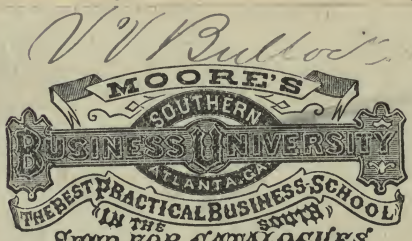
Charles Burrows

R R Burrows.

V V Bullock



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EARLY AMERICAN MATHEMATICS BOOKS

CONCISE

MERCANTILE ARITHMETIC,

FOR

COMMERCIAL COLLEGES,

AND A

HAND-BOOK FOR THE COUNTING-ROOM,

CONTAINING ALL THE MORE USEFUL AND PRACTICAL CALCULATIONS OF EVERY-DAY APPLICATION, EXPLAINED ON SCIENTIFIC PRINCIPLES.

BY

HENRY A. FABER,

AUTHOR OF "THE STATISTICAL ACCOUNT-BOOK" AND "FABER'S MANUAL."

THIRD EDITION.—REVISED.

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PREFACE.

This treatise has been prepared with special reference to the wants of students of commercial colleges. All questions which tend to perplex the learner, with little or no practical utility, have been carefully excluded.

Rules have been almost altogether omitted. The student must see the operation, and, having seen it, his judgment will enable him to deduce some method of solution for himself.

The subjects are treated of in the order of their simplicity and utility. They are so independent of each other, however, that the teacher may introduce them in whatever order his judgment may dictate.

Numerous exercises will be found on short methods of calculation. In fact, every topic which admits of more than one form of solution, has been treated by the shortest practical method.

The Author.

Cincinnati, 1880.

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MONEYS, WEIGHTS AND MEASURES.

MONEYS.

Federal Money.—The unit of our money is the dollar. Accounts are kept in dollars and cents. The coins are, the double-eagle, eagle, half-eagle, quarter-eagle, and dollar; the trade dollar, half-dollar, quarter dollar, dime, and half-dime; the three, five, two, and one cent pieces. Federal money being decimal currency, ten of a lower denomination make one of a higher: 10 mills = 1 cent, 10 cents = 1 dime, 10 dimes = 1 dollar, 10 dollars = 1 eagle. Signs: m, mills; ¢, cents; $, dollars; E, eagle.

British Money.—The unit of British money is the pound sterling. Accounts are kept in pounds, shillings and pence (farthings are written as fractions of a penny): 4 farthings = 1 penny, 12 pence = 1 shilling, 20 shillings = 1 pound. Signs: d, pence; s, shilling; £, pound. The British coins are, the penny, the shilling, the crown, the sovereign, and the guinea. The value of the crown is 5 shillings; the sovereign, 20 shillings; the guinea, 21 shillings.

German Money.—The unit of the money of the German Empire is the rixmark. 10 pennies (pfennige) = 1 silver-groschen (silbergroschen); 10 silver-groschen = 1 rixmark (reichsmark). Signs: d, pennies; sg, silver-groschen; Rm, rixmark.

French Money.—The unit of French money is the franc. 10 centimes = 1 decime; 10 decimes = 1 franc. Signs: c, centimes; d, decimes; fc, francs.

WEIGHTS.

Mint or Troy Weight, used at the mint, and by jewelers: 24 grains = 1 pennyweight, 20 pennyweights = 1 ounce, 12 ounces = 1 lb. Signs, *gr.* grain, *pwt.* pennyweight, *oz.* ounce.

Apothecaries' Weight.—Used in compounding medicines: 20 grains = 1 scruple, 3 scruples = 1 drachm, 8 drachms = 1 ounce, 12 ounces = 1 lb. Signs, *gr.* grain, ℈ scruple, ʒ drachm, ℥ ounce, ℔ pound. (1 lb.= 5760 gr.)

Commercial Weight, used by Grocers, Druggists, Hardware dealers, etc: 16 ounces = 1 pound, 2000 pounds = 1 tun. Signs, *oz.* ounces, *lbs.* pounds, *cwt.* hundreds, *T* tuns. (1 lb.= 7000 gr.)

Hay is weighed by Commercial Weight.

Avoirdupois—*Old Commercial Weight* of the U. States 16 drachms = 1 ounce, 16 ounces = 1 lb., 28 lbs. = 1 quarter 4 quarters = 1 hundred, 20 cwt. = 1 tun.

Pig Iron (chill mold), Iron Ore, Bituminous Coal, and Hemp are weighed by avoirdupois weight.

The AVOIRDUPOIS WEIGHT is the *Commercial weight* of *Great Britain.*

Metric Weight.—The unit of weights of the metric system is the Gram. The Greek prefixes (*deka,* 10; *hecto,* 100; *kilo,* 1000) form the denominations above the unit. The Latin prefixes (*deci,* 10; *centi,* 100; *milli,* 1000) form the denominations below the unit. 1 kilogram = 10 hectogram = 100 decagrams = 1000 grams. 1 milligram = $\frac{1}{10}$ centigram = $\frac{1}{100}$ decigram = $\frac{1}{1000}$ gram. The weight of a gram is equal to 15.432 grains of Troy weight. Signs: DG., deckagram; HG., hectogram; KG., kilogram; G., gram; dg., decigram; eg., centigram; mg., milligram.

NOTE.—The oz. of the Mint and Apoth. weights are the same, viz: 480 grains. The oz. of the Com'l and Avord. weights are the same, viz: 437½ grains.

Weights of Produce per Bushel, according to usage in Cincinnati, and as fixed by statute in Ohio:

	Usage. lbs.	Stat. lbs.
Apples, dried	25	25
Barley	48	48
Barley malt, weight of bags included	34	
Beans	60	60
Bran	20	
Bran shorts	25	
Broom-corn	30	
Buckwheat	50	50
Coal, bituminous	80	
cannel	70	
Charcoal	30	
Coke	32	
Castor beans	46	
Corn, shelled	56	56
in ear....68 and	70	70
Hair, plastering	8	
wet	16	
Hominy	60	60
Lime, slacked	51	
Malt		34
Meal, corn	50	
Middlings	40	
Oats	32	32
Onions	56	
Onion sets	23	
Peaches, dried	33	33
Peas	60	60
green	24	
Plaster and hair	118	
Peanuts, roasted	22	
Potatoes, Irish	60	60
sweet		50
Rye	56	56
Rye malt, wt. of bags included	40	
Salt	56	
Seed, clover	60	60
timothy	45	45
flax	56	56
hemp	44	44
orchard grass	14	
Hungarian grass	50	50
blue grass	14	
millet	50	50
canary	60	
sorghum	45	
Ship stuff	40	
Shorts	30	
Turnips	60	
Wheat	60	60
Water, distilled	77.6274	

Weight of a Cubic Foot of

	lbs.
Cast iron	450.55
Wrought iron	486.65
Steel	489.8
Copper	565
Lead	708.75
Brass	537.75
Tin	456
White pine	29.56
Loose earth or sand	95
Common soil	124
Strong soil	127
Clay	135
Yellow pine	33.81
White oak	35.2
Live oak	70
Salt water (sea)	64.3
Fresh water	62.5
Air	.07529
Steam	.03689
Clay	135
Sand	113
Cork	15
Tallow	59
Brick	119

MEASURES.

Linear Measure is applied in measuring length and distance: 12 inches = 1 foot, 3 feet = 1 yard, 5½ yds. = 1 rod, perch or pole, 40 rods = 1 furlong, 8 furlongs or 320 rods = 1 mile. Signs, *in.* inches, *ft.* feet, *yd.* yard, *rd.* rod, *fur.* furlong, *mi.* mile. Furlongs are seldom used. 5280 ft. = 1 mile.

1 *palm* = 3 inches, 1 *hand* = 4 inches, 1 *span* = 9 inches, 1 *meter* = 3.28 feet.

Scripture Long Measure.—A *digit* = .912 inches, a *palm* = 3.648 inches, a *span* = 10.944, a *cubit* = 1 foot 9.888 inches, a *fathom* = 7 feet 3.552 inches.

Jewish Long Measure.—A cubit = 1.824 feet, a *Sabbath day's journey* = 3648 feet, a *mile* = 7296 feet, a *day's journey* = 175104 feet, or 33 miles 864 feet.

Cloth Measure.—Cloth is measured by the yard and fractional parts of a yard, as half, quarter, eighth, sixteenth, etc. The yard contains 3 feet, or 36 inches.

Marine Measure.—Used at sea: 6 feet = 1 fathom, 120 fathoms = 1 cable length, 880 fathoms = 1 mile.

Metric Long Measure.—The unit of Long or Lineal Measure of the metric system is the *Meter* (whence the name, Metric.) The Greek prefixes, *deka*, etc.; and the Latin, *deci*, etc., form the other denominations the same as by the gram. The meter is equal to 39.3685 inches of our linear measure. The signs of the meter and denominations above are written with capitals: M. for meter, KM. for kilometer; those of the denominations below the meter, with small letters—dm., decimeter, etc.

Surveyors' Measure.—$7\frac{92}{100}$ inches = 1 link, 25 links = 1 rod or pole, 4 poles or 100 links = 1 chain, 80 chains = 1 mile, 10 sq. chains = 1 acre, 640 acres or 6400 sq. chains = 1 sq. mile or *section of land.*

A *square rod* contains 272¼ sq. feet. An *acre* contains 43560 sq. feet.

Circular Measure.—Used in reckoning latitude, longitude, etc., and in trigonometrical calculation: 60 seconds = 1 minute, 60 minutes = 1 degree, 30 degrees = 1 sign, * 12 signs = 1 circle. Signs.—″ seconds, ′ minute, ° degrees, S. sign, C. circle.

35° 3′ 2″ would read *thirty-five degrees, three minutes, and two seconds.*

Measure of Time.—60 seconds = 1 minute, 60 minutes = 1 hour, 24 hours = 1 day, 7 days = 1 week, 30 days = 1 lunar month, 365 days = 1 year, 12 months = 1 year.

Square Measure is used for measuring surfaces: 144 sq. in. = 1 square foot, 9 square feet = 1 square yard, 30¼ square yards = 1 square rod, 160 square rods = 1 acre, 640 acres = 1 square mile. Signs, *sq. ft., sq. yds., sq. rds., A., M.*

Metric Square Measure.—The unit of measure for large surfaces is the *Are*, from which are derived the *Hectare* and *Centare*. For smaller surfaces the denominations are the same as for measures of length, with the addition of the word square.

Are = 100 sq. meters, or 1 sq. dekameter, or 119.6 sq. yds.

Cubic Measure is applied to solids, and comprises length, breadth, and thickness, or depth. A cubic foot contains 1728 inches, that is 12 times 12 times 12 inches; a cubic yard contains 27 feet, or 3 times 3 times 3 feet.

Metric Cubic Measure.—The *Stere* may be called the unit for cubic measure. It is equal to a cubic meter or 1.308 yards.

Wood Measure.—Wood is sold by the cord, which should contain 128 cubic feet, closely piled, and 138 feet if stowed in a boat or barge. A pile of wood measuring 8 feet long, 4 feet wide and 4 feet high, contains a cord.

Stone Measure is used for measuring masonry, which is sometimes paid for by the foot, but usually by the perch, $24\frac{3}{4}$ or 25 cubic feet 1 perch; the former for private, the latter for public contracts, as railroad or government work.

A wall $16\frac{1}{2}$ feet long, $1\frac{1}{2}$ feet thick and 1 foot high contains a perch.

Bricklayers' Measure.—The common dimensions of a brick are 8 inches long, 4 inches broad, and 2 inches thick. There are 21 bricks in a cubic foot of wall, including mortar.

A wall 8 in. or 1	brick in	thickness	contains	14	bricks	to the	sq. ft.	of	surface.	
12	"	$1\frac{1}{2}$	"	"	"	21	"	"	"	"
16	"	2	"	"	"	28	"	"	"	"

Dry Measure.—Used for measuring grain, fruit, etc.: 8 quarts = 1 peck, 4 pecks = 1 bushel. Signs, *qt.* quart, *pk.* peck, *bu.* bushel.

NOTE.—The bushel is a cylindrical vessel, 8 inches deep and $18\frac{1}{2}$ diameter, inside, and contains 2150.42 cu. in.

Coal Measure.—Coal is usually sold by the bushel, which should contain 2688 cubic inches.

Liquid Measure.—For measuring all liquids, except milk, beer and ale: 4 gills = 1 pint, 2 pints = 1 quart, 4 quarts = 1 gallon. Barrels, tierces, etc., are no longer used as measures of capacity; they are all gauged and reckoned by gallons.

REMARK.—The gallon contains 231 cubic inches.

Ale or Beer Measure.—The gallon contains 282 cubic inches, and the number of pints or quarts in a gallon the same as in Liquid Measure.

Metric Measure of Capacity.—The *Liter* is the unit of measure for capacity, and is equal to a cubic decimeter or 1.0567 quarts of United States liquid measure.

MISCELLANEOUS.

Effects of Coal.—Small coal produces about ¾ the effect of large coal of the same species.

Charcoal.—The best quality is made from oak, maple, beech and chestnut. Wood will furnish, when properly burnt, about 16 per cent. of charcoal. A bushel of charcoal from hard wood weighs about 30, from pine, about 29 lbs.

Coke.—A bushel of the best coke weighs 32 lbs. Coal furnishes from 60 to 70 per cent. of coke by weight.

Relative Heating Power of Different Kinds of Fuel, according to Weight.

Charcoal 100, mineral or stone coal 82½, dry wood 48½. Hence, if a tun of charcoal cost no more than a tun of mineral coal, the former would be the cheaper fuel by 21 per cent.

Relative Heating Power of Different Kinds of Wood, according to Measure.

Shell-bark Hickory,	100	Yellow Oak,	60
Pignut,	93	Hard Maple,	60
White Oak,	81	White Elm,	58
White Ash,	77	Red Cedar,	56
Dog Wood,	78	Wild Cherry,	55
Scrub Oak,	73	Yellow Pine,	54
Witch Hazel,	72	Soft Maple,	54
Appletree,	70	Chestnut,	52
Red Oak,	69	Yellow Poplar,	52
White Beech,	65	Butternut,	51
Black Walnut,	65	White Birch,	48
Black Birch,	63	White Pine,	42

Digging.—23 cubic feet of sand, or 18 cubic feet of earth, or 17 cubic feet of clay make a tun. 18 cubic feet of gravel, or earth, before digging, make 27 cubic feet when dug.

Gas.—1.43 cubic feet of gas per hour give a light equal to that of a candle; 1.96 cubic feet equal 4 candles; 3 cubic feet equal 10 candles.

Horse Power in machinery is reckoned at 33,000 lbs. raised one foot in a minute, but the ordinary work of a horse is only 22,500 lbs. per minute for 8 hours.

Strength of a Man.—The mean effect of the power of a man, unaided by a machine, is the raising 70 lbs. 1 foot high in a second for 10 hours a day $= \frac{1}{5}$ of the power of the horse.

NOTE.—Two men working at a windlass at right angles to each other, can raise 70 lbs. more easily than one man can 30 lbs.

A foot soldier travels 70 yards, making 90 steps in one minute, common time.

In quick time, 86 yards, making 110 steps.

In double quick, 109 yards, making 140 steps.

Average weight of men, 150 lbs. each.

Five men can stand in a space of 1 square yard.

A man without a load travels on a level ground $8\frac{1}{2}$ hours a day, at the rate of 3.7 miles an hour, or $31\frac{1}{4}$ miles a day. He can carry 111 lbs. 11 miles in a day.

A porter going short distances and returning unloaded, can carry 135 lbs. 7 miles a day. He can carry in a wheelbarrow 150 lbs. 10 miles a day.

The muscles of the human jaw exert a force of 534 lbs.

Hay.—10 cubic yards of meadow hay weigh a tun. When the hay is taken out of old, or the lower part of large stacks, 8 to 9 cubic yards will make a tun. 10 to 12 cubic yards of clover, when dry, weigh a tun.

Hills in an Acre.—3 feet apart, there are 4840 hills in an acre.

PAPER.

SIZES OF PAPER MADE BY MACHINERY.

FLAT PAPER.

Letter,	10×16	Tax Duplicate,	17×30
Com'l Letter,	11×17	Medium,	18×23
Packet,	12×19	Royal,	19×24
Foolscap,	13×16	Super Royal,	20×28
Cap,	14×17	Elephant,	23×28
Crown,	15×19	Imperial,	23×31
Demy,	16×21	Columbier,	23×34
Folio,	17×22	Atlas,	26×33
Check Folio,	17×24	Antiquarian,	31×53

FOLDED PAPER.

DESIGNATIONS OF SHEETS ACCORDING TO FOLDS OF PAPER.

Folio.—A sheet folded in two leaves.

Quarto.—A sheet folded in four leaves.

Octavo.—Or 8vo, a sheet folded in eight leaves.

Duodecimo.—Or 12mo*, a sheet folded in twelve leaves.

24 sheets = 1 quire, 20 quires = 1 ream, 2 reams = 1 bundle.

Book-binders count from 16 to 20 sheets to a quire in binding account books.

WRAPPING PAPER.

Wrapping paper is sold by the bundle, which are generally *short count.* The *full count* reams contain 20 qrs. of 24 sheets each.

SUNDRIES.

12 articles = 1 dozen.	14 lbs. of flour = 1 stone.
12 dozen = 1 gross.	14 stones of flour = 1 bbl.
12 gross = 1 great gross.	1 bbl. of flour = 196 lbs.
20 articles = 1 score.	1 barrel = 31½ gallons.
1 barrel = 200 lbs.	1 hogshead = 2 bbls.

* The size of this book is 12mo.

THERMOMETERS.

The *Celsius* or *Centigrade* thermometer has the zero at the freezing point of water, and the distance between that and the boiling point of water divided into 100 degrees,—hence the name Centigrade.

The *Reaumur* thermometer has the zero at the freezing point, and 80° between that and the boiling point of water.

The *Fahrenheit* thermometer has the zero at 32° below the freezing point of water, and has 180° between freezing and boiling point of water.

To convert degrees of Centigrade into degrees of Fahrenheit, multiply the degrees of Centigrade by 9, divide the product by 5, and add 32 to the quotient, the answer will be degrees of Fahrenheit.

To convert degrees of Reaumur into degrees of Fahrenheit, multiply the degrees of Reaumur by 9, divide the product by 4, and add 32 to the quotient, the answer will be degrees of Fahrenheit.

To convert degrees of Fahrenheit into Centigrade, subtract 32 from the degrees of Fahrenheit, multiply the remainder by 5, and divide the product by 9.

To convert degrees of Fahrenheit into Reaumur, subtract 32 from the degrees of Fahrenheit, multiply the remainder by 4, and divide the product by 9.

To convert Centigrade into Reaumur, multiply the degrees of Centigrade by 4 and divide the product by 5.

To convert Reaumur into Centigrade, multiply the degrees of Reaumur by 5 and divide the product by 4.

The sum of the degrees of Centigrade and Reaumur plus 32 will give the degrees of Fahrenheit.

THE CONCISE

MERCANTILE ARITHMETIC.

I. INTRODUCTION.

ARITHMETICAL DEFINITIONS.

ARTICLE 1. Arithmetic is the science of numbers.

ART. 2. The theory of Arithmetic treats of the properties and relations of numbers.

ART. 3. The practice of Arithmetic shows the application of number to business, the mechanics' art, etc.

ART. 4. Quantity is any thing that can be increased or diminished.

ART. 5. Notation is the art of representing numbers by figures.

ART. 6. Numeration is the art of reading figures when arranged to represent numbers.

ART. 7. The four fundamental rules of Arithmetic are: Addition, Subtraction, Multiplication, and Division.

ART. 8. ADDITION is the art of uniting two or more numbers into one. The result obtained by adding is called *Amount* or *Sum*.

ART. 9. SUBTRACTION is the method of finding the difference between two numbers. The result obtained is called, *Remainder.*

ART. 10. MULTIPLICATION is the process of taking one number as many times as there are units in another. The result obtained is called, *Product.*

ART. 11. DIVISION is the method of ascertaining how many times a given number is contained in another. The result obtained is called, *Quotient.*

ART. 12. Percentage is the method of reckoning by *hundredths.*

ART. 13. A FRACTION is a part or a number of parts of a whole.

ART. 14. INTEREST is a percentage allowed for the use of capital.

ART. 15. BANK DISCOUNT is a percentage deducted from capital loaned for the use of such capital.

ART. 16. TRUE DISCOUNT is the difference between the *present worth* of a note and the amount for which it is drawn.

ART. 17. PROPORTION is an expression of equal ratios.

ART. 18. ARITHMETICAL SIGNS.

$=$ is the sign of equality.
$+$ is the sign of addition.
$-$ is the sign of subtraction.
$\times$ is the sign of multiplication.
$\div$ is the sign of division.
. is the decimal sign.
$::$ is the sign of proportion.
$\%$ is the sign of percentage.

II. NOTATION AND NUMERATION.

ARTICLE 1. NOTATION is the art of representing numbers by symbols, called *figures* or *digits*. There are ten of these figures:

0	1	2	3	4	5	6	7	8	9
nought	one	two	three	four	five	six	seven	eight	nine

The first is also called *zero*, or *cipher*.

ART. 2. When a larger number than nine is to be represented, two or more figures are used.

ART. 3. NUMERATION is the method of reading these figures when arranged to represent numbers. For this purpose they are usually divided into periods from the *right*.

COMMON METHOD.

ART. 4. According to the *Common* or *French method* of numeration, the first period on the right contains *units*, *tens*, and *hundreds*.

1	2	3.
hundreds	tens	units.

The second period contains units, tens and hundreds of *thousands;* the third, units, tens and hundreds of *millions;* the fourth, *billions;* the fifth, *trillions;* the sixth, *quadrillions;* the seventh, *quintillions;* the eighth, *sextillions;* the ninth, *septillions;* the tenth, *octillions;* the eleventh, *nonillions;* the twelfth, *decillions.*

The higher denominations are formed by prefixing to decillions the Latin words, *uno*, *duo*, *tre*, *quatuor*, *quin*, *sex*, *septen*, *octo*, *noven*.

ENGLISH METHOD.

ART. 5. According to the *English method*, the first six orders have the same names and signification as

those of the French. Every period, however, consists of six orders. The second period is million; the higher denominations are named the same as by the *Common* method, but have different significations.

REMARK.—It will be noticed that each period, according to the *Common method*, is one thousand times the preceding one, and according to the *English method* one million times. Hence, according to the Common method, a billion is a thousand millions, and according to the English a billion is a million millions.

26,839,506,720,052,005 according to the Common method would read: Twenty-six quadrillions, eight hundred and thirty-nine trillions, five hundred and six billions, seven hundred and twenty millions, fifty-two thousand and five.

The same, according to the English method, would be pointed off thus: 26839,506720,052005, and read, twenty-six thousand eight hundred and thirty-nine billions, five hundred and six thousand seven hundred and twenty-millions, fifty-two thousand and five.

ROMAN NOTATION.

ART. 6. In *Roman* notation numbers are represented by letters, as follows: I, one; V, five; X, ten; L, fifty; C, one hundred; D, five hundred; M, one thousand. A line over a letter increases its value one thousand times: thus, $\overline{D}$ denotes five hundred thousand. A letter of less value placed before one of greater value diminished the latter the amount of the value of the former: thus, CM denotes nine hundred.

$\overline{M}\overline{C}\overline{M}\overline{L}\overline{V}$DXLVII reads, one million nine hundred fifty-five thousand, five hundred and forty-seven. MDCCCLXXVI = 1876.

III. ADDITION.

ART. 1. The process of uniting two or more numbers into one is called *Addition.*

ART. 2. The result obtained is called *sum*, *amount*, *total*, or *footing.*

ART. 3. The sign +, when placed between two numbers, indicates that they are to be added together.

NOTE.—This book being a MERCANTILE ARITHMETIC, it is thought best to omit short examples in addition, as the parties using the same are supposed to be acquainted with the fundamental rules of the science, but need to acquire accuracy and rapidity. The principles will, therefore, be stated, and such hints given, which, when put into practice, will enable the learner to add up rapidly and correctly.

ART. 4. It is necessary, in performing the operations in addition, to place units under units, hundreds under hundreds, etc.

EXAMPLES.

To add forty, three hundred and seventy-two, one thousand eight hundred and sixty-seven, and eight hundred and ninety-five, they should be arranged as follows :

40
372
1867
895
3174

We commence the process by adding the right hand or unit column, beginning with the lower figure, thus: 5 and 7 are 12, and 2 are 14, that being 4 units and 1 teen.

The unit (4) is placed under the unit column as the result, and the teen (1) is added to the second or teens column.

Next the teens are to be added, thus: 1 (the 1 teen obtained by adding the unit column) and 9 are 10, and 6 are 16, and 7 are 23, and 4 are 27; namely, 27 teens or 7 teens and 2 hundred.

The 7 teens are placed under the teens column as the result, and the 2 hundred are added to the third or hundreds column, and the hundred column is added in the same way, resulting in 21 as the answer; namely, 21 hundred, or one hundred and 2 thousand. The one is placed under the hundred column as the result, and 2 thousand is added to the 1 thousand in the example, resulting in three thousand, which 3 is placed under the thousand column as the answer—the whole footing will now read (commencing at the left) three thousand one hundred and seventy-four (3174).

Add the following:

Ex. 1.	Ex. 2.	Ex. 3.	Ex. 4.
2365	92245	925683	7629543
8293	28392	968542	9832965
8769	67268	768656	7629824
2965	63629	329871	4567897
3276	24432	123456	7632851

ART. 5. In order to acquire rapidity, the learner should, from the beginning, avoid counting by their fingers, but should familiarize themselves with the *catch figures*. The *catch figure* is the unit figure of the result of adding two units together; thus: Whenever 5 and 6 are added together, the unit figure in the result (11) is one; whenever 6 and 9 are combined, the unit figure is 5.

Class exercises on giving the *catch figure* and on applying it will be found both interesting and profitable.

The exercise may be conducted as follows:

Where 7 and 9 are combined, the unit figure is—? (6). 17 and 9 are —? (26). 37 and 9 —? 67 and 9 —? 27 and 9—? The teacher putting the questions, and the scholars in concert responding by giving the answer.

5. 13965+6725+68349+76587+9825+99542=*
6. 2592+18596+9382+6732+95876+29326=
7. 8549+8329+6784+7376+92542+93586=
8. 3576+7654+3295+7628+27654+7629=
9. 3733+9258+8975+9268+9327+7652=
10. 6686+8259+9762+3876+8585+7895=
11. 2936+9286+7654+6832+9257+6873=

*The answers will be found at the close of the chapter (page 24).

ART. 6. The process of adding Federal Money differs from the foregoing only in the use of the dollar ($) and decimal (.) signs.

12. $568.32+, $965+, $985.20= $—?

NOTE.—In adding federal money the learner must be careful to place the decimal points (the sign separating the dollars and cents) of the amounts of money to be added under each other, thus:

$568.32
965.
985.20

13. $375.15+ $950.+ $876.51+ $7.57+ $987.56+ $781.+ $659.16+ $286.56+.56+ $185.20=

14. $878.10+ $758.+ $238.68+ $875.+ $658.99+ $878.+ $751.87+ $2.85+ $286+289.54=

15. $751.+ $518.91+ $361.98+ $678.10+ $777.67 + $765.+ $958.+ $392.51+ $682.19+ $775.20=

16. $868.19+ $18.+ $85.88+ $567.50+ $678.96+ $879.+ $759.15+ $894.26+ $824.18+ $982.56=

17. $781.59+ $759.10+ $899.99+ $569.+ $569.78 + $656.71+ $871.+ $326.50+ $98.27+ $976.58=

18. $798.15+ $7.76+ $786.56+ $437.+ $788.15+ $788.88+ $935.62+ $92.52+ $768.92=

19. $889.+ $878.99+ $878.95+ $898.10+ $897.+ $987.54+ $651.25+ $329.77+ $628.95+ $628.92=

20. $18146+$71.25+$641.04+$4501+$87700=

21. $1770.03+$1006.01+ $364.01+$5442.99=

22. $2310.00+$1068.24+$26107.18+$2136.18=

23. $109.79+ $999.99+ $666.56+ $449.99=

24. $777.00+$7999.00+ $6666.00+$6730.15=

25. A merchant has 29 pieces of silk in 1 package, 35 in another, 79 in a third. In the first, there are 1497 yards, in the second, 2173, in the third, 4130. How many pieces, and how many yards in all?

26. $1.23+283+$685.04+$123.45+$78=

27. 31465+2316532+107+3790+465321+3654563+107653+23650+1007+30672+503102+21063 is how much?

28. 18230+476+41034+9875+65432+5678+12090+9387+8276+565+13654+443=how much?

Ans. Sum of 27 and 28, 7344065.

29. 46853+9654+45679+9837+18708+7967+485+78963+84989+12345+7069+8090+7483+96748=?

TAKING TWO AND THREE FIGURES AT A TIME.

To enable scholars to grasp two and three figures at a time, and carry them up as one, they might be exercised on the blackboard in such sums as the following:

1	3	6	3	7	7	4	3	5		1	4	6	7	3	9	2	1	3	6	9	8
9	5	4	1	8	6	9	8	7		7	8	2	1	6	3	8	4	5	6	7	3
										2	1	3	4	1	4	1	3	6	2	1	2

Such exercises ought to be of frequent occurrence and scholars encouraged to answer in concert.

The answers should be given instantaneously, *naming only the unit figure*, as shown in the column below:

3456 / 1345 } 1
3689 / 1563 } 2
9456 / 3689 } 5
8998 / 1898 } 6
9873 / 1678 } 1
1684 / 7893 } 7
1453 / 1763 } 6
2195 / 9876 } 1
7897 / 2536 } 3
8529 / 1438 } 7

After writing on the right of the first column the figures produced by pairing, the teacher may lead the class in adding, thus: 17 and 3? 30 and 1? 41 and 6? 47 and 7? 54 and 1? 65 and 6? 81 and 5? 96 and 2? 108 and 11?

It will be observed that the tens produced in forming the pairs were not named. The same course should be pursued in the class, as the learner is unconscious of making as great an effort as he really does.

When the ten is omitted by mistake, attention should be called to it by giving the full number, as 15 or 11 instead of 5 or 1.

The other columns should be added without the aid of the marginal figures.

After thorough drill in this, the class should be taught to take three figures and even four as rapidly as one.

30. Find the sum of 8934, 16749, 809, 67549, 98697, 746839, 1498, 829555, 9218967, 8347912, 968000, 74685.

Total of the preceding two, 20815046.

Foot up the following columns:

31	32	33	34	35
31645	3454	4213	1565	3654
98760	2136	6314	3657	1095
36875	1364	2316	5437	9014
57893	4633	1369	3457	6789
14567	9897	9306	1234	9687
34564	7879	6039	3421	5764
46387	2164	8109	6789	1567
93178	4163	9876	1746	9139
78163	4569	6789	3456	1456
64518	5496	4567	1378	2345
17514	6428	5679	5932	5432
45678	8297	3263	4567	6542
21364	9287	9457	1657	1395
7198	7928	1459	6574	3642
3165	9872	1455	5638	1365
4124	8729	9375	4932	2315
1345	9314	5976	1397	9365
3146	3162	7639	9765	3510
4165	2136	7938	3765	1096
3216	9364	3959	1456	3765

36. Add together the following numbers: 313, 2109, 6785, 2736, 798, 987, 21363, 316, 4934, 2178, 1009, 396, 298, 2753, 607, 3145, 213, 6709, 6093, 190, 2130, 2160, 716, 213, 9876, 45678, 2137, 2198, 9039, 6789, 3097, 4684, 2136, 2178, 5672, 1987, 6789.

Answers promiscuously arranged: 95368, 77823, 120272, 115098, 667465, 88937, 171411.

The Teacher should not permit his scholars to divide these columns when adding, nor should he allow them to resort to the aid of strokes or practice counting on their fingers.

37	38	39	40	41
3286	2467	34564	46321	3614
6713	109	12345	13632	1364
3654	3178	65435	14567	5436
176	145	87654	53678	7835
3976	6178	34564	86367	4678
6345	4156	13682	85432	8793
9823	7532	75671	36457	701
6023	9890	86317	21836	9804
1367	6821	24328	17354	1306
8965	9854	98713	63542	717
8632	3821	21345	78163	2103
1034	5843	1286	82645	6397
6312	1936	78654	34685	1096
4593	7136	19876	31768	2130
3687	9876	93643	65314	3107
5006	2863	6356	68231	167
7164	123	78397	64037	2109
1763	7436	21602	34685	3678
2139	1567	71346	35962	2176
8236	2563	28653	21363	5432
7860	8432	17648	78636	2137
3613	1345	82351	19854	28639
109	8736	21368	80145	1765
1756	8654	78631	87654	371
6386	1263	17639	12345	71031
9890	1345	82360	78654	1463
8243	3093	45671	12345	3168

Answers: 42838, 48213, 217166, 274993, 162504, 45063, 37293080, 25668, 275966, 3116208, 185140, 378363, 434870, 136751, 126362, 1300099, 181217, 171411, 88937, 77823, 115098, 120272, 66746, 20380194, 57436, 7158925, 1325672, $2518.52, $5617.03, $6557.68, $6660.56, $5109.27, $6508.52, $5403.86, $7668.47, $31-621.60, $2226.33, $43.00, $22172.15, 7800, 2500, 2450, 21621200, 1243883.

IV. SUBTRACTION.

ART. **1.** The process of taking a lesser number or quantity from a greater of the same kind or denomination is called *Subtraction.*

ART. **2.** The result obtained is called *difference, remainder,* or *excess.*

ART. **3.** The sign of subtraction is —, and is called *minus.* 8 — 2 reads eight minus two.

ART. **4.** 1. Find the difference between 786 and 323.

SOLUTION.—We place the smaller number under the larger one, units under units and hundreds under hundreds, etc., and proceed to subtract from the right to the left, viz: 3 from 6 leaves 3; next we subtract the teens: 2 from 8 leaves 6, this we place in the teens place, 3 (hundred) from 7 (hundred) leaves 4 (hundred), which is placed in the hundreds place. The answer (Remainder) is 463,

$$\begin{array}{r} 786 \\ 323 \\ \hline 463 \end{array}$$

2. 23964 — 12853 3. 2986 — 258
4. 6972325 — 4232323 5. 6896542 — 84312
6. 276289995 — 16278585 7. 32987632 — 11976412

ART. **5.** 1. Find the difference between 5354 and 897.

SOLUTION.—In this case we find that the several numbers in the *subtrahand** are greater than those of the *minuend.*† We can not take 7 from 4, so we take one from the teens in the *min.* and add it to the units, making 14 — 7 from 14 leaves 7. Having taken one from the teens, leaves 4. 9 from four we can not take, so we again take one of the next figure in the *min.* (3) and add it to the teens, making again 14. 9 from 14 leaves 5. 8 from 2 (1 having been taken from the 3 to add to the teens) we can not take, so we proceed to take 1 from the thousands, which makes 10 (hundred) + 2 (hundred) gives us 12 less 8 makes 4. One having been taken from the 2 (thousand) leaves 1. Answer, 1457.

$$\begin{array}{r} {}^{1}{}^{10}{}_{2}{}^{10}{}_{4}{}^{10} \\ 2\cdot3\cdot5\cdot4 \\ 8\,9\,7 \\ \hline 1\,4\,5\,7 \end{array}$$

*Subtrahend, the number to be taken from the *minuend.*

†Minuend, the greater number, from which the lesser is to be subtracted.

2.	2345678 — 689829	3.	621129 — 509826
4.	6123546 — 5261862	5.	921654 — 629827
6.	3254298 — 1185169	7.	325627 — 124939

ART. 6. In order to ascertain the difference between the sum of two columns, subtraction may be formally dispensed with by adding the largest column first, and by adding in the difference thus:

1.	$385.42	$286.98
	279.35	385.46
	766.42	928.54
		326.28
	$1927.26	$1927.26

SOLUTION.—Having obtained the sum of the larger column, $1927.26, we proceed to add up the smaller, viz: 2 + 5 + 2 are 9 and **7** (to make the result the same as the units in the larger column) are 16 —— 1 (carried from the result of the units) + 4 + 3 + 4 = 12, the unit figure of this result being the same as that of the dimes in the larger column, **0** is the difference —— 1 (carried from the dimes column) + 6 + 9 + 5 = 21 + **6** (to make the difference) = 27 —— 2 + 6 + 7 + 8 = 23; + **9** (to make the difference) = 32 —— 3 + 7 + 2 + 3 = 15 + **4** to make the difference = 19. Thus giving as the difference, $496.07.

Proof:

$385.42	$1927.26 sum of large column.
279.35	$1431.19 sum of small column.
766.42	$496.07 difference.
$1431.19	

Find the difference, by addition, of the following:

2.	$628.93	$258.72	3. $3852.19	$625.28
	542.69	385.98	6829.16	398.75
	392.75	726.18	9325.18	285.32
	826.37	195.42	2762.29	975.68
	978.62	329.54	3218.75	932.85
	126.58			
	$3495.94			

Ans.: 463, 11111, 2728, 2740002, 6812230, 2600-11410, 21011220, 1457, $496.07, $1600.10, $22769.69, 101303, $25170.26, $17796.82, 200688, 2069129, 291-827, 1655849, 861684.

* This method was suggested to the author by E. P. Goodnough, Esq.

THE COMPLEMENT.

TAKING THE COMPLEMENT, or "making change," is the process of subtracting a lesser number from a "round sum." It is employed, as the second term indicates, in making change or finding the sum to be paid back to the payer out of the amount handed by him in payment. The complete number is always the sum of one or more of the denominations of coin or currency—$1, $5, $10, 50c., 25c., etc. It will be found that the complement of the teens is always in the 80s, the complete number being $1; the payment to be made, 11c.—complement, 89; the payment to be 19c.—complement, 81. The complement of the 20s in the 70s; that of the 30s in the 60s; of the 40s in the 50s; of the 50s in the 40s; of the 60s in the 30s; of the 70s in the 20s, etc.

It will be found to be a very profitable class-drill, to conduct an exercise on making change in the following way:

Teacher. The complete number being $3, what is the complement out of a payment of $1.50? (The class calls out the complement, $1.50.) The drill is conducted with enthusiasm for some time on the same complete number without naming it again, naming a different payment, thus: The complete number being $5, payment $3.25, complement —? payment $1.85? $1.75? $3.55? $4.50? 50c.? 75c.? 85c.? $3.60, etc.

The students should be required to give the denomination of the answer, whether in dollars, cents, etc. In a short time the students will find it an advantage to subtract from the left to the right instead of the reverse, by taking the $, calling $5.00 $4.9^{10}. We do not think it advisable to require the student to thus subtract from the left, but his attention may be called to the practicability, and if he find it of advantage, he should use it. If the habit is once acquired, it will facilitate the taking of the complement materially. We have conducted a class exercise in schools where it had never been taught, and in the course of a half hour the complement was given by the entire class *instanter*.

V. MULTIPLICATION.

ART. 1. Multiplication is a short method of adding. × is the sign. $3 \times 6 = 18$, *reads*, three times six equals eighteen.

MULTIPLICATION TABLE.

1 × 1 = 1	2 × 1 = 2	3 × 1 = 3	4 × 1 = 4
1 × 2 = 2	2 × 2 = 4	3 × 2 = 6	4 × 2 = 8
1 × 3 = 3	2 × 3 = 6	3 × 3 = 9	4 × 3 = 12
1 × 4 = 4	2 × 4 = 8	3 × 4 = 12	4 × 4 = 16
1 × 5 = 5	2 × 5 = 10	3 × 5 = 15	4 × 5 = 20
1 × 6 = 6	2 × 6 = 12	3 × 6 = 18	4 × 6 = 24
1 × 7 = 7	2 × 7 = 14	3 × 7 = 21	4 × 7 = 28
1 × 8 = 8	2 × 8 = 16	3 × 8 = 24	4 × 8 = 32
1 × 9 = 9	2 × 9 = 18	3 × 9 = 27	4 × 9 = 36
1 × 10 = 10	2 × 10 = 20	3 × 10 = 30	4 × 10 = 40
1 × 11 = 11	2 × 11 = 22	3 × 11 = 33	4 × 11 = 44
1 × 12 = 12	2 × 12 = 24	3 × 12 = 36	4 × 12 = 48
5 × 1 = 5	6 × 1 = 6	7 × 1 = 7	8 × 1 = 8
5 × 2 = 10	6 × 2 = 12	7 × 2 = 14	8 × 2 = 16
5 × 3 = 15	6 × 3 = 18	7 × 3 = 21	8 × 3 = 24
5 × 4 = 20	6 × 4 = 24	7 × 4 = 28	8 × 4 = 32
5 × 5 = 25	6 × 5 = 30	7 × 5 = 35	8 × 5 = 40
5 × 6 = 30	6 × 6 = 36	7 × 6 = 42	8 × 6 = 48
5 × 7 = 35	6 × 7 = 42	7 × 7 = 49	8 × 7 = 56
5 × 8 = 40	6 × 8 = 48	7 × 8 = 56	8 × 8 = 64
5 × 9 = 45	6 × 9 = 54	7 × 9 = 63	8 × 9 = 72
5 × 10 = 50	6 × 10 = 60	7 × 10 = 70	8 × 10 = 80
5 × 11 = 55	6 × 11 = 66	7 × 11 = 77	8 × 11 = 88
5 × 12 = 60	6 × 12 = 72	7 × 12 = 84	8 × 12 = 96
9 × 1 = 9	10 × 1 = 10	11 × 1 = 11	12 × 1 = 12
9 × 2 = 18	10 × 2 = 20	11 × 2 = 22	12 × 2 = 24
9 × 3 = 27	10 × 3 = 30	11 × 3 = 33	12 × 3 = 36
9 × 4 = 36	10 × 4 = 40	11 × 4 = 44	12 × 4 = 48
9 × 5 = 45	10 × 5 = 50	11 × 5 = 55	12 × 5 = 60
9 × 6 = 54	10 × 6 = 60	11 × 6 = 66	12 × 6 = 72
9 × 7 = 63	10 × 7 = 70	11 × 7 = 77	12 × 7 = 84
9 × 8 = 72	10 × 8 = 80	11 × 8 = 88	12 × 8 = 96
9 × 9 = 81	10 × 9 = 90	11 × 9 = 99	12 × 9 = 108
9 × 10 = 90	10 × 10 = 100	11 × 10 = 110	12 × 10 = 120
9 × 11 = 99	10 × 11 = 110	11 × 11 = 121	12 × 11 = 132
9 × 12 = 108	10 × 12 = 120	11 × 12 = 132	12 × 12 = 144

Write the multiplication table as follows:

2 times 1 or once 2 is 2.
2 times 2 are 4.
2 times 3 or 3 times 2 are 6.
2 times 4 or 4 times 2 are 8.

