COMPOUND
ENGINES—

POWER

THE

COMPOUND ENGINE.

A SERIES OF LECTURES BY F. R. LOW
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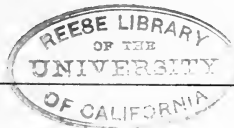
LECTURE X.—THE COMPOUND ENGINE.

BY F. R. LOW.

Before we commence the study of the compound engine let us brush up a little regarding expansion and the gains and losses attending it.

As the volume of steam is expanded its pressure falls and practically in an inverse ratio; that is, if you double the volume you halve the pressure, if you treble the volume you have one-third the pressure, etc., only remember that you must work with absolute pressures, not gage pressures.

If you will consider that statement a little you will find that it means that the product of the volume and the pressure is constant. Suppose you have a cubic foot of steam at 120 pounds (absolute). Expand it to two cubic feet and the pressure will be 60, but you have two of the original volumes and 2×60 is 120. If you expand it to three cubic feet the pressure will fall to 40 pounds, but the product of this pressure and the three volumes is still 120, and however far we may expand it will be true that, counting the original volume as 1, the pressure times the number of volumes to which we have expanded will equal the initial pressure. The number of times that the original volume is contained in the final volume is called the "ratio of expansion." If we take one cubic foot of steam and expand it to four cubic feet the ratio of expansion is four, etc. From this principle you will easily see that we can derive the following:

RULES.

To find the ratio of expansion divide the final volume by the initial volume.

What is the ratio of expansion when two cubic feet of steam are expanded to eleven cubic feet?

$$11 \div 2 = 5.5$$

To find the terminal pressure divide the initial pressure by the ratio of expansion.

EXAMPLE—Steam of 75 pounds gage pressure is expanded to six times its original volume.   What will its pressure be?

A gage pressure of 75 pounds is 75+15=90 absolute, approximately, and 90÷6=15 pounds absolute or zero gage.

To find the initial pressure multiply the terminal pressure by the ratio of expansion.

EXAMPLE—In an engine cylinder steam is expanded to 4.5 times its volume at cut-off, the pressure at the end of the stroke being 18 pounds absolute.   What is the initial pressure?

4.5×18=81 pounds absolute, or 81—15=66 pounds gage.

In Fig. 1 let o *x* represent the absolute zero of pressure and *o A* the zero of volume.   Suppose we have a volume of steam proportional to *A B* or o 1 at 120 pounds absolute.   If it is ex-
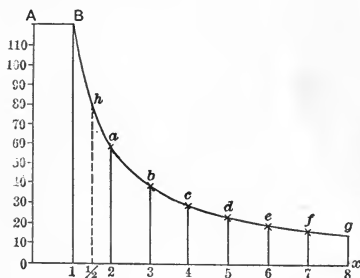

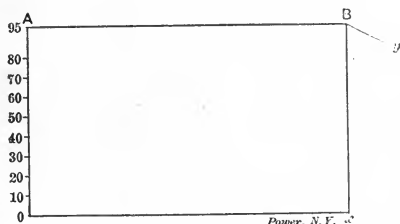FIG. 1.   FIG. 2.

panded to twice its original volume its volume will be represented by the line o 2 and its pressure will be 120÷2=60 pounds. Setting off 60 pounds on the 2d ordinate we have the point *a* as representing its pressure at this volume.   If it is expanded to three times its original size its volume will be bounded by the ordinate 3 and on this ordinate we set off the pressure 120÷3=40 pounds, locating the point *b*.   Locating in the same manner the points *c d e f g,* representing the pressures at successive volumes, we find that if joined together they form a curve which constantly approaches, but never reaches the zero line.   We can locate as many points on this curve as we like.   For instance, when the volume has expanded to 1½ the pressure will be 120÷1.5=80 pounds, which set off on the corresponding ordinate gives us the point *h*.   This curve is called an hyperbola, and represents, when

used in this way, the gradual decrease of pressure when steam is cut off and expanded in an engine cylinder.

Power, you know, is force exerted through space. In Fig. 2 we have a force of 80 pounds gage (95 pounds absolute) exerted throughout the stroke *o x*. The force is proportional to the height of the line *o A*; the space through which it is exerted is proportional to the length of the line *o x*, the power which is the product of the force and space is proportional then to the product of *o A* and *o x*, which is the area of the rectangle *o A B x*.

Now suppose that instead of carrying the steam the full length of the stroke we cut it off at half stroke. Then during the remainder of the stroke the pressure will fall off along the line *B C*, Fig. 3, and the power will be proportional to the area *A B*
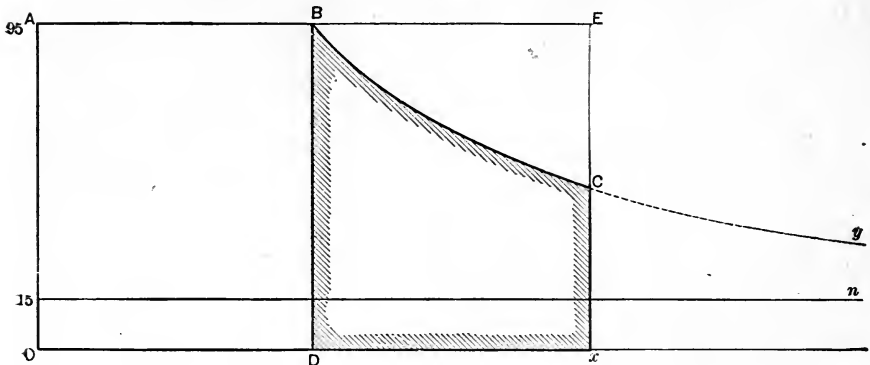

FIG. 3.

C x D O. We have used only half the quantity of steam, but with the exception of the corner *B E C* have got as much power as before. The area *B C x D* is all gain, so you see there is a very great economy in cutting off at half stroke over carrying steam full stroke.

Now suppose that instead of cutting off at one-half stroke as in Fig. 3 we cut off at one-quarter as in Fig. 4. Here again the power is proportional to *A B C x D O*. The area *B C x D* is gain from expansion. We have used half as much steam as in Fig. 3 and lost the area *B F. G C*. There is still a distinct gain, but rather less than before. In Fig. 3 with half the steam we got in the diagram *A B C x D O* 85 per cent. of the power we would have got if we carried steam the full stroke. In Fig. 4, we halve the steam again and still get about 70 per cent. of

what we got with double the quantity in Fig. 3, and about 60 per cent. of what we got with four times as much steam in Fig. 2.

When the steam follows full stroke all its expansive power is sacrificed and we lose the area which would be included between the diagram and the dotted line $B\ Y$ Fig. 1 if that dotted line were extended until it met the back pressure line, which would be somewhat higher than the line $O\ x$ even with a condensing engine. When expansion is introduced this loss is lessened. If $C\ y$ in Fig. 3 were extended to meet a back pressure, line, $m\ n$ at atmospheric pressure, the area would be obviously smaller, and in Fig. 4 the area is shown to be smaller still. In all the diagrams the volume to be filled with steam is proportional to the line $A\ B$, and the power is proportional to the area
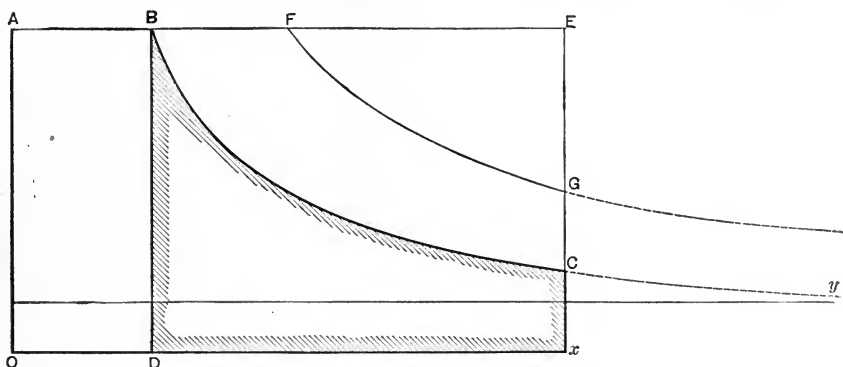

Fig. 4.

of the diagram and theoretically the line $A\ B$ will be shortest per unit of area of diagram, i. e., the volume of steam called for will be least per unit of power developed, when the expansion line just meets the line of back pressure, and the diagram ends in a point as in Fig. 5, in other words when the terminal pressure just equals the back pressure. The number of expansions necessary to do this can easily be found, by dividing the absolute initial pressure by the absolute back pressure.

There are several reasons why this theoretical consideration does not hold good in practice. In the first place, if you had your steam given to you it would still cost you something to run an engine. There is the interest on the investment, depreciation, repairs, attendance, oil, waste, etc. The sum of these fixed charges *per horse power* for a given engine will be least when the engine

is delivering the greatest number of horse-power, or in other words, when it has the greatest mean effective pressure. But the earlier the cut-off, the less the mean effective pressure, the less the horse-power, and the greater the fixed charges per horse power. This factor is not of extreme importance, however, for the fixed charges would not be very much greater for a larger engine out of which we could get the same horse-power, with a lower mean effective, and the earlier cut-off. The principal difficulty comes from cylinder condensation, which was considered somewhat at length in the last lecture. The incoming steam at a temperature due to the boiler pressure strikes against the cylinder

Fig. 5.

Fig. 6.
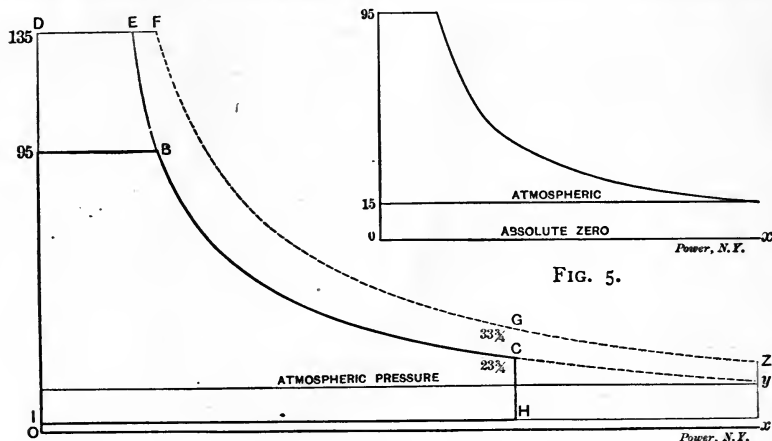
walls which have just been exposed to the exhaust temperature and condenses until enough heat is given up to heat the surfaces up to the temperature of the incoming steam. The steam so condensed is re-evaporated for the most part on the exhaust-stroke, and thus gets through the cylinder without doing any work. The amount of this condensation depends upon the change in the temperature of the cylinder walls. If the cylinder could be made of a non-conducting material, a material which was slow to absorb and radiate heat, there would be little condensation. You know that polished surfaces radiate and absorb less than rough ones, and on several recent high grade engines the inside of the cylinder heads, the piston heads and all the surfaces exposed to live steam in the cylinder have been highly finished with beneficial results. The shorter the time required for the revolution

the less the condensation.   While it is true that whether the engine runs fast or slow the walls will be exposed half of the time to the temperature of the steam and half to the temperature of the exhaust, it is also true that the temperature of the walls will vary least when the revolution is completed in the shortest time.

As the steam expands its temperature decreases and the surfaces begin to cool, continuing to do so throughout the expansion and the exhaust stroke.   The longer it takes to make the revolution the cooler these surfaces will get.   In a pumping engine making 30 revolutions a minute the surfaces are exposed to the exhaust temperature for a full second, and to a temperature below the initial for the greater part of another second during the working stroke.   In an engine running 300 revolutions per minute the surfaces have only one-tenth the time in which to heat and cool, the variation of temperature is less and less steam is condensed in raising their temperature.   The surfaces never get up to the temperature of the steam nor down to that of the exhaust, but vary back and forth through an intermediate range, the magnitude of which depends largely on the time of exposure.   Experiments made by Messrs. Gately and Kletsch upon an unjacketed simple engine at Sandy Hook showed that the condensation varied sensibly inversely as the rotative speed.*   Obviously, too, the emission of heat from the surfaces and the condensation necessary to restore their temperature will be greater with greater differences between the initial and exhaust temperatures, the higher the steam pressure and the lower the back pressure the greater the condensation.

The ratio of expansion has an important bearing on the percentage of loss by cylinder condensation.   Not only is the actual amount of steam lost in this way increased with shorter cut-offs by the fact that the temperature is below that of the initial for a greater portion of the stroke, but as less steam is used per stroke with an early cut-off, the steam condensed in warming up the surfaces is a greater proportion of the total amount. Suppose you have an engine where the stroke is twice the diameter, which is a common proportion.   When it is cutting off at one-quarter stroke the area of the cylinder wall exposed up to

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* A Mauual of the Steam Engine.   R. H. Thurston.   Part 1, page 507.
Journal Franklin Institute, October, 1885.
  Cylinder Condensation.

cut-off will just equal the area of the piston head and the cylinder head.† In addition, there is the counter bore, ports, valve faces, etc., to be heated. Suppose it takes a cubic foot of steam to fill the cylinder up to the point of cut-off, and 20 per cent. more, or one-fifth of a cubic foot, is condensed to warm up the surfaces.

