BOSTWICK-BRAUN COMPANY BUILDING, TOLEDO, OHIO

A. Bentley & Sons, Contractors "Mushroom System"

CONCRETE-STEEL CONSTRUCTION

PART I—BUILDINGS

A TREATISE UPON THE ELEMENTARY PRINCIPLES OF DESIGN AND EXECUTION OF REINFORCED CONCRETE WORK IN BUILDINGS

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PREFACE

When we consider the fact that fire losses in Canada and the United States amount each year to half a billion dollars, and that the question of commercial economy and cost determines whether buildings shall be built of fireproof and incombustible materials such as reinforced concrete, or of inflammable materials such as are used in timber construction, it is evident how important it is to the general public to be able to determine on theoretically correct principles whether safe fireproof buildings can be built at practically no greater cost or even less cost than combustible ones. In case of any uncertainty as to theoretical principles to be applied, the designer is compelled for safety to employ materials in such lavish amounts as to render cost prohibitive.

Engineering writers have heretofore failed to apply the mathematical theory of elasticity to those forms of reinforced concrete that differ from beams in their manner of reinforcement and that depend for their mechanical action on bond shear as it is involved in multiple way systems. The present treatise is devoted to discussion of the elementary principles and the practical problems presented by building work in reinforced concrete. In it an effort is made to dissipate differences of opinion due to lack of familiarity with the mechanics of reinforced concrete such as tend, at the present time, to retard its introduction and to hinder to some extent the rapidity of its progress in the commercial field.

The endeavor is made in this treatise, to bring out these mechanical laws, and to treat at length the restraint imposed upon the elements of the composite structure—the steel and the concrete—in accordance with the fixed principles of physics and mechanics in order that rational rules may be more generally adopted for the safe and economic design of this type of fireproof buildings. Failure on the part of the engineering profession to consider these laws in drawing up building codes, has led to grave errors thereby offering a premium on the more dangerous types of designs in concrete building work, and placing at a disadvantage the more scientific, safe, and conservative types of work as determined by the records of these types in practical construction.

While the mechanical laws above referred to are simple in the extreme when once understood, their application is so far from obvious in a superficial consideration of the subject that it has required continuous investigation and patient study on the part of the authors for years of time to determine and classify their operation according to the mechanical principles and laws of physics, and this supplemented by expert observers engaged for many months in the conduct of experiments and tests to decide the questions which this continued study and consideration had raised.

In this treatise patented as well as unpatented types have been included for the reason that while the consulting engineer without license has no right to design patented types, he is called upon to report upon their strength and should for that reason be as familiar with their analysis as with unpatented constructions.

The hope of the authors, who have devoted so much time and expense to the investigation and presentation of the fundamental principles explained in these pages, is that as these principles become more widely known and understood needless accidents and loss of life in the erection of concrete building work will be avoided and unsatisfactory designs caused by the failure of the engineering profession at large to understand and introduce into practice the proper limitations of steel ratios as depending on the relative thickness or depth of beam and slab to span and the correct arrangement and disposition of the steel in the slab, will disappear from the engineering field.

The frank avowal of this aim, carrying with it as it does a criticism of no small amount of work by the profession at large, might be considered egotistical were the engineer authors unqualified by long experience to speak with some authority, the one as professionally occupied with applied mathematics, having investigated and taught higher mathematics and structural mechanics for nearly fifty years, the other as a bridge and structural engineer engaged in the active practice of his profession for twenty-five years and responsible for the execution of two thousand concrete structures which have been erected without any serious accident to the workmen on them which could be charged to the risk of such erection, and this notwithstanding the fact that much of this work has been carried on in the unfavorable temperature conditions of the northern winter weather.

The requirements of economy in the arrangement and dissemination of the reinforcement have been dealt with at considerable length. There are many who have the idea that if they get the steel into the concrete somehow that is about all that is required, while as a matter of fact, upon the position and arrangement of the reinforcement in the concrete, the strength of a slab may readily vary a hundred percent or more depending upon its lateral distribution, and the stiffness may vary four hundred percent, while with the vertical distribution the strength may vary fifty to eighty percent and the stiffness five hundred percent, and by combining the vertical and lateral arrangement of the same metal the strength may vary four to five hundred percent, and the stiffness three thousand percent.

That these differences are not understood by the layman is not surprising, but they should be understood by the professional engineer who has had every opportunity to observe the deportment of finished structures under load, as have those in charge of building departments thruout the country.

But aside from these questions of relative arrangement and disposition of materials is one underlying advantage possessed by concrete construction whose dominating effect is apt to be overlooked. That advantage inheres in the monolithic character of this form of construction, which imparts to it a stability and strength which has too frequently not been properly taken into the account either by the layman or the responsible designer. Following the ideas current in ordinary structural design where the whole is built up by assembling and joining together a number of independent elements or units these preconceived ideas have led to the attempt to analyse these monolithic structures into separate members which are assumed to act independently as they do in steel structures. Such assumptions lead to conclusions that have little relation to the facts as shown by tests and by experience as well.

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Native laborers placing reinforcement steel—Mushroom System

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Hughes-O'Rourke Co., Contractors, Dallas — Southern State Steel Co., Consulting Engineers

CONCRETE STEEL CONSTRUCTION

CHAPTER 1.

1. Introductory

The history of structural engineering as a science dates from the early part only of the last century. The progress made has been remarkable indeed, and the materials mainly used have varied during well-defined periods. Up to 1860, timber, wrought and cast iron were mainly used; from 1860 on wrought iron with some cast iron was generally employed in bridges and other engineering structures; from 1890 to the present time steel has replaced wrought iron; and while, for long-span bridges, it will perhaps be some time before a more suitable metal is found, yet for short spans, buildings, warehouses and the like, the enterprise of the manufacturers of Portland cement has placed at the disposal of the engineer a new material, reliable, if properly handled, and of reasonable cost, which bids fair to *largely supplant* steel in the construction of minor engineering works. Indeed today, a warehouse designed for a capacity of 400 pounds per square foot of floor, columns 16 to 24 feet centers, can be built more cheaply of reinforced concrete than of wood frame and floor, with similar brick walls. Where the strength required is less, timber at the present rate is slightly cheaper, since the cost of centering is the same for light as for heavy construction. Still, the difference is so slight that, considering the saving in insurance, owners will shortly be convinced that they cannot afford to continue the construction of fire traps if they are to realize the ~~maxium~~ maximum profit on their investment.

The strength of Portland concrete in compression is equal to that of good building stone, with the advantage that it can be placed in monolithic masses. Its tensile strength, like stone, is greatly inferior to that in compression. Concrete yields but little, the stretch being confined to the weak section. When, however, steel is embedded in the concrete and properly disseminated thro it, the deformation or stretch is distributed greatly by the metal.

The conditions leading to the combination of concrete and steel in a beam or girder are these: Concrete is an excellent and trustworthy material for compression and steel for tension. Hence steel should be distributed in such manner as to carry the tensile stresses of the chord and web. To do this economically we can reason from analogy of a truss or beam. The further from the neutral axis the more effective the steel section, hence the reinforcement for tensile chord stress should be at the bottom of the beam or as close to it as satisfactory protection against heat by fire will permit. Now the beams in a building are of constant section, and since a continuous beam is stiffer and stronger than a beam of the same section discontinuous over supports, the ideal concrete-steel beam should be continuous and the top flange reinforced over supports.

As Mörsch states in his treatise:—

> "Practice has been far ahead of theory. The principal question in controversy has been whether the tensile strength of concrete in bending should be considered. Among practical builders this was decided at the start, and decided against its inclusion, because absolutely no attention is paid to it and the steel is stressed to the safe limit. The tensile strength of the concrete is entirely ignored. On this assumption is based the first method of the theoretical computation of slabs devised by Koenen in Berlin in 1866 and his method has been used by the majority ever since."

2. Historical

Steel and concrete as a combination of materials for engineering structures is much older than is generally supposed. Professor Barbour of the University of Nebraska, in a very interesting lecture delivered to the Cement Users of the State of Nebraska, described certain flat arches discovered in the ruins of ancient Rome, which were for a long time a puzzle to engineers and architects until it was found that they were tied together and the thrust in large part resisted by iron tie rods. So far as known, little was done, however, in the way of combining the old Roman concrete with iron, except in the isolated instance just cited, and it was not until 1855 that iron was combined with concrete in a manner similar to that in which it is utilized at the present time.

At the Paris Exposition of 1855, Lambot exhibited a boat made of reinforced concrete, while François Coignet is credited with having built floors and pipes, in the construction of which he had combined steel and concrete to some extent.

In 1867, Scott, a Lieutenant Colonel of Engineers of the British Army, took out a British patent on concrete floor slabs reinforced in one direction, and also in two directions, and in some of the drawings in this early patent woven fabric is found combined with rods.

In France, Joseph Monier took out patents about the same time, and to him, perhaps more than to any other person, is to be given credit for the commercial introduction of reinforced concrete on a large scale. His first use of this type of construction was to fabricate large plant tubs which he found more durable than those of wood, and more readily transported than those of cement without reinforcement. In 1867 he took out his first French patent, which he soon followed with a number of others on reservoirs, floors, and straight and arched beams in combination with the floors, etc. In 1884 the Monier patents were purchased by the firms of Freitag and Heidschuch in Newstad-on-the-Haardt and Martinstein and Josseaux in Offenbach-on-the-Main. Later the patent rights in Germany were sold to Engineer Wayss, under whose supervision tests were made in Berlin, the results of which were published in 1887, and on the basis of these experiments Wayss succeeded in introducing the Monier system into many structures.

The Scott patent is especially significant since the specification states that tie rods and hoop iron were to take the tensile strains and concrete the compressive stresses, showing clearly that Scott understood the basic principle of reinforced concrete. Little was done, however, by the earlier inventors in developing a working theory of design.

Herr Wayss conducted certain tests in Berlin and published his results in a pamphlet entitled "Das System Monier, Eisengerippe mit Zementumhullung." In this pamphlet Wayss expressed the opinion that the steel must be placed where the tensile stresses occurred. The tests were witnessed by government officials as well as by private engineers and architects. Government Architect Koenen, now Director of the Actiengesellschaft fur Beton-und Monierbauten, in Berlin, was commissioned by Wyass to work out methods of computation from these tests, which were published in the volume of the "Zentralblätter der Bauverwaltung" for 1886

Mörsch, Concrete Steel Construction, 1907 thus comments on the introduction of reinforced concrete at this period (1886).

> "Commencing at that time (1886) a theoretical foundation was evolved, according to which the design of reinforced concrete work would be effected, and through these preliminary labors, this method of construction was extensively adopted in Germany and Austria. A turning point in its development was the International Exposition in Paris, in 1900, and the report by von Emperger, published at that time in regard to the position which the subject occupied.
>
> Because of the scientific investigation of reinforced concrete during the past few years, it has made rapid progress in Germany.

> It was specially promoted by the publication, in 1904, through the cooperation of experts and practical men of the "Leitsätze" of the Verbands Deutscher Architekten- und Ingenieurvereine and the Deutschen Betonvereins as well as by the Regulations of the Prussian Government, which abolished many restrictive rules, cleared the way, and inspired in the widest circles confidence in the new method of building."

In 1870 Phillip Brannon made what appears to have been the first application for an English patent on reinforced concrete piles. The patent was granted in 1871, showing reinforced concrete piles with longitudinal reinforcement of angle irons united by bars riveted across them, the whole being wound spirally with wire.

Thaddeus Hyatt between the years 1873 and 1881 took out between thirty and forty different patents relating to reinforced concrete work, pavement lights, floors and slabs. It does not appear, however, that Hyatt made a success of concrete construction commercially altho he did made a success of his paving lights. Hyatt regarded a reinforced concrete beam as one corresponding to a steel beam, and he considered the rods as equivalent to the beam flange and the concrete as the top flange, assuming the neutral axis at mid depth of the beam.

A most important patent was granted to Hyatt in 1874, in England, No. 1715, in which is disclosed spiral and vertical reinforcement for columns, which strangely enough was not appreciated until attention had been called to this type of column by its reinvention at a later date by Considère, to whom the engineering profession is indebted for the attention which he directed to it by the valuable tests carried out by him.

During this time there was considerable activity in the United States. E. L. Ransome was building reinforced concrete warehouses as early as 1884, and patented in the United States a twisted bar reinforcement.

In 1883 John F. Golding secured an American patent for expanded metal which was employed as lathing for plaster in lighter gages, and as reinforcement for concrete slabs with larger mesh and a No. 10 or heavier gage.

In Cassell's Reinforced Concrete, published in 1913, the following sketch is given of the work of the early pioneers of the art before 1900, Edmond Coignet and François Hennebique:—

> "The former of whom, by applying the known principles of mechanics, evolved a system of calculation that has proved remarkably truthful, and the latter of whom, basing his methods of calculation upon results obtained in practice, has also made extremely important contributions to the technical consideration

of the subject. Coignet as the scientific investigator, and Hennebique as commercial organizer, are properly regarded as 'the pioneers of the modern evolution in the art of building.' The story has often been told of the opposition which Coignet had to fight in getting the masonry of the proposed new system of main drainage in Paris in 1892 replaced by reinforced concrete. He promised a large saving of money and of time required for construction, and his system, which was finally adopted, was carried out with complete success. Hennebique, having organized a technical staff and licensed a large number of the most influential contractors to work his system, was able to secure between the years 1892 and 1899 work to the total value of two million sterling, representing three thousand constructions, among the most remarkable of these being the bridge of Chatellerault, 460 ft. long, comprising three arches, two of 133 ft. span and one of 167 ft.

Hennebique's first patent dates from 1892, (British patent, No. 14,530), and in this he demonstrates the utility of stirrups to reinforce beams against shear, in which matters he had to an extent been anticipated by Hyatt in 1877 and Meyenberg in 1891. In 1897 Hennebique introduced cranked-up rods, and placed these one above the other, so as to reduce the width of the beam, following (to some extent) the lines laid down by Hyatt in 1877 and F. G. Edwards in 1892, in which latter year M. Koenen and G. A. Wayss, of Germany, patented in England a method of floor construction with rods cranked-up at the point of contraflexure, "the parts in tension being strengthened by roughened or serrated metal rods or strips embedded in the structure."

We have noted the early work of Ransome in the United States. In his work parallel joists about 3 inches wide, spaced three feet centers were frequently used, while narrow intersecting ribs about ten feet centers in two directions surmounted by a thin slab were also employed.

From 1890 to 1900 while cement was relatively high in price, reduction of mass at increased expense in form work was to be expected.

The expanded metal companies introduced a large amount of short span concrete floors on structural steel frame six to ten feet center to center of beams, competing with hollow tile arches then more commonly employed for fireproof construction.

The Roebling Company, manufacturers of wire, were at the same time putting in a large amount of short span arches and slabs reinforced with wire fabric and rods.

From 1900 to date, reinforced concrete building construction has increased with wonderful rapidity, encouraged by the enterprise of the manufacturers of Portland cement in placing at the disposal of the constructor a reliable and uniform product at so low a cost that a most powerful impetus was thus furnished for the more complete development of the commercial possibilities of reinforced concrete in building construction.

The skepticism of building departments and the natural antagonism of tile interests forced the advocates of concrete construction to make innumerable tests with the final result of creating confidence in the construction when properly designed and executed.

During the six years subsequent to 1900—a short period of time but an epoch from the standpoint of progress in concrete construction—numerous beam theories were proposed and discussed.

Engineering opinion gradually crystallized in the adoption of a modification of the common theory of flexure and the assumption of the linear law of distribution of stress for purposes of computation of beams and slabs, and this opinion has been embodied in nearly all building codes in cities throughout the United States and Canada.

In 1904 a Joint Committee of the American Association of Cement Manufacturers, American Society for Testing Materials, American Society of Civil Engineers, American Railway Engineering and Maintenance of Way Association, was appointed to investigate and report on concrete and reinforced concrete. After eight years, a report was rendered, for which see Eng. News. Feb. 6, 1913.

The report specifically states that it does not go into all types of construction or all the applications to which concrete and reinforced concrete may be put, * * * * * it is not a specification but may be used as a basis for specifications."

Treatment of natural types of reinforced concrete is lacking in this report. The treatment of beams, however, embodies the crystallized opinion above referred to and will be referred to more at length later.

That a theoretical treatment of concrete after the manner of structural iron work, although making allowance for the elastic properties of the two materials, is unsatisfactory was early recognized by practical men and also by some theoretical writers.

Marsh in his treatise on Reinforced Concrete, Edition of 1905, Part V. p. 209, makes the following remarks on this subject:

> "When properly combined with metal, concrete appears to gain properties which do not exist in the material when by itself, and although much has been done by various experimenters in recent years to increase our knowledge on the subject of the elastic behavior of reinforced concrete we are still very far from having a true perception of the characteristics of the composite material.
>
> "It may be that we are wrong from the commencement in attempting to treat it after the manner of structural iron work and that although the proper allowances for the elastic properties of the dual material is an advancement on the empirical formula at first employed and used by many constructors at the present

time, yet we may be entirely wrong in our method of treatment.

"The molecular theory, i. e. the prevention of molecular deformation by supplying resistances of the reverse kind to the stresses on small particles, may prove to be the true method of treatment for a composite material such as concrete and metal. This theory is the basis of the Cottançin construction which certainly produces good results and very light structures, and M.Considère's latest researches on the subject of hooped concrete are somewhat on these lines."

In this statement of Marsh there is some idea of a possible new principle of action; but unfortunately he was unable to form and express any conception of how this might operate in accordance with mechanical principles in a manner which would be of benefit to the industry at large, or aid in the discussion of the stresses operating in reinforced concrete.

The practical constructor has an advantage over the theorist in this respect; having observed the results obtained by a new principle he immediately profits by it by taking advantage of the results through application of the principles of simple proportion leaving the explanation as a matter of academic interest to follow in the wake of his practical accomplishment. In this simple manner two thousand structures of the Mushroom type have been erected and tested for strength and deflection before a comprehensive scientific explanation of the mode of operation was forthcoming.

The constructor wants the least theory possible and that the simplest to meet the specific requirements of the work he has in hand. The scientist with broader and more comprehensive vision sees in the specific performance of the builder only a special case to be treated in conjunction with the species to which it belongs. That the latter treatment is by far the more difficult—and when correctly carried out more valuable and satisfactory—compensates for the almost inevitable position that the scientist occupies in a new art, following in the wake of the practical constructor, whose simple needs require special rather than general solutions of the problems at hand.

As the authors now view the art broad general solutions of the problems in reinforced concrete are in order and the character and nature of the new properties added to concrete by the dissemination of steel through it brought out in the following pages it is hoped may harmonize many differences in engineering opinion existing at the present time relative to the more advanced forms of construction.

In the discussion of concrete-steel construction, we must consider, first, the action of the concrete with the steel, the function of each in the combination, the problems presented by beams, slabs, and columns separately, and finally the mix of the concrete and questions of cost in convenient placing of the reinforcement.

Before taking up these points in detail it would seem in order, however, to turn our attention to the concrete, and the materials entering into it, their characteristics, value and fitness and the proper proportions to use.

3. Materials

Portland Cement only should be used in a reinforced concrete frame or structure. The following specification is recommended by the American Society of Civil Engineers:

Portland Cement

Definition: This term is applied to the finely pulverized product resulting from the calcination to incipient fusion of an intimate mixture of properly proportioned argillaceous and calcareous materials, and to which no addition greater than 3% has been made subsequent to calcination.

Specific Gravity

The specific gravity of cement shall not be less than 3.10. Should the test of cement as received fall below this requirement, a second test may be made upon a sample ignited at a low red heat. The loss in weight of the ignited cement shall not exceed 4%.

Fineness

It shall leave by weight a residue of not more than 8% on the No. 100, and not more than 25% on the No. 200 sieve.

Time of Setting

It shall not develop initial set in less than thirty minutes; and must develop hard set in not less than one hour, nor more than ten hours.

Tensile Strength

The minimum requirements for tensile strength for briquettes one square inch in cross section shall be as follows and the cement shall show no retrogression in strength within the periods specified.

Neat Cement

Age	Strength
24 hours in moist air	175 lbs.
7 days (1 day in moist air, 6 days in water)	500 lbs.
28 days (1 day in moist air, 27 days in water)	600 lbs.

One Part Cement, Three Parts Standard Ottawa Sand.

7 days (1 day in moist air, 6 days in water)	200 lbs.
28 days (1 day in moist air, 27 days in water)	275 lbs.

Constancy of Volume

Pats of neat cement about three inches in diameter, one-half inch thick at the center, and tapering to a thin edge, shall be kept in moist air for a period of twenty-four hours.

(*a*) A pat is then kept in air at normal temperature and observed at intervals for at least 28 days.

(*b*) Another pat is kept in water maintained as near 70° F. as practicable, and observed at intervals for at least 28 days.

(*c*) A third pat is exposed in any convenient way in an atmosphere of steam, above boiling water, in a loosely closed vessel for five hours.

These pats, to satisfactorily pass the requirements, shall remain firm and hard and show no signs of distortion, checking, cracking, or disintegrating.

Sulphuric Acid and Magnesia

The cement shall not contain more than 1.75% of anhydrous sulphuric acid (SO_3), nor more than 4% of magnesia (MgO).

4. Quick Tests

The preceding specifications and methods of investigation presuppose the conveniences of a testing laboratory to be at hand. The constructor, when he comes upon a job, is frequently without such conveniences. He is frequently compelled to decide whether the cement is suitable or not by employing such rough and ready means only as are at hand, and a few words as to such practical methods of investigation as must be used are in order for his benefit.

Fineness

The constructor can readily determine whether the grinding is reasonably coarse or not by feeling of a sample between the thumb and finger without recourse to screens or sieves.

The effect of fine grinding upon the cement is to render samples of mortar made of sand and cement stronger. In other words, it gives the cement a greater sand carrying power; it renders it quicker setting; a stronger concrete is obtained, or a larger proportion of sand can be used with finely than with coarsely ground cement with the same resulting strength.

In making briquettes of neat cement, however, the coarsely ground cement may show higher results, but what the constructor is interested in is the result obtained with the mortar paste of sand and cement in the usual proportions.

Accelerated Test

The object of this test is to bring out and make evident those qualities which tend to destroy the strength and durability of a cement. As it is highly essential to determine such qualities at once, tests of this character are for the most part made in a very short time, and are known, therefore, as accelerated tests. Failure is revealed by cracking, checking, swelling or disintegration, or all of these phenomena. A cement which remains perfectly sound is said to be of *Constant Volume.*

Failure to meet the requirements of the accelerated tests in shipments direct from mill need not be sufficient ground for rejection. The cement may be held for twenty-eight days and a re-test made at the end of that period. But failure to meet the requirements at this time should be considered sufficient cause for rejection.

The accelerated test is a rough and ready means for determining without elaborate equipment whether cement is fit to use. Cement known to have been stored by a dealer for some time should be promptly rejected if it fails in this test.

If a Portland Cement passes the accelerated test it may be used immediately with reasonable certainty as to its ultimate soundness.

Method of Testing

The method of making the accelerated test, is as follows: On a piece of glass about four inches square, take a sample of the cement and mix it to a consistency such that the cement can be readily kneaded without crumbling and at the same time not so soft as to run or lose its shape when pressed into a smooth pat with a thin edge. Place the pat so formed under a moist cloth for a period of twenty four hours in a temperature from sixty to seventy five degrees and then expose it to an atmosphere of steam. Or, if preferred, the specimen after curing as above for twenty four hours may be placed in cold water, which is raised to and maintained at the boiling point for several hours. Three to four hours is the usual period. Under this test the pat should harden without cracking or swelling.

Causes of Unsoundness

Cracking, crumbling, or disintegration of work in Portland Cement concrete properly mixed and laid may be caused by an excess of lime; by under burning or by an excess of magnesia in a thoroly burned cement, producing gradual expansion which will disintegrate the mortar or concrete even after several years.

Care of Cement

The inspector should see that the cement is properly housed when delivered to the job and protected from the elements so that it will not be damaged by moisture. Dampness from insufficient protection will render the cement lumpy and while it may not destroy its setting properties it will greatly reduce its sand carrying power and efficiency or may even render it entirely worthless.

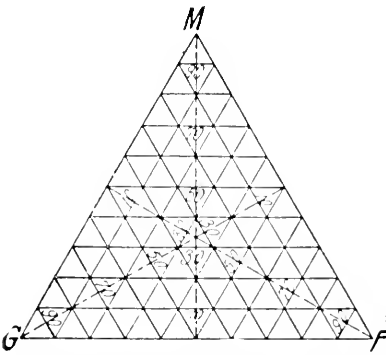
5. Specification for Aggregate

Sand: Sand used should be clean and coarse, or a mixture of coarse and fine grains with coarse grains predominating, which should be free from clay, loam, mica and other impurities.

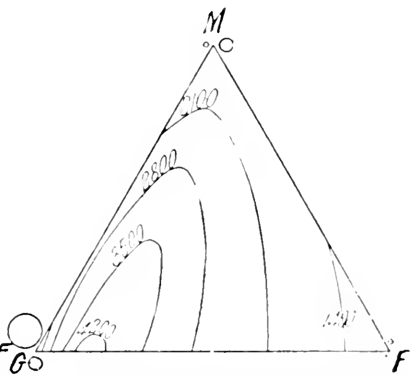
Testing Sand: In order to determine the amount of clay, dirt or other impurities, a simple, practical test is to take an ordinary quart glass preserve jar, put in a pint of sand, fill with water and put on the cap. Shake thoroughly and allow it to settle. The result will be that the coarser grains will go to the bottom in the order of their size, and the silt and light impurities will settle in a layer at the top, giving the observer a means of gaging the amount of the impurities accurately and judging of the character of the sand and the proportion of coarse, medium and fine grains in its make-up. From three and one-half to four percent of clay in the form of finely divided silt will do no harm in a bank sand or gravel for reinforced concrete work. Even higher percentages than this have been claimed to increase the strength of the concrete under test, though where it is exposed to the elements and the action of frost a percentage even as high as this seems to be quite detrimental. However, in building work, which is usually under cover, it does no harm whatever.

The effect of the size of the grains of sand has been investigated by Feret. The accompanying figure from Johnson's Materials of Construction, shows results obtained by Feret on a 1 : 3 mortar after hardening one year in fresh water. The sand used consisted of various proportions of fine grains up to .5 mm, medium .5 to 2mm, and coarse 2 to 5mm, and in the diagram the strength of the mortar is recorded in the triangle at such distances from the base line as represent the proportions of each size of sand used, the line of equal strength being wherever drawn in the diagram. Thus the strength of the mortar in which only fine sand was used was only 1400 pounds per square inch. The maximum strength of 3500 pounds per square inch was obtained from a mixture containing 85 percent of coarse sand and 15 percent of fine with a very little sand of medium size.

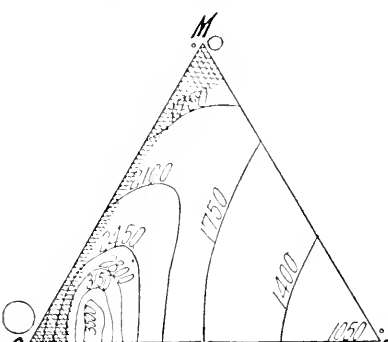
Any point of an entire contour line represents a sand made up of the different sizes G, M, and F in proportions corresponding to its perpendicular distance from the sides opposite each apex but having the same strength as every other point on the same line. This diagram shows that a considerable variation in the proportion of coarse and fine grains will make a mortar of the same strength, but that, in general, the strength of a mortar with fine sand of uniform size of grains is about one half or less than one half that of a mortar made with the same proportion of sand with grains ranging from coarse to fine, and that in general the strongest mortar is secured with a coarse sand with grains ranging from coarse to medium.



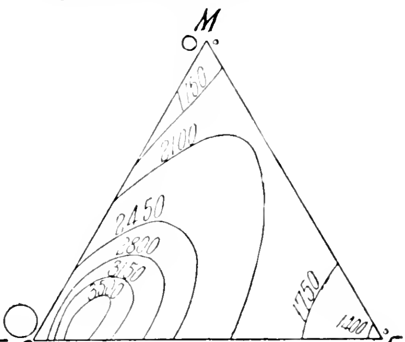
—Showing the Method of Representing Proportionate Mixtures of Three Ingredients. G = coarse sand, 0.2 in. to 0.08 in. in diameter. M = medium sand, 0.08 in. to 0.02 in. in diameter. F = fine sand less than 0.02 in. in diameter.



—Compressive Resistance of Portland-cement Mortars, in pounds per square inch, after nine months in air and then three months in sea-water. Mortar 1 C. : 3 S. in all cases, but the composition of the sand varying according to position in the triangle.



—Compressive Resistance of Portland-cement Mortars, 1 C. : 3 S., in pounds per square inch, *after one year in sea-water.* Shaded part indicates mixtures which were partially disintegrated.



—Compressive Resistance of Portland cement Mortars, 1 C. : 3 S., in pounds per square inch, *after one year in fresh water.*

The effect of an excess of clay, such for instance as the dust from soft magnesium limestone will sometimes greatly retard the hardening of cement, the writer having seen instances where concrete at the age of a month had not attained twenty five percent of its normal strength and where the ultimate strength was reduced perhaps not more than twenty five or thirty percent by the use of this improper mixture.

In many specifications, clean sharp sand is called for in spite of the fact that in many parts of the country sharp sand is not obtainable. Sand with rounded grains such as lake or beech sand is perfectly satisfactory, there being little difference in strength between the mortar made with sand of angular or sharp grains and that with rounded grains. The idea that a sharp, angular aggregate is necessary for strong concrete is the basis for the objection made by some to lake or bank gravel as a coarse aggregate, while as a matter of fact better results and stronger concrete is generally secured with the round pebble than with angular stone, provided the specimen tested is not less than six months old.

Gravel where used, should be composed of clean, hard pebbles and sand free from clay and other foreign matter, such as rotten stone, hardened lumps of clay and the like. A sample having the coarser materials screened out may be tested for impurities in the same manner as was given for the sand.

Broken Stone: Broken stone used should consist of sound crushed stone, such as trap rock, limestone, granite, hard sandstone or conglomerate. If the texture is crystalline and there are no portions of rotten stone or hardened clay such as is sometimes found in oölitic limestone and shale, the crusher run may be used, if a part of the sand content which would otherwise be used in the mix be left out equivalent to the fine particles of crushed stone in the crusher run.

If, however, the stone can be readily reduced under the hammer to a fine, impalpable powder as is the case with some shales and with the oölitic type of limestone referred to, the dust should be entirely removed.

It is better, where possible to use only that stone which is found durable when exposed to the action of the elements and frosts, and the harder the stone the stronger the concrete that may be made when using it as the aggregate.

6. Proportions of Materials

In concrete building construction the proportions which experience indicates most economical in concrete for slab and beam construction, columns and column footings except for the case where the loads to be carried are unusually great, is one part of cement and two parts sand with four parts broken stone or gravel, this being indicated by the expression 1 : 2: 4. These proportions are customarily taken by measure, each bag of cement being estimated as equal to one cubic foot in volume, thus the proportions of 1: 2: 4 mean one sack of cement (94 pounds) two cubic feet of sand and four cubic feet of crushed stone. Too frequently the inexperienced builder interprets a 1 : 2 : 4 concrete to mean a 1 : 6 aggregate or six parts of gravel which may be two thirds sand and one third coarse aggregate. The proportions of 1 : 2 refer to the mortar and mean the ratio of the cement to the volume of sand which runs from about one eighth inch down in size, while coarser material than this may be considered as coarse aggregate or the stone content.

The size of the stone in reinforced concrete work in ordinary building construction should range from one inch down, observing directions for screening as detailed under the specification for broken stone.

The first requirement in proportioning the aggregate for reinforced concrete work is to see that there is an excess of fine material over and above that required to fill the voids in the coarse component of the aggregate. The volume of voids in coarse aggregate is greater with a uniform size of stone than where the sizes of coarse aggregate vary from coarse to fine, and for that reason the crusher run of stone is preferable where the stone is granite, trap or hard crystalline stone. In heavy mass work, however, a larger proportion of coarse aggregate with the size of stone varying from three to four inches in diameter down can be advantageously used, but this is unsuited for reinforced concrete work in the ordinary building line. It is only suitable for bridge piers, and heavy mass work.

7. Analysis of Strength of Concrete

Concrete may be defined as an artificial conglomerate stone in which the coarse aggregate or space filler (generally a hard natural stone, furnace slag or pebble) is held together by a cement mortar matrix. Having selected a given coarse aggregate, the strength of the concrete depends on the strength of the mortar matrix, in other words, on the ratio of cement to sand in the mortar for all samples of the same age, formed under the same conditions.

The Strength of the Concrete depends then:

First, on the grade of sand and the proportion of the cement to the sand in the mortar.

Second, upon the hardness and the character of the coarse aggregate.

Third, on manipulation and the conditions under which the concrete is cured or hardened.

Fourth, on the age of the specimen.

Mortar made with a very fine sand is only about half as strong as that made with coarse and medium grains and for that reason the specification regarding the character of the sand should be given careful attention.

As shown by Feret, quite a variation in the proportion of medium, coarse and fine grains of sand will give nearly the same strength so that the average clean coarse bank sand will generally fill the requirements for a good concrete mortar.

The richer the mortar the stronger the concrete. As noted above, we recommend a one-to-two mortar for reinforced concrete, and where high working stresses are to be used in reinforced concrete columns we would recommend a mortar in the concrete as rich as one cement to one and one half sand, obtaining thereby an increase of twenty five percent in the permissible working stress.

Coarse Aggregate

The effect of the strength of the coarse aggregate upon the strength of the concrete, in tests of concrete made with shale rock crushed to one and one half inches or under, at Duluth, show the shale concrete about sixty-five to seventy percent as strong as trap rock concrete and the trap rock concrete from ninety to ninety-five percent as strong as that made with lake gravel for the coarse aggregate. These tests were made on concrete about four months old.

Manipulation and Conditions of Curing

While the quality of the cement, sand and aggregate have more or less influence on the resulting concrete, with any good brand of first class Portland Cement, clean coarse sand and hard crushed stone, substantially the same results will be secured under identical conditions of mixing and curing. The latter conditions have a most decided influence on the strength of the concrete, viz., whether sufficient water has been used to permit and promote perfect crystallization of the cement, whether an excess amount of water has been used and the fine and coarse materials have been allowed to separate or become segregated, whether the concrete has been thoroly mixed, and whether the conditions for curing were favorable, such as keeping the concrete damp and preventing it from drying out too rapidly or whether it was hardened under unfavorable circumstances in frosty weather. On this account it is difficult to harmonize the large number of isolated tests that have been made by independent investigators under widely varying conditions.

In building work, however, it is a fortunate fact that except in cold weather where the work requires special treatment the general conditions for hardening are most favorable. After one floor has been poured the next is erected thereon within a week or so and the excess water dropping from the upper floors keeps the concrete in the lower properly wet, rendering the conditions more favorable for hardening and curing than those of the ordinary laboratory test.

8. Hardening of Portland Cement.

The hardening of Portland Cement is a chemical process which within certain limits is accelerated by heat and retarded by cold. This is an important consideration for the builder to keep in mind, since when the temperature of the water used in the mix and the aggregate is at or approximately near the freezing point, the cement lies dormant and no fixed rule can be given of a set number of days during which time it is necessary for the concrete to lie in place on the forms before it will attain a certain given degree of strength. In hardening, as in nearly all chemical reactions, heat is generated by the setting of the cement. This heat is radiated away very rapidly where the mass is small or thickness of the part of the concrete work is inconsiderable, while where the mass is large, as in the case of heavy walls, piers, and the like, the heat generated by the setting of the cement is not lost rapidly by radiation and the work tends to cure much more rapidly in heavy work in cold weather than in the case of the thin slabs usually used in the floors of buildings. Special directions for the treatment of concrete at various temperatures will be given under a special section dealing with building work.

Increase of Strength of Concrete with Age

The following table shows compressive strength of concrete as determined by test made at the Watertown Arsenal in 1899. 1 : 2 : 4 mixture.

Brand of cement	7 days	1 month	3 months	6 months
Atlas	1,387	2,428	2,966	3,953
Alpa	904	2,420	3,123	4,411
Germania	2,219	2,642	3,082	3,643
Alsen	1,592	2,269	2,608	3,612
Average	1,525	2,240	2,944	3,904

The above gives a fair idea of the increase in strength of concrete with age under normal temperature above 60° F.

After a period of six months the concrete in ordinary building is found to increase slowly in strength and considerably in hardness and rigidity. Thus it appears that the stiffness of a long span slab will increase about twenty percent between two months and twelve to fifteen months, and the strength perhaps in a lesser ratio, on the assumption that the compression element only in the combination is hardening and increasing in strength.

Coefficient of Expansion

The coefficient of expansion of concrete is practically the same as that of mild steel. Some investigators have made this coefficient per degree of Fahrenheit slightly less and others slightly more than .0000065, which is usually accepted for mild steel, hence ordinary changes of temperature cause no injury to the composite material formed by embedding steel in concrete.

9. Bond between Concrete and Steel.

In the design of any combination of concrete with steel the bond between the two elements is of prime importance. Concrete setting in the air shrinks and grips the reinforced members with a vice-like grip. The richer the mixture the greater this shrinkage stress and the better the bond. In concrete setting in water this shinkage is lacking and in this case deformed reinforcement or mechanical bond is desirable.

The bond between the concrete and steel has a maximum value with a plastic mix of concrete such that the mortar will flow slowly and thoroly surround the metal. It is greatly reduced with a stiff mixture requiring tamping and at the other extreme also is less with too sloppy a mixture of concrete.

With plain round rods embedded 12 inches the bond value may reach, under favorable conditions, a maximum of 600 pounds per square inch with concrete of a 1 : 2 mortar, six months old, but with dry tamped concrete the bond value or adhesion as it is sometimes called may run as low as 200 pounds per square inch of the surface of the bars.

A round bar will give a higher bond value than a flat or rectangular shape, while a polished or cold rolled shaft as it comes from the mill will give a value hardly more than a fourth as great as that. Slight rusting of the surface improves the adhesion or bond since the rust combines chemically with the cement and seems to increase the shrinkage grip. Further, slight rusting tends to remove the black mill scale making the adhesion uniform along the surface of the metal.

Paint, oil or grease, greatly reduces the adhension. With properly arranged reinforcement the designer rarely has occasion to figure upon the bond value between the two materials, since it is amply provided for where due precaution has been taken to render the design safe to execute by properly tying the materials together by an ample lap of the metal over the supports and the use of *sufficient cement.*

Occasionally the designer may be forced to use short stock lengths in beams or slabs, and under such conditions a working stress not exceeding forty pounds per square inch is permissible providing the bars are also hooked at the ends. Even with this additional precaution care must be exercised in keeping the work supported much longer than would be necessary with preferable lengths of rods. The reason for this precaution and the low value recommended is that the bond strength between concrete and steel varies greatly with the age of the concrete and like the shearing resistance it is very low with partly cured concrete but increases with the age and hardness of the work, tho less rapidly than the resistance in compression.

The constructor should keep clearly in mind the important conditions which insure satisfactory bond between the metal and concrete, namely: A mixture of *proper consistency containing sufficient cement* and the preferable use of bars round in section, which are most readily surrounded by the plastic concrete in flowing.

It is interesting to note in the following table of tests of bond that the average adhesion of the $\frac{1}{2}''$ round rods was twenty percent more than that of the square rods with the 1 : 3 mortar; that the adhesion to the steel with the broken stone concrete was greater than with the 1 : 3 mortar or even the neat cement test; and that the adhesion to the quarter inch by one inch averaged only six tenths that of the half inch rounds.

TABLE I

SHOWING ADHESION OF VARIOUS SHAPED RODS TO CONCRETE OF CONSTANT COMPOSITION

1 Part Cement to 3 Parts Sand
*9–10–11–12, 1 Cement, 2 Sand, 4 Broken Stone
(Even numbers 40 days—Odd Numbers 80 Days

No.	Section of Steel	Length Embedment ins.	Perimeter, ins.	Load in lbs. at Failure	Area of contract sq. ins.	Load in lbs. per sq. ins.	Average per sq. in.
1	½″ square...	$6\frac{1}{8}$	.....	.....	12.25	...	...
2		$6\frac{1}{2}$	2.0	5,700	13.00	438	432
3		6	.....	5,200	12.00	433	...
4		$6\frac{1}{4}$	.....	5,300	12.50	424	...
5	½″ round....	6	.....	4,600	9.43	488	...
6		6	1.571	5,000	9.43	530	512
7		6	.....	4,600	9.43	488	...
8		$5\frac{7}{8}$	.....	5,000	9.23	542	...
9	¼″ x 1″....	$6\frac{1}{4}$	.....	4,900	15.63	313	...
10		$6\frac{1}{4}$	2.5	4,400	15.63	282	293
11		$6\frac{1}{8}$	.....	4,800	15.20	314	...
12		$6\frac{1}{4}$	.....	4,400	15.63	282	...
5	1″ square...	$10\frac{1}{4}$	.....	17,400	41.0	424	...
6		$10\frac{1}{8}$	.....	15,800	40.5	390	411
7		$10\frac{1}{8}$	.....	17,000	40.5	420	...
8		$10\frac{1}{4}$	.....	16,800	41.0	410	...
* 9		$10\frac{1}{8}$	.....	21,200	40.5	523	...
*10		$10\frac{1}{4}$	.....	24,600	41.0	600	587
*11		$10\frac{1}{8}$	.....	24,200	40.5	598	...
*12		$10\frac{3}{8}$	.....	26,000	41.5	627	...

Test by Emerson, Eng. News, 1904, p. 222.

That the bond between the concrete and steel is really a shrinkage grip may be easily proved by the simple experiment of molding some concrete and placing a piece of round steel on top, lightly pressing it into the concrete without immersing it more than one third the diameter. When the concrete has cured it will be found that there is very little difficulty in removing the steel. If the piece, however, is pressed into the concrete to a considerable depth and the concrete in its plastic condition allowed to flow around the bar it will be very difficult indeed to remove and it will be found that this is caused by the shrinkage of the concrete around the bar in hardening. That this bond between the concrete and the rod is not due to direct adhesion may be further proved by splitting the concrete about the bar or sawing it down to the side of the bar on each side when it will be found that the bar is readily removed from the concrete.

The following table is given by Professor Hatt. The tests were made by drawing out the rods. In this table it will be noted that the depth of the rod in the concrete is much larger than in the tests by Emerson, while some tests by Feret with the rod embedded $2\frac{3}{4}''$ give values approximately half as great as where the length embedded is from ten to twelve inches.

TABLE II.

Diameter of rod inches	Age of specimen in days	Depth of rod in concrete inches	Adhesion in pounds per square inch of surface in contact		
			Maximum	Minimum	Average
7/16	32	72	735	470	635
5/8	35	76	780	714	756

The adhesive resistance varies somewhat with the depth to which the rod is embedded.

It is greater with a rough than with a smooth surface.

It increases with the proportion of cement up to a certain limit.

It is a maximum with a plastic mix and a minimum with a dry mix and tamped concrete.

It increases with the age of the concrete.

Considère finds that for concrete exposed to air the amount of water used in mixing has a great influence, too dry concrete adhering badly. An excess of water giving the concrete the necessary fluidity for filling up the voids around the reinforcement produced the best results. He considered, however, that this advantage of wet concrete was counterbalanced by a notable diminution of tensile and compressive resistance. This would be true were it not a fact that the excess water in casting reinforced concrete work in building construction is readily disposed of by absorption of the forms and leakage through them.

The low values found by some investigators for adhesion or bond seem to be readily accounted for by the prevalent French custom of tamping dry or stiff mixtures of concrete rather than of pouring, if of a wet or plastic consistancy, as is done by the American constructor today. The early idea was that good concrete could only be made by a dry mix and tamping. Combined with this dry mix the deformed bar was unquestionably an improvement, but as the use of the dry mix has long been abandoned the main advantage in the use of the deformed bar has largely disappeared with its abandonment.

By far the greater number of concrete failures have occurred where deformed bar reinforcement has been used. This is due in part probably to the fact that some types of deformed bar reinforcement are such that with ordinary care the metal is not so well surrounded as in the case with plain rounds, and the shrinkage grip of adhesion is less with the irregular section. Further, the designer seems frequently to place too great confidence in the bond strength in these designs, and has frequently neglected a sufficient lap over the support, to render the design safe and conservative.

Plain reinforcement would no doubt not have done much better than deformed bar reinforcement with the same arrangement and and length of lap, but over confidence in the deformed bar reinforcement has had a tendency to lead to a type design inherently dangerous with any type of reinforcement.

10. Variation in the Strength of Concrete with Variation of Temperature and Moisture.

This question is of the greatest importance to the constructor in putting up work. The concrete, partly cured, may apparently be stiff and rigid when the forms are removed in cold or freezing weather. Then with a sudden change in the temperature, such as may readily be brought about by putting a heating plant into the building, the concrete will sweat and soften and get out of shape. Again, when concrete which has had as long as two to three months in which to cure during the fall season, is exposed all winter, soaked with water and the water frozen, and in the spring a heating plant is put into the building and the slab thawed out, its strength is temporarily greatly reduced, and if the slab is carrying the weight of other stories which are shored from it, permanent deflection and serious trouble may result.

The older and more thoroly cured the concrete, the more rigid it is and the less the variation in strength resulting from the conditions above noted. Concrete which is thoroly soaked with water is less rigid in compression than concrete which is thoroly dried out. This change in strength is due to the fact that the concrete expands with moisture and shrinks or contracts as it dries out, this action being greater with concrete during the hardening stages.

Hence, the constructor should use care and see that his work is not over-loaded, particularly at the time when he is firing up the heating plant in a building in which the floors, though they have had some time to cure in the fall, have been thoroly soaked and frozen.

Undue confidence in the strength of partly cured and frozen concrete arises from observing that when the forms are first removed there is no deflection and the concrete stands up apparently of ample strength and rigidity under the superimposed load of the centering of one or more stories above. A sudden change in temperature, causing the moisture in the slab to thaw and expand in the concrete will so weaken the slab that a large permanent set will result. The cautious builder will keep the floor well shored up until he is sure that it is thoroly dried out by heat so that there is no chance for the work to get out of shape as above explained.

The extent to which the strength of a slab may be reduced even after it has been once fairly well cured, but subsequently exposed to the weather, soaked and frozen is illustrated by a case where it was so softened in thawing out as to deflect eight times as much as it did under identically the same load after drying out and exposure to heat for three weeks, so that the importance of the precaution above outlined should be apparent. In this case the slab was cast in the latter part of August, was fairly well cured but exposed to the weather, flooded by rain and frozen up during the winter. The heating plant was placed under the floor sometime in March and a light load then placed on the slab which had a span of about 22 feet and was 7 inches thick, well reinforced. A deflection resulted of approximately 2 inches. The slab returned to its original shape after removal of the load, and when thoroly dried out the deflection under the same load was hardly 3/16 inches.

Deflections are found to be increased where the slab is wet, and the strength is apparently somewhat diminished.

11. Machine Mixing.

Concrete for a concrete steel building should be machine mixed, preferably in a batch mixer. Some of the continuous mixers do good work where bank gravel is used as the aggregate and fail where crushed stone is used. A batch mixer such as the Smith, Cube, Polygonal or Ransome, is to be preferred because it may be charged with cement, sand and stone by measure and the exact amount of water added. The water content in the mix is best supplied for a large piece of work by a tank which will contain the amount of water needed for a batch arranged with the usual float trap valve so that all the operator needs to do is to pull the string and the tank of water is discharged at once in the mixer. This insures a mixture of uniform consistency and effects a material saving of time.

Where the work is of sufficient magnitude to permit an overhead hopper into which the sand and stone may be elevated, and discharged by gravity into the mixer as desired, a large saving in labor is secured. Where the mixing plant is near a track the hopper may be filled from the cars by a derrick and suitable clam. Where the aggregate is brought to the building by team load, a platform arranged so that the wagon may be driven over it and the stone or sand dumped there-

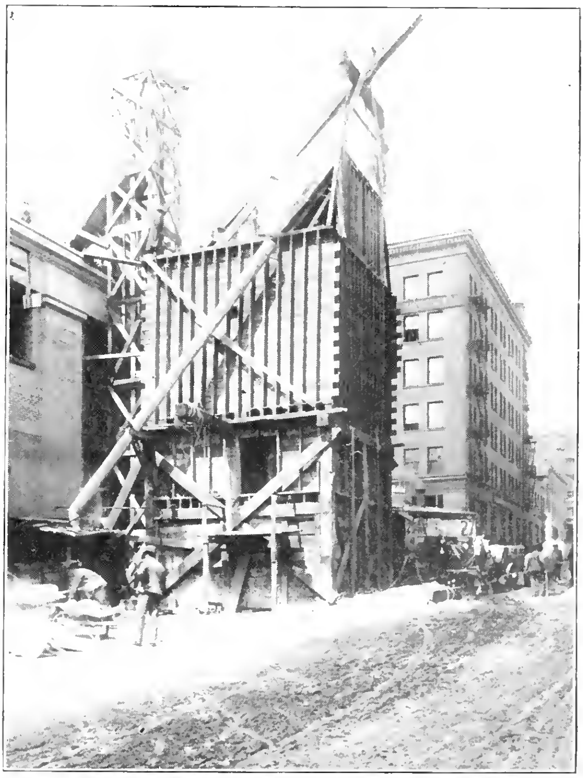


Fig. 1. Mixing Plant, Lindeke-Warner Building, St. Paul.

on and then elevated and discharged into the top of the hopper, is about as economical an arrangement as the writer has seen.

A view of a mixing plant of this kind used in the erection of the Lindeke-Warner building of St. Paul, erected by Butler Bros., is shown in Fig. 1.

In Fig. 2 is shown the mixing plant used in the erection of the John Deere Plow Company's building in Omaha, the hopper in this case being charged by the locomotive crane, using the clam for transferring the materials from the cars to the hopper.

Consistency of Concrete

For building construction and reinforced concrete work generally it is necessary that the concrete shall be mixed so that it will flow slowly and thoroly surround the reinforcement but it should be no more plastic than is required to attain this result. If mixed too dry and tamping is depended upon, voids will be left around the steel and the face of the concrete when the forms are removed will

Fig. 2. Mixing Plant and Hopper, John Deere Plow Company Building, Omaha.

be found rough and full of pockets and the work will present an appearance of weakness which it very likely does not possess.

12. Pouring Concrete.

In pouring concrete, the lowest portions of the forms should be filled first: Thus necessitating the least possible flow of the concrete to reach its final position in the work. In buildings the columns should be filled first, then the beams and finally the slabs, the operation being continuous as far as practicable. If an attempt is made to reverse this program and fill the beam before the column is filled the concrete will flow in an inclined direction to the column and as each batch is deposited the more liquid portions washing over the inclined surface carry the light laitance and fine sand down into the column, and an inferior concrete and one of little strength will be found at the bottom on removal of the column forms.

In pouring columns, especially where closely spaced spiral hooping is used, concrete should be poured over the center. If an attempt is made to fill the column from the side the space between the form and hooping is filled up to a considerable height in advance of the central core, the hooping acting as a screen prevents the coarse aggregate from flowing to the lower central area with the result that the mortar flows to the core and seals to some extent the voids toward the hooping and the next batch cannot flow to fill in the voids thus left in nearly clean coarse aggregate between the hooping and the column form. This leaves rough unsightly work when the forms are removed and while the core may be sound the fireproofing is inferior in character or worthless, and the work presents an appearance of weakness which it does not possess.

Where to Make Joints in the Work and How to Do It

Splicing of concrete in beams and slabs should preferably be made in the center and should be vertical. The reason for this is that where the concrete is allowed to flow out on an inclined plane in the beam the inert material known as laitance comes to the surface, preventing a good bond when the new concrete is added. In fact instances have not infrequently been observed where wedge shaped pieces of concrete three feet long and running from two inches in thickness to one quarter inch at the end dropped away from the beam owing to this manner of placing, the bond being insufficient to carry the weight of the piece. The remedy is to break up the surface of the old concrete and grout it with a neat cement before proceeding to cast the new work.

In very hot weather the crushed stone may readily get so dry and hot that it absorbs the water from the mix and causes the cement to set too rapidly. When this is the case an open crack will appear at a joint. The remedy is to first cool the stone by thoro wetting with the hose.

13. How to Determine when Concrete is Thoroughly Cured.

It may be noted that most failures occur in the cold season, and the need for a certain and simple method of determining whether the concrete has been merely hardened by frost or really cured is obvious. A good way in cold weather work is to cast a few small sample cubes or cylinders when pouring the floor, and allow them to harden under the same conditions as the slab, and these samples may then be examined at any later time. When this has not been done, cut out a small piece of the concrete and place it over a stove or radiator and if the concrete has been merely stiffened by frost it will sweat and soften up, while if cured it will remain firm, dry and hard.

In the previous pages, dealing with the hardening of concrete the fact has been emphasized that curing has no direct relation to the number of days that a floor has been poured. Hardening is to be considered first in connection with the temperature and humidity during the period of curing, and second in connection with the treatment of the materials in cold or chilly weather as to whether the water and aggregate had been properly heated.

In the summer season, during ten or twelve days of continued chilly, rainy weather, concrete frequently hardens so slowly that the early removal of the forms at that period will result in permanent damage to the work, so that the humidity and rain must be taken into consideration as well as the temperature under which the concrete is cured, in deciding the feasibility of removing false work.

In the chilly weather of the spring or fall, if the concrete has been mixed with cold water and subsequently chilled by frost before it has had time to set appreciably it may be very slow in curing. It is exceedingly difficult under such circumstances for the most expert to tell when the false work may be removed and no undesirable results follow. The concrete may after such treatment, (improper mixing with cold water at a time when the weather is chilly and frosty) apparently be hard and subsequently sweat and soften during the continued hardening process and the work get out of shape. That is, the slab or beam may deflect permanently $\frac{1}{2}$ or $\frac{3}{4}''$. Such deflection, while it will not result in permanent weakness after the concrete has finally hardened, will cause the owner to question its strength. It may cause partitions, which were built upon it at its original elevation, to be left unsupported and a year or a year and one half after the inelastic sag has occurred, the partitions will commence to crack and come down to a bearing. The owner will feel certain that the concrete work is weak, although it has hardened up and is of ample strength, tho not in the position in which it was left when the forms were removed. These troubles are entirely obviated by the proper treatment of the materials in mixing, as we have explained heretofore.

As a fair example showing the results that may be secured by the proper treatment of the concrete, we may cite the case of a building in Fort William, Ont., in which the roof slab was cast on the seventh day of December, at a temperature twenty degrees below zero. On the 15th, a fire was started by the carelessness of a workman in placing a kettle of pitch with which cork board was being applied too near a salamander and the centering burned out from under this roof completely and also from beneath a part of the slab below. Notwithstanding the season of the year and the low temperature at which the work was cast, these slabs which were approximately seventeen feet in span, stood up very well. The under side of the concrete, however, was somewhat pitted by the formation of steam in small cavities and the forcing out of small chunks of the concrete thereby. In this case the pitting caused by the fire was readily remedied and the work put in shape at a comparatively small cost.

14. Handling Concrete Above Freezing.

Handling concrete to get the best results requires quite different treatment at different times depending on the temperature. Perhaps the most favorable conditions under which concrete may be placed are at temperatures ranging from 45 degrees to 50 degrees Fahr. Under these conditions the concrete does not dry out too rapidly and while it may set slowly it hardens up better than when the temperature is higher.

In hot, dry weather the moisture dries out of the concrete too rapidly, requiring for the best results, that the work be wet down with a hose, particularly during the first day's exposure to the sun after casting. Wetting should commence as soon as the surface of the concrete has set.

Frequently in the hot sun large cracks will open up due to the rapid evaporation of the water. These can, and should be promptly filled with a bucket of grout. Sometimes the stone, when exposed to the heat of the sun will become so dried out and hot that it will absorb the water rapidly from the mix and the heat of the stone will be sufficient to cause the concrete to set before it can be spread in place. Wetting the rock pile down will eliminate this difficulty.

At all temperatures below 45 degrees Fahr., it is best to warm the water and wake the cement up, otherwise it is liable to set too slowly to enable the forms to be safely removed at the usual intervals common in warm weather.

In putting up a large building at Toledo, the contractor wired to Minneapolis for the writer to visit the building, stating that the cement in the entire third floor was not setting up and had been in two weeks. The writer was unable to leave immediately and arrived at the building three days after receiving the telegram. The weather had turned warm in the meantime and the cement had started to set. There were places, however, where a twenty-penny nail could be pushed into the concrete with the thumb to its full length. The cement had been mixed with water ice cold, and had lain dormant during the chilly weather which succeeded the two weeks after placing the concrete. For the remaining stories, after instructing the foreman to see that the water with which the concrete was mixed was heated to about 120 or 130 degrees there was no difficulty and the forms were promptly removed every ten or twelve days per story although the weather was much colder as the season advanced.

Concrete Below Freezing

In freezing weather it is desirable to wake up the cement by using hot water. Water may be heated to 160 or 180 degrees and the sand and stone mixed with water in the machine before adding the cement. The water will thus warm up the stone and when the cement is dumped into the mixer the temperature will probably be in the neighborhood of 120 degrees.

When the temperature is below zero, boiling hot water is sometimes used. The sand and stone are first placed in the mixer, then the boiling water is added to warm up the sand and stone, and finally the cement, when the sand and stone has been warmed up and the water has been cooled down to a temperature of not more than 120°. This method of procedure is necessary in order to not kill the cement by the boiling water.

At low temperatures salt may be advantageously used. The proportion of salt which it is desirable to use for temperatures between zero and 25 above zero is a pint and one-half of salt to each batch containing two bags of cement. Below zero, a little more than a pint per sack, and extra care is to be taken in heating the materials and seeing that the concrete gets into place *hot*.

For these very low temperatures it is much better to heat the stone and sand over a coil of steam pipe if such is available. In one case concrete was placed at 28 below zero and the work in this case was executed in a very satisfactory manner. In addition to the use of hot water the gravel used as an aggregate was thoroly heated over a coil of steam pipes. There is much less difficulty in placing concrete in large masses in cold weather than in thin slabs and the like.

In large masses such as thick walls, thick slabs, and the like, the cement generates heat in setting, sufficient to keep the body of the material warm, whereas in thin slabs this is not always the case and the concrete may and frequently does freeze.

It comes then to a question as to how to handle the concrete in the best and most practical manner. In a building having exterior bearing walls the walls are built up, then the slab is cast and artificial heat should be promptly applied on the under side of the slab to sweat out the concrete and enable it to harden up promptly. Window openings may be readily closed with canvas or light cloth.

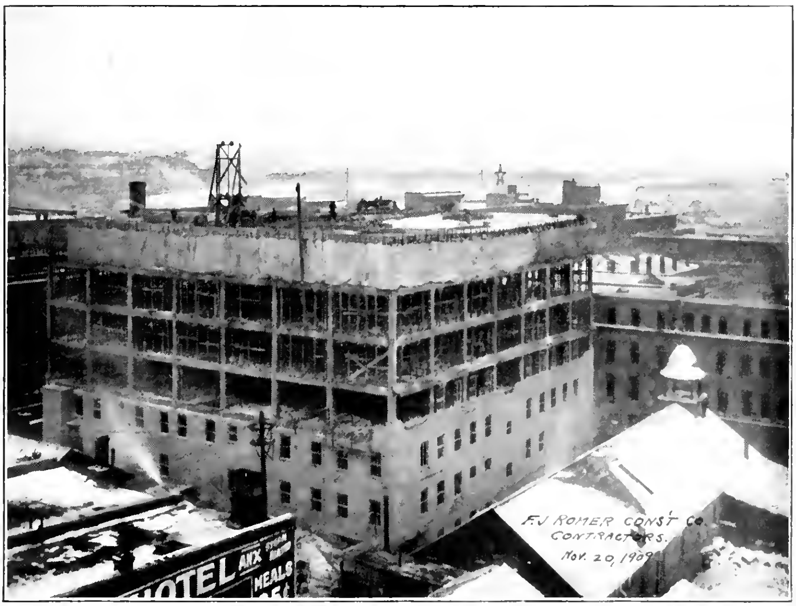


Fig. 3. Canvas curtain protecting an upper story of the Strong-Warner Building, St. Paul, under construction in winter.

Where the building is a skeleton concrete frame it should be protected in winter from outside temperatures by help of canvas curtains in lieu of the walls as shown in Fig. 3. Then the concrete may be artifically heated and hardened.

In a large piece of work the most economical method of heating is to put in a small fan with the usual steam coil, and heat the building by blowing in heated air in the usual manner. In a small building this is too expensive and the resort is had to salamanders and coke for heating.

15. Action of Salt

The action of salt on concrete is three-fold:

First, it lowers the freezing point and by so doing gives the concrete a better opportunity to attain its initial set before freezing.

Second, it tends to retard the setting and allows the materials, the cement, water and aggregate to be heated to a somewhat higher temperature than would be permissible were it not for the salt in the mix.

Third, since salt has an affinity for water, it retains in the concrete the necessary moisture required for perfect crystallization. In other words, it prevents the concrete from drying out before it has had time to set when it thaws after freezing.

Calcium chloride has also been used to some extent to prevent the freezing of concrete in cold weather, but owing to the fact that common salt is so much less expensive and more readily obtained, it is almost universally used by those accustomed to do work in the winter season.

The use of salt in the concrete does not appear to impair its strength in the least, nor does it appear to have an injurious effect on the metal, provided the metal is well covered with wet concrete.

The use of brine in the mixture is particularly advantageous in placing mass concrete in preventing scaling of the surface from frost action and while it may somewhat retard the setting and hardening the ultimate result appears to be a concrete of even greater strength than that hardened under nominally more favorable conditions of warm weather.

Concrete in setting generates considerable heat after the action of setting has started. In cold weather it requires artificial heating to start this chemical action. An experiment was tried on one piece of work when the temperature outside was about 25 below zero. A piece of gas pipe was inserted into a column 36″ square, just cast. A thermometer was dropped down the pipe three feet and the upper end sealed with a cork. Upon removing the thermometer twelve hours afterwards it registered 95 degrees Fahr.

16. Curing Concrete Where Proper Precautions Have Not Been Taken

The engineer is frequently called upon to pass upon concrete which has been placed and the precautions heretofore recommended have not been followed.

We have known of cases where the concrete was placed in December, mixed with cold water, frozen as fast as placed, and when this same material thawed out in March it was as soft as the day when first cast. After the concrete had been kept thoroly wet for two and one-half weeks and then allowed to dry out, a good hard concrete was secured which after eight months stood an exceptionally satisfactory test.

Concrete unless promptly thawed out after it has been frozen, sets so slowly that its hardening may be condemned as altogether too slow for practical purposes if it is expected to clean up the work and get it finished within a reasonable time, and for this reason it pays the constructor well to heat the materials so that the centers may be removed promptly and the work finished up nearly, if not quite, as rapidly as it is ordinarily done in the summer.

Too great care, however, cannot be exercised in handling work during the winter season since frozen or partly frozen concrete may stand well when the forms are first removed and then as soon as it commences to thaw it will begin to get out of shape and look badly if it does not entirely collapse.

Many mistakes in judgment are made in handling work in the cold season of the year, although by the exercise of care and good judgment there is no reason why the work cannot be executed in a thoroly first class and satisfactory manner.

In working during the winter season snow and ice frequently get on the forms. This can be readily removed by the use of a steam hose, melting the snow and ice in advance of placing the concrete.

17. Precautions in Splicing, Mixing, Heating, Etc.

Attention is called pointedly to the necessity of melting snow and ice on old work and on forms before casting concrete and it remains to call attention to the necessity of special care in the splicing work.

The old concrete may be frozen and not hardened. It will be killed or disintegrated by heating with hot water as some thoughtless foremen have tried to do. Splices in the work should be made with great care and in a vertical plane both for beams and slabs. The old concrete should be cleaned of snow and ice with a steam hose, but no hot water used, then the new concrete may be cast against it and the moderate temperature of the new concrete will gradually soften the old work if frozen and the result will be a satisfactory bond between the two. It is preferable where practicable to continue casting until a whole floor is complete unless the work is of too great magnitude.

Inclined splices and irregular joints are very decided sources of weakness in work cast in cold weather, in fact, they can hardly be made good unless by digging out some of the concrete and thoroly grouting the joint after the work has hardened.

The foreman should be cautioned against killing cement by mixing with boiling hot water. Mixing the sand and stone first with boiling water will take the frost out of the stone and sand and warm it up and reduce the temperature of the mix down to 120 or 130 degrees which will not injure the cement. If there is ample salt this temperature may be even ten or fifteen degrees higher for a few minutes and not materially damage the mix.

18. Caution Regarding Removal of Forms.

A word of caution to the builder may not be amiss under this heading. The rapidity of the setting of concrete and hence the time at which it is safe to remove the forms varies materially, depending on the humidity of the atmosphere. In damp, rainy, wet, chilly weather, concrete is liable to set very slowly indeed. In dry weather, and particularly in high altitudes the concrete sets much more rapidly. In central Minnesota for example, under usual conditions, concrete may be counted upon to set more rapidly at temperatures ten or fifteen degrees lower than in situations close to the Great Lakes or in the South where there is a large difference in humidity. The experienced superintendent soon becomes familiar with these conditions for a given locality, but if he moves about it is well to bear these general facts in mind since he will find a marked difference in different sections even with the same cement.

As to the time of removal of the forms, the builder should bear the fact clearly in mind that it is not the number of days time that the concrete has been in place or has stood upon the forms that determines whether it is safe to remove the forms, but the degree of hardness that has been attained during that period. Concrete may remain on the forms for four months in a northern climate, freeze and thaw out in spring and be as soft as the day on which it was placed, if the foreman has been so lacking in judgment as to use cold water to mix the concrete and then allow the material to freeze after it has been placed.

Frequently as far south as southern Kansas, damp, chilly weather so retards the setting of cement not mixed with warm water that after the forms have remained in place a month the construction will not hold its shape, but will sag materially, owing to its half-hardened condition. This will never occur where the simple, inexpensive precaution has been used of warming up the water at all times when the temperature is below 45 degrees Fahr.

It should be borne in mind by the builder, that the slightest sag in the construction, while it may not affect the strength in the least, usually causes the owner to be suspicious of the integrity of the whole work, and as the constructor depends upon satisfied customers in a large measure for future business, and for the prompt payment of the contract price for the work, these matters should receive careful attention.

The danger of accident with half-hardened concrete is comparatively remote with multiple way systems, as this type of construction will almost invariably give the workman ample time to note its yielding and to prop it up before excessive deflection has occurred. Unfortunately, this is not the case with one-way reinforcement. When it once starts yielding as a rule the whole construction goes by the run, and hence from the stand point of safety to workmen, the superintendent should exercise extreme care with this type of construction.

The question of determining the hardness of the concrete and whether it is safe to remove the forms is one which the builder must decide. As a rough test the concrete should be so hard that a twenty penny nail will double over and cannot be driven into it more than three-quarters of an inch. A good idea can be obtained as to its hardness by trying it with a hammer and seeing how readily it can be indented, as well as by driving a nail into it and finding out its condition under the surface. Examining the concrete around openings, etc., will enable the experienced foreman to form a correct judgment as to whether it is safe to remove the centering. In any case, these rough tests are sufficient to determine whether the removal of the forms is safe for the workmen.

In long span slabs or beams there may be some sag owing to compression of the concrete if it has not set sufficiently hard altho no accident may result. Such deflection or sagging tends to destroy the owner's confidence in the work altho it may have no material effect from the standpoint of strength. In fact, where a long span slab or beam has sagged a moderate amount before the concrete is thoroly hard there is generally little loss of strength.

In the case of a slab, if it is afterwards leveled up with additional concrete, it is stronger than that portion of the work which has kept its shape. The owner considers this an evidence of weakness.

The builder knows that if he has filled up the depression in a panel which has sagged slightly it is probably the strongest slab in the building and a test of it will give exceptionally fine results.

The time during which the centering should remain in place varies for different spans. With a slab sixteen or seventeen feet square and seven inches thick under favorable drying conditions, it should be possible to remove the forms in eight or ten days. Where the span is longer, say twenty or twenty-five feet, two to three weeks at least should be allowed unless the slab is extra thick. For example, a slab eight inches in thickness and twenty-five feet in span should be allowed three weeks under the most favorable conditions to thoroly harden if it is to keep its shape immediately after removal of the supporting forms. Whereas a span of the same length, thirteen inches thick would only need practically the same time as the shorter span on account of the additional thickness. These are practical points which it is well to bear in mind as upon them commercial success in a measure depends.

The superintendent should bear in mind, in freezing weather, that concrete is as readily stiffened up by frost as by the true chemical action of hardening and that when thus hardened by frost it only remains for a rise in temperature to occur for the work to get out of shape, if it does not actually collapse. The test in freezing weather to determine whether the concrete has been merely stiffened up by cold or is actually cured, is to dig out a small sample and place it upon a hot stove, then if it sweats and softens the forms must remain in place. If, on the other hand, it remains hard and rigid and does not sweat and soften up then the concrete may be depended upon to retain its shape and forms may be safely removed.

19. Reinforcing Steel

Steel for reinforcement should be tough, homogeneous metal, preferably of structural steel grade, where bending is required, or of harder grade for slab rods when bending is unnecessary.

The following are the specifications adopted by the Association of American Steel Manufacturers, 1910, governing the chemical and physical properties of concrete reinforcing bars:

Standard Specifications for Concrete Reinforcement Bars

1. *Manufacture.* Steel may be made by either the open-hearth or Bessemer process. Bars shall be rolled from billets.

2. *Chemical and Physical Properties.* The chemical and physical properties shall conform to the following limits:

Properties Considered	Structural Steel Grade		Hard Grade		Cold-Twisted Bars
	Plain Bars	Deformed Bars	Plain Bars	Deformed Bars	
Phosphorus, maximum,					
Bessemer.........	.10	.10	.10	.10	.10
Open-hearth......	.06	.06	.06	.06	.06
Ultimate tensile strength, pounds per sq. in.........	55–70,000	55–70,000	80,000 min.	80,000 min.	Recorded only
Yield point, minimum pounds per sq. in .	33,000	33,000	50,000	50,000	55,000
Elongation, per cent in 8″, minimum...	$\frac{1{,}400{,}000}{\text{T. S.}}$	$\frac{1{,}250{,}000}{\text{T. S.}}$	$\frac{1{,}200{,}000}{\text{T. S.}}$	$\frac{1{,}000{,}000}{\text{T. S.}}$	5%
Cold bend without fracture:					
Bars under $\frac{3}{4}''$ in diameter or thickness............	180°d.=1t.	180°d.=1t.	180°d.=3t.	180°d.=4t.	180°d.=2t.
Bars $\frac{3}{4}''$ in diameter or thickness and over.............	180°d.=1t.	180°d.=2t.	90°d.=3t.	90°d.=4t.	180°d.=3t.

The hard grade will be used only when specified.

3. *Chemical Determinations.* In order to determine if the material conforms to the chemical limitations prescribed in paragraph 2, herein, analysis shall be made by the manufacturer from a test ingot taken at the time of the pouring of each melt or blow of steel, and a correct copy of such analysis shall be furnished to the engineer or his inspector.

4. *Yield Point.* For the purposes of these specifications, the yield point shall be determined by careful observation of the drop of the beam of the testing machine, or by other equally accurate method.

5. *Forms of Specimens.* (*a*) Tensile and bending test specimens may be cut from bars as rolled, but tensile and bending test specimens of deformed bars may be planed or turned for a length of at least 9 inches if deemed necessary by the manufacturer in order to obtain uniform cross-section.

(*b*) Tensile and bending test specimens of cold-twisted bars shall be cut from the bars after twisting, and shall be tested in full size without further treatment, unless otherwise specified as in (*c*), in which case the conditions therein stipulated shall govern.

(*c*) If it is desired that the testing and acceptance for cold-twisted bars be made upon the hot rolled bars before being twisted, the hot rolled bars shall meet the requirements of the structural steel grade for plain bars shown in this specification.

6. *Number of Tests.* At least one tensile and one bending test shall be made from each melt of open-hearth steel rolled, and from each blow or lot of ten tons of Bessemer steel rolled. In case bars differing $\frac{3}{8}$ inch and more in diameter or thickness are rolled from one melt or blow, a test shall be made from the thickest and thinnest material rolled. Should either of these test specimens develop flaws, or should the tensile test specimen break outside of the middle third of its gauged length, it may be discarded and another test specimen substituted therefor. In case a tensile test specimen does not meet the specifications, an additional test may be made.

(*d*) The bending test may be made by pressure or by light blows.

7. *Modifications in Elongation for Thin and Thick Material.* For bars less than 7/16 inch and more than $\frac{3}{4}$ inch nominal diameter or thickness, the following modifications shall be made in the requirements for elongation:

(*e*) For each increase of $\frac{1}{8}$ inch in diameter or thickness above $\frac{3}{4}$ inch, a deduction of 1 shall be made from the specified percentage of elongation.

(*f*) For each decrease of 1/16 inch in diameter or thickness below 7/16 inch, a deduction of 1 shall be made from the specified percentage of elongation.

(*g*) The above modifications in elongation shall not apply to cold-twisted bars.

8. *Number of Tests.* Cold-twisted bars shall be twisted cold with one complete twist in a length equal to not more than 12 times the thickness of the bar.

9. *Finish.* Material must be free from injurious seams, flaws, or cracks, and have a workmanlike finish.

10. *Variation in Weight.* Bars for reinforcement are subject to rejection if the actual weight of any lot varies more than 5 percent over or under the theoretical weight of that lot.

20. Quality of Steel

Unfortunately there is an idea prevalent that almost any grade of metal is good enough for reinforcement. Where the contractor or engineer is responsible for the test strength and permanence of the work he needs to see that the steel is of suitable quality.

The product of what is called a fagot mill is generally very undesirable. The trade term, sometimes applied to this product is "Bushel Steel." A fagot is formed using muck bar iron flats for bottom and sides and filling in with miscellaneous scrap steel, iron, etc., heating up and rolling into billets and bars. This grade of metal has an ultimate strength of about 45,000 pounds per square inch and a commercial yield point of 25,000 pounds. It will bend easily but when nicked and broken will show a dull fracture of a ragged and coarse texture in strong contrast with the bright, fine crystalline or silky texture of a good grade of steel.

Rerolled rail stock is sold to a large extent for reinforcing metal. Small rods rolled from the flange and stem of the rails make excellent slab reinforcement while those which are rolled from the heads are liable to be brittle and unreliable. This grade of steel runs from 80,000 to 125,000 pounds per square inch ultimate strength and is much too hard to be safely bent cold.

Cold twisting is safe only with a soft or medium soft grade of metal. The effect of cold twisting is to raise the yield point, reduce toughness and elongation thus rendering the metal more brittle and unreliable and for that reason objectionable. Where the mechanical bond of a twisted bar is demanded hot twisting is preferable for the above reasons.

Hard grade steel has a decided advantage for slab rods of small diameter, since the harder the metal the less liable they are to kink in the handling and shipping. Specifications for small bars such as 5/16 to $\frac{1}{2}$ inch, should require that these bars be shipped in bundles of a dozen to fifteen rods per bundle well wired together, so that they will be less liable to be bent in shipment and can be more readily handled on the work.

21. Cold Bending With Mild Steel

In bending rods for beams, columns or slabs, the method used depends somewhat on the character of the steel. With the kind of metal reinforcement recommended, namely, medium steel, nearly all of the work of bending is done cold and at a comparatively insignificant cost. For instance, bending the column rods for the Mushroom system on one of the large pieces of work cost about fifty cents per ton. In this case a bending machine was arranged using gears similar to those of the ordinary crab hoist, bending the bars by means of a crank pin on the driven shaft. Bars are not damaged to any considerable extent provided that the radius of the bend is not too sharp and that the moving part bending the bar does not jam the metal so that its flow is confined to a short section.

This is one of the difficulties with quite a number of the lever bending machines which we have investigated. Bars were found at one building which were cracked at the bend. Knowing the metal to be good, tough, medium steel, the bending machine was immediately investigated. It was found that a die with a sharp corner had been used around which to bend the bar. One or two bars were broken in handling after bending. Fortunately none of them were in a position where direct tensile stress came upon the metal. This die was immediately ordered to be cut over to a reasonable radius of one and one half diameters of the bars.

22. Bending Machines

Light rods, such as $\frac{3}{8}$ inch and $\frac{1}{2}$ inch and the like, may be readily bent by the use of tongs or a short piece of pipe slipped over the end of the rod. Such tongs are illustrated in the accompanying Fig. 4, and also a lever bender for long rods.

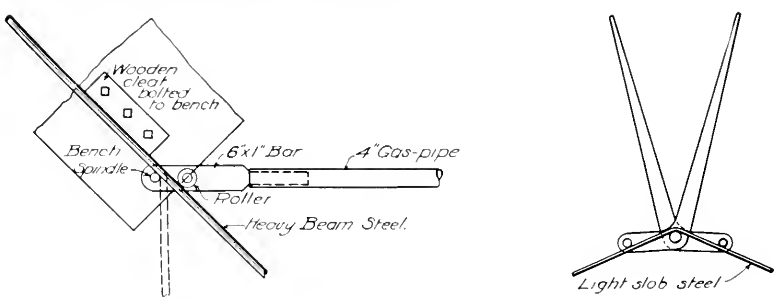


Fig. 4. Cut of Lever Bender and Tongs.

In general, a good detail for a lever bender is to arrange a small roller on the moving part of the bending lever so that the pressure is brought against the bar by a roll and so that there may be no tendency to localize the stretch of the metal at one place by friction, thereby seriously injuring the bar.

For ring rods and the like such as are used in the mushroom system, an ordinary set of blacksmith's tire rolls is the most convenient equipment.

For spirals the same set of rolls is frequently used.

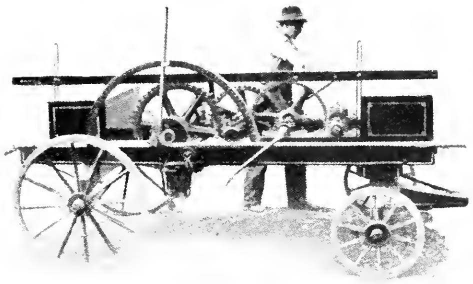


Fig. 5-a.
Hand Power, Star Bender, Bending Column Rods.

The accompanying three figures show a very convenient bending machine, manufactured by Kardong Bros., of Minneapolis, Minn.

Fig. 5a shows the bending of a mushroom column rod, the stop on the circular segment fixing the angle of the bend in a hand power machine.

Fig. 5b shows a form of the machine arranged with a gas engine power for rapid operation.

Fig. 5c is a view showing the bending of beam rods and spiral hooping with this machine.

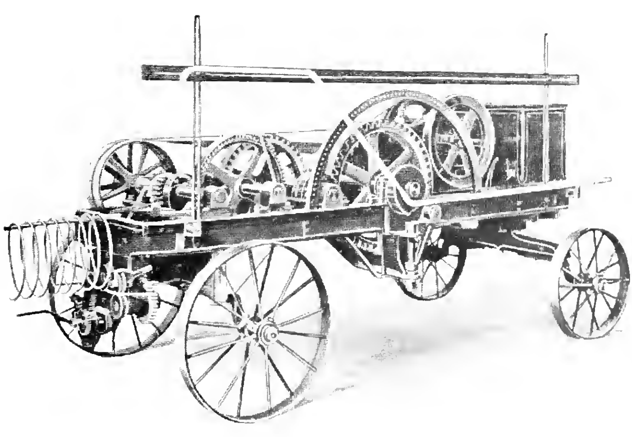


Fig. 5b.
Gasoline Power, Star Bender, Bending Beam Rods and Spirals.

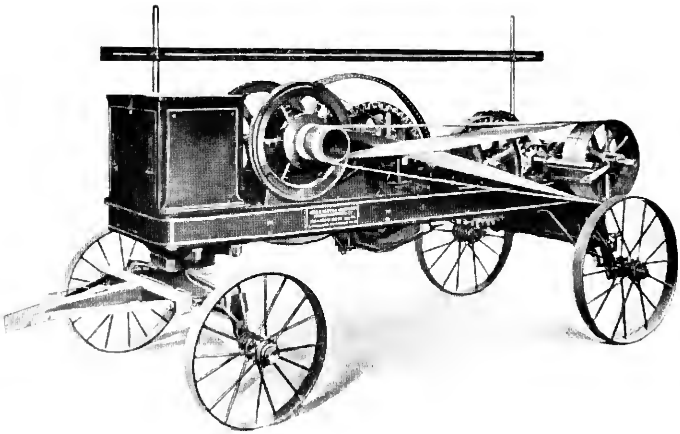


Fig. 5c.
Showing Belt Side of Star Bender.

23. Hot Bending and Precautions with High Carbon Steel

In bending bars hot, which is done commonly where the bars are hard steel over 1 inch diameter, there is sometimes carelessness in overheating the steel. Heating up to a low cherry red is the highest which should be permitted by the foreman in charge of such work.

High carbon steel is more readily injured by overheating than mild steel. It is too hard to be worked cold and can only be bent to the desired form by heating. In heating it is much more liable to be severely damaged than mild steel and hence extra care should be taken when using this grade of metal, to see that it is not burned by the smith.

In bending cold twisted bars, where specified by the architect, they should be invariably heated, otherwise in endeavoring to bend them cold the damage done to the bar by the cold twisting will manifest itself in brittleness of the bar and the tendency to crack or break at the point where the bend is being made.

24. Centering

Centering, or false work, is one of the largest elements of cost in reinforced concrete construction, and accordingly one that should receive careful consideration from the standpoint of cost, and also from the standpoint of the safety both of the work and of the men putting up the work.

We will first consider centering for flat slab and column construction. Seven-eights inch lagging with 2x6″ or 2x8″ joists requires the least lumber and is cheaper with lumber prices from $18 to $22 per thousand such as rule in the middle and Eastern states at present, but with lumber at from $8 to $12 per thousand, plank from ledger to ledger would be more economical, the latter price being common at such Pacific Coast points as Vancouver, Seattle, etc.

Wide boards are undesirable for lagging, since they warp in the sun and swell excessively so that they get out of shape, leaving an uneven surface; a 1x6″ lagging of No. 1 common fencing S1S2E is preferable for the foregoing reason. Matched boards should not be used as the grooves are readily broken out and leave a rough surface.

Joists for an 8″ slab should be not less than 2x6″, sized and about 22″ centers for spans of 6 feet from ledger to ledger, and 2x8″ sized for spans of 7 feet to 8 feet center to center.

Spacing of ledgers should be arranged with reference to the column spacing, so that the line of columns will come approximately at the center of a span between ledgers. Thus for columns 20 feet centers, (the joists running in the direction of the 20 foot span) a spacing of 6′ 8″ for ledgers is economical, using 2x8″ joists, while if the span is 18 feet, 2x6″ joists with 6 foot spacing of ledgers is best.

Working stress for pine and common lumber should not exceed 800 pounds per square inch fiber stress, and due regard must be had for the stiffness of the work, while for Douglas fir and long leaf yellow pine forty percent higher stresses are permissible.

Ledgers and Posts

Evidently the fewer the points of support the less will be the work of leveling up the forms. Four by four posts based on wedges 12 inches long, cut by splitting a 4x4″, three inches at the thick end by one inch at the thin end is the simplest practical adjustment.

For economy the ledgers should be deep, but attention must be paid to the tendency of a narrow but deep beam to cripple by buckling of the compression side or top face. Many centering failures occur from this cause, where a 2x10″ or a 2x12″ has been selected by a table of loads as having a sufficient capacity. Then the supports are placed six or eight feet between centers, and unstayed, when if the ledger twists the failure is so sudden as to give little if any warning.

The unsupported ~~width~~ length ought not to exceed thirty times the thickness, as a practical dimension, the support of the bearing joist not being counted unless the ledger is held laterally by cleats to the joist.

When a deep ledger is used, as two 2x10″ or two 2x12″, it should be double; the verticals should be stayed at the underside of the ledger with a light strip of 1x4″ running transversely to the ledger from post to post and continuous through the extent of the centering.

Formulas for Proportioning Beams and Posts

A convenient formula for the capacity of joists and ledgers as simple beams is, for pine or hemlock.

$W = 100\ b\ d^2 / L$, in which

W = capacity of the joist or ledger in pounds for uniform loading.

b = breadth in inches.

d = depth in inches

L = span in feet.

When full continuity is secured over two spans, twenty-five percent can be safely added to this capacity.

The above formula is applicable to plank flatwise as far as the fiber stress is concerned, but the same fiber stress for a plank or board flatwise will give too great a deflection, so that the plank or board must be figured or selected for stiffness in keeping with span.

For Douglas fir or yellow pine, fifty percent increase in the above safe load is permissible.

Douglas fir or yellow pine timber of 4x4″ vertical posts may be figured as safe for 800 lbs. per sq. in., and Norway pine or spruce for 600, if stayed laterally in each direction by stays six feet apart center to center vertically.

A convenient formula for fir or yellow pine posts is

$$P = 1000 - 10L / B$$

in which P = safe load in lbs. per square inch, L = unsupported length between lateral stays in inches, B = the least breadth in inches. For Norway pine or hemlock take six tenths of the above values.

Fig. 6 shows the centering used in the Minneapolis Knitting Company building, a structure which we have termed type III. The joists used were 2x6s, 20 inch centers, with 1x6″ fencing for the floor. For studding 4x4s are usually used, spaced about seven feet apart, capped by 2x8s double and resting on wedges by means of which the centering can readily be adjusted to the desired level of the finished floor.

For square columns of small section 2x4s spiked together, forming the square ties, are about as cheap as any method of putting the boxes together.

For columns some use 4x4″ side pieces, slotted at the end and $\frac{1}{2}$″ bolts. This allows the same frame to be adjusted for different size columns and makes a very substantial form, but somewhat expensive in first cost. For beam boxes $1\frac{3}{8}$″ plank for bottom and $\frac{7}{8}$″ boards for sides are preferable. For plain slab forms the following is the writer's preference, where lumber is used:

Joist 2x8″, 20 to 22″ centers, 1x6″ fencing for sheathing, 2x10s double for ledgers spaced eight to nine feet apart. Vertical

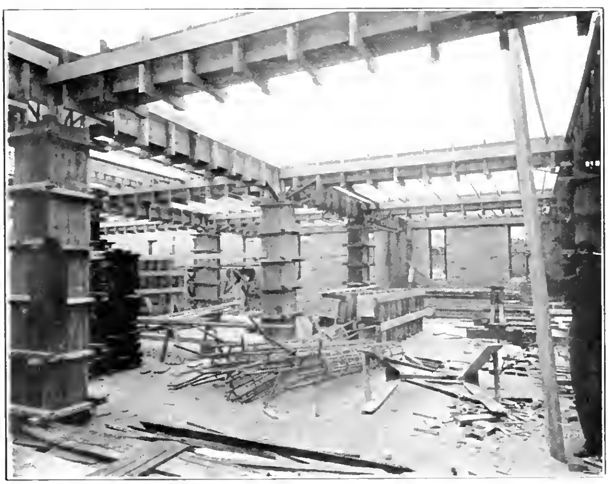


Fig. 6. Centering Northwestern Knitting Company Building.

posts seven to eight feet centers. The 4x4″ verticals butted under the ledger pieces and the ledger was prevented from turning on top of the 4x4s by short pieces of $\frac{7}{8}$x4″, nailed to both ledger and top of the 4x4s with 8s″ nails. The bottom of the posts are best adjusted by wedges 12″ long, cut from 4x4s. This will allow the leveling up of the centering very readily.

In centering shown in Fig. 6 the column boxes are $1\frac{1}{2}$″ stock banded by 2x4″ lapped and fastened together with wire spikes. Beam boxes were made up of $\frac{7}{8}$″ boards and 2x4s forming vertical frame and 1x6″ bottom of same. A light ledger is nailed along the side of the beam box to receive the joist for supporting the slab. The beam box was then braced up and two lines of supports placed under the 2x6″ joist.

Sometimes it is desirable to center by using sizes of lumber which can be worked into boxing such as is used for hardware storage purposes, implement house requirements and the like. In this case verticals can readily be made of 2x6s, but will require additional lateral staying. Verticals are usually stayed every four to six feet in height with 1x4″ ribbons in both directions.

Leveling up Centering

Leveling should be done by using an architect's or an engineer's level.

Evidently the fewer verticals there are the more readily the form can be leveled up and placed in proper condition for casting concrete. After leveling up, the wedges should be nailed so that there will be no slipping. The vertical studs should be stayed along the line of the joist at the top and longitudinally and transversely midway for stories ten to twelve feet in height, so there will be no danger of the stud kicking or buckling and the centering going down should a heavy car run off the track. Where the area to be centered is large it sometimes pays to cleat the sheathing in sections two feet or more wide. This eliminates the necessity for the larger part of the nailing to the joist and enables the taking down of the forms a little more readily.

Wide boards should not be used for sheathing for the reason that they curl and split badly in the sun and swell excessively when wet. For that reason 1x6″ square edge fencing is best. Yellow pine and wood which will stand considerable hard usage is preferable to hemlock or the softer grades of white pine.

Wetting down Wood Centering

Where wood sheathing is used for the forms it should be thoroly wet down from one to two hours in advance of placing the concrete to give the timber which has probably dried out in the sun, a chance to swell and close the cracks so that there will be the least possible loss of cement grout as the casting proceeds.

Inspection of Centering before Casting Concrete

As a general rule the foreman should be instructed to inspect carefully all centering before starting to pour the concrete for the reason that many of the stays and sometimes some of the verticals are left out temporarily for convenience in erection, with the expectation of putting them in before pouring concrete commences.

Column Forms

For octagonal forms we have adopted the standard shown in the Fig. 8 with a cast iron or adjustable sheet metal form for the head.

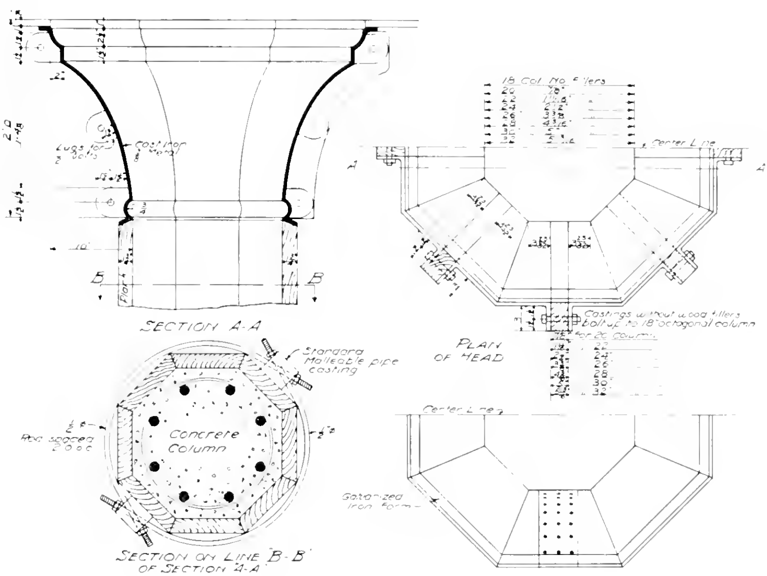


Fig. 8. Column Forms.

The column box is bound together by ½″ rods bent in semi-circular form, with a long thread and nut at the end. These are passed through standard malleable clamps used for wood stove pipe and screwed up.

Another method of making up column forms is to use sheet metal forming adjustable round or octagonal heads, see Fig. 9.

This is one of the most economical types of column forms. It is readily handled, weighs but little and costs but little to transport and is reasonable in first cost.

In general, a light sheet metal form consists of sections which are adjustable by being lapped and are held rigidly by heavy bands of quarter-inch metal at intervals of about two feet.

Sheet Metal for Slab Forms

To save the cost of sheathing and facilitate rapid handling a large amount of corrugated steel in place of fencing has been used for decking.

Ceiling of this type is shown in Fig. 10. This type of floor centering is not suitable where it is desired to plaster, but for a wholesale building or in fact any kind where special decorative finish is not desired it is substantial and neat.

Cost of handling sheet metal is about one-third that of laying boarding. Greasing it with paraffine oil prevents the concrete from adhering and facilitates ready removal and rehandling.

Advantages that are claimed for sheet metal forms are as follows:

That the sheet metal holds the moisture and prevents the concrete from drying out too rapidly. It prevents loss by leakage of the liquid cement mortar, such as sometimes occurs where board forms are used, and leaves a clean, smooth job.

The sheet metal centering can be used over and over again and should it be battered out of shape it is a comparatively inexpensive matter to repress the sheets. At first cost it is at a disadvantage as compared with wood centering, but in the long run it is much cheaper for the reason that it is lighter, requires less labor to handle and involves less labor in carting from point to point. The guage of metal should not be lighter than No. 20.

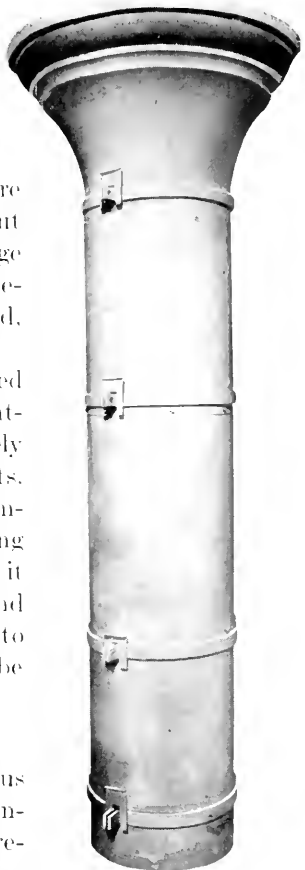


Fig. 9.
Sheet Metal Column Forms.

Improper Specifications for Centering

Many architects have a totally erroneous idea as to the proper requirements for centering. For example, the architects frequently specify matched and surfaced lumber for forms, with the vague expectation that by so doing they will get an exceptionally smooth job. Unless the lumber is over two inches thick, which would involve an unreasonably great expense, the tongue and groove will be soon broken, ragged joints and edges will be a frequent rather than a rare occurrence, and on the whole the work will not present so smooth an appearance as though ordinary square edge fencing was used for the work.

If it is required that the work be finished with a smooth surface the most inexpensive method is to give it a skin coat of plaster as recommended in the sections covering the subject of plastering on reinforced concrete.

Partial Removal of the Forms

It is evident that the earlier the centering can be removed for use in the upper stories the less material will be necessary in handling the work and the lower the cost if successfully executed. In the mushroom system it is customary in good drying weather to remove the forms in from twenty-four to forty-eight hours from columns.

Fig. 10. View showing Ceiling made with Corrugated Steel Forms, Con. P. Curran Building, St. Louis, Mo.

In this type of construction columns carry little weight until after the removal of the slab forms, and by handling the work in this manner a very much smaller number of forms can be used on a large job. Where, however, beam and slab forms are used the column generally supports the beam boxes and the writer is not in favor of removing the centering in part, but prefers to see the whole left standing together except perhaps a few of the stays until the concrete has thoroly cured.

In our illustrations of rapidity of erection of reinforced concrete a number of examples appear which indicate clearly the number of floors under which the centering is left in the conduct of work under favorable conditions.

Handling and making up of forms is more a question of craft than of figures. As to the question of ingenuity the brightest engineer can as a rule learn something from any foreman and even a good carpenter with whom he comes in contact in this line of work. Frequently, however, we see workmen who lack ingenuity and a conception of the simple requirements of form work. For example, we occasionally see a gang of carpenters putting up an expensive braced form for a thin wall, where all that is necessary to do is to set up the cleated boards and tie them together with No. 10 wire. The pressure on the two sides balance and the need of bracing is practically nil.

Special forms, such as are used for chimneys, are very advantageously made up with sheet metal and arranged to be slipped upward as the work advances. It is hardly, however, within the scope of this work to go into special constructions of this character.

CHAPTER II

GENERAL TYPES OF CONCRETE FLOOR CONSTRUCTION

1. Classification.—The history of the development of structural work shows that the engineer has been largely influenced in his first efforts to design any new type by the forms of construction

Fig. 11. Type I.

to which he has been previously accustomed. For example, when wrought iron began to be used in place of timber for railroad trestles the longitudinal bracing was identical with that used in timber construction; indeed it was at first gravely questioned whether these braces ought not to be made of timber for fear of the unknown dangers that might arise from the unequal expansion of these braces of iron and the foundation on which the trestle was supported, and today not a few of our concrete theorists are deeply concerned regarding equally insignificant questions.

The common types of concrete steel floor construction may be classified as follows:

I. The earliest type of timber construction has been followed or imitated closely in some of the pioneer structures in concrete steel and also in not a few of our buildings even today. This type may be described as employing columns to support parallel main girders with joists in one direction only extending crosswise from girder to girder and a thin floor covering the joists. See Fig. 11.

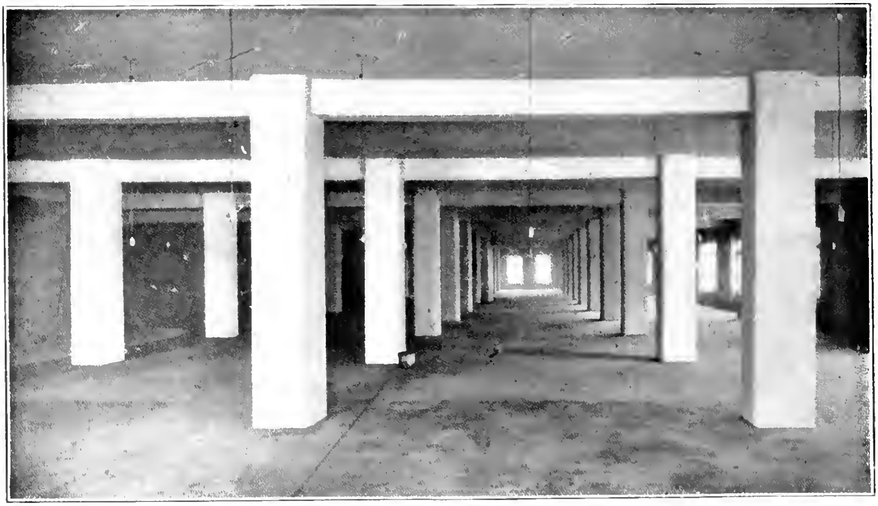


Fig. 12. Type II

II. Similar to I, except the substitution of a slab without joists from girder to girder, similar to mill construction of beams and thick plank flooring from beam to beam. See Fig. 12.

III. A natural concrete type, a true monolith, departing from the characteristics of timber and steel construction in the employment of concrete beams from column to column in two directions and slabs with panels supported on four sides. See Fig. 13.

IV. A second distinctively concrete type, in which the centering is simplified to the limit and consists only of a temporary flooring on which to pour the concrete. The elements involved are two only: column supports, and a continuous flat slab supported directly by the columns and integral therewith. See Fig. 14.

A modification of types I and II is sometimes employed in which arches spring from girder to girder. This modification is not a common type of construction however.

As to safety, these types must be rated with reference to their deportment under overload, whether failure can occur suddenly and without warning, or slowly and gradually.

Failure is more rapid where the flexure under load has a single curvature only under load than where there is double curvature. For example, a slab supported on two sides is deformed in a cylindrical

Fig. 13. Type III

surface. The slab supported on four sides, on the other hand, dishes or bags down from all directions and cannot fail suddenly for this reason.

Ample lap of reinforcement over the supports, thoroly tying the work together, enhances the safety of all types.

Most failures entailing loss of life have occurred with reinforcement in one direction only and of these failures the greater part of them have occurred where insufficient lap of reinforcement has been provided.

From the fireproof standpoint, that form which exposes the least area to heat, which presents the most uniform distribution of metal to provide for the temperature stresses resulting from unequal heating will rank first, and on this basis the natural concrete types III and IV consequently are to be preferred.

In the above types, we have the following five problems in design:

(*a*) Beams, simple, continuous, partially continuous, etc.

(*b*) Slab with panels supported on two sides.

Fig. 14. Type IV.

(*c*) Slab with panels supported on four sides.

(*d*) Slab with panels supported on four posts or corners.

In each of the slab problems we also must consider the conditions of the ends or edges of the panels as in the treatment of beams.

(*e*) Columns similar for all types.

The relative economy of the several types will appear from the methods of computation to follow.

2. Utility of the Theory of Action of Structures—No theory can be devised which will take into consideration all of the phenomena presented by an actual structure. In structural work it is usual to treat for example, beams connected by flange angles or resting on top of girders as simple beams. The stresses in such beams, however, differ somewhat from what they would be on the theoretical assumption that the supports are knife edge bearings without friction. The useful theory then is that which takes into consideration the predominant phenomena presented by the structure under load. The unnecessary refinement of taking into account small, subsidiary actions, such as the restraint of connection angles, and restraint of beams which are not supported by frictionless knife edges, is for practical purposes ignored.

Thus, the direct tensile resistance of the concrete as a tension chord being small, is disregarded in practical computations. In reinforced concrete beams the tensile flange resistance offered by the steel alone as a flange is that considered and counted upon for safety as the predominant action.

The theory of work has this striking advantage over other methods of analysis of such structures, that it indicates this predominant action almost at once in a manner so clear that it requires little or no computation to arrive at a correct method of treating the structure under discussion.

3. **Principle of Proportion**—In the early development of reinforced concrete work, constructors were obliged to experiment in order to ascertain the most suitable proportions and arrangement of materials for supporting a given load on a given span, and from experiments of this kind determine by proportion the carrying strength for other loads and other spans. This principle of proportion is indeed a most useful one, and was employed largely by the builders in the middle ages in the construction of masonry work in the form of arches in the great cathedrals which command our admiration today, which work is not excelled by modern constructors with advanced knowledge of mathematics and mechanics.

The law of proportion, as applied to a slab or beam of reinforced concrete, may be stated as follows:

4. **Variation in Strength with Thickness**—It is known from elementary principles that for a given percentage of steel and a given arrangement of reinforcement, the strength of a slab or beam increases directly as the square of the depth, and for small differences in the percentage of steel, as the product of the steel area times the depth, providing, of course, the steel is not increased to such an extent that the steel element is stronger than the concrete element.

In the combination of concrete and steel, it should be observed that as between the two elements, the steel is more homogeneous, more reliable and dependable from the standpoint of uniformity of strength. Hence, the combination should be made in such manner that should failure occur it would occur in the steel and not in the concrete, and when this principle of design is followed out the reliability and safety of the structure depends on the steel element, and hence no greater factor or margin of safety is needed than in structural steel work. In fact there would be less uncertainty in this case if this principle were carried out than in structural steel work, for the reason that in structural work the members are cut by rivet holes and there is less dependence to be placed upon the large steel shapes so worked and cut than in the case of rods of uniform section and not so nicked or cut.

5. Variation in Strength with Span—The strength decreases inversely as the span for the same total load W, and the same moment of resistance of steel and concrete. If the strength is to be compared on the basis of a unit load per foot of span length then for the same unit load the strength decreases inversely as the square of the span.

These fundamental principles of proportion enabled the earlier constructors Coignet and Hennebique, to build successfully before the development of the theory involving the relation of the elastic properties of the two materials, concrete and steel, and enabled Turner to successfully build many great structures on the Mushroom flat slab system, prior to the development of a rational theory based upon the elastic properties of the materials. It enabled him, not only to guarantee the strength, but also to guarantee the deflection of his structures under load.

The law of proportion as to deflection may be stated as follows:

Within practical limits, including proper percentages of steel, the deflection for a given load W increases as the cube of the span and decreases inversely as the product of the steel area times the square of the depth from the center of the steel to the top of the slab or beam.

These proportionate relations are sufficient to enable the practical constructor, having exact knowledge of the tested strength and deportment of a reinforced beam or slab of a given design, to design a similar beam or slab for a larger or smaller load or span with certainty as to the result which can be obtained with the same grade of workmanship and the same kind of concrete.

The method of proportion applied to full sized structures has this advantage over all other methods. It takes for its foundation or starting point, the tested strength of a member approaching in size that which it is proposed to construct and a comparison is made involving a narrower range for the application of the principle of proportionality than is possible where the theorist undertakes to develop from the elastic properties of a minute sample or unit cube of the materials employed the properties of a full sized structure made of these materials. On the other hand the method of proportion has the distinct disadvantage of limitation in its scope. It cannot be applied to any form of structure which differs except within narrow limits from the proportions of the specimen with which it is compared, and hence this method is defective as compared with a general solution which enables broad conclusions to be drawn as to the generic type under consideration rather than limited conclusions specific to one form only of the genus.