Continue this to 12.

1. To find the *sum* of 123+123+123, we would enter the three amounts as in addition, and *add* for the result.

In *multiplication* we write 123 as in the margin, and say, 3 times 3 are 9; put 9 in the unit's place. Three times 2 are 6; put 6 in the ten's place. Three times 1 are 3; which put in the hundred's place. The result is 369, as it would have been by addition.

123
3
369

TERMS.

ART. 2. The number 123 is called the *multiplicand*, the number 3 the *multiplier*, and 369 the *product*. The multiplicand and multiplier are also called *factors*.

2. To find the product of 1496 by 7.

Here we say 7 times 6 are 42; write 2 under the 7. Then 7 times 9 are 63, and the 4 we carried make 67; write 7 and carry 6. 7 times 4 are 28 and 6 are 34; write 4 and carry 3. 7 times 1 are 7 and 3 are 10.

Ans. 10472.

1496
7
10472

3. 2146×2= 4292
3178×3= 9534
4167×4=16668
5189×5=*
7864×6=
2875×7=

Total, 123748

4. 21007× 5=*
31497× 6=
17843× 7=
41679× 8=
98765× 9=
73149×12=

Total, 2519023

*The pupil will fill the blanks.

Observe to point off the cents in the products of the following:

5.		6.	
$21.37 × 7 =	*	$10.73 × 9 =	*
117.49 × 8 =		117.07 × 6 =	
317.00 × 9 =		307.49 × 7 =	
671.49 × 10 =		678.39 × 11 =	
857.37 × 11 =		467.28 × 12 =	
1096.49 × 12 =		999.99 × 9 =	
Total,	$33246.36	Total,	$25021.08

7. 2785 × 357.

We have here three multipliers—seven, fifty, and three hundred.

2785 × 7 =	19495	19495
2785 × 5 tens =	13925 tens, or	139250
2785 × 3 hundreds =	8355 hundreds, or	835500
	Total products,	994245

This operation might be contracted by arranging the figures as in the margin, and writing the *first* figure of the products of the units in the unit's place and the others to the left of it; the first figure of the product of the tens in the ten's place, or under its own multiplier, 5; and the product of the hundreds in the hundred's place.

```
 2785
  357
-----
19495
13925
 8355
-----
994245
```

8.			
3170 × 178 =	564260	1578 × 753 =	*
6184 × 1794 =	*	9409 × 6781 =	
3867 × 3784 =		2783 × 4679 =	
2896 × 6789 =		8976 × 7659 =	68747184
7109 × 9998 =	71075782	3968 × 6483 =	
2345 × 3979 =		7689 × 2197 =	
6789 × 2164 =		6784 × 7898 =	
Total product,	141049961	Total product,	242956813

NOTE.—Either factor may be used as a multiplier in the above exercises.

* The pupil will fill the blanks.

9. 420001000	10. 109608
109608	420001
3360008	109608
2520006	219216
3780009	438432
420001	
Product, 46035469608000	Product, 46035469608

The multiplier of the ten's place in the first operation being 0, we passed it, and multiplied by the 6 hundreds. In the second operation we passed the ten's, hundred's, and thousand's places for the same reason.

NOTE.—If the learner will simply observe to write the first figure of each product under its own multiplier, he will have no difficulty in multiplying where there are ciphers. For instance, the first figure of the product by 2, in the second example, is immediately *under* the 2.

11.			12.	
12346× 30010=	370503460		2000× 7010=*	
7684× 10900=*			3160×10096=	
6787× 3009=			2178×90909=	197999802
4967× 6007=	29836769		1009×90910=	
5896×900707=			21678×21006=	
7649× 66080=			31784× 7009=	
Total,	6320532304		Total,	1013793476

ART. 3. To multiply by 10, 100, 1000, etc., we have only to annex as many ciphers (0) to the multiplicand as there are in the multiplier:

$$35 \times 10 = 350$$

EXPLANATION.—Multiplying by the 1 and the 0, we say 0 time 5 is 0; write 0 under 5, then 1 time 5 is 5, 1 time 3 is 3=350.

```
  35
  10
 ---
 350
```

NOTE.—*One time* the number is simply the number repeated, so we may as well annex the cipher to the original, as above.

* The pupil will fill the blanks.

$165 \times 10 = 1650$
$165 \times 100 = 16500$
$165 \times 1000 = 165000$

$25 \times 20 = 500$
$25 \times 200 = 5000$
$25 \times 2000 = 50000$

2. $374 \times 10 =$
$268 \times 100 =$
$189 \times 1000 =$
$267 \times 10000 =$

Total, 2889540

3. $749 \times 2000 =$
$836 \times 16000 =$
$341 \times 21000 =$
$876 \times 92000 =$

Total, 102627000

PRINCIPLES OF MULTIPLICATION.

ART. **4.** When two numbers are to be multiplied together, we use for the multiplier that which will produce least figures in the operation. This will be accomplished by selecting the smaller number, except where there are many ciphers, as in Ex. 3.

ART. **5.** If a *number* of articles and the price of *one* article be multiplied together, the product will be the price of *all* at the same rate.

If the price of *one* be in *cents*, the price of *all* will be in *cents*. If in *dollars*, the price of all will be in *dollars*.

NOTE.—Cents are easily converted into dollars, by inserting the separating point. Those on the left will be dollars, the others cents.

ART. **6.** The number of articles contained in any box, bale, package, etc., multiplied with the number of boxes, bales, etc., each containing a like number, will give the whole number of articles in all.

ART. **7.** The interest, discount, premium, commission of one dollar multiplied with the number of dollars, will give the interest premium, etc., of the whole number of dollars.

ART. 8. Any number multiplied by itself, is said to be squared or raised to the *second power* — any number multiplied by itself, and that number again multiplied by the first, is said to be *cubed* or raised to the *third power*.

ILLUS.—$2\times2=4$, or 2d power of 2, or 2^2.
$2\times2\times2=8$, or 3d power of 2, or 2^3.
$2\times2\times2\times2=16$, or 4th power of 2, or 2^4.

ART. 9. Feet multiplied by feet, yards multiplied by yards, etc., produce *square feet*, *square yards*, etc.

ART. 10. Any number of feet multiplied by the number of inches in one foot, will give the number of inches in all the feet. Pounds multiplied by the number of ounces in 1 pound, will give the number of ounces in all the pounds.

ART. 11. Halves, thirds, fourths, multiplied on whole numbers, produce halves, thirds, and fourths.

1. What is the price of 37 bushels of corn at 37 cents per bushel?

2. What should I pay for 357 yards of broadcloth at $2.75 per yard?

3. Find the cost of 325 acres of land at $57 per acre?

Total, $19522.44

4. In 320 bales of cotton there are 460 lbs. each, how many in all? *Ans.* 147200 lbs.

5. In 557 pieces of muslin there are 35 yards each, how many in all? *Ans.* 19495 yds.

6. A ship laden with flour has 7950 barrels on board, and in each barrel there are 196 lbs., how many pounds in all? *Ans.* 1558200 lbs.

* The pupil will fill the blanks.

7. The premium on a dollar is .03 or 3 cents, how much on $149? *

8. At 6 cents on the dollar, how much interest should be received on $1750?

NOTE.—Six cents on the dollar, is the same as 6 per cent.

9. At 8 per cent. premium, how much should be paid on $3764?

Total, $410.59

10. Find the square of the following numbers:

37 =	1369	376 *
570 =	324900	219
10960		109
Total,	120447869	Total, 201218

11. 103^2† is how much?
107^3 is how much?
19^4 is how much?

Total, 1365973

12. How many square feet are in a room measuring 15 feet long, and 14 feet wide? *

13. How many square feet in a board 16 feet long, with an average breadth of 2 ft?

Total, 242 feet.

14. How many feet are in 573 yards? *Ans.* 1719.
15. How many inches in 573 yards? *Ans.* 20628.
16. How many rods are in 374 acres? *Ans.* 59840.
17. If 146 is multiplied by $\frac{3}{7}$, what is the product? *Ans.* $43\frac{8}{7}$.

* The pupil will fill the blanks.

† The small figures are called *indices*. They are used to indicate to what power the numbers are to be raised. See Art. 8.

ART. 12.—*Practical Questions:* 1. Find the price of 87 bushels of wheat at 84 cents a bushel.

2. If I pay 25 cents a cwt. for freight, what should I pay on 2307 cwt.?

3. What will 35 acres of land cost at $25 an acre?

4. How many yds. of muslin in 6 cases, each case containing 20 pieces, and each piece 35 yds.; and what will be the cost of the whole at 12 cents a yard?

5. 250 boards 12 feet long and 1 foot broad were sold at 2 cents a foot, what was the cost?

6. A floor measures 25 feet long and 23 broad, how many square feet does it contain?

7. A merchant failing in business can pay only 37 cents on the dollar, how much will the creditor receive to whom he is indebted $7587?

8. How many quarts are in 25 bushels?

9. In a day there are 24 hours, how many seconds are there?

10. How many pints are there in 17 bushels 2 pecks?

11. How many inches are there in 3 yards 2 feet?

12. In a ream of paper, how many sheets are there?

13. A commission merchant receives 2 %, or 2 cents on the dollar, how much should he receive on $1425?

14. At $3.75 a dozen, what will 7 dozen of chisels cost?

Answers to the above: $576.75, $73.08, $875, 480, 132, 800, 86400, 1120, $504, $68, 575, $2807.19, 4200, $28.50. $26.25, $19.50.

VI. DIVISION.

ART. 1. Division is the method of calculation used to separate a number into equal parts.

ART. 2. The sign is $\div$. When placed between two numbers it indicates that the one on the *left* is to be divided by the one on the *right*.

$6 \div 3$, reads *six divided by three.*

ART. 3. Division is also indicated as follows:

3)6. Which indicates that 6 is to be divided by 3.

$\frac{6}{3}$. This is a *Common Fraction*, and indicates that 6 is to be divided by 3 also.

$\frac{\frac{1}{2}}{\frac{2}{3}}$. This is also a fraction, and indicates that the fraction $\frac{1}{2}$ is to be divided by the fraction $\frac{2}{3}$.

.5, .05 are called *Decimals*, and signify that the first is divided by 10, and the second by 100.

NOTE.—The separating point between dollars and cents is a decimal sign, and indicates that the figures on the right are so many *hundredths of a dollar.* \$4.25 is $\$4\frac{25}{100}$.

TERMS.

ART. 4. The number by which we divide is called the *divisor.*

The number to be divided is the *dividend.*

The number produced by dividing is the *quotient*

The number left, the *remainder.*

dividend.

divisor 3)16784

quotient 5594 — 2 *remainder.*

DIVISION TABLE.

The pupil can make a division table of the multiplication table, by reciting it as follows: 3 times 2 are 6, 3 in 6, 2 times; 3 times 3 are 9, 3 in 9, 3 times.

To divide 346 by 2.

EXPLANATION 1.—Commencing at the *left*, we say 2 in 3, 1 time and 1 left; write the 1 (time) under the 3.
2. Carrying the 1 that was left, we suppose it to stand before the 4, which will raise that number to 14; then 2 in 14 7 times; write 7 under the 4.
4. Then 2 in 6, 3 times. *Ans.* 173.

$$2)346$$
$$173$$

REMARK.—Until he becomes familiar with the process, the learner might write the remainders in small figures, as in the following example. When he has mastered the operation, he can dispense with them.

2. To divide 13076837617 by 8.

$$8)1\ 3\ {}^{5}0\ {}^{2}7\ {}^{3}6\ {}^{4}8\ 3\ {}^{3}7\ {}^{5}6\ 1\ {}^{1}7$$
$$1\ 6\ 3\ 4\ 6\ 0\ 4\ 7\ 0\ 2\text{—}1$$

EXPLANATION.—Commencing at the left, we say, 8 in 13, 1 time and 5 left; place 5 before the next figure, then 8 in 50, 6 times and 2 left; place 6 below, and 2 before the 7 and so on.

Divide the following:

		Quotients.	*Rem.*
3.	134615379÷2	67307689	1
	21637298452÷3	7212432817	1
	59368217755÷4	14842054438	3
	1416823687949÷5	283364737589	4

		Rem.			*Rem.*
4.	13645217÷6	5	5.	361745731÷10	1
	23176841÷7	2		213764952÷11	5
	47896739÷8	3		178961521÷12	1
	89765432÷9	8		345678900÷12	0
Total quotients,		=21546207			=99327785

NOTE.—The value of the remainder may be expressed in a fractional form: $168\div9 = 18\frac{6}{9}$; which signifies that 9 is contained in 168 eighteen and *six-ninths* times.

*6. Divide $4537.25 between 7 persons,

Ans. Each person will have 648.17\frac{1}{7}$.

Omit the remainders in the following:

7. $21372.00÷3	8. $67849132.87÷ 8
13744.00÷8	16493178.00÷ 7
73176.35÷5*	23610934.10÷ 9
14537.07÷9	12310987.47÷11
Total quotients, $25092.50,	$14, 579,927.67.

When flour is $1 a barrel, the loaf will weigh 8 times as much as when it is $8 a barrel: $8\times 9=72$ oz. at $1 a barrel. At $6, it will weigh only $\frac{1}{6}$ as much. $\frac{72}{6}=12$ oz.

9. If 6 men do a piece of work in 11 days, how long will it take 4 men to do it?

10. If 27 men in three days, do a piece of work, how long should it take 25 men?

11. The interest on $367 for 60 days at 6 %, is $3.67; what should be the interest on $1687.25 for the same length of time, and at the same rate per cent.?

12. In a square foot there are 144 square inches, how many square inches are there in a room 15 feet 6 inches by 18 feet 6 inches?

ART. 5. The quotient of a number divided by 2 is the $\frac{1}{2}$ (one-half) of it, divided by 3 it is $\frac{1}{3}$, by 4 the $\frac{1}{4}$; hence to find the $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc., of a number, we have only to divide by 2, 3, 4, etc.

1. $\frac{1}{3}$ of $3716=1238\frac{2}{3}$ 2. $\frac{1}{7}$ of $34161143764=$——$\frac{6}{7}$

Answers: 4880163394$\frac{6}{7}$, 47371476545/8, 1570$\frac{4}{11}$, 1874-7057964/9, 2880348786, 146$\frac{3}{6}$, 162$\frac{1}{2}$, 3$\frac{6}{25}$, 3060, 1238$\frac{2}{3}$, 341$\frac{3}{7}$, 648.17\frac{4}{7}$, $7124, $1718, $14635.27, $1615.23, $16.87, $8481141.61, $2356168.28, $2623437.12, $1119-180.68.

*Remove the decimal point before dividing, and replace it in the quotient.

NOTE.—The learner will observe that we did not *divide* by the fractions in the preceding exercises; on the contrary, we *multiplied* by them. Omit the fractions in the answers of questions in the following groups:

3. $9\frac{1}{3}$ times \$14567.85 is how much?

Ans. \$135966.60.

4. \$ 345.78×37$\frac{1}{3}$ Total, \$24897.37$\frac{1}{2}$.

5. \$4563.28×45$\frac{1}{6}$, 16$\frac{1}{3}$, 18$\frac{1}{2}$ Total, \$3624130.43.

6. If I pay $12\frac{1}{2}$ cents on the dollar for a loan of money, how much should I pay for the use of \$4527?

NOTE.—On 4527 dollars I would pay 4527 times as much as on 1 dollar—4527 times $12\frac{1}{2}$ cents; or $12\frac{1}{2}$ times 4527=56587$\frac{1}{2}$ cents; or \$565.88.

What should I pay on the following amounts at the rates specified?

7. \$3146@2$\frac{1}{2}$, 6$\frac{1}{2}$
1567@3$\frac{1}{3}$, 5$\frac{1}{6}$
7864@6$\frac{1}{4}$, 7$\frac{1}{5}$
Total, \$1474.04.

8. \$71684.25@8$\frac{1}{3}$, 7$\frac{1}{2}$, 5$\frac{1}{2}$
89647.87@1$\frac{1}{2}$, 2$\frac{1}{2}$, 3$\frac{1}{2}$
79943.57@ $\frac{1}{4}$, 6$\frac{1}{3}$, 7$\frac{1}{2}$
Total, \$33274.96.

9. If a steamboat be worth \$155367, what would $\frac{1}{2}$ be worth? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{7}$? $\frac{1}{8}$? $\frac{1}{9}$? $\frac{1}{12}$?

Total, \$297108.56.

ART. 6. To divide by 10, we cut off the right hand figure, then the figures on the left will be the quotient, and those on the right the remainder.

1. Divide 25 by 10. *Operation*—2|5, or 2.5=2$\frac{5}{10}$.

2. Divide 6498 by 100. *Ans.* 64|98, or 64.98=64$\frac{98}{100}$.

NOTE.—We divide by 104, 1000, etc., in the same way, only, instead of cutting off one figure, we cut off as many figures as there are ciphers—for 100, two figures; for 1000, three figures, etc.

Answers: \$135966.60, \$12966.75, \$1223.50, \$11707, 125, \$565.88, 364$\frac{49}{50}$, 2$\frac{1}{5}$, \$78.65, \$204.49, \$52.23, \$80.96, \$491.50, \$566.21, \$5973.687, \$5376.318, \$3942.633, \$1344.718, \$2241.196, \$3137.675, \$199.878, \$5063.092, \$5995.767, \$307791, \$206108.14, \$74533.56, \$84420.68, \$330072.16, \$636567.75, \$582183.44, \$73166.24, \$297-108.56.

NOTE.—If the 36498 had been dollars, then the answer would have been $364.98; or 364 dollars, 98 cents.

3. 43645÷ 10
71987÷100
81674÷100
21362÷100
Total, 6114.73

4. $168938÷ 10 and by 100
678476÷100 and " 10
396889÷100 and "1000
798755÷100 and " 10
$185444.369

ART. 7. When there are cents, the division may be performed by removing the decimal point toward the left. To divide by 10 we remove it *one* figure, to divide by 100 we remove it *two* figures, by 1000 *three* figures, etc.

$55.10 ÷ 10=$5.510, $167.56 ÷100=$1.67,56.

NOTE.—The value of each and all of the figures decreases *ten*-fold for every figure the decimal point is removed to the left. The $5 of first example become 50 cents, and the 10 cents become 10 mills or 1 cent; making the *answer* 5 dollars 51 cents; *not* 5 *dollars* 510 *cents*. The second answer is 1 dollar 67 cents $5\frac{6}{10}$ths mills.

Divide the following, omitting the remainders:

5. $ 457.87÷ 10
1677.45÷ 100
6109.88÷1000
14999.99÷ 100
Total answers, 218 dols. 66 cts.

6. $473.04÷1000 and 100
15.17.÷ 10 and 100
16.57÷ 100 and 10
106.07÷ 100 and 1000
9 dols. 85 cts. 9 mills.

7. Divide the following sums of money by 100: $645, $1678.25, $87493.57, $16453.27, $1998.38, $643.-24, $2168, $4137.54. Total answer, $1152.16,9

ART. 8. It often happens that there are not as many figures to cut off, as there are ciphers in the divisor. In such cases we annex ciphers to the left of the dividend to make up the number.

Divide $5. by 100. *Ans.* .05.

EXPLANATION.—This is the same as removing the decimal point two places to the left, as above. The $5 had the decimal point

on the right of the 5, it is now two places farther to the left, and therefore is divided by 100. The cipher in this case, as elsewhere, possesses no value.

8. $ 5÷ 10 = .5
3÷ 100 = .03
4÷1000 = .004
50÷1000 = .05
457÷1000 = .457

9. $ .03÷ 10
.02÷100
.14÷100
3.16÷100
Ans. $0.0362

10. Divide the following sums by 100: 3 cents, 33 cents, $3.33, $33.33, $333.33, $3333.33.

Total, $37.03,68

ART. 9. To divide by 20, 300, 5000, etc., we point off as many figures in the dividend as there are ciphers in the divisor, and divide by the 2, 3, 5, etc. The figures pointed off will form part of the remainder.

1. Divide 317745 by 500.

5|00)3177|45
635–245
635 $\frac{245}{500}$

2. 467831÷ 20=23391$\frac{11}{20}$
716893÷300=2389$\frac{193}{300}$
417368÷500=834$\frac{368}{500}$

3. 716849÷700=
897653÷900=
49673÷ 80=

Total quotients, 2083.58

Note.—To divide dollars and cents, first reduce the dollars to cents or mills.

4. Divide $36147.59 by 500.

500)36147|59
7229 $\frac{259}{500}$
or, $72.29 $\frac{259}{500}$

5|00)361475|90
72295 $\frac{90}{500}$ mills
or, $72.29,5

In the last solution the answer is given in dollars, cents, and mills—72 dollars, 29 cents and 5 mills. To reduce cents to mills we have only to annex a cipher to the right of the cents, as in this case.

The answers to the following are required in dollars, cents, and mills, omitting the remainders:

5. $13764.75÷50=
73968.23÷60=
37437.18÷90=
18964.20÷80=

6. $16789.37÷ 80=
67859.67÷900=
54168.23÷700=
78910.00÷600=

7. Divide the following sums by 20, and give the answers as above: $1367.25, $3143.57, $2345.87, $34.57, $45670.44.

8. Divide $34567.25 by 10, 12, 20, 100, 30, 50, 70, and 90. Total, $11132.84,6.

9. Divide $367897.87 by 100 and the quotient by 10, 20, 30, 40, 50, 60, 70, 80, 90. Total, $4719.74,4.

10. Divide $17654.37 by 100 and the quotient by 3, 10, 7, 40, 30, 50, 70, 90 and 80. Total, $298.78.

11. Divide $314937 by 100 and multiply the quotient by 7, then divide the quotient by 30, 60, 40, 12, 9, 80, 90. Total, $26117.89,9.

12. Find the sum of $\frac{1}{2}$, $\frac{1}{5}$, $\frac{1}{12}$, $\frac{1}{20}$, $\frac{1}{30}$, $\frac{1}{40}$, $\frac{1}{60}$, of the $\frac{1}{100}$th part of $6739.45. $61.21,3.

13. The $\frac{1}{20}$, $\frac{1}{30}$, $\frac{1}{10}$, $\frac{1}{80}$, $\frac{1}{50}$, $\frac{1}{40}$th of $16894.39 divided by 100, is how much? Total, $41.38,8.

ART. 10.—*Practical Questions:* 1. At 16 cents a bushel for coal, how many bushels can be purchased for 127 feet of lumber at $3.75 a hundred?

2. Three persons invest $6000 in business. The first, $3000; the second, $2000; and the third, $1000; and their gains are $2400; what was each man's share?

Answers: $494.16,9, $2161.11,8, $11132.84,6, $2628.53,3, $4719.74,4, $26117.89,9, $298.78, $41.38,8, $61.21,3; $29\frac{3}{4}$, $1200, $800, $400.

VII. EASY FRACTIONS.*

ART. 1. A FRACTION is a part or number of parts of any thing considered as a whole. Fractions are of two kinds, *common* and *decimal.* A common fraction is written with two numbers, called *terms*, having a line between them, as $\frac{1}{2}$; a decimal fraction with one number, having a period at the left, as .5 (five-tenths).

ART. 2. A common fraction indicates division, the upper number being the dividend and the lower the divisor. In treating of fractions, the dividend is called the *numerator* and the divisor the *denominator.*

The denominator indicates the number of parts into which the whole is divided, and the numerator the number of such parts under consideration.

ART. 3. VALUE OF A FRACTION.—The lowest value of a fraction is expressed by the figure 1 for a numerator, and the highest value a number as great as the denominator less 1.† $\frac{1}{9}$ represents the lowest value of fractions of the denomination of ninths, while $\frac{8}{9}$ represents the highest value of that denomination.‡

*This chapter is introduced for the benefit of that large class of scholars who leave school before completing the study of Arithmetic. The subject of fractions is treated of at length in the latter part of this book.

†This does not apply to *improper* fractions, which, as the name indicates, are not strictly fractions.

‡1. Since this is the case, it is evident that fractions decrease in value as their denominators increase, the numerators remaining the same. $\frac{1}{3}$ is less than $\frac{1}{2}$, $\frac{1}{4}$ than $\frac{1}{3}$, $\frac{1}{5}$ than $\frac{1}{4}$.

2. It is also evident that the value of a fraction depends on the relation of the numerator to the denominator, or, in other words, the number of times the numerator is contained in the denominator. $\frac{3}{8}$ is equal to $\frac{4}{8}$, because the numerator 3 is contained in its denominator, 6, the same number of times that the numerator 4 is contained in the denominator 8.

When a number is divided into two parts, each part is called a *half;* into 3 parts, each part is called a *third;* into 4 parts, each part is called a *fourth;* into 5, a *fifth;* into 12, a *twelfth;* into 18, an *eighteenth;* into 25, a *twenty-fifth;* into 100, a *hundredth;* into 476, a *four hundred and seventy-sixth part.*

1. When a number is divided into 10 parts, what is each part called? Into 11? Into 20? Into 33? Into 45? Into 97? Into 62?

2. When divided into 31, what? Into 69? Into 103? Into 364? Into 155? Into 1000? Into 3144?

3. Which is greater, $\frac{2}{3}$ or $\frac{3}{4}$? *Ans.* $\frac{3}{4}$.

REASON.—Because it will take less to make it a whole number. The first fraction requires $\frac{1}{3}$ to make it a whole number, while this one requires only $\frac{1}{4}$.

4. Which is greater, $\frac{7}{8}$ or $\frac{6}{7}$? $\frac{5}{6}$ or $\frac{5}{5}$? $\frac{3}{4}$ or $\frac{4}{5}$? $\frac{8}{9}$ or $\frac{7}{8}$? $\frac{2}{3}$ or $\frac{7}{8}$? $\frac{7}{9}$ or $\frac{8}{10}$? $\frac{12}{13}$ or $\frac{11}{12}$? $\frac{7}{8}$ or $\frac{7}{9}$?

Since the value of a fraction depends upon the relation of the numerator to the denominator, [note 2, page 43,] both terms may be multiplied or divided by the same number without altering its value.

$$\frac{1\times 2}{4\times 2}=\frac{4}{8} \qquad \text{and} \qquad \frac{2\div 2}{4\div 2}=\frac{1}{2}$$

Now, $\frac{4}{8}$ and $\frac{1}{2}$ possess the same value as $\frac{2}{4}$, because their respective numerators are contained the same number of times in their denominators.

5. Change $\frac{3}{4}$ to twentieths.

$\frac{3\times 5}{4\times 5}=\frac{15}{20}$ EXPLANATION.—By multiplying the 4 by 5, we change the denominator to twentieths; and by multiplying the numerator by the same number we preserve the original value.

6. Change $\frac{3}{4}$ to 8ths; $\frac{1}{2}$ to 12ths; $\frac{1}{5}$ to 20ths; $\frac{2}{7}$ to 14ths; $\frac{3}{4}$ to 12ths; $\frac{5}{6}$ to 18ths; $\frac{4}{5}$ to 30ths.

Answers: $\frac{7}{8}$, $\frac{5}{5}$, $\frac{4}{5}$, $\frac{8}{9}$, $\frac{7}{8}$, $\frac{8}{10}$, $\frac{12}{13}$, $\frac{7}{9}$, $\frac{6}{8}$, $\frac{6}{12}$, $\frac{4}{20}$, $\frac{4}{14}$, $\frac{9}{12}$, $\frac{15}{18}$, $\frac{24}{30}$, $\frac{3}{4}$.

ART. 4. To *reduce a fraction* to its lowest terms is to divide the numerator and denominator by such a number or numbers as will do so without a remainder. When the terms can not be exactly divided by any number greater than 1, the fraction is in its simplest form.

$$6)\frac{6}{12}(\frac{1}{2}$$

1. Reduce $\frac{5}{100}$, $\frac{7}{14}$, $\frac{3}{27}$, $\frac{9}{189}$, $\frac{2}{22}$, $\frac{6}{42}$, to their lowest terms.

When a single number will not reduce the fraction, other numbers may be used, as below.

2. Reduce $\frac{55}{9900}$ to its lowest terms.

$$5)\frac{55}{9900}(11)\frac{11}{1980}(\frac{1}{180}.$$

3. Reduce to their lowest terms, $\frac{48}{272}$, $\frac{46}{116}$, $\frac{176}{484}$, and $\frac{48}{160}$.

Fractions may be *Proper*, *Improper*, *Simple*, *Compound* or *Complex*. We shall treat of only the three former at present.

A *proper* fraction is one whose numerator is less than its denominator, as $\frac{1}{2}$. An *improper* fraction is one whose numerator is equal to or greater than its denominator, as $\frac{5}{5}$, $\frac{9}{7}$.

ART. 5. A *simple* fraction is a single fraction, and may be proper or improper, as $\frac{3}{7}$, $\frac{9}{8}$.

ART. 6. When a whole number and fraction appear together, they are called a *mixed number*, as $5\frac{3}{4}$.

ART. 7. *Improper fractions may be changed to whole or mixed numbers by dividing the numerator by the denominator.**

Answers: $\frac{1}{180}$, $\frac{7}{34}$, $\frac{23}{58}$, $\frac{44}{141}$, $\frac{3}{10}$, $2\frac{3}{5}$, $\frac{1}{20}$, $\frac{1}{2}$, $\frac{1}{9}$, $\frac{1}{21}$, $\frac{1}{11}$, $\frac{1}{7}$, $\frac{3}{17}$.

* This is simply acting on the principle that the numerator is the dividend and the denominator the divisor.

To change $\frac{13}{5}$ to a mixed number.

5)13
$2\frac{3}{5}$

EXPLANATION.—There are 5 fifths in one whole number; in 13 fifths there are as many 1s as the number of times 5 is contained in 13, which is two times, with 3 fifths over, making $2\frac{3}{5}$.

1. Change $\frac{2}{3}$, $\frac{9}{6}$, $\frac{5}{3}$, $\frac{18}{6}$, $\frac{271}{9}$, $\frac{834}{12}$ to whole or mixed numbers.

ART. 8. *To change whole or mixed numbers to improper fractions* is an operation the reverse of the last, which scarcely needs explanation.

1. Change $9\frac{4}{5}$ to an improper fraction.

$9\frac{4}{5}$
5
$\frac{49}{5}$

EXPLANATION.—In 1 whole number there are 5 fifths; in 9 there are 9 times 5 or 5 times 9 fifths, to which we add 4 fifths, and we have $\frac{49}{5}$.

2. *Change the following mixed numbers to improper fractions: $3\frac{5}{6}$, $97\frac{1}{8}$, $8\frac{6}{7}$, $54\frac{2}{5}$, $41\frac{3}{4}$, $97\frac{1}{8}$, $16\frac{2}{3}$.

ART. 9. *To multiply a fraction by a whole number* is simply to multiply the numerator without altering the denominator, or to divide the denominator without altering the numerator.

To multiply $\frac{7}{12}$ by 6.

$$\frac{7}{12}\times 6=\frac{42}{11}=3\frac{6}{12} \text{ or } 3\frac{1}{2}.$$

REASON.—Assuming that 7 is a whole number, multiplying it by 6 gives 42; but since it is *not* a whole number, but *twelfths*, the 42 is $\frac{42}{12}=3\frac{6}{12}$ or $3\frac{1}{2}$.

2. $6)\frac{7}{12}(\frac{7}{2}=3\frac{1}{2}$.

By decreasing the denominator, the fraction is increased (as it takes fewer of the small parts to make a whole number); hence, the 7 represents *halves* instead of twelfths. $\frac{7}{2}=3\frac{1}{2}$.

Answers: $1\frac{1}{2}$, $1\frac{1}{2}$, $1\frac{2}{3}$, 3, $30\frac{1}{9}$, $69\frac{1}{2}$, $\frac{49}{5}$, $\frac{23}{5}$, $\frac{79}{8}$, $\frac{62}{7}$, $\frac{29}{5}$, $\frac{167}{4}$, $\frac{777}{8}$, $\frac{50}{3}$, $3\frac{1}{2}$, $\frac{2}{3}$, $\frac{35}{6}$, $69\frac{7}{12}$.

* The learner should prove the accuracy of his work by last article.

1. $\frac{3}{4} \times 7 = ?$ 3. $\frac{9}{10} \times 12 = ?$ 5. $\frac{13}{15} \times 11 = ?$
2. $\frac{5}{8} \times 9 = ?$ 4. $\frac{15}{19} \times 6 = ?$ 6. $\frac{21}{23} \times 12 = ?$

ART. **10.** *To multiply a whole number by a fraction,* we multiply the numerator without altering the denominator.

1. Multiply 25 by $\frac{3}{4}$.

25×3 fourths=75 fourths, or $\frac{75}{4}$, which, changed to a mixed number, [Art. 7] $=18\frac{3}{4}$.

2. $35 \times \frac{4}{5} = ?$ 3. $134 \times \frac{1}{20} = ?$ 4. $16 \times \frac{3}{10} = ?$

ART. **11.** *To multiply a mixed number by a whole number.*

Multiply $7\frac{3}{4}$ by 9.

$7\frac{3}{4}$
9
$69\frac{3}{4}$

EXPLANATION.—3 fourths multiplied by 9=27 fourths, or $6\frac{3}{4}$; and the 7 multiplied by nine=63, plus the 6=69, making the product $69\frac{3}{4}$.

Or thus:

$$7\frac{3}{4}$$
$$\frac{4}{}$$
$$\frac{31}{4} \times 9 = \frac{279}{4} = 69\frac{3}{4}.$$

1. $18\frac{3}{4} \times 5 = ?$ 2. $29\frac{3}{4} \times 8 = ?$ 3. $83\frac{1}{3} \times 7 = ?$

ART. **12.** *To multiply a whole number by a mixed number.*

1. Multiply 29 by $8\frac{2}{3}$.

29
$8\frac{2}{3}$
$19\frac{1}{3}$
232
$251\frac{1}{3}$

EXPLANATION.—Multiplying 29 by 2 thirds, we have 58 thirds, or $19\frac{1}{3}$, which we write in the first line. Then 29×8=232, which, added to $19\frac{1}{3}=251\frac{1}{3}$.

Or thus: $29 \times \frac{26}{3} = \frac{754}{3} = 251\frac{1}{3}$.

2. $15 \times 3\frac{1}{3} = ?$ 3. $12 \times 12\frac{1}{2} = ?$ 4. $14 \times 17\frac{3}{5} = ?$

Answers: $5\frac{1}{4}$, $4\frac{3}{8}$, $10\frac{4}{5}$, $41\frac{4}{19}$, $9\frac{8}{15}$, $10\frac{22}{23}$, 28, $6\frac{7}{10}$, $4\frac{4}{5}$, $69\frac{3}{4}$, 60, 150, $6\frac{2}{3}$, $93\frac{3}{4}$, 238, $583\frac{1}{3}$, 20, $251\frac{1}{3}$, 150, $246\frac{2}{5}$, $18\frac{3}{4}$, $53\frac{1}{3}$, $15\frac{1}{3}$, $72\frac{1}{2}$, $5\frac{5}{8}$.

The Teacher will find it important to require the learner to preserve the process, as he will be apt to adopt clumsy methods of solution.

To multiply a fraction by a fraction.

3. Multiply $\frac{3}{4}$ by $\frac{5}{6}$.

Assuming the numerator 5 to be a whole number, $\frac{3}{4}\times 5=\frac{15}{4}$; but 5 is not a whole number, but 5 *sixths;* hence $\frac{15}{4}$ is 6 *times too much.* $\frac{15}{4}$ divided by $6=\frac{15}{24}$, or $\frac{5}{8}$. [Note 1, page 43.]

ART. **13.** Hence, to multiply a fraction by a fraction, *we multiply the numerators together for a new numerator, and the denominators for a new denominator.*

$\frac{3}{4}\times\frac{5}{6}=\frac{15}{24}$, which, reduced to its lowest terms $=\frac{5}{8}$.*

2. $\frac{3}{4}\times\frac{7}{8}=?$ 3. $\frac{7}{8}\times\frac{8}{9}=?$ 4. $\frac{1}{2}\times\frac{21}{28}=?$

ART. **14.** *To multiply a mixed number by a fraction or a mixed number.*

1. Multiply $15\frac{3}{4}$ by $\frac{9}{10}$.

$15\frac{3}{4}=\frac{63}{4}$, which, multiplied by $\frac{9}{10}=\frac{567}{40}$ or $14\frac{7}{40}$.

2. Multiply $8\frac{3}{4}$ by $16\frac{2}{3}$.

$8\frac{3}{4}=\frac{35}{4}$ and $16\frac{2}{3}=\frac{50}{3}$. $\frac{35}{4}\times\frac{50}{3}=\frac{1750}{12}=145\frac{10}{12}=145\frac{5}{6}$.

3. $12\frac{1}{2}\times 16\frac{2}{3}=?$ 4. $14\frac{2}{7}\times\frac{9}{10}=?$ 5. $18\frac{1}{2}\times 12\frac{1}{2}=?$

ART. **15.** *To divide a whole number by a fraction or a mixed number.*

1. Divide 315 by $\frac{3}{4}$, or, in other words, find how often $\frac{3}{4}$ is contained in 315.

SOLUTION.—Before we can measure 315 by fourths, we must change it to fourths. In 1 there are 4 fourths; in 315 there

Answers: $\frac{21}{32}$, $\frac{7}{9}$, $\frac{21}{46}$, $14\frac{7}{40}$, $145\frac{5}{6}$, $208\frac{1}{3}$, $12\frac{6}{7}$, $231\frac{1}{4}$, $\frac{5}{8}$, $2\frac{2}{27}$.

*It will be observed that to multiply by a fraction does not increase the multiplicand, as in whole numbers; but, on the contrary, *decreases* it, the $\frac{5}{8}$ being less than $\frac{3}{4}$.

To account for this, it is only necessary to remember that a whole number is reduced to the denomination of a fraction by being multiplied by it. $6\times\frac{3}{5}=18$ fifths, or $3\frac{3}{5}$. Much more is a fraction reduced in value if multiplied by a fraction. From this we readily infer,

2. That to divide by a fraction increases the dividend.

are 315 times 4 or 1260 fourths, which, divided by 3=430. Hence, $\frac{3}{4}$ is contained in 315 420 times.

OPERATION. 315 *or* $\frac{315}{1}\times\frac{4}{3}=\frac{1260}{3}=420$

4

3)1260

420

2. $320\div\frac{7}{8}=?$ 4. $541\div\frac{2}{9}=?$ 6. $987\div\frac{1}{20}=?$
3. $27\div\frac{5}{6}=?$ 5. $684\div\frac{3}{10}=?$ 7. $136\div\frac{6}{7}=?$

8. Divide 25 by $5\frac{1}{2}$.

OPERATION. 25×2 halves$=\frac{50}{2}$ and $5\frac{1}{2}\times2=\frac{11}{2}$. $50\div11=4\frac{6}{11}$, *or* $\frac{50}{2}\times\frac{2}{11}=\frac{100}{22}=4\frac{12}{22}=4\frac{6}{11}$.

ART. **16.** Hence, to *divide* by a fraction, we multiply by the denominator and divide by the numerator, or invert the divisor and proceed as in multiplication.

