48^s$

Apparent time of observation $=$ . . . . $11^h\ 9^m 57^s$
Time of observation, per watch, $=$ 11. 9. 29

Watch *slow* for apparent time $=$ . . . . $0^m 28^s$

Example 2.

January 1st, 1825, in latitude $39^\circ 20' 30''$ S., and longitude $75^\circ 40'$ E., the mean of several altitudes of Procyon, east of the meridian, was $27^\circ 15' 47''$, that of the corresponding times, per watch, $9^h 30^m 23^s$, and the height of the eye above the level of the sea 19 feet; required the apparent time of observation, and the error of the watch?

Procyon's reduced R. A. $= 7^h 30^m 8^s$, and its reduced dec. $= 5^\circ 39' 58''$ N.

Observed alt. of Procyon $= 27^\circ 15' 47''$, hence, its true alt. is $= 27^\circ 9' 46''$

Latitude = . .	39°20′30″S.	. . . . . .	Log. secant=0.111607
Star's red. dec. =	5. 39. 58 N.	. . . . . .	Log. secant=0.002128

Procyon's m.z.dis.=45° 0′28″ Nat. vers. S.=292989
Procyon's true alt.=27. 9.46 Nat. co-V.S.=543480

Remainder = 250491 Log. = 5.398792

Procyon's horary dist., east of the mer.=	3^{h}10^{m}20^{s}	Log.rising=5.51252.7
Procyon's right ascension = ...	7.30. 8	
Right ascension of the meridian = .	4^{h}19^{m}48^{s}	
Sun's right asc. at noon, Jan. 1st, =	18.47.19	
Approximate time = ...	9^{h}32^{m}29^{s}	. . . . . 9^{h}32^{m}29^{s}
Longitude 75°40′ E., in time = .	− 5. 2.40	
Greenwich time = ...	4^{h}29^{m}49^{s}	
Correction of approximate time, answering to Greenwich time 4^{h}29^{m}49^{s}, and variation of sun's right ascension 4^{m}24^{s}.8 =		— 0^{m}50^{s}
Apparent time of observation = ...		9^{h}31^{m}39^{s}
Time of observation, per watch, = ...		9.30.23
Watch *slow* for apparent time = ...		1^{m}16^{s}

Note.—When the star's horary distance *east* of the meridian exceeds the right ascension, the latter is to be increased by 24 hours, in order to find the right ascension of the meridian.

In finding the error of a watch by sidereal observation, two or more stars should be observed, and the error of the watch deduced from each star separately. And, if an equal number of stars be observed on different sides of the meridian, and nearly equidistant therefrom, it will conduce to still greater accuracy; because, then, the errors of the instrument and the unavoidable errors of observation will have a mutual tendency to correct each other. The mean of the errors, thus deduced, should be taken for the absolute error of the watch.

PROBLEM V.

Given the Latitude and Longitude of a Place, and the Altitude of a Planet, to find the Apparent Time of Observation.

RULE.

Reduce the estimated time of observation to the meridian of Greenwich, by Problem III., page 297; to which time let the planet's right ascension and declination be reduced, by Problem VII., page 307; and let the sun's right ascension, at noon of the given day, be also reduced to that time, by Problem V., page 298. Reduce the observed central altitude of the planet to its true central altitude, by Problem XVI., page 325.

Then, with the latitude of the place, the planet's reduced declination, and its true central altitude, compute its horary distance from the meridian, by any of the methods given in Problem III., pages 384 to 392. Let the planet's horary distance from the meridian, thus found, be applied to its reduced right ascension, by addition or subtraction, according as it may be observed in the western or in the eastern hemisphere, and the right ascension of the meridian will be obtained; from which (increased by 24 hours, if necessary,) subtract the sun's reduced right ascension, and the remainder will be the apparent time of observation.

Note.—When the planet's horary distance *east* of the meridian exceeds its right ascension, the latter is to be increased by 24 hours, in order to find the right ascension of the meridian.

Example 1.

January 3d, 1825, in latitude 50°30′ N., and longitude 48°45′ W., the mean of several altitudes of Jupiter's centre, east of the meridian, was 23°41′55″, that of the corresponding times, per watch, 9ʰ 1ᵐ, and the height of the eye above the level of the sea 16 feet; required the apparent time of observation?

Time of observation, per watch, = .	9ʰ 1ᵐ 0ˢ
Longitude 48°45′ W., in time =	+ 3. 15. 0
Greenwich time = ...	12ʰ16ᵐ 0ˢ

Sun's right ascension at noon, January 3d, =	18ʰ56ᵐ 8ˢ
Correction of ditto for 12ʰ16ᵐ = ...	+ 1.39
Sun's reduced right ascension = ...	18ʰ57ᵐ47ˢ

Obs. cent. alt. of Jupiter = 23°41′55″; hence, its true cent. alt. is 23°35′56″

Jupiter's right ascension, Jan. 1st, =	$8^h 58^m 0^s$
Correction of ditto for $2^d 12^h 16^m$ =	− 0.50
Jupiter's reduced right ascension =	$8^h 57^m 10^s$
Jupiter's declination, January 1st, =	17°56′ 0″N.
Correction of ditto for $2^d 12^h 16^m$ =	+ 5. 1
Jupiter's reduced declination = . .	18° 1′ 1″N.
Ditto north polar distance = . .	71°58′59″

Jupiter's true central altitude =	23°35′56″		
Jupiter's north polar distance =	71. 58. 59	Log. co-secant =	0. 021836
Lat. of the place of observation =	50. 30. 0	Log. secant =	0. 196489
Sum = ...	146° 4′55″	Constant log. =	6. 301030
Half sum = ...	73° 2′25½″	Log. co-sine =	9. 464933
Remainder = ...	49°26′29½″	Log. sine = .	9. 880667
Jupiter's horary dist., east of the mer. =	$4^h 58^m 0^s$	Log. rising =	5. 86495,5
Jupiter's reduced right ascension = . .	8. 57. 10		
Right ascension of the meridian = .	$3^h 59^m 10^s$		
Sun's reduced right ascension = . .	18. 57. 47		
Apparent time of observation = . .	$9^h 1^m 23^s$		

Example 2.

January 16th, 1825, in latitude 34°45′ S., and longitude 80°30′ E., the mean of several altitudes of Venus' centre, west of the meridian, was 22°53′25″, that of the corresponding times, per watch, $7^h 20^m 45^s$, and the height of the eye above the level of the sea 18 feet; required the apparent time of observation?

Time of observation, per watch, = .	$7^h 20^m 45^s$
Longitude 80°30′ E., in time = .	− 5. 22. 0
Greenwich time = ...	$1^h 58^m 45^s$

Sun's right ascension at noon, January 16th, =	$19^h 52^m 41^s$
Correction of ditto for $1^h 58^m 45^s$ = . . .	+ 0. 21
Sun's reduced right ascension = ...	$19^h 53^m 2^s$

Venus' right ascension, January 13th, = $22^h 23^m 0^s$
Correction of ditto for $3^d 1^h 58^m 45^s$ = . + 13.52

Planet's reduced right ascension = . . $22^h 36^m 52^s$

Venus' declination, January 13th, = . $11^\circ 39' 0''$ S.
Correction of ditto for $3^d 1^h 58^m 45^s$ = − 1.29.24

Planet's reduced declination = . . $10^\circ 9' 36''$ S.

Observed central altitude of Venus = $22^\circ 53' 25''$; hence, her true central altitude is $22^\circ 47' 24''$, on the assumption that her horizontal parallax, at the time of observation, was 18 seconds of a degree.

Latitude = . . $34^\circ 45' 0''$ S. . . . . . . Log. secant=0.085315
Planet's red. dec.=10. 9.36 S. . . . . . . Log. secant=0.006864

Planet's m.z.dist.=$24^\circ 35' 24''$ Nat. co-sine=909309
Planet's true alt.=22.47.24 Nat. sine = 387355

Remainder = 521954 Log. = 5.717632

Venus' horary dist., west of the merid.=$4^h 36^m 55^s$ Log. rising=5.80981.1
Venus' reduced right ascension = . 22.36.52

Right ascension of the meridian = $3^h 13^m 47^s$
Sun's reduced right ascension = . 19.53. 2

Apparent time of observation = . $7^h 20^m 45^s$

Remark.—Should the horizontal parallaxes of the planets be ever given in the Nautical Almanac, the mariner may *then* deduce the apparent time from their altitudes, by the above Problem, to a very great degree of accuracy, provided the longitude of the place of observation be known within a few minutes of the truth, or that there be a chronometer on board to indicate the time at Greenwich. However, even admitting that those parallaxes are still to remain unnoticed, the apparent time, computed as above, will always be sufficiently near the truth for the purpose of determining the longitude at sea.

Problem VI.

Given the Latitude and Longitude of a Place, the estimated Time at that Place, and the Altitude of the Moon's Limb; to find the apparent Time of Observation.

Rule.

Reduce the estimated time of observation to the meridian of Greenwich, by Problem III., page 297; to which let the sun's right ascension be reduced, by Problem V., page 298; and let the moon's right ascension, declination, semi-diameter, and horizontal parallax be reduced to the same time, by Problem VI., page 302. Reduce the observed altitude of the moon's limb to the true central altitude, by Problem XV., page 323; then,

With the latitude of the place of observation, the moon's reduced declination, and her true central altitude, compute her horary distance from the meridian, by any of the methods given in Problem III., pages 384 to 392. Now, let the moon's horary distance from the meridian, thus found, be applied to her reduced right ascension, by addition or subtraction, according as she may have been observed in the western or eastern hemisphere, and the right ascension of the meridian will be obtained; from which (increased by 24 hours, if necessary,) subtract the sun's reduced right ascension, and the remainder will be the apparent time of observation.

Note.—When the moon's horary distance, east of the meridian, exceeds her right ascension, the latter is to be increased by 24 hours, in order to find the right ascension of the meridian.

And it is to be borne in mind, that the moon's right ascension and declination must be corrected by the equation of second difference, Table XVII., as explained between pages 33 and 38.*

Example 1.

January 4th, 1825, in latitude 50°10′ N., and longitude 60°W., the mean of several observed altitudes of the moon's lower limb, east of the meridian, was 29°25′23″, that of the corresponding times, per watch, 7h28m18s, and the height of the eye above the surface of the water 17 feet; required the apparent time?

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* For the effects resulting from the equation of the mean second difference of the moon's place in right ascension and declination, see "The Young Navigator's Guide to the Sidereal and Planetary Parts of Nautical Astronomy," page 171.

Time of observation, per watch, = .	7ʰ28ᵐ18ˢ
Longitude 60° W., in time = .	+ 4. 0. 0
Greenwich time = ...	11ʰ28ᵐ18ˢ

Sun's right ascension at noon, January 4th, =	19ʰ 0ᵐ32ˢ
Correction of ditto for 11ʰ28ᵐ18ˢ = . . .	+ 2. 6
Sun's reduced right ascension = ...	19ʰ 2ᵐ38ˢ

Moon's right ascension at noon, January 4th, =	98° 6′53″
Corrected prop. part of ditto for 11ʰ28ᵐ18ˢ =	+7. 17. 28
Moon's corrected right ascension = . . .	105°24′21″

Moon's semi-diameter at noon, January 4th, =	16′ 9″
Correction of ditto for 11ʰ28ᵐ18ˢ = . . .	+ 5
Augmentation, Table IV., = ...	+ 8
Moon's true semi-diameter = ...	16′22″

Moon's declination at noon, January 4th, =	22°35′39″N.
Corrected prop. part of ditto for 11ʰ28ᵐ18ˢ =	−1. 10. 18
Moon's corrected declination = ...	21°25′21″N.

Moon's horizontal parallax at noon, January 4th, =	59′17″
Correction of ditto for 11ʰ28ᵐ18ˢ = ...	+ 16
Moon's true horizontal parallax = ...	59′33″

Observed altitude of the moon's lower limb = 29°25′23″; hence, her true central altitude is 30°27′55″.

Latitude = . .	50°10′ 0″N.	. . . . .	Log. secant = 0.193442
Moon's corr. dec. =	21.25.21 N.	. . . . .	Log. secant = 0.031091
Moon's mer. z. dist. =	28°44′39″	Nat. vers. sine = 123225	
Moon's true alt. =	30.27.55	Nat. co-V. S. = 492988	
		Remainder = 369763	Log. = 5.567923

Moon's horary dist., east of the merid. = 4ʰ30ᵐ41ˢ Log. rising = 5.79245.6

Moon's horary dist. east of the merid.=	4h 30m 41s
Moon's cor. R. A. 105° 24′ 21″, in time=	7. 1. 37
Right ascension of the meridian = . .	2h 30m 56s
Sun's reduced right ascension = . . .	19. 2. 38
Apparent time of observation = . . .	7h 28m 18s

Example 2.

January 30th, 1825, in latitude 10° 20′ S., and longitude 100° 50′ E., the mean of several altitudes of the moon's lower limb, west of the meridian, was 7° 23′ 30″, that of the corresponding times, per watch, 13h 33m 20s, and the height of the eye above the surface of the water 20 feet; required the apparent time?

Time of observation, per watch, =	13h 33m 20s
Longitude 100° 50′ E., in time =	— 6. 43. 20
Greenwich time = ...	6h 50m 0s
Sun's right ascension at noon, Jan. 30th, =	20h 51m 25s
Correction of ditto for 6h 50m	+ 1. 10
Sun's reduced right ascension = ...	20h 52m 35s
Moon's right ascension at noon, Jan. 30th, =	76° 21′ 55″
Corrected prop. part of ditto for 6h 50m =	+ 4. 13. 38
Moon's corrected right ascension = ...	80° 35′ 33″
Moon's semi-diameter at noon, January 30th, =	15′ 46″
Correction of ditto for 6h 50m = ...	+ 5
Augmentation, Table IV., = ...	+ 2
Moon's true semi-diameter = ...	15′ 53″
Moon's declination at noon, January 30th, =	23° 57′ 46″ N.
Corrected prop. part of ditto for 6h 50m =	— 3. 44
Moon's corrected declination = ...	23° 54′ 2″ N.
Moon's horizontal parallax at noon, Jan. 30th, =	57′ 51″
Correction of ditto for 6h 50m = ...	+ 17
Moon's true horizontal parallax = ...	58′ 8″

Observed altitude of the moon's lower limb = 7°23′30″; hence, her true central altitude is 8°25′54″.

Latitude = . .	10°20′ 0″S.	. . . .	Log. secant=0.007102
Moon's corr. dec.=	23.54. 2. N.	. . . .	Log. secant=0.038935
Moon's m. z. dist.=	34°14′ 2″	Nat. co-sine= 826748	
Moon's true alt. =	8.25.54	Nat. sine = 146630	
		Remainder = 680118	Log.=5.832584

Moon's horary dist. west of the merid.= 5ʰ 3ᵐ33ˢ Log. rising=5.878621
Moon's cor. R. A. 80°35′33″, in time=5.22.22

Right ascension of the meridian =	10ʰ25ᵐ55ˢ
Sun's reduced right ascension = .	20.52.35
Apparent time of observation = .	13ʰ33ᵐ20ˢ

Remark.—If there be a chronometer on board to indicate the time at Greenwich, the apparent time of observation, at any given place, may be very correctly ascertained by the above problem. But, since the chronometer shows the equable or mean time at Greenwich, this time must be reduced to apparent time, by applying the equation of time thereto with a contrary sign to that expressed in the Nautical Almanac. Thus, in the above example, if the chronometer give the mean time at Greenwich = 7ʰ3ᵐ44ˢ, then the reduced equation of time, viz., 13ᵐ44ˢ, being subtracted therefrom, shows the apparent time at that meridian to be 6ʰ50ᵐ0ˢ. Hence, when the equation of time in the Nautical Almanac is marked *additive*, it is to be applied by subtraction; but when marked *subtractive*, it is to be applied by addition to the mean time (per chronometer) at Greenwich, in order to reduce it to apparent time.

SOLUTION OF PROBLEMS RELATIVE TO FINDING THE ALTITUDES OF THE HEAVENLY BODIES.

It sometimes happens at sea, particularly in taking a *lunar observation*, that the horizon is so ill-defined as to render it impossible to observe the altitudes of the objects to a sufficient degree of exactness; or, perhaps, that one or both of the objects are directly over the land, at the time of measuring the lunar distance, and the ship so contiguous thereto as to render the absolute value of the horizontal dip uncertain: in such cases, therefore, the altitudes of the objects must be obtained by computation, as in the following problems; the principles of which will be found amply illustrated in "The Young Navigator's Guide to the Sidereal and Planetary Parts of Nautical Astronomy," page 237.

Problem I.

Given the Latitude and Longitude of a Place, and the Apparent Time at that Place; to find the true and the apparent Altitude of the Sun's Centre.

Rule.

Reduce the given apparent time to the meridian of Greenwich, by Problem III., page 297; to which let the sun's declination be reduced, by Problem V., page 298.

If the latitude of the place and the sun's declination are of different names, let their *sum* be taken; otherwise, their *difference:* and the meridional zenith distance of that object will be obtained. Then,

To the logarithmic rising answering to the sun's distance from the meridian, (that is, the interval between the given apparent time and noon,) add the logarithmic co-sines of the latitude of the place and of the sun's reduced declination: the sum, rejecting 20 from the index, will be the logarithm of a natural number; which, being added to the natural versed sine of the sun's meridian zenith distance, found as above, will give the natural co-versed sine of its true altitude.

To the sun's true altitude, thus found, let the correction corresponding thereto in Table XIX., be added; and the sum will be the apparent altitude of the sun's centre.

Example 1.

Required the true and apparent altitude of the sun's centre, January 10th, 1825, at $3^h 1^m 45^s$ apparent time, in latitude 40°30′ N., and longitude 59°2′30″ W.?

Apparent time at ship or place = . . .	$3^h\ 1^m 45^s$
Longitude 59°2′30″ W., in time = .	+ 3.56.10
Greenwich time = ...	$6^h 57^m 55^s$

Sun's declination at noon, January 10th, = 21°57′50″ S.
Correction of ditto for 6h57m55s = . — 2.40

Sun's reduced declination = . . . 21°55′10″ S.

Sun's hor. mer. dist. = 3h 1m45s . . . . . Log. rising = . 5.474670
Sun's reduced dec. = 21.55.10 S. . . . . . Log. co-sine = 9.967412
Lat. of the place = 40.30. 0 N. . . . . . Log. co-sine = 9.881046

Sun's mer. z. dist. = 62°25′10″ Nat. vers. sine = 537005
Nat. number = 210440 Log. = 5.323128

True alt. of sun's cen. = 14°37′43″ Nat. co-V.S. = 747445
Correc., Table XIX. = + 3.26

App. alt. of sun's cen. = 14°41′ 9″

Example 2.

Required the true and apparent altitude of the sun's centre, January 20th, 1825, at 19h16m18s apparent time, in latitude 37°20′ S., and longitude 49°45′ E.?

Apparent time at ship or place = . . 19h16m18s
Longitude 49°45′ E., in time = . . — 3.19. 0

Greenwich time = ... 15h57m18s

Sun's declination, January 20th, = . 20° 7′11″ S.
Correction of ditto for 15h57m18s = — 8.48

Sun's reduced declination = . . . 19°58′23″ S.

Sun's hor. dist. fr. mer. = 4h43m42s* . . . Log. rising = 5.828140
Sun's reduced dec. = 19.58.23 S. . . . Log. co-sine = 9.973060
Latitude of the place = 37.20. 0 S. . . . Log. co-sine = 9.900433
Sun's mer. zen. dist. = 17°21′37″ Nat. v. sine = 045552
Nat. num. = 503075 Log. = 5.701633

True alt. of sun's cent. = 26°49′55″ Nat. co-V.S = 548627
Reduc. of do., Tab. XIX., = + 1.44

App. alt. of sun's centre = 26°51′39″

* 24 hours — 19h16m18s = 4h43m42s, the sun's horary distance from the meridian.

Problem II.

Given the apparent Time at a known Place, to find the true and apparent Altitude of a fixed Star.

Rule.

Reduce the given apparent time to the meridian of Greenwich, by Problem III., page 297; to which let the sun's right ascension, at noon of the given day, be reduced, by Problem V., page 298.

Let the star's right ascension and declination (Table XLIV.) be reduced to the given period, by the method shown in page 115. To the sun's reduced right ascension let the given apparent time be added, and the sum will be the right ascension of the meridian; the difference between which and the star's reduced right ascension will be the horary distance of the latter from the meridian. Now, with the star's horary distance from the meridian, thus found, its reduced declination, and the latitude of the place, compute the true altitude of that object, by the last problem. Then, to the star's true altitude, thus found, let the correction corresponding thereto, in Table XIX., be added; and the sum will be the star's apparent altitude.

Example 1.

Required the true and apparent altitude of α Arietis, January 1st, 1825, at $11^h 9^m 29^s$ apparent time, in latitude 40°29′ N., and longitude 59°45′ W.?

Apparent time at ship or place = . . .	$11^h\ 9^m 29^s$
Longitude 59°45′ W., in time = .	+ 3. 59. 0
Greenwich time = ...	$15^h\ 8^m 29^s$

Sun's right ascension at noon, January 1st, =	$18^h 47^m 19^s$
Correction of ditto for $15^h 8^m 29^s$ = . . .	+ 2. 47
Sun's reduced right ascension = ...	$18^h 50^m\ 6^s$
Given apparent time = ...	11. 9. 29
Right ascension of the meridian = ...	$5^h 59^m 35^s$

Star's reduced declination = ... 22°37′50″N.

Star's reduced R. A. = 1h 57m 19s
R. A. of the merid. = 5. 59. 35

Star's hor. dis fr. mer. = 4h 2m 16s . . . . . Log. rising = 5.706360
Star's reduced dec. = 22° 37′ 50″ N. . . . Log. co-sine = 9.965204
Lat. of the place = 40. 29. 0 N. . . . . Log. co-sine = 9.881153

Star's mer. zen. dist. = 17° 51′ 10″ Nat. vers. sine = 048153
Nat. number = 357040 Log. = 5.552717

True alt. of the star = 36° 29′ 56″ Nat. co-V. S. = 405193
Reduc. of do. Tab. XIX = + 1. 17

App. alt. of giv. star = 36° 31′ 13″

Example 2.

Required the true and apparent altitude of Procyon, January 1st, 1825, at 9h 31m 39s apparent time, in latitude 39° 20′ 30″ S., and longitude 75° 40′ E.?

Apparent time at ship or place = ... 9h 31m 39s
Longitude 75° 40′ E., in time = ... 5. 2. 40

Greenwich time = ... 4h 28m 59s

Sun's right ascension at noon, January 1st, = 18h 47m 19s
Correction of ditto for 4h 28m 59s = . . . + 0. 49

Sun's reduced right ascension = ... 18h 48m 8s
Given apparent time = ... 9. 31. 39

Right ascension of the meridian = ... 4h 19m 47s

Procyon's reduced declination = ... 5° 39′ 58″ N.

Procyon's red. R. A = 7h 30m 8s
R. A. of the meridian = 4. 19. 47

Star's hor. dis. fr. mer. = 3h 10m 21s . . . . . Log. rising = 5.512600
Star's reduced dec. = 5° 39′ 58″ N. . . . Log. co-sine = 9.997872
Latitude of the place = 39. 20. 30 S. . . . Log. co-sine = 9.888393

Star's mer. z. dist. = 45° 0′ 28″ Nat. vers. sine = 292990
Nat. number = 250533 Log. = 5.398865

True alt. of giv. star = 27° 9′ 36″ Nat. co-V. S. = 543523
Reduc. of do. Tab. XIX. = + 1. 50

App. alt. of giv. star = 27° 11′ 26″

Problem III.

Given the Latitude and Longitude of a Place, and the apparent Time at that Place; to find the true and apparent Altitude of a Planet.

Rule.

Reduce the given apparent time to the meridian of Greenwich, by Problem III., page 297; to which time let the sun's right ascension be reduced, by Problem V., page 298; and let the planet's right ascension and declination be reduced to the same time, by Problem VII., page 307.

To the sun's reduced right ascension let the given apparent time be added, and the sum will be the right ascension of the meridian; the difference between which and the planet's reduced right ascension will be the horary distance of the latter from the meridian. Now, with the planet's horary distance from the meridian, thus found, its reduced declination, and the latitude of the place, compute the true altitude of that object, by Problem I., page 404. Then, with the planet's true altitude, thus found, by computation, enter Table XIX., and take out the quantity corresponding to the *reduction of a star's true altitude;* the difference between which and the planet's parallax in altitude, Table VI., will leave a correction, which, being added to the *true,* will give the *apparent* altitude of the planet.

Example 1.

Required the true and apparent altitude of the planet Jupiter, January 3d, 1825, at $9^h 1^m 23^s$ apparent time, in latitude 50°30′ N., and longitude 48°45′ W.?

Given apparent time at ship or place = . .	$9^h\ 1^m 23^s$
Longitude 48°45′ W., in time = ...	+ 3. 15. 0
Greenwich time = ...	$12^h 16^m 23^s$
Sun's right ascension at noon, January 3d, =	$18^h 56^m\ 8^s$
Correction of ditto for $12^h 16^m 23^s$ = . . .	+ 1.39
Sun's reduced right ascension = ...	$18^h 57^m 47^s$
Given apparent time = ...	9. 1.23
Right ascension of the meridian = ...	$3^h 59^m 10^s$

Jupiter's declination at noon, January 1st, = 17°56′ 0″N.
Correction of ditto for $2^d 12^h 16^m 23^s$ = . . + 5. 1

Jupiter's reduced declination = ... 18° 1′ 1″N.

Jup.'s R.A. at noon, Jan. 1 = $8^h 58^m\ 0^s$
Cor. of do. for $2^d 12^h 16^m 23^s$ = − 0. 50

Jupiter's reduced R. A. = $8^h 57^m 10^s$
R. A. of the meridian = 3. 59. 10

Planet's hor. dist. fr. mer. = $4^h 58^m\ 0^s$ . . . Log. rising = 5. 864960
Planet's reduced dec. = 18. 1. 1 N. . . Log. co-sine = 9. 978164
Latitude of the place = 50. 30. 0 N. . . Log. co-sine = 9. 803511

Planet's mer. zen. dist. = 32°28′59″ Nat.V. S. = 156449
Nat. num. = 443236 Log. = 5. 646635

Jupiter's true central alt. = 23°35′52″ N.co-V.S. = 599685
Red. Tab. XIX. 2′ 11″ } Diff. = + 2′ 9″
Par. Tab. VI. = 0. 2 }

Jupiter's app. central alt. = 23°38′ 1″

Note.—Jupiter's horizontal parallax is assumed, in *the present instance*, at 2 seconds of a degree.

Example 2.

Required the true and apparent altitude of the planet Venus, January 16th, 1825, at $7^h 20^m 45^s$ apparent time, in latitude 34°45′ S., and longitude 80°30′ E., admitting her horizontal parallax, at that time, to be 18 seconds?

Apparent time at ship or place = ... $7^h 20^m 45^s$
Longitude 80°30′ E., in time = ... 5. 22. 0

Greenwich time = ... $1^h 58^m 45^s$

Sun's right ascension at noon, January 16th, = $19^h 52^m 41^s$
Correction of ditto for $1^h 58^m 45^s$ = . . . − 0. 21

Sun's reduced right ascension = ... $19^h 53^m\ 2^s$
Given apparent time = ... 7. 20. 45

Right ascension of the meridian = . . . $3^h 13^m 47^s$

Venus' declination, January 13th, = ...	11°39′ 0″S.
Correction of ditto for 3d 1h 58m 45s . . .	– 1. 29. 24
Venus' reduced declination = ...	10° 9′36″ S.

Venus' R. A., Jan. 13th, =	22h 23m 0s		
Cor. of do. for 3d 1h 58m 45s =	+ 13. 52		
Venus' reduced R. A. =	22h 36m 52s		
R. A. of the meridian =	3. 13. 47		
Planet's hor. dist. fr. mer. =	4h 36m 55s . . .	Log. rising =	5. 809810
Planet's reduced dec. =	10° 9′36″ S. . .	Log. co-sine =	9. 993136
Latitude of the place =	34. 45. 0 S. . .	Log. co-sine =	9. 914685
Planet's mer. zen. dist. =	24°35′24″ Nat. V. S. = 090691		
	Nat. num. = 521953	Log. =	5. 717631
Venus' true central alt. =	22°47′24″ N. co-V. S = 612644		
Red. Tab. XIX. 2′15″ / Par. Tab. VI. = 0. 16	Diff. = + 1. 59		
Venus' app. central alt. =	22°49′23″		

Remark.—In these problems, a cipher is annexed to the logarithmic rising taken from Table XXXII.: this is done with the view of reducing it to six places of decimals; so that there may be no mistake in properly applying thereto the logarithmic co-sines of the latitude and of the declination.

Problem IV.

Given the Latitude of a Place, and the apparent Time at that Place, with the Longitude; to find the true and apparent Altitude of the Moon's Centre.

Rule.

Reduce the given apparent time to the meridian of Greenwich, by Problem III., page 297; to which let the sun's right ascension be reduced, by Problem V., page 298; and let the moon's right ascension, declination, and horizontal parallax be reduced to the same time, by Problem VI., page 302.

To the sun's reduced right ascension let the given apparent time be added, and the sum will be the right ascension of the meridian; the difference between which and the moon's reduced right ascension, will be the horary distance of the latter from the meridian.

Now, with the moon's horary distance from the meridian, her corrected declination, and the latitude of the place, compute her true central altitude, by Problem I., page 404. Then,

From the moon's true central altitude, thus found, subtract the correction corresponding thereto and her reduced horizontal parallax, in Table XIX., and the remainder will be the apparent central altitude.

Note.—The moon's right ascension and declination must be corrected by the equation of second difference contained in Table XVII., as explained between pages 33 and 37.

Example 1.

Required the true and apparent altitude of the moon's centre, January 4th, 1825, at 7ʰ 28ᵐ 18ˢ apparent time, in latitude 50° 10′ N., and longitude 60° W.?

Apparent time at ship or place = ...	7ʰ 28ᵐ 18ˢ
Longitude 60° W., in time = ...	4. 0. 0
Greenwich time = ...	11ʰ 28ᵐ 18ˢ
Sun's right ascension at noon, January 4th, =	19ʰ 0ᵐ 32ˢ
Correction of ditto for 11ʰ 28ᵐ 18ˢ = . . .	+ 2. 6
Sun's reduced right ascension = ...	19ʰ 2ᵐ 38ˢ
Given apparent time = ...	7. 28. 18
Right ascension of the meridian = ...	2ʰ 30ᵐ 56ˢ
Moon's horizontal parallax at noon, January 4th, =	59′ 17″
Correction of ditto for 11ʰ 28ᵐ 18ˢ = ...	+ 16
Moon's true horizontal parallax = ...	59′ 33″
Moon's declination at noon, January 4th, =	22° 35′ 39″ N.
Corrected prop. part of ditto for 11ʰ 28ᵐ 18ˢ =	− 1. 10. 8
Moon's corrected declination = ...	21° 25′ 31″ N.
Moon's right ascension at noon, January 4th, =	98° 6′ 53″
Corrected prop. part of ditto for 11ʰ 28ᵐ 18ˢ =	+ 7. 17. 28
Moon's corrected R. A. = 7ʰ 1ᵐ 37ˢ = ...	105° 24′ 21″

Moon's corrected R. A.=7h 1m37s
R. A. of the meridian = 2. 30. 56

☽'s horary dis. fr. mer. = 4h30m41s . . . Log. rising = 5.792450
Moon's corrected dec.=21°25′31″ N. . . Log. co-sine = 9.968909
Latitude of the place = 50. 10. 0 N. . . Log. co-sine = 9.806558

Moon's mer. zen. dist.=28°44′39″ Nat.V. S. = 123225
Nat. num. = 369757 Log.=5.567917

True alt. of ☽'s centre=30°27′55″ Nat. co-V. S.=492982
Reduc. of do. Tab. XIX.= −50. 8

App. alt. of ☽'s cent.= 29°37′47″

Example 2.

Required the true and apparent altitude of the moon's centre, January 30th, 1825, at 13h33m20s apparent time, in latitude 10°20′ S., and longitude 100°50′ E.?

Apparent time at ship or place = ... 13h33m20s
Longitude 100°50′ E., in time = . . . − 6. 43. 20

Greenwich time = ... 6h50m 0s

Sun's right ascension at noon, January 30th, = 20h51m25s
Correction of ditto for 6h50m = ... + 1. 10

Sun's reduced right ascension = ... 20h52m35s
Given apparent time = ... 13. 33. 20

Right ascension of the meridian = ... 10h25m55s

Moon's horizontal parallax at noon, January 30th,=57′51″
Correction of ditto for 6h50m = ... + 17

Moon's true horizontal parallax = ... 58′ 8″

Moon's declination at noon, January 30th, = 23°57′46″N.
Corrected prop. part of ditto for 6h50m = . . − 3. 44

Moon's corrected declination = ... 23°54′ 2″N.

Moon's right ascension at noon, January 30th,=76°21′55″
Corrected prop. part of ditto for 6h50m = + 4. 13. 38

Moon's corrected R. A.=5h22m22s = ... 80°35′33″

Moon's corrected R. A. = $5^h 22^m 22^s$
R. A. of the meridian = 10. 25. 55

☽'s hor. dist. fr. merid. = $5^h\ 3^m 33^s$ . . . Log. rising = 5.878620
Moon's corrected dec. = 23°54′ 2″ N. . . Log. co-sine = 9.961065
Latitude of the place = 10. 20. 0 S. . . Log. co-sine = 9.992898

☽'s mer. zen. distance = 34°14′ 2″ Nat. V. S. = 173252
Nat. num. = 680116 Log. = 5.832583

True alt. of ☽'s centre = 8°25′54″ Nat. co-V.S. = 853368
Reduc. of do. Tab. XIX. = −50. 47

App. alt. of ☽'s centre = 7°35′ 7″

Remark.—The natural sines may be used in the solution of the four preceding problems, instead of the versed sines: in this case, if the natural number be subtracted from the natural co-sine of the object's meridional zenith distance, the natural sine of its true altitude will be obtained. Thus, in the last example, the moon's meridian zenith distance is 34°14′2″. Now, the natural co-sine of this is 826748; from which let the natural number 680116 be subtracted, and the remainder = 146632 is the natural sine of that object's true altitude; the arch corresponding to which is 8°25′54″.

These problems are, evidently, the converse of those for finding the apparent time, as given in pages 383, 394, 397, and 400.

SOLUTION OF PROBLEMS RELATIVE TO THE LONGITUDE.

The *Longitude* of a given place on the earth, is that arc or portion of the equator which is intercepted between the first or principal meridian and the meridian of the given place; and is denominated east or west, according as it may be situate with respect to the first meridian.

The *first or principal meridian* is an imaginary great circle passing through any remarkable place and the poles of the world: hence it is entirely arbitrary; and, therefore, the British reckon their first meridian to be that which passes through the Royal Observatory at Greenwich; the French esteem their first meridian to be that which passes through the Royal Observatory at Paris; the Spaniards, that which passes through Cadiz, &c. &c. &c. Every part of the terrestrial sphere may be conceived to have a meridian line passing through it, cutting the equator at right angles: hence there may be as many different meridians as there are points in the equator.

Every meridian line, with respect to the place through which it passes, may be said to divide the surface of the earth into two equal parts, called the eastern and western hemispheres. Thus, when the face of an observer is turned towards the north pole of the world, the hemisphere which lies on his right hand is called east, and that on his left hand west; and, *vice versa*, when the face is directed towards the south pole of the world, the hemisphere which lies on the left hand is called east, and that on the right hand west.

The longitude is reckoned both ways from the first meridian, east and west, till it meets with the same meridian on the opposite part of the equator: hence the longitude of any place on the earth can never exceed 180 degrees. The difference of longitude between two places on the earth is an arc of the equator contained between the meridians of those places, showing how far one of them is to the eastward or westward of the other, and can never exceed 180 degrees, or half the earth's circumference.

All places that are situated under the same meridian have the same longitude; but places which lie under different meridians have different longitudes: hence, in sailing due north or due south, since a ship does not change her meridian, she keeps in the same parallel of longitude; but, in sailing due east or due west, she constantly changes her meridian, and therefore passes through a variety of longitudes.

When the meridian of any place is brought, by the diurnal revolution of the earth round its axis, to point directly to the sun, it is then noon or mid-day at that place.

The motion of the earth on its axis is, at all times, equable and uniform; and, since it turns round its axis *eastward* once in every 24 hours, all parts of the equator, or great circle of 360 degrees, will pass by the sun, or star, in equal portions of time: therefore the twenty-fourth part of the equator, viz., 15 degrees, will pass by the sun in one hour of time: for, $24^h \times 15^\circ$ or 1 hour, $=$ 360 degrees; and, conversely, 360 degrees $\div$ 24 hours $=$ 15 degrees or 1 hour.

Every place on the earth, whose meridian is 15 degrees east of the Royal Observatory at Greenwich, will have noon and every other hour *one hour sooner* than at the meridian of that observatory; if the meridian be 30 degrees east of Greenwich, it will have noon and every other hour *two hours sooner* than at the meridian of that place, and so on; the time always differing at the rate of 1 hour for every 15 degrees of longitude, 1 minute of time for every 15 minutes of longitude, and 1 second of time for every 15 seconds of longitude. Again, every place whose meridian is 15 degrees west of the Royal Observatory at Greenwich will have noon and every other hour *one hour later* than at the meridian of that observatory; if the meridian be 30 degrees to the westward of Greenwich, it will have noon and every other hour *two hours later* than at the meridian of that place, and so on. Hence it is evident, that if the time at the meridian of a ship or place be *greater* than the time, at the same instant, at the meridian of Greenwich, such ship or place will be to the *eastward* of Greenwich; but if the time at a ship or place be *less* than the time, at the same instant, at Greenwich, such ship or place will be to the *westward* of Greenwich.

Since the longitude of any place on the earth is expressed by the difference of time between that place and the Royal Observatory at Greenwich; therefore, to determine the longitude of a given place, we have only to find the time of the day at that place, and also at Greenwich, at the same instant; then, the difference of these times being converted into motion, by allowing 15 degrees for every hour, &c., or, more readily, by Table I. in this work, the longitude of such given place will be obtained.

The readiest, and, indeed, the most simple method of finding the longitude at sea, *in theory*, is by a chronometer, or other machine, that will measure time so exactly true as to go uniformly correct in all places, seasons, and climates: for, such a machine being once regulated to the meridian of the Royal Observatory at Greenwich, would always show the true time under that meridian, though removed in a ship to the most distant parts of the globe,—even to the utmost extent of longitude.

Although such a perfect piece of mechanism can scarcely be hoped for or expected to result from the ablest and best applied course of human industry,—yet, on the supposition that the chronometers used at sea are sufficiently correct for the measurement of time in *short voyages*, we will now proceed to show how the longitude is to be found by means of those instruments.

Problem I.

To convert apparent Time into mean Time.

Rule.

Reduce the equation of time, as given in page II. of the month in the Nautical Almanac, to the time and place of observation, by Problem V., page 298; then, let this reduced equation be applied to the given apparent time of observation, by addition or subtraction, according to the sign expressed against it in the Ephemeris, and the sum or difference will be the corresponding mean time.

Example 1.

January 24th, 1825, in longitude 75° W., the apparent time of observation was $3^h 40^m 10^s$; required the mean time?

Equation of time at noon, January 24th, =	$+\ 12^m 29^s.5$
Correction of ditto for $8^h 40^m 10^s$ = ...	$+\ 5\ .1$
Reduced equation of time = ...	$+\ 12^m 34^s.6$
Apparent time of observation = ...	$3^h 40^m 10^s.0$
Mean time, as required, = ...	$3^h 52^m 44^s.6$

Example 2.

October 6th, 1825, in longitude 80° E., the apparent time of observation was $20^h 10^m 40^s$; required the mean time?

Equation of time at noon, October 6th, =	$-\ 11^m 49^s.5$
Correction of ditto for $14^h 50^m 40^s$	$+\ 10\ .5$
Reduced equation of time = ...	$-\ 12^m\ 0^s.0$
Apparent time of observation	$20^h 10^m 40^s.0$
Mean time, as required, = ...	$19^h 58^m 40^s.0$

Problem II.

To convert mean Time, at Greenwich, into apparent Time.

Rule.

Reduce the equation of time, page II. of the month in the Nautical Almanac, to the given mean time at Greenwich, by Problem V., page 298; then, let this reduced equation be applied to the mean time, with a contrary sign to that which is expressed against it in the Ephemeris; that is, by addition when the sign is negative, but by subtraction when affirmative; and the corresponding apparent time will be obtained.

Example 1.

January 1st, 1825, the mean time at Greenwich, per chronometer, was $10^h 13^m 45^s$; required the apparent time?

Equation of time at noon, January 1st, =	$+\ 3^m 56^s.7$
Correction of ditto for $10^h 13^m 45^s$ = . . .	$+\ 12\ .0$
Reduced equation of time = ...	$-\ 4^m\ 8^s.7$
Mean time at Greenwich = ...	$10^h 13^m 45^s.0$
Apparent time at Greenwich = ...	$10^h\ 9^m 36^s.3$

Example 2.

September 19th, 1825, the mean time at Greenwich, per chronometer, was $18^h 45^m 30^s$; required the apparent time?

Equation of time at noon, Sept. 19th, =	$-6^m 14^s.3$
Correction of ditto for $18^h 45^m 30^s$ = . .	$+16.4$
Reduced equation of time = ...	$+6^m 30^s.7$
Mean time at Greenwich = ...	$18^h 45^m 30^s.0$
Apparent time at Greenwich = . . .	$18^h 52^m 0^s.7$

PROBLEM III.

Given the Latitude of a Place, the observed Altitude of the Sun's Limb, and its Declination; to find the Longitude of that Place by a Chronometer or Time-Keeper.

RULE.

Let several altitudes of the sun's limb be observed, at a proper distance from the meridian,* and the corresponding times, per chronometer, noted down; of these take the means respectively.

Let the mean altitude of the sun's limb be reduced to the true central altitude, by Problem XIV., page 320.

To the mean of the times of observation apply the original error of the chronometer, by addition or subtraction, according as it was slow or fast for mean time at the meridian of Greenwich, when its rate was established; to which let its *accumulated rate* be applied affirmatively or negatively, according as the machine may be losing or gaining, and the result will be the mean time of observation at Greenwich, which is to be converted into apparent time, by Problem II., page 416.

To the apparent time at Greenwich, thus found, let the sun's declination be reduced, by Problem V., page 298. Then, with the sun's true central altitude, reduced declination, and the latitude of the place, compute the apparent time of observation, by any of the methods given in Problem III., page 383; the difference between which and the apparent time at Greenwich will be the longitude of the place of observation in time;—east, if the former time be greater than the latter; otherwise, west.

* See remarks on the most favourable times for observation, page 387.

Note.—If the meridian of the place where the error of the chronometer was determined be different from that of Greenwich, let its longitude in time be applied to the mean time of observation, per chronometer, by addition or subtraction, according as it is west or east, and the mean time of observation at Greenwich will be obtained.

Example 1.

April 7th, 1825, in latitude 48°43′ N., the mean of several altitudes of the sun's lower limb was 9°11′42″, and that of the corresponding times $9^h 37^m 55^s$, by a chronometer, the error and rate of which had been established at noon, January 1st, when it was found $4^m 37^s$ fast for mean time at Greenwich, and gaining $1^s.75$ daily; the height of the eye above the level of the horizon was 20 feet; required the longitude of the place of observation?

Mean time of observation at Greenwich =	$9^h 37^m 55^s$
Original error of the chronometer = . .	− 4. 37
Accumulated rate = $1^s.75 \times 96$ days = .	− 2. 48
Mean time at Greenwich = ...	$9^h 30^m 30^s$
Reduced equation of time = ...	− 2. 5
Apparent time of observation at Greenw. =	$9^h 28^m 25^s$

Sun's declination at noon, April 7th = .	6°49′38″N.
Correction of ditto for $9^h 28^m 25^s$ = . .	+ 8. 53
Sun's reduced declination = ...	6°58′31″N.

Obs. alt. of the sun's lr. limb = 9°11′42″; hence, its true cent. alt. is 9°18′0″

Lat. of the place = 48°43′ 0″N. . . .	Log. secant = 0.180599	
Sun's reduced dec. = 6. 58. 31 N. . . .	Log. secant = 0.003226	
Sun's mer. z. dist. = 41°44′29″	Nat. vers. S. = 253843	
True alt. of sun's cen. = 9. 18. 0	Nat. co-V.S. = 838396	
	Remainder = 584553	Log. = 5.766824
Apparent time at the place of observation = $5^h 35^m 20^s$		Log. ris. = 5.950649
Apparent time of observation at Greenw. = 9. 28. 25		
Longitude of the place of obs., in time = $3^h 53^m 5^s$ = 58°16′5″ west.		

Example 2.

May 1st, 1825, in latitude $30^\circ 15'$ S., the mean of several altitudes of the sun's lower limb was $11^\circ 17' 14''$, and that of the corresponding times $13^h 23^m 10^s$, by a chronometer, the error and rate of which were established at noon, February 1st, when it was found $3^m 25^s$ slow for mean time at Greenwich, and losing $0^s.97$ daily; the error of the sextant was $2' 30''$ subtractive, and the height of the eye 23 feet; required the longitude?

Mean time of observation at Greenwich = .	$13^h 23^m 10^s$
Original error of the chronometer = . . .	+ 3. 25
Accumulated rate = $0^s.97 \times 89\frac{1}{2}$ days = .	+ 1. 27
Mean time at Greenwich = ...	$13^h 28^m\ 2^s$
Reduced equation of time = ...	+ 3. 8
Apparent time of observation at Greenwich =	$13^h 31^m 10^s$
Sun's declination at noon, May 1st, = . . .	$15^\circ\ 4' 19''$ N.
Correction of ditto for $13^h 31^m 10^s$ = ...	+ 10. 9
Sun's reduced declination = ...	$15^\circ 14' 28''$ N.

Obs. alt. of the sun's l. limb $= 11^\circ 17' 14''$; hence, its true cent. alt. is $11^\circ 24' 5''$.

Lat. of the place =	$30^\circ 15'\ 0''$ S. . . .	Log. secant =	0.063569
Sun's reduced dec. =	15. 14. 28 N. . . .	Log. secant =	0.015550
Sun's mer. z. dist. =	$45^\circ 29' 28''$	Nat. co-sine = 701020	
True alt. of sun's cen. =	11, 24. 5	Nat. sine = 197681	
		Remainder = 503339 Log. =	5.701860

Sun's horary dist. from the merid. or noon $= 4^h 26^m 40^s$ Log. ris. = 5.780979

Apparent time at the place of observation $= 19^h 33^m 20^s$
Apparent time of observation at Greenw. = 13. 31. 10

Long. of the place of observ., in time $= 6^h\ 2^m 10^s = 90^\circ 32' 30''$ east.

Problem IV.

Given the Latitude of a Place, and the observed Altitude of a known fixed Star; to find the Longitude of the Place of Observation, by a Chronometer or Time-Keeper.

Rule.

Let several altitudes of the star be observed, at a proper distance from the meridian,* and the corresponding times, per chronometer, noted down; of these, take the means respectively.

Let the mean altitude of the star be reduced to the true altitude, by Problem XVII., page 327.

To the mean of the times of observation apply the original error of the chronometer, by addition or subtraction, according as it was slow or fast for mean time at the meridian of Greenwich when its rate was established; to which let its *accumulated rate* be applied affirmatively or negatively, according as the machine may be losing or gaining, and the result will be the mean time of observation at Greenwich; which is to be converted into apparent time, by Problem II., page 416.

To the apparent time at Greenwich let the sun's right ascension be reduced, by Problem V., page 298; and let the star's right ascension and declination, as given in Table XLIV., be reduced to the period of observation. Then, with the star's true altitude, its declination, and the latitude of the place, compute its horary distance from the meridian, by any of the methods given in Problem III., page 383.

Now, if the star be observed in the western hemisphere, its horary distance from the meridian, thus found, is to be added to its reduced right ascension; but if in the eastern hemisphere, subtracted from it: the sum, or remainder, will be the right ascension of the meridian; from which, (increased by 24 hours, if necessary,) subtract the sun's reduced right ascension, and the remainder will be the apparent time at the place of observation; the difference between which and the apparent time at Greenwich will be the longitude of the place of observation in time:—east, if the computed apparent time be the greatest; if otherwise, west.

Example 1.

January 29th, 1825, in latitude 40°30′ N., the mean of several altitudes of the star Aldebaran, west of the meridian, was 24°57′0″, and that of the

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* See Note, page 417.

corresponding times $16^h56^m3^s$, by a chronometer, the error and rate of which were determined at noon, January 1st, when it was found 7^m29^s fast for mean time at Greenwich, and losing $1^s.53$ daily; the error of the sextant was $3'10''$ additive, and the height of the eye above the level of the horizon 22 feet; required the longitude?

Mean time of observation at Greenwich = .	$16^h56^m\ 3^s$
Original error of the chronometer = . . .	− 7.29
Accumulated rate = $1^s.53 \times 28\frac{1}{2}$ days = .	+ 0.44
Mean time at Greenwich = ...	$16^h49^m18^s$
Reduced equation of time = ...	− 13.38
Apparent time of observation at Greenwich =	$16^h35^m40^s$
Sun's right ascension at noon, January 29th, =	$20^h47^m19^s$
Correction of ditto for $16^h35^m40^s$ = . . .	+ 2.50
Sun's reduced right ascension = ...	$20^h50^m\ 9^s$
Aldebaran's reduced right ascension = . .	$4^h25^m54^s$
Aldebaran's reduced declination = ...	$16^\circ\ 8'57''$N.
Aldebaran's north polar distance = ...	$73^\circ51'\ 3''$

Observed altitude of Aldebaran =

$24^\circ57'0''$; true altitude = .	$24^\circ53'38''$		
Aldebaran's north polar distance=	73. 51. 3	Log. co-secant=	0. 017484
Lat. of the place of observation=	40. 30. 0	Log. secant =	0. 118954
Sum = ...	$139^\circ14'41''$	Constant log. =	6. 301030
Half sum = ...	$69^\circ37'20\frac{1}{2}''$	Log. co-sine =	9. 541836
Remainder = ...	44. 43. $42\frac{1}{2}$	Log. sine = .	9. 847417
Star's horary distance, west of the mer.=	$4^h43^m10^s$	Log.rising=	5. 82672.1
Star's reduced right ascension = . .	4. 25. 54		
Right ascension of the meridian . .	$9^h\ 9^m\ 4^s$*		
Sun's reduced right ascension = . .	20. 50. 9		
Apparent time at the place of observ.=	$12^h18^m55^s$		
Apparent time of observ. at Greenwich=	16. 35. 40		

Longitude of the place of obs., in time= $4^h16^m45^s=64^\circ11'15''$ west.

* The right ascension of the meridian is to be considered as being increased by 24 hours, because it is less than the sun's reduced right ascension.

Example 2.

January 29th, 1825, in latitude 39°15′ S., the mean of several altitudes of the star Regulus, east of the meridian, was 10°28′48″, and that of the corresponding times $3^h36^m46^s$, by a chronometer, the error and rate of which had been established at noon, December 1st, 1824, when it was found 4^m37^s slow for mean time at Greenwich, and gaining $1^s.17$ daily; the error of the sextant was 1′34″ subtractive, and the height of the eye above the level of the sea 21 feet; required the longitude of the place of observation?

Mean time of observation at Greenwich = . .	$3^h36^m46^s$
Original error of the chronometer = ...	+ 4.37
Accumulated rate=$1^s.17$ × 59 days = . .	− 1. 9
Mean time at Greenwich = ...	$3^h40^m14^s$
Reduced equation of time = ...	− 13.33
Apparent time of observation at Greenwich =	$3^h26^m41^s$
Sun's right ascension at noon, January 29th, =	$20^h47^m19^s$
Correction of ditto for $3^h26^m41^s$ = . . .	+ 0.35
Sun's reduced right ascension = ...	$20^h47^m54^s$
Star's reduced right ascension = ...	$9^h59^m\ 3^s$
Star's reduced declination = ...	12°49′10″N.
Star's south polar distance = ...	102°49′10″

Observed altitude of Regulus = 10°28′48″; true altitude = .	10°17′46″	
Regulus' south polar distance =	102. 49. 10	Log. co-secant=0.010962
Latitude of the place of observ. =	39. 15. 0	Log. secant = 0.111039
Sum = ...	152°21′56″	Constant log. = 6.301030
Half sum = ...	76°10′58″	Log. co-sine = 9.378080
Remainder = ...	65. 53. 12	Log. sine = . 9.960347
Star's horary distance, east of the merid.=$4^h20^m\ 0^s$		Log. rising=5.76145.8

Star's horary distance, east of the merid. =	4ʰ 20ᵐ 0ˢ
Star's reduced right ascension = . .	9. 59. 3
Right ascension of the meridian = .	5ʰ 39ᵐ 3ˢ
Sun's reduced right ascension = . .	20. 47. 54
Apparent time at the place of observ. =	8ʰ 51ᵐ 9ˢ
Apparent time of observ. at Greenwich =	3. 26. 41
Long. of the place of observ., in time =	5ʰ 24ᵐ 28ˢ = 81° 7′ 0″ east.

Problem V.

Given the Latitude of a Place, and the observed Altitude of a Planet; to find the Longitude of the Place of Observation, by a Chronometer or Time-Keeper.

Rule.

Let several altitudes of the planet be observed, at a proper distance from the meridian,* and the corresponding times, per chronometer, noted down; of these take the means respectively.

Let the mean altitude of the planet be reduced to its true central altitude, by Problem XVI., page 325.

To the mean of the times of observation apply the original error and the accumulated rate of the chronometer, as directed in the last Problem: hence the mean time of observation at Greenwich will be obtained; which is to be converted into apparent time, by Problem II., page 416.

To the apparent time of observation at Greenwich let the sun's right ascension be reduced, by Problem V., page 298; and let the planet's right ascension and declination be reduced to the same time, by Problem VII., page 307. Then, with the latitude of the place, the planet's reduced declination, and its true central altitude, compute its horary distance from the meridian, and, hence, the apparent time at the place of observation, by Problem V., page 397.

Now, the difference between the computed apparent time of observation and the apparent time at Greenwich will be the longitude of the place of observation in time;—east, if the former exceed the latter; otherwise, west.

* See Note, page 417.

Example 1.

February 4th, 1825, in latitude 39°5′ N., the mean of several altitudes of Jupiter's centre, east of the meridian, was 31°25′29″, and that of the corresponding times 12h 6m 47s, by a chronometer, the error and rate of which were determined at noon, January 1st, when it was found 3m 7s fast for mean time at Greenwich, and gaining 0s.71 daily; the error of the sextant was 1′30″ subtractive, and the height of the eye above the level of the horizon 19 feet; required the longitude of the place of observation?

Mean time of observation at Greenwich = .	12h 6m 47s
Original error of the chronometer = ...	− 3. 7
Accumulated rate = 0s.71 × 34 days = .	− 0.24
Mean time at Greenwich = ...	12h 3m 16s
Reduced equation of time = ...	− 14.20
Apparent time of observation at Greenwich =	11h 48m 56s
Sun's right ascension at noon, February 4th, =	21h 11m 45s
Correction of ditto for 11h 48m 56s = ...	+ 1.59
Sun's reduced right ascension = ...	21h 13m 44s
Jupiter's right ascension at noon, February 1st, =	8h 43m 0s
Correction of ditto for 3d 11h 48m 56s = . . .	− 1.45
Jupiter's reduced right ascension = ...	8h 41m 15s
Jupiter's declination at noon, February 1st, =	19° 3′ 0″N.
Correction of ditto for 3d 11h 48m 56s = . .	+ 7.34
Jupiter's reduced declination = ...	19°10′34″N.
Observed central altitude of Jupiter=31°25′29″; hence, the true central altitude of that planet is . . .	31°18′16″
Zenith distance at time of observation = . .	58°41′44″

Lat. of the place = . 39° 5′ 0″N. . . . . Log. secant=0.110010
Planet's red. dec. = . 19.10.34 N. . . . Log. secant=0.024792

Planet's mer. z. dist. = 19°54′26″ . . . . Const. log.=6.301030

Zenith dist. by obs. = 58.41.44

Sum = ... 78°36′10″Half=39°18′ 5″Log.sine=9.801678
Difference = . . . 38.47.18 Half=19.23.39 Log.sine=9.521223

Jupiter's horary dist., east of the merid.=4ʰ19ᵐ 5ˢ Log. rising=5.75873.3
Jupiter's reduced right ascension = . . 8.41.15

Right ascension of the meridian = . 4ʰ22ᵐ10ˢ
Sun's reduced right ascension = . . 21.13.44

Apparent time at the place of observ. = 7ʰ 8ᵐ26ˢ
Apparent time of obs. at Greenwich = 11.48.56

Longitude at the place of obs., in time= 4ʰ40ᵐ30ˢ = 70°7′30″ west.

Example 2.

October 1st, 1825, in latitude 26°40′ S., the mean of several altitudes of Saturn's centre, east of the meridian, was 10°25′40″, and that of the corresponding times 6ʰ36ᵐ24ˢ, by a chronometer, the error and rate of which had been established at noon, August 1st, when it was found 3ᵐ51ˢ slow for mean time at Greenwich, and losing 0ˢ.49 daily; the error of the sextant was 2′20″ subtractive, and the height of the eye above the surface of the sea 18 feet; required the longitude?

Mean time of observation at Greenwich = 6ʰ36ᵐ24ˢ
Original error of the chronometer = . . + 3.51
Accumulated rate = 0ˢ.49 × 61½ days . + 0.30

Mean time at Greenwich = ... 6ʰ40ᵐ45ˢ
Reduced equation of time = ... + 10.24

Apparent time of observation at Greenwich = 6ʰ51ᵐ 9ˢ

Sun's right ascension at noon, Oct. 1st, = 12ʰ29ᵐ21ˢ
Correction of ditto for 6ʰ51ᵐ9ˢ = . . + 1. 2

Sun's reduced right ascension = . . . 12ʰ30ᵐ23ˢ

Saturn's right ascension at Greenwich time = 5ʰ25ᵐ 0ˢ

Saturn's declination at Greenwich time = 21°41′ 0″N.

Observed altitude of Saturn's centre = 10°25′40″; true alt. = 10°14′12″

Zenith distance = ... 79°45′48″

Lat. of the place = . 26°40′ 0″S. . . . Log. secant=0.048841
Saturn's declination = 21.41. 0 N. . . . . Log. secant=0.031872

Saturn's mer. z. dist. = 48°21′ 0″
Zenith dist. by obs. = 79.45.48 . . . . . Const. log.=6.301030

Sum = ... 128° 6′48″ Half=64° 3′24″ Log. sine=9.953869
Difference = ... 31.24.48 Half=15.42.24 Log. sine=9.432508

Saturn's horary dist., east of the merid.=4ʰ22ᵐ15ˢ Log. rising=5.76812.0
Saturn's right ascension = . . . 5.25. 0

Right ascension of the meridian = . 1ʰ 2ᵐ45ˢ
Sun's reduced right ascension = . . . 12.30.23

Apparent time at the place of observ.=13ʰ32ᵐ22ˢ
Apparent time of obs. at Greenwich = 6.51. 9

Longitude of the place of obs., in time=5ʰ41ᵐ13ˢ = 85°18′15″ east.

Problem VI.

Given the Latitude of a Place, and the observed Altitude of the Moon's Limb; to find the Longitude of the Place of Observation, by a Chronometer or Time-Keeper.

Rule.

Let several altitudes of the moon's limb be observed, at a proper distance from the meridian,* and the corresponding times, per chronometer, noted down; of these take the means respectively.

To the mean of the times of observation apply the original error and the accumulated rate of the chronometer, as directed in Problem III., page 417: the result will be the mean time of observation at Greenwich, which is to be converted into apparent time, by Problem II., page 416.

To the apparent time of observation at Greenwich let the sun's right ascension be reduced, by Problem V., page 298; and let the moon's right ascension, declination, semi-diameter, and horizontal parallax be, also, reduced to that time, by Problem VI., page 302. To the moon's reduced semi-diameter apply the augmentation, Table IV., and the true semi-diameter will be obtained.

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* See Note, page 417.

Let the mean altitude of the moon's limb be reduced to the true central altitude, by Problem XV., page 323.

Then, with the latitude of the place, the moon's corrected declination, and her true central altitude, compute her horary distance from the meridian, and, hence, the apparent time at the place of observation, by Problem VI., page 400. The difference between the computed apparent time of observation and that at Greenwich, will be the longitude of the place of observation in time;—and which will be east, if the computed time be the greatest; if otherwise, west.

Example 1.

April 21st, 1825, in latitude 50°48′ N., the mean of several altitudes of the moon's lower limb, west of the meridian, was 29°30′26″, and that of the corresponding times 12h6m58s, by a chronometer, the error and rate of which had been established at noon, February 1st, when it was found 7m46s fast for mean time at Greenwich, and losing 0s.79 daily; the error of the sextant was 2′25″ additive, and the height of the eye above the level of the horizon 17 feet; required the longitude of the place of observation?

Mean time of observation at Greenwich =	12h 6m58s
Original error of the chronometer = . .	− 7.46
Accumulated rate = 0s.79 × 79½ days =	+ 1. 3
Mean time at Greenwich = ...	12h 0m15s
Reduced equation of time = ...	+ 1.27
Apparent time of observ. at Greenwich =	12h 1m42s
Sun's right ascension at noon, April 21st,=	1h55m41s.5
Correction of ditto for 12h1m42s = . .	+ 1.52 .3
Sun's reduced right ascension = . . .	1h57m33s.8
Moon's semi-diameter at midnt., April 21st =	15′14″*
Augmentation of ditto, Table IV., = . . .	+ 7
Moon's true semi-diameter = ...	15′21″

* The apparent time at Greenwich being so very close to midnight, and the variation in the moon's declination, semi-diameter, and horizontal parallax but trifling, no correction for these elements becomes necessary in the present instance.

Moon's right ascension at midnight, April 21st	=	70°57′59″
Corrected proportional part of ditto for 0h 1m 42s	=	+ 0.56
Moon's corrected right ascension =	. . . .	70°58′55″

Moon's declination at midnight, April 21st, = . 23° 9′28″N.*

Moon's horizontal parallax at midnight, April 21st = 55′54″*

Observed altitude of the moon's lower limb=29°30′26″; hence, her true central altitude is 30°31′19″.

Lat. of the place . .	50°48′ 0″N.	. . . .	Log. secant=0.199263
Moon's corrected dec.=	23. 9.28 N.	. . . .	Log. secant=0.036483
Moon's mer. zen. dist.=	27°38′32″	Nat. vers. S.=114138	
Moon's true cent. alt.=	30. 31. 19	Nat. co-V. S.=492131	
	Remainder =	377993	Log.=5.577484
Moon's horary dist., west of the mer.=	4h 38m 10s		Log. rising=5.81323.0
Moon's corrected right ascension = 70°58′55″, in time = ...	4.43.56		
Right ascension of the meridian = .	9h 22m 6s		
Sun's reduced right ascension = . .	1.57.34		
Apparent time at the place of observ.=	7h 24m 32s		
Apparent time of obs. at Greenwich=	12. 1.42		

Longitude of the place of obs., in time=4h 37m 10s = 69°17′30″ west.

Example 2.

September 2d, 1825, in latitude 40°10′ S., the mean of several altitudes of the moon's lower limb, east of the meridian, was 9°8′36″, and that of the corresponding times 6h 39m 0s, by a chronometer, the error and rate of which were determined at noon, May 1st, when it was found 4m 10s slow for mean time at Greenwich, and gaining 1s.37 daily; the error of the sextant was 1′20″ subtractive, and the height of the eye above the surface of the sea 14 feet; required the longitude?

* See Note, page 427.

Mean time of observation at Greenwich = ...	$6^h 39^m 0^s$
Original error of the chronometer = ...	+ 4.10
Accumulated rate = $1^s.37 \times 124\frac{1}{2}$ days = . .	− 2.50
Mean time at Greenwich = ...	$6^h 40^m 20^s$
Reduced equation of time = ...	+ 0.33
Apparent time of observation at Greenwich =	$6^h 40^m 53^s$

Sun's right ascension at noon, September 2d, =	$10^h 44^m 53^s.5$
Correction of ditto for $6^h 40^m 53^s$ = . . .	+ 1. 0 .5
Sun's reduced right ascension = ...	$10^h 45^m 54^s.0$

Observed altitude of the moon's lower limb = $9°8'36''$; hence, the true central altitude of that object is $10°6'23''$.

Moon's right ascension at noon, September 2d, =	$30°58'\ 8''$
Corrected prop. part of ditto for $6^h 40^m 53^s$ =	+ 3. 19. 9
Moon's corrected right ascension = . . .	$34°17'17''$

Moon's declination at noon, September 2d, =	$16°12'13''$N.
Corrected prop. part of ditto for $6^h 40^m 53^s$ =	+ 54. 3
Moon's corrected declination = ...	$17°\ 6'16''$N.

Moon's semi-diameter at noon, September 2d, =	$14'46''$
Correction of ditto for $6^h 40^m 53^s$ = ...	+ 1
Augmentation, Table IV., = ...	+ 2
Moon's true semi-diameter = ...	$14'49''$

Moon's horizontal parallax at noon, Sept. 2d, =	$54'11''$
Correction of ditto for $6^h 40^m 53^s$ = ...	+ 4
Moon's true horizontal parallax = ...	$54'15''$

Lat. of the place . .	$40°10'\ 0''$S.	Log. secant=0.116809
Moon's corrected dec.=	17. 6.16 N. . . .	Log. secant=0.019647
Moon's mer. z. dist. =	$57°16'16''$	Nat. vers. S.=459336
Moon's true cent. alt.=	10. 6.23	Nat. co.V.S.=824524
	Remainder =	365188 Log.=5.562516
Moon's horary dist., east of the merid.=	$4^h\ 0^m\ 0^s$	Log. rising=5.69897.2

Moon's horary dist., east of the merid. =	$4^h\ 0^m\ 0^s$
Moon's reduced right ascension $34°17'17''$, in time = ...	2. 17. 9
Right ascension of the meridian = .	$22^h 17^m\ 9^s$
Sun's reduced right ascension = .	10. 45. 54
Apparent time at the place of observ. =	$11^h 31^m 15^s$
Apparent time of obs. at Greenwich =	6. 40. 53

Longitude of the place of obs., in time $=4^h 50^m 22^s = 72°35'30''$ east.

Remark 1.—The longitude, thus deduced from the true central altitude of the moon, will be equally as correct as that inferred from the sun's central altitude, provided the moon's place in right ascension and declination be carefully corrected by the equation of second difference, as explained between pages 33 and 38. Whatever little extra trouble may be attendant on this particular operation, will be infinitely more than counter-balanced by the pleasing reflection that it affords the mariner an additional method of finding the longitude of his ship, either by night or by day, with all the accuracy that can possibly result from the established rate or going of his chronometer.

Remark 2.—It frequently happens at sea, that, owing to clouds, rains, or other causes, ships are whole days without profiting by the presence of the sun, or obtaining an altitude of that object for the purpose of ascertaining either latitude or longitude; but it must be remembered, that there are few nights, if any, in which some fixed star, a planet, or the moon, does not present itself for observation, as if intended by Providence to relieve the mariner from the great anxiety which the doubtful position of his ship must naturally excite in him, particularly when returning from a long voyage, and about to enter any narrow sea, such as the English Channel. Under such circumstances, the *three* preceding problems will be found exceedingly useful; because they exhibit safe and certain means of finding the true place of a ship, so far as the going of the chronometer used in the observation can be depended upon. In this case, since a knowledge of the heavenly bodies becomes indispensably necessary, the reader is referred to "The Young Navigator's Guide to the Sidereal and Planetary Parts of Nautical Astronomy," where a familiar code of practical directions is given for finding out and knowing all the principal fixed stars and planets in the firmament.

Problem VII.

To find the Longitude of a Ship or Place by celestial Observation, commonly called a lunar Observation.

The direct progressive motion of a ship at sea is so liable to be disturbed by various unavoidable and often imperceptible causes,—such as a frequent aberration from the true course, by the ship's continually varying a little, in contrary directions, round her centre of gravity; high seas with heavy swells, sometimes with and at other times against, or in directions oblique to the true course; storms, sudden shifts of wind, unknown currents, *local magnetic attraction*, unequal attention in the helm's-men, with many other casualties which cannot possibly be properly provided for,—that the place indicated by *the dead reckoning* is frequently so erroneous as to be whole degrees to the eastward or westward of the actual position of the ship. Of this every person must be fully aware, who has navigated the short run between England and the nearest of the West Indian Islands.

As the best account by *dead reckoning* is evidently but a very imperfect kind of guess-work, it should be employed only as an auxiliary to the elementary parts of navigation, and never confided in but with the utmost caution. Hence it is that celestial observation should be constantly resorted to, because it is the only certain way of detecting the errors of dead reckoning, and of ascertaining, with any degree of precision, the actual position of the ship.

If a chronometer or time-keeper could be so constructed as to go uniformly correct in all seasons, places, and climates, it would immediately obviate all the difficulties attendant on a ship's reckoning, and thus render the longitude as simple a problem as the latitude; for, such a machine being once regulated to the meridian of Greenwich, would always show the absolute time at that meridian; and, hence, the longitude of the place of observation, as has been illustrated in the four preceding problems; but those pieces of mechanism are so exceedingly complicated, and so extremely delicate, that they are liable to be affected by the common vicissitudes of seasons and climates, and also by any sudden exposure to a higher or lower degree of atmospheric temperature than that to which they have been accustomed: the celestial bodies ought, therefore, to be consulted, at all times, in preference to machines so subject to mutability, and should ever be confided in by the mariner, as the only immutable and unerring time-keepers.

Of all the apparent motions of the heavenly bodies, in the zodiac, with which we are acquainted, that of the moon is by far the most rapid; it being, at a mean rate, about 13°10′ in 24 hours, or nearly half a minute of a degree in one minute of time. Hence, the quickness of the moon's motion seems to adapt her peculiarly to the measurement of small portions of corresponding time; and, therefore, careful observations of the angular distance of that object from the sun, a planet, or a fixed star lying in or near the zodiac, afford the most eligible and practicable means of determining the longitude of a ship at sea: for the true distance deduced from observation, being compared with the computed distances in the Nautical Almanac, will show the corresponding time at Greenwich; the difference between which and the apparent time at the place of observation will be the longitude of that place in time; and which will be east if the time at the place of observation be greater than the Greenwich time, but west if it be less.

The method of finding the longitude at sea, by *lunar observations*, is very familiarly explained, by geometrical construction and by spherical calculation, in "The Young Navigator's Guide to the Sidereal and Planetary Parts of Nautical Astronomy," between pages 172 and 212, where it will be seen that in a lunar observation there are two oblique angled spherical triangles to work in, for the purpose of finding the true central distance; in the first of which the three sides are given, viz., the apparent zenith distances of the two objects, and their apparent central distance, to find the angle at the zenith,—that is, the angle comprehended between the zenith distances of those objects; and, in the other, two sides and the included angle are given, to find the third side, viz., the true zenith distances of the objects; and their contained angle, to find the side opposite to that angle, or the true central distance between those objects. The solution of the first triangle falls under Problem V., page 207, and that of the second under Problem III., page 202. This is the direct spherical method of reducing the apparent central distance between the moon and sun, a planet, or a fixed star, to the true central distance; or, in other words, that of clearing the apparent central distance between those objects of the effects of parallax and refraction: but, this being considered by some mariners as rather a tedious operation, the following methods are given, which, being deduced directly from the above spherical principles, will be always found universally correct; and, since they are not subject to any restrictions whatever, they are *general* in every case where a lunar observation can be taken. Besides this, they will be found remarkably simple and concise, particularly when the operations are performed by the Tables contained in this work.

Method I.

Of reducing the apparent to the true central Distance.

Rule.

Take the auxiliary angle from Table XX., and let it be corrected for the sun's, star's, or planet's apparent altitude, as directed in pages 44 and 45.

Find the difference of the apparent altitudes of the objects, and, also, the difference of their true altitudes.

Then, to the natural versed sines supplement of the sum and the difference of the auxiliary angle and the difference of the apparent altitudes, add the natural versed sines of the sum and the difference of the auxiliary angle and the apparent distance, and the natural versed sine of the difference of the true altitudes: the sum of these five numbers, abating 4 in the radii or left-hand place, will be the natural versed sine of the true central distance.

Example 1.

Let the apparent central distance between the moon and sun be 66°48′34″, the sun's apparent altitude 60°15′35″, the moon's apparent altitude 17°15′15″, and her horizontal parallax 59′43″; required the true central distance?

Sun's apparent alt. = 60°15′35″ − Correc. 0′29″ = true alt. = 60°15′ 6″
Moon's apparent alt. = 17. 15. 15 + Correc. 54. 0 = true alt. = 18. 9. 15

Diff. of the app. alts. = 43° 0′20″ Diff. of the true alts. = 42° 5′51″
Auxiliary angle = . 60. 9.′27
Apparent central dist. = 66. 48. 34

Sum of auxiliary angle and diff. of ap. alts. = 103° 9′47″	Nat. versed sine sup. =	0.772277
Difference of ditto = 17. 9. 7	Nat. versed sine sup. =	1.955526
Sum aux. ang. & ap. dist. 126. 58. 1	Nat. versed sine =	1.601354
Difference of ditto = 6. 39. 7	Nat. versed sine =	0.006730
Diff. of the true alts. = 42. 5. 51	Nat. versed sine =	0.257995
True central distance = 66° 2′20″	Nat. versed sine =	0.593882

General Remarks.

1. The *correction of the moon's apparent altitude* is contained in Table XVIII., and is to be taken out therefrom agreeably to the directions given in page 39.

2. The *correction of the sun's apparent altitude* is the difference between the refraction and the parallax corresponding to that altitude in Tables VIII. and VII.

3. The *correction of a planet's apparent altitude* is the difference between the refraction and the parallax answering to that altitude in Tables VIII. and VI. And,

4. The *correction of a star's apparent altitude* is the refraction corresponding thereto in Table VIII. The fixed stars have not any sensible parallax.

Example 2.

Let the apparent central distance between the moon and a fixed star be 37° 12′ 40″, the star's apparent altitude 11° 27′ 50″, the moon's apparent altitude 40° 55′ 15″, and her horizontal parallax 54′ 10″; required the true central distance?

Star's apparent alt.=11° 27′ 50″	−Correc. 4′ 35″	=true alt.=11° 23′ 15″
Moon's apparent alt.=40. 55. 15	+Correc. 39. 51	=true alt.=41. 35. 6

Diff. of the app. alts.=39° 27′ 25″	Difference of the true alts.=30° 11′ 51″
Auxiliary angle= 60. 19. 30	
Apparent cent. dist.=37. 12. 40	

Sum of auxiliary angle and diff. of ap. alts.=89° 46′ 55″	Nat. versed sine sup. =	1. 003806
Difference of ditto =30. 52. 5	Nat. versed sine sup. =	1. 858352
Sum aux. ang. & ap. dist. 97. 32. 10	Nat. versed sine = . .	1. 131151
Difference of ditto =23. 6. 50	Nat. versed sine = . .	0. 080274
Diff. of the true alts.=30. 11. 51	Nat. versed sine = . .	0. 135703
True central dist. =37° 44′ 52″	Nat. versed sine = . .	0. 209286

Remark 1.—Instead of the natural versed sines supplement of the first two terms in the calculation, the natural versed sines of the supplements of those terms to 180° may be taken: for it is evident that the natural versed sine of *the supplement of an arch* is the natural versed sine supplement of that arch. Thus, in the above example, the supplement of 89°46′55″ is 90°13′5″, the natural versed sine of which is 1.003806; and the supplement of 30°52′5″ is 149°7′55″, the natural versed sine of which is 1.858352, the same as above. By this transformation of the first two terms, all the tabular numbers that enter the calculation will become affirmative.