The method of proportion, based as it is on elementary relations, may be used very advantageously to verify the accuracy of more complex and scientific methods of analysis. The relations above outlined follow at once from the fundamental principles governing the strength and flexure of beams and were developed in substantially the following manner by Turner in his treatise on Concrete Steel Construction, published in 1909.

6. Theoretical Treatment—A slab or beam supported at intervals either on points or on walls, if loaded, deflects or bends, and if the load is excessive the concrete cracks first from the lower or tension side upward in a plane normal to lines joining the supports. Since the reinforcing metal acts by tension along its length, it is evident in general that so far as the steel is concerned, whether the reinforcement is in single layers or in multiple layers, the action must be similar in character to the flanges of a beam, and hence the strength of the beam or slab, regardless of the distribution of the stress in the concrete, depends on the tensile stress in the steel. The mathematical expression for deflection and bending would be of identically similar form to those for beams.

W = the total load on the beam or slab, taken for convenience in thousand pound units.
L = the span in feet.
d = the distance from the top of the concrete to the center of steel in inches.
A_s = area of one reinforcing rod in square inches.
f_s = the intensity of actual stress in the steel in thousand pound units.
Σ = the usual sign of summation.

M_1 = moment of resistance for stress in the steel.

$\triangle$ = deflection at the center of beam or slab for any load.

(B) = a coefficient which may be variable or constant in value, to be so determined experimentally that $(B)\ WL$ shall equal M_1.

(D) = a coefficient similarly obtained for deflection formula.

Then by the laws of beams, we have the following equations:

$$M_1 = (B)\ W\ L = \frac{1}{12}\ .85\ d\ f_s\ \Sigma\ A_s \ldots\ldots\ldots\ldots (1)$$

$$\triangle = (D)\ \frac{W\ L^3}{\Sigma A_s d^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

In formula (1) it will be noted that $.85d$ is assumed as a close approximation to the effective lever arm of the steel jd, or the distance from the centroid of tension to the centroid of compression in the beam or slab. This, of course, varies slightly with different percentages of steel, but for practical purposes it may be assumed that this value does not involve material error, and is on the safe side.

The coefficient (B) for the simple beam is $\frac{1}{8}$. For the continuous beam it is customary to take this as $1/12$ at the support and $1/16$ at mid span, while for the slab supported at four sides, (B) is taken as the reciprocal of 30, and where the reinforcing metal is more closely spaced at the center third, the reciprocal of 36, while (B) for a mushroom slab is taken as the reciprocal of 50. It is assumed in these formulas that $f_s = 13$, which is expressed in thousand pound units.

The deflection $\triangle$ of slabs will follow the same laws as the deflection of beams so far as factors depending upon W, L, and I are concerned, and will consequently be equal to some multiple of WL^3/EI. But I varies as $\Sigma A_s d^2$. Hence $\triangle$ varies as $WL^3/\Sigma A_s d^2$, which is expressed above in equation (2) in which the constant multiplier (D) is to be determined experimentally, but could supposedly be derived analytically in case a sufficiently general theory should be developed to express correctly the manner in which it depends upon the known arrangement and properties of the materials composing the slab.

For simple beams, (D) is taken as the reciprocal of 850 and for continuous beams cast integrally with a heavy slab, as the reciprocal of 5,000.

(D) is taken as the reciprocal of 10,000 in the slab supported on four sides, and for the Mushroom system as the reciprocal of 7,000.

The application of the principle of proportion in the determination of the working stresses is based on the assumption (which is on the side of safety) that steel stress under working load is proportional to the steel stress at the elastic limit of the steel under a load which would produce this stress. Now the elastic limit of

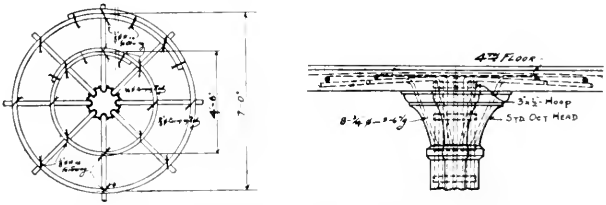


Fig. 15. Mushroom Column Reinforcement in Curtiss Building. Panels approximately 16 ft.

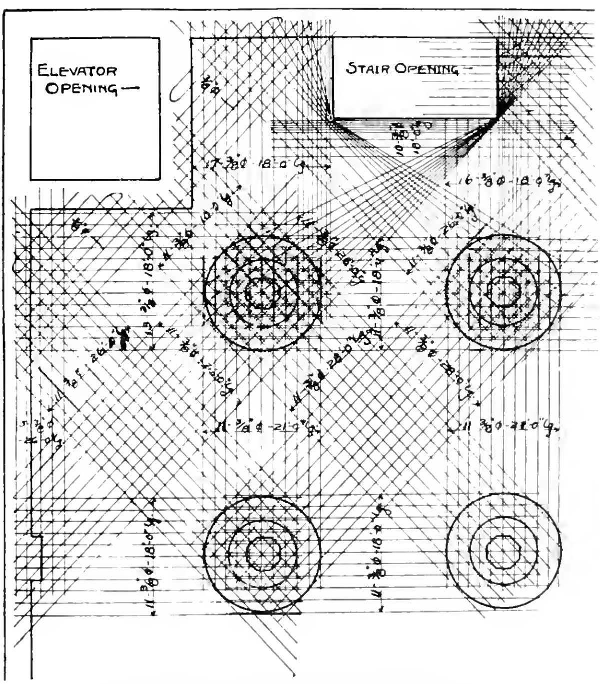


Fig. 16. Slab Reinforcement, Mushroom System, in a wall bearing building; panels about 16 ft.

medium open hearth steel has a practically fixed value which varies little and which can readily be determined by test.

Having constructed a panel or beam reinforced with this known grade of metal, by loading until the first yielding of the steel occurs, we then know the steel stress under the applied load, and may, by proportion, determine closely the steel tension for any smaller working load.

The elastic limit above referred to is the limit of elasticity of shape as determined by a slowly applied load, and is to be distinguished from rigid proportionality of shape. By this method of investigation, coefficients (B) may be determined for all types of construction. Coefficients (D) are figured from the measured deflection of the member or panel tested. This method of determining the coefficients used is, of course, limited to those designs in which the rational method is followed of proportioning the structure so that the steel element determines the safety of the structure.

No formulas for strength, based on the elastic properties of the materials can be accepted as correct unless the corresponding ~~corrected~~ formula for deflection can be depended upon also. In other words, an elastic theory in order to be acceptable must demonstrate its accuracy by agreement of the entire elastic deportment of the structure to which it is intended to be applied including both stresses and deflections. The determination of the respective coefficients for strength and deflection having been derived independently of each other, empirically or by experiment, their general agreement can be established by a concordance of the deflections observed with those computed in structures which have been designed for strength, using the same coefficients (B), for both.

We have noted the method of determining the coefficients (B) and (D) for a specific type of construction. These coefficients can be used for any size or thickness of panel or percentage of reinforcement if they remain constant in value and are not variables. Their values may be rendered constant by fixing the arrangement of reinforcement in strict proportion to the sample tested. We will illustrate this proposition by its application to the Mushroom type of reinforcement shown in Figs. 15 and 16. In this construction the values of the coefficients are made constant by fixing the diameter of the Mushroom head and width of belt as identically or approximately the same fraction of the span of the test panel from which the coefficient was determined. In other words, if the diameter of the head and corresponding width of belt be kept within the limits of 7/16 to 1/2 the distance between column centers in the case of square panels, or 7/32 to 1/4 the sum of the long and short spacing in a rectangular panel, then the coefficients remain practically constant. Otherwise, they become extremely variable, increasing in value several hundred percent as the width of belt is reduced to 50 percent of the above proportions. From this statement it becomes evident that coefficients of this character must be applied rigidly to similarly

Fig. 17. Interior Stock House Hamm Brewing Company Building, showing 50 ton tanks being placed in position covering full area of floor.

proportioned types of reinforcement until the law of their variations is accurately determined.

L in the formulas for bending and deflection is taken as the longer direct distance between column centers, the shorter direct distance appearing in the case of a rectangular panel in the determination of the diameter of the head and in the determination of the load W from the unit load per square foot.

In the treatment of the steel area it may be noted that the line of weakest section at failure as determined experimentally, cuts across the four way belts practically at right angles. Hence the resisting moment is taken at this, the weakest section, as the elastic limit of the steel is approached, although the maximum stress in the steel does not occur actually at this section on the diagonal belts under lesser load.

7. **Example**—An illustration of the application of these formulas is given herewith for the Mushroom system in the floor of the Hamm Brewing Company Stock house, illustrated in Fig. 17.

Take the case of a panel of the Hamm Brewing Company's building, shown in Fig. 17, panel 22′10″ by 26′0″ loaded with four tanks 10′ in diameter, 15′ high full of water. The load is equivalent to 200 tons of uniformly distributed load. The floor slab is 14″ thick at the outer edge and pitches upwards 3″ to the center and is reinforced with twenty-five $\frac{5}{8}''$ rounds each way. Taking an equivalent depth of $15\frac{1}{4}''$ as the distance from center of steel to top, we have the following equation:

$$\triangle = \frac{1}{7000} \times \frac{(400)\ (26)^3}{(4 \times 25 \times .3)(15.25)^2} = .144''$$

Another panel in the same building, 20′10″ by 20′8″. Same loading, thickness and reinforcement.

$$\triangle = \frac{1}{7000} \times \frac{(400)\ (20.83)^3}{(4 \times 25 \times .3)(15.25)^2} = \text{a full } 1/16''.$$

These figured deflections proved exactly equal to the measured deflections as nearly as the engineer of the brewery could determine by marking the same with a knife edge.

We will take another case. Test of the State Factory Building at Stillwater, Minn., Mr. C. H. Johnston, Architect, shown in Fig. 18. Size of panel 19′9″ by 20′8″. Thickness of slab 8″. Reinforcement seventeen $\frac{3}{8}''$ rounds each way. Test load, 450 lbs. per square foot over the full area.

$$\triangle = \frac{(180)(20.66)^3}{(7000)(4 \times 17 \times .11)(7.25)^2} = .573'' = 9/16''$$

the reported deflection.

The Hoffman Building, Milwaukee. Test load 142 tons. Panel 17′0″ by 16′8″. Reinforcement seventeen $\frac{3}{8}''$ rounds each way. Slab $8\frac{1}{2}''$ ($7\frac{1}{2}''$ rough and 1″ finish).

$$\triangle = \frac{(284)\ (17)^3}{(7000)\ (4 \times 17 \times .11)(7.8)^2} = .437 = 7/16''$$

the measured deflection.

Another example: Test of the John Deere Plow Company's building in Omaha. Fig. 18. Panel 18′9″ square. Reinforcement sixteen $\frac{3}{8}''$ rounds diagonally and fourteen $\frac{3}{8}''$ rounds directly from

Fig. 18. Test of John Deere Plow Company Building, Omaha, Neb. 550 pounds per square foot.

column to column. 7″ slab in rough, with strip fill added later about $2\frac{1}{4}''$ thick and $\frac{7}{8}''$ finish floor of maple. This we find an equivalent to a slab of about $8\frac{3}{4}''$ concrete as far as deflection is concerned, the strip being a $1:3\frac{1}{3}:4$ mixture.

$$\triangle = \frac{(160)(18.75)^3}{(7000)\ (6.6)\ (8)^2} = .356'' \text{ or } \tfrac{3}{8}'', \text{ the measured}$$

deflection.

8. Slab Supported on Girders—The treatment of a rectangular slab supported on four sides and cast integrally with the supporting beams by the method of proportion will also be illustrated.

The common arrangement of such slabs is shown in Figs. 19 and 20. For square slab the load, of course, is divided equally between the four supporting beams and coefficient (B) is taken as the reciprocal of 30 where the rod spacing is uniform, and as the reciprocal of 36 where the rods are spaced twice as closely for the middle third as they are for the outer third. The coefficient (D) is taken as the reciprocal of 10,000 for both types. In the case of a rectangular panel on the assumption that the load transferred to the beams is in proportion to the lengths of the sides, a and b, a mean length for moment and deflection is derived by taking the quotient $(a^2+b^2)/(a+b)$ so that the formulas become:

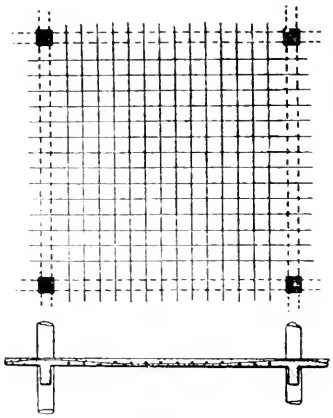


Fig. 19.

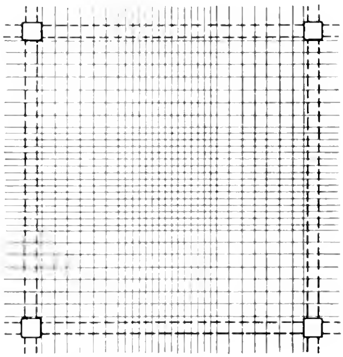


Fig. 20.

Common Type Slabs, supported on four sides, reinforced two ways.

$$M_1 = (B)\ W(a^2+b^2)/(a+b) = .85\ d\ f_s\ \Sigma A_s/12$$

$$\text{and}\quad \triangle = (D)\frac{W\ (a^2+b^2)^3}{(a+b)^3\ \Sigma A_s\ d^2}$$

9. Computation of Deflection Applied to Practical Examples—We will now proceed to apply these formulas to two cases:

First, take the Minneapolis Paper Company's building, details shown in Fig. 21, photograph of test load in Fig. 22. Slab 7″ in the rough, 15′4″ by 21′6″ center to center of columns, strip fill $1\frac{3}{4}''$ and $\frac{7}{8}''$ finished floor. Now taking the strip fill as effective for one half of its actual thickness, we find for a load of 234,000 lbs:

$$\triangle = \frac{1}{10{,}000} \times 234 \times \frac{\left(\dfrac{21.5^2+15.3^2}{21.5+15.3}\right)^3}{(75 \times .41) \times (8)^2} = .30''$$

=deflection of slab at center as measured less the beam deflection.

Take for example the test made at the Smythe block at Wichita, Kans. Test load consisted of fifteen tons concentrated at the center over an area of 7 feet square equivalent to about 45,000 lbs. uniform load. Size of slab 20′9″ by 24′9″, $6\frac{1}{2}''$ in the rough, $1\frac{3}{4}''$ strip fill. Reinforcement, $\frac{3}{8}''$ rounds 5″ centers for the central third of the panel

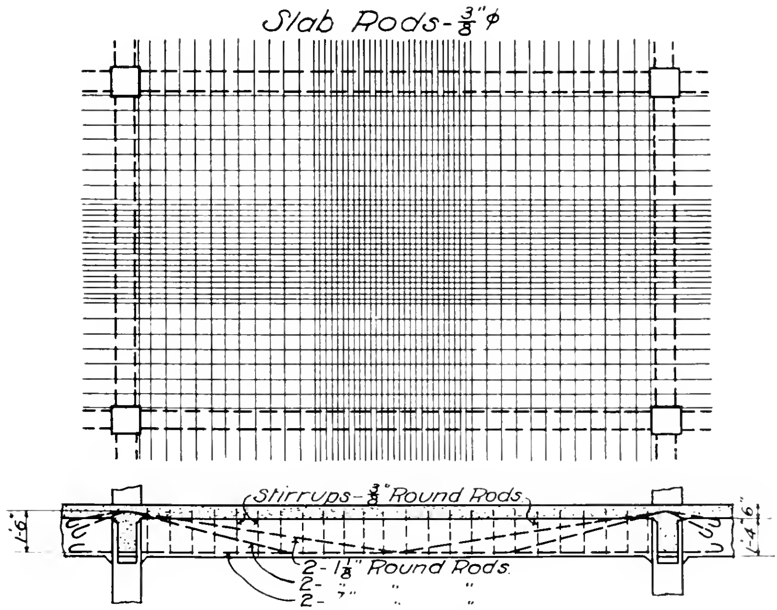


Fig. 21. Details of Reinforcement in Panel of Minneapolis Paper Company Building.

each way and 8″ centers outside third width each way. Then—

$$\triangle = \frac{1}{10{,}000} \times (45) \frac{\left(\frac{20.75^2 \times 24.75^2}{45.5}\right)^3}{9. \times 7.25^2} = .11'' \text{ or}$$

a full 3/32″ the deflection measured.

In the test at Wichita, the beam deflection, owing to the small load on the panel was practically negligible, and does not need to be considered in arriving at the true slab deflection.

Test of the Minneapolis Knitting Co.'s Building:

Slab $5\frac{1}{2}''$ thick with $1\frac{3}{4}''$ strip fill, panel 16′4″ x 15′8″. Reinforcement $\frac{3}{8}''$ round rods 4″ centers each way.

Fig. 22. Photograph of Test Load, Minneapolis Paper Company Building.

$$\triangle = \frac{1}{10500} \text{ x } 160 \text{ x } \frac{\left(\frac{16.33^2 + 15.66^2}{32}\right)^3}{(10.4) \text{ x } (6)^2} = .167'' = 5/32'' +$$

The measured deflection agreed identically with that figured.

This formula is based on the assumption that a large fraction of the internal work of deformation is performed by lateral action after the manner of a uniform continuous plate and while not strictly accurate it is in much closer approximation to the actual condition of stress than irrational formulae based upon a distribution of stress and load in proportion to the fourth power of the respective sides as derived in the irrational treatment on the basis of independent beam strips through the center of the panel, or upen an inapplicable approximate solution of the general differential equation of such a slab as derived by Grashof. The agreement of the formula for deflection gives closely approximate results from the practical standpoint and its mathematical deviation from correct values will be discussed later.

The method of design by proportion, based on the steel stresses as explained in the preceding pages, presupposes that the concrete element resisting compression is of greater strength than the tensional steel element as it should be in conservative design. The limiting steel percentages for the various types of structure will be discussed later rather than under this present heading.

10. Short Span Slabs and Arch Action that may be Counted upon in Their Use.—We have heretofore discussed at some length long span slabs. A long span slab will be defined as a slab, the ratio of whose thickness to length is so small that the possibility of its acting effectively as an arch is eliminated.

It is to long span slabs where the arch action is negligible that formulas for bending apply with a high degree of precision. When, however, we come to test a short span slab which is part of a continuous floor there may be quite a large amount of arching in the slab by which the load is carried to the support without causing tension in the steel to the extent that it would do provided there was no rigid skew-back to sustain the thrust. Where the span of the slab does not exceed ten times its thickness it is permissible and good practice to figure the bending moment on the slab at $WL/16$, and this moment is to be increased to $WL/10$ where the thickness is one-sixteenth of the span, with intermediate values for intermediate ratios of thickness to span. Where the span is more than sixteen thicknesses of slab it is to be figured as heretofore provided.

11. Value of Finish Coat, Strip Fill and Wood Floor from the Standpoint of Deflection—Many engineers have an idea that a finish coat or strip fill or the like cannot act in connection with the slab to good purpose for the reason that the bond between the old concrete of the slab and that which is added at a later date will not be equal to the strength of the original concrete.

While this is true to some extent, nevertheless, where the rough concrete is washed off and scrubbed with a steel brush and then given a coat of neat cement grout immediately before adding the finish coat or laying the strips and strip filling, the concrete is nearly as efficient as though it were all cast at the same time, provided that the top coat or strip fill is given a reasonable time to set up hard before the test load is applied.

A $1\frac{3}{4}''$ strip fill with strips $16''$ on centers and a $\frac{7}{8}''$ wood floor generally deflects the same as a $\frac{7}{8}''$ or $1''$ finish coat of concrete. The strip fill generally, however, if the strips used are $1\frac{3}{4}''$ will somewhat exceed this normal thickness since it is impracticable to leave the top surface of the rough slab perfectly level, and we count as a rule that the actual thickness of the nominal $1\frac{3}{4}''$ strip fill will not be less than $2\frac{1}{4}''$ in the center of the slab tho it may be a little less around the columns if the columns are poured in accordance with our standard practice.

If we assume that the bond between the finish coat and the old concrete is an even 30 percent of the strength of the original concrete we would still have a very large factor of safety in view of the great area of the slab to take care of the horizontal shear between the two layers. This is a fact which is generally disregarded by those who are dealing with reinforced concrete. If a slab the depth of which has been increased by perhaps 20 percent by strip fill and the finish be figured on the basis of the actual thickness of the rough slab only, a surprisingly high degree of strength will apparently be developed on this basis by test, but a more conservative computation taking into consideration the actual value as about one half of this added thickness will estimate the construction at its true worth.

For strip fill, where strength becomes an object, identically the same grade of concrete should be used as in the slab, instead of the weak, indifferent mud filling of cinders or natural cement or brown lime which is sometimes employed. Further, by using Portland cement in the strip fill, the contractor will find that this filling hardens up and dries out much more promptly than any mixture of natural cement, brown lime, or Portland cement and lime, thus permitting the finished floor to be laid at an earlier date without danger of having the hard wood swell, buckle and rise up from the cleats to which it is nailed by reason of the moisture absorbed from the uncured filling.

In a case where lime was used with an idea of economy in a building completed in the fall which the owners were in a hurry to occupy, this filling dried very slowly, seeming to have an affinity for moisture, and when the finished floor was laid it swelled so as to buckle up in places eighteen and twenty inches high, due to the swelling of the boards longitudinally, while laterally the swelling of this kiln dried maple was over fifteen inches in a width of fifty feet. Six widths had to be taken out of the floor, and the floor taken up and entirely relaid. The saving in first cost of fill thus proved very expensive in the end.

CHAPTER III

BEAMS

1. Elastic Properties of Materials, Concrete and Steel. In the design of a composite structure, such as a reinforced concrete beam or member by an elastic theory, it is necessary to know the relative stresses under like deformations. These will depend upon the ratio of the moduli of elasticity of the two respective materials.

For safe design we need to know the range or limits between which the ratio assumed holds true. For the steel, Young's modulus is $E_s = 3\text{x}10^7$. The elastic limit of medium steel may be taken as 35,000 pounds per square inch and for hard steel 50,000 pounds per square inch. (See Standard Specifications.)

The resistance which concrete offers to crushing is variable as we have pointed out in our analysis of the strength of concrete, and depends upon the proportions of the mixture, the character of the sand, gravel and stone, as well as the conditions of hardening and age of the concrete. The form and size of test specimens also influences the apparent strength. The age of the specimens has a marked effect upon the strength as well as upon the modulus of elasticity.

As to increase of strength with age, Mörsch quotes tests in connection with a bridge erected over the Danube at Munderkingen, with one part cement, two and one half parts sand and five parts pebbles. Test cubes developed the following strength:

After	28 days,	average strength compression	3613	lbs. per sq. in.
"	5 months,	" " "	4722	" " " "
"	2 yrs. 8 mo	" compressive strength	7396	" " " "
"	9 years,	compressive strength.........	8107	" " " "

These values are materially higher than the average value of broken stone concrete, which may be accounted for by the excellent quality of sand and the hardness and grade of pebbles used.

Since the strength of concrete increases with time, it is permissible to use higher working stresses when making an addition to an old building constructed of good concrete.

2. Tensile Strength. Results of tensile tests are more variable than those of compression. In most cases, tensile tests are made on mortar specimens; that is those composed of cement and sand only. Few tests have been made on ordinary concrete specimens with coarse aggregate. The latter exhibit less tensile resistance than the specimens of mortar.

In general it may be stated that the tensile strength of concrete may be taken as having a value between one tenth and one twelfth of the ultimate compressive strength.

3. Elasticity of Concrete. It is impossible to assign a definite value for the modulus of elasticity of concrete since all of the factors entering into the breaking strength influence its elastic behavior and make it difficult to compare the results obtained by different observers.

Concrete, unlike steel, has no definite elastic limit, the stress strain curve of a block when first tested in compression, being a curved line from the beginning, due in part to shrinkage stresses induced in the process of hardening. Considering only the stress strain curve obtained the first time it is loaded we might say that the modulus is not practically a constant quantity like that for steel but has only an instantaneous value which varies for any given specimen with the load.

Concrete further differs from steel in taking permanent sets under very light loads, and if these permanent sets are not deducted from the total deformation under gradually increasing load the result does not represent the true elastic deformation. This was pointed out by Professor Bach of Stuttgart in 1895. He found for a given maximum loading of less than half the ultimate strength that repeated loading eliminates the permanent set and gives a fairly constant modulus for subsequent loadings not exceeding this maximum.

As illustrating the variation of the modulus of elasticity with the age of the specimen, the results noted in the following table according to Mörsch, are of interest:

TESTS OF OLD CONCRETE FOR MODULUS OF ELASTICITY

	Unit Stress lbs./in²	Three Months Old E_c	Two Years Old E_c	Remarks
Compression	1223.1	....	3655000	Average of three tests
	1048.2	....	3741000	
	871.9	2973000	3812000	
	697.0	3072000	3869000	
	523.4	3158000	3954000	
	435.2	3229000	3983000	
	348.5	3342000	4025000	
	260.3	3428000	4068000	
	173.5	3613000	4125000	
	.....	3769000	4330000	
Tension	0	....	....	One Single test each
	22.8	3271000	4836000	
	44.1	2944000	4495000	
	65.4	2845000	4423000	
	88.2	2759000	4409000	
	109.5	2489000	4381000	
	130.8	....	4310000	
	153.6	....	4310000	
	174.9	....	4281000	
	196.3	....	4239000	

The accompanying diagram Fig. 23, gives a fair idea of the stress strain curve plotted from test results at one and 5 months. Fig. 24 shows the stress strain curve arrived at by repeated loading.

When the load is applied gradually, the shortening of the specimen which is at first small, increases more and more rapidly as the load increases as shown in the concave curve Fig. 24, plotted with the applied loads as ordinates and the deformation as abscissas. As the loading is gradually removed the curve YO' takes a convex form and shows a permanent set OO' on a horizontal axis. On again applying the same load, the new stress strain curve starting from the new origin O' is still of concave form looked at from the right just as the original curve OY was, but to a lesser degree and for the same load as at Y the point Y' shows a smaller relative set than the set OO'. On unloading

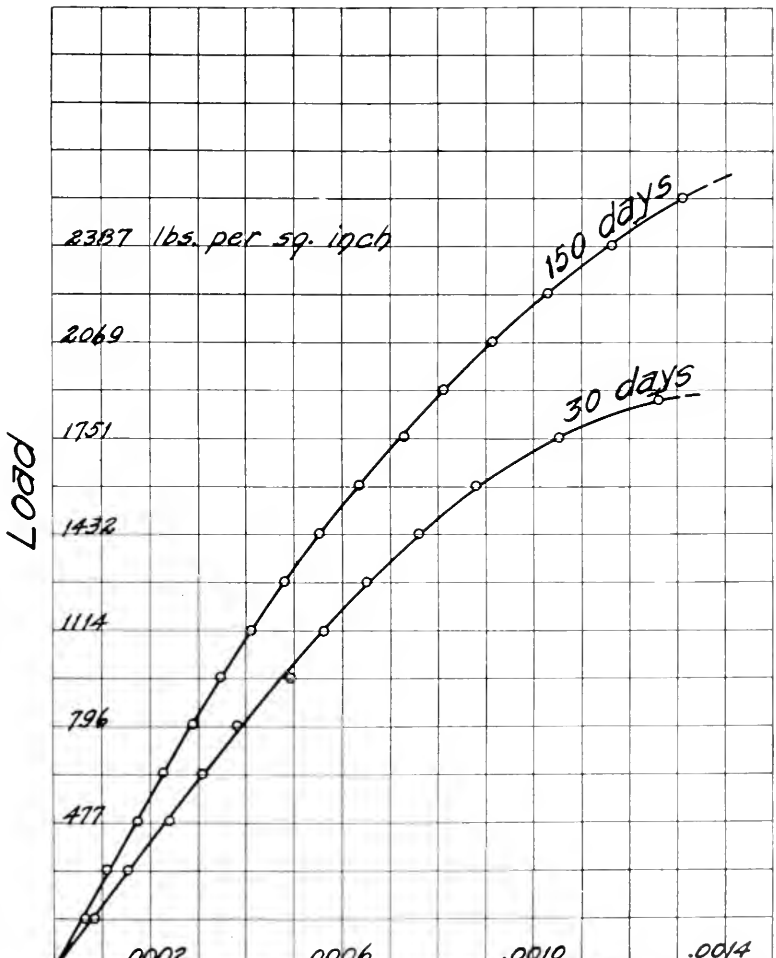


Fig. 23. Stress Strain Curves in Compression from 1 : 2 : 4 Cylinders, thirty days and one hundred fifty days old, respectively.

again the origin is moved slightly to O''. With several successive applications and removals of the same load, the origin is continually removed slightly to the right, but the movement becomes less and less until there is no additional permanent set. The permanent set of concrete appears then to be in a great measure reached under the first loading and for subsequent applications of the same load the concrete acts more and more nearly as a perfectly elastic material.

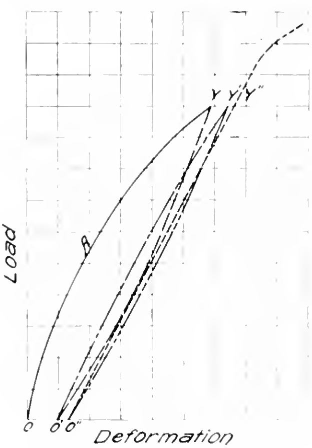


Fig. 24. Stress Strain Curves in Compression of Concrete under Repeated Loading.

4. **Concrete Beams.** Where comparatively large sections of metal are used for the purposes of directly resisting tensile stress due to bending the ratio of the modulus of elasticity of the steel to that of concrete is for practical purposes generally taken as one to fifteen, and the tensile strength of the concrete is entirely neglected This is for the usual 1:2:4 concrete. For a rich mix such as 1:1½:3 this ratio is sometimes taken as one to ten or twelve.

As pointed out in the historical sketch, engineering opinion has crystalized in the adoption of the linear law of distribution of stress for purposes of computation of beams and slabs and the assumptions involved in this modification may be stated as follows:

(a) Adhesion between the concrete and steel shall be sufficient to make the two materials act together.

(b) The steel is to take all direct tensile stress.

(c) The stress strain curve of the concrete in compression is a straight line for the range of working stress.

(d) A plane cross-section of an unloaded beam will still be plane after bending.

(e) The material in the beam will obey Hooke's law in that stress is proportional to strain.

From the above it follows that unit deformations of the fibers at any section are proportional to their linear distance from the neutral surface, and the unit stress in the fiber at any section of the beam is proportional to the distance of the fiber from the neutral surface. The linear law above stated is the basis of all practical formulas of flexure except some which have been developed for reinforced concrete beams applicable to the conditions as failure is approached rather than the condition for safe loads or for safe working stress.

The Joint Committee (American Society Civil Engineers, etc.) on Concrete and Reinforced Concrete, recommended that calculations be made with reference to working stresses and safe loads rather than with reference to ultimate strength and ultimate loads,—an endorsement of customary practice of experienced builders in this respect. It also endorsed current practice with regard to the modulus of elasticity and to commonly accepted formulas for beams which are reproduced herewith as follows:

5. Modulus of Elasticity. "The value of the modulus of elasticity of concrete has a wide range, depending on the materials used, the age, the range of stresses between which it is considered, as well as other conditions, It is recommended that in computations for the position of the neutral axis and for the resisting moment of beams and for the compression of concrete in columns it be assumed as:

(a) One-fifteenth of that of steel, when the strength of the concrete is taken as 2200 lbs. per sq. in. or less.
(b) One-twelfth of that of steel, when the strength of the concrete is taken as greater than 2200 lb. per sq. in. or less than 2900 lb. per sq. in., and
(c) One-tenth of that of steel, when the strength of the concrete is taken as greater than 2900 lb. per sq. in.

Altho not rigorously accurate, these assumptions will give safe results. For the deflection of beams, which are free to move longitudinally at the supports, in using formulas for deflection which do not take into account the tensile strength developed in the concrete, a modulus one-eighth of that of steel is recommended."

6. Formulas for Reinforced Concrete Construction as recommended by the Joint Committee.

(a) Standard Notations

1. Rectangular Beams.

The following notation is recommended:

f_s = Tensile unit stress in steel,

f_c = Compressive unit stress in concrete,

E_s = Modulus of elasticity of steel,

E_c = Modulus of elasticity of concrete,

$n = E_s / E_c$

M = Moment of resistance, or bending moment in general,
M_s for steel, M_c for concrete,
A = Steel area,
b = Breadth of beam,
d = Depth of beam to center of steel,
k = Ratio of depth of neutral axis to effective depth d,
z = Depth of resultant compression below top,
j = Ratio of lever arm of resisting couple to depth d,
$jd = d - z$ = Arm of resisting couple,
$p = A/bd$ Steel ratio (not percentage).

2. T-Beams.
b = Width of flange,
b' = Width of stem,
t = Thickness of flange.

3. Beams Reinforced for Compression.
A' = Area of compressive steel,
p' = Steel ratio for compressive steel,
f_s' = Compressive unit stress in steel,
C = Total compressive stress in concrete,
C' = Total compressive stress in steel,
d' = Depth to center of compressive steel,
z = Depth to resultant of C and C'.

4. Shear and Bond.
V = Total shear,
v = Shearing unit stress,
u = Bond stress per unit area of bar,
o = Circumference or perimeter of bar,
Σo = Sum of the perimeters of all bars.

5. Columns.
A = Total net area,
A_s = Area of longitudinal steel,
A_c = Area of concrete,
P = Total safe load.

(b) *Formulas*

1. *Rectangular Beams.*

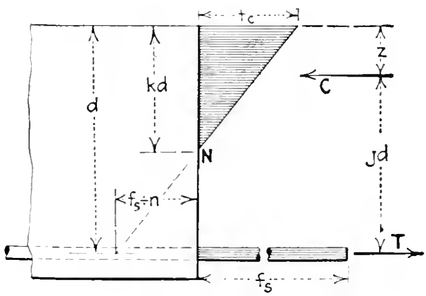
Position of neutral axis,

$$k = \sqrt{2pn + (pn)^2} - pn \qquad \text{.......................} (1)$$

Arm of resisting couple,

$$j = 1 - \tfrac{1}{3}k \qquad (2)$$

(For $f_s = 15{,}000$ to $16{,}000$, and $f_c = 600$ to 650, j may be taken at $\frac{7}{8}$.)



$$f_s = \frac{M}{Ajd} = \frac{M}{pjbd^2} \qquad (3)$$

$$f_c = \frac{2M}{jkbd^2} = \frac{2pf_s}{jk} \qquad (4)$$

Steel ratio, for balanced reinforcement,

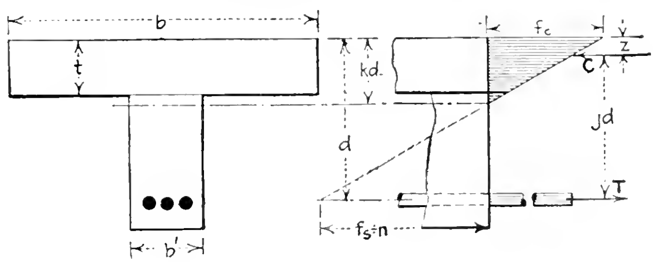
$$p = \frac{1}{2} \frac{1}{\frac{f_s}{f_c}\left(\frac{f_s}{nf_c} + 1\right)} \qquad (5)$$

2. T-*Beams.*

Case I. When the neutral axis lies in the flange: Use the formulas for rectangular beams.

Case II. When the neutral axis lies in the stem.

The following formulas neglect the compression in the stem:



Position of neutral axis,

$$kd = \frac{2\,ndA + bt^2}{2\,nA + 2\,bt} \qquad (6)$$

Position of resultant compression,

$$z=\frac{3\,kd-2\,t}{2\,kd-t}\cdot\frac{t}{3}\quad\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(7)$$

Arm of resisting couple,

$$jd=d-z\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(8)$$

Fiber stresses,

$$f_s=\frac{M}{Ajd}\quad\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(9)$$

$$f_c=\frac{Mkd}{bt\left(kd-\frac{1}{2}t\right)jd}=\frac{f_s}{n}\cdot\frac{k}{1-k}\quad\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(10)$$

(For approximate results, the formulas for rectangular beams may be used.)

The following formulas take into account the compression in the stem; they are recommended where the flange is small compared with the stem:

Position of neutral axis,

$$kd=\sqrt{\frac{2\,ndA+(b-b')t^2}{b'}+\left(\frac{nA+(b-b')t}{b'}\right)^2}-\frac{nA+(b-b')t}{b'}\quad(11)$$

Position of resultant compression,

$$z=\frac{\left(kdt^2-\frac{2}{3}t^3\right)b+\left[(kd-t)^2\left(t+\frac{1}{3}(kd-t)\right)\right]b'}{t(2kd-t)b+(kd-t)^2\,b'}\quad\dots\dots(12)$$

Arm of resisting couple,

$$jd=d-z\quad\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(13)$$

Fiber stresses,

$$f_s=\frac{M}{Ajd}\quad\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(14)$$

$$f_c=\frac{2\,Mkd}{\left[(2\,kd-t)\,bt+(kd-t)^2\,b'\right]jd}\quad\dots\dots\dots\dots\dots\dots\dots\dots(15)$$

3. *Beams Reinforced for Compression.*

Position of neutral axis,

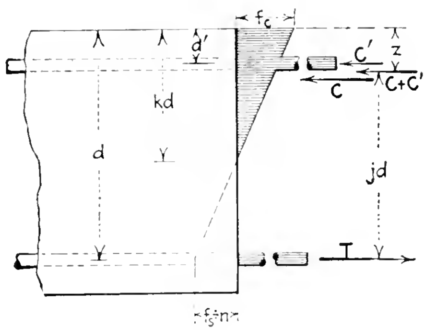
$$k=\sqrt{2\,n\left(p+p'\frac{d'}{d}\right)+n^2(p+p')^2}-n\,(p+p')\dots\dots\dots\dots(16)$$

Position of resultant compression,

$$z=\frac{\frac{1}{3}k^3d+2\,p'nd'(k-d'/d)}{k^2+2\,p'n\;(k-d'/d)} \quad\text{.......................(17)}$$

Arm of resisting couple,

$$jd=d-z \quad\text{.......................................(18)}$$



Fiber stresses,

$$f_c=\frac{6M}{bd^2\left[3k-k^2+\frac{6\,p'n}{k}(k-d'/d)\;(1-d'/d)\right]} \quad\text{.........(19)}$$

$$f_s=\frac{M}{pjbd^2}=nf_c\frac{1-k}{k} \quad\text{.................................(20)}$$

$$f'_s=nf_c\,(k-d'/d)/k \quad\text{...........................(21)}$$

4. *Shear, Bond and Web Reinforcement.*

In the following formula, Σo refers only to the bars constituting the tension reinforcement at the section in question, and jd is the lever arm of the resisting couple at the section.

For rectangular beams,

$$v=\frac{V}{bjd} \quad\text{...(22)}$$

$$u=\frac{V}{jd\;\Sigma o} \quad\text{...(23)}$$

(For approximate results, j may be taken at $\frac{7}{8}$.)

The stress in web reinforcement may be estimated by the following formulas:

Vertical web reinforcement,

$$P=\frac{Vs}{jd} \qquad (24)$$

Web reinforcement inclined at 45° (not bent-up bars),

$$P=0.7\frac{Vs}{jd} \qquad (25)$$

in which P = stress in single reinforcing member, V = amount of total shear assumed as carried by the reinforcement, and s = horizontal spacing of the reinforcing members.

The same formulas apply to beams reinforced for compression as regards shear and bond stress for tensile steel.

For T-beams,

$$v=\frac{V}{b'jd} \qquad (26)$$

$$u=\frac{V}{jd\Sigma o} \qquad (27)$$

(For approximate results, j may be taken at $\frac{7}{8}$.)

5. *Columns.*

Total safe load,

$$P=f_c\,(A_c+nA_s)=f_cA\,(1+(n-1)p) \qquad (28)$$

Unit stresses,

$$f_c=\frac{P}{A\,(1+(n-1)p)} \qquad (29)$$

$$f_s=nf_c \qquad (30)$$

7. Determining Moment. In case the steel element in a reinforced concrete beam is weaker than the concrete, the determining resistance is that of the steel, and

$$M_s=Ajdf_s=pjbd^2f_s \qquad (a)$$

If, on the other hand, the beam be over reinforced, the determining moment is that of the concrete, and

$$M_c=\tfrac{1}{2}\,jkbd^2f_c \qquad (b)$$

For approximate calculations it will be sufficiently correct to assume $j=0.85$ and $k=0.40$, these being fair average values for steel percentages from 0.75 to 1.25. Equations (a) and (b) then become

$M_s=0.85\ Adf_s$

and $M_c=0.17\ bd^2f_c$.

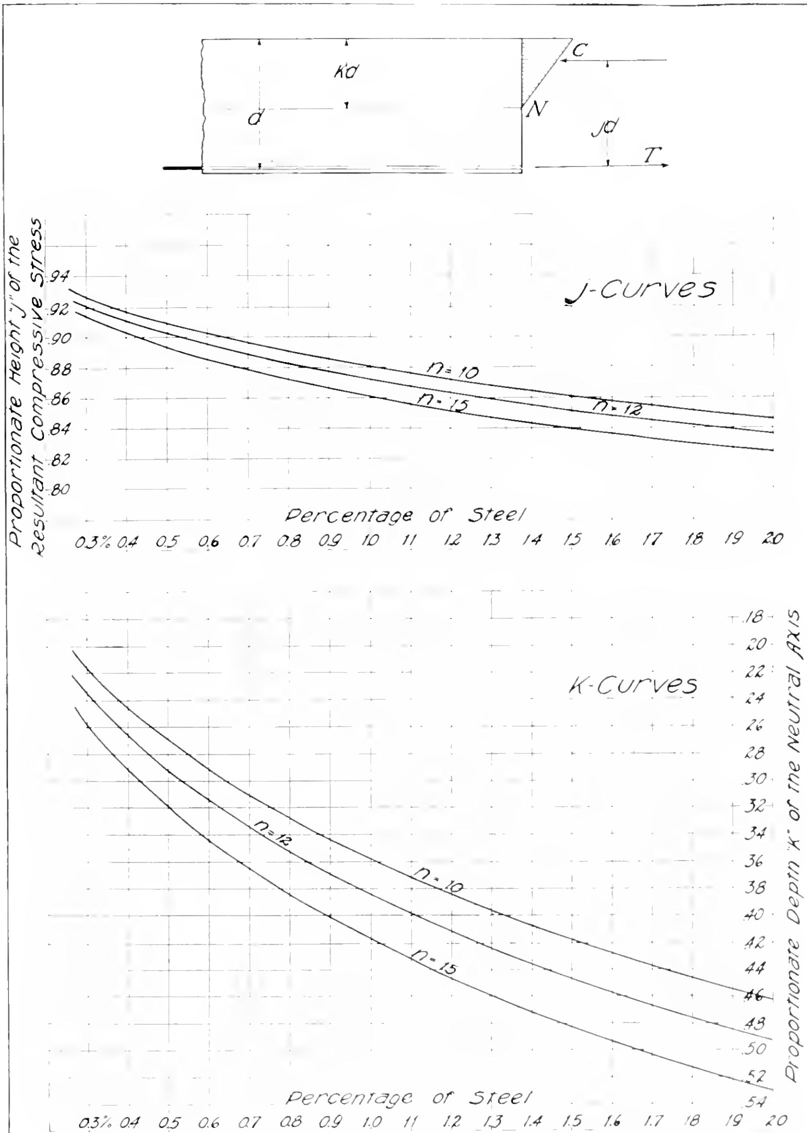


Fig. 25. *j* and *k* curves.

More exact values of j and k for various values of p and n may be obtained from the accompanying j and k curves Fig. 25, which have been computed and plotted from formulas (1) and (2) of Section 6. Also values of f_s and f_c plotted in accordance with equations (a) and (b) above, are shown in Fig. 26a and 26b.

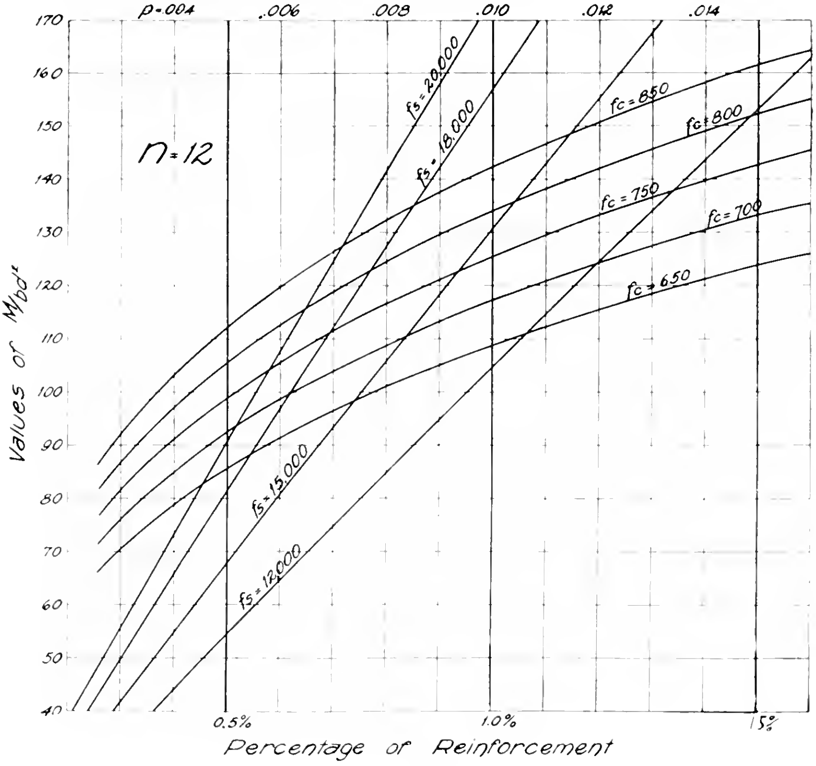


Fig. 26a.

The theory of the flexure of reinforced concrete beams, assumes that for all practical purposes they may be assumed to obey, with sufficient accuracy, the so-called straight line theory of stresses and strains, a theory under which according to Hooke's law the stress is proportional to the strain and varies directly as the distance from the neutral axis. But the assumption is more or less inaccurate because the stress strain curve of the concrete in the compression zone is not a straight line since the rate of deformation is greater where the stress is larger. This is equivalent to saying that the modulus of elasticity E_c of concrete in compression becomes smaller the larger the unit stress f_c becomes and vice versa. This variation of the modulus E_c as well as any small initial permanent deformation of the concrete will cause some deviation of reinforced concrete beams from perfectly elastic flexure. But this deviation will be less appreciable the deeper the beam, because any slight increase of deformation in the upper fiber will have less effect upon the sharpness of

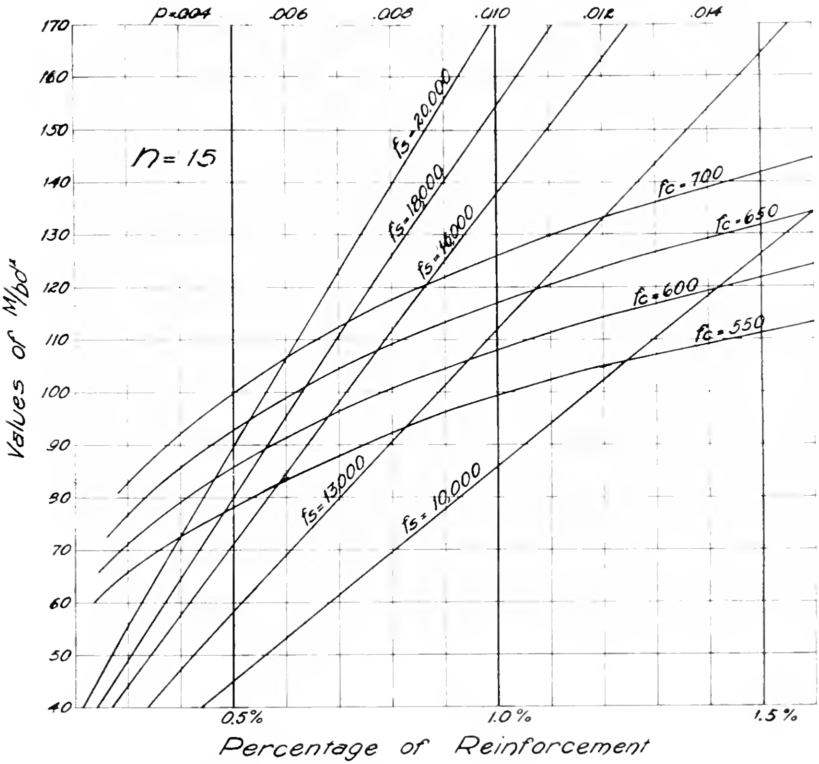


Fig. 26b. Diagram of Steel Percentages for Different values of $M \div bd^2$

bending in a deep beam, and consequently have less effect in causing cracks or checks in the concrete on the under side of it than in a shallow beam. In other words, the deviations of concrete from perfect elasticity have less effect the deeper the beam, and their effect in increasing sharpness of curvature and cracking of the concrete will be more pronounced the more shallow the beam.

How these deviations affect the position of the neutral axis and the sharpness of the bending may be made evident from Fig. 27 which is intended as a representation on a large sale of the deformations, etc., occurring in a unit length of two different beams of the same depth, the two beams being superimposed on each other in the Fig. 27 to assist in the comparison.

Let the two beams have different percentages of reinforcement, but be so loaded that the unit steel stress f_s is the same in both. A larger load will be necessary to produce the same unit steel stress in the beam with the larger percentage of steel, but the actual unit deformation of the steel will be the same in both beams, viz:

$$AO = e = f_s / E_s.$$

In beam No. 1, with the lighter load and smaller value of the steel ratio, assume that f_c the compressive stress in the extreme fiber is so moderate that the concrete in compression may for practical purposes

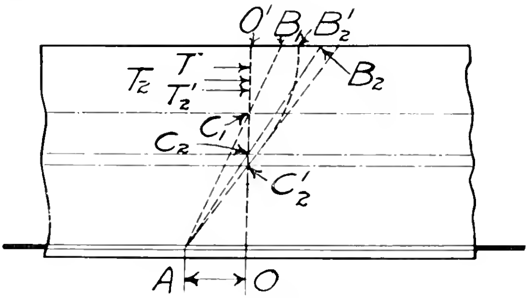


Fig. 27.

be regarded as obeying Hooke's law. Then the line AB_1 by its horizontal distance from OO' at different levels represents the relative unit deformations at those levels. But by Hooke's law these distances would also represent unit stresses when measured in a suitable scale, because stress is deformation multiplied by modulus of elasticity which latter is taken to be constant in this case. In beam No. 1 with light load and small steel ratio p_1 the total tension in the steel and the total compression in the concrete will each be

$$T = A_1 f_s = A_1 e E_s = \text{area } O'C_1B_1.$$

The corresponding neutral axis at C_1 and point of application of compressive stress T will be nearer the top of the beam the smaller OB_1 is.

In beam No. 2 with heavier load and larger steel ratio, p_2, while AO the elongation is unchanged the line of deformations will assume some new position AB_2, and if the compressions near the top surface are large enough in this case to make the modulus of elasticity less than it is at points nearer the neutral axis, the horizontal distances

*See Merriman's Mechanics of Materials p. 273.

of this line of deformations AB_2 will no longer also correctly represent the stresses to scale. Those near the top of the beam will be smaller when plotted to the same scale as in No. 1. But that would reduce the area T_2 between it and OO' if the neutral axis at C_2 remains fixed. In fact, however, the total steel tension

$$T_2 = A_2 f_s = A_2\, e\, E_s = \text{area } O'\, C_2 B_2$$

is a fixed quantity and the neutral axis must be moved to some lower position C_2' in order that the total compression represented by the area

$$T_2 = O'C_2B_2 = O'C_2'B_2' = T_2'$$

may remain constant. It is evident then that there is first a lowering of the position of the neutral axis from C_1 to C_2 by reason of the increase of p_1 to p_2 and next a lowering of it from C_2 to C_2' by reason of the decrease of the modulus of elasticity E_c under large values of the unit stress f_c. This explains more fully why high values of f_c should be avoided in shallow beams.

On the other hand, deviations of concrete from perfect elasticity are less, the less the actual compressive unit stress f_c acting upon the concrete. In other words, concrete under the smaller stresses behaves more nearly like perfectly elastic material. An effective method, therefore, of reducing the sharpness of bending and consequent exaggerated tendency of shallow beams to check and crack is to make the compressive stress f_c in the concrete small compared with f_s the given stress in the steel. This is equivalent to making the stress ratio f_s / f_c larger for shallow beams than for deep beams, as is the practice among experienced builders.

There is still another way of stating the consideration which leads to the adoption of small values of f_c for shallow beams. It is desirable to limit deflections under working loads to a figure not much in excess of $L/1000$. With f_s given, the adoption of large values of f_s / f_c will make the compressions in the concrete so moderate as to prevent excessive deflections even tho there should be some small initial deflection due to non-elastic compression of the concrete. Values of f_s / f_c from 16 to 35, for values of $n = 12$ and $n = 15$ which have been computed from formula (5) section 6, have been plotted in the accompanying Fig. 28.

Since the steel element in the combination is more dependable than the concrete from the standpoint of uniformity of strength, the safety of the structure is made by experienced builders to depend on the steel. In order to effect this the working strength of the concrete should be taken at a smaller fraction of its ultimate strength than the working strength of the steel is of its ultimate strength. For a 1:2:4 mix 650 pounds per square inch is a safe working stress to resist compression in concrete arising from bending. Both tension and compression are developed in concrete by flexure and by bond shear. The resistance, however which it offers to tensile stress is small compared with that which it offers to compression. Forty pounds per square inch is a safe value of the working tensile resistance.

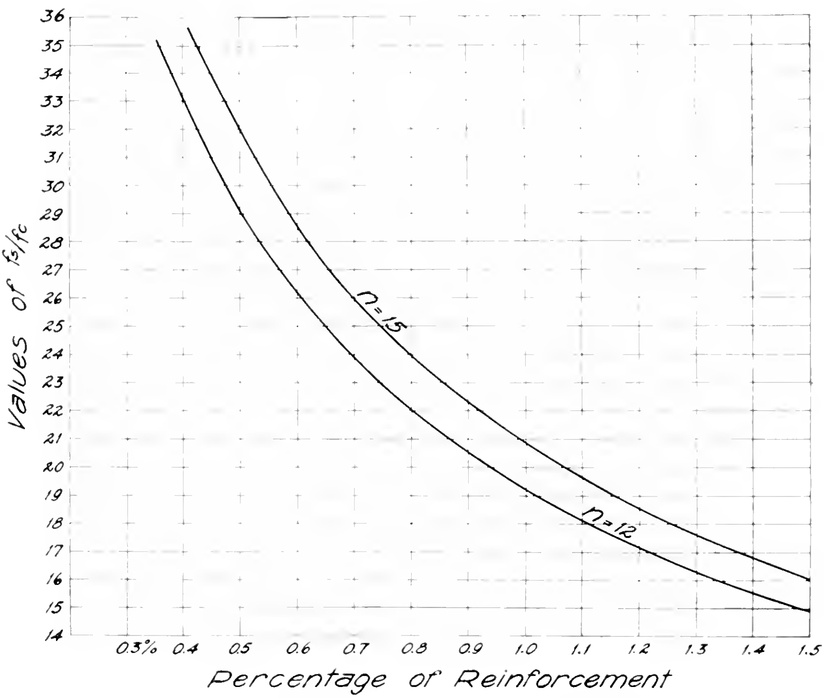


Fig 28. Limiting per cent of Steel for Different values of f_s/f_c.

Now if a beam is to depend for its stability upon the stress in the steel, that stress must not exceed a certain assigned value dependent upon the quality of the steel, or what amounts to the same thing a given quality of steel must not suffer a stress or a corresponding elongation at any point in excess of an assigned value. In order to compare beams of different depths and the same assumed maximum stress f_s or elongation e of steel, draw two plane sections of the beam under consideration at right angles to the neutral axis and at a distance of one unit apart. Before bending occurs, the two sections are parallel to each other, but after bending they make the same elementary angle $\triangle\theta$ with each other. Draw thru one extremity of the unit of length at the neutral axis (which has been unchanged in length by bending) a plane parallel to the section at the other extremity of the unit length. It consequently makes an angle $\triangle\theta$ with the original plane section at this extremity. Any horizontal shearing deformation may be disregarded in this comparison because it will affect both sections to practically the same amount. The unit elongation of the steel due to the bending between these two unit sections will be $e=(1\text{-}k)d\triangle\theta$.

This investigation is made upon the assumption that e has a value which is constant and the same for different beams, but with the proviso that under this steel elongation neither the shearing distortion nor the compression of the concrete anywhere shall exceed permissible limits, questions which will have to be separately investigated since they depend on the steel stresses in too complex a manner to be readily introduced into consideration at the same time with the effect of the constancy of the steel stresses.

Now other things being equal $\triangle\theta$ decreases as p, the percentage of the reinforcement, increases; i. e. $\triangle\theta=c/p$ where c is an experimental constant whose value is dependent upon the grade of concrete, etc. Substitute this value of $\triangle\theta$ in the previous expression, then

$$e/c=(1-k)d/p=\frac{(1-k)L}{pN}$$

in case d be assumed to be some known fraction $1/N$ of the span L, i. e. or $d=L/N$.

It thus appears that the last member of this equation will be found to be an experimental constant for reinforced beams of the same span and grade of concrete; and in case the numerical value of this constant be determined for any given beam not liable to excessive deformation at the center, it will have the same value for a beam of different depth, span, and percentage of steel, provided, as before stated that sufficient resistance to compression and diagonal tension be supplied.

For example, assume $f_s=13{,}000$ and $f_c=650$, then $f_s/f_c=20$. Referring to the curve for f_s/f_c Fig. 28 the corresponding steel ratio is $p=0.0105$, and taking the corresponding value of k from the k curve, it appears that $(1-k)=0.573$. It is known by experience that a beam whose depth is 1/12 of the length should have this amount of reinforcement, or $N=12$ when $p=0.0105$.

Hence $$\frac{(1-k)L}{pN}=\frac{0.573\ L}{0.0105\text{x}12}=4.55\ L$$

is the constant for such beams.

The same curve shows that for $f_s/f_c=26$, $p=0.007$ and $(1-k)=0.635$; hence, using these and the constant 4.55, we find $N=20$.

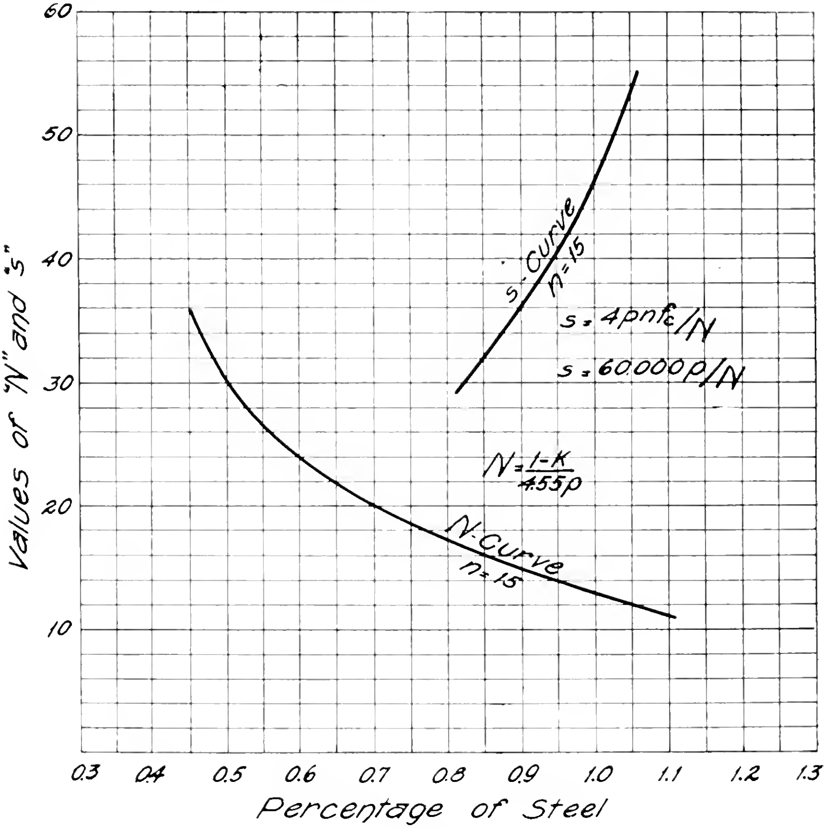


Fig. 29.

Again, for $f_s/f_c=32$, $p=0.005$ and $(1-k)=0.68$ and $N=30$.

The large values of f_s/f_c which have been assumed above for the shallow beams $N=20$ and $N=30$, reduce the working stress f_c and the steel ratio p below the values for $N=12$ in accordance with good practice. The accompanying diagram Fig. 29 gives values of $L/d=N$ for usual values of p computed from the equation $N=(1-k)/4.55\ p$ and $n=15$.

Now the total tension T in the steel at mid span of a simple beam is $T = WL / 8jd$. But $T = bdpf_s$ in which f_s is the unit steel stress at mid span, hence $f_s = \dfrac{WL}{8\,jbd^2p}$.

But the identical elongations e of both the concrete and the steel at mid span may be written

$$e = f_s / E_s = f_c / E_c, \quad \text{or } f_s / f_c = n$$

and
$$f_c' = \frac{WL}{8\,jbd^2pn}$$

in which f_c' is the apparent direct tensile stress in the extreme fiber of the concrete at mid span as shown by its elongation e while in contact with the reinforcement. The so called apparent stress f_c' may or may not correspond to an actual stress of some considerable amount. It is used here simply as another way of expressing the actual elongation e. The experiments of Considère* show that concrete when well reinforced may remain intact under elongations not only far in excess of any possible for concrete without reinforcement but in fact remain intact under elongations several times as great. The reasoning here employed is however entirely independent of any question of actual checking or not, for $f_c' = eE_c$ is simply a convenient unit of comparison computed as the product of elongation and modulus.

Next obtain the shearing stresses and the diagonal tension in the concrete at the extremity of a simple beam. The total horizontal shear between a unit of length of the reinforcement and the concrete is such that a segment of the beam lying between two vertical planes which are one unit apart is held in equilibrium by the total vertical shear $\frac{1}{2}W$ acting with the arm unity and the total horizontal shear S acting with the arm jd.

Hence $\frac{1}{2}W = S\,jd$, or $S = \frac{1}{2}W / jd$.

This makes the unit horizontal shear on any horizontal plane below the neutral axis

$$s = S / b = \tfrac{1}{2}W / jbd$$

provided the total shear in a unit length be regarded as uniformly distributed thruout the breadth b of the beam. This is equal to the unit diagonal tension at the end of the beam which is produced by the shear alone.

Hence $s = 4\,pn\,f_c / N$

Take $f_c = 1000$ and $n = 15$ then $s = 60{,}000\,p / N$

*Experimental Reasearches on Reinforced Concrete, McGraw Pub. 2nd Ed., p. 224

By using corresponding values of p and N given previously we find

p	N	s
.0085	16	32
.009	15	36
.0095	14	40.7
.01	13	46

Plotting the values of s corresponding to the assumed values of p it appears that s will reach a safe limiting value of 40 lbs. per sq. inch when $p = .0094$ nearly and when the span is somewhat more than fourteen times the thickness, or $L / d = N = 14.25$, as may also be seen from the diagram Fig. 29. Beams more shallow than this will have smaller values of s, but deeper beams where $N < 14.25$ will require reinforcing to resist diagonal tension at the ends, when there is a working stress of 16,000 lbs. per sq. inch on the steel at mid span.

Reinforcement for the purpose of increasing the resistance to diagonal tension consists of diagonal or vertical rods toward the ends of the beam. The reinforcement may be introduced in such amount as to make unit steel stresses greater or less at mid span than at the ends. The total resultant diagonal tension at any point of the beam per unit of length of the reinforcement is compounded of the total direct stress in the steel and the total shearing stress per unit of length of the steel, and is

$$R = \tfrac{1}{2} T + \sqrt{\tfrac{1}{4} T^2 + S^2}\ *$$

in which the letters T and S designate the total direct steel tension due to bending and the total shear at the point considered respectively and do not signify as previously the total tension at mid span and the shear at the end. The inclination i of this resultant tension R to the horizon is found from the expression

$$\cot 2\,i = \tfrac{1}{2}\, T \,/ S$$

The value of R at any point of the span may be readily constructed graphically as shown in the accompanying diagram Fig. 30, in which the ordinates of the parabola called the T curve represent the total steel tension at any point of the span due to the load band and the ordinates of the straight line called the S curve represent the total band shear per unit of length of the steel at any point. Then at any point P the resultant $R = PP'$ is constructed and laid off vertically in two segments $PP'' = \tfrac{1}{2}\,T$, and the hypothenense

$$P''S = P''P' = \sqrt{\tfrac{1}{4} T^2 + S^2}.$$

* See Merriman's Mechanics of Materials, Page 273.

The ordinates of the locus of P' give the total diagonal tension R.

The total vertical force in the beam per unit of length of span which must be resisted by vertical reinforcement or tension in the concrete or both is $S = V/jd$ as given by equation (24) Section 6. This is shown by the S curve. At a safe value of forty pounds per square inch of vertical tension in the concrete, the safe vertical resistance of the concrete per unit of length of span is $40b$ pounds, Draw a horizontal line on the diagram thru some point Q at this height above P. Then vertical or diagonal reinforcement is necessary at all points of the span where the S curve lies above QQ and the

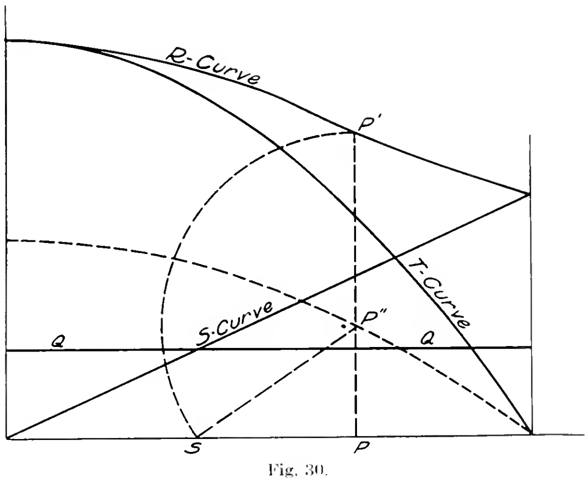


Fig. 30.

total amount of the tension to be resisted by the vertical reinforcement at any point per foot of span is represented by the vertical distance of the S curve above QQ at that point.

It is evident that the point at which a beam will first fail by diagonal tension depends upon $S - 40b$ as compared with the amount and distribution of the reinforcement, and lies at the point where the maxium unit stress occurs. This may occur at any point, depending upon the amount of the vertical steel. In beams with the reinforcing rods turned upward at the ends of the beam and securely anchored there the point is usually removed to some distance from the ends.

The treatment just given of the vertical stress in the concrete assumes that the verticals consist of stirrups or the like at some considerable distances apart horizontally, say 10″ or more.

The case, however, is different if the required vertical steel consists of rods or wires so near together as to prevent checking or cracking of the concrete until the vertical steel has a working stress of 13,000 to 16,000 pounds per square inch. In that case, diagonal tension failure need not occur before the concrete has an elongation e corresponding to an apparent tensile stress of at least 600 pounds per square inch, instead of 40 pounds; but in that case the vertical steel should be designed to resist the entire vertical tensions.

If in addition to distributing the vertical steel along the span so completely as to fully reinforce the concrete vertically, some of the

Fig. 31. View Showing Beam Failure by Diagonal Tension near the End.

longitudinal steel be run parallel to the neutral axis so as to fully reinforce the concrete below the neutral axis longitudinally as well this will introduce coaction of the vertical and horizontal steel in such a way as to materially reduce the steel stresses in the web, in the same manner as occurs in the steel stresses in slabs. This is the explanation of the striking results obtained by the beam designs of Maciachini* and of Cottançin as shown in Fig. 44.

The theoretical deductions which have been reached in the preceding pages may be confirmed by reference to a great mass of test data. It will be sufficient, however, at present to refer to certain of the tests reported in Bulletin No. 197, of the University of Wis-

* See Marsh, Reinforced Concrete, 2nd Edition, Page 77.

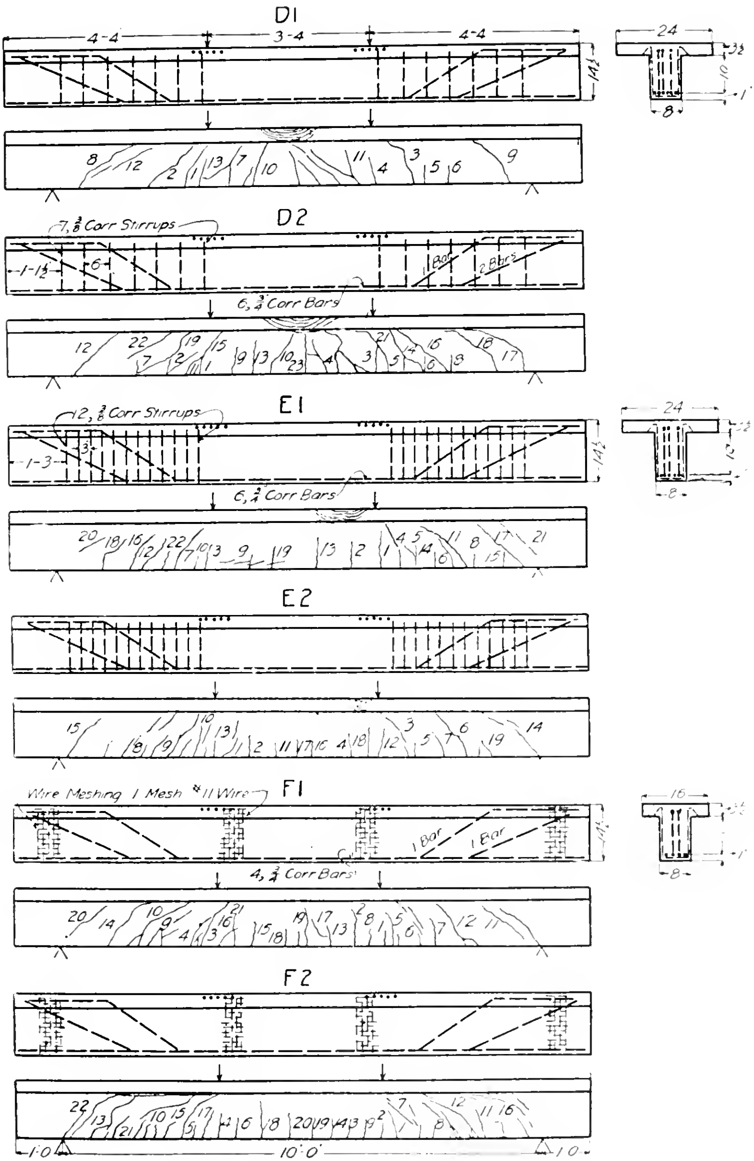


Fig. 32. Reinforcement of Test Beam and Cracks

consin,* from which Figs. 31, 32 and 33 have been taken. They show the details of the yielding and failure of several beams with the checking of the concrete as well as the amount and arrangement of the reinforcement. The beams represented are all T-beams supported at the ends with practically one percent of reinforcement, and $N = L/d < 9$. The top flange of the beams of these beams afforded sufficient resistance to make any moment failure occur by yielding of the steel in the bottom of the beam. Moreover, there was sufficient reinforcement against diagonal tension to prevent failure of that kind in the beams of series D, E, and F, but not enough in Series G. Every failure by yielding of the steel at mid span caused an amount of deflection and a sharpness of bending that crushed the concrete in the flange. The first tension cracks

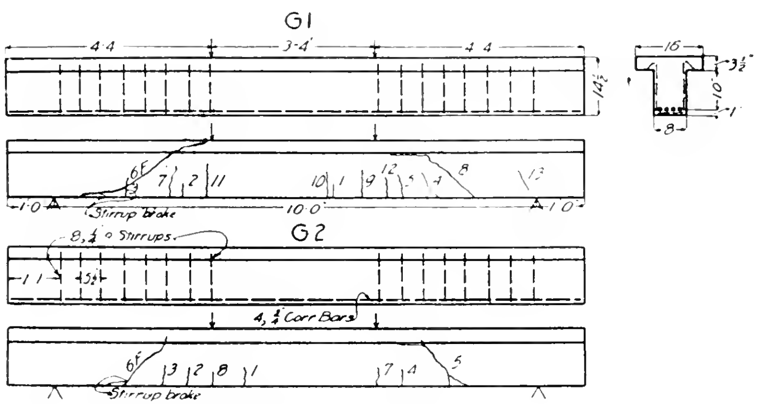


FIG. 33. View Showing Reinforcement and Cracks Test Beams, Diagonal Failure

in the middle third began to be visible on the bottom at a stress of 12000 to 15000 lbs. per square inch in the steel. In Series F, the reinforcement against diagonal tension contained no verticals such as occurred in Series D and E, but wire mesh was used instead as represented. The cracks show that the beams of series F while actually failing by direct tension in the steel were nevertheless appreciably nearer failure by diagonal tension than were those of series D and E. It is stated the mean tensile strength of 6 inch test cylinders of concrete cast at the same time as the beams was 187 lbs. per square

*Tests on Plain and Reinforced Concrete, Series of 1907, by Morton Owen Withey, C. E.

inch and that the beams began to crack by diagonal tension when the unit vertical tensile stress computed by the formula

$$S = \tfrac{1}{2} W / j\,d\,b$$

reached a mean value of 179 lbs. per square inch for all of them. This is in good agreement with the direct tensile strength of the concrete as just quoted, since it differs from it by less than five per cent. But were the steel well spaced at such short intervals along the span as to fully reinforce the web vertically as was done by Maciachini previously referred to, and were web steel introduced all the way from the ends to the loads where the shears began, we should expect to find the concrete take much larger deformations and apparent stresses than this without cracking.

It will be noticed that these beams were all loaded with equal concentrated loads at the one third points of the span. Such a loading makes the moment and shear curves very different from those given in Fig. 30. The moment curve will be horizontal thruout the middle third. It also makes the shear curve zero in the middle third and horizontal in the end thirds. These beams were consequently especially liable to diagonal tension failure at some distance from the ends where the anchorage of horizontal steel at the ends exerts no effect.

Such a failure is shown in Fig. 31 where a $\frac{1}{4}$ inch stirrup is broken just below the arrow, and as a consequence, because the concrete was unable to resist the vertical stress the crack then extended along the horizontal steel at the bottom and along the flange at the top.

It should be noted that the age of these beams at the time of testing was only 28 days, at which time the resistance of the concrete to shear and diagonal tension was probably not more than 40 percent of the ultimate, a fact that would be likely to make their behavior when fully cured materially different from that exhibited at the time of test, so far as shearing and diagonal tension are concerned.

8. **Discussion of the Elastic Properties of Beams and Assumptions involved in the Preceding Theory.** In discussing the elastic properties of concrete it was shown that the modulus of elasticity of the concrete is not constant for different loads and further that the modulus changes with the age of the concrete, it being only $\frac{2}{3}$ as great at the age of thirty to forty days as it is at the age of two or more years, and furthermore that while this modulus is usually considered the same for tension and compression, this is open to some question. Accordingly a reasonable approximation to the conditions which

occur in bending when the building is first ready for occupancy is attempted and in nearly all building codes the ratio of the modulus of elasticity of steel to concrete in bending is assumed at 1 to 15. A difference in this ratio would affect the position of the neutral surface to some extent and the effective lever arm to a still smaller extent, as shown by comparison of the j and k curves in the diagram in Fig. 25. The assumption of the value of n as 15 for bending is accordingly on the safe side, and the error involved is not great. On the other hand, this divergence of practical conditions from the assumptions used in the computation do not justify a high degree of mathematical precision in the work of practical design, for if the computations are carried to a degree of nicety unwarranted by the accuracy or agreement of the assumption with practical conditions it is a mere expenditure of time without commensurate results. Accordingly, it may be stated that the approximate formulas for beams and slabs are sufficiently accurate for practical purposes.

In T-beams where the beam is integral with the slab, it is customary to assume a width of slab not exceeding four times the slab thickness as forming a part of the compressive flange of the beams. This assumption, of coarse, is conservative. It is evident that the compression in the outside edge of that portion of the slab which is regarded as useful section is less than portions nearer the axis of the beam, and that the slab beyond this imaginary division is also restrained in compression, the condition approaching what has been designated as the "twilight zone" between exact knowledge and conjecture as to the actual conditions. Evidently in a case like this, exact computations beyond the limits of accuracy of the assumption is a waste of time and the effective depth jd of a T-beam may be for practical purposes determined at once by the assumption of $(1-j)d = \frac{3}{8}\ t$ without material error when d lies between 2.5 t and 4 t.

In case of the rectangular beam with $\frac{5}{8}$ to $1\frac{1}{4}$ percent reinforcement, the assumption of $jd = .85\ d$ is sufficiently accurate for practical purposes, and for percentages of steel less than $\frac{5}{8}$ percent the assumption of $jd = .9d$ is sufficiently accurate.

The case of the doubly reinforced beam is one which the designer rarely is called upon to make use of. In it the compression steel should preferably be about $2\frac{1}{2}''$ to $3''$ from the top surface of the beam. Unless the percentage of tensile reinforcement is very high, say 3 percent or more, and the compressive reinforcement very low, say less than 0.75 percent, the neutral plane is nearer the compressive steel. Assuming d' to equal $d/10$ when p' is 2 percent, it is nearer the compressive steel for all values of p'. Thus it follows, since the unit stresses in the compressive and tensile reinforcements are as the distances of these reinforcements from the neutral plane, that the unit stress in the compressive steel is for these percentages less than that in the tensile steel. For very rough approximate computations, taking $n=15$ and the average value of $j=.85$, $k=.45$, we have

$$f_s=\frac{1.17M}{p\,b\,d^2} \quad \text{and} \quad f_c=\frac{M}{(.19-10.5\,p')\,bd^2}$$

The above formula for f_s is a fair approximation. The formula for f_c with different percentages of steel is by no means a close approximation.

A much more satisfactory method of computation is as follows: From equations (20) we find

$$f_c=\frac{f_s k}{n(1-k)} \qquad \text{(19a)}$$

To determine f_c we cannot assume an arbitrary value of k in this equation since that would be tantamont to assuming that the amount of compressive steel would make no difference in f_c, hence this equation cannot be used as an approximate method of determining f_c, but it may be employed to determine f_c in an accurate manner by plotting the curve from the values of $n(1-k) \div k$ for different percentages of tensile and compressive steel from which we may derive f_c by dividing f_s by the value taken from the diagram.

The accompanying Fig. 34 shows the curves of different percentages of tensile steel for different percentages of compressive steel reinforcement noted at the bottom of this Fig. The values of j are given at the left of the diagram and the values of $f_s \div f_c$ for the different percentages of tensile and compressive reinforcing steel given at the right.

9. Classification of Beams: This is based upon the manner in which the beam is supported:

A simple beam is one which is merely supported at the ends, and its mathematical treatment is based upon the consideration that the beam is free at the end to turn and that the supports offer no resistance to rotation. Such a beam, does not, of coarse, exist in practice, but all beams which rest upon supports and are not rigidly restrained or have only a small degree of restraint at supports are treated from the practical standpoint as simple beams and figured as such.

Concrete beams or slabs which are apparently continuous over supports but which have reinforcing metal at the bottom thruout, offer so slight resistance to negative moment at supports that they are treated on the theory of predominant action as simple beams unless their depth be sufficient and the longitudinal restraint offered by the construction such that may be treated somewhat on the arch principle. A slab supported on parallel walls and reinforced in one direction is merely a wide beam and with reinforcement in the bottom thruout is to be treated merely as a wide simple beam.

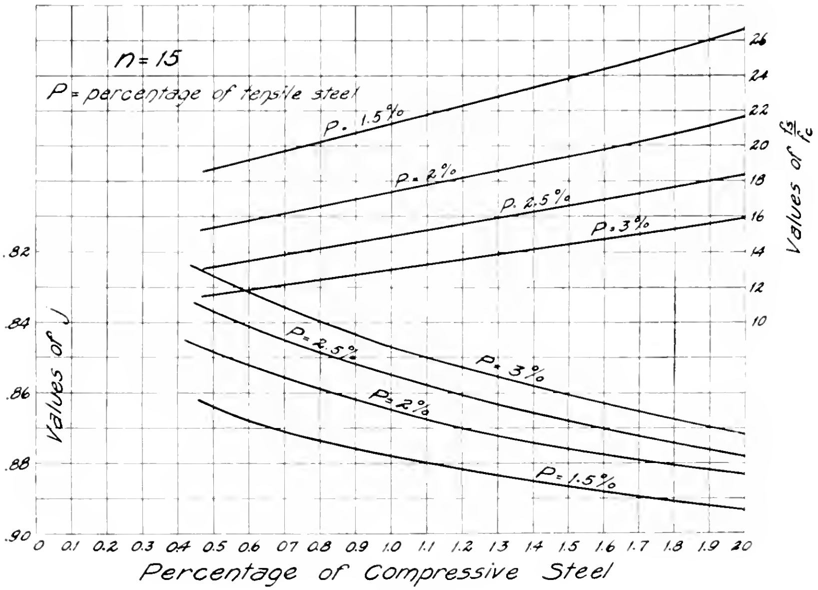


Fig. 34.

Where, however, a beam is continuous thruout, and rigidly built into and integral with a series of columns, and suitable reinforcement is provided at the top of the beam over the support and extending outward to the line of inflection, and then thruout the bottom of the beam, we have a true continuous beam in which the bending moments over the support follow the laws of continuous beams except as they are modified by the rigidity of their integral union or connection with the columns.

The effect of this monolithic connection is to cause the deportment of the beam to approach more and more nearly, for all spans, to the condition of a continuous beam extending through an indefinite number of spans; in other words, to cause the moment over the support for uniform load to become $WL/12$ and the moment at mid span $WL/24$. An end span of the series, however, except in heavy warehouse construction will not receive this full degree of restraint. In a heavy warehouse with the large columns, 26 inches and over in diameter, this degree of restraint is for practical purposes fully secured, but with smaller columns it may be less, and its amount is to be determined approximately by the designer from comparison of the relative rigidity of the columns and beams, so that in the case of light columns the end spans should be somewhat more heavily reinforced for moment at mid span.

The case of an unloaded span with both adjacent spans fully loaded is not uncommon in a warehouse and this, too, must be provided for.

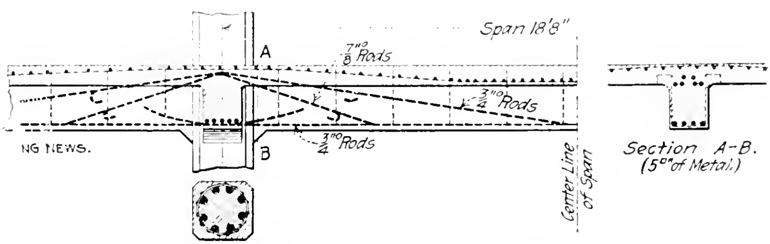


Fig. 35. Continuous Beam, Turner System.

Where the construction consists of beams in but one direction with the slab spanning from beam to beam and with insufficient metal parallel to the beam in the slab to fully reinforce the beam to resist the negative moment under the circumstances stated, which may result from the excess of live load stress over and above the dead load stress, the beam should be treated, in determining the central moment, as continuous for dead load only and as a simple beam for the live load.

The preferable arrangement, however, is the provision of beams in both directions from column to column where beam and slab constructions are used, making the floor a true monolith or a natural concrete type. For this type of construction with ordinary spans, beam reinforcement consisting of say five rods arranged as indicated in Fig. 35 is preferable, in which the beam rods consist of two which extend thruout the length of the beam at the bottom and into the adjacent span, two which are bent up from the quarter point to the top of the beam and extend over into the adjacent span, and one which while extending into adjacent spans slopes gradually from the top of the support to the bottom of the beam near the center of the span.

An arrangement of this kind, after the manner of the Bollman truss, furnishes liberal provision for shear at the support, while the inclined rods, under bending strain resist shear thru their inclination at the support. It is only necessary to figure the moment over the support as $WL/12$ and provide therefor by the cross section of the

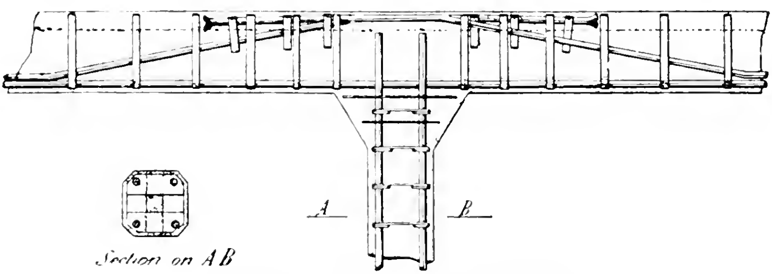


Fig. 36. Continuous Beam, Hennebique System.

six rods crossing it while the four rods in the bottom of the beam and the five rods at the center are ample for all possible conditions of loading. The lap of rods at the support and beyond the support both at the top and bottom render sudden failure or collapse practical-

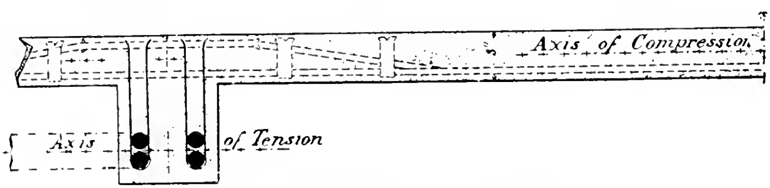


Fig. 37. Cross Section of Beam and Slab.

ly impossible after the concrete has had even a few hours under normal temperature conditions in which to harden.

In addition to enhanced safety there is very material economy in such an arrangement, since compared with beams of constant section, the continuous beam is more than five times as stiff and one and one half times as strong as the simple beam having the same cross section of metal thruout the bottom of the beam as the continuous beam has at the top over the support.

Fig. 36 shows the Hennebique continuous beam which has an enviable record from the standpoint of safety by virtue of the liberal lap of reinforcement and stirrup verticals employed.

The fact that concrete is well adapted to be placed in a monolithic mass renders continuous construction the natural type to use for the reason that it combines the highest degree of safety with the maxium of stiffness and economy.

Even tho settlement of the supports should occur and the concrete should check by reason thereof, the well designed continuous floor does not become dangerous and unsafe. So long as the concrete is hard and rigid, the checked segments can take compression, and the steel while the bond is intact can furnish the full resistance to tension which was originally figured upon disregarding the direct tensile strength of the concrete itself. This statement is true, of

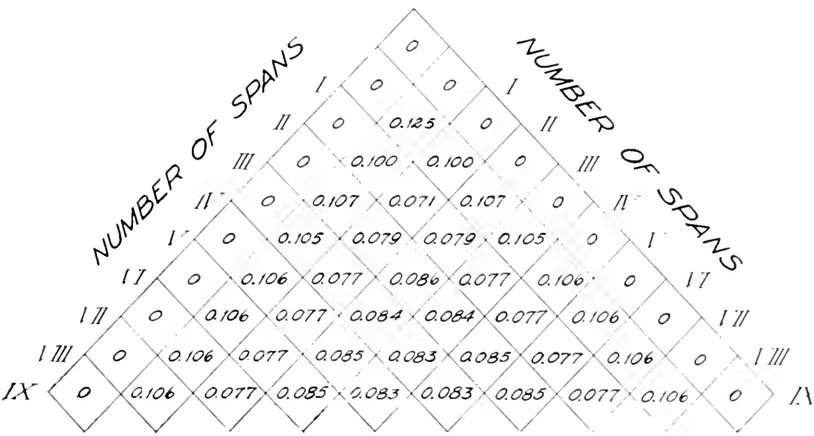


Fig. 38. Table of Coefficients of Moments over the Supports Beams Freely Supported.

course, only where ample lap of the rods has been provided over the support which should always be attended to in order to ensure safe and satisfactory results in beam design.

Independent beam construction has been used to a small extent, that is separate beams cast in the shop and sent to the job as individual units. Difficulties in handling made up units in erection offset some saving in cost of forming, while their lack of joint action in carrying load puts constructions of this kind at a serious disadvantage compared with monolithic work wherever the loads to be carried are of considerable magnitude. Independent units, however, may be quite economically employed for roof construction and the lightest kind of work where suitable facilities are at hand for carrying on the work of erection economically.

For convenient reference, Fig. 38 gives the bending moment over the supports in a series of equal spans uniformly loaded up to nine spans, while Fig. 39, gives the maximum moment near the mid span for a similar series of continuous beams freely supported and uniformly loaded. From these moments, taking into consideration the relative rigidity of the column and the beam in question, such cases as those above suggested where the rigidity of the column is small in comparison to the beam, may be correctly treated. The reactions over the support are shown in Fig. 40. The amount of bending moment and the reactions are to be determined, of course, by multiplying the coefficients in the table by W the total load per

NUMBER OF SPANS

Number of Spans										Number of Spans
I	0.125									I
II	0.070	0.070								II
III	0.080	0.025	0.080							III
IV	0.077	0.036	0.036	0.077						IV
V	0.077	0.033	0.046	0.033	0.077					V
VI	0.078	0.034	0.043	0.043	0.034	0.078				VI
VII	0.078	0.033	0.044	0.040	0.044	0.033	0.078			VII
VIII	0.078	0.034	0.044	0.041	0.041	0.044	0.034	0.078		VIII
IX	0.078	0.033	0.044	0.041	0.042	0.041	0.044	0.033	0.078	IX

Fig. 39. Table of Coefficients, Maximum Positive moment between Supports for Continuous Beam Freely Supported.

span by the length of span to obtain the bending moment, while the reactions are the ~~tabulor~~ tabular coefficients of W the total load on a single span.

It is here in order to call attention to the fundamental relation of moment magnitudes which may be observed in these tables.

For any span, half the sum of the moments over the supports plus the moment at mid span is a constant, equal to $\frac{1}{8}$ $W L$, in which W is the total load uniformly applied and L the span. This relation is of use in treating many problems. The variation of the maximum positive moment from the moment at mid span is greatest in the case of a continuous beam of two spans where it is $12\frac{1}{2}$ per cent. This difference is much less in all other cases and nothing at all whenever the moment over both supports is of equal magnitude.

It will be noted that the greatest moment at mid span for uniform load in the case of the continuous beam of two spans is 9/128 WL, and the maximum positive moment for a span of indefinite length is $WL/24$. Further it will be noted in case of alternate spans unloaded that the usual moment provided for as recommended for live load is 1/16 WL. Hence it is apparent that where the columns are fairly rigid and the beams are integral with the columns, little attention need ordinarily be paid as to whether it is an intermediate or an end span with which we are dealing.

10. Economic Design for Beams. As already noted, the coefficient for bending of a continuous beam is one-third as great

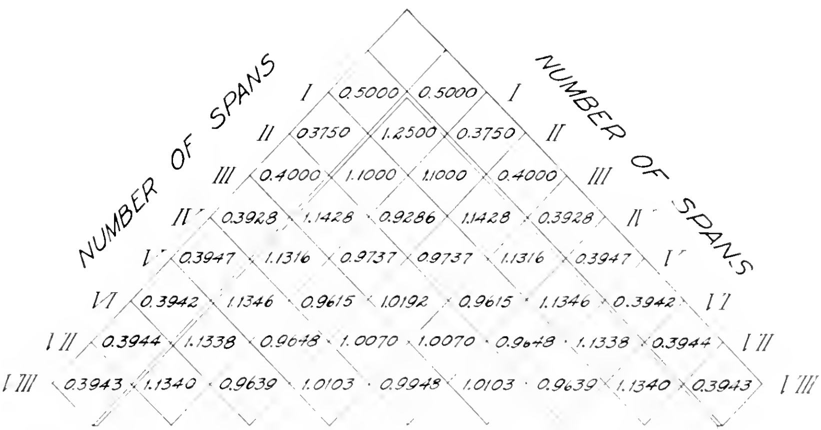


Fig. 40. Table of Reaction Coefficients for Continuous Beams of Equal Spans Uniformly Loaded and Freely Supported.

at the center as in the case of a simple beam, and two-thirds at the support. Now for safety ample lap of the bars is needed, hence by carrying a part of the reinforcing rods required at the center up over the support and by carrying them to about the point of contraflexure or so far that the negative moment in case of a single panel load will be taken care of by slab reinforcement parallel to the beam we have need theoretically (considering moment only) two-thirds the section of steel for about one-third of the length and one-third of the section of metal for two-thirds the length of this beam of that required for a simple beam. In other words, we have the following comparison from the standpoint of theoretical economy. That the metal required for a continuous beam is one-half that required for a simple beam and further that the construction with a continuous beam is safer to erect since the work is more securely tied together and it can be depended upon with a good concrete not to fail suddenly but only by the actual stretching out of the metal to the point of ultimate fracture in case of loading equal to three or four times that which it was calculated to sustain.

This theoretical economy however cannot be fully realized. Two-thirds in place of one-half would be nearly the limit attainable.

Evidently the greater the depth the less steel will be required to carry a given load. Usually, however, the depth to be used in an ordinary building is determined from the standpoint of appearance and the extra cost of walls for a given clear story height rather than from the theoretically economical proportions of steel and concrete alone.

A mistake which is frequently made is in building beams too narrow and deep especially where they are spaced closely. Such construction is lacking in resistance to high temperatures since too great an area is exposed and it should preferably be avoided on that account.

A minimum width of ten to twelve inches should be adhered to for reinforced concrete beams in a building that is intended to be fireproof to a high degree, and such a width for moderate spans of sixteen or eighteen feet will usually give ample concrete to properly surround the reinforcement in the beams.

In general there should be sufficient width to allow one inch of concrete between the bars or a width not less than one and one-fourth times the diameter of the bar if the bars are parallel for any considerable length. Where the bond shear is small and they are bunched as at the top of the beam where there is ample spread beyond this point in the beam this requirement becomes of no especial importance.

Relative to the economic proportion of concrete and steel, the general relation to be observed, is that the amount of steel decreases with the depth of the beam, while the amount of the concrete increases. With one percent of steel as the element of reinforcement, it is evident that the concrete element will cost more than the steel element on the basis of five dollars a yard for the concrete and steel at fifty dollars per ton. Hence with ordinary values of steel and concrete, the limiting permissible percentages and relation of safe working stresses fix economic proportions. In the continuous beam, however, where there is double reinforcement over the support, a nearer approximation of the balance of the cost of concrete and metal may be approached than in the simple beam.

However, in the T-beam which is the usual construction in buildings this percentage would be based upon the area of the beam below the slab plus the area of that part of the slab above and on each side of the rib which it is permissible to consider as forming part of the compression flange of the beam, and the economic proportions would have a smaller proportion of steel, since the portions of the slab figured in with the beam are not added material as far as the beam is concerned, and hence the comparison should be based more properly upon the area of the rib below the slab of the concrete added to form the beam. Hence the economic proportion of steel would in general be reduced below that of the limiting proportion fixed to secure conservative working stresses in the concrete.

This conclusion, that the cost of the steel in the T-beam should be less than the concrete is strengthened by the consideration that the cost of the centering increases with increase of depth of the beam. These practical considerations seem to have been entirely overlooked in the discussion of the paper presented to the American Society of Civil Engineers in 1906 by Capt. Sewell, on the subject of economic construction of reinforced concrete floors.

No mathematical formula can be devised which will take into consideration all of the variable elements of cost. Trial designs and the practical judgment of the constructor enable him to find an approximate and satisfactory solution of this complex problem. The intimate relation of horizontal shear to permissible percentage of steel which we have pointed out earlier in the discussion is but another of the complex elements entering into the problem.

11. Safe Loads for and Tests of Reinforced Concrete Construction. The Joint Committee of the American Society, etc., in their treatment of working stresses lay down this commendable rule:

> "In selecting the permissible working stress to be allowed on concrete, we should be guided by the working stresses usually allowed for other materials of construction so that all structures of the same class but composed of different materials may have approximately the same degree of safety."

12. True and Nominal Factor of Safety. A popular misconception regarding the meaning of the term factor of safety as applied to steel construction has exerted an influence from the economic standpoint adverse to the rapid introduction of concrete construction.

Many have the mistaken idea that the factor of safety of four in steel construction means that the construction may be safely loaded to four times the rated working capacity; but this is not the case, since the yield point of steel is only about twice the working load; hence the actual factory of safety is practically two against the nominal factor of four.

In other words, the nominal factor of safety of four in structural steel work is based on the ultimate carrying strength in tension of the metal which is about four times the working load, but after the load has reached a little more than double the working load the yield point value of the steel has been nearly or quite reached and it commences to stretch, pulling out in case of mild steel before breaking sometimes as much as twenty percent or more of its total length. Evidently when this plastic distortion commences in a beam or column the member is soon so deformed that we cannot figure its strength in the frame, thus limiting the ultimate strength to practically a little more than twice the working load for this nominal factor of four.

In properly designed concrete construction the concrete is made stronger than the steel, for one reason because it is generally economical so to do, and hence the strength of the steel is the strength of the reinforced concrete construction and it would not be reasonable to expect to subject the steel to higher stresses in the case of concrete construction than is permissible in structural work; hence twice the working load is a fair test for this type of work. In reality, in view of the fact that the cement improves with age, if it will stand this test at an age of from three to four months the owner can rest assured that the factor of safety is greater than with structural steel construction.

Referring to the specifications for reinforcing bars, page (34), it will be noted that for structural grade bars (recommended for beams and bent work) the yield point is 33,000, or two and one-fifth times the working stress of 16,000 pounds allowed by nearly all building codes, while for hard grade, 50,000 pounds per square inch is the yield point value. Accordingly, higher test loads can be applied where the reinforcement is of hard grade steel than with the softer grade. However, greater care is necessary in bending the hard grade metal; the structure is not so tough and the results of the use of this grade of steel are more uncertain.

Excessive tests are not to be recommended, since some permanent set and weakening of the structure may result therefrom. Elastic deportment in accord with theory under tests of one and three quarters to two times the working stress for heavy work should suffice.

13. Method of Loading for Tests. To secure results of scientific value the material used for test loads should be piled in such a manner that its action on the slab or beam under consideration shall not be masked by arch action.

A misleading type of test is shown in Fig. 41, consisting of cast iron piled up in a manner which enables it to arch readily to a large

Fig. 41. Test in which Arch Action occurs from Main Beam to Main Beam, giving a Misleading indication as to the Strength of the Floor.

extent from main beam to main beam. In this case the construction is practically type I, with the joist girders five feet to six feet apart. The load shown was actually 1,500 pounds per foot, but so far as the girder on which it rested was concerned it was probably not equivalent to more than 1,000 pounds uniform load placed in a manner which would prevent arch action from main girder to main girder.

A material such as gravel in bulk may arch somswhat, perhaps to the extent of five to six percent. With cement sacks there may be also a small amount of arch action, but in view of the fact that the material is not rigid in form, as in the case of pig iron, this action can amount to very little unless special pains be taken to lay the bags in a manner to secure such action, and even with the greatest pains it is doubtful whether the bags can be placed on a large panel in such a manner as would make the arch action amount to more than twice the above limits.

In considering the degree or amount of arch action which may exist in a pile of material it may be noted in first place that the arch action will be greater the greater the height of the pile as compared with its base.

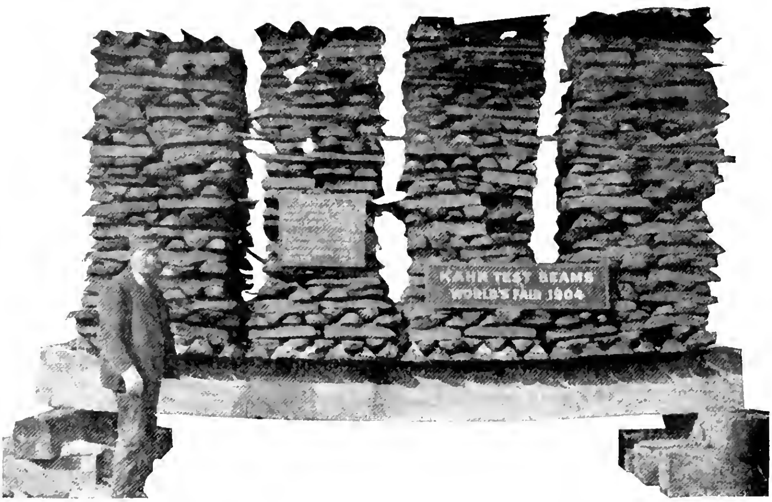


Fig 42. Test in which Arch Action is Eliminated using Pig Iron.

Thus a pile of gravel seventeen feet square held in by a wall of sacks filled with gravel on each side eight feet high might reduce the actual bending on the slab five to eight percent. If the pile were one-third of this height probably the arch action would not exceed one-half to three-quarters of one percent.

With a pile of pig iron carefully built the amount of arch action might readily become large, since the pigs are rigid, and if laid up carefully a quite perfect Hindu arch could readily be built which would carry over half the load to the support or main beam without straining the girder or slab which it is nominally the intent to test.

Fig. 42 is a test made at the St. Louis Exposition, using cast iron, in which there can be no doubt as to the distribution of the load.

In general, the contractor desires to use for loading the materials about the work which can be conveniently placed upon the panel or area to be tested and he should, of course, be allowed to do this, since the expense of making a reasonably conclusive test on a floor will frequently amount to several hundred dollars.

Brick, cement in sacks, sand or gravel, stone, plaster, and the like will frequently be used by the contractor if he has them at hand, instead of carting in pig iron from a distance, unless there is some object to be gained from a misleading test.

The area necessary to be covered in making a satisfactory test of a building, will, of course, depend on the type of construction and the unit which it is desired to investigate. In the case of the slab between parallel beams, Type II, the loaded length parallel to the beams, should be not less than two times the distance between the beams in order to induce a condition of maximum stress at the center of the loaded area approximately equal to that which would occur if the full area of the slab were loaded.

In the case of continuous slabs, Types I and II, the most severe positive stress is determined by loading placed upon single panels or alternate panels.

In the Mushroom construction, the maximum deflection at the diagonal center of the panel is secured by loading one panel only. The maximum possible stress over the column for a given unit intensity of load occurs when four panels are loaded, tho this being a uniform stress all around the column it frequently is not in excess of the unit stress in compression at the underside of the cap with the unbalanced load of the single panel loaded, and as the steel is usually in excess the test of four panels leads to no more knowledge of the deportment of the structure than would be obtained by the single panel test.

In a flat slab on spaced supports, reinforced in two directions, the maximum deflection at the diagonal center of the panel is obtained when five panels are loaded, the panel under consideration and the four panels adjacent to its sides. This difference in deportment is brought about by the fact that the two way reinforced slab throws the shear on the side belts whereas the four way reinforcement tends to transfer it more directly to the column center. In other words, while the mode of operation of the two types is substantially the same as regards cantilever head and the character of the stress about the diagonal center of the panel, it is otherwise with the distribution of shear in cross sections of the direct belts which act after the manner of beams in both constructions, the two way reinforcement not taking advantage of all of the advantageous characteristics of the four way system.

14. Shears in Beams. Shearing stress at and near the supports of a cantilever or continuous beam or slab is an action of an essentially different kind from the shear accompanying bending in a simple beam. At the support of a uniformly loaded continuous beam for example, where the negative moment reaches its greatest numerical value, the beam resists a sliding stress on its vertical cross section equal to the load transmitted by the beam to the support. This is accompanied by no horizontal shearing stress across this section, and no diagonal tensional stress is called into play by this sliding shear, which may be otherwise designated as punching shear, nevertheless diagonal shearing deformation occurs here as will be shown later.