1. $157\div3\frac{1}{2}=?$ 4. $345\div6\frac{3}{4}=?$ 7. $195\div16\frac{2}{3}=?$
2. $22\div12\frac{1}{2}=?$ 5. $39\div15\frac{1}{3}=?$ 8. $39\div12\frac{4}{5}=?$
3. $16\div16\frac{3}{4}=?$ 6. $79\div37\frac{1}{2}=?$ 9. $87\div31\frac{1}{4}=?$

To divide one fraction by another.

10. Divide $\frac{3}{4}$ by $\frac{5}{6}$. Operation. $\frac{3}{4}\times\frac{6}{5}=\frac{18}{20}=\frac{9}{10}$.

EXPLANATION.—By inverting the divisor, we obtain $\frac{18}{20}$, the terms of which, being divided by 2, give $\frac{9}{10}$.

11. $\frac{9}{10}\div\frac{3}{4}=?$ 14. $31\frac{1}{2}\div\frac{6}{7}=?$ 17. $\frac{3}{7}\div1\frac{1}{2}=?$
12. $\frac{5}{13}\div\frac{7}{8}=?$ 15. $31\frac{3}{4}\div\frac{9}{11}=?$ 18. $\frac{9}{10}\div3\frac{1}{2}=?$
13. $\frac{21}{25}\div\frac{0}{7}=?$ 16. $13\frac{1}{2}\div\frac{5}{6}=?$ 19. $\frac{5}{6}\div6\frac{3}{4}=?$

ART. **17.** *To divide when either divisor or dividend is a mixed number and the other term a whole number*, both terms may be reduced to the same denomination. [Art. 15.]

1. Divide $3457\frac{1}{4}$ by 13.

Answers: $32\frac{2}{5}$, $265\frac{5}{7}$, $2434\frac{1}{2}$, 19740, 2280, $151\frac{2}{3}$, $24\frac{2}{3}$, $1\frac{1}{5}$, $\frac{40}{91}$, $\frac{49}{50}$, $38\frac{29}{36}$, $36\frac{3}{4}$, $16\frac{1}{5}$, $\frac{10}{18}$, $\frac{9}{35}$, $\frac{7}{9}$, $\frac{1}{9}$, $\frac{10}{81}$, $44\frac{6}{7}$, $1\frac{19}{25}$, $\frac{64}{69}$, $51\frac{1}{9}$, $2\frac{255}{460}$, $2\frac{8}{75}$, $51\frac{17}{20}$, $3\frac{8}{64}$, $\frac{2}{5}$, $\frac{2}{7}$, $2\frac{98}{125}$.

$3457\frac{1}{4}$
4
52)13829(565
104
342
312
300
269
49

EXPLANATION.—The dividend containing the fraction of $\frac{1}{4}$, both terms are reduced to fourths, and division performed as in whole numbers. The result shows that the divisor is contained in the dividend 265 times, with a remainder of 49 *fourths* [Art. 2, Principles of Division, page 46], or $265\frac{49}{52}$ times.

The same by short division.

13)3457 $\frac{1}{4}$
$265\frac{49}{52}$

EXPLANATION.—13 is contained in 3457 265 times, with a remainder of 12, which, reduced to fourths, including the $\frac{1}{4}$ of the dividend, is 49 fourths. 13 not being contained in this an even number of times, the denominator is increased 13 times, (which is the same as to decrease the numerator,) which gives the same fraction as by long division, $\frac{49}{52}$.

2. $1398\frac{1}{3} \div 56 = ?$
3. $256\frac{1}{2} \div 7 = ?$
4. $1939 \div 8\frac{1}{3} = ?$
5. $1255\frac{5}{6} \div 350 = ?$
6. $796\frac{1}{3} \div 421 = ?$
7. $467\frac{2}{5} \div 12 = ?$

ART. 18. *To subtract a fraction from another of the same denomination* is simply to subtract the less numerator from the greater.

1. From $\frac{7}{10}$ take $\frac{3}{10}$.

$\frac{7}{10} - \frac{3}{10} = \frac{4}{10}$ *or* $\frac{2}{5}$.

2. $\frac{13}{15} - \frac{11}{15} = ?$
3. $\frac{37}{42} - \frac{25}{42} = ?$
4. $\frac{16}{22} - \frac{5}{22} = ?$
5. $\frac{9}{10} - \frac{3}{10} = ?$
6. $\frac{12}{21} - \frac{9}{21} = ?$
7. $\frac{16}{45} - \frac{7}{45} = ?$
8. $\frac{9}{18} - \frac{4}{18} = ?$
9. $\frac{16}{39} - \frac{3}{39} = ?$
10. $\frac{11}{17} - \frac{6}{17} = ?$

ART 19. *To subtract a fraction or mixed number from a whole number.*

1. From 9 take $3\frac{3}{4}$.

Answers: $275\frac{49}{52}$, $241\frac{63}{168}$, $36\frac{9}{14}$, $32\frac{247}{420}$, $11\frac{1126}{1263}$, $232\frac{17}{25}$, $38\frac{57}{60}$, $\frac{2}{7}$, $\frac{1}{7}$, $\frac{1}{3}$, $\frac{1}{15}$, $\frac{1}{5}$, $\frac{1}{2}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{5}{13}$, $\frac{2}{15}$, $\frac{5}{17}$.

The following formula will render the operation simple:

Whole number.	Fourths
9	0.
3	3
5	1
or $5\frac{1}{4}$	

EXPLANATION.—Arranging the less under the greater, we find we can not take 3 fourths from 0 fourths; so a whole number or 1 is added to both terms. In 1 there are 4 fourths, from which we take 3 fourths, and we have a remainder of 1 fourth. To the 3 add 1 and we have 4, which, subtracted from 9, leaves 5, giving for the answer $5\frac{1}{4}$.

2. $13 - 4\frac{1}{4} = ?$ 5. $11 - 2\frac{1}{3} = ?$ 8. $52 - 27\frac{1}{7} = ?$
3. $15 - 5\frac{1}{6} = ?$ 6. $7 - \frac{7}{8} = ?$ 9. $13 - 12\frac{1}{6} = ?$
4. $29 - 12\frac{1}{2} = ?$ 7. $14 - 1\frac{3}{12} = ?$ 10. $89 - 75\frac{1}{4} = ?$

ART. **20.** *To subtract one fraction from another of a different denomination,* it will be necessary first to reduce both to the same or a common denominator.

1. From $\frac{7}{8}$ take $\frac{6}{7}$.

By Art. 3, $\frac{7}{8}$ can be changed to 56ths by multiplying both terms by 7, and $\frac{6}{7}$ can be changed to 56ths by multiplying both terms by 8, giving $\frac{49}{56}$ and $\frac{48}{56}$, the difference between which is $\frac{1}{56}$, the answer.

It will be observed that the multipliers used in this case were the denominators, 7 and 8, which, multiplied together, give a *common denominator,* and multiplied into the numerators of each other give the new numerators.

OPERATION: $\frac{7}{8} - \frac{6}{7} = \frac{49}{56} - \frac{48}{56} = \frac{1}{56}$.

2. From $\frac{6}{7}$ take $\frac{1}{5}$. 5. $\frac{3}{4} - \frac{4}{7} = ?$ 8. $\frac{7}{8} - \frac{5}{9} = ?$
3. From $\frac{2}{5}$ take $\frac{1}{6}$. 6. $6\frac{1}{2} - \frac{3}{4} = ?$ 9. $\frac{4}{12} - \frac{1}{6} = ?$
4. From $\frac{2}{3}$ take $\frac{5}{12}$. 7. $1\frac{1}{2} - \frac{7}{8} = ?$ 10. $\frac{3}{13} - \frac{1}{15} = ?$

ART. **21.** *To add fractions of the same denomination,* the numerators only are added, and the sum reduced to a mixed number or its lowest terms

Answers: $\frac{5}{6}$, $6\frac{1}{8}$, $9\frac{5}{6}$, $4\frac{1}{2}$, $24\frac{6}{7}$, $8\frac{2}{3}$, $8\frac{3}{4}$, $16\frac{1}{2}$, $13\frac{3}{4}$, $13\frac{3}{4}$, $\frac{5}{24}$, $5\frac{3}{4}$, $\frac{23}{35}$, $\frac{5}{8}$, $\frac{32}{195}$, $\frac{1}{6}$, $\frac{23}{72}$, $\frac{1}{4}$, $\frac{5}{28}$, $\frac{3}{5}$. $12\frac{3}{4}$, $\frac{1}{12}$.

1. Add $\frac{3}{8}+\frac{6}{8}+\frac{5}{8}+\frac{7}{8}$.

3
6
5
7

EXPLANATION.—Here the four numbers are added together, making 21 eighths, which, reduced to a mixed number, are equal to $2\frac{5}{8}$.

$\frac{21}{8}=2\frac{5}{8}$

2. $\frac{1}{12}+\frac{5}{12}+\frac{3}{12}+\frac{6}{12}+\frac{4}{12}+\frac{7}{12}+\frac{8}{12}=?$
3. $\frac{3}{9}+\frac{4}{9}+\frac{7}{9}+\frac{8}{9}+\frac{6}{9}+\frac{5}{9}+\frac{1}{9}=?$
4. $\frac{3}{15}+\frac{5}{15}+\frac{7}{15}+\frac{12}{15}+\frac{13}{15}+\frac{2}{15}+\frac{4}{15}=?$

ART. 22. *To add fractions of different denominations*, they should first be reduced to a common denominator, as in subtraction.

$\frac{1}{2}+\frac{3}{4}=?$ $\frac{1}{2}+\frac{3}{4}=\frac{4}{8}+\frac{6}{8}=\frac{10}{8}=1\frac{2}{8}$ *or* $1\frac{1}{4}$.

When three or more fractions of different denominations are to be added together, they may be reduced to a common denominator by multiplying all the denominators together, as above, and then by multiplying each numerator by all the denominators except its own.*

1. Find the sum of $\frac{1}{2}+\frac{3}{4}+\frac{5}{6}$.

$2\times4\times6=48=$*Common denominator.*
$1\times4\times6=24=$*First numerator.*
$3\times2\times6=36=$*Second numerator*
$5\times2\times4=40=$*Third numerator.*
$100=$*Sum of numerators.*

Hence, $\frac{100}{48}=2\frac{4}{48}=2\frac{1}{12}$.

The $\frac{1}{2}$ in the example was multiplied by 24, giving $\frac{24}{48}$; the $\frac{3}{4}$ by 12, giving $\frac{36}{48}$; and the $\frac{5}{6}$ by 3, giving $\frac{40}{48}$.

2. $\frac{3}{5}+\frac{6}{5}=?$
3. $\frac{1}{5}+\frac{7}{8}=?$
4. $\frac{3}{12}+\frac{2}{6}=?$
5. $\frac{3}{4}+\frac{4}{8}+\frac{5}{6}=?$
6. $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}=?$
7. $\frac{1}{12}+\frac{1}{6}+\frac{3}{12}=?$
8. $2\frac{3}{4}+\frac{3}{4}+\frac{5}{6}=?$
9. $5\frac{1}{2}+6\frac{1}{4}+\frac{2}{8}=?$
10. $\frac{7}{8}+2\frac{3}{4}+\frac{3}{12}=?$

Answers: $2\frac{5}{6}$, $3\frac{7}{8}$, $1\frac{13}{30}$, $1\frac{3}{4}$, $2\frac{1}{12}$, $\frac{11}{12}$, $4\frac{1}{3}$, 12, $3\frac{1}{15}$, $1\frac{1}{5}$, $\frac{36}{40}$, $\frac{7}{12}$, $\frac{1}{2}$, $4\frac{7}{24}$.

*This is simply multiplying both terms by the same number.

VIII. DECIMALS.

ART. 1. A Decimal Fraction expresses its value in one term, and is known from a whole number by its having a period called a *decimal point* at the left. .5 is a decimal.

The value of a decimal is more easily ascertained than that of a common fraction; while operations in decimals are performed with nearly the same ease as those in whole numbers.

ART. 2. The numerator only, of a decimal fraction is written, the denominator always being 10 or some power of 10. A decimal composed of one figure as .5, will have 10 for a denominator. One composed of two figures, as .75, will have 100 for a denominator, etc.; hence to find the denominator, we have only to write as many ciphers as decimals, and annex a 1 to the left.

REMARK.—Ciphers on the extreme right of a decimal possess no value. .500 is the same as .5.

The value of .073 is how much, expressed as a common fraction? *Ans.* $\frac{73}{1000}$

What is the fractional value of the following:

.23, .007, .013, .75, .12, .11, .4, .42, .710, .0076, 10.7?

ART. 3. The removal of the decimal point one place to the right or left, increases or diminishes the fraction 10 times.

$31.67 \times 10 = 316.7$, and $316.7 \div 10 = 31.67$.

ADDITION AND SUBTRACTION.

ART. 4. Operations in Addition and Subtraction of decimals are performed in the same way as those in addition and subtraction of simple numbers. The pupil who has calculated Federal money, is already

acquainted with these operations. He has only to observe that units be placed under units, etc., or what is still more simple, to place the decimal points directly under each other, and proceed as in simple numbers. 1.07 + .001 + 37.045 + 10.06 + .0007 would be done thus:

1.07
.001
37.045
10.06
.0007
48.1767

1. Find the sum of

.007 + 31.06 + .1009 + 100.07 = *
710.34 + 2.406 + 67.709 + .0006 =
314.60 + .0006 + .0027 + .001 =
714.06 + .003 + 8.007 + 800. =

Total, 2748.3678

2.

123.006 + .18532 + .0185 + 1672.3 + 1865.01 = *
184.003 + .0185 + 11.10 + 18639.01 + 1657.003 =
0.005 + 2683.17 + 2.95 + 6892.02 + .0031 =

Total, 33729.80242

3. Find the difference between 107.06 and .213.

617.07 — 41.7106 = *
10.06 — .9092 =
36.84 — 6.672 =
118.09 — 7.009 =

Total, 725.7592

107.06
.213
106.847 *Ans.*

4.

21.80 — .0503 = *
364.2 — 128 9 =
6.295 — 2.654 =

Total, 260.6907

NOTE.—For Multiplication, Division, and Reduction of Decimals, see chapter XXXIII, pages 173–176.

* The pupil will fill the blanks.

IX. SHORT METHODS.

PROPERTIES OF NUMBERS.

ART. 1. Numbers ending with 5 or 0 are divisible by 5 without a remainder.

ART. 2. If the *two* right hand figures of a number are divisible by 4 without a remainder the whole number is divisible by 4.

ART. 3. If the *three* right hand figures of a number are divisible by 8 without a remainder the whole number is divisible by 8.

ART. 4. If the *sum* of the figures of any number be divisible by 3 or 9 the whole number will be divisible by 3 or 9.

SHORT METHODS OF MULTIPLYING.

ART. 5. An even part of 10, 100, 1000, etc., can be multiplied *mentally* by division.

ALIQUOT PARTS OF 100.		ALIQUOT PARTS OF 1000.
$50 = \frac{1}{2}$	$14\frac{2}{7} = \frac{1}{7}$	$333\frac{1}{3} = \frac{1}{3}$
$33\frac{1}{3} = \frac{1}{3}$	$12\frac{1}{2} = \frac{1}{8}$	$250 = \frac{1}{4}$
$25 = \frac{1}{4}$	$10 = \frac{1}{10}$	$166\frac{2}{3} = \frac{1}{6}$
$20 = \frac{1}{5}$	$8\frac{1}{3} = \frac{1}{12}$	$125 = \frac{1}{8}$
$16\frac{2}{3} = \frac{1}{6}$	$6\frac{1}{4} = \frac{1}{16}$	$83\frac{1}{3} = \frac{1}{12}$

To multiply by a part of 100, we suppose two ciphers to stand at the right of the number and divide by the part. To multiply by a part of 1000, we suppose three figures, etc.

1. To multiply 176 by $12\frac{1}{2}$.

Operation 8)17600

Ans. 2200

2. To multiply 379 by 250.

Operation $\frac{379000}{4}$ = 94750 *Ans.*

3. To multiply $49.75 by 125.

Operation $\frac{4975000}{8}$ = 621875 cents,
or $6218.75 *Ans.*

NOTE.—Only a few answers will be given in the following, as the pupil can prove the accuracy of his calculations by multiplying in the ordinary way.

4. $140 × $12\frac{1}{2}$ = $1750
5. 3767 × $8\frac{1}{3}$ =
6. 9987 × 25 =
7. 949 × $333\frac{1}{3}$ =
8. 179 × $2\frac{1}{2}$ =
9. 769 × $3\frac{1}{3}$ =

10. 675 yards @ $37\frac{1}{2}$ cents.

Operation 675 at a dollar = $675.00

at 25*c.* = $\frac{1}{4}$	168.75
at $12\frac{1}{2}$*c.* = $\frac{1}{2}$	84.37$\frac{1}{2}$
Ans.	253.12$\frac{1}{2}$

11. 715 × $62\frac{1}{2}$ cents.
12. 947 × $87\frac{1}{2}$
13. 194 × $18\frac{3}{4}$
14. 9876 × 2.18\frac{3}{4}$ cents.
15. 719 × 3.62$\frac{1}{2}$
16. 965 × 4.37$\frac{1}{2}$

NOTE.—The multiplier of the 12th Ex. wants only $12\frac{1}{2}$ cents, or $\frac{1}{8}$ of being a dollar; so we find the cost of 947 at a dollar, and take off $\frac{1}{8}$ of it. Other examples may be solved in the same way.

ART. **6.** To find the cost when there are fractions in both factors: $18\frac{3}{4}$ lbs. @ $12\frac{1}{2}$ cents.

Operation. $18\frac{3}{4}$ lbs. @ a dollar = $18.75

at $12\frac{1}{2}$ cents = $\frac{1}{8}$ or $2.34 *Ans.*

2. $37\frac{1}{2}$ lbs. @ $18\frac{3}{4}$ cents.

Operation. $37\frac{1}{2}$ @ a dollar = $37.50

at $12\frac{1}{2}$ = $\frac{1}{8}$ or	4.687
" $6\frac{1}{4}$ = $\frac{1}{2}$ "	2.343
Ans.	$7.03

1. To multiply 424 by 97:

Operation. $424\times100=42400$
$424\times\ 3=\ 1272$
41128

ART. 7. When the multiplier wants from 1 to 12 of being 100, 200, 3000, etc., the work can be contracted by multiplying by one of these, and subtracting as many times the multiplicand as the multiplier is short of them.

ART. 8. When the multiplier is 29, 39, 49, etc., we multiply by the next higher number and subtract the multiplicand.

1. To multiply 176 by 59:

Operation. $176\times60=10560$
176
10384

2. $671\times\ 39$
3. 59×689
4. 89×784
5. $167\times\ 29$

ART. 9. *To square numbers that end with* 9.

What is the square of 29?

Operation. 29
29
841

EXPLANATION.—Writing 1 for the first figure of the product, we add 1 to the ten's place of the multiplier, and multiply the sum on the multiplicand less 1: $3\times28=84$, with the 1 annexed $=841$.

Find the square of the following numbers mentally:

99, 59, 119, 79, 19, 69, 129, 89.

ART. 10. To square any number of 9's instantaneously, and without multiplying.

Commencing at the *left*, we write as many 9's less one, as the number to be squared, an 8, as many 0's as 9's, and a 1.

The square of 9999999 is 99999980000001.

REMARK.—The square of any number of 3's will be the $\frac{1}{9}$ of the square of the 9's.

ART. 11. *To multiply by* 375, 625, 750, or 875, we first multiply by 125, (Art. 5, Ex. 3,) and that product by 3, 5, 6, or 7.

$$1649 \times 625 = \frac{1649000 \times 5}{8} = \frac{8245000}{8} = 1030625$$

ART. 12. *To square numbers under* 135, *ending with* 5

The first two figures on the right of the product will always be 25; and to find the others, we add 1 to the ten's place and multiply it on the ten's and hundred's places above.

To square 115: *Operation.*

11
12
13225

ART. 13. To square a number containing a half, as $12\frac{1}{2}$, we multiply the *whole* number by the next higher number and add a fourth. $8\frac{1}{2}$ squared $= 8 \times 9 + \frac{1}{4} = 72\frac{1}{4}$.

ART. 14. To multiply by numbers of two figures containing the figure 1.

1. Multiply 346 by 15. *Operation.*

346
1730
5190

2. Multiply 346 by 51. *Operation.*

346
1730
17646

These operations might be performed mentally. Taking the 2nd, for instance, we say one time 6=6; then 5 times 6=30 and 4 of the upper line=34; 5 times 4=20 and 3 of the upper line and 3=26; 5 times 3=15 and 2=17. Product 17646.

SHORT METHODS OF DIVIDING.

ART. 15. *To divide by a number composed of two or more factors,** as 96, which is composed of the factors 12 and 8, or 648 which is composed of $9\times8\times9$. This operation is performed by using the factors instead of the whole number.

1. Divide \$7854 by 32. *Operation.*

$$\begin{array}{r} 4)7854 \\ \hline 8)1963\text{—}2 \\ \hline 245\text{—}3 \end{array}$$

How the true remainder is found:

The first remainder is 2 dollars, because it is left from the *dollars* that were divided. The second remainder is *four times* as great as if it were from the first line, because every figure of the second line is four times as great as if it stood in the first line. Four times $3=12$, and the 2 of the first remainder equal 14, the *true remainder.* *Ans.* $245\frac{14}{32}$.

2. Divide 6371 by 336. *Operation.*

$$\begin{array}{r} 6)6371 \\ \hline 7)1061\text{—}5 \\ \hline 8)\ 151\text{—}4 \\ \hline 18\text{—}7 \end{array}$$

REMARK.—The true remainder of this example is found by multiplying the last remainder by 7, to make it of the same value as if it were from the line above, and that by 6, to make it of the same value as if it were from the upper line: $7\times7\times6=294$, to which, add $6\times4+5$ or 29. The true remainder is 323. *Ans.* $18\frac{323}{336}$.

3.	Divide	1463 by	28	7.	$4571\div441$
4.	"	7714 "	72	8.	$1987\div378$
5.	"	1943 "	49	9.	$9843\div720$
6.	"	8765 "	343	10.	$1456\div729$

Answers: $103\frac{3}{4}$, $25\frac{190}{343}$, $*5\frac{97}{378}$, $17\frac{227}{729}$, $52\frac{1}{4}$, $39\frac{32}{49}$, $10\frac{19}{49}$, $13\frac{161}{180}$, $107\frac{5}{36}$.

* Such numbers are called *composite.* When a number can not be divided into factors, it is called a *prime* number.

DIVISION BY CANCELLATION.

Art. 16. To cancel, signifies to blot out or make void. Division by cancellation is performed by writing the terms in fractional form, and dividing them by any number that will do so without a remainder.

1. To divide 1463 by 28.

$$\frac{\overset{209}{\cancel{1463}}}{\underset{4}{\cancel{28}}}=52\tfrac{1}{4}$$

Explanation.—The first terms were canceled by dividing by 7, leaving $\frac{209}{4}$, which finished, is $52\frac{1}{4}$.

2. 3465 ÷ 35	5. 1962 ÷ 22
3. 2763 ÷ 81	6. 6876 ÷ 152
4. 6545 ÷ 245	7. 5436 ÷ 144

Art. 17. *To divide by aliquot parts of* 100, 1000, *etc.*

This process is the reverse of that under Art. 62, page 69.

1. To divide 7654 by 25. *Operation.*

$$\begin{array}{r} 76.54 \\ 4 \\ \hline 306.16 \end{array} = 306\tfrac{4}{25}$$

Note 1.—The *decimal value* of the remainder is always obtained first by this process. If we divide the 16 by the multiplier, it will give the true remainder or 4.

2. If the aliquot part is of 100, there will be two places; if 1000, there will be three places of decimals.

2. To divide 19765 by 125. *Operation*

$$\begin{array}{r} 19.765 \\ 8 \\ \hline 158.120 \end{array}$$

or $158.\frac{15}{125}$

3. $17630 \div 33\frac{1}{3}$, $74910 \div 12\frac{1}{2}$, $87396 \div 125$, $824 \div 2\frac{1}{2}$.

X. SIMPLE PERCENTAGE.

ART. 1. PERCENTAGE embraces all those calculations in which 100 is made the basis of comparison.

ART. 2. *Per cent.* signifies *by the hundred.* 6 per cent. signifies 6 for every 100. % is the sign, and is written thus: 6%, which reads, *six per cent.* The 6 is called the *rate.*

ART. 3. One per cent. of a number is that number divided by 100. One per cent. of $320 is $3.20. To find any other percentage of a number, we multiply one per cent of it by the rate.

ART. 4. Percentage may be divided into *simple* and *complex.*

ART. 5. SIMPLE PERCENTAGE embraces all those calculations in which both the principal and the rate are known and is applied to Premium, Discount, Exchange, Taxes, Commission, Brokerage, Insurance, Insolvency, Loss and Gain, etc.

1. Find 6% of 758.

7.58=1 per cent.
6
45.48=6 per cent.

2. 3% of 215=6.45.
3. 9% of $756=
4. 4% of 788=
5. 7% of $179=

NOTE 1.—To find 1% of the following, remove the dec. point two places to the left, and point off four figures in the product.

2. Give your answers in dollars and cents. If you have 5 milis or more, add a cent to the cents; if less than 5, omit them.

6. $768.15@6%=
7. $566.75@13 %=
8. $196.55@5%=
9. $789.15@33⅓%=

NOTE.—When the percentage of sums under $10 is required, it will be more simple first to multiply by the rate and then divide by 100.

10. 25% of $0.97=
11. 50 % of $0.17=
12. 65% of 0.08=
13. 33⅓% of 0.76=

ART. 6. *Premium* is a percentage to be *added; discount*, a percentage to be *subtracted* from the face or par value of a bill, note, etc.

ART. 7. The method of settling accounts between persons in distant places by draft is called *Exchange.* If between persons in the same country it is called *Inland* or *Domestic Exchange.*

ART. 8. Exchange may be *par*, or it may be at a *premium* or at a *discount.*

1. What will be the cost of a draft on New Orleans for $3200 at $\frac{1}{4}\%$ discount?

2. What amount will pay for a bill of exchange on New York for $1860 at $\frac{3}{8}\%$ premium?

3. At $\frac{1}{8}\%$ discount, what should I receive for New York exchange calling for $728?

4. At $\frac{1}{4}\%$ premium, what will a bank pay for a draft on Chicago for $276?

ART. 9. *Commission* or brokerage is the percentage charged by a *commission merchant, factor, agent*, or *broker*, for transacting business for another.

ART. 10. Commission is usually reckoned on the whole amount of *sale, purchase, or collection.*

1. At $2\frac{1}{2}$ per cent., what is the commission on $17640?

2. A merchant sells goods for another to the amount of $4371.87, what is his commission at 5 per cent?

3. A broker receives $\frac{1}{4}$ per cent. for selling $2500 worth of merchandise for a commission merchant, what was the amount of his brokerage?

4. A of New Orleans buys sugar for B of Cincinnati, to the amount of $7100, what is the amount of his commission at $1\frac{1}{2}$ per cent.?

Answers: $441.00, $218.59, $6.25, $106.50, $3192.00, $1866.98, $727.09, 276.69.

ART. **11.** Insurance is a guarantee against the loss of property by fire or the dangers of transportation.

The amount paid for guarantee is called the *Premium.* It is a certain percentage on the estimated value of the property insured. This percentage or rate is large or small, in proportion to the risk.

1. How much should be paid to insure a house valued at $1674, premium $1\frac{1}{2}$ %, and policy $1.50?

2. At $2\frac{1}{2}$ % premium, what should I pay on $6710 worth of goods, including policy at $2.50?

3. At 4 % premium and policy $1.75, what should I pay on a freight of furniture worth $2200?

ART. **12.** The term *Stocks* signifies shares in incorporated companies, while *Bonds* represent government and municipal securities, and mortgaged securities of corporations.

1. What is the value of 10 shares of railroad stock, at 5 % premium, par value per share being $50?

2. Bought $1500 of bonds at 1.02 %, what did I pay for them?

3. Sold $500 in railroad stock at a premium of 5 %, and received sugar at 10 cents per pound, how many pounds did I receive?

4. Purchased 50 shares of Pacific railroad stock at a discount of $2\frac{1}{2}$ %, par value being $100, what did they cost me?

5. Sold $5000 in bonds at a premium of $2\frac{1}{2}$ %, what did I gain after paying a broker $\frac{1}{4}$ % for selling?

6. Bought $2600 of canal stock at 3 % below par, and sold it at 4 % above par; how much did I pay, and what did I gain by both transactions?

Answers: $26.61, $170.25, $89.75, $525.00, $1530.00, 5250, $4875.00, $112.50, $182.00, $2522.00.

ART. **13.** *Tax* is a sum imposed or levied upon society to defray its expenses.

ART. **14.** *Poll tax* is a specific sum assessed on male citizens.

ART. **15.** *Duty* is a tax levied by the general government upon imported goods.

ART. **16.** *Specific tax* is a fixed sum levied upon specific things without regard to value.

ART. **17.** A *general levy* is an assessment upon property according to its value.

1. The general levy of a county was as follows: 8 *m* %* for school fund, 4 *m* % for specific purposes, 6 *m* % for sinking fund; how much tax will I have to pay on $750 personal property and on $2800 real estate?

SOLUTION.—We first add the several rates together to ascertain the whole assessment, viz: $8 + 4 + 6 = 18$ *m* %, or 1.8. $1\frac{8}{10}$ % — $750 + $2800 = $3550 taxable property. $1\frac{8}{10}$ % of $3550 = $63.90.

2. Under the same levy what tax would a farmer have to pay on $256 personal property and $8200 real estate, including poll tax for three male persons @ $2 a person, and a special tax of $5 for a piano and $3 for a gold watch?

3. What would be the duty on an invoice of imported silks, costing $560 in gold (including dutiable charges), at 35 % less 10 %?

NOTE.—The 35 % is to be reckoned on $560, and the 10 % to be taken from the result. It will be observed that 35 % less 10 % is not 25 %, but $31\frac{1}{2}$ %.

Answers: $176.40, $63.90, $166.21.

* One *m.* per cent. is one mill on one hundred dollars.

ART. 18. *Marking Goods.*—This is done by selecting *samples* of each kind or quality of goods, and putting on them a private mark, indicating the cost price, the selling price, or both.

Every house has got its own peculiar marks, which generally consist of the letters of some word or phrase, instead of figures. For instance, if we take the word *importance*, we will have a letter for every figure, and can readily substitute the former for the latter:

1	2	3	4	5	6	7	8	9	0
i	*m*	*p*	*o*	*r*	*t*	*a*	*n*	*c*	*e*

When a figure is to be repeated, an additional letter is used. Take *g* in this case.

The *selling price* is commonly found by adding to the cost price a certain amount per cent. to cover the freight and other charges, and allow a remunerative profit.

To facilitate labor, some merchants make this percentage an even part of 100, and add the same part of the cost price to itself.

1. *To add* $12\frac{1}{2}$ per cent. to the cost price of goods at $1.20 per yard.

$12\frac{1}{2}$ being $\frac{1}{8}$ of 100, add $\frac{1}{8}$ of 120 (15c.) to itself, which will make the *selling price* $1.35.

2. To add 25 % to books at $4.80 per doz.

25 being $\frac{1}{4}$ of 100, add $\frac{1}{4}$ of 480 (120c.) to itself, which makes the *selling price* $6.00 per doz., or 50cts. each.

3. To 30c. add 20 % profit.
4. $1.20 " 5 % charges and 20% profit.
5. 1.75 " $2\frac{1}{2}$% freight and 10% "

6. Add 14 % to $1.75
7. Add 15 % to $0.16
8. " 12 % to 0.87
9. " 25 % to 0.05
10. " 53 % to 1.67
11. " 18 % to 3.16

Answers: $1.35, $6.00, $1.50, $1.97, $2.00, $0.97, $0.18, $2.56, $0.06, $0.36, $371.00, $3.73.

ART. 19. MISCELLANEOUS EXERCISES.

1. Received a consignment of tea, which I sold for $1678, how much should I return the consignor, after deducting charges $150, and commission $2\frac{1}{2}$ %?

2. Sold a consignment of cloths for $6750, with the assistance of a broker, who charged me $\frac{1}{2}$ %; what amount of money did I make, com. $2\frac{1}{4}$ %?

3. A commission merchant sells goods for his principal to the amount of $3000, and charges $2\frac{1}{2}$% com., what does he make by the operation, after paying a broker $\frac{1}{4}$% for his services in effecting sales?

4. Insured $\frac{3}{4}$ of a steamboat worth $18000, at $1\frac{1}{2}$ % premium: what did it cost me?

5. How much should an insurance company pay to an insured who hold a policy for $2000 on his dwelling, the damages being estimated at 68%.

6. What is the amount of loss on a policy insuring mdse., $1200; fixtures, $300; building, $4000; damage on building, $37\frac{1}{2}$%; mdse. saved, $300; bal. of mdse. damaged, 75%; loss on fixtures, $83\frac{1}{3}$%.

7. A merchant holds three policies of insurance, as follows: Ætna, $1000 on leaf tobacco, $1500 on cigars, $100 on fixtures, and $200 on retail stock; Aurora, $2000 on leaf tobacco; American, $1500 on leaf tobacco and $1500 on cigars. Damage on leaf tobacco, $3000; cigars, $2500; retail stock, $300; fixtures, $200. What must each company pay?

8. After reserving 5% commission on sales, amounting to $520.75. how much should I return to my principal?

What is the commission on the following amounts:

9. $364.15@3\frac{1}{2}$%= 10. $36.21@1\frac{1}{2}$%=

Answers: $67.50, $202.50, $1360, $2425, $2216.67, $1333.33, $2250, $494.71, $12.75, $4.91, $60.36, $83.29, $0.54, $5.80, $135.49, $4.30, $1486.05, $118.12,5.

XI. BILLS—INVOICES.

When goods are sold, it is the duty of the merchant, or one of his clerks, to make out a statement of the quantity, kind, and price, of each article, for the satisfaction of the purchaser, and to enter at the foot of such statement the whole amount of the purchase, with the payment received, if any, or the terms of settlement. If the goods are bought to sell again, this statement is commonly called an *Invoice*, otherwise it is called a *Bill*, especially by the purchaser.

A bill or invoice is sometimes delivered to the buyer at the time of purchase, but it is usually sent with the goods, or if the buyer resides at a distance, by mail.

An invoice should specify the place and date of sale, the names of buyer and seller, a description of the goods, the prices of boxes, etc., used for packing, and in some kinds of business, the terms of sale.

When goods are received, the quality and quantity are compared with the invoice, and the selling prices made out from it, after which it is filed away or pasted in a book for future reference.

An *Account* is a statement of goods sold at different periods of time. Accounts are taken from the ledger, and often contain items in favor, as well as against the buyer.

Finding the cost of a number of articles at a certain price, and placing the amount opposite, is called, in bill-making, *extending;* adding the columns, *footing up.*

In making out bills, the three requisites are rapidity, legibility and accuracy. The principal is accuracy.

REMARK.—The bills that come in are usually called invoices

Rec'd.

DRY GOODS.

NEW ORLEANS, March 4, 1876.

Mr. W. A. Dickey,

Bought of Charles Shannon.

26	pieces	Calico, 825 yds.	@ 14c	$115.50
2	"	120 "	@ 9c	10.80
12	"	Twilled Muslin, 340 yds	@ 10c	34.00
3	cases		@ 75c	2.25
				$161.05

Received in payment his note at 90 days.

CHARLES SHANNON,
Per H. U.

CINCINNATI, July 31, 1876.

Mr. R. Nelson,

Bought of Haselting, Macfarland & Co

60	12	15 Braid Bonnets,	@$0.62½	$
68	6	Swiss Straw Bonnets,	" 1.25	
70	4	7 Braid "	" 1.50	
80	2	7 " "	" 3.00	
86	2	7 " "	" 3.75	
	6	Pcs. No. 1 Tafft. Ribbon,	" 15	
	5	" " 2 " "	" 28	
	3	" " 4 " "	" 48	
	2	" " 6 " "	" 75	
	1	" " 12 " "	" 1.10	
	3	" Bonnet Ribbon,	" 2.00	
	2	" " "	" 2.50	
3	1	Box Ruches,	" 1.50	
415	1	" "	" 2.25	
210	⅓	Doz. Bunches Flowers,	" 18.00	
	⅓	" " Feathers,	" 36.00	
	1	Pc. Black Silk, 20 yds	" 87½	
				$80.47

NOTE.—The numbers in the column on the left are those marked on the boxes and packages.

GROCERIES AND LIQUORS.

CINCINNATI, Sept. 1, 1875.

Mr James O'Shaughnessy,

Bought of King & Daly.

1	Hhd. N. O. Sugar,	*1200 † 120 **1080 lbs.	@$0.07	$
4	Brls. N. O. Molasses,	‡44 42 42 41 169 gals.	"	35
1	Trs. Rice,	700 70 630 lbs.	"	4
20	Bags Rio Coffee,	3200 lbs.	"	11
2	Half chests Black Tea,	*50–14† 50–14 / 100–28 72 lbs.	"	25
3	" " Yng. Hyson do.	150 lbs.	"	50
1	" " Imperial do.	60 lbs.	"	40
2	" " Gunpowder do.	110 lbs.	"	60
1	" " Colong Blk. Tea,	45 lbs	"	40
6	Doz. ground Cinnamon,		"	40
6	" " Allspice,		"	40
6	" " Pepper,		"	40
4	" " Mustard,		"	75
1	Box 5 lump Tobacco,	132 24 108 lbs.	"	25
1	" pound lump do.	147 23 124 lbs.	"	20
1	" Va. pound do.	140 20 120 lbs.	"	35
1	" 8 lump do.	150 25 125 lbs.	"	22
20	Brls. Rect. Whisky,	800 gals.	"	17
4	" Ginger Wine,	160 gals.	"	60
¼	Cask French Brandy,	40 gals.	"	4.00
¼	" Port Wine,	45 gals.	"	2.00
10	Brls. Bourbon Whisky,	405 gals.	"	1.00
½	Brl. Holland Gin,	20 gals.	"	1.50
				$1761.45

* *Gross Weight*—weight of hogshead, etc., and contents.

† *Tare* or weight of bag, box, etc. Ten per cent. is usually deducted for sugar.

‡ Gallons in each barrel.

** *Net Weight*—weight of goods in hogsheads, etc.

Messrs. Gaff & Baldwin, CINCINNATI, June 8, 1876.

Bought of Straight, Deming & Co

100 Boxes cheese, $\frac{4}{410}\frac{100}{}$ 3690,.....	@ $	.08	$
30 Firkins Butter, $\frac{3}{540}\frac{360}{}$ 2820,.....	"	.15	
100 Boxes* $20.00 Starch, 4810,.....	"	.05	
100 " $25.00 Star Candles, 4000,...	"	.20	
20 bbls.* $25.00 Lard Oil, 810 gals.,.....	"	.85	
50 " Mess Pork,.....	"	16.00	
10 Tierces S. C. C. Hams, $\frac{3}{350}\frac{350}{}$ 3000,.....	"	.11	
30 Kegs Lard, $\frac{1}{376}\frac{710}{}$ 1334,.....	"	.12½	
15 bbls. Mess Beef,.....	"	15.00	
Com. for purchasing, $1521.75,.....	"	2½ %	
Drayages,.....			16.00
Insurance on $5000.00,.....			59.88
QUEENSWARE.			$4152.87

PHILADELPHIA, May 17, 1858.

Mr. W. Anderson, To Samuel Asbury & Co.

W. A. [c] 140 @ 163.	23 Crates Queensware, per ship Lancaster, as per invoice rendered, ... £115 16*s.* 4*d*	$514.14
	Exchange 10 % prem.,	51.41
	Ins. 2¼ % @ 5½ per £,	14.33
		$579.88
	Int. 47 days, from Mar. 31 to May 17, 1858,.....	3.76
	Cash,.....	$583.64

Duties, etc., on the above,
Invoice, £115 16*s* 4*d*
Com. 2½ % 2 17*s* 11*d* £118 14*s* 3*d*

@ $4.84 per £, is $574.57, duties 24 %	$137.90
Custom, House Fees,.....	1.00
Freight, £10 17 3 @ $4.80,.....	52.14
Drayage,.....	9.50
Cash,.....	$200.54

* Price of empty boxes and barrels.

CINCINNATI, December 1, 1876.

Mr. Newton Thompson, Germantown, Ohio,
Bought of William Anderson.

W A [C]	3	doz.	Edged Plates,	@$0.40	$
115	10	"	" "	" 50	
	5	"	CC "	" 50	
	½	"	" Dishes, ea. $1.75, 2.25, $2.75.†		
	1	"	" Bakers, ea. $1.50, 2.00, $2.50.		
	1	"	" Beaded Nappies, each $1.75, 2.25.		
	½	"	" Chambers,	" 3.50	
	2	"	" Bowls,	" 80	
	2½	"	" "	" 60	
	3	"	" "	" 50	
	½	"	" Pitchers,	" 3.50	
	1	"	colored " ea. $2.50, 4.00.		
	4	"	" Bowls,	" 87½	
	5	"	" "	" 65	
	6	"	" "	" 55	
	9	Sets	CC Teas,	" 20	
	36	"	Painted Teas,	" 20	
			Crate,	" 1.00	

$56.73

HATS, CAPS, AND FURS.