Remark 2.—When the sum of the auxiliary angle and the apparent central distance exceeds a semi-circle, or 180 degrees, the natural versed sine supplement of its excess above that quantity is to be taken, or, which is the same thing, the natural versed sine of its supplement to 360 degrees.

Remark 3.—Instead of using the natural versed sines supplement of the first two terms in the calculation, as above, or the natural versed sines of their supplements to 180°, as mentioned in Remark 1, the natural versed sines of those terms may be employed directly; as thus:—Let the sum of the natural versed sines of the first two terms be subtracted from the sum of the natural versed sines of the last three terms, and the remainder will be the natural versed sine of the true distance.

Example.

Let the apparent central distance between the moon and sun be 119°53′58″, the sun's apparent altitude 22°10′35″, the moon's apparent altitude 15°51′22″, and her horizontal parallax 58′40″; required the true central distance?

Sun's apparent alt. = 22°10′35″—Correc. 2′11″=true alt.=22° 8′24″
Moon's apparent alt.=15. 51. 22 +Correc. 53. 7 =true alt.=16. 44. 29

Diff. of apparent alts.= 6°19′13″ Diff. of true altitudes = 5°23′55″
Auxiliary angle = . 60. 8. 25
Apparent cent. dist.=119. 53. 58

Sum of the aux. angle
and diff. of ap. alts.=66°27′38″ Nat.V.S.=0.600620 }
Difference of ditto = 53. 49. 12 Nat.V.S.=0.409676 } Sum=1.010296
Sum of auxiliary angle
and app. dist. = 180. 2. 23 Nat.V.S.=2.000000 }
Difference of ditto= 59. 45. 33 Nat.V.S.=0.496365 } Sum=2.500800
Diff. of true alts. = 5. 23. 55 Nat.V.S.=0.004435 }

True central dist. = 119°22′25″ Nat. versed sine = . . . 1.490504

METHOD II.

Of reducing the apparent to the true central Distance.

RULE.

Take the auxiliary angle from Table XX., and let it be corrected for the sun's, star's, or planet's apparent altitude, as directed in pages 44 and 45.

Find the sum of the apparent altitudes of the objects, and, also, the sum of their true altitudes; then,

To the natural versed sines of the sum and the difference of the auxiliary angle and the sum of the apparent altitudes, add the natural versed sines of the sum and the difference of the auxiliary angle and the apparent distance, and the natural versed sine supplement of the sum of the true altitudes; the sum of these five terms, abating 4 in the radii or left-hand place, will be the natural versed sine of the true central distance.

Example 1.

Let the apparent central distance between the moon and Venus be 53°49′54″, the apparent altitude of Venus 19°10′40″, and her horizontal parallax 23″; the moon's apparent altitude 37°40′20″, and her horizontal parallax 59′47″; required the true central distance?

Venus' apparent alt.=19°10′40″	−Correc. 2′21″	=true alt.=19° 8′19″
Moon's apparent alt.=37. 40. 20	+Correc. 46. 5	=true alt.=38. 26. 25

Sum of the app. alts.=36°51′ 0″	Sum of the true alts. =	57°34′44″
Auxiliary angle = 60. 20. 14		
Apparent cent. dist.= 53. 49. 54		

Sum of auxiliary angle & sum of ap. alts.=117°11′14″	Nat. versed sine = . .	1.456899
Difference of ditto = 3. 29. 14	Nat. versed sine = . .	0.001851
Sum aux.ang.&ap.dis.114. 10. 8	Nat. versed sine = . .	1.409428
Difference of ditto = 6. 30. 20	Nat. versed sine = . .	0.006439
Sum of the true alts.=57. 34. 44	Nat. versed sine sup. =	1.536138
True central dist. = 53°53′48″	Nat. versed sine = . .	0.410755

Note.—For the corrections of the apparent altitudes of the objects, see remarks, page 434.

Example 2.

Let the apparent central distance between the moon and sun be 119°57′56″, the sun's apparent altitude 18°10′50″, the moon's apparent altitude 10°30′10″, and her horizontal parallax 60′37″; required the true central distance?

Sun's apparent alt. = 18°10′50″ − Correc. 2′44″	= true alt. =	18° 8′ 6″
Moon's apparent alt. = 10. 30. 10 + Correc. 54. 35	= true alt. =	11. 24. 45
Sum of the app. alts. = 28°41′ 0″.	Sum of the true alts. =	29°32′51″
Auxiliary angle = . 60. . 5. 34.		
Appar. central dist. = 119. 57. 56		

Sum of auxiliary angle & sum of app. alts. = 88°46′34″	Nat. versed sine = . .	0.978640
Difference of ditto = 31. 24. 34	Nat. versed sine = . .	0.146535
Sum aux.ang.&ap.dis.180. 3. 30	Nat. versed sine = . .	1.999999
Difference of ditto = 59. 52. 22	Nat. versed sine = . .	0.498078
Sum of the true alts. = 29. 32. 51	Nat. versed sine sup. =	1.869948
True central dist. = 119°33′ 4″	Nat. versed sine = . .	1.493200

Remark 1.—Instead of using the natural versed sine supplement of the sum of the true altitudes, the natural versed sine of that term may be employed: in this case, if from the sum of the natural versed sines of the first four terms in the calculation, the natural versed sine of the last term be taken, the remainder, abating 2 in the radii or left-hand place, will be the natural versed sine of the true central distance.

Remark 2. When the sum of the auxiliary angle and the apparent central distance exceeds a semi-circle, or 180°, the natural versed sine supplement of its excess above that quantity is to be taken, or, which amounts to the same, the natural versed sine of its supplement to 360°, as in the above example. The same is to be observed in the event of the aggregate of the auxiliary angle and the sum of the apparent altitudes exceeding 180 degrees: this, however, will but very rarely happen.

Method III.

Of reducing the apparent to the true central Distance.

Rule.

Take the logarithmic difference from Table XXIV., and let it be corrected for the sun's, star's, or planet's apparent altitude, as directed in pages 49, 51, and 52.

Find the difference of the apparent altitudes of the objects, and, also, the difference of their true altitudes.

Then, from the natural versed sine of the apparent distance, subtract the natural versed sine of the difference of the apparent altitudes; to the logarithm of the remainder let the logarithmic difference be added, and the sum (abating 10 in the index,) will be the logarithm of a natural number; which, being added to the natural versed sine of the difference of the true altitudes, will give the natural versed sine of the true central distance.

Example 1.

Let the apparent central distance between the moon and Mars be 83°10′23″, the apparent altitude of Mars 17°10′20″, and his horizontal parallax 15″; the moon's apparent altitude 31°20′30″, and her horizontal parallax 58′53″; required the true central distance?

Mars' apparent alt. = 17°10′20″ — Correc. 2′49″ = true alt. = 17° 7′31″
Moon's appar. alt. = 31. 20. 30 + Correc. 48. 44 = true alt. = 32. 9. 14

Diff. of appar. alts. = 14°10′10″ Diff. of true altitudes = 15° 1′43″

Apparent distance = 83°10′23″ Nat. V.S. = 881129
Diff. of appar. alts. = 14. 10. 10 Nat. V.S. = 030436 Log. diff. = 9.996299

Remainder = 850693 Log. = 5.929773

Natural number = ... 843476 Log. = 5.926072
Diff. of the true alts. = 15° 1′43″ Nat. V.S. = 034204

True central dist. = 82°58′26″ Nat. V.S. = 877680

Note.—For the corrections of the apparent altitudes of the objects, see remarks, page 434.

Example 2.

Let the apparent central distance between the moon and sun be 118°56′40″, the sun's apparent altitude 16°40′10″, the moon's apparent altitude 9°39′50″, and her horizontal parallax 59′19″; required the true central distance?

Sun's apparent alt. = 16°40′10″ − Correc. 3′ 0″ = true alt. = 16°37′10″
Moon's apparent alt. = 9.39.50 + Correc. 53. 3 = true alt. = 10.32.53

Diff. of the app. alts. = 7° 0′20″ Diff. of the true alts. = . 6° 4′17″

Apparent distance = 118°56′40″ N.V.S. = 1.483961
Diff. of appar. alts. = 7. 0.20 N.V.S. = .007466 Log.diff. = 9.998919

Remainder = 1.476495 Log. = 6.169232

Natural number = ... 1.472825 Log. = 6.168151
Diff. of the true alts. = 6° 4′17″ Nat.V.S. = .005609

True central dist. = 118°35′ 1″ Nat.V.S. = 1.478434

Method IV.

Of reducing the apparent to the true central Distance.

Rule.

Take the logarithmic difference from Table XXIV., and let it be corrected for the sun's, star's, or planet's apparent altitude, as directed in pages 49, 51, and 52.

Find the sum of the apparent altitudes of the objects, and, also, the sum of their true altitudes; then,

From the natural versed sine supplement of the sum of the apparent altitudes, subtract the natural versed sine of the apparent distance; to the logarithm of the remainder let the logarithmic difference be added, and the sum (abating 10 in the index,) will be the logarithm of a natural number; which, being subtracted from the natural versed sine supplement of the sum of the true altitudes, will leave the natural versed sine of the true central distance.

Example 1.

Let the apparent central distance between the moon and sun be 110°53′34″, the sun's apparent altitude 38°11′59″, the moon's apparent altitude 15°51′22″, and her horizontal parallax 58′40″; required the true central distance?

Sun's apparent alt.=38°11′59″ — Correc. 1′15″=true alt.=38°10′54″
Moon's appar. alt.= 15. 51. 22 + Correc. 53. 7 =true alt.=16. 44. 29

Sum of the ap. alts.=54° 3′21″ Sum of the true altitudes = 54°55′23″

Sum of ap. alts.=54° 3′21″Nat.V.S.sup.=1.586997
Appar. dist. = 110. 53. 34 Nat. vers. S.= 1.356620 Log.diff.=9.998150

Remainder = .230377 Log. = 5.362439

Natural number = ...229398 Log. = 5.360589
Sum of true alts. 54°55′23″Nat.V.S.sup.=1.574676

True cent. dis.=110°11′56″Nat. vers. S.= 1.345278

Note.—See remarks, page 434, relative to the corrections of the apparent altitudes of the objects.

Example 2.

Let the apparent central distance between the moon and a fixed star be 41°11′7″, the star's apparent altitude 43°10′20″, the moon's apparent altitude 56°48′16″, and her horizontal parallax 59′25″; required the true central distance?

Star's apparent alt.=43°10′20″ — Correc. 1′ 1″=true alt.=43° 9′19″
Moon's appar. alt.=56. 48. 16 + Correc. 31. 56 =true alt.=57. 20. 12

Sum of the app. alts.=99°58′36″ Sum of the true altitudes=100°29′31″

Sum of the app. alts.=99°58′36″ N.V.S.sup.=826753
App. central dist.= 41. 11. 7 N. vers. S.=247416 Log.diff.=9.993895

Remainder = 579337 Log. = 5.762931

Natural number = ... 571250 Log. = 5.756826
Sum of true alts.=100°29′31″Nat.V.S. sup.=817902

True cent. dist. = 41° 7′ 8″Nat. vers. S.= 246652

Method V.

Of reducing the apparent to the true central Distance.

Rule.

To the logarithmic sines of the sum and the difference of half the apparent distance and half the difference of the apparent altitudes, add the logarithmic difference, Table XXIV., and the constant logarithm 6.301030: the sum of these four logarithms (rejecting 30 in the index,) will be the logarithm of a natural number; which, being added to the natural versed sine of the difference of the true altitudes, will give the natural versed sine of the true central distance.

Example 1.

Let the apparent distance between the moon and a fixed star be 37°56′43″, the star's apparent altitude 19°32′, the moon's apparent altitude 56°33′, and her horizontal parallax 61′16″; required the true central distance?

Star's apparent alt. = 19°32′ 0″ − Correc. 2′40″ = true alt. = 19°29′20″
Moon's appar. alt. = 56. 33. 0 + Correc. 33. 9 = true alt. = 57. 6. 9

Diff. of the app. alts. = 37° 1′ 0″ Diff. of the true altitudes = 37°36′49″

Half diff. of ap. alts. = 18°30′30″
Half the appar. dist. = 18. 58. 21½

		Log. diff. = ...	9.993713
Sum = ...	37°28′51½″	Log. sine = ...	9.784259
Difference = . .	0. 27. 51½	Log. sine = ...	7.908677
		Constant log. = ...	6.301030

Natural number = ... 9720 Log. = 3.987679
Diff. of the true alts. = 37°36′49″ Nat. vers. S. = 207855

True central dist. = 38°31′ 1″ Nat. vers. S. = 217575

Example 2.

Let the apparent central distance between the moon and sun be 106°22′48″, the sun's apparent altitude 39°25′, the moon's apparent altitude 19°56′, and her horizontal parallax 58′0″; required the true central distance?

Sun's apparent alt. = 39°25′ 0″—Correc. 1′ 3″=true alt.=39°23′57″
Moon's apparent alt.=19. 56. 0 +Correc. 51. 56 =true alt.=20. 47. 56

Diff. of the app. alts.=19°29′ 0″ Diff. of the true altitudes=18°36′ 1″

Half diff. of app. alts.= 9°44′30″
Half the app. dist. = 53. 11. 24

		Log. diff. = ...	9. 997672
Sum = ...	62°55′54″	Log. sine = ...	9. 949616
Difference = . . .	43. 26. 54	Log. sine = ...	9. 837399
		Constant log. = ...	6. 301030

Natural number = ... 1. 218196 Log.=6. 085717
Diff. of the true alts.=18°36′ 1″Nat. vers. S.= 052234

True central dist.=105°41′24″Nat. vers. S.=1. 270430

Method VI.

Of reducing the apparent to the true central Distance.

Rule.

To the logarithmic co-sines of the sum and the difference of half the apparent distance and half the sum of the apparent altitudes, add the logarithmic difference, Table XXIV., and the constant logarithm 6. 301030: the sum of these four logarithms (rejecting 30 in the index,) will be the logarithm of a natural number; which, being subtracted from the natural versed sine supplement of the sum of the true altitudes, will leave the natural versed sine of the true distance.

Example 1.

Let the apparent central distance between the moon and a fixed star be 69°21′25″, the star's apparent altitude 27°32′37″, the moon's apparent altitude 22°28′56″, and her horizontal parallax 56′17″; required the true central distance?

Star's apparent alt.=27°32′37″—Correc. 1′49″=true alt.=27°30′48″
Moon's appar. alt.= 22. 28. 56 +Correc. 49. 43 =true alt.=23. 18. 39

Sum of the ap. alts.=50° 1′33″ Sum of the true altitudes=50°49′27″

Half sum of ap. alts.=25° 0′46½″

Half sum of ap. alts. = 25° 0′ 46½″
Half ap. cent. dist. = 34. 40. 42½

		Log. diff. = ...	9.997468
Sum = ...	59°41′29″	Log. co-sine = ...	9.702997
Difference = . .	9. 39. 56	Log. co-sine = ...	9.993791
		Constant log. = . . .	6.301030

Natural number = ... 989204 Log. = 5.995286
Sum of true alts. = 50°49′27″ Nat. V. S. sup. = 1.631703

True cent. dist. = 69° 3′ 11½″ Nat. vers. S. = .642499

Example 2.

Let the apparent central distance between the moon and Jupiter be 116°40′28″, Jupiter's apparent altitude 10°40′20″, and his horizontal parallax 2″, the moon's apparent altitude 15°10′30″, and her horizontal parallax 59′13″; required the true central distance?

Jupiter's appar. alt. = 10°40′20″ − Correc. 4′54″ = true alt. = 10°35′26″
Moon's appar. alt. = 15. 10. 30 + Correc. 53. 41 = true alt. = 16. 4. 11

Sum of the ap. alts. = 25°50′50″ Sum of the true altitudes = 26°39′37″

Half sum of ap. alts. = 12°55′25″
Half app. cent. dist. = 58. 20. 14

		Log. diff. = ...	9.998220
Sum = ...	71°15′39″	Log. co-sine = ...	9.506857
Difference = . . .	45. 24. 49	Log. co-sine = ...	9.846327
		Constant log. = ...	6.301030

Natural number = ... 449194 Log. = 5.652434
Sum of true alts. = 26°39′37″ Nat. V. S. sup. = 1.893683

True cent. dist. = 116°23′26″ Nat. vers. S. = 1.444489

METHOD VII.

Of reducing the apparent to the true central Distance.

RULE.

To the apparent central distance add the apparent altitudes of the objects, and take half the sum; the difference between which and the apparent distance, call the *remainder*; then, To the logarithmic difference, Table XXIV., add the logarithmic co-sines of the above half sum and *remainder:* the sum of these three logarithms (rejecting 20 in the index,) will be the logarithm of a natural number. Now, twice this natural number being subtracted from the natural versed sine supplement of the sum of the true altitudes, will leave the natural versed sine of the true central distance.

Remarks.—If the remaining index of the three logarithms (after 20 is rejected) be 9, the natural number is to be taken out to six places of figures; if 8, to five places of figures; if 7, to four places of figures; if 6, to three places of figures,—and so on.

The logarithmic difference is to be corrected for the sun's, star's, or planet's apparent altitude, as directed in pages 49, 51, and 52;—this, it is presumed, need not be again repeated.

Example 1.

Let the apparent distance between the moon and a fixed star be 48°20′21″, the star's apparent altitude 11°33′29″, the moon's apparent altitude 11°10′35″, and her horizontal parallax 55′32″; required the true central distance?

Star's apparent alt. = 11°33′29″ − Correc. 4′33″ = true alt. = 11°28′56″
Moon's appar. alt. = 11. 10. 35 + Correc. 49. 46 = true alt. = 12. 0. 21

Appar. central dist. = 48. 20. 21 Sum of the true altitudes = 23°29′17″

Sum = ... 71° 4′25″
Log. diff. = ... 9.998827
Half sum = . . . 35°32′12½″ Log. co-sine = ... 9.910487
Remainder = . . 12. 48. 8½ Log. co-sine = ... 9.989067

Natural number = ... 791372 Log. = 9.898381

Twice the natural number = ... 1.582744
Sum of true alts. = 23°29′17″ Nat. V. S. sup. = 1.917143

True cent. dist. = 48°16′17″ Nat. vers. S. = .334399

Example 2.

Let the apparent distance between the moon and sun be 108°42′ 8″, the sun's apparent altitude 6°28′, the moon's apparent altitude 54°12′, and her horizontal parallax 55′19″; required the true central distance?

Sun's apparent alt. = 6°28′ 0″ − Correc. 7′45″ = true alt. = 6°20′15″
Moon's apparent alt. = 54. 12. 0 + Correc. 31. 40 = true alt. = 54. 43. 40

Appar. central dist. = 108. 42. 3   Sum of the true altitudes = 61° 3′55″

Sum = ... 169°22′ 3″

Log. diff. = ... 9.994507
Half sum = . . . 84°41′ 1½″ Log. co-sine = ... 8.966858
Remainder = . . 24. 1. 1½ Log. co-sine = ... 9.960673

Natural number = ... 83568 Log. = 8.922038

Twice the natural number = ... 167136
Sum of true alts. = 61° 3′55″ Nat. V. S. sup. = 1.483813

True cent. dist. = 108°27′43″ Nat. vers. S. = 1.316677

Method VIII.

Of reducing the apparent to the true central Distance.

Rule.

To the logarithmic sines of the sum and the difference of half the apparent distance and half the difference of the apparent altitudes, add the logarithmic difference: half the sum of these three logarithms (10 being previously rejected from the index,) will be the logarithmic sine of an arch. Now, half the sum of the logarithmic co-sines of the sum and the difference of this arch and half the difference of the true altitudes, will be the logarithmic co-sine of half the true central distance.

Example 1.

Let the apparent central distance between the moon and a fixed star be 41°24′22″, the star's apparent altitude 12° 4′27″, the moon's apparent altitude 7°47′47″, and her horizontal parallax 57′24″; required the true central distance?

Star's apparent alt. = 12° 4′27″ − Correc. 4′22″ = true alt. = 12° 0′5″
Moon's appar. alt. = 7.47.47 + Correc. 50. 13 = true alt. = 8.38.0

Diff. of the app. alts. = 4°16′40″   Diff. of the true altitudes = 3°22′5″

Half diff. of app. alts. = 2° 8′20″   Half diff. of the true alts. = 1°41′2½″

Half diff. of ap. alts. =	2° 8′20″		
Half the ap. cent. dis. =	20. 42. 11		
		Log. diff. = ...	9.999201
Sum = ...	22. 50. 31	Log. sine = ...	9.589045
Difference = . .	18. 33. 51	Log. sine = ...	9.502927
		Sum =	19.091173
Arch = ...	20°33′44½″	Log. sine = ...	9.545586½
Half diff. of true alts. =	1. 41. 2½		
Sum = ...	22°14′47″	Log. co-sine = . . .	9.966406
Difference = . .	18. 52. 42	Log. co-sine = . . .	9.975987
		Sum =	19.942393
Half the true dist. =	20°38′15″	Log. co-sine = . . .	9.971196½
True central dist. =	41°16′30″		

Example 2.

Let the apparent central distance between the moon and Saturn be 110°14′34″, Saturn's apparent altitude 9°40′48″, and his horizontal parallax 1″, the moon's apparent altitude 15°40′6″, and her horizontal parallax 58′43″; required the true central distance?

Saturn's apparent alt. = 9°40′48″ − Correc. 5′24″ = true alt. = 9°35′24″
Moon's apparent alt. = 15. 40. 6 + Correc. 53. 11 = true alt. = 16. 33. 17

Diff. of the app. alts. = 5°59′18″ Diff. of the true altitudes = 6°57′53″

Half diff. of app. alts. = 2°59′39″ Half diff. of the true alts. = 3°28′56½″

Half app. cent. dist. =	55. 7. 17		
		Log. diff. = ...	9.998176
Sum = ...	58° 6′56″	Log. sine = ...	9.928966
Difference = . .	52. 7. 38	Log. sine = ...	9.897284
			19.824426
Arch = . . .	54°47′ 2″	Log. sine = ...	9.912213
Half diff. of true alts. =	3. 28. 56½		
Sum = . . .	58°15′58½″	Log. co-sine = ...	9.720963
Difference = . .	51. 18. 5½	Log. co-sine = ...	9.796035
		Sum =	19.516998
Half the true dist. =	55° 0′30½″	Log. co-sine = ...	9.758499
True central dist. =	110°. 1′ 1″		

METHOD IX.

Of reducing the apparent to the true central Distance.

RULE.

To the logarithmic co-sines of the sum and the difference of half the apparent distance and half the sum of the apparent altitudes, add the logarithmic difference: half the sum of these three logarithms (10 being previously rejected from the index,) will be the logarithmic co-sine of an arch. Now, half the sum of the logarithmic sines of the sum and difference of this arch and half the sum of the true altitudes, will be the logarithmic sine of half the true central distance.

Example 1.

Let the apparent central distance between the moon and a fixed star be 41°29′58″, the star's apparent altitude 11°31′2″, the moon's apparent altitude 8°44′35″, and her horizontal parallax 57′24″; required the true central distance?

Star's apparent alt.=11°31′ 2″—Correc. 4′34″=true alt.=11°26′28″
Moon's appar. alt. = 8. 44. 35 +Correc. 50. 46 =true alt.= 9. 35. 21

Sum of the app. alts.=	20°15′37″	Sum of the true altitudes=	21° 1′49″
Half sum of ap. alts.=	10° 7′48½″	Half sum of the true alts.=	10°30′54½″
Half ap. cent. dist.=	20. 44. 59		
		Log. diff. = ...	9. 999083
Sum = . . .	30°52′47½″	Log. co-sine = ...	9. 933612
Difference = . .	10. 37. 10½	Log. co-sine = ...	9. 992497
			19. 925192
Arch = . . .	23°26′23″	Log. co-sine = ...	9. 962596
Half sum of tr. alts.=	10. 30. 54½		
Sum = ...	33°57′17½″	Log. sine = ...	9. 747053
Difference = . .	12. 55. 28½	Log. sine = ...	9. 349604
		Sum =	19. 096657
Half the true dist. =	20°41′54½″	Log. sine = ...	9. 548328½
True central dist.=	41°23′49″		

Example 2.

Let the apparent central distance between the moon and sun be 101°54′51″, the sun's apparent altitude 39°34′35″, the moon's apparent altitude 29°23′2″, and her horizontal parallax 58′53″; required the true central distance?

Sun's apparent alt.=	39°34′35″	−Correc. 1′ 3″=true alt.=	39°33′32″
Moon's appar. alt. =	29. 23. 2	+Correc. 49. 38 =true alt.=	30. 12. 40
Sum of the app. alts.=	68°57′37″	Sum of the true altitudes=	69°46′12″
Half sum of ap. alts.=	34°28′48½″	Half sum of the true alts.=	34°53′ 6″
Half app. cent. dist.=	50. 57. 25½		
		Log. diff. = ...	9.996517
Sum = . . .	85°26′14″	Log. co-sine = ...	8.900647
Difference = . . .	16. 28. 37	Log. co-sine = ...	9.981789
			18.878953
Arch = ...	74° 1′57″	Log. co-sine = . . .	9.439476½
Half sum of true alts.=	34. 53. 6		
Sum = ...	108°55′ 3″	Log. sine = ...	9.975885
Difference = . . .	39. 8. 51	Log. sine = ...	9.800249
		Sum =	19.776134
Half the true dist.=	50°36′22″	Log. sine = ...	9.888067
True central dist.=	101°12′44″		

METHOD X.

Of reducing the apparent to the true central Distance.

RULE.

To the logarithmic sines of the sum and the difference of half the apparent distance, and half the difference of the apparent altitudes, add the logarithmic difference, its index being increased by 10: from half the sum of these three logarithms subtract the logarithmic sine of half the difference of the true altitudes, and the remainder will be the logarithmic tangent of an arch; the logarithmic sine of which, being subtracted from the half sum of the three logarithms, will leave the logarithmic sine of half the true central distance.

Example 1.

Let the apparent central distance between the moon and a fixed star be 55°4′53″, the star's apparent altitude 10°8′6″, the moon's apparent altitude 8°1′25″, and her horizontal parallax 58′1″; required the true central distance?

Star's apparent alt.= 10° 8′ 6″ − Correc. 5′ 11″ = true alt. = 10° 2.55″
Moon's apparent alt. = 8. 1. 25 + Correc. 50. 58 = true alt. = 8. 52. 23

Diff. of the app. alts. = 2° 6′ 41″ Diff. of the true altitudes = 1° 10′ 32″

Half diff. of app. alts. = 1° 3′ 20½″ Half diff. of true altitudes = 0° 35′ 16″
Half the ap. cent. dis. = 27. 32. 26½
Log. diff. = 19. 999162
Sum = ... 28° 35′ 47″ Log. sine = 9. 680006
Difference = . . 26. 29. 6 Log. sine = 9. 649299

Sum = ... 39. 328467

Half sum = ... 19. 664233½ . 19. 664233½
Half diff. of true alts. = 0° 35′ 16″ Log. sine = 8. 011083

Arch = ... 88° 43′ 36″ Log. tan. = 11. 653150½ Log. si. 9. 999893

Half the true distance = ... 27° 29′ 44″ Log. sine = 9. 664340½

True central distance = ... 54° 59′ 28″

Example 2.

Let the apparent central distance between the moon and sun be 91° 26′ 8″, the sun's apparent altitude 14° 45′ 41″, the moon's apparent altitude 53° 41′ 1″, and her horizontal parallax 58′ 29″; required the true central distance?

Sun's apparent alt. = 14° 45′ 41″ − Correc. 3′ 26″ = true alt. = 14° 42′ 15″
Moon's appar. alt. = 53. 41. 1 + Correc. 33. 56 = true alt. = 54. 14. 57

Diff. of the ap. alts. = 38° 55′ 20″ Diff. of the true alts. = . 39° 32′ 42″

Half diff. of ap. alts. = 19° 27′ 40″ Half diff. of the true alts. = 19° 46′ 21″
Half ap. cent. dist. = 45. 43. 4
Log. diff. = 19. 994220
Sum = ... 65° 10′ 44″ Log. sine = 9. 957905
Difference = . . 26. 15. 24 Log. sine = 9. 645809

Sum = ... 39. 597934

Half sum = ... 19. 798967 . . 19. 798967
Half diff. of true alts. 19° 46′ 21″ Log. sine = 9. 529285

Arch = ... 61° 44′ 43″ Log. tan. = 10. 269682 Log. sine 9. 944902

Half the true distance = ... 45° 36′ 38½″ Log. sine = 9. 854065

True central distance = ... 91° 13′ 17″

METHOD XI.

Of reducing the apparent to the true central Distance.

RULE.

To the logarithmic difference (its index being increased by 10,) add the logarithmic co-sines of the sum and the difference of half the apparent distance and half the sum of the apparent altitudes; from half the sum of these three logarithms subtract the logarithmic co-sine of half the sum of the true altitudes, and the remainder will be the logarithmic sine of an arch; the logarithmic tangent of which, being subtracted from the half sum of the three logarithms, will leave the logarithmic sine of half the true central distance.

Example 1.

Let the apparent central distance between the moon and a fixed star be 68°52′40″, the star's apparent altitude 10°52′17″, the moon's apparent altitude 6°39′28″, and her horizontal parallax 58′31″; required the true central distance?

Star's apparent alt. = 10°52′17″ − Correc. 4′50″ = true alt. = 10°47′27″
Moon's appar. alt. = 6.39.28 + Correc. 50.26 = true alt. = 7.29.54

Sum of the ap. alts. = 17°31′45″ Sum of the true altitudes = 18°17′21″

Half sum of ap. alts. = 8°45′52½″ Half sum of the true alts. = 9°8′40½″
Half ap. cent. dist. = [illegible].26.20
Log.diff. 19.999326
Sum = ... 43°12′12½″ Log.co-si. 9.862684
Difference = . . 25.40.27½ Log.co-si. 9.954856

Sum = ... 39.816866

Half sum = ... 19.908433 . . 19.908433
Half sum of true alts. = 9° 8′40½″ Log.co-si. 9.994445

Arch = ... 55° 7′ 4″ Log.sine = 9.913988 Log.T. = 10.156675

Half the true distance = ... 34°22′34″ Log. sine = 9.751758

True central distance = ... 68°45′ 8″

Example 2.

Let the apparent central distance between the moon and sun be 120°10′40″, the sun's apparent altitude 13°30′0″, the moon's apparent altitude 6°10′0″, and her horizontal parallax 61′12″; required the true central distance?

Sun's apparent alt. = 13°30′ 0″	− Correc. 3′45″ = true alt. =	13°26′15″	
Moon's appar. alt. = 6. 10. 0	+ Correc. 52. 36 = true alt. =	7. 2. 36	
Sum of the ap. alts. = 19°40′ 0″	Sum of the true altitudes =	20°28′51″	
Half sum of ap. alts. = 9°50′ 0″	Half sum of the true alts. =	10°14′25½″	
Half ap. cent. dist. = 60. 5. 20			
	Log. diff. = 19. 999345		
Sum = ... 69°55′20″	Log. co-sine 9. 535668		
Difference = . . 50. 15. 20	Log. co-sine 9. 805749		
Sum = ...	39. 340762		
Half sum = ...	19. 670381	.	19. 670381
Half sum of true alts. 10°14′25½″	Log. co-sine 9. 993026		
Arch = . . . 28°44′23″	Log. sine = 9. 677355		Log. T. 9. 733070
Half the true distance = ...	59°56′59″		Log. sine = 9. 937311
True central distance = ...	119°53′58″		

Method XII.

Of reducing the apparent to the true central Distance.

Rule.

From the natural versed sine supplement of the sum of the apparent altitudes, subtract the natural versed sine of their difference, and call the remainder *arch first.* Proceed in a similar manner with the true altitudes, and call the remainder *arch second;* and from the natural versed sine supplement of the sum of the apparent altitudes, subtract the natural versed sine of the apparent distance, and call the remainder *arch third.*

Now, to the arithmetical complement of the logarithm of *arch first* add the logarithms of arches second and third, and the sum (rejecting 10 from the index,) will be the logarithm of a natural number; which, being subtracted from the natural versed sine supplement of the sum of the true altitudes, will leave the natural versed sine of the true central distance.

Example 1.

Let the apparent central distance between the moon and a fixed star be 83°15′19″; the star's apparent altitude 7°39′4″, the moon's apparent altitude 10°57′36″, and her horizontal parallax 58′55″; required the true central distance?

✱'s ap.alt.= 7°39′ 4″	−Cor. 6′45″	=True alt. 7°32′19″	
☽'s ap.alt.=10.57.36	+Cor.53. 3	=True alt.11.50.39	
Sum = 18°36′40″ N.V.S. sup. 1.947707		Sum = 19°22′58″ N.V.S. sup. 1.943322	
Diff. = 3.18.32 N.V.S. .001668		Diff. = 4.18.20 N.V.S. .002822	
Arch first = 1.946039		Arch second = 1.940500	

Sum of ap.alts.=18°36′40″ N.V.S. sup. = 1.947707
Ap. cent. dist.=83°15′19″ Nat. V. S. = .882554

Arch third = ...	1.065153 Log. =	6.027432
Arch second = ...	1.940500 Log. =	6.287914
Arch first = ...	1.946039 Log.ar.co.	3.710848
Natural number = ...	1.062169 Log. =	6.026194

Sum of true alts. 19°22′58″ N.V.S. sup. = 1.943322

True cent. dist. 83°10′28″ Nat.vers.sine= .881153

Example 2.

Let the apparent distance between the moon and sun be 111°27′1″, the sun's apparent altitude 24°40′16″, the moon's apparent altitude 16°52.31″, and her horizontal parallax 54′56″; required the true central distance?

☉'s ap.alt.=24°40′16″	−Cor. 1′56″	=True alt.24°38′20″	
☽'s ap.alt.=16.52.31	+Cor.49.28	=True alt.17.41.59	
Sum = 41°32′47″ N.V.S. sup. 1.748419		Sum = 42°20′19″ N.V.S. sup. 1.739177	
Diff. = 7.47.45 N.V.S. .009242		Diff. = 6.56.21 N.V.S. .007325	
Arch first = 1.739177		Arch second = 1.731852	

Sum of ap.alts.=41°32′47″ N.V.S. sup.=1.748419
App. central dist. 111°27′1″ Nat.V.S.=1.365694

Arch third = ...	.382725 Log. = .	5.582887
Arch second = ...	1.731852 Log. = .	6.238511
Arch first = ...	1.739177 Log. ar. co.=	3.759638
Natural number = ...	.381097 Log. = .	5.581036

Sum of true alts. 42°20′19″ N.V.S. sup.=1.739177

True central dist. 110°58′56″ N.V.S.=1.358080

METHOD XIII.

To the apparent distance add the apparent altitudes of the objects; take half the sum, and call the difference between it and the apparent distance the *remainder*. Then,

To the logarithmic difference (its index being augmented by 10,) add the logarithmic co-sines of the half sum and the remainder; from half the sum of these three logarithms subtract the logarithmic co-sine of half the sum of the true altitudes, and the remainder will be the logarithmic sine of an arch. Now, the logarithmic co-sine of this arch, being added to the logarithmic co-sine of half the sum of the true altitudes (rejecting 10 from the index), will give the logarithmic sine of half the true central distance.

Example 1.

Let the apparent central distance between the moon and Spica Virginis be 37°12'40", the star's apparent altitude 11°27'50", the moon's apparent altitude 40°55'15", and her horizontal parallax 54'10"; required the true central distance?

Star's apparent alt.=11°27'50" – Correc. 4'35" = true alt.=11°23'15"
Moon's appar. alt.=40.55.15 + Correc. 39.51 = true alt.=41.35. 6
Appar. cent. dist. = 37.12.40

Sum = . . . 89°35'45"
Log. diff.=19.995703
Half sum = . . . 44°47'52½" Log. co-sin. 9.851012
Remainder = . 7.35.12½ Log. co-sin. 9.996181

Sum = . . 39.842896

Half sum = 19.921448
Half sum of true alts. 26°29'10½" Log. co-sin. 9.951844 . . . 9.951844

Arch = . . . 68°48'45" Log. sine=9.969604 Log. co-si. 9.558014

Half the true distance = ... 18°52'26½" Log. sine=9.509858

True central distance = ... 37°44'53"

Example 2.

Let the apparent central distance between the moon and sun be 117°42'28", the sun's apparent altitude 10°19'19", the moon's apparent altitude 42°55'1", and her horizontal parallax 60'2"; required the true central distance?

Sun's apparent alt.=10°19′19″—Correc. 4′56″= true alt.=10°14′23″
Moon's app. alt. = 42.55. 1 +Correc. 42.57 = true alt.=43.37.58
Appar. cent. dist.=117.42.28

Sum = . . .	170°56′48″		
		Log.diff.=19.995005	
Half sum = . .	85°28′24″	Log.co-sin. 8.897204	
Remainder = .	32.14. 4	Log.co-sin. 9.927305	
		Sum = 38.819514	
		Half sum 19.409757	
Halfsum of true alts.	26°56′10½″	Log.co-s.=9.950127 . . .	9.950127
Arch = . . .	16°44′52″	Log. sine=9.459630	Log.co-si.9.981177
Half the true distance = ...	58°36′58″		Log. sine=9.931304
True central distance = ...	117°13′56″		

Note.—There are some curious properties peculiar to the lunar observations, with which the mariner ought to be acquainted, but which the general tenor of this work will not allow of being touched upon here:—these properties or peculiarities may, however, be readily seen, by making reference to the *General Remarks* contained between pages 208 and 212 of "The Young Navigator's Guide to the Sidereal and Planetary Parts of Nautical Astronomy."

PROBLEM VIII.

Given the apparent Time and the true central Distance between the Moon and Sun, a fixed Star, or a Planet; to determine the Longitude of the Place of Observation.

RULE.

If the true central distance can be found in the Nautical Almanac, the corresponding apparent time at Greenwich will be seen standing over it at the top of the page; but if the true central distance cannot be exactly found, which in general will be the case, take out the two distances from the Nautical Almanac, one of which is next greater and the other next less than the true central distance, and find their difference; find, also, the difference between the true central distance and the *first* of the two distances so taken from the Nautical Almanac; then, from the proportional logarithm of this difference, subtract the proportional logarithm of the former difference, and the remainder will be the proportional logarithm of a portion of time, which, being added to the time corresponding to the first of the two distances taken from the Nautical Almanac, will give the apparent time of observation at Greenwich. Now, the difference between the apparent time at Greenwich, thus found, and the apparent time at the place of observation, being turned into degrees, will be the longitude of the latter place;—and which will be east, if the time at the ship be greater than that at Greenwich; if otherwise, west.

Example 1.

At sea, January 9th, 1825, in longitude (by account) 54°48′ east, at $23^h40^m47^s$ apparent time, the true central distance between the moon and sun was 107°19′56″; required the corresponding apparent time at Greenwich, and the longitude of the place of observation?

True cent. dist. at ship = 107°19′56″	Diff. = 1° 7′32″		Prop. log. = 4258
Distance at 18 hours = 108. 27. 28			
Distance at 21 hours = 106. 48. 12	Diff. = 1. 39. 16		Prop. log. = 2585
Portion of time = ...		$2^h\ 2^m27^s$	Prop. log. = 1673
Time corresponding to first distance = .		18. 0. 0	
Apparent time of observ. at Greenwich =		$20^h\ 2^m27^s$	
Apparent time of observation at ship =		23. 40. 47	
Longitude of the ship, in time = . . .		$3^h38^m20^s$ = 54°35′ east.	

Example 2.

At sea, March 3d, 1825, in longitude (by account) 47°55′ west, at $10^h12^m43^s$ apparent time, the true central distance between the moon and Spica Virginis, was 50°3′23″; required the corresponding apparent time at Greenwich, and the longitude of the place of observation?

True central distance = 50° 3′23″	Diff. = 0°53′20″		Prop. log. = 5283
Distance at 12 hours = 50. 56. 43			
Distance at 15 hours = 49. 2. 43	Diff. = 1. 54. 0		Prop. log. = 1984
Portion of time = ...		$1^h24^m13^s$	Prop. log. = 3299
Time corresponding to first distance =		12. 0. 0	
Apparent time of observ. at Greenwich =		$13^h24^m13^s$	
Apparent time of observation at ship =		10. 12. 43	
Longitude of the ship, in time = . . .		$3^h11^m30^s$ = 47°52′30″ west.	

Problem IX.

Given the Latitude of a Place and its Longitude by account, the observed Distance between the Moon and Sun, a fixed Star, or a Planet, and the observed Altitudes of those Objects; to find the true Longitude of the Place of Observation.

Rule.

Reduce the apparent time of observation to the meridian of Greenwich, by Problem III., page 297; to this time let the moon's horizontal parallax and semi-diameter be reduced, by Problem VI., page 302, and let the moon's reduced semi-diameter be increased by the augmentation (Table IV.) answering to her observed altitude.

Find the apparent and the true altitude of each object's centre, by the respective problems, for that purpose, contained between pages 320 and 327.

To the observed distance between the nearest limbs of the moon and sun, corrected for index error, if any, add their respective semi-diameters, and the sum will be the apparent central distance. But, if the distance be observed between the moon and a fixed star or planet, then the moon's true semi-diameter is to be applied to that distance by addition when it is measured from the nearest limb, but by subtraction when it is measured from the remote limb: in either case, the result will be the apparent central distance. With the apparent and the true altitudes of the objects, and their apparent central distance, compute the true central distance, by any of the methods given in Problem VII., between pages 433 and 454; and find the apparent time at Greenwich corresponding to this distance, by Problem VIII., page 456.

Now, the difference between the apparent times of observation at the ship and at Greenwich, being converted into degrees, will be the longitude of the place of observation; which will be east or west, according as the time at the ship is greater or less than the Greenwich time.

Remarks.

If the watch be not well regulated to the time of observation, the apparent time may be deduced from the true altitude of the sun, moon, star, or planet, used in the computation, provided the object made choice of for this purpose be sufficiently far from the meridian at the time of measuring the lunar distances; if not, the error of the watch must be inferred from the true altitude of one of those objects, when in a more favourable position with respect to the meridian: then the error of the watch, thus found, being applied to the mean time of measuring the lunar distances, by addition or subtraction, according as it is slow or fast, the sum or difference will be the apparent time of taking the lunar observation, agreeably to the meridian under which the error of the watch was obtained. The error of the watch is to be found by Problems III., IV., V., or VI., between pages 383 and 400, according as the object may be the sun, a fixed star, a planet, or the moon.

In taking a lunar observation, it is necessary that several distances be measured,—that the corresponding times, per watch, be carefully noted down,—and that the altitudes of the objects be observed at the same instants with the distances: then, the respective sums of the times (per watch) of the observed distances and of the altitudes, being divided by their common number, will give the mean time of observation, the mean observed distance, and the mean observed altitude of each object.

Example 1.

January 9th, 1825, in latitude 19°30′ N., and longitude 5°45′ E., by account, the following observations were taken; the index error of the sextant by which the distances were measured was 2′30″ subtractive, and the height of the eye above the level of the sea 20 feet; required the longitude of the place of observation?

Apparent time of observation.	Observed distance between nearest limbs of Moon and Sun.	Altitude of Sun's lower limb.	Altitude of Moon's lower limb.
$19^h 10^m 45^s$	107° 48′ 30″	7° 9′ 0″	53° 13′ 30″
. 11. 50	. 47. 50	7. 22. 30	53. 2. 0
. 12. 55	. 47. 15	7. 36. 0	52. 50. 30
. 14. 0	. 46. 40	7. 49. 30	52. 39. 0
. 15. 5	. 46. 0	8. 3. 0	52. 27. 20
Mean.... $19^h 12^m 55^s$	Mean...... 107° 47′ 15″	Mean.. 7° 36′ 0″	Mean 52° 50′ 28″
Longitude in time } − 23. 0	Index error.. − 2. 30	Moon's semi-diameter	+ 16. 25
	☽'s semi-diam. + 16. 25	Dip of the horizon ..	− 4. 17
	☉'s semi-diam. + 16. 18		
Red. time $18^h 49^m 55^s$	Appar. dist... 108° 17′ 28″	Moon's appar. altitude	53° 2′ 36″
		Correction of ditto....	+ 35. 1
		Moon's true altitude..	53° 37′ 37″

Observed altitude of sun's lower limb =	7°36′ 0″
Sun's semi-diameter = ...	+16. 18
Dip of the horizon = ...	− 4. 17
Sun's apparent altitude = ...	7°48′ 1″
Correction of ditto = ...	− 6. 29
Sun's true altitude = ...	7°41′32″

Moon's reduced horizontal parallax = . 59′.25″

Diff. of the app. alts. =	45°14′35″		
Auxiliary angle = .	60. 26. 25		
App. central dist. =	108. 17. 28		
Sum of aux. ang. and diff. of app. alts. =	105°41′ 0″	Nat. versed sine sup. = .	.729680
Difference of ditto =	15. 11. 50	Nat. versed sine sup. = .	1.965029
Sum aux. ang. & ap. dis.	168. 43. 53	Natural versed sine = . .	1.980722
Difference of ditto =	47. 51. 3	Natural versed sine = . . .	.328937
Diff. of true alts. =	45. 56. 5	Natural versed sine = . . .	.304522
		Natural versed sine = . .	1.308890

True central dist. =	107°59′32″		
Dist. at 18 hours =	108. 27. 28	Diff. = 0°27′56″	Prop. log. = 8091
Dist. at 21 hours =	106. 48. 12	Diff. = 1. 39. 16	Prop. log. = 2585

Portion of time = ...	$0^h50^m39^s$	Prop. log. = 5506
Time corresponding to first distance =	18. 0. 0	
Apparent time of observ. at Greenwich =	$18^h50^m39^s$	
Apparent time at the place of observ. =	19. 12. 55	
Longitude at the place of obs., in time =	$0^h22^m16^s$ = 5°34′ east.	

Example 2.

February 1st, 1825, in latitude 45°40′ N., and longitude 59°10′ W., by account, the following observations were taken; the height of the eye above the level of the horizon was 22 feet, and the index error of the sextant by which the distances were measured 1′30″ additive; required the true longitude?

Apparent time of observation.	Observed distance of Moon's remote limb.	Altitude of Regulus.	Altitude of Moon's lower limb.
$8^h50^m10^s$	35° 7′ 40″	28° 50′ 50″	10° 44′ 40″
. 51. 25	. 6. 50	29. 4. 30	11. 40. 0
. 52. 40	. 6. 10	29. 18. 20	12. 35. 20
. 53. 55	. 5. 20	29. 32. 0	13. 30. 50
. 55. 10	. 4. 40	29. 45. 50	14. 26. 10
Mean.... $8^h52^m40^s$	Mean...... 35° 6′ 8″	Mean 29° 18′ 18″	Mean 12° 35′ 24″
Long. in time + 3. 56. 40	Index error.. +. 1. 30	Moon's semi-diameter..	+16. 26
	☽'s semi-diam. −16. 26	Dip of the horizon	− 4. 30
Red. time $12^h49^m20^s$	Apparent dist. 34° 51′ 12″	Moon's appar. altitude..	12° 47′ 20″
		Correction of ditto	+54. 27
		Moon's true altitude....	13° 41′ 47″

Observed altitude of Regulus = ...	29°18′18″
Dip of the horizon for 22 feet = . .	− 4.30
Apparent altitude of Regulus = ...	29°13′48″
Correction of ditto = ...	− 1.41
True altitude of Regulus = ...	29°12′ 7″
Moon's reduced horizontal parallax = . . .	60′3″

Sum of the app. alts. = 42° 1′ 8″
Auxiliary angle = 60. 6.49
App. central dist. = 34.51.12

Sum of aux. ang. and sum of app. alts. =	102° 7′57″	Natural versed sine = .	1.210173
Difference of do. =	18. 5.41	Natural versed sine = .	.049456
Sum aux. ang. & ap. dist.	94.58. 1	Natural versed sine = .	1.086581
Difference of do. =	25.15.37	Natural versed sine = .	.095622
Sum of the true alts. =	42.53.54	Natural versed sine sup. =	1.732563
		Natural versed sine = . .	.174395

True central dist. = 34°21′ 0″			
Dist. at 12 hours = 34.51.16	Diff. = 0°30′16″	Prop. log. =	7743
Dist. at 15 hours = 33. 2.35	Diff. = 1.48.41	Prop. log. =	2191

Portion of time = ...	0ʰ50ᵐ 8ˢ Prop. log. = 5552
Time corresponding to first distance = .	12. 0. 0
Apparent time of observation at Greenwich =	12ʰ50ᵐ 8ˢ
Apparent time at the place of observation =	8.52.40
Longitude at the place of observ., in time =	3ʰ57ᵐ28ˢ = 59°22′0″ west.

Example 3.

March 9th, 1825, in latitude 43°17′ S., and longitude 57°55′ E., by account, at 20ʰ14ᵐ per watch, *not regulated*, the mean of several observed distances between the moon and sun was 107°28′17″; at the same time the mean of several altitudes of the sun's lower limb was 26°39′40″, that of the moon's upper limb 39°30′45″, and the height of the eye above the level of the sea 19 feet; required the longitude of the place of observation?

Time, per watch, = 20h 14m 0s	Dist. of nearest limbs of moon and sun = 107° 28′ 17″
Long. 57° 55′ E. = −3. 51. 40	Sun's semi-diameter = +16. 8
Reduced time = . 16h 22m 20s	Moon's semi-diam. = +16. 0
	Apparent distance = 108° 0′ 25″
Alt. sun's lower limb = 26° 39′ 40″	Alt. ☽'s lower limb = 39° 30′ 45″
Sun's semi-diameter = +16. 8	Moon's semi-diameter = +16. 0
Dip of the horizon = − 4. 11	Dip of the horizon = − 4. 11
Sun's apparent alt. = 26° 51′ 37″	Moon's apparent alt. = 39° 42′ 34″
Correction = . . − 1. 44	Correction = . . +43. 33
Sun's true alt. = . 26° 49′ 53″	Moon's true altitude = 40° 26′ 7″

Moon's reduced horizontal parallax = 58′ 4″

App. cent. dist. = 108° 0′ 25″ N.V. S. = 1. 309132
Diff. of ap. alts. = 12. 50. 57 N.V. S. = . 025041 Log. diff. = 9. 995485

Remainder = 1. 284091. Log. = . 6. 108596

Natural number = ... 1. 270809 Log. = . 6. 104081
Diff. of true alts. = 13° 36′ 14″ N.V.S. = . 028054

True cent. dis. = 107° 23′ 21″ N.V.S. = 1. 298863
Dist. at 15 hrs = 108. 6. 55 Diff. = 0° 43′ 34″ Prop. log. = 6161
Dist. at 18 hrs = 106. 32. 42 Diff. = 1. 34. 13 Prop. log. = 2811

Portion of time = ... 1h 23m 14s Prop. log. = 3350
Time corresponding to first distance = 15. 0. 0

Apparent time of obs. at Greenwich = 16h 23m 14s

To find the apparent Time at the Place of Observation :—

Lat. of the place of obs. = 43° 17′ 0″ S. . . . Log. secant = 0. 137885
Sun's reduced declination = 4. 13. 10 S. . . . Log. secant = 0. 001179

Sun's merid. zen. dist. = 39° 3′ 50″ Nat. V. S. = 223556
Sun's true central alt. = 26. 49. 53 N. co-V. S. = 548634

Remainder = 325078 Log. = 5. 511988

Sun's horary distance from noon = 3h 45m 55s Log. rising = 5. 651052

Apparent time at the place of obs. = 20h 14m 5s

Apparent time at the place of obs. = $20^{h}\,14^{m}\,5^{s}$
Appar. time of obs. at Greenwich = 16. 23. 14

Long. of the place of obs., in time = $3^{h}\,50^{m}\,51^{s}$ = 57°42′45″ east.

Example 4.

April 1st, 1825, in latitude 49°30′ S., and longitude 61°30′ E., by account, at $11^{h}\,29^{m}$ per watch, *not regulated,* the mean of several distances between the moon's remote limb and the star Antares was 76°48′27″, and, at the same time, the mean of an equal number of altitudes of the moon's lower limb was 39°10′12″, and that of the star, east of the meridian, 37°56′43″; the height of the eye above the level of the horizon was 23 feet; required the longitude of the place of observation?

Time, per watch, =	$11^{h}\,29^{m}\,0^{s}$	Dist. ☽'s remote limb =	76°48′27″
Long. 61°30′ E. =	− 4. 6. 0	Moon's semi-diam. =	−16. 50
Reduced time =	$7^{h}\,23^{m}\,0^{s}$	Apparent distance =	76°31′37″
Altitude of Antares =	37°56′43″	Alt. ☽'s lower limb =	39°10′12″
		Moon's semi-diam. =	+16. 50
Dip of the horizon =	− 4. 36	Dip of the horizon =	− 4. 36
Apparent altitude =	37°52′ 7″	Apparent altitude =	39°22′26″
Correction =	− 1. 14	Correction =	+46. 9
Star's true altitude =	37°50′53″	Moon's true altitude =	40° 8′35″

Moon's reduced horizontal parallax = 61′11″

Sum of app. alts. = 77°14′33″ N.V.S. sup. = 1. 220825
App. cent. dist. = 76. 31. 37 Nat.V. sine = .767012 Log.diff. = 9. 995271

Remainder = .453813 Log. = 5. 656877

Natural number = ...448898 Log. = 5. 652148
Sum of true alts. = 77°59′28″ N.V.S. sup. = 1. 208063

True central dist. = 76° 3′55″ } Nat.V.S. = .759165
Dist. at 6 hours = 76. 57. 9 } Diff. = 0°53′14″ Prop. log. = 5291
Dist. at 9 hours = 75. 3. 29 } Diff. = 1. 53. 40 Prop. log. = 1996

Portion of time = ... $1^{h}\,24^{m}\,17^{s}$ Prop. log. = 3295
Time corresponding to first distance = 6. 0. 0

Apparent time of obs. at Greenwich = $7^{h}\,24^{m}\,17^{s}$

To find the apparent Time at the Place of Observation :—

Lat. of the place of obs. =	49°30′ 0″S. . . .	Log. secant=0.187456
Star's reduced declin. =	26. 2. 2 S. . . .	Log. secant=0.046465
Star's merid. zenith dist.=	23°27′58″	Nat. co-sin. 917296
Star's true altitude = .	37.50.53	Nat. sine =613570
	Remainder =	303726 Log.=5.482482
Star's horary distance, east of the mer.=	$4^h\ 5^m 23^s$	Log. rising=5.716403
Star's reduced right ascension = .	16.18.42	
Right ascension of the meridian = .	$12^h 13^m 19^s$	
Sun's reduced right ascension = . .	0.43.19	
Apparent time at the place of observ.=	$11^h 30^m\ 0^s$	
App. time of observ. at Greenwich =	7.24.17	

Longitude of the place of obs., in time= $4^h\ 5^m 43^s$ = 61°25′45″ east.

Example 5.

April 22d, 1825, in latitude 40°10′ N., and longitude 55°17′ W., by account, at $0^h 23^m$ per watch, *not regulated*, the mean of several observed distances between the nearest limbs of the sun and moon was 48°47′46″, and, at the same time, the mean of an equal number of altitudes of the sun's lower limb was 61°26′44″, and that of the moon's upper limb 48°46′32″; the height of the eye above the level of the horizon was 21 feet; required the longitude of the place of observation ?

Time, per watch, = $0^h 23^m\ 0^s$	Dist. nearest limbs of moon and sun = 48°47′46″
Long. 55°17′W. = +3.41. 8	Sun's semi-diameter = +15.56
Reduced time = . $4^h\ 4^m\ 8^s$	Moon's semi-diameter = +15.32
	Apparent distance = 49°19′14″
Alt. sun's lower limb = 61°26′44″	Alt. of ☽'s upp. limb = 48°46′32″
Sun's semi-diameter = +15.56	Moon's semi-diameter = −15.32
Dip of the horizon = − 4.24	Dip of the horizon = − 4.24
Sun's app. altitude = 61°38′16″	Moon's apparent alt. = 48°26′36″
Correction = . . − 0.27	Correction = . . +36.29
Sun's true altitude = 61°37′49″	Moon's true altitude = 49° 3′ 5″

Moon's reduced horizontal parallax = 56′ 16″

Half the apparent central distance =	24° 39′ 37″		
Half difference of the app. alts. =	6. 35. 50		
		Log. diff. =	9.994848
Sum = ...	31° 15′ 27″	Log. sine =	9.715071
Difference = ...	18. 3. 47	Log. sine =	9.491451
		Const. log. =	6.301030
Natural number = ...	317980	Log. = . .	5.502400
Diff. of true alts. = 12° 34′ 44″ N.V. sine	024003		

True cent dist. = 48° 51′ 4″	N.V. sine 341983		
Dist. at 3 hours = 48. 19. 26	Diff. = 0° 31′ 38″	Prop. log. =	7551
Dist. at 6 hours = 49. 47. 57	Diff. = 1. 28. 31	Prop. log. =	3082
Portion of time = ...	1h 4m 19s	Prop. log. =	4469
Time corresponding to first distance =	3. 0. 0		
Apparent time of obs. at Greenwich =	4h 4m 19s		

To find the apparent Time at the Place of Observation:—

Note.—Since the apparent time cannot be inferred with sufficient accuracy from the sun's altitude, on account of the proximity of that object to the meridian, it must be deduced from the moon's true central altitude; as thus:—

Lat. of place of obs. = 40° 10′ 0″ N.	Log. secant =	.116809
Moon's correct. dec. = 23. 13. 46 N.	Log. secant =	.036716
☽'s mer. zen. dist. = 16° 56′ 14″	Nat. V. S =	043376
Moon's true cent. alt. = 49. 3. 5	Nat. co-V. S =	244702
Remainder = 201326	Log. =	5.303900

Moon's horary dist., east of the mer. =	2h 57m 59s Log. rising = 5.45742.5
Moon's corrected right ascension = .	2. 51. 1
Right ascension of the meridian = .	2h 23m 2s
Sun's reduced right ascension = . . .	2. 0. 4
Apparent time at the place of observ. =	0h 22m 58s
Apparent time of observ. at Greenwich =	4. 4. 19

Longitude of the place of obs., in time = 3h 41m 21s = 55° 20′ 15″ west.

Example 6.

May 6th, 1825, in latitude 34°45′ S., and longitude 33°30′ E., by account, at $21^h 30^m$ per watch, *not regulated*, the mean of several observed distances between the nearest limbs of the sun and moon was 119°50′38″; and, at the same time, the mean of an equal number of altitudes of the sun's lower limb (imperfectly observed, owing to an obstructed horizon,) was 27°13′27″, and that of the moon's upper limb (also imperfectly observed,) 19°24′12″; the index error of the sextant used in measuring the distances was 2′25″ subtractive, and the height of the eye above the level of the horizon 19 feet; required the true longitude of the place of observation?

Time, per watch, =	$21^h 30^m 0^s$	Dist. nearest limbs of moon and sun =	119°50′38″
Long. 33°30′ E., =	−2. 14. 0	Index error = . .	− 2. 25
Reduced time = .	$19^h 16^m 0^s$	Sun's semi-diameter =	+15. 52
		Moon's semi-diameter =	+15. 30
		Apparent distance =	120°19′35″
Alt. of sun's low. limb =	27°13′27″	Alt. of ☽'s upp. limb =	19°24′12″
Sun's semi-diam. =	+15. 52	Moon's semi-diameter =	−15. 30
Dip of the horizon =	− 4. 11	Dip of the horizon =	− 4. 11
Sun's apparent alt. =	27°25′ 8″	Moon's apparent alt. =	19° 4′31″
Correction = . . .	− 1. 41	Correction = . .	+50. 44
Sun's true altitude =	27°23′27″	Moon's true altitude =	19°55′15″

Moon's reduced horizontal parallax = 56′34″

Half the app. central distance =	60° 9′47½″	
Half sum of the appar. altitudes =	23. 14. 49½	Log. diff. = 9. 997841
Sum = ...	83°24′37″	Log. co-sine = 9. 059787
Difference = ...	36. 54. 58	Log. co-sine = 9. 902827
		Constant log. = 6. 301030
Natural number = ...	182593	Log. = 5. 261485
Sum of true alts. = 47°18′42″	N.V.S. sup. = 1. 678010	

True cent. dist. = 119°41′50″	Nat. V. S. = 1. 495417	
Dist. at 18 hours = 120. 19. 47	Diff. = 0°37′57″	Prop. log. = 6761
Dist. at 21 hours = 118. 50. 22	Diff. = 1. 29. 25	Prop. log. = 3039
Portion of time = ...	$1^h 16^m 24^s$	Prop. log. = 3722
Time corresponding to first distance =	18. 0. 0	
Apparent time of observ. at Greenwich =	$19^h 16^m 24^s$	

Since the obstruction of the horizon prevented the altitudes of the objects from being taken to that degree of accuracy which is so essentially necessary to be observed when the apparent time is to be inferred from their altitudes (though sufficiently exact to be employed in the reduction of the apparent to the true central altitude),—therefore, in the afternoon, that is, on May 7th, at 1h 53m per same watch, the sun's altitude was again observed, and, when reduced to the true, was found to be 31° 44′ 28″; at that time the latitude of the ship was 34° 50′ S. Hence the apparent time, the error of the watch, and the longitude of the place of observation, are obtained as follows :—

Time of observing the sun's altitude, per watch, = 1h 53m 0s
Time of observing the lunar distance, per watch, = 21. 30. 0

Interval = ... 4h 23m 0s
Apparent time of lunar observation at Greenwich = 19. 16. 24

App. time at Greenwich of observing the sun's alt. = 23h 39m 24s

Sun's declination at noon, May 6th, = ... 16° 31′ 51″ N.
Correction of ditto for 23h 39m 24s = ... +16. 27

Sun's reduced declination = ... 16° 48′ 18″ N.

Lat. of place of obs. = 34° 50′ 0″ S. . . . . . Log. secant = 0. 085754
Sun's reduced dec. = 16. 48. 18 N. . . . . . Log. secant = 0. 018955

Sun's mer. zen. dist. = 51° 38′ 18″ Nat. V. S. = 379376
Sun's true altitude = 31. 44. 28 N. co-V. S. = 473918

Remainder = 94542 Log. = 4. 975625

App. time of observing the sun's altitude = 1h 53m 35s Log. rising = 5. 08033.4
App. time at Greenw. of obs. sun's alt. = 23. 39. 24

Longitude, in time = ... 2h 14m 11s = 33° 32′ 45″ east.

Note.—This is the longitude of the meridian where the sun's altitude was observed for the purpose of finding the apparent time.

Remark.—In place of finding the interval between the time of observing the lunar distance and that of taking the sun's altitude, as above, this part of the operation may be performed as follows; which, perhaps, may be more intelligible to those who are not very conversant with this subject.

Apparent time of observing the sun's altitude =	$1^h 53^m 35^s$
Time of observing ditto, per watch, = . .	1. 53. 0
Watch slow for apparent time = ...	$0^m 35^s$
Time, per watch, of obs. the lunar distance =	21. 30. 0
Apparent time of observing the lunar dist. =	$21^h 30^m 35^s$
Apparent time of ditto at Greenwich = .	19. 16. 24
Longitude of the place where the error of the watch was found, in time = . . .	$2^h 14^m 11^s = 33°32'45''$ E.

Example 7.

June 22d, 1825, in latitude 50°10' N., and longitude 45°7' W., by account, at $3^h 0^m 5^s$ apparent time, the mean of several observed distances between the moon's remote limb and the planet Venus was 118°41'48", and, at the same time, the mean of an equal number of altitudes of the moon's upper limb was 30°18'25", and that of the planet's centre 15°11'47"; the index error of the sextant used in measuring the distances was 2'10" additive, and the height of the eye above the level of the sea 18 feet; required the true longitude of the place of observation?

Apparent time = .	$3^h 0^m 5^s$
Long. 45°7' W. =	3. 0. 28
Reduced time = .	$6^h 0^m 33^s$
Alt. of Venus' centre =	15°11'47"
Dip of the horizon =	− 4. 4
Venus' apparent alt. =	15° 7'43"
Correction = . .	− 3. 9
Venus' true altitude =	15° 4'34"

Dist. of Moon's remote limb from Venus =	118°41'48"
Index error = . .	+ 2. 10
Moon's semi-diameter =	−16. 16
Apparent distance =	118°27'42"
Alt. of ☽'s up. limb =	30°18'25"
Moon's semi-diameter =	−16. 16
Dip of the horizon =	− 4. 4
Moon's apparent alt. =	29°58' 5"
Correction = . .	+49. 40
Moon's true altitude =	30°47'45"

Moon's reduced horizontal parallax = 59'13"

Apparent distance = ...	118°27'42"
Venus' apparent altitude = .	15. 7. 43
Moon's apparent altitude = .	29. 58. 5
Sum = ...	163°33'30"

Sum = ...	163°33′30″	
		Log. diff. = 9.996430
Half sum = ...	81°46′45″	Log. co-sine = 9.155302
Remainder = ...	36.40.57	Log. co-sine = 9.904152
Natural number = ...	113732	Log.=9.055884
Twice the natural number = ...	227464	
Sum of true alts.=45°52′19″ Nat. V. S. sup.=	1.696264	

True cent. dist.=117°57′23″	Nat. V. sine=1.468800	
Dist. at 6 hours=117.56.42	Diff. = 0° 0′41″	Prop. log. = 2.4206
Dist. at 9 hours=119.39. 7	Diff. = 1.42.25	Prop. log. = .2449
Portion of time = ...	0ʰ 1ᵐ12ˢ	Prop. log. = 2.1757
Time corresponding to first distance =	6. 0. 0	
Apparent time of obs. at Greenwich =	6ʰ 1ᵐ12ˢ	
Apparent time at the place of observ. =	3. 0. 5	
Longitude of the place of obs., in time =	3ʰ 1ᵐ 7ˢ	= 45°16′45″ west.

Remark.—In taking a *lunar observation,* it is customary to have three assistants, two of whom are to observe the altitudes of the objects at the moment that the principal observer measures the distance; the third is to be provided with a watch, showing seconds, and to note down carefully the respective times of observation, with the corresponding distances and altitudes. But, since it sometimes happens, particularly in small ships, that the necessary assistant observers cannot be in readiness, or at liberty to attend, the following example is given, by which it will be seen how one person may take the whole of the observations himself, without any other assistant than merely a person to note down the times of observation, per watch, with their respective distances and altitudes.

Example 8.

July 6th, 1825, in latitude 49°13′ N., and longitude 42°22′ W., by account, the following observations were made, in order to determine the true longitude; the index error of the sextant used in measuring the distances was 1′40″ subtractive, and the height of the eye above the level of the sea 17 feet.

Appar. Time.			Mean Time.	Mean Altitude.
21ʰ 8ᵐ32ˢ	Alt. of sun's low. limb=	46°58′ 0″	21ʰ 9ᵐ32ˢ	47° 7′ 0″
21. 9.32	Ditto	47. 7. 0		
21.10.32	Ditto	47.16. 0		

Appar. Time.			Mean Time.	Mean Altitude.
21h 11m 37s	Alt. of ☽'s upp. limb=	23° 51′ 30″		
21. 12. 37	Ditto	23. 42. 20	21h 12m 37s	23° 42′ 20″
21. 13. 37	Ditto	23. 33. 10		
21. 14. 50	Observed distance =	98. 58. 50		
21. 16. 0	Ditto	98. 58. 20		
21. 17. 10	Ditto	98. 57. 50	21. 17. 10	98. 57. 50
21. 18. 20	Ditto	98. 57. 20		
21. 19. 30	Ditto	98. 56. 50		
21. 20. 43	Alt. of ☽'s upp. limb=	20. 9. 40		
21. 21. 43	Ditto	20. 0. 0	21. 21. 43	20. 0. 3
21. 22. 43	Ditto	19. 50. 30		
21. 23. 48	Alt. of sun's low. limb=	49. 14. 0		
21. 24. 48	Ditto	49. 22. 40	21. 24. 48	49. 22. 40
21. 25. 48	Ditto	49. 31. 20		

To find the Sun's Altitude at the Time of taking the mean Distance :—

1st time 21h 9m 32s 1st alt. 47° 7′ 0″ 1st time 21h 9m 32s 1st alt. 47° 7′ 0″
2d time 21. 24. 48 2d alt. 49. 22. 40 Time of mean dist. } 21. 17. 10

As 0h 15m 16s is to 2° 15′ 40″ so is 0h 7m 38s to + 1. 7. 50

Reduced observed altitude of the sun's lower limb = . .	48° 14′ 50″
Sun's semi-diameter = ...	+15. 46
Dip of the horizon = ...	− 3. 57
Sun's apparent central altitude = ...	48° 26′ 39″
Correction of the sun's apparent altitude = ...	− 0. 44
True altitude of the sun's centre = ...	48° 25′ 55″

To find the Moon's Altitude at the Time of taking the mean Distance :—

1st time 21h 12m 37s 1st alt. 23° 42′ 20″ 1st time 21h 12m 37s 1st alt. 23° 42′ 20″
2d time 21. 21. 43 2d alt. 20. 0. 3 Time of mean dist. } 21. 17. 10

As 0h 9m 6s is to 3° 42′ 17″ so is 0h 4m 33s to − 1. 51. 8

Reduced observed altitude of the moon's upper limb = . .	21° 51′ 12″
Moon's true semi-diameter = ... ,	−14. 52
Dip of the horizon = ...	− 3. 57
Moon's apparent central altitude = ...	21° 32′ 23″
Correction of the moon's apparent altitude = ...	+48. 4
True altitude of the moon's centre = ...	22° 20′ 27″

Moon's reduced horizontal parallax = ... 54′ 14″

Obs.dist.betw. ☽ & ☉=98°57′50″	App. time of observ.=21ʰ17ᵐ10ˢ
Index error of sextant= − 1.40	Longitude 42°22′W.,
Sun's semi-diameter = +15.46	in time = . . + 2.49.28
Moon's semi-diameter= +14.52	
	Reduced time past noon,
Appar. central dist. = 99°26′48″	July 7th, = . . 0ʰ 6ᵐ38ˢ

Half the app. central distance=	49°43′24″		
Half the diff. of the app. alts. =	13.27. 8		
		Log. diff. = .	9.997660
Sum = ...	63°10′32″	Log. sine = .	9.950556
Difference = ...	36.16.16	Log. sine = .	9.772033
		Sum =	19.720249
Arch = ...	46°26′21″	Log. sine = .	9.860124½
Half. the diff. of the true alts.=	13. 2.44		
Sum = ...	59°29′ 5″	Log. co-sine =	9.705665
Difference = ...	33.23.37	Log. co-sine =	9.921639
		Sum =	19.627304
Half the true distance = .	49°22′30″	Log. co-sine =	9.813652

True central distance = . .	98°45′ 0″	Diff. 0° 3′ 3″ P.log.=1.7710
Distance at 0 hour, or noon, =	98.48. 3	
Distance at 3 hours = . .	97.26.27	Diff. 1.21.36 P.log.= .3436

Portion of time = ... 0ʰ 6ᵐ44ˢ Prop. log.=1.4274
Time corresponding to first distance = . 0. 0. 0

Apparent time of observ. at Greenwich = 0ʰ 6ᵐ44ˢ
Apparent time at the place of observation=21. 17. 10

Longitude of the place of observ., in time = 2ʰ49ᵐ34ˢ=42°23′30″ west.

Note.—Proportional logarithms will be found very convenient in the reduction of the altitudes of the objects to the time of taking the mean lunar distance: thus, to the arithmetical complement of the proportional logarithm of the first term, add the proportional logarithms of the second and third terms; and the sum, abating 10 in the index, will be the proportional logarithm of the reduction of altitude.—See Example, page 75 or 76.

Remarks.—In taking the means of the several observations, those which are evidently doubtful or erroneous ought to be rejected. A doubtful altitude or distance may be readily discovered, by observing if the successive differences of altitude or distance be proportional to those of the times of observation. If, however, the time (which is supposed to be accurately noted,) and two of the observations be correct, the erroneous observation may be easily rectified by the rule of proportion.

In order to attain to the greatest accuracy in deducing the mean from a series of observations, these ought to be taken at equal intervals of time, as nearly as possible; such as, *one minute, one minute and a half, or two minutes.*

Problem X.

Given the apparent Time, the observed Distance between the Moon and Sun, a fixed Star, or a Planet, the Latitude, and the Longitude by account; to find the true Longitude.

Rule.

Compute the true and the apparent altitude of each object's centre, by Problem I., II., III., or IV., between pages 404 and 410, according as the moon is compared with the sun, a fixed star, or a planet.

Reduce the observed to the apparent central distance, by the rule to Problem IX., page 456; with which, and the computed altitudes of the objects, let the true central distance be determined, by any of the methods given in Problem VII., between pages 433 and 453; and find the apparent time at Greenwich answering to the true central distance, thus computed, by Problem VIII., page 454. Then, the difference between the apparent times of observation at the ship and at Greenwich will be the longitude of the ship or place, in time; which is to be called east or west, according as the apparent time at the place of observation is greater or less than that at Greenwich.

Example 1.

August 4th, 1825, in latitude 40°25′ N., and longitude 56°36′ W., by account, at 19h10m35s, apparent time, the mean of several observed distances between the nearest limbs of the sun and moon was 107°3′47″; required the true longitude of the place of observation?

Appar. time of obs.= 19ʰ10ᵐ35ˢ	Obs. dist. between moon and sun = 107° 3′47″
Long. 56°36′ W., in time = . . . 3. 46. 24	Sun's semi-diameter= +15. 48
	Moon's semi-diam. = +14. 57
Reduced time = . 22ʰ56ᵐ59ˢ	Appar. central dist.=107°34′32″

To find the Sun's true and apparent Altitude:—

Sun's dist. from merid.=4ʰ49ᵐ25ˢ . . . . . Log. rising = 5. 843150
Lat. of place of obs. = 40°25′ 0″N. . . . Log. co-sine = 9. 881584
Sun's reduced dec. = 17. 1. 43 N. . . . Log. co-sine = 9. 980530

Sun's mer. zen. dist.= 23°23′17″ Nat. V. S.=082163
Natural number=507299 Log.=5. 705264

Sun's true central alt.=24°14′19″N. co-V. S.=589462
Correction of altitude= + 1. 59

Sun's apparent alt. = 24°16′18″

To find the Moon's true and apparent Altitude:—

App. time of observ.=19ʰ10ᵐ35ˢ	Moon's red. horiz. par. = 54′16″
Sun's reduced R. A. = 9. 0. 28	
R. A. of the merid. = 4ʰ11ᵐ 3ˢ	Moon's red. semi-diam.= 14′47″
Moon's red. R. A. = 1. 28. 35	Augmentation of ditto = + 10
	Moon's true semi-diam.=14′57″

Moon's dist. from mer.=2ʰ42ᵐ28ˢ . . . Log. rising = 5. 381870
Lat. of place of obs. = 40°25′ 0″N. . . . Log. co-sine=9. 881584
Moon's reduced dec.= 13. 44. 4 N. . . . Log. co-sine=9. 987402

Moon's mer. zen. dist.=26°40′56″ Nat. V. S.=106489
Natural number = 178174 Log.=5. 250856

Moon's true cent. alt.=45°40′15″ N.co-V. S.=284663
Correction of altitude= −37. 0

Moon's apparent alt.= 45° 3′15″

To find the true central Distance, and, hence, the Longitude of the Place of Observation:—

Half the app. cent. dist.=	53°47′16″		
Half sum of the ap. alts.=	34. 39. 46½		
		Log. diff. = ...	9.995327
Sum = ...	88°27′ 2½″	Log. co-sine = . . .	8.431961
Difference = . . .	19. 7. 29½	Log. co-sine = . . .	9.975343
		Sum =	18.402631
Arch = ...	80°51′10″	Log. co-sine = . . .	9.201315½
Half sum of true alts.=	34. 57. 17		
Sum = ...	115°48′27″	Log. sine = . . .	9.954369
Difference = . . .	45. 53. 53	Log. sine = . . .	9.856186
		Sum =	19.810555
Half the true distance =	53°31′ 3″	Log. sine = . . .	9.905277½

True central distance=	107° 2′ 6″	Diff.=0°52′56″	Prop. log.= .5315
Dist. at 21 hours =	107. 55. 2	Diff.=1. 21. 32	Prop. log.= .3439
Dist. at 24 hours, or noon	106. 33. 30		
Portion of time = ...		1h56m52s	Prop. log.= .1876
Time corresponding to first distance = .		21. 0. 0	
Apparent time of observ. at Greenwich =		22h56m52s	
Apparent time at the place of observation=		19. 10. 35	
Longitude of the place of observ., in time =		3h46m17s	=56°34′15″ W.