Now suppose that instead of cutting off at 1-4th, we cut off at 1-8th, we have nearly as much surface to heat up as before, and that surface will be cooler on account of the lesser temperature of the cylinder during the greater expansion, so that we shall still condense our one-fifth of a cubic foot to warm up the surfaces, but, having warmed them up, we let only one-half a cubic foot through to do work and of this half a cubic foot one-fifth is 40 per cent. while it was only 20 per cent. of the cubic foot passed at quarter cut-off. The percentage of loss from cylinder condensation thus increases very rapidly with early cut-offs, and more than equals the gain from increased expansion. Experiments show that for simple engines at about 80 pounds pressure the least amount of steam will be required per horse-power when the cut-off takes place between one-fifth and one-quarter stroke.

If you will consult a steam table you will find that a pound of steam at 80 pounds gage, or say 95 pounds absolute pressure, contains 1180.7 heat units. A pound of steam of 120 pounds gage or 135 pounds absolute contains 1188.7 heat units. In other words, we have only to put 1188.7—1180.7=8 heat units more into a pound of steam to increase its pressure from 80 to 120. This is a very small percentage of the heat used, but see how much we have added to the power-producing possibilities of the steam. In Fig. 6 the heavy diagram represents the power theoretically obtainable from steam of 80 pounds gage 95 absolute, cut-off at one-quarter stroke with an absolute back pressure of 3 pounds corresponding to a vacuum of 24 inches. If by the addition of only 8 heat units per pound we raise the pressure to 120 pounds gage or 135 absolute, we can with the same terminal pressure and using the same amount of steam, add the area

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†The area of the two heads $= \dfrac{D^2\pi}{2}$. The stroke equals 2 diameters so

¼ stroke $= \dfrac{D}{2}$ and this multiplied by the circumferance $D\pi = \dfrac{D^2\pi}{2}$ also.

A D E B to our diagram; or, retaining our quarter cut-off, we can get out of the same engine, the greater power represented by the diagram *D F G H I*. In this case we take the same initial volume as with the 80 pound steam, but the weight per stroke is greater on account of the greater density of the higher pressure. The range in temperature between the initial and exhaust has been increased, but having warmed the surfaces we can get through the volume which they enclose a larger amount of steam on account of the greater density and this greater weight of steam developing a greater number of horse power the condensation per horse power is reduced. If we carry back our cut-off to *E* we greatly reduce the additional power from the higher pressure, and with the increased range of temperatures make a less saving on the condensation per unit of power produced.

In raising the initial pressure, however, with the same ratio of expansion the terminal pressure is also raised. With 95 pounds absolute and a ratio of expansion of 4 we get a terminal pressure of $95 \div 4 = 23.75$ pounds. With an initial of 135 and the same ratio of expansion we get a terminal of $135 \div 4 = 33.75$ pounds, and the loss by free expansion is raised from the area included between the line *C y* and the back pressure line, to the area between the line *G Z* and the same back pressure line, which latter area would be considerably longer as well as higher. It is not so very long ago that engines were run with initial pressures as low as this new terminal pressure. When William Coutie started his engine works at Troy, N. Y., he made a contract with a neighbor who was running a non-condensing engine whereby he was entitled to the use of the exhaust steam. He turned this exhaust steam into his own engine at about atmospheric pressure, connected his engine to a condenser and ran his shop with his neighbor's exhaust for several years. When the stroke is completed in Fig. 6 we have a cylinder full of steam at considerably above atmospheric pressure. Instead of exhausing it into the atmosphere or the condenser, suppose we complete its expansion in another cylinder. We shall then have a compound engine.

In *D F G H I* of Fig. 7 we have the corresponding diagram of Fig. 6 reduced for convenience to half its length without changing its vertical scale, just as though the reducing motion had been shortened one-half. At the end of the forward stroke we have in the cylinder a mass of steam of a volume proportional to

the length of the line *J G* and a pressure proportional to the height of the same line. This steam has been expanded to four times its original volume in the first cylinder. Suppose we conclude to expand it to four times its present bulk in another cylinder. Then the volume of the second or low pressure cylinder must be four times the volume of the first, for at each stroke the first cylinder empties its own volume into the low pressure cylinder, or into the receiver from which it takes its steam, and the low pressure cylinder must take this volume and expand it to four times its present size. The diagram in the low pressure
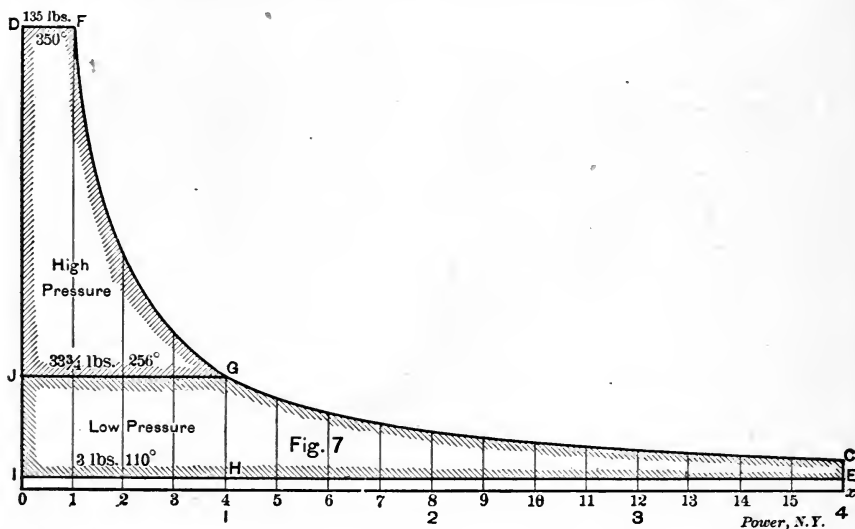
FIG. 7.

cylinder would be *J G C E I*. The combined effect so far as the expansion is concerned is the same as though we had cut off in a single cylinder at one-sixteenth of the stroke, making the diagram *D F C E I*. We have reduced the range of temperature in the first cylinder by raising the exhaust temperature from 110° to 256°, and we have made the distribution of pressure much more uniform through the stroke. Steam of 135 pounds admitted against a piston having but three pounds on the other side would occasion quite a shock, and if this pressure was continued but for one-sixteenth of the stroke, and fell away as rapidly as the line *F G C* shows, the greater portion of the stroke would be executed with comparatively low pressure, and the effort on the

crank-pin would be very jerky.   By maintaining a back pressure
of 33.75 pounds on the high pressure piston we reduce the un-
balanced thrust during the first part of the stroke; by maintain-
ing the initial pressure for a quarter stroke we get no greater
variation of effort than with the ordinary engine, and by setting
the cranks of the two cylinders at right angles, we can greatly in-
crease the uniformity of rotative effort in the shaft.   We have,
moreover, expanded the steam to $135 \div 16 = 8$ 7-16 pounds, lower
than we got even the 80 pound steam in the simple engine, and
this without going below a quarter cut-off, and with a less differ-
ence between the maximum and minimum temperatures in either
cylinder.
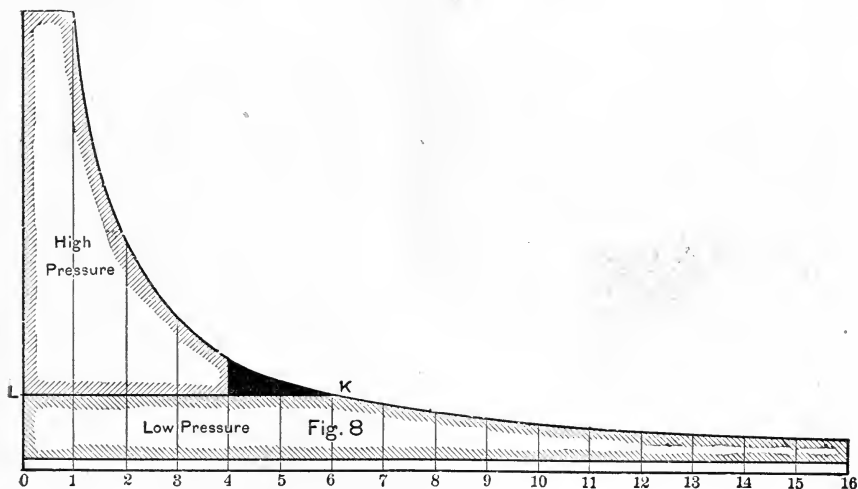
FIG. 8.

Notice that the total ratio of expansion is the volume of the
low pressure cylinder divided by the volume of the high pressure
up to cut-off.   For each stroke the high pressure cylinder meters
off a certain volume of steam; this is the initial volume.   What-
ever we may do to it in the meantime it will eventually occupy the
entire volume of the low pressure cylinder, and the final volume
divided by the initial volume, gives you your total ratio of ex-
pansion.   This makes it plain that the cut-off on the low pres-
sure cylinder has no influence on the ratio of expansion, or the
number of times the steam is expanded.   All that the cut-off on
the low pressure cylinder can do is to regulate the receiver pres-

sure.   The low pressure cylinder has to take away as much
steam as the high pressure cylinder delivers, stroke by stroke,
and it will do it whether the cut-off is long or short.

If you want your high pressure diagram to end in a point as
at G, Fig. 7, the low pressure cylinder must cut off when the vol-
ume of steam admitted just equals the volume of the high pres-
sure cylinder.   If you lengthen the cut-off so as to take away
each stroke a greater volume than the high pressure cylinder
delivers, the delivered steam will expand into the greater space
afforded with a consequent reduction of pressure.   If, for in-
stance, the low pressure cylinder cut off at 3-8ths as in Fig. 8,
instead of at one-quarter as in Fig. 7, each volume J G expelled
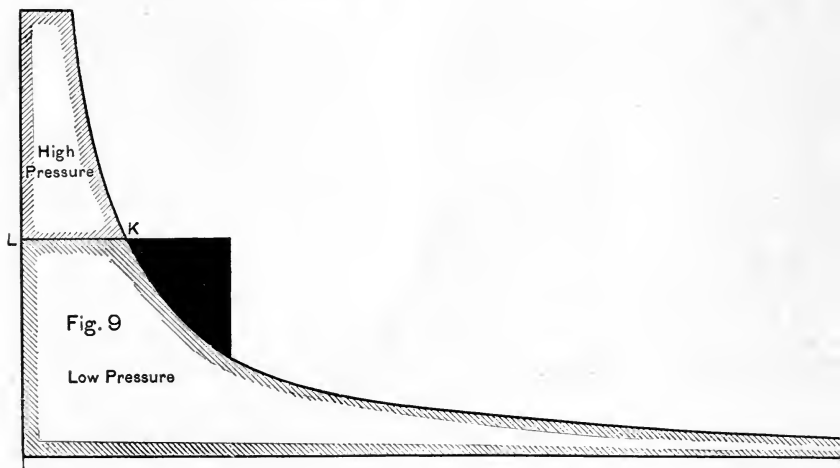
FIG. 9.

from the high pressure cylinder would find a space proportional
to L K to receive it, and would expand to that volume with the
reduction of pressure shown.   This would reduce the back pres-
sure on the high pressure piston, and the initial on the low (that
is the receiver pressure) and cause the loss of a little shaded tri-
angular area.   If, on the other hand, we cut off at one-eighth
of the stroke, as in Fig. 9, the cylinderful of steam which comes
from the high pressure cylinder has to be compressed into a
space less than its own volume, with a consequent increase in
pressure, raising the receiver pressure to K L, Fig. 9, and mak-
ing a loop upon the high pressure diagram, such as we get on
a non-condensing engine when the expansion is carried below the

atmospheric pressure. The area of this loop is represented by the black portion in Fig. 9. It represents back pressure or negative work and is equivalent to the loss of an equal amount of area inside the diagram. You see then that changing the point of cut-off in the low pressure cylinder simply changes the receiver pressure, and determines the apportionment of the load between the cylinders, and you will notice the paradoxical fact that the earlier the cut-off in the low pressure cylinder the greater the amount of work which that cylinder does.

The total ratio of expansion is the ratio of expansion in the high pressure cylinder multiplied by the cylinder ratio. By the cylinder ratio I mean the quotient of the volume of the low pressure cylinder divided by the volume of the high. For equal strokes these volumes will be as the squares of the diameters and the cylinder ratio will be the square of the quotient of the diameter of the low pressure divided by the diameter of the high. For instance, if you have cylinders 24 and 48 inches in diameter with the same stroke the cylinder ratio will be

$$48 \div 24 = 2 \text{ and } 2 \times 2 = 4.$$

If the cylinders were 24 and 60 the ratio would be

$$60 \div 24 = 2.5 \text{ and } 2.5 \times 2.5 = 6.25.$$

In Fig. 7 we had a cylinder ratio of 4. The steam having been expanded 4 times in the high pressure cylinder was expanded four times more in the low. It is a common error to take the sum of the expansions in the two cylinders as the total expansion instead of their product. It is a natural mistake to say that, having expanded four times in one cylinder and 4 in another, you have expanded $4+4=8$ times in all, but this is not so, because the initial volume which you expand in the second cylinder is the already expanded volume from the first. Fig. 10 will make this plain. We start with a volume of steam represented by the black square, and in the high pressure cylinder expand it to 4 times its original volume as represented by the rectangle $A\ B\ C\ D$. In the low pressure cylinder we expand this larger volume $A\ B\ C\ D$ to four times its size, represented by $A\ B\ F\ E$, and the final volume is 16, not 8 times, the original. The total ratio of expansion then is the ratio of expansion in the high pressure cylinder multiplied by the cylinder ratio. Do not make the mistake of multiplying by the ratio of expansion as determined by the cut-off in the low. The terminal pressure and total

ratio of expansion is the same in Figs, 7, 8 and 9, notwithstanding the wide difference in the low pressure cut-off.   The real ratio of expansion in the low pressure cylinder, that is, the continuation of the expansion from the terminal in the high, is the cylinder ratio, because that tells how much bigger the volume of steam is at the end of the low pressure than at the end of the high pressure stroke.