Messrs. D. W. Fairchild & Co., NEW YORK, Aug. 31, 1875.
Bought of Frost & Griffin.

‡1328 1336 1337	3c.	18	doz. 2 x B. C. Shanghai,	@ $13.00	
1522 1543	2c.	6	" Fr. felt Hungarian,	" 21.00	
1491	1c.	6	" B. C.,	" 18.00	
			6 cases 75 cents each, cooperage		
			12½ cents each, drayage 37½ cts.,		5.63

$473.63

* "CC" signifies cream color. "Teas, cups and saucers.
† Three qualities. ‡ Numbers on cases.

CINCINNATI, Sept. 16, 1876.

Messrs. Lewis Evans & Co.,

Bought of D. W. Fairchild.

½	doz.	Men's black cass. Hungarian,	@ 21.00	$10.50
⅓	"	" " " "	" 27.00	
5½	"	" " "	" 33.00	
½	"	" " "	" 24.00	
½	"	" " broad brim wool,	" 14.00	
1	"	" " wove Senate,.....	" 12.00	
½	"	" " cashmerette,.....	" 15.00	
½	"	Boy's " wool Hungarian,.	" 7.00	
½	"	" caps assorted,.....	" 12.00	
½	"	" " "	" 9.75	
¼	"	" cloth caps,.	" 9.00	
¼	"	children's fancy caps,.....	" 8.00	
¼	"	" " "	" 13.00	
1/12	"	men's cloth caps,.....	" 14.00	
1/12	"	boy's " "	" 10.00	
1	Case at 75 and 1 at 50 cents,....			$264.63

BOOKS.

PHILADELPHIA, Oct. 9, 1876.

Messrs. Applegate & Co., Bought of Childs & Peterson.

300	Kane's Arctic Explorations, cloth,	$4.22	
100	" " " sh.,	5.07	
50	" " " hlf. antique	7.19	
50	" " " full "	8.45	
50	Bouvier's Law Dictionary, 2 vols.,	8.45	
30	" Institutes, 4 "	12.67	
400	Shepherd's Constitutional Text Book,	.63	
	18 Cases and drayage,.....	24.17	
			$3633.77
	Exchange 1 %,.....	$36.33	
	Insurance,.....	44.50	
	Freight,.....	85.41	
			$3800.01

HARDWARE.

CINCINNATI, June 3, 1876.

Messrs. A. C. Morris & Co.,

Bought of Tyler Davidson & Co

6 Mouse hole Anvils

*1. 1. 13. 0. 3. 22. 0. 2. 12.
1. 2. 12. 1. 3. 16. 1. 1. 14.

873 lbs. @ 14¼ $124.40

1 Case best cast steel, assorted, 1×½

1¼×½ 4. 0. 26. 474 lbs. @ 17½ 82.95 $207.35

Pocket Knives:

Nos.	1212	1518	22	29	32	37
Doz.	3	3	12	3	6	8
Price,	4 s.	4s. 6d.	8s.	4s. 9d	10s.	3s. 9d.

£11 5s. 9d. @ $5.00 56.44

5 doz. narrow Butts, each, 3 in. .85

" " " " 3½ " $1.15 10.00

Less 10 % discount,..... 1.00

9.00

Less Extra 12 %,..... 1.08

7.92

Package and Drayage,..... 3.25 67.61

$274.96

CINCINNATI, July 3, 1876.

Mr. James Brown,

Bought of Guiou & Kizer

10 Kegs 10d. nails,..... @ $3.50
6 " 8d. " ..... " 3.75
5 " 6d. " ..... " 4.25
2 " 4d. " ..... " 4.75
4 " 8d. fence nails,..... " 3.75
3 " 8d. brads,..... " 3.75
6 " 10d. finishing nails,..... " 4.50

Amount forwarded $141.50

* Long weight, *cwt. qrs. lbs.*

Amount brought forward,.....				$141.50
4 Kegs 8d nails, ...		@	$4.75	
2 Doz. No. 1 tinned bottom coffee mills,		"	12.00	
2 " " 2 " " " " "		"	5.00	
3 " " 3 " " " " "		"	3.50	
2 " Britannia hoppers,.....		"	7.00	
4 " Brass "		"	4.00	
6 " Iron "		"	3.00	
4 " No. 8 x Janus faced locks,.....		"	13.00	
2 " " " a " " "		"	15.00	
				$335.00

TAILORING.

Richmond, Va., Feb. 3, 1876.

Mr. Michael Tracey,

To James Humble.

Sep. 9, For making and trimming one overcoat,.....			$12.00
" 1 silk velvet vest,.....			10.00
" ½ doz. pair socks,.....	@	$2.37½	1.19
Feb. 5 1876. Settled,			$23.19

James Humble.

FARMING.

Lexington, Ky., Oct. 29, 1876.

Mr. W. R. Henderson,

Bought of R. S. Wharton.

Jan. 3, 3 Durham heifers,.....	@	$25.00	
1 Bay 2 yr. colt,.....	"	50.00	
Apr. 11, 140½ Bushels corn,.....	"	.50	
5⅓ Doz. chickens,.....	"	1.37½	
Aug. 9, 123½ lbs. butter,.....	"	.18¾	
			$225.97

Cr.

Apr. 4, By 25 Hogs, 3147 lbs. @ $0.3½

" 1 Plow,. ....$15.00.

Oct. 30, " Cash for balance,.....

R. S. Wharton.

Med.

EXERCISES FOR PEN AND PAPER.

The pupil should provide himself with bill paper, and make out bills from the following transactions, using his own name as clerk or principal, as he prefers.

1. Apr. 3, 1876, Sold to Mrs. E. Nelson, 22 yds. black silk @ $1.25, 12 yds. black silk velvet @ $4.87, 15 yds. linen @ 75 cts., 47 yds. W. flannel @ 62½ cts., to be charged to account of Richard Nelson.

2. Sold to Mr. H. Schnicke, 1 overcoat @ $17.50, ½ doz. shirts @ $32.50, ⅓ doz. pocket hkfs. @ $5.75, ½ doz. pairs socks @ $3.75. Received payment in cash, A. B. (clerk). Feb. 3, 1876.

3. June 9, 1876. Sold to Cyrus Wright on order of A. J. Rice, 2 doz. Gillot's pens @ 12½ cts, 1 box F. envelopes @ $1.50, ½ doz. penholders @ 50 cts., 1 copy Byron @ $1.50, 2 copies Shakespeare's plays @ $2.50. Indorsed on order.

NOTE.—This transaction is a very common one, and should be thoroughly understood by the learner. It supposes us to be indebted to A. J. Rice, who, in his turn, is indebted to C. Wright, in whose favor, he (Rice) draws the following order on us:

"CINCINNATI, June 8, 1876.

Messrs. Nelson, Kizer & Co. will please let C. Wright or bearer have goods to the amount of Twenty dollars, and charge to my account.

$20.00 A. J. RICE."

The amount of the bill being less than that of the order, Wright is permitted to keep the latter, after we write across the back of it, "June 9, paid $8.50, N., K. & Co."

The amount, $8.50, is then charged on our books to Rice.

A better way is, take Rice's order, charge him with the $20.00, and give Wright a due bill for the balance, $11.50.

4. May 13, 1876. Sold to H. J. Minor, Louisville, 3 chests Congou tea, marked H. J. M.—21, 22, 23. No. 21, 102 lbs. gross, tare 21 lbs.; No. 22, 103 lbs. gross, tare 22 lbs.; No. 23, 99 lbs. gross, tare 20 lbs., @ 75 cts.

5. Sept 9, 1876. Sold to Robert O'Brien 125 yds. carpet @ $1.12, 10 pieces Irish linen, 198 yards @ 26 cts., 6 pieces muslin, 71 yds. @ 12½ cts., 5 pieces French merino, 175 yds. @ 87 cts., ¼ doz. silk umbrellas @ $51.50, 12 pieces black silk velvet, 250 yds., @ $3.25. Paid drayage $1.00, and insurance @ 1½%.

6. Jan. 9, 1876. Sold to Andrew Spence, Pittsburgh, 6 hhds. sugar, and shipped same on steamboat Bostona, Miles, master. The hhds. were marked and numbered "A. S., 5, 6, 7, 8, 9, 10." No. 5 weighed 1424 lbs., tare 27 lbs.; No. 6, 1573 lbs., tare 31 lbs.; No. 7, 1397 lbs., tare 35 lbs.; No. 8, 1576 lbs., tare 37 lbs.; No. 9, 1498 lbs., tare 30 lbs.; No. 10, 1675 lbs., tare 36 lbs. @ 12½ cts. Paid drayage, $2.50, insurance 1½ %.

7. Dec. 3, 1876. Sold to Mrs. Sophia Dodd, 20 yds. ⅝ muslin @ 15 cts., 18 yds. French merino @ 87½ cts., 1 silk bonnet @ $5. Rec'd in payment B. R. Gooley's due bill, W. Dodd's favor for $30, and gave our due bill for balance of note unpaid.

EXERCISES IN MAKING EXTENSIONS.

The following exercises can all be solved by the short methods explained in another part of this work: They are designed for the blackboard.

$67\frac{1}{2}$	@	$0.83\frac{1}{3}$	4197	@	$1.25	$619\frac{1}{2}$	@	$3.20
157	"	$0.12\frac{1}{2}$	$464\frac{1}{2}$	"	$7.62\frac{1}{2}$	$116\frac{5}{12}$	"	$3.37\frac{1}{2}$
216	"	$1.87\frac{1}{2}$	$119\frac{3}{4}$	"	1.45	$265\frac{7}{8}$	"	$1.82\frac{1}{2}$
$917\frac{1}{2}$	@	6.55*	$367\frac{1}{2}$	@	1.35	197	@	3.13
$119\frac{3}{4}$	"	1.50	$488\frac{1}{8}$	"	1.65	$682\frac{2}{3}$	"	1.25
175	"	2.17	$771\frac{1}{3}$	"	$2.12\frac{1}{2}$	†798	"	5.17
$143\frac{1}{3}$	"	6.55	$167\frac{1}{2}$	"	2.50	$677\frac{1}{2}$	"	$4.87\frac{1}{2}$
$216\frac{3}{4}$	"	1.75	$719\frac{2}{3}$	"	1.63	35	"	0.35
$116\frac{1}{3}$	"	$1.37\frac{1}{2}$	$711\frac{1}{2}$	"	1.35	$115\frac{1}{4}$	"	$3.12\frac{1}{2}$
$718\frac{1}{6}$	"	0.45	125	"	0.65	417	"	$0.18\frac{3}{4}$

1. Find the cost of 15 yards muslin @ $12\frac{1}{2}$ cents, 22 yards silk @ $82\frac{1}{2}$ cents, 45 yards ticking @ 25 cents, 150 yards satin @ $2.25, 45 yards calico @ $8\frac{1}{3}$ cents, 125 yards M. Delaine @ 27 cents, 121 yards French merino @ $62\frac{1}{2}$ cents, 6 pieces sheetings, 197 yards @ $33\frac{1}{3}$ cents, 12 pieces shirtings, $376\frac{1}{2}$ yards, @ $12\frac{1}{2}$ cents, 15 gross spools, $2.25.

2. Find the cost of 2 cases assorted cassimeres, 175 yards, @ $1.75, 8 pieces blue cloths, 216 yards,

---

*$917\frac{1}{2}$ at a dollar = $917.50

$917.50		† 800 @ 5.17 =	$4136.00
6		2 @ 5.17 deducted	10.34
5505.00			$4125.66
458.75	$= \frac{1}{2}$ of the cost at a dollar.		
$45.87\frac{1}{2}$	$= \frac{1}{10}$ of the cost at 50 cts.		
$6009.62\frac{1}{2}$			

XII. LONG DIVISION.

ART. 1. The previous operations in division have been performed *mentally*, the learner *writing* only the quotient. This is called *Short Division*, and is to be preferred when the divisor is a small number, or can be reduced to a small number, as 200, 12000, which by pointing off the ciphers are reduced to 2 and 12 (Art. 9, p. 41.) But when the divisor is 19, 23, 79, 536, etc., the operation would be too difficult and tedious to perform mentally. In such case the greater part of the process has to be *written*, and is known by the name of *Long Division*.

EXAMPLE 1.—To divide 3147 by 6.

SHORT METHOD.

$$6)3147$$
$$524\tfrac{3}{6}$$

LONG METHOD.

dividend. quotient.

$$6)3147(524\tfrac{3}{6}$$
$$30$$
$$14$$
$$12$$
$$27$$
$$24$$
$$\tfrac{3}{6}$$

EXPLANATIONS.—1. To perform this operation we say, 6 in 31, 5 times, and write 5 in the quotient and multiply it on 6, which makes 30. This we *write* under the 31.

2. We now subtract the 30 from the 31 as we would perform an operation in subtraction. The remainder is 1. Instead of *supposing* this 1 to stand before the 4 in the dividend, we bring down the 4 to it, which makes 14.

3. Six in 14, 2 times, we write 2 in the quotient, multiply it by 6 and write the product underneath.

4. Subtracting this 12 from the 14 we find a remainder of 2, to which we annex 7, brought down from the dividend, making the number 27

5. Six in 27, 4 times, we write 4 in the quotient, multiply it by 6, and write the product 24 underneath.

6. Subtracting as before we find a remainder of 3, under which we write the divisor 6, making $\tfrac{3}{6}$.

2. To divide 834716 by 723.

723) 834716(1154$\frac{374}{723}$

723	[1] This is the product of 1×723.
[2] 1117	[2] The remainder after subtracting 723 from 834, with 7 brought down.
[3] 723	[3] The product of 1×723.
[4] 3941	[4] The remainder of 1117—723, and 1 brought down
[5] 3615	[5] The product of 723×5.
[6] 3266	[6] The remainder of 3941—3615, and 6 brought down.
[7] 2892	[7] The product of 723×4.
[8] 374	[8] The last remainder=$\frac{374}{723}$

NOTE.—The pupil should put a dot under each figure brought down, to prevent its being taken twice.

3. To divide 67314968 by 163000.

163|000)67314|968(412$\frac{158968}{163000}$

652 *

211

163

484

326

158

REMARKS.—1. Instead of using the *whole divisor* in finding a quotient figure, it will generally do to use only the first one or two figures. For instance, in Ex. 2, say 7 in 8, instead of 723 in 834; and in Ex. 3, say 16 in 67, instead of 163 in 673.

2. The products should never exceed the numbers above them; (number 3 should not exceed number 2;) if they do, a smaller number should be put in the quotient.

3. For every figure brought down from the dividend, there should be one in the quotient. Where the divisor is not contained in the small dividends, as the 310 in the 68 (Art. 2, Ex. 1), a cipher should be written in the quotient, and another figure taken down.

4. The divisor can not be contained more than 9 times in the new dividends.

* These three figures are a part of the remainder, as shown in the quotient.

ART. 2. *To divide dollars and cents,* we reduce them to cents, then our quotient or answer will be cents, which are easily converted into dollars, by inserting the decimal point.

1. To divide $3168.20 by 310.

310)316820(1022 or $10.22
310
682
620
620
620

2. Divide 9765837 " 65. *Ans.* $150243\frac{42}{65}$.

3. 1763÷76=
7964÷87=
89737÷98=
77168÷19=
Total remainders, 138

4. 3167÷119=
71438÷320=
67898÷764=
78637÷892=
Total remainders, 958

5.* $10000÷7109=
7185÷1990=
67416÷ 144=
3784÷ 642=
Total quotients, $479.06

6. $140.98÷671=
730.45÷126=
164.87÷144=
1710.14÷166=
Total quotients, $17.44

PRINCIPLES OF DIVISION.

ART. 3. If we divide the price of a number of things of equal value by the number, we obtain the price of one.

ART. 4. The quotient will always be in the same name with the dividend or number to be divided. If the dividend be dollars, the quotient will be dollars; if it be rods, the quotient will be rods.

* Reduce these to cents before dividing: $10000=1000000 cents (See Note page 38,) and omit the remainders.

EXERCISES IN MULTIPLICATION AND DIVISION.

1. If 23 yds. of muslin cost $3.45, what will one yard cost?*

2. If 117 men can do a piece of work in 48 days, how long will it take three times that number to do it?

3. How many men can do a piece of work in 5 days, that took 10 men 25 days?

4. If a case hold 29 pieces of muslin, how many will it take to hold 7250 pieces?

5. If 15 men can do a certain piece of work in 75 days, how long will it take 1 man to do it?

6. If 7 dozen silver spoons cost $35.35, what will 3 dozen cost?

NOTE.—Find the cost of 1 dozen, then the cost of 3.

7. If two-sevenths of a ship cost $14602, what will the seven-sevenths, or the whole ship cost?

METHODS OF PROOF.

ART. 5. Division and Multiplication being *converse* operations, the one is proved by the other.

DIVISION. PROOF.

38)3715(97 97=quotient.
342 38=divisor.
295 776
266 291
29 rem. 3686+ the rem. 29=3715=dividend

MULTIPLICATION. PROOF.

multiplier. product. multiplicand.

465 25)11625(465
25 100
2325 162
930 150
11625 125
125

XIII. TIME.

TO RECKON TIME.

ART. 1. Business men usually reckon 30 days to the month; but when a note is given at one, two or three months, it falls due on the same day of the month it was given, plus the days of grace.

Some notes and bonds draw interest from date. When such is the case, the time is computed as follows:

1. What is the difference of time between January 3, 1878, and February 9, 1879?

	yrs.	mos.	days.
Operation.	1879	2	9
	1878	1	3
Ans.	1	1	6

EXPLANATION.—We call January the first month, February the second, etc.

2. What is the difference in time between April 3, 1878, and January 1, 1879?

	yrs.	mos.	days.
Operation.	1879	1	1
	1878	4	3
Ans.		8	28

Find the difference of time between the following dates:

January 1, 1874, and April 2, 1876 =
October 9, 1871, and Jan. 1, 1875 =
June 23, 1875, and Dec. 9, 1878 =

Total, 8 yrs. 11 mo. 9 ds.

XIV. SIMPLE INTEREST.

ART. 1. Interest is a percentage charged for the use of capital. It is regulated by the year or month. 6 per cent. (per annum) signifies *six* dollars on every *hundred* dollars for a year.*

Interest may be divided into simple and compound.

Simple Interest is percentage on *capital* alone.

Compound Interest is interest reckoned on both *capital* and *interest.*

TERMS.

The terms are *Principal, Rate,* time and *Amount.*

The *Principal* is the sum or capital loaned.

The *Rate* is the percentage charged.

The *Amount* is the *sum* of *principal* and *interest.*

NOTE.—The legal interest of the United States is 6 per cent. When no per cent. is named in this book, 6 per cent. is understood.

2. *Mills* are omitted in the answers.

ART. 2. *The interest on any sum of dollars for* 60 *days, is equal to as many cents as there are dollars.*†

The int. on \$100 for 60 days is 100 cents or \$1.00.
" " " 1250 " " " 1250 " " 12.50.

Find the interest on the following:

1. \$1749 for 60 ds.	2. \$1009 for 60 ds.
785 " " "	719 " " "
9000 " " "	5000 " " "
Total, \$115.34	Total, \$67.28

* It is customary to reckon interest for all rates at 6 per cent., and afterward to increase or diminish as necessary. See Art. 6.

† Since the interest on \$100 for 360 days is \$6 (Art. 2), for 60 days, it is *one-sixth* as much or \$1.00; but \$1 is 100 cents or as many cents as there are dollars in the principal.

ART. 3. *To find the interest for any number of days*, we take that part of the *interest* at 60 days, that the *number of days* is of 60.

To find the interest of $120 for 30, 20, 15, 12, or 10 days.

The interest on $120 for 60 days, is $1.20.

For 30 days it is $\frac{1}{2}$ of $1.20, or 60 cents.
For 20 days it is $\frac{1}{3}$ of 1.20, or 40 cents.
For 15 days it is $\frac{1}{4}$ of 1.20, or 30 cents.
For 12 days it is $\frac{1}{5}$ of 1.20, or 24 cents.
For 10 days it is $\frac{1}{6}$ of 1.20, or 20 cents.

Reason.—Since 30 days is $\frac{1}{2}$ of 60, the interest for 30 days will be $\frac{1}{2}$ of that for 60 days; 20, 15, 12, and 10, are also equal parts of 60.

NOTE.—When the days are not even parts of 60, we divide them into even parts. For 18, we take 15 and 3; for 27, take 12 and 15; for 37, take 30, 5, and 2; for 110, take 60, 30, and 20.

2. Find the interest on $211 for 93 days.

2	11	= int. for 60 days.
$\frac{1}{2}$	1 055	= " " 30 "
$\frac{1}{20}$	105	= " " 3 "
	$3.27	= " " 93 "

The student, after some practice, should not lose time by writing the divisors, or the days on the right, as in this example.

TABLE.

ALIQUOT OR EVEN PARTS OF SIXTY DAYS.

To be committed to Memory by the Pupil.

30 days	$= \frac{1}{2}$	12 days	$= \frac{1}{5}$	5 days	$= \frac{1}{12}$	2 days	$= \frac{1}{30}$
20 "	$= \frac{1}{3}$	10 "	$= \frac{1}{6}$	4 "	$= \frac{1}{15}$	1 "	$= \frac{1}{60}$
15 "	$= \frac{1}{4}$	6 "	$= \frac{1}{10}$	3 "	$= \frac{1}{20}$		

3. $797.00 for 10 ds. = $1.33

4. $1000 for 2 ds. =

* Interest is seldom reckoned on cents. If less than 50, reject them, otherwise add a dollar to the dollars.

	days.		days.
5.	$1000.00 for 27=	6.	$1799.14 for 93=
	71.97 for 47=		387.66 for 67=
	61.80 for 45=		199.44 for 41=
	190.27 for 16=		450.22 for 29=
	Total, $6.03		Total, $35.75

	days.		days.
7.	$719.99 for 11=	8.	$1997.00 for 13=
	55.18 for 9=		7.88 for 54=
	88.17 for 69=		17.97 for 35=
	466.00 for 78=		10.00 for 120=
	Total, $8.47		Total, $4.71

	days.		days.
9.	$1000.00 for 97=	10.	$1999.20 for 23=
	650.00 for 67=		361.74 for 18=
	10.70 for 13=		78.93 for 23=
	127.57 for 51=		1467.20 for 34=
	368.17 for 118=		7100.18 for 77=
	718.57 for 125=		29.00 for 99=
	Total, $46.76		Total, $108.96

ART. 4. *To find the interest for years and months.* In a year there are 6 sixty days; therefore we multiply the interest for 60 days by six times the number of years, and as there are half as many sixty days as months, we multiply the interest for 60 days by half the number of months.

RECAPITULATION.—*Consider the dollars cents, and multiply by 6 times the number of years plus half the number of months, and for the days take aliquot parts as before.*

1. To find the interest of $120 for 1 year 4 months and 20 days.

EXPLANATION.—The interest for 60 days is 120 cents; for 1 year and 4 months it is 8 times 120 or 960 cents; and for 20 days it is $\frac{1}{3}$ of 120, or 40 cents, making the sum $10.00—the interest required.

1.20
8
9.60
40
Ans. $10.00

2. Find the interest of $240 for 3 years 4 months and 10 days. *Ans.* $48.40.

3. What is the interest of $1467.45 for 2 years 6 months and 17 days. *Ans.* $224.21.

Find the interest of the following:

	yrs.	mos.	days.	
4. $321.00 for	2	3	15.*	$44.14
1767.00 for	7	4	21.	783.66
897.25 for	3	6	27.	192.41
898.57 for	2	7	25.†	143.09
716.27 for	2	1	9=	90.57
810.98 for	1	6	7=	73.94
50.00 for	3	7	18=	10.90
8.00 for	9	3	27	4.48

	yrs.	mos	days.
5. $3140.79 for	1	7	7=
795.17 for	2	1	1=
3.90 for	3	5	15=
1057.57 for	1	11	11=

Total, $526.01

	yrs.	mos.	days.
6. $2674.57 for	1	8	21 =
7143.45 for	2	1	18 =
1742.67 for	1	9	13 =
2100.00 for	2	1	1 =
4109.85 for	1	6	17 =

Total, $2022.35

* Find the interest for 2 years 4 months, and deduct the int. for 15 days.

† Call this 2 years 8 months, and deduct the int for 5 days.

		yrs.	mos.	days.	
7.	$7856.00 for	1	1	29 =	
	677.19 for	3	3	3 =	
	287.17 for	1	7	16 =	
	97.19 for	5	10	14 =	
	10.10 for	1	3	19 =	
					$743.95

		yrs.	mos.	days.	
8	$57.87 for	2	6	14 =	
	120.14 for	7	7	7 =	
	340.00 for	9	1	24 =	
	1657.00 for	1	3	24 =	
	769.75 for	2	3	18 =	
					$487.40

ART. 5. *Having the interest at 6 per cent. to find the interest at any other rate.*

This is done by taking aliquot parts of 6, and increasing or diminishing the interest, as the rate is more or less than 6 per cent.

At 2 per cent. the interest is $\frac{1}{3}$ of that at 6 per cent.
At 4 " " it is $\frac{1}{3}$ less than at 6 per cent.
At 8 " " it is $\frac{1}{3}$ more than at 6 per cent.
At 10 " " it is $\frac{10}{6}$ of that at 6%; so we have only to move the decimal point in the 6% interest one place to the right, and divide by 6. For 15 %, we move the decimal point in the same way, and divide by 4; and for 20 % by 3.

1. Let the interest at 6 % be $240.

At 2 %	**it will be $\frac{1}{3}$ as much, or**	**$ 80**
" 8 %	" " $\frac{1}{3}$ more, or	320
" 10 %	" " 10 times $\frac{1}{6}$ of $240, or	400

2. To find $1\frac{1}{2}\%$, $7\frac{1}{4}\%$, $8\frac{1}{3}\%$, and $10\frac{1}{2}\%$ of \$350 for 60 days.

4)3.50	int. at 6 %	6)3.50	int. at 6 %
.87,5	" " $1\frac{1}{2}\%$	.58 3	" " 1 %
		.14 5	" " $\frac{1}{4}\%$
		\$4.23	" " $7\frac{1}{4}\%$

\$3.50	int. at 6 %	\$3.50	int. at 6 %
1.16.6	" " 2 %	1.75	" " 3 %
.19.4	" " $\frac{1}{3}\%$	.87 5	" " $1\frac{1}{2}\%$
\$4.86	$8\frac{1}{3}\%$	\$6.13	$10\frac{1}{2}\%$

	yrs.	mos.	days.	
3. \$798.18 for	6	1	6 @ 9% =	\$438.10
1000.00 for	4	2	4 @ 7% =	292.44

	yrs.	mos.	days.
4. \$340 for	2	2	20 @ $2\frac{1}{2}\%$ =
600 for	3	4	15 @ $6\frac{1}{2}\%$ =
850 for	1	2	12 @ $8\frac{1}{2}\%$ =

Total, \$237.22

5. Find the interest of

	mos.	days.
\$617.18 for	3	18 @ 15% =
460.74 for	2	5 @ 18% =
765.12 for	8	16 @ 20% =

Total, \$151.55

6. Find the interest on the following at 10%

\$710 for 92 days =	7. \$496 for 91 days =
1978 for 27 " =	671 for 86 " =
8889 for 128 " =	100 for 104 " =
75 for 117 " =	269 for 73 " =
Total, \$351.47	Total, \$36.91

ART. 6. It is customary for bankers to lend money and discount by the *month* instead of by the *year*.

This percentage is easily converted into 6% interest, and the work performed with as much ease as before:

1 % per month is 12% per year, or 2 times 6%
1½% " " is 18% " " or 3 " 6%
2 % " " is 24% " " or 4 " 6%

1. Find the interest of the following:

$65.00 for 80 days @2 % per month =
40.00 " 33 " @1½% " " =
190.00 " 63 " @2 % " " =
700.00 " 93 " @3 % " " =

Total, $77.20

ART. 5. The work, when computing interest, can often be abbreviated. Sometimes advantage may be taken of the aliquots of hundred; at other times it will be of advantage to transpose the terms and consider the days as dollars and the dollars as days; or the rate (if it is some other rate than 6%) may be reduced mentally to 6%. For instance, in the second question in the last group, the $40 may be considered $120, and transposing the term and the 33 multiplied by 2, making 66c the answer.

It will materially abridge the operation and expedite the labor, if the learner will observe to avoid the use of all lines, figures or marks that are not absolutely necessary. As, for instance, when using aliquot parts, to write only the results of division, as shown in the following example:

Interest of $321 for 2 years, 1 month and 22 days at 10% per annum.

EXPLANATION.—Mentally it is found that there are 12½ 60 days in 2 years and 1 month, to multiply by which we divide by 8. The division by 6 and the multiplication by 10 were performed simultaneously, giving $68.836 or $68.84 as the answer.

3.21
40.125
1.07
107
41.302
68.836

XV. COMPOUND INTEREST.

ART. 1. In Compound Interest the interest is converted into principal every quarter, half year or year. Capital is thus more rapidly increased, than by simple interest.

Any person acquainted with the principles of simple interest will readily understand how to compute compound interest.

1. What is the compound interest of \$1000 for $1\frac{1}{2}$ years at 6 %, payable semi-annually (half-yearly)?

The interest of \$1000 for 6 mos.=	\$30.00
Add the principal,	1000.00
Amount for 6 mos.	\$1030.00
Interest on \$1030 for 6 mos.,	30.90
Amount for 1 year,	\$1060.90
Interest on \$1060.90 for 6 mos.,	31.827
Amount for 18 mos.,	\$1092.727
Principal,	1000.00
Compound interest for $1\frac{1}{2}$ years,	\$92.73

2. Find the compound interest and amount of \$1865 for 3 years, 3 months, at 8 %, payable tri-monthly.

3. What is the compound interest and amount of \$486 for 4 years, at 10 %, payable annually?

4. What is the compound interest and amount of \$672 for 4 years, at 6 % per annum?

Answers: \$1092.73, \$848.38, \$2412.56, \$711.55, \$92.73, \$176.38, \$547.56, \$225.55.

REMARK.—At 6 per cent. money will double itself in 11 years, 10 months and 21 days. At 5 per cent., in 14 years, 2 months, and 15 days. At 3 per cent., in 23 years, 5 months, and $10\frac{1}{4}$ days.

XVI. ANNUAL INTEREST.

ANNUAL INTEREST is the term applied to interest on a note that is drawn with the clause "interest payable annually." When this interest is not paid at the end of the year, it draws simple interest till paid.

1. A note for $300 at 3 years, 6% interest, payable annually, had nothing paid on it at maturity. How much was due?

Int. on $300 at 6% = $18 = int. for 1 year.
3
$54 = int. for 3 years.
Int. on $18 for 2 years, 2.16
" " 1 year, 1.08
Principal, 300.00
$357.24 amount.

2. What is the amount of a note, at the end of 4 years for $368, for 2 years, 8% interest, payable annually, that had nothing paid on it until settlement?

NOTE.—When a note is overdue interest is calculated up to the date of maturity, as in Ex. 1, and simple interest is calculated on the amount from maturity till paid.

3. What should I pay at maturity to redeem my note for $800, payable 3 years after date with 10% interest, payable annually, nothing having been paid at maturity?

4. A note for $720, at 4 years, 6% interest, payable annually, had nothing paid at maturity. How much was due?

5. What is the amount of a note, at the end of 3 years, for $1268, payable 2 years after date, bearing 8% interest, payable annually, no payments having been made until settlement?

Answers: $497.91, $1064, $738.84, $1597.32, $357.24, $908.35, $497.92.

XVII. PARTIAL PAYMENTS.

ART. 1. Notes, bonds, etc., drawing interest, are sometimes paid by installments, and the amounts thus paid, indorsed on them. The legal rule for computing interest on installments, may be expressed thus:

Apply the payment to the discharge of the interest, and if there is a remainder, subtract it from the debt. When the payment is less than the interest due, it is not applied to the discharge of the interest or debt, but is indorsed on the note until the installments exceed the interest; then the sum of the payments are computed as below.

1. $576. CINCINNATI, Oct. 9, 1875.

On demand, I promise to pay Robert Ingles, or order, five hundred and seventy-six dollars, with interest, value rec'd.

SAMUEL DUNNING.

On the note are the following indorsements:

Rec'd Dec. 16, 1875, $100
" Feb. 28, 1876, 3
" July 27, " 150

Required the amount due Sept. 3, 1878.

	yr.	mos.	ds.	
From	1875	12	16	
Take	1875	10	9	
Difference,		2	7,	or 67 days.
Amount of note,			$576.00	
Interest on $576 for 67 days,			6.43	
Total amount due,			$582.43	

Total amount due,	$582.43
Installment to be subtracted,	100.00
Balance due,	$482.43

The second payment is less than the interest due, and no calculation is required.

From Dec. 16th, 1875, to July 27th, 1876, is 7 mos., 11 days.

Balance due,	$482.43
Interest for 7 mos. 11 days,	17.75
Amount due,	500.18
Amount of payments,	153.00
Balance due,	$347.18

From July 27 to Sept. 3d, is 38 days.

Balance,	347.18
Interest for 38 days,	2.19
Amount due Sept. 3, 1876,	$349.37

2. $650. BOSTON, June 3, 1868.

For value rec'd, I promise to pay on demand t· H. Crooks, or order, six hundred and fifty dollars, with interest at 6 % per annum.

J. F. DAVIS.

Indorsements.

Jan. 6, 1870, $95
Oct. 13, 1870, 350
Jun. 3, 1875, 12

Sept. 7, 1877, paid the balance, how much was it?

Ans. $405.92.

3. On a note drawn Sept. 3, 1877, for $650 with legal interest, there are the following indorsements:

Oct. 4, $100
Nov. 3, 2
Dec. 19, 210

Apr. 3, 1878. the balance; how much was it?

Ans. $354.32.

XVIII. BANK DISCOUNT.

ART. 1. Discounting notes consists in buying them at less than their nominal value, or the amount for which they are drawn. The difference between the nominal value and the price paid is called *discount*.

There are two kinds of discount: *True Discount*, which is interest paid in advance on the *present value* of a note, and *Bank Discount*, which is interest paid in advance on the *face* of the note. The latter resembles compound interest, as it is interest on both interest and principal.*

When a note is discounted in bank, the interest of the note for the time it has to run, and at the banker's rates, is deducted from the sum called for by the note. This species of discount is therefore reckoned in the same way as interest.

1. How much discount should be deducted from a note of $500 at 90 days?

$5.00 = int. for 60 days.
2.50 = " " 30 "
.25 = " " 3 " (*grace*)

Ans. $7.75

2. $1500. COLUMBUS, Jan. 8, 1879.

Sixty days after date, I promise to pay Messrs. M'Ewen and Banfill one thousand five hundred dollars, value received.

WILLIAM DODD.

Required the discount at 6 %. *Ans.* $15.75.

* The present worth of a note drawn for $100, payable in a year at 6 per cent., is $94.34, and the interest is $5.66; that is, the principal and interest together, are equal to $100, or the face of the note; so when a banker discounts from the face of a note, he discounts on both principal and interest.

Bankers prefer lending money on short time, and by the day, instead of by the month. Notes are usually drawn for 30, 60, or 90 days; and interest is always charged on the days of grace.

1. What is the bank discount on a note of $120 at 60 days, at $\frac{1}{2}$ % per month? *Ans.* $1.26.

2. Find the discount on a note of $575.75 at 90 days, at the same rate. *Ans.* $8.92.

3. What is the bank discount on a note of $450 for 60 days at 2 % per month? *Ans.* $18.90.

REMARK.—The discount on $450 at 2 per cent. per month, is the same as the discount of 4 times $450, or $1800 at 6 per cent per annum.

4. How much money should be paid by a banker who discounts a note of $350 at 30 days, at $1\frac{1}{3}$ % per month? *Ans.* $344.22.

5. What will be the proceeds of a note drawn for $670 at 60 days, at 2 % per month? *Ans.* $641.86.

6. At $1\frac{1}{2}$ % per month, how much proceeds should be recovered on a note of $1749.57, drawn at 90 days? *Ans.* $1668.12.

7. Find the discount on a note of $1678.25, drawn at 90 days at $1\frac{1}{4}$ % per month?

8. At $2\frac{1}{4}$ % per month, what is the discount on a note of $688 at 90 days?

9. At $1\frac{3}{4}$ % per month, what will be the proceeds of a note drawn for $6784, at 60 days?

Answers to the foregoing: $47.99, $65.03, $6534.69.

10. Find the discount on the following:

$1310.00	for	60	days	@	2	%	per	mo.
746.87	"	90	"	"	$1\frac{1}{2}$	"	"	"
219.56	"	30	"	"	1	"	"	"
1867.25	"	20	"	"	$2\frac{1}{2}$	"	"	"
1367.00	"	15	"	"	3	"	"	"

$152.57

11. $1673 for 30 days @ 1 % per mo.
6789 " 3 mos. " 2 " " "
1987 " 9 mos. " 1 " " " .
6745 " 10 days " 2½ " " "

$693.21

Find the amount of *proceeds* of the following:

12. $3768 for 10 days @ 4 % per mo.
1767 " 15 " " 3 " " "
8767 " 6 " " 1½ " " "

$14165.43

13. $167.39 for 2 mos. @ 2½ % per mo.
978.00 " 6 " " 20 " " an.
897.87 " 3 " " 15 " " "

$1900.25

NOTE.—As a review exercise the pupil might calculate interest on cents as well as dollars.

ART. 2. Bankers frequently discount notes that are partly matured; when such is the case, the following table will assist the accountant in computing the discount:

A TABLE

Showing the number of days from any day in one month, to the same day in any other month, throughout the year.

MONTHS.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
January,	365	31	59	90	120	151	181	212	243	273	304	334
February,	334	365	28	59	89	120	150	181	212	242	273	308
March,	306	337	365	31	61	92	122	153	184	214	245	275
April,	275	306	334	365	30	61	91	122	153	183	214	244
May,	245	276	304	335	365	31	61	92	123	153	184	214
June,	214	245	273	304	334	365	30	61	92	122	153	183
July,	184	215	243	274	304	335	365	31	62	92	123	153
August,	153	184	212	243	273	304	334	365	31	61	92	122
September,	122	153	181	212	242	273	303	334	365	30	61	91
October,	92	123	151	182	212	243	273	304	335	365	31	61
November,	61	92	120	151	181	212	242	273	304	334	365	30
December,	31	62	90	121	151	182	212	243	274	304	335	365

Use of the Table.—To find the time from Feb. 13 to March 23, in the following example: In the left hand column we find February in the second line, and running the eye along till we come under "March," we find the number 28; hence from Feb. 13 to March 13 is 28 days; to March 23d, will therefore be 10 days more, or 38 days. The discount will be reckoned for 93 days minus 38 days, equal 55 days.

1. A note drawn on Feb. 13, 1878, for $900, at 90 days, was discounted on March the 23d, at 2 % per month, how much was paid by the borrower?

Ans. $867.

2. What proceeds should be paid on a note of $346 at 90 days, drawn on Nov. 3d and discounted on Dec. the 7th, at 1½ % per month? *Ans.* $335.79.

3. A note of $689, made Sept. 9, payable in 60 days, was discounted on Oct. 5th, at 2 % per month, what was the discount? *Ans.* $16.99.

NOTE.—If the decimals be carried out to three or four places, the cents may differ slightly from the following totals.

(4.)

Face of Note.	Date.	Time.	When Disc'td.	Rate of Disc't.	
$167.50	Jan. 3, 1879	60 days	Feb. 7,	2 %	per mo.
9876.00	Feb. 7, "	90 "	Mar. 12,	2½ %	" "
789.00	Jun. 18, "	30 "	July 3,	1½ %	" "
1897.00	Feb. 21, "	90 "	Apr. 1,	1½ %	" "
				Total,	$555.24

(5.)

$676.37	Apr. 3, 1879	90 days	May 9,	2 %	per mo.
679.39	Mar. 9, "	30 "	Apr. 3,	2½ %	" "
7168.00	Jun. 13, "	60 "	July 9,	1½ %	" "
816.37	Aug. 12, "	30 "	Sep. 6,	2¼ %	" "
				Total,	$167.74

(6.)

Face of Note.	Date.	Time.	When Disc'td.	Rate of Disc't.	Discount
2676.00	Jan. 9, 1879	90 days	Feb. 1,	$1\frac{1}{4}$ % per mo.	
7187.00	Feb. 3, "	60 "	Mar. 13,	$1\frac{1}{2}$ % " "	
768.21	Mar. 6, "	30 "	Apr. 3,	2 % " "	
314.00	Apr. 7, "	90 "	May 15,	2 % " "	
				Total,	$181.98

7. A note of $1675 drawn on Nov. 3, at 3 months, was discounted on Dec. 2, at $1\frac{1}{2}$ %, what was the discount?

8. What amount of proceeds will arise from discounting a six months' note, drawn for $197, at 2 % per month?

9.