Example 2.

September 27th, 1825, in latitude 36°15′ S., and longitude 47°30′ E., by account, at 14h58m10s apparent time, the mean of several observed distances between the moon's remote limb and the star Aldebaran was 55°17′36″; required the true longitude of the place of observation?

Apparent time of observation = ...	14h58m10s
Longitude 47°30′ E., in time = . .	− 3. 10. 0
Reduced time = ...	11h48m10s
Observed distance of moon's remote limb =	55°17′36″
Moon's true semi-diameter = ...	−14. 49
Apparent central distance = ...	55° 2′47″

Sun's right ascension at noon, Sept. 27th, = $12^h 14^m 54^s$
Correction of ditto for $11^h 48^m 10^s$ = ... + 1.47

Sun's reduced right ascension = ... $12^h 16^m 41^s$

To find the true and apparent Altitude of the Star Aldebaran:—

Apparent time of observation = ... $14^h 58^m 10^s$
Sun's reduced right ascension = ... 12. 16. 41

Right ascension of the meridian = ... $3^h 14^m 51^s$
Aldebaran's right ascension = ... 4. 25. 55

Aldebaran's distance from the meridian = . $1^h 11^m 4^s$

Aldebaran's dist. from merid. = $1^h 11^m 4^s$ . .	Log. rising = 4. 678460
Lat. of the place of observ. = $36° 15' 0''$S. .	Log. co-sine = 9. 906575
Aldebaran's reduced dec. = 16. 9. 1 N. .	Log. co-sine = 9. 982513
Aldebaran's mer. zen. dist. = $52° 24' 1''$ N.V.S. = 389859	
Natural number = 36944	Log. = 4. 567548

Star's true altitude = $34° 58' 24''$ Nat. co-V. S. = 426803
Correction of alt. = + 1. 21

Star's appar. alt = $34° 59' 45''$

To find the true and the apparent Altitude of the Moon:—

Moon's reduced semi-diameter = ... $14' 42''$
Augmentation of ditto = ... + 7

Moon's true semi-diameter = ... $14' 49''$

Moon's reduced horizontal parallax = $53' 55''$

R. A. of the meridian = . $3^h 14^m 51^s$
Moon's reduced right ascen. = 0. 42. 28

Moon's dist. from the merid. = $2^h 32^m 23^s$ . . .	Log. rising = 5. 328420
Lat. of the place of observ. = $36° 15' 0''$S. .	Log. co-sine = 9, 906575
Moon's reduced declination = 9. 19. 30 N. .	Log. co-sine = 9. 994223
Moon's merid. zenith dist. = $45° 34' 30''$ N.V.S. = 300025	
Natural number = 169519	Log. = 5. 229218

Moon's true altitude = $32° 2' 10''$ N. co-V. S. = 469544
Correction of altitude = − 44. 32

Moon's apparent alt. = $31° 17' 38''$

To find the true central Distance, and, hence, the Longitude of the Place of Observation :—

Half the ap. cent. dist. =	27°31′23½″		
Half diff. of app. alts. =	1.51. 3½		
		Log. diff. 19.996651	
Sum = ...	29°22′27″	Log. sine 9.690648	
Difference = . . .	25.40.20	Log. sine 9.636711	
Sum = ...		39.324010	
Half sum = ...		19.662005 . .	19.662005
Half diff. of true alts. =	1°28′ 7″	Log. sine 8.408737	
Arch = ...	86°48′20″	Log. tan. 11.253268	Log. sin. 9.999325
Half the true distance =	27°22′54½″		Log. sine = 9.662680

True central distance = 54°45′49″ } Diff. = 1°22′18″ Prop. log. = .3399
Distance at 9 hours = 56. 8. 7 }
Dist. at 12 hrs, or midnt. 54.40. 8 } Diff. = 1.27.59 Prop. log. = .3109

Portion of time = ... 2ʰ48ᵐ22ˢ Prop. log. = .0290
Time corresponding to first distance = 9. 0. 0

Apparent time of observ. at Greenwich = 11ʰ48ᵐ22ˢ
Apparent time at the place of observ. = 14.58.10

Longitude of the place of observ., in time = 3ʰ 9ᵐ48ˢ = 47°27′ east.

Example 3.

December 25th, 1825, in latitude 39°13′ N.. and longitude 42°55′ W., by account, at 14ʰ49ᵐ27ˢ apparent time, the mean of several observed distances between the moon's nearest limb and the planet Mars was 94°40′22″; required the longitude of the place of observation?

Apparent time of observation = ...	14ʰ49ᵐ27ˢ
Longitude 42°55′ W., in time = . . .	+ 2.51.40
Reduced time = ...	17ʰ41ᵐ 7ˢ
Observed distance of moon's nearest limb =	94°40′22″
Moon's true semi-diameter = ...	+15.41
Apparent central distance = ...	94°56′ 3″

Sun's right ascension at noon, December 25th, =	$18^h 15^m 13^s$
Correction of ditto for $17^h 41^m 7^s$ = . . .	+ 3. 16
Sun's reduced right ascension = ...	$18^h 18^m 29^s$
Apparent time of observation = ...	14. 49. 27
Right ascension of the meridian = . . .	$9^h\ 7^m 56^s$

To find the true and the apparent Altitude of the Planet Mars :—

Horizontal parallax of Mars = 5 seconds.

R. A. of the meridian = $9^h\ 7^m 56^s$
Reduced R. A. of Mars = 13. 12. 28

Mars' dist. from merid. = $4^h\ 4^m 32^s$ . . .	Log. rising =	5. 713680
Lat. of place of obs. = 39°13′ 0″N. . .	Log. co-sine =	9. 889168
Reduced dec. of Mars = 8. 52. 53 S. . .	Log. co-sine =	9. 994761

Mars' mer. zen. dist. = 48° 5′53″ Nat. V. S. = 332142
Natural number = 395921 Log. = 5. 597609

True altitude of Mars = 15°46′46″N. co-V. S. = 728063
Correction of altitude = + 3. 14

App. alt. of Mars = 15°50′ 0″

To find the true and the apparent Altitude of the Moon's Centre :—

Moon's reduced horizontal parallax = ... 56′48″
Moon's reduced semi-diameter 15′28″ + augmentation 13″ = 15′41″

R. A. of the merid. = $9^h\ 7^m 56^s$
Moon's red. R. A. = 7. 1. 30

☽'s dist. from merid. $2^h\ 6^m 26^s$	Log. rising =	5. 171280
Lat. of place of obs. 39°13′ 0″N.	Log. co-sine =	9. 889168
Moon's red. dec. = 19. 49. 23 N.	Log. co-sine =	9. 973472

☽'s mer. zen. dist. 19°23′37″ Nat. vers. sine = 056740
Natural number = 108123 Log. = 5. 033920

☽'s true cent. alt. = 56°37′48″ Nat. co-V. S. = 164863
Correction of alt. = —31. 7

Moon's ap. alt. = 56° 6′41″

To find the true central Distance, and, hence, the Longitude of the Place of Observation:—

Half ap. cent. dist. = 47°28′ 1½″
Half sum of ap. alts. = 35. 58. 20½

		Log. diff. 19.994221		
Sum = ...	83°26′22″	Log. co-sin. 9.057868		
Difference = . .	11.29.41	Log. co-sin. 9.991201		
Sum = ...		39.043290		
Half sum = ...		19.521645	. .	19.521645
Half sum of true alts. =	36°12′17″	Log. co-sin. 9.906826		
Arch = ...	24°19′33″	Log. sine = 9.614819	Log. tang.	9.655197
Half the true dist. =	47°19′48½″	Log. sine =	. . .	9.866448

True central dist. =	94°39′37″			
Dist. at 15 hours =	96. 4. 2	Diff. = 1°24′25″	Prop. log. =	.3288
Dist. at 18 hours =	94. 30. 7	Diff. = 1. 33. 55	Prop. log. =	.2825
Portion of time = ...		2ʰ41ᵐ47ˢ	Prop. log. =	.0463
Time corresponding to first distance =		15. 0. 0		

Apparent time of observ. at Greenwich = 17ʰ41ᵐ47ˢ
Apparent time at the place of observ. = 14. 49. 27

Long. of the place of observ., in time = 2ʰ52ᵐ20ˢ = 43°5′ west.

Example 4.

December 30th, 1825, in latitude 46°30′ S., and longitude 84°15′ E., by account, at 21ʰ10ᵐ15ˢ apparent time, the mean of several observed distances between the nearest limbs of the sun and moon was 107°2′7″, and, at the same time, the mean of an equal number of altitudes of the moon's upper limb was 15°40′24″; but, for want of the necessary assistants, the sun's altitude could not be taken; the height of the eye above the level of the horizon was 18 feet; required the true longitude of the place of observation?

Apparent time of observation = ...	21ʰ10ᵐ15ˢ
Longitude 84°15′ E., in time = ...	5.37. 0
Reduced time = ...	15ʰ33ᵐ15ˢ

Observed distance between the moon and sun = .	107° 2′ 7″
Sun's semi-diameter = ...	+16. 18
Moon's semi-diameter = ...	+16. 5
Apparent central distance = ...	108°34′30″

Observed altitude of the moon's upper limb = .	15°40′24″
Moon's true semi-diameter = ...	−16. 5
Dip of the horizon = ...	− 4. 4
Moon's apparent altitude = ...	15°20′15″
Correction = ...	+53. 16
True altitude of the moon's centre = ...	15°13′31″
Moon's reduced horizontal parallax = ...	58′47″

To find the true and the apparent Altitude of the Sun's Centre:—

Sun's horary dist. from mer. = 2h49m45s . .	Log. rising =	5.418280
Lat. of the place of observ. = 46°30′ 0″ S. .	Log. co-sine =	9.837812
Sun's reduced declination = 23. 8. 14 S. .	Log. co-sine =	9.963583

Sun's mer. zenith distance = 23°21′46″ N.V.S. = 081988
Natural number = 165835 Log. = 5.219675

Sun's true cent. alt. = 48°46′46″ Nat. co-V. S. = 247823
Correction of ditto = + 0.43

Sun's apparent alt. = 48°47′29″

To find the true central Distance, and, hence, the Longitude of the Place of Observation:—

☉'s ap. alt.	48°47′29″		☉'s true central alt.	48°46′46″	
☽'s ap. alt.	15. 20. 15		☽'s true central alt.	16. 13. 31	
Sum =	64° 7′44″	N.V.S. sup. 1.436349	Sum	64°59′59″	N.V.S. sup. 1.422622
Diff. =	33. 27. 14	N.V.S. .165671	Diff.	32. 33. 15	N.V.S. .157117
	Arch first = . .	1.270678		Arch second =	1.265505

Sum of app. alts.=64° 7′44″Nat.V.S.=1.436349
App. cent. dist.=108°34′30″Nat.V.S.=1.318546

Arch third = ...	.117803	Log. =	5.071156
Arch second = ...	1.265505	Log. =	6.102264
Arch first = ...	1.270678	Log.ar.co=	3.895965
Natural number = ...	.117323	Log. =	5.069385

Sum of true alts.=64°59′59″N.V.S.sup.1.422622

True cent. dist.=107°46′34″	N.V.sine 1.305299		
Dist. at 15 hours=108. 4.31	Diff. = 0°17′57″	Prop. log.=	1.0012
Dist. at 18 hours=106. 27.28	Diff. = 1.37. 3	Prop. log.=	.2683

Portion of time = ...	$0^h33^m17^s$	Prop. log.=	.7329
Time corresponding to first distance =	15. 0. 0		
Apparent time of observ. at Greenwich=	$15^h33^m17^s$		
Apparent time at the place of observ. =	21. 10. 15		

Longitude of the place of obs., in time = $5^h36^m58^s$ = 84°14′30″ east.

Remark.—In Problem XXIX, page 320, of "The Young Navigator's Guide to the Sidereal and Planetary Parts of Nautical Astronomy," there is an interesting method given for reducing the apparent central distance between the moon and sun, or a fixed star, to the true central distance, by an instrumental operation; it being a correct mechanical mode of working the lunar observations by Gunter's scale and a pair of compasses.

Problem XI.

To find the Longitude of a Place by the Eclipses of Jupiter's Satellites.

First,

To know if an Eclipse will be visible at a given Place.

Rule.

Convert the mean time of the eclipse at Greenwich (as given in page III. of the month in the Nautical Almanac,) into apparent time, by Problem II., page 416; and let this time be reduced to the meridian of the place of observation, by Problem IV., page 297.

Now, if at this reduced time Jupiter be not less than 6 degrees above the horizon of the given place, and the sun be as many below it, or stars of the *third magnitude* be visible to the naked eye, the eclipse may be observed at that place: this, it is presumed, does not require to be illustrated by an example.

SECOND,

To find the Longitude of the Place of Observation of an Eclipse.

RULE.

Reduce the mean time of the eclipse at Greenwich into apparent time, by Problem II., page 416. Then, to the observed time of the eclipse, at the given place, apply the error of the watch for apparent time, deduced from observations of the sun's altitude, or from those of a fixed star, a planet, or the moon: hence the apparent time at the place of observation will be known. Now, the difference between this time and the apparent time at Greenwich will be the longitude of the place of observation in time; which will be east or west, according as the former is greater or less than the latter.

Example 1.

January 8th, 1825, in latitude 39°5′ N., and longitude 28°3′ W., by account, an immersion of the first satellite of Jupiter was observed at 8h.12m.59s., by a watch which was 1m.46s. fast for apparent time; required the true longitude of the place of observation?

Mean time of the eclipse at Greenwich =	10h.10m.29s.
Equation of time = ...	− 7.16
Apparent time of the eclipse at Greenwich =	10h. 3m.13s.

Time of observation, per watch, = . . .	8h.12m.59s.
Watch fast = ...	− 1.46
Apparent time at the place of observation =	8h.11m.13s.
Apparent time at Greenwich = ...	10. 3.13
Longitude of the place of observ., in time =	1h.52m. 0s. = 28°.0′.0″ west.

Note.—If Jupiter be far enough from the meridian at the time of observing an immersion or an emersion of one of his satellites, the apparent time of observation may be inferred directly from his altitude; and, if the altitude be taken at the same instant of observing the immersion or emersion of the satellite, the use of a watch will then become unnecessary.

Example 2.

January 2d, 1825, in latitude $39°51'10''$ N., and longitude $4°15'$ E., by account, an emersion of the first satellite of Jupiter was observed; and, at the same instant, the altitude of that planet's centre, east of the meridian, was found to be $28°49'30''$; the height of the eye above the level of the sea was 20 feet; required the true longitude of the place of observation?

Mean time of the emersion at Greenwich = .	$7^h\ 3^m 50^s$
Equation of time = ...	-14.31
Apparent time at Greenwich = ...	$6^h 49^m 19^s$
Observed altitude of Jupiter's centre = . .	$28°49'30''$
Dip of the horizon for 20 feet = ...	$-\ 4.\ 17$
Jupiter's apparent altitude = ...	$28°45'13''$
Refraction = $1'43''$ / Parallax = $-0.\ 2$ } Difference = ...	$-\ 1.41$
True altitude of Jupiter's centre = . . .	$28°43'32''$

Lat. of place of obs. = $39°51'10''$ N.	Log. secant = 0. 114812
Jupiter's red. dec. = 19. 5. 48 N.	Log. secant = 0. 024583
Jupiter's mer. z. dist. $20°45'22''$ Nat. V. S. = 064902	
Jupiter's true cent. alt. 28. 43. 32 N. co-V. S. = 519385	
Remainder = 454483	Log. = 5. 657518
Jupiter's horary dist., east of the merid. = $4^h 32^m 16^s$	Log. rising = 5. 796913

Jupiter's reduced right ascension = .	8. 43. 39
Right ascension of the meridian = .	$4^h 11^m 23^s$
Sun's reduced right ascension = . . .	21. 4. 52
Apparent time at the place of observ. =	$7^h\ 6^m 31^s$
Apparent time at Greenwich = . .	6. 49. 19

Longitude of the place of obs., in time = $0^h 17^m 12^s$ = $4°18'0''$ east.

Remarks.

An *immersion of a satellite* is, the instant of its entrance into the shadow of Jupiter; and an *emersion* is that of its re-appearance out of the shadow. The instant of an immersion is known by the *last appearance of the satellite;* that of an emersion, by its *first appearance.*

The eclipses of Jupiter's satellites afford the readiest, and, for general practice, the best method of determining the true longitudes of places on shore: but, since those eclipses cannot be distinctly observed except by means of telescopes of a high magnifying power,—and since these cannot possibly be used at sea, on account of the incessant motion of the vessel, which continually throws the object out of the field of view,—this method, therefore, though the very best at land, will be but of little, if any advantage to the mariner. It is to be observed, however, that this method of finding the longitude is not always available; because Jupiter passes so apparently close to the sun at certain intervals, that, for about six weeks in every year, both himself and his satellites are entirely lost in the superior splendour of the solar rays.

Problem XII.

To find the Longitude of a Place by an Eclipse of the Moon.

Rule.

Observe the times, per watch, (regulated to apparent time,) of the beginning and the end of the eclipse: the mean of these times will be the apparent time of the middle of the eclipse; the difference between which and that given in the Nautical Almanac, will be the longitude of the place of observation in time; which will be east or west, according as it is greater or less than the time at Greenwich.

Note.—If only the beginning or the end of the eclipse be observed, the apparent time of observation must be compared with the time answering to the corresponding phase in the Nautical Almanac; but, it must be remembered, that it will always be conducive to greater accuracy to observe the instants of both phases.

Example 1.

May 31st, 1825, in latitude 38°24′ N., and longitude 26°0′ E., by account, the beginning of the lunar eclipse was observed at $13^h 35^m 32^s$ per watch, and the end at $14^h 4^m 47^s$; the error of the watch was $2^m 13^s$ slow for apparent time; required the true longitude of the place of observation?

Beginning of the eclipse, per watch, = . .	$13^h 35^m 32^s$
End of ditto ditto = . .	14. 4. 7
Middle of the eclipse, per watch, = ...	$13^h 50^m 9\frac{1}{2}^s$
Error of the watch = ...	+ 2. 13
Apparent time of the middle of the eclipse =	$13^h 52^m 22\frac{1}{2}^s$
Apparent time of ditto at Greenwich = . .	12. 8. 30
Longitude of the place of observ., in time =	$1^h 43^m 52\frac{1}{2}^s = 25°58′7\frac{1}{2}″$E.

Example 2.

November 25th, 1825, in latitude 16°40′ N., and longitude 54°40′ E., by account, the beginning of the lunar eclipse was observed at $7^h 6^m 40^s$, and the end at $9^h 0^m 55^s$ per watch, not regulated. In order to find the error of the watch, the altitude of Aldebaran, east of the meridian, was taken at $8^h 7^m 30^s$, and found to be 28°42′30″; the height of the eye above the level of the horizon was 22 feet; required the true longitude of the place of observation?

Time, per watch, of observing the star's altitude =	$8^h\ 7^m 30^s$
Longitude 54°40′ E., in time = ...	3. 38. 40
Reduced time = ...	$4^h 28^m 50^s$
Sun's right ascension at noon = ...	$16^h\ 3^m 44^s$
Correction of ditto for $4^h 28^m 50^s$ = ...	+ 0. 45
Sun's reduced right ascension = ...	$16^h\ 4^m 29^s$
Observed altitude of Aldebaran = ...	28°42′30″
Dip of the horizon for 22 feet = ...	− 4. 30
Star's apparent altitude = ...	28°38′ 0″
Refraction = ...	− 1. 44
Star's true altitude = ...	28°36′16″

Lat. of the place of obs. = 16°40′ 0″N. . . . Log. secant = 0. 018639
Aldebaran's red. dec. = 16. 9. 5 N. . . . Log. secant = 0. 017489

Aldebaran's mer. z. dist. = 0°30′55″ Nat. V. S. = 000040
Aldebaran's true alt. = 28. 36. 16 N. co-V. S. = 521240

Remainder = 521200 Log. = 5. 717004

Star's horary distance, east of the mer. = $4^h 17^m 13^s$ Log. rising = 5. 753132
Star's reduced right ascension = . 4. 25. 57

Right ascension of the meridian = . $0^h\ 8^m 44^s$
Sun's reduced right ascension = . 16. 4. 29

Apparent time of obs. the star's alt. = $8^h\ 4^m 15^s$
Time of observation, per watch, = . 8. 7. 30

Watch fast for apparent time = . $3^m 15^s$

Beginning of the eclipse, per watch,	=	$7^h\ 6^m 40^s$
End of ditto ditto	=	$9.\ 0.55$
Middle of the eclipse, per watch,	=	$8^h\ 3^m 47\frac{1}{2}^s$
Error of the watch = ...		$-\ 3.15$
App. time of the middle of the eclipse	=	$8^h\ 0^m 32\frac{1}{2}^s$
Apparent time of ditto at Greenwich	=	$4.22.\ 0$

Longitude of the place of obs., in time $= 8^h 38^m 32\frac{1}{2}^s = 54^\circ 38' 7\frac{1}{2}''$ east.

Remarks.

From the two preceding examples, it is evident that the beginning and the end of the eclipse are the principal phases from which the longitude is to be found. If the observer be provided with a sextant, those phases may be observed to a tolerable degree of accuracy, by means of the largest telescope belonging to that instrument; or they may be observed with a good night telescope.

This method of finding the longitude at *sea* is evidently the most simple of any of the astronomical methods that have been proposed for that purpose; however, since the lunar eclipses happen so very seldom, there are but few opportunities of carrying it into practice: nevertheless, whenever such eclipses take place, the prudent mariner will do well to avail himself thereof, and to determine his longitude by them accordingly.

SOLUTION OF PROBLEMS RELATIVE TO THE VARIATION OF THE COMPASS.

Definitions.

The *variation of the compass* is the deviation of the points of the mariner's compass from the corresponding points of the horizon, and is denominated east or west variation accordingly.

East variation is, when the north point of the compass is to the eastward of the true north point of the horizon; *west variation* is, when the north point of the compass is to the westward of the true north point of the horizon.

The variation of the compass may be found by various methods, such as amplitudes, azimuths, transits, equal altitudes, rising and setting of the celestial objects, &c.

The *true amplitude* of any celestial object is, an arch of the horizon intercepted between the true east or west point thereof, and the object's centre at the time of its rising or setting.

The *magnetic amplitude* of an object is, the arch of the horizon that is intercepted between its centre, and the east or west point of the compass, at the time of its rising or setting; or, it is the compass bearing of the object when in the horizon of the eastern or western hemisphere.

The true amplitude of a celestial object is found by calculation; and the magnetic amplitude is found by an azimuth compass.

The *true azimuth* of a celestial object is, the angle contained between the true meridian and the vertical circle passing through the object's centre.

The *magnetic azimuth* is, the angle contained between the magnetic meridian and the azimuth, or vertical circle passing through the centre of the object; or, in other words, it is the compass bearing of the object, at any given elevation above the horizon.

The true azimuth of a celestial object is found by calculation; and the magnetic azimuth by an azimuth compass.

Problem I.

Given the Latitude of a Place, and the Sun's magnetic Amplitude; to find the Variation of the Compass.

Rule.

Reduce the apparent time of the sun's rising or setting to the meridian of Greenwich, by Problem III., page 297; to which time let the sun's declination at noon of the given day be reduced, by Problem V., page 298. Then, to the logarithmic secant of the latitude, add the logarithmic sine of the sun's reduced declination; and the sum, abating 10 in the index, will be the logarithmic sine of the true amplitude,—to be reckoned north or south of the true east or west point of the horizon, according to the name of the declination. Now, if the true amplitude, thus found, and the magnetic amplitude, observed per azimuth compass, be both north or both south, their *difference* is the variation; but if one be north and the other south, their *sum* is the variation:—and to know whether it be east or west, let the observer look directly towards that point of the compass representing the *true amplitude;* then, if the magnetic amplitude be to the *left hand* of this, the variation is easterly; but if to the *right hand,* it is westerly.

Example 1.

May 20th, 1825, in latitude 48°50′ N., and longitude 6°30′ W., at about $7^h 40^m$, the sun was observed to set W. 56°42′ N.; required the variation of the compass?

Estimated time of observation = ...	$7^h 40^m$
Longitude 6°30′ W., in time = ...	+26
Reduced time = ...	$8^h\ 6^m$

Sun's declination at noon, May 20th, ...	19°58′43″N.
Correction of ditto for $8^h 6^m$ = ...	+ 4.11
Sun's reduced declination = ...	20° 2′54″N.

Latitude of the place of observ.= 48°50′N.	Log. secant=	10.181608
Sun's reduced declination = . 20° 2′54″N.	Log. sine =	9.535057
True amplitude = . . . W. 31°23′ 8″N.	Log. sine =	9.716665
Magnetic amplitude = . . W. 56.42. 0 N.		

Variation = ... 25°18′52″; which is *west*, because the magnetic amplitude is to the *right hand* of the true amplitude.

Example 2.

July 10th, 1825, in latitude 18°40′ N., and longitude 73°45′ W., at about $17^h 29^m$, the sun was observed to rise E. 30°12′ N.; required the variation of the compass?

Estimated time of observation = ...	$17^h 29^m$
Longitude 73°45′ W., in time = ...	+ 4.55
Reduced time = ...	$22^h 24^m$

Sun's declination at noon, July 10th, . .	22°16′16″N.
Correction of ditto for $22^h 24^m$ = ...	− 7.13
Sun's reduced declination = ...	22° 9′ 3″N.

Latitude of the place of observ. =	18°40′ 0″N.	Log. secant =	10.023468
Sun's reduced declination = .	22. 9. 3 N.	Log. sine =	9.576395
True amplitude = ...	E. 23°27′ 7″N.	Log. sine =	9.599863
Magnetic amplitude = .	E. 30. 12. 0 N.		

Variation = ... 6°44′53″; which is *east*, because the magnetic amplitude is to the *left hand* of the true amplitude.

Example 3.

October 17th, 1825, in latitude 42°10′ N., and longitude 14°30′ W., at about 5h27m, the sun was observed to set W. 7°33′ N.; required the variation of the compass?

Estimated time of observation = . . .	5h27m 0s
Longitude 14°30′ W., in time = . .	+58. 0
Reduced time = ...	6h25m 0s

Sun's declination at noon, October 17th, =	9°15′19″ S.
Correction of ditto for 6h25m = . . .	+ 5.52
Sun's reduced declination = ...	9°21′11″ S.

Latitude of the place of observ. =	42°10′ 0″N.	Log. secant =	10.130067
Sun's reduced declination = .	9.21.11 S.	Log. sine =	9.210901
True amplitude = . . .	W. 12°39′57″ S.	Log. sine =	9.340968
Magnetic amplitude = .	W. 7.33. 0 N.		

Variation = ... 20°12′57″; which is *west*, because the magnetic amplitude is to the *right hand* of the true amplitude.

Remark.

In finding the variation of the compass by this method, the sun's amplitude should be taken, with an azimuth compass, when the altitude of his lower limb is equal to the sum of his semi-diameter and the dip of the horizon. Thus, if the sun's semi-diameter be 16′5″, and the dip of the horizon 4′17″ (for 20 feet), the sum = 20′22″ is the height which the lower limb of that object should be above the horizon, at the time of observing its amplitude.

If the index of the quadrant be set to the altitude, thus determined, the sun's magnetic amplitude may be taken when his lower limb attains that altitude, either at rising or setting; for, although the sun is apparently so elevated, yet, on account of the atmospherical refraction, his centre is actually then in the horizon of the place of observation.

Note.—For the principles of finding the variation of the compass by the amplitude of a celestial object, see "The Young Navigator's Guide to the Sidereal and Planetary Parts of Nautical Astronomy," page 261.

Problem II.

Given the Latitude of a Place, the Sun's Altitude, and his magnetic Azimuth; to find the Variation of the Compass.

Rule.

Reduce the apparent time of observation to the meridian of Greenwich, by Problem III., page 297; to which time let the sun's declination, at noon of the given day, be reduced, by Problem V., page 298.

Find the true central altitude of the sun, by Problem XIV., page 320; now,

To the sun's polar distance, add its true central altitude and the latitude of the place of observation; take half their sum, and call the difference between it and the polar distance *the remainder.*

Then, to the logarithmic secants, less radius, of the true central altitude and the latitude, add the logarithmic co-sines of the half-sum and the remainder: half the sum of these four logarithms will be the logarithmic co-sine of an arch; which, being doubled, will be the true azimuth, to be reckoned from the north in north latitude, but from the south in south latitude; towards the east in the forenoon, and towards the west in the afternoon.

Now, if the true azimuth, thus found, and the magnetic azimuth, observed per azimuth compass, are on the *same side of the meridian,* their *difference* is the variation; but if *on different sides,* their *sum* is the variation:—and to know whether it be east or west, let the observer look directly towards that point of the compass which represents the *true azimuth;* then, if the magnetic azimuth be to the *left hand* of this, the variation is easterly; but if to the right hand, it is westerly.

Example 1.

April 15th, 1825, in latitude 39°40′ N., and longitude 14°0′ W., at 4ʰ 10ᵐ per watch, the observed altitude of the sun's lower limb was 27°11′, and the bearing of his centre, by azimuth compass, N. 80°37′30″ W.; the height of the eye above the level of the sea was 24 feet; required the variation of the compass?

Time of observation, per watch, = . . .	4ʰ 10ᵐ 0ˢ
Longitude 14°0′ W., in time = . . .	+56. 0
Reduced time = ...	5ʰ 6ᵐ 0ˢ

Observed altitude of sun's lower limb =	27°11′ 0″
Sun's semi-diameter = ...	+15.57
Dip of the horizon for 24 feet = . .	− 4.42
Sun's apparent altitude = ...	27°22′15″
Refraction 1′50″ − Parallax 8″ = .	− 1.42
Sun's true central altitude = . . .	27°20′33″

Sun's dec. at noon, April 15th, = 9°45′46″N.
Correction of ditto for 5ʰ6ᵐ = + 4.32

Sun's reduced declination =	9°50′18″N.		
Sun's north polar distance =	80° 9′42″		
Sun's true central altitude =	27. 20. 33	Log. secant =	0.051451
Latitude of the place of obs. =	39. 40. 0	Log. secant =	0.113638
Sum = ...	147°10′15″		
Half sum = ...	73°35′ 7½″	Log. co-sine =	9.451150
Remainder = ...	6. 34. 34½	Log. co-sine =	9.997133
		Sum =	19.613372
Arch = ...	50° 9′ 9″	Log. co-sine =	9.806686

True azimuth = . . . N. 100°18′18″ W.
Magnetic azimuth = . . N. 80. 37. 30 W.

Variation = ... 19°40′48″; which is *west*, because the magnetic azimuth is to the *right hand* of the true azimuth.

Example 2.

March 10th, 1825, in latitude 42°41′ S., and longitude 148°5′ E., at 19ʰ25ᵐ per watch, the observed altitude of the sun's lower limb was 18°3′, and the bearing of his centre, by azimuth compass, S.108°37′30″E.; the height of the eye above the level of the sea was 19 feet; required the variation?

Time of observation, per watch, = . .	19ʰ25ᵐ 0ˢ
Longitude 148°5′ E., in time = . . .	9.52.20
Reduced time = ...	9ʰ32ᵐ40ˢ

Observed altitude of the sun's lower limb =	18° 3′ 0″
Semi-diameter = ...	+16. 7
Dip of the horizon = ...	− 4.11
Apparent altitude = ...	18°14′56″
Refraction 2′51″ − Parallax 8″ = . .	− 2′.43
Sun's true central altitude = ...	18°12′13″

Sun's dec. at noon, March 10th, =	4° 5′43″ S.		
Correction of ditto for 9ʰ32ᵐ40ˢ =	− 9.21		
Sun's reduced declination = . .	3°56′22″ S.		
Sun's south polar distance = .	86° 3′38″		
Sun's true central altitude = .	18.12.13	Log. secant =	0.022298
Latitude of the place of observ. =	42.41. 0	Log. secant =	0.133647
Sum = ...	146°56′51″		
Half sum = ...	73°28′25½″	Log. co-sine =	9.454014
Remainder = ...	12.35.12½	Log. co-sine =	9.989435
		Sum =	19.599394
Arch = ...	50°54′42″	Log. co-sine =	9.799697
True azimuth = ... S.	101°49′24″ E.		
Magnetic azimuth = . . . S.	108.37.30 E.		

Variation = ... 6°48′ 6″; which is *east*, because the magnetic azimuth is to the *left hand* of the true azimuth.

Note.—After this manner may the variation be deduced from the true altitude and magnetic bearing of a fixed star, a planet, or the moon, as will be seen by referring to "The Young Navigator's Guide to the Sidereal and Planetary Parts of Nautical Astronomy," page 263; where the principles of this method are familiarly explained by a stereographic projection.

A new Method of computing the true Azimuth of a celestial Object, and, thence, finding the Variation of the Compass.

RULE.

From the natural versed sine supplement of the sum of the latitude and the true altitude, subtract the natural versed sine of the object's polar distance: to the logarithm of the remainder add the logarithmic secants of the latitude, and the true altitude: the sum of these three logarithms, rejecting 20 from the index, will be the logarithm of the natural versed sine supplement of the true azimuth; to be reckoned from the north in north latitude, but from the south in south latitude; the difference between which and the magnetic azimuth will be the variation of the compass, as before.

Example 1.

October 17th, 1825, in latitude 42°10' N., and longitude 14°30' W., at 3h2m per watch, the mean of several observed altitudes of the sun's lower limb was 23°39'34", and the mean of an equal number of his central bearings, by azimuth compass, N. 109°28'56" W.; the height of the eye above the level of the horizon was 17 feet; required the variation of the compass?

Time of observation, per watch, = . . .	3h 2m 0s
Longitude 14°30' W., in time = . .	+58. 0
Reduced time = ...	4h 0m 0s
Sun's declination at noon, October 17th, =	9°15'19" S.
Correction of ditto for 4h0m0s = . .	+ 3. 39
Sun's reduced declination = ...	9°18'58" S.
Sun's north polar distance = ...	99°18'58"

Observed altitude of sun's lower limb =	23°39′34″
Semi-diameter = ...	+16. 5
Dip of the horizon = ...	− 3. 57
Apparent altitude = ...	23°51′42″
Refraction 2′9″ − Parallax 8″	− 2. 1
Sun's true central altitude = ...	23°49′41″

Lat. of place of obs.=42°10′ 0″ . . . . . Log. secant = 10.130067
Sun's true cent. alt.=23. 49. 41 . . . . . Log. secant = 10.038692

Sum = ... 65°59′41″ N.V.S.sup.=1.406821
Sun's N. polar dist.=99. 18. 58 Nat.V. sine=1.161881

Remainder = .244940 Log.=5.389060

True azimuth=N.129°41′53″W. N.V.S. sup.=361259 Log.=5.557819
Magnetic do.= N.109. 28. 56 W.

Variation = . 20°12′57″; which is *west*, because the magnetic azimuth is to the *right hand* of the true or computed azimuth.

Example 2.

December 9th, 1825, in latitude 19°40′ N., the true altitude of the star Capella was 20°10′., and his bearing, by azimuth compass, N. 41°0′ E.; required the variation?

Lat. of place of obs.=19°40′ 0″ . . . . . Log. secant = 10.026103
Capella's true alt. = 20. 10. 0 . . . . . Log. secant = 10.027476

Sum = ... 39°50′ 0″N.V.S. sup.=1.767911
Capella's N. pol. dis.=44. 11. 24 N.V. sine = .282968

Remainder = 1.484943 Log.=6.171710

True azimuth=N.47° 9′45″E. N.V.S.sup.=1.679922 Log.=6.225289
Magnetic do.= N.41. 0. 0 E.

Variation = . 6° 9′45″; which is *east*, because the magnetic azimuth is to the *left hand* of the true or computed azimuth.

Remark.—Instead of finding the natural versed sine supplement of the sum of the three logarithms, that sum may be considered as a logarithmic rising. In this case, if the supplement of the time corresponding thereto be taken from Table XXXII., and converted into degrees, by Table I. or otherwise, the result will be the true azimuth. Thus, in the last example, the sum of the three logarithms is 6.225289; the time corresponding to this, in the Table of Logarithmic Rising, is 8ʰ51ᵐ21ˢ, which, taken from 12 hours, leaves 3ʰ8ᵐ39ˢ; and this, being converted into time, gives 47°9′45″ for the true azimuth, which is precisely the same as above.

Problem III.

To find the Variation of the Compass by Observations of a circumpolar Star.

Rule.

From the log. co-sine of the star's declination, (the index being increased by 10,) subtract the logarithmic co-sine of the latitude: the remainder will be the logarithmic sine of the star's greatest eastern or western azimuth (according as it may be situated with respect to the meridian); to be reckoned from the north in north latitude, but from the south in south latitude. Then,

From the logarithmic sine of the latitude, (the index being increased by 10,) subtract the logarithmic sine of the star's declination, and the remainder will be the logarithmic sine of the star's true altitude when at its greatest eastern or western azimuth. Set the index of the quadrant to this altitude, and, when the star has attained it, let its bearing be taken by the azimuth compass; the difference between which and the computed azimuth, when they are of the same name, or their sum when of contrary names, will be the variation; which will be *east*, if the observed or magnetic azimuth be to the *left* of the computed azimuth; otherwise, *west*.

Example 1.

January 1st, 1825, in latitude 41°53′ S., the greatest eastern azimuth of the star Canopus, by azimuth compass, was S. 72°50′ E.; required the variation of the compass?

To find the Star's Altitude when at its greatest Azimuth:—

Latitude of the place of observ. =	41°53′ 0″ S.	Log. sine=9.824527
Reduced declination of Canopus =	52.36.10 S.	Log. sine=9.900063
Star's alt. at greatest azimuth =	57°10′40″	Log. sine=9.924464

To find the Star's greatest eastern Azimuth :—

Reduced declination of Canopus = 52°36′10″ S. Log. co-sine = 9.783430
Lat. of the place of observation = 41.53. 0 S. Log. co-sine = 9.871868

Greatest eastern azimuth = S. 54°39′45″ E. Log. sine = 9.911562
Magnetic azimuth = . . S. 72.50. 0 E.

Variation = ... 18°11′45″; which is *east*, because the magnetic azimuth is to the *left hand* of the computed azimuth.

Example 2.

December 31st, 1825, in latitude 43°45′ N., the greatest western azimuth of the star Dubhe, by azimuth compass, was N. 16°56′ W.; required the variation of the compass?

To find the Star's Altitude when at its greatest Azimuth:—

Latitude of the place of observ. = 43°45′ 0″N. Log. sine = 9.839800
Reduced declination of Dubhe = 62.41.19 N. Log. sine = 9.948670

Star's altitude at greatest azimuth = 51° 6′ 8″ Log. sine = 9.891130

To find the Star's greatest western Azimuth :—

Reduced declination of Dubhe = 62°41′19″N. Log. co-sine = 9.661648
Lat. of the place of observation = 43.45. 0 N. Log. co-sine = 9.858756

Greatest western azimuth = N. 39°25′58″W. Log. sine = 9.802892
Magnetic azimuth = . . N. 16.56. 0 W.

Variation = ... 22°29′58″; which is *west*, because the magnetic azimuth is to the *right hand* of the true or computed azimuth.

Remarks.

In the above method of finding the variation of the compass, the star's declination must be *greater* than the latitude of the place of observation, and of *the same name.*

A star, or other celestial object is said to be circumpolar when its distance from the elevated pole is less than the latitude of the given place (the declination and latitude being of the same name); because, under such circumstances, the object comes within the circle of perpetual apparition, and revolves round the celestial pole without ever setting, or going below the horizon of that place.

The variation of the compass may be found by equal altitudes of the fixed stars; as thus:—

Let the star's altitude be observed in the eastern hemisphere, when it is at least two hours distant from the meridian; and, at the same instant, let its bearing be taken with an azimuth compass: then, when the star comes to the same altitude in the western hemisphere, let its azimuth be again taken. Now, half the difference between the eastern and western azimuths will be the variation; which, when the observations are reckoned from the *south point* of the compass, will be east or west according as the eastern or western azimuth is the greatest; but if they be reckoned from the *north point* of the compass, a contrary process is to be observed: that is, the variation is to be called *east* if the *western azimuth* be the greatest; but *west* if the *eastern azimuth* be the greatest. The variation also may be found, by observing the points of the compass upon which a fixed star rises and sets; then, half the difference between those points will be the variation of the compass, as before.

Note.—The above method of finding the variation of the compass by observations of a circumpolar star, is clearly illustrated in "The Young Navigator's Guide to the Sidereal and Planetary Parts of Nautical Astronomy," between pages 267 and 271.

Problem IV.

To find the Variation of the Compass by the magnetic Bearing of a fixed Star, or Planet, taken at the Time of its Transit, or Passage over any known Meridian.

Rule.

Find the apparent time of the star's transit or passage over the meridian of the given place, by Problem XII., page 317; but if the object selected for observation be a planet, its apparent time of transit, as given in the Nautical Almanac, is to be reduced to the meridian of the place of observation, by Problem XI., page 315. Let the watch be well regulated to apparent time under the meridian of the given place, and it will show the instant of the star's or planet's transit over that meridian; at which instant its bearing, by azimuth compass, is to be carefully taken: the difference between which and the north or south point of the compass (according to the hemisphere in which the star may be posited), will show the deviation of the needle from the true corresponding point of the horizon; then, if the observed or magnetic azimuth be to the *left hand* of the meridian, the variation is easterly; but if to the *right hand*, it is westerly.*

Example 1.

January 2d, 1825, in latitude 20°10′ N., and longitude 165°30′ E., at 11ʰ28ᵐ15ˢ apparent time, the star Canopus was on the meridian, and bore, by azimuth compass, S. 9°30′ E.; required the variation?

Solution.—The observed or magnetic bearing of the star 9°30′ is the variation; and is *easterly*, because it is to the *left hand* of the meridian.

Example 2.

January 1st, 1825, in latitude 34°25′ S., and longitude 18°52′ E., at 14ʰ8ᵐ15ˢ apparent time, the planet Jupiter was on the meridian, and bore, by azimuth compass, N. 25°36′ E.; required the variation?

Solution.—The observed bearing of the planet, 25°36′, is the variation; which is *west*, because the magnetic bearing or azimuth is to the *right hand* of the meridian.

Remarks.

The less the altitude of the star or planet, and the greater its declination, the more accurately will the variation be obtained. When the north polar star is in the same vertical circle with the star Alioth or the star Cor Caroli, it will be on the meridian, or nearly so; and if its azimuth be observed at that time, the variation will be obtained as before. If two stars be observed to be vertical, whose right ascensions are either equal or differ 180 degrees, they will be on the meridian: the azimuth of either may then be taken, but that which is nearest to the elevated pole should be preferred; whence the variation may be inferred, in the same manner as if its apparent time of transit had been computed.

* In like manner may the variation be determined at noon; viz., by observing the magnetic bearing of the sun at the time of its being on the meridian: and, if the place of observation be considerably distant from the equator, a very rigid degree of accuracy is not necessary in the moment of observing the sun's bearing; since, in such a place, an error of 5 minutes in the time, before or after noon, will only produce an error of about *half a quarter of a point* in the variation; which comes sufficiently near the truth for most nautical purposes. Hence, this method may often prove useful in cases where the mariner is prevented, by clouds or other unavoidable causes, from ascertaining, in the forenoon, the true value of the magnetic variation.

The variation may also be deduced from the magnetic azimuth of a fixed star at the apparent time of its transit below the pole: this time may be always known, by adding 12 hours, *diminished by half the variation of the sun's right ascension on the given day*, to the computed apparent time of the star's superior transit above the pole.

The number of brilliant stars which pass over the meridian of a ship at night, and the readiness with which their respective times of transit may be found, render the above method of finding the variation of the compass at sea both desirable and convenient to the practical navigator.

PROBLEM V.

Given the true Course between two Places, and the Variation of the Compass; to find the Magnetic or Compass Course.

RULE.

When the variation is westerly, let it be allowed to the right hand of the true course; but when easterly, to the left hand: in either case, the magnetic or compass course will be obtained.

Example 1.

Required the course, per compass, from Scilly to Cape Clear, the true course being N. 52°55′ W., or N.W. ¾ W. nearly, and the variation 2½ points westerly?

Solution.—The variation 2½ points, being allowed to the right hand of the true course, because it is westerly, shows the magnetic course to be N.N.W. ¼ W.; which, therefore, is the course which a ship must steer by compass from Scilly to Cape Clear, provided the variation be as above.

Example 2.

Required the course, per compass, from Port Royal, Jamaica, to Santa Martha, Columbia; the true course being S. 21°42′ E., or S.S.E. nearly, and the variation about half a point easterly?

Solution.—The variation ½ a point, being allowed to the left hand of the true course, because it is easterly, shows the magnetic course to be S.S.E. ½ E.; which, therefore, is the course which a ship must steer by compass from Port Royal to Santa Martha, provided the variation be as above,—and independent of currents.

Problem VI.

Given the magnetic Course, or that steered by Compass, and the Variation; to find the true Course.

Rule.

If the variation be westerly, it is to be allowed to the left hand of the course steered by compass; but if easterly, to the right hand: in either case, the true course will be obtained.

Example 1.

Let the magnetic, or course steered by compass, be E. by N. ½ N., and the variation 1¼ point westerly; required the true course?

Solution.—The variation 1¼ point, being allowed to the left hand of the compass course, because it is west, shows the true course to be N.E. by E. ¼ E.

Example 2.

Let the course steered by compass be N.W. ¾ W., and the variation one point and three-quarters easterly; required the true course?

Solution.—The variation 1¾ point, being allcwed to the right hand of the magnetic or compass course, because it is easterly, shows the true course to be N.W. by N.

AZIMUTH COMPASS;

The card being graduated on an improved principle, so as to be more particularly adapted to the taking of amplitudes and azimuths, the measuring of horizontal angles, &c. &c.; being thus rendered far more applicable to nautical purposes in general than that which is now in common use at sea.

The azimuth compass, as well as the mariner's compass, is an artificial representation of the horizon of any place on the terrestrial globe: it consists of a circular card, divided into 32 equal parts, called points or rhumbs; and, since the circle contains 360°, each point is equal to

11°15′: for 360° ÷ 32 = 11°15′.* The four principal points of the compass, viz., N., E., S., and W., are called *cardinal points;* the others are compounded of these, and are named according to the quarter in which they are situated.

To the under side of the card, and in the direction of its north and south line, a bar of hardened steel is attached, called the NEEDLE, which, being touched by a load-stone, acquires the peculiar property of pointing north and south, and thus directs the different points on the card to the correspondent points of the horizon. In the centre of the needle there is a small socket, by means of which it is placed, with its attached card, on an upright pin called the *pivot* or *supporter,* which is fixed in the bottom of a circular or conical brass box: on this pin the needle turns freely, and, by its magnetic property, the several points of the compass card keep always in the same direction, very nearly; though these do not always indicate the true correspondent points of the horizon, because of the aberration which the needle suffers, owing to that secret and unknown agency which causes its north and south poles to deviate more or less from the respective correspondent poles of the world.

However, since the compass is an instrument with which mariners are well acquainted, it is not deemed necessary, in this place, to enter any farther into its description. Hence, I shall merely point out some of the many advantages which a compass card, graduated on the above principle, possesses over those now in general use at sea. In this card, the circular ring of silvered brass is to be sufficiently broad to admit of four concentric spaces. The outer edge of the ring is to be graduated, *mathematically correctly,* to every 20th minute of a degree (though, for want of room, the present card is only graduated to every 30th minute of a degree), to which a vernier is to be adapted, containing 20 divisions on each side of its *nonius* for the purpose of subdividing the divisions on the card into minutes of a degree.

The interior surface of the vernier should be ground concave to the segment of a circle, whose radius is equal to that of the card. The remote edge of the inner concentric space, on the silvered brass flat ring, may be graduated similarly to that of the outer edge, so as to render it more convenient in reading off amplitudes according as they may be reckoned from the prime vertical, or from the meridian.

The first space on the broad ring of silvered brass, viz., that next the points of the compass, is particularly adapted to taking amplitudes when the observations are reckoned from the east or the west points of the

* Table XXXIII. contains the different angles which every point and quarter-point of the compass makes with the meridian; and Table XXXIV. contains the logarithmic sines, tangents, and secants of every point and quarter-point of the compass.

horizon; and, therefore, it is numbered both ways, from those points, towards the meridian: that is, from 0° to 90°. The second space being adapted to *horizontal azimuths*, viz., to amplitudes reckoned from the meridian, is therefore numbered both ways, from the north and south points of the horizon towards the east and west points thereof: that is, from 0° to 90°, in a contrary order to the last. The third space is intended for the accommodation of an azimuth when the observation is reckoned from the south in north latitude, or from the south in south latitude: hence, it is numbered both ways from the south to the north point of the compass, or from 0° to 180°. The fourth, or outer space, is designed for azimuths reckoned from the north in north latitude, or from the north in south latitude, according to the will of the observer; and, therefore, it is numbered both ways from the north to the south, or from 0° to 180°, &c.—See the Frontispiece to this volume.

Besides the evident uses of a compass card, graduated after this manner, in observing amplitudes and azimuths, it will also be found of the greatest utility in taking correct surveys of coasts and harbours, and in settling the *true positions* of places on shore from a *known position at sea*. It may, moreover, be applied successfully to many astronomical purposes; nay, it may even be applied to the determination of the longitude by lunar observations, as thus:—Let two observers, with two good compasses of the above description, take the azimuths of the moon and sun, or a fixed star, &c., at the same instant; then, if those two azimuths be reckoned from the same point of the horizon, their *sum*, subtracted from 360°, will be the angle at the zenith comprehended between the zenith distances of the objects; with which, and the true zenith distances of the objects, the true central distance may be found by oblique angled spherical trigonometry, Problem III., Remark 1 or 2, page 203 or 204; and, hence, the longitude of the place of observation, by Problem VIII., page 454.

An azimuth compass of this description would be of real advantage to the practical navigator; whereas, the one now in common use at sea is so very ill adapted to the important purposes for which it is designed, that it is very seldom resorted to for those purposes; and, therefore, it is scarcely ever seen upon deck, except for the simple purpose of comparing its parallelism with that of the binnacle, or steering compass.

SOLUTION OF PROBLEMS RELATIVE TO FINDING THE APPARENT TIMES OF THE RISING AND SETTING OF THE CELESTIAL BODIES.

Problem I.

Given the Day of the Month, the Latitude of a Place, and the Height of the Eye above the Level of the Horizon: to find the apparent Times of the Sun's Rising and Setting.

Rule.

Let the sun's declination, at noon of the given day, be reduced to the meridian of the given place, by Problem V., page 298; then, to the logarithmic tangent of this reduced declination, add the logarithmic tangent of the latitude; and the sum (abating 10 in the index,) will be the logarithmic co-sine of an arch; which, being converted into time, will be the approximate time of the sun's rising, and its supplement to 12 hours will be that of the sun's setting, the latitude and the declination being of the same name; but if these elements be of contrary names, the above arch, reduced into time, will be the approximate time of the sun's setting, and its complement to 12 hours that of the sun's rising.

Reduce the approximate times of rising and setting, thus found, to the correspondent times at Greenwich, by Problem III., page 297; to which times, respectively, let the sun's declination be reduced, by Problem V., page 298; then,

To the aggregate of 90 degrees,* the horizontal refraction,† and the dip of the horizon, diminished by the sun's horizontal parallax,‡ add the sun's polar distance, and the co-latitude of the place of observation: take half the sum; the difference between which and the first term, call the *remainder*.

Now, to the logarithmic co-secants, less radius, of the polar distance, and the co-latitude, add the logarithmic sines of the half sum, and of the remainder: half the sum of these four logarithms will be the logarithmic sine of an arch; which, being doubled, and converted into time, will be the apparent time of the sun's rising. In the same manner the apparent time of the sun's setting is to be computed; but, in this case, the half sum of the four logarithms is to be considered as a logarithmic co-sine.

Example 1.

Required the apparent times of the sun's rising and setting, July 13th, 1824, in latitude 50°48′ N., and longitude 120° W., the height of the eye above the level of the sea being 30 feet?

* The sun's distance from the zenith when his centre is in the horizon.

† The horizontal refraction of a celestial object is 33 minutes of a degree.

‡ The sun's horizontal parallax is about 9 seconds.

Sun's dec. at noon, July 13th, = 21°49'51"N.
Cor. of do. for long. 120° west = – 2.59

Sun's reduced declination = 21°46'52"N. Log. tangent = 9.601613
Lat. of the given place = . 50.48. 0 N. Log. tangent = 10.088533

Arch = ... 60°39'47" Log. co-sine = 9.690146

Approx. time of sun's rising = $4^h\ 2^m 39^s$ Appr. time ☉'s set. $7^h 57^m 21^s$

To find the apparent Time of the Sun's Rising :—

Approximate time of the sun's rising = $4^h\ 2^m 39^s$
Longitude 120° W., in time = . . + 8. 0. 0

Greenwich time past noon of given day = $0^h\ 2^m 39^s$

Sun's declination at noon, July 13th, = 21°49'51"N.
Correction of ditto for $0^h 2^m 39^s$ = . . – 0. 1

Sun's dec., reduced to Greenwich time = 21°49'50"N.

Sun's north polar distance = ... 68°10'10"

90° + 33' + 5'15" – 9" = 90°38' 6"
Sun's polar distance = . . 68. 10. 10 Log. co-secant = 0.032317
Co-latitude = ... 39. 12. 0 Log. co-secant = 0.199263

Sum = ... 198° 0'16"

Half sum = ... 99° 0' 8" Log. sine = . 9.994617
Remainder = ... 8. 22. 2 Log. sine = . 9.162914

Sum = 19.389111

Arch = ... 29°39'57½" Log. sine = . 9.694555½

Arch doubled = ... 59°19'55" = $3^h 57^m 20^s$; which, therefore, is the apparent time of the sun's rising.

To find the apparent Time of the Sun's Setting :—

Approximate time of the sun's setting = $7^h 57^m 21^s$
Longitude 120° W., in time = . . . + 8. 0. 0

Greenwich time past noon of given day = $15^h 57^m 21^s$

Sun's declination at noon, July 13th, = 21°49′51″N.
Correction of ditto for 15h57m21s = . — 5.58

Sun's dec., reduced to Greenwich time = 21°43′53″N.

Sun's north polar distance = ... 68°16′ 7″

90° + 33′ + 5′15″ — 9″ = 90°38′ 6″

Sun's polar distance = . .	68. 16. 7	Log. co-secant=	0. 032017
Co-latitude = ...	39. 12. 0	Log. co-secant=	0. 199263
Sum = ...	198° 6′13″		
Half sum = ...	99° 3′ 6½″	Log. sine = .	9. 994558
Remainder = ...	8. 25. 0½	Log. sine = .	9. 165461
		Sum =	19. 391299
Arch = ...	60°15′ 6″	Log. co-sine =	9. 695649½

Arch doubled = . . . 120°30′12″ = 8h2m1s; which, therefore, is the apparent time of the sun's setting.

Example 2.

Required the apparent times of the sun's rising and setting, October 1st, 1824, in latitude 40°30′ N., and longitude 105° E.; the height of the eye above the level of the sea being 29 feet?

Sun's declination at noon, Oct. 1st,=3°16′ 6″S.
Correc. of ditto for long. 105° E. = — 6.48

Sun's reduced declination = .	3° 9′18″S.	Log. tang.=	8. 741316
Latitude of the given place = .	40. 30. 0 N.	Log. tang.=	9. 931499
Arch = ...	87°18′ 6″	Log. co-sine	8. 672815

Approx. time of the sun's setting = 5h49m12s. Appr. time ☉'s ris. 6h10m48s.

To find the apparent Time of the Sun's Setting:—

Approximate time of sun's setting = . . . 5h49m12s.
Longitude 105° E., in time = ... — 7. 0. 0

Greenwich time past noon, September 30th, = 22h49m12s.

Sun's declination at noon, September 30th, = 2°52′46″ S.
Correction of ditto for 22h49m12s = . . . +22.11

Sun's declination, reduced to Greenwich time = 3°14′57″ S.

Sun's north polar distance = ... 93°14′57″

90° + 33′ + 5′10″ − 9″ =	90°38′ 1″		
Sun's polar distance = . .	93. 14. 57	Log. co-secant =	0. 000699
Co-latitude = ...	49. 30. 0	Log. co-secant =	0. 118954
Sum = ...	233°22′58″		
Half sum = ...	116°41′29″	Log. sine = .	9. 951065
Remainder = ...	26. 3. 28	Log. sine = .	9. 642739
		Sum =	19. 713457
Arch = ...	44° 1′41½″	Log. co-sine =	9. 856728½

Arch doubled = ... 88° 3′23″ = 5h52m13½s; which, therefore, is the apparent time of the sun's setting.

To find the apparent Time of the Sun's Rising:—

Approximate time of sun's rising = . . .	6h10m48s
Longitude 105° E., in time = ...	− 7. 0. 0
Greenwich time past noon, September 30th, =	11h10m48s
Sun's declination at noon, September 30th, =	2°52′46″ S.
Correction of ditto for 11h10m48s = . . .	+ 10. 52
Sun's declination, reduced to Greenwich time, =	3° 3′38″ S.
Sun's north polar distance = ...	93° 3′38″

90° + 33′ + 5′10″ − 9″ =	90°38′ 1″		
Sun's polar distance = . .	93. 3. 38	Log. co-secant =	0. 000620
Co-latitude = ...	49. 30. 0	Log. co-secant =	0. 118954
Sum = ...	233°11′39″		
Half sum = ...	116°35′49½″	Log. sine = .	9. 951423
Remainder = ...	25. 57. 48½	Log. sine = .	9. 641274
		Sum =	19. 712271
Arch = ...	45°53′28″	Log. sine = .	9. 856135½

Arch doubled = ... 91°46′56″ = 6h5m8s; which, therefore, is the apparent time of the sun's rising.

See Examples 1 and 2, page 125; and, also, the Example, pages 126, 127.

Remark.—If the equated or *mean* times of the sun's rising and setting be required, then, to the apparent times, found as above, apply the reduced equation of time, as directed in Problem I., page 415; and the result will be the *mean* times of that object's rising and setting.

Problem II.

Given the Latitude of a Place, and the Height of the Eye above the Level of the Horizon; to find the apparent Times of the Rising and the Setting of a fixed Star.

Rule.

Compute the apparent time of the star's transit, or passage over the meridian of the given place, by Problem XII., page 317; then,

To the aggregate of 90 degrees,* the horizontal refraction,† and the dip of the horizon, add the star's polar distance and the co-latitude of the place of observation: take half the sum; the difference between which and the first term, call the *remainder*.

Now, to the logarithmic co-secants, less radius, of the polar distance, and the co-latitude, add the logarithmic sines of the half sum and of the remainder: half the sum of these four logarithms will be the logarithmic co-sine of an arch; which, being doubled and converted into time, will be the star's semi-diurnal arc, or half the time of its continuance above the horizon; which is to be reduced to apparent *solar time*, by subtracting therefrom the proportional part corresponding to it and the variation of the sun's right ascension for the given day: this is done by Problem V., page 298. Now, the apparent semi-diurnal arc, thus found, being applied by subtraction and addition to the apparent time of the star's transit over the given meridian, will give the respective apparent times of its rising and setting at that meridian: these may be reduced to the mean times of rising and setting, by Problem I., page 415.

Example 1.

Required the apparent times of the rising and setting of the star α Arietis, January 1st, 1824, in latitude 50°48′ N., and longitude 30°0′ E.; the height of the eye above the level of the sea being 16 feet?

---

* This is the star's distance from the zenith when its centre is in the horizon.

† The horizontal refraction of a celestial object is 33 minutes of a degree. The fixed stars have no sensible parallax.

*'s dec., red. to given day, $22^\circ 37' 33''$N., & its R.A.$=1^h 57^m 16^s$
Sun's right ascension at noon of the given day $=$ 18. 43. 58

Approx. time of star's transit over the meridian $=$ $7^h 13^m 18^s$.. $7^h 13^m 18^s$
Longitude of the given place $30^\circ 0'$ E., in time$=$ $-$ 2. 0. 0

Corresponding time at Greenwich $=$ . . . . $5^h 13^m 18^s$
Reduc. of trans. ans. to $5^h 13^m 18^s$ & $4^m 24^s$ the var. of sun's R.A.$=$ $-$ 0. 57

Apparent time of star's transit over merid. of the given place $=$ $7^h 12^m 21^s$

90 degrees $+ 33' + 3' 50'' = 90^\circ 38' 50''$
Star's north polar distance $=$ 67. 22. 27 Log. co-secant$=$0. 034781
Co-latitude of the given place$=$39. 12. 0 Log. co-secant$=$0. 199263

Sum $=$ . . . . $197^\circ 13' 17''$

Half sum $=$ . . . . . $98^\circ 36' 38\frac{1}{2}''$ Log. sine $=$. 9. 995077
Remainder $=$ . . . . . 7. 57. $48\frac{1}{2}$ Log. sine $=$. 9. 141580

Sum $=$ 19. 370701

Arch $=$ . . . . $61^\circ 0' 58''$ Log. co-sine $=$ 9. $685350\frac{1}{2}$

Star's semi-diurnal arc $=$. $122^\circ 1' 56''$, in time $=$ . . $8^h 8^m 8^s$
Prop. part of variation of sun's R. A. $4^m 24^s$ ans. to $8^h 8^m 8^s$ $=$ $-$ 1.30

Star's apparent semi-diurnal arc $=$ . . . . $8^h 6^m 38^s$
Apparent time of the star's transit over the given meridian $=$ 7. 12. 21

Apparent time of the star's rising past noon, Dec. 31st, 1823,$=23^h 5^m 43^s$
Apparent time of the star's setting past noon of the given day$=15^h 18^m 59^s$

Example 2.

Required the apparent times of the rising and setting of the star Sirius, January 1st, 1824, in latitude $40^\circ 30'$ N., and longitude $120^\circ 0'$ W.; the height of the eye above the level of the horizon being 46 feet?

*'s dec., red. to given day, $16^\circ 28' 53''$S., & its R.A.$=6^h 37^m 23^s$
Sun's right ascension at noon of the given day $=$ 18. 43. 58

Approximate time of star's transit over the merid.$=$ $11^h 53^m 25^s$ $11^h 53^m 25^s$
Long. of the place of observ. 120°W., in time$=$ $+$ 8. 0. 0

Corresponding time at Greenwich $=$ . . . . . $19^h 53^m 25^s$
Reduc. of trans. ans. to $19^h 53^m 25^s$ & $4^m 24^s$ the var. of sun's R.A.$=$ $-$ $3^m 39^s$

Apparent time of the star's transit over the given meridian $=$ $11^h 49^m 46^s$

90 degrees + 33′ + 6′30″ = 90°39′30″

Star's north polar distance =	106. 28. 53	Log. co-secant =	0.018220
Co-latitude of the place = .	49. 30. 0	Log. co-secant =	0.118954
Sum = ...	246°38′23″		
Half sum = ...	123°19′11½″	Log. sine = .	9.922008
Remainder = ...	32. 39. 41½	Log. sine = .	9.732132
		Sum =	19.791314
Arch = ...	38° 8′50½″	Log. co-sine =	9.895657

Star's semi-diurnal arc = . 76°17′41″, in time = . . 5h 5m 11s
Prop. part of var. of sun's R. A. 4m24s answering to 5h5m11s = − 0.56

Star's apparent semi-diurnal arc = ... 5h 4m 15s
Apparent time of the star's transit over the given meridian = 11. 49. 46

Apparent time of the star's rising past noon of the given day = 6h45m31s
Apparent time of the star's setting past noon of the given day = 16h54m 1s

See Example 2, page 129; and, also, the Example, page 130.

Problem III.

Given the Latitude of a Place, and the Height of the Eye above the Level of the Horizon; to find the apparent Times of a Planet's Rising and Setting.

Rule.

Compute the apparent time of the planet's transit over the meridian of the given place, by Problems X. and XI., pages 313 and 315; reduce this time to the meridian of Greenwich, by Problem III., page 297; to which let the planet's declination be reduced, by Problem VII., page 307; then,

To the logarithmic tangent of the latitude add the logarithmic tangent of the planet's reduced declination, and the sum (abating 10 in the index,) will be the logarithmic sine of an arch; which, being converted into time, and added to 6 hours when the latitude and the declination are of the same name, but subtracted from 6 hours when of contrary names, the sum or difference will be the planet's approximate semi-diurnal arc, or half the time of its continuance above the horizon.

Let this time be applied, by subtraction and addition, to the apparent time of transit; and the approximate times of the planet's rising and setting will be obtained.

Reduce the approximate times of rising and setting, thus found, to the correspondent times at Greenwich, by Problem III., page 297; to which times, respectively, let the planet's declination be reduced, by Problem VII., page 307; then,

To the aggregate of 90 degrees,* the horizontal refraction,† and the dip of the horizon, diminished by the planet's horizontal parallax,‡ add the planet's polar distance at the approximate time of rising or setting, and the co-latitude of the given place: take half the sum; the difference between which and the first term, call the *remainder*.

Now, to the logarithmic co-secants, less radius, of the polar distance and the co-latitude, add the logarithmic sines of the half sum and of the remainder: half the sum of these four logarithms will be the logarithmic co-sine of an arch; which, being doubled and converted into time, will be half the time of the planet's continuance above the horizon, or its semi-diurnal arc.

Find the proportional part of the variation of the planet's transit over the meridian, answering to half its continuance above the horizon, by Problem XI., page 315, in the same manner as if it were the reduction of transit to a different meridian that was under consideration. Now, this proportional part being added to half the time of the planet's continuance above the horizon when the planet's transit is increasing, but subtracted therefrom when decreasing, the sum or difference will be the planet's apparent semi-diurnal arc; which being applied by subtraction to the apparent time of transit, the remainder will be the apparent time of the planet's rising. In the same manner let the apparent semi-diurnal arc for the time of setting be computed; which, being added to the apparent time of transit, will give the apparent time of the planet's setting. The apparent times of rising and setting, thus found, may be reduced to the *mean* times of rising and setting, if necessary, by Problem I., page 415.

Example 1.

Required the apparent times of Jupiter's rising and setting, January 4th, 1824, in latitude 36° N., and longitude 135° W.; the height of the eye above the level of the horizon being 23 feet?

The apparent time of Jupiter's transit over the meridian of the given

* This is the zenith distance of a planet when its centre is in the horizon.

† The horizontal refraction of a celestial object is 33 minutes.

‡ For the parallaxes of the planets, see page 326.

place is 11ʰ 20ᵐ 30ˢ; his declination, being reduced to this time and the given longitude, is 23° 18′ 55″ north.

Latitude of the given place =	36° 0′ 0″ N.	Log. tangent =	9.861261
Jupiter's reduced declination =	23. 18. 55 N.	Log. tangent =	9.634461
Arch = ...	18° 14′ 52″	Log. sine = . .	9.495722

Arch, converted into time, = 1ʰ 12ᵐ 59ˢ; this, being added to 6 hours, gives the approximate semi-diurnal arc = . . 7ʰ 12ᵐ 59ˢ
Apparent time of Jupiter's transit over the given meridian = . 11. 20. 30

Approximate time of Jupiter's rising = ... 4ʰ 7ᵐ 31ˢ
Approximate time of Jupiter's setting = ... 18ʰ 33ᵐ 29ˢ

The longitude, in time, being applied to those times by addition, because it is west, shows the approximate time of the planet's rising at Greenwich to be 13ʰ 7ᵐ 31ˢ past noon of the given day, and that of its setting 3ʰ 33ᵐ 29ˢ past noon, January 5th. The declination reduced to these times respectively, is 23° 18′ 46″ N. at the time of rising, and 23° 19′ 4″ N. at the time of setting.

To find the apparent Time of Rising:—

90° + 33′ + 4′ 36″ − 2″ = 90° 37′ 34″

Jupiter's polar distance = .	66. 41. 14	Log. co-secant =	0.036988
Co-latitude of the place = .	54. 0. 0	Log. co-secant =	0.092042
Sum = ...	211° 18′ 48″		
Half sum = ...	105° 39′ 24″	Log. sine = . .	9.983580
Remainder = ...	15. 1. 50	Log. sine = . .	9.413860
		Sum =	19.526470
Arch = ...	54° 34′ 3″	Log. co-sine = .	9.763235

Semi-diurnal arc = . . . 109° 8′ 6″, in time = . . 7ʰ 16ᵐ 32ˢ
Variation of transit = 30ᵐ *decreasing;* the proportional part of which, answering to 7ʰ 16ᵐ 32ˢ, is = ... − 1.31 *

Apparent semi-diurnal arc = ... 7ʰ 15ᵐ 1ˢ
Apparent time of Jupiter's transit = ... 11. 20. 30

Apparent time of Jupiter's rising = ... 4ʰ 5ᵐ 29ˢ

* If the transit had been progressive, or increasing, the proportional part would be *additive.*

To find the apparent Time of Setting :—

$90^\circ + 33' + 4'36'' - 2'' =$	$90^\circ 37' 24''$		
Jupiter's polar distance = .	66. 40. 56	Log. co-secant =	0. 037004
Co-latitude of the place = .	54. 0. 0	Log. co-secant =	0. 092042
Sum = ...	$211^\circ 18' 20''$		
Half sum = ...	$105^\circ 39' 10''$	Log. sine = . .	9. 983588
Remainder = ...	15. 1. 46	Log. sine = . .	9. 413829
		Sum =	19. 526463
Arch = ...	$54^\circ 34' 5''$	Log. co-sine =	9. 763231½

Semi-diurnal arc = . . . $109^\circ 8' 10''$, in time = . . $7^h 16^m 33^s$
Variation of transit = 30^s, *decreasing;* the proportional part
of which, answering to $7^h 16^m 33^s$, is = ... $- 1^m 31^s$*

Apparent semi-diurnal arc = ...	$7^h 15^m 2^s$
Apparent time of Jupiter's transit = ...	11. 20. 30
Apparent time of Jupiter's setting = ...	$18^h 35^m 32^s$

Example 2.

Required the apparent times of the rising and setting of the planet Mars, January 16th, 1824, in latitude 40° N., and longitude 140° E.; the height of the eye above the level of the sea being 26 feet?

The apparent time of Mars' transit over the meridian of the given place is $16^h 43^m 37^s$; now, his declination, being reduced to this time and the given longitude, is $0^\circ 55' 45''$ south.

Latitude of the given place=	$40^\circ 0' 0''$N.	Log. tangent =	9. 923814
Mars' reduced declination =	0. 55. 45 S.	Log. tangent =	8. 210009
Arch = ...	$0^\circ 46' 47''$	Log. sine = .	8. 133823

Arch, converted into time = $0^h 3^m 7^s$; this, being subtracted
from 6 hours, leaves the approximate semi-diurnal arc = $5^h 56^m 53^s$
Apparent time of Mars' transit over the given meridian = . 16. 43. 37

Approximate time of Mars' rising = ...	$10^h 46^m 44^s$
Approximate time of Mars' setting = ...	$22^h 40^m 30^s$

* See Note, page 508.

The longitude, in time, being applied to those times by subtraction, because it is east, shows the approximate time of the planet's rising at Greenwich to be $1^h 26^m 44^s$ past noon of the given day, and that of its setting $13^h 20^m 30^s$ past the same noon. The planet's declination reduced to these times, respectively, is 0°54′20″ S. at the time of rising, and 0°57′9″ S. at the time of setting.

To find the Planet's apparent Time of Rising :—

90° + 33′ + 4′52″ − 10″ = 90°37′42″

Planet's polar distance = .	90.54.20	Log. co-secant = 0.000054
Co-latitude of the place = .	50. 0. 0	Log. co-secant = 0.115746
Sum = ...	231°32′ 2″	
Half sum = ...	115°46′ 1″	Log. sine = . 9.954517
Remainder = ...	25° 8′19″	Log. sine = . 9.628195
		Sum = 19.698512
Arch = ...	45° 1′49″	Log. co-sine = 9.849256

Semi-diurnal arc = . . . 90° 3′38″, in time = . . $6^h 0^m 15^s$
Variation of transit = 20^s, *decreasing*; the proportional part of which, answering to $6^h 0^m 15^s$, is = ... − 0.50 *

Apparent semi-diurnal arc = ... $5^h 59^m 25^s$
Apparent time of transit = ... 16.43.37

Apparent time of Mars' rising = ... $10^h 44^m 12^s$

To find the Planet's apparent Time of Setting :—

90° + 33′ + 4′52″ − 10″ = 90°37′42″

Planet's polar distance = .	90.57. 9	Log. co-secant = 0.000060
Co-latitude of the place = .	50. 0. 0	Log. co-secant = 0.115746
Sum = ...	231°34′51″	
Half sum = ...	115°47′25½″	Log. sine = . 9.954432
Remainder = ...	25° 9′43½″	Log. sine = . 9.628573
		Sum = 19.698811
Arch = ...	45° 0′38″	Log. co-sine = 9.849405½

* See Note, page 508.

Arch = ...	45° 0′38″		
Semi-diurnal arc = . . .	90° 1′16″, in time = . .		6ʰ 0ᵐ 5ˢ
Variation of transit = 20ᵐ, *decreasing;* the proportional part of which, answering to 6ʰ0ᵐ5ˢ, is = ...			− 0.50 *
Apparent semi-diurnal arc = ...			5ʰ59ᵐ15ˢ
Apparent time of transit = ...			16.43.37
Apparent time of Mars' setting = ...			22ʰ42ᵐ52ˢ

Problem IV.

Given the Latitude of a Place, and the Height of the Eye above the Level of the Horizon; to find the apparent Times of the Moon's Rising and Setting.

Rule.

Compute the apparent time of the moon's transit over the meridian of the given place, by Problems VIII. and IX., pages 309 and 312; and reduce it to the meridian of Greenwich, by Problem III., page 297; to which let the moon's declination and horizontal parallax be reduced, by Problem VI., page 302; then,

To 90 degrees, diminished by the difference between the moon's horizontal parallax and the sum of the horizontal refraction and the dip of the horizon, add the moon's polar distance and the co-latitude of the given place. Find the difference between half the sum and the first term, which call the *remainder*.

Now, to the logarithmic co-secants, less radius, of the polar distance, and the co-latitude, add the logarithmic sines of the half sum and of the remainder: half the sum of these four logarithms will be the logarithmic co-sine of an arch; which, being doubled, and converted into time, will be the moon's approximate semi-diurnal arc: this being subtracted from and added to the apparent time of the moon's transit, the respective approximate times of her rising and setting will be obtained.