I want to make plain to you the fact that the mean effective pressure due to the total ratio of expansion represents the total horse-power of the engine when considered as acting on the low pressure piston alone.

Suppose, as in the previous illustration, we have an initial
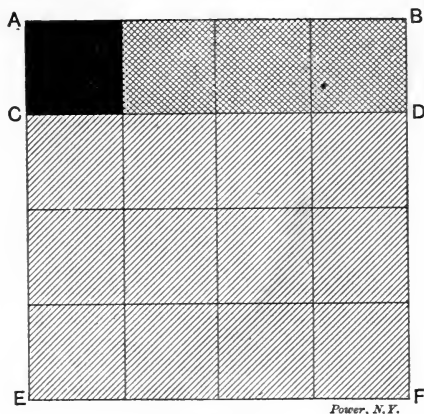
FIG. 10.

pressure of 120 pounds absolute, and a cylinder ratio of 4, that is, that the volume of the low pressure cylinder is 4 times that of the high.   Suppose further that the cut-off in the high pressure cylinder is at quarter-stroke.   At quarter stroke, or with a ratio of expansion of 4, the mean pressure per pound of initial is .59658*.   The mean pressure in the first cylinder would be then 120×.59658=71.5896 pounds.   The back pressure equals the terminal pressure in this cylinder, and would be 120÷4=30 pounds.   Remember we are dealing with absolute pressures all the time, and the mean effective pressure would be the mean pressure minus the back pressure=71.5896—30=41.5896 pounds.

* See table page 5, POWER, July, 1895, or page 122 " The Steam Engine Indicator."

The conventional diagram which would be made is shown in Fig. 11.

Since the low pressure cylinder has four times the volume of the high, it should cut off at one-fourth to take the same volume of steam that the high pressure delivers, and continue the expansion without loop or drop. Here again the ratio of expansion is 4, the mean pressure per pound of initial .59659 the initial 30, giving a mean pressure of 30 × .59658 = 17.8974 pounds. Subtract from this say, 3 pounds absolute back pressure, giving 14.-8974 pounds of mean effective. Fig. 12 gives the conventional diagram. Now the 41.5896 pounds in the high pressure cylinder will do only one-quarter as

FIG. 11

much work as though it acted in the low, because the cylinder is only one-quarter the size. To find the pressure which acting in the low would do an equal amount of work we must divide by 4 and

$$41,5896 \div 4 = 10.3974$$

and the total work is the same as though 10.3974 + 14.8974 = 25.2958 pounds acted only in the low pressure cylinder.

In these diagrams you know the length is proportional to the volume, and as in a single cylinder, the volume is proportional

FIG. 12.

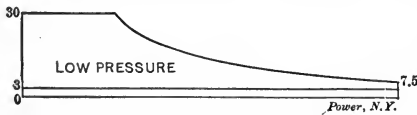to the stroke, the length of the diagram represents also the length of the stroke. When we wish to compare two diagrams, however, they must be reduced no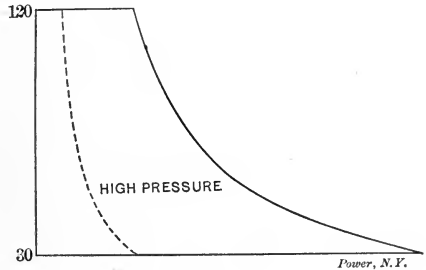t only to the same vertical or spring scale, but to the same scale of volumes. Having been expanded four times in the high pressure cylinder and four times in the low, the volume at the end of the low pressure stroke will be 4×4=16 times that at cut-off in the high. If the length of the low pressure diagram represents 16 volumes the length of the high pressure diagram can represent but four, because the volume at the end of the high pressure stroke was only four times that at cut-

off. To make it comparable with the low pressure diagram we must reduce it to one-quarter of its length, keeping the same vertical scale, as shown by the dotted diagram in Fig. 11. In doing this we divide its area, which is proportional to the power it represents by 4, which is just what we did when figuring the power above. This diagram can now be placed upon the low pressure diagram as shown in Fig. 13, and the combination represents the action of the steam with the total ratio of expansion 16, and its area is proportional to the power developed when the mean effective pressure represented by that area is considered as acting on the low pressure piston. The mean pressure per pound of initial for 16 expansions is .23579 and

$$120 \times .23579 - 3 = 25.2948$$

as before.

Another way of looking at it is this: To find the mean effective pressure of a diagram, you multiply its area by the scale of the spring and divide by the length. In the combined diagram the area of the high pressure portion would by this process be divided by 16 instead of by 4, which is equivalent to dividing its mean effective considered with reference to the cylinder in which it was first expanded by 4. In combining diagrams thus the length of the low pressure diagram must equal that of the high multiplied by the

FIG. 13.

cylinder ratio. It is usually easier to reduce the high pressure diagram in this proportion than to lengthen the low.

We have seen that in the simple engine it does not do to expand until the terminal equals the back pressure. With a compound engine we can carry the expansion much further than in a simple engine, and if we could profitably bring the combined diagram to a point in all cases, the question of compound engine design would be much simplified. Suppose again that we have 120 pounds gage=135 absolute initial and run non-condensing, exhausting at 15 pounds absolute. Then in order that the terminal pressure may equal the back pressure we must expand $135 \div 15 =$
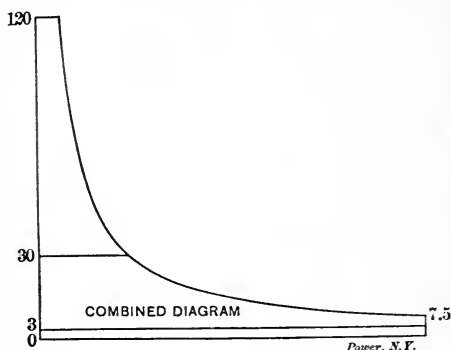
9 times. The diagram is shown in Fig. 14. How shall we divide it up between the high and low pressure cylinder? The usual object is to divide the work equally between the two. This will be done by dividing the diagram into equal areas, by a line like *CD* so located that the area above it representing the high pressure diagram will equal the area of the low pressure diagram below it. Where both diagrams end in a point, as in the case under consideration, this will occur when the ratio of expansion is the same in both cylinders, that is when *CD* is just as many times as long as *AB* and *EF* is as long as *CD*. In Fig. 14, for instance, *CD* is 3 times as long as *AB* and *EF* is 3 times as long as *CD*. This is accomplished by making the ratio between the cylinders the square root of the total ratio of expansion.   To get
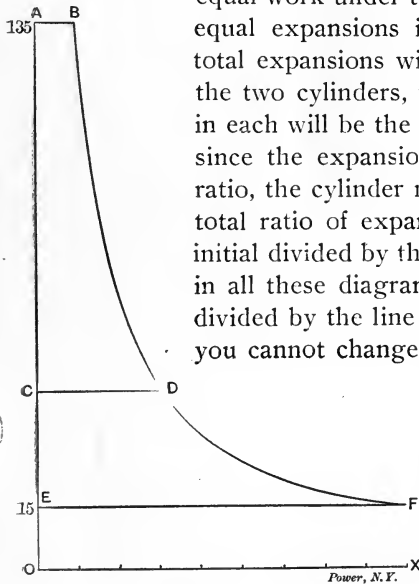
FIG. 14.

equal work under these conditions we must have equal expansions in the two cylinders. The total expansions will be the product of those in the two cylinders, then obviously the expansion in each will be the square root of the whole, and since the expansion in the low is the cylinder ratio, the cylinder ratio is the square root of the total ratio of expansion, or in this case, of the initial divided by the back pressure. Notice that in all these diagrams the length of the line *EF* divided by the line *CD* is the cylinder ratio, and you cannot change the length of the high pressure part of the diagram without changing the ratio between your cylinder volumes.

Now with an engine proportioned on these lines suppose the load to increase and steam to be carried one-half instead of one-third stroke. Either of three things may happen, according to how we manage the low pressure cut-off. If we have a fixed cut-off at one-third stroke, the receiver pressure will go up to the terminal in the high pressure cylinder, as indicated by *CD*, Fig. 15, making the high pressure diagram still end in a point, but disturbing the balance of the load, much more of the work being on

the low pressure than upon the high. To make the low pressure cut-off earlier would add to the evil. Suppose we extend the cut-off in the low pressure cylinder to the same extent as that in the high, cutting off at one-half in both, as in Fig. 15. The receiver pressure will then remain constant and the load will remain equally distributed between the cylinders. The high pressure cylinder no longer ends in a point, but there is some loss from free expansion in the receiver as at A. This, however, is just equal to the loss from free expansion at B. So that so long as the ratio between the cylinders is the square root of the quotient of the absolute initial divided by the absolute back pressure, the load will remain equally distributed between the cylinders if we vary the cut-offs equally, no matter how the load may vary.* You will notice one thing, that if we proportion our cylinder ratio to keep the expansion curve smooth at a given load, we have either got to lose some area from the theoretical diagram by free expansion when more load comes on as at A in Fig. 15, or by
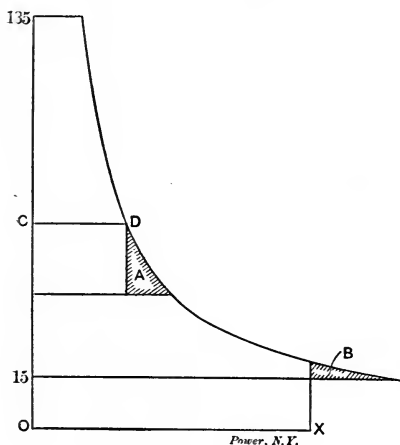

FIG. 15.

running the receiver pressure up to the terminal in the high pressure cylinder, to throw a disproportionate part of the load on the low pressure cylinder. The designer of a pumping or marine engine where the point of cut-off in the first cylinder is fixed, can simply determine upon the total number of expansions he wants to employ, lay out his theoretical combined diagram, locate the line CD where it will divide the load between the cylinders as he wishes it divided, and his problem is solved. But a stationary engine subjected to varying loads must change the point of cut-off in the first cylinder in order to control the speed. If we have fixed the cylinder ratio to give a smooth expansion curve and equal distribution of load at one total ratio of expansion,

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* See A Method of Proportioning the Cylinder of Compound Engines by E. C. Knapp, Trans. Ame.. Soc. Mech. Engrs. Vol. XVI.

the same cylinder ratio will not harmonize them at any other ratio of expansion. We must, therefore, determine what ratio of total expansion we will use at average load, and make our engine large enough to develop the average load with the mean effective that we can realize with the given initial and ratio of expansion. We can then arrange our cylinder ratio to give equal loads at this point of cut-off if we desire, or to have a smooth expansion line or a drop in the first cylinder as may seem best.

Theoretically, at least, the engine will require the least amount of steam per indicated horse-power when the actual indicator diagrams taken from the cylinders combined, as in Fig. 13 most nearly fill the area of the theoretical diagram. We shall get the same losses of area here as with the simple engine, from a failure to realize the full boiler pressure in the high pressure cylinder, from the steam line falling off, from rounded corners and from back pressure; also in the low pressure cylinder from a failure to realize as initial the full receiver pressure. In addition we may lose by free expansion in the receiver as at Fig. 8, or by looping the high pressure diagram as at Fig. 9. Just as soon as a certain point of cut-off in the high pressure cylinder is exceeded, the terminal in the low pressure will exceed the line of counter pressure, and there will be a theoretical loss there by free expansion analogous to that in the simple diagram shown in Figs. 3 and 4.

This terminal pressure will be just the same whatever the point of cut-off in the low pressure cylinder, and there is no way of dividing it between the cylinders. We may, as in Figs. 14 and 15, so proportion the cylinders that there will always come to the high pressure cylinder a loss by free expansion equal to the loss upon the low pressure cylinder from the same cause, but this will be in addition to the loss at the low pressure cylinder, and will not lessen it at all. It simply keeps the load between the cylinders equal, and since with anything but a constant load we must have free expansion in the receiver, it may be arranged as described above to vary equally. If the loss by receiver expansion or "drop," as it is usually called, is serious, we want to so proportion our cylinders that the drop shall be least at the load at which the engine runs most of the time. The diagram shown in Fig. 14 must represent the minimum work of the engine, for if the cut-off were any shorter the low pressure diagram would loop. An

engine designed on these lines then would have some drop when running at anything above its minimum load. If, instead of proportioning the cylinders by the rule there given, i. e., the cylinder ratio equals the square root of the quotient of the initial divided by the back pressure, we lay out a theoretical diagram with the number of expansions which we want to use at the average load, or the most frequent load, and then draw a horizontal line dividing its area equally, we shall have an equal division of work with no drop at the load at which the engine is most used, but the load would not remain evenly balanced when the low pressure cut-off was varied the same as the high.

From what has been said I would like you to catch and to remember particularly

First.—That the total ratio of expansion, that is the total number of times the steam is expanded, is the ratio of expansion in the high pressure cylinder multiplied by the cylinder ratio.