Am't. of Note.	Date.	Time.	When discounted.	Rate of Discount	Proceeds
$6785	Dec. 6, 1878,	6 mo.	Dec. 29, 1878,	2 % per mo.	
3748	Jan. 3, 1880,	5 "	Feb. 3, 1880,	$2\frac{1}{2}$ % " "	
6983	Mar. 9, "	4 "	Jun. 8, "	$1\frac{1}{2}$ % " "	
				Total,	$16277.22

10.

$3784	May 6, 1880,	2 mo.	July 3, 1880,	2 % per mo.	
6987	Jun. 8, "	3 "	Aug. 27, "	$1\frac{1}{2}$ % " "	
7854	July 24, "	4 "	Sep. 17, "	1 % " "	
				Total,	$18371.58

11. What amount of money should I receive on a note of $675, discounted at 35 days (having 35 days to run,) $1\frac{1}{2}$ % per month?

12 June 3d, discounted my note of $350 at 10 %, having 30 days to run, required the discount?

13. Feb. 6, 1878, had A. Seers' note of $500, dated 20th Dec., 1877, discounted at $1\frac{1}{2}$ % per month, time to run 33 days, what were the proceeds?

XIX. TRUE DISCOUNT.

ART. 1. *True Discount* is the difference between the *present worth* of a note and the amount for which it is drawn.

The *present worth* of a note or bill due at a future time without interest, is such a sum as would, if put at interest for the same time and rate per cent., amount to the debt; and the difference between this sum and the debt is the *discount.*

1. What is the true discount on a note of $700 for 90 days at 6 %?

The amount of a dollar for 93 days is $1.0155, by which, if we divide $700, we will find the present worth.

Operation, $1.0155)700.0000*(689.31

60930

90700

81240

94600

91395

32050

30465

15850

10155

PROOF.—The interest on $689.31 92 days, is $10.68, or $10.69 nearly, which, if added to the principal, will give $700.

NOTE.—The interest of $1 for 90 days is .0155. The present value of $1.0155 for 93 days is, therefore, $1, and accordingly the present value of $700 for 93 days is $700.00, divided by $1.0155 or $689.31, and the discount $700, $689.31, or $10.69.

2. What is the true discount on a note of $575 for 90 days at 6 %?

3. What is the true discount on a note for $880 for 120 days at 8 %?

Answers: $23.39, $11.55, $10.69.

* Four ciphers are annexed to the $700 to correspond with the divisor; the quotient from this will be dollars; by annexing two more ciphers, the answer will appear in cents.

XX. DISCOUNTING INT.-BEARING NOTES.

ART. 1. There are three methods of reckoning discount on interest-bearing notes: *Bankers' Method*, *Brokers' Method*, and *Equitable Method.*

ART. 2. Bankers calculate interest on the face of an interest-bearing note up to the date of maturity (including days of grace), and discount the amount (principal and interest) for the unexpired time.

ART. 3. Money brokers reckon interest on the face of an interest-bearing note up to the date of discount, and discount on the face of the note from the date of discount to the date of maturity, at the difference between the rate of interest and rate of discount.

ART. 4. By the Equitable method, interest is reckoned on the face of the note up to maturity, and true interest is calculated on the amount.

1. What amount of proceeds will arise from a note dated January 3, 1875, for $1200, payable 2 years after date, with 6% per annum discounted April 18, 1876 @ 10%?

BANKERS' METHOD.

$12.00	
12	
144.00	int. 2 yrs.
.60	int. 3 days
144.60	int. @ 6% for 2 yrs. 3 ds.
1200.00	principal
$1344.60	amount

The unexpired time from date of discount, April 18, 1876, to maturity, January 6, 1877, is 264 days.

$13.44 60
4

53.78 4 int. for 240 ds.
5.37 8 int. for 24 ds.

59.16 int. for 264 ds. @ 6% per an.
19.72 " " " " " 2%
19.72 " " " " " 2%

98.60 " " " " 10%

1344.60 amount
98.60 discount

$1246.00 proceeds

BROKERS' METHOD.

$12.00			1876	4	18
07			1875	1	3
84.00	int. 1 yr. 2 mos.		1	3	15
6.00	" 1 mo.				
3.00	" 15 ds.				
$93.00	" 1 yr. 3 mos. 15 ds. @ 6% per an.				

$12.00			1877	12	30
4.00			1876	1	6
				4	18
48.00	int. 8 mos. 6%				
3.00	" 15 ds. 6%				
.60	" 3 ds.				
51.60	" 8 mos. 18 ds. 6%				
17.20	" 8 " 18 " 2%				
$34.40	" 8 " 18 " 4%				

Interest in favor of borrower,	$93.00
Discount against "	34.40
Difference in favor of "	58.60
Principal,	1200.00
Proceeds,	$1258.60

Equitable Method.

The interest the same as by the Bankers' method: $144.60; amount, $1344.60.

The amount of $1 for 264 days (the unexpired time) at 10% per annum is $1.0716.

$1.0716) 1344.60

Proceeds, $1254.68

It will be noticed that the Bankers' method is more favorable to the lender; the Equitable method, to the borrower.

2. What is the present value (Jan. 6, 1876) of a note, dated March 18, 1875, for $268.50, payable 3 years after date, with 8% interest per annum; rate of discount, 12% per annum?

Note.—These examples should be worked by all three methods.

3. What amount of money should I receive on a note, dated May 8, 1875, for $668.35, payable 2 years after date, with 6% interest per annum; discounted Sept. 11, 1875; discount, 8% per annum?

Answers: $243.84, $262.57, $263.42, $647.66, $680, $659.93, $68, $261.97, $263.34, $659.85.

XXI. COMPLEX PERCENTAGE.

ART. 1. COMPLEX Percentage embraces all those calculations of percentage the result of which can not be ascertained by the process explained under SIMPLE PERCENTAGE.

ART. 2. To find the GAIN PER CENT. when the actual gain, the principal, and amount are known.

1. What is the *gain per cent.* on goods bought at \$2.50 and sold at \$2.75?

SOLUTION: \$2.75 selling price.
2.50 cost price.
25 actual gain.

The actual gain on the investment or cost price (\$2.50) is 25 cents. To ascertain the gain per cent., *i. e.*, the gain on the 100 cents, the first step will be to find the gain on one cent—viz: if 250 cents bring 25 cents, one cent will gain as much as 250 is contained in 25, viz: $\frac{1}{10}$ cent; if we gain $\frac{1}{10}$ cent on one cent, we gain on 100 cents, 100 times $\frac{1}{10}$, viz: 10 cents; *i. e.*, 10 %

FORMULA: $\frac{25 \times 100}{250} = 10\ \%.$

In the formula we multiply the actual gain by 100 and divide the product by the cost price, the result gives us the gain per cent.

NOTE.—This, like nearly all examples in complex percentage may be worked by proportion, which would read: The cost price (250) is to 100 as the gain on the cost price is to the gain on 100 or the gain per cent.

2. What is the gain per cent. on goods bought at \$1.60 and sold for \$1.96?

3. Bought one bbl. flour for \$8.25 and sold it for \$8.91. What was my gain per cent.?

Answers arranged promiscuously on page 118.

ART. 3. To find the PRINCIPAL, when the rate and amount are known.

1. How much of $2448 can I invest in grain after retaining 2 % commission for buying?

SOLUTION.—The amount $2448 includes the cost of the grain (principal) and the commission. The commission is to be paid on the principal and not on the amount, and as the principal is unknown we can not ascertain the commission by reckoning simple percentage. Every principal is 100 per cent. of itself, plus 2 % commission makes the amount equal 102 per cent. If $2448 = 102 %, and the principal = 100 %, we find the principal by multiplying the amount by 100 and dividing the result by 102, viz:

$$102 \overline{)\$244800}$$

2400 the principal.

PROOF: $2400 the principal.
2 % rate of commission.

4800
2400

$2448.00 the amount received for investment.

NOTE.—The same result may be ascertained by fractions, thus: The amount is equal to the principal and commission. The commission is 2 % or $\frac{2}{100}$ of the principal The principal is $\frac{100}{100}$, and the amount is as much as the principal and commission, viz: $\frac{102}{100}$ or $\frac{51}{50}$. Hence, we have to divide $2448 by $\frac{51}{50}$, thus:

$$\$2558 \div \frac{51}{50} = \$2448 \times \frac{50}{51} = \$2400.$$

2. Allowing $2\frac{1}{2}$ % commission on a sale of 300 bbls. of flour for $2567.60, how much can I invest in sugar after reserving 2 % commission for buying?

NOTE.—The commission for selling must be ascertained by *simple percentage*. By deducting the same from the proceeds ($2567.60), we obtain the amount. The remainder of the example can be solved by either of the foregoing solutions.

3. What would be the face of a draft at a discount of $1\frac{1}{2}$ % to cover investment of $858.92?

NOTE.—In this example the amount is smaller than the principal, The discount is to be taken from the principal—$1 of the principal contains $1\frac{1}{2}$ cents discount and $98\frac{1}{2}$ cents face. Hence, $98\frac{1}{2}$ cents, or 98.5, is to be divided into $858.92 $\times 100$ to ascertain the principal.

4. At a premium of 2 % what should be the face of a draft to cover an investment of $749.70?

ART. 4. To find the RATE when the *principal* and the *gain* or *loss* are known.

1. What is the *rate per cent.* of a dividend of $42 on an investment of $600?

SOLUTION.—To obtain the *rate per cent.* of dividend means to ascertain the amount of dividend on the hundred. If $600 bring $42, $100 (being $\frac{1}{6}$ of 600) will bring $\frac{1}{6}$ of $42, viz: $7 on $100, which is 7 %.

FORMULA: $\frac{42 \times 100}{600} = \$7$ or 7 %.

2. Received a dividend of $63 on an investment of $840; what was the rate per cent.?

3. With an investment of $4944, what rate per cent. will bring $618?

ART. 5. To find the RATE OF INCOME for a given investment obtained at a discount or for a premium.

1. If I invest in 6 % interest-bearing bonds, paying 10 % premium, what per cent. of income will I receive?

SOLUTION.—If I buy at 10 % premium, I pay $110 for a bond of $100. The investment of $110 brings me but $6—

HENCE: $\frac{6 \times 100}{110} = \frac{60}{11} = 5\frac{5}{11}$ %.

2. Bought 5–20 (6 %) bonds @ 12 % premium; whate rate per cent. income will I receive, including ½ % brokerage?

NOTE.—The premium and brokerage added will give what was paid above the par value.

2. Bought railroad bonds bearing 8 % interest at a discount of 4 %; what will be the rate per cent. of my income?

ART. 6. To find the COST PER CENT. of interest-bearing stocks in order to make a certain per cent. income.

1. What should I pay on the dollar of a 6 % bearing bond to make an income of 10 %?

SOLUTION.—A $100 6 % bond will pay $6, and $6 is 10 % of $60. Hence, I must buy 6 % bearing bonds @ 60 cents on the dollar to make 10 %.

FORMULA: $\frac{6 \times 100}{10} = 60.$

2. At what rate should I buy a $7\frac{3}{10}$ % bearing bond to make an income of 10 % on my investment?

3. How much should I pay on the dollar of an 8 % bond to make an income of 12 %?

ART. 7. To find the COST of an investment bought at a premium or discount and sold at a premium or discount, when the gain and rates of premium and discount are known.

1. Bought stock at 4 % discount and sold it at 6 % premium, gaining thereby $200; what was the amount of my investment?

SOLUTION.—Buying at 4 % discount and selling at 6 % premium gives a profit of 10 %.

FORMULA: $\frac{20\cancel{0} \times 100}{1\cancel{0}} = \$2000$, face of bonds.

4 % of \$2000 = \$80; that taken from the par value gives \$1920 as the investment.

2. Bought stock at 10 % discount and sold it at 2½ % premium, realizing \$87½; what was the par value of the stock, and how much did I invest?

3. Bought stock at 2 % premium and sold it at 2½ % discount, thereby losing \$450; what was the par value, and how much did I invest?

ART. 8. To ascertain the TIME a note has to run, the discount, etc., being known, so that a certain rate will be equal to the interest at another certain rate.

1. How long will a note, discounted at 20 % per annum, have to run to make 22½ % interest per annum.

SOLUTION.—The discount at 20 % per annum is equal to the interest @ 25 % per annum (PROOF: Interest on \$100 for 1 yr. @ 20 % = \$20, giving \$80 as the proceeds, 25 % of which will be \$20). Hence, in 1 year, or 360 days, a discount of 20 % will equal the interest at 25 %. To ascertain in how many days a discount would bring 22½ % interest, use the following—

$$\text{FORMULA: } \frac{360 \text{ days} \times 25\% \times 2\frac{1}{2}}{22\frac{1}{2}\% \times 5}$$ which

stands canceled: $\frac{\overset{8}{\cancel{360}} \times 25 \times \overset{1}{\cancel{2\frac{1}{2}}}}{\underset{2}{\cancel{22\frac{1}{2}}} \times \underset{1}{\cancel{5}}} = 200$ days.

PROOF.—The discount for 200 days on \$100@20% = \$11.11, giving as proceeds \$88.89, the interest on which for 200 days @ 22½% is \$11.11.

The numbers used in the formula are 360, the number of days in a year; 25, the rate of interest per year; 22½, the rate of interest for the time desired; 2½, the difference between the rate of discount and the rate of interest for the desired time; and 5, the difference between the rate of discount per year and the rate of interest per year.

2. How long will a note, discounted at the rate of

20 % per annum, have to run to make 24 % interest per annum?

3. To make $11\frac{1}{4}$ % interest per annum, how long would a note, discounted at 10 % per annum, have to run?

ART. 9. To ascertain the RATE OF GAIN on articles which, by being bought at a certain lower rate, will produce a certain higher rate of gain.

1. If an article be bought at 10 % less, and the rate of gain thereby increased 15 %, what would be my rate of gain?

SOLUTION.—By buying at 10 % less, we pay 90 cents on the dollar. Hence, by buying the cost is 90 cents on the dollar. 10 cents gained on 90 cents is a gain of $11\frac{1}{9}$ %. If $11\frac{1}{9}$ cents gain on the dollar in the cost will make 15 cents on the dollar in the gain, the entire gain per cent., the following formula will work out the selling price:

FORMULA: $$\frac{15 \times 100}{11\frac{1}{9}} = \frac{15 \times 9 \times 100}{100} = 135.$$

$1.35 being the amount an investment of $1.00 will produce. Hence, we gain 35 cents on the dollar, or 35 %.

NOTE.—The example may be worked by proportion, thus: $11\frac{1}{9} : 15 :: 100 : 135$. For $11\frac{1}{9}$ is to 15 as 100 is to 135; or, the gain per cent. buying, is to the gain per cent. selling, as the cost price, is to the selling price.

2. If an article cost me $12\frac{1}{2}$ % less, my rate of gain was increased 16 %; what was my rate of gain?

3. What will be my rate of gain, if I buy an article at 10 % less, and thereby gain $16\frac{2}{3}$ % more.

Answers: 50 %, 12 %, 36 %; 400 days, 300 days, 200 days; $10200, $10000, $700, $630, $1920; $66\frac{2}{3}$ cts., 73 %, 60 cts.; $8\frac{1}{3}$ %, $5\frac{1}{3}$ %, $5\frac{5}{11}$ %; $12\frac{1}{2}$ %, $7\frac{1}{2}$ %, 7 %; $872, $735, $2454.32, $2400; $24\frac{1}{2}$%, 8 %, 10 %, $22\frac{1}{2}$%.

ART. 10. *To find the* AMOUNT *for which a note may be drawn to realize a certain sum after being discounted.*

1. Required the face of a 90-day note which will realize $275.23, after being discounted at 2% per month.

Interest on $1 for 93 days at 2% per month = .062.

Proceeds of $1 = $1.000 — .062 = .938.

Since there are as many dollars in the principal as the proceeds of $1 is contained times in the proceeds given, $275.230 ÷ .938 will give the principal required, $293.42 +.

PROOF.—Interest on $293.42 for 93 days at 2% per month = 18.19 +, which, subtracted from $293.42, leaves $275.23, the proceeds.

2. The proceeds are $212.60, time 63 days, rate 1½% per month; required the principal. *

3. What principal will realize $120 proceeds in 6 months at 10% per annum?

4. The time is three months, rate 10% per an., proceeds $168.97; what is the principal?

5. The rate is 12% per annum, proceeds $693.75, time 4 months; required the principal.

ART. 11. *To find the* RATE PER CENT., *when the principal, interest, and time are given.*

1. The principal is $300, time 60 days, interest $5; required the rate.

Interest on $300 for 60 days at 6 % = $3. At 1 % = .50. It is obvious that the rate will be as great as the number of times 1 % is contained in the interest given. Hence, $5.00 ÷ 50 = the rate, 10 %.

PROOF.—Interest on $300 for 60 days at 10 % = $5.

Answers: $293.42, $219.52, 10 %, $173.30, $126.32, $722.66.

*The learner can prove his work by computing interest on the principal found.

2. The principal is $396.15, time 13 months 9 days, interest $26.34,3; required the rate.

3. What is the rate per cent. on $144 for 5 days, when the interest is 24 cents?

4. Required the rate on $250 for 60 days, when the interest is $3.50.

5. The principal is $820, time 30 days, interest $6.15; what is the rate?

ART. 12. *To find the* TIME, *when the principal, rate per cent. and interest are given.*

Grace being allowed only on notes and drafts, where neither is named it is not reckoned.

1. The principal is $1440, rate 10 % per annum, interest $37.50; required the time.

Interest on $1440 for 1 day at 10 % = 40 cents.

Since there are as many days as the interest for 1 day is contained times in the interest given, $37.50 ÷ 40 = $93\frac{3}{4}$, or 94 days.

PROOF.—Interest on $1440 for 94 days at 10 % per annum = $37.60.*

2. The principal is $1674, rate 2 % per month, interest $59.87; required the time.

3. In what time will a note for $600, at 6 % per annum, draw $27.50 interest?

4. A note for $375 drew $21 interest at 6 % per annum; how long did it require to do it?

5. A merchant wishes to know the time it will take a balance of $917.50 to make $60.80, with interest at 10 %.

Answers: $8\frac{2}{5}$ %, 12 %, 6 % 11 months 6 days, 54 days, 108 days, 239 days, 94 days, 275 days. 9%.

*Interest is never reckoned on the fraction of a day, hence the difference.

ART. 13. MISCELLANEOUS EXERCISES.

1. What is the gain per cent. on goods bought at \$1.20 and sold at \$1.35?

135=selling price.
120=cost price.

15=actual gain.

And $*\frac{15}{120}$c.=gain on one cent. On 100 cents there will be 100 times $\frac{15}{120}$ or $\frac{1500}{120}=12\frac{6}{12}$ or $12\frac{1}{2}$.

NOTE.—If the gain is an even part of the first cost take the same part of 100. This is the reverse of operation 1st in last Article.

In the present Ex. 15 is $\frac{1}{8}$ of 120, therefore the gain per cent. is $\frac{1}{8}$ of 100 or $12\frac{1}{2}$.

	FIRST COST.	SELLING PRICE.	GAIN PER CENT
2.	\$2.00	\$3.00	
3.	1.25	1.50	
4.	0.75	1.00	
5.	0.10	$0.12\frac{1}{2}$	
		Total,	$128\frac{1}{3}$

What was the first cost of the goods marked as follows?

6.	115.87 @ $12\frac{1}{2}$ % gain.	9.	3.75 @ 25 % loss.
7.	14.54 " $33\frac{1}{3}$ % "	10.	$0.87\frac{1}{2}$ " $12\frac{1}{2}$ % "
8.	00.87 " $16\frac{2}{3}$ % "	11.	$0.12\frac{1}{2}$ " 50 % "

12. A bill of \$1687.75 had been reduced 10 %, what was the original amount?

13. A carpenter puts in an estimate at 25 % off the bill of prices, and another puts one at 10 % off the first; how much per cent. off the bill was his discount?

Answers: $12\frac{1}{2}$ %, 50 %, 20 %, $33\frac{1}{3}$ %, 25 %, \$102.97, \$10.90, \$0.74, \$5, \$0.25, \$1, 1875.28, $32\frac{1}{2}$ %.

14. Bought a bbl. of apples for $1.75, and sold it for $2.25; what did I gain per cent?

15. Sold 25 bbls. potatoes for $39; how much did I gain per cent. if they cost me $1.25 per barrel?

16. Bought 150 bbls. of flour @ $5.25, paid for drayage $7.50, and porterage $1; at what per barrel should I sell it to gain 15 %.

17. Bought 15 horses at $125 each and sold the lot for $3500; what was my gain per cent. after paying $25 for their feed?

18. Sold a safe which cost me $80 for $75; what was my loss per cent.?

19. Bought a bill of goods for $350, paid freight $15.20; insurance, $5; drayage, $3; and sold them for $425. What was my *actual gain*, and what my gain per cent.?

20. Bought Henry Ullhorn's note for $750 at a discount of 15 %; what did I pay for it?

21. Sold Henry Hazin's note of $320 for $300; what was the rate of discount?

22. A jeweler, whose business capital is $10000, makes 100 % on his goods, and takes in on an average $20 a day. A grocer, whose capital is $1000, profits 15 %, and takes in $35 a day. The jeweler's expenses being $1000, and the grocer's $300 per year; what does each one *gain* % on the capital invested?

23. A pork merchant receives a quantity of pork to be sold on commission, at $2\frac{1}{2}$ %; or he may have the whole on his own account at $7\frac{1}{2}$ cents per pound; should he sell on commission, or buy, supposing he can get $8\frac{1}{8}$ cents a pound?

Answers: $28\frac{4}{7}$ %, $6.10, $84\frac{4}{19}$ %, $6\frac{1}{4}$ %, 14 %, $51.80, $637.50, $6\frac{1}{4}$ %, $6500, $111\frac{3}{5}$ %, buy.

24. A huckster commences business on $50, turns his money every 3 days, making 2 cents on every 10, how much does he make in the year, provided he spends $15 a month for rent, and puts out his gains at 6 % interest at the end of every month?

25. A bookkeeper who receives a salary of $1500 a year, and loans his employer $2000 at 10 %, is offered a fourth of the profits on $8000 for five years, for his capital, influence and services; would he gain or lose by accepting the offer, the profits of the business being 20 % per annum?

26. The assignee of an insolvent debtor reports to the court that preferred claims (which must be paid in full before the general creditors are entitled to a dividend) have been proven to the amount of $386; other claims, $40630; that he has realized from collections and sales, $8650. The costs of court to date are $8.50; the fee for assignee's counsel, $50; assignee's commission on the cash reported, 5%; auctioneer's commission, 2% on $3260 sales. Give the assignee's and auctioneer's commissions, the *per cent.* of dividend (without a fraction) that can be paid to the general creditors, and the balance of cash that will remain in the hands of the assignee after paying costs, fees, commissions, preferred claims, and dividend to general creditors.

27. The final report of the assignee in the above case, shows that all the property of the assignor has been reduced to cash; that there is in his hands $10680. The unpaid costs are $18.60; assignee's commission on the money reported, less the balance on hand at last report, 5%; assignee's attorney's fee, $200; sundry expenses of the trust, $196.92. What is the assignee's commission at this settlement, and how much will the general creditors receive on the dollar?

Answers: $65.20, $432.50, 18 %, $394.40, $412.78, 28 %, $514.28, Lose $6500, $1048.05.

XXII. TIME TABLE

FOR COMPUTING INTEREST AND AVERAGE.

Number of days from 1st of January to any other day of the year. In leap-years, add 1 to the days after 28th of February.

Day of Mo.	January	February	March	April	May	June	Day of Mo.	July	August	September	October	November	December	Day of Mo.
1	0	31	59	90	120	151	1	181	212	243	273	304	334	1
2	1	32	60	91	121	152	2	182	213	244	274	305	335	2
3	2	33	61	92	122	153	3	183	214	245	275	306	336	3
4	3	34	62	93	123	154	4	184	215	246	276	307	337	4
5	4	35	63	94	124	155	5	185	216	247	277	308	338	5
6	5	36	64	95	125	156	6	186	217	248	278	309	339	6
7	6	37	65	96	126	157	7	187	218	249	279	310	340	7
8	7	38	66	97	127	158	8	188	219	250	280	311	341	8
9	8	39	67	98	128	159	9	189	220	251	281	312	342	9
10	9	40	68	99	129	160	10	190	221	252	282	313	343	10
11	10	41	69	100	130	161	11	191	222	253	283	314	344	11
12	11	42	70	101	131	162	12	192	223	254	284	315	345	12
13	12	43	71	102	132	163	13	193	224	255	285	316	346	13
14	13	44	72	103	133	164	14	194	225	256	286	317	347	14
15	14	45	73	104	134	165	15	195	226	257	287	318	348	15
16	15	46	74	105	135	166	16	196	227	258	288	319	349	16
17	16	47	75	106	136	167	17	197	228	259	289	320	350	17
18	17	48	76	107	137	168	18	198	229	260	290	321	351	18
19	18	49	77	108	138	169	19	199	230	261	291	322	352	19
20	19	50	78	109	139	170	20	200	231	262	292	323	353	20
21	20	51	79	110	140	171	21	201	232	263	293	324	354	21
22	21	52	80	111	141	172	22	202	233	264	294	325	355	22
23	22	53	81	112	142	173	23	203	234	265	295	326	356	23
24	23	54	82	113	143	174	24	204	235	266	296	327	357	24
25	24	55	83	114	144	175	25	205	236	267	297	328	358	25
26	25	56	84	115	145	176	26	206	237	268	298	329	359	26
27	26	57	85	116	146	177	27	207	238	269	299	330	360	27
28	27	58	86	117	147	178	28	208	239	270	300	331	361	28
29	28		87	118	148	179	29	209	240	271	301	332	362	29
30	29		88	119	149	180	30	210	241	272	302	333	363	30
31	30		89		150		31	211	242		303		364	31

FOR COMPUTING INTEREST AND AVERAGE.

Number of days from 1st of July to any other day of the year. In leap-years, add 1 to the days after 28th of February.

Day of Mo....	July.....	August.....	September...	October.....	November...	December...	Day of Mo....	January.....	February....	March.....	April.....	May.....	June.....	Day of Mo....
1	0	31	62	92	123	153	1	184	215	243	274	304	335	1
2	1	32	63	93	124	154	2	185	216	244	275	305	336	2
3	2	33	64	94	125	155	3	186	217	245	276	306	337	3
4	3	34	65	95	126	156	4	187	218	246	277	307	338	4
5	4	35	66	96	127	157	5	188	219	247	278	308	339	5
6	5	36	67	97	128	158	6	189	220	248	279	309	340	6
7	6	37	68	98	129	159	7	190	221	249	280	310	341	7
8	7	38	69	99	130	160	8	191	222	250	281	311	342	8
9	8	39	70	100	131	161	9	192	223	251	282	312	343	9
10	9	40	71	101	132	162	10	193	224	252	283	313	344	10
11	10	41	72	102	133	163	11	194	225	253	284	314	345	11
12	11	42	73	103	134	164	12	195	226	254	285	315	346	12
13	12	43	74	104	135	165	13	196	227	255	286	316	347	13
14	13	44	75	105	136	166	14	197	228	256	287	317	348	14
15	14	45	76	106	137	167	15	198	229	257	288	318	349	15
16	15	46	77	107	138	168	16	199	230	258	289	319	350	16
17	16	47	78	108	139	169	17	200	231	259	290	320	351	17
18	17	48	79	109	140	170	18	201	232	260	291	321	352	18
19	18	49	80	110	141	171	19	202	233	261	292	322	353	19
20	19	50	81	111	142	172	20	203	234	262	293	323	354	20
21	20	51	82	112	143	173	21	204	235	263	294	324	355	21
22	21	52	83	113	144	174	22	205	236	264	295	325	356	22
23	22	53	84	114	145	175	23	206	237	265	296	326	357	23
24	23	54	85	115	146	176	24	207	238	266	297	327	358	24
25	24	55	86	116	147	177	25	208	239	267	298	328	359	25
26	25	56	87	117	148	178	26	209	240	268	299	329	360	26
27	26	57	88	118	149	179	27	210	241	269	300	330	361	27
28	27	58	89	119	150	180	28	211	242	270	301	331	362	28
29	28	59	90	120	151	181	29	212		271	302	332	363	29
30	29	60	91	121	152	182	30	213		272	303	333	364	30
31	30	61		122		183	31	214		273		334		31

FOR COMPUTING INTEREST AND AVERAGE.

Number of days from January 1st to any day of the previous year. In leap-years add one day before February 28th.

Day of Month.	January	February	March	April	May	June	Day of Month.	July	August	September	October	November	December	Day of Month.
1	365	334	306	275	245	214	1	184	153	122	92	61	31	1
2	364	333	305	274	244	213	2	183	152	121	91	60	30	2
3	363	332	304	273	243	212	3	182	151	120	90	59	29	3
4	362	331	303	272	242	211	4	181	150	119	89	58	28	4
5	361	330	302	271	241	210	5	180	149	118	88	57	27	5
6	360	329	301	270	240	209	6	179	148	117	87	56	26	6
7	359	328	300	269	239	208	7	178	147	116	86	55	25	7
8	358	327	299	268	238	207	8	177	146	115	85	54	24	8
9	357	326	298	267	237	206	9	176	145	114	84	53	23	9
10	356	325	297	266	236	205	10	175	144	113	83	52	22	10
11	355	324	296	265	235	204	11	174	143	112	82	51	21	11
12	354	323	295	264	234	203	12	173	142	111	81	50	20	12
13	353	322	294	263	233	202	13	172	141	110	80	49	19	13
14	352	321	293	262	232	201	14	171	140	109	79	48	18	14
15	351	320	292	261	231	200	15	170	139	108	78	47	17	15
16	350	319	291	260	230	199	16	169	138	107	77	46	16	16
17	349	318	290	259	229	198	17	168	137	106	76	45	15	17
18	348	317	289	258	228	197	18	167	136	105	75	44	14	18
19	347	316	288	257	227	196	19	166	135	104	74	43	13	19
20	346	315	287	256	226	195	20	165	134	103	73	42	12	20
21	345	314	286	255	225	194	21	164	133	102	72	41	11	21
22	344	313	285	254	224	193	22	163	132	101	71	40	10	22
23	343	312	284	253	223	192	23	162	131	100	70	39	9	23
24	342	311	283	252	222	191	24	161	130	99	69	38	8	24
25	341	310	282	251	221	190	25	160	129	98	68	37	7	25
26	340	309	281	250	220	189	26	159	128	97	67	36	6	26
27	339	308	280	249	219	188	27	158	127	96	66	35	5	27
28	338	307	279	248	218	187	28	157	126	95	65	34	4	28
29	337		278	247	217	186	29	156	125	94	64	33	3	29
30	336		277	246	216	185	30	155	124	93	63	32	2	30
31	335		276		215		31	154	123		62		1	31

FOR COMPUTING INTEREST AND AVERAGE.

Number of days from July 1st to any day in the past year.

Day of Month.	July.....	August.....	September....	October.....	November.....	December.....	Day of Month.	January	February.....	March.....	April.....	May.....	June	Day of Month.
1	365	334	303	273	242	212	1	181	150	122	91	61	30	1
2	364	333	302	272	241	211	2	180	149	121	90	60	29	2
3	363	332	301	271	240	210	3	179	148	120	89	59	28	3
4	362	331	300	270	239	209	4	178	147	119	88	58	27	4
5	361	330	299	269	238	208	5	177	146	118	87	57	26	5
6	360	329	298	268	237	207	6	176	145	117	86	56	25	6
7	359	328	297	267	236	206	7	175	144	116	85	55	24	7
8	358	327	296	266	235	205	8	174	143	115	84	54	23	8
9	357	326	295	265	234	204	9	173	142	114	83	53	22	9
10	356	325	294	264	233	203	10	172	141	113	82	52	21	10
11	355	324	293	263	232	202	11	171	140	112	81	51	20	11
12	354	323	292	262	231	201	12	170	139	111	80	50	19	12
13	353	322	291	261	230	200	13	169	138	110	79	49	18	13
14	352	321	290	260	229	199	14	168	137	109	78	48	17	14
15	351	320	289	259	228	198	15	167	136	108	77	47	16	15
16	350	319	288	258	227	197	16	166	135	107	76	46	15	16
17	349	318	287	257	226	196	17	165	134	106	75	45	14	17
18	348	317	286	256	225	195	18	164	133	105	74	44	13	18
19	347	316	285	255	224	194	19	163	132	104	73	43	12	19
20	346	315	284	254	223	193	20	162	131	103	72	42	11	20
21	345	314	283	253	222	192	21	161	130	102	71	41	10	21
22	344	313	282	252	221	191	22	160	129	101	70	40	9	22
23	343	312	281	251	220	190	23	159	128	100	69	39	8	23
24	342	311	280	250	219	189	24	158	127	99	68	38	7	24
25	341	310	279	249	218	188	25	157	126	98	67	37	6	25
26	340	309	278	248	217	187	26	156	125	97	66	36	5	26
27	339	308	277	247	216	186	27	155	124	96	65	35	4	27
28	338	307	276	246	215	185	28	154	123	95	64	34	3	28
29	337	306	275	245	214	184	29	153		94	63	33	2	29
30	336	305	274	244	213	183	30	152		93	62	32	1	30
31	335	304		243		182	31	151		92		31		31

XXIII. AVERAGE.

ART. 1. When several payments have to be made at one time, or when one bill has to be paid with several notes of different lengths of time, an average has to be sought, the process of finding which is called AVERAGE, or EQUATION OF PAYMENTS.

1. A merchant sells a bill of goods amounting to $4000, to be paid as follows: $400 in 30 days, $600 in 60 days, $1000 in 90 days, and the balance in 4 mos., or 120 days, what would be a mean or average time of payment for the whole?

A credit of	$400	for	30 ds.,	is the same as a credit on	$1	for	12000 ds.
"	600	"	60 "	"	"	"	"	1	"	36000	"
"	1000	"	90 "	"	"	"	"	1	"	90000	"
"	2000	"	120 "	"	"	"	"	1	"	240000	"
	4000									378000	

On one dollar there is a credit for 378000 days.

On $4000, there is a credit for $\frac{378000}{4000} = 94\frac{1}{2}$ days.

That is, the $4000 might be paid in $94\frac{1}{2}$ days, or on the 95th day, without either party sustaining loss by interest.

2. A merchant sells goods to the amount of $1700, $500 payable in 60 days, $300 payable in 90 days, and $900 payable in 30 days, what is the average time of payment of the whole?

3. Sold a bill of goods, amounting to $700, $\frac{1}{4}$ of which is payable in 90 days, $\frac{1}{4}$ in 4 mos., and $\frac{1}{2}$ in 6 mos.; required the average time of payment.

Answers: 49 days, 143 days.

ART. 2. *To find the average date of purchase.*

1. Purchased goods as follows, what was the average date of purchase?

Dec. 31, a bill of $300, Jan. 3, a bill of $100, Jan. 9, a bill of $200, Jan. 18, a bill of $800, Jan. 23, a bill of $500. *Ans.* Jan. 15th.

REMARK.—If the amounts above were equal, and the intervals also equal, the average date of purchase would be on Jan. 9th; because it is midway between the first and last dates.

EXPL.—The first was due at the time of purchase; the second, 3 days after; the third, 9 days after, etc.

$$
\begin{array}{rl}
300\times & 0= \\
100\times & 3= 300 \\
200\times & 9= 1800 \\
800\times & 18=14400 \\
500\times & 23=11500 \\
\hline
19\text{ø}\text{ø} &)28\text{ø}\text{ø} \\
\hline
 & 14\tfrac{14}{19},
\end{array}
$$

or 15 days after Dec. 31, the date of first purchase, which brings the time up to Jan. 15th.

If these debts had been contracted on a credit of three months, a note dated Jan. 15 would be given to settle the bill.

2. What is the average date of purchase of the following?

Jan. 1, Mdse., $360, Feb. 6, Mdse., $325, March 8, Mdse., $180, April 3, Mdse., $65, May 13, Mdse., $275, June 8, Mdse., $70.

Ans. Jan. 15, March 3, Feb. 26.

3. The following goods were sold on a credit of 90 days:

NEW YORK, Apr. 3, 1876.

Mr. James Callen, Bought of Robt. Boggs.

Jan.	1,	Invoice of	Coffee,	$1000.00
"	6,	"	" Sugar,	3500.00
Mar.	9,	"	" Sunds.,	9734.00
"	13,	"	"	976.50
Apr.	3,	"	"	1037.00
				$16247.50

Required the average date of purchase, or date of note.

4. PHILADELPHIA, Dec. 3, 1859.

Mr. Henry Higgins,

Bought of James Niel,

Sept.	3,	Invoice of	Calicoes,	$3150.00
"	19,	"	" Muslins,	1174.00
"	20,	"	" Silks,	3500.00
Oct.	19,	"	" Sundries,	1743.00
				$9567.00

Required the *date of maturity* of a 3 months' note, grace included.

Find the equated time of payment for the following:

5.		6.	
Apr. 3,	$167.25*	May 7,	$674.40
" 9,	374.00	Jun. 7,	168.37
" 19,	176.00	" 10,	370.20
" 20,	371.00	" 15,	167.00
" 25,	197.87	" 19,	679.60
" 30,	300.00	July 23,	679.45
May 9,	150.57	Aug. 18,	993.18
" 23,	720.18	" 19,	875.57

Answers: Feb. 22, April 30, July 11, Dec. 23.

* When the cents are under 50, reject them, otherwise add a dollar to the dollars.

ART. 3. *When goods are purchased at different dates and on different lengths of credit.*

1. Purchased the following bills of merchandise; required the average date of maturity, or the equated time of payment for all:

Apr.	3,	a bill of	$250	on	3	months'	credit.
"	9,	" "	157	"	6	"	"
May	7,	" "	250	"	4	"	"
Jun.	9,	" "	320	"	2	"	"

If we substitute the *date of maturity* of each of these bills for the *date of purchase*, and arrange them in the order of time, we shall have a problem in all respects similar to those under last Art.

The first bill falls due July 3d,* the second Oct. 9th, the third Sept. 7th, the fourth Aug. 9th. Arranged in the order of time, they appear thus:

July 3, $250
Aug. 9, $320 \times 37 = 11840$
Sept. 7, $250 \times 66 = 16500$
Oct. 9, $157 \times 98 = 15386$

977 | 43726 (44 $\frac{738}{977}$ or 45 days.
3908
4646
3908
$\frac{738}{977}$

Hence the date of maturity is 45 days after July 3d, or on Aug. 17th; from which time till the day of settlement, interest is due on the whole amount.

* Days of grace are not allowed on invoices.

When some of the purchases are at cash price.

2. What is the equated time of payment for the following: Jan. 1, $600 on 3 mos., Feb. 3, $670 at cash price, Mar. 3, $950 on 6 mos., May 3, $550 for cash?

The first payment falls due on April 1st, the second, being for cash, was due at the time of purchase, Feb. 3d, the third Sept. 3d, and the last May 3d. Arranging the dates and amounts in the order of time, as before, we have

Feb. 3,	$670 × 00	
Apr. 1,	600 × 57	34200
May 3,	550 × 89	48950
Sept. 3,	950 × 212	201400
	2770	284550 (103 days nearly, or 17*th May*.

NOTE.—The amount due on Sept. 3d, is $2770 plus the interest on that amount, from May 17th.

From May 17th, to Sept. 3d, = 109 days. Interest on $2770 for 109 days = $50.32, which, added to $2770 = $2820.32.

Find the equated time of payment of the following;

3. July 1, $675 on 3 mos.; 13th, $619.54 on 2 mos.; 19th, $147.67 at cash rates; 23d, $678.44 on 5 mos.

4. Sept. 3d, $937.15 on 30 days' credit; 9th, $897.78 on 90 days' credit; 17th, $619.18 at cash prices; Oct. 3d, $777 on 60 days.

Required the amount due on each of the following on July 1st.

5. Jan. 9th, $678.44 @ 60 days; 20th, $419.88 at cash price; 29th, $789.14 at 3 mos.

Answers: October 17, Nov. 2, May 17, $1919.85, $4708.49.

6. April 9th, $1678 on 3 mos.; June 18, $1000 at cash prices; 21st, $879.55 on 60 days; 23d, $371.19 cash; 20th, $785.25 cash. *Ans.* $4708.49.

ART. 4. APPLICATION TO ACCOUNT SALES.

An Account Sales is a detailed statement of goods sold by a commission merchant, on account of the party who sent them.

The person or party who sends goods to another to be sold for himself, is called the *consignor*, the person to whom they are sent, the *consignee*, and the goods sent, the *consignment*.

COMMISSION HOUSE OF STRAIGHT, DEMING & CO.

Shipment 18. *No.* 7828.

Sales for account of MESSRS. GAFF & BALDWIN.

By sundries,

Jun. 4, T. B. Colgan & Co. @ 60 days, 8 hhds Sugar.

1095	1020				
1100	1120				
1080	1240	8965			
1200	1110	896	8069	@ $7\frac{7}{16}$	$600.13

Jun. 6, G. Newton & Co., @ 60 days, 10 hhds. Sugar.