Reduce the approximate times of the moon's rising and setting, thus found, to the correspondent times at Greenwich, by Problem III., page 297; to which times, respectively, let the moon's declination and horizontal parallax be reduced, by Problem VI., page 302; and let the moon's

* See Note, page 309.

declination, at each time, be corrected by the equation of second difference; then,

With 90 degrees, *diminished as before*, the moon's respective polar distances, and the co-latitude, compute the approximate semi-diurnal arcs corresponding to the times of rising and setting.

Find the proportional part of the daily variation of the moon's transit answering to *each semi-diurnal arc*, and 24 hours augmented by the variation of transit, by Problem IX., page 312, in the same manner as if it were the reduction of transit to a different meridian that was under consideration. Now, these proportional parts, being *added* to their corresponding semi-diurnal arcs, will give the apparent semi-diurnal arcs at the times of the moon's rising and setting: the former being subtracted from the apparent time of transit, and the latter added thereto, the respective apparent times of the moon's rising and setting will be obtained. These may be reduced to the *mean* times of rising and setting, by Problem I., page 415, if necessary.

Example 1.

Required the apparent times of the moon's rising and setting, January 17th, 1824, in latitude 51°28′40″N., and longitude 75°W.; the height of the eye above the level of the horizon being 30 feet?

The computed apparent time of the moon's transit over the meridian of the given place is 13ʰ43ᵐ46ˢ; now, her declination, being reduced to this time, and the given longitude, is 10°29′27″N., and her horizontal parallax 60′59″.

90° − 60′59″ + 33′ + 5′15″ = 89°37′16″

Moon's north polar distance =	79. 30. 33	Log. co-secant = 0. 007321
Co-latitude of the given place =	38. 31. 20	Log. co-secant = 0. 205639
Sum = ...	207°39′ 9″	
Half sum = ...	103°49′34½″	Log. sine = . 9. 987230
Remainder = ...	14. 12. 18½	Log. sine = . 9. 389864
		Sum = 19. 590054
Arch = ...	51°24′28″	Log. cosine = 9. 795027

☽'s approx. semi-diurnal arc = 102°48′56″, in time = . . 6ʰ51ᵐ16ˢ
Moon's apparent time of transit over the given meridian = . 13. 43. 46

Approximate time of the moon's rising = ... 6ʰ52ᵐ30ˢ
Approximate time of the moon's setting = ... 20ʰ35ᵐ 2ˢ

The longitude, in time, being added to those times, because it is west, shows the approximate time of the moon's rising at Greenwich to be $11^h 52^m 30^s$ past noon of the given day, and that of her setting $1^h 35^m 2^s$ past noon of the 18th. Now, the moon's declination and horizontal parallax, reduced to these times respectively, (the former being corrected by the equation of second difference,) gives the declination at the time of rising 12°12′0″N., and the horizontal parallax 61′7″; and the declination at the time of setting 8°46′8″N., and the horizontal parallax 60′49″.

To find the apparent Time of Rising :—

90° − 61′7″ + 33′ + 5′15″ = 89°37′ 8″			
Moon's polar distance = .	77.48. 0	Log. co-secant =	0.009921
Co-latitude = ...	38.31.20	Log. co-secant =	0.205639
Sum = ...	205°56′28″		
Half sum = ...	102°58′14″	Log. sine = .	9.988775
Remainder = ...	13.21. 6	Log. sine = .	9.363475
		Sum =	19.567810
Arch = ...	52°33′17½″	Log. co-sine =	9.783905

Semi-diurnal arc = . . . 105° 6′35″, in time = . . .	$7^h\ 0^m 26^s$
Variation of transit = 53^m, the proportional part of which, answering to $7^h 0^m 26^s$, is	+14.55
Apparent semi-diurnal arc = ...	$7^h 15^m 21^s$
Apparent time of moon's transit = ...	13.43.46
Apparent time of the moon's rising = ...	$6^h 28^m 25^s$

To find the apparent Time of Setting :—

90° − 60′49″ + 33′ + 5′15″ = 89°37′26″			
Moon's polar distance . .	81.13.52	Log. co-secant =	0.005106
Co-latitude = ...	38.31.20	Log. co-secant =	0.205639
Sum = ...	209°22′38″		
Half sum = ...	104°41′19″	Log. sine = .	9.985570
Remainder = ...	15. 3.53	Log. sine = .	9.414824
		Sum =	19.611139
Arch = ...	50°16′30½″	Log. co-sine =	9.805569½

Arch = ... 50°16′30½″

Semi-diurnal arc = . . . 100°33′ 1″, in time = . . 6h42m12s
Variation of transit = 53m, the proportional part of which, answering to 6h42m12s, is = ... +14. 17

Apparent semi-diurnal arc = ... 6h56m29s
Apparent time of moon's transit = ... 13. 43. 46

Apparent time of the moon's setting = ... 20h40m15s

Example 2.

Required the apparent times of the moon's rising and setting, January 20th, 1824, in latitude 40°30′N., and longitude 80°E.; the height of the eye above the level of the horizon being 30 feet?

The computed apparent time of the moon's transit over the meridian of the given place is 15h55m5s; now, her declination, being reduced to this time, and to the given longitude, is 5°55′35″S., and her horizontal parallax 58′53″.

90°— 58′53″ + 33′ + 5′15″=89°39′22″
Moon's north polar distance = 95. 55. 35 Log. co-secant= 0.002329
Co-latitude of the given place=49. 30. 0 Log. co-secant= 0.118954

Sum = ... 235° 4′57″

Half sum = ... 117°32′28½″ Log. sine = . 9.947767
Remainder = ... 27. 53. 6½ Log. sine = . 9.669968

Sum = 19.739018

Arch = ... 42°13′42″ Log. co-sine = 9.869509

☽'s approx. semi-diurnal arc = 84°27′24″, in time = . . 5h37m50s
Apparent time of the moon's transit over the given meridian =15. 55. 5

Approximate time of the moon's rising = ... 10h17m15s
Approximate time of the moon's setting = ... 21h32m55s

The longitude, in time, being subtracted from those times, because it is east, shows the approximate time of the moon's rising at Greenwich to be

$4^h 57^m 15^s$ past noon of the given day, and that of her setting $16^h 12^m 55^s$ past the same noon. Now, the moon's declination and horizontal parallax, reduced to these times respectively, (the former being corrected by the equation of second difference,) gives the declination at the time of rising $4^\circ 32' 3''$S., and the horizontal parallax $59' 6''$; and the declination at the time of setting $7^\circ 18' 12''$S., and the horizontal parallax $58' 40''$.

To find the apparent Time of Rising :—

$90^\circ - 59' 6'' + 33' + 5' 15'' = 89^\circ 39' 9''$

Moon's polar distance = .	94.32. 3	Log. co-secant=	0.001361
Co-latitude = ...	49.30. 0	Log. co-secant=	0.118954
Sum = ...	$233^\circ 41' 12''$		
Half sum = ...	$116^\circ 50' 36''$	Log. sine = .	9.950484
Remainder = ...	27.11.27	Log. sine = .	9.659874
		Sum=	19.730673
Arch = ...	$42^\circ 49' 42''$	Log. co-sine =	$9.865336\frac{1}{2}$

Semi-diurnal arc = . . .	$85^\circ 39' 24''$, in time = . .	$5^h 42^m 38^s$
Variation of transit = 49^m, the proportional part of which, answering to $5^h 42^m 38^s$, is = ...		+11.16
Apparent semi-diurnal arc = ...		$5^h 53^m 54^s$
Apparent time of the moon's transit = ...		15.55. 5
Apparent time of the moon's rising = ...		$10^h 1^m 11^s$

To find the apparent Time of Setting :—

$90^\circ - 58' 40'' + 33' + 5' 15'' = 89^\circ 39' 35''$

Moon's polar distance = . .	97.18.12	Log. co-secant=	0.003538
Co-latitude = ...	49.30. 0	Log. co-secant=	0.118954
Sum = ...	$236^\circ 27' 47''$		
Half sum = ...	$118^\circ 13' 53\frac{1}{2}''$	Log. sine = .	9.944998
Remainder = ...	$28.34.18\frac{1}{2}$	Log. sine = .	9.679664
		Sum =	19.747154
Arch = ...	$41^\circ 37' 51''$	Log. co-sine =	9.873577

Arch = ...	41°37′51″	
Semi-diurnal arc = . . .	83°15′42″, in time = . .	5h33m 3s
Variation of transit = 49m, the proportional part of which, answering to 5h33m3s, is = ...		+10.57
Apparent semi-diurnal arc = ...		5h44m 0s
Apparent time of the moon's transit = ...		15.55. 5
Apparent time of the moon's setting = ...		21h39m 5s

See Examples 1 and 2, pages 134 and 136; and, also, the example or work, pages 137 and 138.

Problem V.

Given the Latitude and Longitude of a Place, and the Day of the Month; to find the Time of the Beginning and of the End of Twilight, and the Length of its Duration.

Rule.

Reduce the sun's declination, at the midnights preceding and following the noon of the given day, to the meridian of the given place, by Problem V., page 298; then,

Add together the constant quantity 108 degrees,* the sun's polar distance, and the co-latitude of the given place: take half the sum; the difference between which and the constant quantity call the *remainder*. Now,

To the logarithmic co-secants, less radius, of the polar distance, and the co-latitude, add the logarithmic sines of the half sum and of the remainder: half the sum of these four logarithms will be the logarithmic sine or logarithmic co-sine of an arch; which, being doubled, and converted into time, will be the apparent time of the beginning or of the end of twilight accordingly.

Compute the apparent times of the sun's rising and setting, by Problem I., page 500; then, the interval between the time of the commencement of twilight and that of sun rising, will be the duration of the morning twilight; and the interval between the time of sun setting and the end of twilight, will be the duration of the evening twilight.

Note.—If much accuracy be required, the sun's declination must be reduced to the meridian of the given place, at the respective times of the

* 90° + 18° = 108°. See Remarks, page 518.

commencement and of the end of twilight, found as above; then, the operations being repeated, the correct apparent times of the beginning and of the end of twilight will be obtained. This degree of accuracy may, however, be dispensed with,—unless in cases of mere speculative inquiry, or where some philosophical object is under consideration.

Example.

Required the apparent times of the beginning and of the end of twilight, and its duration, October 1st, 1824, in latitude 40°30′ north, and longitude 105° east?

To find the Beginning of Twilight:—

Sun's declination at midnt. September 30th, =	3° 4′26″ S.
Reduction of ditto for longitude 105° E. =	− 6.48
Sun's reduced declination = ...	2°57′38″ S.

Constant quantity =	108° 0′ 0″		
Sun's polar distance =	92.57.38	Log. co-secant =	0.000580
Co-latitude = . . .	49.30. 0	Log. co-secant =	0.118954
Sum = ...	250°27′38″		
Half sum = . . .	125°13′49″	Log. sine = . .	9.912137
Remainder = . . .	17.13.49	Log. sine = . .	9.471604
			19.503275
Arch = ...	34°21′54½″	Log. sine = . .	9.751637½

Beginning of twilight = 68°43′49″, in time = . . .	4h34m55s
Apparent time of sun-rising on the given day = ...	6. 5. 8
Duration of morning twilight = ...	1h30m13s

To find the End of Twilight:—

Sun's declination at midnight, October 1st, =	3°27′45″ S.
Reduction of ditto for longitude 105° E. =	− 6.48
Sun's reduced declination = ...	3°20′57″ S.

Constant quantity =	108° 0′ 0″		
Sun's polar distance =	93. 20. 57	Log. co-secant =	0. 000743
Co-latitude = . . .	49. 30. 0	Log. co-secant =	0. 118954
Sum = ...	250°50′57″		
Half sum = ...	125°25′28½″	Log. sine = . . .	9. 911093
Remainder = . . .	17. 25. 28½	Log. sine = . . .	9. 476324
			19. 507114
Arch = ...	55°27′40″	Log. co-sine = . .	9. 753557

End of twilight = . . 110°55′20″, in time = ... $7^h 23^m 41^s$
Apparent time of sun-setting on the given day = ... 5. 52. 13

Duration of evening twilight = ... $1^h 31^m 28^s$

Remarks.

Twilight, technically called the *crepusculum,* is that faint light which we perceive before the sun rises and after he sets. It is produced by the rays of light being refracted in their passage through the earth's atmosphere, and reflected from the different particles thereof.

The morning twilight commences when the sun wants 18 degrees of appearing in the horizon of the eastern hemisphere, and the evening twilight ends when he is depressed 18 degrees below the horizon of the western hemisphere.

When the sun's declination exceeds the difference between the co-latitude of any given place and 18 degrees, there will be no *real darkness or* night at that place, but continual day and twilight; as is the case at London, from the 22d of May to the 21st of July.

When the sun is on the same side of the equinoctial with the elevated pole, the duration of twilight will constantly increase as he approaches that pole, till he enters the tropic; at which time the duration of twilight will be the longest. It will then decrease until some time after the sun passes the equinox, but will increase again before he arrives at the opposite tropic: hence, there must be a point within the tropics where the duration of twilight is the shortest. This point may be found by the following problem.

Problem VI.

Given the Latitude of a Place; to find the Time of the shortest Twilight, and its Duration.

Rule.

To the logarithmic tangent of the half of 18 degrees, add the logarithmic sine of the latitude; and the sum (abating 10 in the index,) will be the logarithmic sine of the sun's declination at the time of the shortest twilight, of a *contrary name to the latitude:* the day corresponding to this declination will be that required.

Again, to the logarithmic sine of the half of 18 degrees, add the logarithmic secant of the latitude; and the sum (abating 10 in the index,) will be the logarithmic sine of an arch, which, being doubled and converted into time, will be the duration of the shortest twilight.

Example.

Required the time of the shortest twilight, and its duration, in the year 1824, in latitude 50°48′ N.?

Half of 18 degrees	= 9° 0′ 0″	Log. tangent	= 9.199713
Latitude of the place	= 50.48. 0	Log. sine	= 9.889271
Sun's declination	= 7° 3′ 1″	Log. sine	= 9.088984;

which is *south*, of a contrary name to the latitude.

Half of 18 degrees	= 9° 0′ 0″	Log. sine	= 9.194332
Latitude of the place	= 50.48. 0	Log. secant	= 10.199263
Arch = ...	14°19′49″	Log. sine	= 9.393595

Duration of twilight = 28°39′38″, in time = 1ʰ54ᵐ39ˢ.

The days, in the Nautical Almanac, corresponding to the sun's declination 7°3′1″ S., are March 2d and October 11th, which, therefore, are the days of the shortest twilight in the year 1824, in latitude 50°48′ north; and the duration of the twilight, on those days, is 1ʰ54ᵐ39ˢ.

Problem VII.

Given the Latitude of a Place between 48°32′ and 66°32′ (the Limits of regular Twilight); to find when real Night or Darkness ceases, and when it commences.

Rule.

The complement of the latitude, diminished by 18 degrees, will be the declination of the sun, of the *same name as the latitude*, at the time when it ceases to be real night, and also when real night commences.

Example.

Required the interval of time, in the year 1824, during which there will be no real darkness or night, in latitude 50°48′ north?

Solution.—The complement of the latitude 39°12′ N.−18°=21°12′N. = the sun's declination. Now, the days answering to 21°12′ of north declination are, May 26th and July 17th. Upon the first of these days, therefore, real night ceases, and it commences upon the last. During this interval there is no real darkness, because the sun is less than 18 degrees below the horizon; and so on for any other latitude within the limits.

Problem VIII.

Given the Sun's Declination and Semi-diameter; to find the Interval between the Instants of his lower and upper Limbs being in the Horizon of a known Place.

Rule.

Find the approximate time of the sun's rising or setting, by Problem I., page 124; to which time let the sun's declination be reduced, by Problem V., page 298.

To the logarithm of the sun's semi-diameter, expressed in seconds, add the constant logarithm 9.124939, and call the sum a *reserved logarithm;* then,

To the logarithmic co-sine of the sum of the latitude and declination, add the logarithmic co-sine of their difference: half the sum of these two logarithms, being subtracted from the *reserved logarithm,* will leave the logarithm of the interval of time, in seconds, between the instants of the sun's lower and upper limbs being in the horizon of the given place.

Example 1.

Required the interval between the instants of the sun's lower and upper limbs being in the horizon, at the time of its setting, July 13th, 1824, in latitude 50°48′ N., and longitude 120° W.?

Apparent time of setting in Table L., to latitude 50°48′ N., and declination 21°49′51″ N. = ...	7h57m12s
Longitude 120° west, in time = ...	8. 0. 0
Greenwich time of sun's setting = ...	15h57m12s

Sun's declination at noon, July 13th, 1824, =	21°49′51″N.
Correction of ditto for 15h57m12s = . . .	− 5.58
Sun's reduced declination = ...	21°43′53″N.

Sun's semi-diameter 15′45″.8=945″.8 Log.=		2.975799	
Constant logarithm = ...		9.124939	
Reserved logarithm = ...		12.100738 . .	12.100738
Sun's red. dec. = 21°43′53″N.			
Lat. of the place=50.48. 0 N.			
Sum = . . . 72°31′53″	Log. co-sine =	9.477387	
Difference = . 29. 4. 7	Log. co-sine =	9.941531	
Sum = ...		19.418918	
Half sum = ...		9.709459 . . .	9.709459
Interval, in seconds, = 246.195 = ...		Log. =	2.391279

Hence, the interval between the instants of the sun's limbs touching the horizon, is 4 minutes and 6 seconds.

Example 2.

Required the interval between the instants of the sun's upper and lower limbs touching the horizon, at the time of rising, October 1st, 1824, in latitude 40°30′ N., and longitude 105° E.?

Apparent time of rising, in Table L., to latitude 40°30′ N., and declination 3°16′6″ S. = ...	6h10m48s
Longitude 105° east, in time = ...	7. 0. 0
Greenwich time past noon, September 30th, ...	11h10m48s

Sun's declination at noon, Sept. 30th, 1824,=		2°52′46″ S.
Correction of ditto for 11h 10m 48s = . .		+10.53
Sun's reduced declination = ...		3° 3′39″ S.

Sun's semi-diameter=16′1″.2=961″.2 Log.=	2.982814		
Constant logarithm = ...	9.124939		
Reserved logarithm = ...	12.107753	. .	12.107753
Sun's red. dec. = 3° 3′39″S.			
Lat. of the place=40.30. 0 N.			
Sum = . . 43°33′39″ Log. co-sine =	9.860124		
Difference = . 37.26.21 Log. co-sine =	9.899820		
Sum = ...	19.759944		
Half sum = ...	9.879972	. . .	9.879972
Interval, in seconds, = 168.958 = ...		Log. =	2.227781

Hence, the interval between the instants of the sun's limbs touching the horizon, is 2 minutes and 49 seconds.

Note.—The constant logarithm made use of in this problem is the arithmetical complement of the proportional logarithm of 24 hours esteemed as minutes. If the sun's diameter be used, instead of its semi-diameter, it must be expressed in minutes and decimal parts of a minute: in this case, the same result will be obtained by employing the constant logarithm 8.823909; viz., the arithmetical complement of the common logarithm of 15 degrees, or the motion corresponding to one hour of time.

SOLUTION OF PROBLEMS IN GNOMONICS OR DIALLING.

Dialling, or *Gnomonics*, is a branch of mixed mathematics, which depends partly on the principles of geometry and partly on those of astronomy; and it may be defined as being the method of projecting on the surface of any given body, whether plane or otherwise, a figure called a *sun-dial*,—the different lines of which indicate, by the shadow of a style or gnomon, when the sun shines thereon, the apparent time of the day.

The upper edge of the *style* or *gnomon*, which projects the sun's shadow on the plane of the dial, must be parallel to the earth's axis: hence, it is sometimes called the *axis of the dial*.

The plane of the *gnomon* must be perpendicular to that of the dial, The plane on which it is erected is called the *sub-style*: in horizontal dials it may be called the meridian, or 12 o'clock line.

The angle comprehended between the *style* and the *sub-style,* is called the *elevation of the style:* this angle, in horizontal dials, is always equal to the elevation of the pole, or the latitude of the place for which it is computed; but, in erect direct north or south dials, it is equal to the complement of the latitude of such place.

Those dials whose planes are parallel to the plane of the horizon, are called *horizontal dials*; but such as have their planes perpendicular to the plane of the horizon, are called *vertical* or *erect* dials.

Those vertical dials whose planes are either parallel or perpendicular to the plane of the meridian, are called *direct erect dials.* One of these must always face one of the cardinal points of the horizon, according as it may be a north, south, east, or west, *erect dial.*

All other erect dials are called *declining dials.* Those dials whose planes are neither parallel nor perpendicular to the plane of the horizon, are called *reclining dials.*

In this place, however, we shall only show the method of constructing a horizontal dial, and, also, that of a north or south erect direct dial; these being by far the most useful, and, indeed, the most common of all the varieties in dialling.

Problem I.

Given the Latitude of a Place; to find the Angles which the Hour Lines make with the Sub-Style or Meridian Line of a Horizontal Sun-Dial.

General Proposition.

In every right angled spherical triangle, *radius is to the sine of one of the legs containing the right angle, as the tangent of the angle adjacent to that side is to the tangent of the other containing side of the triangle.* This is merely a variation of the equation for finding the leg B C, in Problem IV., page 189: hence the following

Rule.

To the logarithmic sine of the latitude, add the logarithmic tangent of the sun's horary angle from noon; and the sum (abating 10 in the index,) will be the logarithmic tangent of the angle comprehended between the corresponding hour line and the sub-style, at the centre of the dial.

Note.—Since the sun's apparent motion in the ecliptic is at the rate of 15 degrees to an hour, therefore at one hour from noon the sun's horary angle is 15°; at two hours from noon it is 30°; and so on.

Example.

Required the angles which the hour lines make with the sub-style, or meridian line of a horizontal dial, in a place situated in 50°48′15″ north latitude?

To find the Angle at one Hour from Noon:—

Latitude of the place = . .	50°48′15″	Log. sine =	9.889296
Sun's horary ang. at 1ʰ from noon=	15. 0. 0	Log. tangent=	9.428053
Hour line of 1, or 11 o'clock = .	11°43′52″	Log. tangent=	9.317349

To find the Angle at two Hours from Noon:—

Latitude of the place = . . .	50°48′15″	Log. sine =	9.889296
Sun's horary angle at 2ʰ from noon=	15. 0. 0	Log. tangent=	9.761439
Hour line of 2, or 10 o'clock =	24° 6′20″	Log. tangent=	9.650735

Proceeding in this manner, the several angles which the respective hour lines make with the meridian will be found to be as follows; viz.,

Hour lines of	I.	and	XI.	=	11°43′52″
Ditto	II.	and	X.	=	24. 6.20
Ditto	III.	and	IX.	=	37.46.31
Ditto	IV.	and	VIII.	=	53.18.53
Ditto	V.	and	VII.	=	70.55.39
Ditto	VI.	and	VI.	=	90. 0. 0

The hour lines of VII. in the evening and V. in the morning, make the same angles with the meridian, on *the opposite side of the* VI. *o'clock hour line*, as the hour lines of VII. in the morning and V. in the evening. In the same manner the hour lines of VIII. in the evening and IV. in the morning make the same angles with the meridian as the hour lines of VIII. in the forenoon and IV. in the afternoon; and so on.

The angles for the halves, quarters, or other subdivisions of the hours, are to be determined in the above manner.

The angles which the different hour lines, &c. make with the meridian, being thus determined, the dial may then be very readily constructed, by means of a pair of compasses, and the line of chords on a common Gunter's scale, or of that on a *Sector:* the latter, however, should be preferred, because the degrees thereon are generally divided into halves, and sometimes quarters, which gives it a decided advantage, in point of accuracy, over that on Gunter's scale.

CONSTRUCTION.

On the proposed plane draw the meridian, or XII. o'clock hour line, *a b*; parallel to which, at a distance equal to the intended thickness of the gnomon or style, draw the line *c d*: perpendicularly to these draw the VI. o'clock hour line *e f*. Open the Sector to any convenient extent, and take the transverse distance 60° to 60° (on the line of chords) as a radius in the compasses, and, from *a* as a centre, describe the arc *g h*: with the same radius, and from *c* as a centre, describe the arc *i k*; and, since the hour lines are less distant from each other about noon than in any other part of the day, it is advisable to have the centres of those quadrants or arcs at a little distance from the centre of the plane of the dial, on the side opposite to XII., so as to allow of the hour distances being enlarged near the meridian under the same angles in the plane of the dial: thus, the centre of the plane is at A; but the centres of the quadrants or arcs are taken a little below it, at the points *a* and *c*.

Take the transverse distance 11°43′52″ to 11°43′52″, in the compasses, from the line of chords, and set it off from *g* to 1, and, also, from *i* to 6: take the transverse distance 24°6′20″, in the compasses, and set it off from *g* to 2, and from *i* to 7; and proceed in the same manner with the remaining horary angles.

Now, from the centre *a* draw the forenoon hour lines *a* 1 XI., *a* 2 X., *a* 3 IX., *a* 4 VIII., *a* 5 VII.; and, from *c* as a centre, draw the afternoon hour lines *c* 6 I., *c* 7 II., *c* 8 III., *c* 9 IV., *c* 0 V.: produce *a* 5 VII. and *a* 4 VIII. for the hour lines of VII. and VIII. o'clock in the evening; and produce *c* 9 IV. and *c* 0 V. for the hour lines of IV. and V. in the morning. In the same manner may the quarter and half-hour lines be drawn (and minutes if necessary), by setting off the computed corresponding angles from the meridian: these, however, have been omitted in the above diagram, with the view of preventing embarrassment.

Take the latitude 50°48′15″ in the compasses, viz., the transverse distance 50°48′15″ to 50°48′15″, and set it off from g to L, and draw the hypothenuse line a L P for the axis of the style or gnomon.

The style may have any shape the artist pleases, provided its edge a L P be a perfectly straight line. It should be a metallic substance, and must be of an equal thickness with the breadth of the space comprehended between the two parallel straight lines $a\,b$ and $c\,d$; in which space it must be erected truly perpendicular to the plane of the dial: then, since the angle B a P is equal to the latitude, the straight edge of the style = a L P will be directed to the elevated pole of the world, and, hence, parallel to the earth's axis when the dial is truly set; the shadow of which, when the sun shines, will indicate the hour of the day.

Note.—Since the hour of the day indicated by a sun-dial is expressed in apparent solar time, it must be reduced to *mean time*, by Problem I., page 415, so as to make it correspond with that shown by a well-regulated watch or clock.

Problem II.

To find the Angles on the Plane of an erect direct south Dial for any proposed north Latitude, or on that of an erect direct north Dial for any proposed south Latitude.

Rule.

To the logarithmic co-sine of the latitude, add the logarithmic tangent of the sun's horary angle from noon; and the sum (abating 10 in the index,) will be the logarithmic tangent of the angle comprehended between the corresponding hour line and the sub-style, at the centre of the dial.

Example.

Required the angles which the hour lines on an erect direct south dial make with the sub-style or 12 o'clock line, in latitude 50°48′15″ north?

To find the Angle at one Hour from Noon:—

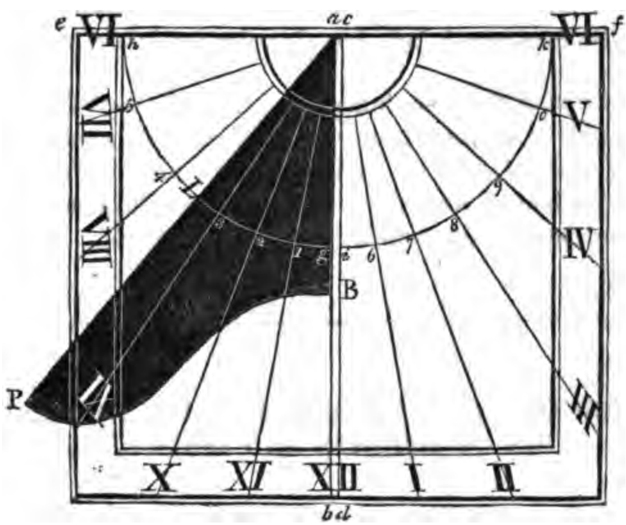
Latitude of the place = . . .	50°48′15″	Log. co-sine = 9.800699
Sun's horary ang. at 1h from noon =	15. 0. 0	Log. tangent = 9.428053
Hour angle of 1, or 11 o'clock =	9°36′40″	Log. tangent = 9.228752

To find the Angle at two Hours from Noon:—

Latitude of the place = . . .	50°48′15″	Log. co-sine = 9.800699
Sun's horary angle at 2h from noon =	30. 0. 0	Log. tangent = 9.761439
Hour angle of 2, or 10 o'clock =	20° 2′44″	Log. tangent = 9.562138

Proceeding in this manner, the several angles which the respective hour lines make with the meridian will be found to be as follows; viz.,

Hour lines of	I.	and	XI.	=	9°36′40″
Ditto	II.	and	X.	=	20. 2.44
Ditto	III.	and	IX.	=	32.17.30
Ditto	IV.	and	VIII.	=	47.35.10
Ditto	V.	and	VII.	=	67. 1.25
Ditto	VI.	and	VI.	=	90. 0. 0



CONSTRUCTION.

On the proposed plane draw the XII. o'clock hour line *a b*; parallel to which, at a distance equal to the intended thickness of the style, draw the line *c d*: at right angles to the sub-style, or XII. o'clock line, draw the VI. o'clock hour line *e f*. Open the sector to any convenient extent, and take the transverse distance 60° to 60° (on the line of chords) as a radius in the compasses, and, from *a* as a centre, describe the arc *g h*; with the same radius, and from *c* as a centre, describe the arc *i k*. Take the transverse distance 9°36′40″ to 9°36′40″ in the compasses, and set it off from *g* to 1, and, also, from *i* to 6. Take the transverse distance 20°2′44″ to 20°2′44″ in the compasses, and set it off from *g* to 2, and from *i* to 7; and proceed in the same manner with the remaining horary angles. Then, from the centre *a*, draw the forenoon hour lines *a* 1 XI., *a* 2 X., &c. &c.; and, from *c* as a centre, draw the afternoon hour lines *c* 6 I., *c* 7 II., &c. &c.

Take the complement of the latitude in the compasses, viz., the transverse distance 39°11′45″ to 39°11′45″; set it off from *g* to L, and draw the hypothenuse line *a* L P for the axis of the style or gnomon.

Now, when the dial is placed vertically, with its plane duly facing the south, the VI. o'clock hour line *e f* will be parallel to the plane of the horizon; and the style B *a* L P, directed downwards, making an angle with the sub-style or XII. o'clock hour line equal to the complement of the latitude, will be truly parallel to the earth's axis.

Since the sun cannot shine any longer on a dial of this description than from VI. in the morning until VI. in the evening, it is not necessary to describe hour lines upon it before or after those periods of time.

Note.—An erect direct north dial for a place in north latitude, is constructed exactly in the same manner as an erect direct south dial; but the position of the dial must be reversed: that is, the VI. o'clock hour line must be at the bottom instead of the top of the dial; and the style or gnomon must be directed upwards instead of downwards.

SOLUTION OF PROBLEMS RELATIVE TO THE MENSURATION OF HEIGHTS AND DISTANCES.

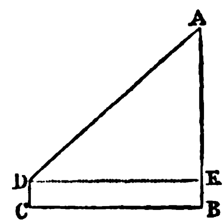
Since it is frequently of the greatest importance to the mariner, but *at all times to the engineer* or other *military officer*, to be able to ascertain the heights and distances of remote objects with precision, the following problems are given for their general guidance in such cases. In solving these problems, it is the logarithmical mode of calculation that will be attended to, with the view of showing the direct application of the principles of plane trigonometry to such cases. To the imagination of the ingenious, however, many other modes of obtaining an approximate value for the heights and distances of remote objects will soon present themselves: such as, by means of shadows, mirrors, unequal vertical staves, &c. &c.; but, since these methods entirely depend upon the principles of similar triangles (as demonstrated in Euclid, Book VI., Prop. 4), they admit of direct solutions without the assistance of trigonometrical tables: hence, no notice can be taken of them in this work.

Problem I.

To find the Height of an accessible Object.

Rule.

Let A B, in the annexed diagram, be the object: from B measure any convenient distance to C; take, at C, with a quadrant or other instrument, the angle A D E; then, in the triangle A D E, given the side D E = B C, and the angle at D; to find the side A E: to which let the height of the observer's eye above the horizontal plane = C D or B E be added, and the sum will be the true height of the object A B.



Example.

Let the horizontal distance B C be 250 feet, the angle of elevation A D E = 41°45′, and the height of the eye C D = 5 feet; required the height of the object A B?

This comes under Problem II. of right angled plane trigonometry, page 172; and, by making D E radius, it will be

As radius = ...	90°	Log. co-secant =	10.000000
Is to the distance D E = C B =	250 feet	Log. = . . .	2.397940
So is the angle of elevation A D E =	41°45′	Log. tangent = .	9.950625
To the part A E = ...	223.13	Log. = . . .	2.348565
Height of the eye C D = . .	5.		
Height of the object A B = .	228.13 feet, as required.		

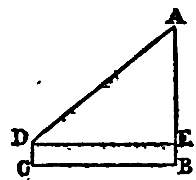
Remark.—By removing either towards or from the object, until the quadrant shows the angle of altitude to be 45 degrees, the measure of the distance between the foot of the observer and that of the object, augmented by the height of the eye, will become the altitude or height of that object.

PROBLEM II.

Given the Angle of Elevation, and the Height of an Object; to find the Observer's horizontal Distance from that Object.

RULE.

At any convenient distance, as at C, let the angle of elevation A D E be taken; then, in the triangle A D E, given A E = the height of the object A B, diminished by the height of the eye C D, or its equal B E, and the angle at D; to find the horizontal distance D E = C B.



Example.

Let the height of the object A B be 175 feet, the angle of elevation A D E 37°20′, and the height of the observer's eye C D = 5 feet; required the horizontal distance B C?

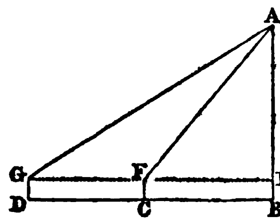
This falls under Problem II., of right angled plane trigonometry, page 172; and by making A B radius, the proportion will be

As radius = ... 90°	Log. co-secant =	10.000000
Is to height of the object A B 175 ft.—B E 5 ft.=170	Log. =	2.230449
So is the angle of elevation A D E=37°20′	Log. co-tangent=	10.117637
To the horizontal dist. D E=C B=222.89	Log. = . . .	2.348086

Problem III.

To find the Height of an inaccessible Object, as A B.

Rule.



At any convenient points, as C and D (these being in the same vertical plane with A B), observe the angles of elevation A F E and A G E; and measure the distance C D: then, because the exterior angle A F E is equal to the two interior and opposite angles A G F and G A F (Euclid, Book I., Prop. 32), if from the angle A F E the angle A G E be subtracted, the remainder will be the angle G A F. Now, in the oblique angled triangle A G F, given the side G F = D C, and the angles A and G; to find the side A F: and, in the right angled triangle A F E, given the hypothenuse A F, found as above, and the angle A F E; to find the perpendicular A E: to which let the height of the observer's eye above the horizontal plane be added, and the sum will be the height of the object A B.

Example.

In the above diagram let the angle of elevation at C = A F E be 49°28′, and, after receding 200 feet in the same vertical plane, to the point D, let the angle of elevation A G E be 31°20′; now, admitting the height of the observer's eye above the horizontal plane = D G or B E to be 5 feet, it is required to determine the height of the object A B?

The angle A F E 49°28′ − the angle A G F 31°20′ = the angle G A F 18°8′.

Now, in the oblique angled triangle A G F, since the angles and one side are given, the side A F is found by oblique angled plane trigonometry, Problem I., page 177; and, in the right angled triangle A E F, since the hypothenuse A F is now known, and the angle at F given, the perpendicular A E is found by right angled plane trigonometry, Problem I., page 171. Hence,

To find the Side A F:—

As the angle G A F = ...	18° 8′	Log. co-secant =	10.506919
Is to the side G F = D C = . .	200	Log. = . . .	2.301030
So is the angle A G F = . . .	31°20′	Log. sine = .	9.716017
To the side A F = ...	334.17	Log. = . . .	2.523966

To find the Perpendicular A E :—

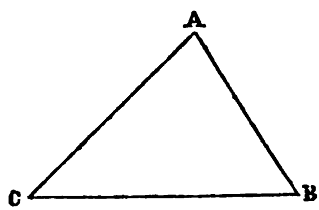
As radius = ...	90°	Log. co-secant=	10.000000
Is to the hypothenuse A F = . . .	334.17	Log. = ...	2.523966
So is the angle A F E = ...	49°28′	Log. sine = .	9.880830
To the perpendicular A E = . . .	253.98	Log. = . . .	2.404796
Height of the eye B E = ...	5.		
Height of the object A B = . . .	258.98 feet, as required.		

Remark.—If it be required to know the horizontal distance B C or B D, it may be readily determined by means of the last problem.

Problem IV.

To find the Distance of an inaccessible Object which the Observer can neither advance towards nor recede from in its vertical Line of Direction.

Rule.



Let the point A be any inaccessible object, and B and C two stations from which the distance of that object is to be determined: measure the distance B C, and, with a sextant or other instrument, observe the horizontal angles A B C and A C B; then, in the triangle A B C, the angles and the side B C are given; to find the other two sides, viz., A B and A C.

Example.

Let the horizontal angle A B C, measured with a sextant, be 59°15′, the angle A C B 42°45′, and the measured base line B C 350 yards; required the respective distances A B and A C?

The angle A B C 59°15′ + the angle A C B 42°45′ = 102°; and 180° − 102° = 78°, the angle C A B.

Now, the angles and one side being thus known, the remaining sides are to be determined by oblique angled trigonometry, Problem I., page 177. Hence the following proportions :—

To find the Distance A C :—

As the angle C A B = ...	78°	Log. co-secant =	10.009596
Is to the side B C = ...	350	Log. = . . .	2.544068
So is the angle A B C = . . .	59°15′	Log. sine = . .	9.934199
To the distance A C = . . .	307.51	Log. = . . .	2.487863

To find the Distance A B :—

As the angle C A B = ...	78°	Log. co-secant =	10.009596
Is to the side B C = ...	350	Log. = . . .	2.544068
So is the angle A C B = . . .	42°45′	Log. sine = .	9.831742
To the distance A B = ...	242.89	Log. = . . .	2.385406

Remark.—This problem will be found of very essential service to His Majesty's ships and vessels of war, on many hostile occasions: for, when it is intended that a squadron of those ships should cannonade a fort to effect, or batter a breach in the sea-defences of a town, the distance at which the ships should be placed, abreast of such fort or town, with the view of opening their fire to the greatest advantage, may be readily determined in the above manner. Thus, let two competent persons, provided with sextants, in two ships, observe the angles subtended between the fort and each ship respectively; and let the distance between the two ships be carefully ascertained, which is readily done by Problem II., page 530, provided the height of the masts be known; or it may be found by means of a boat sent from one ship to the other, with instructions to pull at an uniform rate: then, if the interval, per watch, be noted between the time of the boat's pulling off from one ship and that of her arrival at the other, and her velocity or hourly rate of sailing be duly determined by the log, the distance between those ships may be easily obtained by the rule of proportion.

Now, with the distance between the two ships as a base line, thus found, and the angles subtended between the fort and each ship, the respective distances of those ships from the fort may be very readily computed, agreeably to the principles of the present problem.

Note.—The most convenient distance for commencing a cannonade, is about 300 yards; that is, about a cable and a quarter's length from the object at which the guns are directed. On such occasions, however, the captains of His Majesty's ships of war always make choice of a much closer position, provided there be a sufficient depth of water.

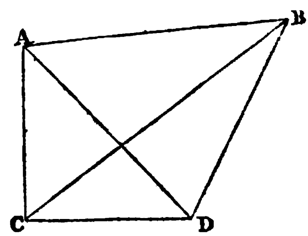
This problem is also extremely useful in military movements: because, when a general is determined on the reduction of a town or garrison, his engineer is thus enabled to apprize him of his absolute distance from any point of the enemy's defences against which he may be desirous of commencing operations, and of the most advantageous position for throwing up batteries which may produce the greatest possible effect on the fortified works of the besieged.

In military operations, the battering guns are generally placed at about 375 paces ($312\frac{1}{2}$ yards) from the works intended to be breached.—A military pace is reckoned at 30 inches.

Problem V.

To find the Distance between two inaccessible Objects.

Rule.



Let A and B be any two inaccessible objects, the distance between which is required. Measure any base line, as C D; at the point C observe the angles A C B, B C D; and, at the point D, observe the angles B D A, A D C. Now, in the triangle A C D, in which the angles and the side C D are given, compute the side A D, by oblique angled trigonometry, Problem I., page 177. In like manner, in the triangle B C D, where the angles and the side C D are given, compute the side B D by the above-mentioned problem. Now, in the triangle A B D, the sides A D and B D, and the included angle A D B, are given; with which the distance A B is to be computed, by oblique angled trigonometry, Problem III., page 179.

Example.

Wanting to know the distance between the two inaccessible objects A and B, in the above diagram, I measured a base line C D of 360 yards: at C, the horizontal angle A C B was observed with a sextant, and found to be 53°30′, and the angle B C D 38°45′; at D, the horizontal angle B D A was 67°20′, and the angle A D C 44°30′; required the distance between A and B?

Angle A C B 53°30′ + angle B C D 38°45′ = angle A C D 92°15′; and angle A C D 92°15′ + angle A D C 44°30′ = 136°45′. Now, 180° − 136°45′ = the angle C A D 43°15′.

Again: angle B D A 67°20′ + angle A D C 44°30′ = angle B D C 111°50′; and angle B D C 111°50′ + B C D 38°45′ = 150°35′. Now, 180° − 150°35′ = the angle C B D 29°25′.

In the Triangle A C D, to find the Side A D:—

As the angle C A D = . . .	43°15′	Log. co-secant =	10.164193
Is to the side C D = ...	360	Log. = . . .	2.556303
So is the angle A C D = . . .	92°15′	Log. sine = . .	9.999665
To the side A D = ...	525.0	Log. = . . .	2.720161

In the Triangle B C D, to find the Side B D:—

As the angle C B D = . . .	29°25′	Log. co-secant =	10.308779
Is to the side C D = . . .	360	Log. = . . .	2.556303
So is the angle B C D = . . .	38°45′	Log. sine = . .	9.796521
To the side B D = ...	458.78	Log. = . . .	2.661603

In the Triangle A B D, to find the Angle D A B or D B A, and the Side A B:—

180° − the angle B D A 67°20′ = 112°40′ ÷ 2 = 56°20′ = half the sum of the angles D B A and D A B.

As the sum of the sides A D and D B =	983.78	Log. ar. comp. =	7.007102
Is to their difference = ...	66.28	Log. = . .	1.821388
So is ½ sum of angles D B A and D A B =	56°20′	Log. tangent =	10.176476
To half difference of ditto = . .	5°46′33″	Log. tang. =	9.004961

Angle D B A = ... 62° 6′33″
Angle D A B = ... 50°33′27″

To find the Distance A B:—

As the angle D A B = . . .	50°33′37″	Log co-secant =	10.112218
Is to the side B D = . . .	458.78	Log. = . .	2.661603
So is the angle A D B = . .	67°20′ 0″	Log. sine = .	9.965090
To the side A B = . . .	548.16	Log. = . . .	2.738911

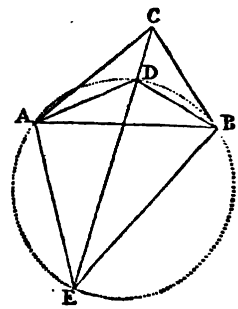
which, therefore, is the required distance.

Note.—This problem is very useful in taking surveys of coasts, harbours, bays, islands, &c.

Problem VI.

Given the Distances between three Objects, and the angular Distances between these Objects taken at any Point in the same horizontal plane; to find the Distance between that Point and each of the Objects.

Rule.



Let A, B, and C be any three objects whose distances from each other are given, and E the place of the observer: at E, observe the angles C E A and C E B; connect the points A, B, and C, by right lines; make the angle A B D equal to the observed angle C E A, and make the angle D A B equal to the angle C E B: hence the point D is found; then, through the three points A, D, and B, describe the circle A D B E; join C D, and produce this line till it meet the circle at the point E, the place of the observer; join E A and E B.

Now, in the triangle A B C, of which the three sides are given, find the angle B A C. In the triangle A B D, in which the angles and the side A B are known, find the side A D. In the triangle A C D, of which two sides, A C and A D, and the included angle C A D are known, find the angle A C D. In the triangle A E C, of which the angles and the side A C are given, find the sides E A and E C. And in the triangle A B E, the sides A B, A E, and the angles A E B and E A B are given; to find the side E B.

Example.

Let the points A, B, and C, in the above diagram, be three known objects: the distance between A and B, 290 yards; between B and C, 195 yards; and between A and C, 240 yards: let E be the place of an observer, where the angle C E A was measured with a sextant and found to be 30°5′, and the angle C E B 25°45′; required the distances E A, E C, and E D?

In the triangle A B C, the three sides are given; to find the angle B A C. Hence, by oblique angled plane trigonometry, Problem IV., page 180,

Side BC (opposite the required angle) = 195		
Side AC (containing the required angle) = 240		Log. ar. co. = 7.619789
Side AB (containing the required angle) = 290		Log. ar. co. = 7.537602
Sum = ... 725		
Half sum = ... 362.5	Log. = .	2.559308
Remainder = ... 167.5	Log. = .	2.224015
	Sum =	19.940714
Arch = ... 20°55′46″	Log. co-sine =	9.970357
Angle BAC = ... 41°51′32″		
Angle BAD = the angle CEB = 25.45. 0		
Angle DAC = ... 16° 6′32″		

In the triangle ABD, the angles and the side AB are given; to find the side AD: thus, the angle ABD (= the angle CEA) = 30°5′ + the angle DAB (= the angle CEB) = 25°45′ = the angle AEB 55°50′; and 180° − 55°50′ = the angle ADB = 124°10′: for, the angle ADB is evidently the supplement of the angle AEB; because the opposite angles of every quadrilateral figure described in a circle are equal to two right angles.—Euclid, Book III., Prop. 22. Hence, by trigonometry,

As the angle ADB =	124°10′	Log. co-secant =	10.082281
Is to the side AB =	290	Log. = . . .	2.462398
So is the angle ABD =	30° 5′	Log. sine = .	9.700062
To the side AD =	. 175.69	Log. = . . .	2.244741

In the triangle ADC, the two sides AC, AD, and the included angle DAC, are given; to find the angle ACD: hence, by oblique angled trigonometry, Problem III., page 179,

As side AC 240 + side AD 175.69 =	415.69	Log. ar. co. 7.381230
Is to side AC 240 − side AD 175.69 =	64.31	Log. = 1.808279
So is 180° − angle DAC 16°6′32″ = 163°53′28″ ÷ 2 = . . .	81°56′44″	Log. tang. 10.849213
To half diff. of angles ADC and ACD =	47.33. 3	Log. tang. 10.038722
Angle ACD = ...	34°23′41″	

In the triangle AEC, the angles and the side AC are given; to find the sides EA and EC: thus, the angle CEA 30°5′ + the angle ACE 34°23′41″ = 64°28′41″; and 180° − 64°28′41″ = the angle EAC 115°31′19″. Hence, by oblique angled trigonometry, Problem I., page 177,

To find the Side EA:—

As the angle AEC =	30° 5′ 0″	Log. co-secant =	10.299938
Is to the side AC =	240	Log. = . . .	2.380211
So is the angle ACE =	34°23′41″	Log. sine = . .	9.751965
To the side EA = .	270.47	Log. = . . .	2.432114

To find the Side EC:—

As the angle AEC =	30° 5′ 0″	Log. = . . .	10.299938
Is to the side AC =	240	Log. = . . .	2.380211
So is the angle EAC =	115°31′19″	Log. sine = . .	9.955407
To the side EC =	432.07	Log. = . . .	2.635556

In the triangle ABE, the sides AB, AE, and the angles AEB, EAB, are given; to find the side EB: thus, from the angle EAC 115°31′19″, take the angle BAC 41°51′32″, and the remainder is the angle EAB = 73°39′47″. Hence, by trigonometry,

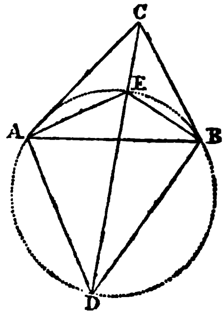
As the angle AEB =	55°50′ 0″	Log. co-secant =	10.082281
Is to the side AB =	290	Log. = . . .	2.462398
So is the angle EAB =	73°39′47″	Log. sine = . .	9.982101
To the side EB =	336.34	Log. = . . .	2.526780

Hence the distance of the object A from the observer at E, is 270.47 yards; that of C, 432.07 yards; and that of B, 336.34 yards.

Problem VII.

Given the Distances between three Objects, and the angular Distances between these Objects, taken at any Point within the Triangle formed by the right Lines connecting the Objects; to find the Distance between that Point and each of the Objects.

Rule.



Let A, B, and C be any three objects whose distances from each other are given, and E the place of the observer: complete the triangle A B C; at E, observe the angles A E C, A E B, and B E C; make the angle B A D equal to the supplement of the angle B E C; in like manner, make the angle A B D equal to the supplement of the angle A E C: hence the point D is found. Through the three points A, B, and D, describe a circle; join D C, and it will cut the circle in E, the place of the observer; connect the points A E, B E, and the construction will be completed; the calculations in which will be nearly similar to those in the preceding problem.

Example.

Let A, B, and C, in the above diagram, be any three known objects whose distances from each other are as follow: viz., A B, 620 yards; A C, 570 yards; and B C, 460 yards. At a point E, within the triangle formed by those objects, the angle A E C was measured with a circle, and found to be 125°15′; the angle A E B, 124°15′; and the angle B E C, 110°30′; required the distances E A, E C, and E B?

In the triangle A B D, the angles and the side A B are given; to find the side A D: thus, the angle B A D 69°30′ = the supplement of the angle B E C; the angle A B D 54°45′ = the supplement of the angle A E C; and the angle A D B 55°45′ = the supplement of the angle A E B. Hence, by trigonometry,

As the angle A D B =	55°45′	Log. co-secant = 10.082710
Is to the side A B =	620	Log. = . . . 2.792392
So is the angle A B D =	54°45′	Log. sine = . 9.912032
To the side A D =	612.54	Log. = . . . 2.787134

In the triangle A B C, all the sides are given; to find the angle B A C: which, being added to the angle B A D, will give the obtuse angle C A D. Hence, by trigonometry, Problem IV., page 180.

Side BC =	460		
Side AC =	570	Log. ar. comp. =	7.244125
Side AB =	620	Log. ar. comp. =	7.207608
Sum = .	1650		
Half sum =	825	Log. = . .	2.916454
Remainder =	365	Log. = . .	2.562293
		Sum =	19.930480
Arch = .	22°37′ 9″	Log. co-sine =	9.965240

Angle CAB = 45°14′18″ + angle BAD = 69°30′ = angle CAD 114°44′18″.

In the triangle ACD, the sides AC, AD, and the included angle CAD are given; to find the angle ACD: hence, by oblique angled trigonometry, Problem III., page 179,

As the side AD 612.54 + the side AC 570 =	1182.54	Log. ar. co.	6.927184
Is to the side AD 612.54 − the side AC 570 =	42.54	Log. =	1.628798
So is 180° − angle CAD 114°44′18″ = 65°15′42″ ÷ 2 = . . .	32°37′51″	Log. tang.	9.806374
To half diff. of angles ACD and ADC =	1.19.10	Log. tang.	8.362356
Angle ACD = ...	33°57′ 1″		

In the triangle ADC, all the angles and the side AC are given; to find the sides AE and EC: thus, the angle AEC 125°15′ + angle ACE 33°57′1″ = 159°12′1″; and 180° − 159°12′1″ = the angle CAE 20°47′59″. Hence, by oblique angled trigonometry, Problem I., page 177,

To find the Side AE:—

As the angle AEC =	125°15′ 0″	Log. co-secant =	10.087968
Is to the side AC =	570	Log. = . . .	2.755875
So is the angle ACE =	33°57′ 1″	Log. sine = .	9.747002
To the side AE =	389.80	Log. = . . .	2.590845

To find the Side EC:—

As the angle AEC =	125°15′ 0″	Log. co-secant =	10.087968
Is to the side AC =	570	Log. = . . .	2.755875
So is the angle CAE =	20°47′59″	Log. sine = .	9.550359
To the side EC =	247.86	Log. = . . .	2.394202

In the triangle BEC, given the sides BC, CE, and the angle BEC; to find the angle BCE, and, thence, the side BE: the angle BCE is found by oblique angled trigonometry, Problem II., page 178; and the side BE by Problem I., page 177. Hence,

To find the angle BCE:—

As the side BC =	460	Log. ar. comp. =	7.337242
Is to the angle BEC =	110°30′	Log. sine = .	9.971588
So is the side EC =	247.86	Log. = . . .	2.394202
To the angle CBE =	30°18′41″	Log. sine = .	9.703032
Angle BEC = . . .	110.30. 0		

Sum = ... 140°48′41″; and 180° − 140°48′41″ = the angle BCE = 39°11′19″

To find the Side BE:—

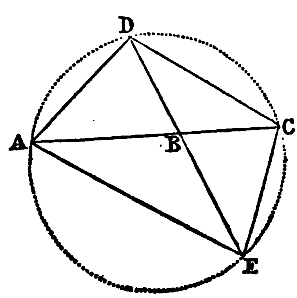
As the angle BEC =	110°30′ 0″	Log. co-secant =	10.028412
Is to the side BC =	460	Log. = . . .	2.662752
So is the angle BCE =	39°11′19″	Log. sine = .	9.800631
To the side BE =	310.31	Log. = . . .	2.491795

Hence the required distances are, EA, 389.80 yards; EB, 310.31 yards; and EC, 247.86 yards.

Problem VIII.

Given the Distances between three Objects situated in a straight Line, and the angular Distances of these Objects taken at any Point in the same horizontal Plane; to find the Distance between that Point and each of the Objects.

Rule.



Let the points A, B, and C be any three objects situated in a straight line: make the angle ACD equal to the observed angle AEB, and make the angle DAC equal to the observed angle BEC: hence the point D is found. Through the three points A, D, and C describe a circle; join DB, and produce it till it cuts the circle in E: then E will be the place of the observer, and EA, EB, and EC the required distances.

Example.

Let A, B, and C, in the above diagram, be any three known objects situated in a straight line, whose distances from each other are as follow: viz., A B, 490 yards; B C, 300 yards; and A C, 790 yards: at a point E, the angle B E C was observed, and found to be 43°, and the angle B E A 33°45′; required the distances E A, E B, and E C?

In the triangle A D C, all the angles and the side A C are given; to find the side A D: thus, the angle D A C 43° = the observed angle B E C; the angle A C D 33°45′ = the observed angle B E A; and, consequently, the angle A D C = 103°15′: hence the side A D may be found.

As the angle A D C =	103°15′	Log. co-secant = 10.011718
Is to the side A C =	790	Log. = . . . 2.897627
So is the angle A C D =	33°45′	Log. sine = . 9.744739
To the side A D =	450.94	Log. = . . . 2.654084

In the triangle A B D, given the sides A B, A D, and the included angle D A B; to find the angle A B D: hence, by trigonometry, Problem III., page 179,

As the side A B 490 + the side A D 450.94 =	940.94	Log. ar. co. 7.026438
Is to the side A B 490 − side A D 450.94 =	39.06	Log. = 1.591732
So is 180° − angle D A B 43° = 137° ÷ 2 =	68°30′ 0″	Log. tang. 10.404602
To half diff. of angles A D B and A B D =	6° 0′57″	Log. tang. 9.022772

Angle A B D = ... 62°29′ 3″; and, since the two straight lines A C and D E intersect each other in the point B, the opposite angles are equal to one another (Euclid, Book I., Prop. 15): therefore the angle E B C is 62°29′3″, equal to the angle A B D. In like manner, the angles D B C and A B E are equal to one another; and because D B C is the supplement of the angle D B A, it is equal to 117°30′57″: hence the angle A B E is also equal to 117°30′57″.

In the triangle A B E, all the angles and the side A B are given; to find the sides E A and E B: thus, the angle B E A 33°45′ + the angle A B E 117°30′57″ = 151°15′57″; and 180° − 151°15′57″ = the angle B A E 28°44′3″. Hence, by trigonometry, Problem I., page 177, To find the Side E A :—

As the angle B E A =	33°45′ 0″	Log. co-secant=	10.255261
Is to the side A B =	490	Log. = . . .	2.690196
So is the angle ABE=	117°30′57″	Log. sine = .	9.947867
To the side E A =	782.21	Log. = . . .	2.893324

To find the Side E B :—

As the angle B E A =	33°45′ 0″	Log. co-secant=	10.255261
Is to the side A B =	490	Log. = . . .	2.690196
So is the angle BAE=	28°44′ 3″	Log. sine = .	9.681917
To the side E B =	424.01	Log. = . . .	2.627374

In the triangle E B C, given the sides E B, B C, and all the angles; to find the side E C.

As the angle B E C =	43° 0′ 0″	Log. co-secant=	10.166217
Is to the side B C =	300	Log. = . . .	2.477121
So is the angle E B C =	62°29′ 3″	Log. sine = .	9.947866
To the side E C =	390.13	Log. = . . .	2.591204

Hence the required distances are, E A, 782.21 yards; E B, 424.01 yards; and E C, 390.13 yards.

Remark.—The above problem, together with that given in page 536, will be found exceedingly useful to a general or other officer employed in conducting the military operations of a *siege*; because, if he can only procure a correct map of the town or garrison which he may have occasion to invest, so as to ascertain the relative distances between any three desirable positions, the above problems will enable him to find his absolute distance from those positions without the trouble of measuring a base line: nor is it necessary to resort to trigonometrical calculation for this particular purpose, since the distances may be readily determined by geometrical projection, to every degree of accuracy desirable in such operations.

PROBLEM IX.

Given the Height of the Eye; to find the Distance of the visible Horizon.

RULE.

Let the earth's diameter, *in feet*, be augmented by the height of the eye; then, to the logarithm thereof, add the logarithm of the height of the eye; from half the sum of these two logarithms subtract the constant logarithm 3.783904,* and the remainder will be the logarithm of the distance in nautical miles; which is to be increased by a twelfth part of itself, on account of terrestrial refraction.

Example.

Chimboraço, the highest part of the Andes, is said to be 20633 feet above the level of the sea: now, admitting that an observer be placed upon its summit, at what distance can he see the visible horizon, allowing a twelfth part of that distance for the effects of refraction?

Diameter of the earth, in feet, = . . .	41804400	
Elevation of Chimboraço 20633 + 5 feet, the height of the observer's eye =	20638	Log. = 4.314668
Sum = ...	41825038	Log. = 7.621436
Sum = ...		11.936104
Half sum = ...		5.968052
Constant log. = ...		3.783904
Distance uncorrected by refraction = . . .	152.81	Log. = 2.184148
Allowance for terrestrial refraction = . . .	12.40	

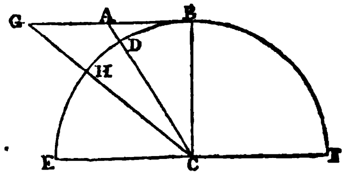
Dist. at which the visible horizon may be seen = 165.21 miles.

* This is the logarithm of 6080, the number of feet in a nautical mile.

Problem X.

Given the measured Length of a base Line; to find the Allowance for the Curvature or spherical Figure of the Earth.

Rule.



Let E B F represent the arc of a great circle on the earth; C, the earth's centre; C B its semi-diameter; and A B the measure of a base line, on an apparent level or horizontal plane on the earth's surface: join C A, and it will cut the arc of the great circle in D; then A D will be the excess of the apparent level of the horizon above its true level.

Now, in the right angled plane triangle A B C, given the perpendicular B C and the base A B; to find the hypothenuse A C: which is readily determined by Euclid, Book I., Prop. 47. Then, the difference between A C, thus found, and C D = C B, will be equal to D A, or the absolute value of the true level *below* the apparent level: and, if this value be expressed in miles and decimal parts of a mile, it may be reduced to inches, if necessary, by being multiplied by 63360 = the number of inches in an English mile.

Example.

Let the base line A B, in the above diagram, be 1 English mile, and the earth's semi-diameter B C = 3958.75 miles; required the allowance for the earth's curvature answering to that base line, or the difference between the true and apparent levels on the earth's surface expressed by the measure of the line A D?

B C 3958.75 × B C 3958.75 = 15671701.5625
A B = 1 × A B = 1 = . 1.

Sum of the squares = . . 15671702.5625; the square root of which = C A, is 3958.7501263
Subtract C D = C B, the earth's semi-diameter, = . . 3958.7500000

Remainder = the line A D, the allowance for curvature, = 0000.0001263
Multiply by the number of inches in an English mile, = 63360.

Number of inches which the true level is below the apparent level in one mile = ... 8.0023680

2 N

Now, since the curvature answering to A B is known, that corresponding to any other base line on the earth's surface may be readily determined by the following proportion:—

As the square of A B, is to A D; so is the square of B G, to G H: whence it is manifest, that the curvature answering to any given distance, as B G, is in the *duplicate ratio* of that distance to A B:

And, since A B is expressed by unity or 1, and that A D is a constant quantity, the proportion may be reduced to a logarithmical expression; as thus:—

To twice the logarithm of the given base line, expressed in miles and decimal parts of a mile, add the constant logarithm 0.903219 (the log. of 8.002368 inches); and the sum will be the logarithm of the number of *inches and decimal parts of an inch* which the true horizontal level at sea is below its apparent level.

Example 1.

Required the curvature of the earth answering to a distance of 2 miles on its surface?

Distance = 2 miles; twice the log. = . . .	0.602060
Constant log. = ...	0.903219
Curvature, in inches, = 32.009 Log. =	1.505279

Hence, the curvature answering to a distance of 2 miles on the surface of the earth, is 32.009 inches; or $2\frac{2}{3}$ feet, nearly.

Example 2.

Required the curvature of the earth answering to a distance of 15 miles?

Distance = 15 miles; twice the log. = . .	2.352182
Constant logarithm = ...	0.903219
Curvature, in inches, = 1800.533 Log. =	3.255401

Hence, the curvature answering to a distance of 15 miles on the earth's surface, is $1800\frac{1}{2}$ inches; or 150 feet and half an inch.

Remark.—If to twice the logarithm of the given base line, in miles, the constant logarithm 9.824037 be added, the sum (abating 10 in the index,) will be the logarithm of the excess of the apparent above the true level, in feet.

Example.

Required the curvature of the earth, or the excess of the apparent above the true level, answering to a base line of 15 English miles in length?

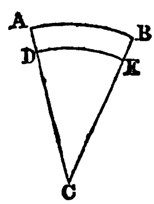
Given base line = 15 miles; twice the logarithm = . . .	2.352182
Constant log. = log. of 8.002368 inches, diminished by the log. of 12 inches, = ...	9.824037
Excess of the app. above the true level, in ft., = 150.044 Log. =	2.176219

Note.—This problem will be found useful to land-surveyors, engineers, and others employed in the art of levelling, cutting canals, and conducting water (by means of pipes, &c.) from one place to another.

Problem XI.

Given the measured Length of a Base Line on any elevated Level; to find its true Measure, when referred to the Level of the Sea.

Rule.



In the annexed diagram, let the arc A B represent the measured length of a base line, at any given elevation above the level of the sea expressed by the arc D E; let C D be the radius of the earth, or the distance from its centre to the surface of the sea; and let C A be the earth's radius referred to the level of the measured base line A B. Now, because the arcs A B and D E are concentric and similar, and that similar arcs of spheres are to each other as their radii, we have the following analogy; viz.,

As the radius C A, is to the radius C D; so is the arc A B, to the arc D E: that is, as the earth's semi-diameter, augmented by the height of the base line above the level of the sea, is to the earth's true semi-diameter; so is the measured length of the given base line, to the true measure of that line at the surface of the sea.

Example.

Given a base line of 36960 feet in length, measured on a horizontal plane which is elevated 120 feet above the level of the sea; required the measure of that base line at the surface of the sea?

As CD = 20902200 + DA = 120 = CA 20902320 = AB 36960 :: CD 20902200 : DE = 36959.787813. Hence the given base line, reduced to the level of the sea, is 36958.787813 feet; which is about $2\frac{1}{2}$ inches less than the measure on the elevated horizontal plane.

But, since the probable elevation of any horizontal plane on the earth, above the level of the sea, can bear but a very insignificant proportion to the earth's semi-diameter,—if, therefore, the product of the measured base line by its height above the level of the sea be divided by the earth's radius, the quotient will be the excess of the measured base above the corresponding arc at the surface of the sea. This may be reduced to a logarithmical expression, in the following manner; viz., to the constant logarithm 2.679808, add the logarithms of the base line and of its elevation above the level of the sea, both expressed in feet: the sum will be the logarithm of a natural number, which, being taken from the measured base line, will leave the measure of that line at the surface of the sea, sufficiently near the truth for all practical purposes. Thus, to work the last example,

Constant log. = ar. co. of the log. of the earth's semi-diam. in ft. = 2.679808
Elevation of given base line above level of sea = 120 feet. Log. = 2.079181
Measured length of the given base line = 36960 feet. Log. = 4.567732

Excess of the given base line above the
arc at the surface of the sea = . . . — 0.212188 Log. 9.326721

Given base line, reduced to level of sea, = 36959.787812; which approximates so very closely to the true result by the direct method of computation, as scarcely to admit of any sensible difference.

Remark.—In consequence of the spherical figure of the earth, no two points on its surface can be situated exactly on the same horizontal plane; for it is the chord of the arc, and not the arc itself, that measures the horizontal distance between two points. Hence, when philosophical inquiries are under consideration, it becomes necessary to apply a small correction to the measured base line on a horizontal plane, so as to reduce it to the corresponding terrestrial arc; though, in general, this correction is so very inconsiderable, that, even in the most extensive trigonometrical surveys, it may be safely disregarded. If, however, it be deemed necessary to find its value, or (which amounts to the same thing) if the excess of the arc over its chord be required, it may be very readily determined by the following rule, to every desirable degree of accuracy; viz.,

From thrice the measured length of the base line, in feet, subtract the constant logarithm 16.020595: the remainder will be the logarithm of the excess of the arc over its corresponding chord, expressed by the given base line.

Let it be required to find the excess of the terrestrial arc over its chord, answering to a measured base line of 36960 feet in length, or seven English miles?

Given base line = 36960 feet; thrice its logarithm = . . 13.703196
Constant log. = log. of 24 times the square of the earth's radius, in feet, = ... 16.020595

Excess of the arc over its chord, in feet, = 0.004815 Log. = − 7.682601

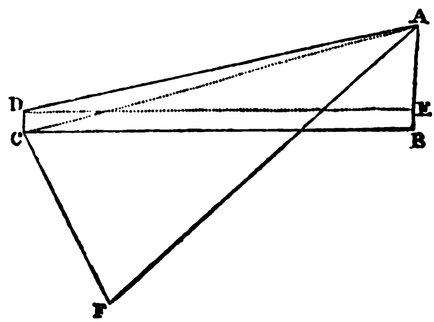
Hence it is evident, that the extent by which a terrestrial arc of 36960 feet exceeds the chord of the same arc, is only the small decimal fraction .004815 of a foot,—an excess so very trivial, as to be scarcely worth taking into account, even where the greatest accuracy is required: for, in the present instance, though the base line is 7 English miles in length, it amounts to no more than the *two hundred and sixteenth part of an inch.*

Note.—The index of the logarithm of the excess comes out a negative quantity, because the index of the constant logarithm is greater than that of the term from which it is subtracted.

Problem XII.

To find the Height and Distance of a Hill or Mountain.

Rule.



Let the point A, in the annexed diagram, be the summit of a hill, the height of which, A B, is to be determined; and let the point C be the place from which its distance is to be found: at C, observe the vertical angle A D E; then measure any convenient distance for a base line, as C F; at the point C, observe the inclined angle A C F, and, at F, observe the angle A F C. Now, in the inclined triangle A C F, given the angles and the side F C; to find the side A C, which may be considered as being essentially equal to the side A D: and, in the vertical or right angled triangle A E D, the angle A D E and the hypothenuse or side A D are given; to find the perpendicular A E: to which, the height of the eye B E = C D being added, gives the required height A B.

Example.

Wanting to know the height of the hill A B, and the distance of its summit A from the point C, the vertical angle A D E was observed, and found to be 14°30′; at the points C and F, 500 feet asunder, the inclined angles A C F and A F C were measured, and found to be 80°5′, and 73°30′ respectively; required the distance of the point A from the observer at C, and its height above the level of the horizontal plane C B?

In the inclined triangle A C F, the angles are given, and the side F C = 500 feet; to find the side or distance A C: thus, the angle A C E 80°5′ + the angle A F C 73°30′ = 153°35′; and 180° − 153°35′ = 26°25′, the measure of the angle C A F. Hence, by trigonometry,

To find the Distance A C = the Side A D:—

As the angle C A F =	26°25′	Log. co-secant =	10.351742
Is to the side C F = .	500	Log. = . .	2.698970
So is the angle C F A =	73°30′	Log. sine = .	9.981737
To the distance A C =	1077.55	Log. = . .	3.032449;

and, since A C and A D are essentially equal, the side A D is also 1077.55 feet.

To find the Height A B:—

As radius = . . .	90°	Log. co-secant =	10.000000
Is to the side A D = A C	1077.55	Log. = . .	3.032449
So is the angle A D E =	14°30′	Log. sine = .	9.398600
To the perpendicular A E =	269.84	Log. = . . .	2.431049
Height of the eye B E = C D	5.		
Height of A B = . .	274.84 feet.		

PROBLEM XIII.

To find the Height of a Mountain, by means of two Barometers and Thermometers.

RULE.

Let two observers (provided with barometers and thermometers of equal construction,) carefully note down, at the same instant, the respective heights of the barometers at the top and bottom of the mountain, or other eminence intended to be measured, with the temperature of the quicksilver in each instrument by means of attached thermometers, and also the temperature of the air, in the *shade*, by means of detached thermometers; then,

Find the difference of the logarithms of the observed heights of the barometers, the first four figures of which, besides the index, are to be considered as whole numbers. To this difference apply the product of 0.454, by the difference of the altitudes of the two attached thermometers, by subtraction if the temperature of the quicksilver at the bottom station exceed that at top; otherwise, by addition: and the sum or difference will be the approximate height, in fathoms, English measure.

Now, to the logarithm of the approximate height, thus found, add the logarithm of the difference between the mean of the two temperatures of the detached thermometers and 32°, and the constant logarithm 7.387390: the sum of these three logarithms will be the logarithm of a correction, which being added to the approximate height when the mean temperature exceeds 32°, but subtracted if it be less, the sum or difference will be the true elevation of the mountain, expressed in fathoms; which may be reduced to feet, if necessary, by being multiplied by 6.

Note.—This rule is deduced from that given by Dr. Hutton, in the second volume of his "Course of Mathematics," page 255.

Example 1.

Let the observations at the top and bottom of a mountain be as follow; required its height?

	Attached thermometer.		Detached thermom.	Barometer.	
Obs. at bottom =	57		57	29.68	Log. = 1.472464
Ditto at top =	43		42	25.28	Log. = 1.402777
Difference =	14	Sum =	99	Difference =	0.0696.87
Multiply by	.454	Mean =	49½	Product =	− 6.36
Product =	−6.356		32	Approx. alt. =	690.51 Log. 2.839170

Diff. = 17½ or 17.5 Log. = . . . 1.243038

Constant log. = ... 7.387390

Correction of the approximate altitude = ... + 29.48 Log. 1.469598

True altitude of the mountain, in fathoms, = . 719.99, or 4319.94 ft.

Example 2.

Let the observations at the top and bottom of a mountain be as follow; required its height?

	Attached thermometer.		Detached thermom.	Barometer.		
Obs. at bottom	=38		31	29.45	Log.=1.469085	
Ditto at top =	41		35	26.28	Log.=1.428459	
Difference =	3	Sum =	66	Difference =	0.0406.26	
Multiply by	.454	Mean=	33	Product =	+ 1.36	
Product =	+1.362		32	Approx. alt. =	407.62	Log.2.610255
		Diff. =	1	Log. =	. . . .	0.000000
		Constant logarithm =		. . . .		7.387390
Correction of the approximate altitude =				. .	+ 0.99	Log.9.997645
True altitude of the mountain, in fathoms, =				.	408.61, or 2451.66 ft.	

Problem XIV.

To find the Distance of an Object, by observing the Interval of Time between seeing the Flash and hearing the Report of a Gun or of a Thunder-Cloud.

Rule.

To the logarithm of the number of seconds elapsed between seeing the flash and hearing the report, add the constant logarithm 9.273762*; and the sum (abating 10 in the index,) will be the logarithm of the distance in nautical miles; or, if the constant logarithm 9.335032† be made use of, it will give the distance in English statute miles.

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* This is the sum of the arithmetical complement of the logarithm of 6080, the number of feet in a nautical mile, and the logarithm of 1142, the number of feet which sound travels in one second of time.

† This is the sum of the arithmetical complement of the logarithm of 5280, the number of feet in an English mile, and the logarithm of 1142 feet, the established velocity of sound.

Example 1.

A ship at sea was observed to fire a gun, and 43 seconds afterwards the report was heard; required the distance of the ship in nautical miles?

Interval betw. seeing the flash and hearing the gun=43ˢ		Log.=1.633469
Constant logarithm = ...		9.273762
Distance in nautical miles = ...	8.076	Log.=0.907231

Example 2.

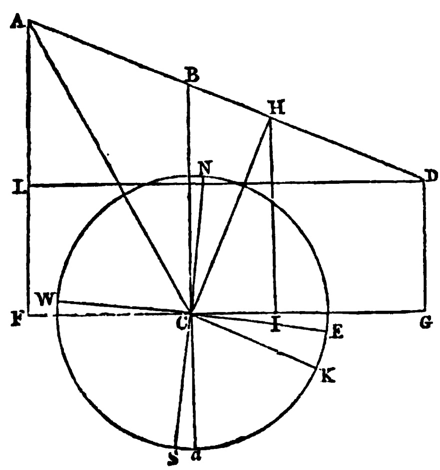
A flash of lightning was seen, and after a lapse of 18 seconds the report reached the ear of the observer; required the distance of the thunder-cloud in English miles?

Inter. betw. seeing the flash and hearing the thunder=18ˢ		Log.=1.255273
Constant logarithm = ...		9.335032
Distance of the thunder-cloud in English miles=3.894		Log.=0.590505

Problem XV.

Given three Bearings of a Ship sailing upon a direct Course, and the Intervals of Time between those Bearings; to find the Course steered by that Ship, and the Time of her nearest Distance from the Observer.

Rule.



Describe the circle NESW, and from the centre C, the place of the observer, draw the lines CA, CB, and CD, to represent the three bearings of the ship: through the centre C draw the line FG, at right angles to the *second bearing* CB; make CF equal to the first interval, and CG equal to the second, each being expressed in minutes, and taken from any convenient scale of equal parts; from the points F and G draw the lines F A, G D, parallel to the second bearing C B, and meeting C A and C D in the points A and D; join A D, and it will represent the ship's track; through C draw C K, parallel to A D, and the arch S K will be the measure of the ship's course. From C let fall the perpendicular C H upon the line A D, produced if necessary; and from H let fall the perpendicular H I upon the line F G, produced also, if necessary; then the measure of C I will give the interval between the time of the second bearing and that when the ship was nearest to the observer.

Make A L equal to the difference between the perpendiculars A F and D G; then, in the right angled triangle A L G, given the perpendicular A L and the base L D; to find the angle L A D, which is evidently equal to the angle *a* C K; to this let the inclination of C B to a parallel be applied, and the result will be the apparent course of the ship.

Example.