We will now consider how it may be true that there is no horizontal shearing stress in this case, a conclusion which is entirely opposed to the principles underlying ordinary bending shear where statical equilibrium requires the intensity of shear on vertical and horizontal planes to be equal at all points of the material.

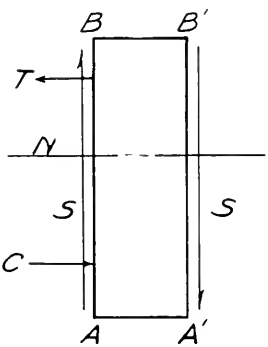


Fig. A

In Fig A let AB represent a vertical section at the edge A of the support of a continuous beam at the left of A, while $A'B'$ is a neighboring vertical section of the beam. This element of length of the beam is subjected to unequal tensile stresses on those points of its vertical faces lying above the neutral axis N and to unequal compressive stresses on the faces below N.

Let T represent the difference of the tensile stresses and $C = —T$ the difference of the compressive stresses on the opposite faces. This difference is greater per unit of length at the support than elsewhere, as appears from the greater slope of the moment curve here. These differences or resultant horizontal forces on the faces form a couple which acts on the element $ABB'A'$. This couple is held in equilibrium by the couple arising from the vertical shearing stresses on the opposite faces AB and $A'B'$. The stresses T and C do not, however, cause shears between the horizontal fibers but merely cause differences between the tensions or compressions at their extremities which determine the law of distribution of the intensity of the total vertical shear S on AB and $A'B'$ and make it increase as T and C do, viz: proportionally to distance from the neutral axis N.

So long as the reinforcement at the top or tension side of the beam or slab at the support preserves the concrete perfectly intact it will compel the concrete to act in the manner just indicated. We shall designate this action as punching shear altho it does not conform to the description of punching shear as used by the Joint Committee, since they do not allow any compressions upon AB or $A'B'$. It is doubtful whether such a state of stress is possible as that described in the definition of the Joint Committee.

The diagonal tensional deformations of punching shear are not the same as in ordinary bending shear as may be seen from the accompanying representation, Fig. B. In this diagram if a vertical shearing stress of given intensity on AB will cause DC situated at a distance of one unit from AB to be displaced to $D'C'$, then in homogeneous material an equal horizontal shear such as occurs in bending shear will displace AC an equal amount, so that the total diagonal displacement CC''

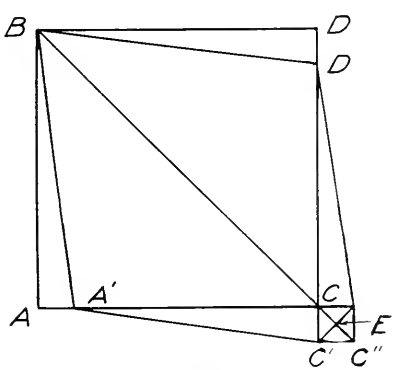


Fig. B

$=CC'\sqrt{2}$ is half of it due to each. Consequently a punching shear in homogeneous materials not accompanied by horizontal shearing stress, causes a diagonal deformation only one-half as great as is caused by bending shear where the intensity on the horizontal plane is equal to that on the vertical plane. Hence only half as much diagonal reinforcement would be needed to restrain diagonal elongation in the one case as would be required in the other. If the mean unit resistance to horizontal shear, however, is less than to vertical shear, the horizontal and vertical deformations CC' and $C'C''$ will be unequal, as well as their diagonal components CE and EC''. But they may be readily computed when the mean moduli of vertical and horizontal resistance are known.

Experimental determinations of the strength of concrete in resistance to direct sliding shear not involving diagonal tension show that in general it exceeds 50 percent of its compressive strength but its strength is largely dependent upon the age of the concrete, especially in resistance to punching shear at supports.

Taylor and Thompson* quote Spofford's Experiments on sliding shear in concrete from 24 to 32 days old and say "these experiments gave a shearing strength ranging from 60 to 80 percent of the compressive strength of the concrete, which agrees substantially with the experiments of Prof. Arthur N. Talbot in 1906."

In order that any such values of direct shear should exist in a beam or slab the anchorage of the tension steel must be absolutely secure and its amount sufficient. In this lies the difference between continuous beams and slabs where such anchorage exists, and footings where anchorage is relatively insecure.

As stated previously, punching shear, which is the shear at or near supports of continuous beams, is distributed on the vertical section in such a manner as to be greatest at the extreme fiber and it is entirely unsafe to trust to the stability of unreinforced concrete to resist it. Steel rods should always be put in both the top and bottom layers of continuous beams, more in the top than in the bottom and continued entirely across the top of the support and well anchored at some distance into the next span. The direct shear of such steel like that of rivets can be counted on with certainty, as concrete cannot.

With a 1 : 2 : 4 mix, 28 days old, cured under laboratory conditions and containing .75 of 1 percent of reinforcement, the section may be counted on to resist safely a shearing stress of 6 percent of the compressive strength of the concrete, and with over 2 percent of tensile steel and two-thirds as much in compression, this may be increased 30 to 50 percent. But no 28 day concrete should be subjected to severe bending and shearing stress if it can be avoided. Resistance to combined bending and shearing develops much more slowly and much later than to compression. But when a beam has been well cured for 90 days in good drying weather, the shearing strength at the supports is much more than double that just stated for 28 day concrete.

Under the conditions which have been outlined it is evident that the integrity of the section is primarily dependent on the sufficiency

*Concrete Plain and Reinforced, 2nd Ed., p. 382.

and proper distribution of the reinforcing steel. It is necessary therefore to introduce the reinforcement as a principal element of strength into the computation of the safe vertical resistance to shear at the supports of continuous beams, which has been done as follows:

For reinforcement arranged as shown in Figs. 35, 36 and 37, so that it has ample anchorage on each side of the support, when the working stress in the concrete is assumed to be merely its resistance to diagonal tension of 40 lbs. per square inch for a 1 : 3 : 5 mix, of 50 lbs. for a 1 : 2 : 4 mix, or of 65 lbs. for a 1 : $1\frac{1}{2}$: 3 mix, the steel in the upper flange may be safely counted on for a working stress in shear of 10,000 lbs. per square inch, and that in the bottom for half as much.

This method is illustrated in the following computations of the allowable working stresses in the continuous beam in Fig. 21, illustrating the Minneapolis Paper Company building, tested after the concrete was well cured for more than 90 days:

Area of Section in Square Inches		Stress lbs. per Square Inch	Working Resistance in Lbs.
Concrete	240	50	12000
Top steel 6 rods	6	10000	60000
Bottom steel 4 rods	2.4	5000	12000
Stirrups 4 rods	.44	15000	6600
			90600

With a design unit load of 500 lbs. upon this floor, the load may be assumed to have been carried by the beams nearly in proportion to their lengths and then the total load upon one of the longer beams would be

$$15.33 \times 21.5 \times 500 \times 21.5 \,/\, (21.5+15.33) = 92.450 \text{ lbs.}$$

and the shear at a support one half this, or 46,225 lbs. The beam was tested to nearly double this amount, or 92,450 lbs., which agrees with the working stress previously computed. But this according to the above computations was not half what the beam would have carried safely.

It will appear from this investigation that the critical section for shear in a beam is not at the support in a continuous beam, neither is it at the support of a properly designed simple beam with steel carried past the supports both at the top and bottom and properly anchored. Failure from diagonal shear will occur in the continuous beam nearer the points of inflection and in the simple beam a little way from the supports, dependent upon the arrangement of the sloping reinforcing rods and stirrups.

In the construction of the simple beam type, the recommendations of the Joint Committee quoted herewith are conservative, tho these rules cannot be reasonably applied in determining the shearing resistance of scientifically designed continuous beams such as those outlined:

> **15. Shear and Diagonal Tension.**—"In calculations on beams in which the maximum shearing stress in a section is used as the means of measuring the resistance to diagonal tension stress, the following allowable values for the maximum vertical shearing stress are recommended:
>
> (a) For beams with horizontal bars only and without web reinforcement, calculated by Formula (22): 2% of the compressive strength. (i. e. for bottom reinforcement only.)
>
> (b) For beams thoroly reinforced with web reinforcement: the value of the shearing stress calculated as for (a), (that is, using the total external vertical shear in Formula (22) for shearing unit-stress), must not exceed 6% of the compressive strength. The web reinforcement, exclusive of bent-up bars, in this case, shall be proportioned to resist two-thirds of the external vertical shear in Formulas (24) or (25).
>
> (c) For beams in which part of the longitudinal reinforcement is used in the form of bent-up bars distributed over a portion of the beam in a way covering the requirements of this type of web reinforcement: the limit of maximum vertical shearing stress (the stress calculated as for (a)), 3% of the compressive strength.
>
> (d) Where punching shear occurs, that is, shearing stress uncombined with compression normal to the shearing surface, and with all tension normal to the shearing plane provided for by reinforcement: a shearing stress of 6% of the compressive strength may be allowed."

But since we are of the opinion that these recommendations are not applicable to columns, working stresses for columns will be specially treated under that heading. The committee failed to recognize the action of bond shear in column and multiple way slab construction, which action is here regarded as an essential factor in assigning their working stresses. Such recognition was perhaps not to be expected in view of the fact that this subject has not heretofore been adequately treated in the literature of reinforced concrete.

> **16. Working Stresses**—General Assumptions: The following working stresses are recommended for static loads. Proper allowances for vibration and impact are to be added to live loads where necessary to produce an equivalent static load before applying the unit stresses in proportioning parts.
>
> In selecting the permissible working stress to be allowed on concrete, we should be guided by the working stresses usually allowed for other materials of construction, so that all structures

of the same class but composed of different materials may have approximately the same degree of safety.

The following recommendations as to allowable stresses are given in the form of percentages of the ultimate strength of the particular concrete which is to be used; this ultimate strength is to be that developed in cylinders 8 in. in diameter and 16 in. long, made and stored under laboratory conditions, at an age of 28 days. In the absence of definite knowledge, in advance of construction, as to just what strength may be expected, the Committee submits the following values as those which should be obtained with materials and workmanship in accordance with the recommendations of this report.

Although occasional tests may show higher results than those here given, the Committee recommends that these values should be the maximum used in design.

TABLE OF STRENGTHS OF DIFFERENT MIXTURES OF CONCRETE

(In pounds per square inch)

Aggregate	1:1:2	1:1½:3	1:2:4	1:2½:5	1:3:6
Granite, trap rock	3300	2800	2200	1800	1400
Gravel, hard limestone and hard sandstone	3000	2500	2000	1600	1300
Soft limestone and sandstone	2200	1800	1500	1200	1000
Cinders	800	700	600	500	400

Bearing: When compression is applied to a surface of concrete of at least twice the loaded area, a stress of 32.5% of the compressive strength may be allowed.

Axial Compression: For concentric compression on a plain concrete column or pier, the length of which does not exceed 12 diameters, 22.5% of the compressive strength may be allowed.

Compression in Extreme Fiber: The extreme-fiber stress of a beam, calculated on the assumption of a constant modulus of elasticity for concrete under working stresses, may be allowed to reach 32.5% of the compressive strength. Adjacent to the support of continuous beams stresses 15% higher may be used.

Bond: The bond stress between concrete and plain reinforcing bars may be assumed at 4% of the compressive strength, or 2% in the case of drawn wire.

Reinforcement: The tensile or compressive strength in steel should not exceed 16,000 lb. per sq. in.

In structural-steel members, the working stresses adopted by the American Railway Engineering Association are recommended."

Under the heading of working stresses the report of the Joint Committee deals only with permissible values for stresses in one direction. Now in concrete work constructed as a continuous monolith the material is frequently strained in multiple directions—for example in Type IV floor construction the bottom portion of the continuous slab near the column is under compression radially toward the column and circumferentially about the column.

Morley in his excellent work on Strength of Materials has discussed the question of compound stress very fully. He shows that failure in elastic materials under stress results not from balanced hydraulic stresses but from the unbalanced shearing stresses.

Considère in a valuable series of tests of the compressive strength of concrete cylinders found that the endwise compressive resistance might be almost indefinitely increased by increase of lateral hydraulic pressure. These tests were carried to the extent of increasing the crushing resistance of cylinders endwise four to five fold. This increase appeared to be limited only by the amount of hydraulic pressure applied laterally.

On this principle the safe radial compression in the lower part of a continuous flat slab may be very conservatively taken at values double those for direct axial compression provided suitable provision is made for shear.

This view is borne out by practical experience with thousands of such cases in which no evidence of weakness has been observed with good concrete thoroly cured.

Shear and tension failures are, however, more liable to occur on the under side of the slab near the cap than elsewhere when the cement is partly cured and the forms have been prematurely removed. In this case the inspector's duty is to first investigate this zone for soundness and remove and recast any damaged material.

As in the case of beams, full advantage of the maximum compressive bending resistance can be taken to the limit only of a certain ratio of thickness to span and proper reduction made for smaller ratios as discussed in the treatment of permissible steel ratios and shearing stresses for this type of construction.

17. Compound Tensile Strength. The same reasoning applies to tensile stress that applies to compressive working stress when the steel is distributed in the form of small rods closely spaced. One of the facts in favor of multiple-way reinforcement in the natural concrete types is that the direct tensile reistance of the concrete is increased somewhat by strain in multiple directions. But in view of the fact that the direct tensile resistance of concrete is only one tenth or one twelfth its compressive resistance, an addition of forty to fifty percent to this direct tensile resistance of concrete does not render its dependable value of sufficient magnitude to be worthy of consideration as a safe practical element of strength, and as in the case of beams it should be disregarded for this reason.

Further the coefficient of expansion or contraction being .0000065, it is obvious that a drop of temperature of 25 degrees will overcome ordinary direct tensile reistance of concrete assuming both ends to be rigidly restrained, and as concrete work in Northern latitudes, at least, is frequently subjected to a range of temperature much greater than this below the temperature at which the concrete has hardened, we are not justified in considering direct tensile resistance as a dependable element of strength even under the more favorable condition under discussion.

It is a favorable condition in building construction that there is more or less chance for adjustment of moderate temperature effects and that columns give and bend in and out by small amounts thus accommodating expansion and contraction of flooring, and the same action occurs with walls, etc., otherwise the combination in the same structure of different materials such as stone, brick, steel, terra cotta, etc., having widely different coefficients of expansion in the same building would not give satisfactory results.

18. The Reinforced Concrete Beam as a Mechanism. The combination of the two elements, the concrete and the steel in the beam constitutes a device consisting of two relatively constrained parts which by certain predetermined intermotions serve to transmit force and motion in such a manner as to produce the effect of carrying the load to the respective supports while the arrangement of the metal in its position vertically and horizontally with reference to the supports determines the general law of operation of the device. This operation, nevertheless must conform to certain fixed or fundamental natural laws. These fundamental laws form the basis of the theory of work which is well understood and generally applied by the engineering profession in the treatment of bridge and frame structures but which seems to have been to some extent ignored in case of such a mechanical device as a concrete beam or floor.

The fundamental laws upon which the theory of work is based are derived primarily from the general principle known as the law of conservation of energy. This law is expressed in Merriman's Civil Engineers' Pocket Book, in the following statement:

> "If the system of bodies neither receives nor gives out energy, then its total store of energy, all forms included, remains constant. There may be a transfer of energy from one part of the system to the other, but the total gain or loss in one part is exactly equivalent to the loss or gain in the remainder."

We may also state the law in a general way as follows: Energy can be transformed or changed in form but it cannot be destroyed.

When we load a floor, we have an arrangement or device by which the load placed on the floor is gradually lowered from its original position to the lower position assumed by the slab as it bends, and if the slab is elastic the actual mechanical energy developed by the downward motion of the load under the law stated must be stored as potential energy of elastic deformation within the substance of the floor. This direct relation is ordinarily expressed in the statement that the external work of the load is transformed into internal work of deformation. This principle is worked out in great detail in designing bridge structures to determine deflections by work done, and is of the utmost value to the engineer in its various applications.

The above relation was expressed in 1866 in the theorem of Clapeyron, (See Lamé, "Lecons sur la théorie mathematique de l'élasticitè des corps solides," deuxième édition Paris, 1866), and is stated as follows:

> "The exterior force applied, multiplied by the displacement in the direction of its point of application, equals the sum of all the internal work of a body elastically deformed."

This theorem is a direct corollary of the fundamental law of conservation of energy.

In applying this method of work, we of course consider only the elastic deformation of the beam relative to its points of support.

When a newly cured beam is first loaded the deflections up to a point where the reinforcement is strained to as much as four or five thousand pounds per square inch, are about half or less than half of the corresponding deflection for the same increment of load where the steel is stressed from ten to twelve thousand pounds per square inch. This difference in measured steel stress and deflection under the laws noted indicates a difference in the mode of operation of the mechanical device with which we are dealing which it is now in order to investigate. As the stress in the steel approaches four to six thousand pounds per square inch the direct tension in the concrete corresponding to the stress in the steel ranges upwards of 250 or 360 pounds per square inch, or approximately the ultimate direct tensile strength of the concrete matrix. As the load is increased further, a yielding of the concrete takes place and cracks begin to develop. This yielding of the concrete results in a dissipation or loss of the mechanical energy stored by the elastic deformation of the concrete and new energy is developed by the motion of the load thru increased deflection until equilibrium is re-established, which energy is stored up in a stable manner in the steel.

The action which we have outlined does not constitute a transfer of the stress in the concrete to the steel as some incorrectly may consider it, but it represents the dissipation of energy thru the over-strain of the concrete, the development of new energy by the motion of the load thru increased deflection and the storage of this new energy in the steel so that when the load is removed the internal work then stored in the beam is only that stored by elastic deformation which may be given back as the load is removed. For perfectly elastic deformation then we may say that thru indirect stress there is an amount of energy stored by lines of indirect tension and compression emanating from bond shear at the surface of the steel equal to that stored in the steel and that the lines of principal stress in the reinforced beam are for this reason somewhat analogous to those of a beam of homogeneous material. When, however, energy is lost or dissipated thru inelastic stretching or cracking of the concrete, a different distribution of internal stress results.

The indirect stresses in tension are greatly decreased in this case while those for compression are increased. The action of the combination of the steel and concrete then approaches more and more nearly to that of the flat arch with the tie rod in the bottom in which the lines of compression arch more and more from the end upward toward the center, for those bars at least which are arranged in the bottom of the beam thruout. If, however, there are a number of bars in the beam, part of which are in the bottom layer thruout and other bars which bend upward from the quarter point or toward the end of the beam, then we have an arrangement such that the bond shears from the bent up bars toward the end of the beam bring the upper layers of the concrete toward the end into direct compression and co-action, which the bars in the bottom of the beam cannot effectively do. Hence in the beams in which there are bent-up bars a higher percentage of steel can be used without overstraining the beam by indirect tension than is the case where the metal is placed in the bottom layer thruout.

Stirrups in the form of small rods extending vertically from the bottom of the beam to the top and crossing the lines of indirect stress at an angle assist in resisting indirect stresses and add materially to the shearing strength of the beam. This additional strength afforded by the stirrups has been very thoroly investigated by the German firm of Wayss and Freytag, and the results have been given quite completely in "Der Eisenbetonbau" by Emil Mörsch, which has been translated by E. P. Goodrich, and published by the Engineering News Publishing Co. under the title "Concrete Steel Construction"

Fig. 43. Cottançin Beam Construction in the Church of St. Jean de Montmartre

in which the reader will find an exhaustive discussion of these valuable tests.

The dissipation of energy in the combination of concrete and steel in the beam and the inequality of the deflections after the applied loads stress the steel beyond 4000 lbs. have been already noted as due to the overstrain or checking of the concrete largely by indirect tensions or bond shears. An arrangement of the metal in the web may be made which so reinforces it that there is no loss of energy from this cause and in such manner that the bending resistance of the steel may be largely increased if not indeed doubled. Since every pound of pull on the bar is induced by bond shear it follows that the energy stored by internal work of indirect stress in the concrete is equal to that stored in the steel. When, however, yielding of the concrete occurs and energy is dissipated thereby the stress in the steel is found by experiment to largely increase until the stress in the steel is substantially that which we compute on the basis that the entire tensile resistance is furnished by the steel element.

In the Cottançin construction the entire web of the beam (which is very thin) is reinforced by a net work of small rods in such manner that this yielding of the concrete is prevented by thoro dissemination of steel both vertically and horizontally, and any deformation of the concrete brings this net-work of steel into action in such manner that the indirect stresses from the bond shears of the vertical and horizontal rods coact with each other, and deflection of the beam is for all loads light and heavy more nearly proportional to the loads.

Marsh in his work on Reinforced Concrete has called particular attention to this deportment of the Cottançin beam construction which he considers is based upon the theory of prevention of molecular deformation by supplying resistances of the reverse kind to stresses on small particles which produce notably good results and very light structures.

The mechanical operation of the Cottançin beam will become apparent from more complete consideration of the action of the bond stresses and the storage of energy of the internal work in the web of the beam by the indirect stresses resulting from bond shear in a scientific and stable manner which it is impossible to effect in a one way reinforced construction lacking thoro dissemination of the steel to resist web stress in such manner that the low tensile resistance of the concrete may not be over-taxed.

The carrying capacity of the Cottançin beam as Marsh has pointed out gives results not accounted for on the basis of the usual theory but which may be accounted for readily when we consider the scientific manner of reinforcing the web so that the indirect stresses are provided for in a manner which does not over-tax the concrete and so the loss of energy accompanying the inelastic deformation of the ordinary beam is avoided.

The same general principle differently applied is brought into

Fig. 41. Cottançin Beam Construction in the Church of St. Jean de Montmarte.

play in multiple way slabs, where the arrangement of the metal is such that the energy stored by indirect stress is stored in a dependable or stable manner.

In column construction this principle may be employed by a proper combination of hooping and vertical steel. If hooping only is used, the fireproofing or outside shell scales early under test, whereas if the combination of vertical steel and hooping in proper proportion and arrangement is used 85 percent of the ultimate strength of the specimen may be developed before scaling and chipping of the outside shell occurs. It is to be borne in mind that the checking or cracking under load is an inelastic deformation accompanied by the dissipation of energy and followed by the development of new energy by the downward motion of the load thru increased deformation before equilibrium can be reestablished. Thus the hooped column, without verticals, is objectionable on the ground of excessive deformation, whereas with a proper combination of vertical steel and hooping this excessive deformation is prevented since the energy of internal work is stored in a dependable manner thru a scientific arrangement of the reinforcement which induces coaction of the indirect stresses in the concrete matrix which are generated on the one hand by the bond shear between the matrix and the verticals

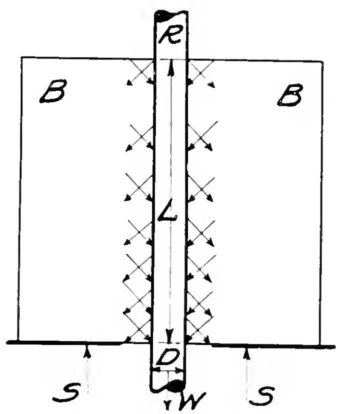


Fig. 45.

and on the other between the hooping and the matrix, as will be explained more at length in succeeding pages.

A fundamental principle of concrete design from the standpoint of work, may be stated in the following words:

The most perfect design is one in which the potential energy of internal work is stored in the most stable manner. Such a design, from the commercial standpoint, will be economical in the quantity of material required and for a given depth of beam or slab will carry a greater load with less deflection, and may be expected to prove most satisfactory under severe conditions of repeated loading.

Indirect Stresses Generated by Bond Shear and the Laws of Distribution Governing These Mechanical Actions

19. Bond Shear in Blocks. Let R Fig. 45 represent a cylindrical metallic rod embedded in a block of concrete B B in which it has been cast. Let it rest on a ring shaped support *SS* having a central apperture somewhat larger than the rod. Then the weight *W* applied to the rod will induce a bond shear at the surface of the rod, whose intensity will depend upon several factors, among which will be the magnitude of the weight *W*, the diameter *D* of the rod, the length *L* of the embedment, and the distance of the point considered from the beginning of the embedment. Other things being equal, the intensity of the bond shear will be greatest at the point of embedment nearest the point of application of *W*, whether the rod be in tension or in compression. Since the concrete which surrounds the rod is in a state of shearing stress along the surface of the rod, it is in a state of stress that may also be defined otherwise by saying that there is indirect or induced tension in the concrete along lines sloping at 45° towards the axis of the rod, represented on Fig. 45 by the arrows with heads at the surface of the rod, and also compression induced in the concrete along lines sloping at 45° away from the axis, the intensity of each of these induced stresses at the surface of the rod being the same as that of the bond shear at the surface and growing less at greater distances from the axis nearly inversely as the distance. The arrows may be regarded as representing lines of force or stress in the concrete in such a manner that the intensity of the stress is proportional to their nearness to each other. The total bond shear between rod and concrete amounts to *W*, and is so distributed on the surface of the rod that it is small at the point of embedment most distant from *W* and is greatest at the points nearest to *W*. The bond shear at any point is the increment of the tension in the rod at that point and there is no element of tension in the rod which is not balanced and transmitted to the concrete by a corresponding equal element of bond shear.

The block tends to decrease in length by reason of the load thus transmitted to it at the same time that the part of the rod embedded in the concrete tends to increase in length, so that the point where the first slipping will occur is at the point nearest to *W*. But if no actual slipping occurs the distortion is greatest at this point. This is the reason that the greatest intensity of bond shear occurs at this point.

In the space where the arrows are represented it might at first be thought that the radial components of the induced tensions and compressions would neutralize each other. But such is not the case because it is a fundamental property of stress in any material that a shear in any plane (as for example the vertical plane in this case) in order to maintain equilibrium is necessarily accompanied by an equal opposite shear on a plane at right angles with it, so that these induced vertical stresses act to produce radial shears of equal intensity on each horizontal plane.

This horizontal shear in the block diminishes somewhat as the radius increases but does not vanish until we reach the inner edge of the ring *SS*. Beyond the inner edge of *SS* a vertical compression exists in the block which causes a pressure upon the ring *SS*, whose intensity is greatest at the inner edge of *SS*, but which diminishes beyond the inner edge of *SS* in a ratio which need not here be considered.

The matter of importance in this discussion is the manner in which the stress is transmitted thru the concrete around the rod,

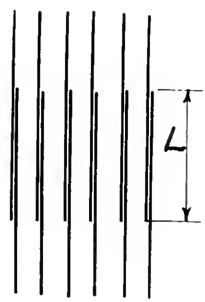


Fig. 46

where the concrete has lines of stress in it similar to those in the web of a built beam which cause the two flanges to coact as do the rod and exterior part of the block in this case.

The induced tensions which have just been described are the same as those frequently designated as diagonal tensions. Failure arising from excessive stress of this kind would evidently give rise to cracks nearly at right angles to the arrows representing the induced tensions, i. e. to cracks along the lines of compression, but such cracks could not occur without excessive elongation along the rod, nor would they be expected to occur in a block at all under ordinary circumstances, because they would be preceded by slipping of the bond, after which the diagonal shear would be relieved where the slipping took place and transferred to some point further along the rod.

20. Bond Shear in Splices. Let Fig. 46 represent a splice in a belt of reinforcing rods with laps of length *L*. The entire effectiveness of the splice depends upon the bond shear at the surfaces of the belt rods in which the action of the embedment is in many particulars like that already discussed, but has several new features especially by reason of the dissymmetry of the embedment laterally, altho longitudinally it has perfect symmetry such as the block did not possess.

The lateral arrangement is such that the stresses in the concrete between the rods on the sides where they are nearest together is of much greater intensity than elsewhere, so that the diagram of the stresses about a rod would be much like that shown in Fig. 45, except that on one side of the rod the arrows would cover the space very closely, while on the other side the arrows would be few and far between. Moreover the successive arrows would be just as near each other at one end of L as at the other end, but nearer together at each end than nearer the middle of L. The bond shear would fail first

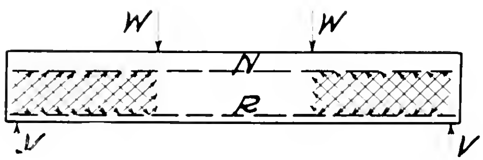


Fig. 47.

on each rod at that end of L furthest from the end of the rod. The lines of force in the horizontal plane of the belt would be as just described, but lines of force which start out from the rods in a direction above or below that plane would curve around spirally from one rod to the other, for each line of force must have one end on one rod and the other on some other rod near by.

The strength of the splice depends upon the bond shear at the surfaces of the rods, and on resistance to the induced tensions and compressions along the lines of stress afforded by the concrete. The integrity of the concrete embedment under the action of these stresses is essential to the transmission of force along the splice, and were the limiting values of either bond shear, indirect tension, or compression, exceeded in the concrete the splice would fail.

21. Bond Shear in Beams. Let Fig. 47 represent a reinforced concrete beam of length L supported at the ends and loaded with two equal weights W each placed symmetrically at the same distance, nL from the end nearest to it. Then the total vertical shear at any point between the equal weights is zero and the bending moment is nWL, while at any point between either load W and the end nearest to it the total vertical shear is W and the moment is Wx, where x is the distance of the point from the end nearest to it. In such a beam where there is no vertical shear at any point of the middle segment there is no horizontal shearing stress at any point of this segment, consequently the bond shear stress is zero thruout this segment.

Let N represent the position of the neutral axis. The concrete above N is in direct compression and that below N is subject to direct tension along horizontal lines none of which are represented in Fig. 47. The entire length of the reinforcing rod R is under tension, the central segment between the applied weights being under a uniform tension thruout which will be determined ultimately by the bending moment WL, for if the weights W are sufficiently great or other contingencies such as rapid setting, or long continued loading have occurred, enough vertical cracks will have developed at the bottom of the beam in this segment to have practically destroyed its direct tensile resistance to horizontal force. Provided there is sufficient horizontal reinforcement to safely resist this horizontal tension and the concrete above N is sufficient to resist the compression the existence of vertical cracks in this segment of the beam at the bottom is not to be regarded as a sign of structural weakness or ultimate failure of the beam. These may under these conditions be regarded as harmless characteristic cracks. But in case the reinforcement is insufficient, yielding will occur in this part of the beam where the moment is greatest and such yielding will cause failure.

In the end segments of the beam the moment of the bond shears S per unit of length of reinforcement represented by Sjd must be equal to the increment of the bending moment, i. e. equal to the total vertical shear W provided it be assumed that the reinforcement furnishes the entire direct tensile resistance and the concrete none. Assuming that case for safe design, the bond shear is represented in Fig. 47, in the same manner as in Fig. 45, viz: the arrows with heads at the reinforcing rods represent the lines of indirect tension in the concrete, and those with heads at the neutral axis N represent the lines of compression induced by the bond shear. The concrete is abundantly able to afford the necessary resistance to the indirect compression, but may be entirely unable to afford the necessary resistance to the indirect or diagonal tension, which may consequently cause cracks perpendicular to its direction, that is along the lines of indirect compression, and at the same time produce rupture of the bond shear. In beams so heavily reinforced as to prevent central yielding, failure due to rupture of diagonal tension and rupture of bond along the steel ordinarily occurs under overload. These two accompany each other because they are of practically equal intensity as noticed before, and the cracks due to slipping of the steel are continuous with those due to diagonal tension. Additional reinforcement to resist diagonal tension consists of stirrups and the like, which are enabled to do this by their own secondary system of bond shears and anchorage.

The system of induced or diagonal tensions which have just been treated are necessarily modified somewhat in amount and direction by their combination with the direct horizontal tensions in the concrete due to the bending where these direct tensions are not eliminated by vertical cracks. But the existence and effectiveness of the induced tensions depending on the bond shear remains practically unimpaired so long as the bond is intact.

It should be noticed that the distribution of the bond shear on the surface of the reinforcement must be more intense on the upper sides of the rods by reason of the greater rigidity of the concrete on that side, due to its backing above, and lack of backing below.

In case of a beam carrying a load distributed differently from that assumed in Fig. 47, there may be no segment of any finite length where the shear vanishes, but such a segment of infinitesimal length will exist wherever the bending moment is constant, i. e. wherever it is a maximum or a minimum. Moreover the total values of the bond shear per unit of length of the reinforcement, multiplied by jd will equal the total vertical shear in case the reinforcement takes the entire direct tension. In such a case the characteristics of the middle and end segments of Fig. 47, merge into each other somewhat, in a manner not difficult to understand.

It will be readily seen that the manner in which the stresses are distributed in any case of reinforcement depends entirely upon the applied forces and the distribution of the rigidities due to the amount and distribution of the steel, but that the coaction of concrete and steel is brought about by stresses communicated from one to the other thru the medium of bond shear which is vital to the discussion of any such question, and that together the steel and concrete form a combination or machine which has properties as a whole which are different from the properties of either of the constituents. What those properties may be in any case must be determined from a careful analysis of the particular case under consideration.

It should be further noticed that the lines of tension that originate or are generated in the bond shear at any given element of the surface of a rod are to be thought of as physically independent of the lines of compression originating at the same point in such fashion that the tension starting at a given point *a* say, is held in equilibrium at the other end of that line by a tension at some point *b*, while the compression which originates at the same point *a*, is resisted and held in equilibrium by a resistance to compression at some entirely different point *c*. Moreover, it may be that *b* and *c* are on the surfaces of entirely different bodies, as frequently occurs in case of splices in a belt.

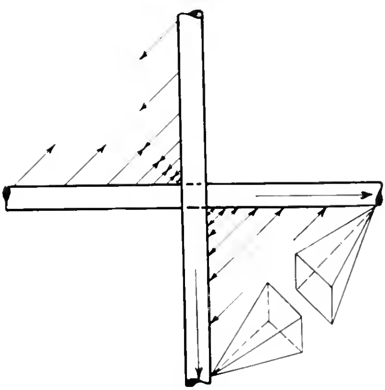


Fig. 48.

22. Bond Shear in Slabs. Let Fig. 48 represent the crossing of two reinforcing rods under tension in a multiple way reinforcement as for example in a slab where the shearing stress of the bond resists any tendency of the rods to slip longitudinally in the direction of the long arrows. Then the mutual action of the lines of tension and compression arising from the bond shear is that represented in the figure where the arrows with points against the rods represent lines of tension and those pointed away from the rods lines of compression. There exist other lines due to the bond shears besides those here represented which are similar in their disposition and action to those found in beam action which has been already discussed. But such other lines are here omitted from discussion because by the principle of rigidities the lines which are here represented as affording short and direct connections thru the concrete are necessarily the ones which include the predominant action of the bond shear. This predominant action is here seen to resemble in its general nature that of a splice, but has laws of distribution peculiarly its own by reason of the relative situation of the rods in the concrete, which is such that the intensity of the stress developed is greatly intensified in the immediate vicinity of the point of contact of the rods.

The principle of rigidities here referred to as determining the predominant action is one susceptible of exceedingly simple illustration yet one whose final implications and applications are of such a nature as to make the layman think for the instant that it is impossible to establish with certainty the definite conclusions that can nevertheless be at once conclusively demonstrated.

To illustrate this point, let there be two springs or spring-balances, one of which is twice as stiff or rigid as the other, and if side by side they together support a given weight, the more rigid spring will carry the larger proportion of the load, two pounds to each pound of the other. Just so in any structure where there are several parts of the structure, each of which is to carry its part of the load the question as to how the load will be divided between them is determined by their relative rigidity or stiffness, that part which lacks stiffness and yields easily will carry little of the load and will leave the brunt of the work to those parts that are rigid and unyielding. Furthermore, since the deformation multiplied by the mean force (or half the final load) is the work done during deformation, it is evident that for a given applied load the more rigid parts carry their part of the load with less deformations than the other parts could carry it. Hence the work of deformation is less by this arrangement than it would be by supposing less rigid parts to carry it. This is the principle of least work, which states that of all the various ways in which a given elastic structure could be supposed to be deformed in carrying a load, the way in which it will actually be deformed is that in which the load will perform the least work in deforming the structure.

Fig. 48, showing, as it does, merely a ground plan of the lines of force cannot represent fully the distribution of the points of attachment of the lines of force which are generated all over the surfaces of the rods and start out everywhere like the quills of a hedgehog, in greatest number at points where the two rods are nearest to each other, until in the immediate neighborhood of the point of contact between the rods, the concrete grips the rods and holds them with a firmness and strength that would seem incredible did not experience demonstrate its safety and reliability.

It is this dominant action which is greatly increased in four way reinforcement which distinguishes slab action from beam action, and brings into play the dependable indirect stresses in the concrete in contradistinction from the direct tensile stresses due to beam action. A failure to understand the physics and mechanics of this predominant action of slab reinforcement as depending on the fundamental role which bond shear plays in this case has prevented those who have discussed this subject from grasping the real relations that here exist. It has been denied most vehemently that it is possible for the stress in any rod to influence in any way the stress in any rod that crosses it, whereas such action is not only unavoidable when bond shear exists on both, but is the key to the situation and explains the otherwise inexplicable phenomena here observed. The mechanism of lateral interaction between the rods in a slab is such as to make the slab have properties which imitate in many respects those of a continuous plate, and the efficiency of this action can be treated as in such a plate by help of a coefficient.

Why should anyone deny that a reinforced slab acts in a manner similar to a continuous plate? The only basis for such a denial is the assertion that in order to so act the direct tensile strength of the concrete will have to be relied on which cannot be relied on in a beam. That assertion involves a fallacy. All that need be relied on to make this action effective is the indirect tensile strength of the concrete which is certainly reliable for this purpose to the required extent as appears from the fact that the bond does not in fact suffer rupture in well cured slabs.

Any analytical method of dealing with the phenomena of plate action or slab action by which the stresses in various directions affect each other depends upon the introduction of the so-called Poisson's ratio to take account of this interaction, so that the numerous and emphatic denials in technical papers of the applicability of this ratio to slab action show in fact an entire ignorance or misunderstanding of what that action is.

It may in general be stated that stresses in steel reinforcement, especially in slab steel are imparted to the steel by the bond shear and not by any other kind of anchorage in the concrete, such as occurs near the ends of simple beams. From this it follows that the total stresses in the steel at any point must have existed previously in the concrete. Since a given distributed force or stress considered as a single thing has a certain magnitude thruout its length, at such places as it is present in the concrete it is not present in the steel and vice versa. The mean stress to be carried by the steel is consequently diminished by that in the concrete.

It comes about, therefore, that the stresses in the steel are reduced by the action of the concrete, or the steel has less stress to carry by reason of its embedment than if it supplied the total tensile resistance itself. It is thus protected by the action of the embedment from some portion of the tensile stress without calling upon the embedment for such direct tensile resistance as is shown to be liable to failure in beams, but is calling instead merely for such indirect tensile stresses as are known to be safe and dependable from what we know of splices and slabs. These two things then are the basis of slab theory: the reliability of bond shear in slabs and the reduction that it produces in the stresses that without it would exist in the rods constituting the multiple way reinforcement. As seen above this reduction may amount in certain cases to 50 percent of the total force acting. A very instructive experiment in this connection was made at the Minneapolis Court House of a slab 6 inches thick, and 7 by 10 feet horizontally, reinforced in two directions lengthwise and crosswise with $\frac{3}{8}$ inch round rods 8 inches between centers. When tested as a beam supported at the ends, the crosswise steel not being subject to bond shear or tensile stress, did not increase the resistance to flexure, nor prevent the longitudinal steel from receiving the full effect of the bending moments.

23. Mechanics of Embedment. The mechanics of reinforced concrete under flexure may be summarized as follows:

The co-operation or combined action of the two materials, concrete and steel, to resist bending, depends solely on the bond between the two, which has been discussed briefly in a preceding article.

In the case of plain rods, this bond is in reality a shrinkage grip which prevents the steel from sliding thru the hardened matrix in which it is embedded, and the resistance afforded by this bond is subject to well defined laws which may be stated as follows:

The bond shear is zero wherever the tension in the steel is constant. It passes thru zero where the increment of the moment passes thru a maximum or minimum. It must be depended upon whether the reinforcement is in one direction only as in a beam, or in multiple directions in the slab.

Bond shear generates stresses emanating from the surface of the bars which may be treated as lines of force. These lines of force follow the general laws of distribution of force thru any medium, that is, their intensity is inversely as the square of the distance from the surface of the steel on which they are generated.

These general laws enable us to investigate or follow out the part played by bond shear in the mechanics of a slab or beam. In the case of a simple beam in accordance with the law stated, the intensity of the bond shear is zero at the center for uniform load and a maximum toward the end of the beam, and it is to the bond shear or the lines of force generated thereby to which we may attribute the difference in the failure of an over and an under reinforced beam.

In the case of the beam with light reinforcement, failure takes place at the center by the yielding of the steel. With heavier reinforcement, on the contrary, failure is more liable to occur toward the end by indirect tension induced by the bond shear which is greatest toward the end of the beam and which may be resisted only by the direct tensile strength of the concrete itself.

The deportment of the simple beam as affected by the stresses set up by the bond shear is of interest. In a newly cast beam, in the preliminary stages of the loading, the stress in the steel as determined by the extensometer is much less than that figured on the assumption that the steel only resists tension. In fact, it is only about half as great as we should compute the stress to be on the above basis, until the steel is stressed up to four or five thousand pounds per square inch. When this point has been reached, there is a rapid increase in the stress in the steel with no corresponding increase in the load until when the steel is stressed up to twelve or fifteen thousand pounds per square inch, the concrete has relieved itself of a large portion of its tensile resistance and the measured stress in the steel corresponds closely to the computed stress in the steel, assuming the steel not to be assisted by the concrete in tension.

With the slab reinforced in two directions, however, the phenomenon differs from that observed in the beam. Take for example the case of the slab reinforced in two directions, bent in such a manner that the rods in both directions are brought into tension at the same time. The indirect stresses generated by the two sets of rods will under this condition react upon each other. Since the lines of force diverge from each rod they may meet and coact thru the concrete as a medium of transmission of the stress, which is not possible in the beam with one way reinforcement, since in the beam these stresses cannot coact with each other, there being one kind only and not two kinds acting in different directions. This fundamental difference in the stress induced by the bond shear in the case of a beam from that in a slab renders the two types of structure mechanically different and necessitates their treatment in a manner which takes into consideration the difference in the mechanical operation of the indirect stresses referred to.

A crack would not materially interfere with the operation of these indirect stresses in a multiple way reinforced slab, because the paths of the lines of force will still be able to find other passage ways such as avoid the cracks. But a crack in a beam which is normal to the direction of the steel would intercept the indirect tensions induced by the bond shear at the section checked, and prevent the accumulated resistances offered by the indirect stresses from being effective in direct resistance to moment.

In treating the combination of the two materials, it has been customary to consider their combined action as determined by the elastic properties of each taken separately, namely by the ratio of the modulus of elasticity of the concrete in compression and tension to the modulus of elasticity of the steel in tension and compression. In a homogeneous elastic slab such as steel in the form of a plate there must needs be taken into consideration in addition to the modulus of elasticity of the metal in tension and compression in one direction, the additional coefficient or modulus of lateral deformation known as Poisson's ratio. This ratio, or lateral effect in a combination of steel and concrete which is sufficiently fine grained to be regarded as acting as a homogeneous material, as is the case with reinforced concrete, cannot be correctly considered as an elastic property of either the concrete or the metal, but on the contrary must be treated as a coefficient expressing the efficiency of the lateral action of the indirect stresses induced by the bond shear in the case of multiple way reinforcement in the slab, which coefficient, for the reasons above explained, must be zero in the case of the beam type with reinforcement in but one direction or in case of the slab in which the reinforcement under strain runs in but one direction only. Altho transverse reinforcement may be introduced in a beam it can perform no useful function in reducing the stress on the carrying rods since the indirect stress induced by one series of rods under stress cannot converge to react upon another set of rods not under stress, but can react with that set of rods only when both are generating indirect lines of force from bond shear.

24. **Bond With Deformed Bars.** Mörsch discusses the action of deformed bar reinforcement thus:*

* Concrete Steel Construction p. 17.

"In America various forms of reinforcement are employed all of which are designed to prevent slipping of the rod in the concrete. In the Ransome rod, this is secured by twisting the square steel bar; in the Johnson bar, elevations on the surface of the rods are produced in the rolling; and the Thacher or knotted bar is provided with swellings, while maintaining a constant sectional area. These 'knots' may well have the desired effect when the rod is anchored in a large mass of concrete, but they will act in an opposite manner in the small stems of T-beams, especially at their bottoms, where they will have a splitting effect and thus cause premature failure of bond. It will be shown later that the adhesion in the case of ordinary round rods with hooked ends is ample to transfer all actual stresses, and furthermore, the arrangement of the principal reinforcement may be so designed with respect to the shearing stresses that no occasion should arise to make up any deficiency through the use of those costly special bars."

As regards the mechanics of embedment, the effect of the "knots" as Mörsch terms them, would cause slight irregularities in the intensity of the indirect stresses, assuming that there was any over-strain tending to cause the bar to slip and to bring into action the mechanical bond. This variation from the condition of uniformity of bond shear with the plain bars would not be material under loads producing stresses not exceeding those which are safe, tho the disadvantage suggested by Mörsch in the case of the thin ribs might be looked for under loads approaching the ultimate strength.

CHAPTER IV.

BEAM ACTION AND SLAB ACTION COMPARED THROUGH APPLICATION OF THE LAWS OF BOND SHEAR AND THE THEORY OF WORK.

1. Introductory. The preceding discussion of the mechanics of the indirect stresses generated by bond shear indicates the laws which govern the distribution of metal required in order to secure effective continuous plate action in type IV floors, i. e. flat slabs supported by columns.

At the diagonal center of a panel, the moment of the applied forces passes through a maximum and the bond shear accordingly passes through zero, hence an arrangement of narrow strips of reinforcement on diagonal lines from column to column is of no utility in securing lateral efficiency by the operation of indirect stresses. Wide spreading reinforcement covering substantially the area between lines of inflection is the prime requisite for efficiency.

The force of this remark will be better appreciated from a detailed consideration of the moment curve of the circular suspended plate under uniform load.

The area of a floor of this type may be subdivided according to deformation into three divisions:

a. Cantilever area about the columns convex upward and approaching spherical curvature for equal column spacing.
b. Suspended circular plate concave upward about the diagonal center of the panel.
c. Saddle shaped areas between the cantilever areas and the suspended central plate.

Considering the suspended plate, the moment curve differs from the parabolic curve of a beam uniformly loaded in that the center of gravity of that part of the load transferred to the support is nearer to the support than in the case of the beam by one third. This renders the moment curve very flat at the center and the moment increment at and for a considerable distance each side of the diagonal center very small, and on the other hand it renders the moment curve correspondingly sharper toward the support, and hence effective lateral action of indirect stress must occur toward the the outer edge of the plate since the intensity of bond shear depends on the moment increment. The moment increment being nearly negligible for a considerable distance about the diagonal center and zero at the center, the efficiency of a narrow belt is negligible from the standpoint of plate action as will be shown experimentally in tests to be discussed later of the Mushroom and Norcross type of slabs.

In the cantilever area about the column the following significant conditions are to be considered:

The critical section is at the support and the resisting section decreases toward the support, hence the necessary radial resistance at the support should be reduced so far as possible by circumferential resistance as the support is approached from the line of inflection.

Fortunately the shear at the line of inflection is about seventy ~~seventy-~~ five percent of that at the support and, as the increment of moment

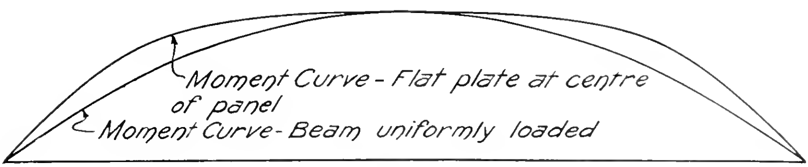


Fig. 49.

depends on the vertical shear, effective action of indirect stress up to the line of inflection is secured by the use of wide belts of reinforcement.

Thus the arrangement of the reinforcing steel laterally as well as vertically vitally affects the characteristics of the structure in such wise that when the belts of rods which run in multiple directions from support to support are spread out sufficiently to make the metal cover the entire area with crossed reinforcement permitting coaction of bond stresses it imparts the property of plate action to the slab in which circumferential resistance occurs in wide areas about the columns where the metal is at the top and also in the central area of each panel where the metal is at he bottom.

Without such wide spreading belts full and effective plate action is impossible. Narrow belts, or belts part of them in the bottom of the slab at the columns, do not realize results that compare favorably with the success of the Mushroom structure.

2. Slab as a Mechanism. Treating the Mushroom slab as a mechanism to carry the load to the supports, the result of its operation must be measured and compared with other structures in terms:

First, of the amount of load it can carry with a given quantity of material:

Second, in terms of its deflection or stiffness under the given load with a given amount of steel and a given depth of slab.

Clayperon's theorem that "The exterior force applied multiplied by the displacement in the direction of its point of application

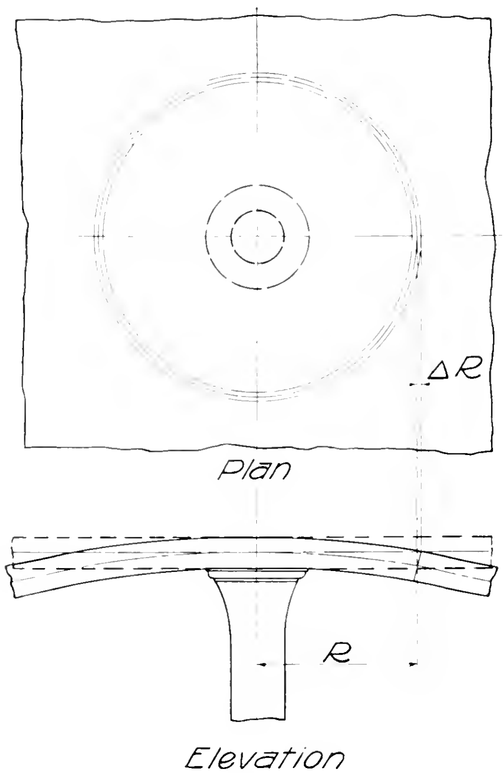


Fig. 50

equals the sum of all the internal work of a body elastically deformed" gives a basis for ascertaining the manner of the storage of potential energy in a reinforced structure by which it is possible to demonstrate the difference between the circumferential cantilever action in a Mushroom slab with four way belts, Type IV, and linear cantilever action such as that of the Hennebique beam type. In making this comparison let the same thickness of slab and cross section of steel be assumed in the two cases and equal column spacing in both directions.

Take a circular arc of radius R about a column as a center, then in case it has a radial deformation $\triangle R$ it will have a circumferential deformation $2\pi\triangle R$ which makes these deformations equal per linear unit. But since the steel runs in multiple directions in the belts of the Mushroom slab with substantially uniform spacing, equal deformations of equal reinforcement radially and circumferentially requires the same work of deformation. Hence the work stored circumferentially and radially is substantially the same. This proof assumes the same unit deformation at all distances from the center such as would occur in stretching a perfectly flat sheet. It will therefore be proper to give a proof by integration of elements applicable to the case of a bent slab where the deformation is not necessarily the same at different distances r from the center.

Referring now to Fig. 50, the position of the slab bent downward about the column is shown in an exaggerated manner. An elongation of the top surface or fiber of the slab is noted as $\triangle R$, for a radius R and an elemental sector included between radii making an angle with each other $d\ \theta$.

Treating the reinforcement as equivalent to a sheet of uniform thickness, we deduce the relation between radial and circumferential unit stresses in the following manner:

Problem: In case of uniform stretching of a circular plate one unit thick, to compute the radial and the circumferential work against the elastic forces having a final intensity of f per square unit of cross section.

Resilience = (mean unit stress) × (cross section) × (elongation).

Mean unit stress during deformation = $\frac{1}{2}f$.

a. *Radial Resilience:* By definition of modulus E,

$$R : \triangle R :: E : f, \therefore f = \frac{E\triangle R}{R}$$

Mean cross section of elementary sector = $\dfrac{R\ d\ \theta}{2}$

Hence the radial resilience of an elementary sector

$$= \frac{E \triangle R}{2R} \times \frac{R\ d\ \theta}{2} \times \triangle R = \tfrac{1}{4} E\ (\triangle R)^2\ d\ \theta.$$

Total radial resilience of circular plate

$$= \frac{E\ (\triangle R)^2}{4} \int_0^{2\pi} d\ \theta = \frac{\pi}{2} E\ (\triangle R)^2$$

b. *Circumferential Resilience* of elementary ring of radius r. By geometry $2\pi r : 2\pi R :: 2\pi \triangle r : 2\pi \triangle R :: E : f$, $\therefore f = \frac{E \triangle R}{R}$

Hence the circumferential resilience of elementary ring

$$= \frac{E \triangle R}{2R} \times dr \times 2\pi \triangle r, \text{ but } \triangle r = \frac{r \triangle R}{R}$$

$\therefore$ Total circumferential resilience of circular plate

$$= \frac{\pi E (\triangle R)^2}{R^2} \int_0^R r\,dr = \frac{\pi}{2} E (\triangle R)^2$$

$\therefore$ Total circumferential resilience = total radial resilience.

In considering the geometry of the convex areas over the head of the columns, or the concave areas about the diagonal center of the panels, produced in a slab by loading it, we notice that the deflections in such areas are completely determined and measured when we know the deflection of the meridian or radial curves of those areas. The deflection of these meridian curves involve certain elongations in the reinforcement in radial directions. They also involve lateral or circumferential elongations. But while these lateral deformations accompany the radial deformations, and are connected with them, the deflections may be regarded geometrically as wholly independent of them, and taken as dependent only on the radial deformations.

Now the energy of deformation stored in these areas under load has been shown to be equally divided between that done radially and that done circumferentially; or that done longitudinally and that done laterally. In beam action all the energy is stored longitudinally, and we proceed to make a comparison of this kind of action with the slab action which we have been considering, where it is half longitudinal and half lateral.

Suppose a given amount of energy Q is stored in a wide portion of a slab extending, let us say, diagonally from column to column, and compare its deportment under load, by beam action, and by slab action. According to beam action, this energy Q is due to a certain load W_1, gradually applied to the beam, which has a mean deflection of h_1, say, so that $Q = \frac{1}{2} W_1 h_1$.

But if the same amount of energy be stored in this area regarded as a slab, only half of it will be stored longitudinally so that the work thus stored will be $\frac{1}{2} Q = \frac{1}{4} W_2 h_2$, half of it in the steel and half in the concrete.

But with half as much energy stored longitudinally, the stress in the longitudinal rods will be one fourth that in regular beam action, and the deflection also one fourth as great, or $h_2 = \frac{1}{4}h_1$.

But if the deflection in the slab is only one fourth as great under the same load as on the beam, the load on the slab would have to be four times the load on the beam in order to produce the same deflection in both.

3. **Relative Stiffness of Beam and Slab.** The significance of this result for slab theory may be set in a clear light by taking account of the fact that a continuous beam is five times as stiff as a simple beam, but a continuous slab is four times as stiff as a continuous beam, consequently a continuous slab supported on points should be twenty times as stiff as a simple beam of the same cross section on knife edge supports. But since it is not practicable to support a slab on points we must take into consideration the diameter of the usual bearing provided therefor.

Taking the breadth of the bearing into consideration would have a tendency to increase the relative stiffness of the plate to more than twenty times that of the continuous beam or plate supported on parallel supports; but this will be referred to in due order.

Now consider further the circumferential action, which has such a remarkable effect upon the deflection as that which has just been deduced. In this treatment of the circumferential frames or the belts of reinforcement, it was assumed that the rods were the equivalent of a sheet of metal of corresponding section. The actual fact, however, is that they approach that when bound together in the concrete matrix. Taking a diametral section across a column head it appears that the pull in the rods across the section cut, especially in the portion of the belt inside the cap, is in position to resist bending in the direction of these rods, and hence the rods forming the frames outside the cap act largely in virtue of the bond values of the concrete in which they are embedded, to resist stresses in the same direction as the radial rods parallel to them inside the cap.

This is confirmatory of the relation which is brought out by comparing the reinforcement to an equivalent sheet of uniform cross section.

4. **Indirect Tension.** The law of the combination of concrete and metal to resist flexure may be stated in the following terms:

Every pound of tension or stress thruout the area of the cross section of the reinforcement, is induced in the bar by the indirect tension or bond value of the adhesion of the concrete to the bar, which causes the rod to coact, or work with the concrete matrix.

Thus it appears that without calling for any resistance to direct tensile stress in the concrete, the embedment of the steel has induced **indirect** tensions in the concrete matrix whose total amount is equal in magnitude to the tensile stress in the bars of the circumferential frames. This indirect tension is developed by the shearing bond between the bar and the matrix, and these stresses are in reality shears extending along the surface of the bar, equivalent to indirect tensions and compressions at 45° thereto. These indirect tensions may be thought of as lines of force radiating from the surface of the bar thru the mass of the concrete, and engaging in action with and held in equilibrium by similar forces generated at the bond surface of some other system of bars reinforcing this zone of the slab at an angle to the first system. This indirect tensile strength differs from direct tensile resistance in a most important particular, viz: that a check or crack in the concrete does not seriously interfere with its continued action.

It can only be destroyed by the complete disintegration of the matrix, and thus it differs from the direct tensile strength of concrete in the important particular that it is dependable and permanent.

The preceding explanation may not appear to be clearly established without a comparison of the action of these indirect tensile stresses with those occurring in the case of the slab reinforced two ways and bent under load in one direction only, as a simple beam. In a test of a two way slab made for the purpose, and as the result of the test by simple bending on end supports it was found that no appreciable interaction of indirect tension occurred between the longitudinal bars and lateral bars. The reason for this becomes evident when we consider the manner of the distribution of the lines of force in the mass of the matrix. As they leave the surface of the bar these lines of force are disseminated thru the mass, and can only be engaged in action and held in equilibrium by similar lines of force emanating from the surface of another bar or bars also under tension at an angle thereto, for the reason that as these forces proceed from the surface of the bar thru the mass, their intensity decreases inversely as the square of the distance, following the usual law of transmission of force thru mass. Thus the area of the surface of a transverse bar not under stress is relatively too small for these forces to act upon and coact with, when the bar is not under tension, since it does not generate similar forces which can coact with the first set, except when it is itself under stress. It is further evident that otherwise there is no reason for these forces to converge, but every reason for them to continue to diverge as they proceed from the generating surface.

The conditions existing in the concrete and steel of a simple beam under small loads, amply justifies this conclusion. The indirect tensile stresses emanating from the surface of the rods finds no other resistance than the direct tensile strength of the concrete to react against. Consequently, as the concrete itself is also elongated in tension, cracks occur early, leaving these indirect tensions to coact with, or react upon separated sections of the concrete relieved from the action of direct tensions. The only function of the concrete in the tension zone after these tension cracks occur, is to transmit shears horizontally and vertically, and it can store up no other potential energy except that of shear. The fact that the beam acts as a beam at all shows that the horizontal shearing stresses essential to beam action are in full operation. It is thus that the simple beam retains its load resisting capacity under steel stresses far above those under which the first cracks in the concrete appear.

This change of function of the concrete, differentiates sharply between direct tensile stress in the concrete, which may not be relied upon as an element of strength, and indirect stresses, which necessarily exist in all reinforced beams and slabs up to the point of complete disintegration of the concrete. Indirect stresses are the necessary basis of beam action in all cases, and are depended on by all constructors.

For example, it has been shown in numerous tests that a simple beam can be loaded to a point where the computed stress on the steel, disregarding the concrete, would be approximately nine thousand pounds to the square inch, with measured elongations in the steel representing only five thousand pounds per square inch. But after the concrete cracks under the direct tension, the stress in the steel rapidly rises to the amount computed by the usual formula, and the beam may be further loaded until the yield point value of the steel is developed.

It thus appears that the useful energy stored up in one way reinforcement by indirect tension is limited by the fact that these tensions must react against, and be held in equilibrium by the direct tensile reistance of the concrete; whereas, in multiple way reinforcement, on the other hand, these tensions react upon similar indirect tensions disseminated through the concrete and induced by one or more systems of reinforcement inclined thereto.

During the period that the tensile resistance of the concrete in a beam is in full operation, or, in other words, its modulus of elasticity in tension remains constant, as much potential energy is stored up by indirect tensions in the concrete as is stored up within the steel, and also an additional amount by direct tension in the concrete.

Hence during this period the measured stress upon the steel is less than half that computed according to the usual beam formula by disregarding the tensile stress on the concrete.

We may illustrate this point by reference to Bulletin No. 28, University of Illinois, by Talbot, Table 3, page 16, describing tests of large reinforced concrete beams.

TABLE

Applied Load in Pounds	Deflection in Inches	Stress in Steel in Pounds Per Square Inch	
		From Deformation	From Bending Moment
67,000	.01	300	4,100
103,000	.02	1,200	6,300
131,000	.03	2,100	8,000
159,000	.05	3,600	9,700
189,000	.10	6,000	11,500
220,000	.11	7,500	13,400
253,000	.13	10,500	15,300
285,000	.17	14,700	17,400
350,000	.24	20,100	21,400
414,000	.31	25,800	25,300

The working load for above beam was 290,000 lbs. impact included; so that for the working load proper no considerable energy of tensile resistance is stored in the concrete. In other words, the storage reservoir of energy in the concrete of a one way reinforced slab or beam is a leaky one which, because it involves stress in tension upon the concrete approaching the ultimate strength of the concrete, allows the stored energy to escape or be dissipated by the permanent yielding of the concrete or its final cracking; in either event, unloading the stress upon the steel which, because the steel unlike the concrete not being in a condition of overstrain, is the more rigid element. This result, the shifting of work from concrete to steel, is accomplished either by semi-plastic local stretching in the matrix, or by actual cracking.

This test by Talbot lacks an interesting feature brought out by tests we have recently had made, and that is this: that if a given load be left upon the beam for a long time there is a leaking away of the energy stored in the concrete, even with a perfectly quiescent load of low intensity; since concrete cannot endure a stress much exceeding twenty-five percent of its ultimate direct tensile strength without some increase in strain with no change in the applied load.

This test beam of Professor Talbot was 25′ 0″ out to out and 2′ 10″ deep, a larger ratio of depth to span than is usual, and the effects above noted are less marked than would be the case with a beam having a smaller depth relative to length of span.

The values observed for the three lower loads indicate the masking effect of shrinkage stress in curing rather than true elastic de-. portment.

Returning now to the comparison of slab and beam action, the difference in mechanical operation is such that the continuous flat slab, Mushroom type, requires the same amount of steel to carry a given load as a beam construction having beams of a depth equal to twice that of the slab. The slab construction has the further advantage over the beam in that its reinforcement covers the area of the floor while additional slab reinforcement is requisite to complete the beam and slab floor. This relation gives the slab type its greatest advantage for heavy loads, tho its utility is not limited to heavy construction alone.

The deflections compared have been limited to that of a simple beam supported on knife edge supports with that of the continuous slab supported on points. The actual support must, of course, have considerable breadth, usually about 0.22 of the span for ordinary slabs. Such supports would reduce the net span to about 0.8 of the distance center to center of columns directly and to 1.214 times the direct span in a diagonal direction, the diagonal of the square being 1.414 L less .2L, the net diameter of the cap.

Accordingly, on this basis the stiffness of the diagonal as compared with that of the simple beam on knife edge supports, having the same span as the diagonal of the panel, should be inversely as the cubes of the ratio of the spans. But it has already been shown that a slab supported on points at the corners is theoretically twenty times as stiff as a simple beam of span L. But the cap increases this stiffness in the ratio of 1.414^3 to 1.214^3 or 1.58 to 1. But $1.58 \times 20 = 31.6$, hence the cap would increase the stiffness to nearly 32 times that of a simple beam of span L, were it not for the disparity in the resisting section as the support is approached in case of the continuous slab as compared with the slab of constant section. The critical section for bending is at the support or around it where the negative bending moment is greatest and where the resisting material presents not a constant section as regards the concrete but one reducing or growing smaller as the support is approached. With an effective cap 0.2 of the span in diameter, the resisting section is approximately $0.2\pi L = 0.63L$ in circumference and the resisting section at the line of inflection taken roughly as circular is about 1.4 times the span in circumference which combined with the massing of the steel at the support would require but a slight reduction in this ratio of 32 to perhaps 27 in order to arrive at the true deflection.

A continuous slab, such as one of the Mushroom type, is not in a strict sense an imitation of a homogeneous plate because the steel percentage varies through wide limits in different parts of the slab. It approaches $1\frac{1}{2}$ or $1\frac{3}{4}$ per cent at and close to the column, which is permissible on account of the manner in which the concrete is strained under compression at the bottom as the support is approached, while it is reduced in the thin slab to the limit of .2 percent at mid span in the side belts where there is one layer only of rods and to .4 percent at the diagonal center, so that its moment of resistance as regards reinforcement is varied thru wide limits in order to proportion the steel to the total stress in the various parts of the panel as required by slab action, and hence such a slab is more scientific and economical than a mere imitation in the form of a homogeneous plate of uniform thickness. This difference is comparable to the difference between a truss and a beam of constant section, a truss providing the most material where it is most needed, while the beam of constant section is wasteful of material in that it is not proportioned with reference to the stresses brought on it by the applied forces.

5. Comparison of Deflections at Mid Span of the Diagonal Belt and the Direct Belt. Having shown the manner of storage of the potential energy of internal work in the cantilever portion of the slab and in the suspended circular plate about the diagonal of the panel to be such that it is equally divided so far as the slab rods are concerned between radial and circumferential deformations, and having derived an approximate value for the deflection at this point based upon these relations, and upon the reduction of span by the diameter of the cap, it is in order to compare the relative magnitude of the deflection at mid span of the direct and diagonal belts respectively.

Considerations of symmetry and equal width of belt for square panels with the wide over-lap of these belts where their width is 7/16 to 1/2 the length of span, would indicate substantially equal division of the load in shear resistance between the four respective belts and on this basis we may proceed to compare the deflections at mid span directly and diagonally between columns. There is no plate action by the indirect stress at the center of the direct belt. The resistance, however, of the direct belt is largely augmented by the over-lapping of the diagonal belts when they are of sufficient width to intersect within or approximately at the edge of the direct belt. Their assistance to the direct belt may be considered then as the addition of an effective section which may be without material error considered as in proportion to the extent of over-lap. Thus with $\frac{1}{2}$ L for the width of belt in a square panel, the over-lapping of the diagonal belts may be considered to increase the resisting section of the direct belt by approximately .7 while if the width of belt is 7/16 L the increase in effective section may be considered as .5 the section of the direct belt.

The formula for stiffness previously derived shows that the stiffness for the same load is inversely as the cube of the span and inversely as the cross section of the resisting steel. In the diagonal direction plate action doubles the efficiency of the steel. Hence we may compare the deflections of the direct and diagonal belts on the basis of the cube of the diagonal span between lines of inflection, divided by the cube of the direct span between lines of inflection over the effective steel sections for the respective belts. The effective section for the direct belt as we have pointed out being from 1.5 to 1.7 of the section of the direct belt, these ratios are reduced approximately or roughly to that of the ratios of the squares of the spans in the direct and diagonal directions. This gives us as a rough computation a deflection at the diagonal center about 1.4 times that at mid span of the direct belts where the diameter of the cap $= .2\ L$ and all panels are loaded and the amount of these deflections we can figure from the ratios previously derived and compare with the computed deflection of the simple beam of constant section. The deflection at mid span of a direct belt is not the same when a single panel only is loaded, tho that at the panel center is unchanged.

6. Bending Moments. The moment of the applied forces in the case of a uniform load acting on an interior panel of a continuous four way slab assumed to be supported on columns uniformly spaced is determined as follows:

From the fundamental relations of moment magnitudes we know that for uniform loads half the sum of the moments over the supports, plus that at mid span, equals a constant, $WL/8$. The division of this moment between the supports and mid span depends on the nature of the design in respect to the relative rigidity of the construction at the support and mid span respectively.

The relative rigidities of these sections, (presupposing of course, moment resisting capacity in the slab thruout) fixes the position of the line of inflection which divides the cantilever portion from the suspended span portion of the construction.

If now these relative rigidities are made such that the position of the line of inflection is located the same distance from the support as in a beam fixed at its ends, then the apparent moment over the support is $WL'/12$ and $WL'/24$ at mid span. But if L denoting the distance center to center of columns rather than the distance L' center to center of supports near the edge of the cap, is to be used in the above expression then these coefficients must be reduced in like ratio to the reduction of span by the cap diameter in order to substitute L for L' in the moment values. Thus for an effective cap of $.2L$ these moments in terms of L, (the distance center to center of column) become $W\ L/15$ over the support and $W\ L/30$ at mid span, which values are to be modified in like manner for other values of cap diameter.

It has been shown that the manner of storage of the potential energy of internal work in the cantilever portion of the slab is such that it is to be equally divided so far as the slab rods are concerned between the radial and circumferential deformations, and this method of distribution would require, under the above proportions of cap diameter to span that provision be made to resist a moment in a radial direction of $W\ L/30$ at the edge of the cap.

In proportioning the steel, the critical section for bending should be taken as a circle about the capital and of a diameter for the effective cap on which the steel rests, of the diameter of the cap plus $1\frac{2}{3}$ times the slab thickness minus 4 to 6 inches, depending on the size of the panels.

The consideration of symmetry and equal width of belt for square panels would indicate a substantially equal division of the total load in the four belts respectively, and on this basis it is possible to determine the resisting moment required of the steel at mid span of the diagonal belt and the direct belt.

On the basis of this division, the apparent moment to be resisted by each of the belts at mid span is $WL/120$ and the true moment of stress in the steel times its lever arm is to be determined from the apparent moment and the mechanics of indirect stress heretofore discussed at length. At the center of the diagonal belts, slab action doubles the efficiency of the true moment of the steel stress. Hence the true moment of steel resistance at the center of the diagonal belts is $WL/240$. The conditions to be met at mid span of the direct belt still remain to be considered.

In referring to the diagram, Fig. 16, showing the plan of the reinforcement of this type of construction, it is observed that the diagonal belts over-lap the direct belts and increase by this over-lap the efficiency of the direct belt by .5 to .7, depending on whether the width of belt is $7/16$ or $\frac{1}{2}$ the span in width. If the efficiency of the direct belt be increased by the over-lap of the diagonal belt .7, then the moment to be resisted by steel at mid span of the direct belt is

$$WL/1.7\times120, \text{ or } WL/204$$

Whereas if the over-lap is less and increases the resisting steel area in this direction only .5 then the moment at mid span is

$$WL/1.5\times120, \text{ or } WL/180$$

The difference in assistance between the manner of coaction of one diagonal belt with another and the coaction of the diagonal belt with the direct belt in resisting moment at mid span should be specifically noted. The coaction of one diagonal belt with another is brought about by the coaction of the steel in one belt under strain with the steel in the other belt under similar strain through the indirect stress generated by bond shear while the assistance rendered the direct belt by the over-lap of the diagonal belt at mid span is largely the assistance rendered by the addition of greater cross sectional area, two radically different methods of working together and yet each adding to the efficiency of the combination structure.

Having explained the method of arriving at the stress at mid span, and the stress over the support for uniform load, and different diameters of cap, it is next in order to discuss the effect on the continuous beam of loads on single spans as contrasted with all spans loaded uniformly.

In a continuous beam of indefinite length, the loading of alternate spans has a very marked effect upon the condition of stress in unloaded spans. The top of the beam is heavily strained in tension

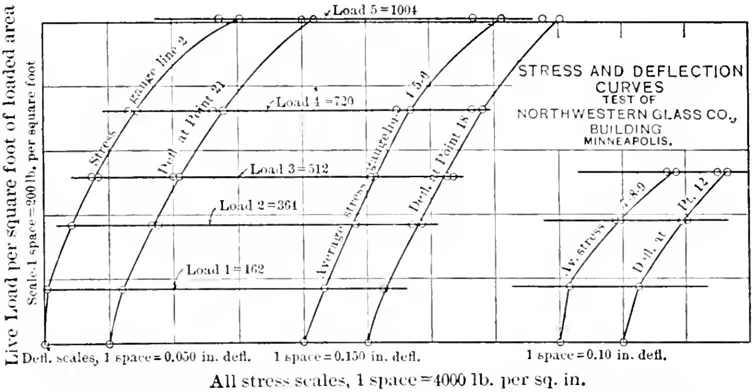


Fig. 51. Showing Agreement of Curves of Measured Deflection and Measured Stress, Mushroom Floor Slab.

thruout the unloaded span and the bottom flange is strained in compression, the negative moment for live load being constant in the top flange thruout the unloaded span and equal to the moment at the support if the dead load of the beam itself be neglected.

This condition of strain results in a negative deflection at mid span of the unloaded span in magnitude equal to a large fraction of the deflection in the loaded span. Accordingly a comparison of the positive deflection of a heavily loaded span of Mushroom construction with the negative deflection in adjacent panels will bring out and show in strong contrast the difference in the mechanics of internal stress of a continuous beam and a continuous slab supported on spaced columns.

The principle of conservation of energy fixes rigidly the relation of proportion between stress and deflection at mid span regardless of the method of reinforcement, as is discussed more at length in the comparative test of the Mushroom and Norcross type test slab, Chapter VI and also illustrated in test in the diagram Fig. 51.

In the test of the Hoffman Building, Milwaukee, Wis., under a load of one thousand pounds per square foot of sand restrained by sacks in the outer edge, a positive deflection of 7/16 inch was measured. The panel was 16′ 8″ by 17′ 0″, the basement columns 24″ in diameter, the slab $8\frac{1}{2}$″ thick. Careful measurements were made at the center of adjacent panels directly and diagonally and no appreciable negative deflection could be observed.

Considering the negative moment of resistance at mid span in comparison with its positive moment of resistance, it is obvious that a negative moment should produce a negative deflection enormously larger than would be produced by an equal positive moment, because while the slab is well reinforced to resist positive moment in the bottom layer at mid span, there is no steel at all in the top at mid span to resist negative moment.

Accordingly, the conclusion is unavoidable in view of the test data cited and many similar tests, that the effect of loading one span is almost negligible upon adjacent spans; and that Mushroom slabs must be consistently treated as acting in a large measure independently of each other as regards the transmission of negative bending moment across column heads and sides into unloaded panels, thus differing radically from a continuous beam construction. Such tests do not, however, show that a considerable amount of circumferential and radial stress is not transmitted through the portion of the slab around the head. This deportment of the Mushroom slab permits the consideration of any panel as operating to a large extent independently of adjacent panels. In other words, panels connected integrally with the columns are far more nearly self contained than are panels cast integrally with T-beams.

In the treatment of beams attention was called to the fact that the continuous beam cast integrally with the column, acts more nearly as a restrained beam than a continuous beam on knife edge supports. So the Mushroom panel acts in a larger measure as a panel restrained at the columns than as one truly continuous, permitting the design of all panels with much less attention to the difference in continuity of the slab than would otherwise be permissible.

Careful measurements of deflection show that when the concrete work is old, and thoroly cured, little difference in the elastic deportment is to be noticed between an end panel and an interior panel.

On the other hand, before the concrete has become thoroly dry, hard and rigid, and in all cases where the forms are prematurely removed, the rigidity of the end panel is much less than that of an interior panel and special care should be exercised in conservatively supporting wall panels until the concrete is thoroly rigid and hard. This consideration has led to the practice of increasing the steel in the direct belts of wall panels longer than eighteen feet where half Mushroom heads are used next to the wall by ten percent, and where the wall end of the slab is framed into a beam or a wall an increase in the direct belt of fifteen percent is made standard practice. No increase, however, is needed in the diagonal belts since the object is to secure such a reduction of the deflections of those side belts which are perpendicular to the wall as to reduce the trough of the panel which is parallel to the wall in somewhat the same ratio that the trough perpendicular to the wall has been reduced by the wall itself, thus insuring ordinary plate or slab action in these wall panels by the approximate equality of the troughs crossing each other in them.

The rigidity of the columns affects the deportment of both the continuous beam and the continuous slab, as we have heretofore pointed out. In a continuous slab, supported by columns continuing upward for one or more stories above, the amount of restraint is evidently greater than can be secured in a floor which is merely supported by columns, since in the latter case the bending resistance of the columns is reduced fully fifty percent. Accordingly, in the case of slabs thus supported it has been made standard practice to increase the steel in the direct belts of such construction by ten percent, making no increase whatever in the diagonal belts, with a corresponding addition in wall panels dependent on the manner in which they are supported, whether they are integral with beams or with half Mushroom heads.

7. Continuity in Flat Slabs and in Thin Slab on Beams, Contrasted Experimentally. In a one-way slab on continuous beams freely supported, the negative moment at the support in the case of alternately loaded panels is transferred across the column to the unloaded panel with no dimunition except that due to the positive moment of the dead load. Where the continuous beam is of concrete and rigidly built into the column this negative moment transferred across to the unloaded span is not only reduced by the dead load of the span not covered by live load, but is also reduced by a moment due to the stiffness and rigidity of the columns into which the beam is framed and acts as a monolith. This reduction in the negative moment depends for its amount evidently on the relative rigidity of the columns and beams.

In the case of the heavy warehouse, where the columns are large and stiff, the negative moment transferred to mid span of the unloaded panel would be relatively small but where the beam is framed into a girder the only resistance offered to reduce the transfer of this moment at mid span of the panel is the twisting resistance offered by the girder, which is relatively small. Hence large negative moments may be transferred from the loaded to the unloaded panel in beam construction where the columns are light or the continuous beam frames into girders. The modifying effect of these supports upon the distribution of moments in continuous beams is in strong contrast with that which occurs in continuous slab floors of either of the natural types of reinforced concrete III or IV.

In the test of the Hoffman Building in Milwaukee, a load of 1000 pounds per foot on a single panel 16′8″ by 17′0″, or a total load of 142 tons, gave a deflection at the diagonal center of the panel of 7/16″. No appreciable negative deflection, however, was observed at the center of either of the panels adjacent to the loaded panel laterally or diagonally. The supporting column in this case was 24″ diameter, reinforced with twelve 1-$\frac{1}{4}$ inch rounds. Story height from basement to the first floor was 10′6.″ The second tier of columns were not connected to the basement columns, but a cast iron base was provided to form a bearing between the second tier and the basement tier. The panel was reinforced with seventeen $\frac{3}{8}$″ round rods in four directions and was 8$\frac{1}{2}$″ thick, including one inch finish coat.

It is evident that columns of this diameter and arranged in this manner would be of insufficient rigidity to resist so great a test load as this and prevent the transference of moment to the adjacent panel if acting on the principle of the beam. Predominating circumferential action, however, prevents the linear transference of moments in the continuous flat slab in the same manner as in the continuous beam, and this is the reason that no negative deflections were observed at the centers of the panels adjacent to the loaded panel laterally or diagonally.

Similar action occurs in square panels reinforced in two directions and supported on beams running from column to column in both directions. A very heavy test load placed upon a single panel has little effect upon adjacent panels because the resistance is in a large measure circumferential, which prevents moments from being transferred from one panel to another.

Bearing in mind that circumferential stresses are coincident with and dependent for their magnitude upon radial stress, it is evident that stresses of this kind cannot be transferred from panel to panel after the manner of stress transferred longitudinally in a member, as in the case of a beam. Numerous tests of wall panels and interior panels in two-way beam contruction by Turner prove beyond question that substantially the same formula applies both to an interior and to a wall panel of type III, even where this wall panel at one side is merely built into the brick side wall of the building. Further that a corner panel built into the brick wall in the same manner exhibits the same degree of elastic resistance to deflection under load as an interior panel forming one of a continuous monolith extending for a number of panels each way and supported on four sides by beams giving the same clear span as in the case of the wall panel. While deflections of wall panels and interior panels are almost identical, it does not follow that there is no difference in the distribution of stress in the two cases which will be brought out more at length in considering the deformations which occur in such cases.

8. Variation in the Position of the Line of Inflection in Continuous Flat Slab Construction. The law of rigidities previously discussed fixes the position of the line of inflection, subject, however, to the proviso that the slab be able to resist both positive and negative moments in the zone thru which this change takes place. But wide variations in the position of the line of inflection cannot occur in continuous flat plate construction because its position is fixed approximately by the amount of metal in the case of the Mushroom system and by the dip or sharp bending down of the slab steel in some other forms, so that the lines of inflection are near the points where the belts cross from above to below the neutral axis. This limits the possible variations in position of the lines of inflection to relatively small distances.

However, the law of variation of the position of the lines of inflection with variations in the moments over the supports and at mid span follow quite a different law in continuous flat slabs from what it follows in beams. What this law is will appear by considering the magnitudes of the moments as expressed in terms of the load w on unit area instead of expressing it in terms of the total load W. The sum of half the moment over the support, plus that at mid span is a constant, $= W L / 8$. For a beam of breadth b this is $w\ b\ L^2 / 8$, but for a square panel it is $w\ L^3 / 8$. This additional power of L is introduced in the case of the continuous slab with equal column spacing in both directions because L is the breadth of the panel, so that its load per foot of length is $w\ L$, and the total load $w\ L^2$. Thus the ratio of the lengths of the suspended spans between the lines of inflection will vary for given applied moments as the cube root of the ratio of the given moments determined by the relative rigidities. Thus in the preceding example, if the moment at mid span be increased twenty-three percent, since the cube root of 1.23 is 1.07 there would be a corresponding increase in the length of the suspended span .577 L of approximately seven percent or four percent of the span L, and a variation in position of the line of inflection of two percent of the span L, a variation which is not great, and one which in view of the fact that the line of inflection is not clearly defined by a true hinge might readily occur.

In finding the moment magnitudes at the support and mid span, we have made use of the fundamental relation of moment magnitudes and assumed that the lines of inflection are located in the same position as in fixed beams of constant section.

In the early constructions of Mushroom floors, the rolling mills were not prepared to furnish small rods in long lengths and hence all belts of rods, as a rule, were lapped or spliced over the column. With such an arrangement of metal, there would be twice the section of the slab steel over the column that would occur with long lengths of rods and no splices at the support. This difference of one hundred percent in slab steel area at the support is a variation which may occur, and demands investigation.

If we over-reinforce the concrete at the support, the concrete element determines to a large extent the relative rigidity of the cantilever at the support. The shifting of the neutral plane would be by no means in proportion to the increase in the percent of steel as may be observed by reference to the k curves in the diagram shown in Fig. 25. Accordingly a large excess of steel over and above that necessary would add a relatively small amount to the rigidity of the cantilever. On the other hand, however, the cantilever must be so designed that the steel provided is not over-strained therein.

With the column cap approximately 0.2 of the span length, the applied moment of $W\ L / 15$ in 180° at its edge may be considered as a fair average value and the steel proportioned accordingly, for a slab of uniform thickness.

9. Effect of Adding Finish to the Rough Slab. A finish coat does not add to the resistance of the cantilever. But the finish coat adds to the resistance at mid span, since it increases the effective depth of the slab and its corresponding rigidity, and it is now in order to discuss the effect which this change in rigidity has upon the distribution of bending moment.

Take as an example, a floor supported on columns of equal spacing, say 17 feet center to center, with an 8 inch rough slab and assume a thickness of finish coat of $1\frac{1}{4}$ inches added thereto. With slab rods $\frac{3}{8}$ inches diameter in four layers, the distance from center of the slab steel at the support to the bottom of the slab would be approximately 7 inches, while at mid span with the rough slab the distance from the center of the slab steel tothe top of the slab would be about 7.3 inches, or with the finished slab $8\frac{1}{2}$ inches the relative rigidity of the central plate would be as the squares respectively of 7.3 and 8.5 or as 49 is to 72.25, or a ratio of 1.47. Now the moment will be distributed between mid span and the supports in such manner that it is divided in proportion to the rigidities of the cantilever and suspended span and in such manner that half the sum of the moments over the supports, plus the moment at mid span is constant, and equals $WL/8$. Consequently this change in relative rigidity would reduce the applied moment and the unit steel stresses at the support by twelve percent, and would increase the applied moment to be resisted at mid span by approximately 23 percent. But this increase of the applied moment at mid span would increase the unit stress in the steel at mid span either not at all or very slightly, because the increase of slab thickness due to the finish will not only increase d but also j so that the arm jd of the steel and its moment of resistance will be so increased as to leave the unit stress in the steel substantially the same.

10. Illustrative Example. Compute the bending moment over the column in the case of the Northwestern Glass Company Building*, at Minneapolis:

Diameter of the basement columns 30 inches. Virtual cap 48 inches; extreme outside diameter 54 inches. Slab reinforcement, four belts of fifteen $\frac{3}{8}$ inch rounds each way, 7′ 9″ wide. The central line of columns had no lap of slab rod belts

The diameter between bearings is 13′ net. Net span equals

*Described in a paper by H. T. Eddy, to the American Society of Civil Engineers, Vol. LXXVII, p. 1338, 1914.

.77 full span, center to center of columns. Therefore, the applied moment at the support

$$= .77\ W\ L\ /\ 120 = W\ L\ /\ 15.6$$

Radial moment is half the applied moment, $= W\ L\ /\ 31.2$. The load was 95,000 pounds.

$$M = \frac{95{,}000 \times 17 \times 12}{31.2} = 620{,}000 \text{ inch pounds.}$$

The center of action of steel was 6.4 inches from the bottom of the slab, and jd equals 5.5 inches. The heavy radial rods are four $1\frac{1}{8}$ inch rounds. Section of slab steel over the cap is four square inches. Total eight square inches. $620{,}000 / 8 \times 5.5 =$ the average of 14,000 pounds per inch on the steel. The neutral plane is found to be $2\frac{3}{4}''$ from the bottom. The center of the slab steel is 0.6 inches above the center of action of steel stress, so that the mean slab steel stress is as

$$2.75 : 3.35 \text{ or as } 14{,}000 \text{ lbs.} : 16{,}000 \text{ lbs.}$$

and the steel stress in the large rods is to the mean stress as

$$2.75 : 2. \text{ or as } 14{,}000 \text{ lbs.} : 10{,}300 \text{ lbs.}$$

These computations are fair approximations within the limit of error involved in the assumptions made, namely, that the coaction of the large rods with the concrete is equivalent to uniform distribution of their area about the circumference. This cannot be precisely true because the stresses from bond shear are distributed thro the mass in proportion to the square of their distance. This relation indicates an interesting variation of stress about the head. The form of the neutral plane is no longer a true plane but is scalloped in form, approaching nearer the middle of the slab under the large rods and being further removed from the middle of the slab between the large rods. Investigation of the error involved in the assumption that the neutral surface is a true plane, however, will show that the error is not large, making a reduction of not more than ten per cent in the steel stress in the large rods, making a small increase in the concrete compression and very little change in the stress in the slab steel. The results above obtained check closely with experimental determinations.

Thus it is obvious that with full lapping of every belt there is a large excess of steel in the standard Mushroom type and this excess of steel would throw the line of maximum unit stress in the slab steel to another point than that where the greatest moment to be supported occurs.

11. Depressed Head or Drop. The variation in the moment at the support by reason of the addition of strip fill and by modification of the diameter of the cap have been considered and it is next in order to investigate the modification in the moment when the slab thickness is increased at the column over and above that at mid span.

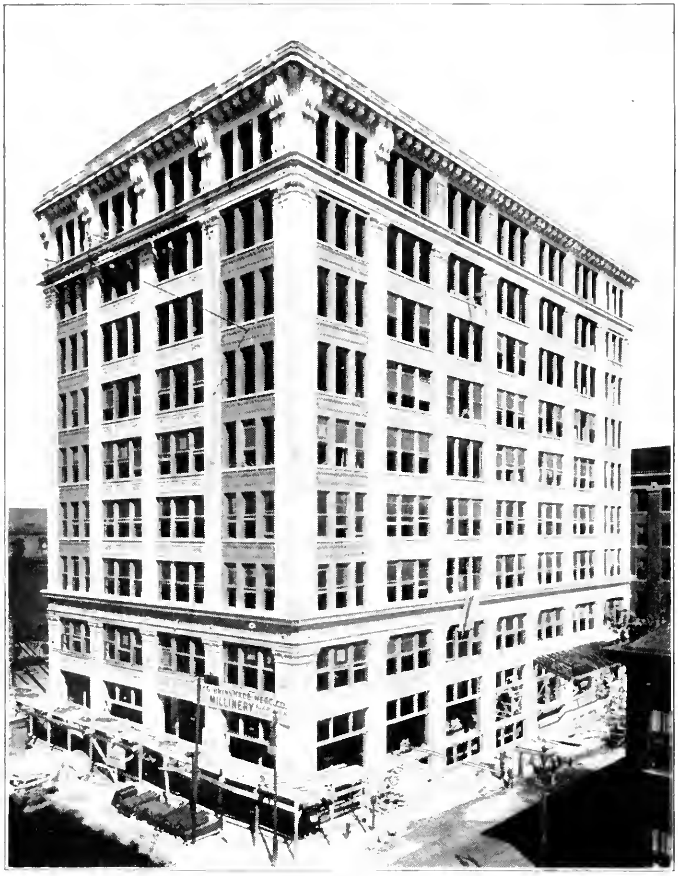
If the thickness of the slab be increased by some sixty percent as is commonly done by a square block, or drop head over the capital, having a size of $0.4L \times 0.4L$ with little or no steel except the belt rods without laps, then the rigidity is increased by reason of the increase of thickness in such a way that the lines of inflection are thereby removed somewhat from the column centers. The slab belts should not be reduced in width below $7/16\ L = 0.44L$ because this width is necessary to make the reinforcement cover the panels properly and they should be located at the top of the slab as far as to the extreme edge of the head, viz: as far as to $0.2L$ or $0.22L$ from the column center. This will fix the lines of inflection at about $0.27L$ from column centers instead of at $0.22L$ or $0.23L$ as in the Mushroom panel. Such a position of the lines of inflection which enlarges the cantilever areas will decrease the moments at mid span and increase those at the edge of the cap. The thickness of the slab at mid span may be safely reduced by ten percent for this reason, but the applied moments at the edge of the cap should not in this case be taken as less than $WL/12$, instead of $WL/15$ previously used for slabs.

With designs of this character little or no increase in steel has usually been provided over and above the slab rod steel and such designs are likely to show weakness over the cap. This weakness has too frequently been assumed to apply also to the Mushroom system which is almost invariably over-reinforced at this important critical section.

The effect of moving out the line of inflection is evidently to increase the bending moment brought upon the column by unbalanced loads and to decrease the toughness of the construction by the introduction of a sharp angle where the depressed head, as it is called, joins the body of the slab. As just stated, these depressed heads or thickening up of the slab, are generally made about 0.4 of the span length in size in each direction and approximately .6 of the depth of the slab in thickness. They permit a reduction of perhaps ten percent of the thickness in the main slab, which is almost exactly off-set by the addition of concrete for the depression.

The forms for this design will cost more than for the Mushroom design, while some saving in steel can be effected where the columns are of sufficient size, this saving in steel is more than off-set by increased cost of centering and reduced toughness or ability to withstand unbalanced load without serious over-strain.

The developments in this chapter are applications of the theory of work to bring out the nature and characteristics of the predominant mechanical actions in flat slabs. This method, while well suited to elucidate the general nature of these actions and explain their general properties, is lacking in precision in its present form and is not so well fitted to be the basis for exact calculation of structures as analysis based on the equilibrium and deformation of the elements of the slab. The following chapters will therefore be devoted to a more rigorous analysis of the flat slab based on the conditions of equilibrium and deformation of its infinitesimal elements.

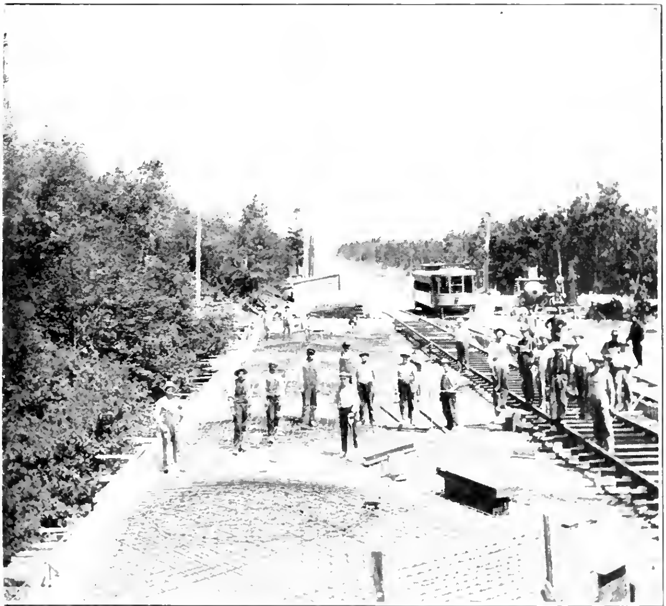


O'REILLY ESTATE BUILDING, ST. LOUIS, MO.

A. B. Groves, Architect — Murch Bros. Construction Co., Contractors

G. S. Bergendahl, M. AM., Soc. C. E., Engineer, St. Louis Representative, Mushroom System

TISCHERS CREEK BRIDGE, DULUTH, MINN.
Spans are 26 feet longtitudinally
This type is built with spans up to 50′0″



View of Reinforcement in Place
TISCHERS CREEK BRIDGE, DULUTH, MINN.
Designed by C. A. P. Turner Geo. H. Lounsbury, Contractor

CHAPTER V.

THEORY OF THE STRENGTH AND FLEXURE OF THE STANDARD MUSHROOM TYPE.

1. The superiority of flat slab floors supported directly on columns, over other forms of construction when looked at from the standpoint of lower cost, better lighting, greater neatness of appearance, and increased safety and rapidity of construction, is so generally, or rather so universally conceded as to render any reliable information relative to the scientific computation of stresses in this type of construction of great interest. Heidenreich, in his Engineer's Pocket Book on Reinforced Concrete, page 89, classifies this type as floors without beams and girders—"Mushroom System."

Since "mushroom," as applied to concrete, is an arbitrary or fanciful term, and indeed, almost a contradictory one, a word of explanation as to its origin may be of interest. The term was originated by C. A. P. Turner, of Minneapolis, and applied to his flat plate construction, more particularly because of the fancied resemblance to the mushroom, of the column and column head reinforcement of that particular form of his flat plate construction which he seemed to prefer by reason of certain practical advantages. Another fancied resemblance is the rapidity of erection, comparable to the over-night growth of the mushroom. Here the resemblance ceases, since the construction, once erected, is enduring and permanent.

The Mushroom System is a continuous flat plate of concrete supported directly on columns, and reinforced in such a maner that circular and radial tensile stresses concentric with the column are provided for by metal reinforcement in the tension zone above the columns, and similar provision is made for tensile stresses in the lower portion of the slab concentric with the center of the panel, diagonally between the columns. Since all forces in a plane may be resolved into equivalent components along any pair of axes at right angles to each other, it is possible to provide reinforcement to resist any horizontal tensile stresses in the slab by various arrangements of intersecting belts of rods at zones where these stresses occur.

All arrangements of this kind are by no means equally effective. A system of wide reinforcing belts from column to column com-

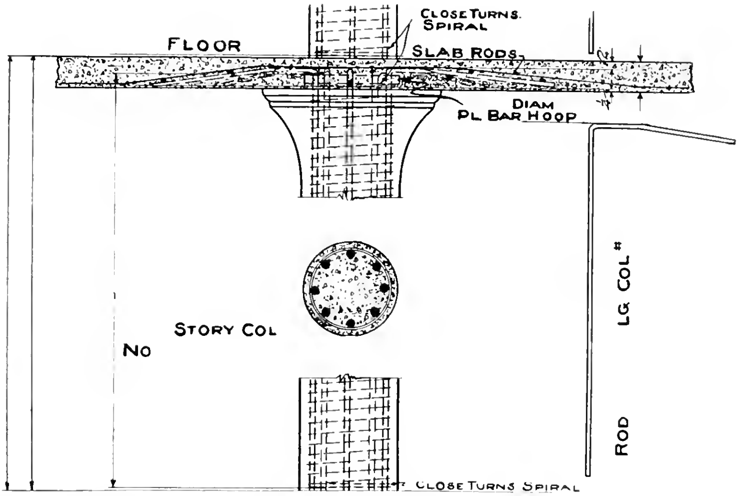


Fig. 53. Vertical Section of Standard Mushroom Head showing position of Radial and Ring Rods, and Slab Rods, Vertical and Horizontal Sections of Spirally Hooped Column, with Plain Bar Hoop Collar Band, Vertical Reinforcing Rods and Elbow Rods.

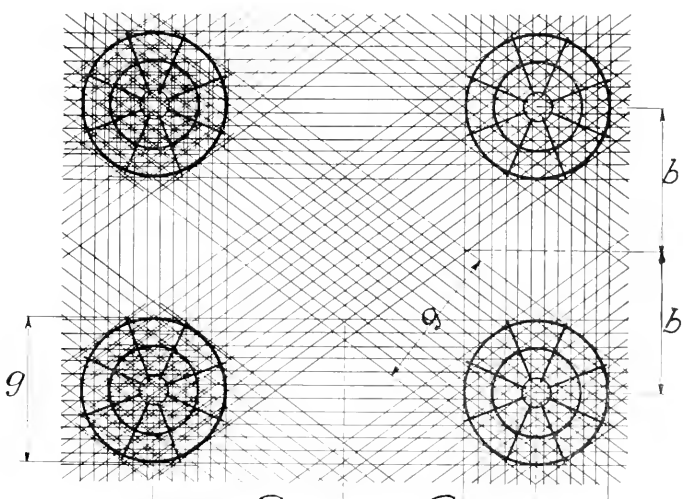


Fig. 54. Plan of Reinforcement in Standard Mushroom System. Radial and Ring Rods, Collar Band and Slab Rods. Diameter of Head $= g = \frac{7}{16}(a+b)$.

bined with a system of radial and ring rods to constitute a large, substantial cantilever mushroom head at the top of each column provides a very effective and economical arrangement for controlling the distribution of the stresses in the slab, and furnishes the resistance necessary to support these stresses by placing the steel where it is most needed. It not only has the same kind of advantage that the continuous cantilever beam has over the simple girder for long spans, but combines with it the kind of superiority that the dome has over the simple arch by reason of circumferential stresses called into play, which adds greatly to the carrying capacity of the slab.

In the standard mushroom type, which is quite fully discussed in this paper, the heavy frame work, concentric with the column, supports the slab reinforcement at a fixed elevation, furnishes a high degree of resistance to shear, and secures a high degree of safety during construction. It extends as a cantilever approximately one fourth of the way to the next column as shown in Figs. 53 and 54 on page 159. Arranged upon the radial rods of this frame rest two or more large hoops and upon these rest the wide spreading belts of rods which extend both directly and diagonally from column to column. Over the columns these belts lie near the upper surface of the slab, but they run near the lower surface as they approach points midway between columns.

The cantilever slab thus formed, not only has the same advantages for this form of construction that the cantilever construction has for long span bridges, but it causes the slab to have greater stiffness and gives it greater resistance to shear in the neighborhood of the columns; it removes the locus of zero bending moment to a much greater distance from the column than would otherwise be the case, thus dimininishing the area of that part of the slab which tends to become concave on its upper face and enlarging the convex area.

The cantilever frame-work further, not only moves the locus of zero bending outward from the column, but it also fixes the locus of zero bending moment at a known position so that it does not vary with increase and decrease of the load or change of the load from one span to an adjacent span as would be the case were the mass of metal in the frame and its stiffness largely reduced. This is accomplished as follows:

The locus of zero bending moments is fixed by the dip of the reinforceing rods as they leave the upper surface of the slab near the edge of mushroom and pass below the neutral surface to a level near the bottom of the slab. Such change of tensile resistance in the slab necessarily localizes at these points the zero bending moments.

In addition to the advantages just mentioned, which are of so self-evident a character as to be readily appreciated even by the layman, there is another of such an obscure and apparently inexplicable a nature that it was for years denied as incredible and regarded as non-existent by practical builders, and engineers as well, unless they had opportunity to be convinced of its reality by experiment. We here refer to the additional strength and stiffness which is imparted to a belt of rods in a given direction in a slab by another belt at right angles to the first belt, or at various angles with it. This should be designated as slab action proper in distinction from cantilever action. It depends for its amount upon the value of Poisson's ratio of the lateral effect due to direct elongation in the slab, and is the basis of the so called circumferential stresses, which make the strength and stiffness of such reinforced flat slabs much greater than they are estimated to be when these are neglected, as they usually have been. This mistaken view has in the past constituted the most serious obstacle to the adoption of this form of structure, and has been the ground of conscientious opposition to its introduction on the part of consulting engineers. It is the object of this investigation to remove so far as possible all reasonable uncertainty as to the rational theory of this form of structure.

The following partial bibliography of this subject may be useful to those unfamiliar with what has been done in this field.

Concrete Steel Construction, (305pp)
By C. A. P. Turner, M. Am Soc. C. E.
816 Phoenix Bldg., Minneapolis, 1909.

Reinforced Concrete Construction, (259pp)
By Turneaure and Maurer, University of Wisconsin
Wiley, N. Y., 1907.

Concrete, Plain and Reinforced, (483pp)
By Taylor and Thompson,
Wiley, N. Y., 1911.

Trans. Am. Soc. C. E.
Vol. LVI. June 1906.

Engineering News:—
Oct. 4, 1906, p. 361.
Feb. 18, 1909, p 176.
Dec. 23, 1909, p. 694.

Engineering Record:—
March 28, 1908, p 374.
May 2, 1908, p 575.
Oct. 10, 1909, p 411.
April 3, 1909, p 408.
April 10 1909, p 492.

Notation.

2. All lengths and areas are measured in inches, and all weights in pounds.

A = area of cross section of steel reinforcement per unit width of slab, in case it be assumed to be replaced by a uniform sheet of equal weight.

A_1 = area of cross section of all the rods in one side belt.

A_2 = area of cross section of all the rods in one diagonal belt.

a = one half the longer side of a panel from center to center of columns.

b = one half the shorter side of a panel.

B = the shortest distance along one side of a panel from the edge of a column cap to the edge of the next cap.

C_1 and C_2 are constants depending on the relative lengths of the sides of any panel, which reduce to unity for any square panel.

D_1 = the deflection of the middle of the longer side of the panel below the edge of the cap.

D_2 = the deflection of the center of the panel below the edge of the cap.

d = the effective thickness of the slab at any point, being the vertical distance from the center of action of the reinforcement to the compressed surface of the concrete.

d_1 = the vertical distance from the center of the rods in the side belt at mid span to the top surface of the concrete.

d_2 = the distance at the center of the panel from the center of the rods in the second or upper diagonal belt to the top of the concrete.

d_3 = the distance at the edge of the cap from the center of the third belt of rods from the top, to the compressed surface of the concrete.

E or E_s = Young's modulus for steel = 3×10^7.

E_c = Young's modulus for concrete.

e_1 = elongation in steel parallel to long side belt.

e_2 = elongation in steel parallel to short side belt.

e_1 = elongation in steel parallel to diagonal belt

F = modulus of elastic resistance to shearing.

f_s = Ec = intensity of actual stress in steel.

f_c = intensity of stress in concrete.

g = $7/16\ (a+b)$ = the diameter of the mushroom head and width of belts.

h = the total actual thickness of concrete slab.

id = vertical distance from center of tension of steel to neutral surface of slab.

jd = vertical distance from center of tension in steel to center of compression in concrete.

kd = vertical distance from neutral surface to compressed surface of concrete, hence $i + k = 1$.

K = Poisson's ratio of lateral effect due to longitudinal resistance in reinforced concrete slabs.

L_1 = $2a$ = long side of panel between column centers.

L_2 = $2b$ = short side of panel between column centers.

l = distance from collar band at top of column to edge of cap.

m_1 = true moment of resistance of the tensile stresses in steel parallel to the long side per unit of width of slab.

m_2 = true moment of resistance of steel parallel to short side per unit of width.

m_1 and m_2 = apparent moments per unit of width of forces applied parallel to the long and short sides respectively.

n = the apparent moments per unit of width of the equal twisting couples parallel to either side.

p_1 = intensity of the forces applied parallel to the long side.

p_2 = ditto for short side.

p = intensity of stress in extreme fiber of radial rods.

q = load on slab in pounds per square inch.