Second.—That this expansion may be effected in three ways.

a. In the high pressure cylinder.

b. In the receiver.

c. In the low pressure cylinder.

Third.—That changing the point of cut-off in the low pressure cylinder does not affect the work done by the engine as a whole, nor even the terminal pressure in that cylinder.

Fourth.—Shortening the cut-off in the low pressure cylinder raises the receiver pressure and throws more of the load on the low pressure cylinder.

Fifth.—Lengthening the cut-off on the low pressure cylinder lowers the receiver pressure and throws load off from the low pressure cylinder onto the high.

Sixth.—When the cut-off on the low pressure cylinder is fixed the receiver pressure will vary with the cut-off in the high pressure cylinder, and the greater the load the greater the proportion of it which the low pressure cylinder will carry.

Seventh.—When the cut-off on the low pressure cylinder varies the same as that on the high the receiver pressure will be constant, and the load remain more evenly distributed.

Eighth.—When there is no drop the cut-off in the low pressure cylinder must be the reciprocal of the cylinder ratio, i. e., if the cylinder ratio is 4 the cut-off in the low pressure cylinder must be one-fourth, etc.

You see, too, that the best ratio between the cylinders depends to a great extent upon the total number of expansions we desire to effect, which will depend upon the boiler pressure, the back pressure, the character of the engine and the use to which it is to be applied. Considering the question only from the standpoint of steam efficiency there is a wide difference in opinion as to the number of expansions which with a given set of conditions will produce a horse-power with the consumption of the least amount of steam.

B. F. Isherwood, in an exhaustive review of a compound engine test in the Jour. Franklin Inst. for October, 1885, points out, page 268, that the heat units consumed per hour per horse power were almost identical, and that no economy of fuel followed increasing the measure of expansion with which the steam was used from 6.26 to 9.64 times. It was a slide valve engine approximately 11 and 19 by 19 with about 90 pounds pressure, run condensing and partially steam jacketed.

Dr. C. E. Emery deduces from the results of his experiments with the U. S. Revenue steamers that the most economical ratio of expansion of steam in two cylinder compound engines where the pressure varies from 75 to 79 pounds absolute is

$$\text{ratio of expansion} = \frac{22 + abs. \text{ initial pressure}}{22}$$

Dr. Thurston, in his Manual of the Steam Engine, Part 1, page 596, says : The first step in designing the compound engine is the determination of the best ratio of expansion under the assumed conditions of operation and for a given type of engine, for a single cylinder ; then the best ratio of expansion for the series ; *   *   *   * The extent of economical expansion in a single cylinder will vary with the working range of temperature and pressure and with the physical condition of the working fluid, but it may be taken as determined by experience as perhaps not above two and a half expansions for unjacketed engines with wet steam or not over three or four for good practice with the better class of engines. The total expansion ratio thus becomes for the several types of multiple-cylinder engines as below :

No. of Cylinders.............	1	2	3	4
Ratio of expansion............	2.5 to 3	6.25 to 9	16 to 27	40 to 81
Initial pressure...............	25 to 30 lbs.	60 to 100	120 to 300	350 to 800

The Pawtucket pumping engine tested by Prof. J. E. Denton gave a horse power on 13.47 pounds of steam per hour with 127 pounds boiler pressure, 27.9 inches vacuum and 16 expansions. The ferry boat Bremen, with 98 pounds boiler pressure, 26.4 inches of vacuum and ten expansions required 18.1. A Wheelock triple expansion engine tested by Mr. Rockwood when running as a compound with the middle cylinder cut out required only 12.9 pounds of steam per hour with 142 pounds boiler pressure condensing, and about 25 expansions. (Trans. A. S. M. E., vol. XIII, page 656.)

Prof. R. C. Carpenter tested a 100 horse-power McEwen engine at 112 pounds boiler pressure, 22 inches vacuum, steam jacketed, and found the lowest steam consumption between 9 and 10 total expansions. (Trans A. S. M. E., vol. XIV, p. 426.)

Mr. F. H. Ball thinks that with 150 pounds pressure and a good vacuum at least 32 expansions should be realized in a triple expansion engine. (Trans. A. S. M. E., vol. XV, page 776.)

The North Point, Milwaukee, triple expansion pumping engine used 19.55 expansions with 121.45 pounds initial pressure, 13.84 pounds vacuum, while making its record-making run on 11.678 pounds of dry steam per hour per horse power. (Trans. A. S. M. E., vol. XV, page 377.)

The Laketon, Indiana, pumping engine tested by Prof. J. E. Denton, used 13.5 pounds of dry steam per hour at 150 pounds boiler pressure and 20 expansions. (Trans. A. S. M. E., vol. XIV, page 1371.)

A Sulzer triple expansion Corliss engine in Germany does a horse power on 12.73 pounds of feed water per hour with 24 expansions. "The foregoing results, as a whole, support the theory," Prof. Denton says, in reporting this test, " that with condensing engines up to at least 24 expansions the economy increases with increase in the ratio of expansion."

Mr. John T. Henthorn reports a triple expansion with 128 pounds boiler pressure, 26.5 inches vacuum, 16 expansions, 12.9 pounds of steam per hour per horse power.

Another triple in an iron works with 145 pounds boiler pressure, 28 inches vacuum and 22 expansions used 12.6.

The real effect of "drop" in the receiver is still a matter of discussion. Notwithstanding there are some designers who attach little importance to it, I think it may safely be said that no

one considers it a desirable thing, and all would like to avoid it
if it could be done without introducing more serious complications
and losses.   It is said that drop cannot be detrimental to econ-
omy, because steam expanding freely in this way loses no heat,
but becomes superheated, and at the lower pressure contains ev-
ery unit of heat which it contained at the high.   This would be
true if the free expansion continued to the back pressure, and the
receiver pressure were no higher than the back pressure in the
low pressure cylinder.   The expanded steam would still contain
every unit of heat that it had when it entered the receiver, but it
could do no work.   It has lost its "potential," as the electrical
men would say, and however superheated it may be it cannot
do any work.   By reducing the receiver pressure it has allowed
the high pressure cylinder to do more work (all the work, in fact),
but it has reduced it to the condition of a simple engine, its tem-
perature ranging from the initial to the ultimate back pressure.
We might equally well regard drop between the boiler and en-
gine as harmless, because the steam is superheated by such drop
or throttling.   I think we would prefer to keep our expansion line
free from loops and drops if we could.   The fact that we have
superheated steam for use in the second cylinder does not wipe
out the loss we have already sustained in the first.   The total
loss by cylinder condensation in a compound engine is not the
sum of the losses in the two cylinders, but only the greater of
these losses.   Suppose that in the high pressure cylinder ten per
cent of the entering steam was condensed and passed through
as water.   Before the end of the exhaust stroke all this water
will be re-evaporated and the second cylinder will get practically
the same quality of steam that the first got.   Let us get this plain.
Each cylinder must give up to the out-going steam exactly as
much heat as it takes out of the incoming steam.   If it gave up
any more it would become a refrigerating machine, and if it gave
up any less heat would accumulate in it and melt it down, so that
when the steam goes to the receiver from the high pressure cylin-
der it carries all the heat that it brought from the boiler with the
exception of the small amount which has been transformed into
work in the high pressure cylinder, as explained in Lecture 1*,
and what has been lost by radiation.   If, then, the loss through
initial condensation has been ten per cent. in the first cylinder,

*See Lecture on Heat, November, 1894, issue of POWER.

and ten per cent. of the same volume of steam is similarly con-
densed in the low, the loss by cylinder condensation in the en-
gine regarded as a whole has been ten, not twenty, per cent.
So that, as Dr. Thurston shows us*, the loss from this cause is
best avoided when it is equally divided between the cylinders.
Drop increases the temperature range in the first cylinder, and
the condensation that we may naturally expect there, and it is
doubtful if we can get square by the avoidance of condensation in
the second cylinder by the consequent superheating.

Suppose now we had an initial pressure of 175 pounds gage,
190 absolute. To get a terminal pressure of about 7 pounds we
should need 190÷7=27 (about) expansions. This would re-
quire about 5.2 expansions in each cylinder if we divide them
equally. If we use less than 5.2 in one we must use more than
5.2 in the other; but we have seen that 5 expansions is too many
to use in a single cylinder for the best economy.   Suppose, then,
we make the high pressure cylinder of such a size that *CD*, Fig.

16, will represent its vol-
ume, and *AB* will be one-
third of *CD*. Now, if *OX*,
as before, represents the
volume of the low pressure
cylinder, and we cut off in
this cylinder also at one-
third, we shall get the drop
shown in the diagram and
a wide range of tempera-
ture in the first cylinder.
This can be avoided by put-
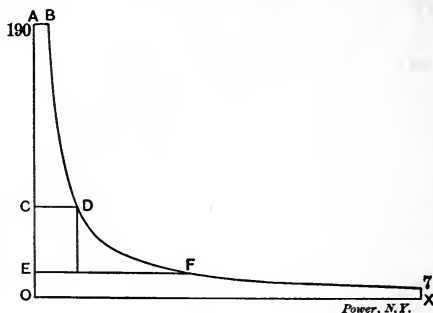
FIG. 16.

ting in a third or intermediate cylinder to effect the expansion
from *D* to *F* instead of letting it take place freely in the receiver.
This gives us a triple expansion engine. Mr. George I. Rock-
wood claims that it is better to leave out the middle cylinder,
using two cylinders with a large ratio, and tolerate the drop†, and
he has succeeded in producing some efficiencies closely approach-
ing those of the triple expansion engine, with engines which were
practically triples with the intermediate cylinder left out.   If it

* Manual Steam Engine, part I, p. 594.
† Railroad and Eng. Jour., Dec., 1891.   Trans. A. S. M. E., Vol., XIII,
p. 647, Vol. XIV, 462.

were simply a question of saving the triangular area the call for
the third cylinder would be less imperative.   If the increased
range of temperature in the first cylinder induces excessive initial
condensation there, which is only partially recovered in the sec-
ond cylinder, the intermediate cylinder may be worth the cost of
installation, maintenance and complication.

Now a few words in regard to the receiver.   From the in-
quiries I receive there appears to be an impression that there is a
certain fixed ratio between the volume of the receiver and that of
the cylinders.   If it were not for the cost and loss by radiation
I would say "the larger the better."   Let us study its effect, and
we can better judge of its required size.   Suppose, first, we have a
pair of cylinders without any receiver, one exhausting directly
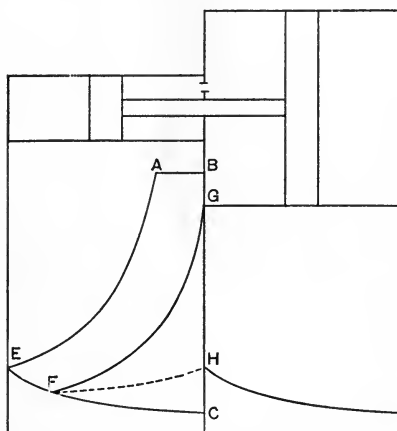
FIG. 17

into the other as in Fig. 17. Suppose that when the piston moved
from right to left the steam was cut off at quarter stroke, making
the expansion line shown beneath the cylinder. When the exhaust
valves are opened there will be no immediate fall of pressure, for
the steam must go into the low pressure cylinder, and that piston
has not yet begun to move and make room for it.   As the pistons
move to the right the steam finds a constantly increasing space
with a consequent gradual decrease of pressure, so that the for-
ward pressure upon the large piston is shown by the expansion
line beneath that cylinder, and as this is also the back pressure in
the small cylinder we can complete the diagram for the high

pressure cylinder by transferring this line to it as shown by the line *EC*. With this arrangement the pressure in the high pressure cylinder goes away down to the terminal in the low, and the entering high pressure steam finds a chilly reception, not so chilly as though we had used but one cylinder and got the diagram *A B C E*, but nevertheless the range is too high. If we cut off say at one-quarter in the low pressure cylinder the expansion in that cylinder will go on, but we have shut off the exhaust from the high pressure cylinder and the back pressure line will run up along the line *F G*.

In our consideration of the engine we have regarded the back pressure line of the high pressure cylinder as practically straight (see *C D* in Figs. 13, 14, 15, 16). Here we find it extremely crooked, as *E F G*. If there were no clearance the line *F G* would go infinitely high, for we should be compressing three-quarters of a cylinder full of steam into nothing, but every time you double the room into which you compress it you reduce the resultant pressure one-half. Now, suppose instead of exhausting one cylinder directly into the other we exhaust it into a receiver. Then when the cut-off in the low pressure cylinder occurs the steam will continue to be exhausted into the receiver, but the pressure will rise in inverse proportion to the size of the receiver. If the receiver were only three-quarters the size of the high pressure cylinder the pressure at *G* would be double that at *F* because two volumes of steam, the three-quarter cylinderful and the equal volume in the receiver, would be compressed into one. If, however, it were ten times as large as the high pressure cylinder the pressure at *G* would be only 10 $3\text{-}4 \div 10 = 1.075$ times that at *F*, and the line would run like *F H*. But we should never get down to F, for when the high pressure exhaust opened it would find in the receiver the steam of the previous stroke, and would have to exhaust again at that pressure. The low pressure cylinder takes out in a quarter stroke as much steam as the high pressure delivers during the whole stroke, but the volume of the receiver is so large in proportion to the amounts added to and taken from it that the fluctuation of pressure is not great. The larger the receiver the nearer the back pressure line of the high pressure cylinder and the steam line of the low will approach to a straight line.