At 1h 20m past noon a ship, sailing upon a direct course, was observed to bear N.W. b. N.; at 2h 10m, she bore N. ½ W.; and at 3h 25m, the bearing was N.E. b. E.; required the apparent course steered by that ship, and the time when she was nearest to the observer?

Solution.—The circle being described and quartered, and the three given bearings laid down as above directed, through C draw F G perpendicular to the second bearing C B; make F C equal to 50 minutes, the interval between the first and second bearings, and C G equal to 75 minutes, the interval between the second and third bearings: these may be taken from any scale of equal parts. Then proceed with the other parts of the construction, agreeably to the rule; now, the ship's apparent course, represented by the angle S C K, being applied to the line of chords, will be found to measure 72½ degrees: hence the course is S. 72° 30′ E., or E. b. S. ½ S. nearly. The perpendicular G D, being applied to the scale of equal parts from which the intervals were taken, will be found to measure 40, and the perpendicular F A 93½; the difference between which = 53½, is the measure of A L. Then C I, measured upon the same scale, gives 26 minutes; which is evidently, by the construction, past the time of the second bearing: hence the time of the ship's nearest approach to the observer at C, is 2h 10m + 26m = 2h 36m past noon. Now, the figure being thus completed, the required parts may be obtained by trigonometrical calculation, in the following manner:—

In the right angled triangle A F C, given the angles and the base F C 50 minutes; to find the perpendicular F A. Thus, since the straight line A C falls upon the two parallel straight lines C B and F A, it makes the alternate angles equal to one another (Euclid, Book I., Prop. 29): therefore the angle F A C is equal to the angle A C B; but the angle A C B is given, being equal to $2\frac{1}{2}$ points, viz., the difference between N.W. b. N. and N. $\frac{1}{4}$ W.: hence the angle F A C is also equal to $2\frac{1}{2}$ points.

In the same manner it may be shown (in the right angled triangle D G C, where the angles and the base C G 75 minutes are given; to find the perpendicular G D,) that the angle G D C is equal to the angle B C D; and since B C D is given, being equal to $5\frac{1}{2}$ points, viz., the sum of N.E. b. E., and N. $\frac{1}{2}$ W., therefore the angle G D C is also equal to $5\frac{1}{2}$ points. Hence,

To find the Perpendicular G D:—

As radius = ...	90°	Log. co-secant =	10.000000
Is to the base C G = .	75'	Log. = . . .	1.875061
So is the angle G D C = $5\frac{1}{2}$ points,		Log. co-tangent =	9.727957
To the perpendicular G D =	40.09	Log. = . . .	1.603018

To find the Perpendicular F A:—

As radius = ...	90°	Log. co-secant	10.000000
Is to the base F C . .	50'	Log. = . . .	1.698970
So is the angle F A C = $2\frac{1}{2}$ points,		Log. co-tang. =	10.272043
To the perpendicular F A =	93.54	Log. = . . .	1.971013
Perpendicular G D =	40.09		
Difference = ...	53.45, which is equal to the part A L.		

In the right angled triangle A L D, given the base L D = F G 125 minutes, and the perpendicular A L 53.45; to find the angle L A D: therefore,

As the perpendicular A L =	53.45 minutes	Log. ar. comp. =	8.272052
Is to radius = ...	90.0	Log. sine =	10.000000
So is the base L D = . .	125 minutes,	Log. = . .	2.096910
To the angle L A D = .	66°50'54"	Log. tangent =	10.368962

Now, since C K is parallel to A D, and C *a* to A L, the angle *a* C K is equal to the angle L A D; but the angle L A D is found, by computation, to be 66°50'54"; wherefore the angle *a* C K is also equal to 66°50'54": to this let the angle *a* C S = the angle N C B $0\frac{1}{2}$ point, or 5°37'30", be added; and the sum 72°28'24" = the angle S C K is the apparent course of the ship between the south and the east, viz., S. 72°28'24" E, or E. b. S. $\frac{1}{2}$ S. nearly.

We have now to determine the measure of the base C I, in the right angled triangle C I H; to do which, we must first find the value of the hypothenuse A C in the right angled triangle A F C, and that of the base C H in the right angled triangle A H C. Thus,

To find the Hypothenuse A C :—

As radius =	. . . . . 90°	Log. co-secant =	10.000000
Is to the base F C =	. . 50^m	Log. = . . .	1.698970
So is the angle F A C =	$2\frac{1}{2}$ points,	Log. co-secant =	10.326613
To the hypothenuse A C =	106.07	Log. = . . .	2.025583

To find the Base C H :—

As radius =	. . . . . 90°	Log. co-secant =	10.000000
Is to the hypothenuse A C =	106.07	Log. = . . .	2.025583
So is L A D − F A C = C A H =	38°43′24″	Log. sine =	9.796269
To the base C H =	. . 66.35	Log. = . . .	1.821852

Now, in the right angled triangle C I H, given the hypothenuse C H = 66.35 minutes, and the angle C H I; to find the base C I. The measure of the angle C H I is thus determined. In all quadrilateral or four-sided figures, the sum of the four angles is equal to four right angles, or 360 degrees. Now, in the quadrilateral figure A H I F, since three of the angles are given, the remaining or obtuse angle A H I is known by subtracting the sum of the given angles from 360 degrees: thus, the angle H I F 90° + I F A 90° + F A H 66°50′54″ = 246°50′54″; and 360° − 246°50′54″ = 113°9′6″, is the measure of the angle A H I; from which take away the right angle A H C 90°, and the remainder = 23°9′6″ is the absolute measure of the angle C H I. Hence C I may be readily found; as thus:—

As radius =	. . . . 90°	Log. co-secant =	10.000000
Is to the hypothenuse C H =	66.35 minutes,	Log. = . . .	1.821852
So is the angle C H I =	. 23°9′6″	Log. sine =	. 9.594572
To the interval or base C I =	26.09 minutes.	Log. = . . .	1.416424

Time of second bearing = $2^h 10^m.0$

Sum = ... $2^h 36^m.09$; which is the time of the ship's nearest approach to the observer.

Note.—This interesting problem is thus worked at length, trigonometrically, with the view of adapting it to the use of mariners in general; though, indeed, in such cases, calculation need not be resorted to, since the solution deduced from geometrical construction will always be sufficiently near the truth.

SOLUTION OF PROBLEMS IN PRACTICAL GUNNERY.

Gunnery is the art of projecting balls and shells from great guns and mortars; of finding the ranges and times of flight of shot and shells; and of determining the different degrees of elevation at which those bodies should be projected, so as to produce the greatest possible effect.

PROBLEM I.

Given the Diameter of an iron Ball; to find its Weight.

RULE.

The diameter of an iron ball of 9 lbs. weight is 4 inches, very nearly; and, since the weights of spherical bodies, composed of the same materials, are as the cubes of their diameters, (Euclid, Book XII., Prop. 18,) it will be,—as the cube of 4, is to 9 lbs.; so is the cube of the diameter of any other iron ball, to its weight. Hence the following rule:—

To thrice the logarithm of the diameter of the given ball, add the constant logarithm 9.148063; and the sum (abating 10 in the index,) will be the logarithm of the required weight in lbs.

Example 1.

Required the weight of an iron ball, the diameter of which is 6.7 inches?

Given diameter = 6.7; thrice its log. =	2.478225
Constant log. = ...	9.148063
Weight in pounds = 42.295 Log. = .	1.626288

Example 2.

Required the weight of an iron ball, the diameter of which is 5.54 inches?

Given diameter = 5.54; thrice its log. =	2.230530
Constant log. = ...	9.148063
Weight in pounds = 23.91 Log. = .	1.378593

Note.—The constant logarithm used in this problem is expressed by the arithmetical complement of the logarithm of the cube of 4, added to the logarithm of 9.

PROBLEM II.

Given the Weight of an iron Ball; to find its Diameter.

RULE.

This problem being the converse of the last, we obtain the following logarithmical expression:—

To the logarithm of the weight of the given ball, add the constant logarithm 0.851937; divide the sum by 3, and the quotient will be the logarithm of the required diameter.

Note.—The constant logarithm given in this rule is expressed by the arithmetical complement of the logarithm of 9, added to the logarithm of the cube of 4.

Example 1.

Required the diameter of a 42 lb. iron ball?

Given weight = 42 lb. Log. = . . .	1.623249
Constant log. = ...	0.851937
Divide by 3)	2.475186
Diameter in inches = 6.685 Log. =	0.825062

Example 2.

Required the diameter of a 24 lb. iron ball?

Given weight = 24 lb. Log. = . . .	1.380211
Constant log. = ...	0.851937
Divide by 3)	2.232148
Diameter in inches = 5.547 Log. =	0.744049¼

PROBLEM III.

Given the Diameter of a leaden Ball; to find its Weight.

RULE.

A leaden ball of 1 inch in diameter, weighs $\frac{3}{14}$ of a lb.; which, reduced to a decimal fraction, is .2143, very nearly: and, as the weights of spherical bodies are as the cubes of their diameters, it will be,—as the cube of 1, is to .2143; so is the cube of the diameter of any other leaden ball, to its weight in lbs. Whence the following logarithmical rule:—

To thrice the logarithm of the diameter of the given leaden ball, add the constant logarithm 9.331022; and the sum (abating 10 in the index,) will be the logarithm of the required weight.

Example 1.

Required the weight of a leaden ball, the diameter of which is 6.68 inches?

Given diameter = 6.68; thrice its log. =	2.474331
Constant log. = ...	9.331022
Weight in pounds = 63.88 Log. =	1.805353

Example 2.

Required the weight of a leaden ball, the diameter of which is 5.32 inches?

Given diameter = 5.32; thrice its log. =	2.177736
Constant log. = ...	9.331022
Weight in pounds = 32.26 Log. =	1.508758

Note.—The constant logarithm used in this problem is the logarithm of the decimal fraction .2143.

Problem IV.

Given the Weight of a leaden Ball; to find its Diameter.

Rule.

Since this problem is merely the converse of the last, we obtain the following logarithmical expression; viz., to the logarithm of the weight of the given leaden ball, add the constant logarithm 0.668978; divide the sum by 3, and the quotient will be the logarithm of the required diameter.

Example 1.

Required the diameter of a 64 lb. leaden ball?

Given weight = 64 lb. . . . Log. =	1.806180
Constant log. = ...	0.668978
Divide by 3)	2.475158
Diameter in inches = 6.68 Log. =	0.825052$\frac{2}{3}$

Example 2.

Required the diameter of a 32 lb. leaden ball?

Given weight = 32 lb.	. . .	Log. =	1.505150
Constant log. =	. . . .		0.668978
		Divide by 3)	2.174128
Diameter in inches = 5.305		Log. =	0.724709$\frac{1}{3}$

Note.—The constant logarithm made use of in this rule is the arithmetical complement of that used in the preceding rule.

Problem V.

Given the internal and external Diameters of an iron Shell; to find its Weight.

Rule.

Find the difference of the cubes of the internal and external diameters of the shell, to the logarithm of which add the constant logarithm 9.148063; and the sum (abating 10 in the index,) will be the logarithm of the required weight in pounds.

Note.—The constant logarithm used in this rule is the same as that given in Problem I., page 557.

Example 1.

Let the external diameter of an iron shell be 12.8 inches, and its internal diameter 9.1 inches; required its weight?

12.8 × 12.8 × 12.8 = 2097.152, cube of the external diameter.
9.1 × 9.1 × 9.1 = 753.571, cube of the internal diameter.

Difference = . .	1343.581	Log. = ...	3.128264
Constant log. =	. . . .		9.148063
Weight in pounds =	188.94	Log. = ...	2.276327

Example 2.

Let the external diameter of an iron shell be 9.8 inches, and its internal diameter 7 inches; required its weight?

9.8×9.8×9.8=941.192, cube of the external diameter.			
7 ×7 ×7 =343. cube of the internal diameter.			
Difference = . . 598.192	Log. =	. . . .	2.776841
Constant log. =		. . . .	9.148063
Weight in pounds=84.12	Log. =	. . . .	1.924904

PROBLEM VI.

To find how much Powder will fill a Shell.

RULE.

To thrice the logarithm of the internal diameter of the shell, in inches, add the constant logarithm 8.241845; and the sum (abating 10 in the index,) will be the logarithm of the pounds of powder.

Example 1.

How much powder will fill a shell, the internal diameter of which is 9.1 inches?

Internal diameter, 9.1 inches; thrice its log. = ...	2.877123
Constant log. = ...	8.241845
Powder, in pounds, = 13.15 Log. = ...	1.118968

Example 2.

How much powder will fill a shell, the internal diameter of which is 7 inches?

Internal diameter, 7 inches; thrice its log. = ...	2.535294
Constant log. = ...	8.241845
Powder, in pounds, = 5.986 Log. = ...	0.777139

Note.—The constant logarithm made use of in this problem is the arithmetical complement of the logarithm of 57.3, the established divisor for filling shells.

PROBLEM VII.

To find the Size of a Shell to contain a given Weight of Powder.

RULE.

This problem being the converse of the last, we obtain the following logarithmical expression:—

To the logarithm of the given weight of powder, in pounds, add the constant logarithm 1.758155; divide the sum by 3, and the quotient will be the logarithm of the internal diameter of the shell, in inches.

Example 1.

Required the internal diameter of a shell that will hold 13.15 lbs. of powder?

Given weight = 13.15 lbs.	Log. = . .	1.118926
Constant log. = ...		1.758155
	Divide by 3)	2.877081
Internal diameter, in inches, = 9.1	Log. =	0.959027

Example 2.

Required the internal diameter of a shell that will hold 5.986 lbs. of powder?

Given weight = 5.986 lbs.	Log. = . .	0.777137
Constant log. = ...		1.758155
	Divide by 3)	2.535292
Internal diameter, in inches, = 7.0	Log. =	0.845097⅓

PROBLEM VIII.

To find how much Powder will fill a rectangular Box.

RULE.

To the logarithms of the length, breadth, and depth of the box, in inches, add the constant logarithm 8.522879; and the sum (abating 10 in the index,) will be the logarithm of the pounds of powder.

Example 1.

How much powder will fill a box, the length of which is 15, the breadth 12, and the depth 10 inches?

Length = 15 inches.	Log. =	. . .	1.176091
Breadth = 12 do.	Log. =	. . .	1.079181
Depth = 10 do.	Log. =	. . .	1.000000
Constant log. =		. . . .	8.522879
Powder, in pounds, = 60.0	Log. =	. :	1.778151

Example 2.

Required the quantity of powder that will fill a cubical box, the side of which is 12 inches?

Side of the cubical box = 12 inches.	Log. =	1.079181
	Multiply by	3
Logarithmical cube of 12 =	. . . .	3.237543
Constant log. =	. . . .	8.522879
Powder, in pounds, = 57.6	Log. = . .	1.760422

Note.—The constant logarithm made use of in this problem is the arithmetical complement of the logarithm of 30, the established divisor for filling rectangular powder boxes.

Problem IX.

To find the Size of a cubical Box to contain a given Weight of Powder.

Rule.

To the logarithm of the given weight of powder, in pounds, add the constant logarithm 1.477121; divide the sum by 3, and the quotient will be the logarithm of the side of the box, in inches.

Example 1.

Required the side of a cubical box that will hold 60 lbs. of gunpowder?

Given weight = 60 lbs.	Log. = ...	1.778151
Constant log. =	. . . .	1.477121
	Divide by 3)	3.255272
Side of the box, in inches, 12.16	Log. =	1.085090$\frac{2}{3}$

Example 2.

Required the side of a cubical box that will hold 120 lbs. of gunpowder?

Given weight = 120 lbs.	Log. =	. . .	2.079181
Constant log. =	. . . .		1.477121
		Divide by 3)	3.556302
Side of the box, in inches, 15.32	Log. =	.	1.185434

Note.—Since this problem is the converse of the last, the constant logarithm made use of is the logarithm of 30, the established divisor and multiplier for filling rectangular boxes.

Problem X.

To find how much Powder will fill a Cylinder.

Rule.

To twice the logarithm of the diameter of the cylinder, add the logarithm of its length and the constant logarithm 8.417937; the sum (abating 10 in the index,) will be the logarithm of the pounds of powder.

Example 1.

How much powder will a cylinder hold, the diameter of which is 13 inches, and the length 26 inches?

Diameter of the cylinder = 13 inches;	twice its log. =		2.227886
Length of ditto = . . 26 ditto.	Log. =	.	1.414973
Constant log. =	. . . .	.	8.417937
Powder, in pounds, = . 115.02	Log. =	.	2.060796

Example 2.

How much powder will a cylinder hold, the diameter of which is 4 inches, and the length 12 inches?

Diameter of the cylinder = 4 inches;	twice its log. =		1.204120
Length of ditto = . . 12 ditto.	Log. =	.	1.079181
Constant log. =	. . . .	.	8.417937
Powder, in pounds, = 5.026	Log. = ...	.	0.701238

Note.—The constant logarithm made use of in this problem is the arithmetical complement of the logarithm of 38.2, the established divisor for filling cylinders with gunpowder.

Problem XI.

To find what Length of a Cylinder will be filled with a given Weight of Gunpowder.

Rule.

To the arithmetical complement of twice the logarithm of the diameter of the cylinder, or caliber of the gun, add the logarithm of the given weight of powder in pounds, and the constant logarithm 1.582063: the sum (abating 10 in the index,) will be the logarithm of the length of the cylinder, in inches.

Example 1.

What length of a 24-pounder gun, of 5.66 inches caliber, will be filled with 8 lbs. of gunpowder?

Caliber of the gun = 5.66	Ar. comp. of twice its log. =	8.494368
Given weight of powder = 8 lbs.	Log. = ...	0.903090
Constant log. = ...		1.582063
Length, in inches, = 9.539	Log. = ...	0.979521

Example 2.

What length of a 42-pounder gun, of 6.23 inches caliber, will be filled with 10⅔ lbs. of gunpowder?

Caliber of the gun = 6.23	Ar. comp. of twice its log. =	8.411024
Given weight of powder = 10.666	Log. = ...	1.028002
Constant log. = ...		1.582063
Length, in inches, = 10.497	Log. = ...	1.021089

Note.—This problem being the converse of the last, the constant logarithm is the logarithm of 38.2, the established divisor and multiplier for filling cylinders with gunpowder.

PROBLEM XII.

To find the Number of Balls in a triangular Pile.

RULE.

To the logarithm of the number of balls in the bottom row, add the logarithm of that number increased by 1, and also its logarithm increased by 2, and the constant logarithm 9.221849: the sum (rejecting 10 in the index,) will be the logarithm of the required number of balls.

Example 1.

Required the number of balls in a triangular pile, each side of its base containing 30 balls?

Balls in one side of the base =	30	Log. =	1.477121
Ditto, increased by 1, = . .	31	Log. =	1.491362
Ditto, increased by 2, = . .	32	Log. =	1.505150
Constant log. = ...			9.221849
Number of balls = . . .	4960	Log. =	3.695482

Example 2.

Required the number of balls in a triangular pile, each side of its base containing 20 balls?

Balls in one side of the base =	20	Log. =	1.301030
Ditto, increased by 1, = . .	21	Log. =	1.322219
Ditto, increased by 2, = . .	22	Log. =	1.342423
Constant log. = ...			9.221849
Number of balls = . . .	1540	Log. =	3.187521

Note.—The constant logarithm employed in this problem is the arithmetical complement of the logarithm of 6, the established divisor for triangular, square, and rectangular piles of shot.

To find the Number of Balls in a square Pile.

RULE.

To the logarithm of the number of balls in one side of the bottom row, add the logarithm of that number increased by 1, the logarithm of twice the same number increased by 1, and the constant logarithm 9.221849: the sum (abating 10 in the index,) will be the logarithm of the required number of balls.

Example 1.

Required the number of balls in a square pile, each side of its base containing 30 balls?

Balls in one side of the base =	30	Log. =	1.477121
Ditto, increased by 1, =	31	Log. =	1.491362
Twice ditto, increased by 1, =	61	Log. =	1.785330
Constant log. = ...			9.221849
Number of balls = . .	9455	Log. =	3.975662

Example 2.

Required the number of balls in a square pile, each side of its base containing 20 balls?

Balls in one side of the base =	20	Log. =	1.301030
Ditto, increased by 1, = . .	21	Log. =	1.322219
Twice ditto, increased by 1, =	41	Log. =	1.612784
Constant log. = ...			9.221849
Number of balls = . . .	2870	Log. =	3.457882

PROBLEM XIV.

To find the Number of Balls in a rectangular Pile.

RULE.

From three times the number of balls contained in the length of the bottom row, subtract the number of balls, less by 1, contained in the breadth of that row; then, to the logarithm of the remainder, add the logarithm of the number of balls contained in the breadth of the bottom row, the logarithm of that number increased by 1, and the constant logarithm 9.221849: the sum (rejecting 10 in the index,) will be the logarithm of the required number of balls.

Example 1.

Required the number of balls in a rectangular pile, which contains 46 balls in the base row of its longest side, and 15 balls in that ofi ts shortest side?

Balls in length	46×3=138			
Balls in breadth	15−1= 14			
Remainder = . . .	124		Log. =	2.093422
Balls in breadth row =	15		Log. =	1.176091
Ditto, increased by 1, =	16		Log. =	1.204120
Constant log. = ...				9.221849
Number of balls = .	4960		Log. =	3.695482

Example 2.

Required the number of balls in a rectangular pile, which contains 59 balls in the base row of its longest side, and 20 balls in that of its shortest side?

Balls in length	59×3=177			
Balls in breadth	20−1= 19			
Remainder = . . .	158		Log. =	2.198657
Balls in breadth row =	20		Log. =	1.301030
Ditto, increased by 1, =	21		Log. =	1.322219
Constant log. = ...				9.221849
Number of balls =	11060		Log. =	4.043755

PROBLEM XV.

To find the Number of Balls in an incomplete triangular Pile.

RULE.

Find the number of balls in the whole pile, considered as complete, by Problem XII., page 566; and find also, by the same problem, the number of balls answering to the triangular pile, the side of whose base is represented by the number of shot in the side of the top course of the incomplete pile diminished by 1; then, the difference of the two results will be the number of shot remaining in the pile.

Example.

Required the number of shot in an incomplete triangular pile; each side of its bottom course containing 40 balls, and each side of its top course containing 20 balls?

To find the Number of Balls in the complete Pile:—

Balls in one side of bottom course =	40	Log. =	1.602060
Ditto, increased by 1, = ...	41	Log. =	1.612784
Ditto, increased by 2, = ...	42	Log. =	1.623249
Constant log. = ...			9.221849
Number of balls for the whole pile =	11480	Log. =	4.059942

To find the Number of Balls deficient:—

Balls in each side of top course = 20 − 1 =	19	Log. =	1.278754
Diminished course, or 19, increased by 1, =	20	Log. =	1.301030
Ditto, increased by 2, = ...	22	Log. =	1.322219
Constant log. = ...			9.221849
Number of shot wanting = . . .	1330	Log. =	3.123852

Now, 11480 − 1330 = 10150 is the number of shot in the incomplete pile.

Problem XVI.

To find the Number of Balls in an incomplete square Pile.

Rule.

Find the number of balls in the whole pile, considered as complete, by Problem XIII., page 567; and find also, by the same problem, the number of balls answering to the square pile, each side of whose base is represented by the number of shot in each side of the top course of the incomplete pile diminished by 1; then, the difference of the two results will be the number of shot remaining in the pile.

Example.

Required the number of shot in an incomplete square pile; each side of its bottom course containing 24 balls, and each side of its top course 8 balls?

To find the Number of Balls in the complete Pile:—

Balls in one side of the base = .	24	Log. =	1.380211
Ditto, increased by 1, = . . .	25	Log. =	1.397940
Twice ditto, increased by 1, = .	49	Log. =	1.690196
Constant log. = ...			9.221849
Number of balls for the whole pile =	4900	Log. =	3.690196

To find the Number of Balls deficient:—

Balls in each side of top course = 8−1=7		Log.=0. 845098
Diminished course, or 7, increased by 1,=8		Log.=0. 903090
Twice ditto, increased by 1, = . .	15	Log.=1. 176091
Constant log. = ...		9. 221849
Number of balls wanting = . .	140	Log.=2. 146128

Now, 4900−140 = 4760 is the number of shot in the incomplete pile.

Problem XVII.

To find the Number of Balls in an incomplete rectangular Pile.

Rule.

Find the number of balls in the whole pile, considered as complete, by Problem XIV., page 567; and find also, by the same problem, the number of balls answering to the rectangular pile, whose sides are represented by the respective sides of the top course of the incomplete pile, the number of shot in each side being diminished by 1; then, the difference of the two results will be the number of shot remaining in the pile.

Example.

Required the number of shot in an incomplete rectangular pile; the length of its bottom course being 40 balls, its breadth 20, and the length of its top course 29 balls, and its breadth 9?

To find the Number of Balls in the complete Pile:—

Bottom course, 40 × 3 =	120	
Breadth, 20 − 1 =	19	
Remainder = ...	101	Log.= 2. 004321
Balls in breadth row = .	20	Log.= 1. 301030
Ditto, increased by 1, = .	21	Log.= 1. 322219
Constant log. = ...		9. 221849
Number of balls for whole pile=	7070	Log.=3. 849419

To find the Number of Balls deficient:—

Top row, 29−1 = 28 × 3 =	84	
Breadth, 9−1 = 8 − 1 =	7	
Remainder = ...	77	Log.= 1. 886491
Balls in breadth row = . .	8	Log.= 0. 903090
Ditto, increased by 1, = . .	9	Log.= 0. 954243
Constant log. = ...		9. 221849
Number of balls wanting =	924	Log.= 2. 965673

Now, 7070 − 924 = 6146 is the number of shot in the incomplete pile.

Note.—In triangular and square piles, the number of horizontal rows or courses is always equal to the number of balls in one side of the bottom row; and, in rectangular piles, the number of horizontal rows is equal to the number of balls in the *breadth* of the bottom row. In these piles, the number of balls in the top row or edge is always one more than the difference between the number of balls contained in the length and the breadth of the bottom row.

Problem XVIII.

To find the Velocity of any Shot or Shell.

Rule.

From the logarithm of twice the weight of the charge of powder, in pounds, subtract the logarithm of the weight of the shot: to half the remainder add the constant logarithm 3.204120, and the sum (rejecting 5 in the index,) will be the logarithm of the velocity in feet, or the number of feet which the shot or shell passes over in a second.

Example 1.

With what velocity will a 24-pounds ball be projected by 8 lbs. of powder?

Twice the charge =	16 lbs.	Log. =	1.204120
Weight of the shot =	24 lbs.	Log. =	1.380211
Remainder =			9.823909
Half the remainder =			$4.911954\frac{1}{2}$
Constant log. =			3.204120
Velocity of shot, in feet, =	1306	Log. =	$3.116074\frac{1}{2}$

Example 2.

With what velocity will a 13-inch shell, weighing 196 lbs., be discharged by 9 lbs. of powder?

Twice the charge =	18 lbs.	Log. =	1.255273
Weight of the shell =	196 lbs.	Log. =	2.292256
Remainder =			8.963017
Half the remainder =			$4.481508\frac{1}{2}$
Constant log. =			3.204120
Velocity of shell, in feet,	485	Log. =	$2.685628\frac{1}{2}$

Note.—The constant logarithm made use use of in this problem is the logarithm of 1600 feet, which is the velocity acquired by a 1 lb. ball, when fired with 8 ounces of powder.

PROBLEM XIX.

To find the terminal Velocity of a Shot or Shell; that is, the greatest Velocity it can acquire in descending through the Air by its own Weight.

RULE.

For *Balls.*—To half the logarithm of the diameter of the ball, in inches, add the constant logarithm 2. 244277; and the sum will be the logarithm of the terminal velocity of the ball.

And, for *Shells.*—To half the logarithm of the external diameter of the shell, in inches, add the constant logarithm 2. 168203; and the sum will be the logarithm of the terminal velocity of the shell.

Example 1.

Required the terminal velocity of a 24 lbs. ball, its diameter being 5. 6 inches?

Diameter of the ball =	5. 6	Log. =	0. 748188
Half the log. =	. . . .		0. 374094
Constant log. =	. . . .		2. 244277
Terminal velocity =	. 415	Log. =	2. 618371

Example 2.

Required the terminal velocity of a shell weighing 196 lbs., its external diameter being 12. 8 inches?

Diameter of the shell =	12. 8	Log. =	1. 107210
Half the log. =	. . . .		0. 553605
Constant log. =	. . . .		2. 168203
Terminal velocity =	527 . .	Log. =	2. 721808

Note.—The constant logarithms made use of in this problem are the respective logarithms of 175. 5 and 147. 3, the established multipliers for shot and shells. It is by this problem that the terminal velocities contained in Tables A and B, following, have been computed.

PROBLEM XX.

To find the Height from which a Body must fall, IN VACUO, *in order to acquire a given Velocity.*

RULE.

Since the spaces descended by falling bodies are as the squares of the velocities, and as a fall of $16\frac{1}{12}$ feet produces a velocity of $32\frac{1}{6}$ feet,—therefore, as the square of $32\frac{1}{6}$ feet, is to $16\frac{1}{12}$ feet; so is the square of any other given velocity, to the altitude from which it must fall, to acquire such velocity. Hence the following logarithmical expression:—

To twice the logarithm of the given velocity, in feet, add the constant logarithm 8. 191564; and the sum, (abating 10 in the index,) will be the logarithm of the required altitude, or height.

Example 1.

From what height must a body fall, in order to acquire a velocity of 1340 feet per second?

Given velocity = 1340;	twice its log. =	6. 254210
Constant log. =	. . . .	8. 191564
Altitude, or height, = 27911	Log. =	4. 445774

Example 2.

From what height must a body fall, in order to acquire a velocity of 1670 feet per second?

Given velocity = 1670;	twice its log. =	6. 445434
Constant log. =	. . . .	8. 191564
Altitude, or height, = 43352	Log. =	4. 636998

Note—It is by this problem that the altitudes in Tables A and B, following, have been computed; but, since the fractional parts beyond 16 and 32 were omitted, and the constant logarithm, in consequence thereof, assumed at 8. 193820, the respective altitudes, in these Tables, are something beyond the truth.

CONCISE TABLES

FOR DETERMINING THE GREATEST HORIZONTAL RANGE OF A SHOT OR SHELL, WHEN PROJECTED IN THE AIR WITH A GIVEN VELOCITY;

TOGETHER WITH THE ELEVATION OF THE PIECE TO PRODUCE THAT RANGE.

TABLE A.—*For Great Guns.*

Weight of Shot.	Diameter, in inches.	Terminal Velocity.	Logarithm.	Altitude.	Logarithm.
1	1.94	244	7.612610	930	2.968483
2	2.45	275	7.560667	1182	3.072618
3	2.80	294	7.531653	1360	3.133539
4	3.08	308	7.511449	1482	3.170848
6	3.53	330	7.481486	1701	3.230704
9	4.04	353	7.452225	1958	3.291813
12	4.45	370	7.431798	2139	3.330211
18	5.09	396	7.402305	2450	3.389166
24	5.60	415	7.381952	2691	3.429914
32	6.17	436	7.360514	2970	3.472756
36	6.41	444	7.352617	3080	3.488551
42	6.75	456	7.341035	3249	3.511750

TABLE B.—*For Mortars.*

Size of Shell, in inches.	Weight of Shells filled.	Diameter, in inches.	Terminal Velocity.	Logarithm.	Altitude.	Logarithm.
4⅖	9	4.53	314	7.503070	1541	3.187803
5½	18	5.72	352	7.453457	1936	3.286905
8	47	7.90	414	7.383000	2678	3.427811
10	91½	9.84	462	7.335358	3335	3.523096
13	201	12.80	527	7.278189	4340	3.637490

TABLE C.—*For Great Guns and Mortars.*

Initial Velocity, divided by Terminal Velocity	Logarithm.	Elevation.	Range divided by Altitude.	Logarithm.
0.6910	9.839478	44° 0′	0.3914	9.592621
0.9445	9.975202	43.15	0.5850	9.767156
1.1980	0.078457	42.30	0.7787	9.891370
1.4515	0.161817	41.45	0.9724	9.987845
1.7050	0.231724	41. 0	1.1661	0.066736
1.9585	0.291924	40.15	1.3598	0.133475
2.2120	0.344785	39.30	1.5535	0.191311
2.4655	0.391905	38.45	1.7472	0.242343
2.7190	0.434409	38. 0	1.9409	0.288003
2.9725	0.473122	37.15	2.1346	0.329317
3.2260	0.508664	36.30	2.3283	0.367039
3.4795	0.541517	35.45	2.5220	0.401745
3.7330	0.572058	35. 0	2.7157	0.433882
3.9865	0.600592	34.15	2.9094	0.463803
4.2400	0.627366	33.30	3.1031	0.491796
4.4935	0.652585	32.45	3.2968	0.518093
4.7470	0.676419	32. 0	3.4905	0.542888
5.0000	0.698970	31.15	3.6842	0.566343

Note.—These Tables are deduced from those given in the third volume of Dr. Hutton's "Course of Mathematics."

Problem XXI.

To find the greatest Range of a Ball or Shell, and the Elevation of the Piece to produce that Range.

Rule.

Enter Table A or B, and take out the logarithm of the terminal velocity answering to the given ball or shell, as the case may be, and also the logarithm of the corresponding altitude; then,

To the logarithm of the velocity with which the ball or shell is projected, add the logarithm of its terminal velocity; and the sum (abating 10 in the index,) will be the logarithm of the quotient of the initial velocity of the ball or shell, divided by its terminal velocity. With this logarithm, enter the second column of Table C, and in the adjoining or middle column will be found the corresponding degree of elevation to produce the greatest range; abreast of which, in the last column of the same table, will be found the logarithm of the range divided by the altitude. Now, to this logarithm add the logarithm of the altitude taken from Table A or B, as above directed; and the sum will be the logarithm of the greatest range.

Note.—If *great accuracy* be required, proportional parts must be taken for the excess of the given above the next less tabular numbers in Table C.

Example 1.

Let it be required to find the greatest range of a 24 lb. ball, when discharged with a velocity of 1640 feet, and the elevation of the piece to produce that range?

Log. of terminal velocity of a 24 lb. ball, Table A, =	7.381952
Given velocity of the ball = 1640	Log.=3.214844
Answering to which, in Table C, is 34°15′	Log.=0.596796
Log. of corresponding altitude, Table A, = ...	3.429914
Abreast of 34°15′, in last column of Table C, stands	0.463803
Greatest range, in feet, = 7829	Log.=3.893717

Hence the greatest range of a 24 lb. ball, when projected with a velocity of 1640 feet, is 7829 feet, which is nearly an English mile and a half; and the elevation to produce that range, is 34°15′.

Example 2.

Let it be required to find the greatest range of a 13-inch shell, when projected with a velocity of 2000 feet per second, and the elevation to produce that range; the diameter of the shell being 12.80 inches?

Log. of terminal velocity of a 13-inch shell, Table B,		=7.278189
Given velocity of the shell = 2000 . . .	Log.	=3.301030
Answering to which, in Table C, is 34°49′	Log.	=0.579219
Log. of corresponding altitude, Table B, = . . .		3.637490
Corresponding to 34°49′, in Table C, is		0.441196
Greatest range, in feet, = 11986	Log.	=4.078686

Hence the greatest range of a 13-inch shell, when projected with a velocity of 2000 feet, is 11986 feet, which is 2¼ miles and 106 feet; and the elevation to produce that range, is 34°49′.

Note.—In this example, proportion is made for the excess of the given above the next less numbers in Table C.

Problem XXII.

Given the Range at one Elevation; to find the Range at another Elevation.

Rule.

As the logarithmic sine of twice the first elevation, is to the logarithm of its corresponding range; so is the logarithmic sine of twice the other elevation, to the logarithm of its corresponding range.

Example 1.

If a 13-inch shell be found to range 11986 feet, when discharged at an elevation of 34°49′, how far will it range when the elevation is 45 degrees; the charge of powder being the same at both elevations?

As twice 34°49′ = 69°38′	Log. co-secant =	10.028036
Is to its range = 11986 feet,	Log. = . . .	4.078674
So is twice 45°0′ = 90° 0′	Log. sine = . .	10.000000
To the required range, in feet, = 12785	Log. =	4.106710

Example 2.

If a shell be found to range 4760 feet, when discharged at an elevation of 45 degrees, how far will it range when the elevation is 30°45′; the charge of powder being the same at both elevations?

As twice 45° = ...	90° 0′	Log. co-secant =	10.000000
Is to its range = ...	4760 feet,	Log. = . .	3.677607
So is twice 30°45′ = ...	61°30′	Log. sine = .	9.943899
To the required range, in feet, =	4183	Log. = . .	3.621506

Problem XXIII.

Given the Elevation for one Range; to find the Elevation for another Range.

Rule.

As the logarithm of the first range, is to the logarithmic sine of twice its corresponding elevation; so is the logarithm of any other given range, to the logarithmic sine of an arch. Now, the half of this arch will be the elevation required.

Example 1.

If a shell be found to range 11986 feet when projected at an elevation of 34°49′, at what elevation must it be discharged to strike an object at the distance of 12785 feet, with the same charge of powder?

As the first range =	11986	Log. ar. comp. =	5.921326
Is to twice 34°49′ =	69°38′	Log. sine = .	9.971964
So is the other range =	12785	Log. = . .	4.106710
To arch = ...	90° 0′	Log. sine = .	10.000000

Half the arch = . 45° 0′, the elevation required.

Example 2.

If a shell be found to range 4760 feet when discharged at an elevation of 45°, at what elevation must it be projected to strike an object at the distance of 4183 feet, with the same charge of powder?

As the first range =	4760	Log. ar. comp.=	6.322393
Is to twice 45° =	90°	Log. sine =	10.000000
So is the other range=	4183	Log. sine =	3.621488
To arch =	61°29′45″	Log. sine =	9.943881

Half the arch = 30°44′52½″, the elevation required.

Problem XXIV.

Given the Charge for one Range; to find the Charge for another Range.

Rule.

Since the ranges at the same elevation are *nearly* proportional to the charges, therefore—As the logarithm of the first range, is to the logarithm of its corresponding charge; so is the logarithm of the other range, to the logarithm of the charge corresponding thereto.

Example 1.

If, with a charge of 12 lbs. of powder, a shell range 5334 feet, what charge will be sufficient to throw it 2667 feet; the elevation being 45° in both cases?

As the first range =	5334	Log. ar. comp.=	6.272947
Is to its charge =	12 lb.	Log. =	1.079181
So is the other range =	2667	Log. =	3.426023
To the required charge, in lbs.=	6.0	Log. =	0.778151

Example 2.

If, with a charge of 9 lbs. of powder, a shell range 4000 feet, what charge will be sufficient to throw it 3000 feet; the elevation being 45° in both cases?

As the first range =	4000	Log. ar. comp.=	6.397940
Is to its charge =	9 lbs.	Log. =	0.954243
So is the other range =	3000	Log. =	3.477121
To the required charge, in lbs.=	6.75	Log. =	0.829304

Problem XXV.

Given the Range for one charge; to find the Range for another Charge.

Rule.

As the logarithm of the first charge, is to the logarithm of its corresponding range; so is the logarithm of the other charge, to the logarithm of its corresponding range; the elevation being the same in both cases.

Example 1.

If a shell be projected 5334 feet by a charge of 12 lbs. of powder, at what distance will it strike an object when discharged with 6 lbs. of powder; the elevation being the same in both cases?

As the first charge = .	12 lbs.	Log. ar. comp.=	8.920819
Is to its range = . . .	5334	Log. = . .	3.727053
So is the other charge =	6 lbs.	Log. = . .	0.778151
To the required range, in ft.,=	2667	Log. = . .	3.426023

Example 2.

If a shell be projected 4000 feet by a charge of 9 lbs. of powder, at what distance will it strike an object when discharged with 6¾ lbs. of powder; the elevation being the same in both cases?

As the first charge = . .	9 lbs.	Log. ar. comp.=	9.045757
Is to its range = ...	4000	Log. = . .	3.602060
So is the other charge = .	6.75	Log. = . .	0.829304
To the required range, in ft., =	3000	Log. = . .	3.477121

Problem XXVI.

Given the Range and the Elevation; to find the Impetus.

Rule.

As the logarithmic sine of twice the angle of elevation, is to the logarithm of half its corresponding range; so is radius, or the logarithmic sine of 90°, to the impetus.

Example 1.

With what impetus must a shell be discharged at an elevation of 34°49′, to strike an object at the distance of 2986 feet?

As twice 34°49′ = .	69°38′	Log. co-secant=	10.028036
Is to half the range =	1493	Log. = ...	3.174060
So is radius, or . . .	90°	Log. sine = .	10.000000
To the impetus, in feet, =	1592	Log. = ...	3.202096

Example 2.

With what impetus must a shell be discharged at an elevation of 25°, to strike an object at the distance of 2760 feet?

As twice 25° = ...	50°	Log. co-secant=	10.115746
Is to half the range = . .	1380	Log. = . . .	3.139879
So is radius, or	90°	Log. sine = .	10.000000
To the impetus, in feet, =	1804	Log. = . . .	3.255625

PROBLEM XXVII.

Given the Elevation and the Range; to find the Time of the Flight.

RULE.

As radius, is to the logarithmic tangent of the elevation; so is the logarithm of the rahge, in feet, to a logarithmic number; which, being divided by 2, will give the logarithm of 4 times the number of seconds taken up in the flight.

Example 1.

In what time will a shell range 11986 feet, at an elevation of 34°49′?

As radius = ...	90° 0′	Log. co-secant=	10.000000
Is to the elevation = .	34°49′	Log. tangent =	9.842266
So is the range = . .	11986	Log. = . . .	4.078674
		Divide by 2)	3.920940
Four times the flight =	91.30	Log. = . . .	1.960470

Number of seconds = . 22.825, = the time of flight.

Example 2.

In what time will a shell range 3250 feet, at an elevation of 32 degrees?

As radius = ...	90°	Log. co-secant=	10.000000
Is to the elevation = . . .	32°	Log. tangent =	9.795789
So is the range = ...	3250	Log. = ...	3.511883
		Divide by 2)	3.307672
Four times the flight = .	45.06	Log. = ...	1.653836
Number of seconds = .	11.26, = the time of flight.		

Note.—From this it is manifest that when the elevation of the piece is 45°, half the logarithm of the range will be the logarithm of 4 times the number of seconds taken up in the flight.

Problem XXVIII.

Given the Range and the Elevation: to find the greatest Altitude of the Shell.

Rule.

As radius, is to the logarithmic tangent of the elevation; so is the logarithm of one-fourth of the range, to the logarithm of the required altitude.

Example 1.

If a shell range 11986 feet, when projected at an elevation of 34°49′; required the greatest altitude which it acquires during its flight?

As radius = ...	90° 0′	Log. co-sec.=	10.000000
Is to the elevation =	34°49′	Log. tangent =	9.842266
So is ¼ of the range =	2996.5	Log. = . .	3.476614
To the altitude, in feet, =	2084	Log. = . .	3.318880

Example 2.

If a shell range 4760 feet, when projected at an elevation of 45°; required the greatest altitude which it acquires during its flight?

As radius = . .	90° 0′	Log. co-secant =	10.000000
Is to the elevation =	45° 0′	Log. tangent =	10.000000
So is ¼ of the range =	1190	Log. = . . .	3.075547
To the altitude, in feet, =	1190	Log. = . . .	3.075547

Note.—From this it is manifest, that when the elevation of the mortar is 45 degrees, one-fourth of the range will be equal to the greatest altitude at which the shell can arrive.

Problem XXIX.

Given the Inclination of the Plane, the Elevation of the Piece, and the Impetus; to find the Range.

Rule.

To twice the logarithmic secant of the inclination of the plane, add the logarithmic sine of the elevation of the piece above the plane, the logarithmic co-sine of the elevation of the piece above the horizon, and the logarithm of 4 times the impetus: the sum of these four logarithms (rejecting 40 in the index,) will be the logarithm of the required range.

Example.

How far will a shell range on a plane which ascends 10°15′, and also on another plane which descends 10°15′; the impetus being 2000 feet in both cases, and the elevation of the mortar 31°45′?

Solution. 31°45′ − 10°15′ = 21°30′, the elevation of the piece above the ascending plane;

and, 31°45′ + 10°15′ = 42° 0′, the elevation of the piece above the descending plane.

To find the Range on the ascending Plane:—

Inclination of the plane =	10°15′	Twice the log. secant =	20.013974
Elevation above the plane =	21.30	Log. sine = . . .	9.564075
Elevation above the horizon =	31.45	Log. co-sine = . . .	9.929599
Four times the impetus =	8000	Log. = ...	3.903090
Range, in feet, = ...	2575	Log. = ...	3.410738

To find the Range on the descending Plane:—

Inclination of the plane =	10°15′	Twice the log. secant =	20.018974
Elevation above the plane =	42. 0	Log. sine = . . .	9.825511
Elevation above the horizon =	31.45	Log. co-sine = . .	9.929599
Four times the impetus =	8000	Log. = ...	3.903090
Range, in feet, = ...	4701	Log. = ...	3.672174

Problem XXX.

Given the Inclination of the Plane, the Elevation of the Piece, and the Range; to find the Impetus.

Rule.

To twice the logarithmic co-sine of the inclination of the plane, add the logarithmic co-secant of the elevation of the piece above the plane, the logarithmic secant of the elevation of the piece above the horizon, and the logarithm of the one-fourth of the range: the sum of these four logarithms (abating 40 in the index,) will be the logarithm of the impetus.

Example.

With what impetus must a shell be discharged to strike an object at the distance of 2575 feet, on an inclined plane which ascends 10°15′, and, also, another object at the distance of 4701 feet, on an inclined plane which descends 10°15′; the elevation of the piece being 31°45′ in both cases?

Solution. 31°45′ − 10°15′ = 21°30′, is the elevation of the piece above the ascending plane;
and, 31°45′ + 10°15′ = 42° 0′, is the elevation of the piece above the descending plane.

To find the Impetus on the ascending Plane:—

Inclination of the plane =	10°15′	Twice the log. co-sine =	19.986026
Elevation above the plane =	21.30	Log. co-secant = .	10.435925
Elevation above the horizon =	31.45	Log. secant = . .	10.070401
One-fourth of the range =	643.75	Log. = ...	2.808717
Impetus, in feet, = . . .	2000	Log. = ...	3.301069

To find the Impetus on the descending Plane :—

Inclination of the plane =	10°15′	Twice the log. co-sine =	19.986026
Elevation above the plane =	42. 0	Log. co-secant = .	10.174489
Elevation above the horizon =	31.45	Log. secant = . .	10.070401
One-fourth of the range =	1175.25	Log. = ...	3.070130
Impetus, in feet, = . . .	2000	Log. = ...	3.301046

PROBLEM XXXI.

Given the Weight of a Ball, the Charge of Powder with which it is fired, and the known Velocity of that Ball; to find the Velocity of a Shell, when projected with a given Charge of Powder.

RULE.

To the arithmetical complement of half the logarithm of the weight of the shell, add half the logarithm of twice the weight of the charge, in pounds, and the constant logarithm 3.204120 : the sum (abating 10 in the index,) will be the velocity of the shell answering to the given charge.

Example.

If a ball of 1 lb. weight acquire a velocity of 1600 feet per second, when fired with 8 ounces of powder, it is required to find with what velocities the several kinds of shells will be projected by the respective charges of powder expressed against them in the following table ?

For the 13-inch Shell :—

Weight of the shell = . . .	196	Ar. comp. of ½ its log. =	8.853872
Twice the weight of the charge =	18	Half its log. = . . .	0.627636½
Constant log. = ...		. . . .	3.204120
Velocity, in feet, = ...	485	Log. = ...	2.685628½

For the 10-inch Shell :—

Weight of the shell	90	Ar. comp. of ½ its log. =	9.022879
Twice the weight of the charge =	8	Half its log. = . . .	0.451545
Constant log. = ...		. . . .	3.204120
Velocity, in feet, = ...	477	Log. = ...	2.678544

For the 8-inch Shell:—

Weight of the shell = ...	48	Ar. comp. of ½ its log.=	9.159380
Twice the weight of the charge =	4	Half its log. = . . .	0.301030
Constant log. = ...			3.204120
Velocity, in feet, = ...	462	Log. = ...	2.664530

Note.—The same results will be obtained by computing agreeably to the rule in Problem XVIII., page 571.

TABLE D.—*Showing the Velocities of the different sized Shells, when projected with given Charges of Powder.*

Size of Shell, in inches.	Weight of Shell, in pounds.	Charge of Powder, in lbs.	Logarithm.	Velocity, in feet.	Logarithm.
13	196	9	0.477121*	485	7.314258†
10	90	4	0.301030	477	7.321482
8	48	2	0.150515	462	7.335358
5½	16	1	0.000000	566	7.247184
4⅖	8	0½	0.349485	566	7.247184

PROBLEM XXXII.

Given the Elevation and the Range; to find the Impetus, Velocity, and Charge of Powder.

RULE.

Find the impetus, by Problem XXVI., page 579; to the logarithm of which add the constant logarithm 1.206376‡: take half the sum, and it will be the logarithm of a natural number; which, being doubled, will be the required velocity. Now, to the logarithm of the velocity, thus found, add the logarithms from Table D answering to the charge and the velocity of the given shell: the sum of these three logarithms (abating 10 in the index,) being doubled, will give the logarithm of the required charge of powder, in pounds.

* The numbers in this column are the logarithms of the square roots of the respective charges.

† The numbers in this column are the arithmetical complements of the logarithms of the respective velocities.

‡ This is the logarithm of 16$\frac{1}{12}$ feet, the descent of a falling body in the first second of time.

Example.

With what impetus, velocity, and charge of powder, must a 13-inch shell be fired at an elevation of 34°49′, to strike an object at the distance of 11986 feet?

To find the Impetus and the Velocity:—

Twice the elevation = ...	69°38′	Log. co-secant =	10.028036
Half the range = ...	5993	Log. = . . .	3.777644
Impetus, in feet, = ...	6392	Log. = . . .	3.805680
Constant log. = ...			1.206376
		Divide by 2)	5.012056
Natural number = ...	321	Log. = . . .	2.506028
Velocity, in feet, = ...	642		

To find the Charge of Powder:—

Velocity, in feet, = ...	642	Log. = . . .	2.807535
Log. of charge for a 13-inch shell, from Table D, = . . .			0.477121
Log. of velocity for a 13-inch shell, from Table D, = . . .			7.314258
		Sum =	0.598914
Charge, in pounds, = ...	15.77	Log. = . . .	1.197828

Hence the impetus is 6392 feet, the velocity 642 feet, and the charge of powder, 15.77 lbs., or 15 lbs. 12½ oz. nearly.

Problem XXXIII.

Given the Inclination of the Plane, the Elevation of the Piece, and the Range; to find the Charge of Powder.

Rule.

Find the impetus, by Problem XXX., page 583; with which proceed as directed in the last problem.

Example 1.

How much powder will throw a 10-inch shell 6760 feet, on an inclined plane which ascends 7°30′; the elevation of the mortar being 33°14′?

above the ascending plane.

Inclination of the plane = .	7°30′	Twice the log. co-sine=	19.992538
Elevation above the plane =	25.44	Log. co-secant = . .	10.362327
Elevation above the horizon =	33.14	Log. secant = . .	10.077562
One-fourth of the range =	1690	Log. = ...	3.227887
Impetus, in feet, = ...	4574	Log. = ...	3.660314
Constant log. = ...		. . . .	1.206376
		Divide by 2)	4.866690
Natural number = ...	271	Log. = ...	2.433345
Velocity = ...	542 feet.		

To find the Charge of Powder :—

Velocity = ... 542 feet,	Log. = ...	2.733999
Log. of charge for 10-inch shell, from Table D, = ...		0.301030
Log. of velocity for 10-inch shell, from Table D, = ...		7.321482
	Sum =	0.356511
Charge, in pounds, = ... 5.164	Log. = ...	0.713022

Hence the charge of powder is 5.164 lbs., or 5 lbs. 2½ oz.

Example 2.

How much powder will throw a 10-inch shell 6760 feet, on an inclined plane which descends 7°30′, the elevation of the mortar being 33°14′ ?

Solution. 33°14′ + 7°30′ = 40°44′ is the elevation of the mortar above the descending plane.

Inclination of the plane =	7°30′	Twice the log. co-sine=	19.992538
Elevation above the plane=	40.44	Log. co-secant = .	10.185393
Elevation above the horizon=	33.14	Log. secant = . .	10.077562
One-fourth of the range =	1690	Log. = ...	3.227887
Impetus, in feet, = . . .	3044	Log. = ...	3.483380
Constant log. = ...		. . . .	1.206376
		Divide by 2)	4.689756
Natural number = . . .	221	Log. = ...	2.344878
Velocity, in feet, = . .	442		

To find the Charge of Powder:—

Velocity = ...	442	Log. = ...	2.645422
Log. of charge for 10-inch shell, from Table D, = ...			0.301030
Log. of velocity for 10-inch shell, from Table D, =. . . .			7.321482
		Sum =	0.267934
Charge, in pounds, = ...	3.434	Log. = ...	0.535868

Hence the charge is 3.434 lbs., or 3 lbs. 7 oz. nearly.

Problem XXXIV.

Given the Inclination of the Plane, the Elevation of the Piece, and the Impetus; to find the Time of Flight.

Rule.

To the logarithmic secant of the inclination of the plane add the logarithmic sine of the elevation above the plane, and half the logarithm of the impetus: the sum (abating 20 in the index,) will be the logarithm of twice the time of flight, in seconds.

Example.

In what time will a 10-inch shell strike an object on an inclined plane which ascends 7°30′, when discharged with an impetus of 4574 feet, the elevation of the mortar being 33°14′; and in what time will it strike another object on a descending plane, with the same impetus and elevation?

Solution. 33°14′−7°30′=25°44′ is the elevation of the mortar above the ascending plane;

and, 33°14′+7°30′=40°44′ is the elevation of the mortar above the descending plane.

To find the Time of Flight on the ascending Plane:—

Inclination of the plane = . . . ,	8°30′	Log. secant=	10.004797
Elevation above the plane = . . .	25.44	Log. sine =	9.637673
Impetus = ...	4574	Half its log. =	1.830148
Twice the time of flight = . . .	29.69	Log. = . .	1.472618
Time of flight = ...	14.845 seconds.		

To find the Time of Flight on the descending Plane:—

Inclination of the plane = ...	8°30′	Log. secant = 10.004797
Elevation above the plane = . . .	40.44	Log. sine = 9.814607
Impetus = ...	4574	Half its log. = 1.830148
Twice the time of flight = . . .	44.62	Log. = . . 1.649552
Time of flight = ...	22.31 seconds.	

Problem XXXV.

Given the Impetus and the Elevation; to find the horizontal Range.

Rule.

To the logarithm of the impetus add the logarithmic sine of twice the angle of elevation: the sum (abating 20 in the index,) will be the logarithm of a natural number; which, being doubled, will give the required range on *the horizontal plane.*

Example 1.

Let a shell be discharged with an impetus of 1592 feet, at an elevation of 34°49′; required its range on the horizontal plane?

Impetus = . . .	1592	Log. = . 3.201943
Twice the elevation =	69°38′	Log. sine = 9.971964
Natural number = .	1492.5	Log. = . 3.173907
Horizontal range =	2985 feet.	

Example 2.

Let a shell be discharged with an impetus of 1804 feet, at an elevation of 25°; required its range on the horizontal plane?

Impetus = . . .	1804	Log. = . 3.256237
Twice the elevation =	50°	Log. sine = 9.884254
Natural number = .	1382	Log. = . 3.140491
Horizontal range =	2764 feet.	

PROBLEM XXXVI.

Given the Impetus and the Elevation; to find the Time of Flight on the horizontal Plane.

RULE.

With the impetus and the elevation compute the horizontal range, by the last problem; then, with the horizontal range, thus found, and the elevation of the piece, compute the time of flight, by Problem XXVII., page 580. Or, the time of flight may be computed directly, by Problem XXXIV., page 588.

Example.

In what time will a 13-inch shell strike an object on a horizontal plane, when discharged with an impetus of 6392 feet, the elevation of the mortar being 34°49′?

Impetus = ...	6392	Log. = .	3.805637
Twice the elevation =	69°38′	Log. sine =	9.971964
Natural number =	5992.4	Log. = .	3.777601
Horizontal range =	11984.8 feet.	Log. =	4.078631
Elevation = . . .	34°49′	Log. tangent =	9.842266
		Divide by 2)	3.920897
Four times the flight =	91.30	Log. = . .	1.960448½
Flight = ...	22.825 seconds.		

To find the Time of Flight, by Problem XXXIV., page 588:—

Inclination of the plane = ...	0° 0′	Log. secant =	0.000000
Elevation above the *plane of the horiz.* =	34.49	Log. sine =	9.756600
Impetus = ...	6392	Half its log. =	1.902818½
Twice the time of flight = . . .	45.64	Log. = . .	1.659418½

Time of flight = ... 22.82 seconds; which agrees with the time of flight found by the last rule, as above.

Problem XXXVII.

Given the Time of Flight of a Shell; to find the Length of the Fuze.

Rule.

To the logarithm of the time of flight add the constant logarithm 9.342423, for 13 and 10-inch shells,—or 9.380211, for 8, 5½, and 4⅖-inch shells: and the sum (abating 10 in the index,) will be the logarithm of the length of the fuze, in inches.

Example 1.

Let the time of flight of a 13-inch shell be 31.75 seconds; required the length of the fuze?

Time of flight, in seconds, =	31.75	Log. = 1.501744
Constant log. =	. . . .	9.342423
Length of the fuze, in inches, =	6.985	Log. = 0.844167

Example 2.

Let the time of flight of an 8-inch shell be 21.5 seconds; required the length of the fuze?

Time of flight, in seconds, =	21. 5	Log. = 1.332438
Constant log. =	. . . .	9.380211
Length of the fuze, in inches, =	5. 16	Log. = 0.712649

Note.—The fuzes for a 13 and a 10-inch shell are so constructed as to burn .22 of an inch in one second; and those for the smaller kind, viz., 8, 5½, and 4⅖-inch shells, .24 of an inch in the same space of time. Now, the logarithms of these two decimal numbers, viz., 9.342423 and 9.380211, are therefore the constant logarithms made use of in the above rule.

Fuzes are generally marked off, by circular lines, into seconds and fractional parts of a second, so that no time may be lost in measuring and adapting them to the shells for which they are intended.

SOLUTION OF PROBLEMS IN THE MENSURATION OF PLANES, &c.

Problem I.

Given the Base and perpendicular Height of a Plane Triangle; to find its Area.

Rule.

To the logarithm of the base add the logarithm of half the perpendicular height, and the sum will be the logarithm of the area, or superficial content of the triangle.

Example.

Let the base of a plane triangle be 37.6 yards, and its perpendicular height 29.8 yards; required its area, or superficial content?

Base of the triangle = ... 37.6 yards	Log. =	1.575188
Perpen. height = 29.8 yards ÷ 2 = 14.9 yards	Log. =	1.173186
Area, or superficial content, = . . 560.24 . . .	Log. =	2.748374

Problem II.

Given two Sides and the contained Angle of a plane Triangle; to find its Area.

Rule.

To the logarithmic sine of the contained angle, add the logarithms of the containing sides: and the sum (abating 10 in the index,) will be the logarithm of twice the area of the triangle.*

Example.

Let the two given sides of a triangle be 109.5 yards and 168.2 yards respectively, and the contained angle 79°16′; required the area, or superficial content of that triangle?

* Or, to the logarithmic sine of the contained angle, add the logarithm of one of the containing sides and the logarithm of *half the other containing side*: the sum of these three logarithms (abating 10 in the index,) will be the logarithm of the area of the triangle.

Contained or included angle = . . .	79° 16′	Log. sine =	9.992335
One of the containing sides = . . .	109. 5	Log. = .	2.039414
The other containing side = . . .	168. 2	Log. = .	2.225826
Twice the area of the triangle = .	18095. 7	Log. = . .	4.257575
Area of the triangle = ...	9047.85 yards, as required.		

Note.—The above problem will be found exceedingly useful in the practice of land-surveying.

Problem III.

Given the three Sides of a Triangle; to find its Area, or superficial Content.

Rule.

Add the three sides together, and take half their sum; subtract each side severally from that half sum, noting the remainders: then,

To the logarithm of the half sum add the logarithms of the three remainders; now, the sum of these four logarithms, being divided by 2, will give the area of the triangle.

Example.

Let the three sides of a triangle be 433, 312, and 205 yards respectively; required its area?

First side = 433	First remainder =	42	Log. =	1.623249
Second side = 312	Second remainder =	163	Log. =	2.212188
Third side = 205	Third remainder =	270	Log. =	2.431364
Sum = . . 950				
Half sum = 475	Log. = ...			2.676694
			Divide by 2)	8.943495
Area of the triangle = 29631.08	Log. = ...			4.471747½

PROBLEM IV.

Given the Diameter of a Circle; to find its Circumference, and conversely.

RULE.

To the logarithm of the diameter add the constant logarithm 0.497150, and the sum will be the logarithm of the circumference. And, to the logarithm of the circumference add the constant logarithm 9.502850, and the sum will be the logarithm of the diameter.

Example 1.

The earth's diameter is 7917.5 miles; required its circumference?

Diameter of the earth = ...	7917.5 miles	Log. =	3.898588
Constant log. = ...			0.497150
Circumference of the earth = . .	24873 miles	Log. =	4.395738

Example 2.

If the circumference of the earth be 25000 miles, what is its diameter?

Circumference of the earth = ...	25000 miles	Log. =	4.397940
Constant log. = ...			9.502850
Diameter of the earth, in miles, = .	7957.7	Log. =	3.900790

Note.—The diameter in the first example, viz., 7917.5 miles, appears to be the true diameter of the earth, on the spherical hypothesis. The constant logarithms used in this problem will be found in the Table of Miscellaneous Numbers, at the end of the second volume.

PROBLEM V.

Given the Diameter, or the Circumference of the Earth; to find the whole Area of its Surface.

RULE.

To twice the logarithm of the earth's diameter add the constant logarithm 0.497150, and the sum will be the logarithm of the area of the earth's surface, in square miles. Or, To twice the logarithm of the earth's circumference add the constant logarithm 9.502850, and the sum will be the logarithm of the earth's surface, in square miles.

Example 1.

Required the area or superficial measure, in square miles, of the whole of the earth's surface, allowing its diameter to be 7917.5 English miles?

Diameter of the earth =	7917.5	Twice its log. =	7.797176
Constant log. =	. . . .		0.497150
Area, in square miles, =	196936545.5	Log. = .	8.294326

Example 2.

Required the area or superficial measure of the whole of the earth's surface, in square miles, allowing its circumference to be 24873 English miles?

Circumference of the earth =	24873	Twice its log. =	8.791476
Constant log. =	. . . .		9.502850
Area, in square miles, =	196936545.5	Log. = .	8.294326

PROBLEM VI.

To find the Length of any Arc of a Circle.

RULE.

To the logarithm of the degrees in the given arc, considered as a natural number, add the logarithm of the radius of that arc, and the constant logarithm 8.241878; the sum will be the logarithm of the length of the arc.

Example.

Required the length of an arc of 45 degrees, the radius being 9 inches?

Length of the arc, in degrees, =	45	Log. =	1.653213
Radius of the arc, in inches, =	9	Log. =	0.954243
Constant log. =	. . . .		8.241878
Length of the arc, in inches, = .	7.0686	Log. =	0.849334

SOLUTION OF PROBLEMS IN GAUGING.

Gauging is the art of finding the number of gallons, &c., contained in any vessel.

By a recent Act of Parliament, there is to be but one general standard gallon throughout His Majesty's dominions of Great Britain and Ireland; which gallon is to contain 10 lbs. (avoirdupois weight) of distilled water, each pound of which is to weigh 7000 grains (troy weight): hence the new standard gallon is to contain 70000 grains (troy weight) of distilled water. Now, since a cubic inch of distilled water weighs 252.458 grains (troy weight), the contents of the new standard gallon may be readily reduced to cubic measure, by the following proportion; viz., As 252.458 grains : 1 inch :: 70000 grains : 277.27384357 inches; which, therefore, is the number of cubic inches in the new standard gallon. And because the measure of the present or old standard wine gallon is 231 cubic inches, and that of the old standard ale gallon 282 such inches, we have sufficient data for obtaining proper multipliers for the reduction of the old standard wine and ale measure into the new general standard measure, and conversely. Hence,

$277.27384357 \div 231 = 1.20031967$, Log. $= 0.079297$ — is the general multiplier for reducing the new standard measure into the *old standard wine measure*; and,

$231 \div 277.27384357 = 0.83311140$, Log. $= 9.920703$ — is the general multiplier for reducing the old standard wine measure into the *new standard measure*.

$277.27384357 \div 282 = 0.98324058$, Log. $= 9.992660$ — is the general multiplier for reducing the new standard measure into the *old standard ale measure*; and,

$282 \div 277.27384357 = 1.01704508$, Log. $= 0.007340$ — is the general multiplier for reducing the old standard ale measure into the *new standard measure*.

Now, the respective multipliers and their corresponding logarithms being thus obtained, the reduction of the old standard wine and ale measure into the new general standard measure, and conversely, may be very readily performed, by means of the following problems.

Problem I.

To reduce the old standard Wine Measure into the new Imperial Measure.

Rule.

To the logarithm of the old standard wine gallons add the constant logarithm 9.920703, and the sum will be the logarithm of the new standard gallons.

Example 1.

Reduce 400 gallons of the old standard wine measure into the new general standard measure.

Given number of gallons = 400	Log. =	2.602060
Constant log. = ...		9.920703
New standard gallons = 333.245	Log. =	2.522763

Example 2.

Reduce 9864 gallons of the old standard wine measure into the new general standard measure.

Given number of gallons = 9864	Log. =	3.994053
Constant log. = ...		9.920703
New standard gallons = 8217.8	Log. =	3.914756

Problem II.

To reduce the new Imperial Measure into the old standard wine Measure.

Rule.

To the logarithm of the new standard gallons add the constant logarithm 0.079297, and the sum will be the logarithm of the old standard wine gallons.

Example 1.

Reduce 400 gallons of the new general standard measure into the old standard wine measure.

Given number of new standard gallons = 400	Log. =	2.602060
Constant log. = ...		0.079297
Old standard wine gallons = . . . 480.128	Log. =	2.681357

Example 2.

Reduce 9864 gallons of the new general standard measure into the old standard wine measure.

Given number of new standard gallons =	9864	Log. =	3.994053
Constant log. = ...			0.079297
Old standard wine gallons = . . .	11839.95	Log. =	4.073350

Note.—From the above problems it appears that the new general standard gallon is, very nearly, *one-fifth* greater than the present or old standard wine gallon.

Problem III.

To reduce the old standard Ale Measure into the new Imperial Measure.

Rule.

To the logarithm of the old standard ale gallons add the constant logarithm 0.007340, and the sum will be the logarithm of the new general standard gallons.

Example 1.

Reduce 400 gallons of the old standard ale measure into the new general standard measure.

Given number of ale gallons =	400	Log. =	2.602060
Constant log. = ...			0.007340
New standard gallons =	406.82	Log. =	2.609400

Example 2.

Reduce 9864 gallons of the old standard ale measure into the new general standard measure.

Given number of ale gallons =	9864	Log. =	3.994053
Constant log. = ...			0.007340
New standard gallons =	10032.13	Log. =	4.001393

Problem IV.

To reduce the new Imperial Measure into the old standard Ale Measure.

Rule.

To the logarithm of the new standard gallons add the constant logarithm 9.992660, and the sum will be the logarithm of the old standard ale gallons.

Reduce 400 gallons of the new general standard measure into the old standard ale measure.

Given number of new standard gallons =	400	Log. =	2.602060
Constant log. =			9.992660
Old standard ale gallons =	393.296	Log. =	2.594720

Example 2.

Reduce 9864 gallons of the new general standard measure into the old standard ale measure.

Given number of new standard gallons =	9864	Log. =	3.994053
Constant log. =			9.992660
Old standard ale gallons =	9698.7	Log. =	3.986713

Note.—From the two last problems it appears that the new general standard gallon is, very nearly, *one-sixtieth* less than the present or old standard ale gallon.

Problem V.

Given the Dimensions of a circular-headed Cask; to find its Content in Ale and in Wine Gallons, and also agreeably to the new general standard or Imperial Gallon.

Rule.

Divide the head diameter by the bung diameter, to two places of decimals in the quotient; then,

Add together the logarithm for ale or wine gallons corresponding to this quotient in the first part of Table LVII., the logarithm of the bung diameter in the second part of that table, and the common logarithm of the length of the cask: the sum (abating 10 in the index,) will be the logarithm of the content of the cask in ale or wine gallons. Now, to the logarithm, thus found, add the constant logarithm 0.007340 for ale gallons, or 9.920703 for wine gallons; and the sum will be the logarithm of the true content of the cask in gallons, agreeably to the new general standard or imperial measure.

Example 1.

Let the bung diameter of a cask be 25 inches, its head diameter 19.5 inches, and length 31 inches; required its content in ale and wine gallons, and also in gallons agreeably to the new general standard measure?

19.50 ÷ 25 = 0.78, quotient of the head diameter divided by the bung diameter.

First,—For Ale Gallons :—

Quotient = ...	0.78	Log. for ale gallons =	7.362671
Bung diameter = . . .	25 inches	Corresponding log. =	2.795880
Length of the cask = . .	31 inches	Common log. = . .	1.491362
Content in ale gallons = . .	44.66	Log. = ...	1.649913
Constant log. = ...			0.007340
Content in imperial gallons =	45.42	Log. = ...	1.657253

Second,—For Wine Gallons :—

Quotient = ...	0.78	Log. for wine gallons =	7.449340
Bung diameter = . .	25 inches	Corresponding log. =	2.795880
Length of the cask = . .	31 inches	Common log. = . .	1.491362
Content in wine gallons =	54.52	Log. = ...	1.736582
Constant log. = ...			9.920703
Content in imperial gallons=	45.42	Log. = ...	1.657285

See the example for illustrating the use of Table LVII., page 153, and also page 154.

Note.—In gauging a cask, it is to be remembered that the dimensions of the bung diameter, the head diameter, and the length of the cask, be all taken within the cask. In measuring these dimensions, it must be carefully observed that the bung-hole be in the middle of the cask, and that the bung-stave and the stave directly opposite thereto be both regular and even within the cask; also, that the heads of the cask be equal and truly circular: if so, the distance between the inside of the chimb to the outside of its opposite stave will be the *head diameter within the cask*, very nearly.

Example 2.

Let the bung diameter of a cask be 31.25 inches, its head diameter 23.75 inches, and length 39 inches; required its content in ale and wine gallons, and also in gallons agreeably to the new general standard or imperial measure?

23.75 ÷ 31.25 = 0.76, quotient of the head diameter divided by the bung diameter.

First,—For Ale Gallons:—

Quotient =	0.76	Log. for ale gallons =	7.355087
Bung diameter =	31.25 inches	Corresponding log. =	2.989699
Length of the cask =	39 inches	Common log. =	1.591065
Content in ale gallons =	86.268	Log. =	1.935851
Constant log. =			0.007340
Content in imperial gallons =	87.74	Log. =	1.943191

Second,—For Wine Gallons:—

Quotient =	0.76	Log. for wine gallons =	7.441742
Bung diameter =	31.25 inches	Corresponding log. =	2.989699
Length of the cask =	39 inches	Common log. =	1.591065
Content in wine gallons =	105.32	Log. =	2.022506
Constant log. =			9.920703
Content in imperial gallons =	87.74	Log. =	1.943209

Remark.—The above problem will be found exceedingly useful to Pursers in the Royal Navy, to Commissaries in the Army, and to other officers in charge of government stores, who may have occasion to purchase beer, wine, or spirits, on His Majesty's account, in foreign countries; because it enables them to ascertain, in a few minutes, the absolute number of gallons contained in any given quantity of liquor, of the old measure, agreeably to the newly-established standard, or imperial measure.

PROBLEM VI.

Given the Content of a Cask lying in a horizontal Position, its Bung-Diameter, and the Depth of the Ullage or wet Inches; to find the Quantity of Liquor in the Cask.

RULE.

Conceive the bung diameter to be represented by unity or 1 inch, and that it be divided into 10000 equal parts; then the half of this, viz., .5000, is to be considered as a *constant decimal.*

Divide the *wet inches*, or depth of the ullage, by the bung diameter, to four places of decimals in the quotient; find the difference between this quotient and the constant decimal. Now, one-fourth of this difference being subtracted from the quotient, if the latter be less than the constant decimal, or added thereto if it be more than that decimal, the difference or sum will be the *multiplier.*

Then, to the logarithm of the multiplier, thus found, add the logarithm of the content of the cask, in wine measure; and the sum will be the logarithm of the ullage, or number of gallons of liquor in the cask, in wine measure. And if to this logarithm the constant logarithm 9.920703 be added, the sum will be the logarithm of the ullage, agreeably to the imperial measure.

Note.—If the content of the cask be given in ale measure, the constant logarithm will be 0.007340.

Example 1.

Let the bung diameter of a cask be 31.25 inches, its content in wine measure 105.32 gallons, and the depth of the ullage, 11.5 inches; required the quantity of liquor in the cask?

Depth of the ullage, or wet inches, 11.5 ÷ 31.25
inches (B. D.) = .3680 quotient, .3680, which is less than the constant
Constant decimal = .5000 decimal.

Difference = ...1320 ÷ 4 = . . 330, subtractive.

Multiplier = ...	.3350	Log. =	9.525045
Content of cask, in wine measure, =	105.32 gallons	Log. =	2.022506
Content of ullage, in wine gallons, =	35.28	Log. =	1.547551
		Constant log. =	9.920703
Content of ullage, in imperial galls. =	29.39	Log. =	1.468254

Note.—If the content of the cask be given in imperial measure, let the logarithm thereof be added to the logarithm of the multiplier; and the sum will be the logarithm of the ullage.

Thus, in the above example, let the content of the cask be given agreeably to the new general standard or imperial measure; viz., 87.74 gallons; then,

Multiplier, as above, = ...	.3350	Log. =	9.525045
Content of the cask, in impl. meas. =	87.74 gallons	Log. =	1.943209
Content of ullage, in imperial galls. =	29.39	Log. =	1.468254

Example 2.

Let the bung diameter of a cask be 25 inches, its content in wine measure 54.52 gallons, and the depth of the ullage 15.75 inches; required the quantity of liquor in the cask?

Depth of the ullage, or wet inches, 15.75 ÷ 25 inches (B. D.) = .6300 quotient, .6300, which is more than the constant decimal.
Contant decimal = .5000

Difference = . . .1300 ÷ 4 = . . .325, additive.

Multiplier = ...	.6625	Log.	=	9.821186
Content of the cask, in wine meas. =	54.52 gallons	Log.	=	1.736582
Content of ullage, in wine gallons, =	36.12	Log.	=	1.557768
		Constant log.	=	9.920703
Content of ullage, in imperial galls. =	30.09	Log.	=	1.478471

But if the content of the cask be given agreeably to the imperial standard measure, viz., 45.42 gallons, then the latter part of the operation will be as thus:—

Multiplier, as above, = ...	.6625	Log.	=	9.821186
Content of the cask, in impl. meas. =	45.42 gallons	Log.	=	1.657285
Content of ullage, in imperial galls. =	30.09	Log.	=	1.478471

Remark.—If the dry inches of the bung diameter be made use of instead of the wet, the result of the operation will express the vacuity in the cask; and if this vacuity be added to the ullage, the sum will be the content of the cask, which will be a proof that the work is right.

Thus, in the last example, where the bung diameter is 25 inches, and the depth of the ullage 15.75 inches, the difference of these is 9.25, which, therefore, is the number of dry inches.