R_1 and R_2 = the radii of curvature of vertical sections of the slab parallel to the long and short sides respectively.

s_1 and s_2 = the vertical shearing stresses per unit of width of slab respectively perpendicular to the long and short sides of the slab.

s = the intensity of vertical shearing stress in radial rods.

t = either of the equal horizontal tangential or shearing stresses parallel to the sides of the panel.

t	=	the thickness of a radial rod.
u and v	=	deformations parallel to the long and short sides respectively.
V	=	total vertical shearing stress in radial rod.
$x\ y\ z$	=	horizontal and vertical coordinates parallel to sides of panel.
$\triangle z$	=	difference of two vertical coordinates.
z_1 and z_2	=	deflections of radial rods.
δ	=	sign of partial differential.
$\dfrac{\delta z}{\delta x}$	=	partial differential coefficient of z with respect to x.

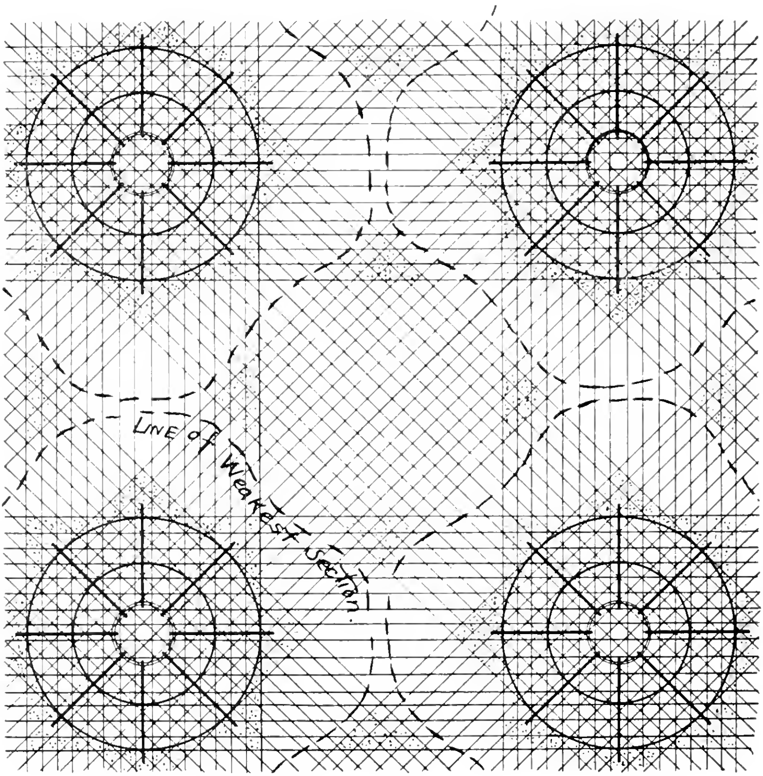


Fig. 55. Plan of Reinforcement Mushroom System. Square Panel, $g=\frac{1}{2}L$ (as drawn). Line of Ultimate Weakness.

3. As preliminary to a general investigation of the rational analysis of the flat slab, it seems desirable in the first place to make a brief exposition of the relationship between the true bending moments and the apparent bending moments in the flat slab as follows:

The fundamental equations of extensional stress and strain in thin flat plates and slabs, established a generation ago and accepted by Grashof* and by all authorities on the subject since then, may be written in the forms:

$$\left.\begin{aligned} Ee_1 &= p_1 - Kp_2 \\ Ee_2 &= p_2 - Kp_1 \end{aligned}\right\} \qquad (1)$$

$$\left.\begin{aligned} (1 - K^2)p_1 &= E(e_1 + Ke_2) \\ (1 - K^2)p_2 &= E(e_2 + Ke_1) \end{aligned}\right\} \qquad (1a)$$

in which p_1 and p_2 are the external applied or apparent stresses per unit of area of cross section of the plate, or of the reinforced slab, which act parallel to the axes of x and y respectively if these latter lie in the neutral plane of the slab; and e_1 and e_2 are extensometer elongations of plate or slab reinforcement per unit of length parallel to x and y respectively. E is Young's modulus of elasticity, and K is Poisson's ratio of the lateral effect due to linear elongation. Any piece of material which is subjected to stress, and is of such shape that more than one of its dimensions is considerable, as compared with its remaining dimension, must have its stresses and strains considered with reference to the lateral effect produced. This is the case in plates and slabs, as it is not in case of rods and beams.

In the above equations Ee_1 and Ee_2 are the true stresses per square inch of section of reinforcement acting along lines parallel to x and y respectively, whatever p_1 and p_2 may be. These latter are the cause of true stresses, but are not themselves the values of the true stresses, as they are in case of rods, etc., where one dimension only is large.

These equations show that the elongation e_1 in the direction of x is not dependent alone upon the tension p_1 applied in that direction, for it is diminished by any tension acting along y, but is increased by any compression acting along y. It thus appears that any tension p_2 along y assists the piece in resisting elongation along x and makes it able to endure safely a larger applied stress p_1 with the same degree of safety, *i. e.*, with the same percentages of elongation or true stress. But it is also equally true that any compression of amount p_2 reduces the safe value of p_1 which may be applied to

*Theorie der Elasticität und Festgkeit, F. Grashof Berlin 1878.

it. These principles are not in accordance with those which hold in ordinary computations for rods and bars, whose lateral dimensions are small compared with their lengths, and whose lateral stresses are negligible. This divergence between the true stresses as shown by actual deformations, and the apparent or applied stresses, is a fruitful source of error in the attempted computation of slabs.

Equations (1) in their present form apply to simple extensional or compressive stresses and strains but may be extended to apply to bending of slabs in the following manner:

Take A as the cross section of the reinforcement per unit of width of slab when the actual reinforcement is regarded as distributed into a thin sheet of uniform thickness, and let jd be the vertical distance from the center of the reinforcement to the center of compressional resistance of the concrete regarded as a fraction j of d, d being the distance from the center of the steel to the top of the slab. Then

$$\mathrm{m}_1 = Ap_1\, jd, \text{ and } \mathrm{m}_2 = Ap_2\, jd, \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

are the apparent bending moments per unit of width of slab, of the applied apparent stresses p_1 and p_2, tending when positive, to cause lines which before bending are straight and parallel to x and y respectively, to become concave upwards.

$$\text{Again } m_1 = Ee_1\, Ajd, \text{ and } m_2 = Ee_2\, Ajd, \ldots\ldots\ldots\ldots\ldots (3)$$

are the true bending moments of the actual resistance stresses in the reinforcement per unit of width of slab, as shown by extensometer strains in the steel parallel to the axes of x and y respectively.

Multiply equations (1) thru by Ajd and substitute the values given in equations (2) and (3), from which we obtain the following relations between the true and apparent bending moments in the slab.

$$\left.\begin{aligned} m_1 &= \mathrm{m}_1 - K\mathrm{m}_2 \\ m_2 &= \mathrm{m}_2 - K\mathrm{m}_1 \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$$\left.\begin{aligned} (1 - K^2)\mathrm{m}_1 &= m_1 + Km_2 \\ (1 - K^2)\mathrm{m}_2 &= m_2 + Km_1 \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4a)$$

These equations bring out in a striking manner the essential divergence of the correct theory of slab action from that of beam action in which latter case we have the well known equations

$$\mathrm{m}_1 = m_1, \text{ and } \mathrm{m}_2 = m_2$$

i. e., in beams the moment of the applied forces is equal to the moment of the internal resistance, which is not true in slabs.

All attempts to base computations of the deflection of slabs upon beam action are therefore necessarily erroneous. Such computations are inapplicable and misleading, hence deflections and stresses in slabs cannot be correctly computed by any form of simple or compound beam theory.

Equations (4) show:

1st That at points where m_1 and m_2 are of the same sign, (as for example in the convex part of the mushroom near the columns and also near the center of the panel) the true bending moments m_1 and m_2, which determine the actual stresses in the reinforcement are less than the apparent bending moments, which latter have been ordinarily assumed, according to the beam theory, to determine those stresses.

2nd That the compressive stresses in the concrete around the column cap are determined on the same principles as the tensile stresses and are consequently reduced in accordance with the value of K by a considerable percentage below values corresponding to m_1 and m_2 of the beam theory.

3rd That at points where m_1 and m_2 have different signs, as they have for example in the middle part of the span at the side of the panel directly between mushroom heads, the values of the true bending moments are larger than the apparent moments as found by the beam theory.

4th One deduction from this (which is also confirmed by extensometer tests) is, that in slabs having equal side and diagonal belts of reinforcing rods the greatest actual extensions and true stresses in the steel occur at the mid points of those reinforcing rods which run directly between the mushroom heads parallel to the sides of the panel, and do not occur at the center of the panel where m_1 and m_2 have their greatest values. Further, the true stresses in the reinforcement are not so large at the edge of the column caps as at the points just indicated. Neither of these conclusions is in accordance with the beam theory as implied in ordinary formulas such as have been frequently adopted in practice in computing slabs.

5th In making any statement or specification respecting the bending moments at any point of a slab, it is essential to state which bending moments are contemplated, the true bending moments or the apparent moments, with the understanding that the true bending moments only are to be used in determining cross sections and stresses of steel. Any statement omitting this distinction is ambiguous, and any requirement seeking to proportion cross sections of steel to apparent stresses and apparent moments is incorrect.

4. Poisson's ratio K plays an important rôle in the theory of flat slabs and plates, as is evident from equations (1) and (4). Few attempts have been made to determine K by directly measuring the amount of the lateral effect accompanying the elongation of test specimens, and, were such measurements made, the relative dimensions of the cross section of the specimen would need to be considered as affecting in a very complicated way the true value of K to be derived from observation. Reliable determinations of K usually depend upon observations of Young's modulus of elasticity E and the shearing modulus of elasticity F.

It is proven in the general theory of the deformation of isotropic elastic solids that all the elastic properties of any such solid are determined without excess or defect by its values of E and F, and that Poisson's ratio is a function of E and F expressed by the equation

$$K + 1 = \tfrac{1}{2}\, E/F \qquad (5)$$

There is evidence to show that for concrete K is approximately 0.1*. For steel it is known that $K = 0.3$ nearly.

Now it is evident that a horizontal slab of reinforced concrete, in which the reinforcement consists of rods, differs from one in which its reinforcement is considered to be a simple uniform sheet of metal in this, that the former has much less shearing rigidity in resisting horizontal forces than the latter, for in it all stresses transmitted from one band or belt of rods to any other belt crossing it are transmitted thru concrete only, as is not the case if the reinforcement consists of a continuous sheet. It is evident therefore that the value of K which must be employed in applying the foregoing equations to reinforced concrete slabs must exceed 0.3, the value required in case the reinforcement is a sheet of steel.

This analysis of the conditions affecting the value of K for a reinforced flat slab differs radically from assuming at random that because $K = 0.3$ for steel alone and $K = 0.1$ possibly, for concrete alone, that therefore some intermediate value of K may be correct for these two materials combined in a slab. Such an assumption is merely a blind guess and has no rational basis.

As already partly stated, the view here put forth is this: Since in any homogeneous, isotropic, elastic material the experimental values of E and F perfectly define all its elastic properties, and since we are evidently at liberty to assume our flat slab as sufficiently fine grained in its structure to act nearly like a slab constructed of some sort of homogeneous materials, it will be possible to determine

* Turneaure and Maurer's Reinforced Concrete Construction 2nd Ed. 1907, p. 210.

certain mean values of E and F which will define its elastic properties. It is moreover evident that in a slab, where two kinds of elastic solids are combined as they are here, the mean value of F for the combination is much more affected by the concrete than is E, which latter may be taken as that applying to the steel alone, and, consequently as unchanged by the combination. It is otherwise, however, with F, because the arrangement of the combination is such as to require the assumption of a value of F lying somewhere between that for steel and that for concrete. Since the latter value is much less than the former, the mean value of F is smaller than for steel alone.

This reasoning and other independent theoretical and kinematical considerations have led to the same conclusion, viz: that the correct value of K for the slab is larger than 0.3.

Assuming $E = 30{,}000{,}000$, we may compute corresponding values of K and F from (5) as follows:—

If $K = 0.1$, $F = 13{,}600{,}000$
If $K = 0.3$, $F = 11{,}600{,}000$
If $K = 0.5$, $F = 10{,}000{,}000$

Were a perfectly complete and accurate mathematical theory of the flat slab at our disposal, we might consider every experimental test of the deflection of such a slab, and every extensometer measurement of its reinforcing rods as an experiment for determining the numerical value of K, since deflections and extensions would then all be known functions of K. Having brought such a rational theory to a somewhat satisfactory degree of perfection, we have found that, in the light of all known tests of slabs, the value that best satisfies all conditions is $K = 0.5$ . . . . (6)

It is possible that this value of the constant K for slabs may need some slight modifications hereafter, but for the present this may be regarded as substantially correct for mushroom slabs. It may be found necessary to assume a somewhat different value for other forms of structure, as for example, beam and girder construction. That, however, must be determined later. Moreover it must be said that this value of K applies to tests made upon slabs from 2 to 4 months old, and under loads which have been applied to such relatively soft concrete as this for a period of usually not longer than one or two days, and of an intensity such as to cause a maximum stress in the steel of from 10,000 to 16,000 lbs. per square inch. Less loads on better cured concrete, or longer time under load, may show considerable deviation from this value of K.

How important a factor K is in slab theory is evident on considering equations (4) which show that in a square panel uniformly loaded the true moments as shown by the elongations of the reinforcing rods at the center of the panel and over the centers of the columns are only one half the corresponding apparent moments derived from considering the moments required to hold the applied forces in equilibrium, this being on the assumption of course that $K = 0.5$. (See further remarks in Section 1, Chapter VII).

5. In order to derive the general differential equation of shears and moments in any rectangular panel in an extended horizontal plate or slab, take the axes of x and y in the neutral plane of the plate and parallel respectively to the longer and shorter sides of the panel with the origin at its center before flexure occurs, and assume that they remain fixed with reference to the points of support of the panel. Then during flexure the center of the panel and all other points of the slab or plate not in contact with the fixed points of support will attain some deflection z, of amount to be determined later. Take z positive downwards.

Then $\delta x \delta y$ is the horizontal area of an element of the slab bounded by vertical planes, and if d be the effective thickness of the slab or plate, the areas of the sides of this element which are respectively perpendicular to x and y are $d\delta y$ and $d\delta x$, while $d\delta x\delta y$ is the volume of the element.

We proceed to obtain the equations of equilibrium of this element of the slab as follows:—

Let s_1 and s_2 be the total vertical shearing stresses per unit of width of slab for sections perpendicular to x and y respectively. In case these shears are variable, as they are in a continuously loaded slab, they respectively contribute elementary forces tending to move the element vertically, of the following amounts:

$$\frac{\delta\ s_1}{\delta\ x}\ \delta y \delta x, \qquad \text{and} \qquad \frac{\delta\ s_2}{\delta\ y}\ \delta x \delta y$$

Assume that the slab carries a uniformly distributed load of q pounds per square unit of area. Then the load upon the elementary area $\delta x \delta y$ is $q\delta x \delta y$, and the equation of equilibrium of the vertical forces acting on the element reduces to this:

$$\frac{\delta\ s_1}{\delta\ x} + \frac{\delta\ s_2}{\delta\ y} + q = 0 \qquad \text{.................(7)}$$

in which s_1 and s_2 are taken as positive when they are such as would be produced in the slab by the loading q in case it were supported at the origin only.

Let m_1 and m_2 be the apparent moments per unit of width of slab of the applied forces which tend to bend those lines in the slab which before bending are parallel to x and y respectively. Take them as positive when they tend to make those lines respectively concave upwards. These are the moments obtained by multiplying the total applied tension per unit of width of slab by the vertical distance jd from the center of the reinforcement of the slab to the center of compression in the concrete as given in (2). These moments are not identical in a slab with the true resisting moments m_1 and m_2 in the same directions, which latter are the moments obtained by multiplying jd by the actual tension in the steel per unit of width of slab, which last is to be correctly computed by taking the product of the area of steel per unit of width and its elongation multiplied by E its modulus of elasticity as shown in (3).

Again, let n be the twisting moment per unit of width of vertical section of slab cut by planes perpendicular to either x or y, and acting about either x or y, which moment n is regarded as due to the variation of the vertical shearing stress s_1 when y varies, and to the variation of s_2 when x varies. The moment n is held in equilibrium by horizontal shearing stresses in these same sections, which are opposite in sign above and below the neutral surface. Let t be the total horizontal shearing stress per unit of width of slab in the reinforcement on one side of the neutral plane, then:

$$\mathrm{n} = \mathrm{t}\ A\ j\ d \dots\dots\dots\dots\dots\dots (8)$$

At any point xy this horizontal shearing stress t must be the same for the section perpendicular to x, as for the section perpendicular to y, because in every state of stress the tangential components are equal and of opposite sign on any two planes mutually at right angles. Consequently the moment n is the same about x as about y, as has been assumed in (8).

It is implicitly assumed in (2) and (3) that the concrete on the same side of the neutral plane as the reinforcement is ineffective and that its resistance is negligible, so that on that side the resistance of the reinforcement alone counts. This condition actually occurs only after a state of quite considerable stress obtains, and of itself affords a sufficient reason why the formulas based on it fail of accurately representing deflections and elongations at small loads and low stress.

The elementary couples acting on the vertical faces of the element which are in equilibrium with those arising from the shearing stresses are:—

$$\left(\frac{\delta\, m_1}{\delta\, x} + \frac{\delta\, n}{\delta\, y}\right) \delta x \delta y \quad \text{about } y, \text{ and}$$

$$\left(\frac{\delta\, m_2}{\delta\, y} + \frac{\delta\, n}{\delta\, x}\right) \delta x \delta y \quad \text{about } x;$$

while those arising from the shears themselves are:—

$$s_1\ \delta x\ \delta y \quad \text{and} \quad s_2\ \delta x\ \delta y.$$

Consequently the equations of equilibrium of the couples acting on the element reduce to the following:

$$\left.\begin{aligned} \frac{\delta\, m_1}{\delta\, x} + \frac{\delta\, n}{\delta\, y} + s_1 = 0 \\ \frac{\delta\, m_2}{\delta\, y} + \frac{\delta\, n}{\delta\, x} + s_2 = 0 \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Differentiate equations (9) with respect to x and y respectively and substitute in (7), and we obtain

$$\frac{\delta^2\, m_1}{\delta\, x^2} + 2\frac{\delta^2\, n}{\delta x\, \delta y} + \frac{\delta^2\, m_2}{\delta\, y^2} = q \ldots\ldots\ldots\ldots\ldots (10)$$

which is a general differential equation of the apparent moments of the applied forces which exist in a uniformly loaded slab in terms of rectangular coordinates. From it the differential equation of the deflections may be derived as follows:—

6. To obtain the general differential equation of the deflections of a slab, note that from geometrical considerations such as are familiar in the theory of beams we have

$$R_1\, e_1 = i\, d = R_2\, e_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

in which R_1 and R_2 are the radii of curvature of sections of the neutral surface by vertical planes parallel to x and y respectively; and id is the distance from the center of the reinforcement to the neutral surface. In equations (1a) replace p_1 and p_2 by values given in (2), and e_1 and e_2 by values taken from (11) and we have:—

$$\left.\begin{aligned}(1 - K^2)\mathrm{m}_1 &= E\,A\,ijd^2\left(\frac{1}{R_1} + \frac{K}{R_2}\right)\\ (1 - K^2)\mathrm{m}_2 &= E\,A\,ijd^2\left(\frac{1}{R_2} + \frac{K}{R_1}\right)\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (12)$$

But from the theory of curvature

$$\frac{1}{R_1} = \pm \frac{\delta^2 z}{\delta x^2}, \text{ and } \frac{1}{R_2} = \pm \frac{\delta^2 z}{\delta y^2} \quad \ldots\ldots\ldots\ldots (13)$$

Also write for brevity $\quad I = A\,i\,j\,d^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$

Then we have from (12), (13) and (14):

$$\left.\begin{aligned}(1 - K^2)\mathrm{m}_1 &= \pm EI\left(\frac{\delta^2 z}{\delta x^2} + K\frac{\delta^2 z}{\delta y^2}\right)\\ (1 - K^2)\mathrm{m}_2 &= \pm EI\left(\frac{\delta^2 z}{\delta y^2} + K\frac{\delta^2 z}{\delta x^2}\right)\end{aligned}\right\} \quad \ldots\ldots (15)$$

By the fundamental equations of elasticity we also have

$$\mathrm{t} = Fe_3 = F\left(\frac{\delta u}{\delta y} + \frac{\delta v}{\delta x}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

in which F is the shearing modulus, e_3 is the horizontal shearing deformation of the reinforcement for two vertical planes one unit apart horizontally, and

$$u = \pm i\,d\frac{\delta z}{\delta x}\quad,\quad v = \pm i\,d\frac{\delta z}{\delta y} \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

are the deformations along x and y respectively, due to the vertical distance $i\,d$ of the reinforcement from the neutral surface.

From (16) by help of (17) we have

$$\mathrm{t} = \pm 2\,F\,i\,d\frac{\delta^2 z}{\delta x \delta y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

In (18) replace F by its value obtained from (5), and then substitute the resulting value of t in (8):—
we then have

$$n = \frac{E\ I}{1 + K} \cdot \frac{\delta^2\ z}{\delta x \delta y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

From (15) and (19) obtain values of the second differential coefficients of the moments appearing in (10), which on being introduced into (10), transform that equation into the required general differential equation of deflections as follows:—

$$\frac{\delta^4\ z}{\delta\ x^4} + 2\frac{\delta^4\ z}{\delta x^2 \delta y^2} + \frac{\delta^4\ z}{\delta\ y^4} = \frac{(1 - K^2)}{EI} q \ \ldots\ldots\ldots (20)$$

which is a partial differential equation of the fourth order that must be satisfied by the coordinates $x\ y\ z$ of the neutral surface of any uniform plate or slab initially flat, when deflected by the application of a uniformly distributed load of intensity q, and supported in any manner whatever.

It may be shown that any deviations from strict accuracy by reason of local stretching of the neutral surface (here neglected) are small compared with corresponding deviations in beam theory.

7. The solution of the general differential equation of deflections (20) for the case of a horizontal slab carrying a uniformly distributed load and supported on rows of columns placed in rectangular array and having the points of support all on the same level, will now be considered.

The integration or solution of (20) would, since it is a partial differential equation, introduce arbitrary functions of the independent variables x and y whose forms would need to be so determined as to cause the solution to satisfy the conditions imposed by the position and character of the supports at certain points, or along certain lines. It would be possible to expand these functions in terms of ascending whole powers and products of x and y, and, in case the supports are symmetrically situated with respect to the axes, the expansions will contain no odd powers of x or y, because the value of z must remain unchanged by changes of sign of either x or y, or both x and y. Any form of polynomial expansion which satisfies (20), and also all the conditions of any given case, must be the correct solution for that case, for, the solution of any given case must be unique.

Instead therefore of carrying thru the tedious analytical development involved in solving (20) mathematically and then applying it to the case we are treating, we shall at once write down the form of solution that applies to the case in hand and verify the fact that it satisfies (2) and all the required geometrical conditions. It will therefore be the solution sought for, which might also have been obtained by the somewhat intricate analytical processes involved in the intregation of such differential equations as (20).

Assuming at first that the slab is unlimited in extent and uniform thruout in the distribution of its reinforcement and loading, and that the parallel rows of supporting columns divide the slab into equal rectangular panels, we shall find a solution in which every panel is deformed precisely in the same manner as every other. Modifications made later will render it possible to take account of variations and irregularities in the distribution and arrangement of the reinforcement, and to estimate to some extent at least the effect of loading only one or more panels.

Let $2a$ be the length and $2b$ be the breadth of a panel; then the equation of its neutral surface, referred to axes parallel to its sides and to an origin fixed in space at the center of the neutral surface of the panel before deflection, is:—

$$48\,EIz = q(1-K^2)\left[(a^2-x^2)^2+(b^2-y^2)^2\right] \quad \ldots\ldots (21)$$

This is the correct solution of (20) not only because it satisfies (20), as it will be found to do by trial, (and just as many other functions of x and y do also) but it also satisfies all the other conditions required by the case proposed, viz.:

1st $z = 0$ when both $x = \pm a$ and $y = \pm b$;

because there must be no deflection at these points of support which are on the same level as the origin.

2nd $dz/dx = 0$, when $x = 0$, and also when $x = \pm a$; as well as $dz/dy = 0$, when $y = 0$, and also when $y = \pm b$; because straight lines drawn in space to touch the slab across its edges, and across its mid sections parallel to those edges, must all be horizontal by reason of the symmetry of the slab on each side of its edges and mid sections. That these conditions hold is evident from the following equations derived from (20):

$$\left.\begin{aligned} \frac{\delta z}{\delta x} &= \frac{(1-K^2)}{12\,E\,I}\,q\,x\,(x^2-a^2) \\ \frac{\delta z}{\delta y} &= \frac{(1-K^2)}{12\,E\,I}\,q\,y\,(y^2-b^2) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (22)$$

It is of interest to note that the sections of this surface made by all vertical planes parallel to the axes of y, *i. e.*, by x = constant, are precisely the same except in position, since their equations differ by a constant only. The same is true of sections parallel to x. It thus appears, that, in a square panel where $a = b$, the surface may be regarded as a ruled surface described by using the two of these curves on a pair of parallel sides of the panel as directrices and a third one of these curves as a ruler sliding on the first two in such a manner as to remain parallel to the other pair of parallel sides.

The deflections at the center of the panel and middles of the sides are:

$$\begin{array}{ll} \text{At } x = 0 = y, & 48\, E\, I\, z = q\,(1-K^2)\,(a^4 + b^4) \\ \text{At } x = \pm a,\ y = 0, & 48\, E\, I\, z = q\,(1-K^2)\, b^4 \\ \text{At } x = 0,\ y = \pm b, & 48\, E\, I\, z = q\,(1-K^2)\, a^4 \end{array}$$

so that in a square panel the center deflection is twice the mid edge deflection.

Differentiating equations (22) we have by help of (11), (13), (14) and (3):

$$\left.\begin{aligned} e_1 &= \frac{id}{R_1} = \pm\, i\, d\, \frac{\delta^2 z}{\delta x^2} = \frac{(1-K^2)}{12\, E\, A\, j\, d}\, q\,(3x^2 - a^2) \\ e_2 &= \frac{id}{R_2} = \pm\, i\, d\, \frac{\delta^2 z}{\delta x^2} = \frac{(1-K^2)}{12\, E\, A\, j\, d}\, q\,(3y^2 - b^2) \end{aligned}\right\} \ldots \ldots (23)$$

$$\left.\begin{aligned} m_1 &= \frac{(1-K^2)}{12}\, q\,(3x^2 - a^2) \\ m_2 &= \frac{(1-K^2)}{12}\, q\,(3y^2 - b^2) \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots (23a)$$

in which the ambiguous signs are to be so taken that m_1 and m_2 in (15) will be positive at $x = 0 = y$, and negative at $x = \pm a$ and $y = \pm b$.

From (23) it appears that extensions vanish and contra-flexure occurs at lines lying in vertical planes whose equations are

$$x = \pm \tfrac{1}{3}\, a\sqrt{3} \quad \text{and} \quad y = \pm \tfrac{1}{3}\, b\sqrt{3} \ldots\ldots\ldots\ldots (24)$$

It thus appears that the slab is subdivided by these lines (24) drawn parallel to the edges into a pattern which consists of a rectangle occupying the middle part of each panel, of a size $\frac{2}{3}\, a\sqrt{3}$ by

$\frac{2}{3} b\sqrt{3}$, *i. e.*, of the same relative dimensions as the panel itself, and bounded by lines (24), which rectangle is concave upward thruout.

On all four sides of this central rectangle are rectangles of saddle shaped curvature directly between the central rectangles of adjoining panels, while each point of support is situated in a rectangle which is convex upward over its entire area, of dimensions

$$2a(1 - \tfrac{1}{3}\sqrt{3}) \text{ by } 2b(1 - \tfrac{1}{3}\sqrt{3}).$$

From (22) we obtain the equation

$$\delta^2 z \,/\, \delta x \delta y = 0 \ldots\ldots\ldots\ldots (25),$$

hence by (18) and (19) it follows that

$$\mathrm{t} = 0 = \mathrm{n}, \ldots\ldots\ldots\ldots (26),$$

from which it appears that there is no horizontal shear in the steel, and no twisting moment in vertical planes perpendicular to x or y. This would be otherwise evident from considerations of symmetry. It will be shown that this is not true of all other vertical planes.

Again from (15) and (23) we have

$$\left.\begin{array}{l} \mathrm{m}_1 = \frac{1}{12}\, q\, [3x^2 - a^2 + K(3y^2 - b^2)\,] \\ \mathrm{m}_2 = \frac{1}{12}\, q\, [K(3x^2 - a^2) + 3y^2 - b^2\,] \end{array}\right\} \ldots\ldots\ldots\ldots (27)$$

in which we have omitted the sign $\pm$ as superfluous.

From (9) by help of (26) and (27), we have

$$-\mathrm{s}_1 = \frac{\delta \mathrm{m}_1}{\delta x} = \tfrac{1}{2}\, q\, x, \quad \text{and} \quad -\mathrm{s}_2 = \frac{\delta \mathrm{m}_2}{\delta y} = \tfrac{1}{2}\, q\, y \ldots\ldots (28)$$

from which it appears that any strip of the panel parallel to x or y, and one unit wide exerts a shear at its ends such as it would if it were an isolated beam loaded uniformly with an intensity of $\frac{1}{2}q$ per unit of length. According to this, a total shear of $q\, a\, b$, which is one fourth of the total load carried by the panel, appears at each edge of the panel, this total shear on each edge being uniformly distributed along it.

It is seen therefore that the form of solution which we are investigating implicitly assumes that at each edge of the panel there is some auxiliary form of structure that will bear the shears coming to it from each side and at the same time assume the curvatures and deflections contemplated in (21). This will immediately engage our further attention.

8. In order to investigate more fully the deflections, stresses and strains in the side belts of any panel directly between the mushroom heads, let us consider the results just reached somewhat more fully.

The conclusion drawn from (28) was, that a panel with reinforcement distributed with perfect uniformity thruout would require to be supported by a narrow auxiliary girder extending from column to column along each side, and of such resisting moment as to take on, under its load, the precise curvature required by the neutral surface in (21), which curvature must be produced by a uniformly distributed load of $2\,q\,a\,b$, one half of it coming from each of the two panels beside it.

It seems then, that up to this point, we have in reality been treating the theory of the continuous uniform slab with specially designed continuous beams supporting its edges, without as yet investigating those beams in detail. But since no such beams in fact exist under the flat slab, it is clear that the side belts of the slab lying directly between the extended heads of the columns must discharge the functions which would be discharged by the auxiliary beams just spoken of. Such functions must necessarily be added to those already discharged by those belts in supporting the loading which rests directly upon them. In order that this may occur in a manner readily amenable to analysis, the extended stiffened headings of the columns which constitute the mushrooms should in general be approximately of the diameter required to support the ends of a belt of reinforcing rods forming a flat beam which fills the width along the edge of two adjacent panels between the two lines of contra-flexure on each side of that edge, as given in (24).

This requires that the mushroom head should have a width of at least $(1 - \frac{1}{3}\sqrt{3}) = .423$ of the width of the slab between columns. For reasons that will appear later, it is current practice to make these heads not less than $\frac{7}{16} = .437$ of this width.

The lines of contra-flexure in (24) have a fixity of position, (in a flat slab constructed with mushroom heads of this size and stiffness,) under single panel loads, that does not exist in a uniform slab, or where the headings are not so stiff. It may be readily shown by Mohr's theorem respecting deflection curves as second moment polygons, that where there are large sudden changes in the magnitude of the moment of inertia I, such as exist in this case at the lines of contra-flexure at the edges of the mushroom, the lines of contra-flexure remain fixed. But in systems where the diameter of the head is smaller than given above, or its stiffness is much reduced, these lines may be removed to greater distances from the center in loaded panels surrounded by those not loaded than when all are loaded, thereby increasing the deflections and stresses in a single loaded panel over that of a uniformly loaded slab of many panels.

The lines of contra-flexure in (24) separate the slab into areas which are largely independent of each other, since no bending moments are propagated from one to another. The only forces crossing these lines of section are the total vertical and horizontal shearing stresses. The horizontal shears (which are unimportant so far as deflections go) will be considered later so far as may be necessary, but the vertical shears found by (28) are of prime importance. Let us then consider one of these side belts.

In any extended slab with its panels all loaded uniformly thruout, the vertical shear must vanish at all points along sections made by vertical planes thru the centers of columns at each side of any panel, as appears by reason of symmetry of loads. Let the edges of the side belts be situated at some given distances, say x_1 and y_1 on each side of the centers of all the panels, where x_1 and y_1 are not necessarily the values of x and y given in (24), altho those values are also included in this supposition. Then by (28) there is a uniformly distributed vertical shear of intensity $\frac{1}{2}\ q\ y_1$ along the edge of the belt at $y = y_1$, even tho the reinforcement in the side belt may be greater than that in the central rectangle, for the deviations caused by the irregularity of its distribution may be regarded as unimportant and practicably negligible.

It may then be assumed that any side belt parallel to x must carry, in addition to that already provided for in (21), a total loading of $q\ y_1$ per unit of length, uniformly distributed along the two edges that are parallel to x. Now since the width of this belt is $2(b - y_1)$, the load already provided for in (21) is $\frac{1}{2}q$ per unit of area, or $q(b\text{-}y_1)$ per unit of length parallel to x, which added to that arising from the shears just mentioned makes a sum total of $q\ b$ per unit of length of belt, which it will be noticed is independent of the width of the belt. In other words, any such belt must support a load of one fourth of the total load on the two panels of which it forms a part, or one half of all that lies between the panel center lines which are parallel to it on either side. The other half may be regarded as carried to the heads by the diagonal belts. This in effect transfers the entire loading of the slab to the side belts by the agency of the shearing stresses. It does this in such a way that one half of the total loading of the entire slab is carried by one set of side belts, and the other half by a second set which crosses the first at right angles.

In those parts of the slab area where these sets of belts cross, forming the heading of the columns, the loading is superposed also.

The preceeding investigation of the shears at the edges of side belts and their loading is independent of their width and of the position of the lines of contra-flexure, but their width will be assumed in what follows to be determined by the position of those lines as shown in (24) on account of the independence of action of belts of their width, as previously explained, where it was shown that no bending moments are propagated across those lines.

The question now arises, how the vertical shears at the edges of the side belts are distributed across their width and carried by them. Since by symmetry of loading, etc., there is no vertical shear at the edge of the panel where $y = b$, the shear must diminish from each edge of a belt to zero at that line. If it be assumed to diminish uniformly, that is equivalent in its action to a uniformly distributed load on the belt, which may be assumed in computation to replace the shears at the edges. Whether it will be so distributed or not depends upon the stiffness of the mushroom head and the smallness of its flexure. Extensometer measurements on the rods of the side belt of the floor slab of the St. Paul Bread Company Building by Prof. Wm. H. Kavanaugh show beyond question that in the mushroom system the load is so distributed. Other extensometer measurements to which the writer has access also show that in systems in which the heading of the column is not so stiff as this the distribution of loading cannot be taken as uniform over the side belts.

Now the belt parallel to x was shown to carry a load per unit of length of $q\ b$ and to have a width $2(b - y_1)$, in general, or a width $2b(1 - \frac{1}{3}\sqrt{3})$ for the belt between the lines of contraflexure; hence the intensity of the loading on this belt is $q\ b \diagup 2(b - y_1)$, instead of q, as it would be in a uniformly loaded panel duly supported at its edges by beams from column to column. Let ΣA, designate the area of the effective right cross section of the steel in the entire width of a side belt regarded as forming a single sheet of metal of the width of the belt; then $\Sigma A \diagup 2(b - y_1)$ is the effective right cross section per unit of width of belt, and we may write (14) in the form

$$I = i\ j\ d^2\ \Sigma A \diagup 2(b - y_1) \qquad (29)$$

We shall consider admissible values of ΣA later.

Since the deflection of the side belts may be taken independently of the rest of the slab, let those values for the intensity of loading and the moment of inertia (29) be introduced into (21).

We then obtain an expression for the law governing the deflection of that part of the side belts parallel to x which lies between the mushroom heads, and is bounded by lines of contra-flexure, viz:

$$z = \frac{(1 - K^2)\, q\, b}{48\, E\, i\, j\, d^2\, \Sigma A}\left[\, (a^2 - x^2)^2 + (b^2 - y^2)^2 \,\right] \ldots\ldots (30)$$

with a corresponding equation for the side belts parallel to y, which may be obtained by replacing $q\,b$ in (30) by $q\,a$. Call this second equation (31). Now (30) and (31) would hold thruout the entire length of these belts from column to column were they entirely separate from each other and from the diagonal belts where they cross each other. It will be necessary later to obtain the equation which holds true where these belts cross and combine with each other.

9. Practical formulas for the stresses in the steel and concrete of side belts between the lines of contra-flexure will now be obtained from (30) and (31).

In order to do this, consider the summation in (30) expressing the effective cross section of the steel in the mid area of the side belt regarded as forming a single uniform sheet, that mid area being bounded on all sides by lines of contra-flexure.

It is to be noticed that the factor $(1 - K^2)$ of (30) takes into account the fact that the lattice of rods forming the reinforcement is less effective than the same amount of metal in the form of a sheet, the only question left being this: Will the great irregularity of distribution of the reinforcement in this area cause it to act differently to any noticeable extent from the manner in which the same amount of metal would act were it possible to distribute it uniformly over the entire area? There are strong reasons which go to sustain the view that this irregularity of distribution is negligible in the standard mushroom slab, at least for loads less than those that stress the steel below the yield point, or do not stress the concrete for too long a time while it is imperfectly cured. On examining a diagram of the reinforcing rods of a slab made with square panels of such proportions that the width of the belts is one half the distance between columns, then the pattern previously mentioned into which it would be divided by these belts will be seen to consist of equal squares whose edges are equal to the width of the belts, with one central square in each panel concave upwards, and one half of each of the saddle shaped squares which border it, also lying within the same panel, and one quarter of each of the four convex squares at the head of each of the columns at the corners of the panel, also lying within the same panel, see Fig. 55, page 164.

Each side square will be found in this case to have double (or two belt) reinforcement over one half or its area, single belt reinforcement over a diamond occupying one fourth of its mid area, and triple reinforcement over four triangular areas along its sides which together cover one fourth of the square. This gives a mean value of $\Sigma A = 2 A_1$ in which A_1 is the total right cross section of the rods in the side belt.

The belts in the standard mushroom are, however, not so wide as this, since that system simply requires that the edges of the side and diagonal belts intersect in a single point, Fig. 54, instead of forming four areas of triple reinforcement on the sides. This makes the width of the singly reinforced diamond sufficient to just reach across the side belt. In this practical case we find that very approximately

$$\Sigma A = 1.5 A_1 \qquad (32)$$

in which, as before, A_1 is the total right cross section of the side belt in square inches. It is evidently impossible for this single side belt of rods which crosses the diamond, to elongate without a corresponding equal elongation of the double reinforcement on all its sides, or at least it is impossible for readjustments to take place in any short time such as will make these direct deformations within the diamond larger than those in the areas along side of it, or before somewhat more permanent deformations have taken place in the concrete.

In cases where the column heads are smaller than the standard, and the side belts still narrower, not only may ΣA become much less than $1.5 A_1$ but the belt become so weakened near the central diamond as to render it very questionable whether the irregularity of distribution of steel in the area considered may be safely disregarded. Diminution of the size of the heading thus not only diminishes cantilever action, but reduces the effective resistance of the reinforcing steel. Not much diminution of the size of head would be required to reduce the value of ΣA to an amount as small as A_1.

Introducing the estimate given in (32) for the standard mushroom into (30) we derive by (23), (23a) and (3), for that part of the side belt parallel to x between $x = + \frac{1}{3} a\sqrt{3}$ and $x = - \frac{1}{3} a\sqrt{3}$,

$$\left.\begin{aligned} f_s &= E\,e_1 = \pm E\,i\,d\frac{\delta^2 z}{\delta x^2} = \pm\frac{(1-K^2)q\,b}{18\,j\,d\,A_1}(3x^2 - a^2) \\ M_1 &= 1.5\,A_1\,j\,d\,f_s = \pm\frac{(1-K^2)}{12}\,q\,b\,(3x^2 - a^2) \end{aligned}\right\} \qquad (33)$$

in which M_1 is the total true moment of resistance of the side belt, f_s is the true stress per square unit of the reinforcement in the side belt, and $1.5\ A_1$ is the effective right cross section of the reinforcement. This is independent of y as before noted, showing that the values of f_s and e_1 are the same for one rod as for another, but they attain their greatest values at the mid length where $x = 0$. If units be pounds and inches, and we assume $j = 0.91$ for the very small percentage of reinforcement of the standard mushroom system, then by (33) and (6) the practical formulas for design are:

$$\left.\begin{aligned} f_s &= \frac{3\ q\ a^2\ b}{4 \times 18 \times 0.91\ d_1\ A_1} = \frac{W\ L}{175\ d_1\ A_1} \\ M_1 &= 1.5\ A_1\ j\ d_1\ f_s = \frac{W\ L}{128} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (34)$$

in which f_s is the true stress in the steel, and M_1 is the true bending moment of the effective cross section $1.5\ A_1$ of the steel in the entire belt as shown by the elongation (at mid span) of the rods in a side belt of length L, where L is either $2a$ or $2b$, and $W = 4\ q\ a\ b$ is the total load on the panel in pounds, where d_1 is the vertical distance from the center of the rods in the single belt at mid span to the top surface of the slab.

While the values obtained from (34) are conservative for $j = 0.91$, corresponding to a percentage of reinforcement for one belt of less than 0.25%, (34) should be regarded merely in the light of a specimen equation for that percentage, and any slab where the percentage differs materially from that assumed value should be submitted to separate computation in the same manner.

Values of j are given for beams by ~~Turneaure~~ Turneaure and Maurer in their "Reinforced Concrete Construction," page 57, for different percentages of reinforcement on the straight line theory, which latter is now accepted usage. As already stated, standard mushroom design makes the percentage of reinforcement for warehouse floors where the panels are, say 20′ x 20′, as low as 0.25% or less, at the middle of the side belts, reckoned on the beam theory. But in heavier and larger construction it may reach 0.33%.

We have taken the mean available steel in the belt as $1.5\ A_1$, hence the mean slab reinforcement will not be less than $1.5 \times 0.23 = 0.4\%$ in the side belt areas between lines of contra-flexure.

In case we assume the ratio of E_s for steel to E_c for concrete to be 15, as is often prescribed, we find the above stated value of j as a good mean value, which will be less in cases where the percentage of steel is greater. The small percentage of steel and great relative thickness of concrete is one of the distinguishing features of the standard mushroom design.

We may write (34) in the form:

$$f_s = \frac{W\ L}{175\ d_1\ A_1}, \text{and } M_1' = A_1\ j\ d\ f_s = \frac{W\ L}{192} \quad \ldots . (34a)$$

in which M_1' is the true bending moment of the actual cross section A_1 at mid belt. We have written this modification of (34), not for use in design, but merely for the purpose of instituting a comparison with empirical formulas obtained by Mr. Turner to express the results of numerous tests made by him. On pages 26 and 28 of his "Concrete Steel Construction" he has given equations expressing the values of stresses and moments in mushroom slabs which in our notation may be written as follows:—

$$M_1' = A_1\ j\ d\ f_s = \frac{W\ L}{200}, \text{ and } f_s = \frac{W\ L}{200 \times 0.85\ d\ A_1} = \frac{W\ L}{170\ d\ A_1} . (35)$$

in which he has assumed 0.85 as a mean value of j.

It is seen that equations (34a), obtained from rational theory alone, are in practical agreement with (35), which were deduced from experimental tests of mushroom slabs, where the numerical coefficient introduced is entirely empirical.

As will be seen later, (34) is the equation which ultimately controls the design of the slab reinforcement; so that the agreement of these two entirely independent methods of establishing this fundamental equation cannot but be regarded with great satisfaction as affording a secure basis for designs that may be safely guaranteed by the constructor, as has been the custom in constructing standard mushroom slabs.

The slab theory here put forth diverges so radically from the results of beam theory that we introduce here the following comparative computation of the smallest values of true bending moment and stress in steel, which can be obtained by beam theory for the side belt parallel to x, as follows:—

That part of the side belt between the lines of contra-flexure is simply supported at its ends by shearing stresses, and so may be taken to be a simple beam resting on supports at these end lines.

Hence the true stress f_s and the true bending moment M' at the middle of this simple uniformly loaded beam may be computed from the equation,

$$M' = A_1 \, j \, d \, f_s = \tfrac{1}{8} \, W' \, L' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (36)$$

in which M' is the total moment of resistance.

A_1 is the total right cross section of the reinforcement, W' is the total uniformly distributed load, and L' is the length of the beam. The length of the simple beam in that case is evidently the distance along x between lines of contra-flexure, viz, $L' = \frac{2}{3} a \sqrt{3} = \frac{1}{3} L \sqrt{3}$, where L is the edge of the panel, and the total load at most will be that already proven to be carried by the side belt viz, $q \, b$ per unit of length, or a total for a span L' of $W' = q \, b \, L' = \frac{2}{3} q \, a \, b \, \sqrt{3} = \frac{1}{6} W \sqrt{3}$ where $W = 4 \, qab$ is the total load on the panel, hence

$$M' = A_1 \, j d f_s = \frac{W \, L}{48} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (36a)$$

It thus appears that according to simple beam theory the true stress, or the cross section of steel required in the belt, is four times that obtained by slab theory as shown by (34a). Since (34a) is in good accord with experimental tests, this comparison justifies the statements made near the beginning of this paper respecting the inapplicability of beam theory to the computation of slab design.

The floor of the St. Paul Bread Co. Building, previously mentioned,is a rough slab 6″ thick, and has panels 16′ x 15′, with ten 3/8″ round rod reinforcement in each belt, built for a design load of 100 pounds per square foot; constructed in winter and frozen, the final test was not made until the end of its first summer after unusually complete curing, such as might make the value of K given in (6) not entirely applicable. In one long side belt, extensometer measurements were made at the mid span on three rods, (1) a middle rod, (2) an intermediate rod and (3) an outside rod of the belt, with the following values of f_s in pounds per square inch for the given live load in pounds per square foot:

Live Loads	108.4#	316.8#	416.8#
$f_s = E \, e_1$ (1)	7650	15000	17940
" (2)	7080	14190	16470
" (3)	7320	13920	17160
Average	7350	14370	17200
f_s by (34)	5000	14440	19000

The observed results are seen to be in excellent agreement with those computed from (34) for the heavier loads, while any disagreement is on the safe side. Agreement is not expected for light loads.

The accuracy and applicability of (34) and preceeding formulas is dependent on the fixity of the lines of contra-flexure (24) which were previously stated to be practically immovable because of the sudden large change of the moment of resistance of the slab at those lines. That fact may be put in a more definite and convincing form than has been done so far. Consider for a moment that form of continuous cantilever bridge where there are joints between the cantilevers over the successive piers (which are in the form of a letter T) and the intermediate short spans which connect the extremities of the cantilevers. At such joints the resisting moments vanish, and they form in a sense artificially fixed points of contra-flexure. The same thing approximately occurs at the edge of the mushroom, because there the reinforcing steel rapidly dips down from a level above the neutral plane to one below it, and the sign of the moment of resistance changes thru zero at that edge.

Furthermore, it may be proper to state in this connection that the foregoing theory has been developed in consonance with the general principles of elasticity, and that somewhat different conditions and relations are thought to exist when the steel at the middle of the side belts reaches its yield point, as it does in advance of the rest of the reinforcement. As the yield point is reached equations (34) no longer hold; for, as will be seen more clearly later, the single belt of reinforcing steel, which crosses the circumference of an approximately circular area of radius $L/2$ about the center of each column, will everywhere reach the yield point at practically the same instant, and if loaded much beyond this will develop a continuous line of weakness there. The equations that hold in this case will be approximately those due to the actual cross section A_1 of the belt, in place of (34), which contain the effective cross section, viz:

$$\left.\begin{aligned} f_s &= \frac{3\, q\, a^2\, b}{4 \times 12 \times 0.91\, d_1\, A_1} = \frac{W\, L}{117\, d_1\, A_1} \\ M_1 &= A_1\, j\, d_1 f_s = \frac{W\, L}{128} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (37)$$

which may be regarded as expressing the relations that exist at the limit of the elastic strength of the slab and the beginning of permanent deformation, tho not necessarily of collapse.

The percentage of reinforcement in standard mushroom slabs is small enough to make their elastic properties depend upon the resistance of the steel. The stresses in the concrete may then be be computed from those in the steel, but many uncertainties attend any such computation. It is usage, fixed by the ordinances of the building codes of most cities to require the application of the so called "straight line theory" in such computations, not because that will give results which will be verified by extensometer tests of compressions in the concrete, for it will not, but because it is definite and on the side of safety. Furthermore it is usually prescribed that the ratio of the modulus of elasticity of steel divided by that of concrete shall be assumed to be 15, where the moduli are unknown by actual test of the materials. This is usually far from a correct value. The consequence is that the results of computation of the stresses in concrete are highly artificial in character, and should not be expected to be in agreement with extensometer tests. With this understanding the computed stress in the concrete at the middle of the side belt will be found as follows:

Let *id* be the distance from the center of the steel to the neutral plane. (It happens to be more convenient in this investigation to use this distance *id* here and in our previous formulas than to introduce the distance from the neutral axis of the slab to the compressed surface of the concrete, as is done by many writers, under the designation $k\ d$. These quantities are so related that $i + k = 1$).

Then, as is well known from the geometry of the flexure of reinforced concrete beams, in case tension of concrete is disregarded,

$$f_c = \frac{k}{i} \cdot \frac{E_c}{E_s} f_s \quad \text{.............................} (38)$$

where the subscripts c and s refer to concrete and to steel respectively.

Applying (38) to the greatest computed stress $f_s = 19000$ in the St. Paul Bread Co's Building, gives a computed stress $f_c = 492$; but taking the greatest observed stress $f_s = 17940$ gives $f_c = 465$ lbs. per sq. inch, as the greatest computed compressive stress in the concrete at the middle of the side belt, if $i = 0.72$.

The tensile stress across the middle of the side belt at the extreme fiber of its upper surface is fixed by the curvature of the vertical sections of the slab in planes that cut the side belt at right angles. As stated previously all such planes make cross sections of the side belt that are identical in shape. That is a consequence of the conclusion reached previously, that all the rods in the side belt are subjected to equal tensions. The curvature of these sections is controlled by the stiffness of the mushroom heads, which is so great as to make the curvature very small. No considerable tensile cross stresses are consequently to be apprehended; but in case the stiffness of the head were to be decreased, stresses might arise such as to develop longitudinal cracks over the middle rod of the side belts.

10. In order to obtain practical formulas for the deflections and stresses in the steel thruout the areas at and near the tops of the columns where all the belts cross each other, and lying between lines of contra-flexure, we shall have recourse to (30) and (31) which are here superimposed on each other, and combined together. Were there no steel here in addition to the side belts, that superposition could be correctly effected by writing a value of z whose numerator would be the sum of the numerators of (30) and (31), for that would superpose the loads of the two side belts, and thus place the total required loading upon this area as previously explained; and then by writing for a denominator the sum of the denominators of (30) and (31), for that would superpose and combine the resistance of all the steel in both belts. But such a result would leave out of account the reinforcement arising from the diagonal rods, and the radial and ring rods, which should also be reckoned in as furnishing part of the resistance.

Supposing this additional steel to be distributed in this area in the same manner as is that of the side belts, a supposition which is very close to the fact, we may write

$$z = \frac{(1 - K^2)\, q\, (a + b)}{48\, E\, i\, j\, d^2\, \Sigma A} [(a^2 - x^2)^2 + (b^2 - y^2)^2] \quad . . (39)$$

in which ΣA is the cross section of the total reinforcement in this area regarded as forming a uniform sheet, i and j stand for mean values that have to be determined by the percentage of reinforcement and its position, while d is the mean distance of the center of action of the steel above the lower compressed surface of the concrete at the point xy.

We may conservatively assume in the standard mushroom that the center of action of the steel is at the center of the third layer of rods from the top, as will appear more clearly later. This defines d, which we shall consequently designate by d_3.

It remains therefore to estimate the amount of the total reinforcement ΣA, and then find mean values of i and j.

In case of reinforcing rods which are all of them continuous over a head without laps, the percentage of reinforcement falls only slightly below 4 times that at the middle of a side belt; but on the other hand were none of them continuous for more than one panel and each lap reached beyond the center of the column to the edge of the mushroom, the percentage of reinforcement would not be less than 7 times that at the middle of a side belt, and to this must be added that due to the steel in the radial and ring rods. Thus the percentage of reinforcement here may be varied not only by reason of the larger or smaller number of laps over each mushroom, but by reason of the length of the laps, from perhaps 3.75 to 7 times that at the middle of a side belt. For standard mushroom construction using long rods, it may be taken conservatively as a 4.25 times that at the middle of a side belt.

It is impossible to make an estimate that will be accurate for all cases, but commonly the 8 radial rods of a 20′ x 20′ panel are equivalent in amount to a single $1\frac{1}{8}''$ round rod, or a 1″ square bar circumscribing the area under consideration, that is to 4 square inches of additional reinforcement to be distributed in the width of a single side belt.

The two rings rod, of which the larger is commonly 7/8″ round, and the smaller 5/8″ round, may be taken to increase the reinforcement of this area by at least one square inch of cross section, giving all told some five square inches of cross section additional, equivalent forty-five 3/8″ round rods, or twenty-one 1/2″ rods. It thus appears that the increased reinforcement from this source reaches from 2 to 4 times A_1, and we may safely assume a mean total reinforcement over this area of

$$\Sigma A = 7.5\, A_1 \qquad (40)$$

of which the center of action may be pretty accurately stated to be at the middle of the third layer of reinforcement rods from the top.

In the standard design of mushroom floors for warehouses with panels about 20′ x 20′, the mean percentage of reinforcement for a single belt A_1 being about 0.23%, may be taken by (40) for a reinforcement $7.5\, A_1$ as

$7.5 \times 0.23 + = 1.75\%$ The corresponding value of j is 0.83, and we shall have

$$j\, \Sigma\, A = 0.83 \times 7.5\, A_1 = 6A_1 \qquad (41)$$

As previously stated, these equations (containing estimated mean numerical values) are given as a specimen computation for the purpose of making comparisons. In actual design, computations like these should be made which introduce the exact values appearing in the design under consideration.

We now derive from (39) and (40) by the help of (23) the following equations for this area where the belts all cross:—

$$\left.\begin{aligned} f_s &= Ee_1 = \pm Eid_3 \frac{\delta^2 z}{\delta x^2} = \frac{\pm(1 - K^2)\, q\,(a + b)}{90\, j\, d_3\, A_1}\,(3x^2 - a^2) \\ M_1 &= 7.5\, A_1\, j\, d_3\, f_s = \frac{(1 - K^2)}{12}\, q\,(a + b)\,(3x^2 - a^2) \end{aligned}\right\} \quad ..(42)$$

in which j and d_3 are less than in (33) and (34), as has been stated previously.

Apply (42) to find the stresses at the edge of the column cap on the long side L_1.

Let $B = 2x$ be the shortest distance along the middle of the side belt parallel to x between the edges of the caps of two adjacent columns, and introduce the values $j = 0.83$, $K = 0.5$, and $W = 4qab$, then:

$$\left.\begin{aligned} f_s &= \frac{W\, L_1\,(L_1 + L_2)\,(3B^2/L_1^2 - 1)}{800\, d_3\, A_1\, L_2} \\ M_1 &= 7.5\, A_1\, j\, d_3\, f_s = \frac{W\, L_1\,(L_1 + L_2)\,(3B^2/L_1^2 - 1)}{128\, L_2} \end{aligned}\right\} \quad .(43)$$

in which $7.5\,A_1$ is the effective cross section of the steel in this area, and M_1 is the true resisting moment of the total steel derived from the elongation, and d_3 is as stated after (39).

It has been found that the foregoing theoretical expressions which neglect to take account of the local stresses induced in the slab just outside the edge of the cap by reason of the rigidity of the cap itself are incapable of giving results in accordance with experimental data.

Equation (43) implicitly assumes that the slab while supported on the cap is nevertheless so separate and independent of it as to have no great rigidity over the cap than elsewhere and is consequently unaffected by the mass of the cap. But this assumption is not in accordance with the fact because the cap is integral with the slab and forms a nearly rigid boss on the slab which largely prevents bending and stretching over the cap so that inside the edge of the cap much smaller extensions occur in the belt rods than would otherwise occur, and less than do occur in parallel rods outside the cap. But the total extensions of the several rods of a belt from one line of inflection to another must accompany one another, and those rods that cross the cap and have their extensions mostly prevented inside the cap must suffer correspondingly greater extensions in the remaining parts of their lengths by way of compensation for this loss. This effect will be most accentuated just where the rods cross the edges of the cap and will cause abnormal local extensions especially in the rods along the middle of the belts that cross the edge of the cap nearly perpendicularly. It is just here that the greatest stresses are observed as might be expected. The stresses in rods tangent to the cap are less than these, and in the other rods which are nearer the edges of the belts the stresses are smaller also, partly by reason of the low level at which they are usually placed. These circumstances all conspire to accentuate the stresses of the rods at the edge of the cap.

It is not possible to make an exact mathematical analysis of the resulting abnormal stresses. But the form of (43) has suggested an expression which will enable us to fix a limiting value to those stresses with considerable assurance, since the greatest steel stresses which have been observed around the cap in extensometer tests do not exceed the values thus computed.

We feel sure that ample allowance for this increased stress is included in the following amended expression for the greatest stresses in the side belts at the edge of the cap:

$$\left.\begin{aligned} f_s &= \frac{W L_1 (L_1 + L_2)(3L_1^2/B^2 - 1)}{800\, d_3 A_1 L_2} \\ M_1 = 7.5\, A_1\, jd_3\, f_s &= \frac{W L_1 (L_1 + L_2)(3L_1^2/B^2 - 1)}{128\, L_2} \end{aligned}\right\} \quad \ldots\ldots\ldots (44)$$

This equation gives the same values of f_s and M_1 as (43) at the column center, i. e. when $B = L_1$. But at other points (43) and (44) diverge from each other, for at any points between the column center and the line of inflection (43) gives smaller values of f_s and M_1 than at the column center, but (44) gives larger values for the stresses, and thus makes allowance for abnormal stresses. Moreover (44) makes the stresses greater the smaller B is and the larger the cap. It is evident that these abnormal stresses should increase with the size of the cap which causes them. But it should be stated again and emphasized that the values thus obtained are an outside limit, and that (44) may give values considerably in excess of observed stresses.

(44) consequently should not be used to compute the amount of reinforcement required in any proposed design. But with a given design it is possible to say with considerable confidence that the greatest stress will not exceed those computed by (44).

It should be further stated in this connection that abnormal stresses in a few rods at the center of a belt at the edge of the cap is a local phenomenon of no serious import for the stability of the slab because such stresses if sufficient will cause a slight yielding at the point which will bring the other parallel slab rods into play to assist them.

In the same way as (44) has been obtained from (43) and (43) from (42), a similar expression may be obtained for the abnormal stress in the rods of diagonal belts at the edge of the cap,* but the numerical values thus obtained differ little from those resulting from (44). This expression has therefore been omitted as unimportant.

Tests and long experience show that much higher compressive stresses may be safely permitted in the concrete around column caps where the compression is in two directions at once, circumferential and radial, than in ordinary direct one way compression. Aside from this additional resistance to these converging compressions the Joint Committee has recognized a greater capacity for resistance to compression at supports than elsewhere in recommending that in general "the extreme fiber stress of a beam may be allowed to reach 32.5 percent of the compressive strength," while "adjacent to the support of continuous beams stresses 15 percent higher may be used."

If we use no higher stresses around the cap than those allowed by this recommendation for beams we obtain for a concrete whose compressive strength is taken as 2000 lbs. per square inch a working stress of 2000 x .325 x 1.15 = 747.5 lbs., or 822 lbs. for a concrete having a strength of 2200 lbs.

Now various published and unpublished extensometer tests of slabs show that the unit deformation in the concrete immediately adjacent to the edge of the cap designated by e_c has a mean value of not more than seven tenths (0.7) of e_s the unit deformation of the steel in the top belt immediately above it, this ratio varying under the heavier loads between 0.5 and 0.85. Hence the working steel stress f_s corresponding to a working stress in the concrete of f_c = 747.5 lbs. per square inch is

* See Proc. Am. Soc. C. E. Jan. 1914, p. 77.

$$f_s = \frac{E_s e_s}{E_c e_c} f_c = 15 \times 747.5/0.7 = 16000 \ldots \quad \ldots\ldots\ldots\ldots (45)$$

lbs. per square inch.

It thus appears that ordinary working stresses in the steel at the edge of the cap will be accompanied by safe stresses in the concrete. As a matter of experience, it has known that failure arising from compression around the column is practically unknown in concrete that has had opportunity to become reasonably hard.

It will be noticed that in order to make f_s and f_c as small as possible in this area d_3 must be made as large as possible, *i. e.*, the steel at the edge of the cap must be raised as near the top of the slab as possible. Neglect of this is to invite failure and weakness such as has overtaken certain imitators of the mushroom system.

A final remark is here in place respecting the values of j and d_3 in this area. The stresses f_s and f_c diminish very rapidly towards the lines of contra-flexure, where they vanish, and the fact that the steel also rapidly increases its distance from the top of the slab at the same time might be regarded at first thought as requiring some modification of the assumptions we have made as to the values of j and d_3, which are approximately correct at the edge of the cap where the steel is placed as near the top surface as due covering will permit. But the fact is this: the only consideration of importance is the one respecting the position of the steel in that part of this area where the moments and stresses are large. The effect of the position of the steel near the lines of contra-flexure is negligible, and the fact that the amount of reinforcement may be somewhat smaller near these lines than elsewhere may also be neglected, so that the mean effective reinforcement previously estimated is likely to be an underestimate rather than the reverse. Further, the fact that the slab is practically clamped horizontally either at the edge of the cap or the edge of the superposed column, instead of at its center as assumed in our formulas, renders the results given thus far slightly too large.

Good average values of the size of steel used in the standard mushroom system of medium span would make the radial rods 9/8″ round, the outer ring rod 7/8″ round, the inner ring rod 5/8″ and the belt rods 3/8″ round. The importance of having the belt rods small is that for a given thickness of slab the smaller these rods are the larger is d in both (34) and (43) and consequently the smaller is f_s and A_1.

11. In attempting to consider the stresses in the diagonal rods of the central rectangle between the side belts of a panel, it will be

noticed, as stated before, that no true bending moments are propogated across the vertical planes or lines of contra-flexure (24) which bound it, and since the vertical shearing stresses at these lines are uniformly distributed along them, as already shown, (28), there are no true twisting moments in these planes. The curvatures of this rectangle will consequently depend upon its own loading and the resistance of its own moment of inertia, regarded as uniformly distributed, independently of that of other parts of the slab.

Hence (21) may be correctly applied to this area, regardless of the values which I (and q) may assume elsewhere, provided only that the values of I in other areas may be assumed to have constant values thruout those areas, and, further, that those areas are symmetrically disposed, so that all central rectangles have one and the same given value of I thruout, all side belts also have one given value of I, and the mushroom heads have a given value also, each of these three sorts of areas being independent. The truth of this proposition has been heretofore tacitly assumed in applying (21) to these latter areas as has been done.

It will be seen however, that the values of z obtained from such diverse equations express deflections of any point xy on the supposition that all the areas considered have the same value of I; but these separate equations, each with its own peculiar value of I, can be used separately to find the difference of level $z_1 - z_2$ between any two points $x_1\,y_1$ and $x_2\,y_2$ which lie in an area where I may be regarded as constant. We shall return to this point when we come to the derivation of practical deflection formulas.

For convenience in computing stresses in the rods of the diagonal belt, let the direction of the coordinates be changed so that in square panels they will lie along the diagonals which make angles of 45° with those used thus far. In (21) let

$$x = \tfrac{1}{2}\sqrt{2}(x' + y'), \qquad y = \tfrac{1}{2}\sqrt{2}(x' - y'), \text{ then}$$

$$z = \frac{(1 - K^2)\, q\, g}{24\, E\, i\, j\, d^2 \Sigma A}\left[a^4 - a^2(x'^2 + y'^2) + x'^2 y'^2 + \tfrac{1}{4}(x'^2 + y'^2)^2\right] \ldots (47)$$

in which the panel is square and the axes of x' and y' lie along its diagonals, while the value of $\Sigma A / g$ is the effective cross section per unit of width of all the reinforcement in this area regarded as a single uniform sheet of metal, and $g = 7/8\, a$, is the width of a diagonal belt, and is equal to the diameter of the mushroom head. In rectangular panels $g = 7/16\, (a + b)$.

From (34) we have

$$\frac{\delta z}{\delta x'} = \frac{(1 - K^2)\, q\, g}{24\, E\, i\, j\, d^2\, \Sigma A}\, [\, x'\, (x'^2 + 3y'^2) - 2a^2 x'\,] \quad \ldots\ldots\ldots\ldots\ldots (48)$$

$$e_1' = e_2' = -\, i\, d\, \frac{\delta^2 z}{\delta x'^2} = -\, id\, \frac{\delta^2 z}{\delta y'^2} = \frac{(1 - K^2)\, q\, g}{24\, E\, j\, d\, \Sigma A}\, [2a^2 - 3(x'^2 + y'^2] \quad (49)$$

and $$\frac{\delta^2 z}{\delta x'\, \delta y'} = \frac{(1 - K^2)\, q\, g\, x'\, y'}{4\, E\, i\, j\, d^2\, \Sigma A} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (50)$$

These expressions satisfy (20) as they should, for (20) is independent of the directions of the rectangular axes x and y.

From (49) it appears that $e_1' = 0 = f_s$, on the circumference of the circle $x'^2 + y'^2 = \frac{2}{3}\, a^2$, which passes thru the points where the lines of contra-flexure intersect.

By (19), which holds for any rectangular axes, and by (50), we find

$$\mathrm{n}' = \tfrac{1}{4}(1 - K)\, q\, x'\, y' \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)'$$

From $(26)'$ it appears that n sections by all vertical planes parallel to the diagonals, the twisting increases uniformly with the distance from the diagonal.

Hence by (9) we have

$$\left.\begin{aligned} -s_1' &= \left(\frac{\delta \mathrm{m}_1'}{\delta x'} + \frac{\delta \mathrm{n}'}{\delta y'}\right) = \tfrac{1}{2} q x' \\ -s_2' &= \left(\frac{\delta \mathrm{m}_2'}{\delta y'} + \frac{\delta \mathrm{n}'}{\delta x'}\right) = \tfrac{1}{2} q y' \end{aligned}\right\} \quad \ldots\ldots (28)'$$

It thus appears that the same law holds for vertical shearing stresses on planes parallel to the diagonals, as holds in (28) for planes parallel to the edges of the panel.

In standard mushroom designs the edges of the diagonal belts intersect on or very near to the edges of the side belts. That makes the middle half of the central square to be covered by double belting, and the remainder of it by single belting, so that $\Sigma A = 1.5 A_2$ or perhaps $1.6\ A_2$, and the mean value of A, the reinforcement per unit of width of slab here, is to be found by dividing this by the width of a belt, which is $7/8\ a$. We should then find $A = 1.5\ A_2 / 7/8\ a = 1.7\ A_2/a$. But this mean value of A is not its mean effective value for this area, because the reinforcement is so disposed as to furnish the larger values of I in the central diamond just where the largest true applied moments and stresses occur. The mean value of A in the central diamond is $2A_2/7/8\,a = 2.3A_2/a$. The mean effective value lies between these two extremes, probably nearer the latter than the former. A similar question was discussed in connection with (40) and (41). We shall assume as the mean effective reinforcement in this central rectangle,

$$A = 2A_2/a, \text{ and } I = 2A_2\, i\, j\, d_2^2/a$$

or in case of rectangular panels

$$I = 4A_2\, i\, j\, d_2^2/(a + b) \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (51)$$

In case of rectangular panels the term $2a^2$ in (49) should be replaced by $a^2 + b^2$ as a mean value to make it depend the dimensions of the panel symmetrically, as it must. Making these substitutions in (49) we have at $x' = 0 = y'$ the center of the panel.

$$\left.\begin{aligned} f_s &= Ee' = \frac{W\,(L_1 + L_2)\,(L_1^2 + L_2^2)}{1024\, L_1\, L_2\, A_2\, j\, d_2} = \frac{C_1\, W\, L_1}{256\, A_2\, j\, d_2} \\ M_1' &= 2A_2\, j\, d_2\, f_s = \frac{W(L_1 + L_2)\,(L_1^2 + L_2^2)}{512\, L_1\, L_2} = \frac{C_1\, W\, L_1}{128} \end{aligned}\right\} \ldots\ldots (52)$$

where $C_1 = \frac{1}{4}(L_1/L_2 + 1)\,(1 + L_2^2/L_1^2)$. Take $j = 0.89$.

If $1 > L_2/L_1 > 0.75$ then $1 < C_1 < 1.042$, hence C_1 varies less than 5% while L_2/L_1 varies by 25% between its extreme permissible values. C_1 may ordinarily be taken as unity, or may be found with sufficient precision by interpolation between the values just given.

The steps by which these equations (52) were deduced may not seem conclusive, since they are not rigorous. They need be only good, working approximations for the purpose for which they will be here used, viz, to show that the stresses at the center of the panel are less than those at the mid span of the side belts in case $A_1 = A_2$.

The value of d_2 in (52) is less than d_1 in (34), but always more than 90% of it. We may define d_2 as the vertical distance from the center of the second and upper of the two diagonal belts to the top surface of the concrete. We may assume $d_2 = 0.9d_1$ and $j = 0.89$ in (52), and then we may compare these stresses for a square panel as follows:—

$$f_s' = \frac{175}{205} f_s \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (53)$$

where f_s' refers to the center of the panel. Even were the smaller value for the mean reinforcement, $1.7\,A_2/a$, used in deriving (52) and (53), the stress given by these equations would not exceed that given by (34). The compressive stress f_c in the concrete at the center of the panel may readily reach a dangerous value in case the forms are removed too soon. It should therefore be carefully considered in each case. Here, we have an approximate value of $i = 2/3$ and (38) then becomes $f_c = f_s/30$ with no possible assistance from steel reinforcement since that is all on the bottom of the slab. An estimate that the elastic stress in the steel at the center of the panel does not much exceed 80% of that at the middle of the side belt cannot be far from the truth.

While this is undoubtedly the fact, it will appear on further consideration that local stresses and strains which exist at incipient failure are of such magnitude as to make the weakest points of the diagonal belts to lie ultimately not at the center, but, instead, just outside the diamond where they cross each other.

Take the standard case where the central diamond reaches just across to the side belts. For square panels imagine a circle to be drawn concentric with each column of radius $L/2$. Any circle at a column will be tangent to the edges of four diagonal belts across the tops of the four columns adjacent to it, and then the octagon circumscribing it, whose sides cut at right angles all the belts that cross this column head, intersects but a single belt of rods as every point of its perimeter. It is evident that, so far as reinforcement is concerned, such a line or section cuts less steel per unit of perimeter than any other regular figure concentric with the column and that the reinforcement is entirely symmetrically disposed about the column center, so that in case of equal diagonal and side belts, it would be impossible from their geometry to distinguish the one from the other by anything inside the octagon. That fact would make it inherently probably that the stresses and strains of the rods where they cross any one side of this octagon should be approximately the same ultimately as in those that cross any other side, whether they be rods in a diagonal belt or in a side belt. And what will be attempted to be shown immediately is that ultimately the stresses and strains in these several belts approach equality. If that should be established, it will follow from the conclusion already reached as to the excess of the stresses and strains of the side belt over those at the center of the panel, that ultimately those at the edges of the ocatgon exceed those in the same rods at the center of the panel.

The qualification implied above in affirming that this is what will occur ultimately, is for the purpose of conveying the idea that this is the approximate distribution of stresses and strains which will take place when the slab is sufficiently loaded to bring the steel at the middle of the side belt to the yield point. At less stress than this there is so much lag in the distribution of the effect of loading that it penetrates to the various parts of the slab unequally.

Taking up now the deferred proof that the diagonal rods where they cross the edge of the octagon are subject ultimately to the same local stresses and strains as the direct rods of the side belts; note that these diagonal rods lie in a triangular area between two side belts, which latter experience equal elongations e_1 in directions at right angles to each other. The edges of the triangle in which the single layer of diagonal rods lie are continuous with the side belts and necessarily experience the same elongations, which are propogated from the side belts into the triangle by the agency of horizontal shears on its edges. Such equal elongations at right angles imply the same elongation in every direction in the triangle, as appears from the fundamental properties of equal principal stresses and strains. Hence we have the same elongations along the diagonal rods as along the rods of the side belts at the edges. The existence of an ultimate stress and strain in the diagonal belt equal to that in the side belt would require that the cross sections A_2 and A_1 of the two belts should be equal, altho so far as the elastic value of f_s at the center of the panel is concerned A_2 might be less than A_1, as has been already shown in (52) and (53). The relationships of stress, load, etc., for this ultimate condition, have been already given in (37).

Besides the stresses and strains in the diagonal belts, just investigated, those due to the local stretching (arising from the deflections themselves) exert their greatest effect on the rods of the diagonal and side belts just in the region of the line of weakest section, and partly because of that fact. While these local stresses may not exceed 10% in addition to those already present, their existence should prevent any thought of taking ΣA larger than A_1 in (37) when deriving the ultimate stresses at the yield point. Similar results may be formulated to cover cases where g is greater or less than $7/16\ L$.

It is perhaps desirable at this point to consider a little more at length the matter of local stretching in a slab. It is impossible for a continuous flat floor slab to undergo the deflections which we are treating, consisting of convexities, concavities, etc., without local stretching to allow this to occur. A floor slab of many panels does not undergo any change of its total linear dimensions which would account for these corrugations. A continuous beam under flexure would have its extremities drawn toward each other. But not so to any such extent with a slab. Such contractions are resisted by local circumferential strains which result in true stresses. An investigation of such stresses leads to the conclusion just stated that in general they cannot exceed 10% of the ordinary stresses due to slab bending when they are left out of the consideration. For this reason a single panel alone will not function precisely in the same way as a panel in a floor of many panels.

12. Actual deflections are distances which any given points of a slab sink down by reason of the application of a given load, and their theoretical values are to be computed by help of the formulas which have been developed for z in the various areas into which the panel has been divided.

We shall now make a slight modification in our definition of the level of the origin of coordinates, and shall take it at the upper or lower plane surface of the flat slab before flexure, in which surface the axes of x and y are assumed to lie. It is of no consequence whether it be the upper or the lower surface which is assumed, the equations will be the same in either case. The reason for this new definition of the position of the origin is this: Each kind of partial area into which the slab has been supposed to be subdivided has its neutral surface at a different depth in the slab, and so it does not furnish a single suitable level from which to reckon deflections, as does the upper or lower surface of the slab. None of the equations which have been derived in this paper will undergo any modification by reason of this change of definition. It has been assumed that each kind of area has a separate value of I which remains constant thruout, so that the neutral surfaces of different areas do not join at their edges. As previously explained this is of no consequence mechanically by reason of the zero true moments that exist at these edges. The modification just introduced avoids the geometrical perplexities arising from this discontinuity of neutral surfaces.

Deflections in the side belt area between the lines of contra-flexure (24) are to be found from (30), or (31), and (32). To find the deflection or difference of level in the mid side belt between $x = 0$, $y = b$, and $x = \frac{1}{3} a\sqrt{3}$, $y = b$, substitute these values in (30), take $i = 0.71$, $j = 0.91$, $K = 0.5$ and subtract the value z at the second point from that at the first point, which gives the following value of the deflection of the one point below the other:

$$\triangle z_1 = \frac{W L_1^3}{10.7 \text{ x } 10^{10}\, d_1^2 A^1} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (54)$$

in which d_1 is the vertical distance from the center of the single belt of rods at the mid span of the side belt to the effective top of the slab, considering the strip fill or other concrete finish at its effective value.

In the same manner take the difference of level in the central rectangle bounded by the lines of contraflexure between the center point at $x=0$, $y=0$ and the corner $x=\frac{1}{3}a\sqrt{3}$, $y=\frac{1}{3}b\sqrt{3}$ by using (21) and (51) and introducing the values $i = 2/3$, $j = 0.89$, etc., and $C_2 = 1/4(L_1/L_2+1)(1+L_2^4/L_1^4)$, then:

$$\triangle z_2 = \frac{C_2 W L_1^3}{6.56 \times 10^{10} d_2^2 A_2} \quad \ldots\ldots (55)$$

in which A_2 is the cross section of one diagonal belt and d_2 is the vertical distance from the center of the upper or second diagonal belt to the effective upper surface of the panel at its center.

On evaluating C_2 above, we find

when $1 > L_2/L_1 > 0.75$

then $1 > C_2 > 0.77$

hence we may with sufficient accuracy for practical purposes assume

$$C_2 = L_2/L_1 \quad \ldots\ldots (56)$$

Deflections in the mushroom area between lines of contraflexure (24) are to be derived from (39) (40) and (41) by introducing $i = \frac{1}{2}$, $j = 0.83$, $K = 0.5$ and $\Sigma A = 7.5 A_1$. Assuming the diameter of the cap to be $0.2L_1$ we have, at its edge where $x = 0.8a$, $y = b$, from (39)

$$z = \frac{W L_1^3 (L_1/L_2+1)}{19.1 \times 10^{10} d_3^2 A_1}\left(\frac{36}{100}\right)^2 \quad \ldots\ldots (57)$$

The value of z at the edge of the mushroom area, where $x = \frac{1}{3}a\sqrt{3}$, $y = b$, is to be obtained from (57) by replacing the last factor by $4/9$; and the deflection between the edge of the cap and the edge of the mushroom obtained by taking the difference of these quantities is as follows:

$$\triangle z_3 = \frac{W L_1^3 (L_1/L_2+1)}{60 \times 10^{10} d_3^2 A_1} \quad \ldots\ldots (58)$$

in which h_3 is the vertical distance of the center of the third layer of reinforcing rods over the edge of the cap above the bottom surface of the slab.

Similar expressions may be obtained for the values of z and $\triangle z$ on the side parallel to y, where $x = a$ at $y = 0.8b$, and $y = \frac{1}{3} b\sqrt{3}$, by exchanging L_1 and L_2 in (57) and (58).

Take half the sum of (57) and the corresponding values so obtained at $x = a$, $y = 0.8b$, as the value of z at the edge of the cap where it is intersected by the diagonal of the panel, viz.

$$z = \frac{W\,(L_1 + L_2)\,(L_1^4 + L_2^4)}{38.2 \times 10^{10}\, L_1\, L_2\, d_3^2\, A_1} \left(\frac{36}{100}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots (59)$$

and subtract this from the value of z on the diagonal at the corner of the mushroom area where $x = \frac{1}{3} a\sqrt{3}$, $y = \frac{1}{3} b\sqrt{3}$ and we have

$$\triangle z_4 = \frac{C_2\, W\, L_1^3}{12.5 \times 10^{10}\, d_3^2\, A_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (60)$$

as the deflection along the diagonal between the edge of the cap and the intersection of the lines of contraflexure, in which C_2 and h_3 are as previously defined.

Expressions (58) and (60) somewhat exceed the true values of these deflections because the slab has no slope at the edges of the cap as tacitly assumed. A close estimate requires that the denominators be increased by 60 percent on this account, thereby changing the factors 60 to 96 and 12.5 to 20 respectively. These amended values of the deflections will be used hereafter instead of (58) and (60).

$$\left.\begin{aligned} \text{Let}\quad D_1 &= \triangle z_1 + \triangle z_3 \\ \text{and}\quad D_2 &= \triangle z_2 + \triangle z_4 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (61)$$

in which D_1 is the deflection of the mid point of the side belt below the edge of the cap, and D_2 is the deflection of center of the panel below the edge of the cap.

The proportionate deflections of these points are obtained by dividing by the spans, viz: D_1 / L_1 and $D_2 / \sqrt{L_1^2 + L_2^2}$.

13. Estimated proportionate deflections may be obtained from (61) under such circumstances as to convey reliable information respecting what may be reasonably expected. Let h = the total thickness of the slab. The limiting values of the thickness of standard mushroom construction are expressed as follows:

$$L_1 / 20 > h > L_1 / 35, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (62)$$

and assuming that the reinforcing rods are 1/2″ rounds with 1/2″ covering of concrete we shall have from the definitions of d_1, d_2 and d_3, already given

$$h = d_1 + 0.75 = d_2 + 1.25 = d_3 + 1.75 \qquad (63)$$

Substituting these in (62) etc. we have

$$\left.\begin{array}{l} L_1/20 - 0.75 > d_1 > L_1/35 - 0.75 \\ L_1/20 - 1.25 > d_2 > L_1/35 - 1.25 \\ L_1/20 - 1.75 > d_3 > L_1/35 - 1.75 \end{array}\right\} \qquad (64)$$

If it be assumed that we are dealing with medium sized panels about 20′ x 20′ (64), may be written in the form:

$$(1 - 0.062)\ L_1/20 > d_1 > (1 - 0.11)\ L_1/35$$

$$(1 - 0.1)\ L_1/20 > d_2 > (1 - 0.18)\ L_1/35$$

$$(1 - 0.15)\ L_1/20 > d_3 > (1 - 0.255)\ L_1/35$$

or,

$$\left.\begin{array}{l} \dfrac{0.94}{20} > \dfrac{d_1}{L_1} > \dfrac{0.89}{35} \\[2ex] \dfrac{0.90}{20} > \dfrac{d_2}{L_1} > \dfrac{0.82}{35} \\[2ex] \dfrac{0.85}{20} > \dfrac{d_3}{L_1} > \dfrac{0.745}{35} \end{array}\right\} \qquad (65)$$

In (54), (55), (58) and (60) replace $W\ L_1$ by its value given in (34), viz, $175\ d_1\ A_1\ f_s$, and we have

$$\left.\begin{array}{l} \triangle z_1 = \dfrac{L_1^2\ f_s}{6.11 \times 10^8\ d_1} \\[2ex] \triangle z_2 = \dfrac{C_2\ d_1\ L_1^2\ A_1\ f_s}{3.75 \times 10^8\ d_2^2\ A_2} \\[2ex] \triangle z_3 = \dfrac{d_1\ L_1^2\ (L_1/L_2 + 1)\ f_s}{55 \times 10^8\ d_3^2} \\[2ex] \triangle z_4 = \dfrac{C_2\ d_1\ L_1^2\ f_s}{11.4 \times 10^8\ d_3^2} \end{array}\right\} \qquad (66)$$

in which f_s is the greatest stress in the steel, *i. e.*, at the mid side belt, employed here to express deflections instead of expressing them in terms of panel load as was done previously.

Introduce into (66) the numerical values given in (65) which will then express limiting values of deflection for medium spans. For simplicity let $L_1 = L_2$ then:

$$\left.\begin{array}{l} 287 > \dfrac{L_1 f_s}{10^5 \triangle z_1} > 155 \\[2ex] 162 > \dfrac{L_1 f_s}{10^5 \triangle z_2} > 81 \\[2ex] 1056 > \dfrac{L_1 f_s}{10^5 \triangle z_3} > 498 \\[2ex] 440 > \dfrac{L_1 f_s}{10^5 \triangle z_4} > 203 \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (67)$$

By (61) we have the proportionate deflection of the side and diagonal belts as follows:—

$$\left[\frac{1}{287} + \frac{1}{1056}\right]\frac{f_s}{10^5} < \frac{D_1}{L_1} < \left[\frac{1}{155} + \frac{1}{498}\right]\frac{f_s}{10^5},$$

$$\left[\frac{1}{162} + \frac{1}{440}\right]\frac{f_s}{10^5\sqrt{2}} < \frac{D_2}{L_1\sqrt{2}} < \left[\frac{1}{81} + \frac{1}{203}\right]\frac{f_s}{10^5\sqrt{2}}$$

$$\left.\begin{array}{l} \dfrac{f_s}{225 \times 10^5} < \dfrac{D_1}{L_1} < \dfrac{f_s}{118 \times 10^5} \\[2ex] \dfrac{f_s}{167.4 \times 10^5} < \dfrac{D_2}{L_1\sqrt{2}} < \dfrac{f_s}{82 \times 10^5} \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (68)$$

$$\left.\begin{array}{ll} \text{If } f_s = 16000, & \dfrac{1}{1046} < \dfrac{D_2}{L_1\sqrt{2}} < \dfrac{1}{512} \\[2ex] \text{If } f_s = 24000, & \dfrac{1}{697} < \dfrac{D_2}{L_1\sqrt{2}} < \dfrac{1}{341} \\[2ex] \text{If } f_s = 32000, & \dfrac{1}{523} < \dfrac{D_2}{L_1\sqrt{2}} < \dfrac{1}{256} \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (69)$$

Larger spans then $20'$, or smaller steel than $1/2''$ round, or $L_2 < L_1$ will reduce the above values somewhat, while smaller spans or larger steel will increase these values, all of which can in each case be submitted to calculation by the methods here developed.

To recur at this point to the expression for the deflection D_2 in terms of the panel load W by help of (55), (60) and (61)

$$D_2 = \frac{C_2\ W\ L_1^3}{10^{10}\ A_1}\left[\frac{1}{6.56\ d_2^2} + \frac{1}{20\ d_3^2}\right] \ldots\ldots\ldots\ldots (70)$$

By (65) we find

$$\frac{90}{85} < \frac{d_2}{d_3} < \frac{82}{74.5}, \text{ or } 1.1 > \frac{d_2}{d_3} > 1.06$$

and using this inequality to eliminate d_3 from (70) we find after reduction

$$\frac{C_2\ W\ L_1^3}{4.8 \times 10^{10}\ d_2^2\ A_1} < D_2 < \frac{C_2\ W\ L_1^3}{4.7 \times 10^{10}\ d_2^2\ A_1}$$

from which we may write as a mean value

$$D_2 = \frac{C_2\ W\ L_1^3}{4.75 \times 10^{10} d_2^2 A_1} \ldots\ldots\ldots\ldots (71)$$

The empirical deflection formula given on page 29 of Turner's Concrete Steel Construction, when written in these units, is

$$D_2 = \frac{W\ L_1^3}{4.84 \times 10^{10}\ d_2^2\ A_1} \ldots\ldots\ldots\ldots (72)$$

This is identical with (71) when $C_2 = 0.98$, and diverges from it slightly for other admissible values of C_2. The practical agreement of (71) and (72) affords a second confirmation of the theoretical deductions made thus far, and this taken in conjunction with the practical identity of formulas (34) and (35), the theoretical and empirical expressions for the maximum tensile stresses in the reinforcement, furnishes what on the theory of probabilities may be regarded as so strong a probability of the general trustworthiness of the entire theory as to exclude any rational supposition to the contrary.

The various formulas for stresses and for deflections which have been developed in this paper have been obtained under the express proviso that the panel under consideration was assumed to be one of a practically unlimited number of equal panels constituting a continuous slab, all of which are loaded uniformly and equally. The question at once arises as to the amount and kind of deviations from these formulas which will occur by reason either of discontinuity of slab or loading, such as occurs at the outside panels of a slab or at panels surrounded partly or entirely by others not loaded. The answer to this question depends very largely upon the construction of the flat slab itself.

In the standard mushroom construction it has been found that the stresses and deflections of any panel are almost entirely independent of those in surrounding panels. This is due to the fact that the mushroom head is an integral part of the supporting column in such a manner that it is impossible for it to tilt appreciably over the column under the action of any eccentric or unequal loading of panels near it. When single panels have been loaded with test loads, no appreciable deflections have been discoverable in surrounding panels, and no greater stresses and deflections have been discovered than were to be expected in case surrounding panels were loaded also. Future careful investigation of this may reveal measureable effects of this kind, but they must be small.

A like statement cannot be made of other systems of flat slab construction where the reinforcement over the top of the column is not an integral part of the column reinforcement itself. Tests on these systems have shown clearly the effects of the tipping of the part of the slab on the top of the column, and lack of stiffness of head, in the increase of the deflection of the single loaded panel over the deflection to be expected in case of multiple loaded panels, and especially in the disturbance of the equality of the stress in the otherwise equal stresses in the rods of the side belts. Such distrubance, by increasing the stress in part of these rods, would necessitate larger reinforcement in the side belts of such systems than would be required in mushroom slabs. The great stiffness of the mushroom head is also of prime importance in taking care of accidental and unusual strains liable to occur in the removal of forms from under insufficiently cured slabs.

14. In considering the design of the ring rods and radial cantilever rods of the mushroom head, it should be borne in mind that they occupy a position in such close proximity to the level of the neutral surface as to prevent them from being subjected to severe tensile or compressive stresses by reason of the bending of the slab as a whole. Their principal function as slab members is to resist shearing stresses and the bending stresses due to

local bending. Their total longitudinal stresses are too small in comparison to require consideration.

Let a cylindrical surface be imagined to be drawn concentric with a column to intersect the slab, then the total vertical shearing stress which is distributed on the surface of intersection is equal to the total panel load W diminished by the amount of that part of the panel load lying inside the cylinder. If the cylinder be not large, the total shear may be taken as approximately equal to W itself.

It is evident that the smaller the diameter may be that is assumed for this cylinder, the greater will be the intensity of the vertical shear on its surface and that for two reasons: First, because the total load thus carried to the column will be greater the smaller the diameter, and second because the surface over which the total shear will be distributed decreases with its diameter.

The result of this is that the dangerous section for shear is the cylindrical surface at the edge of the cap. For cylinders smaller than this the increased vertical thickness of the cap diminishes the intensity of the shear. We proceed therefore to consider the manner in which the total vertical shearing stress of approximately W in amount is distributed in the material of the cylindrical surface at the edge of the cap.

In a beam or slab the horizontal shearing stresses due to bending reach a maximum at the neutral surface. It is a fundamental condition of equilibrium that shearing stresses on planes at right angles shall be equal, and it is this condition that determines the distribution of the vertical shears, which are at right angles to the horizontal shears resulting from bending the slab as a whole. From this we have the well known fact that the vertical shear varies from zero at the upper and lower surfaces to a maximum at the neutral surface, and this is necessarily the manner in which the total shear is distributed at the edge of the cap. The top belt of rods will be subjected to comparatively small shearing stresses, and successive layers of rods will be under larger and larger shearing stresses by reason of their greater nearness to the neutral surface, while the total shear borne by the radial rods near the neutral surface will be much larger than that upon the others. The shearing stress in the concrete will need to be considered also.

It is to be noticed that all the steel of the belts and mushroom head act together without the necessity of supposing large compressive stresses in the concrete to transmit vertical forces, because the belts of reinforcement rest directly upon each other, and these in turn upon the ring rods and radial rods, all in metallic contact with each other, in the mushroom head, and so they transmit and adjust the distribution of stresses within the system to a very large extent independently of the concrete.

We can then safely assign moderate values of the shearing stress to each of the elements that constitute the slab at the edge of the cap, with the assurance that they will each play a part in general accordance with the distribution which has been already explained.

The mushroom is constructed of great strength and stiffness not merely to effect the results which have appeared previously in the course of the investigation but also to ensure the stability of the slab in case of unexpected or accidental stresses due to the too early removal of the forms, before the slab is well cured, at a time when the only load to which it is subjected is due to the weight of the structure itself.

The working load to be assumed in designing the mushroom may be taken as the dead load of a single slab plus the design load, provided sufficiently low values of the shearing stresses be assumed in the cross sections of steel and concrete at the edge of the cap for the support of this working load, as follows:

For slabs having a thickness of $h = L/35$ a mean working shearing stress of 2000 lbs. per square inch at the right cross section of each reinforcing rod which crosses the edge of the cap, a mean shearing stress of 40 lbs. per square inch in the vertical cylindrical section of the concrete at the edge of the cap, and 8000 lbs. per square inch of right cross section of each radial rod.

For slabs having a thickness of $h = L/20$ the intensities just given may be safely increased by 50 per cent for reasons that will be explained later. For slabs of intermediate thickness increase the intensities proportionately.

These values are sufficiently low to enable the structure to support itself before the concrete is very thoroughly cured, and the head so designed will be found after it is well cured to be so proportioned as to carry safely a test load of double the live and dead loads for which it was designed.

In this connection it seems desirable to investigate what takes place in case of overloading and incipient failure of an insufficiently cured slab, or one unduly weakened by thawing of partially frozen concrete. Suppose that under such circumstances a shearing crack were formed extending completely thru the head at the edge of the cap, and we wish to investigate the stresses and behavior of the rods that cross the crack at which shearing deformation has begun to take place. Designate the position of the crack by X.

The total vertical shearing stress on a radial rod at X is the sum of two parts found as follows: First, the vertical reaction at the top of a column is made up of the vertical reaction of the concrete core of the column and the reactions of its vertical reinforcing rods. Call the vertical reaction of one of these rods V_1. The rod is bent over radially and V_1 expresses also the amount of the vertical shear in that rod where it starts out radially from the column. Between this point and X for a distance which measures usually from 9 to 12 inches, the rod experiences the supporting pressure of the concrete in the cap under it to a total amount which we will designate by V_2. The total shear in the radial rod at X will then amount to

$$V = V_1 + V_2 \quad \text{.............................(73)}$$

provided we neglect the weight of that small part of the actual load of the slab which lies directly over this piece of the rod and may be regarded as resting upon it. This portion of the radial rod of length l is a cantilever fixed at one end in the top of the column, and carrying a load V at the other end with a supporting pressure underneath of total amount V_2 whose intensity is greatest at X and gradually decreases along l from X to the fixed end. The rod has a point of contraflexure and zero moment at X. The portion of the rod outside the crack has a fixed point in the slab at the place where it supports the inner ring rod, at a distance from X which should not exceed l as just defined. Similar conditions hold for this length; *i. e.* there will be a total shear in the radial rod at a point just inside the inner ring, rod due to its total shear outside this ring rod and to the vertical loading imparted to it by the ring rod itself. To this must be added the downward pressure of the concrete between the inner ring rod and X. All these, together, constitute the total shear $-V$ at X, in equilibrium with the reaction $+V$ already obtained at that point.

We shall discuss separately the action of V_1 and V_2 upon a radial rod. A load V_1 at the end of a cantilever of length l causes a deflection of amount $z_1 = \frac{1}{3} V_1 l^3 / EI$ ....(74)
in which $I = \pi t^4 / 64$ where t = the thickness of the rod.

$$\text{Also } V_1 = s_1 A \quad , \qquad A = \pi t^2 / 4$$

in which s_1 = the mean shearing stress per square unit of cross section and A is the cross section of the rod. Hence

$$s_1 = 3 z_1 E t^2 / 16 l^3 \quad \text{..........................(75)}$$

which shows that so far as V_1 is concerned, for any given displacement z_1 the shearing stress carried per square unit of rod will be proportional to the square of its diameter, and up to its permissible limiting shearing resistance, each unit of section of such a rod will be effective in proportion to the square of its diameter. For economical construction, this will require the radial rods to be few and large, rather than numerous and small. The bending moment is greatest at the distance l from X and amounts to $V_1\,l$. The stress in the extreme fiber due to the bending moment $V_1\,l$ in the rod is

$$p_1 = V_1\,l\,t / 2I = 8s_1\,l / t \ldots\ldots\ldots\ldots (76)$$

This equation shows that the stress in the extreme fiber is so very large at the fixed end of the rod compared with the shear at X that so far as V_1 is concerned the rod will suffer permanent deformation by bending long before there is any danger of its shearing. V_1 is so large compared with V_2 that this conclusion will not be altered when we come to consider the combined action of V_2.

Incipient failure of this kind will therefore cause distortion and sag without collapse. In case such sag as occurs in this case is detected underneath the head around the cap, the slab should be blocked up at once and the concrete picked out at all parts showing facture. This should then be refilled with a stronger concrete which will set rapidly. Such repair should not weaken the slab.

Whenever the intensity with which a radial rod presses upon the concrete at the edge of a crack at X passes the compressive strength f_c of the concrete, it must begin to yield. At this instant we shall have a pressure of the concrete against the rod which gradually diminishes as we pass along the rod from X to the distance l, where it becomes zero. We shall assume that the pressure diminishes uniformly with this distance. This may not be precisely correct, but cannot be much in error. If the shear V_2 at X is the sole cause of this pressure, then $V_2 = \frac{1}{2}\,t\,l\,f_c$, and $\frac{1}{3}\,V_2\,l = \frac{1}{6}\,t\,l^2\,f_c$ is the bending moment in the rod at the distance l, due to V_2 at X and the pressure distributed along l.

It will be found that these produce a deflection

$$z_2 = 3\,f_c\,l^4 / 20EI = 0.3\,l^3\,V_2 / EI \ldots\ldots\ldots\ldots (77)$$

a unit shear of

$$s_2 = V_2 / A = z_2\,E\,t^2 / 4.8\,l^3 \ldots\ldots\ldots\ldots (78)$$

and a stress on the extreme fiber at a distance l amounting to

$$p_2 = V_2\,t\,l / 3I = 16s_2\,l / t \ldots\ldots\ldots\ldots (79)$$

It thus appears that the equations expressing the action of V_2 are precisely similar to those for V_1, differing only in their numerical coefficients, and consequently all the statements as to the resistance of the radial rods under the action of V_1 hold for the action of V_1 and V_2 together in the case of given initial deformations, $z_1 = z_2$ at X.

While the preceding investigation has, in order to make ideas explicit, ostensibly assumed a crack at X and an initial small shearing deformation at X, the investigation applies equally well to the elastic shearing deformation of the concrete at the dangerous section in which case the total shearing stress will consist of an additional component due to the resistance of the concrete, which however may for additional safety be neglected. If the assumed deformation be confined within limits so small that the concrete is able to endure it without cracking then the preceding investigation may properly be applied to it. It is right here that the thickness of the radial rods is able to render its most effective service, for it appears from (75) and (78) that any permissible intensity of shear may be developed in the radial rods by making them of suitable thickness, even tho the deflection be kept within the elastic limits of the concrete.

As already stated we must not overlook the fact that the major stresses here are those under the head of V_1, which are due to the direct metallic contacts of the steel rods resting one upon another, where large stresses are transmitted and pass independently of the concrete except for the distortions of the steel which meet resistance, and the secondary reactions such as have been treated in a single aspect while investigating the action of V_2.

It is due to this fact that large shearing stresses may be safely borne by the slab at and near the edge of the cap, which the concrete mostly escapes, it merely furnishing some lateral stiffening to the steel. On this principle the outer ring rod should have a cross section not much less than one half that of the radial rods on which it rests. For, this arrangement provides for the transferal to the radial rods of all the shear the ring rod is able to carry, it being in double shear compared with the radial rod it rests on.

It is impossible to determine the cross section of the inner ring rod, with the same definiteness as that of the radial rods, but that is unimportant. Its position has already been fixed as not more than l from the edge of the cap, where l is the distance from the top hoop or collar band of the column to the edge of the cap.

The vertical shearing stresses may be regarded as sufficiently resisted outside the mushroom by the concrete alone. The critical cylindrical surface separating those areas where the shear may be assumed to be safely carried by concrete alone, from those areas where the steel may be relied on to carry as much of the shear as may be required, should evidently be taken somewhat inside the outer ring rod, but just where is of no particular consequence.

The supposition of the existence of a crack at X, either actual or potential, on which our computation of the stresses in the radial rods has been based, is sufficiently satisfactory so far as the rods themselves are concerned; but it seems desirable to consider in more detail the phenomena attending the development of the stresses in the concrete at and near the edge of the cap, especially in soft concrete when the limit of its compressive resistance is reached in this region.

The horizontal compressive resistance of the concrete at the lower surface of the slab is that already treated in (38), and it is our present object to consider how that is to be combined with the vertical supporting pressures under the radial rods, and with the horizontal and vertical shears in the slab due to bending. These latter are greatest in the neutral surface, as has been previously stated, and according the general theory of stresses are equivalent to, and may be replaced by, a compression and a tension in the material respectively at 45° with the vertical (and mutually at right angles) of the same intensity as the shear. It is evident that the combination and resultant of these three compressive stresses would form the dangerous element in the stress, since the single tensile element would be relatively unimportant, and it would find assistance in its resistance from the steel running in a direction thru the concrete such as to afford it substantial support. This direction is that of the straight lines on the surface of a right cone whose vertex is above the center of the column and whose slope is 1 to 1.

Consider now two of the elements of the compression in the concrete around the cap, viz, the horizontal compression which is a maximum at the lower surface and zero at the neutral surface, and that due to shear which is parallel to the sides of a right cone with vertex downward, whose sides have an upward and outward slope of 1 to 1, while its intensity is so distributed that it is zero at the bottom of the slab and greatest at the neutral surface. It appears consequently that the lines of greatest compression in the concrete due to the combination of these two elements of compression would lie in vertical planes on a bowl or saucer-shaped surface that is horizontal at the edge of the cap and inclined at a slope of 45° at the neutral surface; and if the concrete were to crush under these stresses alone, the surface of fracture would have the shape indicated instead of that of the cylindrical surface previously assumed. This change would not, however, materially affect the computations we have made of stresses in steel; it merely serves to fix more definitely the position of the points of contra-flexure of the radial rods.

But there is still one further element or component of the total compression in the concrete to be considered and combined with those just treated in order to arrive at the resultant or total compression. This componenent is that due to the concentrated pressures underneath each of the radial rods. These rods are at some distance apart circumferentially and so do not exert a pressure that is uniformly distributed circumferentially. Any concentrated stress, such as that in the concrete supporting a rod, diffuses itself in the material in such a manner that its intensity rapidly diminishes with the distance from the surface of the rod, in accordance the same law as exists in case of centers of attraction. Since the supporting compression under the rods is vertical, we can imagine the lines of greatest compression in the concrete, when this component is combined with those already mentioned, to lie in vertical planes on a bowl or saucer-shaped surface which has as many indentations or scollops around its edge as there are radial rods, at which indentations the slope of the sides is such more nearly vertical than a slope of 45°. At such parts of the surface the intensity is also more severe, and especially is this the case if the slab is thin so that the concentrated pressure has small opportunity to distribute itself by radiating into a considerable body of material before it reaches the bottom of the slab. It thus comes about that thick slabs are enabled to carry safely larger intensities of shearing stress around the cap than can thin slabs, which is in accordance with and in justification of the statements already made as to permissible shears around the cap.

The resulting surface of fracture due to shear and compression around the cap would be of irregular conical shape starting from the edge of the cap and extending thru the entire thickness of the slab, were this not interfered with in the upper part of the slab by the mat of reinforcing rods, which are so tenacious as to tear to pieces and fracture the upper surface to a considerable distance in all directions whenever any such fracture occurs around the column.

Nevertheless such fracture as here described does not under any ordinary circumstances result in a dangerous collapse of the slab, or one that cannot be repaired without much difficulty, for, the radial rods and the reinforcing rods will at most have suffered some individual deformation by bending and are still far from being broken. This will become evident later where an experimental attempt to load a full-sized slab to failure is described in detail, and full account of the results reached is explained and illustrated.

It is stated on good authority that in experience with many hundreds of buildings constructed on this system, no case of shear failure or even of incipient shear failure or fracture has occurred in a well cured slab near the column and while a few cases of incipient failure have occurred in floors where forms were prematurely removed, no injury or fatality has resulted therefrom to any person.

It appears that the line of weakest section in the cured slab of the standard mushroom type is that discussed previously in obtaining (37) and shown in Fig. 55, page 164. This is brought out later by a test to destruction of a fairly well cured slab. The line of weakest section in a partly cured slab is on the other hand not definitely fixed, but may be and sometimes is, shearing weakness near the column as has been discussed and pointed out. Provision against such weakness or carelessness is a safeguard which, while costing a small amount in the matter of steel, is an insurance against serious accident well worth the investment involved. It is secured by making the radial and ring rods sufficiently stiff and strong.

15. This section will be devoted to a consideration of the mushroom system, and to several more or less similar flat slab systems, in order to comment on the modifications in mechanical action that are produced by the particular modifications of the arrangement of the reinforcement in these systems.