LECTURE XV.—COMBINING DIAGRAMS.

When a volume of steam is taken from the boiler, expanded a given number of times and exhausted against a given back pressure, either above or below that of the atmosphere, we can make a theoretical diagram which will show the ideal action of the steam under the assumed conditions. Comparing with this ideal diagram the diagram actually obtained, we have in the difference a measure of the imperfection of the machine as a work of human art. For instance, in Fig. 18 let vertical distances represent pressures, and horizontal distances volumes. To start with, we have a volume $A B$ representing the amount of steam which under theoretical conditions, that is if the given pressure could be realized and maintained, would be contained in the cylinder at the point of cut-off including that in the ports and clearance spaces below the cut-off valve, in other words, the volume of steam shut into the engine between the piston and the cut-off valve at the instant of cut-off. When the piston has moved to C this volume will be doubled and its pressure halved. At this distance from $O A$ then representing the doubled volume, set off the point C at a height representing 30 pounds. When the piston is at D the volume will be three times the original, and the pressure one-third of 60, or 20 pounds. When the piston arrives at E the volume is 4 times the original and the pressure one-quarter or 15 pounds. Locate the point E four times as far away from the vertical line $O A$ as is the point B and 15 pounds above the line of zero pressure $O X$. Connecting these points by the curve $B C D E$ we have a curve illustrating the relation of volumes and pressures when the steam expands under ideal conditions. At the end of the stroke the exhaust valve should open and the pressure fall on the line $E F$ to that of the space into which the engine exhausts, which we will assume to

be a vacuum of 26 inches, or a pressure of 2 pounds absolute.
The counter pressure should ideally continue constant along the
line $F G$ until the admission of steam at the end of the backward
stroke sends the pressure up to 60 again on the line $G A$. We
have then as the ideal diagram producible by a volume of steam
at this pressure working through the given expansion $A B C D$
$E F G$. Comparing this with the actual diagram it is seen that
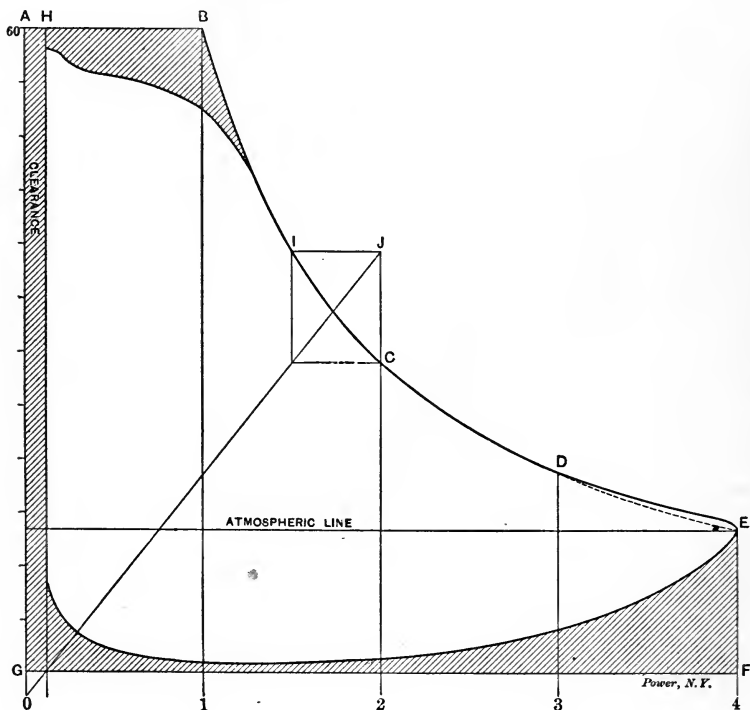
FIG. 18.

all this area has not been realized in practice, but that through
clearance, failure to realize the boiler pressure, tardiness of ex-
haust, etc., a considerable proportion is lost, while on the other
hand between D and E the actual expansion line overruns the
theoretical probably on account of re-evaporation at the lower
pressure, of heated water in the cylinder, as I explained in the
lecture on Steam.

The ideal diagram is usually laid out on the actual diagram itself. After the expansion line has become well established choose two points on the curve as *I* and *C,* and through them
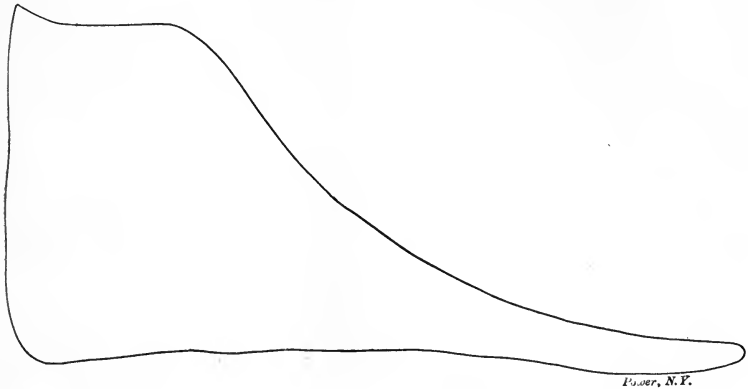
FIG. 19.

draw horizontals and verticals to form the rectangle as shown. Then draw the diagonal *J O* and extend it until it cuts the line of absolute vacuum or zero pressure *O X* 14.7 pounds below the atmospheric line. From *O* erect *O A* as the zero line of volumes and the space between *O A* and the actual diagram will represent the clearance. Then divide the length of the diagram
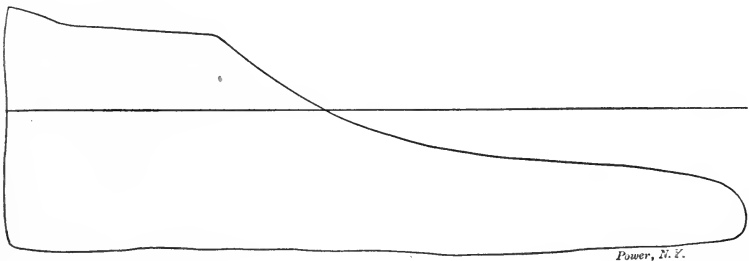
FIG. 20.

including the clearance as *G F* into any number of equal parts, and lay out the theoretical curve as already explained, working backwards to locate the point *B,* of theoretical cut-off at boiler pressure.

In a compound engine we have a volume of steam admitted to the high pressure cylinder finally expanded to fill the entire volume of the low pressure cylinder and exhausted to the condenser or atmosphere. We can plot very simply the diagram which would show the ideal action of the steam with the given number of expansions, initial and back pressure, but how shall we arrange the diagrams from the two cylinders of a compound to see how nearly their combined effect approaches the ideal?

Suppose, for instance, we have the two diagrams shown in Figs. 19 and 20 taken from the high and low pressure cylinders of an engine in which the low
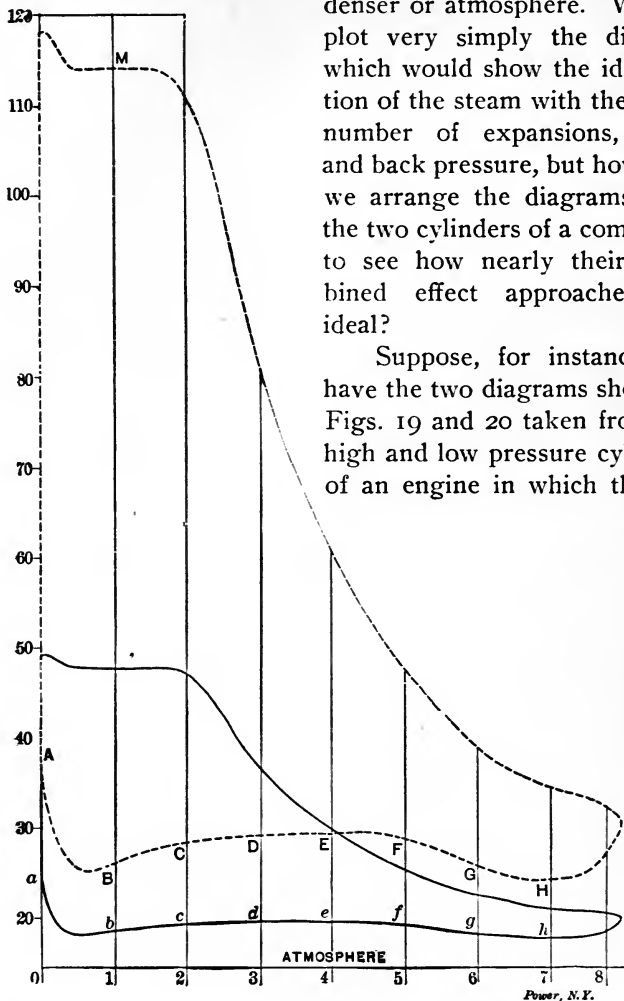
FIG. 21.

pressure has twice the diameter of the high with the same length of stroke. The scales are 60 for the high and 20 for the low.

In order to be at all comparable the diagrams must be re-

duced to the same scale, and we must either redraw the high pressure to a 20 scale or the low to a scale of 60. It will be more satisfactory to enlarge than to reduce, for a slight inaccuracy of measurement is of less importance when working upon a coarse than upon a fine scale. On a 20 scale each point of the high pressure diagram will be $60 \div 20 = 3$ times as far from the atmospheric line as on a 60 scale, for it takes three inches now to represent the pressure that one inch represented before. This process is shown in Fig. 21, where OA is 3 times

Power, N. Y.
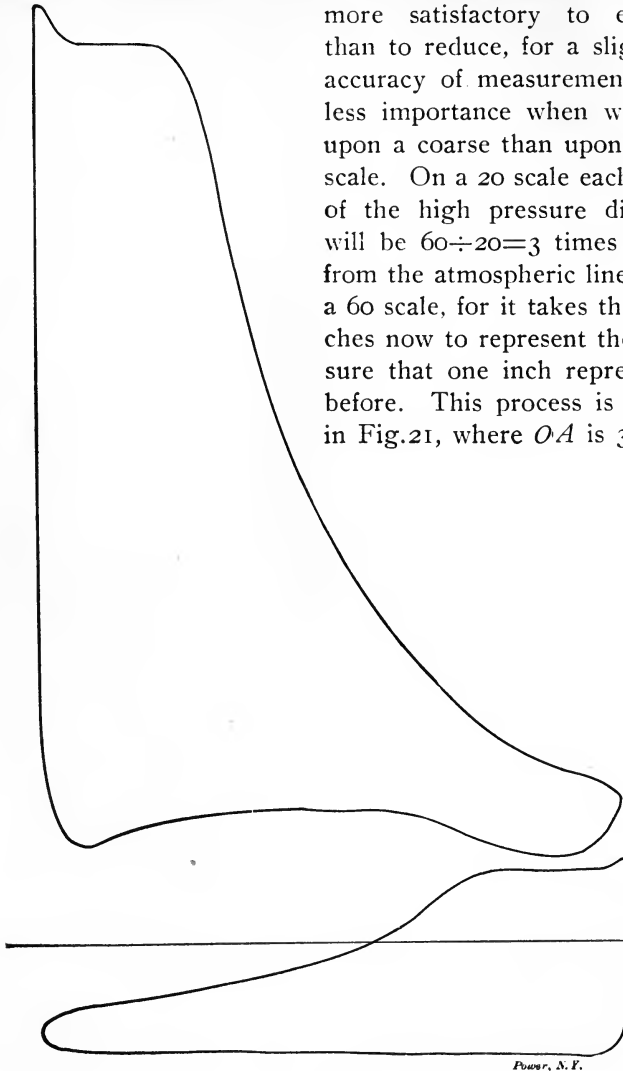
FIG. 22.

Oa, $1B$ 3 times $1b$, $1M$ 3 times $1m$, etc., and the dotted outline gives the diagram as it would have appeared if taken with a 20 spring.

Having now our diagrams upon the same scale we can compare them, in a way, by reversing one of them and placing them together so that the atmospheric lines coincide, as in Fig. 22. This, while it shows us nothing of the continuous action of the steam as compared with the theoretically plotted diagram, does show the loss between the cylinders, the rise of pressure in the high pressure cylinder after the cut-off takes place in the low, etc., and is particularly instructive in engines of the Wolff or receiverless type, where there is no cut-off on the low pressure cylinder and the steam passes directly from one cylinder to the other.
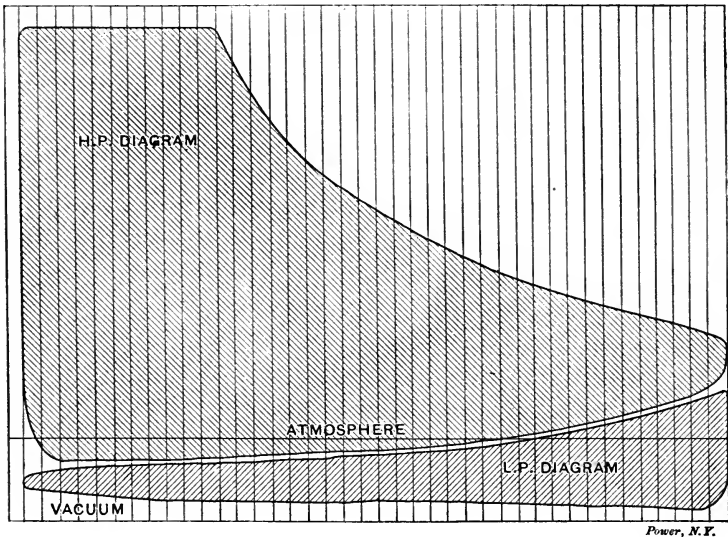
FIG. 23.