Then, dry inches 9.25 ÷ 25 inches (B. D.). = .3700 quotient, .3700, which is less than the constant decimal.
Constant decimal = .5000

Difference = ...1300 ÷ 4 = . . .325, subtractive.

Multiplier = ...	.3375	Log.	=	9.528274
Content of the cask, in impl. meas. =	45.42 gallons	Log.	=	1.657285
Vacuity in the cask = ...	15.33	Log.	=	1.185559
Content of the ullage = ...	30.09			
Content of the cask = ...	45.42; which proves the work is right.			

PROBLEM VII.

Given the Content of a Cask standing in a vertical or upright Position, its Length, and the Depth of the Ullage or wet Inches; to find the Quantity of Liquor in the Cask.

RULE.

Conceive the length of the cask to be represented by unity or 1 inch, and that it be divided into 10000 equal parts; then the half of this, viz., .5000, is to be considered as a *constant decimal.*

Divide the *wet inches*, or depth of the ullage, by the length of the cask, to four places of decimals in the quotient; find the difference between this quotient and the constant decimal: now, one-tenth of this difference being subtracted from the quotient, if the latter be less than the constant decimal, or added thereto if it be more than that decimal, the difference or sum will be the multiplier.

Then, to the logarithm of the multiplier, thus found, add the logarithm of the content of the cask, in wine measure; and the sum will be the logarithm of the ullage, or number of gallons of liquor in the cask, in wine measure. And if to this logarithm the constant logarithm 9.920703 be added, the sum will be the logarithm of the ullage agreeably to the imperial standard measure.

Note.—If the content of the cask be given in ale measure, the constant logarithm will be 0.007340.

Example 1.

Let the length of a cask, between the heads, be 39 inches, its content in wine measure 105.32 gallons, and the depth of the ullage 16.5 inches; required the quantity of liquor in the cask?

Depth of ullage, or wet inches, 16.5 ÷ 39
inches (length) = .4231 quotient, .4231, which is less than the constant decimal.
Constant decimal = .5000

Difference = ... 769 ÷ 10 = . . 77, subtractive.

Multiplier = ...4154	Log.	=	9.618467
Content of the cask, in wine meas. = 105.32 gallons	Log.	=	2.022506
Content of ullage, in wine gallons, = 43.75	Log.	=	1.640973
	Constant log.	=	9.920703
Content of the ullage, in imperial galls. = 36.45	Log.	=	1.561676

Note.—If the content of the cask be given agreeably to the imperial standard measure, let the logarithm thereof be added to the logarithm of the multiplier; and the sum will be the logarithm of the ullage. Thus, in the above example, let the content of the cask be given in imperial measure; viz., 87.74 gallons; then,

Multiplier =	4154	Log. =	9.618467
Content of the cask, in impl. meas. =	87.74 gallons	Log. =	1.943209
Content of ullage, in imperial galls. =	36.45	Log. =	1.561676

Example 2.

Let the length of a cask, between the heads, be 31 inches, its content in wine measure 54.52 gallons, and the depth of the ullage 18.5 inches; required the quantity of liquor in the cask?

Depth of ullage, or wet inches, 18.5 ÷ 31 inches (length) = .5968 quotient, .5968, which is more than the constant decimal.
Constant decimal = .5000

Difference = ...968 ÷ 10 = . .97, additive.

Multiplier = ...	.6065	Log. =	9.782831
Content of the cask, in wine galls.=	54.52	Log. =	1.736582
Content of ullage, in wine galls. =	33.07	Log. =	1.519413
		Constant log. =	9.920703
Content of ullage, in imperial galls.=	27.55	Log. =	1.440116

But if the content of the cask be given in imperial measure, viz., 45.42 gallons, then the latter part of the operation will be as thus:—

Multiplier, as above, =	6065	Log. =	9.782831
Content of the cask, in imperial meas.=	45.42 gallons	Log. =	1.657285
Content of ullage, in imperial galls. =	27.55	Log. =	1.440116

Remark.—If the dry inches of the length of the cask be made use of instead of the wet, the result of the operation will express the vacuity in the cask; and if this vacuity be added to the ullage, the sum will give the content of the cask: but this, it is presumed, does not need to be elucidated by an example.

Note.—The following Table, which is particularly adapted to the reduction of the old-established wine and ale measure into the new general standard or imperial measure, and the contrary, will be found of very considerable use in the event of purchasing wine or spirits in places out of His Majesty's dominions.

A Table

For readily finding the Number of Wine or Ale Gallons which is actually equivalent to any given Number of Gallons of the newly-established general standard or Imperial Measure, and conversely.

Impl. Meas.	Wine Measure				Impl. Meas.	Ale Measure.				Wine Meas.	Imperial Meas.				Ale Meas.	Imperial Meas.			
	G.	Q.	P.	Gills.		G.	Q.	P.	Gills.		G.	Q.	P.	Gills.		G.	Q.	P.	Gills.
1gill	0	0	0	1.200	1gill	0	0	0	0.983	1gill	0	0	0	0.833	1gill	0	0	0	1.017
2do.	0	0	0	2.401	2do.	0	0	0	1.966	2do.	0	0	0	1.666	2do.	0	0	0	2.034
3do.	0	0	0	3.601	3do.	0	0	0	2.950	3do.	0	0	0	2.499	3do.	0	0	0	3.051
1pt.	0	0	1	0.801	1pt.	0	0	0	3.933	1pt.	0	0	0	3.332	1pt.	0	0	1	0.068
1qt.	0	1	0	1.603	1qt.	0	0	1	3.866	1qt.	0	0	1	2.665	1qt.	0	1	0	0.136
2qts.	0	2	0	3.205	2qts.	0	1	1	3.732	2qts.	0	1	1	1.330	2qts.	0	2	0	0.273
3qts.	0	3	1	0.808	3qts.	0	2	1	3.598	3qts.	0	2	0	3.995	3qts.	0	3	0	0.409
G.1	1	0	1	2.410	G.1	0	3	1	3.464	G.1	0	3	0	2.660	G.1	1	0	0	0.545
2	2	1	1	0.820	2	1	3	1	2.927	2	1	2	1	1.319	2	2	0	0	1.091
3	3	2	0	3.231	3	2	3	1	2.391	3	2	1	1	3.979	3	3	0	0	1.636
4	4	3	0	1.641	4	3	3	1	1.855	4	3	1	0	2.638	4	4	0	0	2.182
5	6	0	0	0.051	5	4	3	1	1.318	5	4	0	1	1.298	5	5	0	0	2.727
6	7	0	1	2.461	6	5	3	1	0.782	6	4	3	1	3.957	6	6	0	0	3.273
7	8	1	1	0.872	7	6	3	1	0.246	7	5	3	0	2.617	7	7	0	0	3.818
8	9	2	0	3.282	8	7	3	0	3.710	8	6	2	1	1.277	8	8	0	1	0.364
9	10	3	0	1.692	9	8	3	0	3.173	9	7	1	1	3.936	9	9	0	1	0.909
10	12	0	0	0.102	10	9	3	0	2.637	10	8	1	0	2.596	10	10	0	1	1.454
20	24	0	0	0.205	20	19	2	1	1.274	20	16	2	1	1.191	20	20	1	0	2.909
30	36	0	0	0.307	30	29	1	1	3.911	30	24	3	1	3.787	30	30	2	0	0.363
40	48	0	0	0.409	40	39	0	2	2.548	40	33	1	0	2.383	40	40	2	1	1.818
50	60	0	0	0.511	50	49	0	1	1.185	50	41	2	1	0.978	50	50	3	0	3.272
60	72	0	0	0.614	60	58	3	1	3.822	60	49	3	1	3.574	60	61	0	0	0.727
70	84	0	0	0.716	70	68	3	0	2.459	70	58	1	0	2.170	70	71	0	1	2.181
80	96	0	0	0.818	80	78	2	1	1.096	80	66	2	1	0.765	80	81	1	0	3.635
90	108	0	0	0.921	90	88	1	1	3.733	90	74	3	1	3.361	90	91	2	0	1.090
100	120	0	0	1.023	100	98	1	0	2.370	100	83	1	0	1.956	100	101	2	1	2.544
200	240	0	0	2.046	200	196	2	1	0.740	200	166	2	0	3.913	200	203	1	1	1.089
300	360	0	0	3.069	300	294	3	1	3.110	300	249	3	1	1.869	300	305	0	0	3.633
400	480	0	1	0.092	400	393	1	0	1.479	400	333	0	1	3.826	400	406	3	0	2.177
500	600	0	1	1.115	500	491	2	0	3.849	500	416	2	0	1.782	500	508	2	0	0.721
600	720	0	1	2.138	600	589	3	1	2.219	600	499	3	0	3.739	600	610	0	1	3.266
700	840	0	1	3.161	700	688	1	0	0.589	700	583	0	1	1.695	700	711	3	1	1.810
800	960	1	0	0.184	800	786	2	0	2.959	800	666	1	1	3.652	800	813	2	1	0.354
900	1080	1	0	1.207	900	884	3	1	1.329	900	749	3	0	1.608	900	915	1	0	2.898
1000	1200	1	0	2.229	1000	983	0	1	3.699	1000	833	0	0	3.565	1000	1017	0	0	1.443

Note.—In using the above Table, if the given number of gallons cannot be exactly found, or if it fall without the limits of the Table, the sum of the different quantities corresponding to the several terms which make up the given number of gallons is, in such cases, to be taken; as in the following examples:—

Example 1.

In 1736 gallons, imperial measure, how many gallons of wine measure?

			G. Q. P. Gills.	
1000	galls., impl. meas.,	are equal to	1200. 1. 0. 2. 229	W. M.
700	ditto	ditto	840. 0. 1. 3. 161	ditto.
30	ditto	ditto	36. 0. 0. 0. 307	ditto.
6	ditto.	ditto	7. 0. 1. 2. 461	ditto.

Hence, 1736 galls., impl. meas., are equal to 2083. 3. 0. 0. 158 W. M.

Example 2.

In 1839 gallons, wine measure, how many gallons imperial measure?

			G. Q. P. Gills.	
1000	galls, wine meas.,	are equal to	833. 0. 0. 3. 565	impl. meas.
800	ditto	ditto.	666. 1. 1. 3. 652	ditto.
30	ditto	ditto	24. 3. 1. 3. 787	ditto.
9	ditto	ditto	7. 1. 1. 3. 936	ditto.

Hence, 1839 galls., wine meas., are equal to 1532. 0. 0. 2. 940 impl. meas.*

SOLUTION OF MISCELLANEOUS PROBLEMS.

Problem I.

Given the Circumference of a Cable, and its Length; to find its Weight.

Rule.

To twice the logarithm of the circumference of the given cable, add the logarithm of its length, and the constant logarithm 9.734967: the sum of these three logarithms (abating 10 in the index,) will be the weight of the given cable, in pounds, avoirdupois.

Example.

Let the circumference of a cable be 21 inches, and its length 110 fathoms; required its weight?

Circumference or girt of given cable = 21 inches		Twice its log. =	2. 644438
Length of ditto, in fathoms, = . 110		Log. = . .	2. 041393
Constant log. = ...			9. 734967
Weight of the given cable, in pounds, = 26351		Log. = . .	4. 420798

* A general Victualling Table is given at the end of the second volume, which will be found of considerable utility to the Pursers of the Naval service, in making out their annual accounts, or in completing the provisions of their ships to any given time.

Remark.—It has been found, by actual experiment, that 1 fathom of a hemp cable which measures 9 inches in circumference weighs 44 lbs. avoirdupois. Now, since cylinders of equal lengths are as the squares of their circumferences,—therefore, as the square of 9 inches (the circumference of the experimented cable), is to the weight of 1 fathom thereof, viz., 44 lbs.; so is the square of the circumference of any other cable, to the weight of 1 fathom of such cable: which, multiplied by the length of the cable, will give its whole weight. The constant logarithm 9.734967 is found by adding the arithmetical complement of twice the logarithm of 9 inches to the logarithm of 44 pounds.

Problem II.

Given the Diameter of a Circle; to find its Circumference.

Rule.

To the logarithm of the diameter of the given circle add the constant logarithm 0.497150, and the sum will be the logarithm of the circumference of that circle.

Example 1.

Let the diameter of a circle be 78.41 yards; required its circumference?

Diameter of the given circle = . . 78.41 yards	Log. =	1.894372
Constant log. = ...		0.497150
Circumference of the given circle, in yards, = 246.33	Log. =	2.391522

Note.—The circumference of a circle whose diameter is unity or 1, is 3.14159265; and, since the circumferences of circles are to each other as their diameters, or radii,—therefore, as the diameter 1, is to its circumference 3.14159265; so is the diameter of any other circle, to its circumference: and hence the above rule. The constant logarithm is expressed by the logarithm of 3.14159265.

Example 2.

If the diameter of the earth be 7917.5 miles, what is its circumference?

Diameter of the earth = ... 7917.5 miles	Log. =	3.898588
Constant log. = ...		0.497150
Circumference of the earth, in miles, = 24873.5	Log. =	4.395738

The converse of this problem, viz., deducing the diameter from the circumference, is obvious.—See Problem IV., page 594.

PROBLEM III.

Given the Diameter of a Circle; to find its Area, or superficial Content.

RULE.

All circles are to one another, as the squares of their diameters; and as the area of a circle whose diameter is unity or 1, is .7853982, the logarithm of which is 9.895090,—therefore, to twice the logarithm of the given circle, add the constant logarithm 9.895090; and the sum (abating 10 in the index,) will be the logarithm of the area, or superficial content of that circle.

Example.

If the diameter of a circle be 78.41 yards, what is its area or superficial content?

Diameter of the given circle = 78.41 yards Twice its log. = 3.788744
Constant log. = ... 9.895090

Area of the given circle, in yards, = 4828.8 Log. = . . . 3.683834

PROBLEM IV.

Given the Area or superficial Content of a Circle; to find its Diameter.

RULE.

As this problem is evidently the converse of the last,—therefore, to the logarithm of the area of the given circle, add the constant logarithm 0.104910 (the arithmetical complement of 9.895090): divide the sum by 2, and the quotient will be the logarithm of the diameter of the given circle.

Example.

Let the area of a circle be 4828.8 yards; required its diameter?

Area or superficial content of given circle=4828.8 yards Log.=3.683834
Constant log. = ... 0.104910

Divide by 2) 3.788744

Diameter of the given circle, in yards, = 78.41 Log. = . . . 1.894372

PROBLEM V.

Given the Diameter of a Circle; to find the Side of a Square equal in Area to that Circle.

RULE.

To the logarithm of the diameter of the given circle, add the constant logarithm 9.947545 (the logarithm of the square root of .7853982); and the sum (abating 10 in the index,) will be the logarithm of the side of a square equal in area or superficial content to that circle.

Example.

If the diameter of a circle be 78.41 yards, what is the side of a square equal in area to that circle?

Diameter of the given circle = . . .	78.41 yards	Log. =	1.894372
Constant log. = ...			9.947545
Side of the required square, in yards, =	69.49	Log. =	1.841917

PROBLEM VI.

Given the Diameter of a Circle; to find the Side of a Square inscribed in that Circle.

RULE.

To the logarithm of the diameter of the given circle, add the constant logarithm 9.849485; and the sum (abating 10 in the index,) will be the logarithm of the side of a square inscribed in that circle.

Example.

If the diameter of a circle be 78.41 yards, what is the side of a square inscribed in that circle?

Diameter of the given circle = . . .	78.41 yards	Log. =	1.894372
Constant log. = ...			9.849485
Side of the inscribed square, in yards, =	55.44	Log. =	1.743857

Problem VII.

Given the transverse and the conjugate Diameters of an Ellipsis; to find its Area.

Rule.

To the logarithms of the longer and the shorter diameters of the ellipsis, add the constant logarithm 9.895090: the sum (abating 10 in the index,) will be the area of that ellipsis.

Example.

Let the transverse diameter of an ellipsis be 616 yards, and its conjugate diameter 445 yards; required the area or superficial content of that ellipsis?

Transverse diameter = ...	616 yards Log. =	2.789581
Conjugate diameter = ...	445 yards Log. =	2.648360
Constant log. = ...		9.895090
Area of the given ellipsis, in yards, =	215294 Log. =	5.333031

Problem VIII.

Given the transverse and the conjugate Diameters of an Ellipsis; to find the Diameter of a Circle equal in Area to that Ellipsis.

Rule.

To the logarithm of the longer diameter, add the logarithm of the shorter diameter; divide the sum by 2, and the quotient will be the logarithm of the diameter of a circle equal in area to the ellipsis.

Example.

Let the transverse diameter of an ellipsis be 616 yards, and its conjugate diameter 445 yards; required the diameter of a circle equal in area to that ellipsis?

Transverse diameter of the given ellipsis =	616 yards Log. =	2.789581
Conjugate diameter of ditto = . . .	445 yards Log. =	2.648360
	Divide by 2)	5.437941
Diameter of a circle equal in area = 52.356 yards	Log. =	2.718970½

PROBLEM IX.

Given the transverse and the conjugate Diameters of an Ellipsis; to find its Circumference.

RULE.

Square the two diameters; add those squares together: take half the sum, and find the logarithm corresponding thereto. Now, the half of this logarithm will be the logarithm of a natural number, which, being added to half the sum of the two diameters, will give the corrected mean diameter.

To the logarithm of the corrected mean diameter, thus found, add the constant logarithm 0.196121; and the sum will be the logarithm of the circumference of the ellipsis.

Example.

Let the transverse diameter of an ellipsis be 616 yards, and its conjugate diameter 445 yards; required its circumference?

Tr.diam.616 × 616 = 379456, the square.
Conj.do.445 × 445 = 198025, ditto.

Divide by 2) 577481, sum of the squares.

Half sum of squares = 288740½ Lg.5.4605077

Half the logarithm = . . . 2.7302588½ Nt.No.537.3519
Half the sum of the two diameters = . . . 530.5

Corrected mean diameter = ... 1067.8519 Lg.3.028511
Constant log. = ... 0.196121

Circumference of the ellipsis, in yards, = 1677.38 Log. = . 3.224632

PROBLEM X.

Given the Diameter of a Sphere, or Globe; to find its Solidity.

RULE.

To thrice the logarithm of the diameter of the given sphere, add the constant logarithm 9.718999; and the sum (abating 10 in the index,) will be the solid content of such sphere.

Example.

If the diameter of the earth be 7917.5 miles, what is its solidity?

Diameter of the earth = 7917.5 miles	Thrice its log. =	11.695764
Constant log. =	. . . .	9.718999
Solidity of the earth, in miles, = 259874059701.5	Log. =	11.414763

Note.—It has been found that the solidity or solid content of a sphere, whose diameter is unity or 1, is .5235988; and since spheres are to one another, as the cubes of their diameters,—therefore, as the cube of the diameter 1, is to its solidity .5235988; so is the cube of the diameter of any other sphere or globe, to the solidity of such sphere or globe: and hence the above rule. The constant logarithm is expressed by the logarithm of .5235988.

Remark.—For the method of finding the number of square miles contained in the earth's superficies, see Problem V., page 594.

PROBLEM XI.

Given the Earth's Diameter; to find the Height to which a Person should be raised to see one-third of its Surface.

RULE.

From twice the logarithm of the earth's semi-diameter, subtract the logarithm of its one-third: the remainder will be the logarithm of the height to which a person should be raised above the earth's centre, to see one-third of its surface; from which let the earth's radius or semi-diameter be taken, and the remainder will be the required height above its surface.

Example.

How high above the earth must a person be raised, that he may see one-third of its surface?

Earth's semi-diameter = . . .	3958.75 miles	Twice its log.=	7.195116
One-third of ditto = . . .	1319.5833 miles	Log. =	3.120437
Height above the earth's centre =	11876.25 miles	Log. =	4.074679
Deduct the earth's semi-diameter =	3958.75 miles.		
Remainder = ...	7917.50 miles;		

which is the true height to which a person should be raised above the earth, to see one-third of its surface.

PROBLEM XII.

Given the Earth's Semi-Diameter, and the Sun's mean horizontal Parallax; to find the Earth's Distance from the Sun.

RULE.

To the logarithm of the earth's semi-diameter, add the logarithmic co-tangent of the sun's mean horizontal parallax; and the sum (abating 10 in the index,) will be the logarithm of the sun's mean distance from the earth.

Example.

By the transits of Venus over the sun's disk in the years 1761 and 1769, the sun's mean horizontal parallax appears to be about 8.65 seconds of a degree; now, if the earth's semi-diameter be 3958.75 miles, its mean distance from the sun is required?

Semi-diameter of the earth =	. 3958.75 miles	Log. =	3.5975581
Mean horizontal parallax of the sun=	8″.65	Log. co-tang.=	14.3780860
Earth's mean distance from the sun=	94546196 miles	Log. =	7.9756441

PROBLEM XIII.

Given the Sun's mean Distance from the Earth, and his apparent Semi-Diameter, at a mean Rate; to find the true Measure of his Diameter, in English Miles.

RULE.

To the logarithm of the sun's mean distance from the earth, add the logarithmic tangent of his semi-diameter; and the sum (abating 10 in the index,) will be the logarithm of the sun's semi-diameter, in English miles; the double of which will be the measure of his whole diameter.

Example.

If the sun's mean distance from the earth be 94546196 English miles, and his mean apparent semi-diameter 16′ 1″.65, the true measure of his diameter is required?

Sun's mean distanee from the earth=	94546196 miles	Log. =	7.9756441
Sun's apparent semi-diameter =	16′ 1″.65	Log. tangent=	7.6685950
Sun's true semi-diameter = . . .	440797.5 miles	Log. =	5.6442391

True measure of the sun's diameter=881595 English miles.

PROBLEM XIV.

Given the Diameters of the Earth and the Sun; to find the Ratio of their Magnitudes.

RULE.

Since the magnitudes of all spherical bodies are as the cubes, or triplicate ratio, of their diameters (Euclid, Book XII., Prop. 18),—therefore, from thrice the logarithm of the sun's diameter subtract thrice the logarithm of the earth's diameter, and the remainder will be the logarithm of the ratio of their magnitudes.

Example.

If the earth's diameter be 7917.5 English miles, and that of the sun 881595 such miles, required the ratio of their magnitudes?

Sun's diameter, in English miles, =	881595	Thrice its log. =	17.8358076
Earth's diameter, in ditto, = . .	7917.5	Thrice its log. =	11.6957643
Ratio of the magnitudes of the earth and sun =	1380522	Log. =	6.1400433

PROBLEM XV.

Given the Circumference of the Earth; to find the Rate, per Hour, at which the Inhabitants under the Equator are carried, in consequence of the Earth's diurnal Motion round its Axis.

RULE.

To the arithmetical complement of the logarithm of 24 hours, add the logarithm of the earth's circumference, and the logarithm of 1 hour: the sum of these three logarithms (abating 10 in the index,) will be the logarithm of the rate per hour at which the inhabitants under the equator are carried by the earth's diurnal motion on its axis.

Example.

Let the circumference of the earth be 24873.5 miles; required the rate per hour at which the inhabitants under the equator are carried, in consequence of the earth's diurnal motion?

One day, or 24 hours,		Arith. comp. of its log. =	8.6197888
Earth's circumference =	24873.5 miles	Log. =	4.3957369
Given time, or	1 hour	Log. =	0.0000000
Rate per hour, in miles, =	1036.396	Log. =	3.0155257

Problem XVI.

To find the Rate at which the Inhabitants under any given Parallel of Latitude are carried, in consequence of the Earth's diurnal Motion on its Axis.

Rule.

The circumference of the earth under the equator is 24873.5 miles; and since the circumference under any parallel of latitude decreases in proportion to the co-sine of the latitude of such parallel,—therefore, to the logarithm of the earth's circumference, under the equator, add the logarithmic co-sine of the latitude of the given parallel; and the sum (abating 10 in the index,) will be the logarithm of the earth's circumference under that parallel: with which proceed as directed in the last problem.

Example.

Let the circumference of the earth be 24873.5 miles; required the rate per hour at which the inhabitants under the parallel of London are carried by the earth's motion on its axis?

Circumference of the earth = 24873.5 miles	Log. = . .	4.3957369
Latitude of the parallel of London = 51°31′	Log. co-sine =	9.7939907
Circumference under given parallel = 15478.45	Log. = . .	4.1897276
One day, or 24 hours, Arith. comp. of its log. =	. . . .	8.6197888
Rate per hour, in miles, as required, = 644.93	Log. = . .	2.8095164

Problem XVII.

To find the Length of the tropical or solar Year.

Rule.

It has been found, by observation, that the sun *apparently* advances in the ecliptic 59′8″.33 of a degree every day at a mean rate; that is, from the time of his leaving any given meridian to the time of his returning to the same meridian. Now, since the ecliptic is a great circle of 360 degrees, —therefore, as the sun's apparent diurnal motion in the ecliptic, is to 1 day, or 24 hours; so is the great circle of 360 degrees, to the true length of the tropical or solar year; that is, to the time of the sun's periodical revolution round the ecliptic from any equinoctial or solstitial point to the same point again. Hence, by logarithms,

Example.

The sun's daily motion in the ecliptic is $59'\ 8''.33$ in every natural day, or 24 hours, at a mean rate; required the length of the tropical or solar year?

Sun's app. diur. motion $59'\ 8''.33$, in secs. =	3548.33	Log. ar. co.	6.4499760
One day, or 24 hours, in seconds = . .	86400	Log. =	4.9365137
Ecliptic, or great circle of 360°, in secs. =	1296000	Log. =	6.1126050
Length of the tropical year, in seconds =	31556928	Log. =	7.4990947

Hence the tropical or solar year consists of $365^d\ 5^h\ 48^m\ 48^s$, as required.

PROBLEM XVIII.

To find the Rate at which the Earth moves in the Ecliptic during the Time of its annual or periodical Revolution round the Sun.

RULE.

Since the earth's mean distance from the sun is 94546196 miles (Problem XII., page 614), the diameter of the orbit in which it moves round that great luminary is 189092392 miles; and since the diameter of a circle is to its circumference in the ratio of unity or 1, to 3.14159265, the circumference of the earth's orbit is 594051320 miles. Now, as the earth describes this circumference in $365^d\ 5^h\ 48^m\ 48^s$ (last problem), or 8766 hours nearly, we have the following computation by logarithms:—

As the length of the year, in hours, =	8766	Log. ar. comp. =	6.0571985
Is to the circumf. of the earth's orbit =	594051320 miles	Log. =	8.7738239
So is	1 hour	Log. =	0.0000000
To the earth's hourly motion in its orbit =	67768 miles	Log. =	4.8310224

PROBLEM XIX.

Given the Moon's mean Distance from the Earth, and her apparent Semi-diameter, at a mean Rate; to find the true Measure of her Diameter, in English Miles.

RULE.

It is shown in page 9, under the head "Augmentation of the Moon's Semi-diameter," that the moon's mean distance from the earth is

236692.35 miles. Now, since her apparent semi-diameter is 15′43″ at a mean rate,—therefore, to the logarithm of her mean distance from the earth, add the logarithmic tangent of her apparent semi-diameter; and the sum (abating 10 in the index,) will be the logarithm of the moon's semi-diameter in English miles: the double of which will be the measure of her whole diameter.

Example.

Let the moon's distance from the earth be 236692.35 miles, and her semi-diameter 15′43″; required the true measure of her diameter in English miles?

Moon's mean distance from the earth = 236692.35 miles Log. = 5.3741842
Moon's apparent semi-diameter = 15′43″ Log. tangent = 7.6600896

Moon's true semi-diameter = . . . 1082.1 miles Log. = 3.0342738

True measure of the moon's diameter = 2164.2 English miles.

Problem XX.

Given the Diameters of the Earth and the Moon; to find the Ratio of their Magnitudes.

Note.—This is performed by Problem XIV., page 615.

Example.

If the earth's diameter be 7917.5 English miles, and that of the moon 2164.2 such miles, required the ratio of their magnitudes?

Diameter of the earth = . . 7917.5 Thrice its log. = 11.6957643
Diameter of the moon = . . 2164.2 Thrice its log. = 10.0058922

Ratio of the magnitudes of the earth and moon = 48.96 Log. = 1.6898721

Problem XXI.

To find how much larger the Earth appears to the lunar Inhabitants than the Moon appears to the terrestrial Inhabitants.

Rule.

Since the distance between the earth and the moon is such as to cause their opposing hemispheres to appear, reciprocally from each other, like flat circles; and since circles are to one another as the squares of their diameters (Euclid, Book XII., Prop. 2,) or, which is the same thing, since spherical surfaces are to each other as the squares of their radii,—therefore, from twice the logarithm of the earth's diameter, subtract twice the logarithm of that of the moon; and the remainder will be the logarithm of the number of times that the earth appears larger to the inhabitants of the moon than the moon does to the inhabitants of the earth.

Example.

The diameter of the earth is 7917.5 miles, and that of the moon 2164.2 miles; required how much larger the earth appears from the moon than the moon does from the earth?

Diameter of the earth = . . .	7917.5	Twice its log.=7.7971762
Diameter of the moon = . . .	2164.2	Twice its log.=6.6705948
Number of times the earth is larger than the ☽ =	13.38	Log.=1.1265814

Problem XXII.

To find the Rate at which the Moon revolves round her Orbit.

Note.—This is performed by Problem XVIII., page 617; as thus:—

Since the moon's mean distance from the earth is 236692.35 miles, the diameter of her orbit must be twice that distance, or 473384.70 miles: hence its circumference is 1487182 miles. And since the moon goes through this circuit, or orbit, in 27ᵈ 7ʰ 43ᵐ 5ˢ, her hourly motion may be determined in the following manner; viz.,

As one lunation=27ᵈ 7ʰ 43ᵐ 5ˢ, in secs.=	2360585	Log.ar.co.=3.6269804
Is to the circumference of the moon's orbit=	1487182	Log.=6.1723641
So is one hour, in seconds = ...	3600	Log.=3.5563025
To the moon's hourly motion in her orbit =	2268 miles	Log.=3.3556470

Problem XXIII.

To find the mean Distance of a Planet from the Sun.

Rule.

It has been demonstrated, by the celebrated *Kepler*, that if two or more bodies move round another body as their common centre of motion, the squares of their periodical times will be to each other in the same proportion as the cubes of their mean distances from the central body; and hence the following rule:—

As the square of the earth's periodical or annual motion round the sun, is to the cube of its mean distance from that luminary; so is the square of any other planet's periodical revolution round the sun, to the cube of its mean distance therefrom; the root of which will be the distance sought.

Example.

The earth's periodical or annual motion round the sun is completed in 365 days, 5 hours, 48 minutes, 48 seconds, and that of Venus in 224 days, 16 hours, 49 minutes, 11 seconds. Now, if the earth's mean distance from the sun be 94546196 miles, what is Venus' distance from that luminary?

Earth's periodical revolution
365ᵈ 5ʰ 48ᵐ 48ˢ, in secs. 31556928 Ar. co. of twice its log. 5.0018106
Earth's mean dist. from sun, in miles, = 94546196 Thrice its log. 23.9269323
Venus' per. rev. 224ᵈ 16ʰ 49ᵐ 11ˢ, in secs. 19414151 Twice log. 14.5762368

Reject 20 from the index; and, to extract the root, divide by 3) 23.5049797

Venus' mean distance from sun, in miles, = 68390098 Log. = 7.8349932½

Problem XXIV.

To find how much more Heat and Light the Planets adjacent to the Sun receive from that Luminary than those which are more remote.

Rule.

Since the effects of heat and light are reciprocally proportional to the squares of the distances from the centre whence they are generated,— therefore, from twice the logarithm of the remote planet's distance from the sun, subtract twice the logarithm of the adjacent planet's distance therefrom; and the remainder will be the logarithm of the number of times that the planet adjacent to the sun is hotter and more luminous than that which is more remote.

Example.

If the earth's distance from the sun be 94546196 miles, and that of Venus 68390098 miles, required how much more heat and light the latter planet receives from the sun than the former?

Earth's mean dist. from sun = 94546196 miles	Twice its log. =	15.9512882
Venus' ditto = 68390098 miles	Twice its log. =	15.8699865
Heat and light Venus receives more than the earth = 1.205	Log.	0.0813017

PROBLEM XXV.

Given the apparent Diameter of a Planet; to find the Measure of its true Diameter.

RULE.

Find the difference between the earth's and the planet's mean distances from the sun, and it will show the planet's mean distance from the earth; with which and the planet's apparent semi-diameter, compute her true diameter, by Problem XIX., page 617.

Example.

Let the earth's distance from the sun be 94546196 English miles, that of Venus 68390098 such miles, and her apparent diameter 58″.79; required the true measure of her diameter, in English miles?

Earth's distance from the sun = 94546196 miles		
Venus' ditto = 68390098 miles		
Venus' mean dist. from earth = 26156098 miles	Log. =	7.4175729
Venus' apparent semi-diameter = 29″.395	Log. tang. =	6.1537885
Venus' true semi-diameter = 3727 miles	Log. =	3.5713614

True measure of Venus' diam. = 7454 English miles.

Note.—If the ratio of the magnitudes of the earth and a planet be required, it may be determined by Problem XIV., page 615; thus in the case of Venus:—

Diameter of the earth = 7917.5 miles	Thrice its log. =	11.6957643
Diameter of Venus = 7454 miles	Thrice its log. =	11.6171682
Ratio of the magnitudes of the earth and Venus = 1.198	Log. =	0.0785961

The velocity or rate at which a planet moves round its orbit may be determined by Problem XVIII., page 617.

Problem XXVI.

To find the Time that the Sun takes to turn round its Axis.

Rule.

If the bright face of the sun be carefully observed through a good telescope, large black spots will be found to make their appearance on its eastern limb: from this they gradually advance to the middle of the disk, and thence to the western limb, where they disappear. After being absent for nearly the same period of time that they were visible, they will be observed to appear again on the eastern limb as at first; thus finishing their career in 27 days, 12 hours, and 20 minutes. Call this time the *observed interval.*

Find the number of degrees and parts of a degree that the earth has moved eastward in the ecliptic during that interval, in the following manner; viz.,

As the earth's annual motion
round the sun=365d.5h.48m.48s., in secs. 31556928 Log. ar. co. 2.5009053
Is to eclip., or great circle of 360°, in secs. 1296000 Log. = 6.1126050
So is the obs. interv.=27d.12h.20m., in secs. 2377200 Log. = 6.3760657

Earth's advance in ecliptic during obs. interv. = 97628″ Log. = 4.9895760

Ditto, in degrees and parts of a degree, = 27°.7′.8″

Now, as 360 degrees, augmented by the earth's advance in the ecliptic during the observed interval, thus found, is to the observed interval; so is the great circle of 360 degrees, to the absolute time of the sun's rotatory motion on its axis; thus:—

As 360° + 27°.7′.8″ = 387°.7′.8″, in secs. = 1393628 Log. ar. co. 3.8558532
Is to the obs. int. = 27d.12h.20m., in secs. = 2377200 Log. = 6.3760657
So is the great circle of 360° in secs. = 1296000 Log. = 6.1126050

To the time of the sun's rotatory motion = 2210670s. Log. = 6.3445239

Ditto, in days and parts of a day = 25d.14h.4m.30s.; which, therefore, is the true time that the sun takes to turn round *once* upon its axis, as required.

Problem XXVII.

To find the Length of a Pendulum for Vibrating Seconds in the Latitude of London.

Rule.

It has been found by actual experiment that a heavy body let fall in the latitude of London, will descend, by the force of gravity, $16\frac{1}{12}$ feet in 1 second of time;—now, since the circumference of a circle, whose diameter is unity or 1, is found by computation to be 3.14159265; and since the pendulum vibrates in the arc of a circle, or cycloid, the radius of which is equal to the length of the pendulum from the centre of oscillation; therefore if twice the space passed through by a falling body in one second of time, be divided by the square of the computed circumference of a circle, as above, the quotient will be the length of the pendulum for vibrating seconds in the parallel of London.

Thus. $16\frac{1}{12}$ feet = 193 inches × 2 = 386 inches, Log. = 2.5865873
Circumf. of circle to diam. 1 = 3.14159265, twice its Log. = 0.9942998

Length of the pend. in inches = 39.11 Logarithm = 1.5922875

Note.—By actual experiment the length of a pendulum for vibrating seconds in London is $39\frac{1}{8}$ inches, or 39.125.

Problem XXVIII.

To find the Length of a Pendulum for vibrating Half-Seconds.

Rule.

To the arithmetical complement of twice the logarithm of 120 (the number of vibrations in a minute for the half-seconds' pendulum), add twice the logarithm of 60 (the number of vibrations in a minute for the seconds' pendulum), and the logarithm of the length of the latter pendulum: the sum of these three logarithms (abating 10 in the index,) will be the logarithm of the length of the pendulum for vibrating half-seconds.

Example.

Let the length of a pendulum for vibrating seconds be 39.125 inches; required the length of a pendulum that will vibrate half-seconds?

Vibrations for ½-secs.' pendulum = 120 Ar. co. of twice its log. = 5.841638
Ditto for the seconds' pendulum = 60 Twice its log. = . . . 3.556302
Length of the pendulum for secs. = 39.125 inches Log. = 1.592454

Length of half-seconds' pendulum = 9.781 inches Log. = 0.990394

Hence the length of a pendulum for vibrating half-seconds, is $9\frac{3}{4}$ inches.

A COMPENDIUM OF PRACTICAL NAVIGATION; including the direct manner of making out a *Day's Work* at sea; intended for the use of persons unacquainted with the elements of Geometry and Trigonometry.

Problem I.

To reduce the Sun's Declination, as given in the Nautical Almanac, to the Time of apparent Noon under any known Meridian.

Rule.

From page II. of the month in the Nautical Almanac take out the sun's declination for noon of the given day, and note whether it is increasing or decreasing; and, at the same time, take out the variation of the sun's declination between the noons of the given and preceding days if the longitude be east, but between those of the given and following days if the longitude be west. Then, with this variation, or difference of declination, enter Table XV, at top, and the longitude of the given meridian in the right hand column;—in the angle of meeting will be found a correction, which being applied to the declination, taken from the Nautical Almanac, agreeably to the directions expressed at the bottom of the Table, will give the sun's correct declination at the noon of the given place.

Note.—When the longitude of the given meridian, and the variation of declination cannot be exactly found in the Table; then, the sum of the proportional parts, corresponding to the several terms which make up the whole longitude and the whole variation, will be the correction of declination required.

Example 1.

Required the sun's declination at noon, August 10th., 1825*, in longitude 100°30′ East?

Variation of declination between given and preceding noons (the longitude being east) is 17′26″

Sun's declination at noon of the given day per Nautical Almanac (*decreasing*) =					15°36′30″ N.
Pro. pt. to lon.	90° 0′	and var.	17′ 0″	= 4′15″ 0‴	
Ditto . . .	90.. 0	ditto	0.26	= 0. 6.30	
Ditto . . .	10.30	ditto	17. 0	= 0.29.45	
Ditto . . .	10.30	ditto	0.26	= 0. 0.45.30	
Correction of dec. additive = ...				4′52″ 0‴30⁗	+ 4′52″
Sun's reduced, or corrected declination = ...					15°41′22″ N.

* It is the nautical or sea day that is made use of in this and the following Examples:—this day, like the civil, begins at midnight, and ends at the following midnight:—it is divided into two parts, of 12 hours each; the first part or that contained between midnight and noon is called A.M. or ante meridiem, and the other part, or that between noon and midnight, P.M. or post meridiem.

Example 2.

Required the sun's declination at noon, April 3d., 1825 *, in longitude 75°.45′. West?

Variation of declination between given and following noons (the longitude being west) is 22′.54″.

Sun's declination at noon of the given day, per Nautical Almanac. (*increasing*) =						5°.18′.40″. N.
Pro. pt. to lon.	75° 0′.	and var.	22′. 0″.	= 4′.35″. 0‴.		
Ditto	0.45	ditto	22. 0	= 0. 2.45		
Ditto	75. 0	ditto	0.54	= 0.11.15		
Ditto	0.45	ditto	0.54	= 0. 0. 6.45		
Correction of declination, additive =				4′.49″. 6‴.45″″.	+	4′.49″.
Sun's reduced, or corrected declination =						5°.23′.29″. N.

Problem II.

Given the Sun's Meridian Altitude, to find the Latitude of the Place of Observation.

Rule.

Reduce the sun's declination to the meridian of the given place by the preceding Problem.

Then, to the observed altitude of the *sun's lower limb* add the difference between its semi-diameter (page III. of the month in the Nautical Almanac,) and the dip of the horizon, (Table II.) and the sum will be the apparent altitude of the sun's centre; from which, let the difference between the parallax and refraction answering thereto (Tables VII. and VIII.) be subtracted, and the remainder will be the sun's true central altitude; which being taken from 90 degrees will leave the sun's meridional zenith distance of a contrary denomination to that of its observed altitude. Now,

If the sun's meridian zenith distance and its reduced declination are both north, or both south, their sum will be the latitude of the place of observation: but if one be north and the other south, their difference will be the latitude, and always of the same name with the greater term.

Example 1.

April 10th, 1825, in longitude 75° west, the meridian altitude of the sun's lower limb was 57°.40′.30″. south, and the height of the observer's

* See Note, page 624.

eye above the level of the sea 22 feet; required the latitude of the place of observation?

Variation of the sun's declination between the given and following noons, (the longitude being west,) is 22′.6″.

Sun's declination at noon of the given day per Nautical Almanac (*increasing*) = ...	7°56′42″ N.
Propl. part to long. 75°0′ and var. 22′. 0″=4′.35″ 0‴.	
Ditto . . . . . 75.0 ditto 0. 6 =0. 1.15	
Correction of declination, additive = . . . 4′.36″15‴	+ 4′36″
Sun's reduced, or corrected declination = ...	8° 1′18″ N.
Observed altitude of the sun's lower limb = . . .	57°40′30″ S.
Sun's semi-diameter = . . . 15′.59″ } Dip of the hor. for 22 feet = 4.30 } difference, add	11′29″
Apparent altitude of the sun's centre = ...	57°51′59″ S.
Parallax 0′.5″ refrac. 0′.35″, diff. = 0′.30″ subtractive =	0′.30″
Sun's true central altitude = ...	57°51′29″ S.
Sun's meridional zenith distance = ...	32° 8′31″ N.
Sun's reduced declination = ...	8. 1.18 N.
Latitude of the place of observation = ...	40° 9′49″ N.

Example 2.

September 21st., 1825, in longitude 60° east, the meridian altitude of the sun's lower limb was 56°26′ north, and the height of the observer's eye above the level of the sea 26 feet; required the latitude of the place of observation?

Variation of the sun's declination between the given and preceding noons, the longitude being east, is 23′.22″.

Sun's declination at noon of the given day per Nautical Almanac (*decreasing*) = ...	0°43′34″ N.
Propl. part to long. 60°0′ and var. 23′. 0″=3′.50″ 0‴	
Ditto . . . . . 60.0 ditto 0.22 =0. 3.40	
Correction of declination, additive = . . . 3′.53″40‴	+ 3′.54″
Sun's reduced or corrected declination = ...	0°47′.28″ N.

Observed altitude of the sun's lower limb = . . .	56°26′ 0″	N.
Sun's semi-diameter 15′58″ dip of the horizon for 26 feet = 4′52″ difference = ...	+ 11′ 6″	
Apparent altitude of the sun's centre = ...	56°37′ 6″	N.
Parallax 0′5″ refrac. 0′37″ diff. = 0′32″ subtractive =	0′32″	
Sun's true central altitude = ...	56°36′34″	N.
Sun's meridional zenith distance = ...	33°23′26″	S.
Sun's reduced declination = ...	0.47.28	N.
Latitude of the place of observation = ...	32°35′58″	S.

Problem III.

Given the difference of Longitude between two Places, both under the same Parallel of Latitude, to find their Distance.

Rule.

To the logarithmic co-sine of the latitude, add the logarithm of the difference of longitude, in miles; and the sum, abating 10 in the index, will be the logarithm of the distance.

Example.

Required the distance between Portsmouth, in longitude 1°6′ west, and Green Island, Newfoundland, in longitude 55°35′ west, their common latitude being 50°47′ north?

Long. of Portsmouth = ...	1°. 6′ W.		
Long. of Green Island = . . .	55.35 W.		
Difference of Longitude = . .	54°29′ = 3269 ms.	Log.	3.514415
Latitude of the parallel = 50°47′ N.		Log. co-sine = . .	9.800892
Distance, in miles = ...	2066.8	Log. = . .	3.315307

Problem IV.

Given the Distance between two Places, both under the same Parallel of Latitude, to find their Difference of Longitude.

Rule.

To the logarithmic secant of the latitude, add the logarithm of the distance, and the sum, abating 10 in the index, will be the logarithm of the difference of longitude.

Example.

A ship from Cape Clear, in latitude 51°25′ north, and longitude 9°29′ west, sailed due west 1040 miles; required the longitude at which she then arrived?

Lat. of the parallel = . . .	51°25′	Log. secant = . . .	10.205057
Distance sailed = ...	1040	miles, Log. = ...	3.017033
Difference of long. = . . .	27°48′ W.	= 1667.6 miles, Log.	3.222090
Longitude sailed from =	9.29 W.		
Longitude arrived at =	37°17′ W.		

Note.—The above two Problems are essentially useful when a ship sails upon a parallel of latitude; that is, when she steers either due east, or due west.

PROBLEM V.

Given the Latitudes and Longitudes of two Places, to find the Course and Distance.

RULE.

From the logarithm of the difference of longitude, the index being augmented by 10, subtract the logarithm of the meridional difference of latitude; the remainder will be the logarithmic tangent of the course:—then, to the logarithmic secant of the course, thus found, add the logarithm of the difference of latitude, and the sum, abating 10 in the index, will be the logarithm of the distance.

Example.

Required the course and distance between Cape Bajóli, in latitude 40°3′ north, longitude 3°52′ east, and Cape Sicie, in latitude 43°2′ north, and longitude 5°58′ east?

Lat. of C. Bajoli	40° 3′ N.	Merid. pts.	2626.6,	Longitude	3.52 E.
Lat. of C. Sicie =	43. 2 N.	Merid. pts.	2865.8,	Longitude	5.58 E.
Diff. of latitude	2°59′	Merid. diff. lat.	239′.2.	Diff. long.	2° 6′
	= 179 miles.				= 126 miles.

To find the Course:—

Difference longitude 126 miles, . .	Logarithm =	2.100371
Merid. difference of latitude 239 miles	Logarithm =	2.378398
Course N. 27°47′53″ E. = . . .	Log. tang. =	9.721973

To find the Distance :—

Course 27°47′53″	Log. secant = .	10.053254
Difference latitude = 179 miles,	Logarithm = .	2.252853
Distance in miles = 202.3 . .	Logarithm = . .	2.306107

Hence the true course is N. 27°47′53″ E., or N. N. E. ½ E. nearly, and the distance 202¼ miles.

Problem VI.

Given the Latitude and Longitude of the Place sailed from, with the Course and Distance; to find the Latitude and Longitude of the Place come to.

Rule.

To the logarithmic co-sine of the course, add the logarithm of the distance; the sum, abating 10 in the index, will be the logarithm of the difference of latitude; which being applied to the latitude left by addition or subtraction, according as the latter is increasing or decreasing, the sum, or difference will be the latitude come to. Now, to the logarithmic tangent of the course, add the logarithm of the meridional difference of latitude; the sum, abating 10 in the index, will be the logarithm of the difference of longitude; which being applied by addition or subtraction to the longitude left, according as the latter is increasing or decreasing, the sum or difference will be the longitude come to.

Example 1.

A ship from Cape Ortegal, in latitude 43°47′ N. and longitude 7°49′ W., sailed N. W. ½ N. 560 miles; required the latitude and longitude of the place come to?

To find the Difference of Latitude :—

Course steered = 3½ points	Log. co-sine =	9.888185
Distance sailed 560 miles	Logarithm =	2.748188
Difference of latitude 432.8 miles . .	Logarithm =	2.636373

To find the Latitude come to :—

Latitude of Cape Ortegal	43°47′ N.	Meridional parts . .	2927.8
Diff. of lat. 432.8 N. =	7.13 N.		
Latitude come to . . .	51° 0′ N.	Meridional parts .	3568.8
Meridional difference of latitude =	. . . .		641.0

To find the Difference of Longitude, and hence the Longitude come to :—

Course steered = ...	$3\frac{1}{2}$ points.	Log. tang. . .	9.914173
Meridional difference of lat. = .	641 miles.	Logarithm . .	2.806858
Difference of long. = 8°46′ W. =	526 miles.	Logarithm . .	2.721031
Long. of C. Ortegal = 7.49 W.			
Long. come to = . 16°35′ W.			

Remarks.—When a ship decreases her latitude; that is, when the difference of latitude made good is of a different name to the latitude sailed from; then, if the difference of latitude, expressed in degrees, be greater than the latitude left, their difference will be the latitude come to; which will be of a contrary denomination to that sailed from; because, in this case it is evident that the ship must have crossed the Equator.

And, when a ship decreases her longitude; that is, when the difference of longitude made good is of a contrary name to the longitude sailed from; then, if the difference of longitude, expressed in degrees, be greater than the longitude left, their difference will be the longitude come to; which will be of a contrary name to that sailed from; because, in this case the ship will have crossed the meridian whence the longitude is reckoned.

Again.—When a ship increases her longitude; that is, when the difference of longitude made good, expressed in degrees, is of the same name with the longitude sailed from, their sum will be the longitude come to; but, if this sum exceeds 180 degrees, then, its difference to 360 degrees will express the longitude come to, which will be of a contrary denomination to that sailed from; for, in this case, also, the ship will have crossed the meridian that the longitude was reckoned from:—see Problems, Rules, and Remarks, between pages 211 and 217.

Example 2.

A ship from the Island of Annabona, in latitude 1°23′ S., and longitude 5°34′ E., sailed W. N. W. 546 miles; required the latitude and longitude of the place at which she arrived?

To find the Difference of Latitude:—

Course steered = 6 points . . .	Log. co-sine . .	9.582840
Distance sailed 546 miles	Logarithm . .	2.737193
Difference of latitude 208.9 miles =	Logarithm . .	2.320033

To find the Latitude come to:—

Latitude sailed from = . .	1°23′ S.	Merid. parts = . .	83.0
Diff. lat. = 208.9 miles =	3.29 N.		
Latitude come to = . .	2° 6′ N.	Merid. parts = . .	126.0
Meridional difference of latitude =	. . . .		209.0

To find the Difference of Longitude, and hence the Longitude come to:—

Course steered = ...	6 points.	Log. tang. =	10.382776
Meridional diff. of lat. = . .	209 miles.	Logarithm =	2.320146
Diff. of long. = . 8°25′ W. =	504.6 miles	Log. =	2.702922
Long. sailed from 5.34 E.			
Long. come to = . 2°51′ W.			

Hence, the latitude come to is 2°6′ N. and the longitude 2°51′ W.

Example 3.

A ship from Pitt's Island, in latitude 2°54′ N. and longitude 174°30′ E. sailed S. E. by E. ½ E. 760 miles; required the latitude and longitude of the place at which she arrived.?

To find the Difference of Latitude:—

Course steered = . .	5½ points.	Log. co-sine = . . .	9.673387
Distance sailed 760 miles.		Logarithm = . . .	2.880814
Diff. of lat. = 358.4 miles.		Logarithm . . .	2.554201

To find the Latitude come to:—

Latitude sailed from = . .	2°54′ N.	Merid. parts = . .	174.1
Diff. of lat. 358.4 miles =	5.58 S.		
Lat. come to = ...	3° 4′ S.	Merid. parts = .	184.1
Meridional difference of latitude = ...			358.2

To find the Difference of Longitude, and hence the Longitude come to:—

Course steered = . . .	5½ points.	Log. tangent =	10.272043
Meridional diff. of lat. =	358.2 miles.	Logarithm = .	2.554126
Diff. of long. made good = 11°10′ E. =	670 miles	Log.	2.826169
Longitude sailed from = 174.30 E.			
Sum = 185°40′ E.			
Longitude come to = . 174°20′ W.			

Hence, the latitude come to is 3°4′ S., and the longitude 174°20′ west.

Problem VII.

Given both Latitudes and the Course; to find the Distance Sailed and the Longitude come to.

Rule.

To the logarithmic secant of the course, add the logarithm of the difference of latitude; the sum, abating 10 in the index, will be the logarithm of the distance.—Then,

To the logarithmic tangent of the course, add the logarithm of the meridional difference of latitude; the sum, abating 10 in the index, will be the logarithm of the difference of longitude; which being applied to the longitude left by addition or subtraction, according as it is increasing or decreasing, the sum or difference will be the longitude come to.

Example.

A ship, from a place in latitude 3°4′ S., and longitude 174°20′ W., sailed N. W. by W. ½ W. until she was found, by observation, to be in latitude 2°54′ N.; required the distance sailed, and the longitude at which the ship arrived?

Lat. sailed from =	3° 4′ S.	Mer. parts = .	184.1 miles.
Lat. come to = .	2.54 N.	Mer. parts = .	174.1 miles.
Diff. of lat. = . .	5°58′ = 358 ms.	Mer. diff. lat. =	358.2 miles.

To find the Distance Sailed:—

Course = ...	5½ points.	Log. secant = . .	10.326613
Diff. of lat. . . .	358 miles.	Logarithm = . .	2.553883
Distance sailed =	759.4 miles.	Logarithm = . .	2.880496

To find the Difference of Longitude:—

Course = ...	5½ points.	Log. tang. = . .	10.272043
Merid. diff. lat. . .	358.2	Logarithm = . .	2.554126
Diff. of long. = . .	670.1 ms.	Logarithm = . .	2.826169

Long. sailed from = ...	174°20′ W.
Difference of long. made good 670 miles = .	11.10 W.
Sum = . .	185°30′ W.
Longitude come to = ...	174°30′ E.

Note.—The three last Problems comprehend all the cases that usually occur in the practical part of Mercator's sailing;—for the speculative cases, see pages from 236 to 248, inclusive.

Problem VIII.

To find the Course, Distance, Difference of Latitude, and Difference of Longitude made good upon compound Courses, and also the Bearing and Distance from a Ship to the Place to which she is bound, viz:—To make out a Day's Work at Sea.

Rule.

Make a Table of any convenient size, and divide it into six columns:—in the first of these place the several courses, taken from the log board (corrected for lee-way, if any, and also for variation), and in the second place their corresponding distances.—The third and fourth columns are to contain the differences of latitude, and, therefore, to be marked N. S. at top; and the fifth and sixth the departures, or meridian distances, which are to be marked at top, also, with the letters E. W.—Now,

Enter the general Traverse Table, and take out the difference of latitude and departure answering to each *corrected course* and distance, and place them in their respective columns:—then, the difference between the sums of the N. and S. columns will be the whole difference of latitude made good, of the same name with the greater; and the difference between the sums of the E. and W. columns will be the whole departure made good, of the same name with the greater term.

Remark.—The courses, taken from the log board, are to be corrected for variation, and lee-way, if any, in the following manner, viz.

If the variation be easterly, it is to be allowed to the right hand of the course steered by compass; but to the left hand if it be westerly.—And,

If the larboard tacks be aboard, the lee-way is to be allowed to the right hand of the course steered by compass; but, to the left hand if the starboard tacks be aboard.

To find the Course and Distance made good:—

From the logarithm of the departure, the index being increased by 10, subtract the logarithm of the difference of latitude; the remainder will be the logarithmic tangent of the course.—Then, To the logarithmic secant of the course, thus found, add the logarithm of the difference of latitude, and the sum, abating 10 in the index, will be the logarithm of the distance.

To find the Latitude in, by Account, or Dead Reckoning:—

If the difference of latitude, and the latitude of the place from which the ship's departure was taken, or the yesterday's latitude, be of the same name; their sum will be the latitude in, by account: but if of contrary names, their difference will be the latitude in, of the same name with the greater term.

To find the Difference of Longitude; and thence the Longitude come to:

To the logarithmic tangent of the course made good, add the logarithm of the meridional difference of latitude (by observation); the sum, abating 10 in the index, will be the logarithm of the difference of longitude.—Now, if the difference of longitude, and the longitude of the place from which the ship's departure was taken, or the yesterday's longitude be of the same name; their sum will be the longitude in, by account, when it does not exceed 180 degrees; otherwise it is to be taken from 360 degrees, and the remainder will be the longitude in, of a contrary name to that left:—but, if the difference of longitude, and the longitude left be of contrary names, their difference will be the longitude come to, of the same name with the greater term.

To find the Bearing and Distance of the Ship to the Port, or Place to which she is Bound:—

From the logarithm of the difference of longitude between the ship and the place to which she is bound, the index being increased by 10, subtract the logarithm of the meridional difference of latitude; the remainder will be the logarithmic tangent of the course. Then,—To the logarithmic secant of the course, thus found, add the logarithm of the difference of latitude, and the sum, rejecting radius, will be the logarithm of the distance.

Note.—The true bearing, or course thus found, may be reduced to the magnetic, or compass course, if necessary, by allowing the value of the variation to the right hand thereof if it be westerly; but, to the left hand, if easterly:—this being the *converse* of reducing the course steered by compass to the true course.

And this rule comprises the substance of that nautical operation, which is generally termed making out a day's work at sea.

Example 1.

A ship from Cape Espichell, in latitude 38°25′ N. and longitude 9°13′ W. bound for Porto Santo, in latitude 33°3′ N. and longitude 16°17′ W., by reason of contrary winds was obliged to sail upon the following courses: viz., (with the larboard tacks aboard,) W. by S. 56 miles; N. W. by W. 110 miles; W. N. W. 95 miles; (and then with the starboard tacks aboard,) S. by E. ½ E. 50 miles; S. by W. ¾ W. 103 miles; and S. S. W. 116 miles, when she was found by observation to be in latitude 34°17′ N. and longitude 13°42′ W.; the lee-way on each of the courses was about half a point; the variation was two points westerly on the three first courses, and 1¾ point on the three last; required the true course and distance made good; the latitude and longitude at which the ship arrived by account; and the direct course and distance between her true place, by observation, and the port to which she is bound?

Traverse Table.					
Corrected Courses.	Distances.	Difference of Latitude.		Departure.	
		N.	S.	E.	W.
S.W.b.W.½W.	56	—	26.4	—	49.4
W.byN.½N.	110	31.9	—	—	105.3
W.½N.	95	9.3	—	—	94.5
S.E.¼S.	50	—	37.0	33.6	—
S.½E.	103	—	102.5	10.1	—
S.¼E.	116	—	115.9	5.7	—
		41.2	281.8	49.4	249.2
			41.2		49.4
		Diff. lat.	240.6	Departure.	199.8

To find the Course made good:—

Departure = ... 199.8 miles. . . Logarithm = 2.300596
Difference of lat. = 240.6 miles. . . Logarithm = 2.381296

Course made good S. 39°42′25″ W. . . Log. tang. = 9.919300

To find the Distance made good:—

Course made good = S. 39°42′25″ W. . Log. sec. = 10.113892
Difference of lat. = 240.6 miles. . . Logarithm = 2.381296

Distance made good = 312.7 miles. . . Logarithm = 2.495188

Hence, the course made good is S. 39°42′25″ W. or S.W. ¾ S. nearly, and the distance 313 miles nearly.

To find the Latitude and Longitude come to by Account, or Dead Reckoning :—

Latitude sailed from 38°25′ N. . . . . 38°25′ N. M. pts.= 2500.1 ms.
Diff. of lat. made
good = 240.6 ms.=4° 1′ S.

Lat. come to by acc.=34°24′ N. By ob.=34°17′ N. M. pts.= 2192.0 ms.

Meridional difference of latitude by observation = ... 308.1 ms.

Meridional difference of lat. =	308.1 miles.	Logarithm =	2.488692
Course made good = . .	S. 39°42′25″ W.	Log. tang. =	9.919300
Diff. of long. made good =	4°16′ W. = 255.8 ms.	Log. =	2.407992
Longitude sailed from =	9.13 W.		
Long. come to by acct. =	13°29′ W.		

To find the Course and Distance from the Ship to her intended Port :—

Lat. of ship by ob.=34°17′ N.	Mer. pts. = 2192.0	Long. =13°42′ W.
Lat. of Porto Santo=33. 3 N.	Mer. pts. = 2103.1	Long. =16.17 W.
Diff. of lat. = . . 1°14′ = 74 miles.	Mer. diff. lat. 88.9	Diff. long. 2°35′ = 155 miles.

Difference of longitude =	155 miles.	Logarithm = . . .	2.190332
Mer. diff. latitude =	88.9 miles.	Logarithm = . . .	1.948902
Course = . . .	S. 60°9′49″ W.	Log. tang. . . .	10.241430
Course = ...	S. 60°9′49″ W.	Log. secant = . .	10.303185
Difference of latitude	74 miles. . .	Logarithm = . .	1.869232
Distance = . . .	148.7 miles.	Logarithm = . .	2.172417

Hence,—The course made good is S. 39°42′25″ W. or S.W. ¾ S. nearly.
Distance made good = ... 313 miles.
Latitude come to by account = ... 34°24′ N.
Latitude by observation = ... 34°17′ N.
Longitude come to by account = . . . 13°29′ W.
Longitude by observation = ... 13°42′ W.
Porto Santo bears S. 60°9′49″ W. or S.W. by W. ¼ W. nearly.
Distant. . . . 149 miles.

Note.—If the variation be one point and three-quarters west, the ship must steer W. b. S., by compass.

Example 2.

A ship from Port Royal, Jamaica, in latitude 17°58′ N., and longitude 76°53′ W., got under weigh for Hayti, St. Domingo, in latitude 18°30′ N., and longitude 69°49′ W., and sailed upon the following courses; viz.,— S. 40 miles, S.E. b. S. 97 miles, N. b. E. 72 miles, S.E. ½ S. 108 miles, N. b. E. ½ E. 114 miles, S.E. 126 miles, N.N.E. 86 miles; and then by observation was found to be in latitude 16°55′ N., and longitude 72°30′ W.; the lee-way on each of those courses was a quarter of a point (the wind being between S.E. b. E. ¼ E. and E. b. N. ½ N.), and the variation of the compass half a point easterly; required the true course and distance made good, the latitude and longitude at which the ship arrived by account, with the direct course and distance between her true place by observation and the port to which she is bound?

Traverse Table.

Corrected Courses.	Distances.	Difference of Latitude.		Departure.	
		N.	S.	E.	W.
S. ¾ W.	40	—	39.6	—	5.9
S.S.E. ¼ E.	97	—	87.7	41.5	—
N.b.E. ¼ E.	72	69.8	—	17.5	—
S.S.E. ¾ E.	108	—	92.6	55.5	—
N.b.E. ¾ E.	114	107.3	—	38.4	—
S.E. ¾ S.	126	—	101.2	75.1	—
N.N.E. ¼ E.	86	77.7	—	36.8	—
		254.8	321.1	264.8	5.9
			254.8	5.9	
		Diff. of Lat.	66.3	258.9 =	Departure.

To find the Course made good:—

Departure = ... 258.9 miles Log. = ... 2.413132
Difference of latitude = . . . 66.3 miles Log. = ... 1.821514

Course = ... S. 75°38′10″ E. Log. tangent = 10.591618

To find the Distance made good :—

Course made good = . . S. 75°38′10″ E. Log. secant = 10.605409
Difference of latitude = . . 66.3 miles Log. = . . 1.821514

Distance = ... 267.3 miles Log. = . . 2.426923

To find the Latitude and Longitude come to by Account, or Dead Reckoning :—

Lat. sailed from = 17°58′N. . . . 17°58′N. Mer. pts=1096.1 miles.
Diff. of lat. made
good 66.3 miles=1. 6 S.

Lat. come to by acc.16°52′N. By obs. 16°55′N. Mer. pts=1030.1 miles.

Meridional difference of latitude, by observation, = . . . 66.0 miles

Meridional difference of latitude = 66 miles Log. = . . 1.819544
Course made good = . . S. 75°38′10″ E. Log. tangent=10.591618

Difference of long. made good=4°18′ E.=257.7 miles Log.=2.411162
Longitude sailed from = . 76.53 W.

Longitude come to by account=72°35′ W.

To find the Course and Distance from the Ship to her intended Port :—

Lat. of ship by obs.16°55′N. Merid. pts=1030.1 Long. = 72°30′W.
Lat. of Hayti = 18.30 N. Merid. pts=1129.8 Long. = 69.49 W.

Diff. of latitude 1°35′ Mer. diff. lat. 99.7 Diff. long. 2°41′

= 95 miles. = 161 miles.

Difference of longitude = . . . 161 miles Log. = . 2.206826
Meridional difference of latitude = 99.7 miles Log. = . 1.998695

Course = ... N. 58°13′55″ E. Log. tang = 10.208131

Course = ... N. 58°13′55″ E. Log.secant=10.278617
Difference of latitude = ... 95 miles Log. = . 1.977724

Distance = ... 180.4 miles Log. = . 2.256341

Hence,—The course made good is S. 75°38′10″ E., or E. b. S. ¼ S. nearly.
Distance made good = ... 267½ miles.
Latitude come to, by account, = . 16°52′ north.
Latitude by observation = . . . 16°55′ north.
Longitude come to, by account, = 72°35′ west.
Longitude, by observation, = . . 72°30′ west.
The true course from the ship to Hayti is N. 58°13′55″ E., or N.E. b. E. ¾ E. nearly.
The course, by compass, is N.E. ¾ E.
And the distance 180½ miles nearly.

Note.—For the method of making out a day's work by *inspection*, see Problem IX., page 249.

OF THE LOG-BOOK.

A Log-Book is a true and correct register of all the various transactions which happen on board of a ship, whether at sea or in harbour: such as, coming to an anchor, getting under weigh, loosing or furling sails, mooring or unmooring, making or shortening sail, mustering at quarters or by divisions, exercising great guns and small arms, &c. &c. &c. This book should be a faithful transcript of the log-board.

The sea day, like the civil, begins at midnight in the Royal Navy, and ends at the midnight following: it is, however, divided into two parts, each consisting of 12 hours. The first 12 hours, or those contained between midnight and noon, are denoted by A.M., which signifies ante meridiem, or *before mid-day;* and the other 12 hours, or those from noon to midnight, are denoted by P.M., which signifies post meridiem, or *after mid-day*. The reckoning, however, is kept from noon to noon, the same as in the merchant service.

When a ship is bound to a distant port or place, the bearing and distance of that port or place must be previously computed, by Problem V., page 628. The bearing or true course, thus determined, must be reduced to the compass course, by applying the variation to the right hand thereof if it be westerly, but to the left hand if easterly:—(see Problem V., page 496). If islands, capes, or head-lands intervene, it will be necessary to find the several courses and distances between each successively; making proper allowance for the variation.

At the time of leaving the land, the bearing of some point or place is to be carefully observed, whose latitude and longitude are known; which, together with the estimated distance of the ship from such point or place, is to be noted down on the log-board. This is called taking a *departure.*

As the distance inferred from estimation is very susceptible of error, particularly in hazy weather, or when that distance is considerable, it will be advisable to make use of the following method in taking a *departure;* viz., Let the bearing of some well-known place be observed, and, when the ship has run a convenient distance, on a direct course, let the bearing of the same well-known place be again observed; then there will be a triangle formed, in which there is one side given: that is, the distance sailed between the times of observation, and all the angles, to find the distance between the ship and the place observed. This may be done by Problem I., Oblique Sailing, page 256; or it may be very readily determined by means of a good chart. In like manner may a departure be taken from a light-house at night.

In making out the first day's work after leaving the land, especial care must be taken, in setting down the bearing and distance of the departure in a traverse table, to make use of the opposite point of the compass to that bearing; and, also, to make due allowance for the variation. Thus, if the object from which the departure was taken bore N.E. b. E., and the variation of the compass be 2 points westerly, then the true course for the traverse Table is S.W. b. S.; abreast of which, in the proper column, is to be placed the estimated or computed distance.

The course steered, is indicated by the compass; the distance sailed, in a given time, is determined by the log-line and the half-minute or quarter-minute glass. In His Majesty's Royal Navy, the log is hove once in every hour; and so it is on board ships belonging to the East India Company.

The several courses and distances sailed during the interval of 24 hours, or from noon to noon, together with all the remarks and occurrences that are worthy of notice, are generally marked down with chalk on a board, painted black, called the log-board. This board is usually divided into six columns: the first column on the left hand contains the hours from noon to noon, viz., from noon to midnight, and then from midnight to noon; the second and third columns contain the knots and fathoms sailed every hour; the fourth contains the courses steered; the fifth the winds; and in the sixth the various remarks are written,—such as, the state of the weather, the sails set or taken in, the observations for ascertaining the ship's place, the variation of the compass, and whatever else may be deemed necessary. The log-board is transcribed every day at noon (under the direction of the Master,) into the log-book, which is divided into columns exactly in the same manner.

The form of the log-book which is now made use of in the Royal Navy, will be shown presently.

The courses steered must be corrected for the variation of the compass, and also for lee-way, if any. If the variation be westerly, it must be allowed to the left hand of the course steered; but if easterly, to the right hand thereof, in order to obtain the true course.—See Problem VI., page 497.

The lee-way is to be allowed to the right hand of the course steered, if the larboard tacks be on board; but to the left hand, if the starboard tacks be on board.

The variation of the compass should be determined twice a day (every morning and evening,) if possible. The method of doing this is shown in the several problems contained between pages 483 and 495.

With respect to the lee-way, its nature or effect may be thus explained:—

When a ship is close-hauled, and the wind blowing fresh, that part of the wind which acts upon the hull and rigging, together with a considerable part of the force which is exerted on the sails, tends to drive her immediately from the direction of the wind, or, as it is termed, to leeward. But since the bow of a ship exposes less surface to the water than her side, the resistance will be less in the first case than in the second; the velocity, therefore, in the direction of her head, will, in most cases, be greater than in the direction of her side; and the ship's real course will be between those two directions. Hence the angle contained between the line of the ship's apparent course and the line she actually describes through the water, is termed the *angle of lee-way*, or, simply, the *lee-way*.

The *angle of lee-way* that a ship makes may be very readily determined in the following manner; viz., Draw a semi-circle on the taffrail, with its diameter at right angles to the ship's keel, and its circumference divided into points and quarter-points; then let the angle be observed which is contained between the semi-diameter pointing right aft, or parallel to the keel, and that which points in the direction of the wake, and it will be the lee-way required. Or, after heaving the log, if the line (before it is drawn in) be applied to the centre of the semi-circle, the points and quarter-points contained between its direction and the fore and aft radius of the semi-circle will be the lee-way, as before.

Many writers on navigation have given rules for ascertaining the quantity of lee-way which a ship makes, independent of observation. These are as follow; viz.,

1. When a ship is *close-hauled*, has all her sails set, the water smooth, with a light breeze of wind, she is then supposed to make little or no lee-way.
2. Allow one point when the top-gallant-sails are handed.
3. Allow two points when under close-reefed top-sails.
4. Allow two points and a half when one top-sail is handed.
5. Allow three points and a half when both or the three top-sails are handed.
6. Allow four points when the fore-sail or fore-course is handed.
7. Allow five points when under the main-sail or main-course only.

8. Allow six points when under a balanced mizen.

9. Allow seven points when under bare poles.

As these rules depend entirely upon the quantity of sail set, without any regard to the model of the ship, or to the nature of the way in which she may be trimmed for sailing, it is evident that they are far from being general, and that they are, in reality, little more than mere probable conjectures. But since the accuracy of a ship's reckoning depends, in some measure, upon the truth of the lee-way, it ought to be deduced, at all times, from actual observation, as above directed; and then its value should be carefully noted down, in a separate column, on the log-board: so that all concerned may be thereby enabled to correct the courses steered, in making out their days' works at noon.

In very strong gales, with a contrary wind and a high sea, it is not prudent to attempt working to windward: in such cases, the grand object is, to avoid, as much as possible, losing ground, or being driven back. With this intention, it is customary to lay the ship to, under no more sail than may be barely sufficient to check that violent rolling which she would otherwise acquire, to the endangering of her masts, yards, and rigging. When a ship is brought to, the helm is kept about three parts alee, which brings her head gradually round to the wind. The force of this element having then very little power on the sails, the ship consequently loses her way through the water, which ceasing to act upon the rudder, her head falls off from the wind; the sail which she has set fills, and gives her fresh way through the water, which, acting on the rudder, brings her head again gradually round to the wind; and thus she obtains a kind of vibratory motion, coming up to the wind and falling off from it alternately.

Ships lie-to under different sails, according to circumstances; and one vessel will lie-to considerably better under some particular sail than another. But, in general, a close-reefed main-top-sail is, perhaps, the most eligible sail to lie-to under; because of its being nearly over the centre of motion, and, also, because of its elevated position, which renders it far less susceptible of being becalmed in the trough of the sea than either the courses or storm-stay-sails.

When a ship is lying-to, observe the points of the compass upon which she comes up and falls off, and take the middle point for her apparent course: to which let the variation and the lee-way be applied, and the true course will be obtained. Thus, suppose a ship lying-to under a close-reefed main-top-sail, with her larboard tacks on board, comes up S.S.W., and falls off to S.W. b. W.; then, allowing the variation to be 1½ point west, and the lee-way to be 2½ points, the course made good is S.W. ½ W.: for the middle point between S.S.W. and S.W. b. W. is S.W. ½ S.; to which, 1½ point westerly variation being allowed to the left, and 2½ points lee-way to the right, makes the true course S.W. ½ W.

The setting and drift of currents, with the heave and drift of the sea, should be set down as courses and distances upon the log-board: these are to be corrected for variation only.

The computation made from the several corrected courses, and their corresponding distances, is called a *day's work;* and the ship's place, deduced therefrom, is called her place by account, or dead reckoning.

If the course and distance made by a ship could be correctly ascertained, by means of the compass and the log, nothing more would be necessary in determining her true place at sea; for the absolute course and distance being known, the latitude and longitude could be readily computed, by Problem VI., page 629. But, in consequence of the irregularities to which the heaving of the log is subject, particularly during the night, with many unforeseen and unavoidable causes, such as sudden squalls, imperfect compasses, unequal care in the helms-man, inaccurate allowances for variation and lee-way, &c. &c., the latitude and longitude of the ship, as inferred from dead reckoning, will very seldom agree with the truth, or with those immediately deduced from celestial observation. In consequence of this discrepancy, several writers on navigation have proposed to apply a conjectural correction to the departure or meridian distance, in order to find the true longitude. Thus, if the course be near the meridian, the error is wholly attributed to the distance, and the departure is to be increased or diminished accordingly; if it be near a parallel, that is, near the east or west point of the compass, the course only is supposed to be erroneous; and if the course be towards the middle of the quadrant, viz., near four points, the assumption is that both course and distance are wrong. These corrections, being computed and applied according to the rules given by different authors, will generally place the ship upon different sides of her meridian by account: hence, since the corrections arising from these rules are evidently founded upon a vague kind of guess-work, they ought to be absolutely rejected.

When the latitude by account differs from that by observation, the log-line and half-minute glass should be carefully examined, and, if found erroneous, the distance sailed, as indicated thereby, should be corrected accordingly, by the Problems given for that purpose, between pages 272 and 276. If the corrected distance, thus found, with the course, does not produce a coincidence in the latitudes by account and observation, the mariner should then consider whether the variation has been properly determined and allowed upon the courses steered by compass; if not, these courses are to be again corrected; but no other alteration whatever should be made in them. If the latitudes by account and observation be still found to disagree, the navigator should next consider whether the ship's place has been affected by a current or by the heave of the sea, and allow for their course and drift to the best of his judgment. By carefully applying these corrections, a new difference of latitude and departure, and a new course and distance, will be obtained; which will, in general, produce an approximation in the latitudes: beyond this, no alteration whatever should be made in the departure with the view of finding the longitude by account.