Fig. 53, page 159 represents the section of a standard mushroom head by a vertical plane thru the axis of the column. In this the elbow rods are shown, the vertical portions of which are embedded for such distances as may be necessary in the columns or are themselves column rods. One of these is represented separately at the right side of Fig. 53. They are confined just under the elbow at the top of the column by a steel neck band, and are bent over at the elbow to extend radially into the slab. This bent over portion is formed to scale as to length and slopes in accordance with the size and thickness of the slab in which it is to be used, in such a way that when the ring rods and four layers of slab rods rest upon it and are tied in place, the top of the upper layer will be

0.75 inch below the top of the slab at a distance of the thickness of the slab outside the edge of the cap, and at the same time the extremities of the radial rods will be 0.5 inch above the bottom of the slab. In order to accomplish this, the radial portions of these rods must be nearly horizontal over the cap, and have a suitable slope outside the cap as shown in Fig. 53, page 159.

Fig. 55, page 164, shows the ground plan of the reinforcement of the mushroom slab when the panel is square so that $L_1 = L_2 = 2a = 2b$. In Fig. 55 the diameter of the mushroom head is assumed to be of the extreme size $g = L/2$, a size which would increase the cantilever beyond that in usual practice to an extent not adopted except in the case of very unusual intensity of loading. It will be observed that the areas where the reinforcement consists of a single belt or layer are thereby rendered small, and the slab action due to the mutual lateral action of belts which cross each other exists over nearly the whole slab.

In Fig. 54, the dimensions of the rectangular sides are so taken that $L_1/L_2 = 0.75$, which is assumed to be the limiting or smallest value of that ratio for constructional purposes. Further, the diameter of the mushroom is made as small as will permit the reinforcing belts to cover the entire panel, viz. $g = 7(a + b)/16$. For example if $L_1 = 20$, and $L_2 = 15$, we have $g = 7.65+$. This may be considered to represent standard practice, where the edges of the diagonal belts intersect on the edges of the side belts. This was the case assumed for treatment in deriving the formulas of the preceding investigations. Those formulas could be modified to apply to larger values of g, by taking lines of contra-flexure at the edges of the head nearer the panel center than given by (24), and by taking larger values of the effective cross section of steel than those employed in (32), (40) and (51).

Now it is evident that systems similar to this may differ from it in several ways:—

1st. The design of the frame-work at the top of the column may be different from this without any change in the belts of reinforcing rods. It is hardly possible for any other form of framework to be substituted for this which will exhibit the same rigidity of connection between it and the column as do the elbow rods embedded in the column and bent over radially in the slab so as to make the column and slab integral with each other by means of this common reinforcement. Any reduction of the stiffness of connection between column and frame-work of head results in increased tipping of the head under eccentric loading of the slab.

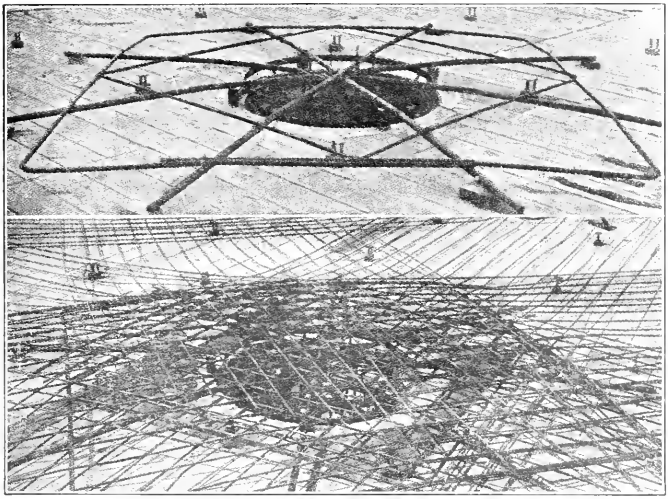


Fig. 56

Eccentric loading is any loading of one panel differently from another. Tipping of the head increases some deflections at the expense of others, and increased stresses in some of the reinforcing rods at the expense of others, and so requires some additional reinforcement. Such a frame-work is illustrated in Fig. 56, which merely rests upon the top of the column without the support of metallic connection with the vertical column rods. It consequently affords less resistance to tipping under eccentric loads than when stiffened by such metallic connection.

2nd. The ground plan of the reinforcing belts may remain unchanged but part only of the belt rods may be carried at the top of the slab over the column head, while the rest of them are carried thru under the head at the bottom of the slab. This modification of design, when a sufficient number of rods go over the head to resist the negative bending moments there, is very uneconomical of steel, because in the case where they all go over the head, it is the fact that altho the mean tension of the steel is not so great as at mid span, nevertheless, by reason of the overlapping of the belts in crossing, the stresses in the rods at the top reach a value not much less than at mid span, and cannot be safely diminished in number. It thus appears that the rods carried thru on the bottom are largely superfluous. Of these two mats of rods at top and bottom, one of them is necessarily in tension and the other in compression. But it is a mistake to use steel to resist compression when concrete can be better used for this purpose. The lower mat is superfluous for this reason.

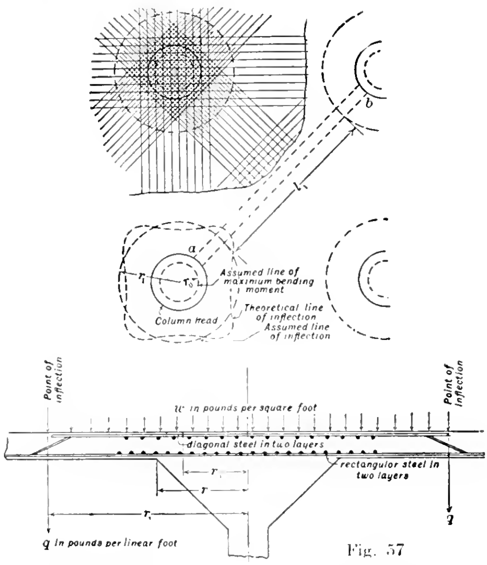


Fig. 57

There is still another and, if possible, more serious objection to this arrangement of rods to form a mat or double layer of rods at the top and at the bottom of the slab near the columns. This is because they are too far removed from each other in the slab for the elongations of the steel in one mat to be resisted by lateral contractions in the other. The reinforcement does not therefore conspire to produce the slab action expressed by Poisson's ratio, which requires that the interacting steel concerned should lie approximately in the same zone or level.

This arrangement is illustrated in Fig. 57, copied from Taylor and Thompson's Concrete Plain and Reinforced, p. 484. In this design the size of the head is small enough to reduce the width of the belts so greatly that not only are the areas where we have a single layer of rods on the plan much enlarged, but we find that nowhere do more than two layers lie in metallic contact with each other, and the areas where even this occurs are limited to one relatively small square over each column, and one of equal size at the middle of each panel. The remaining areas are subject to the law of single rod reinforcement, where we must assume lateral action to be such as greatly to diminish K for the combination, a fact very injurious to the efficiency of the reinforcement. This as has been said, is due partly to the smallness of the head and partly to the separation of the layers between the top and the bottom of the slab.

3rd. Another modification of design without change of ground plan is that where the rods that are carried over the head at the top of the slab are given a sudden steep dip at the line of contraflexure to carry them to the bottom of the slab at that line. This is also illustrated in Fig. 57. Such sudden bends or kinks anywhere in the rods may give rise to very serious fractures because of straightening out under tension, especially when the forms are removed. Such bends give rise to great differences of stress in the extreme fibers of the rods, thus diminishing their resistance also. All sudden bends in rods embedded in concrete should be sedulously avoided as tending very effectively to crack the concrete, whether the rods are part of the belts or in the frame-work of the head, as shown in Fig. 55, in which are many such angles and elbows unsupported except by concrete, and therefore objectionable.

It seems fair to conclude that the cracks shown in the plan of the floor of the Deere & Webber Company Building, Minneapolis, tested by Mr. Arthur R. Lord, and occuring along the edges of some of the loaded panels at the upper surface, where none usually appear, were due to the elbows in the frame work of the head, like that in Fig. 56, in conjunction with the comparatively small resistance to bending in a vertical plane offered by the rods forming this projecting elbow.

In the mushroom head the only bend permitted is that at the elbow of the radial rods where a strong steel neck band prevents any such bad effect as has just been pointed out.

Fig. 58

4th. We may notice a form of design in which the diagonal belts are omitted and the entire panel is covered by rods parallel to the sides of the panel. This, while apparently very different in ground plan from those just considered does not differ from it materially in principle. It is clear that the lattice pattern of the web in this case is in many parts of the panel not woven so close as where diagonals exist, while in other parts of the mesh the number of layers in contact with each other has been decreased. Experimental results do not as yet enable us to determine with certainty whether Poisson's ratio for this combination is as great as for the mushroom. Upon that depends in part the relative efficiency of the two arrangements. A form of this design is seen in Fig. 58.

The maximum deflections at the center of a loaded panel of the system of Fig. 58, would occur when the panels touching its four sides were also loaded. In this particular it differs from a loaded panel in a mushroom slab which would theoretically have its deflection slightly decreased by loading surrounding panels, tho this is too insignificant to have been observed as yet.

Deflections shown by tests of this system of two way reinforcement are wholly inconsistent with simple beam theory, and can only be explained on the basis of slab theory. Nevertheless, some of its advocates attempt to design its reinforcement and compute its strength on the basis of beam theory, which actual deflections show to be untenable. Such attempts should be entirely abandoned as erroneous and misleading.

All considerations which have been discussed under the three previous counts are to be taken as applying equally to this plan of arranging the reinforcing rods, especially as to carrying of part of the belts thru on the bottom surface at columns.

5th. Another element of design is the relative number of rods in the side and diagonal belts. We have previously adduced reasons to show that in a square panel the same number of rods is required ultimately in the diagonal belts as in the side belts, tho for stresses less than the yield point of the steel, it would be possible to diminish the number of rods in the diagonal belts somewhat. Equation (34) shows that for equal stresses in the steel of the side belts the number of rods should have the same ratio as the lengths of the sides.

A different rule from this has been erroneously proposed, viz., that the ratio of the number of rods in the side belts should be equal to the ratio of the cubes of their lengths. The only foundation for this rule is that according to the beam strip theory as developed in Marsh's Reinforced Concrete, p. 283, a rectangular plate carried by a level rigid support around its perimeter, would divide the load per unit of area which is carried by two unit-wide rectangular strips that cross each other, as the fourth power of their lengths, and hence would carry to the edges of the rectangle loads proportional to the cubes of the lengths of those edges. Were this so, the case of a horizontal rigid support around the entire perimeter of the panel is wholly different from support on columns at the corners, and such a rule would be wholly inapplicable therefore to a floor slab so supported. This rule was, however, evidently adopted in the design of the Larkin Building, Chicago, as shown by a photograph of its reinforcement in place before the concrete was poured, to which the writer has access and published in Cement Era for February, 1913. The very exhaustive tests of this building made by the Concrete Steel Products Company of Chicago, and published in the Cement Era, for January 1913, show that this ratio of rods caused the stresses for the larger loads to be more than twice as great at the middle of the short side belts as at the middle of the long side belts. This was assuredly an uneconomical distribution of steel, since correct design would require these stresses to be equal, when in fact one exceeded the other by 120 to 140 per cent. This discrepancy would be largely rectified by making the number of rods directly proportional to the lengths of the sides, as required by (34).

It also appears that the diameter of the mushroom head and the width of belts of slab rods in the Larkin Building is less than the limiting size in the standard mushroom system, viz. $g = 7(a+b)/16$. This makes the intersection of the diagonal belts fall nearer the center of the panel than the edges of the side belts. The very considerable effect of a very inconsiderable change of this width has been mentioned on p. 182. The result would be that the steel would for this reason be far less effective, and its resistance would be more nearly in accordance with (37) than with (34), a loss of perhaps 25 to 30% in its effectiveness.

Tischers Creek Bridge, Duluth

Test of Tischers Creek Bridge with 30 ton construction cars, each loaded with 20 tons of rails
Deflection less than one twenty thousandth part of the span

CHAPTER VI

CALCULATIONS OF STRESSES AND DEFLECTIONS VERIFIED BY TESTS

1. Specimen Computations of Stresses and Deflections in Several Slabs.

Ford Building, Los Angeles, Calif.

$L_1 = 28 \text{ x } 12 = 336''$

$L_2 = 25' \ 10'' = 310''$

Thickness of panel tested varied from $11\frac{3}{8}$ to $12\frac{3}{4}$ inches. Effective thickness taken conservatively at $11\frac{3}{4}$ inches $= L_1/28 = L_2/26.4$

By (56) $C_2 = L_2/L_1 = 0.92$ nearly.

Diameter of head $g = 7\ (L_1 + L_2)/32 = 141''$.

Diameter of cap $L_1 - B = 0.2L_1 = 67''$. $B = 0.8L_1 = 268.8''$.

Each belt of twenty-five, $7/16''$ round rods.

Cross section of each belt, $A = 25 \times 0.15+ = 3.76$ sq. inches.

Depth of center of mid side belt with $\frac{1}{2}$ inch concrete covering $d_1 = 11.75 - 0.5 - 0.2 = 11''$.

Depth of center of second layer of slab rods at panel center, $d_2 = 11.75 - 0.5 - 0.64 = 10.6''$.

Depth of bottom surface below third layer of slab rods at edge of cap with $\frac{3}{4}''$ covering, $d_3 = 11.75 - 0.75 - 1.1 = 9.9''$.

Design load per square foot $= 150$ lbs.

Dead load per square foot $= 130$ lbs.

Total design load, $W = 280 \times 28 \times 25\frac{5}{6} = 202{,}550$ lbs.

Test load $W = 300 \times 28 \times 25\frac{5}{6} = 217{,}000$ lbs.

A maximum tension is found in the slab rods at the middle of the long side belt, and is to be computed from (34) as follows:

$$f_s = \frac{217000 \times 336}{175 \times 11 \times 3.76} = 10{,}050 \text{ lbs. per sq. in.}$$

Any other loading within elastic limits of the steel would produce proportionate stresses.

The tension in the steel at the center of the panel is computed by (52), as follows:

$$f_s = \frac{1.02 \times 202550 \times 336}{256 \times 3.76 \times 0.89 \times 8.86} = 8184 \text{ lbs. per sq. in.}$$

The radial tension at the edge of the cap by (44) is less than

$$f_s = \frac{217000 \times 336 \times 646\ (3 \times 1.61 - 1)}{800 \times 9.9 \times 3.76 \times 310} = 19550 \text{ lbs.}$$

per sq. in.

Hence by (45) the unit compression in the concrete at the edge of the cap is less than

$$f_c = 19550 \times 0.7 / 15 = 912 \text{ lbs.}$$

The compression in the concrete lengthwise of the longer side belt at its middle is to be computed from f_s and (38) as follows, by taking the percentage of belt reinforcement at 0.24%, the corresponding value of $i = 0.76$, and $E_s / E_c = 15$:

$$f_c = \frac{0.24 \times 10050}{0.76 \times 15} = 211 \text{ lbs. per sq. in.}$$

The compression at the center of the panel, where the percentage of slab reinforcement may be conservatively assumed at 0.6% and $i = 0.67$, may be computed thus:

$$f_c = \frac{8184}{2 \times 15} = 273 \text{ lbs. per sq. in.}$$

If a test load of twice the design load, viz., in this case of 300 lbs. per square foot, be placed upon the slab, the deflections which will be produced by the addition of this total load of 217,000 lbs. may be computed as follows:

$$\text{By (54),} \quad \triangle z_1 = \frac{217000 \text{ x } 336^3}{10.7 \text{ x } 10^{10} \text{ x } 11^2 \text{ x } 3.76} = 0.169''$$

$$\text{By (55),} \quad \triangle z_2 = \frac{0.92 \text{ x } 217000 \text{ x } 336^3}{6.56 \text{ x } 10^{10} \text{ x } 10.6^2 \text{ x } 3.76} = 0.268''$$

$$\text{By (58),} \quad \triangle z_3 = \frac{217000 \text{ x } 336^3 \text{ x } 2.084}{96 \text{ x } 10^{10} \text{ x } 9.9^2 \text{ x } 3.76} = 0.0485''$$

$$\text{By (60),} \quad \triangle z_4 = \frac{0.92 \text{ x } 217000 \text{ x } 336^3}{20 \text{ x } 10^{10} \text{ x } 9.9^2 \text{ x } 3.76} = 0.101''$$

By (61), $D_1 = 0.22$ inch, and $D_2 = 0.37$ inch.

The observed deflection was $\frac{3}{8}$ in. $= 0.375$ in. The deflection computed by the less exact equation (71) is $D_2 = 0.378$.

$$\frac{D_1}{L_1} = \frac{1}{1530}, \quad \text{and} \quad \frac{D_2}{\sqrt{L_1^2 + L_2^2}} = \frac{1}{1235}$$

Any loading differing from this would produce deflections proportionate to its intensity.

The observed results of quite a number of tests of mushroom slab floors are to be found on pp. 32 and 44 of Turner's Concrete Steel Construction. These are there compared with results computed according to Turner's empirical formula, which translated into our present notation has been reproduced in equation (72). The observed and computed results show a very close agreement. The results given by (72) are in close agreement, as has been seen, with those derived from (61).

Some of these test slabs present peculiarities of reinforcement such as need to be individually considered in order to make exact computations of their deflections. It is thought that the specimen computation already given will afford sufficiently guidance in the methods to be employed.

Having considered the stresses and deflections of a slab which is near the minimum thickness for the standard mushroom system, viz. $L_1/35$, it will be instructive to consider a specimen or two near the maximum thickness $L_1/20$.

The Bridge Over Tischer's Creek, Duluth, Minn., is such an example. See cuts opposite page 157 and page 220. It is supported on three rows of columns crossing the gorge, at a distance apart of 27 feet center to center of columns, the two street car tracks being over the side belt that lies along the center line of the bridge lengthwise. Each of these rows consist of six columns lengthwise of the bridge, at a distance apart of 26 feet from center to center, so that

$$L_1 = 27 \times 12 = 324''$$
$$L_2 = 26 \times 12 = 312''$$

The size of the mushroom heads and width of the belts is 12 feet, which is in excess of $7(L_1 + L_2)/32 = 139\ 1/8'' = 11.6'$, thus giving great stiffness. The object to be obtained by maximum thickness and large head is to secure great stiffness and so reduce vibrations as well as decrease deflections. There are twenty 9/16 inch round slab rods in each belt, or a total cross section in each belt of $A_1 = 5$ square inches of metal. The slab is 15″ deep at its thinnest part at the gutter on each side of the roadway, and the steel is kept down to that level throughout the slab, altho at the crown of the roadway under the tracks and over the center row of columns the slab is 5″ thicker, or 20″, with the same thickness over the side rows of columns where the sidewalks are. The mean thickness is somewhat in excess of $L_2/20$. This makes $d_1 = 19''$ for the short side belts, $d_1 = 17''$ for the long side belts and $d_3 = 14''$ approximately for the heads. The design load per square foot = 150 pounds. The dead load of the slab per square foot = 300 pounds. Hence W = 450 x 26 x 27 = 315,900 pounds. The effective cross section of slab steel is so great by reason of large heads that instead of (34) we may take

$$f_s = \frac{W\ L}{200\ d_1\ A_1} \qquad \ldots\ldots\ldots\ldots(34)'$$

For the long side belt this gives f_s = 6,033 pounds per square inch. The total load imposed on the slab might be made six times as great without causing the steel to reach its yield point, and the live load might become 900 pounds per square foot without causing f_s to exceed 16,000 pounds.

This slab was tested as shown in the cut, page 220, by running two construction cars loaded with 20 tons of rails each over the bridge at the same time along one track of the short side belt 26

VIEW OF REINFORCING STEEL

Flat Slab Bridge, Denver, Colo. Spans 43 ft. 6 in. Carries Heavy Interurban Cars feet long. Weight of each car = 60,000 pounds. Weight of rails 40,000 pounds. Total weight of train = 200,000 pounds extending over several spans. The deflections were too small to be discovered by observations with level and rod. It is useless to attempt to compute the deflection of this slab under the test load because the four steel rails of the railway tracks across the bridge were so fastened to the steel cross ties which were embedded in the concrete as to make the rails a part of the reinforcement of the slab. They furnish a cross section of reinforcement equal perhaps to $7\ A_1$, which would effectually bar the application of our deflection formulas and reduce deflections to very small quantities.

In so thick a slab as this the action of any contemplated load is widely distributed by the slab itself, and such loads, as well as all shocks and vibrations are largely dissipated or absorbed by the body of slab itself without causing observable local stresses as they do in steel structures.

The Curtis Street Bridge, Denver, Colorado, is one of four bridges across Cherry Creek, shown by the cut on page 224, constructed on the mushroom system. It has three rows of three columns each crossing the stream, the middle column of each row in mid stream with spans of 42 feet between columns centers lengthwise of the bridge, thus obstructing the waterway as little as possible. It has a width of 28 feet between column centers. The slab is 17 inches thick at the gutters, 26.5 inches at the sidewalks outside the gutters, and 21″ over the center row of columns. The sidewalk is stiffened with fourteen 3/8″ round rods lengthwise just below its top surface as supplementary reinforcement, and there is an outside parapet giving added stiffness. There are also three stiffening rods 24″ apart across the bridge midway between columns. There are three ring rods, and the width of the belts is 16′. This is in excess of $7\ (L_1 + L_2)/32 = 183.75'' = 15.5/16'$. The heads are exceptionally stiff each having twelve 1 3-8″ round radial rods. Each belt has twenty-six 5/8″ round rods, hence $A_1 = 26 \times 0.3 = 8$ square inches nearly.

$L_1 = 42 \times 12 = 504''$, $L_2 = 28 \times 12 = 336''$.

The dead load per square foot = 300 pounds.

The design load per square foot = 150 pounds.

$W = 450 \times 42 \times 28 = 529{,}200$ pounds.

$d_1 = 20''$ for long side belt.

Compute the stress in the steel by (34) modified to (34)′ by reason of exceptional stiffness, and we obtain $f_s = 13{,}320$ pounds.

Compute the central deflection due to a test load of 100 pounds per square foot. Let $d_3 = 16''$. Then in (71) $L_2/L_1 = 2/3$: hence $C_2 = 3/4$, and we have $D_2 = 0.125''$. This is probably considerably in excess of the correct deflection, since the slab is stiffer than the one considered in equation (71), which was derived for 20 foot spans. More correct values are to be computed from (54), (58) and (61). Moreover for such comparatively light stresses in the concrete, the deflections, as we have seen previously fall short of those computed by the formula, which agrees with experiment for stresses nearer the yield point of the steel. $D_2 = 0.125''$ is less than one four-thousandth of the span, and the deflection under the working load would undoubtedly be less than one sixth-thousandth of the span.

A word is here in place respecting working stresses and the factor of safety in the reinforcement of slabs, to the effect that the same values of these quantities in slabs affords a greater degree of security than in ordinary structural steel construction, and that occurs for several reasons:

1st. Steel rods such as are used in slabs have a higher yield point by perhaps 25% than the steel of other structural members. Furthermore, it is quite possible and desirable to use a higher carbon steel for these rods than the mild steel necessarily used in structural work, where it must be manipulated in such ways that high carbon steel cannot be used. But in these rods which suffer no usage tending to impair their condition, there is good reason to use a steel of higher yield point and greater ultimate strength. This yield point may readily be 70% greater than that of ordinary mild steel for structural purposes.

2nd. Rods embedded in concrete do not yield as do bare single rods in a testing machine or elsewhere by the formation of a neck and drawing out at that point. The concrete embedment prevents that.

3rd. In a reinforcement consisting of multiple parallel rods acting together, no single rod can become overstrained and yield to any appreciable extent before bringing into play adjacent rods. This makes the construction tough, and not liable to sudden collapse, as well as obviates concentration of stresses thus ensuring a high degree of security.

2. Further Calculations of Test Slabs. A verification of the calculated stresses and deflections by discussion of test data in case of several slabs appeared in the Trans. Am. Soc. C. E., for 1914, Vol. LXXVII, page 1338 to page 1453. Some of the more important calculations and verifications are here inserted, but for details consult the original paper.

Test of the Northwestern Glass Company Building, Minneapolis, made May, 1913, by F. R. McMillan.

$L_1 = 17 \times 12 = 204''$. $L_2 = 16 \times 12 = 192''$. Rough slab $8''$ thick.

Diameter of cap $= 50''$; diameter of head $= 87''$.

Each belt consisted of fifteen $\frac{3}{8}''$ round rods with no other laps than merely splices. Standard Mushroom heads.

Design live load 400 pounds per square foot.

Test load taken to be equivalent to a total panel load of 200,000 pounds uniformly distributed.

The unit stress at mid span of the shorter side belts by equation (34), page 183, is

$$f_s = \frac{200{,}000 \times 16 \times 12}{185 \times 7.31 \times 1.6567} = 18000 \text{ lbs. per sq. inch.}$$

Two of the observed readings were 17000 pounds, and it is probable that if all the panels instead of only four of the slab had been loaded at once each with a uniformly distributed load of 200,000 pounds, all the rods in these side belts would have shown nearly or or quite the foregoing computed values of f_s.

The observed stresses at mid span of the longer side belts in this test were somewhat less than at mid span of the shorter belts, a divergence from (34) which seems to be due to the effect of the wall support on the long side of the wall panels.

By equation (52), page 196, the unit stress in the middle rod of the diagonal belt at the center of the panel is

$$f_s = \frac{200{,}000 \times 204}{256 \times 0.89 \times 6.94 \times 1.6567} = 15{,}570 \text{ lbs. per sq. inch.}$$

The observed unit stress at center of interior panel $= 14{,}200$ lbs. which was larger than in the rods on either side of it, as required by theory, but the unit stress at the center of one of the wall panels was 20,500 lbs., probably due to the fact that there is less cantilever support at the wall than is exerted by interior column heads.

The greatest stresses occur at the edge of a cap when the four panels around it are all loaded. Such a distribution of load occurred in this test when there was a load on each of the four panels which was assumed to be equivalent to 106,250 lbs. The limiting unit stress at the edge of the cap calculated by Equation (44) is 20,700 lbs., on the side belt and the similar equation for the diagonal belt gives 19,500 lbs.

The two highest unit stresses observed at the edge of the central cap were in the diagonal belt on opposite edges of the cap and were 17,500 lbs. and 20,000 lbs. respectively. There was also an abnormal unit stress of 22,400 lbs. in a middle diagonal rod of a cap at the edge of the loaded area due to a forcible bending of a radial rod.

By equation (71) the deflection in the diagonal center of the panel is

$$D_2 = \frac{200{,}000 \times 16 \times (17)^2 \times 1728}{4.75 \times 10^{10} \times 1.6567\ (6.9375)^2} = 0.422 \text{ in.}$$

While this load rested on the slab for 18 hours the deflection gradually increased from 0.416 in. to 0.456 in., so that the calculated deflection is entirely satisfactory.

By Equations (61), (54) and (58), as corrected by the introduction of 96 in place of 60 the values of the deflections at mid span of the side belts are:

At point 12: Computed 0.229 in.: Observed 0.22 in.

At point 20: Computed 0.228 in.: Observed 0.20 in.

The observed side deflections of a single loaded panel are necessarily less than where surrounding panels are equally loaded.

The deformations and stresses observed in the vertical steel in one of the columns at a corner of a panel loaded with some 800 lbs. per sq. foot, have by careful analysis given a probable limiting value of that part of the unbalanced moment due to this load which was resisted by the column itself. The diameter of the column core was 27 inches and the column was continued to the upper stories. Analysis shows that of the total unbalanced moment $W\ L/12$ which is to be resisted at a support only about one fifth took effect upon the column so far as shown by differences of the deformations of steel and concrete in the opposite sides of the column.

Test of the Deere and Webber Company Building, Minneapolis,* made by A. R. Lord, Nov. 1910.

Design live load 225 lbs. per square foot.

$L_1 = 19'1'' = 229$ inches. $L_2 = 18'8'' = 224$ inches.

Slab 9 and 3/8 inches thick; concrete 40 days old at beginning of test. All slab rods 7/16 inch round, with 12 rods in each side

*Reported by Mr. Lord. Proceedings Nat. Assoc. Cement Users, Vol. VII Philadelphia, 1911.

belt and 15 in each diagonal; all slab rods $7\frac{1}{4}$ inches between centers, so that the side belts may be taken to be 80 inches wide and the diagonal nearly 90 inches. The head was a rectangular diamond frame with four rods extending entirely across it. Column caps 54 inches in diameter. Mean size of cap = 0.24 L.

Take d_1 = 8.5 inches, d_2 = 8 inches and d_3 = 7.6 inches. The calculated unit stress at mid span on the short side by equation (34) under the test load of 350 lbs. per square foot or a total load of $350 \times 356.2/9 = 124{,}678$ lbs. is

$$f_s = \frac{124678 \times 224}{175 \times 8.5 \times 12 \times 0.15} = 10{,}000 \text{ lbs.}$$

The observed unit stress at one mid span was 10,400 lbs. and at others somewhat less than 10,000, so that this is a satisfactory determination of the greatest stress at mid span of the short side.

Equation (34) gives for the unit stress at mid span of the longer side 10,220 lbs. The mean observed result was only 6600 lbs. There was some reason in the arrangement of loaded and unloaded panels to expect a result of this kind, tho larger stresses would hardly be expected on the short side than on the long side unless the bulkhead cracks in the slab had a considerable influence.

The unit stress at the panel center calculated by equation (52) is

$$f_s = \frac{124678 \times 229}{256 \times 0.89 \times 14 \times 0.15 \times 8} = 7440 \text{ lbs.}$$

This is larger than the mean of four observed values, and larger than all but one of them which has an appreciably abnormal observed value.

The limiting unit stress at the edge of the cap by equation (44) in case we assume a mean of 13 rods to a belt is

$$f_s = \frac{124678 \times 229 \times 453}{800 \times 7.6 \times 13 \times 0.15 \times 224}\left(\frac{3 \times 52441}{30625} - 1\right) = 2015 \text{ lbs.}$$

The largest observed unit stress in a side belt was 20,000 lbs. A similar computation for the diagonal belt gives 22440 lbs. There were two observed stresses larger than this, one of 23400 lbs. and one of 24,200, due no doubt to the lighter head used in this construction.

Applying Equation (71) to the calculation of the deflection at the panel center we have

$$D_2 = \frac{124678 \times 224 \times 229 \times 229}{4.75 \times 10_{10} \times 6_4 \times 14 \times 0.15} = 0.23 \text{ inches.}$$

The mean value of seven readings was 0.224.

The mean deflections of adjacent panels were 0.291, 0.271, or 0.306 inches, these all being somewhat larger because adjacent panels were not loaded.

The interaction of contiguous panels across the column heads was evidently of considerable amount in this slab, which differed from mushroom construction in having no stiff connection with columns such as is afforded by elbow rods. Where the greatest stresses in steel and concrete occurred around the cap there appears to be a deformation ratio e_c/e_s of 0.5 to 0.6.

The Larkin Building, Chicago, tested by A. R. Lord.* The design of the belt reinforcement is of the usual four-way type employed in the Mushroom system, but the column heads omit much of the steel and in place of it use a depressed head or "drop" to resist shear and flexure, which is 8 ft. square and 6.75 inches thick, and is placed on top of each column cap under the slab and integral with it. The panels are 24′2″ by 20′, or $L_1 = 290''$ and $L_2 = 240.''$ The thickness of the slab is 9 inches except at the drop where it is 15.75 inches. The diameter of the cap = 60 inches. The width of side belts is 90 to 93 inches and diagonals 105 to 108 inches. All belt rods are $\frac{1}{2}$ inch rounds, 13 in each short side belt, 22 in each long side belt and 21 in each diagonal belt.

$$d_1 = 8 \text{ inches}, d_2 = 7.75 \text{ inches}, d_3 = 14 \text{ inches}.$$

The floor was designed for a dead load of about 120 lbs. per square foot and a live load of from 225 to 250 lbs. with a maximum test load of twice the sum of these, or actually 739 lbs. per square foot.

The total panel load producing stress was

$$W = 738 \times 20 \times 24\ 1/6 = 356{,}700 \text{ lbs.}$$

By (34) the unit stress at mid span of the shorter side belt is $f_s = 24{,}000$ lbs. The observed stress was 24,200 lb. between two loaded panels, which would be decreased slightly if adjacent panels were equally loaded as contemplated in equation (34).

For the long side belt with 22 rods (34) gives the computed unit stress $f_s = 17{,}000$. No observed value was so large as this, because all long sides were at the edge of the loaded area.

By (52) the calculated value of the unit stress in the middle rod of the diagonal at the panel center is $f_s = 14{,}070$ lbs. The ob-

* The results of this test were presented by Mr. Lord to the Ninth General Convention of Cement Users. Extracts from his paper appeared in the Cement Era, Jan. 1913, page 53.

served value is stated as 12,900 which is probably a mean stress in the rods of the diagonal belt and perhaps less than the stress in the middle rod.

To find the unit stress in the steel at the edge of the cap, it is evident that owing to the drop the effect of the cap in causing abnormal stresses at its edge will be much reduced. Assume that the stress will not exceed that obtained from (43) at the column center where $B = L_1$, then

$$f_s = \frac{W\ L_1\ (L_1 \ \ L_2 + 1)}{400\ d_3\ A_1} = \frac{356{,}700 \times 290 \times 2.21}{400 \times 14 \times 19.25 \times .19635} = 10{,}800 \text{ lbs.}$$

in which a mean belt of 19.25 rods is assumed for purposes of computation because all the belts are intimately combined in their action in the head. Under this test load no one of the columns was completely surrounded by loaded panels, and this computed stress would be approached only very exceptionally. There was one observation, however, of 10,400 lbs. and others of 7300 lbs. and 7000 lbs.

If, moreover, we suppose that no noticeable abnormal action is to be looked for at the edge of the drop, then the stress would be calculated by (43) by taking $B = 290 - 96 = 194''$, and $d_3 = 7''$, with a mean belt of 19.25 rods which gives $f_s = 14{,}400$ lbs. The largest observed unit stress in any rod at the edge of the drop was 14,500 lbs. in a side belt and 14,200 lbs. in a diagonal belt.

The test load for deflections was 618 lbs. per square foot. Hence

$$W = 618 \times 24\ 1 \ \ 6 \times 20 = 298700 \text{ lbs.}$$

$$\text{By (54)} \quad \triangle z_1 = \frac{298700\ (290)^3}{6.56 \times 10^{10}\ (7.75)^2 \times 21 \times .19635} = 0.2463''.$$

$$\text{By (55)} \quad \triangle z_2 = \frac{298700 \times 240\ (290)^2}{6.56 \times 10^{10}\ (7.75)^2 \times 21 \times .19635} = 0.3711''.$$

$$\text{By (58)} \quad \triangle z_3 = \frac{298700\ (290)^3\ 2.21}{96 \times 10^{10} \times 14^2 \times 22 \times .19635} = 0.0188''.$$

$$\text{By (60)} \quad \triangle z_4 = \frac{298700 \times 240\ (290)^2}{20 \times 10^{10} \times 14^2 \times 21 \times .19675} = 0.0375''$$

By (61) $D_1 = 0.265''$ and $D_2 = 0.408''$.

The corresponding observed values were 0.26″ and 0.40″, respectively, an agreement which is surprising in view of existing circumstances as to reduction of steel in the head and displacement of lines of inflection in this slab compared with the Mushroom construction for which the equations were deduced.

The St. Paul Bread Company Building. The floor of the St. Paul Bread Company Building is a rough slab, 6 in. thick, and has 16 by 15 ft. panels, reinforced with $\frac{3}{8}$ in. round rods, ten in each belt. The design load was 100 lb. per sq. ft. The slab, Fig. 59, was constructed in winter and frozen, but, as the final test was postponed until August, 1912, the slab was very fully cured, considerably more so, in fact, than most slabs when subjected to test. The test was made by Professor W. H. Kavanaugh, of the University of Minnesota, in the following manner:

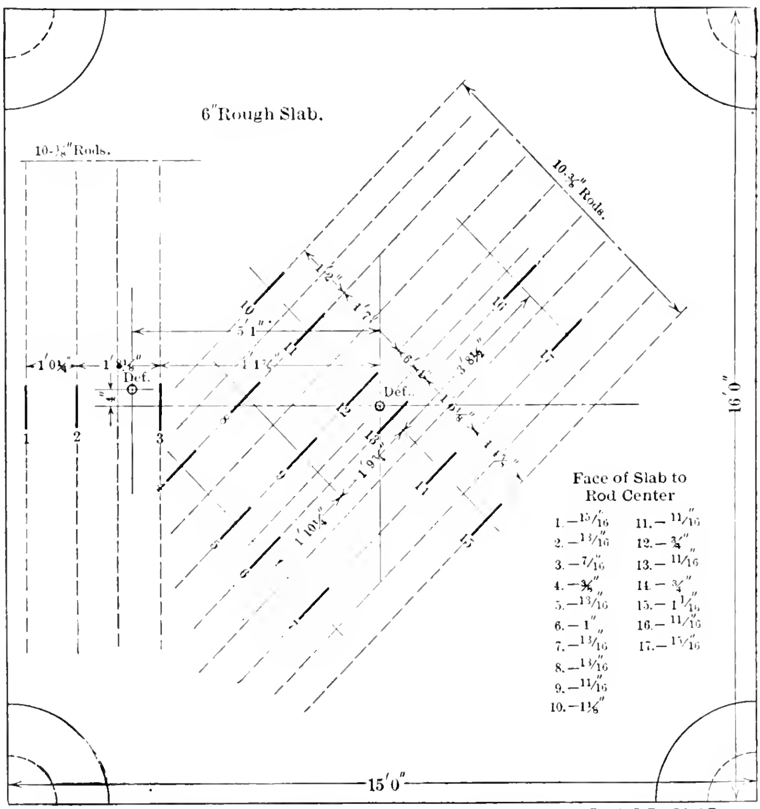


DIAGRAM SHOWING LOCATION OF TEST POINTS IN FLOOR SLAB.
SAINT PAUL BREAD COMPANY BUILDING.
"TURNER MUSHROOM SYSTEM"

Fig. 59

First, extensometer measurements were made on seventeen 8 in. lengths of slab rods, which were exposed under a single loaded panel, three of these, Nos. 1, 2, and 3, being at the middle of three rods of one long side belt at the edge of the load, and the remaining fourteen distributed over the central area of one diagonal belt. Second, measurements of deflections were made at two points, one at the center of the panel and one near the middle of the interior edge of one long side belt. Third, the embedment of the rods was tested. Table 1 contains the observed elongations due to change of loading at all the seventeen positions for each of the test loads, 108.4, 316.8, and 416.8 lbs. per sq. ft., as well as after the removal of the load. A comparison of the observed elongations at symmetrical points reveals such discrepancies in the observations as to require some preliminary discussion.

Observations Nos. 5 and 6 were on a pair of diagonal rods on each side of and adjacent to the diagonal line of the panel, there being no rod exactly on the diagonal, and situated just beyond the edge of the other diagonal belt. No reason can be discerned for any difference between these elongations, but the wide difference that appears must be due to some peculiarity in one of the rods, such as a crook or bend, or some lack of homogeneity in the concrete. Comparing Observations Nos. 5 and 6 with Nos. 4, 10, 15, and 16, which, being at about the same distance from the center, should, by Equation (49), have about the same elongations, it is found that No. 5 is abnormally large, and at the same time No. 16 is abnormally small. No. 8 is another set of abnormal results, which is evident from the fact that, being midway between Nos. 4 and 11, its elongations should lie between them; it is larger than it should be, with a final compression after removing the load. Nos. 7 and 17 should be the same, and Nos. 4 and 17 should be alike. The varying embedments of the portions of this rod which were observed show that there was probably a kink in it, which might account for the observed discrepancies. It is possible, however, that some such differences may appear when the loading is piled on one side of the panel before piling it on the other. No such explanation, however, will fit the case of Nos. 12 and 13, which are at the middle of the two rods adjacent to the diagonal line at the center of the panel. There appears to be no question that No. 13 is abnormally large, for No. 12 agrees well with others, being only a little larger than those at the nearby positions 9, 11, and 14, and the values at No. 13 are in wide disagreement with them. The very considerable differences between results which should apparently be equal makes it evident how in-

TABLE 1

OBSERVED ELONGATIONS, IN INCHES PER MILLION INCHES UNDER GIVEN LOADS PER SQUARE FOOT

Note.—Obtain actual unit stresses by multiplying by 30.

Observation No.	Elongations under the Following Loads, in Pounds.			
	108.4	316.8	416.8	0
1	255	500	598	80
2	236	473	549	−3
3	244	464	572	−57
4	63	207	283	145
5	64	262	387	164
6	66	168	271	114
7	45	152	220	69
8	218	370	421	−64
9	89	266	372	152
10	68	120	159	46
11	122	263	347	146
12	153	327	400	28
13	276	534	653	93
14	60	204	282	80
15	71	164	165	18
16	11	40	22	−30
17	31	70	124	17

exact single determinations must often be by reason of bends in the rods, lack of homogeneity in the concrete, etc., and emphasizes the importance of carefully laying belt rods straight and having them spaced uniformly, as well as embedded equally, before pouring the concrete, if consistent results are desired. It also shows the importance of checking all readings by readings at symmetrical positions.

It may be stated in general that the observed unit stresses and the deflections in this test are less than they would be for a slab tested at the stage of curing at which tests are usually made, a stage to which the equations apply more precisely. In consequence of this, all the computed results will exceed to some extent those actually observed.

Apply Equation (34) to compute the unit stress at Nos. 1, 2, and 3 of the long side belt. Assuming $d_1 = 5.3$ in.,

$$f_s = \frac{100{,}000 \times 192}{175 \times 5.3 \times 1.1} = 18{,}800 \text{ lb. per sq. in.}$$

in case of many panels equally loaded. The mean observed unit stresses for three rods at the edge of the load was 17,190 lbs., the stress in each of the rods being practically the same, a fact that speaks well for the stiffness of the head. The computed unit stress at the center of the panel is

$$f_s = \frac{100{,}000 \times 192}{246 \times 0.9 \times 5 \times 1.1} = 15{,}150 \text{ lb. per sq. in.}$$

The observed unit stress at No. 12 was 12 000 lb., tho at No. 13 the abnormal value of 19 600 lb. was found.

As the observations of stresses and deflections were made when only a single panel was loaded, and the computations assume that all the panels are equally loaded, any very close agreement of Table 1 with the observed results is not to be expected; nevertheless, comparison with that table shows that the computed results agree with the observations far better than the observations agree among themselves at such symmetrical points as admit of any comparison.

The deflection at the center of the panel under a load of more than double the design load plus once the dead load, namely, 316.8 lb. per sq. ft., was 0.32 in., which is less than 1/800 of the diagonal span. To compute the deflections at the panel center, apply Equation (71), as follows:

$$D_2 = \frac{100\,000 \times 180 \times 192^2}{4.75 \times 10^{10} \times 5^2 \times 1.1} = 0.50 \text{ in.}$$

Computation of the deflection at the point near the middle of the interior edge of a long side belt gives a deflection for a load of 100 000 lb. on each panel of approximately 0.4 in.

TABLE 2

Deflections, in Inches, Under Given Loads per Square Foot

		Test Loads, in Pounds			
		108.4	316.8	416.8	0
Center of panel.	Observed	0.077	0.320	0.437	0.155
" " " ..	Computed	0.130	0.380	0.500	
Edge of side belt	Observed	0.065	0.247	0.332	0.124
" " " " .	Computed	0.093	0.271	0.357	...

As above stated, the somewhat large excess of computed over observed deflections is due to two circumstances: first, and principally, to the age and consequent stiffness of the slab; and secondly, at the edge, to the fact of a single panel load instead of many, neither of which circumstances is taken account of in the equations used.

3. Comparative Test of Norcross and Mushroom Slabs. This section will be devoted to a detailed consideration of a test to destruction of two slabs, 12′ x 12′ between column centers, constructed for experimental purposes. The tests were made by Professor Wm. H. Kavanaugh, in November and December, 1912, and the results he obtained, together with a mathematical discussion based upon them, will be here given. One slab was constructed in accordance with the plans and specifications of the U. S. Patent No. 698,542 issued to O. W. Norcross for a slab for flooring of buildings, and the other was a Turner Mushroom slab under U. S. Patent No. 1,003,384. The test serves to bring out in a striking manner not only how two slabs, which present a superficial resemblance in the plan of arrangement of reinforcement, differ from an experimental and practical standpoint, but it also makes evident their radical divergence of action mechanically and mathematically.

That two slabs of the same span, thickness and amount of reinforcement should on test show that one of them was more than twenty times as stiff, and more than five times as strong as the other, and that the failure of the weaker one was a sudden and complete collapse, with little or no warning to the inexperienced eye, while the other gave way by slowly pulling apart little by little, thus gradually getting out of shape without any final break down, are phenomena that deserve the close attention of the designer, and are of the highest interest scientifically as well as practically. The enormous differences in the deflections and in the stresses in the reinforcement as shown by extensomoter measurements, and in the character of the failure in respect of safety and its relation to the line or zone of weakest section, as well as in the difference of design loads and breaking loads amounting to 500%, all illustrate what scientific design will accomplish and what results are possible by an ingenious arrangement of the reinforcement.

These slabs were each of the same thickness, viz 6″, and were supported by columns placed at the corners of a square 12′ x 12′ from center to center of columns. The slabs projected 2′ to 3′ beyond the centers of the columns on each side, and had precisely the same number and size of reinforcing rods in each belt, viz eleven 3/8 inch round rods. The concrete was of a 1 : 2 : 4 mix, and while only about four weeks old at the time of the test, it had been poured warm and kept warm by steam heat under such unusually favorable conditions as to have become well cured at the time of the test. The steel used showed by test a stress at yield point of 51,000 to

55,000 pounds per square inch, and an ultimate strength of 76,000 to 80,000 pounds, with an elongation of twenty to twenty-five per cent.

The first slab was made in accordance with the specifications of the Norcross patent already referred to except that belts of rods were substituted for the netting mentioned by the patentee. This design was selected as one of the two for this comparative test, not because it is a good design, or one that any engineer would to-day care to employ, but because it exhibits, according to the express intention of the patentee, simple tension on its lower surface, everywhere between columns, and simple compression everywhere on its upper surface between columns; this being in direct contrast to the other design, which is arranged not only to resist direct tensions over the supports, which the first does not, but also to resist circumferential stresses both around the supports and around the panel centers, as any truly continuous flat slab must.

This test may then be viewed in the light of an experimental demonstration of the difference between a reinforced flat slab constructed in accordance with the beam theory and one constructed in accordance with correct slab theory, where true and apparent moments differ radically as shown at the beginning of this investigation, but are wholly contradictory to any form of simple or continous beam theory. This test may be regarded as settling once for all the question of applying simple beam theory to a cantilever flat slab, reinforced throughout practically its entire area with a lattice of rods crossing each other and in contact. It shows that it is impossible to compute the deflections of such a slab by beam theory. Furthermore this impossibility makes it certain that the stresses in such a slab cannot be computed by beam theory, for to do this is to commit an inconsistency such as has heretofore too often been committed, but one which should hereafter be carefully avoided.

Norcross in his patent already referred to describes his construction as consisting "essentially, of a panel of concrete having metallic network encased therein, so as to radiate from the posts on which the floor rests....... The posts are first erected, and a temporary staging built up level with the tops of posts. Strips of wire netting are then laid loosely in place on top of the staging.... The concrete is then spread upon or moulded in place on the staging to enclose the metallic network. In practice I have sometimes laid the concrete in layers of different quality, the lower layer of the floor which encloses the wire being laid with the best concrete available....... If the forces acting upon a section of flooring supported between two posts be analyzed it will be found that the tendency of the floor section to sag between its supports will cause the lower layers of the flooring to be under tension while the upper layers of the flooring will be under compression, these stresses being, of course, the greatest at the top and bottom layers, respectively."

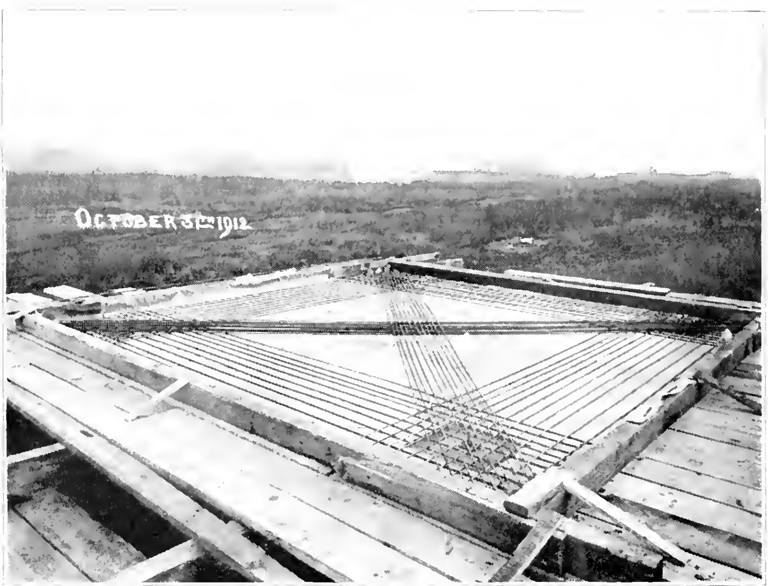


Fig. 60. Reinforcement of Norcross Slab

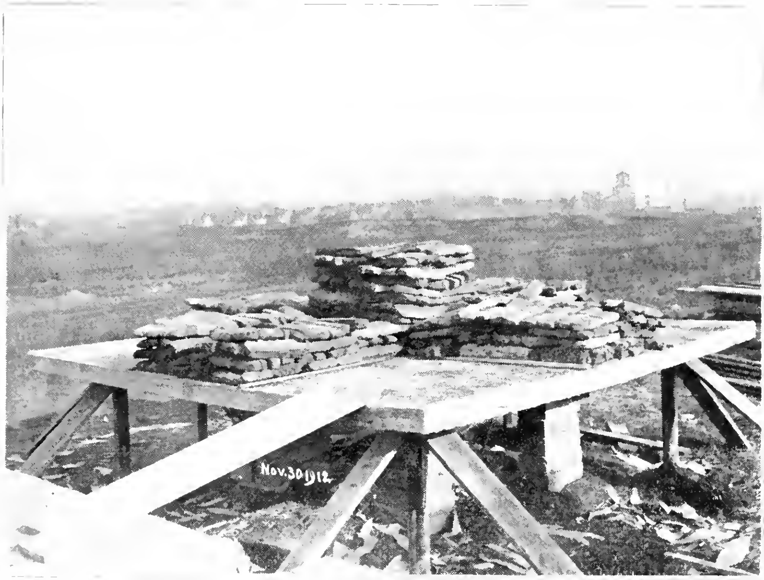


Fig. 61. Norcross Slab Carrying Load 3

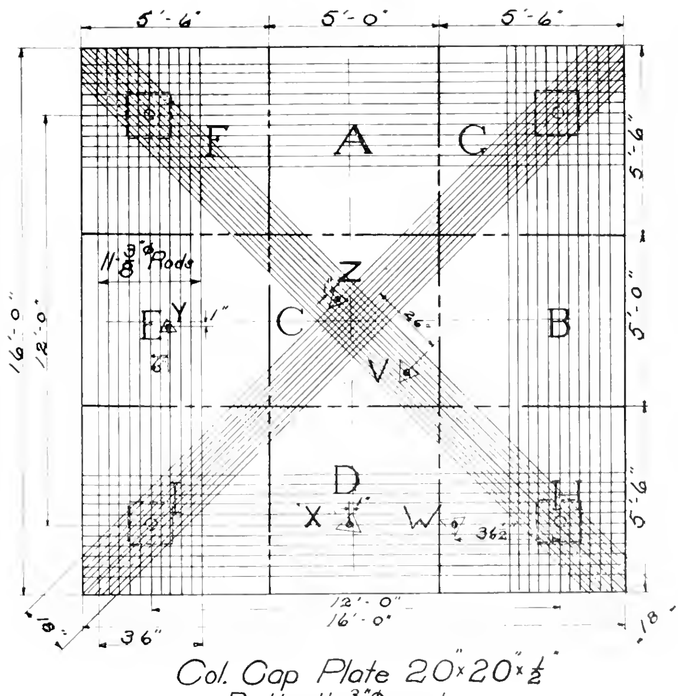


Fig. 62. Norcross Slab

Table 3. Loads on Norcross slab in pounds

No.	1		2		3		4	
Areas	per sq. ft.	Total	per sq. ft.	Total	per sq. ft.	Total	per sq. ft.	Total
A	114.1	3138	228.7	6288	228.7	6288	228.7	6288
B	114.1	3138	228.7	6288	228.7	6288	228.7	6288
C	114.1	2852	240.1	6002	456.8	11420	456.8	11420
D	114.1	3138	228.7	6288	228.7	6288	228.7	6288
E	114.1	3138	228.7	6288	228.7	6288	228.7	6288
F							250	7560
G			Corner Areas Not Loaded.				250	7560
H							250	7560
I							250	7560
Slab	60	15404	121	31154	143	36572	276	66812

The number and arrangement of the reinforcing rods in the Norcross experimental slab, (eleven 3/8″ round rods in each side and diagonal belt) is clearly shown in the view of Oct. 31st, Fig. 60, which shows the forms ready for pouring the concrete. Steel plates 20″ x 20″ x 0.5″ carry the rods and rest on the tops of the columns, which last in this case consisted of steel pipes about $5\frac{1}{2}$″ in diameter filled with concrete and embedded at their lower ends in large concrete blocks. A vertical central bolt in the concrete at the upper end of each pipe permitted the plates to be firmly secured to the tops of the columns. The view of Nov. 30th, Fig. 61, clearly shows the manner of placing the pig iron on the slab for load 3. This slab is 16′ x 16′. The loading at first covered an area having the form of a Greek cross whose central square was five feet on a side with arms 5′ 6″ long, as represented in accompanying diagram of loaded areas A, B, C, D, E, Fig. 62, and of amounts shown in Table 3.

Fig. 63. Collapse of Norcross Slab

When 10,000 pounds had been piled on the central part of the slab in addition to load No. 4, of 66,812 pounds, the slab suddenly failed. In anticipation of such failure timber blocking had been placed under the slab to prevent its falling more than possibly ten or twelve inches.

Fig. 64 Collapse of Norcross Slab

The two views of Dec. 2d, Fig. 63 and Fig. 64, show the condition of the slab after removing part of the final loading in order to render the nature of the failure visible. Careful extensometer measurements of the elongations of the steel rods at the middle of the side and diagonal belts were made under the action of loads 1, 2, 3 and 4, and also similar extensometer measurements in the concrete both on the top and the bottom of the slab along the center line of the side and diagonal belts near those edges of two of the steel plates which were nearest the center of the belts. Besides these, certain other measurements of the concrete were made at right angles to the diagonals. Deflections were also measured under these loads at the middle of the diagonal belt and of two of the side belts at V, W, X, Y, Z.

These measurements all show beyond question that the side and diagonal belts act like simple beams in this form of construction, since the stresses in the steel and concrete on the under side of the slab in the direction of the rods is invariably tensile, while the stresses in the same directions on top of the slab are always compressive. It was the avowed intention of Norcross to reinforce the slab in this manner since he regarded the upper part of the slab as being subjected everywhere to compression and the lower part to tension only, as stated in his specifications as already quoted.

The following computation, Table 4, shows a good approximate agreement of the results of this test with the beam theory of flexure, assuming for simplicity that the stiff steel supporting plate and interlacing of the ends of the belts diminishes the effective span of the side belts by 12″, and the diagonals in the same proportion, and further assuming that the loading was all applied at the middle of the side and diagonal belts.

The extensometer measurements made were for a length of 8″, consequently the stress in the steel per square inch would be computed thus:

$f_s = 1/8$ (elongation in 8″) x 30,000,000;. . . . . . . $(1)_1$

and, this being known from observation, it will be possible to compute the load W carried by the beam in which the given elongation occurs, as follows:

The bending moment due to a concentrated load W at the middle of a beam of length L is $M = \frac{1}{4}\ W\ L$,. . . . . $(2)_1$
and the equal moment of resistance of the reinforcement by which it is held in equilibrium is $M = A\ j\ d\ f_s$. . . . . $(3)_1$
in which A is the total cross section of the steel in the belt = 11 x 0.11 = 1.215 sq. in., and the distance from the center of the steel to the center of compressive resistance of the concrete is assumed to be, $j\ d = 0.9 \times 5.75$
when $d = 5.75$ is taken as the distance from the center of action of the steel to the top of the slab,

Hence $W = 4\ A\ j\ d\ f_s / L$. . . . . $(4)_1$

is the load required to cause the stress f_s in the steel. In the side belts we assume the span L to be 132″, and in the diagonals $132\sqrt{2}$.

In Table 4, which follows, it will be noticed that loading No. 1 is too small to develop sufficient elongations or deflections to overcome the initial compressions in the concrete in which the reinforcement is embedded, so that the load carried by the steel is only about one half of the actual load, the other half being evidently carried by the concrete in which it is embedded. This is in complete accord with other similar experiments. But in case of loads No. 2 and No. 3, where the steel is stressed close to the yield point, the sum of the loads as shown by the stresses in the steel is very close to the total actual load. It is assumed that these total actual loads are carried by the various belts in the same proportion as the computed loads, since there is no other way of dividing the total load between the belts. This may be stated mathematically, as follows:

Table 4, Loads and deflections of the Norcross Slab computed on the simple beam theory from the observed elongations of the steel reinforcement, compared with the observed loads and deflections.

	Load No.	1	2	3	4
Side Belts	Average observed elongation in 8″	.00205	.00746	.00922	.01133
	f_s in pounds by $(1)_1$	7690	27980	34575	42500
	W_1 computed by $(4)_1$	1465	5330	6590	8098
Diag. Belts	Average observed elongation in 8″	.00195	.00722	.00939	.01229
	f_s in pounds by $(1)_1$	7313	27070	35200	45900
	W_2 computed by $(4)_1$	985	3650	4750	6184
$4W_1$	Load in 4 side belts comp. by $(4)_1$	5860	21320	26360	32392
$2W_2$	Load in 2 diagonal belts comp. by $(4)_1$	1970	6700	9490	12368
$4W_1 + 2W_2$	Total load on Slab Comp. by $(4)_1$	7830	28020	35850	44760
$4W'_1 + 2W'_2$	Total actual load on slab observed	15404	31154	36572	66812
Side Belts	W'_1 actual load by $(5)_1$	2880	5800	6690	12090
	D_1 mid deflection by $(6)_1$	0.186	0.37	0.43	0.77
	D'_1 observed mid deflection	0.128	0.41	0.52	0.81
Diag. Belts.	W'_2 actual load by $(5)_1$	1940	3970	4820	9230
	D_2 mid deflection by $(6)_1$	0.35	0.72	0.865	1.66
	D'_2 observed mid deflection	0.22	0.707	1.02	1.49

Let W_1 = the computed load on a side belt.
and W_2 = the computed load on a diagonal belt.
Let W_1' = the actual load on a side belt.
and W_2' = the actual load on a diagonal belt.
Then $4W_1 + 2W_2$ = total computed load on slab.
and $4W_1' + 2W_2'$ = total actual load on slab.

$$\text{Then } \frac{4\,W_1' + 2\,W_2'}{4\,W_1 + 2\,W_2} = \frac{W_1'}{W_1} = \frac{W_2'}{W_2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (5)_1$$

from which W_1' and W_2' can be computed, W_1 , W_2 and $4W_1' + 4W_2'$ being already known.

The distribution of load No. 4 is not such as to render this method of computing it so applicable as to other loads. Having found the actual distribution of loading W_1' and W_2' the center deflections of the belts have been computed by simple beam theory from the formula.

$$D_2 = \frac{W'\,L^3}{48\,E\,A\;i\;j\;d^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (6)_1$$

in which $i\,d$ = the distance from the steel to the neutral axis and the value of i has been assumed to be 0.69; W' is the actual load on the belt and L is its span as previously stated.

It appears from Table 4, that the effect of the reinforcement is accounted for to a reasonably close approximation by considering the belts to act as a combination of simple beams, at least within the range of loading near the yield point of the steel.

It appears that the steel reached its yield point under a total load on the slab of from 15 to 18 tons and final collapse occured under a total load of a little over twice the latter amount not distributed uniformly but piled more in the general form of a pyramid.

It was observed that the application of the relatively small loading on the corner areas F, G, H, I, had a very injurious effect upon the slab, tending to break it across the tops of the columns.

The results of the test may be summarized in the Norcross system as follows:

1st. This slab is of the simple beam type, and the test shows no cantilever action and no circumferential slab action.

2nd. The narrow belts running diagonally leave large areas without reinforcement, and there is consequently no provision for resisting circumferential tensions as required in slab action.

3rd. The concrete showed compressive stresses on the upper surface of the slab in the direction of all the reinforcing rods.

4th. The concrete showed tension at the bottom surface in the direction of all the reinforcing rods, in agreement with Norcross' own analysis.

5th. This slab deflected 1.6″ under 33 tons and then broke down completely under 38 tons.

6th. The first crack appeared under a load of 15 tons and deflection of 0.7″.

7th. The slab, not being reinforced on the top surface over the columns, inevitably cracks at a column when the slab is loaded around the column.

8th. At failure the steel had passed its yield point. The percentage of reinforcement in the diagonal belt if we regard the belt as about 18″ wide is very nearly 1%, but since a width of concrete somewhat greater than that may be assumed to act with this steel, the percentage of reinforcement is somewhat less than 1%. Similarily, the side belts of width 36″ have a reinforcement less than 0.5%. The full strength of the steel in both belts was developed by the concrete, which fact demonstrates that the concrete was of high grade and well cured. The steel was also of good standard quality, and the test was therefore in every way fair to the Norcross slab, since it was so loaded as to cause the stresses in the side and diagonal belts to be practically equal, thus using the steel most economically. The slab failed because the steel yielded near the middle of the spans, thus causing the concrete above the steel to crack and break.

The second slab was made according to the Turner Mushroom System, under the patent already referred to.

Since all forces in a plane may be resolved into components along any pair of axes at right angles to each other it is possible to provide reinforcement to resist any horizontal tensile stresses in the slab by various arrangements of intersecting belts of rods at zones where these stresses occur. The combination of such belts with radial and ring rods to constitute a large and substantial cantilever mushroom head at the top of each column affords a very effective and economical arrangement for controlling the distribution of the stresses in the slab, and it places the reinforcement where it is most needed. It not only has the same kind of advantage that the continuous cantilever beam has over the simple girder for long spans, but combines with it the kind of superiority that the dome has over the simple arch by reason of circumferential stresses called into play, which greatly adds to the carrying capacity of the slab.

Fig 65. Reinforcement of Mushroom Slab

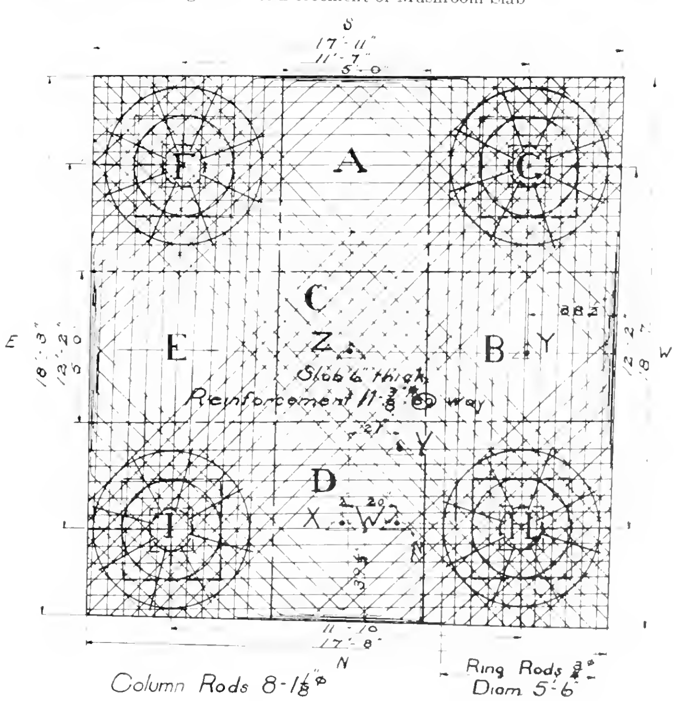


Fig. 66. Mushroom Slab

The mushroom test slab was six inches thick, and was supported on four 18″ by 18″ square reinforced concrete columns distance 12′ from center to center. These had square capitals, 42″ x 42″. The slab was appromimately 18′ x 18′, and the diameter of the outer ring rod of the Mushroom was 66″, while the inner ring was 42″. These were supported on eight 1-1/8″ round radial column rods.

Fig. 67. Mushroom Slab, Load 4.

This will be clearly understood from the view dated October 31st, Fig. 65, which shows the reinforcement and forms ready for pouring the concrete. The remaining views are explained by their accompanying legends.

The diagram of loaded areas for the mushroom slab Fig. 66, is like that already given for the Norcross slab in every particular except that the size of the mushroom slab being 18′ x 18′, while the Norcross slab was 16′ x 16′, the arms of the Greek cross in the mushroom slab are each 6′ 6″ long and 5′ wide.

Table 5. Loads on Mushroom Slab in pounds.

Load	1		2		3		4		5	
Area	per sq. ft.	Total	per sq. ft.	Total	per sq. ft.	Total	per sq. ft.	Total	per sq. ft.	Total
A	100.8	3276	201.6	6552	201.6	6552	425.3	13822	625.3	20322
B	100.8	3276	201.6	6552	201.6	6552	425.3	13822	625.3	20322
C	100.8	2532	201.6	5040	418.3	10458	861.8	21546	1261.8	31546
D	100.8	3276	201.6	6552	201.6	6552	425.3	13822	625.3	20322
E	100.8	3276	201.6	6552	201.6	6552	425.3	13822	625.3	20322
				Corner	areas not	loaded.				
Slab	48.2	15624	96.4	31248	113	36666	237	76835	348	112836
Inside the panel		9576		19152		24570		51317		75318
On the overhang		6048		12096		12096		25518		37578

Table 5—Cont. Loads on Mushroom Slab in pounds

Load	6		7		8		9		10	
Area	per sq. ft.	Total	per sq. ft.	Total	per sq. ft.	Total	per sq. ft.	Total	per sq. ft.	Total
A	736	23922	722.9	25122	899	29222	899	29222	899	29222
B	736	23922	772.9	25122	899	29222	899	29222	899	29222
C	1480.2	37006	1569.4	39236	1889.4	47236	2689.4	67236	3329.6	83236
D	736	23922	772.9	25122	899	29222	899	29222	899	29222
E	736	23922	772.9	25122	899	29222	899	29222	250	29222
F	.....	.....	250	10560	250	10560	250	10560	250	10560
G	.....	.....	250	10560	250	10560	250	10560	250	10560
H	.....	.....	250	10560	250	10560	250	10560	250	10560
I	.....	.....	250	10560	250	10560	250	10560	250	10560
Slab	409	132695	564.5	181965	637	206365	700	226365	748	242365
Inside the panel		88535		104591		121425		141425		157425
On the overhang		44160		77374		84940		84940		84940

Fig. 68. Mushroom Slab, Load 7.

Fig. 69. Mushroom Slab, Load 9.

The accompanying Table 5, exhibits the loads per square foot of each of the subsidiary areas shown in the diagram as also the total loads on each of those areas. The view of Dec. 3, Fig. 67, shows load 4, and that of Dec. 13, Fig. 68, load 7, while that of Dec. 16, Fig. 69, shows load 9.

Elongations of steel were measured by Berry extensometers in two of the side belts and in one of the diagonal belts until the concrete began to fail under loads Nos. 7 and 8. Deflections were also measured. In Table 6, these will be considered so far as they relate to the middle points of the belts. Loads 8, 9, 10, are of great interest as exhibiting the behavior of the slab under excessive loads, showing, as they do, yielding and large permanent deformation without dangerous collapse.

By (52) the uniformly distributed load per square foot of panel area when the stress in the diagonal belt is f_s is found for a square panel from the expression

$$144q = w = W/144 = \frac{256\, j\, d_2\, A}{144\, L} f_s \ldots\ldots\ldots\ldots\ldots\ldots (52a)$$

which applied to this slab gives us

$$w = \frac{256 \times 0.89 \times 5.125 \times 1.215}{144 \times 144} f_s = f_s/14.6 \ldots\ldots\ldots (52b)$$

The values of this uniformily distributed load w is tabulated in table 4, for each of the observed values of the f_s in the diagonal belts. The values of w so computed tend to become identical, in case of the heavier loads, with the loads per square foot on the central area C, as might reasonably be expected, w being the uniformly distributed load which is equivalent so far as the stress on the diagonal belt is concerned to the action of the actual loads which are not uniformly distributed.

Now compute by (54), (55), (58), (60) and (61), the deflections at the mid side belt and at center of the panel, due to a uniform load. These results are given in Table 4, and accord closely with those actually observed, as they should, because the irregularity of distribution does not produce deflections that differ much from the equivalent uniform load as computed above.

In these computations it is assumed that $d_1 = 5.5''$, $d_2 = 5.125''$, $d_3 = 4''$

The double set of values under loads 4 and 5 is due to the fact that readings were had under load 4, immediately after the load was applied, and again 7 days later before applying load 5. The second set of readings were the larger as shown. The second set of readings under load 5, were taken four days subsequently to the first set.

It appears from Table 6, that the observed results are accounted for by the slab theory to a good degree of approximation so long as the concrete was intact.

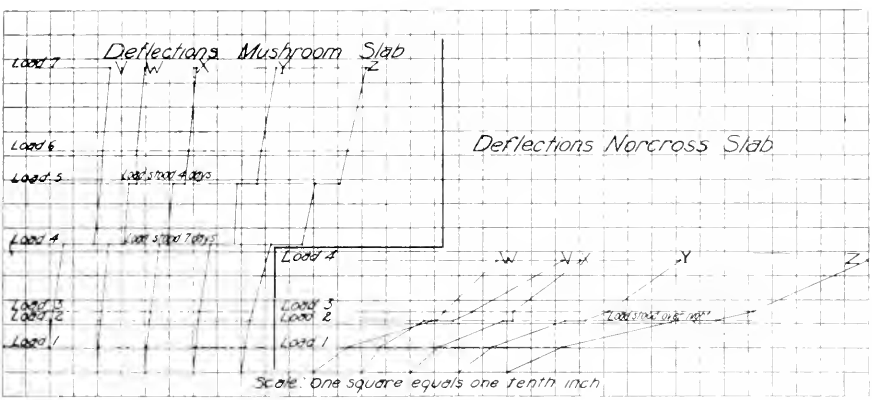


Fig. 70. Comparative Deflections of Norcross and Mushroom Slabs.

A graphical representation of the experimental observations in the deflections at the points V, W, X, Y, Z, of the two slabs is found in Fig. 70, which shows in a striking manner how small the loads and how great the deflections were in the Norcross slab on the one hand, and how large the loads and how small the deflections were in the mushroom slab on the other hand.

Table 6. Loads and Deflections of the Mushroom Slab computed on the Slab Theory from observed Elongations of the Steel Reinforcement, compared with the Observed Loads and Deflections.

	Load No.	1	2	3	4	5	6	7
Diag. Belt	Obs. elong. of mid rod in 8″	.00026	.00058	.00071	.00140 .00270	.00373 .00610	.00628	.00708
	f_s by $(1)_2$	975	2175	2662	5250 10120	14000 22900	23550	26550
	w the effective load per sq.ft. by (52b)	65.5	149	182	356 692	958 1550	1610	1816
	D_2 center deflection comp. by (61)	.0206	.046	.057	.111 .215	.300 .488	.50	.573
	D_2' obs. mid deflection	.0170	.0353	.0463	.121 .251	.305 .408	.439	.517
Side Belts	Ave. obs. elong. in 8″	.00017	.00031	.00039	.00126 .00272	.00401 .00607	.00681	.00758
	f_s by $(1)_1$	637.5	1160	1460	4725 10200	15000 12762	25500	28400
	D_1 mid deflection computed by (61)	.0095	.0215	027	.052 111	.140 228	231	.268
	D_1' mean of obs. deflection	.0100	.0202	.0250	.0653 .155	.173 .241	.256	.303

It will be seen from Tables 3 and 5, that the first three loads were practically the same for both slabs. In the Norcross slab load 3, of 18 tons, stressed the steel up to the yield point, but in the mushroom slab the stress was so small, (being in fact less than ten per cent of the former) as probably not to remove all the compression from the concrete in which it was embedded. Indeed the load on the latter slab became five times as much, 90 tons, without stressing its steel out to the yield point, at which time it was carrying about twice the load which caused the complete failure of the Norcross slab.

Moreover the deflection of the Norcross slab under load 3, was twenty-two times that of the mushroom slab under the same load. This result is in full accord with slab theory which shows that the central deflection of a continuous diagonal beam with fixed ends uniformly loaded with one sixth of the total load on the slab and having the same thickness and reinforcement as the diagonal belt, would have more than six times the central deflection of the slab, while the stress in its steel would be three or four times as much. This gives a measure of the effect of slab action.

By the phrase "slab action" we designate the increased strength and stiffness of the slab by reason of its resistance to circumferential stresses around the columns and around the center of the panel.

Furthermore, if this continuous beam be compared with a simple beam uniformly loaded and having the same reinforcement, the latter would have five times the deflection of the continuous beam, or thirty times that of the slab, while the stress in the steel would be one and one-half times that in the continuous beam, and six or seven times that in the slab. This last exhibits the effect of cantilever action combined with slab action.

The apparent discrepancy between the observed ratio of deflections in these two slabs of 22 and the just computed deflections of 30, is to be accounted for by the fact that the computation assumed equal spans, whereas the Norcross span was assumed to be diminished from 144″ to 132″ by the column plate. A reduction of the span of this amount will change the computed deflections in the ratio of $144^3 : 132^3 :: 30 : 23$ which is in practical agreement with the observed result of 22.

By the phrase "cantilever action" we designate the increased strength and stiffness which is due to the continuity of the beam or slab at its supports so that it is convex upwards at such points.

While the concentration of the loading toward the middle of the panel, such as was the case in this test, may prevent any precise agreement of these numerical estimates based on uniform loading with the results of the tests, they cause the general agreement shown in the tables and tend strongly to sustain our confidence in the validity of the analysis from which these concordant approximate estimates are obtained.

The amazing difference in the strength and stiffness of these two slabs, which contain practically the same amount of concrete and steel, is due to the difference of principle of their construction, which may be summarized for the mushroom system by considering its slab action and its cantilever action under the following counts, viz:

1st. Circumferential slab stresses around the column heads are most economically and effectively provided for by the ring rods and the octagonal interlacing of the slab rods.

2nd. The size of the mushroom heads is such as to make the belts so wide as to provide reinforcement over the entire area of the slab, thus securing slab action in the central part of the panel where the belts lie near the lower surface.

3rd. The reinforcing belts cover a wide zone at the top of the slab over the columns and mushroom head, which thus provides resistance to tension, and ensures effective cantilever and slab action.

4th. Concrete is thus stressed in compression at the bottom of the slab for a wide zone around the columns.

5th. Under a load equal to the breaking load of the Norcross slab, amounting to thirty-eight tons, the mushroom slab deflected at first only 1/8″, but after exposure to rain and great changes of temperature for seven days had somewhat softened the concrete the deflection increased to 1/4″.

6th. The first crack appeared underneath the edge of the slab across the side belt under load No. 5, of fifty-six tons, with a center deflection of 0.4″ and an average deflection at the middle of side belts of 0.25″.

7th. No cracks appeared on the upper side of slab at the edge, nor were any seen elsewhere, until load No. 7, of 90 tons was applied, when a center deflection of 1/2″ was reached.

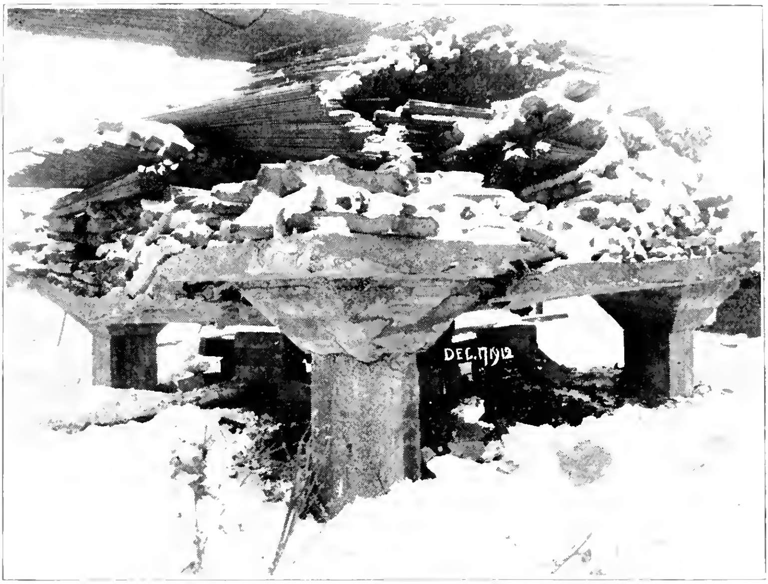


Fig. 71. Failure of Mushroom Slab.

Fig. 72. Failure of Mushroom Slab. Load Removed.

8th. The slab carried its final load of over 120 tons for twenty-four hours without giving way. It demonstrated the impossibility of its sudden failure by gradually yielding until it reached a final deflection of some nine inches, as seen in the views of Dec. 17th and 24th, Figs. 71 and 72.

9th. While the slab steel in each belt was the same as in the Norcross slab, the crossing of the belts increased the percentage of slab reinforcement so much above that of the simple belt reinforcement that stress in the steel did not pass the yield point and the failure was largely due to the giving way of the concrete around the cap, but partly to some yielding at the line of weakest ultimate resistance, both of which statements are confirmed by the view of Dec. 24th, Fig. 72, where the removal of the loading permits the irregular circular line previously mentioned to be made out at a distance from the center of each column of somewhat less than $L/2$.

Less steel is required in this system than in the Norcross slab for the same limiting stresses. Since the steel in this slab did not pass the yield point any greater percentage of reinforcement would be useless and would not increase the strength of the slab. It has been found that good practice requires a percentage of steel dependent in the following manner upon the thickness of the slab:

If $d = L/35$ the belt reinforcement $= 0.2\%$

If $d = L/24$ the belt reinforcement $= 0.3\%$

If $d = L/20$ the belt reinforcement $= 0.4\%$

Comparision of the steel in the test slabs:	Norcross.	Mushroom.
Size of slab	16′ x 16′	18.4′ x 17.8′
Area of slab	256 sq. ft.	328 sq. ft.
Length of 3/8″ rods in the slab	1188 ft.	1750 ft.
Weight of 3/8″ rods in the slab	446 lbs.	650 lbs.
Weight of Plates or Heads in the slab	228 lbs.	435 lbs.
Total weight of steel in the slab	674 lbs.	1085 lbs.
Weight of steel per square foot of slab	2.6 lbs.	3.3 lbs.
Area of Panel 12 x 12 ft.	144 sq. ft.	144 sq. ft.
Length of slab rods per panel	638 ft.	638 ft.
Weight of slab rods per panel	239 lbs.	239 lbs.
Weight in plates or heads per panel	57 lbs.	109 lbs.
Total weight of steel per panel	296 lbs.	348 lbs.
Weight of steel per square foot of panel.	2.06 lbs.	2 5/12 lbs.

The results of this comparative test of two slabs lead to several interesting applications of the principles previously cited under the treatment of such slabs as a kind of mechanism amenable to analysis by the theory of work.

The law of conservation of energy teaches that for elastic deflections the average stress for instance at mid span of a floor slab of one kind compared with the average stress at mid span of another kind of the same uniform thickness and percentage of steel at the sections compared, will bear the same ratio as the respective deflections under identical loads producing these stresses. In other words, a radical difference in mode of action brought about by a different arrangement of the reinforcement in the slabs will not affect the validity of this deduction. The experimental data supplied by the two respective slabs, the Norcross and the Mushroom type slab, is of interest in this connection.

Compare the loads which were applied in Greek cross form, the areas A, B, D, E, loaded alike, but the area C loaded at times, with approximately double the load on the arms of the cross —the idea being to throw upon the respective belts crossing the area C approximately the same load as on each direct belt as shown in the following table:

TABLE 7.

Type	Load			Steel Stress			
	On each Area A B E D	On Area C	Total	Average on belts at mid span	Ratio	De-flection	Ratio
Norcross.....	3138	2852	15404	7690		.2207	
Mushroom....	3276	2532	15624	643	12	.017	12.8
Norcross.....	6288	6002	31154	27133		.707	
Mushroom....	6552	5040	31248	1154	23.5	.0353	20.
Norcross...	6288	11420	36572	34894		1.020	
Mushroom....	6532	10458	36666	1472	23.4	.0463	22.

The equality of the ratios above compared for the respective steel stresses and deflections is in most satisfactory agreement with the theory when the lack of perfect identity of application of the load as regards quantitive distribution and the failure of the observers to measure stresses and deflections simultaneously is considered.

It will be noted that these ratios remain substantially the same, as nearly as can be expected considering the fact that the measurements of the deflections and the measurements of the steel stresses were not simultaneous, there being, however, some time effect as indicated by the diagrams.

These tests show a surprising difference in stiffness for the same cross section of steel in diagonal belts, and substantially the same difference so far as its efficiency is concerned in direct belts. The enormous difference in stiffness—the one being 22 times as stiff as the other—is a difference little short of amazing, when we consider the fact that a continuous beam is only five times as stiff as a simple beam. Hence the test shows clearly that the mode of operation must involve a far more radical difference than mere continuity of reinforcement.

The difference that is most striking in the arrangement of reinforcement in these two slabs, lies in the difference in the width of the belts, the material in the Norcross type slab being concentrated in narrow strips diagonally, while the material in the Mushroom type is spread out from three to four times as wide, and covers the area of the slab fully.

A comparison, then, of the average stress in the respective belts at mid span, should bring out in a striking manner, the difference in mode of action, because of the difference in the width of the belts in the two areas. This comparison should show the difference in effect at mid span of spreading the belts as against concentrating the metal in relatively narrow diagonal beam strips. The following table of average stresses is submitted in order to show the ratio of the stress in the diagonal belt compared to that in the direct belt:

NORCROSS SLAB

Load	Stresses in Direct Belts in lbs. per sq. inch.		Stresses in Diagonal Belts	Ratio diag. to direct belt. Stress average.
3	34,650	34,690	35,213	1.02
4	42,800	42,050	45,880	1.08

MUSHROOM SLAB

Load	Stresses in Direct Belts in lbs. per sq. inch.		Stresses in Diagonal Belts	Ratio diag. to direct Belt. Stress Average
4	9,677	12,627	3,977	.36
7	25,380	31,490	13,713	.484

The ratio of the diagonal belt stress to the direct belt stress is what was to be expected in the Norcross type slab on the beam strip theory; and this is to be accounted for by the unreinforced triangular areas in the arrangement of the metal.

The spreading out of the belt reverses the ratio found in the Mushroom slab and changes it so that the relative stress in the diagonal belt at mid span of the Norcross slab, when compared with the stress in the direct belts is one to two hundred percent greater than the similar ratio in the case of the Mushroom slab; and this striking difference between the two, must account then, in a large part, for the amazing difference in deportment of the two respective tests slabs. It should be noted that the Mushroom test slab was at a great disadvantage on account of the Greek cross type of loading suggested by Mr. Eddy, which was very advantageous to the Norcross type slab, since it would cause the greatest stress at that section of the slab best able to resist the same; while in the case of the Mushroom slab, a uniform load over the column areas would produce cantilever action which would tend to neutralize the stress at mid span.

Refer now to the diagram in Fig. 70, page 253, where the deflections of the Mushroom and Norcross test slabs have been plotted.

The Norcross type slab with the steel at the bottom is known by the application of the law of rigidities to exhibit in its action the predominant phenomena and characteristics of the simple beam. If this deduction be correct, a rapid rise in steel stress should be shown by the yielding of the matrix under the larger loads thru over-strain by the action of indirect tensions combined with direct tensions. The following table is instructive in regard to these ratios.

NORCROSS TEST SLAB

Load in Terms of Load 1.	Steel Stresses in Ratios of that for Load 1.	Deflections in Ratios of that for Load 1
Load 1	7690	.2207
Load 2—Load 1 x 2.01	7690 x 3.54	.2207 x 3.21
Load 3—Load 1 x 2.38	7690 x 4.53	.2207 x 4.65

Observe from this table the leaking out of the potential energy stored in the concrete, as shown by the increase in the steel stress by two hundred and fifty percent for a one hundred percent increase in load; or under Load 3, an increase of three hundred and fifty percent for an increase in load of one hundred and forty percent.

Now compare this with the deportment of a true multiple-way reinforced slab, which, by the law of rigidities must act as a true continuous plate, instead of an aggregation of simple beam strips.

If the preceding theory of storage of potential energy be correct, the storage reservoir of indirect tensile energy found in this case should be dependable or without leak in the slab and a uniform increase in stress and deflection with increase in loads should be the phenomena found.

MUSHROOM TEST SLAB

Load in terms of Load = 15624	Average Steel Stress at mid span in terms of that for Load 1.	Deflection in terms of that for Load 1
Load 1 = 15624..................	643	.017
Load 2 = 15624 x 2...............	643 x 1.8	.017 x 2.07
Load 3 = 15624 x 2.38...........	643 x 2.3	.017 x 2.7
Load 4 = 15624 x 4.9	643 x 6.34	.017 x 7.12

Loads 1, 2, and 3, on the Norcross test slab, and loads 1, 2, 3, and 4, on the Mushroom test slab, are strictly comparable, since for these loads, there was identity in location, and approximate equality in the time element of loading.

The proportionality of increase of stress and deflection with the load in this table of the Mushroom test, contrasts strongly with the absence of such proportionality in the Norcross test slab deportment. Each test slab was supported upon columns, (see Figures 63 and 67) and the stress and deflection in each test is affected to some extent by the rigidity of the columns. Were the columns perfectly rigid, the proportionality of stress to deflection would be unaffected by the columns, but in case they participate in the bending of the slab, the exactness of this proportionality will not be entirely preserved. But as loads increase, a divergence will appear between the rate of the increase in stress and rate of the increase in deflection.

In the Norcross test slab, where the column effect is a slight unresisted tipping, this is nearly negligible, amounting under load 4, to nearly three percent, while under Load 4, Mushroom test, it amounts to six times as much, or to an even eighteen per cent.

The large variation under Load 4, between the stress ratio and deflection ratio in the Mushroom test, indicates an intimate relation of column stiffness to stress and deflection. The continuity of the slab being secured by the integral connection of the column and slab, a certain amount of potential energy is stored within the columns by flexure; and as we are tracing out and locating all the leaks in our storage system of energy, these columns must be considered. As restraint of connection is secured in part by the columns, any column bending reduces the amount or degreė of fixity, and increases the slab deflection and stress at mid span. But increased slab deflection and increased stress at the panel center involve increase of work done on the slab and columns by the load, in greater amount than merely in proportion to the increase of stress in the slab represented by divergence noted in the increase in stress and in deflection. For consider the analogous case of a beam fixed horizontally at supports. If these supports yield sufficiently to cause it to act as a simple beam, the deflection is increased five times, and the work done by the load is multiplied by a somewhat larger number than that. Similarly, when the columns supporting a slab yield somewhat by tipping they must increase the central deflection of the slab and so increase the energy stored in the slab, in addition to what they themselves absorb in the column flexure by tipping. The energy stored in the column itself, like that of any beam, may afford a leak which is a fraction of its energy of flexure, and tho it may not be a large fraction of the total energy expended upon the structure, it may nevertheless be the cause of some increase in the total energy of deformation.

The jogs in the seven day period after Load 4, Fig. 70 and in the four day period after Load 5, may be accounted for in part by the fact that the ground was frozen and there was perhaps some heaving affecting the bench marks at which the levels were taken.

In the curves of deflections, Fig., 70 the influence of the direct tensions is observed in the slight curvature of the line between Loads 1 and 3, and the small percent of loss from this source is indicated by the fact that the curves of deflection continue parallel and show little or no divergence in direction under higher loads than under load of lower intensity, which is in strong contract to curves of deflection of the Norcross test slab.

The views which have been put forth in the foregoing pages to account for the radical experimental difference between the deportment of beams and that of slabs by a rigid application of the fundamental laws which have been already enunciated have been vigorously opposed by an attempt on the part of certain members of the engineering profession to explain the wide divergence of actual slabs from the results of beam strip theory by a pretended belief in the efficacy and sufficiency of the direct tensile resistance in concrete as sufficient to account for the phenomena observed in the flexure of slabs.

In order to determine what, if any, basis in fact there might be for any such view, a test slab twenty-five feet square, and approximately five inches thick, was constructed, which was supported at its edges by walls and at its center by a masonry pier 20 inches square and reinforced with wire netting radiating from the center in the bottom of the slab. The netting was ordinary poultry netting and 2 inch mesh, galvanized. The applied load was 15,000 pounds in the form of concrete barrels partly filled with water and arranged in circular formation at mid span around the central pier. When the load was first applied the slab carried the load by tensile resistance of the concrete without apparent over strain, no cracks of any kind appearing. The load was left in place, and owing to the leaking of the barrels which were imperfect, gradually diminished by 25 or 30 percent. In the course of about five days cracks began to develop in the slab, these extending in the top radially and circumferentially about the center pier and in the bottom of the slab at approximately the same time from one corner along mid spans. The slab was left undisturbed for six days longer, and these cracks continued to increase until finally the whole structure collapsed completely. The concrete was found, after the collapse, to be nearly $5\frac{3}{4}''$ thick at the center as against 5 inches at the edge.

This test is of value as showing somewhat the effect of time, combined with temperature changes upon the endurance of tensile stresses in the concrete. The test was made in the fall of the year and the drop in temperature from mean conditions under which the slab was cured may be stated as approximately 35 to 40 degrees.

The reinforcement of this slab was designed with the purpose of making it substantially like the wire netting of certain old floors of similar span which had, however, a thickness of from 12 to 15 inches the outside edges of these latter slabs being supported vertically and laterally by heavy masonry retaining walls which formed substantial abutments and in their action as retaining walls caused a certain amount of thrust to act upon the slab.

The great thickness of these floors relative to span caused arch action to predominate rather than slab action and their permanent stability in contrast with the slab tested is readily accounted for on this principle, and sharply differentiates slab action from arch action.

The predominance of arch action is dependent upon a large ratio of thickness to span and vanishes practically in a thin slab of long span.

Mr. Arthur R. Lord, in a paper published in the "Engineering and Contracting," January 29, 1913, reports interesting data relative to the test of the Larkin Building in Chicago, and concludes that there is a marked degree of arch action in a slab the span of which is approximately twenty times its thickness. He reasons that there is such action because where the line of inflection should be, he observed compressions in the concrete both in the top and bottom of the slab, and infers from this that these compressions are a measure of arch action. Now, it is a fundamental principle of flexure that the sum of the horizontal compressions must equal the sum of the tensions at any section thru a plate in bending, if no arch action be present. Examining the cross section of this design, we find the slab rods, at the deflection line, crossing the neutral plane of the slab at an angle, and turning downward at a considerable inclination. Now since there is shear across this section, these rods must be in tension at the line of inflection, by virtue of vertical shear, and the horizontal component of the tension in the steel must be balanced, to fulfill the laws of flexure, by compressions in the concrete; and hence this supposed arch action is thus readily accounted for as a phenomena of flexure. Moreover the magnitude of the thrust was wholly insufficient to account for the carrying capacity of the slab in excess of beam theory.

4. Investigation of Structures by the Berry Extensometer and Interpretation of Results. In the investigation of concrete structures with the Berry Extensometer, or similar instrument, it is usually possible to secure measurements on one side only of the reinforcing rod, and hence the measurement is primarily a measurement of fiber stress rather than that of the average stress across the section of the bar. Any kink in the bar, due to careless handling before placing in the structure, is liable to induce a bending stress under working conditions, which will mask in a large measure the character of the average stress in the bar and its true mechanical action in the structure. This difficulty might be obviated if we could get at both the top and bottom of the bar, and take observations on both the lower and upper fibers; but it is generally impracticable to do this, and far better to check up the accuracy of the readings by careful comparison with observed deflections.

The next difficulty in the experimental solution of the problem of stresses in flat slabs by the strain gage lies in the masking of the true action of the material by the stresses induced in the process of casting. These stresses naturally vary thru a wide range, dependent on the temperature conditions at which the concrete was cast and the temperature conditions, humidity and barometic pressure of the air under which the concrete was cured.

In practical work, it is frequently the case that where the work is executed in hot weather, the steel and the concrete materials are heated by the sun and are quite warm when the concrete is mixed, so that very rapid setting and hardening results. Such hardening is accompanied in the chemical process of curing with a considerable evolution of heat, and the steel is thus heated to a temperature as high as 130° Fahrenheit or even more during this process. If the hardening is sufficiently rapid to form a rigid bond between the steel and the concrete during this stage of hardening, (and it frequently does form such bond) the final result is that as the mass cools down, the steel is thrown into tension by the cooling and the concrete to a considerable degree into compression, this compression being distributed over the cross section of the slab. The result of the combined temperature and shrinkage stresses induced in hot weather, is such as at times to cause the slab to be practically self supporting and remove its weight largely or entirely from the supporting forms so that these in hot weather are frequently found to be really loose, and may be knocked out with little resistance.

This condition, to some extent, may, of course, be accounted for by the shrinkage of the lumber forms which are wet in casting, but this shrinkage is insufficient to account for the difference in conditions observed in warm weather work contrasted with cold weather work.

The presence of such shrinkage stresses in the material, cause its apparent deportment to be materially different under loads of low intensity from its action under loads of higher intensity where the mechanical operation of the combination is not masked by extraneous influences.

The effect of casting stresses and shrinkage stresses which have been referred to above gradually disappears of course with time and continuance of the chemical process of hardening, and under the repeated changes in form of the structure caused by temperature variations and changes of load. Accordingly it must be kept clearly in mind that positive conclusions as to the mechanical operation of the slab cannot be deduced under loads that are too small to permit the character of the stresses induced by the load to be distinguished from stresses originally induced by the weather conditions while casting.

Measured stresses on newly cured work can be given weight, accordingly, only after the loads applied become materially greater than the working load. The true action of the structure commences, then, to become dominant, because this action is not masked by the influence above discussed. These influences have been, by some, improperly credited to an impossible direct tensile resistance of concrete.

Moreover, measurements of the deformations in the concrete by the extensometer are frequently erroneously interpreted. In the practical testing of a building applied loads remain upon the concrete a considerable period of time, since a comprehensive survey of the stresses in the slab cannot be executed short of several day's continuous work. When it is attempted to interpret extensometer measurements of the concrete which has been subjected to a given load continuously for several days or a week on the basis or the modulus of elasticity determined by measurements made on test cylinders of concrete which are loaded with given loads for very short periods only a considerable error is involved in such comparison. First, because an 8 inch cylinder 16 inches long cast at the same time that the floor of the building was cast, has a better opportunity to dry out and become hard and rigid before testing than the concrete work of the practical structure. Second, because the short period of time in which the load is applied to the cylinder in the ordinary method of making tests does not correspond to the time element involved in making tests of the work in the finished structure, and neglect of these conditions involves a fundamental error lost sight of frequently in the experimental determination of concrete stresses. The correct method would be to determine the residual set of the concrete prism under a continued load of the intensity which it is desired to interpret, then deduct this set from the measurements made on the practical structure and determine the true modulus of the specimen by repeated loadings. Scientific results may be thus secured which would be of value in checking the mathematico-elastic theory.

The great difficulty with extensometer tests aside from the labor and expense involved, is due to the great uncertainty arising from the causes which have been mentioned and to the fact that measurements taken on corresponding rods at corresponding points where like results would be expected differ so greatly as to show that accidental differences of construction have so large an effect upon the measurements as to make precise deductions very difficult.

Such inequalities would, however, evidently not be dangerous to the structure because overstrain on any rod would ultimately be relieved by others coming into action nearby.

The measurement and interpretation of deflections under load is not beset by uncertaintities and difficulties of this character. A deflection is the result of the combined action of all the elements of the slab and not of any single one exclusively and so has a degree of reliability which cannot attach to any result derived from measurements on single elements however numerous. If deflections and stresses are mathematical elements of a comprehensive slab theory the measurement of either one is sufficient to determine the other just as in the theory of beams. When the profession shall have become convinced of the validity and sufficiency of slab theory, there will be little use for extensometer tests. Deflections are sufficient.

CHAPTER VII

MOMENTS IN TWO-WAY AND FOUR-WAY FLAT SLABS

1. Simple Approximate Theory of Four-Way Slabs. In order to investigate approximately the applied bending moments and resulting stresses in a four-way flat slab in a more elementary manner and dispense with the use of higher mathematics, assume that each of the four-way reinforcing belts supports one-fourth of the total uniformly distributed panels loads W. This is very nearly the fact in the central portion of the slab where the curvature is concave upward and the side and diagonal belts are to a considerable extent separate from each other.

Assume that the central portion of each side belt for example at least as far as the lines of inflection, is uniformly loaded with a part of $W/4$ proportional to its length and that the position of the lines of inflection is the same as would be found in a uniform cantilever beam, viz: at a distance $\frac{1}{2}L \sqrt{3} = .288L$ each way from mid span. The assumption however that so far as the central portion of the side belt is concerned the load $W/4$ may be taken as uniformly distributed is only approximate, for the load is concentrated somewhat toward mid span as may be seen from Fig. 73 where the load

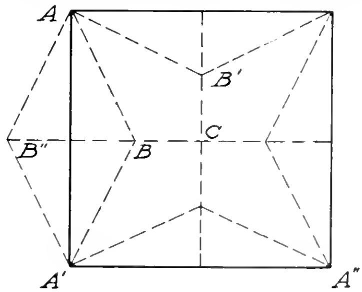


Fig. 73.

upon half a side belt of a panel may be taken as that resting on the triangle $A\ B\ B''$, and that upon half a diagonal belt as that on the quadrilateral $A\ B\ C\ B'$. This assumes that there are no vertical shearing stresses in the slab on the lines $A\ B$, $A\ B'$ etc., which would not necessarily be exactly the fact, especially for oblong panels.

But the lines of inflection have been assumed above to be at somewhat greater distances from mid span than occurs in a slab where the caps have a diameter of $0.2L$ or more, so that the applied moment at mid span of a uniformly loaded continuous beam of length L will be approximately that of a side belt in a standard mushroom slab. It should however be noticed that the position of the lines of inflection is a matter which is determined by the designer and within practical limits is within his control, tho so far as known they are universally assumed to occur where they would be situated in a plate which opposes to the applied moment a uniformly distributed moment of inertia. This is not the case with a reinforced cantilever slab, any more than it is with a continuous cantilever bridge where the moment of inertia is reduced to zero at the ends of the suspended span by joints. The resisting moment of inertia is practically reduced to zero in the reinforced slab at the lines where the reinforcement dips below the neutral axis and thus the lines of inflection are fixed at these loci.

Designs which have definite bends in the slab rods where they make a somewhat steep descent from the top to the bottom of slab are to be avoided, for any severe stress at such a bend is apt to make cracks in the concrete, while there is nothing in slab construction to forbid a very gradual dip from top to bottom of slab at the lines of inflection where the moments gradually approach zero.

But designs in which some of the belt rods dip suddenly at one distance from the column center, and others at a different distance and still others at another distance, are especially reprehensible because they mechanically obliterate any definite lines of inflection and put them in a different position for each different unbalanced load, and so introduce uncertainty in place of certainty in design. Especially is this true in case of any accidental subsidence of column under load where the same principles obtain as in a cantilever bridge with joints, as compared with a continuous bridge, where the latter is liable to dangerous stresses in case of subsidence from which the cantilever is measurably free.

The designer of a slab thus having control of the size of his cantilever and consequently of the position of his lines of inflection naturally removes these lines as far as circumstances will permit from column centers when by so doing the stresses in the concrete around the column cap are not too greatly increased.

Assume in the first place for the purposes of computation that the part of the side belt lying brtween the points of inflection is a simple beam of length $0.577L$, loaded with a proportionate load of $0.577W/4$. Then the applied moment at its center will be (as in any simple beam) one eighth the product of these quantities, viz:

$$(0.577)^2 \, W\,L\,/32 = W\,L\,/\,96.$$

In order to derive from this applied moment the resisting moment of the rods of the side belt the effect of the diagonal belts which cross the side belts diagonally and increase the cross section of the steel resisting the moment by about 50 per cent on the average must be allowed for, as well as the mutual effect of the reinforcing rods which cross each other under tension at the edges of the belt and the embedment which have an effect to reduce the stresses in the side belt, an effect which is dependent upon Poisson's ratio K. thru the action of bond shear already discussed in this paper. As already shown in Chapter V, both the stresses in the steel and the deflection in the slab are reduced in general by the factor $(1—K^2)$ in case of a Poisson ratio $=K$.

No direct determinations of K for such a composite material as reinforced concrete are available, but every test for stresses or deflection may be regarded in the light of a determination of K provided the formulas for these quantities are completely known otherwise.

A general value of $K=0.5$ brings a good agreement between the formulas previously given and the observed data in a very large number of tests, some of which have already been detailed in Chapter VI entitled **Steel Stresses in Flat Slabs.** It is probable that K would have somewhat different values for different arrangements of belts with reference to each other. Now the value of $K=0.5$ is one which would necessarily hold for any incompressible solid i. e., a solid of constant volume, while $K=1$ is a value which would apply to a sheet of constant area without regard to thickness. Great objection has been raised to adopting so large a value of K as 0.5 but its total effect, depending as it does upon the factor $(1—K^2)$, is at most to make the stress 75 percent of what it otherwise would be and that is believed not to be an over estimate of the effect of the bond shear which has been previously discussed.

Introducing therefore the effect of the increase of the amount of reinforcement due to the overlapping of the diagonal belts, and also that due to the lateral effect into the expression for the part of the resisting moment exerted by the direct rods in the side belts it becomes

$$2/3\,(1—K^2)\,W\,L\,/\,96 = W\,L\,/\,192 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which the first factor takes account of the fact that the direct rods constitute on the average only two thirds of the tension reinforcement actually present, and $(1-K^2)=0.75$ if $K=0.5$, takes account of the reduction of stress due to the lateral action expressed by Poisson's ratio. This is precisely the same result that was reached in equation (34a) Chapter V, which was derived by the application of exhaustive mathematical analysis to a continuous uniform slab square or oblong and supported at the corners, where L is the length of the side belt under consideration.

There is one other question in this connection which needs consideration, viz: the irregularity of the lapping of the belts over the area of the side belts. The question is as to what amount of irregularity of distribution may exist without materially interferring with or changing the action of the total amount of steel. All designers and investigators agree that a belt of rods is practically equivalent in its action lengthwise of the rods to a sheet of metal of equal width and weight if all questions of bond be disregarded, and the question is whether other large irregularities of distribution such as occur in the over-lapping of side and diagonal belts may be disregarded, and whether the mean weight of metal present is the only significant factor. Such would seem to be the fact within limits of area which are comparatively small fractions of the total panel area. This may be stated more convincingly perhaps by saying that it is impossible to elongate the central portions of the side belts without at the same time elongating the steel of the diagonal belts that lies along the edges of the side belt. Tests show what is otherwise evident that the elongations in all the rods across the side belts are practically the same. Hence the diagonal rods at the edges of the side belts participate in the same elongations. And this is the basis of the assumption of an average reinforcement of 50 percent in addition to the side belts themselves.

Next compute the moments at the middle of the diagonal belts, each under a total assumed load of $W/4$ uniformly distributed. If the distance between inflection points on the diagonal be taken to be 2 times that on the side belts, then the applied moment at the center becomes $WL\sqrt{2}/96$ and the resisting moment of the steel at mid span in one diagonal belt may be written

$$\tfrac{1}{2}\sqrt{2}\,(1-K^2)\,WL/96 = WL/180 \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which the factor $\frac{1}{2}$ takes account of the fact that one diagonal belt comprises only one half of the reinforcing steel present, and $(1-K^2)$ takes account of the reduction of stress due to the lateral action expressed by Poisson's ratio. No account however has been taken of the reduced concentration of the load at mid span of the diagonals as shown in Fig. 73, which in fact makes the stresses at mid span of the diagonal belts not only less than those computed from (2) but somewhat less even than those computed from (1) for the side belts, a fact which is established by the observed results of all available tests of four-way slabs in buildings. The same fact appears mathematically from the results of the more exact analysis given in Chapter V, so that with equal belts in four-way reinforcement greater stresses occur at mid span of the side belts than at the center of the panel in the diagonals.