1080	1040				
1090	1340				
1120	1020				
1240	1100	11440			
1200	1210	1144	10296	@ $6\frac{7}{8}$	$707.85

Sales for account of Messrs. Gaff & Baldwin—continued.

Jun. 10, B. Vilgers & Co., @ 60 days, 20 hhds. Sugar.

1060	1240				
1210	1110				
1180	1005				
1055	1285				
1240	1100				
1185	1210				
1300	1325				
1010	1140				
1120	1205	23185			
1205	1000	2318	20867	@6 7/16	1343.31

$2651.29

CHARGES.

Jun. 1,	P'd cash st'mr Landis for fr't,	$ 87.18	
" "	Drayage,	9.50	
" 14,	Insurance,	4.63	
" "	Storage,	9.50	
" "	Commission and guarantee,	132.56	243.37
	Net proc'ds due by equat'n, Aug. 13,		$2407.92

E. O. E.

CINCINNATI, June 14th, 1876.

STRAIGHT, DEMING & Co.,
per F. Jelke.

Recapitulation of answers of Art. 1–4 inclusive: Jan. 15, Feb. 22. April 30, May 17, July 11, Aug. 7, Aug. 17, Oct. 17, Nov. 2, Dec. 23. 45 days, 49 days, 95 days, 143 days, $1919.85, $4708.49.

ART. 5. When payments are made before a note or bill is due, to find how long after maturity it should run, to balance the interest on the advanced payments.

1. A merchant holds a note of $500 at 6 months. Three months before it is due, he receives $100, and one month before it is due, he receives $300, how long should he allow the balance to run, to equal the interest on the advance?

The int. on $100 for 3 mos.=int. on $1 for 300 mos.
" " 300 " 1 " = " " 1 " 300 "
600 mos.

Hence, the interest on the advanced payments is equal to the interest of $1 for 600 months; that is, a balance of $1 should have run 600 months, but the balance due on the note is $100; therefore, it should run $\frac{600}{100}$ months = 6 mos.

Proof.—The int. of the $100 *that is to run* 6 *mos.*=$3.

The int. of $100 (the first pay't,) for the 3 mos.=$1.50
" " 300 (the sec. pay't,) " " 1 " = 1.50
Total interest on advance=$3.00

2. A note of $600 given on Jan. 3, 1876, payable in 6 months. 4 months before it was due, $100 was paid on it, and 3 months before it was due, $200 was paid; how long in equity should the balance run?

3. A merchant owes $700 due 8 months from the time he contracted the bill; 5 months afterward, he pays $200, and 2 months after that, $300, how long should the balance remain unpaid?

Answers: 3 mo. 10 days, 4 mo. 15 days, 4 mo. 12 days.

4. If I borrow \$600 from A at one time, and \$500 at another, each for 4 months, how long should I lend him \$1000 to return the favor?

Ans. 4 months 12 days.

APPLICATION TO ACCOUNTS CURRENT.

ART. **6.** In applying equation to accounts current, both the debit and credit sides of an account have to be averaged, for which reason, the operation is usually called Compound Equation.

DR. J. DOE *in acc't with* R. ROE. CR.

			\$	c.				\$	c.
1876.					1876.				
July	3	To Mdse.,	1000	00	Aug.	1	By Cash,	500	00
"	7	" "	500	00	"	12	" "	500	00
Aug.	18	" "	250	00					

Required the amount due on Aug. 18th, supposing the sales to have been made at cash rates.

We first find the average of the debit side, because it contains the earlier date. The credit side is averaged from the same date.

July 3, 1000× 0
" 7, 500× 4= 2000
Aug. 18, 250×46=11500

1750)13500(7$\frac{25}{175}$, or 8 days from July 3.
1225
125

Aug. 1, 500×29=14500
" 12, 500×40=20000

1000)34500, or 35 days from July 3.

The debit side averages July 11th, that being the *mean time of purchase.* The credit side shows that the payments average Aug. 7th; hence the *whole debt* was due from July 11th to Aug. 7th (27 days), for which, interest should be charged; and the balance, $750, was due from Aug. 7th to Aug. 18th, for which interest should also be charged.

Interest on	$1750	for 27	days	=$7.88
"	" 750	" 11	"	1.38
				$9.26

Which added to the debit side of Doe's account, gives a balance against him of $759.26.

ART. 7. APPLIED TO STATEMENTS.

When settlement is made by note, an average date of payment is found by dividing the difference of the products by the balance of debt, and counting backward or forward from the assumed date.

34500—13500=21000 days, which divided by 750 gives 28 days, to be counted *backward* from July 3d, giving *June 5th.*

EXPLANATION.—In the preceding operation we assumed that the whole debt was due on July 3d, making the sales subject to a discount equal to that on $1 for 13500 days (favor of buyer), and the payments to a discount of 34500 days (against buyer), showing a balance against him of 21000 days on $1, or 28 days on his debt of $750.

To be against him we must count backward, for to count forward would give him longer time to pay the note.

PROOF.—Interest on $750 from June 5th to Aug. 18th (74 days), $9.25.

The difference of 1 cent is caused by the fraction of a day reckoned as a full day, on preceding page.

MESSRS. GAFF & BALDWIN, in acc't current and Interest acc't., with STRAIGHT, DEMING & Co., to June 14, 1876.

DATE.		DESCRIPTION.	TIME. m.	d.	INT. $	c.	AMOUNT. $	c.
1877.								
Jan.	4	To Mdse. as per bill rendered,.....	5	11	11	40	425	00
"	10	" " " " "	5	5	20	93	810	00
Feb.	8	" Your draft on us at 60 days sight, favor, H. King & Co.,..	2	3			500	00
"	20	" Paid your draft on us at sight,	3	24			1000	00
Mar.	2	" Mdse. as per bill rendered,....	3	14			125	00
"	10	" Exchange on $500 at 1 per cent., ...					5	00
Apr	18	" Mdse. as per bill rendered,.....					434	00
"	20	" " " " "					504	00
"	25	" Our draft on G. Wright at 3 days, your favor,.....					150	00
"	26	" Mdse. as per bill rendered, ...					1101	00
May	4	" N. Davis & Co., note at 30 days, our favor,.....					500	00
"	8	" Discount on $300 uncurrent money, at 1⅔ per cent., ...					5	00
"	25	" Mdse. as per bill rendered,....					275	00
June	5	" Net proceeds, sales 100 bbls. flour,.....					495	00
"	8	" Mdse. as per bill rendered,.....					4151	00
"	10	" Amount paid for your telegram, ...					5	00
		" *Interest, in our favor,*					30	03
					84	17	10515	03
June	14	*To Balance,* $3976.03.						

DATE.		DESCRIPTION.	TIME. m.	d.	INT. $	c.	AMOUNT. $	c.
1877.								
Jan.	20	By Draft on New York, ½ per cent. premium, ...	4	25	24	29	1005	00
"	28	" Cash,.....	4	17			100	00
Mar.	18	" Your sight draft on Moore & Adams, our favor, ...					800	00
"	29	" Draft on New York, ¾ per cent. premium,.					403	00
Apr.	10	" Net proceeds, sale 50 bbls. molasses, ...					624	00
May	8	" Cash, ...					200	00
"	29	" "					1000	00
June	14	" Net proceeds, sale 38 hhds. sugar, ...					2407	00
		" *Balance of Interest,*.....			30	03		
		" *Balance, our favor,*.....					3976	03
					84	17	10515	03

Dr.			A. Mills.					Cr.	
1876.			$	c.	1876.			$	c.
Feb.	18	To Mdse,	600	00	Apr.	3	By Cash,	500	00
"	28	" "	700	00	"	12	" "	400	00
Mar.	17	" "	800	00					

Required the amount due July 1st; also, the equated time at which a note would have been dated.

ART. 8. APPLICATION TO STORAGE.

Storage is usually charged for by the box, barrel, etc., for the month.

		bbls.	days. products.
Received July	1,	200	$\times 3 = 600$
Received "	4,	300	$500 \times 2 = 1000$
Delivered "	6,		100
Balance "		400	$\times 4 = 1600$
Received "	10,	300	$700 \times 6 = 4200$
Received "	16,	200	$900 \times 2 = 1800$
Delivered "	18,		600
Balance "		300	$\times 3 = 900$
Received "	21,	500	
Balance on hand		800	

$30)\overline{10100}$

$336\frac{2}{3}$

The products divided by the number of days in a month, give the number of barrels chargeable for a month.

Answers: $1229.20, 337 days.

ART. 9. GENERAL EXERCISES.

1. A bill of $1000 is to be paid in five equal installments, at 3, 4, 5, 6, and 7 months, what time should be allowed, if the individual will pay it all at once?

2. The following bills of goods have been purchased at different periods; required the average time of payment, allowing 30 days credit on each:

$1347 on Jan. 1, $167 on Feb. 3, $1794 on Feb. 8, $6783 on Feb. 10, $1076 on Feb. 19, $319, on Mar. 6, and $1674 on April 9.

3. What should be the date of a 60 day note for the following bills: $168 purchased on April 6, $3196 Apr. 9, $1668 May 3, $6847 June 1?

4. When would a 90 day note fall due, given for the following bills: $673 on June 3, $710 on July 6, $415 on July 9, $678 on Aug. 3?

5. What is the storage of the following account, closed Aug. 4, at 5 cents per bbl. per month?

Apr.	3,	received	167	July	1,	delivered	200
"	9,	"	145	"	3,	"	150
"	17,	"	450	"	7,	"	190
May	18,	"	198				

6. At 5 cents per barrel, what will the storage of the following amount to on Oct. 24?

July	9,	received	167	Oct.	1	delivered	125
Aug.	5,	"	378	"	3	"	500
"	9,	"	780	"	19	"	450
"	31	"	178				

(Answers on next page.)

7. How much was due on the following account on Sept. 13, 1859; bills sold on 60 days time:*

Dec. 8, 1858, $1676, Jan. 9, 1859, $1675, Feb. 14, paid $500, Apr. 16, paid $1000.

No credit being allowed on the following bills, required the balance due at date of last purchase or payment.

	Debit side.	Credit side.
8.	Apr. 9, $600	May 13, $700
	" 9, 500	" 15, 100
	" 15, 700	
	Balance due May 15,	
9.	June 3, $365	June 20, $300
	" 19, 784	" 29, 500
	July 18, 594	" 30, 100
	Balance due July 18,	
	1858.	1859.
10.	Dec. 7, $1874	Jan. 7, $1000
	1859.	
	Jan 3, 1678	April 14, 900
	" 21, 712.53	

What will be average date of maturity of the following, and what will be the balance due Mar. 7?

11. Jan. 1, $1673 on 3 mos. credit, Mar. 4, $1000
" 9, 740 on 2 " " " 7, 500
" 29, 500 on 4 " "

Answers: March 16th, May 13th, Oct. 6th, 5 mo., $141.80, $168.96, $846.23, $1009.93, $1921.65, $2426.41, May 8th, $1398.40.

* Some reckon every *fraction* of a day as a whole day; others only fractions that are over ½.

METHODS OF AVERAGING.

There are two methods of averaging known to accountants, viz.: the *Interest* Method and the *Product* Method. By the first interest is fully computed upon every item up to the day of settlement. By the Product Method the time is usually reckoned from the date of the first item, and multiplied into the various amounts. From this arises discount.

A new method, now introduced for the first time, is a modification of the latter, which we shall call the *Interest-Product Method.* By it the time is reckoned from the last item of the account, or the day of settlement, which results in giving the interest direct without further calculation, except to divide by 6000.

The Time Tables on pages 114 and 115 are used for the Discount-Product Method, and those on pages 116 and 117 for the Interest-Product Method.

Dr. 12. C. A. WALWORTH. *Cr.*

1876.			1876.		
Jan. 1,	To Mdse,	$300.00	Feb. 9,	By Mdse,	$200.00
Mar. 3,	" "	500.00	23,	" "	100.00
	300× 0			200×39=7800	
	500=61=30500	*Dr. products.*		100×53=5300	
	—	13100 *Cr. products.*		—	—
	800	—		300	13100
	300	5ØØ)174ØØ			
	—				
Bal. due	500	34⅘ *or* 35 *days from Jan.* 1 *or Feb.* 5.			

EXPLANATION.—Assuming both purchases and sales to be due on January 1, W. would be entitled to a discount on his purchases equal to that on $1 for 30500 days, and I would be entitled to a discount on my purchases equal to that on $1 for 13100 days, making a difference of 17400 days in W.'s favor on the balance, $500. The discount on $1 for 17400 days is equal to the discount on $500 for 17400 days÷500=34⅘ days from January 1, to be reckoned forward, which will give February 5. Had the discount been against him, it would have shown that the balance was past due on January 1, which

would indicate that the time would be counted backward from that date. Should the balance, instead of the equated time, be required, the difference between the Dr. and Cr. products may be divided by 6000 to find the discount on the balance to the day of settlement at the rate of 6 per cent per annum. Any amount being 100 per cent of itself, when multiplied by a number of days, will be 100 per cent per day. Six per cent per annum being the 1-6000th part of 100 per cent per day; hence by dividing the difference as aforesaid, we have the discount on the balance up to the assumed day of settlement at the rate named. 17400 : 6000 = $2.90 discount. This process we shall name the *Discount-Product Method.*

13. J. C. HINTZ.

Dr.			*Cr.*
1869.		1869.	
July 3, To Mdse.	$1000.00	Aug. 1, By Cash,	$500.00
7, " "	500.00	" 13, " "	500.00
Au. 18, " "	250.00		

Assuming the day of settlement to be July 1, we have.

1000 × 2 =	2000	*Days.*	500 × 31 =	15500	*Days.*
500 × 6 =	3000	"	500 × 43 =	21500	"
250 × 48 =	12000	"			
			1000	37000	*Cr. products.*
1750	17000	"		17000	*Dr. products.*
1000					
				20000	*Difference of do.*

$750 *Balance due.*

75)2000(26 *ds.*
150
500
450
50

EXPLANATION.—The sum of the credit products being greater than that of the debit products, shows that the discount is in my favor. Hence, in order to settle on the assumed date, his payments would be at a discount of 27 days for the balance; but as it would be impossible to settle on a past date, we will have to charge him interest from 27 days *prior to July* 1 (June 4) to the real day of settlement, whatever that may be, or else take his note, dated June 4, bearing interest from date. Say the date of the settlement is January 1, 1870. Interest on $750 from June 4, 1869, to January 1, 1870 (211 days) is $26.37, which, added to $750=$776.37, balance due with interest, January 1, 1870. This result, however, may be ascertained directly by the *Interest-Product Method,* as shown by the same example on page 134.

INTEREST-PRODUCT METHOD.

ART. **10.** To ascertain the balance due on the day of settlement the *Interest-Product Method* may be employed. By it the actual date of settlement is used instead of an assumed date, as by the Discount-Product Method.

14. *Find the balance due January* 1*st*, 1870, *of the account of J. C. Hintz, page* 133, *by the Interest-Product Method.*

1869			1869		
July 3...	$1000×182=	182000	Aug. 1.....	$500×153=	76500
July 7...	500×178=	89000	Aug. 13.....	500×141=	70500
Aug. 18...	250×136=	34000			
				Cr. Interest-Product,	147000
Dr. Interest-Product,		305000			
Cr. " "		147000			
		60)1580.00			
		$ 26.33	Interest due January 1, 1870.		
		750.00	Balance of account.		
		$776.33	Balance, including interest.		

EXPLANATION.—A purchase made on July 3—terms cash—should pay interest up to the date of settlement. That being, in this case, Jan. 1, 1870 the debtor is to be charged with interest up to that date, viz.: for 182 days. The interest on $1000 for 182 days is equal to the interest on $182,000 for one day, (or on $1 for 182000 days.) The same with every item on the Dr. side, making the Dr. Interest-Product equal to the interest on $305,000 for one day. The day of settlement being January 1, 1870, Hintz paid $500 on August 1, 1869, 153 days before the day of settlement, and $500 on August 13, 1869, 141 days before the day of settlement, and is therefore entitled to a credit for interest on the respective amounts. The Cr. Interest-Products, 147000, being equal to the interest on $147,000 for one day. The amount on which Hintz is entitled to interest for one day being $147,000, and the amount upon which he is chargeable with interest for one day being $305,000, he is chargeable with interest on $158,000 for one day more than the amount upon which he is entitled to receive interest. The interest on $158,000

for 60 days at the rate of 6 per cent per annum is $1580.00—that being 1 per cent of the amount—1 day is 1-60th of 60 days; hence, by dividing $1580.00 by 60 we have the interest for 1 day on the balance of the products, viz.: $26.33.

The advantage of this method over the Discount-Product Method is obvious. By this nothing is assumed. Interest is actually reckoned from the date of the first item of account to the day of settlement, and the accrued interest obtained without further calculation. Should the *time* be required it is readily found by dividing the difference of the product by the balance of the account. By the Interest-Product Method the interest is simply charged to the side of the account on which is the greater product, irrespective of the balance of the account; whereas, by the Discount-Product Method the time has to be reckoned backward or forward from the date obtained, and the interest computed and applied afterward.

15–16. *Find the balance due July 1st, 1877, by the Interest-Product Method, of the accounts of R. H. Langdale and Edw. Witte, page* 137.

17–18. *Find the balance due January 1st, 1877, by the same method, of the account of N. J. Jones, page* 137. *Also of C. A. Walworth's account, page* 132.

19. *July 1st, 1873.* R. H. Langdale's account is as follows:

1873			1873		
Jan'y 3,	To Mdse,	$300.00	Feb. 6,	By Cash,	$ 100.00
" 4,	" "	250.00	May 3,	" "	1,000.00
Feb. 9,	" "	730.00	June 3,	" "	160.00
May 6,	" "	800.00			

20. Charge him with the interest up to July 1, 1873, close his account and bring down the balance.

Charge him with goods bought since the day of last settlement, as follows: July 3, $500; Aug. 6, $100; Nov. 8, $100. Credit him with cash, paid as follows: Aug. 30, $800; Dec. 1, $600. Find balance due, with interest, January 1, 1874.

COMPOUND METHOD.

ART. 11. *Find the balance due, July* 1, 1874, *of the following:*

21. THEODORE BAUR.

1874			1874		
Jan. 12,	To Cash,	$ 500.00	Jan. 3,	By Mdse, 90 ds,	$1000.00
Feb. 5,	" Acc'p 60 ds....	120.00	June 30,	" " 60 ds,...	150.00
Mar. 8,	" Mdse,.....	500.00			$1150.00
June 3,	" Note, 3 mos...	100.00			
		$1220.00			

Arranged according to the dates when the items are due.

	Dr.		Int.	Dis.	Dr	Cr.
Jan.	12	$500×170			85000	
Apr.	9	120× 83			9960	
Mar.	8	500×115			57500	
Sept.	6	100×		67		6700
	Cr.		Dis.	Int.		
Apr.	3	$1000×		89		89000
Aug.	29	150×59			8850	
					161310	95700
					95700	
				60	656.10	
					$10.93	int.Dr

Dr. side, of ac.	$1220.00
Cr. " "	1150 00
Difference	70.00
Interest Dr.....	10.93
Balance due.....	$80.93

EXPLANATION.—It will be seen that both the Discount and Interest-Product Methods are employed in this solution. B is Dr. for the interest on the three first items of the Dr. side of his account, because it was due before the day of settlement, (July 1, '74), and he is credited with the discount on the fourth item, that being due (Sept. 6, '74) 67 days after the settlement. He is credited with the interest on the $1000 paid 89 days before the settlement, and charged with the discount on the $150 due (Aug. 29th,) 59 days after the day of settlement; he is therefore chargeable with the interest on $1613.10 for one day, less the interest on $957.00 for that time, viz: $10.93.

22–24. *Find the balance due January* 1, 1877, *by the Compound Method, of the accounts of Langdale and Witte, Ex.* 25 *and* 27, *also the balance due July* 1, 1876, *of the account of N. J. Jones, Ex.* 26.

25. R. H. Langdale.

1876				1876			
July	3,	To Mdse, 6 mos,	560.87	July	1,	By Balance,	127.15
	15,	" " 3 "	149.50		30,	" Accept. 60 ds,	300.00
Aug.	21,	" " 3 "	2000.00	Aug.	29,	" Cash,	460.00
Sep.	18,	" " Cash,	396.40	Oct.	20,	" Note, 3 mos,	1000.00
Oct.	15,	" " "	175.20		31,	" Cash,	100.00
	21,	" " "	425.16		31,	" Mdse Ret.,	250.00
	27,	" Cash,	100.00	Nov.	30,	" Cash,	450.00
	31,	" Mdse, 3 mos,	506.18		30,	" Balance,	2144.77
Nov.	28,	" " 4 "	197.45				
	30,	" " 4 "	321.16				
			4831.92				4831.92
Dec.	1,	To Bal.,	2144.77				

26. N. I. Jones.

1876				1876			
Jan.	1,	To Balance,	650.00	Jan.	8,	By Mdse, 3 mos,	160.00
Feb.	3,	" Cash,	245.00		15,	" " 6 "	710.87
	15,	" Note, 60 ds,	416.87	Feb.	14,	" " 2 "	910.14
Mar.	18,	" Accept. 30 ds,	1000.00	Apr.	16,	" " Cash,	1000.00
June	4,	" " 60 "	750.14	June	8,	" " 4 mos,	900.00
	16,	" Note, 3 mos,	987.64		15,	" " 4 "	2500.00
	30,	" Cash,	500.00		17,	" " 6 "	1215.00
	30,	" Mdse, abate,	200.00		30,	" Sunds,	700.00

27. Edward Witte.

1876				1876			
July	3,	To Balance,	1500.00	Aug.	3,	By Cash,	1000.00
	18,	" Mdse, 4 mos,	750.40	Sept.	7,	" Accpt, 60 ds,	500.00
Aug.	29,	" " 4 "	128.80	Nov.	5,	" " 60 "	750.00
Sep.	30,	" " 4 "	916.84	Dec.	14,	" Cash,	2000.00
Oct.	10,	" " 3 "	500.00				
	30,	" Cash,	675.14				
Nov.	18,	" Sunds,	561.18				

Answers: $3253.15; $2145.12; $776.33; $62.50; $2209.79; $807.63; $831.31; $3355,76; $848.11; $173.02; $3252.93; $3355.54: $527.53.

XXIV. RATIO.

ART. 1. *The relation that one number bears to another is called ratio.* The quotient arising from dividing one number by another of the same denomination, is the ratio between them.

And as two quotients can be obtained from comparing any two numbers, it follows that two ratios can also be obtained. The relation that 1 bears to 2 is $\frac{1}{2}$, and that which 2 bears to 1 is $\frac{2}{1}$.

The sign of ratio is the colon. The above ratios would be expressed thus: 1 : 2 and 2:1, and would be read *one is to two* and *two is to one.* French mathematicians divide the first term by the second; English the second by the first. The English method is used here, 3 : 6 will equal $\frac{6}{3}$ or 2, $\frac{1}{2} : \frac{3}{4} = \frac{\frac{3}{4}}{\frac{1}{2}} = \frac{3}{8}$.

ART. 2. Numbers or quantities of different denominations can not have a ratio. We can not compare 3 trees with 5 books. But if the numbers are capable of being reduced to the same denomination, they can be compared; for we can say 3 feet is to 2 inches, as it is the same as to say, 36 inches is to 2 inches.

Each number is called a term of the ratio. The first term is called *antecedent;* the second, *consequent.*

The value of a ratio depends upon the relative size of its terms.

Every ratio may be formed into a fraction by making the *consequent* the numerator and the *antecedent* the denominator, thus: $4:8 = \frac{8}{4} = 2$; and $8:4 = \frac{4}{8} = \frac{1}{2}$.

XXV. PROPORTION.

ART. 1. Two ratios may be equal to each other. 2 : 4, = 4 : 8.

2 bears the same relation to 4 that 4 does to 8.

ART. 2. When ratios are equal, the numbers or *terms* which compose them, are said to be in *proportion*, and are written thus: 2 : 4 : : 3 : 6, and read 2 *is to* 4 *as* 3 *is to* 6.

The first and last terms, as the 2 and 6, are called *extremes*, and the second and third the *means*.

ART. 3. The same ratio may arise by comparing 4 quantities, two of which are different in denomination from the other two.

tuns tuns $ $
3 : 6 :: 6 : 12. The ratio is 2.

ART. 4. If the extremes are multiplied together, the product will be equal to the product of the means.

$$3 \times 12 = 36$$
$$6 \times 6 = 36$$

Hence, when any 3 terms are given, we can readily find the fourth, by dividing the product by the odd term. If we had only the three first terms of the above proportion: that is, 3 tuns : 6 tuns : : 6 $, the fourth term would be found by dividing the product of 6×6, or 36 $ by 3, = 12 $, or the fourth term as above.

To apply this in practice, we have only to suppose the 3 tuns and 6 tuns to be coal, and the $6, the price of 3 tuns. Then 3 tuns is to 6 tuns, as the price of 3 tuns is to the price of 6 tuns.

2. What will 35 lbs. of sugar cost, if 7 lbs. cost 77 cents?

Statement.—7 : 35 : : 77 is to the price of 35.

lbs. lbs. cents.
7 : 35 : : 77
35
385
231
7)2695
$3.85

The above operation might have been abridged by cancellation.

$$\begin{array}{c|l} 7 & \$\$\ \ 5 \\ & 77\times 5=3.85\ \textit{Ans.} \end{array}$$

3. If 6 lbs. of tea cost $4.75, what will 15 lbs. cost?

4. Find the price of 37 horses, when 16 cost $1500?

5. What will 120 bbls. of potatoes cost, if 21 cost $67?

6. 42 bushels of beans cost $87.50, what will 3 bushels cost?

7. If 610 bushels of wheat cost $1670, what will be the price of 27?

8. If 3 bushels 1½ pecks of beans cost $12.50, what will 5 bushels 4 qts. cost?

bu. pks. bu. qts.
Statement.—3 1½ : 5 4 : : 12.50
4 4
13 20
8 8
108 164

$$\frac{164\times 12.50}{108}=\$18.98$$

The bushels, pks., and qts. had to be reduced to qts., in order to be of the same denomination. Art. 175.

Answers.—$3468.75, $11.87½, $6.25, $382.86, $73.92, $3.85.

9. If $27\frac{1}{2}$ lbs. of butter cost \$3.75, what will $16\frac{1}{2}$ lbs. cost?

10. Find the price of $12\frac{1}{2}$ dozen of chickens at 30 cents a pair.

11. The price of 21 tuns, 13 cwt., 3 qrs., and 15 lbs. of hemp is \$1680.55, what will 15 cwt. cost?

12. What will 54 lbs. $7\frac{1}{2}$ oz. of tea cost, if $15\frac{1}{4}$ lbs. cost \$8.47?

13. If $\frac{6}{7}$ of a ship cost \$7000, what will $\frac{9}{10}$ cost?

These fractions need not be reduced to the same denomination.

14. If 6 men do a piece of work in 7 days, how long will it take 5 men to do it?

In stating the previous question, we compared quantity with quantity and cost with cost. In this question there is nothing relating to cost, so we must adopt another method of making the statement. Perhaps the simplest is the following:

1. Inquire what is wanted, and put the term of that name to the right. In the question above, *time* is wanted, so we put the term of that name to the right.

2. Ascertain by reasoning, whether the quantity wanted will be greater or less than that given; if less, put the smaller of the two numbers for the middle term; if greater, put the greater of the two terms for the middle term. In the above question, we reason, that it will take 5 men a greater time than 6 men, so we put the greater of the two terms (6) in the second place.

men. men.

Statement.—5 : 6 :: 7 days. The *answer* is $8\frac{2}{5}$ days, or 8 days 4 hours, reckoning 10 hours to the day.

15. If 2 men plow a field in 3 days, how long will it take 3 men to do it?

Answers.—\$22.50, \$2.25, \$7350, \$58.09, \$30.25, 2.

16. If 26 yards of linen cost \$13.50, what will 10 yards cost?

17. If 3 coats can be made from $10\frac{1}{2}$ yards of cloth, how many can be made from $31\frac{1}{2}$ yards?

18. If the interest of \$750 for 3 years, 4 months, and 10 days be \$151.25 (360 days to the year), what is it for one year?

19. The interest of £100, from 3d of April to 25th February, is £6 5s. $9\frac{1}{3}$d., what is it per year?

20. A, B, and C are in partnership, and their gains for the year are \$6757, what is each man's share, suppose A invested \$1567, B \$2600, and C \$3798?

The sum of their investments is to each man's investment, as the total gains to each man's gain.

21. M invests \$6500, N \$1487, O \$3654; in three months, it is found that their gains are \$1678, what is each man's share?

22. A lends B \$1000 for 13 months 10 days, how long should B lend A \$8271, to return the favor.

23. If the shadow from a two foot rule be 6 in., what is the hight of the tree that throws a shadow of 75 feet?

24. If 7 men can build 21 perches of masonry in a day, how many men will it require to build 156 perches in a day.

25. The shadow of a tree being 87 feet; two nails being driven in the tree 3 feet apart, show a distance on the tree of $4\frac{1}{2}$ feet, what is the height of the tree?

26. The net profits of a concern being \$1860; A's interest is \$8750, and B's interest is \$8190; what is each man's gain?

Answers: 9, 7, 300, 58, 52, 49, \$5.19, \$5.90, \$4.50 \$45, \$910, \$936.89, \$950, \$1329.34, \$936.94.

XXVI. PARTNERSHIP.

ART. 1. When two or more persons associate together to carry on a business, they are said to be in partnership, and are called a *firm*, *house*, or *company*.

The funds, property, and merchandise furnished by partners for carrying on business, are called stock or capital, and the gains are called *dividends*.

The *liabilities* of a partnership or individual business are the debts, and the *assets* their available means, including the indebtedness of others to them.

An *inventory* is a list or statement of those things which constitute *assets*.

ART. 2. In keeping partnership accounts, each member of the firm should be credited with all that he brings into the concern or business, and be charged or debited with all he takes out, just the same as if he had no interest in it.

ART. 3. The calculations peculiar to partnership, relate to the division of property and profits.

1. A, B, and C have been in business one year, and find they have made a net gain of \$3476, which is to be divided as follows: A is to have $\frac{1}{2}$, B $\frac{1}{4}$, and C $\frac{1}{4}$; required the share of each.

$\$3\frac{4}{2}\frac{7}{}\frac{6}{}=\$1738$, A's share; $\$3\frac{4}{4}\frac{7}{}\frac{6}{}=\$869=$B's share; and \$869=C's share.

2. X, Y, and Z purchase a tract of land for \$2000; X giving \$600, Y \$900, and Z the remainder. In one year afterward, they sell it for \$5500; required each person's share of the proceeds.

3 A, B, and C invest \$2000 each. In 3 months their gross gains are \$2000; expenses, including \$250 for additional services of C, \$600, what will be each man's share of the gain?

4. D's interest in a partnership is $\frac{5}{16}$. What is his share of a gain to the firm of $3467.18?

5. E, F and G own a steamboat worth $35,000, their respective shares being $\frac{1}{2}$, $\frac{3}{16}$, $\frac{5}{16}$. What is the profit of each after deducting $1350 expenses, from $5450. gross profits?

ART. 4. *Interest on Investment.*

6. H, I and J invest in partnership $3400, $2900, and $1500 respectively, and at the end of the year find a net gain of $2600. Allowing 6% on their investments, what amount is each entitled to in proportion to the capital advanced?

Interest on Investment, $468.
Net profits, $2600, minus $468=$2132, to be divided pro rata.
$2132 : 7800=.2733 gain on $1.00.
.2733×3400=$929.33 H's share.
.2733×2900= 792.67 I's share.
.2733×1500= 410.00 J's share.

$2132.00, whole gain.

The respective shares of gain may be ascertained by the following proportion: The whole investment is to H's investment as the whole gain is to H's gain, thus:

$7800 : 3400 : : 2132 : H's gain.
39 17 × 2132

39)36244

$929.33

$7809 : 2900 : : 2132 : 792.67 (I's gain)
$7800 : 1500 : : 2132 : 410.00 (J's gain)

7. K, L and M engage in partnership with a capital of $15000, to share equally, K investing $10000, L $3000, and M $2000, and L and M to receive salaries of $1500 and $1200 a year respectively; allowing interest on their investments, which remained intact, what is each partner's share in a gross gain of $5700, expenses being $1950, exclusive of partner's salaries?

ART. 5. *Winding up a Losing Concern.*

8. R, S and T, equal partners, with a capital of $30000, finding that they are losing money, agree to dissolve, and on March 4, 1874, leave the property in the hands of T to settle. At this time the effects were cash on hand

$500, merchandise $17500, bills receivable $1300, and book accounts $1000, and their liabilities were bills payable to the amount of $2100. On September first T reports as follows: sales of merchandise $14000, on hand $1500, cash on hand $13000, notes $300, uncollected bills $750, liabilities extinguished; expenses $650. Of the remaining effects T proposes to take the Mdse at a discount of 50% if his partners take notes and accounts at the same rate. Failing to agree, they sell the goods at auction for $900, and T agrees to take the bills receivable in payment for the collection of the unsettled bills which he thus guarantees. Required the amount coming to each, allowing T 1% commission for settling the business?

ART. 6. *Average Capital.*

9. U, V and W engage in business January 1, 1874, investing respectively $3000, $2000, and $1000, and agreeing to share the gains and bear the losses in the ratio of their average capital. April first U draws $100, May first V draws $200, and July first W draws $100. Assuming the gains to be $1500 at the end of the year, what was each partner's share?

U.
Int. on $3000 for 12 mos. $180
" " 100 " 9 " 4.50
$175.50

V.
Int. on $2000 for 12 mos. $120.00
" " 200 " 8 " 8.00
$112.00

W.
Int. on $1000 for 12 mos. $60.00
" " 100 " 6 " 3.00
$57.00

$175.50
112.00
57.00
$344.50

344.50 : 175.50 : : 1500=U's share.
: 112.00 : : 1500=V's "
: 57.00 : : 1500=W's "

The question may also be solved by Products.

Answers: $768.75, $1281.25, $5281.00, $1083.49, $4834.50, $1738, $860, $487.66, $764.15, $248.19, $2050, $466.66, $1650, $1375, $2475, $4100, $50.

XXVII. JOINT STOCK COMPANIES.

ART. 1. A *Joint Stock Company* is a body of men associated together in a species of partnership, to carry out some heavy undertaking requiring the investment of more capital than individuals or partnership companies commonly possess. Joint stock companies are usually incorporated by act of legislature, with certain privileges. Railroads, canals, bridges, etc., are generally constructed by this species of combined interest, and many banking and insurance houses, scholastic institutions, etc., are owned and managed by joint stock companies.

When an association of this kind is to be formed, a few leading persons make an estimate of the probable amount of capital required, divide it into equal shares of from $10 to $100, or $500, according to the nature of the undertaking, and issue certificates of ownership for each share. These are called *certificates of stock*, and are transferable. Persons owning certificates, are called *stockholders*.

Joint stock companies are usually managed by a *president* and *board of directors*, elected for the purpose, by the stockholders.

When shares sell for the price named in the certificate, the stock is said to be at *par;* if above this value, they are said to be *above par;* if below it, *below par*.

Besides the stocks of companies, there are *government stocks*, which consist of bonds that have been issued by state officers, for the purpose of borrowing money. These draw interest at a specified rate.

In dividing the profits of joint stock companies, it has been found more convenient to declare the dividend as so much per cent.

1. What is the cost of 10 shares of railroad stock at 5 % below par, the original cost being $100 per share?

Find the cost of 10 shares, at $100 and deduct 5 %.

2. A banking institution declares a dividend of 18 % on a capital of $376198, what amount of money should a stockholder receive, who holds 5 shares valued at $200 each?

3. I hold 15 shares (each of $100) of stock, in gas works, which have declared a dividend of 20 %, how much am I entitled to after my gas bill of $20, is deducted?

4. How many shares of United States stocks at 2 % above par, can I buy for $1224, the original cost being $100 per share?

5. What amount of stock can I buy for $1687, if I am allowed 2 % commission on the amount invested?

The amount I am to receive is to be $\frac{2}{100}$, or $\frac{1}{50}$ of the amount of stock purchased—not $\frac{1}{50}$ of $1687, for that would be commission *on commission* and investment.

Let the amount to be invested be represented by $\frac{50}{50}$, and to this add $\frac{1}{50}=\frac{51}{50}$; then we discover that $1687 is $\frac{51}{50}$ of the amount to be invested. $1\frac{68}{51}7=33.078=\frac{1}{50}$, or my commission, which if we multiply by 50, will give us the amount to be spent, $1653.90.

To *prove* this, find the com. on $1653.90 at 2 %.

6. A broker receives $6785, which he is desired to invest in State stocks, how much should he invest, and allow himself $2\frac{1}{2}$ % on the investment?

7. What amount of stock can a broker buy for 16700, and allow himself $\frac{1}{4}$ % on the investment?

Answers: $180, $950, $280, $16658.35, $6619.49, 12, $1780.

XXVIII. COMPOUND NUMBERS.

The application of the fundamental rules to numbers of different denominations, *gallons, quarts,* and *pints; hundreds, quarters,* and *pounds,* etc., will, it is presumed, be sufficiently taught in the following examples:

BRITISH MONEY.

ART. 1. *To add compound numbers.*

What is the amount of the following sums of British money?

£	s.	d.
18	17	$4\frac{1}{2}$
19	6	$7\frac{1}{4}$
17	7	$8\frac{3}{4}$
55	11	$8\frac{1}{2}$

SOLUTION.—We first add the fractions, calling them all *farthings*, which makes 6 farthings; these we reduce to pence, by dividing them by 4. $\frac{6}{4}=1\frac{2}{4}$ or $1\frac{1}{2}$. Write $\frac{1}{2}$, and add the 1 penny to the column of pence, which makes 20 pence; this number divided by 12 (the number of pence in a shilling)=1 shilling and 8 pence. Write the 8 under the pence, and add 1 to the units of the shilling's place, which makes 21; write 1, and add the 2 to the ten's column =3 or 31 shillings, which divided by 20=£1 and 11 shillings left. Write the latter under the shillings, and add the 1 pound to the pound's column =£55. *Ans.* £55, 11s. $8\frac{1}{2}$d.

2. Add the following:

£ 17 18 $11\frac{3}{4}$ + £ 14 17 $2\frac{1}{2}$ + £ 16 14 8 =
£ 17 19 $0\frac{1}{4}$ + £ 45 0 $11\frac{3}{4}$ + £111 10 $2\frac{1}{2}$ =
£116 16 6 + £320 14 $5\frac{1}{4}$ + £ 38 18 8 =

Total, £700, 0*s.* 8*d.*

ART. 2. *To subtract compound numbers.*

Subtract £14, 7*s.* $6\frac{1}{4}$*d.* from £19, 4*s.* 3*d.*

£	s.	d.
19	4	3
14	7	$6\frac{1}{4}$
4	16	$8\frac{3}{4}$

SOLUTION.—We can not take $\frac{1}{4}$ from nothing, so we add a penny to both numbers; then subtracting the $\frac{1}{4}$ from a penny, or $\frac{4}{4}$, we have $\frac{3}{4}$ left. Adding 1d. to the 6d., we have 7d., which we can not subtract from the 3d. above, and accordingly add 1s. to both numbers; 7 from

1s. 3d. or 15d., leaves 8d. Adding 1s. to the shillings, we have 8s., which can not be taken from 4s. without adding £1 to both numbers; £1 to 4s.=24s.; 8s. from 24s.=16s. Then adding £1 to the 14, we have £15, which, taken from £19=£4, making the *answer* £4 16s. $8\frac{3}{4}$d.

2 Subtract the following:

£	s.	d.		£	s.	d.	
17	10	$8\frac{1}{4}$	—	14	5	3	=
119	7	6	—	17	19	$5\frac{1}{2}$	=
500	0	0	—	20	18	8	=
176	14	$7\frac{1}{4}$	—	129	15	$7\frac{1}{2}$	=

Total, £620, 13*s.* $9\frac{1}{2}$*d.*

ART. 3. *To multiply compound numbers.*

Multiply £17, 4*s.* $9\frac{1}{4}$*d.* by 8.

Operation.

£17	4	$9\frac{1}{4}$
		8
£137	18.	2.

After performing operations in addition, the learner will readily see how this is done.

£	s.	d.		
17	18	$8\frac{1}{2}$	× 7	=
120	16	$6\frac{1}{4}$	×12	=
365	0	$7\frac{3}{4}$	× 9	=

Total, £4860, 15*s.* $0\frac{1}{4}$*d.*

ART. 4. *To divide compound numbers.*

Divide £157, 13*s.* $6\frac{1}{2}$*d.* equally between 25 persons:

Operation. 25)£157 13*s.* 6½ (£6, 6*s.* 1½*d.*, or
150 £6, 6*s.* 1¼*d.*, nearly.

£7=remainder.
20

153=shillings in £7, with 13*s.* of the
150 [dividend added.