However, since there are many mariners who, from long-established practice, are not willing to depart from the common system of correcting the dead reckoning by the rules laid down for that purpose in certain Epitomes of Navigation; and since these rules are exceedingly complicated, and admit of a *variety of cases*, the following general rule is given for the use and guidance of such persons, which reduces those various cases into one very concise method, and thus does away with the necessity of consulting several complex rules before the desired correction can be obtained.

A general Rule for correcting the Dead Reckoning:—

Augment the distance sailed by *two-thirds* of the difference between the latitude by account and that by observation, when the observed latitude is before or *ahead* of that by account; but diminish the distance sailed in the same proportion, when the observed latitude is *astern* or behind that by account. Then,

Enter the general Traverse Table with this corrected distance and the difference of latitude by observation, and find the corresponding departure. Now, with the departure, thus found, in a latitude column, and the middle latitude as a course, find the corresponding distance, and it will be the corrected difference of longitude.

Example 1.

Suppose a ship, from a place in latitude 47°49′ N. and longitude 9°29′ W., sailed S. 43° W. 160 miles, and then finds her latitude by account to be 45°54′ N., but by observation her true latitude is 45°39′ N.; required the longitude come to by account, or dead reckoning?

Solution.—The difference between the latitude by account and that by observation, is 15 miles; the two-thirds of which is 10 miles. Now, this being added to the distance sailed, because the observed latitude is before or *ahead* of that by account, makes the corrected distance = 170 miles: with this corrected distance and the difference of latitude by observation, viz., 2°10′ or 130 miles, the corresponding departure, in the general Traverse Table, is 109.3 miles. Then, with this departure, in a latitude column, and the middle latitude (between the latitude sailed from and that arrived at by observation), viz., 46°44′ as a course, the difference of longitude corresponding thereto, in a distance column at the top or bottom of the page, is 159 miles, or 2°39′ W.; which, being added to the longitude left, shows the longitude at which the ship arrived to be 12°8′ west.

Example 2.

Suppose a ship from Porto Santo, in latitude 33°3′ N. and longitude 16°17′ W., sailed N. 47° E. 210 miles, and then finds her latitude by account to be 35°26′ N., but by observation her true latitude is only 35°8′ N.; required the longitude come to by account, or dead reckoning?

Solution.—The difference between the latitude by account and that by observation, is 18 miles; the two-thirds of which is 12 miles. Now, this being subtracted from the distance sailed, because the observed latitude is *astern* or behind that by account, makes the corrected distance = 198 miles: with this corrected distance and the difference of latitude by observation, viz., 2°5′ or 125 miles, the corresponding departure, in the general Traverse Table, is 153.5 miles. Then, with this departure, in a latitude column, and the middle latitude (between the latitude sailed from and that come to by observation), viz., 34°5½′ as a course, the difference of longitude corresponding thereto, in a distance column at the top or bottom of the page, is 185 miles, or 3°5′ E.; which, being subtracted from the longitude left, shows the longitude at which the ship arrived to be 13°12′ west.

Remark.—Although the above general rule for correcting the dead reckoning is the most simple, and, perhaps the most accurate of any that have been as yet devised for that purpose, yet the author has frequently found, on making the land after a long voyage, that the longitude deduced therefrom *differed several degrees from the truth*: hence it is evident, notwithstanding the easy and specious feasibility of this method, that the prudent mariner will do well to be extremely cautious in applying it to practice; nor should he ever place any manner of faith in the longitude so deduced, particularly if he has been any considerable time from the land. From this it is manifest that the navigator should determine the longitude of his ship, as often as possible, both by the lunar observations and by a chronometer; and from the true longitude, thus found, the reckoning of this element is to be carried forward, in the same manner as that of the latitude, from the last observation. A separate account, however, should be kept of the longitude by dead reckoning: such account is not only very satisfactory, but it often proves highly useful as a reference; particularly in comparing the computed velocity and drift of a current with those deduced from actual experiment.

The following is the form of the log-book which is now used in His Majesty's Royal Navy, and from which we will make out a *practical day's work.*

Log-Book of His Majesty's Ship ———, *Wednesday, June 4th,* 1823.

H.	K.	F.	Courses.	Winds.	No. of Signals.	Remarks and Occurrences.
1						A.M. Moderate breezes and hazy weather.
2				S.E.		
3						
4						At 4^h 30^m employed washing decks.
5						
6						
7						At 7^h 30^m unmoored ship, and hove short on the best
8						bower.
9						At 8^h 40^m weighed and made sail.
10						
11						At noon light winds and hazy weather.
12				S.E.		

Course.	Distance.	Lat. by Account.	Latitude by Observ.	Long. by Account.	Long. by Lunar Observ.	Long. by Chron.	Variation at Noon.	Bearings and Dist. at Noon.
			34° 20′ S.				25° 20′ W.	Cape of Good Hope E b. S. 15 miles.

H.	K.	F.	Courses.	Winds.	No. of Signals.	Remarks and Occurrences.
1	5	4	N.W.b.W.	S.E.		P.M. Moderate breezes, with thick hazy weather.
2	5	4				
3	6	0	N.W.¾W.			At 2^h 30^m shook a reef out of the top-sails, and set
4	6	2				the top-mast and top-gallant studding-sails.
5	6	2				
6	6	2	N.W.¼W.	S.E.		At 5^h beat to quarters, and exercised great guns
7	7	6				and small arms.
8	8	0				
9	8	6	N.W.			At 8^h fresh breezes and cloudy weather.
10	9	0				
11	9	4				At midnight fresh breezes and clear weather.
12	9	6		S.E.		

Log-Book of His Majesty's Ship ———, *Thursday, June* 5*th,* 1823.

H.	K.	F.	Courses.	Winds.	No. of Signals.	Remarks and Occurrences.
1	9	4	N.W.	S.E.		A.M. Fresh breezes and clear weather.
2	9	4				
3	9	4				
4	9	6				At 4^h fine clear weather; employed washing decks.
5	9	6				
6	9	6	N.W.b.N.	S.E.		At 6^h 30^m set the lower studding-sails.
7	10	0				
8	10	0				At 8^h 40^m in lower studding-sails.
9	9	6				
10	9	6				At 10^h 30^m mustered by divisions, and inspected the
11	9	4				people's clothing.
12	9	2		S.E.		At noon fresh breezes and fine clear weather.

Course.	Distance.	Lat. by Account.	Latitude by Observ.	Long. by Account.	Long. by Lunar Observ.	Long. by Chron.	Variation at Noon.	Bearings and Dist. at Noon.
N. 69 W.	214 miles.	33° 6′ S.	33° 4′ 30″ S.	14° 24′ E.	14° 28′ E.	14° 24′ E.	23° 10′ W.	St. Helena N. 46° 51′ W., dist. 1565 ms.

Note.—The departure is taken from the Cape of Good Hope; and as this place bore, at noon, E. b. S. from the ship, distant 15 miles, the compass bearing or course of the ship from the Cape was, therefore, W. b. N. Now, the variation, 2¼ points west, being allowed to the left-hand of W. b. N., shows the true bearing or course to be W. b. S. ¾ S. The other courses are, in like manner, to be corrected for variation; but since the value of this element is not the same at both noons, it is advisable to allow 25°20′, or 2¼ points west, on the courses in the first 12 hours, or between noon and midnight, and 23°10′, or 2 points west, on the courses in the other 12 hours, or between midnight and noon; then, these corrected courses, with their respective distances, being inserted in a Traverse Table, after the following manner, the difference of latitude and departure corresponding thereto, with the course and distance made good, may be readily determined by calculation, agreeably to the rule given in Problem VIII., page 633; or, perhaps more readily, by the general Traverse Table.—See Problem II., page 108.

TRAVERSE TABLE.

Corrected Courses.	Distances.	Difference of Latitude. N.	Difference of Latitude. S.	Departure. E.	Departure. W.
W.b.S.¾S.	15	—	3.6	—	14.6
W.¾N.	11	1.6	—	—	10.9
W.b.N.¼N.	18	4.4	—	—	17.5
W.b.N.½N.	22	6.4	—	—	21.1
W.b.N.¾N.	37	12.5	—	—	34.8
W.N.W.	48	18.4	—	—	44.3
N.W.b.W.	68	37.8	—	—	56.5
		81.1	3.6	—	199.7
		3.6			0.0
	Diff. of lat.	77.5		Departure =	199.7

To find the Course and Distance made good:—

The difference of latitude 77.5, and the departure 199.7, are found to agree nearest abreast of 69 degrees, and under or over 214 miles distance.

Hence the course made good is N. 69° W., or W. b. N. ¾ N. nearly, and the distance 214 miles.

To find the Latitude and Longitude come to by Account :—

Lat. Cape Good Hope 34°23′40″ S. . 34°23′40″ S. Mer. pts.=2200.1
Diff. Lat. 77.5 ms. . . 1.17.30 N.

Lat. in by acc. = . 33° 6′10″S. By ob.33° 4′30″ S. Mr. ps.=2104.9

Meridional difference of latitude, by observation, = ... 95.2 [miles.

Meridional difference of latitude = 95.2 miles Log. = . . . 1.978637
Course made good = ... N. 69° W. Log. tang. = 10.415823

Diff. of long. made good = . 4° 8′ W.=248 ms. Log.=2.394460
Long. of C. of Good Hope = 18°32′15″ E.

Long. come to by account = 14°24′15″ E.

To find the Course and Distance from the Ship to St. Helena :

Lat. of ship by obs. 33° 4′ S. Mer. pts. = 2104.3 Long.=14°28′ E.
Lat. of St. Helena 15.55 S. Mer. pts. = 967.5 Long.= 5.44 W.

Diff. of lat. = . 17° 9′ Mer. diff. lat.= 1136.8 Diff. long. 20°12′
= 1029 miles. = 1212 miles.

Difference of longitude = 1212 miles. Log. = . . . 3.083503
Merid. diff. lat. = . . . 1136.8 miles. Log. = . . . 3.055684

Course = ... N. 46°51′ W. Log. tang. = . 10.027819

Course = ... N. 46°51′ W. Log. secant = 10.165001
Diff. of lat. = . . . 1029 miles. Log. = . . . 3.012415

Distance = ... 1505 miles. Log. = . . . 3.177416

Hence,—The course made good is N. 69° W. or W. by N. ¾ N. nearly.
Distance made good = . . . 214 miles.
Latitude come to by account = 33° 6′10″ S.
Latitude by observation = . . 33. 4.30 S.
Longitude come to by account = 14. 24. 15 E.
Longitude by lunar observation = 14. 28. 0 E.

Longitude by chronometer = . . 14. 24. 0 E.
Variation at noon . . . . . . 23. 10. 0 W.*
St. Helena bears N. 46°51' W. or N. W. ¼ W. nearly, independent of variation.
Distant . . . . 1505 miles.

OF THE MEASURE OF A KNOT ON THE LOG LINE.

It has been remarked, page 272, in the introduction to the Problems for correcting the distance sailed on account of any errors that may be discovered in the log line and half-minute glass, that the distance between any two adjacent knots on the log line should bear the same proportion to a nautical mile that half a minute does to an hour, viz. *the one hundred and twentieth part;* that a nautical mile contains 6080 feet; and that this number divided by 120, gives the true measure of a knot, viz. 50 feet and 8 inches.—But, since the young navigator may be desirous of being made acquainted with the principles upon which this measure has been determined, the following considerations are, therefore, submitted to his attention; which, besides satisfying him in that particular, may do something towards giving him a just idea of the true figure of the earth;—and, without which idea he can never clearly comprehend the principles upon which the art of navigation is founded.

The earth is a planet, and the next, in the solar system, above Venus.—Our senses assure us of its opacity;—and that it is of a globular or spherical figure will appear evident from the arguments which follow:—

A lunar eclipse is occasioned by the moon's passing through the earth's shadow; and since this shadow, when projected on the lunar disc, is observed to be always circular in every different position of the earth, it necessarily follows that the earth, which casts the shadow, must be spherical, since nothing but a sphere, when turned in various positions with respect to a luminous body, can project a circular shadow.—Again,

A lunar eclipse is observed sooner by those who live eastward than by those who live westward; the difference of time being always proportional to the difference of longitude between the places of observation.

Now, if the earth were an extended plane, as the primitive fathers asserted, the eclipse would happen at the same instant in all places:—but this is so far from being the case, that the inhabitants of Jamaica will not

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* The variation of the compass may be very readily determined at noon (sufficiently correctly for nautical purposes,) by the second part of the Rule to Problem IV. page 494, reading *sun* instead of star or planet.—see note at bottom of page 495.

see an eclipse of the moon until about five hours after it takes place at Greenwich;—therefore the figure of the earth must be spherical, or very nearly so.

If the earth were an extended plane, the meridian zenith distance of any one fixed star would be the same in all parts of the world; because the measure of the earth's diameter bears no more proportion to the immeasurable distance of the nearest fixed star than an indivisible point does to the diameter of the earth.—But, since the meridian zenith distance of the same fixed star is found to differ with the latitude, the difference in the zenith distance being always proportional to the intercepted arch of the meridian; and since it is the known property of a curve that the arches are proportional to their correspondent angles, therefore the surface of the earth and sea is of a curvilinear form.—Hence the earth must be of a spherical figure.

The earth has been circumnavigated by many persons, at different periods, who, by sailing in a westerly direction, allowance being made for promontories, &c. arrived at the place whence they sailed.—Hence, the earth must be either of a cylindrical, or a spherical figure;—but that it is not of a cylindrical figure will appear obvious by considering that the difference of longitude and meridional distance between two places would, on the cylindrical hypothesis, be equal;—whereas, experience and actual observation demonstrate that the very reverse of this takes place:—therefore the earth must be of a spherical form from west to east.

If a ship in north latitude sails southerly, the north polar star will be found gradually to decrease in altitude till the vessel reaches the Equator; at which place the star will be seen immersed in the horizon.—After crossing the Equator, and as the ship advances in the southern hemisphere, the stars in the neighbourhood of the south celestial pole will be seen gradually emerging from the southern horizon, and increasing in altitude, whilst those about the north celestial pole will be entirely lost sight of; being hid below the horizon:—hence the earth is spherical from north to south; but it is also spherical from west to east, as appears from its circumnavigation; therefore the figure of the earth is that of a sphere.

When two distant ships are approaching each other, at sea, the royals and top-gallant sails only of each are visible at first; the lower sails and hulls being concealed by the convex surface of the water:—but as they draw nearer towards each other, the parts that were so concealed by the convexity of the sea's surface, will be seen to rise gradually above the horizon.—Now, if the sea were an extended plane, the hulls or bodies of the ships would be the first parts seen; and because they are the largest, they would, evidently, be seen at the greatest distance; nor would the small sails near the masts' heads be visible until the approach of the ships brought them considerably nearer.

In making the land the most elevated parts are first seen, such as mountains, &c.; then tops of light houses and steeples, and shortly afterwards the coast, or beach:—this plainly demonstrates that the surface of the sea is convex.

The sun is observed sooner at rising and later at setting by a person at the mast-head of a ship than by one on deck; and so is the moon and all other celestial objects.—These phenomena evidently arise from the spherical figure of the earth; and are, therefore, most convincing and satisfactory proofs of its globularity.

Again.—The continual presence of the sun, above the horizon, during the space of several months in the neighbourhood of one terrestrial pole, while at a place equally distant from the other, he is as long absent, affords another convincing proof that the earth is of a spherical figure.

The spherical figure of the earth may be also inferred from the method of *levelling*, or the art of conveying water from one place to another;—for, in this operation it is always found necessary to make an allowance between the true and the apparent levels on account of the rotundity of the earth; the true level being a curve line which falls below the straight line of apparent level about 8 inches in 1 mile; 32 inches in 2 miles; 128 inches in 4 miles, &c., the curvature always augmenting in proportion to the square of the distance. See Problem X., between pages 545 and 547.

Finally,—All the planets are observed to be of a spherical figure; and since the earth is a planet, subject to the same laws, and revolving round the sun in the same manner as the other planets, it must, therefore, by analogy, be also spherical.

The irregularities on the earth's surface, occasioned by mountains and vallies, are very inconsiderable compared with its magnitude; and take off no more from its actual rotundity than the little risings on the coat of an orange do from the rotundity of that fruit:—for the highest eminence or mountain bears a less proportion to the magnitude of the earth than the smallest grain of sand does to an 18-inch globe.—Thus,

The summit of Chimboraço, one of the Andes Mountains, and the highest in the known world, is only 20280 feet above the level of the sea, or not quite 4 miles in perpendicular height.—Now, the radius of the earth is 20902200 feet, and that of an 18-inch globe 9 inches;—hence, by the rule of proportion, as 20902200 feet : 20902200 feet + 20280 feet = 20922480 feet :: 9 inches to 9.0087 inches; from which deduct 9 inches (the radius of the artificial globe,) and the remainder 0.0087 is the relative elevation of Mount Chimboraço on an 18-inch globe; and as this is scarcely *the one hundred and fifteenth part* of an inch, it is, therefore, considerably less than a common grain of sand.—Hence it is evident that the highest mountains, and deepest vallies, take little or nothing from the earth's rotundity.

Although when speaking of the earth in general terms, it may be considered as a globe, or sphere; yet, in strictness it is not a perfect sphere, but rather an oblate spheroid; which is a solid generated by the revolution of a semi-ellipse about its shorter axis or diameter;—and actual admeasurements, in sundry places, have clearly proved that the polar axis, or diameter is about 35 miles less than the equatorial diameter.—However, since the earth differs so very little from a globe or sphere, it may, therefore, be very safely considered as being perfectly spherical in all nautical calculations whatever.

The spherical figure of the earth being thus satisfactorily established, its magnitude may be determined by measuring a small portion of a meridian, and observing the zenith distances of one or more stars at the extreme stations; then, the difference between the zenith distances of the same star gives the correspondent celestial arch.—Now,

As the celestial arch, thus found, is to the measured or intercepted portion of the meridian; so is one degree, to its absolute length in the same measure in which the portion of the meridian was taken.

In this manner the celestial arch of one degree has been found to contain 69.093 English miles; and since the earth's circumference, like that of all other spheres, contains 360 degrees; therefore 360 degrees × 69.093 miles = 24873.48, is the true measure of the earth's circumference in English miles.—Hence, its diameter is 7917½ miles, English measure.

Now, since the nautical arch is, in every respect, equal to the celestial arch, the length of a degree in the one being precisely equal to the length of a degree in the other, each containing 60 geographical miles; and since the measure of a degree of this arch in English miles, is 69.093, or 364815 English feet;—therefore 364815 feet ÷ 60 miles = 6080 feet; which, evidently, is the true length of a nautical mile, expressed in English measure.—And, if 6080 feet be divided by 120 (the number of half minutes in an hour,) the quotient 50 feet and 8 inches will be the true measure of a knot.—And, hence the principles upon which the measure of a knot upon the log line has been determined.

But, because it is safest to have the reckoning a-head of the ship, 48 feet, or 8 fathoms are, therefore, commonly allowed between every two adjacent knots on the log line:—and this measure is to correspond to a glass running 30 seconds; or, rather 29½ seconds, so as to make up for any time that may be unavoidably lost in the act of turning the glass.

Remark.—The instruments made use of for measuring angles at sea, and for ascertaining the latitude and longitude, are quadrants, sextants, and reflecting circles. Since, however, space cannot be afforded in this work for giving particular descriptions of these instruments, and the manner of adjusting and using them; the reader is, therefore, respectfully referred to an ocular examination of the instruments, and to a few explanatory remarks from some person practically acquainted with the various uses to which they may, or can be applied.—A few hours practical instruction will convey more real information to a person on these subjects, than if he were to spend a whole year in poring over the voluminous descriptions which have been published, by different authors, relative to the use of the quadrant and sextant.

Instead, therefore, of wasting time and paper with descriptions that may be well omitted, we will here endeavour to describe that which is of far more importance to the practical navigator, viz.

The true Method of finding the Index Error of a Sextant, so as to guard against the Errors arising from the Flexibility and the Friction of the Index Bar.

The customary method of finding the index error of a quadrant or sextant (as directed by writers on the use of these instruments,) is by measuring the vertical diameter of the sun to the right and left of 0 on the arch, with a motion of the index in contrary directions (that is, by bringing the reflected image to touch the lower and upper limbs of the direct object alternately), and then taking half the difference of those measures for the index error of the instrument.—This method, it must be observed, is very far from being correct; because it is the horizontal diameter of the sun, and not its vertical diameter, that should be measured; for while the former remains invariably the same, the latter is subject to continual alterations owing to the effects of atmospherical refraction, as will appear evident by an inspection of the last column of Table V.—Moreover, since the index is not an inflexible bar, and since it does not turn upon its centre without suffering some slight degree of friction; it is therefore evident that the measure of the sun's diameter taken by the progressive motion of the index will, in most cases, be *more* than the truth: whilst that taken by the contrary or retrogressive motion will, in general, be *less* than the truth:—hence, the index error established upon the above principles must frequently mislead the mariner by rendering inaccurate what, otherwise, might be a very correct observation. And this accounts for the result of the evening observations, taken on shore by means of an artificial horizon, so very seldom agreeing with the result of those taken in the morning; even though all imaginable care be used, and though the observer keeps the same plane and roof of the horizon directed to him during the time of both observations.

Now, to guard against the errors arising from the bending and the friction of the index bar, as well as that proceeding from the contraction of the sun's vertical diameter; let the following observations be attended to, in finding the index error of a quadrant or sextant, and the conjoint effect of those errors will be obviated.

First.—To find the Error for a Progressive Motion of the Index :—

Screw the inverting telescope into its place. Slack the index. Turn the tangent screw *backward* to nearly as far as it will go. Put the nonius of the index about 1°15′ to the *right* of 0 on the arch, and then fasten the index sufficiently tight for observation.—Hold the sextant so that its plane may be parallel to the horizontal diameter of the sun: direct the sight to that object, and turn the tangent screw *forward* until the limbs of the sun seen by reflection and direct vision make a perfect contact.—Note down the angle and it will express the measure of the sun's diameter to the right of 0 on the arch.—Direct the sight again to the sun, and turn the tangent screw *still forward* until the opposite limbs are in perfect contact: note down the angle, and it will be the measure of the sun's diameter to the left of 0 on the arch.—Now, if both measures of the diameter are the same, there is no error in the angles shown by the progressive motion of the index; but if those measures do not correspond, half their difference is to be taken as the index error of the instrument, which error will be additive when the diameter measured to the right of 0 exceeds that measured to the left; otherwise, subtractive.—Then, this error is to be considered as a constant quantity (so long as the instrument does not meet with any accident), and to be applied to all *increasing angles*, either of altitude or distance, which may be taken by the progressive motion of the index.

Again.—To find the Error for a Retrogressive Motion of the Index :—

Slack the index. Turn the tangent screw *forward* to nearly as far as it will go. Put the nonius of the index about 1°15′ to the *left* of 0 on the arch, and then fasten the index sufficiently tight for observation.—Hold the sextant as before; direct the sight to the sun, and turn the tangent screw *backward* until the limbs of the sun seen by reflection and direct vision make a perfect contact:—note down the angle, and it will express the measure of the sun's diameter to the left of 0 on the arch.—Direct the sight again to the sun, and turn the tangent screw *still backward* until the opposite limbs are in perfect contact; read off the angle, and it will be the the measure of the sun's diameter to the right of 0 on the arch.—Now, if both measures of the diameter are the same, there is no error in the angles shown by the retrogressive motion of the index: but if those measures do not correspond, half their difference is to be taken as the index error of the instrument; which error will be additive when the diameter measured to the right of 0 exceeds that measured to the left; otherwise, subtractive.

Then, this error is to be considered as a constant quantity (so long as the instrument does not meet with any accident), and to be applied to all *decreasing angles*, either of altitude or distance, which may be taken by the backward or retrogressive motion of the index.

Hence it is very probable that *two errors* may be established for the same instrument; the one for increasing, and the other for decreasing angles. The true values of those errors should be noted down (for the future guidance of the observer,) with a black-lead pencil on the inside of his sextant case in the following manner, viz.:—

Error for the forward or progressive motion of the index 0′ 10″ subtractive.

Error for the backward or retrogressive motion of the index 1′ 40″ additive.

Or whatever the errors may be.

And thus the correct values of the index error will be properly determined, whilst the errors arising from the spring and the friction of the bar, together with that proceeding from the contraction of the sun's vertical diameter will be all safely provided against.

OF TAKING THE ALTITUDE OF A CELESTIAL OBJECT BY MEANS OF AN ARTIFICIAL HORIZON.

Since the generality of nautical persons do not appear to be sufficiently acquainted with the manner of applying the necessary corrections to angles of altitude taken by means of an artificial horizon; the author is, therefore, induced to make a few observations touching the direct application of those corrections; in doing which some hints will be thrown out for the guidance of young observers, relative to the nature and use of the astronomical instrument now under consideration.

Of the Artificial Horizon.

In settling the positions of places in-land in an astronomical manner, or in ascertaining the error and the rate of a chronometer on shore where there is not an open and commanding view of the sea horizon, the observer must, in all such cases, have recourse to an artificial horizon for the purpose of taking the necessary angles of altitude.

Although there is a great variety of artificial horizons now extant, yet, for the sake of conciseness, I shall only touch upon the two that are in my own possession.—The first of these consists of a plane speculum, or polished plate of glass (4 inches long by 3 inches broad,) fixed in a brass frame, and standing upon three adjusting screws: by means of these and a spirit level, placed in different positions on its surface, it may be made perfectly parallel to the plane of the horizon: observing that the adjusting screws are to be turned until the air-bubble rests in the middle of the spirit level on the surface of the speculum.—The other is the common, or quicksilver horizon;—this simply consists of a small wooden trough, about half an inch deep, 3½ inches long, and 2½ inches broad;—into this trough a few pounds of mercury or quicksilver are poured; the surface of which assumes when settled, agreeably to the nature of fluids, an exact horizontal plane.—To prevent the mercury from being ruffled or agitated by the action of the wind, a roof is placed over it, in which are fixed two plates of glass, the two sides of each plate being ground mathematically plane and parallel to one another:—And, of all artificial horizons an instrument of this description is the very best that can be employed in taking the altitudes of the heavenly bodies.

Of the Use of the Artificial Horizon; that is, to observe the Altitude of the Sun, or other Celestial Object, with a Sextant, and an Artificial Horizon.

In taking the altitude of the sun, or other luminary, the observer is to place his artificial horizon betwixt him and the object selected for observation; and at such a convenient distance as to see the image of that object reflected from the middle of the quicksilver as well as the real object in the heavens:—then, having screwed the plain tube, or the natural telescope of the sextant into its place in the socket; and placed one or two of the dark screens, according to the brightness of the sun, to intervene on each side of the horizon glass; the lower limb of the reflected image of the sun, as seen through the erect or natural telescope, is to be brought into contact with the upper limb of the image reflected from the artificial horizon:—but, if the altitude of the upper limb of the object be required, it must be brought into contact with the lower limb of the image as seen in the artificial horizon.—Now, the angle on the arch of the sextant being read off, and the index error, if any, applied to it, the result will be the double of the sun's, or other object's altitude above the horizontal plane: to the half of which, if the object be the sun, let the semi-diameter, refraction and parallax be applied, and the true central altitude will be obtained.

Remarks.

Since neither the plain tube, nor the natural or erect telescope can be depended upon in taking observations when rigorous exactness is required;

the inverting telescope should, therefore, be invariably made use of in all cases where angles of altitude are to be measured with astronomical precision :—and here, perhaps, it may not be unnecessary to state that when the inverting telescope is used, the lower limb of the sun, or moon, will appear to be the upper limb, and conversely.—Hence, in observing the altitude of the lower limb of the sun or moon, the *apparent upper limb of the object*, as seen in the horizon glass through the inverting telescope, is to be brought into contact with the *lower limb of the image in the artificial horizon* :—in this case the reflected image in the artificial horizon will appear to be uppermost.—Again, in observing the altitude of the upper limb of the sun or moon, the *apparent lower limb of the object*, as seen in the horizon glass of the sextant though the inverting telescope, is to be brought into contact with the *upper limb of the image in the artificial horizon* :—in this case the reflected image in the artificial horizon will appear to be undermost.

If an observer be placed as remote from, or as near to, an artificial horizon as possible, the rays of light passing from the sun or other celestial object to his eye, and from that object to the surface of the artificial horizon, will, on account of the immense distance of such object from the earth, be physically equal and parallel in every respect to each other :—hence, it is easy to perceive that it is immaterial whether the artificial horizon be placed high or low, remote or near with respect to the observer, provided he can but see the object's reflected image therein.

When an angle of altitude is taken by means of an artificial horizon, its measure on the limb of the sextant will always be the double of the true value thereof above the horizontal plane :—this will appear evident by considering that if a person places himself at any distance before a plane mirror, or common looking-glass, his reflected image will appear just as far behind such looking-glass as he is before it :—and, upon this simple principle it is that the reflected image of the sun, or other object, will appear to be as far below the surface of the artificial horizon as the real object is above it ;—but since the limb of the real object, as reflected from the index glass of the sextant, is to be brought into contact with that of the image apparently reflected below the surface of the artificial horizon, it is therefore manifest that the contained angle, as expressed on the arch of the sextant, must be equal to twice the measure of the observed angle of altitude above the plane of the horizon :—and from this we may readily perceive that angles of altitude taken in the above manner are not affected by the angle of horizontal depression, commonly called "the dip of the horizon." *

* See page 387.

Now, the double angle of altitude being thus obtained, the true altitude of the object is to be deduced therefrom in the following manner, viz. :—

First.—To apply the Corrections when the Sun is observed :—

Correct the observed angle for the index error of the sextant, if any ;—to the half of which apply the sun's semi-diameter by addition if the lower limb be observed, but by subtraction if it be the upper limb, and the sun's apparent central altitude will be obtained; from which let the difference between the refraction and parallax corresponding thereto be subtracted, and the remainder will be the sun's true central altitude.

Second.—To apply the Corrections when the Moon is observed :—

Find the moon's apparent central altitude in the same manner as if it were the sun that was under consideration; observing to correct her semi-diameter by the augmentation contained in Table IV.;—then, to the apparent altitude, thus found, let the correction in Table XVIII. be added, and the sum will be the true altitude of the moon's centre.

Third.—To apply the Corrections when a fixed Star is observed :—

Correct the observed angle for the index error of the sextant, if any; from the half of which subtract the refraction corresponding thereto, and the remainder will be the star's true altitude.

Example 1.

Let the measure of the observed angle between the lower limb of the sun reflected from the index glass of a sextant, and the upper limb thereof reflected from an artificial horizon be 103°14′40″; the index error of the sextant 3′10″ additive, and the sun's semi-diameter 16′18″; required the true altitude of the sun's centre?

Measure of the observed angle = ...	103°14′40″
Index error = ...	+ 3.40
Corrected observed angle = ...	103°18′20″
The half of which is the correct observed altitude of the sun's lower limb above the plane of the hor. =	51°39′10″
Sun's semi-diameter = ...	+ 16.18
Apparent altitude of the sun's centre = ...	51°55′28″
Refraction answering to ditto = 0′44″ } difference =	
Parallax, ditto ditto 0. 5 }	− 0.39
Sun's true central altitude = ...	51°54′49″

Example 2.

Let the measure of the observed angle between the upper limb of the moon reflected from the index glass of a sextant and the lower limb thereof reflected from an artificial horizon be 39°50′20″; the index error of the sextant 1′50″ subtractive, the moon's semi-diameter 14′46″, and her horizontal parallax 54′13″; required the true altitude of the moon's centre?

Measure of the observed angle = ...	39°50′20″
Index error = ...	− 1.50
Corrected observed angle = ...	39°48′30″
The half of which is the correct observed altitude of the moon's upper limb above the plane of the horizon =	19°54′15″
Moon's semi-diameter = . . 14′46″ ; Augmentation of do. Tab. IV. = 0. 5 } Sum = . . .	− 14.51
Apparent altitude of the moon's centre = ...	19°39′24″
Correction of ditto, Table XVIII. = ...	+ 48.25
Moon's true central altitude = ...	20°27′49″

Example 3.

Let the measure of the observed angle between the centre of a fixed star reflected from the index glass of a sextant and the centre thereof reflected from an artificial horizon be 71°16′10″, and the index error of the sextant 2′30″ subtractive; required the true altitude of the star?

Measure of the observed angle = ...	71°16′10″
Index error = ...	− 2.30
Corrected observed angle = ...	71°13′40″
The half of which is the correct observed altitude of the star's centre above the plane of the horizon = . .	35.36.50
Refraction corresponding to ditto = ...	− 1.19
True altitude of the star = ...	35°35′31″

Note.—When the altitude of a celestial object exceeds 60 degrees, it cannot be taken by means of a sextant and an artificial horizon; because, in this case, the measure of the double angle of altitude will exceed the limits of the graduated arch of the former instrument.

Remarks.

1. In observing equal altitudes by means of an artificial horizon, or in taking a continued series of altitudes for the purpose of determining the error and the rate of a chronometer; it will be essentially necessary to keep the same plane of the glass roof of the horizon towards the observer in each observation; so that in the event of there being any trifling defect in the parallelism of the surfaces of the two plates of polished glass, which form the roof of the horizon, the error arising therefrom may equally affect each observed altitude. To make certain of always having the same side of the roof next the observer, it will be advisable to make a small mark in the wooden part thereof: then, this mark being kept towards the observer, in every observation, the altitudes will thus be prevented from being unequally affected by any want of parallelism that may chance to be in the planes of the glass part of the roof.

2.—In calm weather the altitudes may be taken by reflection from the quicksilver without making use of the glass roof:—in like manner they may be taken, during such weather, by reflection from a bason of water; or, by reflection from a cup of tar, treacle, oil, or other fluid and viscous substance.

3.—Mariners frequently supply themselves with, what may be termed, a *home-made*, or *ship-built* artificial horizon; the quicksilver in which they shelter under a roof formed by two squares of the thick glass with which ships are usually furnished:—this is, to say the least of it, a poor substitute:—it is vainly endeavouring to accomplish that, by means of a couple of squares of common glass, which can scarcely be effected by the most highly-finished and parallel planes that can possibly be produced by the labour and ingenuity of the most eminent optician, or mathematical instrument maker;—indeed, it is a most absurd and mistaken contrivance; and a pertinacity in its use betrays an evident deficiency of useful knowledge on the part of the proprietor:—for, since the surfaces of those squares are not rendered mathematically accurate by being ground perfectly plane and parallel to one another, they cannot possibly refract and reflect the rays of light in an exact uniform manner:—hence, the angle of incidence will not be equal to the angle of reflection; and thus the angles of altitude observed in such a *make-shift and defective* horizon must and will be always *erroneous.*

A NEW AND CORRECT METHOD OF FINDING THE LONGITUDE ON SHORE, AND, IN SOME CASES, AT SEA;

Which, besides being remarkably simple, will be found equally as strict as the common Method by the Lunar Distances, and, at the same time, considerably more practicable than that Method: for most Mariners must be aware of how extremely difficult it is to take an accurate Lunar Observation at Sea, particularly when the Ship rolls rapidly, or pitches with any degree of violence; but few find any difficulty whatever in measuring the Altitude of a Celestial Object, provided the Horizon be sufficiently clear;—and, these points being premised, we will now proceed to the Solution of the following

Problem.

Given the Latitude of a Place or Ship, the observed Altitude of the Moon's well-defined Limb, and the apparent Time of Observation: to find the Longitude of that Place or Ship.

Rule.

To the apparent time of observation (always reckoned from the preceding noon), *add* the longitude by account, in time, if it be west, or *subtract* it if east, and the sum or remainder will be the estimated time at Greenwich; to which let the sun's right ascension, at the noon preceding the Greenwich time, be most carefully reduced; and let the moon's declination, at the period preceding the Greenwich time, be also carefully reduced to the same time.

To the sun's reduced right ascension, add the apparent time of observation (rigidly determined); and the sum (abating 24 hours, if necessary,) will be the right ascension of the meridian; which convert into motion or degrees.

Reduce the observed altitude of that part of the moon's well-defined limb which is either nearest to or farthest from the horizon, according as the lower or upper limb may be observed, to the true central altitude, by Problem XV., page 323. Then, with the moon's true central altitude, thus found, her corrected declination, and the given latitude, compute her angular distance from the meridian. Now, if the moon were in the eastern hemisphere at the time of observation, let her angular distance from the meridian be added to the right ascension of the meridian; and the sum (diminished by 360 degrees, should it exceed this quantity,) will be the moon's correct right ascension; but if she were in the western hemisphere, her angular distance is to be subtracted from the right ascension of the meridian (increased by 360 degrees, if necessary): the remainder will be the moon's correct right ascension at the time and place of observation.

With the moon's correct right ascension, thus found, enter page VI. of the month in the Nautical Almanac, opposite to the given day, or to that which immediately precedes or follows it, and take out the next less and the next greater tabular right ascensions; find their difference, and find, also, the difference between the computed and the next less tabular right ascension. Now, from the proportional logarithm of the last difference, subtract the proportional logarithm of the first difference: the remainder will be the proportional logarithm of a portion of time, which being added to the hour standing over the least tabular right ascension, the sum will be the apparent time at Greenwich; the difference between which and the apparent time of observation, will be the required longitude in time;—east, if the time at ship or place be the greatest; otherwise, west.

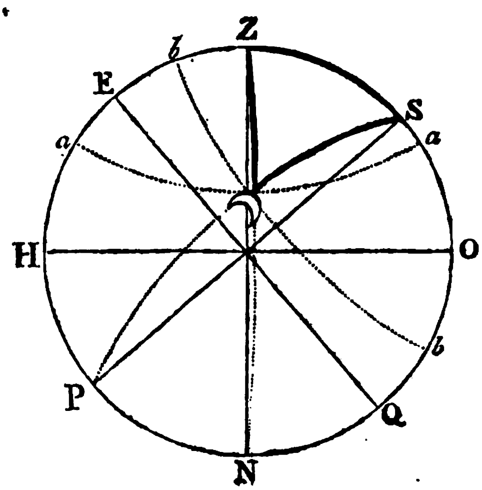
Note.—The above rule is made out on the assumption that the moon's place in right ascension and declination is given in page VI. of the month in the Nautical Almanac, agreeably to the form exhibited in the page at the end of this problem. It is evident that this rule may be very considerably contracted, but the author gives it thus in detail with the view of making himself the more clearly understood,—so that the reader may not mistake his meaning.

Example 1.

September 8th, 1826, in latitude 40°27′30″ south, and longitude, by account, 30°40′ west, at 1h10m17s apparent time, the true altitude of the moon's centre, east of the meridian, was 31°55′23″; required the true longitude of the place of observation?

Illustration.

In the annexed diagram (projected stereographically upon the plane of the meridian,) let the primitive circle ZHNO represent the plane of the meridian of the place of observation; SP the earth's axis; EQ the equator; HO the horizon; and ZN the prime vertical, in which Z represents the zenith. Let the small circle *a a* represent the parallel of the moon's altitude, and that expressed by *b b* the parallel of her declination.



The intersection of those small circles in ☽ shows the moon's place in the heavens at the time of observation. Draw the oblique circles S☽P and Z☽N. Then, in the oblique angled spherical triangle Z☽S, there are given the three sides, to find the hour angle ZS☽; viz., the side ☽Z = the co-altitude or zenith distance of the moon, the side ☽S = the moon's distance from the elevated pole of the equator, and the side SZ = the co-latitude of the place; to find the moon's distance from the meridian expressed by the angle at S; which angle is easily determined by calculation, and which being compared with the right ascension of the meridian (by addition or subtraction, according as the object may have been in the eastern or western hemisphere at the time of observation,) the moon's correct right ascension will thus be obtained, and, hence, the apparent time of observation at Greenwich.

Computation.

Apparent time of observation = ...	$1^h 10^m 17^s$
Longitude by account $30°40'$ W., in time =	$+2.\ 2.40$
Estimated time at Greenwich = ...	$3^h 12^m 57^s$

Sun's reduced right ascension =	$11^h\ 6^m 11^s.2$
Apparent time of observation =	$1.10.17$
Right ascension of the meridian =	$12^h 16^m 28^s.2 = 184°7'3''$

Moon's reduced declination = $20°49'3''$ south.

Moon's true central altitude =	$31°55'23''$		
Moon's south polar distance =	$69.10.57$	Log. co-secant =	0.029319
Latitude of the place of obs. =	$40.27.30$	Log. secant = .	0.118685
Sum = ...	$141°33'50''$		
Half sum = ...	$70°46'55''$	Log. co-sine =	9.517413
Remainder = ...	$38.51.32$	Log. sine = .	9.797547
		Sum =	19.462964
Half the hour angle = . .	$32°36'24''$	Log. sine = .	9.731482
☽'s angular dist. east of merid.	$65°12'48''$		

Moon's angular dist. E. of merid. 65°12′48″
Right ascension of the merid. = 184. 7. 3

☽'s R.A. at time & place of obs. 249°19′51″ } Diff. 0° 8′ 5″ P. log. 1.3477
☽'s R.A. per Naut. Al. at III hrs 249. 11. 46 }
☽'s ditto ditto at VI hrs 251. 4. 53 } Diff. 1. 53. 7 P. log. .2017

Portion of time = ... $0^h 12^m 52^s$ Prop. log. = 1. 1460
Time corresponding to least tabular R.A. = 3. 0. 0

Apparent time of observ. at Greenwich = $3^h 12^m 52^s$
Apparent time at the place of observ. = 1. 10. 17

Long. of the place of observ., in time = $2^h\ 2^m 35^s$ = 30°38′45″ west.

Note.—The latitude and the altitude are made use of directly, in the above calculation, instead of their complements, so that the hour angle may come out a logarithmic sine.

Example 2.

September 14th, 1826, in latitude 47°12′20″ north, and longitude, by account, 38°47′ east, at $14^h 58^m 38^s$ apparent time, the true altitude of the moon's centre, west of the meridian, was 16°24′18″; required the true longitude of the place of observation?

Apparent time of observation = ... $14^h 58^m 38^s$
Longitude by account 38°47′ east, in time = – 2. 35. 8

Estimated time at Greenwich = ... $12^h 23^m 30^s$

Sun's reduced right ascension = $11^h 29^m\ 8^s.3$
Apparent time of observation = 14. 58. 38 .0

Right ascension of the merid. = $2^h 27^m 46^s.3$ = 36°56′34½″

Moon's reduced declination = 4°24′0″ south.

Moon's true central altitude = 16°24′18″
Moon's north polar distance = 94. 24. 0 Log. co-secant = 0.001282
Lat. of the place of observ. = 47. 12. 20 Log. secant = . 0.167893

Sum = ... 158° 0′38″

Half sum = ... 79° 0′19″ Log. co-sine = . 9.280393
Remainder = ... 62. 36. 1 Log. sine = . . 9.948324

Sum = 19.397892

Half the hour angle = . . . 29°59′53″ Log. sine = . . . 9.698946

Half the hour angle = . .

☽'s angular dist. west of mer.
Right ascension of the merid.

☽'s R. A. at time & place of obs
☽'s R. A. per Naut. Al. at XII. h
☽'s ditto ditto at XV. h

Portion of time = . . .
Time corresponding to least ti

Apparent time of observ. at G
Apparent time at the place of

Long. of the place of observ.,

Note.—The two last exam
is to be applied to the right
observation may be made in
and thus the reader is prese
longitude, *in all cases on sh*
sufficiently clear to admit of
being taken with the necessa
that the apparent time of ol
known.

We will here give anothe
actual observation at the auth
longitude of that place to eve

September 18th, 1826, in l
account, 1°5′ west, at 19h2(
moon's centre, west of the m
longitude of the place of obse

Apparent time of obse
Longitude by account

Estimated time at Gre

Sun's reduced right as
Apparent time of obser

Right ascension of the

Moon's reduced c

Moon's true central altitude =	17°41′40″		
Moon's north polar distance =	76. 22. 22	Log. co-secant =	0.012401
Lat. of the place of observ. =	50. 48. 15	Log. secant = .	0.199301
Sum = ...	144°52′17″		
Half sum = ...	72°26′ 8½″	Log. co-sine = .	9.479685
Remainder = ...	54. 44. 28½	Log. sine = . .	9.911984
		Sum =	19.603371
Half the hour angle = . .	39°18′ 8″	Log. sine = .	9.801685½
☽'s angular dist. west of merid.	78°36′16″		
Right ascension of the merid.	106. 8. 10½		

☽'s R.A. at time & place of obs. =	27°31′54½″	Diff. 0°41′45½″	P.log. .6346
☽'s R.A. per Naut. Al. at XVIII. hs	26. 50. 9		
Moon's ditto ditto at XXI. hrs	28. 19. 16	Diff. 1. 29. 7	P.log. .3053

Portion of time = ...	1ʰ24ᵐ20ˢ	Prop. log. = .3293
Time corresponding to least tabular R. A. =	18. 0. 0	
Apparent time of observation at Greenwich =	19ʰ24ᵐ20ˢ	
Apparent time at the place of observation =	19. 20. 0	

Longitude of the place of observ., in time = 0ʰ 4ᵐ20ˢ = 1°5′0″ west; which is the correct longitude of the given place.

Remarks.

In finding the longitude by the proposed method (which is evidently founded on the most natural and unerring principles,) the moon's right ascension becomes the principal element in the calculation; and since this must be deduced from her true central altitude, the observed altitude of her limb must be taken with all imaginable care: for which purpose the observation should necessarily be made with a sextant or circular instrument, and the inverting telescope ought invariably to be used, particularly if the altitude be observed at night; moreover, it is that part of the moon's round or well-defined limb (upper or lower, as the case may be,) which is perpendicular to the plane of the horizon, that must be brought down to the surface of the water or sea.

The most favourable time for observing the moon's altitude, so as to obtain the longitude to the greatest possible degree of exactness, is when her change of altitude is the quickest; and this always happens when she is in or near to the prime vertical: that is, the east or west point of the horizon. The altitude, however, should not be less than 5 degrees, on account of the uncertainty of the atmospherical refraction near the horizon; nor should the object's distance from the meridian be less than 3 hours, or 45 degrees.

Since an error of 1 minute of a degree in the moon's computed right ascension will have the same effect upon the deduced longitude that an error of the same value has in the computed distance by the method of the lunar observations,* it is essentially requisite that several altitudes of the moon's round or well-defined limb be most carefully observed, and the corresponding apparent times per watch noted down: the sums of these, divided by their number, will give the mean altitude and the mean corresponding apparent time; which should be depended upon in preference to any single observation. If there be an assistant observer to take the altitude of the sun, a planet, or a fixed star, at the same moment that the principal observer takes the altitude of the moon's well-defined limb, the apparent time may then be deduced from the altitude of such object; and thus any imperceptible irregularity in the going of the watch, since the last time of ascertaining its error, will be provided against.

In a very rough sea, and when the ship rolls or pitches considerably, it will be most highly advisable to multiply the observations, and to take the mean as the true result.

It is to be noted, however, that the sextant must be properly adjusted, or the value of its index error very carefully determined, by the method in page 653; the instrument must then be held in a direct vertical position, so that its plane, if produced, would meet with an imaginary plumb-line passing through the moon's centre, and let fall from that part of her well-defined limb which is exactly perpendicular to the plane of the horizon, and which is either the nearest to or the most remote from the said plane: any deviation from this position, either to the right or left, will make the angle of altitude something *more* than the truth.

As it may be rather difficult to make a perfect contact between that part of the moon's well-defined limb, thus indicated, and the horizon immediately under, when she is so posited in the heavens that an imaginary straight line joining the cusps of her horns is at right angles, or perpendicular to the plane of the horizon, (a position in which she is at times during the two or three days preceding and following her conjunctive Syzygia, or change,†) it will therefore be advisable to take her altitude at

* This, at a mean rate, will be about 27½ miles in places under the equator; but since the error decreases in proportion to the co-sine of the latitude, it will only amount to about 17½ miles in the parallel of Portsmouth.

† The moon is never exactly in this position, except when she arrives at the nonagesimal degree; that is, the 90th degree of the ecliptic above the horizon; and then she is too near to the meridian for observation.

some convenient moment before or after the time of her being in this unfavourable position; observing, however, that the moment so chosen be sufficiently far from the period of her passing over the meridian of the given place.

It will appear very perceptible, from what has been thus adduced, that the proposed method of finding the longitude possesses a most decided advantage over that by the lunar distances; because, while most mariners are found competent to take a very correct altitude of a celestial object, few are found sufficiently qualified to measure the angular distance between the moon and sun, or a fixed star, to that degree of precision which is so indispensably necessary to the obtaining of the true longitude; particularly when the object seen by reflection is to the *left-hand* of, and considerably *lower* than, that seen by direct vision. Besides, there is, at times, a very considerable degree of uncertainty attendant on the admeasurement of a lunar distance from the sun or a fixed star; for the objects approach or recede from each other so very slowly, that the eye of the observer is very frequently deceived, unless aided by a high magnifying power; and this cannot always be used, when the ship suffers a violent degree of agitation: hence it commonly happens, after making the contact between the limbs of the objects apparently perfect, that, on directing the sight to them again, their limbs will appear to be separated, or, perhaps, entered upon each other; and this separation, or entering, in direct opposition to the absolute motion of the moon. But if the horizon be clear, and the moon at a *proper distance from the meridian*, all uncertainties vanish in bringing her well-defined limb in contact with the visible horizon expressed by the convex surface of the sea; because she then rises or falls so very rapidly, that a careful observer may take the altitude of her limb to the sixth part of a minute; and this is a degree of exactness that can very rarely, if at all, be introduced into the measured lunar distances taken at sea.

Though what has been said here is strictly true in theory, yet it may often fail in practice, owing to the uncertainty of the sea horizon: hence it is evident that the method in question is *only properly adapted to the determination of the longitude of places on shore, where the altitudes can be correctly taken by means of an artificial horizon;* and, certainly, with this view, it will be found to be one of the very best methods for settling the true positions of places inland or along the coast.

Were the moon's place in right ascension and declination computed in the Nautical Almanac agreeably to the form in page 670, it would very considerably facilitate the finding of the longitude by the method now under consideration; for then the mariner would be provided with all the necessary elements that enter into the calculation, with the bare exception of the moon's correct central altitude and the apparent time; but these, it is supposed, his own diligence will always furnish. Moreover, the longitude could then be as readily inferred from an altitude of the moon's limb and the corresponding time indicated by a chronometer, as it is now by that of the sun and the same delicate piece of mechanism.

Note.—Page 670 shows the manner in which the moon's right ascension and declination should be computed and arranged, so as to answer the intention of the present problem; these right ascensions and declinations may be readily deduced from the elements given, at noon and midnight, in page VI. of the month in the Nautical Almanac, by means of the problem for illustrating the use of Table XVII., page 34. Thus, to find the moon's right ascension at III., VI., and IX. hours, September 1st, 1826:

		First Diff.	Second Diff.	Mean 2d Diff.
Moon's R. A. at midnt., Aug. 31st,	142°43′12″			
		6°32′ 4″		
Moon's do. at noon, Sept. 1st, =	149. 15. 16		0. 38	
		6. 32. 42		1′0½″
Moon's do. at midnt., Sept. 1st, =	155. 47. 58		1. 23	
		6. 34. 5		
Moon's do. at noon, Sept. 2d, =	162. 22. 3			

The variation of right ascension between noon and midnight of the given day, is 6°32′42″; the proportional part of which corresponding to

III. hours, is	1°38′10″.5
Equation of second difference = . .	− 5. 6
Proportional part corrected = ...	1°38′ 4″.9
Moon's R. A. at noon, 1st September, =	149. 15. 16
Moon's R. A. at III. hours = ...	150°53′20″.9

The propor. part of 6°32′42″ corresponding to VI. hours, is	3°16′21″.0
Equation of second difference = ...	− 7. 5
Proportional part corrected = ...	3°16′13″.5
Moon's R. A. at noon, 1st September, = ...	149. 15. 16
Moon's R. A. at VI. hours = ...	152°31′29″.5
The propor. part of 6°32′42″ corresponding to IX. hours, is	4°54′31″.5
Equation of second difference = ...	− 5 .6
Proportional part corrected = ...	4°54′25″.9
Moon's R. A. at noon, 1st September, = ...	149. 15. 16
Moon's R. A. at IX. hours = ...	154° 9′41″.9

The moon's declination is to be determined in the same manner precisely; observing, however, to apply the corrected proportional part by addition or subtraction, according as the declination at the preceding noon or midnight is increasing or decreasing.

Month Days	Noon, or 0h	III.h	VI.h	IX.h	XII.h	XV.h	XVIII.h	XXI.h
1	149°15′ 16″	150°53′ 21″	152°31′ 30″	154° 9′ 42″	155°47′ 58″	157°26′ 19″	159° 4′ 47″	160°43′ [illegible]
2	162. 22. 3	164. 0. 53	165. 39. 53	167. 19. 4	168. 58. 25	170. 37. 58	172. 17. 46	173. 57. 4[illegible]
3	175. 38. 2	177. 18. 33	178. 59. 22	180. 40. 28	182. 21. 51	184. 3. 34	185. 45. 37	187. 28. [illegible]
4	189. 10. 45	190. 53. 52	192. 37. 22	194. 21. 16	196. 5. 32	197. 50. 13	199. 35. 19	201. 20. 5[illegible]
5	203. 6. 46	204. 53. 8	206. 39. 56	208. 27. 8	210. 14. 46	212. 2. 50	213. 51. 18	215. 40. 1[illegible]
6	217. 19. 26	219. 19. 6	221. 9. 8	222. 59. 32	224. 50. 17	226. 41. 23	228. 32. 47	230. 24. 2[illegible]
7	232. 16. 25	234. 8. 38	236. 1. 3	237. 53. 39	239. 46. 26	241. 39. 22	243. 32. 22	245. 25. 2[illegible]
8	247. 18. 36	249. 11. 46	251. 4. 53	252. 57. 56	254. 50. 56	256. 43. 48	258. 36. 29	260. 28. 5[illegible]
9	262. 21. 17	264. 13. 20	266. 5. 4	267. 56. 29	269. 47. 36	271. 38. 20	273. 28. 38	275. 18. 31
10	277. 7. 59	278. 56. 58	280. 45. 27	282. 33. 25	284. 20. 54	286. 7. 49	287. 54. 11	289. 39. 59
11	291. 25. 14	293. 9. 53	294. 53. 56	296. 37. 25	298. 20. 18	300. 2. 34	301. 44. 15	303. 25. 2[illegible]
12	305. 5. 50	306. 45. 43	308. 25. 2	310. 3. 48	311. 41. 57	313. 19. 33	314. 56. 37	316. 33. [illegible]
13	318. 9. 8	319. 44. 36	321. 19. 35	322. 54. 5	324. 28. 6	326. 1. 39	327. 34. 46	329. 7. 28
14	330. 39. 44	332. 11. 36	333. 43. 7	335. 14. 16	336. 45. 4	338. 15. 32	339. 45. 43	341. 15. 37
15	342. 45. 13	344. 14. 34	345. 43. 42	347. 12. 36	348. 41. 18	350. 9. 49	351. 38. 10	353. 6. 23
16	354. 34. 27	356. 2. 24	357. 30. 16	358. 58. 3	0. 25. 46	1. 53. 26	3. 21. 4	4. 48. 42
17	6. 16. 18	7. 43. 55	9. 11. 35	10. 39. 17	12. 7. 1	13. 34. 50	15. 2. 44	16. 30. 43
18	17. 58. 48	19. 26. 57	20. 55. 17	22. 23. 46	23. 52. 26	25. 21. 12	26. 50. 9	28. 19. 16
19	29. 48. 34	31. 18. 4	32. 47. 45	34. 17. 39	35. 47. 45	37. 18. 4	38. 48. 36	40. 19. 21
20	41. 50. 19	43. 21. 31	44. 52. 57	46. 24. 37	47. 56. 30	49. 28. 39	51. 0. 59	52. 33. 35
21	54. 6. 23	55. 39. 25	57. 12. 41	58. 46. 9	60. 19. 50	61. 53. 44	63. 27. 50	65. 2. 7
22	66. 36. 37	68. 11. 18	69. 46. 9	71. 21. 11	72. 56. 23	74. 31. 45	76. 7. 15	77. 42. 53
23	79. 18. 40	80. 54. 34	82. 30. 34	84. 6. 41	85. 42. 54	87. 19. 13	88. 55. 36	90. 32. 4
24	92. 8. 35	93. 45. 10	95. 21. 48	96. 58. 28	98. 35. 11	100. 11. 55	101. 48. 40	103. 25. 27
25	105. 2. 14	106. 39. 1	108. 15. 49	109. 52. 36	111. 29. 24	113. 6. 11	114. 42. 58	116. 19. 45
26	117. 56. 31	119. 33. 17	121. 10. 2	122. 46. 48	124. 23. 34	126. 0. 20	127. 37. 6	129. 13. 54
27	130. 50. 42	132. 27. 32	134. 4. 23	135. 41. 19	137. 18. 15	138. 55. 16	140. 32. 21	142. 9. 32
28	143. 46. 47	145. 24. 8	147. 1. 37	148. 39. 14	150. 16. 58	151. 54. 51	153. 32. 56	155. 11. 11
29	156. 49. 38	158. 28. 17	160. 7. 11	161. 46. 20	163. 25. 44	165. 5. 24	166. 45. 23	168. 25. 40
30	170. 6. 16	171. 47. 12	173. 28. 31	175. 10. 11	176. 52. 14	178. 34. 41	180. 17. 34	182. 0. 52

The Moon's Declination, September, 1826.

Month Days	Noon, or 0h	III.h	VI.h	IX.h	XII.h	XV.h	XVIII.h	XXI.h
1	7° 10′ 0″ N	6° 34′ 34″ N	5° 58′ 45″ N	5° 22′ 33″ N	4° 45′ 38″ N	4° 8′ 37″ N	3° 31′ 21″ N	2° 53′ 49″ N
2	2. 16. 2 N.	1. 38. 3 N.	0. 59. 56 N.	0. 21. 41 N.	0. 16. 41 S.	0. 55. 3 S.	1. 33. 27 S.	2. 11. 51 S.
3	2. 50. 17 S.	3. 28. 34 S.	4. 6. 41 S.	4. 44. 38 S.	5. 22. 25 S.	5. 59. 57 S.	6. 37. 10 S.	7. 14. 4 S.
4	7. 50. 39	8. 26. 50	9. 2. 33	9. 37. 48	10. 12. 34	10. 46. 47	11. 20. 22	11. 53. 21
5	12. 25. 42	12. 57. 21	13. 28. 14	13. 58. 22	14. 27. 43	14. 56. 14	15. 23. 52	15. 50. 36
6	16. 16. 26	16. 41. 18	17. 5. 9	17. 27. 59	17. 49. 49	18. 10. 34	18. 30. 12	18. 48. 44
7	19. 6. 9	19. 22. 26	19. 37. 32	19. 51. 25	20. 4. 7	20. 15. 25	20. 25. 30	20. 34. 24
8	20. 42. 5	20. 48. 40	20. 54. 6	20. 58. 23	21. 1. 31	21. 1. 19	21. 1. 4	21. 0. 45
9	21. 0. 23	20. 57. 1	20. 52. 28	20. 46. 43	20. 39. 47	20. 31. 40	20. 22. 35	20. 12. 2
10	20. 0. 32	19. 47. 56	19. 34. 16	19. 19. 34	19. 3. 49	18. 47. 3	18. 29. 21	18. 10. 41
11	17. 51. 5	17. 30. 34	17. 9. 13	16. 47. 2	16. 24. 1	16. 0. 12	15. 35. 40	15. 10. 24
12	14. 44. 25	14. 17. 46	13. 50. 30	13. 22. 37	12. 54. 8	12. 25. 6	11. 55. 35	11. 25. 34
13	10. 55. 4	10. 24. 8	9. 52. 49	9. 21. 8	8. 49. 4	8. 16. 41	7. 44. 2	7. 11. 7
14	6. 37. 56 S.	6. 4. 32 S.	5. 30. 59 S.	4. 57. 16 S	4. 23. 24 S.	3. 49. 25 S.	3. 15. 23 S.	2. 41. 17 S.
15	2. 7. 8 S.	1. 32. 58 S.	0. 58. 50 S.	0. 24. 45 S.	0. 9. 19 N.	0. 43. 17 N.	1. 17. 8 N.	1. 50. 51 N.
16	2. 24. 27 N.	2. 57. 53 N.	3. 31. 6 N.	4. 4. 6 N.	4. 36. 54 N.	5. 9. 26 N.	6. 18. 41 N.	6. 13. 38 N.
17	6. 45. 18	7. 16. 38	7. 47. 36	8. 18. 12	8. 48. 27	9. 18. 18	9. 47. 42	9. 56. 41
18	10. 45. 13	11. 13. 17	11. 40. 50	12. 7. 54	12. 34. 28	13. 0. 30	13. 25. 58	13. 50. 52
19	14. 15. 13	14. 38. 58	15. 2. 5	15. 24. 36	15. 46. 29	16. 7. 43	16. 28. 17	16. 48. 11
20	17. 7. 24	17. 25. 55	17. 43. 44	18. 0. 48	18. 17. 10	18. 33. 6	18. 47. 37	19. 2. 1
21	19. 15. 0	19. 27. 31	19. 39. 14	19. 50. 9	20. 0. 16	20. 9. 33	20. 18. 0	20. 25. 37
22	20. 32. 23	20. 38. 18	20. 43. 21	20. 47. 32	20. 50. 52	20. 52. 22	20. 53. 38	20. 54. 36
23	20. 55. 20	20. 53. 49	20. 51. 40	20. 48. 54	20. 45. 31	20. 40. 49	20. 35. 13	20. 28. 52
24	20. 21. 18	20. 13. 1	20. 3. 49	19. 53. 42	19. 42. 40	19. 30. 34	19. 17. 35	19. 3. 45
25	18. 49. 3	18. 33. 45	18. 17. 35	18. 0. 35	17. 42. 45	17. 23. 53	17. 4. 10	16. 43. 35
26	16. 22. 10	15. 59. 58	15. 36. 58	15. 13. 10	14. 48. 35	14. 23. 13	13. 57. 7	13. 30. 16
27	13. 2. 41	12. 34. 23	12. 5. 25	11. 35. 46	11. 5. 27	10. 34. 29	10. 2. 56	9. 30. 46
28	8. 58. 1	8. 24. 42	7. 50. 52	7. 16. 32	6. 41. 41	6. 6. 22	5. 30. 40	4. 54. 33
29	4. 18. 2 N.	3. 41. 10 N.	3. 4. 0 N.	2. 26. 34 N.	1. 48. 51 N.	1. 10. 54 N.	0. 32. 48 N.	0. 5. 26 S.
30	0. 43. 50 S.	1. 22. 19 S.	2. 0. 48 S.	2. 39. 17 S.	3. 17. 46 S.	3. 56. 11 S.	4. 34. 27 S.	5. 12. 31 S.

The author is of opinion, that, for the purpose of determining the true longitude of places on shore, the method in question will be found preferable to any other, particularly when the altitudes are taken by means of an artificial horizon.

To Reduce the Moon's Right Ascension and Declination to any given Time under a known Meridian.

RULE.—To the apparent time of observation (always reckoned from the preceding noon,) add the longitude, in time, if it be west, but subtract it if east; the sum, or remainder will be the time at Greenwich.

With this time enter page VI. of the month in the Nautical Almanac, opposite to the given day, or to that which immediately precedes or follows it, and take out the right ascensions and declinations at the periods directly preceding and following the said time; find their difference, and, also, the difference between the said or Greenwich time and the preceding period: then,

To the proportional logarithm of the last difference, add the proportional logarithm of the first difference, and the sum will be the proportional logarithm of a correction which is always to be added to the right ascension at the preceding period; but, to be applied by addition, or subtraction to the declination at that period, according as it may be increasing or decreasing.—And, thus, the right ascension and declination will be obtained, sufficiently near the truth for all nautical operations, independent of interpolations, or second differences.

This rule is founded on the assumption that the moon's right ascension and declination are given in the Nautical Almanac agreeably to the form in page 670.

Example.

Required the moon's right ascension and declination, Sept. 18th, 1826, at $19^h 20^m 0^s$ apparent time, in long. $1^\circ 5'$ W. of the meridian of Greenwich?

Apparent time	$19^h 20^m 0^s$
Longitude $1^\circ 5'$ west, in time = . . .	+ 4.20
Greenwich time	$19^h 24^m 20^s$

To find the Moon's Right Ascension:—

Ap. time at Gr. = $19^h 24^m 20^s$	difference = . .	$1^h 24^m 20^s$	P. L. = .3293
☽'s R.A. Sep. 18, at 18 hours = $26^\circ 50' 9''$	diff.	$1^\circ 29' 7''$	P. L. = .3053
☽'s ditto ditto at 21 hours = 28. 19. 16			
Correction of right ascension = ...		$0^\circ 41' 45''$	P. L. = .6346
R.A. at the period preced. the Greenwich time =		26. 50. 9	
Moon's correct R.A. at time and place of obs. =		$27^\circ 31' 54''$	

To find the Moon's Declination:—

Ap. time at Gr. = $19^h 24^m 20^s$	difference = . .	$1^h 24^m 20^s$	P.L. = .3293
☽'s dec. Sep. 18, at 18 hours = $13^\circ 25' 58''$ N.	dif.	$0^\circ 24' 54''$	P.L. = .8591
☽'s do. do. at 21 hours = 13. 50. 52 N.			
Correction of declination = ...		+ $0^\circ 11' 40''$	P.L. = 1.1884
Declin. at period preced. the Greenwich time =		13. 25. 58 north.	
Moon's correct dec. at time and place of obs. =		$13^\circ 37' 38''$ north.	

Hence, it is evident, that if the moon's right ascension and declination be given in the Nautical Almanac to every third hour, the reduction of those elements, to a given time under a known meridian, will become a more simple operation than the reduction of the sun's right ascension and declination to a given time and place.

SOLUTION OF PROBLEMS relative to finding the Latitudes and Longitudes, Right Ascensions and Declinations of the Heavenly Bodies, and to the computing of the Central Distances between the Moon and Sun, a fixed Star, or a Planet.

Problem I.

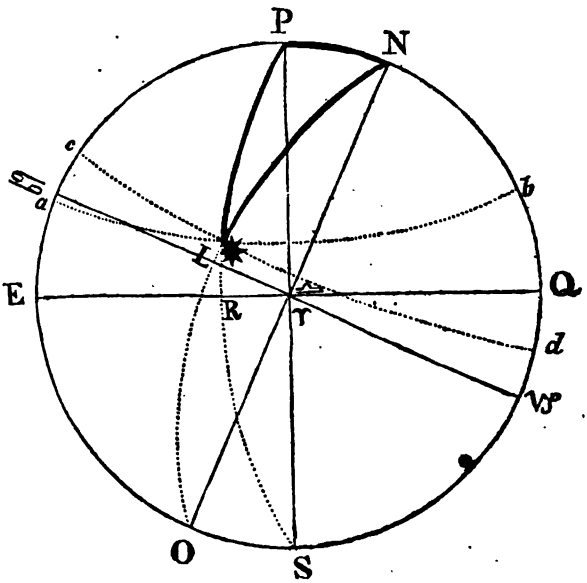
Given the Right Ascension and Declination of a Celestial Object, to find its Latitude and Longitude.

Example.

The apparent right ascension of α Arietis, August 1st. 1825, was $1^h 57^m 23^s.176$, and its declination $22^\circ 38' 4''.44$ north; required its apparent latitude and longitude, the obliquity of the ecliptic being $23^\circ 27' 42''.875$?

α Arietis, right ascen. = $1^h 57^m 23^s.176$, in degrees = $29^\circ 20' 47''.64$, and north polar distance = $67^\circ 21' 55''.56$.

Construction.



Describe the primitive circle PESQ, with the chord of 60° on the plane of the solstitial colure:—draw the Equator EQ, and, at right angles thereto, the axis PS; in which P, represents the N. celestial pole. Make E a, Q b = $22^\circ 38' 4''.44$, the star's north declination; and with the tangent of its complement, viz. $67^\circ 21' 55''.56$, describe the parallel of declination $a\,b$.—Take the star's right ascension, viz. 29°20′47″.64, in the compasses, from the scale of semi-tangents, and lay it off on the Equator from ♈ to R, and with the secant of its complement, viz. 60°39′12″.36, describe the circle of right ascension PRS; the intersection of which with the parallel circle *a b*, at ∗, will be the apparent place of the star in the heavens.—Make PN, SO = 23°27′42″.875 the obliquity of the ecliptic:—draw the polar line NO, and, at right angles thereto, the ecliptic line ♑ ♈ ♋:—through the intersecting point ∗, draw the circle of longitude N ∗ O, cutting the ecliptic in L;—then, the arc ♈ L, will be the longitude of the star, and the arc L ∗, its latitude; the former being taken in the compasses, and applied to the scale of semi-tangents, will be found to measure about 35¼ degrees:—the angle PN ∗ (measured by the arc L ♋ = 54¾ degrees,) represents the co-longitude of the star, and the arc N ∗ its co-latitude; the latter being reduced to the primitive circle will be found to measure about 80 degrees.

Now, in the oblique angled spherical triangle PN ∗, given the side PN = 23°27′42″.875, the obliquity of the ecliptic; the side P ∗ = 67°21′55″.56, the star's north polar distance, and the included angle NP ∗ = 119°20′47″.24; to find the side N ∗ = the co-latitude of the star, or its distance from the north pole of the ecliptic, and the angle PN ∗ = its co-longitude.

Note.—The circle of right ascension which passes through ♑, viz. PQS, is always equal to, or expressed by 270 degrees; and that which passes through ♈, or ♎, by 0, or 360 degrees; the difference, therefore, between ♑ and ♈, or, which is the same, between Q and ♈, is 90 degrees; which being added to the arc ♈ R, 29°20′47″.64, makes the whole arc QR = 119°20′47″.64, which is the true measure of the angle RPQ; that is, the angle NP ∗, comprehended between the two given sides.

Hence, by spherical trigonometry, Problem III., Remark 1, page 203,

An. NP ∗ = 119°20′47″.64 ÷ 2 =	59°40′23″.82	tw. L.sin.	19.8721827.68
Side PN = obl. of the ecliptic =	23.27.42 .875	L.sin.	9.6000350.88
Side P ∗ = star's N. polar dis. =	67.21.55 .56	L.sin.	9.9651914.48
		Sum =	39.4374093.04
Difference of the two sides = . .	43°54′12″.685	Hf. S.	19.7187046.52

[+

Diff. of the two sides=43°54′12″.685	Half S. of logs.		19.7187046.52 +
Half difference . .	21°57′ 6″.3425	Log. sine =	9.5726693.15
Arch = ...	54°27′23″.53	Log. tang.=	10.1460353.37
Log. sine of arch, subtract from half sum of logs. =			9.9104508.95 −
Half the side P ∗ =	40° 1′14″.26 =	Log. sine =	9.8082537.57

The whole side P ∗ =80° 2′28″.52; which is the co-latitude of the given star, or its distance from the north pole of the ecliptic; hence the latitude of α Arietis is 9°57′31″.48 north.

Now, in the oblique angled spherical triangle PN ∗, the three sides are given, and the angle P; to find the angle N = the co-longitude of the star.—Hence, by spherical trigonometry, Problem I., page 198,

As the side N ∗ = .	80° 2′28″.52	Log. co-secant =	10.0065934.92
Is to the ang. NP∗ =	119.20.47 .64	Log. sine = . .	9.9403527.32
So is the side P ∗ =	67.21.55 .56	Log. sine = . .	9.9651914.48
To the angle PN ∗ =	54°46′11″.38	Log. sine = . .	9.9121376.72

The angle N, thus found=54°46′11″.38, which is the co-longitude of the star, and which is measured by the arc of the ecliptic L ♋, being taken from 90 degrees; that is, from ♈ ♋, leaves the arc ♈ L=35°13′48″.62; which, therefore, is the apparent longitude of the star *a* Arietis.—Hence, the apparent longitude of the given star is 1ˢ5°13′48″.62, and its apparent latitude 9°57′31″.48 north.

Now, from the above Problem we obtain the following

General Rule

for computing the latitude and longitude of a celestial object, viz.

Find the difference between the object's right ascension, expressed in degrees, and 270 degrees; *— to twice the logarithmic sine of half this difference, add the logarithmic sines of the object's north polar distance, and of the obliquity of the ecliptic: from half the sum of these three logarithms subtract the logarithmic sine of half the difference between the object's north polar distance and the obliquity of the ecliptic; the remainder will be the logarithmic tangent of an arch, the logarithmic sine of which being subtracted from the half sum of the three logarithms will leave the logarithmic sine of an arc, which being doubled will give the object's distance from the north pole of the ecliptic; the difference between which and 90 degrees will be the latitude of the object, which will be north when the ecliptic polar distance is the least; otherwise, south.

* In all cases, whenever this difference exceeds 180 degrees, it must be subtracted from 360 degrees.

To find the Longitude :—

To the logarithmic secant of the object's latitude, add the logarithmic sine of the difference between its right ascension and 270 degrees, * and the logarithmic sine of its north polar distance; the sum of these three logarithms, abating 20 in the index, will be the logarithmic sine of an arch, which being subtracted from 90 degrees, will leave the object's true longitude when its right ascension is less than 6 hours or 90 degrees; but which is to be increased by 6 signs when the right ascension is between 12 and 18 hours, that is, between 180 and 270 degrees.

Again.—If the right ascension lies between 6 and 12 hours; that is, between 90 and 180 degrees, the arch, so found, is to be augmented by 3 signs; but if the right ascension is between 18 and 24 hours, viz. between 270 and 360 degrees, it is to be augmented by 9 signs; in either case the true longitude of the object will be obtained.

Example 1.

The apparent right ascension of Aldebaran, August 3rd., 1825, was $4^h 25^m 56^s.115$, and its declination 16°9′5″.35 north; required its apparent latitude and longitude, the obliquity of the ecliptic being 23°27′42″.875?

Aldeb's. R.A. = $4^h 25^m 56^s.115$ = 66°29′ 1″.725

Diff. between R.A. and 270° = 203°30′58″.275

And 360° − 203°30′58″.275 = 156°29′ 1″.725

The half of which is = ... 78°14′30″.862 Tw.L. sin. 19.981580
Aldebaran's. N. polar distance = 73.50.54 .650 Log. sin. = 9.982511
Obliquity of ecliptic = . . . 23.27.42 .875 Log. sin. = 9.600035

Sum = 39.564126

Diff. bet. P. dist. and ob. of eclip. = 50°23′11″.775 Half S. = 19.782063 +

Half ditto = ... 25.11.35 .887 Log. sin. 9.629077

Arch = ... 54°53′21″ Log. tang. = 10.152986
Log. sine of arch, subtract from half sum of logs. = ... 9.912775 −

Arc = ... 47°44′22″.5 Log. sine = 9.869288

Twice the arc = ... 95°28′45″.0; which is Aldebaran's distance from the north pole of the ecliptic; its latitude, therefore, is 5°28′45″ south.

* See Note, page 674.

To find Aldebaran's Longitude :—

Lat. of Aldebaran = ... 5°28′45″ S. Log. sec.=10.001989
Diff. betw. R. A. and 270 deg.=156. 29. 1 .725 Log. sine= 9.600982
Aldebaran's N. polar distance = 73. 50. 54 .65 Log. sine = 9.982511

Aldebaran's co-longitude = . . . 22°38′41″.6 Log. sine = 9.585482

Aldebaran's longitude = . . . 67°21′18″.4; or 2^s.7°21′18″.4.

Example 2

The apparent right ascension of Pollux, September 3rd., 1825, was $7^h 34^m 38^s.74$, and its declination 28°26′19″.48 north; required its apparent latitude and longitude, the obliquity of the ecliptic being 23°27′42″.875 ?