Consider in the next place the applied moments at the column heads. In a uniformly loaded cantilever beam such as has been assumed for the purposes of computation, each side belt will have an applied moment at each end which is twice that at mid span viz.: $WL/48$, making a total applied moment for the four belts in $180°$ around the column center of $WL/12$. Owing to the somewhat greater concentration of stresses in the center rods of the belt by reason of their being at a level above those at the edges of the belt, as well as by reason of the concentration of stress at the middle rods of the belt at the edge of the cap due to its rigidity the decrease of belt stresses arising from the shortening of the clear span by the caps will be disregarded in obtaining this roughly approximate value of the stresses at the edge of the cap. Disregarding therefore any reduction of the moment due to shortening the span by the breadth of the support afforded by the column caps and assuming that each belt is carried across the column as a continuous beam the question arises as to what reduction of stress will arise from other steel with which it is in contact by its coaction therewith. Assume as a safe basis of computation that each belt coacts with one other belt as do each of the diagonal belts at the panel center.

The resisting moment of each side belt at the edge of the cap will then be written

$$\tfrac{1}{2}\ (1-K^2)\ WL/48 = WL/128 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which the factor $\frac{1}{2}$ takes account of the steel other than the belt itself in assisting the belt, and $(1-K^2)$ gives the additional reduction due to the lateral action in the slab of Poisson's ratio.

From this it is evident that with steel at the same distance from the neutral axis, 50 per cent more steel would be required according to this computation in each belt over the column head than at mid span, which increase is to be provided for by laps or otherwise in the belts over the head. But these laps need not be distributed equally among the belts. Any or all the laps may occur equally well in two belts only by extra rods placed between the belt rods which individually extend several spans. The laps or extra rods will be more effective the nearer they are to the top of the slab and also the nearer they are to the middle of the belt, because the edges of the belts are at a somewhat lower level than the middle of the belts.

The steel of the ring and radial rods has been left out of the account in this rough computation as well as the breadth of the cap, in order to offset the smaller arm with which part of the belts act when cob piled one on another at the top of the column as well as to compensate for the lower level of the belts at their edges.

2. Simple Approximate Theory of Two-Way Slabs. To investigate in a similar manner the flat slab with two-way reinforcement suppose the lines of inflection to be situated as before at a distance of $.288L$ each way from mid span. Then the width of the central area between lines of inflection is $.577L$ and the width of the side belts is $.423L$.

Let the loading upon each central area of a panel between the side belts be transmitted symmetrically sidewise to the side belts by the median belts. Each central median belt parallel to the sides may be regarded as constituting a simple beam of length $.577L$ and carrying a uniform load of $W/6$ or half that on this central area

There will consequently be a positive central applied moment in each median belt at mid span amounting to one eighth the product of the load and span, which is

$$\frac{1}{8}\,\frac{W}{6}\,\frac{L}{\sqrt{3}}=\frac{W\,L}{83}=.012\;W\,L$$

The resisting moment of the steel in one median belt at mid span will be

$$\tfrac{1}{2}\;(1-K^2)\;W\,L\,/83=W\,L\,/222 \qquad \text{(4)}$$

Where the factor $\frac{1}{2}$ takes account of the fact that one belt is only half of the reinforcement present and $(1-K^2)$ makes allowance for lateral action of the other belt. This will give the mean stress in the slab rods of the median belt. The middle rods of this belt however have greater stresses than this. The negative moment applied to the median belt across the edge of the panel at the middle of the side belt will be one eighth of the product of its load $W/6$ by the width of the side belt regarded as the length of the simple beam transmitting this load to the side belt and uniformly supported by it. Hence the moment is

$$\frac{1}{8}\cdot\frac{W}{6}\,.423L = \frac{W\,L}{112} = .009\;W\,L.$$

This is also the resisting moment of the median belt at this point because there is no steel in the top of the slab coacting with it. This resisting moment is the greatest in this belt. It consequently determines the cross section of the steel in the entire belt which should not be less at the panel center since the stress in the middle rods at the panel center is greater than at the edges. The effect of the median belts is to transfer that portion of the load actually covering the central area between the side belts, viz. $W/6$, and place it upon those belts, so that the load acting upon each side belt of length L between columns is $\frac{1}{2}W$, irrespective of its width and the size of the central area.

It will be assumed that this load is uniformly distributed along the side belt, tho its apparent distribution has a somewhat greater concentration toward mid span, as may be seen by considering the situation of the square areas included between the panel diagonals of several panels, for on drawing these diagonals the square load areas supported by each side belt have corners at column centers and at panels centers. The median belts will have some effect to transmit loads diagonally as well as laterally and it is not far from correct to assume uniform distribution of load upon the side belts, tho that assumption reduces their central moments somewhat, as was the case to a less extent for the side belts of the four-way slab. With twice the load of the side belt of the four-way slab upon each side belt of the two-way slab the applied moment at mid span of each side belt will be twice that in the four-way side belt, viz.: $W\,L/48$ at mid span of a side belt. This is also equal to the moments of resistance of the steel in the side belt without the benefit of any assistance from the steel that crosses this belt. There is no such assistance here because the median steel that is in tension lies across the top surface of slab and cannot coact to any appreciable extent with the steel of the side belt at the bottom, neither can it coact with any steel that might be continued across the bottom in compression as is sometimes done. The applied bending moment in each side belt where it crosses the column center may be assumed to be twice that at mid span, viz.: $W\,L/24$ giving a total moment of $W\,L/12$ in 180° about the column center.

The resistance afforded by the steel in each belt at the support combined with the lateral action of that crossing it at right angles will be

$$\tfrac{1}{2}(1-K^2)\ WL/24 = WL/64 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

which is the same as that in two belts of the four-way slab. This requires less steel in the belt where it crosses the top of the column than at mid span and permits a fraction of the side belts rods to be carried thru on the bottom of the slab at the column when so desired It would not be good practice to reduce the total cross section of the side belts at the columns, below that required at mid span, whatever theory may be accepted respecting shearing stresses in reinforcing rods around the columns.

3. Weight of Steel in Two-Way and Four-Way Slabs Compared. In making a comparison of the weight of slab steel required in a two-way panel with that in a four-way panel of the same size and thickness, it will be noticed that the cross section of the steel required in each belt will be proportionl to its resisting moment and its weight will be proportional to the product of cross section by length.

Now omitting common factors of W and L the weights will be proportional to the following numbers:

In a four-way panel:

Two side belts together give by (3)	$2/128$
Two lapped belts half length give	$1/128$
Two diagonal belts give	$2\sqrt{2}/128$
Making a total of about	$1/22$

In a two-way panel:—

Two side belts together give	$2/48$
Two median belts together give	$2/112$
Making a total of	$1/16.8$

exclusive of laps. This shows an excess of weight of belt rods in the two-way panel of somewhat more than 30 percent over that required in the four-way panel, but does not take account of any head steel used in supporting the belts in the two-way panel over the heads of the columns, nor of the Mushroom heads in the four-way panel.

The above simplified analysis shows how this excess arises in the main, viz.: from lack of suitable arrangements in the two-way reinforcement to take advantage of coaction of belts, and besides that the excess due to the round about indirect transmission of the loads thru the median belts to side belts instead of direct transmission to columns thru diagonals.

The question of the weight of steel required in a 20′ by 20′ panel designed to carry a total live and dead load of 300 lbs. per square foot has been discussed recently by the writer*, and it is shown that such a panel would require about 1000 lbs. of steel according to several authorities on slab design, while several others who would reject the foregoing slab theory as inadvisable and insist on beam theory as alone applicable to slabs and essential for safe design would require about 2,000 lbs. of steel per panel.

It has been tacitly assumed in the foregoing computations and comparisons that reinforcing rods across the top of the side belts in four-way slabs are unnecessary and superfluous, and that the cracks occasionally observed extending along the middle of the side belts do not indicate any structural weakness. Such is the fact, since the necessary reinforcing steel to reisst the negative moment occurring across the side belts is to be found in the parallel side belts across the column heads. The cracks where they exist allow sufficient deformation and twisting moment to act in the slab to bring this belt steel into play in this way. Cross reinforcement on top of the side belts viewed from the standpoint of mechanics, only serves to increase the load upon them and so increase the stress in them, and at the same time relieves to some extent the stress in the diagonals, thus making the method of slab operation to resemble the uneconomical action of two way reinforcement.

4. Panels Reinforced Unequally Lengthwise and Crosswise. The particular solution of the general partial differential equation (20) of Chapter V which was developed in that Chapter was one that has special reference to slabs resting on separate supports or columns at the corners of the panels. It is a solution in which the deflections at mid span of the sides of a square panel are more than half as great as at the panel center, and one in which the ratio of the deflections at mid span of the sides of a rectangular panel varies as the fourth power of the lengths of the sides so that for the extreme case of $L_2 \quad L_1 = .75$ the deflection at mid span of the long side would be more than three times that on the short side. It is evident therefore that such a solution as that is entirely inapplicable to the case of a slab where the edges of the panels are supported on walls which deflect not at all or on beams which are so stiff that their deflections are small compared with slab deflections.

* Proc. Am. Soc. C. E., May, 1914, p. 1513.

Grashof has proposed the following equation as the best he was able to invent to represent the surface of the middle layer of a uniform rectangular plate fixed horizontally at the edges:

$$24EI(a^4+b^4)\,z=q\,(a^2-x^2)^2(b^2-y^2)^2 \quad\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

This equation was proposed by Grashof on the analogy of the equation for a beam with fixed ends, and not as a solution of the differential equation, which in fact it is not, tho it has often been quoted as if it were in some way so affiliated with the differential equation as to derive some validity from it. Such is not the fact however. No solution of this differential equation of the fourth order can contain terms of higher degree than the fourth, since otherwise the last member would not be constant. We can dismiss Grashof's equation as simply an invention of an ideal nature. He was aware that the equation sought must contain the two quantities $(a^2-x^2)^2$ and $(b^2-y^2)^2$, and he in fact proposed that the result contain their product as stated above. But as just shown, that is impossible because the degree of the result would prevent it from satisfying the differential equation of which it purports to be a solution.

It can in fact be readily shown that no exact algebraic solution of this differential equation is possible that will fit the case of a slab resting on relatively stiff beams at the edges of the panels. In equations (9), (10), (11), (12), however, a novel solution is obtained which will be used as a basis for approximate equations applying to a slab resting on beams. It is evident since side beams are designed of arbitrary cross sections to carry the slab, that their deflections which depend upon their design as to stiffness relative and absolute is the determining factor not only of the slab deflections but of the shears and steel stresses of the slab. Beams and slab are consequently independent members of the combination and are not readily amendable to treatment as a simple system.

The general partial differential equation of the surface of the middle layer of a continuous flat slab loaded uniformly is, see equation (20) Chapter V.

$$\frac{\delta^4 z}{\delta x^4}+2\frac{\delta^4 z}{\delta x^2\delta y^2}+\frac{\delta^4 z}{\delta y^4}=\frac{(1-K^2)\,q}{E\,I} \quad\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

in which $x\ y\ z$ are the coordinates of the deflected surface, q the intensity of the uniform loading, K is Poisson's ratio, E is Young's modulus of elasticity, and I is the moment of inertia per unit of width of vertical cross section of the slab in any plane parallel to z

In deriving (8) it is assumed that during the small flexure, which occurs by reason of the loading, z only varies, and that the coordinates x and y of any given point of the slab remain unchanged, which assumption undoubtedly is sufficiently in accordance with fact for technical purposes.

A somewhat more general form of solution of (8) than that given in Chapter V may be written as follows:—

$$24EI(c_1+c_2)\,z=q\,(1-K^2)[c_1\,(x^2-a^2)^2+c_2\,(y^2-b^2)^2] \quad . \quad . (9)$$

in which c_1 and c_2 are any arbitrary constants whatever. That (9) in fact satisfies (8) and is consequently a particular solution of (8), may be readily verified by trial.

A form of solution less general than (9) is the following, which involves but one arbitrary constant n in place of the two found in (9).

$$24EI\left\{\left(\frac{b}{a}\right)^{n}+\left(\frac{a}{b}\right)^{n}\right\}z=q(1-K^2)\left\{\left(\frac{b}{a}\right)^{n}(x^2-a^2)^2+\left(\frac{a}{b}\right)^{n}(y^2-b^2)^2\right\} (10)$$

which is a form of solution especially applicable to the single panels of a continuous slab divided into rectangular panels of size $2a$x$2b$, where the origin of coordinates is at the point occupied by the center of the panel before deflection, and the axes of x and y are parallel to the edges $2a=L_1$ and $2b=L_2$ respectively.

It will be noticed that the corners of the panel, $x=a$ and $y=b$, are fixed points of zero deflection, whatever may be the load. These consequently are points of support of the panel with reference to which other points $x\ y$ undergo the deflection z.

It is to be observed that solutions (9) and (10) differ in effect from solution (21) Chapter V, in introducing into the solution unit moments of inertia which are not the same for x as for y. For let the solutions (9) and (10) be written in the following form:

$$z=\frac{q\,(1-k^2)}{48\,E}\left\{(x^2-a^2)^2 / I_1+(y^2-b^2)^2 / I_2\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . (11)$$

which is identical with (9) and (10) provided

$$I_1=(c_1+c_2)\,I / 2c_1=(a^{2n}+b^{2n})\,I / 2b^{2n}$$

$$I_2=(c_1+c_2)\,I / 2c_2=(a^{2n}+b^{2n})\,I / 2a^{2n}$$

Hence the modified moments of inertia I_1 and I_2 of unit widths of slab perpendicular respectively to x and y have the ratio to each other

$$\frac{I_1}{I_2} = \frac{c_2}{c_1} = \frac{a^{2n}}{b^{2n}}, \quad \text{or} \quad \frac{1}{I} = \frac{1}{2}\left\{\frac{1}{I_1} + \frac{1}{I_2}\right\} \ldots\ldots\ldots\ldots\ldots (2)$$

in which $1/I$ is the mean of the reciprocals of the moments of inertia I_1, and I_2

For $n>0$ and $a>b$ we have $I_1>I_2$, and the slab is stiffer lengthwise than crosswise. This decreases the deflections on the long side compared with those on the short side more and more the larger n becomes in such wise that they are equal in case $n = 2$.

By sufficiently increasing n the case may be treated where the stiffness along x is any required multiple of that along y. Solutions (9), (10), (11) then all refer to a slab which in case n is positive is stiffer and has more steel along x per unit of width of slab than along y, a case which is inconceivable in a homogeneous plate but perfectly realizable in a slab, especially in a slab with two way reinforcement. As shown by equation (13) the deflection on the stiffer long side will be reduced so as to become equal to that on the short side or in other words $z_{01} = z_{02}$ when the stiffness along x is so increased that $n = 2$ or $I_1/I_2 = b^4/a^4$; whereas in the mushroom slab in which $I_1 = I_2$ and $n = 0$ we find the ratio of the deflections at mid span to be $z_{02}/z_{01} = b^4/a^4$.

Assume that the longer side is $2a$, so that $a>b$; and designate the deflection at mid span on the longer edge where $x=0$ and $y=b$ by z_{01}, and by z_{02} the deflection at mid span on the shorter edge where $x=a$ and $y=0$ as shown in the diagram Fig. 74 showing a plan of a panel. Also let z_0 be the deflection at the panel center where $x=0=y$. Then in case I has the same value at all points of the slab, we find,

$$\left.\begin{aligned} z_0 &= \frac{q\,(1-K^2)\,(a^4b^{2n}+a^{2n}b^4)}{24\,E\,I\,(a^{2n}+b^{2n})}, &\qquad \frac{z_{02}}{z_{01}} &= \left(\frac{b}{a}\right)^{4-2n}, \\ z_{01} &= \frac{a^4\,b^{2n}}{a^4b^{2n}+a^{2n}b^4}\,z_0, &\qquad z_{02} &= \frac{a^{2n}b^4}{a^4b^{2n}+a^{2n}b^4}\,z_0, \end{aligned}\right\} \ldots\ldots (13)$$

It will be noticed that in case of square panels, where $a=b$, all values of n lead to identically the same results, viz.: those already discussed in Chapter V, where in fact the case of $n=0$ was treated. It was there applied to flat slabs devoid of stiffening girders other than those forming part of the slab itself, and having a moment of inertia no greater than the rest of the slab. In fact the moment of inertia at the side belts was taken as somewhat less than its mean value in those parts of the panel subject to the maximum applied bending moments. This case of (10) where $n=0$ has been shown by detailed tests to agree well with such a flat slab for panels of an oblateness as great as $b/a=0.75$.

In case $n=0$ the deflection at the mid spans of the longer and shorter sides of a uniform slab without stiffening beams are as the fourth powers of those sides, so that for $b/a=.75$ the deflection on the shorter side is not quite one third that on the longer side, while one fourth of the total load on the panel is theoretically carried to each edge by the shears, regardless of the relative lengths of the edges. Quite an important portion of the discussion of flat slabs was devoted to the investigation of how this shear is distributed in the beamless uniform slab without producing prohibitive stresses. Now it appears from (11) that in case $n=1/2$ the deflections at the panels edges are as the third powers of their lengths, just as occurs in beams with equal moments of inertia loaded with loads of the same unit intensity. It will be shown later that in case $n=1/2$ the shear at the edges of the panel is the same per unit of edge thruout both the longer and shorter sides.

The results just considered as well as those for intermediate cases will be found together with other matter in Table 1, page 284.

The discussion of the theory of the beamless flat slab, already referred to, was rendered possible by introducing into the formulas which were employed such values of the moment of inertia I as were shown to exist in the various parts of the panel by reason of the amount and position of the reinforcement, on the assumption that the action of the reinforcement could be replaced without noteworthy error by a uniform sheet of metal of the same weight as the actual reinforcement.

But, in case of a slab with stiffening beams, it would manifestly be incorrect to assume that the effective moments of inertia may be taken as approximately the same both along and across an edge. Hence along edges the value of I will be large compared with I elsewhere, and along the edges $y=\pm b$ the value of $(x^2-a^2)^2/I_1$ will nearly or quite vanish by reason of the largeness of I_1, the moment of inertia in the girder along the edge; and similarly along the edges $x=\pm a$ the value of $(y^2-b^2)^2/I_2$ will also vanish. It is in this manner that the slab becomes nearly level along the panel edges. As previously stated, the effect of this stiffening of the edges will need to be considered and allowed for in obtaining practical formulas for deflection and stress in case of a slab resting on side beams.

If we compare the value of z_0 in (11) in case $n=0$ with z_0 in case $n=2$ we find the latter is the smaller when $a>b$. As will be seen from (10), all sections of that surface made by vertical planes parallel to an edge are curves that are identical in shape, and such that $z_0=z_{01}+z_{02}$ in which z_{01} and z_{02} may be regarded either as the mid deflections of the edges or the mid deflections of the meridian curves of the surface made by vertical planes respectively parallel to the edges.

Now as n increases from 0 to 2, the deflection z_0 decreases somewhat, but the deflection z_{01} of the longer side decreases more rapidly than z_0, while the deflection z_{02} of the shorter side, or the crosswise deflection the short way of the slab actually increases. This appears from inspection of Tables 1 and 2. This may be regarded as due to the increase of the portion of the total load carried by the shears to the longer edges, so that the loading of the crosswise reinforcement increases with the increase of stiffness along x.

It appears from (10) that

$$\left.\begin{aligned} \frac{\delta z}{\delta x} &= \frac{q\,(1-K^2)\,b^{2n}\,(x^3-a^2x)}{6\,E\,I\,(a^{2n}+b^{2n})} \\[2ex] \frac{\delta z}{\delta y} &= \frac{q\,(1-K^2)\,a^{2n}\,(y^3-b^2y)}{6\,E\,I\,(a^{2n}+b^{2n})} \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots (14)$$

Hence $\dfrac{\delta z}{\delta x}=0$ at $x=0$, and at $x=+a$

and $\dfrac{\delta z}{\delta y}=0$ at $y=0$, and at $y=+b$

consequently the panel is horizontal across its meridian sections and across its edges, regardless of n.

$$\left.\begin{aligned} \text{Again}\quad \frac{\delta^2 z}{\delta x^2} &= \frac{q\,(1-K^2)\,b^{2n}\,(3x^2-a^2)}{6\,E\,I\,(a^{2n}+b^{2n})} \\[2ex] \text{and}\quad \frac{\delta^2 z}{\delta^2 y} &= \frac{q\,(1-K^2)\,a^{2n}\,(3y^2-b^2)}{6\,E\,I\,(a^{2n}+b^{2n})} \\[2ex] \text{Also}\quad \frac{\delta^2 z}{\delta x\,dy} &= 0 = \frac{\delta^4 z}{\delta x^2\,\delta y^2} \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots (15)$$

Let $I = i\,j\,d^2 A$ in which $1/A = 1/2\,(1/A_1 + 1/A_2)$ where $1/A$ is the mean of the reciprocals of the steel areas

$$\left.\begin{aligned} \text{Hence}\quad e_1 &= +\,i\,d\frac{\delta^2 z}{\delta x^2} = \frac{q\,(1 - K^2)\,b^{2n}\,(3x^2 - a^2)}{6\,EA\,j\,d\,(a^{2n} + b^{2n})} \\ \text{and}\quad e_2 &= +\,i\,d\frac{\delta^2 z}{\delta y^2} = \frac{q\,(1 - K^2)\,a^{2n}\,(3y^2 - b^2)}{6\,EA\,j\,d\,(a^{2n} + b^{2n})} \end{aligned}\right\}\ldots\ldots(16)$$

Note that e_1 is independent of y, and e_2 independent of x.

Designate the unit elongation along x at $x=0$ by e_{01}, and along y at $y=0$ by e_{02}, then $e_{02}/e_{01} = (b/a)^{2-2n}$ ....(17)

In case $n=0$, the elongations (and unit stresses) in the reinforcing rods are proportional to the squares of their lengths, but in case $n=2$ they are inversely as the square of their lengths, and the short cross rods are under the greater stresses. Other intermediate cases are shown in Table 1.

It will be noticed that the signs of the e_1 and e_2 change at lines of inflection situated at the same positions as in the mushroom slab giving negative bending moments across the edges of the panels twice as large as the positive moments across the meridian lines parallel to those edges. These negative moments do not in general require any reinforcement at the top of the slab across the panel edges because they are resisted sufficiently by the reinforcement running perpendicular to these edges, to which the entire negative moments are transferred laterally by twisting moments induced in the panel. This is the same kind of action that occurs in the Mushroom slab, by which the applied negative moments across the panel edges are carried laterally by twisting moments toward the columns, until they are held in equilibrium by the side belts running at the top of the slab over the columns.

The twisting moments here mentioned are not to be derived analytically from (3) because that equation contemplates the case of reinforcement distributed thruout the panel to resist negative as well as positive moments wherever they may occur. In case of distributed reinforcement the magnitude of the twisting moment in any vertical plane parallel to the edges per unit of width of slab is

$$n = \frac{E\,I}{1 + K}\cdot\frac{\delta\,z}{\delta\,x\,\delta y}\ \ldots\ldots\ldots\ldots\ldots(18)$$

therefore n = 0 by (17), and the only twisting moment in vertical planes parallel to the edges is that due to the action just stated, viz. that the steel required to resist negative moments is not to be found distributed across the panel edges but instead is concentrated in parallel positions at the edges of the panel. Such twisting moments induce shearing stresses in steel and concrete in vertical planes parallel to the edges, but in amounts and with a distribution such as not to require investigation here.

The intensities of the shearing stresses in (10) per unit of width of slab found by equations (9) and (15) Chapter V, are

$$\left.\begin{aligned} -s_1 &= \frac{\delta m_1}{\delta x} = \pm \frac{E\,I}{1-K^2}\,\frac{\delta}{\delta x}\left(\frac{\delta^2 z}{\delta x^2} + K\frac{\delta^2 z}{\delta y^2}\right) = \frac{q\,b^{2n}x}{a^{2n}+b^{2n}} \\ -s_2 &= \frac{\delta m_2}{\delta y} = \pm \frac{E\,I}{1-K^2}\,\frac{\delta}{\delta y}\left(\frac{\delta^2 z}{\delta y^2} + K\frac{\delta^2 z}{\delta x^2}\right) = \frac{q\,a^{2n}y}{a^{2n}+b^{2n}} \end{aligned}\right\} \quad \ldots\ldots (19)$$

Hence the shear at any distance x from the center is independent of y, and vice versa. The total shears at the edges $x = a$ and $y = b$ are:

$$\left.\begin{aligned} &\text{on edge } 2b, \quad -2\,b\,s_1 = 2\,q\,a\,b^{2n+1}/(a^{2n}+b^{2n}) \\ &\text{on edge } 2a, \quad -2\,a\,s_2 = 2\,q\,a^{2n+1}\,b/(a^{2n}+b^{2n}) \\ &\qquad s_2/s_1 = (b/a)^{1-2n}, \quad a\,s_2/b\,s_1 = (b/a)^{-2n} \end{aligned}\right\} \quad \ldots\ldots (20)$$

This distribution of shear on the edges when $n > 1$ is very different from that occurring in the uniform slab supported on columns where $n = 0$. The total shear on the four edges is in any case twice the sum of the shears on one short and one long edge as just obtained, and amounts to $W = 4\,q\,a\,b$, the total load on the panel.

TABLE 1.

n	0	1/2	1	3/2	2
z_{02}/z_{01}	$(b/a)^4$	$(b/a)^3$	$(b/a)^2$	b/a	1
e_{02}/e_{01}	$(b/a)^2$	b/a	1	$(b/a)^{-1}$	$(b/a)^{-2}$
s_2/s_1	b/a	1	$(b/a)^{-1}$	$(b/a)^{-2}$	$(b/a)^{-3}$
as_2/bs_1	1	$(b/a)^{-1}$	$(b/a)^{-2}$	$(b/a)^{-3}$	$(b/a)^{-4}$

The diagram in Fig. 74 shows a plan of a single panel arranged to show the notation for deflections, shears and elongations in connection with the formulas of this paper. Table 1 expresses the ratios of these quantities for various values of n between 0 and 2 inclusive, and Table 2 gives numerical values of such ratios in case $b/a = 0.75$. From these the truth of previous statements as to the relative magnitudes of various deflections, shears, elongations, etc., will be apparent.

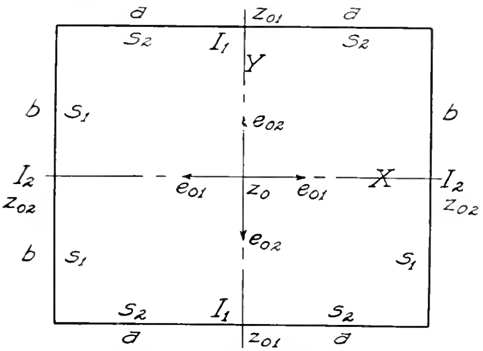


Fig. 74. Diagram of Notation.

TABLE 2.

$a/b=0.75$

n	0	1/2	1	3/2	2
z_{02}/z_{01}	0.32	0.42	0.56	0.75	1
z_{02}/z_0	0.24	0.3	0.36	0.43	0.50
z_{01}/z_0	0.76	0.7	0.64	0.57	0.50
e_{02}/e_{01}	0.56	0.75	1	1.33	1.77
s_2/s_1	0.75	1	1.33	1.77	2.37
as_2/as_1	1	1.33	1.77	2.37	3.16

5. Slab with Rectangular Panels Supported on Beams. It is usual practice for architectural and other reasons to make the beams on the long and short sides of panels of the same depth. In case the slab is uniformly loaded thruout assume that the steel will have equal unit stresses in both beams. In order that this may occur it is necessary to know the loads that are transmitted to the beams from the slab.

It may be shown that the load which comes upon the beams is nearly uniformly distributed when the slab is uniformly loaded thruout and that the load per unit of length is nearly the same for both side beams even for considerable variations of the relative stiffness of the side beams.

While the same relative distribution of load would continue to hold in case of the beams at the edges of a single loaded panel, the total loads upon these beams would be reduced to one half those of a slab loaded uniformly thruout. But the twisting of the beams due to the unbalanced load on one side of the beam would induce unequal stresses in the several reinforcing rods of the beam so that the stresses in the rods next to the loaded area might experience little or no reduction of stress from removal of load from all except one panel.

In case of three adjacent panels loaded with their long sides in common, the reduction of the deflection at their ends or short sides to about one half of that in case of uniform loading thruout will have some effect to increase the stresses in the longitudinal reinforcement to the relief of the crosswise reinforcement, which latter will be shown later to be under the more severe stresses. This case therefore does not need special consideration, and the case of uniform load thruout alone needs be provided for.

The fact that in case of full load on the slab all the side beams have practically the same unit load is due to the peculiar action of the beams in producing a kind of flexure in the slab which is very different in its nature from what occurs when it is supported on columns.

In case of column support and stiff heads, the surface of the slab which is convex upward about the column center has a uniformity of curvature that ensures cantilever action and concentric circumferential stresses which are practically uniform completely around the column. If any slight deviation from this actually occurs it may be assumed to be represented by a slightly greater extension of the cantilever area along the diagonals than along the sides, but the difference is practically negligible.

With deep side beams and a comparatively flexible slab all this is changed under heavy loads. Large parts of what in case of no side beams would be cantilever area is changed by the introduction of beams into hollow valleys running up toward the column centers at an angle of 45 degrees with the beams. The bending moments across these valleys which in the simple cantilever slab were negative have changed sign, and this change has profoundly modified the mechanical action of the slab. The surface about the column center instead of being practically uniformly convex now has four valleys and four ridges radiating from the column, and it is evident as previously stated that no expression is possible which is of algebraic form merely, that will express such relations. It would require certain trigonometric expressions of multiple angle about the column center to express this scallop-shaped surface.

At the diagonal center of the panels, however, conditions are sufficiently unchanged by the side beams to admit of approximate algebraic expression of the mechanical and geometrical relations.

The condition of the slab may be regarded as having been brought about from the initial condition of a uniform slab supported on columns with the usual side and center deflections by the deformations which would be produced in it by stiffening or jacking up the sides sufficiently to reduce their deflections by two thirds or three quarters of their initial amounts. This would bend the edges upward enough to form the valleys before mentioned, and at the same time greatly reduce the width of the saddle across each side. These reductions make radical changes in the nature of all the curvatures near the sides which, as has been said, cannot be readily expressed algebraically.

But certain aspects of these phenomena admit of satisfactory graphical expression. It is known from a considerable body of experimental work on slabs with wall supports or deep beams at the panel edges that for heavy loads the valley lines will occur at angles of practically 45 degrees with the sides regardless of the relative lengths of the sides or of the continuity of the panels. The valley lines are of necessity lines of maximum moments positive across them and consequently define at the same time the lines of zero shear. The loads that go to the sides may consequently be computed approximately from consideration of these lines across which no loads are transmitted.

Draw lines from the ends of the short sides of the panel at angles of 45 degrees with them thus forming two right angled isosceles triangles. The apex of each of these triangles lies on the meridian section of the panel made by a vertical plane midway between the long sides of the panel. The areas of these two triangles and the two halves of the remaining area of the panel lying on either side of the meridian section consequently show approximately the relative loads carried to the beams on the short and long sides of the panel. The load on each short side beam will be $W b \;\; 2a$ and on each long beam $W [b^2 + 2 (a - b) b] \;\; 2 a b$ giving us a total load of $2 W$ on the four side beams of which W comes from the panel itself and W from the surrounding panels. This distribution of loads may be assumed to be exact in case of rigid wall supports and approximate for stiff beams. The relative values of these quantities for varying proportions of sides are shown graphically in the diagram Fig. 75. Fig. 75 also shows the relative values on the assumption that the panel loads are uniformly distributed along the four sides as expressed in equation (21) given later.

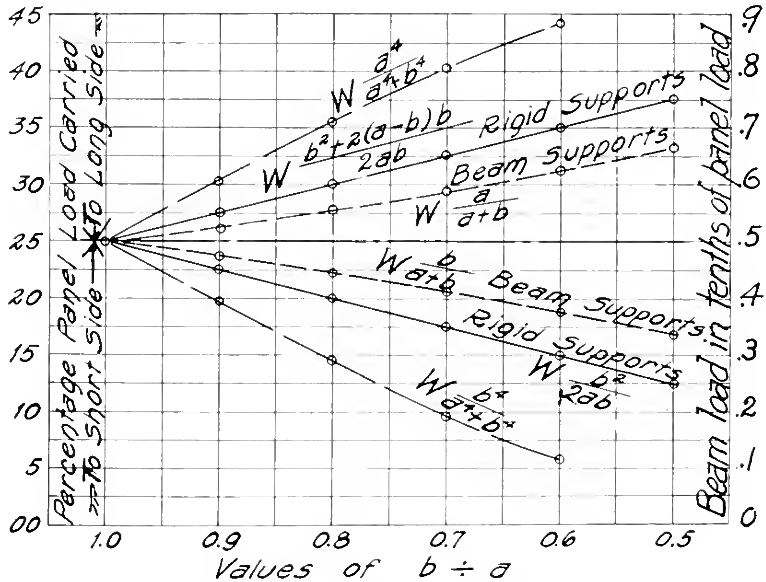


Diagram of Loads on Side Beams on Various Assumptions
Fig. 75

The general agreement of these two assumptions with each other is sufficient to enable us to adopt a uniform distribution on the perimeter as a convenient and closely correct basis of distribution. It might, however, at first be thot that the loads would be more concentrated toward the middle of the sides, but the twisting moments such as have already been discussed in case of mushroom slabs largely prevents such inequality. These twisting moments transfer the applied negative moments along the sides even tho there is little or no reinforcement in the top of the slab across the beams to resist them and apply them to the side beams at the columns, which beams are so stiff that it seems to be of little importance whether the reinforcement which is carried across the side beams is at the top surface or not. The exact distribution, however, is dependent to some extent upon the relative stiffness of the side beams. The assumption of a uniform distribution is, however, sufficiently exact for practical purposes.

It would seem appropriate at this point to refer to another proposed distribution of loading which was put forth by Marsh in his Reinforced Concrete, pages 282 et seq. where it was attempted to be shown on the basis of beam strip theory that the loads on the sides are proportional to the fourth powers of the sides. This result has been incorporated into building codes and text-books almost universally. But in the opinion of the writers has not a scintilla of evidence to support it either in correct theory or experiment.

Any correct theory would have to find a separate equation for the curvatures of the parts of the panel into which it is separated by the valleys which are known to exist. These valleys are of such a nature as to prevent continuity of beam action such as was assumed to exist in order to arrive at these erroneous conclusions. Since it has been sufficiently shown that no algebraic expression is possible which will exactly express the relationship here existing it is clear that were this result correct for the case of supporting walls it could hardly hold at the same time for supporting beams also. The wide divergence of this theory from the previous nearly concordant estimates appears from the diagram in Fig. 75 where the relative beam loads on this theory are shown in a manner comparable with the uniform distribution here adopted. Such a theory would render longitudinal reinforcement practically useless in any slab whose width is more than a few percent less than its length. The inherent improbability of this may be regarded as a sufficient disproof of this so called law.

This is evident, because the fact of the existence of 45° valleys at which maximum moments exist seems in itself to make it certain that the steel stresses diagonally across these valleys, or in other words the stresses parallel to the long and short sides of the panel must be equal. This would require practically the same reinforcement per unit of width of slab lengthwise as crosswise of the panel at the valleys where the stresses may be regarded as critical. It would thus appear that the reinforcement instead of being largely superfluous longitudinally should be practically of the same amount per unit of width lengthwise and crosswise of the slab, a requirement in the most striking contrast with the common but erroneous theory just mentioned. As the loading of such a slab becomes more severe and the point of failure is approached the stresses at the valleys apparently increase more rapidly than elsewhere and the phenomena accompanying them become more pronounced. This ultimate condition is the controlling condition and due provision for it is essential in correct design.

If the unit shear at the edges of the panel be taken as constant and the width of the beams be disregarded then the load that is supported on each unit of length of a side beam of a panel in a slab which is uniformly loaded thruout with a total load of W per panel will be twice as much as comes to that side from the panel itself. The total perimeter is $4\ (a + b)$ and the total load on the sides is $2W$. Hence $w = W/2\ (a + b)$ is the load per unit on the side beams.

$$\left.\begin{array}{l} \text{The total load on long side} = 2aw = W_a = Wa/(a + b) \\ \text{" " " " short " } = 2bw = W_b = Wb/(a + b) \end{array}\right\} \ldots (21)$$

The deflection formulas for these continuous beams will be

$$\left.\begin{array}{l} 24\ E\ I_a z_a = w\ (x^2 - a^2)^2 = W\ (x^2 - a^2)^2/2\ (a + b) \\ 24\ E\ I_b z_2 = w\ (y^2 - b^2)^2 = W\ (y^2 - b^2)^2/2\ (a + b) \end{array}\right\} (22)$$

Let I_a and I_b denote the moments of inertia of the long and short side beams respectively. Let these side beams be of equal depth as usually designed and let them have the same unit stress in the steel. Then $I_a/I_b = A_a/A_b = a^2/b^2$ . . . . (23) since the moments of resistance are proportional in that case to the cross sections A_a and A_b of the reinforcements and the applied moments at mid span are proportional to the squares of the spans.

$$\text{By (23)} \quad \frac{I_a}{I_a + I_b} = \frac{a^2}{a^2 + b^2} \text{ and } \frac{I_b}{I_a + I_b} = \frac{b^2}{a^2 + b^2} \ldots (24)$$

$$\left.\begin{array}{l} \text{Hence } z_a = \dfrac{W\ (a^2 + b^2)\ (x^2 - a^2)^2}{48\ E\ (I_a + I_b)\ a^2\ (a + b)} \\[2ex] \text{and } z_b = \dfrac{W\ (a^2 + b^2)\ (y^2 - b^2)^2}{48\ E\ (I_a + I_b)\ b^2\ (a + b)} \end{array}\right\} \ldots\ldots\ldots\ldots (25)$$

These equations permit us to compare the deflections at mid span z_{oa} and z_{ob} with each other and with the deflections that occur in mushroom slabs at the same points.

$$z_{ob}/z_{oa} = b^2/a^2$$

In case, $b/a = 0.75$, we have $z_{ob}/z_{oa} = 0.565$.

Compare the deflection at mid span of the long side beam z_{oa} with the mid span deflection z_{ob} in the long side belt of a Mushroom slab having the same total load per panel and the same amount of steel in its four belts as in the two side beams, that is $A_a + A_b = 4A_1$. By (30) Chapter V, the deflection at mid span of the long side of the mushroom panel is

$$z_{01} = \frac{(1 - K^2)\, W\, a^3}{48\, E\, i\, j\, d_1^2\, 6\, A_1}$$

By (25) above the deflection at mid span of the long side beam is

$$z_{oa} = \frac{W\,(a^2 + b^2)\, a^2}{48\, E\, i\, j\, d^2\,(A_a + A_b)\,(a + b)}$$

$$\therefore \frac{z_{o1}}{z_{oa}} = \frac{(1 - K^2)a(a + b)d^2}{1.5\,(a^2 + b^2)\, d_1^2} = \frac{(1 + b/a)}{2\,(1 + b^2/a^2)} \left(\frac{d}{d_1}\right)^2 \quad \ldots\ldots\ (26)$$

If $d/d_1 = 2.5$, $\quad (d/d_1)^2 = 6.25$

" $d/d_1 = 3$, $\quad (d/d_1)^2 = 9$.

" $b/a = .75$, then $z_{o1}/z_{oa} = 0.56\,(d/d_1)^2$

In case $(d/d_1) = 2.5$, then $z_{o1}/z_{oa} = 3.5$.

" " $(d/d_1) = 3$, " $z_{o1}/z_{oa} = 5$.

from which it appears that the deflection of the side beam is from one third to one-fifth that of a mushroom slab, dependent upon relative depths of slab and beam.

It should be noticed that reinforcing steel of the slab which is near by and parallel to either side beam lies at a level compared with that in the beams at which it is able to offer resistance to the negative moments in the beams near the columns. In particular it largely prevents the propagation of moments across the column heads due to unbalanced loads such as occur with single or alternate panels loaded. It therefore assists that part of the beam reinforcement which is near the top of the slab over the columns.

Having treated the side beams consider now the deflections of the slab supported on side beams. It is evident from the preceding discussions that while the curvatures of the meridian or central sections of the slab by vertical planes parallel to the sides are greatly increased near the sides the curvatures of these sections near the panels centers are not so much changed. The changes which do occur, however, are such as increase the curvature of the cross-wise section and flatten that part of the lengthwise meridian section which lies between the apices of the valleys. This is equivalent to increasing the ratio $z_{02} : z_{01}$, in (11) if we suppose that the same form of (10) will approximately represent the actual surface for the central portion of the slab. It is evident that the surface $n = 2$ will not make sufficient allowance for the effect of the valleys, because with $n = 2$ the surface would have the deflections at midspans of the long and short sides equal, whereas the actual surface has a deeper crosswise deflection than this by reason of reduction of its saddles and also has a flatter central portion between valley apices.

Designate the deflection of the panel center below the mid span of the long side beam by D and assume, since z_{02} from (13) is not the total deflection of the center below mid span of the long beam, that an approximate value may be obtained by increasing this value of z_{02} in the ratio of the sides $a : b$.

Assume that the central deflection z_{02} for $n = 2$ will be increased in this manner to

$$D = z_{02}\, a : b = \frac{(1 - K^2)\, q\, a^5\, b^3}{24\, E\, I\, (a^4 + b^4)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

an assumption that will need to be verified by experiment as it seems in fact to be so verified.

This approximate expression for the deflection D is intended to express the difference of level between the mid span of the long side beam and the panel center. It is not thot possible, however, to obtain any closely approximate expression for the difference of level between the mid span of the short side beam and the panel center because of the great discontinuities of curvature that occur at the apices or points of intersection of the valleys.

In order to obtain a practical and convenient form of this proposed deflection formula in which the percentage of reinforcement parallel to each side is assumed to be the same let

$$I = i\, j\, d^2\, A \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

in which A is the cross section of one unit of width of a uniform sheet of metal whose weight is equal to that of the reinforcing rods.

Take the case of two way reinforcement parallel to the edges of the panel

Let $\Sigma A_1 =$ the total across section of all the rods running the long way of the panel.

and $\Sigma A_2 =$ the cross section of these running the short way across the panel.

then $\Sigma A_s = \Sigma A_1 + \Sigma A_2 =$ the total right cross section of slab steel in square inches.

Let $A_0 =$ mean right cross section of slab steel per unit of width of each single belt.

then $A_0 L_2 = \Sigma A_1$, and $A_0 L_1 = \Sigma A_2$.

Hence $A_0 (L_1 + L_2) = \Sigma A_s$, and $L_1 L_2 =$ area of panel.

Again $A_0 L_1 L_2 =$ total volume of steel in each belt,

and $V = 2A_0 L_1 L_2 =$ total volume of both belts.

Therefore $V/L_1 L_2 = 2A_0 =$ thickness of equivalent uniform sheet.

But by definition $A = V/L_1 L_2$, hence $A = 2A_0$.

Hence $\Sigma A_s = \frac{1}{2} A (L_1 + L_2)$,

$$\text{or } A = \frac{2 \Sigma A_s}{L_1 + L_2} = \frac{\Sigma A_s}{a + b} \qquad (29)$$

Substitute (29) in (28), and (28) in (27),

and put $4\, q\, a\, b = W$, $K = 0.5$, $i = 2/3$, $j = 0.89$, $E = 3 \times 10^7$, and $(1 + b^4/a^4)/(1 + b/a) = b/a$. This last is an approximate numerical value, true for $b/a = 1$, and $b/a = 0.75$. We then have the practical deflection formula

$$D = \frac{W L_2 L_1^2}{1.82 \times 10^{10}\, d^2\, \Sigma A_s} \qquad (30)$$

an expression in which the numerator may also be written $W C_2 L^3$, where $C_2 = L_2/L_1$. The deflections D consequently vary directly as $C_2 = L_2/L_1 = b/a$.

The empirical formula in Turner's Concrete Steel Construction pages 55 and 56 and found also on page 62 above gives practically the same deflection as (30) in square panels, and slightly greater deflections for $a > b$.

Computed deflections may be compared with the following test data 1, 2, and 3, taken from Turner's Concrete Steel Construction, pages 56 and 57:

Deflections

Building	Computed by (8)	Observed
1 Minneapolis Paper Co	0.298″	0.30″
2 Smythe Block	0.1148″	0.11″
3 Minneapolis Knitting Co	0.1696	0.167
4 Minneapolis Armory	0.657	0.75
5 Nicollet Associates Bldg	0.2497	0.25

The Minneapolis Armory panel had one edge on a wall with a wall above and three edges on beams with the steel raised slightly; panels 20′ by 20′ from center to center of girders and 19′ clear spans. Thickness 5.25″ at center, 6.5″ at edge. Reinforcement $\frac{1}{2}$ inch rounds at 9″ between centers. Load 40 pounds per square foot. Observed deflection $\frac{3}{4}$″.

$$D = \frac{160{,}000\ (240)^3}{1.82 \times 10^{10} \times 4.25 \times 4.25 \times 52 \times .196} = 0.657$$

The discrepancy in case of this panel is due to several causes: It was not built into the wall it rested on. Unusually large variations of thickness occurred in it. By reason of scant thickness it was over reinforced and stresses in concrete were excessive.

The Nicollet Associates Building: Panels 20′ 5.5″x24′ 2.5″. Thickness of rough slab 7″ and 1.75″ strip fill. Mean thickness $7\frac{7}{8}$ inches. Reinforcement 7/16″ rounds, hard grade high carbon steel, 7″ center to center in the middle third of spans, and 9″ center to center in the rest of the span. Load 200 pounds per square foot. Deflection $\frac{1}{4}$ inch.

$$D = \frac{99058 \times 245.5\ (290.5)^5}{1.82 \times 10^{10} \times 6.72 \times 6.72 \times .15 \times 67} = .2497''$$

We consequently feel justified in asserting that (30) is in good agreement with experimental data.

Now assume that by reason of the increased curvature the greatest elongation e_{02} at the panel center is increased in the same ratio a/b as the center deflection. Then the greatest unit stress in the cross reinforcement at the center derived by taking $n = 2$ will be given by the expression

$$\left.\begin{aligned} f_{s2} &= E\, e_{02}\, a/b = \frac{q\,(1 - K^2)\, a^5\, b\,(a + b)}{6\, j\, d\,(a^4 + b^4)\, \Sigma A_s} \\ \text{or}\quad f_{s2} &= \frac{W\, L_1^2/L_2}{57\, d\, \Sigma A_s} = \frac{W\, L_1/C_2}{57\, d\, \Sigma A_s} \end{aligned}\right\} \quad \ldots\ldots\ldots (31)$$

provided the same substitutions be used in deriving this final practical formula as were employed in obtaining (30).

The steel stresses obtain by (31) vary as $1/C_2 = L_1/L_2 = a/b$.

Applied to the buildings previously mentioned in connection with measured deflections, (31) would give

Building	f_{s2} Computed by (31)
Minneapolis Paper Co.	22500 lbs. per sq. in.
Smythe Block	4287 " " "
Minneapolis Knitting Co.	9190 " " "
Minneapolis Armory	15500 " " "
Nicollet Associates Bldg.	8890 " " "

The stresses computed by (31) are somewhat smaller than those given by Turner's empirical formula for safe design in his Concrete Steel Construction page 55, and also found on page 62 above.

It is not certain, however, that the stresses across the line joining the apices as above computed are greater than or even so great as those across the valleys at points near the apices. It is believed, however, that any larger stresses at such points will by reason of their concentration yield sufficiently to bring into play nearby rods in a way that will afford relief from any dangerous stresses.

Without having been able to make exhaustive tests sufficient to completely establish the practical accuracy of the theoretical evaluation of stresses as here proposed the writers nevertheless have highly corroborative experimental evidence in support of the substantial correctness of equation (31) for stresses.

6. Steel Ratios and Minimum Thickness of Slab. Since slabs should be so designed that the steel would yield before the concrete, it is important to determine how large a percentage of steel may be introduced without passing this limit. This, however, is dependent upon the relative thickness of the slab. In case of the continuous slab, the ratio of the depth to side span involves somewhat different considerations from that of the ordinary beam since the minimum thickness of the slab on a diagonal span is relatively less than is permissible in beam construction. That part of the thickness of the slab which serves as fireproofing and which is constant for any given size of rods bears a greater ratio to the depth than is the case in beam construction. Furthermore, the depth necessary for proper embedment over the support and the massing of the steel at the support reduces the effective lever arm of the steel in the cantilever to j_3d, which reduces it to a relatively greater extent than at mid span where the effective depth becomes j_1d as it does in the case of one way slab construction also. This reduction in effective depth for fireproofing, embedment and massing of the steel is, however, a constant for a given size and arrangement of reinforcement and for the continuous slab it is found that we may accordingly take this constant as follows:

2 " for 3/8" rod reinforcement (or smaller sizes)
$2\frac{1}{4}$" " 7/16" " "
$2\frac{1}{2}$" " 1/2" " "
3 " " 5/8" " "

The limiting minimum thickness of slab then may be stated as $L/48$ plus the constant just tabulated for the various sizes of rods.

In case square twisted rods are used, the same constant should be used for 3/8" square twisted as for 1/2" rounds, and for 1/2" square twisted the same constant as for 5/8" rounds. This will give us a satisfactory minimum thickness of slab for all spans of this type, L being the longer direct distance between column centers.

For square panels supported on beams or walls at the edges, the constant for the various size of reinforcing rods should be one inch less than that tabulated above, and the minimum thickness taken as $L/48$ plus this new constant.

Where the panel is rectangular and not square, and supported on the sides by beams, the same rules will hold by using $2(a^2+b^2)/(a+b)$ in place of L for the span of the square panel, in which a and b are the half spans in the two directions respectively.

Determination of reinforcement required in slabs. In case of beams with different values of N the ratio of depth to span, the proper values of the steel ratio p were obtained in Chapter III page 86. In case of slabs the ratios so obtained should not be exceeded, but in reckoning the reinforcement of slabs account must be taken of the fact that the steel is in multiple directions so that the reinforcement per belt will be only a fraction of the total permissible steel.

In the continuous slab with four way reinforcement and Mushroom heads, having given the ratio of the thickness to span taken center to center of columns, determine the percentage of steel from the diagram for beams, page 86, and divide this by 2 to find the limiting percentage of steel for each belt.

In the slab with two way reinforcement, supported on columns, the side belt may be made a little heavier than in the four way belt type, .6 of the percentage given for beams being permissible.

In slabs not reinforced with a supplemental cantilever frame, but with a depressed head instead, the minimum thickness should not be less than the minimum thickness allowed for the former type, but the steel ratio may be made .55 that for beams instead of .5 which is permissible with mushroom heads.

In square slabs supported on beams or walls the percentage of steel in the strip occupying the middle third of the panel each way should not exceed that for beams of similar proportions of depth to span. In case the panel is a rectangular panel, the percentage of steel should not exceed that for square panel having a side equal to $2(a^2 + b^2)/(a + b)$ which we have previously used in determining the minimum thickness of span for this form of panel.

With the above interdependent limitations as to the minimum thickness and maximum steel ratio in mind the designer has to determine how far in any given design he shall deviate for any reason from them. By making N smaller and the slab consequently thicker he will be able to reduce the deflection to such figures as may be desired or required. But when such deviations are made the proper relationship between steel ratio and slab thickness is still to be determined from the diagram page 86 as has just been done for the case of minimum thickness.

The erroneous requirement is made in some building codes that the maximum deflection of a slab under test shall be a no greater percentage of length of span than that permitted or allowed for a deep beam. A deep beam might be broken and seriously injured under a deflection which would do no damage whatever to a long thin slab. A certain degree of stiffness, however, is required under working loads wherever there are partitions in a building which may be damaged by undue deflections. The deflection in a thin slab should not exceed 1/700th of the span under working load, and preferably less. In an office building or an apartment house, where there are partitions which might be cracked by deflection, the deflection should be limited for working loads to 1/1000th of the span.

The inappropriateness of any requirement limiting the deflection to a given fraction of the span will appear from the following investigation of the elementary relations between the deflections and stresses in the steel of reinforced beams, since similar considerations apply to slabs. Using the notation previously employed we obtain from the well known expression for the steel stress f_s, the equation

$$W\ L\ (1 - k)\ d = n\ I\ f_s,$$

in which $n = 4$, 8, 8 or 12 for four different cases, namely simple and restrained beams carrying a load W either concentrated at mid span or uniformly distributed.

Again the well known expression for the deflection D gives us the equation.

$$W L^3 = m E I D$$

in which m = 48, 76.8, 192 or 384 for the four cases mentioned.

By combining these equations to eliminate W the following relation between f_s and D is obtained:

$$D = \frac{n L^3 f_s}{m (1 - k) d E}$$

This shows that the maximum deflection D in beams of different depths d, and the same length and unit steel stress f_s varies as 1/d while the coefficient n and m make further wide variations, which make it absurd to limit the permissible deflection to any given fraction of the span.

For test loads, however, greater deflections are permissible than for the working loads discussed previously. Actual test under common conditions of partial restraint shows that in a span of forty times the thickness or depth of the slab *i. e.*, $L = 40\ d$, a deflection equal to $L/250$ will not materially injure the construction. Consequently in case $L = 10\ d$, a deflection of $L/1000$ should not be exceeded.

Since a practical constructor desires to test only to safe limits rather than to anything approaching the ultimate limit he will not object to regulations twenty percent more rigid than those just mentioned provided the time of making the test is not less than four months after casting the concrete.

7. Size and Spacing of Rods in Flat Slab Construction. The plate action which we have been treating, brought about by indirect stress, depends for its efficiency upon the dissemination of the steel through the mass of the concrete. Large rods and wide spacing should accordingly be avoided. Where the steel percentage is very small it sometimes happens that 5/16 or 3/8 inch rods, the smallest practical sizes, will be spaced as far apart as eight or nine inch centers. While reasonable results will be secured with such spacing where the percentage of steel in the belt is as small as .22, where the percent of steel is larger the spacing should be closer. Three-eighths rods 4″ centers are preferable to 7/16″ rods at six inch centers and far better than half inch rods at eight inch centers.

One of the common errors in flat slab design is the use of such rods as 1/2 inch or even 5/8 inch from eight to twelve inches between centers with the expectation of securing results in keeping with a more scientific and uniform dissemination of metal.

8. Rectangular Panels of Hollow Tile and Concrete with Two-Way Reinforcement and Supported on Side Beams. This combination has been used primarily with the idea of reducing dead weight and of securing greater depth of slab. The construction consists in reality of a network of narrow concrete beams or ribs filling the spaces between hollow tile blocks which usually are about 12 x 12 inches horizontally. These beams should preferably be not less than five inches in width and should be reinforced with at least two rods each, one at the bottom thruout and lapping completely over the supporting beams while the other is bent up over the top and given a lap of at least a foot or two beyond the beams.

It is customary to put the same reinforcement in each of the ribs regardless of its position in the slab, thus giving a uniform reinforcement thruout the slab.

Such a construction will be properly figured as a beam construction, treating it as continuous so far as dead load is concerned provided it has ample laps, and as a simple beam as respects its live loads.

In one form of two-way tile construction the ends of the hollows in the blocks are closed by U-shaped pieces of terra cotta thus leaving a rectangular net-work of crisscross channels to be filled in with reinforced concrete which also spreads over the entire top surface in a layer several inches in thickness. This forms a rectangular net work of T-beams or ribs, but the continuity of the lower part of the slab is so imperfect as to transmit no more than a negligible amount of indirect tensile stress from rod to rod, especially under heavy loads. What it may do under light loads is of no account in design. Tho some little coaction might possibly occur thru bond shear at the intersections of the ribs, the tile blocks are as a rule twelve inches in width which puts the reinforcing rods some sixteen to seventeen inches apart so that the structure with this wide spacing is not sufficiently fine grained or uniform in texture to approximate in effect to the properties or characteristics of a homogeneous plate. It seems conservative therefore to treat this combination on the beam strip theory as provided in most building codes.

In case of a live load W_1 uniformly distributed on a square panel, consider the four triangular areas into which it is divided by the diagonals. Two of these may be taken to be transmitted to the sides by one set of ribs, and two by the other. Since the center of gravity of each triangle is at a distance $L/6$ from the edge and its load is $W_1/4$ the total applied moment across the slab at mid section is

$$\tfrac{1}{4} W_1 (L/2 - L/3) = W_1 L/24.$$

instead of $W_1 L/16$ as would occur in case the load $W_1/2$ carried by this parallel set of ribs were uniformly distributed along them. The several parallel ribs do not resist this moment equally, but for a considerable portion of the width there is little difference in this respect. The basis of computation, being that of a simple beam, is so liberal that the applied moment $W_1 L/24$ is ample.

Again, in case of a dead load of W_2 on the panel the building codes of various cities prescribe that it shall be computed on the same basis as the live loads. But since the results so obtained are not thot to be in good accordance with experiment it would seem preferable to apply the equations already derived for the case of a concrete slab supported on side beams to this case. Stiff side beams cause the formation of valleys in this case as they do in any slab supported in this manner. But with tile the reinforcing rods in the two directions act practically independently of each other at heavy loads. It will therefore not be possible to assume that they in effect form a single sheet of steel as has been done where belts of rods cross each other in solid embedment. The steel will in this case therefore be only one half as effective. Moreover the value of K will be so small that its square is negligible.

Make these modifications in equations (30) and (31) by first multiplying them by 4/3 in order to remove the effect of the embedment which was included in them and then replace ΣA_s by $\frac{1}{2} \Sigma A_s$ because the belts act independently, and we have

$$D = \frac{W_2 L_2 L_1^2}{.68 \times 10^{10} d^2 \Sigma A_s} \quad \ldots \ldots (32)$$

$$f_{s2} = \frac{W_2 L_1^2 / L_2}{24 j d \Sigma A_s} \quad \ldots \ldots (33)$$

For a square it will be noticed that on this basis the unit stress per pound of live load is twice that per pound of dead load in case of $A_1 = A_2 = \frac{1}{2} \Sigma A_s$, because the former is taken to be carried by simple beam action and the latter by beams that are mutually restrained.

We may then on analogy of (33) write the total unit stress as follows:

$$f_s = (W_1 + \tfrac{1}{2} W_2) \frac{L_1^2 / L_2}{12 j d \Sigma A_s} \quad \ldots \ldots (34)$$

in which the effect of the relative length of the sides is introduced for both live and dead loads in the same manner. It is not advisable in this construction to make L_2/L_1 less than 0.8.

The observed deflections under a moderate test load of any amount W_2 will be less than those computed from equation (32), and the results observed will not reach those computed until the stress in the steel reaches approximately eight-tenths of its yield point value.

9. Expanded Metal as a Form of Reinforcement for Slabs. Expanded metal is formed by slashing sheets of steel in such manner that it may be stretched or expanded laterally to form a diamond mesh. The junction of one mesh to another beside it is called a bridge, and in forming the bridge the metal is not only distorted in a lateral direction but twisted at the bridge. The diamond meshes thus formed are usually exactly or approximately twice the length of the short diameter of the mesh. This proportion gives a right cross section of metal half as great laterally as longitudinally and in view of the twisting the stiffness laterally is very small indeed in comparison with the stiffness in a longitudinal direction. When embedded in concrete expanded metal forms a very convenient means of reinforcement for short spans and light loads. It must be placed with the long direction of the mesh in the line of greatest stress since if it were placed in a lateral instead of in a longitudinal direction not only would the effective cross section of the metal be reduced one half but its efficiency would be further reduced one half because of the unfavorable angle of inclination of the strand to the direction of the stress and even this value is further much reduced because the consecutive diamonds or meshes do not pull in line laterally on account of the offset at the bridge. The net effect of the form of the mesh, the reduced section of metal resisting strain in a lateral direction and the offset at the bridge, accordingly is to reduce the lateral efficiency of the sheet as a means of reinforcement to less than 25% of its value longitudinally and probably to about ten or twelve percent in view of the bridge. All catalogues published by manufacturers of expanded metal give explicit instructions that the long dimension of the diamond shall be placed in the direction of the greater stress wherever this material is used as a means of reinforcing concrete slab construction.

The coaction of expanded metal with concrete differs greatly from that of a diamond mesh formed of straight rods in two layers crossing each other diagonally for the reason that in the case of the rods shear resistance between the two layers is furnished by bond shear between the concrete and the respective rods, whereas, in the case of expanded metal the bridge forms a rigid connection and this shear is developed largely, or almost entirely in the steel itself. Thus the coaction of expanded metal with concrete differs radically from that of rod reinforcement in the manner in which energy is stored by the deformations of the concrete and metal. The bond between expanded metal and the concrete is formed by interlocking blocks formed within the mesh of the metal and as stress is brought upon the metal in the direction of the length of the sheet these blocks are compressed laterally and the tendency of one mesh to slide upon another is prevented by the rigid steel section of the bridge.

Efforts were made early in 1901 to construct flat slab floors reinforced with expanded metal. Sheets of expanded metal were laid nearly covering the bottom of the slab from column to column and sheets were placed at the top of the slab over the columns in the form of a cross, and it is claimed that sheets were also placed in parallel position over column tops. Arranged in the form of a cross the sheets were not effective as a circumferential cantilever reinforcement. Since the ends of the sheets forming the cross were arranged to project far beyond the over lapping area it was manifestly impossible to approximate plate action in this manner so that the few structures so erected developed no greater carrying capacity then would be expected on beam strip theory.

Expanded metal in coaction with the concrete, adds, however, a greater increase in strength than the mere cross section and yield point value of the metal would indicate. This would appear to be due to the storing up of energy in the slab by the lateral compression of the concrete thru longitudinal strain in the mesh.

Wire woven fabrics also are often a most convenient form of reinforcement and frequently the best possible to be used for wrapping as reinforcement for beams, light slabs, roof work and the like, and altho their cost per pound is greater than rods convenience in handling and placing may more than offset this difference. The fact that thoro dissemination of metal in small units thru the concrete is conducive to better results than the use of the same sectional area in larger units makes their use for many purposes almost indispensible.

CHAPTER VIII

REINFORCED CONCRETE COLUMNS

1. General Considerations. The requirements for suitable design for reinforced concrete columns in building construction may be briefly stated as follows:

First, that the longitudinal reinforcing metal should be toward the outer portion of the column in order to properly resist any tendency to bend or deflect.

Second, that the bars should be banded or tied together to maintain them in their desired position and add toughness to the column.

Third, that the bands or ties should not so cross the core as to interfere with placing the concrete and securing a monolithic solid core.

Fig. Type A, shows one of the old forms of Hennebique type of column. In this type the principal reinforcement consists of heavy vertical or longitudinal bars tied across laterally from one to the other by comparatively small ties. It will be noted that these ties cross and recross the core of the column, and require considerable care in filling to make sure that there are no voids in the finished work.

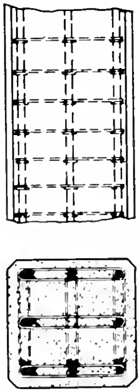


Fig. Type A Column

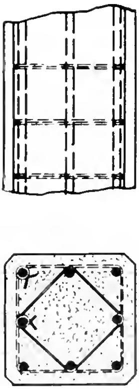


Fig. Type B Column

Cases have occurred with this type where the concrete has been arrested part way down in pouring the column and on removal of the form an open space was found of perhaps as much as two feet between the concrete above and below, so that the load above was carried by the vertical bars only. Evidently such an arrangement of metal is somewhat dangerous, but with unusual care it may prove satisfactory from the standpoint of strength if the work be properly executed.

Fig. Type B, shows an improved form of tying together the eight vertical bars forming the vertical reinforcement with horizontal ties in the form of squares, one inscribed within the other. The advantage of this type over that previously shown lies in the fact that the central core of the column, or inscribed square, is clear and unobstructed thruout.

Fig. Type C, shows a column reinforcement consisting of four vertical rods with wrapping or ties holding them together at intervals. This is suitable for very light loads where the concrete is more than sufficient to take the entire compression without excessive stress.

Fig. Type D, shows a column section of the Considère type in which the vertical rods are hooped with spiral reinforcement. Considerable work has been executed using hooped columns that omit the vertical steel. This, as the authors view it, is a very grave mistake.

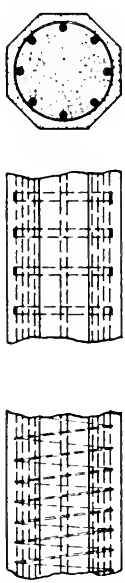


Fig. Type D Column Considère Type.

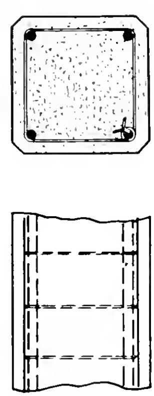


Fig. Type C Column Suitable for Light Loads Only.

Hooping may be of two types: First, a spiral coil in which the wire is wound around the core of the column in the form of a continuous spiral, and second that in which separate independent hoops are placed at intervals and attached to the vertical reinforcement.

The strength of Types A, B, C, and D, all depend, first upon the strength of the concrete, second, upon the amount of vertical steel used, and, third, upon the amount of ties or hooping holding the rods in position and bringing lateral restraint upon the concrete.

Theoretical formula based on the ratio of the moduli of elasticity of concrete and steel alone cannot be depended upon for a satisfactory solution of the problem presented by the third element noted and we must depend largely upon experimental investigation to determine reasonable and safe practical values to use for our working stress.

In deciding upon these values we need to consider the column, first, from the standpoint of its ultimate strength in the finished building, second, from the standpoint of its strength and safety during construction, and third, from a consideration of the relative values of the various types in securing strength at a minimum cost.

Type D, with a proper proportion of vertical steel combined with the hooping ranks first, from the standpoint of safety and economy.

Type B, second.

Type D, with hooping but with no vertical steel third; and types A and C fourth.

It may be stated that type A is now rarely used and discussion concerning it may be omitted.

For Type C, the allowance permissible for working stress on a 1 : 2 : 4 concrete is 350 pounds per inch of the core area between rods, and 10,000 pounds per square inch on the vertical steel and besides this the volume of metal in the ties is to be treated as forming imaginary verticals with a working stress equal to that allowed for the vertical bars, the ties to be spaced not further apart than ten times the diameter of the vertical bars in case the bars are one inch section, but where smaller bars are used the spacing should not exceed 9″ nor the size of the tie to be less than one quarter inch round.

Type B. The allowable working stress for a 1 : 2 : 4 mix is 600 pounds per square inch on the concrete of the core, 10,000 pounds on the vertical steel, and one and one half times the volume of the ties treated as imaginary verticals. These ties should not be spread further apart than 9″, and, if they are to be considered of value, they should be put not more than ten diameters of the vertical bars apart.

Type D, the Considère type, is by far the most economical type of column reinforcement that has been invented. It was brought prominently to the attention of the public by Armand Considère.

The principle involved is this: by restraining the concrete laterally its strength in compression is greatly increased. Just as an ordinary piece of stove pipe filled with sand will carry a load several times greater than the pipe itself would be able to do, so will a hooped column owing to the fact that the metal is strained in tension, while the filling held in position by the restraint of the pipe, carries the weight of the load. For strength see Section 5.

There have been quite a number of experiments on hooped concrete using spiral hooping only. In these experiments it has been found that after the ultimate strength of plain concrete has been developed, splitting and sealing of the outside shell occurs, combined with a large vertical deformation and considerable lateral bending before ultimate failure.

2. Considerations of Safety Determining Carrying Loads. If it is expected to develop the core of the concrete to a point beyond its normal strength we must evidently prevent its lateral distortion or bulging and also the sliding or flow of the concrete between consecutive bands or turns of the spiral, hence a certain proportion of vertical steel must be used in connection with the hooping to secure the best results.

In determining the degree of safety of the various types of column design, an investigation of the manner of failure of the respective types is in order as to whether it occurs suddenly and without warning, or gradually, accompanied by indications of approaching failure long before failure occurs, and there is the further question as to whether the conditions of strain in the column are proportional or comparable between the column under ordinary working stresses and the column as it approaches the breaking down point and ultimate strength. The following general observations may be made as to these questions:

In columns shown in Fig. Type C, the failure occurs with little warning, the vertical bars bending outward and the ties yielding.

In the hooped column without vertical steel, when it is loaded from forty to fifty percent of its ultimate strength, the portion of the concrete outside the hooping commences to check and crack, and later to scale. From this point, the rate of deformation with addition of the load increases rapidly owing to dissipation of energy by the cracking and scaling of the shell. Further loading is accompanied by large lateral deformations up to the final failure. Such a column gives ample warning but the point at which the outer shell or fire protection commences to fail is but little higher than the point at which the ordinary vertically reinforced column fails, so that little advantage in the way of increased working stress is secured unless the hooping is combined with vertical steel.

The well hooped column vertically reinforced shows a large increase in strength over that of the vertically reinforced column with ties and a great increase in toughness. Its failure is not sudden and without warning as in the former type, while the point at which checking and scaling of the outside shell occurs is raised to eighty or eighty-five percent of the ultimate strength, thus giving a large margin of safety to the fireproofing between the working load and the load where the failure of the shell is in evidence.

3. Experimental Data. A partial report on tests on full sized columns, made at Phoenixville, Pa., for C. A. P. Turner, engineer, by Mason D. Pratt, is given in the following table:

Test No. 1

Marks on column: None.

Reinforcement: Eight $1\frac{1}{8}$ inch round vertical bars.

Bands: Spaced 9 inches vertically, $\frac{1}{2}$ inch rivets, cross section $1\frac{3}{4}$ x $\frac{1}{4}$ inches, inside diameter 14 inches.

Hooped with 7 32 inch wire spirals, about 2 inch pitch.

Total load at failure, 1,360,000 lbs.

Remarks: Point of failure was about 22 inches from the top. Little indication of failure until ultimate load was reached.

Some slight breaking off of concrete near the top cap, due possibly to the cap not being well seated in the column itself.

Test No. 2

Marks on columns: Box 4.

Reinforcement: Eight $1\frac{1}{8}$ inch round vertical bars.

Bands: Spaced 13 inches vertically, $\frac{1}{2}$ inch rivets, cross section $1\frac{3}{4}$ x $\frac{1}{4}$ inches, inside diameter 14 inches.

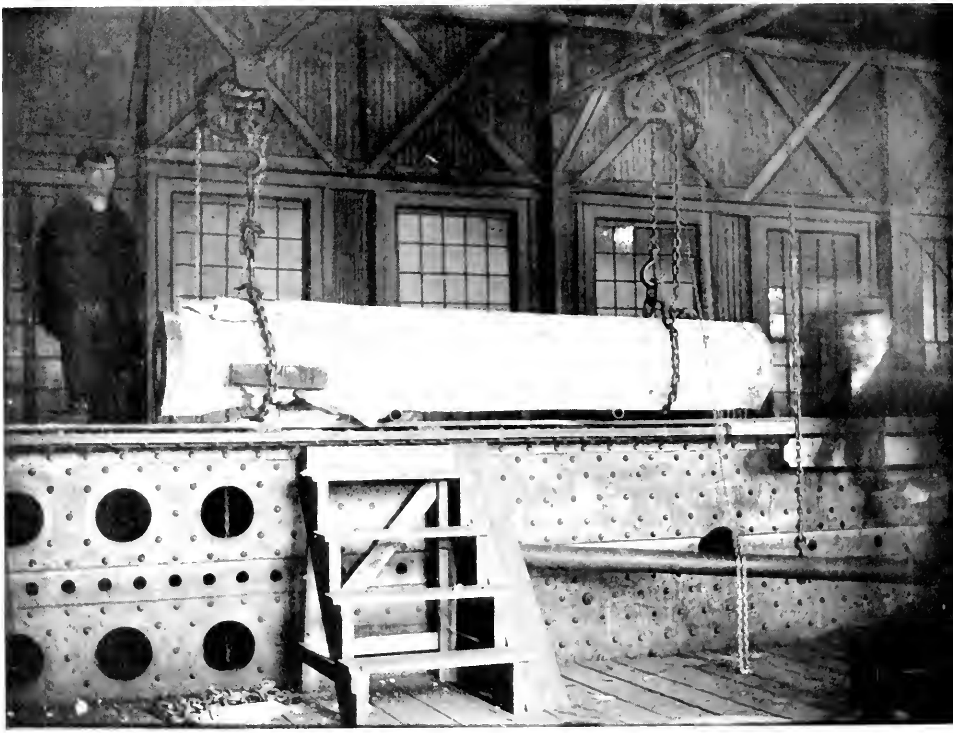
Hooped with 7 32 inch wire spiral, about 3 inch pitch.

Point of failure: About 18 inches from top.

Top of cast iron cap cracked at four corners.

Ultimate load: 1,260,000 lbs.

Remarks: Both caps apparently well seated, as was the case with all the subsequent tests.



Test No. 1
Showing Column as it Came from Testing Machine.

Test No. 3

Marks on column: 4-b.

Reinforcement: Eight 7/8 inch round vertical bars.

Bands: Spaced 13 inches vertically, $\frac{1}{2}$ inch rivets, cross section $1\frac{3}{4}$ x 3/16 inches, outside diameter 14 inches.

Ultimate load: 900,000 lbs.

Point of failure: About 2 feet from top.

Remarks: Concrete at failure, considerably disintegrated, probably due to continuance of movement of machine after failure.

Test No. 4

Marks on columns: Box 4-c.

Reinforcement: Eight 1 inch round vertical bars.

Bands: Spaced 8 inches vertically, $\frac{1}{2}$ inch rivets, cross section $1\frac{3}{4}$ x $\frac{1}{4}$ inches, inside diameter 14 inches.

Hooped with 7/32 inch wire spirals, about 3 inch pitch.

Total load at failure: 1,260,000 lbs.

Remarks: First indications of failure were nearest the bottom end of the column, but the total failure was as in all columns, within 2 feet of the top. Large cracks in the shell of the column extended from both ends to very near the middle. This was the most satisfactory showing of all the columns, as the failure extended over nearly the full length of the column.

Column No. 4, after Test

Test No. 5

Marks on column: None.

Reinforcement: Eight $\frac{7}{8}$ inch vertical bars.

Bands: Spaced 10 inches vertically, $\frac{1}{2}$ inch rivets, cross section $1\frac{3}{4}$ x $\frac{1}{4}$ inches, $14\frac{1}{2}$ inches outside diameter.

Hooped with 7/32 inch wire spiral as before, 3 inch pitch.

Load at failure: 1,100,000 lbs.

Ultimate load: 1,130,000 lbs.

Remarks: The main point of failure in this as in all other columns was within two feet of the top altho this column showed some scaling off at the lower end.

This set of tests were not conducted with any considerable degree of refinement but were a practical test of ultimate strength and the yield point value of specimens of full sized members, which lends greater value to the determination than laboratory tests on small specimens.

The concrete mixture was one part Portland cement, one part sand, and one and one-half parts buckwheat gravel, and three and one-half parts gravel ranging from one-quarter inch to three-quarter inch in size.

It should be noted that in these tests the cracking of the shell did not occur until the hoops were over-strained, and that the strength of the hooping closely defined the ultimate strength of the column with the proportions of vertical steel used.

1. In these columns, pressures were developed on the core more than three times the ultimate strength of plain concrete at 2600 pounds per square inch.

2. Incipient failure occurred only by the stretching or bursting of the bands.

All columns were approximately octagonal in shape, 10′6″ long and 18″ diameter. Final failure occurred toward the upper end of the column. Mr. Pratt accounts for the regularity with which the columns failed at the upper end on the ground that the concrete at the lower end was more dense, owing to its being under considerable hydraulic pressure, while setting. The rods were all shorter than the concrete shaft. Examination of the columns after removal from the testing machine showed in all cases a bulging out of the vertical reinforcement at the principal point of failure with the nearest hoop ruptured and in every case the wire spiral was broken in one or more coils at the point where the vertical rods were bent out. The vertical bars in nearly every case bulged as a column with fixed ends. Where the hoop spacing was six to nine inches, the deformed length of the bar would extend over the space of two hoops. Where the hoop spacing exceeded nine inches the deformed length of vertical bars was confined to the space between one pair of hoops.

It may be well to review at this point two series of carefully conducted tests upon plain and reinforced concrete prisms. Fig. 76, shows the dimensions and reinforcement of a series of prisms tested by Bach* for Wayss and Freitag. These specimens are Type C columns.

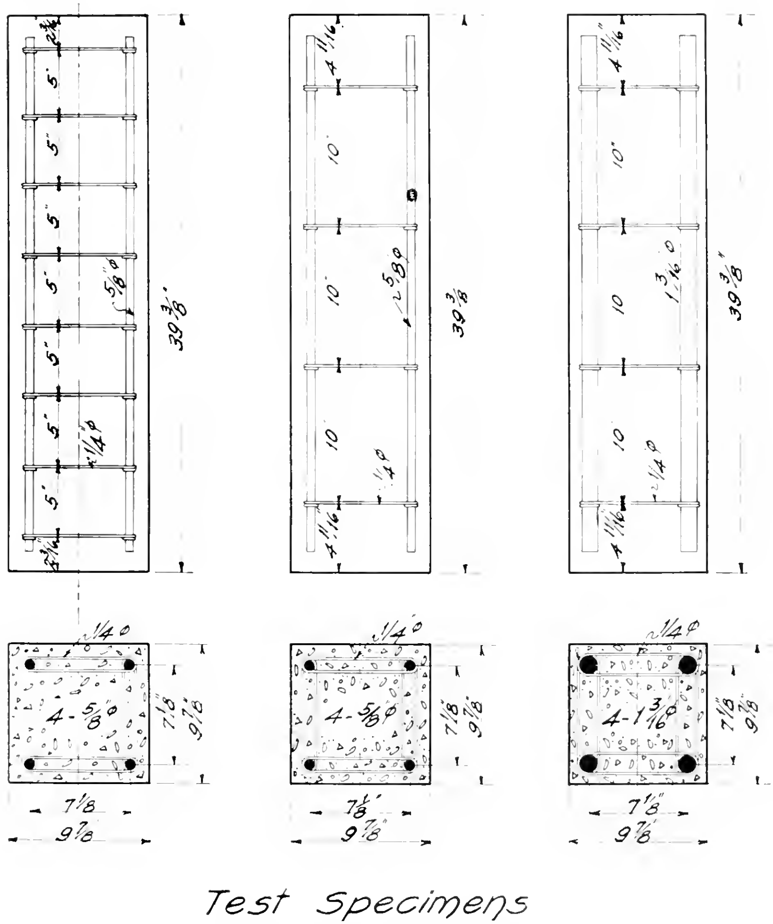


Fig. 76 Showing Dimensions and Reinforcement.

The first table for elasticity test of Type C columns, shows the reinforcement, tie spacing, and elastic deportment under different loads.

The second table for Type C columns, shows the breaking strength of this series.

*See Het Cement-Ijzer, Saunders, p. 91.
Eisenbetonbau, Mörsch.

It will be noted under the table showing the breaking strength that the specimens with 1 3/16 inch round verticals and 4.6 percent reinforcement, do not show as great an increase in strength as the specimens with 1.14 percent reinforcement, but with much closer spacing of the ties, the specimens with the large rods having ties four times as far apart as the specimens with the small rods.

COL. TABLE I

Elasticity Test of Columns (Bach)

Stresses	4 Rods of Diameter	Tie Spacing	Shortening in Millionths of the length		
Lbs. sq. in.	Inch	Inches	Total	Elastic Diff.	Permanent Set
459	Plain Concrete		133	7	126
459	5/8 in. rd.	10	114	5	109
459	5/8 " "	5	110	2	108
459	5/8 " "	$2\frac{1}{2}$	106	4	102
919	Plain Concrete		333	37	296
919	5/8 in. rd.	10	267	20	247
919	5/8 " "	5	264	18	246
919	5/8 " "	$2\frac{1}{2}$	241	13	228
1380	Plain Concrete		709	164	545
1380	5/8 in. rd.	10	488	63	425
1380	5/8 " "	5	473	58	415
1380	5/8 " "	$2\frac{1}{2}$	421	42	379

COL. TABLE II.

Breaking Strength of Columns (Bach)

Specimen 3 mo. old.		Breaking Strength				
Diameter of rods.	Tie Spacing	Each			Average lbs. per sq. in.	Percent of Reinforcing
Plain Concrete		2076	1963	1977	2006	0
5/8 in. rd.	10 in.	2432	2290	2447	2390	1.14
5/8 " "	5 "	2389	2660	2489	2512	1.14
5/8 " "	$2\frac{1}{2}$ "	3015	2845	2887	2915	1.14
3/4 " "	10 "	2404	2404	2446	2418	2.04
$1\frac{3}{16}$ " "	10 "	2474	2830	2802	2702	4.60
Test Cubes.		2389	2401 2631	2432 2617	2490	0

Looking at the elasticity tests of the specimen reinforced with the same vertical steel the marked difference in toughness, increase of modulus of elasticity and the reduction of permanent set produced by the increase in the number of ties, is strikingly shown by this series of tests.

This series of tests show conclusively that where the vertical bars do not bear directly upon the face plate of the machine and the load is brought upon the reinforcement under the usual condition of the column in the reinforced concrete structure, that is the steel is strained by the load brought on it thru the concrete, that a formula which does not take into consideration the amount and spacing of the ties fails to account for the deportment of the column both as regards elasticity and ultimate resistance. That is, merely an increase in the total cross section of the longitudinal reinforcement does not produce an increase in the breaking strength to the extent which would be expected by the formula

$$P = f_c (A_c + nA_s)$$

Hence in inexperienced hands, this formula may produce unsafe designs by increasing the percentage of longitudinal reinforcement disproportionately in order to secure columns of small diameter. This procedure gives a column with a calculated margin of safety which it does not possess.

When the increase in the resistance is compared for equal longitudinal reinforcement, there being a difference in the number of ties, it is shown by these experiments that the steel used as ties is much more effective than the longitudinal steel. As noted, however, the columns reinforced with vertical steel and ties do not develop that degree of strength which is secured by columns properly reinforced vertically and hooped with bands or spirals. Hence their use has been largely discontinued where loads are at all heavy and is confined to those cases where loads are relatively light and some slight saving may be made in the use of ties over the cost of spiral hooping.

4. Reinforced Columns Classified by the Manner in Which Loads are Applied. We have called attention to the type of prisms which were tested by Bach in which the steel was cut short at the end of the concrete prism so that the load applied to the prism was brought upon the longitudinal steel thru the bond shear between the steel and the concrete. This is the common mode of arrangement of the steel in the reinforced concrete column in the practical building. Sometimes, however, in order to secure relatively small columns for heavy loads very high percentages of vertical steel are used and the load is brought largely upon the steel by direct bearing of steel upon steel. Evidently the coaction of the elements of the composite structure differ widely under these radically different conditions and correspondingly different formulas must be applied to these different conditions.

We have cited the interesting series of tests by Bach on rectangular prisms, plain and reinforced with longitudinal steel and ties and we may profitably consider the corresponding series of tests on hooped and longitudinally reinforced prisms.

Fig. 77 gives the dimensions and type of reinforcement used while table III shows the results of the tests.

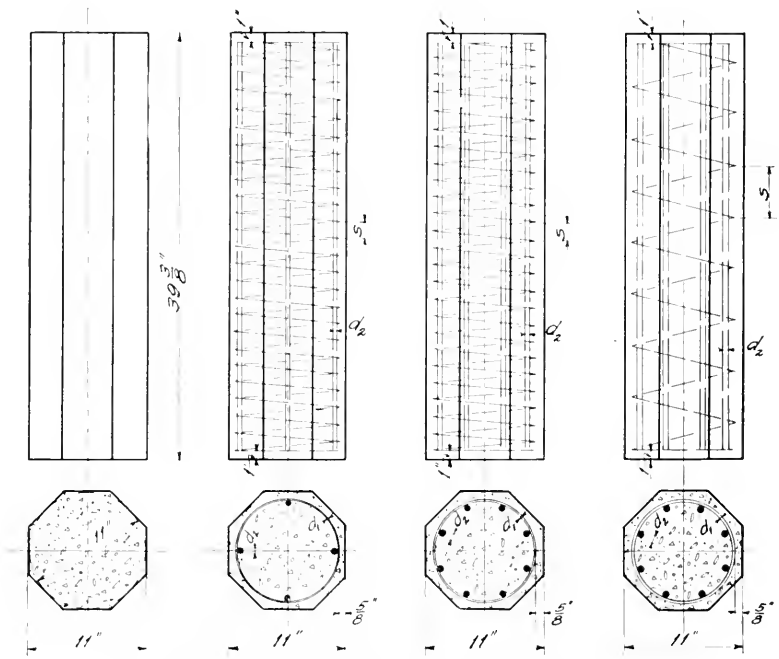


Fig. 77 Dimensions of Columns and Type of Reinforcement.

As in the first series the longitudinal steel was cut short of the ends of the prisms so as to secure normal coaction between the steel and the concrete. Percentage of hooping varied thru a wide range and the pitch of the spiral hooping was also varied between wide limits. Unfortunately the series is lacking in a corresponding variation in the percentage of vertical steel.

On the whole, these specimens correspond approximately with results indicated by the Considère formula and exceed these results for those specimens which approach desirable proportions in point of spacing of the spiral and relation of the vertical steel to the hooping.

COL. TABLE III

Bach's Tests of Plain and Hooped Concrete Prisms, Longitudinal Steel Cut Short of End of Prism.

Col No.	Description	Spirals Dia. wire in.	Spirals Pitch	Verticals No.	Verticals Dia. in.	Area Core sq. in.	% Reinforcement Spiral	% Reinforcement Vert.	% Reinforcement Total	Load lbs. per sq. in. when first cracks started.	Increased st. due to reinf. Lbs. per sq. in. at yield pt.	Increased st. lbs./sq. in. per 1% reinf. at yield pt.	Ultimate strength lbs. per sq. inch	Increased ult. st. lbs./sq. in. due to reinf.	Increased ult. st. in. lbs. per sq. in. per 1% reinf.	Load when considering core only lbs./sq. in.
I	4 spec.	Unreinforced					0	0	0	1,890			1,890			
II	3 "	.2	1.52	4	.28	69.69	.88	.35	1.23	2,260	370	301	2,260	370	301	3,270
III	3 "	.28	1.48	4	.28	68.51	1.78	.36	2.14	2,290	400	187	2,550	640	299	3,660
IV	3 "	.4	1.68	4	.28	66.76	3.24	.37	3.61	2,420	530	147	3,420	1,525	422	4,930
V	3 "	.2	1.52	8	.44	69.69	.88	1.75	2.63	3,190	1,300	495	3,210	1,320	502	4,650
VI	3 "	.28	1.48	8	.44	68.51	1.78	1.77	3.55	3,270	1,380	389	3,270	1,380	389	4,720
VII	3 "	.4	1.72	8	.44	66.76	3.17	1.82	4.99	3,460	1,570	315	4,000	2,110	424	5,770
VIII	3 "	.28	1.24	4	.28	68.51	2.12	.36	2.48	2,790	900	363	2,850	960	387	4,100
IX	3 "	.40	1.60	4	.28	66.76	3.41	.37	3.78	2,420	530	140	3,000	1,110	294	4,340
X	3 "	.48	1.64	4	.28	65.61	4.82	.38	5.20	2,560	670	129	3,640	1,750	336	5,260
XI	3 "	.56	1.48	4	.28	64.47	7.36	.38	7.74	2,250	360	47	3,500	1,610	208	5,050
XII-I	3 "	.28	1.60	8	.20	68.51	1.64	.37	2.01	2,320	430	214	2,320	430	214	3,360
XII-II	3 "	.40	1.60	8	.28	66.76	3.41	.74	4.15	2,330	440	106	3,270	1,380	333	4,720
XII-III	3 "	.56	1.60	8	.40	64.47	6.82	1.56	8.38	2,620	730	87	4,290	2,400	286	6,200
XIII-I	3 "	.28	3.20	8	.28	68.51	.83	.72	1.55	2,300	410	264	2,300	410	265	3,330
XIII-II	3 "	.40	3.20	8	.40	66.76	1.71	1.51	3.22	2,550	660	205	2,580	690	214	3,720
XIII-III	3 "	.56	3.20	8	.48	64.47	3.43	2.25	5.68	2,650	760	134	2,830	940	166	4,240
XIV-I	3 "	.28	4.80	8	.40	68.51	.55	1.47	2.02	2,200	310	153	2,210	320	158	3,190
XIV-II	3 "	.40	4.80	8	.48	66.76	1.15	2.17	3.32	2,600	710	214	2,600	710	214	3,750
XIV-III	3 "	.56	4.80	8	.56	64.47	2.30	3.11	5.41	2,950	1,040	193	2,940	1,050	195	4,250

Specimens had an octagonal section with a diameter of 10.8 inches and had a length of 39.37 inches. Mix 1 part of Heidelberg Portland Cement to 4 parts of Rhine Sand and gravel. Specimens 5 to 6 mo. old when broken.

Specimens XIII and XIV, in which the pitch of the spiral was exaggerated gave results less than the formula, pointing to the necessity for closer spacing.

In specimens II, III, IV, VIII, IX, X, XI and XII, the sectional area of the longitudinal rods was small and the results were consequently indifferent, but the greater the total weight of spiral reinforcement the higher were the values.

The tests show that when the spirals are increased in strength, their pitch must be decreased and the cross section and number of vertical rods must be increased, for with the increase of spirals the concrete is in a condition to resist heavier pressure and its tendency to force its way out between the longitudinal rods and the spiral bands increase.

Examining the table of the values obtained, the increase in strength and yield point value per one percent of total steel is found to be greatest in column specimen V, in which the spiral reinforcement is .88 percent and the vertical reinforcement was 1.75. In columns XII and XIII, with the spiral reinforcement 6.82 percent and the vertical reinforcement 1.56 percent, the increase percent of total reinforcement at the yield point was only 87 pounds and the increase of the ultimate strength per percent of the ultimate reinforcement 286 pounds per square inch, showing an improper proportion of the spiral and vertical reinforcement.

On noting the pitch of the spiral it is quite noticeable that the specimen with a small pitch gives the highest resistance per percent of steel. In none of these specimens, however, are the longitudinal rods of sufficient size to secure the best results. Had 5/8, 3/4 and 1 inch rods been used in place of 1/4 inch, 7/16 and 1/2 inch, much higher values would have resulted from the stiffness of the verticals which must act as beams from coil to coil in resisting outward or bulging pressures.

The following table gives the percentage of vertical steel and hooping in the Phoenixville tests, see Section 3.

COL. TABLE IV

Phoenixville Column Tests. Specimens 3 mo. old

No. of Test	% Verticals	% Spiral	% Hooping	% Spiral and Hooping	% Total	Ultimate Tested Strength	Ultimate Strength Per Sq. In.	Computed Increase per % of Steel	Strength by Considère Formula
1	5.17	1.54	.99	2.53	7.70	1,360,000	8,850	785	1,291,000
2	4.82	1.35	.66	2.01	6.83	1,260,000	7,630	710	1,279,000
3	3.12	0	.72	.72	3.84	900,000	5,800	792	898,300
4	4.08	1.36	1.12	2.48	6.56	1,260,000	8,200	820	1,216,000
5	3.12	1.36	.90	2.26	5.38	1,130,000	7,350	842	1,127,000

From these tests it may be observed that the best results are to be secured when the value of the vertical steel as an element of strength is approximately the same as the hooping according to Considère's formula, Section 5. In other words, the reinforcement should be so proportioned that the volume of the hooping for this type of column should be between the limits of 35 and 45 percent of the volume of the vertical steel to secure the best results in increasing the yield point and ultimate strength of the column. Between these limits the yield point value of the columns should vary from 75 to 85 percent of the ultimate strength of the specimen, so that due warning of approaching failure is given by the member while ample margin exists between the safe working stress and that point at which the fireproofing will commence to scale and chip.

5. Considere Formula. As we have pointed out, the Considère formula is conservatively applicable to the vertically reinforced and hooped columns provided that the hooping and vertical steel are properly proportioned. With no vertical steel, the yield point value of the column is not sufficiently raised to warrant very material increase in safe working stress over that of ordinary types notwithstanding the fact that there is an increase in the ultimate resistance and large deformations occur before ultimate failure.

The Considère formula for column resistance is as follows:

$$P = 1.5\,A_c f_c + (\,f_s A_s + 2.4 f'_s A'_s)$$

in which A_c is the area of the core, A_s the area of the longitudinal steel, A'_s is the area obtained by dividing the volume of the hooping by the length of the column. The coefficient 1.5 is a coefficient which Considère considers represents the effect of the hooping in increasing the strength of the core, and that this coefficient is a maximum at 1.5 and that it is less than this value for a percentage of hooping which does not furnish a resistance equivalent to 700 pounds per square inch lateral pressure on the column.

Mörsch, however, and also Saunders, in discussing the Bach tests where the percentage of steel was low, seem to treat this coefficient of Considère as the ratio of the core to the total area of the column (fireproofing and all). In most building ordinances this coefficient, however, is taken as unity. 2.4 times the volume of the hooping is figured as the effect of the stress on the hooping in increasing the crushing resistance of the core by the lateral pressure brought about by the stress in the hooping and represents the resistance which would be developed were the hooping in the form of cylinders and filled with a granular mass such as sand subjected to pressure. Hence $2.4A'_s$ equals the cross section of the equivalent imaginary verticals.

Le Genie Civil, Feb. 9, and 16, 1907, reported a very extensive series of tests by Considère on 260 columns with lengths varying from 3.15 in. to 13 ft. $1\frac{1}{2}$ in. and diameters from 1.8 in. to 27.5 in. The percentage of reinforcement varied from 1 to 14, various methods of spiral reinforcement, including concentric spirals were tried. The effects of richness of mixture, age, percentage of water, ramming and irregularities in workmanship were also studied. One specimen having 12.9 percent spiral and 1.2 percent longitudinal reinforcement, and made from a mixture containing 1830 lb. of cement per cu. yd. of sand sustained 25,600 lb./in.^{2} at rupture. A similar specimen having only 6.6 percent of spiral reinforcement failed at 17,200 lb./in.^{2} at rupture, with a deformation of 12 percent. From these and previous tests Considère deduced the conclusions that the rupture load of a spirally reinfored concrete column exceeds the sum of the three following factors:

1. The strength of the plain concrete times 1.5.
2. The strength of the longitudinal rods stressed to their yield point.
3. The strength of a longitudinal reinforcement of 2.4 times the weight of the spiral stressed to its yield point.

Turner, in the design and guarantee of strength for working several hundred thousand columns has used the Considère formula slightly modified,allowing 800 pounds per square inch on a 1 : 2 : 4 mix, 12,000 pounds per square inch on the vertical steel, 16,000 pounds per square inch on the hooping, treating the spirals as imaginary verticals having a volume 2.4 times the volume of the hooping. For these values the ratio of the length of the column to its diameter should not exceed 12. For a longer column increased vertical steel should be used to resist flexure. Where the vertical resistance of the concrete developed by the hooping exceeds 2000 pounds per square inch the proportion of the mix should be increased from 1 : 2 : 4 to 1 : $1\frac{1}{2}$: 3, and extra care should be used in the selection of the stone aggregate to see that it is hard and satisfactory. Screened gravel is preferable where high working pressures are used in the columns. To be efficient, spiral hooping should not exceed a 4″ pitch when combined with not less than four vertical rods and these should be limited to a minimum diameter of 3/4 of an inch for columns carrying moderate loads, or to a pitch not exceeding 3″ if the full value above recommended is to be used in considering the hooping. A reduction of the allowance on the vertical steel should be made where the spacing of the vertical bars exceeds 9″ center to center in proportion to fifty percent of the increase in spacing of the bars.

6. Safe Ultimate Limit of Compression. A wonderful degree of strength which may be developed by properly hooped and longitudinally reinforced concrete columns is established by the Considère experiments and it becomes a question as to how great values it is permissible to use. Turner has found it permissible, where small diameters are desired, to use as the approximate limit of safe design a working stress as high as 4000 pounds per square inch of cross section of the core including the vertical steel. Under these ultimate conditions it is recommended that the column should have

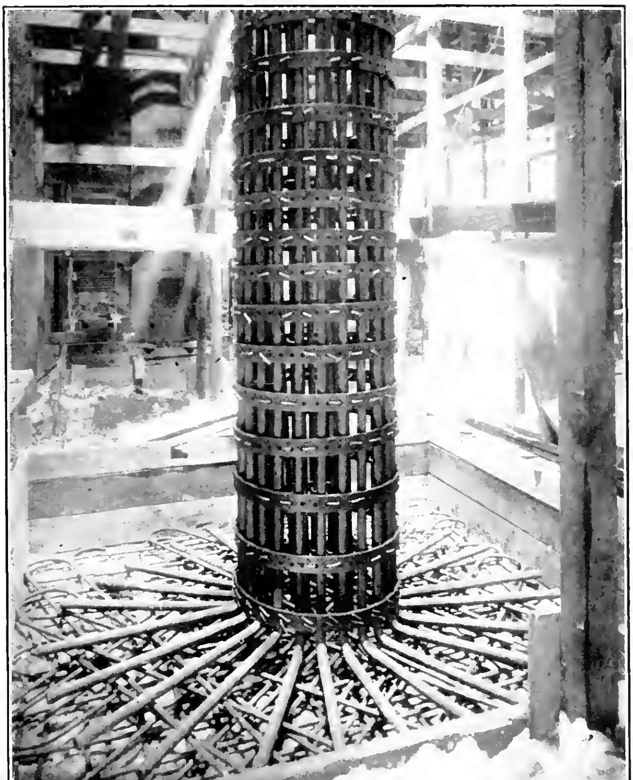


Fig. 78. Cut showing reinforcement of column carrying working pressure of 4000 pounds per inch on the core.

enough vertical steel to carry the entire load at little more than the yield point value, say at 50,000 pounds per square inch; that there be sufficient hooping to develop the value of the load figured at 40,000 pounds per square inch on imaginary verticals corresponding to 2.4 times the volume of the hooping; and that the gross area of the column inside the fireproofing should be sufficient to carry the load at 4000 pounds per square inch.

7. The Mode of Operation of the Reinforcement in Concrete Columns. In a practical concrete structure, as a rule, the load is brought on the column thru the beams or the concrete of the slab and the load is transferred to the vertical steel thru the medium of the concrete by the bond between the concrete and the steel.

In the tests by Withey and those of some other American investigators, the load on the test specimens was transferred directly to the steel by direct bearing between the longitudinal reinforcement and the face plate of the testing machine. This makes an important difference in the mode of operation of the column. The vertical steel is a more rigid element under compression than the concrete and if the load be not applied directly to the steel but be brought on the steel thru the surrounding matrix, the coaction of the two materials is brought about by bond shear and indirect stress generated by the bond shear between the steel and the concrete. Now, since there is no slipping of the steel in the concrete within the yield point of the column, the same amount of potential energy or work of deformation is stored up by the indirect stress of the bond shear between the concrete and the steel as is stored directly in the steel. Hence it appears that in this case the steel by help of bond shear operates to store energy more efficiently up to the point where cracks and checks occur in the concrete than a mere comparison of the relative moduli of steel and concrete would indicate.

This phase of the co-operation of the steel and the concrete in the column requires, for its greatest efficiency, a strong, rich, concrete, in order that the most dependable bond may be secured.

If, however, the load be brought upon the steel by a rigid bearing between the face plate of the testing machine and the steel of the longitudinal reinforcement and in like manner on the concrete core, such a bearing makes the elastic deformation of the steel the same as that of the concrete thruout, and no elastic efficiency by coaction of the concrete and metal exists, such as occurs in the practical structure.

8. Effect of Hooping. The crushing of solid bodies cannot take place without lateral swelling. Therefore, by resisting any such swelling, the compressive resistance of the column is increased. In the practical column this resistance to lateral bulging or swelling is furnished by the hooping in which the spacing should be limited generally to four inches for light pressures and closer spacing for higher pressures. As the hooping is brought into play by lateral swelling or bulging, the degree of restraint furnished by the hooping alone is not uniform and this lack of uniformity causes the concrete to check and crack outside of the hooping under pressures as large approximately as the ultimate strength of plain concrete.

The addition of vertical steel distributes the bulging pressure from band to band or hoop to hoop to such an extent that the vertical steel which has been added forms beams spanning the spaces between coils or bands and does so to an extent measured by the more or less close spacing of the vertical bars. In other words, the vertical reinforcement receives lateral pressures between the hoops and transfers it to the hoops as supports. This action destroys the equilibrium of uniform pressure outward upon the hoops and tends to deform the configuration of the circular hoop from circles to polygons having apices at the point of bearing of the vertical steel against the hoops. This action between the vertical steel and the hooping induces indirect stresses between the hoops and the vertical bars similar to those in the slab with two way reinforcement.

The outer shell of the concrete is subjected to direct compression vertically and circular tensions horizontally, brought about by the bulging tendency. These circular tensions are reduced between verticals by the compression brought about by the tendency to change configuration in the hooping just mentioned, while the ring tension opposite verticals is increased by this action. The lateral reinforcement by its compressive action exerts a powerful effect to resist bulging and to prevent circumferential elongations so that the reinforcement (both lateral and longitudinal), enables it to withstand much greater deformations without cracking or checking than would be otherwise possible.

Let us review the action of the column as the load is applied with reference to the manner in which the potential energy of internal work is stored within the structure. As the load is gradually applied we have a certain elastic deformation and the internal work is the mean weight times the deformation. If yielding of the material occurs or scaling of the shell, a certain amount of energy is dissipated, equilibrium is destroyed, and new energy is developed by downward motion of the load thru increased deformation until a new condition of stability of equilibrium in established.

Scaling and cracking of the shell means a loss of potential energy stored and correspondingly larger deformations which are inadmissible in the practical structure. The addition of vertical steel prevents this checking and scaling and dissipation of energy because it provides a storage system of energy which is stable and in which the storage of energy by indirect stress can be depended upon.

Coaction between the hoops and the vertical steel reduces the deformation and hence the quantity of potential energy stored for a given load and correspondingly increases the efficiency of the structure as a load carrying mechanism.

9. **Comparison of Test Data.** Having pointed out in detail, the difference in action between these two kinds of columns, it is in order to compare test data. Column Table V, gives the reinforcement, percentage of steel, and test results by Withey* of a series of vertically reinforced and hooped columns with vertical steel flush with the ends and resting against bearing plates. These may be compared with the tests by Bach, and the tests at Phoenixville.

Taking up the comparison, first, with the series by Bach, the following point of difference is noticeable. The test results obtained by Withey can be substantially accounted for by considering the influence of the concrete and vertical steel alone. The test results by Bach, cannot be accounted for in this manner. Both series show that hooping adds toughness. Both indicate that larger vertical rods have more effect in raising the yield point than rods of too small diameter, which offer little resistance to lateral bending. The Bach tests, where the spiral pitch of is not too great, and where larger vertical rods are used with close spacing of spirals, are in good accordance with the Considère formula; while the tests by Withey are not in agreement but are in more close agreement with a different formula, which considers the manner in which the load is brought upon the steel, namely, by direct pressure instead of by bond shear as in usual operation of the column in practical building.

Comparing the tests by Withey with those at Phoenixville, the longitudinal reinforcement of the columns, in specimens, *J*, *N*, *P*, *O*, *R*, and *Q*, is substantially the same; while the hooping in the Phoenixville tests was much greater. The increase in ultimate strength in the Phoenixville test, for one percent of total steel, ranges from two to three times as much as in the tests by Withey. This

*Bulletin of the University of Wisconsin, No. 466.