3=remainder in shillings.
12

42=pence in 3 shillings, and 6 pence
25 [from the dividend.

17=remainder in pence.
4

70=farthings in 17 pence and ½.
50

20=remainder, or $\frac{20}{25}$ farthings.

RECAPITULATION.—We first divided the £157 by 25; then 153 shillings by 25; then 42 pence by 25; and, lastly, the 70 farthings.

	£	*s.*	*d.*			
Divide	167	18	6¾	by	25	=
	768	14	3¼	by	125	=
	17	11	3½	by	875	=

Total, £12, 17*s.* 8¾*d.*

	£	*s.*	*d.*			
Divide	25	18	4	by	5	
	76	12	8	by	4	
	1	15	9	by	3	
	162	12	6	by	30	

Total, £30 7*s.* 2*d.*

XXIX. FOREIGN EXCHANGE.

ART. 1. In calculating *Foreign Exchange* the money of one country has to be expressed in that of another. A bill drawn in New York on an English house, will be expressed in *pounds, shillings*, and *pence.*

The relative value of moneys of different countries depends on the *par of Exchange*, and the *course of Exchange.*

The *Par of Exchange* is the comparative value of the coins of the different countries, and is fixed, while the relative purity of the coins is the same. The par of exchange between the United States and Great Britain is $4.8665 to the pound sterling.

The *Course of Exchange* usually depends upon the relative state of indebtedness of the merchants of the different countries, and the supply of gold and silver; accordingly, the course of exchange will sometimes be above, and sometimes below par.

FORM OF A FOREIGN BILL.

Exchange for £1567. CINCINNATI, June 3, 1876.

Thirty days after sight of this first of Exchange (second and third of the same tenor and date unpaid,) pay to the order of J. H. Story, the sum of one thousand five hundred and sixty-seven pounds sterling, value received, and place to my account as advised.

To William Morgan, Esq., C. H. GUIOU.
Liverpool, England.

NOTE.—Foreign Bills are generally drawn in *sets* of two, three, or four; that is, copies of the same bill are made out and transmitted by different conveyances to the payee, one of which being received and accepted, or paid, the others to be void.

BRITISH OR STERLING EXCHANGE.

ART. 2. *British or Sterling Money Reduced to Federal Money or United States Currency.*

The calculations relating to sterling money have been reduced to simple operations. In the daily papers we find quoted in gold or currency the precise value of the pound sterling in dollars and cents, as in the following example, the operation of which we give below.

1. Required the value of £157 9 2 in Federal Money, when sterling exchange is quoted at 4 86 in gold, and gold at 10% or 110.

By Aliquots.

	486
	157 9 2
	3402
	2430
	486
s. d.	76302
6 8=$\frac{1}{3}$	162
2 6=$\frac{1}{8}$	607
	765.247=cost in gold.
	76.524=10%
	$841.771=cost in currency.

By Decimals.

12)2.0

20)9.166

£157.4583

486

9447498

12596664

6298332

$765.247338=cost in gold.

ART. 3. *To assist the learner we give the following Table of*

ALIQUOTS OF A POUND.

s.	d.	s.	d.	s.	d.	s.	d.
10	0=$\frac{1}{2}$	4	0=$\frac{1}{5}$	2	0=$\frac{1}{10}$	1	3=$\frac{1}{16}$
6	8=$\frac{1}{3}$	3	4=$\frac{1}{6}$	1	8=$\frac{1}{12}$	1	0=$\frac{1}{20}$
5	0=$\frac{1}{4}$	2	6=$\frac{1}{8}$	1	4=$\frac{1}{15}$		6=$\frac{1}{40}$

2. Sterling at 4 87½ in gold, and gold at 110¼, required the currency for £147 6 8.

3. The quotation for sterling being 540 in currency, how much will buy a bill for £652 10?

4. Required the currency for the following bill at 3 days sight: 486½, gold at 110; £376 4 6.

5. What will pay for a sight bill for £319 4 9, with the market at 489 in gold, and gold at 9½ premium?

6. How much will a bill for £794 5 4, cost in currency, sterling exchange being quoted at 486½, and gold at 110?

7. Required the cost of £113 3 3 at the same quotations. (Ex. 6.)

ART. 4. *To Reduce Federal to Sterling Money.*

8. How much British Money can be bought for $841.77, exchange being quoted at 486 in gold, and gold at 110?

In other words, if £1 cost $4.86+10%, what sum in the same currency can be bought for $841.77?

841.77 : 4.86+.486=157.458
20
9.160
12
1.920=£157 9 2. (See Ex. 1.)

9. Sterling at 4 89 in gold, and gold at 107½, what amount of a bill can be bought for $1051.35?

10. Required the amount of a bill that can be bought for $31.49, sterling quotations being 4 86¾, in gold, and gold being 111½.

11. Sterling quotations being 520, in currency, what sight bills can be bought for $650?

12. Required the face of a sight bill that can be bought for $50, sterling quotations being 489 in gold, and gold at 10% premium.

13. Sterling at 487½ in gold, and gold at 109, what amount of a bill can be bought for $79.56?

14. Required the amount of a bill that can be bought for $47.20 currency, sterling at 487, gold at 108?

Answers: $841.77, $1709.37, $791.87, $3523.50, $2013.37, $4250.52, $605.58, £157 9 2, £200, £14 19 5, £5 16, £8 19 4, £1 10, £125, £246 13, 4. £9 5 11.

GERMAN EXCHANGE.

ART. 5. The money of the whole German Empire is *Reichmark* and *pfennige*.

Signs: *Rm.* Reichmark, *d.* pfennig. 1 *Rm.*=100*d.*

COMPARATIVE TABLE.

	Rm.		*Rm.*
1 Prussian Thaler (30 Silbergroschen,) . . .	= 3	10 Mark-Banco, (Hamburg,)	= 15
1 Florin, Austrian Coinage,	= 2	100 Florins, (Holland,) .	=169
7 Florins, S'th Ger. Currency, (Süddeutsche Währung,)	=12	5 Francs, (France,) .	= 4
		10 £, (British,) . . .	=203
		97 Cents, (Federal Money,)	= 4

ART. 6. *To Reduce German to Federal Money.*

1. Required the value of *Rm.* 1264 in United States currency when exchange is at par, and gold is quoted at 110.

First Method.

1264
24.25
316
5056
2528
$306.52 in gold.
30.65=10% cost of gold.
$337.17 in currency.

Second Method.

Rm. 4 : *Rm.* 1264 : : 97*c* : *x*

1 316
97
2212
2844
$306.52 in gold.
30.65
$337.17 in currency.

EXPLANATION—*First Method.*—If one *Rm.* is equal to 24¼c in gold, *Rm.* 1264 are equal to 1264 times 24¼c, viz.: $305.62 in gold. Gold being quoted at 110; *i. e.*, 10 per cent above par, we add 10 per cent to the value in gold to obtain the value in currency.

The Second Method is by proportion, which see page 139.

2. The quotations for German Exchange being 98⅛, and for gold 110½, find the cost in currency of a bill for *Rm.* 1892.

3. German Exchange at 100, and gold at 110½, what will be the cost of a bill for *Rm.* 720?

4. What will be the cost of a bill for *Rm.* 58, German exchange being quoted at 98, and gold at 112½.

ART. 7. *To Reduce Federal to German Money.*

How much German Exchange can be bought for $125.40 in currency, when the quotation is 4:96 and gold 110?

```
24
 2.4
26.4)12540.0(475 Rm.
   1056
   1980
   1848
   1320
   1320
```

EXPLANATION.—If four *Rm.* are equal to 96c in gold, one *Rm.* is equal to 24c in gold, gold being at 10 per cent premium, one *Rm.* will cost 10 per cent more in currency or 26 4.10th cents. $125.40 in currency will bring as many *Rm.* as 26.4 is contained therein, viz: *Rm.* 475.

5. Required the amount of a bill that can be bought for $1862, exchange being quoted at 95, to *Rm.* 4, and gold at 112?

6. German Exchange at 96, and gold at 110, what amount of a bill can be bought for $125?

7. Required the amount of a bill that can be bought for $42.25 currency, exchange being quoted at 96 and gold at 111?

8. What is the face of a bill that costs $666.68 in currency, exchange 100, gold 115?

Answers: Rm. 475, Rm. 473.45, Rm. 7000, Rm. 158.60, $15.98, $512.86, $198.90, $337.17, Rm. 2318.89.

FRENCH EXCHANGE.

ART. 8. The unit of French Money is the *franc,* (a silver coin equal in quality to our silver coins.) 1 *fc.*= 10 *decimes,* 1 *dec.*=10 *centimes.*

The par value of the *fc.* is about 19½ cents, or *fcs.* 512½ to $100 in gold.

ART. 9. *To Reduce French to Federal Money.*

1. French Exchange being quoted at 510 and gold at

111, required the cost in United States currency of a bill for *fcs.* 2465.

510)2465.00(4.83·	4.83·
2040	111
4250	4.83·
4080	48.33·
1700	483.33·
1530	$536.50 in U. S. currency.

It will be noticed that the amount of the bill is divided by the quotation of the French Exchange, and the result multiplied by the gold quotation.

French Exchange may be worked by proportion.

fcs. 510 : *fcs.* 2465 ∷ $111 : $536.50.

2. Required the cost in currency of a bill for *fcs.* 727.6, exchange being quoted at 520, and gold at 110.

3. What will be the cost of a bill for *fcs.* 226.66, French Exchange 518, gold 112½?

4. French Exchange at 518½, and gold at 115, what will be the cost of a bill for *fcs.* 52.5?

ART. 10. *To Reduce Federal to French Money.*

5. The quotation being 510 for French Exchange and 111 for gold, required the amount of a bill that can be bought for $536.50 currency.

The process is the reverse of the above. The proportion would be $111 : $536.50 : : *fcs.* 510 : *fcs.* 2465.

6. Required the amount of a bill that can be bought for $260 currency, French Exchange being quoted at 515, and gold at 110.

7. French Exchange at 516½, and gold at 109½, what amount of a bill can be bought for $1410 currency?

8. The quotation being 512 and 108½, what will be the face of a bill that costs $682.75?

Answers: $37.57, $49.23, $153.92, $3253.15, $536.50, *fcs.* 367.57, *fcs.* 1217.27, *fcs.* 2465, *fcs.* 3221.82, *fcs.* 6650.82, $11.64.

XXX. IMPORTING.

ART. **1.** IMPORTING is the business of buying goods in a foreign, to sell in the home market. A tax, under the name of *Duties* or *Customs*, is imposed by government on most imported articles of commerce.

Such taxes are levied for the purpose of creating revenue to defray the expenses of government, or to protect home manufacturing and agricultural interests. Duties are regulated by a scale of rates called a *Tariff*, and are altered according to the exigencies of the times.

A *high tariff* signifies high rates of duties; a *low tariff*, low rates of duties. In the United States, a high tariff is called for, when the expenditure of government exceeds the revenue. In Great Britain, it is advocated when imported articles sell so cheap as to interfere with the sale of home products.

The persons appointed to examine imported goods and collect duties, are called *Custom House Officers*, and their place of business, the *Custom House*.

ART. **2.** Duties are of two kinds: *ad valorem* and *specific*. *Ad valorem* duties consist of a rate per cent. on the *value* of the goods, as stated in the invoice; *specific* duties, of a stated sum of money on the *quantity imported*, without regard to value, as $1 a gallon, $20 a ton.

Certain allowances are made on goods charged with specific duties. These are *draft*, *tare*, *leakage*, and *breakage*. These allowances sometimes consist of a percentage of the weight or quantity, and sometimes of a specified deduction.

Tare is an allowance made for the weight of the

box, barrel, bag, crate, etc., which contains the goods, and is usually calculated by percentage, etc., after the deduction for draft is made.

Draft or *tret* is an allowance made for loss by weighing in small quantities, and for impurities to which some goods are subject.

On 112 lbs., or less, it is 1 lb.; from 112 lbs. to 224 lbs., 2 lbs.; from 224 lbs. to 336 lbs., 3 lbs.; from 336 lbs. to 1120 lbs., 4 lbs.; from 1120 lbs. to 2016 lbs., 7 lbs.; more than 2016 lbs., 9 lbs.

NOTE.—The *draft*, though not stated in the question, is to be deducted before other allowances are made.

Leakage is an allowance of 2 % on liquids, in casks, paying duties by the gallon.

Breakage is an allowance on bottled liquors, usually 5 %, but on ale, beer, and porter, 10 %.

Gross Weight is the total weight of goods and box, barrel, etc.

Net Weight is what remains after all deductions are made.

ART. 3. Goods imported may be placed in Government warehouses and the duty paid on withdrawal therefrom, or duties may be paid at once and the goods taken by the importer.

2. Entries should be made within twenty-four hours of the arrival of the goods.

3. Each class of merchandise should be entered by itself, without regard to rate of duty, and goods designed for warehouse entered by the case.

4. All charges incurred before shipment at the foreign port should be added to the invoice value; and every invoice is subject to duty on at least $2\frac{1}{2}$ %

commission. Except the fee for Consul's Certificate, ocean insurance, and freight, which are not dutiable.

5. Fractions are omitted in reckoning duties. Half a dollar is considered $1; under that the fraction is rejected.

6. On the back of the blank form of entry is an affidavit to be made by the owner or owners of the merchandise, or some one acting for them under power of attorney, stating that the invoice produced is the only invoice received for the goods; that the entry contains a true account of said goods; that nothing has been concealed whereby the United States may be defrauded; that if any mistake is discovered in the future the affiant will make it known to the Surveyor of Customs, etc.

7. There are three separate entries on the blank—next page—which, in practice, would be made out on separate papers.

8. The first entry represents merchandise subject to both specific and *ad valorem* duties, and a reduction of 10 % on the rates.

ART. 4. *Custom House values of foreign currencies.*

Crown of Sweden, Norway and Denmark..... .268	Pagoda of Madras.....1.84
Dollar, Egypt.....1.0039	Patacans of Uruguay ..... .9498
Dollar, Mexican.....1.0475	Peso of Cuba..... .9258
Dollar, Central America.. .965	Peso of Chili..... .9123
Florin of Austria..... .476	Peso of Venezuela..... .7773
Florin, Southern Germany.	Peso of Columbia..... .965
Florin of Netherlands..... .405	Pound Sterling, Gr. Brit..4.8665
Franc, France & Belgium. .193	Piastre, Turkish..... .0439
Lira, Italy..... .193	Rix mark, Germany..... .2382
Mahbub, Tripoli..... .8909	Rupee of India..... .4584
Mark reichs(rix),Ger.Em. .2382	Ruble of Russia..... .7717
Milreis of Brazil..... .5456	Tale of China.....1.61
Milreis of Portugal 1.0847	Thaler (see dollar).....
	Yan of Japan..... .997

ENTRY FOR CONSUMPTION.

Port of New York, December 20th, 1876

Entry of Merchandise by John Jones & Co, imported into New York by them in the "Ocean," from London, on the twentieth day of December, 1876.

MARKS.	Numbers	PACKAGES AND CONTENTS.	QUANTITY.	Per Cent.	Per Cent. 35%—10%	TOTAL DUTY.	Dutiable Value.
S	1/5	Five Cases Woolens, £150.10.6	500 lbs. @ 50c less 10%			$250.00	
		Charges, 1. 9.6					
		£152. 0.0					
		Commission 2½% 3.16.0					
		£155.16.0	@$4.8665		$758.	265.30	$758.
						$515.30	
						Less 10% 51.53	
						Duty in gold, $463.77	
B	10	One Case Linens, *Rm.*376.40	F. O. B.*	40%			
		Commission 2½% 9.41					
		*Rm.*385.81	@.2382	$92.		Duty in gold, $36.80	$92.
W	7/10	Four Cases Orna'ts, *fcs.*785.70			35%		
		Charges, 50.30					
		*fcs.*836.00					
		Commission 5% 41.80					
		*fcs.*877.80	@.193		$169.	$59.50	$169.

* Free on board.

JOHN JONES & CO.

ART. 5. FOREIGN INVOICES.

1. Invoice of five cases woolens, shipped on steamer "Ocean," London, November 18, 1876, to John Jones & Co., New York.

S 1/5. Five cases woolens, weighing 500 lbs.

	£	s.	d.	£	s.	d.
847½ yds. @ 16d.	56	10	0			
819 " " 14d.	47	15	6			
740 " " 15d.	46	5	0	150	10	6
Shipping charges, ...	1	2	0			
Cases, ...		7	6	1	9	6
Commission @ 2½ %, ...				3	16	0
(*amount chargeable with duty,*)				£155	16	0
Consular Certificate, ...	0	10	0			
Ocean Insurance on £150@32s. %,	2	8	0			
Freight, etc., ...	4	10	0	7	8	0
				£163	4	0

a. What is the duty @ 50c. per pound less 10 %, specific, and @ 35 % less 10 % *ad valorem* duty?

b. What is the entire cost, in currency, of the invoice, gold quoted @ 1.02 and exchange @ 4.90?

NOTE.—On receipt of an invoice the accountant should examine every item and ascertain whether or not it is correct. The first item of the invoice is 847½ yds. @ 16d.= 13560 pence or £56 10; the second item is 819 yds. @ 14d.= 11466 pence or £47 15 6; the third item is 740 yds. @ 15d.= 11100 pence or £46 5.

The next item for examination is the ocean insurance, £150@32s. per cent. (32 shilling per cent. is 32 shillings on every £100; *i. e.*, if we multiply the shilling per cent. on one per cent. of the £ the answer will be in shillings). one per cent. of £150 is £1 50-100. $1.50 \times 32 = 48$; *i. e.*, 48 shilling, or £2 8s.

We now proceed to ascertain the duty. The *ad valorem* duty on the first three extensions of the invoice,* aggregating £155 16 0, which multiplied on 4.8665 the custom-house value of a £) = $758. @ 35 per cent = $265.30, less 10 per cent. = $238.77. The *specific* duty on 500 lbs. @ 50 cents = $250, less 10 per cent.= $225. $238.77 + $225 = $463.77, duty in gold.

In order to ascertain the entire cost, we must reduce the amount of the invoice to Federal money, and add the duty. £163 4 0 reduced to Federal money (according to Art. 2, p. 152), $4.90 being the value of a £, would be $799.68 in gold, plus the duty, $463.77 = $1263.45. Gold being worth $1.02 in currency, the entire cost in currency is $1288.72.

*See Art. 3, ¶ 4, page 158.

2. Invoice of queensware consigned to Mr. W. H. Hall, Cincinnati, O., by W. Anderson, Liverpool, shipped on steamer "Star," March 16, 1876.

WA[c] 140	72 doz Twiflers, $\frac{1}{8}$ soups, blue edge,.....$^{1}/_{8}$			6 0 0
141	100 doz. Muffins, 7 in. flat, blue edge,.....$^{1}/_{6}$			7 10 0
142	One crate same,.....			7 10 0
143	60 doz. unh. London teas, neatly painted,.....$^{2}/_{3}$			6 15 0
144	One crate same, only another pattern,...			6 15 0
145	One crate same, only blue sponged marble, ...			6 15 0
146	One crate same,.....			6 15 0
147	One crate same, only pink sponged marble,.....			6 15 0
148	24 doz. jugs $6^{3}/_{6}$, $12^{4}/_{-}$, cone blue dipt,.....	4 8 0		
	40 doz. unhd. toy teas, fancy sponged,.....$^{1}/_{3}$	2 10 0		6 18 0
149	38 doz. bowls, 6, 9, 24, 30, 36, blue dipt,. .....$^{3}/_{3}$			6 3 6
			£	
	Packages....			1 10 6
	Commission, 2 %.....			
	Freight and Shipping Charges...			2 12 0
	Ocean Ins. @ 41s. % on £70.....			

What is the duty @ 45 less 10 %, including com. @ $2\frac{1}{2}$ %, and what is the entire cost of the invoice in currency, gold 1.02, exchange $4.87\frac{1}{2}$?

*16 doz. sixes at 3 shillings and 6 pence.

3. Memorandum of goods shipped on steamer "Gladiola," from Liverpool, March 18, 1876, and consigned to R. H. Langdale, Cincinnati, Ohio.

154	38 doz. bowls, $^{6}6, ^{6}9, ^{8}24, ^{10}30, ^{8}36$, C. C.,.....$^{2}/_{9}$		5	4 6
155	24 doz. jugs, $^{5}12, ^{13}6^{3}/_{-}, ^{6}12^{3}/_{6}$, cone C. C.,.....	3 15 0		
	40 doz. unhd. toy teas, fancy sponged,.....$^{1}/_{3}$	2 10 0	6	5 0
156	60 doz. unh. London teas, C.C., $^{1}/_{10}$		5	10 0
157	60 " " " " "		5	10 0
158	One crate same,.....		5	10 0
159	50 doz. nappies, $^{8}5^{1}/_{4}, ^{10}6^{1}/_{6}$, $^{8}7^{2}/_{-}, ^{12}8^{2}/_{6}, ^{7}9^{3}/_{6}, ^{5}10^{4}/_{6}$, beaded C. C.,.....		5	18 8
160	72 doz. twiflers, flat French, " C. C.,.....$^{1}/_{7}$		5	14 0
161	100 doz. muffins, 7 in. flat, French C. C.,.....$^{1}/_{5}$		7	1 8
162	One crate same,.....		7	1 8
			*	
	Discount 5 %		*	
	{ Exch $485 to £ Gold 110. }		*	
	Packages,		3	16 0
	Charges,		8	7 0

Cash, March 31, 1876,

What is the duty @ 35 less 10 %, including com., and what is the entire cost in currency of the invoice?

* The pupil will fill the blanks.

4. Invoice of six hogsheads of tobacco, shipped on board the "Leviathan," Davis, master, and consigned to Edwin Kessinger, on his account and risk.

	No.	†cwt.	qr.	lbs.	cwt.	qr.	lbs.
E. K.	1	18	0	23	1	2	11
1 to 6	2	19	1	12	1	3	5
	3	18	3	15	1	2	26
	4	18	1	26	1	2	19
	5	19	2	24	1	3	24
	6	12	2	17	1	2	17
		*			*		
		96	3	15	net, @ 7d. per lb.		

Charges.

Bond and custom house entry,	£0	10	6			
Cost of empty hogsheads,	4	16	0			
Lighterage and small charges,	1	4	0			
Bills of lading,		6	6			
Brokerage £316 9 9 @ ½ %,	*			8	8	8
Com. on £324 18 5 @ 2 %,	*					
Insurance on £350 @ 42s. %,						
Com. on same @ ½ %,						
Policy duty,	1	1	0			
				£341	11	5

Exch. $5 currency to the £;

Errors excepted

C. H. Guiou

Liverpool, Aug. 3, 1876.

What is the duty @ 40 less 10% on this invoice?

Answers: Duty on the several invoices, $140.13, $99.23, $463.77, $583.56. Cost in currency, $514.73, $446.52, $2320.56, $1288.72.

* The pupil will fill the blanks. † Avoirdupois weight.

XXXI. FARMING.

ART. 1. The business of a farmer sometimes comprises several of the mechanic arts, in which a knowledge of arithmetic is necessary. It also sometimes embraces surveying, engineering, etc., professions that require a familiar knowledge of this science.

ART. 2. To find the number of acres in a field or tract of land having 4 square corners,* we multiply the length by the breadth, and divide the result by 160, if the measure was taken in rods; or by 43560, if taken in feet.†

1. The length of a field is 125 rods, and its breadth 112 rods, how many acres are in it?

2. A lot of land is 400 ft. long by 110 ft. broad, how many acres does it contain? *

(For answers see end of chapter.)

ART. 3. *To lay off a given quantity of land.*

What should be the length of a strip of land 30 rods broad, to contain 6 acres?

In 6 acres there are 960 rods, which, divided by 30=32 rods.

ART. 4. *To find the contents of a triangular field having a square corner,* (a right-angled triangle,) we multiply the two shorter sides together, and take one-half the product.

Reason.—A right-angled triangle is half a square or parallelogram, formed by drawing a line between opposite corners.

† In a sq. rod there are 272¼ sq. feet. When there are feet remaining to be reduced to rods, it will be sufficiently accurate to divide by 272.

* A figure having square corners, and all its sides equal, is a *square;* one having its opposite sides equal, a *rectangle* or *parallelogram.*

1. The shorter sides of a right-angled triangle are 45 and 60; required the contents.

ART. 5. *To find the quantity of grain or coal in a bin or wagon*, we multiply the length, breadth and hight together; and for *grain*, divide the product by 1.2444,* if the dimensions were given in feet; or by 2150.42,† if given in inches. For *coal*, by 1.555 or 2688.

To find the number of bushels of unshelled *corn in a bin*, we multiply the cubic feet by $4\frac{1}{2}$ and divide the product by 10.

1. A wagon is 8 feet long, 5 feet broad, and 18 in. deep, how many bushels of corn does it contain?

$8 \times 5 \times 1\frac{1}{2} = 60$, *the number of cubic feet.*

$60 : 1.2444 = 48.21$ or $48\frac{1}{3}$ bushels.

2. How many bushels of grain in a bin measuring 4 feet every way?

ART. 6. *To find the quantity of wood or bark in a pile*, we multiply the three sides given in feet as before, and divide by 128, the number of feet in a cord.

1. How many cords of wood in a pile 40 feet long, 7 feet high and 4 feet broad?

ART. 7. *Having two sides and the contents of a box, to find the third side*, we divide the cubical contents by the product of the two sides.

Reason.—Since the product of the three sides equals the contents, then the contents divided by two of the sides will give the third side.

* Feet in a bushel † Inches in a bushel.

1. A box is 2 feet wide and 3 feet high, how long should it be to hold 25 bushels of coal?

In 25 bushels there are 2688×25 or 67200 cu. in.
In 2 feet there are 24 inches.
" 3 " " " 36 " $24 \times 36 = 864 =$ area of the end

$864)67200(77$ inches, or 6 feet $5\frac{3}{4}$ in. length of box.
6048
6720
6048
672

2. What must be the hight of a bin that will hold 300 bushels of wheat, if its length is 30 feet, and its width 4 feet?

3. What must be the depth of a box 16 inches square to hold a bushel; a box 10 inches square to hold a peck; one 8 inches square to hold half a peck?

To find the side of a cube that will hold a certain quantity.

ART. 8. *To find the quantity of grain when heaped against a wall or partition.* Take half the perpendicular hight for one side and multiply it by the length and breadth, as in Art. 5.

ART. 9. To find the number of cubic feet in a round log.

How many feet are in a log 12 feet long and 30 inches diameter?

In 30 inches there are $2\frac{1}{2}$, or 2.5 feet: $2.5 \times 2.5 \times .7854 = 4.9087$, *the area of the end.* $4.9087 \times 12 = 58.9044$, or $58\frac{9}{10}$ feet, *the solid content.*

NOTE.—This method of calculating, though correct, is seldom used for practical purposes. It is customary for lumber merchants to throw off $\frac{1}{3}$ of the diameter, and consider the remainder the side of a square log. A log of the dimensions named in the preceding question, would thus measure only $33\frac{1}{3}$ feet, or $\frac{1}{3}$ of 100 feet; and is thereby taken as the standard of measurement in some of the Western States. See Lumber Business page 169.

ART. **10.** *Trade or Barter.*

1. How many cords of wood at \$3.75 a cord should I get for 50 bushels of wheat at $\$1.12\frac{1}{2}$ a bushel

$50 \times 1.12\frac{1}{2} = \$56.25$, which divided by \$3.75, will give the number of cords.

$5625 : 375 = 15$ cords.—Proof—15 cords @ $\$3.75 = \$56.25$.

2. How many pounds of sugar at 8 cents a pound, should I get for 127 lbs of butter, at $12\frac{1}{2}$ cents a pound?

3. How many days work of a man at 75 cents a day, will be equal to 45 days work of a man at \$1.25?

4. How many cords of wood at \$2.25, will be equal to 150 cords at \$3.50?

TO DETERMINE THE WEIGHT OF LIVE CATTLE.—Measure in inches girth around breast just behind shoulder blade, and the length of back from tail to fore part of shoulder blade. Multiply girth by length and divide by 144. If girth is less than three feet, multiply quotient by 11; if between three and five, by 16; between five and seven, by 23; between seven and nine, by 31. If animal is lean, deduct one-twentieth from result; or, take girth and length in feet, multiply square of girth by length, and multiply product by 3.36. Live weight multiplied by .005 gives net weight—nearly.

Answers: 15, $198\frac{1}{2}$, 75, $3\frac{1}{9}$, $37\frac{1}{3}$, $51\frac{1}{2}$, 77, $8\frac{3}{4}$, 1350, 1 ac., 1.7 rds., $48\frac{1}{5}$, 350, $93\frac{3}{4}$, $87\frac{1}{2}$.

XXXII. LUMBER MEASURE.

ART. 1. Lumber measure comprises solid and superficial measure. Round logs are measured by deducting *one-third* of the diameter for waste, and calling the remainder the side of a square log.

To find the contents of a round log 24 inches in diameter, and 30 feet in length.

SOLUTION.—Deducting $\frac{1}{3}$ from 24 for waste, we have 16, which squared =256in. and multiplied by the length =640 ft. ***board measure.***

In some places, only $\frac{1}{4}$ is deducted for pine lumber.

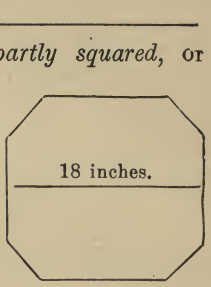
NOTE.—Inch measure is taken as the standard for lumber. If a board is under an inch, it is measured as a full inch; and if over an inch, it is reduced to inch measurement. A plank 2 inches thick, would be considered as 2 boards one inch thick.

Planks or joists are sometimes reckoned by *face measure,* that is, the dimensions of one side of the board are taken instead of the solid content. A 16 feet board 2 inches thick by 12 inches broad, would measure 32 feet, *board measure,* or 16 feet *face measure.*

In some places, the *saw log* is taken as a standard of measurement for round timber. A log 12 feet long and 30 inches in diameter, is the standard in some parts of the west. In Pennsylvania, a *saw log* is one that will cut into 200 feet of lumber.

Boards of different widths are measured with a tape-line, as they are put on the wagon, by passing one hand to the other in measuring each board. If the boards are 12 ft. long, the number of inches measured will be the number of feet of lumber; in measuring 16 ft. boards, one-third must be added.

ART. 2. *To measure timber partly squared*, or having its ends of the form of the diagram, it is customary to deduct the "wane" (the length of the corner), from the thickness of the log, and call the remainder one side. A log 18 inches thick, with a "wane" 3 inches, would be called one of 18 by 15 inches.



1. In an octagonal log 25 feet long, 20 inches thick, with a wane 4 inches, how many solid feet are there?

2. There are 150 logs, the average length and breadth of which, are 20 feet by 22 inches, wane 3 inches; required the number of solid feet they contain.

3. In a raft, there are 450 boards 16 feet long and $1\frac{1}{2}$ inches thick, and measuring in the aggregate 757 feet broad; how many feet of lumber (board measure) does it contain? How many *face measure?*

4. How much lumber can be cut from a tree measuring 20 feet long and 14 inches diameter at the smaller end, allowing for waste, one-fourth of the diameter?

5. The average length of 50 logs is 21 feet, and the average thickness 24 inches, wane 2 inches; required the number of solid feet they contain.

Answers: $8708\frac{1}{3}$, $55\frac{5}{9}$, 18.168, 12.112.

XXXIII. FRACTIONS.

ART. 1. When numbers are written as follows, they are called fractions: $\frac{1}{2}$, $\frac{7}{3}$, .05.

A *common fraction* is composed of two terms or numbers, one above the other, with a line between them, as $\frac{1}{2}$, $\frac{3}{4}$; which signify *one part* of something which has been divided into *two* parts, and *three* parts of something which has been divided into *four* parts.

The upper term is called the *numerator*, the under one, the *denominator*.

$\frac{1}{2}$, $\frac{2}{13}$, $\frac{67}{520}$, $\frac{11}{2000}$, would be read *one-half*, *two-thirteenths*, *sixty-seven five hundred and twentieths*, *eleven two thousandths*.

ART. 2. Common fractions are divided into *simple*, *proper*, *improper*, *compound*, *complex*, fractions, and *mixed numbers*.

A *simple fraction* is a single fraction, as $\frac{2}{3}$, $\frac{8}{7}$.

A *proper fraction* is a single fraction whose numerator is less than the denominator, as $\frac{1}{6}$.

An *improper fraction* is a single fraction whose numerator is equal to or greater than the denominator, as $\frac{3}{3}$, $\frac{9}{6}$.

A *compound fraction* is a fraction of a fraction or fractions, as $\frac{1}{2}$ of $\frac{2}{3}$, $\frac{4}{5}$ of $\frac{11}{13}$ of $\frac{45}{60}$.

A *complex fraction* is one having a fraction in the numerator or denominator, or in both, as $\frac{\frac{2}{3}}{7}$, $\frac{1}{\frac{7}{8}}$.

A *mixed number* is composed of a fraction and whole number together, as $7\frac{3}{4}$.

ART. 3. REDUCTION OF FRACTIONS.

Reduction of fractions consists in changing the terms without altering their values; thus, $\frac{6}{12}$ can be reduced to $\frac{3}{6}$, and that to $\frac{1}{2}$, without altering the value of the fraction.

To reduce a fraction to a lower denomination, we divide the two terms by such a number as will do so, without a remainder.

1. Reduce $\frac{24}{96}$ to its lowest terms.

$$8)\frac{24}{96}\left(\frac{3}{12}\right. \text{ and } 3)\frac{3}{12}\left(\frac{1}{4}\right.$$

EXPLANATION.—24 and 96 were divided by 8, giving the quotients 3 and 12. Then 3 and 12 were divided by 3, giving 1 and 4, or $\frac{1}{4}$. See Prop. of Numbers, page 82.

2. Reduce $\frac{875}{1000}$ to its lowest terms.

$$5)\frac{875}{1000}=5)\frac{175}{200}=5)\frac{35}{40}=\frac{7}{8}.$$

3. Reduce the following fractions to their lowest terms:

$\frac{24}{60}$, $\frac{184}{728}$, $\frac{21}{196}$, $\frac{125}{1000}$, $\frac{6874}{24682}$, $\frac{7216}{36080}$, $\frac{234}{6188}$, $\frac{126}{6666}$, $\frac{2168}{4682}$, $\frac{936}{2808}$, $\frac{9876}{79008}$.

Answers arranged promiscuously $\frac{1}{8}$, $\frac{1}{3}$, $\frac{21}{1111}$, $\frac{1084}{2341}$, $\frac{2}{5}$, $\frac{3}{28}$, $\frac{1}{8}$, $\frac{23}{91}$, $\frac{491}{1763}$, $\frac{1}{5}$, $\frac{9}{238}$

ART. 4. *To reduce a mixed or whole number to an improper fraction.* This is done in the same way that we would reduce feet and inches to inches.

1. Reduce $5\frac{7}{8}$ to an improper fraction; that is, in $5\frac{7}{8}$ how many eighths?

$5\frac{7}{8}$
8
$\frac{47}{8}$

EXPLANATION.—In every whole number there are 8 eighths, and in 5 whole numbers there are 8 times 5, or 40 eighths, to which add 7 eighths, and the result is 47 eighths.

Reduce the following numbers to improper fractions: $7\frac{3}{7}$, $6\frac{4}{5}$, $51\frac{2}{3}$, $17\frac{7}{34}$, $113\frac{16}{80}$, $16\frac{21}{50}$.

Answers.—$\frac{52}{7}$, $\frac{34}{5}$, $\frac{155}{3}$, $\frac{585}{34}$, $\frac{9056}{80}$, $\frac{821}{50}$.

ART. 5. *To reduce improper fractions to whole or mixed numbers*, is an operation the reverse of the last.

1. Reduce $\frac{47}{8}$ to a mixed number.

$$\frac{47}{8}=5\frac{7}{8}.$$

2. Reduce the following to whole or mixed numbers, and the remaining fractions to their lowest terms:

$\frac{146}{6}$, $\frac{456}{9}$, $\frac{364}{2}$, $\frac{161}{15}$, $\frac{1196}{25}$, $\frac{100}{8}$, $\frac{4160}{6}$, $\frac{3179}{185}$, $\frac{7854}{864}$, $\frac{11000}{85}$

NOTE.—When fractions are to be reduced to their lowest term, and the learner should be unable to see what number both the denominator and the numerator can be divided by, the GREATEST COMMON DIVISOR may be found by dividing the denominator by the numerator, and the numerator by the remainder, and the old remainder by the new remainder, etc. The last divisor—*i. e.*, the one which can be divided into the dividend without a remainder is the *Greatest Common Divisor.*

130)169(1
130
39)130(3
117
13)39(3
39

To find the greatest common divisor of $\frac{130}{169}$, we proceed as follows:

13 being the divisor, that divided 39 without a remainder, is the greatest common divisor: $\frac{130\div13=10}{169\div13=13}$

MULTIPLICATION OF DECIMALS.

ART. 6. In this rule we multiply as in whole numbers, and mark off as many places of decimals in the product as there are in the two factors.

1. 5.7×6.107

6.107 There are 3 places in this factor,
5.7 and 1 place in this "

42749
30535

34.8099 so we mark off 4 in the product.

Answers: 182, $10\frac{10}{15}$, $24\frac{1}{3}$, $50\frac{2}{3}$, $12\frac{1}{2}$, $693\frac{1}{3}$, $17\frac{34}{185}$, $129\frac{7}{17}$, $9\frac{13}{144}$, $47\frac{21}{25}$.

ART. 7. When the product contains fewer figures than there are decimals in the factors, we make up the number by annexing ciphers to the left.

2. $100 \times .0005$.

100.
.0005
500
or .0500
Ans. .05

The product contains only 3 figures (500), so we annex one more cipher to make up four, the number contained in the factors.

$.107 \times .05 =$*
$61.04 \times .0007 =$
$.7103 \times .004 =$

Total, .0509192

DIVISION OF DECIMALS.

ART. 8. Division of decimals is effected in the same manner as division of whole numbers, with the difference in using the decimal point. *The divisor must contain as many places of decimals as the dividend.*

1. To divide 34.8099 by 6.107

6.1070)34.8099(5.7
30 5350
42749
42749

The dividend contains 4 decimals, and the divisor only 3; so we point off *one* in the quotient. For proof, see Ex. 1, in Multiplication.

NOTE.—When a remainder occurs, we may annex ciphers indefinitely, and carry out the quotient to as many places as we desire.

ART. 9. When the *dividend* does not contain as many decimals as the divisor, annex ciphers to the right of the former, until it contains the same number; the quotient will then appear in whole numbers. If a remainder occurs, annex ciphers, and the result will be decimals.

2. Divide 3066 by .1783.

```
.1783)3066.0000(17195.7
      1783
      ----
      12830
      12481
      -----
       3490
       1783
       ----
       17070
       16047
       -----
        10230
         8915
        -----
         13150
         12481
         -----
```

Four ciphers have been annexed to the dividend, and a fifth annexed in finding the 7 of the quotient; so we point off 1 decimal.

When there are not figures enough in the quotient to make up the number of decimals in the dividend, annex ciphers to the left of the former.

3. Divide 10.70067 by 370.4.

```
370 4)10.70067(.0288
      7408
      -----
      32926
      29632
      -----
       32947
       29632
       -----
```

Here the quotient produced only three figures (288), which, with the one in the divisor makes only four decimals; so to make the number equal to the decimals in the dividend we annex a cipher to the left

314.06÷10.73 =29.2693
17600÷785.4 =
3170.09÷2.4014=
417.456÷31.145=

Total, 1385.1825

30.640÷493.67 =
10.8739÷117.406=
6.342÷22.973 =
1467.06÷196.04 =

Total, 7.91420

REDUCTION OF DECIMALS.

ART. **10.** *To reduce a common fraction to a decimal,* we annex ciphers to the right of the numerator, and proceed as in division.

Reduce $\frac{1}{2}$ to a decimal, $\frac{3}{4}$ to a decimal.

2)10 4)300

.5 .75

Reduce $\frac{1}{3}$ to a decimal.

3)100000

.3333 This quotient might be carried out indefinitely. It is called a *repeating* decimal. Such a decimal is marked thus, $.\dot{3}$. Its fractional value is restored by using 9 instead of 10 for the denominator. $\frac{3}{9}=\frac{1}{3}$.

Reduce $\frac{1}{7}$ to a decimal.

7)1000000000000

142857142857

This is called a *circulating* decimal, and is marked thus, $.\dot{1}42857\dot{1}=\frac{142857}{999999}=\frac{1}{7}$.

Its fractional value is restored in the same manner as that of the $.\dot{3}$ in the preceding example.

Express the following fractions decimally:

$\frac{33}{140}$
$\frac{5}{9}$
$\frac{16}{54}$
$\frac{1}{32}$

Total, 1.1186

$\frac{1}{9}$
$\frac{7}{8}$
$\frac{1}{371}$
$\frac{4}{17}$

Total, 1.2239

ART 11. *To find the value of the decimal part of a denominate number, as* £0.75, 0.33\frac{1}{3}$, etc.