Pollux's R.A.=$7^h 34^m 38^s.74$=113°39′41″.1

Diff. bet. R.A. and 270° = . 156°20′18″.9

The half of which is = ... 78°10′ 9″.45, Tw. L.S. 19.981350
Pollux's N. polar distance = . 61.33.40 .520 L. sin. = 9.944150
Obliquity of ecliptic = ... 23.27.42 .875 L. sine = 9.600035

Sum = . 39.525535

Diff.bet. P.dis. and ob. of ecl.= 38° 5′57″.645 Halfsum 19.762767½+

Half ditto = ... 19° 2′58″.822 Log. sine 9.513733½

Arch = ... 60°35′41″ L. tang. = 10.249034

Log. sine of arch, subtract from
half sum of Logs. = ... 9.940102 —

Arc = ... 41°39′50″.5 Log. sine= 9.822665½

Twice the arc . . . . . . 83°19′41″.0; which is Pollux's distance from the north pole of the ecliptic :—hence, its latitude is 6°40′19″ north.

To find Pollux's Longitude :—

Latitude of Pollux = . . .	6°40′19″ N.	Log. secant = 10. 002951
Diff. bet. R. A. and 270 deg. =	156. 20. 18 .9	Log. sine = . . 9. 603503
Pollux's north polar distance =	61. 33. 40 .52	Log. sine = . . 9. 944150
Arch = ...	0s. 20°48′44″. 2	Log. sine = . . 9. 550604
Add	3. 0. 0. 0 .0.	

Sum = ... 3s. 20°48′44″. 2; which is the true longitude of Pollux.—Hence, the latitude of Pollux is 6°40′19″ north, and its longitude 3s. 20°48′44″. 2, as required.

In like manner may the latitudes and longitudes of the moon and planets be deduced from their respective right ascensions and declinations.

Problem II.

Given the Latitude and Longitude of a Celestial Object; to find its Right Ascension and Declination.

Example.

The apparent long. of *a* Arietis, August 1st, 1825, was 1s. 5°13′48″. 62, and its latitude 9°57′31″. 48 north; required its right ascension and declination, the obliquity of the ecliptic being 23°27′42″. 875 ?

The construction of this Problem is exactly like that of the preceding: thus, lay the longitude of the given star off on the ecliptic line from ♈ to L, and draw the circle of longitude N L O.—Make ♑ *d*, ♋ *c* = the star's latitude, and draw the parallel circle *c d;* the intersection of which with the circle of longitude, at ✱, will represent the apparent place of the star in the heavens.—See the last *projection.*

Now, in the oblique angled spherical triangle N P ✱; given the side PN = 23°27′42″. 875, the obliquity of the ecliptic; the side N ✱ = 80°2′28″. 52, the star's distance from the north pole of the ecliptic, and the included angle P N ✱ = 54°46′11″. 38, the complement of the star's longitude (measured by the arc L ♋); to find the side P ✱ = the star's north polar distance, and the angle N P ✱ (measured by the arc R Q); the difference between which and ♈ Q, viz. 90 degrees, expressed by the arc ♈ R = will be the star's right ascension.

Hence, by spherical trigonometry, Problem III. Remark 1, page 203,

Angle PN ∗ = ...	54°46′11″.38		
Half ditto = ...	27°23′ 5″.69	Tw. L.sin.	19.3254514.58
Side P N = obliquity of the ecliptic = ...	23.27.42 .875	Log.sin.=	9.6000350.88
Side N ∗ = star's ecliptic polar distance = . .	80. 2.28 .250	Log.sin.=	9.9934065.08
		Sum =	38.9188930.54
Difference of the two sides =	56°34′45″.645	Half S.	19.4594465.27 +
Half difference = . . .	28°17′22″.822	Log. sin.	9.6757137.08
Arch = ...	31°17′22″.56	Log. T.	9.7837328.19
Logarithmic sine of arch = ...			9.7154718.60 −
Half the side P ∗ = . .	33°40′57″.76	Log. sine	9.7439746.67

The whole side P ∗ = . . 67°21′55″.52; which is the co-declination of the given star, or its north polar distance; hence, the declination of α Arietis is 22°38′4″.48, north.

Now, in the oblique angled spherical triangle N P ∗; the three sides are given, and the angle N; to find the angle P = the arc R Q; or the sum of the star's right ascension and 90 degrees.

Hence, by spherical trigonometry, Problem I., page 198,

As the side P ∗ = . .	67°21′55″.52	Log. co-sec. =	10.0348085.84
Is to the angle P N ∗ =	54.46.11 .38	Log. sine = . .	9.9121376.70
So is the side N ∗ = .	80. 2.28 .52	Log. sine = .	9.9934065.08
To the sup. of ang. NP∗ =	60.39.12 .38	Log. sine = .	9.9403527.62

Hence, the ang. NP∗ is = 119.20′47″.62 = the arc RQ; from which take the arc ♈Q, 90 degrees; and the remaining arc ♈ R = 29°20′47″.62, is the star's right ascension.—The apparent right ascension of α Arietis, on the given day, was therefore, 29°20′47″.62, and its declination 22°38′4″.48 north.

Now, from the above Problem the following general rule is deduced for computing the right ascension and declination of a celestial object, viz.

Find the difference between the object's longitude and three signs (♋)

ence, add the logarithmic sines of the object's distance from the north pole of the ecliptic, and of the obliquity of the ecliptic; from half the sum of these three logarithms subtract the logarithmic sine of half the difference between the obliquity of the ecliptic and the object's ecliptic polar distance, and the remainder will be the logarithmic tang. of an arch; the log. sine of which being subtracted from the half sum of the three logarithms, will leave the logarithmic sine of an arc, the double of which will be the object's distance from the north pole of the equator:—now, the difference between this distance and 90 degrees will be the declination of the object; which will be north when the first term is less than the latter; otherwise south.

To find the Right Ascension:—

To the logarithmic secant of the object's declination, add the logarithmic sine of the difference between its longitude and 90 degrees,* and the logarithmic sine of its distance from the north pole of the ecliptic; the sum of these three logarithms, abating 20 in the index, will be the logarithmic sine of an arch, which being subtracted from 90 degrees will leave the object's correct right ascension when its longitude is less than 3 signs or 90 degrees; but which is to be increased by 180 degrees when the longitude is between 6 and 9 signs, or between 180 and 270 degrees. Again, if the longitude is between 3 signs and 6 signs, that is between 90 and 180 degrees, the arch, so found, is to be augmented by 90 degrees; but if the longitude lies between 9 and 12 signs, viz. between 270 and 360 degrees, it is to be augmented by 270 degrees; in either case the correct right ascension of the object will be obtained, which may be converted into time, if necessary, by Problem I., page 296.

Example 1.

The apparent longitude of Aldebaran, Aug. 3rd, 1825, was 2ˢ 7° 21′ 18″. 4 and its latitude 5° 28′ 45″, south; required its apparent right ascension and declination, the obliquity of the ecliptic being 23° 27′ 42″. 875?

Aldebaran's longitude = . . .	67° 21′ 18″. 4		
Difference to 90 degrees = . .	22° 38′ 41″. 6		
The half of which is = . . .	11° 19′ 20″. 8	Tw. L. sine	18. 585974
Aldebaran's ecliptic polar dist. =	95. 28. 45 . 0	Log. sine	9. 998011
Obliquity of ecliptic = . . .	23. 27. 42 . 875	Log. sine	9. 600035
		Sum =	38. 184020
Difference betw. ob. of eclip. and star's eclip. polar distance = .	72° 1′ 2″. 125	Half S.	19. 092010 +

* In all cases, whenever this difference exceeds 180 degrees, it must be subtracted from 360 degrees.

Difference betw. ob. of eclip. and
star's eclip. polar distance = . 72° 1′ 2″.125 Half S. 19.092010+

Half ditto = ... 36° 0′31″.062 Log sine 9.769309
Arch = ... 11.52.21 .27 L. tang. 9.322701
Log. sine of arch, subtract from
half sum of logs. = ... 9.313310−

Arc = ... 36°55′27″.3 Log. sine 9.778700

Twice the arc = ... 73°50′54″.6; which is Aldebaran's distance from the north pole of the equator; hence its declination is 16°9′5″.4 north.

To find the Right Ascension :—

Declination of Aldebaran = 16° 9′ 5″.4 Log. secant = 10.017490
Diff. bet. long. and 90 deg. = 22.38.41 .6 Log. sine = 9.585481
Aldebaran's ecliptic P. dis. = 95.28.45 .0 Log. sine = 9.998011

Arch = ... 23°30′58″.3 Log. sine = 9.600982

Aldebaran's right asc. = 66°29′ 1″.7, or 4h25m56s.1

Example 2.

The apparent longitude of Pollux, Sep. 3rd, 1825, was 3s20°48′44″.2, and its latitude 6°40′19″ north; required its apparent right ascension and declination, the obliquity of the ecliptic being 23°27′42″.875?

Pollux's longitude = ... 110°48′44″.2

Difference to 90 degrees = . . . 20°48′44″.2

The half of which is = . . . 10°24′22″.1 Tw.L.sin. 18.513353
Pollux's ecliptic polar distance = 83.19.41 .0 Log. sin. 9.997049
Obliquity of the ecliptic = . . 23.27.42 .875 L. sine 9.600035

Sum 38.110637
Difference between ob. of ecliptic
and star's eclip. pol. distance = 59°51′58″.125 Hf. S. 19.055318½+

Difference between ob. of ecliptic
and star's eclip. pol. distance = 59°51′58″.125 Hf. S. 19.055318½+

Half ditto = ... 29°55′59″.062 L. sin. 9.698090

Arch = ... 12°49′25″.32 L. tang. 9.357228½

Log. sine of arch, subtract from
half sum of Logs. = ... 9.346258½−

Arc = ... 30°46′50″.03 Log. sin. 9.709060

Twice the arc = ... 61°33′40″.06; which is Pollux's distance from the north pole of the equator;—hence, its declination is 28°26′19″.94 north.

To find the Right Ascension :—

Declination of Pollux = .	28°26′19″.94	Log. secant	=	10.055850
Diff. betw. lon. and 90 deg. =	20.48.44 .20	Log. sine	=	9.550603
Pollux's ecliptic polar dis. =	83.19.41 .00	Log. sine	=	9.997049
Arch = ...	23°39′41″	Log. sine	=	9.603502
Add	90. 0. 0			

Right ascension of Pollnx = 113°39′41 = 7h.34m.38s.73

Note.—In the same manner may the right ascensions and declinations of the moon and planets be deduced from their respective latitudes and longitudes.

Problem III.

Given the Latitudes and Longitudes of the Moon and Sun, Moon and fixed Star, or Moon and Planet; to find the true Central Distance between them.

Note.—If this Problem be projected stereographically on the plane of the circle of longitude passing through one of the objects, it will be found, in every respect, like Problem XXIV., page 273, of "The Young Navigator's Guide to the sidereal and planetary parts of Nautical Astronomy;" reading, however, difference of longitude for difference of right ascension, and ecliptic polar distances for polar distances :—hence, it is evident that there is a spherical triangle to work in, where two sides and the included angle are given to find the third side, or central distance between the objects, and which may be computed by the following

General Rule.

To twice the logarithmic sine of half the difference of longitude between the two objects, add the logarithmic co-sines of their latitudes; from half the sum of these three logarithms subtract the logarithmic sine of half the difference or half the sum of the latitudes, according as they are of the same or of contrary names; the remainder will be the logarithmic tangent of an arch, the logarithmic sine of which being subtracted from the half sum of the three logarithms, will leave the logarithmic sine of half the true central distance between the two given objects.

Example. 1.

September 3rd, 1825, the moon's apparent longitude, at noon, was 1ˢ.16°.19′.29″, aud her latitude 2°.33′.30″ north; at the same time the apparent longitude of Pollux was 3ˢ.20°.48′.38″, and its latitude 6°.40′.19″ north; required the true central distance between those two objects?

Longitude of Pollux = . .	110°.48′.38″		
Longitude of the moon = .	46. 19. 29		
Difference of longitude =	64°.29′. 9″		
The half of which is = . .	32°.14′.34½″	Tw. the log. sin.	19. 454285
Latitude of Pollux = . . .	6. 40. 19 N.	Log. co-sine=	9. 997049
Latitude of the moon = . .	2. 33. 30 N.	Log. co-sine=	9. 999567
		Sum =	39. 450901
Difference of latitude . . .	4°. 6′.49″	Half sum .	19. 725450½ +
Half difference = ...	2°. 3′.24½″	Log. sine .	8. 554977½
Arch = ...	86°. 8′.11″	Log. tang. =	11. 170473
Log. sine of arch, subtract from half sum of logs. =	. . . .		9. 999012 −
Half true distance = . .	32°.11′. 4″	Log. sine =	9. 726438½

Hence, the true central distance between the moon and Pollux, at the given time, was 64°.22′.8″; which corresponds exactly with the computed distance in the Nautical Almanac.

Note.—It is evident that the same result will be obtained by making use of the right ascensions and declinations of the objects.

Example 2.

August 22, 1825, the moon's apparent longitude, at noon, was 8ˢ.19°.6′.15″, and her latitude 0°.3′.21″ north; at the same time the apparent longitude of Spica Virginis was 6ˢ.21°.24′.32″, and its latitude 2°.2′.25″ south; required the true central distance between these objects?

Longitude of the moon = . 259° 6′ 15″

Longitude of Spica Virginis = 201. 24. 32.

Difference of longitude = . 57° 41′ 43″ this divided

by 2 gives 28° 50′ 51½″ Twice L. sin. = 19. 366962

Lat. of moon = 0. 3. 21 N. Log. co-sine 10. 000000

Lat. of Spica Virg. 2. 2. 25 S. Log. co-sine 9. 999725

Sum = . . 39. 366687

Sum of lats. = . 2° 5′ 46″ Half sum = . . 19. 683343½ . . 19. 683343½

Half ditto = . 1° 2′ 53″ Log. sine = . 8. 262237½

Arch = . . 87° 49′ 42″ Log. tang. 11. 421106 L. sin. 9. 999688

Half true dist. = 28° 51′ 37″ Log. sine = ... 9. 683655½

Hence, the true central distance between the moon and Spica Virginis is 57° 43′ 14″; which is 1″ more than that given in the Nautical Almanac.

Example 3.

August 4th, 1825, the moon's apparent longitude, at noon, was 0ˢ 14° 13′ 32″, and her latitude 4° 38′ 41″ north; at the same time the sun's longitude was 4ˢ 11° 43′ 46; required the true central distance?

Note.—Since the sun apparently moves in the ecliptic, he has, therefore, *no latitude.*

Moon's longitude = 14° 13′ 32″

Sun's longitude = 131. 43. 46

Difference of long. = 117. 30. 14 this divided

by 2 gives 58° 45′ 7″ Twice the L. sine = 19. 863860

Moon's lat. 4. 38. 41 N. Log. co-sine = 9. 998571

Sun's lat. = 0. 0. 0 Log. co-sine = . 10. 000000

Sum = ... 39. 862431

Diff. of lat. = 4° 38′ 41″ Half sum = . . 19. 931215½ 19. 931215½

Half ditto = 2° 19′ 20½″ Log. sine = . . 8. 607688

Arch = . 87° 16′ 55″ Log. tang. = . 11. 323527½ L. sin. 9. 999511

Hf. req. dis. 58° 42′ 10½″ Log. sine = ... 9. 931704½

Hence, the true central distance between moon and sun is 117° 24′ 21″; which corresponds with that in the Nautical Almanac.

Remark.—Since the co-latitudes of the sun and moon and the comprehended angle (expressed by their difference of longitude,) form a quadrantal spherical triangle; therefore the true central distance between these particular objects may be more readily determined by the following concise method than by the above general Rule, viz.

To the logarithmic co-sine of the difference of longitude, add the logarithmic co-sine of the moon's latitude; the sum of these two logarithms, abating 10 in the index, will be the logarithmic co-sine of the true central distance between the sun and moon.

Example 1.

August 6th, 1825, the moon's longitude, at noon, was 1ˢ 8° 0′ 34″, and her latitude 3° 23′ 20″ north; at the same time, the sun's longitude was 4ˢ 13° 38′ 46″; required the true central distance?

Moon's longitude =	38° 0′ 34″		
Sun's longitude = . .	133. 38. 46		
Difference of long. =	95° 38′ 12″	Log. co-sine = ...	8.992199
Moon's latitude = . .	3. 23. 20	Log. co-sine = ...	9.999240
True central distance =	95° 37′ 36″	Log. co-sine = ...	8.991439

which is precisely the same as that given in the Nautical Almanac.

Example 2.

August 7th, 1825, the moon's longitude, at noon, was 1ˢ 20° 4′ 42″, and her latitude 2° 30′ 42″ north; at the same time the sun's longitude was 4ˢ 14° 36′ 18″; required the true central distance?

Moon's longitude =	50° 4′ 42″		
Sun's longitude = .	134. 36. 18		
Difference of long. =	84° 31′ 36″	Log . co-sine = ...	8.979468
Moon's latitude = .	2. 30. 42	Log. co-sine = ...	9.999583
True central dist. =	84° 31′ 55″	Log. co-sine = . . .	8.979051

which exactly corresponds with the computed distance in the Nautical Almanac.

Example 3.

Required the true central distance between the moon and sun at noon, August 8th; at midnight, August 8th; at noon, August 9th, and at midnight, August 9th, 1825?

Moon's long. noon Aug. 8th =	62°22′38″		
Sun's longitude ditto = . .	135. 33. 51		
Difference of longitude = . . .	73°11′13″	Log. co-sine =	9. 461273
Moon's latitude = ...	1. 30. 9	Log. co-sine =	9. 999851
Distance at noon, Aug. 8th,	73°11′34″	Log. co-sine =	9. 461124

Moon's long. mid. Aug. 8th =	68°38′20″		
Sun's longitude ditto = . .	136. 2. 38½		
Difference of longitude = . .	67°24′18½″	Log. co-sine .	9. 584572
Moon's latitude = ...	0. 57. 33	Log. co-sine .	9. 999939
Distance at midnight, Aug. 8th=	67°24′31″	Log. co-sine =	9. 584511

Moon's long. noon, Aug. 9th =	74°59′18″		
Sun's longitude ditto = . . .	136. 31. 26		
Difference of longitude = . .	61°32′ 8″	Log. co-sine =	9. 678166
Moon's latitude = ...	0. 23. 49	Log. co-sine =	9. 999990
Distance at noon, Aug. 9th, =	61°32′11″	Log. co-sine =	9. 678156

Moon's long. at mid. Aug. 9th =	81°25′59″		
Sun's longitude at ditto = . .	137. 0. 14		
Difference of longitude = . .	55°34′15″	Log. co-sine =	9. 752346
Moon's latitude = ...	0. 10. 42	Log. co-sine =	9. 999998
Distance at midnight Aug. 9th =	55°34′16″	Log. co-sine =	9. 752344

Now, from these four consecutive lunar distances, the distances at the intermediate periods, or every third hour may be readily determined in the following manner, viz.

Find the proportional parts of the difference at the middle interval between the four distances (that is, between the second and third distances,) answering to 3 hours, 6 hours, and 9 hours: correct these proportional parts by the equation of second differences agreeably to the rule given, for that purpose, in page 34;—then, these corrected proportional parts being applied to the second lunar distance by addition or subtraction, according as the distances are increasing or decreasing, the sum or difference will be the true distances at the given periods:—thus,

	1st diff.	2d diff.	Mean 2d diff.
Aug. 8, 1825, dis. at N. = 73°11′34″	5°47′ 3″	0°5′17″	5′26″
Ditto . . . do. at M. = 67. 24. 31.	5. 52. 20	0. 5. 35	
Aug. 9, . . do. at N. = 61. 32. 11.	5. 57. 55		
Ditto . . . do. at M. = 55. 34. 16.			

The proportional parts of 5°52′20″ (the middle first difference) answering to the intermediate periods, viz.

To 3 hours it is = ...	1°28′ 5″;
Equation of second difference = ...	− 31
Proportional part corrected = ...	1°27′34
Distance at midnight, Aug. 8th = ...	67. 24. 31
Distance at 15 hours, Aug. 8th = ...	65°56′57″.
To 6 hours it is = ...	2°56′10″
Equation of second difference = ...	− 41
Proportional part corrected = ...	2°55′29″
Distance at midnight Aug. 8th = ...	67. 24. 31
Distance at 18 hours Aug. 8th = ...	64°29′ 2″
And to 9 hours it is = ...	4°24′15″
Equation of second difference = . . .	− 31
Proportional part corrected = ...	4°23′44″
Distance at midnight Aug. 8th = ...	67. 24. 31
Distance at 21 hours Aug. 8th = ...	63° 0′47″

The distances for the intermediate periods corresponding to the first and to the last 12 hours; that is, for every third hour between the first and second distances, and between the third and fourth distances, may be also very readily determined by means of the Formulæ which are given in page 117 of the Nautical Almanac for 1825.

APPENDIX.

SHOWING the direct application of logarithms to the solution of problems connected with the doctrine of compound interest; which develops the extraordinary powers of logarithmical arithmetic more than any other department of science which has been touched upon in this work.

Definition.—COMPOUND INTEREST is that which is deduced not only from the sum of money lent as the principal, but also from the interest arising thereon; which interest, as it becomes due at the stated times of payment, is added, or supposed to be added, to the principal.

Although it is illegal to lend money at compound interest, yet in purchasing annuities, pensions, or leases in reversion, it is usual to allow the purchaser compound interest for the use of his ready money.—And, these points being premised, we will proceed to the solution of the most interesting problems relating to this department of science;—for which purpose the following Tables have been computed.

Note.—The rates, or ratios, of £1 sterling contained in these tables were computed by the rule of proportion in the following manner, viz.—As £100 : £3 :: £1 to £.0300;—hence, the amount of £1 for one year is £1.0300; which, therefore, is the ratio; and so on for the rest.—The respective numbers annexed to these ratios are expressed by the common logarithms corresponding thereto:—thus, the logarithm of 1.0300 is 0.0128372; and the logarithm of .0300 is 8.4771213, and so on of others.—In this part of the work it has been deemed advisable to take out the logarithms to seven places of decimals; though for ordinary purposes six places of decimals will be found amply sufficient.

CONCISE TABLES

For facilitating the various Logarithmical Calculations connected with the Doctrine of Compound Interest.

Rates per Cent.	Table A. Amount, or Value of *One Pound* Sterling, for — Yearly Payments.		Half Yearly Payments.		Quarterly Payments.		Table B. Decimal part of the Value of *One Pound* Sterling for — Yearly Payments.		Half Yearly Payments.		Quarterly Payments.	
£.	Ratio.	Logarithm.	Ratio.	Logarithm.	Ratio.	Logarithm.	Dec. of Ratio.	Logarithm.	Dec. of Ratio.	Logarithm.	Dec. of Ratio.	Logarithm.
3.	1.0300 =	0.0128372	1.01500 =	0.0064660	1.007500 =	0.0032450	.0300 =	8.4771213	.01500 =	8.1760913	.007500 =	7.8750613
3¼	1.0325 =	0.0138901	1.01625 =	0.0070006	1.008125 =	0.0035144	.0325 =	8.5118834	.01625 =	8.2108534	.008125 =	7.9098234
3½	1.0350 =	0.0149403	1.01750 =	0.0075344	1.008750 =	0.0037835	.0350 =	8.5440680	.01750 =	8.2430380	.008750 =	7.9420081
3¾	1.0375 =	0.0159881	1.01875 =	0.0080676	1.009375 =	0.0040525	.0375 =	8.5740313	.01875 =	8.2730013	.009375 =	7.9719713
4.	1.0400 =	0.0170333	1.02000 =	0.0086002	1.010000 =	0.0043214	.0400 =	8.6020600	.02000 =	8.3010300	.010000 =	8.0000000
4¼	1.0425 =	0.0180761	1.02125 =	0.0091321	1.010625 =	0.0045900	.0425 =	8.6283889	.02125 =	8.3273589	.010625 =	8.0263289
4½	1.0450 =	0.0191163	1.02250 =	0.0096633	1.011250 =	0.0048585	.0450 =	8.6532125	.02250 =	8.3521825	.011250 =	8.0511525
4¾	1.0475 =	0.0201540	1.02375 =	0.0101939	1.011875 =	0.0051269	.0475 =	8.6766936	.02375 =	8.3756636	.011875 =	8.0746336
5.	1.0500 =	0.0211893	1.02500 =	0.0107239	1.012500 =	0.0053950	.0500 =	8.6989700	.02500 =	8.3979400	.012500 =	8.0969100
5¼	1.0525 =	0.0222221	1.02625 =	0.0112532	1.013125 =	0.0056630	.0525 =	8.7201593	.02625 =	8.4191293	.013125 =	8.1180993
5½	1.0550 =	0.0232525	1.02750 =	0.0117818	1.013750 =	0.0059309	.0550 =	8.7403627	.02750 =	8.4393327	.013750 =	8.1383027
5¾	1.0575 =	0.0242804	1.02875 =	0.0123098	1.014375 =	0.0061985	.0575 =	8.7596678	.02875 =	8.4586378	.014375 =	8.1576079
6.	1.0600 =	0.0253059	1.03000 =	0.0128372	1.015000 =	0.0064660	.0600 =	8.7781513	.03000 =	8.4771213	.015000 =	8.1760913

Problem I.

Given the Principal, Rate of Interest, and Time; to find the Amount.

Rule.

Multiply the logarithm of the ratio, Table A, by the time, or number of payments, according as the instalments may be reckoned in years, half years, or quarters; to the product of those two numbers, add the logarithm of the principal, and the sum will be the logarithm of the amount.

Remark.—The logarithm of the ratio, contained in Table A, must always be taken out so as to correspond with the nature of the instalments;—thus, if the payments are to be made yearly, the logarithm is to be taken out of the first column; if half yearly, it is to be taken out of the second column; but if quarterly, it must be taken out of the third column.—The same is to be observed of Table B;—and, in order to avoid repetition, it must be remembered that this remark is applicable to the various cases of compound interest, &c. which may be given.

Example 1.

What will a principal of £240000 sterling amount to in 45 years, by annual payments, at the rate of 5 per cent. per annum, compound interest?

Rate, 5 per cent.; ratio = 1.0500, logarithm, Table A. =	0.0211893
Multiply by the number of payments = ...	45
	1059465
	0847572
Product = ...	0.9535185
Principal = £240000, the logarithm of which is = ...	5.3802112
Amount, or improved principal = . £2156402 Log. =	6.3337297

Example 2.

What will a principal of £240000 sterling amount to in 45 years, by half-yearly payments, at the rate of 5 per cent. per annum, compound interest?

Rate, 5 per cent.; ratio = 1.02500; logarithm, Table A, =	0.0107239
Multiply by the number of payments = 45 years × 2 = . .	90
Product = ...	0.9651510
Principal = £240000, the logarithm of which is = . . .	5.3802112
Amount, or improved principal, = £2214941 Log. = .	6.3453622

Example 3.

What will a principal of £240000 sterling amount to in 45 years, by quarterly payments, at the rate of 5 per cent. per annum, compound interest?

Rate, 5 per cent.; ratio = 1.012500; log., Table A, = .	0.0053950
Multiply by the number of payments = 45 years × 4 = . .	180
	4316000
	053950
Product = ...	0.9711000
Principal = £240000, the log. of which is = ...	5.3802112
Amount, or improved principal, = £2245490 Log. = .	6.3513112

Note.—The above examples will show the reader the great evil which is attendant upon other than yearly payments of interest. Thus, in the present instance, while the principal is augmented to nine times its original value, the excess of the half-yearly above the yearly instalments is £58539; but by quarterly instalments, it is full £89088: which excess is evidently to the manifest injury of the borrower or debtor. This will appear still more evident from the following

Example.

Admitting that the national debt of Great Britain was £206590000 sterling in the year 1786, what would its probable amount be in the year 1814 (being a lapse of 28 years), by yearly, half-yearly, and quarterly payments, at the rate of 5 per cent. per annum, compound interest?

First,—For Yearly Payments:—

Rate, 5 per cent.; ratio = 1.0500; log. Table A, = . .	0.0211893
Multiply by the number of payments = ...	28
	1695144
	0423786
Product = ...	0.5933004
Amount of debt in 1786 = £206590000, the log. of which is=	8.3151093
Amount of ditto in 1814 = £809859533, very nearly Log.=	8.9084097

Second,—For Half-yearly Payments:—

Rate, 5 per cent.; ratio = 1.02500; log., Table A, = . .	0.0107239
Multiply by the number of payments = 28 years × 2 = .	56
	0643434
	0536195
Product = ...	0.6005384
Amount of debt in 1786 = £206590000, the log. of which is=	8.3151093
Amount of debt in 1814 = £823469720, very nearly Log.=	8.9156477

Third,—For Quarterly Payments:—

Rate, 5 per cent.; ratio = 1.012500; log., Table A, = .	0.0053950
Multiply by the number of payments = 28 years × 4 = .	112
	0107900
	0593450
Product = ...	0.6042400
Amount of debt in 1786 = £206590000, the log. of which is=	8.3151093
Amount of ditto in 1814 = £830518462, very nearly Log.=	8.9193493

Hence it appears, that if the national debt were £206590000 in the year 1786, and that it were allowed to multiply by the accumulation of interest upon interest, it would amount, at the end of 28 years, viz., in 1814, by annual payments, to the sum of £809859533; by half-yearly payments, to the sum of £823469720; but, by quarterly payments, to the enormous sum of £830518462 sterling! And this is a sum of such magnitude, that it could not be liquidated by all the gold and silver now in circulation amongst the different kingdoms, states, and empires, in the known world.

We will only adduce one more example in order to apprize the reader of the almost incredible manner in which a sum of money may be improved by the accumulation of interest; for which purpose, let us suppose that sixpence sterling is the sum put out at compound interest, that the rate is 6 per cent. per annum, payable by half-yearly instalments, and that the time is 450 years.—Then,

Rate, 6 per cent; ratio = 1.0300; log., Table A, = . .	0.0128372
Multiply by number of payments = 450 years × 2 = . .	900
Product = ...	11.5534800
Principal = 6*d.* sterling, or .025 £, the log. of which is = .	8.3979400
Amount, or improved principal, = £8941700000 Log. =	9.9514200

Hence it is evident, that sixpence sterling put out at *interest upon interest*, agreeably to the given rate and time, would amount to the amazing sum of eight thousand nine hundred and forty-one millions and seven hundred thousand pounds sterling; which sum could not be made up by all the gold and silver that have been dug out of the bowels of the earth from the creation of the world to the present day!

Problem II.

Given the Amount or improved Principal, Rate of Interest, and Time; to find the original Principal.

Rule.

Multiply the logarithm of the ratio, Table A, by the time or number of payments; subtract the product from the logarithm of the amount, and the remainder will be the logarithm of the original principal or sum put out at interest.

Example.

What principal put to interest for 31 years, and payable half-yearly, will amount to £29876 sterling, at 5 per cent. per annum, compound interest?

Rate, 5 per cent.; ratio = 1.02500; log., Table A, = . . .	0.0107239
Multiply by number of payments = 31 years × 2 = . . .	62
	0214478
	0643434
Product = ...	0.6648818
Amount, or improved principal, = £29876, the log. of which is =	4.4753225
Original principal, or sum put to interest, = £6463 Log. =	3.8104407

Problem III.

Given the original Principal or Sum lent, the Time, and the Amount or improved Principal; to find the Rate of Interest.

Rule.

From the logarithm of the amount or improved principal, subtract the logarithm of the original principal; divide the remainder by the number of payments, and the quotient will be the logarithm of the ratio; with which enter the proper column of Table A, according to the modes of payment, and opposite thereto, in the left-hand column, will be found the required rate of interest.

Example.

At what rate per cent. per annum will £2360 amount to the sum of £4792 in 15 years, the payments being made quarterly?

Amount, or improved principal, = £4792, log. of which is = 3.6805168
Original principal or sum = £2360, the log. of which is = 3.3729120

Divide by the number of payments = 15 years × 4 = 60) 0.3076048

The ratio to which, in Table A, is 1.011875, = quotient = 0.0051267.5
Hence, the rate of interest is 4¾ per cent. per annum.

Problem IV.

Given the original Principal or Sum lent, the Rate of Interest, and the Amount or improved Principal; to find the Time.

Rule.

From the logarithm of the improved principal, subtract the logarithm of the original principal; divide the remainder by the logarithm of the ratio, Table A, and the quotient will be the time or number of payments in years, halves, or quarters, as the case may be.

Example.

In what time will the deposits or funds in the savings' banks, which are now (June, 1826,) estimated at £14500000 sterling, amount, by half-yearly payments, to the sum of £107415024 sterling, at the rate of 4½ per cent per annum, compound interest?

Amount, or improved principal, £107415024, the log. of which is 8. 0310457
Principal, or sum funded in June, 1826, = £14500000 Log. = 7. 1613680

Rate, $4\frac{1}{2}$ per cent.; ratio = 1.02250; divide by log. of this,
Table A, = ... 0.0096633) 0.8696777
= 90 half-yearly payments: hence, the required time is 45 years.

From the above result, an abstract reasoner would be apt to imagine that the savings' banks are more of individual than of national utility.

ANNUITIES IN ARREARS, AT COMPOUND INTEREST.

Definition.—An *annuity* is said to be in *arrears* when the debtor keeps it in his hands for any certain time after the term or period of payment becomes due. The sum of all the single payments, together with the interest due upon each payment, from the time of its becoming payable to the time that the whole is paid off, is called the amount of such annuity.

PROBLEM I.

Given an Annuity, the Time or Number of Payments, and the Rate per Cent. per Annum; to find the Amount.

RULE.

Multiply the logarithm of the ratio, Table A, by the number of payments; find the natural number answering to the product, and diminish it by the integral part of the ratio, viz., 1; then, to the logarithm of this diminished natural number, add the logarithm of the annuity (proportioned to the modes of payment): from the sum of these two logarithms, subtract the logarithm of the decimal part of the ratio contained in Table B; and the remainder will be the logarithm of the amount of the annuity.

Note.—If the payments be yearly, take the logarithm of the whole annuity; if half-yearly, take the logarithm of half the annuity; and if quarterly, take the logarithm of the one-fourth part of the annuity.

Example.

If an annuity of £560 per annum be unpaid for 9 years, what will it amount to, at $4\frac{1}{2}$ per cent. compound interest, by yearly, half-yearly, and quarterly payments?

First,—For Yearly Payments :—

Rate, 4¼ per cent.; ratio = 1.0425;
log., Table A = 0.0180761
Mul. by no. of paym. = 9

Product = . . 0.1626849 Nat. no. = 1.4544033
Subtract the integral part of the ratio, viz., 1

Diminished natural number = . . . 0.4544033 Log. = 9.6574415
Annuity = £560, the log. of which is = ... 2.7481880

Sum of the two logarithms = ... 12.4056295
Decimal part of the ratio = .0425; log., Table B, = . . . 8.6283889

Amount of annuity by yearly payments = £5987.432 Log. = 3.7772406

Second,—For Half-yearly Payments :—

Rate, 4¼ per cent.; ratio = 1.02125;
log., Table A, = 0.0091321
Multiply by no. of
paym. = 9 years × 2 = 18

0730568
0091321

Product = . . 0.1643778 Nat. no. = 1.4600838
Subtract the integral part of the ratio, viz., 1

Diminished natural number = . . . 0.4600838 Log. = 9.6628370
Annuity, £560; one half of which is £280: log. of this = . 2.4471580

Sum of the two logarithms = ... 12.1099950
Decimal part of the ratio = .02125; log., Table B, = . . 8.3273589

Amount of the annuity by half-yearly paym. = £6062.382 Log. 3.7826361

Third,—For Quarterly Payments :—

Rate, 4¼ per cent.; ratio = 1.010625;
log., Table A, = 0.0045900
Multiply by no. of
paym. = 9 yrs × 4 = 36

0275400
0137700

Product = . . 0.1652400 Nat. no. = 1.4629855

Product = . . . 0.1652400 Nat. no. = 1.4629855
Subtract the integral part of the ratio, viz., 1

Diminished natural number = . . . 0.4629855 Log. = 9.6655674
Annuity, £560; one fourth of which is £140: log. of this = 2.1461280

Sum of the two logarithms = ... 11.8116954
Decimal part of the ratio = .010625; log., Table B, = . . 8.0263289

Amount of the annuity by quarterly payments = £6100.515 Log. 3.7853665

Note.—Instead of subtracting the logarithm of the decimal part of the ratio (Table B) from the sum of the logarithms of the diminished natural number and the annuity, its arithmetical complement may be added to these two logarithms; which, perhaps, will render the operation a little more concise, or, at least, apparently so. Thus:—

Diminished natural number = 0.4629855 Log. = . . 9.6655674
Annuity, £560; one fourth of which is £140 Log. = . . 2.1461280
Decimal part of ratio = .010625; log., Table B, arith. comp. = 1.9736711

Amount of the annuity by quarterly paym. = £6100.515 Log. 3.7853665

Problem II.

Given the Time or Number of Payments, the Rate per Cent. per Annum, and the Amount; to find the Annuity.

Rule.

Multiply the logarithm of the ratio (Table A) by the number of payments; find the natural number answering to the product, and diminish it by the integral part of the ratio. Then,

To the arithmetical complement of the logarithm of the diminished natural number, add the logarithm of the amount and the logarithm of the decimal part of the ratio (Table B): the sum of these three logarithms (abating 10 in the index,) will be the logarithm of the annuity for yearly payments, but of half the annuity for half-yearly payments, or of one fourth thereof for quarterly payments.

Example.

A certain annuity, at the rate of 4¼ per cent. per annum, amounted, at the end of 9 years, by annual payments, to the sum of £5987.432; by half-yearly payments, it amounted to the sum of £6062.382; but by quarterly payments, to the sum of £6100.515; required the yearly value of that annuity?

First,—For Annual Payments:—

Rate, 4¼ per cent.; ratio = 1.0425;
log., Table A, = 0.0180761
Mult. by no. of paym. = 9

Product = . . 0.1626849 Nat. no. 1.4544033
Subt. the integral part of the ratio, viz., 1

Diminished natural number = . . 0.4544033 Log. ar. co. 0.3425585
Amount of annuity by annual payments = £5987.432 Log. = 3.7772406
Decimal part of the ratio = .0425; log., Table B, = . . 8.6283889

Yearly value of the annuity = £560 Log. = ... 2.7481880

Second,—For Half-yearly Payments:—

Rate, 4¼ per cent.; ratio = 1.02125;
log., Table A, = 0.0091321
Mult. by no. of paym.
= 9 years × 2 = 18

0730568
0091321

Product = . . 0.1643778 Nat. no. 1.4600838
Subt. the integral part of the ratio, viz., 1

Diminished natural number = . . 0.4600838 Log. ar. co. 0.3371630
Amount of annuity by half-yearly payments = £6062.382 Log. 3.7826361
Decimal part of the ratio = .02125; log., Table B, = . . 8.3273589

Half the value of the annuity = ... £280 Log. = 2.4471580

Yearly value of the annuity = ... £560, as required.

Third,—For Quarterly Payments :—

Rate, $4\frac{1}{4}$ per cent.; ratio = 1.010625;
log., Table A, = 0.0045900
Mult. by no. of paym.
= 9 years × 4 = 36

00275400
00137700

Product = . . 0.1652400 Nat. no. 1.4629855 Log. ar. co. 0.3344326
Amount of annuity by quarterly payments = £6100.515 Log. = 3.7853665
Decimal part of the ratio = .010625; log., Table B, = 8.0263289

One fourth the value of the annuity = . . . £140 Log. = 2.1461280

Yearly value of the annuity = ... £560, as required.

Problem III.

Given the Annuity, the Rate per Cent. per Annum, and the Amount, with the Modes of Payment; to find the Time, or Number of Payments.

Rule.

To the logarithm of the amount, add the logarithm of the decimal part of the ratio (Table B); find the natural number answering to the sum of these two logarithms, and increase it by the whole, the half, or the fourth of the annuity, according as the instalments may be annual, half-yearly, or quarterly. From the logarithm of this increased natural number, subtract the logarithm of the whole, the half, or the fourth of the annuity, according to the manner in which the instalments may be made payable: divide the remainder by the logarithm of the ratio (Table A), and the quotient will express the time, or number of payments; which will be in years, if those be annual; otherwise, in half years or quarters, as the case may be. The latter expressions are to be divided by 2 or 4, to find the time.

Example.

An annuity of £560, at $4\frac{1}{4}$ per cent. per annum, amounted, by annual payments, to the sum of £5987.432; by half-yearly payments, to the sum of £6062.382; but, by quarterly payments, to the sum of £6100.515; required the time corresponding to each mode of payment?

First,—For Yearly Payments :—

Amount by yearly payments
=£5987.432 Log. 3.7772406
Dec. part of the ratio,
Tab. B, .0425 Log. 8.6283889

Sum of the two logs. = 2.4056295 Nat. no. 254.46585
Whole annuity for yearly payments = £560

Increased natural number = . . . 814.46585 Log. 2.9108729
Whole annuity = £560, the log. of which is = . . . 2.7481880

Rate, 4¼ per cent.; ratio = 1.0425; divide by log. of this,
Table A, = ... 0.0180761) 0.1626849 = 9
hence the time, by annual payments, is 9 years.

Second,—For Half-yearly Payments :—

Amount by half-yearly payments
=£6062.382 Log. 3.7826361
Dec. part of the ratio,
Tab. B, .02125 Log. 8.3273589

Sum of the two logs. 2.1099950 Nat. no. 128.82347
Half the given annuity = ... £280

Increased natural number = . . . 408.82347 Log. 2.6115358
Half the given annuity = £280, the log. of which is = 2.4471580

Rate, 4¼ per cent.; ratio = 1.02125; divide by log. of this,
Table A, = ... 0.0091321) 0.1643778 = 18

Now, 18 half-yearly payments, divided by 2, show the time to be 9 years.

Third,—For Quarterly Payments :—

Amount by quarterly payments
=£6100.515 Log. 3.7853665
Dec. part of the ratio,
Tab. B, .010625 Log. 8.0263289

Sum of the two logs. = 1.8116954 Nat. no. 64.81797

Sum of the two logs. = 1.8116954 Nat. no. 64.81797
One fourth of the given annuity = . £140

Increased natural number = . . . 204.81797 Log. 2.3113680
One fourth of the annuity = £140, the log. of which is = 2.1461280

Rate, 4¼ per cent. ; ratio = 1.010625 ; divide by log. of
this, Table A, = ... 0.0045900) 0.1652400 = 36

Now, 36 quarterly payments, divided by 4, give 9 years; which, therefore, is the required time.

PRESENT WORTH OF ANNUITIES IN ARREARS, AT COMPOUND INTEREST.

Definition.—When an annuity, to be entered on immediately, is sold for ready money, the price which ought to be paid for it is called its *present worth.*

Problem I.

Given an Annuity, the Time of its Continuance, and the Rate per Cent. per Annum; to find the present Worth of that Annuity.

Rule.

Multiply the logarithm of the ratio (Table A) by the number of payments; subtract the product from the logarithm of the whole, the half, or the fourth of the annuity, according to the mode of payment: find the natural number of the remainder, and subtract it from the whole, the half, or the fourth of the annuity, according as the instalments may be yearly, half-yearly, or quarterly; find the logarithm of this difference, from which let the logarithm of the decimal part of the ratio (Table B) be subtracted, and the remainder will be the logarithm of the present worth of the annuity.

Example.

An annuity of £365 is to be continued 7 years; required the present value thereof, by yearly, half-yearly, and quarterly payments; allowing the purchaser 5 per cent. per annum, compound interest, for the use of his ready money?

First,—For Yearly Payments :—

Rate, 5 per cent.; ratio = 1.0500;
log., Table A, = . 0.0211893
Mult. by no. of payments= 7

Product = ... 0.1483251
Whole annuity, £365 Log. 2.5622929 . . £365

Remainder = . . . 2.4139678 Nat. no. 259.39869

Difference between nat. number and annuity = 105.60131 Log. 2.0236693
Decimal part of the ratio, Table B, = .0500 Log. = . 8.6989700

Present worth of given annuity by yearly paym. £2112.0267 Lg. 3.3246993

Second,—For Half-yearly Payments :—

Rate, 5 per cent.; ratio = 1.02500;
log., Table A, = . 0.0107239
Multiply by number of
payments = 7 × 2 = 14

0428956
0107239

Product = ... 0.1501346
Half annuity, £182.5 Log. 2.2612629 . . £182.5

Remainder = ... 2.1111283 Nat. no. 129.16009

Diff. between nat. no. and half the annuity = 53.33991 Log. 1.7270523
Decimal part of the ratio = .02500 Log., Table B, = . 8.3979400

Present worth of annuity by half-yearly paym. £2133.5966 Log. 3.3291123

Third,—For Quarterly Payments :—

Rate, 5 per cent.; ratio = 1.012500;
log., Table A, = . 0.0053950
Multiply by number of
payments = 7 years × 4 = 28

0431600
0107900

Product = ... 0.1510600

Product = ... 0.1510600
One 4th of ann. £91.25 Lg. 1.9602329 . . £91.25

Remainder = . . . 1.8091729 Nat. no. 64.442582

Diff. betw. nat. num. and one 4th of annuity = 26.807418 Log. 1.4282550
Decimal part of the ratio = .012500 Log., Table B, = 8.0969100

Present worth of the ann. by quarterly paym. £2144.5936 Log. 3.3313450

Problem II.

Given the present Worth of an Annuity, the Time of its Continuance, and the Rate of Interest; to find the yearly Value of that Annuity.

Rule.

Multiply the logarithm of the ratio (Table A) by the number of payments; take the arithmetical complement of the product, find the natural number corresponding thereto, and let its arithmetical complement or *difference to zero* be noted. Then, to the arithmetical complement of the logarithm of this *difference to zero*, add the logarithm of the purchase-money, or present worth of the annuity, and the logarithm of the decimal part of the ratio (Table B): the sum of these three logarithms (abating 10 in the index,) will be the logarithm of the annuity, or of its half, or quarter, according as the mode of payment may be by yearly, half-yearly, or quarterly instalments.

Example.

The purchase-money, or present worth of an annuity, which is to continue 7 years, amounts, by yearly payments at 5 per cent. per annum, to the sum of £2112.0267; by half-yearly payments, to the sum of £2133.5966; but by quarterly payments, to the sum of £2144.5936; required the yearly value of the annuity, agreeably to each mode of payment?

First,—For Yearly Payments:—

Rate, 5 per cent.; ratio = 1.0500;
log., Table A, = 0.0211893
Mult. by no. of paym. 7

Product = . . 0.1483251

Product = . . . 0. 1483251

Arith. comp. = 9. 8516749 Nat.no. 0. 7106

Ar. co. of nat. num., or diff. to zero = 0. 2893
Purchase-money, or present worth of the annuity
payments, = £2112. 0267 Log. = .
Decimal part of the ratio, Table B, = .0500

Yearly value of the annuity = £365 Log.

Second,—For Half-yearly Pa

Rate, 5 per cent.; ratio = 1. 02500;
log., Table A, = 0. 0107239
Multiply by no. of
paym.=7yrs × 2= 14

0428956
0107239

Product = . . . 0. 1501346

Arith. comp. = 9. 8498654 Nat. no. 0. 70

Ar. co. of nat. num., or diff. to zero, = 0. 29
Purchase-money, or present worth of the ann
yearly payments, £2133. 5966 Log. =
Decimal part of the ratio, Table B, = . .0250

Half the annuity = . . . £182. 5 Log

Yearly value of the annuity=£365, as require

Third,—For Quarterly Pa

Rate 5 per cent; ratio = 1. 012500;
log., Table A., = 0. 0053950
Multiply by no. of
paym.=7yrs × 4= 28

0431600
0107900

Product = . . . 0. 151060

Product = . . 0.151060

Arith. comp. = 9.8489400 Nat. no. 0.7062200

Ar. co. of nat. num., or diff. to zero, = 0.2937800 Log. ar. co. 0.5319778
Purchase money or present worth of the annuity by quarterly payments, £2144.5936 Log. = ... 3.3313450
Decimal part of the ratio, Table B. = .012500 Log. . . 8.0969100

One fourth of the annuity = £ 91.25 Log. = ... 1.9602328

Yearly value of the annuity = £365, as required.

Problem III.

Given an Annuity, the present worth of that Annuity, and the Rate per Cent. per Annum; to find the Time of its Continuance.

Rule.

To the logarithm of the present worth of the annuity, add the logarithm of the ratio, Table A., and find the natural number corresponding to their sum;—augment the present worth of the annuity by the *yearly value of the annuity*, its *half*, or *quarter*, according to the modes of payment; and find the difference between it and the natural number found as above:—find the logarithm of this difference, and subtract it from the logarithm of the given annuity, its half or quarter, as the case may be:—divide the remainder by the logarithm of the ratio, Table A, and the quotient will be the number of payments; which will be in years if those payments be annual; otherwise in halves, or quarters according to the modes of instalment.

Example.

An annuity of £365, can be purchased for the sum of £2112.0267, by annual payments; for £2133.5966, by half-yearly payments, or for £2144.5936, by quarterly payments; the purchaser is to be allowed 5 per cent. per annum for the use of his ready money; required the time of the continuance of that annuity agreeably to each mode of payment?

First,—For Yearly Payments:—

Present worth of the ann. = £2112.0267 Log. = 3.3246993
Rate, 5 per cent; ratio, Tab. A. = 1.0500 Log. = 0.0211893

Sum of the two logs. = 3.345886

Sum of the two logs.=3.3458886 Nat.N.=2217.6031
Present worth of the annuity =
£2112.0267 + whole ann. £365 = 2477.0267

Difference = ... 259.4236 Log.=2.4140095
Given annuity £365, the log. of which is = ... 2.5622929
Rate 5 per cent; ratio = 1.0500; divide
by logarithm of this, Table A. = ... 0.0211893) 0.1482834
= 7 payments;—hence the time, or continuance of the annuity is 7 years.

Second;—For Half Yearly Payments:—

Present worth of the ann. =
£2133.5966 Log.=3.3291123
Rate 5 per cent;
ratio, Tab. A. =
1.02500 Log.=0.0107239

Sum of the two logs.=3.3398362 Nat.N.=2186.9367
Present worth of the annuity =
£2133.5966 + half ann. £182.5 =2316.0966

Difference = ... 129.1599 Log.=2.1111277
Half given ann.=£182.5 the log. of which is 2.2612629
Rate 5 per cent; ratio = 1.02500; divide
by logarithm of this, Table A. = . . . 0.0107239) 0.1501352
= 14 payments; and 14 ÷ 2 = 7 years, is the time or continuance of the annuity, as required.

Third,—For Quarterly Payments:—

Present worth of the ann. =
£2144.5936 Log.=3.3313450
Rate 5 per cent; ratio
1.012500
Log. Tab. A.= 0.0053950

Sum of the two logs.=3.3367400 Nat.N.=2171.4010
Present worth of the annuity =
£2144.5936+one fourth
of the ann. £91.25 = ... 2235.8436

Difference = ... 64.4426 Log.=1.8091730
One fourth of the given ann. = £91.25, the log. of which is=1.9602329
Rate 5 per cent; ratio = 1.012500: divide
by logarithm of this, Table A. = . . . 0.0053950) 0.1510599
= 28 payments; now 28 ÷ 4 = 7 years, which, therefore, is the time required.

PRESENT WORTH OF ANNUITIES IN REVERSION, AT COMPOUND INTEREST.

Definition.—An annuity is said to be in *reversion* when it is not to be entered upon until some particular event has happened, or until some certain period has elapsed after the time of its sale :—the ready money which should be paid down for an annuity of this description is called its *present worth.*

Problem I.

Given an Annuity in Reversion, the Time of its Continuance, the Period at which it is to be entered upon, with the Rate of Interest; to find its present Worth.

Rule.

Find the logarithm of the present worth of the annuity agreeably to the time of its continuance, as if it were to be entered upon immediately, by Problem I., page 700; from this logarithm subtract the logarithm of the ratio, Table A., multiplied by the time which is to elapse before the purchaser enters upon the annuity, and the remainder will be the logarithm of the present worth of the annuity in reversion.

Note.—If the annuity be made payable by half-yearly, or quarterly instalments, the ratio, &c. &c. are to be proportioned accordingly, the same as in the preceding Problems.

Example.

What is the present worth of the reversion of a lease of £475 per annum, by yearly payments, to continue 25 years, but not to be entered upon till the end of 7 years after the time of sale; allowing the purchaser $5\frac{1}{2}$ per cent. for present payment?

Rate $5\frac{1}{2}$ per cent.; ratio, Tab. A. = 1.0550 Log. = ...			0.0232525
Multiply by number of payments = ...			25
			1162625
			0465050
Product = ...			0.5813125
Annuity, or lease = £475, ...		Logarithm =	2.6766936
Natural Number =	124.5607	Logarithm =	2.0953811
Difference = . .	350.4393		

and annuity, or lease = . . 350.4393 Log. = 2.5446128
Decimal part of the ratio, Tab. B=.0550 Log. = 8.7403627
Logarithm of the present worth of the lease, supposing it were to be entered upon immediately =3.8042501=£6371.6
Rate 5½ per cent.; ratio, Tab. A.=1.0550 Log.=
0.0232525 × 7 years = ... 0.1627675

Present worth of the rev. = £4380.0848 Log.=3.6414826

Note.—The latter part of the operation may be abridged in the following manner, viz:—

To the logarithm of the difference between the natural number and the given annuity or lease, add the arithmetical complement of the logarithm of the decimal part of the ratio, Table B., and the arithmetical complement of the product of the logarithm of the ratio, Table A., by the time which is to elapse before the purchaser enters upon the annuity; the sum of these three logarithms, abating 10 in the index, will be the logarithm of the present worth of the annuity or lease in reversion.

Thus. Diff. betw. nat. numb. and ann. or lease=350.4393 L.=2.5446128
Dec. part of ratio, Tab. B.=.0550 Log. arith. comp.= 1.2596373
Ratio, Tab. A.=1.0550 L.=0.0232525 × 7 ys. Ar. com. 9.8372325

Present worth of the reversion =£4380.0848 Log.= 3.6414826

Problem II.

Given the present Worth of an Annuity in Reversion, the time of its continuance, and the Period at which it is to be entered upon, with the Rate of Interest; to find the yearly value of the Annuity.

Rule.

Multiply the logarithm of the ratio, Table A., by the number of payments or time of continuance: take the arithmetical complement of the product, and find the natural number corresponding thereto, and let its arithmetical complement, or *difference to zero* be noted:—then, to the arithmetical complement of the logarithm of this *difference to zero,* add the logarithm of the decimal part of the ratio, Table B., the product of the logarithm of the ratio, Table A. by the time that is to elapse before entering upon the reversion, and the logarithm of the present worth of the reversion; the sum of these four logarithms, abating 10 in the index, will be the logarithm of the yearly value of the annuity.

Note.—If the annuity be made payable by half-yearly, or quarterly instalments, the ratio, &c. &c. are to be proportioned accordingly, the same as in the preceding Problems.

Example.

What is the yearly value of an annuity or lease, to be entered upon 7 years hence, and then to continue 18 years, that is 25 years continuance, and which may be bought for the sum of £4380.0848, ready money, allowing the purchaser $5\frac{1}{2}$ per cent. compound interest?

Rate $5\frac{1}{2}$ per cent; ratio, Tab. A. = 1.0550 Log. = . . .			0.0232525
Multiply by number of payments = ...			25
			1162625
			0465050
Product = ...			0.5813125
Arithmetical complement = ...			9.4186875
The natural number answering to arithmetical complement, considered as a Logarithm, is =0.2622331			
Difference to zero = . . .	0.7377669	Log. ar. comp.	0.1320808
Dec. part of the ratio, Tab. B. =	.0550	Logarithm=	8.7403627
Ratio, Table A.=1.0550 Log.=	0.0232525 × 7 years =		0.1627675
Present worth of the reversion =	£4380.0848	Logarithm =	3.6414826
Yearly value of the annuity or lease =	£475	Logarithm =	2.6766936

Problem III.

Given an Annuity in Reversion, its present Worth, the Period at which it is to be entered upon, and the Rate of Interest; to find the Time of its Continuance.

Rule.

To the logarithm of the present worth of the reversion, add the product of the logarithm of the ratio, Table A., by the time that is to elapse before entering upon the annuity, and the logarithm of the decimal part of the ratio, Table B; the sum of these three logarithms, abating 10 in the index, will be the logarithm of a natural number. Take the difference between the natural number, thus found, and the yearly, half-yearly, or quarterly value of the annuity, according to the nature of the instalments; subtract the logarithm of this difference from the logarithm of the yearly, half-yearly, or quarterly value of the annuity, as the case may be: divide the remainder by the logarithm of the ratio, Table A., and the quotient will be the whole time of the continuance of the given annuity or lease; which will be in years, halves, or quarters, according to the mode of payment.

An annuity or lease worth £475 per annum (to be entered upon at the end of 7 years after the time of sale,) may be purchased for the sum of £4380.0848; required the time of the continuance of that annuity, allowing the purchaser $5\frac{1}{2}$ per cent. for the use of his ready money?

Present worth of the annuity or lease=£4380.0848		Log. =	3.6414826
Ratio, Tab. A.=1.0550 Log.=0.0232525×7 yrs. (Rate $5\frac{1}{2}$ per cent.) = ...			0.1627675
Decimal part of the ratio, Tab. B. =	.0550	Logarithm =	8.7403627
Natural number = . . .	350.4393	Logarithm =	2.5446128
Yearly value of given ann.=	475.	Logarithm =	2.6766936
Difference = ...	124.5607	Logarithm =	2.0953811

Rate $5\frac{1}{2}$ per cent; ratio, Table A.=1.0550; divide by logarithm of this = ... 0.0232525) 0.5813125 = 25 payments;—hence the time, or continuance of the annuity is 25 years, as required.

Note.—Should it be required to find the time that should elapse between the periods of purchasing and entering upon the annuity or lease, it may be very readily determined by an indirect solution of Problem I., page 706;—as thus:

From the logarithm of the present worth of the annuity determined as if it were to be entered upon immediately, subtract the logarithm of the present worth thereof in reversion:—divide the remainder by the logarithm of the ratio, Table A., and the quotient will be the time required;—this, it is presumed, is so very obvious as not to require the illustration of an example.

PRESENT WORTH OF FREEHOLD ESTATES, OR PERPETUAL ANNUITIES,

At an Assigned Rate per Cent. Compound Interest, to be entered on immediately.

Definition.—Freehold Estates, or *perpetual Annuities* signify any interest of money, rents, pensions, grants, &c. payable yearly, half yearly, or quarterly, and to continue for *ever:*—in buying these, the purchaser is allowed a certain per centage for his ready money; which money is called the *present worth* of the perpetual annuity, to be entered on immediately.

Problem I.

Given the Yearly Rent of a freehold Estate, or perpetual Annuity, and the Rate per Cent.; to find the present Worth thereof.

Rule.

From the logarithm of the yearly, half-yearly, or quarterly value of the given perpetual annuity, according to the mode of payment (the index being increased by 10), subtract the logarithm of the decimal part of the ratio corresponding to such payment, Table B., and the remainder will be the logarithm of the present worth of the given perpetual annuity, &c.

Example.

What is the present worth of a freehold estate, or perpetual annuity of £360, per annum, payable half yearly, allowing the purchaser 4¾ per cent. compound interest, for his ready money?

Given perpetual annuity=£360; half of which is £180 Log.=2.2552725
Rate, 4¾ per cent.; dec. part of ratio, Tab. B=.02375 Log.=8.3756636

Present worth of the given freehold estate=£7578.93 Log.=3.8796089

Problem II.

Given the present Worth of a freehold Estate, or perpetual Annuity, and the Rate per Cent.; to find the yearly Value of that Estate, or perpetual Annuity.

Rule,

To the logarithm of the present worth of the given perpetual annuity, add the logarithm of the decimal part of the ratio (Table B), according to the mode of payment; and the sum will be the logarithm of the yearly value of the annuity, or of its half or quarter, as the case may be.

Example.

If a freehold estate, or perpetual annuity, be bought for £7578.93, what ought the yearly value thereof to be, allowing the purchaser 4¾ per cent. compound interest, for his ready money; the rent being payable half-yearly?

Present worth of the given perpetual annuity=£7578.93 Log. 3.8796089
Rate, 4¾ per cent.; decimal part of the ratio, Table B, =
.02375 (half-yearly payments) Log. = ... 8.3756636

Half-yearly value of the annuity = £180 Log. = ... 2.2552725

Yearly rent of the freehold estate=£360, as required.

PROBLEM III

Given the yearly Value of a freehold Esta the present Worth thereof; to fin

RULE.

From the logarithm of the yearly, half-y given perpetual annuity, according to the being increased by 10), subtract the logari that annuity, and the remainder will be the of the ratio: with this enter Table B., and th thereto will be found standing abreast there Table A.; which will be the rate per cent. re

Example.

If a freehold estate, or perpetual annuity half-yearly, be sold for £ 7578. 93, what i interest, allowed to the purchaser?

Yearly value of the annuity, £360, half of wl
Present worth of ditto = £7578. 93. the lo
Rate, 4¾ per cent., Table A, = decimal par
Table B, = .02375 Log. = . .

PRESENT WORTH OF FREEHOLD E ANNUITIES IN RE

At an assigned Rate per Cent.,

Definition.—Freehold estates, or *perpetu* any interest of money, rents, pensions, gra yearly, or quarterly, and to continue for e entered upon until some particular event specified time has elapsed after the time of purchaser holds the freehold, &c., after he *the reversion;* and the money which he called its present worth.

Problem I.

Given the yearly Rent of a freehold Estate, or perpetual Annuity, the Time at which it is to be entered upon, and the Rate per Cent.; to find the present Worth of the Reversion of that Estate.

Rule.

To the product of the logarithm of the ratio (Table A) by the time that is to elapse before entering upon the reversion, add the logarithm of the decimal part of the ratio (Table B): subtract the sum of these two logarithms from the logarithm of the yearly value of the given perpetual annuity (the index being increased by 10), and the remainder will be the logarithm of the present worth of that perpetual annuity, or freehold estate.

Example.

The reversion of a freehold estate, or perpetual annuity of £490 per annum, to be entered upon 7 years hence, is to be sold; what is its present worth, allowing the purchaser 5 per cent., compound interest, for his ready money?

Rate, 5 per cent.; ratio, Table A, = 1.0500 Log. = .	0.0211893
Mult. by given time, viz., the time before entering upon the revers.	7
Product = ...	0.1483251
Decimal part of the ratio, Table B, = .0500 Log. = .	8.6989700
Sum of the two logarithms	8.8472951
Yearly value of the estate, or perpetual annuity, = £490 Log. =	2.6901961
Present worth of the reversion = £6964.6774 Log. = .	3.8429010

Problem II.

Given the present Worth of a freehold Estate, or perpetual Annuity in Reversion, the Time at which it is to be entered upon, and the Rate per Cent.; to find its yearly Value.

Rule.

To the product of the logarithm of the ratio (Table A) by the time that is to elapse before entering upon the reversion, add the logarithm of the decimal part of the ratio (Table B), and the logarithm of the present worth of the reversion of the given perpetual annuity: the sum of these three logarithms (abating 10 in the index,) will be the logarithm of the yearly value of that perpetual annuity.

If a freehold estate, or perpetual annuity, to be entered upon 7 years hence, be sold for £ 6964. 6774, what is the yearly value thereof, allowing the purchaser 5 per cent., compound interest, for his ready money?

Rate, 5 per cent.; ratio, Table A, = 1.0500	Log. = .	0.0211893
Mult. by given time, viz., the time before entering upon the revers.		7
Product = ...		0.1483251
Decimal part of the ratio, Table B, = .0500	Log. = .	8.6989700
Present worth of the reversion=£6964.6774	Log. = .	3.8429010
Yearly value of the given perpetual annuity = £490		Log.=2.6901961

Problem III.

Given the yearly Value of a freehold Estate, or perpetual Annuity in Reversion, the Rate per Cent., and its present Worth; to find the Time that must elapse before entering upon the Reversion.

Rule.

To the logarithm of the present worth of the reversion, add the logarithm of the decimal part of the ratio (Table B); subtract the sum of these two logarithms from the logarithm of the yearly value of the given perpetual annuity: now, the remainder being divided by the logarithm of the ratio (Table A), the quotient will express the time that must elapse before entering upon the reversion of that perpetual annuity.

Example.

If a freehold estate, or perpetual annuity of £490 per annum, be sold for £6964.6774, in what time hence will the purchaser be entitled to enter thereon, allowing him 5 per cent., compound interest, for the use of his ready money?

Pres. worth of the revers. of given annuity=6964.6774		Log. 3.8429010
Rate, 5 per cent; dec. part of ratio, Table B, 0.500		Log. 8.6989700
Sum of the two logarithms = ...		2.5418710
Yearly value of the given perpetual annuity £ 490		Log. = 2.6901961
Rate, 5 per cent.; ratio, Table A,=1.0500; divide by log. of this = ...	0.0211893)	0.1483251=7

Hence, the purchaser will be entitled to enter upon the *reversion* at the end of 7 years.

Problem.

To find in how many Years any Principal or Sum of Money will double itself, at compound Interest, by yearly, half-yearly, or quarterly Payments.

Rule.

Let the logarithm of the ratio (Table A) be considered as the *decimal part of a natural number*; find the logarithm corresponding thereto, and subtract it from the constant logarithm 9.4786098 : the remainder will be the logarithm of the time in which a given sum of money will double itself at any proposed rate of interest within the limits of Table A: if the payments of interest be annual, the time will be expressed in years; otherwise, in half years, or quarters of years, as the case may be.

Note.—The constant logarithm is thus determined:—Let the *double* of any given sum of money be represented by the number 2, the logarithm of which is 0.3010300; consider this as the decimal part of a natural number: then, the logarithm corresponding thereto is 9.4786098; which, therefore, becomes a constant expression for all modes of payment and rates of interest.

Example.

Required the number of years in which any given sum of money will double itself, at compound interest, by yearly, half-yearly, and quarterly payments; the rate being 5 per cent. per annum?

First,—For Yearly Payments:—

Constant log. = ... 9.4786098
Rate, 5 per cent.; log., Table A, = 0.0211893; consider this as the decimal part of a natural number, the log. of which is = 8.3261167

Number of years, as required, = 14.2067 Log. = . . 1.1524931

Second,—For Half-yearly Payments:—

Constant log. = ... 9.4786098
Rate, 5 per cent.; log, Table A, = 0.0107239; consider this as the decimal part of a natural number, the log. of which is = 8.0303528

Time, in half years, = . . . 28.07094 Log. = . 1.4482570

Number of years, as required, = 14.03547; which, therefore, is the time in which a sum of money will double itself, at 5 per cent. compound interest, by half-yearly payments.

Third,—For Quarterly Pa

Constant log. = ...
Rate, 5 per cent.; log., Table A,=0. 005395(
the decimal part of a natural number, the log

Time, in quarters of years, = 55. 79797

Number of years, as required, 13. 94949; whi
which a sum of money will double itsel
interest, by quarterly payments.

It is after this manner that the followin
excepting, however, the last column, or that
merely expressed by the quotient of £100, d
cent.: thus, £100 ÷ £5 = 20 years; which,
a given sum of money will double itself, at 5
interest.

A Table,

Exhibiting the Time in which any Sum of several given Rates per Cent. per Annum, at Simple Interest.

Rates per Cent.	Time, for compound Interes	
	Yearly Payments.	Half-yearly Payments.
£	Years.	Years.
3	23. 4498	23. 2779
3¼	21. 6723	21. 5003
3½	20. 1488	19. 9770
3¾	18. 8284	18. 6567
4	17. 6730	17. 5013
4¼	16. 6535	16. 4820
4½	15. 7473	15. 5759
4¾	14. 9365	14. 7652
5	14. 2067	14. 0355
5¼	13. 5464	13. 3753
5½	12. 9461	12. 7752
5¾	12. 3981	12. 2273
6	11. 8957	11. 7249
6½	11. 0067	10. 8361
7	10. 2448	10. 0744
8	9. 0065	8. 8365
9	8. 0432	7. 8736
10	7. 2725	7. 1033

By the above table it is evident, that if a
compound interest, at the rate of 5 per ce
itself, by yearly payments of the interest,

payments, in 14 years; and by quarterly payments, in $13\frac{9}{10}$ years; whilst at simple interest, it will not double itself in less than 20 years. Hence, if the given sum be £1250, it will amount, in $13\frac{9}{10}$ years, by quarterly payments, to the sum of £2500; in $27\frac{9}{10}$ years, to the sum of £5000; in $41\frac{8}{10}$ years, to the sum of £10000; in $55\frac{8}{10}$ years, to the sum of £20000; and so on in geometrical progression: while at simple interest, the same sum would only amount, in the same space of time, viz., $55\frac{8}{10}$ years, to the sum of £4737. 7s. 6d.

Remark.—The preceding problems and examples contain all that is essentially necessary to be known, independently of theory, in the doctrine of compound interest. The author is not aware that the direct logarithmical solutions of the various complex cases connected with this subject have been given by any other writer: many, indeed, have published theorems for this purpose; but these theorems (such as those given by the late ingenious Dr. Maskelyne, in his very learned Introduction to Taylor's Logarithms, under the head *Compound Interest*,) are expressed in such a scientific manner as to be of little use, in a mere practical point of view; being much better adapted for employing the minds of the curious in mathematical researches and investigations, than for abridging the labour attendant on arithmetical computations.

THE USE OF THE GENERAL VICTUALLING TABLE,

Contained in Vol. II., Page 661.

As this Table contains the exact daily proportion of sea provisions for any given number of men within the ordinary limits of victualling, it will be found of considerable utility to the Pursers of the Royal Navy, in closing their annual accounts, and in completing the ship's provisions to any specified time: it will also be of great use to officers serving as Commanders and Pursers;—and, perhaps, to those gentlemen in the Victualling Department of His Majesty's service who are employed in the examining and auditing of the Naval Victualling accounts of the above-mentioned officers.

Remarks.

1.—As the size of the page would not admit of separate columns being employed for the salt beef and salt pork, and as the allowance of each of these species is precisely the same; one column only has been introduced into each page of the Table on account of those articles of victualling;—which column contains the exact proportion of each.—Hence, in taking out the proportions from this column, corresponding to any given number of men, care must be taken to put down such proportions *twice*; that is, first for salt beef, and then for salt pork; or, otherwise, to *double* those proportions, at once, for salt meat generally.

2.—In like manner, as the size of the page would not admit of separate columns being employed for bread and beer, and for oatmeal and vinegar; it will be necessary, in taking out the proportions of those species, corresponding to any given number of men, to put down as many gallons of beer as the *second column* expresses pounds of bread; and as many gallons of vinegar as the last column expresses gallons of oatmeal.

The following problems will illustrate the principal uses to which this Table may be applied.

Problem I.

Given the Number of Men victualled for one Day, to find the corresponding Proportion of each Species of Provisions.

Rule.

If the given number can be found in the left-hand column of the table, the corresponding proportion of each species of provisions will be found abreast of it in the same horizontal line; but if it cannot be exactly found, which in general will be the case, write down any two or more of the *tabular numbers* that will make up the given one, opposite to which put down the corresponding quantities of provisions: then, the sums of these quantities will be the true proportion of each species of provisions.

Example.

Let the number of men victualled for one day be 45685; required the true proportion of each species of provisions corresponding thereto?

Men.		Bread.	Beer.	Salt Meat.	Flour.	Pease.	Sugar.	Cocoa.	Tea.	Oatmeal.	Vinegar.
45000	give	45000	45000	33750.0	16875.0	1406.2	4218.12	2812.8	703.2	401.6.4	401.6.4
600	do.	600	600	450.0	225.0	18.6	56.4	37.8	9.6	5.2.12	5.2.12
85	do.	85	85	63.12	31.14	2.5¼	7.15½	5.5	1.5¼	0.6.1	0.6.1
45685	give	45685	45685	34263.12	17131.14	1427.5¼	4282.15½	2855.5	713.13¼	407.7.3	407.7.3

Note.—In making out the Purser's annual victualling account, the proportions of salt beef and pork, as given in the table, are to be doubled, or thrown into one sum under the head of *salt meat*, as above.

If there be any *fresh meat* issued during the period of the account, subtract the amount thereof from the number victualled, and then take out the proportion of salt meat, flour, and pease, corresponding to the remainder: thus, suppose the quantity of *fresh meat* issued to be 22238 pounds.

No. vic. for one day is 45685
Lbs. fr. meat issued = 22238

				Salt meat.	Flour.	Pease.
Remainder = .	23447.	Now,	23000 gives	17250.0	8625.0	718.6
			400 do.	300.0	150.0	12.4
			47 do.	35.4	17.10	1.3¾
			23447 gives	17585.4	8792.10	732.5¼

See Pursers' Instructions (Appendix), No. 21, page 117.

Problem II.

Given the Complement of Men, and the Number of Days for which they are to be victualled; to find the Proportion of each Species of Provisions.

Rule.

Multiply the complement of men by the given number of days for which they are to be provisioned, and the product will be the number to be directed in the last problem.

Example.

Let the complement of a ship be 275 men, and the time for which they are to be victualled 4 lunar months or 112 days; required the proportion of each species of sea provisions?

Given complement of men = 275
Multiply by time, in days, = 112

Product = ... 30800, which is the number to be victualled for one day.

Men.		Bread.	Spirits.	Salt Beef.	Salt Pork.	Flour.	Pease.	Sugar.	Cocoa.	Tea.	Oatmeal.	Vinegar
30000	give	30000	937.4	11250	11250	11250	937.4	2812.8	1875	468.12	267.6.12	267.6.12
800	do.	800	25.0	300	300	300	25.0	75.0	50	12.8	7.1. 2	7.1. 2
30800	give	30800	962.4	11550	11550	11550	962.4	2887.8	1925	481.4	275.0.0	275.0.0

And these are the exact proportions of the different species of provisions for the given complement and time.

Remarks.

1. Since the salt beef is generally cut up in 8 lbs. pieces, and the pork in 4 lbs. pieces, the pounds of salt beef are to be divided by 8, and the pounds of salt pork by 4: the respective quotients will be the number of pieces of each species. Thus, in the above case, the number of pieces of salt beef is 1443¾, and of pork 2887½.

2. It being customary to substitute a proportion of raisins and suet for a part of the flour, a deduction is to be made from the full allowance of the latter, on account of such quantities of those substitutes as it may be deemed advisable to demand from the victualling stores; observing that one pound of raisins is equal to one pound of flour, and that half a pound of currants or half a pound of suet is to be considered as being equal to one pound of raisins or one pound of flour.

3. As tobacco and soap are directed to be issued to the ship's company, in the proportion of two pounds of the former and one pound of the latter per man per lunar month, the Purser is to include those articles in his demand for provisions from the Victualling Agents; but it is to be observed, that he must only demand as much tobacco and soap as will answer for the two-thirds of the complement, agreeably to the time for which the ship is ordered to be provisioned.

If the complement of men be multiplied by 2, and the product divided by 3, the quotient will be the two-thirds of the complement: this, being multiplied by the number of months for which the ship is ordered to be victualled, will give the number of pounds of soap; the double of which will be the number of pounds of tobacco. Thus, in the above case, where the complement is 275 men, and the time 4 lunar months;—

Now, 275 men × 2 ÷ 3 = $183\frac{1}{3}$, which is the $\frac{2}{3}$ of the complement.
Multiply by no. of months = 4

Product . . . . $733\frac{1}{3}$ = pounds of soap.

Double of ditto . . . . $1466\frac{2}{3}$ = pounds of tobacco.

THE END OF VOLUME I.

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ERRATA

Page 141, *Note*, line 4, *for* "lat. 50° 3

Page 463, line 6 from the bottom, *for* "n
2. 51. 1" *read*, moon's corrected ri