COL. TABLE V

Withey's Tests of Hooped Columns Longitudinal Steel Full Length of Shaft

Col. No.	Reinforcement: Round Vertical Rods: No.	Reinforcement: Round Vertical Rods: Size	Reinforcement: Spiral: Size	Reinforcement: Spiral: Pitch	Percent Reinforcement: Longitudinal	Percent Reinforcement: Lateral	Core area sq. in.	Age in days	Mix	Ultimate strength lbs.–sq.in P A	Stress at yield pt. in core lbs.–sq.in P1 A	P1 P	Compressive strength of cylinders lbs sq.in
1	2	3	4	5	6	7	8	9	10	11	12	13	11
C 1	0	0	$\frac{1}{4}''$ rd.	1″	0	2.00	78.5	62	1-2-4	4,660	2,720	0.58	2,280
C 2	0	0	$\frac{1}{4}''$ "	1″	0	2.00	78.5	62	1-2-4	4,390	2,330	0.53	2,190
C 3	0	0	$\frac{1}{4}''$ "	1″	0	2.00	78.5	64	1-2-4	3,660	1,950	0.53	2,180
C 4	0	0	$\frac{1}{4}''$ "	1″	0	2.00	78.5	65	1-2-4	3,410	2,330	0.68	2,150
D 1	9	$\frac{5}{8}''$	$\frac{1}{4}''$ "	1″	3.50	2.00	78.5	58	1-2-4	4,470	3,290	0.74	2,150
D 2	9	$\frac{5}{8}''$	$\frac{1}{4}''$ "	1″	3.50	2.00	78.5	60	1-2-4	4,200	3,480	0.83	2,130
D 3	9	$\frac{5}{8}''$	$\frac{1}{4}''$ "	1″	3.50	2.00	78.5	61	1-2-4	4,970	3,860	0.77	2,380
D 4	9	$\frac{5}{8}''$	$\frac{1}{4}''$ "	1″	3.50	2.00	78.5	62	1-2-4	5,360	3,670	0.68	2,350
H 1	0	0	No. 7	2″	0	0.50	78.5	57	1-2-3.5	2,330	1,950	0.84	2,040
H 2	0	0	No. 7	2″	0	0.50	78.5	57	1-2-3.5	2,140	1,760	0.82	*1,460
G 1	8	$\frac{1}{2}''$	No. 7	2″	2.00	0.50	78.5	19	1-2-3.5	3,320	2,710	0.82	2,100
G 2	8	$\frac{1}{2}''$	No. 7	2″	2.00	0.50	78.5	58	1-2-3.5	3,280	2,710	0.83	*1,420
I 1	8	$\frac{11}{16}''$	No. 7	2″	3.78	0.50	78.5	57	1-2-3.5	4,240	3,660	0.87	2,240
I 2	8	$\frac{11}{16}'$	No. 7	2″	3.78	0.50	78.5	57	1-2-3.5	4,080	3,280	0.80	2,120
J 1	8	$\frac{7}{8}''$	No. 7	2″	6.11	0.50	78.5	58	1-2-3.5	5,190	4,240	0.82	2,110
J 2	8	$\frac{7}{8}''$	No. 7	2″	6.11	0.50	78.5	58	1-2-3.5	5,050	4,240	0.84	2,000
L 1	0	0	No. 7	2″	0	0.50	78.5	57	1-2-3.5	2,680	1,370	0.51	1,780
L 2	0	0	No. 7	2″	0	0.50	78.5	58	1-2-3.5	2,600	1,370	0.53	1,760
K 1	8	$\frac{1}{2}''$	No 7	1″	2.00	1.00	78.5	57	1-2-3.5	4,050	2,710	0.67	2,100
K 2	8	$\frac{1}{2}''$	No. 7	1″	2.00	1.00	78.5	57	1-2-3.5	3,760	2,520	0.67	1,890
N 1	8	$\frac{11}{16}''$	No. 7	1″	3.78	1.00	78.5	57	1-2-3.5	4,050	3,470	0.86	1,880
N 2	8	$\frac{11}{16}''$	No. 7	1″	3.78	1.00	78.5	58	1-2-3.5	4,340	3,280	0.76	1,720
M1	8	$\frac{7}{8}''$	No. 7	1″	6.11	1.00	78.5	57	1-2-3.5	4,790	3,860	0.81	1,660
M2	8	$\frac{7}{8}''$	No. 7	1″	6.11	1.00	78.5	58	1-2-3.5	1,580	3,660	0.80	1,710
P 1	8	1″	No. 7	1″	8.00	1.00	78.5	57	1-2-4	6,760	5,560	0.82	2,380
P 2	8	1″	No. 7	1″	8.00	1.00	78.5	60	1-2-4	7,090	5,760	0.81	2,350
O 1	8	$\frac{7}{8}''$	$\frac{1}{4}''$ rd	1″	6.11	1.96	78.5	57	1-2-4	6,510	1,240	0.65	2,270
O 2	8	$\frac{7}{8}''$	$\frac{1}{4}''$ "	1″	6.11	1.96	78.5	57	1-2-4	6,650	1,620	0.70	2,690
R 1	8	1″	$\frac{1}{4}''$ "	1″	8.00	1.96	78.5	53	1-2-4	7,250	5,000	0.69	2,310
R 2	8	1″	$\frac{1}{4}''$ "	1″	8.00	1.96	78.5	53	1-2-4	6,680	5,380	0.80	2,470
Q 1	8	$1\frac{1}{8}''$	$\frac{1}{4}''$ "	1″	10.12	1.96	78.5	53	1-2-4	6,190	5,190	0.84	2,280
Q 2	8	$1\frac{1}{8}''$	$\frac{1}{4}''$ "	1″	10.12	1.96	78.5	53	1-2-4	7,990	6,340	0.79	2,330
W1	0	0	0	0	0	0	86.6	52	1-2-4	2,660			2,600
W2	0	0	0	0	0	0	86.6	52	1-2-4	2,660			2,400
W3	0	0	0	0	0	0	86.6	51	1-2-4	2,480			2,250

Note: All spirals 10″ in diameter. Cols. C1-D4 are 120″-lg. All other cols. 102″-lg.

wide discrepancy, apparently can be accounted for only as above outlined, since the discrepancy is far too great for it to be possible to account for it on the ground of difference in the strength of the concrete. The concrete in the test of the Phoenixville specimens is undoubtedly closely comparable to the concrete of the specimens tested by Bach, and it is undoubtedly a somewhat better grade than the concrete in the specimens tested by Withey.

The conclusions that the authors draw from these tests are:

That it is bad practice to splice longitudinal reinforcing bars by bearing of one bar upon the other.

That it is better to lap bars of consecutive columns at the floor line in order that a natural adjustment may take place between the materials as the load is brought upon the concrete during erection.

That a column constructed by lapping the bars, so that the natural relations between the reinforcement and the concrete are conserved, that is, that indirect stress of the bond shear may be effective, substantially doubles the efficiency of the vertical steel; and hence this detail of reinforcing should be used. If this efficiency is to be counted upon, the diameter of the vertical bars should, for like reason, be limited preferably to $1\frac{1}{8}$ inch diameter, or in special cases of heavy work to $1\frac{1}{4}$ inches to $1\frac{3}{8}$ inch round bars.

That the hooping should not be spaced further apart than four inches for light pressures, and closer spacing should be used for heavy pressures.

That the spacing of the vertical bars should not exceed nine inches between centers, but if spaced further apart their efficiency should be considered to be reduced by fifty percent of the relative increase in spacing.

10. Formula for Columns, where the Load is Brought upon the Steel by Direct Bearing on Metal. Tests of this type of column show, up to the yield point, so-called, that the effect of the hooping is relatively small; that beyond the yield point the hooping is brought into play, giving the column a degree of toughness and ability to stand increased load without sudden failure, altho accompanied by considerable deformation. Neglecting the circumferential reinforcement and the concrete outside of it,

Let A = area of core occupied by concrete and longitudinal reinforcement having a steel ratio of p, so that

$A_s = pA$ = cross section of longitudinal steel.
$A_c = (1 - p)\ A$ = cross section of concrete core.
W = total load to yield point that column would carry without circumferential steel.

e_1 = .00125″ = average deformation per unit of length at yield point of column.

$$W = (A_s\ E_s + A_c\ E_c)e_1. \quad \text{Let } E_s/E_c = n, \text{ then}$$
$$w = W/A = E_s\ e_1\ [\ (1 - p)/n + p]$$

is the load at yield point per unit of cross section of concrete.

Let E_s = 30,000,000, and n = 15; then
$$w = 37{,}500\ [\ (1 - p)/15 + p'\ ] = 2500 + 35{,}000\ p$$

The point here designated on the diagram of unit deformations and load is at intersection P of the tangents to the two slopes of the curve, Fig. 78.

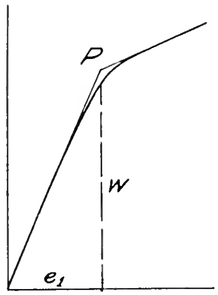


Fig. 78. Diagram Showing Elastic Curve of Column

Equation (1) is in good agreement with the series of tests made by Professor M. O. Withey, C. E., in 1910, at the University of Wisconsin,* with longitudinal reinforcement from 1 percent to 10 percent and circumferential reinforcement from 0.5 percent to 2 percent, given in Col. Table V. Professor Withey has written empirical formulas for several different percentages of circumferential reinforcement which differ slightly from this and from each other; but they do not refer precisely to the point P as above defined, but to a slightly smaller value of W. The point P does not actually fall on the test curve of unit deformations and loads, but is so related to it geometrically as to make its use advisable. The actual deformation at this load will be somewhat in excess of the assumed value of e_1.

On page 49, in Bulletin 466, University of Wisconsin, the following conclusions are drawn from the results of these tests regarding the behavior of columns with rigid metallic bearing for the longitudinal reinforcement built of concrete similar to those tested by Withey.

> 1. "Closely spaced spiral reinforcement will greatly increase the toughness and will considerably increase the ultimate strength of a concrete column, but will not materially affect the yield point. The ultimate strength under dead load will doubtless be somewhat less than the value obtained from a testing machine. Since spiral reinforcement should be employed principally as a factor of safety against sudden collapse, more than one percent does not appear to be necessary. On account of lack of stiffness in columns made from this grade of concrete and reinforced with spirals only, it seems necessary to use some longitudinal steel.
>
> 2. "Longitudinal steel in combination with such spiral reinforcement raises the yield point and ultimate strength of a column and increases its stiffness. As was shown in Bulletin No. 300 columns reinforced with longitudinal steel only are brittle and fail suddenly when the yield point of the steel is reached; but they are considerably stronger than plain concrete columns made from the same grade of concrete.
>
> 3. "From behavior under test of the columns reinforced with spirals and vertical steel and the results computed, it would seem that a static load equal to 30 to 40 percent of the yield point would be a safe working load."

*Bulletin No. 466.

These tests are conclusive as regards the deportment of the column without vertical steel. Spiral reinforcement gives a large increase in the ultimate strength and while providing safety against sudden collapse, does not increase the yield point at which the fireproofing commences to scale, and hence does not permit material increase in the working stress over that of the vertically reinforced column types with ordinary ties.

11. Working Stresses. (*a*) *The case of no direct metallic bearings.* In the vertically reinforced and hooped column, in which the hooping follows closely the percentage of the vertical steel recommended previously, namely, .35 to .45 of the volume of the verticals, and the verticals are not less than three quarters of an inch in diameter, and spaced approximately as recommended, the yield point of the column may be taken as eighty percent of the ultimate strength. This would give a yield point value, for a column to which the Considère formula is applicable of .80 x 1.5 = 1.2 times the crushing strength of the concrete times the core area, plus eighty percent of the yield point value of the steel in verticals and hoops.

Since a column of this character may be depended upon with certainty, if ordinary care is used in erecting the work, the factor of safety of 2.5 may be employed. This would give, in round numbers, 800 pounds per square inch for the concrete of the core, 12,000 pounds per inch on the verticals, and 16,000 pounds per square inch on the hooping, treated as imaginary verticals, for the working stress, the hooping being assumed to be a drawn wire, and possessing a higher yield point than the material of the verticals.

With a $1 : 1\frac{1}{2} : 3$ mix, twenty percent higher values may be assigned to the core.

(*b*) *Metallic bearings.* Where vertical reinforcement is used, and the load is brought on the vertical steel by bearing on metal, there is much less certainty of uniformity of joint action between the matrix and the reinforcement than where the longitudinal bars are lapped; and hence a somewhat higher margin of safety should be allowed. Taking this margin of safety as the reciprocal of $3\frac{1}{2}$, the writers would recommend a working stress not exceeding 700 pounds per square inch on the concrete plus 10,000 pounds per square inch on the vertical steel for this type of column with rigid end bearing, limiting the percentage of hooping to not less than one half of one percent nor less than twenty percent of the vertical steel, and the vertical steel limited to not more than eight percent of the column area. This factor of safety is based on the yield point value of the column.

12. Structural Columns Filled with Concrete. As steel is more rigid than concrete, the higher the percentage of the steel the less the assistance we may expect the concrete to be to the steel, and while the formulas given heretofore for the column in which the vertical steel is loaded in large part by direct bearing of metal on metal agrees with tests closely, it should be noted that the percentage of steel in these specimens was not high. Accordingly, it would seem conservative to reduce the allowance on concrete in cases where the percentage of steel exceeds eight percent of the combined area of the concrete and steel in proportion to the increase in steel above this percentage.

13. Concrete Columns Compared to Structural Steel. From the Phoenixville tests, it would seem that there is a higher degree of uniformity in the tested strength of reinforced concrete columns made with ordinary care and with well designed reinforcement than with average structural steel columns. The reason is that in the concrete column, we have a solid core. The larger the column the greater the strength, in strong contrast with some structural steel columns. In the structural steel columns, we have the uncertainty due to the form and make-up of the section. In the make-up of most small struts and columns constructed of steel and iron, we have a specific ratio of the area of the web and flanges consisting of channels, or in other forms, a complete circular or box section, such as the Phoenix column or the box made up of two channels and two plates. The semi-empirical formula for struts has been worked out for the radius of gyration of those forms of sections which are comparable in the case of laced channel struts to a certain distribution of metal between the web and the flange and a certain ratio of flange width to depth, which ratios, under standard sections, are comparable to the box and Phoenix sections. When there is a wide variation from these proportions, the formula based on the radius of gyration is inapplicable as is proved in the original design of the columns of the Quebec Bridge. In this case the area of the flange as compared to the web was about one tenth of this standard proportion between the area of the web and the channel, and there is no experimental data in existence covering the action of the latticed bars and secondary stress for such a combination.

In concrete work such uncertainties are eliminated by the uniform solid core. With ordinary care there can be no doubt about securing a solid casting, if the type of reinforcement selected is that recommended in this chapter, namely, one in which there is no obstruction whatever in the core to prevent the concrete from flowing freely and filling the same completely and so securing a solid casting. Improper handling of the concrete might lead to considerable reduction of the strength of the section, but the damage which might occur thru bad workmanship is if anything much less in concrete work than in steel construction and the confidence in the builder in the integrity of his concrete work should be correspondingly greater.

14. Wall Columns and Interior Columns in Skeleton Construction. Wall columns differ from the interior column in that the load comes to them from one side instead of equally or approximately equally from all directions as it does under a uniform load on the floors.

Whether the floor is beam and girder construction or flat slab construction supported directly upon columns in view of the monolithic connection between the floor and the columns certain eccentric loads or bending stresses are brought upon the wall columns to a greater extent than is the case with interior columns.

The amount of bending induced in the column will depend, first on the rigidity of the floor, and second, upon whether the resistance of the floor to deformation is furnished by beam action or circumferential slab action. If the resistance is by circumferential slab action, the effect upon the column is far less for the same deflection than it would be in case of beam action, because the slab tends to twist from all directions and so in a large measure the effect of the load in producing bending and deformation of the column is counter balanced by the loads coming to the column from the sides instead of being augmented by these loads so much as is the case in the beam action of one-way reinforcement.

Favoring Conditions. The walls of a building are made vertical and as the number of stories increases, the columns are reduced in diameter, but kept flush or vertical on the outer face. This produces an eccentric application of the column load from the upper stories upon the columns of the lower stories which in some large measure off-sets and holds in equilibrium the bending moment of the floor loads.

Little attention is usually paid to this difference in conditions. The wall column is commonly made approximately the same as the interior column except in the upper stories where it is possible to carry the wall columns up for the last three stories without reduction in size, making thereby a provision for the columns of these upper stories which would be most affected by the eccentric bending moment referred to.

Good practice would limit the dimensions of columns to sixteen inches as a minimum size in office buildings and in warehouse construction, eighteen inches as a minimum.

Provision of material in the beam to take the entire bending resistance figured as simply supported at the wall column and continuous over the interior columns will in no wise eliminate this bending stress in the column itself, and does not excuse failure to make such provision.

In fact if the beams are treated as beams fixed at the inner ends and free at the outer end, and the five rod type of reinforcement used in the Turner beam is adopted, the positive moment provided for near mid span is $W\,L/14.4$ nearly, or $.07W\,L$, while the maximum positive moment in the beam freely supported and continuous over a number of spans in the end span requires a coefficient of from 0.07 for two spans to .08 for three spans, and .077 or .78 for a greater number, so that in this type of design there is nearly sufficient provision without considering the stiffness of the wall column. Nevertheless, in the view of the authors the provision recommended should be followed in practice.

Bending from Wind Pressure. In view of the monolithic character of reinforced concrete construction, wind has much less effect on a structure of this kind than is the case with the steel frame, since each floor forms a monolith of enormous lateral rigidity causing columns all to act in perfect unison. Where the building is narrow the columns should be fully spliced at the floor level which doubles the resistance of the column to flexure. Where the buildings are broad and only six to eight stories in height, provision of this character is unnecessary except in wall columns which should be well spliced.

15. Temperature Effect on Columns. Changes in temperature cause expansion and contraction of the concrete floor. This is taken up by bending in the column and the in and out motion of the walls where they are above the ground line. There is, however, a tendency for the basement walls to crack every thirty or forty feet because of the restraint of the surrounding earth. There is a further tendency for brick walls to crack where there is expansion if brick piers are used in place of well reinforced concrete piers in a long building. In a building over 300 feet in length, no dependence should be placed upon either a plain brick or a plain concrete pier without reinforcement to withstand temperature stresses of expansion and contraction of the floors since unsightly cracks and checks will very likely occur from this cause. In a building over 300 feet in length either a full, well reinforced concrete skeleton should be provided or if bearing walls are used the building should be cut at 250 foot intervals so that it will allow expansion and contraction without damage to the brick work or undue strain on the concrete of the floors and columns.

16. Economic Column Design. With reinforcing steel in the column figured at $2\frac{1}{2}$ cents a pound and concrete at \$6.00 per cubic yard, we may estimate the proportion of steel and concrete needed to obtain the most economic support from the standpoint of cost. Since a cubic foot of concrete weighs 150 pouds and a cubic foot of steel 485 pounds, a cube of concrete weighing one pound would have a volume 3.2 times as great as a cube of steel of the same weight, and have a face nearly one and one-half times as great. On this basis, for equal cost in carrying load, f_s f_c = 37, but in a fairly rich concrete n = 10 to 15, so that a load can be carried with less cost on a concrete pier than upon one of reinforced concrete, but in carrying the load in this manner, the diameter of the columns are greater, which is objectionable in that they occupy valuable room. Unbalanced bids are sometimes received on this basis. One puts in a bid using a column of a small diameter as specified, thus using more steel, but saving space; another makes a column larger by two or three inches and uses less steel and puts in a lower price. The owner is frequently persuaded to accept the large columns without giving the regular bidder an opportunity to revise his bid on the same basis.

It should be noted that the efficiency of the hooping varies as the diameter of the core, while the core area varies as the square of the diameter; hence a small increase in the diameter of the core permits a large decrease in the amount of hooping and vertical steel. Hooping costing 1.6 times as much as the vertical bars in a column in place has an efficiency 2.4 times that of the same cost of metal in vertical steel. On the other hand, certain relative proportions between the amount of the vertical steel and hooping are necessary to secure the best results, and these proportions should be adhered to if the formulas recommended are to be applied.

CHAPTER IX

FOUNDATIONS

1. Bearing Value on Soil. In a building the floor loads are carried to the columns, or to the walls in case bearing walls are used, and the weight is concentrated on small areas of ground at the footings of the columns and walls. Evidently if we are to avoid settlement the weight must be distributed over a sufficient area. The following are suggestions for safe loading for foundations where the material can be clearly defined.

Ordinary ledge rock, such as good shale, limestone and the like, twenty to thirty tons per square foot. Granite, trap where the ledge is not shattered, fifty to seventy tons per square foot. Hard pan, seven tons per square foot. Gravel, five tons. Clean, coarse sand, four tons. Fine sand, with a little clay, three to three and one-half tons. Hard clay, three tons. Clay, such as is to be found in Regina, which is rather soft, not over one and one-half tons. Blue clay of Winnipeg, two to two and one-half tons.

In each case, however, it is well for the engineer to look into the conditions carefully unless thoroly conversant with the locality. His judgment as to the bearing power of the soil should be checked, if doing business in a strange city, by a careful examination of the buildings resting on similar foundation and general inquiry among experienced members of his profession. This caution may prove of value to the engineer doing business over an extended area, particularly if he is acting in a consulting capacity for a contracting firm assuming responsibility for the design.

In cases where there is filled ground, marsh, quick-sand and the like it is frequently necessary to use a pile foundation or distribute the weight over the entire area. In general in reinforced concrete construction, it is economical to use a comparatively thin footing and thoroly reinforce it and make the concrete a rich mix.

2. Column Footings and Method of Figuring. Fig. 80 shows a form of footing which seems somewhat objectionable for the reason that as usually placed the concrete is worked rather dry in order to make the slope without top forms, and the contractor does not ordinarily get it in place so that it can be depended upon with certainty.

A preferable form is shown in Fig. 81, in which the footing is

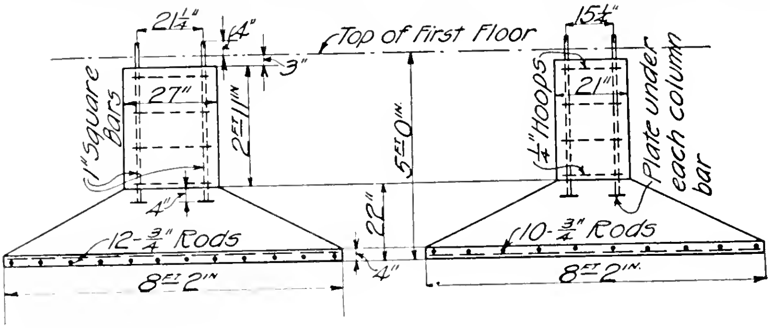


Fig. 80.

made in two layers. The bottom layer is cast with the rods at the bottom, then the column assembled and the top layer cast with the column. Its advantage from a practical standpoint is first, that

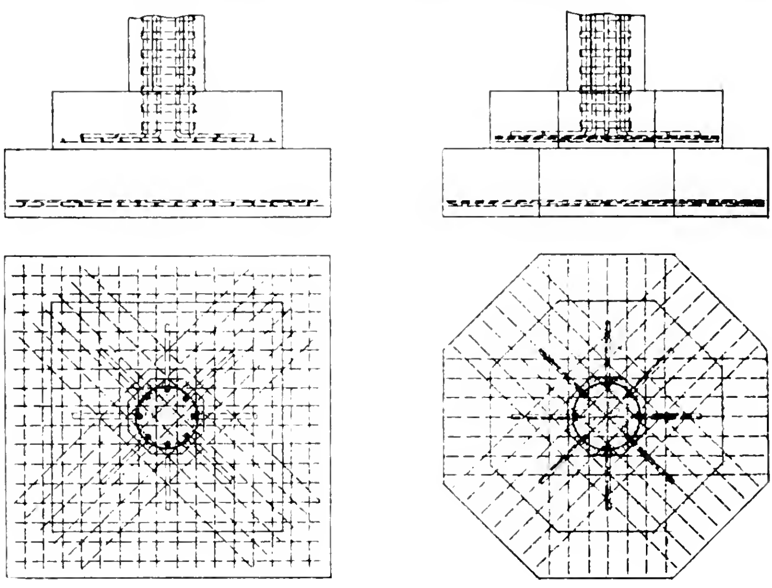


Fig. 81.

the upper layer of rods assist the footing in resisting the shearing strain of the column which tends to punch thru the footing and secondly it also assists in distributing the load out over the lower plate. The bottom plate being cast with a wet mix we can depend with certainty on the bond between the steel and the concrete by the shrinkage of the wet mixture as we could not in the footing first mentioned.

In computing the lower plate, the cross sectional area of the steel should be such as to provide for a bending moment of the upward pressure on the under side of the plate distributed uniformly over it with the column as the point of support.

The arm of the total pressure either side of the center may be taken roughly as five-eighths of the half diameter of the footing, hence the actual bending moment is $5/16\ Wb$.* The metal acting as a flat plate would be as we have shown for the square panel twice as efficient as single way reinforcement and as the sectional area of each bar comes into play on each side of the center of the footing we can use four times the area of all rods crossing the footing times .85 of the total thickness of the two layers times the working stress on the steel as the resisting moment.

The amount of steel placed in the top layer is more a matter of practice than that of exact computation. The rods are generally made the same size as those used in the lower layer and no time is wasted in computing the stresses thereon. It is preferable to take long rods and bend them hair-pin style for the footings rather than to use short rods.

There is a further advantage in ordering the rods this way, that in case the steel for the footings is delayed stock steel can be used while the steel ordered for footings may be used elsewhere in the building later.

3. Pile Foundations. Where piles if used will be continually wet and there is no possibility of changing conditions from that to alternate drying out and wetting, there is no type of reinforced concrete pile that can compete with timber piles. However, where the piles are liable to be above the permanent water line or it becomes necessary to excavate thereto, then and there concrete piles become an economic method of building up the foundation.

Practical concrete piles may be divided into two classes, one, those which are made up first and driven afterwards, and second, those in which a core or form is used and the hole filled in with concrete reinforced or otherwise.

4. Driven Piles. Hennebique was one of the first to use reinforced concrete piles that were made up separately and then driven.

*In this notation b is the breadth or diameter of footing.

In design they were similar to his columns which have been previously illustrated, with longitudinal reinforcement coupled with either of several arrangements of lateral ties.

Considère in his pile design, shown in the accompanying Fig. 82, commonly made use of spirals for lateral reinforcement in place of Hennebique's ties. His form of piles is used to a large extent on the continent and is similar as regards the point at its lower extremity to Hennebique's.

In driving, it is customary to use a water jet for loosening up the sand or earth at the bottom and rap or jar the pile into place with a

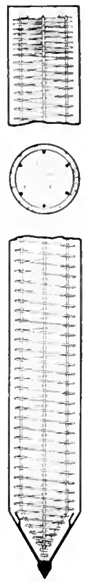


Fig. 82 Showing Reinforcement of the Considère pile.

hammer. A steam hammer should be used in driving concrete piles rather than a drop hammer, since with the heavy ram and short stroke of the steam hammer there is less shock.

5. Piles Cast in Place. Coming under the second class are the Raymond and Simplex piles.

The Raymond piles are made tapering to secure greater resistance against settling. The manner of constructing Raymond concrete piles is as follows: A steel pile-core the size and shape of pile desired is encased in a thin, closely fitting shell. The core and shell are then driven into the ground by means of a pile-driver in the usual manner. By a simple and ingenious device the core is collapsed or shrunken slightly, so that it loses contact with the shell, and is easily withdrawn, leaving in the ground a clean, perfectly formed hollow tube of the size and depth required, which has only to be filled in with the best Portland cement concrete to complete the pile.

Fig. 83, shows the Raymond pile expanded ready to be driven.

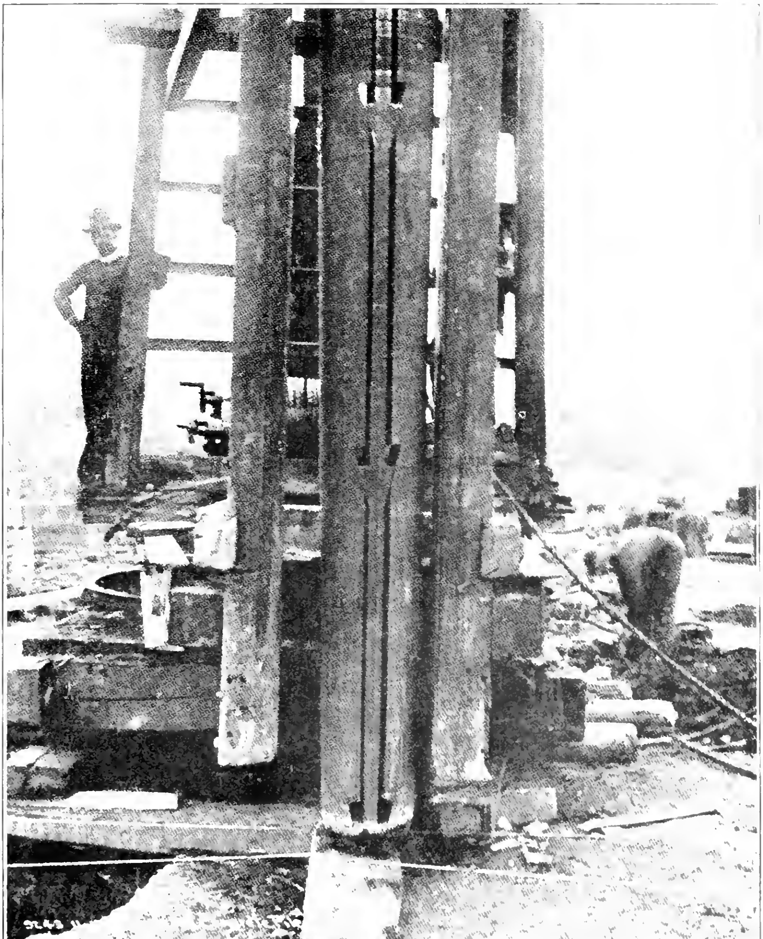


Fig. 83. Raymond Pile Core. This core is used to drive to rock and tapers from 20 to 13 inches. The cut shows core full size as it is driven.

Fig. 84, shows the core in the leaders with the shell on the right.

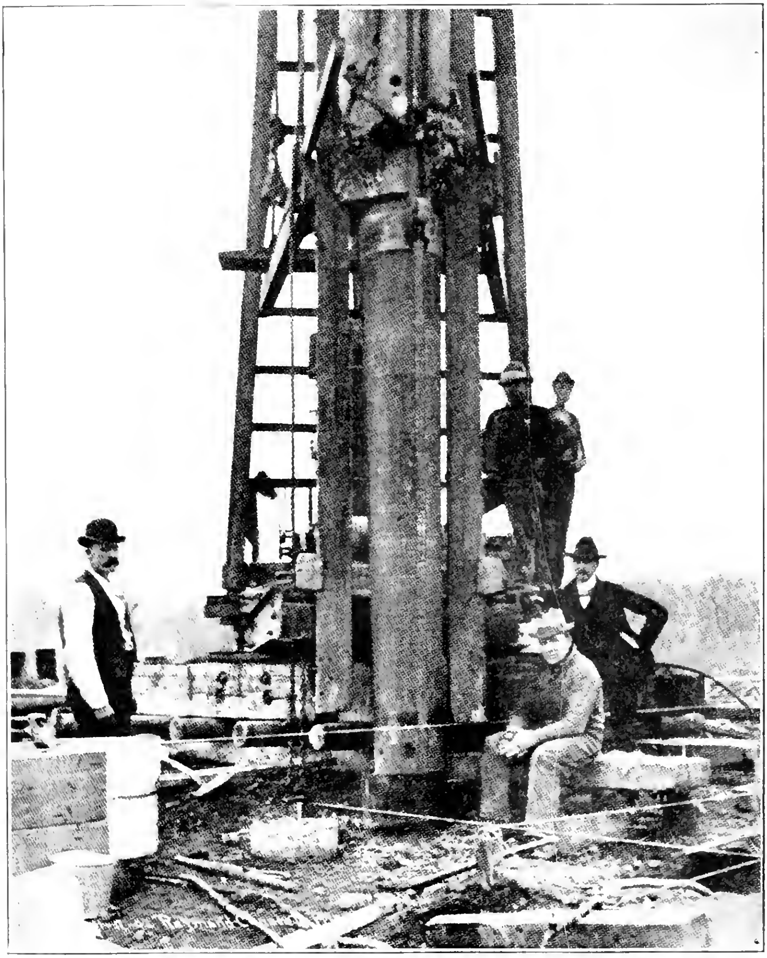


Fig. 84. Pile Core Collapsed or Shrunken, and partly withdrawn from the shell.

The core is expanded when driven and collapsed to be withdrawn from the shell.

The relative merits of the various kinds of piles would apparently depend on outside conditions. A pile like the Raymond pile should be most suitable for a clay soil where the consistency or cohesiveness of the clay is such that the pile core can be driven and the shell omitted.

Raymond piles are made of various lengths, tapering generally from 20″ at the top to 6″ at the botton, making a symmetrical cone affording material resistance to soil penetration by friction.

For the Simplex pile a casing is first driven, and then as the casing is pulled up, concrete is deposited and rammed in place, forcing it out to a somewhat greater diameter than the shell that has been driven.

Figs. 85 and 86, show the make-up of the Simplex pile. With the steel casing is driven a point, either of steel or concrete, and afterwards the shell is gradually withdrawn and the hole filled with concrete as the shell is filled up.

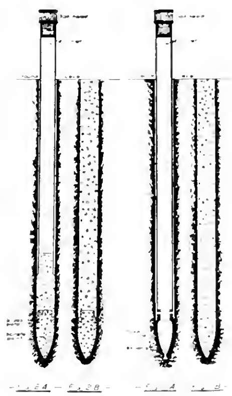


Fig. 86 Fig. 85
Simplex Pile.

For use in earth that is reasonably firm in its texture and free from water, the preparatory removable pile, (see Fig. 85) is used. This pile form consists of a length of extra heavy wrought iron pipe, fitted with a suitable driving head of oak, and a conical steel point of a somewhat larger diameter than the pipe, and fitted with an automatic air valve. This preparatory tube is driven into the ground to the required depth, and then withdrawn without difficulty, and the hole so produced is filled with well rammed concrete. This form of pile can be constructed of any desired length, as the preparatory tube can be driven and removed with but a fraction of the force required in the planting or removal of the ordinary pile. It can also be driven thru ground of a density quite impenetrable by any wooden pile and to almost any desired depth, as there is no appreciable frictional resistance, as the depth increases, either in driving or withdrawing the tube.

The ramming process forces the larger pieces of the aggregate into the sides of the hole, materially adding to the frictional hold of the pile on all parts of its surface.

Where the earth is soft, marshy, or where quicksand or water is encountered, a detachable "point" of concrete, (see Fig. 87) is substituted for the fixed one of steel. This concrete point is driven to the required depth, and as the pipe is being lifted off, concrete is gradually filled in and rammed home thru the pipe, care being taken that a head of the concrete be maintained inside of the pipe while it is being thus gradually withdrawn. By this system all water is displaced and the possibility of the sides of the aperture closing in is entirely removed.

Fig. 87. Concrete Points for Simplex Piles.

Corrugated piles have been patented by Frank B. Gilbreth. One of the claims advanced in their favor is that the corrugations assist in jetting the piles into place.

Fig. 89 shows a driver handling one of the corrugated piles.

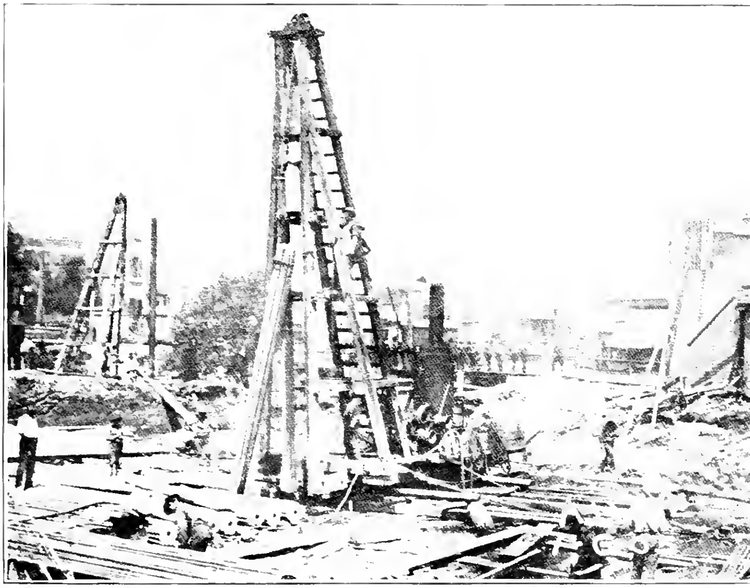


Fig. 89. Driving Corrugated Concrete Piles.

Fig 90 shows the cushion cap used in driving the pile.

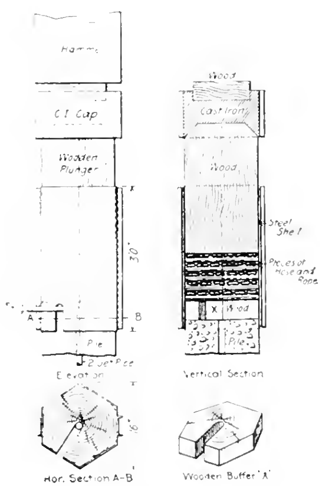


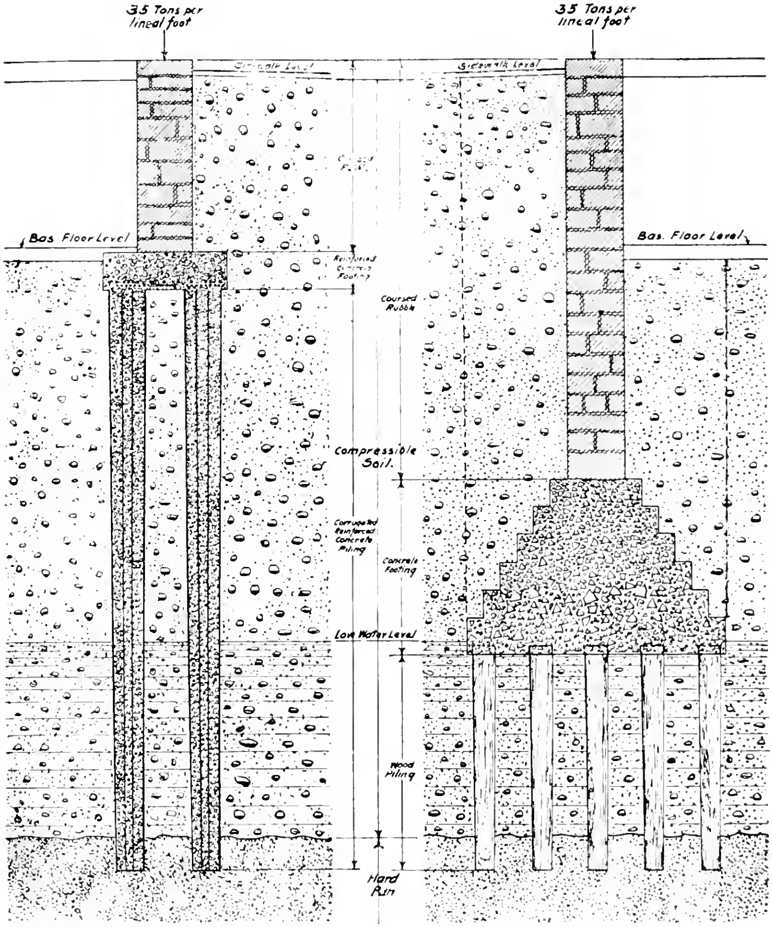
Fig. 90. Cushion Cap used in driving the corrugated concrete pile.

Concrete piles have this advantage over wood piles; they do not decay, are not subject to destruction by insects and furnish a durable foundation regardless of soil conditions; they can be used in dry filled ground as well as in wet soil, which may dry out and cause wood piles to decay.

A pile foundation generally tends to prevent settlement by the packing and settling of the soil in the vicinity of the pile, frequently rendering the bearing value of the soil around the piles materially greater. See Fig. 91.

Fig. 91. A Trench Filled with Concrete Piles.

Fig. 92, gives a fair idea as to the advantage of a reinforced concrete pile foundation over the wood pile which must be cut off in the vicinity of the low water.



Reinforced Concrete Piles. Wood Piles.

Fig. 92. Showing the advantage of concrete over wood pile foundations.

6. Safe Bearing Loads for Piles. Two cases are to be distinguished; that of piles, the lower end of which rests upon a hard strata and that of the ordinary pile which is supported largely by skin friction of the material into which it is driven. The capacity of the former is determined by the strength of the pile as a column thru the upper or soft strata while the bearing power of the latter is some function of the penetration under a given drop of a ram of a given weight.

Many formulas have been proposed, but the only formulas in anything like general use are known as the Engineering News Formulas. They are:

For a pile driven with a drop hammer $P=\dfrac{2\,W\,h}{S+1}$

For a pile driven with a steam hammer $P=\dfrac{2\,W\,h}{S+0.1}$

to which P is the safe load in pounds, W the weight of the hammer in pounds, h the fall of the hammer in feet, and S the penetration or sinking in inches under the last blow, assumed to be at an approximately uniform rate. They are deduced for wood piles; but are the best there are for concrete piles.

Mörsch gives the Brix formula as usually employed on the Continent:

$$p=\frac{h\,Q^2\,g}{2e\,(Q+g)^2}$$

wherein h is the fall of the hammer;

Q the weight of the hammer;

g the weight of the pile;

e the penetration of the pile under the last blow;

p is double the safe allowable load for the pile.

The quantity e will naturally be the average of the last few blows.

CHAPTER X

ELEMENTS OF ECONOMIC CONSTRUCTION AND COST OF REINFORCED CONCRETE WORK

1. **Introductory.** Reinforced concrete construction of buildings presents problems which from an economical standpoint are so complicated that they cannot well be investigated as questions of maxima and minimia by means of equations which show the cost of the several variable items and determine their proper relation by mathematical treatment. For this reason it is necessary to pursue the investigation along simpler lines, by taking up questions of general arrangement, column spacing, spacing of beams, choice of type of reinforcement, cost of centering for each kind, cost of aggregate, etc.

2. **Column Spacing.** Eighteen feet center to center of columns in each direction usually costs less than for shorter spans while the increase in cost with increase of column spacing up to twenty feet is very little, if the loads are heavy, and the building high. In wholesale hardware buildings, the customary requirement is that of lines of shelving and boxing about twelve feet centers. For such a building, diagonal spacing of columns with flat slab construction, making the columns about seventeen feet between centers, will provide the desired spacing in a multiple of twelve feet for the boxing and shelving, i. e. twenty-four feet in directions parallel to the sides of the building.

3. **Floors.** In treating the strength of floors by comparison on the principle of proportion it has been noted that the coefficient of bending with continuous flat slab type is smaller than the corresponding coefficient for beam construction. This advantage is offset for light loads by the disadvantage of the smallness of depth which is a factor in the moment of resistance. Further, stiffness varies as the square of the depth, but the coefficient for deflection is smaller than for simple beams. Hence it is evident that for stiffness under light loads, lower percentages of steel are requisite than in the beam type of construction. Where the loads are heavy the continuous flat slab type becomes more and more economical compared with other types of floor as the depth of slab increases by reason

of the relatively small coefficient of bending. A little computation as to the cost of concrete and steel shows that considerable variation from the exact economical proportions of the two materials for a given strength will make comparatively little difference in the ultimate cost, whereas in the flat slab types using a smaller amount of steel and more concrete does make a large increase in the stiffness.

Flat slab types have the advantage of simpler centering and lower cost of placing the steel, but aside from that they possess little advantage over the two-way beam system for light loads, tho requiring less material both of concrete and steel for heavy loads and moderate spans up to twenty-five or thirty feet, and they compare favorably for greater spans where the loads exceed five hundred pounds per square foot.

Slabs reinforced in one direction, being supported on two sides only, are at a disadvantage, since the coefficient of bending is three times as high with one-way reinforcement, as it is for the slab reinforced in two ways and supported on four sides.

In comparing Types III and IV, it should be observed that if the beams of Type III are of twice the thickness of the slab of Type IV, the same weight of reinforcement is required in the beams of Type III that is required in the whole floor of Type IV.

Usual proportions would be a depth of beam equal to three times the thickness of the slab of Type IV requiring two thirds the steel for the beams.

The apparent moment to be resisted in the slab supported on four sides Type III is but two thirds that to be borne by slab of Type IV. Hence a slab may be used of less thickness than with Type IV tending to equalize the concrete quantities and leaving the difference in cost of forms and steel to be considered.

Type II is frequently made of a combination of hollow tiles and thin reinforced concrete beams between the hollow tiles, in order to secure economic depth and reduced weight for long spans, with decreased deflection under load. Where this type is used, the cost of centering is kept down to a figure approaching that of the flat slab types but extra care is required in placing the steel and the labor of putting tile in position and the cost of the tile is added. Where tile is low in cost and concrete material is high in cost, tile and concrete for a light building may be more economical. On the other hand, the risk of erection is greater with this type on account of the brittleness of tile and the character of failure of one way reinforced slabs as pointed out in the discussion of beams. For heavy loads, however, this type cannot compete with the natural concrete types, III and IV.

4. Centering. As we have noted, centering is one of the important considerations since the cost of centering runs to twenty-five to thirty-five percent of the total cost of the ordinary floor.

In the selection of the type of floor to be used, the cost of centering is so large an item that it should be given careful consideration in any approximate solution of an economic design.

Where the spans are long and the loads are light and the cost of concrete materials, stone, gravel and cement are very high, flat slab types cannot compete with the beam type. Type III as a beam and slab type is more economical than other beam types for heavy loads and panels of sixteen to eighteen feet. Where the columns are equally spaced, the economy is greatest. Where the spacing is such that the panels are rectangular and one side is less than six-tenths of the longer side, its economy disappears and it is preferable to use an intermediate beam, thus dividing the slab into panels more nearly square.

For joist and girder construction surmounted by light slabs, the spacing of the ribs is governed by the character of the centering used. If the centering is arranged in panels so that it is easily handled as such, six to ten foot spacing of beam joists works out very economically. Considerable increase, however in the cost of centering is brought about by the additional framing of the joists to the main beams or girders.

No exact rule can be given covering all cases, as the conditions of the problem, such as size of the building, manner in which it divides up for the purposes intended, etc., fix so many of the conditions that any rule disregarding these conditions might readily be misleading. The number of stories figures largely in the cost of centering. The cost of framing beam boxes as a rule must be figured for three stories. Additional stories, if the framing is worked out so that the same boxes can be used over and over again reduces the cost per foot greatly and these are conditions which must be taken into consideration in any practical comparison of different types.

5. Columns. The hooped column is the safest type to erect and where the loads are at all heavy the most economical type to adopt. The economy of the reinforcement depends largely upon the adoption of the proper proportion of hooping and vertical steel. The hooping adds to the toughness of the column and to its ultimate strength but does not raise the point where the column shell commences to scale and chip unless the hooping be combined with the proper proportion of vertical steel. When this has been done, very high values indeed can be safely developed. The carrying capacity, however, of the column is secured at a minimum cost by concrete rather than by reinforcement. Still a certain amount of steel is necessary to secure toughness and resist flexure. Again, consideration of the value of floor space frequently limits the size of the column which the architect or owner is willing the designer should employ.

As to what may be done with the reinforced concrete column, Turner has used a twenty-seven inch core, heavily banded and reinforced vertically, for working loads from eleven to twelve hundred tons. This pressure being a little over 4000 pounds per square inch of core area. The development of such high working values by heavy reinforcement, tho unobjectionable from the standpoint of safety and desirable from the standpoint of occupying small floor space is not economical from the standpoint of first cost of providing a post of proper capacity for a given load. A rich concrete is more economical than a lean mixture, and where the loads are heavy a $1:1\frac{1}{2}:3$ mixture is recommended in place of the usual $1:2:4$ employed under ordinary conditions.

6. Bearing Walls or Full Concrete Skeleton. A very important question in economic design is the question as to whether bearing walls are to be preferred to a full concrete skeleton.

For such a low building as four to five stories, bearing walls are generally cheaper than a full concrete skeleton. For buildings higher than five stories a full concrete skeleton with curtain walls costs less than heavy bearing walls.

In putting up a concrete skeleton, the additional cost involved in making provision for two or three additional stories is generally so small that it is advisable for the owner to make this provision if there is a reasonable probability that he may use the additional floor space in the future and the value of the real estate warrants such investment.

7. Concrete, or Brick Exterior Walls. In some cases where the concrete aggregate is cheap and union brick layers' wages are high it is better to use a concrete exterior wall. Generally, however, exterior walls may be constructed much more cheaply of brick, or of some material that can be laid up without the necessity of using forms, since the forms for exterior wall construction run into money quite rapidly.

8. Rich Mixture. Economic construction in reinforced concrete requires a rich mixture. This is a necessity, first, from the standpoint of certainty of computation, second from the standpoint of quick hardening which enables the early removal of the forms with economy incident to repeated use of the same form lumber, and from the standpoint of economy due to the fact that we can use less material of good quality which we can absolutely depend upon than we can of material of an inferior quality and uncertain character which is liable to be discredited by reason of its slow hardening through the lack of necessary amount of cement. Further, where the material used has been of good quality the construction can be increased in strength to any desired degree by the addition of more good concrete tho the strength so secured will not be at so low a cost as if the original design was for heavier construction.

9. Economy in Selecting Aggregate. Good bank gravel when obtainable makes an excellent aggregate. Its adaptability for the purpose should be determined by screening out the sand and pebbles which are under $\frac{1}{8}$ inch diameter and comparing the volume of sand with the volume of coarse aggregate. If these proportions depart from those desired, that is, one cement, two sand, and four of the coarse aggregate, then the cement content must be increased to make the mortar of the proper proportion, or else crushed stone must be added to the mixture.

Where the expense of securing crushed stone is high and the cost of cement is low, the addition of more cement to keep the mortar a true 1 : 2 mix is cheaper than the addition of stone, and in a way preferable, because with the excess of mortar there is less liability of voids and poor work. Sometimes crushed stone and gravel is not available and a good hard furnace or smelter slag may be secured. Slag should be examined for chemical impurities which might injure the cement and for hardness which determines its fitness as a good aggregate.

10. Cinders. Cinders are sometimes used as an aggregate for concrete. Cinder from the soft Iowa coal is generally very injurious to the cement. In fact it may be stated as a general rule that the only cinder fit to make a permanent concrete is that which is a hard or more or less vitrified clinker such as generally results from burning soft coal with a mechanical stoker. Too great care cannot be exercised in this respect as upon the character of the aggregate and its freedom from sulphur or other injurious chemical elements which would damage the cement, depends the permanence and integrity of the work.

In general a clinker concrete should not be used where a high degree of strength is required. It is desirable to use it for such work as roof work where the spans are short and it is desired to nail a tile or slate cover to the concrete roof slab. For such purposes the concrete should not be mixed too rich, otherwise it will be difficult to nail into it.

11. Adaptability. Reinforced concrete is not adapted for long spans and light loads. For instance, in a span of fifty or sixty feet in a shop or factory building, having only a light roof load, reinforced concrete is not an economical material to use. Structural steel costs far less and is generally employed. For bridges of long spans and light loads reinforced concrete is not economical in first cost. Where the loads are heavy, as for a city bridge, and the grade such that there is opportunity for ample rise, a reinforced concrete arch may be built at a cost not greatly exceeding that of a good structural steel bridge and when the maintenance charges are considered the concrete will be the least expensive.

For office buildings and ordinary business blocks reinforced concrete will in general save the owner from one-half to three-quarters of the cost of a structural steel skeleton.

Reinforced concrete is particularly well adapted for school buildings. The difference in cost between the ordinary timber floor and a reinforced concrete floor will frequently not exceed ten or fifteen cents per square foot. With this fact in mind it is really astonishing to observe how frequently dangerous fire traps are erected to serve as school buildings on which expensive and ornamental exteriors have been used when plainer buildings with fireproof construction could be erected for the same money.

The architect for a school building, in order to make a show, frequently specifies a fancy brick exterior, terra cotta or stone trimmings and other external frills and then economizes in the interior construction of the building by the use of timber joists $1\frac{1}{2}$ or $1\frac{3}{8}$ inches thick by 16″ covered with $\frac{7}{8}$ inch rough floor and $\frac{7}{8}$ inch hardwood finished floor, with wood lath and plaster on the under side, electric wires, heating flues, etc., between the joists. This is a construction in which if a fire once started there would be hardly time for the occupants of the building to escape before the collapse of the floor with the probable loss of life incident thereto.

The Collinwood disaster well illustrates this fact. The state of Wisconsin has passed a law making it compulsory to build school buildings of fireproof materials and other states may well follow her example.

For buildings subject to the vibration of heavy machinery concrete steel construction has many advantages. Properly designed, the joints (connections of floor to columns) are far more rigid than in any of the old types of construction, hence a rigid building costs least in concrete steel.

The Forman-Ford Company, plate glass dealers, etc., make the statement that twenty-five percent more work is done in their cutting and polishing department in their new concrete building with the same men than in the old timber framed structure previously occupied owing to the increase in rigidity of the structure.

Impact or shock at any point of a steel structure is propagated longitudinally along elastic members extending in a linear direction from the point and it goes practically undiminished to the far ends of these members where it is subdivided among other members and propagated still further. An allowance up to eighty or ninety per-cent is usually added for impact to the static effect of a moving load in bridges.

Impact, or the dynamic effect upon any point of a reinforced concrete slab, however, is entirely different from this. In the first place, the effect of the blow does not travel in one direction only but in all directions radially from its point of application, so that in a very thin slab its effect at any other point would be inversely as the distance and in a very thick slab inversely as the square of the distance. This would make the allowance for impact in the thick concrete floor of a bridge or building very small in comparison with that inevitable in steel construction.

Secondly, the effect of impact must be inversely proportioned to the weight of the body receiving the blow. Now in monolithic concrete construction the mass affected is of far greater weight than would be the case in a steel frame which is made up of independent steel members and tile as in the old style buildings and for that reason the effect of impact on a monolithic concrete floor would be reduced by a large percentage.

Third, the continuity and stiffness of the floor greatly reduces its vertical, lateral and torsional deformations below those of the steel structure. The work done during an impact and its effect depend on the amplitudes of the deformations. In particular the horizontal resistance of the slab is many thousand times that of the steel members in a structure. The vibratory energy absorbed by the slab during impact is consequently small.

Fourth, the small amount of energy which is absorbed is not transmitted (as it is in a highly elastic and resilient structure) to a considerable distance in the slab, but owing to the nature of the concrete, is dissipated near its source, transformed into heat, and rapidly absorbed.

Fifth, concrete slabs are tough and not brittle, like terra cotta, for example, so that, in cases where great weights have fallen on them, little effect has been produced, whereas brittle slabs such as those of concrete and tile have been smashed under such circumstances, and have failed.

The concrete of the compression zone is such a shock-absorber as to protect the tension zone from jarring and vibration, both as regards steel, in tension and the bond of the concrete to the steel as well.

For all these reasons the shock which a rolling load imparts to a slab is inconsiderable, and is absorbed and dissipated so readily that it is a negligible factor, rendering reinforced concrete adaptable for use in railroad structures and in buildings where the service is most severe from shock of machinery such as beater floors in paper mills and hammers in stamping and working metal.

As illustrating these statements, an interesting accident occurred in Winnipeg where a heavy cornice block of hard Tyndale limestone, weighing between a quarter and a half ton, was dropped seventy feet, striking a $5\frac{1}{2}$ inch concrete slab 19 feet square in the center. The effect of this blow was a small dent half inch deep where the corner of the stone struck the slab and the stone itself was badly broken and shattered. The slab, however, was uninjured and showed no cracks or evidence of over-strain.

In Minneapolis, a steel water-tower failed under wind pressure and a fifty ton tank dropped ten feet on a light roof slab of concrete reinforced after the manner of the Mushroom system in four directions. The slab was approximately 22 feet by 23 feet clear span, seven inches thick, reinforced with 7/16 inch rounds averaging eight inches centers in two directions with diagonal belts of fourteen

7/16 inch rounds. The shock of the fifty tons falling ten feet dented the slab down about eight inches and produced some shear cracks but it carried the load and caused the contents of the tank to fall outside of the building, demolishing two freight cars and the awning over the loading platform. The toughness of this slab saved a wreck of the side of the building and the elevator machinery.

Other interesting cases might be cited, but it is thought that these unusual examples illustrate the remarkable toughness and dependability of concrete reinforced in multiple directions and its great adaptability from the standpoint of capacity to withstand severe usage with high degree of safety for all purposes when scientifically designed and properly executed.

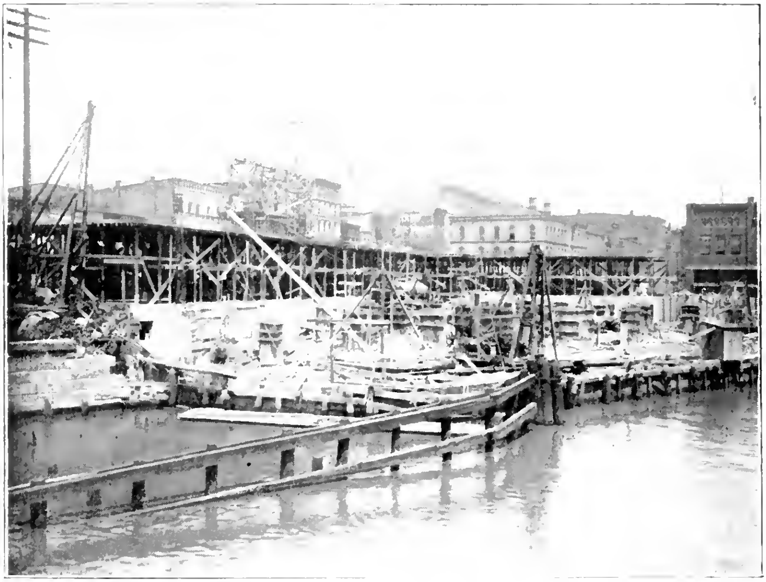
12. Rapidity of Erection and Ease of Securing Material. No type of building can be so rapidly or quickly designed, detailed and erected, as the natural types of reinforced concrete construction. If we take Type IV, for instance, a single computation is sufficient for a panel and where panels are tabulated for various loads two hours' work is sufficient to make the computation for a given size of factory building or manufacturing plant, including an estimate of the cost of reinforcement, quantities of concrete and centering with sufficient precision for bidding purposes.

In no type of building construction can the materials be secured so promptly as for the reinforced concrete structure. An ordinary four or five-story building can sometimes be erected complete in the time ordinarily required to get out the shop details for a structural steel frame. Especially where the building is irregular in form there is this advantage with reinforced concrete, that the joints are made with the cement in plastic form, that the rods can be lapped more or less over the supports, avoiding the necessity for the large amount of figuring required for the skew connections of structural work, hence the engineer's end in this line of building construction is greatly simplified.

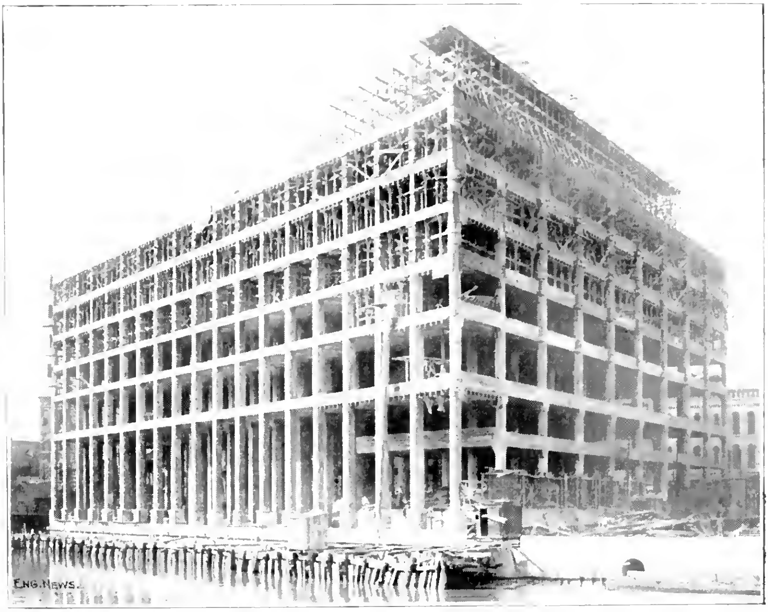
The accompanying figure shows the rapidity of construction of the Bostwick Braun Building; its condition August 1st showing the sea wall and adjacent footings incomplete; November 11th, showing the centering nearly completed for the roof, or eight stories in place.

This is merely a usual example in the construction of a large building. A floor per week of the concrete skeleton and rough slab can easily be erected under favorable weather conditions.

The is little difference between a large building and a small building in this respect, since with a larger building it is possible to rig up in



Aug. 1st, Showing Seawall and Adjacent Footings Incomplete.



Nov. 11th, Showing Centering Nearly Complete for the Roof, Eight Stories in Place.

a manner that will facilitate the handling the work more rapidly. In fact where there is a large area it permits the employment of more men and makes it possible to keep them at work on the various features such as centering, pouring concrete, placing steel, in one continuous operation, centering going ahead for one floor as the steel is being placed, the concrete men in turn following up those placing the steel. When the carpenters are thru with this floor they immediately proceed to erect the forms for the next on that portion of the floor where the concrete has been cast.

Where the amount of work to be handled runs eight to ten thousand yards, it pays well to rig up with overhead bins and mixing plant and to arrange for the use of half-yard dump cars in placing the concrete. Where the yardage is much less the wheelbarrow or two-wheeled truck and scale hoist becomes an economic method of handling material. The later mixers are arranged with a charging device which saves wheeling material up an incline as was customarily done in earlier work.

13. Analysis of Items of Cost. In arriving at a detail estimate of cost we have the following items to consider:

Basis of Labor

Materials and cost of handling:

$$\begin{array}{l}\text{Concrete,}\\ \text{Unit price}\end{array}\left\{\begin{array}{l}\text{Cement}\\ \left.\begin{array}{l}\text{Sand}\\ \text{Stone}\end{array}\right\}\text{or gravel}\\ \text{Water}\\ \text{Common labor}\\ \text{Cost of plant}\end{array}\right\}\text{Quantities and base prices.}$$

$$\text{Steel}\left\{\begin{array}{l}\text{Cost of metal}\\ \text{Cost of unloading}\\ \text{Labor, cost of bending (union or common)}\\ \text{Labor, cost of placing (union or common)}\end{array}\right.$$

$$\text{Centering}\left\{\begin{array}{l}\text{Cost of lumber}\\ \text{Cost of framing beam boxes, columns, etc.}\\ \text{Cost of erecting and rehandling}\\ \text{Slab forms, beam forms and column forms.}\end{array}\right.$$

Season of the year.
Floor finish or strip fill.
Dead expense.
General data on costs per foot of floor and items entering into it.

14. Labor, Unit Prices, Quantities of Material. Under the general heading of labor the contractor must consider, first, the wages per hour; second, the character and efficiency of the labor, whether the labor is union or non-union, probability of strikes and delay of work into the unfavorable season when artificial heat must be used.

Where trade unions are strong the specialist in reinforced concrete can never tell when his work will be tied up by some disagreement between master plumbers and walking delegates or other trades with which he has no relation whatever further than that a sympathetic strike may be called without notice or grievance at any time and his operations brought to a standstill.

This condition means idle equipment and sometimes cost of heating materials and may mean readily an additional cost of five to ten per cent in the work.

In Chicago the bricklayers' union demands that the contractor shall keep employed an extra brick foreman who is supposed merely to watch the placing of concrete wherever there may be brick work on the job.

While a labor union should prove of benefit to employer and employee alike in case its motto is efficiency and skilled service, it loses both public sympathy and support when it cultivates inefficiency and loads the work up with men who are useless as is the case with an extra foreman. In many cases concrete is used for exterior walls in place of brick where, were it not for this short-sighted policy, brick would be used from an economic standpoint.

For purpose of discussion and comparison we will take the following costs: Common labor, $2.25 per day of ten hours; carpenters, $3.00 per day of eight hours; steel to be placed by common labor at $2.25 to $3.00 per day.

Unit Price of Concrete

1 : 2 : 4 Mix

Material for one cubic yard of concrete, wet mix:

Cement	$1\frac{1}{2}$ bbls.
Crushed stone	.9 cubic yards.
Sand	.45 cubic yards.

Where a crusher run, including dust of good hard crystalline stone is used the sand may be readily reduced to one-third yard.

The above amounts required to make a cubic yard of wet mixed concrete in place may vary somewhat on the character of the crushed stone or gravel, but for estimating purposes they are conservative.

$1 : 1\frac{1}{2} : 3$ Mix

Material for one cubic yard of concrete, wet mix:

Cement	2 bbls.
Sand	.43 cubic yards.
Stone	.85 cubic yards.

1 : 3 : 5 Mix

Material for one cubic yard of concrete, wet mix:

Cement	$4\frac{1}{2}$ sacks = 1.125 bbls.
Sand	.52 cubic yards.
Stone	.85 cubic yards.

Labor of Handling

Given an ordinary equipment such as a half-yard Smith, Cube or Ransome machine the labor cost of handling concrete may be stated as follows:

Wheelbarrow gang, from \$1.25 to \$1.50 per cubic yard, including cost of coal or gasoline for the engine. In walls, footings or where there is quite a mass this may be reduced to \$1.00 per cubic yard.

On a large job where one-half yard cars are used and overhead bins for handling the aggregate by gravity, the labor cost may be reduced to 35c to 40c per cubic yard. To this must be added, however, the cost of fitting up the plant which will increase this figure to sixty or even seventy cents per cubic yard.

The labor costs will increase or decrease as the price of common labor is above or below twenty-two and one-half cents per hour, figured upon.

Cost of cement varies with the market and distance of the work from the nearest mill from eighty cents or a dollar per barrel to two or three dollars.

Cost of crushed stone varies with the locality and distance of the work from railroad or crushing plant.

In Minneapolis and St. Paul, from \$1.25 to \$1.75 on the work. Milwaukee, \$1.25 to \$1.59. Washed gravel, Ohio river points, \$1.00 to \$1.25.

Cost of carting and hauling must be investigated in each individual case. In many of the smaller towns good concrete gravel can be secured as low as thirty to fifty cents per cubic yard and in order to give a clear idea as to the general questions of cost these variables must be carefully considered and investigated by the bidder if figuring reasonably close.

In securing this essential information the conservative business man will secure quotations in writing, especially when not personally acquainted with the reliability of the parties quoting; then if the work is secured he may at his option hold the bidder to his price or seek redress by suit.

In giving the foregoing average values it should be noted that the cost of placing varies eight to ten per cent with the character of the reinforcement. Where there are numerous beam boxes and stirrups and increased work of puddling the concrete, the cost may readily run five or six per cent above the average while where there is a plain flat slab such as the mushroom system the cost will readily run five or six per cent lower than the average given.

15. Cost of Steel. Medium Steel, Open Heath or Bessemer, Manufacturers' Standard specification is at present writing at $1.20 base, Pittsburg.

The base price is given in all the engineering and iron trade papers. All bars from $\frac{3}{4}''$ rounds to three inches are base. Smaller bars are sold at base plus card extras.

The following is the standard steel classification:

	Extra	
3/4″ to 3″	Base	Rounds or square.
5/8″ to 11/16″	.05	
1/2″ to 9/16″	.10	
7/16″	.20	
3/8″	.25	
5/16″	.30	
1/4″ to 9/32″	.35	
1 to 6″x3/8 to 1″	Base	Flats and heavy bands.
1 to 6″x1/4 to 5/16″	.20	

The above are full extras and such sizes as we can ordinarily use to advantage in concrete steel construction.

In figuring, take base price plus extras plus freight to destination. Freight rates to all points in the United States and the Dominion of Canada are given in compact form in a book published by the American Steel & Wire Company.

Cost of deformed bars rolled to standard specification at the present writing, one dollar per ton above plain bars. Special reinforcement sold with design from eight to twelve dollars per ton additional.

16. Cost of Bending. Medium steel with proper equipment rods for the mushroom system can be bent cold for fifty to sixty cents per ton; where high carbon steel is used and rods are heated, one and a half to two dollars per ton for bending.

Beam rods, such as are used in Turner beam system, can be bent for from two to two and a half dollars per ton; where more complicated bends are employed, from two and a half to three and a half per ton.

Cost of placing steel, including handling and bending, in Mushroom flat slab work has run from six to ten dollars per ton. With a beam system in beams spaced four to six feet centers a cost of ten to twelve dollars would be a fair basis upon which to figure.

17. Cost of Hooping for Columns. Spirals made at the shop can, at the present time, be furnished at a less cost than they can be fabricated in the field, except in parts of Canada where this statement may not be true. Shop work is better done than field work and should be preferred other things being equal.

18. Cost of Centering. No item in concrete construction is so generally underestimated as the cost of false work for reinforced concrete. In fact, so generally is this the case that the contractor inexperienced in this kind of work is more than likely to underbid those possessing both equipment and experience by reason of underestimating this item of cost.

Cost of the centering per foot of floor, including columns and beams, will vary anywhere from six to more than twenty cents per square foot, depending on the following items:

1. The number of beam boxes, whether they frame into each other or into column boxes only.
2. The number of columns for a given floor area.
3. The number of stories or floors that are alike.
4. The rapidity with which it is desired to push the work and whether the weather conditions are favorable for the prompt removal of the forms.

Where the building has a full concrete skeleton, centering costs generally are a third more than where bearing walls are used. Evidently the greater the number of stories the more times the lumber may be moved up and used over. Where the work is to be rushed rapidly in cold weather a larger amount of lumber is required.

The practical constructor is inclined to check his estimate of cost on the basis of so much per foot of floor for centering for reinforcement and concrete, and estimating in this rough way will generally detect any error of more than four or five per cent in an elaborate detailed estimate.

Attention may be called to the following elements necessary in any estimate:

1. Lumber required, nails and fastenings.
2. Carpenter labor of framing beam boxes, column boxes, etc., per thousand feet.
3. Labor of setting plain slab forms.
4. Labor of taking down forms and moving up to upper story per thousand feet B. M.
5. Waste of lumber and value of old centering.

Under 1, the amount of lumber required, it should be observed that the amount will vary with the type of design. In such a type as the Mushroom system, Type IV, Fig. 14, there must be for the sheathing approximately one foot B. M. for each flat foot of floor area. For the joist, $\frac{3}{4}$ of a foot B. M. per flat foot of floor area. For ledgers, one-third of a foot, B. M. for each flat foot of floor. For uprights, $\frac{5}{8}$ of a foot B. M. for each flat foot of floor. For columns, spacing 18′ centers, from one-third to one-half foot B. M. for each flat foot of floor. Total, about three and one-quarter or three and one-half feet B. M. per foot of floor.

If the work is to be pushed rapidly it is necessary to figure, under favorable conditions for centering, not less than two complete floors of centering plus waste. If the weather conditions are unfavorable there should be enough lumber for centering for three to four floors. On a building having eight stories we would ordinarily figure enough centering for three floors, plus waste. With the flat slab system there is no waste with the joists as they are simply lapped by and the waste in the boards would amount to about two per cent each time they are used. There will be some waste in the uprights if the stories are of different heights, which must be figured in each individual case.

Where a beam system is used the waste will be much greater as the loss from breakage and cutting the lumber to the size of the panels will generally run the waste up to ten to twenty per cent of the lumber in each floor, and sometimes much more than this. Also the surface contact is increased by the area of the sides of all beams requiring additional lumber.

Cost of Framing. Labor for framing beam boxes, column boxes, etc., will generally run about twelve dollars per thousand feet B. M. Labor of placing plain slab forms, carpenter's wages, being figured at $37\frac{1}{2}$c per hour, will run about five to six dollars per thousand feet. The cost of taking down the forms and moving them up should run about three dollars per one thousand feet B. M., for the flat slab type and five to seven dollars per thousand where there are a large number of beam boxes, etc. Nails and fastenings are generally a small item.

Where sheet metal is used for the sheathing the cost per foot of laying it and greasing it with paraffine is about one-third the cost of placing boards, altho the first cost of the metal is considerably higher.

Mr. L. C. Wason, president of the Alberthaw Construction Company, of Boston, at the fifth annual convention of the National Association of Cement Users at Cleveland, Ohio, presented a paper on costs from which the following table is condensed, giving the cost of handling and some very interesting costs of centering. It would be well for the reader to look up this paper which is reprinted in part in the Engineering News, January 14, 1909, and a number of the other engineering papers.

The following table, condensed by the Engineering News, from the original paper, is given as a fair indication of the variation in cost of different designs and different conditions. The author states that only typical cases are given where the items of cost were accurately known. Enough are given for a fair average except in the case of long span flat slab which appears to him by comparison a recent type of construction.

By reference to the general averages on form work in the accompanying tables the cost of forms per square foot of surface contact, namely: Columns, \$0.13; floors with reinforced concrete beams, \$0.116; flat floors without beams, \$0.111; short span slabs between steel beams including the fireproofing on the side of the beams, \$0.05; walls exposed to view above ground, \$0.093; the writer believes are all higher in price than usually believed to be a fair cost by most builders. It is upon the success of handling forms that good results financially depend. In regard to concrete, labor is

TABLE 1.—SHOWING COST OF FORMS AND CONCRETE ON VARIOUS MEMBERS IN REINFORCED-CONCRETE STRUCTURES

PLAIN CONCRETE COLUMNS

Location	Forms per sq. ft.				Concrete per cu. ft.						
	Carpenter Labor	Lumber	Nails and Wire	Total	Concrete Labor	General Labor	Cement	Aggregate	Team and Misc.	Plant	Total
Office building, Portland, Me...	$ .133	$ .039	$ .011	$ .173	$ .064	$ .004	$ .087	$ .084	$ .012	$ .022	$ .273
Coal pocket, Lawrence, Miss...	.057	.024	.001	.082	.166	.003	.073	.041	.008	.016	.307
Mill, Southbridge, Mass.......	.097	.082	.002	.181	.073	.056	.107	.035	.027	.030	.328
Mill, Attleboro, Mass..........	.093	.022	.001	.116	.110	.014	.062	.038	.013	.034	.271
Mill, Southbridge, Mass.......	.080	.056	.001	.137	.108	.048	.100	.037	.013	.031	.340
Coal pocket, Hartford Ct......	.098	.047	.002	.147	.089	.043	.069	.055	.017	.013	.286
Garage, Brookline, Mass......	.071	.051	.002	.124	.070	.028	.072	.058	.041	.020	.289
Warehouse, Portland, Me.....	.118	.016	.001	.135	.087	.027	.087	.070	.039	.025	.335
Textile mill, Lawrence, Miss...	.061	.013	.001	.075	.095	.019	.109	.027	.018	.015	.283
Highest........	.133	.082	.002	.181	.166	.056	.109	.084	.041	.034	.340
Lowest........	.057	.013	.001	.075	.064	.003	.062	.027	.008	.013	.271
Average of 9.....	.082	.036	.001	.130	.096	.027	.085	.019	.021	.023	.301
REINFORCED-CONCRETE BEAM FLOORS											
Highest...	.165	.107	.004	.275	.186	.035	.194	.101	.052	.055	.470
Lowest...	.037	.027	.001	.067	.017	.004	.071	.037	.007	.010	.202
Average of 18	.070	.045	.002	.116	.111	.020	.106	.063	.025	.024	.354
FLAT SLAB FLOORS											
Highest .	.078	.039	.003	.118	.146	.017	.109	.084	.026	.039	.374
Lowest......	.067	.037	.001	.106	.013	.001	.087	.053	.012	.010	.252
Average of 3. .	.071	.038	.002	.111	.097	.009	.096	.070	.019	.024	.315
REINFORCED-CONCRETE SLABS BETWEEN STEEL BEAMS											
Highest	.110	.071	.003	.184	.144	.018	.208	.080	.064	.046	.428
Lowest..........	.028	.012	.001	.049	.073	.005	.076	.026	.004	.010	.272
Average of 13	.061	.032	.002	.095	.102	.019	.128	.068	.024	.017	.359
BUILDING WALLS ABOVE GRADE											
Highest	.136	.073	.005	.176	.146	.052	.105	.187	.077	.055	.446
Lowest..	.046	.016	.001	.079	.042	.004	.034	.043	.007	.005	.174
Average of 17.	.085	.036	.002	.128	.090	.016	.073	.076	.025	.019	.301
FOUNDATION WALLS											
Highest .	.134	.018	.004	.193	.213	.037	.203	.116	.057	.040	.599
Lowest..	.032	.009	.001	.056	.040	.002	.038	.027	.003	.010	.148
Average of 14.	.068	.033	.002	.103	.076	.015	.080	.062	.019	.017	.269
FOOTINGS AND MASS FOUNDATIONS											
Highest	.119	.077	.003	.198	.081	.020	.098	.099	.013	.049	.275
Lowest.	.016	.006	.001	.018	.025	.001	.047	.043	.003	.010	.181
Average of 10.	.057	.031	.002	.093	.015	.007	.071	.077	.007	.021	.229

the variable item which must be carefully considered. Any person of intelligence can make a careful estimate of the materials to be used, but note the average prices of labor per cubic foot of concrete, namely: For columns, $0.123; beam floors, $0.131; flat floors, $0.106; floors between steel beams, $0.121; walls, $0.106; foundations, $0.091; and mass work in connection with buildings, $0.052. Not until the last item is a price reached which according to observation and experience must be expected to obtain in ordinary building work. Many who have had wide experience in handling large quantities of concrete in mass have occasionally attempted a lighter type of construction and have been greatly surprised at the large expense connected therewith. Men with this experience have frequently added fifty to one hundred per cent to the cost of mass work and only by doing so have they felt that they were sufficiently covered for light structural work.

Table II is an exact copy of a "master card" which gives the complete financial history of the job, when it is finally completed. The first column, which is blank, is occasionally used for an estimate of the first cost, the proposal including the profit as well as the estimated actual cost. It will be seen that on some items, a loss was incurred, as well as a profit on others, showing that it is difficult to reach the right price on everything, even on work on which a company is fairly experienced, and also that when slight changes are made by the owner or architect they often entail heavy loss even though the changes appear to be extremely trivial. Take the case of the external walls. The owners furnished the window frames and sash, which were all of metal. The original design was for a frame with two sash, which could easily be put into a six-inch wall.

TABLE II.—TYPICAL "MASTER CARD."

Job No. 747. Date, May 24th, 1906. Mill, Tappan Bros., Attleboro, Mass.

	Proposal	Actual Cost	Per cu. ft.	Profit	Loss	%
Total	$35,164.55	$31,330.48		$3,834.07		11
Excavate	790.00	823.18	$0 021		$33.18	
Footings and Fn	1,738.00	1,033.57	.137	704 43		
			Per sq. ft.			
Exterior walls	1,955.00	2,162.02	.190		207 02	
Wall and Fn. centers	1,520.00	3,630.08	125		2,110 08	
Floors $6\frac{3}{8}$ ins. thick	8,883.00	6,544.16	.339	2,338.84		
Roof $5\frac{1}{4}$ ins. thick	2,869.00	1,713.51	237	1,155.49		
			Per lin. ft.			
Columns, 20 ins. x 20 ins	832.00	676.65	1.470	155.35		
Stairs	883.00	910.35	.912		27.35	
			Per sq. ft.			
Tool surface	469.00	636.53	.056		167.53	
Ornaments and cornice	348.00	164.33		183.67		
Ventilators on roof	44.00	35.64		8.36		
			Each			
Set windows and door frames	852.00	929.99	2 19	122.01		
			Per sq. ft			
Interior partitions	1,770.25	1,656.35	.189	113.90		
Bolts and iron work	253.00	257.06			4.06	
Stair railing and grill	387.00	654.00			267.00	
			Per M.			
Screeds and settings	1,086.00	835.12	52 17	250.88		
2-in. Spr. plank and laying	2,839.00	1,431.69	33 30	1,40731		
$\frac{7}{8}$-in. maple	1,738.00	1,788.88	89 44		50.88	
Motor shaft	379.50	533.19	98 89		153.69	
Motor shaft found	98.00	70.07		27.93		
Roofing and conductors	1,255.00	1,026.06		228.94		
			Per sq. ft			
Paving	1,009.00	747.54	094	361.16		
Retaining wall centers, per sq. ft			211			
Retaining wall, concrete per c. ft	429.00	316.90	175	112.10		
Painting	400.00	375.00		25.00		
Steel footings and walls	300.00	218.91		81.09		
Plant, frt., etc	1,860.00	2,271.73			111.73	
Bond	100.00	120.00			20.00	
Extras	77.80	67.97		9.83		

They later decided, for greater fire protection, to use four sash. This required an eight-inch wall instead of a six inch, and the form work on the inside had to be built inward and then the space under the windows paneled to save material. To save making a very narrow panel at the side of the window, which would cost more than the concrete saved, the space was filled up solid so that the columns appear to be wider than they were actually figured. This slight change, which did not appear great at the time, when the job was entirely complete showed that the concrete of the walls showed an actual loss instead of profit because the form work cost more than twice what was originally estimated that it should cost.

19. Season of Year. The season of the year has to be considered in its relation to the cost of reinforced concrete work. In the summer season when the concrete dries out rapidly the forms may be removed every ten to twelve days, while in the fall and early spring during frosty weather the water must be heated or the forms left in place longer, requiring more lumber for centering. In the winter when the materials must be heated by artificial heat and artificial heat used in sweating out the concrete, the cost of work will be increased from ten to twelve per cent. Additional cost of merely heating the water is of course small. In the chilly weather of fall or spring good results may be frequently obtained merely by turning the exhaust steam into the water barrel and warming the water up so that the concrete will set quickly notwithstanding the chilly temperature.

20. Dead Charges. No contracting firm can do business without a considerable general expense, which must be distributed over all work executed by them. This expense includes office expense, advertising, soliciting work, estimates on not only the work taken but the work which the concern fails to secure, depreciation of the plant, freight, storage and equipment, the cost of keeping the organization together in slack periods. This expense may readily vary with various concerns from five to seven per cent of the cost of the work executed. In addition to this dead expense and the actual cost of labor there must be included the item for liability insurance which the contractor cannot afford to neglect to carry. Frequently the owner requires a surety bond for the faithful execution of the work and the payment of bills, the cost of which must be added to the incidental charges in the estimate of cost.

21. General Data on Cost. The architect is in the habit of figuring the building as so much per cubic foot. For heavy warehouses with the plainest kind of finish and large size the cost per cubic foot may run as low as six and one-half to seven cents up to ten and twelve cents for the smaller size of buildings with office fixtures, plumbing and the like. No approximate cost per cubic foot of any value can be given for office buildings, hotels and the like, since this item would vary greatly with the character and difference in the quality of the finish, fittings and the like.

For the concrete end of the building, however, a rough approximate estimate can be made very readily by figuring a unit price per square foot of floor area. In a large building of six or seven stories having a floor area of twenty to thirty thousand feet, panels approximately eighteen feet square, labor as outlined, sand at $1.00 per yard, cement, $1.20, crushed stone, $1.40, capacity of floors three hundred pounds per foot; rough slabs, columns and footings may be erected at an approximate cost of the contractor of about forty cents per square foot of floor area. Where the building is narrow and there are more columns in proportion to the floor area, on the same basis fifty cents per square foot would be a reasonable price.

Reduction in the floor load carried makes a relatively small reduction in the cost of the construction, since the centering would be the same for the light and the heavy building.

Where the load is increased fifty per cent above these requirements the additional cost would be increased over eight per cent. While doubling the load would not increase the cost over about ten or eleven per cent.

This is the general type of information the shrewd contractor carefully figures out for himself and which enables him quickly and accurately to check up estimates made by his assistants or even to take work on an approximate estimate of this kind without going into details. Turner, when pressed for time once took a $60,000 contract on a twenty-minute estimate based on a computation only of the floor area and a knowledge of the conditions covering labor and cost of materials.

Where there are plain reinforced floors resting on walls and the panels are of large size such as in court house work and many other public buildings and where gravel can be cheaply obtained the cost per foot of floor may run as low as 30 cents per square foot. In other localities forty cents per foot under less favorable conditions would be a reasonable figure.

CHAPTER XI

1. Fireproof Properties of Concrete and the Protection of Steel from Heat. The value of concrete as a fireproof material has been pretty well demonstrated in a large number of severe conflagrations and also in many fire tests by the building departments of various cities. In fact, it may be stated that concrete ranks as the best fireproof building material and it is to this quality together with its low cost that the enormous increase in its use is due.

Intense heat injures the surface of the concrete, but it is so good a non-conductor that if sufficiently thick it provides ample protection for the steel reinforcement and the interior of the mass is unaffected even in unusually severe fires.

For efficient fire protection in slabs under ordinary conditions with one-way reinforcement the lower surface of the steel rods should be $\frac{3}{4}''$ above the bottom of the slab. With two-way reinforcement this may be reduced to $\frac{1}{2}''$, for in case one layer of rods should become overheated the upper layer is still amply protected.

Structural beams, girders and columns should have at least $2\frac{1}{2}''$ of good concrete for efficient protection. In beams having large rods the thickness of the concrete coating outside of the rods should never be less than $1\frac{1}{4}''$ nor less than the diameter of the largest rod used in the beam. In columns the shell outside the reinforcement should be considered as fire protection and no dependence placed upon it in figuring the strength of the section, in carrying the working load.

These limitations are sufficient for ordinary purposes. Where, for example, a factory building is to be erected in which there will be scarcely any inflammable materials to be stored, it is a waste of money to provide a thick concrete protection to resist possible fire. On the other hand, where the building is to be used for storage of material capable of creating not only a hot fire but an intense heat of long duration, special provision may be made by using an excessive thickness of concrete for fire protection tho in such a situation a sprinkler system would be preferable.

A most severe practical test occurred in a fire at the Pacific Coast Borax Refinery at Boyanne, N. J. This building was a four-story factory built entirely of reinforced concrete except the roof. The contents of the building, the roof and interior wood trim were destroyed, but the walls and floors remained intact except where an eighteen ton tank fell thru the roof and cracked some of the floor beams. The heat was so intense that brass and iron castings were melted to junk. A small annex built of structural steel frame was completely wrecked and the metal bent and twisted into a tangled mass.

In general, the fire resistance of Portland cement concrete is governed or affected by the character of the aggregate and the amount of cement in the mortar.

First, we may state it as a general rule that the richer the mortar or the greater the amount of cement used the greater the fire resisting properties of the concrete. Rich mortar makes a stronger concrete better able to resist severe temperature stresses while the high proportion of cement when dehydrated on the exposed surface makes a very perfect non-conducting material, preventing the uninjured parts from further or rapidly progressive injury.

Second, as regards the aggregate, the smaller the stone the better the fire resisting qualities.

Trap rock will make a concrete offering greater resistance to extreme heat than limestone or granite.

In a series of experiments to determine the effect of very high temperatures on concrete, with the acetylene oxygen blow-pipe, interesting results were secured. The heat of this flame is approximately 6500° Fahr. Applied to concrete paving block 2 inches thick and five years old, made of sand and gravel, the heat under the flame was sufficient to melt the silica sand and form a little puddle of glass. Pebbles of feldspar or granite under this intense heat popped, but the little puddle of glass once formed did not seem to increase under the continued application of the flame and hardened up as soon as the flame was removed.

This series of experiments was continued by using a concrete of silica sand mixed with a higher percentage of brine and it was found with such concrete possible to glaze the surface in this manner, while the concrete back of the glazing did not seem to be materially injured in point of strength. Whether a concrete block of selected materials can be glazed in this manner uniformly is, of course, open to question, but any effort to cut concrete by intense heat as steel is cut, was proved by this work to be impracticable.

2. Fire Tests. Building departments sometimes require fire tests of the finished construction. A test required in the Railway Exchange Building, Denver, Colo., is of interest from the fact that two tests were made, one on thoroly cured concrete and the other on concrete not well cured nor dried out. The first test was a fire consisting of a cord of pine wood split in faggots about two inches square and soaked with oil, applied to the under side of the first floor slab. The fire gave a very intense heat and as it dried down a fire hose was turned on the white hot surface. The damage to the slab consisted in the spalling of an area about two feet in diameter to a depth of about one and one-half inches. Spalling was accompanied by reports described as being as sharp as pistol shots. The cause of this spalling was at first somewhat puzzling. An examination of the aggregate showed it to be a good hard sandstone which had been at some period metamorphosed by heat. There were, however, numerous porous veins running thru the stone and it seems that these veins having absorbed considerable water in mixing the concrete which had not been dried out in the curing, offered an opportunity for the generation of steam in the small cavities under the heat of the fire resulting in the bursting of the stone with a sharp report. The fractures noted were clean cut, as would be expected from such a cause.

Allowing the slab to thoroly dry out for an additional period of five weeks, a second test was made, similar to the first, with absolutely no spalling and no apparent injury to the slab. Cut showing this test is shown in Fig. 93.

3. The Theory of Fire Protection. The theory of fire protection is given by Mr. Newberry as follows:

"Two principal sources from which cement concrete derives its capacity to resist fire and prevent transference of the heat to the steel are its combined water and porosity. Portland cement takes up in hardening a variable amount of water, depending on surrounding conditions. In a dense briquette of neat cement the combined water may reach twelve per cent. A mixture of cement with three parts sand will take up water to the amount of about eighteen per cent of the cement contained. This water is chemically combined, and not given off at the boiling point. On heating, a part of the water goes off at about five hundred degrees Fahr., but the dehydration is not complete until nine hundred degrees Fahr., is reached.

Fisher Bros., Architects, Denver, Colo.
Martin Carroll, Contractor, Kansas City, Mo.
Fig. 93.

This vaporization of water absorbs heat and keeps the mass for a long time at a comparatively low temperature. A steel beam or column embedded in concrete is thus cooled by the volatilization of water in the surrounding cement. The principle is the same as in the use of crystallized alum in the casings of fireproof safes; natural hydraulic cement is largely used in safes for the same purpose.

The porosity of concrete also offers great resistance to the passage of heat. Air is a poor conductor, and it is well known that an air space is a most efficient protection against conduction. Porous substances, such as asbestos, mineral wool, etc., are always used as heat insulating material. For the same reason cinder concrete, being highly porous, is a much better non-conductor than a dense concrete made of sand and gravel or stone, and has the added advantage of lightness. In a fire the outside of the concrete may reach a high temperature but the heat only slowly and imperfectly penetrates the mass, and reaches the steel so gradually that it is carried off by the metal as fast as it is supplied."

In regard to cinder concrete it may be added, first, that it is not a desirable material to use from the standpoint of strength. Second, that as usually employed, insufficient cement is used to make a good fire resisting material. Thus Prof. Norton compares the action of stone and cinder concrete in the Baltimore fire as follows:

"Little difference in the action of the fire on stone and cinder concrete could be noted and as I have earlier pointed out the burning of bits of coal in poor cinder concrete is evenly balanced by the splitting of stone in the stone concrete. I have never been able to see that in the long run either stood fire better or worse than the other. However, owing to its density, the stone concrete takes longer to heat through."

Perhaps if the relative proportion of cement were the same in each, the cinder concrete, if the cinders are real clinker, would prove the better fire resisting material as Mr. Newberry assumes. This point cannot be too much emphasized.

A concrete must be rich in cement to make a first class fireproof material and for this reason alone a leaner mixture than 1 : 2 : 4 should not be allowed in an important building.

Thus far our attention has been primarily directed to the fireproof qualities of concrete as such. In considering the fire resisting properties of the composite material known as concrete steel or reinforced concrete, the effect of the unequal heating of different parts of the construction must be considered. It has been previously noted that the coefficient of expansion of steel and concrete are practically identical. Their coefficients of heat capacity and conductivity, however, differ widely and for this reason the distribution of the metal in the form of small bars rather than in large units will give a more satisfactory result from the fireproof standpoint.

4. Terra Cotta and Tile Compared with Concrete. The difficulty with the combination of tile or terra cotta and structural steel as a fire resting material lies largely in the fact that the coefficient of expansion of the two materials is different.

This is well illustrated in Fig. 94 showing the effect of heat in breaking and cracking tile between steel beams after exposure to a severe fire.

Professor Norton, in his report on the Baltimore fire to the Insurance Engineering Experiment Station, states:

"Where concrete floor arches and concrete steel construction received the full force of the fire it appears to have stood well, distinctly better than the terra cotta. The reasons I believe are

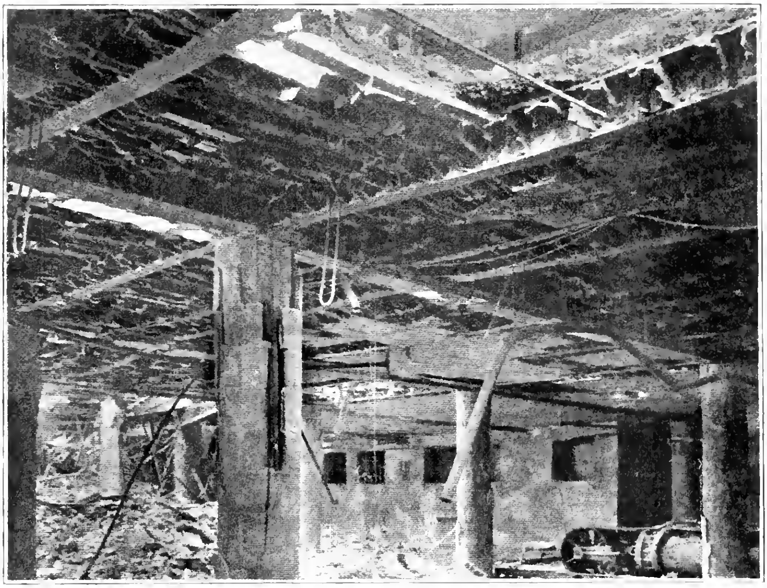


Fig. 94 Effect of Fire on Tile Construction

these: first, because the concrete and steel expanded at sensibly the same rate, and hence when heated do not subject one another to stress, but terra cotta usually expands about twice as fast with increase in temperature as steel, and hence the partition and floor arches soon become too large to be contained by the steel members which under ordinary temperature properly enclose them. Under this condition the partition must buckle and the segmental arches must lift and break the bonds, crushing at the same time the lower surface member of the tiles.

"When brick or terra cotta are heated no chemical action occurs, but when concrete is carried up to about 1,000 degrees Fahr., its surface becomes decomposed, dehydration occurs, and water is driven off. This process takes a relatively large amount of heat. It would take about as much heat to drive the water out of this outer quarter inch of the concrete partition as it would to raise that quarter inch to 1,000 degrees Fahr. Now, a second action begins. After dehydration the concrete is much improved as a non-conductor and yet thru this layer of non-conducting material must pass all the heat to dehydrate and raise the temperature of the layers below, a process which cannot proceed with great speed."

In the composite material of concrete and steel in the form of a continuous concrete monolith there are severe temperature stresses set up by the unequal heating of different parts of a floor during a fire and the manner in which the material will withstand these stresses will depend in a large measure on how thoroly the steel is disseminated thru the concrete to enable it to take up the tensile stresses induced by this unequal expansion in the various parts caused by the unequal heating, hence that type of construction which is reinforced practically in all directions is best calculated to withstand the severe stresses so produced. Further, since the concrete is injured or disintegrated on its surface the smaller the surface exposed the less will be the damage, and the fewer irregularities in the form of the construction, the less it will be injured.

Looking at the question from this standpoint then, the flat slab type of construction would rank first from the fireproof standpoint and Type III second. In other words, the natural concrete types which are in no wise imitations of older types of construction are far better adapted to resist the severe conditions of a conflagration than those types which are merely imitations of older forms of construction.

In reporting to the Chief of Engineers, U. S. A., regarding one of the reinforced concrete buildings which passed through the Baltimore fire, Capt. Sewell writes:*

"It was surrounded by non-fireproof buildings, and was subjected to an extremely severe test, probably involving as high temperature as any that existed anywhere. The concrete was made with broken granite as an aggregate. The arches of the roof and the ceiling of the upper story were cracked along the crown, but in my judgment very slight repairs would have restored any strength lost here. Cutting out a small section— say an inch wide—and caulking it full of good strong cement mortar would have sufficed. The exposed corners of columns and girders were cracked and spalled, showing a tendency to round off to a curve of about three inches radius. In the upper stories, where the heat was intense, the concrete was calcined to a depth of from $\frac{1}{4}$ to $\frac{3}{4}$ inch, but it showed no tendency to spall, except at exposed corners. On wide, flat surfaces, the calcined material was not more than $\frac{1}{4}$ inch thick, and showed no disposition to come off. In the lower stories, the concrete was absolutely unimpaired, tho the contents of the building were all

*Eng. News, March 24, 1904.

burned out. In my judgment, the entire concrete structure could have been repaired for not over twenty to twenty-five per cent of its original cost. On March 10th, I witnessed a loading test of this structure. One bay of the second floor, with a beam in the center, was loaded with nearly three hundred pounds per square foot superimposed, without a sign of distress, and with a deflection not exceeding $\frac{1}{4}$ inch. The floor was designed for a total working load of 150 pounds per square foot. The sections next to the front and rear walls were cantilevers, and one of these was loaded with 150 pounds per square foot, superimposed, without any sign of distress, or undue deflection."

In concluding the subject of the fireproof qualities of concrete it may be well to call attention to the stock argument of the burned clay advocate.

A small specimen of burned clay or terra cotta if subjected to a temperature of 2,000 degrees and then immersed in water will remain undamaged.

A small sample of concrete subjected to similar treatment will be totally disintegrated. Hence the burned clay advocate argues that concrete is not a suitable fireproof material.

The fallacy in this plausible argument as has been pointed out in an excellent editorial in the Engineering News lies in the fact that the conditions in a building during a fire and in the furnace are radically different.

In a fire in a building the concrete is not exposed to heat on all sides, nor is it exposed continuously for any long time to very high temperatures. The greatest heat is generally near the ceiling when the surface, as noted in Capt. Sewall's report, may be dehydrated slightly and protect the material back of the injured portion. The net result is that less damage results than to the terra cotta or hollow tile, since the latter does not expand in unison with the supporting steel frame, and is crushed and broken by the severe temperature stress resulting from this cause.

Combination structures of hollow tile and concrete are open to the same criticism from the fireproof standpoint, namely, the combination of two elements in a composite structure having radically different coefficients of expansion. Evidently the expectation that the combination will, under severe conditions, prove satisfactory cannot be realized.

5. Rates of Insurance on Concrete Buildings and Contents. Boards of fire underwriters representing the older line companies, have been somewhat slow in recognizing concrete as a fireproof material and it seems to the concrete constructor frequently that they do not recognize the great differences that exist in this material as dependent on the character of the mixture and dissemination of the metal reinforcement.

The position that some of these boards have taken in rating the mill building with a sprinkler system lower than a concrete building without a sprinkler is a position hard to explain except that possibly members of these boards are financially interested in sprinkler system equipments.

On the other hand, the mutual companies appear to have been more progressive and are writing policies at rates that appeal to the constructor as far more consistent and rational.

Comparing the lowest rate which has come to the writer's attention for a timber building, mill construction, used for mercantile purposes, equipped with sprinkler system, A. D. T. watchman service, etc., with the lowest rate which has come under his notice for a reinforced concrete building similarly equipped with a sprinkler system, the rate for the concrete building was less than one-half that for the timber building, being a six-cent rate for the concrete structure against a twelve and one-half-cent rate for the timber building. The advantages from the fireproof standpoint may be stated as follows:

(1) A well designed reinforced concrete building offers security against disastrous fire and total loss of structure.

(2) It reduces the danger of damage to the contents by preventing the spread of fire from floor to floor.

(3) It prevents damage to the contents by water from story to story.

(4) It renders sprinklers unnecessary in buildings whose contents are not especially inflammable.

(5) It reduces the danger of panic and loss of life incident thereto among employes or occupants of the building.

Evidently in order to prevent the spread of fire from floor to floor, the floors should be continuous, or have openings properly protected by automatic shutters or doors. Thus, if we are to protect the goods or contents on the floors above from fire below, it is necessary to have the elevator shafts protected by automatic fire doors and stairways cut off in a similar manner. This can be done at a comparatively small expense.

Protection from exterior exposure may be readily made by the employment of wire glass, metal frames and the like, in place of wood frames and ordinary glass windows.

A good concrete floor is practically waterproof and a slight pitch with suitable scuppers would practically eliminate water loss in floors below from flooding a floor in which fire has broken out in the contents or goods stored thereon.

In the ordinary factory or mercantile building with wood floors, loss from water is frequently greater than the loss by actual fire where an incipient blaze has been extinguished.

In the concrete building, on the other hand, each floor becomes almost a waterproof roof. Frequently a tenant moves into the lower stories of a concrete building before the upper portion is complete, the floors above acting as a roof.

According to Mr. Kunhardt, vice-president of the Boston Manufacturers Mutual Fire Insurance Company, these mutual companies take a business-like stand regarding the extent of fire protection required in each individual case. While the value of the automatic sprinkler is recognized and the general rule specifies its installation the Factory Mutual Companies do not require it in the concrete building except where there is sufficient inflammable material in the contents to furnish fuel for a fire.

An essential feature in good factory construction includes not only consideration of the building but protection adequate to its needs only. The extent to which the above is faithfully carried out will eventually be the determining feature in the cost of insurance.

Mr. Kunhardt gives the following table:

	All Concrete		Brick Mill Construction or Open Joists		Wood Mill Construction or Open Joists		Add for Brick or Wood Buildings in Small Towns and Cities without Best of Water and Fire Departments
	Bldg.	Cont's	Bldg.	Conts'	Bldg.	Cont's	
General Storehouse	20c	45c	60c	100c	100c	125c	25c
Wool Storehouse	20c	35c	40c	60c	75c	100c	25c
Office Building	15c	30c	35c	50c	100c	125c	25c
Cotton Factory	40c	100c	100c	200c	200c	300c	50c
Tannery	20c	40c	75c	100c	100c	100c	25c
Shoe Factory	25c	80c	75c	100c	150c	200c	50c
Wollen Mill	30c	80c	75c	100c	160c	200c	50c
Machine Shop	15c	25c	50c	50c	100c	100c	25c
General Merchandise Bldg.	35c	75c	50c	100c	100c	150c	25c

These costs are based on the absence of automatic sprinklers and other private fire protective appliances of the usual completely equipped building. They are not schedule rates, but may be an approximation to actual costs under favorable conditions based on examples in various parts of the country.

As illustrating the value of fire protection, Mr. Kunhardt states, that in the Boston Manufacturers' Mutual Companies, the average cost of insurance on the better class of protected factories has now for some years averaged, excluding interest, less than seven cents on each hundred dollars of risk taken, and on first class warehouses connected with them, one-half of this amount. These figures can be compared with the table as illustrating the gain by the installation of proper safeguards for preventing and extinguishing fire.

In these same protected factories and warehouses the *actual fire and water loss* is less than four cents on each $100 of insurance and he regards it possible to reduce this loss materially, practically along the lines above outlined.

Where sprinkler systems are installed in concrete buildings, and in particular where these buildings are of the flat slab type which does not interfere with the most perfect operation of the sprinkler, rates quoted as low as 10 or 12 cents for the building and 15 cents for the contents are not uncommon at the present time (1914), providing that 80 percent of the insurable value of the building and contents is covered and further that the policy is written for the term of five years.

CHAPTER XII

1. Protection of Steel and Iron from Corrosion by Portland Cement. Deterioration of steel by corrosion or rusting is one of the difficult problems in nearly all structures intended to be permanent.

Paint of linseed oil combined with some pigment is ordinarily used for the protection of structural steel and its efficiency depends on the complete removal of rust before painting. Further, this coat of paint must be renewed at frequent intervals.

Fortunately in concrete steel construction we have in the cement itself the most perfect protective coating known for iron and steel. If bars that are somewhat rusty be placed in wet concrete and removed after one week they will be found to be perfectly clean, the rust having been chemically destroyed by the cement.

The bond between cement and steel is formed better with bars that are somewhat rusty when placed in the concrete than with bars new from the mill. The reason seems to be that a small amount of rusting removes the black mill scale and allows the cement to come in contact with the solid bar. Paint, oil or grease tend to weaken the bond of the concrete.

Bars, removed from cement after over twenty years' exposure of the specimens to the elements, have been found bright and as good as when first placed in the work.