Such a diagram is shown in Fig. 23, from a pumping engine where the steam line of the low pressure diagram follows the back pressure line of the high pressure diagram remarkably close, the difference representing the frictional resistance to the flow of steam through the ports.

We have seen that the length of the diagram represents volumes. The lengths of the cylinders being the same, their volumes will be proportional to the squares of their diameters and as the diameter of the large cylinder is twice that of the small, its volume will be 4 times the volume of the small. Now if the

total length of the diagram represents the volume of the low pressure cylinder it is clear that the diagram of the high pressure cylinder representing only one-quarter of that volume should be only one-quarter as long.   When the piston of the high pressure cylinder reaches the end of its stroke the steam, cut-off at about a quarter, occupied 4 of its original volumes at a pressure of about 30 pounds.   At the end of the low pressure stroke it occupied 16 volumes at a pressure of about 7½ pounds. We must then reduce the diagrams to the same scale of volumes as well as of pressures, by making the length of the high pressure diagram bear the same proportion to the length of the low that the volume
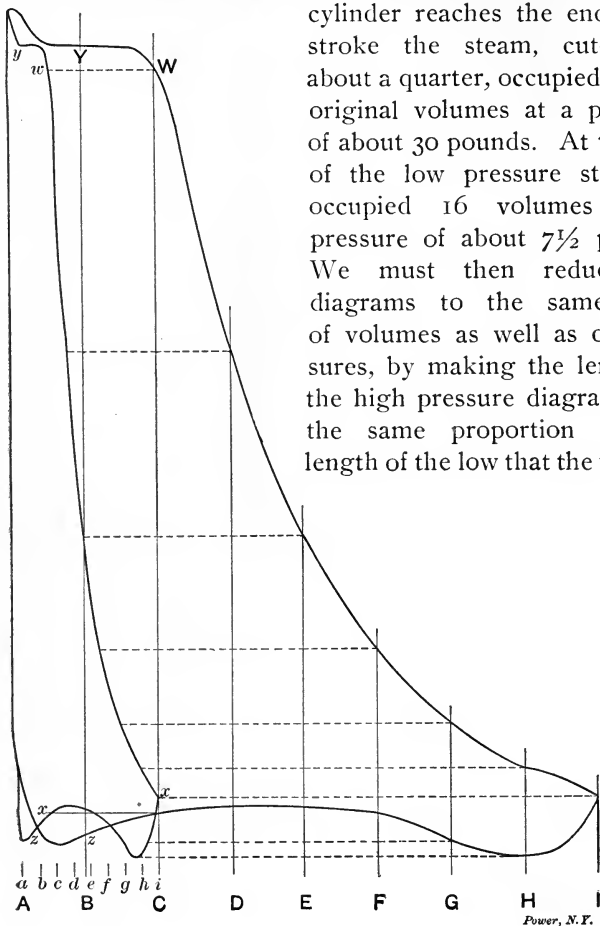
Fig. 24.

of the high pressure cylinder bears to the volume of the low; in this case by reducing it to one-quarter of the length of the low pressure diagram.   This is done in Fig. 24, where the diagram is divided into eight equal parts by the ordinates $A\ B\ C\ D\ E\ F\ G\ H\ I$. One quarter of its length is then divided into the same number
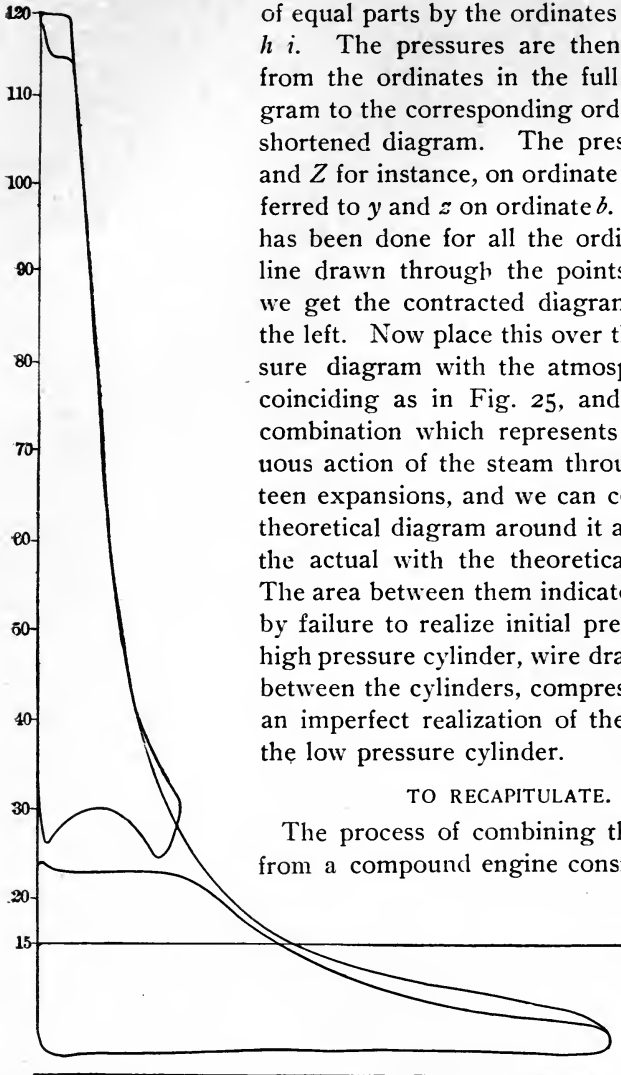
of equal parts by the ordinates *a b c d e f g h i*. The pressures are then transferred from the ordinates in the full length diagram to the corresponding ordinates in the shortened diagram. The pressures on Y and Z for instance, on ordinate B are transferred to y and z on ordinate b. When this has been done for all the ordinates and a line drawn through the points so located we get the contracted diagram shown on the left. Now place this over the low pressure diagram with the atmospheric lines coinciding as in Fig. 25, and we have a combination which represents the continuous action of the steam through the sixteen expansions, and we can construct the theoretical diagram around it and compare the actual with the theoretical efficiency. The area between them indicates the losses by failure to realize initial pressure in the high pressure cylinder, wire drawing, losses between the cylinders, compression and by an imperfect realization of the vacuum in the low pressure cylinder.

TO RECAPITULATE.

The process of combining the diagrams from a compound engine consists:

FIG. 25.

First, in reducing the diagrams to the same scale. Divide the scale of the high pressure diagram by the scale of the low and multiply the distance of every point in the high pressure diagram from the atmospheric line by the quotient.

Second, in reducing the diagrams to the same scale of volumes. Divide the volume of the low pressure cylinder by the volume of the high and divide the length of the high pressure diagram or multiply the length of the low by the ratio of the cylinders thus obtained. When the stroke is the same in both cylinders the ratio may be obtained by dividing the diameter of the low by the diameter of the high and squaring the quotient.

You will notice that I have not complicated this demonstration of the combination of diagrams by the introduction of clearance. When clearance is involved locate the clearance line on each diagram in the usual way, either by increasing the length of the diagram in accordance with its known percentage of clearance or graphically as explained above, and in combining make the clearance lines as well as the atmospheric lines coincide.

LECTURE XIV.—RECEIVERS.

I have received several letters lately which indicate that the above subject is under discussion, and in looking it up for the purpose of replying I find that there is more to it than there seems to be and that it is not well understood. Even the books are misleading. Seaton, in his Manual of Marine Engineering, says, page 131 :

"The space between the valve of the high pressure cylinder and that of the low pressure cylinder into which the steam exhausts from the high pressure cylinder should be from 1 to 1.5 times the capacity of the high pressure cylinder when the cranks are set at an angle of from 120 to 90 degrees. When the cranks are opposite, or nearly so, this space may be very much reduced. The pressure in the receiver should never exceed half the boiler pressure and is generally much lower than this. * * * * The receiver of three crank engines need not be nearly so large as the cranks are usually at angles of 120 degrees ; in the case of triple compound engines with the middle pressure leading the high pressure a very small receiver will do."

This, if I read it correctly, means that a cross compound with cranks at 90 degrees would require a larger receiver than a tandem where we have the same relation between the exhausts from the high pressure cylinder to the receiver and the drafts by the low pressure cylinder from the receiver that we have in an engine with cranks at 180 degrees. Also that an engine with cranks at 120 degrees would require a much smaller receiver than either. Let us see whether these conclusions are borne out by analysis.

We will assume an engine with the low pressure twice the diameter of the high. With the same stroke its volume would be four times that of the high. Let there be no clearance in either cylinder and let the receiver equal half the capacity of the high pressure cylinder. The initial pressure is 120 pounds absolute,

and cut-off takes place at quarter stroke in both cylinders. Let
the expansion be isothermal or hyperbolic; that is, doubling the
volume will halve the pressure, increasing the volume to three
times its original size will divide the pressure by 3, etc. Under
these conditions the product of the volume and pressure will be
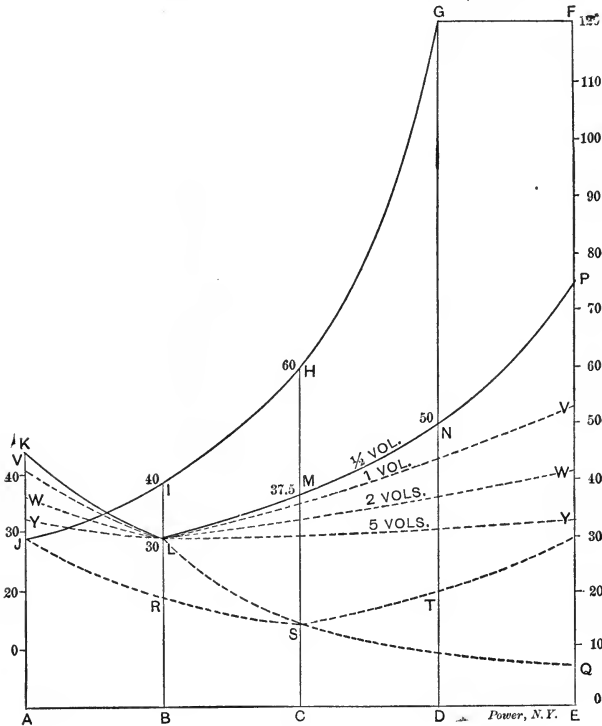
FIG 26.

constant and the pressure at any volume can be found by multi-
plying the original pressure and volume together and dividing by
the new volume, always remembering to use the absolute pres-
sure. For example, in the diagram, Fig. 26, we have at the point
of cut-off G what we will call one volume of steam at 120 pounds
absolute. At H this has been expanded to 2 volumes and we
should have

$$\frac{1 \times 120}{2} = 60 \quad \text{lbs.}$$

At I the original 1 volume has been expanded to 3 volumes and the pressure would be

$$\frac{1 \times 120}{3} = 40 \quad \text{lbs.}$$

At the completion of the stroke J we have 4 volumes and the pressure would be

$$\frac{1 \times 120}{4} = 40 \text{ lbs.}$$

In each case, you see, the product of the pressure and volume is constant.

$$2 \times 60 = 120$$
$$3 \times 40 = 120$$
$$4 \times 30 = 120$$
$$1 \times 120 = 120$$

Let us first consider the case of the tandem compound, and first let me remind you that the low pressure cylinder must take out of the receiver at each stroke as much steam as the high pressure cylinder delivers to it. It obviously cannot continuously take out of the receivver more than is put into it, and if it did not take out as much the steam would accumulate in the receiver and the pressure increase until the high pressure cylinder cound not exhaust into it. It may take out a greater volume than the high pressure cylinder delivers but at a lesser pressure, or it may take out a lesser volume at a greater pressure, but the product of the volume and the pressure of steam taken out by the low pressure cylinder must equal the product of the volume and the pressure of the steam delivered by the high pressure.

When the high pressure piston was at the right hand end of the cylinder in the position $E\,F$ on the diagram Fig. 26, steam was admitted through the valve represented by the black square at F in position A, Fig. 27, and attaining a pressure of 120 pounds absolute was continued until the piston reached the position $G\,D$, where it was cut off, giving us the volume represented by the rectangle $G\,F\,E\,D$, in both Figs. 26 and 27, to work with. When the piston reached the end of the stroke as at position A, Fig. 27, this had been expanded to 4 volumes at 30 pounds, as explained before. These 4 volumes at 30 pounds will be delivered at each stroke to the receiver. Pass now to position B, where the quarter stroke in the other direction has been reached, and both cylin-

ders are ready to cut off. The total volume of the low pressure cylinder is 4 times that of the high. Then one-quarter of its volume is just equal to the whole volume of the high. The high pressure cylinder delivers to the receiver 4 volumes at 30 pounds pressure, the low pressure cylinder at quarter cut-off takes out

four volumes, and necessarily, as I have just explained, at the same pressure. The high pressure cylinder, receiver and low pressure cylinder are all open to each other, as shown in position B, so the pressure throughout at this point must be 30 pounds. On ordinate B, Fig. 26, representing the position of the piston, at this time set off at L 30 pounds as the back pressure in the high pressure cylinder and the receiver at this point. While the pistons are still in position B, the valve between the low pressure cylinder and the receiver closes, shutting into the receiver and high pressure cylinder $2\div3=5$ volumes of steam at 30 pounds pressure. There is now no outlet from the receiver and the further movement of the high pressure piston to the right must compress this steam.