1. What is the value of .5 of a yard?

If we wanted to know the value of 5 yards in a lower denomination, we would multiply by 4, as $5 \times 4 = 20$; that is 20 quarters. The same principle applies in decimals.

.5
4

2.0 *Ans.* 2 quarters.

£0.345 is how much?

.345
20

6.900 shillings.
12

10.800 pence.
4

3.2ØØ farthings

EXPLANATION.—The next lower denomination to pounds is shillings; so we multiply by 20, and point off three figures to correspond with the number in the factors. The next lower denomination is pence; so we multiply by 12, and the next farthings, which we multiply by 4. The *answer is* 6 *shillings,* 10 *pence* $3\frac{1}{5}$ *farthings.*

NOTE.— 1. The ciphers on the right need not be used, as they possess no value.

2. Observe that the *whole numbers* are not multiplied, else the shillings and pence would be reduced to farthings.

Find the value of

3. .625 of a gallon.	*Ans.* 2, 1.
4. .1425 of a year.	*Ans.* 1, $21\frac{3}{10}$.
5. .8323 of a £.	*Ans.* 16, $7\frac{3}{4}$.
6. .1374 of a tun.	*Ans.* 274, 12.8.
7. .0037 of a lb. Troy.	*Ans.* 21.3.

ART. 12. *To reduce denominate values to decimals.*

Reduce 6 shillings, 10 pence, $3\frac{1}{5}$ farthings to the decimal of a pound sterling.

12

1. 5) 10
4) 3.2
12)10.8
20) 6.9
£ .345

This operation is the reverse of the last. Observe that the 3 in the second line was annexed after reducing $\frac{1}{5}$ to a decimal; so with the 10 and 6.

2. Reduce 3 quarters to the decimal of a yard.
3. Reduce 6 lbs. 3 oz. to the decimal of a cwt.
4. Reduce 12s. $6\frac{3}{4}$d. to the decimal of a £.
5. Reduce 12 lbs. to the decimal of a tun.
6. Reduce 1 foot $3\frac{1}{8}$ in. to the decimal of a yard.
7. Reduce 16 oz. to the decimal of a tun.

The pupil can prove his calculations by last Art.

PRACTICAL QUESTIONS.

1. At 56 cents a pound, what will 127 lbs. 6 ounces of tea come to?

16)60
.375 decimal part of a pound.
127.375
56
764 250
6368 75
7133.000 or $71.33

REMARK.—This is not the shortest method of computing the above; the object being merely to show the practical application of decimals.

2. At $5 for a pound sterling, what will be the value of £16 8s. 10d.? *Ans.* $82.21.

3. What will be the value of the following sums of money, at the same rate?

£167 10s. 3¼d., £19 2s. 6d., £10 10s. 10½d.

Total, $985.91

4. At $75 per hundred (112 lb.), what will 14 cwt., 3 qrs. and 15 lbs. cost?

5. Find the cost of 3 tuns 15 cwt. of hemp, a $140 per tun?

Answers.—$1116.29, $525.

MULTIPLICATION OF COMMON FRACTIONS.

ART. 13. A fraction is multiplied by a whole number, by simply multiplying the numerator without altering the denominator. $\frac{3}{4}\times 7=7\times 3$, or $\frac{21}{4}$, which reduced to a mixed number, equals $5\frac{1}{4}$.

ART. 14. Fractions can also be multiplied by dividing the denominator, without altering the numerator. $\frac{4}{15}\times 5=\frac{4}{5)15}=\frac{4}{3}$, or $1\frac{1}{3}$.

Multiply the following fractions:

1. $\frac{2}{9}\times 5=1\frac{1}{9}$
2. $\frac{1}{8}\times 4=3\frac{1}{2}$
3. $\frac{2}{3}\times 12=8$
4. $\frac{3}{14}\times 11=2.357$
5. $\frac{7}{19}\times 9=3.316$
6. $\frac{8}{17}\times 6=2.824$

ART. 15. Mixed numbers are multiplied by whole numbers, as compound numbers are multiplied. Let it be required to multiply $4\frac{5}{8}$ by 7.

Illustration,

Whole Nos.	Eighths.
4	5
	7
32	3

or $32\frac{3}{8}$

EXPLANATION.—Seven times 5 eighths equal 35 eighths, or 4 whole numbers and 3 eighths. Seven times 4=28 and 4 make 32. *Ans.* $32\frac{3}{8}$.

It will not be necessary for the pupil to write his work in so formal a manner as in this illustration.

2. Multiply $6\frac{7}{9}$ by 12.

$$\begin{array}{r} 6\frac{7}{9} \\ 12 \\ \hline 81\frac{3}{9} \end{array}$$

Answers.

3. $6\frac{3}{4} \times 8 = 54$
4. $7\frac{1}{5} \times 7 = 50\frac{2}{5}$
5. $8\frac{7}{8} \times 6 = 53\frac{1}{4}$
6. $1\frac{12}{113} \times 12 = 13\frac{31}{113}$
7. $914\frac{2}{3} \times 120 = 109760.$
8. $63\frac{1}{2} \times 15 = 952.5$
9. $127\frac{2}{11} \times 20 = 2543.\dot{6}3\dot{6}$
10. $110\frac{1}{6} \times 14 = 1542.333$

ART. **16.** *To multiply fractions together,* we multiply the numerators together for a new numerator, and the denominators together for a new denominator.

1. $\frac{2}{3} \times \frac{3}{7} = \frac{6}{21}$, or $\frac{2}{7}$.
2. $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} = \frac{120}{360} = \frac{1}{3}$.

These operations might have been abbreviated by what is called *cancellation.* In the first example, for instance, $\frac{2}{3}$ is to be multiplied by $\frac{3}{7}$, that is, the numerator 2, is to be *multiplied by* the numerator 3; but the 2 is also *to be divided by* 3, for $\frac{2}{3}$ signifies that 2 is to be divided by 3; therefore, since the 2 is to be multiplied by 3, and divided by 3, it remains exactly the same, and the 3 of the denominator is said to cancel or make void the 3 of the numerator. In the following operations, the canceled figures will be known, by having a line drawn across them.

$$\frac{2}{\cancel{3}} \times \frac{\cancel{3}}{7} = \frac{2}{7}$$

$$\frac{2}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{\cancel{5}} \times \frac{\cancel{5}}{6}$$

$$\frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{6}} \quad 3$$

In the second operation, the 2 of the numerator and the 6 of the denominator are uncanceled, making $\frac{2}{6}$, which reduced by dividing both by 2, equals $\frac{1}{3}$.

The 2 and 6 might have been canceled also, by dividing both where they stood in the question, as in the 3d example, placing only 3 as a denominator, and 1 as a numerator.

1 is always to be understood, where a number has been canceled.

2. $\frac{1}{2}\times\frac{18}{24}\times 7\frac{1}{2}\times 9\frac{3}{8}$.

$$\begin{array}{r|l} 2 & 1 \\ 8\ 24 & 18\ 9\ 3 \\ 2 & 15 \\ 8 & 74\ 37 \end{array}$$

Some prefer arranging the terms of canceling fractions as in the margin, with the denominator or divisor on the left, and the numerator on the right.

EXPLANATION.—The first 2 was canceled in the 18, leaving 9; the 24 and 9 were canceled by dividing both by 3, leaving 8 and 3; the 74 was canceled by the second 2.

The fractions arranged in the usual order are

$$\frac{3}{8}\times\frac{15}{1}\times\frac{37}{8}=\frac{1665}{64}=26\frac{1}{64}.$$

	Answers.		*Answers.*
3. $\frac{1}{5}\times\frac{7}{8}\times\frac{5}{7}=$	$\frac{1}{8}$.	7. $1\frac{1}{2}\times 2\frac{1}{2}\times\frac{9}{10}=$	3.38
4. $\frac{2}{3}\times\frac{4}{7}\times\frac{14}{15}\times\frac{5}{16}=$	$\frac{1}{9}$	8. $6\frac{1}{2}\times\frac{4}{5}\times\frac{12}{13}=$	4.8
5. $\frac{1}{9}\times\frac{8}{15}\times\frac{5}{24}\times\frac{7}{21}=$	.0041	9. $87\frac{1}{2}\times\frac{2}{175}\times\frac{1}{20}=$	.05
6. $\frac{1}{2}\times 1\frac{1}{9}\times\frac{2}{7}\times\frac{1}{6}$	.0265	10. $\frac{161}{211}\times 52\frac{3}{4}\times\frac{1}{7}=$	5.75

ART. 17. *Compound fractions are reduced to simple ones, by multiplication.* Let it be required to reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ to a simple fraction. We know by inspection, that one-half of $\frac{2}{3}$ is $\frac{1}{3}$, and that $\frac{1}{3}$ of $\frac{3}{4}$ is $\frac{1}{4}$, the answer.

By multiplication $\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}=\frac{6}{24}=\frac{1}{4}$.

By cancellation $\frac{1}{\cancel{2}}\times\frac{\cancel{2}}{\cancel{3}}\times\frac{\cancel{3}}{4}=\frac{1}{4}$.

	Answers.		*Answers.*
2. $\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{9}{21}=$	$\frac{1}{4}$	6. $\frac{7}{8}$ of $\frac{8}{9}$ of $9\frac{1}{2}=$	7.389
3. $\frac{1}{7}$ of $\frac{14}{21}$ of $3\frac{1}{2}=$	.3	7. $\frac{2}{7}$ of $\frac{1}{6}$ of $\frac{5}{35}=$	.0068
4. $\frac{8}{3}$ of $\frac{6}{7}$ of $\frac{8}{16}=$	.286	8. $4\frac{1}{2}$ of $1\frac{1}{3}$ of $\frac{1}{4}=$	1.5
5. $\frac{1}{2}$ of $1\frac{1}{2}$ of $\frac{7}{3}=$	1.75	9. $\frac{1}{2}$ of $\frac{2}{3}\times\frac{6}{7}\times 1\frac{1}{2}=$	.4286

10. At $11\frac{1}{4}$ cents a pound, what will $147\frac{1}{2}$ lbs. of coffee cost?

11. What will $7\frac{1}{2}$ lbs. of cheese cost, at $9\frac{1}{2}$ cents per lb.?

12. At $12\frac{1}{3}$ cents a pound, what will 120 lbs. of sugar cost?

13. What will $14\frac{1}{2}$ lbs. of beef cost, at $6\frac{3}{4}$ cents a pound?

14. Fifteen and a half yards of muslin at $9\frac{1}{4}$ cents, will cost how much?

Answers.—71 cents, $14.80, $16.59, 98 cents, $1.43.

DIVISION OF COMMON FRACTIONS.

ART. **18.** Division being the reverse of multiplication, *to divide a fraction by a whole number,* we divide the numerator or multiply the denominator.

$\frac{6}{21} \div 3 = \frac{3)\ 6}{21} = \frac{2}{21}$. $\quad \frac{6}{21 \times 3} = \frac{6}{63} = \frac{2}{21}$.

Divide $\frac{16}{18}$ by $4 = \frac{2}{9}$ $\quad \frac{37}{28} \div 7 = .1887$

$\frac{14}{16}$ by $7 = \frac{1}{8}$ $\quad \frac{16}{21} \div 10 = .076$

$\frac{18}{36}$ by $3 = \frac{1}{6}$ $\quad \frac{18}{27} \div 4 = .16$

ART. **19.** *To divide mixed numbers.*

1. $21\frac{3}{5} \div 6$.

Whole Nos.	Fifths.
6)21	3
3	3
or $3\frac{3}{5}$	

EXPLANATION.—6 is contained in 21 3 times and 3 left. In the 3 of remainder, there are 15 fifths, which added to the 3 fifths in the question, make 18 fifths. 6 in 18, 3 times. *Ans.* $3\frac{3}{5}$.

2. $12\frac{1}{7} \div 8$.

$8)12\frac{1}{7}$

$1\frac{29}{56}$

EXPLANATION.—In this example, we had 4 remainder, in which were 28 sevenths, and the one in the question made 29. Then as 8 would not divide 29 without a remainder, we multiplied it n the denominator, which made 56. *Ans.* $1\frac{29}{56}$.

	Answers.		Answers.
3.	$677\frac{7}{8} \div 7 = 96\frac{29}{56}$	7.	$167\frac{1}{9} \div 25 = 6.684$
4.	$44\frac{5}{8} \div 3 = 14\frac{17}{18}$	8.	$21\frac{1}{3} \div 14 = 1.524$
5.	$119\frac{2}{3} \div 6 = 19\frac{17}{18}$	9.	$16\frac{1}{7} \div 7 = 2.306$
6.	$118\frac{1}{2} \div 12 = 9\frac{7}{8}$	10.	$22\frac{1}{3} \div 12 = 1.861$

ART. **20.** One fraction is divided by another, by inverting the terms of the divisor. $\frac{1}{2} \div \frac{4}{1} = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$.

Proof: $\frac{1}{2} \div \frac{4}{1}$ is $\frac{1}{2}$ divided by 4, because $\frac{4}{1}$ is 4, and by Art. we find that $\frac{1}{2} \div 4 = \frac{1}{2 \times 4} = \frac{1}{8}$.

Caution.—The pupil will observe not to invert the terms of the number to be divided.

Complex fractions are unsolved questions in division.

$\frac{2\frac{1}{2}}{\frac{5}{7}} = 2\frac{1}{2} \div \frac{5}{7} = \frac{5}{2} \times \frac{7}{5}$, which canceled

$$= \frac{\cancel{5}}{2} \times \frac{7}{\cancel{5}} = \frac{7}{2} = 3\frac{1}{2}.$$

1. $\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$
2. $\frac{7}{9} \div \frac{1}{7} = 5\frac{4}{9}$
3. $\frac{6}{7} \div \frac{1}{3} = 2.571$
4. $\frac{1}{3} \div 1\frac{1}{2} = .36$
5. $\frac{1}{2}$ of $\frac{2}{3} \div \frac{5}{8} = .53\dot{}$
6. $1\frac{1}{2} \times \frac{1}{6} \div \frac{3}{4} = .\dot{3}$
7. $2\frac{1}{3} \times \frac{1}{7} \div \frac{2}{3}$ of $\frac{3}{4} = .\dot{6}$
8. $\frac{3\frac{1}{9}}{5\frac{1}{4}} \times \frac{\frac{1}{2}}{\frac{1}{3}} \div \frac{6}{7}$ of $\frac{7}{8} = 1.185$

9. If $120\frac{1}{2}$ lbs. of cheese cost \$14.80, what will 1 lb. cost?

10. Find the cost of 1 lb. of coffee, when $15\frac{1}{2}$ lbs. cost \$1.43.

11. If $11\frac{1}{4}$ yds. of cassimere cost \$16.59, what will one yard cost?

12. If $9\frac{1}{2}$ yds. of muslin cost 71 cents, what will 1 yard cost? *Answers.*—$7\frac{9}{19}$, $9\frac{7}{31}$, $147\frac{7}{15}$, $12\frac{68}{241}$.

SUBTRACTION OF COMMON FRACTIONS.

ART. **21.** Fractions and mixed numbers can be ubtracted from whole numbers, in the same way hat one denominate number is subtracted from an- ther.

From 87 take $25\frac{3}{7}$.

	Whole Nos.	Sevenths.
Illustration.	87	0
	25	3
	61	4

or $61\frac{4}{7}$.

EXPLANATION.—The 3 sevenths could not be taken from the number above, so we borrow 1 from the whole numbers, in which there are 7 sevenths; 3 from 7 leaves 4. Then 1 to 5 of the whole numbers makes 6, which subtracted from 7 leaves 1; and 2 from 8 leaves 6, making the remainder $61\frac{4}{7}$.

Answers.

2. From 210 take $37\frac{1}{2}$ = $172\frac{1}{2}$
3. " 119 " $82\frac{1}{8}$ = 36.875
4. " 61 " $4\frac{8}{9}$ = 56.$\dot{1}$
5. " 54 " $5\frac{1}{8}$ = 48.875
6. 1063—$819\frac{1}{9}$ 243.8
7. 3785—$10\frac{5}{7}$ 3774.14
8. 2168—$14\frac{3}{7}$ 2153.571
9. 1765—$777\frac{1}{2}\frac{5}{5}$ 987.4

ART. 22. *To subtract one fraction from another,* it is necessary that both be of the same denomination. We can subtract $\frac{1}{4}$ from $\frac{3}{4}$, but can not conveniently subtract $\frac{1}{3}$ from $\frac{3}{4}$, without first altering the denomination of one or both of the fractions. Let us alter the denomination, and reduce both fractions to twelfths:

$$\frac{1\times4}{3\times4}=\frac{4}{12}, \quad \frac{3\times3}{4\times3}=\frac{9}{12}.$$

Subtracting these four-twelfths from nine-twelfths, we have five-twelfths for a remainder.

REMARK.—The 12 being common to both of the new fractions, is called the *common denominator.*

ART. 23. *To find a common denominator*, it is only necessary to multiply the denominators all together. The common denominator of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, is $4\times3\times2=24$, and to raise fractions to a common denominator, we simply take the fractional part of the denominator, as $\frac{1}{2}$ of $24=12$, that is $\frac{12}{24}$, $\frac{2}{3}$ of $24=16$, that is $\frac{16}{24}$. $\frac{3}{4}$ of $24=18$, that is $\frac{18}{24}$. The fractions reduced to a common denominator, are $\frac{12}{24}$, $\frac{16}{24}$, $\frac{18}{24}$.

1. From $\frac{8}{9}$ take $\frac{7}{8}$. $8\times9=72$ com. denominator.

$\frac{7}{8}$ of $72=63$, $\frac{8}{9}$ of $72=64$. $\frac{64}{72}-\frac{63}{72}=\frac{1}{72}$ *Ans.*

NOTE.—A shorter method of finding the new numerators when there are only two terms, is to multiply the first numerator on the second denominator, and the second numerator on the first denominator.

2. From $5\frac{1}{8}$ take $\frac{2}{5}$.

$$4\tfrac{1}{8}-\tfrac{2}{5}=\frac{(41\times5)-(8\times2)}{8\times5}=\frac{205-16}{40}=\tfrac{189}{40}=4\tfrac{29}{40}$$

3. $\frac{176}{213}-\frac{145}{213}=\frac{31}{213}$
4. $\frac{4}{7}-\frac{5}{12}=\frac{13}{84}$
5. $\frac{2}{3}-\frac{4}{7}=.095$
6. $14\frac{1}{2}-\frac{6}{11}=13.95$
7. $12\frac{1}{6}-8\frac{1}{4}=3.92$
8. $2\frac{1}{2}-1\frac{1}{10}=1.4$
9. $\frac{1}{2}$ of $\frac{3}{4}-\frac{1}{8}=.25$
10. $\frac{6}{7}$ of $.\frac{1}{10}-\frac{1}{2}$ of $.\frac{2}{180}=.078$
11. $2\frac{1}{2}\times\frac{4}{5}-\frac{2}{3}$ of $\frac{6}{7}=1.429$
12. $13\frac{1}{2}-1\frac{1}{3}\times5\frac{1}{2}=9\frac{13}{24}$

$\frac{4}{5}$

ADDITION OF COMMON FRACTIONS.

ART. 24. *Fractions of the same denomination are added together, by finding the sum of the numerators.*

1. $\frac{2}{3}+\frac{5}{3}+\frac{7}{3}=2+5+7=14$ thirds, or $4\frac{2}{3}$.
2. $\frac{6}{7}+\frac{7}{7}+\frac{8}{7}+\frac{9}{7}=4\frac{2}{7}$
3. $\frac{5}{11}+\frac{6}{11}+\frac{7}{11}+\frac{8}{11}=2\frac{4}{11}$
4. $\frac{3}{12}+\frac{2}{12}+\frac{13}{12}+\frac{1}{12}=1\frac{7}{12}$
5. $\frac{4}{160}+\frac{15}{160}+\frac{98}{160}=\frac{117}{160}$
6. $\frac{51}{1000}+\frac{82}{1000}+\frac{718}{1000}=.851$
7. $\frac{16}{119}+\frac{18}{119}+\frac{125}{119}=1.336$

ART. 25. *Fractions of different denominations are added together, by finding a common denominator, as in subtraction*, and proceeding as under last Art.

1. Let it be required to add together $\frac{2}{4}+\frac{5}{6}+\frac{7}{8}$.

$4\times6\times8=192$ common denominator.

4)192

$48\times2=96$

6)192

$32\times5=160$

8)192

$24\times7=168$

$\frac{4}{1}\frac{2}{9}\frac{4}{2}=2\frac{5}{24}$

EXPLANATION.—The common denominator was found as in subtraction, Art. 22, and the numerators by taking the fractional parts of the common denominator, as $\frac{1}{4}$ of 192=48 and 2 fourths=2×48=96, that is $\frac{96}{192}$, etc.

2. $\frac{3}{7}+\frac{4}{9}+\frac{1}{4}=1\frac{31}{252}$

3. $\frac{1}{6}+\frac{1}{7}+\frac{1}{8}=\frac{73}{168}$

4. $\frac{2}{3}+\frac{2}{9}+\frac{2}{11}=1\frac{7}{99}$

5. $6\frac{1}{2}+7\frac{2}{3}+8\frac{1}{9}=22\frac{5}{18}$

6. $8\frac{1}{2}+6\frac{1}{4}+12=26.75$

7. $2\frac{1}{3}+\frac{17}{21}+\frac{16}{19}=3.98$

8. $\frac{11}{11}+9+\frac{1}{11}=9.61$

9. $6\frac{4}{7}+1\frac{3}{5}+2\frac{1}{5}=10.37$

REMARK.—The process of adding fractions can be abridged by using the *least* common denominator. In the last example, 35 may be made a common denominator: $\frac{20}{35}$, $\frac{21}{35}$, $\frac{7}{35}=\frac{48}{35}=1\frac{13}{35}$ or 1.37, which added to the whole numbers =10.37.

ART. 26. *To find the least common denominator*, the process is as follows: to add $\frac{1}{2}+\frac{3}{4}+\frac{5}{6}+\frac{6}{7}+\frac{7}{8}$.

2)2 4 6 7 8

2)1 2 3 7 4

3 7 2

$2\times2\times3\times7\times2=168$

$\frac{1}{2}$ of 168= 84

$\frac{3}{4}$ " " =126

$\frac{5}{6}$ " " =140

$\frac{6}{7}$ " " =144

$\frac{7}{8}$ " " =147

$\frac{6}{1}\frac{4}{6}\frac{1}{8}=3\frac{137}{168}$

EXPLANATION.—We divide all the denominators by such a number as will divide into most of them without a remainder, bringing down those that are not divisible, as the 7. We then divide the result, and each successive result in the same way, until the numbers can be reduced no lower; after which, all the divisors and the remaining numbers are multiplied together.

REMARK.—The *least common denominator* is also the LEAST COMMON MULTIPLE, as it is the least number which can be divided by the several denominators without a remainder.

4. $\frac{3}{7}+\frac{7}{14}+\frac{7}{21}+\frac{8}{28}+\frac{16}{35}=2\frac{1}{210}$

5. $\frac{5}{6}+\frac{6}{7}+7\frac{1}{2}+\frac{8}{9}+4\frac{1}{5}=14\frac{88}{315}$

6. $\frac{3}{10}+\frac{4}{25}+5\frac{1}{5}+8\frac{1}{4}+6=19.91\cdots$

7. $2\frac{1}{9}+6\frac{1}{7}+5\frac{3}{8}+67+\frac{2}{7}=80.9146$

8. $\frac{1}{2}$ of $\frac{2}{3}+\frac{3}{4}$ of $\frac{4}{5}+\frac{6}{7}+\frac{2}{9}=2.01269$

9. $\frac{8}{9}\times\frac{9}{12}$ of $\frac{2}{7}\times\frac{6}{7}$ of $\frac{7}{8}+1=1.14285$

10. $2\frac{1}{2}\times6\frac{1}{4}+8\frac{7}{8}+\frac{3}{4}$ of $\frac{6}{7}=25.1428$

11. $1\frac{1}{2}\times2\frac{1}{3}+\frac{6}{7}$ of $\frac{7}{8}$ of $1=4.25$

PRACTICAL QUESTIONS.

1. In an invoice of goods, there are the following items, required the amount.

$27\frac{1}{2}$	doz. @	$9\frac{1}{4}$c.	$13\frac{2}{3}$	doz. @	$5\frac{1}{2}$c.
$18\frac{7}{12}$	" "	12	$16\frac{5}{6}$	" "	$3\frac{1}{4}$
$16\frac{5}{12}$	" "	$12\frac{1}{4}$	$118\frac{1}{6}$	" "	$2\frac{1}{4}$

Ans. $10.65.

Answers will not be given to the following, as the pupil can easily prove the accuracy of his own calculations.

2. $\frac{2}{3}$ of a merchant's goods were destroyed by fire, and what remained was worth $1637.50, what was his loss?

3. A owns $\frac{2}{3}$ of a steamboat, B $\frac{1}{8}$, and C the remainder, which is worth $1000; what is the value of the boat?

4. $\frac{1}{4}$ of a saw mill belongs to A, $\frac{1}{8}$ to B, $\frac{3}{16}$ to C, the remainder to D, and the profits for the year amount to $1680; what is each man's share?

5. The par value of the pound sterling is $\$4\frac{4}{9}$, required the value of £1674, at 10 % premium.

6. A can do a piece of work in 8 days, B in 7 days, and C in 6 days; in what time can they do it if all work together?

Solution.—A can do $\frac{1}{8}$, B $\frac{1}{7}$, and C $\frac{1}{6}$ of the work in a day. The sum of these fractions is $\frac{73}{168}$. If $\frac{73}{168}$ can be done in a day, $\frac{168}{73}$ (the whole), can be done in $\frac{168}{73}=2\frac{22}{73}$ or 2 days $3\frac{4}{7}$ hours.

7. There are 3 pumps placed in a coffer dam; one will empty it in 10, another in 15, and the third in 20 hours; in what time can it be emptied by working all three at once? *Ans.* $4\frac{8}{13}$ hours.

ART. 27. To find fractional parts of denominate or compound numbers, as of pounds, shillings and and pence; days, hours, minutes and seconds.

1. Express $\frac{3}{7}$ of a day in hours, minutes, etc.

$\frac{3}{7}$ of a day is the same as $\frac{1}{7}$ of 3 days.

Days.	Hours.	Min.	Sec.
7)3	0	0	0
	10	17	$8\frac{4}{7}$

NOTE.—As 7 is not contained in 3 days, we reduce them to hours=72 hours, which divided by 7=10 hours and 2 left, etc.

2. In $\frac{5}{6}$ of a pound (British money) how many shillings and pence? *Ans.* £0 16s. 8d.

3. In $\frac{7}{8}$ of a bushel, how many pecks quarts, etc.? *Ans.* 3 pecks 4 quarts.

4. In $\frac{1}{6}$ of a tun (long weight), how many hundreds, etc.? *Ans.* 3 cwt. 1 qr. 9 lbs. $5\frac{1}{3}$ oz.

5. Find the $\frac{3}{5}$ of £167 18s. 6d.

First find $\frac{1}{5}$, and multiply it by 3.

6. $\frac{3}{4}$ of 41 bushels, 3 pecks, 2 quarts, is how much?

7. $\frac{9}{10}$ of 114 tuns, 8 cwt. 2 qrs. 14 lbs. long weight, is how much?

8. £168 18s. 8d., is how much in American currency—old standard, $4\frac{4}{9}$? *Ans.* $750.81.

Reduce 18 shillings and 8 pence to the decimal of a pound. Page 152.

9. Find the cost of a draft on London for £246 14s. 10d. at 9 % premium. *Ans.* $1195.33.

10. Reduce $1687.25 to British currency, at 9 %. *Ans.* £348 5s. $8\frac{1}{2}$d.

ART. **28.** *To find what part one number is of another*, we place the one above the other in fractional form.

1. 3 is what part of 4? *Ans.* $\frac{3}{4}$.

2. $\frac{3}{4}$ is what part of $\frac{7}{8}$? *Ans.* $\frac{\frac{3}{4}}{\frac{7}{8}}$ or $\frac{6}{7}$.

3. 25 lbs is what part of 45 lbs.?

4. 3 doz. chickens is what part of 42?

5. 1 peck is what part of a bushel?

6. $\frac{2}{4\frac{8}{3}}$ of a pound is what part of a penny?

$\frac{2}{4\frac{8}{3}}$ of a pound is the same as $\frac{1}{4\frac{8}{3}}$ of £2. Reducing these two pounds to pence, we have $2\times20\times12=480$. *Ans.* $\frac{480}{4\frac{8}{3}}$.

7. $\frac{75}{20}$ of a minute is what part of a day?

SOLUTION.—Reduction from a higher to a lower denomination, is performed by multiplication; therefore reduction from a lower to a higher denomination will be performed by division. To divide a fraction by a whole number, we either multiply the denominator or divide the numerator; in this case, we multiply the denominator.

$$\frac{75}{20\times60\times24}=\frac{75}{28800}=\frac{1}{384}\text{ day.}$$

XXXIV. DUODECIMALS.

Mechanics make most of their calculations in feet and inches by duodecimals.

ART. 1. DUODECIMALS like *Decimals*, is a species of calculation which enables the operator to compute fractional quantities as whole numbers.

12'''' *fourths* make 1 third.
12''' *thirds* make 1 second.
12'' *seconds* make 1 prime or inch.
12' *primes* or in. make 1 foot.

1 *inch* is the $\frac{1}{12}$ of a foot.
1 *second* is the $\frac{1}{12}$ of an inch, or $\frac{1}{144}$ of a foot.
1 *third* is the $\frac{1}{12}$ of a second, $\frac{1}{1728}$ of a foot.
1 *fourth* is the $\frac{1}{12}$ of a third, or $\frac{1}{20736}$ of a foot.

1. Multiply the following dimensions together: 10 ft. 7 in. $\times$ 3 ft. 8 in. $\times$ 7 ft. 9 in.

10	7			
3	8			
30	21			
	80	56		
30	101	56		1*st pro.*
7	9			
210	707	392		
	270	909	504	
300	8	11	0	

We commence to multiply by the left-hand figure (3), and write the result without reducing to a higher denomination. 3×10 ft. $= 30$ ft., and 7 in. $\times 3 = 21$ in. Then multiplying by the 8, we write the first product under itself as the multiplier, and the second product, 56, one place further to the right. Adding these, we have the product of two divisors.

Proceeding in the same way with the 7 and 9 of the third dimension, we add together the products and reduce them to higher denominations, by which we get 300 ft. 8′ 11″, or $300\frac{8}{12}$ ft. $+\frac{11}{144} = 300\frac{3}{4}$ ft., nearly.

	ft.	in.	ft.	in.	ft.	in.	″
2.	2	5	$\times$ 3	4	= 8	0	8
3.	17	1	$\times$ 3	4	= 56	11	4
4.	14	6	$\times$ 7	8	= 111	2	0
5.	21	9	$\times$ 14	11	= 324	5	3
6.	18	8	$\times$ 16	7	= 309	6	8

	ft.	in.	ft.	in.	ft.	in.	ft.	in.	″	‴
7.	4	8×6	4×17	2						
8.	3	9×2	6×11	0						
9.	21	11×6	7×17	8						
					3159	6	3	8		

10. How many squares of flooring in 3 rooms measuring 18 ft. 6 in.×15 ft. 8 in., and what is the cost of laying, at 50 cents per square?

18×6	
15×8	
277	6
12	4
289	10
	3
869	6

or $869\frac{1}{2}$ sq. feet, which reduced to squares of 100 feet = 8.695 squares. 8.695 × 50 cts. = 4.347, or $4.35.

11. What is the cost of laying 4 floors of the following dimensions, at 75 cents per square: 18 ft. 9 in. ×17 ft. 3 in.?

12. What will be the cost of shingling a roof which measures 53 ft. 6 in. long, and 5 ft. 8 in. from the ridge to the outer edge of the wall, at $1.50 per square?

13. The average breadth of a board is 1 ft. 4 in., and the length 23 ft. 9 in., what number of feet does it contain?

14. How many solid feet in a log measuring as follows: 45 ft. 4 in.×1 ft. 6 in.×1 ft. 3 in.?

15. What will it cost to shingle a roof 26 feet long, rafters 14 feet, at $1.25 per square?

16. How many square feet of lumber in a staircase 12 ft. wide, with 23 steps 8 in. high, and steps 1 ft. 2 in. front to back?

17. How much will it cost to floor a house of 6 rooms, with ash lumber, ready for laying, at $5.25 per hundred, and $1.25 per square for laying? The rooms measure as follows: 2 rooms 18 ft. 6 in. by 16 ft. 8 in., 3 rooms 16 ft. 7 in., by 14 ft. 6 in., and one 16 ft. 6 in. square?

18. How many squares in a partition that measures 22 ft. 6 in. long, and 15 ft. 4 in. high?

19. What will be the expense of shingling a roof 120 ft. long, and 18 ft. 6 in. from the ridge to the side wall of the house, at $2.50 per square?

20. How many cubic feet of timber are in 17 logs of the following dimensions: 3 logs 40 ft.$\times$2$\times$2; 5 logs 28 ft.$\times$16 in. square, and the balance 54 ft.$\times$22 in. square?

21. What is the cost of laying 3 floors of the following dimensions, at 75 cents per square? 16 ft. 9 in.$\times$17 ft. 3 in.

22. What will be the cost of shingling a roof which measures 62 ft. 6 in. long, and 8 ft. 8 in. from the ridge to the outer edge of the wall, at $1.50 per square?

23. The average breadth of a board is 1 ft. 6 in., and the length 14 ft. 9 in.; what number of feet does it contain?

Answers: $9.09, 85 ft., $9.70, 31$\frac{2}{3}$ sq. ft., $9.10, 506 ft., $104.66, $3.45, $111, 2362$\frac{1}{3}$, 22\frac{1}{3}$, $72.23, $90.28.

XXXV. COMPOUND PROPORTION.

ART. 1. When there are more than three terms in a proportion, it is said to be *compound*.

1. If 3 men in 5 days, by working 8 hours a day, dig a cellar 15 feet long, 12 feet wide, and 7 feet deep, in how many days will 2 men dig one 17 feet long, 14 feet wide, and 6 feet deep, by working 10 hours a day?

In this problem, there are 11 terms and 5 ratios: the ratio between men and men, that between hours and hours; between feet and feet of the length; feet and feet of the width, and feet and feet of the depth. In arranging these terms, we proceed as in Simple Proportion, Ex. 14 page 141.

		ds.
men,	2 : 3 : :	5
hours,	10 : 8	
length in ft.	15 : 17	
width in ft.	12 : 14	
depth in ft.	7 : 6	

Days are wanted, write days as the right hand term.

2. Comparing men with men, we find that it will take 2 men a *longer* time to do the job, than it took 3 men, so we write the greater of the two terms (3) in the second place.

3. Comparing hours with hours, we reason that it will take less time to do the job, by working 10, than by working 8 hours a day, so we write the smaller number on the right, and under the second term.

4. Comparing length with length, we reason that it will take a longer time to dig a cellar 17 feet long, than it did to dig one 15 feet long; so we write the greater (17) term under the second term.

5. Comparing breadth with breadth, it will take a longer time to dig a cellar 14 feet wide, than it did to dig one 12 feet wide; so we write the greater (14) under the second term.

6. Comparing depth with depth, it will take less time to dig a cellar 6 feet deep, than it did to dig one 7 feet deep; so we write the smaller number under the second term.

The terms on the left being divisors of those on the right, this statement resolves itself into a fraction, which can be solved with great ease by cancellation.

		5
	2	3
5	10	8 4
5	15	17
3	12	14 2
	7	6 2

$$\frac{17\times 2}{5}=6\tfrac{4}{5}, \text{ or } 6 \text{ days } 8 \text{ hours.}$$

The example may be reasoned out thus: If 3 men work 5 days 8 hours per day, that is equal to the work of one man for $3\times 5\times 8=120$ hours; $15\times 12\times 7$ feet is equal to 1260 cubic feet, $17\times 14\times 6$ feet is equal to 1428 cubic feet. 2 men working 10 hours per day is equal to 1 man working 20 hours per day. It takes 120 hours to dig 1260 cubic feet, hence $10\frac{1}{2}$ feet per hour. In 20 hours 210 feet can be dug. 1428 feet : 210 feet $= 6\frac{4}{5}$ days or 6 days 8 hours.

2. If 6 men in 15 days dig a trench 18 feet long, 7 feet wide, and 5 feet deep, in how many days will 21 men dig a trench 125 feet long, 9 feet wide, and 4 feet deep? *Ans.* 30.61 days.

3. What is the interest of \$6784 for 2 years 6 months, and 15 days, at 6 % per annum (365 days)?

Statement.

		\$6
days,	365	927
dollars,	100	6784

Ans. \$1033.77

4. The interest of \$1467 for 3 years, 4 mos., and 12 days, is \$450.72, what is the rate per cent?*

5. The interest on \$786.55 at 10 % is \$176.44, what is the time?

6. The interest of a certain sum of money for 4 years, 2 months, and 20 days at 6 % is \$100, required the principal?

* The pupil can prove his own work by computing the interest by the method taught in the first part of this book.

XXXVI. GAUGING.

The process of finding the capacity of barrels, etc., is callen GAUGING.

ART. 1. *To find the capacity of a vessel in the form of a cylinder*, square the diameter in inches, multiply by the length in inches, and the product by 34, then point off four figures from the right, and you have the capacity in wine gallons.

1. Find the capacity in gallons of a cistern measuring 8 feet in diameter and 10 feet in depth.

SOLUTION.— 8 ft. = 96 inches; 10 ft. = 120 inches. $96 \times 96 \times 120 \times 34 = 3760.1280$, or $3760\frac{128}{1000}$ gal.

NOTE.—To find the capacity in bbls., divide the number of gal. by $31\frac{1}{2}$ (the number of gal. to a bbl.).

2. Find the capacity, in gallons and barrels, of a cistern measuring 10 feet in diameter and 12 feet in depth.

ART. 2. *Having the head and bung diameters, to find the mean diameter* add two-thirds of the difference to the head diameter. *To find the capacity of a barrel or cask*, ascertain the mean diameter and proceed to solve as under Art. 1.

1. A cask, having for the head and bung diameters 30 and 36, and length 40 inches, holds how many wine gallons?

$36 - 30 = 6$. $\frac{2}{3}$ of $6 = 4$. $4 + 30 = 34$ *mean diam.*
$34^2 = 1156 \times 40 \times 34 = 157.2160$ *gallons.*

2. Find the capacity of a barrel measuring 17 inches at the head, 21 inches, bung, and being 2 feet 3 inches long.

3. What is the capacity of a barrel, having the head diameter 36 inches, bung diameter 40 inches, and length 46 inches?

ART. 3. *Having the top and bottom diameter* of a vessel in the form of a frustrum of a cone, to find the mean diameter add half of the difference to the smaller.

1. Find the capacity in gallons of a vat, in the form of a frustrum of a cone, the diameter at the top being 5 feet, and at the bottom 7 feet, and the depth 6 feet.

SOLUTION.—5 ft. = 60 inches. 7 ft. = 84 inches. 60 from 84 = 24, half which (12) added to 60 (the smaller diameter) = 72 inches mean diameter. $72 \times 72 \times 72$ (depth in inches) $\times$ 34, etc.

2. What is the capacity in gallons of a vat, the top and bottom diameters being 4 and 6 feet, and the depth $6\frac{1}{2}$ feet?

3. How many gallons will a vat hold, measuring 6 feet at the top, $6\frac{1}{2}$ feet at the bottom, and 7 feet in depth?

ART. 4. To find the number of gallons of linseed oil in a barrel, add one-third of the number of pounds to the net weight in pounds of the barrel, and divide the sum by 10 (there are $7\frac{1}{2}$ pounds of linseed oil to the gallon).

1. How many gallons of linseed oil are contained in a barrel weighing 315 pounds net?

SOLUTION.—$\frac{1}{3}$ of 315 = 105 + 315 = 420. 420 divided by 10 = 42 gallons.

2. Find the contents in gallons of a barrel linseed oil weighing 324 pounds.

3. In a barrel of linseed oil weighing 298 pounds, are how many gallons?

Answers: 157.216 gal., $35\frac{1}{2}$ gal., 119.369 barrels, 3760.128 gal., 1269.0432 gal., 42 gal., 43.2 gal., 39.733 gal., 233.835 gal., 7050.24 bbls. 223.82 bbls. 954.72 gal., 1359.74 gal, $32\frac{1}{10}$ gal.

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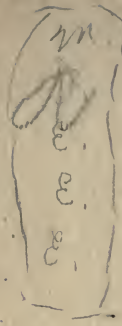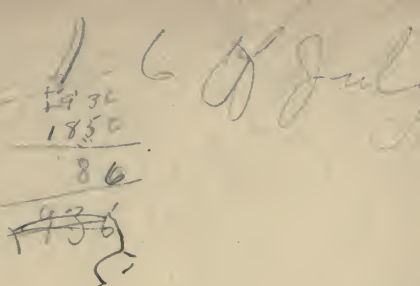
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