This protection however is dependent entirely on the thoro covering of the steel by the wet concrete and hence the importance of using a plastic mixture, or one that will flow slowly and thoroly surround the steel, and require only puddling rather than tamping to secure substantial work.

It may be noted incidentally also that exactly the same kind of mixture is essential if we are to secure smooth work, neat in appearance, that is, work without ragged patches, and showing no rough stone that expose voids not filled with mortar.

A. B. Newberry states the chemical theory of protection of iron embedded in concrete as follows:

"The rusting of iron consists of oxidation of the metal to the condition of hydrated oxide. It does not take place at ordinary temperatures, in dry air or in moist air free from carbonic oxide. The combined action of moisture and carbonic acid are necessary. Ferrous carbonate is first formed; this is at once oxidized to ferric oxide and the liberated carbon dioxide acts on a fresh portion of metal. Once started the corrosion proceeds rapidly, perhaps on account of the galvanic action between the oxide and the metal. Water holding carbonic acid in solution, if free from oxygen, soon acts as an acid and rapidly attacks iron. In lime water or soda solution the metal remains bright. The action of cement in preventing rust is now apparent. Portland cement contains about sixty-three percent lime. By the action of water it is converted into a crystalline mass of hydrated calcium silicate and calcium hydrate. In the hardening it rapidly absorbs carbonic acid and becomes coated on the surface with a film of carbonate. Cement mortar thus acts as an efficient protector of iron and captures and imprisons every carbonic acid molecule that threatens to attack the metal. The action is, therefore, not due to the exclusion of the air, and even tho the concrete be porous, and not in contact with the metal at all points, it will still filter out and neutralize the acid and prevent its corrosive effect."

2. Permanence of Concrete Construction when made with Proper Materials. The best grade of Portland concrete made with the first class cement selected aggregate, properly mixed and cured is indeed a most permanent material, fully justifying all that can be said in its favor. It will withstand the action of the elements as well as granite and quartzite, and will withstand the heat of fire better than granite, while in small samples is not equal to the granite in point of strength, in large masses it may be said that it may be depended upon with great certainty because there are no seams, flaws or planes of weakness such as are liable to be found in masses of natural stone.

A good concrete increases in strength with age and grows harder and stronger as time continues. The increase in strength is rapid for the first three months and continues at a gradually decreasing rate for the next six or eight months, and then very slowly as time goes on, perhaps thru a period of twenty-five years or more.

Where the concrete, however, is not made from suitable aggregates, is not properly mixed and cured, it is by no means a permanent material when exposed to the action of the elements. Concrete having for an aggregate a soft stone, such as some of the oölitic limestones, shale or one which is made with sand which is fine and containing considerable clay will inevitably be affected materially by frost in the severe climate of the north.

Concrete Made with Improper Materials. In building work concrete is under cover and in the main protected from the elements, and hence some contractors have an idea that this being the case almost anything can be utilized as aggregate. Thus cinders in which there is quite a large percentage of ash, partly burned coal and the like, have been used in some cases and with exceedingly bad results.

In one case where the concrete had been made from cinders from Southern Iowa coal, the concrete after it was cast in the form of slabs expanded to such an extent that it pushed the face brick out of the side of the building and the slabs checked and cracked to a considerable extent owing to this same action.

The following is from an articles by Mr. D. B. Butler in the Engineering Record.*

"EXPANSION OF CONCRETE MADE WITH COKE BREEZE."

"On account of a number of failures of roof and floor slabs, made of coke breeze concrete, which were called to his attention, Mr. D. B. Butler undertook a series of experiments to determine the expansion of such concrete since an examination of the faulty structures indicated that such action was responsible for the failures. His conclusions were presented in a paper before a recent meting of the Society of Architects, England, from which these notes are taken.

"In nearly all samples of so-called breeze concrete examined by Mr. Butler, a very considerable quantity of material other than pure coke was noticeable in the aggregate, such as clinkers, stones, shale and ashes, together with, in some instances, a noticeable amount of coal. Whatever may be the disadvantages of other extraneous material found in breeze, coal is not, in Mr. Butler's opinion a desirable constituent for concrete; in the first place, on account of its smooth, shiny surface, the adherence of the cement would be extremely poor; in the second place, it is worse than useless as a fireproof material, an account of its tendency to decompose on heating. The question arose, however, whether apart from being undesirable for the reasons aforesaid, either coke breeze or coal was in any way dangerous as being likely to cause expansion of the concrete.

"The first experiment was of a somewhat rough and ready nature, and was made with coal. An ordinary bituminous house coal was crushed and sifted about the fineness of standard sand; with this coal a 1 to 3 mortar was made, and two small 2-ounce glass bottles filled with the mixture; one bottle was filled quite full, and the other was filled to within a quarter inch of the top and sealed down with a paste of neat cement, the object of the sealing being to ascertain whether the imprisonment of any hydrocarbons set free from the coal

*June 19, 1909, page 767.

would have any bursting effect. For comparative purposes similar bottles were also filled with a paste of neat cement and 1 to 3 mortar of standard sand.

"The whole of the bottles eventually cracked, with the exception of one filled with standard sand-cement mortar. But while those entirely filled with the coal mortar generally cracked within two or three days, and with a very few exceptions continued to expand until the bottles burst away into several pieces, those filled with the neat cement and the sand mortar frequently did not develop any cracks whatever till several months, and it was the exception rather than the rule for them to continue expanding sufficiently to burst the bottle. Both the neat cement and sand cement mortar bottles remained perfectly sound after eleven months and then only developed very minute cracks, whereas the coal mortar bottle was cracked in twelve days and burst right off in forty two days. This suggests that the cause of the cracking after such protracted periods might be due to unequal expansion of the glass and the mortars at varying temperatures.

"The subsequent experiments were made with rectangular bars 100 mm long and 22 mm square in cross-section, the expansion and contraction of which were accurately measured in the Bauschinger micrometer calliper apparatus. By the use of this a minute variation of 0.005mm, or 0.005 per cent in the length of the prism, may be detected.

"Eight bars or prisms were made with satisfactory cement, four being made with neat cement and four with 1 to 3 standard sand cement mortar. Two of each series were kept entirely in air and two placed in water after twenty-four hours and kept therein during three months. The test pieces numbered 300 and involved 5,000 measurements.

A noticeable feature of the experiments in that many of the specimens which show very marked expansion when placed under water as soon as set expand very much less when left entirely in air. It therefore seemed a point worth determining as to whether exposure to damp or moisture would in any way affect these air-set specimens at the end of the three months' test, after they had become thoroughly seasoned. One of the duplicate air bars from each series was therefore placed under water, the time elapsing between the date of moulding and placing under water ranging from 91 to 292 days. Immersion had practically no effect upon those specimens which had previously shown no expansion which kept under water, but it caused almost immediate expansion of a very serious nature with those fractions of breeze which had previously developed expansion when placed under water in the first instance. This clearly shows that the expansive agent, whatever it may be, is more or less dormant in the dry air-set block, and only requires to become damped to constitute a serious element of danger.

"Taken as a whole, the experiments as far as they go seem to point to the fact that as regards subsequent expansion there is not much danger to be apprehended from good, cleam coke or clinkers, or or even anthracite coal, but that some kinds of ashes and furnace refuse are highly dangerous, while any considerable quantity of bituminous coal is absolutely fatal. One noticeable feature of the experiments, however, was, that most of the coke-breeze mortars had a tendency more or less seriously to attack the iron moulds, causing them to rust during the short space of twenty-four hours between the moulding of the specimens and their removal from the moulds. Mr. Butler is unaware if such results have been found to any appreciable extent in actual practice, but samples of breeze concrete sent him for examination a short time ago showed distinct marks of considerable rusting having taken place where the concrete had been in contact with the rolled joists."

Mr. Butler's experiments quoted above coincide with our observations. In general, cinder, if fit to use, should be free from ash and should be well burned stoker clinker. Concrete made from good hard clinker has proved a good and substantial fire proofing material.

There is this difficulty in its use, however, that the contractor too frequently furnishes cinders rather than hard clinker.

3. Concrete Mixed Dry and Tamped. Concrete mixed dry and tamped in the old fashioned way is more or less porous, and liable to disintegrate under severe conditions of exposure, as for example, whenever the concrete is soaked with water, frozen and thawed repeatedly. Such conditions may occur in an aggravated form in retaining walls.

The government sea wall at the ship canal, Duluth, made in the old fashioned manner, mixing the concrete dry and tamped is showing the effect of exposure to a far greater degree than we should expect had the work been executed in accordance with the present standard practice.

In this sea wall it should be noted that in the cold season the wall was alternately wet and dry as the waves washed against it; that moisture is alternately frozen and thawed in the exposed surface, and owing to the fact that the method of mixing leaves the concrete slightly porous some disintegration naturally results.

In general, the best concrete to withstand such severe conditions is that which is most dense, is strongest, and made from the hardest and most durable stone as an aggregate and with clean, coarse sand.

Where brick or building stone is made of a fairly dry or moist mixture and is not exposed to the severe conditions above described it proves very durable material.

4. Hair Cracks, Map Checks and Crazing. In troweling a finished surface on concrete the moisture is brought to the surface by the working of the material, resulting in somewhat unequal conditions of moisture. The exposure promotes the rapid drying out of the surface and causes what is known as hair cracks, map checks and the like. These are generally only of very slight depth and mean little as to the permanence of the material, providing the concrete is made of good cement and a first class aggregate is used.

A peculiar fact concerning this defect in concrete finished surfaces is that on some blocks it will not appear at all, while others made under almost identical conditions will be badly affected. Perhaps the difference in part may be accounted for by the thoroness with which the concrete has been mixed, the time expended in mixing, as well as the conditions of drying and curing.

Concrete which has been thoroly mixed in a machine for double or triple the ordinary time will be a little stronger than concrete which has been mixed for only fifteen or twenty revolutions. If the mixing is continued for twenty minutes there will be less tendency towards rapid setting and shrinkage and the development of checks and cracks, much on the order of the results obtained by skilled mechanics by retempering cement mortar in patching old work.

In the treatment of concrete which is finished with a troweled surface to prevent checking it is desirable, where it is exposed, to keep it protected by burlap soaked in water and to keep the direct rays of the sun from it by an additional cover of canvas. In this way steps and similar work may be executed with the least trouble from this source, provided care has been used in the selection of both sand and stone used as aggregates.

In the manufacture of cast stone this difficulty is one which the worker in this field is forced to meet and it is essential that the mixing should continue without intermission until the material is run into the mold.

In general, cast stone made by the sand mold process will keep its color better than such natural stone as Bedford, altho it may discolor in streaks and blotches known as crazing. Efforts made to overcome crazing may be summarized as follows:

First, by the addition of other ingredients to the cement in mixing with the intent to render the material more perfectly waterproof and more uniform in setting.

Second, to coat or waterproof the material after it has been cast, with a compound repellent to moisture.

Third, to remove a thin layer of the surface of the stone and concrete and get below the depth of the hair lines or depressions which form in casting and cause this peculiar marking or discoloration when exposed to the weather.

The first two methods have apparently been successful in somewhat mitigating this difficulty, while the third method has been successful as practiced by the Roman Stone Company, of Toronto. Their method is to use a carborundum wheel, dressing and tooling the surface therewith.

5. Temperature Effects. Changes of temperature cause changes in the volume in concrete as in all materials with which we have to deal. The difficulty which is encountered is the cracking of the concrete as it is brought into tension by change in volume. Massive walls, unless cut at intervals of thirty feet or thereabouts will crack thru from this cause. Where openings, such as windows, are cut thru a solid wall of concrete cracks are liable to develop at the corners unless the concrete is well reinforced by steel rods crossing the corners in such a manner as to take care of this tension and prevent the development of cracks.

In slabs reinforced in one direction there should be used not less than eight-hundredths percent of metal for temperature stress if it is expected to prevent the development of unsightly checks.

6. Disintegration of Concrete by Oil, Grease, etc. In factory buildings, machine shops, etc., oil and grease are liable to come in contact with the concrete and it is important to know what effect it will have upon the material. Certain kinds of oils are known to be positively injurious to concrete in the earlier stages of hardening and to disintegrate it to a considerable extent. Where, however, the concrete has had ample time to harden there seems to be little if any damage resulting from lubricating oils such as are ordinarily employed in a factory or machine shop. Where it is desired to use a floor which has not had at least two months in which to thoroly harden we would recommend coating the concrete with some good waterproofing compound or floor paint, thereby protecting it until after it has had opportunity to become thoroly cured and hardened thruout.

The question of disintegration of Portland cement briquettes and experiments to prevent it have been quite fully discussed by Mr. James D. Hain, Assoc. M. Am. Soc. C. E., in the Engineering News, March 16, 1905. His conclusions may be summarized as follows:

1. Most oils penetrate concrete mortar, which makes them dangerous.

2. Concrete is more liable to be disintegrated when saturated with oils and fats if not thoroly set.

3. A good quality of concrete is less liable to be damaged by oil than a poorer quality, such as a porous, poorly mixed or improperly seasoned concrete.

4. Ordinary concrete work is rarely subjected to continued

doses of oil. It is more often only occasionally spattered. Disintegration under the latter conditions seems remote, especially in the case of the first class, well seasoned concrete.

Last, even tho subjected to the equivalent of continued saturation, this disintegration would be long drawn out if the concrete were properly made and well set. Even under ordinary conditions it seems desirable to use a wash for oil spattered concrete to prevent the oil from penetrating it.

Mr. Hain in his experiments tried the following wash: Five per cent solution of alum and a seven percent solution of castile soap, and also experimented with paraffine. None of these proved satisfactory where the briquettes were immersed in oil.

The following table shows the result of some of these experiments of Mr. Hain:

No. Briquettes made	Class of Portland Cement	Mixture Portland Cement and Sand	Extract Lard Oil	Whale Oil	Castor Oil	Linseed Oil	Petroleum Oil (Crude)	Signal Oil
			Time applied before disintegration					
18	Stone and clay	Neat...	3 mos.	*	*	*	*	*
12	Stone and clay	1:3 sand.	*	*	*	*	*	*
18	Marl and clay.	Neat....	$2\frac{2}{3}$ mos.	*	*	*	*	6 mos
12	Marl and clay.	1:3 sand	*	*	*	*	*	*
18	Slag and stone	Neat......	1 mo.	3 mos.	4 mos.	*	*	$1\frac{1}{4}$ mos
12	Slag and stone	1:3 sand...	7 mos..	$4\frac{1}{4}$ mos.	$6\frac{1}{2}$ mos	*	*	4 mos

*Sound after applying oil nine months at which tests were discontinued
All briquettes set seven days in air before applying oil.

Mr. Reid in his work on concrete states that one of the briquettes tested with signal oil was sent to the laboratory of Toch Brothers, Long Island City, and a careful analysis was made. Mr. Maximilian Toch states that a determination of the soluble substances in the briquette showed that the disintegration was due to the formation of oleate and stearate of calcium. To reduce this to its simplest expression, the animal oils contain acids which combine with the lime and crystals and stearate and oleate of lime are formed. It is very likely that these crystals in the process of formation have increased in bulk in the briquette and the bond which has been formed by the lime in the set cement has been totally disintegrated and ruptured. These crystals were isolated and verified under the microscope.

Mr. Toch also states that machine oils are almost all paraffine oils, do not contain animal fats, and hence do not affect concrete.

Silicate of magnesia, sold under the name of fluate, has often been used as a wash to protect concrete against the action of oil. When this wash is applied to concrete, silica is liberated and fills up the pores. The magnesium fluate acts as a binder, and the cement becomes excessively hard after a few months. Limestone and building stone have been treated with this material in Europe with great success. This compound is, however, expensive.

7. Disintegration of Reinforced Concrete by Electrolysis. Laboratory experiments by Toch, Knudson and Langsdorf in 1906 and 1907 show that under certain circumstances passage of direct electric current from the reinforcing metal into the concrete gives rise to corrosion of the metal and to cracking and splitting of the surrounding concrete which seems to be brought about by the mechanical pressure developed by the oxides which occupy a volume over twice as great as the metal from which they are formed.

The conditions under which reinforced concrete may be seriously injured by electrolysis are fortunately exceptional rather than the rule. That it may be injured even in exceptional cases presents a problem of importance which requires a statement of the conditions under which injury may occur and the method of controlling them.

The conditions under which electrolysis may occur and the concrete suffer by electrolysis are moisture and difference of potential between the electrodes and contact with the mass of the concrete. Perfectly dry concrete below grade level is seldom found, while there are few places in our cities at the present time where some appreciable differences of potential cannot be found between two points which are more than a few yards apart. On the other hand, concrete has to be very wet in order to possess a maximum of conductivity. Any reduction of the moisture content below the saturation point causes an increase in its resistance and consequent decrease in the current which will flow thru the concrete under a given potential difference.

The resistance of ordinary air-dried concrete is usually about ten times that of wet concrete, and for this reason concrete above grade level is less susceptible to electrolitic damage than if located where it is permanently wet. While air-dried concrete is not immune from electrolysis troubles, difference of potential due to stray currents is rarely sufficient to produce trouble and in the absence of special conditions electrolitic damage to concrete at any level above grade is extremely rare.

Special emphasis should be laid upon the conditions that are liable to produce damage by the flow of the electric current between electrodes in contact with the concrete. The conduction being electrolitic the reactions take place only at the electrodes and in the absence of such electrodes no reactions occur within the concrete. The only effect therefore would be the slow removal of the constituents which are soluble in water and hence the effect on plain concrete would not be essentially different from that of slow water seepage.

Sources of Stray Currents. The sources of potential differences in concrete structures may be classified under two heads: those due to direct contact between conductors of direct current lighting or power circuits in some part of the building and those which have their origin in stray currents from electric railways or other grounded power lines.

The former may happen in any building containing direct current electric wires thru defective insulation. It is not necessary that both sides of the line be grounded in the building itself, since if one side of the line is grounded on the building and the other grounded in some remote quarter of the system those portions of the building near the wire may be subjected to a considerable difference of potential. If the wire be grounded directly on the concrete and not on the reinforcement the comparatively small section of the path of the current near the point of contact between the concrete and the wire will cause most of the total drop of potential to the ground to occur within the restricted region near the wire and it is only here that any damage may be expected, and since the current will be small the damage if any will be small.

Ultimately in case both sides of the line are not grounded in the structure any current that leaks from the wire would pass into the earth thru the footings and foundations and thru pipe systems entering the building. The cross section of these paths is so large in the aggregate that the potential gradients would be insufficient to raise the temperature appreciably and no appreciable damage is likely to occur, since the corrosion above referred to occurs in wet concrete only and not to any considerable extent until at least a temperature of 113° Fahr. is reached. According however, to determinations made by the Bureau of Standards, if the power wire be grounded directly on a portion of the reinforcing metal the condition is more serious. The extent of the damage will be greater if such large area of the reinforcement is in metallic contact with the electric wires, as to reduce the resistance across this area to a small amount. In case the ground is on the positive side, the potential gradient near the reinforcement may become high enough to cause rapid corrosion and consequent destruction of the reinforcing metal. If on the other hand the reinforcing metal be the negative electrode a softened condition of the concrete would be developed near the surface of the iron which would tend to destroy the bond and this would probably be the more serious condition of the two since it will not manifest itself by producing local cracks in the concrete and might not become known until a large portion of the building had become weakened.

While such a condition as this might possibly occur, and if neglected might become serious, it is nevertheless a trouble that can be readily guarded against.

The other source of current that, under certain circumstances, might possibly give rise to trouble is the grounded current of electric railways. Various electrolysis surveys show that a potential difference exceeding two volts due to stray currents between any two parts of the building is extremely rare and this would almost inevitably be distributed over so great a distance that the potential gradient would not be great enough to cause appreciable trouble. Stray currents may enter a building thru water pipes, gas pipes, lead cable sheathes and the like. Differences of potential considerably larger than two volts may be brought about between the different portions of the building in this manner or between parts of the building and the earth. If the pipe systems come in contact with the concrete only and not with the reinforcing metal any damage that may occur will be slight and confined to the immediate vicinity of the pipes or cables, but if the pipes come into metallic contact with the reinforcement the latter comes to the same potential as the pipes and may become anode or cathode according to the direction of flow in the circuit. Cases may arise where a difference of potential of serious magnitude may be produced in this way.

Conclusions from Laboratory Experiments. Laboratory experiments show that the corrosion of iron even in wet concrete is very slight at temperatures below 113 degrees Fahr. For any fixed temperature the amount of corrosion for a given number of ampere hours is independent of the current strength. The rapid destruction of anode specimens of moist concrete at high voltage (60 to 100 volts or more) is made possible mainly by the heating effect of the current which raises the temperature above the limit above stated. If the specimens be artifically cooled no appreciable corrosion occurs and no cracking results. The potential gradient necessary to produce a temperature rise to 113 degrees Fahr., with consequent corrosion was 60 volts per foot in the specimens tested by the U. S. Bureau of Standards. For air dried concrete it is much higher. This indicates that under actual conditions corrosion caused by stray currents may be expected only under very unusual and special conditions.

Specimens of normal wet concrete carrying currents increase their resistance 100 fold or more in the course of a few weeks which fact further lessens danger of trouble. The presence of a small amount of salt greatly increases the initial conductivity of wet concrete thus allowing more current to flow and it also destroys the passive conditions of iron at ordinary temperatures increasing the rate of corrosion and consequent tendency of the concrete to crack.

Concrete structures built in contact with salt water or in salt marshes are more susceptible to electrolysis than concrete not subjected to such influences.

Conditions may arise in practice which give rise to damage due to stray currents but the danger from this source has been greatly over-estimated. While precautions are necessary under certain conditions there is no cause for serious alarm.

It may be here noted that alternating currents have largely displaced direct currents for lighting and power purposes because of reduced transmission losses in alternating circuits. It should be further noted that office buildings with structural steel skeletons which have passed thru the period during which direct current was used almost entirely for lighting, have suffered little or not at all from this cause and it may be stated that there is little or no reason to expect electrolysis trouble in reinforced concrete buildings generally, and that only in special classes of buildings such as ice cream factories, cold storage plants, packing houses, and the like, where steam and ammonia or acid fumes come in contact with the floors may serious trouble be expected from this cause.

Protective Measures. In all cases it is conservative to forego the use of salt or chloride of calcium in winter on reinforced construction below grade level regardless of the character of the building This would demand greater care in protecting the work and heating material for this part of the structure. Also careful attention should be given to the insulation of gas pipes, water pipes and soil pipes from all contact with the reinforcement. Proper insulation of the wires should be provided so that any leakage to reinforcement may be prevented. Finally in that class of buildings in which the conditions are favorable to damage from electrolysis, alternating currents for lighting and power should preferably be adopted. In bridge work for carrying electric lines and power lines fiber conduits are to be preferred to metal conduits. Any possible damage from alternating currents may be considered insignificant or negligible in comparison to that by direct currents transmitting the same power.

Mr. H. P. Brown, Engineering News, June, 1911, offers the following suggestions in regard to electrical currents in damp reinforced concrete buildings:

> Do not depend for insulation upon even the best rubber covered wire nor upon japanned conduits in rooms subjected to fumes and vapors.
>
> Do not permit the grounding of the intermediate wire in three-wire systems.
>
> Do not permit any grounding of the secondary circuit of a transformer.
>
> In the vicinity of electric power houses or substations use wooden pipe for gas or water serving pipes from the street mains. Use insulating tubes around the gas, water or steam pipes where they pass thru concrete floors or walls.

The following are recommendations from Bulletin No. 18, U. S. Bureau of Standards on "Electrolysis in Concrete" by E. B. Rosa, Burton McCollum, and O. S. Peters.

> In order to insure safety of reinforced concrete from electrolysis the investigation shows that potential gradients must be kept much lower in structures exposed to the action of salt waters, pickling baths, and all solutions which tend to destroy the passive state of iron.
>
> All direct current electric power circuits within the concrete building should be kept free from grounds. If the power supply comes from a central station the local circuits should be periodically disconnected and tested for grounds and incipient defects in the insulation. In the case of isolated plants ground detectors should be installed and the system kept free from grounds at all times.
>
> All pipe lines entering concrete building should, if possible, be provided with insulating joints outside the building. If a pipe line passes thru a building and continues beyond, one or more insulating joints should be placed on each side of the building. If the potential drop around the isolated section is large, say, 8 or 10 volts or more, the isolated portion should be shunted by means of a copper cable.
>
> Lead-covered cables entering such buildings should be isolated from the concrete. Wooden or other nonmetallic supports which prevent actual contact between the cable and the concrete will give sufficient isolation for the purpose. Such isolation of the lead-covered cable is desirable for the protection of the cable as well as the building.

Partial Bibliography of Electrolysis in Concrete:

Max Toch, German Electric Chemical Society Vol. 9, page 77, 1906
" " Iron Trade Review, Vol. 46, page 1007, 1910.
Engineering News, Vol. 60, page 710, A. J. Nicholes.
" " Vol. 63, page 372, O. L. Eltinge.
" " Vol. 65, page 684, H. P. Brown.
" " Vol. 66, page 10, Barker & Upson.
" " Vol. 66, page 152, B. C. Worth—H. P. Brown.
" " Vol. 66, page 207.
Engineering Record, Vol. 62, page 132.
" " Vol. 63, page 272.
U. S. Bureau of Standards, Bull. No. 18, Electrolysis in Concrete.
Am. Electric Ry. Practice, Herrich & Boynton, (Electrolysis and its Prevention), page 387, 397.

CHAPTER XIII

1. Floor Finish. The rough concrete slab in a warehouse, factory or office building may be finished in a variety of ways. The requirements for factory purposes and office buildings frequently demands a wood finished floor, while in a warehouse and some classes of manufacturing buildings, the concrete finished floor is preferable.

2. Strips and Strip Fill for Wood Floors. Preparation of the rough slab is made for the wood floor as follows: Parallel strips are laid at about sixteen inch intervals, embedded in concrete and the flooring nailed thereto. The better time to apply the strip fill is immediately after the rough slab is sufficiently hardened to work upon for the reason that at this time the strips can be readily spiked down to the partly hardened concrete and wedged and aligned to the proper level without difficulty. Then the strip fill can be put in with the same equipment that has been used to cast the floor slab. For the reason that a hard strip fill adds materially to the strength and stiffness of the slab it is preferable to make the mix approximately the same as that for the slab except where the loads are light and strength is no object. Then a $1:3\frac{1}{2}:4$ mix is ample for all purposes. In this mix, however, the coarser aggregate should be in the form of gravel in size from $\frac{3}{8}$ inch down, or crushed stone from $\frac{1}{2}$ inch down.

No natural cement or lime should be used in the mix since where it is used trouble almost inevitably results by reason of its slow hardening. The moisture in the strip fill swells and expands the wood flooring to such an extent that it springs away from the fastenings to the strips necessitating the entire relaying of the floors in many cases.

Strips are economically and conveniently made by splitting up old centering lumber, such as 4 by 4s, ripping them thru the center and then ripping the 2 by 4s with a beveled cut giving a strip $1\frac{1}{2}$ inches wide at top, $2\frac{1}{4}$ inches wide on the bottom and $1\frac{3}{4}$ inches deep. This is a good way to work off the old lumber.

3. Width of Flooring. Narrow widths of maple flooring are preferable to the wider widths. Two and one-half inches is as wide as can be recommended where the floor is $\frac{7}{8}$ inches in thickness. Where $1\frac{1}{4}$ or $1\frac{3}{4}$ thickness is used $3\frac{1}{4}$ to $3\frac{1}{2}$ inches is the preferable limit.

4. **Cement Finish Coat.** In no part of concrete construction has there been so much difficulty in securing first class and satisfactory work as in putting down concrete floor finish. A good bond is desired between the finish coat and the concrete of the rough slab. To secure this bond, some endeavor to apply the finish integrally with the rough slab. The difficulty attending this method is the shrinkage of the body of the work, checking and injuring the top coat together with the fact that if this finish is applied while the rough slab is still plastic the hardening of the surface in getting its initial set will be slow. This means that some partly hardened cement will be broken up in troweling and the finish will be brittle and will dust badly altho it looks well when the work is first completed.

Again, working on the finish before it has sufficient time to dry in placing the centering thereon for the succeeding stories is quite likely to scratch and mar the finish and leave it in bad shape when the building is done.

A further difficulty occurs from unequal moisture conditions about the base of the column. When the columns for the next story are cast the excess of water in the mix wets down and swells the concrete surface about their bases and expands it and as this dries more slowly than the rest of the slab shrinkage checks and spider web cracks will very likely be found in the finish about the column bases when it dries, if the work has been executed in this manner.

The application of the finish coat before casting the succeeding story has this advantage: The dripping from the floor above does not coat the rough slab and prevent securing a good bond thereto, as sometimes happens where this dirt and cement wash is not removed before the application of the finish. The trouble of cleaning the floor thoroly is thereby obviated.

If it is desired to lay the finish before carrying up the next story, it is recommended that after the concrete has stiffened up and before it is thoroly hard, the surface be roughened with a rake to secure a better bond for the finish, and that the rough slab be then allowed to stand for not less than twenty-four hours in good drying weather and longer where the weather is chilly so that the rough slab becomes thoroly rigid. Then the finish may be successfully applied, cutting it back from the foot of the column in squares at least a foot from the column base, and applying this part of the finish at a later date. The surface then should be protected preferably by sawdust thoroly wet down, while the centering for the floors above should be well supported on planks so that the finish coat will not be abraded during the process of curing.

If the finish is to be applied after the rough work of the building is complete, which is the usual manner, the surface of the slab should be first thoroly cleaned of dirt, and the drippings from the upper floor removed. It should then be prepared to receive the finish, by thoroly soaking with water.

5. The Mix of Finish. The mix of the finish should preferably be one part Portland cement, to one and one-half parts clean coarse sand. A good siliceous sand is a better aggregate for this purpose than crushed granite.

The finish coat should be thoroly mixed as a stiff paste, screeded to the proper level as it is applied, and as soon as it has taken its initial set, troweled and rubbed to a smooth surface.

The cause of much of the difficulty with floor finish is due to the mistaken idea that it requires a very wet mix to secure a good bond to the old concrete. With this sloppy mix more or less separation occurs and the inert material and laitance comes to the surface. Then when it is leveled off the workmen are obliged to wait until some of the cement has attained its final instead of its initial set before they can proceed to trowel off the finished surface. The cement in the finish that has attained its final set is broken up and does not recover its strength while that which has not progressed so far hardens in a normal manner. The result is that the portion of the cement which has attained its final set combined with inert material brought to the surface by troweling forms a dust which is readily rubbed up on the finish. The condition of the floor finished in this wise is better or worse dependent on the following conditions:

Where temperature conditions are such that the cement hardens very slowly, as in the fall of the year, and the finish is allowed to stand five or six hours before it gets sufficiently hard to work upon, the resulting finish is most inferior. Where, however, the temperature conditions are such that the cement sets more rapidly, a much better surface results, sometimes one that is fairly satisfactory.

6. Hardening Compounds. A number of compounds have been placed on the market to harden floor finish and render it tougher under wear. Good results with any of these compounds depend, as in the case of the cement finish, upon proper workmanship and attention to the mixture, and that particularly in the cool season a stiff mixture is used. Steel filings and a small percentage of carborundum in the proportion of 16 pounds to the sack of cement produce good results.

7. Treatment of Floors. A concrete floor may be treated in a manner somewhat similar to a wood floor. It may be shellaced and waxed or varnished and painted if desired. Where a floor has been put down and the finish is unsatisfactory from the standpoint of dusting, if not too bad the trouble may be remedied by a coat of floor paint made with a thin varnish as the base. The thinner the paint of the first coat the greater its penetration and the better the result from the standpoint of reducing the tendency to dust.

Where, however, the surface is unusually bad there is no remedy except by rubbing it down with the carborundum wheel in a manner similar to that in which the finish is secured in terrazzo floors.

8. Concrete Stairs. Reinforced concrete provides an inexpensive means for building stairways which are far more nearly fireproof than any other type which can be constructed.

The accompanying typical detail of reinforcement Fig. 95, shows the usual method of reinforcing employed for this purpose.

For ordinary runs, such as twelve feet, $\frac{3}{8}''$ rounds, $6''$ on centers are ample for the inclined slab. The inclined slab is generally built $4\frac{1}{4}$ or $5''$ thick for ordinary runs and the horses are cast on top of the incline. Where fancy treads are desired they are sometimes made of white Portland cement and crushed quartz. This makes a very durable tread and a material which in its good appearance ranks next to marble and will wear somewhat better.

When marble or slate treads are used they can be readily bedded on the concrete horse and the riser brushed up, rubbed and painted or varnished as preferred. Frequently it is desirable to suspend stair platform supports from above. This can very readily be done by dropping the slab rods down to the level of the platform from the slab above on one or more sides and encasing the suspender rods in an ordinary $2''$ partition of cement plaster for fire protection. Fastenings for metal hand rails can be readily cast in the end or top of the stair treads as the work is placed.

9. Insulation of Roofs. Those not familiar with reinforced concrete frequently make the mistake of designing roof slabs in a cold climate without insulation. The result is that the moisture in the warm air in the room below the roof slab is condensed on the underside of the cold slab and drips continually whenever the slab is colder than the air within the room. This is readily remedied by a cinder filling from four to six inches thick. In fact, we frequently recommend to our clients that instead of putting on a roof slab proper the ceiling slab be cast level which may at some future date be used for a floor should a story be added and on this slab to build up with cinders sufficiently to give the standard pitch-and-gravel

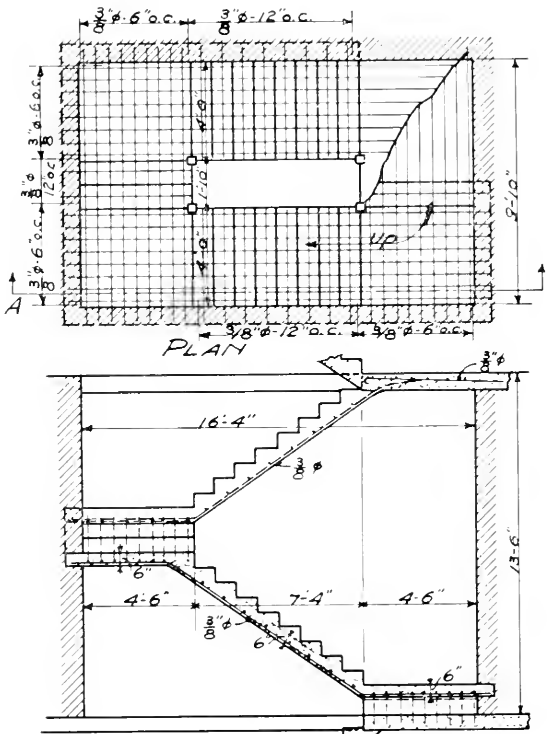


Fig. 95. Typical Detail of Reinforcement Concrete Stairs

roof the usual slope. This slope should preferably be in the neighborhood of 5/16 to 3/8 inch to a foot of run. On the top of the cinder insulation a one inch coat of cement mortar mixed about one cement to three sand is spread which forms a good base for the pitch-and-gravel roof. In place of the cinder filling a false roof is frequently built up using old centering lumber. Where this is done all openings thru the ceiling and roof should be encased or protected by a concrete fire wall, then no further damage can occur than the burning up of the comparatively inexpensive false roof should the same catch fire from above. The ceiling slab, should the roof burn away, would protect the goods or business carried on beneath it until the false roof could be replaced.

Insulation is of the utmost importance in concrete roofs in the climate of the United States, north of Southern or Central Kansas. We would hesitate to allow any work to be executed with our guarantee against this difficulty in latitudes north of this. On the Pacific Coast insulation may be omitted as far north as Portland without serious difficulty.

10. Protection and Provision for Plumbing. In plumbing fixtures a considerable amount of lead piping is used. This should be either entirely eliminated where it comes in contact with concrete or well protected by a heavy coating of tar or asphalt paint since pieces of lead piping uncoated when removed from concrete are often found transformed almost completely into red oxide.

Cast iron, wrought iron or steel and brass fittings are not injuriously affected.

In general, provision should be made in the pouring for all large fixture pipes, since digging large holes in the concrete should not be allowed because frequently these come at points where they may seriously weaken the construction.

Cases have occurred where the plumber thotlessly dug a hole right thru the center of a beam, leaving an insignificant amount of concrete each side of the hole to take care of the shear and, in doing so, cut one of the main beam rods, thus forcing the slab to carry a load provided for safely only by the portion of the beam cut away.

11. Placing Electric Conduits, Gas Pipes, etc. The most convenient disposition in reinforced concrete work of pipes, conduits, and the like, is to bury them in the middle of the slab with outlets at desired points. So buried, conduits if of moderate size, in no wise weaken the construction. They should not, however, be placed beneath the reinforcement. This is a mistake that is too often made.

Sometimes conduit pipes are placed right along on top of the centering with perhaps $\frac{3}{8}''$ of concrete under them in the finished work, and reinforcing bars resting on top of the conduit pipe dip downward in the slab on each side of the conduit to a greater or less extent. Then, as soon as the centering is struck and the strain comes upon the rods, there is a tendency to straighten out under pull, and to cause the slab to deflect or sag and open up large unsightly cracks near the bottom of the conduit pipe. The reduction in strength due to this position of the pipe may be as much as from ten to twenty-five percent of the strength of the slab. In the ceiling the crack, from the standpoint of appearance is unsightly and leads to somewhat unwarranted suspicion of extreme weakness. This should be avoided.

Standard outlet boxes as furnished by electric supply companies are unfortunately usually much too shallow. They should be deep enough so that the pipe connections can be readily made without interference with the reinforcement. The writer has frequently had wood plugs turned up and put in these boxes in order to keep them at the proper elevation and give an opportunity to place the conduits without bending and kinking them as they enter the box.

Provision for openings in floors for steam pipes, soil pipes, leaders and the like, may be made most economically by placing thimbles of sheet metal (filled with sand) on the forms in the desired location, thus saving the expense of cutting later.

When holes have to be cut thru the slab the cutting should commence from the bottom. If the hole is cut thru from the top, as soon as the drill or chisel strikes the bars a large unsightly chunk will be broken out of the underside of the slab. Since it is quite difficult to patch these places with plaster the architect should not allow the work to be done in this manner.

12. Plastering on Reinforced Concrete Work. This is a feature of concrete building construction which is of considerable interest to the architect. It is decidedly annoying for a client to come to the architect and state to him that a large section of the plaster has dropped off from certain sections of his building. This happens far more frequently than the advocate of reinforced concrete likes to admit, altho when the causes of the failure of plaster to adhere to the work are fully investigated and the work properly executed there is little trouble from this cause.

Plasterers are in the habit of plastering on wire lath, wood lath or the like. With such a base upon which to work there is ample opportunity for a lean mortar to clinch in a firm and satisfactory manner. When plastering on concrete, however, plaster is held to the concrete by adhesion only. There is little or no chance for efficient clinch or mechanical bond such as occurs when plastering on lath or wire cloth. The materials, the concrete and the plaster, which do not have exactly the same coefficient of expansion are held together by adhesion between the two, and evidently this will be greater the richer the plaster mortar. It will be greater when the surface of the forms used for centering are rough sawed lumber than with surfaced lumber. The tendency to drop off will be less the thinner the plaster coat and less damage can result from the fall of any given section of plaster; hence, a thin coat of plaster is to be preferred to a thick scratch coat and finish coat thereon.

Lime mortar well gaged with Portland cement just before use will adhere better to reinforced concrete than the gypsum or patent plasters. Any plaster will adhere to concrete best when the concrete is thoroly set and dry. Trouble almost invariably results from the attempt to plaster on concrete before it has had a chance to thoroly dry out and set hard, as it seems that the moisture from the concrete prevents the plaster from drying and setting properly.

Washing the surface of the concrete before plastering with a solution of one part vinegar to three parts clean water greatly improves the bond between the two materials, since it removes the inert matter from the surface of the concrete.

Some plasterers prefer to coat the concrete work with R. I. W., or other tar paint in advance of applying the plaster in order to secure a more satisfactory bond.

Considerable trouble has occurred with plaster upon reinforced concrete, tho in all cases, on investigation, it has been found either that the plaster was applied upon partially cured concrete or improperly put on.

Sometimes the plasterer will endeavor to put on a thick coat, get air bubbles between the new plaster and the concrete and these expanding and contracting with each change of temperature will gradually loosen up quite a large area of the plaster coat and after six or eight weeks it will drop off in large chunks.

The remedy for this difficulty is as follows:

First, see to it in centering the floor that the rough side of the lumber is placed next to the concrete, giving a rough surface rather than a smooth surface for the plaster to stick to.

Second, see to it that the concrete work is thoroly dried before attempting to plaster it.

Third, thoroly wash the under side of the surface of the slab with the vinegar solution recommended.

Fourth, see to it that a rich mortar is used.

Fifth, make the finish as thin as possible, a skin coat 1/16 to 1/32 inch thick being ample to make a good finish.

Sixth, avoid the use of soap, grease or benzine to prevent the concrete from adhering to the centering.

Nearly all of the patent gypsum plasters will, when applied wet to steel or iron, badly corrode the metal. Fortunately this corrosion seems to continue only until the plaster sets but it is sufficient to stain the plaster badly in the vicinity of the metal. It may be prevented in the manner recommended for the protection of lead in concrete.

13. Suspended Ceilings. Frequently a slab is put up where it is desired to suspend a ceiling below, either to conceal pipes, flues and the like, or as insulation for the roof. This is readily arranged in the following manner:

Take ordinary $\frac{1}{4}''$ round wire, make a $3''$ loop on the upper end and drop it thru a hole in the form. It will then be anchored in the concrete as soon as the concrete is cast, and the free end may be used to tie up angles, tees or groove irons which are to be used for the ceiling frame.

CHAPTER XIV

ARTISTIC AND COMMERCIALLY PRACTICABLE CONCRETE SURFACE FINISHES.

1. **Stipple Coat.** It is found that by applying a stipple coat, either rough or smooth as desired, a very pleasing effect can be readily obtained. An example of this treatment is shown in the figure of the Smythe block, Wichita, Kansas. The writer has adopted this finish for most of his bridge work, as it gives a greater appearance of strength, readily covers up the minor imperfections in the centering, and affords a pleasing contrast with the highly ornamental stone railings with which he prefers to finish his work.

The stipple coat is usually applied with a broom-corn brush and consists of a thoroly mixed grout of a neat cement one part, and sand one part. Treatment of the surface should be as follows:

Wet down the face of the wall thoroly with a hose. Then apply the stipple coat, spattering it on. This method of treatment is being adopted to a large extent by architects in the finishing of exterior cement plaster walls for residences. A very neat effect indeed is obtained in this manner at a low cost. Expanded metal wire lath is nailed to the studs, plastered with a Portland cement mortar mixed usually with same ten percent. of hydrated lime, the mixture being practically one cement to one and one-half sand and finished with a stipple coat as outlined.

Failure of cement plastered walls may be attributed in the main to failure of the contractor to use enough cement. Too many workmen put forward the argument that the concrete will not be good if it is made too rich, while as a matter of fact it requires a rich, strong mixture to withstand the frost and severe climate in all northern states. Plaster work which would stand without injury in Cuba and Arizona would fall to pieces in short order in Minnesota or Manitoba. A properly applied cement coat of a rich mortar, however, will stand the climatic conditions in the north while in the south also a rich mixture is to be decidedly preferred.

2. **Plaster Coat on Rough Cast Concrete.** A very expensive effort was made to secure a good surface finish on the Grand avenue

viaduct in Milwaukee. The specifications required that the inside of the forms be lathed with expanded metal and plastered with a

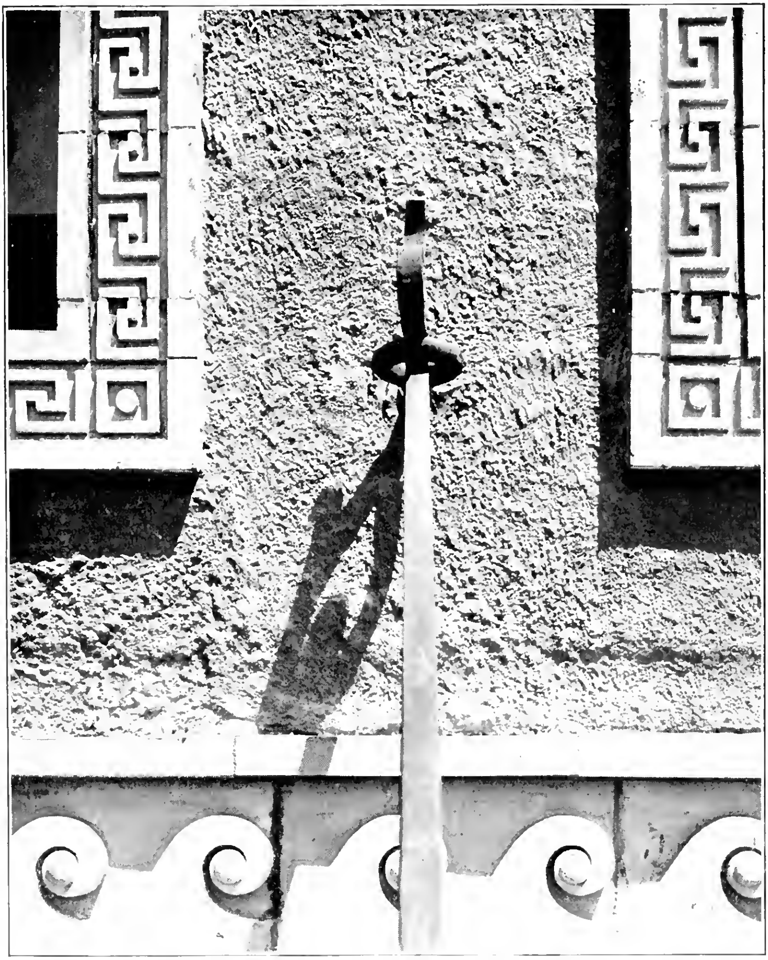


Fig. 96. Stipple finish of Smythe Block, Wichita, Kans. Louis Curtis, Architect.

mixture of plaster of Paris and lime. This plaster coat was to be oiled in advance of placing the concrete and the concrete to be placed and tamped in layers. On removal of the forms, notwithstanding the greatest care on the part of the contractor, the line of demarkation between the several layers was plainly visible and it was found impossible to put up the work without blemish as required in the specifications.

The work was finally carried out by removing the forms on the exposed surface as soon as practicable and plastering with a thin coat of rich cement grout. This is a practice which is not to be recommended because wherever the mass of the concrete has had time to get fairly hard this plaster coat is liable to check and scale off, tho occasionally it has been applied with a fair degree of success before the concrete has had time to thoroly harden.

3. Finish Obtained by Brushing and Washing. Mr. Henry H. Quinby of Philadelphia appears to have been one of the first to introduce a method of brushing and washing the concrete surfaces, to bring into relief the aggregate used.

The process consists of removing the forms after the material has set, but while it is still friable, and then immediately washing and rinsing the cement which has formed against the mold and thereby expose the particles of sand and stone. The appearance then depends upon the character of the aggregate in the concrete as respects its color and the uniformity of its distribution in the mixture.

The time to be allowed for setting before washing must depend upon the nature of the cement and the temperature conditions. Quick setting cement and warm weather call for the removal of the forms in from seven to ten hours. The appearance may be controlled somewhat by the extent of washing which may be to the point of leaving the stone aggregate in decided relief producing a rough coarse texture much admired by most architects.

An interesting article on this subject will be found in a book entitled "Concrete Factories," by Leslie, published by Bruce and Banning of New York, and in some of the older numbers of the "Cement Age."

A well written paper on the same subject has been published by the Universal Portland Cement Company in their trade bulletins, numbers 54, 55 and 56, which, thru the courtesy of the company, is reproduced in part herewith:

The ordinary concrete surface, it must be admitted, is anything but pleasing in appearance, being usually a comparatively smooth, lifeless surface of a somber grayish color. It makes but little difference what cement, sand or aggregate is used, or in what proportions they are mixed, the general aspect of the unfinished form surface is the same. There may be the greatest difference in color, shade and texture of the aggregate used in two separate concrete surfaces, yet unless they are so treated as to bring out and expose the aggregate, the resulting surfaces will look alike.

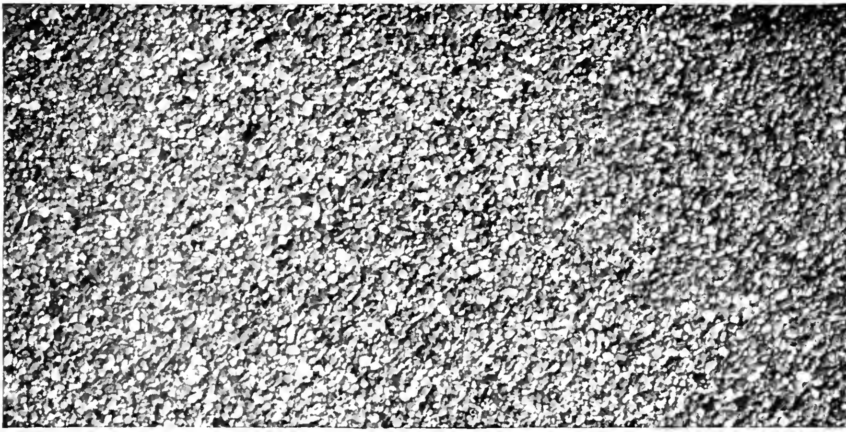


Figure I

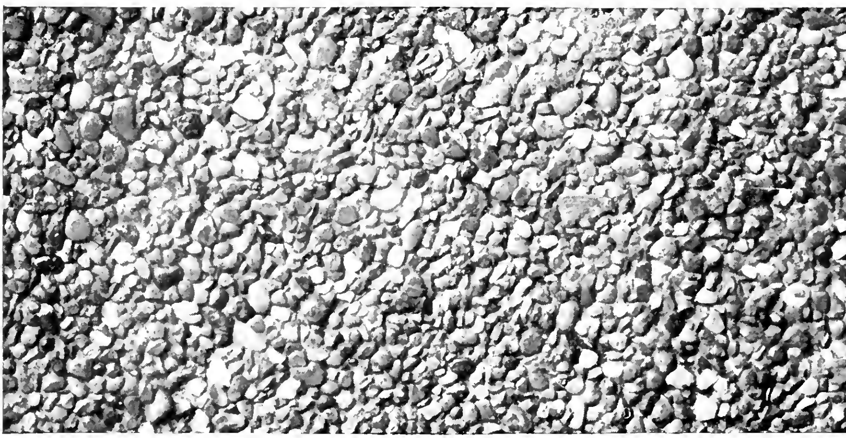


Figure II

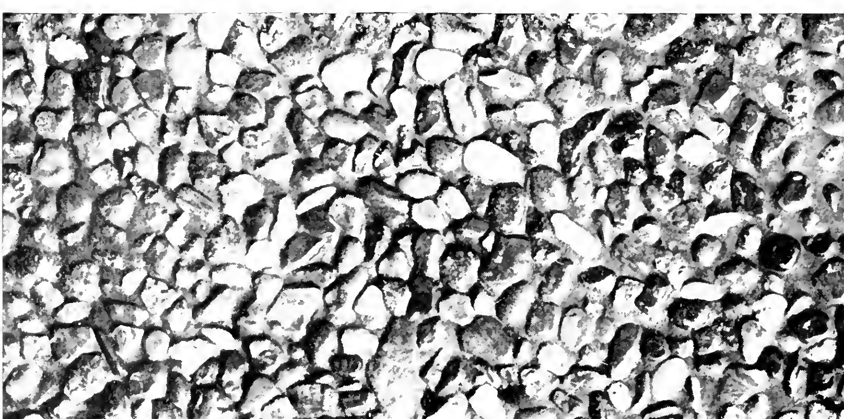


Figure III
(Surfaces reduced one-half of original.)

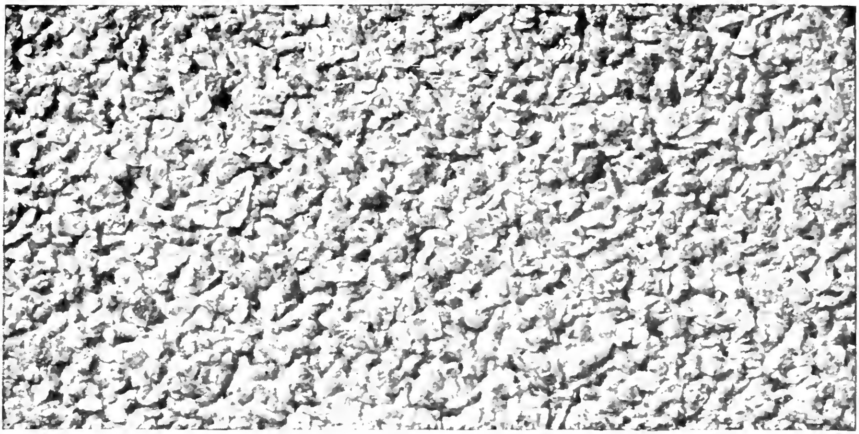


Figure IV.

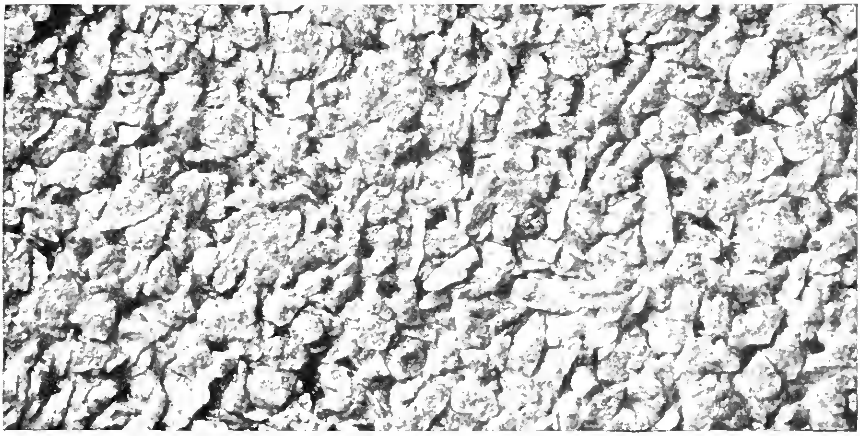


Figure V.

Figure VI.
Reproductions are actual size.

It is quite difficult to distinguish an ordinary unfinished concrete surface in which bank gravel is the aggregate from one in which crushed red granite is used, but the same surfaces, if subjected to any one of a number of different methods of surface treatment, will present a marked and pleasing contrast in appearance. It is the monotonous sameness in the appearance of concrete work that architects object to so strongly. To show what can be accomplished in producing pleasing, artistic and commercially practicable surface finishes for concrete work is the object of this article.

On the preceding pages are found photographic reproductions of brushed concrete surfaces. The difference between these surfaces and that of ordinary gravel concrete is very striking, yet they are all practical, commercial finishes, and can be obtained by the use of material from ordinary gravel banks.

Figure I shows a comparatively fine, even-grained surface, composed of one part Portland cement and three parts of fine sand all of which passed a No. 8 and was retained upon a No. 50 mesh screen. Figure II is very much like Figure I in general appearance and color, but of a rougher, more uneven texture. This surface is a 1 : mixture, with coarse sand, passing thru a No. 4 and retained on a No. 8 screen. Figure III represents a finish made from a 1 : 3 mixture of cement, and $\frac{1}{4}''$ to $\frac{1}{2}''$ pebbles. Thus these surfaces are identical in every respect, except as to size of aggregate. The three surface finishes were all produced by the same method of treatment.

The cuts give but a poor idea of the appearance of the actual surfaces, as the color and texture which give life and individuality to the surfaces are lacking. To appreciate the value of this finish for concrete work, the surfaces from which these cuts were made should be seen.

Figures IV, V and VI are three cuts from photographic reproductions of concrete surfaces similar as to surface treatment to those previously shown, but differing from them in the aggregates used.

Figure IV shows a decidedly pleasing, even-grained surface composed of one part Portland cement and two and one-half parts red granite screenings, all of which passed a No. 8 and was retained on a No. 16 seive. Figure V is a reproduction of a surface composed of one part cement to two and one-half parts ordinary, quarter inch, granite screenings, the material passing a No. 8 sieve being rejected. Both these surfaces are quite similar in every respect in texture, that represented by Figure V being of a rougher texture than the other. As the cement is barely perceptible on these surfaces both look very much like rough, undressed red granite, the color being practically the same as that of the screenings of which they were made. Figure VI represents a treated surface composed of one part cement to two and one-half parts of black pebbles, varying in size from those retained on a No. 10 sieve to those passing a $1\frac{1}{4}''$ mesh. The cut gives but a poor idea of the pleasing contrast between the light colored cement background and the black pebbles which stand out in bold relief from the surface.

Comparing these cuts and those in the preceding page, quite a variation in general aspect and texture is to be noted, and an examination of the actual surfaces would reveal a still greater difference in appearance owing to the striking variation in color and size of the aggregate used. Had these six surfaces been left untreated, they would have looked practically alike.

By varying the kind, size and proportions of the aggregates, surface finishes of practically any desired color and texture can be obtained, the possibilities being limited only by the number of different aggregates available and the combinations of same. A great variety of finishes may be produced by using red and black granite and limestone screenings, black and white marble chips and different colored pebbles and sands.

All the cuts shown represent brushed concrete surfaces, the process consisting of simply brushing the surfaces with a stiff brush, permitting it to harden for a few days and then treating it with a dilute solution of hydrochloric acid, the method of procedure being as follows:

Having decided upon the general color scheme and texture of the desired surface the first step is the making and treating of small sample surfaces. A limited amount of experimenting with the materials available will always prove profitable. The color and texture of the finished surface depends upon the color, size and proportions of the aggregates used, and the successful reproduction of the desired surface is dependent upon the proper selecting, grading, proportioning and mixing of the materials and the proper placing and finishing of the surface. After determining by experiment the proper size and proportions of aggregates to produce the desired effects and the proper consistency of the mix, adhere strictly to them; that is, take the trouble to measure the materials for each batch of concrete and to gauge them with a measured amount of water. The results obtained will more than justify the extra expense this will entail over the all too prevalent method of measuring material by wheelbarrow loads and adding the water with a hose; in fact, uniform results cannot be obtained unless the work is done as pointed out.

The slightest imperfections and irregularities in form surface are transferred to the concrete, producing unsightly surfaces when left untreated, and a pleasing surface cannot be obtained by a nicety of form construction alone. For brushed surfaces, all that is required of the forms is that the face lagging be kept true to surface and the joints be tight. For surfaces too large to concrete in one day the forms should be so constructed as to permit of the removal of sections of the face form. This can be accomplished by setting the studs or uprights back a few inches from the face lagging and connecting both by means of cleats and wedges. The face forms also should be well oiled to prevent the concrete sticking to the forms. In large areas the introduction of buttresses and panels or the breaking up of the surface by horizontal joints or courses will add greatly to the appearance, the joints being simply indentations in the surface produced by beveled beads fastened to the forms. It is extremely hard to join two different days' work so that the joint is not perceptible and unsightly, and the breaking up of the surface as indicated will greatly assist in the concreting if care be taken to end and start each succeeding day's work at a course or joint.

The facing material should be from one to one-and-a-half inches thick, the remaining thickness of the work being composed of ordinary concrete, but the facing and backing must be deposited at the same time so as to make one solid mass, thereby insuring perfect bond. The facing material may be applied to the forms just ahead of the backing, which is placed against and rammed into it, or the backing first and then brushed back from the form with a spade and the facing material deposited between the backing and the form. Both these methods have been successfully used. A third and possibly the best method of placing the facing material consists of the use of what might be called a metal facing form or mold, constructed and used as follows: To short lengths of 3/16″ iron plates 8 or 10 inches wide and 6 feet long, three 1 or $1\frac{1}{2}''$ angles are riveted, placing an angle at the center of the plate and one about six inches from each end. One edge of the plate should be slightly flared to assist in depositing the material and this edge provided with handles. The metal facing plate is placed against the wall form with the handles up and the angles tight against the form. The space between it and the back of the wall filled with the concrete backing and the 1 or $1\frac{1}{2}''$ space between the metal form and the face form filled with the facing material. The metal form is drawn almost out, and after thoroly tamping the backing against the facing the process is repeated.

For brushed surfaces the forms must be removed from the work as soon as possible and the concrete surface brushed while still green. It is not possible to state how old the work should be before removing the forms and brushing the surface. This will depend upon a number of conditions, the character of the work, cement and aggregate used, consistency of the mixture, and very much upon the weather conditions. As a rule in hot weather the forms can be removed the next day and the surface brushed, but in cold weather the facing form cannot be removed so soon, several days perhaps a week being required for the concrete to attain the necessary hardness and strength. Care must be taken that the brushing is not done too soon, as little particles of aggregate will be removed, resulting in a pitted, unsightly surface. On the other hand the longer the surface stands before being brushed the more brushing it will require to remove the film of material that has flushed to the surface. Brushing should be done just as soon as it can be without removing particles of aggregate. When this can be done, can only be determined by experimenting with the particluar surface. An ordinary scrubbing brush with stiff palmetto fibers or a metal wire brush will answer for the work. Two or three days after the brushing the surface should be washed down with a dilute solution of commercial hydrochloric acid, one part acid to two or three parts water. The acid should be applied with an ordinary calcimining brush and the walls thoroly rubbed, while wet with the acid, with a stiff vegetable fiber brush. The acid should not be allowed to remain on the surface for any length of time—not over half an hour— and should be washed off with a hose and clean water. It is important that the surface be thoroly washed after the acid treatment, for if it is not it will have a mottled, streaky appearance.

A desirable surface can be obtained by simply brushing and then washing with a hose and clean water, but the final acid treatment in connection with the brushing will produce a still better surface.

This method of treatment removes the film of mortar that has flushed to the surface, exposes the aggregate, erases all traces of form markings and produces a rougher, more artistic surface. The roughness of the surface breaks up the light, the color of the aggregate adds variety and life, and we have a pleasing, artistic, true concrete surface.

4. Finish by Tooling. Where the architect is not limited in point of cost, an excellent effect can be secuerd by tooling the surface of the concrete either by hand or using pneumatic tools. The effect will depend largely on the character of the aggregate and

Fig. 97. Cast Stone Railing, bridge at Fergus Falls, Minn. John Lauritzen, Contractor. C. A. P. Turner, Engineer.

where this has been carefully selected the finish is quite attractive, especially when the surface is broken into blocks by rustication or grooves.

The expense of tooling ranges from five to ten cents per surface foot, depending on the equipment used, while that of brushing and washing should not run more than one-fifth of this amount.

5. Cast Stone. Where suitable aggregate is available an excellent building material is made by casting concrete in sand molds.

The process is similar to the iron molders' art. Wood or plaster patterns are used, a sand mold prepared and the concrete which is to be cast is mixed to about the consistency of cream. When the resulting material has been tooled it is hard to distinguish it from the natural stone.

Fig. 98. Ornamental Cast Stone Railing.

In cost it cannot be manufactured to compete with the natural stone where there is little freight to pay, but where the work is at all complicated and there is a duplication of the parts and quarries of good building stone are not situated convenient to the locality, there is a good field for this product.

It has been very successfully manufactured in Toronto, St. Louis, and other parts of this country and also in Germany.

Success in cast stone work depends, first, on a rich mixture, second upon the selection of the proper aggregate, which must be a crushed stone or hard pebble which will weather without disintegration, and third, on the proper method of mixing and agitating the mixture until it is desposited in the sand mould.

The mixture must be semi-fluid so that it will flow and fill the mould and must be kept continually agitated until in place in the mould. Otherwise separation occurs with an inferior casting as the result.

Details of the process of manufacture are beyond the scope of our present purpose. The preceding statement, however, comparing the mode of casting to the iron moulders art gives a clean cut idea as to the method pursued.

CHAPTER XV.

1. The Execution of Work. Construction work of any kind involves a great responsibility, not only on the part of the designer, but also on the part of those in charge of the work, and that responsibility is for the safety of those erecting the work.

Perhaps the erection of no type of building is so free from hazard and risk to the lives of those erecting it as reinforced concrete construction when scientifically designed and intelligently executed.

During the last ten or twelve years, the manufacturers of Portland Cement, have through improvements in methods of manufacture and great reduction in cost, placed this material on the market at such reasonable rates that it has given a remarkable impetus to the construction of concrete work in all lines. Since, as a material of construction, it has but recently come into general use, it is not surprising that a large part of the engineering and architectural profession have not yet become so familiar with its characteristics, but that designs lacking in conservatism from a scientific standpoint have been frequently made, and this combined with the execution of the work by unskilled contractors, has resulted in a number of instances in needless sacrifice of life and large property losses, such as a more thoro knowledge and study of the characteristics of the material should entirely prevent.

It would be neglect of duty to fail to briefly summarize and to call attention pointedly to those properties and characteristics of concrete which must be known and appreciated by the engineer and constructor in order that he may avoid the serious disasters into which those ignorant or forgetful of them have been too frequently led.

The Hardening of Concrete. Concrete may be defined as an artificial conglomerate stone in which the coarse aggregate or space-filler is held together by the cement matrix. The cement should conform to the Standard Specifications for Cement, recommended by the American Society for Testing Materials.*

The contractor and architect should, at least, see to it that the cement is finely ground, and that it meets the requirements of the

*Substantially the same specifications are adopted thruout England and America.

boiling test. This last may be readily made by forming pats of the cement of $3\frac{1}{2}$ to 4 inches in diameter on a piece of glass, kneading them thoroly with just enough moisture to make them plastic, so that they will hold their shape without flowing, and taper to a thin edge. Store the pats under a moist cloth at a temperature of sixty-five to seventy-five degrees Fahr. for a period of 24 hours. Then place the pats in a kettle or pan of cold water, and after raising the temperature of the water to the boiling point, continue boiling for a period of four hours. If the pats do not then show cracks, and if they harden without cracking or disintegrating, the constructor may be satisfied that the cement is suitable for use in the work. Coarse grinding reduces the sand-carrying capacity of the cement, and its consequent efficiency.

The function assigned to the concrete element in the combination of reinforced concrete is to resist compressive stresses in bending; but when first mixed the concrete is nothing more than mud, and in order for it to become the hard, rigid material necessary to fulfill its function in the finished work it must evidently pass in the process of hardening thru all stages and varying degrees of hardness from mud and partly cured cement to the final stage of hard, rigid material. This curing or hardening being a chemical process, does not occur in any fixed period of time, save and except the temperature conditions are absolutely constant. Hence the time at which forms may be safely removed is not to be reckoned by a given number of days, but rather it must be determined by the degree of hardness attained by the cement. In other words, during warm summer weather, concrete may become reasonably well cured in twelve or fifteen days. If the weather, however, is rainy and chilly, it may not become cured in a month. In the cold, frosty weather of the spring and autumn, unless warm water is used in the mix, the concrete may require two or three months to become thoroly cured, while by heating the mixing water, whenever the temperature is below 50 degrees Fahr., the concrete will harden approximately as it does during the more favorable season.

Concrete which has been chilled by the use of ice cold water, or that has become chilled within the first day or two of the time it is cast, has this peculiarity, that it is very difficult indeed for the most expert to determine when it is in such condition that it will retain its shape after the removal of the forms. Once having been chilled in the early stages, it goes thru successive stages of sweating with temperature changes, and during these periods it sometimes happens that the concrete diminishes in compressive strength, and if the props are removed it sags and gets out of shape. Such deformation will generally result in checks and fine cracks, though there may not be any serious diminution of the ultimate strength. These checks may be prevented as explained above by the simple method of heating the mixing water whenever the temperature has dropped below 50 degrees Fahr. In colder weather, that is below the freezing point, not only must the water be heated, but as a rule the sand and stone too, also a little salt may be advantageously used, as discussed in Chapter I, Section 15, Page 30. The work must then be properly housed and kept warm for at least three weeks subsequent to pouring.

Pouring Concrete. Bad work frequently results from improper pouring, or casting of the work. In filling the forms, the lowest portion of the forms must be filled first. A column should be filled from the center and not from the side of the cap. Filling from the center will insure a clean smooth face when the forms are removed. Filling from the side will frequently give a bad surface because the mortar will flow into the center of the column through the hooping, leaving the coarse aggregate with voids unfilled at the outside. As more concrete is then poured in, the voids between the core and the out-side portion will become filled, and the soft mortar will not be able to flow back to completely fill the voids between the hooping and the casing. Where the spacing of the hooping is wide, this is not so important, but it becomes very important where the spiral used has close spacing. It is better to cast the column and mushroom frame complete, continuing to pour the concrete over the center of the column so that it always flows from the column into the Mushroom slab rather than the reverse. All splices must be made in a vertical plane, in a beam preferably at the middle of the span, and in a slab at a center line of a panel.

Separation of Materials. In pouring concrete where the mix is too sloppy, separation of the material is liable to occur. This is particularly the case in filling columns where with too sloppy a mix layers of sand and gravel and cement may result instead of a concrete of uniform composition.

In spouting concrete, careful attention should be given to the mixture at the point of discharge, since if the inclination of the spout is too great, considerable separation occurs and inferior concrete is the result, and where such separation occurs the concrete should be re-mixed before allowing it to be deposited in its final position in the work.

Test of Hardness in Warm Weather. We have pointed out that the criterion governing the safe removal of forms is the hardness or rigidity of the concrete. A test of hardness in concrete not frozen may be made by driving a common eight-penny nail into it; the nail should double up before penetrating more than half an inch. The concrete should further be hard enough to break like stone in knocking off a piece with the hammer. Noting the indentation under a blow with the hammer, gives a fair idea of its condition to those having experience.

Sub-centering is a desirable method of preventing deformation, where the use of the forms is desired for upper stories before the concrete is fully cured.

Test for Hardness in Cold Weather. Concrete freshly mixed and frozen hard will not only sustain itself but carry a large load in addition, until it thaws out and softens, when collapse in whole or in part is inevitable. Partly cured concrete if frozen, sweats and softens with a rise in temperature, hence in cold weather there is danger of mistaking partly cured concrete made rigid by frost for thoroly cured material. In fact the only test that can be depended upon with certainty in cold, frosty weather, is to dig out a piece of concrete, place a sample on a stove or hot radiator, and note whether, as the frost is thawed out of it, it sweats and softens. This gives the builder and engineer a perfectly conclusive test of the condition of the concrete as to whether it is cured or merely stiffened up by frost.

Lap of Reinforcement over Supports. Thoroly tying the work together by ample lap of the reinforcement is a prime requisite for safety in any form or type of construction. This general precaution insures toughness, and prevents instantaneous collapse, should the workmen exercise bad judgment in prematurely removing forms.

2. Responsibility of the Engineer. The steps which it is possible for the engineer to take in securing a safe construction are limited in the first place to the production of a conservative design, and one which will present toughness, so that its failure under overload or under premature removal of the forms will be slow and gradual This he can do, and this it is believed he is morally bound to do. On the other hand, he cannot design reinforced concrete work which will hold its shape without permanent deformation, unless it is properly supported until the concrete has had time under proper conditions to become thoroly cured.

The engineer is accountable for the selection of a type of design which is safe to erect. That is, a design in which a sudden collapse cannot readily occur. He should so design his work that it can be executed by the exercise of ordinary care. He should design it so that there shall be a minimum chance of bad work or disastrous results thru lack of care on the part of workmen.

We have called attention to the fact that concrete is a material naturally best fitted for monolithic construction, that the natural concrete types are best tied together by so reinforcing the construction that it will act as a continuous monolith. To do this ample lap of the bars is essential over all supports whether bearings or supporting columns.

Every failure in concrete construction is detrimental to all who are engaged in this line of business, regardless of the system, type of construction, or particular reason for the collapse, and accordingly all engaged in this line have a like interest in tracing out the cause and profiting by the lesson of every mishap which occurs.

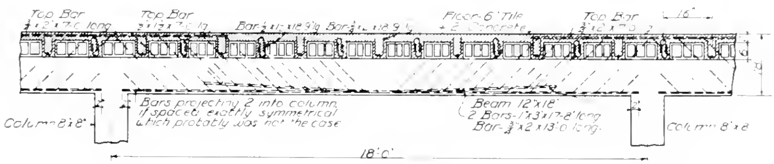


Fig. 99 Detail of beam causing trouble through insufficient lap of reinforcement over support.

The accompanying detail shows the beam and slab reinforcement of a structure which collapsed during erection and the characteristics of this failure are worthy of note, as due to insufficient laps over the supports. This failure started in an upper story where the small diameter of the column gave little or no lap of the steel over the supporting concrete while the lower stories where the columns provided greater overlap were erected without mishap until broken by the fall of the upper stories. Where there is insufficient lap, owing to the shrinkage of the concrete in setting, we have not only the shear on partly hardened concrete but also tensile shrinkage stresses tending to crack the concrete thru at the point where the bars are not sufficiently lapped over the supports.

A further weakness in the detail illustrated lies in the fact that none of the reinforcement for positive moment was carried up over the support to resist negative moment as in the Hennebique and Turner types of continuous beams, illustrated in Chapter III. This detail, viz. the carrying of a portion of the reinforcement in a continuous bar over and beyond the support, enables a portion of the load to be carried by such bars as in a swing and greatly relieves the shear stress from such loads as may be brought upon partly hardened concrete, and thus increases the safety of the work during the critical period of construction, or renders failure slow and gradual under these conditions instead of sudden and without warning, should the partly cured work be over-loaded. This safe-guard it is within the province of the engineer to provide. No excuse can be made for failure so to do on the ground of increased cost or special patent monopoly standing in the way.

Shearing and tensile resistance, as has been noted, is developed during curing, at a less rapid rate than compressive resistance and hence any reasonable safe-guard of value such as that just described should not be neglected by the engineer.

It has been almost invariably the case where combinations of tile and concrete have been used, that a failure starting on one floor has carried with it one floor after another to the basement. Such failures do not occur in well designed reinforced concrete structures. Hence where economic conditions permit, the engineer is accountable to a large extent for the selection of the safer, tougher types of construction in place of a fragile construction which may be readily destroyed by the impact of any large mass accidentally falling upon it.

In most cases where failures have occurred, had the centering been left in place for a period of from four to six months, in the authors' judgment, the work would have stood and no serious trouble would have resulted. On the other hand, they are unable to regard that kind of design as legitimate which must necessarily be treated with this extreme degree of care, there being no excuse for designing in a manner which leaves an opportunity for sudden and complete failure of the work.

The engineer designer is responsible for failure to provide a type of column design in which there are no obstructions in the shaft of the column to interfere with securing a solid casting. Column failure in a number of structures under construction have occurred where the longitudinal column reinforcement was arranged with prongs projecting into the body of the column but with no ties binding the longitudinal steel together. These prongs interfered with the flow of the concrete material in the shaft of the column, interrupt the concrete in its descent, leaving large voids which in some cases caused failure when the forms are removed. The use of this type has fortunately to a large extent been discontinued.

In the case of an eleven story building, some years ago, the designer used vertical reinforcing bars in the columns and tied them across the shaft with numerous quarter inch ties. In pouring the concrete in several columns these ties blocked the flow of the concrete and when the forms were removed large voids were found two or three feet in length in several columns thru the interference of the ties in pouring the column. There is no excuse for the employment of such details.

A slab reinforced in two directions and supported on four sides may be loaded until it is cracked thru and if the slab is a large one may be loaded until the deflection is twelve or fifteen inches and still carry the load which broke the construction down at this point and strained the steel beyond the yield point value.

A slab reinforced in one direction only will on the other hand break down completely and sometimes let go quickly and almost without warning. This is especially true where forms are prematurely removed.

We have noted in Chapter X under "Elements of Economic Construction," that the true concrete types which are continuous monolithic construction have the lowest coefficient of bending, hence there is little excuse on the part of the designer for failure to adopt the safest type of construction, particularly in view of the fact that it may be figured with greater certainty and a higher degree of scientific accuracy than the types of simple beam or one way reinforcement that have been used in this composite type of structure.

While the engineer may be held responsible for accurate computation and for features that bear upon the safety of the design, he cannot, unless on the ground, prevent the inexperienced foreman from knocking centers at too early a period. He cannot prevent the deflection of reinforced concrete work where the material has not been allowed sufficient time to properly harden before the removal of the forms. If, however, his design is one of the two natural concrete types of construction there is little danger of a sudden collapse and the worst that can happen will probably be the necessity or digging out and replacing some work which has got out of shape owing to lack of judgment and haste on the part of the erection superintendent.

The superintendent on the job should make a special point of inspection of all points of the construction where cantilever action is depended upon for stiffness and strength. Wherever such action is required, the steel should be at the top and its position at the top should be made certain by such inspection. Far too frequently carelessness has been exhibited in this respect and unsatisfactory results secured from the standpoint of strength and service.

3. Responsibility of the Constructor and Engineer Superintendent. The constructor is primarily responsible—

For the honest execution of the work.

For the proper housing of the cement and the simple methods of determining the fineness and quality as recommended in Chapter I.

He is resposible for the use of sufficient cement.

Securing a proper aggregate and seeing that the mixture of concrete has the proper consistency to produce good work, *i. e.*, that the stone is of proper hardness, suitable size and free from dirt and mud. That reasonably clean, coarse sand is used.

He should inspect the centering and see that it is erected of proper strength and that ledgers and posts are properly braced so that collapse cannot occur during erection.

He should be responsible for the exercise of care in leaving the forms in place until the concrete has become properly cured.

He should see that splices are properly made between old and new work and that segregation and separation of the concrete does not occur in pouring.

He should see that the reinforcement is placed as required by the engineer's plans and while he cannot be held responsible for the design he should exercise greater care in putting up the less conservative types which consist of one-way reinforcement than is essential in putting up multiple way systems or natural concrete types.

He can determine whether the steel furnished is of reasonable quality by the bending test and by nicking and breaking so that the fact is ascertained whether the metal is of the undesirable material known as bushel steel or a uniform quality of good metal. This is of special importance in tensile reinforcement in slab and beam steel, hooping and the like, and of less consequence in reinforcement for compression.

4. Significance of Cracks in Reinforced Concrete. Concrete in setting shrinks, and sometimes cracks by reason of this shrinkage, particularly when it hardens rapidly, as it does in hot weather.

This shrinkage sets up certain stresses in the concrete, which, combined with temperature changes, occasionally manifest themselves by subsequent cracks in the work. Such checks or cracks do not of necessity indicate weakness, providing the concrete is hard and rigid, since the steel is intended to take the tensile stresses and the concrete the compressive. Such checks sometimes cause an unwarranted lack of confidence in the safety and stability of the work arising from the common lack of familiarity with the characteristics of the material. For example, the owner of a frame building would never imagine it to be unsafe because he found a few season checks in the timber. He is sufficiently familiar with the seasoning of timber to understand how these checks occur, and that in most instances they do not mean a loss of strength, since, as the timber hardens by thoroly drying out, it becomes stronger, as a rule, to an amount in excess of any slight weakness which might be developed by ordinary season cracks or checks. So in concrete, when the general public becomes more familiar with its characteristics they will regard as far less important than they now do, checks which are produced by temperature and shrinkage stresses, or possibly by slight unequal settlement of supports.

Like timber, concrete grows harder and stronger with time, so that the ordinary temperature check does not reduce the strength of the work as much as the hardening of the concrete with age increases it as the steel is the tensile element and the compressive element, the concrete, having grown stronger with time, the strength of the combination has usually increased more than the decrease in strength brought about by the check.

Taking the modulus of elasticity of the concrete at 2,000,000, and the tensile strength of concrete at 300 pounds per square inch, with coefficients of expansion of .0000065, if the ends of a slab or beam are rigidly fixed it would require a drop of 24 degrees below the mean temperature at which the concrete hardens to stress the concrete in tension up to its ultimate capacity. The ends of our beams and slabs are rarely absolutely fixed, as the walls can generally go and come slightly and accommodate some temperature change. A certain amount of temperature reinforcement, however, should always be provided where the reinforcement is in one direction only. Eight hundredths of one percent should be the minimum in slabs. Even with this reinforcement or with multiple way reinforcement, shrinkage combined with temperature will occasionally cause cracks in the work. The season of the year and the temperature conditions at which the work is cast or the condition of the cement, all play their part in producing this phenomenon and while the constructor can guarantee safe work from the standpoint of strength, he cannot guarantee with certainty that temperature cracks will not occur.

Their occurrence is less frequent by far with the natural types of concrete and multiple-way reinforcement than with one-way slab and girder construction, such as Type II, or the combination of tile and concrete discussed elsewhere.

Where structural steel frame work is fireproofed with concrete, or concrete slabs are built in or supported by and made integral with structural shapes and beams, temperature cracks are much larger and more noticeable than with true reinforced concrete types for the reason that altho the coefficient of expansion of concrete and steel is substantially the same, the specific heat and conductivity of the two materials is widely different. Hence where the steel is placed in the concrete in large sections it responds more quickly to changes in temperature than does the concrete envelop and accordingly large checks and cracks are a common occurrence. Further, the large section of steel separates the concrete surrounding it, causing a weakness which is manifest by the concentration of the temperature effect at the weak section, accounting for the results noted.

5. Encouragement to Progress in the Concrete Industry by Patents. The Federal Constitution, Art. I, Section 8, provides that Congress shall have the power to encourage the progress of science and promote the useful arts by securing to authors and inventors, for limited periods, exclusive rights to their inventions and discoveries. In accordance with this authorization and in persuance of the object mentioned, Congress, by the enactment of Section 4886, Revised Statutes of the United States, provides that any person who has invented or discovered any new and useful art, machine, manufacture or composition of matter, or any improvement thereof, may obtain a patent therefor under certain prescribed rules and restrictions.

Statute law identical with this has been in force since April 10, 1790, except that the conditions and limitations relating to it have been modified somewhat from time to time.

The worker in concrete-steel construction is naturally interested in coming to a complete understanding of the scope and the extent of protection afforded him by this statute.

*The word "discovery" is not used either in the Constitution or the Statute, with its broadest significance. In these documents it is a synonym for the word "invention," and in them it means nothing else. The discoveries of inventors are inventions. The same man may invent a machine and may discover a law of nature. For doing the first of these things the patent laws may reward him because in so doing he is an inventor, but under those laws he cannot be rewarded for discovering a law of nature because he has originated or invented nothing by this act.

A discovery, or the devising of some means to utilize a discovered law of nature in a new and novel manner is an invention.

†The Statute provides that a grant of a patent may be made, and it says that the grant shall be limited in three respects:

1. For respective discoveries, and hence to the inventor and no one else.

2. For limited times, and hence no perpetual monopoly.

3. For useful art, and hence every patent must possess utility.

The character of inventions are broadly divided into six classes:

1. A machine. 2. A manufacture. 3. A composition of matter. 4. An art. 5. An improvement in a machine or process. 6. A design.

Those engaged in the industry of reinforced concrete construction are not interested from the standpoint of patent protection in reinforced concrete as an art, for as such the embedment of iron or metal in a concrete matrix has been practiced, as we have pointed out in our historical sketch, since the time of the Roman Empire, and in modern times to a considerable extent since 1850. As used in a building or bridge, a patent for a design would offer little protection.

Reinforced concrete cannot be logically classified as a composition of matter for there would be no means of distinguishing between different concrete designs from this standpoint, as having different degrees of utility and strength. If it is treated from the standpoint of a manufacture, the same process, the same machine, the same tools and the same general classes of material are used in the manufacture of all concrete structures. The mere form of a building and shape of a room is not patentable if the decision of the Court of Appeals in the Folding Bed Company case is considered

*See Walker on Patents, Art. 1, Chap. 2.
†Macomber, Hand Book of Patents.

conclusive. Hence the mere external form of a structure cannot form the basis of a valid claim.

If we treat the reinforced concrete structure from the standpoint of its mode of operation as a mechanism, we have here a means of differentiating between the mechanical efficiency and operation of different arrangements of reinforcement in the same matrix, which results in differences in the strength and stiffness of the structures using the same quantity of metal and concrete. Hence improvement in the design of reinforced concrete structures can be rewarded by our patent laws only as viewed in the light of an improvement in their mode of operation which enables us to differentiate one type or genus from another and to differentiate between different forms of the same species. It is on this theory that the fixed practice of the United States Patent Office is founded in granting patents on the different types of design of reinforced concrete members and structures.

This statement is substantiated by the many decisions of the Primary Examiner in charge of the concrete division of the Patent Office in numerous motions for dissolution in interference proceedings. Differentiation between the case under consideration and the prior art cited in the motion to dissolve the interference is effected by considering the mode of operation of the structure as a machine or mechanism on the general principles elucidated in the preceding chapters.

The word "machine" as used in the patent statute is not confined to the popular idea of a mechanism consisting of pulleys, shafts, levers, etc., in which the motion is quite obvious, but is to be interpreted in accordance with the broader, general definition of machine, as given in Webster's Dictionary, as follows-

> "Any device consisting of two or more resistant, relatively constrained parts, which, by a certain predetermined intermotion, may serve to transmit and modify force and motion so as to produce some given effect or to do some desired kind of work. According to the strict definition, a crowbar abutting against a fulcrum, a pair of pliers in use, or a simple pulley block with its fall, would be a machine."

In applying this definition it is evident that there are resistant parts in the composite structure of a beam or slab, to wit: the steel and the concrete. When any load is brought upon this combination, deformations occur in both steel and the concrete and the relative motion of these parts is constrained by the shrinkage grip of the concrete on the steel operating thru bond shear. This combination performs the desired work of carrying the load to the support. It modifies the motion of the load and transmits motion in its parts until equilibrium is established between the load and the supporting combination, the steel and the concrete.

That the movements in a slab or beam are not noticeable to the ordinary observer in no wise affects the application of this definition. We might state for example, that in the machine or mechanism for transmission of sound, known as the telephone, the motion of the disc is too slight to be observed by the eye. We, however, readily measure the rapidity and extent of its motion by the sound it produces. So in the beam or slab where these motions are enormously greater than they are in the telephone disc, they are readily measured by the deflectometer and strain gage.

There are many patents upon reinforcement as a manufacture, *per se,* such that when combined with concrete in the finished structure they do not produce a load-carrying mechanism differing in any wise in principle from others which have preceded them. Such patents, protect an invention, the utility of which is limited to more convenient handling of the materials in the erection of the structure or more economical method of placing the material in the desired position in the finished structure without the introduction in any wise of anything new or novel in the mode of operation of the structure itself.

Scope of Patents. The "scope" of a patent or its power to secure to its owner the limited monopoly, or control of the invention which forms its subject depends on the relation of the invention to the state of the art, or what had been done at the time of the appearance of the invention, and also upon the skill with which the claims have been drawn. Having reference to the chronology, or the time relation of the invention to the state of the art, a patent may be basic, or "pioneer" as it is usually termed, or it may be a specific or narrow patent. Putting it otherwise, a patent may be generic, that is for a genus; or specific, that is for a particular species of the genus which forms the subject of the generic or pioneer patent, and, obviously, this relation of genus and species requires that the thing constituting the species must be within the control or dominion of the genus patent.

Even tho a given invention may not, in the most general aspect, be new, and hence a pioneer in the broadest sense, yet by virtue of its practical value and importance in the art it may be regarded by the courts as, in a sense, a pioneer and to such a patent the courts have applied the term of "limited pioneer," and the scope of its protection is broad. Such a patent may be regarded, so to speak, as a sub-genus, and obviously the invention thereof can exist in the form of various species falling within this sub-genus and hence proper to be dominated by the limited pioneer patent.

To illustrate the matter by the subject in hand, at the time of advent of the "Mushroom" or true flat slab type of reinforced concrete construction, reinforced concrete was an old thing in building construction. The "Mushroom" invention, therefore, could not be covered by a patent which would dominate or control any and all combinations of concrete and steel combined to utilize the compressive strength of concrete and the tensile strength of steel. The patent covering it could not be a pioneer patent in the broadest sense. But the "Mushroom" invention being the first instance in the art of a true continuous flat slab resting on columns, and resisting flexure by circumferential cantilever action about the head of the column as explained in Chapters IV and V, and which by flexure between the columns about the diagonal center, secures plate action by wide spreading reinforcement of substantially equal strength in all directions, a patent therefor is to that extent a pioneer, and is for a sub-genus in reinforced concrete construction, and all subsequent patents having characteristics of the sub-genus are merely species thereof and hence proper to be dominated or controlled by the sub-genus patent if the claims of this patent are finally sustained by decisions of the Courts as they have been in repeated contests in the Patent Office.

The Importance of Investigating the Scope of a Patent. The scope of a patent is determined by the breadth of its claims. Any limitations or conditions placed in its claims narrow and restrict its scope.

The scope, and consequently the value of a patent as a means of controlling a given construction, while depending principally on the character of the patent claims, is also affected by the state of the art at the time of the advent of the invention, and by the effect of proceedings in the Patent Office to secure the claims. A patent attorney, thru a misunderstanding of the invention, or from want of skill or experience, may draw or word the claims so that, tho in fact, the invention is a pioneer, the patent itself is narrow or specific, and limited to a particular species of a genus, instead of dominating all species of that genus. The state of the art may be such that the field for a new construction is so circumscribed that the scope of the claims may be restricted even to the identical construction shown in the drawings of the patent. At this point, it seems proper to explain that the state of the art embraces everything relating to the subject, whether found in books or other publications, and patents here and abroad, and what has been done in this country in actual use.

The Patent Office, by reason of its limited force and facilities rarely does more than search thru United States and foreign patents before deciding whether or not to grant a patent. It has absolutely no facilities for ascertaining what has been done in the way of actual use, and, hence innocently and excusably, at times grants patents for what has already long been in actual use, but such patents, of course, have no validity.

Revised Statute, Section 4886, covers this point in the following words:

> "A patent may be obtained by any person who has invented or discovered any new and useful art, machine, manufacture, or composition of matter, or any new and useful improvement thereof, *not known or used by others in this country before his invention or discovery thereof, and not patented or described in any printed publication in this or any foreign country before his invention or discovery thereof, or more than two years prior to his application, and not patented in a country foreign to the United States on an application filed more than twelve months before his application, and not in public use or on sale in the United States for more than two years prior to his application*, unless the same is proved to have been abandoned, upon payment of the fees required by law and other due proceedings had."

The effect of proceedings in the Patent Office and the importance of looking into the matter when determining the value of a patent is well shown in a recent case. A patent was submitted to an engineer for purchase, for which he was willing to pay $50,000, providing it was what it was purported to be. An examination of the application and the records of the Office soon disclosed, that by reason of amendments and disclaimers filed by the inventor in response to rejections of the Patent Office, the claims of the patent were of such narrow scope as not to cover even the actual commercial form of the invention, and so the patent was valueless.

Prior Art. The art necessarily includes all prior patents, applications, domestic and foreign, and all domestic use prior to actual date of invention. It may be said that the only difference between the limitations by prior art and anticipation is that the former limits the scope of the claims while the latter kills them and it is not infrequently a fact that the limitation of a claim by the prior art is such as to all intents and purposes destroys its utility.

Regarding such limitations, the Court of Appeals of the Second Circuit says:

> "Where the patentee specifies a special form by which the effect of the invention is produced or otherwise confines himself to the particular form of what is prescribed he is limited thereby in his claims for infringement."

And in the Keystone Bridge Co. case, 95 Fed. U. S. 274, at page 278, the Supreme Court said:

> "They (the patentees) cannot expect the courts to wade thru the history of the art, and spell out what they might have claimed and have not claimed. . . . But the courts have no right to enlarge a patent beyond the scope of its claims as allowed by the Patent Office. . . As patents are procured *ex parte*, the public is not bound by them, but the patentees are. And the latter cannot show that their invention is broader than the terms of their claim, or, if broader, they must be held to have surrendered the surplus to the public."

*Genus and Species Patents.** It seems desirable to correct a widespread error as to the scope of the grant of the patent by the Patent Office in its relation to other patents (either earlier or of later date of issue) as far as the right to use the construction of such patent is concerned.

The grant of a patent confers no right to use the construction shown in the patent. It simply gives the right to the owner of the patent to prevent others from using that construction. The legal proposition may be illustrated in this way:

A patent is issued in 1910 to C on a given type of construction. All that this patent gives to C is the right to stop other persons from making use of or selling the construction set forth in the claims of that patent. It gives no such right, as the right to use the subject matter of the patent and the patent office has no authority in law to give the right actually to use the construction shown in the patent for the reason that because of some dominating genus patent the owner of the latter B has the right to prevent others from using the construction which forms the subject of the patent to C to which patent that of C stands in the relation to that of B of a species to a genus. The fact that the construction of C and B belong to the same family, the B genus, in reality are merely different species thereof or different expressions of the same idea, explains the relation of a broad to a specific or narrow grant.

Consequences of Infringement. For the infringement of a patent, the law provides redress in two forms: One in compensation in money, which covers the profits made by the infringer and the

*Reinforced Concrete Patents, Williamson.

damages (which the Court may treble) to the owner of the patent, together with the costs of the suit. The other is an injunction prohibiting further infringement, which in the case of a building would be the prohibition of further use of it. Among the persons liable for infringement in the case of a building are the builder, the owners and the user.

Indeed, the Courts have gone so far as to order the destruction of the infringing thing. Thus the Supreme Court of the United States in Birdsell v. Shaliol, 112 U. S. 485, said:

> "But an infringer does not, by paying damages for making and using a machine in infringement of a patent, acquire any right himself to the future use of the machine. On the contrary, he may, in addition to the payment of damages for its infringement, be restrained by injunction from its further use, and when the whole machine is an infringement of the patent, be ordered to deliver it up to be destroyed."

Some idea of the great favor which the law gives to the owner of a patent in enforcing his rights may be gathered from a few decisions of the courts. The Court of Appeals of the Seventh Circuit speaks of the patent owner as a "czar," so great is his power under the law, and other Courts describe his power in language equally as strong. Said that Court of Appeals (which sits at Chicago) in Victor Talking Machine Co. v. The Fair, 123 Fed. Rep. 424:

> "All that the government can and does grant, is the right to exclude others from practicing his invention without his consent. Within his domain, the patentee is czar. The people must take the invention on the terms he dictates, or let it alone for seventeen years. This is a necessity from the nature of the grant. . . . . The field being his own property and there is no law for seizing it and adjudging his damages, he cannot be compelled to part with his own except on inducements to his liking."

Said the Court in General Electric v. Wise, 119 Fed. Rep. 922:

> "No time will be used in answering this suggestion, except to say that if complainant's patents are valid, it is entitled to protection by injunction against all the world. No other person or company can use its property of this description without its consent and relegate it to an action for damages. If this patent is valid complainant has an absolute right under the laws of our country to the use of the patent and to designate the parties on whom it will confer the right to use it."

While the preceding statements clearly illustrate the position taken by the Court in the case of machines used to turn out a product, the decisions are less numerous and are somewhat conflicting with reference to structures or load-carrying machines. In a building, looking at it as a load-carrying mechanism from the viewpoint taken by the Supreme Court in the Birdsell v. Shaliol case, the Court took the stand that the infringer may be restrained by injunction from the further use of it as a machine and when the whole machine is an infringement of the patent, to order him to deliver it up to be destroyed. Here the Court seems to apply what was termed the "Rule of Reason" which created much discussion in the rulings relative to the scope of the Sherman law.

While no ultimate conclusions have been reached in the cases involving building construction where patents have been sustained, this rule would indicate they should be so construed as to do justice to the holders of such patents and to fairly carry out the contract entered into by the government when it issued such patents.

In view of the fact that the finished building contains much more than the mere load carrying skeleton or mechanism of reinforced concrete, an injunction against its use without qualification would be inequitable to the owner and in a measure unreasonable. A choice in extreme cases between an injunction and the payment of three times a contractor's profit of fifteen percent of the value of the cement work in the case of contested cases and of the usual fee in cases not contested would seem to be the limit of reasonable protection to which the holder of a broad patent may be entitled, even tho this amount might not pay in a single instance for the expense of carrying a suit to conclusion.

In cases where the patent covers merely a convenient form of make-ready for the reinforcement without involving a new mode of operation differing materially from other and older forms, it would seem that the redress should be logically limited to a judgment for damage to be recovered from those making or putting up the structure who have been directly benefited by the economy resulting from the form of manufactured material used. Certainly in this second class of inventions, classification as a new and novel load-carrying device or mechansim would not hold good.

Were an injunction unqualified as suggested issued against the owner of an infringing structure, unconditionally, it would place the owner in the serious position of being forced to pay any amount which the patentee might demand. The inequity of such a position as this has apparently deterred some members of the Judiciary from deciding in favor of the patent. One Federal Judge made the assertion that he would not issue an injunction which apparently was the only remedy because in the case of a building he did not consider it to be equitable and construed the claims of the patent in an extremely narrow manner, which decision enabled him to escape the dilemma.

The Court of Appeals was apparently dissatisfied with this decision when it was confronted with similar considerations. Instead of upholding the decision of the Lower Court on the grounds on which it was rendered it stated that in its opinion a concrete floor slab was merely an aggregation. In other words, the bending resistance of the slab was the sum of the bending resistance of the steel and the concrete acting separately. In this decision, altho furnished with a complete library of all American treatises on reinforced concrete the Court over-looked the connecting link between the concrete and the metal known as adhesion or bond which causes the two materials to act together and form a true combination. This legal point may need some discussion.

Referring to ~~Curtiss~~ Macomber Fixed Law of Patents:

> "AGGREGATION: The distinction between an aggregation and a true combination is not always clear. The main test lies in examination of the result—the function performed. If that result is the sum of the several actions of the elements, it is an aggregation; if it is the product of those actions—if the action of one element so modifies the action of another that the resultant action differs from the sum of the separate actions—it is a true combination."

The Circuit Court of the Eighth Circuit, No. 3801, thus explains the difference between an aggregation and a combination.

> "For example, it is not invention to take a fire pot from an old stove, a flue from another and a coal reservoir from a third and assemble them where each merely performs its old function in its new location." Hailes v Van Wormer, 20 Wall. 353.

The error of the view that this is the state of the case with a concrete beam or slab may be illustrated as follows: Consider the case of two planks, one superimposed upon the other, and load them in this position; then the resistance supplied by the two planks in bending under load is the aggregate resistance of the two planks and is accompanied by the phenomena of the lower corners of the upper plank sliding by the upper corners of the lower plank, as the planks bend under load. In this case the bending resistance of the planks is the aggregate of the bending resistance of the two elements, and this phenomena of sliding must occur when the connecting link of bond shear is lacking between the two planks. But when by bolting and glueing the two planks together so that sliding is prevented we secure shearing resistance between them, the stiffness of the two planks so joined becomes four fold the aggregate stiffness of the two under load and the strength is increased one hundred percent. With this arrangement the two planks no longer form an aggregation, they have become a combination with greatly increased efficiency as a load carrying mechansim, and were the combination novel to the art would be patentable.

Now we have pointed out in earlier chapters that the joint action of the two materials in a Mushroom slab thru bond shear produces a result widely different from the aggregate resistance of the two elements and the same quantities of the two elements have widely different efficiencies dependent upon the manner and arrangement of the reinforcement in the matrix vertically and horizontally, which determines the law or mode of operation of the structures. This consequently is a combination and patentable, as held by the experts of the Patent Office.

In the decision handed down by the Lower Court in an Eastern Circuit, the Judge concluded that a patent on a reinforced concrete structure could only be granted as a patent for an art. The art of burying metal in concrete being older than Christian civilization, if this decision is concurred in by the higher courts, the United States Patent Office is placed in the position of having accepted fees in a wholesale manner for patents on a branch of science or industry not patentable under the Constitution, and the trained experts of the Patent Office are open to criticism not for having erred excusably in granting patents for what had already been in actual use, but for inaugurating a policy of accepting fees upon an industry which the ruling of the Courts holds to be an improper subject matter for patent.

Looking at the decision of the Court of Appeals, referred to in another light, if the strength of concrete and reinforcement is an aggregation and bond shear plays no part in the mode of operation of the structure then the technical men of the Patent Office are in error in granting patents on alleged improvements which must from their very nature as an aggregation be absolutely useless.

This conclusion would seem, however, to be incorrect, for the reason that the Examiners in Chief and the Commissioner of Patents inaugurating this practice, are trained experts in their particular branches, while the training of the Federal Judiciary is along legal rather than along technical lines. Further, the hundreds of millions of dollars of reinforced concrete construction show that it is commercially valuable and useful. Any opinion which involves a hypothesis to the contrary is accordingly erroneous and untenable.

In the specific instances cited, while these decisions are not in accord with the general tenor of rulings by Federal Courts, nevertheless, a comparison of the efficiency of the present method of court procedure in deciding technical causes where the judge is trained along legal lines in contradistinction to one trained along technical lines in the particular branch under which the case comes may not be amiss.

One engineer thus describes the operation of the Federal Courts: After two years' time, at an expense of fifteen to twenty thousand dollars, the probability is that the litigant will secure in a complicated case, a decision from some Court of Appeals which by implication conveys to the expert the unavoidable conclusion that the law of gravitation and conservation of energy is held by the Court to be inoperative or unconstitutional in this particular branch of science. Perseverence and continued effort for two or three years more may secure an opposing decision in another circuit and then by carrying the matter to the Supreme Court of the United States, in the course of eight or ten years from the date of filing the original suit, a decision will very likely be rendered in accordance with fixed natural laws.

The judiciary, however, are not really to be blamed for this state of affiairs. They perform their appointed task as best they may.

Should a structural engineer be requested by his employer, first to report on a complicated question of inorganic chemistry, then upon a metallurgical question, and then on a mechanical proposition entirely outside of his special line of business, in the same month, he would decline to undertake the work and conclude that his employer was *non compos mentis*, and resign his engagement. The employer of the Federal Judges in this case, is the United States Congress which has exhibited in its provision for the adjudication of technical cases a lack of capacity in keeping with its lack of technical knowledge. The system inaugurated by our Congress is incomparably bad. It provides the same judge or man learned in law to try a case involving complicated questions of structural design as to decide complicated questions in the chemistry of dyes. It provides a judge qualified by learning in law only to decide metallurgical questions and to decide all manner of special technical and mechanical questions which it would seem must try his patience to the extreme, for he is supposed to find the time to study and digest all the scientific principles that may be involved in the question at issue presented by the one side and the clever misleading evidence of experts on the other side, specially designed to confuse rather than clear up the question of fact at issue.

In this connection a comparison of the relative efficiency of a scientific expert as a judge, with the judge qualified by ordinary legal training will emphasize the point above made.

The discussion of the entire prior art and the technical principles involved are brought into question in the course of an interference by a motion for dissolution in the Patent Office. Such a motion is usually disposed of in from three to six hours argument before the expert examiner as judge. It is needless to say that no principles of elementary mechanics nor any digest of the theoretical principles need be presented to such a judge. He would be thoroly familiar with those principles, and hence in two to four hours the same ground would be covered as would be covered in two to four years' time in deciding the same questions before the Federal Judiciary.

The relative efficiency from the standpoint of cost may be compared without exaggeration by saying that where it would cost dollars in deciding a question before a technical judge, it costs an equal number of thousands of dollars to decide the points at issue before the Federal Judiciary in any case which involves mechanical principles that are in the least complicated.

Thus by establishing an unscientific system, Congress has to a large degree nullified the intent of the Constitution in its attempt to encourage the poor but worthy inventor and by its provisions for his benefit, has rendered a patent a luxury for the wealthy only, a privilege entitled to respect only in proportion to the bank account of the holder. The wealthy piratical infringer under the present system expects to wear out the deserving inventor by the expense which is necessarily involved in carrying out this cumbersome system of enforcing his rights, and in the opportunity which this system offers for bringing fake suits on patents which are worthless and merely alleged to apply to the subject matter of the improvement for the purpose of crippling the inventor from the financial standpoint.

As tho the apparent total absence of systematic provision for rendering just judicial decisions of technical questions as outlined in the preceding discussion were in fact lacking in some particulars that might prevent it from being as bad as could possibly be devised, Congress has provided that there shall be nine different independent federal circuit courts each to be local and provincial in its character, and omnipotent in its own district. But the decisions of no one court are of binding force in any other district. No patent court of appeals exists which may render a decision in a single suit once and for all for the country at large in the interest of simplification of legal controversies about patents, except in case of conflicting decisions as previously stated. This arrangement tho well devised for the emolument of the legal fraternity is objectionable in the extreme from the standpoint of the interests of the common citizen.

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