At position C, or half-stroke, we have 2 volumes in the receiver and 2 in the high pressure

FIG. 27.

cylinder, 4 in all. What is the pressure? Multiply the former volume 5 by the corresponding pressure 30, and divide by the new volume.

$$\frac{5 \times 30}{4} = 37.5 \text{ lbs.}$$

This is the receiver pressure and the back pressure in the smaller
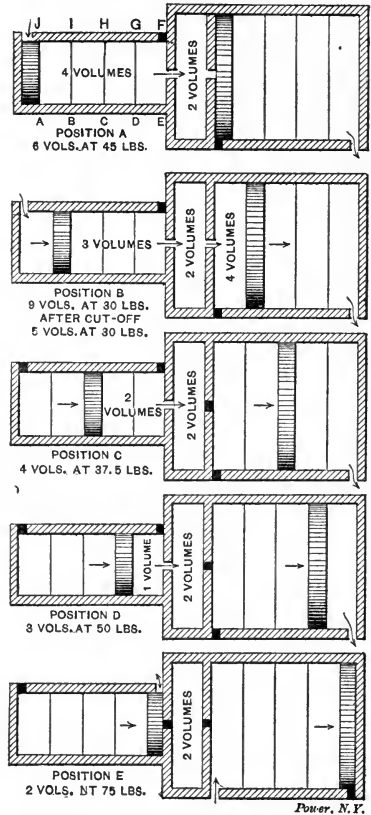
cylinder at the middle of the stroke. We will set it off at M on the ordinate C H representing that position.

At position D the five volumes at 30 pounds have been compressed to $2+1=3$ volumes. The pressure is

$$\frac{5 \times 30}{3} = 50 \quad \text{lbs.}$$

We will set this off at N on ordinate D, representing this position on the diagram.

At position E the five volumes at 30 pounds have been compressed to the 2 volumes of the receiver, the pressure is

$$\frac{5 \times 30}{2} = 75 \quad \text{lbs.}$$

Set this off on P on the ordinate, E representing this position on the diagram. Connecting the points L M N P we have the curve representing the increase of pressure in the receiver and high pressure during the last three-quarters of the stroke.

Now we have a receiver containing two volumes of steam at 75 pounds pressure. At the end of the stroke this will be opened to a high pressure cylinder containing 4 volumes at 30 pounds. The conditions will be practically as at position A only that it is the other ends of the cylinders that are involved. We must multiply each volume by its own pressure and divide the sum of the products by the sum of the volumes. This will give us the pressure of the mixture

$$
\begin{array}{r}
75 \times 2 = 150 \\
30 \times 4 = 120 \\
\hline
6) \quad 270 \\
\hline
45
\end{array} \text{ lbs.}
$$

This will be the pressure at position A. Setting it off at K on ordinate A, Fig. 26, we see that when the exhaust valve opened between the high pressure cylinder and the receiver the back pressure increased to 45 pounds on account of the higher pressure in the receiver as a result of the compression during the last three-quarters of the stroke. As the pistons move to position B, the steam will expand to 30 pounds on the line KL, completing the counter pressure line of the high pressure cylinder, which line also shows the variation of pressure in the receiver. KL is the steam line of the low pressure cylinder, and the expansion would

continue along the dotted line, making the diagram *KLQEA* for
the low pressure. Since the low pressure piston has 4 times the
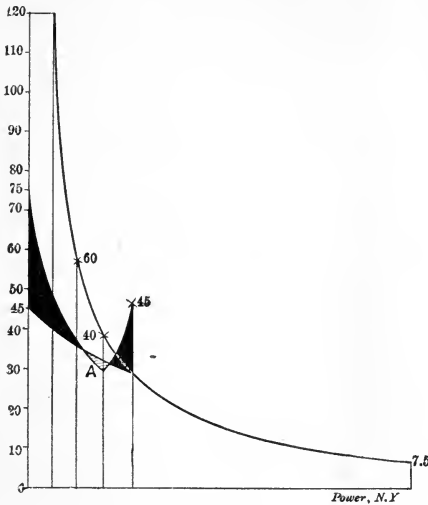
FIG. 28.

area of the high, one foot
of movement will generate
4 times as much volume in
the larger cylinder. In or-
der to be comparable then
the high pressure diagram
must be reduced one-
fourth the length of the low.
Reducing its length, leav-
ing the vertical scale the
same, and placing it
over the low pressure dia-
gram as in Fig. 28, we have
the combined diagram
showing the complete ex-
pansion from 120 to 7½
pounds. The two black
portions are lost work but

the cross hatched area at *A* where the diagrams overlap, repre-
sents double that area of useful work.

$$
\begin{array}{lllll}
\text{At } B & 5 \text{ vols. at } 30 & = 150 \\
\text{At } C & 4 \text{ `` `` } 37.5 & = 150 \\
\text{At } D & 3 \text{ `` `` } 50 & = 150 \\
\text{At } E & 2 \text{ `` `` } 75 & = 150 \\
\text{At } A & \left\{ \begin{array}{lll} 2 \text{ `` `` } 75 & = 150 \\ 4 \text{ `` `` } 30 & = 120 \end{array} \right\} \\
& 6 \text{ `` `` } 45 & = 270 \\
\text{At } B & 9 \text{ `` `` } 30 & = 270 \\
\end{array}
$$

You see in Fig. 26 that the high pressure diagram loops at
the end of the expansion line. To avoid this loop altogether we
should have to cut off at one-half stroke in the low pressure cyl-
inder, with the ratio giving us the back pressure line *JRSTU* for
the high pressure cylinder, and giving that cylinder a large varia-
tion of temperature and by far the most of the load. As the size
of the receiver is increased, the line of back pressure becomes
more nearly straight, *VV* showing its appearance when the re-
ceiver is equal to the volume of the high pressure cylinder, *WW*
when it equals two such volumes and *YY* when it equals five

such volumes, but notice that only when the receiver becomes infinitely large shall we get rid theoretically of the end loop because the pressure in the receiver at the end of the stroke is bound under our assumed conditions to be greater than that in the high pressure cylinder.

Now for the cross compound with crank at 90°, and here too

POSITION A. 6 VOLS. 31.36

POSITION B. 5 VOLS. 37 62 LBS.

POSITION C. 4 VOLS. 47.025 LBS.

POSITION D. 4.08 VOLS. 46.1 LBS.

POSITION E. 6.27 VOLS. 30 LBS, AFTER CUT-OFF 2.27 VOLS. AT 30 POUNDS

POSITION F. 2 VOLS. 34.05

Power, N.Y.
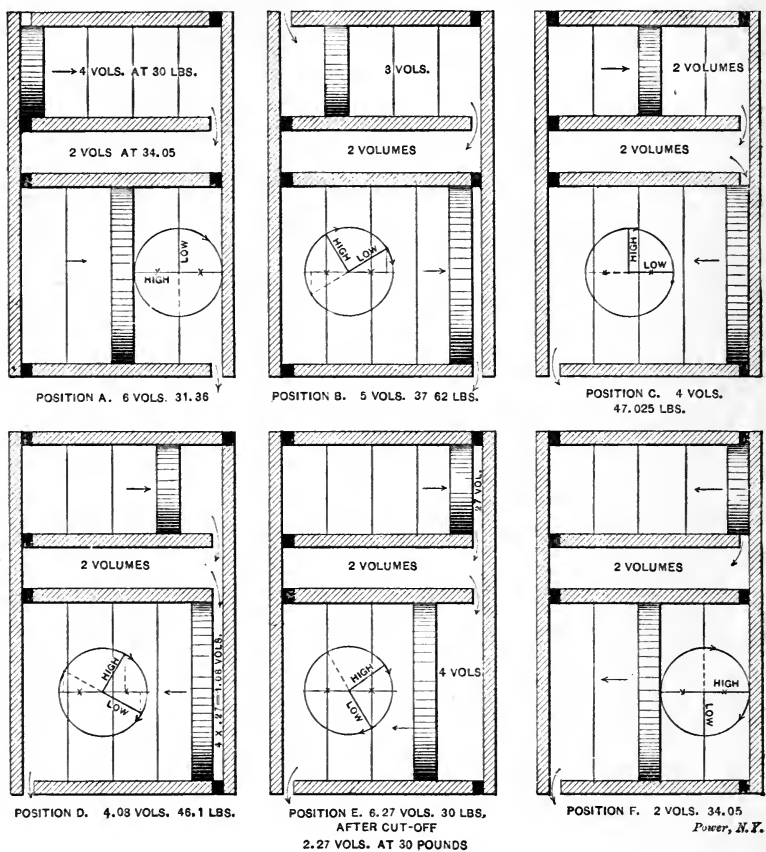
FIG. 29.

we must commence at the point of cut-off in the low pressure cylinder as this is the only point at which we are sure of the receiver pressure. This will be at position E, Fig. 29, when the low pressure piston is at quarter-stroke. If you will look at the relative position of the cranks for this position you see that the high

pressure piston will have nearly completed its stroke, leaving only, .27.

The high pressure crank is 30° from the horizontal. The versed sine of 30 (=1—the cosine) is 1—.866025=.133975. This would be the distance of the piston from the end of the stroke if the crank was 1, but the length of the crank represents 2 of our units of volume, the stroke being 4, so the volume between the piston and cylinder head in position E is .133975 × 2 =.297950 or .27 nearly, of one of our original volumes in that cylinder.
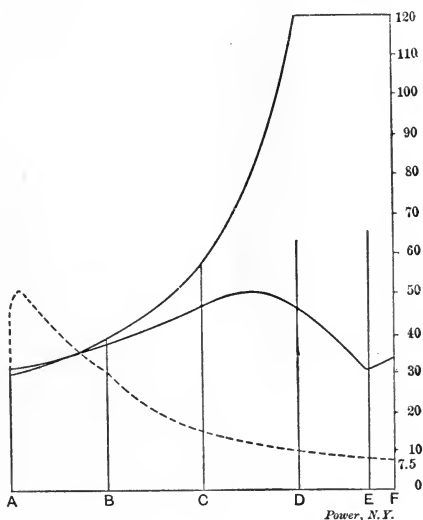
FIG. 30.

The valve between the receiver and the low pressure cylinder closing at this point leaves 2 × .27 =.2.27 volumes at 30 pounds.

Now on diagram Fig. 30 locate ordinate E, representing the position of the high pressure piston at this time. The length of the stroke represents 4 volumes, the uncompleted portion of one volume, so the piston will be .27×.25=.0875 of the stroke from the end. On this ordinate when located, set off 30 pounds, representing the pressure in the receiver, and high-pressure cylinder at this point.

When the high pressure piston reaches the end of its stroke the 2.27 volumes will be compressed to 2 as shown in position F and the pressure will be

$$\frac{2.27 \times 30}{2} = 34.05 \text{ lbs.}$$

Set off this pressure on ordinate F. When the high-pressure cylinder releases on the other side by opening the valve A at position F we add to this 4 volumes at 30 pounds, with a resulting pressure of

$$
\begin{array}{rl}
34.05 \times 2 = & 68.1 \\
30 \quad \times 4 = & 120. \\
\hline
6 & \overline{188.1} \\
& \overline{}\; 31.35 \text{ lbs.}
\end{array}
$$

Set off this pressure on ordinate A.

At B this 6 volumes at 31.35 pounds will be reduced to 5 volumes and the pressure will be

$$\frac{6 \times 31.35}{5} = 37.62 \text{ pounds.}$$

which we set off on ordinate B.

At C the volume is reduced to 4 and the pressure

$$\frac{6 \times 31.35}{4} = 47.025$$

Set this off on ordinate C.

At position D the low pressure piston has moved as far from the cylinder head as the high pressure was in position F giving 4 times the volume the high pressure gave in the same position.

$$4 \times .27 = 1.08 \text{ volumes}$$

There is still one volume in the high pressure cylinder and the total volume including the receiver is

$$1.08 + 1 + 2 = 4.08$$

The pressure is

$$\frac{6 \times 31.35}{4.08} = 46.1 \text{ lbs.}$$

Set this off on D.

Between C and D we have had at first compression because the low pressure piston while near the center moved away less than one-quarter as fast as the high. When for an instant it moved just one-quarter as fast as the high the volume was constant and the pressure neither rose nor fell. Then as the low pressure piston gained in speed relatively to the high, the volume increased and the pressure fell to 46.1 pounds at D. I have figured the intermediate volumes and pressures and plotted the

curve as you see it in Fig. 30, but will not weary you with the
details.

At E we have 6.27 volumes and the pressure is as it should be

$$\frac{6 \times 31.35}{6.27} = 30 \text{ lbs.}$$

Connecting the various points which we have set off on the ordi-
nates with the curve shown we get the contour of the back press-
ure line and complete the high pressure dia gram. Notice that
the exhaust goes direct to the low pressure c ylinder, that is the
receiver is open to both cylinders from po sition C to posi-
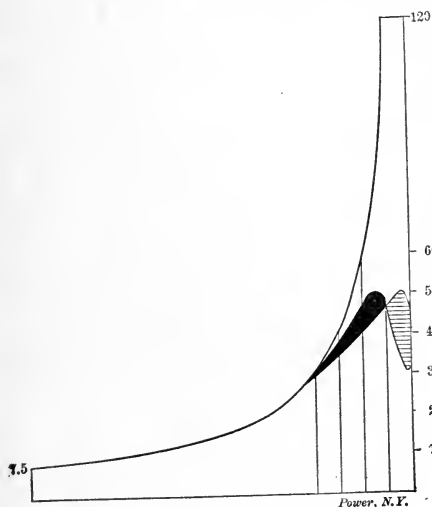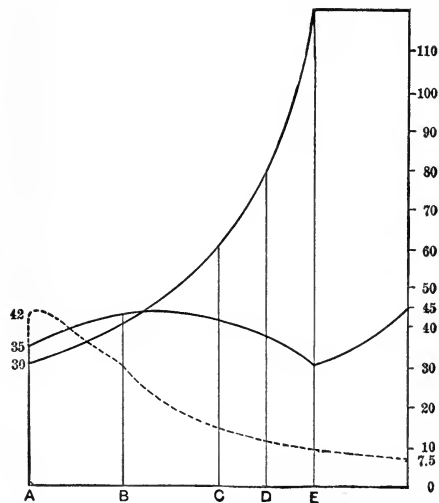
FIG. 31.   FIG. 32.

tion E, nearly half the stroke. The steam line of the low pres-
sure diagram is that portion of the counter pressure line of
the high pressure diagram which lies between ordinates C and
E, and this, somewhat changed in shape by being referred to
the ordinates which represent the correspo nding volumes in
the first quarter of the low pressure stroke, is shown by the
dotted steam line of the low pressure cylind er between A and
B, the expansion continuing regularly from B as before, giving
us the complete low pressure diagram. These are combined in
Fig. 31. The blackened area is minus, the shaded area
where the diagrams overlap has double value. They appear to
be so nearly equal that there can be little loss.

At E 2.27 vols. at 30   = 68.1
At F 2   "   " 34.05 = 68.1
At A $\begin{cases} 2 \\ 4 \end{cases}$ "   " 34.05 = 68.1 $\Big\}$
                 "   " 30   = 120
$\overline{6}$ "   " 31.25 = $\overline{188.1}$
At B 5   "   " 37.62 = 188.1
At C 4   "   " 47.025= 188.1
At D 4.08 "   " 46.1 = 188.1
At E 6.27 "   " 30   = 188.1

cross compounds at 90 degrees.

Let us look now at the cross compound with cranks at 120° the low pressure leading. Cut-off in the low pressure cylinder will take place when the pistons are in position E Fig. 33, and when the valve closes we shall have 3 volumes at 30 pounds, which at position F is compressed to 2 volumes at

$$\frac{3 \times 30}{2} = 45 \text{ lbs.}$$

Now the valve at A opens allowing 4 volumes at 30 pounds to mingle with 2 volumes at 45 pounds, and we get a resulting pressure of

$$\begin{array}{r} 45 \times 2 = 90 \\ 30 \times 4 = 120 \\ \hline 6 \;) \; 210 \\ \hline 35 \text{ lbs.} \end{array}$$

At position B the low pressure cylinder has not yet taken any steam from the receiver and we have 5 volumes with a pressure of

$$\frac{5}{6 \times 35} = 42 \text{ lbs.}$$

At C the low pressure crank is 30° below the horizontal and the same volume will have been generated as in the similar position D Fig. 29. This we found to be 1.08. The total volume at this point then is 5.08 and the pressure

$$\frac{6 \times 35}{5.08} = 41.34 \text{ lbs.}$$

At position D the high pressure piston has advanced another eighth, and the low has opened 2.294 volumes. I will ask you to take my word for this and not bother you with the details of its calculation. The total volume will be 5.794 and the pressure

$$\frac{6 \times 35}{5.794} = 36.26 \text{ lbs.}$$

At E we have one volume in the high and two in the receiver, which, with the 4 in the low, make 7 in all and the pressure will be

$$\frac{6 \times 35}{7} = 30 \text{ lbs.}$$

Setting off these pressures on their respective ordinates and

POSITION A. 6 VOLS. 35 LBS.   POSITION B. 5 VOLS. 42 LBS.   POSITION C. 5.08 VOLS. 41.34 LBS.

POSITION D. 5.794 VOLS. 36.24   POSITION E. 7 VOLS. 30 LBS.   POSITION F. 2 VOLS. 45 LBS.
                                   AFTER CUT- OFF 3 VOLS. 30 LBS.
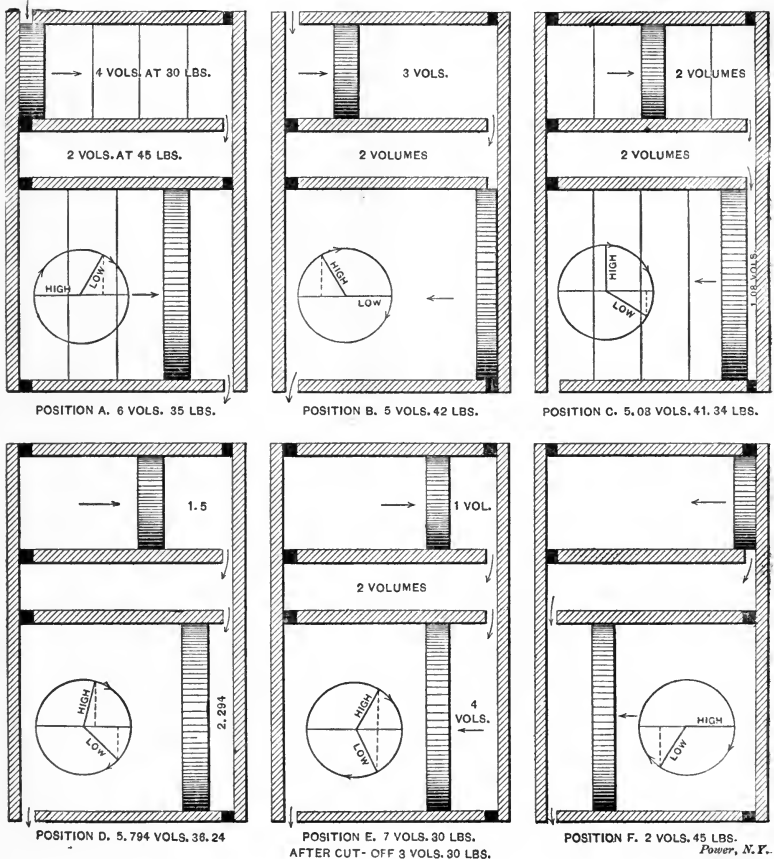
FIG. 33.

drawing the curves through them we get the back pressure line shown in Fig. 32. The receiver is open to both cylinders during a full half of the high pressure stroke, from position B to position

E, and that part of the back pressure line is contracted to form the dotted steam line of the low pressure cylinder. The combined diagrams are shown in Fig. 34, where as before, the back spaces represent minus area and the cross-hatched spaces area of double value. The high pressure diagram loops more and there
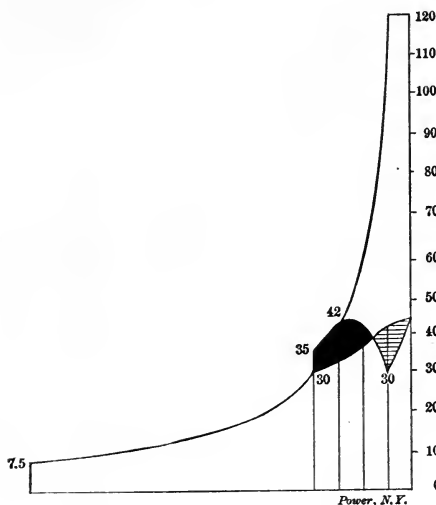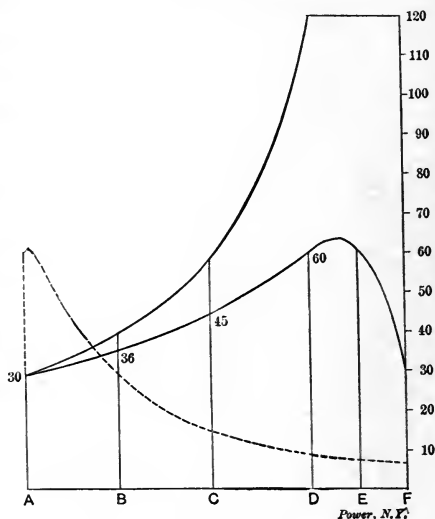
FIG. 34.   FIG. 35.

is an excess of black space, but the variation, from 30 to 43 pounds, is not so great as in either of the other cases.

$$
\begin{array}{llll}
\text{At } E & 3 \text{ vols. at } 30 & = 90 \\
\text{At } F & 2 \text{ `` `` } 45 & = 90 \\
\text{At } A & \left\{\begin{array}{l} 2 \text{ `` `` } 45 = 90 \\ 4 \text{ `` `` } 30 = 120 \end{array}\right\} \\
& \overline{6} \text{ `` `` } 35 & = \overline{210} \\
\text{At } B & 5 \text{ `` `` } 42 & = 210 \\
\text{At } C & 5.08 \text{ `` `` } 41.34 & = 210 \\
\text{At } D & 5.794 \text{ `` `` } 36.26 & = 210 \\
\text{At } E & 7 \text{ `` `` } 30 & = 210
\end{array}
$$

With cranks opposite neither can be said to lead for one is as much ahead of the other as the other is ahead of it. With cranks at 90° it apparently makes no difference which leads for as

shown by the dotted cranks in Fig. 29 the low pressure crank
would be in the same position with reference to the other end of
the cylinder that it bears to the end as shown. With cranks at
120°, however, it does make a difference, for if instead of leading
the high pressure crank as in Fig. 33 the low pressure crank fol-
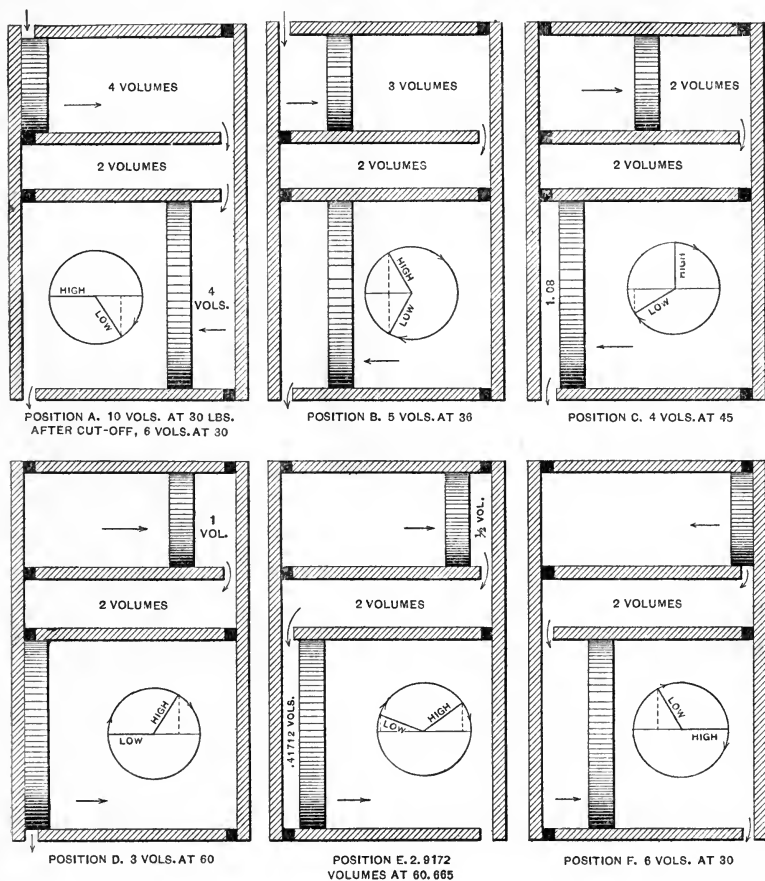
FIG. 36.

lows the high as in Fig. 36, the high pressure piston will be
commencing its stroke when cut-off occurs in the low as at posi-
tion A Fig. 36, instead of having one-quarter to go as at E Fig. 33.
Calculating the pressures for the various positions of the pistons
as in the other cases, we get the receiver pressure line shown in

Fig. 35 and the combined diagram shown in Fig. 37, where as before the black area is minus and the shaded area of double value.

Time does not permit of a discussion of the comparative desirability of these different methods of distribution nor a consider-
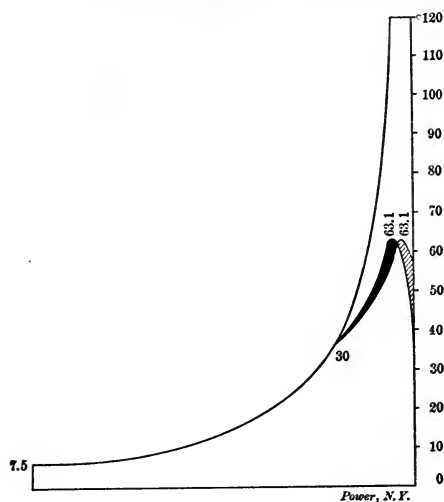
FIG. 37.

ation of the effect of an increase of cylinder ratios to avoid the loop. I should say, however, that the diagram shown by the tandem was the worst of all, and it is not apparent how it can get along with a smaller receiver than the others.

INDEX.